A HEDGE EXTENSION OF THE CAPM MODEL: AN EMPIRICAL INVESTIGATION

by

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ABSTRACT

Extending the CAPM with practical hedging strategies presents an innovative approach to managing portfolios. This study extends CAPM through incorporation of a Hedge Extension that utilizes cointegration-based pairs trading within a core satellite portfolio framework. While CAPM remains foundational in asset pricing and portfolio optimization, its reliance on market efficiency assumptions and the exclusion of idiosyncratic risk undermines its efficacy in volatile markets. This paper bridges these gaps by constructing a core satellite portfolio where the JALSH functions as the passive core, while a dynamically managed satellite component employs cointegration-based pairs trading that provides an uncorrelated source of returns due to relative mispricing. Using daily closing prices of Financial and Basic Material sector stocks listed on the JSE between 2nd January 2015 and 31st December 2024, this study tests the Hedge Extension's efficacy. The pairs were selected using the Engle-Granger two-step approach, where positions were traded when normalized cointegration spreads exceeded ±2 standard deviations, and unwound upon mean reversion. The hedge portfolio was constrained to a gross exposure limit of 200% and 2.5% net exposure cap per pair on entry.

In terms of risk-adjusted returns, the hedge portfolio notably outperformed the market portfolio. The performance yielded a Jensen's alpha of 4.28%, a Sharpe ratio of 0.1582 (in contrast to -0.1754), along with a Modigliani Squared ratio of 11.39% (compared to the markets return of 5.49%). Cumulative returns for the hedge portfolio reached 111.15%, substantially exceeding the 68.50% achieved by the market. The hedge portfolio's systematic risk decreased significantly due to the hedging effect of cointegrated pairs, while idiosyncratic risk increased marginally due to higher gross exposure and residual pair volatility. These findings validate the hedge extension of CAPM's efficacy and reveal how cointegration-based pairs trading is used as a mechanism to enhance CAPM's practical utility. Thus, the strategy offers a comprehensive framework for portfolio construction, equipping portfolio managers with practical insights into how this hedging strategy can serve as an effective alpha generator within a diversified portfolio. This study advances portfolio management theory by demonstrating that integrating a dynamic hedging strategy with equilibrium models can reconcile theoretical frameworks with real world market complexities.

Keywords: Capital Asset Pricing Model, Portfolio Management, Cointegration-Based Pairs Trading, Mean Reversion

DECLARATION

I, Donovan Shaun Waring, declare that this research report is my own work except as indicated in the references. It is submitted in partial fulfilment of the requirements for the degree of Master of Management in Finance and Investment at the University of the Witwatersrand, South Africa. It has not been submitted before for any degree or examination in this or any other university.

……………………………… Date:

Donovan Shaun Waring 27th February 2025

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1. INTRODUCTION

1.1 Background to the study

Using the Capital Asset Pricing Model (CAPM), the optimal portfolio is determined as the point at which the efficient frontier is tangent to the risk-free rate. Following the efficient market hypothesis, holding the optimal portfolio maximizes expected returns at a certain risk level. A portfolio is closest to the optimal portfolio the more diversified it is, normally holding between 50-100 selected assets (Treynor & Black, 1973). CAPM's practical applications have cemented its status as a fundamental tool in investment management. CAPM aids in optimizing the risk-return tradeoff within a portfolio. Fama and French (1993) stated that CAPM's attraction is in its offering of intuitively powerful predictions of risk measurement and its relation to expected return. Discounted cash flow models require the determination of discount rates which is derived using CAPM, thus assisting portfolio managers with asset selection. As pointed out by Fama and French (2004), the significance and presence of CAPM in practice and research has been widely accepted.

CAPM provides a benchmark for portfolio performance assessment. Comparing the portfolio's expected return rooted in its beta, a sensitivity measure to market volatility, against the portfolio's actual return. It allows for portfolio evaluation relative to the market. Jensen's alpha evaluates the additional return earned by a portfolio's active component over the expected CAPM return (Jensen, 1968). This is an essential metric in the evaluating the performance of different portfolios. Jensen's alpha serves as a widely employed measure of performance attribution in the asset management industry.

CAPM is criticized largely for its fundamental assumptions that exclude idiosyncratic risk and presuppose market efficiency. As indicated by Banz (1981), idiosyncratic factors further explain cross-sectional variation in asset returns than systematic risk. Chen (2021) noted that investor irrationality leads to mispricing, thus contradicting CAPM's market efficiency assumption. Volatile market conditions have made the limitations of CAPM more noticeable. Alternative models, such as Arbitrage Pricing Theory (APT) due to Ross (1976), encompass additional components of risk above CAPM to account for the fluctuations in asset returns. However, demand for advanced risk management is overlooked in these theoretical models. Abderisak and Göran (2019) noted that technological advancements in risk management allow for the dynamic mitigation and identification of risks instead of reliance on static theoretical models.

The study aims to extend CAPM by integrating the cointegration-based pairs trading strategy within a core satellite portfolio framework, testing whether this hybrid approach enhances risk-adjusted returns compared to traditional CAPM. It addresses CAPM's limitations, specifically its reliance on market efficiency assumptions and exclusion of idiosyncratic risk, by proposing a dynamic hedging mechanism that exploits relative mispricing between cointegrated assets.

1.2 Significance of the study

By incorporating cointegration-based pairs trading using the core satellite approach, this research extends CAPM's framework. This extension addresses the limitations of CAPM by providing a more encompassing understanding concerning the risk-return relationship. The hedge extension of CAPM presents an effective solution to portfolio construction during volatile periods, bridging the gap between industry practice and theory. This extension introduces advanced risk management processes within CAPM, and it increases the effectiveness of portfolio management.

The proposed extension encompasses the changes within financial markets by constructing sound portfolios that are better positioned to be robust to the volatility in financial markets. The strategy achieves uncorrelated returns at reduced risk levels to the market. The hedging strategy takes up a small portion of the portfolio as pairs mostly finance themselves due to the nature of going long on one asset whilst shorting another simultaneously. The core satellite approach increases the probability of generating superior risk-adjusted returns than CAPM. This is a result of the core consisting of the market portfolio and the satellite portion being the alpha generator.

1.3 Research Problem

Behavioral finance highlights the impact of investor psychology on market efficiency, challenging CAPM's reliance on the efficient market hypothesis. Empirical evidence has shown that idiosyncratic risk is prevalent in financial markets and should be considered in portfolio construction. This has led to several papers criticizing CAPM's problems. As noted by DeBondt and Thaler (1985), inefficiencies and abnormalities in financial markets are not explained due to the equilibrium assumption of CAPM. As noted by Odean (1998), investors expose themselves to idiosyncratic risk as they are inclined to hold concentrated portfolios which they also frequently trade. As pointed out by Jegadeesh and Titman (1993), assets with elevated idiosyncratic risk levels display patterns that deviate from the expectations of CAPM. Simplistic assumptions such as an efficient market and the exclusion of idiosyncratic risk in CAPM are problems which are commonly criticized. A critical gap persists between equilibrium-based models and real-world portfolio management needs, particularly in emerging markets like South Africa where liquidity constraints amplify mispricing. This has resulted in gaps between theory and practical applications, especially during periods of market uncertainty, necessitating an integrated framework that reconciles CAPM's theoretical rigor with dynamic risk mitigation tools.

1.4 Research Question

The research question is as follows:

Does the incorporation of cointegration-based pairs trading using the core satellite approach increase the risk-adjusted portfolio return of CAPM?

1.5 Hypothesis

The hypothesis is as follows:

H_0 : The incorporation of cointegration-based pairs trading using the core satellite approach does not increase the portfolio's risk-adjusted return of the hedge extension relative to CAPM.

H_1 : The incorporation of cointegration-based pairs trading using the core satellite approach increases the portfolio's risk-adjusted return of the hedge extension relative to CAPM.

