CLASSIFICATION OF GRADE 11 LEARNERS' RESPONSES TO FUNCTIONS EXAMINATION ITEMS IN TERMS OF WEBB'S DEPTH OF KNOWLEDGE

By

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DECLARATION

I, the undersigned, declare that the thesis which I hereby submit for the Master's Degree to the University of the Witwatersrand contains my independent work. I have fully documented references to the works of others. I have not previously submitted this thesis for any degree at this or any other tertiary institution.

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ABSTRACT

This study was aimed at determining learners’ depth of knowledge in the topic of functions in Grade 11. The 2019 Grade 11 question paper examined by the Gauteng Department of Education (GDE) and learners’ written responses to examination items that tested conceptual or procedural knowledge or a combination of both on the topic of functions were analysed and classified in terms of Webb’s Depth of Knowledge (DOK) model. The study participants were 44 learners from two schools who had written the Grade 11 Mathematics Paper I in the final standard examination of 2019. The study followed a qualitative methodology involving aspects of document analysis. The study results show that most items in the Grade 11 examination paper are pitched at **DOK Level-2**, which addresses the basic skills or concepts. However, learners perform better on questions that require **DOK Level-1** and poorly on questions that require **DOK Level-3**. The findings of this study also reveal that procedural knowledge was the most tested knowledge in the question paper. From 19 questions that were analysed, 11 questions required procedural knowledge to respond to it, 2 questions required procedural and conceptual knowledge, and another 3 questions required conceptual or procedural knowledge, while only 3 questions required conceptual knowledge. It was observed that learners struggle with questions that require conceptual knowledge. The study recommends integrating technologies in the teaching and learning of functions because the use of technology in mathematics, more specifically in the concept of functions, has been recognised by many researchers to improve learners' conceptual and procedural knowledge. Improving conceptual knowledge and procedural knowledge is necessary for learners to perform better in all questions consisting of different DOK levels.

DEDICATION

This research report is dedicated to my wife, Lindiwe Buthelezi, who has been a constant source of support and encouragement during the challenging times of writing this research report. I am truly thankful for having you in my life.

I also dedicate my research report in memory of my mother Makhosazane Buthelezi and my mother in law Nokuthula Shabalala. May their souls rest in eternal peace. They will always be in our hearts.

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ATP –	Annual Teaching Plan
CAPS –	Curriculum and Assessment Policy Document
DBE –	Department of Basic Education
DOK –	Depth of Knowledge
FET –	Further Education and Training
GDE –	Gauteng Department of Education
ICT –	Information Communication Technologies
NCS –	National Curriculum Statement
SSIP –	Secondary School Intervention Programme
STAAR –	State of Texas Assessment of Academic Readiness
TIMSS –	Trends in International Mathematics and Science Study
WAEC –	West African Examination Council
WASSCE –	West African Senior School Certificate Examination

1 CHAPTER 1: CONCEPTUALISATION OF THE STUDY

1.1 INTRODUCTION

This chapter introduces the study that aimed to classify Grade 11 learners' responses to functions examination items that tested conceptual or procedural knowledge or a combination of both in terms of Webb's Depth of Knowledge (DOK) as a lens. Webb's DOK model is also used as a lens in classification of functions examination items in the Mathematics Paper I 2019 final examinations question paper. Section 1.1 presents the study's background. It incorporates a systematic review of the literature in the concept of functions and conceptual and procedural knowledge which leads to this study's topic. Section 1.2 describes the motivation relative to the researcher's experience in teaching Grade 11 mathematics in the current South African curriculum and attending the Gauteng Department of Education (GDE) workshops. Section 1.3 presents the problem statement that the researcher investigated. Section 1.4 outlines the aim of the study which expresses the intentions of the study and objectives which outline the specific steps taken to achieve the aims of the study. Section 1.5 outlines the research questions. Section 1.6 provides an idea of how this research might enhance the learning and teaching of mathematics and who might benefit from this study's results. Section 1.7 provides the conclusion as well as the review of critical aspects and claims discussed in Chapter 1. Finally, Section 1.8 outlines the summary of what other chapters entailed.

1.2 BACKGROUND TO THE STUDY

During the last few years, ever-increasing resources have been allocated to pursue innovation in mathematics teaching (Sfard, 1991). However, the findings are still far from satisfying (Sfard, 1991). South Africa is in the same situation as many countries globally (Engelbrecht, Bergsten & Kagesten, 2009). The teaching of mathematics, especially in high school, emphasises procedural skills at the expense of enhancing deep conceptual knowledge. Most of what has been taught in the characterisation of modern mathematics is taught and learned instrumentally (Skemp, 1976). Sfard (1991) claims that to enhance instruction in mathematics classrooms, teachers and mathematical researchers should address the most fundamental epistemological questions about the essence of knowledge in mathematics.

Over the past decades, the knowledge outcomes in mathematics teaching and learning have been described using multiple terminological frameworks (Star & Stylianides, 2013). Skemp

(1976) believed that there are two forms of understanding. The first is relational understanding, which includes a deep conceptual understanding of the content. The second is instrumental understanding, including knowledge of rules without justification. However, initially, Skemp (1976) claimed that he did not recognise instrumental understanding as understanding, until recently, when he found out that many teachers and their learners claimed that the ability to use a principle is what they meant by understanding.

However, ever since the mid-eighties, procedural and conceptual knowledge have dominated these frameworks. Even though the roots of this framework are not easy to pinpoint accurately, it became commonly known after the publication of a book edited by Hiebert (1986) (Star & Stylianides, 2013). Hiebert and Lefevre (1986) typically define conceptual knowledge as knowledge-rich in relationships. "We can think of it as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions, so it links all pieces of information to some network" (Hiebert & Lefevre, 1986, p. 3). They defined procedural knowledge in two forms of knowledge: “One form of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second form of procedural knowledge consists of procedures or rules for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbols” (Hiebert & Lefevre, 1986, p. 7-8). In referring to knowledge, procedural knowledge and conceptual knowledge are used in this paper.

Sfard (1991) claimed that such dichotomies between procedures and concepts contribute to the decomposition of mathematical knowledge instead of complementing each other. She offers a framework for the acquisition of a mathematical notion. Sfard (1991) proposed an operational and a structural conception of mathematical notion. An operational conception can be realised if a learner can operate with a notion such as process, action and algorithm. In contrast, structural conception can be realised if a learner recognises the notion as an abstract mathematical object (Sfard, 1991). Sfard believes that the structural conception lags on operational conception.

The concept of functions in modern mathematics has become one of the most significant and fundamental mathematical concepts as a mode to organise and brand mathematical relationships (Denbel, 2015). The importance of functions has led to its introduction in the school curriculum in many countries during the new mathematics reform in the era of the 1950s to 1960s (Denbel, 2015). In the Oxford Dictionary, functions are defined as a quantity of which the value depends on the varying values of others. In the statement $y = 2x$, y is a function of x. In the Curriculum and Assessment Policy Statement (CAPS), the Department of Basic Education (DBE) outlines that under the concept of functions, Grade 11 learners must work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations in the form of tables, graphs, words and formulae. In the South African curriculum at the end of secondary schooling, learners are expected to know the function concept in general and be familiar with specific types of functions, including linear, quadratic, exponential, cubic, hyperbolic and trigonometric (DBE, 2011). However, various studies disclose that learners have hitches in learning the concept of functions (Denbel, 2015). Hence, these challenges encountered by learners on the concept of functions motivated the researcher to conduct this study by looking at the conceptual and procedural knowledge requirements on the concept of functions in the test items. The concept of functions was chosen because functions form one of the most important unifying ideas in mathematics education (Mellor, Clark & Essien, 2018).

1.3 MOTIVATION FOR THE STUDY

During nine years of experience as a mathematics teacher, the researcher observed Grade 11 mathematics learners performing poorly in questions that examined the concept of functions. In assessments, most learners can confidently work with algebraic calculations which lead to the sketching of a function such as calculating the intercept (questions that require procedural knowledge). However, learners struggle with questions that require interpreting the function, such as reading information from the function (questions that require conceptual knowledge). Learners seem to have proper procedural knowledge but lack conceptual knowledge linked with the interpretation of a function.

Various research on learners' development and understanding of functions concepts have been explored. However, they vary in focus, level of learners' schooling and theoretical perspectives employed (Abdullah, 2010). During the research process, it was noted that there were limited articles on mathematics education that focused on the depth of knowledge, specifically, of functions. Some articles focused on integrating software, such as GeoGebra and SmartGraphs, to teach functions (Zucker, Kay & Staudt, 2014; Zulnaidi, Oktavika & Hidayat, 2020). Others focused on the assessment of conceptual knowledge (Boote, 2014; Orhun, 2013;). Furthermore, others concentrated on integrating the real-world context when teaching functions (Font, Bolite & Acevedo, 2010; Gower, 2015; Jones, 2019). Based on this, there is limited research available that addresses the depth of knowledge of functions using Webb's DOK model as the lens to guide the study. More research is needed to address learners' depth of knowledge of functions. In acknowledging this research gap, the researcher found it vital to classify learners' responses to examination questions that require conceptual or procedural knowledge or a combination of both forms of knowledge in the concept of functions using Webb's DOK model as a lens to classify their responses.

The questions meant to assess procedural knowledge of functions are usually questions that require learners to determine the values of a specified function for different values of the arguments, where *x*-value is substituted, and they must sketch a function (Lauritzen, 2012). Typically, these types of questions are answered by applying a step-by-step routine with no need for in-depth knowledge of functions. An example of procedural questions in functions is where learners are given an equation and required to sketch a linear function represented by the given equation, for example, $y = 2x + 3$. The learner can follow well-known steps when answering these questions. First, they find the *x*-intercept and *y*-intercept of the function. Then, they plot those points in the Cartesian plane and join them. In so doing, the learner is following a step-by-step routine in solving a given problem. Hence, these kinds of questions can be classified as questions that assess procedural knowledge (Lauritzen, 2012).

In contrast, questions meant to assess conceptual knowledge of functions are usually questions where learners must execute operations on a function, and it does not specify the matching algebraic expressions. Posing conceptual questions aims to test the learners' capacity to work on functions as an abstract concept with no routine steps for the provided functions (Lauritzen, 2012). These questions are intended to assess learners' capacity to reflect on the provided function rather than to execute algorithmic procedures (Lauritzen, 2012). For example, given the equation $y = x^2 + 2x - 3$, the function is drawn for learners and they are required to answer the question by reading the information from the function. The question could be to determine the *x*-values where the function is increasing. Hence, these kinds of questions can be classified as questions that assess conceptual knowledge (Lauritzen, 2012).

1.4 PROBLEM STATEMENT

Over two decades, the concept of functions has been universally regarded as a unifying theme in mathematics curricula. However, learners encounter many instructional hitches when developing an understanding of functions (Panaoura, Michael-Chrysanthou, Gagatsis, Elia, & Philippou, 2017). The importance of the concept of functions has also been witnessed in the South African mathematics curriculum for secondary schooling. Its weight is ± 30% (± 45 out of 150) for Mathematics Paper I based on the CAPS document (DBE, 2011). Functions play a crucial part in trigonometry and algebra, leading to the learning of calculus (Kilpatrick, Swafford & Findell, 2001). Therefore, learners need to acquire both conceptual and procedural knowledge of functions to lay a robust foundation for other topics such as trigonometry and calculus and become proficient in mathematics (Kilpatrick et al., 2001).

The current situation in the South African context does not create a conducive environment for learners to develop both conceptual and procedural knowledge of functions. For example, the researcher has attended several workshops meant to train teachers on mathematics content, including the Secondary School Intervention Programme (SSIP). The SSIP workshops are organised by the GDE. These workshops put more emphasis on procedural knowledge at the expense of conceptual knowledge. The workshops' main intention is to equip teachers to help all learners achieve more than 30% as a pass mark. These workshops focus primarily on providing teachers with skills to drill learners using the past question papers with a focus on procedures rather than developing deep understanding. These workshops should emphasise the depth of knowledge in mathematics classrooms (teaching and assessment) for a deep understanding of mathematics content rather than focusing more on procedures.

Teachers need to know how they will teach and assess for both conceptual and procedural knowledge. Those skills should be prioritised in these workshops. Consequently, if only procedural knowledge is emphasised, learners will experience mathematics as a subject that consists of procedures with limited practice on conceptual knowledge. Using the DOK model for both the question paper and the learners' written responses, this paper will explore a form of knowledge learners seem to possess when answering questions based on the concept of functions.

The workshops seem to disregard conceptual knowledge, although conceptual knowledge is believed to be more genuine, powerful and satisfactory for the learner and can help them to demonstrate proper mathematical perception (Kent & Foster, 2015). Such workshops must focus on assisting teachers to develop learners' conceptual and procedural knowledge of mathematics concepts. It is imperative to stress that the two forms of knowledge are not mutually exclusive. However, they are complementary. A learner is considered to have acquired mathematical concepts if both forms of knowledge are understood (Sfard, 1991).

Functions are an essential topic in mathematics, and their usage relies on both forms of knowledge. However, many studies have highlighted that learners encounter challenges when working with functions. For example, a number of national assessments such as the National Senior Certificate results for Grade 12 and the Annual National Assessment results for Grade 9 have indicated that the topic of functions is a repetitive area of weakness for learners in South Africa (Mellor et al., 2018).

Limited research addresses the depth of knowledge of functions using Webb's DOK model as the lens to guide the study. Few studies have explored the depth of knowledge that learners display when responding to questions that require conceptual knowledge or procedural knowledge or the combination of both. Hence, this study seeks to investigate learners' written responses to questions that require procedural knowledge, conceptual knowledge, and a combination of both forms of knowledge on the concept of functions in a provincial Grade 11, November 2019 Mathematics Paper I final examination for the National Curriculum Statement (NCS) in South Africa.

1.5 AIM AND OBJECTIVES

This study aims to present a classification of learners' written responses to functions examination items (Grade 11 2019 Mathematics Paper I) that tested conceptual knowledge or procedural knowledge or a combination of both in terms of the DOK model as a lens. The depth of knowledge that learners seem to possess when answering questions on the concept of functions is classified. To achieve the stipulated aims of the study, the research objectives are as follows:

- To identify the DOK levels evident when analysing the Mathematics Paper I examination question paper based on the questions that required conceptual knowledge, procedural knowledge or a combination of both when tested on functions.

- To identify learners' DOK levels based on their written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions.

1.6 RESEARCH QUESTIONS

The research questions for this study are based on the question paper analyses and learners' written interpretations. The research questions are:

- Which DOK levels are evident when analysing the Mathematics Paper I examination question paper based on the questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?
- Which DOK levels do learners seem to respond well to based on their written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?

1.7 SIGNIFICANCE OF THE STUDY

This study aimed to analyse the 2019 Grade 11 Mathematics Paper I final examination question paper using the DOK model. The result of this study identifies the most presented DOK level. These result should be significant to teachers during teaching and learning practice to consider the most assessed DOK level and allocate more time for that DOK level. The result should also help curriculum developers and textbook writers to allocate more time to the most assessed DOK level. Furthermore, the study aimed to analyse learners' written responses to questions where the concept of functions was examined using the DOK model. The results of this study indicate the DOK level which learners seem to struggle with and the DOK level at which learners seem to perform better. The results should be significant to teachers who are teaching Grade 11 learners to consider learners' challenges and acknowledge their strengths and use learners' strengths and weaknesses to prepare for lessons. Researchers could also use these results to research and understand why learners struggle with certain DOK levels and propose recommendations for improvements. This study's results could also help the GDE improve their content training workshops to incorporate both forms of knowledge. The results for this research could also be beneficial to novice researchers who might want to do extensive quantitative studies using examination scripts.

1.8 CONCLUSION

Chapter 1 outlined the background of the study by reviewing the literature leading to selecting the topic. The study's motivation relative to the researcher's mathematics teaching experience was discussed. The problem statement was highlighted and the aims and objectives of the study were outlined. From the literature background and the basis of the study, the research questions were formulated. The researcher highlighted the experience of attending SSIP workshops and those workshops' primary objectives. The study's significance also addressed how the study's findings could enhance learners' understanding of mathematics.

1.9 OVERVIEW OF THE CHAPTERS

Chapter 1: This chapter introduces the study by outlining the background of literature that led to the topic's formulation. It also presents the motivations, problem statement, research questions and significance of the research.

Chapter 2: This chapter outlines a literature review that presents a systematic overview of previous research that has been performed in the context of conceptual and procedural knowledge and the concept of functions. The chapter also discusses results from research papers and identifies gaps in the literature.

Chapter 3: This chapter provides the theoretical framework that guides this study. The notion of Webb's Depth of Knowledge (DOK) model is used as a lens to interpret forms of knowledge manifested in the Grade 11 Mathematics Paper I question paper and learners' written responses regarding the concept of functions.

Chapter 4: This chapter outlines the research design, including the procedures and methods used in the research study. It also discusses the sampling of data used for data collection and the instrument used for data analysis.

Chapter 5: This chapter presents the analysis of the Mathematics Paper I Grade 11 2019 final examination question paper using the DOK model.

Chapter 6: This chapter presents the analysis and interpretation of the Mathematics Paper I Grade 11 2019 final examination learners’ scripts using the DOK model. A summary of critical findings is also presented.

Chapter 7: This chapter provides answers to the research questions and proposes recommendations for future studies. It also presents the meanings and shortcomings of the research.

2 CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents a systematic overview of previous research which was conducted in the context of conceptual and procedural knowledge and the concept of functions and possible links to the research questions of this study in Section 1.6. The results from recent research papers are discussed and gaps in the literature are identified. The literature is discussed under the following headings:

- Linking procedural and conceptual knowledge.
- Learners' preferences of procedural and conceptual knowledge.
- Assessing procedural and conceptual knowledge of functions.
- Challenges in teaching and learning of functions.
- Teaching and learning of functions.
- Analysis of mathematics textbook activities and assessment in terms of knowledge levels.

2.2 LINKING PROCEDURAL AND CONCEPTUAL KNOWLEDGE

Hiebert and Carpenter (1992) emphasised that we should not ask which form of knowledge is essential. Instead, attention should be given to the fact that both forms of knowledge are necessary for mathematical competence. In the concept acquisition model by Sfard (1991), both forms of knowledge tend to be vital because mathematical concepts are formed entirely only if they can be realised both operationally and structurally. For learners to be competent in mathematics, they need to know concepts, symbols, procedures and have awareness of how these types of knowledge are related (Engelbrecht et al., 2009; Hiebert & Lefevre, 1986; Kilpatrick et al., 2001). Learners undoubtedly require the development of both conceptual and procedural knowledge. Rittle-Johnson, Schneider and Star (2015) acknowledge that learners must advance procedural and conceptual knowledge to improve mathematical proficiency.

However, the question arises on how these knowledge forms are related (Rittle-Johnson et al., 2015). Hiebert and Carpenter (1992) claimed that both the theory and the available data support the sequencing and emphasising of conceptual knowledge before procedural knowledge when teaching. In contrast, Engelbrecht et al. (2009) argued that most of the time, mathematicians primarily claim that it is essential to study procedures first to gain conceptual knowledge.

Rittle-Johnson et al. (2015) asserted that the relationship between conceptual and procedural knowledge is often bidirectional with procedural knowledge enhancements frequently promoting advances in conceptual knowledge and vice versa.

In the development of mathematical competence, procedural knowledge is essential as it allows mathematical tasks to be performed quickly and efficiently without mental effort. Often, well-practiced procedures capture the kind of mathematical power that exploits the consistency, accuracy and patterns of the mathematical notation and facilitates the effortless solution of routine problems (Hiebert & Carpenter, 1992). Conceptual knowledge is also essential for developing mathematical competence. It enables learners to easily remember (since it was not memorised) what they have learned and transfer it or apply it to new situations. Furthermore, conceptual knowledge leads to mathematical competence through its interaction with procedural knowledge. For example, the method for sketching the exponential function is a step-by-step sequence of actions, such as finding intercepts, asymptotes, etcetera. In reality, those actions are connected to mathematical principles, often represented as richly linked networks (Hiebert & Carpenter, 1992).

The benefit of procedures relating to conceptual knowledge is the increase in versatility (Hiebert & Carpenter, 1992). Once procedures are linked to connected networks of knowledge, they will gain access to all the network knowledge. Once confronted with problems that differ from those for which the procedure was initially taught, the associated conceptual knowledge might recognise functional similarities and differences between the problems. As a result, the conceptual knowledge will then inform the protocol for the correct changes. In this way, conceptual knowledge broadens the applicability procedure scope (Hiebert & Carpenter, 1992).

From the literature, it is evident that learners need both forms of knowledge to acquire proficiency in mathematics. Each form of knowledge has its benefits. Hence, conceptual and procedural knowledge should be complementary to each other instead of being mutually exclusive. Acquiring conceptual and procedural knowledge is essential for learners to perform well in mathematics assessments. For example, if learners manifest both forms of knowledge, it will allow them to respond to all questions that require different Depth of Knowledge (DOK) levels. However, limited research addresses how learners can use these forms of knowledge to respond to various questions that require different DOK levels. Thus, this study aims to analyse written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions.

2.3 LEARNERS' PREFERENCES OF PROCEDURAL AND CONCEPTUAL KNOWLEDGE

Jarvela and Haapasalo (2005) proposed three kinds of learner: a procedurally bounded learner, a procedurally oriented learner and a conceptually oriented learner. The conceptually oriented learner develops from conceptual knowledge to procedural knowledge, while the procedurally oriented learner develops from procedural knowledge to conceptual knowledge (Jarvela & Haapasalo, 2005). However, the procedurally bounded learners are only motivated on procedures and often show no conceptual learning progress (Jarvela & Haapasalo, 2005). For procedurally bounded learners, Skemp (1976) sees some mathematical mismatches that might occur. When learners whose preference is to understand procedurally are taught by a teacher who wants them to understand conceptually, such learners would not show interest. Those procedurally bounded learners will be looking forward to a rule that will help them get the answer. As soon as the answer has been attained, procedurally bounded learners will concentrate on the rule and disregard the rest. If the teacher asks a question that does not fit the rule, they will provide an incorrect solution (Skemp, 1976).

However, for learners to acquire competency in mathematics, both conceptual and procedural knowledge is a necessity. Conceptually oriented learners and procedurally oriented learners acquire both forms of knowledge by using different techniques. These learners have the capacity to confront questions that require different DOK levels. Procedurally bounded learners can only confront questions that require DOK Level-1 and DOK Level-2 (see Section 3.6 for more details on DOK levels) and will have a problem with questions that require DOK Level-3 due to the lack of conceptual knowledge.

2.4 ASSESSING PROCEDURAL AND CONCEPTUAL KNOWLEDGE OF FUNCTIONS

Much research has been done based on procedural and conceptual knowledge in mathematics. Most of the research has focused on procedural and conceptual knowledge of calculus (Chappell & Killpatrick, 2003; Engelbecht et al., 2009). For example, Engelbecht et al. (2009)

explored how third-semester engineering students at the University of Pretoria in South Africa and Linkoping University in Sweden solved problems that are intended to be overcome with either a conceptual or procedural approach in calculus. Their results revealed that students attempted to answer questions that required the application of conceptual knowledge by using procedural methods. This indicates that many types of problems can be solved procedurally, and the problem is only conceptual in the framework of learners' experience. Since the conceptual questions are not absolute, conceptual problems may become procedural if learners are continuously subjected to the same form of questions (Engelbrecht et al., 2009).

In their research, Mofolo-Mbokane, Engelbrecht and Harding (2013) aimed to examine areas of difficulty throughout learning about volumes of solids of revolution Their results indicated that many students cannot draw graphics and interpret the region confined by the specified graphs. However, students' manipulation skills were moderately developed even though they showed failure to use integration techniques and mathematical errors. Students appear to have difficulty when working with problems that are conceptual in nature, such as selecting the strip that approximates the bounded region, rotates it and develops the correct formula for volume. Their results concur with Ivanjek, Susac, Planinic, Andrasevic and Milin-Sipus (2016) whose findings indicated that students have difficulty with graph interpretations.

In their study, Ivanjek et al. (2016) aimed to examine first-year university students’ challenges in physics, mathematics and graph interpretation methods. Students were requested to give mathematical procedures or mathematical explanations in their responses. Their results revealed that students performed better on items that are regarded as low-difficulty problems, such as the calculation of the area under a graph using geometry formulas. However, the same students appeared to struggle with questions regarded as high-difficulty problems. These questions expected learners to solve problems by first interpreting the area under a graph as the point of concern. That part of interpretation becomes the central basis of difficulty in those problems. It is worth noting that findings by Mofolo-Mbokane et al. (2013) and Ivanjek et al. (2016) indicate that students can work with procedural items although they are not fully competent. However, they continue to struggle with items that require conceptual knowledge. Even the teachers appear to be competent in solving procedural items but struggle with conceptual items (de Kock, 2015).

In her study on probability, de Kock (2015) aimed to examine teachers' mathematics content knowledge under the topic of probability by evaluating the performance of the Grade 12 and Grade 11 learners on the topic. de Kock (2015) used the CAPS taxonomy as the framework derived from the TIMSS report on the international benchmark test by Mullis, Martin, Gonzalez, and Chrostowski (2004). Her findings were that mathematics content knowledge on probability is low, as teachers were proficient in solving items on the level of complex procedures. However, they struggled to respond to items where they could not recall the formulas and instead employed problem-solving knowledge. Her results suggest that teachers are competent in applying procedural knowledge but struggle to solve items that require conceptual knowledge.

Research was done on the assessment of conceptual knowledge and procedural knowledge, as seen in work by Engelbrecht et al., 2009. The findings revealed that learners seem to respond better on items that required procedural knowledge over items that required conceptual knowledge (Engelbrecht et al., 2009; Ivanjek et al., 2016; Mofolo-Mbokane et al., 2013). In some instances, learners were even using procedural knowledge to respond to items that require the use of conceptual knowledge (Engelbrecht et al., 2009). However, there is limited research on examining the conceptual and procedural knowledge under the concept of functions using the DOK model as the lens to guide the study.

