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A STOCHASTIC CUT-OFF GRADE OPTIMISATION ALGORITHM

Joseph Muchiri Githiria

A thesis submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering.

Johannesburg, 2018

DECLARATION

I declare that this thesis is my own, unaided work. Where use has been made of the work of others, it has been duly acknowledged. It is being submitted for the Degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before in any form for any degree or examination in any other University.

Signed:

_____

Joseph Muchiri Githiria

This 2nd day of October 2018

ABSTRACT

Cut-off grade is the grade used to distinguish between ore and waste in a mining complex. The cut-off grade is a function of economic parameters, grade distribution within the deposit, metallurgical recovery, and mining, milling and refinery capacities. A cut-off grade is used to determine the quantity of material mined, quantity of ore processed and lastly, the quantity of product (e.g. metal) produced in a mine. The resultant sequence of cut-off grades generated from the cut-off grade calculations over the life of mine (LOM) defines the cut-off grade policy for that specific operation. The cut-off grade policy generated from these calculations determines the future cash flows hence the present value (NPV) of a mining project.

Optimisation of cut-off grade is paramount in most mines and aims to maximise NPV subject to economic, geological and operational parameters in a mining complex. There are different cut-off grade optimisation approaches applied in mine planning. These include break-even, Lane's and heuristic approaches. The most common approach used in the mining industry is based on Lane's cut-off grade theory. Lane developed a three-dimensional process that calculates cut-off grades that considers the economic parameters, geological parameters and production capacities. It uses mathematical derivations to generate six cut-off grades that are sorted using an algorithm to obtain an optimum cut-off grade. Lane's framework assumes that the input parameters are deterministic, yet their values are stochastic in actual operations. Therefore, Lane's approach fails to reflect the reality hence the need for its modification.

This thesis developed a stochastic approach using Lane's theory by considering the dynamic nature of the variables and constraints governing a mining operation. The stochastic cut-off grade approach which was developed in this thesis aims at maximising NPV by concurrently varying metal price and grade-tonnage distribution in the algorithm. Variation of these variables was necessary to capture the inter-temporal fluctuations experienced in these two variables for most mining operations. The metal price was varied using a random number generator developed in the code for generating values within a specified range while multiple realisations of grade-tonnage distribution were factored into the algorithm.

The algorithm for the stochastic cut-off grade approach was coded using C++ on a Microsoft Visual Studio integrated development environment (IDE) to develop a computer-aided application named *NPVMining*. Using a dataset from a gold mine, the algorithm generated a set of possible solutions with the maximum NPV against a specific time range as the optimal solution to a given block model. The resultant cut-off grade policies were compared and analysed using linear regression analysis. The analysis

indicated that metal price has a higher effect on NPV compared to the grade-tonnage distribution as these had correlation coefficients of 0.97 and 0.13 with NPV, respectively.

Using the same dataset from the gold mine, a comparison was between *NPVMining* and other cut-off grade calculation approaches. *NPVMining* produced economically superior NPV over other methods: 7% better than Cut-Off Grade Optimiser; 13% better than Maptek Evolution; 35% better than heuristic cut-off grade and 185% better than break-even cut-off grade approach. A risk analysis of the NPV generated from *NPVMining* against the input parameters using @Risk software from Palisade Corporation indicated that metal price has the highest effect on the NPV. Sensitivity analysis was undertaken on the results by factoring a range of -15% to +15% on the input variables to determine the stability of the solutions. The NPV values from the sensitivity analysis ranged from about US$460 to US$472 million, which is a relatively stable solution and can therefore be accepted with confidence when undertaking the project.

In conclusion, the study contributed to knowledge by developing a stochastic cut-off grade optimisation model. The study demonstrated the advantage of modifying Lane's cut-off grade theory to accommodate the two stochastic input parameters. For this reason, it is prudent for mining companies to apply stochastic-based techniques such as the stochastic cut-off grade optimisation algorithm developed in this thesis.

DEDICATION

To my dear family: my parents (Eng. D.G. Muchiri and Mrs. Dianah Githiria) and my sisters (Wambui Githiria, Muringe Githiria and M. Muthoni Muchiri) who always supported my academic life and journey.

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- Staff from the School of Mining Engineering in their assistance in relation to my postgraduate study and solving any problems and difficulties that I experienced while undertaking this thesis.
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- Fellow academic colleagues (Mr. Salim Faraj and Mr. Gift Khangamwa) for assisting me to code the algorithm developed in this thesis into a working computer-aided application.
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May God bless them abundantly!

LIST OF PUBLICATIONS

The following are peer-reviewed or referred publications that have already emanated from this thesis:

1. Githiria, J., Muriuki, J., and Musingwini, C., 2016. Development of a computer-aided application using Lane's algorithm to optimise cut-off grade, *Journal of the Southern African Institute of Mining and Metallurgy,* Vol. 116, No. 11, pp 1027-1035.
2. Githiria, J., and Musingwini, C., 2018. Comparison of cut-off grade models in mine planning for improved value creation based on NPV, *Proceedings of the 6th Regional Conference of the Society of Mining Professors (SOMP)*, Johannesburg, South Africa, pp. 347–362.
3. Githiria, J., Nhleko, A.S., 2018. Generic approach to developing an algorithm for mining related problems, *Proceedings of the 4th Young Professionals Conference*, Focus Rooms, The Core, Sunninghill, Johannesburg, South Africa, pp. 137–150.

TABLE OF CONTENTS

DECLARATION i

ABSTRACT ii

DEDICATION iv

ACKNOWLEDGEMENTS v

LIST OF PUBLICATIONS vi

LIST OF FIGURES xi

LIST OF TABLES xiii

1 INTRODUCTION 1

1.1 Chapter overview 1

1.2 Research background 2

1.3 Problem statement 6

1.4 Research objectives 10

1.5 Scope of the study 11

1.6 Research question(s) and relevance 11

1.7 Outline of chapters 12

2 LITERATURE REVIEW 14

2.1 Chapter overview 14

2.2 Input parameters used in cut-off grade calculations 16

2.3 Break-even cut-off grade approach 17

2.4 Heuristic cut-off grade approach 18

2.5 Lane's cut-off grade approach 20

2.5.1 Basic equations 21

2.5.2 Limiting cut-off grades 24

2.5.3 Balancing cut-off grades 25

2.5.4 Optimum cut-off grade 27

2.5.5 General steps of Lane's algorithm 27

2.6 Application of cut-off grade approaches in mine planning 29

2.7 Stochastic and dynamic cut-off grade approaches 31

2.8 Summary of Chapter 2 33

3 DEVELOPMENT OF THE STOCHASTIC CUT-OFF GRADE OPTIMISATION ALGORITHM 34

3.1 Chapter overview 34

3.2 Development of the cut-off grade optimisation model 37

3.2.1 Steps of the modified approach 42

3.2.2 Implementation plan 45

3.3 *NPVMining* application 46

3.3.1 Graphic user interface (GUI) 48

3.3.2 Calculation of optimum cut-off grade 52

3.4 Summary of Chapter 3 53

4 APPLICATION OF THE STOCHASTIC CUT-OFF GRADE ALGORITHM 55

4.1 Chapter overview 55

4.2 Input parameters used in the cut-off grade optimisation models 57

4.2.1 Grade-tonnage distribution 57

4.2.2 Economic, design and operational parameters 60

4.3 Cut-off grade policy generated from the proposed stochastic model (*NPVMining*) 63

4.3.1 Results from the application (NPVMining) using constant economic and technical parameters 63

4.3.2 Results generated using NPVMining with price variations 63

4.3.3 Results generated using NPVMining under multiple realisations of the grade-tonnage distribution 64

4.4 Linear regression analysis of the results 65
4.5 Summary of Chapter 4 68
5 RESULTS AND VALIDATION 69
5.1 Chapter overview 69
5.2 Validation of the proposed model 69
5.2.1 Application of break-even cut-off grade approach 70
5.2.2 Application of traditional heuristic cut-off grade approach 72
5.2.3 Application of a deterministic approach based on Lane's theory (Cut-off Grade Optimiser) 73
5.2.4 Application of a modern heuristic cut-off grade approach using Maptek Evolution® 74
5.2.5 Application of the proposed stochastic Lane's cut-off grade approach using NPVMining 75
5.2.6 Comparison of the cut-off grade models 76
5.3 Risk and decision analysis 78
5.3.1 Probabilistic analysis 79
5.3.2 Sensitivity analysis 81
5.4 Summary of Chapter 5 82
6 CONCLUSIONS AND RECOMMENDATIONS 84
6.1 Introduction 84
6.2 Research contributions 84
6.3 Research limitations 86
6.4 Recommendations 86
7 REFERENCES 87
8 APPENDICES 91
8.1 Appendix 1: Summary of mathematical programming techniques 91

8.2 Appendix 2: Monte Carlo integration as applied in RStudio 95

8.2.1 Monte Carlo integration code 95

8.3 Appendix 3: *NPVMining* user guide 96

8.3.1 Data input: Price criteria 96

8.3.2 Data input: Other economic and operational parameters and the grade-tonnage category 101

8.3.3 Selection of limiting capacity and cut-off grade calculation method 104

8.3.4 Output 105

8.3.5 Output based on price variation generated from the NPVMining for all the grade-tonnage curves (GTC1-GTC6) 106

8.3.6 Components of the computer-aided application (NPVMining) 112

8.4 Appendix 4: *NPVMining* code in C++ 114

8.4.1 Stripping ratio 114

8.4.2 Average grade 114

8.4.3 Optimum cut-off grade 115

8.4.4 Limiting cut-off grade 115

8.4.5 Break-even cut-off grade 116

8.4.6 Grade category 116

8.4.7 Sub category 117

8.4.8 Creating an array of price values 117

8.4.9 Annual profit 122

8.4.10 NPV 122

LIST OF FIGURES

Figure 1.1: Open pit long-term production planning variables (Osanloo *et al*, 2008a) 1

Figure 1.2: Layout of a mining complex 4

Figure 1.3: 5–Year Gold Prices and Price Charts (InfoMine, n.d. http://www.infomine.com/investment/metal-prices/gold/5-year/) 7

Figure 1.4: 5–Year Platinum Prices and Price Charts(InfoMine, n.d. http://www.infomine.com/investment/metal-prices/platinum/5-year/)..... 7

Figure 2.1: Model described by Lane's algorithm (Lane, 1964; Hustrulid *et al.*, 2013) 21

Figure 3.1: Simple flow diagram of a cut-off grade algorithm (Asad, 1997; Bascetin and Nieto, 2007; Githiria *et al.*, 2016) 35

Figure 3.2: General flow of the proposed algorithm..... 36

Figure 3.3: Flow diagram of the modified cut-off grade algorithm 45

Figure 3.4: Component files of *NPVMining* on solution explorer in VC++ 47

Figure 3.5: Diagram showing features of the application as displayed in the user interface 48

Figure 3.6: NPVMining user interface 49

Figure 3.7: Grade category window 50

Figure 3.8: Price criteria window..... 50

Figure 3.9: Other parameters (economic and operational parameters) window 51

Figure 3.10: Selection of the limiting capacity and cut-off grade policy used 52

Figure 3.11: Output exported to a Microsoft Excel spreadsheet..... 52

Figure 4.1: Cross-section of sheeted gold vein in the orebody (Enderlin, 2002)..... 57

Figure 4.2: RStudio user interface 59

Figure 4.3: Flow of material through the mining operation 61

Figure 4.4: Yearly gold price projections (InfoMine, n.d. http://www.infomine.com/investment/metal-prices/gold/5-year/) 62

Figure 4.5: Linear relationship between metal price and NPV..... 67

Figure 4.6: Linear relationship between grade-tonnage distribution and NPV 67

Figure 5.1: A comparison of cut-off grade per year from the cut-off grade models 77

Figure 5.2: Probabilistic analysis of *NPVMining* results using @Risk software..... 79

Figure 5.3: Normal probability distribution of NPV against input parameters 80

Figure 5.4 Spider graph showing percentage change of input variables on NPV 82

Figure 8.1: Entering price category 97

Figure 8.2: Entering price sub-category 97

Figure 8.3: Entire price category list 98

Figure 8.4: Downloaded Microsoft Excel spreadsheet for price value entry 99

Figure 8.5: Saving the Microsoft Excel spreadsheet with price values 99

Figure 8.6: Uploading the saved Microsoft Excel spreadsheet with price values 100

Figure 8.7: Popup window confirming the array of price values read by the application 101

Figure 8.8: Other economic and operational parameters 102

Figure 8.9: Grade category showing lower limit, upper limit and quantity with the grade-tonnage distribution entry 102

Figure 8.10: Appended list of grade-tonnage data 103

Figure 8.11: Removing a grade-tonnage entry 103

Figure 8.12: Deleting the grade-tonnage entry 104

Figure 8.13: Limiting capacity and method used to calculate the optimal solution 105

Figure 8.14: Output based on price variation 105

LIST OF TABLES

Table 2.1: Brief comparison among break-even, heuristic and Lane's cut-off grade approaches 15

Table 2.2: Notations of parameters in Lane's cut-off grade algorithm (Hustrulid *et al.*, 2013) 16

Table 3.1: Notations of parameters in the modified cut-off grade algorithm 38

Table 4.1: Summary of the original block model 55

Table 4.2: Grade-tonnage distribution of the McLaughlin gold deposit 56

Table 4.3: Simulated grade-tonnage distribution of the McLaughlin gold deposit 60

Table 4.4: Economic, design and operational parameters used in the model 61

Table 4.5: Price variations used in the model 62

Table 4.6: Results as reported from *NPVMining* for grade-tonnage curve (GTC1) at a price of US$1250 per ounce 63

Table 4.7: Summary of the output generated from six possible metal price ranges for grade-tonnage curve (GTC1) 64

Table 4.8: Summary of the output generated from six equally probable simulated grade-tonnage curves 65

Table 4.9: Net present values generated using *NPVMining* when varying price and grade-tonnage distribution 66

Table 5.1: Cut-off grade policy using the break-even cut-off grade model 71

Table 5.2: Cut-off grade policy using a traditional heuristic model (fixed costs included) 73

Table 5.3: Results of cut-off grade policy from Cut-off Grade Optimiser (Githiria *et al*., 2016) 74

Table 5.4: Cut-off grade policy as reported from Maptek Evolution® per period 75

Table 5.5: Cut-off grade policy as reported from *NPVMining* per period 76

Table 5.6: Comparison of the mine life, cash flow and NPV from the cut-off grade models 78

Table 5.7: Probability distribution of the NPV as reported using @Risk 80

Table 8.1: Chronological summary of deterministic cut-off grade optimisation techniques used in the mining industry 91

Table 8.2: Chronological summary of uncertainty-based cut-off grade optimisation techniques used in the mining industry 93

Table 8.3: Output based on price variation for McLaughlin gold deposit (GTC1)..... 106

Table 8.4: Output based on price variation for McLaughlin gold deposit (GTC2)..... 107

Table 8.5: Output based on price variation for McLaughlin gold deposit (GTC3)..... 108

Table 8.6: Output based on price variation for McLaughlin gold deposit (GTC4)..... 109

Table 8.7: Output based on price variation for McLaughlin gold deposit (GTC5)..... 110

Table 8.8: Output based on price variation for McLaughlin gold deposit (GTC6)..... 111

1 INTRODUCTION

1.1 Chapter overview

This chapter provides background information on the significance of optimising cut-off grade in relation to mining project value, the research studies undertaken and the relevance of undertaking this thesis. Cut-off grade is defined as the grade that is normally used to distinguish between ore and waste within a given orebody (Dagdelen, 1992, Asad, 2002, Gholamnejad, 2009). It also differentiates between various ore types before processing takes place for different metallurgical processing options. The main objective of mining companies is to maximise value over the life of mine (LOM). The value is normally measured using a proxy called the net present value (NPV). The NPV expected throughout the LOM from an operation is dependent on interrelated variables such as mining and milling capacities, extraction sequence, and cut-off grade. These interdependent variables interact in a complex manner in defining the NPV of a project. Cut-off grade interacts with other variables in open pit long-term production planning as shown in Figure 1.1. The figure shows how cut-off grade is relevant in mine planning. Mine planning is the process that defines sets of values for each of the decision variables during the LOM. The efficiency of the optimisation model allows it to solve the optimisation problem to maximise the NPV of the mine, without violating any mining constraints.

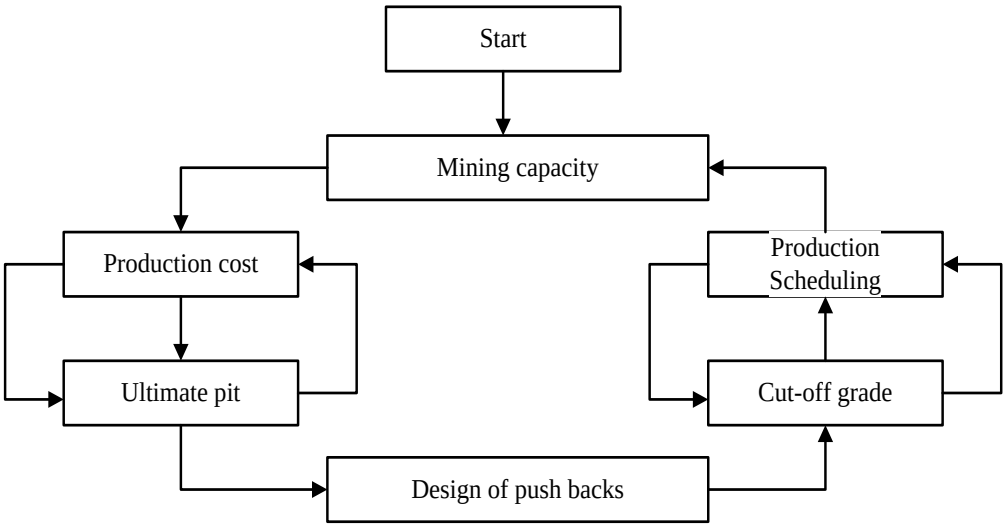


Figure 1.1: Open pit long-term production planning variables (Osanloo *et al*, 2008a)

The choice of an optimal cut-off grade in any given production period for a mining project varies significantly over time hence affecting the mining operation. This will consequently affect the economic feasibility of such a mining project. Most research studies such as those undertaken by Taylor (1972, 1985), Dagdelen (1992, 1993), Dagdelen and

Kawahata (2008), King (2001, 2011), Whittle and Wharton (1995), Asad (1997, 2002, 2005), Minnitt (2004), Osanloo *et al.* (2008b), Gholamnejad (2008, 2009) and Githiria *et al.* (2016) have focussed on optimising cut-off grade using deterministic variables. These variables include cost, selling price, recovery, and grade distribution of the deposit. This contrasts with what happens in actual mining operations. These variables change with time depending on factors influencing the global mining industry. For example, when commodity prices fluctuate over time in the markets the cut-off grade selection should ideally be responding accordingly. Most mining operations do not consider these variations when selecting their cut-off grades, thus, leading to sub-optimal cut-off grade policies and sub-optimal NPVs. This thesis, therefore, developed an approach to define the capacity of a mining system that is in harmony with the grade distributions of the deposit and all mining variables through the planned extraction sequence and cut-off grade policy. This approach optimises cut-off grades under uncertainty.

The mine planning process includes production planning and scheduling, within the mine design, considering such aspects as geological features and mineralisation, associated mining infrastructure and other constraints (the South African Mineral Resource Committee (SAMREC) code, 2016). A Mineral Resource is the basis on which mine planning can be undertaken. A Mineral Resource is defined by the SAMREC code (2016) as a concentration or occurrence of material of economic interest in or on the Earth's crust in such form, grade or quality and quantity that there are reasonable prospects for eventual economic extraction. The geological characteristics of a Mineral Resource are known, estimated or interpreted from geological evidence and knowledge, including sampling results. According to the SAMREC code (2016), a Mineral Reserve is the economically mineable part of a Measured and/or Indicated Mineral Resource. It includes diluting materials and allowances for losses, which may occur when the material is mined or extracted. It is defined by studies at Pre-Feasibility or Feasibility level as appropriate that include the application of modifying factors. Such studies demonstrate that, at the time of reporting, technical and economic extraction could reasonably be justified. The accuracy range or order of magnitude as described in the SAMREC code (2016) is defined to be at ±25 to 50% when undertaking a Scoping Study, ±15 to 25% for the Pre-feasibility Study and ±10 to 15% when carrying out a Feasibility Study. This thesis uses Mineral Reserves to define the grade-tonnage distribution in the orebody that is employed in the cut-off grade calculations.

1.2 Research background

According to Minnitt (2004), NPV is the commonly used criterion that determines the economic value of mining projects. It is the sum of all future cash flows discounted at a

specific interest rate. The relationship between cut-off grade and NPV provides a means by which cut-off grades are optimised.

There are several methods that are used in calculating cut-off grade such as the break-even model, traditional or heuristic methods, Lane's cut-off grade model (Lane, 1964; 1988) and extensions to Lane's model. A detailed description and comparison between these methods is provided later in Chapter 2. The break-even cut-off grade model is a one-dimensional calculation approach. It is widely used in mining operations due to its simplicity unlike Lane's model, which involves mathematical derivations of Lane's formulae. It is one-dimensional model because it accounts only for economic parameters and ignores the grade-tonnage distribution of the deposit. Break-even cut-off grade models are not focused on maximising cash returns hence are not an optimisation approach. The cut-off grades are held constant throughout the life of mine at mine planning or feasibility study stages. The model ensures that every tonne classified as ore pays for itself at the time that it is treated. There are different ways in which mining companies have treated costs in the calculation of break-even cut-off grades. There are no industry-wide standard definitions on the costs and each one of the stakeholders uses their own (Hall, 2014, Poniewierski and Hall, 2016). The traditional calculation includes only the mining and milling costs (Bascetin *et al.* 2011). Capital costs could potentially be included as well (Hall 2014).

Heuristic cut-off grade optimisation approaches have been incorporated in the mining industry. This is because they use cut-off grades higher than break-even grades during the early years of the mine operation for faster recovery of capital investment (Dagdelen, 1992; Asad, 2002). The approaches then lower down the break-even grades during the later years of the mine to maximise extraction. Cut-off grade calculation in these approaches includes depreciation, fixed cost and minimum profit per tonne of ore for a period. This is done to get a much higher cut-off grade during the early years of the mine's life. Towards the end of the mine's life the minimum profit requirement is removed from the equation to lower the cut-off grades further to pay off the plant's costs. Then the depreciation charges are removed from the equation, thus, leaving the mine to operate on the milling cut-off grades. The whole mining process involves dynamic cut-off grades which tend to be higher in early years and then lower in the later years of the mining project or operation. Calculated cut-off grades change along the mine's life in intervals to define a cut-off grade policy. Heuristic approaches tend to produce sub-optimal NPVs for any given deposit therefore, returns on investment need to be improved (Dagdelen, 1992).

Lane (1964, 1988) came up with a three-dimensional process for calculating cut-off grades. Lane's process accounts for economic parameters, geological parameters and

production capacities in the calculation of cut-off grades. Lane's theory uses mathematical derivations to get six cut-off grades, which are then sorted using an algorithm to get an optimum cut-off grade. The mathematical derivations of Lane's formulae are described in Chapter 2.

Three of the cut-off grades are limiting cut-off grades, derived by assuming that the three stages; mining, processing and refining are each independently constraining throughput. The limiting cut-off grades are based upon capacities, costs, and price. The other three are balancing cut-off grades that are determined by assuming that two of the three stages are operating in unison at their capacity limits. They are based on the grade, and capacities of the production stages. The rate of production in a mining system is determined by the production capacities as shown in Figure 1.2. They include mining capacity, milling or concentrator capacity and refinery capacity, denoted by M, C and R, respectively.

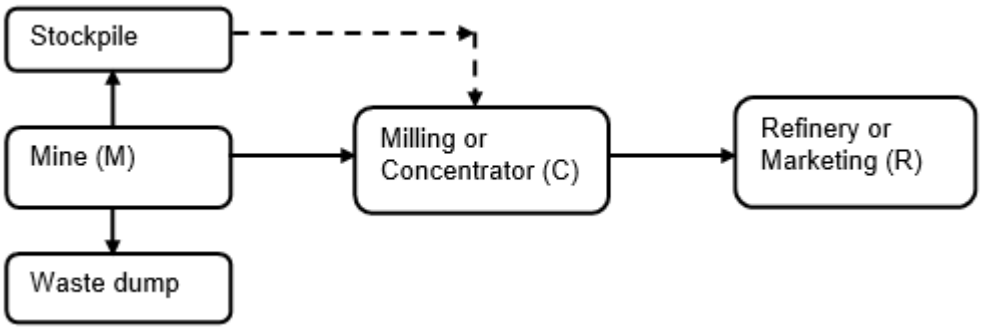


Figure 1.2: Layout of a mining complex

Lane's cut-off grade theory is a major development on how cut-off grade is calculated. It encompasses the concept of opportunity cost that is accounted for to indicate the extent to which future production can be deferred to immediately treat additional material as ore. It also incorporates balancing cut-off grades that ensure that the grade-tonnage curve and production capacities are accounted for. These two aspects show how important and feasible this method is in calculating cut-off grades.

Economic cut-off grades are determined using approaches that aim to obtain the highest NPV from any mine. There are several approaches based on Lane's theory that are used in the optimisation of cut-off grade, that have continually been revised to account for most of the actual circumstances involved in mining operations. These approaches are discussed comprehensively in Chapter 2, Section 2.6. Optimisation formulae developed by Lane (1964, 1988) represent an all-inclusive approach, which is counter-intuitive from a mining perspective, but provides good first pass approximations to optimal cut-off grade. Financial and technical modelling, and optimisation simulation have taken over to

accommodate most of the actual circumstances that mining operations face. Lane's approach has been modified in several ways to incorporate other complications that are involved in a mining complex.

The aim of optimising cut-off grade policy is to select a sequence cut-off grades over time to maximise the NPV for any mine. To meet this objective, cut-off grade is considered in all the stages of a mining operation. The three major stages (mining, processing, and refining or marketing) involved in resource extraction are all defined in Lane's approach. Using Lane's approach, the economics involved in each stage are identified independently and interact to provide an optimum cut-off grade (Lane, 1964, 1988). The sequence of extraction is dependent on the rates of production, the grade distribution of the deposit, the cut-off grades and the mining method selected. The sequence of extraction is more dependent on the mining method selected in underground mining.

Cut-off grade optimisation is necessary for operational decision making in the many different circumstances affecting mine operations hence the need for the creation of several cut-off grades that are appropriate to the intertemporal circumstances. The cut-off grades are supposed to reflect circumstances of specific mines to achieve the maximum economic return over the life of the mine. This implies that there is a need to consider the time value of money meaning that the maximum economic return is achieved when the highest NPV of future cash flows is achieved. This requires the mining of higher-grade ore before lower grade ore where possible. In doing so cut-off grades of a specific mine tend to decline over the life of the mine.

Lane's approach shows that the change in value that is associated with depletion of ore is the difference between the cash flow from mining the Mineral Reserves and the opportunity cost. Opportunity cost is the loss incurred by a mining firm for having invested their money in the present mining operation. The firm will have to forego the profits they would otherwise have obtained if they had invested in another mining investment and hence incurring an opportunity cost (Minnitt, 2004).

Githiria *et al.* (2016) developed a computer-aided application (Cut-off Grade Optimiser) using Lane's theory and had the results for a hypothetical mine presented in a Microsoft Excel spreadsheet. The production schedule reported from the model shows the NPV generated, quantities of material mined (Qm), material milled (Qc) and the metal product (Qr) from the entire mining process. This production schedule guides the mine planning department for either the long-term or short-term by specifying the period to be used in the cut-off grade policy. The cut-off grades are dynamic since Lane's approach accounts for the change in the grade-tonnage curve. This ensures that the cut-off grades used by the mine planners are proportional to the rate of ore depletion.

1.3 Problem statement

Mining has come to a stage where mining engineers are now embracing optimisation in planning their operations. A few mining companies are using mathematical models to optimise their daily operations. Both open pit and underground mine planning have the same objective of maximising the NPV of future cash flows, thus, the need to use computer programming to optimise mining processes to manage the intensity of the computational work required. Recently, there has been a decline in the prices of most commodities leading to the closure of some mid-sized mines around the world. This has further strengthened the paradigm shift in the way most mining companies are embracing optimisation methods to improve the current practices in mining. Further research is being done on optimisation of the global mining system to minimise costs and maximise profits.

Potential mining projects stopped their exploration activities and evaluation procedures due to a decline in the prices of commodities such as gold and platinum. These two precious metals have seen a decline in the past five years as shown in Figures 1.3 and 1.4. In a report done by Johnson Matthey on the platinum group metals (PGM) market, the global primary supplies of platinum in 2017 declined by 2% from the previous year by 6 million ounces. There were two mine closures during 2017 in South Africa: Maseve mine owned by Platinum Group Metals Limited that ceased producing in mid-year and Bokoni mine owned by Atlatsa Resources at the end of September (Matthey, 2018).

Gold mines around the world have also suffered the same fate, with several mines closing in the past years. The gold sector's profitability started declining in the early 21st century as mines started experiencing the effects of the global recession. South Africa's gold industry has for decades being a leading source of the precious metal, but it has declined significantly due to the falling grades, old and deep mines, rising costs, declining metal prices and productivity. According to the Minerals Council South Africa, three-quarters of gold mines are unprofitable or barely making money due to the weak rand price of gold (Seccombe, 2018). This has forced several mines to close such as Evander gold mine belonging to Pan African Resources and the Cooke mines owned by Sibanye-Stillwater mine company (Seccombe, 2018).



Figure 1.3: 5–Year Gold Prices and Price Charts (InfoMine, n.d. http://www.infomine.com/investment/metal-prices/gold/5-year/)



Figure 1.4: 5–Year Platinum Prices and Price Charts(InfoMine, n.d. http://www.infomine.com/investment/metal-prices/platinum/5-year/)

A mining complex constitutes a mine, an ore processing plant and a metal refinery. To avoid creating losses in a mining system, these areas should be optimally planned.

Mathematical optimisation techniques have been used in the mining industry to develop several computer-aided applications currently used in the mining industry to maximise profits. However, there is a great need for improvement in mine planning through integrated optimisation of the whole process. Examples of the mathematical optimisation techniques used in mining include linear programming techniques (mixed integer programming, integer programming, binary integer programming), dynamic programming, greedy method and heuristic methods such as local search, Tabu search, simulated annealing, and genetic algorithms. Common cut-off grade optimisation techniques used in the mining industry are summarised in Appendix 1 (Tables 8.1 and 8.2).

The development and implementation of mathematical formulations to solve complex optimisation problems in mine planning has been progressive, especially in open pit mining. Cut-off grade optimisation is important in mining since it assists to determine the economic feasibility of a project. A cut-off grade policy enables risk-quantified decision-making process streams that create sustainable utilisation of Mineral Resources through major investments (Asad and Dimitrakopoulos, 2013).

Although there have been successive strides in the optimisation of cut-off grade, there is a need to come up with an algorithm that incorporates all aspects involved in a mining operation. Most approaches developed to optimise cut-off grade tackle individual aspects, hence, making it impossible to apply in a mining operation. They focus on optimising cut-off grade under deterministic variables while, these variables change in value along the LOM.

Variables such as cost, selling price, recovery, and grade-tonnage distribution of the deposit change with time depending on various factors including factors influencing the global mining industry. Therefore, this thesis developed an extension of Lane's approach in optimising cut-off grade by incorporating dynamic mining variables and constraints in a single commodity mine (a gold mine). The gold mine case study was used to enable comparison of results from the developed stochastic cut-off grade model and other cut-off grade approaches used in the mining industry.

Mining firms should advocate and develop global strategic mine plans that optimize both open pit and underground operations to manage the impact of uncertainties such as fluctuating commodity price and grade uncertainty. For mine plans to incorporate such uncertainties, optimization methods need to change from being deterministic to stochastic (Musingwini, 2016).

This study considers a stochastic approach from the supply of ore to the ore-processing stream and further through a refining or marketing option. It aims to address the risk involved in a mining operation where the capacities of the mine, mill and refinery are

directly related to the cut-off grades and extraction sequences. Previous studies on cut-off grade optimisation have not fully incorporated the various mining uncertainties in the calculation of cut-off grades. This study considers the ever-changing mining variables such as costs, selling price, recovery, and geological parameters.

The cut-off grade policy is based on a stochastic integer programming approach to account for the uncertainties. Stochastic integer programming is an Operations Research (OR) technique that models optimisation problems that have uncertainty. OR is a mathematical science commonly applied to optimisation problems that are concerned with improving the performance of a real-world system. It determines the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost) of some real-world objective (Musingwini, 2016; Githiria and Nhleko, 2018).

There are five main categories of OR models namely: optimisation model, simulation model, network model, multi-criteria decision-making (MCDM) model and global optimisation model. The model applied in this thesis is the optimisation model, which refers to a mathematical formulation that is solved using an exact algorithm to produce a single optimal solution by either maximisation or minimisation (Musingwini, 2016). Developing an algorithm is the first step when coming up with a mathematical model to optimise mining operations. An algorithm is a systematic procedure to accomplish a specific task. It is the idea behind any computer program. Application of programming languages such as C++, Python, among others are used to solve algorithm design problems.

