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# From textual problems to mathematical relationships: case studies of secondary school students and the discourses at play in interpreting word problems

**Tobias, Bruce**

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# The initial case study

## Doing and talking about word problems

### Analysis of phase 1, step 1

In the analysis of this phase I ally what the students do in solving the problems with what they say about their experience of the problem. This is to say that I look at where the students are positioned on a continuum ranging from algorithmic to heuristic discourse with respect to the progress that they make in solving the problem as discussed in my analytical framework.

I attempted a 'graphic' rendition of the analysis of each student so as to bring more clarity to my discussion. The data is plotted on a two dimensional plane. Horizontally we see the algorithmic-heuristic continuum and vertically we see a classification of the student working in a four-point scale. The vertical axis ranges from 'cannot proceed' to 'some working, but not correct' to 'makes some progress' and culminates in 'correct solution' (see figure 1). For analytical purposes it is tempting to dichotomise student work as 'algorithmic' or 'heuristic', but in reality it does not work this way. For my analysis I have simply located the student discourse

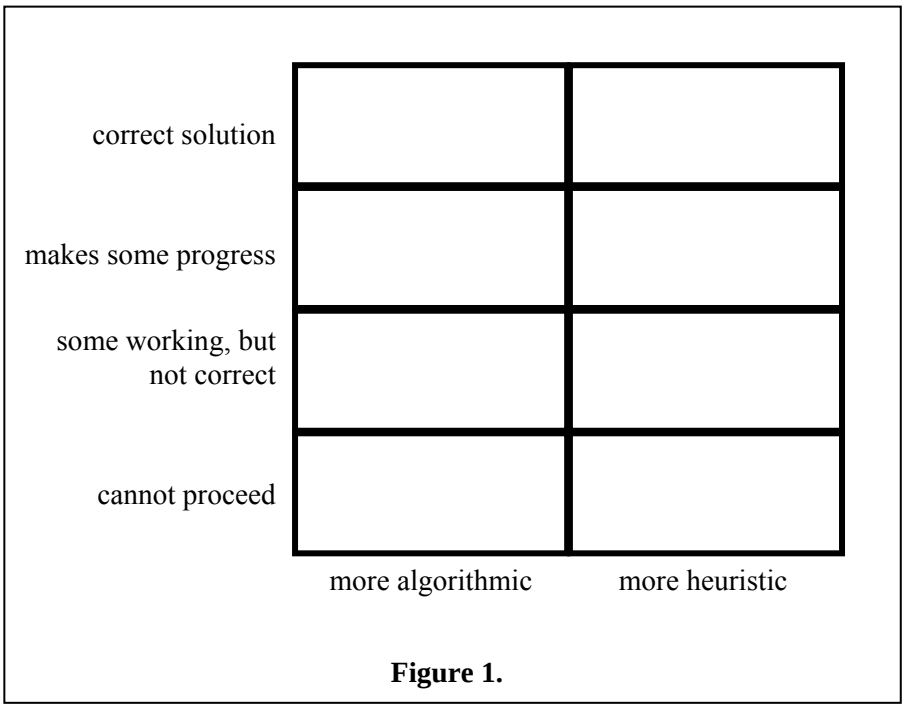



**Figure 1.**

along the horizontal axis as being 'more algorithmic' or 'more heuristic' without pinpointing the degree to which it tends in either direction. Also, students often attempt more than one solution strategy and where this occurs the problem number (e.g. ②) will be reflected more than once.

This grid allows an analysis of the data by horizontal split with respect to vertical split. The horizontal split reflects the success with which the student attempts the problem and the vertical split reflects the solution strategy as being more algorithmic/heuristic. Thus an overview of what a student does with all the problems can be obtained.

**Karim**

In the first problem Karim clearly used an arithmetic approach with trial and error (see figure 2). He starts by assuming that there was an equal number of each type of watch and then by a numeric process of increasing the number of R24 watches and reducing the number of R36 watches he arrives at values that 'work'.

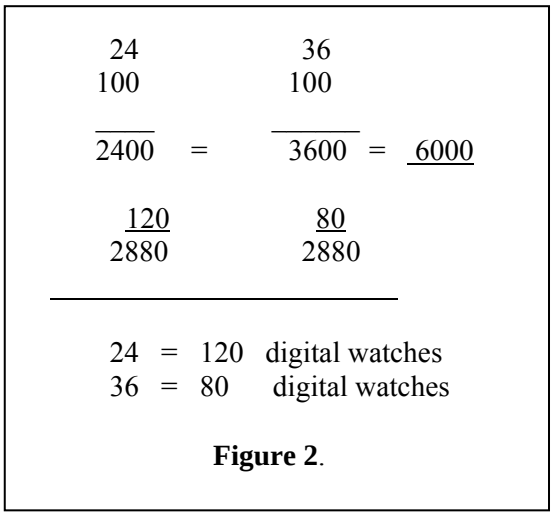



**Figure 2**.

His comments in phase 1, step 3 and phase 3 confirm this:

> "Look at this. This is all guesswork."
>
> (Phase 1, step 3, line 85)

> "I found that one [Problem 1] quite easy because we had done similar ones in the exams. I worked it out like I worked it out in the exam, guessing and so forth."
>
> (Phase 3, stanza 1, line 2)

Even though Karim is working numerically his reasoning reflects a more heuristic discourse. The 'guesswork' that he refers to in phase 1, step 3 is actually designed to give him insight into the problem and is not simply a means to an end. This is evident in the way that Karim firstly assumes that there were an equal quantity of the two types of watches and then proceeds to use these figures to better his estimate. This is not a 'taught' procedure, but rather one that employs individual thinking designed to establish the answer by a process of better approximation.

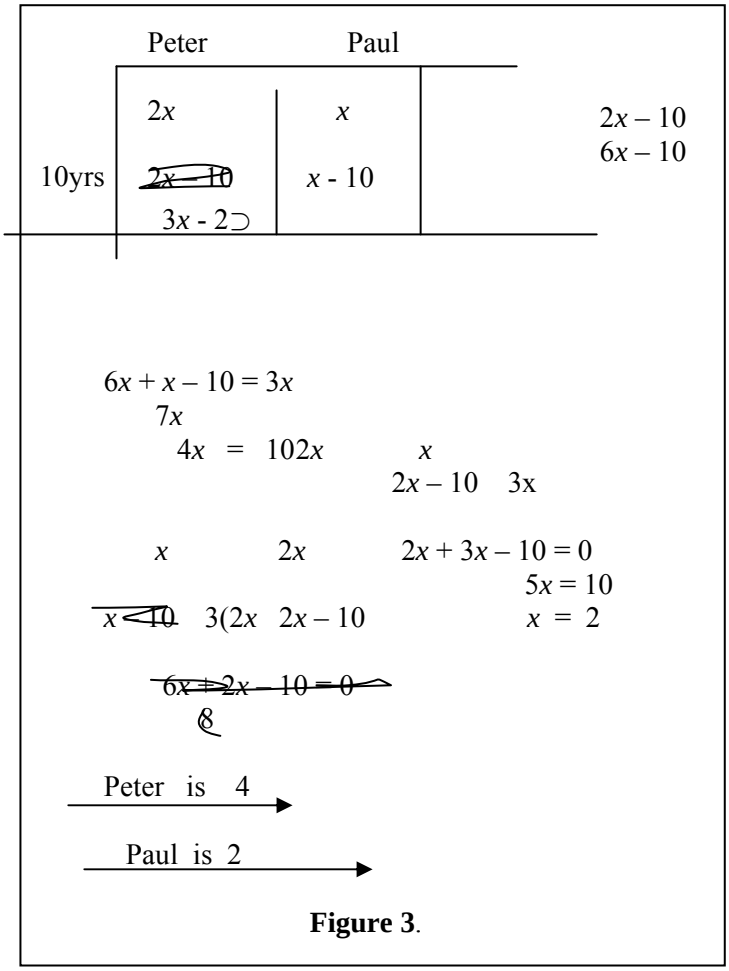



**Figure 3**.

In problem 2 Karim assigns his variables using a table but seems to be unable to establish the correct relationships between the variables. He is using the relationships from the wording in the question and tries to apply these to the expressions in his table in an almost random fashion. For example, in the equation below the first table (see figure 3) Karim has used the relationship "three times as old as" to treble Peter's present age whereas he should be trebling Paul's age ten years ago to establish the correct equation. His equation also does not make

any sense since he adds treble Peter's current age to Paul's age ten years ago and gets treble Paul's present age. In phase 3 Karim explained it as follows:

BT How about the age problem?

Karim I always find it difficult. Because of the table setting out and like the last sentence always confuses me.

BT Do you understand what the question is asking, though?

Karim Ja, I understand the question but like to put it in an equation is difficult.

(Phase 3, Stanza 2, lines 1 – 4)

For Karim, the table is a standardised or 'taught' procedure and it does not help him to establish meaning. This is in line with what the students did in the study conducted by Boaler (1997) and she attributed this to 'inert knowledge' or the inability to apply what they knew. Karim demonstrates a tendency to use intuitive attempts to solve the problem which, according to De Bock *et al*. (2002), is compelling. When intuition fails him he resorts to the table but the underlying structure of the problem that should be reified through the table (Kutscher and Linchevski, 1997) is not helpful to Karim. Therefore, it can be said that Karim is demonstrating algorithmic discourse when he uses the table. However, he does establish an equation that yields a solution (Peter is 4 and Paul is 2) but he does not relate this to the data in the table (Peter's age ten years ago was $x - 10$, viz. $2 - 10$, according to his solution). De Bock *et al*. (2002) found that students accepted the incorrect linear solution to a problem even when a strong case was given to show that it was incorrect. In a similar way Karim here accepts his answer despite the blatant nonsense that it purports in the context of the question. This further confirms for me that Karim is not using the table to generate understanding and it is thus an expression of algorithmic discourse.

Problem 3 shows more clearly that Karim is randomly relating the expressions that he generates and he confirms this in phase 1, step 3:

Warren What did you get? You got 98, dude.

| | |
|---|---|
| Karim | I got 86 and 146. |
| Warren | Ok, how did you get your answer? Because we've all got conclusions, that's why. |
| Karim | I just said $x$ plus $x$ is $2x$, equals 195, plus 45 minus 60. $2x$ equals 175. $x$ equals 86. |
| Warren | It works for me. |
| Karim | I just added all the numbers they gave us. |

(Phase 1, step 3, lines 32 – 37)

Karim correctly assigns $x$ and $x + 60$ as the speeds of the planes but he seems to be unable to gather any further meaning from the question. He therefore comes up with two equations that make no sense (see figure 4). In the first instance he writes an equation that means, "the sum of the two speeds equals the time travelled" and in the second it means, "the sum of the two speeds equals the total distance travelled added to the time travelled". (I am sure that "$x + x + 6$" was meant to be "$x + x + 60$".) He is literally just adding all the numbers that they gave which confirms that he does not know how to relate these algebraically. His perception of the solution strategy is that one needs to use all the information that they give and in doing this he therefore shows an algorithmic discourse.

| A | B |
|---|---|
| $x$ | $x + 60$ |
| $x + x + 60 = {}^{4}45^{60}$ | $= 45$ |
| $2x = -60 + 45$ | |
| $2x = -15$ | $x + x + 6 = 195 + 45 - 60$ |
| $x = \frac{15}{2}$ | $2x = 175$ |
| $x = 7\frac{1}{2}$ | $x = 86$ |
| $A = x = 86$ | $B = x + 60 = 146$ |

**Figure 4**.

In my field notes during phase 1, step 1 I noted that Karim finished well before any of the others. The other three all went back to their work to check and redo some of the problems and I made the following note:

> "Karim did not return to the problems, but spent the time watching Warren complete his working."
>
> (Field notes, 24 July, 2003; 1)

It seems that Karim sets up his equations quickly and solves them, giving little thought to the validity or plausibility of the solutions and this further confirms for me that he is working algorithmically. He uses 'guesswork' as his preferred strategy and this can be seen as heuristic discourse since it enables Karim to make meaning from the data by a process of improved approximation.

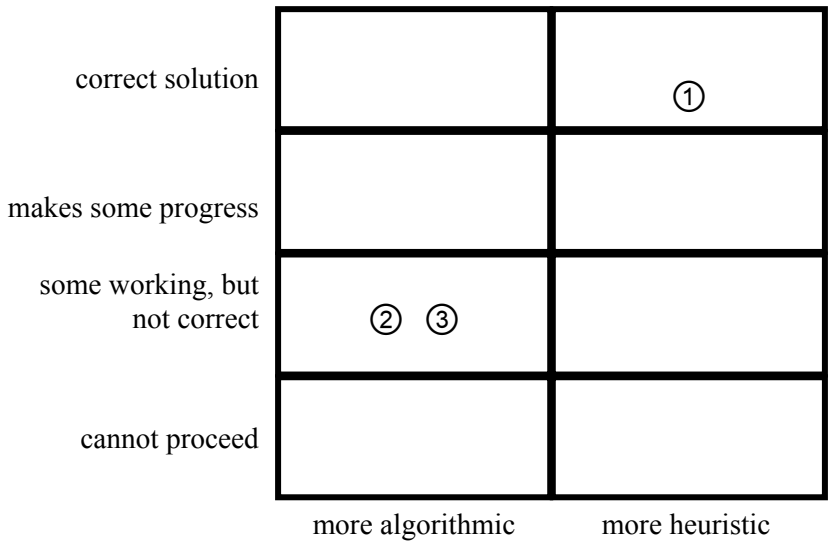



**Figure 5** – Karim's solution strategies.

However, when the problem becomes too difficult for guesswork Karim resorts to a table, and it is clear from his working that this has little meaning for him and can thus be seen as algorithmic discourse. Whenever Karim resorts to algorithmic methods (wittingly or otherwise) he is not able to solve the problem successfully.

Figure 5 shows that Karim is generally not successful with algorithmic approaches, but is successful when he can use heuristic methods. Thus the graph shows a very definite split between the two strategies that Karim uses. It is also interesting to note that he only makes one attempt at each problem and, if he cannot use his heuristic method (viz. 'guesswork'), he resorts to 'taught' procedures almost immediately. This horizontal split in the graph is tied to the vertical split, which reflects the success that Karim has in solving the problem with respect to the method that he adopts. It appears that a heuristic solution strategy is more successful for Karim when he can use it but that algebraic means do not work for him. This

gives a very different picture of word problems to that described in the literature that I discussed earlier. I shall elaborate on this different picture later on.

**Gary**

In assigning unknowns it appears as though Gary does not understand what the unknowns represent, even though *he* has assigned them. In Problem 1 he starts by writing, "Let *x* = R24 watches" and "Let *y* = R36 watches", but does he mean number of watches or price per watch? His uncertainty in this regard can be seen in his working (see figure 6).

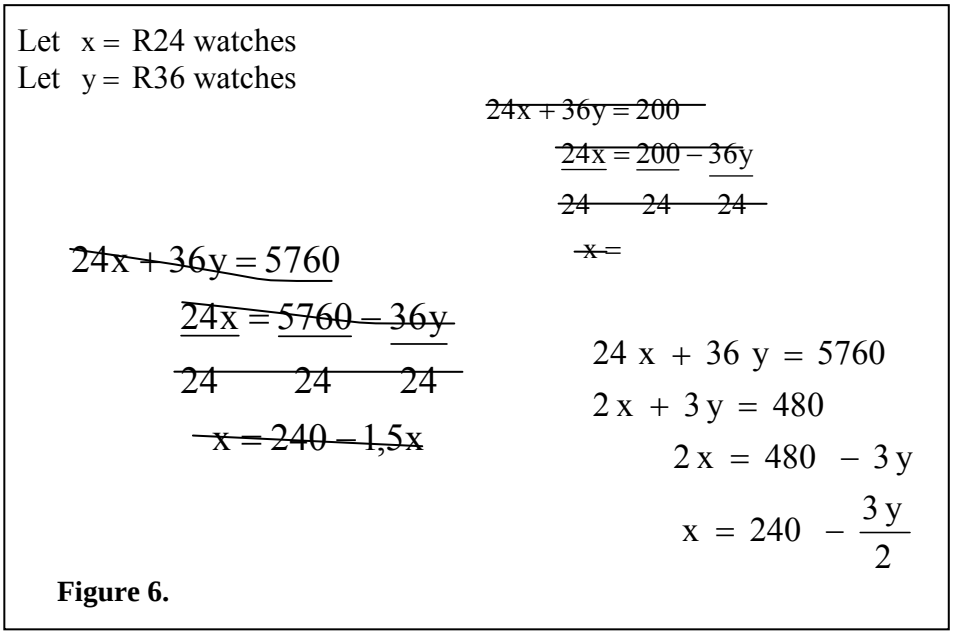



**Figure 6.**

Gary seems to be following a 'taught' procedure since he knows that he needs to assign unknowns, but he is unable to do this so that it creates meaning for him. Also he seems to know that in assigning two unknowns he needs to establish two equations and he comes up with $24x + 36y = 200$ and $24x + 36y = 5760$, where he is clearly using the two unknowns in different contexts. This also appears to be a 'taught' procedure since Gary is unable to relate the data in two different ways according to the contextual meaning in the problem. Thus it is reasonable to conclude that Gary demonstrates a degree of algorithmic discourse in his solution to Problem 1. However, Gary's working is not entirely incorrect and it would be wrong to conclude that he is working purely in an algorithmic manner. By this I mean that he

is able to use his unknowns to establish meaningful equations and to an extent this is indicative of heuristic discourse. Thus, for Gary in Problem 1 algorithmic procedures have enabled him to make some progress in solving the problem.

In Problem 2 Gary uses a table correctly to capture the data, but then seems to be unable to establish meaning that will enable him to set up equations to solve the problem. His working

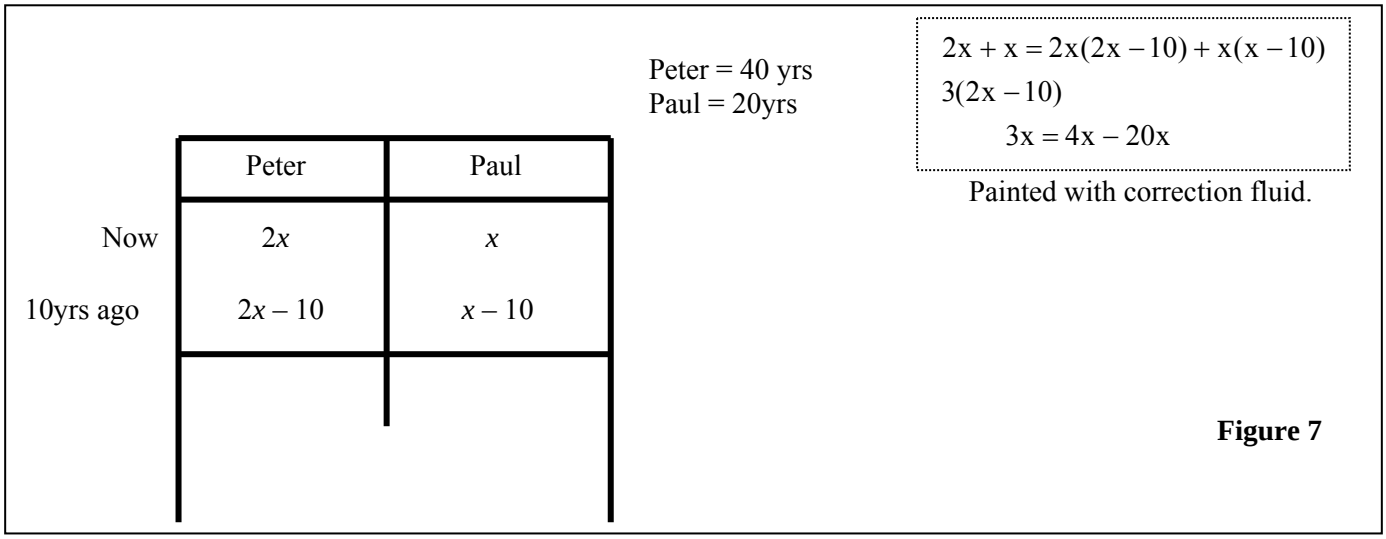



**Figure 7**

suggests that he has a limited understanding of what the data in the table represents and the fact that he has blocked most of his working out with correction fluid suggests that he does not have much confidence in what he has written (see figure 7). In the first step he has added the ages whereas the problem relates the ages multiplicatively. In the second step he has trebled Peter's age instead of Paul's (a quotidian error in my experience). Gary again shows algorithmic discourse in his use of the table. Although he sets up the information in the table correctly, he lacks the understanding of the meaning of the data in the table, and this inhibits him from deriving suitable equations to solve the problem. It seems, however, that Gary has been able to solve the problem arithmetically, and I assume that this has largely been accomplished by trial and error procedures. I noted the following whilst the boys were working the problems:

> "Gary seems a bit worried about Problem 2 and has spent quite a lot of time pondering his working, paper lifted off the desk."

(Field notes, 24 July 2003; 1)

The following excerpt from phase 1, step 3 throws some light on the matter:

| | |
|---|---|
| Imran | How did you manage that one? I started the table… |
| Gary | I got the answer, but I didn't do any maths. |
| Warren | What's the answer, dude? |
| Gary | Peter is forty years old and Paul is twenty years old. |
| Warren | Ok, I'm out of the loop. |

(Phase 1, step 3, lines 16 – 20)

Gary appears to use 'taught' procedures in assigning unknowns and placing these in a table but, in this problem this does not establish meaning for him that enables him to generate equations to solve the problem. Furthermore, Gary sees this as a 'mathematical' method, which confirms for me that his working reflects an algorithmic discourse. I return to a more in-depth discussion of 'mathematical' as opposed to 'non-mathematical' methods in the phase 3 analysis.

