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Developing Early Number Learning using Maths Recovery principles

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Declaration

I hereby declare this PhD thesis, and the work presented in it, to be my own and to have been generated by me as result of my own original research. It has not been submitted for degree purposes to any other university.

A handwritten signature in black ink, reading "S. S. Morrison". The signature is written in a cursive style with a large, stylized 'S' at the beginning.

S. S. Morrison

17th September 2018

Dedication

To my mother,
Helen Mavis Goliath (*nee Hoods*)

Abstract

National, regional and international comparative data all paint the same dismal picture of South African learners' low levels of numeracy (Schollar, 2008; DBE, 2012; Mullis, Martin & Fay, 2008) which has a detrimental knock-on effect on their mathematics learning in higher grades. One of the recommendations made by Spaul (2013, 2016) and others for the remedying of this mathematics 'crisis' in our country is for the development and implementation of a structured, evidence-based intervention programme in the midst of a lack of teacher knowledge on the ground related to how children learn early number. Bearing this recommendation in mind, this doctoral study was aimed at improving a sample of South African children's early number learning using principles based on the Mathematics Recovery (MR) programme (Wright et al., 2006). I opted to investigate the efficacy of a short-term intervention programme using Mathematics Recovery because this programme sets out a detailed, research-based trajectory for children's early number learning while also providing explicit teaching and assessment guidelines. The scale of the problem on the ground, and my belief that learning mathematics is a social enterprise, led me to adapt the MR programme for grouped intervention. To this end, I used the Emergent Approach (Cobb & Yackel, 1996) which incorporates both cognitive and sociological perspectives of learning as the theoretical underpinning of my study. I employed a teaching experiment using 20 mid-attaining Grade 2 learners that were split into a matched intervention and control group. The 10 intervention learners were then further divided: 2 learners received individual intervention and 8 learners received grouped intervention (two groups of 4 each). The different layers of cognitive analyses allowed me to determine two things. Firstly, learners who received intervention based on MR principles moved further along the early number learning trajectory across all aspects of MR's Learning Framework in Number (LFIN) than the control group who continued with their normal classroom teaching; and secondly, comparable learning gains were made between learners who participated in different formats of the same intervention (individual and grouped). From the sociological perspective, analysis based on aspects of interaction – which focused on learners' oral discourse and solution strategies used across different representational settings – provided insight into the emergent and more specific focus on 'base-ten thinking', which is one part of the LFIN framework. This emergent specificity led to further development of the Base Ten aspect of the LFIN related to progressions and reversions in learners' use of base-ten as inferred from their oral discourse and flexibility in enacting base-ten strategies across various representational settings.

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List of Abbreviations

ACR	Africa Competitiveness Report
ANA	Annual National Assessment
BNWS	Backward Number Word Sequence
CAPS	Curriculum and Assessment Policy Statement
CBO	Count-by-one
CGI	Cognitively Guided Instruction
C2005	Curriculum 2005
CPV	Conceptual Place Value
DBE	Department of Basic Education
DoE	Department of Education
ENL	Empty Number Line
ESL	English Second Language
FNWS	Forward Number Word Sequence
GDP	Gross Domestic Product
IFEN	Instructional Framework for Early Number
IRE/F	Initiation Response Evaluation/Feedback
LFIN	Learning Framework in Number
LoLT	Language of Learning and Teaching
MAB	Multibase Arithmetic Blocks
MR	Mathematics Recovery (or Maths Recovery)
MRAP	Maths Recovery Assessment Purposes
NCTM	National Council of Teachers of Mathematics
NWA	Number Word After
NWB	Number Word Before
PEI	President's Education Initiative
RME	Realistic Mathematics Education
RNCS	Revised National Curriculum Statement
RRTI	Record of Response per Task Item
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
Scr	Screened
SEAL	Strategies for Early Arithmetical Learning
SES	Socio Economic Status

STEM	Science, Technology, Engineering and Mathematics
TIMSS	Trends in Mathematics and Science Study
USA	United States of America
WEF	World Economic Forum
WMC-P	Wits Maths Connect – Primary project

Chapter One – Introductory Chapter

1.1 Introduction

This study reports the processes and outcomes of a withdrawal-based intervention model that sought to investigate the possibilities for shifting early number competences at Grade 2 level in South Africa. The intervention lesson sequence, topped and tailed by individual interview-based assessments, drew from approaches developed in Wright, Martland and Stafford's (2006a) Mathematics Recovery programme. Wright et al.'s (2006a) programme was designed originally to work in a classroom withdrawal model with individual tailored development based on the initial assessment and then ongoing diagnoses of progress with learners in the early primary grades who start behind the 'mainstream', with accumulated evidence pointing to the gap between these learners and the mainstream growing over time (Spaull, 2013) without development-focused intervention. Given the extent of the 'lags' between specified curriculum content and learner performance for many learners in South Africa (DBE, 2012), I used a small group withdrawal model in this study alongside an individual learner withdrawal model in order to gain insight into comparative shifts over time. The scale of the problem of poor mathematical performance in South Africa renders individual withdrawal models impractical for working at larger scale, and hence my interest in the comparative model.

A small cluster of studies located within the two South African Numeracy Chair research-based development projects in South Africa have now conducted intervention studies based on the Mathematics Recovery (MR) model, using parts of their battery of assessments (Mofu, 2013; Ndongeni, 2013; Stott, 2014; Wasserman, 2015; Young, 2016). These studies have already shown that in both individual and small group withdrawal models, gains in attainment between pre- and post-testing, based on the MR Learning Profile in Number (LFIN) summary, are possible. This raises the question about the substantive contribution to knowledge that this study offers. I answer this question in a number of key ways. On the methodological side, my assessments and intervention lessons were located in Grade 2, rather than the later grades that have been in focus in the earlier South African studies, and I explicitly included a group/individual comparative element through working with two groups of four learners, and two single learners. Additionally, a control group of ten learners was given the individual assessments at the same times as the intervention learners, thus building in an

intervention/control comparison element. In addition to this contribution, there are two further features:

- the comparison between learners who received one-on-one intervention and those who received 'grouped' intervention (the former designed around the particular number learning needs of individuals while the later was designed around the broad number learning needs of the group);
- the 'base-ten thinking' displayed by the intervention group during intervention lessons that provided broader openings into learners' strategies and oral discourse used across different representational settings.

The second feature was linked to Grade 2 curricular content in number that incorporates an expansion of number range for additive calculation from the 1-20 range that is predominantly in focus in Grade 1 to the 1-100 range. The literature base on early number learning, which I deal with later in this study, points to extensive connections between the number relationships and facts that are developed in the 1-20 range that can be combined with emergent place value understandings for use in calculations in the 1-100 range and beyond (Anghileri, 2006). Further, writing on early number also points to place value awareness developing in the context of calculating in the 1-100 range (Fuson, 1990; Wright, Ellemor-Collins & Tabor, 2012).

The location of my study with Grade 2 learners led to an emergent and more specific focus on base-ten thinking, which is one part of Wright et al.'s (2006) Learning Framework In Number (LFIN) profile (the LFIN being a 'map' of early number aspects and progressions in these aspects devised by these authors, and explained in more detail in Chapter Two). This emergent specificity of focus on base-ten thinking led into further development, related to questions that I asked about the profile of gains that I was able to report based on the pre- and post-assessment responses. In particular, I was concerned with some of the contradictions between the claims that I could make based on the 'snapshot' pre- and post-assessment data, and what I had seen in terms of learners' discourse within particular task and representational settings during the intervention lessons. These discrepancies led me to add an additional layer to the Base-Ten Arithmetical Strategies profile available within the LFIN model. This layer is based on documenting the progressions and reversions in participants' base-ten thinking, as displayed in the solution strategies and oral discourse they used when solving additive tasks in different representational settings, and monitoring changes in these features over time. Over and above the methodological adaptations then,

these two features – both related to understanding of shifts in base-ten thinking in the South African context over time – represent the specific contribution that I believe this study offers.

In line with the previous studies, this study also shows that it IS possible to improve learners' competence in early number through a carefully structured sequence of responsive and diagnostic intervention lessons. Specifically, improvements in base-ten thinking – which have particular resonance in a South African context marked by the ongoing use of highly rudimentary and inefficient counting-by-ones strategies (Hoadley, 2007; Schollar, 2008; Taylor & Vinjevoold, 1999) – are shown to be possible to achieve. But what is also marked and needing further attention is 'localization' within particular representational settings of some of these gains in the shorter term, as well as the important marker of appropriation provided by learners' discourse. In this thesis, I share my analyses of the journey that led to these insights.

In this opening chapter I provide broader background information about the post-apartheid changes in the South African educational landscape that act as antecedents to this study before painting a picture of the current state of mathematics learning in South African government schools. I then provide a succinct explanation of the particular research problem that provided the impetus for this study and the specific research questions that developed from the problem. This is followed by a brief outline of the theoretical and analytical frameworks that shaped the intervention programme and informed my analysis of the data generated. I conclude my introduction by providing an outline of the other chapters that make up this thesis.

1.2 Broader historical background

This study is located within the context of post-apartheid South Africa where the advent of democracy ushered in great change in many aspects of society that were previously plagued by inequality and exclusion. Arguably, one of the biggest changes was the calibration of the education system towards democratic ideals (Chisholm, 2003). South Africa's education system underwent many sweeping changes since 1994 to ensure equal access to education and these changes necessitated, amongst other things, national curriculum reform. Thus, a new curriculum, Curriculum 2005 (C2005), based on principles of outcomes-based education was introduced in 1997 (DOE, 1997). However, the body of work done by Jansen and colleagues (amongst others) showed that the large-scale discrepancies in resources and capacity between the few privileged schools and the large number of disadvantaged schools

with respect to implementation meant that the new curriculum reinforced racial injustice and inequity instead of mitigating it (Jansen & Christie, 1999; Jansen & Taylor, 2003). Empirical research conducted for the President's Education Initiative (PEI) was highly critical of C2005 and linked poor learner performance to the curriculum and the quality of teaching and learning (Taylor & Vinjevold, 1999). So, before the new curriculum ran its course (through 2005), a revised version of the curriculum had already been introduced – that was the Revised National Curriculum Statement (RNCS) implemented in 2004 in the Foundation Phase and in 2006 for Grades 10-12 (Jansen & Taylor, 2003).

Research findings at the time pointed to South African learner achievement remaining very poor despite the widespread curriculum changes. Various factors such as 'time on task' and 'poor curriculum coverage' were cited as some of the reasons why certain sectors of the population (mostly poorer learners in township and rural schools) were still not receiving the quality of education envisioned for the new South Africa. All these factors led to yet another national curriculum change that saw the Curriculum and Assessment Policy Statement (CAPS) being implemented in 2011 (DBE, 2011a). CAPS is a prescriptive curriculum based on the level of detail it provides in its content specification (see 'Clarification notes or teaching guidelines' for Grades 1-3, p.93 - 484 in CAPS). Thus, this latest curriculum reform stands in stark contrast to our first 'democratic' curriculum in that it is characterised by a significantly increased specification of content, sequencing and pacing.

1.3 Mathematics learning in South Africa

Researchers have rightly described mathematics learning in South Africa as being in a state of 'crisis' (Fleisch, 2008; Spaull, 2013). This crisis is evidenced by South African learners' poor performance in international, regional and national studies. Our country started participating in the international Trends in Mathematics and Science Study (TIMSS) study in 1995. The TIMSS studies are aimed at determining the nature of Grade 4 and Grade 8 learners' achievement in mathematics and science across various countries. South African learners (Grade 8 and 9) who took part in TIMSS studies from 1995 to 2003 performed poorly but showed marked improvement between 2011 and 2015. Despite this improvement the country remained in the lower end of the rank order (Reddy, 2017). In 2015 (twenty years after democracy) South African Grade 5 learners participated in TIMSS-Numeracy for the first time with their results indicating very low performance (Reddy et al., 2016). One of the main

findings from the Grade 5 TIMSS study was that, "South Africa is one of the lower performing countries in mathematics in comparison to other participating countries and the national average falls short of the lowest international benchmark set by TIMSS" (Reddy et al., 2016, p.16).

Performance in the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) cross-national studies, which are an important comparative tool in the sub-Saharan region (Spaull, 2013), echoes this picture. According to the World Economic Forum (WEF), South Africa spends more per capita on education than other sub-Saharan countries but to little effect. Poorer African countries like Zimbabwe and Uganda spend a smaller percentage of their GDP on education yet their education systems are considered better than that of South Africa (Spaull, 2013). Despite the significant changes made to the national curriculum and the high per capita expenditure on education in South Africa, the quality of primary education was ranked 126th out of 138 countries in the WEF's Africa Competitiveness Report (ACR) for 2017 (WEF, 2017). Whilst the ranking has gone up seven places since the ACR 2015, mathematics performance at primary level remains a concern.

South African learners' poor performance in the 2010-2012 Annual National Assessments (ANAs) showed that many learners lagged behind the expected curriculum levels and this lag increased sharply from lower to higher grades (DBE, 2012). The poor ANA results confirmed findings from international and regional assessments: that the quality of our primary education system is one of the poorest in Africa; and by Grades 3 and 4 South African children are already below the international benchmark set by TIMSS. These results form the bases for researchers' argument that the poor learner performance evident at secondary level in South Africa is underpinned by weak numeracy foundations in the early grades (DBE, 2014; Schollar, 2008; Spaull, 2013).

1.4 Specific challenges related to teaching and learning

Numeracy in the South African context

A number of research projects have been undertaken in South Africa with the aim of unpacking (and finding ways of mitigating) the factors that fuel the low numeracy levels that constrain mathematics learning in higher grades and leads to exclusion from Science, Technology,

Engineering and Mathematics (STEM) disciplines at tertiary level (Wang, 2013). Here 'numeracy' refers to the early number concepts children learn during their first few years of schooling, like: counting, numbers and number names, number patterns, solving simple additive tasks, etc. In the South African context, Numeracy – which focuses largely on developing number – is one of the three learning areas that constitute the curriculum for the Foundation Phase, which is comprised of Grades 1 – 3 (and recently also the entry Grade R). This phase caters for children between the ages of 6 and 9 years old. There is growing consensus that an overarching problem with the learning of early number in the majority of South African public schools that cater to poorer communities is linked to persistent counting-by-ones (Schollar, 2008; Ensor et al., 2009; Hoadley, 2012). Different researchers provide insight into how this phenomenon – evident in both teaching and learning practices – affects various aspects of number learning. Here I overview the body of research conducted by Schollar (2008) and in the next chapter I look at other national bodies of research conducted by Hoadley (2008; 2012) and Ensor et al. (2009) which shed more light on this topic.

Based on his analysis of three overlapping data sets from 154 South African primary schools (collected from 1998 to 2004), Eric Schollar reports on the actual solution strategies employed by 7 028 randomly selected children. Schollar's (2008) overarching finding was:

learners at all primary grade levels in all provinces routinely reduced all addition, subtraction, multiplication and division tasks to counting forwards and backwards, usually in single units. (p.5)

Another interesting finding from this research was that about 80% of Grade 5 and 60% of Grade 7 learners from the sample relied on concrete counting-by-ones to some degree in their solution strategies. Schollar (2008) believes that the major cause of poor learner performance, which is linked to overreliance on counting-by-ones in South African primary schools, is learners' poor grasp of the base-ten numeration system and the related understanding of place value. These examples of actual learner workings below (Figure 1.1 and 1.2) depict ways in which Intermediate Phase learners (10 - 13 years old) reduced calculations to unit or skip (multiples-based) counting in Schollar's (ibid) dataset:



This example, taken from a Gr5 script, shows separate calculations that are performed using counting-by-ones until the page is filled. Taken from Schollar (2008, p6)

Figure 1.1



This is an example of a Gr7 learner's attempt to use repeated addition of 20 to solve $1420 \div 20$. A tick shows every 20 that is added and these ticks are then counted in ones to get to an answer. Taken from Schollar (2008, p6)

Figure 1.2

Now that I have painted a clear picture of the current problem with primary mathematics achievement on the ground, and situated it in the broader historical South African context, I next highlight different aspects of early number learning that were important in my study.

1.5 Early Number Learning

Children's early number learning is considered by many researchers as the key foundation upon which all understandings of mathematics are built (Reys et al., 1999). The term 'number sense' is generally accepted as one that reflects a changed view regarding number learning over recent years – from an over-emphasis on computational procedures to a focus on children's flexible and efficient mental calculations underpinning written approaches. Thus the intent to develop children's number sense is a stated national education priority for many nations. Australia, the USA and England have undertaken systemic initiatives in early mathematics and numeracy, namely, the National Numeracy Benchmarks for 8-year-olds, Principles and Standards for School Mathematics, and the National Numeracy Strategy's Framework for Teaching Mathematics, respectively (Wright et al., 2006). Although South Africa's Curriculum and Assessment Policy Statement (CAPS) attempts at developing children's

number sense in the Foundation Phase (Grades R – 3, age appropriate learners aged between 5 and 9) by allocating the bulk of mathematics notional time to the teaching and learning of number in Grades 1-3, that is, 65%, 60% and 58%, respectively (DBE, 2011a), this curriculum has been criticised for not placing enough emphasis on the important role of mental computation within the push for greater number sense across the primary grades (Graven, Venkat, Westaway, & Tshesane, 2013).

