involves a flow regime intermediate between the above two extremes, and mathematical analysis becomes extremely complicated. Simulation of intermediate mixing regimes is possible, however, by replacing the real system by a series of two or more completely mixed reactors (Marais & Ekama, 1976).

Since dynamic behaviour of the activated sludge process is beyond the scope of this work, steady state behaviour is assumed throughout. The layout for a completely mixed activated sludge process with recycling is shown diagrammatically in figure 3.1.

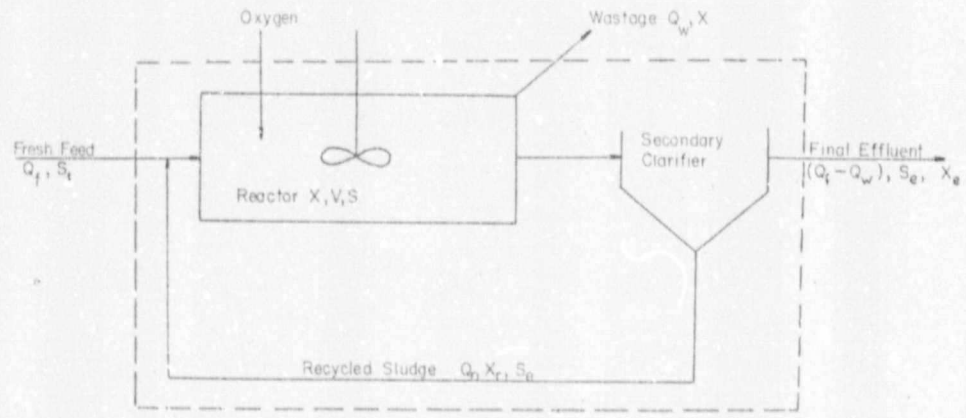

Figure 3.1 Completely mixed activated sludge process with recycle

Q_f	=	Wastewater flow rate	m^3/day
Q_w	=	Rate of sludge wastage	m^3/day
Q_r	=	Flow rate of recycled sludge	m^3/day
S_1	=	Substrate concentration of fresh feed	mg BOD/l
S_e	=	Substrate concentration after biological treatment	mg BOD/l
V	=	Reactor volume	l
X	=	Concentration of organisms in the reactor	mg VSS/l
X_e	=	Concentration of organisms in the final effluent	mg VSS/l
X_r	=	Concentration of organisms in the recycled sludge	mg VSS/l

For purposes of an optimisation model the major cost components of the activated sludge plant were considered to be the reactor size, recycle ratio (pumping cost), oxygen requirements (aerator costs), and sludge wastage (sludge separation and disposal costs). The costs of primary treatment was not considered as these would be fairly standard and not dependent on the biochemical oxygen demand (BOD) of the wastewater and effluent.

3.2.2 Kinetics relationships

Studies of substrate degradation in batch reactors have indicated that the rate of substrate removal follows zero-order kinetics at high substrate concentrations (BOD values greater than 500 mg/l) and first-order kinetics at low substrate concentrations. Since in a reactor the BOD values are generally considerably lower than 500 mg/l, first-order kinetics are assumed for mathematical modelling of the process.

$$\text{i.e. } \frac{dS}{dt} = -KS \qquad 3.1$$

where S is substrate concentration (mg BOD/l)
K is the removal rate constant (day^{-1})
and t is time

Substrate removal rate is usually expressed per mg/l of mixed liquor volatile suspended solids (MLVSS) so that equation 3.1 becomes

$$\frac{dS}{dt} = -k X S \qquad 3.2$$

where X is the concentration of MLVSS in the reactor (mg/l)
and k = maximum rate of waste utilisation per weight of micro-organisms (day^{-1}/mg/l)
= K/X

3.2.3 Design relationships

The design of activated sludge plants has tended recently to be based on the work of Lawrence and McCarty who proposed the following relationships for a complete mix reactor with recycle (Lawrence and McCarty, 1970).

Specific efficiency $E_s = \frac{100\ (S_o - S_e)}{S_o}$ 3.3

Effluent waste concentration $S_e = \frac{K_s\ (1+b\ \theta_c)}{\theta_c(YK-b)-1}$ 3.4

Micro-organism concentration in the reactor $X = \frac{Y(S_o-S_e)\theta_c}{1 + b\theta_c}\ \frac{\theta_c}{\theta}$ 3.5

Excess micro-organism production rate $P_x = \frac{YQ\ (S_o-S_e)}{1 + b\theta_c}$ 3.6

Hydraulic retention time $\theta = \theta_c(1+r-r\ \frac{X_r}{X})$ 3.7

Solids retention time $\theta_c^{-1} = \frac{YKS_e}{K_s + S_e} - b$ 3.8

Where K_s = the waste concentration at which the rate of waste utilisation per unit weight of micro-organisms is one half the maximum rate. (mg/l)

b = micro-organism decay coefficient (day^{-1})

Y = growth yield coefficient, being the mass of organisms synthesised per unit mass of substrate utilised.

θ_c = sludge age.

θ = hydraulic retention time.

r = recycle ratio.

S_o = substrate concentration of combined feed (mg/l)

It is apparent from the above equations that the solids retention time or sludge age θ_c is fundamental to the theory, whereas the hydraulic retention time is only incidental. The importance of the sludge age had been established earlier by McKinney (1962), but in his approach he implies that the effluent quality is a function of the hydraulic retention time and independent of the sludge age (Marais and Ekama, 1976). Most recent papers appear to favour Lawrence and McCarty's approach, but Marais and Ekama have reported that the differences in effluent quality when applying the two methods, are inconsequential (Marais & Ekama, 1976).