1.6 Contribution

The incorporation of hedging mechanisms and advanced portfolio construction methods into CAPM has not previously been done. The increasing integration of financial markets presents an opportunity to develop an extension of CAPM that is reflective of these changes, closing a gap in the existing literature. The proposed research of a hedge extension of the CAPM is significant because it bridges the gap between industry practical application and theory. Li (2024) stated that traditional models like APT and CAPM provide a sound foundation however, they fall short in addressing real world financial markets complexities and increasing dynamic changes in asset prices. CAPM assumes the relationship linking return and risk is linear without considering the dynamic interactions between correlated assets that pairs trading exploit.

The motivation of this research is CAPM's inadequate assumptions of market efficiency and the exclusion of idiosyncratic risk. Cointegration-based pairs trading strategies as an uncorrelated alpha generator and the core satellite approach have independently surfaced as credible risk management strategies. As noted by Andrade et al. (2005), pairs trading achieves effective hedging by taking an offsetting position with homogeneous factor exposures, resulting in low exposure to risk. Lemishko et al. (2024) noted that cointegration-based pairs trading displays a stable and consistent rising trend in cumulative returns coupled with relatively shallow drawdowns that recover swiftly, resulting in an effective and resilient tool to manage risk. Welch (2008) stated that the core satellite approach to portfolio construction balances risk and return effectively by capitalizing on an active strategy to create alpha and a passive strategy to stabilize volatility. The integration of these strategies into CAPM bridges the gap in existing literature.

The research extends CAPM's framework by combining it with cointegration-based pairs trading, using the core satellite approach to portfolio construction. The incorporation of cointegration-based pairs trading creates a source of uncorrelated returns as a function of market mispricing. This strategy benefits investors by reducing exposure to market volatility. This extension not only enhances CAPM but also broadens its applicability in conditions where market uncertainty is high. The research provides portfolio managers with a practical strategy by enhancing CAPM through advanced risk management strategies. The findings also provide insights into enhancing portfolio resilience against market volatility and optimizing performance, which are essential for effective portfolio management.

The hedge portfolio generated a statistically significant Jensen's alpha of 4.28%, indicating outperformance relative to CAPM's expected returns. Risk-adjusted returns, measured by a Sharpe ratio of 0.1582, substantially exceeded the market portfolio's -0.1754, while cumulative returns reached 111.15% versus the market's 68.50% over the sample period. Systematic risk exposure decreased by 22% compared to the market benchmark, attributed to the hedging effect of cointegrated pairs, though idiosyncratic risk rose marginally due to residual spread volatility. The Modigliani Squared ratio of 11.39%, double the market's 5.49%, confirmed the strategy's efficacy in enhancing risk-adjusted performance during market dislocations. These findings demonstrate that integrating cointegration-based pairs trading into CAPM's framework successfully mitigates its theoretical limitations while capitalizing on transient mispricing opportunities.

2. LITERATURE REVIEW

In this chapter, we present and critique CAPM and substantiate how its shortcomings have resulted in subsequent extensions being developed. The core satellite approach is briefly discussed. We explain where it comes from and how it functions. We delve into the limited literature on the pairs trading strategy in its various forms, explaining how its performance is not attributable to systematic risk. The review is instrumental to the understanding of both CAPM and the pairs trading strategy. This sets up how and why these concepts should be linked through the core satellite approach to create a risk-adjusted optimal portfolio.

2.1 Uses of the CAPM Model

The portfolio manager's purpose is the management of securities to achieve predetermined objectives by constructing portfolios that increase returns while reducing risk, using modern portfolio theory (Mazzola & Gerace, 2015). This optimal portfolio can consist of several blends of risk and return depending on an investor's objectives and time horizon. These portfolios fall on the efficient frontier through optimizing methods by prioritizing efforts to reach the highest expected return and solving for the associated risk level and vice versa. (Perold, 2004, pp. 10 - 11) stated that, "portfolio theory prescribes that investors choose their portfolios on the efficient frontier, given their beliefs about expected returns and risk. The Capital Asset Pricing Model, on the other hand, is concerned with the pricing of assets in equilibrium". CAPM provides a foundation for asset pricing in equilibrium by extending the framework of mean-variance optimization developed by Markowitz (1952). Assuming equilibrium in the market, CAPM asserts that expected returns and market risk have a linear relationship.

The market compensates investors for taking specific risks by providing a return, however risk that can be diversified away does not warrant compensation when following the theory of CAPM (Perold, 2004). Ansari (2000) noted that the implementation of CAPM is beneficial when allocating capital to real and financial assets. The model has been extensively used in corporate finance with respect to financing, capital expenditure and restructuring. Companies use CAPM to calculate their expected return to determine their equity value, which guides them in evaluating which source of financing, equity or debt, is most beneficial (Ansari, 2000). As noted by Ansari (2000), CAPM is used in discounting future cash flows through calculating the appropriate cost of capital when making capital expenditure decisions. Company restructuring requires the resulting risks to be determined using CAPM to ensure value is generated, as diversification does not necessarily result in an increase in expected return (Ansari, 2000).

CAPM is fundamental in asset management as it is used to measure portfolio performance (Perold, 2004). The model has allowed for the quantification of risk and the associated compensation for bearing the risk by economists (Ansari, 2000). As noted by Merton (1973), the theoretical foundation of asset pricing and portfolio optimization was enhanced by introduction of CAPM, which provided a systematic framework for understanding the risk and expected return relationship. CAPM clarified how diversification can mitigate unsystematic risk and established a benchmark for portfolio performance assessment. The development of our understanding regarding the operation of financial markets through the integration of risk and return has resulted in the advancement of asset pricing models.

2.2 Criticism of the CAPM Model

CAPM has been criticized for a number of reasons. Firstly, investors at large do not exploit all arbitrage opportunities to correct mispricing. Practically, they are discouraged to correct some of the mispricing because it may have resulted from idiosyncratic risk (Ansari, 2000). This clarifies that investors are not solely concerned with systematic risk but seek compensation for the total risk borne. Secondly, tax focused investors manage portfolios to realize losses early and defer any gains made. This results in an increase in idiosyncratic risk because investors adjust their portfolios for tax purposes, see Constantinides (1983) and Perold (2004). Thirdly, Brennan and Lo (2010) show that shorts arise in the mean-variance efficient frontier, suggesting that the relationship between efficiency and equilibrium is a primary contradiction in CAPM. Fourth, Campbell et al. (1997) and Ansari (2000) stated that the attitude of investors towards returns and risks are not linear as proposed by CAPM along with markets being characterized by chaotic conditions associated with a disequilibrium.

These criticisms have resulted in researchers developing extensions to CAPM to resolve its unrealistic assumptions (Mazzola & Gerace, 2015). CAPM’s assumptions of perfect capital markets, rational investors, similar expectations, and no taxes or transaction costs are widely viewed as unrealistic. Black (1972) noted that deviations from CAPM’s assumptions in financial markets will lead to structural biases in the expectations formed using the model. Jagannathan and Wang (1996) also argued that assuming a risk premium is constant over time in CAPM is not practical. The proposal of a time-varying risk premium integrated into a conditional CAPM has shown significant performance advantages. Najand, Yan, and Fitzgerald (2006) found that the conditional CAPM significantly outperformed the traditional CAPM as it provided more accurate return expectations during periods of market uncertainty. As indicated by Vendrame, Guermat, and Tucker (2018) incorporating a risk premium that is time-varying into CAPM results in predictive accuracy and improved model fit when compared to the traditional CAPM. Psychological aspects such as investor conduct are excluded from CAPM. Kahneman and Tversky (1979) found that investors risk preference is shown to vary, deviating from CAPM’s static assumption.

2.3 Extensions of the CAPM Model

The absence of congruence to the real world, coupled with the failure to validate the model in research-based studies have resulted in several rejections of CAPM (Basu & Chawla, 2012). This has prompted several extensions of CAPM to be developed. These extensions aim to accommodate real world complexities (Perold, 2004). Investors use various observational methods when pricing assets, and alternative optimization methods when choosing portfolios depending on their subjectivity (Surtee & Alagidede, 2023). Some of the alternative methods most commonly used in practice are the Intertemporal CAPM due to Merton (1973), Fama-French Three-Factor Model due to Fama and French (1993), and Arbitrage Pricing Theory due to Ross (1976).

