8

LÁSZLÓ'S PAPERS ON TESSELLATED STRESSES: A REVIEW

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SYNOPSIS.

Four recent papers by László on " Tessellated Stresses, Parts I–IV ", published in the *Journal of the Iron and Steel Institute*, are reviewed in detail.

The present review contains no new results.

PART I.

F. LÁSZLÓ has published a series of four papers [1, 2, 3, 4] under the general title of " Tessellated Stresses ". It seems to be impossible to study this subject in any detail without tedious calculations, and the aim of this summary is to present the meat of László's general methods and results without the skeleton of calculation which holds them together. I have added a few comments, enclosed in square brackets, thus : [ ]. Where I have been unable to follow Dr. László's arguments, I have given my own views. Dr. László kindly sent me his comments on the first draft of this summary, but owing to the difficulties of communication a few points remain unresolved.

The paper by Dr. Orowan in the present volume emphasizes the complex nature and diverse origin of internal stresses, and touches briefly on the effect of these stresses on the mechanical properties and on the process of precipitation. The effects of strain on diffusion and precipitation are discussed in another paper. The scope of László's four papers coincides roughly with that of Orowan's contribution, but László includes a formal mathematical treatment of each problem and tables showing the numerical result of applying these considerations in special cases of practical importance. The resulting combination of general discussion, mathematical analysis, and detailed application is not easy reading, especially as war conditions enforced a compression of the manuscript. The mathematical problems arising are mostly intractable unless some simplifying assumptions are made. In some cases there is no doubt that the original simplifications do not seriously alter the problem; in other cases assumptions made in early papers of the series are reviewed in a later paper. I feel myself that in this preliminary

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attack on so large a field László has sometimes used mathematical methods or models of physical processes which further investigation will have to reject. This in no way alters the fact that he has made a most original, stimulating, and comprehensive contribution to an important subject which has received far too little attention.

Part I discusses (pp. 173–175) the various sources of internal stress in a polycrystalline material. Stresses due to external forces or to self-strain spread uniformly over regions containing many grains are called " body-stresses ". Other stresses are due to the failure of neighbouring grains to fit together properly. These stresses, which change in magnitude and direction from grain to grain, are described by the somewhat sibilant expression " tessellated stresses ".

Even a perfectly annealed specimen made of grains having cubic symmetry [iron or copper] develops tessellated stresses if it is loaded within the elastic limit. For (pp. 175–176) although the grains of a metal may have cubic symmetry, the elastic properties of a single grain are far from the same in all directions. Young's modulus for an iron crystal is more than twice as great in the (111) direction as it is in the (100) direction. This means that in a polycrystalline aggregate under tension the grains in which a (111) axis lies along the direction of the applied tension will [extend less, and] carry a larger stress, than their neighbours. Localized stresses are set up along the grain boundaries as a result of this. [Orowan and Nye [5] have recently demonstrated this photo-elastically in silver chloride.] By simplifying assumptions, these internal stresses are shown to be about 35% of the applied stress in iron.

The thermal expansion of cubic metals is isotropic, and if the grains fit together at the temperature of annealing they will still fit at room temperature. This no longer holds for hexagonal metals and others of low symmetry (pp. 176–177), and a change of temperature of 1° C. in zinc causes stresses between grains of the order of 200 lb./in.^{2}. [This has been investigated experimentally by Boas and Honeycombe.[6]]

Further analysis of stresses in aggregates of grains all of the same kind, but not isotropic, is left to the third paper, and the first paper considers two-phase systems in which the included phase may form laminar, rod-like or spherical particles (pp. 177–188). Both matrix and inclusion are now treated as isotropic. Two sources of internal stress exist. During the formation of the particles of inclusion stresses are set up because their volume generally differs from the volume of the metal they replace. If these stresses are removed by annealing, further stresses are set up on cooling because the thermal expansions of the matrix and the inclusion are different. The ratio (difference of linear dimensions of hole and inclusion)/(linear dimensions of inclusion) is

termed the strain potential of structural tessellation. For thermal strains this potential is usually of order $\frac{1}{10}$ to 1%; in precipitation processes it is often of order 1%, with an upper limit of about 15%.

