PRIMARY MATHEMATICS

By

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I propose to talk in two sections, the first section being a general one, not concerned directly with mathematics teaching, but more with the kind of situation, the environment, the conditions, which I would suggest are necessary for the proper teaching of mathematics. Then I shall move on to mathematics as such.

There is a general feeling that mathematics is easy. Among the excellent collection of books on display in this hall is one called "Arithmetic Made Easy". You will quite often meet people who will, by implication, if nothing else, suggest that mathematics can be made easy. Mathematics, in my view, is a rigorous, disciplined, progressive study.

The apparatus in the classroom that I would regard as of first importance in the teaching of mathematics is the teacher. No other apparatus has greater significance than the teacher and I would submit that the value, the use, of any other apparatus, Cuisenaire and the rest, will lie directly in proportion to the skill of the teacher using them. They have little built-in teaching value. The wise teacher can get a lot from them. The other sort of teacher may possibly do harm.

For the teacher to have the best conditions for his teaching, he must have firstly the freedom to work out his own salvation and that of the children. He must have the freedom to choose his own methods, and any restriction through timetables, schemes, syllabuses, text-books, on that teacher's freedom, is bad unless the limits are pretty wide. I know that if you give this freedom, there are dangers of abuse. We must accept that; there may be a number of teachers who will abuse their freedom. But there will be many more who will make good use of it. As Mr. Sealey said last night, "this sort of thing is an act of faith", and if we have not got faith in our teachers, then I do not think any progress of consequence is likely.

The freedom of the teacher does carry obligations, of course. It carries the obligation on the teacher to study, to hear as many points of view on the teaching of mathematics as is possible. It is my firm belief that there is no one way of teaching mathematics. There are as many ways almost as there are teachers. We have got to find the way which is the way for us.

I can well understand the confusion of teachers today, and this is true all over the world. Brought up as they were in their traditional methods, they are now finding those methods openly suspect. They are finding on all sides alternatives beguiling them with their virtues and when those alternatives are being sold by high pressure salesmanship by expert enthusiasts, they are mighty attractive and very difficult to resist. There is a great danger of adopting a method before you have made it your own; that, I suggest, you must do, and it will involve a lot of hard work on the part of the teacher. He may decide to adopt one of the methods that are being advocated; he may make a blend of a number of them. His lot is even more difficult if his head teacher happens to be a traditionalist, and the teacher is one of those wishing to take a more progressive line. He has the difficulty of choosing between loyalty to his head or loyalty to his principles. How you solve that, I frankly do not know.

There always have been some exciting, excellent features of mathematics. I would like to underline this, because there are dangers in thinking that in the past everything was awful. That is sheer nonsense. Nearly all of us were brought up, so far as maths is concerned, under the old, traditional, rigid schemes. We have not all become frustrated. I do think there is a danger in leaving the traditional without giving it real thought. What mattered, of course, in the past was the teacher. We hear an awful lot about experience, but the finest experience any child has is contact with a first-class, inspired teacher. There is no experience that can equal that.

I would like to move on to some general principles underlying the teaching of mathematics, and I am going to put first this business of words; it is so important. To think of the older type of arithmetic text-book in my country is almost repulsive — pages and pages of utterly indigestible sums, usually set down in the wrong form, set down one above the other, which is not the right form for a mathematical statement. The only correct form for a mathematical statement is the basic mathematical sentence which is the equation, and text-books, when they do use these mechanical examples, should more often than not put them as an equation. Why should a text-book in mathematics not be exciting to handle? Why should it not have colour? Why should it not have pictures? It ought to be as attractive to the child as his reading books. The day of the dull books, I hope, has gone — and there are some books for young children that are a joy to handle.

I used to be asked occasionally by the headmistress of the infant school attached to my junior school: "What would you like me to do in the infants' department that will fit them best for their mathematics learning in the junior school?" My reply was: "Teach them to read. If you have any time left over, give the children as wide, as comprehensive, as exciting, experiences with number, with quantity, as you can. But I don't want them to come to me having done little sums."