2.5 CHALLENGES IN TEACHING AND LEARNING OF FUNCTIONS

In Grade 10 in the South African curriculum, learners are introduced to different functions, such as linear functions (initially introduced in Grade 9), the quadratic (parabola) function, the hyperbolic function and the exponential function. The Platinum textbook and the Handbook and Study Guide series for Mathematics are the popular options often used in the mathematics classroom. The latest version of the Handbook and Study Guide for Mathematics defines functions as "a set of ordered pairs (x; y) in which there are no two ordered pairs that have the same x-value" (Smith, 2020, p. 96). In the Platinum textbook, a function is perceived when the values of one set of numbers (y) depend on the values of another set of numbers (x) (Campbell & McPetrie, 2011). Functions can be presented in different ways, in words as the value of y is equal to three times the value of x or in a formula as $y = 3x$ (see Figure 2.1 in tabular form and Figure 2.2 in graphic form).

x	-3	-2	-1	0	1	2	3
$y = 3x$	$3(-3) = -9$	-6	-3	$3(0) = 0$	3	6	9

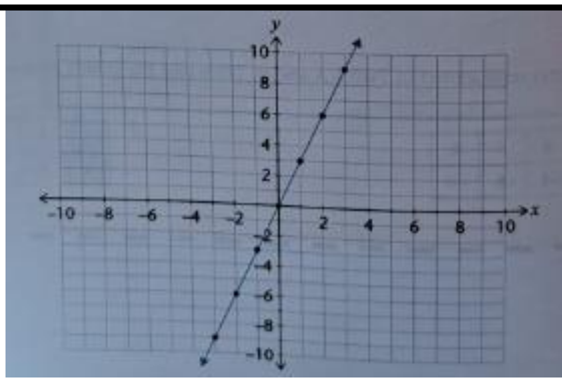


Figure 2.1: Tabular presentation of a function

Figure 2.2: Graphic presentation of a function

Wang (2015) defined functions as a relationship between two variables, such that changes in one variable result in changes in the other. "A function is a relationship between two variables in which the value of the independent variable uniquely determines the value of the dependent variable" (Insook, 1999, p. 50). Sfard (1991), drawing on Euler (1755), proposed another meaning of the function in which the term variable was not stated. They stated that a quantity may be called a function unless it relies on another quantity so that, if the latter is altered, the former is modified by itself. For example, if there is a set of numbers, then call the first set input values and the second set output values. The output value depends on the input value and the relationship or rule that exists between the two sets. Hence, if the input value is altered, the output value is modified. The concept of function is one of the basic mathematical concepts (Saraiva & Teixeira, 2009). It focuses on the ranges of its interpretations and representation .(Chazan & Yerushalmy, 2003).

In the South African secondary school curriculum, learners are expected to understand functions (DBE, 2011). However, learners encounter many challenges when attempting to understand functions (Patahuddin & Lowrie, 2019; Denbel, 2015). Mpofu and Mudaly (2020) stated that the Department of Basic Education (DBE) Grade 12 examination reports validate the notion that learners experience difficulty in learning mathematical functions. Mpofu and Mudaly (2020) acknowledged that there is evidence of increasing levels of learner underperformance in mathematics in the public school system in South Africa. Hence, they conducted a study to investigate the mathematical discourses of Grade 11 learners related to the word, algebraic and graphic representations of asymptotes of the hyperbola and exponential functions. Their findings indicated that learners could comfortably work with familiar questions, such as writing the equation of the asymptotes. However, learners showed limited understanding when they represented the same asymptote in the Cartesian plane. Their results also indicated that the mathematical discourse of learners is based on ritualised routines. Learners would correctly name a mathematical object but fail to explain the association between a function and its parent function. Learners could work efficiently on procedure tasks but struggled with action-oriented tasks (Mpofu & Mudaly, 2020).

Makonye (2014) highlighted the reason that might lead to learners' poor performance in functions. He claimed that the concept of functions is frequently taught without connections to the daily environment of learners. Patahuddin and Lowrie (2019) claimed that in sociological perspectives of functions, interpretation is regarded as a contextually-dependent socialisation process. If a function concept is introduced using a mathematical context that learners do not understand, learners can find it challenging to understand (Makonye, 2014). For example, if the formal mathematical language of functions such as $f(x)$ is introduced prematurely to learners, it can lead to some learners gaining misconceptions about the concept as the teaching method does not connect to learners' everyday context (Makonye, 2014). Dubinsky and Wilson (2013) asserted that various studies on learners' problems with functional notation exist in the literature. These difficulties show that many learners are unfamiliar with the mathematical notion of functions' conceptual dimensions. Learners sometimes do not understand the concept of variable and $f(x)$ notation. For example, "they may not understand the distinction between $f(a)$ and finding the values of x for which $f(x)$ = a" (Dubinsky & Wilson, 2013, p. 86).

Sajka (2003) and Lauritzen (2012) claimed that the concept of function is challenging for learners because of its duality in nature. This duality means that structurally, the function is a group of ordered pairs, and operationally, it is a well-defined method of moving from one system to another (Sfard, 1991). The equation $f(x) = x^2 + 3x - 10$ offers two things simultaneously how to determine the value of a function for a given argument and it embodies the whole definition of the function for any given argument. These two ways of understanding functions should complete each other and establish a coherent unity (Saraiva & Teixeira, 2009). Kalchman and Fuson (2001) suggested that learners struggle to grasp conceptual knowledge of functions. In some instances, learners' problems are induced by the reality that mathematics, specifically functions, must be understood concurrently both as a process and as a concept (Kalchman & Fuson, 2001), a procept in Gray and Tall (1993).

Procept is defined as a combined mental object consisting of a process, a concept produced by that process, and a symbol that may be used to denote either or both. A symbol is called precept if it evokes either a process or the product of that process (Gray & Tall, 1993, p. 2). Based on Gray and Tall (1993) functions can be regarded as procept, for example, given the f(x) it donates the process of where x is in the input and is processed to get the output. The symbol f(x) it also donates the concept of functions. Such symbol f(x) stands dually for both a process and a concept. In Sfard (1991) such a symbol stands for both operational conception and structural conception. Hence, the function needs to be understood as both operational or structural conception (Sfard, 1991). If functions donate both operational and structural conceptions, then are considered as procept in Gray and Tall's (1993) notions. However, it has become the common practice to give precise definitions for mathematical concepts which focus on the object at the expense of the inner process. This makes matters particularly difficult for the learner.

Saraiva and Teixeira (2009) conducted a study with Grade 11 learners on the topic of Functions II which aimed to identify the learners' difficulties concerning the concept of functions. Their results showed that learners were unsuccessful in linking the function definition with the graph representing the functions. For example, the relationship of dependence among the variables was highlighted by definitions given by learners. In the function, there is a dependent variable (y) and an independent variable (x). However, this relation of dependence seems to be confined to symbols x and y. Learners do not identify the dependent and independent variables if variables are not x and y (Saraiva & Teixeira, 2009).

From the preceding discussion, it is evident that learners are struggling with functions. However, functions are vital to fulfilling the Science, Technology, Engineering, and Mathematics (STEM) disciplines' job criteria. For students to start and enhance a career in the STEM disciplines, they are expected to perform better in college calculus (Adhikar & Graysay, 2019). Students are better prepared for tertiary calculus if they possess a solid conceptual knowledge of functions and algebraic skills aligned with functions (Adhikari & Graysay, 2019). However, there is limited research on how procedural and conceptual knowledge of functions link and the possible pedagogical consequences (Haapasalo & Kadijevich, 2000). This research report tries to close that gap by looking at how learners respond to the questions that require conceptual knowledge, procedural knowledge or both when learning functions.

The study also tries to understand the relation between these forms of knowledge in learners' written responses.

2.6 TEACHING AND LEARNING OF FUNCTIONS

The concept of functions is used in every single branch of mathematics, such as algebraic operations on numbers, transformations on points in the plane, intersection and union of pairs of sets, to name a few. Hence, the concept of function is regarded as a unifying concept in all mathematics concepts (Denbel, 2015). The concept of function has a vital role to play in the school mathematics curriculum. Mathematics textbook content has been identified to have a crucial influence on teaching and learning mathematics in the classroom (Mellor, Clark & Essien, 2018). Textbooks represent the proposed curriculum by interpreting it into a series of concepts. In this interpretation, textbooks shape the implemented curriculum and influence classroom teaching and learning practices by specifying the items to be covered during mathematics lessons (Hadar, 2017).

It is difficult for learners to understand the concept of functions (Zulnaidi & Zakaria, 2012), mainly to build a conceptual knowledge of this topic (Orhun, 2013). Most learners, however, do acquire procedural knowledge but struggle with conceptual knowledge (Eraslan, 2008; Orhun, 2013; Zucker, Kay & Staudt, 2014). Panaoura, Michael-Chrysanthou, Gagatsis, Elia and Philippou (2016), citing Kieran (1992), raised the question of whether a learner's failure to conceptually understand functions is related to the teaching technique or the learner’s ineptitude at approaching function tasks. Scholars such as Hadar (2017) and Mellor et al. (2018) indicated that the mathematics textbook used during teaching and learning mathematics can influence learners' understanding. Mellor et al. (2018) aimed to analyse and compare how linear functions are presented in the South African and German mathematics textbooks. The Grade 9 Classroom Mathematics textbook was used in South Africa, which is one of the most used textbooks in teaching and learning mathematics in the context of South Africa. Their results showed that 78 of the analysed content blocks indicated that the majority of tasks available in the textbook promote procedural knowledge (Mellor et al., 2018). The dominance of procedural knowledge items at the expense of conceptual knowledge could have strong implications in learners’ proficiency in mathematics. According to Kilpatrick et al. (2001), both conceptual knowledge and procedural fluency are vital for learners to attain mathematical proficiency. Thus, the two cannot be taught independently. Therefore, teachers need to pay extra attention to choosing a relevant textbook for teaching and learning mathematics since their selection of textbooks affect what learners learn, how they learn, the cognitive level at which they learn and the form of knowledge they employ.

While there are challenges with learners' performance in the concept of functions, teachers in classrooms still have to make decisions on how to overcome those challenges. Such decisions are not easy to take and are made more difficult when the goals for including functions in the curriculum are not clear (Denbel, 2015). Consequently, there are many challenges to be addressed by teachers. The first one is that textbook writers do not present the notion of function in ways that connect function to the rest of the curriculum, leaving this task to the teacher. The teacher must ensure that, eventually, learners develop a concept image that matches a comprehensive definition of a function. The second challenge for a teacher is that it takes time to allow learners to explore and develop a deep and rich understanding of functions. The use of technology can help, but the optimal allocation of classroom time provides a difficult challenge (Denbel, 2015).

Zucker et al. (2013) perceived two significant factors which could contribute to the difficulty of understanding functions teachers and textbooks which provide insufficient clarification of functions. Zulnaidi and Zakaria (2012) and Parrot and Leong (2018) argued that teachers should also consider effectively using technology in their teaching and learning practices to benefit learners. Through advancements in information and communication technology (ICT), technology's effect on mathematical practice is unavoidable (Parrot & Leong, 2018). Zulnaidi, Oktaviki and Hidayat (2019) argued that the optimistic procedure to encourage learners to understand mathematical concepts, such as functions, integrates ICT in the teaching and learning practices. However, the impact of technology integration varies and depends on education technology practiced in the classroom. Thus, teachers need to choose the latest technologies carefully to bolster their classroom instruction (Cheung & Slavin, 2013). These technologies might include GeoGebra software (Zulnaidi et al., 2019), graphing calculators (Parrot & Leong, 2018) and Smart Graphs software (Zucker, et al., 2013).

The use of these software packages, like GeoGebra, in the mathematics classroom, specifically in teaching functions, can enhance procedural and conceptual knowledge since learning functions using GeoGebra permits students to express their opinions in increasing their knowledge of functions (Zulnaidi et al., 2019). Zulnaidi and Zakaria (2012) conducted a study that aimed to examine the impact of GeoGebra on enhancing conceptual and procedural knowledge of function. Their findings showed that using GeoGebra in mathematics classrooms could develop conceptual knowledge in learners. Using graphical presentation could make it easier for learners to learn about the concept of function. Zulnaidi and Zakaria (2012) found that learners in the experiment group had higher procedural knowledge than the control group. This suggests that GeoGebra can also enhance procedural knowledge. Such technology helps to understand the relationship between conceptual and procedural knowledge. Their research findings are aligned with the result of the study conducted by Zulnaidi et al. (2019).

Parrot and Leong (2018) have claimed that problem-solving is a crucial component of understanding and learning in mathematics, specifically in the concept of functions. The growing focus has been on enhancing the ability to solve problems in mathematics. Thus, it is crucial to explore ways of developing learners' problem-solving skills. A graphing calculator is one of the technologies that can improve these skills (Parrot & Leong, 2018). Hence, Parrot and Leong (2018) conducted a study to investigate the effect of the graphing calculator on students' ability to solve problems involving linear equations. Their findings suggested that the experimental group with access to the graphing calculators during the lesson had a substantially higher test score than the control group. Using a graphing calculator positively affected students' ability to solve problems. These findings are consistent with the results of other scholars such as Zulnaidi et al. (2019) and Zulnaidi and Zakaria (2012), which confirm that the integration of technologies in mathematics practices enhances learners' performance.

Numerous studies have been conducted with the intention to increase learners' competency on the concept of functions (Naidoo & Govender, 2014; Ngwabe & Felix, 2020). However, there is still much work needed to enhance this topic. For example, much work has been done in linking the concept of function to technology (Zulnaidi & Zakaria, 2012), how it should be taught (Makonye, 2014) and learners' difficulties with the concept of functions (Saraiva & Teixeira, 2009). However, limited studies tried to find the depth of knowledge learners possess when responding to the questions that require conceptual or procedural knowledge or both forms of knowledge. Hence, this study is a necessity in trying to address this gap. Webb's DOK model is used as the framework to guide this study and help analyse the depth of knowledge learners possess in responding to questions that require conceptual knowledge, procedural knowledge or a combination of both forms of knowledge.

2.7 ANALYSIS OF MATHEMATICS TASKS OR TEXTBOOK ACTIVITIES IN TERMS OF KNOWLEDGE LEVELS

The assessment of students' achievement is a critical component of educational systems universally (Dogbey & Dogbey, 2018). In several nations, assessment in education is used to gather information about a learner's skills, attitudes or knowledge. Such assessments are frequently employed to decide on students' grade retention or promotion, provide feedback to students regarding their weaknesses and strengths and judge curricular suitability and instructional efficacy (Patterson et al., 2013; Webb, 1997). Different educational systems administer various types of assessments for several reasons (Dogbey & Dogbey, 2018). Students' success in those assessments rely on the textbook used as an effective teaching and learning resource. For example, the tasks that mathematics textbooks entail can stimulate different levels of mathematical thinking and learning (Mellor et al., 2018).

Hadar (2017) conducted a study to examine the connections between the cognitive learning opportunities offered by mathematics textbooks and students' cognitive achievements in national standardised assessments in the Israeli national curriculum. Hadar (2017) analysed two Grade 8 mathematics textbooks used by students in the Arab community in Israel. The findings of the study showed that students who used different textbooks, performed differently on standardised assessments. If a textbook engages students with problems that require a higher degree of comprehension, students who use this book performed better. These results show that for learners to perform better on questions that require conceptual knowledge, the mathematics textbooks should engage learners with conceptual activities. However, for the textbook to engage learners in both conceptual and procedural activities equally, the assessment policies also need to promote equal procedural and conceptual knowledge assessment. Therefore, there should be no contradiction between the standardised assessment and what is entailed by the mathematics textbook. Viera (2020) claimed that a solid alignment in the proposed mathematics curriculum implemented through textbooks and student assessments is beneficial to policymakers and educators in evaluating students’ performance and making suitable policy decisions about students’ learning.

In her thesis, Viera (2020) employed the Mathematical Task Framework developed by Stein and Smith (1998) with its four levels, namely, (1) doing mathematics, (2) procedures with connections, (3) procedures without connection, and (4) memorisation (in the order from the highest to the lowest level). These levels were used in the study to examine the cognitive demand of the mathematical tasks on functions, linear equations and inequalities in the State of Texas Assessment of Academic Readiness (STAAR) assessment and the mathematics textbooks for the Grade 7, 8 and Algebra 1 students' levels. This was done to determine the level of alignment between the mathematics curricular tasks and the related items on the STAAR Test for the selected grade levels in the chosen topics (Stein & Smith, 1998). The result of the thesis, specifically on functions, showed that the cognitive demand analysis of each selected grade-level textbook contained tasks at the four levels of cognitive demand. However, there were limited tasks set at the doing mathematics level of cognitive demand. In comparison, the STAAR assessments contained questions on the procedures without connections and procedures with connections levels of cognitive demand. The results showed that linear functions were aligned correctly for all three grade levels.

Dogbey and Dogbey (2018) conducted a study to examine the Core Mathematics Assessment of the West African Senior School Certificate Examination (WASSCE) by the West African Examination Council (WAEC) in Ghana over a period of 20 years in terms of the depth of knowledge of the assessment questions and the contexts in which the questions are rooted. According to the DOK findings, roughly 80% of the questions on the Core Mathematics Assessment measured students' ability to either remember basic facts or execute routine procedures, the two lower levels of DOK. However, Dogbey and Dogbey (2018) argued that the dominance of recall and routine procedures in the WASSCE Core Mathematics Assessment is concerning. Because these assessments might give learners the impression that doing mathematics entails recalling factual facts and performing well-known routine procedures, it diverges from learning mathematics as engaging in critical thinking and solving non-routine and routine problems. Their findings imply that mathematics learning in schools in Ghana prioritises low-level thinking skills over desired higher-order thinking skills.

There is tangible evidence that students' experiences of classroom learning contexts and their learning outcomes are interrelated (Fraser, 2014). Thus, the investigation of students' interactions in mathematics classrooms remains critical for mathematics education research to better understand how effective teaching and learning practices contribute to desirable learning outcomes (Hatisaru, 2020). Teaching and learning practices that promote mathematical proficiency can take several forms, such as implementing the relevant task. Thus, Hatisaru (2020) conducted a study to explore Turkish Grade 6 to 8 learners' insight into their mathematics classroom experience. In his study, the nature of the mathematical task and the kinds of mathematical representations illustrated in students' drawings were analysed. The findings of his study showed that student responses were dominated by procedural tasks, whereas contextual, representational and open-ended tasks were limited. The majority of the students' responses imitated their perception of mathematics classroom practice that emphasised applying facts and procedures to solve routine problems and problem repetition. Since the level of student thinking needed by those tasks applies learned procedures to a series of similar problems, these tasks are regarded as low-level tasks involving procedures with no connections. Students are most likely expected to use memorised or learned procedures in their classes, leaving no space for uncertainty about what they need to do to solve the particular task.

From the literature, it is worth noting the importance of assessment in mathematics teaching and learning. Assessments are employed to provide feedback to students regarding their weaknesses and strengths. The topic of functions has been indicated as one of the challenging topics for learners from assessments. The high failure rate on functions shows that learners are struggling to attain proficiency in the function topic. Mathematical proficiency relies on acquiring both procedural and conceptual knowledge. The textbooks used during mathematics lessons should engage learners in conceptual and procedural knowledge to acquire both forms of knowledge and reflect it during the assessment. However, limited research examines the depth of knowledge in terms of conceptual and procedural knowledge learners should have during mathematics assessments. Thus, this study aimed to examine learners' response to questions that require conceptual knowledge, procedural knowledge or the combination of both using the DOK model as the lens to guide the study.

2.8 CONCLUSION

From the literature, it is apparent that there is a duality of knowledge regarding conceptual knowledge and procedural knowledge. This duality is the source of debate, as various scholars argue about which knowledge is superior amongst the two (Star & Stylianides, 2013). However, attention should be given to both forms of knowledge as both are essential for learners to be competent in mathematics (Hiebert & Carpenter, 1992). Learners have their preferences when it comes to acquiring these forms of knowledge. Some learners prefer to acquire conceptual knowledge before procedural knowledge, others prefer vice versa. Others are procedurally bounded learners and are only interested in procedural knowledge with no interest in conceptual knowledge (Jarvela & Haapasalo, 2005).

Conceptual and procedural knowledge have been examined in different levels of schooling and various topics. Various studies on conceptual and procedural knowledge revealed that learners performed better on procedural items than conceptual items. Even teachers in the study by De Kock (2015) appeared to be more competent in procedural items than conceptual items. For learners to acquire mathematical concepts, they should understand them structurally and operationally (Sfard, 1991). Thus, learners must acquire conceptual and procedural knowledge to be competent in mathematics, especially in topics like functions.

It is evident from the literature that learners appear to have challenges with functions (Lauritzen, 2012). The root of its challenges is its duality in nature (Sajka, 2003). Structurally, the function is a group of ordered pairs, and operationally, it is a well-defined method of moving from one system to another (Sfard, 1991). However, scholars like Zulnaidi et al. (2019) proposed integrating technology in teaching and learning mathematics, especially in concepts like functions. There is tangible evidence that using ICT, such as GeoGebra software, graphing calculators and SmartGraphs software, can progress conceptual and procedural knowledge simultaneously (Parrot & Leong, 2018; Zucker et al., 2013; Zulnaidi et al., 2019). Hence, teachers should employ those technologies in their lessons to enhance the development of both forms of knowledge.

The literature has highlighted the necessity of assessment for enhancing learners' performance. Teachers should use the textbook as a central resource for teaching and learning mathematics. However, scholars like Hadar (2017) proposed that for learners to reflect conceptual knowledge and procedural knowledge during assessments, they should use textbooks that engage them in both forms of knowledge during lessons.

A systematic overview of previous research in the context of conceptual and procedural knowledge of functions has assisted the researcher in identifying the gap in the literature. There is little work that examines the depth of knowledge in terms of conceptual and procedural knowledge in the literature, specifically on the topic of functions. Thus, this study aimed to use the DOK model to analyse the question paper and learners' written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both.

3 CHAPTER 3: THEORETICAL FRAMEWORK

3.1 INTRODUCTION

This chapter outlines the theoretical framework that guided the study. It introduces Webb's Depth of Knowledge (DOK) model as the lens to view the knowledge and the DOK levels presented in the questions and in learners' written responses in the 2019 Grade 11 Mathematics Paper I November examination regarding the concept of functions. The DOK model is relevant for this study because it relates more strictly to the depth of content understanding and range of learning activity, which shows the skills required to complete the task from initiation to end (Hess, Jones, Carlock & Walkup, 2009). Ahead of discussing the DOK model and relating it to the research questions of this study, it is necessary to discuss the duality of knowledge by drawing on the perspectives of different scholars, including Skemp (1976), Sfard (1991) and Hiebert and Lefevre (1986) under the following headings:

- The duality of mathematical understanding.
- Instrumental and relational understanding (Skemp, 1976).
- Operational and structural conception (Sfard, 1991).
- Conceptual and procedural knowledge (Hiebert & Lefevre, 1986).
- Discussing the depth of knowledge in terms of conceptual and procedural knowledge.

3.2 THE DUALITY OF MATHEMATICAL UNDERSTANDING

Most research and policy implementation in mathematics education has been aimed at encouraging understanding learning. There are different interpretations of the concept of understanding in mathematics (Hiebert & Carpenter, 1992). The term understanding in mathematics has an alternative meaning assigned to it, which may be the origin of many mathematics education obstacles (Skemp, 1976). Thus, there is a duality of mathematical knowledge, 'knowing how' and 'knowing why' (Lauritzen, 2012). This duality can be seen in verbal statements and through multiple symbolic presentations (Sfard, 1991).

In mathematics education, there are diverse opinions about the relation between this duality of knowledge regarding 'relational and instrumental understanding' (Skemp, 1976), 'operational and structural conceptions' (Sfard, 1991) and 'procedural and conceptual knowledge' (Hiebert

& Carpenter, 1992; Hiebert & Lefevre, 1986). These diverse opinions raise questions about how learners should learn mathematics, which type of knowledge is essential, or what might be applicable stability between them (Hiebert, 1986; Rittle-Johnson, Schneider, & Star, 2015). There has been an ongoing debate about how the two types of knowledge are related (Star & Stylianides, 2013).

Rittle-Johnson, Schneider and Star (2015) argued that some scholars agreed that conceptual knowledge often supports and often leads to procedural knowledge. In contrast, other scholars agreed that learners slowly derive conceptual knowledge from using procedures through the abstraction process. Rittle-Johnson et al. (2015) provided evidence to show the relations between conceptual and procedural knowledge. They claimed that the relationship is often bidirectional, with enhancement in procedural knowledge often supporting improvements in conceptual knowledge and vice versa.

3.3 INSTRUMENTAL AND RELATIONAL UNDERSTANDING (SKEMP, 1976)

Most teachers teach mathematics instrumentally where they emphasise procedures, algorithms, and rules without justifications. They might be teaching this way because there could be certain advantages. Skemp (1976) highlighted three benefits of teaching instrumentally. Firstly, commonly, it is easier to understand mathematics when it is taught instrumentally. If the correct answer is expected in examinations or tests, understanding mathematics instrumentally can efficiently give the answer without justification. For example, a negative number (-3) times a positive number (+3) is equal to a negative number (-9). This can be seen as a rule and is easily remembered. A learner could apply this rule during the assessment and quickly arrive at the correct answer without justification. Secondly, the benefits of understanding mathematics instrumentally can be seen as immediate. This is useful for learners who want to restore their self-confidence and develop a positive attitude towards mathematics. This goal can be achieved faster and more efficiently in instrumental mathematics than in relational mathematics. Thirdly, less integration of knowledge is involved in instrumental thinking. So, learners can frequently get more reliable and correct answers faster without integrating knowledge from various topics.

Figure 3.1 is the parabola function taught in Grade 11 in the South African curriculum (DBE, 2011). In this function, learners might be given that line DE is parallel to the *y*-axis and provided with the coordinate of point D and point E. They could be asked to determine the length of DE. The learner with instrumental understanding will apply rules without justification, $(DE)^2 = (x_d - x_e)^2 + (y_d - y_e)^2$. This formula will lead to the correct answer, and it is not complex to understand. A learner needs to substitute the values correctly. Multiple rules can be included in the execution process using an instrumental understanding. In this case, the learner should be able to follow the rules for substitution and simplification. As multiple rules are involved, there is a chance for error. For example, a learner might miss a negative sign during substitution. Hence, possessing instrumental understanding alone is not enough. Another form of knowledge is necessary to avoid such mistakes, and that understanding is relational (Skemp, 1976). The learner with relational understanding will recognise that if DE is parallel to the *y*-axis, then the *x*-value at point D should be the same at point E. Hence, the length of DE should be the *y*-value at D minus the *y*-value at E. The exact answer can be reached without following multiple rules but with reasoning.