In Problem 3 Gary again set up a table. He then correctly uses the formula for speed being distance with respect to time, but initially works with minutes and only later resorts to hours

| | D | S | T |
|---|---|---|---|
| Plane A | 195 | $x$ | ¾ |
| Plane B | 195 | $x + 60$ | ¾ |

195 km = $x \times 45$
195 = $45x$
4⅓ = $x$

$$x = \frac{195}{0{,}75}$$

= 260 km/h $x$
∴ Plane B is 260 + 60
= 320 km/h

**Figure 8.**

as the unit of time (see figure 8). He changed his unit because the result of 4⅓ as a speed did not seem plausible to him. However, Gary did not realise that 4⅓ km/min is in fact plausible in the context of the question, even though it is not the correct speed. When he changed the

unit of time to hours he obtained a speed of 260 km/h, which he accepted as plausible, even though it is also incorrect. (In fact, 4⅓ km/min = 260 km/h.) Of course, his error is the incorrect entry in his table of the distance travelled by each plane. It is interesting to note that, had Gary chosen to work with the data from plane B in his table he would have arrived at $x = 200$km/h, which he would most probably have also accepted as plausible. What I am suggesting here is that Gary has not established meaning from his algebraic working in the table and that algorithmic procedures are therefore being employed.

Thus Gary again demonstrates algorithmic discourse in his use of the table. There is a contradiction with the entry of the speeds (even though he has shown that he knows the formula) in that his data reflects that $x = x + 60$. Despite this entry error, he is able to proceed from there and obtain a solution to the problem.

So in each of the problems Gary's initiation of the solution reflects algorithmic discourse whilst the solution strategy thereafter demonstrates a degree of heuristic discourse. In each of the problems, either the table or Gary's assigning of unknowns (both of which stem from algorithmic procedures) impedes a correct solution to the problem, and this is as a result of the lack of insight suggested by the algorithmic discourse that Gary displays in initiating the problem.

Figure 9 shows an interesting split in Gary's solution strategies. In Problems 2 and 3 he

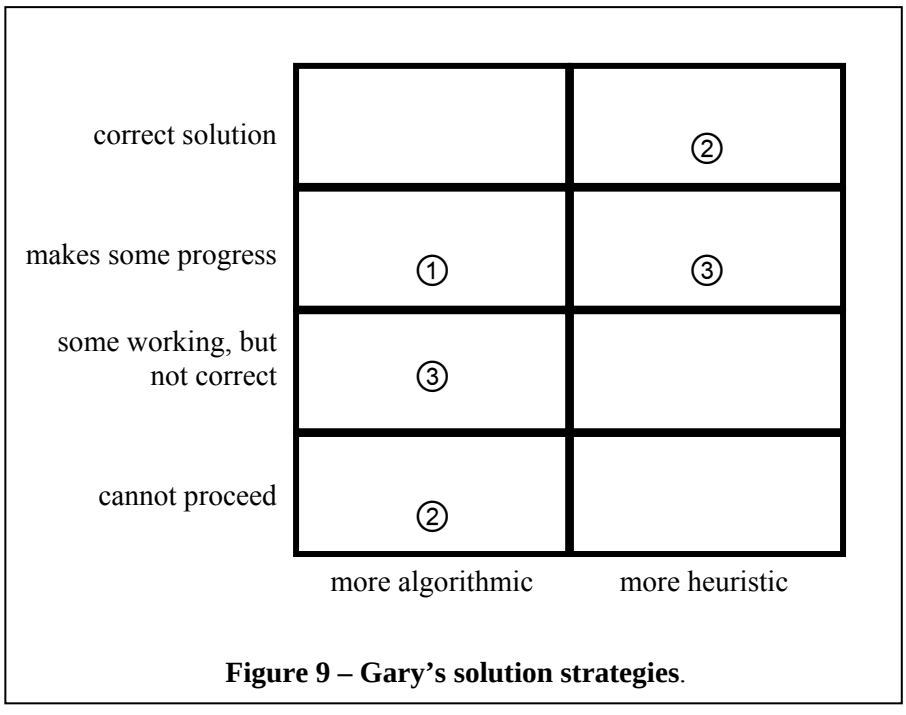



**Figure 9 – Gary's solution strategies**.

attempted the solution in two different ways. When he uses his own heuristic methods he is generally successful but algorithmic methods yield a mix of results for him. However, algorithmic methods did not enable Gary to fully solve any of the problems and only with Problem 1 was his working mathematically correct and could possibly have led to a solution. There is a vertical split (note the two attempts of problems 2 and 3) that indicates that Gary experiences more success when he works with his own methods rather than the 'taught' procedures (see the analysis of phase 1, step 3).

**Imran**

In Problem 1 Imran uses a table but the way in which he fills in the information shows us that this is a cue-based response (Boaler, 1997). He has probably been taught to use a table as a means of organising and understanding the data, much in the same way as Kutscher and Linchevski (1997) used their table, but the table does little to achieve this for him (see figure 10). For example, it does not take into account that there were 200 watches and it does not enable him to establish a quantity for each type of watch. The table can therefore be seen as algorithmic discourse.

| watches | price |
|---|---|
| Cheap | 24 |
| expensive | 36 |
| | **Total** 5760 |

**Figure 10**.

In explaining his reasoning to the other participants during phase 1, step 3, Imran states the following:

> "…I had 100 times 24 is 2400 and 100 times 36 is 3600. That comes to R6000, minus the total cost of 200 watches, which is 5760, gave me R240. Then I divided that by 24… And now the watches is worth R24, and that equals to ten like, 10 extra – that gives R6000. Then I…because the cost is 5760 minus, like I just assumed, like 100. If you bought 100 of the R36 watches then, uh, so you, so I took 5760 minus 3600…gave me 2160, and I divided that by 24

> and got 90. So my final answer was 100 of the R36 watches, and 90 of the R24 watches gave me 5760." (Phase 1, step 3, line 6)

In his working Imran therefore 'guessed' at 100 of each type of watch and found that the total cost then exceeded the given total cost by R240, which he decided was the cost of ten R24 watches. The table has done nothing to guide his translation of the problem and his working is arithmetic and based on what appears to be a trial and error approach. It is an expression of heuristic discourse to some degree since it reflects Imran's own reasoning (for example, he divided by 24, even though he was 'assuming' that these were all R24 watches).

His attempt to use a table has proved fruitless for Imran but his arithmetic reasoning has resulted in more meaningful organisation of the data and could have yielded a correct result had he realised that the R240 was made up of both types of watches and that it represented an amount in excess of the given cost. Thus Imran demonstrates algorithmic discourse in that he uses a 'taught' method (the table) and this does not enable him to solve the problem, but in working arithmetically he demonstrates a measure of heuristic discourse in which he is better able to make some progress towards a solution.

In Problem 2 the use of the table is also clearly indicative of algorithmic discourse (see figure 11). One column heading is missing and the third gives a very scant description. His attempt to use the table to organise the data is very procedural. He uses an unknown and manages to correctly relate the information in the first column, but is unable to relate the information in the second column. In other words, the table has not helped Imran to relate the data in the problem meaningfully. He has attempted to use the table as a means to an end and this has not proved successful for him.

In his first and second attempts in his working it is clear that Imran is using "three times as old" and "ten years ago" when he writes "$2x = -10 \times 3$" and "$2x - 10 = 3$", both of which come directly from the problem statement.

| Names | | $-10$ |
|---|---|---|
| Peter | $2x$ | $3x$ |
| Paul | $x$ | |

$100 \times R24 = R2400$

$100 \times R36 = \underline{R3600} +$

$\underline{R6000}$

$R6000 - R5760 = R240$

$= \frac{R240}{24} = 10$

cost: $\underline{R5760} - \underline{\underline{R3600}}$ $(\underline{\underline{100 \times \exp\ watches}})$

$= \frac{2160}{24} = 90$ $\qquad 100 \times 36 + 90 \times 24 = R5760$

$\therefore$ if the watches cost him R5760, he bought 90 R24 watches and 100 R36 watches.

**Figure 11.**

He has clearly abandoned the table as a means of helping him organise the information (see figure 12).

In my field notes I recorded the following:

> "Imran spent a lot of time on Problem 2 in which he did extensive calculator work. He took by far the longest to get to Problem 3."
>
> (Field notes, 24 July, 2003; 1)

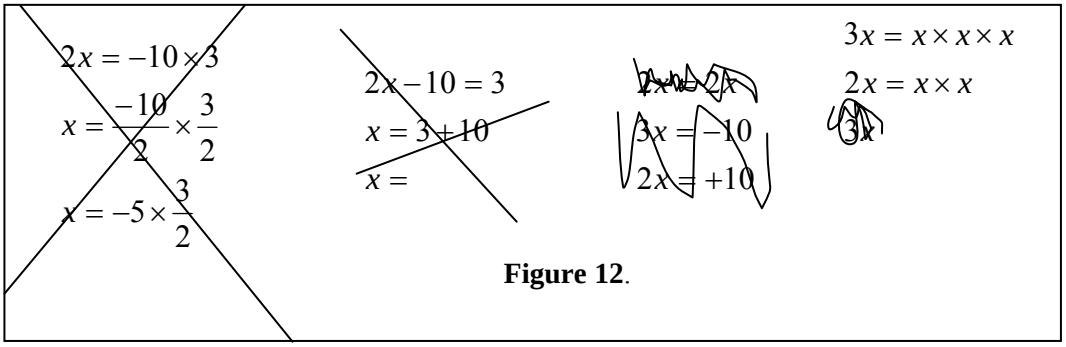



**Figure 12**.

Imran tries to use the table as a first resource but is unable to proceed. His attempt to make sense of the data from the problem also proves fruitless and he then resorts to a trial and error technique. The table again indicates algorithmic discourse since it is procedural and does nothing to assist Imran in achieving his end, viz. solving the problem. In this case his trial and error approach can also be seen as indicating algorithmic discourse to an extent. Unlike Karim, Imran is randomly choosing numbers in an attempt to somehow *stumble* upon the answer and the lack of reasoning that this implies precludes it from being seen as heuristic discourse.

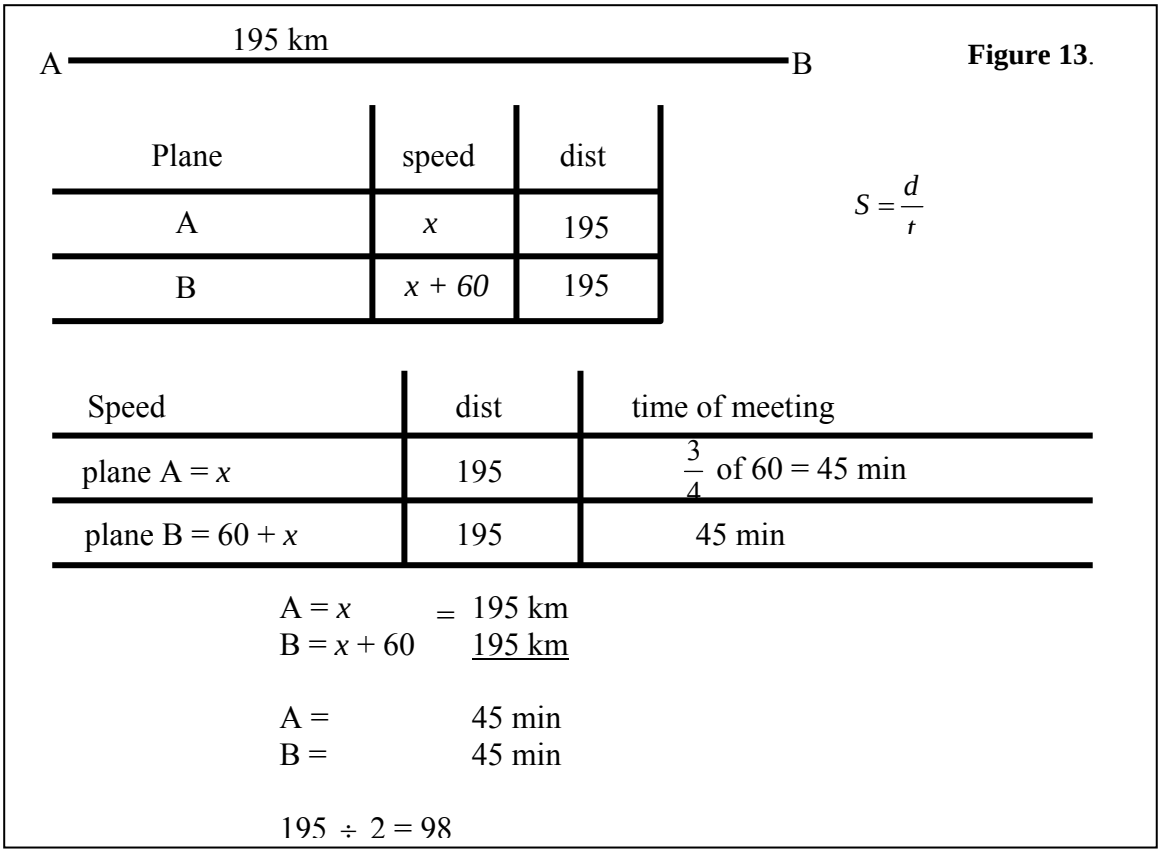



**Figure 13**.

In Problem 3 Imran used a diagram, two tables and a formula, but again these seem to be indicative of algorithmic discourse (see figure 13). In the first table he is able to relate the speeds of the planes correctly but the use of two tables suggests that he does not know how to relate speed, distance and time simultaneously. In addition to this his working shows

arbitrary equations gleaned from the table, but they make no sense in terms of the problem. He tries to equate speed with distance and his positioning of the 'is equal to' sign indicates that he is unsure of how to do this, and that the formula that he has written next to the first table has no real meaning for him. The last line clearly shows a reversion to arithmetic techniques (and is a repeat of the procedure in the first problem). Imran appears to have assumed that the two planes travelled equal distances. In my field notes I observed the following:

> "Imran, on Problem 3, again seems to spend most of his time on the calculator. He seems to be trying different combinations of numbers, often punching in numbers and looking visibly disappointed with the result."
>
> (Field notes, 24 July, 2003; 1)

Imran attempts to use a table in each of the problems and his inability to make any meaningful progress is indicative of algorithmic discourse. In other words, it appears as though he has been taught a tabulation technique for compiling his data but that he does not have a sufficient understanding of the method to do this correctly and in such a way that he can make progress in solving the problem. Further evidence of this can be seen in his working, which is very seldom based on the organisation of the data in the tables, and this

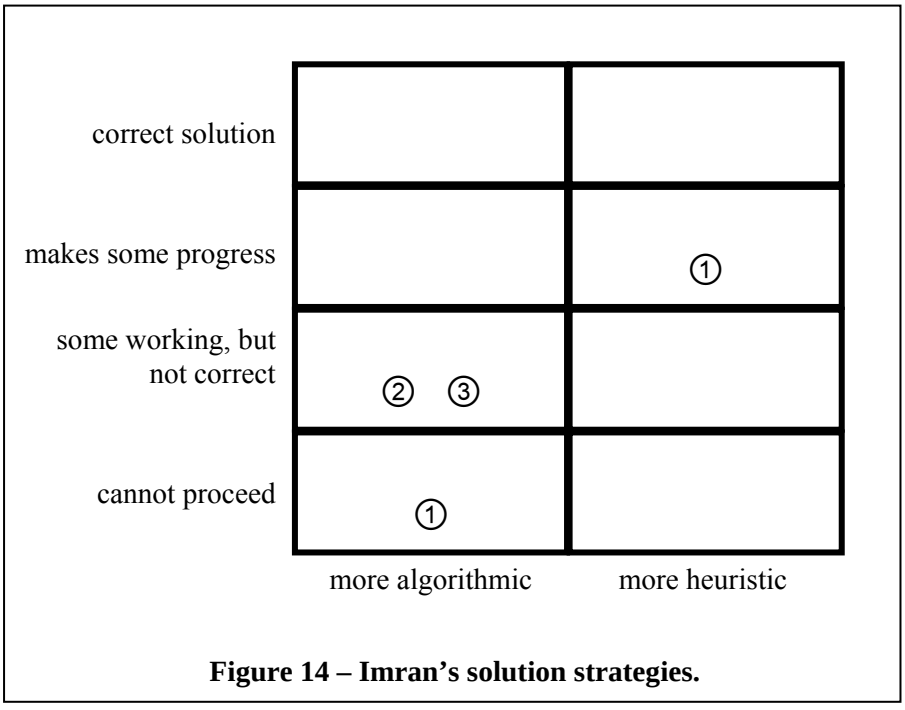



**Figure 14 – Imran's solution strategies.**

working reflects Imran's real solution strategy, which is arithmetic and largely based on trial and error. Interestingly, Imran is able to make more progress by using his 'unstructured' methods, and it is almost as if the 'table-method' is hindering him.

Figure 14 shows the graph for Imran's solution strategies. The graph shows a vertical split, which indicates that he nearly always attempts algorithmic methods. The horizontal split shows that he is not very successful with these techniques. However, when he did try to work from his own intuition he did experience a measure of success.

**Warren**

In Problem 1 Warren shows in his working that he is aware of two conditions implied in the problem, but that he is unable to express both of these algebraically. His working from phase 1, step 1 is shown in figure 15.

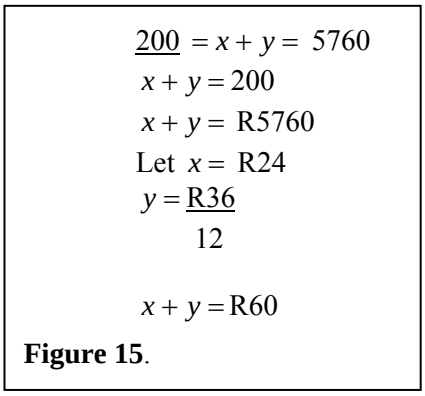



**Figure 15**.

From the underscoring of 200 in his first step it appears that Warren originally assigned his unknowns to be the quantity of each type of watch. However, he then construes the sum of these same unknowns to be the *total cost* of the watches (see steps one and three). He then goes on to assign these unknowns to be the unit cost of each type of watch. This last equation represents assigning unknowns to known values. Warren seems to have learnt to assign unknowns that will describe the relationships, but he lacks the ability to attach meaning to what he is doing – the first two equations are both correct, but not simultaneously. His working is therefore reflective of algorithmic discourse since he is assigning unknowns as a means to an end, but he has little understanding of what he is doing.

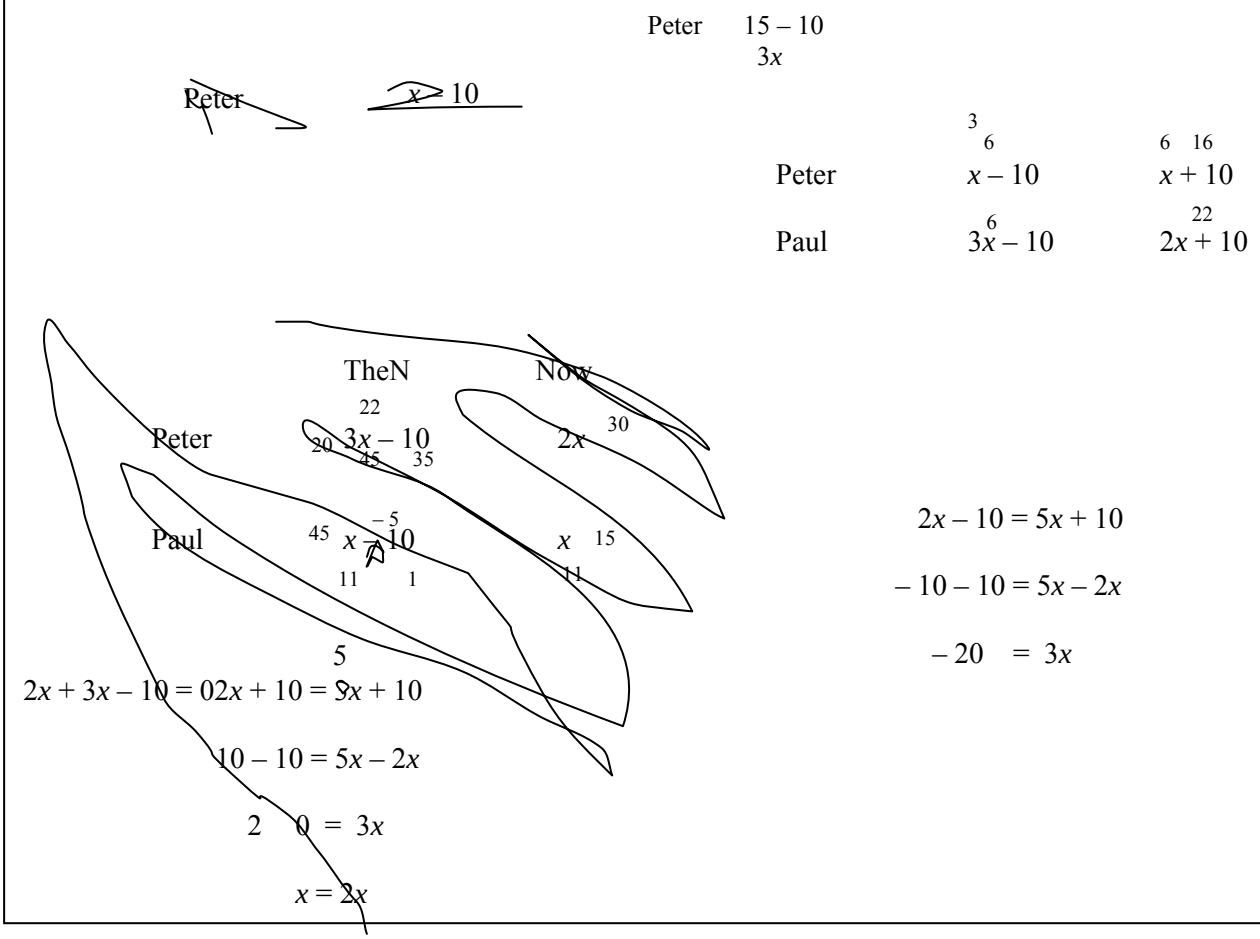
The following excerpt from phase 1, step 3 helped me to analyse Warren's interpretation of the problem:

| | |
|---|---|
| Warren: | Ok, the first one is wrong because I didn't do it out of 200, dude. So my one you can just ignore. |
| Karim: | [Referring to Gary] And your one was about what…? |
| Gary: | I said $24x$ plus $36y$ is 5760. |
| Warren: | I also did that. I started off… No! I started off with exactly what you're doing, and then I said let $x$… $y$ equal, let $x$ equal the R24 and let $y$ equal the R36, and then I said let $x$ plus $y$ equal R60… which is all good except that when you times it… when you divide 5760 by 60, you get 96 exactly. So, 96 plus 96 equals 180…err…192. I'm eight watches short! |

(Phase 1, step 3, lines 1 - 4)

Warren believes that he has done the same as Gary, whereas his working reflects differently, and his explanation shows a markedly different equation to what Gary has given. This is further evidence that Warren has learnt to assign unknowns but he is unable to do this in a way that will help him solve the problem. In any event, it seems that Warren has procedurally assigned unknowns in as many ways as he can and these have proved to be counter-productive to finding a solution, and thus this assigning of unknowns can be seen as algorithmic discourse.