Those of you who know that excellent book of Miss Adams "A Background to Primary Mathematics" will remember that she makes this statement. (Miss Adams, by the way, had been an inspector of schools all her working life.) "After forty years experience of looking at mathematics and arithmetic in schools, I have come to the conclusion that from the time they begin little sums, be it at 6, 7 or 8 years of age, be they bright, average or dull, their mathematics steadily deteriorates." That is a statement of a person who spent a lifetime observing mathematics teaching.

The place of words, particularly, I think, is in the end stage. The procedure would go something like this, if we are dealing with experimental work. We may decide to do a simple experiment which we hope will involve the children in their mathematical thinking — and once a child becomes involved, he becomes highly receptive. We talk about the experiment we are going to do and get as many opinions from the children as we can. We set up the apparatus, making it if possible, because home-made apparatus is far better than the glossy bought stuff. We do the experiment which will involve recording, which is the basis of the mathematics, then we discuss the findings.

I would like to give you two very simple experiments that can be done. Neither needs apparatus costing more than a few pence and you will see what I mean by this technique. It shows the place of words and it shows also the integration of mathematics into the whole day. You cannot pigeon-hole mathematics any longer.

The first simple experiment is to dig a hole in the ground about 18 in. deep, have a cover against the weather, which possibly you would not need in South Africa, one thermometer in the hole, another thermometer hanging in the air. Record the temperature of both thermometers at a chosen time of the day — ten o'clock in the morning, for example. That will involve the ability to read the scale of the thermometer. You will not find all children can do that; there is a bit of mathematics there. You record these readings. What will be the best form of recording for such temperature readings? Obviously graphic. So we shall get simple graphs showing fluctuations in temperature. Having got the answers, we talk, and if your children have experience of this, you will very soon get: "Oh, please sir, now I know why dormice burrow a little under the ground in the colder countries." "Now I know why wine is stored in cellars below the ground." "Now I can see why the water and the services which come to the house are buried a few inches under the ground," and so on.

Another more amusing one — and this I have not done myself. I was in a school some years ago and the children said: "We are having fun in our maths." I replied: "Good, maths should be fun, what are you doing?" "We are weighing birds." I said: "Lovely, how do you do it?" Against a wall or a tree or some rigid object, attach a flexible stick. Towards the end of that stick, erect another stick from the ground vertically. Attach a pan to the end of that flexible stick. We have got to calibrate the stick — here is the maths coming in. Mark the zero point, that is with no weight at all, then put a 1 oz. weight in, which will depress the bar to a certain point; mark it, 1 oz., 2 oz. and so on. Now we get to work; we put some dainties for birds on the pan. We find the birds do not come. Why not? We are standing too close, so we go back a good distance. The birds do come, but we cannot read the calibrations, we are too far away, so now we get the binoculars out and a simple study of optics will follow. With the binoculars, we are able to observe the birds coming on to the lid, resting for a moment, depressing the bar and recording their weight. But we want to know what sort of birds they are and so the children have to know something about ornithology or they have to have a bird book handy. The amount of material you can get out of a simple experiment like this is quite extraordinary. There is almost unlimited scope for this kind of experimental work. Normally, you will have to use such environmental factors as you have. A town school will not do the same experiments as a country school.

Words are used traditionally in another sense in mathematics — the problem. And it has always puzzled me why in our teaching we have had a kind of hierarchy in maths — mechanical for the ones who are not much good; mental, well, that is a bit harder, a bit higher up the scale; but the real stuff, the real legion-of-honour stuff is the problem — if they can do problems, they are mathematicians.

I should like to give you two of the traditional types of problem and two of what I would call mathematical problems today, just for you to make a comparison in your minds.

One of the commonest types of problem when I was a boy was a curious kind of problem, the type I call the "libel-scandal problem". For some reason, they would not print the names of the people taking part in the problem. They called them A, B and C. That is why I have called them libel-scandal problems. And they took this pattern: A was a jolly good chap, enterprising, with initiative, he really got on. B was quite a decent sort of chap; he would never make a particular mark, but he was steady and reliable. But C was a complete clot. Everything that A and B did, C would sabotage. It does not matter what he did — you will know the types of things that they did in these lovely problems, the A B C type.