Equations 3.3 to 3.8 indicate that the cost components are functions of the design variables as follows:

Reactor volume	=	$f(\theta_c, X, S_o)$
Pump size	=	$f(r, X_r)$
Oxygen requirement	=	$f(\theta_c, X, V)$
Sludge yield	=	$f(\theta_c, S_o)$

Since the oxygen requirement is a function of the aerator volume V, it is not independent and is thus not considered as a design variable.

3.2.4 Nitrification

Nitrification is the process by which free and saline ammonia in the wastewater is oxidised to nitrites and nitrates, and is discussed in more detail in chapter 4. The process results in an oxygen demand that can be substantially higher than that due to the carbonaceous oxygen demand acting alone and, in addition,

nitrification can cause a drop in pH in the system (due to the formation of nitric acid) with a resultant adverse effect on the biological process.

Consequently the tendency in overseas countries has been to design to avoid nitrification, provided that the receiving waters can tolerate the oxygen demand exerted by the ammonia in the effluent. Since in temperate climates nitrification only occurs after a sludge age of about ten days, this forms another reason for designing for short sludge ages. In the warmer regions where nitrification may occur at shorter sludge ages, special techniques have been developed for the suppression of nitrification.

3.3 Design of activated sludge plants in overseas countries

Metcalf and Eddy (1974, p.493) present a design precedure for the activated sludge process, which may be considered as typical for most overseas designs, based on Lawrence and McCarty's relationships.

The design uses preselected values for sludge age (θ_c), return sludge concentration (X_r) and reactor micro-organism concentration (X). The values chosen for X and X_r are based on experience and are usually in the ranges 2 000 to 4 000 mg/l and 10 000 to 15 000 mg/l respectively. The determination of the reactor volume is dependent on the effluent quality required and on the sludge age according to the following relationship derived by Lawrence and McCarty:

$$XV = \frac{YQ_f\theta_c (S_o - S_e)}{1 + b\theta_c} \qquad 3.9$$

However the food to micro-organism ratio (F/M), defined as mass of substrate in influent per day per mass of VSS in the reactor, must also be checked in order to ensure that good flocculation

and settling will occur. According to Eckenfelder (1970, page 167) the settling characteristics of a sludge can be determined experimentally and will tend to follow a curve of the type shown in fig. 3.2 in which the sludge volume index (SVI) is plotted against the F/M ratio. The sludge volume index is the volume (in cm^3) occupied by one gram of dry weight sludge after settling for thirty minutes.

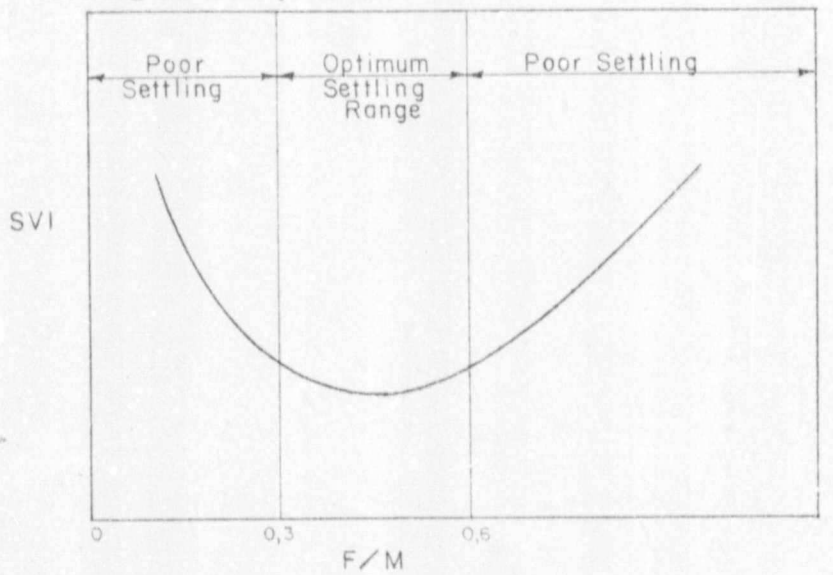

Figure 3.2 Typical plot of SVI against F/M ratio

Ramalho (1977) states that for most wastewaters optimum settling conditions fall within the range $0,6 > F/M > 0,3$

The F/M ratio (or substrate utilisation rate, SUR) is related to the sludge age by the following equation (Marais and Ekama, 1976, page 183):

$$\frac{1}{\theta_c} = Y.\,SUR - b \qquad 3.10$$

From figure 3.2 it would therefore appear that the settling characteristics of a sludge will deteriorate with increasing sludge age. Consequently, it is usual in overseas countries to work with short sludge ages in order to obtain satisfactory settlement.

3.4 Design of activated sludge plants in South Africa

In South Africa, local conditions have tended to result in significant differences in the design approach and consequently considerable amount of local research has been done in this field, particularly by Professor G.v.R. Marais at the University of Cape Town.

Local standards for rivers in South Africa require that the concentration of ammonia in the effluent should not exceed 10 mg/l (Republic of South Africa, Government Gazette, 1962). Since municipal sewage typically contains ammonia concentrations in the region of 20 to 30 mg/l it is therefore necessary to include nitrification in the treatment process in order to meet this requirement.

As mentioned earlier, a minimum sludge age of about ten days is required to ensure nitrification where the water temperature falls below $10^{o}C$ (which it may certainly do during the winter months in the Highveld region). Since increasing the sludge age up to 20 days or more results in a more stable sludge better suited to direct discharge onto drying beds, and since this additional aeration period requires only a small percentage increase in oxygen requirements (Marais & Ekama, 1976, p.195) activated sludge plants in South Africa have tended to be designed for extended aeration with sludge ages of 20 to 25 days.