The International CAPM is focused on the consumption requirements of investors particular to their country or residence. The investors purchasing power in a specific currency is of concern when pricing assets and making investment decisions as it affects portfolio risk. International investors place significant emphasis on currency risk as a contributing factor to portfolio risk when making investment decisions. The expected return of the model involves the portfolio's beta that is exposed to market risk as well as the beta that is exposed to real currency fluctuations.

The Intertemporal CAPM of Merton (1973) is focused on the opportunities to consume the return during the investment period and not only the final return. Therefore, investors consider current and expected economic variables at the initial investment period when selecting a suitable portfolio. Consequently, these portfolios are considered multifactor efficient as they offer the highest expected return for the associated variance and covariance of returns with the applicable economic variables (Fama & French, 2004). This suggests a relationship between the market beta and additional economic variable betas tied to expected returns.

The expectation that investors hold a fully diversified portfolio imitating the market is an impractical CAPM assumption. Its inability to capture size and value effects is considered an irregularity, resulting in alternative models being created. As indicated by Fama and French (1993) the size of companies, excess market return, and book-to-market ratio explain returns more extensively. These variables generate undiversifiable risks that are not explained by systematic risk in estimating asset pricing and explaining returns (Fama & French, 2004). Fama and French (1993) supported their finding by showing that the returns of large companies and value stocks co-vary more with each other than with growth stocks. They uncovered that the proposed three-factor model captures more fluctuations in portfolio returns than CAPM.

APT explains that a linear relationship exists between expected return and multiple macroeconomic variables, known as factor loadings (Ross, 1976). The APT model demands more realistic assumptions than that of CAPM and is based on the law of one price. The theory behind the law of one price is based on a market in equilibrium where rational investors eliminate arbitrage opportunities that exist between two assets with an identical payoff as they should have identical prices. (Basu & Chawla, 2012). The identification of appropriate macroeconomic variables that can be considered as a substitute for systematic risk is considered as both a limitation and a benefit. The APT allows its user to decide on which macroeconomic factors to utilize that best explain the variation in asset returns (Basu & Chawla, 2012). This is useful as the model argues that individual assets react differently depending on the type of news.

A commonality amongst most CAPM extensions is that investors have different optimal portfolios, depending on their unique circumstances and requirements (Perold, 2004). It is for this reason that investors allocate their capital across different risky portfolios which aggregate to form the market portfolio. Beta, as a risk measurement of the market, is used to determine the expected return as per CAPM and is questioned by this observation. Thus, suggesting that beta may not adequately capture the dynamic risk preferences and diverse investment strategies of investors.

2.4 Core Satellite Approach

Portfolio construction is the process of asset allocation and weighting within a portfolio, and how these variables interact will result in a specific risk and return profile. Markowitz (1952) introduced Modern Portfolio Theory (MPT) which is the principal theory used in the construction of asset portfolios. The MPT places significant emphasis on diversification as a tool to enhance portfolio returns on a risk-adjusted basis. The amalgamation of passive and active management in a single portfolio to reduce risk and improve returns was introduced by Treynor and Black (1973). An example of this is the core satellite approach which emerged from modern portfolio theory, advocating for diversification to reduce risk. The core satellite method is used to determine asset allocation when constructing a portfolio which consists of a secure diversified core, coupled with an alpha generating satellite (Nielsen et al., 2012). The amalgamation that constitutes the core satellite method has been shown to reduce tracking error without sacrificing outperformance (Blake et al., 1999). The core satellite method aligns with the advancements seen in financial markets through adjustability of asset allocation within the portfolio. The satellite is constituted by a pairs trading strategy, where it is expected to generate alpha whilst simultaneously reducing market risk.

2.5 What is Pairs Trading?

Hedge funds are known to capitalize on arbitrage opportunities, caused by market inefficiencies, through the use of statistical arbitrage (Perlin, 2009). Statistical arbitrage constitutes a degree of market neutrality that uses statistical models to find, and subsequently exploit mispricing between assets (Rad et al., 2016). Relative value strategies capitalize on disequilibrium in financial markets by simultaneously buying one asset while selling another (Andrade et al., 2005). These strategies take advantage of relative pricing discrepancies that aim to capitalize on market inefficiencies while maintaining market neutrality. They are intrinsically linked to statistical arbitrage, that involves the use of statistical methods to identify and exploit statistical mispricing between assets. Relative value strategies strong connection to statistical arbitrage lies within their shared reliance on quantitative methods and mean-reversion principles to manage risk and identify trading opportunities.

Pairs trading is considered a hedging strategy that entails taking both long (buying) and short (selling) positions on assets that are positively correlated concurrently, attempting to decrease market risk and exploit short-term price discrepancies (Gatev et al., 2006). Andrade et al. (2005) stated that mispricing between assets is largely a result of uninformed demand shocks, causing divergence between assets that have moved in tandem historically. Gatev et al. (2006) noted that risk-adjusted pair trading returns should be negative if the market was continuously efficient.

Pairs trading consists of two steps, first is forming of pairs and second is trading of the pairs (Rad et al., 2016). The formation stage applies statistical methods to detect asset prices that follow a long run equilibrium relationship, then calibrating the spread to follow a mean-reverting process subject to the assets being cointegrated (Krauss, 2017). The Engle-Granger (1987) theory of cointegration has been employed substantially within the pairs trading strategy framework. This theory is used to recognise assets that share a long-term relationship in equilibrium, which experiences short-term deviations. The pairs trading strategy was pioneered by the quantitative trading group and Gerry Bamberger in the 1980's headed by Morgan Stanley's Nunzio Tartaglia, see Bookstaber (2007) and Rad et al. (2016). The pair is traded when the spread widens consequent to price divergence, the trade is then unwound when the spread converges to its equilibrium mean (Krauss, 2017). The trade consists of buying the asset that is relatively undervalued and selling the other asset that is relatively overvalued (Do et al., 2006). To execute a pairs trade successfully, the investor must determine the strength of mean reversion and evaluate the degree of mispricing in ascertaining appropriate trading rules (Do et al., 2006).

The market neutral portfolio optimization strategy pursues the gathering of two alphas in a single pair trade, by going long on one asset and shorting another concurrently, while achieving negligible systematic risk exposure (Do et al., 2006). The sizing of the short and long position in the spread may be calibrated by using the beta of each respective asset. This can result in pair spread that is market neutral, meaning it has a beta of zero (Elliot et al., 2005). Pairs trading is not necessarily a direct representation of market neutrality, however it is often perceived as a distinct application of market neutrality, see Jacob and Levy (1993). As stated by Do et al. (2006), pair trading is a popular hedge fund strategy however, due to the restrictive nature of the industry, published research has been limited.

2.6 Approaches to Pairs Trading

Several approaches to execute the pairs trading strategy exist that are unique in both formation and trading, however this study focuses on the main approaches that are commonly used in the available literature. The distance approach discovers pair trading possibilities by employing nonparametric distance metrics (Krauss, 2017). The cointegration approach employs cointegration testing to determine if assets are cointegrated of the same order and if the spread is a stationary time series (Krauss, 2017). The main differentiator is the econometric determination of the equilibrium relationship of the pairs in the cointegration approach, which holds in the long run (Krauss, 2017). These two approaches have had different outcomes regarding the number of pair trading possibilities found and the success of the proposed pairs during the same period.

Do et al. (2006) stated that distance-based pairs trading is based on price level exploitation of statistical associations of the pair. Distance criteria are used in ascertaining assets that move in correlation during the formation of pairs (Krauss, 2017). The distance approach selects complementing assets that have the lowest sum of squared deviations (SSD) spread's when forming tradeable pairs (Rad et al., 2016). Pairs trading rules are determined using standard deviation of the spread, which is triggered when the spread diverges more than a specified standard deviation (Do et al., 2006). The opening positions of the pair are dependent on the direction of the divergence and is subsequently closed when the spread converges to zero (Rad et al., 2016). As pointed out by Do et al. (2006) the distance-based pairs trading advantage is that it does not require an economic model, resulting in the approach not being exposed to estimation and specification errors. Krauss (2017) concluded that the large-scale adoption of the approach is a result of its simplicity and transparency.