Integer programming is applied in this model because the variables in this mathematical model are integer values. Stochastic optimisation problems have probability distributions governing the data. The choice of technique applied in this thesis, stochastic integer programming, is due to the incorporation of uncertainty into the parameters. The probability distributions in this technique are known or can be estimated. In this case probability distributions of both economic and geological uncertainties are known and/or can be estimated.

Deterministic optimisation problems are solved with known parameters, which is not the case in real mining optimisation problems. The mining parameters are known in specified bounds whereby stochastic programming can be applied. It takes the advantage of the fact that the probability distributions governing the data are known or can be estimated reliably. In this case, the cut-off grade optimisation problem would have parameters with specified bounds which are known or estimated probability distributions.

The main aim is to get an optimal cut-off grade policy that is feasible for all the possible data instances, while, maximising the NPV based on the decision variables. These

mathematical models are useful in providing useful information to the decision maker and easing the computational effort required to solve large problems. In the stochastic approach which was developed in this thesis, each ore grade in the orebody is a random variable following a distribution, while, commodity price and costs are varied with time.

When formulating mathematical programming models, it is important to consider the activities involved and how they are sequenced in the problem. When models are formulated, the number of variables reduces and in turn the solution time, hence solving large and complex problems that aim to maximise the NPV of the mine.

The mathematical model which was developed in this study provides an optimal platform on which mine planners can use easily in routine planning activities. The stability or robustness of the cut-off grade policy is measured using sensitivity analysis to indicate how the variables affect the NPV distribution curve.

1.4 Research objectives

The objectives of this thesis were to:

- Review the existing algorithms on cut-off grade optimisation and justify the need for a stochastic extension to Lane's algorithm.
- Develop a mathematical model using the OR technique (stochastic integer programming) to optimise cut-off grade policy for a mining complex that constitutes of three stages; a mine, ore processing stream, and refinery or market, under uncertain geological, mining and economic variables.
- Create possible strategies in the modified algorithm to generate solutions to the model without violating any physical mining constraints. The algorithm which was developed was coded using C++ programming language in Visual Studio 2017 Integrated Development Environment on a standard computer.
- Using C++ to assemble a graphical user interface (GUI) that displays results in a user-friendly way. This application will have an interface that will enable mining engineers to calculate optimum cut-off grades with ease. C++ has a highly-developed design capability that enables the assembly of the entire GUI for an application graphically and has the code that creates it, generated automatically.
- Application and implementation of the developed model code named *NPVMining* on a gold mine deposit to ascertain its benefits in any operational mine.
- Benchmarking of the results generated from *NPVMining* through comparisons with other conventional cut-off grade optimisation approaches. The output was exported to Microsoft Excel for comparison and analysis. Risk and decision

analysis was done on the output generated from *NPVMining* and other computer-aided applications in mine planning optimisation such as Maptek Evolution® and Cut-off Grade Optimiser (Githiria *et al.*, 2016).

1.5 Scope of the study

The purpose of this thesis was to determine the optimal cut-off grade policy for a mining complex that constitutes a mine, ore processing stream and refinery given geological, mining and economic uncertainties. The cut-off grade policy is based on a stochastic approach that incorporates dynamic interdependent variables such as mining, processing and refining costs, selling price, recovery and grade-tonnage distribution of the deposit. This approach enables mine planners to estimate the suitable cut-off grade for the mine in their routine planning. This thesis considered Lane's algorithm for a single mineral and modified it to incorporate the uncertainty in price and grade-tonnage distribution.

1.6 Research question(s) and relevance

The mining industry has recently been experiencing a volatile shift in commodity prices, labour costs and mining costs thus triggering the need to apply OR techniques in optimising cut-off grades and consequently, maximising NPV. It is evident that mine planners need to calculate cut-off grades under these dynamic variables.

Though significant strides have been made in mine planning, most approaches in cut-off grade optimisation have not tackled the complexity of a mining operation in totality. The geological input should be depicted as reliably as possible, thus having multiple realisations of grade-tonnage curves as an input to the optimisation problem (Asad and Dimitrakopoulos, 2013). The same applies to the dynamic economic parameters that affect daily operation of the mining industry.

The study considered an optimal cut-off grade policy under a range of geological, mining and economic variables that change with time due to factors influencing the global mining industry. The mining variables such as costs, selling price, recovery, and geological parameters are affected either by the economics or by the geological formation of the orebody. These two factors vary with time thus affecting the valuation of the minerals being extracted along the mine life.

In view of several research developments, there is a need to develop a mathematical algorithm or model that will solve the complex cut-off grade optimisation problem. The model should simultaneously determine the cut-off grades of a mining operation involving most of these complications. This study therefore attempts to answer the question that *"Is it possible to optimise the cut-off grade policy of a mining operation under uncertain*

geological, mining and economic variables using a stochastic integer programming approach?".

Optimum cut-off grades are important in determining the economic feasibility over the life of a mining project. The choice of cut-off grades in a mine depends on the capacities of the mine, mill and refinery in place and the associated costs. These factors are influenced by the economic and geological parameters that fluctuate due to several factors including the dynamic state of the mining industry. This indicates the need to optimise cut-off grades in the mining industry. The capacities of the three stages and the costs associated with them are chosen in this study by benchmarking with existing similar operations.

1.7 Outline of chapters

This thesis is structured as follows:

- Chapter 1 presents an introduction to existing approaches on cut-off grade optimisation based on the major areas of concern in mining, research background or scope, objectives, methodology and relevance of the project to the mining industry.

- Chapter 2 gives an insight on the literature review of cut-off grade optimisation and existing algorithms depending on different scenarios. The literature review helps in justifying the need for a stochastic programming approach.

- Chapter 3 explains the methodology used in the thesis. It describes the different OR techniques and their application in the mining industry, specifically in cut-off grade optimisation. It further explains the choice of the OR technique used in this thesis and the application of computer programming in coding the algorithm developed in this study. It explains the model formulation and the modifications made to the model to incorporate the scenarios involved in a mining complex. This involves developing a mathematical model using stochastic integer programming to optimise cut-off grade policy of a mining complex under geological, mining and economic variables.

- Chapter 4 describes how the proposed algorithm is applied on a real gold mine case study. It presents implementation of the algorithm for an authentic Mineral Resource model from McLaughlin gold deposit in Northern California, USA. Application of the algorithm on a gold mine deposit is done to ascertain its benefits in an operational mine. The applicability of the algorithm is illustrated based on different mining scenarios.

- Chapter 5 explains the results and analyses the output of the algorithm developed. The computer-aided application developed using this algorithm is compared with traditional, break-even, heuristic cut-off grade approaches and other existing computer-aided applications such as Cut-off Grade Optimiser and Maptek Evolution®. The developed stochastic approach is compared with several deterministic approaches to validate this thesis.

- Chapter 6 concludes, recommends and illustrates how the model applies the complex situations facing any mine operation into its search algorithm and describes ways of incorporating it into mine planning of any given mining operation.

2 LITERATURE REVIEW

2.1 Chapter overview

Production scheduling determines the quantities of ore that can be mined, processed, and refined in any given period under specified mining, processing, and refining capacities. The cut-off grade policy is part of the operation's production scheduling. The cash flows generated are discounted at a predetermined rate to give the NPVs of the specific mining project. Other than the economic conditions, there are other two distinct categories of parameters that govern the existing cut-off grade models. These are geological and operational parameters.

The main challenge mining engineers face when calculating cut-off grade is identifying the costs that should be included in the calculations. Costs are divided into three categories: fixed, variable and semi-variable costs. Fixed costs refer to the expenses that remain constant within a specific period or scale of production, while variable costs change over time or in relation to the level of production. Semi-variable costs are composed of a mixture of both fixed and variable costs. They are fixed for a set level of production, and become variable after this production level is exceeded, but not in direct proportion. Fixed costs are defined in the calculations to include general administration, initial capital expenditures, equipment depreciation, property taxes, and marketing. Variable costs included in cut-off grade calculations are incurred during drilling, sampling, blasting, loading, crushing, grinding, flotation, concentrate drying, filtering and shipping, smelting and refining. Semi-variable costs include maintenance costs and labour costs.

There are several cut-off grade methods used to calculate cut-off grades used in mine planning. Sections 2.2 to 2.7 review existing cut-off grade models used in the mining industry. They describe break-even and Lane's cut-off grade optimisation models and illustrate their differences. A brief comparison among break-even, traditional or heuristic and Lane's cut-off grade approaches is shown in Table 2.1 indicating the difference between the three major cut-off grade models in mine planning. In addition to these approaches, Lane's approach has been modified in different ways with the aim of improving and widening its scope and applicability to mining operations.

Table 2.1: Brief comparison among break-even, heuristic and Lane's cut-off grade approaches

Break-even cut-off grade approach	***Heuristic cut-off grade approach***	***Lane's cut-off grade approach***
• It ignores maximising undiscounted cash flows.	• It maximises undiscounted cash flows.	• It maximises discounted future cash flows.
• The cut-off grades calculated are constant throughout the life of operation at feasibility mine planning stage unless the commodity prices or costs of operation change.	• The cut-off grades are dynamic.	• The cut-off grades are dynamic at feasibility mine planning stage. They are generally higher during early years and lower during later years of operation.
• It ignores the grade-tonnage curve of the deposit and production capacities.	• It does not accommodate the grade-tonnage curve and capacity constraints in its calculation.	• It accommodates the grade-tonnage curve and satisfies production capacity constraints.
• It gives sub-optimal NPV for a given deposit.	• NPV is sub-optimal since it does not reflect the actual situation of the mining operation.	• Due to higher cut-off grades in the first years, it results in a higher NPV. The change in cut-off grade is relative to the decline in NPV with the depleting Mineral Reserves.
• Has higher mine life thus decreasing the recovery rate of the capital invested.	• The dynamic cut-off grades increase the rate of mining proportionally thus reducing the mine life.	• The mine life is shorter to allow fast recovery of the capital investment.
• It does not allow flexibility in the mining stages. It does not allow stockpiling of potential ore and blending.	• The change in cut-off grade along the mine life helps the mine to stockpile ore.	• Due to the dynamic cut-off grades the mining allows the mine to stockpile potential ore.

2.2 Input parameters used in cut-off grade calculations

Input parameters in any cut-off grade policy include economic, geological and operational parameters. Economic parameters include the price of metal, mining cost, processing cost, refining or marketing cost, fixed or period cost, and discount rate. Geological input is represented by the grade-tonnage curve inside the ultimate pit limit or pushbacks on an orebody model. Operational parameters include mining, processing and refining capacities and metallurgical recovery (Asad *et al.* 2016). Table 2.2 shows the notations of both input and output parameters in Lane's cut-off grade algorithm.

Table 2.2: Notations of parameters in Lane's cut-off grade algorithm (Hustrulid *et al.*, 2013)

Notation	Explanation	Unit
i	Year	-
T	Mine life	Years
s	Price	U$/g
m	Mining cost	U$/ton
c	Milling cost	U$/ton
r	Refining cost	U$/ton
f	Annual fixed costs	U$/year
y	Recovery	%
d	Discount rate	%
M	Mining capacity	ton/year
C	Milling capacity	ton/year
R	Refining capacity	ton/year
Qm	Material mined	ton/year
Qc	Ore processed	ton/year
Qr	Concentrate refined	ton/year

Definitions of the capacities, unit costs and quantities involved in Lane's algorithm are presented as (Hustrulid *et al.*, 2013):

i. Costs

- *m* is mining costs expressed in $/t of material moved.
- *c* is processing costs expressed in $/t of material milled.
- *r* includes all costs incurred at the product and selling stages. These are expressed in terms of $/unit of finished product.

- *f* represents the fixed costs which include all costs such as rent, administration, etc. which are independent of production levels that are within normal limits of variation.
- *s* is the selling price which is expressed in terms of the price per unit of product.
- *y* is the recovery which is that proportion of the mineral contained in the original ore feed retained in the finished product.

ii. Capacity

- *M* is the maximum amount of material (ore and waste) that a mine can produce in a given period. It is therefore the limit on the maximum rate of production from the orebody.
- *C* is the maximum amount of ore that is put through the mill (concentrator) in a given period assuming unlimited availability of input ore from the mining stage.
- *R* is the maximum amount of final product produced in the given period assuming unlimited availability of concentrate from the concentrator.

iii. Quantities

- Q_m is the quantity of material mined, Q_c is the quantity of ore sent to the processing plant and Q_r is the amount of product produced over that specific period.
- *T* is the length of the production period being considered.

2.3 Break-even cut-off grade approach

The break-even cut-off grade approach is a technique that calculates cut-off grade based on economic parameters only. Costs and prices are accounted for in the calculation, but the calculation ignores both the grade-tonnage curve of the deposit and production capacities (mining, processing and refining capacities). It ignores maximising undiscounted cash flows. The cut-off grades calculated using this model are constant throughout the life of a mining operation at a planning stage. Most mining operations use break-even analysis to calculate cut-off grade hence creating sub-optimal mine plans. Fluctuating metal prices and the use of this cut-off grade analysis are probably the main reasons why mining companies have not been generating large returns for investors (Hall, 2014). The break-even formula used in determining the grade at which revenue obtained is equal to the cost of producing that revenue is given by (Hall, 2014):

$$Break-even\ cut-off\ grade\ \left(\frac{g}{t}\right) = \frac{cost\ \left(\frac{US\$}{ton}\right)}{selling\ price\ \left(\frac{US\$}{ton}\right) x\ recovery} \quad (2.1)$$

The United States customary units or imperial units can also be applied equivalently in the same Equation 2.1 where the units would be oz/ton instead of gram/tonne; US$/ton instead of US$/tonne and US$/oz instead of US$/gram. The main challenge encountered by many mining companies when using this formula is deciding on the costs that should be included in the break-even calculations. The mining companies must first justify why they choose the costs in the calculations. Each one of the stakeholders in the mining industry uses their own definition of costs since there are no industry-wide standard definitions on the costs (Poniewierski and Hall, 2016). The following steps are used in calculating cash flows generated using a break-even cut-off grade model (Hall, 2014):

1. Determine cut-off grade.
2. Determine quantity of ore (*qo*) and quantity of waste (*qw*), stripping ratio (*SR*), and average grade ($\bar{g}$) of ore.
3. Determine *Qm*, *Qc*, and *Qr* as follows:

$$Q_m = Q_c\left[1 + \frac{qw}{qo}\right] = Q_c[1 + SR] \tag{2.2}$$

$$Q_c = \frac{Q_m}{(1 + SR)} \tag{2.3}$$

$$Q_r = (Q_c)(\bar{g})(y) \tag{2.4}$$

4. Determine cash flow for year *i, CF_i*.

$$CF_i = (s - r)Q_{ri} - cQ_{ci} - mQ_{mi} - f_i \tag{2.5}$$

where s is selling price; *y* is recovery; *m* is mining costs; *c* is processing costs; *r* is refining costs; *f* is fixed costs per period, *Qm* is material mined; Q_c *is* ore processed; Q_r is concentrate refined; *i* is the specific period in a mine life*(T)*.

2.4 Heuristic cut-off grade approach

Heuristic cut-off grade optimisation approaches have been incorporated in the mining industry since they use cut-off grades that are higher than break-even cut-off grades during the early years of the mine operation for faster recovery of capital investment. Then they lower down to break-even cut-off grades during the later years of the mine to maximise extraction. Heuristic cut-off grade calculation methods have been applied in the mining industry to achieve different results depending on the specific mining conditions.

Heuristic approaches were modified from the traditional approaches such that the NPVs improved significantly. Rendu (2014) discussed several ways of calculating minimum cut-off grades using heuristic approaches. However, they are not optimal since they only consider direct operating costs. Capacity constraints and opportunity costs are not taken into consideration. The economic changes affecting the cut-off grades do not affect the mining and processing plans and cash flows. Rendu (2014) described how to calculate the minimum cut-off grade by calculating cut-off grades in the following scenarios:

- Cut-off grade for material at the bottom of an open pit mine.
- Cut-off grades in underground mines.
- Cut-off grade to choose between processes.
- Cut-off grade between ore and waste.
- Cut-off grade between waste and low-grade stockpile.
- Cut-off grade with variable recoveries.
- Opportunity cost of not using the optimum cut-off grade.

Rendu (2014) described the ways in which cut-off grade for polymetallic deposits can be calculated without compromising the significance of one metal to another. Polymetallic deposits are deposits that contain more than one metal of economic value. The formulae, described in his approach, are used in calculating the cut-off grade for polymetallic deposits by considering the contribution of each metal to the material mined and processed.

Cut-off grade calculation in one of the heuristic approaches includes depreciation, fixed cost and minimum profit per tonne of ore for a period. This is done to get a much higher cut-off grade during the early years of the mine life. At the end of the mine life the minimum profit requirement is removed from the equation to lower the cut-off grades further to pay off the plant's costs. Then the depreciation charges are removed from the equation thus leaving the mine to operate on the milling cut-off grades.

Cut-off grade calculation changes along the mine life in intervals as shown in Equations 2.6, 2.7, and 2.8. The whole mining process involves dynamic cut-off grades which tend to be higher in early years and then lower in the later years of the mining project. However, the approach gives sub-optimal NPVs for the given deposit (Dagdelen, 1992).

Milling cut-off grade applied in early years:

$$= \frac{milling\ cost + minimum\ profit + fixed\ cost + depreciation}{[price - (refining\ cost + marketing\ cost)] * (recovery)} \quad (2.6)$$

Milling cut-off grade applied in mid years:

$$= \frac{milling\ cost + fixed\ cost + depreciation}{[price - (refining\ cost + marketing\ cost)] * (recovery)} \quad (2.7)$$

Milling cut-off grade applied in later years till depletion:

$$= \frac{milling\ cost + fixed\ cost}{[price - (refining\ cost + marketing\ cost)] * (recovery)} \quad (2.8)$$

Rendu (2014) described the effect of capacity constraints on cut-off grade by showing the relation of cut-off grade and mining capacity, processing capacity and refining capacity when one of these capacities is fixed. Several research studies have been done to determine the relationship between cut-off grade and mine selectivity, deposit modelling and ore flow control. This is important in mine planning of both open pit and underground mining.

Some studies, as shown in the summary in Appendix 1 (Section 8.1), have applied modern heuristics and integrated them with mathematical techniques to develop cut-off grade optimisation models which maximise NPV. Such models employ evolutionary algorithms in developing the cut-off grade model that caters for the dynamic nature and complexity of mining parameters (Githiria and Musingwini, 2018).

2.5 Lane's cut-off grade approach

A mine produces ore (valuable) and waste material. Ore is transported to an appropriate processing stream to produce a concentrate. A refinery plant processes the concentrate to produce marketable metal product (Lane 1964, 1988). Lane (1964) stated that cut-off grade is a function of grade distribution and capacities of the various components in the mining operation. Each component of the mine has its own set of costs and capacities and thus, its own optimum cut-off grade. The overall optimum cut-off grade is one that balances the capacities of all components of a mining operation (mining, milling and refinery).

Lane's optimum economic cut-off grade is the grade that maximises the NPV of future cash flows. Lane's approach considers a mining operation that has a combination of three stages namely; mine, mill or concentrator, and refinery. Each of these three components has its own limiting capacity to handle either ore or product and has its own set of associated costs. In addition, the whole operation has a set of fixed costs, which do not necessarily relate to any one stage.

In summary, Lane's approach satisfies production capacity constraints and accommodates the grade-tonnage distribution curve of the deposit. The cut-off grades

are dynamic in that they are higher during the early years and lower during later years of operation. Lane (1964) argued that the best criterion for optimising cut-off grade is by deriving a cut-off grade that maximises the NPV of the cash flows of the whole operation. This, therefore, considers the time value of money invested in the mining project. It reflects the costs and benefits of bringing forward or delaying extraction of any one block of ore in the orebody.

Most mining operations involve three common stages namely: mining, concentrating and refining with each stage having its own associated costs and a limiting capacity. The nomenclature used in this mathematical formulation is described in Lane's model as shown in Figure 2.1.

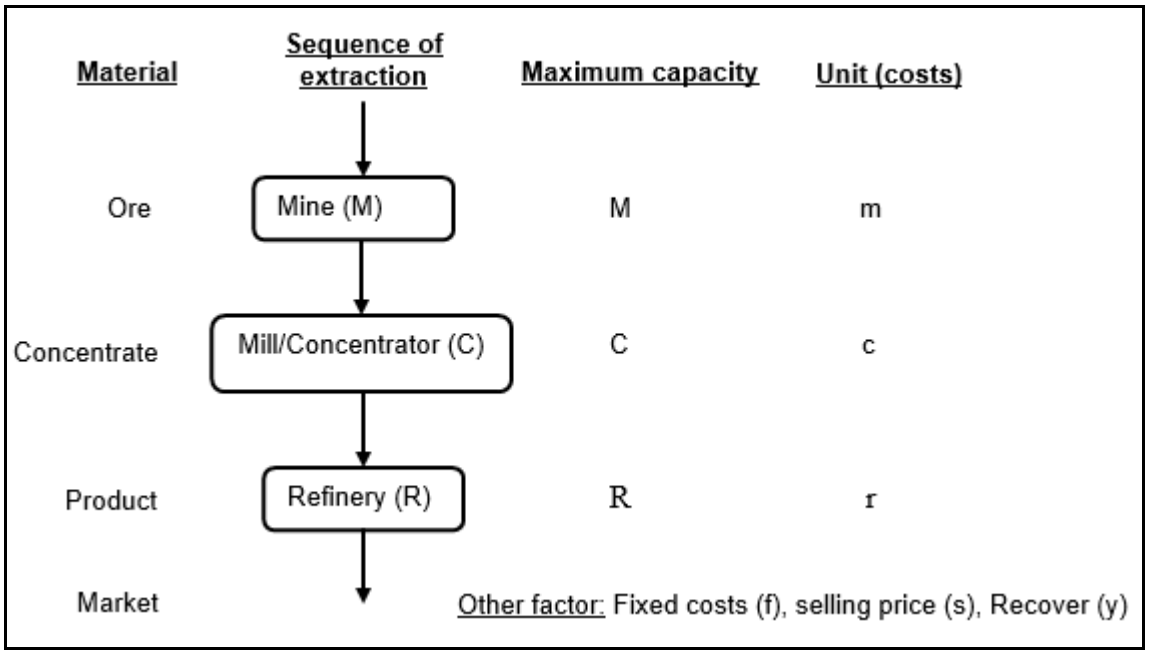


Figure 2.1: Model described by Lane's algorithm (Lane, 1964; Hustrulid *et al.*, 2013)

2.5.1 Basic equations

The basic equations used in cut-off grade optimisation can be developed using the definitions described in this section.

Total costs, T_C are $Tc=mQ_m +cQ_c+ rQ_r+fT$ (2.9)

Revenue, R is $R=sQ_r$ (2.10)

Profit, P is given by $P=R-T_c=sQ_r – (mQ_m+ cQ_c+ rQ_r+fT)$ (2.11)

Combining terms, yields: (profit expression)

$P=(s-r)\ Qr – cQ_c - mQ_m –fT$ (2.12)

It can be used to calculate the profit, P from the next Q_m of material mined. The profit expression is formulated by considering the next unit of material to be mined, which is Q_m. Therefore, the cost of mining the material will be mQ_m. Some of the mined material will be sent to the concentrator depending on the capacity of the concentrator, which is denoted as Q_c. The cost incurred in processing this material is cQ_c. The refining costs are incurred on the final product, Q_r thus having a cost of rQ_r. Total fixed costs, fT, are incurred over a period, T.

Lane's approach uses mathematical formulation in selecting cut-off grades by maximising NPV subject to mining, concentrating and refining constraints. This is represented mathematically as follows (Asad, 2005):

$$\text{Max NPV} = \sum_{i=1}^{N} \frac{P_i}{(1+d)^i} \tag{2.13}$$

Where Profit, $P_i=(s_i-r_i)\, Q_{ri} - cQ_{ci} - mQ_{mi} - f_i$ (2.14)

d represents the discount rate

Subject to: $Q_{mi} \leq M$ for i=1… T, $Q_{ci} \leq C$ for i=1… T and $Q_{ri} \leq R$ for i=1… T

The first question to be asked is whether some other combination of mine production rate and concentrator cut-off grade would yield a better NPV from this deposit or not. The variations are done on the constraints to optimise the NPV. The constraints (mine, concentrator and refining capacities) are varied to determine the optimum set of capacities to be used over the mine life. The net present value, V_i is calculated using the following basic equation, using the notation in Figure 2.1:

$V_i = (s-r)\, Qr - cQ_c - mQ_m - T\,(f+Vd)$ (2.15)

where s is the selling price; y is recovery; m is mining costs; c is processing costs; r is refining costs; f is fixed costs, Qm is material mined; Q_c *is* ore processed; Q_r is concentrate refined; T is time; V is net present value; d is discount rate.

Assume that prior to mining increment Q_m, the present value of all remaining profits is V. This is composed of two parts:

- PV_p is from the profit P realized at time T by mining the quantity Q_m.
- PV_w is obtained from profits realized by mining the material remaining after time T.

The value of all these remaining profits for mining conducted after *t=T* as expressed at time *T* is *W*. The present values of *W* and *P* respectively, discounted to time *t=0* are given by:

$$PV_w(t=0) = \frac{W}{(1+d)^T} \tag{2.16}$$

$$PV_p(t=0) = \frac{W}{(1+d)^T} \tag{2.17}$$

Therefore, the present value at time *t=0* is:

$$V = \frac{W}{(1+d)^T} + \frac{p}{(1+d)^T} \tag{2.18}$$

Since the present value at time *t=T* of the remaining Mineral Reserves is *W*, the difference *v* between the present values of the remaining Mineral Reserves at time *t=0* and *t=T* is:

$$v = V - W \tag{2.19}$$

Equation 2.18 can be written as:

$$W + P = V(1+d)^T \tag{2.20}$$

Applying the binomial expansion to the term $(1+d)^T$ one finds that

$$(1+d)^T = 1 + Td \tag{2.21}$$

Therefore, Equation 2.20 results in

$$V - W = P - VTd \tag{2.22}$$

Comparing Equations (2.19) and (2.22) one finds that

$$v = P - VTd \tag{2.23}$$

The profit *P* obtained through the mining of Q_m in time *T* is given as before by

$$P = (s-r)Q_r - cQ_c - mQ_m - fT \tag{2.24}$$

Combining Equations (2.23) and (2.24) yields

$$V = (s - r)Q_r - cQ_c - mQ_m - T(f + Vd) \quad (2.25)$$

2.5.2 Limiting cut-off grades

The limiting cut-off grades are based upon capacities, costs, and price. The cut-off grades are determined based on the operations involved such as mining, concentrating and refining. One of these constraints is used as the governing constraint i.e. controlling factor in the system while the rest of the constraints are varied with respect to the grade distribution. This approach is in line with the Theory of Constraints (TOC). TOC is a continuous improvement methodology used to drive improvement in both strategic and tactical planning. It is based on the principle that there is one constraint that is controlling the rate at which the output is generated. The theory states that addressing this constraint first will deliver improvement to the whole operation in terms of profitability (Elkington and Finch, 2016).

The basic expression used to calculate the economic cut-off grade based on mining as the governing constraint (Lane, 1964) is as follows:

$$v_m = (s - r) * Q_r - c * Q_c - \left(m + \frac{f + d * V}{M} \right) * Q_m \quad (2.26)$$

$$T = \frac{Q_m}{M} \quad (2.27)$$

From the equations 2.26 and 2.27, the optimum cut-off grade is given when:

$$(s - r) * y * g_m - c = 0 \quad (2.28)$$

Therefore, $$g_m = \frac{c}{(s - r) * y} \quad (2.29)$$

If milling throughput is the governing limitation, the optimum cut-off grade is given by Equation 2.32 as derived from Equations 2.30 and 2.31:

$$T = \frac{Qc}{C} \quad (2.30)$$

$$v_c = (s - r) * Q_r - \left(c + \left(\frac{f + d * V}{C} \right) \right) * Q_c - m * Q_m \quad (2.31)$$

$$g_c = \frac{c + \frac{f + d * V}{C}}{(s - r) * y} \tag{2.32}$$

If refining throughput is the governing limitation, the optimum cut-off grade is given by Equation 2.35 as derived from Equations 2.33 and 2.34:

$$T = \frac{Qr}{R} \tag{2.33}$$

$$v_r = \left(s - r - \left(\frac{f + d * V}{R} \right) \right) * Q_r - c * Q_c - m * Q_m \tag{2.34}$$

$$g_r = \frac{c}{\left(s - r - \frac{f + d * V}{R} \right) * y} \tag{2.35}$$

2.5.3 Balancing cut-off grades

For the balancing cut-off grades, two constraints are to be in balance i.e. operating in full capacity. The calculations are based on the distribution of the grades of the mined material. The average grade of the treated material can be found as a function of the chosen cut-off. Balancing cut-off grades can be calculated analytically using a technique developed by Dagdelen (1992). The balancing cut-off grades do not depend on economic factors entirely and they are dynamic such that they vary as the grades vary in the deposit. They depend on the grade distribution of the deposit.

If the grade-tonnage distribution of a given mineral deposit consists of *"K"* individual grade values, then there exists a set of grade values as:

[{g (1), g (2)}, {g (2), g (3)}, ----------, {g (K-1), g (K)}]

By using the lower grade limit *g (k)* for a given set of grade values *{g (k), g (k+1)}* as the cut-off grade representing interval k^*, calculate the tonnes above cut-off grade i.e. ore tonnes, tonnes below cut-off grade i.e. waste tonnes, and the average grade of ore above cut-off grade as shown in the following Equations 2.36, 2.37 and 2.38 (Asad, 1997).

$$qo(k^*) = \sum_{k=k^*}^{K} q_k \tag{2.36}$$

$$qw(k^*) = \sum_{k=1}^{k^*} q_k \tag{2.37}$$

$$g_{avg}(k^*) = \frac{\sum_{k=k^*}^{K} \left(q_k * \left[\frac{g(k) + g(k+1)}{2} \right] \right)}{qo} \tag{2.38}$$

Where *qo* represent the quantity of ore, *qw* represent the quantity of waste and q_k represent the quantity of ore in a specific grade interval k^*.

The ratio of ore tonnes to total tonnes mined *"mc"*, the ratio of quantity of metal to total tonnes *"mr"*, and the ratio of quantity of metal to ore tonnes *"rc"* can be calculated if ore tonnes, waste tonnes and average grade of ore are known as shown in Equations 2.39, 2.40, and 2.41.

$$mc(g) = \frac{qo(g)}{qo(g) + qw(g)} \tag{2.39}$$

$$mr(g) = \frac{(qo(g))(g_{avg}(g))(y)}{qo(g) + qw(g)} \tag{2.40}$$

$$rc(g) = (g_{avg})(g)(y) \tag{2.41}$$

In the calculation of mine and concentrator balancing cut-off grade, the ratio *(C/M)* is determined and the grade interval is located as shown in Equation 2.42.

$$mc(g') \geq (C/M) \geq mc(g) \tag{2.42}$$

The optimum cut-off grade is then determined using Equation 2.43.

$$g_{mc} = \frac{\frac{C}{M} - mc(g)}{\frac{mc(g') - mc(g)}{g' - g}} + g \tag{2.43}$$

This is repeated for the other balancing cut-off grades using Equations 2.44 and 2.45.

$$g_{rc} = \frac{\frac{R}{C} - rc(g)}{\frac{rc(g') - rc(g)}{g' - g}} + g \tag{2.44}$$

$$g_{mr} = \frac{\frac{R}{M} - mr(g)}{\frac{mr(g') - mr(g)}{g' - g}} + g \tag{2.45}$$

2.5.4 Optimum cut-off grade

The optimum cut-off grade policy determines the optimum cut-off grade among the limiting economic and balancing cut-off grades that maximise the NPV of an operation (Lane, 1964; 1988). The three limiting economic cut-off grades (g_m, g_c, g_r) change with changes in economic parameters. They are based upon the capacities, costs and price of the commodity. The other three (g_{mc}, g_{rc}, g_{mr}) are based on the grade distribution of the mined material and capacities.

The objective is to find the cut-off grade which produces the overall maximum profit subject to mining, concentrating and refining constraints. An optimum grade will be determined for each of the three pairs of operations. For example, when the mine and the concentrator are considered as a pair, there are three possible options for the optimum cut-off grade G_{mc}. These are g_m, g_c and g_{mc}. The overall effective optimum cut-off grade is the middle value among G_{mc}, G_{rc}, and G_{mr} and subsequently Q_m, Q_c, and Q_r and the NPV can be computed (Lane, 1964; 1988).

2.5.5 General steps of Lane's algorithm

The milling and refining cut-off grades depend on the estimates of NPV of the future profits in year *i* through the end of mine life from the operation. The NPV_i of the future profits cannot be calculated until the optimum cut-off grades are chosen. Therefore, an iterative approach is used where the initial NPV is set to zero at first (if the initial NPV_i is not known) and after several iterations the NPVs converge to a stable optimum answer which will be used in the cut-off grade calculations.

