

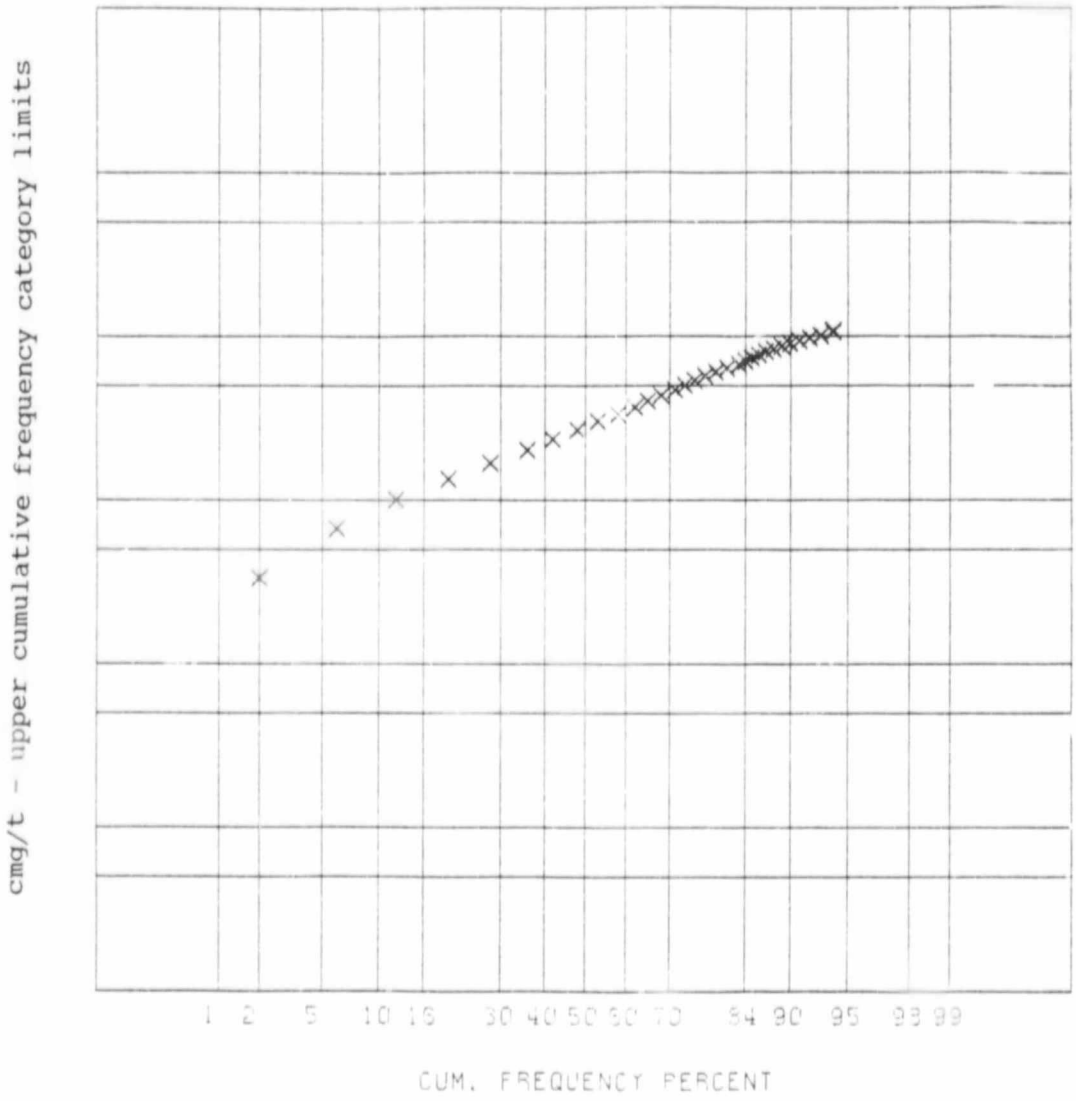
https://dspace.org/

A Study To Optimize Ore Evaluation At Western Deep Levels Gold Mine, Witwatersrand, By Using Geostatistics And Geological Subdivision

Braun, Rolf

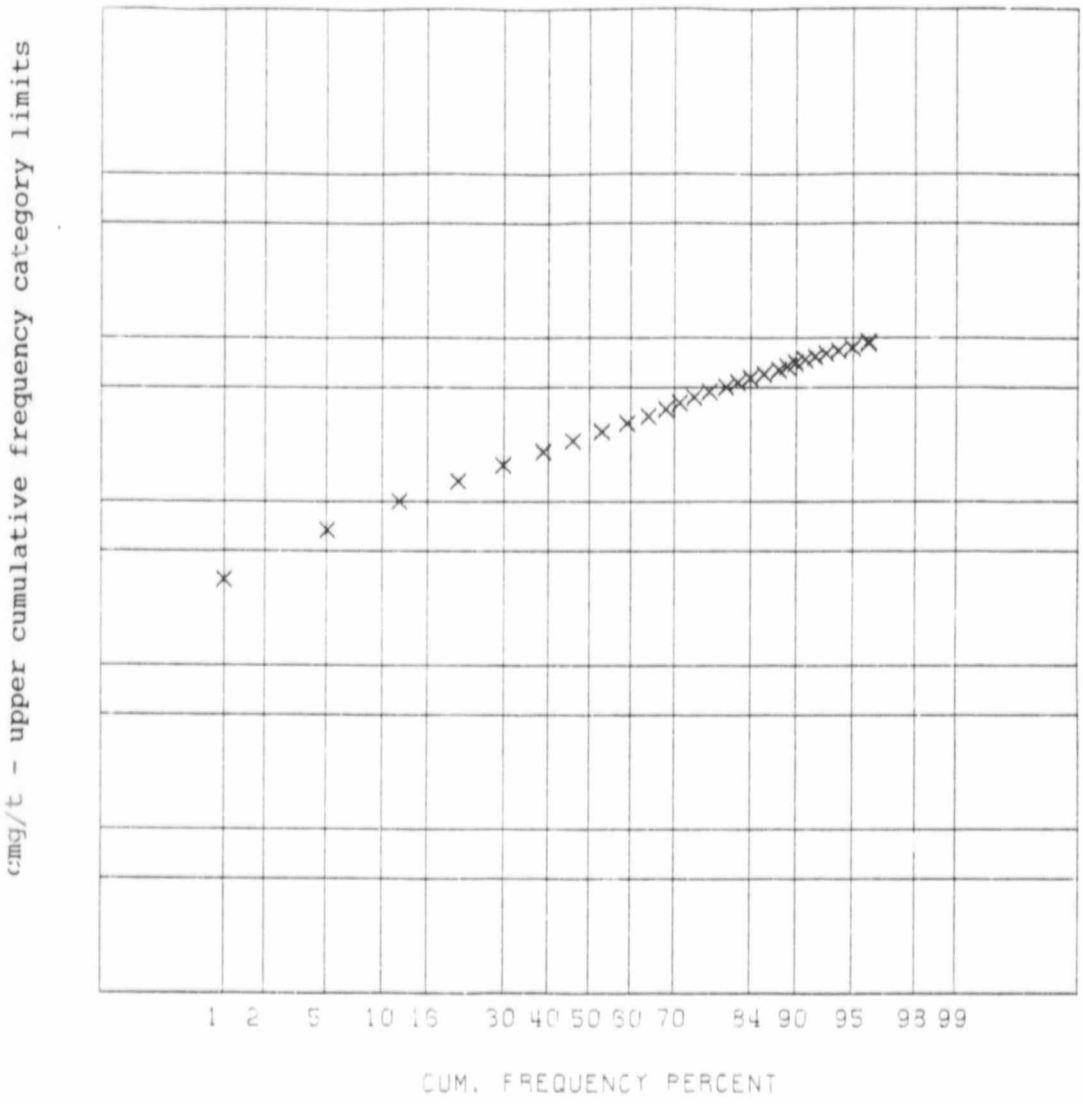
2015-01-12
http://hdl.handle.net/10539/16439

Downloaded from a DSpace repository. https://dspace.org/



× = ACTUAL VALUES + CONST.
ARITHM. MEAN=
GEOM. MEAN=
SICHELS t-ESTIMATOR
VARIANCE= 114506265.6
LOG. VARIANCE= 0.936
STD. DEV.= 10700.750
NUMBER OF SAMPLES= 9404
ADD. CONSTANT: 171.000

Figure 4.21: Logarithmic probability plot of 9404 chip samples of subarea S2.1 .



× = ACTUAL VALUES + CONST.
ARITHM. MEAN=
GEOM. MEAN=
SICHELS T-ESTIMATOR
VARIANCE= 13678192.0
LOG. VARIANCE= 0.680
STD. DEV.= 3698.403
NUMBER OF SAMPLES= 4611
ADD. CONSTANT: 230.000

Figure 4.22: Logarithmic probability plot of 4611 chip samples of subarea S2.2 .

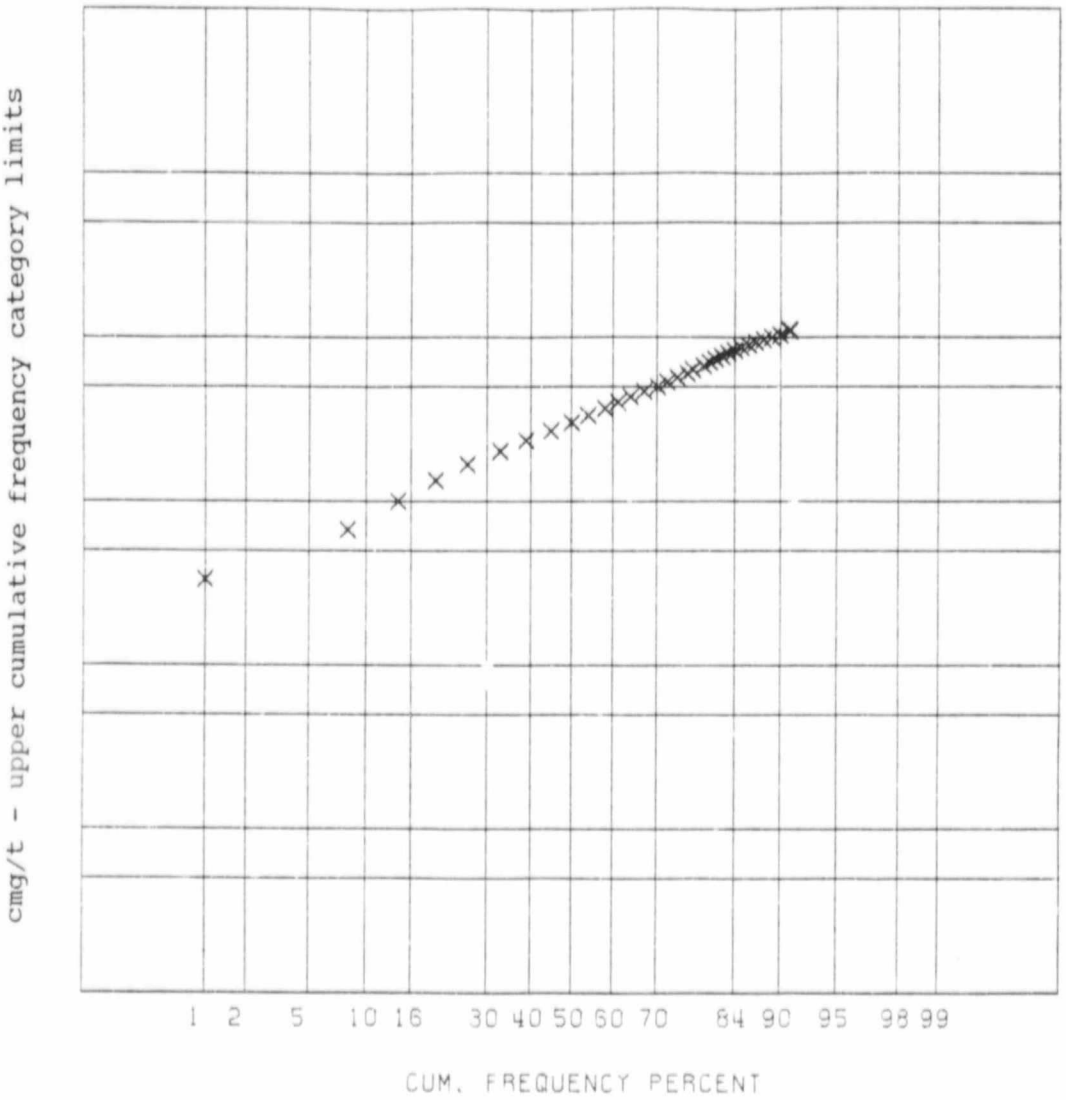


Figure 4.23: Logarithmic probability plot of 5519 chip samples of subarea S2.3 .

Fig 4.24a: N-S Logtransformed Semivariogram S1.1

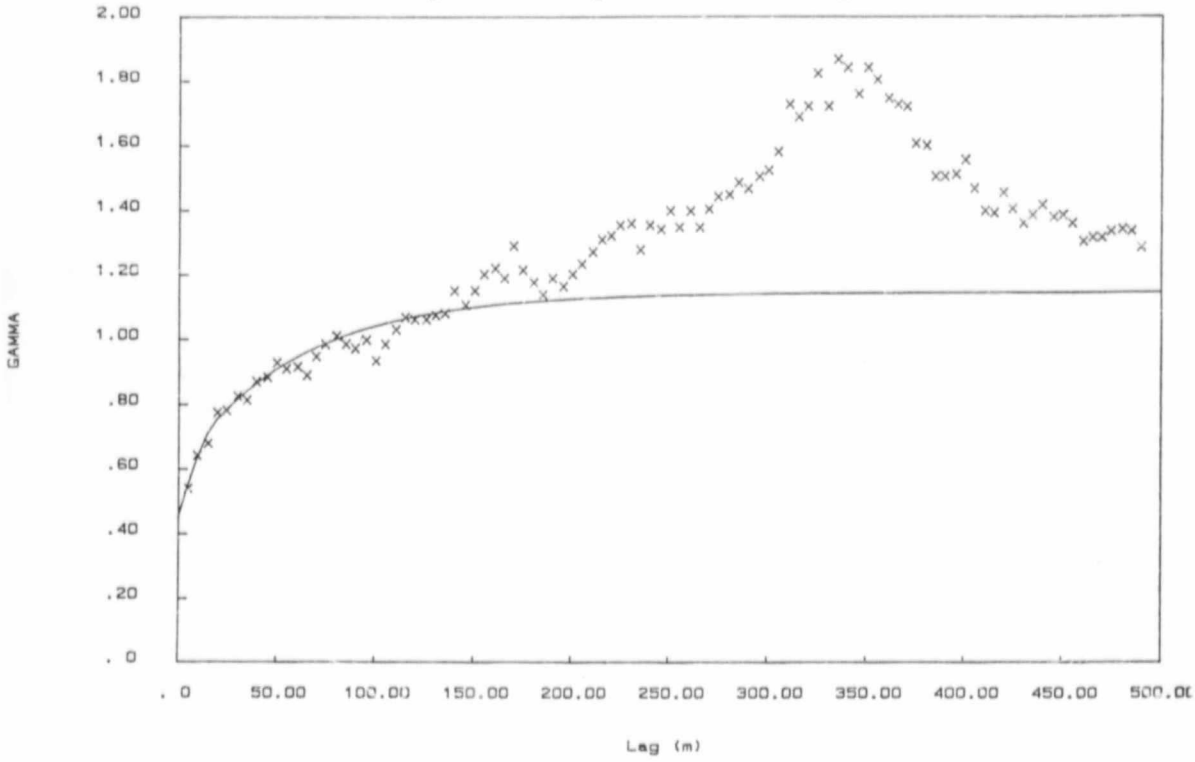
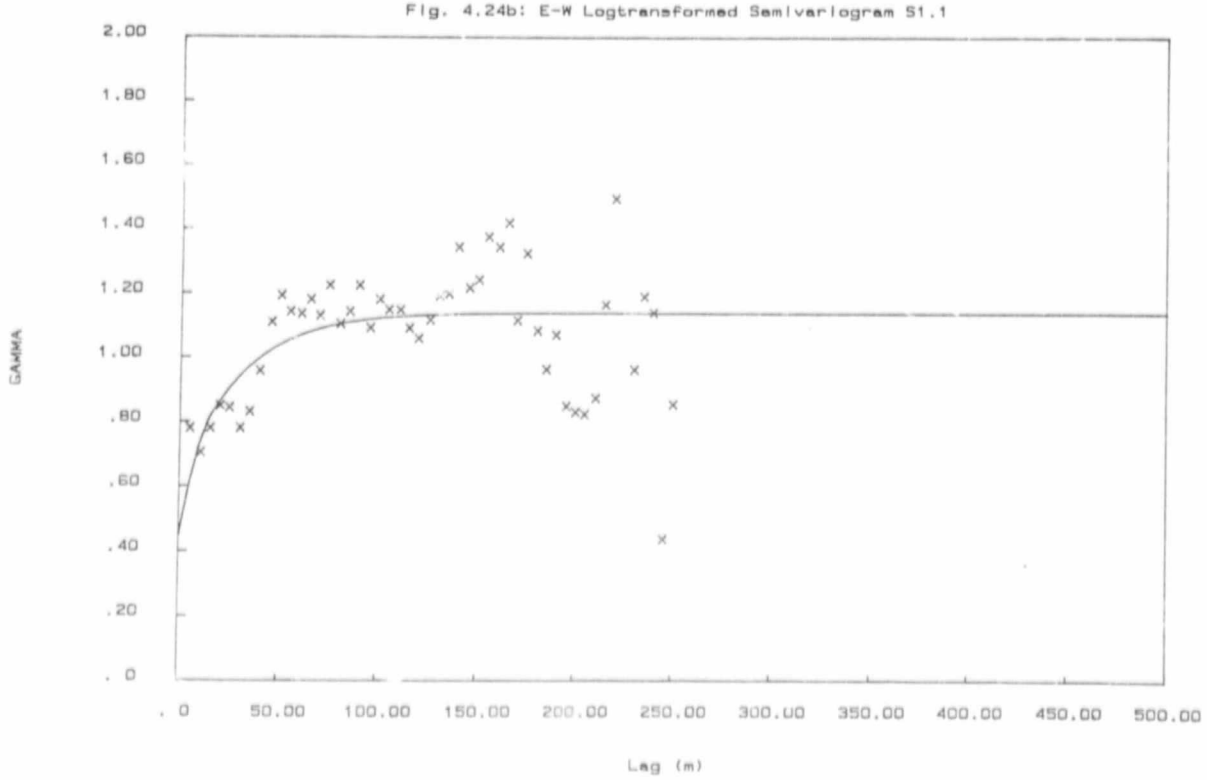


Fig. 4.24b: E-W Logtransformed Semivariogram S1.1



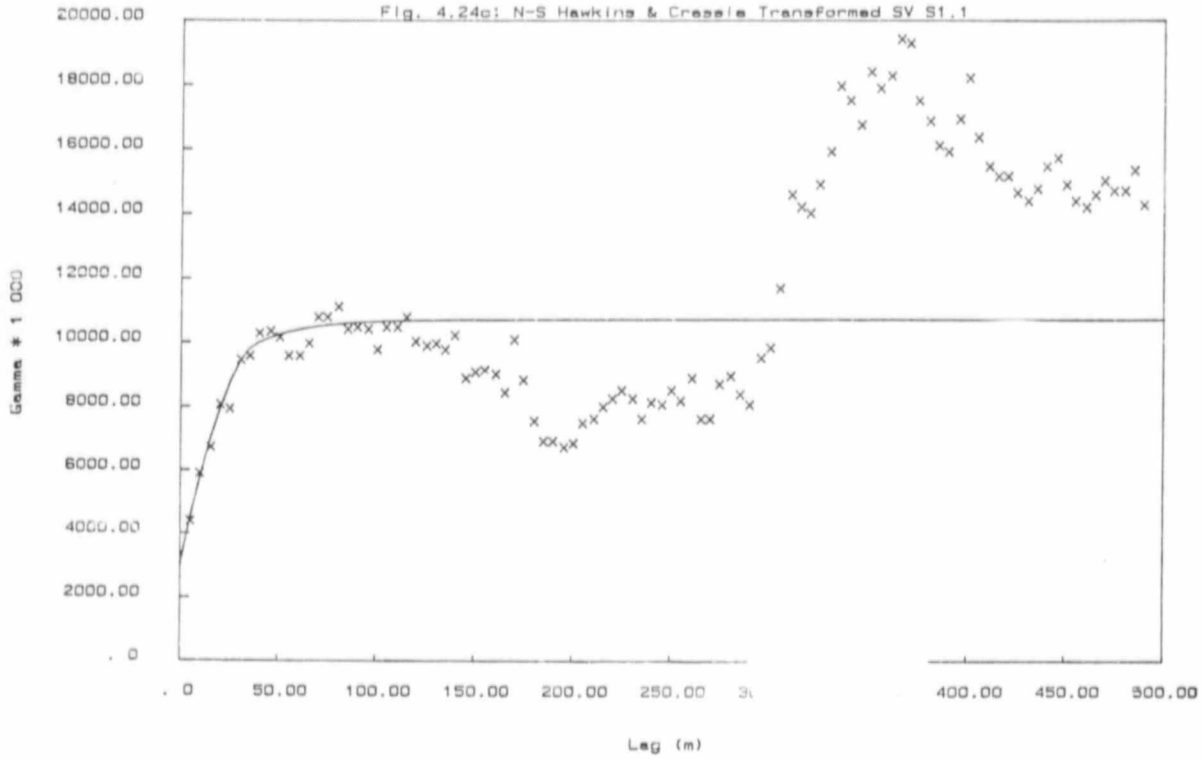


Fig. 4.24c: N-S Hawkins & Cressie TransFormed SV S1.1
Gamma * 1 000
20000.00
18000.00
16000.00
14000.00
12000.00
10000.00
8000.00
6000.00
4000.00
2000.00
. 0
. 0
50.00
100.00
150.00
200.00
250.00
400.00
450.00
500.00
Lag (m)

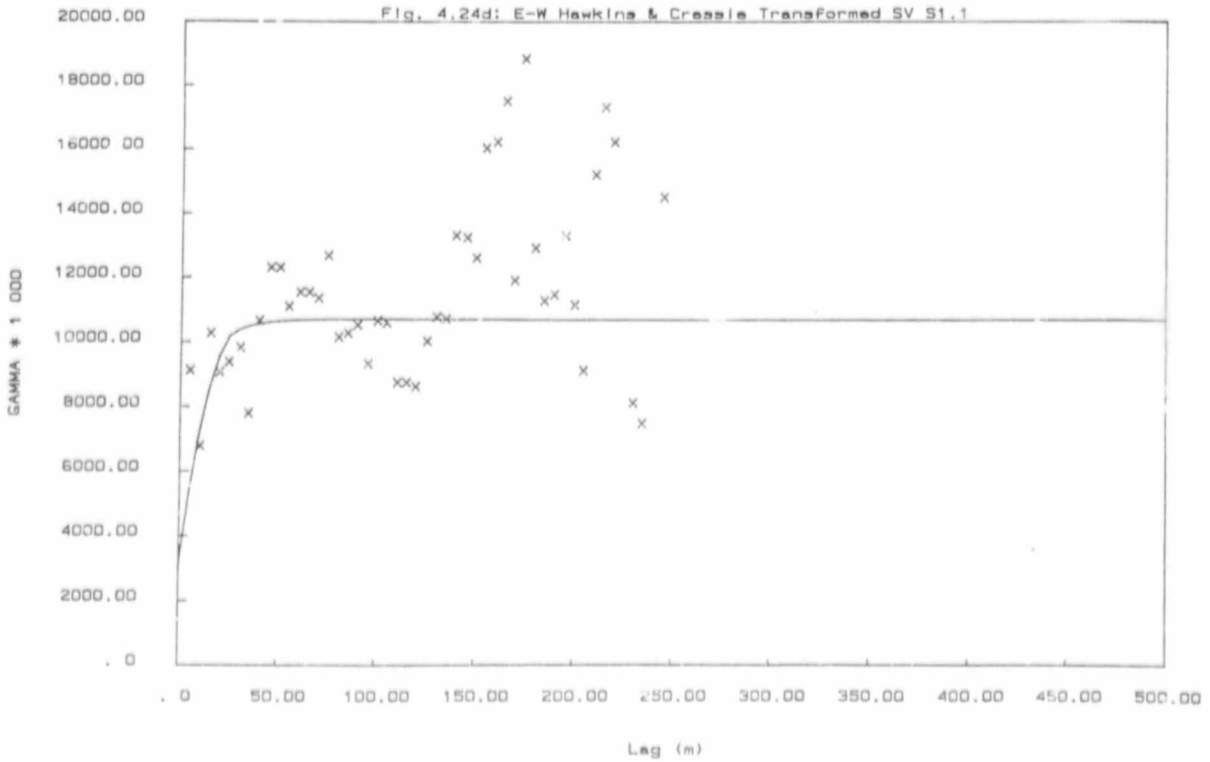


Fig. 4.24d: E-W Hawkins & Cressie TransFormed SV S1.1
GAMMA * 1 000
20000.00
18000.00
16000.00
14000.00
12000.00
10000.00
8000.00
6000.00
4000.00
2000.00
. 0
. 0
50.00
100.00
150.00
200.00
250.00
300.00
350.00
400.00
450.00
500.00
Lag (m)