Number sense broadly refers to an intuitive feel for how numbers are related and behave, as well as the ability to be 'inventive and efficient' with chosen calculation strategies (Anghileri, 2006). Thus, number sense stems from well-interconnected knowledge about numbers, number relationships and how numbers can be operated upon (Baroody, Lai, & Mix, 2006). Whilst individuals who possess 'number sense' have been described in different ways over the years, that is, as having an "at-homeness with numbers" (Cockroft, 1982, p.39) or as having a "feel for numbers" (Anghileri, 2006), there is general consensus that number sense refers to:

a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for dealing with numbers and operations (McIntosh, Reys & Reys, 1992, p.322).

Opinions on how children learn to be numerate vary. Some researchers like (Engel, Claessens, & Finch, 2013) advocate for children's number learning to be premised on counting because a host of research points to children's counting ability being the strongest predictor of later mathematics achievement (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Clements & Sarama, 2011). Other researchers argue that early experiences of structuring number (underpinned by number relationships) and generality, not counting, should be the origin of children's arithmetic skills (Neuman, 1987; Schmittau & Morris, 2004). There are also researchers in the field of mathematics education, like Ellemor-Collins and Wright (2009), who argue that a combined focus on structuring number and sophisticated counting skills, is needed for children to become numerate.

I now present an overview of the debates around whether counting, or structuring number, or counting *and* structuring should form the basis of children's early number learning and

highlight my stance which is synonymous with that of Ellemor-Collins and Wright (2009) who advocate for the use of a 'hybrid approach' that encompasses counting and structure. Following this – and given that the focus of my study is on learners at Grade 2 level where the curriculum focusses on a move from the 1-20 to 1-100 number range for additive working – I pay attention to strategies and representations that develop from the activity of counting and those that build on the structuring of number (including mental and place value strategies) as these speak to efficient additive calculating.

1.5.1 Counting

Many researchers in the field of mathematics education regard counting as one of the most important foundations for the development of early number skills (Anghileri, 2006) and as the fundamental mathematical process that needs to be addressed from the child's earliest experiences in school (Thompson, 2008). Learners' initially solve simple additive relations tasks by counting in ones, either forward or backward (Fuson, 1988; Steffe, Cobb & von Glasersfeld, 1988) thus, developing this facility is an important aspect of early number learning. In this view, from this starting point learners can then progress to 'advanced counting-by-ones strategies' such as 'count-on-from' or 'count-up-to' on missing addend/subtrahend problems (Wright et al., 2006), prior to developing strategies based on attention to numerical structure and relations. The research base drawn on in the counting-based position shows that children who have difficulty counting tend to have later difficulties in mathematics (Clements & Sarama, 2011; Geary, Hoard, & Hamson, 1999). It is for this reason that children's struggles with counting should not be considered insignificant and overlooked but rather developed early on. Our knowledge about how children learn to count is largely based on the seminal work of Gelman and Gallistel (1978), that is, the *one-to-one*, *stable order*, *cardinality*, *abstraction* and *order irrelevance* principles; whilst the widely accepted trajectory for developing counting skills is based on the work done by Carpenter and Moser (1984), that is, *count all*, *count on from the first number*, *count on from the larger number*, *using known number facts* and *using derived number facts* (both bodies of work are detailed in the next chapter).

There is general agreement in the literature on the development of children's counting skills that the act of counting and saying a forward number word sequence (also called oral recitation or acoustic counting) are two distinct concepts (Steffe et al., 1988; Ellemor-Collins, Wright and Lewis, 2007). The former involves incremental enumeration of a quantity while

the latter is likened to reciting the alphabet. Wright, Ellemor-Collins, and Tabor (2012) argue for the need for building fluency with number word sequences and numerals ahead of working on additive tasks in the recitation range.

A further aspect of the literature on early counting is attention to the representational format of the objects being counted, with children having to learn to work with different countable items. Steffe, von Glasersfeld, Richards, and Cobb (1983) have ranked these items in order of sophistication from *perceptual items* to *abstract items*. Similarly, Carpenter, Fennema, Franke, Levi, and Empson (1999) speak of the different types of countable items children work with as they try to find solutions to mathematical problems: starting with the manipulation of concrete items as they try to act out the problem situation and progressing to mental manipulation of numerals.

This overview of the literature on early counting thus points to two aspects that proved important within my study: attention to learners' facility with shifting from concrete counting to more sophisticated calculating strategies, as well as learners' ability to use increasingly abstract representations of number. Both these aspects (which are discussed in greater detail in the next chapter) are important building blocks for acquiring early number skills as these enhance *efficiency* and *flexibility* in calculating – both of which are hallmarks of number sense.

Whilst counting is considered by some as the cornerstone of children's early number development, another school of thought argues for children's number sense to be premised on the structuring of number rather than counting. This school of thought is further divided into those who argue for children's number learning to be solely based on structuring and generality (Schmittau, 2004) and those who argue for a dual focus on structuring *and* counting (Ellemor-Collins and Wright, 2009). A deeper discussion of these different views related to how children learn to be numerate is laid out in the next chapter. Here, I present an overview of the term *structuring number*.

1.5.2 Structuring Number

Whilst children usually start solving simple additive problems by counting-by-ones they also develop strategies more sophisticated than counting-by-ones, such as bridging-through-ten (e.g. $6+8=6+4+4$ or $13-7=13-3-4$) using base-five (e.g. $6+7=5+5+1+2$), and near-doubles (e.g. $6+7=6+6+1$). Developing these more sophisticated strategies builds on learners'

knowledge of structuring number, that is, knowing how to combine and partition numbers in various ways and seeing the relationship between numbers (Bobis, 1996; Treffers, 1991b; Gravemeijer & Terwel, 2000). Knowledge of the different ways in which numbers can be structured is critical if children are to move beyond the limited realm of calculation-by-counting. Splitting numbers in the 1-10 range using base-five is a form of structuring that is learnt early on: seeing that 6 is 5 and 1; 7 is 5 and 2; 8 is 5 and 3; and 9 is 5 and 4. Learning to use the base-five number structure is supported by the five-structure embodied in our fingers as seen below (although this does not imply that children do not need explicit instruction in seeing and using the five-structure of number).

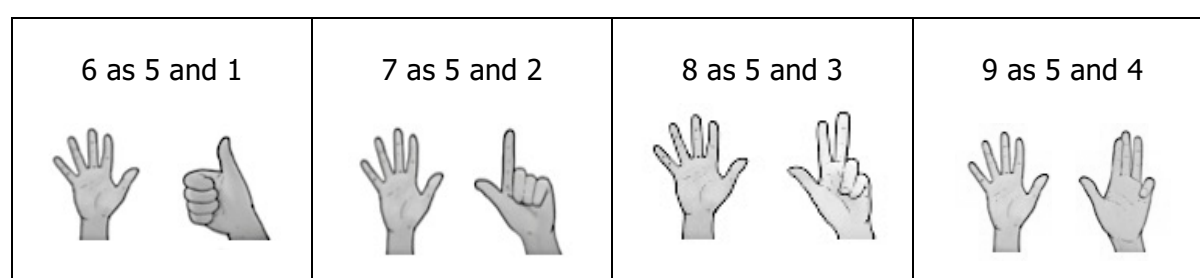


Figure 1.3

A key form of structuring number that children start learning fairly early on in their number learning journey relates to the use of the base-ten structure of the decimal number system (Cobb & Wheatley, 1988; Wright, 1994). This is arguably the most important number structure children need to learn as we use a decimal or positional number system which employs ten as a base. Early primary grades' work with multi-digit numbers are primarily structured around ones, tens and hundreds (Ellemor-Collins & Wright, 2009). These basic units of our numeration system (also called place values) represent a power of 10 which increases by one for every place to the left. For example, the numeral 645 represents the sum of $(6 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$. It is widely accepted that children need to be facile in the use of the composite unit in base-ten representations of number and this facility is considered to be well within the reach of children in the first years of school, if not earlier (Beishuizen, van Putten & van Mulken, 1997; Perry & Dockett, 2002).

Although different forms of counting featured within my intervention study, having 'my eye on the mathematical horizon' (Deborah Loewenberg Ball, 1993) necessitated the need for a parallel focus on structuring number. Thus, my intervention made use of the 'hybrid' orientation (mentioned earlier) where counting *and* number structure were in focus. To this

end my study made use of structured representations as literature points to the need to work with structured representations of number in the 1-20 and 1-100 ranges from the outset in order to support learners' move from calculation-by-counting to calculation-by-structuring (van den Heuvel-Panhuizen, 2008).

The literature on strategies for solving additive relations tasks is overviewed next and detailed in the next chapter. Important within this aspect of early number learning are moves into the use of structured representations for efficient calculation. In looking at the nature and extent of a learner working with structure, attention to learners' interactional 'math talk' during the intervention lessons provided a window into the nature and extent of their moves towards more formal (or generalized) mathematical discourse, and away from more concrete, setting-specific discourse.

1.5.3 Strategies for solving Additive Relations Tasks

The ways in which children solve mathematical problems or tasks is often referred to as strategies or methods. In my study I use these terms interchangeably following Wright et al.'s, (2006, p.191) sentiment that the term strategy is a "...generic label for a method by which a child solves a task". The term 'additive relations tasks' refers to both addition and subtraction problems that children typically encounter in their early years of schooling. Some researchers argue that children develop an understanding of numbers and operations as they learn simple addition and subtraction, hence:

Learning arithmetic begins with learning to add and subtract in the range 1 – 20. (Ellemor-Collins, Wright, & Lewis, 2007).

Even though addition and subtraction are complementary operations, the body of work done by Baroody and colleagues (1983, 1999) shows that – with the exception of doubles facts and their related subtraction combinations – the complementary relation between addition and subtraction is not obvious to children. Based on his research, Baroody (1999) argues that a key to becoming proficient with additive number combinations is through making full use of various patterns and relationships in number.

The developers of Cognitively Guided Instruction (CGI), a professional development programme which advocates for a learner-centred approach to teaching mathematics, have

developed a hierarchy of calculation strategies commonly used by learners when solving additive relations problems, namely, direct modelling → counting → calculating (Carpenter et al., 1999). These authors argue that young children intuitively try to solve problems by using direct modelling of the situation which usually involves physical or concrete representations. Whilst children's intuitive ways of thinking are valued, proponents of the CGI programme believe that teachers should build on the informal strategies learners use and shift them to more sophisticated ways of thinking and working (Carpenter et al., 1999). Wright et al. (2012) and van den Heuvel-Panhuizen (2008) state a similar trajectory for the development of young children's whole number calculations.

Van den Heuvel-Panhuizen (2008) groups children's whole-number calculations in Grades 1 to 3 around three levels (calculation-by-counting; calculation-by-structuring and pure calculation) with a related increase in number range (Kindergarten or Grade R – 1-10; Grade 1 – up to 20; Grade 2 – up to 100; and Grade 3 – up to 1000). This is described in terms of a learning-teaching trajectory characterised by a recurring pattern of inter-locking transitions to a higher level from one grade to the next. This learning-teaching trajectory is detailed in the next chapter.

Wright and colleagues (2012) group all the additive relations between numbers in the range 1-20 and refer to this as the 'domain' of *Structuring Numbers 1-20* (which includes the significant sub-domain of structuring numbers 1-10). These authors similarly group the additive relations of numbers in the range 1-100 – especially those relations organized around tens and decuples – and refer to this as the 'domains' of *Conceptual Place Value* and *Addition and Subtraction to 100*. In the next chapter I elaborate on additive strategies based on: counting, the mental structuring of number and formal place value, respectively.

1.5.4 Representational Settings

The importance of representations in the teaching and learning of number (and mathematics on the whole) has been widely acknowledged (Ball & Bass, 2003; Cobb, Yackel, & Wood, 1992; Heinze, Star, & Verschaffel, 2009; Pape & Tchoshanov, 2001; Terwel, van Oers, van Dijk, & van den Eeden, 2009). A study where children were asked to describe 'what was in their head' when they calculated answers to number problems showed the extent to which their mental images were influenced by the physical representations teachers introduced into the learning space. Reporting on this research, Askew and Brown (2003, p.7) noted that:

“Children need to be encouraged to develop efficient mental images and such a range will be influenced by physical representations offered by the teacher”.

A key theme in the literature on mathematical representations is the idea of *representational flexibility* (Heinze et al., 2009) which speaks to learners’ facility with seamlessly transitioning between different representations in their attempt to explain or construct mathematical understanding.

I draw on the work of Graham, Pfannkuch, and Thomas (2009), amongst others, to talk about different facets of the flexible use of representations, and in particular establishing meaningful links between and amongst representational forms and translating from one representation to another. Establishing *connections* between various representational forms is another key idea around the use of representations in the teaching and learning of early number.

Some representational settings used in the classroom to aid the teaching and learning of early number can be labelled as ‘unstructured’ because they have no number relationship explicitly inlaid into their design (Askew, 1998). Counters, beans and unbundled sticks are examples of representations or teaching-and-learning materials that are unstructured. Such settings are frequently used in pre-school/kindergarten or a Grade R context where young children are learning how to count. However, unstructured representations of number must be used judiciously as these settings can perpetuate counting-by-ones even when children are ready to move beyond that rudimentary strategy to a more efficient counting strategy as discussed by Ensor et al. (2009).

Several researchers agree that learning how to structure number can be supported by the use of structured representations, initially presented in the form of concrete materials and gradually ‘faded’ until these representations merely need to be imagined by the learner to support their reasoning process (Anghileri, 2006; Treffers, 2008; Wright et al., 2012; Askew, 2012). Structured settings are those settings that embody some form of number-structure, like doubles, base-five or base-ten. For example, the pair-wise ten-frame in Figure 1.4 is a setting that shows the structure of double 4 (and half of 8). So, using a structured representation of number like the one shown can initiate children into using

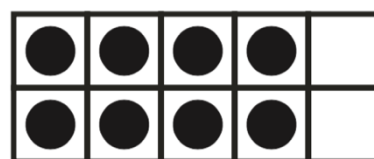


Figure 1.4

knowledge of doubles when solving additive tasks. This setting also encourages children to see eight in relation to ten: that eight is two away from ten – which is linked to $8+2$ as a bond of ten. In the next chapter I elaborate on the structure embodied within various settings and how these structured settings could be used to enculturate learners into using structuring, specifically the structure of base-ten.

1.5.5 Oral Discourse

Vygotsky regarded language as the most powerful psychological tool available to humans because it enables us to do or achieve certain things while at the same time also has an effect on us developmentally (Kozulin, 2003). However, what is not made very clear in Vygotsky's ideas about language and communication is that the exchange and negotiation of meanings that takes place during interaction is not a neat and orderly process, but rather a messy and complicated one (Van Oers, 2002).

Mathematical understanding involves progression from practical experiences to talking about these experiences, first using informal language, and then more formal language. Later, children will learn to use the symbols that characterize the conciseness and precision of a mathematically reasoned argument (Anghileri, 2006, p. 8-9).

Drawing on the body of work produced by (Sfard, 2007) – who describes language as a tool and discourse as an activity in which the tool is used – I regard language as being a medium or part of oral discourse. In my thesis I use the terms 'mathematical discourse' and 'math talk' (described as *thinking aloud with others* by Monaghan, 2005) interchangeably, as I believe that both terms refer to conversations about mathematical thinking and reasoning (Hufferd-Ackles, Fuson, & Sherin, 2004).

I use the term 'interaction' to refer to the words, actions and/or inscriptions used by members of a group as they work collectively on mathematical tasks. Language and discourse play a major role in this interaction because people's actions or inscriptions do not have inherent meaning for everyone in the group – meaning is given to these components of interaction as group members make use of language to participate in discourse or 'math talk'. Thus my focus is primarily on the talk that takes place as participants interact while learning number within a social setting. My interest in the discourse that takes place during interaction is fuelled by

emerging learning theories that highlight the “importance of mathematical discourse as a tool to learn mathematics” (Dunphy, Dooley, & Shiel, 2014, p.10).

Recent work in Mathematics education and in other areas of cognitive development suggests that argumentation is central to the mathematics development of young children (Perry & Dockett, 2008, p.84).

A dialogic pedagogy that encourages discussion, explanation and argumentation is seen by many as an important cog in effective conceptual learning (Dunphy et al., 2014). Research shows a link between the quality and frequency of mathematical language used by teachers and carers as they interact with young children and children’s development of important aspects of mathematics (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Gentner, 2003). Here reference is made to mathematical terms such as ‘more/less than’, ‘as much as’ and spatial terms like ‘above’ and ‘next to’ that children encounter early on in their learning journey as they play with building blocks and build puzzles. However, it is not enough to merely introduce children to mathematical vocabulary – they also need to be engaged in mathematical discourse.