In calculating the stresses, each particle of inclusion is considered to be embedded, not in an infinite block of matrix, but in a block of such a size that the ratio of the volume of the inclusion to the total volume has its correct value. The surface of this block is considered to be free from stress, and the solid is built up by assembling a number of such blocks. [The fact that the elastic calculations are carried out for cylindrical or spherical blocks, whereas only prismatic or polyhedral blocks would pack together properly, is of no essential importance.] It follows (pp. 180–181) that the coefficient of thermal expansion of a tessellated body is not simply a weighted mean of the coefficients for the components. [Consider, for example, particles of a material having a large thermal expansion, and very compressible, embedded in a firm matrix. A very small elastic strain in the matrix will produce enough pressure to prevent the expansion of the included particles on heating.]

Stresses of this kind are reduced at the surface of the specimen, because the shape of the surface adjusts itself in such a way as to reduce the elastic strain energy. The resulting reduced stresses are (pp. 181–182) about 70% of those in the interior of the body.

These methods are applied to estimate the stresses set up in the heat-treatment of iron and steel. Steel of eutectoid composition is considered as a laminated slab of pearlite : hypo-eutectoid steels are considered as having spherical grains of pearlite embedded in a matrix of ferrite, and hyper-eutectoid steels are considered as having spherical grains of pearlite embedded in a matrix of cementite. At 6·67% carbon the material is pure cementite. The stresses are calculated (pp. 183–185) on the assumption that the steel is annealed at 700° C., and thermal stresses are set up by cooling to 20° C.

The stresses in this rather complicated system, where the particles of inclusion are themselves compound and internally stressed, are calculated by the following [reasonable, but rough] method. First the internal stresses in the pearlite are calculated by considering the cooling of an isolated grain of pearlite. The principal stresses are + 10,450, +10,450 and 0 lb./in.^{2}. Then the hydrostatic stress set up by the different thermal expansions of the ferrite or cementite matrix and the pearlite inclusions is calculated on the assumption that matrix and inclusion are homogeneous materials with equal elastic constants but different thermal expansions. These are then added. For hypo-eutectoid steels most of the stresses are of the order of 7000–10,000 lb./in.^{2}, with greater stresses up to 70,000 lb./in.^{2} in the lamellæ of cementite. Stresses of this high order are also found in the cementite matrix in hyper-eutectoid steels.

Spheroidized steel is treated (pp. 185–186) as a dispersion of spheres of cementite in a matrix of ferrite. These are again annealed at 700° C. and cooled to 20° C. The ferrite matrix contracts more than the cementite, which, if the ferrite did not yield, would be under a hydrostatic pressure of about 50,000 lb./in.^{2}. Since the shear stress on the ferrite at the surface of each grain of cementite is over 40,000 lb./in.^{2}, there is little doubt that the ferrite yields, reducing the stresses both in itself and in the cementite. Yielding begins at the surface of each cementite particle, and is only likely to spread throughout the whole matrix if the carbon content is at least 2%.

In grey cast iron (pp. 187–189) the carbon separates as graphite, which has a much smaller thermal expansion (3×10^{-6}/° C.) than ferrite (15×10^{-6}/° C.) or cementite (12×10^{-6}/° C.). Consequently the stresses set up round a spherical particle of temper carbon on cooling are very large, of the order of 200,000 lb./in.^{2}. If the graphite forms lamellæ, the stresses are smaller by a factor of ten. [This is said to account for "the trend towards the formation of random lamellæ of graphite during solidification". The metal at high temperature cannot be influenced by the stresses which will ultimately be set up if the temperature is altered: it is not the strain energy due to thermal expansion, but the strain energy due to the change of volume in the reaction $Fe_3C \rightarrow 3Fe + C$ that causes the graphite to adopt a lamellar form.] For lamellar graphite it is assumed that "hydrostatic locked-up effects cannot develop, and two-dimensional stresses are set up in the graphite and the metallic matrix", and the calculated stresses are all less than 20,000 lb./in.^{2}. [The lamellar form of the graphite undoubtedly allows a reduction of hydrostatic pressure, but it is difficult to see how graphite can support any appreciable stress other than a pure hydrostatic pressure. The case in which the graphite yields is treated in Part IV.]

The volume change in the decomposition of cementite to ferrite and graphite corresponds to a change of linear dimensions of 4·39%. The resulting strain energy is so great that cementite embedded in iron is much more stable than it is when isolated. Similar considerations may show why it is cementite and not graphite which is originally formed when carbon separates from solid solution. [It is also suggested that cementite rather than graphite is formed in the carburization of iron because the volume changes are smaller. However, during the actual process of carburization the carbonized layer at least is austenitic, and the carbon is largely in solid solution.]

