

https://dspace.org/

Communicating mathematics reasoning in multilingual classrooms in South Africa.

Aineamani, Benadette

2011-06-20
http://hdl.handle.net/10539/10133

Downloaded from a DSpace repository. https://dspace.org/

Communicating Mathematics Reasoning in Multilingual classrooms in South Africa

By

Benadette Aineamani

(Student Number: 386418)

A research report submitted in partial fulfilment of the requirements for the degree of

Master of Science

School of Education, Faculty of Science

University of the Witwatersrand

JOHANNESBURG

May 2011

<u>Supervisors</u>

Dr. Clement Dlamini

Anthony Essien

Declaration

I declare that this research report is my own work, except as indicated in the acknowledgements, the text and the references. It is being submitted in fulfilment of the requirements for the degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other institution.



Signed

Benadette Aineamani

13th DECEMBER, 2010

Date

ii

Abstract

This is a qualitative research that draws Gee's Discourse analysis to understand how learners communicate their mathematical reasoning in a multilingual classroom in South Africa. The study involved a Grade 11 class of 25 learners in a township school East of Johannesburg. The research method used was a case study. Data was collected using classroom observations, and document analysis. The study has shown that learners communicate their mathematics reasoning up to a certain level. The way learners communicated their mathematical reasoning depended on the activities that were given by the textbook being used in the classroom, and the questions which the teacher asked during the lessons. From the findings of the study, recommendations were made: the assessment of how learners communicate their mathematical reasoning should have a basis, say the curriculum. If the curriculum states the level of mathematical reasoning which the learners at Grade 11 must reach, then the teacher will have to probe the learners for higher reasoning; mathematics classroom textbooks should be designed to enable learners communicate their mathematical reasoning. The teacher should ask learners questions that require learners to communicate their mathematical reasoning.

Key Words: Mathematical reasoning, deductive reasoning, inductive reasoning, classroom Discourse, classroom discourse, social language, ordinary language, mathematical language, formal language, informal language, multilingual classroom.

Acknowledgements

Firstly, I would like to thank God for giving me the opportunity to complete this Masters degree. It is by his grace that this Masters has been completed.

I wish to thank most deeply my supervisors, ***Dr. Clement Dlamini*** and ***Anthony Essien*** who have provided, without hesitation, and with unflagging support, constructive comments, and suggestions, all of which provided a strong backing for this Masters Research work. I would like to thank them for their support and the intellectual energy that they gave to this thesis and for their ability to listen with care and critique with rigour. Thank you is not enough!

I would like to thank the participants in the study for their patience and support during the process of data collection.

To my family- my dad- C. B Wandera, my sisters and brothers who supported me even when it felt like it was impossible, Thank you very much.

Dedication

This thesis is dedicated to my wonderful Dad- **C. B. Wandera**, who, as a single parent saw me through school and supported me unconditionally. Thank you very much Dad for the love and kindness you showed me!

Table of contents

Abstract iii

Acknowledgements iv

Dedication v

Table of contents vi

List of Tables x

List of Figures xi

Abbreviations xii

1. Chapter One: Introduction of the Study 1

1.1. Introduction 1

1.2. Research Problem Statement 1

1.3. Aim of the Study 4

1.4. Purpose of the Study 4

1.5. Objectives the Study 4

1.6. Research Questions 5

1.6.1. Main research question 5

1.6.2. Justification for Choosing the Particular Questions 5

1.7. Rationale 6

1.7.1. Why the Focus on Mathematics Reasoning? 6

1.7.2. Why is the Study Important? 8

1.7.3. Why South Africa? 9

1.7.4. Why in a Multilingual Classroom? 9

1.7.5. Who Will Benefit From the Study? 10

1.8. Conclusion 10

2. Chapter Two: Theoretical Framework and Literature review 12

2.1. Introduction 12

2.2. Theoretical Framework 12

2.3. Literature Review 17

2.3.1. Multilingualism in South Africa 17

2.3.2. Communicating Mathematical Reasoning 18

2.3.3. Communication in Group Discussions 20

2.3.4. The Teacher's Role in Communication 21

2.3.5. Barriers Associated With Communicating in a Second Language 21

2.4 Conclusion 22

3. Chapter Three: Mathematical Reasoning 23

3.1. Introduction 23

3.2. Mathematics Knowledge and Reasoning 23

3.3. Mathematical Reasoning and Proof 23

3.4. What Counts as Mathematical Reasoning? 24

3.5. Types of Mathematical Reasoning 28

3.5.1. Inductive Reasoning 28

3.5.2. Deductive Reasoning 29

3.6. Forms of Mathematical Reasoning Progression 30

3.6.1. Empirical Evidence 30

3.6.2. Preformal Explanation 30

3.6.3. Formal Argumentation 31

3.7. The South African National Curriculum Statement and Mathematical Reasoning 31

3.8. Ways of Enabling or Restricting Mathematical Reasoning 33

3.9. Discursive Practices 35

3.10. Conclusion 37

4. Chapter Four: Research Design and Methodology 38

4.1. Introduction 38

4.2. Research Approach 38

4.3. Research Method 40

4.4. Sampling 40

4.4.1. The School 41

4.4.2. The Participants 42

4.3. Data Collection 43

4.3.1. Classroom Observations 43

4.3.2. Piloting 45

4.3.3. Documentation 46

4.3.4. The Mathematics Textbook 47

4.4. Data Analysis 48

4.4.1. The Analytical Framework 49

4.5. Validity and Reliability Issues 59

4.5.1. Reliability 59

4.5.2. Validity 61

4.6 Ethical Considerations 62

4.7. Conclusion 64

5. Chapter Five: Data Analysis and Presentation 65

5.1. Introduction 65

5.2. Back ground of the Study 66

5.3. The Process of Data Analysis 67

5.3.1. Transcription 68

5.3.2. Learners' Note Books 70

5.4. Communication Legitimised by the Textbook 71

5.5. The Classroom Discourse 76

5.5.1. Oral Communication Legitimised by the Teacher 78

5.5.2. Written Communication Legitimised by the Teacher 81

5.6. How Learners Communicate their Mathematical Reasoning 83

5.6.1. Oral Communication of Learners' Mathematical Reasoning with the Teacher . 84

5.7. The Languages Learners Use to Communicate their Mathematical Reasoning 92

5.8. Conclusion 94

6. Chapter Six: Conclusions and Recommendations 95

6.1. Introduction 95

6.2. Summary of the Findings 95

6.4. Implications of the Study 101

6. 5. Recommendations 104

6.6. Limitations 105

6.7. Further Research 105

6.8. Reflections 105

References 107

Appendix A: Permission Letters 113

Appendix B: Lesson Transcripts 123

Appendix C: Analysis of Textbook- Classroom Mathematics 147

Appendix D: Analysis of Learners' oral communication 148

Appendix E: Types of responses given by the learners 153

Appendix F: Levels of Mathematical reasoning 155

Appendix G: Responses Legitimised by the teacher 161

Appendix H: Responses not marked by the teacher 165

Appendix I: Levels of classroom discourse 167

List of Tables

Table 4.1: Steps in Typological Analysis 48

Table 4.2: Analytical Framework 50

Table 4.3: Levels of Mathematical Reasoning 57

Table 4.4: Levels of Language used in the Classroom 59

Table 5.1: Conventions in the transcribed data of the study 69

List of Figures

Figure 5.1: General discussion section from the textbook Classroom mathematics 72

Figure 5.2: An example of a question in the textbook activity that requires procedural manipulation 73

Figure 5.3: Learner's work legitimised by the teacher 82

Figure 5.4: Learner's work that was not marked by the teacher..... 83

Figure 5.5: Nicole's response to Question 2a and b 90

Figure 5.6: Joy's response to Question 4 91

Figure 5.7: Peter's response to Question 4 92

Figure 5.8: Peter's response of Question 2a 92

Abbreviations

IR	**:**	Inductive reasoning
DR	**:**	Deductive reasoning
OL	**:**	Ordinary language
ML	**:**	Mathematical language
L1IR	**:**	Level one inductive reasoning
L2IR	**:**	Level two inductive reasoning
L3IR	**:**	Level three inductive reasoning
L1DR	**:**	Level one deductive reasoning
L2DR	**:**	Level two deductive reasoning
L3DR	**:**	Level three deductive reasoning
Cd0	**:**	Classroom discourse zero
Cd1	**:**	Classroom discourse one
Cd2	**:**	Classroom discourse two
RNCS	**:**	The revised national curriculum statement
DoE	**:**	Department of Education

1. Chapter One: Introduction of the Study

1.1. Introduction

In this chapter, I will discuss the reason why the study was conducted and what inspired the researcher to do a study on mathematical reasoning. Based on the results from the previous study, this study sought to investigate how learners reason and communicate mathematics reasoning in a multilingual school in South Africa. A qualitative study was conducted in one multilingual classroom in South Africa.

1.2. Research Problem Statement

Aineamani (2010) conducted a study on reasoning and communicating mathematically. The aim of the study was to understand why students regard some solutions as the best representation and solution of a word problem in their own view and in their teacher's view, and how these students construct a proof. The participants were required to explain and justify their choices and their proofs using mathematical language. Questionnaire and semi structured interviews were used as methods for data collection. The first phase of the study required the students to write down their answers, and also write down the proof to the statement that was given to them. Four students were purposefully selected from the sample for interviews based on the responses they gave and the proof that they constructed. The data was analysed and the findings revealed that the students could not orally express their thinking. Some other students failed to write down mathematical ideas that made sense to the reader especially when it came to constructing a proof. The students in the study were constrained by the fact that they could not communicate their mathematical reasoning. Could these learners reason and communicate mathematically? In order to answer this question, I refer to literature to define what reasoning and communicate mathematically means.

Reasoning mathematically refers to being able to formulate and represent a given mathematics problem, explain and justify the solution or argument to the mathematics problem (Kilpatrick, Swafford & Findell, 2001). Reasoning mathematically also involves finding out what is it that is true in a mathematics conjecture, constructing an argument to convince oneself that the result is true and thereafter find out why the conjecture is true

(Brodie, 2000). Communicating mathematically is a way of talking about mathematical activities using a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings (Pimm, 1991).

In the study that was conducted by Aineamani (2010), about reasoning and communicating mathematically, learners were required to choose the best model and solution of a word problem according to them and according to their teacher. They were expected to justify why they chose the answer from a list of the given answers. One of the learners chose a solution and then wrote down a justification which was simply an explanation of what the answer was all about. In other words, she simply wrote down the steps taken to solve the problem. When the student was interviewed, she said "I do not know how to justify this answer that I chose. I chose it because it has algebra in it". Another student said "I cannot explain it in words but I think I can write it down." When he was given a paper to write the justification, he failed to write down anything. This learner could neither write down her reasoning nor orally communicate the mathematical reasoning.

There may have been many reasons to explain the inability of the learners to explain their reasoning in the study that was conducted. The learners could not verbalise their reasoning clearly. In other words, they failed to speak and mean like mathematicians. This means that these learners could not talk about the mathematical activities using a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings (Pimm, 1991).

The research above was limited to looking at only the difficulties that the learners face and not how learners communicate their mathematical reasoning in relation of factors such as language. Therefore, the interest of conducting a research about communicating mathematical reasoning emerged from the findings of the study about reasoning and communicating mathematically that was discussed above. The students in the study were struggling with English because it is their second language, and also the language of mathematics. But, this was not made explicit because of the research questions which

guided the study. Though language difficulty was a problem that was identified in the study, it was not well discussed and given much attention as it should have been. Therefore, a study about how learners communicate their mathematical reasoning would be more interesting and meaningful in a context where learners cannot verbalise their mathematical reasoning.

Another reason for learners' failure to reason and communicate mathematically may be that the learners were struggling with mathematics concepts (Aineamani, 2010). Therefore, if reasoning and communicating mathematically was researched in relation to language, the study that was conducted by Aineamani (2010) would have been very interesting. The form of language that would be of interest in reasoning and communicating mathematically is that of the mathematics Discourse.

Mathematics as a subject has its own Discourse. A mathematics Discourse is has an accepted way of communicating mathematics. This includes a learner's point of view, beliefs, and thoughts about mathematics (Gee, 2005; Moschkovich, 2003). As learners take part in the mathematics Discourse, language is involved. The language that the learners use should be in such a way that they make a socially acceptable meaning of mathematics as a subject (Moschkovich, 2003). In the study discussed above, most learners that tried to explain their reasoning were using language that may not be socially acceptable in a mathematics community. For example, four of the learners referred to the variable 'x' as many 'xs'. One learner said "I don't know why they have to use many 'xs' when solving the problem. Why can't they use numbers? When I see the 'xs', I just get confused and I hate mathematics". This learner did not understand the concept of algebra and variables, as it is used in the mathematics Discourse.

Lemke (cited in Cleghorn & Rollinick, 2002) argues that learners find difficulties in reading, writing and talking about science because the Discourse is new to those learners. This also applies to mathematics. The mathematics Discourse is new to learners because they do not use it home. The study carried out by Setati and Adler (cited in Cleghorn & Rollinick, 2002) found that for learners to take part in a mathematics Discourse, they have to leave behind their informal ways of speaking and learn to use the formal

language that is used in the mathematics Discourse. The learners in Aineamani's (2010) study were struggling with leaving behind their informal ways of speaking and this may have been because they are not yet used to the language of mathematics.

In the study that is discussed in this report, the researcher investigated how learners participate in a mathematics Discourse, the focus being the ways how learners communicate mathematical reasoning within the mathematics Discourse. A mathematics Discourse requires learners to be able to read, write and talk about mathematics. However, it does not mean that children must master formal technical language of mathematics (Moschkovich, 2003), but they should be able to communicate mathematically in an acceptable way.

1.3. Aim of the Study

The aim of the study was to understand how learners in a multilingual school in South Africa communicate their mathematical reasoning.

1.4. Purpose of the Study

The purpose of the study was to investigate how learners in a multilingual school in South Africa communicate their reasoning. In a mathematics discourse, learners are expected to read, write and talk about mathematics (Moschkovich, 2003). This investigation intended to provide an understanding of their communication practices and insights into what hinders and enhances their communication of mathematics reasoning.

1.5. Objectives the Study

The objectives of the study were to:

- investigate learners' language difficulties and the reasons behind those difficulties
- investigate how learners carry out their interactions in the classroom to ascertain what they struggle with in a mathematics discourse.
- make recommendations on language use and communicating mathematical reasoning in multilingual classrooms.

1.6. Research Questions

1.6.1. Main research question

How do learners in a multilingual school in South Africa communicate their mathematical reasoning?

Critical Questions:

1. What are the communication practices of the classroom of mathematics learners in a multilingual school in South Africa?
    i) What communication practices are legitimised by the textbook?
    ii) What communication practices are legitimised by the teacher?
2. How do these learners communicate their reasoning in
    i) Written texts?
    ii) Oral texts with their:
        a) Teacher?
        b) Classmates?
3. What languages do they use in communicating their mathematical reasoning in formal and informal mathematical discussions?

1.6.2. Justification for Choosing the Particular Questions

The main research question is the broad perspective of the study. This question was answered by looking at the answers to the critical questions.

The first critical question is about communication practices in the classroom. This question is important because in order to understand how learners communicate their mathematical reasoning, their background must be investigated first. In other words, the classroom practices that learners are exposed to may or may not develop their ability to reason and communicate mathematically. From my experience as a teacher, communication practices in mathematics classrooms are dictated by the teacher most of the time and the teacher may follow a given textbook in order to decide on which communication practices to promote. Therefore, it is important in my study to look at both the practices legitimised by the teacher and the textbook.

In a mathematics classroom, learners are expected to write, and talk mathematically (Moschkovich, 2003). As learners talk and write about mathematics, they are communicating mathematically (Pimm, 1991). In order to investigate how learners communicate their mathematical reasoning, both oral and written communications have to be considered. This is because some learners maybe able to orally communicate their reasoning and fail to write it down while others are able to write down their reasoning and they may not be able to orally communicate their reasoning.

The language used to communicate mathematically is also very important. Clark and Ramahlape (cited in Cleghorn & Rollinick, 2002) carried out a study in which they found out that learners participate more lively and freely when they are allowed to use their home languages to talk about mathematics than when required to use English. This means that language is very important when learners are required to communicate their mathematical reasoning. Therefore, the third question is useful for my study because it helped me to focus on the language that learners were more comfortable with when communicating their reasoning.

1.7. Rationale

1.7.1. Why the Focus on Mathematics Reasoning?

Reasoning mathematically refers to being able to formulate and represent a given mathematics problem, explain and justify the solution or argument to the mathematics problem (Kilpatrick et al., 2001). Reasoning mathematically also involves finding out what constitutes the truth in a mathematics conjecture, constructing an argument to convince oneself that the result is true and thereafter find out why the conjecture is true (Brodie, 2000). According to Martin and Kasmer (2010), reasoning is a process whereby one is required to draw conclusions on the basis of evidence or stated assumptions. Reasoning has a very important and particular role that it plays in mathematics. It includes logical deduction, formal reasoning and proof. Reasoning also involves informal observations, conjectures and explanations and these are used in lower grades (Martin & Kasmer, 2010). Therefore, it is very important that learners develop mathematical

reasoning at lower grades so that they do not find difficulties in higher grades since reasoning is part of mathematics (Martin & Kasmer, 2010).

Over the years, mathematics teaching has moved from a mechanical way towards a view which encourages teachers to teach learners mathematics by emphasising problem solving, understanding and communicating mathematically with others (McKenzie, 2001). The reforms in mathematics education invite teachers to provide a learning environment that encourages learners to connect mathematics ideas with the real world, explore mathematics ideas and deepen their understanding (McKenzie, 2001). Stein, Grover and Henningsen (1996) also contends that there is an increased emphasis in 'doing mathematics'. According to Stein et al. (1996), doing mathematics requires learners to be able to understand so that they can take part in the process of mathematics thinking, and be able to do what mathematicians actually do. However, Sfard (2001) argues that finding ways of making the idea of learning mathematics with understanding work in the classrooms is extremely difficult to achieve.

The new move from viewing mathematics as static to viewing mathematics as dynamic means that learners have to engage in activities such as making conjectures, looking for patterns, inventing, explaining, justifying and challenging other people's views on mathematics (Stein et al., 1996). As learners take part in activities such as making conjectures, looking for patterns, inventing, explaining, justifying, they are reasoning mathematically (Brodie, 2000).

In order for learners to be able to communicate their reasoning with their teacher, and peers, they must have developed a language of mathematics that enables them to express their thinking (McKenzie, 2001). Most of the time, teachers expect their learners to be able to communicate their reasoning effectively and this may not be the case in some instances (McKenzie, 2001). As learners take part in discussions amongst themselves, and with their teacher, they are provided with a chance to take part in a social interaction and as a result, their understanding is negotiated and developed (Bicknell, 1999).

The Revised National Curriculum Statement (RNCS) states that one of the unique features of teaching and learning mathematics is investigating, explaining and justifying (DoE, 2002). These unique features of mathematics require reasoning. As learners explain and justify in mathematics, they are communicating mathematical reasoning in the process. Therefore, mathematical reasoning is very important because it helps in bringing out the unique features of mathematics as a subject.

Once learners are able to apply the unique features of mathematics, they are then able to solve problems that require them to generalise, to apply abstract thinking and to also simplify. Therefore, mathematical reasoning is very useful because it enables learners to solve problems which they have not come across before by using justification, and generalisation techniques in the process of answering a given question (Kilpatrick et al., 2001).

1.7.2. Why is the Study Important?

The essence of mathematics as a subject lies in the fact that all claims can be justified. Epistemologically, all knowledge that we hold should have a basis and we should be in position to explain and justify the knowledge that we hold (Johnston, 2002). Once an individual is able to give an explanation of why something is the way it is, a well based understanding of the knowledge develops and then the individual is able to refer to such knowledge as his personal knowledge since he is able to justify it.

Reasoning mathematically forms the foundation of mathematical understanding (McKenzie, 2001). Therefore, mathematical understanding depends on reasoning and reasoning is very important for a learner to grow in mathematical knowledge (Muller & Maher, 2009). Once a learner is able to reason mathematically, she is able to apply the mathematical ideas to new situations and hence problem solving skills are developed (Muller & Maher, 2009).

Sfard (2001) argues that placing communication at the centre of mathematics education is most likely to change the ways people think about the process of learning mathematics

and about what is being learnt in mathematics classrooms. Communication is not simply an aid to thinking but it is a requirement for one to reason mathematically (Sfard, 2001).

1.7.3. Why South Africa?

In South African classrooms the teacher is looked at as a source of the mathematics knowledge and so the learners wait for the teacher to decide for them what to do each time they are faced with a mathematics problem (Brijlall, 2008). This makes such learners dependent on their teacher. For learners to develop problem solving skills, they have to become independent and critical thinkers (Brijlall, 2008). Therefore, the issue of reasoning and communicating mathematically should be addressed and emphasised in mathematics classrooms so that learners are given the opportunity to become independent and critical thinkers. Once learners are given an opportunity to develop their reasoning, their attitude towards mathematics as a subject may change for the better (Brijlall, 2008). The South African curriculum statement emphasises the idea of reasoning and critical thinkers (DoE, 2003), therefore the classroom should provide a conducive environment for this reasoning to take place.

1.7.4. Why in a Multilingual Classroom?

In South Africa, there is a complex nature of multilingualism. This is reflected in the classrooms whereby learners speak different languages (Brijlall, 2008). The learners in South African classrooms speak English, Afrikaans, isiZulu, seSotho and siSwati among others. In such classrooms, reasoning and communicating mathematically might be affected by the language problems. Reasoning and communicating mathematically may become problematic in a multilingual classroom as shown by the study conducted by Barton and Barton (2005).

Barton and Barton (2005) conducted a study in a multilingual classroom in New Zealand. They found that due to the fact that English was used the language of teaching and learning, students whose home language was not English had difficulties of understanding the vocabulary used in the mathematics discourse as a whole. They also found that some mathematics terms that are used in everyday contexts caused confusion

for the students in the mathematics classroom. They found that the students who are not first language speakers of English had a 10-15% disadvantage due to language difficulties. The worst part of the problem of language was that the students were not aware of their problem. Reasoning and communicating mathematically may become problematic in a multilingual classroom as shown by the study conducted by Barton and Barton (2005). Language is a tool that is required for one to think and communicate mathematically (Setati, 2005b). Therefore language should not be underestimated in the process of teaching learners mathematics.

1.7.5. Who Will Benefit From the Study?

The study will help teachers understand the complexities of a mathematical Discourse. The teachers will be able to see how reasoning and communicating mathematically may or may not be developed in the classroom. Subject advisors will benefit from the study because they will get information on what teachers emphasise as communication practices and what the textbooks emphasise and thus will be able to make judgements about the relationship between the intended and the implemented curriculum. With this information subject advisors would be able to formulate ways of ensuring that communication practices emphasised by both the teacher and the textbook are those that promote reasoning and communicating mathematically.

1.8. Conclusion

In this chapter, I have presented the problem statement that led to the current research study. I have also stated the aim, purpose and objectives of the study. The research questions that guided the study have been stated and justified in this chapter. The rationale for conducting the study has also been presented in detail. In other words, this chapter has highlighted how the researcher came up with the idea of conducting the study on how learners communicate their mathematical reasoning in a multilingual classroom. Mathematical reasoning is very important in the mathematics Discourse as discussed in the rationale. Therefore, learners should be able to communicate their mathematical reasoning. In the next chapter, I will present the theoretical framework that informed the

study and the literature that was reviewed in relation to how learners communicate their mathematical reasoning in multilingual classrooms.

2. Chapter Two: Theoretical Framework and Literature review

2.1. Introduction

In this chapter, I will discuss the Gee's discourse analysis theory that informed the study. Literature that has been written about how learners communicate their mathematical reasoning will also be discussed in this chapter.

2.2. Theoretical Framework

The notion of communication in this study is informed by Gee (2005) who argues that language is situated. He, therefore, says that in order to study any language that is being used to communicate, we must consider more than the language. In other words, for one to study any language, one has to study the Discourse in which that particular language is used (Gee, 2005). Gee (2005) distinguishes between Discourse with a capital 'D' and discourse with a lower case'd'.

Gee (2005: 36) defines Discourses, with a capital 'D' as "ways with words, deeds and interactions, thoughts and feelings, objects and tools, times and places that allow us to enact and recognize different socially situated identities". Mathematics as a subject has its own Discourse and so learners in a mathematics classroom are expected to use ways, deeds and interactions that are part of the mathematics Discourse. Gee defines discourse (with a lower case 'd') as the actual language that is used in the Discourse. For example, mathematics discourse (with a lower case 'd') refers to mathematical language, e.g. in mathematical reasoning it is the language that is used in proofs and mathematical conjectures that learners may formulate in the classroom.

Gee (2005) discusses the idea of *social language* in order to show that language alone is not sufficient for one to participate in a given Discourse. Social languages are "what we learn and what we speak" (Gee, 2005:38) in a given social setting. For every setting that an individual finds herself in, there is a different social language that one has to use in order to participate in that particular setting. Gee (2005) argues that there is a formal and informal setting. If one finds oneself in either a formal setting or an informal setting, one is expected to use a different social language (Gee, 2005). For example, when a learner is

communicating with the teacher, he may use a different social language that is different from the one which the learner uses when he is communicating with the peer about the same idea because the learner is communicating within two different social settings, with the teacher, and with the peer. Pimm (1991) discusses the informal and formal settings that learners find themselves while at school.

Moschkovich (2003) argues that for learners to take part in a mathematics Discourse, they have to move from an everyday way of talking to a more precise way of using mathematical language. For learners in multilingual classrooms the movement also includes moving between languages (Setati & Adler, 2001) and moving between cultures (Zevernbergen, 2000; Cleghorn & Rollinick, 2002). Cleghorn & Rollinick (2002) refer to the movement between the culture of the home and the culture of the school as 'border crossing'. Second language learners have to do a border crossing as well as moving between their home languages to English (the language of teaching and learning) and between informal and formal mathematics language. This, therefore, means they have to navigate between numerous social languages.

According to Pimm (1991), in a mathematics classroom, learners learn to move from informal spoken languages which they use outside the classroom setting (informal setting) to a formal spoken or written activity which is viewed as a requirement for the learners to participate in the mathematics activities. Learners are also required to speak in a formal way in the mathematics classroom because the classroom is a formal setting (Pimm, 1991). For example, a learner may want to talk about variables in a mathematics classroom, this learner has to be explicit and say 'the variable *x*' and not 'letter *x*' because the word variable makes the language more formal in a mathematics classroom. The movement from an informal way of communication to a formal way is not easy for the learners because they come to school when they are fluent in communicating informally (Pimm, 1991).

Within the mathematics Discourse, there is a '*sub*' Discourse- mathematical reasoning, which is a Discourse on its own because learners are expected to have "ways with words,

deeds and interactions, thoughts and feelings (Gee, 2005) within this Discourse so that they can be identified as participating in that Discourse. Learners have to communicate in the process of participating in the Discourse, and in the process, the learners are said to be communicating their mathematical reasoning. When learners are required to communicate their reasoning, they may use different social languages and these different social languages in the mathematics reasoning Discourse are *inductive* and *deductive* reasoning (Yopp, 2010).

Deductive reasoning is a logical process whereby something that is already known and everyone agrees that it is true is applied to a particular case (Johnston, 2002). For example, if a learner is asked to prove that the sum of two square numbers is always a square number, and the learner uses algebra, or a theoretical understanding of even numbers that are squared to prove the statement, such a learner is reasoning deductively. Deductive reasoning is a reasoning Discourse that is highly valued and acceptable in the mathematics Discourse because deductive reasoning is general and therefore applies to all cases being discussed in a given conjecture (Brodie, 2000). Inductive reasoning is a logical process in which a learner proceeds from particular evidence to a conclusion, which is viewed as true (Johnston, 2002). In other words, inductive reasoning is generally used to prove or establish that a given statement is true for some natural numbers. For example, if a learner is asked to prove that the sum of two squares numbers is always a square number and the learners uses two numbers: 4^2 and 3^2 to prove the statement, then such a learner is reasoning inductively. Muller and Maher (2009) argue that there is increasing evidence that learners can challenge claims by either reasoning inductively or deductively.

Therefore, learners may communicate their reasoning in different ways depending on the setting they find themselves in. Muller and Maher (2009) carried out a study and found that learners' reasoning flourishes when they are given the opportunity to share their thinking in a community, for example in the classroom. Muller and Maher (2009) found that learners naturally used different types of arguments in justifying their solutions to the problem solving tasks. Students used direct reasoning, case-based reasoning, reasoning

by contradiction, upper and lower bounds to support their solutions. These forms of reasoning have certain practice orientation in the mathematical reasoning Discourse.

The types of reasoning that the learners use have different social languages in mathematics reasoning Discourse. Some types of reasoning are formal and others are informal. The act of presenting justifications to the community and listening to the argument provided by others and challenging each others' justifications is a characteristic that is within the mathematics reasoning Discourse (Muller & Maher, 2009). Arguments in a mathematics classroom are a sign that learners are participating in a mathematics Discourse because one characteristic of a mathematics Discourse is being able to provide justification for any mathematical claim (Moschkovich, 2003).

The different social languages that are used to communicate within the mathematics Discourse have different sorts of grammar (Gee, 2005). For example social language that is used in a school setting has a different kind of grammar from the social language used in an everyday informal setting. Pimm (1991) discussed two types of languages that maybe used to communicate within the mathematics Discourse: ordinary language and mathematical language. Ordinary language refers to when English that is not combined with mathematical conventional terms is used to communicate mathematical knowledge, without using the formal terms that are conventional within the mathematics Discourse (Pimm, 1991). For example, if a learner uses words like average instead of mean and number in the middle instead of median, he is using ordinary language. Mathematical language refers to when a learner uses mathematics symbols and terms to communicate mathematical knowledge (Pimm, 1991). For example, if a learner uses terms such as median, mean, mode when communicating mathematical knowledge, then he is using mathematical language. Mathematical language maybe symbolic, in prose or diagrammatic while ordinary language can only be in prose. These two types of languages arise from the different grammar that is used to construct meaning in the process of communication. These two types of languages discussed above can be referred to as social languages within the mathematics Discourse, which learners may use in different social settings within the mathematics classroom.

Gee (2005) also defines Grammar Two as the different expressions that individuals use when communicating. For example, facial expressions, signs and movement of the hands are categorized as Grammar Two. An example of Grammar Two is when a learner uses two fingers to mean number two. In my study, learners may use different facial expressions, signs and movements depending on the social setting they find themselves in. For example a learner may use two signs to refer to one thing depending on who the learner is talking to, the teacher or a peer. Communication of mathematics reasoning may become problematic because of the different Grammar Twos, and the different meanings of the same word in different contexts. Gee (2005) argues that in order for a new social language to be used, the situated meaning of the words must be clearly explained.

In a mathematics Discourse, learners are expected to communicate their mathematical reasoning in two ways, either by writing it down or by orally communicating their reasoning. Gee (2005) argues that an oral or written text may be in one social language or it can switch between different social languages and be able to mix the different social languages appropriately. The process of incorporating different social languages in a text, in an appropriate way is called intertextuality (Gee, 2005). An example of intertextuality in a mathematics Discourse is when learners use information other texts, e.g. their textbooks or from their teacher to support their arguments. In this study, I investigated how learners communicate their mathematical reasoning in oral and written texts.

The construct such as social language, Grammar Two, and the different forms of mathematical reasoning were used in the analytical framework. In other words, the constructs from the theoretical framework guided the study in the process of data collection.

2.3. Literature Review

2.3.1. Multilingualism in South Africa

A classroom is multilingual if any of the learners or teacher is able to draw on more than one language in the process of carrying out activities in the classroom (Setati & Barwell, 2006; Halai, 2009). When learners or the teacher draws on more than one language to make sense of what is required in an activity, it does not follow that in such a classroom, language diversity is an asset (Setati & Barwell, 2006).

South Africa is one of the countries in the world that faces the challenge of providing quality mathematics education for its multicultural society of 43 million people. The rich diversity of this society is reflected by the large number of official languages of the country. English is spoken as a first language by less than ten percent of the population and is the language of business and government. It is also one of two languages usually used at schools although it is not the most widely spoken language at home (Howie, 2003). Setati and Adler (2000) observed the dominance of English in non-urban primary schools in South Africa. They found that in rural areas English is only used in formal settings, e.g. in the school when the learner is answering a question paused by the teacher. Within the multilingual classroom, the language which the learner understands best should be used to help the learner access the knowledge being taught and also the learner should use that language when communicating his mathematical knowledge. In other words, the language which the learner understands best should be used a form of linguistic capital for the learner to be able to understand and contribute during the mathematics lesson (Zevenbergen, 2000).

However, some teachers are faced with dilemmas of deciding when the learners should be given an opportunity to use the languages which they understand best as linguistic capital (Adler, 2001). Within a South African mathematics classroom, some teachers want the children to use English when explaining their mathematical reasoning because the teachers feel that if these learners are denied the chance of becoming fluent in English, which is an international language, then they may not be able to participate with other people outside the classroom setting (Setati, 2005a). The question is: how will the

learners communicate their mathematical reasoning in a language they do not understand?

2.3.2. Communicating Mathematical Reasoning

Brandt and Tatsis (2009) argue that teaching of mathematics must focus on encouraging collective mathematics argumentation and support learners to express their reasoning. It is very crucial that learners are encouraged to verbalise their ideas and thoughts. In other words, the way learners are involved in explaining, reasoning and justifying content related actions in the mathematics classroom is crucial for their learning. In the process of argumentation, explaining and reasoning in the mathematics classroom, learners are encouraged to communicate their mathematics reasoning. However, Brandt and Tatsis (2009) do not give suggestions on how learners should be encouraged to communicate their mathematics reasoning. The issue of language is not acknowledged by Brandt and Tatsis (2009) and yet it is very crucial when learners are being encouraged to communicate their mathematical reasoning.

Involving learners in communicating mathematics reasoning is not straight forward and this is an issue that should be considered when learners are encouraged to communicate their mathematics reasoning. In other words, when learners are encouraged to explain, justify and argue about mathematics content related ideas, they cannot follow the same way of expressing their mathematics reasoning. The learners are most likely to follow different paths in an attempt to communicate their mathematical reasoning as shown by the study conducted by Muller and Maher (2009).

When learners are required to communicate their reasoning, they use different forms of arguments. In a 3-year study that investigated the forms of reasoning used by 24 urban, middle school minority students who worked collaboratively in an informal, after-school program to construct and justify solutions to problems, Muller and Maher (2009) found that students naturally used different types of arguments in justifying their solutions to the problem solving tasks. Students used direct reasoning, case-based reasoning, reasoning by contradiction, upper and lower bounds to support their solutions. They also

found that the act of presenting justifications to the community and listening to the argument provided by others led to students challenging each others' justifications and this led to even stronger arguments (Muller & Maher, 2009). They suggested that students should be encouraged to communicate their reasoning, be given enough time to work on problems and that they should be encouraged to communicate with others and also listen to what others have to say and this is in agreement with Brandt and Tatsis's (2009) argument that learners should be encouraged to take part in collective mathematics argumentations. However, Muller and Maher (2009) do not explain the language structure of the learners in their study. Therefore, the question about whether the language which the learners understand best helps them to communicate their reasoning cannot be answered from their study.