Whilst establishing a culture of ‘math talk’ amongst learners can be a daunting task in any classroom, research shows that for some learners – mainly English second language (ESL) speakers – it is a greater challenge because they “have to *use* the mathematics register while *learning* the register” (Schleppegrell, 2010, emphasis mine). I believe that this point is even more profound in the South African context beyond the early grades, where the majority of learners in government schools learn Mathematics in English whilst they, and their teachers, do not speak English as their home language. This phenomenon forms one element of a broader culture of instruction across all phases, including the early grades, in which ESL learners’ talk tends to be very limited, and in chorused Initiation-Response-Evaluation (IRE) mode (Sinclair & Coulthard, 1975). Many researchers, building on the seminal work of Halliday (1978), highlight the distinction between different ‘registers’ used in the teaching and learning of mathematics and how ESL learners can be supported to participate in math talk communities that make use of such registers (Christie, 1991, 2005; Neuman, Newman & Dwyer, 2011). In the next chapter I elaborate on issues related to language and discourse in early number learning as well as my understanding of shifts in learners’ base-ten thinking using their ‘math talk’ as a marker of appropriation.

1.6 Purpose of this study

The main purpose of my study was to explore the effectiveness of a classroom withdrawal based model for developing the early number learning of a sample of Grade 2 learners in a South African government school, with a focus on understanding participants' gradual move from counting-by-ones to 'base ten thinking' as evidenced by the solution strategies they used and the oral discourse they engaged in when solving additive tasks in different representational settings. To this end my study implemented a short-term intervention programme based on principles of Mathematics Recovery (MR) – a programme that has been used successfully in developed countries such as Australia, Britain, the USA and Canada in classroom teaching and individual development to shift low attaining learners towards more sophisticated strategies for working with number (Wright et al., 2006).

Following from this overall purpose, the over-arching question that guided this study was:

To what extent does short term intervention based on Wright et al.'s (2006) Mathematics Recovery programme prove effective in developing a sample of South African learners' understanding of early number?

With this broad question in mind the following specific questions were formulated – the first three questions drawing on data from learners' LFIN profiles while the final question drew on data gathered during intervention lessons:

1. What was the profile of the sample of Grade 2 learners' early number understanding prior to and immediately after receiving intervention based on Mathematics Recovery principles?
2. Following the intervention pathway for learners who made up the intervention group, what learning gains (if any) were produced at the end of the intervention, and how does this compare to learning gains made by the control group?
3. What were the differences and similarities in the nature of learning gains made by 'singletons' compared to gains made by learners who received grouped intervention?
4. What was the nature of learners' progress (if any) in base-ten thinking during intervention as evidenced by the strategies used and the nature of oral discourse across different representational settings across work on additive tasks?

1.6.1 Motivation

Early number learning is an important building block for future mathematics learning and is considered by some researchers as the area of early math development most predictive of later mathematical success (Missall, Mercer, Martinez, & Casebeer, 2012). My focus on early number learning is motivated by literature within the international landscape which points to the importance of young children acquiring early number skills (Perry & Dockett, 2008, Anghileri, 2006; Askew & Brown, 2003), and the current situation in local South African primary schools concerning learners' poor numeracy levels (DBE, 2012b; Ensor et al., 2009). This lack of early numeracy skills is cause for concern because mathematics is frequently described as a sequential subject in which new concepts build on previous learning.

Whilst poor numeracy levels are commonly accepted as the broad-brush concern with regard to teaching and learning mathematics in South Africa's primary school system, research shows that persistent counting-by-ones (Hoadley, 2012; Ensor et al., 2009; Schollar, 2008), which ultimately rests on children's poor grasp of the base-ten numeration system (Schollar, 2008), is the specific problem that needs to be addressed from a learning and teaching perspective. Research also shows that children's over-reliance on counting-by-ones can carry on well into their upper primary grades (Schollar, 2008) and that this phenomenon has a greater prevalence (and thus greater impact) in the bottom 60% of public schools that cater to the majority of children in South Africa (Ramadiro & Porteus, 2017).

The difference between children who are progressing well in mathematics and those who are struggling with maths concepts can be seen in early childhood and this gap becomes wider as children grow older (Geary, 2011; Mulligan, 2011). However, some studies suggest that young children's arithmetical difficulties can be highly susceptible to structured intervention (Dowker & Sigley, 2010). This means that if children's difficulties with mathematics are identified and developed early on, these children can go on to greater participation and success in learning mathematics in the higher grades.

Given that the ANAs (which were the DBE's primary means of identifying children who need further support) pointed to many learners not acquiring the necessary proficiency in numeracy by the end of the Foundation Phase and given too, the evidence of gaps in primary teachers' mathematical content knowledge, Spaul (2013) suggested that the DBE should implement targeted remedial interventions that are highly structured and sequenced. These

recommendations made by Spaul (2013) were similar to those made by the National Planning Commission (2012, p.38) – which have subsequently been taken up as part of the road map towards future progress in South Africa's schooling system by the National Development Committee.

Whilst a host of information is available with regard to children's language and literacy difficulties, much less research has been conducted on the causes and/or development of children's mathematical difficulties (Dowker & Sigley, 2010; Wright et al., 2006). The dearth of information regarding children's learning of early number means that the little we do know has been gleaned from international literature as very little is known about South African learners' early number development (other than that their numeracy levels are low), or how their learning of early number concepts could be improved (Ensor et al., 2009). This study aimed at addressing this gap by exploring the extent to which short term intervention based on principles of Wright et al.'s (2006) Mathematics Recovery programme might be effective in developing a sample of South African learners' early number skills.

The extent of learners' numeracy backlog in South Africa is higher than that seen in countries like Australia where Mathematics Recovery in an individual model has been successfully implemented, leading to the need for an adaptation to this model in the South African context. Thus the scale of the problem necessitated working with groups of learners, which leads to questions of the interaction within these groups and their potential role in individual learning gains – with international literature suggesting that attention to 'tasks, tools and talk' have a role to play here (Askew, 2016).

Therefore, this investigation into the extent to which a short term intervention programme could develop the early number learning of a sample of South African learners was warranted for several compelling reasons: widespread low attainment in early number learning among South African learners, and more specifically, the persistent use of counting-by-ones; the importance of early number learning for future success in maths; the need for a structured intervention programme on the ground; research which shows that children's mathematics difficulties can be successfully remediated; and literature regarding the benefits of collaborative mathematics learning.

1.7 Methodology

In order to answer these research questions, twenty middle-attaining Grade 2 learners from a South African primary school (ten intervention and ten control learners, evenly distributed) made up the purposive sample for the study. The intervention approach for this study was a 'teaching experiment' which has been used to explore children's mathematical learning and explain the progress they make over time in the context of interactive teaching (Steffe & Thompson, 2000; Steffe et al., 1983). Intervention learners were withdrawn from normal teaching (two groups of four and two singletons) for 40 minutes twice a week for ten weeks to receive grouped and individual intervention while control learners continued with classroom learning. The intervention model took the form of individual pre-tests followed by grouped and individual intervention and ended with individual post-tests. This process is described in greater detail in Chapter 4.

Using the Emergent Approach (Cobb & Yackel, 1996) which is framed by a social constructivist paradigm, I coordinated an individual constructivist perspective with a social interactionist perspective to analyze the different data sets generated by my study. The social constructivist perspective allowed me to focus on different aspects of the data in order to answer the different questions that guided my research. On one hand, I brought individual learner's acquisition of number skills into focus to determine the effectiveness of using Mathematics Recovery principles for developing early number learning in the South African context whilst the interactively constituted situation remained unarticulated in the background. Thus, to answer the first research question I analyzed participants' video-recorded pre-test interviews using the indicators set out in the Learning Framework in Number (LFIN) to get an overview of their early number knowledge prior to intervention. I then analyzed participants' post-test interviews (also video-recorded) and compared these results to those of the pre-test to answer the second and third questions. Additionally, I focused on interaction within the intervention space – particularly, on the 'math talk', solution strategies and settings used – that linked to participants' base-ten thinking. So, to answer the fourth research question I analyzed intervention lessons (videos and transcripts) to gain insight into learners' moves from counting-by-ones to base-ten thinking made evident by their oral discourse and choice of solution strategies across different representations of number.

1.8 Outline of the Thesis

In this opening chapter I presented an overview of the broader South African educational landscape which provides the backdrop for this study; as well as evidence that supports the notion that mathematics education in South Africa is in a state of 'crisis'. I then introduced the specific challenges facing the teaching and learning of early number skills in this country – which is underpinned by an overemphasis on counting-by-ones. Further, I outlined the importance of developing children's number skills for further mathematics learning and explained how my interest in developing early number learning evolved. I highlighted the different facets pertaining to early number learning – additive relations tasks, counting, structuring number, the use of representations and discourse – and touched on the challenges that many low-attainers face when learning early number skills. In this first chapter I also detailed the rationale and purpose of this study and located it within the local and international literature base. Finally, I briefly outlined the data gathered and the analysis process followed and I introduced the theoretical framework on which this study is based.

The remaining chapters of this thesis are organized as follows:

In *Chapter Two* I contextualize the study within the broader literature base related to early number learning. Here I provide a more in-depth discussion of the important aspects related to early number that I have briefly touched on in Chapter One.

In *Chapter Three* I explain the theoretical underpinnings of this study as well as the constructs that make up the analytical framework. I further exemplify how bodies of literature reviewed in Chapter Two were used to create the frameworks used to analyse the different data sets generated.

In *Chapter Four* I provide a detailed account of the specific research methodology I used to gather, organize and analyse data that I believed would answer my overarching and specific research questions. I also explain the research setting and participants that made up my sample as well as the measures I took to ensure that rigour and ethical considerations were in place.

In *Chapter Five* I analyze data generated by MR pre- and post-test interviews using the LFIN framework in order to answer the first research question. I present intervention learners' profiles, explaining gains made across pre- and post-test interviews. I further provide a

tabulated overview of gains made by the intervention and control groups in order to answer the second research question. A comparison of gains made by 'singletons' and those who received grouped intervention is included in this chapter in answer to the third research question.

In *Chapter Six* I report on formative assessment which focused on the development of intervention learners' base-ten thinking inferred from the nature and progress of their solution strategies and oral discourse used while solving 2-digit by 2-digit additive tasks during the intervention period. The analysis of intervention learners' solution strategies and oral discourse answers my fourth research question.

In *Chapter Seven* I discuss the findings related to the research questions and identify links between findings based on different data sets. Finally, this chapter outlines the implications for developing early number in the South African context using Mathematics Recovery principles.

Underpinning the whole study, there is the extensive literature base on **early number learning** (including what constitutes early number, learner counting and calculating strategies and progression), the **representational settings** in which early number learning can develop, and the **oral discourse** accompanying this learning. It is to this literature base and to the specific orientations of the **Mathematics Recovery** programme (and its use internationally and locally to develop children's early number learning) that I turn in the next chapter.

Chapter Two – Literature Review

2.1 Introduction

Any research-based study focused on developing early number learning needs to be cognisant of the evidence base relating to the focus area. With this in mind, my review of international literature is focused on children's early number learning, with particular emphasis on what constitutes early number learning and how children learn to be numerate through becoming facile with additive strategies that involves counting, structuring number and understanding of early place value. The importance and use of representational settings and oral discourse in learning number, as well as Wright et al.'s (2006) Mathematics Recovery (MR) programme are also in focus here. An emergent focus on aspects pertaining specifically to structuring number using base-ten, combined with curricular moves from the 1-20 number range to 1-100 in Grade 2, led to a focus on 'base-ten thinking'.

Because Mathematics Recovery principles formed the nexus of my intervention study I do not only describe the tenets of the programme in this literature review but also draw links between the important aspects related to early number learning in the broader literature base and how these are accounted for (or not) in the MR programme. Insights gleaned from previous studies conducted in South Africa, which were also based on aspects of the Mathematics Recovery programme, are also discussed and related to my study. I thus offer a critical consideration of the Mathematics Recovery programme, and provide the reasons why I regarded this programme as potentially useful to develop the early number learning of a sample of Grade 2 learners in the South African context.

I start this chapter by locating my study within the national literature base, considering what is known about South African contextual challenges related to early number learning.

2.2 South African Contextual Challenges

Research shows that many South African children do not have a strong numeracy foundation upon leaving primary school – a problem (noted in the previous chapter) linked to persistent counting-by-ones and poor understanding of base-ten (Schollar, 2008). In this section, I outline key aspects regarding early number learning in South Africa that were important to be aware of when considering intervention content and approaches.

2.2.1 Overemphasis on Concrete Counting

The body of research conducted by Hoadley (2007, 2012) contributes to our understanding of how highly inequitable learning outcomes between poorer and more privileged students in South Africa are reproduced through pedagogy. Using observation data taken from Grade 3 classrooms in four South African schools (two each from middle-class¹ and working-class² schools), Hoadley (2007) shows how a pedagogical emphasis on 'context-dependent knowledge' like concrete counting limits working-class learners' access to more generalized forms of knowledge, which is made readily available to middle-class learners. Examples of learners' work taken from this study contrast the concrete counting based strategy (Fig.2.1) used by a working-class learner (which is time consuming and error-prone) with the more abstract, symbolic working (Fig.2.2) of a middle-class learner:

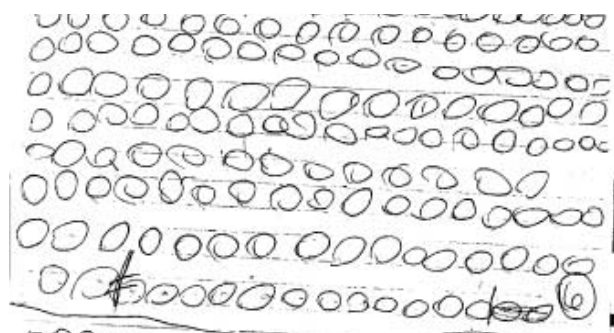


Figure 2.1
Student's drawing of counters for: $214 + 12$
Taken from Hoadley (2007, p.699)

dresses: $54 - 17 = 37$
 $54 - 10 = 44$
 $44 - 7 = 37$

Figure 2.2
Student's calculation for: There are 54 dresses in a shop. Some are sold. There are 17 dresses left. How many dresses were sold? Taken from Hoadley (2007, p.703)

In another study conducted in eighteen disadvantaged Foundation Phase classrooms Ensor, et al. (2009) report on the prevalence of concrete representations of number across Grades

¹ Roughly the top 20% of public schools, charging relatively high school fees, known as 'ex-Model C' schools. Enjoying relatively high levels of learner performance, they reflect the following typology: historically privileged, high mean school SES, and at least 20% of children who speak English as a home language or as one of their home languages (Ramadiro & Porteus, 2017, p.15)

² Poor urban and rural working class schools, roughly the bottom 60% of public schools, not charging school fees. Marked by low levels of learner performance, they reflect the following typology: historically neglected, low mean school SES, and few if any children who speak English as a home language. (Ramadiro & Porteus, 2017, p.16)

1 – 3, as well as the lack of progression from counting to calculating strategies. These authors argued that teachers' persistent use of concrete representations of number had a negative effect on children's conceptual understanding thereof. Based on their empirical research, Ensor et al. (2009) reported that the majority of pedagogic time in the classrooms observed was spent on counting or, in van den Heuvel-Panhuizen's terms: 'calculating-by-counting' – linked largely to children's informal strategies for solving additive tasks that may depend on counting materials, with little or no pressure on Grade 2 and 3 learners to move to what they described as 'pure calculating' – based on reified numerals and attention to, and use of, number relations and structure. These researchers argued that South African teachers could improve learners' understanding of number in the Foundation Phase by 'specializing' the number content being taught – which are, to move from counting to calculating, and to 'specialize' the forms of number representations they make available to learners by encouraging progression from concrete to more abstract numerals (Ensor et al., 2009). So, whilst concrete or countable representations of number play an important role in the development of children's number sense in that they perform an essential bridging function between concrete counting and formal calculation (van den Heuvel-Panhuizen, 2008), these same representations can actually hamper learners' progression if not used judiciously, and 'faded' over time.

These studies show how pedagogical emphases on context-dependent knowledge like concrete counting, the prevalence of concrete representations of number and the lack of progression from counting to calculating strategies can limit South African children's success in early number (Schollar, 2008; Hoadley, 2007, 2012; Ensor et al., 2009). Whilst the constraints related to concrete representations of number are known, further complexities in selecting appropriate junctures in which more abstract models can be introduced are noted in Askew, Venkat and Mathews' (2012) work – where the teacher's jump to a more abstract symbolic manipulation of a missing addend task (like $8 + \square = 15$) left learners confused. The research conducted by Schollar (2008), with images of children's work presented in Chapter 1, cites children's lack of base-ten understanding as the root of the pervasive counting-by-ones strategies seen even among learners in the Intermediate Phase (Grades 4-6).