During the hardening of steel (pp. 189–193) "needles" [usually

plates, unless the carbon content is low [7]] of martensite form in the matrix of austenite. The resulting stress is calculated on the assumption that it depends only on the difference in the specific volumes of martensite and austenite, and the maximum shear stress is over 240,000 lb./in.^{2}. [From studies of lattice orientations and habit planes, it seems clear that the stress on a plate of martensite includes large shears in addition to the simple hydrostatic tension.] As the transformation proceeds, the model with dispersed needles of martensite embedded in a matrix of austenite is replaced successively by models with needles of austenite and with spheres of austenite retained in a matrix of martensite. If there were no yielding, the stresses would remain of the same order throughout the transformation. These stresses are all greater by a factor between two and five than the large-scale stresses caused by quenching, which are about 100,000 lb./in.^{2}. It is stated that the internal stresses caused by the martensite transformation "have an unvarying tendency to complete the transformation, thus assisting the effort due to the free energy of martensite being less at 125° C. than that of austenite".

The retention of austenite is in some way due to the difficulty of initiating the formation of a needle of martensite. If, by rapid cooling, the transformation is made to occur at low temperatures, each needle which is formed grows very violently, since the free energy of austenite is much higher than that of martensite at low temperatures. This violent disturbance initiates the formation of needles in the neighbouring parts of the crystal, and the percentage of retained austenite is therefore reduced. At still lower temperatures the transformation is so violent that regions of austenite, which would be transformed to martensite if a single needle were to spread more slowly, become trapped in the meshes of the growing needles. "These opposing effects seem to explain why there is a certain degree of undercooling, and a corresponding medium rate of transformation (with plain carbon steel), at which there is a maximum percentage of austenite retained."

[The increased violence of the martensite transformation at low temperatures accounts satisfactorily for the observed decrease in the amount of retained austenite with increased rate of cooling. However, László's opposing effect would lead to an increase in the retained austenite at extremely high rates of cooling, which has not been observed. The experimental result is that decreasing the rate of cooling increases the retained austenite until the critical rate of cooling is reached. At slower rates the austenite is able to decompose by processes involving diffusion, and the amount retained decreases as the cooling rate decreases further below the critical value.]

The large strains involved in martensite formation may be relieved by the formation of cracks, especially because the strains develop so quickly that there is little time for plastic flow. The brittleness of martensitic steels is thus easily understood.

Particles of slag (pp. 193–196) or of cementite which fail to dissolve on heating form nuclei of internal stress on cooling, and are likely to act as nuclei for the transformation of austenite to ferrite or martensite.

The first paper concludes (pp. 196–199) with a discussion of flaking caused by hydrogen in steel. Various factors are considered, which may be combined to form the following picture. The energy required to form a flake is stored in internal stresses which are too small to start a crack, but large enough to propagate it. The energy to form the initial crack is stored in a small region of retained austenite. This in turn is stabilized by the difficulty of forming a nucleus for the martensite transformation. The gradually increasing pressure in a small bubble of hydrogen in the austenite enables a nucleus of martensite to form, the martensite grows, increasing the stress so much that a small crack forms, which then spreads.

Part II.

Part II considers in more detail the stresses in a material composed of grains, each of which is composite (pp. 138–141). First a model is treated in which the grains are cubical, equal numbers of cubes being laminated parallel to the x, y, and z axes. As a [reasonable, but certainly inexact] basis for calculation it is assumed that the internal stresses are such that each cubical grain remains cubical when the material is heated. Shear stresses across the surface of each grain and between lamellæ are neglected in comparison with the normal stresses. [The first three of equations (8) are then valid. The fourth is the condition that the mean hydrostatic pressure in the material vanishes, which is a necessary condition. If the mean hydrostatic pressure were p, the corresponding equation, obtained by considering any one of the cubes in Fig. 8, would be $2(u_1f_1 + u_2f_2) + q = 3p$. The conditions of equilibrium for each cube are satisfied identically, since the forces on opposite faces are equal and opposite. To obtain a condition of equilibrium we may imagine the group of three cubes in Fig. 8 to be joined, forming a rectangular block with its long axis parallel to the arrow q in Fig. 8 (c). If we neglect the fact that the internal stresses in this block are not self-compensated we obtain the condition of equilibrium $-(u_1f_1 + u_2f_2) + q = 0$ for translation parallel to the arrow. The incompatibility of this with the last of equations (8) results from the neglect of the shear stresses, and illustrates the roughness of the approximate methods used.] The relief of stress at a free surface is then discussed more fully (pp. 141–143). Even though the two components may have the same elastic constants, there is a surface distortion which results in the ratio of the two components appearing at the surface being different from the ratio in the bulk material (p. 143).