The following steps as outlined by Lane (1964) and modified by Dagdelen (1993) are used in determining the cut-off grade policy for a mineral deposit:

1. Formulate a grade-tonnage distribution curve for the whole deposit.
2. Input the parameters to be used in the cut-off grade policy such as the mining capacity (M), milling capacity (C), refining capacity (R), selling price (s), mining cost

(m), milling cost (c), refining cost (r), recovery (y), milling cost (c), mining cost (m), annual fixed costs (f) and the discount rate (d).

3. Determine the optimum cut-off grade to be used in Year *i* by the cut-off grade equations, if the initial NPV_i is not known set the NPV_i to zero.
4. Determine the tonnes of ore, tonnes of waste and average grades of the ore associated by the optimum cut-off grade. Using the equations below calculate the quantity to be mined (Q_m), milled (Q_c) and refined (Q_r). Find the limiting capacity and the mine life (*T*) from the following:

$$Q_m = Q_c[1 + SR] \tag{2.46}$$

$$Q_c = \frac{Q_m}{(1 + SR)} \tag{2.47}$$

$$Q_r = (Q_c)(\bar{g})(y) \tag{2.48}$$

$$T = \frac{Q_m}{M} \tag{2.49}$$

$$T = \frac{Q_c}{C} \tag{2.50}$$

$$T = \frac{Q_r}{R} \tag{2.51}$$

5. Determine the yearly profit using the basic profit equation.
6. Compute the NPV using the formula below by discounting the profits at a given discount rate (d) for the time calculated as the mine life.

$$NPV = \sum_{i=1}^{N} \frac{P_i}{(1 + d)^i} \tag{2.52}$$

   a)
   b) This value of *V* becomes the second approximation of *V* (the first was *V*=0) for use in the formulae to calculate the optimum cut-off grade.
7. Repeat the computation from Step 4 until the value, V, converges.
8. Adjust the grade-tonnage distribution by subtracting the ore tonnes from the grade-tonnage distribution intervals above optimum cut-off grade and the waste tonnes from the intervals below the optimum cut-off grade in proportionate amounts such that the distribution is not changed.
9. If it is the first iteration then knowing the profits obtained in each year, find the NPV year−by−year by discounting back those profits and go to Step 3. If it is the second iteration, then stop.

10. Use the NPVs obtained in Step 9 as initial NPV for each of the corresponding year for the second iteration.

2.6 Application of cut-off grade approaches in mine planning

The development of cut-off grade models can be traced back to the 1960s in the published work of Henning (1963), Johnson (1969), Lane (1964, 1988), among others. Henning (1963) created a framework of relationships for cut-off grade calculation with varying objectives. The framework generates cut-off grades which are higher during earlier years and lower during later years and finally reducing to the break-even cut-off grade. The whole process aims at maximising the difference between present value of annual profit and associated costs. Johnson (1969) applied the Dantzig-Wolfe decomposition method using mixed-integer programming by considering dynamic cut-off grades that can handle large deposits. However, the solution is not optimal due to the many constraints in the approach such that some blocks are not completely mined.

Past research has seen much development in mine planning through incorporation of mathematical optimisation techniques. The optimal cut-off grade approach (Lane, 1964; 1988) has been modified in several studies by incorporating different mining scenarios. Research studies done by: Taylor (1972, 1985), Dagdelen (1992, 1993), Dagdelen and Kawahata (2008), King (2001, 2011), Whittle and Wharton (1995), Asad (1997, 2002, 2005), Minnitt (2004), Osanloo *et al.* (2008b), Gholamnejad (2008, 2009) and Githiria *et al.* (2016) applied and modified Lane's approach to incorporate certain aspects in a mining operation.

These studies did not consider variations in the economic and mining variables that affect daily mining operations implying that NPVs produced for the mining projects are not realistically optimal. When calculating cut-off grades, the reality encountered in mining operations should be reflected appropriately and as close as possible. This is done by applying mining variables as dynamic variables in the calculation of cut-off grades.

Lane (1964, 1988) came up with an approach that has higher cut-off grades in the first years hence higher NPV and internal rate of return (IRR). This ensures quicker recovery of the capital investment. The change in cut-off grade is relative to the decline in NPV with the depleting Mineral Reserves. Due to these dynamic cut-off grades the mining allows the mine to stockpile potential ore which will increase the LOM by processing the stockpiles later in the LOM. Although Lane's approach has been proven to be applicable in most mining operations, there is a need for its modification. In his approach, commodity prices and costs associated with mining operations are assumed to be stable throughout the LOM. This is a shortcoming of Lane's theory, thus, the need for much more research to develop a general model for cut-off grades in a fluctuating commodity market.

Taylor (1972) used Lane's approach whereby three stages in a mine operation are involved also having balancing and limiting cut-off grades. He introduced a different calculation of the balancing cut-off grades by using statistical parameters to describe the grade distribution of the orebody. Taylor (1985) explained the use of stockpile system through some actual examples. A substantial low-grade stockpile was created at an average grade of 0.6% Cu from the initial high-grade open-pit at the Craig Mont Copper Mine. After that, the operation developed into an underground mine using sublevel caving method which yielded a daily output of about 4500 short tons at higher grade (1.8%Cu). The low-grade stockpile was used for many years by inputting 1500 short tons per day that were needed to feed the mill. However, Taylor did not give any information about the methodology used to create the stockpiles (Taylor, 1985).

Dagdelen (1992) and Asad (2002) discussed the concept of optimum cut-off grade policy as outlined by Lane and formulated an algorithm used to maximise the NPV of a project. Dagdelen (1993) introduced an analytical method to find the balancing cut-off grades instead of using the graphical method by modifying the Lane's approach, which simplified the problem. Dagdelen and Kawahata (2008) applied Lane's algorithm using mathematical programming to optimise cut-off grades. By applying mixed-integer linear programming technique, they came up with a mining software, OptiPit®. This study demonstrated the difference in value creation when calculating cut-off grade using traditional break-even cut-off grade approach and modified cut-off grade optimisation approach.

Whittle and Wharton (1995) introduced optimisation that determined simultaneously for a mining operation involving some or all of the complications such as stockpiling of mined material, blending, multiple minerals and others. This is incorporated in open pit mining software, Geovia Whittle™. The latest version, Geovia Whittle 4-X version, does simultaneous optimisation with the help of a new linear programming engine that provides significant processing speed improvements. It has a comprehensive spreadsheet reporting option that opens in Microsoft Excel for easy analysis of the reports. It has the capability of handling up to 30 processing methods. Additionally, it may be used in blend optimisation under an unlimited number of constraints and variables in several blend scenarios. However, the processing or computing time increases with increase in the number and size of problem.

King (2001) modified Lane's approach by incorporating variations in ore type throughput for multi-metal mineralisation and later in 2011; he had a variation of cut-off grade policies considering operating and administrative costs (King, 2011). Asad (1997, 2005) developed a cut-off grade optimisation algorithm with a stockpiling option for open pit mining operations with two economic minerals. Minnitt (2004) applied Lane's theory to

show how cut-off grade can be determined in a small Wits-type gold mining operation to maximise NPV.

In optimizing cut-off grade, Gholamnejad (2008, 2009) argued that there should be a trade-off between increasing the average mill grade and increasing the rehabilitation cost. Increasing cut-off grade results in increasing the amount of waste material hence the extra rehabilitation cost which will consequently reduce the NPV of the project. The approach incorporated rehabilitation cost in Lane's model by factoring the cost on the quantity of material mined and processed. Osanloo *et al.* (2008b) incorporated environmental issues into cut-off grade optimisation. The environmental-oriented model incorporated costs such as waste disposal operating costs, treating operating costs, and tailings costs.

2.7 Stochastic and dynamic cut-off grade approaches

Githiria *et al.* (2016) focussed on optimising cut-off grade under deterministic variables and developed a computer-aided application using Lane's algorithm. The application is useful to mine planners when coming up with a cut-off grade policy. However, it does not solve the complications involved in a mining operation. The complications facing the mining industry are caused by the ever-changing mining variables that affect the operations daily. The NPV generated from these models are not optimal since the model did not account for the change in prices, costs and grade-tonnage distribution in the deposit. The model did not have the ability to respond to these changes hence the need for a stochastic approach using Lane's algorithm. The dynamic variables considered in a stochastic approach include costs, commodity prices, recoveries and grade-tonnage distribution of the deposit.

Several studies (Dowd, 1976; Krautkraemer, 1988; Cetin and Dowd, 2002; Cetin and Dowd, 2011; Cetin and Dowd, 2013; Cetin and Dowd, 2016; Asad, 2007; Asad and Dimitrakopoulos, 2013; Li *et al.*, 2012; Myburgh *et. al.*, 2014; Thompson and Barr, 2014) have applied stochastic and/or dynamic programming approaches in the calculation of cut-off grades. The models applied the dynamic aspect of certain parameters affecting daily mining operations such as price and costs.

Dowd (1976) developed a stochastic cut-off grade approach with an optimising cut-off grade criterion that aimed at maximising future net profits by incorporating dynamic programming in the calculation of cut-off grade. An extension of stochastic programming is evident in his paper whereby it allowed for the forecasting of probable future market conditions. He analysed the interaction of cut-off grades and fluctuating prices and illustrated the effects of the variations through examples. However, the model did not

account for the change in costs and the ability of the operation to respond to these changes.

Krautkraemer (1988) explained the effect of a stochastic price in the selection of cut-off grade depending upon the rate of metal price change relative to the discount rate. The paper concluded that fluctuating prices are important elements of the cut-off grade selection. Cetin and Dowd (2002, 2013) applied genetic algorithms (GA) in the optimisation of cut-off grade for multi-metal mines. They applied dynamic programming to calculate optimum cut-off grades on a multi-mineral deposit (Cetin and Dowd, 2011).

Later, Cetin and Dowd (2016) compared the use of GA, the grid search method, and dynamic programming in coming up with optimal cut-off grade schedules for deposits with up to three constituent minerals. This study shows the significance of applying stochastic approaches when considering variables in cut-off grade schedules. The results from all the three methods are compared to show their implication when using them. It shows the applicability and robustness of genetic algorithms to multi-mineral cut-off grade optimisation under technical and economic constraints only. It assumes the grade-tonnage distribution is identical for all parts of the orebody and for all blocks of ore.

Asad (2007) optimised cut-off grade for an open pit mining operation through an NPV algorithm considering metal price and cost escalation. This approach accounted for change in economic conditions only. Asad and Dimitrakopoulos (2013) developed a heuristic approach that determines the optimal cut-off grade policy based on a stochastic context that accounts for uncertainty of ore input to the multiple ore processing streams. They considered multiple processing streams that were individually summed up to form an input to the refinery/market. This proposed modification of Lane's approach enhanced the theory to cover open pit mining operations with multiple ore processing streams and accounted for geological uncertainty. However, the proposed heuristic approach was limited to a single mine or material source and refinery/market.

Myburgh *et al.* (2014) applied modern heuristics to come up with a mine planning software package, Maptek™ Evolution, which maximises NPV through cut-off grade optimisation. The approach employed an evolutionary algorithm in developing a cut-off grade model that caters for the dynamic nature and complexity of mining parameters. This hybrid algorithm applies three levels of optimisation consisting of an evolutionary algorithm which manages two other lower level algorithms. The evolutionary algorithm manages the variation of cut-off grades and the extraction sequence, while the linear programming algorithm determines the optimal flow of material through multiple processing streams as well as management of the stockpile policy. Lastly a local search technique is applied to provide the best or fittest schedule. The self-adaptiveness of such

genetic algorithms contributes greatly to its ability to successfully address complex real-world problems. This is due to its agility which allows it to be easily hybridised with other optimisation methods.

The model performs simultaneous optimisation of extraction sequence and cut-off grade for a single element that can be processed in different through multiple processing streams. The evolution algorithm used in this model stochastically produces high-quality results quickly through several iterations. However, the iterations performed do not guarantee convergence and the time taken to complete an optimisation increases with the complexity of the problem, typically in the range of 3-20 minutes depending on the size of the model (Myburgh *et al.*, 2014).

Li *et al.* (2012) and Thompson and Barr (2014) optimised cut-off grade using stochastic programming in open-pit mining but considered commodity price as the only dynamic variable. This did not reflect the reality since there are other variables that change frequently and consequently, affecting the valuation of a mining project.

2.8 Summary of Chapter 2

Lane's cut-off grade optimisation approach has been applied with much success in most mining projects but, there are several shortcomings to it, thus necessitating heuristic modifications as shown in Section 2.3. The algorithm is limited to a single process and a single commodity and cannot be applied in operations with multiple processing streams. The algorithm does not consider the economic impact of stockpiling and blending requirements in ore processing. It does not consider the variability in operating costs, commodity price and grade-tonnage distribution. Previous research studies on cut-off grade optimisation using Lane's approach have focussed on optimising cut-off grade under deterministic variables such as cost, selling price, recovery, and grade distribution of the deposit. This is summarised in Appendix 1 (Tables 8.1 and 8.2) showing the common cut-off grade optimisation techniques used in the mining industry.

Future mine planning activities through the cut-off grade optimisation models should focus on strategies under volatile economic situations by considering alternative economical approaches while, considering the grade-tonnage distribution of the orebody. Multiple realisations of economic and geological parameters incorporated in Lane's approach increases the reliability of the cut-off grades calculated from the policy. The output from such a stochastic cut-off grade optimisation model considers uncertainty in economic and geological parameters of a mining complex. This chapter has made a case for a more holistic stochastic cut-off grade algorithm. The development of such an algorithm is presented in the next chapter (Chapter 3).

3 DEVELOPMENT OF THE STOCHASTIC CUT-OFF GRADE OPTIMISATION ALGORITHM

3.1 Chapter overview

The cut-off grade that maximises the NPV of the cash flows is a function of economic parameters, mining operating capacities as well as the grade distribution within the deposit. The optimisation problem in this thesis aims at maximising NPV by optimising cut-off grade subject to economic and geological parameters with limitations from mining, processing and refinery stages.

This thesis extends Lane's theory by developing a cut-off grade optimisation algorithm that considers variability of operating parameters in a mining complex. The reason for this choice is made based on how effective and comprehensive the approach is, by utilizing all important parameters when calculating optimal cut-off grades. The constraints and variables in the algorithm vary and therefore increase the size of the optimisation problem. They involve two major factors: multiple realisations of grade-tonnage distributions depicting the geological formation of the orebody and probable realisations of the commodity price indicating the economic trend.

Before the development of a cut-off grade optimisation model, there are prerequisites that need to be done. The orebody model is first determined to ascertain the quantity of material mined, quantity of material processed, and quantity of metal refined in several years. The model designed determines the established and available Mineral Reserves within the orebody. These Mineral Reserves describe the multiple probable grade-tonnage curves within the orebody model.

The developed model *(NPVMining)* is tested on an operational mine enabling easy and accurate comparison of results with the current computer-aided applications used in the mining industry. The parameters used in the model such as operating costs, metal price, mining, processing and refining capacities are determined. Lastly, the design of the stages involved in a mining system is important in cut-off grade optimisation to define the calculation path in the algorithm (Myburgh *et al.*, 2014). An example of such a design is illustrated in a simple cut-off grade optimisation algorithm flowchart in Figure 3.1 indicating steps taken when determining the optimum cut-off grade.

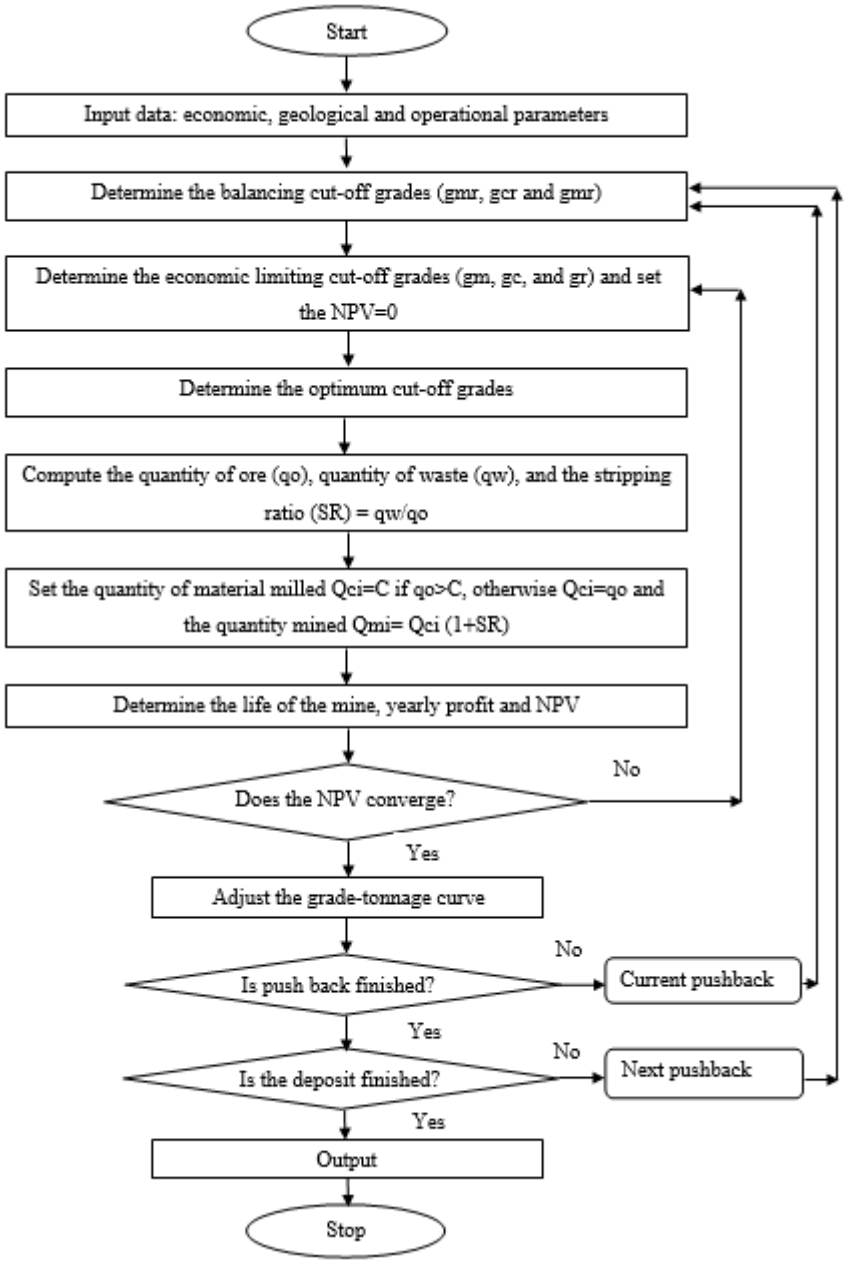


Figure 3.1: Simple flow diagram of a cut-off grade algorithm (Asad, 1997; Bascetin and Nieto, 2007; Githiria *et al.*, 2016)

The general flow of the proposed algorithm has several steps that are implemented sequentially and iteratively until the optimum solution is achieved. The process is described in Figure 3.2, showing the path taken in developing the model. The value of the deposit is calculated using this algorithmic process by determining optimum cut-off grade which is then used to maximise the NPV.

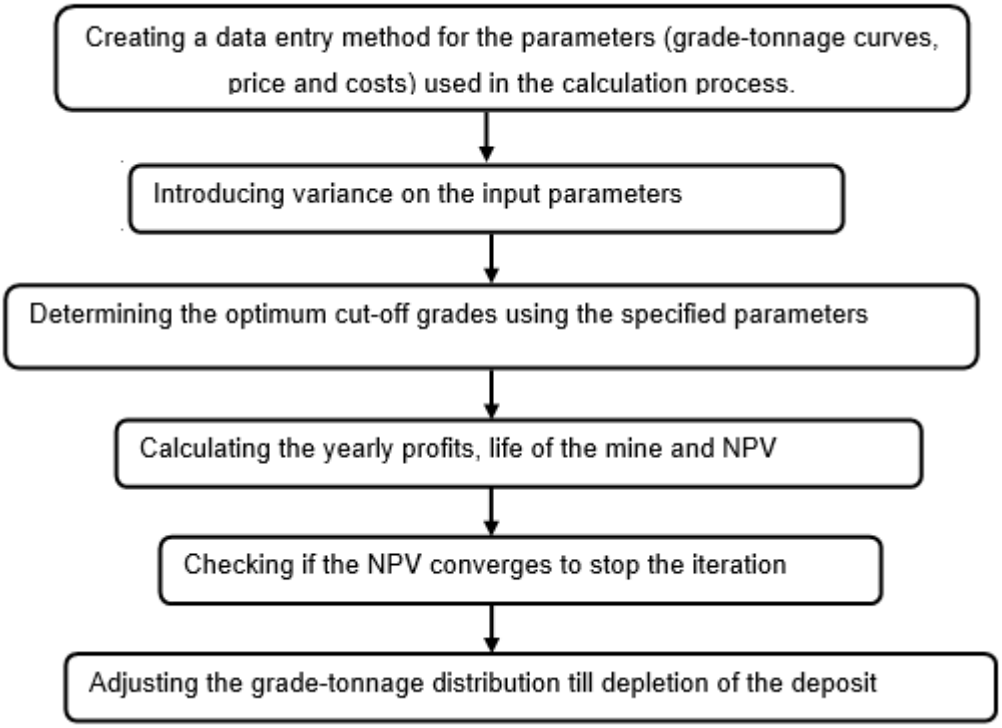


Figure 3.2: General flow of the proposed algorithm

The research methodology to achieve the outlined targets involves: the development of a stochastic integer-programming based mathematical model; creation of possible strategies to generate the solution to this cut-off grade model; and comparing with conventional cut-off grade approaches.

A model is created using mathematical optimisation techniques to find a solution to optimisation problems. There are various components found in most models such as the objective function; decision variables; constraints and data used in the model. The objective function represents the function to be optimised for instance, maximising the NPV or minimising costs. Decision variables are the parameters used in the model. They can be continuous, semi-continuous, binary or integer. The values set in the parameters are varied to optimise the objective function. Constraints consist of equalities and inequalities that limit the values that any decision variable can assume. The mine planner will then come up with data that will be used in the model.

Maximising the NPV of the project in this study is the objective function and is represented as Equation 2.13 where annual profit (p) is consequently discounted using Equation 2.14 in Chapter 2 to produce the NPV. Decision variables/parameters used in the model are divided into economic and geological parameters. Economic parameters

include price of metal (s), mining cost (m_i), processing cost (c_i), refining/selling/marketing cost (r_i), fixed/period cost (f_i), recovery (y) and discount rate (d).

The geological parameters are defined by the grade-tonnage distribution inside the ultimate pit limit or pushbacks. They include optimum cut-off grade (G), average grade (g), quantity of material corresponding to a specific grade (q), total quantity of material inside the pit or pushback (Q), quantity of material mined (Q_{mi}), quantity of material processed (Q_{ci}) and quantity of metal refined (Q_{ri}).

The constraints that bound the decision variables in this study consist of equalities and inequalities represented as follows: ($Q_{mi} \leq M$ for i=1… T), ($Q_{ci} \leq C$ for i=1… T), and ($Q_{ri} \leq R$ for i=1… T). Q_{mi} represents quantity of material to be mined, Q_{ci} represents quantity of material mined sent to the concentrator and Q_{ri} represents quantity of the final product. M, C and R represent mine, concentrator and refining capacities, respectively. The variables are governed by a set of non-negativity restrictions: $X_1, X_2 \ldots X_n \geq 0$ where $X_1, X_2 \ldots X_n \in [0, 1]$.

3.2 Development of the cut-off grade optimisation model

The model is based on Lane's cut-off grade theory with modification that accounts for the uncertainty in the economic, operational and geological conditions involved in a mining operation. The notations of the input parameters used in the proposed cut-off grade algorithm are shown in Table 3.1.

Table 3.1: Notations of parameters in the modified cut-off grade algorithm

Notation	Explanation	Unit
i	Period (year) indicator	-
T	Mine life	Years
w	Grade-tonnage curve indicator	
k	Grade-tonnage indicator	
K	Number of multiple realisations of grade-tonnage distribution in a grade-tonnage curve	
s	Price	US$/g
m	Mining cost	US$/tonne
c	Milling cost	US$/tonne
r	Refining/Selling cost	US$/g
f	Annual fixed costs	US$/year
y	Recovery	%
d	Discount rate	%
M	Mining capacity	tonnes/year
C	Milling capacity	tonnes/year
R	Refining capacity	tonnes/year
$\mathbf{P_{wi}}$	Cash flow at period *i*	US$/year
$\boldsymbol{Qm_{wi}}$	Quantity of material to be mined for grade-tonnage curve *w* during period *i*	tonnes/year
$\boldsymbol{Qc_{wi}}$	Quantity of material mined sent to the concentrator for grade-tonnage curve *w* during period *i*	tonnes/year
$\boldsymbol{Qr_{wi}}$	Quantity of the final product for grade-tonnage curve *w* during period *i*	tonnes/year

The proposed algorithm comprises of several stages that are implemented sequentially for a given resource model. The first stage in the algorithm is the cut-off grade optimisation stage whereby calculation of the cut-off grades is done using specified economic, operational and geological parameters. The variance of these parameters is factored at this stage before they are used in the calculation of the cut-off grades. These cut-off grades are then used in the second stage (the NPV convergence stage) to calculate the NPV in the specific period. The final stage is the grade-tonnage curve

adjustment stage where the grade-tonnage curve is adjusted per the specified period to factor in the depletion of the Mineral Reserves.

Given a set of equally probable grade-tonnage curves (*w*), the model develops the cut-off grade policy by determining cut-off grade (G) from periods 1 to N. The NPV of future cash flows is maximised subject to mining, processing and refining capacity constraints. The objective function of this cut-off grade optimisation model is represented as:

$$Maximise\ NPV = \sum_{i=1}^{N} \frac{P_{wi}}{(1+d)^i} \tag{3.1}$$

However, the cash flow equation *(Pw_i)*, as represented in Equation 3.2, changes to incorporate the uncertainty of both the economic and geological parameters

$$Cash\ flow, P_{wi} = \left(\sum_{i=1}^{N} s_i - r_i\right) * Qr_{wi} - c_i * Qc_{wi} - m_i * Qm_{wi} - f_i \tag{3.2}$$

Subject to: ($Qm_{wi} \leq M$ for *i*=1… T), ($Qc_{wi} \leq C$ for *i*=1… T), and ($Qr_{wi} \leq R$ for *i*=1… T).

The basic expression used to calculate the economic cut-off grade based on mining as the governing constraint (Lane, 1964) is written as follows:

$$V_{mwi} = \left(\sum_{i=1}^{N} s_i - r_i\right) * Qr_{wi} - c_i * Qc_{wi} - \left\{m + \frac{f + d * V_w}{M}\right\} * Q_{mw} \tag{3.3}$$

Qm_w/M in Equation 3.3 represents the specific time, *i'* at which Qm_w of material is mined under the specified mining capacity (M) to generate the present value, Vm_w. The basic equation represented in Equation 3.3 is used to calculate the present value for grade-tonnage curve *w* during period *i*. Following the same procedure in Lane's approach as described in Chapter 2, the limiting cut-off grades are calculated using Equations 3.4, 3.5 and 3.6. If mining throughput is the governing limitation, then the optimum cut-off grade is:

$$g_m = \frac{\sum_{i=1}^{N} c_i}{\sum_{i=1}^{N} (s_i - r_i) * y_i} \tag{3.4}$$

If milling throughput is the governing limitation, the optimum cut-off grade is given by Equation 3.5:

$$g_c = \frac{\sum_{i=1}^{N} c_i + \frac{\sum_{i=1}^{N} f_i + d * V_w}{C}}{\sum_{i=1}^{N} (s_i - r_i) * y_i} \tag{3.5}$$

If refining throughput is the governing limitation, the optimum cut-off grade is given by Equation 3.6:

$$g_r = \frac{\sum_{i=1}^{N} c_i}{\left(\sum_{i=1}^{N} (s_i - r_i) - \frac{\sum_{i=1}^{N} f_i + d * V_w}{R} \right) * \sum_{i=1}^{N} y_i} \tag{3.6}$$

In the calculation of balancing cut-off grades, if the grade-tonnage distribution of a given mineral deposit represented by a grade-tonnage curve (*w*) consists of *"K"* individual grade values, then there exists a set of grade values represented as: *[{g_w (1), g_w (2)}, {g_w (2), g_w (3)}, ----------, {g_w (K-1), g_w (K)}]*

By using the lower grade limit $g_w(k)$ for a given set of grade values *{g_w (k), g_w (k+1)}* as the cut-off grade representing interval k^*, calculate the tonnes above cut-off grade (qo_w). Then, calculate tonnes below cut-off grade (qw_w), and the average grade of ore above cut-off grade (g_{avgw}). This is shown in the following Equations 3.7, 3.8 and 3.9 (Lane, 1964; Asad and Dimitrakopoulos, 2013):

$$qo_w(k^*) = \sum_{k=k^*}^{K} q_w k \tag{3.7}$$

$$qw_w(k^*) = \sum_{k=1}^{k^*} q_w k \tag{3.8}$$

$$g_{avg_w}(k^*) = \frac{\sum_{k=k^*}^{K} q_w k \left[\frac{g_w(k) + g_w(k+1)}{2} \right]}{qo_w} \tag{3.9}$$

Where qo_w represent the quantity of ore, qw_w represent the quantity of waste and q_wk represent the quantity of ore in a grade interval k^*. In all the subsequent equations, $g_w(k)$ is represented as g'.

As described in section 2.4.3, *"mc_w"*, *"mr_w"* and *"rc_w"* can be calculated if qo_w, qw_w and g_{avgw} are known as shown in Equations 3.10, 3.11 and 3.12.

$$mc_w(g') = \frac{qo_w(g')}{qo_w(g') + qw_w(g')} \tag{3.10}$$

$$mr_w(g') = \frac{[qo_w(g')][g_{avg\,w}(g')(y)]}{qo_w(g') + qw_w(g')} \tag{3.11}$$

$$rc_w(g') = (g_{avg\,w})(g')(y) \tag{3.12}$$

The mine and concentrator balancing cut-off grade is calculated by first determining the ratio *(C/M)*, then locating the grade interval as shown in Equation 3.13.

$$mc_w(g') \geq (C/M) \geq mc_w(g) \tag{3.13}$$

The optimum cut-off grade is then determined using Equation 3.14.

$$g_{mcw} = \left[\frac{\frac{C}{M} - mc_w(g')}{\frac{mc_w(g'+1) - mc_w(g')}{g_w(k+1) - g_w(k)}} + g_w(k) \right] \tag{3.14}$$

This is repeated for the other balancing cut-off grades using Equations 3.15 and 3.16.

$$g_{mrw} = \left[\frac{\frac{R}{M} - mr_w(g')}{\frac{mr_w(g'+1) - mr_w(g')}{g_w(k+1) - g_w(k)}} + g_w(k) \right] \tag{3.15}$$

$$g_{rcw} = \left[\frac{\frac{R}{C} - rc_w(g')}{\frac{rc_w(g'+1) - rc_w(g')}{g_w(k+1) - g_w(k)}} + g_w(k) \right] \tag{3.16}$$

The optimum cut-off grade, G is given by:

$$G_{mc} = \begin{cases} g_m \text{ if } g_{mc_w} \le g_m \\ g_c \text{ if } g_{mc_w} \ge g_c \\ g_{mc_w} \text{ otherwise} \end{cases} \quad (3.17)$$

$$G_{mr} = \begin{cases} g_m \text{ if } g_{mr_w} \le g_m \\ g_r \text{ if } g_{mr_w} \ge g_r \\ g_{mr_w} \text{ otherwise} \end{cases} \quad (3.18)$$

$$G_{rc} = \begin{cases} g_r \text{ if } g_{rc_w} \le g_r \\ g_c \text{ if } g_{rc_w} \ge g_c \\ g_{rc_w} \text{ otherwise} \end{cases} \quad (3.19)$$

$$G = middle\ value\ (G_{mc}, G_{mr}, G_{rc}) \quad (3.20)$$

The overall effective optimum cut-off grade (G) is the middle value among G_{mc}, G_{rc}, and G_{mr} and subsequently Q_m, Q_c, and Q_r and the NPV can be computed.