2.2.2 Multilingual Classrooms

Whilst the national curriculum advocates for collaborative learning where critical, constructive dialogue is encouraged, evidence on the ground shows that very little learner-talk occurs in most multi-lingual classrooms (Setati & Adler, 2000). Research conducted by Webb and Webb (2008) in the Eastern Cape (a mostly rural South African province) shows that most schools in that province have opted for English as the language of learning and teaching (LoLT) even though large numbers of the populace speak isiXhosa. Schools whose LoLT differs from most teachers' and learners' home language is a common occurrence across the country and is certainly the case in the school where my study was based. The danger of having a LoLT that differs from the teacher and learners' home language is that the *Initiation Response Feedback* (IRF) pattern – where the teacher asks a question, the learner makes a simple statement as an answer, and the teacher simply agrees or rejects the offer – tends to dominate classroom talk (Sinclair & Coulthard, 1975).

The issue of language policy in South African schools is a contentious debate that for the most part polarizes two positions: those who argue for mother-tongue instruction during Grades 1-3 before shifting to English as the medium of instruction from Grade 4 onwards and those who believe that 'straight-for-English' schools, where English is the medium of instruction from day one, is the best route for children whose mother-tongue is not English (Taylor & Coetzee, 2013). The push for access to English in South Africa is based on the commonly accepted belief that proficiency in English is needed for anyone to compete in the global market – thus more affluent and poorer parents alike push for their children to attend 'straight-for-English' schools. Across both groups of schools, the result is that many African children attend a school where the language of learning and teaching (LoLT) is not their mother-tongue – either from their first day at school or further along in their schooling career.

Christie (1991, 2002) makes a distinction between the 'instructional register' and the 'pedagogical register' as the former is said to speak to the mathematics content being learnt/taught while the latter is more the regulatory register that projects the content. The instructional register is always embedded in and projected by the pedagogical register, thus the pedagogical register can be seen as the vehicle through which the content is made available. This means that if the pedagogical register is not easily accessible to the child then the content that is supposed to be transmitted through that medium will necessarily be lost to that child. Research suggests that establishing a 'math talk' community in the classroom

should build on learners' everyday language – but what if their everyday language (or the pedagogical register in Christie's terms) is not the same as that of the LoLT of the school? (Schleppegrell, 2010). For many South African children attending 'straight-for-English' schools (as in the case of my study) the Language of Learning and Teaching (LoLT) in early primary grades is not the same language they use to communicate with family and close friends at home. Attention to appropriation of mathematical language, important in any context, therefore carries a particular salience in this context. This phenomenon is not only common in South Africa but also in other parts of the world (like Ghana and Botswana) where children from minority or poorer race groups are faced with the added challenge of learning in a language while they grapple with learning the language itself (Alidou et al., 2006).

2.2.3 Common Pedagogical Practices across Foundation Phase classrooms

Through baseline observation data gathered by the Wits Maths Connect – Primary (WMC-P) project (Phase 1) across Grades 2 and 3 in ten public schools (with an even spread of suburban and township schools) during 2011-2012, key practices and resources relevant to my study were found to be prevalent across project-schools, namely, teacher-led pedagogy, choral counting and use of the hundred chart. Whilst these practices can be used in positive ways, the implementation thereof in project schools often had the opposite effect. In discussing how these practices played out in classrooms we observed, I speak to why these key issues are of interest to my study.

Teacher-led pedagogy

Broader evidence on the ground suggests a South African Foundation Phase mathematics pedagogy that is centred around whole-class teaching, where teacher talk dominates lessons and where 'group work' consists of four to ten learners seated around a table with one piece of paper and a pencil which one learner uses while the others watch (Ensor et al., 2009). Thus current pedagogical practices in some South African classrooms have tended not to view learning mathematics as a social enterprise – which research suggests has promise generally for conceptual mathematical learning (Boaler, 1998), and more specifically in the context of early number learning. Hoadley (2012) further reports on a "strongly communal pedagogy" prevalent in poor, rural and working class classrooms that privileges the classroom interaction pattern of 'group chorusing'. These chorused learner responses to teachers' (often low-level) questions usually require little more than a mindless "Yes, Ma'am" from learners who are not

engaged in the lesson. This issue was of interest to my study because literature shows that a teacher-dominated pedagogy does not promote the notion of 'dialogically organized instruction' which research shows to be a means of enhancing learning (Skidmore, 2006). Hence my intention was to purposefully cultivate an intervention space that was learner-centred, where learners actively engaged with content and other interlocutors during lessons.

Choral counting

Forward and backward counting routines were commonly practiced in choral fashion within the first 10 minutes (or 'mental maths' part) of every Foundation Phase project classroom. I viewed the manner in which this practice was carried out as constraining rather than enhancing learning because, instead of being linked explicitly to other parts of the mathematics lesson, this choral counting simply seemed part of the 'communal pedagogy' reported on by Hoadley (2012). One of the common counting routines practiced by learners was skip-counting forward and backward in tens on the multiples of ten (or decuples), that is, from 10, 20, ...100 and 100, 90, ...10. Of interest to my study was that after learners had practiced this counting routine a few times many were not able to answer a question like, "What is ten more than 30?" without having to count-on in ones. This was partly because teachers often conflated saying a forward number word sequence by ten and incrementing by ten – which Ellemor-Collins, Wright and Lewis (2007) argue are two distinct concepts. The latter involves the increment of a quantity while the former is like reciting a poem (akin to the difference between counting and saying a number word sequence). Literature indicates that incrementing and decrementing by ten off the decuple is not an easy skill for young children to master (Beishuizen, 1993). However, the body of work done by Askew and colleagues (1997) shows that learners who are taught with a 'connectionist' pedagogy can produce greater learning gains than those who are not. Based on this insight, I planned on making explicit links between forward/backward number word sequences by ten (on and off the decuple) and adding/subtracting ten from any 2-digit number. By doing this, I intended to assist intervention learners to appropriate the important skill of incrementing and decrementing by ten on and off the decuple.

The Hundred Chart

The hundred chart is a structured-resource because it embodies the tens-structure of counting numbers from 1-100 in a recognizable pattern (Askew, 1998). This resource was widely-used in several of the Grade 2 and Grade 3 classes we observed as the WMC-P team. In most

classes, each child had one hundred chart stuck on their desk and a second was often stuck in the back of their writing book. The relatively low cost of reproducing the hundred chart and its unburdensome nature (it takes very little space) were probably some of the reasons why this representation was so popular across many schools. Because of its inherent structure, the hundred chart could be used to encourage children to recognize and use the tens structure of numbers when calculating (to add ten from any number you move one block down on the chart and vice-versa) but, it could also be used for counting-in-ones because the units are discretely available. Research evidence shows pedagogical practices across the South African terrain where structured representations of number (like the abacus and hundred chart) were used in unstructured ways (for counting-by-ones) – which obviates the ten-ness of the representation (Venkat & Askew, 2012 and 2018). I anticipated intervention learners' familiarity with this resource and decided to enculturate the structured use thereof during intervention so that learners would have an alternate way of working with it (other than counting in ones) during their day-to-day numeracy lessons.

In summary, bearing these realities in mind, I looked at what research advocates for mitigating against the contextual challenges facing many South African classrooms and note these briefly in the next section.

2.2.5 Literature-based suggestions for intervention

Based on her extensive work in the field, Boaler (2008) concludes that learners should be given the opportunity to work on interesting problems that require them to think and not merely follow a prescribed solution pattern. Learners who work in an environment where they are expected to represent and communicate their mathematical ideas with one another; and who are rewarded for different ways of being mathematical had higher learning gains than learners in traditional classrooms who were required to reproduce solution methods while engaged in individual work. Further, evidence drawn from national research studies points to the need for: differentiation in pace for stronger learners (as opposed to the class working at the pace of the slowest learner); less 'communalized learning' that favours the 'chorused' format of responding to teachers' questions; and higher levels of cognitive demand in lessons which sets up greater opportunities to learn (Hoadley, 2012).

According to Boaler (2008), for mixed ability groups to work well two things are needed: firstly, groups need broad multilevel problems so that learners can access these differently

based on their ability levels; and secondly, learners must work respectfully with each other. The first point is taken from the culture in Japanese classrooms where all children are not expected to have the same learning experience during collaborative work, but rather that each child should get as much from the exercise as she can. Boaler's sentiment about the need for respectful interaction within groups is akin to Askew's (2016) notion that 'deep listening' is essential for classroom interaction to be successful. Askew (2012) believes that increased classroom interaction, which is linked to teachers viewing mathematics learning as a collective activity (rather than a solitary one), can help learners in their mastery of mathematics. Based on the body of work conducted by Brynner and Parsons (1997), Askew draws attention to educators' moral obligation to ensure that children achieve the highest levels of mathematics as they can, as a poor mathematical qualification is correlated with poor economic opportunities (Askew, 2016).

It is against this background that I provide a broad overview of what early number learning entails. This context guides the aspects of early number learning which I believe are pertinent to my study: additive strategies, counting, structuring number and early place value. Each of these aspects is discussed extensively and is thus presented in separate sections (even though they all form part of the broad topic of early number learning).

2.3 Early Number Learning

2.3.1 What is early number learning?

Early number learning refers to the number skills children learn during their first years of formal schooling. These skills generally include saying and identifying the number names and symbols in a gradually extending number range; counting different types of objects; ordering objects and numbers in various ways (e.g. smallest to biggest); structuring numbers – combining and partitioning numbers; the ordinal and cardinal aspects of number; and doing basic calculations (addition and subtraction) within a relatively small number range that is gradually extended over time.

In the past, children's earliest experiences with learning mathematics was the 'drill and practice' of basic number bonds and 'the four operations', which was then largely referred to as arithmetic. Recent reforms in mathematics education have seen a shift from the rote

learning of basic number facts without understanding that was prominent in the past, to a focus on sense-making and children seeing the relationship between numbers. This latter approach is often referred to as involving 'number sense' or 'numeracy' in current mathematics curricula (Anghileri, 2007).

2.3.2 The importance of early number learning

Since almost all the mathematics that children will meet in the elementary school, and much of what they will meet beyond this, is firmly based in number, the importance of sound number sense cannot be overstated. (Perry & Dockett, 2008, p.84)

Numeracy is widely regarded as the fundamental building block upon which more sophisticated mathematical learning is built. As a result, many countries (including South Africa) have placed great emphasis on the teaching and learning of number in their mathematics curricula and some have also instituted programmes (as early as kindergarten-level) aimed at supporting children who are lagging behind their cohort in early number skills. The effects of a poor numeracy foundation have far-reaching consequences, as Spaul (2013, p. 8) notes:

The learning deficits that children acquire in their primary school career grow over time to the extent that they become insurmountable and preclude pupils from following the curriculum at higher grades, especially in subjects that are vertically demarcated like mathematics and science.

Learners' academic performance at primary grades has been shown to be predictive of performance at secondary level. Just as 'emergent literacy' during the pre-school years predicts later reading achievement, so 'emergent numeracy' in pre-school (including counting, number knowledge, estimation and number pattern facility) predicts later mathematical competence (Duncan et al., 2007; Welsh, Nix, Blair, Bierman, & Nelson, 2010). Thus intervention in the early grades (referred to as the Foundation Phase in South Africa) is considered by many to be an imperative. Factors such as the cost of intervention and returns on investment also give impetus to the notion of early intervention, discussed next.

2.3.3 The need for early intervention

Early interventions in children's mathematical difficulties are likely to be cost-effective as later learning builds on earlier learning which could result in lower drop-out rates, lower numbers of learners repeating a grade and less need for remedial support in higher grades (Karoly, Kilburn, & Cannon, 2005). Returns on later investments into development for disadvantaged young adolescents tend, therefore, to be lower, while returns on early investments into development for younger children tend to be higher. Two reasons cited for the greater returns seen from early intervention by Heckman and Masterov (2007) are that with younger children there is a longer horizon over which to recover the investments made and because 'skill begets skill'. A similar sentiment regarding 'accumulated advantage' known as the 'Matthew effect' is noted by Neuman and colleagues (2011).

Most researchers in the field agree that early interventions, especially in hierarchical subjects like mathematics, yield better learner gains over time than intense input during the final year/s of schooling. Thus, increased efforts at improving matric marks in Mathematics and Science across the South African landscape have not been entirely well placed for the kinds of systemic change needed on the ground. Some studies even suggest that intervention should start as early as the pre-school years because differences in children's mathematical understanding are already evident at this young age (NCTM, 2000). Many initiatives by government and the private sector have heeded such findings. Therefore, the focus of more recent research and development within mathematics education has largely shifted to the primary school level, and in particular, to a focus on numeracy in the Foundation Phase which, in South Africa, starts with learners' reception year (Grade R, age appropriate learners aged 5-6) and ends after Grade 3 (age appropriate learners aged 8-9).

2.4 Additive Strategies

Children's early number learning journey starts with learning how to add and subtract using relatively small numbers (Ellemor-Collins et al., 2007). Some researchers describe this journey as the process of *progressive mathematization* – where learners initially invent and use their own informal, context-bound strategies, with subsequent guidance from the teacher towards the use of more formal, efficient strategies. The broad agreement in literature regarding the development of children's whole number strategies is that children start with calculation strategies based on counting, then move on to strategies based on the structuring of number

until they are able to use formal calculation strategies (Carpenter et al., 1999; van den Heuvel-Panhuizen, 2008; Wright et al., 2012).

Learning increasingly sophisticated or efficient ways of solving additive tasks (which includes addition and subtraction) is considered an important part of early number learning. To this end, a considerable amount of research was conducted on children's learning of addition and subtraction in the 1980's and early 1990's (Ellemor-Collins & Wright, 2009).

Although the ultimate aim is for instant recall of basic number facts, such as the pairs that make ten and other combinations such as $25+25$, the strategies children develop on the way to knowing these facts will form the basis for calculation with larger numbers. (Anghileri, 2006, p.133)

In the previous chapter I noted the importance of additive relations tasks with such tasks noted as central to assisting learners in their growing understanding of numbers and operations (Ellemor-Collins et al., 2007). I further noted the general agreement concerning the trajectory for children's whole number calculations as calculation-by-counting → calculation-by-structuring → pure calculation (Carpenter et al., 1999; van den Heuvel-Panhuizen, 2008; Wright et al., 2012). Using the work of van den Heuvel-Panhuizen (2008) I next discuss the distinguishing features of each stage in the trajectory which is structured around the different number ranges that young learners (in the international context) typically work at in different grade levels. While South African evidence stands separate from these age ranges for strategic development, I nevertheless outline the literature because it marks the progressions involved.

Number range 1-10

In many countries' curricular systems, children between the ages of 4 and 5 usually work in the 1-10 number range during their Pre-School and Grade R years. Here different types of counting feature prominently: *acoustic (choral) counting* – where the whole class counts aloud together; *synchronous counting* – which involves pointing to objects to be counted while saying the number word sequence (this implies a one-to-one relationship between the objects being pointed to and each number word spoken, called the *one-to-one principle* by Gelman and Gallistel, 1978); and *resultative counting* – an action that results out of synchronous counting, which determines the size of sets (called the *cardinality principle* by Gelman &

Gallistel, 1978). Counting and calculating are considered synonymous for children at this stage as their first calculation tasks are usually solved by counting in ones. For children at this stage three broad levels of counting-and-calculating are evident, namely, 'context-bound' where children need a context to make sense of the operation → 'object-bound' where problems focus directly on the quantities involved → towards pure counting-and-calculating using symbols (van den Heuvel-Panhuizen, 2008). What is described as important in this range is that children gradually appropriate the additions, subtractions and combinations and partitions (or 'splittings') of numbers in the 1-10 range by heart.

Number range up to 20

In the international literature, children aged between 6 and 7 years old are described as calculating in the 1-20 number range during Grade 1 even though their range for choral counting is usually higher. Here learners' success depends mainly on automated knowledge of addition and subtraction around ten. That means that bonds or complements of ten (2 and 8, 4 and 6) as well as knowledge of $10 + [\text{unit}]$ and $[\text{teen}] - \square = 10$ should be known facts. The knowledge and strategies children appropriated in the 1-10 number range are used and built upon during this stage. Here children's number learning once again develops along three levels (broadly linked to the levels in the 1-10 number range): *calculation-by-counting* which links largely to children's informal strategies that may depend on counting materials → *calculation-by-structuring* which involves the use of known facts, doubles, base-five and base-ten related thinking (this is usually initiated by the use of structured representations of number like the arithmetic rack). Van den Heuvel-Panhuizen (2008) describes three basic structuring models that make it possible for learners to calculate without counting, namely, 'line models' like bead strings that link with counting, 'group models' where objects are grouped but are still countable (like tallies) and 'combination models' like the arithmetic rack which can be used to show relationships between numbers involving doubles, base-five and base-ten → *formal calculation* where learners use visualized images of structured representations until they are confident to use purely mental calculations. Research points to the importance of learners' developing a relational understanding of addition and subtraction combinations and further shows that achieving this goal is quite a challenge for weaker attainers (Baroody, 1999). With relational understanding, learners would be able to use a known fact like $5 + \square = 12$ to figure out the answer to $12 - 5$ without counting or calculating. However, this relationship between addition and subtraction as complementary operations is not obvious to young

learners (Baroody and colleagues, 1983, 1999). Within the 1-20 number range it is imperative that learners develop a 'framework of number relations' that they can draw on for flexible mental calculations (A. J. Baroody, Cibulskis, Lai, & Li, 2004), rather than trying to learn isolated facts and solution strategies devoid of any meaningful connections to other aspects of early number.