Krauss (2017) further noted that astute econometric identification of the pair equilibrium relationship provides the cointegration approach with a meticulous framework compared to the distance approach. The cointegration approach encompasses mean reversion that is vital in the implementation of pairs trading, demonstrating how the process diverges from its equilibrium (Rad et al., 2016). As stated in Rad et al. (2016) the formation of pairs in this approach filters all possible pairs on their SSD, followed by testing the pairs with the lowest SSD for cointegration. All cointegrated pairs coefficients are determined and those that are not, are removed (Rad et al., 2016). The Engle-Granger two-step approach is preferred in cointegration testing and estimating the pair coefficient, firstly using Ordinary Least Squares followed by the error correction model (ECM) in step two (Engle and Granger, 1987). Using historical data, the ECM can estimate the dynamics of the pairs spread by correcting of the previous deviations from the spread equilibrium (Do et al., 2006). Significant deviation of the spread from its equilibrium results in the trade opening, where the cointegration coefficient determines the ratio of assets bought and sold. The trade is unwound when the spread converges to the equilibrium relationship consequent to the pair being stationary (Rad et al., 2016).

2.7 Performance of Pairs Trading

As pointed out by Andrade et al. (2005), pairs trading has negligible exposure to systematic risk, superior risk-adjusted returns, offsetting practicable trading fees, and is not derived from reversals in the short-term. Naccarato et al. (2021) found pairs trading has continuously superior risk-adjusted returns than alternative models tested. The annualized Sharpe ratios of pairs trading are found to be greater than one, and the returns across all degrees are statistically significant (Andrade et al., 2005). Gatev et al. (2006) noted that pair trading generated significant and economic excess returns, commonly referred to as alpha. Andrade et al. (2005) stated that notorious sources of systematic risk are incapable of explaining pairs trading returns.

Uninformed trading is found to be a significant factor in explaining pairs trading returns, which suggests that the strategy's profitability is consequent to its ability to discover conditions of short-term price pressures (Andrade et al., 2005). Pairs trading profitability can be viewed as compensation for maintaining prices in equilibrium as per the law of one price theorem (Andrade et al., 2005). Chen et al. (2019) concluded that the pairs trading produces significant alpha. As noted by Andrade et al. (2005), a pairs position is hedged by a balancing position with homogeneous factors, resulting in a lower risk profile. Pairs trading profits considering frictions in the market, such as restrictions on short-selling and cost of trading, are regarded as robust (Krauss, 2017).

Do and Faff (2010) noted pair trading profitability is higher during periods of market downturns, commonly known as a bear market. Jacobs and Weber (2015) and Krauss (2017) found that pairs trading profits are highly linked to arbitrage limitations. Krauss (2017) noted a positive relationship between escalating restrictions to arbitrage and pairs profitability. A transnational analysis showed that emerging markets experience higher pairs trading profitability (Krauss, 2017).

The market's inability to account for pairs trading performance demonstrates the market neutrality of the strategy (Rad et al., 2016). This motivates its use by practitioners in the reduction of market risk exposure and supports academics' use of the strategy in asset allocation as an effectual diversification tool in a portfolio (Rad et al., 2016). The unexplained alpha generated by pairs trading strategies promotes the strategy to one of few market occurrences that have prevailed over time (Krauss, 2017). This market neutral strategy has been used and tested in isolation to determine its viability. Gatev et al. (2006) found that pair trading had a significantly greater risk-adjusted return than the S&P 500, which they defined as the market, as the Sharpe ratio of the pairs trading portfolio was between four and six times larger. This strategy offers an uncorrelated return profile at a lower risk to the market, which, when used as the satellite in a core satellite approach, can offer a risk-adjusted return above the efficient frontier.

As pointed out by Rad et al. (2016), the cointegration approach shows a higher Sharpe ratio than the distance approach, especially during turbulent market conditions, that assist portfolios by increasing alpha and decreasing downside deviation. Cointegration-based pair trading performance is consequent to the exploitation of information derived from the cointegrated spread, which should not be ignored in the estimation of a model that forecasts returns (Naccarato et al., 2021). Gatev et al. (2006) proved the profitability of cointegration-based pair trading, emphasizing the possibility of statistical arbitrage opportunities from mean-reversion. Rad et al. (2016) tested several methods of pairs trading strategies and found that the cointegration method generated significant alpha and is the superior method during significantly volatile markets.

Having considered literature on CAPM and pairs trading, we could not find work that integrates CAPM and pairs trading. Thus, the research seeks to provide an encompassing tool for portfolio construction that appropriately reflects the realities of financial markets. The proposed extension aims to enhance the practical applicability and efficacy of CAPM in portfolio construction. Thereby contributing to the effective risk management and decision-making of the portfolio through the utilization of a pairs trading strategy in portfolio construction. Effectively testing whether integrating cointegration-based pairs trading, using the core satellite approach, increases the risk-adjusted portfolio return of CAPM.

3. METHODOLOGY

CAPM is used for pricing risky assets and in optimizing portfolio construction. Sharpe (1964) and Lintner (1965) CAPM's is the starting point of the extension and is expressed below:

$$E(R_j) = R_f + \beta_j(E(R_m) - R_f) \tag{3.1}$$

Where:

- $E(R_j)$ is the expected return of asset (j)
- R_f is the risk-free rate
- β_j is the beta of asset (j)
- $E(R_m)$ is the market portfolio's expected return
- $(E(R_m) - R_f)$ is the risk premium

Beta is used to adjust the risk premium according to the investment's specific risk level relative to the market. A beta of 1 indicates full correlation with market movements. A beta less than 1 is known as being less responsive to market changes, while a beta greater than 1 is known as being more responsive to market changes. Combining these factors, CAPM explains what compensation percentage is required for a specified level of risk.

The core satellite framework involves splitting the portfolio to comprise a passively managed market tracking core portion and an actively managed alpha generating satellite portion. The core portion of the portfolio will follow CAPM, and it will make use of the market portfolio with a beta of 1 represented by a market index tracker, as expressed below:

$$E(R_c) = R_f^s + \beta_c\left(E(R_m) - R_f^s\right) \tag{3.2}$$

Where:

- $E(R_c)$ is the core portion of the hedge portfolio's expected return
- β_c is the beta of the core portion of the hedge portfolio
- R_f^s is the short-term risk-free rate

Pairs trading involves going long on one asset and short on another to exploit pricing inefficiencies between two assets. The expected return of the long and short positions of the pairs trading strategy respectively is given below:

$$E(R_{long}) = R_f^s + \beta_{long}(E(R_m) - R_f^s)) \tag{3.3}$$

$$E(R_{short}) = R_f^s + \beta_{short}(E(R_m) - R_f^s)) \tag{3.4}$$

Where:

- $E(R_{long})$ is the long position in the pair's expected return
- β_{long} is the beta of the long position in the pair
- $E(R_{short})$ is the short position in the pair's expected return
- β_{short} is the beta of the short position in the pair

Cointegration-based pairs trading involves identifying two assets whose prices move in unison in the long-run and constructing a trading strategy based on short-term deviations from this long-run relationship. This relationship ensures that any divergence between the prices of the paired assets above a specified standard deviation will mean revert through time. The cointegration-based pairs trading strategy is expressed below:

$$E(R_{pair}) = E(R_{long}) - \theta E(R_{short}) \tag{3.5}$$

Where:

- $E(R_{pair})$ is the cointegration-based pairs expected return
- θ is the proportion of the short to the long position, also known as the hedge ratio. It is derived from the cointegration parameter.