It is therefore necessary to review the design criteria mentioned earlier for overseas design practice:

(a) Effluent quality. Since there is little change in the unmetabolised BOD in the effluent for sludge ages greater than 4 days, the effluent quality is no longer a factor in the design (Marais and Ekama, 1976, p.165). The sludge age, upon which the reactor is sized, is determined by nitrification requirements and requirements for a well

stabilised sludge, and so a high quality effluent in terms of BOD is always assured.

(b) F/M ratio. Pitman (1975) has shown that for long sludge ages the relationship shown in fig.3.2 is only part of the story and in fact excellent settling conditions can be achieved at longer sludge ages, with poorest settling conditions occurring at sludge ages of around six days. This is shown in fig. 3.3 which is derived from Pitman's paper.

The portion of the curve for sludge ages less than six days clearly resembles that of figure 3.2 with the F/M axis reversed, since sludge age is related to F/M ratio (or SUR) by equation 3.10. This portion is also similar to Eckenfelder's figure 9.7 (Eckenfelder, 1970, p.166). However, neither figure 3.2 nor Eckenfelder's figure show that the optimum range of F/M ratios for good flocculation (i.e. 0,3 to 0,6 as mentioned earlier) is merely a local optimum and in fact beyond a sludge age of 6 days settling conditions progressively improve. Beyond 25 days, however, settling ability tends to deteriorate sharply and so activated sludge plants are not normally designed for sludge ages beyond this. It is shown in Pitman's paper that dewaterability of the sludge follows a similar curve and so the advantages of long sludge ages in this respect are obvious.

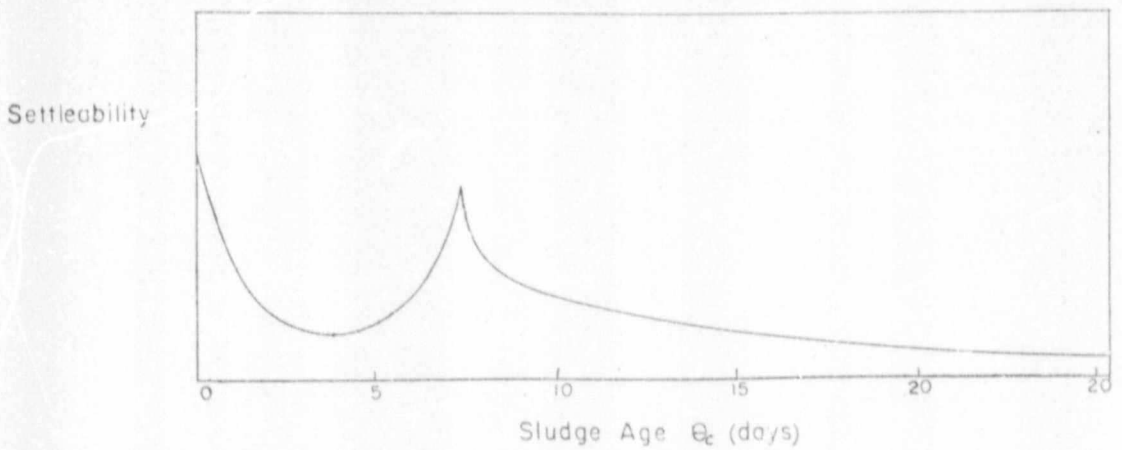

Figure 3.3 Sludge settleability against sludge age

(c) Nitrification

As already stated, a minimum sludge age of ten days is required to ensure nitrification at temperatures below 10°C. In South Africa, water temperatures in summer can rise to higher than 20°C under which circumstances it may be almost impossible to design a plant that will not nitrify in hot weather. When nitrification is included in the design it is necessary to include denitrification in order to avoid discharging nitrates (which are an important nutrient and can lead to excessive algal growth) into the receiving waters.

3.5 Development of a computer model for the activated sludge process

In order to become familiar with the processes involved in activated sludge plant design, a design procedure outlined by Ramalho (1977, Chp.5) was programmed on the computer. The object of this simple exercise was to establish the sensitivity of the major cost components to the various parameters and variables involved. A flow diagram of the model is shown in figure 3.4 (with the relevant equations on the two pages following) and a listing of the program (program SLD1) appears in Appendix D1.

However, shortly after the model had been programmed, further investigations established that Ramalho's design procedure was not entirely applicable to the South African situation and no further work was done using this model. Attention was then directed towards the model outlined by Marais and Ekama (1976) as a basis for optimisation. It was during these investigations, which included discussions with various authorities in the field, that the abovementioned differences between South African and overseas designs came to light.