Number range up to 100

Children's work in this number range lays the foundation for further calculating with whole numbers, percentages, decimals, fractions, and so forth. Van den Heuvel-Panhuizen (2008) argues that even though calculating in the number range up to 100 is largely seen as the focus for children in Grade 2 (usually aged between 7 and 8 years old), it should not be limited to learners in the second grade but extended to higher grades too because of its importance. Children's number learning in this range develops broadly along the same three levels outlined in the previous number range: *calculation-by-counting* → *calculation-by-structuring* → *pure or formal calculation*.

The international literature base largely points to a minority of children using counting to calculate in the 1-100 number range while the majority move to use counting alongside more efficient calculating strategies. The two main obstacles children face when employing *calculation-by-counting* in the 1-100 number range is crossing a decuple (multiple of ten) boundary and counting backwards. When *calculating-by-counting* in this number range children can be encouraged to use a 100-bead string with every ten beads-group in alternate colours to mitigate against the two potential problems just mentioned (Van den Heuvel-Panhuizen, 2008).

To *calculate-by-structuring* in this number range children often make use of 'jumps-of-ten' (also called the N10 strategy) or a 'jump-via-ten' (or 'bridging-through-ten' strategy) which can be elicited through the use of suitable context problems and related models (van den Heuvel-Panhuizen, 2008). Two prerequisites for using the 'jump-via-ten' strategy are knowing what is needed to fill a ten (knowing the complements of ten) and being able to split numbers up to ten with ease. Both these skills are developed in the 1-10 number range – thus the idea that what children learn in smaller number ranges provides the foundation for their strategies in bigger number ranges.

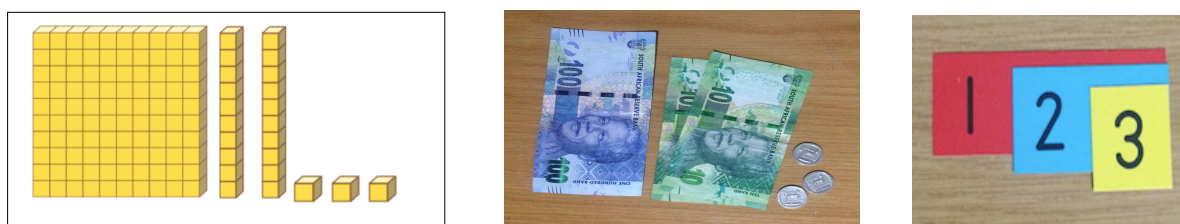
Whilst not mentioned by van den Heuvel-Panhuizen (2008), other literature points out that children may also make use of the 1010 (or split) strategy (Beishuizen, 1993; Anghileri, 2007). Here each addend is split into place value units and the operation is carried out on 'like' units. However, using this strategy to solve a subtraction task can cause a problem for children when decomposition of the tens is needed to subtract, for example $67-39 \rightarrow (60-30 \text{ and } 7-9)$. Here children often subtract the smaller number from the bigger and end up with an erroneous intermediate step like $7-9=2$. Literature indicates that the 1010 strategy is favoured by weaker learners who initially find it easy to use when no carrying or decomposition is required but as soon as the number range gets bigger (and calculations become more complicated) then this strategy can be very limiting (Beishuizen, 1993; Wright et al., 2006).

Children perform *formal calculations* in the 1-100 number range when they can solve written number tasks without being dependent on visual aids or physical representations to structure number. They thus perform calculations 'in their head' and note down interim steps where necessary. Strategies used to perform *formal calculations* include the column method, N10 (or stringing), 1010 (splitting) and/or varying (compensation) strategies. These strategies are explained in detail in an upcoming section.

Number range up to 1000

Learners in Grade 3 (between 9 and 10 years of age) are described in van den Heuvel-Panhuizen's work as mostly having developed an understanding of the tens structure of numbers up to 100 at the start of the year and able to solve additive tasks within the 1000 number range by the end of the year. While working in this number range children's structuring of number improves as they operate on numbers and they get better at calculating as they learn to structure numbers – thus displaying the reciprocal relationship that exists between structuring number and calculating. Van den Heuvel-Panhuizen (2008) argues that whilst numbers were first used in context in earlier grades and then positioned on a number line, numbers are now described as possible to work with in three ways, namely, *splitting*, *factorization* and *characterization*. In previous grades numbers were split into place value units (decimal splitting) for use in N10 and 1010 strategies and in the 1000 number range learners continue to use this process to calculate by *splitting*. The models used here can include Multibase Arithmetic Blocks (MAB) or Dienes blocks, money and/or flard cards. Figure 2.3 shows the number '123' using these models.

Figure 2.3



Learners working within this number range are described as accustomed to seeing '123' as 1 hundred, 2 tens and 3 units but the literature base advocates that they should also be encouraged to see the same number as 12 tens and 3 units or as 1 hundred and 23 units (Anghileri, 2006). It is also pointed out that many learners will not come to this realization on their own; thus, this type of 'flexible structuring' of number should be introduced explicitly.

Factorization speaks to seeing how numbers relate to other numbers in a multiplicative sense which is a more complicated relationship between numbers when compared to relationships based on 'splittings' of numbers as these are based on additive reasoning. *Characterization* involves learners sorting and grouping numbers based on certain criteria like even or odd numbers, prime numbers, triangular numbers, and so on (van den Heuvel-Panhuizen, 2008). The aim of my intervention study was to get participants calculating efficiently in the 1-100 range – thus elements of working in the 1000 number range largely fell outside of the scope relevant for this study.

Calculation-by-counting stands at the first level described in this overview of progressive mathematization, but literature also points to a range of 'sub-progressions' within learning to count. I detail this progression in the next section.

2.5 Counting

The importance of counting to young children's mathematical development is widely accepted and as such many early numeracy programmes are based on the enhancement of children's counting skills (Steffe, et al., 1988; Steffe, et al., 1983; Wright et al., 2006; Perry & Dockett, 2008). The general understanding of how children learn to count is based on the seminal work of Gelman and Gallistel (1978). According to these authors children have mastered counting when they understand the following five principles: the *one-to-one*, *stable order*, *cardinality*, *abstraction* and *order-irrelevance* principles, which I discuss in the next section.

2.5.1 Learning to count

When learning to count children first learn the *one-to-one principle* which means that they have to keep track of items that they have already counted in a collection and those that still have to be counted, and must allocate one number word to each counted item (also referred to as 'one-to-one correspondence'). Secondly, children need to learn the *stable order principle* where they come to understand that the number words used for counting are organised in a stable, repeatable order. This means that 'five' will always follow 'four' regardless of what is being counted or who does the counting. We know that immature counters usually recount a set when asked, "So how many altogether?" (Beckmann, 2014), thus the third principle children need to learn is the *cardinality principle* – which is the understanding that the last number word spoken represents the sum of a whole set. Next, the *abstraction principle* – children must understand that different things can be counted, like objects they can perceive through their senses and things that they can imagine. Lastly, children must learn the *order irrelevance principle* – where they know that they can assign any number word to any of the objects to be counted as long as there is a tag for all the objects (Gelman & Gallistel, 1978). Here again literature tells us that immature counters make mistakes like saying two number words while pointing to one object or skipping objects in the set to be counted (Beckmann, 2014).

The work done within cognitive science by (Le Corre & Carey, 2008) corroborate many of the principles outlined by Gelman and Gallistel's (1978) work relating to how children learn to count (that is, *one-to-one correspondence*, *stable order principle* and *cardinality principle*) but their work also refutes some of the claims made. Firstly, these authors argue that the evidence from their study shows that knowledge of the counting principles delineated by Gelman and Gallistel are not innate, but rather that this knowledge is constructed as children attempt to make sense of the verbal list of number words they initially learnt like a recitation. Secondly, although they fully agree with Gelman and Gallistel (1992) that analog magnitudes (symbols representing neural magnitudes linearly related to number) are part of our innate cognitive resources and that they eventually provide an important part of the meaning of verbal numerals they take data from their study and that of (Condry & Spelke, 2008) to show that knowledge of the verbal counting principles is not constructed out of analog magnitudes. Instead, they argue that this knowledge is provided by a system they call 'enriched parallel individuation' – wherein children map mental models of small sets (1 – 4 items) stored in long-term memory onto the relevant numerals and apply this to sets in their working memory that

can be put in one-to-one correspondence with this long-term model. In rejecting a role of the analog magnitude system in the early development of knowledge of the meaning of numerals, LeCorre and Carey (2008) stand in stark opposition to Gelman and Gallistel's (1992) theory of the acquisition of verbal counting.

This empirical research conducted by LeCorre and Carey (2008) showing that, "the count list is first mastered much as children learn to recite the alphabet, that is, without attributing any significance to the order" (p.1) echoes the body of work done by Steffe and colleagues (1988), who make a distinction between children's acts of 'counting' and that of 'saying a number word sequence'. Steffe et al. (1988) believe that when children are saying number words in the correct order without omitting or repeating any words, they are merely reciting the number words in sequence – much like reciting a poem – whilst the act of counting is seen as 'productive'. So, if a learner says the number words from 'one' to 'ten' in the correct sequence she is considered to be reciting the forward number word sequence from one to ten. But, if she enumerates the number of dots in a 5x2 array by saying the number words 'one' to 'ten' in sequence while coordinating the utterance of each number word with one item among those that are to be counted (*order irrelevance principle*), keeping track of the dots she has already counted (*one-to-one principle*) and knowing that the last number word spoken represents the total number of dots (*cardinality principle*), then she is counting.

2.5.2 Different countable items

In becoming numerate, children have to learn to work with different countable items. This idea, called the *abstraction principle*, was introduced by Gelman and Gallistel (1978) and deals with the different things that can be counted. Steffe et al., (1983) further developed this notion by ranking different countable items in order of sophistication as: *perceptual items* – referring to things that can be perceived through our senses; *figural items* – things that cannot be seen but can be recalled like the number of pets you have at home; *motor items* – movements that one can make like clapping your hands; *verbal items* – oral words spoken like number words; and *abstract items* – including number symbols. In a similar vein, developers of Cognitively Guided Instruction (CGI) show how children's solution strategies usually progress from using direct modelling of problem situations (often with concrete materials) to using more abstract mental calculations that do not rely on direct modelling nor the manipulation of concrete objects (Carpenter, et al., 1999). So, the idea that there is a

hierarchy of different (increasingly abstract) countable entities that children can learn to count (and eventually calculate with) is well established in literature.

2.5.3 Counting Strategies

Counting strategies used to solve additive relations problems have been the focus of many studies within the field of mathematics education (Clarke, McDonough, & Sullivan, 2002; Fuson, 1992; Griffin, Case, & Siegler, 1994; Steffe, et al., 1988). Children's earliest experiences with number usually involves counting, and their understanding of forward and backward counting sequences initially forms the foundation of their understanding of operating on numbers. For example, when adding one to a known collection the next number word in the forward counting sequence will tell me how many in the new collection (so if I have 5 stones in my pocket and I add 1 more stone to it, I do not have to recount all the stones because the next number word will quantify the new set); and likewise the backward counting sequence is linked to subtraction (Anghileri, 2006). The general agreement in the literature base on the trajectory for developing counting skills is based on the work done by Carpenter and Moser (1984), that is, *count all*, *count on from the first number*, *count on from the larger number*, *using known number facts and derived number facts*.

In the counting trajectory linked to addition, the first strategy, *count all*, is the most basic as this follows on from direct modelling of the action described in the problem. For example, if learners are given the problem: 'Ben's mother gave him 6 sweets and his aunt gave him 8 sweets. How many sweets does Ben have now?' - using counters or drawings (Figure 2.4), young learners will show Ben's 6 sweets that he got from his mother (the first count) and the 8 sweets he got from his aunt (the second count) like this:

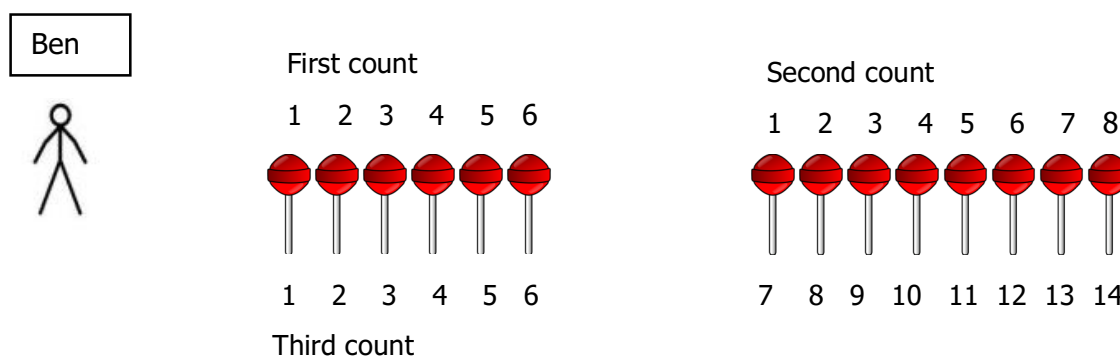


Figure 2.4

Then they will find the total number of sweets Ben now has by counting these sets together from 'one' to 'fourteen' (the third count) – this is also described in literature as a 'triple count' (Anghileri, 2006).

The next strategy, *count on*, is considered a big next step as it involves the child reciting number names in an oral sequence that starts from points other than the first number in that sequence. Here 'reification' of the counting process "one, two... six" takes place (Sfard, 2008). So for the same example used above, a child who can *count on* will keep 6 (as a reified quantity) in her head and count on 8 counts using her fingers to keep track of the number of counts she has made: "seven, eight, nine, ten, eleven, twelve, thirteen, fourteen" – answer 14. When a child uses *count on* to solve a missing addend task like $8 + \square = 14$ she keeps the given addend (objectified) in her head and *counts up to* the sum: "nine, ten, eleven, twelve, thirteen, fourteen" – the number of counts made (6) will be the answer. For the next strategy, *count on from larger*, a child needs to understand the commutative law of addition to know that the answer for, e.g., $4 + 19$ will be the same as for $19 + 4$, the major difference being that adding/counting on 4 from 19 is much more efficient (or quicker) than counting on 19 from 4. In our example used previously, the child who uses *count on from larger* will keep 8 in her head and count on 6 counts.

The next strategy in the counting trajectory, *using known number facts*, sees children using number facts which include bonds of numbers up to ten and doubles (Thompson, 2008) – here the child does no counting or calculating. *Derived number facts* involves working with near doubles, bridging-through-ten, partitioning numbers, and skip counting. So for our previous example, a child who knows that double 6 is 12 could reason that the answer to $6 + 8$ must be two more than 12 which is 14.

Carpenter and Moser's (1984) progression of children's subtraction strategies linked to counting are outlined by Anghileri (2006, p. 56) as: *count out*, *counting back from*, *counting down to*, *count up*, *using known facts and derived facts*. When solving a subtraction problem, the child using the *count out* strategy will count out the minuend (bigger number) using concrete objects or a drawing of these objects, then take away the subtrahend (smaller number) and finally count all remaining objects to find the answer. If given this problem: Mike has 8 marbles and gives 3 marbles to Matt. How many marbles does Mike have now? Young

learners could enact the *count out* strategy by drawing Mike (Figure 2.5a) with his eight marbles like this:



Figure 2.5 a

Thereafter, three of the eight marbles will be scratched out like this (Figure 2.5b) leaving five as the answer:

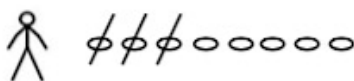


Figure 2.5 b

The next strategy, *counting back from*, also involves 'reification' because the child does not count to establish the minuend as she has 'objectified' this number. She thus counts back from the minuend as many counts as the subtrahend, and the number word said at the end would be the answer. When using this strategy to solve '18-5' the child will put 18 (an object) in her head and count back five counts using her fingers to keep track of the number of counts made: "seventeen, sixteen, fifteen, fourteen, thirteen" – the answer is the number word spoken on the fifth count, i.e. 13. This strategy is not efficient when subtraction tasks involve numbers that are relatively close to one another, like 18-15, because then the child has to make fifteen counts backwards to get the answer.

The next strategy in the trajectory is the *counting down to* strategy – also known as the 'difference' conception of subtraction. This strategy is mostly used to solve 'missing subtrahend' tasks like $18 - \square = 15$ or other subtraction tasks involving two numbers that are relatively close, like $68 - 65 = \square$. Here the child's backward count goes from the bigger number to the smaller number, and tallies of the number of counts are used to get the answer. So to solve $18 - \square = 15$ the child will keep 18 in her head and *count down to* 15: "seventeen, sixteen, fifteen" and the answer would be the number of counts made which in this case is 3 (recorded with tallies or fingers). The *counting up* strategy involves the child counting up from the smaller number to the bigger number. This strategy is efficient when solving tasks like 18-

$\square = 15$ or $68 - 65 = \square$. To solve the latter example the child will keep the smaller number in her head and *count up* from the smaller (65) to the bigger number (68) while keeping track of the number of counts made on her fingers or with tallies. When *using known and derived facts* the child could use knowledge of combinations and partitioning of numbers (especially complements of ten), and her understanding of addition and subtraction as inverse operations to solve the problem (Thompson, 2008). So, for learners to efficiently calculate additive relations tasks, knowledge of additive number relationships and/or properties, such as commutativity ($4 + 9 = 9 + 4$) and inversion (e.g. if $13 + 4 = 17$ then $17 - 4 = 13$), are also important.