**Figure 16.**




In Problem 2 Warren demonstrates a similar tendency to the one shown in Problem 1 in assigning his unknowns (see figure 16): In both tables he assigns $x$ to be an 'age now' as well as an 'age ten years ago'. Warren then introduces another unknown and in doing so changes the meaning of his original $x$ (see figure 17). Using this pair of equations he arrives at a solution of "Peter = 4" and "Paul = 2" and proceeds to check this in the original expressions (see the inserted numbers in figure 16).

$x = 2y$ $x$ Now

$x - 10 = 3y$ $x$ Then

**Figure 17.**

In my field notes I wrote the following:

> "Warren seems to be worried about Problem 2. He has finished all three problems and seems to be reworking Problem 2."
>
> (Field notes, 24 July, 2003; 1)

Later I noted that he was still making amendments to Problem 2 and towards the end of the session (approximately half an hour after the start) I noted the following:

> "Warren, still working on Problem 2 on the back of the page has indicated an affirmative with his index finger to Karim, suggesting that he is on the right track."
>
> (Field notes, 24 July, 2003; 2)

His working on the back of the page is shown in figure 18. Although his equation is incorrect Warren was trying to derive algebraic relationships from the expressions that he has generated. As with Gary, his only error is that he has trebled the wrong age, but mathematically all other working is correct.

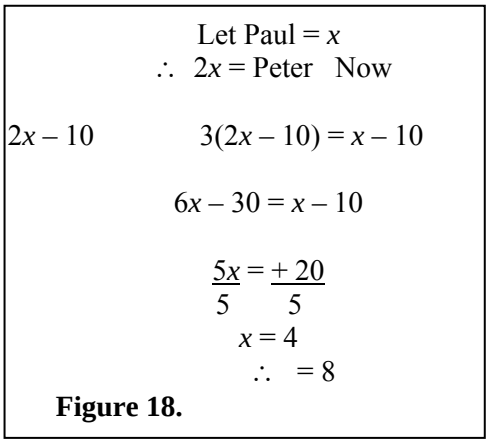



**Figure 18.**

The working of Problem 3 again shows a procedural assignment of the unknown. This is apparent from the way in which Warren assigns *x* and then does not use it again (see figure 19). He fills in a table but also does not use this to supplement his solution, which again can be said to reflect algorithmic discourse. However, there is evidence of an attempt to establish meaning in the first three lines and Warren's reasoning, albeit arithmetic, is correct, except that he misconstrues a distance for a speed.

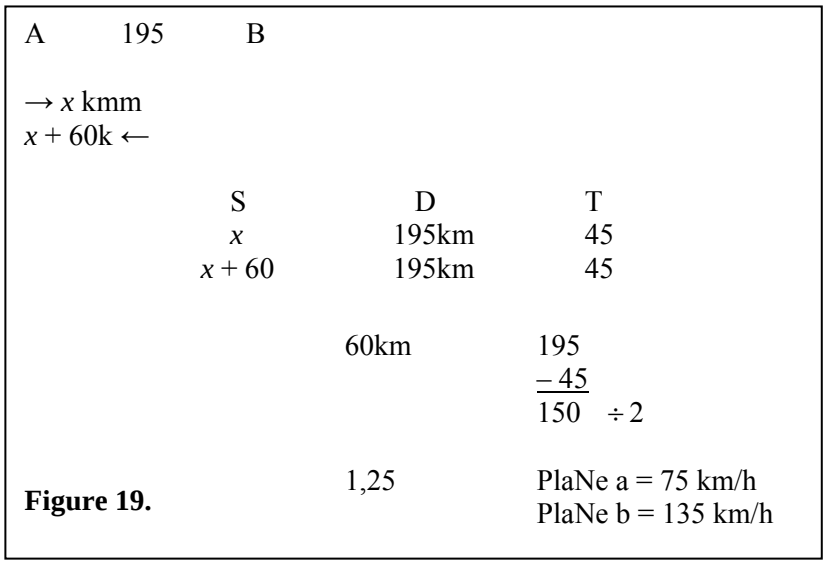



**Figure 19.**

Warren demonstrates algorithmic discourse in assigning his unknowns, which is evident in either the ambiguity of the unknown or the lack of meaning that it has for him. However, in Problem 2 Warren is able to manipulate his unknowns until they give an almost correct equation for the problem, and this is indicative of heuristic discourse. Further evidence of heuristic discourse is the way in which Warren checks his solution and rejects answers that appear meaningless to him.

The assigning of unknowns is a 'taught' procedure for Warren and at this stage does not have full meaning for him. However, in Problem 2 it did ultimately assist him in obtaining a solution, but in Problem 1 it impeded his progress because of the ambiguity of $x$ and $y$, and in Problem 3 it had no bearing on his solution at all.

Figure 20 shows the graph of Warren's solution strategies. Clearly he is more inclined to opt for 'taught' procedures indicated by the very apparent vertical split, but these do not necessarily bring him success. His perseverance with Problem 2 did enable him to establish meaning using the 'table-method'. Kutscher and Linchevski (1997) also found that if the students persisted with the table filling method there was generally more success in achieving a correct solution. The one occasion that Warren tried his own reasoning can be seen in Problem 3, and this led to an incorrect solution.

The graph indicates that the algorithmic methods seem to work better for Warren, which confirms that he does not like "that thing outside of the box" (Phase 3, stanza 2; line 15).

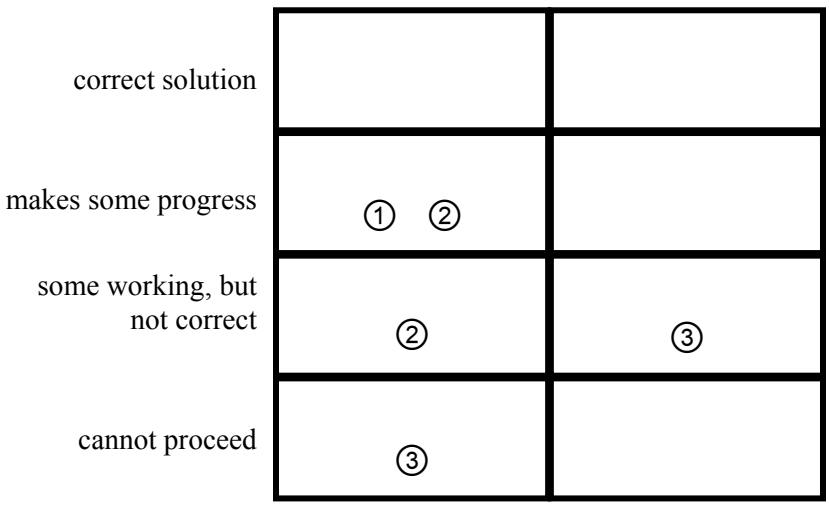



**Figure 20 – Warren's solution strategies.**

**Implications for this study**

This 'doing' phase of the research is similar to what many researchers have examined in the past in that it looks at what the boys actually did with the problems. However, when we look at the discursive aspects of what the boys did the data gives a different slant on the difficulties that the students experienced.

What is evident is that all four of the boys demonstrate a vertical split when it comes to solution strategies. The table (see figure 21) gives an overview of all the solution strategies undertaken in this study. Here the number that appears in the table represents the number of times that all the boys used a particular strategy with respect to the degree of success that they experienced in solving the problem. What is immediately noticeable is that there were only two fully correct solutions and that these were obtained through heuristic means.

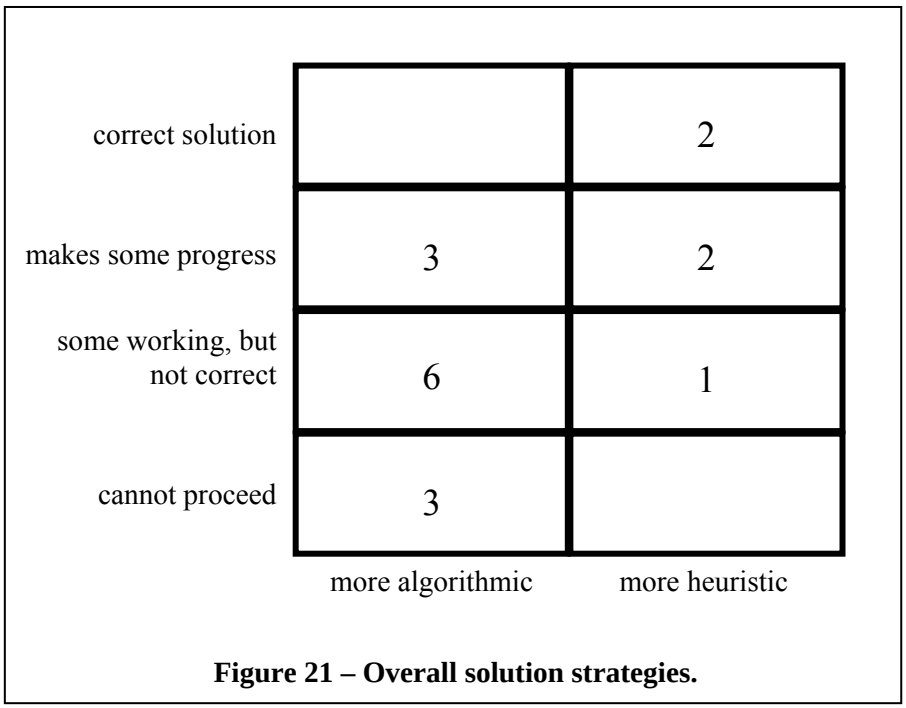



**Figure 21 – Overall solution strategies.**

The vertical split is also noticeable. Algorithmic procedures have yielded nine out of twelve attempts that are incorrect whilst heuristic methods have yielded four out of five that are fully or partially correct. However, these heuristic methods have largely been intuitive whereas the algorithmic techniques have been mostly 'taught' processes. The general lack of success with algorithmic methods demonstrates that these boys are not positioned flexibly (Boaler, 1997) when it comes to word problems. They don't appear to have the confidence to solve the problems algebraically and they seem to make more progress when they use their 'own methods'. Thus it is not surprising that the boys are lured into using their own methods, which is what De Bock *et al*. (2002) describe in their study as being coerced into intuitive reasoning.

To understand what it is that causes the boys to behave in this manner I now turn to an analysis of what they say about the problems and how they experienced them.

**Analysis of phase 1, step 2**

At this stage I reproduce the paragraphs that the boys wrote in phase1, step 2. This happened directly after they had attempted the problems. They are pertinent to the phase 3 analysis since I based quite a number of my questions for the interview on these writings.

> "Word problems in maths are questions without the usual amount of Numbers and containing words used to signify actions ie multiply = $\times$. The purpose the [sic] serve is to make a person think laterally. So that we think out side of Normal maths. I am not one For word problems they are 2 complex for me. [I tackle word problems] with a reluctance and rules that we have Been taught."
>
> (Phase 1, step 2, Warren, 24 July, 2003)

- I think word problems are problems that you can work out in your head but it is a challenge to work them out mathematically.
- I think they help you understand maths a little bit easier.
- They are taught so you can understand maths a little easier.
- They are always a challenge. It feels good when you solve them.
- I try and solve them using variables (e.g. $x - y$)"

(Phase 1, step 2 – Gary, 24 July, 2003)

- word problems are mathematical illusions which cause the human mind to apply different methods and techniques to work out multiple problems and finally get one answer.
- they serve the purpose of creating a better understanding of how to solve mathematical problems.
- to teach the learners how to programme their minds correctly when attending to not only word problems but also to implement word problems in other areas of maths.
- I feel that I have not fully grasped the knowledge of doing problems, but I will be happy to acquire whats needed to do them.
- I start with the things that are given and try to write some thing in the place of the things not given.
- They're quite mind boggling and they are good in terms of programming the mind."

(Phase 1, step 2 – Imran, 24 July, 2003)

- Word problems are maths equations written in words.
- They help with problems in life.
- I think because you use linear equations with them
- I hate word problems. When I see them I just want to move on to the next question.
- 80% luck and 20% skill.
- Why are they so hard?"

(Phase 1, step 2 – Karim, 24 July 2003)

I see two categories of cultural models emerging in these paragraphs. I did not anticipate these in my theoretical and analytical frameworks but they give an interesting perspective on how the boys position themselves with regard to word problems. The first category consists of cultural models that the students inherently ascribe to as a result of their experiences with word problems. I shall refer to these as *empirical* cultural models. The second category

comprises cultural models that define what the students believe teachers and other educational figures ascribe to. These I shall refer to as *putative* cultural models. Putative cultural models reflect the students' perceptions of what word problems are *supposed* to be and do, whereas empirical models indicate what word problems *actually* are or do for the students. These two categories of cultural models are not necessarily mutually exclusive but there does seem to be quite a rift between them in many instances in this study.

I give two examples that for me explicate putative and empirical cultural models. Gary says that word problems "…are taught so you can understand maths a bit easier". This discourse underlies a putative cultural model since, as we see in phases 1, step 3 and phase 3, Gary's discourse indicates that word problems have not *actually* helped him in any way that he can discern. Another example can be seen in Karim's statement, "I hate word problems. When I see them I just want to move on to the next question". This discourse is indicative of an empirical cultural model since it is borne out of Karim's personal experience of the problems.

Gee (1999) states that, "…we need to distinguish between cultural models based on how they are put to use and on the effects they have on us" (p. 68). Where putative and empirical cultural models are in conflict there are likely to be dilemmas. For example, in the paragraphs above the boys express a general putative cultural model that **word problems are good for us**, but we also see a general empirical cultural model that **word problems are too difficult for us**. Each of the boys fully endorses the notion that word problems are supposed to help them but their experience is that they can't do word problems. The conflicting nature of these cultural models creates a tension within the boys that begs the question, why are these things not helping them? The boys then begin to lose confidence in their own ability as we see in

the comments, “Why are they so hard”, “They’re quite mind boggling” and “they are 2 complex for me”.

Applying cultural models to understand mathematical behaviour is a little different to what Gee (1999) describes. He talks about *cultural models in conflict* where he shows that, “…since they [the people he is analysing] fail to identify themselves as actors within that model, they cannot develop the very expertise that would allow and motivate them to practice it” (cf. Gee, 1999: 66 – 68). In the case of this study we find that the boys act upon the empirical cultural model (since experiencing the problem dictates this) whilst the putative cultural model is simply espoused and has no empirical basis since the boys generally *cannot* do the problems. Thus the consequences are different here from the situation described by Gee since the boys do not have a choice regarding which cultural models to act upon whereas the ‘subjects’ in Gee’s example theoretically do have. This complicates issues surrounding how to remediate this problem, but Gee’s concept of cultural models in conflict does in any event help to explicate why students battle with word problems.

Thus we can say that the putative cultural model mentioned above is *espoused* whilst the empirical cultural model is *evaluative* (Gee, 1999). The empirical cultural model is used to form judgements whilst the putative one is the perception of what word problems should be. This, according to Gee, determines how the students will act in the context of word problems. If they are acting in response to their somewhat unpleasant and unsuccessful experiences with a view to what word problems *should* be doing for them, then it is not surprising that the success rate as indicated in the analysis of phase1, step 1 is so low. Furthermore, we can ask ourselves, whose interests does the putative cultural model serve? Certainly it is meant to

serve the interests of the students but, because of its conflict with what the students actually experience, it does not achieve this.

The analysis of empirical and putative cultural models is richer when seen in the light of the phase 3 interview since the students add their viewpoints and this helps to classify the cultural models more accurately. I will therefore incorporate further analysis of this phase together with the phase 3 analysis.

**Analysis of phase 1, step 3**

The phase 1, step 3 discussion took place directly after phase 1, steps 1 and 2, and before I had given any input about the problems. I was also not present during the discussion. The boys were asked to discuss the problems with particular regard to how they tackled them, comparing their methods and answers, and anything else that they wished to discuss about the problems.

| | | |
|---|---|---|
| 1. | Warren | Ok, the first one is wrong because I didn't do it out of 200, dude. So my one you can just ignore. |
| 2. | Karim | And your one was what? |
| 3. | Gary | I said 24*x* plus 36*y* is 5760. |
| 4. | Warren | I also did that. I started off…no! I started off with exactly what you're doing and then I said let *x*… *y* equal… Let *x* equal the R24 and then let *y* equal R36, and then I said let *x* plus *y* equal R60. Which is all good except when you times it, when you divide 5760 by 6 you get 96 exactly. So, 96 plus 96 equals 180…err 192… I'm eight watches short! |
| 5. | Karim | I just done it by lot. I said 100 and 100 watches. 24 times 100 is 2400. 36 times 100 is 3600. Adding that together you get about 6000… I said, and that doesn't work. I said 120, 24 and 80, 36 gave me 288. 288 gives me…what's that answer? [Uses a calculator] 5760. |
| 6. | Imran | I started off the same way as you. I had 100 times 24 is 2400 and 100 times 36 is 3600. That comes to R6000, minus the total cost of 200 watches, which is 5760, gave me R240. Then I divided that by 24… And now the watches is worth R24, and that equals to 10 like, 10 extra - that gives R6000. Then I…because the cost is 5760 minus, like I just assumed, like 100… If you |

bought 100 of the R36 watches then, uh, so you…so I took 5760 minus 3600…gave me 2160, and I divided that by 24 and got 90. So my final answer was 100 of the R36 watches, and 90 of the R24 watches gave me 5760.

7. Warren And you're also ten watches short.

8. Imran 190 watches.

9. Warren 200 watches!

10. Imran Oh yeah.

11. Warren See this is the problem, dude.

12. Gary [In frustration.] No! No…

13. Warren So Dayal [Karim] gets a mark for that one and we all failed.

In the discussion about Problem 1 Imran and Karim are talking in arithmetic terms in which they manipulate numbers until they arrive at what they believe to be the correct answer. Gary and Warren both talk in algebraic terms in which they try to establish relationships between the unknowns that they assign and the given information in the problems. The two boys who use algebraic means are unable to make any progress towards finding a correct solution, whereas the two who use arithmetic methods come remarkably close to a correct solution (in fact Karim actually solves the problem!).

What I find interesting is that nobody questions the techniques used, but the answer is questioned by Warren on two occasions when he says, "I'm eight watches short" (line 4) and "…you're also ten watches short" (line 7). It is clear that Warren does not know how to solve the problem, but the context of the situation tells him that the answers are not likely to be correct. So, is Warren's difficulty here manifesting itself, as Mulligan (1992) suggests, in *structural understanding*? Or is it perhaps developmental as Christou and Philippou (1998a) found? Or possibly, has the table served to obscure as opposed to enlighten as Kutscher and Linchevski (1997) found. I believe that this question is more fully addressed when we examine the cultural models that emerge through the student discourse

The discussion then moved to Problem 2.

| | | |
|---|---|---|
| 14. | Karim | The next one. |
| 15. | Warren | Ok, don't even go there again. |
| 16. | Imran | How did you manage that one… I started the table… |
| 17. | Gary | I got the answer but I didn't do any maths. |
| 18. | Warren | What's the answer, dude? |
| 19. | Gary | Peter is 40 years old and Paul is 20 years old. |
| 20. | Warren | Ok, I'm out of the loop. |
| 21. | Karim | How about 4 and 2? |
| 22. | Warren | Ja, ok. I got like 4 and then, that's…and then I wrote all this and got 4 and then I stopped. That's good enough for me. |
| 23. | Imran | I tried to do it in that way and to retain the…and I had 3$x$. Because $x$ times $x$ times $x$. 2$x$ equals $x$ times $x$. I just stopped - I couldn't go. |
| 24. | Gary | Ja, but I don't know what I'm doing because…[inaudible] |

Imran's statement in line 16 tells us that his first line of action is to use 'taught' procedures and in line 23 we can see that this has not helped him at all, and has probably served to confuse him more. Earlier in line 1 and now in line 20 we can see Warren expressing a lack of confidence in his solution. Even when Karim gives his answer, Warren recognises that he arrived at the same ages, but the hesitation in line 22 and the amount of time that he spent on Problem 2 (cf. analysis of phase 1, step 1) show that he has doubts about the answer, but is unable to see why. Karim, however accepts his answer without question (line 21).