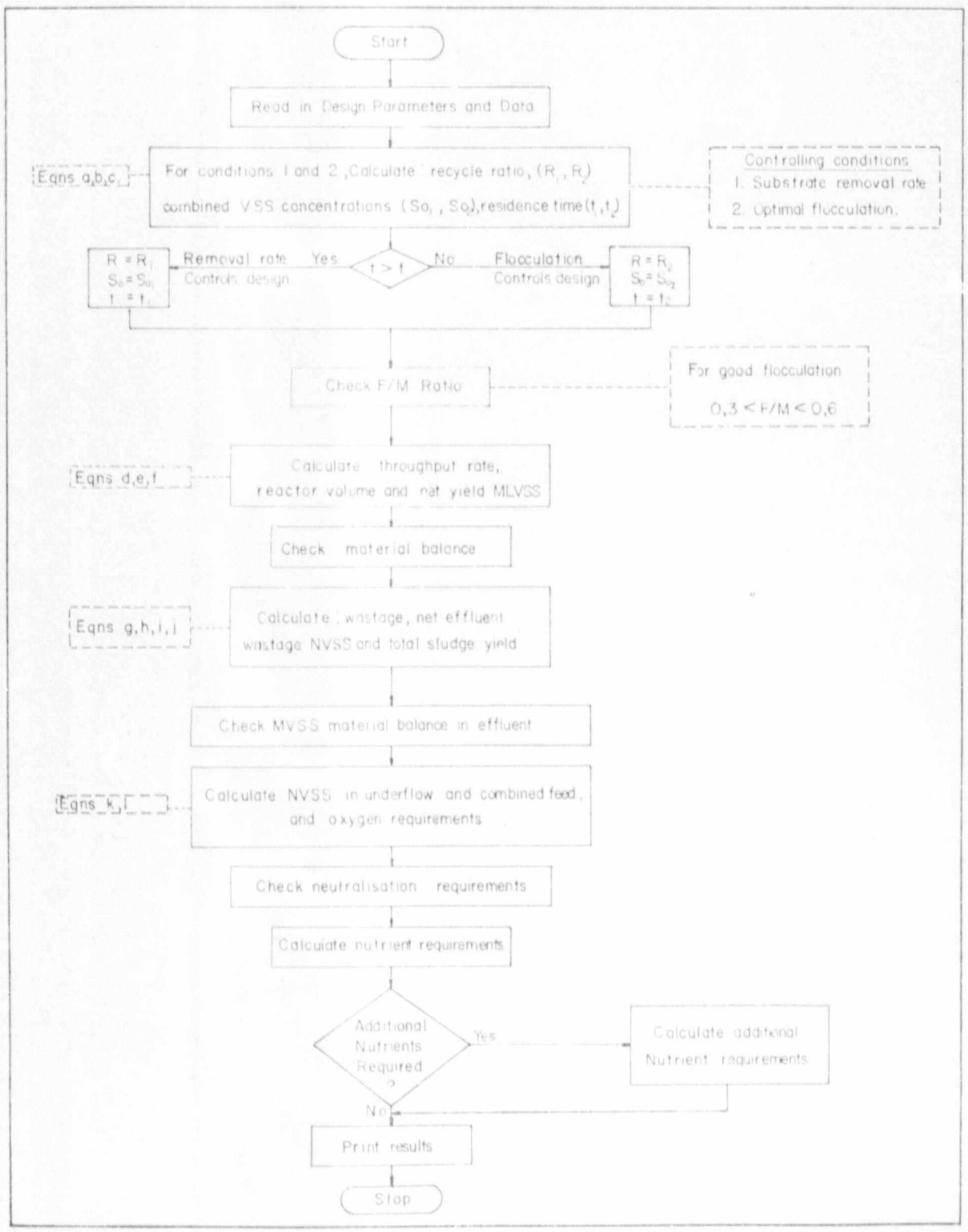

Figure: 3.4 :- SLDI– Flow Diagram.

Equations for Activated Sludge Design Procedure (Ramalho, 1977, Chp. 5). For symbols see list on page viii.

Equation a - Recycle ratio

1. Substrate removal rate controlling

$$r = X - Y(S_1 - S_e) + \frac{b\ (S_1 - S_e)}{K\ (S_e - S_n)} - X \quad / (X_r - X)$$

2. Optimal flocculation conditions controlling

$$r = X - Y(S_1 - S_e)\ (F/M) + b\ S_1 - X\ (F/M)$$
$$/\ (X_r - X)\ (F/M) - b\ S_e$$

Equation b - Concentration of combined VSS

$$S_o = (S_1 + rS_e) / (1 + r)$$

Equation C - Hydraulic retention time

1. Substrate removal rate controlling

$$\theta = (S_o - S_e) / kX\ (S_e - S_n)$$

2. Optimum flocculation conditions controlling

$$\theta = S_o / X\ (F/M)$$

Equation d - Reactor throughout rate

$$Q_a = Q_f\ (1 + r)$$

Equation e - Reactor volume

$$V = Q\theta$$

Equation f - Net yield MLVSS

$$P_x = Y(S_o - S_e)\, Q_f\, (1 + r) - b\, X\, Q_f\, (1 + r)$$

Equation g - Wastage flow rate

$$Q_w = (P_x + Q_f\, X_f) \,/\, X_r$$

Equation h - Effluent flow rate

$$Q_e = Q_f - Q_w$$

Equation i - NVSS in wastage

$$P_w = Q_f\, (X_{nf} - X_{ne}) + Q_w\, X_{ne}$$

Equation j - Total sludge yield

$$P_t = P_x + P_w + Q_f\, X_f$$

Equation k - Combined feed NVSS

$$X_{no} = (X_{nf} + rP_w/Q_w) \,/\, (1 + r)$$

Equation l - Oxygen requirements

$$\text{Oxygen} = a'(S_o - S_e)\, Q_a + b'XV$$

From the above discussion and for the following reasons it was decided not to continue with the development of an activated sludge optimisation model at this stage.

In South Africa activated sludge plants are designed for long sludge ages. These sludge ages are determined by nitrification requirements and are considerably in excess of that required for carbonaceous oxidation alone. Consequently the biochemical oxygen demand of the sewage plant effluent is no longer a decision variable in the sizing of the major components of the sewage works design.

In addition, discussions with Johannesburg municipality's engineers established that unit costs for the major components would be difficult to determine from their records. This is particularly so in the case of sludge disposal costs which may differ considerably from one sewage works to another depending on the method used, and, as noted by Halliday (1976), can amount to as much as 30 to 50 per cent of the total cost of wastewater treatment.

Furthermore, the design of wastewater treatment processes is extremely complex, involving numerous alternatives which must be thoroughly examined on an economic basis for each individual situation. Any general optimisation model using only the four cost components mentioned in section 3.2 and limited cost data must be considered as very crude and could be considerably out in a final comparison.