3.2.1 Steps of the modified approach

The following steps have been modified to accommodate the uncertainty in the input parameters used in determining the cut-off grade policy for a single mineral deposit:

i. Read the input files:
   a. Grade-tonnage distribution for the whole deposit.
   [{g_w (1), g_w (2)}, {g_w (2), g_w (3)}, ----------, {g_w (K-1), g_w (K)}]

   b. Economic parameters to be used in the cut-off grade policy: mining capacity (M), milling capacity(C), refining capacity (R), discount rate(d), recovery (y_i), probability distribution of selling price ($\sum s_i$), mining cost (m_i), milling cost (c_i), refining cost (r_i) and annual fixed costs (f_i) over a certain period *i*.

ii. Set the iteration indicator j to 1.

iii. Set the year indicator i to 1.

iv. Compute the Mineral Reserves available in the ultimate pit limit (Tdep)i and current pushback (Tpb)i, if "(Tdep)i =0", then check if j is not equal to 1 (j ≠ 1), STOP; and if j=1 then go to step xvii, otherwise go to the next step.

v. Set V=NPV_i , the initial NPV_i=0.

vi. Determine g_c, g_r, g_m, g_{mc}, g_{rc}, g_{mr} and the optimum cut-off grade G using Equations 3.4, 3.5, 3.6, 3.14, 3.15, 3.16 and 3.20, respectively.

vii. From the grade-tonnage curve of the deposit compute:
   a. The ore tonnage (qo_w) above the optimum cut-off grade (G) obtained in step (vii) using Equation 3.7.

b. The waste tonnage (qw_w) that is below the optimum cut-off grade (G) using Equation 3.8.

c. Average grade of ore (g_{avg}) using Equation 3.9.

d. The stripping ratio (SR) where $SR = \frac{qw_w}{qo_w}$ (3.21)

viii. Set: $Qc_{wi} = C$, if qo_w is greater than the milling capacity (C), otherwise, $Qc_{wi} = qo_w$. The Qm_{wi} quantity mined, $Qm_{wi} = Qc_{wi}(1+SR)$ and quantity of the final product, $Qr_{wi} = Qc_{wi} * g_{avg} * \sum y_i$

ix. If the iteration indicator j is equal to 1, compute:

a. The life of deposit, T based on the limiting capacity among mine, mill, and refinery.

b. The annual profit (P_{wi}) for the life of mine using:

$$P_{mwi} = \left(\sum_{i=1}^{N} s_i - r_i\right) * Qr_{wi} - c_i * Qc_{wi} - m_i Q_{mw} - f_i \quad (3.22)$$

where:

t=1 if T≥1

t=T if T<1

c. The NPV_i by using Equation 3.1 by discounting the profits at the given interest rate *d*.

x. Compare the 'NPV_i' computed in step (ix) with previous 'V' in step (v). Repeat the computation from step (vi) until the value V converges. If the computed NPV is not converged, then go to step (xv), otherwise, go to the next step.

xi. Set the year indicator i=i+1

xii. Adjust the grade-tonnage distribution by subtracting the ore tonnes from the grade-tonnage distribution intervals above optimum cut-off grade (G) and the waste tonnes (Qm_w - Qc_w) from the intervals below the optimum cut-off grade (G) in proportionate amounts such that the distribution is not changed. Due to multiple realisations of the grade-tonnage distribution the adjustment, the current grade-tonnage curve being used at that specific period will be altered simultaneously. This will cater for the inter-temporal dependencies that alter the grade-tonnage distribution with time.

xiii. Check if Qc_w is less than the milling capacity C, then set mine life N=i and go to step (xv), otherwise, go to step (v).

xiv. Compute the accumulated future NPVs based on the profits P calculated in step (ix) for each year from i to N by using Equation 3.22. For each year *i*=1, T is the total mine life in years.

xv. Set j=j+1, then go to step vi.

xvi. This NPV value becomes the second approximation of V for use in the formulae to calculate the optimum cut-off grade. If it is the first iteration then knowing the profits obtained in each year, find the NPV year−by−year by discounting back those profits.

xvii. Use the NPVs obtained in step (xvi) as initial NPVs for each corresponding year for the second iteration.

Using the optimum cut-off grades obtained in the algorithm, a yearly production schedule that shows the cut-off grade, quantity mined (Qm_w), quantity processed (Qc_w), quantity refined (Qr_w), profit and NPV is calculated.

The flow diagram in Figure 3.3 illustrates the steps involved in the algorithm used in this thesis. The algorithm developed was coded using C++ programming language in Microsoft Visual Studio 2017 Integrated Development Environment on a standard computer. The developed computer-aided application for cut-off grade optimisation was named *NPVMining*. To run on a computer, the computer must have Visual Studio 2017 and Microsoft office 2013 installed on it. Microsoft Visual Studio 2017 (VC++) supports the C++ programming language, which has a highly-developed design capability that enables the assembly of the GUI for an application and has the code that creates it (Githiria *et al.*, 2016). C++ programming language is used in the implementation of the algorithm to enable the execution of the application on windows-based computers with different platforms. This compatibility aspect enables easy access of the application to mine planners. The output from the model is easily exported to Microsoft Excel 2013 for comparison and analysis.

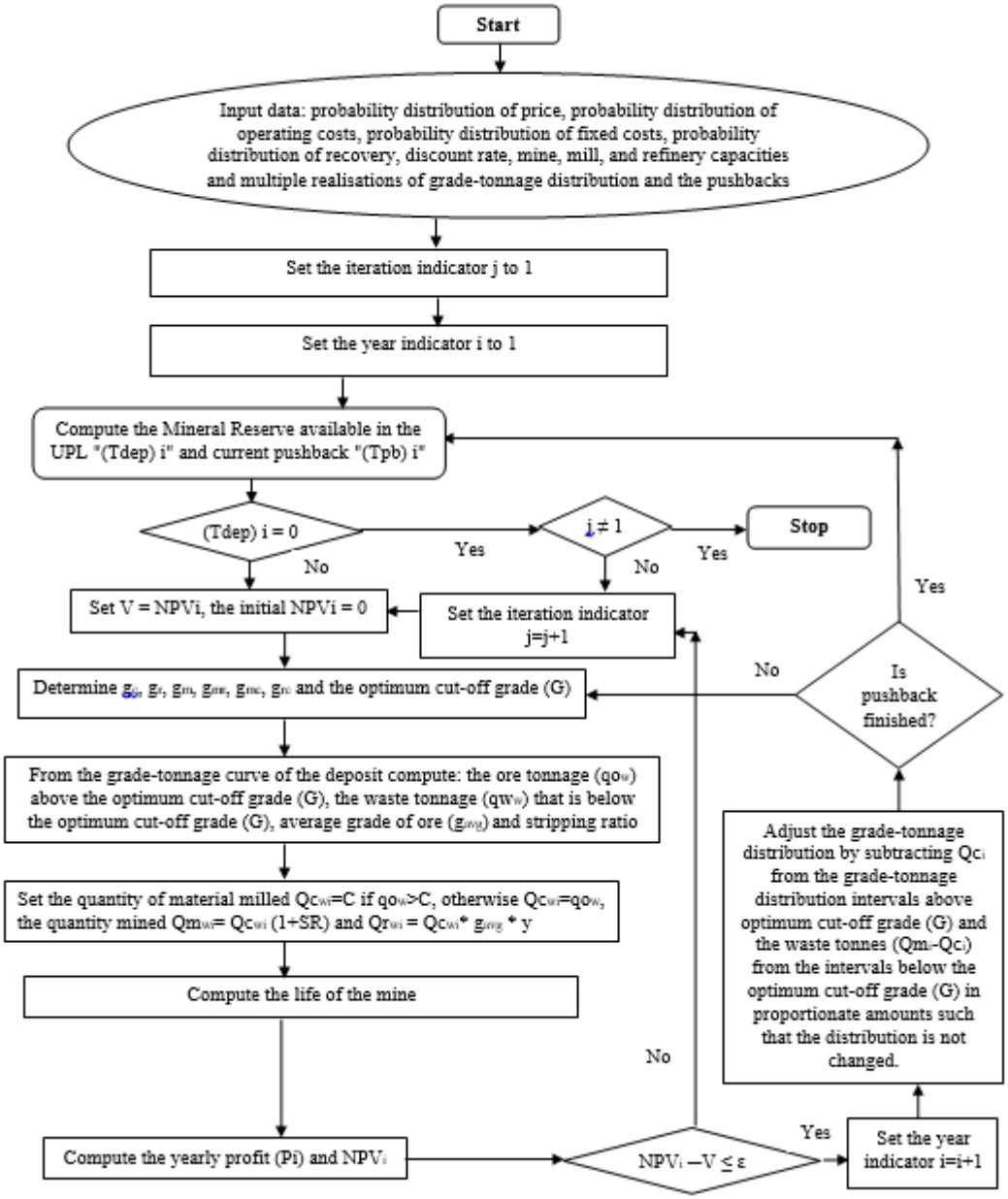


Figure 3.3: Flow diagram of the modified cut-off grade algorithm

3.2.2 Implementation plan

The stochastic approach which was developed in this study applies mining parameters that are randomly determined; having a random probability distribution or pattern that is analysed statistically. The parameters and variables are modelled to have multiple realisations depicting the reality of mining operations. They assume a range of values applied on the current market prices and costs when calculating the cut-off grades in the computer-aided application. The range applied on these economic parameters aims to depict the volatility of the market conditions.

The other input in the computer-aided application, grade-tonnage distribution, is simulated using Monte Carlo simulation due to its easy applicability. Monte Carlo simulation is used to model the grade-tonnage distribution to generate multiple realisations of grade-tonnage curves. They are used to depict the geological characterisations of the deposit.

The implementation plan of the project is as follows:

(i) Identify the methods of data entry for the grade-tonnage curves, price and costs.

(ii) Develop software to input the grade-tonnage curves, price and costs variations and run tests for validation.

(iii) Implement the algorithms for calculating cut-off grade and NPV using the inputs provided.

(iv) Test and validate the output and provide the output in a format that will display the results appropriately.

Linear, quadratic functions or any other mathematical function of price or cost against time and/or any other relevant parameter that these parameters depend on are used in order to achieve the above procedure. Secondly, a random number generator is developed for generating values within a specified range. This is used to create variability in the algorithm.

To achieve the first option, the software needs to be able to provide an input for the parameters of the function along with a list of functions available. The software needs to be able to interpret the functions and calculate them. In this case there could be open source libraries developed that can achieve this. This is how the development of this code is done.

As for the second option, the software generates random values for price or cost which require the range of values to be specified. No pattern is expected here and should be straight forward. The total number of generated values or calculated values should be specified along with the grouping, if any, which also requires ranges of the values to be provided. This affects both value generation methods.

3.3 *NPVMining* application

The three major files used in the execution of the application, *NPVMining*, are Header, Resource and Source files as shown in the solution explorer in Figure 3.4. The header files are sometimes known as ‘include file’ and these have an .h extension. A header file holds declarations for other files to use in this case it holds economic and operational parameters, grade-tonnage distribution. It also holds worksheets and workbooks that

generate Microsoft Excel spreadsheet with the data used in the application. Header files in the application comprise of CApplication.h, CMining.h, ComboCompletion.h, CRange.h, CWorkbook.h, CWorkbooks.h, CWorksheet.h, CWorksheets.h, *NPVMining*.h, *NPVMining*Dlg.h, Resource.h and stdafx.h.

Resource files are interface features that provide information to the user. Bitmaps, icons, toolbars, and cursors are all resources. These resources are used to perform an action such as entering data or selecting from a menu in dialog box. The resource files comprise of *NPVMining*.ico, *NPVMining*.rc and *NPVMining*.rc2. Source files contain a collection of statements or declarations in C++/CLI computer programming language and they are used to specify the actions to be performed by the computer. Source files have .cpp extension and each file is compiled at a time. The source files comprise of CMining.cpp, ComboCompletion.cpp, *NPVMining*.cpp, *NPVMining*Dlg.cpp and stdafx.cpp. *NPVMining*.cpp contains the main code in which the cut-off grades are calculated step by step. These are shown in Figure 3.4 and further described in Appendix 3 (Section 8.3.6).

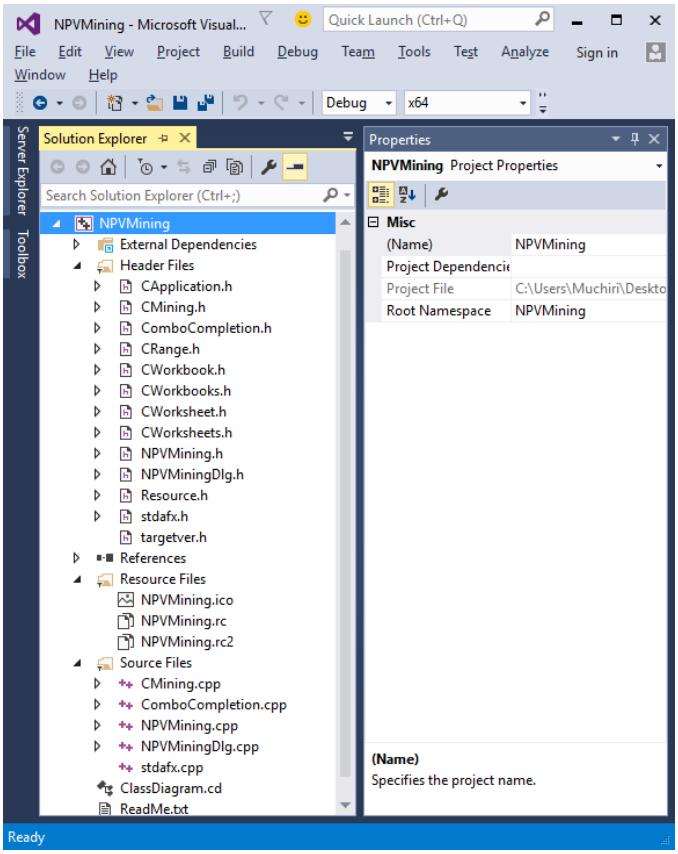


Figure 3.4: Component files of *NPVMining* on solution explorer in VC++

The solution is generated in 14.45 seconds after running the code on Microsoft Visual Studio 2017. The GUI developed from the code is described in the next sections (Sections 3.3.1 and 3.3.2).

3.3.1 Graphic user interface (GUI)

NPVMining user interface has the following input and ouput sections as shown in Figure 3.5 and 3.6:

(i) economic and operational parameters,

(ii) grade-tonnage distribution, and

(iii) the cut-off grade policy or production schedule.

One of the source files, *NPVMining*Dlg.cpp, contains the code that executes the algorithm through the user interface. The input parameters (price and costs) used in the calculations are uploaded to the application using Microsoft Excel spreadsheets. Other parameters are keyed in the user interface via the dialog boxes. The variance on the metal price is realised by factoring a range or factors, either fixed or geometric. After calculating the optimum cut-off grade, a cut-off grade policy is generated showing the three main economic indicators: annual profit, NPV and mine life.

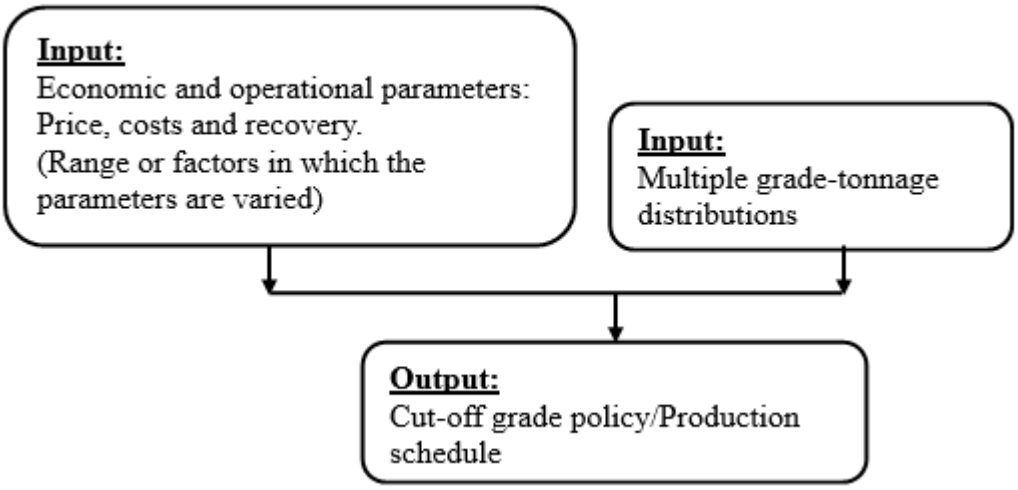


Figure 3.5: Diagram showing features of the application as displayed in the user interface

The *NPVMining* user interface is divided into five window boxes:

(i) grade category window,

(ii) price criteria window,

(iii) other parameters window,

(iv) limiting capacity selection window, and

(v) cut-off grade calculation window.

The graphic user interface developed for the application is shown in Figure 3.6.

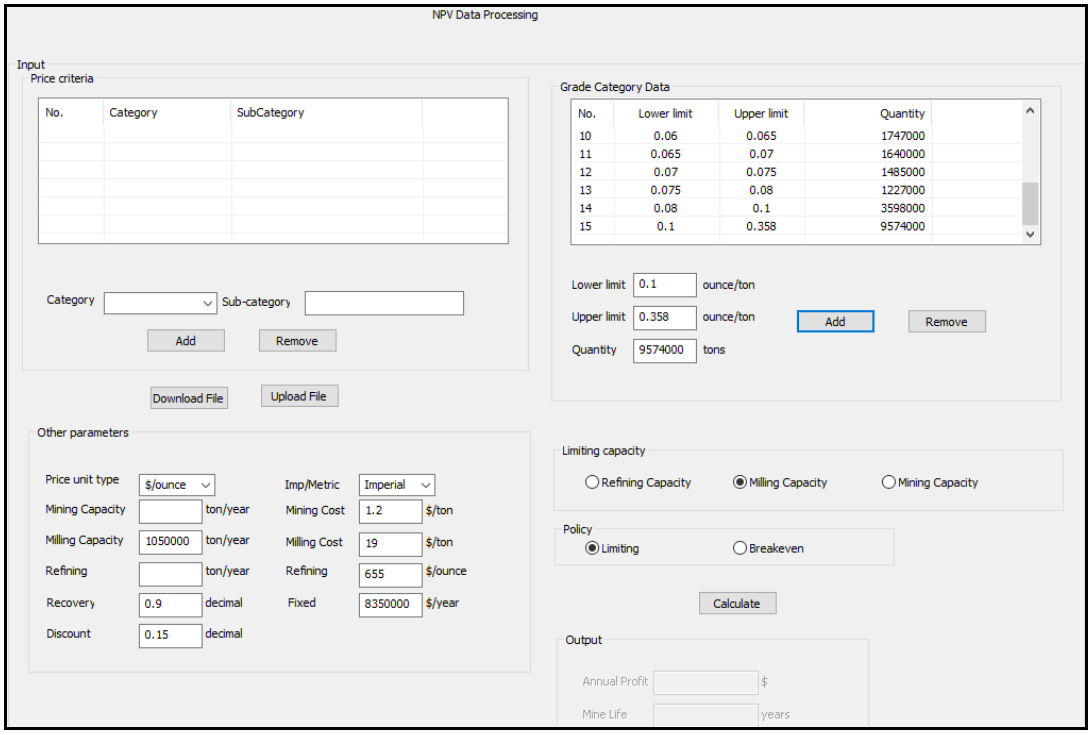


Figure 3.6: NPVMining user interface

The grade-tonnage distribution, economic and operational parameters are the input data used in the computer-aided application. This thesis uses data from a gold mine as shown in the next chapter (Chapter 4). The mining variables can be in either United States customary (or imperial) units and metric units. The user has options to choose from the dropdown unit types. The units can be applied equivalently in the computer−aided application (*NPVMining*) without errors or affecting the results.

The grade-tonnage distribution is keyed into the grade category window as shown in Figure 3.7. The grade category input window has several dialog boxes describing the lower grade limit, upper grade limit and quantity of ore per each increment for the multiple realisations.

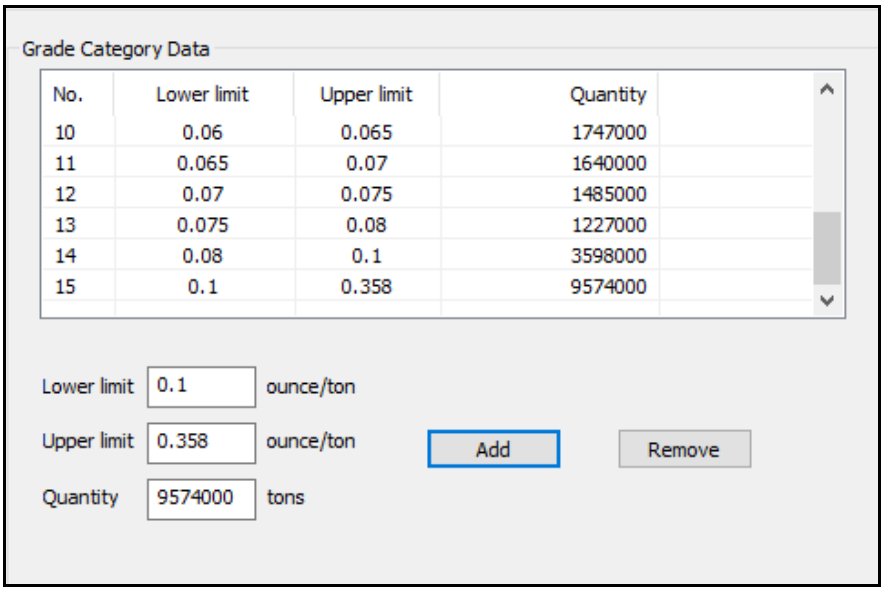


Figure 3.7: Grade category window

The economic and operational parameters are keyed on the user interface using the price criteria and other parameters window as shown in Figures 3.8 and 3.9. This data is uploaded using worksheets from Microsoft Excel spreadsheets prepared by the mining engineer while some are entered in the dialog box as required. This process simplifies the exercise of entering the data which is tiresome if done manually. These two input windows in Figure 3.8 and 3.9 have several dialog boxes that represent economic and operational parameters such as metal price, mining cost, milling cost, refining cost, mining capacity, processing capacity, refining capacity, recovery, discount rate, and fixed cost.

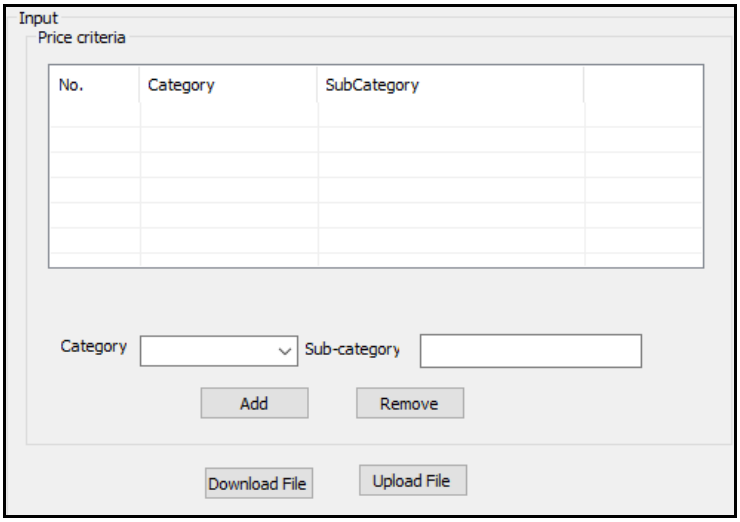


Figure 3.8: Price criteria window

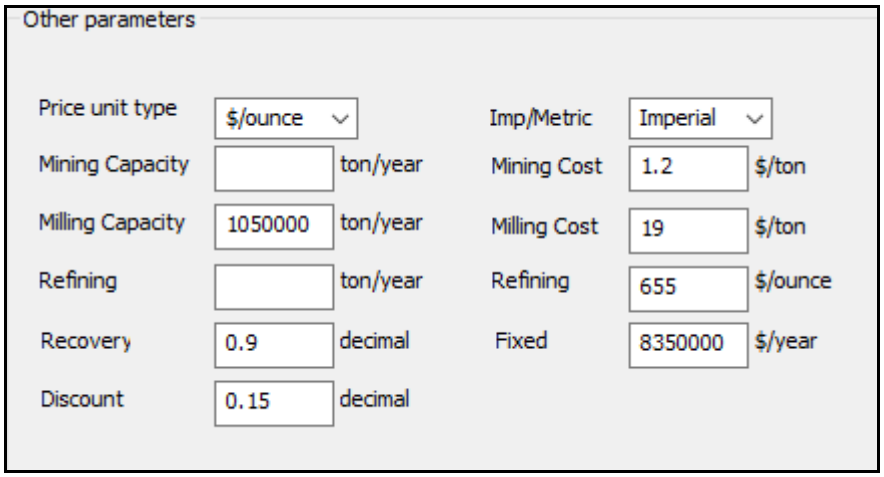


Figure 3.9: Other parameters (economic and operational parameters) window

The break-even cut-off grade and Lane's limiting cut-off grade are selected and calculated as shown in Figure 3.10, and the output is displayed on the output window as shown in Figure 3.11. The main economic indicators of the project (annual profit, NPV and mine life) are displayed on the user interface. After clicking the 'calculate' button, the results of the cut-off grade policy calculations are displayed in the output window and generated on an Microsoft Excel spreadsheet shown in Figure 3.11. It consists of seven main columns namely:

- optimum cut-off grade,
- quantity of material to be mined (Qm),
- quantity of ore to be milled (Qc),
- quantity of product refined (Qr),
- mine life,
- annual profit, and
- NPV.

The resultant cut-off grade policy is exported to Microsoft Excel for ease of use and interpretation.

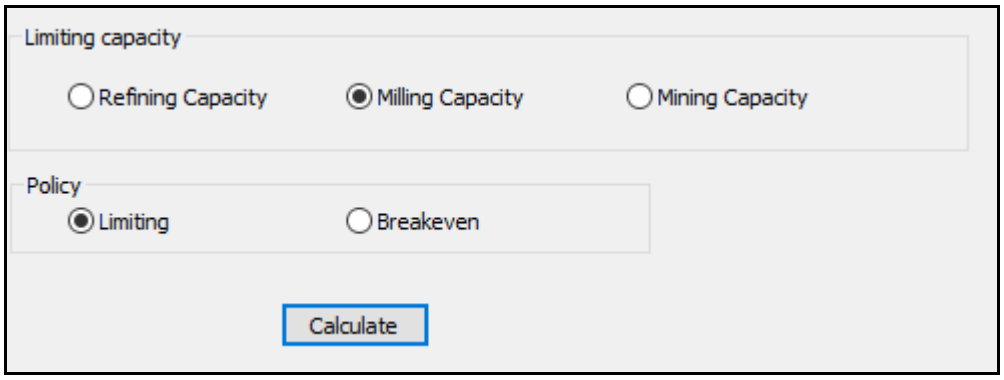


Figure 3.10: Selection of the limiting capacity and cut-off grade policy used

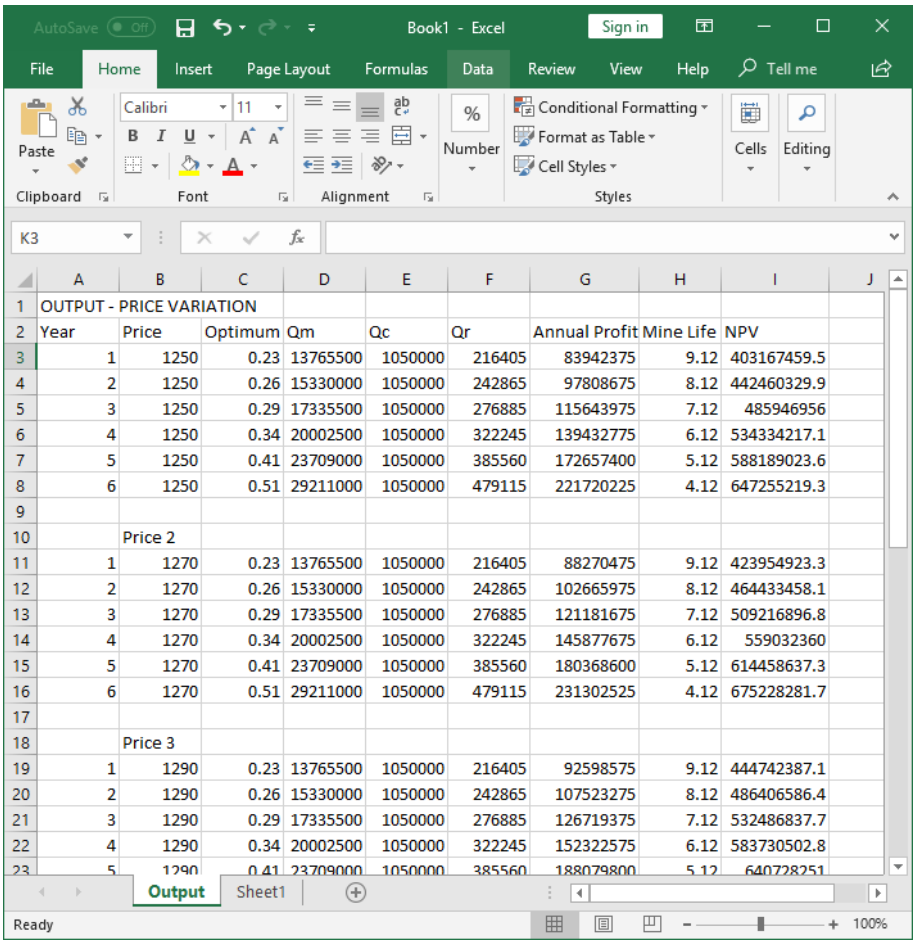


Figure 3.11: Output exported to a Microsoft Excel spreadsheet

3.3.2 Calculation of optimum cut-off grade

The computer-aided application has two options of calculating cut-off grade: break-even cut-off grade approach and proposed stochastic cut-off grade approach. The stochastic approach applies Lane's theory whereby one mine process limits the whole mine operation. Before the application calculates the optimum cut-off grade, the user must

specify which mine process out of the three processes (mine, mill and refinery) is acting as the bottleneck for the entire mine operation. Most mine operations are limited by the processing plant due to its inflexibility to resize its capacity. Hence, most cut-off grade calculations would assume the processing capacity as the limiting capacity.

After selection of the limiting process in the mine operation, the user selects from the two policies (break-even cut-off grade approach and Lane's limiting cut-off grade approach) on which cut-off grade calculation is based. This addition is made on the application to allow the user to apply whichever approach is appropriate to the specific mine.

The ability of the application to handle calculation of break-even cut-off grade policy is a modification that is not necessary in development of the stochastic model (*NPVMining*), but it is included due to the wide usage of break-even cut-off grade policy in the industry. The user will have an option to use and compare the two cut-off grade policies with ease. The application has been configured to upload and download Microsoft Excel 2013 spreadsheets. This allows the capability to upload data and download output in spreadsheets with ease

3.4 Summary of Chapter 3

The proposed stochastic model in this chapter applies modified Lane's theory of cut-off grade to calculate cut-off grades based on varying economic and operational parameters in a mining complex. The cut-off grade algorithm in the model incorporates variance in the economic and geological parameters used. The computer-aided application, *NPVMining*, developed for this model was coded using C++ programming language in Microsoft Visual Studio 2017 Integrated Development Environment (IDE). The user has the capability to upload data in Microsoft Excel 2013 spreadsheets on the graphic user interface to be used in cut-off grade calculation. This makes it easy to access and manipulate the data despite its size. The application also allows the user to download the output for easy use and interpretation.

NPVMining has two cut-off grade policies that the user can select on the user interface. They are (i) Lane's limiting cut-off grade method and (ii) the conventional break-even cut-off grade method. *NPVMining* gives the user the choice to manipulate the data by using either a deterministic or stochastic approach when calculating the cut-off grade. The calculation on the application changes depending on the data entered in the user interface.

The stochastic approach in *NPVMining* employs multiple realisations of probable grade-tonnage distribution and price distribution in its calculations. The application uses these parameters to solve the issue of uncertainty when calculating cut-off grade with the aim of

maximising the output (NPV). Modification of Lane's cut-off grade theory to accommodate uncertainty in the application (*NPVMining*) enables the mine planners to ascertain the risk profile of the mine project. The results from *NPVMining* facilitates a process to quantify the risk involved hence making right decisions when making investments in the mining industry. This application (*NPVMining*) was applied to a gold mine case study as discussed in the next chapter (Chapter 4).

4 APPLICATION OF THE STOCHASTIC CUT-OFF GRADE ALGORITHM

4.1 Chapter overview

The developed cut-off grade optimisation application (*NPVMining*) described in Chapter 3 was tested on real data from a mining project to show its applicability. This project depicts a typical gold deposit with details of the orebody model summarised in Table 4.1. The developed model *(NPVMining)* is applied to a mining complex for a single-element mineral. The data collected from the block model representing the gold deposit contains the following information:

(i) Block model and corresponding documentation indicating the grade-tonnage distribution

(ii) Economic and operational parameters, i.e. metal price, capital and operational costs, etc.

(iii) Design parameters, i.e., processing type, recovery rates, environmental conditions, reclamation requirements, etc.

The data used in this study was based on a McLaughlin gold deposit in Northern California, USA. McLaughlin mine is a gold/silver mine which operated at a former site of the Manhattan mine. Although small amounts of native gold are visible, most of the gold in the deposit is present in nanoparticles. There is no report of disseminated gold; most of it is present in veins. The dataset is provided through a newly established mining library developed by Espinoza *et al.* (2012) to be used by academics for research purposes in their projects. The mining variables were quantified in United States customary (or imperial) units. These units can be applied equivalently with the metric units in the computer−aided application (*NPVMining*) without errors. The common units applied in this cut-off grade calculation would be oz/ton instead of gram/tonne, US$/ton instead of US$/tonne and US$/oz instead of US$/gram.

Table 4.1: Summary of the original block model

Grade variation	From 0 to 0.358 oz/t
Slope profiles	Slope of 45 degrees pit slope
No. of blocks	2,140,342
Block sizes	Blocks of size 25ft x 25ft x 20ft

Drill-hole data and corresponding documentation used to develop the block model describe the tonnages of both ore and waste and the quantity of gold mineable in the pit. The gold grade ranges from 0 to 0.358 ounces/ton with an average grade of 0.110 ounces/ton and a total tonnage of 125,523,000 tons. Geological input used in the proposed cut-off grade model is obtained from the grade-tonnage curve of the deposit. The grade-tonnage distribution shown in Table 4.2 is adopted from the gold mine (McLaughlin gold deposit, Northern California, USA) that is used as a case study to calculate optimum cut-off grade in this chapter.