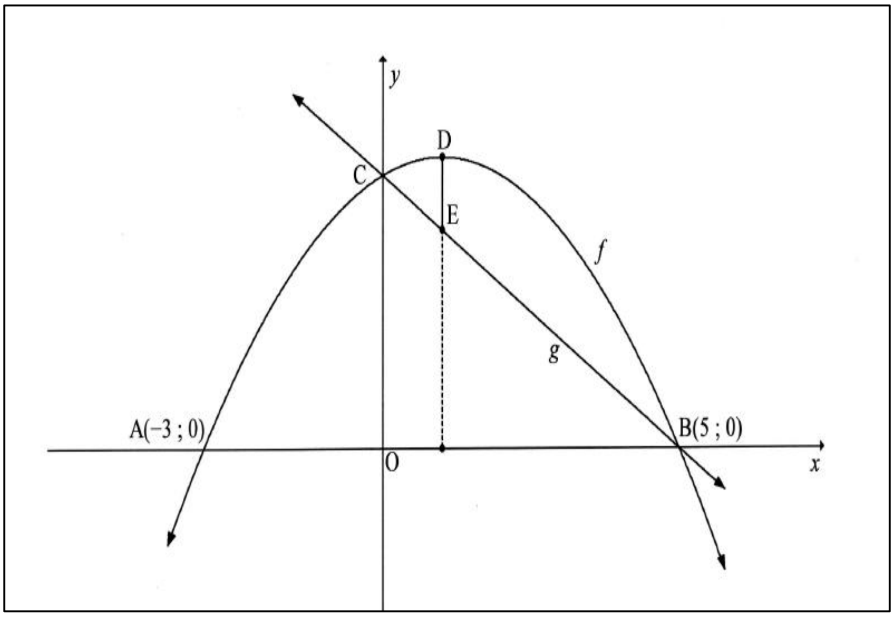


Figure 3.1: Example of the parabola and straight-line functions in Grade 11

Skemp (1976) provided four benefits of a learner's comprehension of relational understanding. Firstly, relational understanding is more adaptable to new tasks. A learner with relational knowledge understands which strategies are working and why they are working, which will allow a learner to adapt the strategy to other questions. With a relational understanding of a domain of functions, the learner can easily understand the range of the function. Secondly, it is easier to recall but more difficult to understand during the acquisition process. It is simple to understand that the formula for the x-value of the turning point is $x = -\frac{b}{2a}$. However, it is not easy to know why it works. Once relational understanding is acquired, it helps one to interrelate and recall quickly. Thirdly, relational understanding can be helpful as a goal, making teaching much more manageable. Lastly, relational schemes make the connection. Once the learner acquires relational understanding, that learner may be willing to pursue other problems and explore new knowledge.

3.4 OPERATIONAL AND STRUCTURAL CONCEPTION (SFARD, 1991)

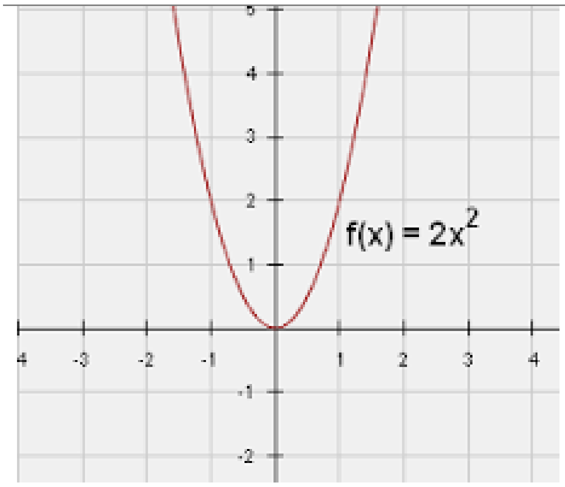
Sfard (1991) suggested two modes of thinking that may not appear so different from some of the dichotomies, such as instrumental and relational understanding proposed by Skemp (1976). Despite all the parallels, there are two essential criteria for the distinction. Firstly, criteria have a mixture of an ontological-psychological nature. Some scholars who proposed a dichotomy hardly paid much attention to the question of implicit philosophic expectations underlying any mathematical activity. Throughout the categorisation, Sfard (1991) sought to enlighten this problem by concentrating on the nature of mathematical structures as viewed by the learner. The second criterium is complementarity, where some of the dichotomies contribute to the decomposition of mathematical knowledge into two different components, such as procedures versus concepts.

The mathematical concept analysis supports the idea that abstract concepts such as functions can be developed in two vitally different forms: structural conception (an abstract object) and operational conception as a process (actions and algorithms instead of objects) (Sfard, 1991). It is also important to note that the mathematical concept's conceptual and operational interpretation are not mutually exclusive. Instead, they are complementary (Kalchman & Fuson, 2001). More frequently, the mathematical notion can be comprehended both structurally and operationally (Sfard, 1991). For example, structurally, a circle can be seen as a locus of all equidistance points from a given point. Operationally, a circle can be seen as a curve obtained by rotating a compass around a fixed point (Sfard, 1991).

Sfard (1991) asserted that the accurate definitions of operational and structural modes of thinking are almost tricky to formulate instantly. The distinction between these two modes of thinking lies in the fundamental, typically implicit, belief-like mathematical entity. Figure 3.2

illustrates two distinct ways in which the function $f(x) = 2x^2$ can be represented in the form of a graph and flow diagram. The flow diagram presentation tends to be consistent with the operational conception, since it presents the function as a computational process and sequential, not as a unified entity. In the graphic representation, all points or components of the function are merged into a smooth line. Hence, these components can be concurrently comprehended as an integrated whole. Therefore, presenting the function in the graph form could be consistent with the structural conception.

Graph



Flow Diagram

Input: x-value representing 1

Process/rule: $2x^2$ representing. 2×1^2

Output: $f(x)$ representing 2

Figure 3.2: Two different representations of $f(x) = 2x^2$

Although these modes of thinking are complementary, Sfard (1991) believes that structural thinking prevails in modern mathematics and has provided sufficient evidence to prove this. The structural conception should be seen as a more elevated level of conceptual development in the concept of function or mathematics in general. In the process of concept development, operational conceptions would precede the structural conception. The notion that structural thinking prevails in mathematics (Sfard, 1991) concurs with the South African mathematics curriculum structure, where flow charts, as operational conception, are taught in Grade 7-9 before learners are exposed to sketching the function. This should also act as evidence that the operational concept precedes the structural concept.

3.5 CONCEPTUAL AND PROCEDURAL KNOWLEDGE (HIEBERT & LEFEVRE, 1986)

Hiebert and Carpenter (1992) and Hiebert and Lefevre (1986) categorised learning using understanding. They also identified two forms of mathematical knowledge procedural and conceptual knowledge. Procedural and conceptual knowledge are hard to define accurately (Hiebert & Carpenter, 1992). In this regard, Hiebert and Carpenter (1992) developed two approaches which might lead to understanding conceptual and procedural knowledge. The first approach is to accept that 'which form of knowledge is most valuable' is the wrong question. Instead, one should ask how these forms of knowledge are related. Both forms of knowledge are essential. The second approach is to accurately describe procedural and conceptual knowledge to clarify the relationships within these forms of knowledge.

In the literature, there are many notions relating to conceptual and procedural knowledge. Conceptual knowledge can be viewed as an integrated and functional grasp of mathematical ideas where learners with conceptual knowledge know more than isolated proofs and methods. These learners can link knowledge into a rational whole, enabling them to learn new ideas. If learners could grasp conceptual knowledge, it could help them reduce critical errors in solving problems (Kilpatrick et al., 2001). For example, if learners are given an equation $y = -(x-2)^2 - 1$ and required to sketch the function, a learner with conceptual knowledge will not attempt to find the *x*-intercept, which may lead to errors. Preferably, a learner would realise that the value of a is negative and that the shape of the graph will be convex and the turning point of the function will be (2; -1). Therefore, the roots are unreal since the turning point is below the *x*-axis, and the shape of the function opens downwards (convex). The function will never cut the *x*-axis.

Conceptual knowledge can be developed by linking the relations between bits of information. This linking process can occur among two bits of knowledge that have previously been stored in memory or among the existing bits of knowledge and newly learned knowledge (Hiebert & Lefevre, 1986). When a learner realises that two previously isolated items are related, the process is characterised as an increase in conceptual knowledge. Conceptual knowledge could also be enhanced by forming a connection between the existing knowledge and new knowledge arriving at the structure (Hiebert & Lefevre, 1986). The process is like assimilation (Piaget, 1964), which is defined as integrating any reality into a structure. If a learner has conceptual knowledge of hyperbolic functions, the learner must relate the bit of information that builds up to the hyperbolic function's connected network structure. For example, in the function $f(x) = \frac{a}{x-p} + q$, a learner should know the influence of all these variables (a, p and q) on the function. A learner should be able to find both intercepts and asymptotes and present them in the Cartesian plane. If a learner manages to link this bit of knowledge about hyperbolic function, a learner can be considered to have a conceptual knowledge(Hiebert & Lefvre, 1986).

Procedural knowledge embraces two kinds of knowledge. The first one consists of the symbol representation system of mathematics. This includes familiarity with the symbols used to represent mathematical ideas and an awareness of the syntactic rules for writing symbols in the acceptable form. The second one comprises rules, algorithms and procedures for completing the mathematics tasks. There are step-by-step instructions that prescribe how to complete tasks (Hiebert & Lefevre, 1986). A key feature of procedures is that they are executed in a predetermined linear sequence (Kilpatrick et al., 2001). For example, if learners are given the equation $f(x) = \frac{3}{x-1} + 4$, they are asked to find the *y*-intercept. A learner with procedural knowledge should know the principle of finding the *y*-intercept and know the first step is to let $x = 0$, followed by substitution rules and simplification. Thus, Hiebert and Carpenter (1992) defined procedural knowledge as a series of actions. The minimum relations required to establish the procedure are the links between the follow-up actions in the procedure. When a learner understands the first step of replacing x with 0, the next step is a simplification, and there is no reason for a learner to think about why x is being replaced with 0.

It is evident from the literature (in Section 3.2 to Section 3.4) that there is duality in mathematics knowledge, such as instrumental versus relational knowledge (Skemp, 1976), operational versus structural knowledge (Sfard, 1991) and conceptual versus procedural knowledge (Hiebert & Lefevre, 1986). However, the term knowledge is intended to encompass all forms of knowledge broadly, which in this study is referred to as conceptual knowledge and procedural knowledge. These two forms of knowledge are classified in terms of the DOK levels from the 2019 final examination question paper and learners' written responses to such items in the functions section. The following section outlines the theoretical framework, which is the notion of Webb's DOK, that guides this study.

3.6 DISCUSSING THE DEPTH OF KNOWLEDGE IN TERMS OF CONCEPTUAL AND PROCEDURAL KNOWLEDGE

Historically, knowledge was first categorised as behavioural by psychologists who tried to capture and define abstract thinking through behaviour. This escalated to the creation of Bloom's Education Taxonomy Objectives as a way to express the required learner's action that is useful for curriculum and test development (Holmes, 2012). Bloom formed a taxonomy model that describes the level of complexity that learners would need to function at when responding to an assessment. This taxonomy model consists of six levels of knowledge, beginning at the simplest level (demanding almost no complexity) to the highest possible level (demanding critical thinking) and a web of integrated schemas (Holmes, 2012). Webb (1997) established a comparable model to Bloom’s Taxonomy, known as Webb’s Depth of Knowledge. However, his model uses four levels instead of six and presents the levels as nominative rather than hierarchical (Branscome & Robinson, 2017). Bloom’s Taxonomy is more about progression from level one to level six as learners become more familiar and competent with relevant content. In Webb’s DOK, however, students interact simultaneously with all four levels of learning (Wyse & Viger, 2011). The focus in the DOK Model is more on how deeply learners interact with content in assessed tasks in addition to the level of complexity of such tasks.

Webb (1997) established a procedure and measures for analytically analysing the order between standards and standardised assessments. Webb's explanation of the DOK is the amount of knowledge that the content curriculum standards and beliefs need. If one goes through objects for the depth of knowledge, Webb reviews the object to determine whether it is as demanding cognitively as what the actual content curriculum standard expects (Webb, 1997). The model is centred around the hypothesis that it may consider curricular components based upon the cognitive demands required to yield an acceptable response. Each grouping of tasks reflects a different level of depth of knowledge necessary to complete the task.

To address the research questions for this study, the DOK model was used as the lens to classify the 2019 Grade 11 Mathematics Paper I questions that examined the concept of functions and to classify learners’ responses to such questions. Bloom’s Taxonomy encounters drawbacks, particularly in this context, where the depth of knowledge presented in the question paper and displayed in the learner’s response is classified. When choosing test or assessment items and formulating questions, Bloom's Taxonomy uses verbs that often occur at several levels and do not clearly express the taxonomy's expected difficulty. For example, verbs like *explain* and *compare* may appear at multiple levels. If such verbs appear in different levels, then it might be difficult to identify the depth of knowledge necessary to react to each question (Hess et al., 2009). Furthermore, Bloom's Taxonomy is better used in measuring the instruction or objective, while DOK is better used in measuring the actual assessment itself (Francis, 2016). In other words, Bloom's Taxonomy provides the instructional framework, while DOK analyses the specifics of the assessment, such as tests or assignments. Hence, Webb's DOK model was used as the lens for this study as its focus is on measuring the actual assessment and this study aimed to classify the knowledge presented in the question paper under the concept of functions. The DOK model is also relevant to this study because it is strongly linked to the depth of content understanding and the nature of learning practice that expresses the skills required to accomplish the task (Hess et al., 2009).

Therefore, the DOK model is more applicable to the context of this study because it pays attention to the depth of content understanding in relation to tasks that require conceptual knowledge and those that require procedural knowledge. Webb's notion of DOK has prompted educational systems to reconsider the nature of the test alignment to include both the content assessed in the test element and the depth to which students are required to demonstrate their understanding of that content (Hess et al., 2009). The DOK model has been used by many scholars (Branscome & Robinson, 2017; Wyse & Viger, 2011; Qhibi, Dhlamini, & Chuene, 2020) as a framework. For example, Qhibi et al. (2020) employed the DOK model as their framework to investigate the strength of alignment between senior phase mathematics content standards and workbook activities on numeric and geometric patterns. They used DOK levels to compare cognitive levels of content standards and assessments. The content focus was verified if the cognitive levels between the workbook activities on numeric and geometric patterns and senior phase mathematics content standards had something in common or when it differed.

To validate and allocate DOK levels, Webb (1999) generated a four-level rubric that is shown in Table 3.1. These levels may seem to scaffold similarly to taxonomies like Bloom's. However, they establish how in-depth learners will direct and share their learning (Hess et al., 2009). Hess et al. (2009) labelled these levels not as stages but as the upper limit that establishes how far or in-depth students will study and share knowledge and thinking. The information in Table 3.1 offers a description of the four depth-of-knowledge levels.

Table 3.1: The Webb's Depth-of-Knowledge (DOK) levels

Level	Description
DOK-1	Recall and Reproduction - recall a fact, term, principle, or concept; perform a routine procedure.
DOK-2	Basic Application of Skills/Concepts – use information, conceptual knowledge; select appropriate procedure for a task; perform two or more steps with decision points along the way; solve routine problems; organise or display data; interpret or use simple graphs.
DOK-3	Strategic Thinking – Reason or develop a plan to approach a problem; employ some decision-making and justification; solve abstract, complex, or non-routine problems.
DOK-4	Extended Thinking – Perform investigations or apply concepts and skills to the real world that require time to research, problem-solve, and process multiple conditions of a problem or task; perform non-routine manipulations across disciplines, content areas, or multiple sources.

Source: Hess et al. (2009, p.4)

DOK levels are allocated to each question to work as general guidelines for analysis:

- The DOK level allocated should show the level of work learners are most commonly required to perform for the response to be deemed acceptable.
- The DOK level should show the difficulty of the cognitive procedures required by the task outlined by the aim rather than its difficulty. Eventually, the DOK level defines the thinking needed for a task, whether the task is complex or not.
- In this research, the DOK level was allocated based on the cognitive demands required by the central performance described in the aim.

3.7 CONCLUSION

In this chapter, it is worth noting that the duality of knowledge in mathematics exists as illustrated by the discussion of instrumental and relational understanding (Skemp, 1976), operational and structural conception (Sfard, 1991) and conceptual and procedural knowledge (Hiebert & Lefevre, 1986). This duality is the root of challenges faced by learners in mathematics (Skemp, 1976). The concept of functions is an excellent example to demonstrate this duality of existence (Sfard, 1991). In this chapter, the theoretical framework was developed based on Webb's DOK model. The four DOK levels were outlined, described.

4 CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

This chapter outlines the research design and methodology of the study. The study investigated learners' written responses to questions that required procedural knowledge, conceptual knowledge and a combination of both on the concept of functions in their Grade 11 Mathematics Paper I final examination. This study used the Webb's Depth of Knowledge (DOK) model to determine the depth of knowledge learners possess when answering questions on function concepts. Section 4.1 outlines the qualitative research approach as the research methodology adopted for the study. Section 4.2 discusses the selection of participants and the sampling procedure. Section 4.3 discusses the instrument, documentary analysis used for data collection and the connection between the research question, theoretical framework and methodology. Section 4.4 discusses the technique used during the analysis of data as well as the presentation of data. Section 4.5 discusses the measures taken by the researcher to ensure the validity of the study. Section 4.6 discusses the reliability of the conclusion made based on the data collected. The ethical considerations are discussed in Section 4.7. Section 4.8 discusses the limitations of the study for generalisation and a summary of this chapter is provided in Section 4.9.

4.2 RESEARCH DESIGN

This study utilised a qualitative research approach in the form of documentary analysis. Qualitative research is a method of social science research that collects and works with non-numerical data. Qualitative research aims to understand meaning from the data and helps recognise social life by studying the marked population (Maxwell, 2013). This type of research is popular with social scientists because it permits the researcher to investigate the meanings that people allocate to their behaviour, actions and interactions with others (Kuckartz, 2014). The research questions for this study (outlined in Section 1.6) could be best answered through a qualitative approach. The research questions sought to appreciate and understand phenomena in a context such as a real-world setting where the researcher does not manipulate the phenomenon of interest. Furthermore, the research questions sought to understand the meaning of learners' written responses when responding to questions where the knowledge of functions had been examined. Figure 4.1 displays the research design for this study.

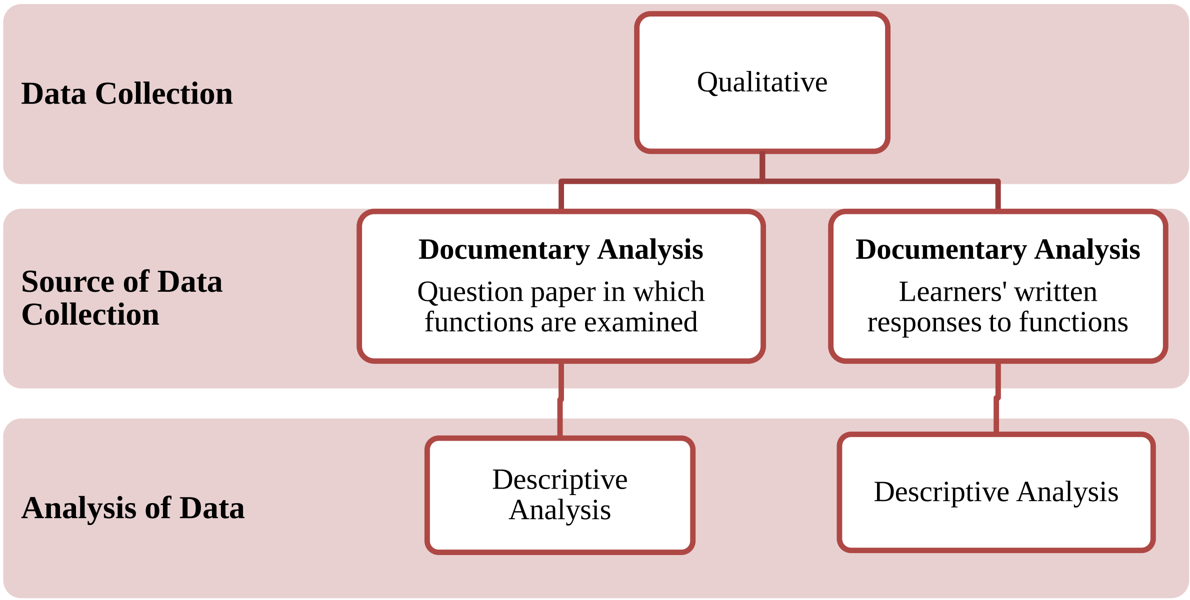


Figure 4.1: The summary of the research design and the research process

4.3 SELECTION OF PARTICIPANTS AND SAMPLING PROCEDURE

The sampled schools comprised of three government schools in the Gauteng Province of South Africa, which were located in the Ekurhuleni North district. The district was chosen because the researcher worked and resides in that district. Hence, it was reasonable, economical and practical to conduct the research in the Ekurhuleni district. The selected schools were chosen based on two criteria. The first criterium was based on the mathematics Grade 12 pass rate (percentage of learners who wrote the examination and scored 30% and above) in the National Curriculum Statement (NCS) examination for the three years (2016 to 2018) preceding the conducting of this study. The reason for choosing schools with different pass rates was to improve the reliability and trustworthiness of the study. The sampled schools belonged to three categories of scholastic pass rates. The first school, School A, had consistently attained a pass rate of 75% or more in the three years preceding the conducting of the study. The second school, School B, had achieved a pass rate of between 50% and 74% for the three years preceding the conducting of the study. The third school, School C, had achieved a pass rate below 50% for the three years preceding the conducting of the study. The second criterium was that schools had to be near the researcher’s workplace at the time of conducting the study. All three selected schools were within a radius of 10 kilometres from the researcher’s workplace, hence, sampling of the participants was purposeful.

The researcher sent these criteria to the Ekurhuleni district research office including the research ethics clearance from the University of the Witwatersrand, before attempting to access the school mathematics pass rate for the three years preceding the conducting of the study. However, the researcher was denied that information but the district officer gave the researcher a list of schools categorised into three groups. The first group consisted of a list of schools that have consistently attained a pass rate of 75% or more for mathematics, the second group was of schools that have consistently attained a pass rate of between 50% and 74% for mathematics and the third group consisted of schools that have been attaining a pass rate of below 50%. The researcher used the list to select the schools that would participate in the study. In all three groups, the researcher selected one school per group and the selected school in each group was the one closest to the researcher's workplace. Thus, the sampling of the study was purposive.

The researcher received research approval from the Gauteng Department of Education (GDE) in October 2019. The research approval letter from the GDE, the ethics clearance from the University of the Witwatersrand and the consent forms to the principal, learners, teachers and parents were used to request the three sampled schools to participate in the study. Initially, all the schools agreed to participate in the research, however, on the day the researcher was supposed to collect scripts (learners' answer sheets with written answers responding to the examination question paper), there were challenges. In School A, there were two classes, Grade 11A and Grade 11B, that wrote the 2019 Mathematics Paper I examination. The subject teacher misplaced the scripts for Grade 11B. Hence, the researcher only got 24 learner scripts that were available for School A. In School B, there were more than forty learners who had written the 2019 Mathematics Paper I examination. On the day the researcher was there to collect the scripts, the school principal refused to give the research scripts. He suggested that the school will photocopy the scripts for the researcher, but the school has limited resources. Therefore, they could only make 20 copies. In School C, the subject teacher said they would still need the scripts in case some learners wanted to submit appeals and that they would provide the scripts in the first week of the second term. By the time schools opened for term two, the COVID-19 pandemic hit South Africa. When the researcher went to School C for the scripts, the subject teacher refused to provide the scripts for photocopying, citing COVID-19 as the reason. By the time the subject teacher refused, it was too late for the researcher to select another school. Therefore, the participating schools were School A with 24 learners who had written the 2019 Grade 11 Mathematics paper I, and School B with 20 learners who had written the 2019 Grade

11 Mathematics paper I. The participants in the study encompassed learners taking mathematics as a subject in the selected schools.

4.4 DATA COLLECTION

This study used documentary analysis as the primary source of data collection. The documentary analysis included examination question paper analysis and learners' examination scripts analysis for the November 2019 NCS Mathematics Paper I examination. The research analysis was only done on questions 5, 6 and 7, where the concept of functions was examined. The documentary analysis consisted of attaining data from existing documents without questioning people through interviews, questionnaires or observing their behaviour (Bowen, 2009). The documentary analysis as a data collection source seems to fit this study's objections since the researcher aimed to obtain the data from the question paper and learner examination scripts without having to interview learners or examiners.

In collecting the data, two sources of data were needed to respond to both research questions the 2019 Grade 11 Mathematics Paper I question paper examined by the GDE and learners' scripts responding to the question paper. The question paper and the memorandum were obtained from the researcher work place, while the learner scripts were obtained from two different schools.

In the process of collecting scripts, ethical protocols were followed (see Section 4.7). The researcher obtained permission to conduct the research on GDE schools. Schools were approached and requested to take part in the study. The principals of the relevant schools, the Grade 11 Mathematics subject teachers, learners who wrote the 2019 Mathematics Paper I examination and their parents were given consent forms and letters detailing the study objectives and how they would be required to take part in the research to grant the researcher permission to access and review learner scripts. In both schools, teachers collected the consent forms from all the stakeholders. They then informed the researcher when to collect the scripts. At School A, the researcher collected the scripts, photocopied them, and took them back to the school within two days. At school B, the scripts were already photocopied. Scripts obtained from both schools were not original scripts but copies. However, before the researcher started analysing the scripts, they were re-marked to verify the accuracy and fairness of the marking.

It was anticipated that the collected data using the question paper would help to respond to the first research question: **Which DOK levels are evident when analysing the Mathematics Paper I examination question paper based on the questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?** (see Section 1.6). The second source of data collection, which was the learners' examination scripts of the 2019 November Mathematics Paper I examination, assisted the researcher to answer the second research question: **Which DOK levels do learners seem to respond well to based on their written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?** (see Section 1.6). Table 4.1 demonstrates the link that existed between the research questions and the theoretical framework and methodology of the study.

Table 4.1: Linking research questions, theoretical framework and methodology

Research Questions	Theoretical Framework	Methodology
Which DOK levels are evident when analysing the Mathematics Paper I examination question paper based on the questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?	Classification of the questions in the Grade 11 examination question Paper I according to DOK levels and knowledge needed.	Documenting the qualitative data from documentary analysis of the question paper.
Which DOK levels do learners seem to respond well to based on their written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?	Classification of the learners' responses in terms of DOK level and forms of knowledge presented in their final examination scripts.	Qualitative data from documentary analysis of learners' examinations and scripts.