CHAPTER 4

THE OXYGEN BALANCE IN NATURAL STREAMS

4.1 Dissolved oxygen as a pollution indicator

The dissolved oxygen (DO) in a body of water is an important pollution indicator as so much of the ecology in the water environment is dependent on it. In South Africa, the acceptable lower limit of DO for fresh water fish is around 4 milligrams per litre (Bruwer, 1980). Under colder conditions such as are found in many overseas countries, this level may be higher whilst certain hardier fish, such as barbel, may be able to survive adequately where DO levels are as low as 3 mg/l. In any event one of the objectives of pollution control is to maintain a suitable level of dissolved oxygen so as not to endanger the natural ecological balance.

4.2 Dissolved oxygen saturation

Depending on certain conditions, viz. temperature, atmospheric pressure and the concentration of certain salts in the water, a body of water has a maximum level of dissolved oxygen called the saturation concentration.

(a) Variation with temperature

The saturation concentration of dissolved oxygen, as with all gases, decreases with increasing temperature. This variation can be expressed by the following empirical relationship for distilled water at 760 mm Hg.

$$C_s = 14{,}652 - 0{,}41022\ T + 0{,}0079910\ T^2 - 0{,}00077774\ T^3 \qquad 4.1$$

where C_s = DO saturation concentration (mg/l)

T = water temperature in ^{o}C

(b) Variation with barometric pressure

Dissolved oxygen saturation concentration decreases with decreasing atmospheric pressure. The correction factor is approximately 0,8 for the Johannesburg area (1 700 m A.M.S.L.)

(c) Variation with salt concentration

This variation is normally only of importance in sea and estuarine conditions and is not considered here.

4.3 The dissolved oxygen sag curve

The concentration of dissolved oxygen (DO) in natural streams is dependent on those factors causing a reduction in DO concentration, i.e. the oxygen demand, and those factors tending to increase or maintain the concentration of DO, the re-oxygenation factors. This is best illustrated by the well known dissolved oxygen sag curve (fig. 4.1) which shows the effect of a point discharge of oxygen demand, e.g. a sewage outfall, into the stream.

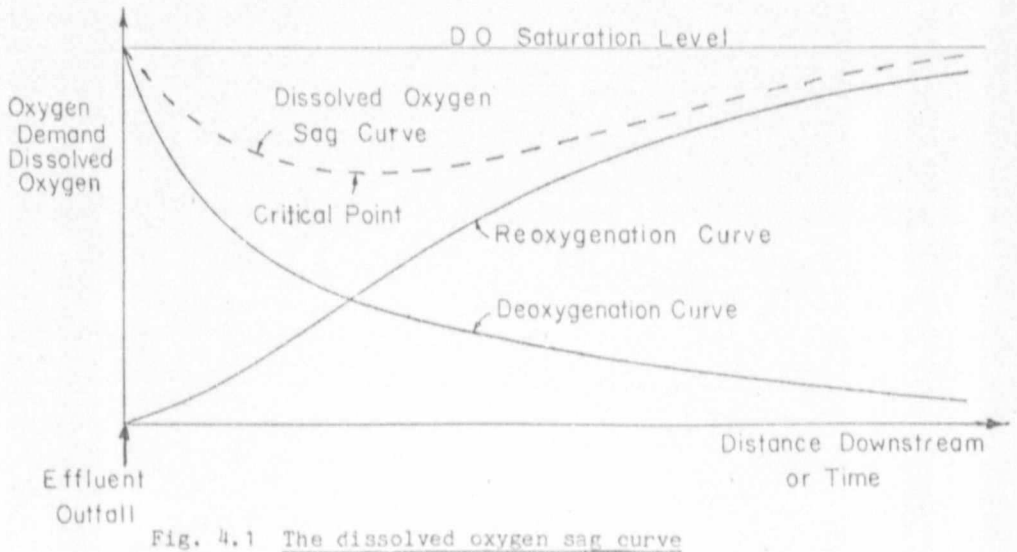

Fig. 4.1 The dissolved oxygen sag curve

The curves in figure 4.1 show the oxygen balance in its simplest form. The DO is at saturation up until the point of effluent discharge and thereafter the level of dissolved oxygen is the resultant of the oxygen demand and the re-oxygenation. Without reaeration the dissolved oxygen concentration curve would be coincident with the oxygen demand curve, but with reaeration the DO attains a critical low point where the oxygen demand and reaeration curves cross, and thereafter re-oxygenation exceeds the oxygen demand and the DO level recovers to attain the saturation level asymptotically.

The remainder of this chapter will deal with the factors which go to make up the oxygen demand and re-oxygenation curves.

4.4 The oxygen demand

The most important factors exerting a demand on the oxygen resources of a stream are:

- decay of biodegradable carbonaceous organic matter
- nitrification
- inorganic chemical reducing agents
- plant respiration

4.4.1 Carbonaceous biochemical oxygen demand

The decay of biodegradable matter is brought about by bacteria which utilise the available oxygen in the water to convert unstable organic matter to stable mineralised forms. In the presence of dissolved oxygen this process is carried out mainly by aerobic forms of bacteria, but, should the DO be completely depleted, the process becomes anaerobic. In the latter case oxygen is obtained by the reduction of molecular oxygen such as in sulphate, resulting in the production of hydrogen sulphide with its associated unpleasant odour.

The rate at which organic matter is stabilised is proportional to the amount of remaining unstabilised matter. Factors which will affect the biochemical oxygen demand of the stream are temperature, sludge scour and deposit, and biological extraction.

(a) Temperature

The rate of stabilisation of organic matter (or decay of carbonaceous biochemical oxygen demand - CBOD) increases with increasing water temperature.