In a study conducted by Edwards (1999) where ten high school algebra students were asked to judge simple statements about combining odd and even numbers, stating whether they were true or false, and give justifications or explanations for their decisions, all the students initially reasoned inductively or empirically, appealing to specific cases and justifying their answers with additional examples. Brodie (2000) argues that children in lower grades are expected to give empirical arguments when communicating their mathematics reasoning. When the students in Edward's (1999) study were prompted for further explanations, seven of the students gave creative coherent arguments without using algebraic notations. Instead the seven students used visual representations of odd and even numbers. The other three students gave informal and partial argument by cases. The argument by cases is similar to the case based proof that Maher and Muller (2009) found in their study. However, Edwards did not describe the languages that learners used while justifying and explaining their reasoning. He also did not state the first language of the learners and the language of teaching and learning.

Healy and Hoyles (2000) conducted a research that focused on proofs at high school level. They found out that students construct better arguments for familiar conjectures than for unfamiliar ones. This means that students always find it easy to communicate

their mathematics reasoning when dealing with mathematical statements that they have seen and used before than one which they have never come across.

Edwards (1999) also found that learners give different forms of arguments when communicating mathematics reasoning but he did not make a distinction between how learners explained their reasoning verbally and in written texts. In a study conducted on communicating mathematics reasoning where first year University student teachers were required to explain and justify their solutions to a word problem, a mathematics task questionnaire was given to learners and they were required to write down their reasoning. After the task questionnaires were analysed, six students were interviewed. The students were required to use English only to communicate their reasoning. The findings showed that these students' mathematics reasoning was different during the interviews (oral communication) from what they had written down (Aineamani, 2010). Therefore, there is a gap in some students' communication of mathematics reasoning when they move from written mathematics to spoken mathematics (Aineamani, 2010). Perhaps if the learners were allowed to communicate their mathematical reasoning in any language of their choice, there would have been a difference in their communication.

2.3.3. Communication in Group Discussions

Learners are most likely to communicate their mathematics reasoning in group discussions than individually. Brijlall (2008) conducted a qualitative study on two Grade 8 mathematics classes (51 learners) at a high school in Pietermaritzburg, South Africa. The school is co-educational, with multilingual classrooms and has mixed socio-economic backgrounds. In his study, learners were divided into two groups, the control group and the experimental group. The control group was given an activity to carry out individually and the learners in the experimental group were allowed to carry out discussions. Data was collected through lesson observation, analysis of learners' worksheets, questionnaires and interviews of eight participants. The study found that learners solve mathematics problems better in groups, and not individually. Learners working in a group were very relaxed and willing to share valuable information with others in order to come to the answer (Brijlall, 2008). Learners easily communicated their

mathematical reasoning while having group discussions. Muller and Maher (2009) also found that learners work better and learn more when taking part in group discussions than when they are doing individual work.

2.3.4. The Teacher's Role in Communication

If learners are given the opportunity to learn and know how to use language to communicate mathematics reasoning, they easily communicate their mathematics reasoning. Mercer and Sams (2006) conducted a study to explore the role of the teacher in guiding the development of children's skills in using language as a tool for reasoning. The study involved 406 children and 14 teachers in schools in Milton Keynes. Observations and formal assessment in experimental and control classes were used to collect data. The experimental classes took part in the thinking together programme while the control classes followed the prescribed National curriculum for year 5. The data gathered included pre and post-intervention video recordings of a focal group in each target class. The study found that providing children with guidance and practice in how to use language for reasoning would enable them to use language more effectively as a tool for working on mathematics problems together. When learners are in a school setting, they struggle to communicate mathematics reasoning in a formal way. Most of the time, the everyday language hinders the formal communication in the school setting (Pimm, 1982). Schleppegrell (2007) argues that the challenges of communicating mathematics reasoning go beyond the language issues but she suggests that linguistic challenges need to be addressed so that learners are helped to construct knowledge about mathematics and be able to communicate the reasoning behind the knowledge that they construct.

2.3.5. Barriers Associated With Communicating in a Second Language

Learners have difficulties in communicating mathematics reasoning in a language that is not their first language. Barton and Barton (2005) conducted a study in four schools and one university over a two year period in New Zealand. One of the aims of the study was to examine the impact and nature of language factors in the teaching of mathematics learners for whom English is an additional language. They used observation,

questionnaires and interviews for data collection. 12 mathematics classrooms were observed and 16 students were interviewed. They found that language features causing difficulties varied across the studies, and appear to depend on the mathematical level as well as the home language and English language proficiency levels. Vocabulary on its own was not the big issue that was anticipated in their study. However, it was a component of the difficulty experienced with understanding mathematical discourse as a whole. Prepositions and word order were key features causing problems at all levels. Learners at senior secondary struggled with communicating mathematics reasoning logically. Mathematics that was integrated with everyday contexts also caused problems for learners that had difficulties with English as a language. Most learners in this study were unable to communicate their mathematics reasoning in English but they were able to communicate their mathematics reasoning in their first languages.

2.4 Conclusion

In this chapter, I have presented the theoretical framework that informed the study, and literature that was reviewed about the study has also been presented. The theoretical framework highlights an important factor of being able to communicate within a given Discourse, and that being able to communicate does not only require language but also other factors such as ways and feelings that are acceptable within a given Discourse. Literature has highlighted different ways in which learners can communicate their mathematical reasoning within the mathematical reasoning Discourse. In the next chapter, I will present literature that has been written about what mathematical reasoning entails and how mathematical reasoning can be enabled or restricted in the mathematics classroom.

3. Chapter Three: Mathematical Reasoning

3.1. Introduction

In this chapter, I will discuss what mathematical reasoning entails, by reviewing literature that has been written about mathematical reasoning. The South African curriculum documents: RNCS and the national curriculum statement (NCS) will also be discussed in detail to show what the documents state about mathematical reasoning in different grades of teaching mathematics.

3.2. Mathematics Knowledge and Reasoning

Mathematics knowledge by its nature has a foundation in reasoning. Reasoning refers to the use of a logical and coherent argument to form conclusions, inferences, or judgments (Ross, 1997). Reasoning can also be defined as the process of drawing conclusions on the basis of evidence or stated assumptions (Martin & Kasmer, 2009). Mathematics relies on logic and it is through this logic that mathematics knowledge can be justified. Without reasoning, mathematicians would not be able to convince other people that their conclusions are true, and make sense (Muller & Maher, 2009). Ross (1997) argues that mathematics lies in proof, yet proof requires reasoning. Therefore, since mathematics as a discipline lies in proof, and since there is no way a proof can be constructed without reasoning, then reasoning is the foundation of mathematics, as argued earlier. Mathematical reasoning refers to thinking through mathematics problems logically in order to arrive at solutions (Selden & Selden, 2003).

3.3. Mathematical Reasoning and Proof

In mathematics, a proof is a convincing argument within the accepted standards of mathematics that some mathematical statement is necessarily true. Proofs are obtained from deductive reasoning, rather than from inductive or empirical arguments (Selden & Selden, 2003). That is, a proof must demonstrate that a statement is true in all cases, without a single exception. In other words, a mathematical proof must be applicable to all cases (Healy & Hoyles, 2000; Selden & Selden, 2003). For example, in mathematics, proving that all odd numbers are not divisible by two, all the odd numbers, and not just one or two, must be included in the proof in order for a proof to be considered as a

mathematical proof. However, mathematical reasoning employs numbers, symbols, inductive, visual, and heuristic inferences (Brodie, 2000). In other words, mathematics reasoning does not have to be a proof, but all forms of explanations that are used to construct a convincing argument can be included in mathematical reasoning.

Mathematical reasoning is very useful in mathematics. Most of the time, in school mathematics, there is solving of problems using conventional algorithms, translating to another setting; looking for patterns; reasoning by analogy; generalizing and simplifying; exploring specific cases; abstracting to remove irrelevant detail. Students are not involved in constructing rigorous proofs (Forman & Steen, 1995). As discussed earlier, mathematical reasoning does not involve construction of only mathematical proofs, but also all the different ways that are discussed above. Packer (1997) argues that constructing mathematical proofs is not what makes mathematics useful but habits such as problem solving and mathematical calculation skills.

Most of the time, people cannot use some form of mathematics without reasoning, e.g. in real life situations such as gambling and investing. The argument that all sorts of activities that involve critical thinking and problem solving are examples of mathematical reasoning can be contested. In school mathematics, the strict sense of constructing formal proofs is not common. Mckenzie (2000) argued that teachers tend to teach their students how to solve problems and not why they are solving problems in the way they are solving those problems. This is the beginning of inhibiting mathematical reasoning in schools. Students who think about what they are doing and why they are doing it are more successful than those who just follow rules they have been taught (Mckenzie, 2000).

3.4. What Counts as Mathematical Reasoning?

In order to answer the question: what counts as mathematical reasoning in school mathematics, it is worthwhile to discuss the nature of mathematical knowledge and what is involved in the learning of mathematics. How an individual thinks about mathematics has important consequences on what the individual will identify as mathematical reasoning. Different authors have given their views about what counts as mathematics

knowledge. For example Kilpatrick, Swafford and Findell (2001) define what counts as mathematical knowledge by five interwoven strands and these strands, they argue, are necessary for learners to successfully learn mathematics.

The first strand is conceptual understanding. This is the ability to comprehend mathematical concepts, operations and relationships (Kilpatrick et al., 2001, p. 116). An example of conceptual understanding in mathematics is when learners are taught algebra, in order to say that they understood the algebra conceptually, they must have understood the meaning of the letters used in algebra, the relationships between the different letters and also the structural meaning of algebraic expressions. If a learner is given an equation such as '$x+4=7$', with conceptual understanding, the learner must be able to recognise the structural and operational relationship between the letter and the numbers, and the algebraic mathematical concept inherent in that equation.

Procedural fluency is another strand. This is the ability to "carry out procedures flexibly, accurately, efficiently and appropriately" (Kilpatrick et al., 2001, p. 116). An example of procedural fluency is when a learner is given an equation such '$x^2+2x+4=0$' and asked to solve for *x*, if the learner has procedural fluency, he will flexibly solve for *x*, following the necessary steps accurately, without encountering any problems.

Strategic competence is the "ability to formulate, represent, and solve mathematical problems" (Kilpatrick et al., 2001, p. 116). An example in mathematics that requires strategic competence is a word problem such as '*Is the mean of the squares of two numbers greater than, or less than, the square of their means*? This problem requires the learner to formulate it using either algebra or theoretical thinking and after that represent the problem using algebraic symbols or mathematical thinking, and then solve it. Therefore, such a problem requires strategic competence.

Adaptive reasoning is the "capacity for logical thought, reflection, explanation, and justification" (Kilpatrick et al., 2001, p.116). An example that requires adaptive reasoning is when a learner is given a problem such as 'In three more years, Jack's grandmother will

be six times as old as Jack was last year. If Jack's present age is added to his grandmother's present age, the total is 68. How old is each one now?', a learner has to think logically before solving this problem, after solving the problem, the learner has to reflect on the answer in order to be check if it makes sense in the context the problem is given. The learner has to be able to explain his answer and give logical justification in an attempt to convince others why his answer is correct.

Productive disposition is a "habitual inclination to see mathematics as sensible, useful and worthwhile and also to see one's self as a doer of mathematics" (Kilpatrick et al., 2001, p.116). An example of productive disposition is when a learner says that he loves mathematics, and he would love to become a real mathematician one day. Such a statement from a learner shows that he sees mathematics as sensible and meaningful, and he sees the importance of doing mathematics.

Mathematics is also discussed in relation to mathematics practices and mathematics is defined as what mathematicians actually do in their work (RAND Mathematics study Panel, 2002). Douady (1997) argues that to know mathematics is a double aspect involvement. It involves firstly, the acquisition of certain concepts and theorems that can be used to solve problems and interpret information at a functional level and the ability to pose new questions. And secondly, knowing mathematics is the ability to identify those concepts and theorems as elements of a scientifically and socially recognised corpus of knowledge. It also involves the ability to formulate definitions, equations, and to state theorems, methods/strategies belonging to this corpus and prove them.

There are types of mathematical knowledge and Piaget (1964) distinguishes three of them:

Social knowledge – depends on the particular culture e.g. culture of mathematics. Social mathematics is 'culture-specific' and can be learned only from other people within one's cultural group." An example of social knowledge is the counting system of the early

Egyptians. The counting system of the Egyptians can only be understood and learnt within the social setting of the Egyptians.

Physical knowledge – gained when one abstracts information about the object itself e.g. colour, shape and behaviour of the object in different situations. It refers to knowledge related to objects in the world, which can be acquired through perceptual properties. The acquisition of physical knowledge has been equated with learning (Piaget, 1964). Physical knowledge is directly related to experience. For example, if a learner is shown the shape of a triangle, he is able to relate to it because he has seen the physical shape which is referred to as a triangle.

Logico-mathematical knowledge – made up of relationships between objects, which are not inherent in the objects themselves but are introduced through mental activity. Logico-mathematical knowledge is abstract and must be invented, but through actions on objects that are fundamentally different from those actions enabling physical knowledge For example, the formation of the concept of a 'triangle' can be shown in triangular shapes such as cardboard cut-outs, three sticks joined to form one, triangular bridge structures or a triangular picture. All these are just representations or models of a triangle not a triangle itself. The concept of a triangle, therefore, resides in the mental representation of the idea that the mind has constructed. More examples of such concepts could be a pi (π), locus or an inequality. So to acquire a concept, a learner needs to experience different situations where an object or element is encountered. Logico-mathematical knowledge is acquired through reflective abstraction depending on the learner's mind and how the learner organises and interprets reality. And an important aspect to note is that, its acquisition without the use of social and physical knowledge as a foundation is bound to be ineffective. So teachers must take into account how these mental representations of the mind are constructed. Kilpatrick et al.'s conceptual understanding which Van de Walle (2004) has termed conceptual knowledge of mathematics is what Piaget (1964) refers to as logico-mathematical knowledge. By its very nature it is the knowledge that is understood.

Drawing from logico-mathematical knowledge, it shows that mathematical concepts have only mental existence because learners cannot see, touch, hear or smell them. So in order

to construct a mathematical concept or relationship the learner has to turn it away from the physical world of sensory objects to an inner world of purely mental objects.

What counts as mathematical reasoning? From the definitions of what counts as mathematical knowledge above, it could be argued that mathematical reasoning is any activity that promotes the five strands given by Kilpatrick et al. (2001), it is any mathematics activity that requires learners to solve problems through argumentation and experimentation, enables learners to acquire logico-mathematical knowledge (mathematical concepts, theorems, strategies, skills, and including beliefs and attitudes). Mathematical reasoning involves carrying out mathematical practices in ways that the makers and users of mathematics do: justifying, making conjectures, explaining, challenging, and solving problems. According to Martin and Kasmer (2009), mathematical reasoning involves informal observations, conjectures, and explanations that are familiar to the learners.

Mathematical reasoning enables learners to interpret information, solve problems and pose new questions. Mathematical reasoning can be provoked by high level demand procedural tasks that seek to enhance understanding and sense-making in learners as they explore relationships and mathematical conceptual understanding and processes (Stein et al., 1996). The high level task questions maybe open ended or explanatory questions which require learners to formulate a way of solving them without relying on already known procedures and calculations (Kilpatrick et al., 2001)

3.5. Types of Mathematical Reasoning

According to Yopp (2010), there are two forms of the reasoning Discourse[1]. These are *inductive* reasoning and *deductive* reasoning.

3.5.1. Inductive Reasoning

[1] Gee (2005: 36) defines Discourses, with a capital 'D' as "ways with words, deeds and interactions, thoughts and feelings, objects and tools, times and places that allow us to enact and recognize different socially situated identities". This is the definition referred to when talking about reasoning Discourse.

Inductive reasoning (IR) is a logical process in which a learner proceeds from particular evidence to a conclusion, which is viewed as true (Johnston, 2002). In other words, inductive reasoning is generally used to prove or establish that a given statement is true for some natural numbers. An example of inductive reasoning is empirical reasoning. In empirical reasoning, the learner uses a particular case to generalize for all cases. For example, if a learner is required to prove that all even numbers are divisible by two, the learner may choose three even numbers say, 6, 8 and 10, and then show that they are divisible by two. Basing on the three numbers, if the learner makes a conclusion, then such a learner is reasoning inductively. Inductive reasoning is viewed as an informal way of justifying a mathematical conjecture. A conjecture refers to reasoning that involves the formation of conclusions from incomplete evidence. It is likely to be true based on available evidence, but it has not been formally proven (Brodie, 2000). Conjectures are powerful tools that allow learners to operate at a higher level of mathematical abstraction.

3.5.2. Deductive Reasoning

Deductive reasoning (DR) is a logical process whereby something that is already known and everyone agrees that it is true, is applied to a particular case (Johnston, 2002). For example, if a learner uses a theory that 'all even numbers are divisible by two' to prove that 20 is an even number, such a learner is reasoning deductively. In other words, when we arrive at a conclusion using facts, definitions, rule, or properties, it is called deductive reasoning. Deductive reasoning is a reasoning Discourse that is highly valued and acceptable in the mathematics Discourse because deductive reasoning is general and therefore applies to all cases being discussed in a given conjecture (Brodie, 2000). For example, 'for all real numbers a and b, $(a + b)^2$ is not equal to $a^2 + b^2$. Justify your answer. In order for a learner to justify his answer, he must use the known convention of difference of two squares. However, if a learner attempts to substitute real numbers for a and b, then the justification cannot be referred to as deductive reasoning. Deductive reasoning follows from a set of already proven conventions. An example of solved problem using deductive reasoning (Highpoints Learning Inc, 2010) is given below:

Prove that $(x \times y) \times z$ is the same as $(z \times y) \times x$ always. Explain your steps.

Solution:

Step 1: $(x \times y) \times z = z \times (x \times y)$, using commutative property of multiplication. **Step 2:** $= z \times (y \times x)$, using commutative property of multiplication again. **Step 3:** $= (z \times y) \times x$, using associative property of multiplication.

From the worked example above, it follows that the proof was constructed using already known mathematical conventions and this solution applies to all cases in mathematics. All the conclusions made follow from a stated rule as shown in the example.

3.6. Forms of Mathematical Reasoning Progression

Martin and Kasmer (2009) argue that student expectations for mathematical reasoning increase in sophistication across grades. They give three forms of progression of mathematical reasoning that students go through, as discussed below.

3.6.1. Empirical Evidence

The role of empirical evidence is to support a conjecture. However, it does not justify a conjecture. Empirical evidence works in a number of cases (Martin & Kasmer, 2009). For example if a conjecture is made such as "The interior angles of a triangle add up to 180 degree", empirically, the student will support this conjecture with an example of considering a triangle with angles 50^0, 40^0 and 90^0, the student will add up the three angles and get a total of 180 degrees. This is an example of empirical evidence. Empirical evidence that is discussed by Martin and Kasmer (2009) is similar to inductive reasoning which is discussed above.

3.6.2. Preformal Explanation

Preformal explanations maybe intuitive, meaning they follow from what a student sees from his common sense (Martin & Kasmer, 2009). Preformal explanations are used by students to give an insight in what is happening in a given mathematics problem. An example of preformal explanation is when a student develops a conjecture about the angle of rotational symmetry for a given regular polygon and may be able to explain why that angle works.

3.6.3. Formal Argumentation

Formal argumentation is based on logic and its role in mathematics is to make statistical inferences. An example of formal argumentation is a mathematical proof. Formal argumentation must apply to all cases (Martin & Kasmer, 2009). Deductive reasoning, as discussed earlier is an example of formal argumentation. Martin and Kasmer (2009) argue that students move through empirical evidence, to preformal explanations and lastly to formal argumentation. Therefore, all the three forms of justification in mathematics are very important.

3.7. The South African National Curriculum Statement and Mathematical Reasoning

The South African curriculum documents: RNCS, and the (NCS) encourage teachers to teach learners mathematical reasoning. This is seen across all the documents of different levels, from Foundation Phase (FP) across to Grade 12.

In the assessment framework for FP (Grade 1 to Grade 3), which derives its knowledge and skills from the learning outcomes of revised national curriculum statement for languages and mathematics (Grades 1-3), there is a clear emphasis on teachers giving learners problems to solve, and giving the learners an opportunity to explain their solutions. As discussed earlier, mathematical reasoning involves the learners explaining and justifying their solutions to mathematical problems. The assessment framework, which was released to focus the system on the improvement of learner performance in literacy and numeracy emphasises the importance of mathematical reasoning. Throughout all the grades, learners are encouraged to solve problems and explain their solutions in all the different sections taught (DoE, 2008).

The RNCS of Grade R-9, which was written before the assessment framework discussed above, defines mathematics and it gives the skills that are required for mathematics learning. The skills given are problem solving, investigation, reasoning and communication among others. This is a clear indication that the RNCS recognises the

importance of mathematical reasoning and communication. The RNCS states that one of the unique features of teaching and learning mathematics is:

"investigating patterns and relationships: describing, conjecturing, inferring, deducing, reflecting, generalising, predicting, refuting, explaining, specialising, defining, modelling, justifying and representing" (DoE, 2002, p. 5).

The statement above shows that the RNCS values mathematical reasoning since activities such as, explaining, justifying, deducing, conjecturing and reflecting are referred to as unique features of mathematics learning and teaching. These activities as discussed earlier are used by Kilpatrick et al. (2001) to explain adaptive reasoning, which is one of the strands of mathematics proficiency. Development of this strand is an indication that a leaner can communicate mathematical reasoning.

The RNCS also states that learners in mathematics classrooms must "display critical and insightful reasoning and interpretative and communicative skills when dealing with mathematical and contextualised problems" (p.5). This implies that learners have to develop adaptive reasoning, strategic competence and procedural fluency (Kilpatrick et al., 2001).

The RNCS emphasises investigation activities because investigations develop mathematical thinking skills such as generalising, explaining, describing, observing, inferring, specialising, creating, justifying, representing, refuting and predicting (DoE, 2002: p.9). Activities that require learners to carry out investigations are viewed as important by the RNCS as activities that promote mathematical reasoning.

In FP (Grade 1-3), learners are required to be able to solve and explain solutions to practical problems. There is an emphasis of learners being able to explain their own solutions to mathematical problems. In intermediate phase (Grade 4- 6), the RNCS states that learners should recognise, describe in own words, reflect, compare and interpret mathematics ideas (DoE, 2002). In this phase, the idea of learners being involved in mathematical reasoning is not so pronounced. Most of the activities are geared towards learners describing without explaining their descriptions. In the Senior Phase (SP) (Grade

7- 9), the RNCS states that the learner should develop the ability to reason effectively and justify appropriately when required. In all the learning outcomes, there is an emphasis of the learners being able to describe, explain and justify observed relationships or rules in own words (DoE, 2002). In the document, there is a clear indication of emphasis of mathematical reasoning. Whether it really takes place in the mathematics classrooms as the document states is an interesting thing to find out.

The NCS of Grade 10 to 12 states that "Competence in mathematical process skills such as investigating, generalising and proving is more important than the acquisition of content knowledge for its own sake" (p.9). This statement implies that mathematical reasoning is an important skill in the process of acquiring mathematics knowledge. Mathematics knowledge becomes meaningful if a learner is able to explain why that knowledge is true. The statement also emphasises that learners must be able to effectively communicate conclusions and predictions made from analysis of data, in the data handling learning outcome. Learners have to reason mathematically in order to make predictions. However, the NCS puts more emphasis on learners being able to solve problems, demonstrate an appreciation of mathematical ideas and calculating and representing mathematics data. The mathematical reasoning is not emphasised in the activities suggested in the NCS, and this is a contradiction of the statement which was quoted earlier, saying that mathematical reasoning is very important (DoE, 2003). Therefore, it may not be a big surprise if teachers do not require their learners to explain and justify their solutions in the classroom since the curriculum statement does not show how important it is, unlike in the RNCS where it is emphasised in every learning outcome, for every grade level.

3.8. Ways of Enabling or Restricting Mathematical Reasoning

There are different ways in which a teacher can enable to restrict mathematical reasoning in the classroom. According to the socio-cultural theory of learning, learning takes place through social interaction (Vygotsky, 1978). If learning is to take place in a social setting, there are certain concepts that have to be fulfilled in order for the learning to be

successful. During classroom interaction, a variety of ways exist in which teachers may go about trying to understand and develop learners' mathematical reasoning.

One of the ways a teacher may use during classroom interaction with the learners is listening to what students are saying. Davis (1997) identified three categories of listening: evaluative listening, interpretive listening and hermeneutic listening. Evaluative listening results from the teacher posing questions that lead to an expected response from the learner. This form of listening focuses attention on hearing a 'correct' response and hence overlooking the important information from the learner's understanding (Davis, 1997). This type of listening classifies learners' answers as correct or wrong and hence reasoning is not encouraged. Interpretive listening is as a result of the teacher seeking information from the students instead of a predetermined correct response. The teacher may ask the learner to interpret a given answer or clarify on the answer given hence mathematical reasoning maybe developed depending on the type of questions asked by the teacher. Hermeneutic listening is where the teacher is part of the learning community and he/she listens and responds to learners using questions that develop thinking (Davis, 1997). The three types of listening will be used as concepts for analysis during data analysis.

The notion of telling may also be used by a teacher during classroom interaction with the learners. Lobato, Clarke and Ellis (2005) discussed the different ways of initiating and eliciting, and the notion of telling in teaching. These are: initiating by describing a new concept; initiating by summarizing student work so that new information is inserted; providing information so that students can test their ideas. An example of initiating by describing a new idea is when a teacher is teaching data handling as a topic and he defines the term 'mode' to the learners as a new concept without asking the learners whether one of them knows what mode means. A teacher is said to be initiating by summarising student work so that new information can be inserted if he listens to what the student is saying and then repeats the same definition or explanation of an idea but this time with clarity and correct terminology. For example, if a student says that 'when we are looking for median, we must arrange the numbers first', the teacher then says

'good, we must arrange the numbers in ascending or descending order first'. A teacher is said to be providing information so that students can test their ideas if for example the teacher, after teaching the difference of two squares, asks learners if 'x^2-y^2 = (x^2-y^2)', with this question, the teacher has already provided learners with the knowledge of difference of two squares and he is now providing them with information in order for them to test their knowledge of difference of two squares.

Depending on how these concepts are used in the classroom, they may help the teacher to develop mathematical reasoning or they may hinder mathematical reasoning in the classroom. The different ways of telling as developed by Lobato et al. (2005) will be used as concepts of analyzing the transcripts during data analysis.

3.9. Discursive Practices

Discursive practices are linguistic and socio-cultural characteristics of recurring episodes of face-to-face interaction and episodes that have social and cultural significance to a community of speakers. Discursive practices that can be found in a mathematics classroom are: verbal expressions which maybe oral or written, verbal exchange in conversation between the teacher and the learners or amongst the learners, and a formal length discussion of an idea in writing or orally (Lerman, 2003). As learners communicate their mathematical reasoning, they may use prose, symbols or diagrams within the mathematics Discourse.

The socio-cultural theory emphasises learning through social interaction. One form of social interaction is classroom conversation. During the lesson, learners are sometimes engaged in mathematics conversations. Not all conversations lead to development of mathematical reasoning (Brijllal, 2008).

The teacher's questions during the lesson may or may not give learners an opportunity to communicate their mathematical reasoning. If the teacher asks questions which require the learner to give a short and direct response, without asking the 'why' question as a follow up, then learners will not communicate their reasoning. The types of questions which the teacher asks in the classroom may be informed by the way he listens to the

learners' responses (Davis, 1997). The RNCS and the NCS do not state the types of questions which the teacher has to ask during the class. However, since the statement shows that learners have to be able to explain their solutions, then teachers have to ask the learners for their explanations in some way that is stated by the RNCS and the NCS (DoE, 2002; DoE, 2003).

Explaining mathematical reasoning requires the learners to give a justification why they think that their solution to the mathematics problem is correct. This explanation may be informal or formal, as discussed earlier. Asking learners to explain their reasoning gives them an opportunity to communicate their mathematical reasoning (Brodie, 2000). However, the kinds of questions that are given to learners to should be formulated in such a way that they give learners an opportunity to communicate their reasoning. Open ended questions are examples of questions that require the learners to communicate their reasoning since there is no predetermined answer that the teacher expects from the learners (Stein et al., 1996). An example of an open ended question is: What does the mean of the given set of data inform you about the students' performance in the mathematics classroom. In order for the learner to be able to answer such a question, he must know what mean is and 'why' mean is calculated the way it is calculated. In the process of the learner answering the question, he is communicating his mathematical reasoning about mean. Explanatory type of questions (Brodie, 2000) also promotes mathematical reasoning. For example if the learner is asked to explain his reasoning about the answer which he has got, he is communicating his mathematical reasoning about that particular concept.

Giving learners responsibility for their learning requires the teacher to let learners be active and take part in the lesson, the teacher should not tell learners everything. Learners should be given a chance to take part in mathematics talk. Group discussions can be used to give learners an opportunity to be responsible for their learning. Brijllal (2008) argues that learners are responsible for their learning if they are in group discussions and they freely communicate their mathematics reasoning. The revised national curriculum statement (RNCS) emphasises group discussions and encourages teachers to use

discussions groups during the lesson (DoE, 2002). However, if group discussions are not monitored, the learners may go off task and this could be time wasting (Brijllal, 2008).

3.10. Conclusion

Reasoning is part of mathematics as a discipline. The way students communicate their reasoning should be an important concern to the teacher. Students should be discouraged from attempting a mathematics problem without communicating their reasoning. Justifying, explaining and conjecturing should be a culture in any mathematics classroom in order for learners to think critically about what they are doing. The ways in which learners communicate their mathematical reasoning helps them to think through mathematics problems logically in order to arrive at solutions. In this chapter, I have presented what mathematical reasoning entails and how it can be restricted or enabled in the mathematics classroom. In the next chapter, I will present the methodology that was used in the study.

4. Chapter Four: Research Design and Methodology

4.1. Introduction

In this chapter, I describe the research design, the methods of data collection and the analytical framework that guided the process of data analysis. I also describe my sample and what informed the choice of the sample, the research design and the methods of data collection in relation to the research questions:

Main research question: How do learners in a multi lingual school in South Africa communicate their mathematical reasoning?

Critical questions:

- What are the communication practices of the classroom of learners who learn mathematics in a first and second language?
  - (i) What communication practices are legitimised by the teacher?
  - (ii) What communication practices are legitimised by the textbook?
- How do these learners communicate their reasoning in
  - i) Written texts?
  - ii) Oral texts with their:
    - a) Teacher?
    - b) Classmates?
- What languages do they use in communicating their mathematical reasoning in formal and informal mathematical discussions?

4.2. Research Approach

The study is informed by Gee's (2005) notion of discourse, in particular, Gee's argument that language is situated. Therefore, in order to investigate how learners communicate their mathematical reasoning, in a classroom situation, I selected a qualitative approach for my research. The approach used a naturalistic paradigm to understand the investigated phenomena in a context-specific setting (real world settings) like a classroom. Schumacher and McMillan (1993) define a qualitative study as "naturalistic inquiry" (p.372) and this inquiry uses noninterfering data collection strategies to discover the

natural flow of events and processes and how the participants interpret them. They argue that a qualitative research views reality as a multilayered, interactive and a shared social experience interpreted by individuals. A qualitative study emphasizes the importance of looking at variables in the natural setting in which they are found (Opie, 2004). Therefore, the main goal of a qualitative study is to understand the social phenomenon from the view of participants. Understanding is acquired by analysing the many contexts of the participants and by narrating the situations and the events. Feelings, beliefs, ideas and thoughts should be captured in order to gain an understanding of the participants' experiences. In other words, a qualitative study seeks to understand people in their real setting and the reality changes with the changes in people's perceptions. The researcher therefore observed how learners communicate mathematics reasoning in their natural setting of the classroom, without trying to manipulate the learners' behaviour during the lessons.

The researcher was a non-participant observer (Opie, 2004), and, therefore, could not manipulate the phenomena of interest (Patton, 2002). It is important to ensure that the classroom context is kept natural for reliability purposes. Schumacher and McMillan (1993) argued that a qualitative research has to use noninterfering data collection strategies in order to keep the naturality of the context.

In a qualitative approach, "the researcher is the instrument" (Patton, 2001, p.14), and the aim of the research is to probe for deeper understanding rather than examining surface features (Patton, 2001). According to Schumacher and McMillan (1993), the researcher should get immersed in the phenomenon being studied and he should assume interactive social roles in which he records observations and interactions taking place among the participants. In this study, the researcher was actively involved in data collection by recording and observing the interaction of the participants.

A qualitative study was appropriate for this study because of the aim of the study which is to understand how learners communicate mathematical reasoning in a multilingual classroom. In order to understand how learners communicate their mathematical

reasoning, they have to be observed in their natural setting, the classroom, and noninterfering data collection strategies have to be used. The learners' interactive and a shared social experience interpreted by the learners had to be observed and recorded for a better understanding of the phenomenon being studied.

4.3. Research Method

The research method used was a case study. A case study was appropriate for the study because this study is an investigation on a real situation, with real people in an environment that is often familiar to the researcher (Opie, 2004). The study took place in a classroom environment, familiar to the researcher who is a mathematics educator, and the classroom was a real lesson, not prepared for only data collection.

In a case study, there is an opportunity to study one aspect in some depth within a limited timescale (Bell, 1999). The aspect being studied is how learners communicate mathematical reasoning. One classroom was chosen in order for the researcher to have enough time to study how the learners communicate their mathematical reasoning, in depth, which may not have been possible if two or more classes were chosen as cases in this study. Schumacher and McMillan (1993) argue that a case study provides a detailed description and analysis of the processes voiced by the participants in the situation being studied.

The focus of the study was not to make generalizations but on understanding the particulars of the case in its complexity. A case study focuses on a bounded system, usually under natural conditions, so that the system can be understood in its own original and not artificial setting (Merriam, 1988). Human relations and interactions are studied in a unique way in a case study (Opie, 2004). The classroom was studied in its natural conditions and analysis of the processes in the classroom was done.

4.4. Sampling

Due to the complexity of following learners who are attempting to communicate their mathematical reasoning in a mathematics multilingual classroom, a manageable sample

was selected to take part in the study. Purposeful sampling was conducted in this study. According to Schumacher and McMillan (1993), purposeful sampling refers to selecting a case which is rich in information and the aim of this sampling is not for finding generalizations. The main advantage of purposeful sampling is that it increases the utility of data collected from a small sample. This is because the sample chosen must be knowledgeable and informative about the phenomenon being studied. Before a sample is selected, researcher has to find information about all the subunits available. From the various subunits, the researcher then selects the case that is rich in information. (Schumacher & McMillan, 1993). Below is a detailed description of how and why the sample was selected.

4.4.1. The School

The sample comprised one school in a township setting. The school is located in a township, East of Johannesburg. Most of the learners in the school come from an informal settlement. From the description by the teacher, the majority of the learners are from child headed families, and they struggle to cope with school work. There is high rate of absenteeism which affects the performance of the school. In the same school, most learners drop out, especially when they reach Grade 11. The school was selected for the following reasons:

- It is well resourced, with dedicated teachers. Therefore, factors such as lack of resources were eliminated in the study.
- The school has a high admission of learners from different language backgrounds. Therefore, since my study is located in a multilingual classroom, there is a high chance of finding different languages in the classroom.
- The school was in close proximity to the researcher, therefore, it was convenient and the researcher was in position to arrive early for the lesson observations.