4.5 DATA ANALYSIS

Figure 4.2 shows how the researcher worked with the data collected. The first step after data collection was scanning and cleaning of data. This process was largely tied to the research questions that were asked.

Figure 4.2: The depiction of the data analysis

In this study, the first step towards analysing the qualitative data was analysing the 2019 November Mathematics Paper I question paper and classifying the questions on the function topic in terms of the DOK model. The study aimed to classify learners' written responses to questions that required conceptual knowledge, procedural knowledge or a combination of both in the concept of functions in terms of the DOK model. Before analysis of the question paper, the researcher started by identifying the questions that examined the concept of functions. In the Grade 11 2019 Mathematics Paper I question paper, the concept of function was examined in question 5 (hyperbola function), question 6 (parabola and straight-line function) and question 7 (exponential function). Hence, in both data source instruments (question paper and scripts), only these three questions were analysed.

Based on this study's second research question, the 2019 Grade 11 Mathematics Paper I examination questions where the concept of function was examined were classified based on the appropriate DOK level and the form of knowledge required to answer the question correctly. By referring to Table 3.1 in Section 3.5, the question was aligned to **DOK Level-1** if a learner was expected to recall a fact, principle or carry out a routine procedure with one step. For example, if the learner was given the equation of a hyperbolic function $f(x) = \frac{2}{x-3} + 2$ and they were asked to write the equation of asymptotes. **DOK Level-2** demands a basic application of skills. A learner is expected to carry out routine problems with more than two steps, display data, and execute the primary application of conceptual knowledge. If learners are asked to sketch the function of $f(x)$, they are expected to carry out the well-known procedure of drawing the function, finding the intercept and representing the information in the form of a graph. These kinds of questions are aligned with **DOK Level-2**.

If the same function is transformed, with certain units vertical and horizontal, then learners are given the new equation as the result of transformation in the form of $h(x)$. If the question is to describe the transformation, these kinds of questions require strategic thinking and can be solved using non-routine techniques. These kinds of questions are aligned to **DOK Level-3**. **DOK Level-4** is irrelevant for this study because it demands extended thinking in which learners must carry out research and apply skills and principles to a real-world context (Webb, 1999). In the mathematics examination, **DOK Level-4** was not presented. It can be assessed in the School-Based Assessment (SBA) in the form of an investigation or project. The lack of **DOK Level-4** also exposes other drawbacks of the CAPS cognitive levels because it does not accommodate such a level of knowledge. Table 3.2 demonstrates how the DOK level may be linked to conceptual or procedural knowledge.

Table 4.2: DOK levels aligned to form(s) of knowledge

LEVEL	DESCRIPTION	KNOWLEDGE
DOK-1	Recall & Reproduction	Procedural
DOK-2	Basic Skills/Concepts	Procedural/Conceptual
DOK-3	Strategic Thinking	Conceptual or conceptual and procedural
DOK-4	Extended Thinking	Procedural/Conceptual

The second step towards answering the second research question of the study was the analysis of the learners' 2019 November Mathematics Paper I examination scripts. The scripts were re-marked and analysed on questions 5, 6 and 7 where the concept of functions was examined. Learners' written responses were analysed by looking at the learner response and the question's demands. Learners' responses were assigned to the relevant code. Each code indicated a certain DOK level and the knowledge manifested in the learner’s response. In this study, five codes were used, ***Entirely Correct*** (EC), ***Nearly Correct*** (NC), ***Slightly Correct*** (SC), ***Not Correct*** (NCO) and ***Not Done*** (ND). A learner response was assigned to EC if the response was executed correctly and the correct answer was displayed in the learner’s script. Such learners were classified as those who accomplished all the requirements of a specific DOK level and was seen to possess the required form of knowledge. Although, in some cases, a learner provided the correct answer but did not have the conceptual knowledge for the answer. For example, a learner might write the correct equation of asymptotes but fail to display that equation in the Cartesian plane. However, for this study, if the question needed the equation of asymptote and the learner was able to present it correctly, it was accepted that the learner presented the required knowledge and was considered to have the necessary DOK level. The learner responses were assigned to NC when the knowledge required was demonstrated but the learner committed some errors during the execution process. For example, if a learner was required to sketch the hyperbola function, there were three steps that needed to be followed finding the intercept, drawing both asymptotes and drawing the function. If a leaner drew the correct asymptotes, indicated the intercept correctly, drew another arm of the hyperbola correctly and the other one incorrectly, then the learner response was assigned to the NC code.

The learner response was assigned to SC when the learner presented basic knowledge of answering the question, such as choosing the correct formula. For example, where a learner was expected to calculate the gradient and the learner selected the correct formula from the datasheet but substituted the wrong points, then that learner showed a basic understanding by choosing the correct formula to calculate the gradient. In the above example, if the learner only drew the correct asymptotes and failed the other two steps (finding the intercept and displaying the function on the Cartesian plane), then that learner's response was assigned to the SC code. The learner's response was assigned to NCO when it showed no knowledge required for the problem. In other words, there was nothing correct in the learner's response. The learner's response was assigned to the ND code when the learner did not respond to the question by leaving a blank space.

The schools in this study were coded as School A and School B. The scripts from School A were coded as SAL1, SAL2 up to SAL24 where SA stands for School A, L for learner and the number for the learner numbers 1 through 24. School B's scripts were coded as SBL1, SBL2 up to SBL20 where SB stands for School B, L for learner and the number for the learner numbers 1 through 20. The data was coded to label, organise and recognise various themes and relations between the scripts. The data organisation permits a researcher to create meaning from the evidence by ordering it into a convenient form (Jansen & Vithal, 2019). Maxwell (2013) stated that there are three analytic options memos, coding and connecting strategies. According to Kuckartz (2014), coding refers to analysing, naming and categorising data.

After assigning questions in the question paper and learners' responses with relevant DOK levels and knowledge required, the data was presented in tables and discussed. Those tables display a summary of the analysis of how learners responded to questions by indicating the raw number and percentage of learners in how they responded to that question. The results show every question and the number of learners who answered the relevant question. There are also selected extracts from the photocopied scripts where interesting findings were identified to exemplify the discussion.

The steps followed can be summarised as follows:

- The Grade 11 learners' scripts for the Mathematics Paper I 2019 final examination as well as the question paper were collected. The scripts were collected from two different schools, while the question paper and memorandum were collected from the researcher's workplace.
- The scripts and questions were analysed and classified using the DOK model to identify the DOK level displayed by learners in their responses and the DOK level presented in the question paper under the topic of functions.
- The knowledge displayed in learners' responses was identified and linked to the form of knowledge as either conceptual or procedural or the combination of both conceptual and procedural knowledge.

4.6 VALIDITY

Validity is an effort to detect whether the meaning and interpretation of the results are comprehensive or to what extent it is an accurate reflection of what was observed (Jansen & Vithal, 2019). Maxwell (2013) used validity in a reasonably common-sense or straightforward manner but consistent with qualitative researchers' general approach. The word "validity" is referred to as credibility or accuracy of the description, argument, interpretation and explanation. Validity relies on the relationship between conclusions and actuality, and no method can entirely declare that it was captured.

To evade intimidations and to test for validity, two strategies were employed. First, tthe researcher used the question paper which the GDE examined. This question paper was written by all learners attending public schools in the Gauteng province. The question paper was set by experts and passed through different levels of moderations, which ensured the quality of the questions in the question paper and the assessment policies were followed precisely. Using the GDE question paper ensured that all complexity levels were addressed in the question paper based on the CAPS taxonomy. This gave the validity that the questions analysed were up to standard for Grade 11 learners. The questions presented in the question paper were analysed to observe if they aligned with the policy document and were of the level of Grade 11 and this was later verified by the supervisor. The researcher also checked if the content examined was covered during the process of teaching and learning by using the annual teaching plan. The annual teaching plan is provided by the GDE, it intended to provide teachers with an overview of curriculum guidelines at a glance and it aligned to CAPS document. The validation process was aimed at ensuring that the Grade 11 Mathematics Paper I was at the required standard and to increase validity of the instrument used (examination items). The other strategy used to increase the validity was data collection from three different schools with different mathematics pass rates as evident in the final results. Due to unforeseen circumstances, the researcher managed to collect data from only two schools.

4.7 RELIABILITY

Reliability refers to the consistency and stability of measurement and is concerned with whether the study results are replicable (Morrison, 2006). Reliability indicates the chances that redoing the research would yield similar findings. It provides the degree of sureness that repeating the process would confirm consistency. A measure is reliable if it gives the same results on two or more occasions when the assumption is made that the object being measured has not changed (Morrison, 2006). If a series of measures, when repeated, give similar results, it is possible to say that it has high reliability. The researcher used the question paper examined by the GDE, which ensures the question paper's high quality as an instrument for data collection. Suppose this question paper was written by learners who were learning the same curriculum. In that case, the researcher is confident that repeating the research in different schools will yield similar results. Learners’ responses were assigned to a certain code based on the knowledge and DOK level displayed in the responses. To ensure that the assigning of codes to learners' responses was not subjective, the researcher and supervisor agreed that with each question, the code assigned to the question must be determined by the knowledge displayed in each learner’s response. Data triangulation was used to ensure the validity and reliability of the study.

4.8 ETHICAL CONSIDERATIONS

Ethical considerations are one of the most vital parts of research. Sales and Folkman (2000) proposed the most important principle related to ethical considerations research participants should not be subjected to harm. Respect for the dignity of research participants should be prioritised. Full consent should be obtained from the participants before the study. The protection of the privacy of research participants must be ensured. A researcher should provide an adequate level of confidentiality of the research data. The anonymity of individuals and organisations taking part in the research must be secured. The researcher must avoid misleading information and representation of primary data findings in a biased way (Sales & Folkman, 2000).

The researcher discussed the proposed research with the Grade 11 Mathematics teachers to analyse learners' final examination scripts. The GDE gave the researcher permission before conducting the investigation (see Appendix B). The University of the Witwatersrand ethical committee provided ethical clearance before the study took place. The protocol number is 2019ECE047M (see Appendix A).

To ensure confidentiality during data collection, the participants' names and marks were not exposed to anyone (see Appendix C). When learners' scripts were photocopied, the names were hidden. During the data analysis and research report phase, anonymity was guaranteed. The participating schools were named School A and School B, while learners were given codes such as SAL1 and SBL1.

4.9 LIMITATIONS AND GENERALISATION OF THE STUDY

The limitations of the study are mainly regarding the study sample. The scope, size and locality make generalisation difficult. The findings will not be easy to generalise because of the sample scope. However, some findings that emerged from the study could be generalised. Only two schools were used for the study as the third school withdrew, citing COVID-19 as the reason. The schools sampled for this study were those close to the researcher's workplace for easy accessibility during the data collection stage. The schools are located in one township where only black learners from middle-class families attend. The question paper analysed was only written in the Gauteng province, limiting the generalisation of the study results.

4.10 CONCLUSION

This chapter presented and discussed the research design and methodology employed to respond to the research questions. The qualitative research approach was selected as the method for this study. The data sources were the questions based on the functions topic from the November 2019 examination paper and learners’ answer scripts. The participants were sampled purposefully from the Ekurhuleni North district in Gauteng. The study employed a descriptive analysis of the data. This chapter outlined how the examination items and learners' written responses were classified in terms of the DOK level required by the relevant question and the knowledge shown by the learner. For ethical considerations, relevant protocols were followed and discussed.

5 CHAPTER 5: DATA ANALYSIS

5.1 INTRODUCTION

This chapter reflects on the analysis of the Mathematics Paper I Grade 11 2019 final examination question paper, focusing on questions 5, 6 and 7, where the concept of function was examined. The questions were analysed and classified in terms of the DOK level presented and the knowledge form required for learners to answer a question. The questions were analysed and classified to respond to research question one. **Which DOK levels are manifested when analysing the Mathematics Paper I examination question paper based on the questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?** Research question two will be addressed in Chapter 6 since it deals with learners' written interpretations.

5.2 GRADE 11 2019 FINAL EXAMINATION QUESTION PAPER

In this section, question 5, 6 and 7 (45 marks in total) from the 2019 Grade 11 Mathematics Paper I examination question paper, contributing 30% of 150 marks of the whole examination, is analysed in terms of the DOK model. Each question is discussed based on the knowledge and DOK level required to answer that question. The question is aligned to **DOK Level-1** if a learner is expected to recall a fact, principle or carry out the routine procedure with one step and the knowledge required to respond is procedural. The question is aligned to **DOK Level-2** when it requires a basic application of skills or concepts and the knowledge required to respond can be procedural or conceptual or a combination of both. The question is aligned to **DOK Level-3** if it requires strategic thinking from learners and can be solved using non-routine techniques. The knowledge form required to respond to the question is conceptual or a combination of both. Figure 5.1 illustrates how the DOK levels were classified as different forms of knowledge.

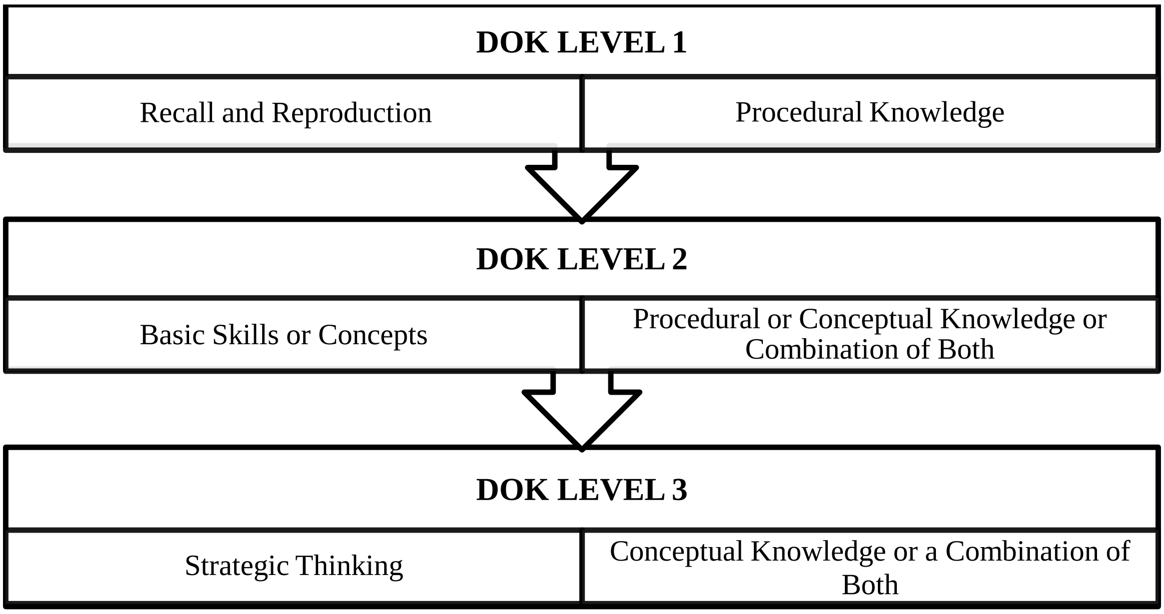


Figure 5.1: DOK levels classified as different forms of knowledge

5.2.1 Question 5

Question 5 of the question paper is illustrated below.

Given $\boldsymbol{f(x) = \frac{1}{x-3} - \frac{2x+6}{x+3}}$

Question 5.1: Show that *f(x)* can be written as $\boldsymbol{f(x) = \frac{1}{x-3} - 2}$ (2)

Solution: $f(x) = \frac{1}{x-3} - \frac{2x+6}{x+3}$

$$= \frac{1}{x-3} - \frac{2(x+3)}{x+3}$$

$$= \frac{1}{x-3} - 2$$

Question 5.1 allowed a learner to take out a common factor and then simplify using two steps. Therefore, the required DOK level for this question was **DOK Level-1**. The basic concept of algebra, such as taking out the common factor, would be the prerequisite form of knowledge required to answer this question. The learner should be able to recall the knowledge required, since taking out the common factor was introduced and taught in Grade 9 and strengthened in Grade 10. Thus, the knowledge required to respond to the question is procedural.

Question 5.2: Write down the equation of asymptotes of *f*. **(2)**

Solution: $x = 3\ and\ y = -2$

The standard equation for a hyperbolic function is $f(x) = \frac{a}{x+p} + q$. Question 5.2 expects a learner to recall what variables p and q represent in the equation and link them to the given constant. Afterward, they would need to execute a one-step routine procedure, $x - 3 \neq 0$, hence, $x \neq 3$. Subsequently, a learner would be able to write both equations of asymptotes of *f*. Thus, this question could be aligned to **DOK Level-1**. Procedural knowledge is required to address the question since a learner is supposed to understand symbolic representations and what variables *p* and *q* stand for in the standard equation.

Question 5.3: Determine the *x*-intercept of *f*. $\boldsymbol{f(x) = \frac{1}{x-3} - 2}$ **(3)**

Solution: $0 = \frac{1}{x-3} - 2$

$$2 = \frac{1}{x-3}$$

$$2(x-3) = 1$$

$$2x - 6 = 1$$

$$x = \frac{7}{2}$$

Question 5.4: Determine the *y*-intercept of *f*. **(2)**

Solution: $y = \frac{1}{0-3} - 2$

$$= \frac{-7}{3}$$

Both questions 5.3 and 5.4 required the same **DOK Level-2** and the same knowledge form. However, they differ in that 5.3 focused on the *x*-axis while 5.4 focused on the *y*-axis, and 5.3 required an additional execution for cross multiplication, which is why it has a higher mark allocation than 5.4. In these questions, a learner had to decide on the correct procedure to determine the intercept. Once the decision was made, a learner had to solve for *x* in 5.3 or *y* in 5.4 by performing more than two steps, where the first step was substituting followed by multisteps of simplification. As a result, this question was aligned with **DOK Level-2**. The concept that was assessed in this question was the knowledge of intercept, which was introduced in Grade 10. Hence, at the Grade 11 level, knowledge demanded by the question cannot be regarded as conceptual knowledge but rather procedural knowledge. When substituting and performing simplification, a learner requires knowledge of rules and procedures.

Question 5.5: Sketch the graph of *f*. Show clearly ALL the intercepts with the axes and the asymptotes. (3)

Solution:

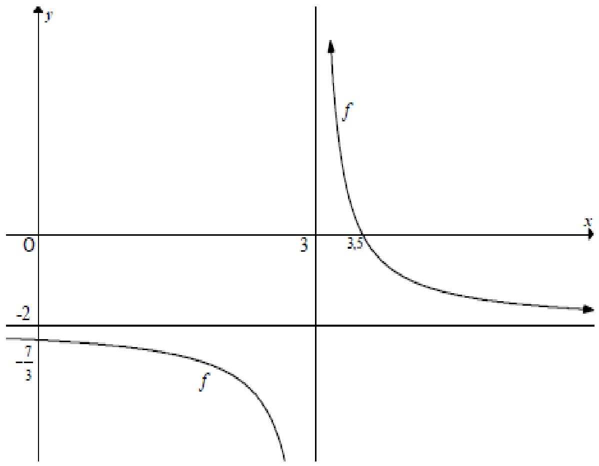


Figure 5.2: Hyperbolic function as an answer to question 5.5

Question 5.5 required a learner to generate the responses obtained in 5.2, 5.3 and 5.4 in the form of a hyperbolic function. A learner is expected to uphold a clearly defined sequence of steps in drawing the hyperbolic function, indicating asymptotes and intercepts in the Cartesian plane. To respond to this question, **DOK Level-2** is required. Here, a learner had to have the procedural knowledge of the hyperbolic function. For example, what 'a' describes in the standard equation and in which quadrants the function should be drawn. However, that knowledge was learned as conceptual knowledge in Grade 10. On the Grade 11 level, therefore, this question demands procedural knowledge.

Question 5.6: Determine the equation of the axis of symmetry of f having a positive gradient. (3)

Solution: $y = x + c$

$-2 = 3 + c$

$c = -5$

$y = x - 5$

Question 5.6 called for a learner to have procedural knowledge to recall from Grade 9 a positive gradient +*x* and negative gradient –*x* to decide on the correct equation to be used between $y = x + c \ or \ y = -x + c$ and to interpret the point of intersection of the asymptotes. Once the correct equation was selected, a learner had to perform two routine steps that would include rules for the substitution of the point, finding the value of c and substituting the value of c in the equation. This would be accomplished by applying procedural knowledge. This question is aligned to **DOK Level-2** since it required execution using basic skills and concepts.

Question 5.7: The graph of *f* is transformed to obtain the graph of $h(x) = \frac{1}{x}$. Describe the transformation from *f* to *h*. (2)

Solution: Translate *f* three units to the left and two units up.

Question 5.7 required a learner to compare the graph of f and h. A learner was expected to gather the difference by making the decision based on the transformation that occurred. Furthermore, a learner had to offer justification for the decision made. The response of a learner should involve a non-routine approach to the solution. The required DOK level for this question could therefore be aligned with **DOK Level-3**. In responding to this question, a learner had to have conceptual knowledge of hyperbolic functions This would help a learner understand the changes in the equation when the graph was transformed horizontally and vertically.

Question 5.8: Write down the domain of h. (2)

Solution: $x \in (-\infty: +\infty); x \neq 0$ **OR** $x \in R; x \neq 0$

In Grade 10, learners are exposed to the knowledge of a hyperbolic function, using a simple equation similar to h. If this question is asked in Grade 11, a learner should recall the hyperbolic function domain that was learned in the previous grade level. Therefore, a learner had to be able to execute a one-step systematic procedure to write the asymptotes, $x \neq 0$, and then include them in domain writing. Therefore, this question could be aligned with **DOK Level-1**. Knowledge required in the Grade 11 level of learning is no longer conceptual knowledge but procedural knowledge.

5.2.2 Question 6

The diagram below shows the graph of $f(x) = -x^2 + 2x + 15$ and $g(x) = -3x + k$. Graph *f* cuts the *x*-axis at A (-3; 0) and B (5; 0), the *y*-axis at C and has the turning point at D. Graph *g* cuts the *x*-axis at B and the *y*-axis at C. E is the point on *g* such that DE is parallel to the *y*-axis.

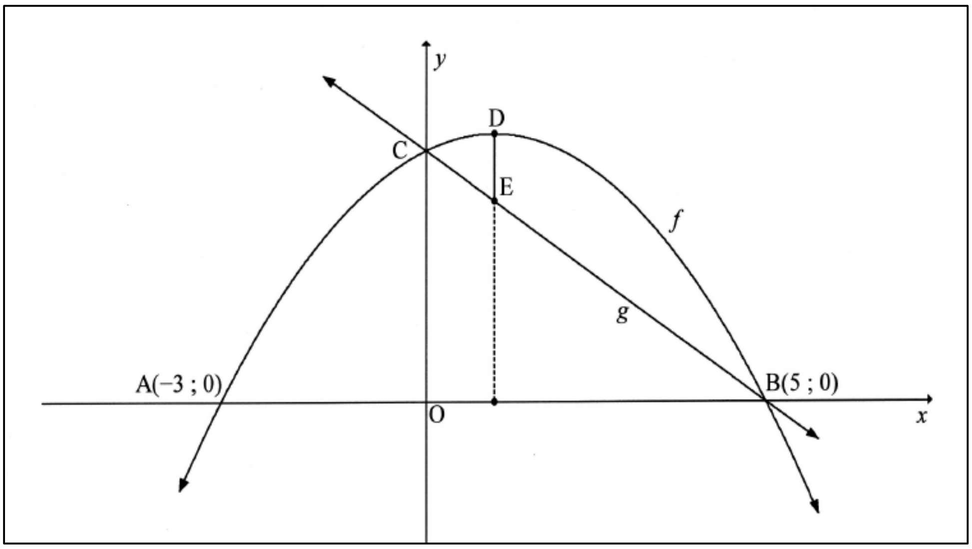


Figure 5.3: Parabola and a linear function

Question 6.1: Show that $k = 15$. **(1)**

Solution: $y = -3x + k$

$0 = -3(5) + k$

$k = 15$

Based on the memorandum of Question 6.1, the researcher presumed that the examiner expected a learner to respond to the question by substituting point B to determine the value of *k*, which is done using procedural knowledge. A learner had to recognise that *k* is the *y*-intercept or unknown and needed to be solved and performed in two steps, first substituting point B and then solving the value of *k*. That moves the question to **DOK Level-2** since the learner had to execute two steps to reach the solution. However, this question could also be answered by using conceptual knowledge. The learner could identify the coordinates of point C (0;15) by looking at function *f*, then recall the intersection principles of functions (when two functions intersect, they share a point). Therefore, the value of *k* could be easily obtained by linking it to the *y*-intercept of the parabolic function. Thus, the knowledge needed to respond to the question could either be procedural or conceptual.

Question 6.2: Determine the coordinate of D, the turning point of *f*. **(3)**

Solution: $x = \frac{-b}{2a}$

$$x = \frac{-2}{2(-1)} \; OR \; x = \frac{-3+5}{2}$$

$$x = 1 \qquad\qquad x = 1$$

$$y = -(1)^2 + 2(1) + 15$$

$$y = 16$$

D (1;16)

Question 6.2 could be answered using conceptual or procedural knowledge. Procedural knowledge could have been employed by utilising the formula $x = \frac{-b}{2a}$ and executed by using well-known principles such as substitution and simplification. If a learner had conceptual knowledge of functions, that learner would realise that the *x*-value of point D is the midpoint of AB or recall the concept of symmetry. Afterward, they could add the *x*-value at point A and *x*-value at point B, then divide by two to get the *x*-value at D. To obtain the coordinate of D, they would have to substitute the *x*-value in the equation of *f*. To respond to this question, a learner was expected to use the basic application of concepts or procedures. Thus, the question was aligned to **DOK Level-2**.

Question 6.3: Determine the values of *x* for which *f* is increasing. **(1)**

Solution:$x < 1 \;\; \boldsymbol{OR} \;\; x \epsilon (-\infty; 1)$

Question 6.3 required a learner to read the information from the function. A learner had to have a conceptual knowledge to analyse the function. Furthermore, a learner would use the function features to decide where the function is increasing to address the question. This question was aligned with the **DOK Level-3** because it required a learner to interpret the function and use non-routine techniques to respond to the question. For this question a learner can only obtain the correct answer by understanding the features of the graph. No calculation are required to answer this question. Hence, non-routine technique should be used to answer the question.