Gary makes a very striking comment in line 17 when he says that he didn't do any maths. I raised a question regarding this in phase 3 and I therefore leave a discussion of this comment to the phase 3 analysis.

| | | |
|---|---|---|
| 25. | Karim | And the last one? |

| | | |
|---|---|---|
| 26. | Gary | No, the last one I think I got. |
| 27. | Karim | What you got – 86 or 8? |
| 28. | Gary | No, 260. |
| 29. | Warren | Holy shit, dude! No! That's going to travel the full distance, you know that? |
| 30. | Gary | Ja. |
| 31. | Warren | I mean ¾ of an hour. |
| 32. | Warren | [To Karim.] What did you get? You got 98, dude. |
| 33. | Karim | I got 86 and 146. |
| 34. | Warren | Ok, how did you get your answer? Because we've all got conclusions, that's why. |
| 35. | Karim | I just said $x$ plus $x$ is $2x$, equals 195, plus 45 minus 60. $x$ equals 175. $x$ equals 86. |
| 36. | Warren | It works for me. |
| 37. | Karim | I just added all the numbers they gave us. |
| 38. | Warren | And divided by the distance. Well this one I just said that, like the distance is $x$ kilometres an hour, and then from A to B…and then from B to A it's $x$ plus 60 kilometres an hour. So that's…you could do exactly 1 kilometre a minute. That's 60 kilometres an hour. So 195 kilometres minus 45 kilometres will give you 150. That answer divided by 2 gives you 75 kilometres an hour. 75 plus 60 equals 135 kilometres an hour. They will meet at 45 minutes. But I don't know if the distance is right. |
| 39. | Gary | Ok…umm. |
| 40. | Warren | But we're all different, dude. |
| 41. | Karim | Hey! |
| | | [All participants contemplate their working.] |

Gary thinks he has solved Problem 2 and he expresses this in line 26. Warren picks up immediately that Gary's answer is incorrect, which he states quite emphatically in line 29. However, when Karim gives his answer in line 33, Warren accepts it, and probes further. Furthermore, he indicates in line 36 that he is happy to accept this result. Warren's discourse here demonstrates that he has a good understanding of the content of the problem since he is able to make informed judgements about the proposed answers. However, he is still unable to

generate equations that will enable him to find the solution. His explanation in line 38 reveals an arithmetic approach to the problem, which is not what Warren demonstrates in his working of the problems (cf. phase 1 – Warren). It appears as if the 'taught' methods have failed him and he is resorting to a more intuitive approach: for example, he says, "…you can do exactly one kilometre a minute. That's 60 kilometres an hour." (line 38). It is important to point out here that Warren's reasoning is not algorithmic since he is generating understanding of the situation and actually uses this to make significant progress towards a solution. (One boy in the second pilot study used the same technique and actually solved the problem.)

From this point on the discussion jumps around somewhat.

| | | |
|---|---|---|
| 42. | Warren | Umm… So in conclusion I rate when we get these in the exam we're all going to fail, verrry quickly. |
| 43. | Karim | I got this one right in the exam. I got this one right. |
| 44. | Warren | You got that one right – you definitely got that one right. |
| 45. | Gary | But, what about this with the ages. It gives you like how old Peter is and then, or how old Paul is… |
| 46. | Warren | Ja, you need an age, a variable. Ja, like you… |
| 47. | Karim | …nothing here to work with. |
| 48. | Warren | I rate…[inaudible]…the question so many times it is not even funny. It's like its all the exact same thing; it's 3$x$ minus 10$x$, 3$x$ minus 10$x$… |
| 49. | Gary | Did any of you guys use distance equals speed times time? |
| 50. | Karim | Ja. I had it distance plus distance all the time. |
| 51. | Gary | That's what I did and then I got this 260 kilometres per hour. |
| 52. | Karim | I didn't think a train can travel so fast. |
| 53. | Gary | Oh, it's a train? No…it's a plane. |
| 54. | Warren | Ok, speed… Speed equals $x$. |
| 55. | Gary | Yes. So $x$ is 195 over 3 over 4, which is point seven five. |
| 56. | Warren | Now, 45 minutes. Ok what's 195 divided by 45. |
| 57. | Gary | No, do it in hours… So like point seven five. |

58. Warren No, No, No! That's assuming you, dude…

59. Gary You're doing it in minutes.

60. Warren Ja! Point seven five is out of 100 minutes. You don't get 100 minutes in an hour.

61. Gary The speed is per hour so if you do it in hours then it's…

62. Warren Ja, but in an hour of 60 minutes.

63. Gary Oh yes. Ok.

64. Warren So 195 divided by 45 equals…
65. Imran So it's like point seven five.

66. Warren It can't be because minutes are in 60s not in 100s.

67. Gary 195…?

68. Warren Ok, I'm duded up on this.

69. Gary 4,33…

70. Warren This does what?

71. Gary Which is 4 and 1/3. It's 4 and 1/3.

72. Warren 4 and 1/3. Ok. So…that doesn't help us in any way.

73. Karim Ja.

74. Warren I think there I got about like the closest, except my distance when, you add them together, I get like 157,5 meters…err, kilometres. Change that…I don't!

75. Imran Big problems, and we all…[inaudible]…in things.

76. Karim This is getting different answers every time.

77. Gary And we don't know whose is right, so…?

78. Warren Ja, this is actually right. We need like the answer pages here, really.

79. Gary Ja, it would make things so much more sense then.

80. Karim Call him then. You're the closest to the door.

81. Warren You check how I get nominated.

82. Karim You're the closest to the door.

83. Warren Fine.

[Warren leaves.]

| | | |
|---|---|---|
| 84. | Gary | It will take long to do these sums. Can you imagine if we had to write a test again right now like this and we don't have the answers. |
| 85. | Karim | Look at this, this is all guesswork. |
| 86. | Warren | He's coming. |
| 87. | BT | Are you happy? |
| 88. | Warren | We are not even close to happy, but we will accept the fact that we are all wrong. |
| 89. | BT | Not necessarily. |

Warren's statement in line 42 indicates that the boys are involved in a results orientated environment. This obviously impacts on how they perceive mathematics, and in particular word problems. Gary reiterates Warren's sentiments in line 84, but he also mentions that 'time taken for the problem' is also a concern, especially in the context of testing. There is an element of anxiety that is being expressed in this discourse and this seems to be impacting on how the boys perceive word problems.

In lines 44 through 48 Warren, Gary and Karim talk about how they feel that something is missing in the problem statement, and that this prevents them from being able to solve the problem. Some interesting discussion of this arose in the phase 3 interview, so I leave discussion of this aspect for the phase 3 analysis.

In the phase 1 analysis I referred to Karim's statement about 'guesswork' and we see this resurfacing in line 85. Looking at this in the context of Gary's statement in line 84, we can say that this discourse is collaborative, but more importantly, it reflects a lack of confidence in his solution strategies, despite the fact that he was successful with these strategies in Problem 1. Warren echoes Karim's sentiments when he emphatically declares that they are *all* wrong (line 88).

Some interesting things happen from line 52 through 73. Firstly, Karim indicates that he thought that they were dealing with a train in Problem 3 (line 52). Gary's reaction indicates to us that his thinking about the problem was initially 'challenged', but he then realises that they were indeed dealing with a plane, and that his answer was still feasible. This interaction shows that the context of the problem has an influence on how the boys respond in terms of what happens in the description of the problem. But, more importantly for this study, Karim's discourse reveals insight into his approach to word problems. In reading (and I assume doing) the problem Karim 'mistook' the plane for a train, and this suggests that he is looking for 'critical' information in the wording of the problem. By 'critical' information I mean data in the problem that Karim believes will help him find a solution. At no stage in his working did he give any indication that he was dealing with a train or a plane, and this did not seem to impact on his solution strategy. Thus, it seems likely that he 'overlooked' this in favour of the 'numbers', since it was the numbers that he believes will possibly lead him to a solution as in the case with Problem 3 where he "added all the numbers they gave" (line 37).

The second interesting aspect of this part of the discussion is the 'different' interpretations of the three quarters of an hour given in Problem 3. Warren is looking at 45 minutes in 60, whilst Gary is talking about 'point seven five' of an hour (lines 55 – 63). These clearly amount to the same thing. This is important to this study as we can see that these two boys have different ways of looking at the information given in the problem, different even to the extent that they cannot recognise that they are *both* correct. (Gary even appears to concede in line 63!) The same type of thing happens when Gary suggests the number 4,33 in line 69. He is actually working with speed as 'distance divided by time' (cf. line 49). Warren does not

see the relevance of this, which again suggests that he is viewing the problem from a different angle. Gary again concedes, even though his working is correct (lines 69 – 73).

At this point we have looked at three situations in which the students express themselves about word problems: doing the problems, writing about the problems and talking about the problems. In the 'doing' stage we see the emergence of algorithmic and heuristic discourse. When the solution strategies were viewed in the tables a definite vertical split between algorithmic and heuristic methods was evident. The table also enabled us to view the success that the students experienced and this reflected inertia when it comes to heuristic methods for the most part.

In the paragraphs that the students wrote we see a different kind of split in the empirical and putative cultural models. The former being models that the students acquire through experiencing word problems and the latter being models about word problems that the students commonly accept on undisputed grounds. These appear to be in conflict and therefore the students are in a quandary about what word problems actually do as opposed to what they supposedly do.

In the discussion we see some of the concerns of the students that they experienced when doing the problems and these concerns lay the foundations for identifying cultural models in the analysis of phase 3. Actually, many of the findings discussed in the analyses of phase 1 are tied to those that emerge in the analysis of phase 3. Only once we pool this data can we see cultural models emerging more clearly, and it is these cultural models that will inform us of the influences that affect the way in which these boys approach and tackle word problems.

**Relating how word problems are experienced**

Phase 2 did not prove productive from a data point of view but this was anticipated in the planning of the study. However, it formed an integral part of the research since it gave the Grade 10 students the opportunity to clarify their thinking about word problems. This development of meta-thinking, I believe, resulted in a richer discussion in phase 3.

I now turn to an analysis of phase 3. This was actually a culminating phase since the questions that guided the discussion were drawn from all of the previous phases and it was therefore logical to use this phase as the primary source for identifying cultural models. For clarity I reiterate the questions that guided this analysis as discussed in my analytical framework:

1. What cultural models are at play?
2. What are the differences/similarities between these cultural models?
3. Are there master models at play?
4. To what extent are these models consistent or conflicting?
5. Whose interests do the cultural models serve?

**Analysis of phase 3**

In order to saturate the data the entire transcript had to be parsed. In the first parse I attempted to divide the transcript into stanzas according to the questioning that I had prepared as a result of the first two phases. For example, the first three stanzas are divided according to the discussions that happened regarding the three problems respectively, and this was intended to unveil discourses at play with respect to how the boys perceive the 'nature of word problems', and what these mathematical problems actually are in their worlds. In terms of my analytical framework, I intended this discussion to reveal the discourses that might

enlighten me about the espoused cultural models that the students hold. However, I found that beyond stanza 3 my approach was too rigid. Too much emerged in the discussions to confine whole areas to one particular cultural model. My stanzas have therefore been defined differently to what I intended in my original interview schedule since it allows for a more in-depth analysis across cultural models, which I now believe is more enlightening about the difficulties that the boys experience in formulating algebraic solutions to the problems. I have therefore prefaced each stanza with an explanation of how I have divided the discussion for the analysis.

**Stanza 1 – discussing Problem 1**

The following is a general discussion of Problem 1:

| | | |
|---|---|---|
| 1. | BT | We are just going to discuss the word problems that you've dealt with and your experiences, and how you feel about them and that sort of thing. There are quite a few things that I have picked out that I actually want to ask you about. So I have got certain things that you have said or done or whatever, and I will go through those as we move along. Let's just start with a general discussion of what you sort of thought about each of the problems. We will start with the watch problem, and let's start here [with Karim]. |
| 2. | Karim | I found that one quite easy because we had done similar ones in the exams. I worked it out like I worked it out in the exam, guessing and so forth. |
| 3. | BT | What did you think of the problem itself? |
| 4. | Karim | What do you mean, like the type of question? |
| 5. | BT | Ja. |
| 6. | Karim | It was a good question, but it was difficult if you are going to use *x* and *y*. I find it difficult if I have to use *x* and *y*. |
| 7. | BT | Ok. Gary. |
| 8. | Gary | Ja. It was quite hard. But then when I saw the solution I thought now why didn't I think of that. It was easy when I saw the answer, and when I was actually sitting there doing it,<br>I thought now what the hell to do now? What the hell is going on? |
| 9. | BT | Ok. Imran. |
| 10. | Imran | First when I looked at it, it looked difficult. But when I turned the page (I normally do that in the exams, turn the page and see what's the next question)… |

| | | |
|---|---|---|
| | | And when I looked at the other two I noticed that those two are more difficult than the first one. So, like I tried to work it out now. It looks like I only got the price of the cheaper one's watches, not the expensive watches. It was difficult for me to get the amounts. |
| 11. | BT | Warren. |
| 12. | Warren | Watches one, yeah, I know. If it is like things like that, I usually get completely lost. I did get completely lost on that one as you can see on my page. I wrote lots of different answers…um, but as [Karim] said, once you do see the answer, you know that you should have seen it earlier. There is always something that you are missing. |

We see in line 2 that Karim seems to find word problems easy if he is able to manipulate figures and arrive at a solution that works. However, in line 6 he states that he finds it difficult if he uses algebraic techniques. It is interesting that he says that the problem is "…difficult if you are going to use *x* and *y*." This confirms that he does not find algebraic techniques helpful in the solution of the problem and prefers to use arithmetic methods.

Gary, in line 8, expresses his experience of the problem as not knowing how to go about obtaining a solution. Unlike Karim, who uses "guessing and so forth" (line 2), Gary does not have a solution strategy. When Gary saw the answer he felt that it became simple for him. In phase 1, step 3 he expresses a similar concern: "It will take long to do these sums. Can you imagine if we had to write a test again right now like this and we don't have the answers?" (line 84). It appears that Gary is reliant on the answers to give him the solution strategy. What he seems to be saying is that when you know the answer, then it's easy (line 8). Without the answers Gary seems to be completely lost and he expresses this very emotively in line 8: "…I thought now what the hell to do now. What the hell is going on?" This shows the helplessness and exasperation that Gary experiences when doing word problems.

Warren states in line 12 that once you know the answers then you realise that you should have been able to work it out before. This is different to what Gary expresses with respect to

knowing the answers in that Warren feels he *should have* been able to solve the problem once he sees the answers. However, his experience in doing the problem seems to have been one of not knowing where to begin. He says that he "usually get[s] completely lost" and that "[t]here is always something that you are missing" (line 12). His use of the words *usually* and *always* tell us that this is not a new experience to him and it is borne out of the idea that there seems to be something *missing* in the problem. However, it is interesting that Warren perceives that the problem doer is missing the information and not that the information is missing in the problem *per se*. This suggests that he perceives some ineptitude within himself as far as being able to solve problems is concerned, and I see this as being the essence of the difference between Warren's and Gary's perceptions of the problem.

Warren also says, "I wrote lots of different answers…" (line 12), but in his working he has not found any answers (cf. analysis of phase 1, step 1). This suggests to me that Warren's notion of the equations is that they *are* the answers, or in other words, if you can find the correct equation(s) you will get the answers. Thus it is not algebraic manipulation that impedes Warren in solving the problem, but rather establishing the correct equation(s).

In line 10 Imran tells us that his initial impression of the problem was that he probably would not be able to do it. He says that he *normally* turns the page to see the next question, which implies that he had given up on the first question before he had made any attempt, and that this is the way in which he usually approaches word problems. Having looked at the other two problems he comes to the conclusion that they are *more* difficult than the first one, yet he still has made no attempt at solving any of them before making this judgment.

All four of the boys express difficulty with Problem 1, but it is the nature of the difficulty that is of interest here. Karim says that he only finds the problem difficult if he has to use algebraic techniques, but his arithmetic methods proved "quite easy" for him (line2). He attributes this to having done similar problems before and it appears that familiarity with the problem enables him to recognise a solution strategy. Gary in line 8 says that having access to the answers makes the problem easy for him; otherwise he finds it quite "hard". Imran has the perception that the problem 'looks difficult' and he expresses an unwillingness to tackle it. Warren's difficulty is expressed as becoming "completely lost" (line 12).

A general cultural model emerges from this discussion and that is **word problems are obfuscatory**. However, we need to look at the nature of difficulty that each boy experiences to gain insight into why they find the problem esoteric. Nested within this general cultural model are the following: **word problems are difficult if you have to use algebraic techniques** since the variables conceal the meaning in the problem; **word problems are difficult if you don't have the answer** because the answer helps you to find a solution strategy; **word problems are difficult if you don't know where to begin** because you can't see where and how to begin; **word problems cause you to get completely lost** and you can then not make any progress. Each of the boys indicates that there is some kind of path to solving the problem and that this is blocked from sight. In other words, the problem is not seen as something that you can think about and then set out to solve. I shall elucidate on the nature of student difficulty in doing word problems in the discussion after stanza 3 at which point all three problems would have been generally discussed in the interview.

**Stanza 2 – discussing Problem 2**

Here each of the boys discusses his experiences with Problem 2:

| | | |
|---|---|---|
| 1. | BT | How about the age problem? |
| 2. | Karim | I always find it difficult. Because of the table setting out and like the last sentence always confuses me. |
| 3. | BT | Do you understand what the question is asking, though? |
| 4. | Karim | Ja, I understand the question but like to put it in an equation is difficult. |
| 5. | Gary | The word sums we've got like that before they normally give us an age of one of the people, so then it is easier to work it out. But with this one they did not give any ages, so… I've never done ones like that before so I found them hard. |
| 6. | BT | I see. So they only gave relationships, one was twice as old and then 10 years ago he was three times as old, that sort of thing. What do you think of that as a concept for the problem? |
| 7. | Gary | Well ja, it's more challenging, and once you see the solution, again it is so much easier. Ja. |
| 8. | BT | Imran. |
| 9. | Imran | I remembered how to put it in the form there to say $x$ equals, $x$…put down $x$ into the table and other person, the other brother equals two times $x$. But the thing which I forgot was how to put it 10 years ago, and until I saw sir explain it me how to put minus a 10 years ago, then after that I realised how to get the final answer. But it was quite hard working it out myself. I went home and I practised it quite a lot of times on scrap paper and after that finally I got the answer. |
| 10. | BT | Back to Warren. |
| 11. | Warren | I felt that the age one, when I first looked at the question, I felt there has *got* to be something missing. You feel as though there is going to be too many possible answers, but after I had started working with it, I used one method, I saw it wasn't going to work, I tried a different one. Eventually, I just gave up on that question. I had to come back, and when I looked back at it the second time I just got it. |
| 12. | BT | And what made you get it at that point? Can you remember? |
| 13. | Warren | I think, I just said why not, why can't it be this, or why should it be this? And if you usually work in reverse order like don't use the one, start with this one and try work it from there. You usually get either an alternative answer or the exact one. |
| 14. | BT | I found that interesting that you said there wasn't enough information in the question, and you thought there was something missing. I can't remember what you said exactly. Do you think word problems often appear like that? |
| 15. | Warren | To me, definitely. Because me I'm a, put it in front of me, if I can see it I'll do it. But as soon as you ask me to think about like abstract things – that's not my forte. I can design, I can break down, I can put back together, and I can't think laterally. That thing outside of the box. |

16 BT How do you guys feel about that? About there not being enough information. You were saying something similar Gary, along the lines that there were no ages given. [The boys nod agreement.] So you would agree with that?

In line 2 Karim uses the word 'always' twice, which indicates that his difficulties with word problems and the confusion caused by the 'table-method' are perennial. Ironically, tabulating data is intended to provide a solution strategy, yet Karim is 'confused' by it. If the table is not helping Karim then it cannot be viewed as flexible knowledge (Boaler, 1997). Kutscher and Linchevski (1997) saw their *table-filling* method as enabling the students but this certainly is not the case for Karim. It appears that in Problem 2 Karim has not been able to use arithmetic manipulation of figures as successfully as he did in Problem 1, and consequently he cannot solve the problem because the 'method' that he has been given (viz. tabulation) confuses, rather than aids him. This suggests that the 'table method' is knowledge that for Karim is inert (Boaler, 1997). In other words, he knows *about* the table method but he is not able to apply it in solving the problem. Gee (1992) attributes this to "(*midlevel*) *situated meaning*" (p. 243). He argues that the inability to apply knowledge can result when students "…either have some induction into a cultural model (theory) without any real feeling for the situated meanings connected to it (this is too general), or they have some feeling for the situated meanings and ability to work with and recognize them in situ, but do not really have much feeling for the larger cultural model that connects and explicates them (this is too specific)" (p. 243)[1]. Karim recognises that tables are used in algebraic solutions to word problems but he fails to see any connection with how this is used as a technique for solving the problem, particularly when it comes to connecting the table with the last sentence in the problem. This, in Gee's words, is too specific. It is this discourse that reaffirms that he holds the cultural model that **word problems are difficult if you have to use algebraic techniques**, i.e. there is

[1] By *situated meaning* Gee(1999 ) offers the following: "…meaning is a matter of situated meanings, customized in, to, and for context, used always against a rich store of cultural knowledge (cultural models) that are themselves "activated" in, for, and by contexts" (p. 63).

a set path for solving word problems (viz. algebra) but he cannot follow that path because he is unable to decipher the route markers.