For the second, I should like to read you one from Pendlebury's "Shilling Arithmetic", under which I suffered as a boy. I will read you one which shows what grand fellows we English are, in case you did not know. "Twenty English navvies, each earning 7s. a day can do the same piece of work in 15 days that it takes 28 foreign workmen, each earning 6 francs a day, to complete in 20 days. If a piece of work done by the navvies costs £3,000, what would be the cost of the same work done by foreign workmen? £1 equals 25 francs."

Now, two problems which I would call "mathematical problems", very simple, very slight, very slender ones. I would suggest as number one: Ask your children one day, "what is the weight of a pint of water?" Do not help them. When they start on it, they will begin to realise that they cannot pour a pint of water on to the scales. They have got to think out the techniques for actually weighing a pint of water. I should call that a real mathematical problem.

I have another one which I find quite delightful. I am going to read to you what a young girl, aged 8, wrote in answer to this problem: "Why is 11 an odd number?" "Eleven is an odd number and it is an odd number because it has an odd number on the end. One is an odd number, 11 has two odd numbers, because 10 has one at the beginning and you take the nothing away from the 10 and you are left with one 10 that looks like a 1 and if you put a real 1 beside the 10 that you took the nothing away from, it looks like 11." The logic is almost perfect. I call that the very wise use of words and quite extraordinary for her age.

The true mathematical problem is concerned with relationships, with the structure, with the pattern of number, in fact with why number acts in the way that it does. I agree with Mr. Sealey that what matters is not the answer, it is the doing. Obviously in those two experiments that I mentioned, they would not get very accurate answers. They could not possibly. You could not trust the weights of the birds that they found. The value lay in the doing, not in the product that they reached at the end of the doing.

Mathematics is so often, so rightly in my view, regarded as the development of logical thinking, of logical thought. I wonder if you have ever stopped to think for yourselves how we can detect logical thought in young children. What are the signs that they would give to us that suggest they are thinking logically? It is not an easy question. I would suggest that probably the best way of judging that is by the sorts of questions that the children ask. They have got to ask questions. Then we know that they are beginning to reason, because questions are the product of reasoning.

Professor Synge, a brother of the dramatist, wrote a book on philosophy, "Science, Sense and Nonsense". It is not maths, it is philosophy, but it is eminently worth reading. He wrote: "Knowledge and clarity of thought are reached mainly in two ways, first by asking questions and second by answering them. Contrary to popular belief, it is harder to ask than to answer, but our whole educational system, the traditional one, and our whole conception of what culture consists of is based on answering and not asking. Answering shuts the box. Asking blows the lid off. A dangerous game. And that is why it is deprecated." There is wisdom distilled in a few words.

In my early teaching, I asked the questions, up went some of the hands and sometimes we got correct answers. That process has got to be reversed. You have already had a considerable number of suggestions from the two earlier lecturers as to how that process can be reversed. The children have got to be provoked, stimulated, involved, so that they just co-operate by asking questions.

One final thought on this question of words. In their discussions, we must encourage children to speak freely. If children are going to say only the things they think we shall like, then the whole thing is a dead loss. They have got to say the things they believe. Mathematics is concerned with truth and falsehood, not really right and wrong at all. And if that is the case, we have got to encourage our children to go for the truth at whatever cost.

Another little book for you — this is a forty-year old book. You see there is nothing new in this business at all. A man named Lambourne wrote a book in the twenties called "Reason in

Arithmetic" and I must read you an extract from it. I think you will find it acceptable. "When I admit a new boy to my school, I usually say to him: 'Three eights!' The answer almost always comes promptly or not at all. It is very rare for a boy to reckon the answer if he fails to remember it. If he says '24', I ask: 'How do you know?' And he generally replies: 'They told me at the other school.' Now you must tell a child that there are three persons in the Trinity because that is a dogma, but it is a sin against a child's mind to tell him what three eights are. When a boy says they told him that three eights are 24, I usually reply: 'Well, here we say that three eights are something else.' It is rarely indeed that he raises any objection. He is usually quite prepared to believe that three eights are whatever the local head teacher chooses to make it."

When children cannot contradict falsehood, I do not think falsehood should be deliberately thrown at them, but they should be encouraged to stand up for what is true against what is false.