A formal analysis is then given (pp. 143–147) of the distribution of stress between the components of an aggregate [again on the reasonable, but inexact, assumption that both components undergo the same homogeneous strain] when the specimen as a whole is subjected to external stress.

Stress concentrations at the edges of lamellæ are important, but would be difficult to calculate. They are not very different from those surrounding a cylinder of diameter equal to the thickness of the lamella (p. 147).

These methods are applied (pp. 147–154) to obtain fresh numerical values for internal stresses in ferrous alloys, and for the internal stresses produced by a homogeneous external stress.

The graphite in cast iron (pp. 154–157) is stressed to its elastic limit, and some flakes are caused to yield by any external stress. Cast iron, therefore, shows imperfect elasticity under small stresses. Under larger stresses, bonding of the graphite to the matrix breaks down, first in the flakes which lie normal to the applied tension, and later in the flakes lying more obliquely. Whiteheart malleable cast iron is similarly considered " as a porous ferrous metal with cavities having the same volume fraction as the previous graphite content ". [It seems generally accepted that the decarburized regions in whiteheart malleable cast iron are not in fact porous.]

It is a general rule (p. 157) that the volume changes accompanying any phase transformation lead to a strain energy which hinders the initial stages of the transformation. This strain energy reaches a maximum when the transformation is about half completed; further progress of the transformation reduces the strain energy, which vanishes when the material is again homogeneous.

Since fatigue and damping are related to internal strains, and internal strains have abnormal values at the surface, it may be possible (pp. 158–159) to explain why fatigue cracks nearly always start in the surface layer. [Since the stresses are *reduced* at a free surface, this does not seem to me a hopeful approach. There is a further discussion on p. 203 of Part III.]

PART III.

Part III of the series returns to problems concerning polycrystalline materials in which the grains are all of the same material, but each grain is anisotropic. The approximation used till now has required that each grain should undergo the same elastic strain (change of shape and size) as the bulk material: the corresponding stresses are not "self-compensated", that is to say, the force which a grain A is required to exert on a neighbouring grain B is not equal and opposite to the force B is required to exert on A (pp. 183–185). This type of difficulty is treated in a thesis by Bruggeman. [Also more satisfactorily in later papers [8–12] by Bruggeman. The application of Bruggeman's "self-consistent" method to determine the elastic constants of polycrystalline aggregates of anisotropic crystals does not seem to have been worked out.] The approximation should be adequate for qualitative work, and is applied to determine the distribution of load among the grains of a metal when it is subjected to an external tensile stress. The strain in each grain is taken to be the strain which would occur if the grain had the elastic properties of the isotropic bulk metal, and the stresses required to produce this strain in the actual anisotropic grain are calculated. For cubic metals (pp. 185–193) the variations of stress from grain to grain are of order $\pm$ 30%, in agreement with the rough estimate in Part I. As a result of this variation of stress, the surface of a polycrystalline specimen will warp from grain to grain when it is subjected to a load even within the elastic range. If a metal has a strong preferred orientation, these concentrations of stress do not occur, but the metal is nevertheless weak because slip is easier if the grains are nearly parallel. The stress concentrations at the grain boundaries may have an influence on stress-corrosion, but the governing factor in securing freedom from corrosion is the nature of the oxide film. The analysis for hexagonal and tetragonal metals (pp. 193–199) is similar.