Question 6.4: Calculate the average gradient between point A and D. (3)

Solution: A(-3;0) and D(1;1 6)

$$ave\ grad = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 16}{-3 - 1}$$

$$4$$

This question can be problematic because there is no line drawn between the two points A and D. A learner should understand that a gradient can be calculated using two points in the presence of a line or without a line. A learner had to decide on the points to be used and on the selection of the relevant formula for an average gradient $ave\ grad = \frac{y_D - y_A}{x_D - x_A}$. Procedural knowledge was necessary to take the required steps, such as substitution and simplification, to get a final solution. Question 6.4 was aligned with **DOK Level-2**.

Question 6.5: Calculate the length of DE. (2)

Solution: At D $y = 16$

At E $y = 12$

DE = 4 units

Question 6.5 required a learner to have conceptual or procedural knowledge. In the statement, a learner was given that DE is parallel to the *y*-axis. The learner had to understand the meaning of this statement (the *x*-value stays constant along the line DE) when responding to the question. Understanding this statement would help a learner determine the coordinate at point E, which is requisite in finding the length of DE. Then a learner could choose to tackle the problem using the conceptual knowledge without using the distance formula in calculating the length. However, if the learner lacks conceptual knowledge, the required knowledge could still be demonstrated through procedural knowledge. A learner would then approach the question using the distance formula, which consists of a few steps. Thus, this question is aligned to **DOK Level-2**.

Question 6.6: If $\boldsymbol{h(x) = f(x+1) - 2}$, determine the equation of $\boldsymbol{h}$ in the form

$$\boldsymbol{h(x) = (x+p)^2 + q.} \qquad \textbf{(4)}$$

Solution: $h(x) = f(x-1) - 2$

$= -(x-1)^2 + 2(x-1) + 15 - 2$

$= -x^2 + 2x - 1 + 2x - 2 + 15 - 2$

$= -x^2 + 4x + 10$

$= -(x^2 - 4x + 4 - 4 - 10)$

$= -(x-2)^2 + 14$

OR

D(1;16) and $a = -1$

$f(x) = -(x-1)^2 + 16$

$h(x) = f(x-1) - 2$

$= -(x-1-1)^2 + 16 - 2$

$= -(x-2)^2 + 14$

OR

D(1;16) and $a = -1$

$f(x) = -x^2 + 2x + 15$

$= -(x^2 - 2x - 15)$

$= -(x^2 - 2x + 1 - 1 - 15)$

$= -(x-1)^2 + 16$

$= -(x-1-1)^2 + 16 - 2$

$= -(x-2)^2 + 14$

In response to question 6.6, a learner had to have a conceptual knowledge of functions before initiating a routine using procedural knowledge. In the function $h(x) = f(x+1) - 2$, a learner had to know what effect +1 and -2 would have on the equation. A learner had to deal with the complicated situation in making decisions on these constants' impact and justify those decisions. Furthermore, a learner also required conceptual knowledge to represent the equation of the parabola function in different forms, such as $f(x) = ax^2 + bx + c \; or \; f(x) = a(x+p)^2 + q$. After breaking down the complexities of the problem, a learner could apply the knowledge of procedures to execute the necessary steps to address the question. This question required strategic thinking because the learner had to establish the strategy to use in approaching the question. Thus, the question was aligned to **DOK Level-3**.

Question 6.7: Determine the maximum value of $p(x) = 3^{f(x)-12}$ **(3)**

Solution: Max value of *f(x)* is 16

Max value of *f(x) -12* is *16-12=4*

Max $p(x) = 3^4$

$=81$

OR

$f(x) - 12 = -x^2 + 2x + 15 - 12$

$New\ Equation = -x^2 + 2x + 3$

$$x = \frac{-b}{2a} = \frac{-(2)}{2(-1)} = 1$$

$y = -(1)^2 + 2(1) + 3 = 4$

$\max value = 3^4 = 81$

When responding to question 6.7, a higher level of thinking was necessary. A learner had to explain the decision made. Thus, a learner had to understand the parabolic and exponential functions and integrate these two functions. This question was aligned to **DOK Level-3** because strategic thinking was required to answer the question.

Question 6.8: Determine the values of x for which $f(x) + k = 0$ will have two distinct real roots. **(2)**

Solution: $-x^2 + 2x + 15 + 15 = 0$

$-x^2 + 2x + 30 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(-1)(30)}}{2(-1)}$$

$x = -4.57\ or\ x = 6.57$

In this question, a learner had to have the conceptual knowledge of functions to understand the meaning of the equation $f(x) + k = 0$. A learner had to substitute *f(x)* and k in the equation $f(x) + k = 0$, then solve the problem by applying well-known procedures. At some point, a learner would realise that there are no factors for the equation. Hence, the quadratic formula should be used. Solving using the quadratic formula would require knowledge of the rules and a learner had to be able to link the variable in the quadratic equation *a, b,* and *c* to the coefficients and constant in the equation. A learner would require both conceptual and procedural knowledge to respond to this question. The question was classified as **DOK Level-3** because learners had to decide how to react to it and perform more than two algorithm steps.

5.2.3 Question 7

The point A (3; 54) lies on the graph of $f(x) = 3^{x+p} - 27$.

Question 7.1: Determine the value of *p*. **(2)**

Solution: $y = 3^{x+p} - 27$

$$54 = 3^{3+p} - 27$$
$$81 = 3^{3+p}$$
$$3^4 = 3^{3+p}$$
$$4 = 3 + p$$
$$p = 1$$

In question 7.1, a learner had to decide to substitute point A in the equation of the function and execute to obtain the value of *p*. Execution should involve well-known procedures in solving the value of *p*, such as performing a substitution and transposing of the constant or the variable. The question was aligned to **DOK Level-2** since it required the basic application of procedural skills.

Question 7.2: Determine the range of *f*. **(2)**

Solution: $range{:}\, y > -27$ ***or*** $y\epsilon(-27;\infty)$

If the function was sketched, a learner would read the information from the graph and make it easier for learners to write the correct response. However, in this question, a learner was expected to recall the impact of q in the exponential functions, which they had done in Grade 10. For example, the standard equation of exponential is $f(x) = ab^{x+p} + q$. A learner should know the impact of *q* in the function to respond to the question. Knowing the impact of *q* would allow a learner to know where the function asymptotes are plotted, which is vital in writing the range of the function. Therefore, this question was aligned with **DOK Level-1** since a learner was required to recall the impact of *q* in writing the range of the function.

Question 7.3: Graph *g* is obtained by reflecting graph *f* about the *x*-axis. Determine the coordinates of the *y*-intercept of *g*. (2)

Solution: $-y = 3^{x+1} - 27$

$g(x) = -1.3^{x+1} + 27$

$y = -1.3^{0+1} + 27$

$y = 24$

y-intercept (0;24)

OR

$f(0) = 1.3^{0+1} - 27 = -24$

y-intercept (0;24)

In Grade 8 in the South African curriculum, learners are introduced to the concept of translation, reflection, reduction and enlargement. Once more, this concept is developed in Grade 10, where they must focus on reflecting with the line $x = 0$ *or* $y = 0$. If the reflection is done by reflecting with the *x*-axis and is the same as reflecting with the line $y = 0$, then a learner should recognise the effect on the point's coordinates. To respond to this question, a learner had to recall basic concepts of reflection, then perform simple procedures to get the *y*-intercept for *f* and reflect that coordinate using the *x*-axis to get the coordinate of the *y*-intercept for *g*. Thus, this question was aligned to **DOK Level-2**.

5.3 DISCUSSION OF THE ANALYSIS

This final examination was a form of assessment since it was used to measure learners' achievement in mathematics and show if the required expectations had been met. Effective schooling is measured through assessment. The learner performance in the assessment relies on aligning three elements of the educational environment: curriculum, instruction and assessment. During the analysis of the question paper, all the questions were evaluated to ensure that they were within the scope of the policy (CAPS). The fairness of the examination relies on pedagogy and curriculum, which must be aligned with the examination's content coverage. Fairness of this 2019 Mathematics Paper I November examination may be achieved if teachers adhere to the CAPS document and use the specified pedagogy. Also, teachers must cover all the content based on their Annual Teaching Plan (ATP), which is closely related to the CAPS document. Therefore, this examination paper under the concept of functions reflected fairness based on the CAPS document and ATP. However, the marks scored by the learner might disagree with these statements.

The total marks allocated to the concept of functions examined in the question paper was 45 marks out of 150 marks, which is equivalent to 30% of the total marks in the Mathematics Paper I question paper. The findings indicate that **DOK Level-2** was the most assessed level with 9 out of 19 questions focusing on this level, which means 47.4% of the questions required **DOK Level-2**. It was also noted that more marks were allocated to questions that required **DOK Level-2** with 22 out of 45 marks, which means 48,9% of the questions focused on this level. The number of questions focused on **DOK Level-1** and **DOK Level-3** was the same. However, results in Table 5.1 show that there were more marks allocated to **DOK Level-3** than to **DOK Level-1**. For **DOK Level-3**, 12 (26.7%) marks were relevant, while 11 (24.4%) marks were focused on **DOK Level-1**. Table 5.1 shows the number of questions under the concept of functions. It also shows the distribution of percentages of the DOK levels among the examination items that addressed the topic of functions in a Grade 11 Mathematics Paper I.

Table 5.1: The distribution of percentages of DOK levels in the Grade 11 Mathematics final examination paper 1

DOK LEVEL	Number of questions that examined DOK	Percentage of questions that examined DOK	Allocated marks in the question paper	Percentage allocation in the question paper
Level 1	5	26.3	11	24.4
Level 2	9	47.4	22	48.9
Level 3	5	26.3	12	26.7
TOTAL	**19**	**100**	**45**	**100**

Table 5.2 shows an overview of the analysis of the results and displays the knowledge required on each question and the DOK level presented for each question.

Table 5.2: Knowledge and DOK levels presented on the concept of functions questions

Question	Form of knowledge required in the question	DOK level required to respond to the questions
5.1	Procedural	Level 1
5.2	Procedural	Level 1
5.3 & 5.4	Procedural	Level 2
5.5	Procedural	Level 2
5.6	Procedural	Level 1
5.7	Conceptual	Level 3
5.8	Procedural	Level 1
6.1	Conceptual or Procedural	Level 2
6.2	Conceptual or Procedural	Level 2
6.3	Conceptual	Level 3
6.4	Procedural	Level 2
6.5	Conceptual or Procedural	Level 2
6.6	Conceptual and procedural	Level 3
6.7	Conceptual	Level 3
6.8	Conceptual and procedural	Level 3
7.1	Procedural	Level 2
7.2	Procedural	Level 1
7.3	Procedural	Level 2

It is evident from the findings in Table 5.2 that procedural knowledge is the most assessed knowledge compared to conceptual knowledge. In question 5, seven out of eight questions required procedural knowledge to answer successfully. There was only one question that required conceptual knowledge. In question 5, **DOK Level-2** was the most required DOK level. Five questions required **DOK Level-2**, two questions required **DOK Level-3** and only one question required **DOK Level-1**.

In question 6, there was only one item that required procedural knowledge and two items that required conceptual knowledge for an accurate answer. However, three questions required procedural knowledge or conceptual knowledge. In those questions, a learner had the chance to respond to the question using the preferred form of knowledge. Two items required a combination of both procedural and conceptual knowledge. In question 6, conceptual knowledge seemsed to dominate procedural knowledge. It will be interesting in Chapter 6 to look at learners' performance on question 6 where conceptual knowledge dominated and how they chose to respond to questions that required conceptual knowledge or procedural knowledge. There was no **DOK Level-1** requirements in question 6, whereas **DOK Level-2** and **DOK Level-3** were equally represented. From eight questions, four required **DOK Level-3** and the other four questions required **DOK Level-2**.

In question 7, two questions out of three required procedural knowledge and only one question required conceptual knowledge. Only **DOK Level-1** and **DOK Level-2** were represented in question 7.

5.4 CONCLUSION

In this chapter, the Mathematics Paper I NCS question paper was analysed using Webb's DOK model as the theoretical framework. The question paper was analysed on questions 5, 6 and 7 where functions were examined. During the analysis, the depth of knowledge required in the question was highlighted and aligned to the adequate DOK level. In question 5 and question 7, results indicated that procedural knowledge was the most assessed form of knowledge instead of conceptual knowledge, and **DOK Level-2** was the most required DOK level compared to the other levels. In contrast, in question 6, conceptual knowledge was the most assessed form of knowledge. Although, in question 6, three questions out of eight required conceptual knowledge or procedural knowledge. Those questions provided a learner with an opportunity to choose the preferred knowledge. In question 6, there was no **DOK Level-1**, while **DOK Level-2** and **DOK Level-3** were equally represented. From these results on the concept of functions assessed in the Grade 11 Mathematics Paper I 2019 November examination, it is clear that procedural knowledge was the most assessed knowledge compared to conceptual knowledge and the **DOK Level-2** was the most represented DOK level.

6 CHAPTER 6: PRESENTATION OF KEY FINDINGS

6.1 INTRODUCTION

This chapter discusses the description and analysis of the learners' written responses in the Grade 11 November 2019 Mathematics Paper I final examination in detail. The analysis is only done for questions 5, 6 and 7 where functions are examined. Learners’ scripts are analysed to find whether the knowledge and DOK level required by the question are manifested in responses. The learners’ scripts are interpreted in a way that responds to research question two. **Which DOK levels do learners seem to respond well to based on their written responses to questions that require conceptual knowledge, procedural knowledge, or a combination of both when tested on functions?**

6.2 LEARNERS’ SCRIPT ANALYSES

There were 44 scripts analysed, 24 from School A and 20 from School B. The learners' scripts were re-marked by the researcher on questions 5, 6 and 7. Questions 5, 6 and 7 added up to 45 marks which contributed 30% towards the total mark of 150 marks for Mathematics Paper I. The presentation and analyses of the responses for questions 5, 6 and 7 are presented on how learners responded to the question and discussed under the DOK model and form(s) of knowledge manifested on the learners’ responses. Five rank scores were used in conjunction with the knowledge required (procedural, conceptual, procedural or conceptual and conceptual and procedural) to analyse the learners' responses. Each code indicates a certain DOK level and the knowledge manifested in the learner’s response. Those rank scores are ***Entirely Correct*** (EC), ***Nearly Correct*** (NC), ***Slightly Correct*** (SC), ***Not Correct*** (NCO) and ***Not Done*** (ND). A learner response was assigned to EC if the response was executed correctly and the correct answer was displayed in the learners’ scripts. For this code, learners managed to demonstrate the required DOK level and the required form of knowledge. The learner response was assigned to NC when the knowledge required was demonstrated but some errors occurred during the execution process. The learner response was assigned to SC when the learner presented basic knowledge of answering the question such as choosing the correct formula. The learner response was assigned to NCO when learner response showed no knowledge required for the problem. In other words, there was nothing correct in the response of the learner. The learner response was assigned to ND when the learner did not respond to the questions by leaving a blank space. The analysis in this chapter also includes selected extracts from the learners' written responses. Extracts were taken from the photocopied scripts. Tables are used to display rank scores for the responses. Table 6.1 shows the results in rank scores for both School A and School B, collectively, with the knowledge and DOK level required for each question.

Table 6.1: Forms of knowledge and rank scores of the learners' written responses

Questions	Required DOK Level	Required Knowledge	SCHOOL A					SCHOOL B				
			EC	NC	SC	NCO	ND	EC	NC	SC	NCO	ND
5.1	Level 2	Procedural	4	0	0	16	4	3	0	0	14	3
5.2	Level 1	Procedural	22	0	2	0	0	10	0	3	7	0
5.3	Level 2	Procedural	10	12	0	2	0	7	8	0	5	0
5.4	Level 2	Procedural	10	12	0	2	0	7	8	0	5	0
5.5	Level 2	Procedural	3	8	9	4	0	3	1	3	13	0
5.6	Level 2	Conceptual and Procedural	13	2	0	7	2	0	0	0	3	17
5.7	Level 3	Conceptual	1	0	0	12	11	1	0	0	6	13
5.8	Level 1	Procedural	1	7	0	7	9	0	4	0	6	10
6.1	Level 2	Conceptual or Procedural	12	0	0	5	7	7	0	0	7	4
6.2	Level 2	Conceptual or Procedural	10	1	5	3	5	9	2	2	7	0
6.3	Level 3	Conceptual	0	0	0	18	6	0	0	0	17	3
6.4	Level 2	Procedural	8	0	10	1	5	13	1	6	0	0
6.5	Level 2	Conceptual and Procedural	2	0	0	9	13	5	0	0	9	6
6.6	Level 3	Conceptual and Procedural	0	0	1	12	11	0	0	0	8	12
6.7	Level 3	Conceptual	0	0	0	6	18	1	0	0	5	14
6.8	Level 3	Conceptual and Procedural	0	1	0	8	15	0	2	0	4	14
7.1	Level 2	Procedural	10	0	4	5	5	10	0	7	3	0
7.2	Level 1	Procedural	2	0	0	15	7	1	0	0	15	4
7.2	Level 2	Procedural	2	0	0	13	9	4	0	0	10	6

From Table 6.1, it is evident that learners performed better in one question that required **DOK Level-1**. For example, question 5.2 required **DOK Level-1** and of the 24 learners, 22 (92%) in School A demonstrated the required knowledge and provided the entirely correct answer. The other 2 (8%) learners showed understanding and none of the learners showed no understanding or did not answer the question. However, in School B, only 10 (50%) of the learners demonstrated the required knowledge and gave the entirely correct answer, while 3 (15%) showed understanding and 7 (35%) showed no understanding. In both schools, learners performed better on this question compared to other questions. However, in School B, learners performed better in question 6.5 than in question 5.2. It is also worth noting that learners did not perform better in all **DOK Level-1** questions. For example, in question 5.8, only one learner from School A provided the entirely correct answer, with no learners in School B providing the entirely correct answer. However, based on the limited questions that required the **DOK Level-1**, it is not feasible to conclude that the **DOK Level-1** was the most represented DOK level.

We observe in Table 6.1 that there were 11 (58%) questions that required **DOK Level-2** and in all these questions, there were learners who provided the entirely correct answer. In these questions, learners performed better compared to questions that required **DOK Level-3**. However, the performance was poor when compared to **DOK Level-1**. In questions 5.7, 6.3, 6.6, 6.7, 6.8 and 7.2, **DOK Level-3** was required to respond to these questions, and it is evident from Table 6.1 that these questions were poorly answered in both schools. For example, in questions 6.3, 6.6, 6.7 and 6.8, none of the learners (from either school) provided the entirely correct answer. However, with question 5.7, one (4%) learner in School A and one (5%) learner in School B provided the entirely correct answer to this question. With question 7.2, 2 (8%) learners in School A and one (5%) learner in School B provided the entirely correct answer. This finding also indicates that **DOK Level-3** was the least demonstrated DOK level in learners' responses.

From Table 6.1, it is also evident that learners were performing better with questions that demanded procedural knowledge compared to those that demanded conceptual knowledge. In three questions that required conceptual knowledge (questions 5.7, 6.3 and 6.7) and in questions 6.3 and 6.7, none of the learners from School A demonstrated the required knowledge by providing the entirely correct response, a nearly correct response, or showing some understanding. In questions 5.7 very few learners demonstrated the required knowledge (one

(4%) learner from School A and 2 (8%) learners from School B). Also, in School B, learners performed poorly on questions that required conceptual knowledge. In question 6.3, none of the learners from School B provided an entirely correct response, while in questions 5.7 and 6.7 only one (5%) learner per question had the entirely correct answer. It is apparent from Table 6.1 that questions that required conceptual knowledge showed poorer performance than all other questions.

6.3 LEARNERS' SCRIPTS ANALYSIS PER QUESTION

This section discusses the analysis of learners' written work results in the 2019 Grade 11 November Mathematics Paper I final examination by using the DOK model and form(s) of knowledge demonstrated in learners' responses. The results are presented using tables and extracts from the photocopied scripts were selected if the response was interesting to note. The learners who participated in this study are referred to as SAL1 (School A Learner 1), SAL2, SAL3, etcetera, and SBL1 (School B Learner 1), SBL2, SBL3, etcetera.

6.3.1 Question 5

Given $f(x) = \frac{1}{x-3} - \frac{2x+6}{x+3}$

Question 5.1: Show that *f(x)* can be written as $\boldsymbol{f(x) = \frac{1}{x-3} - 2}$

This question asked learners to simplify the given equation to a standard equation of the hyperbolic function. For a learner to demonstrate the required **DOK Level-1**, they had to have procedural knowledge. Hence, a learner could demonstrate procedural knowledge by taking out the common factor and recognising the factors that would divide with each other, leading to the correct answer for the question. Table 6.2 shows the percentage of rank scores for the responses and the knowledge demonstrated by learners from both schools on question 5.1.

Table 6.2: Form of knowledge presented and the rank scores for question 5.1

Responses to Question 5.1	Knowledge	School A	%	School B	%
EC	Procedural	4	16.7	3	15
NC	Procedural	0	0	0	0
SC	Procedural	0	0	0	0
NCO	None	16	66.7	14	70
ND	None	4	16.7	3	15

It is apparent from Table 6.2 that in both schools learners performed poorly on this question, with the minority of learners demonstrating the entirely correct answer (4 (16.7%) in School A and 3 (15%) in School B). In both schools, there were no learners who demonstrated a nearly correct answer or a slightly correct answer. All learners who managed the first step of taking out the common factor successfully demonstrated the required knowledge and **DOK Level-1**. Furthermore, the results show that most learners attempted the question with 16 (66.7%) learners from School A and 14 (70%) learners from School B responding to the question. However, their responses failed to indicate the required knowledge. Figure 6.1 shows an example of SAL11 who tried to solve the problem without taking out the common factor, but the technique used was incorrect. The method used by the learner shows a lack of algebraic comprehension because the learner failed to recognise the common factor on the numerator and that, if taken, it would result in two factors that would divide with each other. The learner demonstrated a lack of procedural knowledge needed for this question. Some learners did not answer the question by leaving a blank space 4 (16.7%) learners in School A and 3 (15%) learners in School B.

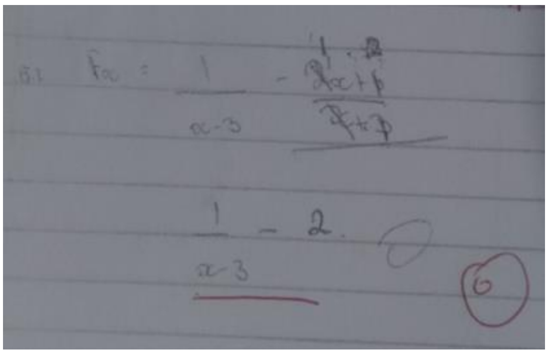


Figure 6.1: SAL11's response to question 5.1

Question 5.2: Write down the equation of asymptotes of *f*.

This question required a learner to recognise the asymptotes from the standard equation of the hyperbolic function and write it in the form of an equation. For a learner to demonstrate the required DOK level, they had to have procedural knowledge of symbols, specifically p and q, in the standard equation. A learner had to link symbols p and q in the standard equation with constants in the given equation and write asymptotes as equations. Table 6.3 shows the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 5.2.

Table 6.3: Form of knowledge presented and the rank scores for question 5.2

Responses to Question 5.2	Knowledge	School A	%	School B	%
EC	Procedural	22	91.7	10	50
NC	Procedural	2	8.3	3	15
SC	Procedural	0	0	0	0
NCO	None	0	0	7	35
ND	None	0	0	0	0

It is evident from the results in Table 6.3 that learners from School A performed exceptionally well on this question, with 22 (91.7%) learners giving the entirely correct response and 2 (8.3%) learners giving nearly correct responses. However, in School B, 10 (50%) learners gave the entirely correct response, and 3 (15%) learners gave an almost correct response. It is apparent from Table 6.3 that most learners in both schools had the required knowledge. These learners demonstrated the required **DOK Level-1** for this question. In both schools, all learners answered the question. In School B, 7 (35%) learners failed to write the correct answer and those learners did not demonstrate the required knowledge of how the hyperbola function's asymptotes are determined. Figure 6.2 is an example of how two learners from School B responded to the question. These learners wrote the number without the variable and the equal sign. Their responses displayed a lack of basic concept understanding. These learners failed to indicates if it is a horizontal or vertical asymptote. If the question is "*write down the equation*", there must be an equal sign to write an equation.

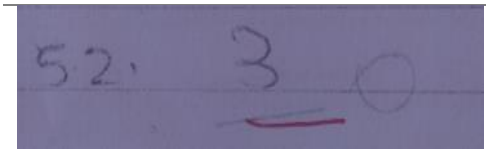


Figure 6.2a: SBL 4

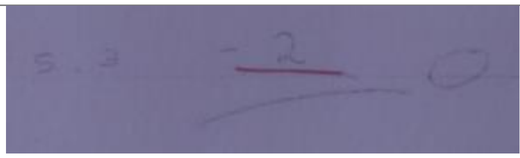


Figure 6.2b: SBL 13

Figure 6.2: Examples of some learners' responses to question 5.2

Question 5.3: Determine the x-intercept of *f* and Question 5.4: Determine the y-intercept of *f*.

Questions 5.3 and 5.4 are discussed simultaneously because they examine the same concept intercept but differ in that 5.3 examined the *x*-intercept while 5.4 examined the *y*-intercept. However, the knowledge required and the DOK level were the same for both questions. A learner had to know procedures such as substitution and solving for the unknown in the equation to demonstrate the required **DOK Level-2**. Table 6.4 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on questions 5.3 and 5.4.

Table 6.4: Form of knowledge presented and the rank scores for questions 5.3 and 5.4

Responses to Questions 5.3 & 5.4	Knowledge	School A	%	School B	%
EC	Procedural	10	41.7	7	35
NC	Procedural	12	50	8	40
SC	Procedural	0	0	0	0
NCO	None	2	8.3	5	25
ND	None	0	0	0	0

From Table 6.4, it is evident that the majority of learners failed to provide the entirely correct response to this question, with 10 (41.7%) learners from School A and 7 (35%) learners from School B providing the entirely correct response to the question. These were the only learners who demonstrated the required knowledge and the **DOK Level-2** in their responses. However, 12 (50%) of learners from School A and 8 (40%) of learners from School B demonstrated the required knowledge but committed some errors in their simplification and their responses were nearly correct. None of the learners provided a slightly correct answer, but all of them answered the question, with 2 (8.2%) learners from School A and 5 (25%) learners from School B providing incorrect responses. These learners failed to demonstrate the required knowledge.