Table 4.2: Grade-tonnage distribution of the McLaughlin gold deposit

Grade (oz/t)	***Quantity of material in the orebody ('000t)***
0.000-0.020	70,000
0.020-0.025	7,257
0.025-0.030	6,319
0.030-0.035	5,591
0.035-0.040	4,598
0.040-0.045	4,277
0.045-0.050	3,465
0.050-0.055	2,438
0.055-0.060	2,307
0.060-0.065	1,747
0.065-0.070	1,640
0.070-0.075	1,485
0.075-0.080	1,227
0.080-0.100	3,598
0.100-0.358	9,574
Total	**125,523**

The sheeted vein zone is viewed in cross-section striking in the NW direction in Figure 4.1. The image shows a profile of average gold concentrations in the zone with a variable grade-tonnage distribution. The block model of the orebody is based on assay results of blast-holes collected during the course of mining. The geology of the deposit is indicated by the bold blue line (marked SCF) referring to the Stony Creek Fault while the surficial volcanics are marked as Qv.

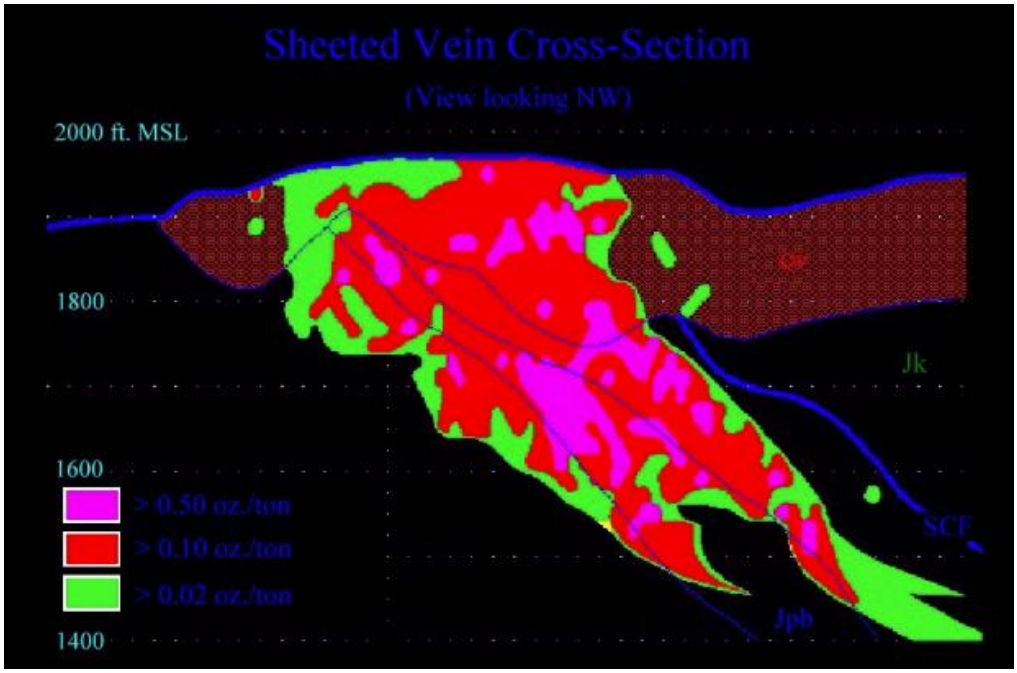


Figure 4.1: Cross-section of sheeted gold vein in the orebody (Enderlin, 2002)

4.2 Input parameters used in the cut-off grade optimisation models

The input parameters for a cut-off grade model typically include economic and technical (geological and operational) parameters. Economic parameters include: metal price, discount rate, fixed cost, mining cost, processing cost, and refining cost. Operational parameters include metallurgical recovery, mining, processing and refining capacities. Geological input is represented by the grade-tonnage curve distribution inside the ultimate pit limit or pushbacks of an orebody model in a mine.

An optimal ultimate pit limit is established indicating the available Mineral Reserves within the pit limit. The multiple grade-tonnage curves used in the model are generated from these available Mineral Reserves. There are a variety of options for pit design, processing, financing as well as influencing economic factors that need to be considered to get the input parameters. The case study data presented in this Chapter is adopted from two studies on cut-off grade optimisation: Dagdelen and Kawahata (2008) and Myburgh *et al.* (2014). This data is used in computation of cut-off grades using the proposed stochastic cut-off grade optimisation model and other existing models.

4.2.1 Grade-tonnage distribution

Grade-tonnage curves are used in the determination of balancing cut-off grades as described in Chapter 2. The balancing cut-off grades are best achieved in practice by tabulating the grade distribution data for the relevant mine. Indeed, the balancing cut-off grades are not calculated in the formulae. They can be achieved by tabulating the grade

distribution data, but they can also be derived from curves of values versus the respective limiting cut-off grades created for the respective stages that need to be balanced (Hall, 2014; Minnitt, 2004). This data is used to calculate the ratios and cumulative values in relation to the different stages in a mining complex. Grade-tonnage distribution in both open pit and underground mine operation is based upon a mineralised orebody. The mineralised material will consist of a range of grades which, in practice, is estimated through a sampling process. The grade distribution is generated from the samples by calculating proportions that exceed a range of grade limits.

Over-estimation or under-estimation of the grade, volume or tonnage and other parameters related to a deposit is common in most conventional and deterministic orebody models. This is detrimental to the planning of the mine which consequently leads to loss of profits. Several statistical methods (Kriging, Multiple-Indicator Kriging (MIK), and Gaussian geostatistical simulation) measure uncertainty of the orebody in relation to the geological characteristics. Stochastic approaches are employed to characterise the geological uncertainty by modelling and estimating the orebody accurately.

In this case, the study employs the Monte Carlo method to simulate the grade-tonnage distribution of the orebody. It uses statistical and graphical techniques to simulate probable distribution of data. The detailed mathematical calculations underlying the whole simulation process are not presented, as they are beyond the scope of this study. The Monte Carlo method is one of the statistical methods with a computational algorithm using randomness to solve problems that might be stochastic. It is mainly used in optimisation, numerical integration and generating draws from a probability distribution. It relies on the possibility of producing a supposedly endless flow of random variables for well-known distributions by modelling and sampling probability density functions (PDFs).

PDFs are the basis for a Monte Carlo simulation whereby the functions define the range of possibilities and the relative probability of those possibilities for a given step in the simulation. They must be non-negative, real-valued functions and the integral over their range must be 1. The cumulative distribution function (CDF) is used for sampling purposes whereby it is defined as an integral of the PDF, for the uniform distribution. Equation 4.1 shows the integral of the function of h and f over a range of x to give estimated random values of the whole sample.

$$E_f[h(X)] = \int_x h(x)f(x)dx \tag{4.1}$$

Where:

x= rfunction (J),

$J=10^8$, represents the whole sample,

rfunction samples x from *f(x)*,

hs = h (X),

(estimate) est = mean (hs),

provided h^2 has finite expectation under *f*.

The CDF indicates the probability that a number sampled randomly from the PDF will be ≤ x. With a range of 0 to 1 in the equation, the range used for the grade-tonnage curve (GTC1) to be simulated is 0.7, 0.07257, 0.06319, 0.05591, 0.04598, 0.04277, 0.03465, 0.02438, 0.02307, 0.01747, 0.01640, 0.01485, 0.01227, 0.03598, and 0.09574.

R programming language is employed in this section to solve the Monte Carlo equations involved in generating simulated realisations of the grade-tonnage distribution. This programming language applies syntax for both graphics and statistical computation. RStudio, a free and open-source IDE for R programming language, is used to interpret the code and generate values from the equations. The user interface shown in Figure 4.2 illustrates basic syntax of R programming language and use of the command-line interface. The process of generating the simulation involves several equations of Monte Carlo integration and getting normal quantiles from the distribution. The equations found in the code are expounded in Appendix 2.

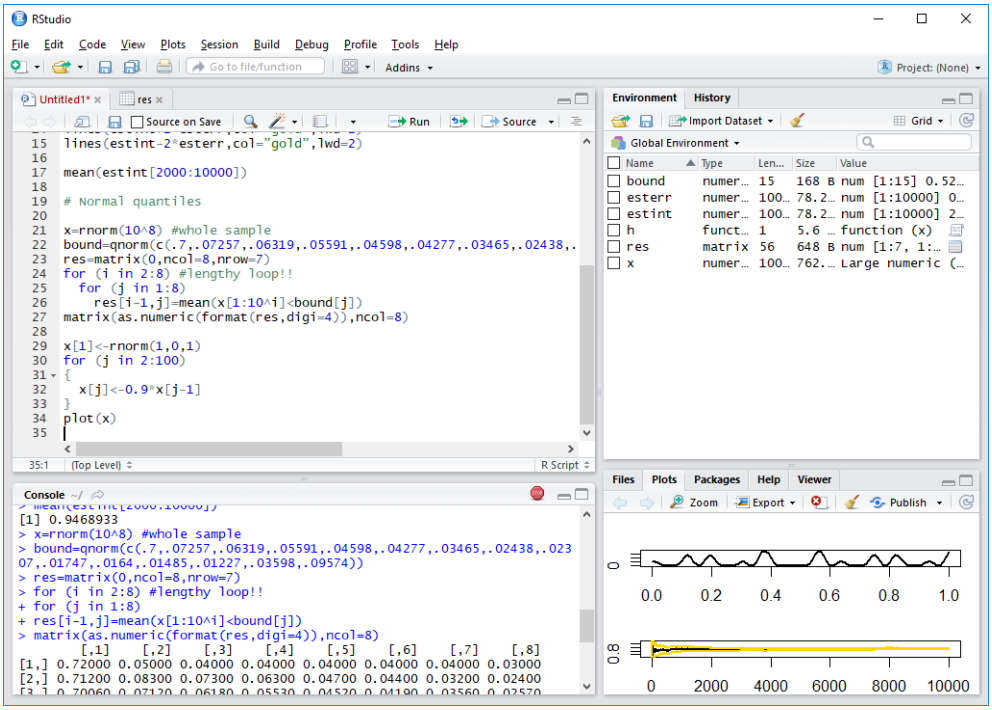


Figure 4.2: RStudio user interface

The simulated multiple realisations of grade-tonnage distribution developed from this exercise are shown in Table 4.3. It is evident that the tonnages decrease as the grade limit increases. A total of six grade tonnage curves were created using Monte Carlo simulation method, resulting in six different but equally probable realisations of the orebody. Simulation of several equally probable realisations of the orebody helps in undertaking risk and decision analysis (sensitivity and probabilistic analysis) on the cut-off grade policy generated and its relation to the key input parameters.

Table 4.3: Simulated grade-tonnage distribution of the McLaughlin gold deposit

	Quantity of material in the simulated orebody realisation ('000t)					
Grade (oz/t)	***GTC1***	***GTC2***	***GTC3***	***GTC4***	***GTC5***	***GTC6***
0.000-0.020	70,000	70,022	70,081	70,003	69,640	70,900
0.020-0.025	7,257	7,263	7,257	7,364	7,890	7,000
0.025-0.030	6,319	6,325	6,332	6,405	6,860	5,700
0.030-0.035	5,591	5,595	5,616	5,632	6,100	5,200
0.035-0.040	4,598	4,603	4,628	4,608	4,950	4,600
0.040-0.045	4,277	4,279	4,306	4,318	4,570	4,300
0.045-0.050	3,465	3,466	3,488	3,502	3,680	3,000
0.050-0.055	2,438	2,439	2,468	2,438	2,630	2,200
0.055-0.060	2,307	2,309	2,335	2,318	2,520	2,100
0.060-0.065	1,747	1,748	1,754	1,740	1,900	1,500
0.065-0.070	1,640	1,643	1,647	1,637	1,770	1,400
0.070-0.075	1,485	1,488	1,494	1,481	1,560	1,200
0.075-0.080	1,227	1,229	1,237	1,246	1,270	800
0.080-0.100	3,598	3,598	3,625	3,636	3,810	3,300
0.100-0.358	9,574	9,578	9,579	9,641	10,330	9,800
Total	**125,523**	**125,585**	**125,847**	**125,969**	**129,480**	**123,000**

4.2.2 Economic, design and operational parameters

A typical mining complex of a specific commodity such as gold consists of a mine, ore processing plant, refinery and/or marketing facility and a waste dump. The mine produces sulphide and oxides ores mixed with waste material in controlled quantities. The ore goes through a processing stage having several processes such as gravity concentration, flotation, and leaching. After this step, the ore is refined to extract gold from the solution while the waste material is sent to the waste dump. Figure 4.3 explains the flow of material from the mine to the processing plant, refinery and waste dump.

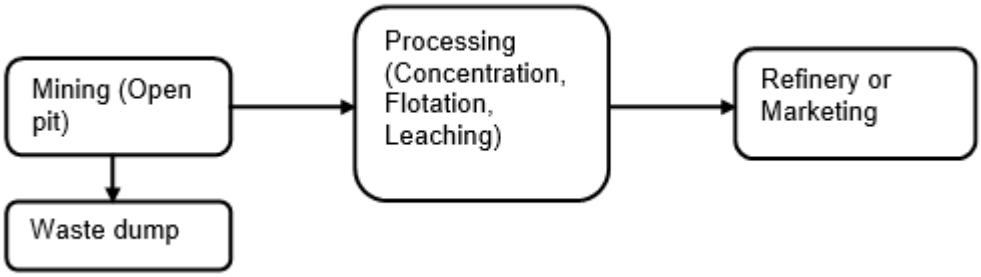


Figure 4.3: Flow of material through the mining operation

The optimisation parameters used in generating the optimal block model for this study tend to depict the parameters based on the McLaughlin gold deposit where economic and operational conditions in the mine are represented in Table 4.4. The economic and operational costs included in the cut-off grade calculations are based on the McLaughlin gold mine operations. The range of values used for the parameters is assumed to have a uniform distribution along the mine life. The uniform distribution is factored into the calculations.

The operation is assumed to have an unrestricted potential to mine and refine or market gold while, the processing capacity is set to be at 1.05 million tonnes of ore which was the capacity of the McLaughlin mine processing plant. Metal price and the operational costs for the mine are based on figures supplied by the specific mine's planning cycle as shown in Table 4.4.

Table 4.4: Economic, design and operational parameters used in the model

Notation	Explanation	Unit	Amount
s	Price	US$/oz	1250
m	Mining cost	US$/ton	1.2
c	Milling cost	US$/ton	19.0
r	Refining/Selling cost	US$/oz	655
f	Annual fixed costs	US$/year	8,350,000
y	Recovery	Decimal	0.9
d	Discount rate	Decimal	0.15
M	Mining capacity	ton/year	Unlimited
C	Milling capacity	ton/year	1,050,000
R	Refining capacity	ton/year	Unlimited

The average metal price used in this study was assumed to be US$ 1250 per ounce as based on projections in Figure 4.4. Metal price sourced from InfoMine website is provided by Xignite Incorporation, a market data company. Xignite provides financial market data application programming interfaces (APIs) that is cloud-based to help companies to generate real-time and reference market data for their own use.



Figure 4.4: Yearly gold price projections (InfoMine, n.d. http://www.infomine.com/investment/metal-prices/gold/5-year/)

Gold price uncertainty applied in this study was assumed to have a range (2% per period) that is increasing yearly as shown in Table 4.5 to be within the generally accepted long-term gold price of around US$ 1300/oz. This range is assumed to factor in the volatility that is experienced in the mining industry.

Table 4.5: Price variations used in the model

No.	Price Category	Price (US$/oz)
1	Year 1Range1	1250
2	Year 2 Range2	1270
3	Year 3 Range1	1290
4	Year 4 Range2	1310
5	Year 5 Range1	1330
6	Year 6 Range2	1356

4.3 Cut-off grade policy generated from the proposed stochastic model (*NPVMining*)

The proposed stochastic cut-off grade model (*NPVMining*) was used to calculate the optimum cut-off grade that maximises NPV in the shortest mine life possible under three setups. One of the setups incorporates constant economic and technical parameters in the calculation while the other two setups have dynamic parameters in their calculation process.

4.3.1 Results from the application (NPVMining) using constant economic and technical parameters

Using data from Tables 4.2 and 4.4, optimum cut-off grade was calculated using *NPVMining* to get results as tabulated in Table 4.6. Results from this dataset show that the mine project has a mine life of 9.12 years and the NPV to be at US$ 467.17million.

Table 4.6: Results as reported from *NPVMining* for grade-tonnage curve (GTC1) at a price of US$1250 per ounce

Economic indicators	***Value***
Quantity of material mined (Q_m) (tons per year)	125,538,000.00
Quantity of ore sent to concentrator (Q_c) (tons per year)	9,450,000.00
Quantity of gold produced (Q_r) (koz)	1931.60
Mine life (years)	9.12
Cash flow or Profits (US$)	887,089,585.49
NPV (US$)	467,172,060.50

4.3.2 Results generated using NPVMining with price variations

Using the price variations in Table 4.5 and the grade-tonnage curve in Table 4.2, the application (*NPVMining*) generates possible outcomes in relation to the change in price. The six possible outcomes on each price are categorised into the different six price ranges as shown in Appendix 3 (Tables 8.3 – 8.8). A summary of these results indicating the output generated from the project when price is varied is shown in Table 4.7. The results indicate that an increase in price increases the annual profit and NPV generated.

Table 4.7: Summary of the output generated from six possible metal price ranges for grade-tonnage curve (GTC1)

No.	*Price (US$/oz)*	*Annual profit (US$)*	*Mine life (yrs.)*	*NPV (US$)*
1	1250	97,268,594.9	9.12	467,172,060.5
2	1270	101,596,694.9	9.12	487,959,524.3
3	1290	105,924,794.9	9.12	508,746,988.1
4	1310	110,252,894.9	9.12	529,534,451.9
5	1330	114,580,994.9	9.12	550,321,915.7
6	1350	118,909,094.9	9.12	571,109,379.5

The results show a difference of 18.23% between the minimum and maximum NPV generated by varying the price from the base value (US$ 1250/oz).

4.3.3 Results generated using NPVMining under multiple realisations of the grade-tonnage distribution

A summary of the calculated annual profit, mine life and NPV for six equally probable grade-tonnage curves is generated as shown in Table 4.8. The results generated are calculated using a metal price of US$ 1250 per ounce whereas keeping the other economic and operational parameters constant. The resultant cut-off grade policies from *NPVMining* shows a difference of 4.34% between the minimum and maximum NPV generated through a set of six equally probable grade-tonnage curves. The grade-tonnage distribution is varied consistently over time in the cut-off grade calculations as shown in Appendix 3 (Tables 8.3 – 8.8). The resultant cut-off grade policies generated by the grade-tonnage curves (GTC1 – GTC6) vary depending on time and quantity of mineable ore in every grade interval.

From the summarised results in Table 4.8, the grade-tonnage distribution affects the cash flow and NPV. Grade-tonnage curve (GTC5) with a high tonnage of 129,480,000 tons generates the highest NPV (US$ 488,180,335.90), although, this does not happen in the rest of the grade-tonnage curves. For example, GTC6 with the least tonnage (123 million tons) has a higher NPV of US$ 475,864,529.70.

The six resultant cut-off grade policies generating NPV in approximately 9 years change with the irregular grade-tonnage distribution of the orebody. This indicates that the level of uncertainty in the geological data is high, hence the need to account for it. Therefore, ignoring the geological uncertainty in mining operations may lead to serious economic effects.

Table 4.8: Summary of the output generated from six equally probable simulated grade-tonnage curves

Grade-tonnage curves (GTCs)	*Average annual profit (US$)*	*Mine life*	*Net present value (US$)*
GTC1	97,268,594.9	9.12	467,172,060.50
GTC2	97,268,594.9	9.12	467,172,060.50
GTC3	97,230,794.9	9.12	466,990,510.60
GTC4	97,318,994.9	9.18	468,922,686.82
GTC5	97,999,394.9	9.84	488,180,335.90
GTC6	97,974,194.9	9.33	475,864,529.70

4.4 Linear regression analysis of the results

The relationship between NPV and the main parameters (price and grade-tonnage distribution) is modelled using a linear regression approach. It is a linear approach for modelling the relationship between a dependent variable and one or more independent variables. The linear regression models in this section are applied to identify the relationship between NPV and the response variables (price and grade-tonnage distribution) when all the other variables in the model are held fixed. The linear relationship is generated using values from Table 4.9.

Table 4.9: Net present values generated using *NPVMining* when varying price and grade-tonnage distribution

NPV (US$)	*Price (US$/oz)*	*Grade-tonnage ('000t)*
467,172,060.50	1250	125,523
467,172,060.49	1250	125,585
466,990,510.59	1250	125,847
468,922,686.82	1250	125,969
488,180,335.90	1250	129,480
475,864,529.70	1250	123,000
487,959,524.30	1270	125,523
487,959,524.30	1270	125,585
487,777,974.39	1270	125,847
489,777,241.30	1270	125,969
509,740,604.86	1270	129,480
496,886,282.14	1270	123,000
508,746,988.10	1290	125,523
508,746,988.10	1290	125,585
508,565,438.18	1290	125,847
510,631,795.78	1290	125,969
531,300,873.82	1290	129,480
517,908,034.58	1290	123,000
529,534,451.90	1310	125,523
529,534,451.88	1310	125,585
529,352,901.98	1310	125,847
531,486,350.25	1310	125,969
552,861,142.78	1310	129,480
538,929,787.03	1310	123,000
550,321,915.70	1330	125,523
550,321,915.68	1330	125,585
550,140,365.78	1330	125,847
552,340,904.73	1330	125,969
574,421,411.74	1330	129,480
559,951,539.47	1330	123,000
571,109,379.50	1350	125,523
571,109,379.47	1350	125,585
570,927,829.57	1350	125,847
573,195,459.21	1350	125,969
595,981,680.69	1350	129,480
580,973,291.91	1350	123,000

The NPV represents the independent variable as shown in Figures 4.5 and 4.6 (on the x−axis), while the dependent variables on the y−axis are price and grade-tonnage distribution. The linear correlation between the two dependent variables is measured using the correlation coefficient. It has a value between +1 and −1, where +1 signifies total positive linear correlation and −1 is total negative linear correlation. If the value is 0 then there is no linear correlation. The correlation coefficient in the relationship between price and NPV is 0.97 and 0.13 for the relationship between grade-tonnage distribution and NPV. The difference shows that price variation has a higher significance to NPV compared to grade-tonnage distribution.

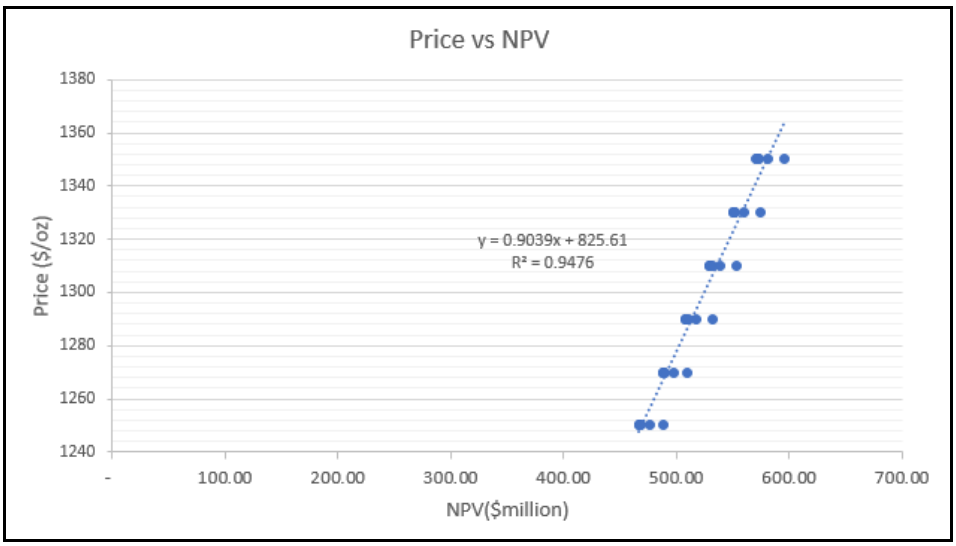


Figure 4.5: Linear relationship between metal price and NPV

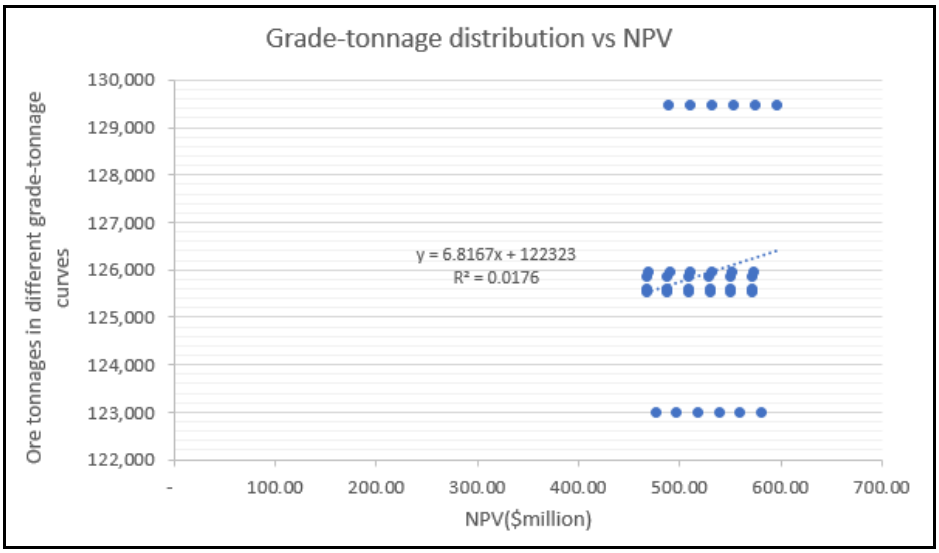


Figure 4.6: Linear relationship between grade-tonnage distribution and NPV

4.5 Summary of Chapter 4

The developed model (*NPVMining*) is applied and implemented on a gold mine deposit to ascertain its benefits in an operational mine. The dataset used in this study is from a defunct gold mine. *NPVMining* has the capability to calculate several cut-off grade policies under economic and geological uncertainties with ease. In this chapter, *NPVMining* is used to generate six cut-off grade policies indicating that a change in grade-tonnage distribution has an overall effect on NPV. The results indicate that the NPV increases when the grade-tonnage distribution curve is high. There are also different scenarios generated by the application when price is varied illustrating the effect on NPV generated along the mine life. From the results, there is an indication that the cash flow and NPV generated increase as price increases.

Consequently, the resultant cut-off grade policies, as presented in Tables 4.7 and 4.8, facilitate a risk-quantified, decision-making process when undertaking major mining investments for sustainable utilisation of Mineral Resources. This process is described in Chapter 5 where risk and decision analyses are done on the output generated from *NPVMining*.

The results indicate that the developed computer-aided application can be applied to optimise cut-off grade policy of a mining complex that comprises of three stages; a mine, ore processing stream and refinery/market, under uncertain geological and economic variables such as selling price. Although there is an improvement in the modified computer-aided application (*NPVMining*) compared to a deterministic approach (such as Cut-off Grade Optimiser by Githiria *et al.*, 2016), there is a challenge in varying costs and recovery.

Using a standard computer running at 3.00 GHz and with 4.00 GB RAM, *NPVMining* generated results for each simulation within 5s. A comparison is later made between *NPVMining* and other existing cut-off grade models in Chapter 5 to indicate the efficiency of the proposed model.

5 RESULTS AND VALIDATION

5.1 Chapter overview

This chapter emphasises and illustrates the need to optimise the cut-off grade used by mining engineers to calculate the production rates and NPV of mining projects. These values are calculated by models designed to ease the work done by the mining engineers when dealing with complicated scenarios. This chapter reviews different models used in calculating cut-off grades in both open pit and underground mining operations. The comparison of existing cut-off grade models to the model developed in this thesis helps to indicate the improved value creation based on NPV and illustrates the efficiency and significance of the developed model *(NPVMining)*. The degree of confidence on the model is measured by analysing the distribution of cut-off grades against the NPV generated.

5.2 Validation of the proposed model

There are several methods used in calculating cut-off grade such as the break-even model, traditional or heuristic methods, Lane's cut-off grade model and extensions to Lane's model. The subsequent discussions in this chapter focus on highlighting the quality of the results that are obtained from several optimization techniques in cut-off grade calculations.

Using data from Tables 4.2 and 4.4, the results are compared based on the output between *NPVMining* and other existing cut-off grade optimisation models. The economic, technical and geological data used in this analysis was obtained from the McLaughlin gold deposit as represented in Tables 4.2 and 4.4. The comparison is based mainly on the NPVs generated and associated life of mine (LOM) to assist in decision making during the mine planning process. The cut-off grade optimisation approaches used in this analysis are classified into two (deterministic and stochastic) approaches as follows:

- Deterministic cut-off grade optimisation approaches
    i. Break-even cut-off grade approach
    ii. Traditional heuristic cut-off grade approach (Dagdelen, 1993; Asad, 1997)
    iii. Deterministic approach based on Lane's theory (Cut-off grade Optimiser)
- Stochastic cut-off grade optimisation approaches
    i. Maptek Evolution® (Myburgh *et al.*, 2014)
    ii. Developed stochastic model (*NPVMining*)

Deterministic cut-off grade optimisation approaches applied in the mining industry have their outcomes determined through known parameters without any room for random variation. This limits the flexibility of a mine plan in case price and operational costs fluctuate along the mine life. The three approaches are applied in this chapter to show this drawback. On the other hand, the self-adaptation in stochastic algorithms contributes greatly to their ability to address successfully most complex real-world problems. This is due to their strategic outlook of mining problems that allows them to be easily applied on complex mining situations. This is evident from the two approaches compared in Sections 5.2.4 and 5.2.5.

5.2.1 Application of break-even cut-off grade approach

Break-even cut-off grade models are not focused on maximising cash returns hence tend to produce sub-optimal mine plans under the reality of fluctuating mineral prices (Hall, 2014). In addition, the cut-off grades are constant throughout the LOM at the planning stage. The steps used by the break-even cut-off grade model in calculating cash flows generated are demonstrated as follows (Hall, 2014):

1. Determine the break-even cut-off grade using Equation 5.1 as follows:

$$Break-even\ cut-off\ grade = ((c+m))/((y(s-r))) = ((19+1.2))/0.9(1250-655) = 0.038\ oz/t \quad (5.1)$$

   Where processing cost (c) = US$ 19/ton; mining cost (m) = US $1.2/ton; recovery (y) = 0.9; price (s) = US$ 1250/oz and refinery cost (r) = US$ 655/oz.

2. Determine the quantity of ore (*qo*) and quantity of waste (*qw*), stripping ratio (*SR*), and average grade (ḡ) of ore. Using this break-even cut-off grade of 0.038 oz/ton, all the material above this cut-off grade is sent to the processing plant while the rest is sent to the waste dump. Ore tonnage(*qo*) in this deposit is the total tons of material between grade intervals 5 and 15 (=36,346 ktons). All the material below the break-even cut-off grade is considered as waste material(*qw*) (=89,167ktons). The average grade of the ore (ḡ) is calculated by taking a weighted average of ore tonnage and average grades of these materials (=0.102 oz/ton). The stripping ratio is calculated as follows: (*qw*/*qo) = 89,167/36,346 = 2.45*

3. Determine the quantity of material to be mined (*Qm*), quantity of ore sent to the concentrator (*Qc*), and amount of product produced over the production period (*Qr*) as indicated in Equations 5.2 to 5.4. With a yearly ore production (Q_c) of 1.05 million tons/year, quantity of mined material (Q_m) and quantity of refined product (Q_r) are calculated as follows:

$$Q_c = \frac{Q_m}{(1+SR)} = 1.05Mt \tag{5.2}$$

$$Q_m = Q_c\left[1+\frac{qw}{qo}\right] = Q_c[1+SR] = 1.05 \times (1+2.45) = 3.62Mt \tag{5.3}$$

$$Q_r = (Q_c)(\bar{g})(y) = 1.05 \times 0.102 \times 0.9 \times 1000 = 96.3koz \tag{5.4}$$

4. Determine cash flow for year *i, CF_i* as indicated in Equation 5.5.

$$CF_i = (s-r)Q_{ri} - cQ_{ci} - mQ_{mi} - f_i \tag{5.5}$$

where *s* is selling price (US$ 1250/oz); *m* is unit mining costs (US$ 1.2/ton); *c* is unit processing costs (US$ 19/ton); *r* is unit refining costs (US$ 655/oz); *f* is fixed costs (US$ 8.35million), *Qm* is material mined (3.62 million tons); Q_c is ore processed (1.05 million tons); Q_r is concentrate refined (96.3 kilo ounces); *i* is the specific time period.

$$CF_i = \left\{(1250-655)*\frac{96.3}{1000}\right\} - (1.05*19) - (1.2*3.62) - 8.35 = 24.7 \tag{5.6}$$

When break-even cut-off grade of 0.038 oz/t is applied to the deposit throughout the mine life, NPV of US$ 163.42 million is generated as shown in Table 5.1. The computer-aided application (*NPVMining*) developed in this study has an additional function that calculates break-even cut-off grade as shown in Figure 3.10 (Chapter 3).