The school has four Grade 11 streams of mathematics. There are two mathematics teachers who teach Grade 11 mathematics classes in the school. One class was selected out of the four Grade 11 streams. The learners in this class were more active than all the other streams, according to the head of the mathematics department in the school. The teacher had taken a university course that stressed the importance of allowing learners to

communicate their reasoning. Therefore, the researcher made an assumption that the teacher would not teach traditionally whereby learners are not given an opportunity to communicate their reasoning. This teacher was selected because there was a possibility that she would apply her new acquired knowledge about enabling mathematical reasoning in the learners during the mathematics lessons. The class was also chosen because the teacher said that among all the four streams of Grade 11, the selected stream was the most active and academically focused stream. Therefore, since the study used purposeful sampling, this class was chosen because it was seen as the case that was rich in information about the phenomenon being studied (Schumacher & McMillan, 1993).

4.4.2. The Participants

Only one class of Grade 11 learners, aged 14 to 16, was selected to take part in the study. One class was selected out of the four classes available because the researcher wanted to carry out a detailed study of the class. All the 25 learners in the class were focused on during the first lesson observations. In other words, all the learners were participants in the study. With time, the learners who were active during the lessons were observed consistently because they were carrying out activities that involved communicating mathematical reasoning. The participants comprised 9 girls and 16 boys. The gender biasness was inevitable since the gender was not a criterion for sample selection. In the classroom, the participants had a variety of home languages as shown below:

- IsiZulu- 14 learners
- IsiXhosa-3 learners
- SiSwati-1 learner
- SiSotho-3 learners
- Sepedi- 4 learners

Grade 11 learners were selected because at this level, they are required to reason and communicate mathematics in a formal way since the learners are about to go to Grade 12. Therefore Grade 11 learners are most likely to be constrained by the mathematics language. Therefore, the study employed a purposeful sampling of the participants. The learners were seated in groups of four during the lesson. The learners were seated in

groups of four during the lessons observed, and the teacher used the following strategies during the lesson:

- Reviewing previous work
- Introducing the work for the lesson
- Giving learners examples while writing on the board
- Giving learners time to discuss problems given in the textbook used in the classroom, in their groups.
- Allowing learners to come to the board to explain to their classmates.

Therefore, the teacher gave the learners an opportunity to participate during the lessons by allowing group discussions and allowing learning to come to the board to the class to explain a given problem.

4.3. Data Collection

Data was collected over a period of two weeks. The methods of data collection included lesson observations and documentation, such as learner textbook and notebooks. All the data collection was conducted by the researcher. Below is a detailed discussion of why these methods of data collection were used in this study.

4.3.1. Classroom Observations

Classroom observations were carried out during the mathematics lessons. Observation was used in this study for data collection because of the following advantages as discussed by Opie (2004):

- The researcher is able to record the information about the environment and the human behaviour of the participants directly.
- Observations allow the researcher to see the familiar as "strange".
- The researcher is able to collect data on the environment and behaviour of the participants that cannot speak or will not be able to speak for themselves.
- Observational research data is a very useful check for supplement data collected using other methods.

However, observational research has disadvantages (Opie, 2004), as shown below:

- The participants may change their behaviour in presence of the researcher and this affects the naturality of the environment.
- It requires a lot of time.
- The observer may misinterpret the activities being observed.

The lessons observed lasted for 60 minutes each, which was the duration for a double period in the school. A video recorder was used to capture the data during observation. The video recorder was used because of the following reasons:

- To answer the main question of the study, which is: How do learners in a multi lingual school in South Africa communicate their mathematical reasoning? I had to record the aspects of behaviour (Opie, 2004) which were be related to communicating mathematical reasoning. For example, when learners were explaining their answers to the teacher and their classmates. I had to focus on the main aspects related to my research questions, during video recording.

- The study was situated in a multilingual classroom and so the issue of language was very important. Plowman (1999) argues that video recording is particularly important in studies where the researcher is interested in language and interaction. My third research question is: What languages do learners use in communicating their mathematical reasoning in formal and informal mathematical discussions? In order to answer this question, I had to observe learners interacting with each other, and with their teacher in the classroom. A video recorder was very suitable for my study because I was able to record the interactions taking place in the classroom exactly as they happened, including the languages being used since the digital video recorder records voices as well.

- The data from the video would also help me answer my question which is: How do learners communicate their mathematical reasoning orally with their teacher, and their classmates? Using an observation form would be an option but some

most of the data would be omitted, especially the verbal interaction among the learners (Opie, 2004).

- According to Opie (2004), using a digital video recorder helps the researcher to make sense of non-verbal activities taking place during the lesson. In my study, the non-verbal activities taking place are very important in the process of analysing how learners communicate their mathematical reasoning. These activities are referred to as a Grammar Two[2] (Gee, 2005) in my theoretical framework. Therefore, in order to capture the different non-verbal activities of learners in the classroom, I had to use the digital video recorder since all the other available instruments such as a questionnaire are not able to capture the activities exactly as they are occurring.

The video was placed at different angles in the classroom, to focus on the learners, and the teacher especially when the teacher was interacting with the learners. All the learners were captured, though the focus was on the learners that were actively taking part in the lesson, through explaining and sharing their views with the teacher and their peers. Because the researcher was the recorder, field notes were taken after every recording. During some lessons, there was a reasonably long period of classroom interaction whereby the teacher gave learners an opportunity to interact with their peers in their groups. In this case, the video recorder was zoomed out to be able to capture some of the interesting group discussions, in relation to the focus of the study, which is communicating mathematical reasoning.

4.3.2. Piloting

Video recording was piloted for two days prior to data collection, in a different class, with the same class size as the class of the participants. This was very helpful because it

[2] Grammar Two refers to the different expressions that individuals use when communicating. For example, facial expressions, signs and movement of the hands are categorized as Grammar Two (Gee, 2005).

informed the researcher of the different positions in which to place the video recorder for clear voice recording and the best view of learners, and the teacher during the lesson. After the piloting, the data were transcribed in order to identify the weaknesses of the video recording. The following points about the video recorder came up from the pilot study:

- The written work on the chalk board had to be included in order to make sense of the interaction that was going on in the classroom.
- Some learners seemed to be discussing actively in their groups, yet they were off task, as shown by the transcriptions.
- The teacher was conscious of the video and he kept on changing to see if the video was still focusing in him. This distorted the naturally of the lesson. On the second day of piloting, the teacher seemed not care about the presence of the video, at least not as he did in the first lesson.
- During some of the interactions in group discussions, I had to re-position the video recorder in order to capture the group discussions that were far from the original position of the video recorder.

However, the data collected from the pilot study were not transcribed.

The researcher observed the classroom continuously for two weeks, though four lessons were recorded. The participants were very cooperative and willing to take part in the study. None of the participants pulled out during data collection. Therefore, the process of data collection was successful and smooth since there was no resistance from the participants during the study.

4.3.3. Documentation

In order to answer my research questions, I had to observe how the learners were communicating their reasoning orally and in written texts. Therefore, learners' mathematics note books were collected, with permission from teacher, and the learners themselves. I selected six notebooks, two from the most active learners in the class, two from the learners that participated, but not so frequently, and the last two books were selected from learners that did not share their views with the teacher and their peers in the

classroom. The active learners' books were selected in order to check how their oral communication and written communication is linked. The books of the learners that participated rarely in the classroom were selected to follow up their few oral communications with their written communication of mathematical reasoning. The books of the learners who did not communicate their reasoning orally in the classroom were selected to find out how their written mathematical reasoning differed from those learners that communicated their reasoning orally in the classroom. Therefore, the notebooks were selected purposefully after observing the learners' participation during the lessons. The notebooks of the learners which were selected were photocopied and the books were given back to the learners on the same day. The mathematics textbook which the teacher and the learners used was also collected for document analysis.

4.3.4. The Mathematics Textbook

The mathematics textbook used in the classroom by the teacher and the learners as reference was necessary for the study because one of the research questions was 'What communication practices are legitimised by the textbook?' To answer his question, I had to analyse the textbook used in this classroom. The use of the textbook by the learners as a reference is known as intertextuality (Gee, 2005). Intertextuality is an important construct for this study because it shows what informs how learners communicate their mathematical reasoning. In the classroom observed, the teacher, and the learners used a textbook known as Classroom Mathematics. The class used only this textbook and it was used all the time in the classroom. The teacher used the book during her teaching; the learners opened each page that the teacher would open during the lesson. The teacher also referred the learners to the page numbers for exercises and home work. The textbook (Classroom mathematics) used by the teacher and the learners was briefly analysed to determine the kinds of communicative strategies it legitimises.

All the data that was collected through classroom observation was transcribed and analysed.

4.4. Data Analysis

As discussed earlier, the study was qualitative and so I analysed the data using qualitative methods. I used the typological analysis method discussed by Hatch (2002) to analyse my data. Typological analysis is where data analysis is started by dividing the collected data into a set of categories that are based on predetermined typologies. The typologies are generated from the theory, common sense and objectives of the research. I used this method because the topics in mind are usually the logical places to start in the process of analysis (Hatch, 2002). The data analysis happens within the generated typological groupings (Hatch, 2002). Therefore, I viewed my data, including the transcribed portions of the video, and "divided it into elements based on predetermined elements" (Hatch, 2002: 152).

In my study, I was interested in how learners communicate mathematical reasoning in multilingual classrooms in South Africa. I went through the video to locate every time a learner or learners were communicating their mathematical reasoning. I then made connections to other factors (Opie, 2004) such as language, social setting to identify the social language (Gee, 2005). For example if learners were communicating mathematical reasoning to their teacher, were they using formal social language? I then followed the steps (see table 4.1) in typological analysis as given by Hatch (2002: 153). The typologies were developed from the theoretical framework. Below is a table showing how the typologies were developed and later used for analysis.

Table 4.1: Steps in Typological Analysis

Step	**Activity**
1	Identify typologies to be analysed e.g. formal social language (deductive and inductive reasoning), informal social language (deductive and inductive reasoning), use of Grammar Two, and language used to communicate mathematics reasoning. These categories were generated from my theoretical framework and common sense (Hatch, 2002).
2	Read the data, marking entries related to my selected typologies

3	Read entries by typology, recording the main ideas in entries on a summary sheet
4	Look for patterns, relationships, themes within typologies
5	Read data, coding entries according to patterns identified and keeping a record of what entries go with elements of your pattern
6	Decide if my patterns are supported by the data, and search the data for no examples of your patterns
7	Look for relationships among the patterns identified
8	Write my patterns as one sentence generalisations
9	Select data excerpts from transcribed data that support my generalisations

Before thinking about the process of analysis, I made backup copies of the video in order to avoid disappointments of losing the data (Plowman, 1999). I used the backup copies to transcribe the video for analysis purposes. Transcribing the video data was labour intensive and it required a lot of time. I spent one month transcribing the data. On the day of recording, I viewed the video while the classroom environment was fresh in my mind and I made some notes on the activities that I recalled from the field. These notes were also used during the process of analysis. The transcribed data, the learners' photocopied notebooks and the textbook were analysed. The analysis was guided by the analytical framework that was developed based on Hatch's (2002) typological analysis, for purposes of data analysis.

4.4.1. The Analytical Framework

The analytical framework was informed by the constructs from Gee's theory of Discourse analysis. Below is a discussion of the analytical framework that was used to analyse the data in the study.

Table 4.2: Analytical Framework

Construct	Definition	Why this specific construct	Indicator	Guiding Questions
Classroom Discourse	Ways with words, deeds and interactions, thoughts and feelings, objects and tools, times and places that allow us to enact and recognize different socially situated identities	Mathematics as a subject has its own Discourse and so learners in a mathematics classroom are expected to use ways, deeds and interactions that are part of the mathematics Discourse to understand what the practice of the classroom is in terms of language and how these practices are manifested or influence how learners communicate their mathematical reasoning	The way learners interact with each other and with their peers during the lesson, -the activities which learners do during the lesson -teacher's explanations -sitting arrangements -teaching method -use or non use of textbooks (how frequent and for what purpose)	• What teaching strategies does the teacher use during the lesson? • How do learners interact with each other? • How they interact with the teacher during the lesson? • What mathematical (reasoning) activities are learners engaged with? • How are learners involved in the classroom activities? • Is the classroom environment conducive for mathematical reasoning? • How are learners inducted into the Discourse of mathematical reasoning? • What mathematical reasoning tasks are learners given? • What formal writing techniques are emphasized? • What teaching strategies does the teacher use? (strategies will include re-voicing)
Classroom discourse	The language used in the classroom	Within a Discourse, there is language-in-use, therefore, there is need to document what languages are used and for what purpose	-The language learners use when communicating their reasoning -language the teacher is using for teaching -Language the teacher is using for keeping order -language learners use for mathematical arguments -language learners use to present their written work -the language learners use in their oral communication in group discussions -language learners in oral public presentations	1. How is language used to construct their arguments? By the learners or the teacher 2. What languages do learners use when communicating their reasoning? 3. Which language is used by the teacher for: (i) keeping order (ii) teaching 4. What ways of communication are legitimized by the teacher?

Construct	Definition	Why this specific construct	Indicator	Guiding Questions
Social language	The different ways of speaking in a given social setting. For every setting that an individual finds him/herself in, there is a different social language that is used in order to participate in that setting.	To ascertain the different ways learners use language to communicate mathematical reasoning with each other and with their teacher and in different settings.	- formal spoken language -informal spoken language -formal written language -informal written language - language spoken with peers or with the teacher (African language or English) -using terminology that seeks to gain solidarity from peers.	• What words do learners use when they communicate mathematics informally? • What language is accepted by the teacher during the lesson? • What language do learners use when communicating with their teacher? • What language do learners use when communicating with their peers? • How do learners present their written texts? Note: There is an overlap between social language and classroom discourse
Grammar Two (collocational patterns)	Different expressions or patterns that individuals use when communicating that which can attribute situated social identities and specific identities to them. For example, facial expressions, signs and movement of the hands	To identify ways of communication through patterns of talk or writing that learners use to communicate their mathematical reasoning. For example, how they use different facial expressions, signs and movements depending on the social setting they find themselves in to make a point.	Nodding head in agreement or disagreement, using hands to illustrate a point, using facial expressions to stress a point. Use of visuals, e.g. pictures, graphs etc.	1. What other "stuff" is used to communicate mathematical reasoning in the classroom? 2. Do learners use facial expressions communicating mathematical reasoning? 3. Do learners use visuals such as pictures and graphs when communicating their mathematical reasoning?

Construct	Definition	Why this specific construct	Indicator	Guiding Questions
	when used to give a certain meaning to an utterance or how certain words in an utterance co-locate with one another to give a situated meaning.			
Deductive reasoning	Logical process whereby something that is already known and everyone agrees that it is true is applied to a particular case.	To document the extent to which this form of reasoning is used and how language is used to communicate it. Deductive reasoning is a reasoning Discourse that is highly valued and acceptable in the mathematics Discourse because deductive reasoning is general and, therefore, applies to all case being discussed in a given conjecture.	-using already known definitions to reason and make an argument -using formulae to explain -using a conventional presentation of an argument (written or oral) -classroom legitimized presentation of mathematical reasoning -informal presentations	• What language (informal or mathematical) is used to communicate this form of reasoning • How are argument presented by the (i) textbook (ii) teacher (iii) learner • Are arguments presented in a mathematically conventional way? • What proofs are legitimized? • How are these proofs presented? Do the presentations conform to conventional ways of presenting?
Inductive reasoning	Logical process in which a learner proceeds from particular evidence to a conclusion, which is viewed as true.	To document the extent to which this form of reasoning is used and how language is used to communicate it. Inductive reasoning is viewed as an informal way of justifying a mathematical conjecture. Therefore, to	generalizations are made - activities used to form a conjecture - conjectures formed - general conjectures (prevalent) -the way textbook leads learners to a conjecture - the legitimized conjecture	• How does the learner justify his/her arguments? • What does the learner use to justify his/her reasoning? • Does the learner make generalisations? • Does the textbook use specific examples to justify a concept? • Does the teacher legitimize generalisations?

Construct	Definition	Why this specific construct	Indicator	Guiding Questions
	An example of inductive reasoning is empirical reasoning. In empirical reasoning, the learner uses a particular case to generalize for all cases.	communicate this type of reasoning, a certain social language is used. However, this social language is informal though it is acceptable in the mathematics Discourse but it does not apply to all cases being discussed in a given conjecture.		
Intertextuality	How language is used to allude to other texts. An example of intertextuality in a mathematics Discourse is when learners use information from different texts to formulate an argument	To establish if and how learners draw from other texts when justifying their arguments.	-use of quotes -use of words or visual representations from other texts (e.g. the textbook) -using words of other people (e.g. teacher, learners, etc) -Writing texts from the teacher's notes and from the textbook. -Quoting the teachers or their peers	• Do learners quote their teacher's words when making an argument? • How do learners use information from the textbook to make their arguments? • Do learners use information they learnt outside the classroom setting?

The analytical framework above was constructed using the Gee's notion of Discourse analysis. According to Gee (2005), in order to analyse the Discourse of a given setting, certain aspects have to be considered. The aspects which were considered for analysis in this study to analyse the classroom Discourse of how learners communicate their mathematical reasoning in a multilingual classroom, as shown in the table above were: classroom discourse, social language, inductive reasoning, deductive reasoning, intertxtuality and Grammar Two (Gee, 2005). The data was analysed based on the predetermined analytical framework shown above. Deductive and inductive reasoning were categorised under different levels as shown below. Classroom discourse was also categorised as shown in table 4.4. The guiding questions in the analytical framework guided the researcher in the process of analysis. The explanations of the categories given in table 4.3 and table 4.4 were also used to analyse the data.

Levels of Mathematical Reasoning

From the definitions of inductive and deductive reasoning given in the table 3 above, when learners communicate their mathematical reasoning, the reasoning can be categorised under different levels as given below.

Inductive Reasoning Levels

Level one (L1IR)

Level one reasoning is when a learner communicates his reasoning without giving a justification which makes sense in relation to the problem being solved. An example of level one reasoning is when a learner is asked to explain why 10 is not the median of the data set: *3, 4, 6, 1, 2, 8, 2, 9,* and the learner says that it is not the number in the middle of the data without giving a convincing justification, then the reasoning is under L1IR. In other words, level one reasoning is a short answer response without a justification. L1IR can be referred to as direct argument (Muller & Maher, 2009).

Level two (L2IR)

Under level two inductive reasoning, the learner gives one or two examples to justify his point. An example of L2IR is when a learner is asked to explain why 10 is not the median of the data set: *3, 4, 6, 1, 2, 8, 2, 9,* and the learner uses another data set such as *4, 6, 7, 9, 1, 2* to show how median should be calculated and then from the two data sets, make a conclusion about why 10 is not the median. In other words, level two inductive reasoning is a response with a partial justification which is not entirely convincing. L2IR can be referred to as lower case bound reasoning (Muller & Maher, 2009).

Level three (L3IR)

Level three inductive reasoning, learners give sufficient empirical examples to justify their mathematical reasoning and they explain their reasoning in detail without leaving out any idea that is used in solving the problem. L3IR can be referred to as upper case bound reasoning (Muller & Maher, 2009) because it is bound to a specific data set and not for all data sets. Martin and Kasmer (2009) also refer to this type of reasoning as empirical reasoning. An example of L3IR is when a learner is asked to explain why 10 is not the median of the data set: *3, 4, 6, 1, 2, 8, 2, 9,* and the learner uses three or more data sets such as *3, 5, 7, 8, 2*; *4, 7, 9, 10, 6, 7*; *7, 1, 8, 2, 4, 3* to show how median should be calculated and after the learner makes a conclusion as to why 10 is not the median of the data given from the various examples given.

Deductive Reasoning

Level one deductive reasoning (L1DR)

Under level one deductive reasoning, the learner gives an explanation of the problem by basing the reasoning on a conventional definition of ideas. For example when a learner is asked why 10 is not the median of the data set: *3, 4, 6, 1, 2, 8, 2, 9,* using level one deductive reasoning, the learner will give the mathematical definition of median and then go on to explain why 10 cannot be a median of this data set using the calculation of this particular set.

Level two deductive reasoning (L2DR)

Level two deductive reasoning is where the learner gives explanations based entirely on the conventional definitions of mathematical terms and he relies entirely on the conventional definition and not on the given problem to make his point clear. For example when asked to explain 10 is not the median of the data set: *3, 4, 6, 1, 2, 8, 2, 9,* the learner will define what median is and then conclude that if 10 does not fall under the definition of median under the given data set, then it cannot be the median of that data set. The difference between level one and level two is that under level one, the learner still relies on the given data set to show that 10 is not the median while under level two, the learner focuses more on the definition than the given problem to justify his reasoning and under level two the learner may not work out the problem to get the median.

Level three deductive reasoning (L3DR)

Level three deductive reasoning involves the learner justifying his reasoning using the appropriate conventional language that is acceptable as formal mathematical reasoning language. Level three deductive reasoning includes only formal proofs as discussed earlier in the chapter. Deductive reasoning, also called Deductive logic, is reasoning which constructs or evaluates deductive arguments. In level three deductive reasoning, the learner justifies his reasoning following deductive arguments which are valid in the mathematics Discourse. The conclusion under L3DR follows from a premise which is logical and predetermined. L3DR is what Martin and Kasmer (2009) refer to as formal reasoning. An example of L3DR is when a learner is asked to explain why 10 is not the median of the data set: 3, 4, 6, 1, 2, 8, 2, 9, and the learner uses the conventional definition of calculating median which is:

If you have values x1, x2, ..., and xn (n is a positive integer) where they are arranged in numerical order (either xi <= x(i+1) for i = 1, 2, ..., n-1 -or- xi >= x(i+1) for i = 1, 2, ..., n-1), a simple formula to calculate the median is to first determine if n is even or odd. If odd, just return the "middle" value - i.e., xj where j is n/2 rounded up to the next integer. If n is even, simply take the mean of xk and x(k+1) where k = n divided by 2 (Gladwell, 2006).

This is an example of how L3DR should be presented by learners, and the example above shows the conventions of formal reasoning in a mathematical Discourse. The reasoning and explanation has to be generalised to cover all cases within the section, in this case median, that is being discussed.

Table 4.3 below shows a summary of the different levels of mathematical reasoning as discussed above.

Table 4.3: Levels of Mathematical Reasoning

Level	**Explanation**
Inductive reasoning	
Level one inductive reasoning (L1IR)	Level one reasoning is when a learner communicates his reasoning without giving a justification which makes sense in relation to the problem being solved. This form of reasoning is categorised under level one reasoning. In other words, level one reasoning is a short answer response without a justification.
Level two inductive reasoning (L2IR)	Under level two inductive reasoning, the learner gives one or two examples to justify his point. In other words, lever two inductive reasoning is a response with a partial justification which is not entirely convincing.
Level three inductive reasoning (L3IR)	Level three inductive reasoning, learners give three or more empirical examples to justify their mathematical reasoning and they explain their reasoning in detail without leaving out any idea that is used in solving the problem. The examples given under level three inductive reasoning should be able to lead to a conjecture or a generalisation.
DEDUCTIVE REASONING	
Level one deductive reasoning (L1DR)	Under level one deductive reasoning, the learner gives an explanation of the problem by basing the reasoning only on a conventional definition of ideas, a concept or a conjecture without elaboration.

Level two deductive reasoning (L2DR)	Level two deductive reasoning is where the learner gives explanations based entirely on the conventional definitions of mathematical terms and he relies entirely on the conventional definition and not on the given problem to make his point clear.
Level three deductive reasoning (L3DR)	Level three involves the learner justifying his reasoning using the appropriate conventional language that is acceptable as formal mathematical reasoning language. Level three deductive reasoning includes only formal proofs as discussed earlier in the chapter. In other words, level three deductive reasoning involves abstract reasoning.

Levels of Language Used in the Classroom

The language that is used by the learners to communicate their mathematical reasoning within the mathematics reasoning Discourse is referred to as discourse, with lower case d (Gee, 2005). Within the mathematics classroom, learners may use formal or informal language to communicate their mathematical reasoning (Pimm, 1991).

Pimm (1991) discusses two types of languages which are ordinary language and mathematical language. Ordinary language refers to the language where the learner uses words, phrases and sentences that do not have mathematical terms to define mathematics ideas. Ordinary language can also be referred to as informal language (Pimm, 1991; Moschkovich, 2003). Mathematical language refers to the language where the learner uses terms that are conventional within the mathematics Discourse to communicate mathematical ideas. Mathematical language is also referred to as formal language within the mathematics Discourse. The two languages were used as constructs during the analysis in order to find out if learners communicate their mathematical reasoning formally or informally. These two constructed were informed by classroom discourse which refers to the language in use in the classroom (Gee, 2005). Below is a table showing how the language used in the classroom was coded for data analysis purposes.

Table 4.4: Levels of Language Used in the Classroom

Level	Comment
Cd0	The response of the learner is referred to as Cd0 if it does not make any sense in relation to the question given and also in relation to the mathematics classroom discourse
Cd1	The response is referred to as Cd1 if the learner uses ordinary language (OL) to respond to the question given. When the learner uses ordinary language to respond to the question, he does not use mathematics terms.
Cd2	The response is referred to as Cd2 if the learner uses mathematics language (ML) to respond to the question asked. This is also referred to as the formal language in the mathematics classroom discourse.

4.5. Validity and Reliability Issues

Although the terms reliability and validity are concepts used in quantitative research, Patton (2001) argues that a qualitative researcher should be concerned about reliability and validity while designing a study, analysing the results and also judging the quality of the study. In qualitative studies, the researcher is the instrument for data collection. Therefore, in qualitative research, validity and reliability means credibility of the research and the credibility of a qualitative research depends on the efforts of the researcher (Golafshani, 2003). Although reliability and validity are treated as separate terms in quantitative research, these terms are not viewed separately in qualitative research. Instead, terms such as credibility and trustworthiness are used in qualitative research (Opie, 2004; Golafshani, 2003).

4.5.1. Reliability

Reliability is used in a number of different ways; however, there are some commonly occurring terms such as consistency and repetition. Opie (2004) defines reliability as the extent to which a method or tool gives constant results each time it is used in the same context by different researchers. On the other hands, Schumacher and McMillan (1993)

define reliability as the extent to which an independent researcher could discover the same phenomena and to which there is an agreement on the description of the phenomena between the researcher and participants. In both definitions, repetition is emphasized. Opie (2004) argues that reliability should be used to judge data gathering processes and not the product. Schumacher and McMillan (1993) argue that reliability in qualitative research refers to the "consistency of the researcher's interactive style, data recording, data analysis, and interpretation of participant meanings from the data" (p.385). Reliability is concerned with whether we would obtain the same results if we could observe the same thing twice (Opie, 2004). Therefore, data is said to be trustworthy if the findings are consistent and hence reliable.

Ensuring Trustworthiness in the Study

Schumacher and McMillan (1993) state the strategies that can be used to minimize threats to reliability and hence ensure trustworthiness in a qualitative research. These strategies are described below and how they were used to minimize threats to reliability in this study.

The social relationship of the researcher with the participants during data collection was stated earlier. In this study, the researcher was a non participant; therefore, to minimize reliability threats, the researcher's role in the data collection process was stated. The criteria, rationale and decision process that was used in purposeful sampling was explained in detail by the researcher. The data collection strategies and data analysis strategies used in the study was explained by the researcher, and the analytical premises were also explained (Macmillan & Schumacher, 1993).

To ensure that the data collected is trustworthy, mechanically recorded data was used (video recorded data), and the data was transcribed verbatim. Quotations were taken from the collected documents and the textbook without modifying them. In the classroom, the learners used some languages which were not known by the researcher. The researcher therefore gave the data to an individual who was proficient in the languages which the learners were using to help with the transcribing and translating the language to English.

After transcribing, the transcribed data was given to two other people who are proficient in the languages to check whether the transcribed data and the translated data were accurate.

4.5.2. Validity

Validity is defined by Macmillan and Schumacher (1993) as the appropriateness of the conclusions that are made from the data collected in a given study. There are different types of validity but not all of them were applicable to this study.

Internal Validity (Credibility)

Internal validity refers to the extent to which the explanations of the phenomena being studied suits the reality of the world. For internal validity to be achieved in a qualitative study, the researcher and the participants must have a mutual meaning of the interpretations of the phenomena being studied. Internal validity is also referred to as the credibility criterion and it involves establishing that the results of qualitative research are believable from the perspective of the participant in the research. As discussed earlier, the purpose of qualitative research is to understand the phenomena of interest from the participant's eyes. Therefore, the participants are the only ones who can truly judge the credibility of the results (Golafshani, 2003).

Ensuring Credibility in the Study

Credibility can be achieved through lengthy data collection periods, carrying out field research, disciplined subjectivity on the part of the researcher (Macmillan & Schumacher, 1993).

In order to minimize threats to credibility, I recorded accurately and I developed the records during rather than after data gathering sessions. I made a rough draft of the study before going to the field to collect the data so that the data gathering process focused on the information that met the specific needs of the study (Wolcott, 1990). I also included primary data in the final report in order to allow the reader to see exactly the basis upon which my conclusions were made (Wolcott, 1990). I carried out field research by asking

the teacher and some learners about their languages spoken in the classroom, and the general performance of learners in the school.

Transferability

External validity in quantitative research refers to the extent to which the results from the study can be generalized to other settings. This is referred to as transferability in qualitative research (Golafshani, 2003), and this in most cases is not the aim of a qualitative case study (Opie, 2004). Transferability may be enhanced by researcher through describing the research context and the assumptions that were central to the research, and then the choice of "transferring" the results to a different context is left to the readers to decide whether it makes sense to transfer the results (Opie, 2004). The researcher thoroughly defined the context and assumptions that were central to the study.

Confirmability

Objectivity refers to the degree to which the obtained results can be checked for truthfulness (Golafshani, 2003). In order to ensure objectivity in this study, the researcher reported all the negative instances that contradicted prior observations during data collection, and the potential for bias or distortion was also acknowledged by the researcher (Opie, 2004).

4.6 Ethical Considerations

Ethical considerations require sensitivity on the part of researcher in order to show respect to the participants in the research. The participants should not be forced to take part in the study. When it comes to observing the classroom of the participants using a video recorder, permission should be sought from the participants and if they are not willing, they should not be forced (Opie, 2004).

A school was selected for the study as discussed earlier in the chapter, and then permission was sought from the principal using a written consent form. When the principal agreed, the researcher then sought permission from the Department of Education to conduct a study in a public school. The Department of Education agreed by

sending a letter which permitted the researcher to go ahead (See Appendix A). The letter from the Department of Education was taken to the school principal. After seeking permission from the Department of Education and the Principal of the school, the researcher went to the classroom where data was to be collected. Written consent forms inviting the learners to take part in the study were given out and the teacher of the class was also requested for permission to observe her lessons. The consent forms included a section explaining the research as well as what would be expected from the learners during the process of data collection. The learners were also given consent forms to be taken to their parents. Permission for video recording was sought from the learners, the parents of the learners and the teacher who was teaching the class (see Appendix A). The consent forms stated clearly that the participants were not to be forced to take part in the study, the participants were free to withdraw at anytime during the study and the learners who never wanted to take part in the study would not be disadvantaged in any way.

Before the process of data collection began, I collected the consent forms from the learners and I went through both the learners and parents' forms to check for any parent or learner who was not interested in the study. All the 25 parents were interested in the study because they signed the forms and agreed their children to be part of the study. The learners in the class also agreed to take part in the study and to be video recorded as well. The teacher also accepted to take part in the study.

The researcher then briefly discussed the issues of confidentiality and anonymity with the learners and the teacher. These issues were stated on the information sheet which was handed out together with the consent forms. The participants were informed that their names would not be used during the report of the finding (see Appendix A).

In the analysis, all the learners were referred to using pseudonyms and the teacher's name was also not stated anywhere in the report. The video that was recorded was not shown to anyone except to the two people that helped with transcribing of the isiZulu words which were in the video.

4.7. Conclusion

In this chapter, I have explained the research design as well as motivation as to why the study was conducted the way it was conducted. I have described some of the issues, at a personal level and as a researcher which I had to grapple with during the process of conducting the study. In the next chapter, I will present the analysis of the data that was collected using the methodologies discussed in this chapter.

5. Chapter Five: Data Analysis and Presentation

5.1. Introduction

This chapter presents the process of data analysis and the findings that emerged from it. Data analysis revealed that learners in a township multilingual school could only communicate their mathematical reasoning within Levels 1 and 2 in a topic on Data Handling that was observed over four days. Although the presentation is done in a linear format, the process involved moving to and fro between the stages of analysis. The research questions that guided the study will be used to present the findings of the study. The research questions that guided the study are:

Main research question: How do learners in a multi lingual school in South Africa communicate their mathematical reasoning?

Critical questions:

- What are the communication practices of the classroom of learners who learn mathematics in a first and second language?
  - (i) What communication practices are legitimised by the teacher?
  - (ii) What communication practices are legitimised by the textbook?
- How do these learners communicate their reasoning in
  - i) Written texts?
  - ii) Oral texts with their:
    - a) Teacher?
    - b) Classmates?
- What languages do they use in communicating their mathematical reasoning in formal and informal mathematical discussions?

The textbook used in the classroom was Classroom Mathematics Grade 11 (Laridon et al., 2006). The textbook legitimised some form reasoning by asking questions that required learners to communicate their mathematical reasoning (See Appendix C). The teacher asked questions and guided the learners in the process of the learners communicating their mathematical reasoning. A few learners in the class seemed uninterested during the lesson and they remained silent throughout all the lessons that

were observed and recorded. The teacher gave learners work to do in their groups during the lesson moved around the class checking what learners were writing down in their notebooks and she marked the learners' work which she found to be correct. The findings of the study that will be discussed in this chapter are: the teacher aided learners in the process of mathematical reasoning, learners communicated their written mathematical reasoning using prose, the learners communicated their mathematical reasoning up to level 1 and 2 under inductive reasoning, and only up to level 1 under deductive reasoning. The learners communicated their mathematical reasoning using classroom discourse one (Cd1) because they used only ordinary language to communicate their mathematical reasoning. During the lesson, learners used both Zulu and English to communicate their reasoning. In this chapter, I present the findings of how learners communicate their mathematical reasoning in a multilingual classroom in South Africa.

5.2. Back ground of the Study

Only one class of Grade 11 learners, aged 14 to 16, was selected to take part in the study. All the 25 learners in the class were participants during the first lesson observations. The class comprised of only black learners. The language of teaching and learning in this class was English. The learners' first languages in this class were: Is Zulu, Is Xhosa, Sepedi, Siswati, and Sisotho. There were five first languages in the classroom. The teacher's first language was Is Zulu though she could speak English, Sepedi and Is Xhosa. The topic of discussion in the class was Data handling and the topic was not observed to the end because the teacher gave the learners a break of one week giving learners exercises to consolidate what she had taught.

The learners were seated in groups of five during the lesson. English was the most used language during the lesson though learners sometimes used Zulu. The teacher used only English throughout all the lessons that were observed. The teacher started her lessons by reviewing the work that had been done in the previous lesson (see transcript in Appendix B). When data collection started, the teacher had covered the introductory part of Data handling, the topic which was being taught in the class. The teacher referred to the textbook- Classroom Mathematics most of the time, during the lessons which were

observed. The teacher asked learners questions and learners were active and they answered the questions, though the teacher did most of the talking in the lessons (see transcripts in Appendix B). The teacher gave learners an opportunity to come up to the chalkboard to explain their answers. When learners came to the board to explain their reasoning, they were assisted by the teacher and the other learners in the class. As the learners explained their reasoning in front of the class, they used facial expressions, body movements, for example movements of the hands and nodding of the head in agreement or disagreement with classmates as they communicated their mathematical reasoning. This is what I referred to earlier as Grammar Two (Gee, 2005). Throughout the four lessons that were observed, the teacher asked a total of 5 questions that required learners to communicate their mathematical reasoning (See Appendix D).