Line 4 suggests to me that Karim loses confidence in his ability to solve the problem when he feels he needs to find an equation as opposed to "guessing and so forth" (Stanza 1, line 2). The need for an equation prompted Karim to say that Problem 2 was "difficult", yet Problem 1 was gauged as "easy" because he could use arithmetic methods. If Karim cannot use intuition he then tries algorithmic methods that don't help him to solve the problem. As he says, "Because of the table setting out and like the last sentence always confuses me" (line 2). The last sentence in a word problem is invariably the question, and Karim confirms that he understands what the question is asking him to do (lines 3 & 4). From this discourse Karim appears to hold an interactional cultural model that **if I have to use algebra I will not be able to solve the word problem**. Because of this Karim tackles word problems algebraically by 'adding all the numbers that they give' and accepting whatever results from that.

In Stanza 2 we again see both Gary and Warren expressing the view that some information is 'missing'. Gary says that "they normally give us an age" (line 5), which is actually what the question is asking the problem solver to find. This ties in with the observation that I made in Stanza 1 regarding Gary looking for the 'answer' (albeit unconsciously) to provide the solution strategy. In addition to this Gary says, "…and once you see the solution, again it is so much easier" (line 7). Clearly, this prevents Gary from finding a solution strategy since he is looking for an 'answer' to guide him to the answer. Warren states in line 13 that he tries to work 'in reverse order', but this is different to what Gary is trying to achieve in that Warren's approach can be seen as a strategy used to find a solution whilst Gary's is searching for a strategy.

Warren initially felt that there was something missing (line 11) but in line 15 he suggests, as with Problem 1, that it was he that was missing the information and not the information missing in the question. Both Warren and Gary use a discourse that reflects an espoused cultural model that **in the presentation of a word problem there appears to be something missing**. However, for Gary the cultural model is **in the presentation of the problem they appear to have left something out** and for Warren it is **in the presentation of the problem I seem to be missing something**.

I see these subtly different perceptions of the problem resulting in markedly different approaches to finding a solution. Gary never really 'solved' the problem, but he managed to write down the correct ages (and I assume here that he used similar techniques to what Karim used in Problem 1). Warren, according to my field notes, returned to the question and spent a lot of time on it. His statement, "I think, I just said why not, why can't it be this, or why should it be this?" suggests that he was looking for the information that he had 'missed', and this enabled him to establish an almost correct algebraic solution. (There was a misinterpretation of which age he should treble for the equation – see phase 1, step 1 – Warren.)

Imran expresses how he attempted to use an algebraic solution but "forgot" how to express the ages ten years ago (line 9). He goes on to say that after I had explained it to him he went home and practised (line 9). This suggests that Imran perceives the mathematical procedures to be rigid, standard and repeatable (i.e. you can *forget* them) and therefore you need to *practise* them in order to master the technique. Also, these procedures are difficult for him to achieve individually and the teacher's input for him is crucial – once this was given he was

able to go and practise the strategy and 'achieve' a correct solution: "I went home and I practised it quite a lot of times on scrap paper and after that finally I got the answer" (line 9). Imran's discourse underlies the espoused cultural model that **word problems can be solved using set procedures** and the interactional cultural model that **set procedures in solving word problems can be mastered through practise**.

As a result of the cultural models discussed above I see a difference between the working of Warren and Imran emerging. Imran appears to believe that there is a 'formula' for solving the problem, which with guidance and practise he can master, whereas Warren goes in search of information in order to establish a solution. Gary is looking for an answer that will guide him to a solution strategy whilst Karim is unable to make progress because he cannot apply his solution strategy of 'guessing'.

**Stanza 3 – discussing Problem 3**

I then directed the discussion to how they generally felt about Problem 3.

| | | |
|---|---|---|
| 1. | BT | Ok, what about the speed problem. |
| 2. | Karim | That one was actually easy after you showed me what to do. What speed equals to distance times time. |
| 3. | BT | And before that? |
| 4. | Karim | I struggled. I got the answers and thought it could be right and yet it was wrong. |
| 5. | BT | Ok, how did you know it was wrong? |
| 6. | Karim | Because of the way I done it. I just added all the numbers and I got *x*. |
| 7. | BT | Oh yes. I've actually got a question about that just now so we will come back to that. Gary what did you think there? |
| 8. | Gary | The question asked you to work out the speed, but you had to work out the distance first to find it out, and then once you found out the distance you tried to work out the speed using that pyramid, distance equals speed times time. |

| | | |
|---|---|---|
| | | But…um…I found out another way where you add the two distances together and it equalled 195. It is much easier. |
| 9. | BT | Ja, you can do it two different ways. |
| 10. | Imran | When we've done it on our own in class I didn't understand it, and even when I stayed at home I didn't understand it. When you explained it to me I still never understood it, until when I had to explain the Grade 9 boy. I don't know how, it just came into my mind and then I just knew where to start, and where to end, and then I found out that this is how you do this type of equation. In fact, initially you don't know where to start, don't know where to end but after you put into the equation, the pyramid, and you work it like that in order, then you get the final answer. |
| 11. | BT | So do you think that actually teaching it, helps you to understand it? |
| 12. | Imran | Ja, sure. |
| 13. | BT | That's why we teachers are so good at these problems, hey? |
| 14. | Warren | Could be that you've done them a couple of times as well. |
| 15. | BT | Could be. And Warren what do you think about the speed problem? |
| 16. | Warren | No, I thought I started off pretty well in the speed one. Um…because I usually like do it when you're travelling, I work out how many k's per minute you're doing, how fast you're going, how far you've gone. But I knew that when they intercepted or they met it would not be a distance of 195. I just hadn't thought about what distance it would be. So I…because I hadn't done that, whatever I had done wasn't based on anything. I just basically worked the numbers out in my head and like there was nothing to base it on…then obviously [inaudible]… |

There is a striking comment by Karim in line 6 in which he reveals his solution strategy: "I just added all the numbers and I got *x*". He made a similar comment in phase 1, step 3 where he said, "I just added all the numbers they gave us" (Phase 1, step 3, line 37). For me this confirms that he uses this as a strategy when arithmetic methods fail him, but also because it enables him to avoid tabulating the data, which confuses him. This approach to word problems is in line with Karim's cultural models discussed in stanzas 1 and 2.

Karim has stated that he is confused by the 'table method', and, judging by his comments with regard to Problem 3, he holds the cultural model that **word problems are confusing if done by algebraic methods**. It is not surprising then that this may elicit a 'do-whatever' strategy. What is of concern is that he then believes that his answers "could be" correct (line

4), but that he fails to check in any way. He states resignedly, "I…thought it could be right and yet it was wrong" (line 4). I see this as resulting from his experience of confusion in the solution strategy and he is arriving at some answer and simply *hoping* that it is the correct one. This could also be as a result of what De Bock *et al*. (2002) refer to as *intuitive cognition*. Citing Fischbein (1987) they say that, "…intuitive cognitions have an obvious, self evident, and coercive character, receive great confidence and persist despite formal learning" (p. 327). Karim tends to go with his intuition and this, in spite of his admission that he suspects his method to be wrong (see line 6), I suggest, results from his aversion to the algebraic techniques.

When Karim worked on Problem 3 in phase 1, step 1 he did not use the formula for speed and it is evident that he is not too familiar with the formula when he says, "What speed equals distance times time" (line 2). Again this points to Karim's aversion to algebraic methods whereas Imran and Gary both express a dependence on the 'pyramid' (cf. lines 8 and 10). In view of the foregoing discussion I believe that the cultural models **word problems are difficult if done by algebraic techniques**, **word problems are confusing if done by algebraic methods** and **I probably wont be able to solve the problem if I have to use algebra** play an enormous role in the way in which Karim tackles word problems. He acts on the cultural models by actively trying to avoid using algebraic methods and, when he cannot use his 'guessing' techniques he finds himself without a solution strategy.

Gary says that he found an 'easier' method (line 8), but his method is based on a misinterpretation of the distance travelled by each plane in the problem. Imran also believes that he came to understand the problem when he taught it to the Grade 9 boy (line 10), but he made the same mistake as Gary (cf. phase 2 – Imran; 7). Warren tells of how he has had

experience with the situation of the word problem in real life. All of these discourses suggest that there needs to be some sort of familiarity with the problem in order to be able to cope with it.

Therefore, I see two interesting dichotomies of discourse taking place in this stanza. We see easy-difficult discourses, and these appear to be linked with familiar-unfamiliar discourses.

Karim in line 2 directly relates familiarity with easiness of the problem and in line 4 he polarises unfamiliarity with difficulty of the problem. Once he had been shown how to do the problem he found it 'easy', but left to his own devices he 'struggled' (line 4). Imran became familiar with the problem in phase 2 when he taught it to the grade 9 boy and he believes that this brought about an understanding of the problem for him (line 10).

Gary's experience of difficulty is linked to an algorithmic approach in solving the problem (viz. using the pyramid, line 8) and he perceives the problem to be "much easier" if he can use 'his own methods' (in this case, adding the two distances, line 8). Algorithmic procedures for Gary are unfamiliar, whilst 'his own methods' are more familiar, and he attributes degrees of difficulty of the problem according to these notions.

Warren's familiarity with the problem in the everyday situation led him to find the problem easy to start with (line 16). Later in line 16 he says, "…because I hadn't done that, what I had done wasn't based on anything". This suggests that he was unfamiliar with establishing the equation, it was something that he had not personally experienced, and so he encountered difficulty.

Again we see that the essential nature of the difficulty that each boy experiences differs, and whilst each can be linked to the notion of familiar-unfamiliar, that 'familiarity' is different for each boy. Thus, these discourses bring about a general cultural model: **the difficulty/easiness of word problems is directly linked with unfamiliarity/familiarity with the situation described in the problem**. Nested within this cultural model are the following individual cultural models: **word problems are familiar if you have done similar problems before**; **you become familiar with word problems if you teach them to someone else**; **there is a familiarity with word problems if you can use your own methods for solving them**; **word problems are familiar if you have experienced a similar situation in the everyday**.

**Stanza 4 – the problems in the everyday**

At this stage the boys were beginning to relax and I tried to encourage a more general discussion along the lines of how they each felt about where these problems may or may not fit their experiences of the everyday. The discussion picks up where we left off discussing Problem 3.

| | | |
|---|---|---|
| 1. | BT | So did you think that was quite a realistic problem in a sense? |
| 2. | Warren | Yes. |
| 3. | BT | So is that the type of calculation you would do when you are travelling? |
| 4. | Warren | Ja. I wouldn't be hitting anything but that would be somewhere pretty close. |
| 5. | BT | Ja. You wouldn't be talking about two things moving towards each other, but one thing. |
| 6. | Warren | Going towards another, ja. It's if it is 100k's away and what speed you are going, how many minutes will it take us to get there. |
| 7. | BT | Ok, which of those problems do you think are realistic, real life type problems? |
| 8. | Karim | That one. |
| 9. | Gary | Ja, the speed one. |
| 10. | BT | Just that one? |
| 11. | Karim | Ja. |

12. BT Why would you say the other two are not?

13. Karim I um…maybe the first one is like a …

14. Gary The scenario of that one is more common. Not everyone goes out and buys 200 watches or works out ages of people and…

15. BT What else do you think about the problems are not realistic, why they are not realistic? Because they look as though they are real, don't they? Talking about real people…

16. Karim The second one, I wouldn't think that is realistic because no one would actually go and think about ages.

17. Warren Ja, who would say to you, no I'm like seven times this oke's age, but three times this oke's age. No, you just say I was born that day, I'm this old. But with the watches…is…you've got the shopkeeper that buys so many watches. Now I would understand it if the guy like owns a flea-market but the average shopkeeper won't buy 200 watches, that have 120 watches that are the exact same, because he will never sell them, full stop.

18. BT Ok. That's quite an interesting aspect, I didn't think of that. But what about the fact that he buys 200 watches, and he doesn't know how many of each he has bought, but he knows how much he has paid.

19. Warren Bad businessman.

The boys have had exposure to speed-distance-time situations in their everyday lives and therefore can relate better to Problem 3, even though it is not a particularly realistic problem. For example, Gary sees Problem 3 as being more 'familiar' than the other two in the everyday context (line 14). However, the other two problems are far more contrived and Karim says, "…no one would actually go and think about ages" (line 16) and Warren says, "…who would say to you I'm like seven times this oke's age, but three times this oke's age. No, you just say I was born that day, I'm this old" (line 17).

From these discourses emerges the cultural model **word problems sometimes bear no resemblance to happenings in the everyday**, and this is tied to the notion of unfamiliarity, and hence meaninglessness in terms of how useful word problems are in the students' everyday lives. This cultural model gives us another dimension of the obfuscatory nature that was discussed earlier in that the problems are real but not *really* real. This is an empirical

cultural model and it conflicts with what Karim, for example, said in his paragraph: **word problems help with problems in life**. This latter cultural model can be seen as putative since Karim does not actually view cultural models as helping him in life.

Generally, this may affect the way in which the boys approach the word problems because they could feel that they are not worthwhile in terms of how useful they will be to them and this suggests that the boys may ascribe to the cultural model that **word problems are of no use if they are not representative of the everyday**.

**Stanza 5 – word problems as mathematical illusions**

In the following lines Imran, Warren and Gary tell of their experiences when attempting word problems and I felt that these were worthy of comparison since there appear to be surprising similarities:

| | | |
|---|---|---|
| 1. | BT | Ok. There are a few general questions I want to ask, starting with Imran. I am going to jump around a bit. You said it earlier, now as well, and you also said it in one of the transcripts…that word problems are mathematical illusions. I found that a very interesting way of looking at it and I wanted you just to explain what you mean by that. |
| 2. | Imran | Thinking of the word illusion like, in primary school when they showed us something, like it wasn't clearly…it was sort of a certain type of a drawing that if you look at it from different angles you see different things. That's what I think about word problems, but just in a mathematical sense. Because maybe, like as I said before, like when I look at one of the word problems, right…then I thought of different ways to put it, like different ways to write it down, different ways to think about it, of getting the final answer. And after that when the final answer does come, you realise that 80% of the things that you wrote down on the paper were wrong and 20% were right. So that creates a whole lot of illusions in your brain, of thinking…what must you do? If you do it like this will it work, will you get to the final answer. If you do it like that, don't you think you will get to the final answer. |
| 3. | BT | That was very nicely explained. Warren, I found a little comment that you made. You said, "…word problems are questions without the usual amounts of numbers". I think we have spoken about that a bit. And then I thought, what do you mean when you say without the usual amount of numbers. |

| | | |
|---|---|---|
| 4. | Warren | Well for me, you give me a word problem, I find a word problem more intimidating than a simple equation. A simple equation I can see four times whatever, whatever, whatever, equals a certain amount. But as soon as you put it into words it is like, you start using times and multiply, and *x* suddenly doubles and it becomes too many terms for me to work out comfortably. I like start making notes, like if it says John is two times as whatever, put like 2 times above the words so that I just write out at the top and I put that into a sum. I find that easier than tackling like the whole string of words. |
| 5. | BT | Right, another one for you Warren. You said that they contain certain words used to signify actions. Can you just explain what you meant by that? |
| 6. | Warren | Ja…um, you use words again like to multiply, to times. |
| 7. | BT | Can you give an example maybe? |
| 8. | Warren | In the watches one he says he has a stock of 200 watches, but…and then at certain prices each. Now, if I were going to set up that I wouldn't use words, because I can't, I'm a good… I don't like the way that you use like multiply and that. It's better if you use maths and keep it as maths. As soon as you start mixing the two, I feel that you lose the general… |
| 9. | BT | So you are saying that he has got a certain number of watches at R24 each, are you saying that those are the words, which signify multiplication? |
| 10. | Warren | Ja. Not necessarily multiplication, um…at, you use words like that. You say it is a definite at that price but not how many. Or…it is difficult for me to explain. Do you have one of the questions? |
| 11. | BT | I don't actually have one on me. But I am just trying to think, the word problem said that he had bought a certain number of R24 watches. |
| 12. | Warren | Yes. |
| 13. | BT | And a certain number of R36 watches, and there was a total of 200 bought. |
| 14. | Warren | Ja then you've got a certain number. |
| 15. | BT | Is that what you are talking about? |
| 16. | Warren | Ja. Then you've got to say a certain number will be let's say *z* and then it is multiplied by 4*xyq* would equal this answer. So now you've got these whole string of letters, and then you relate this one to *x* and this one to *y*. So instead of you saying that this guy's got so many watches, you physically have that many watches, and then this many watches, and it's 200 in total. You've actually got to say he has got *x* amount, *y* amount which equals *q*. I don't like the substituting. |
| 17. | BT | I think I understand what you are saying. Karim, your turn. You said word problems help with problems in life. |
| 18. | Karim | Like the speed one. |
| 19. | BT | You are saying now that some of them do and some of them don't. |

| | | |
|---|---|---|
| 20. | Karim | Like why would you use the age one in life? That just doesn't do anything for me. It's not going to help in any way, but the speed one… Say I want to go from here to Durban in a certain amount of hours. I can work out what speed I should go, and that will also help me. |
| 21. | BT | What time you are going to do, whatever. Yes. Then Imran and Gary both said that word problems help you to understand maths easier. Do you think that that is true? In what way is that true, let's put it that way? |
| 22. | Imran | Ok. It maybe can like help you to understand maths, to make equations and see where they are coming from, and that stuff. But I don't know. |
| 23. | BT | And Gary? |
| 24. | Gary | Well, I think that is so because some people are… it depends what kind of person you are. If you are good at maths then you prefer just seeing an equation, whatever. If you are not really good at maths and you are good at English, you can interpret it better so you can understand it better. So once you've seen the word, and you put it into a sum, you know what you are doing. And sometimes you get an equation and you don't know what is going on. If you have the words there that explain it and you can work it out and it can make you understand it better. |
| 25. | BT | So it's actually giving the maths a bit of meaning, is that what you are saying? |
| 26. | Gary | Ja. |

In line 20 Karim believes that Problem 3 can help him in life whereas Problem 2 is irrelevant in his life. His discourse reflects the interactional cultural model that **for word problems to be meaningful they must be applicable in the everyday**.

Imran used an eloquent metaphor to express his experiences of word problems and he explains this in line 2. He sees word problems as being *mathematical illusions* and that these illusions manifest themselves in different perspectives of the problem, or in "different ways to think of it". But the irony here is that, unlike the joy we experience in seeing the illusions in pictures, for Imran the illusions of the words in the problem are "80%…wrong, and 20%…right". So Imran holds an espoused cultural model that **word problems are mathematical illusions** and as a result of this word problems are more likely to elicit experiences of frustration and failure for him since he is unable to make out which of the illusions is the 'correct' one. Interestingly, in his paragraph in phase 1, step 2 Imran expresses the interactional cultural model **word problems help to give a better**

**understanding of how to solve mathematical problems**. These two cultural models are clearly in conflict and I see the former one as being empirical whilst the latter is putative.

In line 4 Warren expresses views that are not entirely dissimilar to those expressed by Imran. He says, "*x* suddenly doubles", as if some conjurer was involved. He also says, "…it becomes too many words for me to work out comfortably", which tells me that for Warren to discern what the problem is saying is *uncomfortable*, which is similar to Imran's experience of illusions in the problems. However, Warren is more explicit about what causes his confusion – he attributes this to the need for "tackling…the whole string of words" because the words are "more intimidating than a simple equation" (line 4). For Warren, reading the mathematics contained in the words is a difficulty, and in line 8 he again expresses a *discomfort* with mixing 'mathematics' and 'words'. His *discomfort* is made clear again in line 16 where he tells us of how confusing all the different unknowns become in the problem, and here he uses three different unknowns as if to emphasise this point and he says, "I don't like the substituting" (line 16). I see two associated evaluative cultural models emerging here: **word problems are more intimidating than other mathematics** because **in word problems the words blur the mathematics**, which again reflects the model that **word problems are obfuscatory**. These are empirical cultural models, which to an extent are in conflict with the putative cultural model **word problems serve to make a person think laterally** (Phase 1, step 2 – Warren) which is an interactional cultural model. This demonstrates how evaluative cultural models can serve to undermine interactional models and thus inhibit the person from attaining the goals that are underpinned by the interactional model. In this case the intimidating and abscurring perceptions that Warren has of word problems prevents him from applying them to develop his lateral thinking.

In line 24 Gary expresses the belief that a student who is more proficient with language can "…interpret it better [and hence] understand it better". Moreover, he goes on to suggest that this interpretative ability will then enable the student to formulate the "sum" that will solve the problem. His final comment in line 24 sums his perspective for me: "…if you have the words…it can make you understand it better". I believe that in the context it is reasonable to interpret "have the words" to mean *have the ability with words* and "make you understand" as being *give you the ability to understand*. If we accept this, then Gary's perception is that an inability with words precludes any ability to solve word problems. The empirical cultural model that Gary ascribes to here is that **to be able to solve word problems you need proficiency with words**. Gary expresses a putative cultural model that **word problems help you to understand maths a little easier**, which conflicts to a degree with the former model since it suggests that one must first be proficient with language before word problems will help one with mathematics.