Consideration of hexagonal metals leads (pp. 199–203) to a discussion of the stresses set up by changes in the temperature of a polycrystalline sample of a hexagonal metal, because the thermal expansion of hexagonal metals is not isotropic. The thermal expansion of the bulk metal is calculated as the expansion which a single crystal would have if it were acted on by stresses which forced it to maintain its shape (while increasing in size), and were such that the resultant hydrostatic pressure vanished (equation (34)). The volume expansion is not in general equal to the expansion of an unconfined single crystal. [The observed values seem to agree better with the single crystal values a_m than with the calculated values a_{ch}, according to Table XV. Possibly the experimental figures are not reliable.] As the temperature changes, the internal stresses are relieved by slip, and the resulting cold work will cause recrystallization. These effects do not occur in a specimen with a strong preferred orientation. [The suggestion that recrystallization will occur in such a way as to produce a preferred orientation and thereby reduce the internal stresses caused by subsequent change of temperature seems to be unsound, for the recrystallization is an isothermal process which cannot be influenced by possible future changes of temperature.]

Internal strains caused by anisotropic thermal expansion affect the specific heat in two ways (pp. 203–204). If the body is free from stress, and is heated, it acquires elastic strain energy and its specific heat is greater than that of a single crystal. If the body is cooled below its initial temperature it again acquires elastic strain energy, and its specific heat is less than that of a single crystal. In general, the elastic strain is reduced by slip when it reaches a certain value depending on the temperature. As a result, the temperature at which the internal stresses are least, and the specific heat is equal to that of the single crystal, is not the temperature at which the specimen was originally annealed, but is close to the present temperature of the specimen at any time. This results in a hysteresis of the specific heat.

Internal stresses may be removed by cold working (pp. 204–206), when they are replaced by a new set of cold-working stresses. The new stresses may be less liable to cause fatigue failure than the old. For example, they may leave the surface layer in compression and so hinder the formation of surface cracks. [The idea that they may be "of a less order of magnitude" than the old stresses does not agree with the theory that the stresses required for cold working are of the same order of magnitude as the internal stresses originally present, and that the internal stresses produced by cold working are of the same order of magnitude as the applied stresses. There may be cases in which the cold-working stresses lead to recrystallization and softening, while the stresses previously present were stable.] While the effect of surface treatments in improving resistance to static loads is easily understood, the improved fatigue resistance is not so easy to explain, since repeated alterations in loading seem to remove the large-scale compressive stresses in the surface, without causing the improved fatigue resistance to be lost. It may be that the regions of maximum compression are relieved, while the critical regions where the compression is less are unaffected.

Internal stresses may affect the density, but this problem is not discussed in detail (pp. 206–207).

PART IV.

Part IV states some general results which hold when the material consists of grains of an elastically isotropic substance embedded in a matrix of a different elastically isotropic substance with the same rigidity. The effect of the material yielding to its internal stresses is discussed, and spheroidization, precipitation hardening, magnetization, and internal friction are considered.

If the rigidity of the matrix is equal to that of the inclusion (pp. 207–209), the requirement that all the units (laminated cubes, or spheres, or cylinders of the matrix containing cores of precipitate) undergo the same strain ensures that the forces between pairs of neighbouring units are equal and opposite, so that the approximate solutions already obtained become exact. Simple formulæ are obtained for the coefficient of expansion, internal stresses, and strain energy. [These, and an analogous formula for the bulk modulus which Dr. László has recently derived, should hold for any concentration of precipitate, provided the units are of simple symmetrical shapes, and also for particles of precipitate of any shape, provided these occupy only a small part of the total volume, but are not established for units of irregular shapes occupying a large part of the total volume.]

The first case of yield considered is that of laminar graphite, which is taken to be incapable of supporting shear stress. László finds (pp. 209–213) that the yielding of the graphite has little effect in reducing the stresses in the ferrite matrix in the body of the specimen. The effect on the surface stresses depends whether the specimen is machined, annealed, and cooled, or whether it is annealed, cooled, and then machined to expose a new surface. Assuming that the properties of the material near the surface are the same as those of the bulk material, and that the stresses are large enough to cause yield in the metallic matrix, the stresses are greater in a surface layer which has been machined and then annealed than in a layer which has been annealed and then machined. [One might argue that when the specimen is cooled to room temperature it is always stressed just to its yield point around each particle of graphite. If now a new surface is prepared by machining, the grains brought to the surface distort in such a way as to relieve their internal stresses.]

The second problem in yield (pp. 213–217) considers a spherical particle of precipitate which after cooling is too large for the hole in which it formed at high temperature. The surrounding matrix yields and becomes plastic up to a critical radius r_E, outside which it is still perfectly elastic. [Mathematically similar problems were considered at

the same time by Sachs and Lubahn [13] and by Bishop, Hill, and Mott.[14]] Numerical results are given for spherical graphite or cementite in a matrix of ferrite. It is not certain how far this idealized model can be applied to real steels.