5.3. The Process of Data Analysis

My study is qualitative and so I analysed my data using qualitative methods. I used the typological analysis method discussed by Hatch (2002) to analyse my data. Typological analysis is where data analysis is started by dividing the collected data into a set of categories that are based on predetermined typologies. The typologies are generated from the theory, common sense and objectives of the research. I constructed an analytical framework from the theoretical framework as discussed in the earlier chapter. The data analysis happened within the generated typological groupings (Hatch, 2002). All the typologies were generated from the theoretical framework that guided the study.

In my study, I was interested in how learners communicate mathematical reasoning. I went through the video to locate every time a learner or learners are communicating their mathematical reasoning. I then tried to make connections to other factors (Opie, 2004) such as language, social setting to identify the different aspects such as the social language (Gee, 2005) which learners were using or the types of reasoning which learners were using.

In order to answer my research questions, I analysed the mathematics textbook which the learners and the teacher were using in the classroom to find out what forms of

mathematical reasoning the textbook legitimised, I analysed work from the learners notebooks to find out how they communicate their mathematical reasoning in written form and what their teacher legitimised in their notebooks, and then I analysed the transcripts to find out how the learners communicated their reasoning orally with peers and with the teacher, and what the teacher legitimised during the discussions.

5.3.1. Transcription

The process of data analysis proceeded both during and after the process of data collection. The first step that was taken was to transcribe all the classroom observations in order to have written texts. Before thinking about the process of analysis of the transcripts, I made backup copies of the video in order to avoid disappointments of losing the data (Plowman, 1999). I used the backup copies to transcribe the video for analysis purposes. Transcribing the video is labour intensive and it requires a lot of time. However, it is not necessary to transcribe everything that was recorded. It is always important to view the video and then decide on which sections to transcribe according to the aim of the study (Plowman, 1999). I reviewed the video first before transcribing so that I could identify the different sections where learners were communicating their mathematics reasoning. After viewing the video, I decided to transcribe all the lessons that were recorded without picking out some sections because in most sections, the learners were communicating their mathematical reasoning. On the day of recording, I viewed the video while the classroom environment was fresh in my mind and I made some notes on the activities that I recalled from the field.

The process of transcription was done individually, though there were parts where the learners used languages which were not familiar to the researcher. The researcher identified the language used by learners as Zulu by asking two people who could speak the language, and then transcription of those parts was done by first language speakers of the language which the learners were using. Zulu was then translated into English by the same people who transcribed. The transcripts were then given to two other people to check whether the language was transcribed correctly hence ensuring face validity (Opie, 2004). During the process of transcribing, consistency was the main aim in order to

produce accurate re-presentation of the videotaped lessons (Plowman, 1999). Conventions to ensure consistency in the transcripts were constructed. For example, the process of transcription ensured that all the words which the learners said were written as they were said without changing anything. The Zulu words were written in Zulu and the English words were written in English. The table 5.1 below presents the conventions that were constructed by the researcher.

Table 5.1: Conventions in the Transcribed Data of the Study

Symbol	**What the symbol stands for**
()	Action which the speaker is carrying out
[]	Translation from Zulu to English
L1, L2, L3, L4	Lesson names, from lesson one (L2) to lesson four (L4)
...	Interruption from another speaker
(...)	Inaudible

After the process of transcribing, the transcripts were revisited several times to minimise errors and the researcher listened to the video tapes while checking the transcripts in hand. The most common errors were omissions or repeated words, misspelt words and length of silence from the speakers (Gorden, 1980). In the process of reviewing the transcripts and listening to the video tapes over and over again, the researcher was able to reduce the errors and non verbal cues were added wherever they were found to be necessary.

One of the difficulties during transcribing was the issue of learners using languages other than English. This is common in most multilingual classrooms (Setati, 2005a). The language other than English was translated successfully as discussed earlier. The Zulu

words which were used by the learners were short phrases or just a single word such as *ne* meaning ‘okay’. The researcher faced some challenges because the language which learners used was not known to the researcher. Therefore, the researcher had to find ways of overcoming these challenges such as finding people to transcribe the data which had a language that was not familiar to the researcher, and ensuring that the transcribed data is accurate by checking with other people who knew the language.

5.3.2. Learners’ Note Books

From the analysis of the textbook, five questions that required the learners to communicate their mathematical reasoning were identified. Six notebooks of the learners were selected during data collection and an analysis was carried on these books. The notebooks were collected from the two most active learners during the lesson, the two average active learners and the two learners that were silent during the lesson.

However, the data was not sufficient because 4 of the learners out of the 6 learners had not answered the questions that required them to communicate their reasoning. Therefore, I went back to school and collected 10 notebooks from the top learners because I realized from my analysis of the 6 notebooks I had collected earlier, that the top two learners are the ones who had answered the questions. From the 10 notebooks I collected, only 7 learners had answered two questions out of five completely and three learners answered the third question, the 7 learners include the 2 learners whose notebooks I had collected at first.

An analysis of the notebooks collected from the top 7 learners in the classroom was conducted. Learners’ notebooks were analysed to find out how they communicated their mathematics reasoning in response to three questions in the textbook. In other words, using three questions from the textbook that required learners to communicate their mathematical reasoning, I analysed the learners’ notebooks to find out how they communicated their mathematical reasoning with regard to the questions in the textbook. The three questions were selected out of the 5 that were identified because the learners had answered the three questions and left the other three blank. The learners’ notebooks

were also analysed to find out what form mathematical reasoning communication the teacher legitimised in the learners' notebooks. The names used in the analysis are pseudonyms.

5.4. Communication Legitimised by the Textbook

Classroom mathematics Grade 11 (Laridon et al., 2006) is the textbook that was used in the classroom where data was collected. The teacher used the textbook to give learners problems to work on during the lesson. The textbook was analysed to answer the research question about the communication practices legitimised by the textbook. By communication practices legitimised by the textbook, the researcher was interested in finding out what the textbook promotes as a way of communicating mathematical reasoning.

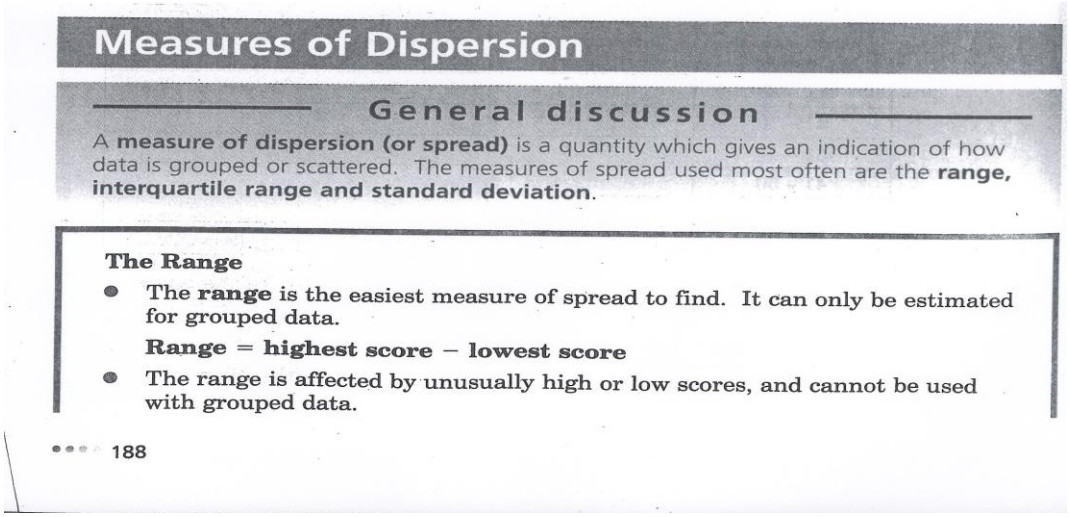
Classroom mathematics is a mathematics textbook which is widely used in South African classrooms. The textbook has three main sections: activities, exercises, and general discussion sections. The textbook uses the general discussion section to define terms that are to be used within a topic, the textbook gives exercises which follow after the general discussion and these exercises are meant for the students to test their understanding of the terms defined in the general discussion. The activities are also used in the textbook to test student understanding of the concepts being discussed in the textbook.

During the lessons observed, Data Handling was the topic that the teacher was teaching. Therefore, the analysis of the textbook focused on the Data Handling chapter in the textbook.

Under the data handling chapter, the textbook has a section of 'general discussion' (see figure 5.1) which gives definitions of the terms, for example, the definitions of *mean* and *range* are given under the General Discussion section. The formulae for finding *mean*, *range* and other concepts being discussed in the data handling topics are also given in the General Discussion section. However, the textbook simply gives a procedural way of working out the problems. The textbook does not go in detail of why the *mean*, the *range*

and the other terms being discussed are solved the way they are solved. Therefore, there are no explanations that might influence certain forms of mathematical reasoning.

Figure 5.1: General discussion section from the textbook- Classroom mathematics



Measures of Dispersion

General discussion

A **measure of dispersion (or spread)** is a quantity which gives an indication of how data is grouped or scattered. The measures of spread used most often are the **range, interquartile range and standard deviation**.

The Range

- The **range** is the easiest measure of spread to find. It can only be estimated for grouped data.

  Range = highest score − lowest score

- The range is affected by unusually high or low scores, and cannot be used with grouped data.

188

The explanation of concepts and the examples given in the textbook that are given under the data handling topic are all procedural as shown in figure 1 above. The explanations are referred to as procedural because they require learners to memorise them and if required, the learners can simply reproduce the explanations (Kilpatrick et al., 2001) as given in the textbook.

The activities and the exercises in the textbook were analysed to find out the extent to which learners were required to communicate their mathematical reasoning when answering the questions in the activities.

While some of the activities in the textbook require the learners to communicate their mathematical reasoning, other questions require learners to carryout procedural manipulations (see figure 5.2) because the questions required the learners to recall a given formula or way of working out a given problem and reproduce it to answer the given question (Kilpatrick et al., 2001).

Figure 5.2: An example of a question in the textbook activity that requires procedural manipulation

1. Find the mode, median and mean of the following values: 1; 5; 7; 3; 5; 9; 5; 8; 10

In the question shown in figure 2 above, the learners are not required to communicate their mathematical reasoning, they are required to manipulate symbols and recall the formula for calculating mode, mean and median in order to come up with the answer. However, not all the questions in the textbook required learners to manipulate symbols in order to find the solution. Some of the questions in the activities and the exercises were formulated in such a way that learners were required to communicate their mathematical reasoning in the process of answering the questions. Below are some of the questions in the activities that required learners to communicate their reasoning.

Questions that require Deductive Reasoning

Deductive reasoning is a logical process whereby something that is already known and everyone agrees that it is true, is applied to a particular case (Johnston, 2002). For example, if a learner uses the conventional definition of mean to solve a question that is asking about mean of a given data set. In the textbook, below are the questions that required learners to communicate their reasoning deductively.

"c. Use the measure of central tendency to answer the following question: "Were the ages of the men in the ward on that night more or less than the ages of the women in the ward on that night?" Give a reason for your answer."

"4. What do the mode, median and mean tell you about the average number of people in the cars during the time that the survey took place?"

"4. What do the mode, median and mean tell you about the width of the leaves on the geranium plant investigated by David?"

In order for the learners to be able to answer the questions, they had to know what mean, mode and median refer to. The definitions of mean, median and mode are conventional in mathematics and therefore, answering these questions required the learners to use the conventional definitions of mean, mode and median to communicate their reasoning. Therefore, these questions above required deductive reasoning (Johnston, 2002). The two question 4s above required learners to communicate their mathematical reasoning about mode, mean and median because they are open ended questions (Brodie, 2000). Open ended questions do not give learners any clue on how to answer the question. Therefore, the learners were given an opportunity to choose a way of answering the question. Such open ended questions require learners to have strategic competence (Kilpatrick et al., 2001) and learners must therefore reason mathematically in order to answer the question, as discussed earlier in the chapter on mathematical reasoning. The questions also required logical thought and hence reasoning on the part of the learner because he has to make his argument convincing (Yopp, 2010). Because the learner has to know what mean, median and mode are, this question requires more deductive reasoning than inductive reasoning (Martin & Kasmer, 2009).

Questions that Require Inductive Reasoning

Inductive reasoning is a logical process in which a learner proceeds from particular evidence to a conclusion, which is viewed as true (Johnston, 2002). In other words, inductive reasoning is generally used to prove or establish that a given statement is true for some natural numbers. An example of inductive reasoning is empirical reasoning. In empirical reasoning, the learner uses a particular case to generalize for all cases. For example, if a learner is required to explain why a certain number is not the mean of a given data set, using inductive reasoning is when the learner uses the given data set to explain why the number is not the mean, instead of using the conventional definition which would make the explanation deductive. The questions from the textbook that required learners to communicate their mathematical reasoning inductively are given below.

> *"d. Do you think that the test shows that the average life span of a bulb is over 200 hours? Explain your answer."*

" a) In 1.a) Sipho gets a mean of 6, 2. Explain why this must be wrong.

"b) In 1. b) Explain how Sipho can write down the correct mean immediately."

Learners were required to explain their thinking and these questions required learners to communicate their mathematical reasoning, because when learners are required to give an explanation of why their answers are wrong or correct, there is some logical thought and reflection that is required hence adaptive reasoning (Kilpatrick et al., 2001). These questions above require inductive reasoning because the learner may use an empirical example to explain why the mean that Sipho got is wrong (Martin & Kasmer, 2009) or give an empirical example of the average span (Brodie, 2000). The questions required learners to communicate their reasoning because the learners were expected to give an opinion and justification. Once learners are required to give an explanation for the answer they get, they have to reflect and think logically about why they answered the way they did (Kilpatrick et al., 2001) and in the process, they communicate their mathematical reasoning. The learners may use inductive reasoning to explain their answer to this question by using specific examples that show how the life span of an item is determined by looking at the mean. A question which asks learners what they think about what the average is showing does not require learners to rely on any conventional definition although the learners may use the definitions to justify their reasoning. Such questions can be said to require high form of reasoning because learners are given the liberty to decide what to use when justifying their mathematical reasoning (Stein et al., 1996). In other words, in such questions, learners' reasoning is not limited to specific ways.

From the questions shown above, the textbook legitimises both deductive and inductive reasoning. Questions that require learners to construct formal proofs are not included in the textbook. For example, questions such as: show that the mean of grouped data is calculated using the formula $\sum fx$ *divide by n*, are not included in the textbook. In other words, the textbook does not require learners to prove the formulae which they use in the process of calculating. This maybe as a result of the demands from the curriculum document as discussed earlier in the mathematical reasoning chapter. The curriculum

emphasizes procedural work more than the reasoning work (DoE, 2003), and therefore there is no point in the textbook having questions about proof construction. However, the openness of the questions given by the textbook is sufficient enough for the learners to answer those using proofs if necessary, though the curriculum at Grade 11 does not require learners to construct proofs to communicate their mathematical reasoning.

5.5. The Classroom Discourse

The classroom Discourse includes ways with words, deeds and interactions, thoughts and feelings, objects and tools, times and places that allow learners and the teacher to enact and recognise different socially situated identities (Gee, 2005). Mathematics as a subject has its own Discourse and so learners in a mathematics classroom are expected to use ways, deeds and interactions that are part of the mathematics Discourse to understand what the practice of the classroom is in terms of language and how these practices are manifested or influence how learners communicate their mathematical reasoning. Within the classroom, the teacher legitimised a certain form of communication orally and in the written work of the learners. To analyse the classroom Discourse, two constructs were considered: Reasoning in the classroom and the language used in the classroom. The reasoning in the classroom was looked at under two forms: Inductive reasoning and Deductive reasoning. The two categories were divided into different levels are discussed earlier in chapter four. Below is a summary showing the levels of inductive and deductive reasoning that were used during the analysis of the classroom Discourse.

Levels of mathematical reasoning

Level one inductive reasoning (**L1IR**)- Level one reasoning is when a learner communicates his reasoning without giving a justification which makes sense in relation to the problem being solved.

Level two inductive reasoning (**L2IR**)- Under level two inductive reasoning, the learner gives one or two examples to justify his point. In other words, lever two inductive reasoning is a response with a partial justification which is not entirely convincing.

Level three inductive reasoning (**L3IR**)- Level three inductive reasoning, learners give three or more empirical examples to justify their mathematical reasoning and they explain their reasoning in detail without leaving out any idea that is used in solving the problem.

Level one deductive reasoning (**L1DR**)- Under level one deductive reasoning, the learner gives an explanation of the problem by basing the reasoning only on a conventional definition of ideas, a concept or a conjecture without elaboration.

Level two deductive reasoning (**L2DR**)- Level two deductive reasoning is where the learner gives explanations based entirely on the conventional definitions of mathematical terms and he relies entirely on the conventional definition and not on the given problem to make his point clear.

Level three deductive reasoning (**L3DR**)- Level three involves the learner justifying his reasoning using the appropriate conventional language that is acceptable as formal mathematical reasoning language.

The language used to communicate mathematical reasoning by the learners was also considered during the analysis. The language used in the classroom was referred to as classroom discourse (Gee, 2005). The language was also categorised under three different levels: classroom discourse zero (Cd0), classroom discourse one (Cd1) and classroom discourse two (Cd2) as shown below.

Levels of Classroom Discourse

Classroom discourse zero (**Cd0**) -The response of the learner is referred to as Cd0 if it does not make any sense in relation to the question given and also in relation to the mathematics classroom discourse.

Classroom discourse one (**Cd1**)- The response is referred to as Cd1 if the learner uses ordinary language (OL) to respond to the question given. When the learner uses ordinary language to respond to the question, he does not use mathematics terms.

Classroom discourse two (**Cd2**)- The response is referred to as Cd2 if the learner uses mathematics language (ML) to respond to the question asked. This is also referred to as the formal language in the mathematics classroom discourse.

Using the levels of reasoning and the levels of classroom discourse discussed above, the data was analysed as discussed below.

5.5.1. Oral Communication Legitimised by the Teacher

The lesson transcripts of the four lessons which were recorded were analysed to find out what is legitimised by the teacher in the classroom. Lesson one, three and four were analysed but the only data that could be identified from the three lessons was that the teacher legitimised short responses and the teacher aided learners in the process of reasoning. There was not much in these three sections where the teacher was legitimising any form of oral communication of the learners. Therefore, lesson two was the focus of the analysis when it came to finding out the forms of mathematical reasoning that the teacher legitimised. From the transcripts that were analysed, below are the findings of what the teacher legitimised in the classroom where data was collected.

The teacher allowed the learners to complete her sentences. For example, out of 53 statements 9 were teacher's completed statements by learners in lesson one (see transcript in Appendix B). In the excerpt below, the teacher allowed learners to complete her sentences and she aided the learners in the process of communicating their mathematical reasoning.

L2:129 Teacher: Write there, what do they say, they say find the mean, ne [okay], you can calculate from here, you don't have to transfer data from stem and leaf to, because already…
L2:130 Nicole: They have given the scores
L2:131 Teacher: Eeh people exercise eight coma ten, ne [okay], you have been given scores there as stem and leaf you don't have to re-write, right, already stem and leaf is giving you what, the…

L2:132 Class: ...The scores

L2:133 Teacher: The scores from the smallest to the…

L2:134 Class: ...Highest

L2:135 Teacher: Highest, do you understand, time management, hullo can I have your attention please, stem and leaf, your data already has been arranged in an ascending order okay, please time management it's also important because if you re-write things that are not necessary you won't finish the question okay, do you understand, so that data there is giving us stem and leaf, just calculate your mean, median, whatever is asked there, okay.

From the excerpt above, the learners were responding in short phrases and one word answers. For example in line 132 and 134 where the whole class was answering using short phrases. The response from the teacher about the learners' answers shows that she did not mind the learners answering using the short sentences and phrases. The learners were also using the only example given to communicate their reasoning. For example in line 130, Nicole said that the scores were given. This shows that she was communicating her reasoning using the given example. This is level one inductive reasoning (L1IR) because the learner communicated her reasoning without giving a justification which makes sense in relation to the problem being solved. In other words, level one reasoning is a short answer response without a justification.

The excerpt above also shows that the teacher aided the learners as they reasoned and communicated their mathematical reasoning. For example, in the excerpt above line 135, the teacher helped the learners to think about why they needed to arrange the values in ascending or descending order before working with the values.

The teacher did not condone short responses because there is nowhere in the transcripts where the teacher is telling the learners to complete their sentences. For example, out of 117 responses from the learners in lesson two, 86 responses are short responses and the teacher did not probe the learners to say more about their responses (see transcript Appendix B). Most of the time in the lessons, the teacher legitimised level one inductive

reasoning (L1IR) in the oral communication of the learners because she did not probe the learners to give justification for their reasoning. The probing the teacher did in this class was to help learners communicate their reasoning and she did not probe them for further explanation of their reasoning. The excerpt below shows the kind of reasoning that the teacher legitimised.

L2:157 Teacher: Why is it thirty five?

L2:158 Nicole: Half is sixteen

L2:159 Teacher: Sixteen, okay and thirty five, okay, let's see you say half its sixteen, sixteen it means the first sixteen and the last sixteen, so let's talk about thirty two, thirty two, is it the even number or the odd number

L2: 160 Nicole: Even number

L2:161 Teacher: So if it's an even number what do we do?

L2:162 Nicole: We add the numbers and divide by two because we want to find the half of it.

L2: 163 Teacher: Ja [yes] you must take score number sixteen plus score number seventeen we add it together and divide by two, is it what you did, is it what you did

L2: 164 Nicole: Yes

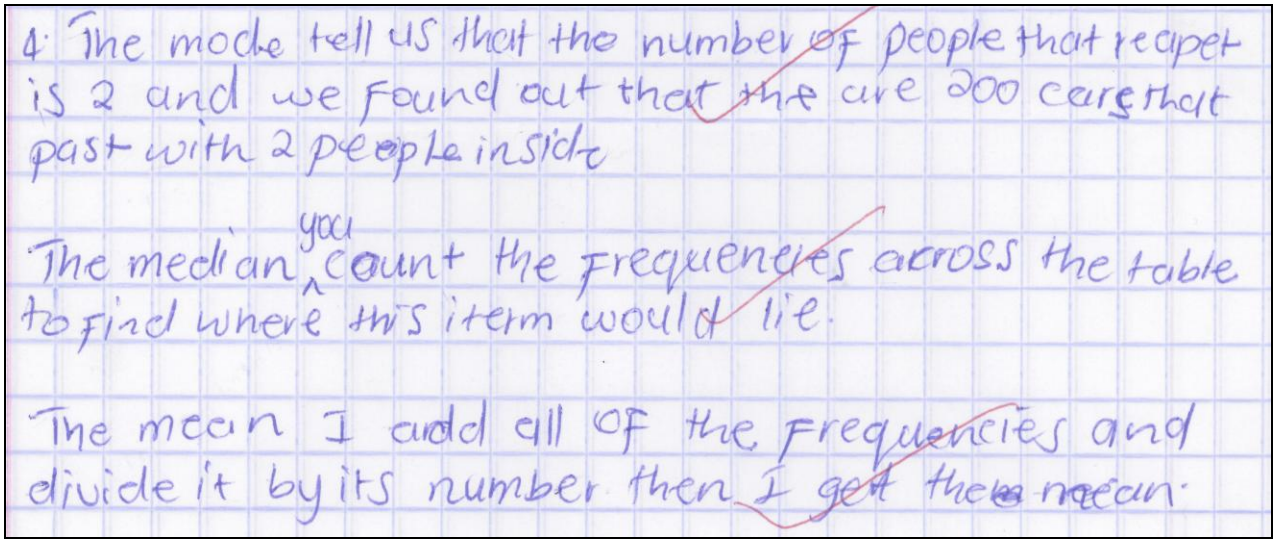
From the excerpt above, the teacher did not probe the learner for further reasoning when the learner communicated her reasoning as shown in line 162. Instead, the teacher repeated the learner's reasoning in agreement and asked her if that is what she did. This shows that the teacher legitimised this kind of reasoning. The learner used classroom discourse one (Cd1) because she used ordinary language (OL) to communicate her reasoning. For example, the learner said "*We add the numbers and divide by two because we want to find the half of it",* from this statement, the learner was trying to communicate her reasoning about how to find the median of the data given, but the learner did not use any mathematical terms in the process of communicating her reasoning hence classroom discourse one (Cd1). The teacher agreed with the learner's communication, showing that she legitimised Cd1 whereby learners were allowed to use ordinary language to

communicate their reasoning all the time. In the oral communication, the teacher legitimised level one inductive reasoning (L1IR) and classroom discourse one (Cd1) most of the time in the classroom because the teacher agreed with learners that gave a justification in relation to the question being answered only and also agreed with the learners that gave answers in ordinary language without pestering for mathematical language use from the learners. Learners were aided to produce the required answer by the teacher through probing. Hence, in this Discourse it is not possible to tell if the learners would have produced the legitimate answer without the teacher's support. For example, out of 66 responses from the learners in lesson one, 45 were as a result of the teacher's probing (see transcript in Appendix B).

5.5.2. Written Communication Legitimised by the Teacher

The learners' notebooks were analysed to find out the kinds of written communication of mathematical reasoning that the teacher legitimised. From the books that were analysed, the learners' work was checked to find out how the teacher deals with what she thinks is correct or wrong in the learners' note books. The teacher used a tick to show that the learner's answer is correct. When the learners' notebooks were to find out what the teacher did not legitimise, it was not obvious because the teacher did not make any comment on the learners' work which was mathematically wrong according to researcher. Therefore, the researcher made an assumption that the teacher did not mark or comment on learners' work which she found to be wrong. This was the classroom Discourse practice (Gee, 2005) of the teacher legitimizing and not legitimising the work in the learners' notebooks. An example of the learner's response to question 4 from the textbook about mean, mode and median and cars, which was legitimised by the teacher, is shown below.

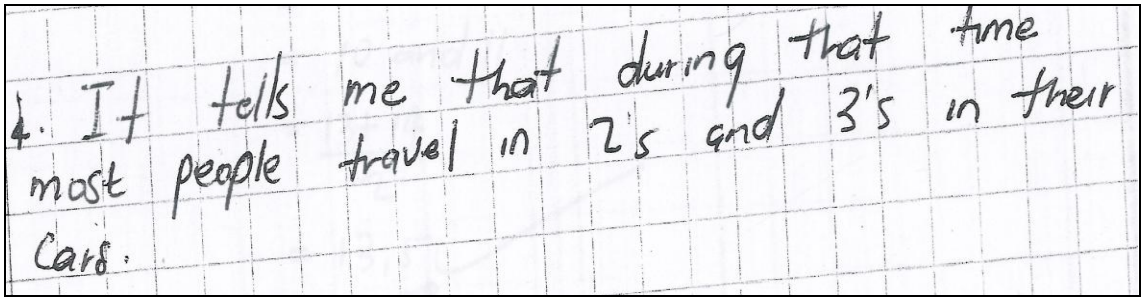
Figure 5.3: Learner's work legitimised by the teacher



In agreement with the learners' work, the teacher used ticks as shown in figure one above. The written communication of mathematical reasoning above is under level two inductive reasoning (L21R) because the learner gave a justification by stating the frequency of 200 and he also explained how the mean and median is calculated using the given data. The learner gave a justification for his reasoning though the justification is limited to only the question that was asked and hence level two inductive reasoning. From the learners' notebooks that were analysed, the teacher legitimised 7 level two inductive reasoning (7 L2IR) and only 2 level one inductive reasoning (2 L1IR), out of the 14 responses that were analysed (see Appendix F).

From 14 items that were analysed all incorrect responses were not marked. This data seemed to suggest that the style of marking was to leave incorrect responses blank. The unmarked responses were 3 responses under level one inductive reasoning (3 L1IR), 1 level one deductive reasoning (1 LIDR) and 1 level two inductive reasoning (1 L2IR) (see Appendix G). Below is an example of a learner's response to question 4 from the textbook, about cars, that was not marked by the teacher and it mathematically wrong in response to the question asked because of the 3 which the learner refers to.

Figure 5.4: Learner's work that was not marked by the teacher



In all the 14 responses of the learners that were analysed, the teacher did not put a comment on the levels of reasoning that the learners were using. She did not probe the learners for higher reasoning as shown by the work she legitimised (see Appendix G).

The analysis of the learners' written work that was legitimised by the teacher shows that the teacher was not looking for a specific way of mathematical reasoning and all the responses that the teacher legitimised were written in ordinary language hence classroom discourse 1 (Cd1) was legitimised by the teacher in the learners' written work (see Appendix I)

5.6. How Learners Communicate their Mathematical Reasoning

The main research question of the study is about how learners communicate their mathematical reasoning. Both oral and written communication of mathematical reasoning was considered in this study. The transcripts, as stated earlier were analysed to find out how learners communicate their mathematical reasoning orally. While the learners' notebooks were analysed to find out how the learners communicate their mathematical reasoning in written form.

For a detailed analysis, three learners were selected out of the seven. The three learners selected are those learners that had answered all the three questions completely, and I was able to follow up their oral communication in the classroom observations that were recorded using the digital video recorder. The other four learners were left out because they had answered the third question partly, and they had not actively participated in the oral communication in the classroom. Being able to follow three learners through their

oral and written communication was advantageous as I was able to make comparisons between the two types of communication- oral and written.

5.6.1. Oral Communication of Learners' Mathematical Reasoning with the Teacher

Three learners, Joy, Nicole and Peter (pseudonyms) as mentioned earlier were followed during the analysis of the video transcripts. The learners' oral communication was analysed as shown below.

Levels of Mathematical Reasoning and Classroom Discourse

As discussed earlier (see analytical framework), the levels of mathematical reasoning as were used during the analysis of the learners' oral communication. The learners communicated their reasoning to the teacher, and their peers. Below is an excerpt of Joy's oral communication to the teacher.

L2: 47 Teacher: Eeh Joy what do you write, what is your lower quartile, how do you find the lower quartile?

L2: 48 Joy: I divided the positive two by two…

L2: 49 Teacher: Right

L2: 50 Joy: Then I find …

L2: 51 Teacher: Remove the lower quartile, just do one thing at a time so that you are able to, good, Eeh start with your median, remember we start with our median

L2: 53 Joy: We divide our data into two, then we calculate the median

L2: 54 Teacher: Right

Joy was able to communicate her mathematical reasoning with the help of the teacher's questions and probing. The teacher also aided Joy's communication as shown in line 51 where the teacher tells Joy what she needs to start with. In other words, the teacher was helping Joy to reason. Joy's oral communication in this excerpt is under level two inductive reasoning (L2IR) because Joy communicated her reasoning using the given question, without referring to any other example in order to make her justification clearer. Joy's oral communication is classroom discourse one because she used ordinary language (OL) to communicate her reasoning and not mathematical language (ML). The teacher

also gave learners an opportunity to correct each other's reasoning as shown in the excerpt between Joy , Peter and the teacher, below.

L2: 57 Teacher: How do you find the lower quartile tell us

L2: 58 Joy: I jumped this number, then I got to (points at 3 in between 33 and 34 in row three)

L2: 59 Peter: No

L2: 60 Teacher: Yes Peter

L2: 61 Peter: Eeh in your, I must do, this is your half of your scores le [this one] and then get the half of the half of your scores and then you say (counts the scores) okay go straight to the half of the eleven, one two three four five six seven eight nine ten eleven four then here you count again (counts again from one to eleven) because your scores are given there. Therefore, you take two numbers to get your middle number and then add them and divide by two and its going to be sixteen plus fifteen then you get thirty one, divide by two which is equal to fifteen coma five and that means end quartile

L2: 64 Teacher: You understand Joy now, can you do the upper quartile for us

Joy's oral communication in this excerpt is still under L2IR and Cd1 because she used the data given to justify her reasoning and she used ordinary language as shown in line 58. The teacher gave Peter an opportunity to help Joy with her reasoning. Peter communicated his reasoning using level two inductive reasoning (L2IR) because he used the data that was given to explain how the quartile is calculated and why the answer is the way it is. Peter's reasoning is under Cd1 because he used ordinary English, for example, he used the word 'end quartile' in line 61 to mean upper quartile.

Nicole was communicating her reasoning about the stem and leaf diagram and the teacher together with the class tried to help her with her communication as shown in the excerpt below.

L2: 129 Teacher: Write there, what do they say, they say find the mean, ne [okay], you can calculate from here, you don't have to transfer data from stem and leaf to, because already…

L2: 130 Nicole: They have given the scores

L2: 131 Teacher: Eeh people exercise eight coma ten, ne [okay], you have been given scores there as stem and leaf you don't have to re-write, right, already stem and leaf is giving you what, the…

L2: 132 Class: ...The scores

L2: 133 Teacher: The scores from the smallest to the…

L2: 134 Class: ...Highest

L2: 135 Teacher: Highest, do you understand, time management, hallo can I have your attention please, stem and leaf, your data already has been arranged in an ascending order okay, please time management it's also important because if you re-write things that are not necessary you won't finish the question okay, do you understand, so that data there is giving us stem and leaf, just calculate your mean, median, whatever is asked there, okay

Nicole was also aided by the teacher in the process of her oral communication. Nicole's oral communication is under level two inductive reasoning (L2IR) because in line 130, she used the given data to justify her reasoning without giving any other example. Nicole communicated here reasoning using Cd1 (see Appendix I)

The three learners that were followed during the analysis of the transcript communicated their mathematical reasoning to the teacher using L2IR and Cd1 as shown above.

Oral Communication of Mathematical Reasoning with Peers

The teacher gave the learners an opportunity to discuss in their groups during the lesson. One of the groups was followed during data collection in order to have a consistent conversation recorded. Peter and Joy were in the group discussion that was followed, but Nicole was not in the group discussion. Therefore, Nicole's oral communication with the peers was not captured.

Joy and Peter were part of the group discussion that was recorded as shown in the excerpt below. The learners were discussing how to calculate quartiles and why they should calculate the quartiles the way they were calculating them.

L4 13 Joy (In her discussion group) I am telling you, I am giving you what it means, it comes as this at the end

L4: 14 Stella : Oh!

L4: 15 Joy: Now you can say it shares it off because you have to share it into two to get the half of it, then you get the first quartile (demonstrating using the data set in her book)

L4: 16 Stella: Now what is it, first quartile?

L4: 17 Joy: Second quartile. You see, if we share it we get the lower quartile.

L4: 18 Stella: What lower quartile?

L4: 19 Peter: What we want that…

L4: 20 Joy: ...Lower quartile is also first quartile because we get it first.

L4: 21 Peter: That's what we want.

L4: 22 Many learners: (laughter)

L4: 23 Stella: What is the first quartile now?

L4: 24 Joy: First quartile, I don't know where I started, one, two, three, four, five… (counting from the data set in her book)

L4: 25 Learners in Joy's group: Forty two, forty three…

L4: 26 Stella: ...Hey guys, forty two, forty three, forty seven plus fifty, forty seven besides, aah…

L4: 27 Peter: Forty seven.

L4: 28 Stella: Forty two, forty seven.

L4: 29 Joy: Divide by two to get it because we want the half of it which is median of the other half (pointing at the data in her book).