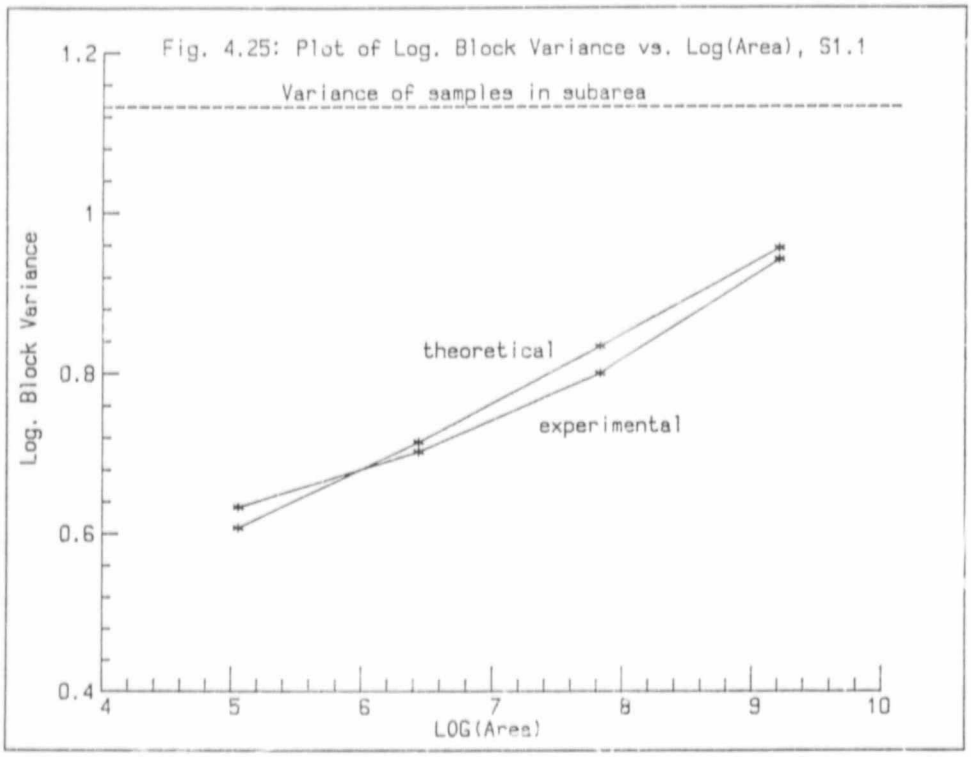


Fig. 4.25: Plot of Log. Block Variance vs. Log(Area), S1.1
Variance of samples in subarea
Log. Block Variance
1.2
1
0.8
0.6
0.4
theoretical
experimental
4
5
6
7
8
9
10
LOG(Area)

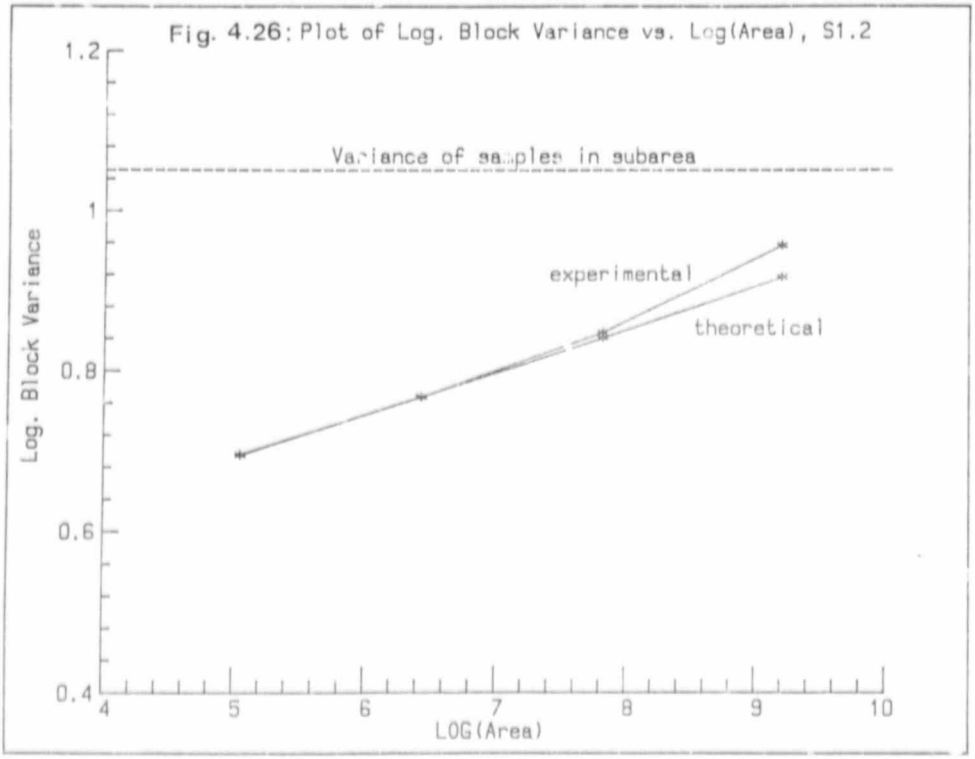


Fig. 4.26: Plot of Log. Block Variance vs. Log(Area), S1.2
Variance of samples in subarea
Log. Block Variance
1.2
1
0.8
0.6
0.4
experimental
theoretical
4
5
6
7
8
9
10
LOG(Area)

Fig. 4.27a: N-S LogtransFormed Semivariogram S1.2

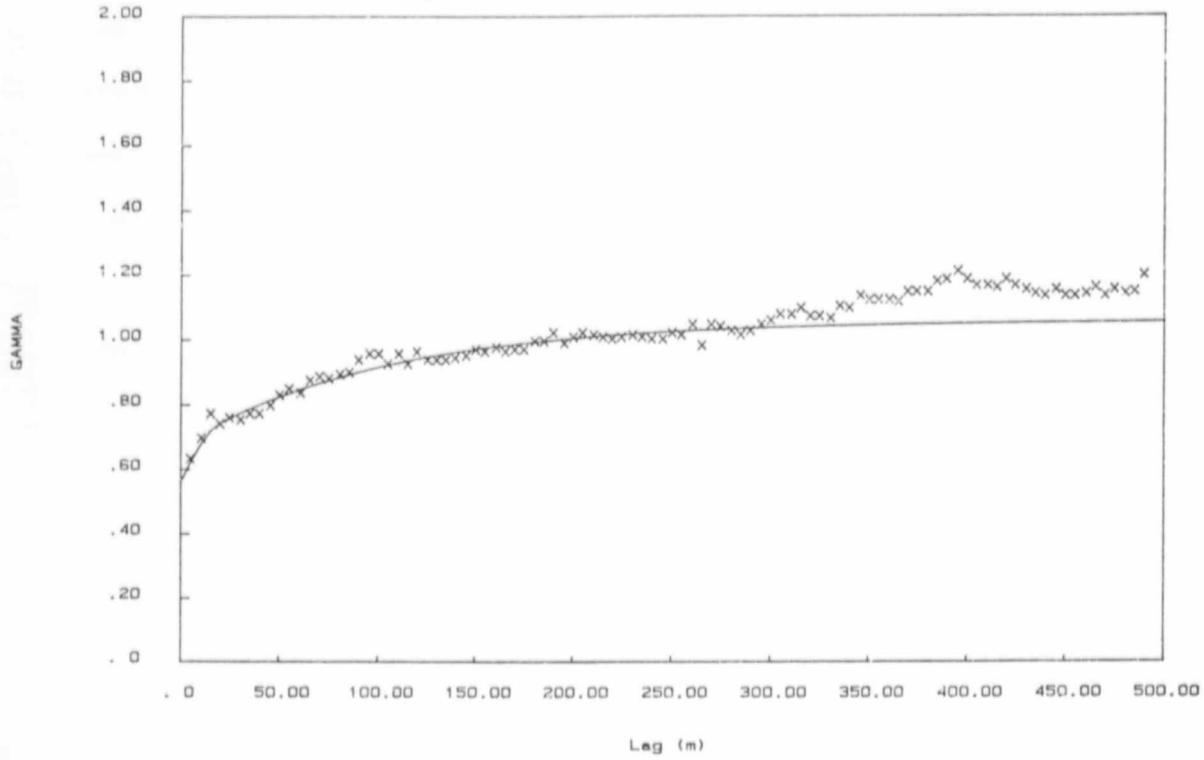
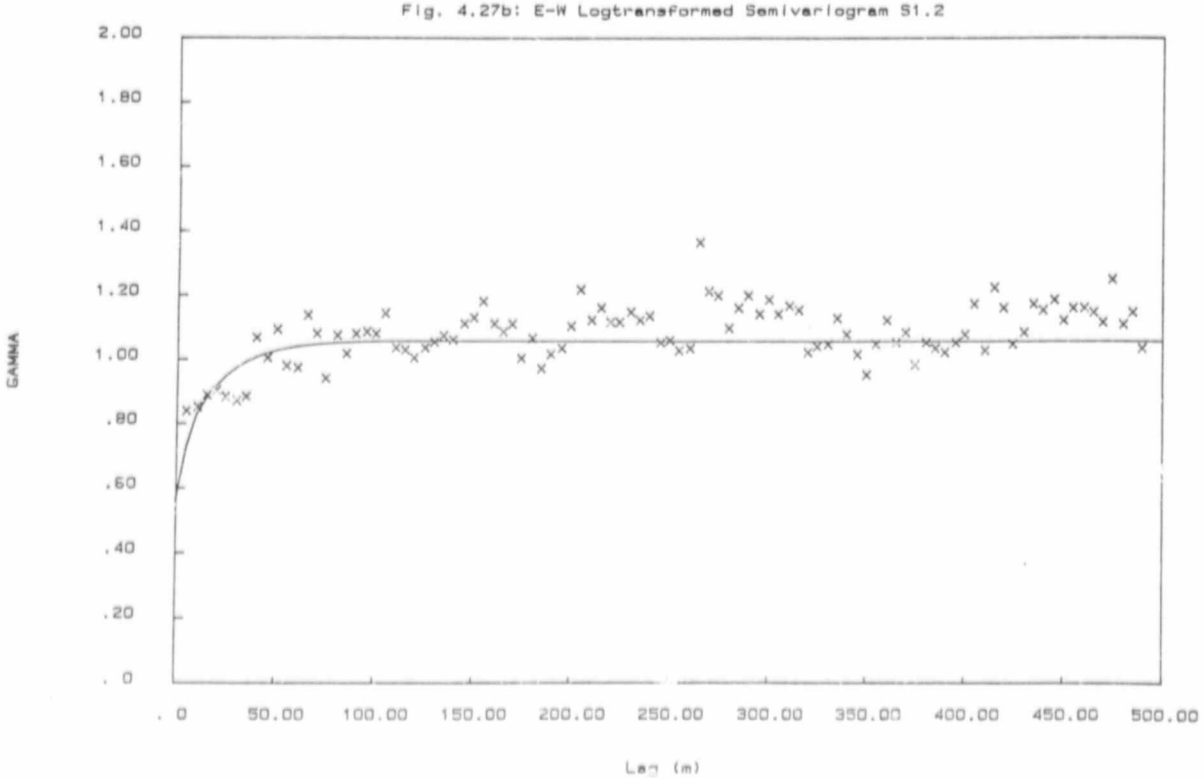


Fig. 4.27b: E-W LogtransFormed Semivariogram S1.2



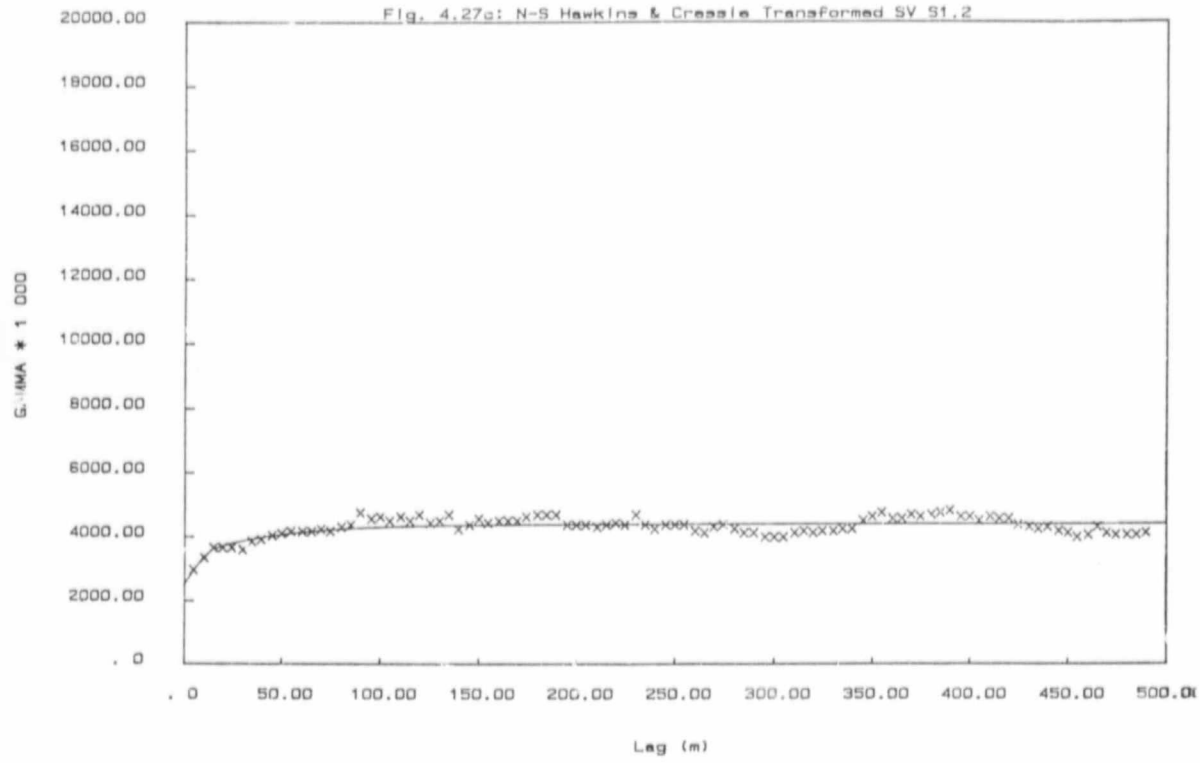


Fig. 4.27c: N-S Hawkins & Cressie Transformed SV S1.2
20000.00
18000.00
16000.00
14000.00
12000.00
10000.00
8000.00
6000.00
4000.00
2000.00
. 0
GAMMA * 1 000
. 0
50.00
100.00
150.00
200.00
250.00
300.00
350.00
400.00
450.00
Lag (m)

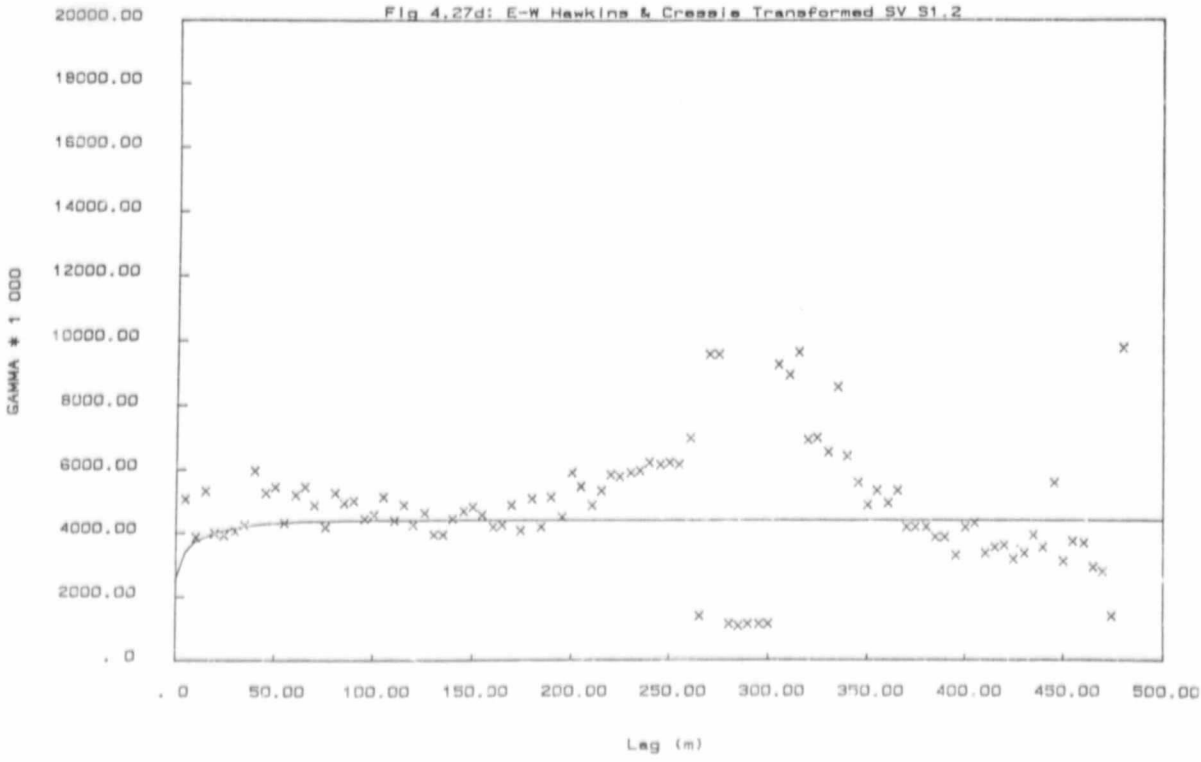


Fig 4.27d: E-W Hawkins & Cressie Transformed SV S1.2
20000.00
18000.00
16000.00
14000.00
12000.00
10000.00
8000.00
6000.00
4000.00
2000.00
. 0
GAMMA * 1 000
. 0
50.00
100.00
150.00
200.00
250.00
300.00
350.00
400.00
450.00
500.00
Lag (m)