Table 5.1: Cut-off grade policy using the break-even cut-off grade model

Period (yr)	*Cut-off grade (oz/t)*	*Avg. grade (oz/t)*	*Quantity of material mined (Qm) (million t)*	*Quantity of ore sent to concentrator (Qc) (million t)*	*Quantity of gold produced (Qr) (koz)*	*Cash flow (US$ million)*
1 to 34	0.038	0.102	3.6	1.05	96.3	24.7
35	0.038	0.102	3.4	1.00	91.8	23.2
Total			**125.8**	**36.7**	**3366.0**	**863.0**
NPV@15%						**163.42**

5.2.2 Application of traditional heuristic cut-off grade approach

With the heuristic approaches, cut-off grade calculation changes along the mine's productive life in intervals. Equations 5.7 to 5.9 show how the calculations are done in one of the heuristic approaches (Dagdelen, 1992 and 1993; Asad, 1997). The traditional cut-off grade calculation is modified to include fixed costs, depreciation, minimum profit per ton required for a period to obtain a much higher cut-off grade during the early years. If a minimum profit of US$ 3.0/t would be imposed to increase the cash flows further during the first 5 years and that the plant's capital cost of US$ 105 million would be depreciated during the first 10 years by the straight-line depreciation method, then:

- Depreciation cost per year = US$ 105 million/10 years= US$ 10.5 million/year
- Depreciation cost per ton = US$ 105 million/1.05 million ton= US$ 10/t of ore

If fixed costs are also included in the equations, the following results can be achieved:

- Fixed cost per ton= US$ 8.35M per year/US$ 1.05Mtons= US$ 7.95/t

Milling cut-off grade for years 1-5:

$$= \frac{milling\ cost + minimum\ profit + fixed\ cost + depreciation}{[price - (refining\ cost + marketing\ cost)] * (recovery)} \quad (5.7)$$

$$= \frac{19 + 3 + 7.95 + 10}{(1250 - 655) * 0.9} = 0.075 oz/t$$

Milling cut-off grade for years 6-10:

$$= \frac{milling\ cost + fixed\ cost + depreciation}{[price - (refining\ cost + marketing\ cost)] * (recovery)} \quad (5.8)$$

$$= \frac{19 + 7.95 + 10}{(1250 - 655) * 0.9} = 0.069 oz/t$$

Milling cut-off grade for years 11-18 (till depletion):

$$= \frac{milling\ cost + fixed\ cost}{[price - (refining\ cost + marketing\ cost)] * (recovery)} \quad (5.9)$$

$$= \frac{19 + 7.95}{(1250 - 655) * 0.9} = 0.050 oz/t$$

The case study results of applying a heuristic cut-off grade model are illustrated in Table 5.2. The table shows an improvement in NPV (=US$347.08 million−US$163.42 million) by approximately 52.9% compared to the break-even cut-off grade model. However, the mine project has a shorter LOM from 35 years to only 18 years to achieve a faster recovery of the capital investment.

Table 5.2: Cut-off grade policy using a traditional heuristic model (fixed costs included)

Period (yr.)	*Cut-off grade (oz/t)*	*Avg. grade (oz/t)*	*Quantity of material mined (Qm) (million t)*	*Quantity of ore sent to concentrator (Qc) (million t)*	*Quantity of gold produced (Qr) (koz)*	*Cash flow ($ million)*
1 to 5	0.075	0.182	9.2	1.05	171.6	62.8
6 to 10	0.069	0.169	8.2	1.05	160.0	57.1
11 to 17	0.050	0.132	5.4	1.05	124.8	39.5
18	0.050	0.132	1.3	0.26	30.5	9.6
Total			**125.8**	**18.11**	**2562.5**	**885.6**
NPV@15%						**347.08**

5.2.3 Application of a deterministic approach based on Lane's theory (Cut-off Grade Optimiser)

Githiria *et al.* (2016) developed the Cut-off Grade Optimiser, an executable computer-aided application that applies Lane's cut-off grade theory without any modifications. Cut-off Grade Optimiser was developed in the C++ programming language in Microsoft Visual Studio 2008 Integrated Development Environment (IDE). The resultant cut-off grade policy generated from this approach is as illustrated in Table 5.3. The cut-off grades are changing along the mine life due to Lane's model that accounts for the dynamic geological data in the calculations.

Table 5.3: Results of cut-off grade policy from Cut-off Grade Optimiser (Githiria *et al*., 2016)

Period (yr.)	*Cut-off grade (oz/t)*	*Avg. grade (oz/t)*	*Quantity of material mined (Qm) (million t)*	*Quantity of ore sent to concentrator (Qc) (million t)*	*Quantity of gold produced (Qr) (koz)*	*Cash flow ($ million)*
1	0.16	0.259	17.93	1.05	244.76	95.81
2	0.15	0.254	17.07	1.05	240.03	94.03
3	0.14	0.249	16.29	1.05	235.31	92.16
4	0.13	0.244	15.57	1.05	230.58	90.21
5	0.12	0.239	14.92	1.05	225.86	88.18
6	0.10	0.229	13.77	1.05	216.41	83.94
7	0.09	0.224	11.58	1.05	211.68	83.75
8	0.07	0.204	8.30	1.05	192.78	76.44
9	0.06	0.181	6.83	1.05	171.05	65.28
10	0.05	0.161	5.49	1.05	152.15	55.64
Total			**127.75**	**10.5**	**2120.61**	**825.44**
NPV@15%						**435.52**

5.2.4 Application of a modern heuristic cut-off grade approach using Maptek Evolution®

Maptek Evolution® performs simultaneous optimisation of extraction sequence and cut-off grade optimisation for a single element. The evolution algorithm employed in this computer-aided application stochastically produces high-quality results quickly through several iterations. However, the iterations performed do not guarantee convergence hence compromising on optimality. Additionally, the time taken to complete an optimisation increases with the complexity of the problem, in the range of 3-20 minutes (Myburgh *et al.*, 2014; Githiria and Musingwini, 2018). Using data in Tables 4.2 and 4.4 (Chapter 4), Maptek Evolution® software generates a cut-off grade policy shown in Table 5.4. This modified heuristic approach produces a 19.2% higher NPV (US$413.84 million−US$347.08 million), but a shorter LOM of 10 years compared to the basic heuristic approach.

Table 5.4: Cut-off grade policy as reported from Maptek Evolution® per period

Period (yr.)	*Cut-off grade (oz/t)*	*Avg. grade (oz/t)*	*Quantity of material mined (Qm) (million t)*	*Quantity of ore sent to concentrator (Qc) (million t)*	*Quantity of gold produced (Qr) (koz)*	*Cash flow ($ million)*
1	0.156	0.257	17.55	1.05	242.7	95.0
2	0.145	0.251	16.66	1.05	237.6	93.1
3	0.137	0.247	16.03	1.05	233.7	91.5
4	0.123	0.240	15.11	1.05	227.2	88.8
5	0.111	0.234	14.37	1.05	221.6	86.3
6	0.105	0.232	14.06	1.05	219.0	85.1
7	0.086	0.200	10.83	1.05	189.0	71.2
8	0.072	0.174	8.55	1.05	164.7	59.4
9	0.061	0.153	6.87	1.05	144.6	49.5
10	0.050	0.134	5.49	1.05	126.2	40.2
Total			**125.3**	**10.5**	**2002.5**	**760.1**
NPV@15%						**413.84**

5.2.5 Application of the proposed stochastic Lane's cut-off grade approach using NPVMining

NPVMining applies a stochastic approach using Lane's cut-off grade theory as shown in Chapters 3 and 4. The results generated from this developed computer-aided application are summarised in Table 5.5. It summarises the major mining economic indicators such as quantity of material mined, quantity of ore processed, quantity of gold produced, mine life, cash flow and NPV.

Table 5.5: Cut-off grade policy as reported from *NPVMining* per period

Period (yr.)	*Cut-off grade (oz/t)*	*Avg. grade (oz/t)*	*Quantity of material mined (Qm) (million t)*	*Quantity of ore sent to concentrator (Qc) (million t)*	*Quantity of gold produced (Qr) (koz)*	*Cash flow (US$ million)*
1	0.161	0.259	18.0	1.05	245.2	111.9
2	0.152	0.255	17.2	1.05	241.0	110.4
3	0.142	0.250	16.4	1.05	236.4	108.6
4	0.131	0.245	15.7	1.05	231.3	106.5
5	0.120	0.239	14.8	1.05	225.7	104.1
6	0.107	0.232	14.1	1.05	219.6	101.4
7	0.092	0.213	12.1	1.05	200.9	92.7
8	0.079	0.188	9.7	1.05	177.9	81.9
9.12	0.065	0.163	7.5	1.05	153.6	69.9
Total			**125.5**	**9.45**	**1931.6**	**$887.09**
NPV@15%						**$467.17**

5.2.6 Comparison of the cut-off grade models

Cut-off grades are lower and constant with the break-even cut-off grade but higher with the other approaches that have dynamic cut-off grades as shown in Figure 5.1. These dynamic cut-off grades are higher during the early years and lower during the later years of operation. Due to the higher cut-off grades in early years, higher NPVs are generated. The change in cut-off grade is relative to the decline in NPV with the depleting Mineral Reserves. The mine life is also shorter in dynamic cut-off grade approaches that ensure quicker recovery of the capital investment. A summary comparison of cut-off grade approaches in relation to the output (cut-off grade per year, mine life, annual profit, NPV) is illustrated in Figure 5.1 and Table 5.6.

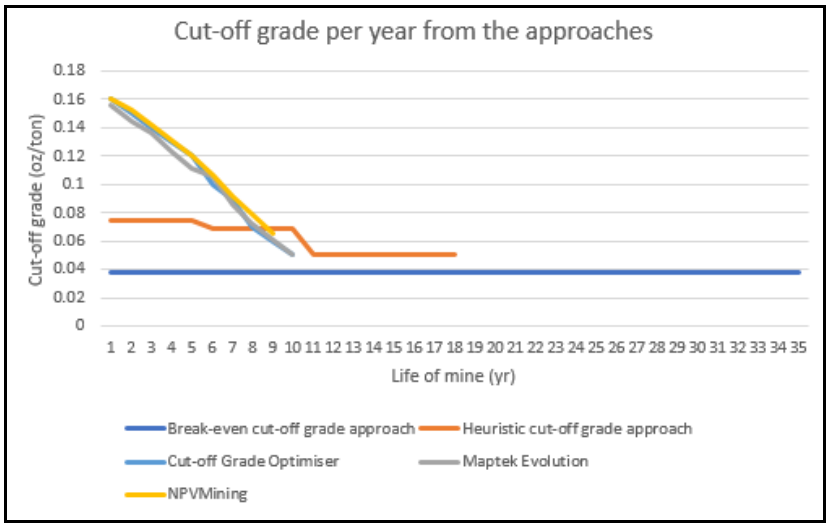


Figure 5.1: A comparison of cut-off grade per year from the cut-off grade models

The cut-off grade models presented in this chapter are useful to mine planners when developing a cut-off grade policy. Four cut-off grade models, both classical and modern, are compared to illustrate the significance and impacts of optimising cut-off grades based on NPV and LOM identified during the mine planning process. The three deterministic cut-off grade models (break-even cut-off grade approach, traditional heuristic cut-off grade approach and Cut-off Grade Optimiser) produce sub-optimal results compared to the other two models (Maptek Evolution® and *NPVMining*) which have better value creation through improved NPVs.

From this comparison, *NPVMining* generates better results than the other four approaches. It generated economically higher NPV over the other methods by: 185% better than break-even cut-off grade approach; 35% better than heuristic cut-off grade; 7% better than Cut-Off Grade Optimiser and 13% better than Maptek Evolution®. In addition, the mine life is lower (9.12 years) compared to the other methods indicating faster recovery of capital invested.

Table 5.6: Comparison of the mine life, cash flow and NPV from the cut-off grade models

	Deterministic cut-off grade approaches			*Stochastic cut-off grade approaches*	
	Break-even cut-off grade model	***Traditional heuristic cut-off grade model***	***Cut-off Grade Optimiser***	***Maptek Evolution®***	***NPVMining***
LOM (years)	35	18	10	10	9.12
Cash flow (US$ million)	863	885.60	825.44	760.10	887.09
NPV (US$ million)	163.42	347.08	435.52	413.84	467.17

5.3 Risk and decision analysis

There are several conventional approaches used in the mining industry for assessing the risk of mineral projects, such as sensitivity analysis and probabilistic analysis. This study uses a risk analysis tool, @Risk, from Palisade Corporation to run Monte Carlo simulations of NPV for many combinations of the input distributions as shown in Figure 5.2.

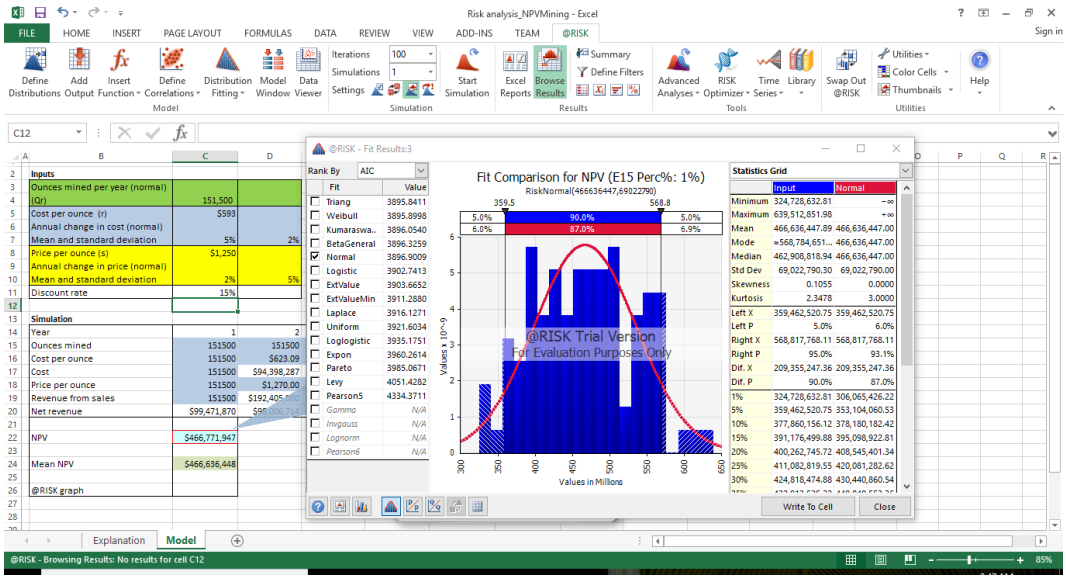


Figure 5.2: Probabilistic analysis of *NPVMining* results using @Risk software

Monte Carlo simulation is a method used to understand the impact of risk and uncertainty in financial models. It runs simulations of various stochastic variables through risk analysis. The outcome is then analysed and interpreted to validate the efficiency of *NPVMining*.

5.3.1 Probabilistic analysis

Key inputs, such as costs, metal prices, recovery and grade-tonnage distribution, are characterised as probability distributions. Monte Carlo simulation is used to simulate the NPV under a range of these key inputs sampled from the primary distributions. From this exercise, the NPV is distributed and appropriate statistics can be collected to characterise the quantities such as skewness, mean, minimum, maximum, standard deviations and average. Using the @Risk software package, this thesis quantified the uncertainty of the NPV of the project against the critical input variables (metal price and grade-tonnage distribution) as probability distributions. The probability density function (pdf) of the normal distribution in Figure 5.3 shows a continuous distribution of the NPV in relation to the input variables.

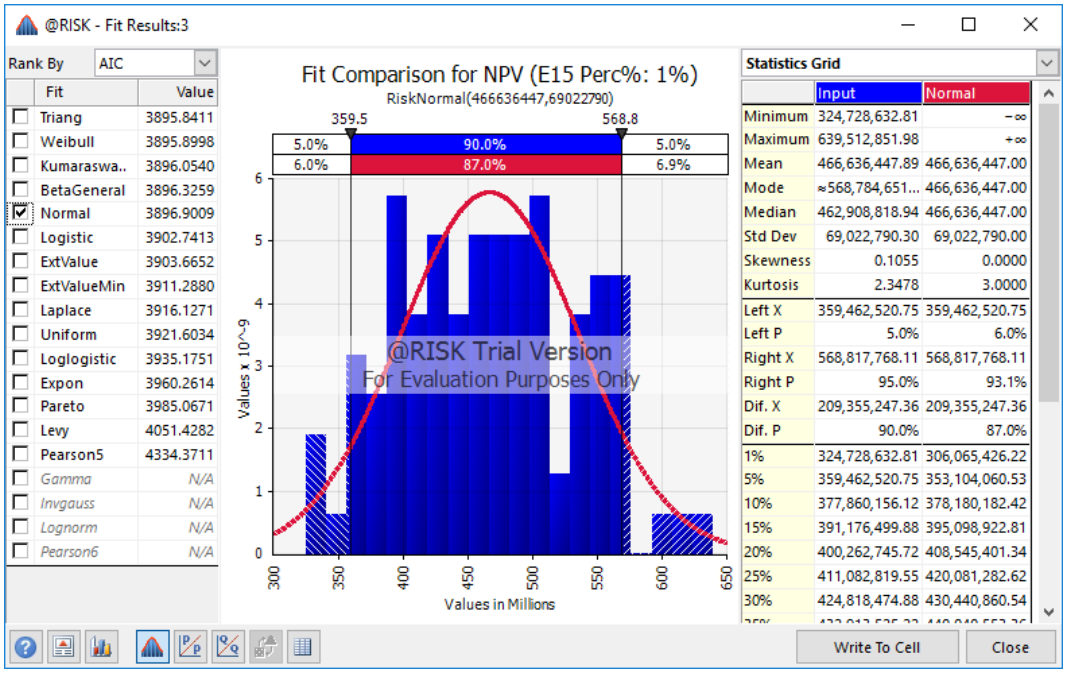


Figure 5.3: Normal probability distribution of NPV against input parameters

The mean or the expected value is defined as the weighted average of the possible values which is relatively the same with the NPV of the project as shown in Table 5.7. The values under the bell curve that range between the minimum and maximum values have a standard deviation of 69,022,790.30. The coefficient of variation (CV=standard deviation/mean) is 0.148 indicating that the distribution has low variance. Skewness is a measure of the extent to which a probability density function leans to one side of its mean as shown in Figure 5.3. The skewness of the values is 0.1055 meaning that the values are positively skewed or slightly skewed right in the graph. The skewness is low (near zero) indicating that the values range around the mean.

Table 5.7: Probability distribution of the NPV as reported using @Risk

Statistic measurement	*Value*
Minimum	324,728,632.81
Mean	466,636,447.89
Maximum	639,512,851.98
Skewness	0.11
Standard deviation	69,022,790.30

5.3.2 Sensitivity analysis

Sensitivity analysis involves varying one or more parameters to understand what effect it has on the NPV. The optimal NPV generated is evaluated with respect to the economic and geological parameters. The parameters are selected randomly in the spider graph tab, and then analysis is done to determine the most critical parameters. Variation can be observed at in both a pessimistic way to check if the project is at risk or optimistically maintaining the optimal input parameters in the cut-off grade policy. The critical parameters considered here include metal price of gold and the multiple realisations of the grade-tonnage distribution.

After testing the variables in the range of -15% to + 15% as shown in Figure 5.4, the results indicated that price of gold and unit cost of refining/selling gold had more influence. The sensitivity analysis on the project show that gold price is the most critical parameter which influences the NPV by a great margin. Other parameters which are critical in this project are recovery factor and refining cost of gold. These critical parameters need to be evaluated before the start of mining operations to curb any mishap in the future. This approach is used in the mining industry to assess the maximum loss possible over a target time or period at a given level of confidence. Sensitivity analysis performed in this study indicated that the optimal range of approximately US$460 to US$472 million is a moderately stable solution and can therefore be accepted with confidence.

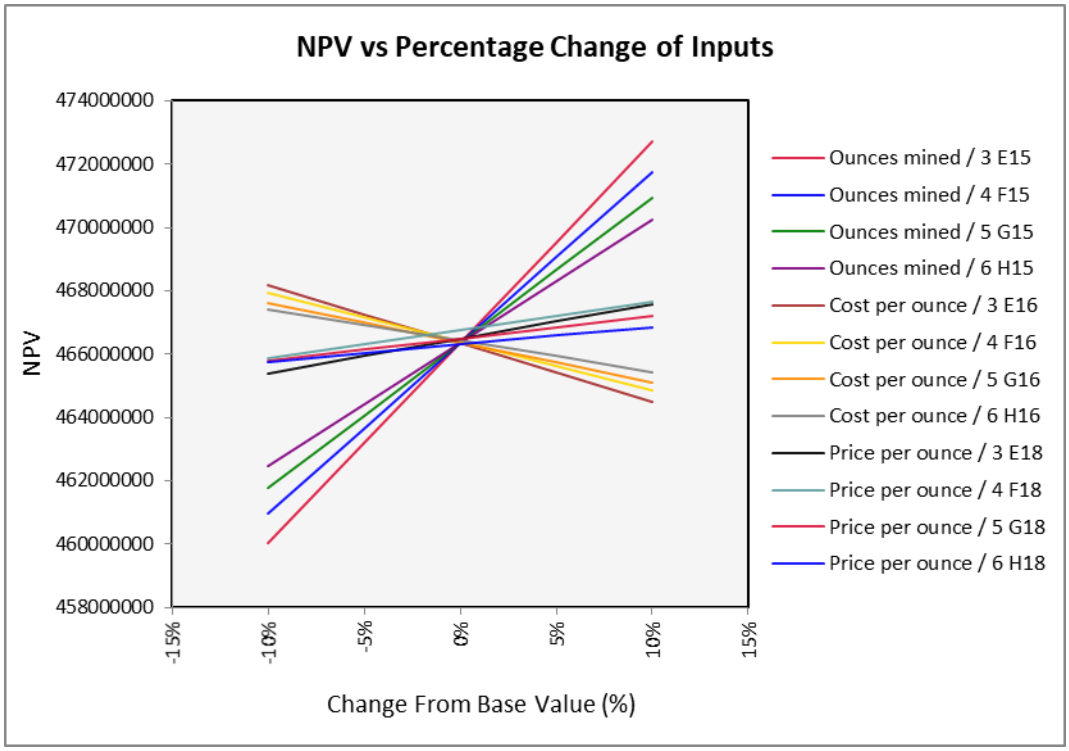


Figure 5.4 Spider graph showing percentage change of input variables on NPV

5.4 Summary of Chapter 5

As shown previously in Table 5.6, two stochastic models (Maptek Evolution®, *NPVMining*) have higher NPVs compared to the deterministic models. The variance of results generated from the two stochastic models is small indicating that the models are appropriate for optimising cut-off grades. The output from such cut-off grade models would be dynamic reflecting the variability experienced in the mining industry. Such stochastic models consider and account for uncertainty in economic and technical parameters in a mining complex.

The most appropriate cut-off grade model is a model that generates the highest NPV in the shortest period possible. Examples of such suitable models applied in the mining industry are generated using mining computer-aided applications such as Geovia Whittle 4X™, Maptek Evolution®, and *NPVMining*. The model should take an orebody model as an input and generate a cut-off grade policy with an optimal mining sequence simultaneously.

Lane's cut-off grade theory has been applied with success in most mining projects because it recognises that to create more value, cut-off grades should be dynamic to account for fluctuating economic and technical conditions over the LOM. However, it has some shortcomings that have necessitated heuristic modifications or extensions to the

theory (Githiria and Musingwini, 2018). With these modifications, it accounts for complex mining scenarios that are encountered in mining, concentrating and refinery stages.

NPVMining modifies Lane's theory to account for the price fluctuations and irregular grade-tonnage distributions. This modification increases the NPV of the project compared to the deterministic Lane's approach. Both Maptek Evolution® and *NPVMining* have high NPVs compared with other existing cut-off grade optimisation approaches due to the stochastic component in the models. However, Maptek Evolution® has less NPV due to its inability to guarantee convergence in the selection of the optimal cut-off grades hence generating a sub-optimal solution.

Results generated from *NPVMining* are validated by comparing with other existing cut-off grade optimisation approaches such as Maptek Evolution® and Cut-off Grade Optimiser (Githiria *et al.*, 2016). Later, risk and decision analyses are done to assess the risk profile of the project based on the output generated from the cut-off grade models. The results generated from *NPVMining* reflects a positive outcome as shown in the probabilistic and sensitivity analysis. The results follow a normal distribution where all the key statistics characterise the NPV against the input variables. The next chapter (Chapter 6) presents the conclusions and recommendations based on the findings from Chapter 5.

6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

There are two major insights that can be drawn from this study. Firstly, all factors (economic, technical, legal, environmental, social and political) must be considered when calculating cut-off grade. Secondly, when evaluation time horizons are longer, cut-off grade values are lower leading to lower NPVs. In general, an optimised LOM schedule tends to have high early grades which eventually decline to the marginal cut-off grade at the end of the LOM. This behaviour is enabled partly by raising the early cut-off grade by present high-grade sources of ore within the mining system hence maximising the overall schedule value (as per Lane's cut-off grade approach).

The developed model (*NPVMining*) extended Lane's cut-off grade theory. The choice of Lane's theory was based on its general acceptance for cut-off grade calculation. The theory has two major concepts in its calculation that help to achieve the optimal cut-off grade. It incorporates the concept of opportunity cost by deferring the costs incurred in future production to immediately treat additional material as ore. Additionally, it accounts for the grade-tonnage curve distribution and production capacities in its calculation. This concept ensures that the cut-off grade generated has accounted for the geological uncertainty and balances the three main stages of any mine operation (mine, mill and refinery). These two features of the theory show how important and feasible this method is in calculating cut-off grades.

The role of a cut-off grade as a balancing parameter is understood but not widely applied in the mining industry as a balancing parameter. The main reason is that cut-off grade is regarded only as a strategic parameter. It does not reflect in the daily mine operations where mining variables change over time, instead plans are worked on including or excluding uncertain amount of ore until a balance is obtained. This trial and error or manual adjustment creates losses in the mining operation. Optimisation of cut-off grade has both the strategic and tactical elements that balance all mining stages in both the short-term and long-term mine plans.

6.2 Research contributions

This thesis developed a stochastic model that optimises cut-off grade when price and grade-tonnage curves are varied concurrently. The developed stochastic model applies the variability in the mining parameters into its algorithm to generate optimal solutions. The main objective of developing the algorithm is to incorporate uncertainty in Lane's cut-off grade theory. The model generates several solutions that can be projected and analysed by the user. Metal price and geological data can be manipulated simultaneously

on the application to generate solutions that can be analysed to ascertain the risk involved when making major mining investments. When undertaking long-term mine planning, the proposed stochastic approach can provide a tool for evaluating alternative policies as part of feasibility studies. Comparison of different cut-off grade policies generated by the approach facilitates a risk-quantified decision-making process when investing in major mining projects.

NPVMining has a GUI that makes it easy to use and displays results in a user-friendly way. It gives the user the choice to manipulate the data depending on the study being undertaken or any other factor affecting the specific mine. It has options of using either a deterministic or stochastic approach when calculating the cut-off grade. The user can use either a break-even cut-off grade method or Lane's limiting cut-off grade method. The calculation on the application changes depending on the cut-off grade method selected on the user interface. Chapters 4 and 5 described different ways of incorporating it into the mine planning of any given mining operation by either varying price and/or geological data.

In summary, this study presents the following major contributions:

1. Development of a new algorithm that optimises cut-off grade where price and grade-tonnage distribution are stochastic. It is an extension of Lane's approach from a deterministic to a stochastic approach**.** It optimises cut-off grades under the ever-changing operating conditions in a mining complex that comprises of three stages (mine, ore processing stream and refinery/market) under uncertain geological and economic variables.
2. Development of user-friendly software that generates cut-off grade policies based on a stochastic cut-off grade optimisation algorithm that is coded using C++ programming language on Visual Studio 2017 Integrated Development Environment (IDE). This application has an interface that enables mining engineers to calculate optimum cut-off grades with ease.
3. Development of a mathematical model using an OR technique that generates several possible solutions in a single run. The capability of the application to generate as many solutions as possible adds an advantage to it when undertaking a feasibility study.
4. The developed computer-aided application has two cut-off grade calculation options. There is an extension on the code that accommodates the break-even cut-off grade method.

Application and implementation of the developed stochastic model was undertaken in Chapters 4 and 5 on a real gold mine deposit to ascertain its benefits in any operational mine. It is benchmarked based on the results generated from *NPVMining* and other existing models. Comparison and risk analysis were undertaken to ascertain and validate the results from *NPVMining*.

6.3 Research limitations

This thesis assumed that the probability distribution of the parameters (gold price and grade-tonnage) is uniform when developing the computer-aided application (*NPVMining*). The other parameters were assumed to be deterministic, yet they are stochastic in nature. The reason for using these deterministic input parameters is that the optimisation problem would be large, making it impossible to code the algorithm hence generating sub-optimal solutions.

A limitation of the optimum solutions generated is also contributed by applying a random number generator developed in the code for generating values within a specified range. This is used to create variability in the algorithm. The software generates random values for gold price which requires the range of values to be specified. There is no pattern as compared to where mathematical functions of gold price against time or any other relevant parameter are used to achieve the variability.

6.4 Recommendations

Lane's cut-off grade theory is limited to a single processing stream and a single commodity and cannot be applied in operations with multiple processing streams and multiple commodities. The algorithm does not consider many aspects affecting the overall mining system such as the economic impact of stockpiling and blending requirements in ore processing. It also does not consider uncertainty in the operating costs, commodity price and grade-tonnage distribution. Therefore, future cut-off grade optimisation models should focus on global strategies under dynamic technical and economic situations.

This study proposed an algorithm that modifies Lane's cut-off grade theory by introducing variability in price and grade-tonnage distribution over time. However, the approach assumes costs and recovery to be deterministic, thus, the need to develop a stochastic approach that encompasses all stochastic parameters in its computation. The algorithm should also be tested and applied in different mining scenarios such as underground mining and with more parameters varying simultaneously. Due to the size of such an optimisation problem, the approach should employ appropriate optimisation techniques that would generate an optimal solution to the problem in the shortest time possible.

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8 APPENDICES

8.1 Appendix 1: Summary of mathematical programming techniques

The summary of mathematical programming techniques used in cut-off grade optimisation shows the different applications developed over the years. Some of the techniques used are linear programming techniques (mixed-integer programming, integer programming, binary integer programming), dynamic programming, greedy method and heuristic methods such as local search, Tabu search, simulated annealing, and genetic algorithms.

Table 8.1: Chronological summary of deterministic cut-off grade optimisation techniques used in the mining industry

Researcher	*Year*	*Solution method*	*Remarks*
Henning	1963	Break-even cut-off grade model	Cut-off grade is higher during earlier years and lower during later years, reducing to the break-even value that corresponds to the objective of maximising the difference between revenue and cost.
Lane	(1964, 1988)	Graphical method	Optimises cut-off grade based on the economic, operational and geological conditions involved in a mining operation. Determines the optimum cut-off grade among the limiting economic and balancing cut-off grades that maximise the NPV of the operation.
Taylor	1972	Statistical method	Calculates balancing cut-off grades by using statistical parameters to describe the grade distribution.
Taylor	1985	Statistical method	Calculates cut-off grade using a system that creates stockpiles.
Dagdelen	1992	Break-even cut-off grade model	Strategy used in this approach applied the value of break-even cut-off grade along the entire mine life of an open pit mining project.
Dagdelen	(1992, 1993)	Analytical method based on Lane's theory	Formulated an algorithm based on Lane's cut-off grade theory to maximise NPV. Finds balancing cut-off grades using an analytical method instead of using the graphical method which simplified the problem.
Whittle & Wharton	1995	Linear programming combined with search algorithm	Simultaneously handles mining problems involving some or all of the complications such as stockpiling of mined material, blending, multiple minerals and others. However, the processing or computing time increases with an increase in the number and size of problem.

Researcher	*Year*	*Solution method*	*Remarks*
King	(1999, 2001, 2011)	Analytical method based on Lane's theory (Linear programming)	Modified Lane's approach by incorporating variations in ore type throughput for multi-metal mineralisation and later had a variation of cut-off grade policies considering operating and administrative costs.
Asad	2002	Analytical method (Linear programming)	Developed an algorithm based on Lane's cut-off grade theory to maximise NPV.
Minnitt	2004	Graphical method using Linear programming	Applied Lane's theory to show how cut-off grade can be determined in a small Wits-type gold mining operation to maximise the NPV.
Asad	2005	Analytical method using Linear programming	Developed a cut-off grade optimisation algorithm with a stockpiling option for open pit mining operations of two economic minerals.
Dagdelen and Kawahata	2008	Mixed-integer programming	Optimises the mine sequence, process flows, and cut-off grades simultaneously for a mining complex.
Osanloo *et al.*	2008b	Analytical method based on Linear programming	Incorporated environmental issues into cut-off grade optimisation. The environmental-oriented model incorporated costs such as waste disposal operating costs, treating operating costs, and tailings costs.
Gholamnejad	(2008, 2009)	Analytical method based on Linear programming	This approach argued there should be a trade-off between increasing the average mill grade and increasing the rehabilitation cost. The approach incorporated rehabilitation cost in Lane's model by factoring the cost on the quantity of material mined and processed.
Hall	2014	Break-even cut-off grade model	Applied the approach to generate break-even cut-off grade for a mining project.
Githiria *et al.*	2016	Analytical method based on Linear programming	Optimises cut-off grade under deterministic variables using Lane's cut-off grade theory.