It was shown earlier (p. 208) that if no yield takes place the thermal expansion of the compound material differs from the weighted mean of the expansions of its components unless the components have the same rigidity. If the matrix yields completely (pp. 217–219), the simple weighted mean formula holds if both components have the same compressibility.

Suppose (pp. 219–223) that two constituents in given proportions can exist either in the form of a matrix of A containing spherical particles of B, or in the form of a matrix of B containing spherical particles of A. Then if A and B have different thermal expansions, the strain energies produced by a given change in temperature are in general different for these two arrangements. There is a similar difference in strain energy if A and B have separated by diffusion from a homogeneous solid solution. László shows that the arrangement which develops least strain energy is that in which the component with the lower shear modulus forms the matrix.

The effect of surface tension is to produce spheroidization and coarsening, with an effort to convert the component occupying the larger volume into the matrix.

Mott and the present writer put forward a theory of precipitation hardening according to which the yield stress was roughly equal to the average shear stress in the matrix. László points out (pp. 223–226) that the average shear stress along the length of a dislocation is zero. [This is correct. Our picture, however, was that a dislocation could not move unless the external shear stress exceeded the opposing internal stress at every point of its length. This is equivalent to the idea that a dislocation is flexible, so that a favourable internal stress over part of its length cannot help the dislocation to overcome opposing internal stresses on other parts of its length. We are developing this idea more fully. A preliminary discussion of the other limiting case where the dislocation is rigid over a length of 1000 atoms has already been published.[15]] Moreover, the simple picture of precipitation starting by the formation of clusters of solute atoms in one lattice plane may not be valid. These atoms do not fit in the lattice, and it seems likely that the strain energy can be reduced if the sheet of foreign atoms does not remain flat, but buckles. These highly localized deformations of the lattice may contribute appreciably to the resistance of the lattice to plastic shear. [Both the existence of buckling and its possible effect on the strength deserve further consideration: neither problem looks easy.]

In a brief discussion of magnetic effects (pp. 226–227), László suggests that internal stresses due to magnetization cannot be large enough to affect the permeability, or to cause plastic strain and so contribute to the hysteresis. [Dr. László has made it clear in correspondence that he is concerned only with magnetostrictive stresses caused by the application of an external field. Becker, Döring, and Kersten [16, 17, 18] have shown that the magnetostrictive stresses set up on cooling through the Curie point usually set an upper limit to the initial permeability.]

REFERENCES.

1. F. László, *J. Iron Steel Inst.*, 1943, **147**, 173.
2. F. László, *J. Iron Steel Inst.*, 1943, **148**, 137.
3. F. László, *J. Iron Steel Inst.*, 1944, **150**, 183.
4. F. László, *J. Iron Steel Inst.*, 1945, **152**, 207.
5. E. Orowan and J. F. Nye, private communication.
6. W. Boas and R. W. K. Honeycombe, *Proc. Roy. Soc.*, 1946, [A], **186**, 57.
7. A. B. Greninger and A. R. Troiano, *Trans. Amer. Inst. Min. Met. Eng.*, 1940, **140**, 307.
8. D. A. G. Bruggeman, *Z. Physik*, 1934, **92**, 561.
9. D. A. G. Bruggeman, *Ann. Physik*, 1935, **24**, 636, 665.
10. D. A. G. Bruggeman, *Ann. Physik*, 1936, **25**, 645.
11. D. A. G. Bruggeman, *Physikal. Z.*, 1936, **37**, 906.
12. D. A. G. Bruggeman, *Ann. Physik*, 1937, **29**, 160.
13. G. Sachs and J. D. Lubahn, *J. Appl. Mechanics*, 1943, **10**, A-147.
14. R. F. Bishop, R. Hill, and N. F. Mott, *Proc. Phys. Soc.*, 1945, **57**, 147.
15. F. R. N. Nabarro, *Proc. Phys. Soc.*, 1946, **58**, 669.
16. R. Becker (ed.), "Probleme der Technischen Magnetisierungskurve". Berlin: **1938**.
17. R. Becker and W. Döring, "Ferromagnetismus". Berlin: **1939**.
18. M. Kersten, "Grundlagen einer Theorie der Ferromagnetischen Hysterese und der Koerzitivkraft". Leipzig: **1944**.