Substituting equation (3.3) and (3.4) into equation (3.5) we get:

$$E(R_{pair}) = R_f^s + \beta_{long}\left(E(R_m) - R_f^s\right) - \theta(R_f^s + \beta_{short}\left(E(R_m) - R_f^s\right)) \tag{3.6}$$

Taking equation (3.6) and simplifying it results in the below expected return for the pairs:

$$E(R_{pair}) = (1 - \theta)R_f^s + (\beta_{long} - \theta\beta_{short})\left(E(R_m) - R_f^s\right) \tag{3.7}$$

Cointegration-based pairs trading makes up the entire satellite portion of the portfolio, therefore we integrate equation (3.7) into the satellite portion below:

$$E(R_S) = E(R_{pair}) \tag{3.8}$$

Using the core satellite approach by combining equations (3.2) and (3.8) results in the below hedge extension of CAPM:

$$E(R_p) = W_c[E(R_c)] + W_s[E(R_s)] \tag{3.9}$$

Where:

- W_c is the weight of the core portion of the portfolio
- W_s is the weight of the satellite portion of the portfolio derived from $(1 - W_c)$

Substituting equation (3.2) and (3.7) into equation (3.9) we get:

$$E(R_p) = W_c[R_f^s + \left(E(R_m) - R_f^s\right) + W_s[R_f^s(1-\theta) + (\beta_{long} - \theta\beta_{short})\left(E(R_m) - R_f^s\right)\,)] \tag{3.10}$$

This model is based on the core satellite approach, making use of cointegration-based pairs trading and CAPM, providing a more comprehensive model for portfolio construction.

The proposed, hedge extension of CAPM is expressed below:

$$E(R_p) = [W_c + W_s(1-\theta)]R_f^s + [W_s(\beta_{long} - \theta\beta_{short})](E(R_m) - R_f^s) \tag{3.11}$$

The proposed hedge extension of CAPM explains how the addition of cointegration-based pairs trading as the satellite in conjunction with the diversified core portfolio will be utilized to calculate the expected return of the portfolio. The expected portfolio return is anticipated to yield a higher risk-adjusted return than the traditional CAPM. This extension solves for CAPM's assumption of efficient market's and excluding idiosyncratic risk, while keeping the core theory of CAPM intact.

4. EMPIRICAL ANALYSIS

4.1 Data description

The research requires high frequency data to test the viability of the proposed hedge extension of CAPM in comparison to CAPM. The Yahoo Finance and Bloomberg databases are used to get historical individual stock and index prices which are required to test the trading strategy and the calculation of returns and the applicable beta. The data is collected at a daily frequency as it incorporates short-term market volatility which can exploit more cointegration-based pairs and the precise calculation of beta over specific time periods. The sample period for the research is from the 2nd of January 2015 to the 31st of December 2024, which covers a 10-year period. This period was chosen to increase the frequency of cointegration-based pairs trading as well as to test the hedge extension of CAPM over a market cycle.

The sample includes the FTSE/JSE All Share index (JALSH) and all listed stocks in the basic material, and financial sectors on the Johannesburg Stock Exchange (JSE) within the sample period. Liquidity and market capitalization are the first level of filtering to exclude highly volatile stocks that are not suitable for pairs trading. Daily closing stock prices are used for return calculations by dividing the entry price into the exit price.

The risk-free rate is obtained using the Alexander Forbes Short-Term Fixed Interest (SteFI) Call Deposit Index, which is collected from the Bloomberg database. It is standard practice in the industry to use the SteFI Call Deposit Index as a risk-free rate proxy due to it being stable and certain, directly linked to the South African repurchase rate, liquid and bearing no duration risk.

The hedge portfolio is constrained to a maximum gross exposure of 200%. The pairs are further constrained to an absolute 2.5% net exposure of the portfolio at each trade entry point to ensure the portfolio's gross exposure is not breached. The hedge portfolio's core has a weight of 92.5% at launch, with no rebalancing occurring in the core portfolio during the sample period.

The selection process of pairs used in cointegration-based pairs trading follows that used in Rad et al. (2016, p.6). The first step involves identifying and sorting pairs of assets based on the normalized price's lowest SSD. The second step involves choosing ten pairs with the lowest SSD values and testing both assets for cointegration to establish whether they share the same order of integration. The cointegration coefficients are estimated for these pairs that are cointegrated of the same order and their residuals are tested for stationarity. This process is repeated until four cointegrated pairs with the lowest SSD are selected for trading. This is done to ensure that a suitable satellite portfolio can be created, consisting of several trades per pair during the sample period.

In relation to performance evaluation, the research uses the Sharpe ratio (Sharpe, 1964) and Jensen's Alpha (Jensen, 1968) as performance metrics to compare the two models, with the benchmark being the market proxied by the JALSH. The Sharpe ratio will be utilized to assess risk-adjusted performance as shown below:

$$M = \frac{R_p - R_f}{\sigma_p} \tag{4.1}$$

Where:

- R_p is the hedge portfolio's return
- σ_p is the hedge portfolio's standard deviation

The Modigliani and Modigliani (M^2) measures risk-adjusted returns that provides a more intuitive explanation than the Sharpe ratio (Modigliani & Modigliani, 1997). Providing a percentage return the M^2 measure provides an easier comparison against the market index. The M^2 measure adjusts a portfolio's return to reflect the identical level of risk as the market as shown below:

$$M^2 = R_f + \frac{(R_p - R_f) - \sigma_m}{\sigma_p} \tag{4.2}$$

Where:

- σ_m is the market portfolio's standard deviation

Jensen's Alpha is used to assess the portfolio's return relative to its expected return as shown below:

$$\alpha = R_p - E(R_p) \tag{4.3}$$

These metrics offer a more comprehensive understanding of risk-adjusted returns and the efficacy of the hedge extension of CAPM relative to that of the traditional CAPM.

4.2 Econometric method

Rad et al. (2016) found in their research comparing several methods that cointegration-based pairs trading was the superior strategy during volatile markets. The econometric method employed to facilitate cointegration-based pair trading in this paper is derived from the research of Rad et al. (2016). Suppose $P_{A,t}$ and $P_{B,t}$ to be two natural log asset prices integrated of order one. If a stationary linear relationship exists among time series $P_{A,t}$ and $P_{B,t}$, then they are considered cointegrated. Therefore, $P_{A,t}$ and $P_{B,t}$ are cointegrated provided there is a real non-zero number θ so that:

$$P_{B,t} - \theta P_{A,t} = \mu_t \tag{4.4}$$

Where θ is the cointegration coefficient and μ_t is the residuals resulting from the cointegrating regression that is a stationary at level. Using Engle-Granger's two-step approach (Engle & Granger, 1987), the cointegrating relationship can also be represented in the structure of an Error Correction Model (ECM), see Vidyamurthy (2004) and Rad et al. (2016). This procedure begins with the estimation of the cointegrating regression using Ordinary Least Squares and then proceeds to determine the ECM in step two. Dependent on the ECM, long-term equilibrium is preserved in the cointegrated series, despite short-term deviations from the equilibrium potentially occurring. These deviations are corrected, through time, with the ECM's error correction term. The cointegrating relationship linking time series $P_{A,t}$ and $P_{B,t}$ can be represented in the ECM as follows:

$$\Delta P_{B,t} = \alpha_{\Delta P_B}\left(P_{B,t-1} - \theta P_{A,t-1}\right) + \delta \Delta P_{A,t} + \varepsilon_{\Delta P_{B,t}} \tag{4.5}$$

Equation (4.5) shows the first difference of time series, $\Delta P_{B,t}$, consisting of white noise, $\varepsilon_{P_{B,t}}$, first difference of time series, $\delta \Delta P_{A,t}$, and an error correction term, $\alpha_{\Delta P_B}\left(P_{B,t-1} - \theta P_{A,t-1}\right)$, that converges to its long-term equilibrium as it progresses with time. The cointegrated series mean reversion is used in the pairs trading implementation.