Warren and Gary both hold a common cultural model that **ability with language is an advantage when solving word problems**, but I see a subtle difference in their perceptions reflected in their respective discourses. Warren's perception is that the words blur the mathematics for him and this is therefore an inadequacy within himself. Gary feels that if one is talented with language then one copes better with word problems, and this is simply a reason why he does not cope with them. I see this as being a reason for the different solution strategies, similar to the situation of missing information in the question that I discussed in stanzas 1 and 2.

**Stanza 6 – word problems aren't normal maths**

This part of the discussion revolved largely around the way in which the boys went about solving word problems and enters the realms of what some of the boys consider 'normal' mathematics to be.

| | | |
|---|---|---|
| 1. | BT | There were three very interesting statements here. I'm going to go through them one at a time because they all gave very different perspectives on the purposes of word problems. We will start with Warren. You said that the purpose they serve is to make a person think laterally, and you've mentioned that already today. What do you mean by laterally, firstly, from the mathematical point of view? |
| 2. | Warren | It gives you the ability to try different things. A word problem, there is no formal setting out of a word problem. So you can try any method you want. You can say that 1 million times by this whatever equals the answer. Whereas if you get a set equation, you know that you've got this rule to follow, and if you don't follow the rules more than likely you won't get the answer. The only thing that I've seen that does something similar is now, we are doing in tech. drawing, we are getting our steps to draw a picture, or a drawing or a plane in words. Now they use words like plane, things that if you weren't really involved with, and I said to you ok, this guy's got like a rectangular plane, you wouldn't know what I am talking about. It's something that you've actually got to go into to understand. So it broadens your knowledge. That is why I said it. |
| 3. | BT | That's very interesting. And then you said so that you think outside of normal maths. Can you just explain to me what you think normal maths is? |
| 4. | Warren | Normal maths in my terms would be that if you had R20 to go and buy a packet of crisps from the shop that cost R2.50 how much do you have left. Things that you will use in everyday life. Trigonometry I don't think I will ever use again. Factorisation I might use if I've got lot's of kids. If you've got R20 and you give it to them, yes. I find that in maths we get taught things we don't use. |
| 5. | BT | So, normal maths you are saying is what you can use in everyday life. |
| 6. | Warren | Everyday life, yes. |
| 7. | BT | And maths that is not normal is…are things that you are not going to use as such, or that you don't see where you are going to use them. |
| 8. | Warren | Ja, specialist people use them. Architects will use them, like certain parts of maths. Your aerodynamicists will use trigonometry to work out the perfect shapes and things. Accountants will use maybe some other types of maths. A plumber won't use maths. A computer programmer maybe, if he doesn't have a calculator. So I don't see why we have to learn to find out K for a sum, where in life we will get so much information that we can instantly determine K. |
| 9. | BT | Right, Imran you said word problems are good in programming the mind. Just explain to me what you mean there by programming the mind. |

10. Imran — What I think is that with word problems, like let's take like you are sitting there in your class. Your teacher and you are paging through word problems. All kinds of things go across your mind. You don't know like where must you start and where must you finish. You look at a sum about ten or twenty times, you try and think now, if I use *x* on this side, you put a number this side, and I get something. Or will I get the proper answer if I put *x* on this side and the number on this side? Will I get the proper answer? So programming the mind, what I means was that like it broadens your capability of working out things and it gives you the ability to finally extract something from that knowledge that you have learnt. To put it onto that piece of paper to get a final answer.

11. BT — In a way are you saying that it is giving you a method for solving word problems?

12. Imran — Yes.

13. BT — And then Karim said because we use linear equations with that as a purpose for word problems.

14. Karim — The R24 one and the R36 one. You use two different equations when you work that one out. Instead I just worked it out in my mind. My method.

15. BT — Gary, you said word problems are problems you can work out in your head, but it is a challenge to work them out mathematically. Can you say what you mean by working them out in your head? I think rather tell me how did you actually work them out in your head?

16. Gary — Ok. With that, like that second problem with the ages, you can like for now he is twice as old as Peter, so you can get some numbers into your head and then you go 10 years ago, and it's three times as old. So I got the ages 40 and twenty in my head automatically, but then to try and work them out with equations and stuff is just more of a challenge, it's hard.

17. BT — What I want to know now is, is that different to working mathematically?

18. Gary — Yes.

19. BT — What is different?

20. Gary — It is much quicker as well.

21. BT — What is the actual difference between that kind of working out in your head as you call it, and working mathematically? What is the exact difference there?

22. Gary — When you work it out in your head, you don't really need equations. You can just use your brain. You can just work it out. When you do it mathematically you have to do it in a set way with an equation. You have to do it in a specific way to get the correct answer.

23. BT — You find those equations difficult to get?

24. Gary — Ja.

25. BT — Once you have got the equation, do you battle with that?

26. Gary — No, then it's easy. Ja.

27. BT Warren, in terms of what you said about normal maths, what do you think of what Gary has just said now about working it out in your head as opposed to working mathematically. Do you think it matches with what you are talking about?

28. Warren Similar, but different people…like you can get some people that can look at a piece of music and know when there is something wrong, and then you get people like Gary who can see a word problem, and work it out in their head without actually having to work out with the formula. I unfortunately am not one of those people. Certain things I can look at it and know that's the answer, but like Gary says, the steps to get to the answer is what eventually one does as well.

In line 10 we see Imran re-describing his experience of illusions discussed in stanza 5 in response to me probing his statement about word problems serving to program the mind. I was interested to establish why he chose the word *program*, which for me implies a rigid, very rule-bound approach. His response towards the end of line 10 says the exact opposite: "…it broadens your capability of working out things…". So we have two conflicting notions of word problems: Imran sees them as *programming the mind*, but he believes that they should *broaden* your understanding. Added to this, word problems *cloud* his understanding (because they are *illusions*), whereas they should give him "…the ability to extract something from the knowledge that [he has] learnt…"(line 10).

There is further evidence of the illusionary effect that word problems have for Imran when he says, "All kinds of things go across your mind" and, "You look at a sum about ten or twenty times" (line 10). The discourse reveals two conflicting cultural models here. Imran believes that **word problems create confusion** but that **word problems help to broaden understanding**. From the discourse we can see that the former cultural model is the one that Imran actually ascribes to, and it can thus be classified as an empirical cultural model, whilst the latter one is interactional and is what he believes word problems *should* be doing for him, which classifies it as a putative cultural model.

In line 2 Warren states that he does not see word problems as having any specific method for their setting out and "…you can try any method you want." This, he believes, induces a type of lateral thinking. Since he sees the actual equation as being very rule-bound, his notion of lateral thinking must apply to interpretation of the problem. In stanza 5 (line 4) Warren talks about how *uncomfortable* he feels with the words in the problem, and here in line 2 we see an analogy with technical drawing of how words are used in a specialised way. Warren actually says, "…things that if you weren't really involved with,…you wouldn't know what I am talking about". Warren supposes that **word problems contain words used in specialised ways** and this requires that the problem solver becomes more involved in the problem. In this regard Warren's discourse reveals the same cultural model as Imran's: **word problems help to broaden understanding**, but in Warren's case it represents an empirical cultural model since it is borne out of his own experience.

In line 14 Karim shows that he believes that he has worked out the problem *by his own method* and he implies that this is different to working it out using equations. His discourse suggests that he 'owns' that method and that there are 'differences' between his method and the 'mathematical' method. He therefore has an interactional cultural model that **if I can use my method I can solve word problems**. Gary also says, "When you work it out in your head you don't really need equations. You can just use your brain." (line 10). As he says, working it out 'mathematically' is different to working it out 'non-mathematically' in that "[y]ou have to do it in a specific way to get the correct answer" (line 10). Also, he implies that his 'non-mathematical' approach is better since it is 'easier' (cf. line 4) and, "It is much quicker as well" (line 8). Thus Gary holds a cultural model that **it is easier to solve word problems in your head**. This dichotomy of what is mathematical and what is not mathematical, I suggest, is an inhibiting factor for these two boys since they believe that their

methods are not maths. But, in Karim's case he was able to solve Problem 1 using *guessing and so forth*, and Gary was also able to come up with the solution for Problem 2 *using his brain* (cf. Phase 1, step 1 – Gary). In both cases the boys must have used some logical reasoning to arrive at the answers, which must have had some mathematical basis to them even if they were more arithmetically orientated. What I am suggesting is that both boys have experienced some success using 'their' methods, but have had very little, if any success when using what they view as being 'mathematical' methods, and the fact that they distinguish between these two 'methods' leads them to feel a lack of confidence in trying to establish an equation. Both of these boys believe that **word problems are more easily solved by non-mathematical methods** and this is tied to the familiarity that they have with the 'non-mathematical' methods as opposed to the unfamiliarity that they experience with the 'mathematical' methods. As the problems become more difficult the equation becomes more necessary in the solution process and they experience even less success. I see this as being the underlying cause of some of their affective responses that we will see in stanza 8. Again we can view these empirical cultural models as being in conflict with Gary's putative cultural model **word problems help you understand maths a little bit easier**, which is also an interactional model.

Warren also refers to 'normal maths' in line 4, but his distinction between this and that which is "outside normal maths" (line 3) is very different to Imran's and Gary's mathematical/non-mathematical dichotomy. It is clear from Warren's discourse that he holds a cultural model that **word problems are not normal maths**, which for Warren is mathematics that only "specialist people use" (line 8) and is of no use in everyday life (lines 5 and 6). By implication then, Warren maintains that **word problems are for specialist people** and that **word problems are not everyday problems**. These perceptions of word problems reflected

in these consistent cultural models probably serves to alienate Warren since he does not see the relevance in doing the problems for himself.

**Stanza 7 – word problems involve luck**

There were a couple of occurrences in this stanza that caught me a little off guard.

| | | |
|---|---|---|
| 1. | BT | Karim, I'm going to come back to you. I'm not sure whether it was Karim or Imran who said this, because you said something about 80% and 20% just now. I might have the name wrong here. Did you write down that it was 80% luck and 20% skill? |
| 2. | Karim | For the first question? |
| 3. | BT | No. When I asked you to write the paragraph. It must have been Imran. You don't recall writing that? |
| 4. | Karim | Didn't I say it the other way, like 20% luck, 80% skill? |
| 5. | BT | No I have it 80% luck and 20% skill. |
| 6. | Karim | I'm sure it was … like 80% skill and 20% luck. |
| 7. | BT | We can check that just now, I think I've got it here. Warren let's just go back to you then. |

Some confusion arose in lines 1 through 6. Imran had spoken of, "…80% of the things…were wrong, and 20% were right" in stanza 5 (line 2) and Karim had written about his tackling of word problems as being "80% luck and 20% skill" in his written paragraph (Phase 1, step 2 – Karim). Since both of them were using a similar manner of expressing themselves I was somewhat thrown during the discussion, but I now believe that it may very well have been as a result of something that transpired in the classroom. In any event, it is clear that both boys initially ascribed to the belief that there is a large element of 'luck' involved in solving word problems and both Karim and Imran display a reliance on 'luck' in their solution strategies. It is interesting that Karim in line 4 feels that he may have said it the other way round (i.e. 20% luck and 80% skill). However, much of what he says in the discussion, and the manner in which he attempted the problems (cf. Phase 1, step 1 – Karim)

point to a technique that does involve a certain reliance on 'luck'. I therefore see both boys holding a cultural model that **solving word problems involves a degree of luck** and this influences their solution strategy. The interesting thing here is that I see this as an interactional cultural model for Karim since he uses this as a strategy, whereas for Imran I view it as an espoused model since he sees this as being the nature of word problems. Karim uses a trial-and-error technique for solving the problems in which he chooses values and then manipulates them until they 'fit' the problem. Whilst I acknowledge that this is not entirely luck, and that there is an element of informed guessing, for me it is compatible with his wording in phase 1, step 2, viz. "80% luck and 20% skill". Imran, I believe, is very reliant upon luck. He describes a 'hit-and-miss' tactic in stanza 6 (line 10) which I see as resulting from the illusions that he perceives in the problem when doing the problems he used a calculator extensively in the hope of stumbling upon values that worked (cf. Field notes; 1).

**Stanza 8 – word problems can affect you**

This stanza reveals some of the affective responses that the boys gave regarding word problems.

| | | |
|---|---|---|
| 1. | BT | [Addressing Warren.] You said you tackle word problems with reluctance and rules that you have been taught. Firstly, tell me what you mean by reluctance. |
| 2. | Warren | Given the choice I wouldn't do them. |
| 3. | BT | Ok, nice and short. But the thing is why are you reluctant, and why would you not do them, let's put it that way. |
| 4. | Warren | I've never really enjoyed them. I've done them since like Standard 1 [Grade 3] and I think it is just that I've never really understood why you had to do it this way, and why you had to do it that way. Who does it like that? Who receives questions the way they set them out? In maths you get like this guy is so far away from this guy and you work that out. But like the problem is here you get a oke's age, he is not going to tell you that I'm so many years older than this guy, and so many years younger than him. I don't find it extremely practical. |
| 5. | BT | Again we are getting back to that it is not realistic. |
| 6. | Warren | Ja. |

| | | |
|---|---|---|
| 7. | BT | Then can I just ask you in what way do the rules help you? These rules that you are talking about. |
| 8. | Warren | Not very much can you say about doing a word problem. |
| 9. | BT | Why is that? |
| 10. | Warren | Because if I had to get given a word problem like a page long and I did not know anything, I would look at it, turn it over… |
| 11. | Karim | Like an exam. |
| 12. | Warren | Exactly like that. Look at it again and think, now what am I supposed to do. And if I didn't have rules I would say ja, I would just start writing anything, anything that I would have thought made sense, but with rules you can say: Step 1: find this argument or sum. Step 2: find that. Look for something that is the same between the two. |
| 13. | BT | Can you give me an example of one of these rules? |
| 14. | Karim | Solve *x*. |
| 15. | Warren | Ja. Very good example. Try and find something that they have in common like in the ages, this guy's age is twice as old as this guy. So we can use *x* as a denominator for the age, or as a substitute, and then you can work out whatever that *x* will then become by working out different methods, or by using your answer, divided by how many *x*'s. |
| 16. | BT | Right, Karim, you said at one stage when you guys were chatting here, you said there is nothing here to work with. |
| 17. | Karim | Oh! We can both explain that with the age. Like I would just go so confused when you never give me one person's age, like I would normally do in class. Normally I couldn't just work it out like with the table and everything. |
| 18. | BT | So you are actually saying something similar to Warren, I think…said that there is not enough information. |
| 19. | Karim | Ja. |
| 20. | Warren | I think we also become defensive, if it is not something we've been taught, because we feel we have not been taught to cope with this sort of stuff. And… |
| 21. | BT | So were any of these problems ones that you haven't been taught? |
| 22. | Warren | I don't know. I might not have been paying attention, but, um, ja, well the ages we have done something similar but not something that's completely removed from where you had to find two separate answers. We've been given like something similar to the watches in an exam, where I just did not know what to do, so I left it out. So it could just be me, I have no idea. |
| 23. | BT | It's possible. Ok, Gary you actually said something quite interesting here. You said they are always a challenge, and it feels good when you solve them. So does this mean that you actually enjoy doing word problems? |

| | | |
|---|---|---|
| 24. | Gary | Not the working out. When you actually get the answer it looks so much easier, and you can enjoy them more. The more word sums you do, I'm sure the easier it will get to do any kind of word sum. |
| 25. | BT | What do you mean by challenge? Let's just go to that firstly. |
| 26. | Gary | Ok, the actual working it out, ok. If it is a really challenging one and you work it out, you come up with an equation in your head, and you actually do it, and you get the right answer. It feels good. It feels like it's so much easier. |
| 27. | BT | And doesn't that motivate you to do more of them? |
| 28. | Gary | More, ja. |
| 29. | BT | And if you are getting them wrong, then you don't get that same satisfaction? |
| 30. | Gary | You just feel like giving up, like what happened to me in the last …[inaudible] |
| 31. | BT | Ok, Karim you said you hate word problems (that is not unusual lots of people do) and that you just want to move on to the next question. |
| 32. | Karim | Like in the exam… I just get the paper and I look at the word problems, and at the top I make 0 out of 10, and I just go to the next question, and if I have like time I'll just read through it, see maybe I can work it out. But it is not likely that I will get full marks for it, and normally I'll get the answer right, but I will get one out of five. |
| 33. | BT | Now what is it about the problem that makes you hate them? |
| 34. | Karim | It's putting them in an equation. And reading through it and then from there putting it into an equation. It just loses me. |
| 35. | BT | So in trying to find the mathematical relationships, you find that difficult? |
| 36. | Karim | Ja. |
| 37. | BT | Imran, you said word problems are mind-boggling. What do you mean by mind-boggling. In what way are the word problems causing this in your mind? |
| 38. | Imran | Actually the way it affects your mind is like, for example if you are standing in a house and there is a fire and the fire is coming at you from all sides, you don't know how to escape because if you are going to run that side, there is still fire, if you go this side, you are still going to get burnt. So if you put it like in that case in terms of your mind. Now you get a word problem and you didn't wait to start thinking, and that's like you've got a certain number of rules that have been drummed into your head for a certain amount of years, and that you know that really, really, really works and gets you to the correct answer, then that's fine. Like in my case now, like when I got those word problems then, I looked at it, and I pick certain terms like I use the $x$ and $y$ method. Put the $x$ as the cheaper ones and the $y$ as the expensive ones, but still I felt there was something missing. Something missing that I couldn't, couldn't just get to the answer. That's why it was like, played on my mind, and that's why, that's what happened in the exams, and that's what causes me to lose time and half of the paper doesn't get done. I waste too much time. |

Warren's response in line 4 indicates that he finds that **word problems are artificial and contrived**, which he gives as his reason for not enjoying them. More specifically he says, "In maths you get…", and this tells us that he sees word problems as being confined to 'maths', which he more than likely sees as being a 'school subject'. (I unfortunately did not delve a bit more into this.) As he says, "Who receives questions the way they set them out?" and, "I don't find them extremely practical." Warren indicates here that **word problems are confined to being 'something that your mathematics teacher gives you**' and that they don't really serve any other purpose. This is interactional in the sense that it casts word problems in the light of not being of any use. Implicit in this, and in his comment in line 4, **word problems don't serve any practical purpose** and he therefore feels a lack of motivation to do the problems, and this in turn impacts on how he tackles the problem. Clearly, from the discourse we can see that Warren also holds the cultural model that **word problems are not enjoyable** and this exacerbates his reluctance to do them.

As we can see from his description of the 'rules' in line 12, he does not believe that he has a solution strategy for word problems and this ties in with his belief that **you can use 'any method' for solving word problems** (cf. stanza 6, line 2). However, Warren suggests that **word problems are solved using specific methods** when he says, "…I've never really understood why you had to do it this way, and why you had to do it that way" (line 4). These are conflicting cultural models and the former ties in with Warren writing "lots of different answers" (stanza 1, line 12). He feels that he cannot think laterally (stanza 2, line 15), and in line 12 he expresses the desire for an algorithmic technique, and this ties in with the latter cultural model discussed above. Interestingly, Warren is aware of the need to derive an equation when he says, "Look for something that is the same between the two" (line 12). In other words, he knows that he needs to find two expressions that he can equate, and in line 15

he describes the process of setting up expressions by using "*x* as a denominator" and working "out whatever that *x* will then become." Although Warren does not like word problems, they make him feel *uncomfortable* and he experiences difficulties with them, he shows a good understanding of the mathematical procedures involved in their solution. According to the findings of Cortes (1998) Warren's *operational variants* of *pragmatic concept of function* and the *concept of equivalence* seem to be established. Even though he does not demonstrate *respect for homogeneity of terms*, according to the study by Cortes (1998), he should be in a position to construct the equations by the implicit processes of *substituting unknowns with linear functions* and by *equating linear functions*. However, this does not seem to be the case.

It appears to me that Warren's lack of motivation impinges on his ability to solve the problem because it inhibits perseverance. We can see this from his working (Phase 1, step 1 – Warren), what I noted down about him returning to Problem 2 (Field notes; 1) and what he says in stanza 2 (line 11), "Eventually, I just gave up on that question. I had to come back, and when I looked back at it the second time I just got it."

Gary had suggested earlier on in phase 1, step 2 that success in solving word problems gives a measure of enjoyment so I asked a 'baited' question in line 23, to which he responds by saying that he does not enjoy "the working out" involved in word problems. His experience of word problems incorporates feeling "good when you solve them" (Phase 1, step 2 – Gary) and, "When you actually get the answer it looks so much easier, and you can enjoy them more" (line 24). In other words, Gary sees that **working out the word problems is not enjoyable** but that **getting the answer to a word problem is satisfying**, both of which are evaluative models Therefore, when he tackles the problem it is likely to be with some anxiety due to his anticipation of the 'un-enjoyable' solution process. The question is, does he

successfully solve a sufficient number of problems to enable him to experience the 'enjoyable' aspect of solving a word problem, which, as he indicates in line 28, would motivate him further? From what he says in line 30 I do not believe that this is the case, and thus his approach to tackling word problems is more likely to be in a negative vein. Furthermore, he says in line 24, "If it is a really challenging one and you work it out, you come up with an equation in your head…". The interesting thing here is that, for Gary, finding an equation is mathematical whilst using your head is non-mathematical. It therefore seems that Gary views the notions of challenge and enjoyment in both a mathematical and non-mathematical light.