(b) Sludge deposit and scour

If the velocity of the water is sufficiently low the suspended organic matter may be deposited on the stream bed resulting in a net decrease in BOD in the water. On the other hand, during periods of high flow with associated high velocities, such deposits may be scoured out thereby increasing the BOD in the overflowing water. The critical mean channel velocity below which sludge deposit is likely to occur is around 0,18 metres per second and scour may occur above a critical velocity of between 0,18 and 0,3 m/s (Velz 1970, pp.134-135).

These sludge deposits will create a localised oxygen demand due to decomposition, but the rate of decomposition is comparatively low (e.g. 0,03 per day according to Velz, 1970, p.167). The reaction may be semi-anaerobic with the formation of hydrogen sulphide. Velz makes a distinction between sludge deposits and benthal deposits, the latter consisting of 'old compacted accumulations of partially stabilised organic residue and river mud from surface wash after flood periods'. He considers benthal deposits to be relatively inactive compared to sludge deposits.

(c) Biological extraction and accumulation

The processes involved in the trickling filter and the activated sludge treatment plants have already been mentioned in chapter 3. Velz (1970) describes similar processes occurring in natural streams, whereby biological growth attached to the bottom can extract organic matter from the overflowing water, as in a trickling filter, or alternatively such growths may be dispersed in the water as biological floc, as in the activated sludge process. The process consists of both normal biochemical satisfaction and biological extraction with the latter dominating and resulting in a BOD removal far in excess of the normal rate.

4.4.2 Nitrification

In a similar manner to that of carbonaceous oxygen demand, the process of converting ammonia to nitrates utilises the dissolved oxygen in the water. This process is carried out by bacteria in two stages:

$$2NH_3 + 3O_2 \longrightarrow 2HNO_2 + 2H_2O \quad \text{(Nitrosomas bacteria)}$$
$$2HNO_2 + O_2 \longrightarrow 2HNO_3 \quad \text{(Nitrobacter bacteria)}$$

Unlike the carbonaceous oxygen demand the bacteria concerned are highly specialised and consequently sensitive to environmental conditions. In the river environment therefore, nitrification may not be a significant factor but it may be present in the laboratory tests for determining the rate of BOD decay. However in wastewater treatment plants, or under optimal river conditions (25-30^{o}C), the effect of nitrification on the overall oxygen demand can be considerable.

In laboratory tests, the nitrification oxygen demand only begins to be significant after a sufficiently long period of time. This lag is attributed to the need for the bacterial numbers to build up sufficiently to give active nitrification (Phelps, 1944, and Thomann, 1972).

4.4.3 Inorganic chemical reducing agents

Certain effluents, especially those from industrial sources can result in a high immediate oxygen demand in the vicinity of the sewage outfall. This demand can be considerably higher than the normal biochemical oxygen demand but is usually a rapid reaction and consequently fairly localised.

4.4.4 Algal respiration

Where algal populations are significant the process of respiration will lead to a continuing oxygen demand which has, in at least one report, been found to be considerable (Gunnerson and Bailey, 1963).

4.5 Re-oxygenation

Natural replenishment of the dissolved oxygen in a stream, other than that from lateral runoff, is achieved in the following ways:

- atmospheric reaeration
- plant photosynthesis

Where the depletion of oxygen in stream has become critical various methods of artificial reaeration have been employed but this is beyond the scope of this study.

4.5.1 Atmospheric reaeration

Reaeration of the stream from the atmosphere depends on the dissolved oxygen deficit, i.e. the level of the dissolved oxygen concentration below the saturation value. Since, as mentioned in paragraph 4.2, the saturation value varies with temperature, this will affect the reaeration taking place; but for constant temperature (or rather constant saturation concentration) reaeration is a linear function of the deficit $C_s - C$, where C is the concentration of dissolved oxygen and C_s the saturation concentration.

Dissolution of oxygen can only take place at the water surface and since without mixing this surface layer would soon become saturated, it is clear that reaeration of the body of water as a whole must depend on the degree of turbulence and molecular diffusion. The latter is very small compared to turbulent mixing in streams and is therefore neglected.

4.5.2 Plant photosynthesis

The effect of plant photosynthesis due to suspended and fixed green aquatic plants, principally algae, in the water can have a significant effect on the DO levels in the stream.

Photosynthesis utilises the energy of sunlight and therefore takes place during daylight hours only. The process basically involves the use of sunlight energy and carbon dioxide to produce plant structure, and is represented by the following reaction:

$$CO_2 + H_2O \quad = \quad HCOH + O_2$$

Camp (1965) has shown that the dissolved oxygen produced by photosynthesis can be the major contributor of DO in the river environment.

4.6 Mathematics of the dissolved oxygen balance in the stream environment

Mathematical models for the above processes are described in detail in the following sections.

4.6.1 Biochemical oxygen demand

The rate of decay of BOD is proportional to the BOD remaining. This can be expressed for low BOD's as found in rivers by means of the first-order reaction:

$$-\frac{dL}{dt} = k_1' \qquad 4.2$$

where L = BOD concentration (mg/l)

k_1' = decay coefficient (day^{-1})

t = time

Integrating yields the relationship

$$L(t) = L_o e^{-k_1' t} \qquad 4.3$$

where L(t) = BOD concentration at time t

L_o = Initial BOD

The decay curve is illustrated in fig. 4.2 in terms of the BOD removed, as obtained from laboratory tests.