Fig. 4.28a: N-S Logtransformed Semivariogram S2.1

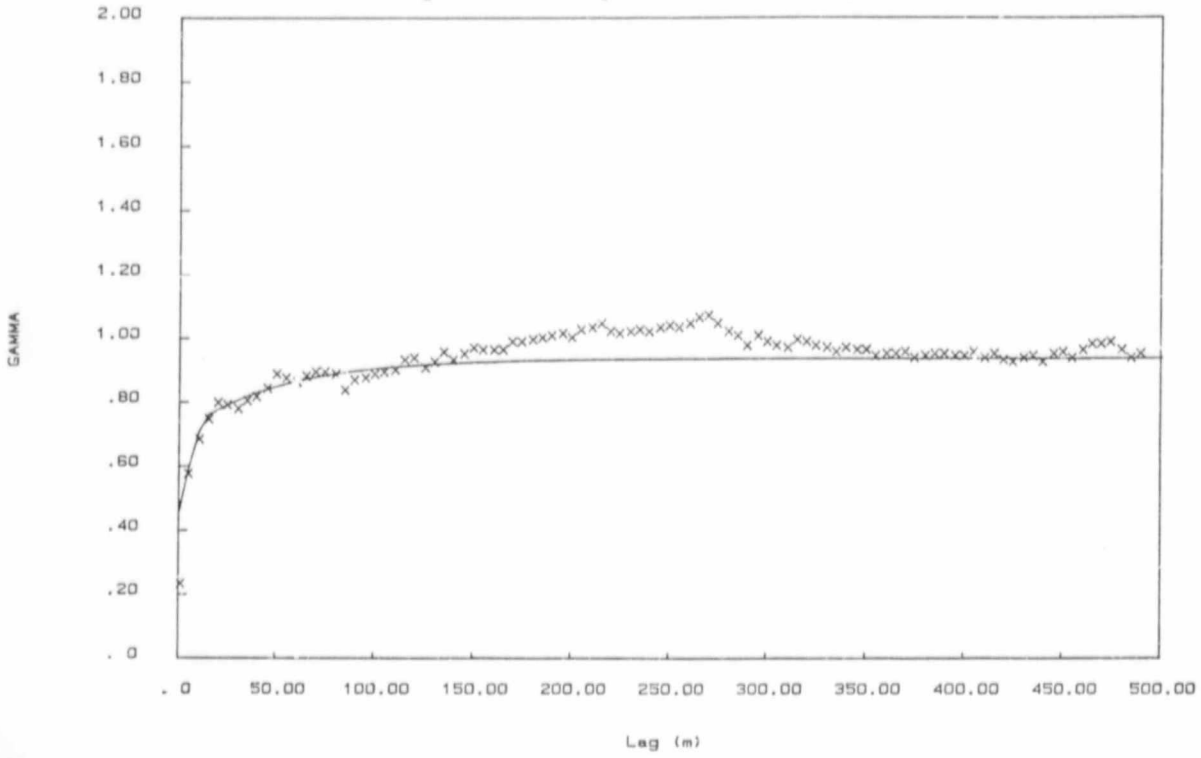


Fig. 4.28b: E-W Logtransformed Semivariogram S2.1

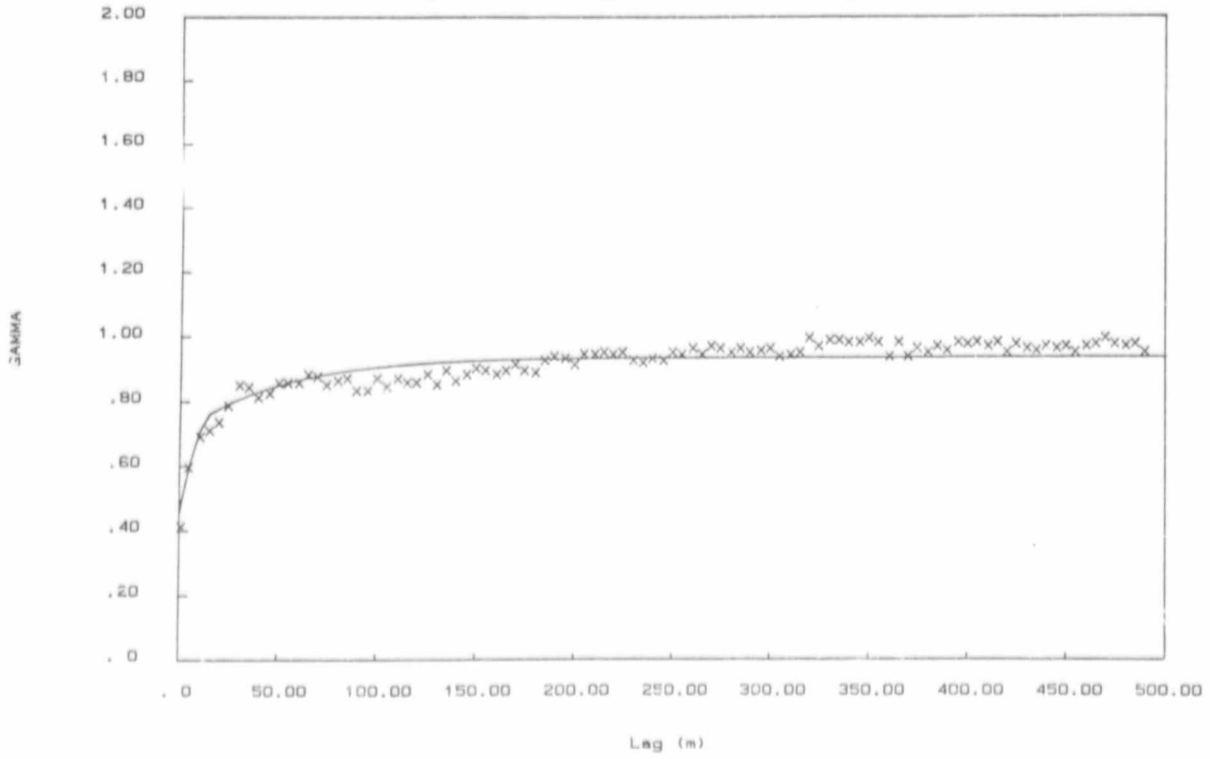


Fig. 4.28c: Omnidirectional Log. Semivariogram S2.1

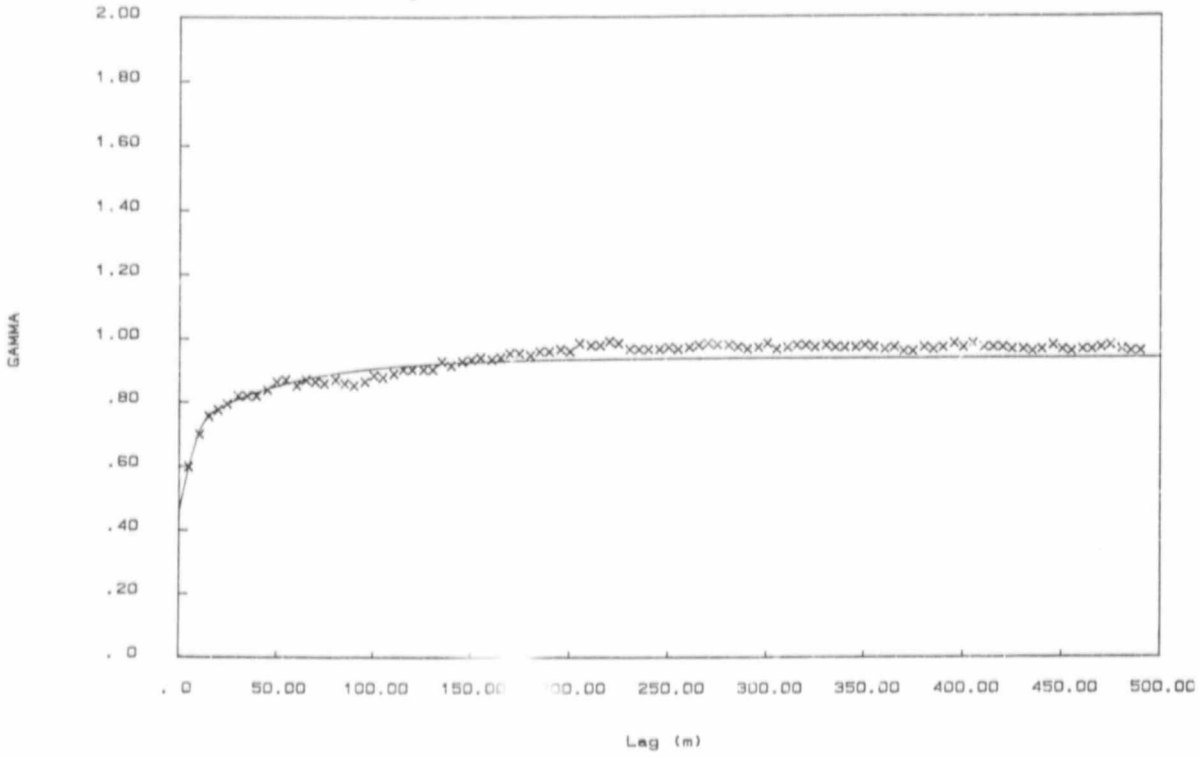


Fig. 4.28d: Omnidirectional H&C - Semivariogram S2.1

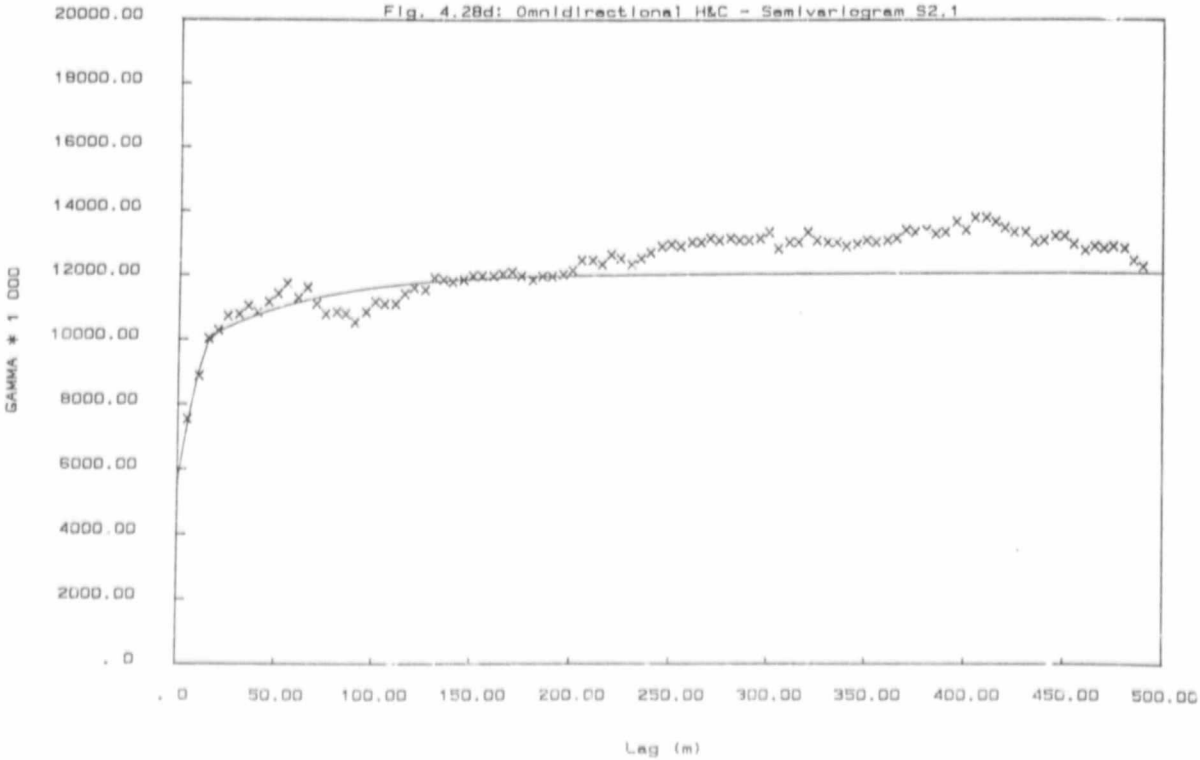


Fig. 4.29: Plot of Log. Block Variance vs. Log (Area), S2.1

Fig. 4.30a: N-S Logtransformed Semivariogram S2.2

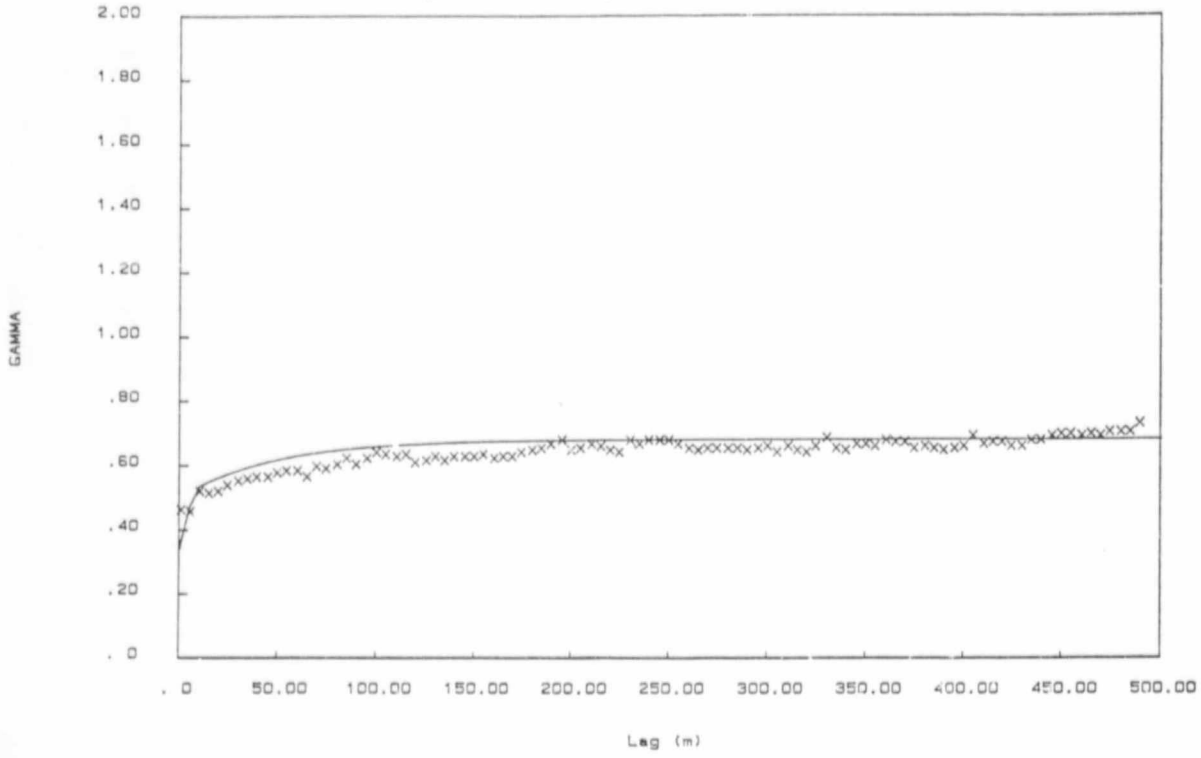


Fig. 4.30b: E-W Logtransformed Semivariogram S2.2

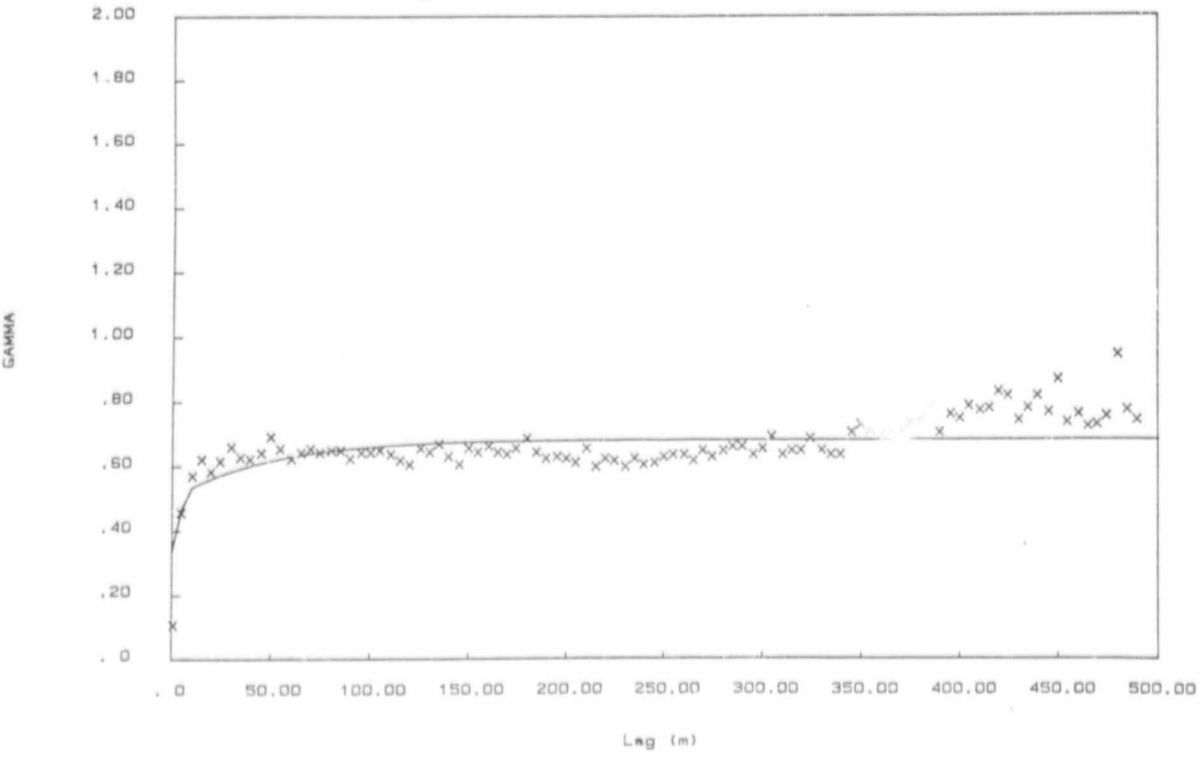


Fig. 4.30c: NE-SW Logtransformed Semivariogram S2.2

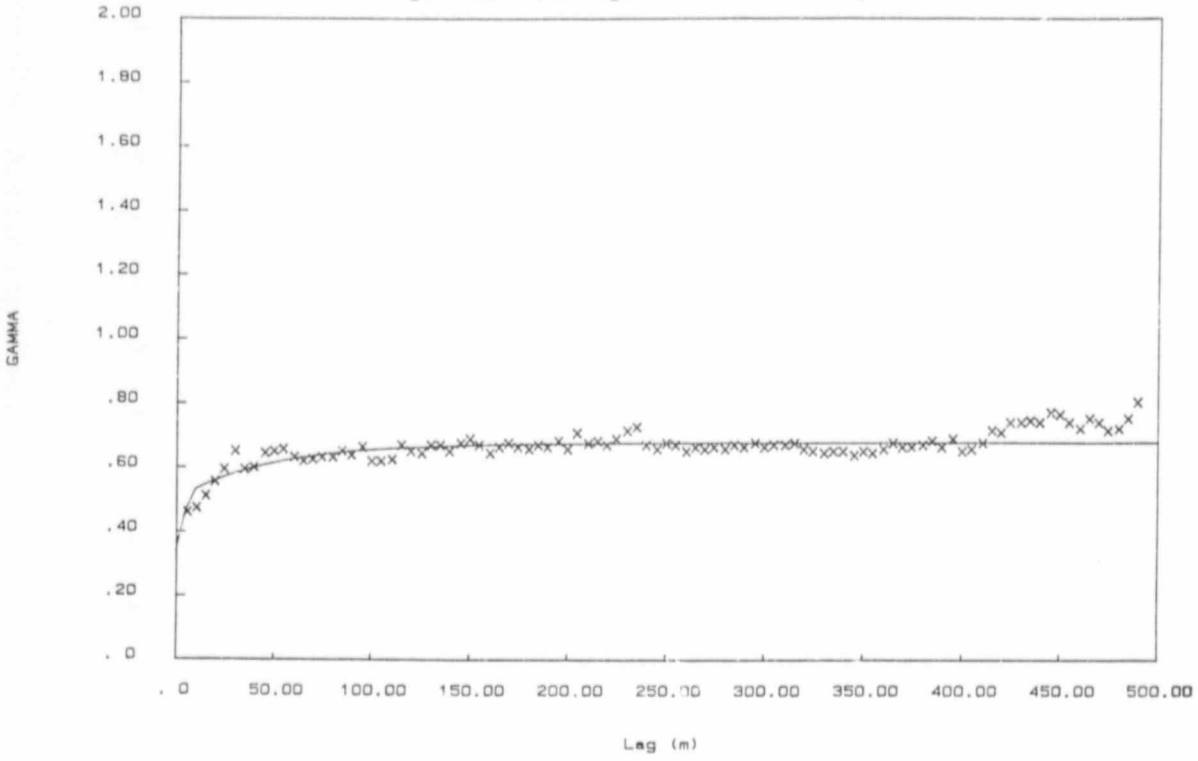


Fig. 4.30d: NW-SE Logtransformed Semivariogram S2.2

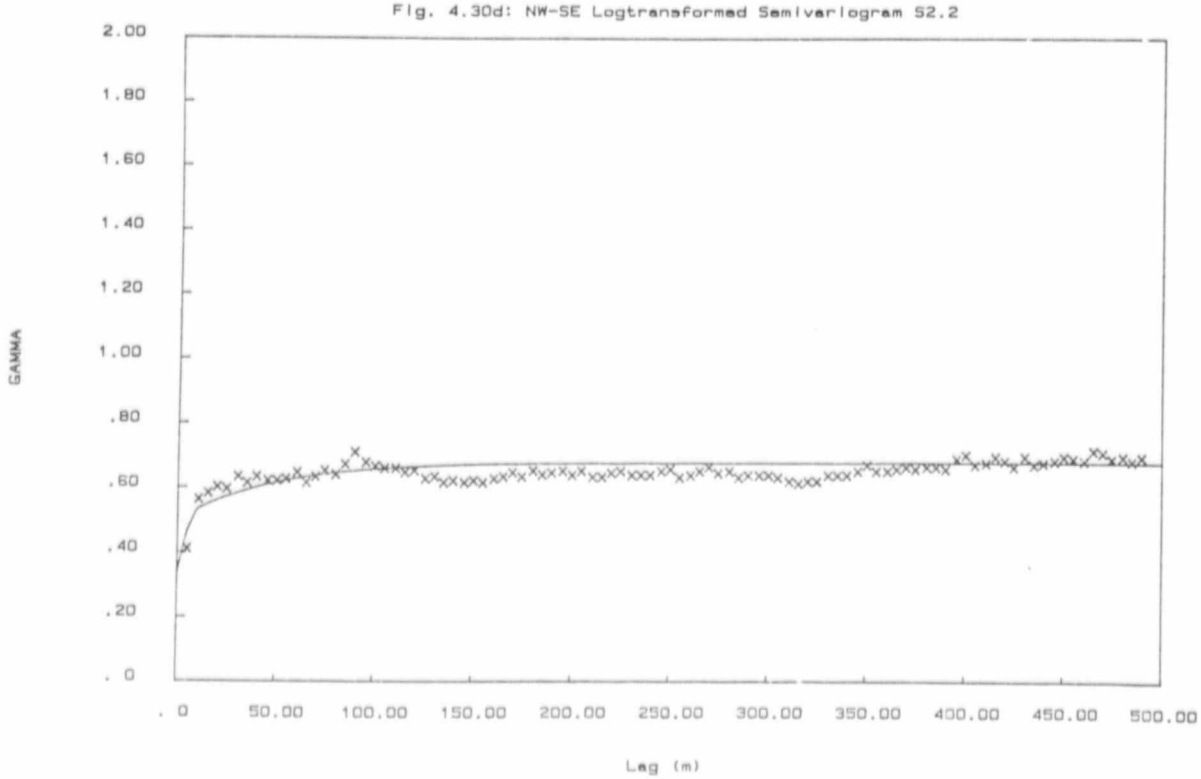


Fig. 4.30e: Omnidirectional Log. Semivariogram S2.2

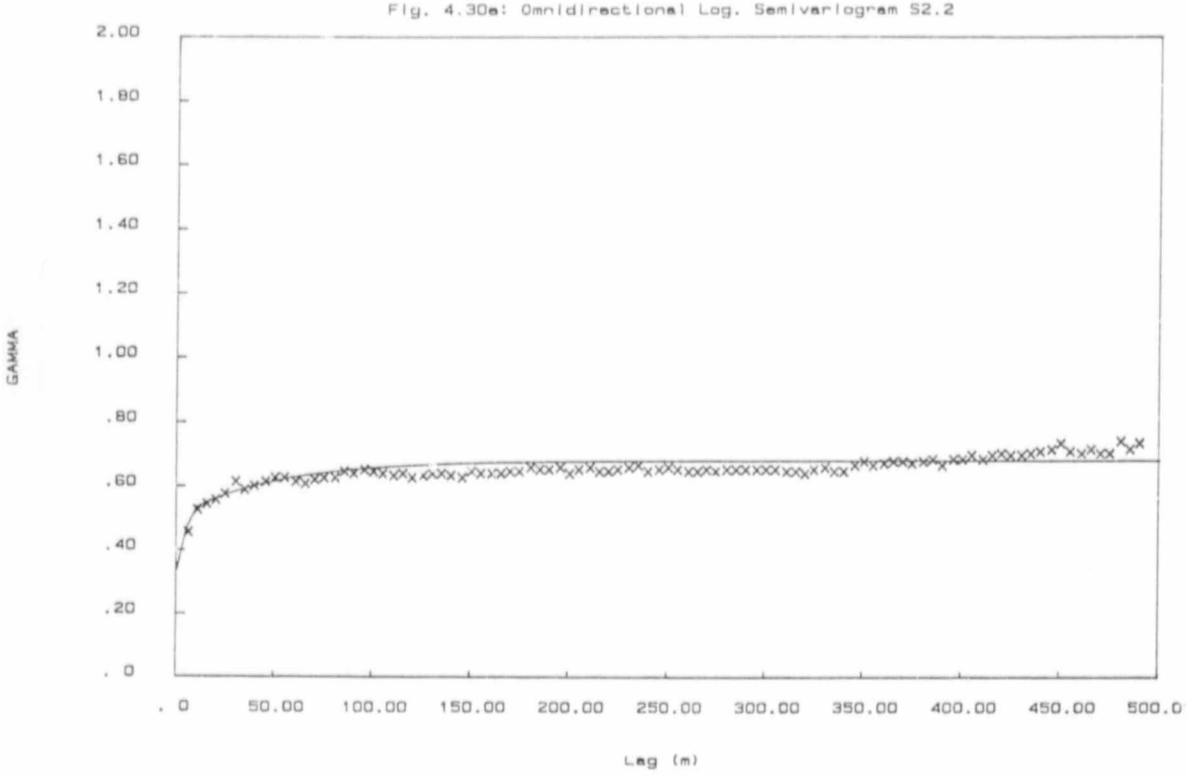
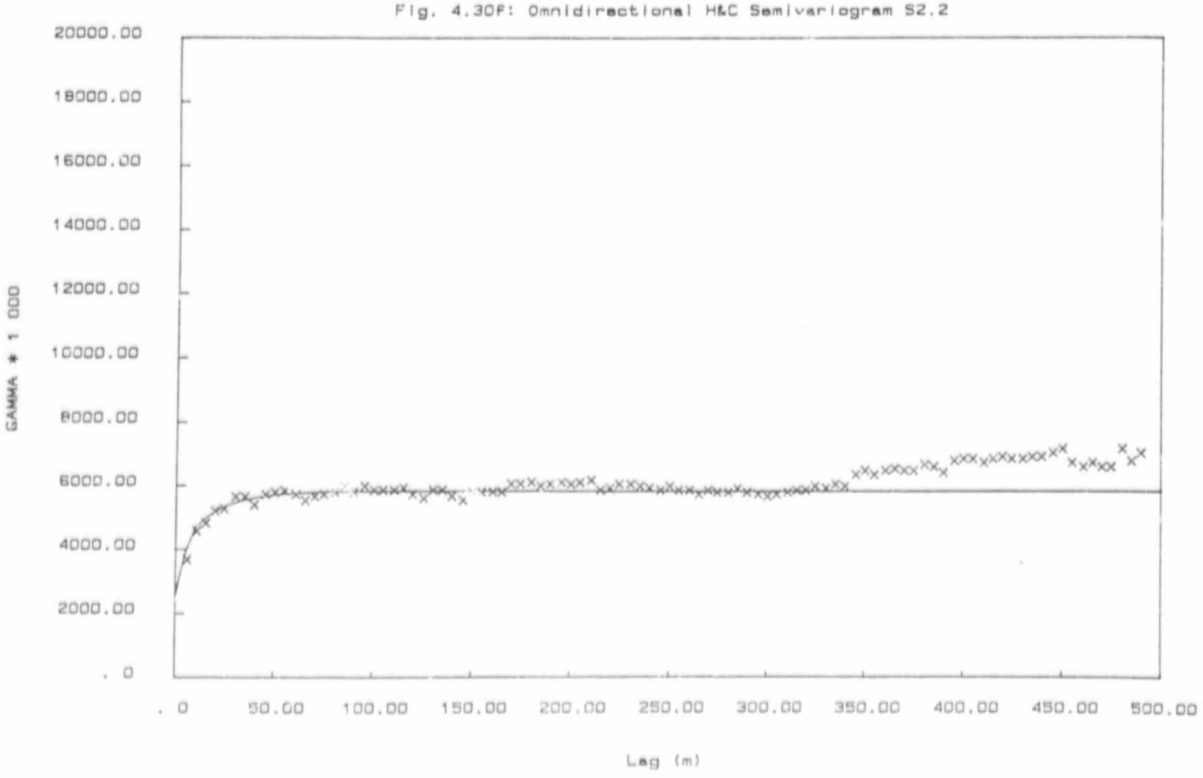