The error correction term from equation (4.5) is split into the cointegrating relationship, $\left(P_{B,t-1} - \theta P_{A,t-1}\right)$, and the correction rate, $\alpha_{\Delta P_B}$. The cointegrating relationship shows the process' deviation from its equilibrium over the long-term. The spread is the price differential between two assets that are cointegrated:

$$\omega_t = P_{B,t} - \theta P_{A,t} \tag{4.6}$$

Where ω_t denotes the spread of the pair. For each pair nominated, equation (4.6) defines the spread, which is established and used to compute the mean μ_t and standard deviation σ_t. These parameters serve in determining the normalized cointegration spread for each nominated pair. From equation (4.4) and (4.6) the spread is given by $\omega_t = \varepsilon_t$ where ε_t is a stationary process at level, with mean μ_t and standard deviation σ_t. The normalized form of the cointegration spread is shown as:

$$\omega_n = \frac{\varepsilon_t - \mu_t}{\sigma_t} \tag{4.7}$$

Where ω_n denotes the normalized cointegration spread of the pair. The pairs strategy takes long and shorts simultaneously once the normalized cointegration spread exceeds 2 standard deviations. If the spread drops below -2 standard deviations, one share of asset B is bought and θ shares of asset A is shorted. Correspondingly, $\frac{1}{\theta}$ share of asset B is shorted and one share worth of asset A is bought, when the spread moves above +2 standard deviations. Both positions are simultaneously unwound once the normalized cointegration spread reverts to zero. Throughout the sample period, the pair undergoes ongoing monitoring for further deviations.

Suppose at time $t-1$ one share of asset B is bought (long) and θ share of asset A is sold (short). $P_{A,t}$ and $P_{B,t}$ represents assets A and B's price series, respectively. The unwound trades profit is:

$$\Delta\omega_{t,} = \left({P_{B,t}}/{P_{B,t-1}}\right) - \left({\theta P_{A,t}}/{\theta P_{A,t-1}}\right) \tag{4.8}$$

The pair's profit for Δt is given by the change in the value of the short being subtracted from the change in the value of the long, as the spread is unwound. From equation (4.4), the spread by definition is stationary and displays mean reversion.

The hedge extended CAPM framework incorporates the alpha generated from cointegration-based pairs trading as a satellite component, improving the traditional CAPM. The process involves integrating cointegration-based pairs trading returns into the overall portfolio construction. The excess return of the hedge extension of CAPM is specified as follows:

$$R_p - R_f^s = \alpha_c + \beta_p R_m + \sum_{i=1}^{4} \theta_{s_i} \alpha_{s_i} + \varepsilon_p \tag{4.9}$$

Where:

- α_c is the alpha generated from the core
- β_p is the beta of the hedge portfolio
- θ_{s_i} is the coefficient capturing the contribution of alpha from the satellite

- α_{s_i} is the alpha generated from the pair in the satellite
- ε_p is the error term of the hedge portfolio

The equation looks at each pair individually as multiple pairs could be active simultaneously and similar pairs can be traded several times during the sample period. The coefficient θ_{s_i} quantifies the impact of cointegration-based pairs trading on the overall portfolio's performance. If $\theta_{s_i}$is statistically significant, it indicates that cointegration-based pairs trading contributed positively to the hedge portfolio's risk-adjusted returns. Following the econometric method employed in this research provides a vigorous framework for integrating cointegration-based pairs trading, using the core satellite approach into CAPM to increase the portfolios risk-adjusted returns.

The equation assists in evaluating the hypothesis through several key steps. First, the alpha of the portfolio α_p is calculated using the formula $\alpha_p = \alpha_c + \sum_{i=1}^{4} \theta_{s_i}\alpha_{s_i}$ where $\alpha_p - \alpha_c > 0$ is expected due to the hedge portfolio's anticipated outperformance in relation to the market portfolio, attributed to the incorporation of cointegration-based pairs trading. Next, the portfolio's systematic risk exposure is examined, where $\beta_p - \beta_c < 0$ is expected as cointegration-based pairs trading lowers the portfolio's systematic risk exposure. Finally, the portfolio's exposure to idiosyncratic risk is analyzed, expecting that $E(\varepsilon_p^2) - E(\varepsilon_c^2) < 0$ due to cointegration-based pairs trading capturing the portfolio's idiosyncratic risk.

4.3 Empirical Results

The construction of the hedge portfolio's satellite and evaluation of its performance in comparison to the market portfolio, proxied by JALSH index, is presented in this section. The results from testing equation (3.11) are discussed and analyzed in this section. The selection process of pairs was done by finding comparable companies that have competitive relationships to create a sample which resulted in 20 viable pairs. The SSD of these pairs were calculated and ranked from lowest to highest as shown below.

Table 1: Comparable Pairs Selection Using SSD

Rank	Pairs	SSD
1	Firstrand Ltd (FSR) / Standard Bank Group (SBK)	10.81
2	Anglo American Platinum Ltd (AMS) / Impala Platinum Holdings Ltd (IMP)	16.01
3	Nedbank Group Ltd (NED) / Standard Bank Group (SBK)	22.24
4	African Rainbow Minerals (ARI) / Anglo American plc (AGL)	23.69
5	Anglo American Platinum Ltd (AMS) / Northam Platinum Holdings Ltd (NPH)	23.88
6	Kumba Iron Ore (KIO) / Anglo American plc (AGL)	30.12
7	Nedbank Group Ltd (NED) / Firstrand Ltd (FSR)	31.69
8	Kumba Iron Ore (KIO) / African Rainbow Minerals (ARI)	34.75
9	Exxaro Resources Ltd (EXX) / African Rainbow Minerals (ARI)	42.02
10	Anglogold Ashanti (ANG) / Gold Fields Ltd (GFI)	45.46
11	Gold Fields Ltd (GFI) / Harmony Gold Mining Company (HAR)	47.34
12	DRDGOLD Ltd (DRD) / Pan African Resources plc (PAN)	49.63
13	Anglo American plc (AGL) / Glencore plc (GLN)	63.39
14	Sanlam Ltd (SLM) / Momentum Metropolitan Holdings (MTM)	65.83
15	Harmony Gold Mining Company (HAR) / DRDGOLD Ltd (DRD)	67.19
16	Remgro Ltd (REM) / JSE Ltd (JSE)	72.89
17	Capitec Bank Holdings (CPI) / Investec Ltd (INL)	73.54
18	Standard Bank Group (SBK) / Investec plc (INP)	78.19
19	Discovery Ltd (DSY) / Momentum Metropolitan Holdings (MTM)	81.20
20	Sanlam Ltd (SLM) / OUTsurance Holdings Ltd (OUT)	89.14

The top 10 pairs log normal prices from Table 1 were tested for stationarity at level and at first difference using the Augmented Dickey Fuller test as both stocks are required to be integrated of the same order. These stocks were then regressed on each other and their residuals were tested for stationarity at level following the Engle-Granger two-step analysis to determine if the pair is cointegrated. The research found four pairs that were cointegrated and their test results are shown below.

Table 2: Augmented Dickey Fuller Stationarity Tests

Pair	Stock	Stationarity at Level t-Statistic	Stationarity at First Difference t-Statistic	Residual Stationarity at Level t-Statistic
1	LnFSR	-2.62 *	-49.92 ***	-3.85 **
	LnSBK	-2.20	-50.26 ***	
2	LnARI	-1.62	-49.53 ***	-3.70 **
	LnAGL	-1.19	-48.00 ***	
3	LnKIO	-1.31	-49.62 ***	-3.15 **
	LnAGL	-1.19	-48.00 ***	
4	LnEXX	-1.73	-49.15 ***	-3.36 **
	LnARI	-1.62	-49.53 ***	

*** reflecting a significance level of 1%.
** reflecting a significance level of 5%.
* reflecting a significance level of 10%.

The stocks in Table 2 are all shown to be stationary at first difference at a 1% level of significance and the residuals of each pair are shown to be stationary at level at a 5% level of significance. Following the confirmation of a cointegrating relationship between the stocks in each pair, the cointegration coefficient is estimated which is used in determining the spread of the pair. Each selected pair's suitability and cointegrating relationship is discussed below.