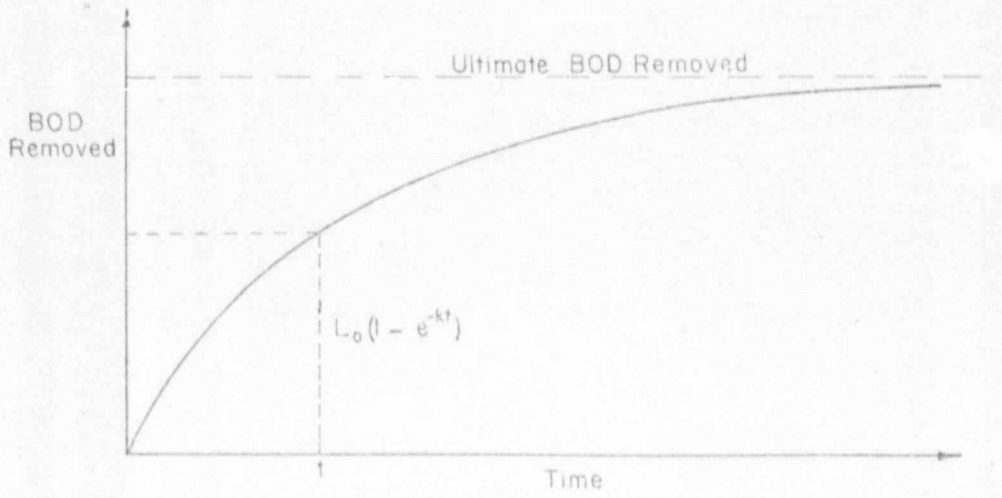

Figure 4.2 BOD removal against time

Referring to figure 4.2 the amount of BOD removed due to biochemical decay at any time t is given by:

$$L_o - L(t) = L_o (1 - e^{-k_1' t})$$

$$\text{or} \quad L(t) = L_o e^{-k_1' t} \qquad 4.4$$

The initial BOD (L_o), or alternatively the ultimate BOD removed, is the amount of BOD removed when all organic matter has been stabilised. Theoretically this value is only approached asymtotically but in practice and in laboratory determinations, the ultimate BOD removal is assumed to be achieved after twenty days at 20°C.

Effect of temperature

The effect of water temperature in the rate of BOD stabilisation, k_1, is given by the relationship (Thomann, 1972):

$$(k_1')_T = (k_1')_{20} \times 1{,}047^{(T-20)} \qquad 4.5$$

where T is the water temperature in degrees Celsius.

Equation 4.5 must be used if variations in temperature occur.

Sludge deposits and scour

Deposition of organic material is, for modelling purposes, regarded as a uniformly distributed sink along the reach of river being studied. Similarly scour of sludge deposits is considered a source of BOD and a single source/sink term is adequate to describe the net effect. Where sludge deposits are very localised, the assumption of a uniform sink along a reach could lead to poor results. However, if the subdivision of the study reach into sub-reaches is carried out taking stream velocities into account, then this effect should be minimal.

The oxygen demand of these sludge deposits is unrelated to the concentration of BOD in the overflowing water. In a mathematical model it is simplest to include this effect as a uniformly distributed sink of oxygen along the reach. Again the effects of localised deposits may be minimised by the judicious selection of sub-reaches.

Biological extraction and accumulation

According to Velz (1970, p.179) 'the rate of removal of organic matter in a unit time of passage through a biologically active reach of stream is proportional to the remaining concentration of organic matter measured in terms of its removability'. In other words a similar function to that for BOD oxidation (equation 4.4) is applicable:

$$L(t) = L_o e^{-k'_s t} \qquad 4.6$$

where L = the concentration of removable BOD
k'_s = the rate of BOD removal by biological extraction
t = time

The effective rate of BOD removal measured in the stream environment is therefore a measure of both the oxidation rate and the biological extraction rate. These two can therefore be incorporated in the same coefficient:

$$K_1 = k'_1 + k'_s$$

4.6.2 Nitrification

The effect of nitrification shows up in a laboratory oxygen demand curve as indicated in figure 4.3 and a relationship similar to equation 4.3 applies.

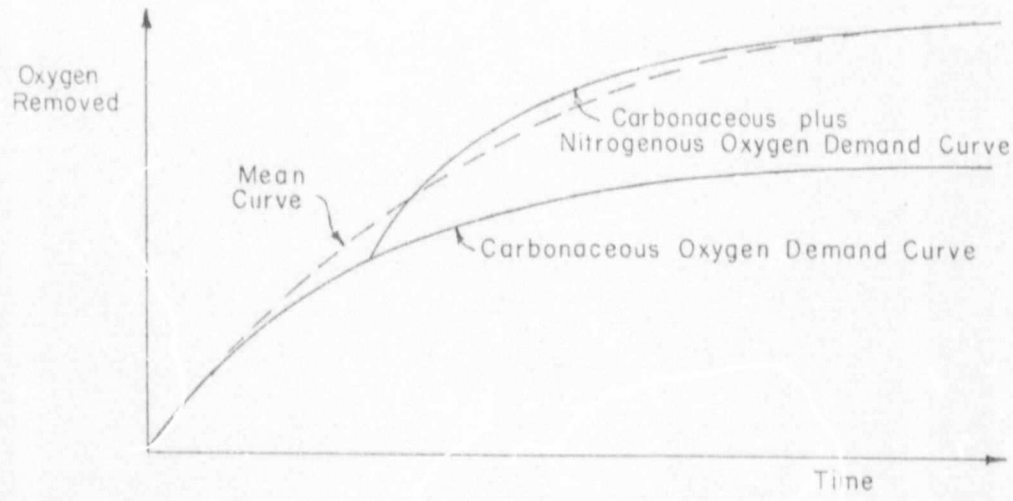

Figure 4.3 Carbonaceous and nitrogenous oxygen demand curves

In stream conditions the nitrification effect can be considerably different from that determined in the laboratory and is therefore difficult to isolate. However for modelling purposes it is adequate to assume a mean curve such as that indicated in figure 4.3. In this way the rate of nitrification is included in the coefficient K_1.