Fig. 4.30f: Omnidirectional H&C Semivariogram S2.2



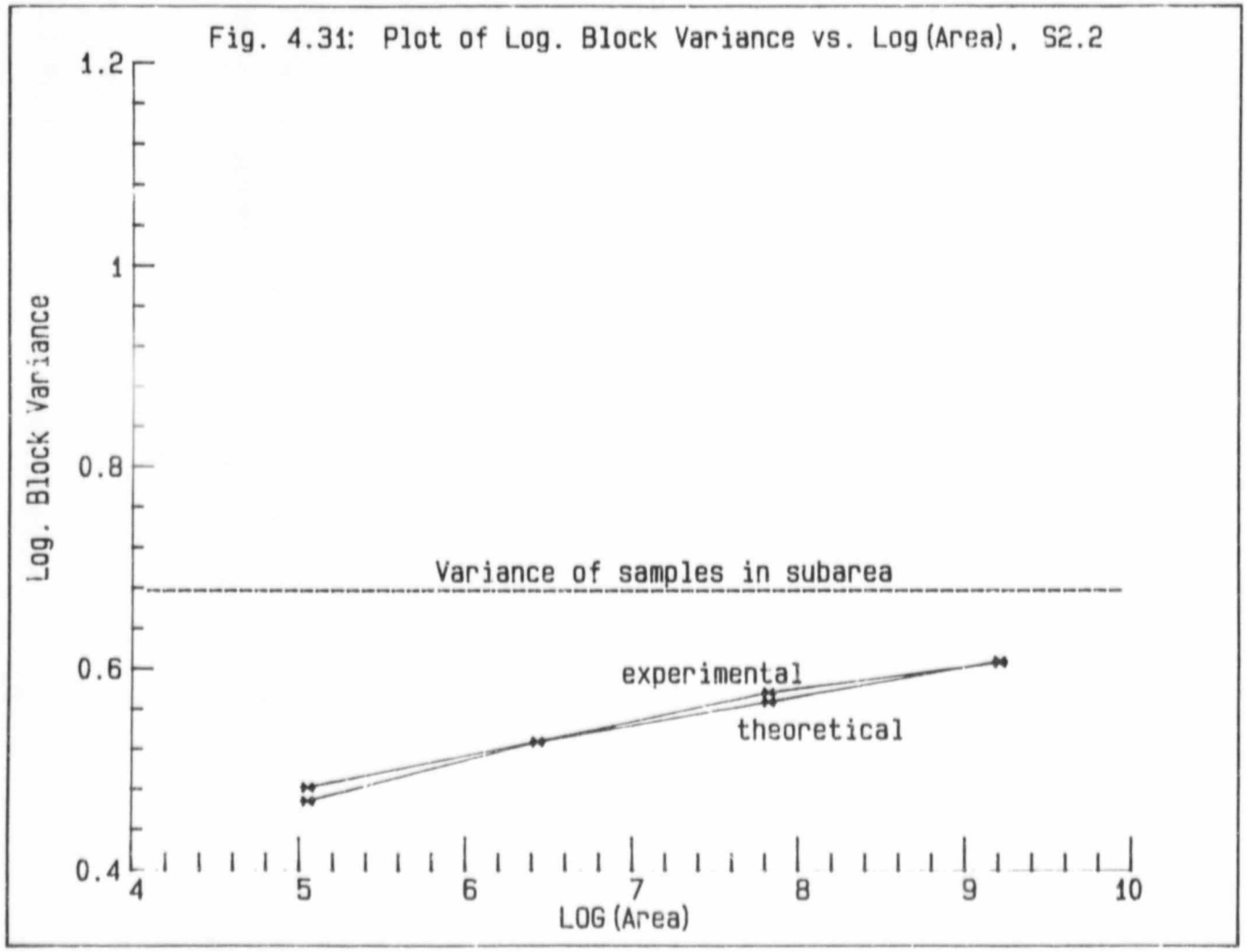


Fig. 4.31: Plot of Log. Block Variance vs. Log(Area), S2.2
1.2
1
0.8
0.6
0.4
Log. Block Variance
Variance of samples in subarea
experimental
theoretical
4
5
6
7
8
9
10
LOG(Area)

Fig 4.32a: N-S Logtransformed Semivariogram S2.3

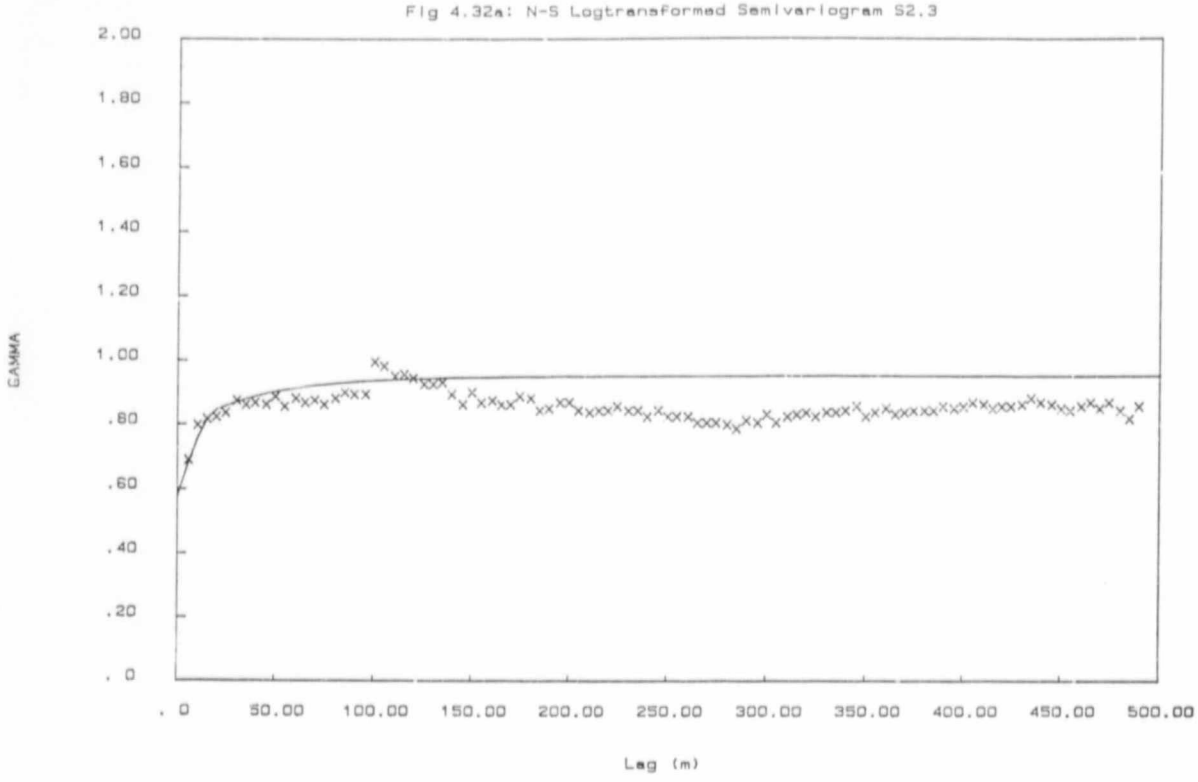


Fig. 4.32b: E-W Logtransformed Semivariogram S2.3

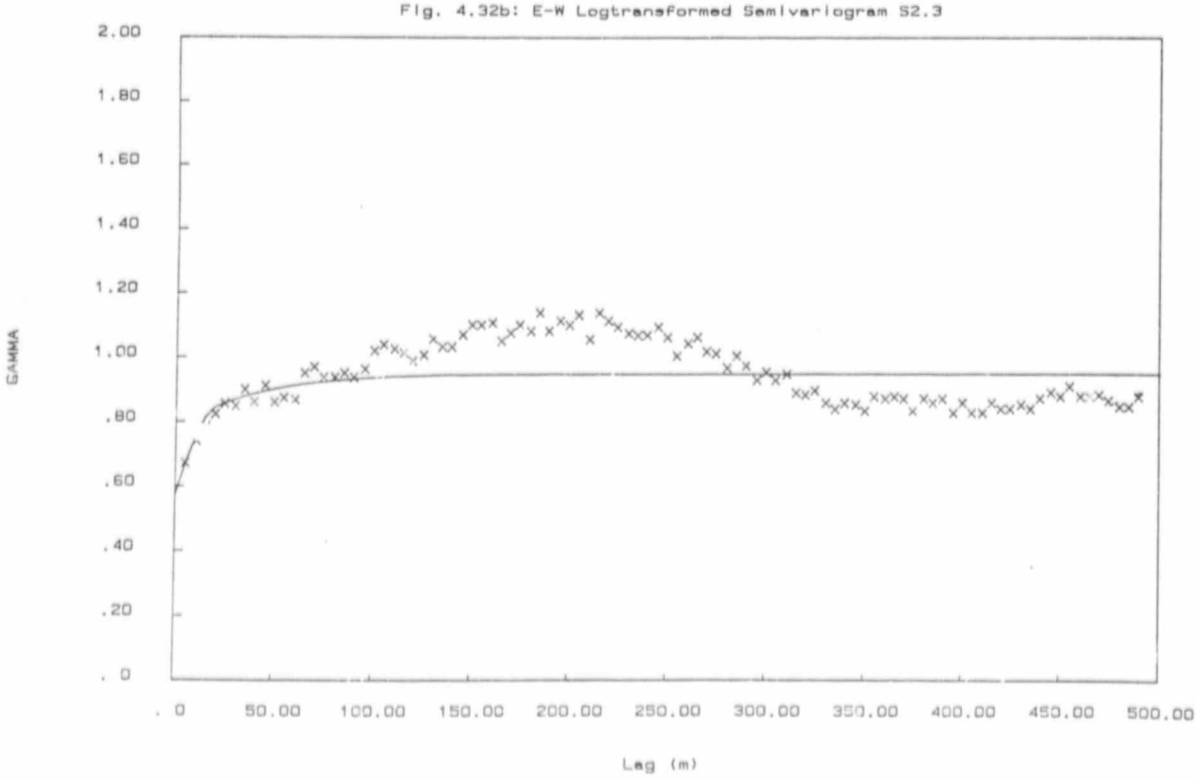


Fig. 4.32c: Omnidirectional Log. Semivariogram S2.3

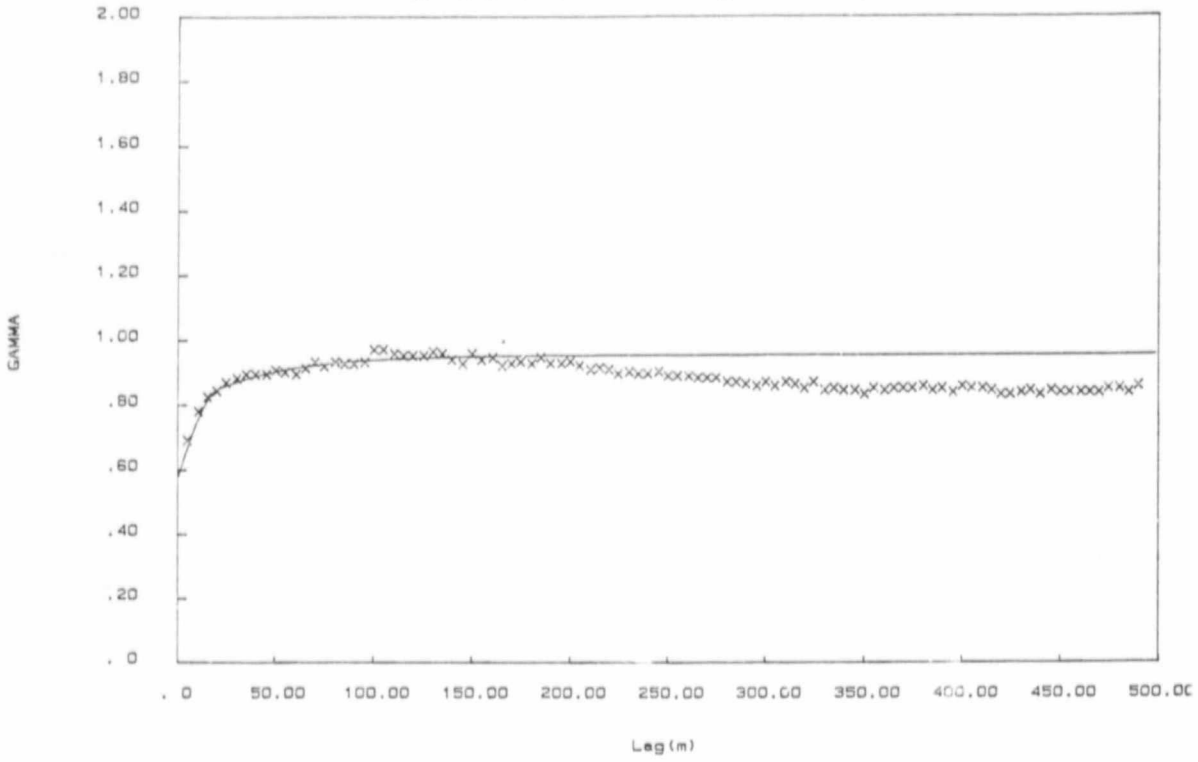
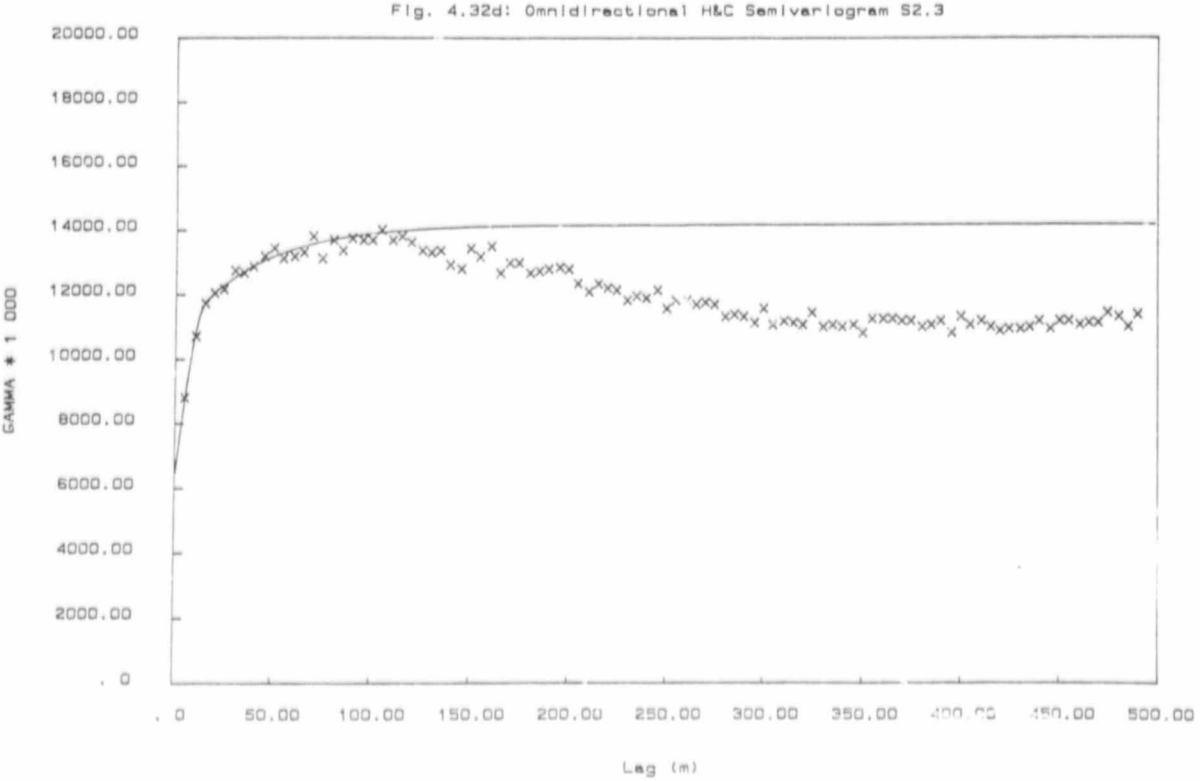


Fig. 4.32d: Omnidirectional H&C Semivariogram S2.3



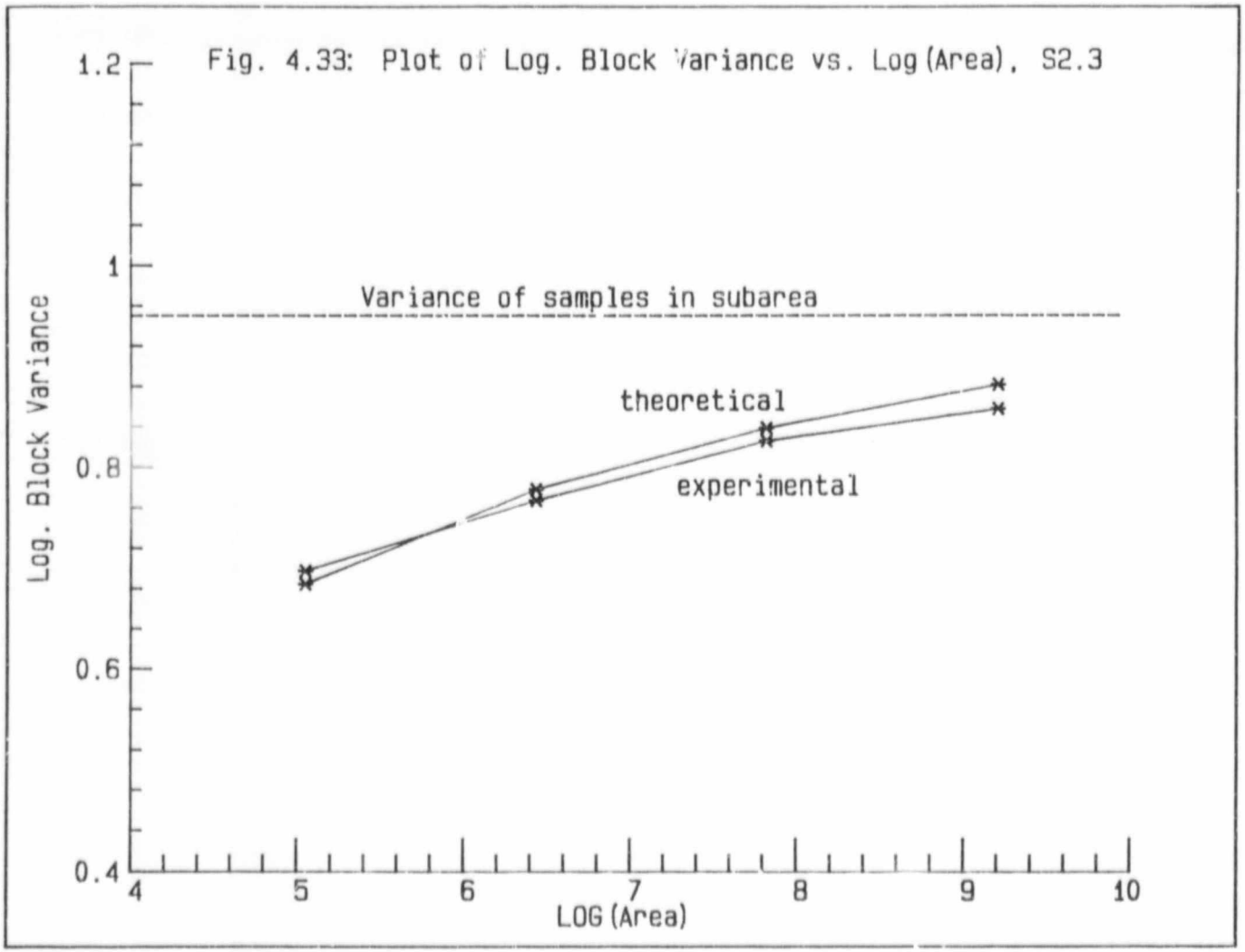


Fig. 4.33: Plot of Log. Block Variance vs. Log(Area), S2.3
1.2
1
0.8
0.6
0.4
Log. Block Variance
Variance of samples in subarea
theoretical
experimental
4
5
6
7
8
9
10
LOG(Area)