Table 3: Cointegrating Regression of all Pairs

Pair	Variables	Coefficients	Standard Errors	t-Statistic
1	LnSBK	0.804247	0.0080	100.5424 ***
2	LnAGL	0.758565	0.0051	148.3151 ***
3	LnAGL	1.102559	0.0080	137.3469 ***
4	LnARI	0.779673	0.0075	103.6738 ***

*** reflecting a significance level of 1%.
** reflecting a significance level of 5%.
* reflecting a significance level of 10%.

As two of the largest commercial banks in South Africa, FirstRand Limited and Standard Bank Group offer a wide array of traditional and investment banking services. These companies exhibit sensitivity to interest rate fluctuations and macroeconomic trends within the South African economy, leading to correlated price dynamics in the long-term, making the pair suitable for pairs trading. The regression for Pair 1 in Table 3 exhibits an Adjusted R-squared of 0.8019 and an F-statistic probability of 0.0000, indicating that the model explains the dependent variable effectively. The explanatory variable is significant at a 1% level.

Anglo American plc and African Rainbow Minerals are diversified mining companies whose revenues are similarly influenced by fluctuations in global commodity prices and prevailing South African economic conditions. The shared exposure to these factors leads to correlated price dynamics in the long-term, making the pair suitable for pairs trading. The regression for Pair 2 in Table 3 exhibits an Adjusted R-squared of 0.8981 and an F-statistic probability of 0.0000, indicating that the model explains the dependent variable effectively. The explanatory variable is significant at a 1% level.

Kumba Iron Ore Ltd, a specialist in iron ore mining, and Anglo American, a diversified miner with significant iron ore production, exhibit a notable connection through their shared involvement in the global iron ore market. This common global iron ore exposure leads to correlated price dynamics in the long-term, making the pair suitable for pairs trading. The regression for Pair 3 in Table 3 exhibits an Adjusted R-squared of 0.8831 and an F-statistic probability of 0.0000, indicating that the model explains the dependent variable effectively. The explanatory variable is significant at a 1% level.

African Rainbow Minerals and Exxaro Resources Limited, prominent entities within the South African mining sector, possess distinct commodity specializations. Exxaro Resources primarily focuses on coal, whereas African Rainbow Minerals maintains a more diversified commodity portfolio. Despite these differences, the companies are influenced by overarching global commodity trends and macroeconomic factors prevalent in South Africa. Leading to correlated price dynamics in the long-term, making the pair suitable for pairs trading. The regression for Pair 4 in Table 3 exhibits an Adjusted R-squared of 0.8115 and an F-statistic probability of 0.0000, indicating that the model explains the dependent variable effectively. The explanatory variable is significant at a 1% level.

The normalization of these spreads involves subtracting the mean and dividing by the standard deviation. The standard deviation and mean of each spread is shown in Table 4 below.

Table 4: Pair spreads normalization metrics

Variables	**Spread Mean**	**Spread Standard Deviation**
Pair 1	-0.021639	0.082332
Pair 2	0.587127	0.144538
Pair 3	-0.699608	0.226860
Pair 4	0.963376	0.170118

The normalized cointegration spread was used to determine the trading signals of the pairs by signaling a long on the spread when +2 standard deviations were exceeded and a short on the spread when -2 standard deviations were exceeded. The trades were subsequently closed when the normalized cointegration spread reverted to 0. The spread and normalized spread for each pair is graphically shown in Figure 1 below.

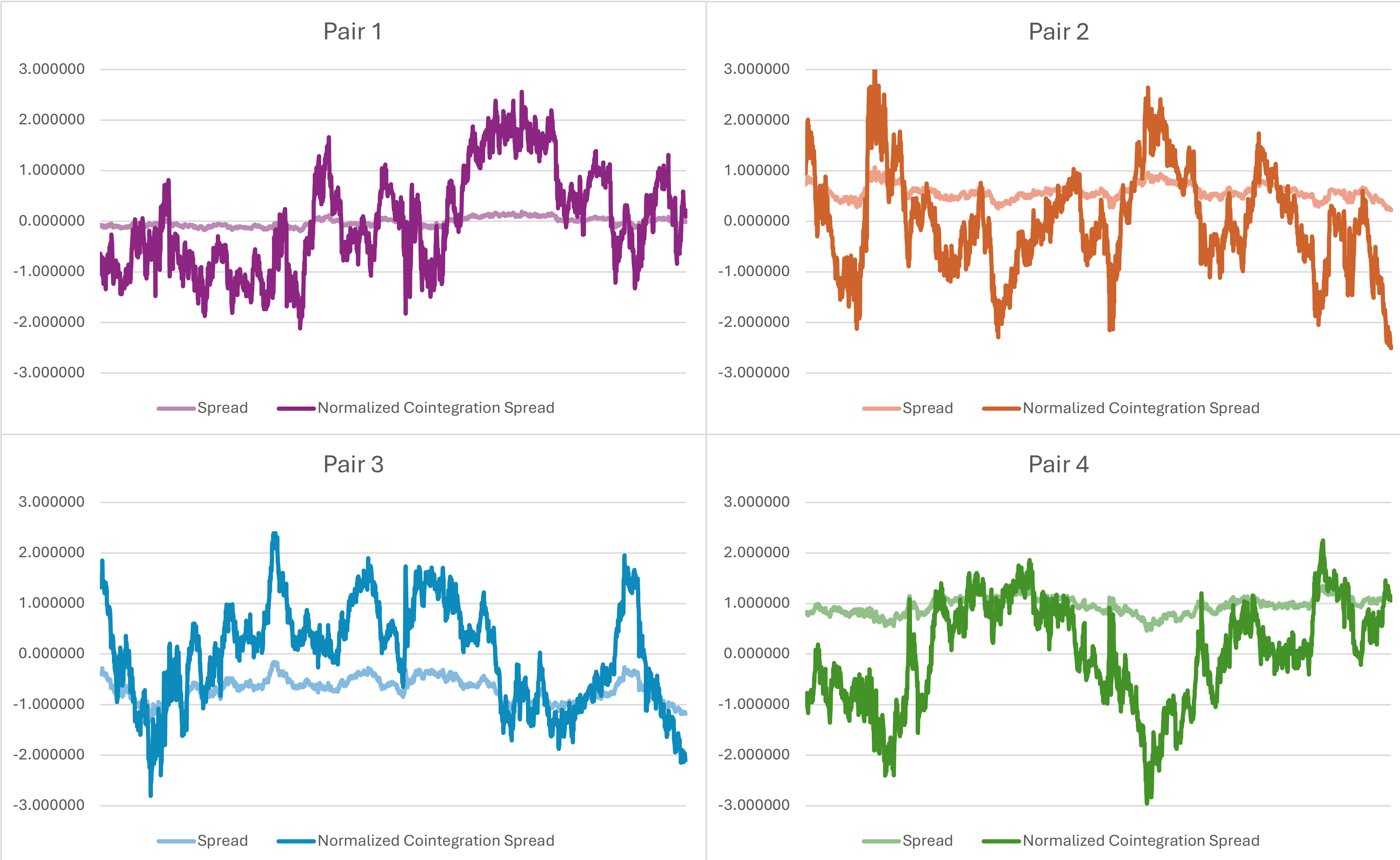

Figure 1: All Trading Pairs Spread and Normalized Cointegration Spread

These pairs represented the hedge portfolio's satellite, with its core consisting of the JALSH index. The JALSH index proxies the market portfolio. Figure 2 below shows the cumulative return of the hedge portfolio and the market portfolio over the sample period.