Karim expresses a complete lack of confidence in line 32. He 'knows' that he will not be able to solve the problem even before he begins, and attributes this to his inability to find the equation that will solve the problem. More specifically, he comments about the wording in the problem, which is similar to what Warren had to say in stanza 5 (line 4). Karim says, "…reading through it and then from there putting it into an equation. It just loses me" (line 34). Karim holds the same interactional cultural model as Warren (stanza 1): **word problems cause me to get completely lost**, and linked to this is **word problems in exams wont get me any marks**. Also, during the problem solving in phase 1, step 1, Karim was the first to finish, well ahead of the other three, and he made no attempt to check or rework his solutions (cf. Field notes; 2). Karim approaches the problems with a 'knowledge' of the inevitable, viz. "0 out of 10" (line 32), and he has no motivation to check or redo the problems, if he even attempts them in the first place, as he suggests in line 32. There is a hopelessness that Karim is expressing that is causing him to give in, often before he even begins, and he attributes this to a difficulty in creating mathematical meaning from the words in the problem (cf. line 34).

Thus he ascribes to the same cultural model as Warren (stanza 5): **in word problems the words blur the mathematics**.

Imran's metaphor at the beginning of line 38 gives some insight into the desperation that he experiences when he is confronted with a word problem. The burning house scenario brings to mind a terrifying situation in which whatever he tries to do to escape he ends up getting burnt. Read this in the context of what Imran said in stanza 6, line 10: "You look at a sum about ten or twenty times, you try and think now, if I use *x* on this side, you put a number this side, and I get something. Or will I get the proper answer if I put *x* on this side and the number on this side. Will I get the proper answer?" What he is saying here is that **word problems purport to having many solution possibilities** but that only one is right, and because he is unable to discern which one is the right solution, **word problems are terrifying**. He is confronted with many options but he has no way of knowing which strategy will work, and they probably all appear to be the wrong one to him, much the same as him not knowing which way to turn in the burning house (line 38).

From the discourse Imran holds two related cultural models, the first evaluative and the second interactional: **word problems are frustrating** and **word problems cause you to waste time in exams**. At the end of line 38 he expresses frustration when he says, "Something missing that I couldn't, couldn't just get to the answer…". The repetition emphasises his exasperation and he relates how this affects him in the exams by causing him to lose time. In his last sentence in line 38 we see how intense he is about how much time he is *wasting* as a result of this perception of word problems.

Imran also said, "…you've got a certain number of rules that have been drummed into your head for a certain amount of years, and that you know that really, really, really works and gets you to the correct answer, and that's fine" (line 38). In other words, **word problems have rule-bound solution strategies**. These 'rules' however do not seem to be working for Imran. He talks about 'the *x* and *y* method' and about assigning unknowns, but he does not speak at all about establishing expressions or equations that describe the relationships in the problem, which is the key to its solution. Thus his *midlevel situated meaning* (Gee, 1992) with regard to the '*x* and *y* method' is too vague for him to meaningfully ascribe to the cultural model that **word problems have rule-bound solution strategies**. Put differently, he knows of the '*x* and *y* method' but not how to implement it.

**Stanza 9 – word problems are not always what they seem**

In this part of the discussion some aspects of misinterpretation of the problems arose along with certain answers that yielded unrealistic situations. Gary and Imran were remarkably quiet in this stanza.

| | | |
|---|---|---|
| 1. | BT | Karim, you said that you didn't think that a train could travel so fast, and we actually had a problem involving planes, actually, flying things. But anyway, I am not criticising you, so don't worry. I'm just saying the fact that you mistook this for a train in the actual problem, do you think that affected your ability to handle the problem? |
| 2. | Karim | Ja, because like he got an answer like some high number. If trains they are, I don't think trains can travel so fast. It is not a realistic answer. |
| 3. | BT | So you would immediately have a worry about that problem, if you came up with an answer like that. |
| 4. | Warren | Ja. It's got to be like worked out rationally in your head. |
| 5. | Karim | Ja, like a train will travel in between this amount and that amount, it can't be like over that. |
| 6. | BT | It was 260 was the number that he came up with. But anyway… In other words you are saying that the problem needs to be realistic. |
| 7. | Karim | Ja. |

| | | |
|---|---|---|
| 8. | BT | Then Warren, the same sort of thing happened with you, when you were dealing with the age problem, you got an answer of 4, and you stopped at that point, well that's what you said. Now tell us why you stopped there. |
| 9. | Warren | Simply because 4 years old, minus 10 years is minus 6. Now if you know someone who is minus 6, that's a bit dodgy. That's way dodgy. So I thought I've got it to here, it's obvious I don't have a clue of what I am doing. |
| 10. | BT | So again you are saying the answer needs to be realistic. |
| 11. | Karim | Exactly. |
| 12. | Warren | Ja. |
| 13. | BT | Because in maths you can get an answer of minus 6. |
| 14. | Karim | You can, but not with the age. |
| 15. | Warren | Ja. |
| 16. | BT | Not with an age, ok. So you feel that a word problem like that needs to be realistic. |
| 17. | Warren | Correct. |
| 18. | Gary | Yes. |
| 19. | Warren | It's like saying that John ran from here to Cape Town in 3 hours. |
| 20. | BT | Good example. And then Gary, we get to that point about you getting an answer of 260 kilometres per hour, and Warren you rejected that answer. Can you tell us a little bit about why you rejected the answer, do you remember the answer? |
| 21. | Warren | Because it was, I don't know if it was 2… What was it, exactly 260? |
| 22. | BT | It was exactly 260. |
| 23. | Warren | Because at 260k's an hour, if memory serves, at three and a quarter hours you will get exactly 195k's. |
| 24. | BT | Three quarters of an hour, yes. |
| 25. | Warren | That would mean that the guy has taken off, flown around in circles for 45 minutes and then collided with this other plane. He hasn't moved at all. So that again is also not, it's not the norm. That's got to be a realistic answer. Ja. If this guy's going this way and this guy's going this way, no way can his speed exceed the distance point in that amount of time. But if you had said that maybe one guy was travelling at 100 and the other one at, like 160, then they could meet, but then they would meet half way. Or just under half way. |
| 26. | BT | That's fine. So that brings me to that particular problem. The actual answer was 100 kilometres an hour and 160 kilometres an hour for the two planes. Now what do you think of those numbers? Just anybody who wants to answer. |

| | | |
|---|---|---|
| 27. | Warren | It's a bit slow, if it's a plane, seriously. I think at 100k's an hour you would pretty much stall a Boeing. So again you think, it's possible but not for some, a plane that size, and I think from here to Durban, they don't fly very many Boeing 747's. They fly the smaller planes because a Boeing is a long haul airplane. It is designed to travel thousands of kilometres. |
| 28. | BT | But this problem said the distance is 195. |
| 29. | Warren | Ja. |
| 30. | BT | Which is not very far. |
| 31. | Warren | Ja, you might as well drive. Why don't you take a helicopter? Why don't you do something else? Why fly? |
| 32. | BT | Ok, but they are flying. |
| 33. | Warren | How rich are you to fly 195k's? |
| 34. | BT | What do you think would be a realistic number then in terms of this particular problem if you are flying? |
| 35. | Karim | A train would like be in between 100 and 160, a plane… |
| 36. | Warren | Ja, a train would work. |
| 37. | BT | Imran is… Ja, but I'm talking about in a plane though. |
| 38. | Karim | A plane. |
| 39. | Warren | The distance would be anything over about 700k's. Because, if it is 500k's more people would rather drive there. |
| 40. | BT | Yes, but what about these little light planes that they fly? |
| 41. | Warren | Like a Cessna? |
| 42. | Karim | One of them is still faster than 260 also. |
| 43. | Warren | I don't think it's just…no I don't know. I think a Cessna gives about 270 on max throttle. But I'm full of useless information, hey? I'm telling you. |
| 44. | BT | Ok, let's leave that one, that's fine. |

In line 9 we again see that Warren explains that he found something in his solution to the problem that was amiss, viz. the age of 4 years was too young to suit the other relationships. However, he sees this as a fault of *his* when he says, "…it's obvious that I don't have a clue of what I'm doing." This reaffirms the cultural model that **in the presentation of word problems I miss something**. If we look at his solution in phase 1, step 1, we see that his

working is all correct and that he made an error in the relationship 'Peter is three times as old as Paul' when setting up his equation. This indicates that Warren does not have the confidence in his ability to solve word problems even though he seems to have a fairly good understanding of how to go about the solution.

In lines 10 through 19 we see that Warren and Karim are adamant that **the solution to a word problem must be realistic**. The fact that word problems have a context has caused these boys to ascribe to this cultural model. However, Warren is the only one of the four who shows any signs of checking solutions (cf. Phase 1, step 1 – Warren, and phase 1, step 3, lines 28 – 29). I find this interesting since there is a general belief that Problem 2 is not a real life problem but there is an insistence on the answer being realistic. I see the cultural model discussed here and the one in stanza 5, **for word problems to be meaningful they must be applicable in the everyday**, as being conflicting cultural models.

**Stanza 10 – word problems influence results**

There were some comments about word problems and passing and failing tests and exams and I felt that it would be necessary to investigate the extent to which these ideas contribute or inhibit performance when solving word problems.

| | | |
|---|---|---|
| 1. | BT | Warren and Gary, when you were having your chat here, you made some arbitrary comments about if you had got these things in the exam you would have failed, and whatever else. Why did you say that to start with? |
| 2. | Gary | Like Karim said, if you get to the word problems, just turn the page and do the rest. Leave that to the last minute because you don't know how long you have to think it through, you don't know how long it is going to take. Whereas, if you are doing normal other maths stuff you can do that quickly if you are good at it. If you, ja…word problems take so long. Just rather do that at the end. They should actually just leave word problems to the end of the exam. They mustn't put it in between. At the end. |
| 3. | BT | Ok, I'll bear that in mind. |
| 4. | Warren | When I say I most probably will fail, the reason I said that is because I did fail. |

| | | |
|---|---|---|
| 5. | BT | So it's a fact? |
| 6. | Warren | That's a fact. I failed my algebra exam, which had a word problem in it. I think I got like half a mark. |
| 7. | BT | What do you think word problems…do you think word problems have to do with passing and failing, though? |
| 8. | Karim | No. Like in this exam we only had like one or two. |
| 9. | Warren | Ja. So it's not really counted as much, but you do get… |
| 10. | Karim | But you do lose marks with it. And you know you lose marks. |
| 11. | BT | But do you see doing word problems… There is an element of passing and failing that comes into play there? |
| 12. | Warren | I think that, especially for some people, that you go, you look… I can answer that question, I can't answer that question, I can't answer that question. And then you get to like, you know that certain one that you know you can answer. Then you get to the word problems, and then it becomes more than likely or not that I can't answer that question. And if you are only reading through your exam, and you haven't done any other questions, or if the word problem is first, that pretty much destroys you for the rest of the exam. Because you are already starting the exam saying I know I can't do this. |
| 13. | BT | How do you think other chaps in your standard etcetera feel about word problems? |
| 14. | Gary | They also hate them. |
| 15. | Warren | Ja. |
| | | [All boys nod agreement.] |
| 16. | BT | Very general. |
| 17. | Gary | Except for two people like Danny de Costa, the guy who always gets 100 percent. Otherwise I think everyone hates them. |

In line 2 we again see Gary making a distinction between word problems and "normal other maths stuff". One of these distinguishing features is that word problems take more time and you cannot predict how long it will take. Gary attributes this to having "to think it through" (line 2), which tells us that he sees this as being peculiar to word problems and that he ascribes to the cultural model that **word problems need to be thought through**.

Warren in lines 4 through 12 addresses the issue of word problems contributing to him failing his algebra exam. What is noteworthy is that initially he seems to attribute this failure almost entirely to the one word problem in the exam (cf. line 6). Karim tempers this in line 8, and Warren acknowledges that word problems don't count for that much, but he is concerned that they *do* count, especially as he has no confidence in his ability to do them. Karim sums this up for both of them when he says, "And you know you lose marks" (line 10). Warren also says in line 10, "Because you are already starting the exam saying I know I can't do this" (line 12). What emerges here are the interactional cultural models that both Warren and Karim hold: **word problems cause you to lose marks in exams** and **knowing that word problems cause you to lose marks in exams causes anxiety**. This is an important factor at play that will affect how these boys approach word problems in an exam. Warren actually says that it "pretty much destroys you for the rest of the exam", which may be why he attributes his failure in the exam to the word problem

Gary seems to be categorical that this dislike of word problems is a general phenomenon (cf. lines 14 and 17), and the other three support this. He uses emotive wording in expressing an evaluative cultural model that **everyone hates word problems.** He suggests an underlying solidarity in this regard, a kind of 'us versus them', and it appears from the afterthought in line 17 that 'everyone' represents the students (or the 'us'), whilst the better able students are classified as 'them' along with educational figures.

**Stanza 11 – what word problems ask**

In this stanza I asked questions that were intended to establish whether the boys were comfortable with understanding the wording of the questions and what the problems were asking them to do.

| | | |
|---|---|---|
| 1. | BT | When you tackle a word problem, do you understand the problem? I'm asking generally, do you actually know what the problem is asking you to do, and do you understand the wording etcetera in it? |
| 2. | Karim | I understand that, but when it comes to put it in an equation I'm lost. |
| 3. | BT | I'm talking about the actual words and… |
| 4. | Warren | I understand what they want from us, but the question is how do you get it. |
| 5. | Gary | It's like an illusion, like Imran said. The answer is there, you just have to try and find it, try and see it. |
| 6. | BT | Right. Now, how did you feel about the teaching? Again I am just asking generally, if you want to make any comments. We did have one comment that the teaching helps you to understand it better. Are there any other things that you found with the teaching? Did you find it different to actually just doing them? |
| 7. | Gary | It is nice to see how other people understand it. How they react to the word problems. |
| 8. | Warren | Ja, and I've always found that when I spell something out, it just makes more sense. You just talk about it like you talk about problems or whatever. You just talk about it; it makes more sense to me. |
| 9. | BT | But from your point of view though? [Warren nods.] Ja. |
| 10. | Imran | I've seen that like when I explain things, it's really just at the last period, maths. I explained this one. I think it was when we were doing linear graphs, and you had to explain how to get the co-ordinates of what A and B and that. Now when I explained to the guy, he never understands, and he told me that he doesn't understand. So I like take a thing like that for granted, like I want the guy to understand and benefit from what I am telling him. Now, but what I noticed was that teaching this experience, was that when I write things down, that's what people understand. Not when I talk. When I talk they kind of miss a certain point and get lost on the way. |
| 11. | BT | That's interesting. |

From what Karim says in line 2, and what Warren says in line 4, it seems that understanding the word problem is not the issue. The difficulty in solving these problems lies more in the formulation of mathematical equations. Gary again indicates that he is 'looking' for the answer in order to solve the problem, and he believes that one can just "see it" (line 5). I suggest that this is a result of his reliance on arithmetic approaches where he 'sees' the answer through a process of intuition as suggested in the study by De Bock *et al*. (2002). This discourse indicates that he believes that one should be able to see the answer to a word

problem and, therefore instead of trying to manipulate the algebra, Gary is almost waiting for that 'moment of recognition' that will enable him to "see" the answer.

**Stanza 12 – general discussion about word problems**

In this last part of the discussion the boys were talking more generally about word problems but some noteworthy comments were made.

| | | |
|---|---|---|
| 1. | BT | Ok, just a couple more questions. What do you thing about word problems in the real life situation? Do you think we find them out there? I'm talking about outside of school. |
| 2 | Gary | Well, like the distance one. |
| 3. | Warren | Ja. Certain aspects you will find it. But it also depends on what you are planning to do. If you are an engineer, you might find like a guy wants a building to be so high and this one is so much, and you could have that. But you won't more than likely that often…you won't find a word problem that complex. It will be just like… |
| 4. | BT | Just explain what you mean by that complex. |
| 5. | Warren | It won't have that many steps in, it will be like the…how far are you away from work? And, that at this point you work it out. Ok, I'm 10 minutes away from work. I've got so many days until this person's birthday. You won't have to figure out where these two things are going to crash, or how old this oke will be in 25 years compared to you. But there is also short cuts for word problems, like if you both, you and let's say Joe Soap, have both ages at the same time. So, there are short cuts, you can say whatever his age is minus my age equals 4. So whatever age it's plus that number, plus 4, so there are things that you can do to counteract lengthy word problems. But it's more often than not that you will never use them. Well that's what I feel. |
| 6. | Imran | I did a very interesting thing in English. This thing we do for SRA. This particular SRA card was dealing with the fact that why is maths real in today's life, and when I finished reading that card, I'm telling you it sounded so realistic. I thought that of it properly and sat there like for 5 minutes and just thinking. Because, like the once it happened to me, we were travelling to Durban and when we just reached the highway here to leave, I asked my father how many…I know everybody says that from here to Durban is about 500 kilometres. Now that's just plus minus. I want to know the exact amount. My father, like he never like uses a mathematical equation to tell me that. He reset the clock, and that car actually said that in today's time the machines that you have, will do the mathematical work for you. So you don't actually need maths because for everything there is a speedometer, a this-meter, a that-meter, a thermometer and a calculator. |
| 7. | Gary | So then bring a calculator to school. |

8. Imran I tried that. They said no.

Warren in line 3 reiterates the belief that **word problems are for specialist people** (cf. also stanza 6, line 8). But what struck me as interesting is what he meant when he said, "…more than likely that often…you won't find a word problem that complex" (line 3). Thus Warren holds a cultural model that **word problems in school are more complex than what you find in everyday life**. These beliefs affect the way in which Warren approaches the problem in that he does not see any benefit in mastering the skills necessary for solving problems because he does not see himself in a specialised position and, for him, everyday problems are far simpler than what is suggested by the 'school' problems. He states in line 5, "…its more often than not that you'll never use them".

Imran tells us of why he feels he will never need mathematics generally, but this implies word problems as well. At the end of line 6 he explains how modern technology will enable him to do any calculations necessary without employing any mathematics. His discourse here reveals the interactional cultural model that **modern technology has negated the need to use maths in everyday life**. This belief will impact particularly on his motivation to master word problems as he will perceive them to be superfluous in terms of the world that he envisages ahead.

From both Warren's and Imran's perspectives in this stanza we are taking about pertinence of word problems. They purport to pose problems that help us in the everyday, but clearly for these two boys, they do not give that impression.

**Discussion of the phase 3 analysis**

Initially I had intended to use Gee's notion of espoused, evaluative and interactional cultural models as the primary classification for cultural models as discussed in my analytical

framework. In using this categorisation of cultural models Gee was giving substance to Strauss' work on cultural models. Gee (1999) says, "…we need to distinguish between cultural models based on how they are put to use and on the effects they have on us" (p. 68). His subsequent categories for distinguishing between cultural models, Gee suggests, are not limited to the three that he gives: he says, "We can distinguish between, *at least*, the following…" (p. 68, my emphasis) and he goes on to give the three discussed above.

In the context of this study further categories were needed, and it was in the analysis of phase 1, step 2 that the notions of empirical and putative cultural models became evident. This also tied in very nicely with conflicting and consistent cultural models (Setati, 2002; Gee, 1999).

I suggest that where one's empirical and putative cultural models are in conflict attaining levels of competence in working with word problems can be impeded. For example, Gary holds the cultural model that **word problems help to make maths easier**. He also holds the cultural model that **to be able to solve word problems you need proficiency with words**. For Gary, these are in conflict. Furthermore, Gary does not *act on* the former cultural model but he does on the latter one, since he presumable has experienced difficulty with the wording in the problems. The influence of the latter cultural model is that Gary's self-esteem with regard to his ability to cope with word problems is lowered since he sees himself as not having sufficient proficiency with words (cf. Gee, 1999: 66 – 67). This is exacerbated by the belief that word problems should be making maths easier but that this is not happening. Gary has *adopted* the putative cultural model (the former one) whilst he *acts upon* the empirical cultural model (the latter one). In this way one of the factors that underlie Gary's lack of success in solving word problems is made 'visible'.

When looking at Imran we see that he holds the same putative cultural model as Gary but his empirical cultural model is that **word problems are mathematical illusions**, and these models are in conflict. When Imran attempts a word problem he expects to encounter multiple solution possibilities because he is acting on the empirical cultural model even though he believes that word problems should be helping him mathematically.

Warren expresses a putative cultural model that **word problems serve to make a person think laterally** whilst he holds an empirical cultural model that **word problems cause you to get completely lost**. We can see that he is not acting on the putative model when he says that he cannot think laterally (phase 3, stanza 3, line 15) or in a manner that will help him to solve the problem (phase 3, stanza 6, line 28). He is acting on the empirical cultural model and, because this is in conflict with the putative model he is unable to achieve the lateral thinking that he believes word problems should be giving him.

The situation with Karim I see as being different from the other three. He holds two putative cultural models that are consistent: **word problems help with problems in life** and **for word problems to be meaningful they must be applicable in the everyday**. However, he also holds the models that **word problems are confusing if done by algebraic techniques** and **I probably wont be able to solve the problem if I have to use algebra**. Karim is acting upon these empirical cultural models which inhibit him with regard to solving the problems. However, he is also acting on the second putative cultural model in that he believes that problems 1 and 2 were very unrealistic and therefore meaningless to him in terms of helping him with everyday problems.

Thus I found the distinction between empirical and putative cultural models to be a useful tool since viewing empirical and cultural models that are in conflict enables us to identify which model is being acted upon and which one is being suppressed. Furthermore, I suggest that with time an empirical cultural model may become more consistent with a putative cultural model, and they may even eventually merge into one empirical model, which could alter the success or lack of success that one has in dealing with word problems. However, research undertaken by Christou and Philippou (1998b) found that students' belief structures about mathematics are established in early schooling and became increasingly difficult to alter as the students progressed through the schooling system. Cultural models are tied to this belief structure and could therefore also be subject to similar rigours. Therefore, it seems that further research is needed to establish whether empirical and putative cultural models can or do become more consistent over time.