Fig. 4.34a: Logtransformed Semivariogram Models

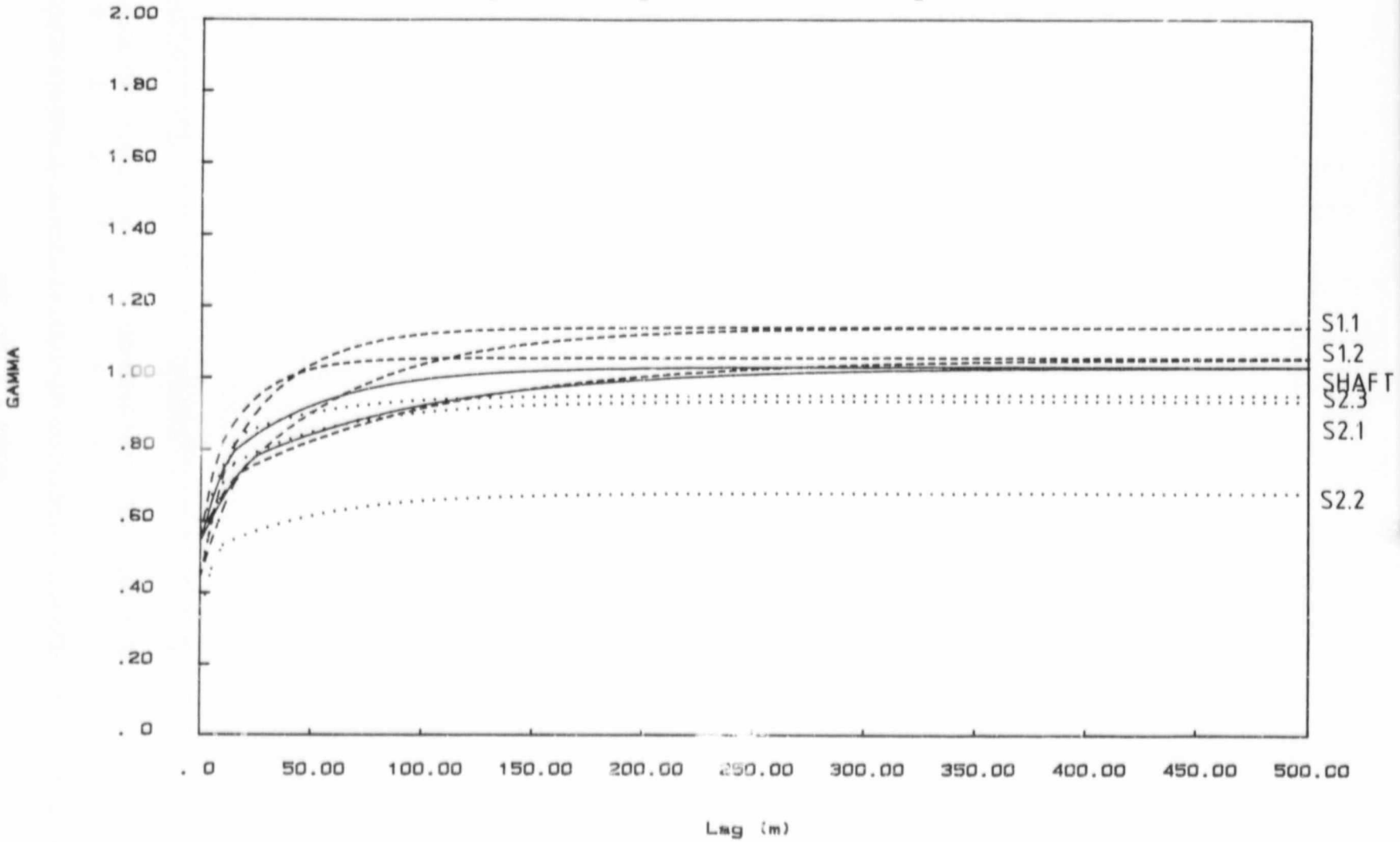


Fig. 4.34b: Backtransformed Semivariogram Models

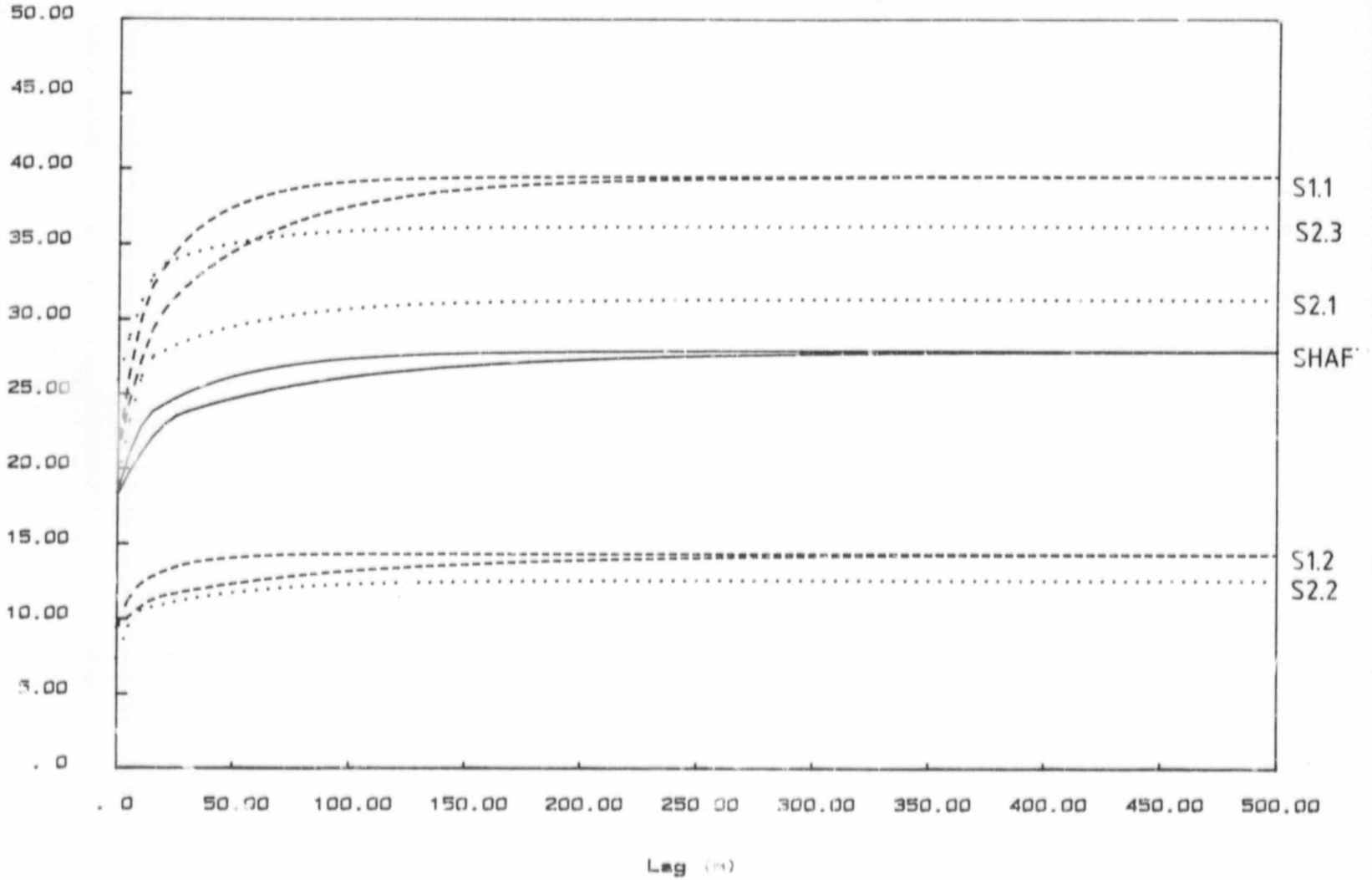
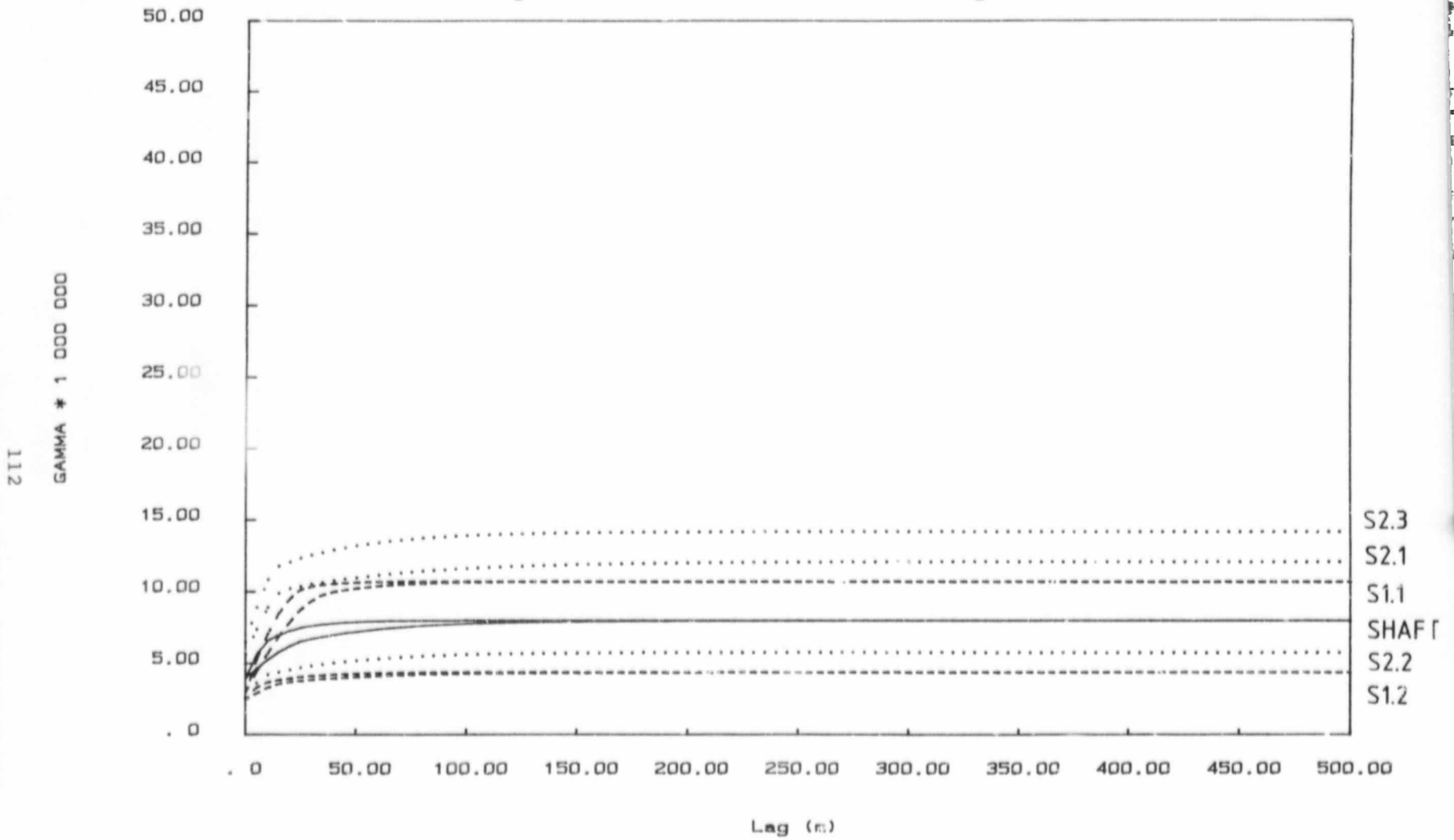


Fig. 4.34c: Hawkins & Cressie Semivariogram Models



4.6.3 Individual Analysis of Mineralisation in Top and Bottom Bands for Subfacies 1

As mentioned in chapter 3.3, geological mapping indicates that the top conglomerate band of Subfacies 1 is the equivalent of the single conglomerate band of Subfacies 2. The spatial structure of the gold mineralisation differs between the two subfacies as shown by the semivariogram analysis. Assuming the top reef band of Subfacies 1 as being equivalent to the Subfacies 2 reef band means that its gold distribution should be similar to that of Subfacies 2 areas. In order to investigate this hypothesis, the accumulations (cmg/t) in a multiple reef bands area in the southeastern quadrant had to be recalculated separately for top and bottom band(s), as the assay plans only contain the combined grade. Distinguishing between top and bottom band or bands on the individual sample sheets is often ambiguous. Only sections where the bands can be differentiated clearly are included in the analysis, resulting in 296 samples.

The lower band(s) contain about twice as much gold than the top band. It is generally observed that the gold content of reef type 2 is low where it overlies reef type 1(see chapter 3.5).

Semivariograms of both the gold data from the top and bottom bands are pictured in Figures 4.35a and 4.35b. Because of the limited number of data these are scattered and unstable. But it still can be seen that in palaeocurrent direction (N-S) the bottom band (Reef Type 1) shows a larger range than across (E-W), thus indicating the presence of a geometric anisotropy. For the top band (Reef Type 2) the ranges in both directions are the same , indicating a homogeneous mineralisation in all directions. Though, because of the few data available, these semivariograms are not as clear as those calculated on the thousands of data of the subareas (Figs. 4.24,4.27,4.28,4.30 and 4.32), the similarities between the top band of Subfacies 1 and the Subfacies 2 mineralisation are apparent.

So the continuity of the gold mineralisation supports the sedimentological observations.

The grades of the bottom bands are responsible for the anisotropy in the mineralisation of the Subfacies 1 subareas, when the overall accumulation, as given on the assay plans is analysed.

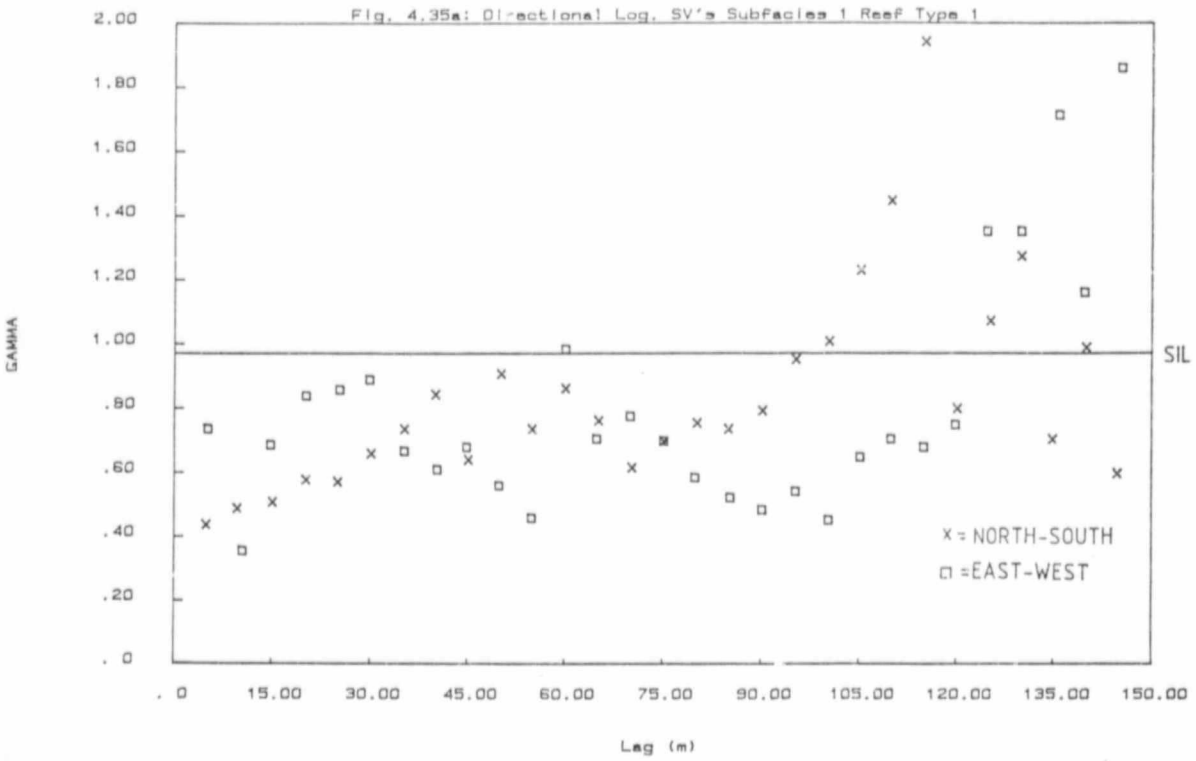


Fig. 4.35a: Directional Log. SV's Subfacies 1 Reef Type 1
2.00
1.80
1.60
1.40
1.20
1.00
.80
.60
.40
.20
. 0
GAMMA
SIL
x = NORTH-SOUTH
□ =EAST-WEST
. 0
15.00
30.00
45.00
60.00
75.00
90.00
105.00
120.00
135.00
150.00
Lag (m)

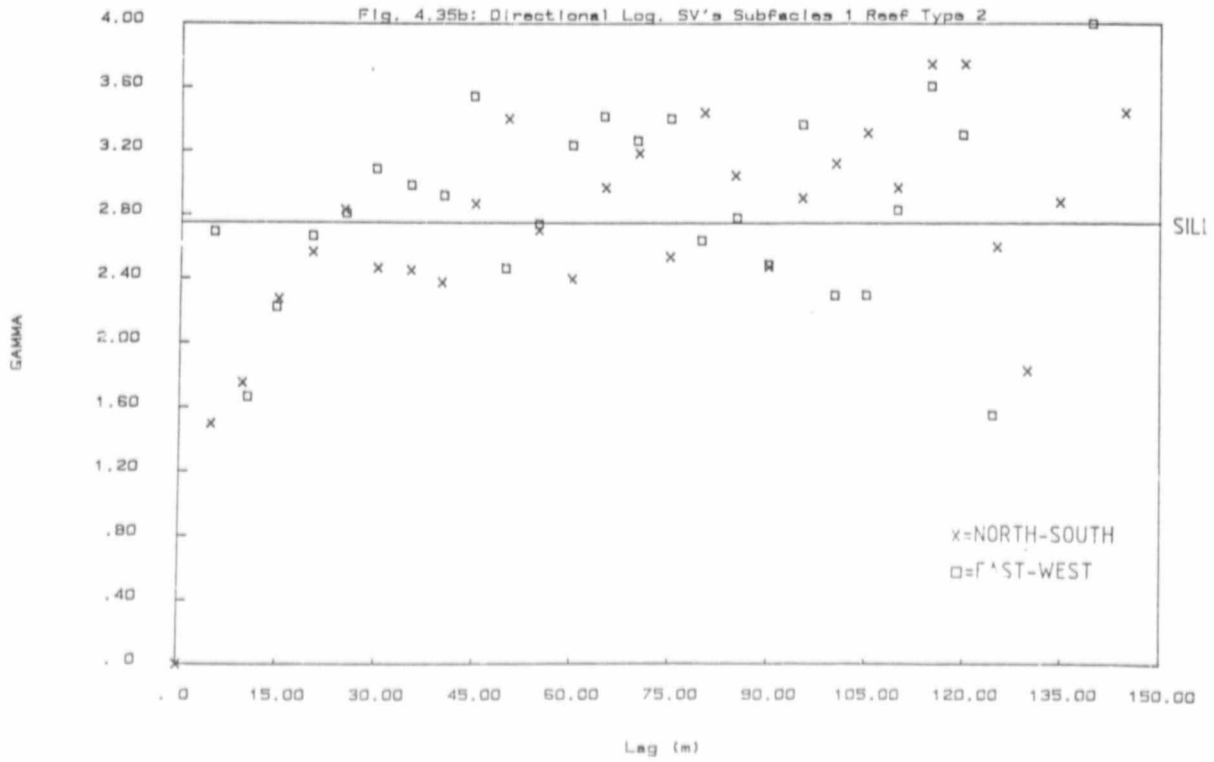


Fig. 4.35b: Directional Log. SV's Subfacies 1 Reef Type 2
4.00
3.60
3.20
2.80
2.40
2.00
1.60
1.20
.80
.40
. 0
GAMMA
SILI
x=NORTH-SOUTH
□=EAST-WEST
. 0
15.00
30.00
45.00
60.00
75.00
90.00
105.00
120.00
135.00
150.00
Lag (m)

4.7 Simulation of ore reserve estimation for mine blocks, with and without subdivision

4.7.1 Ore reserve estimation

Of the five subareas considered, only four are included in the follow-up analysis.

Quite a few assay plans covering subarea S2.3 are missing. They are mostly located so that either not enough follow-up data in an estimated oreblock are available because of the missing data, or that no samples are available close to the oreblock, which is then estimated by samples unrealistically far away.

For this reason subarea S2.3 is excluded from the follow-up study.

Another problem area is subarea S1.1, which gives only 10 data pairs for the regression analysis. But this subarea is still included in the follow-up analysis, as otherwise only one subfacies 1 subarea is left.

The number of data pairs for the other subareas is: 34 for S1.2, 82 for S2.1 and 39 for S2.2.

Oreblocks of $(100m)^2$ are estimated, as described in chapter 4.5.2. Only data inside a subarea are used, both for the estimation and the calculation of the 'true' follow-up grade.

The minimum number of samples an oreblock has to contain to qualify as a 'true' oreblock for the regression analysis, is determined in Appendix 1.1.

Again, fourteen different kriging variations are tested, and compared with follow-up data in a regression analysis (see chap. 4.5.2).

In this case the estimates are based on individual sample sections ('point data') instead of on the data blocks regularised on a $(50m)^2$ grid, as with the reduced size of the subareas the computer hardware limitations are no longer that effective.

The semivariogram models used are described in chapter 4.6.2. The maximal search radius is 600m, and the maximum number of samples used in an estimation is 64.

Actual error variances, as well as estimation variances, are corrected for the additive constant in the case of logtransformed kriging, and transformed to logarithmic variances, if linear kriging is used.

4.7.2 Analysis of results of $(100m)^2$ oreblock estimation in the individual subareas

Figures 4.37 to 4.40 compare the slopes and error variances of the follow-up analysis graphically (the detailed results of the regression analysis are listed in Tables 4.5 to 4.8). Note that the mean grades of the subareas as disguised by a factor are reflected in Table 4.2.

While the horizontal scale remains the same for all figures, the vertical scale varies, and covers the range of occurring error variances, except for universal kriging. Only methods where slope factors are below 2.5 are plotted.

Because of the limited number of data pairs these figures also illustrate the sometimes considerable error of the slope factor. A high error in the slope factor usually happens when simple kriging is used. As some weight is given to the mean for each block, the estimates are grouped closely around the mean. But because the estimates are not widely spread, the regression line can be expected to be unstable. Thus the calculation of the slope of the linear regression line involves a higher error than in cases where the estimates are spread wider.

4.7.2.1 Influence of subdivision on grade estimates

Conditional unbiasedness

The conditional unbiasedness of an estimation method is shown by the deviation of the slope factor of the regression formula from 1.0 (see chapter 4.2). Comparing the slope factors for individual subareas and individual kriging methods (Figure 4.37 to 4.40) shows that there is no significant difference between using local and overall semivariograms. The methods that give conditionally unbiased results (simple linear and lognormal kriging with a global mean), do so already without subdivision (see chapter 4.5.3). Naturally no improvement can be expected here by subdivision.

Mean error of the estimation

The global mean error of individual estimation methods is generally high. However, the mean error depends on the number of data pairs on which the regression analysis is based, and comes close to zero for all methods, if the number of pairs is high. Figure 4.36 illustrates this relation.