From Table 6.4, it is evident that many learners know the rule of finding the intercept of the hyperbola function (EC + NC) 22 (91.7%) learners from School A and 5 (25%) learners from School B. These learners substituted correctly, with 12 (50%) learners from School A and 8 (40%) learners from School B committing mathematical errors in their work. For example, in Figure 6.3, SAL5 committed some errors while answering the question. In step $2x - 6 = 1$, the learner made an error of not changing the sign of six. However, by looking at the learner's response in question 5.2, the learner knows that a sign must change when the number is transposed. Instrumental understanding typically requires a multiplicity of rules, rather than limited concepts of more general implementation, which increases the probability of error during the process (Skemp, 1976). It is evident from Figure 6.3 that when multiple steps are involved, there is a high probability of errors.

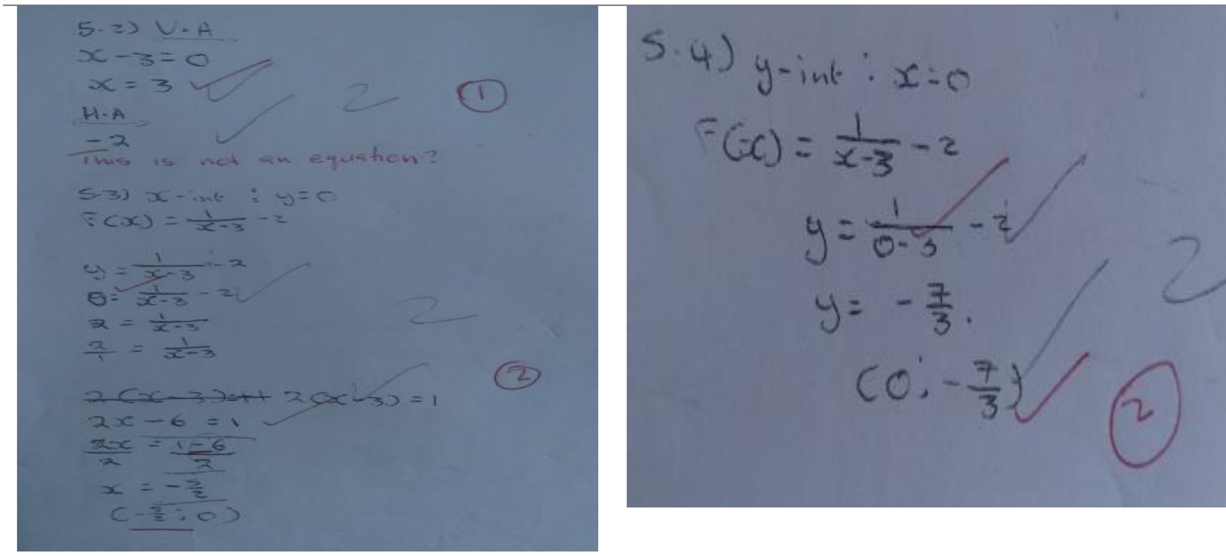


Figure 6.3: SAL5's response to questions 5.2, 5.3 and 5.4

Question 5.5: Sketch the graph of *f*. Show clearly ALL the intercepts with the axes and the asymptotes.

This question required learners to display the responses acquired in questions 5.2, 5.3 and 5.4 in a graph form. A learner had to demonstrate **DOK Level-2** through correctly sketching both asymptotes vertically and horizontally, indicating both the *y*-intercept and the *x*-intercept and sketching the function correctly in the correct quadrants. To respond to this question, a learner had to have procedural knowledge. Figure 6.4 displays an example of SBL7's drawn graph. The learner attained the required information required to sketch the hyperbola function but failed to present the gained information in the graph form. This learner revealed a lack of understanding of what an intercept means in the graph. In question 5.4, the learner calculated the *y*-intercept correctly but allocated it in the *x*-axis, which raises the question of a deep understanding of functions. The learner failed to demonstrate a deep understanding of the function.

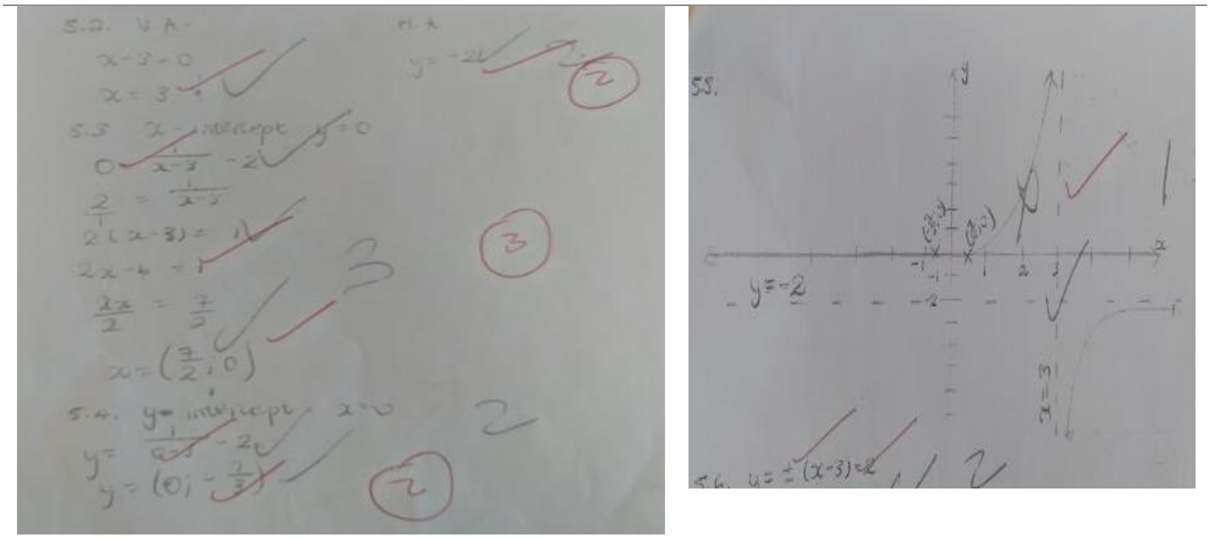


Figure 6.4: SBL7's response to questions 5.2, 5.3, 5.4 and 5.5

Table 6.5 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 5.5.

Table 6.5: Form of knowledge presented and the rank scores for question 5.5

Responses to Question 5.5	Knowledge	School A	%	School B	%
EC	Procedural	3	12.5	3	15
NC	Procedural	8	33.3	1	5
SC	Procedural	9	37.5	3	15
NCO	None	4	16.7	13	65
ND	None	0	0	0	0

The results in Table 6.5 show that most learners failed to sketch the function correctly, with only 3 (12.5%) learners from School A and 3 (15%) learners from School B providing the entirely correct response for the question. However, 8 (33.3%) learners from School A and one (5%) learner School B provided a nearly correct answer. They showed the required knowledge, managed to sketch the asymptotes correctly and indicated the intercept correctly on the axis but failed to sketch the function. Some learners (9 (37.5%) from School A and 3 (15%) from school B) showed understanding in their responses and correctly plotted the asymptotes or the intercept or the shape of the function. All learners tried to respond to the question, although 4 (16.7%) learners from School A and 13 (65%) learners from School B provided incorrect responses. Hence, the required knowledge was not demonstrated in their responses.

The responses of learners on this question indicated that they do not have a deep understanding of the concept of functions. In question 5.2, 22 (91.7%) learners from School A and 10 (50%) learners from School B managed to write the equations of asymptotes correctly. However, not all learners indicated those asymptotes correctly in the Cartesian plane. Only 20 (83%) learners from School A and 7 (35%) learners from School B plotted it. Even with those who plotted it in the Cartesian plane, there were still others who showed a lack of deep knowledge of the asymptotes. In Figure 6.5, SAL2 plotted the asymptotes correctly but showed a lack of deep understanding of the asymptotes. The learner plotted the function which crosses the line of the asymptotes and indicates a lack of deep understanding of the asymptotes because the function should not touch or cross the asymptotes.

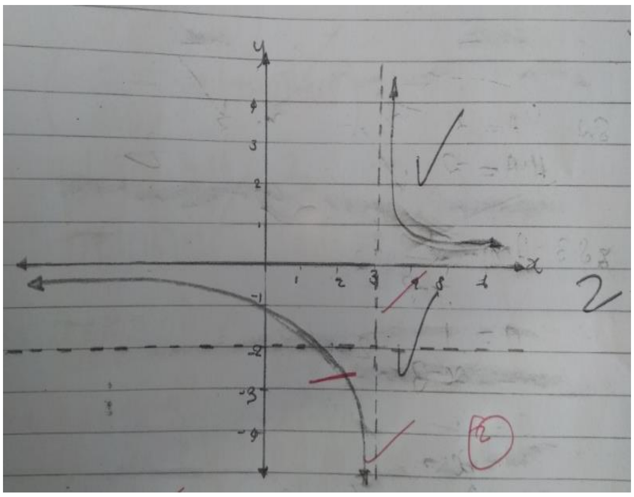


Figure 6.5: SAL2’s response to question 5.5

Question 5.6: Determine the equation of the axis of symmetry of f having a positive gradient.

For a learner to demonstrate the **DOK Level-1** required to answer the question, the correct equation with the positive gradient had to be chosen. A learner had to be able to recall a basic concept of positive and negative gradients from Grade 9. After that, they had to perform the proper substitution and execute the mathematical elements correctly by using procedural knowledge. Table 6.6 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 5.6.

Table 6.6: Form of knowledge presented and the rank scores for question 5.6

Responses to Question 5.6	Knowledge	School A	%	School B	%
EC	Procedural	13	54.2	0	0
NC	Procedural	2	8.3	0	0
SC	Procedural	0	0	0	0
NCO	None	7	29.2	3	15
ND	None	2	8.3	17	85

School B's result is concerning, with no learners providing the entirely correct response, a nearly correct response or showing some understanding. For this question, 3 (15%) learners tried to respond to the question but their responses were incorrect and 17 (85%) did not even try to respond. However, in School A, 13 (54.2%) of the learners provided the entirely correct answer and 2 (8.3%) learners provided a nearly correct answer. There were 7 (29.2%) learners that provided incorrect answers and 2 (8.3%) did not answer the question. Based on School A’s results, most of the learners had the knowledge required for the question. For School B, it is possible that the concept was not taught. In the absence of COVID-19, it would be interesting to interview School B’s mathematics teachers based on their learners' responses to this question.

Question 5.7: The graph of f is transformed to obtain the graph of $h(x) = \frac{1}{x}$. Describe the transformation from f to h.

Question 5.7 required a learner to recognise the transformation from f to h. For a learner to demonstrate adequate **DOK Level-3** in the response, they had to describe the effect of p on the horizontal transformation and q on the vertical transformation. To answer the question, conceptual knowledge was necessary to understand the effect of a, p and q variables in the standard equation. Table 6.7 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 5.7.

Table 6.7: Form of knowledge presented and the rank scores for question 5.7

Responses to question 5.7	Knowledge	School A	%	School B	%
EC	Procedural	1	4.2	1	5
NC	Procedural	0	0	0	0
SC	Procedural	0	0	0	0
NCO	None	12	50	6	30
ND	None	11	45.8	13	65

Table 6.7 shows that, in both schools, learners performed poorly on this question with one (4.2%) learner from School A and one (5%) learner from School B who managed to provide the entirely correct response. There were no learners who showed a nearly correct answer. In both schools, most learners had no understanding of the concept examined in this question, with 12 (50%) learners from School A and 6 (30%) learners from School B who tried to respond. However, their responses did not demonstrate the required knowledge. In School B, the majority 13 (65%) of learners did not answer the question, while in School A, 11 (45.8%) did not answer.

Question 5.8: Write down the domain of *h*.

This question wanted learners to recall the domain of the hyperbola function. A learner could demonstrate the adequate **DOK Level-1** for this question by stating where the function is defined and undefined on the x-axis. Table 6.8 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 5.8.

Table 6.8: Form of knowledge presented and the rank scores for question 5.8

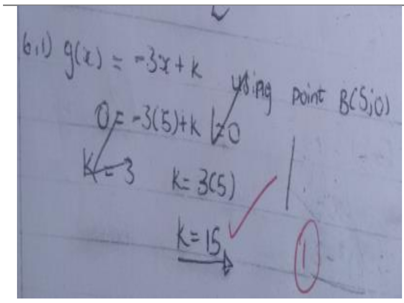
Responses to Question 5.8	Knowledge	School A	%	School B	%
EC	Procedural	1	4.2	0	0
NC	Procedural	0	0	0	0
SC	Procedural	7	29.2	4	20
NCO	None	7	29.2	6	30
ND	None	9	37.5	10	50

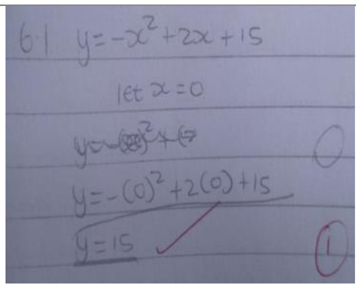
It is evident from Table 6.8 that learners performed poorly on this question, with one (4.2%) learner from School A managing to provide the entirely correct response while none of the learners from School B provided the entirely correct response and none of the learners were almost correct in their responses from either school. However, 7 (29.2%) learners from School A and 4 (20%) learners from School B showed understanding in their responses by indicating where the function was defined but failing to indicate where the function was undefined. Some learners 7 (29.2%) from School A and 6 (30%) from School B showed a lack of the required knowledge by linking the domain with *y*-values, whereas 9 (37.5%) learners from School A and 10 (50%) learners from School B did not respond to the question.

6.3.2 Question 6

Question 6.1: Show that $\boldsymbol{k = 15}$.

Question 6.1 required a learner to prove that the value of the *y*-intercept of the linear function is 15. A learner could demonstrate adequate **DOK Level-2** through executing the question using procedural or conceptual knowledge. By using the procedural knowledge, a learner must substitute the value of *x* that is 0 and solve for *k*. If a learner decide to use conceptual knowledge, then s/he must realise that there is intersection between the parabola and linear function, since there is the *y*-intercept of the parabola function. The interesting part of this question is that most learners solved the question using procedural knowledge. Those who tried with conceptual knowledge failed to demonstrate a complete understanding thereof. Figure 6.6 shows extracts of learners responding to question 6.1 using different forms of knowledge. In Figure 6.6a, SAL6 successfully solved the question procedurally. Figure 6.6b and 6.6c show extracts of SAL21 and SAL13, respectively. These two learners solved the question by using conceptual knowledge. However, their responses indicated that they did not have a complete conceptual knowledge. For example, SAL21 should have known that in the equation $f(x) = -x^2 + 2x + 15$, the constant 15 represents the y-intercept. Hence, there was no need for a learner to solve algebraically. SLA13 understood that at the point of intersection, $f(x) = g(x)$, but failed to link the constant 15 to k.





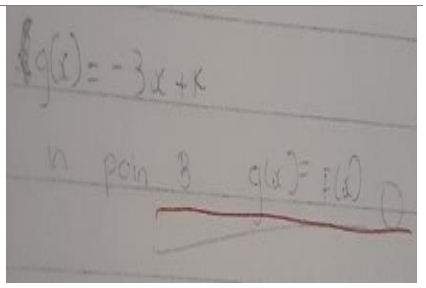


Figure 6.8a: SAL6	Figure 6.8b: SAL21	Figure 6.8c: SAL13

Figure 6.6: Learners' responses to question 6.1

The other interesting point here for SAL21's answer, is that even the subject teacher had mismarked the conceptual response. At the teacher's level, one would assume that even if the solution is not stated in the memorandum, the teacher should pick up on the correct answer from the learner's response. The teacher compromised the use of conceptual knowledge to answer this question. This behaviour could harm learners who are expected to gain a deep understanding of the functions, especially those learners who are conceptually oriented. Table 6.9 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools in question 6.1.

Table 6.9: Form of knowledge presented and the rank scores for question 6.1

Responses to Question 6.1	Knowledge	School A	%	School B	%
EC	Procedural or Conceptual	12	50	7	35
NC	Procedural or Conceptual	0	0	0	0
SC	Procedural or Conceptual	0	0	0	0
NCO	None	5	20.8	9	45
ND	None	7	29.2	4	20

Based on the results in Table 6.9, learners' performance on this question was not pleasing, with 12 (50%) learners from School A and 7 (35%) learners from School B providing the entirely correct response. None of the remaining learners from either school showed some understanding or provided a nearly correct response. Not all learners attempted to answer this question 7 (29.2%) from School A and 4 (20%) School B did not answer the question.

However, 5 (20.8%) learners from School A and 9 (45%) learners from School B answered the question, but their answers indicated none of the required knowledge.

Question 6.2: Determine the coordinate of D, the turning point of *f*.

Question 6.2 asked learners to determine the coordinates of the turning point for a parabola function. Learners could demonstrate the required **DOK Level-2** in their responses using conceptual or procedural knowledge. Using procedural knowledge, a learner must select the correct formula, performing the proper substitution, and executing the calculation to get the *x*-value. Using conceptual knowledge, a learner should know that the *x*-coordinate of the turning point is equidistant from both the *x*-intercept of the function, then add the intercept and divide by two to get the *x* coordinate of the turning point, and then correctly substitute the *x*-value for the corresponding *y*-value. As was observed with question 6.1, learners preferred to solve using procedural knowledge over conceptual knowledge for question 6.2 as well. Figure 6.7 displays learners responding to question 6.2, where SBL8 used procedural knowledge while SBL18 used conceptual knowledge.

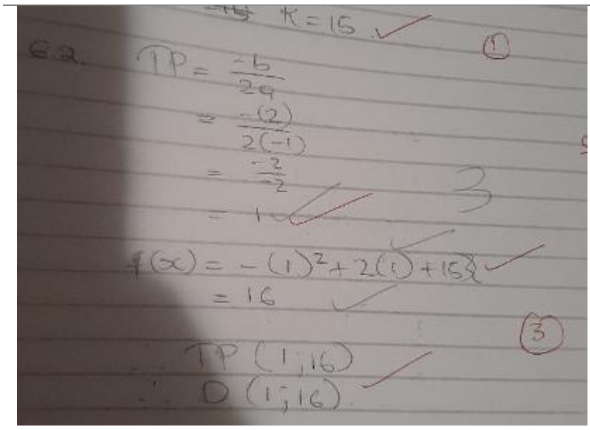


Figure 6.7a: SBL8's procedural response

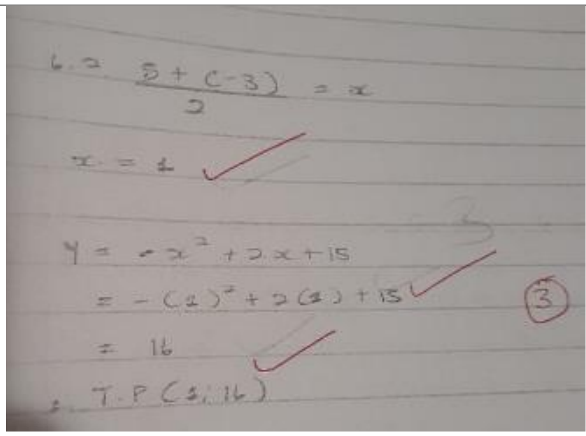


Figure 6.7b: SBL18's conceptual response

Figure 6.7: Learners' responses to question 6.2

Table 6.10 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 6.2.

Table 6.10: Form of knowledge presented and the rank scores for question 6.2

Responses to Question 6.2	Knowledge	School A	%	School B	%
EC	Procedural or Conceptual	10	41.7	9	45
NC	Procedural or Conceptual	1	4.2	2	10
SC	Procedural or Conceptual	5	20.8	2	10
NCO	None	3	12.5	7	35
ND	None	5	20.8	0	0

From the data in Table 6.10, it seems that most learners appeared to have the required knowledge to answer the question. However, some had the knowledge but committed errors and failed to get the entirely correct answer. There were 10 (41.7%) learners from School A and 9 (45%) learners from School B who provided the entirely correct answer, while one (4.2%) learner from School A and 2 (10%) learners from School B provided answers that were nearly correct and 5 (20.8%) learners from School A and 2 (10%) learners from School B showed understanding in their answers. In School B, all learners answered the question, although 7 (35%) learners' answers showed a lack of the required knowledge, while in School A, 5 (20.8%) learners did not answer the question and 3 (12.5%) answered but provided incorrect answers.

Question 6.3: Determine the values of *x* for which *f* is increasing.

This question required the learners to understand the parabola function in order to read the function and answer the question. A learner could demonstrate the required **DOK Level-3** by correctly reading or interpreting the function and presenting the answer accurately. Table 6.11 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 6.3.

Table 6.11: Form of knowledge presented and the rank scores for question 6.3

Responses to Question 6.3	Knowledge	School A	%	School B	%
EC	Conceptual	0	0	0	0
NC	Conceptual	0	0	0	0
SC	Conceptual	0	0	0	0
NCO	None	18	75	17	85
ND	None	6	25	3	15

From Table 6.11, it is evident that learners from both schools performed poorly on this question, with none of the learners providing the entirely correct or a nearly correct response or showing some understanding in their responses. Most learners answered the question 18 (75%) from School A and 17 (85%) from School B. However, their responses showed no required knowledge. Only 6 (25%) learners from School A and 3 (15%) learners from School B did not answer the question. Based on this question's results, none of the learners showed the capability to read the information from the graph. Hence, learners lacked a conceptual knowledge of functions.

Question 6.4: Calculate the average gradient between points A and D.

This question required learners to calculate the average gradient between two points. A learner could demonstrate adequate **DOK Level-2** by selecting the correct formula and deciding which points to substitute. Furthermore, a learner had to substitute the coordinate correctly in the formula and simplify to get the correct response. Table 6.12 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 6.4.

Table 6.12: Form of knowledge presented and the rank scores for question 6.4

Responses to Question 6.4	Knowledge	School A	%	School B	%
EC	Procedural	8	33.3	13	65
NC	Procedural	0	0	1	5
SC	Procedural	10	41.2	6	30
NCO	None	1	4.2	0	0
ND	None	5	20.8	0	0

From Table 6.12, it is evident that the majority of learners from School B performed well on this question, with 12 (65%) learners providing the entirely correct response and one (5%) learner providing a nearly correct response. This learner substituted the correct points and executed them correctly but failed to show the formula used as required by the question paper instructions and information, bullet number four "[c]learly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers." The learner failed to follow this instruction and was penalised with one mark for not indicating the formula. In School A, most learners performed poorly, with only 8 (33.3%) learners managing to provide the entirely correct answer and none of the learners providing a nearly correct answer, while 10 (41.2%) learners from School A and 6 (30%) learners from School B showed some understanding in their answers by selecting the correct formula for calculating the average gradient. In School B, all learners tried to answer the question with all of the learners showing some understanding in their answers, while in School A, one (4.2%) learner failed to demonstrate the required knowledge in their response and 5 (20.8%) learners did not answer the question.

Question 6.5: Calculate the length of DE.

Question 6.5 required a learner to determine the length of DE. However, the coordinates of E were not given. For a learner to demonstrate the required **DOK Level-2**, they had to start by finding the coordinates of E using conceptual knowledge. Thereafter, a learner could decide to execute using conceptual or procedural knowledge. If learners preferred to use conceptual knowledge, they had to know what it means for DE to be parallel to the *y*-axis as given in the statement. If a learner prefers procedural knowledge, then a distance formula should be used. However, it was noted that if the question could be solved using conceptual knowledge or procedural knowledge, the majority of learners chose to solve the problem by using procedural knowledge. Hence, most learners answered the question by using procedural knowledge in this question, as was also the case with questions 6.1 and 6.2. Those questions could be solved using conceptual knowledge or procedural knowledge, however, most learners solved using procedural knowledge.

Figure 6.2 shows the different responses of learners to question 6.5. Some learners responded using conceptual knowledge while others responded using procedural knowledge but all providing the same correct answer. As shown in Figure 6.8a, SBL18 approached the problem using conceptual knowledge, indicating that the learner understood the meaning of parallel. If DE is parallel to the *y*-axis, then the *x*-value at point D should be the same as the *x*-value at point E. If length DE is required, the *y*-value at point E should be subtracted to the *y*-value at point D. As shown in Figure 6.8b, SAL6 approached the question using procedural knowledge. The learner demonstrated the basic concept of parallelism and managed to obtain the coordinate of point E using that knowledge. Instead of subtracting the *y*-value, the learner decided to use the distance formula. The learner knew that if the length is requested, the distance formula should be used, which can be regarded as procedural knowledge.

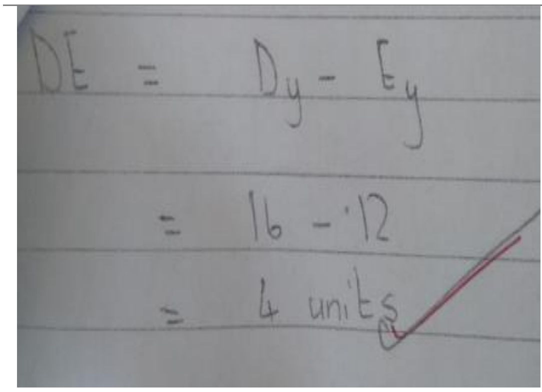


Figure 6.8a: SBL18's conceptual response

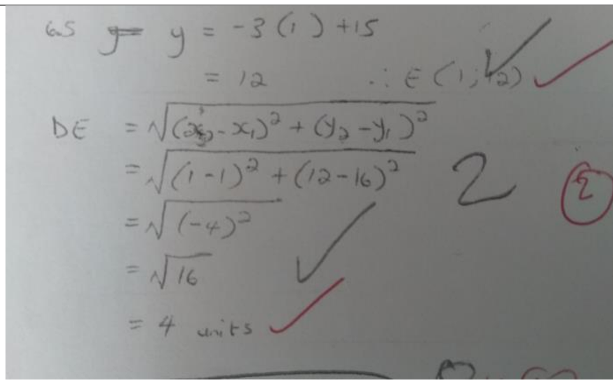


Figure 6.8b: SAL6's procedural response

Figure 6.8: Learners' different responses to question 6.5

Table 6.13 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 6.5.