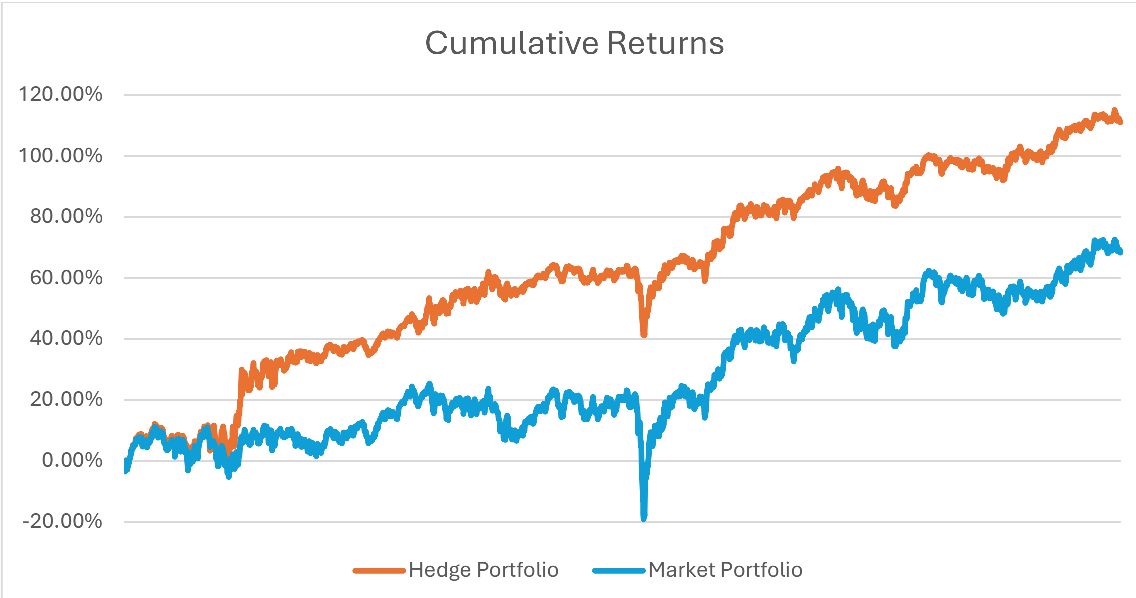

Figure 2: Cumulative returns of the Hedge Portfolio and the Market Portfolio

Figure 2 clearly shows that the hedge portfolio outperforms while providing smoother returns, which aligns with expectations regarding enhanced risk-adjusted performance, supporting the alternative hypothesis. This can further be seen in the performance evaluation metrics shown below in Table 5.

Table 5: Comparative Performance Statistics

Performance Metrics	Hedge Portfolio	Market Portfolio
Annualized Return	10.79%	5.49%
Annualized Standard Deviation	13.91%	17.68%
Sharpe Ratio	0.1582	-0.1754

The hedge portfolio's Sharpe ratio is positive, in contrast to the market portfolio's negative ratio, indicating that the hedge portfolio has a superior risk-adjusted return. This is further confirmed when looking at the hedge portfolio Modigliani Squared Ratio of 11.39% compared to the market portfolio's annualized return of 5.49%. These performance metrics provide initial support for the alternative hypothesis indicating a superior risk-adjusted return of the hedge portfolio when compared to the market portfolio. The portfolio's Jensen's alpha of 4.28% indicates that the portfolio has exceeded its expected return based solely on beta. The proposed hedge extension is tested and discussed below.

Table 6: Hedge Extension of CAPM

Variable	**Coefficient**
Core Alpha	-0.000330 ***
JALSH Return	0.668851 ***
Pair 1 Return	0.125229 ***
Pair 2 Return	0.123880 ***
Pair 3 Return	0.216677 ***
Pair 4 Return	0.087785 ***

*** reflecting a significance level of 1%.
** reflecting a significance level of 5%.
* reflecting a significance level of 10%.

The model's Adjusted R-squared of 0.9487 and an F-statistic probability of 0.0000, indicating that the model explains the dependent variable effectively. The explanatory variables shown in Table 6 are significant at a 1% level.

The hedge portfolio's alpha is thus calculated as 0.012138, therefore after subtracting the alpha of the core, the alpha of the satellite is 0.012468 which is greater than 0. Confirming the hedge portfolio's expected outperformance in relation to the market portfolio, which is attributed to the inclusion of cointegration-based pairs trading. The portfolio's systematic risk exposure after subtracting the core's systematic risk exposure results in -0.3311, which is less than 0. This confirms that cointegration-based pairs trading has lowered the portfolio's systematic risk exposure. The portfolio's variance of the error term, which is derived by squaring the standard error of the regression results in 0.000394% which is greater than the core portfolio's variance of the error term of 0.000157%, thus indicating that cointegration-based pairs trading increased the idiosyncratic risk in the portfolio. This is due to cointegration-based pairs trading not being completely market neutral as all pairs have marginal net exposure to the market and the portfolio had a maximum gross exposure of 180.61%. While this marginal increase in idiosyncratic risk must be acknowledged, the overall results support rejecting the null hypothesis, demonstrating the effectiveness of the hedge extension enhancing the traditional CAPM's risk-adjusted portfolio return through cointegration-based pairs trading.

5. CONCLUSION

The paper aims to address the limitations of CAPM by introducing a hedge extension of CAPM that incorporates cointegration-based pairs trading. This paper evaluates the risk-adjusted performance of the hedge portfolio constructed using the proposed hedge extension of CAPM, compared to the market portfolio. The research is conducted within the South African market, utilizing daily closing prices of JSE listed Financial and Basic Material stocks from 2nd of January 2015 to 31st of December 2024. The hedge portfolio was constructed using a core satellite approach, where the JALSH index served as the core, and cointegration-based pairs trading as the satellite. Pair trades were then initiated when the normalized cointegration spread exceeded +2 or -2 standard deviations and unwound when the normalized cointegration spread reverted to 0. The cointegration approach ensured all pairs maintained a marginal net exposure of 2.5%, which was set as a portfolio limit. However, this approach resulted in pairs not being fully market neutral. Additionally, the portfolio was constrained to not exceed a 200% gross margin.

The hedge portfolio outperformed the market portfolio in terms of both risk-adjusted and cumulative returns over the sample period. Specifically, the hedge portfolio achieved a Jensen's alpha of 4.28%, a superior Sharpe ratio of 0.1582 in contrast to market portfolio's -0.1754, and a Modigliani-Squared ratio of 11.39% compared to the market portfolio's return of 5.49%. The cointegration-based pairs trading strategy effectively reduced systematic risk exposure in the hedge portfolio, aligning with theoretical expectations. However, contrary to expectations, idiosyncratic risk increased in the hedge portfolio. This increase was attributed to higher gross exposure relative to the market portfolio and net exposure within individual pairs contributing to additional idiosyncratic risk. Hedging strategies are often perceived as inherently risky and likely to increase overall portfolio risk, this paper demonstrates that such concerns do not apply to cointegration-based pairs trading strategies. These findings highlight the potential of statistical arbitrage strategies to enhance portfolio performance while managing risk effectively.

The results of this paper should encourage portfolio managers to actively consider incorporating statistical arbitrage-based relative value strategies, such as cointegration-based pairs trading, into their portfolios. Compared to traditional approaches, this strategy has demonstrated a superior capacity to generate risk-adjusted returns. Notably, this paper has shown that portfolios constructed using this method can exist above the efficient frontier, challenging conventional beliefs about hedging strategy's risk-return tradeoffs. By reducing systematic risk exposure while maintaining competitive returns, cointegration-based pairs trading offers a compelling alternative for passive portfolio management. These findings provide significant insights into how innovative extensions of CAPM can improve investment outcomes and contribute to more efficient capital allocation.

Future research could expand on this paper by increasing the sample size, including all listed JSE stocks and testing the strategy's performance out-of-sample through rolling training periods. This further validates its efficacy across different market conditions and ensure robustness in its application. Another promising avenue for exploration could involve testing this strategy alongside a concentrated, actively managed core portfolio while accounting for trading costs. Such research helps determine whether cointegration-based pairs trading offers superior risk-adjusted returns compared to diversified portfolios under real world constraints.

In conclusion, this paper has demonstrated that the hedge extension of CAPM through integrating cointegration-based pairs trading leads to superior risk-adjusted returns compared to traditional CAPM. By achieving enhanced risk-adjusted returns and reducing systematic risk exposure, this approach represents a significant improvement over conventional models of portfolio construction. These findings underscore the importance of exploring innovative financial models that challenge established paradigms and offer practical solutions for modern investment challenges.

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