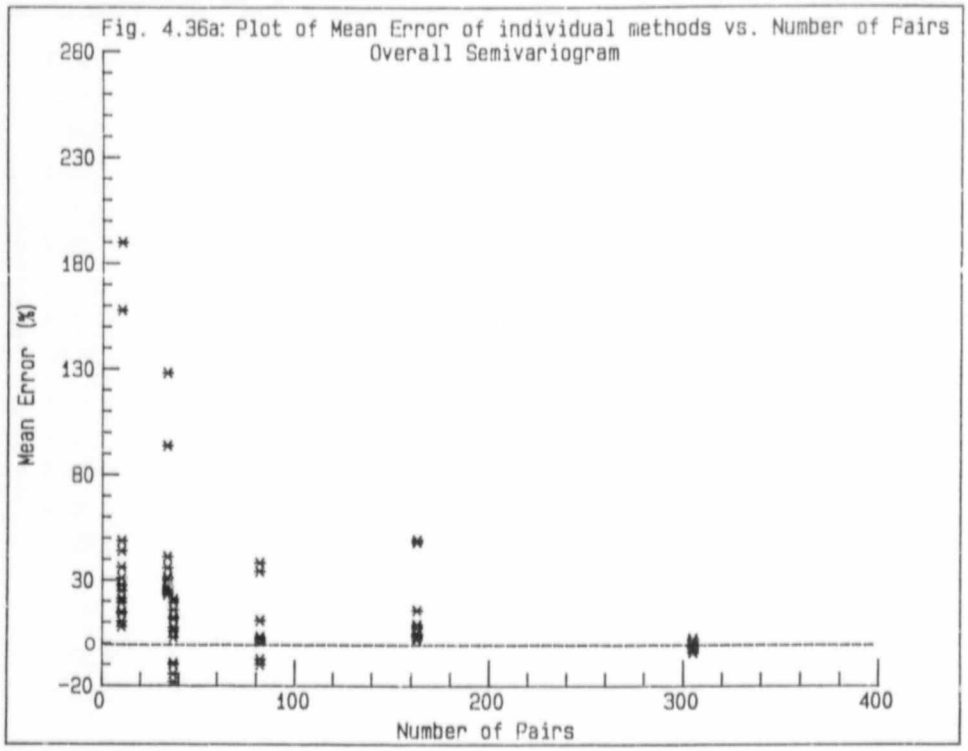


Fig. 4.36a: Plot of Mean Error of individual methods vs. Number of Pairs
Overall Semivariogram
Mean Error (%)
280
230
180
130
80
30
0
-20
0
100
200
300
400
Number of Pairs

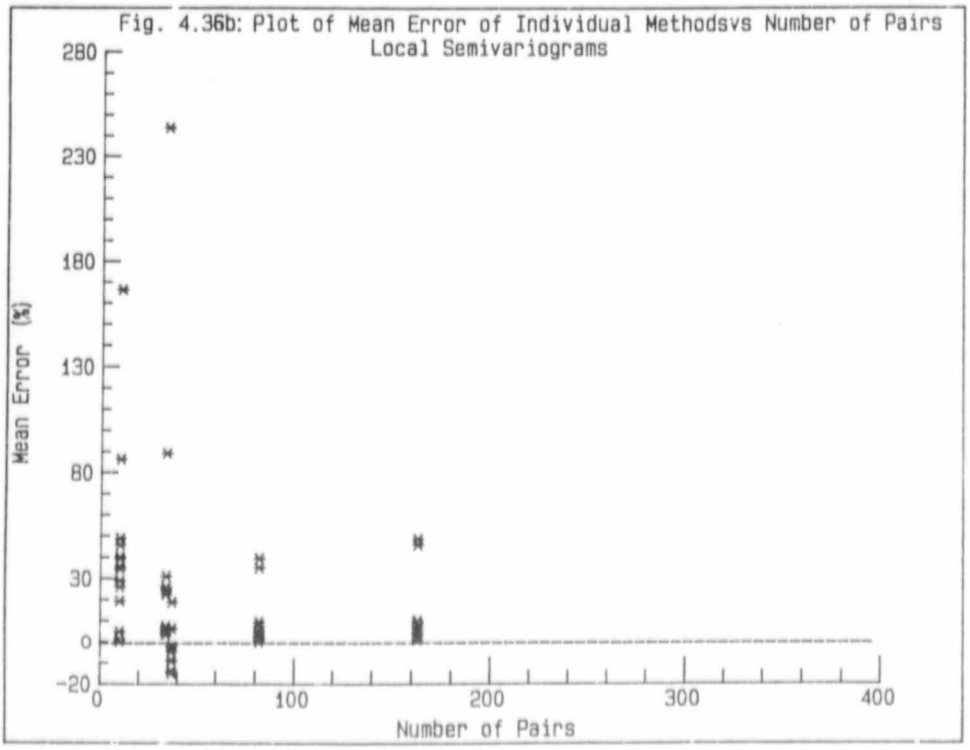


Fig. 4.36b: Plot of Mean Error of Individual Methodsvs Number of Pairs
Local Semivariograms
Mean Error (%)
280
230
180
130
80
30
0
-20
0
100
200
300
400
Number of Pairs

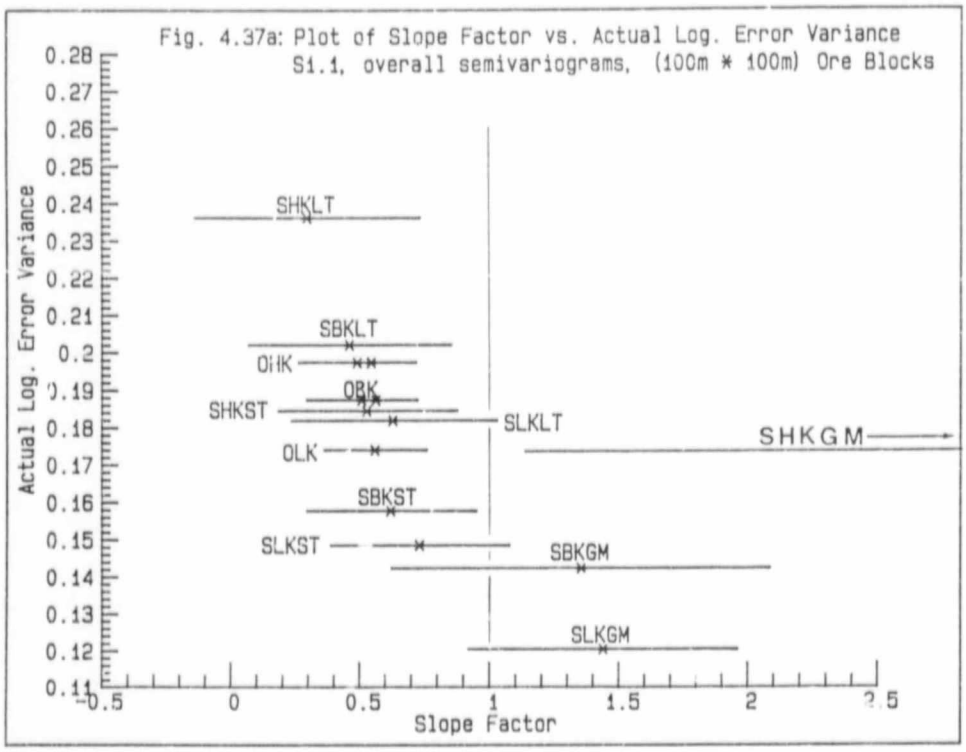


Fig. 4.37a: Plot of Slope Factor vs. Actual Log. Error Variance
Si.1, overall semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
SHKLT
SBKLT
OHK
OBK
SHKST
SLKLT
SHKGM
OLK
SBKST
SLKST
SBKGM
SLKGM
-0.5
0
0.5
1
1.5
2
2.5
Slope Factor

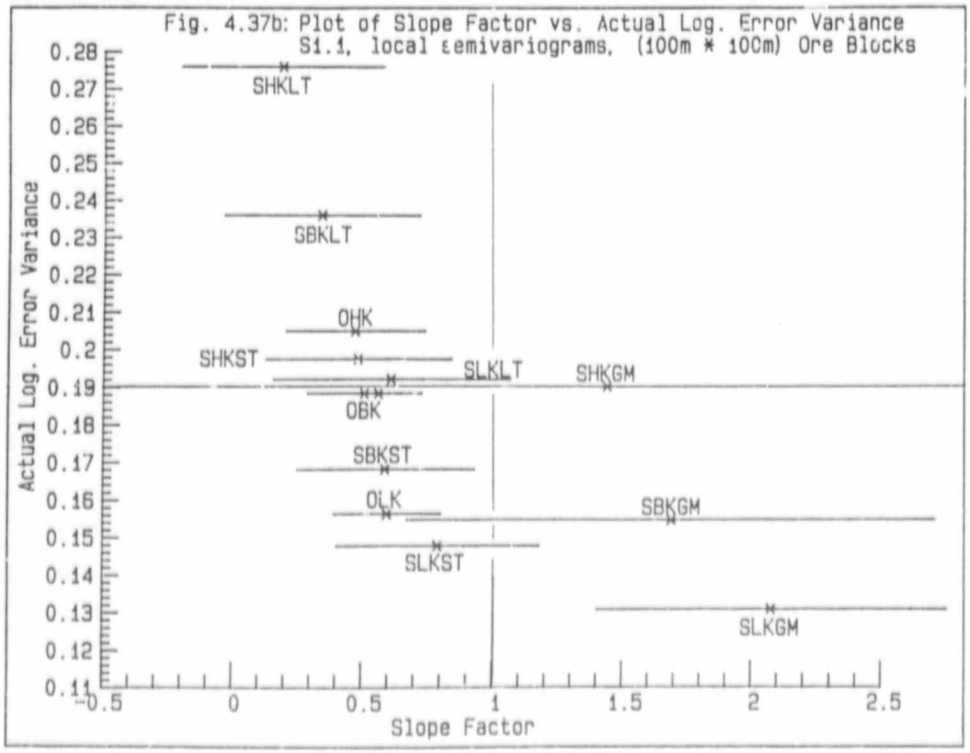


Fig. 4.37b: Plot of Slope Factor vs. Actual Log. Error Variance
Si.1, local semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
SHKLT
SBKLT
OHK
SHKST
SLKLT
SHKGM
OBK
SBKST
OLK
SBKGM
SLKST
SLKGM
-0.5
0
0.5
1
1.5
2
2.5
Slope Factor

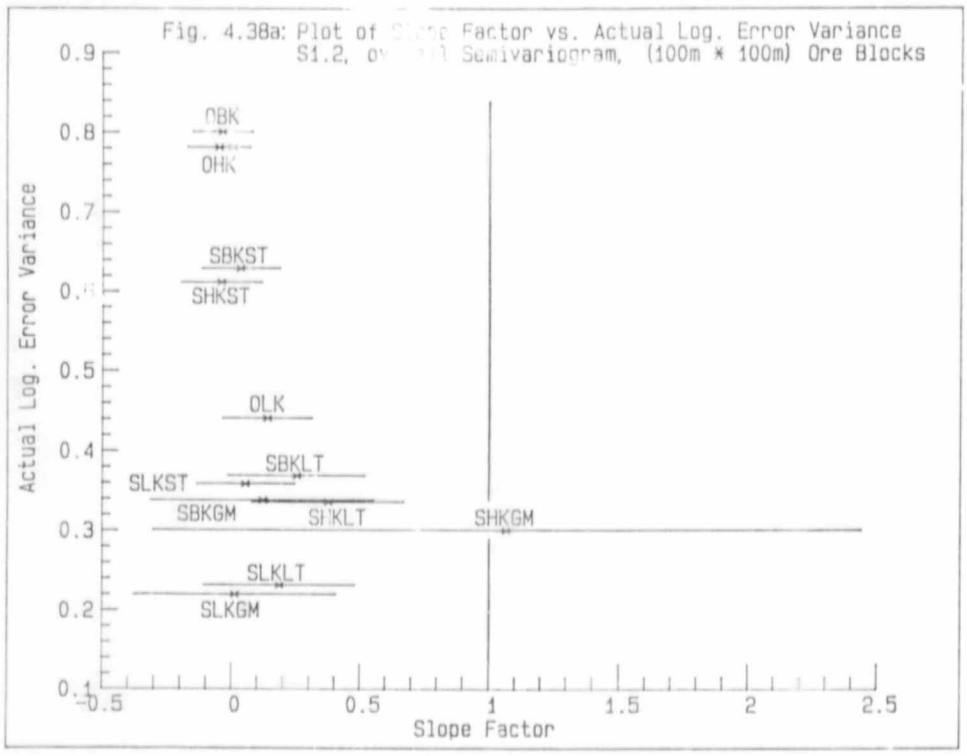


Fig. 4.38a: Plot of Slope Factor vs. Actual Log. Error Variance
S1.2, overall Semivariogram, (100m * 100m) Ore Blocks
Actual Log. Error Variance
Slope Factor
OBK
OHK
SBKST
SHKST
OLK
SBKLT
SLKST
SBKGM
SHKLT
SHKGM
SLKLT
SLKGM
0.9
0.8
0.7
0.5
0.4
0.3
0.2
0.1
-0.5
0
0.5
1
1.5
2
2.5

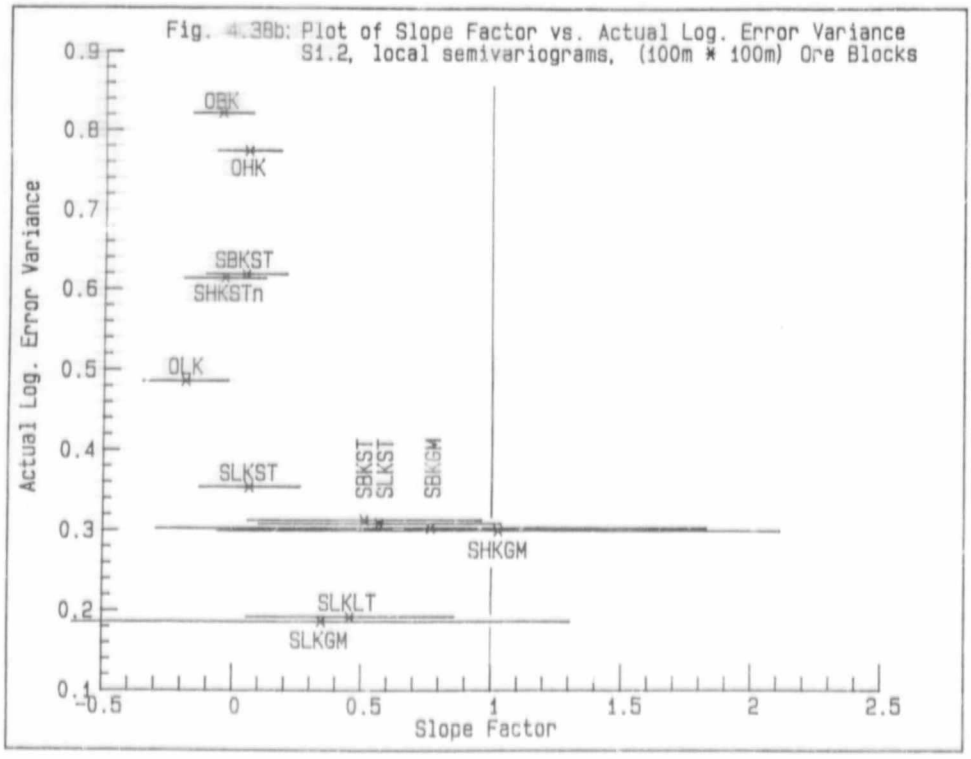


Fig. 4.38b: Plot of Slope Factor vs. Actual Log. Error Variance
S1.2, local semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
Slope Factor
OBK
OHK
SBKST
SHKSTn
OLK
SLKST
SBKST
SLKST
SBKGM
SHKGM
SLKLT
SLKGM
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.5
0
0.5
1
1.5
2
2.5

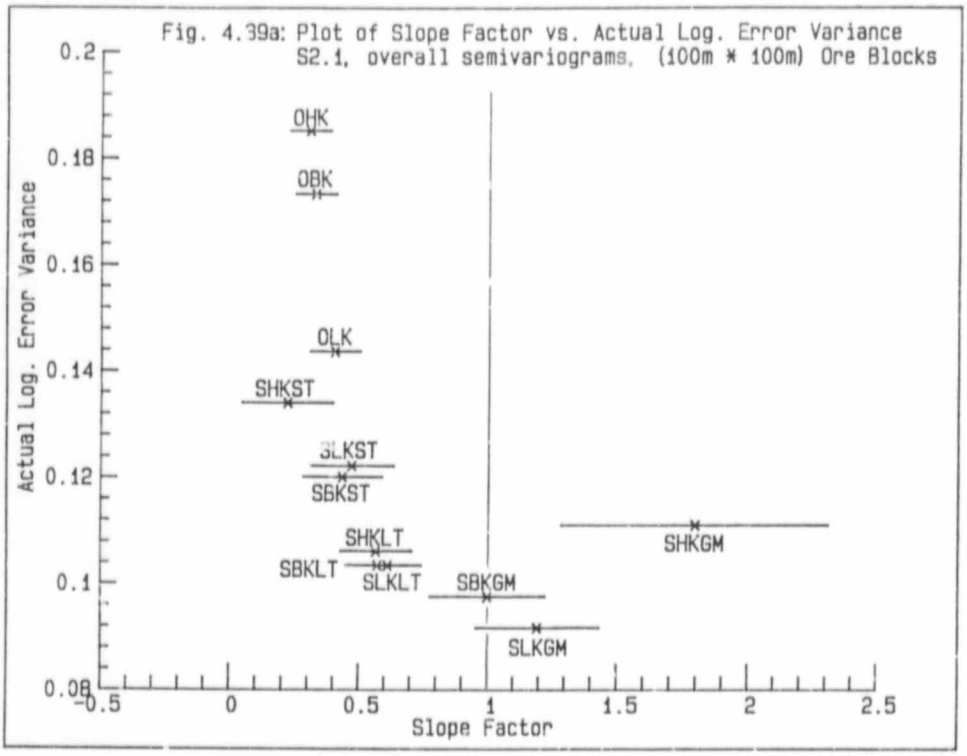


Fig. 4.39a: Plot of Slope Factor vs. Actual Log. Error Variance
S2.1, overall semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
0.2
0.18
0.16
0.14
0.12
0.1
0.08
OHK
OBK
OLK
SHKST
SLKST
SBKST
SHKLT
SBKLT
SLKLT
SBKGM
SHKGM
SLKGM
-0.5
0
0.5
1
1.5
2
2.5
Slope Factor

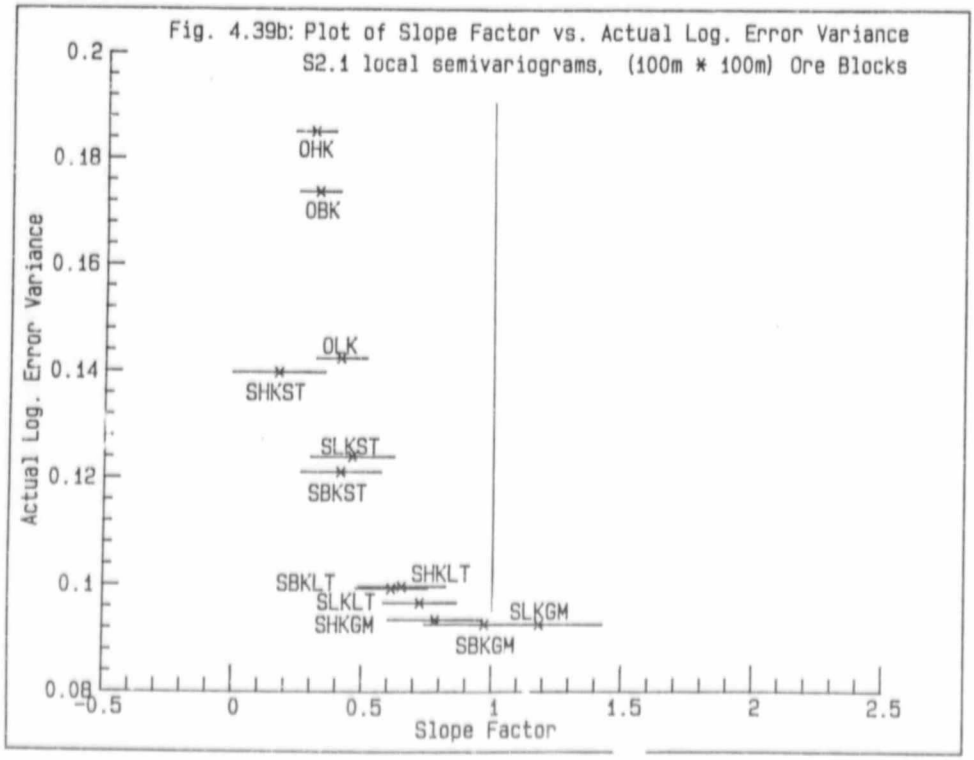


Fig. 4.39b: Plot of Slope Factor vs. Actual Log. Error Variance
S2.1 local semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
0.2
0.18
0.16
0.14
0.12
0.1
0.08
OHK
OBK
OLK
SHKST
SLKST
SBKST
SBKLT
SHKLT
SLKLT
SHKGM
SLKGM
SBKGM
-0.5
0
0.5
1
1.5
2
2.5
Slope Factor

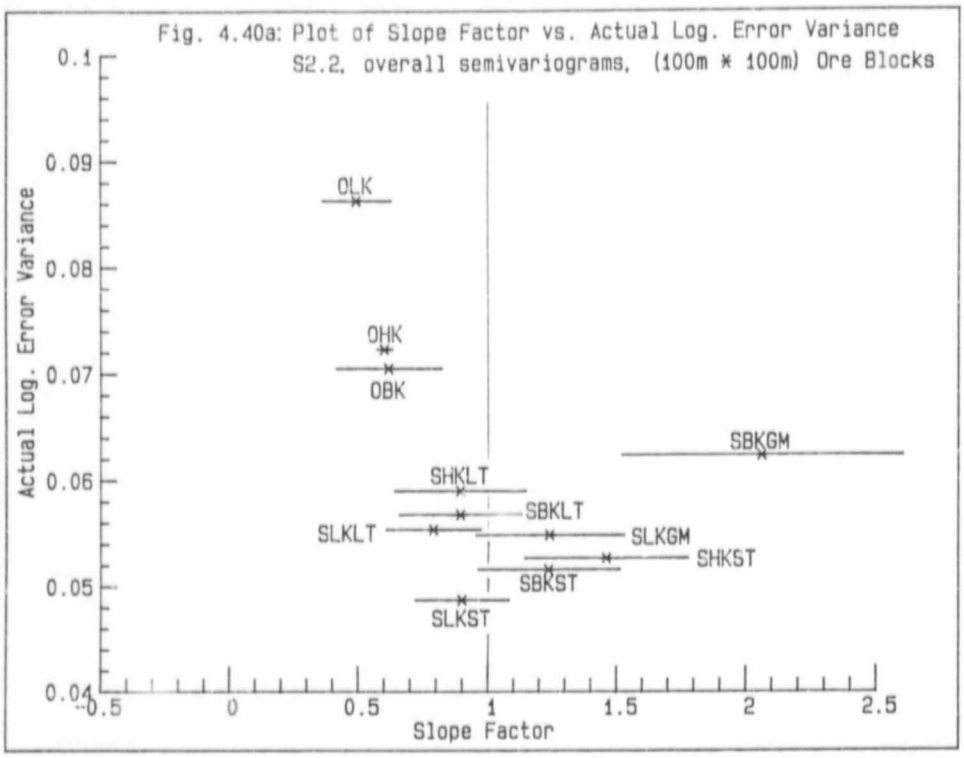


Fig. 4.40a: Plot of Slope Factor vs. Actual Log. Error Variance
S2.2, overall semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
0.1
0.09
0.08
0.07
0.06
0.05
0.04
OLK
OHK
OBK
SBKGM
SHKLT
SBKLT
SLKLT
SLKGM
SHKST
SBKST
SLKST
-0.5
0
0.5
1
1.5
2
2.5
Slope Factor

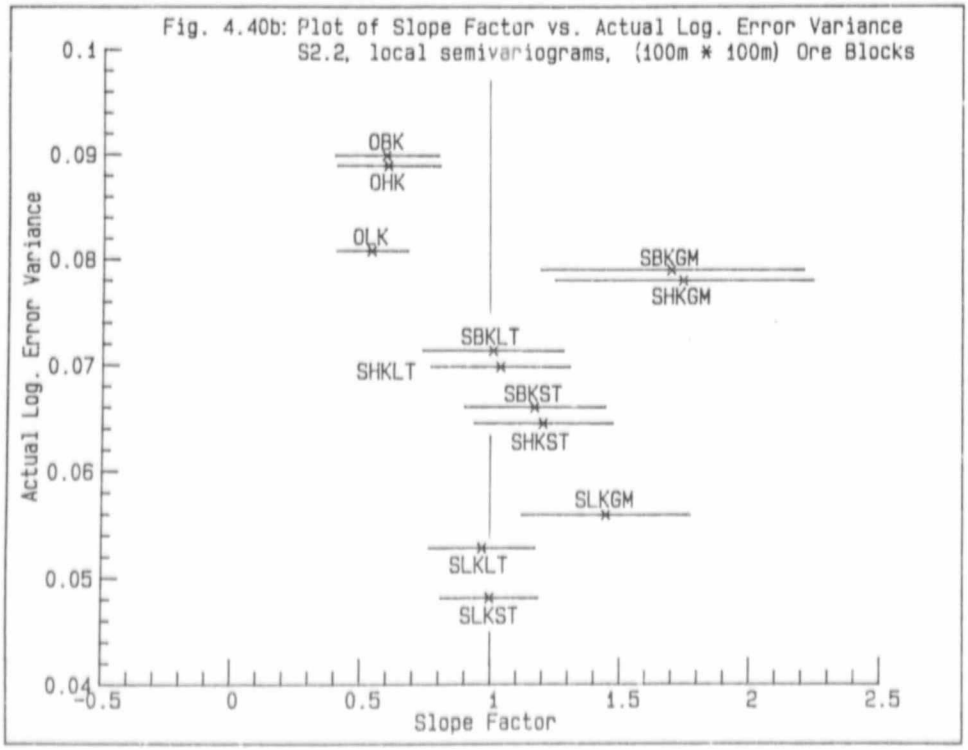


Fig. 4.40b: Plot of Slope Factor vs. Actual Log. Error Variance
S2.2, local semivariograms, (100m * 100m) Ore Blocks
Actual Log. Error Variance
0.1
0.09
0.08
0.07
0.06
0.05
0.04
OBK
OHK
OLK
SBKGM
SHKGM
SBKLT
SHKLT
SBKST
SHKST
SLKGM
SLKLT
SLKST
-0.5
0
0.5
1
1.5
2
2.5
Slope Factor

Table 4.4: Weight given to mean for simple kriging with a global mean, based on local semivariograms and an idealized sample grid of 10m by 5m. In addition, the estimation variance for this ideal setup, and the average estimation variance based on the actual position of the samples (as listed in Tables 4.5 to 4.8), are listed. A comparison of these two variances shows that the average estimation variance is higher than the estimation variance based on the idealized sample grid. This is due to the actual sample grid being on average slightly less favourable for the estimation. This is to be expected (Dykes, fault losses etc.). In general, however, the estimation setup is approximated closely by the idealized sample grid.