2.5.4 A different view

Alongside the importance of learners using increasingly sophisticated counting-based strategies to solve simple additive problems is literature which argues that developing children's concept of number on empirical counting strategies hinders their development of abstract mathematical ideas. Davydov and colleagues developed a curriculum wherein children's concept of number and number operations are built on understanding the relationships between quantities (Schmittau & Morris, 2004). Davydov, following Vygotsky's notion that only scientific concepts are 'true concepts', argues that if mathematics is to be understood at its deepest level, children must learn to think theoretically and go beyond the empirical aspects of phenomena (Schmittau & Morris, 2004). This view argues that establishing children's concept of number on the activity of measurement, rather than counting, enables children to understand mathematics concepts at their most abstract and generalised level (Schmittau, 2003). The Vygotskian curriculum advocated by Davydov and Schmittau is very different to the maths curricula used by most countries, including South Africa, which are grounded in counting-based learning paradigms (Schmittau & Morris, 2004). Whilst the trajectory I focus on is built on counting, I follow Ellemor-Collins and Wright (2009) and others in incorporating a focus on the mental structuring of number – which emphasizes the structural relationships between numbers that figure prominently in the Davydov approach. In the next section I discuss structuring number which is considered vital for early number learning.

2.6 Structuring Number

Literature in the field of mathematics education depicts mental arithmetic or mental computation as an important foundation for all aspects of early number learning and working,

and especially for efficiency in multi-digit calculations (Ellemor-Collins & Wright, 2009; van den Heuvel-Panhuizen, 2008). Proponents of reform mathematics place greater emphasis on children's mental strategies that start out at an informal level and progress to more formal mental strategies based on a deep understanding of the underlying number relationships (Anghileri, 2006). Research evidence supports the development of learners' multi-digit arithmetic based on their informal understandings of number structure, by focusing on the development of mental computations with 2-digit numbers (Beishuizen & Anghileri, 1998; Fuson et al., 1997; Yackel, 2001). But what is mental computation or mental arithmetic?

2.6.1 Mental working

Van den Heuvel-Panhuizen (2008) argues that mental arithmetic should not be seen as the opposite of written arithmetic, nor as 'doing calculations in your head'. She argues instead, that a mental strategy speaks of an insightful calculation where one makes use of memorized facts combined with awareness of the properties of numbers and operations (and the way these are related) in the head and linked with writing down certain interim steps, where necessary (p.90). Many authors agree that "mental computation is an integral part of young children's learning about number" (Perry & Dockett, 2008, p.85) and is vital for efficient computation, numerical reasoning and number sense (NCTM, 2000; McIntosh, Reys, & Reys, 1997; Treffers, 1991b).

The body of work done by Wright and colleagues (2006, 2009) describe structuring numbers as the 'skilled organization of numbers using relationships, patterns and other structures'. These authors also report on children's mental strategies related initially to structuring number, that develop alongside counting strategies. Askew and colleagues (2001; 2003) also show that developing children's mental strategies is vital for becoming numerate. These authors believe that children's mental strategies can be built on the partitioning of numbers in various ways. Understanding the structure of numbers entails, for example, knowing that 7 can be made of 5 and 2, or 3 and 4 – seeing 'numbers within numbers' (Clements & Sarama, 2014). These competencies are not premised on counting; rather they focus on relationships between numbers, which support the deriving of new number facts from known facts.

Research shows that 'knowing by heart' and 'figuring out' support each other in children's learning about numbers (Askew & William, 1995, p. 8)

The quote above implies that the benefits of children working mentally do not stop with the acquisition of a growing bank of known facts but extends to children being able to use a known fact as an intermediary step to get to an unknown solution. For example, knowing doubles facts to ten (like $4+4$) and then to twenty (e.g. $7+7$) are important to 'know by heart'. But then these same rapid recall facts can be used to work out a problem like $8+9$, because a child who knows $8+8=16$ as a recalled fact can reason that $8+9$ must be one more than 16, so the answer is 17. Doubles, halves and 'near doubles' are examples of number structures that young children can learn to use early on.

2.6.2 Benchmarks³ in structuring number

Flexible counting routines (counting in 2s, 5s and 10s from any number) help children gain insight into the relationships between numbers and to recognize patterns in the number system. These counting routines can also provide opportunities for children to create the benchmarks they need for solving problems efficiently and for doing mental calculations (Anghileri, 2006; Thompson, 2008). When children learn to count in multiples of five they soon come to realise that every multiple of five ends either in a 5 or 0. This realisation can help a child know (without calculating) whether *any* number is divisible by five by simply looking at the last digit: 1348 is not divisible by 5 whilst 1345 *is* because with the former the last digit is not 5 or 0 while the last digit in the latter is 5. The value of this kind of flexible counting was the basis for my decision to develop learners' counting skills alongside their facility with structuring number to enhance their early number learning. Benchmarks that are particularly important in the move from the 1-20 to 1-100 range for developing fluency and efficiency in children's mental strategies are: 1-10 number bonds; starting at any number and adding ten; and knowing the distance from any number to the previous multiple or the next multiple of ten.

Number bonds, especially bonds of ten and then bonds to twenty, are important in early number learning as they help children develop more efficient strategies for calculating, like bridging-through-ten (Anghileri, 2006). When children know that 6 and 4 more makes 10, this can form the basis for associating any number that ends in 6 (e.g. 16, 56, 136) with 4 to get to the next multiple of ten or 'decuple'. Learning how to split 'teen' numbers into $10 + \square$ is one

³ I use the term 'benchmark' to refer to important early number knowledge/skills (like knowledge of bonds of 10) and not to specific numbers (like 5 or 10) as used by Van de Walle and Lovin (2007).

of children's earliest experiences in school with regard to structuring number. This splitting of teen numbers is followed by the structuring/splitting of other 2-digit numbers like 36 into 30 and 6. This form of structuring builds knowledge of the distance from any number to the previous decuple (e.g. $36-6=30$), which is a prerequisite for subtraction using bridging-through-ten.

Another benchmark in children's mental structuring relates to building facility with starting at any number and adding ten. Anghileri (2006) believes that this understanding must be automated before children are introduced to 2-digit addition and subtraction calculations. Thus, children who do not know that 32 add 10 is 42 as a known fact will need more practice with tens-based materials to establish these patterns of incrementing and decrementing by ten (on and off the decuple) before attempting additive relations problems like $35+24$ or $46-12$ (Anghileri, 2006). Benchmark facts are considered important because they provide useful reference points that support efficient calculation strategies. Anghileri (2006, p.55) notes that: "confident recall of these number facts will be more important than any formal recording" when children first attempt to calculate-by-structuring.

2.6.3 Using informal understandings of structuring number

Wright and colleagues advocate for learners to use their informal understandings of the structure of number to learn how to flexibly increment and decrement numbers by 1s, 10s and 100s – which they refer to as 'conceptual place value'. They state that a focus on formal place value is (narrowly) aimed at supporting the development of standard, written algorithms (Ellemor-Collins et al., 2007; Wright et al., 2012). The body of work done by Cobb & Wheatley (1988) and Treffers (1991b) supports this view. These researchers posit that young children first reason about numbers in terms of verbal patterns or sequences and quantities, not written symbols. So, for young learners, adding and subtracting by manipulating formal symbols through employing arbitrary rules is not necessarily meaningful. For example, a learner might understand the result of adding ten bundling sticks to those already counted or jumping forward by ten on the number line, but may not understand what happens when 'carrying' one in the tens column to calculate $25+38$ using the written column method. Underlying this attention to structure, there is emphasis on the need to attend to the representation within which competences are manifested – a point I return to later in this chapter.

Structuring using base-ten thinking

To make any progress in mathematics children must be inducted into base-ten thinking, developing a skilful habit of organizing numbers and calculations into 1s, 10s and 100s. Freudenthal emphasized this induction, calling it 'decimalizing' (Wright et al., 2012, p.16).

From the previous quote, I draw a link between Wright et al.'s (2012) notion of 'conceptual place value' and Freudenthal's earlier work where he referred to the organisation of numbers into 1s, 10s, 100s and 1000s (called 'decimalizing') which initiates children into using the base-ten structure of our numeration system – which he called *base-ten thinking*. Freudenthal believed that children needed to be able to skilfully structure numbers and calculations using base-ten thinking in order for them to get a handle on working with numbers larger than twenty (Wright et al., 2012). 'Base-ten thinking' thus draws on children's ability to reason in tens and ones which is linked to very early/informal place value knowledge or 'conceptual place value'. In my study a focus on developing base-ten thinking emerged during intervention when I realised that the learners in my sample did not yet have a firm grasp on structuring numbers into ones, tens and hundreds which was evident in their struggles to increment and decrement a number by ten without counting-by-ones.

Research shows that the development of children's number sense as a whole, and in particular the development of mental strategies, is disrupted by early algorithm introduction (Askew & Brown, 2003). Further research evidence suggests that "undue emphasis on the place value structure may invite a *digitwise* approach to calculating" (Anghileri, 2006, p.40 emphasis in original) which is error-prone. Here, calculations involving multi-digit numbers are reduced to operations on strings of single-digits set out in columns. Therefore, the suggestion from this research evidence is for the teaching of algorithms that focus on a digit column value to be delayed. Proponents of reform mathematics place greater emphasis on children's mental strategies that start out at an informal level and progress to more formal mental strategies based on a deep understanding of the underlying number relationships (Anghileri, 2006). This focus on moves from more informal towards more formal mental strategies related to base-ten thinking was followed in this study.

2.6.4 Strategies based on Mental Structuring of Number

There are various strategies that children can use to solve additive tasks which are based on the mental structuring of number. These mental computation strategies can be broadly classified into three types: sequence strategies; split strategies; and compensation strategies (Ellemor-Collins & Wright, 2009; van den Heuvel-Panhuizen, 2008). In order for young children to use these strategies flexibly they need broad knowledge of number relationships (Threlfall, 2002). In particular, each of these mental strategies depends on ways of relating or structuring multi-digit numbers (Ellemor-Collins & Wright, 2009). Wright and colleagues (2007) further note that these structures of multi-digit numbers are based on tens patterns such as: additive place value (knowing that 34 is made up of 30 and 4); adding ten and small multiples of ten to decuple and non-decuple numbers ($50+20=70$ and $56+20=76$); and adding by 'bridging-through-ten' or first jumping to the nearest decuple (so $47+8$ will be worked out as $47+3=50$ and $50+5=55$).

Together these structures (including other important structures mentioned earlier: doubles, halves, and base-five) form a rich network of number relations which in turn provide the foundation for flexible and efficient mental computation (Foxman & Beishuizen, 2002; Heirdsfield & Cooper 2002; Threlfall, 2002). In this review, I group together those strategies that require seeing ten as a unit and discuss these under the heading *Base-Ten Strategies* while strategies based on other forms of structuring number are discussed separately.

Base-Ten Strategies

Base-ten strategies speak to seeing and using ten as a unit of ten ones when solving multi-digit tasks and organizing numbers and calculations into 1s, 10s and 100s. Children's facility with base-ten thinking influences their facility with 2-digit by 2-digit calculations and working with tasks in higher number ranges.

Base-ten calculation strategies used for additive relations tasks (which could also be considered 'mental strategies') fall into two broad categories, namely, *sequence strategies* and *split tens strategies* – also referred to as *N10* and *1010 strategies*, respectively. Sequence strategies are related to the counting sequence (because children will solve the task by counting-on or back in tens and ones) while split tens strategies are related to early place value understandings (Anghileri, 2006, p. 65). Knowing sequences of ten off the decuple is

an important part of knowing base-ten structures (Ellemor-Collins et al., 2007) and is a prerequisite for mental jump strategies (Fuson et al., 1997).

Sequence or N10 strategies retain the first number as a whole and partition the other number into 'decimalized' chunks before adding or subtracting these partitions. For example, when calculating $36+12$ the first addend is kept whole and the second is added in place value chunks as $36+10=46$ and $46+2=48$ (as seen in Figure 2.6). In a subtraction task like $48-25$ the minuend is kept whole and the subtrahend is subtracted in place value chunks as $48-20=28$ and $28-5=23$ (Figure 2.7). Structured or empty number lines⁴ are representations usually associated with the sequence or N10 strategy (Beishuizen, 1999, Anghileri, 2006) but this strategy can also be enacted on the hundred chart or completed as a number sentence⁵.

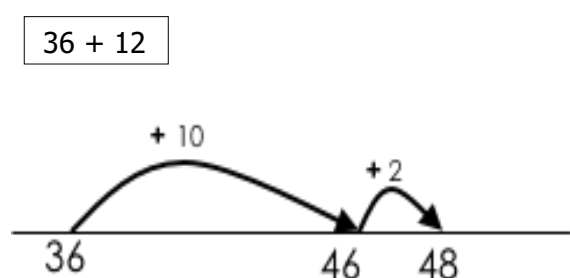


Figure 2.6

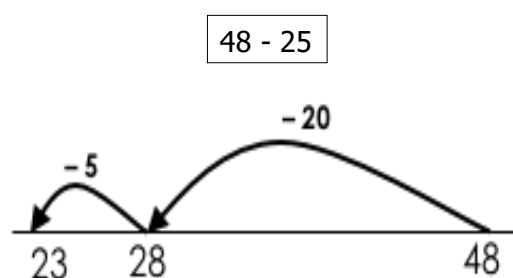


Figure 2.7

'Bridging-through-ten' or 'jumping-via-ten' can be considered a subsidiary of the N10 strategy because here the first number is also kept whole. The only difference is that the second number is not added or subtracted in place value chunks, but rather, the first 'jump' made is to a multiple of ten. So, for the example used previously ($36+12$) the learner will start at 36 (Figure 2.8) and add on 4 (from the 12) to get to 40, then she will add on 8 (what is left of the 12) to get to 48. To solve our previous subtraction example ($48-25$) the learner will start at 48 (Figure 2.9) and jump back 8 to get to 40, then jump back 10 to get to 30 and finally jump back 7 to get to 23.

⁴ An informal recording

⁵ A formal recording

$$36 + 12$$

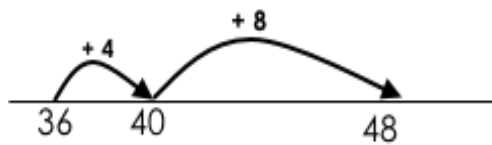


Figure 2.8

$$48 - 25$$

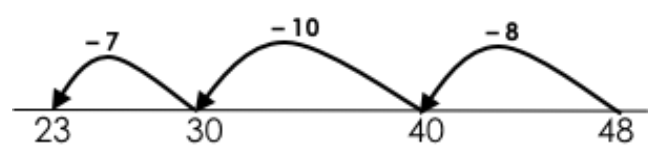


Figure 2.9

The idea here is that calculations are made simpler with an intermediary step to a 'friendly' number like a multiple of ten (or decuple). Facility with structuring number is needed for bridging-through-ten as this strategy involves knowing the distance to the next (addition) or previous (subtraction) multiple of ten and then knowing what the remainder of the second number will be after the necessary partitioning has been done (Anghileri, 2006).

Split tens or 1010 strategies 'decimalize' both numbers into tens and units, which are operated on separately and then joined again at the end to get a final solution. For example, when calculating $36+12$ both addends are split into place value units ($30+6$ and $10+2$), the same place value units are added ($30+10=40$ and $6+2=8$) and finally the sum of each is added together to get the final solution ($40+8=48$). The split tens or 1010 method is more often associated with structured materials like bundling sticks or Dienes blocks (Anghileri, 2006, p. 39). The difficulty with the 1010 strategy arises for most learners when decomposition is needed in a subtraction task like $62-35$ in Figure 2.10:

$$\begin{array}{l}
 62 - 35 \\
 \rightarrow (60 - 30) + (2 - 5) \longrightarrow \text{The difficulty with this strategy usually arises} \\
 \rightarrow 30 + 3 \quad \text{with this step - as many learners will subtract} \\
 = \underline{33} \times \quad \text{the smaller number from the bigger.}
 \end{array}$$

Figure 2.10

Given this complexity, RME-based approaches tend to advocate for the N10 strategy (Beishuizen, 1993). However, given the emphasis within traditional teaching approaches of attention to formal place value associated with 1010 approaches, it was worth being aware of the possibility of learners in my study using 1010 approaches in response to additive tasks.