We have had several definitions of mathematics this week, and here are two more for your consideration.

"Mathematics is a way of thinking and communicating certain aspects of the world around us." Notice the stress of "communicating". "Mathematics is a language." How often we hear that, yet how rarely it is followed up by saying: "Here is an exercise in translation in that language," or "here is a comprehension exercise in that language". It is perfectly easy to give translation exercises and comprehension exercises in maths, just as in English. The technique is the same. In English, you give children a paragraph, they read it, and then answer questions. In mathematics, you give them an equation and ask them to read it, understand it and then answer questions. Some questions can be extremely difficult and thought-provoking. You can make the questions quite up to graduate level. I will give you one I would call a jolly good top junior, 12-year old standard and see what you make of it. This is a comprehension exercise:

$$21,147 \div 87 = 243, \text{ remainder } 6.$$

I am not advocating they do calculations of that nature. That is just fantastically absurd. But if it is a comprehension exercise, the size of the numbers does not matter. An exercise based on that mathematical sentence might be:

$$21,147 \div 87$$

It is not easy, and if a child does not understand the full significance of that, he is not likely to do it. This is a hard one. You can make them as easy as you like or as hard as you like. If a child works it out, he is giving himself incredibly unnecessary labour. There are thousands of possible comprehension exercises based on a mathematical equation.

A child, in my view, cannot be considered proficient in mathematics unless he can communicate, translate, comprehend freely. I would in fact regard that as a better test than the traditional type of test.

Now for my second definition of mathematics. This is one of Professor Goodstein's of Leicester University. In an article he wrote some years ago on primary mathematics, he said: "Mathematics is an exciting adventure in ideas." Let us look at that definition for a moment. Exciting — it stimulates, it creates the right attitude. I would put very high in my priority for what we must aim at in junior school the right attitude. I would prefer a child leaving my school loving mathematics, but being able to do very little — it can happen, though it is not likely — than having a whole lot of indigestible facts — the ability to do sums, but not able to think about what he is doing. Exciting then, that implies a good attitude. Adventure— that implies that there is no one way. It is not an adventure if you know where you are going. It implies that the way is yet unknown. When you get to the situation concerned, you will then meet the challenge and choose the way. Mathematics is not concerned with facts, it is concerned with ideas. Facts are static, dead. We are coming back to Synge again. Ideas are dynamic, they are alive, they take you on to something else.

I would like to suggest the ideas that I think could be the basis of the more formal aspect of primary maths teaching and in my view they are the syllabus, the scheme. You will notice that I shall not mention any of the four processes in the scheme, not how you teach subtraction, addition, multiplication, division at all. The first idea that a child should be acquainted with is number and quantity, and we have neglected that. We have raced on to little sums before they understood what the numbers they were using were. I would suggest a child has no business to be asked to add 127 to 185 until he has some idea of what those two numbers are, some sense of the magnitude.

In number also we have utterly neglected one aspect — ordinal number. A number has two quite different forms. It can be cardinal, that is the size or it can be ordinal, which is its rank, its place in a rank.

I remember about three years ago, an examination I was closely connected with in Birmingham. We put in the paper:

$$4 - 6$$

I think most of the youth in Birmingham thought we had gone mad. A second quotation from Professor Synge's book: "the magic and mystery of number that brings about a suspension of common sense." Common sense is a very rare commodity in traditional maths teaching.

A child should have had the opportunity of some introduction to ordinal number, and for ordinal number I know of nothing so good as a number line, a number track. A classroom without a number track lacks one of the most useful aids in maths teaching that there is. Its size is immaterial. You have got to put on that track a point of origin. You are giving a rather new conception to the zero when you do that and probably a truer conception than the one we often do get across. You have to decide that you number one way and you number the other way. What makes them different? Then the children must be introduced to the convention that movement to the right means plus and movement to the left minus. Once a child has that very simple knowledge, that sum I quoted simply means, take six steps to the left of 4 and we shall obviously stop on a point on the other side of the point of origin —2.