L4: 30 Stella: Divide by two eeh…

In the excerpt above, Peter and Joy were communicating their mathematical reasoning with their peers. The learners used only English in this excerpt. In line 15, Joy was communicating her mathematical reasoning about quartiles. She used L2IR because she used the given data to justify her reasoning. Line 20 shows that Joy was using Cd1 because she referred to lower quartile as first quartile which is ordinary language (Pimm, 1991). Peter on the other hand was adding to what Joy was communicating as shown in line 21 and 27. In this excerpt, the learners were communicating their mathematical reasoning using L2IR because they referred to only the question they were working on. Most of the time when learners were communicating with each other, they used Cd1 and this can be referred to as a social language which they used (Gee, 2005). Another social language which the learners used to communicate their reasoning was code switching (Adler, 2001) to Zulu, another language other than English, especially when communicating to peers (see transcript in Appendix I).

5.6.2. Written Communication of Mathematical Reasoning

Within the mathematics Discourse, written mathematical reasoning can be communicated using prose, symbols or diagrams (Yopp, 2010). From the learners' notebooks that were analysed, all responses were prose (see Appendix E). The prose responses were categorised under levels of mathematical reasoning and classroom discourse as shown in the table below.

Table 5.4: Learners' written communication of mathematical reasoning

Learner	Type of response			Level of language			Level of reasoning		
	4	**2a/b**	**4**	**4**	**2a/b**	**4**	**4**	**2a/b**	**4**
Peter	Prose	prose	prose	Cd1	Cd2	Cd1	L2IR	L1DR	L2IR
Nicole	Prose	Prose	prose	Cd1	Cd1	Cd1	L2IR	L1IR	L2IR
Joy	Prose	prose	prose	Cd1	Cd1	Cd0	L1IR	L2IR	L2IR

From the table 1 above, all the three learners used prose to communicate their mathematical reasoning, the learners all used ordinary language to communicate their reasoning, except for one learner who used mathematical language. Most of the responses were under level two inductive reasoning (L21R) (see detailed analysis in Appendix F and I). Below is a detailed discussion of how the learners communicated their mathematical reasoning.

Levels of Mathematical Reasoning and Classroom Discourse in the Written Communication

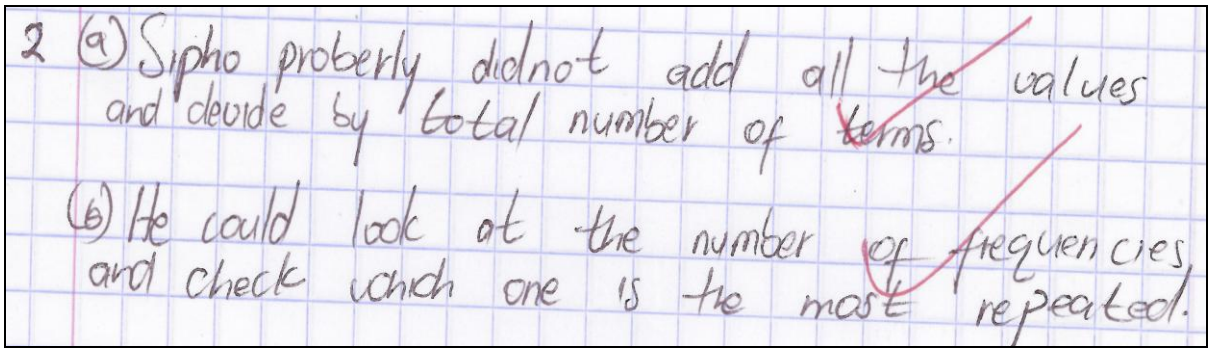
The written responses of the three learners Joy, Peter and Nicole were analysed using the levels of mathematical reasoning.

Nicole's written responses

Three responses from Nicole's notebook were analysed. Nicole answered question 4 about cars, 2a and b and question 4 about leaves from the textbook. Of the three responses, two were under L2IR and one was L1IR. All the three responses from Nicole were marked and hence legitimised by the teacher.

The one response from Nicole was under L1IR because she gave a short answer that was not justified while the other two responses were L2IR because Nicole gave a justification of the response in relation to the question that was asked. For example, in the response of question 2a and b shown below she justified why and how mean should have been calculated in the given problem.

Figure 5.5: Nicole's response to Question 2a and b

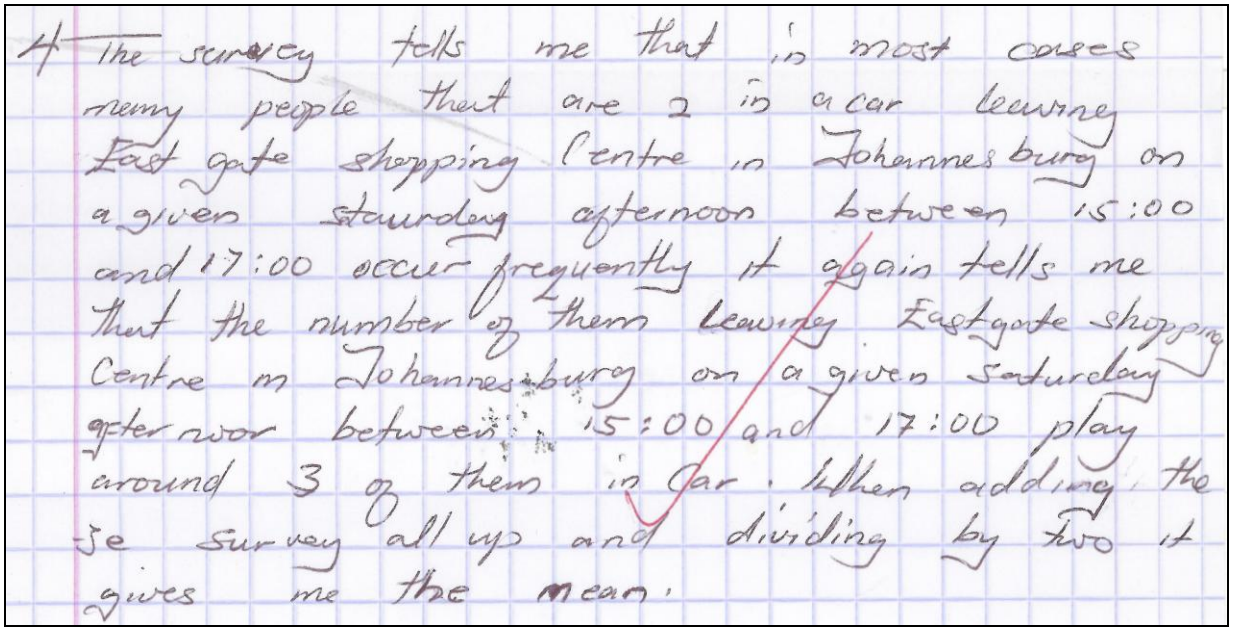


Nicole used ordinary language (Pimm, 1991) to communicate her reasoning. For example, instead of using the actual symbolic formula of calculating mean to justify her reasoning, she used ordinary language such as most repeated instead of frequency, and add all the values instead of saying summation.

Joy's written responses

Three responses from Joy's notebook were analysed. Of the three responses, two were L2IR because the learner used the question given to explain her reasoning, and one was L1IR because she did not give a justification for her reasoning as shown in Figure 6 below.

Figure 5.6: Joy's response to Question 4

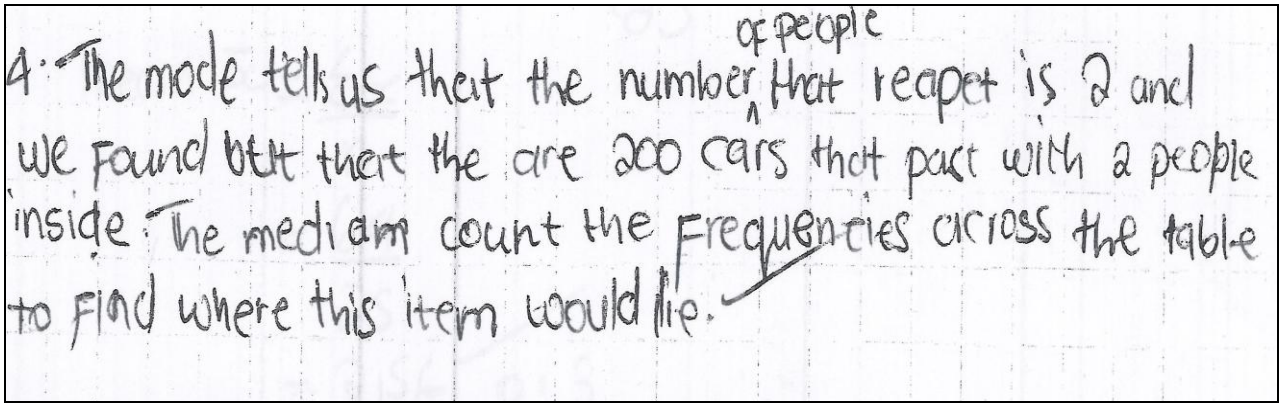


The language which the learner used in the response above is not formal mathematics language (Pimm, 1991) used in the mathematics classroom discourse. Therefore, the learner used ordinary language to communicate her reasoning.

Peter's written responses

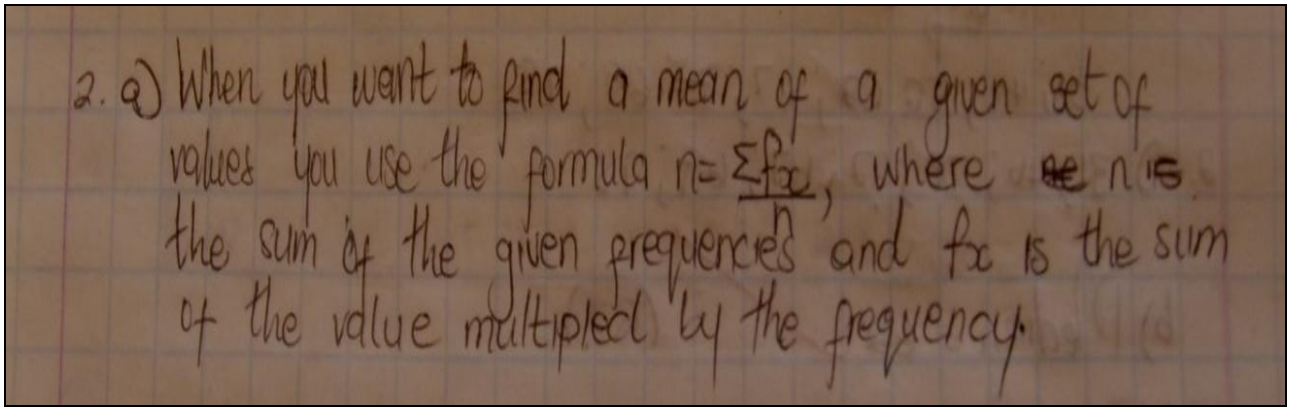
Peter's responses which were also analysed and two of them were under L2IR. In one of the responses, he gave a justification of what the mode is showing in this particular problem by giving the highest frequency which is 200 cars. This was legitimised by the teacher because it is marked correct as shown in figure 7 below.

Figure 5.7: Peter's response to Question 4



One of Peter's responses was level one deductive reasoning (L1DR) because he used only the definition of mean to justify his answer , the teacher did not mark this question probably because of the statement where the learner says that *fx* is the sum of the value multiplied by the frequency, which is not correct, as shown in the figure below.

Figure 5.8: Peter's response of Question 2a



The response above is classroom discourse two (Cd2) because the learner used mathematical language to communicate his mathematical reasoning. For example, he used the symbol of summation, the *x*, *f* and *n* to communicate his reasoning.

5.7. The Languages Learners Use to Communicate their Mathematical Reasoning

During the lessons that were observed, learners communicated their mathematical reasoning with the teacher and with their peers as discussed above. As mentioned earlier,

the main language of communication in the classroom was English. Learners used only English to communicate their mathematical reasoning to the teacher because when they tried to use another language, the teacher would still communicate with them in English (see transcript in Appendix B) and this made the learners use English while communicating their mathematical reasoning to the teacher.

During group discussions, the learners used both English and Zulu to communicate their mathematical reasoning (see transcript in Appendix B) and they felt more relaxed and comfortable than when they were using English. Below is an excerpt of learners communicating their mathematical reasoning using Zulu.

L4: 57 Joy: Yiyo le, siphumile [It's the one which we got]
L4: 58 Peter: Is skewed to the lower
L4: 59 Stella: I-wrong, iwrong [its wrong, it's wrong]
L4: 60 Joy: If the second quartile, if the second quartile…
L4: 61 Peter: Forty two
L4: 62 Joy: I-graph, yakuphi [this graph is for which one]
L4: 63 Stella: Yakuphi [for which one]
L4: 64 Peter: Ngizokhutshela[I will tell you]
L4: 65 Joy: Right, uh question vele [well]…
L4: 66 Stella: Uthini [what are you saying]

In the excerpt above, the learners were communicating in their group discussion. Zulu was dominant in this excerpt and learners were freely asking each other questions and they seemed very relaxed during the group discussion. The learners did not communicate using Zulu predominantly when it came to communicating with their teacher. For example in lesson one, learners were communicating with the teacher and out of 66 responses from the learners, only 6 were said in Zulu. Zulu was one of the social languages which they used in the context where they were communicating their mathematical reasoning with their peers (see transcript in Appendix B).

5.8. Conclusion

In this chapter, I have presented the analysis and findings of the study. From the analysis, the researcher gained an understanding of how learners communicate their mathematical reasoning. The analysis of the textbook has been presented to show what it legitimised, the teacher's practices were also analysed and presented in this chapter to show what the teacher legitimised in this classroom. From the analysis presented above, learners were able to communicate their mathematical reasoning orally and in written form at level 1 and 2 for both forms of mathematical reasoning, i.e. deductive and inductive reasoning. Cd1 and L2IR were the most prominent ways which were used by the learners to communicate their mathematical reasoning orally. In the next chapter, I will present the summary of the findings and the discussion of the results in relation to literature review. The implications and recommendations of the study will also be presented in the next chapter.

6. Chapter Six: Conclusions and Recommendations

6.1. Introduction

In this chapter, I will discuss the findings of the study, the implications of the findings and the conclusions made from them. As discussed earlier in Chapter One, this study was inspired by the findings of a study which showed that second language participants seemed to have been unable to reason and communicate mathematically (Aineamani, 2010). In Aineamani's study the learners were expected to communicate their mathematical reasoning in English only in a situation where they were not aided in their communication of mathematical reasoning. The aim of the study was to understand why students regard some solutions as the best representation and solution of a word problem in their own view and in their teacher's view, and how these students construct a proof. The focus of the study conducted by Aineamani was not on language and how the language affected their reasoning and communication. As stated earlier, the focus of this study was on language, therefore, the conclusions and recommendations are guided by the underlying language assumptions espoused in Chapter Two.

As mentioned earlier, the participants in this study were Grade 11 learners in a multilingual classroom whose first language was not English. The teacher in the class was also not a first language speaker of English. The purpose of the study was to investigate how learners in a multilingual school in South Africa communicate their mathematical reasoning. This classroom was selected out of four Grade 11 streams in the school. The learners in this class were selected because the learners were more active than all the other streams, according to the head of the mathematics department in the school. The teacher had taken a university course that stressed the importance of allowing learners to communicate their reasoning. Therefore, the researcher made an assumption that they teacher would not teach traditionally whereby learners are not given an opportunity to communicate their reasoning.

6.2. Summary of the Findings

Analysis of data in Chapter Five showed that learners were able to communicate their mathematical reasoning orally and in written form.

Orally, the learners were aided by the teacher in the process of communicating their mathematical reasoning. While communicating with peers, the learners were able to communicate their reasoning without being aided by the teacher though they used Zulu more than English. Classroom discourse one (Cd1) and Level two inductive reasoning (L2IR) were the most prominent ways which were used by the learners to communicate their mathematical reasoning orally. By Cd1, learners used ordinary English to communicate their mathematical reasoning and learners communicated their mathematical reasoning by giving short responses and partial justifications hence L2IR.

The learners communicated their mathematical reasoning using prose when it came to writing. Under written communication, L2IR and Cd1 were very prominent. The learners used ordinary English to communicate their mathematical reasoning and they justified their reasoning using the given question hence L2IR. In all the learners' responses that were analysed, there was no level three inductive reasoning (L3IR) or level three deductive reasoning (L3DR) and the teacher did not probe level three reasoning in this classroom. By L3IR, learners were required to communicate their reasoning by giving three or more empirical examples to justify their mathematical reasoning and explanations of their reasoning in detail without leaving out any idea that is used in solving the problem. The examples given under level three inductive reasoning should be able to lead to a conjecture or a generalisation (Brodie, 2000). Under L3DR, learners were required to justify their reasoning using the appropriate conventional language that is acceptable as formal mathematical reasoning language. Level three deductive reasoning includes only formal proofs. In other words, level three deductive reasoning involves abstract reasoning (Muller & Maher, 2009).

6.3. Discussion and Conclusions

The findings from the study show that the teacher wanted the learners to communicate their mathematical reasoning in English during the lesson. The teacher did not tell the learners to speak English but she expected the learners to follow her example of not using any other language other than English. Setati (2005a, 2005b) argues that within a South

African mathematics classroom, some teachers want the children to use English when explaining their mathematical reasoning because the teachers feel that if these learners are denied the chance of becoming fluent in English, which is an international language, then they may not be able to participate with other people outside the classroom setting. This could have been the reason why the teacher in this classroom did not use any other language when teaching and also aiding the learners to communicate their mathematical reasoning.

The teacher asked learners a few questions that required the learners to communicate their mathematical reasoning. The textbook used in the classroom also had a few questions that required learners to communicate their mathematical reasoning. Mathematics teaching should be geared towards supporting learners to express their reasoning (Brandt & Tatsis, 2009). The teacher and the textbook that was being used in the classroom did not help the learners to learn how to communicate their mathematical reasoning. This is shown by the few mathematical reasoning questions and the activities in the textbook, and the teacher asked very few questions that required mathematical reasoning.

The communication practices legitimised by the textbook were procedural because most of the questions and the activities in the textbook did not require learners to give a justification for the solutions. Within the mathematics reasoning Discourse (Gee, 2005), learners are required to provide justification for any response they give to a problem (Muller & Maher, 2009). However, majority of textbooks are designed to teach students particular mathematical techniques and procedures rather than to help students develop thinking skills necessary for the learners to take part in the mathematical reasoning Discourse (Green & Emerson, 2010).

The communication practices legitimised by the teacher were procedural because the teacher rarely asked learners to justify their responses during the lessons observed. The teacher legitimised informal language within the classroom Discourse because she did not probe learners to use formal language when they used informal mathematics language

orally and in written responses. Dawe (1983) argued that learners often follow what the teacher legitimises within the classroom. The informal language which the teacher legitimised is referred to as Ordinary language which is defined as English that is not combined with mathematical conventional terms (Pimm, 1991). For example, if a learner uses words like average instead of mean and number in the middle instead of median, he is using ordinary language. While formal mathematical language is referred to as mathematical language which is defined as using mathematics symbols and terms to communicate mathematical knowledge (Pimm, 1991).

According to Pimm (1991), in a mathematics classroom, learners learn to move from informal spoken languages which they use outside the classroom setting (informal setting) to a formal spoken or written activity which is viewed as a requirement for the learners to participate in the mathematics activities. In this study, the learners used ordinary language which is also referred to as informal language to communicate their mathematical reasoning. Moschkovich (2003) argues that for learners to take part in a mathematics Discourse, they have to move from an everyday way of talking to a more precise way of using mathematical language. For learners in multilingual classrooms the movement also includes moving between languages (Setati & Adler, 2001) and moving between cultures (Zevernbergen, 2000; Cleghorn & Rollinick, 2002). The teacher did not encourage the learners to make the move by probing them for more formal mathematical reasoning communication.

The learners were able to communicate their mathematical reasoning in this study to a certain extent, an extent which limited them to the first two levels of mathematical reasoning as discussed in Chapter Three and Chapter Five. All the learners used prose responses to mathematical reasoning questions that they answered.

Learners used level one inductive reasoning (L1IR) and level two inductive reasoning (L21R), and level one deductive reasoning (L1DR) only. L2IR was widely used both in learners' oral and written communication of mathematical reasoning. Muller and Maher (2009) argue that when learners are required to communicate their reasoning, they use

different forms of arguments. Edwards (1999) also argues that learners give different forms of arguments when communicating mathematical reasoning. The arguments used by the learners maybe formal, preformal or empirical (Martin & Kasmer, 2009). The learners used classroom discourse zero, one and two (Cd0, Cd1 and Cd2). Cd0 and Cd1 are preformal ways of communicating mathematical reasoning because ordinary language is used in the process of communication (Clarkson, 2009; Pimm, 1991).

From the findings of the study, the learners communicated their mathematical reasoning up to level two inductive reasoning (L2IR) and level one deductive reasoning (L1DR). This implies that learners were not able to communicate their mathematical reasoning using formal proofs. This may be caused by the fact that in school mathematics, the strict sense of constructing formal proofs is not common (Mckenzie, 2000). Or perhaps it is as a result of the requirements of the curriculum at that Grade 11 level. The NCS puts more emphasis on learners being able to solve problems, demonstrate an appreciation of mathematical ideas and calculating and representing mathematics data. The mathematical reasoning is not emphasised in the activities suggested in the NCS (DoE, 2003). However, the learners are said to be able to communicate their mathematical reasoning because, according to Martin and Kasmer (2009), mathematical reasoning involves informal observations, conjectures, and explanations that are familiar to the learners.

Barton and Barton (2005) argue that learners have difficulties in communicating mathematical reasoning in a language that is not their first language. This may have been the problem in this class. The learners were not first language speakers of English and therefore they were not able to use English fluently to articulate what they were thinking, and this was reflected in the written work as well. Some of the learners' written work was written in wrong grammatical English. However, the learners work still had meaning to the teacher and to the researcher. The work was marked correct by the teacher because the teacher understood what they were communicating, and researcher was also able to make sense of the learners' communication hence categorising it under different levels of mathematical reasoning.

Within the mathematics reasoning Discourse, as discussed in Chapter Two, there are "ways with words, deeds and interactions, thoughts and feelings (Gee, 2005) within this Discourse so that they can be identified as participating in that Discourse. Learners have to communicate in the process of participating in the Discourse, and in the process, the learners are said to be communicating their mathematical reasoning. Discourse analysis was done in order to find out how learners communicate their mathematical reasoning within the mathematical reasoning Discourse. Different constructs like the language used within the Discourse were analysed. The different languages were referred to as social languages (Gee, 2005) in this study.

In an attempt to communicate their mathematical reasoning, learners may use different social languages. Gee (2005) discusses the idea of *social language* in order to show that language alone is not sufficient for one to participate in a given Discourse. Social languages are "what we learn and what we speak" (Gee, 2005:38) in a given social setting. The social language may be formal or informal within the mathematics reasoning Discourse (Pimm, 1991; Clarkson, 2009). The learners were able to communicate their mathematical reasoning using an informal social language within the mathematics reasoning Discourse.

A formal or informal social language may be used inductively or deductively within the mathematics reasoning Discourse. When used inductively, empirical examples are used to justify the given response, and when used deductively, justification is given based on conventional rules that are already in place within the mathematics Discourse (Martin & Kasmer, 2009). The learners communicated their mathematical reasoning using the informal social language and they used the social language inductively because they based their justification on empirical evidence of the question they were solving (Brodie, 2000).

Therefore, from the study, the learners had a certain way of communicating their mathematical reasoning. By identifying this way of communicating from the learners' responses, the researcher was able to identify how the learners communicated their

mathematical reasoning within the mathematics reasoning Discourse. The learners communicated their mathematical reasoning using informal classroom discourse (Gee, 2005), which is the language they used to communicate their mathematical reasoning within the mathematics reasoning Discourse and the learners communicated their mathematical reasoning under a certain level of mathematical reasoning which was level two inductive reasoning (L2IR) hence the learners communicate their mathematical reasoning in a way that is not formal in the mathematics Discourse but it is acceptable within the mathematics reasoning Discourse for learners at Grade 11 level (Selden & Selden, 2003).

6.4. Implications of the Study

Under implications for study, I will look at two perspectives- the implications of the study on the movement of the learners' communicating of mathematical reasoning from informal to formal mathematical language and the movement from inductive reasoning to deductive reasoning.

Within the mathematics Discourse, learners are expected to move from informal ways of speaking and writing mathematics ideas to more formal ways (Moschkovich, 2003). Therefore, the mathematics teacher should not only accept learners' informal ways of communicating mathematical reasoning but also formal ways. The teacher should probe the learners for more formal mathematics communication in order for the learners to make the move from informal to more formal mathematics talk and writing. If the learners are unable to move from informal ways of communicating their mathematical reasoning to formal ways, then the learners are unable to flexibly take part in the mathematics reasoning Discourse.

Communicating mathematical reasoning in the mathematics classroom can be done using different levels of reasoning as discussed in Chapter Three and Chapter Five. Muller and Maher (2009) argue that learners can communicate mathematical reasoning using different forms. These forms discussed by Muller and Maher (2009) can be referred to as levels of mathematical reasoning in this study. The levels of mathematical reasoning in

this study were: level one inductive reasoning (L1IR), level two inductive reasoning (L2IR), level three inductive reasoning (L3IR), level one deductive reasoning, (L1DR), level two deductive reasoning (L2DR), and level three deductive reasoning (L3DR). The findings in the study showed that the learners were not able to communicate beyond L2IR, and the teacher did not probe the learners for further reasoning as shown by what the teacher legitimised in the classroom. This implies that when learners are not probed for higher mathematical reasoning, they only communicate their reasoning to a certain extent. The teacher should also probe learners for a higher level of mathematical reasoning where necessary and learners should be encouraged to communicate their mathematical reasoning using different forms such as a symbols and diagrams (Lerman, 2003), and not only prose as it was seen in the study. During the mathematics lesson, if the teacher asks the right questions that encourage learners to communicate their mathematical reasoning, the learners respond to such questions by communicating their mathematical reasoning (Aberdein, 2008). If the teacher asks questions that require short and procedural answers, the learners will give answers that are short and procedural as shown in the findings of the study. In other words, the type of responses which the teacher legitimises in the classroom determines how the learners communicate their mathematical reasoning.

The way learners communicate their mathematical reasoning also depends on the activities that are given by the textbook being used in the classroom. The textbook used in the classroom did not promote mathematical reasoning because of the few activities and questions which were included in the textbook. If the textbook used in the classroom does not have activities and questions that promote mathematical reasoning, learners do not learn how to communicate their mathematical reasoning since they are not exposed to activities and questions which require them to communicate their mathematical reasoning (Stein et al., 1996).

The learners in this study are not adequately developed in their reasoning skills by the school curriculum and this has implications for those who would want pursue studies in mathematics because university education is at level 3, which is not in the school

curriculum. The school curriculum requires learners to reason up to a certain extent which may not be adequate for further study within the mathematics Discourse. Selden and Selden (2003) argue that most students at the university cannot construct a formal proof because of the way they were taught at high school. Healy and Hoyles (2000) also argue that few University mathematics students can tell what constitutes a formal mathematics proof which can be used to justify a mathematical statement.

The study has shown that the textbook used in the classroom is very important in determining how learners communicate their mathematical reasoning. The textbook also helps in enabling or restricting learners to communicate their mathematical reasoning. If learners are not asked to communicate their mathematical reasoning, they do not communicate their reasoning as shown in the study. The questions which the teacher asked helped the learners to communicate their mathematical reasoning. However, the questions were very few during the lessons, and this made learners not to communicate their mathematical reasoning where necessary. This has shown that in order for learners to communicate their mathematical reasoning, they should be probed or asked the right questions that enable mathematical reasoning.

The study has added on the research by Muller and Maher (2009) who argue that learners communicate their mathematical reasoning using different levels of mathematical reasoning. The learners may not be able to communicate their mathematical reasoning up to the highest level of reasoning. The learners should be probed and encouraged to reason at a high level where necessary. This can be done by asking them to give more convincing arguments and explanations.

The study also showed that learners communicate their mathematical reasoning informally if not encouraged to use mathematical language when required. Therefore, the teacher should encourage learners to use both ordinary and mathematical language so that they do not end up using only ordinary language to communicate their mathematical reasoning.

6. 5. Recommendations

Mathematics textbooks should be designed to enable learners communicate their mathematical reasoning. For example, the activities and the questions given in the textbooks should require learners to communicate their mathematical reasoning. Asking open ended questions and questions that require learners to justify and give explanations to their answers should be included in the textbook. The teacher should ask learners questions that require learners to communicate their mathematical reasoning. The teacher should also probe learners for higher mathematical reasoning by asking questions that require them to give more explanation and good justifications for their responses in the mathematics classroom.

The curriculum should emphasise the idea of learners communicating their mathematical reasoning, and not simply stating that learners should justify and explain their solutions (DoE, 2003), but instead ways how learners should communicate their reasoning should be stated. A level of mathematical reasoning which the learners are expected to reach in Grade 11 should be agreed upon within the South African curriculum, so that some learners are not disadvantaged when they communicate their mathematical reasoning at a lower level.

The assessment of how learners communicate their mathematical reasoning should have a basis, say the curriculum. If the curriculum states the level of mathematical reasoning which the learners at Grade 11 must reach, then the teacher will have to probe the learners for higher reasoning. Therefore, teachers within the classroom should encourage learners to communicate their mathematical reasoning and they should probe the learners for higher mathematical reasoning where necessary and also probe the learners for more formal mathematical reasoning. From the study, the researcher argues that assessment of the mathematical reasoning of learners should have a frame of reference so that the teacher is aware of the level to which the learners are expected to reach when communicating their mathematical reasoning at every Grade in mathematics teaching. Without a frame of reference, the teacher may legitimise very low levels of mathematical reasoning which are not at the academic standards of the learners.

6.6. Limitations

The data that was collected had to be transcribed. Some of the learners were using Zulu to communicate within the classroom and this was a challenge for the researcher because the researcher was not familiar with Zulu and therefore had to seek help during the process of transcribing, from people that knew Zulu.

Doing research in a foreign country also posed a challenge for the researcher because the researcher had to communicate with different people in the process of acquiring permission to conduct the study and some people were not friendly because they were not willing to communicate with the researcher in English which the researcher was familiar with.

Another limitation the researcher found was not being very familiar with the school system of South Africa since the researcher was not teaching in any South African school at the moment. Therefore, the researcher had to study the curriculum and some textbooks used in Grade 11 from scratch.

The findings from this study cannot be generalised because I used only one classroom in a school. Therefore the sample is not sufficient to generalise for all other settings. However, the purpose of my study was not to come up with generalisations but to get to learn from the one case which was studied.

6.7. Further Research

Having found out how learners communicate their mathematical reasoning, further research should be conducted to find out why learners communicate their mathematical reasoning in the way they do within the mathematics reasoning Discourse and ways in which the learners' mathematical reasoning may be improved.

6.8. Reflections

The process of carrying out the research was not straight forward and obvious. The researcher had to go back to the field on several occasions to collect more data especially

when it came to the notebooks of the learners. The teacher whom I had agreed with on observing his class declined my request and so I had to find another class though this did not affect the data collection process since the teacher showed that he was uninterested before the process of data collection began. The process of data analysis brought in many insights in the study. The study has made some contribution to the mathematics education community in respect to how learners communicate their mathematical reasoning. I found some ideas which confirmed the literature that was reviewed in the study. For example, Muller and Maher's (2009) research about the different levels of communicating mathematical reasoning was confirmed in this study. The process of carrying out the research was very interesting because the research questions that guided the study were answered by the data that was collected, and very many insights were found from the data such as the way learners communicate their mathematical reasoning using informal mathematical language and inductive reasoning.

References

Aberdein, A. (2008). *Mathematics and Argumentation*. Humanities and Communication, Florida Institute of Technology. Retrieved on 23rd October 2010 from http://www.springerlink.com/content/e1306515n3t507g0/

Adler, J. (2001). Teaching and learning mathematics in multilingual classrooms. Dordrecht, Netherlands: Kluwer Academic Publishers.

Aineamani, B. (2010). Reasoning and Communicating Mathematically. In M. de villers (Ed.), *Proceedings of 18th meeting of SAARMSTE* Vol 1, University of Kwazulu-Natal: South Africa, pp 171-183.

Bell, J. (1999). *Doing Your Research Project* (3rd edn), Milton Keynes: Open University Press.

Bicknell, B. (1999). Language and mathematics. In F. Biddulph & K. Carr (Eds.), *SAME papers1999* (pp. 203 - 224). Hamilton, NZ: Centre for Science, Mathematics and Technology Research, University of Waikato.

Brandt, B., & Tatsis, K. (2009). Using Goffman's concepts to explore collaborative interaction processes in elementary school mathematics. *Research in Mathematics Education*, 11 (1), 39-56.

Brijlall, D. (2008). Collaborative learning in a multilingual class. *Pythagoras*, 68, 34-39.

Brodie, K. (2000) *Teaching and learning mathematical reasoning*. Bed (Honours) Course Material. Johannesburg: Wits University.

Clarkson, P. C. (2009). Mathematics teaching in Australian multilingual classrooms: Developing an appropriate approach to the use of classroom languages. In R. Barwell (Ed.) *Multilingual Mathematics Classrooms: Global Perspectives*: (pp 129-145). Bristol, Buffalo, Toronto: Multilingual Matters.

Cleghorn, A. & Rollinick, M. (2002). The role of English in Individual and Society Development: A View From African Classrooms. *Tesol Quarterly*, 36(3), 10-21.

Davis, B. (1997). Listening for differences: an evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28(3) 355-376.

Dawe, L. (1983). Bilingualism and Mathematical Reasoning in English as a Second Language. Educational Studies in Mathematics, 14(4), 325-353.

DoE. (2003). *National Curriculum Statement Grade 10 –12*. Pretoria, National Department of Education.

DoE. (2002). *Higher Education restructuring and transformation: Guidelines for mergers and incorporations*. Pretoria: Department of Education. INSERT in chapter 2.

DoE. (2008). *Assessment procedures Grades R to 9*. Eastern Cape. Department of Education.

Douady, R. (1997). Didactic Engineering, in Nunes, T & Bryant, P. (eds) Learning and Teaching Mathematics: *An international Perspective*. East Sussex: Psychology press Ltd.

Edwards, L. D. (1999). Odds and Evens: Mathematical Reasoning and Informal Proof among High School Students. *Journal of Mathematics Behaviour*, 17(4), 489-504.

Forman, S. L. & Steen, L. A. (1995). Mathematics for Life and Work. In M. Carl (Ed.), *Prospects for School Mathematics* (pp. 219-241). Reston, V A: National Council of Teachers of Mathematics.

Gee, P. J. (2005). *An introduction to discourse analysis: theory and method*. New York: Routledge.

Gladwell, M. (2006). *Blink: The Power of Thinking without Thinking*. South Africa: Penguin Books Ltd.

Golafshani, N. (2003). Understanding reliability and validity in qualitative research. *The Quarterly Report*, 8(4), 597-606. Retrieved on 19th April, 2010, from http://www.nova.edu/ssss/QR/QR8-4/golafshani.pdf.

Gorden, R. L. (1980). *Interviewing: Strategies, techniques and tactics*. Thousand Oaks, CA: Sage.

Green, K. H., & Emerson, A. (2010). Mathematical Reasoning in Service Courses: Why Students Need Mathematical Modeling Problems. *The Montana Mathematics Enthusiast*, 7(1), 113- 140.

Halai, A. (2009). Politics and practice of learning mathematics in multilingual mathematics classrooms: Lessons from Pakistan. In R, Barwell, (Ed.). *Multilingual classrooms, global perspectives*. London: Multilingual matters.

Hatch, J. A. (2002). *Doing qualitative research in educational settings*. SUNNY. New York. Pp. 147- 210.