Related strategies based on number structure

Another mental strategy that children can use for easier calculating of additive problems, particularly when numbers in the problem end in 1 and 2 or 8 and 9, is called compensation or the varying strategy (Anghileri, 2006). With this strategy the original calculation is changed: if a child is required to add 25 to 39, she could instead add 25 to 40 as this would be easier to calculate. After obtaining an interim result (e.g. 65) the child will then compensate her answer – in this case the child will subtract the one she added to the '39', making her final answer 64 (Figure 2.11). Similarly, a subtraction task like 45-28 can be changed to 45-30 and when the interim result is found the two that was added to the subtrahend can be reversed (Figure 2.12). Images of certain representations, like the structured or empty number line, can support children's use of the compensation strategy.

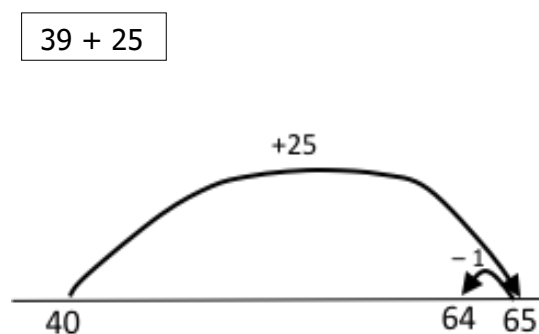


Figure 2.11

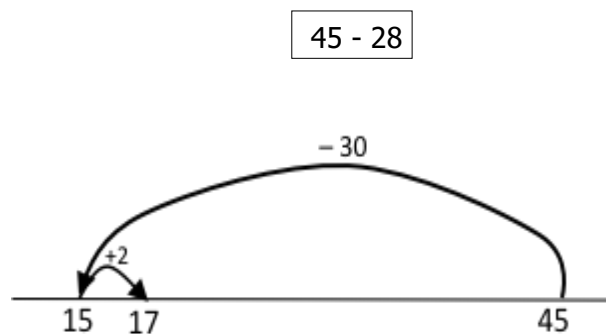


Figure 2.12

2.7 Formal Place Value (PV)

Before children learn about formal (or conventional) place value their understanding of number is derived from and connected to a count-by-ones approach to quantity – so to them the number 16 means sixteen ones. Even when young children can read and write 2-digit numbers correctly, this does not mean that they can meaningfully use groups of ten to represent such quantities. The following quote from Van De Walle, Karp and Bay-Williams (2013), citing Fuson's (2006) work in support, explains this well:

Fuson and colleagues refer to children's pre-place-value understanding of number as 'unitary'. That is, there are no groupings of ten, even though a two-digit number is associated with the quantity. They rely on unitary counts to understand quantities.

'Formal' or conventional place value refers to the positional emphasis of digits (or numerals) in numbers as a means of identifying their size. This is born out of our Hindu-Arabic system of number notation which uses numerals 1-9 and zero (0) to represent all numbers and is based on a decimal number system.

This notion stands in contrast to constructs previously discussed which are based on informal place value understandings, referred to as 'conceptual place value' (Wright et al., 2012) and 'base-ten thinking' (Freudenthal, 1983). Whilst formal place value builds on using the positional value of digits in a multi-digit number, conceptual place value and base-ten thinking build on knowledge of structuring numbers in various ways, that is, seeing and using patterns in numbers (i.e. if ten more than 20 is 30 then ten more than 24 is 34). Another difference between formal place value and conceptual place value is that the former is geared towards using written column addition/subtraction while the latter is focused on developing skills built on knowledge of the structure of number. An example of this kind of connected relation is seen when children who can count "one, two, three" use this awareness to count "one hundred, two hundred, three hundred" or "one thousand, two thousand, three thousand".

Different approaches to the teaching of formal place value at primary school level are evident across different countries. In England, for example, place value understanding is considered essential for the development of children's mental and written strategies and as such is taught from the early grades (Thompson, 2000). In the Netherlands, the opposite is considered to be true: that if children are taught standard algorithms (which is based on knowledge of formal place value) in the first three years of schooling then their mental strategies will have very little chance of developing. Dutch children are rather encouraged to develop their calculation skills gradually through the extension of their intuitive counting strategies (Anghileri, Beishuizen & Van Putten, 2002). The following quote from Treffers (1991b, p.48) states the Dutch position strongly, "In grades 1, 2 and 3 there is no room for the standard algorithm... (or) mental arithmetic does not stand a chance, certainly not for weaker pupils...".

2.7.1 Difficulties associated with formal PV for English speakers

When learning about place value children need to know how two key systems used to represent multi-digit numbers work. These systems are English *number words* (like forty-two) and written symbolic numbers or *numerals* (like 42) and children should also know the differences between them. For example, the ten is not stated explicitly in some English multi-digit number words, like 'eleven' and 'twelve' and the symbol for ten is written first in 'teen' numbers even though the 'teen' sound (which means ten) is said last. So, for the number word 'seventeen' the 'seven' is said (and heard) first and the 'teen' is said (and heard) last but if a child writes the number symbol in the same order that the number is said (or heard) then she will erroneously write '71'.

English number words and written/symbolic numbers are built up of increasingly larger 'multiunits' related to ten and children's understanding of multi-digit numbers requires being able to think about these various sizes of 'multiunits' (Fuson, 1990). Children also need to know how to compose and decompose multi-digit numbers into multiunits in order to carry out the various operations on these numbers. Research evidence shows that children often do extensive work on place value first before doing any work on multi-digit addition and subtraction because understanding place value is thought to be a prerequisite for understanding multi-digit addition and subtraction (Fuson, 1990).

Fuson (1990) clearly states her disagreement with the above view and believes that this pedagogy is what leads to U.S. children not building multiunit conceptual structures for multi-digit numbers, causing them to treat multi-digit numbers as "concatenated single digits". Empirical research conducted by Fuson found that many aspects of teaching and learning place value (and the differences between the English number words and their related written symbols) could be understood through meaningful engagement with multi-digit addition and subtraction in the early grades. Her work suggests that much place value work should be done *within* the multi-digit addition and subtraction setting rather than being done separately before any multi-digit addition and subtraction work is begun (Fuson, 1990). The work done by Thompson (2000) echoes Fuson's views as he proposed a reappraisal of the teaching of place value in primary schools in England. Thompson (2000) argues that the highly sophisticated concept of place value (which he believes is not fully understood by many children at the end

of their primary schooling) is only needed during Year 4 (Grade 3) and thus the teaching thereof should be delayed.

It has been suggested that English number words, which do not explicitly name the tens in 2-digit numbers (in contrast to several Asian languages), may cause problems with children's learning of place value concepts (Fuson, 1990). For example, 12 in many Asian languages is 'ten two' and 58 is 'five tens eight' whereas in English these number words are 'twelve' and 'fifty-eight'. A wide range of literature supports the idea that English-speaking children should be given extended support for constructing multiunit meanings based on ten because English number names offer a distinct disadvantage (Fuson, 1990). I believe this to be even more pertinent for children who learn mathematics in English but who do not speak English at home which is the case with many young learners in South Africa.

2.7.2 Errors related to written Place Value Strategies

"In traditional classrooms, the place value aspect of numbers has been emphasized more than counting", with images relating to tens sticks, unit cubes and Dienes' blocks which are closely associated with column arithmetic that was central in the past (Anghileri, 2006, p.11). This is in contrast to what research advocates for: a focus on mental strategies that often relate, initially at least, to more intuitive ideas based on counting.

Formal place value strategies are usually associated with a vertical layout of the given numerals where the operation (either addition or subtraction) is performed on separate columns starting from the rightmost-column, moving to the left. These strategies differ from other aforementioned strategies in that they do not deal with the value of the 'whole number' but rather deal with the digits in each number. In the example below (Figure 2.13) of an addition task ($38+9$) solved through the use of a column or place value method, one can see why Fuson (1990) refers to this strategy as children making use of "concatenated single digits".

$$\begin{array}{r}
 38 \\
 9+ \\
 \hline
 47 \\
 1
 \end{array}$$

In Figure 2.13 the digits in the column on the right are units and the digits in the column on the left are tens. In some classes learners write T and U above each string of digits to show tens and units, respectively. The solution procedure is: 8 and 9 is 17 which gives 7 units and 1 ten to 'carry', 3 and the 1 carried is 4, to get 4....7 as the answer (which is read as 47).

Figure 2.13

Place value calculations like the one in the example above pose many problems for children. Firstly, some children confuse the placement of the second addend when this number has fewer digits than the first – like writing the 9 under the 3 in the example. Children also struggle with the regrouping or 'carrying' action – they either forget about the amount they carried forward or they write the 2-digit sum for one column without carrying (in this case, getting 317 as the answer).

Underlying the attention to move learners to progressively sophisticated additive strategies that are underlain by counting, structuring and base-ten thinking, there is emphasis in the literature base on the need to attend to the representations within which additive competences are manifested. I turn to a discussion of these representational settings and how learners make use of these to construct increasingly sophisticated strategies for solving additive tasks.

2.8 Representational Settings

Drawing on the work of Wright and colleagues (Wright et al., 2006; Ellemor-Collins & Wright, 2009) I use the terms *settings*, *representations* or *representational settings* to refer to contexts used for posing tasks, for supporting learners' reasoning about a task and for classroom discussions. There is broad agreement across mathematics education literature that young children tend to directly model problem situations with concrete settings (like fingers or counters) that can represent the real objects before they progress to more efficient counting and calculation strategies in which the actions or relationships described in the problem become less explicit (Carpenter et al., 1999; Anghileri, 2006).

Other aspects concerning the use of representations like making connections between different representations and flexibly moving between different representations when teaching and learning early number are widely discussed in literature. For example, the importance of establishing connections between different mathematical ideas when teaching numeracy using a range of representations is highlighted in the work done by Askew et al. (1997) and Ball and Bass (2003). Similarly, Haylock and Cockburn's (2008) work speaks of the importance of establishing connections between language, pictures, symbols and concrete experiences when teaching and learning number in the early primary years.

The body of work done by Heinze et al. (2009) shows that classroom environments wherein learners are exposed to multiple representations of the mathematical concept being taught (including graphical, tabular, algebraic and verbal); and wherein they learn to seamlessly shift between these representations are more effective in developing conceptual understanding and an appreciation for mathematics than environments in which learners do not have the opportunity to work across multiple representations. Thus, the idea of establishing links between and working across different representations – referred to as 'representational flexibility' – is an important aspect of children's early number learning (Graham et al., 2009). Mathematics education literature uses different terms to refer to representational flexibility such as: 'representational fluency' (Lesh, 1999), 'representational competence' (Shafrir, 1999) and 'representational versatility' (Nistal, Van Dooren, Clarebout, Elen, & Verschaffel, 2009).

Certain representations often foreground particular aspects of the given mathematical concept while obscuring other equally important aspects (Ball, 1993; Dreyfus & Eisenberg, 1996). Some commonly used representations that highlight different aspects of learning number include: MAB materials, bead strings and number lines. A 'bead string' is a representation that links more closely with the counting sequence while the number line shows the ordering of numbers and MAB materials highlight the place value aspect of number. The work done by Ensor et al. (2009) in South Africa (following the evidence overviewed earlier of limited moves towards the use of more abstract representations in instruction) ranks the various types of representations commonly used in Foundation Phase classrooms according to levels of abstraction. Concrete representations are at the bottom end of the hierarchy and more abstract representations are at the top. The different categories of representations ranked from concrete to abstract by Ensor et al. (2009) are:

- *concrete representations* which include physical objects like counters

- *iconic representations* are realistic depictions of contexts like a drawing of sweets
- *indexical representations* are generic depictions of everyday objects like tallies
- *symbolic-number based representations* that include number lines and number charts
- *symbolic-syntactical representations* like mathematical number sentences
- *no representation used* where learners perform calculations mentally

A key aspect of early number learning therefore, is facility with transitioning between different representational settings, with the possibilities alongside this, for attention also, to the moves towards increasingly abstract settings over time while retaining the need for flexibility. Schleppegrell's (2010) notion of 'multi-semiotic discourse' (discussed further in section 2.9) underpins the ideas of flexibility and making connections across different modes of representation.

2.8.1 Structured representations vs Unstructured representations

Askew (1998) distinguishes between 'structured' and 'unstructured' representational settings. The former are resources that embody a particular mathematical idea like doubles or the ten-structure of number. Examples of structured representations include the hundred chart, the abacus, ten-frames and the arithmetic rack. In contrast, unstructured settings are mostly everyday materials or objects like counters, stones or blocks that are used for counting in ones. Structured representations like the arithmetic rack that embody mathematical ideas like base-five and base-ten can support children's use of efficient calculation strategies like 'bridging-through-ten' (van den Heuvel-Panhuizen, 2008). Figure 2.14 shows examples of representations that embody the five-structure of number, these are, a hand (with five fingers), a ten bead-string with different coloured beads for each group of five and tallies.

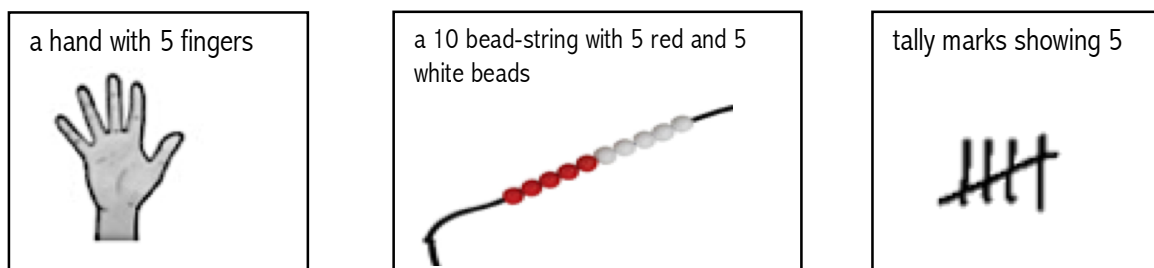


Figure 2.14

Ten-frame cards, the 20-seater double-decker bus (based on the structure of double ten-frames) and the arithmetic rack embody both the five- *and* tens-structure of numbers in the

range 1-20 while the 100 chart (that looks like a stacked number track with 10 numbers in each row) and a 100 bead string (with every group of 10 beads in an alternate colour) embody the tens-structure of numbers in the 1-100 range. By using structured settings learners have a better chance of being enculturated into mathematical reasoning using number structure, as opposed to using unstructured materials (like counters) that encourage counting in ones.

Representations of number can present different levels of abstraction, e.g. showing five with tallies or counters is a representation of five in a countable form, whereas the numeral 5 is a representation that does not contain a countable quantity even though it can refer to five countable things. The concealment of certain representations (physical objects) can help children make the transition from being bound to physical representations of number to using more symbolic representations (Wright et al., 2006; van den Heuvel-Panhuizen, 2008). The idea around using settings in the teaching and learning of early number is not for learners to form a dependence on these settings, but for these settings to initially support learners' reasoning about numbers and then act as a bridge towards independent reasoning (van den Heuvel-Panhuizen, 2008).

So, using RME's terms, the representational settings used in teaching and learning early number should ideally move from being an external *model of* the situation-specific task to an internalized *model for* mathematical reasoning where the numerical relationships within the task are more in focus (Gravemeijer, 1999). Gravemeijer further argues that learners' transition from using representations as 'models of' to using them as 'models for' concurs with a shift in learners' thinking: from thinking about the context situation to a focus on mathematical relations.

2.8.2 Base-Ten Representational Settings

The following structured representational settings have been suggested for use in the classroom for enculturating base-ten thinking which leads to efficient multi-digit addition and subtraction: bundling sticks; base-ten dot materials (10-dot strips, shorter strips with 1-9 dots, and laminated cards with printed dots in either 1 or 2 rows of 5 called 'five frames' or 'ten frames', respectively); flard cards or place-value cards (attributed to Montessori and Gattegno); a numeral track; a semi-structured number line; an empty number line (ENL); the arithmetic rack (or rek-en-rek) and the hundred chart (Ellemor-Collins & Wright, 2009; Ellemor-Collins et al., 2007). I briefly introduce these resources in the following section, noting the approaches suggested for how they can be used in instruction. Children's use of fingers

in developing skill with number combinations has also been discussed within the literature base on early number learning (Jordan, Kaplan, Ramineni, & Locuniak, 2008) so I start my overview with this embodied ten-based representation before moving on to 'external' representations.

Fingers

Studies show that children's use of fingers for counting and doing simple calculations dates back to ancient times. This could be because fingers are thought to provide a natural scaffold for calculation (Jordan et al. 2008). When one considers that the decimal base of our number system is embodied in our fingers then it should come as no surprise that this natural representation of ten, that is readily available, is often used instinctively for representing and operating on numbers.

Finger use for mathematics calculation is natural and intuitive (Jordan et al., 2008, p.667).

Children's proficiency with using their fingers to calculate is a better predictor of calculation skills when compared with their linguistic ability, even though both are cognitive abilities (Asakawa & Sugimura, 2014). Despite the fact that young children's finger use has also been associated with accuracy on number combinations (Jordan and colleagues, 1992, 1994; Siegler & Shipley, 1995) we find that many learners in early grades are discouraged from using their fingers for calculation. This relates to important questions raised by Jordan et al. (2008) about when children are most likely to benefit from finger use and when might fingers get in the way of development.