Most of these studies did not consider variations in the economic and mining variables that affect daily mining operations implying that NPVs produced for the mining projects are not realistically optimal. When calculating cut-off grades, the reality in the mining industry should be reflected appropriately and as close as possible. This is done by applying dynamic mining variables in the calculation of cut-off grades.

Table 8.2: Chronological summary of uncertainty-based cut-off grade optimisation techniques used in the mining industry

Researcher	***Year***	***Solution method***	***Remarks***
Dowd	1976	Dynamic programming (Stochastic programming)	It allowed for the forecasting of future market conditions in terms of probability. It looked at the interaction of cut-off grades and fluctuating prices and illustrated the effects of the variations through examples. However, the model did not account for the change in costs and the ability of the operation to respond to these changes.
Krautkraemer	1988	Stochastic programming	The approach explained the effect of a stochastic price in the selection of the cut-off grade depending upon the rate of metal price change relative to the discount rate. The research paper concluded that fluctuating prices are important elements of the cut-off grade selection.
Cetin and Dowd	2002	Genetic algorithms	Determine the optimal sequence of cut-off grades for multi-mineral deposits that contain up to three valuable minerals by means of genetic algorithms.
Ataei and Osanloo	2004	Combination of Genetic Algorithm and the Grid Search Method	Used to find the optimum cut-off grades of multiple metal deposits that will maximise the NPV. The solution is determined by using the grid search method and genetic algorithm to determine optimum cut-off grades.
Asad	2007	Linear programming	Optimises cut-off grade for an open pit mining operation through an NPV algorithm considering metal price and cost escalation. This approach accounted for change in economic conditions only, ignoring geological uncertainty.
Cetin and Dowd	2011	Dynamic programming	Calculates optimum cut-off grades on a multi-mineral deposit using dynamic programming.
Li, Yang and Lu	2012	Stochastic programming	Optimised cut-off grade in open-pit mining but considered commodity price as the only dynamic variable.

Researcher	*Year*	*Solution method*	*Remarks*
Cetin and Dowd	2013	Genetic algorithms	Applied grid search method in cut-off grade optimization for multi-mineral deposits. The study describes a software implementation of the method that can optimise cut-off grade for mineral deposits that contain up to three economic minerals.
Asad and Dimitrakopoulos	2013	Heuristic approach based on stochastic programming	Determines optimal cut-off grade policy based on a stochastic framework that accounts for uncertainty in the supply of ore to the multiple ore processing streams. It considers multiple processing streams that are individually summed up to form an input to the refinery/market. It covers open pit mining operations with multiple ore processing facilities, and accounts for geological uncertainty. It is limited to a single mine or material source and refinery/market.
Thompson and Barr	2014	Stochastic programming	Considered commodity price as the only dynamic variable
Myburgh *et al.*	2014	Genetic algorithm: Evolutionary algorithm (Combined with both local search and linear programming)	Manages variation of cut-off grades as well as permutations of the extraction sequence. It manages a linear programming algorithm which is responsible for determining the optimal material flow through multiple processing streams as well as management of the stockpiles in the mine. A local search technique is then called upon to provide the best schedule.
Cetin and Dowd	2016	Genetic algorithms, grid search method and dynamic programming	Calculates optimal cut-off grade schedules for deposits with up to three constituent minerals using the three cut-off grade approaches.

8.2 Appendix 2: Monte Carlo integration as applied in RStudio

A Monte Carlo estimate of any problem can be obtained using the basic equation represented in Equation 8.1.

$$E_f[h(X)] = \int_x h(x)f(x)dx \tag{8.1}$$

Where:

x= rfunction (J),

J=10^8, represents the whole sample,

rfunction samples x from *f(x)*,

hs = h (x),

(estimate) est = mean (hs),

provided h^2 has finite expectation under *f*.

Monte Carlo code used in RStudio to simulate a normal grade-tonnage distribution used in this study is described in Section 8.2.1.

8.2.1 Monte Carlo integration code

```
#Monte Carlo integration
h=function(x){(cos(50*x) +sin(20*x)) ^2}
par(mar=c (2,2,2,1), mfrow=c(2,1))
curve(h,xlab="Function",ylab="",lwd=2)
integrate(h,0,1)
x=h (runif (10^4))
estint=cumsum(x)/ (1:10^4)
esterr=sqrt(cumsum((x-estint) ^2))/ (1:10^4)
plot (estint, xlab="Mean and error range",type="l",lwd=2,
    ylim=mean(x)+20*c(-esterr[10^4], esterr[10^4]),ylab="")
lines(estint+2*esterr,col="gold",lwd=2)
lines(estint-2*esterr,col="gold",lwd=2)
mean(estint[2000:10000])
#Normal quantiles
x=rnorm(10^8) #whole sample
bound=qnorm(c(.7,.07257,.06319,.05591,.04598,.04277,.03465,.02438,.02307,.01747,.0164,.01485,.
01227,.03598,.09574))
res=matrix(0, ncol=8,nrow=10)
for (i in 2:8) #lengthy loop!!
  for (j in 1:8)
    res[i-1,j]=mean(x[1:10^i]<bound[j])
matrix(as.numeric(format(res,digi=4)),ncol=8)
x[1]<-rnorm(1,0,1)
```

```
for (j in 2:100)
{
  x[j]<-0.9*x[j-1]
}
plot(x)
```

8.3 Appendix 3: *NPVMining* user guide

This section explains the step-by-step guideline used in executing the *NPVMining* application. For the application to generate an optimal solution, the following process is undertaken before the cut-off grade calculation process:

i. Setting the price categories.

ii. Determining other economic and operational parameters used in the cut-off grade policies.

iii. Generating multiple grade-tonnage curves.

The input data is entered in the user interface as described in Section 8.3.1 to 8.3.2. The user can either choose imperial or metric unit system on the interface depending on the units of the variables.

8.3.1 Data input: Price criteria

Metal price is varied over a selected realistic range depending on the factors affecting current market conditions. The ranges are categorised in several sub-categories as indicated in the step-by-step procedure below:

1. First, type a category name in the category dropdown as shown in Figure 8.1 then press enter key. The dropdown will be cleared but on clicking the dropdown arrow as shown in the Figure 8.1 it will display the entry.

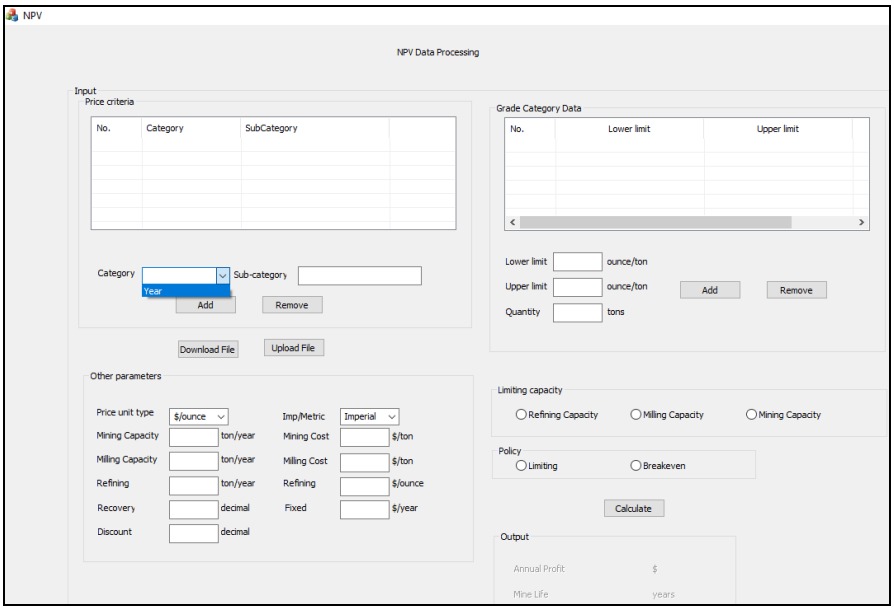


Figure 8.1: Entering price category

2. Type a sub-category name in the sub-category edit box and press enter key. The entry should appear at on the price category list as shown in Figure 8.2.

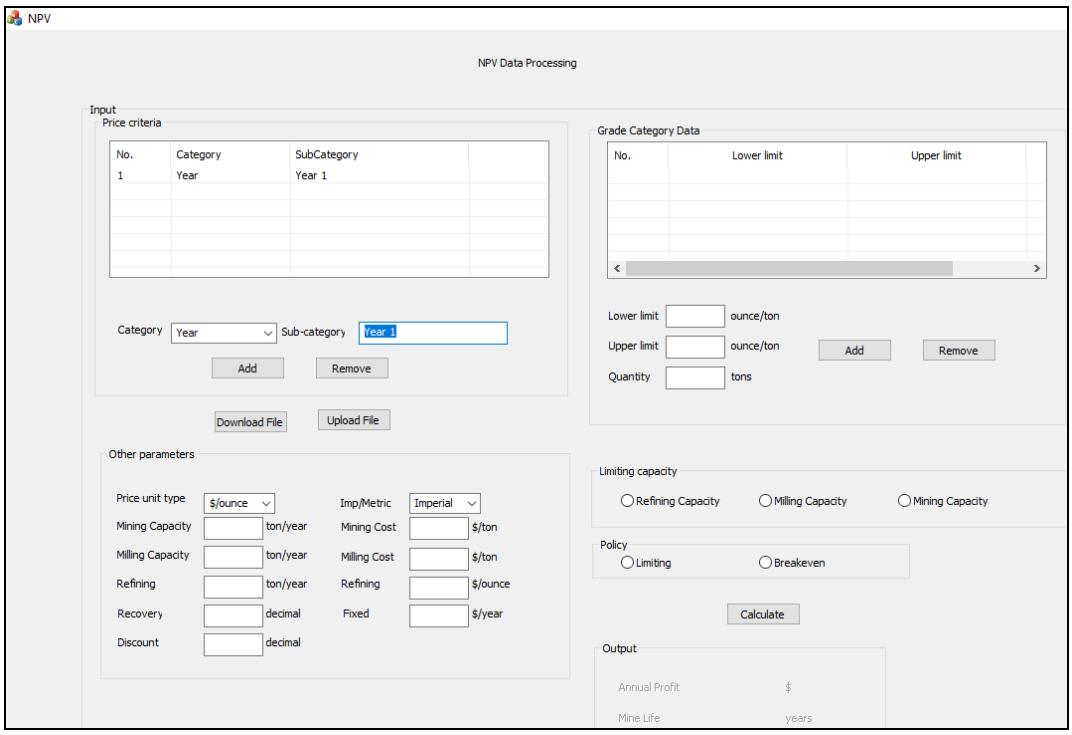


Figure 8.2: Entering price sub-category

3. To enter new entries of the same category just type a new sub-category name while the category name is selected in the category dropdown and press enter key. The

new items will be listed on the price category list as shown in Figure 8.3. To add a new category, repeat step (1). Type the new category name and press enter key. It should be listed in the dropdown as shown in the figure. Then repeat the process of adding new entries using the new category name and the new entries will be listed after pressing the enter key.

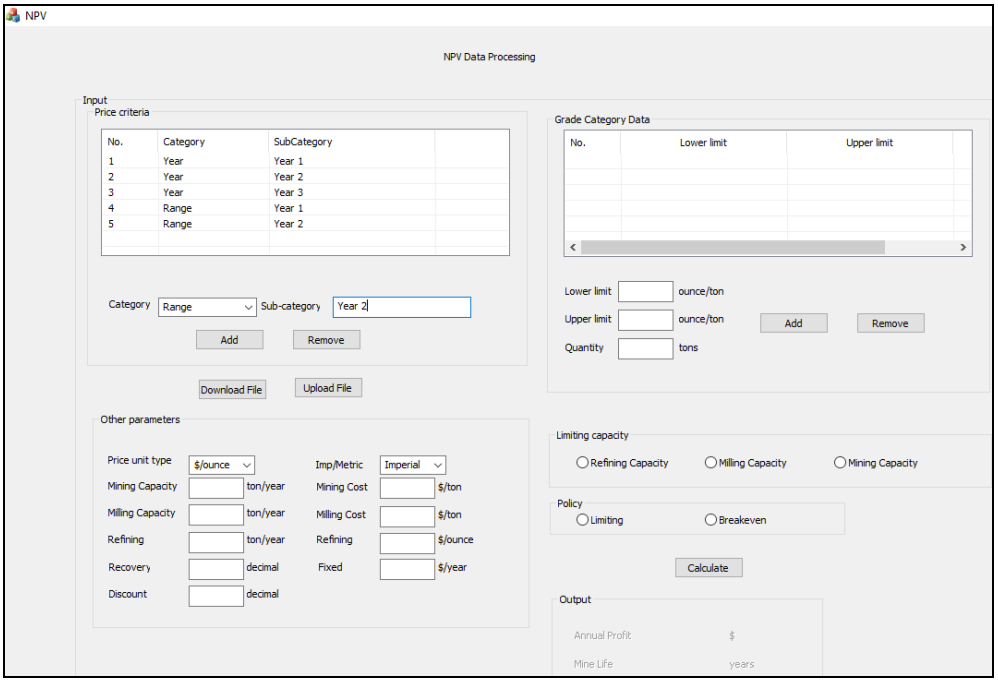


Figure 8.3: Entire price category list

4. Click on the Download file button and a Microsoft excel file will be generated with two spreadsheets as shown in Figure 8.4. The second spreadsheet (price data) has a table that the user can use to enter the price information (value column highlighted).

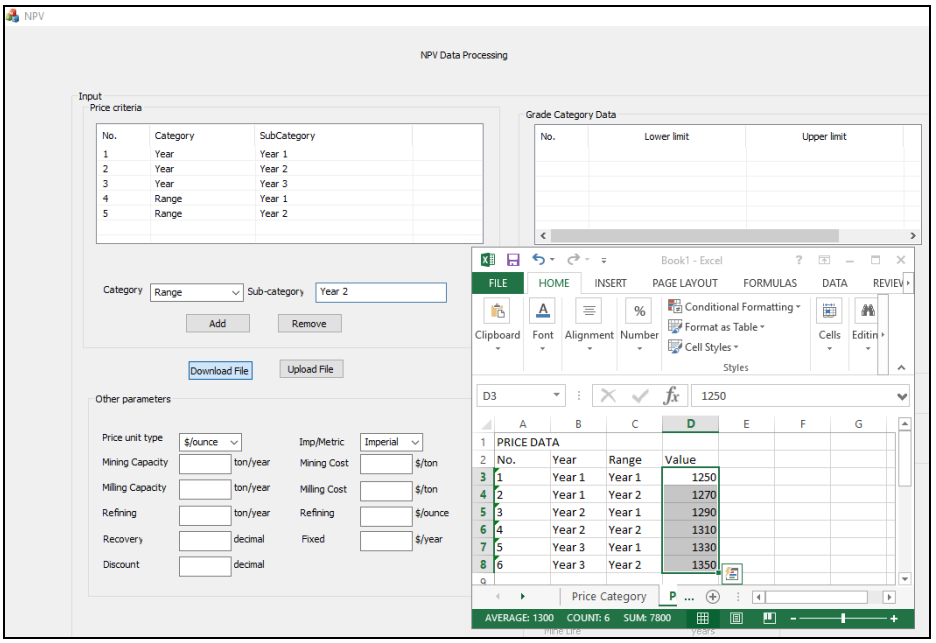


Figure 8.4: Downloaded Microsoft Excel spreadsheet for price value entry

5. Save the Microsoft Excel spreadsheet and close it as shown in Figure 8.5.

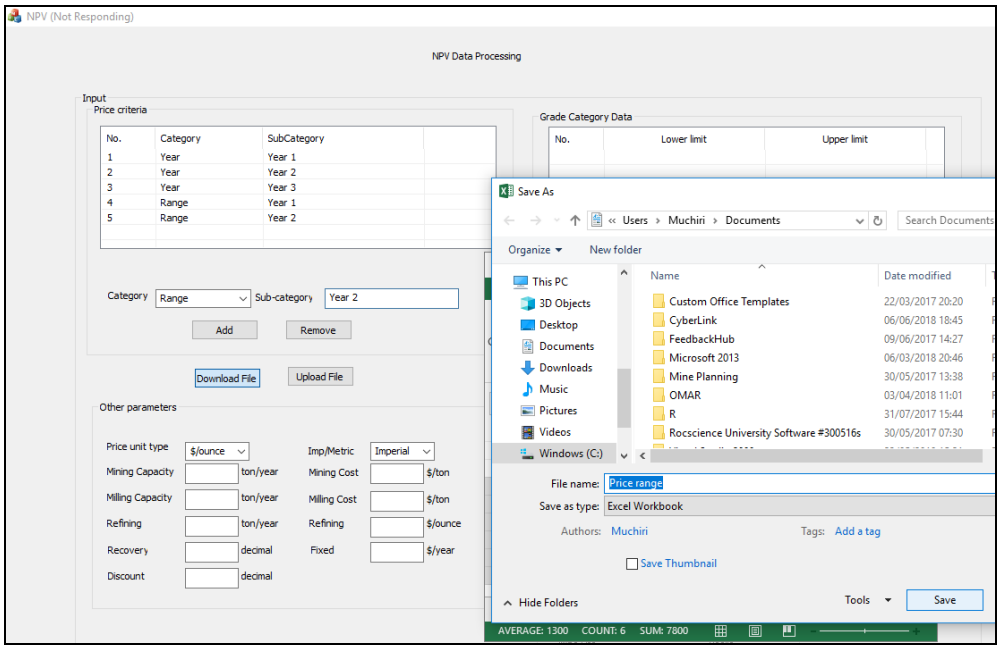


Figure 8.5: Saving the Microsoft Excel spreadsheet with price values

6. Click on the Upload file button shown in Figurer 8.6 to copy the values into the application (*NPVMining*). Select the saved Microsoft Excel spreadsheet file and click open as shown in the figure.

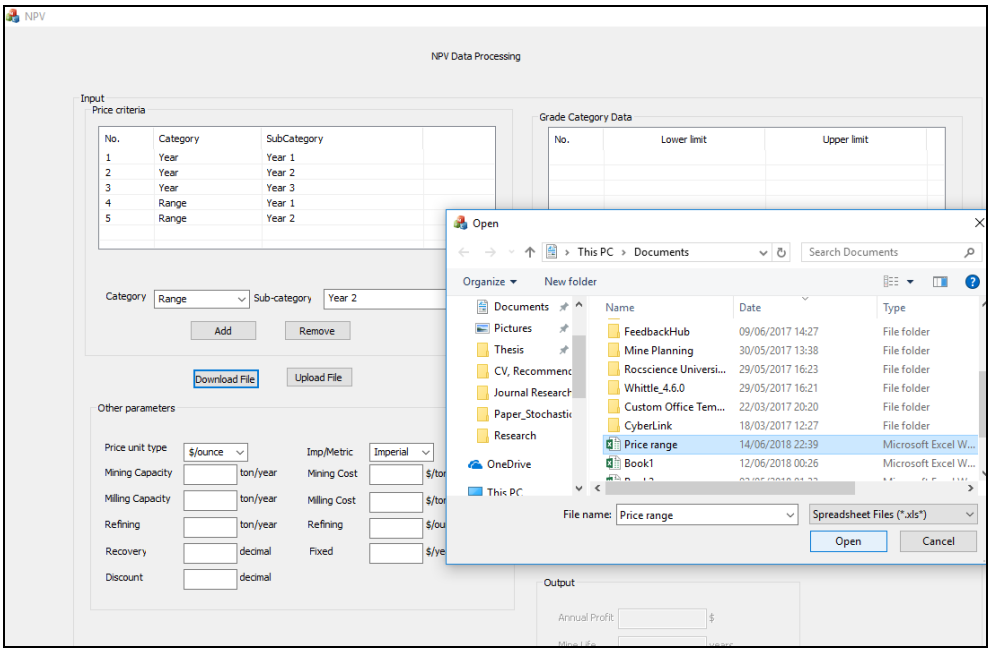


Figure 8.6: Uploading the saved Microsoft Excel spreadsheet with price values

7. A popup window will display the array of values that have been read as shown in Figure 8.7. This array of values will match with ones saved on the Microsoft Excel spreadsheet. Confirm the values and close the popup window.

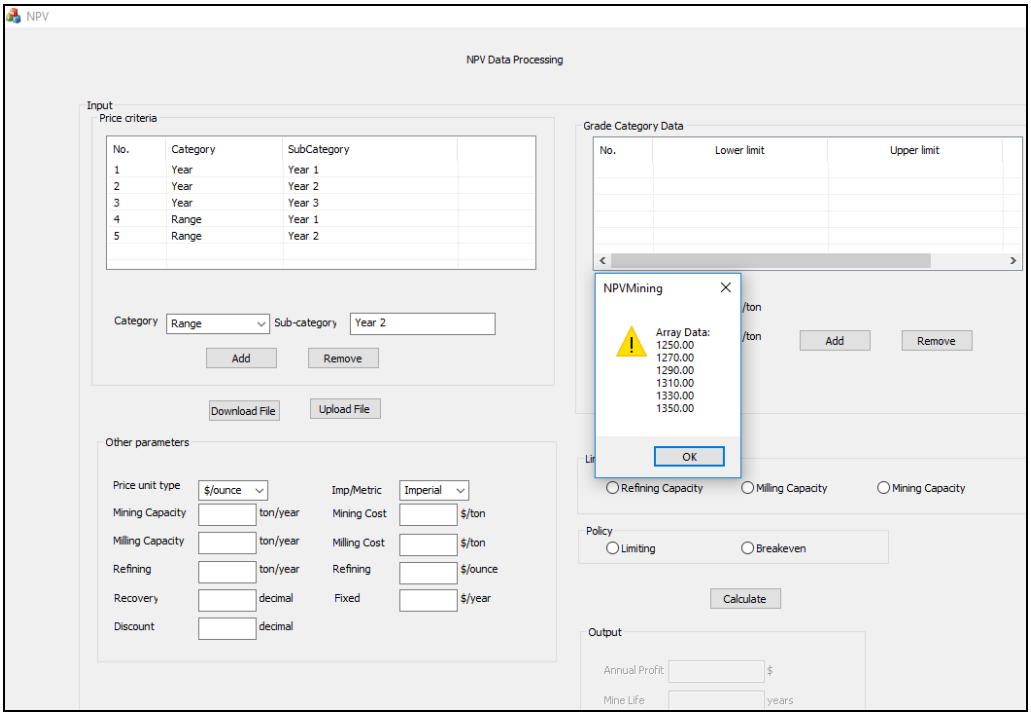


Figure 8.7: Popup window confirming the array of price values read by the application

8.3.2 Data input: Other economic and operational parameters and the grade-tonnage category

1. Type the rest of the economic and operational parameters that will be used in the calculation as shown in Figure 8.8.

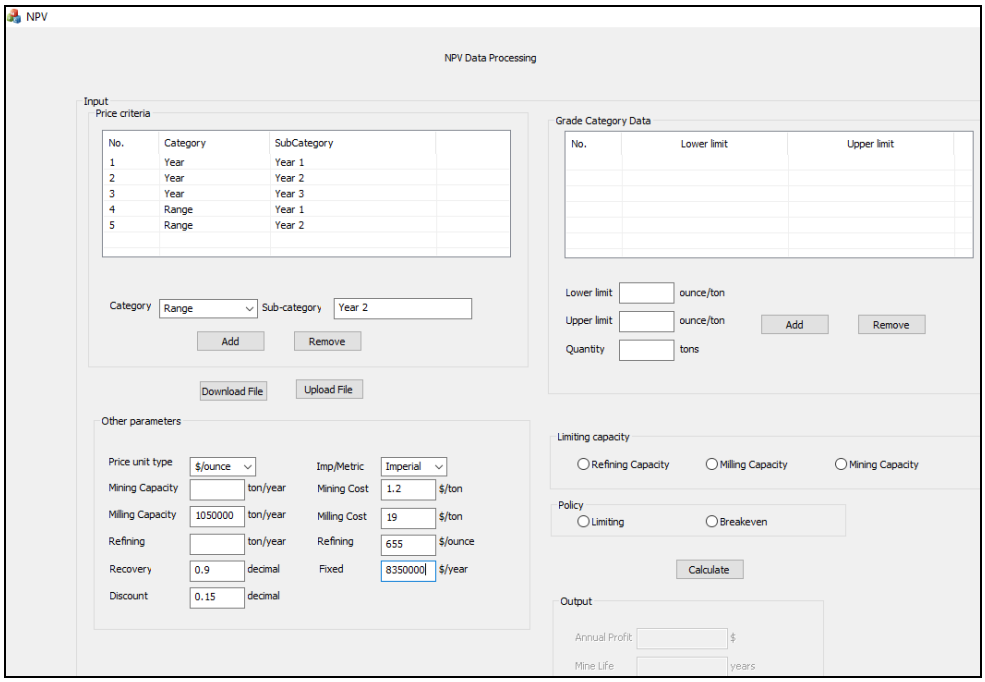


Figure 8.8: Other economic and operational parameters

2. For grade category, type the lower limit, upper limit and quantity dialog boxes and click Add button. The entry should appear as shown in Figure 8.9.

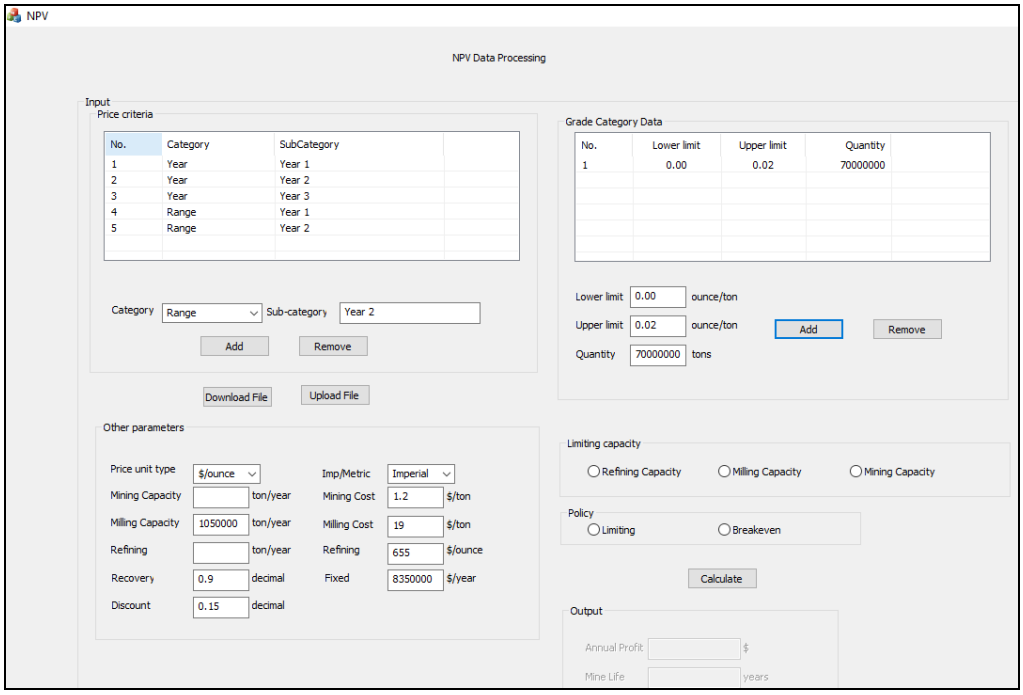


Figure 8.9: Grade category showing lower limit, upper limit and quantity with the grade-tonnage distribution entry

3. Repeat the above step to add subsequent entries and the entries will be appended to the list as shown in Figure 8.10.

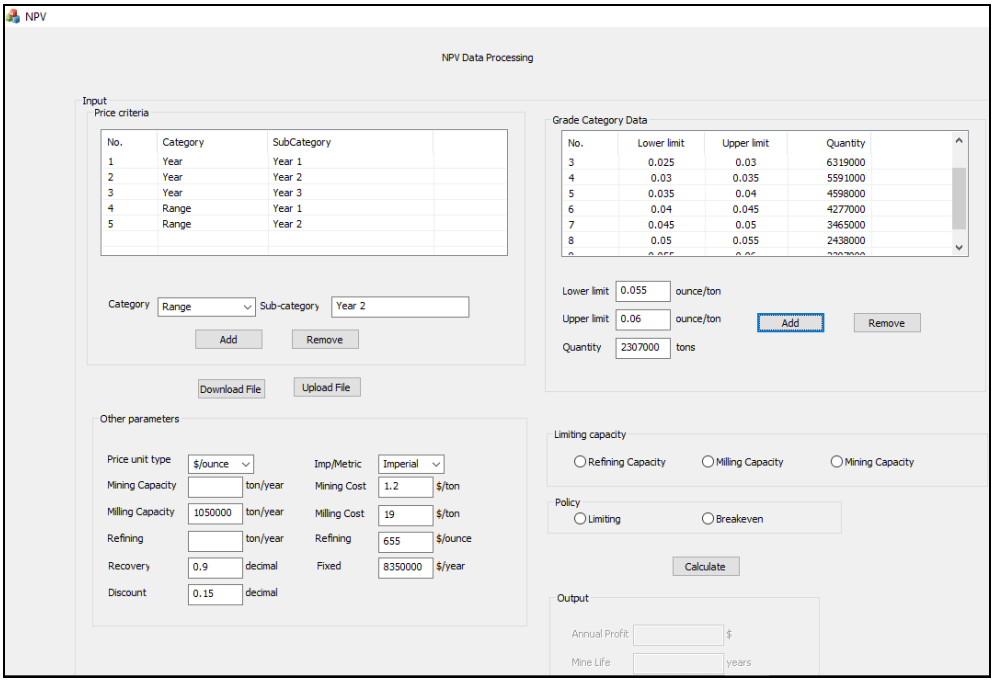


Figure 8.10: Appended list of grade-tonnage data

4. To remove an entry, click on it to highlight it then click Remove button (Figure 8.11).

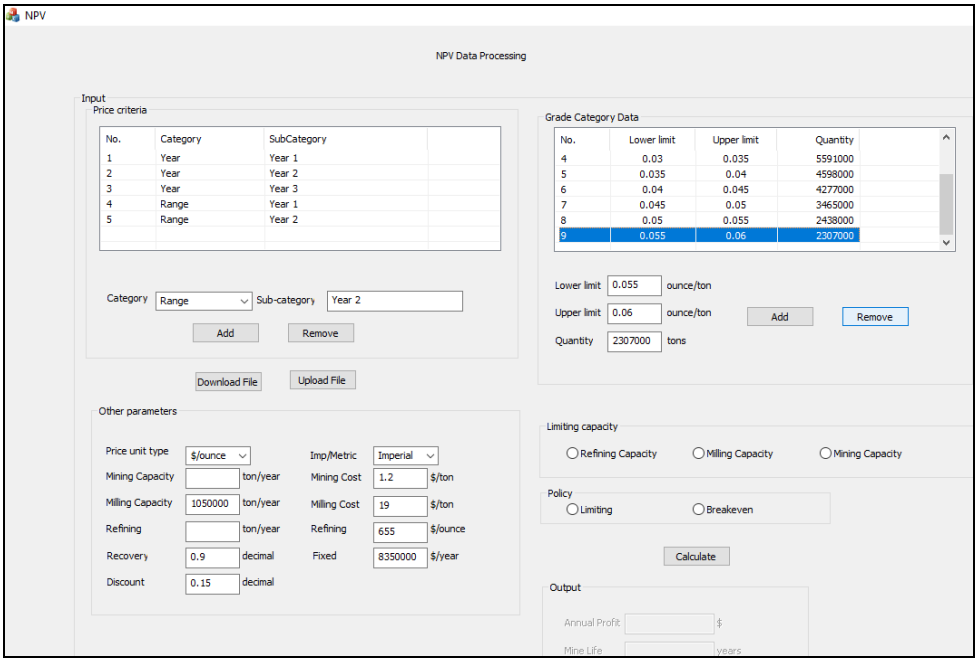


Figure 8.11: Removing a grade-tonnage entry

5. Click Yes button to delete the entry as shown in Figure 8.12.

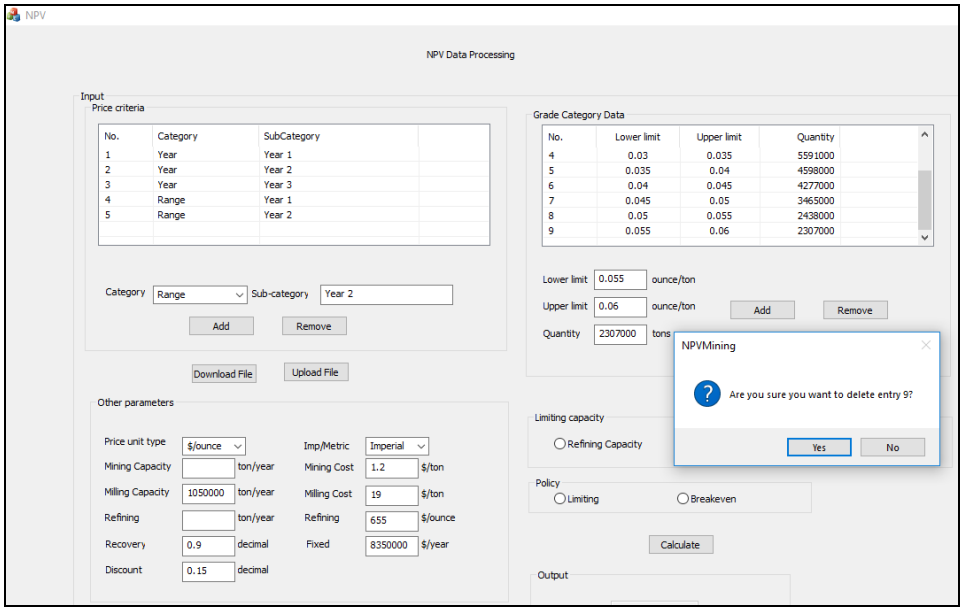


Figure 8.12: Deleting the grade-tonnage entry

8.3.3 Selection of limiting capacity and cut-off grade calculation method

In this section, the user selects the capacity (refining, milling or mining capacity) that is limiting the entire operation and the calculation method used in determining NPV by clicking on the respective radio buttons as shown in Figure 8.13. Then click on the calculate button to determine the NPV.