4.6.3 Respiration of aquatic plants

The effect of respiration on the oxygen demand is difficult to quantify and is best included in a model as a uniformly distributed oxygen sink, although some evidence (Gunnerson and Bailey, 1963) has shown that this may not be necessarily true.

4.6.4 Atmospheric reaeration

As stated in paragraph 4.5.1 the rate of reaeration from the atmosphere is proportional to the dissolved oxygen deficit, i.e.

$$\frac{dC}{dt} = K_2 (C_s - C) \qquad 4.7$$

where K_2 is the reaeration coefficient (per day).

The effect of temperature on this rate is twofold:

(i) The reaeration coefficient varies with temperature according to

$$(K_2)_T = (K_2)_{20}(1,024)^{(T-20)}$$

for T in degrees Celsius, and

(ii) the dissolved oxygen saturation value decreases with decreasing temperature according to equation 4.1

Various formulations have been derived for estimating the reaeration coefficient K_2 (O'Connor and Dobbins, 1958; Churchill, et al, 1962; Owens, et al, 1964). Thomann (1972) suggests that the following formula of O'Connor and Dobbins be used for streams of average depth greater than about 0,75 m:

$$K_2 = \frac{12,9\ U}{H^{1,5}}^{0,5} \text{ per day}$$

where U is the velocity in feet per second and H is the depth in feet.

In metric units this becomes:

$$K_2 = \frac{3,93\ U}{H^{1,5}}^{0,5} \text{ per day}$$

for U in metres per second
and H in metres.

Velz (1970) suggests a method of calculating the quantity of DO added to a body of water during a period of time by means of a standard reaeration curve, and the concept of mix intervals which are dependent on depth and velocity.

4.6.5 Plant photosynthesis

The effect of plant photosynthesis is too complex to model and is normally included as a uniformly distributed source of oxygen along the reach. Generally a mean rate of DO production per day is used but in the case of a short study reach, with a corresponding short time of passage, the diurnal effect becomes important.

If it is assumed that the rate of photosynthesis follows a half sinusoidal variation during the daylight hours, as depicted in figure 4.4, then it is a simple matter to include this variation in a computer model (see Appendix A3).

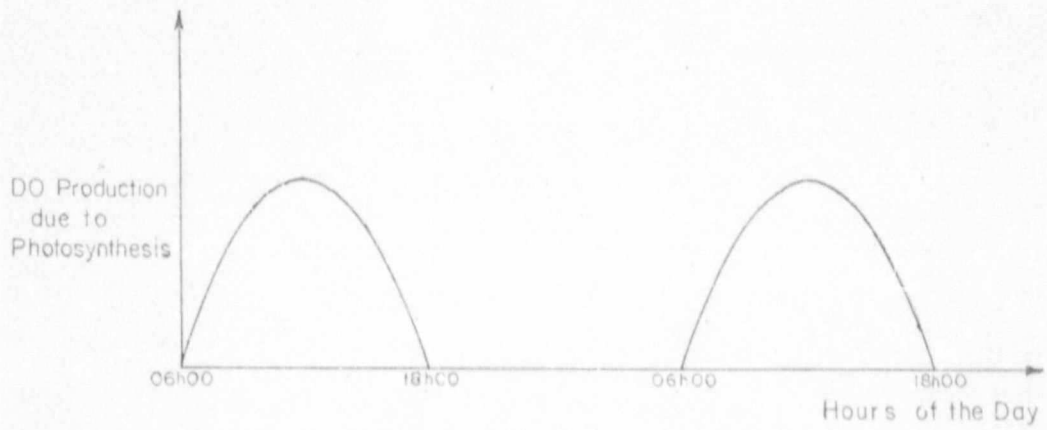

Figure 4.4 Variation of dissolved oxygen production due to photosynthesis

In fact, the diurnal variation is slightly skewed with a minimum just before dawn and a maximum in mid-afternoon (O'Connor and Di Toro, 1970) but the above approximation is considered adequate for this study.

4.7 Models for the BOD and dissolved oxygen balance in rivers

4.7.1 The coupled equations

The partial differential equations for describing the BOD and DO

processes in a river, with flow, cross sectional area and dispersion coefficient all being constant, are derived in Appendix A1 and given by equations 4.8 below.

$$\frac{\partial L}{\partial t} = \frac{\epsilon \partial^2 L}{\partial x^2} - \frac{U \partial L}{\partial x} - K_1 L \pm S \qquad 4.8a$$

$$\frac{\partial D}{\partial t} = \frac{\epsilon \partial^2 D}{\partial x^2} - \frac{U \partial D}{\partial x} + K_1 L - K_2 D \pm P \qquad 4.8b$$

Where L = concentration of BOD (ML^{-3})

ϵ = dispersion coefficient ($L^2 T^{-1}$)

U = stream velocity ($L T^{-1}$)

K_1 = deoxygenation coefficient (T^{-1})

S = net oxygen demand sources and sinks ($ML^{-3} T^{-1}$)

x = distance

t = time

D = dissolved oxygen deficit (ML^{-3})

K_2 = reaeration coefficient (T^{-1})

P = net DO sources and sinks ($ML^{-3} T^{-1}$)

Equation 4.8a represents the variation of BOD at a point with respect to time due to dispersion ($\epsilon \frac{\partial^2 L}{\partial x^2}$), advection ($U \frac{\partial L}{\partial x}$), decay ($K_1 L$), and the net result of sources and sinks as described in section 4.6.1 ($\pm$ S). The second equation for the dissolved oxygen deficit is similar in structure and is coupled to equation 48a by the deoxygenation term $K_1 L$. The term $K_2 D$ represents the reaeration and the DO sources and sinks are contained in the term $\pm$ P.