Healy, L. & Hoyles, C. (2000). A study of Proof Conceptions in Algebra. *Journal for Research in Mathematics Education*, 31(4), 396-428.

Highpoints Learning Inc. (2010). *Mathematics programs online*. Retrieved from http://www.ehighpoints.com/ on 24th May 2010.

Howie, S. J. (2003). Language and other background factors affecting secondary pupils' performance in Mathematics in South Africa. *African Journal of Research in SMT Education*, 7 (4), 1-20.

Johnston, I. (2002). *Essays and Arguments*. British Colombia: Malaspina.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal*, 102 (3), 59-80.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds). (2001). .*Adding it up: Helping children learn mathematics.* Washington: National Academy Press. Pp 115-155.

Laridon, P., Brown, M., Jawurek, A., Mbatha., Mbatha, P., Scheiber, J., Sigabi, M. & van Noort, D. (2006). *Classroom Mathematics Grade 11*. Sandton: Heinnemann publishers.

Lerman, S. (2003). Cultural, Discursive Psychology: A Sociocultural Approach to Studying the Teaching and Learning of Mathematics. *Educational Studies in Mathematics*, 46 (3), 86- 113.

Lobato, J., Clarke, D. & Burns Ellis, A. (2005). Initiating and eliciting in teaching: a reformulation of telling. *Journal for Research in Mathematics Education*, 36(2), 101-136.

McKenzie, F. (2001). *Developing children's communication skills to aid mathematical understanding*, ACE papers (Student Edition), Issue 11.

Mercer, N., & Sams, C. (2006). Teaching Children How to Use Language to Solve Maths Problems. *Language and Education*, 20 (6), 507-528.

Merriam, S.B. (1988). *Case study research in education: A qualitative approach*. San Francisco: Jossey-Bass.

Moschkovich, J. (2003). What counts as mathematical discourse? In N. Pateman, B. Dougherty and J.Z, lliox (Eds) *Proceedings of the 2003 Joint meeting of PME and PMENA*, Volume 3. University of Hawaii: USA, pp 325-331.

Muller, M. & Maher, C. (2009). Learning to reason in an informal Math After-School Program. *Mathematics Education Research Journal*, 21 (3), 109-119.

Barton, P. & Barton, B. (2005).The Relationship Between English Language and Mathematics Learning for Non-native Speakers. *International Journal of Mathematical Education*, 36(7), 721-729.

Opie, C. (2004). Research approaches. In C. Opie (Ed). *Doing Educational Research*. SAGE. London. Pp 73-94.

Packer, A. (1997). Mathematical Competencies that Employers Expect. In L. A. Steen (Ed.), *Why Numbers count: Quantitative Literacy for Tomorrow's America* (pp. 137-154). New York, NY: The College Board.

Patton, M.Q. (2002). *Qualitative evaluation and research methods* (3rd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Piaget, J. (1964). Development and Learning. In Ripple, R. & Rockcastle, V. (Eds) (1964). *Piaget Rediscovered*. Ithaca: Cornell University Press.

Pimm, D. (1982). Mathematics? I speak it fluently. In A. Floyd (Ed.), *Developing mathematical thinking*, (pp. 139 – 150). Wokington: Addison Weslely.

Pimm, D. (1991). Communicating mathematically. In Durkin, K & Shire, B (Ed.) *Language in Mathematical Education: Research and Practice*. Open University Press. Milton Keynes. Pp 17-23.

Plowman, L. (1999). *Using Video for observing interaction in the classroom.* Spotlight 72, Scottish Council for Research in Education. Retrieved on 11 April 2010 from http://www.sfu.ca/media-lab/cmns362/spotlight72.pdf.

RAND Mathematics Study Panel. (2002). *Mathematics Proficiency for all students: Towards a strategic research and development program in mathematics education* (DRU-2773-OERI). Arlington, VA:RAND Education, Science and Technology Policy Institute. Chapter Three: Teaching and Learning Mathematics Practices. (pp. 29-41).

Ross, K. A. (1997). *Second Report from the MAA Task Force on the NCTM Standards. Mathematical Association of America Web site*. Retrieved on 30 July 2010 from URL: www.maa.org/past/nctmupdate.html.

Schleppegrell, M.J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly,* 23 (2), 139-159.

Schuck, S. & Kearney, M. (2004). *Digital Video as a tool in Research Projects: Zooming In On Current Issues.* Retrieved on 11 April 2010 from http://www.ed-dev.uts.edu.au/teachered/research/dvproject/pdfs/Edmedia.pdf.

Schumacher, S. & McMillan, J. (1993). *Research in education* (3rd Ed.) New York: Harper Collins College Publisher.

Selden, A. & Selden, J. (2003). Validations of Proofs Considered as Texts: Can Undergraduates Tell Whether an Argument Proves a Theorem? *Journal For Research in Mathematics Education*, 34(1), 4-33.

Setati, M. (2005a). *Power and access in multilingual mathematics classrooms*. Paper presented at the proceedings of the 4th International Mathematics Education and Society Conference, Australia.

Setati, M. (2005b). Teaching Mathematics in a Mathematics Primary Multilingual Classroom. *Journal for Research in Mathematics Education,* 36(5), 447-466.

Setati, M. & Adler, J. (2000). Between Languages and Discourses: Language Practices in Primary Multilingual Mathematics Classrooms in South Africa. *Educational Studies in Mathematics*, 43(3), 243-269.

Setati, M. & Barwell, R. (2006). Discursive practices in two multilingual mathematics classrooms: An international comparison. *African Journal of Research in SMT Education*, 10(2), 27-38.

Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning,* 2(3), 157-189.

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics,* 46(3), 13-57.

Stein, M.K., Grover, B.W. & Henningsen, M.A. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used

in reform classrooms. *American Educational Research Journal*, 33(4), 455-488.

Van de Walle, J. A. (2004). *Elementary and middle school mathematics: teaching developmentally*. New Jersey: Pearson Education. Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press

Wolcott, H.R. (1990). *Qualitative inquiry in education*: The continuing debate.

Yopp, D.A. (2010). From Inductive Reasoning to proof. *Mathematics Teaching in the middle school*, 15(5), 286- 291.

Zevenbergen, R. (2000). "Cracking the Code" of mathematics classrooms: School success as a function of linguistic, social, and cultural background. In J. Boaler (Ed.), *Multiple perspective on mathematics teaching and learning* (1st Ed.), pp. 201 – 242. London.

Appendix A: Permission Letters

Subject Information Sheet

TO WHOM IT MAY CONCERN

As a requirement for the fulfillment of the Masters degree that is awarded by the University of the Witwatersrand I will undertake a study that will involve participation of learners and mathematics teachers.

The envisaged study will take place in multilingual classrooms where learners learn mathematics in a second language. The aim of the study is to understand how learners communicate their mathematical reasoning in a multilingual classroom in South Africa. One Grade 11 class will be selected to participate in the study. This means that one teacher will also be involved though the teacher will not be the subject for the investigation. A video will be used to record the lessons. Permission to record will be sought from the learners and their parents.

The data will be analysed and a report will be written. Thereafter the data will be destroyed after a period of not more than five years. In the event the data is used in conference presentations the names of the participants and the school will not be used. If the need to mention names arises, pseudo names will be used.

Participants in the study will do so voluntarily. They will be given consent/assent forms to sign as an indication that they agree to participate. Guardians of participants will also be given a form to show their consent for their children's participation. Furthermore, participants will be informed that they can withdraw at anytime from the study. Participants and non participants will neither be advantaged nor disadvantaged by their participation or non participation as far as marks are concerned. These rights will be explained to the participants before data collection begins.

I hope this information is sufficient to give an understanding and the scope of the research. For more information please do not hesitate to contact me. My contact details are as follows:

Cell:0713500817
Email:benadetteaineamani@gmail.com

Yours Faithfully

Benadette Aineamani

…….

Department of Education acceptance letter

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Enquiries: Nomvula Ubisi (011)3550488

Date:	17 May 2010
Name of Researcher:	Aineamani Benadette
Address of Researcher:	21 Symons Road
	Brixton
	Johannesburg
Telephone Number:	0713500817
Fax Number:	N/A
Research Topic:	Communicating Mathematics Reasoning in Multilingual Classrooms in South Africa
Number and type of schools:	1 Secondary School
District/s/HO	Johannesburg East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. ***The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.***
2. ***The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.***
3. ***A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.***

Office of the Chief Director: Information and Knowledge Management
Room 501, 111 Commissioner Street, Johannesburg, 2000 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0809 Fax: (011) 355-0734

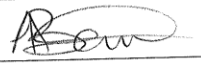
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Director: Knowledge Management & Research with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

24-05-2010

Martha Mashego
ACTING DIRECTOR: KNOWLEDGE MANAGEMENT & RESEARCH

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	24/05/2010

To the Principal

Benadette Aineamani
Wits School of Education

THE PRINCIPAL,
ERASMUS MORANENG SECONDARY SCHOOL,
JOHANNESBURG EAST.

Dear Sir/Madam

RE: REQUEST TO CONDUCT RESEARCH IN YOUR SCHOOL

I hereby request permission to conduct research in your school. My name is Aineamani Benadette. I am currently pursuing my studies in the Master of Science programme at the University of the Witwatersrand. My research is on communicating mathematics reasoning in a multilingual classroom. The study will entail working with mathematics learners and a teacher in Grade 10 to administer research instruments. The whole process is expected to take not more than seven days. I wish to assure you of my commitment to ensure minimal disruptions of the operation of the school and of the classroom where the data will be collected.

Consent will be sought from learners and their guardians, and the classroom teacher. Learners will be told that participating will on voluntary basis and that those who will agree to participate will be guaranteed confidentiality and anonymity and an option to withdraw during the course of the exercise. I promise to abide by the school regulations during the period of data collection.

I hope this request will meet your favourite consideration.

Yours Faithfully
Benadette Aineamani
……

Parent/Guardian consent form

I have read the information and I understand my role in the study. I also understand that:

- My child's participation in the study is voluntary
- My child can withdraw any time during the course of the study
- His/her name will not be used in the study
- All his/her responses will remain confidential
- The data will be destroyed after five years
- His/her participation will not be rewarded by marks or any other enticement

I, therefore, grant him/her permission to participate in the study.

Name:…….. Signature ……..

Date……..

PARENT CONSENT FORM: Videotaping

I …….. (please print your name in full) a parent to……., am aware of all the data collection processes in this study as listed in the information sheet attached. I give consent to the following:

• videotaping my child during the mathematics lesson.

Yes No

(use a cross to indicate your selection)

Name:…….. Signature ……..

Date……..

Teacher's consent form

I have read the information and I understand my role in the study. I also understand that:

- My participation in the study is voluntary
- I can withdraw any time during the course of the study
- My name will not be used in the study
- All my responses will remain confidential
- The data will be destroyed after five years

I, therefore, agree to participate in the study.

Name:…….. Signature ……..

Date……..

Learner's consent form

I have read the information and I understand my role in the study. I also understand that:

- My participation in the study is voluntary
- I can withdraw any time during the course of the study
- My name will not be used in the study
- All my responses will remain confidential
- The data will be destroyed after five years
- My participation will not be rewarded by marks or any other enticement

I, therefore, agree to participate in the study.

Name:…….. Signature ……..

Date……..

LEARNER CONSENT FORM: Videotaping

I …….. (please print your name in full) a mathematics learner at ……., am aware of all the data collection processes in this study as listed in the information sheet attached. I give consent to the following:

• Being videotaped during mathematics lesson.

Yes No

(use a cross to indicate your selection)

Signed …….. Date …….

Appendix B: Lesson Transcripts

Lesson one Transcript

	SPEAKER	Utterance
L1 1	Teacher:	Ooh he is saying the range, what he has found out is that there is a range there. So range is there, is the range part of the five number summary, yes, no
L1: 2	Class:	No
L1: 3	Teacher:	No
L1: 4	Class:	Yes
L1: 5	Teacher:	Okay let's just write it even though, because that's what he has learned about (writes range on the board), saying the range, how do you define the range, what's the range
L1: 6	Class:	Highest score minus lowest score
L1: 7	Teacher:	No, okay I will separate whatever you give me, tell me whether I falls under the five number summary, right, what else, Joy
L1: 8	Joy:	No idea
L1: 9	Nicho:	We only look to three first quartile
L1: 10	Teacher:	Three, why three and not...
L1: 11	Nicho:	Quartile, it's the lower quartile which is the median of the first half
L1: 12	Teacher:	Lower...
L1: 13	Class:	Quartile
L1: 14	Teacher:	What's the abbreviation of the lower quartile
L1: 15	Class:	Q1
L1: 16	Teacher:	Q1 (writes Q1 on the board) okay and the other one?
L1: 17	Class:	The median
L1: 18	Teacher:	The median, right median the abbreviation
L1: 19	Class:	Q2 or M

L1: 20	Teacher:	Q2 or M, *le* [This one] (points at Q2 and M on the board)
L1: 21	Class:	Yes
L1: 22	Teacher:	Which is the median, what is the median, who can define for us, let's go back a little bit
L1: 23	Learner:	The number that divides that divides the data into two parts, right
L1: 24	Teacher:	Next what's the third quartile
L1: 25	Class:	Upper quartile
L1: 26	Teacher:	Upper quartile (writing on the board) how do we abbreviate
L1: 27	Class:	Q3
L1: 28	Teacher:	Q3, okay how many do we have now
L1: 29	Class:	Three
L1: 30	Teacher:	One two three, but what are we looking for
L1: 31	Class:	Five number summary
L1: 32	Teacher:	Two, outstanding...
L1: 33	Class:	Minimum value and maximum value
L1: 34	Teacher:	It's minimum value and maximum value, right, I want us to look at exercise eight point six on page one hundred eighty nine, there it says in your groups, the baker keeps (inaudible) of number of dough nuts sold a day for three weeks, the numbers are (inaudible) there is a data that is listed there, they said find the range using the formula that the range is equal to highest score minus lowest score
L1: 35	Class:	Lowest score
L1: 36	Teacher:	Right, I want you in your groups (inaudible) on the data, find the range and the five number…
L1: 37	Class:	Summary
L1: 38	Class:	Five number summary in your groups, I will be moving in your groups checking , okay, what's the first step, what do you do (teacher moves around the class as the learners are discussing in their groups)

L1: 39	Many learners:	(In one of the groups in class) Arranging in...
L1: 40	Teacher:	Arranging in...
L1: 41	Many learners:	In ascending order
L1: 42	Teacher:	Arranging the data in ascending order, quickly do that, why are you five in the group (asking the learners in their group)
L1: 43	Learner:	(learners discussing in their groups) No, no
L1: 44	Teacher:	Who (inaudible) okay what is the range, define the range, not yet
L1: 45	Class:	Yes
L1: 46	Teacher:	Quickly calculate it
L1: 47	Learner:	M'am
L1: 48	Teacher:	Yes, how did you do it, how many are they, one two three, twenty, you see so it means there is a problem, the first what you check, did you pick all your data, check for that, it have been agreed (Helping one of the groups in the class)
L1: 49	Learner:	Fifty two, the range
L1: 50	Teacher:	What's the range here, what is the range, do you get range here, where is your book
L1: 51	Learner:	It's forty five
L1: 52	Teacher:	Right, what's your range here, right people can we just summarise here what's the range (teacher goes back to the board)
L1: 53	Class:	Forty eight
L1: 54	Teacher:	Forty eight, people did we arrange from the highest to the lowest, yes or no
L1: 55	Class:	No
L1: 56	Teacher:	No, because it would be wasting your time le, look (inaudible) find the highest score, find the lowest score add and, move on, now number two, we have the range there, what is the highest score there
L1: 57	Class:	Eighty two
L1: 58	Teacher:	Eighty two minus the lowest...

L1: 59	Class:	Thirty four
L1: 60	Teacher:	Which is (pointing at the data set on the board)
L1: 61	Class:	Thirty four
L1: 62	Teacher:	Which is (pointing at the data set on the board)
L1: 63	Class:	Forty eight
L1: 64	Teacher:	Forty eight, right who is done with number two a, okay there is a hand (goes to one of the groups in the class), label so that you remember lower quartile, eh next aah what's this, you must write quartile, median, range or what please, don't just write anything you don't know, use both please sometimes you find out ahah of them (inaudible) so don't summarise too much, next
L1: 65	Learner:	I forty six [its forty six]
L1: 66	Teacher:	They say arrange after that number b
L1: 67	Learner:	*Sithole imedian, le ephakhati khuphela forty six* [We have found this median, the one inside is forty six]
L1: 68	Teacher:	The median number, the middle number which one is the middle number
L1: 69	Learner:	Between sixty seven and sixty two
L1: 70	Teacher:	Between sixty seven and sixty two you say the median is between sixty seven and sixty two
L1: 71	Learner:	Yes
L1: 72	Teacher:	Which one is in the middle, okay let's check did you arrange in ascending order, so which one is your median then, okay lets count, count the data, count your scores there, how many scores do you have one two three four five (counting the scores) seven so which one is in the middle
L1: 73	Learner:	fifty
L1: 74	Teacher:	Its true fifty, can you put your hand on top of fifty, count this side then that side, so let's just count seventy eight sixty seven which one is the, you know what your scores are not the same are you working as the whole group
L1: 75	Learner:	Yes
L1: 76	Teacher:	No she got seventy six, she has sixty seven we don't have sixty nine, you are no working in a group, you must discuss as a

		group, you see, let me see seventy eight, sixty nine and which did you add extra seeming you have other one extra, so you see that's why the median is not the same, so how did you define the median, you say the median is the middle number, what does middle mean
L1: 77	Joy:	Number in between
L1: 78	Teacher:	In between, you say your median is what?
L1: 79	Joy:	Number in between the one this side and the other that side
L1: 80	Teacher:	But you did not write the here you have write sixty seven or what is or we want sixty seven and sixty two, right let's discuss two minutes what is the median and give me your answer (inaudible) not at all, the other thing you must write this is a median in brackets Q2, in brackets Q2 something, right next, Q1 what is Q1, the first quartile and the minimum value, are they the same
L1: 81	Joy:	No
L1: 82	Teacher:	You have written Q1 (inaudible) are you working as a group, you must work as a group, so discuss, what is the difference between first quartile and minimum value, third quartile and maximum value, okay, can I check your median, what's this right median (inaudible)
L1: 83	Joy:	Q2
L1: 84	Teacher:	So you have not written anything for today, remember you are all learners no spectator you must write, eeh I'm coming there, lower quartile, the median is correct, you say three to one, *ja* [yes] you have to write it down, after two months you have forgotten, next, lower quartile
L1: 85	Learner:	Three numbers after, find the lower quartile, the next Q1 okay between Q1 and (inaudible)
L1: 86	Teacher:	You understand now, you have divide data into...
L1: 87	Peter:	(Discussing with his group mates while the teacher was listening) Four
L1: 88	Teacher:	Four *ja* [yes]
L1: 89	Peter:	*Sifuna amamedian* [we want medians] i.e. add and divide by two
L1: 90	Joy:	*Ja* [yes] first quartile
L1: 91	Peter:	*I forty six* [its forty six]

L1: 92	Joy:	*Ja* [yes] forty six number three
L1: 93	Teacher:	When you divide into two parts then you get the median from there take the first half find the middle quartile range, lower quartile upper quartile , people write in full in your books, when you study you study it, don't summarise here write it in full
L1: 94	Joy:	*Lesibala ngapa ubale weng mfana i19 ne 17 ne i17 le 17 no i19 lo17 eish 36* [This one which we are reading here, you should write it boy, it's 19 and 17 and 17 and 17 and 19 and 17 oh 36] what's half of 36
L1: 95	Learner:	lower quartile *le* [this one]
L1: 96	Learner:	I first le third quartile [it's first and the third quartile]
L1: 97	Teacher:	Right any group who have done the stem and leaf I just want to check the stem and leaf now, there, there group
L1: 98	Brenda:	Here (raising hand up)
L1: 99	Teacher:	Stem and leaf how did you get this, right thirty two divide by two now, what is the median?
L1: 100	Brenda:	Number between numbers 36,6
L1: 101	Teacher:	Okay 36,6 okay can you count your data here, in a stem 10 good you can arrange stem and leaf in a correct way, where is my pen, I left it there
L1: 102	Brenda:	*Hasifuni lawa four four siarrange le* [we don't like these four fours let us arrange them]
L1: 103	Teacher:	And then first quartile, le [this one] right do the discussion, eeh Brenda you must remind your group mates they must label don't just write eeh also there label that what is that a stem and leaf, *ja* [yes] good uh is this quartile two
L1: 104	Brenda:	No
L1: 105	Teacher:	You see you must always check, Simon this is not quartile two its three, quartile two is there, your data divide into two take the first half divide into two, take the other half and divide into two again so that you get your quartile two so please be careful okay, next stem and leaf
L1: 106	Learners:	*Le* [This one]
L1: 107	Teacher:	Right you have only done the median, something something, what is this twenty three lower quartile range, okay for number two eeh what do we have, suppose to do stem and leaf for number three, you are rectifying number two

L1: 108	Brenda:	Yes
L1: 109	Teacher:	When you did it, with stem and leaves
L1: 110	Learner:	No we did it there stem and leaves *le* [this one]
L1: 111	Teacher:	Okay, you see that's why its confusing when you are studying you label it number two that's number three okay, continuing doing first quartile, third quartile for number three, using stem and leaf, you can also do the the…
L1: 112	Learner:	*Sipumile, imedian 26* [Here is the median, 26]
L1: 113	Learners:	26
L1: 114	Teacher:	Right (inaudible) quickly and do we meet again today
L1: 115	Class:	No
L1: 116	Teacher:	When are we meeting
L1: 117	Class:	Tomorrow and Wednesdays
L1: 118	Teacher:	Tomorrow and Wednesdays
L1: 119	Class:	Yes

Lesson Two Transcript

Speaker	**Utterance**
L2: 1 Teacher:	Which group can present the stem and leaf of data from the previous work, Thabo
L2: 2 Thabo:	(Goes to the chalk board, draws a stem and leaf on the board, writes an incorrect spelling of stem and the class corrects him), *ngu number bani lapha* [what is the number here]
L2: 3 Class:	One
L2: 4 Thabo:	One, here

L2: 5 Class: Two

L2: 6 Thabo: Here

L2: 7 Class: Two

L2: 8 Thabo: Here (for stem 3)

L2: 9 Class: Three, three, three

L2: 10 Joe: Ehee ehee *ngu six lapha bona bo three bayi two* [it is six here, see the threes are two]

L2: 11 Thabo: Oh *ja ja bayi two* [yes yes they are two]

L2: 12 Class: *Bayi two* [they are two]

L2: 13 Joe: So it's six

L2: 14 Class: Six, seven, eight

L2: 15 Thabo: *Wabona* [you see], here

L2: 16 Class: *Kunye, kubili, kubili* [one, two two]

L2: 17 Thabo: (writes on the board)

L2: 18 Class: Haa (laughter)

L2: 19 Thabo: *Kuthathu, kuthathu, kuthathu, kuthathu, Kuthathu, kune, kuhlanu* [three, three, three, three, three, four, five]

L2: 20 Teacher: Eh people I would say you don't say one, two. three you would use eleven, twelve, thirteen just to use the correct numbers then that way helps you to remember this, you then say twenty one, twenty three, okay

L2: 21 Class: Fourteen, fifteen, sixteen, seventeen, eighteen

L2: 22 Thabo: (writes on the board) *sesiqedile la* [We have finished here]

L2: 23 Class: *Ja* [yes] five, six, eight, nine

L2: 24 Joe: *Angiyibalanga* [I did not write it]

L2: 25 Nicole:	*Ehe wena awuyibalanga* [you, you did not write it]
L2: 26 Teacher:	How many scores are there?
L2: 27 Class:	Twenty four
L2: 28 Thabo:	(counts the scores to verify) *ja* [yes]twenty four
L2: 29 Teacher:	Right, thank you, your next session
L2: 30 Nicole:	Nceli [may I have]something
L2: 31 Brenda:	(Goes to the board)The start for our new (inaudible) is five then put
L2: 32 Teacher:	Listen, listen
L2: 33 Joe:	She is not loud
L2: 34 Brenda:	I started like finding the, mean of the scores…
L2: 35 Teacher:	Scores
L2: 36 Brenda:	Then this is the other, have been to, then I found that by have respective and…
L2: 37 Joy:	Mistake three
L2: 38 Nicole:	Three and four
L2: 39 Brenda:	Here…
L2: 40 Class:	Yes, yes
L2: 41 Brenda:	Three and four, I look for my lower quartile then I found its twenty three, then my median is twenty three plus twenty four divide by two because lower quartile is also median
L2: 42 Class:	No it is five
L2: 43 Brenda:	Eh can I talk
L2: 44 Class:	*Hoho siyadlala* [wait wait we are playing] don't rush

L2: 43 Nicole: *Unamanga, ayisi*, [You are lying, it's not] twenty three it's its fifteen plus sixteen

L2: 44 Teacher: Please, please help her

L2: 45 Brenda: I find the median of the whole numbers, then I find the median of the half because median of the lower is the lower quartile

L2: 46 Nicole: (inaudible)

L2: 47 Teacher: Eeh Joy what do you write, what is your lower quartile, how do you find the lower quartile

L2: 48 Joy: I divided the positive two by two…

L2: 49 Teacher: Right

L2: 50 Joy: Then I find …

L2: 51 Teacher: Remove the lower quartile, just do one thing at a time so that you are able to, good, eeh start with your median, remember we start with our median

L2: 52 Joy: We divide our data into two, then we calculate the median

L2: 53 Teacher: Right

L2: 54 Joy: Then I look for the lower quartile I found that its twenty three

L2: 55 Teacher: Twenty three

L2: 56 Joy: Yes

L2: 57 Teacher: How do you find the lower quartile tell us

L2: 58 Joy: I jumped this number, then I got to (points at 3 in between 33 and 34 in row three)

L2: 59 Peter: No

L2: 60 Teacher: Yes Peter

L2: 61 Peter:	Eeh in your, I must do, this is your half of your scores *le* [this one] and then get the half of the half of your scores and then you say (counts the scores) okay go straight to the half of the eleven, one two three four five six seven eight nine ten eleven four then here you count again (counts again from one to eleven) because your scores are giving there, therefore you take two numbers to get your middle number and then add them and divide by two and its going to be sixteen plus fifteen then your get thirty one divide by two which is equal to fifteen coma five and that means end quartile
L2: 62 Stella:	Lower quartile
L2: 63 Peter:	Lower quartile
L2: 64 Teacher:	You understand Joy now, can you do the upper quartile for us
L2: 65 Joy:	Upper quartile, for the upper quartile, I'm going to take there second half (points at the data set on the board)
L2: 66 Teacher:	Good
L2: 67 Joy:	Then I will divide into two and by dividing into two my half is going then (counts the scores) then I find that my middle this time I found its thirty three and thirty three
L2: 68 Nicole:	Which is sixty six
L2: 69 Joy:	Then I divide it by two its thirty three
L2: 70 Brenda:	Yes
L2: 71 Joy:	Then my next here, here, do I leave here (points at the data on the board)
L2: 72 Class:	Space
L2: 73Teacher:	Good, thanks, right can I add additional questions?
L2: 74 Joy:	Yes
L2: 75 Teacher:	Who wants to come and do the range for us, every time you come across questions try and do them on your own the other

	questions that we had gone through, Joe
L2: 76 Joe:	Yes, (Comes to the board) the range I find the highest number, for this one the range will be (inaudible) because it is the highest number that is given here (points at the board)
L2: 77 Teacher:	Eeh yes Brenda, Brenda what is a range?
L2: 78 Brenda:	Forty six
L2: 79 Teacher:	And what is the inter quartile
L2: 80 Brenda:	Range
L2: 81 Teacher:	Range
L2: 82 Brenda:	Fifty one one
L2: 83 Teacher:	What was there (pointing at the data on the board), you are listening
L2: 84 Brenda:	Yes, the interquartile range
L2: 85 Nicole:	(explaining something to the teacher, inaudible)
L2: 86 Teacher:	Ooh u have done everything, right now people any problems with other questions , did you encounter any?
L2: 87 Nicole:	Box and whisker
L2: 88 Teacher:	Okay who can draw that box and whisker for us so that you can move on to, now you can move on to the next question, right box and whisker who wants to come and draw it, can we have someone who wants to come and draw a box and whisker who wants to come and draw the box and whisker, okay Lerato wants to come and…
L2: 89 Lerato:	Try
L2: 90 Teacher:	Try, okay that's why you are here, try, can I just check, how did you draw your box and whisker because there is a problem who doesn't understand, let me see, can I see what how have you drawn it, no Jackie, it can't be, what must I correct, you must draw it so that I can correct it, (Moving around in the

learners' groups) each one of you still okay correct, okay who else has got a problem with box and whisker, which group?

L2: 91 Stella: Us

L2: 92 Teacher: Which group, you, right okay let's look at what Brenda has drawn there, your five number…

L2: 93 Class: Summary

L2: 94 Teacher: With your box and whisker, that's what you show, your five number summary then you say minimum value, the lower quartile, the medium, upper quartile and the maximum value, right, so people try and draw it because most of you didn't draw it, okay I'm going to give you some five minutes to draw, but now people what we should do le eeh try and use a scale, I will be coming to you and show you how to use a scale so that you can actually see whether your box and whisker is symmetrical, is skewed to the left or skewed to the right

L2: 95 Stella: Right

L2: 96 Teacher: Okay, so you can draw it using a free hand, sit down, it looks symmetrical but if, people you must use a ruler try and do some scale, don't, now if you look at the eeh, Brenda has drawn box and whisker using free hand. Right? The information is misleading because look at your box and whisker there, box is symmetrical but if you draw it it's not symmetrical. Right? In other words the difference between the lower quartile and the median is not equals to the difference between the median and the upper quartile.

L2: 97 Class: Upper quartile

L2: 98 Teacher: So, let's use the ruler to draw a scale, okay draw, did you see what I did there?

L2: 99 Nicole: No

L2: 100 Teacher: Oh you have drawn nice, let's see, *ja* [yes] good but you must label, please show them, label, don't put five there, put five number summary, okay next group, they are fine, you are also fine, I expect everyone to be drawing, people remember I told

you, you must give me my, please speak with me, where is the register, the register, where is it?

L2: 101 Learner: This one

L2: 102 Teacher: But it's always the same, right lets read here, some books when they talk about skewed, they are saying a symmetrical data set is balanced or you heard so that it have to be exactly, exactly on either side of the median, note that it doesn't have to be exactly equal on both sides to be called symmetrical. This is good what you are doing then calculate thirty three minus twenty three. It's what? Twenty three coma five minus fifteen, so I want you to read the drawing, you must read about symmetrical, skewed to the right, skewed to the left, when your box is skewed to the left, when it is skewed to the…

L2: 103 Nicole: ...Right

L2: 104 Teacher: But note about symmetrical, you don't have to get to get exactly exactly. Do you understand?

L2: 105 Joy: Yes

L2: 106 Teacher: *Ja* [yes], and then but you must show the number you know why you must show the numbers because when I'm marking your papers how will I be able to see that you should know the minimum, you don't write the lot of numbers just show the minimum based we talking of five number summary show the lower quartile, show the median, show the upper quartile, show the…

L2: 107 Class: Maximum value

L2: 108 Teacher: Maximum value. Do you understand? So please people just go through the symmetrical data, eeh all of you today after you have drawn your box and whisker, can I have your attention please, after you have finish drawing your box and whisker just turn on page one hundred and ninety four and read there about skewed and symmetrical data then analyse your box and whisker, is it symmetrical, is it skewed to the right, is it skewed to the…

L2: 109 Class: ...Left

L2: 110 Teacher: Left. ,Okay please differentiate between the three, who can't draw, who can't draw, please people try and draw in your books, where is your book, you can't just draw, open it, I want to see what is inside your book, ahah he is not writing, where is your stem and leaf, where is your stem and leaf, eeh Peter you are in the same group with Simon sitting next to you, you have the stem and leaf, he doesn't have

L2: 111 Peter: Simon took his book because, explain (asks Simon to explain for himself to the teacher)

L2: 112 Simon: Because I was…

L2: 113 Teacher: Outside during the period here

L2: 114 Simon: I came and, *thatha* [took] my back

L2: 115 Teacher: So for how many days you didn't have your book?

L2: 116 Simon: Um...

L2: 117 Teacher: But now what did I say, if you didn't have a book, what did I say?

L2: 118 Class: Write on a page

L2: 119 Teacher: You write on a page and then slip stick it, you must write on a page you can't sit like this Simon you can't okay, on break you should come to me so that you can write everything, okay, I will be checking before the end of the period, ladies, ladies, ladies enjoy your....

L2: 120 Nicole: Sixty seven exactly (helping his peer)

L2: 121 Teacher: Right, people now let's move on to exercise eight coma ten, exercise eight coma ten, number one, eight coma ten, eight coma ten in your groups, people you must draw those box and whiskers and then analyse the data, i.e. whether it is skewed or symmetrical, people just write on top there five number summary, stem and leaf (inaudible), stem and leaf, five number summary, minimum value, maximum value, write everything to be clearer for you, you must write five number summary,

	bring your book so that I can mark. Sam! Where is your five number summary, still write it here
L2: 122 Sam:	It is here.
L2: 123 Teacher:	*Ja* [yes] but why do you squeeze it here, just write it here, write it neatly here, the five number summary , write here five number summary here, Sam Sam what are you laughing, have you finished eight coma ten?
L2: 124 Sam:	No
L2: 125 Teacher:	So why are you laughing (continues explaining to Sam) so you write five number summary summary, no no you must write you know what you must write in your book such that you look on your information after three months it gives you information because if it's just numbers like this after three months, what did you say, what is this, you will have forgotten about it, you must write five number summary, minimum value, maximum value, the box and whisker, label it box and whisker diagram so that you remember what is this, and don't call it thing and thing and thing, they have got their names
L2: 126 Sam:	Left and right…
L2: 127 Learner:	Five number
L2: 128 Sam:	Five plus fifteen *uyenza kanjani* [how are you doing it], imagine, *i-maximum* [maximum] *number yakho* [your number] seventy eight minus fifty…
L2: 129 Teacher:	Write there, what do they say, they say find the mean, *ne* [okay], you can calculate from here, you don't have to transfer data from stem and leaf to, because already…
L2: 130 Nicole:	They have given the scores
L2: 131 Teacher:	Eeh people exercise eight coma ten, *ne* [okay], you have been given scores there as stem and leaf you don't have to re-write, right, already stem and leaf is giving you what, the…
L2: 132 Class:	...The scores

L2: 133 Teacher: The scores from the smallest to the…

L2: 134 Class: ...Highest

L2: 135 Teacher: Highest, do you understand, time management, hullo can I have your attention please, stem and leaf, your data already has been arranged in an ascending order okay, please time management it's also important because if you re-write things that are not necessary you won't finish the question okay, do you understand, so that data there is giving us stem and leaf, just calculate your mean, median, whatever is asked there, okay

L2: 136 Learner: Yes

L2: 137 Teacher: Right, don't re-write there, you should be finished by now, how to calculate the middle, the middle, calculate, lets read our data, this is twelve, thirteen, fourteen, fifteen, sixteen, this is twenty, twenty, twenty one, twenty etc

L2: 138 Nicole: Yes

L2: 139 Teacher: That's your minimum value, that's your maximum value, *ne* [okay], so find the middle, divide the data into two, divide the data into two, uh, what are you calculating, uh no you don't have to add them you must count, just count one two three four remember what's this, this is twelve, thirteen, the first scores twelve, the second score thirteen, fourteen, fifteen, sixteen, sixteen, seventeen, do you understand

L2: 140 Nicole: Yes

L2: 141 Teacher: And then twenty twenty twenty one twenty two thirty thirty one thirty two thirty five thirty seven thirty nine forty four forty six forty eight forty nine

L2: 142 Nicole: Forty nine

L2: 143 Teacher: Forty nine, so do you see, so what you need is to count how many scores are here and then you know where is the out

L2: 144 Learner: Okay

L2: 145 Teacher: Okay

L2: 146 Nicole: Um

L2: 147 Teacher: Where is your textbook, where is your textbook, please bring the textbook to the class all the time, you see now if you had your textbook, you would be comparing how many did you get, how many did you get, but if you count as one like this it's a problem if you are wrong you are both wrong, do you see, how many did you get

L2: 148 Nicole: Thirty two

L2: 149 Sam: Thirty two, okay it's both thirty two, then what's the half of thirty two

L2: 150 Joe: Sixteen

L2: 151 Teacher: *Ja* [yes] have you found the median

L2: 152 Joy: Yes

L2: 153 Teacher: Okay, try to find the median and then discuss it, check if its correct

L2 154Nicole: Its thirty five

L2: 155 Teacher: It's what…

L2: 156 Nicole: Its thirty five

L2: 157 Teacher: Why is it thirty five

L2: 158 Nicole: Half is sixteen

L2: 159 Teacher: Sixteen, okay and thirty five, okay, let's see you say half it's sixteen, sixteen it means the first sixteen and the last sixteen, so let's talk about thirty two, thirty two, is it the even number or the odd number

L2: 160 Nicole: Even number

L2: 161 Teacher: So if it's an even number what do we do

L2: 162 Nicole: We add the numbers and divide by two because we want to find the half of it

L2: 163 Teacher: *Ja* [yes] you must take score number sixteen plus score number seventeen we add it together and divide by two, is it what you did, is it what you did

L2: 164 Nicole: Yes

L2: 165 Teacher: So which one is it

L2: 166 Mike: Thirty five and thirty seven

L2: 167 Teacher: Thirty five and thirty seven divide by…

L2: 168 Nicole: Two

L2: 169 Teacher: Okay, write it, you must write the formula mean it was two, you write the two numbers an divide by...