Table 6.13: Form of knowledge presented and the rank scores for question 6.5

Responses to Question 6.5	Knowledge	School A	%	School B	%
EC	Conceptual and Procedural	2	8.3	5	25
NC	Conceptual and Procedural	0	0	0	0
SC	Conceptual and Procedural	0	0	0	0
NCO	None	9	37.5	9	45
ND	None	13	54.2	6	30

From Table 6.13, it is evident that most learners failed to demonstrate the required knowledge for this question, with 2 (8.3%) learners from School A and 5 (25%) learners from School B managing to provide the entirely correct response. While none of the learners showed a nearly correct response, most showed some understanding by at least writing the distance formula to indicate that they understood that the distance could be calculated that way. The majority of learners 13 (54.2%) from School A did not answer the question and 9 (37.5%) learners answered but their answers did not show the required knowledge of the concept examined. However, in School B, only 6 (30%) learners did not answer the question. Most learners answered but 9 (45%) learners failed to demonstrate the required knowledge.

Question 6.6: If $h(x) = f(x+1) - 2$, determine the equation of h in the form $h(x) = (x+p)^2 + q$.

This question examined the understanding of the transformation of the parabola function and presenting the transformed equation in the standard form. For a learner to demonstrate adequate **DOK Level-3** in their response, they had to describe the effect of p on the horizontal transformation and q on the vertical transformation. Understanding this would require a learner to have a conceptual knowledge of the parabola function. Table 6.14 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 6.6.

Table 6.14: Form of knowledge presented and the rank scores for question 6.6

Responses to Question 6.6	Knowledge	School A	%	School B	%
EC	Conceptual	0	0	0	0
NC	Conceptual	0	0	0	0
SC	Conceptual	0	0	0	0
NCO	None	13	54.2	8	40
ND	None	11	45.8	12	60

It is apparent from Table 6.14 that none of the learners demonstrated the required conceptual knowledge. All learners from both schools failed to provide the entirely correct answer, a nearly correct answer or show some understanding. This poor performance was observed in question 6.3, which also required conceptual knowedge and **DOK Level-3**. The results from questions 6.3 and 6.6 reveal that learners struggled with the questions that required conceptual knowledge and **DOK Level-3**. The majority of learners 13 (54.2%) from School A answered the question but their responses did not demonstrate the required knowledge, while 11 (45.8%) learners did not answer the question at all. In School B, the majority of learners 12 (60%) did not answer the question, with 8 (40%) answering the question but providing incorrect answers.

Question 6.7: Determine the maximum value of $p(x) = 3^{f(x)-12}$

To demonstrate **DOK Level-3** required for question 6.7, a learner had to understand the maximum meaning in the sense of the functions. Conceptual knowledge was, therefore, essential when responding to this question. While looking at the equation of *p*, a learner had to determine the maximum value for *f*, then subtract twelve from it, substitute the maximum value of *f* in *p* and simplify to get the final answer. The learners had another option. By solving the exponent separately, *f(x)* -12 could be calculated and the answer substituted in the equation of *p*. Table 6.15 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools in question 6.7.

Table 6.15: Form of knowledge presented and the rank scores for question 6.7

Responses to Question 6.7	Knowledge	School A	%	School B	%
EC	Conceptual	0	0	1	5
NC	Conceptual	0	0	0	0
SC	Conceptual	0	0	0	0
NCO	None	6	25	5	25
ND	None	18	75	14	70

It is apparent from Table 6.15 that learners performed poorly on this question, with 18 (75%) learners from School A and 14 (70%) learners from School B not answering the question. Even those learners who answered 6 (25%) from School A and 5 (25%) from School B did not demonstrate the required knowledge. None of the learners from School A provided the entirely correct or nearly correct answer or showed some understanding, while in School B, only one (5%) learner provided a entirely correct response, and none provided a nearly correct response or showed some understanding. This question was the third one that required **DOK Level-3** and conceptual knowledge and learners constantly performed poorly on all these questions. These results indicate that learners struggle to use conceptual knowledge to demonstrate **DOK Level-3**.

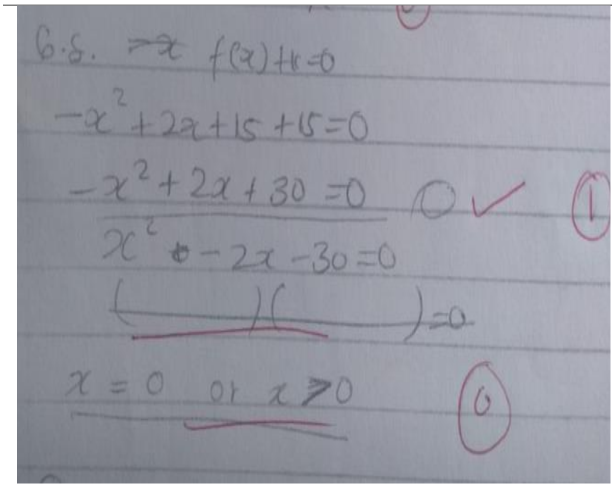
Question 6.8: Determine the values of *x* for which $f(x) + k = 0$ will have two distinct real roots

In this question, a learner could demonstrate the **DOK Level-3** required if they understood the nature of the roots and substituted the function *f* in equation *f(x) + k = 0*, then solved the equation correctly in determining two distinct real roots. Both conceptual and procedural knowledge were required to answer this question. Table 6.16 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools in question 6.8.

Table 6.16: Form of knowledge presented and the rank scores for questions 6.8

Responses to Question 6.8	Knowledge	School A	%	School B	%
EC	Conceptual and Procedural	0	0	0	0
NC	Conceptual and Procedural	1	4.2	2	10
SC	Conceptual and Procedural	0	0	0	0
NCO	None	8	33.3	4	20
ND	None	15	62.5	14	70

From the data in Table 6.16, it is evident that the performance of learners on this question was poor, with no learners from either school providing the entirely correct answer or showing some understanding. There was one (4.2%) learner from School A and 2 (10%) learners from School B who provided a nearly correct answer. These three learners managed to make the correct substitution and provide the equation in the standard form. However, three of them struggled to make the right decision to use the quadratic formula. Figure 6.9a shows SBL17 trying to factorise the equation. The learner failed to get the correct factors but still did not think of the quadratic formula. The other two learners showed that they lacked a conceptual knowledge that confused the nature of roots and discriminants. They calculated the discriminant instead of the root. Figure 6.9b shows SBL9 finding a discriminant instead of a root. The learner started by making the correct decision to use the suitable method. Towards the end, however, the learner cancelled the correct procedure and solved the discriminant, which was not what the question asked.



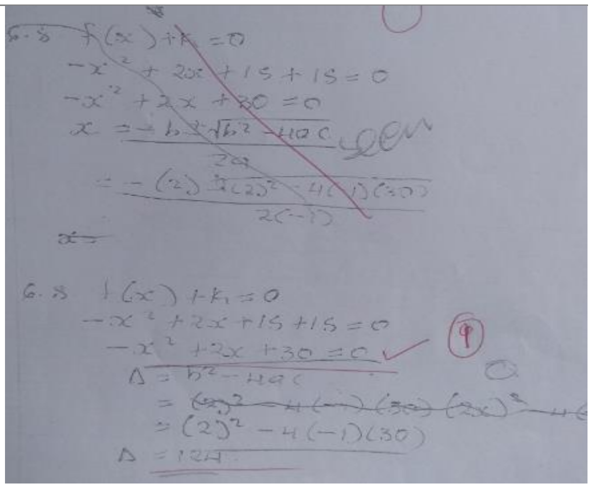


Figure 6.9a: SBL17 **Figure 6.9b: SBL9**

Figure 6.9: Examples of learners' responses to question 6.8

As indicated in Table 6.16, 15 (62.5%) learners from School A and 14 (70%) learners from School B did not answer this question. These learners showed no idea of the examined concept, while 8 (33.3%) learners from School A and 4 (20%) learners from School B answered the question but their answers did not demonstrate the required knowledge. This was the fourth question that required **DOK Level-3** and conceptual knowledge to answer correctly and learners continued to perform poorly on these questions.

6.3.3 Question 7

The point A (3; 54) lies on the graph of $f(x) = 3^{x+p} - 27$.

Question 7.1: Determine the value of *p*.

Question 7.1 asked learners to solve for the unknown *p*. A learner could demonstrate the required **DOK Level-2** by executing three crucial steps. Firstly, the coordinate of point A had to be substituted correctly in the equation. Secondly, 81 had to be changed to the base of 3. Lastly, they had to equate the indices and apply the correct rules of solving an equation to get the right value of *p*. Table 6.17 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools in question 7.1

Table 6.17: Form of knowledge presented and the rank scores for question 7.1

Responses to Question 7.1	Knowledge	School A	%	School B	%
EC	Procedural	10	41.7	10	50
NC	Procedural	0	0	0	0
SC	Procedural	4	16.7	7	35
NCO	None	5	20.8	3	15
ND	None	5	20.8	0	0

From the data in Table 6.17, the performance of learners was not pleasing, with 10 (41.7%) learners from School A and 10 (50%) learners from School B providing the entirely correct response and none of the learners from either school providing a nearly correct response. However, 4 (16.7%) learners from School A and 7 (35%) learners from School B showed understanding in their responses. These learners managed to substitute point A in equation *f* as the first step. Thereafter, they failed to change 81 to the exponential form with a base of 3. In School B, all learners answered the question but 3 (15%) learners showed no required knowledge in their answers. In School A, 5 (20.8%) learners did not answer the question, while 5 (20.8%) learners answered but provided incorrect responses.

Question 7.2: Determine the range of *f*.

This question examined the range of the exponential function. However, the function was not drawn. Hence, learners had to use procedural knowledge of the exponential function, as it was learnt in Grade 10. A learner could demonstrate **DOK Level-1** by relating the range to *y*-values and state where the *y*-values are defined and undefined using the function equation. Table 6.18 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 7.2.

Table 6.18: Form of knowledge presented and the rank scores for question 7.2

Responses to Question 7.2	Knowledge	School A	%	School B	%
EC	Conceptual	2	8.3	1	5
NC	Conceptual	0	0	0	0
SC	Conceptual	0	0	0	0
NCO	None	15	62.5	15	75
ND	None	7	29.2	4	20

It is evident from Table 6.18 that learners performed poorly on this question, with only 2 (8.3%) learners from School A and one (5%) learner from School B providing the entirely correct response. None of the learners provided a nearly correct response or showed some understanding in their answer. The results in Table 6.18 also show that 15 (62.5%) learners from School A and 15 (75%) learners from School B answered the question. However, their answers did not demonstrate the required knowledge. Figure 6.10 shows how the learners tried to respond to question 7.2. Figure 6.10a shows SBL2's response. This learner failed to relate range to *y*-values. In Figure 6.10b, SBAL22 confused the range of the exponential function with the hyperbolic function range.

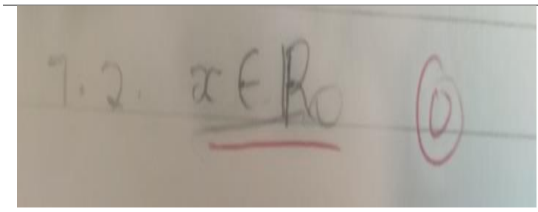


Figure 6.10a: SBL2

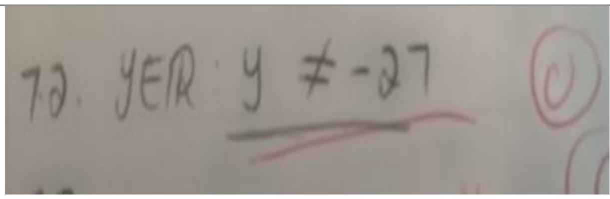


Figure 6.10b: SAL22

Figure 6.10: Examples of learners' responses to question 7.2

Question 7.3: Graph *g* is obtained by reflecting graph *f* about the *x*-axis. Determine the coordinates of the *y*-intercept of *g*.

This question examined the concept of reflection. A learner could demonstrate adequate **DOK Level-2** for this question by reflecting the function of *f* with the *x*-axis, then replacing *x* with 0 to get the *y*-intercept of *g* or replace *x* with 0 to find the *y*-intercept of *f* and reflect the point to get the *y*-intercept of *g*. Table 6.19 displays the raw and percentage rank scores for the responses and the knowledge demonstrated by learners from both schools on question 7.3.

Table 6.19: Form of knowledge presented and the rank scores for question 7.3

Responses to Question 7.3	Knowledge	School A	%	School B	%
EC	Procedural	2	8.3	4	20
NC	Procedural	0	0	0	0
SC	Procedural	0	0	0	0
NCO	None	13	54.2	10	50
ND	None	9	37.5	6	30

From Table 6.19, it is evident that most of the learners from both schools showed no understanding of the concept, with 13 (54.2%) learners from School A and 10 (50%) learners from School B falling into the NCO code category. Several learners did not answer the question at all, with 9 (37.5%) learners from School A and 6 (30%) learners from School B under the ND code. Table 6.19 shows that learners performed poorly on this question, with only 2 (8.3%) learners from School A and 4 (20%) learners from School B managing to provide the entirely correct answer. None of the learners provided a nearly correct answer or showed some understanding.

6.4 LEARNERS' PERFORMANCE ON THE CONCEPT OF FUNCTIONS

The learners' scripts were re-marked only on questions 5, 6 and 7, where functions were examined. The marked scripts were summed up to 45 marks. The performance of learners in the section of functions was not pleasing. Table 6.20 shows the levels that learners obtained in the concept of functions.

Table 6.20: Analysis of learners' results on the concept of function

		LEVELS								
	No. of Scripts	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**No. Passed**	**No. Failed**
SCHOOL A	24	11	3	5	3	2	0	0	13	11
SCHOOL B	20	11	4	5	0	0	0	0	9	11

From the seven-point scale (DBE, 2011), Level 2 is an elementary achievement between 30 and 39 percent. As a result, the learners who fall in and beyond this range are deemed to have passed. In School A, 13 out of 24 learners passed the section of functions, with only 5 learners managing to score convincing marks or reach adequate achievement above 50% for the section. In School B, only 9 learners out of 20 managed to pass the concept of function. None of the learners managed to reach adequate achievement. In both schools, none of the learners managed to score above 70% in the section. Based on the information in Table 6.20, School A's learners performed better on the concept of function as opposed to School B's learners. However, overall, 5 (11%) learners out of 44 learners from both schools performed better. This indicates that learners are still struggling with the concept of function, and much work still needs to be done. Suppose mathematical teachers and researchers are serious about enhancing the performance of learners in mathematics, researchers must conduct studies to identify the challenges and make some recommendations. Teachers must conduct practices that encapsulate the recommendations.

6.5 SUMMARY

Learners' written responses to the Grade 11 November 2019 Mathematics Paper I were carefully analysed on the questions where the concept of function was examined. Four interesting findings were observed during the analysis process. The findings are discussed next.

i. Most learners appeared to perform better in questions that required **DOK Level-1**.

Table 6.21 indicates the average percentage of learners who attained the entirely correct answers in questions requiring different DOK levels. It also shows that learners performed better in questions requiring **DOK Level-1**. An average of 39% of the learners attained the EC answer to questions requiring **DOK Level-1** and 61% of learners failed to attain the EC answer in **DOK Level-1** questions. The level of complexity in the **DOK Level-1** questions is lower than that of the other DOK levels. Hence, this demonstrates that learners are not proficient in working with the concept of functions. Although, 39% is higher than the average percentage of learners attaining the EC in questions that required **DOK Level-2** and **DOK Level-3**. In **DOK Level-2** questions, an average of 29% of learners attained the EC answers, while in **DOK Level-3**, an average of 2% of learners achieved the EC answer in all questions that required **DOK Level-3.** Hence, learners performed better in questions that required **DOK Level-1** and struggled with questions that required **DOK Level-3**.

Table 6.21: The average percentage of learners who scored the maximum marks in questions requiring different DOK levels

DOK Levels	Average Percentage of Learners Who Demonstrated the Entirely Correct Response
Level 1	39
Level 2	29
Level 3	2

ii. Most learners committed errors in procedural questions that demanded multiple steps to reach the solution.

Based on the analysis of the learners' written responses, learners were doing well on questions that required procedural knowledge. However, in some questions that required procedural knowledge, learners struggled to attain the EC answer. For example, questions 5.3 and 5.4 required the same DOK level and procedural knowledge to answer correctly. In Question 5.3, learners were examined on the *x*-intercept and in question 5.4, on the *y*-intercept. The responses indicated that many learners knew the rule of finding the intercept of the function. In question 5.4, more learners attained the EC answer compared to question 5.3. In question 5.3, learners performed more steps than in question 5.4. The responses to question 5.3 and 5.4 indicated that learners knew the principle of finding the intercept. However, if more steps are required in executions, learners commit errors (see Figure 6.3). Skemp (1976) stipulated that instrumental understanding typically requires a multiplicity of rules rather than limited concepts of more general implementations, which increases the probability of error during the process.

iii. Most learners preferred to answer questions using procedural knowledge rather than conceptual knowledge.

The memorandum showed only one possible answer to question 6.5. The marks allocated in the question were suitable for the question to be addressed using conceptual knowledge. Hence, the examiner believed that the answer could be attained using conceptual knowledge only. Engelbrecht et al. (2009) argued that the kind of knowledge necessary to solve a task is not initially determined by the task. Even if the expert categorised the task primarily as conceptual, learners may want to approach it in a strictly procedural manner. It is evident from Figure 6.10 on SAL6's response to question 6.5 that even with a question meant to be answered using conceptual knowledge, learners can attempt to answer that question using procedural knowledge. This learner response suggests that learners prefer to solve problems using procedural knowledge rather than conceptual knowledge. Learners might be preferring procedural knowledge because they are taught more procedural knowledge than conceptual knowledge. This finding concurs with the GDE SSIP content training workshops. It proves that when teachers go back to their schools from workshops, they mainly teach procedural knowledge.

iv. Most learners were struggling to sketch the function, failing to link the knowledge needed to sketch the function.

Question 5.5 asked learners to draw the hyperbola function using the answers attained in questions 5.2, 5.3, and 5.4. The interesting point is that most learners could get the correct answers in questions 5.2, 5.3, and 5.4 but failed to represent the information in the graph form. These results are consistent with the results attained by Mofolo-Mbokane et al. (2013), as it provides evidence that the learners are unable to draw the graph, even though they attained the required information, such as asymptotes and intercepts. Various scholars considered conceptual knowledge as knowledge that is abundant in relationships. It can be thought of as a linked system of knowledge, a network in which relationships are as prominent as discrete pieces of information. Based on the interpretation of this study's findings, there is evidence that learners lack conceptual knowledge of functions. Learners struggle to connect pieces of information to an interconnected network of knowledge. For example, they have the asymptote equation and intercept, but they struggle to connect it to offer a function in the Cartesian plane.

7 CHAPTER 7: INTERPRETATION OF RESULTS

7.1 INTRODUCTION

In this chapter, the results of the study are interpreted. The chapter starts with an overview of the study in Section 7.2. The research questions for this study are answered in Section 7.3 by analysing the Mathematics Paper I question paper on the concept of functions and analysing learners' scripts on their responses to the concept of functions. Recommendations for improving the conceptual knowledge of functions are discussed in Section 7.4. In Section 7.5, the limitations of the study are discussed. Section 7.6 discusses the reflection and conclusion of the study.

7.2 OVERVIEW OF THE RESEARCH

This study aimed to investigate learners' written responses to questions that require procedural knowledge, conceptual knowledge and a combination of both in the concept of functions in their Grade 11 Mathematics Paper I final examination. Chapter 1 introduced the study by outlining the background of literature that led to the topic's formulation. It also presented the motivations, problem statement, research questions and significance of the research. Chapter 2 presented the systematic overview of previous research that was performed in the context of conceptual and procedural knowledge and the concept of functions. The key findings of recent research papers were discussed and the gaps in literature were identified. The questions asked on the concept of function and learners' responses to those questions were interpreted using the notion of Webb's DOK model. The DOK model guided this study as the theoretical framework and was discussed in Chapter 3. The research design incorporating procedures, methods and sampling was discussed in Chapter 4. Chapter 5 analysed the questions where the concept of function was examined in the Mathematics Paper I Grade 11 2019 examination question paper by using the DOK model. Chapter 6 analysed the learners' responses to questions where functions were examined in the Mathematics Paper I Grade 11 2019 examination by using the DOK model.

7.3 ADDRESSING THE RESEARCH QUESTIONS FOR THIS STUDY

The study's purpose resulted in two research questions, which are addressed independently in this section. The results from the data analyses from Chapter 5 and Chapter 6 are discussed and aligned to the literature that was discussed in Chapter 2.

i. Which DOK levels were evident when analysing the Mathematics Paper I examination question paper based on the questions that required conceptual knowledge, procedural knowledge or a combination of both when tested on functions?

To address this research question, the Mathematics Paper I question paper was analysed on only the questions where functions were examined. During the analysis, the researcher focused on two elements, the form of knowledge required to respond to the question and the DOK level demanded by the question. The findings show that both forms of knowledge were presented in the questions. However, procedural knowledge was the most presented knowledge in the question paper. In 19 questions that involved the concept of functions, 11 (57.9%) questions required procedural knowledge to answer correctly. There were 3 (15.8%) questions that required conceptual knowledge. Even the marks allocated to questions that required conceptual knowledge were lower compared to questions that required procedural knowledge. Another 3 (15.8%) questions required conceptual knowledge or procedural knowledge. These questions allowed learners to choose the form of knowledge they wanted to use to solve the problem. In other words, either conceptual knowledge or procedural knowledge could be used to respond to the question. There were only 2 (10.5%) questions that required both conceptual and procedural knowledge. From the results of this study, one should note that procedural knowledge was the most assessed form of knowledge. However, these results are not in isolation as they concur with the findings of the study by Hatisaru (2020), which showed that in the task item, procedural knowledge was the most assessed knowledge compared to conceptual knowledge.

Mellor et al. (2018) analysed Grade 9 Mathematics textbooks on the concept of linear functions in South Africa and Germany. The analysis of the South African textbooks indicated that the majority of tasks available in the textbooks promoted procedural knowledge. Their findings concur with the findings of this study as it reveals that in the mathematics examination under the concept of functions, procedural knowledge was the most assessed knowledge compared to conceptual knowledge. In the process of teaching and learning, mathematics textbooks are used and if those textbooks are dominated by procedural knowledge, the examination will also be dominated by procedural questions. Mpofu and Mudaly (2020) acknowledged that there is evidence of increasing levels of learner underperformance in mathematics in the public school system in South Africa which is validated by the DBE Grade 12 examination report. Kilpatrick et al. (2001) emphasised that for learners to be competent in mathematics, both procedural and conceptual knowledge are a necessity. Therefore, examiners must equally assess procedural and conceptual knowledge.

The researcher did not only focus on the knowledge presented, but also the DOK level needed to respond to the questions. The DOK model consists of four levels of cognitive complexity. However, in this study, only **DOK Level-1** up to **Level-3** were applicable. **DOK Level-4** was not present in the scope of examination as it demands extended thinking in which learners must carry out research and apply skills and principles to a real-world context that require time to do research. The findings showed that the most manifested DOK level was **DOK Level-2**, followed by **DOK Level-3** and the least manifested DOK level was **DOK Level-1**.

ii. Which DOK levels do learners seem to respond well to based on their written responses to questions that require conceptual knowledge, procedural knowledge or a combination of both when tested on functions?

The learners' Mathematics Paper I examination scripts were analysed only on the sections where they responded to functions to address this research question. Before analysis, the researcher started by re-marking the scripts to validate the fairness of marking and consistency and to identify any biases. During the analysis, the researcher's focus was on two elements - the form of knowledge learners seemed to display in their responses and the DOK level to which learners seemed to respond well to. The findings show that in questions that could be answered using procedural knowledge or conceptual knowledge, most learners responded to those questions using procedural knowledge. In most cases, learners responded to questions using procedural knowledge even if the question could be answered using procedural knowledge or conceptual knowledge. This behaviour was observed in question 6.1 where learners were supposed to find the value of k, which was the y-intercept of the straight-line functions. Two functions were drawn (parabola and straight-line function) and these functions intersect on the *y*-intercept. Using conceptual knowledge, learners were supposed to recognise the point of intersection and identify the value of *k*, because the *y*-intercept of the parabola was given. Using procedural knowledge, learners had to substitute the coordinate of the *x*-intercept in the equation of the straight-line function and solve for *k*. Out of 19 learners from both schools who managed to have the entirely correct answer in this question, only one learner used conceptual knowledge to determine the value of *k*. This same behaviour of using procedural knowledge to answer questions that can be solved using conceptual knowledge or procedural knowledge was observed in question 6.2 and question 6.5. This finding concurs with Orhun's (2013) study that learners choose algebraic manipulation over graphical understanding. The results of this study also concur with Engelbecht et al. (2009) that stated that students attempted to answer questions that required the application of conceptual knowledge by using procedural methods.

The findings of this study also show that **DOK Level-1** was the most demonstrated DOK level in learners' responses. Hence, it is safe to say that learners did well in questions that demanded **DOK Level-1**, followed by **DOK Level-2**, while **DOK Level-3** was the least demonstrated DOK level in learners' responses. In most cases, questions that demanded **DOK Level-3**, also required learners to answer using conceptual knowledge and most learners failed to demonstrate conceptual knowledge in their responses. This lack of conceptual knowledge might be the reason why learners failed to demonstrate **DOK Level-3** in their responses. Questions 5.7, 6.3 and 6.7 were the only 3 questions that required conceptual knowledge for learners to respond correctly. From 44 learners from both schools, only 2 learners managed to demonstrate the required knowledge on question 5.7, while the majority of learners (24) did not even attempt to solve the question. In question 6.3, none of the learners from either school demonstrated the conceptual knowledge required to show at least some understanding. While in question 6.7, only one learner managed to demonstrate the required conceptual knowledge. Based on the limitations of data collection, it will be unreasonable to say that learners have no conceptual knowledge. However, the data presented reveal that learners failed to demonstrate conceptual knowledge in all the questions that required it. The results of this study concur with the findings by Mofolo-Mbokane et al. (2013) and Ivanjek, Susac, Planinic, Andrasevic & Milin-Sipus (2016) who indicated that students could work with procedural items even if they are not fully competent, but struggle with items that require conceptual knowledge.