The second form of number track which I have found useful is a broken number line, consisting of decades with one line split up into peaks. You go along in 9 stages, then up to 19, 20, 29, 30, 39 and so on. The child will see something of the structure of number with this line. If you ask him how far we are jumping if we go from one point to another, he should see that it is not necessary to count the lines. You can go up in tens. Have you ever been frustrated because the obvious counting in tens was not obvious to children? I was often frustrated. I used to think: "Well, why can't they say, 13, 23, 33, 43. It's so simple." But it is not simple to a child. There are almost countless uses of the broken track and the single line.

If you want to get into the realm of mystics, of philosophy, ask them one day: "Let's go along to the right, along the number line. What's the biggest number, we can ever get to?" "A million, sir!" "— and one." "A billion, sir!" "— and one." "A billion billion, sir!" "Jolly good, Tommy — and one." They are beginning to get the idea that there is no end. They are getting some groping idea of the term "infinity". And why not? It is a mathematical term and we can start in a humble way. Carry it further, if you like. Say to them: "Between the number 1 on this track and the number 2, there are some other numbers, fractional numbers. Give me some. "$1\frac{3}{8}$, $1\frac{7}{8}$, $1^{19}/_{20}$, $1\frac{2}{3}$ — we go on. How many fractional numbers are there between 1 and 2? There is an infinity of fractional numbers between 1 and 2. This can develop most profound thought. In fact we are almost doing religion more than we are doing mathematics. And why not?

My second theme is grouping, which is the basis of all arithmetic. I have not separated arithmetic at all. Once a child begins to understand that he can group and regroup his numbers in an endless series of ways, he is going forward towards the understanding of mathematics. You can start right at the bottom in the infant school quite easily, and I am sure quite profitably. You would use what I think is another of the most important tools a maths teacher has — the abacus. I should be sorry to see structural material ousting the abacus in schools. Counters have got a place, I think, equally important with the structural material. Get a set of 13 straws, if you like, 13 anything. Say to the children: "I want you to group them first of all into fours, then in fives, you can go on indefinitely, but I want you each time to record what you do. I want you to record it in abacus form. When you group in fours, I want you to write your answer in columns of fours and ones. When you group into fives, I want you to record your answer in fives and ones. When you group in sixes, in sixes and ones." The child will do it. It is not a difficult stage from this to writing thirteen as three ones two threes and two twos. You can write thirteen in a dozen ways. It is not difficult to build on that a true understanding of notation which is the same as this except that the columns are tenths this time and not fours or fives or sixes. From that grouping, a child will begin to understand something of the radix, the base.

I was in a school recently where a child was using the apparatus that Mr. Sealey has had a great hand in constructing. I asked a child: "Can you tell me what three sixes are?" A little girl of about eight looked up at me and said: "In what base?" So I said: "Try it in the 10 base first." She replied: "Eighteen!" and looked at me as if to say: "Give me another one!" So I said: "What about the 12 base?" So she said: "Sixteen!" A child of eight, you see, was thinking in terms of the bases because she had had the experience which equipped her to do so. There is nothing whatever to be afraid of in this question of working in a different base. I would suggest that grouping leads to this notational principle. It leads also, of course, to an understanding of what the actual operations of mathematics are. What is addition, after all? Think of the thousands of children who are scribbling with their pencils, doing addition sums. Let us go in to the classroom and ask the children: "What do you mean by addition? What is addition?" (I do not think they should be doing addition until they have got some idea of what it is they are doing.) You will usually end up with something of this sort: "It is making bigger", which, of course, mathematically, it is not doing. Mathematically, there are no more at the end of an addition than there were at the beginning. And if we start children thinking in terms such as "it makes bigger", then we are making another rod for our backs. Addition is the collecting of a number of groups into one main group. If you have a basket with 17 oranges in, and a basket with 10 in, another one with 12 and another one with 15, then you can collect them all up in one basket — that is an addition sum. But you have not got more oranges at the end than you had at the beginning. You have got exactly the same number. If you have got more, you have somehow materialised oranges out of nothing. The same thing holds with number.

These things are important. Multiplication is a different form of grouping. Five baskets, each one containing 11 oranges — multiplication is the collecting up of groups of equal size. So actually, so far as the operational side is concerned, the two are identical — addition and multiplication. It is in the techniques for the working of them out that we sometimes separate them.