L2: 170 Mike: ...Two

L2: 171 Teacher: Two, okay

L2: 172 Mike: Thirty six

L2: 173 Teacher: Thirty six, write it, okay, don't just write thirty six, write mean equals to, what's the formula for the mean, go back and check the formula, how did you write it the formula, how do we write our formula…

L2: 174 Mike: X bar is equal to the sum of

L2: 175 Teacher: Good x bar is equal to the sum of all squares

L2: 176 Mike: Yes

L2: 177 Teacher: The mean not the median, this is the mean, why is it the mean not the median

L2: 178 Mike: Yes, it is the mean because it is from the formula

L2: 179: Teacher: *Ja ja* [yes yes] but how did you calculate the mean, ja, is it correct

L2: 180 Mike: No

L2: 181 Teacher:	No, how must you calc
	ulate the mean
L2: 182 Mike:	You must use this formula which is x bar is equal to...
L2: 183 Teacher:	You must use this formula
L2: 184 Mike:	Yes
L2: 185 Teacher:	Let's use our calculator, mode right, do you still remember that, mode
L2: 186 Mike:	The steps, the number that is appearing the most because it is the mode of the numbers
L2: 187 Teacher:	Two
L2: 188 Mike:	This one (points at two on his book)
L2: 189 Teacher:	*Ja* [yes] two, then enter your scores, where are your calculators

Lesson Four Transcript

	Speaker	Utterance
L4: 1	Teacher:	Stop concentrating on your book, we all have eeh question papers, this is one of the typical exam questions, I want you to answer after you discuss with people in your group, I will be back after twenty five minutes, okay
L4: 2	Class:	Yes
L4: 3	Teacher:	Try to select a person who is going to present, not all questions I'm going to give a chance to each group to give us a question, Try as many question can you present. are you discussing with your group?
L4: 4	Class:	Yes
L4: 5	Teacher:	Okay
L4: 6	Class:	(Discussing in their groups)
L4: 7	Stella:	*Ja* [Yes]
L4: 8	Joy:	Lower quartile
L4: 9	Peter:	Forty seven
L4: 10	Stella:	Forty five, forty five
L4: 11	Peter:	Forty two
L4: 12	Stella:	Ah *mfana* [boy] lets discuss
L4: 13	Joy:	(In her discussion group) I am telling you, I am giving you what it means, it comes as this at the end
L4: 14	Stella:	Oh
L4: 15	Joy:	Now you can say it shares it off because you have to share it into two to get the half of it, then you

		get the first quartile (demonstrating using the data set in her book)
L4: 16	Stella:	Now what is it, first quartile
L4: 17	Joy:	Second quartile you see, if we share it we get the lower quartile
L4: 18	Stella:	What lower quartile
L4: 19	Peter:	What we want that…
L4: 20	Joy:	Lower quartile is also first quartile because we get it first
L4: 21	Peter:	That's what we want
L4: 22	Many learners:	*(laughter*)
L4: 23	Stella:	What is the first quartile now
L4: 24	Joy:	First quartile, I don't know where I started, One, two, three, four, five… (counting from the data set in her book)
L4: 25	Learners in Joy's group:	Forty two, forty three…
L4: 26	Stella:	Hey guys, forty two, forty three, forty seven plus fifty, forty seven besides, aah…
L4: 27	Peter:	Forty seven
L4: 28	Stella:	Forty two, forty seven
L4: 29	Joy:	Divide by two to get it because we want the half of it which is median of the other half (pointing at the data in her book)
L4: 30	Stella:	Divide by two eeh…
L4: 31	Peter:	(Inaudible)

L4: 32	Stella:	Eeh *i-right* [it is right]
L4: 33	Peter:	Next
L4: 34	Joy:	One, two, three, four, five…
L4: 35	Learner:	*Kusala* one [There is one which is left]
L4: 36	Peter:	Ooh *ja ja* [yes yes]
L4: 37	Joy:	Draw a box and whisker diagram to present a data, *uyazwisisa* [are you understanding]
L4: 38	Stella:	Eeh
L4: 39	Joy:	Whisker diagram, whisker diagram *kupela* [only]
L4: 40	Peter:	Absolutely
L4: 41	Stella:	*Bona elakho ibook*[see your book] (telling Joy to see her book)
L4: 42	Many learners:	Lower quartile
L4: 43	Peter:	Lower quartile
L4: 44	Joy:	*Siyavuma* [we agree]
L4: 45	Many learners:	Max
L4: 46	Joy:	Max
L4: 47	Peter:	Quiet, *thula* [keep quiet] better I leave it
L4: 48	Stella:	Noise pollution
L4: 49	Joy:	*Ruler khati* [use a ruler to draw]
L4: 50	Stella:	*Shaya ngezandla*[use your free hands]
L4: 51	Joy:	Rough sketch

L4: 52	Peter:	Q 1
L4: 53	Joy:	*Ja* [yes]fifty eight
L4: 54	Peter:	Fifty eight
L4: 55	Joy:	*Sijahile* [we are in a hurry]
L4: 56	Stella:	*Yi three* [its three]
L4: 57	Joy:	*Yiyo le, siphumile* [It's the one which we got]
L4: 58	Peter:	Is skewed to the lower
L4: 59	Stella:	*I-wrong, iwrong* [its wrong, its wrong]
L4: 60	Joy:	If the second quartile, if the second quartile…
L4: 61	Peter:	Forty two
L4: 62	Joy:	*I-graph, yakuphi* [this graph is for which one]
L4: 63	Stella:	*Yakuphi* [for which one]
L4: 64	Peter:	*Ngizokhutshela*[I will tell you]
L4: 65	Joy:	Right, uh question *vele* [well]…
L4: 66	Stella:	*Uthini* [what are you saying]
L4: 67	Joy:	But use your calculator to determine the following about
L4: 68	Peter:	Mean thirty three *la* [here]
L4: 69	Stella:	Mean *yanifuti* [which mean are you talking about], so mean that part ...

Appendix C: Analysis of Textbook- Classroom Mathematics

Extract from the textbook	Comment
4. What do the mode, median and mean tell you about the widths of the leaves on the geranium plant investigated by David?	This question requires learners to communicate their reasoning because as they explain what the mode, mean and median tell them about the leaves, they will have to give an explanation to support their answer. The question does not have a specific answer since it is open ended.
d) Do you think that the test shows that the average life span of a bulb is over 200 hours? Explain your answer.	This is a question that requires learners to communicate their reasoning because the learners are expected to give an opinion and justification
2. Answer the following by studying the above data. It is not necessary to do any calculations. a) In 1. a) Sipho gets a mean of 6,2. Explain why this must be wrong. b) In 1. b) explain how Sipho can write down the correct mean immediately.	Learners are required to explain their thinking and this requires learners to communicate their mathematical reasoning
$\bar{x} = \frac{\Sigma x}{n} = \frac{\ldots\ldots\ldots}{\ldots\ldots\ldots} = \ldots\ldots\ldots$ 4. What do the mode, median and mean tell you about the average number of people in the cars during the time that the survey took place?	This question requires learners to communicate their mathematical reasoning about mode, mean and median because it is an open ended question.
c) Use the measures of central tendency to answer the following question: "Were the ages of the men in the ward on that night more or less than the ages of the women in the ward on that night?" Give a reason for your answer.	This question requires learners to communicate their reasoning because the question asks the learners to give a reason for their answer.

Appendix D: Analysis of Learners' oral communication

Speaker	Utterance	Comment
L2 129Teacher	Write there, what do they say, they say find the mean, *ne* [okay], you can calculate from here, you don't have to transfer data from stem and leaf to, because already…	**Level of reasoning:** L1IR, learners are responding in short phrases and one word answers. They are using only the given example to communicate their reasoning- the learner says they have given the scores. This script shows what is legitimised in the classroom. One word and short phrases are encouraged because: (i) the teacher allows them to complete her sentences (ii) the teacher condones short responses because there is no reprimand or any statement to the contrary (iii) Learners are aided to produce the required answer by the teacher through probing. Hence, in this Discourse we cannot tell if the learners would have produced the legitimate answer without the teacher's support.
L2 130Nicole	They have given the scores	
L2 131Teacher	Eeh people exercise eight coma ten, *ne* [okay], you have been given scores there as stem and leaf you don't have to re-write, right, already stem and leaf is giving you what, the…	
L2 132Class	...The scores	
L2 133Teacher	The scores from the smallest to the…	
L2 134Class	...Highest	
L2 135Teacher	Highest, do you understand, time management, hullo can I have your attention please, stem and leaf, your data already has been arranged in an ascending order okay, please time management it's also important because if you re-write things that are not necessary you won't finish the question okay, do you understand, so that data there is giving us stem and leaf, just calculate your mean, median,	

	whatever is asked there, okay	

L2 157Teacher	Why is it thirty five	Comment
L2 158Nicole	Half is sixteen	**Level of reasoning:** L1IR, the learner uses the given question to answer the teacher. **Level of language:** Cd1, the learner uses ordinary English to communicate her reasoning. For example- "We add the numbers and divide by two because we want to find the half of it."
L2 159Teacher	Sixteen, okay and thirty five, okay, let's see you say half its sixteen, sixteen it means the first sixteen and the last sixteen, so let's talk about thirty two, thirty two, is it the even number or the odd number	
L2 160Nicole	Even number	
L2 161Teacher	So if it's an even number what do we do?	
L2 162Nicole	We add the numbers and divide by two because we want to find the half of it.	
L2 163Teacher	*Ja* [yes] you must take score number sixteen plus score number seventeen we add it together and divide by two, is it what you did, is it what you did	
L2 164Nicole	Yes	

Inductive and deductive questions asked by the teacher during the lessons observed

Lesson	**Inductive reasoning questions**	**Deductive reasoning questions**	**Comment**

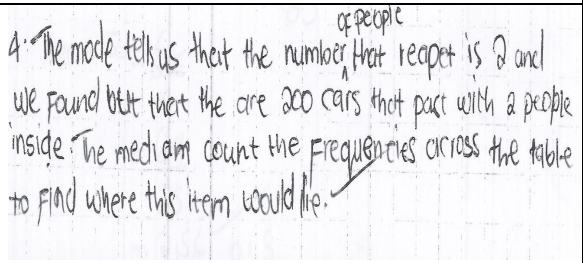
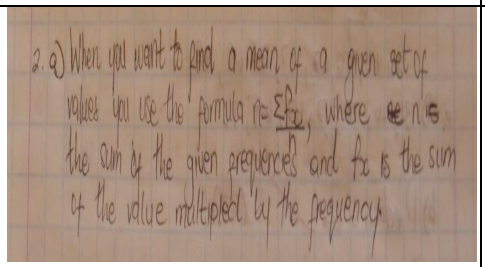
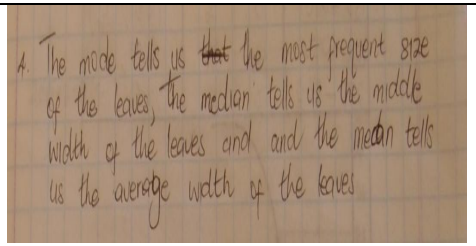
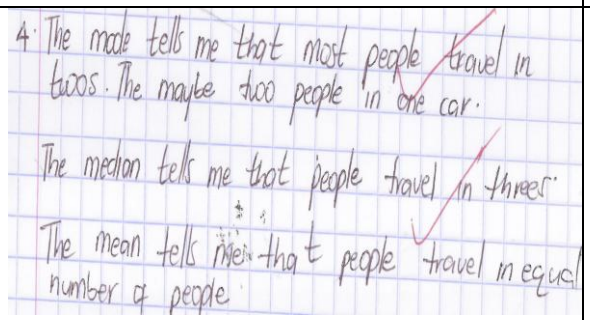
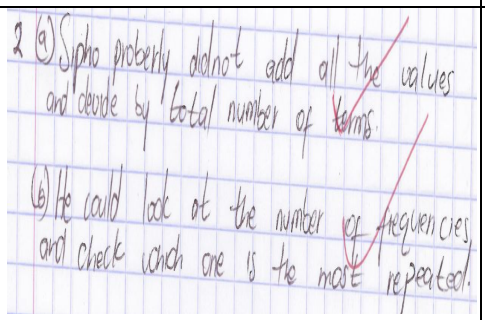
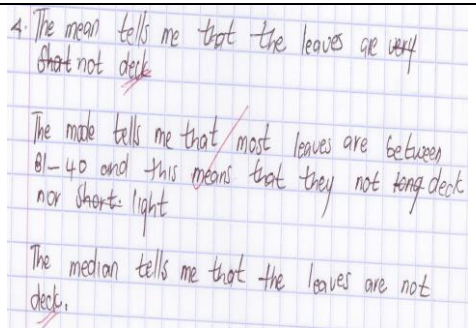
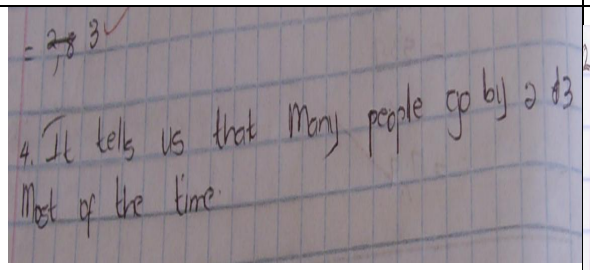
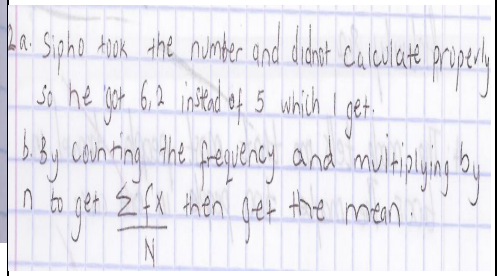
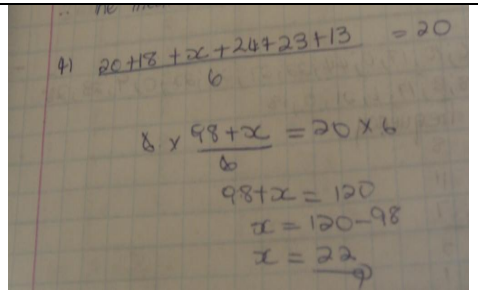
Lesson 1	10: Teacher Three, why three and not…	5: Okay let's just write it even though, because that's what he has learned about (writes range on the board), saying the range, how do you define the range, what's the range?	The teacher did not ask many questions that required learners to reason in this lesson.
Lesson 2	157: Teacher Why is it thirty five	157: Teacher Why is it thirty five	Some questions asked by the teacher were both deductive and inductive because the learners could have answered them using conventional definitions or using empirical data. The questions on line 161 and 171 are open ended and therefore the learner could have answered it inductively or deductively.
	161: Teacher So if it's an even number what do we do?	161: Teacher So if it's an even number what do we do	
	177: Teacher The mean not the median, this is the mean, why is it the mean not the median	177: Teacher The mean not the median, this is the mean, why is it the mean not the median	
Total	**4 inductive questions**	**4 deductive questions**	**5 mathematical reasoning questions**

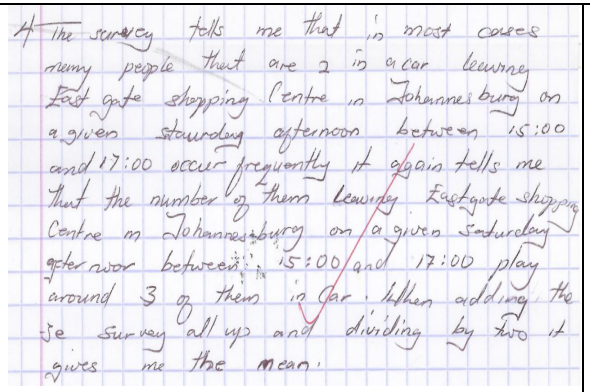
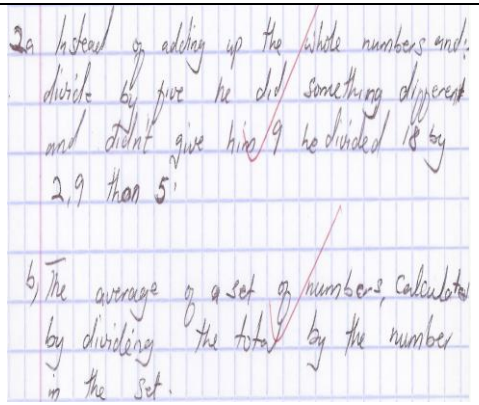
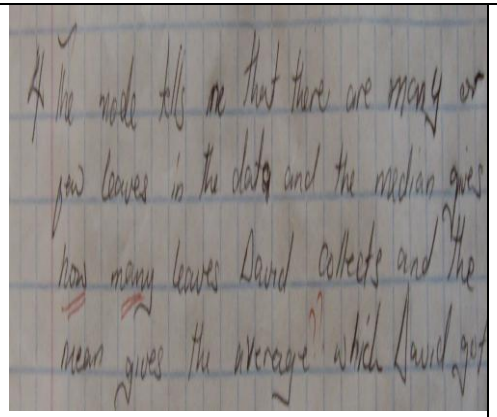
Responses from the learners to the questions which the teacher asked

lesson	Question	Response	Comment
Lesson 1	5: Okay let's just write it even though, because that's what he has learned about (writes range on the board), saying the range, how do you define the range, what's the range	6: Class Highest score minus lowest score	The class answered deductively to a deductive question because they gave the conventional definition of range
	10: Teacher Three, why three and not…	11: Nicho Quartile, it's the lower quartile which is the median of the first half	Nicho answered inductively to an inductive question because he used the given data to explain quartile
	157: Teacher Why is it thirty five	158: Nicole Half is sixteen	Nicole answered inductively by referring to the data given
	161: Teacher So if it's an even number what do we do?	162: Nicole We add the numbers and divide by two because we want to find the half of it	Nicole responded inductively to an inductive question which could also be referred to as a deductive question because a learner can use a conventional definition to explain what to do in the question
	177: Teacher The mean not the median, this is	178: Mike Yes, it is the mean because it is from the	Mike answered deductively to a

	the mean, why is it the mean not the median	formula	question that is both deductive and inductive. The response is deductive because Mike refers to the formula

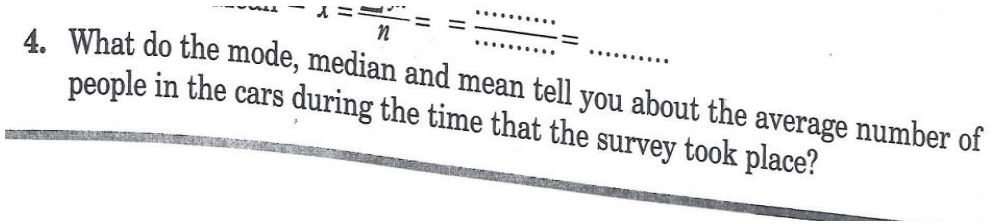
Appendix E: Types of responses given by the learners

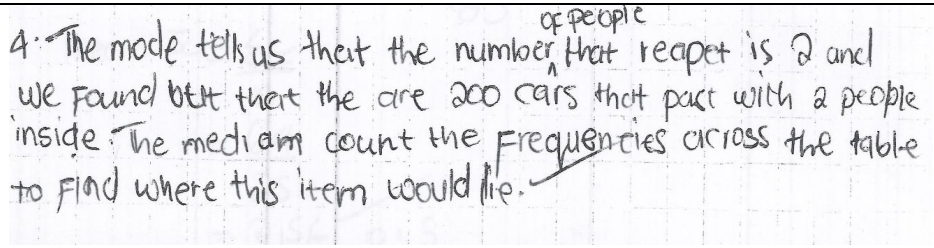
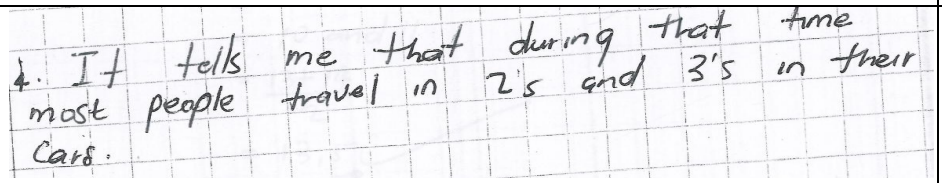
Name	Question 4 (about cars)	Question 2a and b	Question 4 (about leaves)	Total
Peter	 4. The mode tells us that the number of people that reapet is 2 and we found out that the are 200 cars that past with 2 people inside. The mediam count the frequencies across the table to find where this item would lie.	 2. a) When you want to find a mean of a given set of values you use the formula $n = \frac{\Sigma fx}{n}$, where n is the sum of the given frequencies and fx is the sum of the value multipled by the frequency	 4. The mode tells us ~~that~~ the most frequent size of the leaves, the median tells us the middle width of the leaves and and the median tells us the average width of the leaves	3 prose
Nicole	 4. The mode tells me that most people travel in twoos. The maybe two people in one car. The median tells me that people travel in threes. The mean tells me that people travel in equal number of people	 2 (a) Sipho proberly didnot add all the values and devide by total number of terms. (b) He could look at the number of frequencies and check which one is the most repeated.	 4. The mean tells me that the leaves are very ~~short~~ not deck. The mode tells me that most leaves are between 31–40 and this means that they not ~~long~~ deck nor ~~short~~ light. The median tells me that the leaves are not deck.	3 prose
Brenda	 = ~~2,8~~ 3 4. It tells us that Many people go by 2 & 3 Most of the time.	 2a. Sipho took the number and didnot calculate properly so he got 6,2 instead of 5 which I get. b. By counting the frequency and multiplying by n to get $\frac{\Sigma fx}{N}$ then get the mean.	 4) $\frac{20+18+x+24+23+13}{6} = 20$ $6 \times \frac{98+x}{6} = 20 \times 6$ $98+x = 120$ $x = 120-98$ $x = 22$	2 prose 1 symbolic

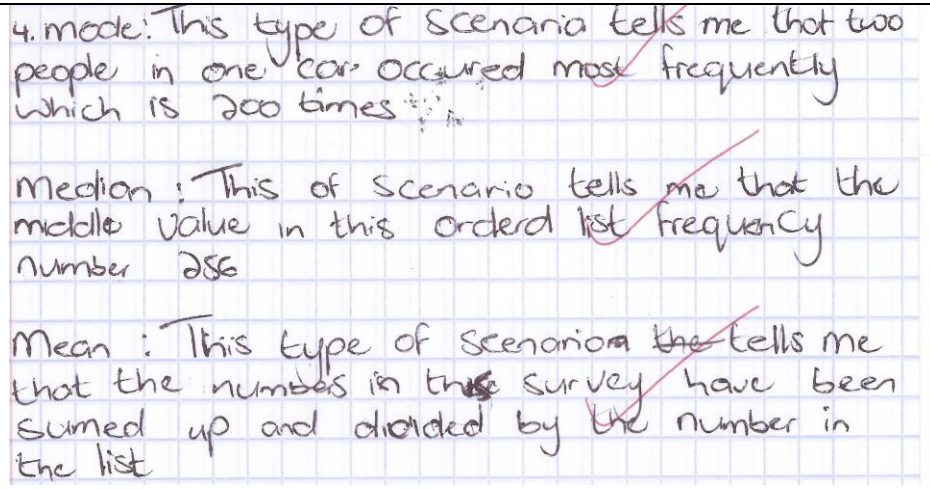
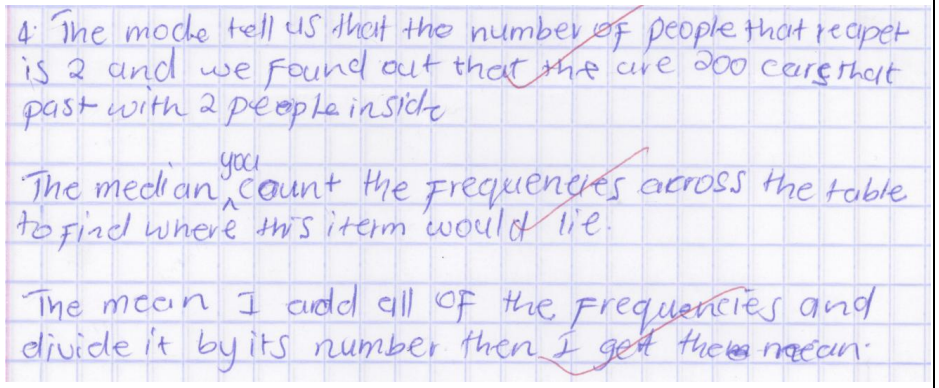
Joy				3 prose
Total	**4 prose**	**4 prose**	**3 prose 1 symbolic**	**11 prose 1 symbolic**

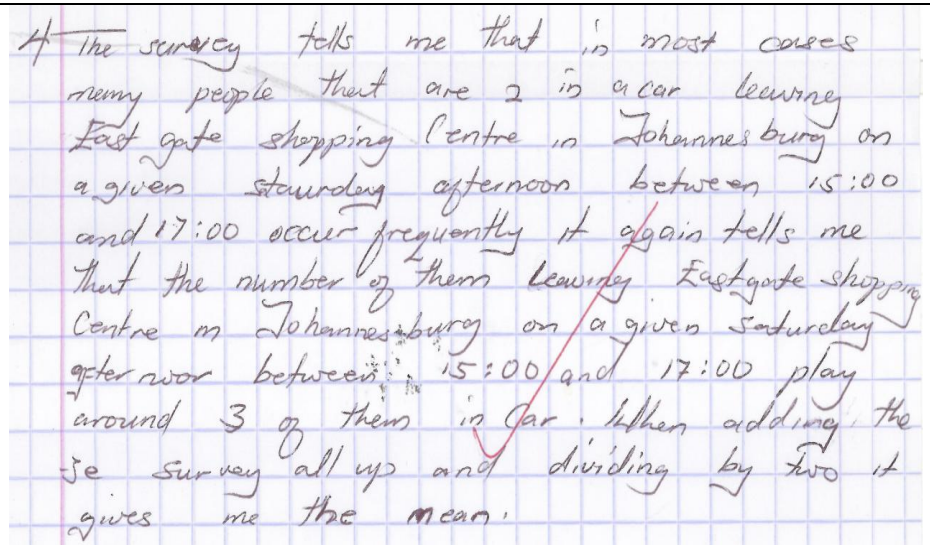
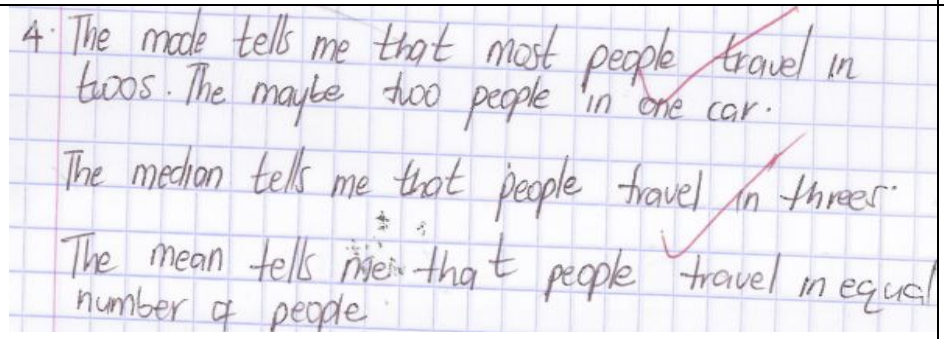
Appendix F: Levels of Mathematical reasoning

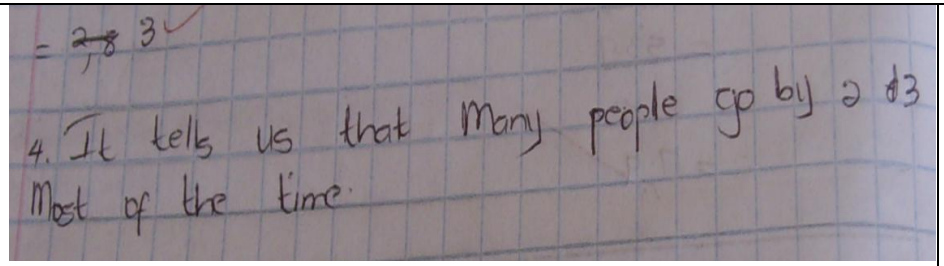
Levels in prose response of reasoning for question four:

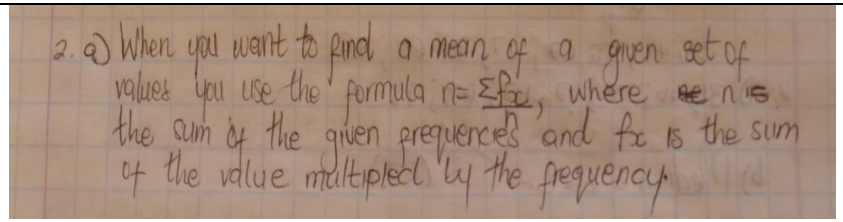
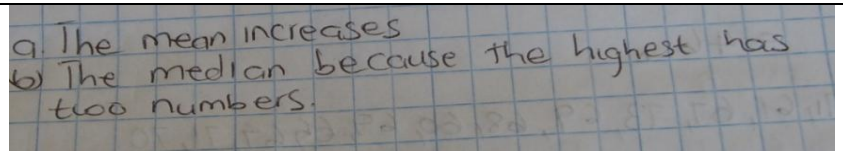


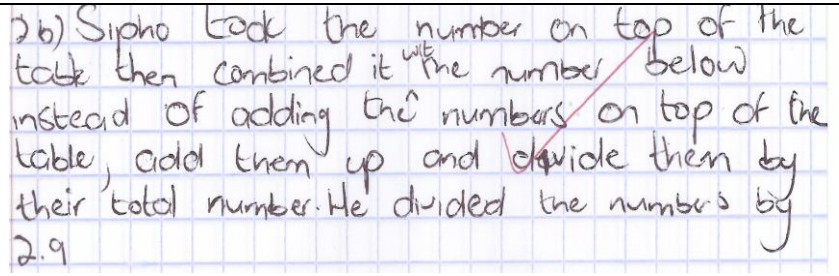
Learner	Question 4 Response	Level	Comment
Peter		L2IR	The learner gave a justification of what the mode is showing this particular problem by giving the highest frequency which is 200 cars. This is legitimized by the teacher because it is marked correct
Stella		L1IR	The learner did not give a justification of what the mode, mean and median are showing in the data.