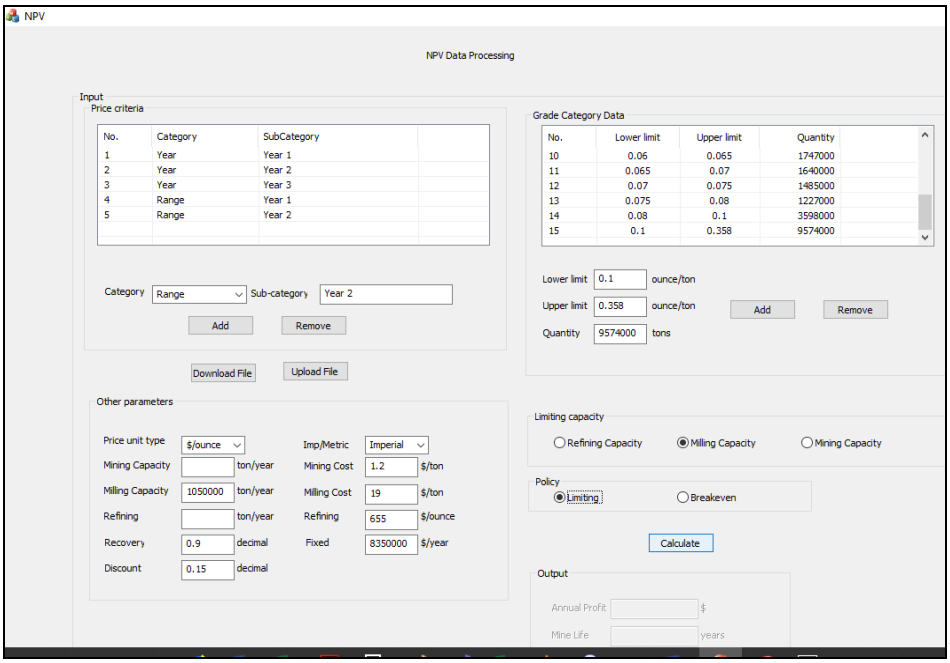


Figure 8.13: Limiting capacity and method used to calculate the optimal solution

8.3.4 Output

The output should generate a Microsoft Excel file as shown in Figure 8.14 indicating the price selected, mine life, annual profit and NPV generated.

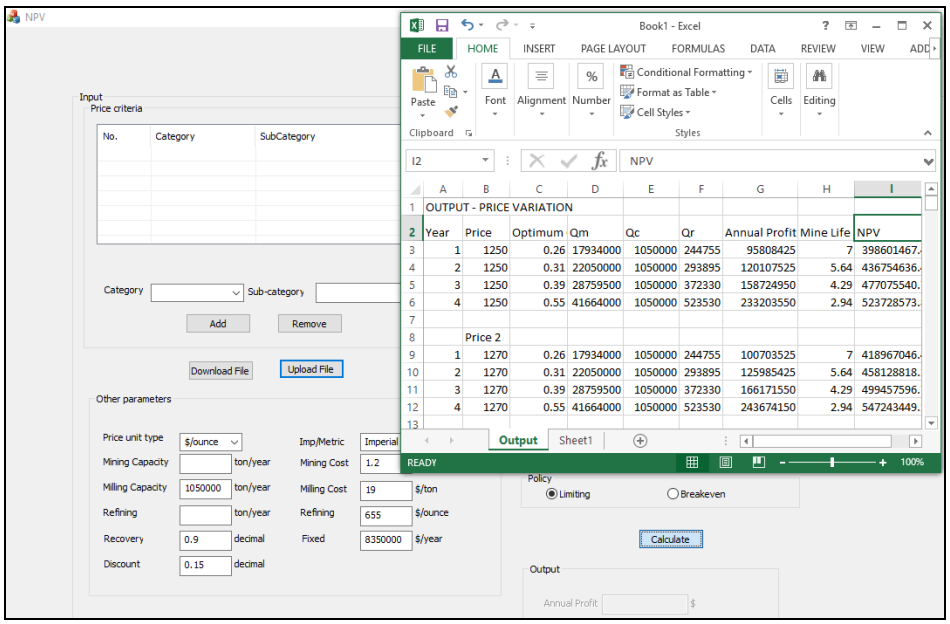


Figure 8.14: Output based on price variation

Lastly, save and close the Microsoft Excel file to calculate again using a different grade category.

8.3.5 Output based on price variation generated from the NPVMining for all the grade-tonnage curves (GTC1-GTC6)

The output shown in Table 8.3−8.8 are generated based on price variation from the developed application (*NPVMining*) for the six grade-tonnage curves (GTC1−GTC6). These multiple grade-tonnage curves are obtained from the McLaughlin gold deposit.

Table 8.3: Output based on price variation for McLaughlin gold deposit (GTC1)

No.	*Price (US$/oz)*	*Annual Profit (US$)*	*Mine Life (yrs.)*	*NPV (US$)*
1	1250	97268594.9	9.12	467172060.5
2	1250	112764301.7	8.12	510115591.8
3	1250	132694553.3	7.12	557595103.8
4	1250	159276622.1	6.12	610379799.0
5	1250	196400184.8	5.12	669073164.2
6	1250	251224126.7	4.12	733384278.4
1	1270	101596694.9	9.12	487959524.3
2	1270	117621601.7	8.12	532088720.0
3	1270	138232253.3	7.12	580865044.7
4	1270	165721522.1	6.12	635077941.8
5	1270	204111384.8	5.12	695342777.9
6	1270	260806426.7	4.12	761357340.8
1	1290	105924794.9	9.12	508746988.1
2	1290	122478901.7	8.12	554061848.3
3	1290	143769953.3	7.12	604134985.5
4	1290	172166422.1	6.12	659776084.7
5	1290	211822584.8	5.12	721612391.6
6	1290	270388726.7	4.12	789330403.2
1	1310	110252894.9	9.12	529534451.9
2	1310	127336201.7	8.12	576034976.5
3	1310	149307653.3	7.12	627404926.4
4	1310	178611322.1	6.12	684474227.6
5	1310	219533784.8	5.12	747882005.3
6	1310	279971026.7	4.12	817303465.6
1	1330	114580994.9	9.12	550321915.7
2	1330	132193501.7	8.12	598008104.8
3	1330	154845353.3	7.12	650674867.2
4	1330	185056222.1	6.12	709172370.4
5	1330	227244984.8	5.12	774151619.0
6	1330	289553326.7	4.12	845276528.0

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1350	118909094.9	9.12	571109379.5
2	1350	137050801.7	8.12	619981233.0
3	1350	160383053.3	7.12	673944808.1
4	1350	191501122.1	6.12	733870513.3
5	1350	234956184.8	5.12	800421232.7
6	1350	299135626.7	4.12	873249590.4

Table 8.4: Output based on price variation for McLaughlin gold deposit (GTC2)

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1250	97268594.9	9.12	467172060.5
2	1250	112764301.7	8.12	510115591.8
3	1250	132694553.3	7.12	557595103.8
4	1250	159276622.1	6.12	610379799.0
5	1250	196412784.8	5.12	669116088.4
6	1250	251249326.7	4.12	733457843.4
1	1270	101596694.9	9.12	487959524.3
2	1270	117621601.7	8.12	532088720.0
3	1270	138232253.3	7.12	580865044.7
4	1270	165721522.1	6.12	635077941.8
5	1270	204123984.8	5.12	695385702.1
6	1270	260831626.7	4.12	761430905.8
1	1290	105924794.9	9.12	508746988.1
2	1290	122478901.7	8.12	554061848.3
3	1290	143769953.3	7.12	604134985.5
4	1290	172166422.1	6.12	659776084.7
5	1290	211835184.8	5.12	721655315.8
6	1290	270413926.7	4.12	789403968.1
1	1310	110252894.9	9.12	529534451.9
2	1310	127336201.7	8.12	576034976.5
3	1310	149307653.3	7.12	627404926.4
4	1310	178611322.1	6.12	684474227.6
5	1310	219546384.8	5.12	747924929.5
6	1310	279996226.7	4.12	817377030.5
1	1330	114580994.9	9.12	550321915.7
2	1330	132193501.7	8.12	598008104.8
3	1330	154845353.3	7.12	650674867.2
4	1330	185056222.1	6.12	709172370.4

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
5	1330	227257584.8	5.12	774194543.2
6	1330	289578526.7	4.12	845350092.9
1	1350	118909094.9	9.12	571109379.5
2	1350	137050801.7	8.12	619981233.0
3	1350	160383053.3	7.12	673944808.1
4	1350	191501122.1	6.12	733870513.3
5	1350	234968784.8	5.12	800464156.9
6	1350	299160826.7	4.12	873323155.3

Table 8.5: Output based on price variation for McLaughlin gold deposit (GTC3)

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1250	97230794.9	9.12	466990510.6
2	1250	112726501.7	8.12	509944594.6
3	1250	132644153.3	7.12	557383318.2
4	1250	159238822.1	6.12	610234941.8
5	1250	196349784.8	5.12	668901467.4
6	1250	251173726.7	4.12	733237148.6
1	1270	101558894.9	9.12	487777974.4
2	1270	117583801.7	8.12	531917722.9
3	1270	138181853.3	7.12	580653259.1
4	1270	165683722.1	6.12	634933084.7
5	1270	204060984.8	5.12	695171081.1
6	1270	260756026.7	4.12	761210211.0
1	1290	105886994.9	9.12	508565438.2
2	1290	122441101.7	8.12	553890851.2
3	1290	143719553.3	7.12	603923199.9
4	1290	172128622.1	6.12	659631227.6
5	1290	211772184.8	5.12	721440694.8
6	1290	270338326.7	4.12	789183273.4
1	1310	110215094.9	9.12	529352902.0
2	1310	127298401.7	8.12	575863979.4
3	1310	149257253.3	7.12	627193140.8
4	1310	178573522.1	6.12	684329370.4
5	1310	219483384.8	5.12	747710308.5
6	1310	279920626.7	4.12	817156335.8
1	1330	114543194.9	9.12	550140365.8

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
2	1330	132155701.7	8.12	597837107.7
3	1330	154794953.3	7.12	650463081.6
4	1330	185018422.1	6.12	709027513.3
5	1330	227194584.8	5.12	773979922.1
6	1330	289502926.7	4.12	845129398.2
1	1350	118871294.9	9.12	570927829.6
2	1350	137013001.7	8.12	619810235.9
3	1350	160332653.3	7.12	673733022.5
4	1350	191463322.1	6.12	733725656.1
5	1350	234905784.8	5.12	800249535.8
6	1350	299085226.7	4.12	873102460.6

Table 8.6: Output based on price variation for McLaughlin gold deposit (GTC4)

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1250	97318994.9	9.18	468922686.8
2	1250	112839901.7	8.18	512461736.1
3	1250	132795353.3	7.18	560778031.5
4	1250	158819954.0	6.18	612425455.0
5	1250	195411248.6	5.18	671262424.5
6	1250	249158054.3	4.18	735159773.7
1	1270	101647094.9	9.18	489777241.3
2	1270	117697201.7	8.18	534521134.9
3	1270	138333053.3	7.18	584163040.3
4	1270	165245954.0	6.18	637204746.7
5	1270	203084648.6	5.18	697621526.8
6	1270	258664754.3	4.18	763210014.6
1	1290	105975194.9	9.18	510631795.8
2	1290	122554501.7	8.18	556580533.7
3	1290	143870753.3	7.18	607548049.1
4	1290	171671954.0	6.18	661984038.5
5	1290	210758048.6	5.18	723980629.1
6	1290	268171454.3	4.18	791260255.4
1	1310	110303294.9	9.18	531486350.3
2	1310	127411801.7	8.18	578639932.5
3	1310	149408453.3	7.18	630933057.9
4	1310	178097954.0	6.18	686763330.2
5	1310	218431448.6	5.18	750339731.3

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
6	1310	277678154.3	4.18	819310496.2
1	1330	114631394.9	9.18	552340904.7
2	1330	132269101.7	8.18	600699331.3
3	1330	154946153.3	7.18	654318066.7
4	1330	184523954.0	6.18	711542621.9
5	1330	226104848.6	5.18	776698833.6
6	1330	287184854.3	4.18	847360737.0
1	1350	118959494.9	9.18	573195459.2
2	1350	137126401.7	8.18	622758730.1
3	1350	160483853.3	7.18	677703075.5
4	1350	190949954.0	6.18	736321913.7
5	1350	233778248.6	5.18	803057935.9
6	1350	296691554.3	4.18	875410977.8

Table 8.7: Output based on price variation for McLaughlin gold deposit (GTC5)

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1250	97999394.9	9.84	488180335.9
2	1250	112480965.5	8.84	531886736.1
3	1250	130269544.7	7.84	578104677.4
4	1250	153670604.9	6.84	630594997.9
5	1250	185446286.6	5.84	689753061.3
6	1250	230031134.6	4.84	753906480.8
1	1270	102327494.9	9.84	509740604.9
2	1270	117300465.5	8.84	554676619.8
3	1270	135693844.7	7.84	602176406.6
4	1270	159888704.9	6.84	656111281.7
5	1270	192741686.6	5.84	716887735.0
6	1270	238838534.6	4.84	782771947.0
1	1290	106655594.9	9.84	531300873.8
2	1290	122119965.5	8.84	577466503.5
3	1290	141118144.7	7.84	626248135.7
4	1290	166106804.9	6.84	681627565.4
5	1290	200037086.6	5.84	744022408.8
6	1290	247645934.6	4.84	811637413.2
1	1310	110983694.9	9.84	552861142.8
2	1310	126939465.5	8.84	600256387.3

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
3	1310	146542444.7	7.84	650319864.9
4	1310	172324904.9	6.84	707143849.2
5	1310	207332486.6	5.84	771157082.5
6	1310	256453334.6	4.84	840502879.4
1	1330	115311794.9	9.84	574421411.7
2	1330	131758965.5	8.84	623046271.0
3	1330	151966744.7	7.84	674391594.1
4	1330	178543004.9	6.84	732660132.9
5	1330	214627886.6	5.84	798291756.2
6	1330	265260734.6	4.84	869368345.6
1	1350	119639894.9	9.84	595981680.7
2	1350	136578465.5	8.84	645836154.7
3	1350	157391044.7	7.84	698463323.3
4	1350	184761104.9	6.84	758176416.6
5	1350	221923286.6	5.84	825426430.0
6	1350	274068134.6	4.84	898233811.8

Table 8.8: Output based on price variation for McLaughlin gold deposit (GTC6)

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1250	97974194.9	9.33	475864529.7
2	1250	112975434.0	8.33	518024519.1
3	1250	132474217.0	7.33	566161645.7
4	1250	158092550.0	6.33	618793299.4
5	1250	193783108.0	5.33	678455579.5
6	1250	244981973.0	4.33	741721258.3
1	1270	102302295.0	9.33	496886282.1
2	1270	117813834.0	8.33	540209960.3
3	1270	137974117.0	7.33	589666841.7
4	1270	164461850.0	6.33	643723507.5
5	1270	201362008.0	5.33	704990126.5
6	1270	254299673.0	4.33	769932053.1
1	1290	106630395.0	9.33	517908034.6
2	1290	122652234.0	8.33	562395401.4
3	1290	143474017.0	7.33	613172037.7
4	1290	170831150.0	6.33	668653715.6
5	1290	208940908.0	5.33	731524673.5
6	1290	263617373.0	4.33	798142847.8

No.	Price (US$/oz)	Annual Profit (US$)	Mine Life (yrs.)	NPV (US$)
1	1310	110958495.0	9.33	538929787.0
2	1310	127490634.0	8.33	584580842.6
3	1310	148973917.0	7.33	636677233.7
4	1310	177200450.0	6.33	693583923.7
5	1310	216519808.0	5.33	758059220.5
6	1310	272935073.0	4.33	826353642.6
1	1330	115286595.0	9.33	559951539.5
2	1330	132329034.0	8.33	606766283.7
3	1330	154473817.0	7.33	660182429.6
4	1330	183569750.0	6.33	718514131.8
5	1330	224098708.0	5.33	784593767.5
6	1330	282252773.0	4.33	854564437.4
1	1350	119614695.0	9.33	580973291.9
2	1350	137167434.0	8.33	628951724.9
3	1350	159973717.0	7.33	683687625.6
4	1350	189939050.0	6.33	743444339.9
5	1350	231677608.0	5.33	811128314.5
6	1350	291570473.0	4.33	882775232.1

8.3.6 Components of the computer-aided application (NPVMining)

The following is a summary explaining what is found in each of the files that make up the *NPVMining* application. Figure 3.4 shows files in the solution explorer used in execution of the application.

i. *NPVMining*.vcxproj

This is the main project file for VC++ projects generated using an application wizard. It contains information about the version of Visual C++ that generated the file, and information about the platforms, configurations, and project features selected with the application wizard.

ii. *NPVMining*.vcxproj.filters

This is the filters file for VC++ projects generated using an Application Wizard. It contains information about the association between the files in the project and the filters. This association is used in the IDE to show grouping of files with similar extensions under a specific node (for e.g. ".cpp" files are associated with the "Source Files" filter).

iii. *NPVMining*.h

This is the main header file for the application. It includes other project specific headers (including Resource.h) and declares the C*NPVMining*App application class.

iv. *NPVMining*.cpp

This is the main application source file that contains the application class C*NPVMining*App.

v. *NPVMining*.rc

This is a listing of all of the Microsoft Windows resources that the program uses. It includes the icons, bitmaps, and cursors that are stored in the RES subdirectory. This file can be directly edited in Microsoft Visual C++. The project resources are in 1033.

vi. res*NPVMining*.ico

This is an icon file, which is used as the application's icon. This icon is included by the main resource file *NPVMining*.rc.

vii. res*NPVMining*.rc2

This file contains resources that are not edited by Microsoft Visual C++. All resources that are not editable by the resource editor are placed in this file.

viii. Dialog class: *NPVMining*Dlg.h and *NPVMining*Dlg.cpp

The application wizard creates one dialog class containing: *NPVMining*Dlg.h, *NPVMining*Dlg.cpp. These files contain the C*NPVMining*Dlg class. This class defines the behaviour of the application's main dialog. The dialog's template is in *NPVMining*.rc, which can be edited in Microsoft Visual C++.

ix. Other standard files:

a) StdAfx.h and StdAfx.cpp: These files are used to build a precompiled header (PCH) file named *NPVMining*.pch and a precompiled types file named StdAfx.obj.

b) Resource.h: This is the standard header file, which defines new resource IDs. Microsoft Visual C++ reads and updates this file.

8.4 Appendix 4: *NPVMining* code in C++

8.4.1 Stripping ratio

```
double CNPVMiningDlg::determineStripRatio(double dCutoff_grade, int iPos)
{
	//determine breakeven cutoff grade policy
	double dSRatio; int g = 0;
	double dCurrentPrice = dValArray[iPos]; //getCurrentPrice();
	POSITION pos = listGradeCat.GetHeadPosition();
	dTotalWaste = 0.0;
	dTotalOre = 0.0;
	while (pos)
	{
		GradeCategory sGradeCat = listGradeCat.GetNext(pos);
		if (sGradeCat.dLowerLimit < dCutoff_grade) dTotalWaste += sGradeCat.dQuantity;
		else dTotalOre += sGradeCat.dQuantity;
	}
	dTotalOre -= dMinedOre;
	dSRatio = dTotalWaste / dTotalOre;
	dSRatio = round(dSRatio*100)/100;

	return dSRatio;
}
```

8.4.2 Average grade

```
double CNPVMiningDlg::average_grade(double dCutoff_grade)
{
	double dResult = 0.0;
	POSITION pos = listGradeCat.GetHeadPosition();
	while (pos)
	{
		GradeCategory sGradeCat = listGradeCat.GetNext(pos);
		if (sGradeCat.dLowerLimit >= dCutoff_grade) {
			dResult += sGradeCat.dMidpoint * sGradeCat.dQuantity;
			dResult = round(dResult * 1000) / 1000;
		}
	}
	dResult /= dTotalOre;
```

```
	dResult = round(dResult * 1000) / 1000;
	return dResult;
}
```

8.4.3 Optimum cut-off grade

```
void CNPVMiningDlg::determineOptimumGrade()
{
	dTotalWaste = 0.0;
	POSITION pos = listGradeCat.GetHeadPosition();
	while (pos)
	{
		GradeCategory sGradeCat = listGradeCat.GetNext(pos);
		sGradeCat.dOreQty = dTotalQty - dTotalWaste;
		sGradeCat.dWasteQty = dTotalWaste;
		dTotalQty -= sGradeCat.dQuantity;
		dTotalWaste += sGradeCat.dQuantity;
		sGradeCat.dmc = sGradeCat.dOreQty / (sGradeCat.dOreQty + sGradeCat.dWasteQty);
		sGradeCat.dG = determineGSum(pos) / sGradeCat.dOreQty;
		sGradeCat.dmr = (sGradeCat.dOreQty * sGradeCat.dG) / (sGradeCat.dOreQty + sGradeCat.dWasteQty);
		sGradeCat.dcr = sGradeCat.dG;
	}
}
```

8.4.4 Limiting cut-off grade

```
double CNPVMiningDlg::limitingcutoff_grade(int iPos)
{
	double dResult = 0.0;
	double dPrice = 0.0;
	double dNPV = cObj->get_NPV();
	double dAFixedCost = cObj->get_annualFixedCost();
	double dMillCapacity = cObj->get_millingCapacity();
	double dMineCapacity = cObj->get_miningCapacity();
	double dRefCapacity = cObj->get_refiningCapacity();
	//using the first value in the array
	dPrice = dValArray[iPos];

	if (m_boolMillCap == true) {
```

```
                dResult = (cObj->get_millingCost() + ((dAFixedCost + (cObj->get_discountedRate() * dNPV)) / dMillCapacity)) / ((dPrice - cObj->get_refiningCost()) * cObj->get_recovery());
        }
        else if (m_boolMineCap == true) {
                dResult = (cObj->get_millingCapacity() + ((dAFixedCost + (cObj->get_discountedRate() * dNPV)) / dMineCapacity)) / ((dPrice - cObj->get_refiningCost()) * cObj->get_recovery());
        }
        else if (m_boolRefCap == true) {
                dResult = (cObj->get_refiningCost() + ((dAFixedCost + (cObj->get_discountedRate() * dNPV)) / dRefCapacity)) / ((dPrice - cObj->get_refiningCost()) * cObj->get_recovery());
        }
        dResult = round(dResult * 100) / 100;

        return dResult;
}
```

8.4.5 Break-even cut-off grade

```
double CNPVMiningDlg::breakevencutoff_grade(int iPos)
{
        double dResult = 0.0;
        double dPrice = 0.0;
        //using the first value in the array
        dPrice = dValArray[iPos];
        dResult = cObj->get_millingCost() / ((dPrice - cObj->get_refiningCost()) * cObj->get_recovery());
        dResult = round(dResult * 100) / 100;

        return dResult;
}
```

8.4.6 Grade category

```
bool CNPVMiningDlg::checkGradeCatDuplicate(double lowerlimit, double upperlimit)
{
        bool bFound = false;

        for (int g = 0; g < m_listGradeCat.GetItemCount(); g++)
        {
                if (_wtof(m_listGradeCat.GetItemText(g, 1)) == lowerlimit || upperlimit == _wtof(m_listGradeCat.GetItemText(g, 2)))
                {
                        bFound = true;
```

```
                        break;
                }
        }

        return bFound;
}
```

8.4.7 Sub category

```
bool CNPVMiningDlg::checksubCatDuplicate(CString cat, CString subcat)
{
        bool bFound = false;

        for (int g = 0; g < m_listPriceCat.GetItemCount(); g++)
        {
                if (m_listPriceCat.GetItemText(g, 1) == cat && subcat == m_listPriceCat.GetItemText(g, 2))
                {
                        bFound = true;
                        break;
                }
        }

        return bFound;
}
```

8.4.8 Creating an array of price values

```
int CNPVMiningDlg::factorial(int n)
{
        return (n == 1 || n == 0) ? 1 : factorial(n - 1) * n;
}

int CNPVMiningDlg::combinationcount(int n, int r)
{
        return (factorial(n) / (factorial(r)*factorial(n - 1)));
}

int CNPVMiningDlg::getusedcategoriesList()
{
        int iTotal = 0, iLast = 0;
```

```
	int iCat = m_listPriceCat.GetItemCount();
	for (int v = 0; v < 30; v++) szUsedCatItems[v] = L"";
	for (int t = 0; t < iCat; t++)
	{
		CString szTempCat = m_listPriceCat.GetItemText(t, 1);
		for (int k = 0; k < 30; k++)
		{
			if (k == iLast)
			{
				if (szTempCat == szUsedCatItems[iLast - 1]) break;
				else {
					szUsedCatItems[iTotal] = szTempCat;
					iTotal += 1;
					iLast += 1;
					break;
				}
			}
			else if (k < iLast) continue;
		}
	}

	return iTotal;
}

int CNPVMiningDlg::combinationtotals()
{
	CString szArrayVals[100][2];
	CString szArrayCat[20];
	int iArrayCatSubItems[20] = { 0 };
	int iArrayCatSubItemsCmb[20] = { 0 };
	int iArrayCatSubItemsCmbTotals = 0;
	int iCurrVal = 0, iCurrPos = 0;
	//determine how many categories exist
	int iCat = m_cmbPriceCat.GetCount();
	for (int u = 0; u < iCat; u++)
	{
		m_cmbPriceCat.GetLBText(u, szArrayCat[u]);
	}
	//determine the number of combinations expected
	int iCatCmb = combinationcount(iCat, 1);
	//loop through all the data and identify the subcategories in each category and store them in an array
	for (int t = 0; t < m_listPriceCat.GetItemCount(); t++)
```

```
		{
			szArrayVals[t][0] = m_listPriceCat.GetItemText(t, 1);
			szArrayVals[t][1] = m_listPriceCat.GetItemText(t, 2);
			for (int k = 0; k < iCat; k++)
			{
				if (szArrayVals[t][0] == szArrayCat[k]) iArrayCatSubItems[k] += 1;
			}
		}
		//determine the number of combinations for each category
		for (int d = 0; d < iCat; d++)
		{
			if (iArrayCatSubItems[d] > 1)
			{
				iArrayCatSubItemsCmb[d] = combinationcount(iArrayCatSubItems[d], 1);
			}
			else if (iArrayCatSubItems[d] == 1)
			{
				iArrayCatSubItemsCmb[d] = 1;
			}
		}
		//get the summation of all the combinations
		for (int j = 0; j < iCatCmb; j++)
		{
			for (int i = 0; i < iCatCmb; i++)
			{
				iCurrVal = iArrayCatSubItemsCmb[iCurrPos];
				if (i <= iCurrPos) continue;
				else iArrayCatSubItemsCmbTotals += iCurrVal*iArrayCatSubItemsCmb[i];
			}
			iCurrPos += 1;
		}
		return iArrayCatSubItemsCmbTotals;
}

void CNPVMiningDlg::fillcombinationlist(COleSafeArray *m_combinationList)
{
		long index1[2];
		int iArrayCatLastSubItems[20] = { 0 };
		int iLastPosArray[20] = { 0 };
		//get a list of the category combinations
		int iCat = iUsedCatNum;
		for (int r = 0; r < 100; r++)
```

```
	{
		for (int e = 0; e < 2; e++)
		{
			szArrayVals[r][e] = L"";
		}
	}
	for (int b = 0; b < 20; b++) iLastPosArray[b] = -1;
	int iItemCount = m_listPriceCat.GetItemCount();
	for (int t1 = 0; t1 < iItemCount; t1++)
	{
		CString szCatItem = m_listPriceCat.GetItemText(t1, 1);
		szArrayVals[t1][0] = szCatItem;
		CString szSubCatItem = m_listPriceCat.GetItemText(t1, 2);
		szArrayVals[t1][1] = szSubCatItem;
		for (int k = 0; k < iCat; k++)
		{
			if (szArrayVals[t1][0] == szUsedCatItems[k])
			{
				iArrayCatLastSubItems[k] = t1;
			}
		}
	}
	int iCurrRow = 2, iCurrCol = 1;
	int i = 0;
	while (1)
	{
		for (int j = 0; j < iCat; j++)
		{
			for (int t = 0; t < iItemCount; t++)
			{
				if (szArrayVals[t][0] != szUsedCatItems[j]) continue;
				if (j == 0)// has a number on its right
				{
					if (iLastPosArray[j] != iArrayCatLastSubItems[j]) {
						int iTemp = j;
						bool bTemp = true;
loopj:
						if (iLastPosArray[iTemp + 1] == iArrayCatLastSubItems[iTemp + 1]) {
							if (iTemp + 1 == iCat - 1)
							{
								if (t <= iLastPosArray[j] && bTemp == true) {
									continue;
```

```
                                        }
                                }
                                else {
                                        iTemp += 1;
                                        if (iLastPosArray[iTemp + 1] < iArrayCatLastSubItems[iTemp + 1]) bTemp = false;
                                        goto loopj;
                                }
                        }
                        if (t < iLastPosArray[j]) continue;
                }
                else {
                        if (t < iArrayCatLastSubItems[j]) continue;
                        else {
                                int iTemp = j;
                                bool bTemp = true;
loopj1:
                                if (iLastPosArray[iTemp + 1] == iArrayCatLastSubItems[iTemp + 1]) {
                                        if (iTemp + 1 == iCat - 1)
                                        {
                                                if (t <= iLastPosArray[j] && bTemp == true) {
                                                        //start from the first sub item
                                                        if (j > 0) t = iArrayCatLastSubItems[j - 1] + 1;
                                                        else return;
                                                }
                                        }
                                        else {
                                                iTemp += 1;
                                                if (iLastPosArray[iTemp + 1] < iArrayCatLastSubItems[iTemp + 1]) bTemp = false;
                                                goto loopj1;
                                        }
                                }
                        }
                }
        }
        else //no number on its right
        {
                if (iLastPosArray[j] != iArrayCatLastSubItems[j]){
                        if (t <= iLastPosArray[j]) continue;
                }
        }
        index1[0] = iCurrRow + i;
        index1[1] = iCurrCol + j;
```

```
                                        if (j == 0)
                                        {
                                                long numpos[2];
                                                numpos[0] = index1[0];
                                                numpos[1] = 0;
                                                CString szNum; szNum.Format(L"%d", numpos[0] - 1);
                                                BSTR bstr = szNum.AllocSysString();
                                                m_combinationList->PutElement(numpos, bstr);
                                                SysFreeString(bstr);
                                        }
                                        BSTR bstr1 = szArrayVals[t][1].AllocSysString();
                                        m_combinationList->PutElement(index1, bstr1);
                                        SysFreeString(bstr1);
                                        iLastPosArray[j] = t;
                                        break;
                                }

                        }
                        i++;
                }
}
```

8.4.9 Annual profit

```
double CNPVMiningDlg::dgetAnnualProfit(int iPos)
{
        double dResult = 0.0;
        double dPrice = 0.0;
        //using the first value in the array
        dPrice = dValArray[iPos];
        dResult = ((dPrice - cObj->get_refiningCost()) * cObj->get_refiningCapacity()) - (cObj->get_millingCapacity() * cObj->get_millingCost()) - (cObj->get_miningCapacity() * cObj->get_miningCost()) - cObj->get_annualFixedCost();
        dResult = round(dResult * 100) / 100;
        return dResult;
}
```

8.4.10 NPV

```
double CNPVMiningDlg::dgetNPV(double dMinelife, double dAnnualProfit)
{
        double dResult = 0.0;
```

```
	dResult = (dAnnualProfit * ((round(pow(1 + cObj->get_discountedRate(), dMinelife) * 1000) / 1000) - 1)) / ((round(pow(1 + cObj->get_discountedRate(), dMinelife) * 1000) / 1000) *
cObj->get_discountedRate());
	dResult = round(dResult * 100) / 100;

	return dResult;
}
```