The same holds for division and subtraction. They are both sharing, but how often do you hear a teacher talk of subtraction as sharing. It is kept for division for some reason which I know not, subtraction being the sharing of one main group into two usually unequal groups. If I take 7 from 12, I have got two groups of 7 and 5 left. There are no fewer left at the end. There are two groups instead of one. Division is the sharing of a main group into a number of equal groups.

Once we start on this grouping, we are led straight into the operations of mathematics and this approach I would like to see developed. Were I not old-fashioned, I should be talking of this as set work.

My next fundamental idea in junior mathematics would be the idea of approximation and estimation, which links very closely with magnitude. I think we have tended in the past to regard approximation as of secondary importance. Sometimes we make an approximation before we do a sum, but the approximation does not matter, it is the sum that matters. That is quite absurd. In life, the majority of the mathematical decisions we take are approximations. They have to be. And yet, we rarely say to a child: "What's the answer, roughly? I don't want the exact answer — it doesn't matter. What is the approximate answer?" — and leave it at that. So I would put approximation and estimation, that is in quantity, as an important aspect of junior maths. An estimation, of course, is only one way. You cannot understand a yard unless you have handled a yard. There is no other way; experience is the only answer for this, in estimation.

Number four in my list of fundamentals, my scheme of work for the junior school, is spatial relationships — geometry, if you like. I do not like the term "geometry" myself for the junior school. It suggests some kind of "proving things", which I do not think is the part geometry should have in the junior schools. Euclid has absolutely no place. The basis for geometry, I suggest, is intuitive. Two years ago I spoke at Exeter, in Devon, at a teachers' course. I had some time to spare before the lecture, so I went into Exeter Cathedral. I stood in that cathedral for ten minutes and looked around. I was surrounded by shape, by form and I was filled with wonder. Why do we not introduce our children to these shape ideas, which is all we should do in the junior school? I went to my lecture uplifted, because of a few moments in a place where I was surrounded by form, surrounded by spatial relationships, to reduce them to horrible terms.

My fifth fundamental would be the laws of mathematics which children should abstract from their work — the algebra of the work.

My sixth would be the study of pattern — one of the most exciting new developments in mathematics teaching that there is. There is great scope in this search for pattern. I will only give you one simple example which I think may be of interest. It is not one of the complicated matrices type of patterns. Try this on your children:

$$16 \div 8 = 2$$

$$16 \div 4 = 4$$

$$16 \div 2 = 8$$

$$16 \div 1 = 16$$

$$16 \div \tfrac{1}{2} = ?$$

If you ask children normally "divide 16 by a half", you will get some funny answers because it is difficult. Give it to them as the end product of a pattern which they must study for themselves. Do not tell them about the relationships here. Do not tell them: "This is getting half as big each time, so the answer is getting twice as big." Let them discover the pattern for themselves. There is, as I say, great scope for this work in pattern. This is really looking behind life and living, at the patterns that underlie life and living.

My last fundamental is not a fundamental in the sense of the others. This is really a by-product. I still feel that there is a place for calculation. I think most certainly there is a place in the transitionary period, that is while we are changing from our old ways of thinking to our newer ways of thinking, but I mean something totally different from the conventional teaching of sums. I mean making every calculation a challenge to the child in its own right. I mean getting rid of the conventional ways of teaching completely, in the early stage — throwing decomposition overboard, throwing equal addition overboard, throwing all the rest of them overboard, finding the way best suited to each particular example. (And this, by the way, is one of those delightful cases where common sense has been utterly lacking in the past.) I well remember going into my third year B class. I put 50 subtraction sums as equations on the board and we talked about subtraction. We did sums in all sorts of ways, not only one way, then I said: "Now, you do those 50 and while you are doing them, I'll put another half-a-dozen in the corner for the clever ones. You mustn't do those extra six until you have shown me the others." A youngster came out and said after about ten or twelve minutes: "I've done the 50, sir," so I went through them. He got about 42 or 43 right I did not say: "Go and correct the others." I think correction is frequently an utter waste of time. I said: "John, that's splendid, go and try the other six." I hope he went away feeling a little bit encouraged, not depressed by this emphasis on the wrong side. He came back two or three minutes later with the other six done. Among those six, I had put this one:

$$321 — 33$$

which, as you will see, conventionally would be something like this — 3 from 1 you can't, off we go, back in the world of magic and mystery! I said to John: "How did you do that one, John?" — just for interest. The answer I got nearly sent me through the roof with joy. He said: "Please sir, I took away 21 and then I took away 12," — making what is traditionally a very difficult subtraction into an absurdly simple one. He made it in effect 12 from 300 because he intelligently regrouped that 33 in the light of this particular sum — nothing else.

I can assure you this sort of work can be most exciting. Youngsters will find ways of doing sums you had never dreamed of and sometimes they are better than the conventional ways. Have we not been in the habit of indoctrinating our children into thinking there is only one way of doing things? I remember going to Scunthorpe three years ago, one Saturday morning. I said to the teachers: "Why don't you sometimes give them a little sum and show them several ways of doing it?" I went back home from Scunthorpe and about ten days later had a letter from the headmaster at whose house I had stayed over the weekend. He wrote: "You will be interested. One of my chaps liked the idea. He went to school on Monday morning, put a little sum on the board, showed them several ways of doing it and said: 'Now, you write down four different ways of doing this sum.' (These were the 11 plus children, scholarship class.) His best pupil wrote this down: '(1) Do it on your fingers. (2) Do it in your head. (3) Do it like you did it on the blackboard. (4) Do it the proper way.' "

Those are my fundamentals. I have not forgotten in these fundamentals the pattern search and the experimental work which I referred to earlier. They are also strands of the general pattern.

If your teaching is to be of value, I would suggest that these are the points which you must bear in mind. Firstly, the teacher must have the freedom to think for himself and not teach to an externally imposed authoritarian prescription. Secondly, the child also must have the freedom to think for himself. He must have the right to work out these little situations in the way that best suits him; it is the child who decides the way and not the teacher. Thirdly we must recognise the place of words in our teaching of mathematics in the primary school. We must give the children adequate opportunities for experimenting. (I have given you two simple examples of the experiment.) Fourthly we must nurture attitudes as well as we possibly can. This is, I think, of first importance and is it not a tragedy that this emphasis on calculation has killed the attitude? Professor Godfrey Thompson wrote in an essay at least forty years ago that if a child did no calculation at all before the age of $12\frac{1}{2}$, he would learn the lot in about three to six months. And yet we sweat and toil with children from the age of 6 upwards until they hate the sight of the stuff and that attitude is very difficult to repair. Let us be clear on this — attitudes do matter.

My final reminder to you is that it is the teacher who matters most.

I read some time ago a book by the late John Buchan, "Memory Holds the Door" in which he wrote this: "For youth to continue into age and bind the years pleasantly together, it must have spaciousness and wide horizons." Is not that just exactly what this new idea in mathematics is aiming at?

Let me conclude by re-stressing the difficulty of mathematics. I must finish with this because it is a most difficult subject. It is an abstract study being given to immature minds. Can that combination ever be a particularly happy one? Thirty years ago a wise lady, a mathematician and a clever one, wrote something which strikes me as singularly prophetic. Miss Renwick in a book called "The Case Against Arithmetic" wrote this: (She foresaw Piaget, she foresaw the structural materials, she foresaw the lot, probably not knowing herself she was foreseeing them.) "In arithmetic, the notions with which we are concerned are abstract and recondite notions which develop only after years of experience. They cannot be hurried into existence. We may not assume that a change in the method of presentation is all that is required, that if only we can invent better didactic apparatus, more interesting or illuminating exercises, clearer ways of explaining difficult points, we shall be able to ensure that every normal child of 11 will understand simple operations involving continuous quantity and abstract number. Where so many teachers have failed, it may be that no simple solution exists."

I would submit that that is the measure of our problem, but I would hate to finish on such a sombre note. I feel that the exciting things that are happening all over the world give us real confidence, give us high hopes, that something is going to happen in the teaching of mathematics very soon. I only wish that I were thirty years younger and could have a hand in it myself.