7.4 RECOMMENDATIONS OF THE STUDY

This study's recommendations may raise awareness about learners' preferred form of knowledge for mathematics teachers, policymakers and curriculum development specialists interested in improving learners' performance in functions. Functions form the centre of mathematical subjects and provide wholeness between the subjects. For example, at the university level, the concept of functions is used in advanced mathematics courses as basic ideas (Orhun, 2013). However, research in mathematics education has shown that learners have difficulty in comprehending the concept of functions (Mpofu & Mudaly, 2020). The findings of this study also revealed that learners indeed have difficulty with the concept of functions.

Table 6.20 in Chapter 6 tells a story about learners' performance in the concept of functions. Out of 44 learners from both schools, none of the learners scored more than 70% out of 45 marks assigned to the concept of functions, with 22 learners failing to score above 30%. The situation should alarm those who have an interest in improving mathematics performance. The results of this study indicate that there is a crisis in the teaching and learning of functions and something needs to be done to prevent the continuation of this situation. This study recommends focus on the vital issue raised by Makonye (2014) who argued that one of the reasons for learners' poor performance on the concept of functions is that functions are frequently taught without a connection to the daily environment and prematurely introduces learners to the concept of functions. Teachers must incorporate learners' everyday knowledge as a steppingstone in learning the concept of functions.

The results of this study have shown that learners are failing to demonstrate conceptual knowledge whenever it is required. Other scholars such as Zulnaidi and Zakaria (2012) and Orhun (2013) indicated that learners have difficulty understanding the concept of functions, and, more specifically, developing conceptual knowledge. Panaoura, Michael-Chrysanthou, Gagatsis, Elia and Philippou (2016) claimed that learners' failure to conceptually understand functions is related to their teaching technique. Mellor et al. (2018) indicated that the mathematics textbook used during teaching and learning can influence learners' conceptual knowledge. This study recommends that textbook writers prepare textbooks that manage to equally develop both procedural and conceptual knowledge. For learners to be mathematically proficient, both conceptual and procedural knowledge are necessary (Kilpatrick et al., 2001).

These forms of knowledge are not mutually exclusive (Sfard, 1991). Thus, the two cannot be taught independently.

The optimistic way to enable students to understand mathematical concepts is using Information and Communication Technology (ICT) in teaching and learning. If learning technologies are used, students will experience distinct learning environments and cognitive obstacles will change (Zulnaidi et al., 2019). The use of technology in mathematics, more specifically in the concept of functions, has been emphasised by many researchers to improve students' conceptual knowledge of functions (Parrot & Leong, 2018; Zucker et al., 2013; Zulnaidi et al., 2019). The study by Zulnaidi et al. (2019) indicated that when learners are taught functions using GeoGebra, their comprehension of functions improves. In the study by Parrot & Leong (2018), similar findings were observed, where learners that were taught using graphing calculators, showed a better understanding than learners who were taught traditionally. Furthermore, similar findings were also observed in the study by Zucker et al. (2013) where learners were taught by using SmartGraphs software to better understand functions. From the above findings, one can comfortably conclude that the best approach for enhancing learners' conceptual knowledge of functions is the use of technology in teaching and learning. Observing the current situation in South Africa, more specifically in the Gauteng province where the study was conducted, the use of technology is feasible as more and more schools are introduced to ICT. For example, all schools surrounding the researcher's workplace, are equipped with smartboards and internet. Thus, the introduction of ICT in teaching and learning mathematics is feasible and is recommended by this study as it has proved in many research studies and findings that it can improve learners' performance in mathematics.

Functions can be taught on a procedural basis by focusing on isolated skills and steps needed in the process or taught conceptually by focusing on the broader context and relationships involved (Boote, 2014). However, students are subjected to procedural knowledge in traditional methods. Procedural knowledge can only be meaningful if it is related to conceptual knowledge (Zulnaidi et al., 2019). Teaching and learning in the classroom should encapsulate the introduction of conceptual and procedural knowledge. Zulnaidi et al. (2019) asserted that the benefit of technologies when integrated into the teaching and learning process is that students can understand the procedures and concepts of mathematics. Thus, it is recommended in this research that if teachers or policymakers are serious about enhancing conceptual knowledge of functions, they must consider the integration of technology in their teaching and learning practices.

7.5 LIMITATIONS OF THE STUDY

The data collected for this study was limited to two schools located in one township, one question paper written in only one province and there were only 44 scripts analysed in the study. During the data collection stages, the researcher experienced some challenges, such as not being allowed to enter the school premises due to COVID-19. This restricted the researcher from communicating with subject teachers and getting a greater number of scripts to analyse.

The COVID-19 pandemic limited the findings of this study. It was challenging to increase the number of schools that would take part in the study and get the opportunity to interview the subject teachers.

Time constraints limited the findings of this study as the researcher required more time to look at the trigonometric functions, which were part of the Mathematics Paper II**.** The time for analysing the errors committed by learners while working with the concept of functions in detail was limited**.** Due to the mentioned factors, the findings of this study could not be generalised. Although there is part of the finding in this study that can be generalised, the findings presented in table 6.20 can be generalised. As it concurs with the number of findings from various scholars.

7.6 REFLECTIONS AND CONCLUSION OF THE STUDY

The researcher's experience in teaching mathematics and research literature motivated the decision to conduct this study. The topic of functions was chosen because it weighs to the South African curriculum, learners' performance on the topic, and it contributes to other topics like calculus and its applicability that rely on both forms of knowledge.

Overall, this study attained that in the 2019 Grade11 Mathematics Paper I on the topic of functions, procedural knowledge was the most assessed form of knowledge and that the **DOK Level-2** was the most required DOK level. Furthermore, the findings on learners' written responses showed that learners' performance was better on questions that required procedural knowledge compared to those that required conceptual knowledge. **DOK Level-3** was poorly manifested in learners' responses compared to the other DOK levels.

Conducting this research has assisted the researcher in many ways. Understanding the duality of knowledge in mathematics, such as conceptual and procedural knowledge, has improved dramatically. The study has also assisted the researcher in enhancing the understanding of the importance of cognitive levels, especially for Webb's DOK model.

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APPENDIX A: WITS SCHOOL OF EDUCATION ETHICS CLEARANCE LETTER

WITS SCHOOL OF EDUCATION

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL OF EDUCATION ETHICS COMMITTEE
CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2019ECE047M**
PROJECT TITLE	Interpreting Grade 11 Learners' Responses to Functions in Terms of their Conceptual and Procedural Knowledge
INVESTIGATOR	Thabani Buthelezi
SCHOOL/DEPARTMENT OF INVESTIGATOR	WITS SCHOOL OF EDUCATION
DATE CONSIDERED	11 November 2019
DECISION OF THE COMMITTEE	Approved unconditionally
EXPIRY DATE	Date of submission of the project report

ISSUE DATE OF CERTIFICATE 18 November 2019 **CHAIRPERSON** (Dr. Paul Goldschagg)

cc: Supervisor: Dr. Batseba Mofolo-Mbokane

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** emailed to the Ethics Office: Matsie.Mabeta@wits.ac.za .

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

____
Signature

_______/_______/_______
Date

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

APPENDIX B: GDE RESEARCH APPROVAL LETTER

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**25 November 2019**
Validity of Research Approval:	**10 February 2020 – 30 September 2020** **2019/338**
Name of Researcher:	**Buthelezi T**
Address of Researcher:	**19 Oaklane Corner, Oaklane Estate**
	Hospital Road, Brakpan North
	Brakpan, 1541
Telephone Number:	**081 315 2345**
Email address:	**thabanib2012@gmail.com**
Research Topic:	**Interpreting grade 11 learners' responses to functions in terms of their conceptual and procedural knowledge.**
Type of qualification	**Masters' Degree**
Number and type of schools:	**Three Secondary Schools**
District/s/HO	**Ekurhuleni North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

26/11/2019

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 26|11|2019

APPENDIX C: TEACHERS INFORMATION SHEET

22 November 2019

Dear Teacher

My name is Thabani Buthelezi, and I am registered for a Masters in Mathematics Education with the School of Education at the University of the Witwatersrand. I am researching to explore learner's responses to questions that require procedural and conceptual knowledge in the concept of functions in grade 11. My supervisor is Dr. B. Mofolo-Mbokane. My research will involve an analysis of grade 11 learners examination scripts for mathematics paper one, to determine the impact of conceptual and procedural understanding on learners' performance in the concept of functions. It will also be interview teachers to accumulate information about their views concerning procedural and conceptual knowledge in the concept of functions in grade 11 mathematics. The interviews will tape-record. I will photocopy the learners' scripts for the researcher to remark and able to analyze the script question by question under the section of functions.

I would like to request you to take part in my research. If you take part in this research, I will request to interview you as a teacher who is teaching grade 11 and I will tape-record the interview. I will keep your name and identity confidential at all times and in all academic writing about the study. Your privacy will be maintained in all published and written data resulting from the study.

All research data scripts will be kept in a locked office in the Wits School of Education and will be destroyed in 5 years after completion of the project. You will not be advantaged or disadvantaged. Your participation is voluntary, so you can withdraw your permission during this project with no penalty. There are no foreseeable risks in taking part and I will not pay you for this study. Please let me know if you require any further information.

Thank you so much for your help. For questions please contact either me or my supervisor

Yours sincerely,

Buthelezi	Mbokane
Mr Thabani Buthelezi	Dr Batseba Mofolo-Mbokane
Student (thabanib2012@gmail.com)	Supervisor (Batseba.Mbokane@wits.ac.za)

APPENDIX D: TEACHER CONSENT FORM

Research Topic: Interpreting Grade 11 learners’ responses to Functions in terms of their Conceptual and Procedural knowledge

Please fill in and return the reply slip below showing your willingness to be a participant in my voluntary research project.

I, ____give my consent for:

☐ I agree to being interviewed in the research study outlined above.

☐ My participation is voluntary, and I can withdraw at any time.

☐ I agree that learners' scripts will be photocopied the interviews will be tape-recorded.

☐ I am aware that my privacy and identity will be kept confidential at all times and in all academic writing about the study.

MY SIGNATURE SHOWS THAT I HAVE DECIDED TO TAKE PART HAVING READ THE INFORMATION PROVIDED ABOVE.

Sign____ Date_______________________

APPENDIX E: LETTER TO PRINCIPAL

22 November 2019

Dear Principal

My name is Thabani Buthelezi, and I am registered for a Masters in Mathematics Education with the School of Education at the University of Witwatersrand. I am researching to explore learner’s responses to questions that require procedural and conceptual knowledge in the concept of functions in grade 11. My supervisor is Dr. B. Mofolo-Mbokane. My research will involve an analysis of grade 11 learners examination scripts for mathematics paper one, to determine the impact of conceptual and procedural understanding on learners’ performance in the concept of functions. It will also be interview teachers to accumulate information about their views concerning procedural and conceptual knowledge in the concept of functions in grade 11 mathematics. The interviews will tape-record. I will photocopy learners’ scripts for the researcher to remark and able to analyze the script question by question under functions.

I request permission to carry out my research in the school that is under your management, the study will be carried out after school hours and research activities will be done within the school environment. The research will require:

- Participation of one teacher who teaches grade 11 mathematics
- Examination script for grade 11 learner mathematics paper one.
- I will photocopy the learners’ scripts and will tape-record the interviews.

There will be an interview conducted with a teacher; the interviews will be for a maximum of 30 minutes. I will conduct the interview after school hours or at any time that will be suggested by a teacher when s/he is free.

The research participants will not be advantaged or disadvantaged in any way. I will reassure them they can withdraw their permission during this project with no penalty. There are no foreseeable risks in taking part in this study. The participants will not be paid for this study. The names of teachers, learners' script, and the name of the school will be kept confidential at all times and in all academic writing about the study. Learners will remain anonymous and information from their scripts confidential. I will maintain your privacy in all published and written data resulting from the study. All research data scripts will be kept in a locked office in the Wits School of Education and will be destroyed in 5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Buthelezi

Mbokane

Mr Thabani Buthelezi — Student (thabanib2012@gmail.com)

Dr Batseba Mofolo-Mbokane — Supervisor (Batseba.Mbokane@wits.ac.za

APPENDIX F: LETTER TO THE PARENT/GUARDIAN

22 November 2019

Dear Parent/Guardian

My name is Thabani Buthelezi, and I am registered for a Masters in Mathematics Education with the School of Education at the University of Witwatersrand. I am researching to explore learner's responses to questions that require procedural and conceptual knowledge in the concept of functions in grade 11. My supervisor is Dr. B. Mofolo-Mbokane. My research will involve an analysis of grade 11 learners examination scripts for mathematics paper one, to determine the impact of conceptual and procedural understanding on learners' performance in the concept of functions. It will also be interview teachers to accumulate information about their views concerning procedural and conceptual knowledge in the concept of functions in grade 11 mathematics. The interviews will tape-record. I will photocopy the learners' scripts for the researcher to remark and able to analyze the question by question under the section of functions.

I would like to request permission for your child to take part in my research. If you allow your child to take part in this research, I will use the child's final examination script. The name of the child will be kept confidential at all times and in all academic writing about the study. Child individual privacy will be maintained in all published and written data resulting from the study.

The research participants will not be advantaged or disadvantaged. I will reassure them that they can withdraw their permission during this project with no penalty. There are no foreseeable risks in taking part in this study. The participants will not be paid for this study. The names of teachers, learners' script, and the name of the school will be kept confidential at all times and in all academic writing about the study. Learners will remain anonymous and information from their scripts confidential. Your privacy will be maintained in all published and written data resulting from the study. All research data scripts will be kept in a locked office in the Wits School of Education and will be destroyed in 5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Buthelezi Mbokane

Mr Thabani Buthelezi Dr Batseba Mofolo-Mbokane

Student (thabanib2012@gmail.com) Supervisor (Batseba.Mbokane@wits.ac.za)

APPENDIX G: PARENT/GUARDIAN CONSENT FORM

Research Topic: Interpreting Grade 11 learners' responses to Functions in terms of their Conceptual and Procedural knowledge

Please fill in and return the reply slip below showing your willingness to allow your child to be a participant in my voluntary research project.

I, ____give my consent for:

☐ I allow my child exam script to be photocopied and analyzed in the research study outlined above only.

☐ My child participation is voluntary, and s/he can withdraw at any time.

☐ I agree that my child script will be photocopied his/her name to hide.

☐ I am aware that my child's privacy and identity will be kept confidential at all times and in all academic writing about the study.

MY SIGNATURE SHOWS THAT I HAVE ALLOWED MY CHILD TO TAKE PART HAVING READ THE INFORMATION PROVIDED ABOVE.

Sign____ Date______________________

APPENDIX H: LETTER TO THE LEARNER

22 November 2019

Dear Learner

My name is Thabani Buthelezi, and I am registered for a Masters in Mathematics Education with the School of Education at the University of Witwatersrand. I am researching to explore learner's responses to questions that require procedural and conceptual knowledge in the concept of functions in grade 11. My supervisor is Dr. B. Mofolo-Mbokane. My research will involve an analysis of grade 11 learners examination scripts for mathematics paper one, to determine the impact of conceptual and procedural understanding on learners' performance in the concept of functions. It will also be interview teachers to accumulate information about their views concerning procedural and conceptual knowledge in the concept of functions in grade 11 mathematics. The interviews will tape-record. I will photocopy the learners' scripts for the researcher to remark and able to analyze the script question by question under the section of functions.

I would like to request you to take part in my research, by permitting me to analyze your final examination script. Your name and identity will be kept confidential at all times and in all academic writing about the study. You will remain anonymous and your information from the scripts confidential. Your privacy will be maintained in all published and written data resulting from the study.

All research data scripts will be kept in a locked office in the Wits School of Education and be destroyed 5 years after completion of the project. You will not be advantaged or disadvantaged. Your participation is voluntary, so you can withdraw your permission during this project with no penalty. There are no foreseeable risks in taking part and you will not be paid for this study. Please let me know if you require any further information.

Thank you so much for your help. For questions please contact either me or my supervisor

Yours sincerely,

Yours sincerely,

Buthelezi　　　　Mbokane

Mr Thabani Buthelezi　　　　Dr Batseba Mofolo-Mbokane

Student (thabanib2012@gmail.com)　　　　Supervisor (Batseba.Mbokane@wits.ac.za)

APPENDIX I: LEARNER CONSENT FORM

Research Topic: Interpreting Grade 11 learners' responses to Functions in terms of their Conceptual and Procedural knowledge

Please fill in and return the reply slip below showing your willingness to be a participant in my voluntary research project.

I, ____give my consent for:

☐ I agree that my exam script can be photocopied and analyzed in the research study outlined above only.

☐ My participation is voluntary, and I can withdraw at any time.

☐ I agree that my scripts will be photocopied and my details will be hidden.

☐ I am aware that my privacy and identity will be kept confidential at all times and in all academic writing about the study.

MY SIGNATURE SHOWS THAT I HAVE DECIDED TO PARTICIPATE HAVING READ THE INFORMATION PROVIDED ABOVE.

Sign_____________________________________ Date______________________

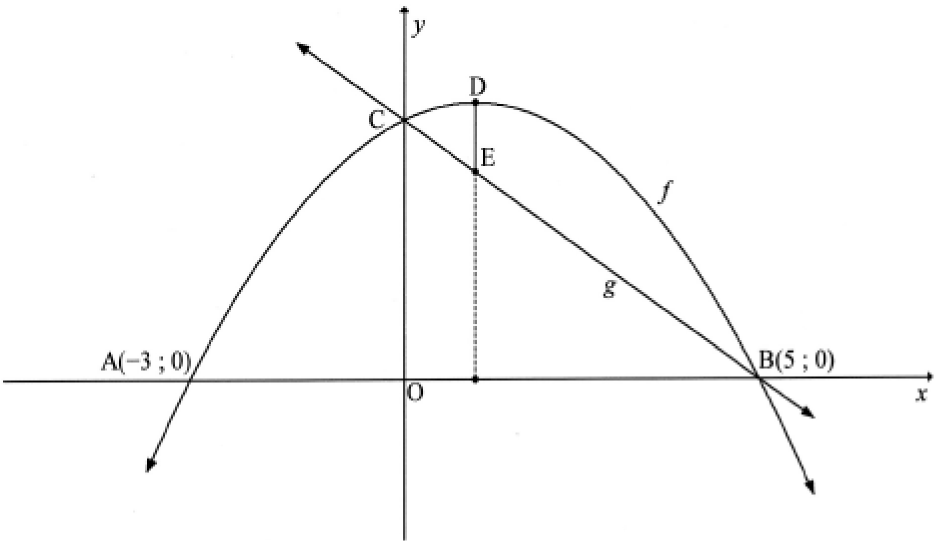
QUESTION 5

Given: $f(x)=\frac{1}{x-3}-\frac{2x+6}{x+3}$

5.1	Show that $f(x)$ can be written as $f(x)=\frac{1}{x-3}-2$	(2)
5.2	Write down the equations of the asymptotes of f.	(2)
5.3	Determine the x-intercept of f.	(3)
5.4	Determine the y-intercept of f.	(2)
5.5	Sketch the graph of f. Show clearly ALL the intercepts with the axes and the asymptotes.	(3)
5.6	Determine the equation of the axis of symmetry of f having positive gradient.	(3)
5.7	The graph of f is transformed to obtain the graph of $h(x)=\frac{1}{x}$. Describe the transformation from f to h.	(2)
5.8	Write down the domain of h.	(2) **[19]**

QUESTION 6

The diagram below shows the graphs of $f(x)=-x^2+2x+15$ and $g(x)=-3x+k$.
Graph f cuts the x-axis at A(–3 ; 0) and B(5 ; 0), the y-axis at C and has a turning point at D. Graph g cuts the x-axis at B and the y-axis at C. E is a point on g such DE is parallel to the y-axis.



6.1	Show that $k = 15$.	(1)
6.2	Determine the coordinates of D, the turning point of f.	(3)
6.3	Determine the values of x for which f is increasing.	(1)
6.4	Calculate the average gradient between points A and D.	(3)
6.5	Calculate the length of DE.	(2)
6.6	If $h(x) = f(x-1) - 2$, determine the equation of h in the form $h(x) = a(x+p)^2 + q$.	(4)
6.7	Determine the maximum value of $p(x) = 3^{f(x)-12}$	(3)
6.8	Determine the values of x for which $f(x) + k = 0$ will have two distinct real roots.	(2)
		[19]

QUESTION 7

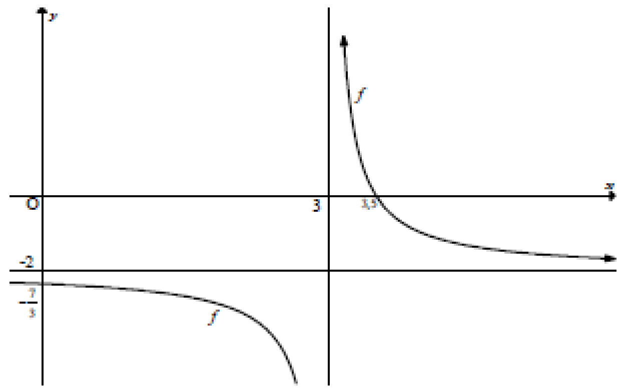
The point A(3 ; 54) lies on the graph of $f(x) = 3^{x+p} - 27$.

7.1	Determine the value of p.	(3)
7.2	Determine the range of f.	(2)
7.3	Graph g is obtained by reflecting graph f about the x-axis. Determine the coordinates of the y-intercept of g.	(2)
		[7]

APPENDIX K: GRADE 11 NOVEMBER 2019 MATHEMATICS PAPER 1 MEMORANDUM

QUESTION/*VRAAG* 5

5.1	$f(x)=\frac{1}{x-3}-\frac{2x+6}{x+3}$ $=\frac{1}{x-3}-\frac{2(x+3)}{x+3}$ $=\frac{1}{x-3}-2$	✓common factor *gemene faktor* ✓simplification/*vereenv.* (2)
5.2	$x=3$ $y=-2$	✓$x=3$ ✓$y=-2$ (2)
5.3	$0=\frac{1}{x-3}-2$ $2=\frac{1}{x-3}$ $2(x-3)=1$ $2x-6=1$ $x=\frac{7}{2}$	✓subst./*verv.* $y=0$ ✓simplification/*vereenv.* ✓answer/*antw.* (3)
5.4	$y=\frac{1}{0-3}-2$ $=\frac{-7}{3}$ **OR/*OF*** $\left(0;\frac{-7}{3}\right)$	✓subst/*verv.* $x=0$ ✓answer/*antw.* ✓✓answer/*antw* (2)

5.5		✓asymptotes/*asimptote* ✓shape/*vorm* ✓x- and y- int. (3)

5.6	$y=x+c$ $-2=3+c$ $c=-5$ $y=x-5$	✓ $m=1$ ✓substitution of/*vervanging van* $(3;-2)$ ✓ $c=-5$ (3)
5.7	Translate f 3 units to the left and 2 units up. *Transleer f 3 eenhede na links en 2 eenhede op.*	✓3 units to the left *eenhede na links* 2 units up/*eenhede op* (2)
5.8	$x\in(-\infty;+\infty)$; $x\neq 0$ **OR/*OF*** $x\in R; x\neq 0$	✓ $x\in(-\infty;+\infty)$ ✓ $x\neq 0$ (2) **[19]**

QUESTION/*VRAAG* 6

6.1	$y=-3x+k$ $0=-3(5)+k$ $k=15$	✓ substitute/*verv.* $(5;0)$ (1)
6.2	$x=\frac{-b}{2a}$ $x=\frac{-2}{2(-1)}$ $x=1$ *or* $x=\frac{-3+5}{2}$ $=1$ $y=-(1)^2+2(1)+15$ $y=16$ $D(1;16)$	✓ $x=1$ ✓substitution/*vervanging* ✓ $y=16$ (3)
6.3	$x<1$ **OR/*OF*** $x\in(-\infty;1)$	✓ answer/*antwoord* (1)
6.4	$A(-3;0)$ $D(1;16)$ Ave grad / *Gemidgrad* $=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{0-16}{-3-1}$ $=4$	✓formula/*formule* ✓subst. into correct formula /*verv. in formule* ✓answer/*antwoord* (3)

6.5	D(1 ;16) E(1;12) DE = 4units	✓ E(1;12) ✓ answer/*antwoord* (2)
6.6	$h(x)=f(x-1)-2$ $=-(x-1)^2+2(x-1)+15-2$ $=-x^2+2x-1+2x-2+15-2$ $=-x^2+4x+10$ $=-(x^2-4x-10)$ $=-(x^2+-4x+4-4-10)$ $=-(x-2)^2+14$ **OR/*OF*** $D(1;16)$ and $a=-1$ $f(x)=-(x-1)^2+16$ $h(x)=f(x-1)-2$ $=-(x-1-1)^2+16-2$ $=-(x-2)^2+14$	✓ $-(x-1)^2+2(x-1)+15-2$ ✓ $-x^2+4x+10$ ✓ $-(x^2+-4x+4-4-10)$ ✓ $h(x)=-(x-2)^2+14$ **OR/*OF*** ✓ subst./*vervang* $D(1;16)$ and $a=-1$ ✓ correct form/*korrekte vorm* ✓ $h(x)=-(x-1-1)^2+16-2$ ✓ $h(x)=-(x-2)^2+14$ (4)
6.7	max value of/*maks wrde van* $f(x)$ is 16 $\therefore$ max value of/*maks wrde van* $f(x)-12$ is $16-12=4$ max/*maks* $p(x)=3^4$ $=81$	✓ max value of $f(x)$ is 16 ✓ max value of $f(x)-12$ is $16-12=4$ ✓ answer/*antw.* (3)
6.8	$x\in R;x\neq 1$	✓✓ answer/*antwoord* (2) **[19]**

QUESTION/*VRAAG* 7

7.1	$y = 3^{x+p} - 27$ $54 = 3^{3+p} - 27$ $81 = 3^{3+p}$ $3^4 = 3^{3+p}$ $3 + p = 4$ $p = 1$	\\ ✓subs/*vervanging* (3 ; 54) ✓ equating indices/*gelykst. eksp.* ✓answer/*antwoord* (3)
7.2	range / *waardeversameling* $y > -27$ or / *of* $y \in (-27; \infty)$	✓✓ answer/*antwoord* (2)
7.3	$-y = 3^{x+1} - 27$ $g(x) = -1.3^{x+1} + 27$ $y = -1.3^{0+1} + 27$ $= 24$ y – intercept/*afsnit*$(0; 24)$	✓new equation/*nuwe verg.* ✓ answer/*antwoord* (2) [7]

APPENDIX L: TURNITIN REPORT

2290217:30_April_2021_Thabani_Research_Final_Draft_with_...

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