Subarea	Method	Weight for Mean	Corrected/Transformed Estimation Variance	Corrected/Transformed Average Est. Variance
S1.1	SLKGM	.6296	.15431	.17159
	SBKGM	.6700	.18924	.21088
	SHKGM	.8631	.05060	.05293
S1.2	SLKGM	.7650	.10440	.11123
	SBKGM	.7876	.13169	.13691
	SHKGM	.7639	.03088	.03338
S2.1	SLKGM	.4945	.06906	.08431
	SBKGM	.5409	.09104	.10512
	SHKGM	.4725	.04191	.05131
S2.2	SLKGM	.4905	.05335	.06484
	SBKGM	.5222	.06689	.07788
	SHKGM	.5338	.04018	.04777

Table 4.5a: Subarea S1.1 , regression analysis of 10 $(100m)^2$ oreblocks, overall semivariograms.

method	regression formula ln(Z+170) / Z =	standard error of slope	corrected/ transformed error variance actual	corrected/ transformed error variance estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	-3.76 +1.4402ln(Z^*+170)	.5204	.12052	.13222	8.1	.11895
SLKST	1.94 + .7325ln(Z^*+170)	.3445	.14856	.13222	36.3	.14052
SLKLT	2.90 + .6308ln(Z^*+170)	.3981	.18196	.13222	15.1	.17952
OLK	3.45 + .5612ln(Z^*+170)	.1986	.17403	.22348	31.0	.17201
SBKGM	-1890 +1.3559 Z^*	.7304	.14236	.18663	14.4	.14235
SBKST	355 + .6220 Z^*	.3275	.15778	.34605	44.1	.15774
SBKLT	1514 + .4613 Z^*	.3925	.20212	.23199	21.3	.20210
OBK	1201 + .5095 Z^*	.2149	.18744	.30365	27.7	.18740
UBK	1806 + .1522 Z^*	.0572	1.52352	1.79904	190.0	1.52154
SHKGM	-9650 +3.4486 Z^*	2.3104	.17364	.03962	10.5	.17355
SHKST	732 + .5288 Z^*	.3468	.18458	.03962	48.8	.18454
SHKLT	2224 + .2953 Z^*	.4371	.23623	.03962	19.4	.23615
OHK	1325 + .4912 Z^*	.2280	.19743	.10190	25.0	.19645
UHK	2620 + .1123 Z^*	.0711	1.48714	1.10637	157.9	1.39902

Table 4.5b: Subarea S1.1 , regression analysis of 10 $(100m)^2$ oreblocks, local semivariograms.

method	regression formula ln(Z+211) / Z =	standard error of slope	corrected/ transformed error variance actual	corrected/ transformed error variance estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	-8.94 +2.0784ln(Z^*+211)	.6756	.13071	.17159	1.4	.13026
SLKST	1.42 + .7901ln(Z^*+211)	.3880	.14764	.17159	40.3	.13953
SLKLT	2.98 + .6118ln(Z^*+211)	.4561	.18207	.17159	29.0	.18834
OLK	3.11 + .5961ln(Z^*+211)	.2054	.15618	.31600	39.5	.15595
SBKGM	-2656 +1.6917 Z^*	1.0180	.15457	.21088	4.8	.15448
SBKST	485 + .5888 Z^*	.3407	.16805	.43370	45.8	.16802
SBKLT	1860 + .3409 Z^*	.3781	.23608	.37436	34.5	.23500
OBK	1234 + .5083 Z^*	.2197	.18824	.36469	26.0	.18726
UBK	2697 + .1007 Z^*	.0714	1.54254	1.94314	166.3	1.45294
SHKGM	-1530 +1.4434 Z^*	3.9700	.19018	.05293	0.2	.19012
SHKST	967 + .4833 Z^*	.3577	.19741	.05293	48.8	.18740
SHKLT	2558 + .1880 Z^*	.3913	.27601	.05293	35.7	.26557
OHK	1496 + .4722 Z^*	.2676	.20494	.11332	19.5	.20392
UHK	2351 + .1693 Z^*	.0802	1.08775	1.40585	86.4	1.01848

Table 4.6a: Subarea S1.2 , regression analysis of 34 $(100m)^2$ oreblocks, overall semivariograms.

method	regression formula ln(Z+170) / Z =	standard error of slope	corrected/ transformed error variance actual	estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	7.68 + .0139ln(Z^*+170)	.3923	.22008	.13218	31.2	.20699
SLKST	8.23 + .0540ln(Z^*+170)	.1894	.35901	.13218	24.3	.34999
SLKLT	6.30 + .1857ln(Z^*+170)	.2951	.23163	.13218	23.6	.23118
OLK	8.91 + .1379ln(Z^*+170)	.1731	.44124	.23063	31.5	.42828
SBKGM	2153 + .1222 Z^*	.4341	.33830	.30150	35.9	.35349
SBKST	2695 + .0334 Z^*	.1515	.62960	.30339	26.4	.62453
SBKLT	1765 + .2525 Z^*	.2681	.36900	.27789	27.4	.36039
OBK	2715 - .0400 Z^*	.1176	.80119	.49307	24.6	.80056
UBK	2753 - .0107 Z^*	.0725	1.63844	2.40441	93.8	1.54802
SHKGM	-1344 +1.0682 Z^*	1.3717	.30050	.06592	41.2	.29445
SHKST	2720 - .0410 Z^*	.1576	.61196	.06592	25.4	.60639
SHKLT	3755 + .3755 Z^*	.2954	.33585	.06592	30.4	.32640
OHK	2755 - .0529 Z^*	.1219	.78155	.16695	23.1	.78050
UHK	2818 + .0020 Z^*	.0061	1.59037	1.64844	128.2	1.57216

Table 4.6b: Subarea S1.2 , regression analysis of 34 $(100m)^2$ oreblocks, local semivariograms.

method	regression formula ln(Z+112) / Z =	standard error of slope	corrected/ transformed error variance actual	estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	5.06 + .3449ln(Z^*+112)	.9596	.18595	.11123	3.2	.12980
SLKST	8.30 + .0621ln(Z^*+112)	.1941	.35298	.11123	24.1	.32068
SLKLT	4.18 + .4567ln(Z^*+112)	.4017	.19172	.11123	5.8	.19016
OLK	9.31 - .1874ln(Z^*+112)	.1638	.48576	.25354	31.1	.49661
SBKGM	509 + .7666 Z^*	1.0607	.30204	.13691	4.6	.29603
SBKST	2720 + .0411 Z^*	.1557	.61846	.24687	26.0	.61308
SBKLT	1176 + .5101 Z^*	.4530	.31181	.15257	7.4	.30172
OBK	2755 - .0524 Z^*	.1158	.82150	.29118	24.5	.81135
UBK	2840 - .0059 Z^*	.0186	1.68147	4.84632	243.8	1.60626
SHKGM	-165 +1.0260 Z^*	1.0853	.29871	.03338	3.6	.29261
SHKST	2721 - .0416 Z^*	.1572	.61342	.03338	25.5	.60789
SHKLT	1028 + .5680 Z^*	.4679	.30748	.03338	6.5	.30727
OHK	2741 - .0490 Z^*	.1228	.77403	.08169	22.5	.76275
UHK	2721 + .0172 Z^*	.0784	1.52018	1.00778	89.1	1.50312

Table 4.7a: Subarea S2.1, regression analysis of 82 $(100m)^2$ oreblocks, overall semivariograms.

method	regression formula ln(Z+170) / Z =	standard error of slope	corrected/ transformed error variance actual	estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	-1.58 +1.1937ln(Z^*+170)	.2387	.09170	.13067	-7.5	.09038
SLKST	4.39 + .4715ln(Z^*+170)	.1619	.12218	.13067	0.9	.10413
SLKLT	3.21 + .6120ln(Z^*+170)	.1278	.10344	.13067	1.2	.10239
OLK	4.93 + .4047ln(Z^*+170)	.0977	.14368	.20421	10.9	.13679
SBKGM	327 + .9992 Z^*	.2238	.09752	.12327	-7.5	.09377
SBKST	2381 + .4348 Z^*	.1530	.12010	.16313	1.3	.11971
SBKLT	1787 + .5718 Z^*	.1232	.10347	.16608	1.4	.10059
OBK	2831 + .3272 Z^*	.0800	.17332	.19040	2.2	.17049
UBK	3687 + .1075 Z^*	.0325	.65496	.90782	34.2	.62464
SHKGM	-2609 +1.7976 Z^*	.5123	.11096	.02530	-10.0	.10404
SHKST	3304 + .2209 Z^*	.1748	.13402	.02530	1.2	.13003
SHKLT	1804 + .5661 Z^*	.1582	.10612	.02530	1.7	.10497
OHK	2916 + .3047 Z^*	.0788	.18521	.05545	3.2	.18504
UHK	3710 + .1043 Z^*	.0334	.63082	.45542	38.3	.60894

Table 4.7b: Subarea S2.1, regression analysis of 82 $(100m)^2$ oreblocks, local semivariograms.

method	regression formula ln(Z+171) / Z =	standard error of slope	corrected/ transformed error variance actual	estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	-1.60 +1.1857ln(Z^*+171)	.2423	.09256	.08431	0	.09256
SLKST	4.52 + .4563ln(Z^*+171)	.1607	.12383	.08431	0	.12383
SLKLT	2.25 + .7235ln(Z^*+171)	.1402	.09655	.08431	5.5	.09560
OLK	4.91 + .4096ln(Z^*+171)	.0976	.14232	.13212	4.6	.14128
SBKGM	40 + .9730 Z^*	.2307	.09256	.10512	1.8	.09173
SBKST	2470 + .4127 Z^*	.1538	.12094	.10851	1.6	.12003
SBKLT	1450 + .6134 Z^*	.1376	.09922	.12492	7.3	.09816
OBK	2872 + .3180 Z^*	.0794	.17379	.16219	2.2	.17052
UBK	3716 + .1020 Z^*	.0318	.68307	.88065	35.0	.66151
SHKGM	855 + .7034 Z^*	.1817	.09340	.05131	1.9	.09059
SHKST	3526 + .1695 Z^*	.1756	.13971	.05131	1.0	.13771
SHKLT	1207 + .6544 Z^*	.1664	.09957	.05131	9.3	.09798
OHK	2943 + .2989 Z^*	.0769	.18514	.06237	3.1	.18327
UHK	3700 + .1015 Z^*	.0317	.66912	.54649	39.3	.66591

Table 4.8a: Subarea S2.2 , regression analysis of 37 $(100m)^2$ oreblocks, overall semivariograms.

method	regression formula ln(Z+170) / Z =	standard error of slope	corrected/ transformed error variance actual	corrected/ transformed error variance estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	-2.09 +1.2428ln(Z^*+170)	.2869	.05487	.13054	7.5	.05467
SLKST	0.89 + .9014ln(Z^*+170)	.1807	.04872	.13054	-9.8	.04819
SLKLT	1.62 + .7915ln(Z^*+170)	.1806	.05538	.13054	12.3	.05512
OLK	4.10 + .4929ln(Z^*+170)	.1317	.08630	.19861	15.8	.08529
SBKGM	-3955 +2.0636 Z^*	.5410	.06235	.15859	2.9	.06115
SBKST	-204 +1.2384 Z^*	.2744	.05161	.09932	-14.4	.05094
SBKLT	144 + .8959 Z^*	.2360	.05678	.17877	7.4	.05484
OBK	1544 + .6187 Z^*	.2032	.07052	.23716	-9.4	.07044
UBK	3239 + .0944 Z^*	.1268	.20111	.99241	19.5	.19427
SHKGM	-15706 +5.1684 Z^*	1.4364	.07166	.03309	5.8	.07102
SHKST	-735 +1.4624 Z^*	.3167	.05267	.03309	-18.2	.05013
SHKLT	18 + .8954 Z^*	.2537	.05902	.03309	11.6	.05737
OHK	1605 + .6023 Z^*	.0296	.07230	.07040	-9.8	.07218
UHK	3316 + .0757 Z^*	.1166	.22841	.51224	21.1	.22303

Table 4.8b: Subarea S2.2 , regression analysis of 37 $(100m)^2$ oreblocks, local semivariograms.

method	regression formula ln(Z+230) / Z =	standard error of slope	corrected/ transformed error variance actual	corrected/ transformed error variance estimated	$\bar{Z}^* - \bar{Z}$ in %	actual error variance adjusted for global bias
SLKGM	-3.67 +1.4468ln(Z^*+230)	.3257	.05587	.06484	-2.0	.05510
SLKST	0.16 + .9970ln(Z^*+230)	.1884	.04809	.06484	-13.7	.04770
SLKLT	0.19 + .9673ln(Z^*+230)	.2050	.05276	.06484	6.2	.05270
OLK	3.78 + .5407ln(Z^*+230)	.1369	.08080	.09765	-2.7	.07826
SBKGM	-2221 +1.7010 Z^*	.5066	.07904	.07788	-4.4	.07227
SBKST	-31 +1.1708 Z^*	.2722	.06599	.05926	-14.5	.06034
SBKLT	-243 +1.0090 Z^*	.2718	.07136	.10296	6.2	.06529
OBK	1603 + .5955 Z^*	.1995	.08990	.11544	-8.7	.08230
UBK	3284 + .0846 Z^*	.1279	.24842	.58538	18.8	.21562
SHKGM	-2356 +1.7469 Z^*	.4975	.07803	.04777	-4.7	.07141
SHKST	-111 +1.2038 Z^*	.2686	.06446	.04777	-15.0	.05894
SHKLT	-343 +1.0368 Z^*	.2684	.06983	.04777	6.1	.06387
OHK	1589 + .6029 Z^*	.1976	.08894	.07306	-9.1	.08120
UHK	3346 + .0699 Z^*	.1219	.26786	.37152	18.9	.23280

Because global unbiasedness can therefore be assumed for all methods, the regression results were adjusted for the global bias. This was done by multiplying each estimate with a correction factor. This correction factor is determined by adjusting the mean of the estimates to the mean of the true grades. Journel & Huijbregts (1978; p.572) propose this technique for the elimination of any global bias resulting from lognormal kriging.

The global bias becomes zero. The error variance decreases after the adjustment.

Simple linear and lognormal kriging with a global mean forms an exception. Because 50% and more of the weight goes to the chosen mean grade (see Table 4.4), ignoring subdivision results in a systematic underestimation for subareas with mean grades above the shaft area mean grade. Similar, a systematic overestimation occurs in subareas with mean grades lower than the shaft area mean grade.

Error variance

As observed with the slope factor, the actual error variance of individual kriging methods is generally very similar for the local and the overall semi-

variogram in each subarea. Keeping the limited number of data in mind, it remains doubtful whether the actual error variance is significantly reduced by subdivision into sedimentologically homogeneous zones (Figures 4.37 to 4.40).

4.7.2.2 Evaluation of different methods

Conditional unbiasedness

Of all the methods tried, only simple kriging with a global mean results consistently in slope factors close to 1.0 (Figures 4.37 to 4.40). All other methods are conditionally biased, and on average underestimate in the lower grade categories, and overestimate in the upper grade categories.

In the case of simple linear and lognormal kriging with a global mean, from between 63% and 86% of all weight is allocated to the mean in Subfacies 1 subareas. In Subfacies 2 subareas, the weight allocated to the mean reduces to between 47% and 54% (see Table 4.4).

Rivoirard (1987) recommends to choose a larger kriging neighborhood for ordinary kriging, as long as the weight for the mean in simple kriging is greater than 20%.

variogram in each subarea. Keeping the limited number of data in mind, it remains doubtful whether the actual error variance is significantly reduced by subdivision into sedimentologically homogeneous zones (Figures 4.37 to 4.40).

4.7.2.2 Evaluation of different methods

Conditional unbiasedness

Of all the methods tried, only simple kriging with a global mean results consistently in slope factors close to 1.0 (Figures 4.37 to 4.40). All other methods are conditionally biased, and on average underestimate in the lower grade categories, and overestimate in the upper grade categories.

In the case of simple linear and lognormal kriging with a global mean, from between 63% and 86% of all weight is allocated to the mean in Subfacies 1 subareas. In Subfacies 2 subareas, the weight allocated to the mean reduces to between 47% and 54% (see Table 4.4).

Rivoirard (1987) recommends to choose a larger kriging neighborhood for ordinary kriging, as long as the weight for the mean in simple kriging is greater than 20%.

The kriging variations without mean base their estimate on 64 samples, which due to the high sampling density cover an area of only about 35m * 100m on one side of the ore block. The sample grades of this areawise relatively small kriging neighborhood are extrapolated to give an estimate of the average grade of the ore block.

Due to the high influence of the mean in simple kriging, the otherwise high grade estimates are decreased. On the other side, low grade estimates are increased. This reduces the overestimation in high grade, and underestimation in low grade categories sufficiently to give conditionally unbiased results.

Enlarging the kriging neighborhood would eliminate or at least substantially reduce the conditional bias for ordinary kriging. The covariance matrix which has to be solved to find the set of optimum weights (e.g. David, 1977), however, quickly becomes extremely difficult to handle with the inclusion of even more samples (with n=64 samples, this matrix is 65 columns by 65 rows).

Including the mean is a much more efficient way to increase the kriging neighborhood. The resulting substantial smoothing of the estimates is acceptable because of the high average grades of the Carbon Leader reef in relation to the pay limits.

Global mean error of estimation

As stated in chapter 4.13.1, differences in the global mean estimation error are due to the limited number of data pairs in the individual subareas. It is no reliable criterion for assessing the performance of the different estimation methods.

Error variance

The lowest actual error variance is generally achieved by using simple kriging with global means (Figures 4.37 to 4.40). The application of simple kriging with local means (SLKST, SLKLT, SKBST, SKBLT, SKHST, SKHLT) results in higher error variances, but these generally stay below those of ordinary kriging. The error variances for universal kriging are very high.

Even though the data are lognormally distributed, linear kriging gives results which are comparable to those of lognormal kriging (Figures 4.37 to 4.40).

But there is an indication that, with increasing deviation from normality, lognormal kriging gives lower error variances: for subarea S2.1, for example, the logarithmic error variance for SLKGM, corrected for the

additive constant, is very similar to the transformed error variance for SBKGM. For subarea S2.2, the corrected lognormal error variance for SLKGM is lower than the transformed error variance of SBKGM. In S2.2, the ratio of the additive constant to the mean is almost twice that of S2.1, indicating a skewer gold grade distribution. As linear kriging is based on the assumption of normally distributed grades, this higher deviation from normality might be the reason for the higher error variance for linear kriging in S2.2.

4.7.2.3 Evaluation of different semivariogram transformations

Conditional unbiasedness

The conditional unbiasedness of individual methods is not much affected by the use of either logtransformed, backtransformed, or Hawkins & Cressie transformed semivariograms (Figures 4.37 to 4.40).

The choice of the estimation method clearly has the dominating influence on the conditional unbiasedness of the estimation.

Global mean error of estimation

As with the slope factor, the global mean error does not vary significantly, if different semivariogram transformations are used in the individual kriging variations (Tables 4.5 to 4.8).

Error variance

One of the advantages of kriging is that an estimation variance is assigned to each estimated grade. As normal error distributions (or lognormal in the case of log-

normal kriging) may be assumed for any practical purposes (e.g. David, in print), the estimation variance can be used for determining confidence limits to each grade estimate.

The correct calculation of the estimation variance is important for obtaining valid confidence limits.

As follow-up data are available in this analysis, the estimation variance can be checked against the actual error variance.

Because the follow-up grades are calculated from a limited number of samples inside each block (see Appendix 1.1), these 'true' grades are themselves subject to errors. The kriging estimation variance should ideally be compared with the error variance based on the estimates and the unknown true grades. But as only the follow-up grades are available, the observed or actual error variance will be slightly larger than the kriging estimation variance.

Initially, only the average estimation variance is considered. Because of the pseudoregular data grid, the estimation variance remains fairly constant for all oreblocks. This can be proved by the following consideration: the average estimation variance for each of the four subareas, calculated from the overall semi-

variograms, are very similar for individual methods (see Tables 4.5 to 4.8 for the overall semivariogram, column 4). For each kriging method, the estimation variance depends only on the relative positions of the samples to the oreblock, and the chosen semivariogram. As the semivariogram used is the same overall one for all four subareas, and as the average calculated estimation variance is very similar, the average relative positions of the samples to the oreblock must be similar in each of the four subareas. The average estimation variance may therefore be used to compare different kriging methods and semivariogram transformations.

The use of the overall semivariogram in no case gives accurate estimates of the actual error variance (Tables 4.5 to 4.8, columns 3 and 4).

By using local semivariograms and simple or ordinary kriging for the subfacies 2 subareas (S2.1, S2.2), the average estimation variance becomes a good estimate of the actual error variance, except if a Hawkins & Cressie transformed semivariogram is used. The Hawkins & Cressie transformed local semivariogram consistently underestimates the actual error variance.

For the subfacies 1 subarea S1.2, the actual error variance is much higher than the estimated one. This is due to one 'true' block value of 3.5 times the subarea mean. After the removal of this outlier value, the average estimation variance based on local semivariograms, again becomes a good estimate of the actual error variance, if simple lognormal kriging with a global mean (SLKGM) or simple linear kriging with a global mean and the backtransformed semivariogram (SBKGM) is used (Table 4.9). The use of the overall semivariogram still does not give accurate estimates of the actual error variance.

Table 4.9: Subarea S1.2, actual and average estimated error variances of 33 $(100m)^2$ ore blocks, 1 outlier removed.

method	local semivariogram mean error variance		overall semivariogram mean error variance	
	actual	estimated	actual	estimated
SLKGM	0.11735	0.10284	0.15448	0.12231
SBKGM	983506	1033478	1405310	2478567
SHKGM	955747	239051	959497	479982

The regression analysis of subarea S1.1 suffers from the limited number of available data (n=10). Here, the actual error variance is not estimated correctly, (too

high using the local semivariogram), and the overall semivariogram even seems to give better results. But the very few data on which this regression analysis is based, makes the results seem doubtful. A plot of the actual error distribution and the estimated one (Figure 4.43) shows no obvious difference within the limitations of the data available.

4.7.2.4 Preliminary conclusions

Because of the limited number of data pairs in the subareas (except for subarea S2.1), the following conclusions are preliminary, and will be finalized in a pooled analysis of all four subareas, which is based on the combined data (see chapter 4.7.3).

After the analysis of all the regression results of the different methods and semivariogram transformations, it becomes clear that simple kriging with a global mean performs best.

There is not much difference in the results between linear and logtransformed simple kriging.