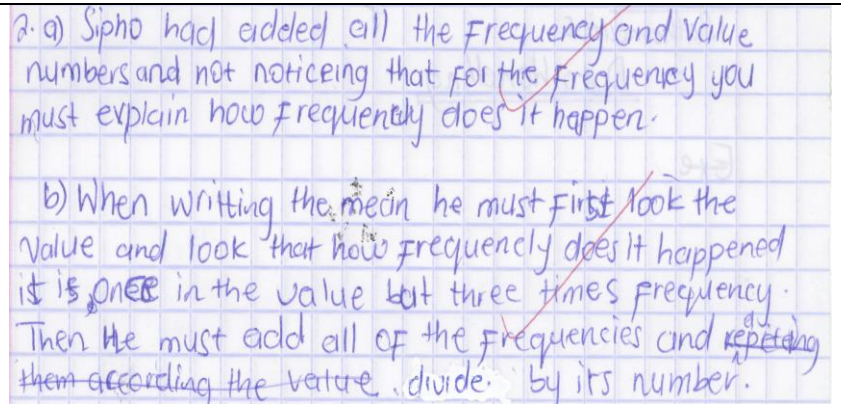
Joe		L2IR	The learner gave a justification of the mode by stating the frequency of 200, he also explains the median as the middle number and also explains what the mean is telling him in the data
Mike		L2IR	The learner gave justification by stating the frequency of 200 and he also explained how the mean and median is calculated in the given data. Therefore he gave a justification of his reasoning

Joy		L1IR	The used her own understanding of a scenario without giving a justification from the given data
Nicole		L1IR	The learner gives short answers that are not justified

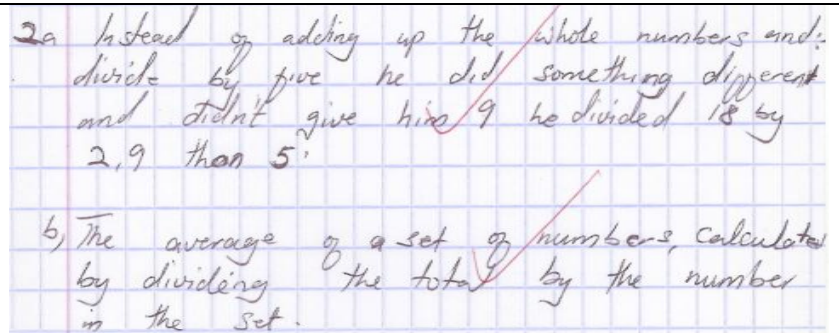
<table>
<tr><td>Brenda</td><td> 
</td><td>L1IR</td><td>The learner gives a short answer without any justification from the given data</td></tr>
<tr><td colspan="4">Levels on prose response of reasoning for question 2a and b:

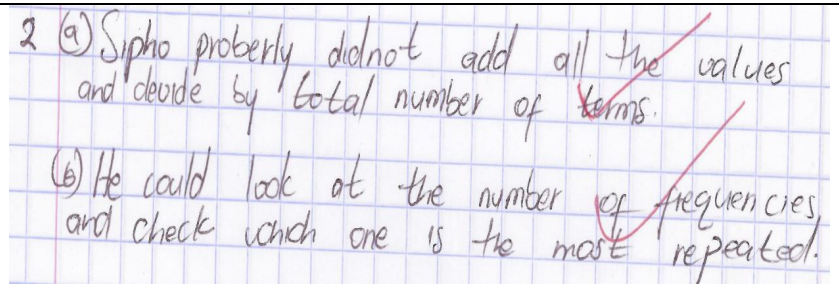
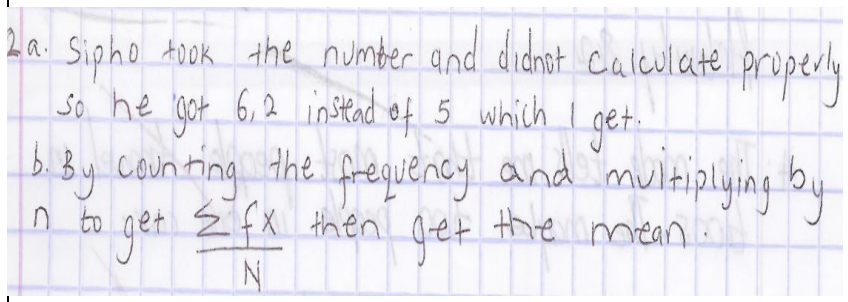
2. Answer the following by studying the above data. It is not necessary to do any calculations.
a) In 1. a) Sipho gets a mean of 6,2. Explain why this must be wrong.
b) In 1. b) explain how Sipho can write down the correct mean immediately.</td></tr>
<tr><td>Peter</td><td> 
</td><td>L1DR</td><td>The learner uses only the definition of mean to justify his answer</td></tr>
<tr><td>Stella</td><td> 
</td><td>L1IR</td><td>The learner gives a short answer without a justification</td></tr>
</table>



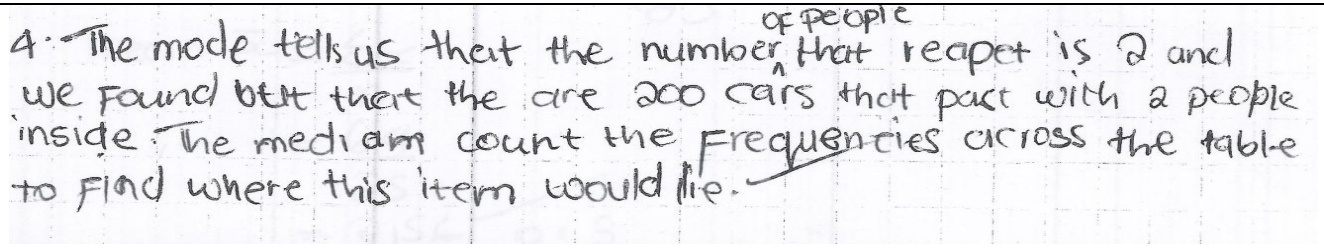
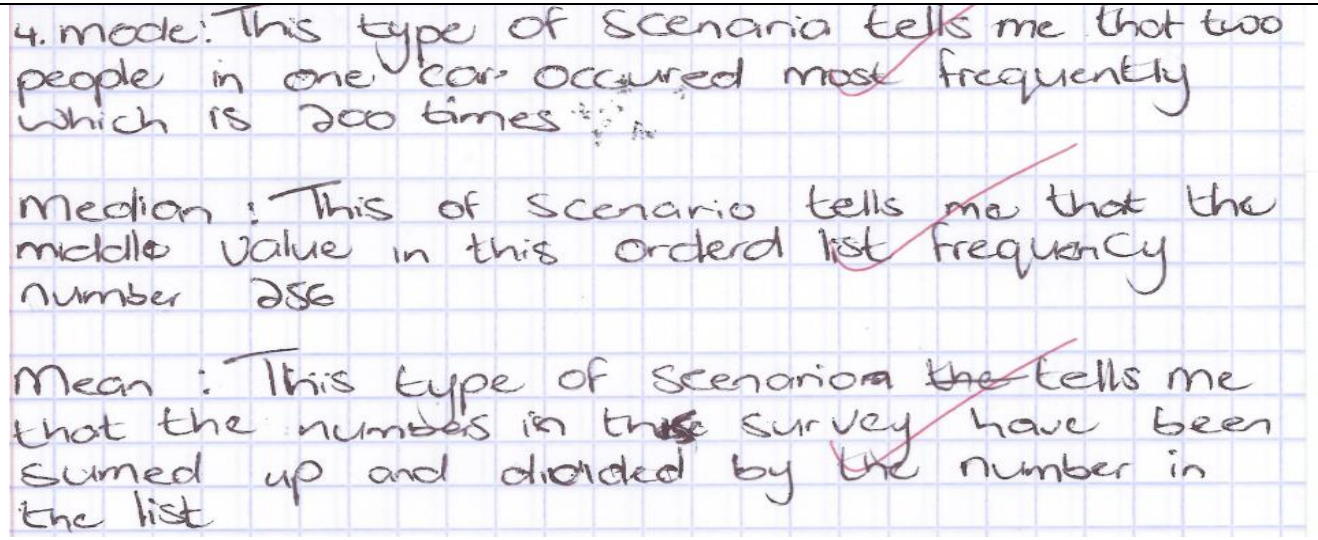


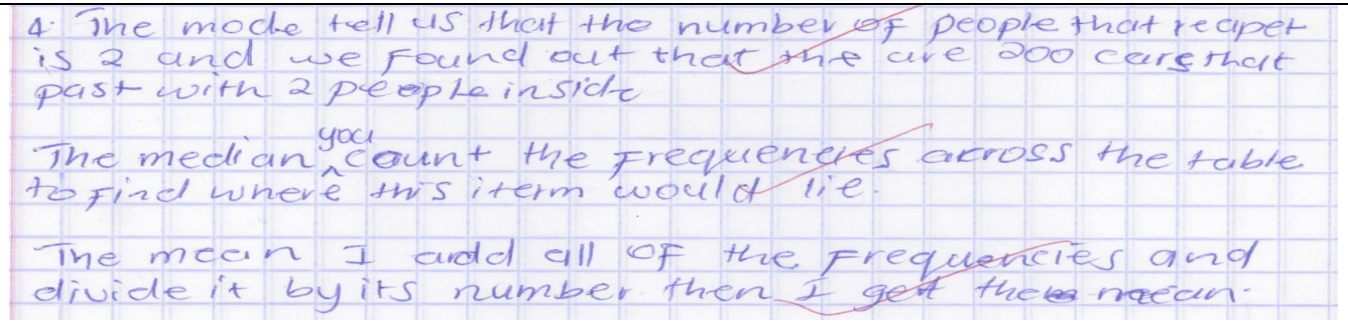
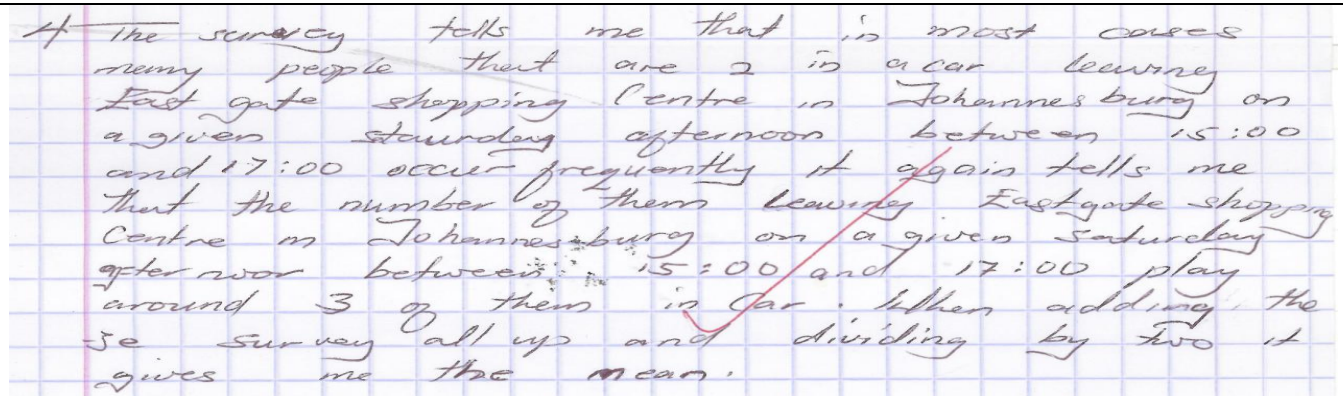
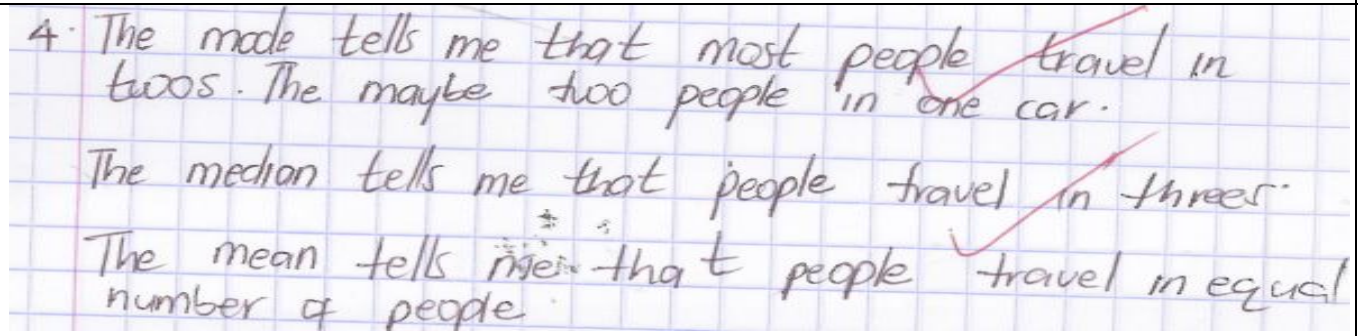
Joe	2 b) Sipho took the number on top of the table then combined it wit the number below instead of adding the numbers on top of the table, add them up and divide them by their total number. He divided the numbers by 2.9	L2IR	The learner uses the scenario of the given problem to justify why the answer given is wrong
Mike	2. a) Sipho had added all the Frequency and Value numbers and not noticeing that for the Frequency you must explain how Frequently does it happen. b) When writting the mean he must First look the Value and look that how Frequencly does it happened it is once in the value but three times frequency. Then He must add all of the Frequencies and ~~repeting them according the value~~ divide by its number.	L2IR	The learner uses the given problem as the example to explain how mean should be calculated
Joy	2a Instead of adding up the whole numbers and divide by five he did something different and didn't give him 9 he divided 18 by 2,9 than 5. b, The average of a set of numbers, calculated by dividing the total by the number in the set.	L2IR	The learner used the problem to explain how the mean should have been calculated



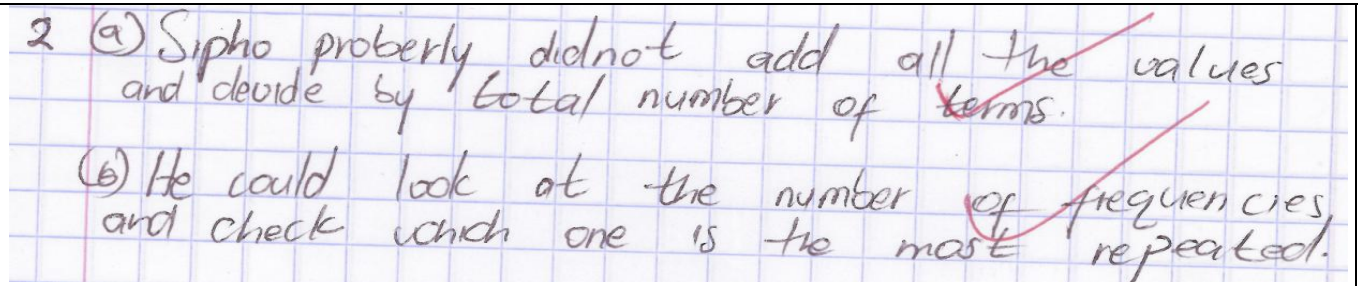
Nicole		L2IR	The learner gave an justification of how mean should have been calculated in the given problem
Brenda		L2IR	The learner justified her answer by explaining how the learner should have worked out this particular problem

Appendix G: Responses Legitimised by the teacher

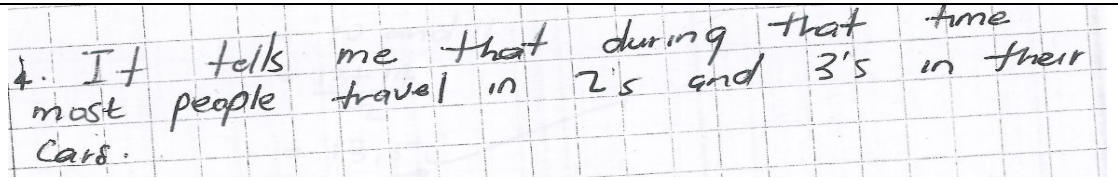
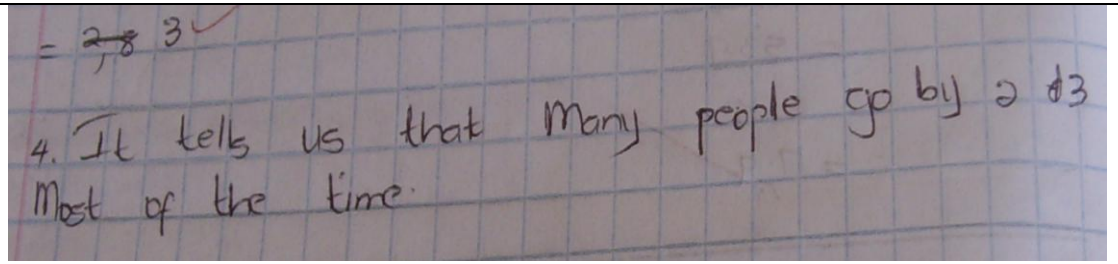
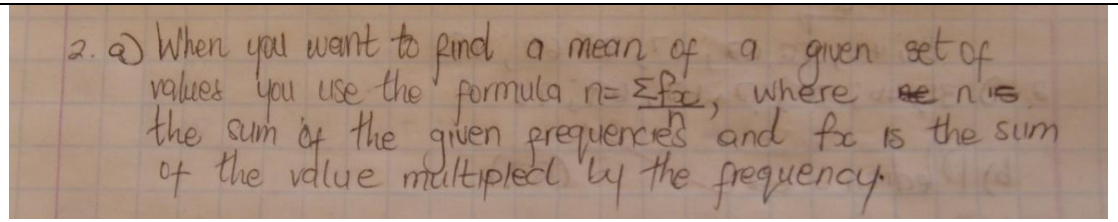
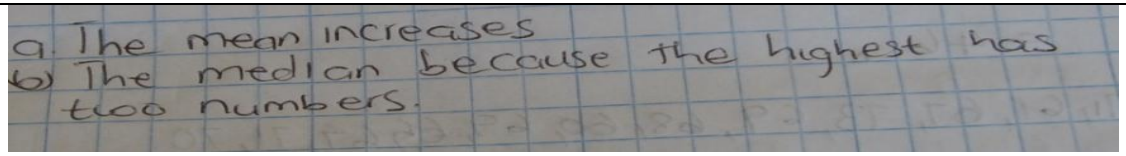
Learner	Response legitimsed by the teacher	Level
Peter		L2IR
Joe		L2IR

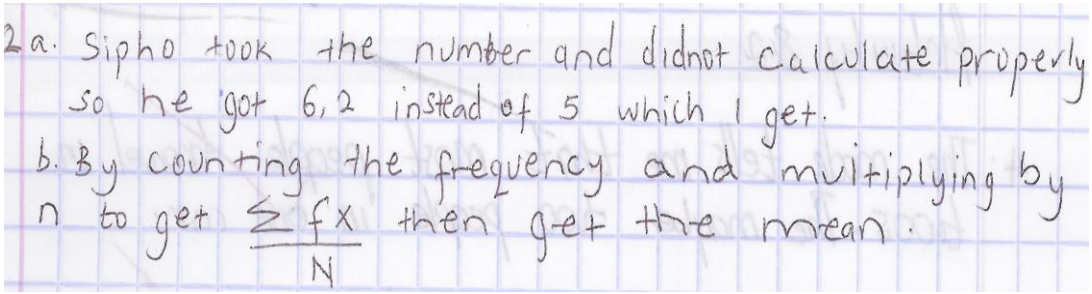
Mike		L2IR
Joy		L1IR
Nicole		L21R

Joe	2 b) Sipho took the number on top of the table then combined it with the number below instead of adding the numbers on top of the table, add them up and divide them by their total number. He divided the numbers by 2.9	L2IR
Mike	2. a) Sipho had added all the Frequency and value numbers and not noticeing that for the Frequency you must explain how Frequently does it happen. b) When writting the mean he must First look the value and look that how Frequency does it happened it is ONCE in the value but three times Frequency. Then He must add all of the Frequencies and repeating them according the vetue. divide by its number.	L2IR
Joy	2a Instead of adding up the whole numbers and divide by five he did something different and didn't give him 9 he divided 18 by 2.9 than 5. b) The average of a set of numbers, calculated by dividing the total by the number in the set.	L2IR

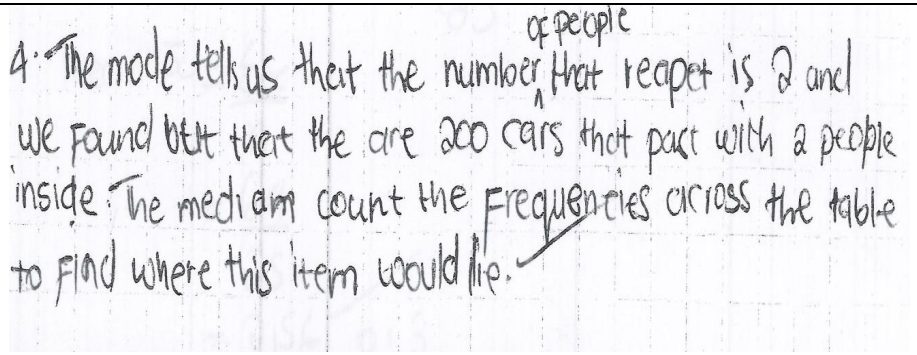
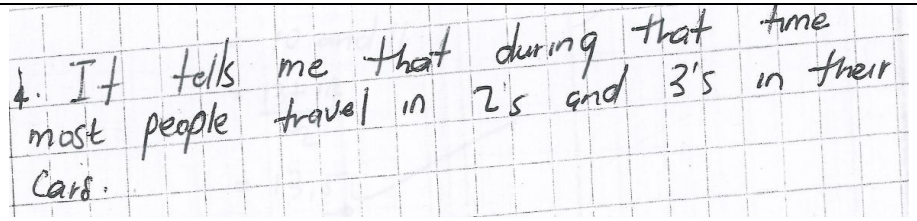
Nicole		L1IR
Total		**7 L2IR and** **2 L1IR**

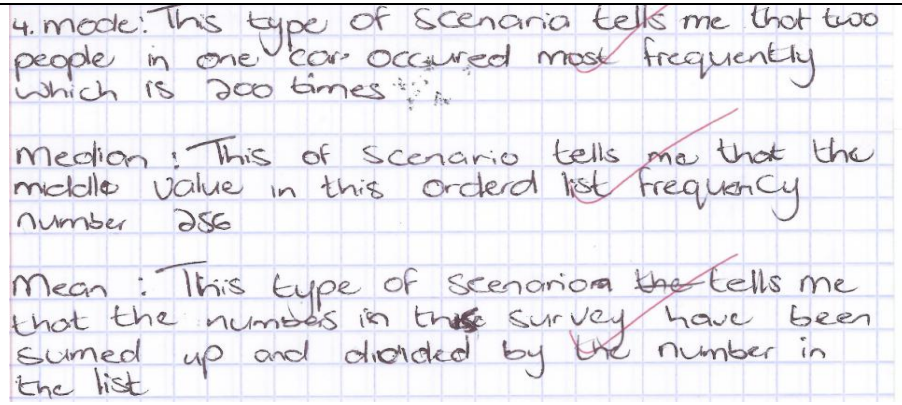
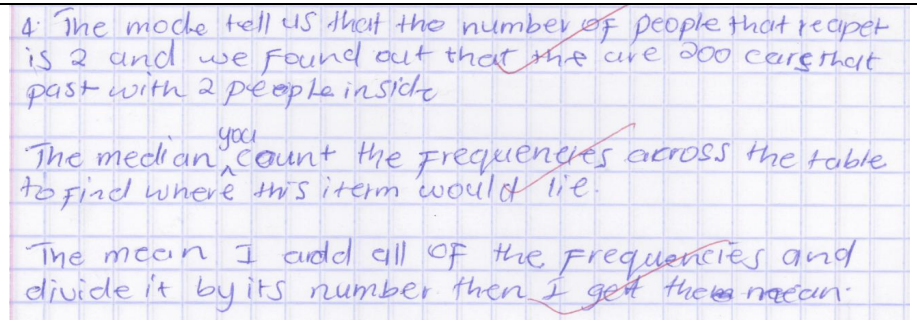
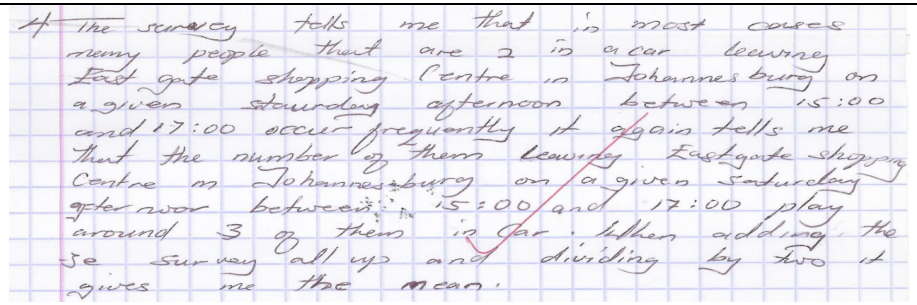
Appendix H: Responses not marked by the teacher

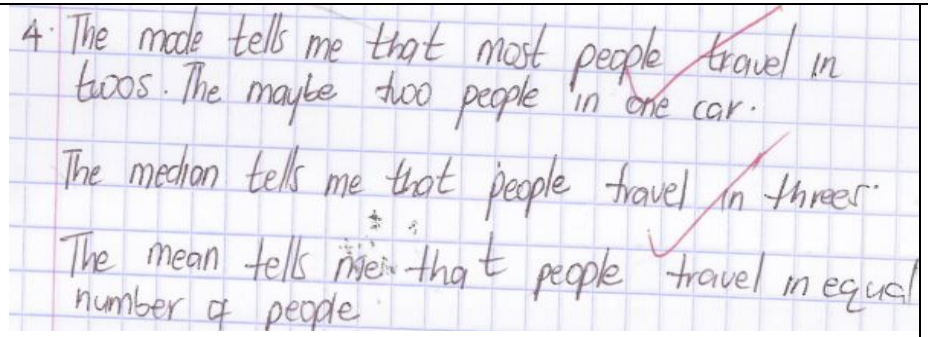
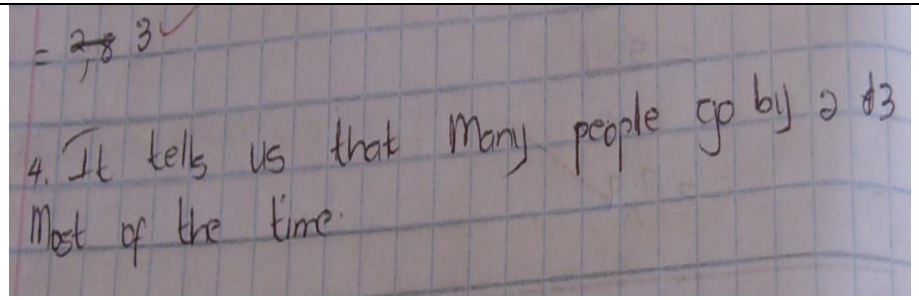
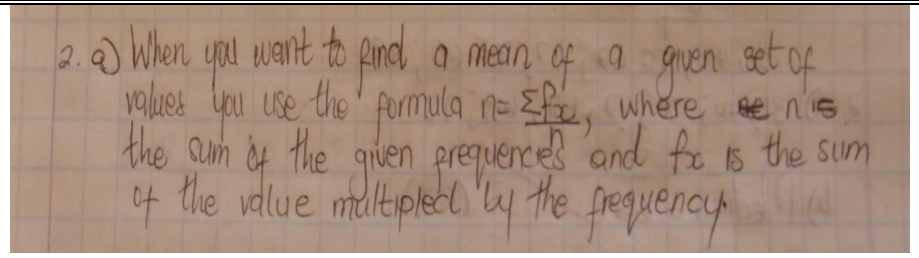
Learner	Response	Level
Stella		L1IR
Brenda		L1IR
Peter		L1DR
Stella		L1IR

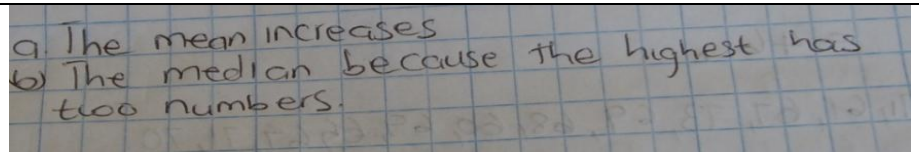
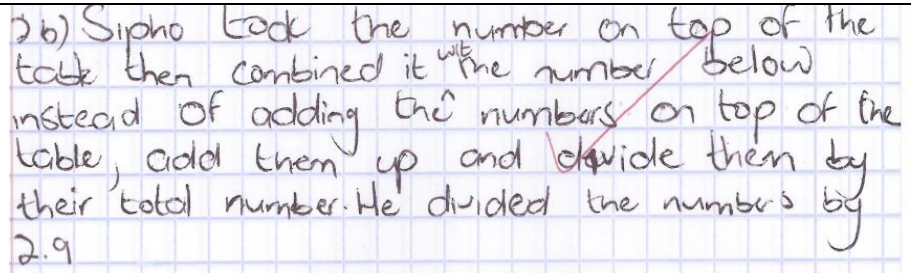
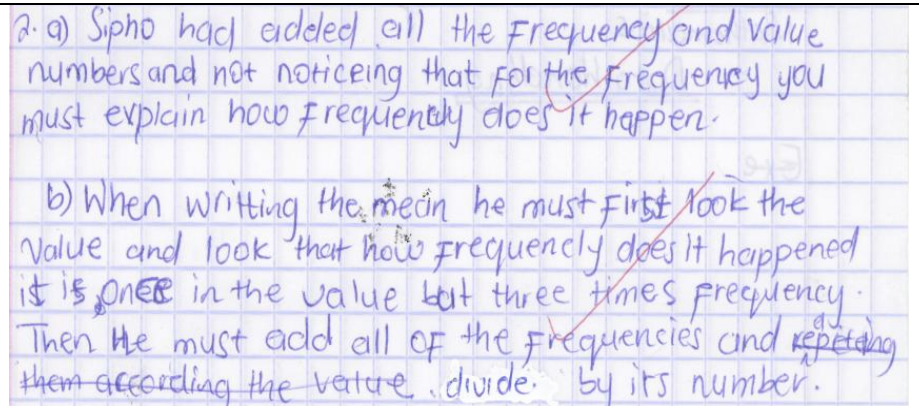
Brenda		L2IR
Total		**3L1IR, 1L1DR, 1L2IR**

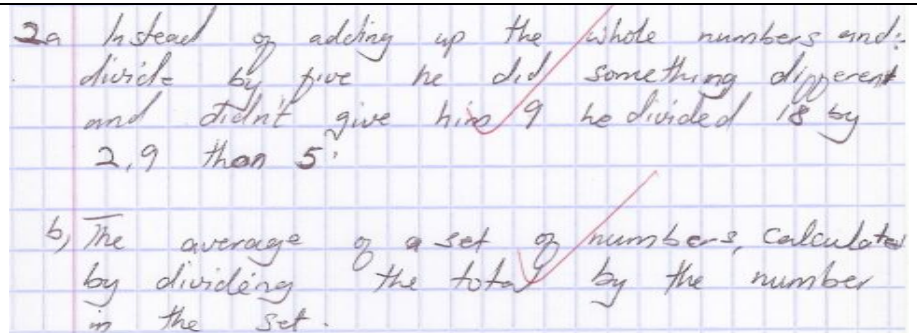
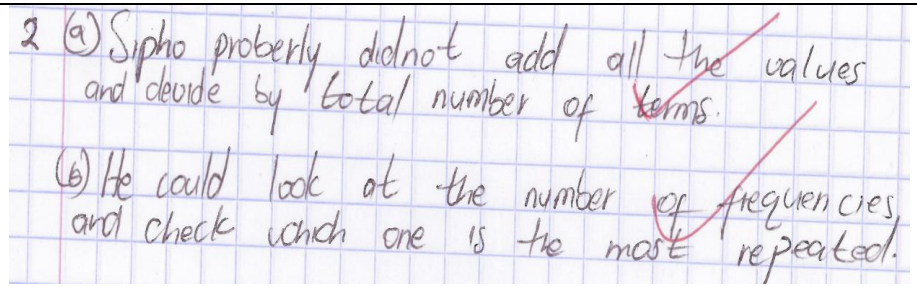
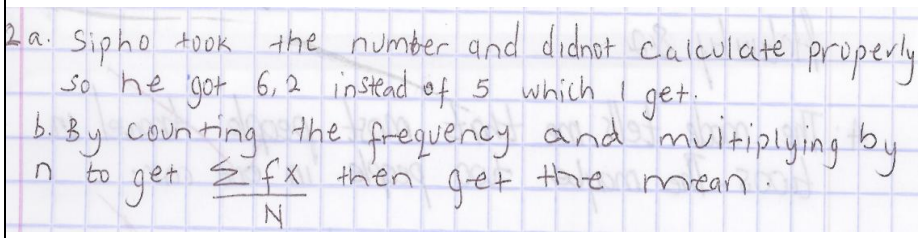
Appendix I: Levels of classroom discourse

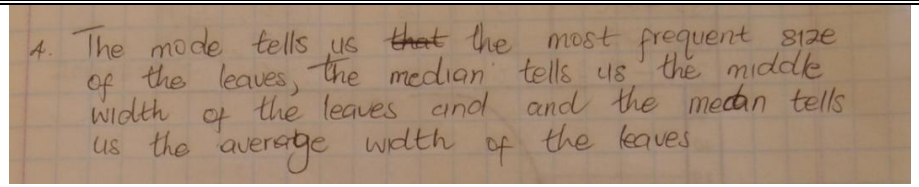
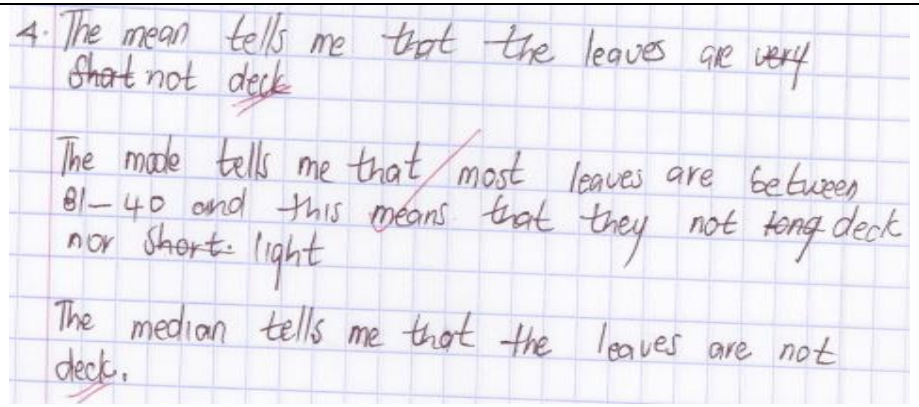
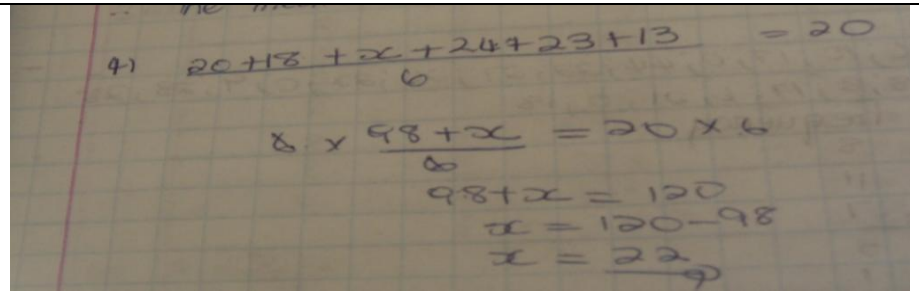
Learner	Response	Language Level	Comment
Peter	 4. The mode tells us that the number of people that reapet is 2 and we found out that the are 200 cars that pass with 2 people inside. The mediam count the Frequencies across the table to find where this item would lie.	Cd1	The learner managed to communicate his reasoning because the point he is making can be identified from the response though the language the learner used is not the formal language used in a classroom discourse. The learner used ordinary language (OL) to communicate his reasoning
Stella	 4. It tells me that during that time most people travel in 2's and 3's in their cars.	Cd1	The learner's point was made explicit by the language she used though the point made does not answer the question given and the learner used OL give answer the question

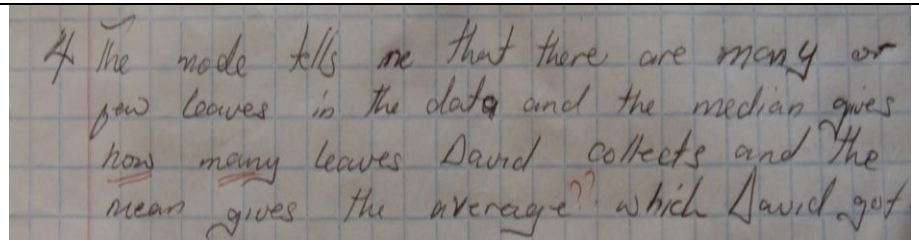
Joe		Cd1	The language that the learner used is not a formal way of communicating within the mathematics classroom discourse. The language the learner used is OL.
Mike		Cd1	The learner uses OL to respond to the question
Joy		Cd1	The learner's response is written in correct grammar though the learner does not address the formal mathematical language used in the classroom discourse. In other words the learner uses OL to answer the question

Nicole		Cd1	The learner uses OL to communicate her reasoning
Brenda		Cd1	The learner uses OL to communicate her reasoning
Peter		Cd1	The learner uses OL to communicate his reasoning

Stella		Cd1	The learner uses OL to communicate her reasoning
Joe		Cd1	The learner uses OL to explain her reasoning
Mike		Cd 1	The learner uses OL to communicate his reasoning

Joy		Cd1	The language which the learner uses is not formal mathematics language used in the mathematics classroom discourse. Therefore the learner is using OL to communicate her reasoning
Nicole		Cd1	The learner uses OL to communicate her reasoning
Brenda		Cd1	The learner uses OL to communicate her reasoning

Peter		Cd1	The learner uses OL to communicate her reasoning
Nicole		Cd1	The learner uses OL to communicate her reasoning
Brenda		Cd2	Discarded (the response does not make sense in relation to the question being answered)

Joy		Cd0	The learner uses OL to communicate her reasoning