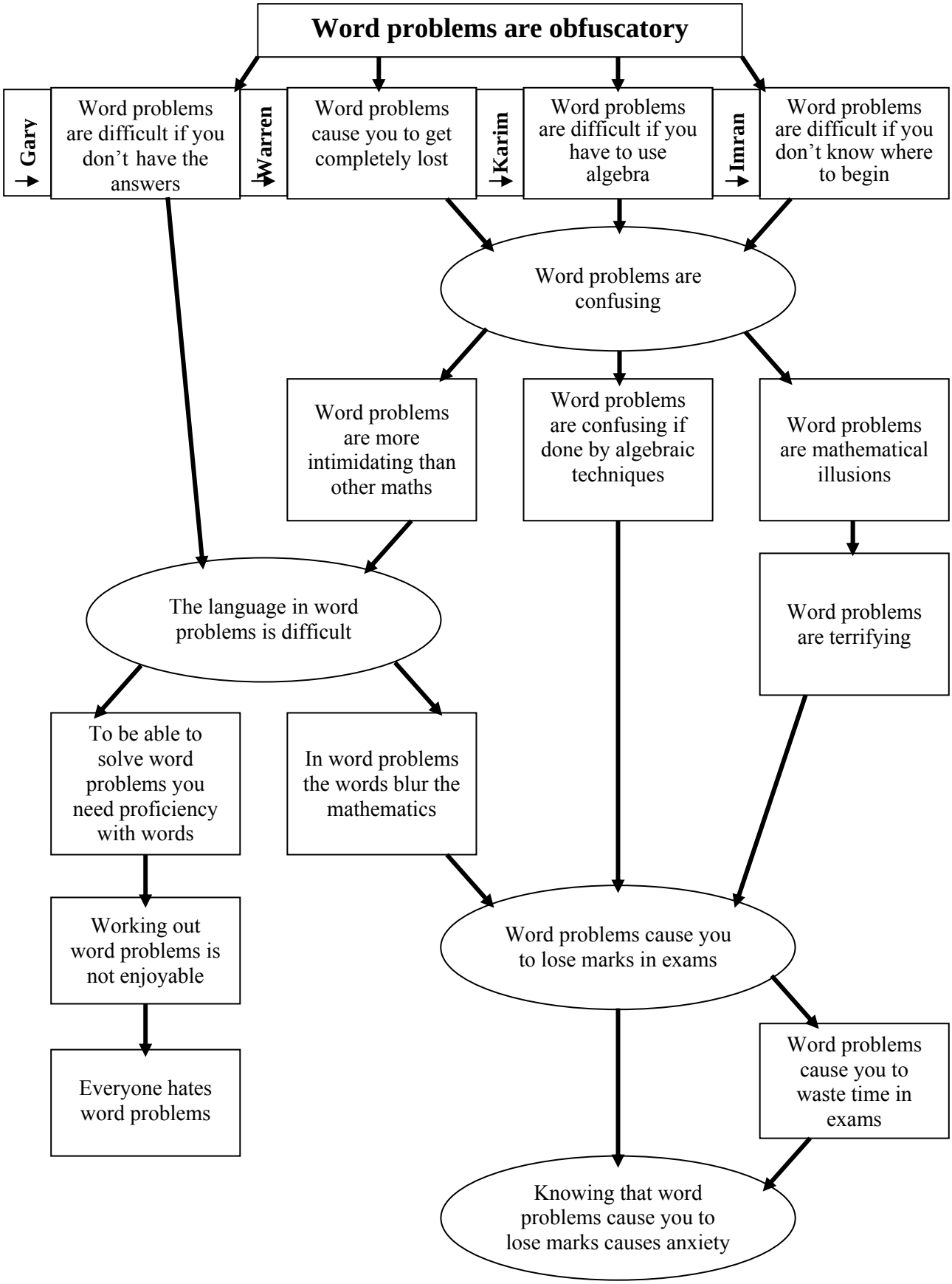

Word problems are obfuscatory
Gary
Word problems are difficult if you don't have the answers
Warren
Word problems cause you to get completely lost
Karim
Word problems are difficult if you have to use algebra
Imran
Word problems are difficult if you don't know where to begin
Word problems are confusing
Word problems are more intimidating than other maths
Word problems are confusing if done by algebraic techniques
Word problems are mathematical illusions
The language in word problems is difficult
Word problems are terrifying
To be able to solve word problems you need proficiency with words
In word problems the words blur the mathematics
Working out word problems is not enjoyable
Word problems cause you to lose marks in exams
Everyone hates word problems
Word problems cause you to waste time in exams
Knowing that word problems cause you to lose marks causes anxiety

**Figure 1.**

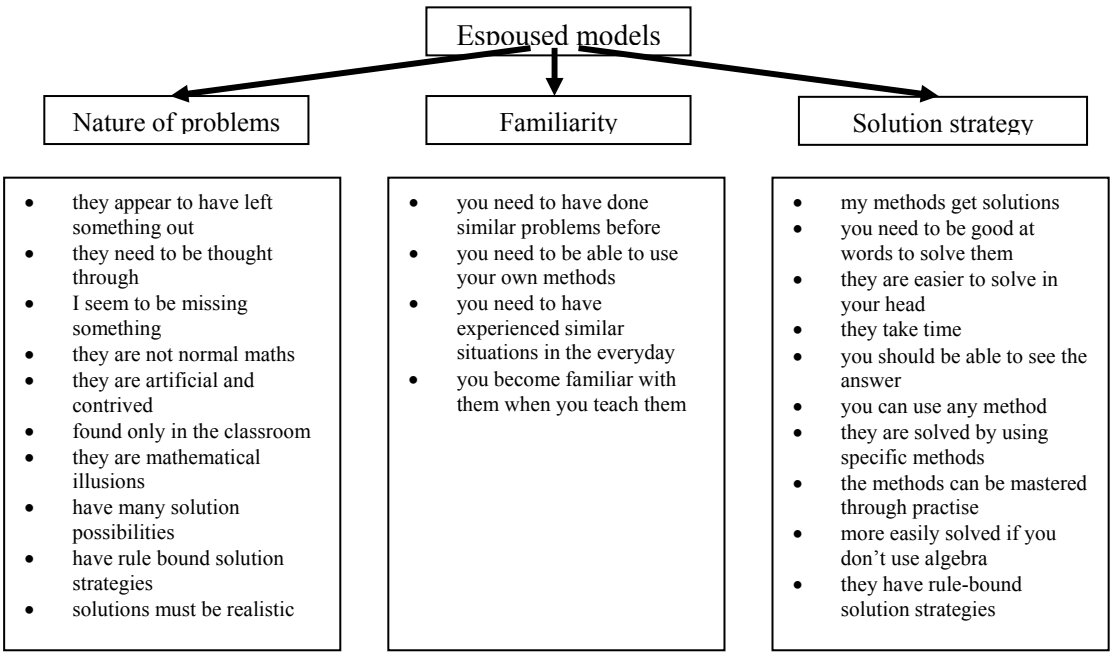
From the analysis a master model is very evident and that is **word problems are obfuscatory**. This is clearly an empirical cultural model for all the boys since the discourse that underpins it is experiential, and quite emotively so.

As we have seen each boy's experience of the perplexing nature of word problems is different. This has resulted in the development of further empirical cultural models, some of which are common to two or three of the boys. In this study it appears that empirical models have 'grown' from previous models, and it seems that this is a result of repeated experiences when looking at the discourse (for example, the use of words like *always* and *usually*). In figure 1 I have attempted to represent this idea diagrammatically. The empirical cultural models 'grow' downwards from the master model. The circled cultural models are shared by two or three boys. The arrows show the development of subsequent cultural models starting with the master model.

What appears to be happening with all four boys is a spiralling effect in which more negative cultural models 'grow' and these are expressed with heightened emotiveness. Imran is a very good example of this as is illustrated in figure 1. For him the difficulty of the word problems manifest themselves in an illusionary quality that frustrates and terrifies him, resulting in a loss of marks in exams and the accompanying anxiety that this brings. Following Warren's and Karim's 'growth' of cultural models we see very similar trends. For Gary the difficulty about word problems is that he feels one needs proficiency in language and this results in him not being able to enjoy the working out of word problems as well as the accompanying notion that they are disliked by everyone.

Another useful way of viewing the cultural models is in the categorisation suggested by Gee (1999) since it enables us to see a 'flow of events' and this helps to give insight as to why students tackle word problems in the way that they do. It appears that there is a flow from espoused models to evaluative models and then to interactive models.

**Figure 2.**




**Espoused models**

The nature of word problems – this is what the students generally think word problems are. From the cultural models we can see that they perceive them to be 'fake' renditions of real life, that are encountered only in the classroom, they are illusionary and they are not normal mathematics.

Familiarity with word problems – each of the boys expresses a need to be familiar with the problems to be able to cope with them. One of these notions was that word problems should be representative of real life, which is in conflict with the perceptions of the nature of word problems.

Solution strategies – here, Warren and Imran especially, each raised two conflicting cultural models: they each said that word problems have a *specific* solution strategy whilst on another occasion they said that they have *many* different solution strategies. Generally, the boys see word problems as being rule-bound but that they are easier when done by their 'own' methods.

The espoused cultural models generate a fairly bleak picture of what the students hold to be normal and typical about word problems. This is tied in very closely with the belief structure that was investigated by Christou and Philippou (1998b) and, if their claim that these become increasingly more difficult to change is true, the way forward for these students with respect to word problems does not look promising.

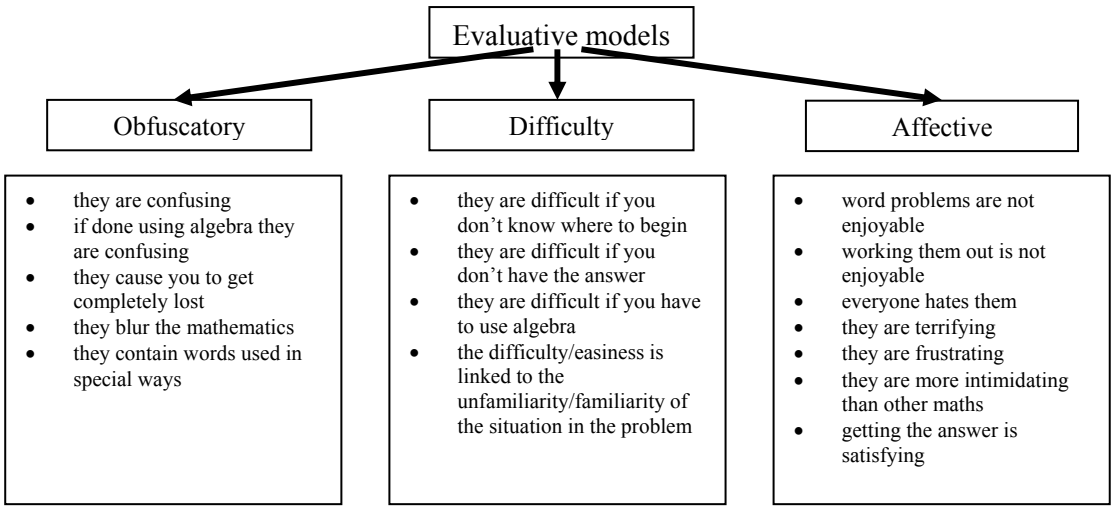



**Figure 3.**

**Evaluative cultural models**

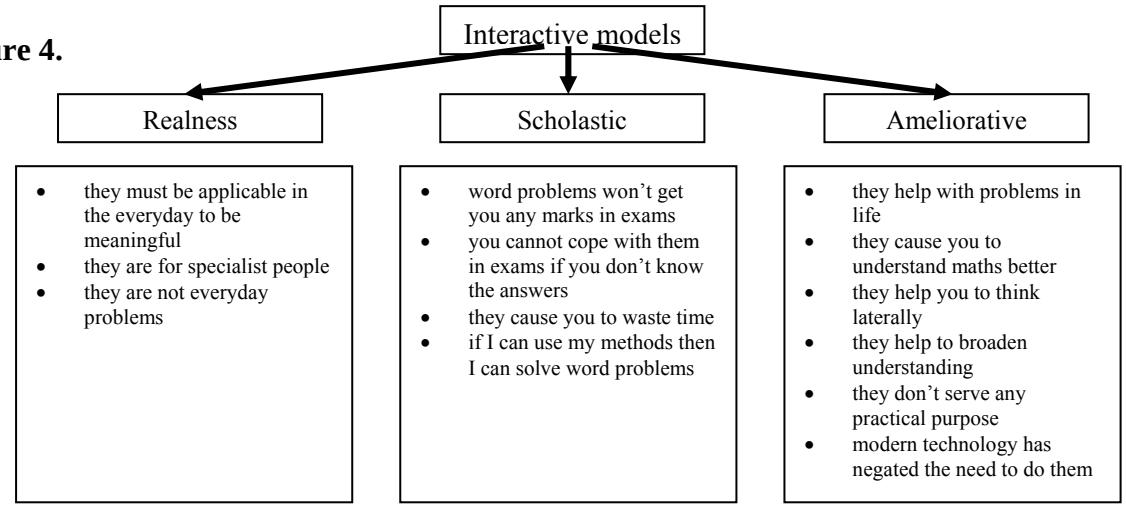
The obfuscatory effect of the models – generally, the students perceive word problems to be confusing, they blur the mathematics and they cause you to get lost.

The difficulty of word problems – each of the boys experienced difficulty with word problems differently but these are largely linked to the obscurity that they experience with the problems.

The affect of word problems – generally they cause anxiety, frustration, fear, intimidation and in Gary's case, they can be satisfying.

Based on their assumptions about what word problems are the boys are experiencing an overwhelming amount of negative affect. The obscurity of the problems brings about a difficulty in dealing with them and the consequences are almost always negative and very emotively expressed.

**Figure 4.**




**Interactive cultural models**

The realness of the problems – there is a need for the problems to be applicable in the every day, but the general perception of them is that they are not every day problems. They are for specialist people.

How word problems contribute to amelioration – they are supposed to help with problems in life, to think laterally and to do maths more successfully, but they are not actually doing this.

They don't serve any real purpose and with the modern devices that exist today and therefore there is no need to do word problems.

Scholastic aspects of word problems – word problems waste time in exams and they cause you to lose marks. There is a sense of failure and inability to cope with them in the test situation. When the answer becomes apparent it tends to make the students feel 'stupid'.

The students don't perceive word problems to be real and applicable and thus cannot identify how they could improve their position with respect to advancement in life. The notion that the problems are for 'specialist' people implies that 'specialist' people are beyond the capabilities of the 'ordinary' student. Consequently, word problems waste your time and jeopardise you in exams.

**In conclusion**

The cultural models that the students in general are acting upon are reflected in the progressive classification of the models (see figure 1). It helps to give a picture of why students have such a poor success rate when tackling word problems. The students firstly formulate opinions about what these problems are (the espoused models) and these form the basis of their belief structure (Christou and Philippou, 1998b) about word problems. They then make judgements about what they have conceptualised about these problems (the evaluative cultural models). Based upon their conceptualisation of the problems themselves and the judgements that they have made the students formulate opinions of how these will affect them in terms of advancement socially (the interactive models).

It appears that these boys have positioned themselves in a situation of relative helplessness mathematically. They 'should' be able to do these problems and the problems are 'supposed' to be helping them to do mathematics and solve everyday problems, yet they cannot solve them for the most part. These feelings of ineptitude broaden into feelings of inadequacy that tend not to be limited only to word problems. The associated feelings of anxiety impinge on their ability to do other mathematics as well as their overall mathematics results.

The picture looks bleak. However, it is not possible that a remedy does not exist and it is hoped that further study of the underlying cultural models that are at play will reveal ways in which we can help students to cope more successfully with word problems.

## Concluding remarks

For many years I have found myself stumped as to why students sometimes produce the most bizarre equations in response to word problems. I have always been of the opinion that the majority of word problems presented at Grade 9 and Grade 10 level are contrived to say the least, but this does not satisfactorily address my concerns about the illogical and meaningless equations that students often conjure up. Over the years I have tried different teaching approaches for secondary school word problems but there has been no noticeable improvement in the students' abilities to cope with these problems.

A while back I became interested in discourse analysis when I encountered the writings of James Gee (1999; 1992) and Mamokgeti Setati (2002). Simplistically put, Gee very elegantly used the notion of cultural models to explain why it is that people act in certain ways, whilst Setati undertook a study in which cultural models were used to understand teacher actions in multilingual classrooms. I began to see possibilities for using Gee's theories to help get to the root of my puzzlement over word problems.

Gee's theories were indeed very alluring. They neatly explain human behaviour in the social setting. However, I was soon to discover that, in the context of student responses to mathematical word problems, the theories were difficult to apply for two main reasons: firstly to elicit student discourse about word problems was a challenge that I had not anticipated and I had to undertake two pilot studies to address these issues; secondly, cultural models that arise from the discourse pertaining to word problems did not fit neatly into certain of Gee's categorisations and I needed to refine these so as to address the mathematical aspects of this research.

I have been reassured. With perseverance Gee's theories have been very informative of the underlying evolution that results in the ridiculous equations that students sometimes produce. It seems that there is a developmental path that is reflected in the discourse: espoused cultural models give rise to evaluative models, which in turn determine interactional models. Further to this it appears that the conflicting or consistent cultural models give a deeper understanding of how this development originates and is shaped. Thus, I see this study as being one from an affective stance, but one that complements the cognitive study undertaken by Christou and Philippou (1998a). Espoused models reflect perceptions of word problems that the students establish (presumably early on). They then evaluate these perceptions, and in so doing formulate a 'picture' that becomes a 'belief' about word problems. This 'belief' has an affective quality and I see this as being closely linked to the belief structure described by Christou and Philippou (1998b). The students then carry this 'belief' through to the formulation of the interactional models, which ultimately determines how they position themselves and how they act when doing word problems.

The central issue in this study is that the students' have a 'belief' that word problems are obfuscatory. This became evident in the espoused cultural models and it results in interactive cultural models that the students are acting upon. The consequences of this process are that the students now position themselves as being mathematically helpless when it comes to word problems, but that this also impinges upon other scholastic activity.

I made mention earlier of the belief structures that appear to be increasingly difficult to change as the students progress through the system (Christou and Philippou 1998b). I suggest that this may not be entirely true when it comes to cultural models and this may open up

avenues for addressing student inability to cope with word problems. There is one very important difference between the belief structures studied by Christou and Philippou (1998b) and the 'beliefs' that have been observed in this study. Belief structures are intrinsic to the individual, having been internalised over time, and this makes accessing and altering them a difficult task (as was observed by Christou and Philippou, 1998b). Cultural models are sociologically constructed and do not exist purely in our minds, but rather they form a part of our everyday functioning in society (Gee, 1999). A belief can be seen as a principle by which someone lives, and it can even be accepted as true without substantiation. A cultural model is a notion that is reduced to essentials and embraces what individuals construe to be normal and acceptable within society. According to Gee (1999) cultural models *usually* have a subconscious effect on behaviour and this is the crux of the difference between beliefs and cultural models. This makes them eminently more accessible in the sense that we have social interaction as a means for identifying them and, consequently, they can be easier to influence. Therefore, the picture that this study portrays is perhaps not as bleak as it may appear. However, eliciting cultural models requires very carefully planned activities and astute analysis, and it is here that we may find obstacles to the use of cultural models as a means of identifying why students struggle with word problems.

If we are able to identify conflicting cultural models we are in a better position to understand the student perception of what word problems *should* be doing for them as opposed to what they actually *are* doing for them. What I am advocating here is intervention from a sociological perspective. I do not believe that it is feasible to 'water down' the mathematics to make word problems more accessible for our students. On the contrary, this serves to remove the very mathematics that it intends to make more accessible. By intervening at the level of cultural models we can impact on experience, thereby affecting conceptualisations. In other words, we identify and target cultural models that students are acting upon (to their

detriment) and steer them into acting upon those cultural models that they hold (but are not acting upon), and which potentially would benefit them.

At this point this study has simply shown that cultural models can be used to provide insight into what it is that students undergo that cause them to have difficulty with word problems. This study has been viewed through a socio-cultural lens (where the emphasis has been on the affective) as opposed to a structured, cognitive or pedagogic lens. It has gleaned data from a discourse perspective, an advantage of which is that it enables a perspective of student experiences, and these come from within the students themselves and embody what they hold to be normal and typical. However, one of the disadvantages is that the data collection is necessarily complex, as it must elicit the type of discourse that will reveal the students' cultural models. What I believe has resulted from this study is that it has cast a different light on the problematic nature of student difficulties with word problems and it suggests that there may be different ways of dealing with the situation. This provides an opportunity for further research.

The cognitive, pedagogic and structural approaches to examining the difficulties that students experience when attempting word problems have been illuminating and go some way towards helping us to understand the these difficulties. However, the difficulties still persist. Without addressing socio-political issues pertaining to word problems it may well be that progress for many of our students remains a bleak prospect. This is an untenable situation.

This study has opened up new angles from which to view the problem. Further investigation from this perspective may serve to reveal ways in which we can ensure that in the future our students experience that much less obfuscation.

Finally, this study does not attempt to generalise these findings but rather to generalise a method for examining student experiences with word problems in order to understand where they are positioned and why they act in certain ways. It can be said that this method enables a different view of student difficulties with word problems to that which has gone before and therefore helps to create a fuller picture. It is only in knowing all the facets of a problem that we might address it properly.