Simple kriging with a global mean comes closest to ensuring conditional unbiasedness. This does, however, not depend on subdivision.

Subdivision on a sedimentological basis and simple kriging with a global (subarea-)mean is necessary to give the lowest absolute mean error.

Simple kriging with a global mean gives the lowest actual error variances. With the limited number of data given, this seems to be independent of subdivision.

Subdivision based on sedimentology, however, is necessary to find the valid semivariogram. This in turn is needed to calculate the estimation variance that predicts the actual error variance well.

Theoretical geostatistical calculations based on the spatial variability of the gold, like optimal oreblock size and shape, or optimal sampling spacing, is therefore to be expected to be meaningful only if the Carbon Leader reef is subdivided into sedimentologically homogeneous areas, and local semivariograms are used.

4.7.3 Analysis of results of $(100m)^2$ ore blocks for the combined subareas

As the number of pairs for the regression analysis of the individual subareas is fairly small, it was decided to pool the pairs of estimates and follow-up grades, and run a regression analysis on the combined data of the four subareas S1.1, S1.2, S2.1 and S2.2. This gives 163 data pairs for the analysis. The results are listed in Table 4.10.

Again it is seen that simple kriging with a global mean, either linear or logtransformed, gives slope factors closest to 1.0, and thus is conditionally least biased. As this is already true without subdividing, subdivision can naturally not improve this.

Simple kriging with a global mean also gives the lowest actual error variances. Considering the subareas combined, it becomes clear that subdivision results in considerably lower actual error variances than for simple kriging with a global mean (SLK-GM, SBK-GM & SHK-GM). The actual error variances are reduced by between 13% and 38%, if local semivariograms and subarea means are used, rather than the overall semivariogram and the shaft area mean. Because of the pooling of the subareas, the estimates of the overall shaft area means also have very low mean errors. As seen in chapter

Table 4.10: Combined regression analysis for four subareas, using local and overall semivariograms; and regression analysis of mine method. Oreblock size is $(100m)^2$, n=163.

	subdivision			no subdivision		
method	Regression formula	corrected error variance	$\bar{Z}^*-\bar{Z}$ in %	regression formula	corrected error variance	$\bar{Z}^*-\bar{Z}$ in %
SLKGM	$-1.94+1.23\ln(Z^*+b)$	0.09571	0.76	$-1.64+1.19\ln(Z^*+170)$	0.13225	2.19
SLKST	$4.08+0.51\ln(Z^*+b)$	0.17399	2.82	$4.05+0.51\ln(Z^*+170)$	0.16882	4.06
SLKLT	$1.00+0.87\ln(Z^*+b)$	0.11194	6.70	$1.97+0.76\ln(Z^*+170)$	0.12925	7.60
OLK	$4.99+0.39\ln(Z^*+b)$	0.21014	8.60	$4.63+0.44\ln(Z^*+170)$	0.19239	15.78
SKBGM	$-82 + 1.01\ Z^*$	0.11920	1.08	$-331 + 1.06\ Z^*$	0.13556	2.38
SKBST	$2364 + 0.34\ Z^*$	0.19174	4.44	$2354 + 0.35\ Z^*$	0.19211	4.06
SKBLT	$895 + 0.70\ Z^*$	0.12061	8.67	$1102 + 0.65\ Z^*$	0.13012	7.60
OKB	$2625 + 0.28\ Z^*$	0.25389	4.74	$2583 + 0.29\ Z^*$	0.24824	4.71
UKB	$3381 + 0.08\ Z^*$	0.71639	48.15	$3304 + 0.09\ Z^*$	0.70149	49.30
SKHGM	$308 + 0.91\ Z^*$	0.12134	0.70	$-5236 + 2.37\ Z^*$	0.14774	2.03
SKHST	$2658 + 0.27\ Z^*$	0.20549	4.11	$2626 + 0.28\ Z^*$	0.20527	3.52
SKHLT	$850 + 0.70\ Z^*$	0.12137	9.75	$955 + 0.68\ Z^*$	0.12871	8.99
OKH	$2607 + 0.28\ Z^*$	0.25228	4.49	$2604 + 0.28\ Z^*$	0.25067	4.81
UKH	$3223 + 0.10\ Z^*$	0.66248	45.48	$3311 + 0.09\ Z^*$	0.65081	48.33
Mine Method; n=153				$3.29+0.61\ln(Z^*+170)$	0.16876	-12.80
Mine Method, regressed, n=153				$0.00+1.00\ln(Z^*+170)$	0.15622	0.00

4.7.2, however, the mean error is large in the individual subareas. It only becomes small in the combined analysis, because with the larger data set the systematic over- and underestimation in low and high grade subareas is balanced out over the shaft area.

The regression results of the three variations of simple kriging based on the combined four subareas and the overall semivariograms (Table 4.10, n=163), are compared with the regression results the same simple kriging methods give over the whole shaft area (Table 4.1, n=305, $(50m)^2$ regularized data blocks),as explained in chapter 4.5.3. This comparison shows a close agreement for the slope factors and actual error variances. This indicates that the estimation situation for simple kriging in the four subareas is representative for the whole shaft area.

At the same time, it justifies the regularisation into $(50m)^2$ data blocks, that was applied in the estimation of the 305 ore blocks of Table 4.1.

The regression results of the mine method, combined for all quadrants, are shown at the bottom of Table 4.10.

By regression of the mine estimates (Krige, 1951), the procedure becomes conditionally unbiased, and gives a mean error of zero. The actual error variance is

reduced by 7.5% if compared with the mine method.

By using simple kriging with a global mean (no subdivision), the actual error variance is decreased by between 12.5% and 21.6% as compared to the mine method.

Subdividing , and using simple kriging with a global (subarea-)mean, gives a reduction of the actual error variance by between 28.1% and 43.3%, compared to the mine method.

Figures 4.41a to 4.41d show a sequence of the regression diagrams of the mine estimation method, and the improvements achieved by regressing these estimates, and using simple lognormal kriging with a global mean before and after subdivision. The scale of both the horizontal and vertical axes are constant for all plots.

The plots show the line X=Y, the regression line of the true grades on their estimates, the means of the true grades and the estimates (dashed lines), and the 90% confidence interval of the estimates.

The stepwise reduction of the 90% confidence interval is clearly visible.

The correlation coefficient between 'true' grades and

estimates is 0.39 for the mine method and the regressed mine method. This becomes 0.48 for simple lognormal kriging with mean, and subdivision improves the correlation coefficient further to 0.67.

Figures 4.42.a and 4.42.b illustrate the regression diagrams for simple linear kriging with mean before and after subdivision. The 90% confidence interval is again reduced by subdivision.

The correlation coefficient in this case is improved from 0.42 before subdivision to 0.54 after subdividing on a sedimentological basis.

In conclusion it can be said that simple kriging with a global mean gives conditionally unbiased estimates, and global mean errors close to zero. The already low actual error variances (compared to other kriging variations, and the mine method), can be further reduced by subdivision on a sedimentological basis.

As seen in chapter 4.7.2, subdivision also has the advantage of allowing the best modelling of the semivariogram, which then results in reasonable estimates of the estimation variances.

Fig.4.41a: Regression Diagram Mine Method (100m * 100)

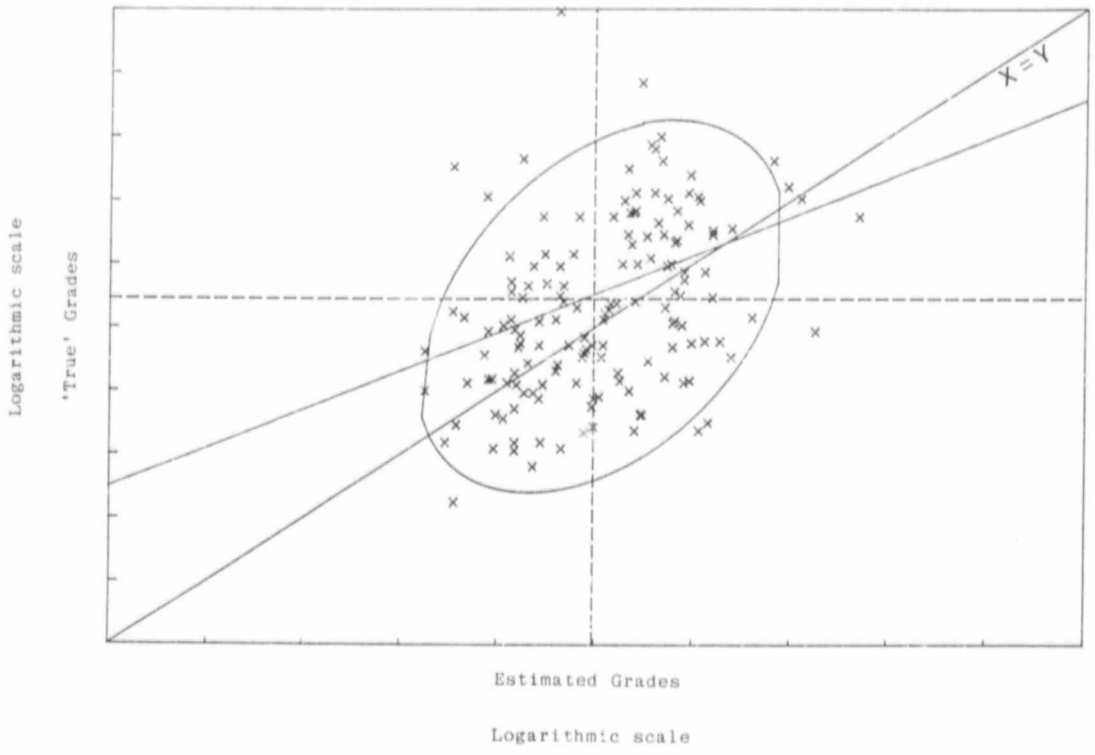


Fig.4.41b: Regressed Mine Estimates (100 * 100m)

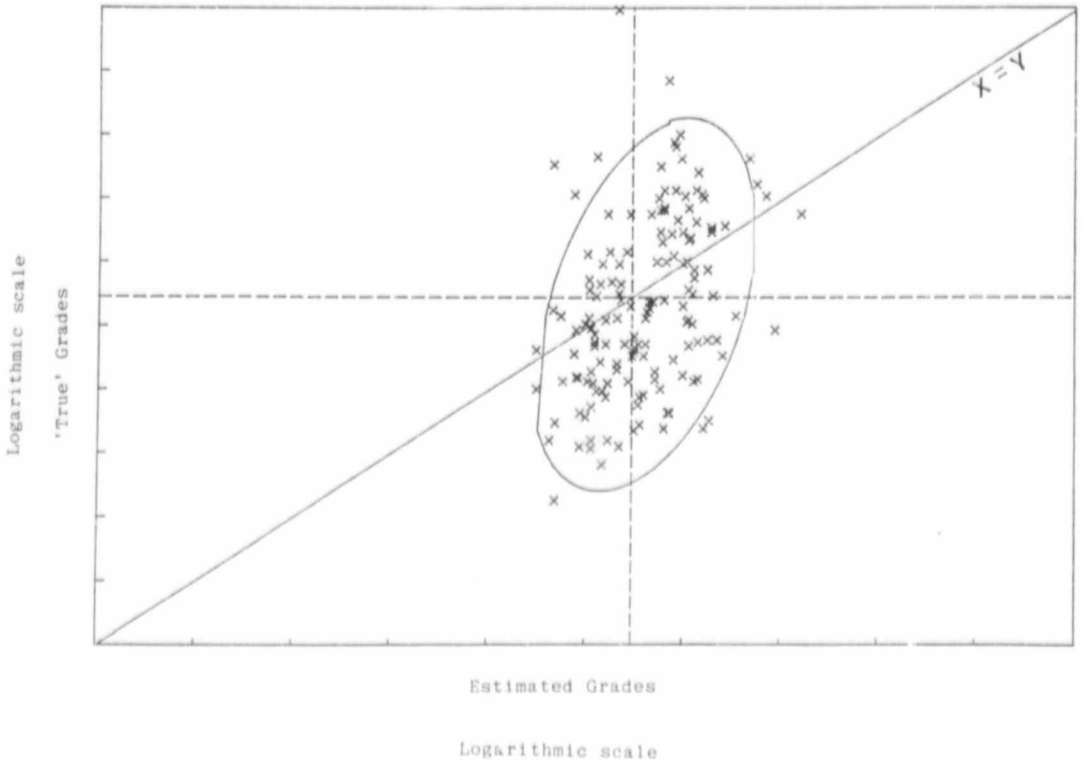


Fig.4.41c: SLKGM - Without Subdivision

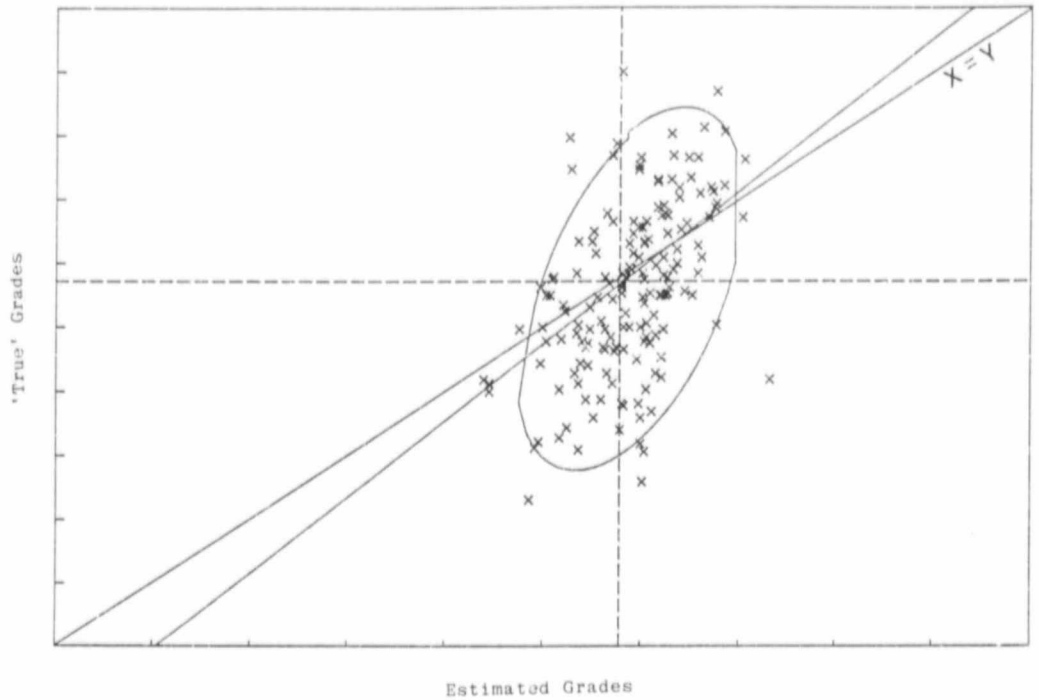


Fig.4.41d: SLKGM - With Subdivision (100m*100)

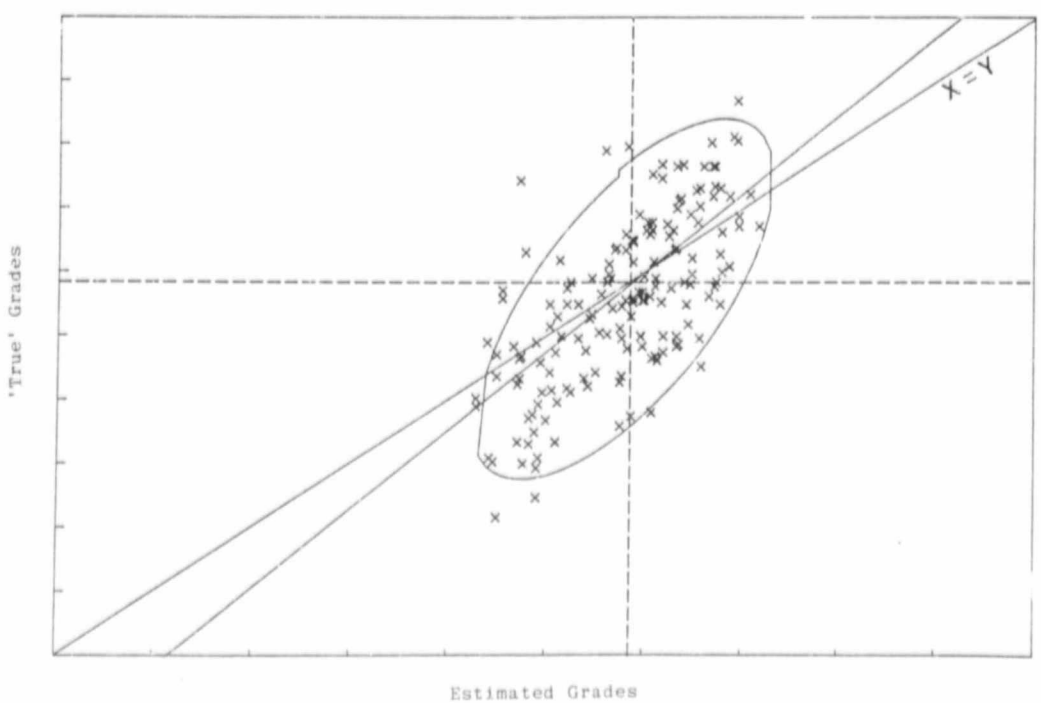


Fig. 4.42a: SBKGM - Without Subdivision

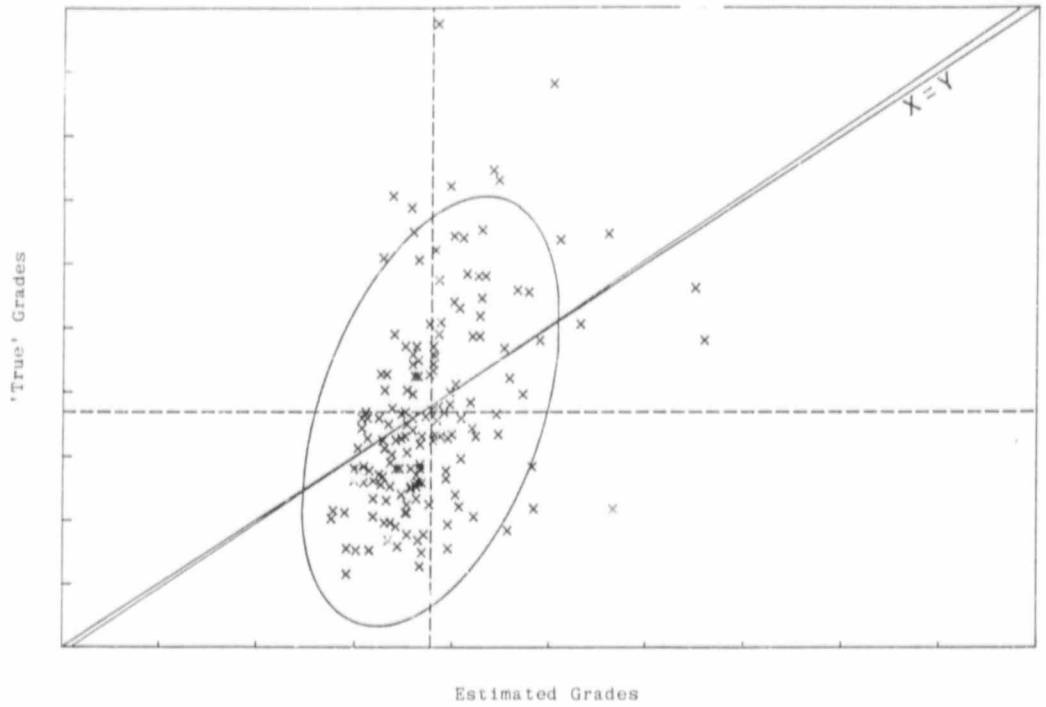
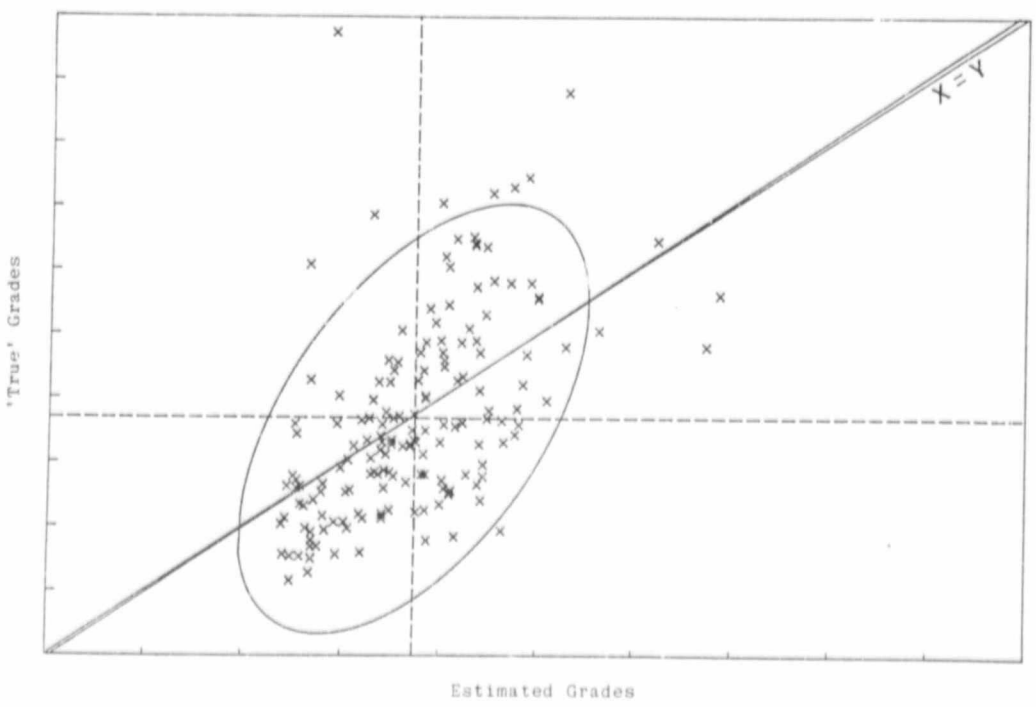


Fig. 4.42b: SBKGM - With Subdivision



4.8 Comparison of actual and estimated errors

The calculations following in chapter 5 are based on the local, (subarea) logtransformed or backtransformed semivariograms. The estimation procedures used are simple lognormal kriging or simple linear kriging, both with a global mean.

Two further tests are conducted to ensure the validity of the chosen semivariograms as representative of the spatial variability of the gold in each subarea:

- the correct prediction of the actual error variance by the estimation variance proves the validity of the semivariogram model, as the estimation variance is calculated from the semivariogram. This proof can be gained by comparing the distribution of the actual errors with their predicted distribution. Assuming normality of the (logarithmic, if lognormal kriging) error distribution, kriging predicts the mean error as zero, and the error variance by the estimation variance. If the data grid is regular or pseudoregular, the estimation variance stays the same, or very similar for each oreblock. Therefore the average estimation variance can be used as prediction of the actual error variance.

Figures 4.43 to 4.46 show histograms of the actual errors, and a distribution model based on a mean of zero,

Author Braun Rolf
Name of thesis A Study To Optimize Ore Evaluation At Western Deep Levels Gold Mine, Witwatersrand, By Using Geostatistics And Geological Subdivision. 1988

PUBLISHER:
University of the Witwatersrand, Johannesburg
©2013

LEGAL NOTICES:

Copyright Notice: All materials on the University of the Witwatersrand, Johannesburg Library website are protected by South African copyright law and may not be distributed, transmitted, displayed, or otherwise published in any format, without the prior written permission of the copyright owner.

Disclaimer and Terms of Use: Provided that you maintain all copyright and other notices contained therein, you may download material (one machine readable copy and one print copy per page) for your personal and/or educational non-commercial use only.

The University of the Witwatersrand, Johannesburg, is not responsible for any errors or omissions and excludes any and all liability for any errors in or omissions from the information on the Library website.