My review on children's use of fingers is drawn largely from the work of Jordan and colleagues (1992, 1994, 2008). Literature indicates that children's finger use is most adaptive when they are first learning number combinations but this benefit lessens over time. Jordan et al.'s (2008) longitudinal study, that examined change over time in frequency of finger use on number combinations in relation to change over time in accuracy, found distinct differences in children from middle- and low-income backgrounds. Middle-income children used their fingers more during pre-school while low-income children almost never used their fingers – this was linked to better accuracy on number combinations for the former group. Further, during the first grade, middle-income children used their fingers less than they did in pre-school which could indicate that early use of fingers helped them shift more quickly to accurate mental calculation and retrieval. On the other hand, "low-income learners started using their

fingers more frequently in the first grade and made substantial gains in calculation accuracy” (p.663). When children worked within a higher number range by the end of the second grade (numbers > 10), a negative correlation between finger use and accuracy was found. Taken together, these results suggest that children would benefit more from using their fingers to calculate during the pre-school years, although this skill would have to be explicitly taught to children from low-income backgrounds. How children are taught to use ‘finger patterns’ – where for example 4 is shown with a single opening action rather than the sequential opening of fingers which links to counting in ones – has been associated with better performance. Further, children should be discouraged from using their fingers for calculating with numbers greater than 10 as this is a laborious activity that leads to greater inaccuracy.

Bundling Sticks

Bundling sticks are usually made with popsicle-sticks that are bound together in groups of ten using rubber bands. The advantage of using this resource is that it is inexpensive to obtain, it is not bulky and it is relatively easy to manipulate. Research conducted by Broadbent (2004) focussed on exploring ways in which teachers could improve students’ understanding of the structure of the base-ten number system using concrete materials and how students could learn to operate on numbers using this base-ten structure. Teachers involved in this action research used bundling sticks and a column representation of place value units (place value board) to play the ‘base ten game’ (Figure 2.15).

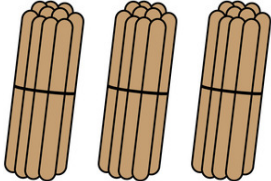
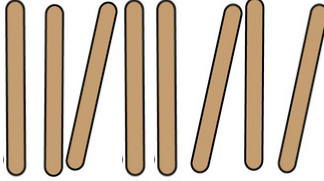
Hundreds	Tens	Units
		

Figure 2.15 Place value board showing 38 bundling sticks.

This game was played by learners who took turns to roll two die, they then added the numbers shown, collected that quantity of sticks and added it to their place value boards. The only rule of the game was that there could be no more than nine items in each column on the board: no more than nine sticks in the units (ones) column, and so forth. Once there were nine sticks in the units column the tenth stick was combined (with the 9 sticks) to make a bundle of ten and this bundle was then placed under the tens column. The same principle applied for the

tens column. The major findings from this study that speak to my particular interest (the use of bundling sticks in children's base-ten learning) were:

- concrete materials *can* significantly contribute to children's conceptual and procedural understanding of the base ten number system (emphasis mine), and
- any one set of concrete materials only highlights a certain aspect/s of the number system.

I believe that this implies two things. Firstly, that the use of concrete materials (in this case the bundling sticks and place value board) in and of themselves do not automatically guarantee enhanced learning of base-ten thinking – which links to the tenet that, 'there is no obvious meaning embedded in any setting or representation (Chandler & Kamii, 2009), teachers or more knowledgeable others have to guide children in attaching meaning to different representations or materials'. This speaks to a mediation-oriented position in relation to instruction and learning in this reported study. Secondly, for children to develop a flexible or deep understanding of the base-ten number system they must be exposed to a range of different materials (and activities) that are carefully selected based on the feature of the number system that they foreground – which is similar to the point made by Askew and Brown (2003): that teachers should use a 'few, well-research and evidence-based representations' in a structured and systematic way.

Ten-frames

The ten-frame, unlike bundling sticks, has the five- and ten-structures of number embodied within the configuration of the frame (Figure 2.16). The ten-frame can be used to highlight the idea that teen-numbers are made of ten and some more, for subitizing small numbers and for 2-digit addition and subtraction. Other uses of ten-frames in the development of early number skills include: it can be used to work on complements of ten and the commutative property of addition ($9+1$ and $1+9$); understanding teen-numbers as ten plus $\square$; addition through compensation (e.g. 9 frame and a 4 frame, children can imagine moving one of the four dots to the 9 frame thereby changing the task to 10 add 3); to explore part-part-whole relationships e.g. show six dots on a ten-frame card and ask children how many more dots are needed to make ten (Shumway, 2011; Van De Walle et al., 2013).

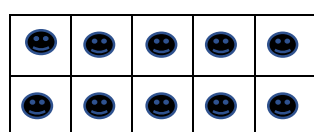


Figure 2.16 Ten Frame

One of the most valuable features of the ten-frame cards is how clearly they illustrate the distance to the next multiple of 10. Figure 2.17 shows 16 on a double ten-frame and the blank spaces on the second ten-frame card shows how many more to the next decuple.

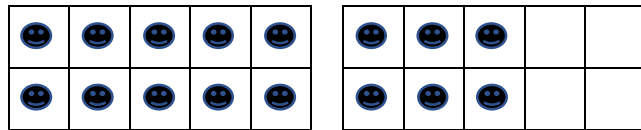


Figure 2.17 Double ten frame

Arithmetic Rack or Rek-en-Rek

The Rek-en-Rek is a Dutch arithmetic rack developed by Adrian Treffers. It has two rows of beads with ten beads on each row that are usually suspended between blocks (Figure 2.18). This setting is also called a 'bead board' (Wright et al. 2006) when the two rows of beads are strung across a piece of cardboard as in Figure 2.19 – which is a more cost-effective version.

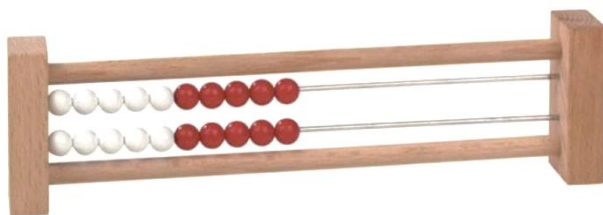


Figure 2.18 Arithmetic Rack

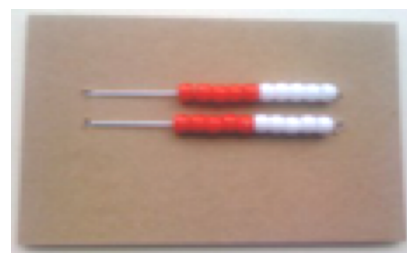


Figure 2.19 Bead board

Each row of beads is made up of five red beads and five white beads. Like the ten-frames, the arithmetic rack highlights the five- and ten-structures but it is different in that it has 20 beads in total and the beads move on the rods giving it a kinesthetic aspect. The arithmetic rack can be used in different ways: to flash images of different quantities from 1-20, to show part-part-whole relations of different numbers and for 'bridging-through-ten' for addition and subtraction (Shumway, 2011).

By using an arithmetic rack learners can 'bridge-through-ten' to solve $8+7$ as follows: start with all 20 beads on the right, move 8 beads over on the top row (this should be done in one move using the five structure of the beads, i.e. 5 white and 3 red beads), move 7 beads over

to the left on the bottom row (also in one move), then fill the 10 on the top row by moving over the 2 remaining red beads, and move 2 of the 7 white beads back to the right on the bottom row; the answer is the ten beads on left side of the top row and the 5 white beads on the left side of the bottom row, which makes 15. This process is depicted in Figure 2.20.

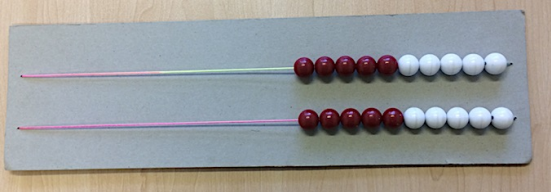
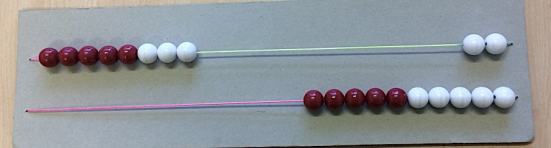
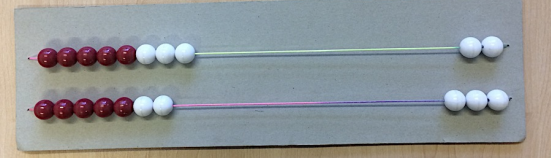
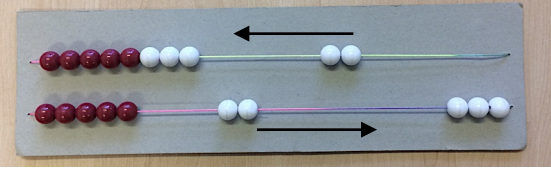
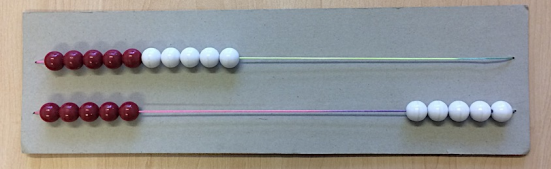
Step 1: Start with all 20 beads on the right.	
Step 2: Move 8 beads over on the top row in one move.	
Step 3: Move 7 beads over on the bottom row in one move.	
Step 4: Move the 2 remaining white beads over on the top row to make ten and move 2 white beads back on the bottom row.	
Step 5: The answer is the number of beads on the left side - 15.	

Figure 2.20

The Hundred (100) Chart

The hundred chart (or hundred square) is considered by many educationists to be an important tool in the development of base-ten understanding (Van De Walle, Karp, Bay-Williams, Wray, & Brown, 2007) and is widely used in many early grades classrooms. However, some critiques have been leveled against the use of this resource like the premature 'overstructuring' of number in the 1-100 range (Beishuizen, 1993). Teachers and learners use

the hundred chart for different activities including counting routines, recognizing 2-digit numbers, discovering number patterns and for solving additive problems. Because of the diverse patterns on a hundred chart, different children may describe one phenomenon in several ways because children's understanding of number does not develop in exactly the same way. Children's varying ideas about number patterns seen on the hundred chart should be accepted and shared with the class to build a richer understanding of numbers and our number system (Van De Walle et al., 2007).

Many different versions of the hundred chart have been used to familiarize learners with the regularity in number patterns or the 'ten-ness' of our number system (Wright et al., 2006). For example,

- a 'pocket-chart' version of the hundred chart can be used to practice a counting routine as certain numbers are hidden by placing a blank card in front of the number in the pocket while other numbers can be highlighted by placing coloured paper (behind numbers) in certain slots;
- 'blank' hundred charts are empty 10x10 grids that can be used for developing learners' fluency in understanding the tens-structure of the hundred chart. Activities like finding the position of certain numbers using the tens-structure of the chart and explaining how these were found gives the teacher insight into how well learners have constructed base-ten thinking; and
- 'crazy squares' which are cutouts of the hundred chart with certain numbers missing can be useful for encouraging base-ten thinking or incrementing and decrementing in ones and tens. The pictures below (Figures 2.21 a, b) are examples of such 'cut-outs' of the hundred chart completed by intervention learners on the 22nd of July 2014.

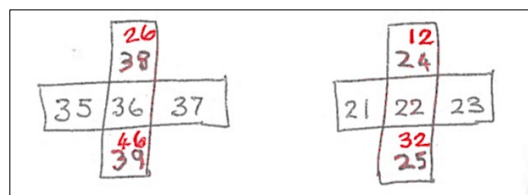


Figure 2.21a 'Crazy squares' completed by Julie

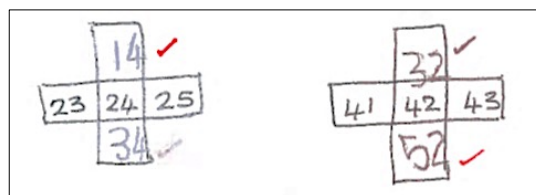


Figure 2.21b 'Crazy squares' completed by Kyle

The background to this next section is provided by the body of work produced by Beishuizen (1993) and Van De Walle et al. (2007).

How the hundred chart can be used to add and subtract

One of the more important ideas embedded in the hundred chart is that it can be seen as a stacked number track – thus showing the distance from any number to the next multiple of ten. The format of the hundred chart allows one to add ten to any number (except 91-100) by jumping one block down and similarly subtract ten from any number on the chart (except 1-10) by jumping one block up. Adding and subtracting one can be achieved by jumping one block to the right and left, respectively, except when one adds to a decuple or subtracts from the first number in the row (here you would move to the next or previous row, respectively). A suggestion from literature is for teachers to initially pose tasks with relatively small second addends where no carrying is required (like $23+12$ and $35+13$) when introducing children to adding 2-digit numbers on the hundred chart. In the same vein, teachers should then also select tasks with relatively small subtrahends (like $47-12$ and $68-14$) where no decomposition is required when introducing subtraction with 2-digit numbers on the hundred chart.

A child who uses ten as a unit might solve $23+12$ on the hundred chart by placing her finger on block 23, add 10 by moving one block down to 33 and add 2 by moving two blocks to the right to land on 35 (the answer) – this would be what literature describes as the N10 or sequence method because the first addend is kept whole while the second is added in place value units. Another way children might add by using ten as a unit is to first add the tens (place her finger on 20 and add 10 by jumping one block down) and then to add the ones mentally and move that number of blocks to the right (from 30 to 35). This is the 1010 or split tens method because both addends are first split into place value units, the 'like' place value units are added before the two interim totals are added together.

Subtracting on the hundred chart for the child who can use ten as a unit might follow the same two methods: either by using the N10 method, that is, keeping the minuend whole and taking away the subtrahend in place value chunks (with coordinated upward and backward jumps on the hundred chart); or by using the 1010 method, that is, subtracting 'like' place value units and adding the two differences to get the final answer. Some children (who understand the inverse relationship between addition and subtraction) may also use the idea of 'adding up to' on the hundred chart to solve a subtraction task. For example, a child may solve $47-12$ by starting at 12, add 30 (move three blocks down) to get to 42 and add 5 (move

five blocks to the right) to get to 47 – the answer would then be the number of blocks added, which in this case is 35.

Caveat

A caveat regarding the use of the 100 chart in early number learning highlighted by proponents of RME is that the pre-arrangement of numbers in rows of ten does not encourage children to use their own intuitive problem solving ideas (Beishuizen, 1993; Klein, Beishuizen, Treffers, 1998). Dissatisfaction with the hundred chart prompted Adrian Treffers (1991a) to search for an alternative model which would not only encourage children to use their own intuitive strategies but also allow them to enact more sophisticated strategies as their early number learning becomes progressively mathematized.

Anghileri (2006), Gravemeijer (2011) and others caution that whilst tactile and visual representations like the hundred chart or the arithmetic rack can support children's mathematical learning, use of such manipulatives is not without risks. One of the obvious pitfalls connected to using representational settings with discrete units (like the arithmetic rack, hundred chart or bead string) is that children may just count-in-ones or start to 'read off' a number relation from what they see on the setting rather than use it to scaffold arithmetical reasoning and communication of such reasoning. Here Askew's (2016) admonition regarding teachers' careful observation of how children use representations is apt, together with his suggestion that children talk about their actions on settings. There is general agreement in mathematics education literature that children must also be encouraged to develop a mental image of the representations they use to learn a particular strategy so that they do not become dependent on that level of support.

Researchers from the RME camp believe that instructional settings can initiate learners into developing informal context-dependent understandings of number which can later become more generalized into formal reasoning about numbers as learners reflect on their actions on the settings (van den Heuvel-Panhuizen, 2008). This reflection usually involves using language as a tool to share individuals' thoughts and workings with others in either a small group or class discussion. The role language and oral discourse play in children's early number learning is thus important and is overviewed next.

2.9 Oral Discourse

It is widely accepted that children's facility with language is very closely tied to their learning of calculation strategies especially in the early primary grades (van den Heuvel-Panhuizen, 2008). The work done by Webb (1991) showed that two kinds of verbal interactions affect mathematical learning in small-groups, namely, giving *elaborate explanations* and *receiving non-responsive feedback*. The former was positively related to learner achievement because explanations were content-related which helped clear-up misunderstandings amongst group members. The opposite was found to be true for the latter because receiving a one-word response as feedback (or simply the correct answer) did not help those who lacked understanding solve similar tasks (Webb, 1991). The weight of research evidence that shows the benefits of oral discourse for developing deeper conceptual understanding has propelled many researchers to advocate for children to have greater participation in classroom 'math talk' (Hufferd-Ackles et al., 2004; Perry & Docket, 2008; Dunphy, Dooley & Shiel, 2014).

In Schleppegrell's (2010) view mathematical discourse is a complex 'multi-semiotic discourse' that encompasses natural language, mathematics symbolism and visual display. The notion of 'multi-semiotic discourse' ties in with an idea expressed in the previous chapter: that language is a medium or a part of discourse (Sfard, 2006) and also echoes the work done by Cobb, Yackel and McClain (2000) and Gutiérrez, Sengupta-Irving and Diekmann (2010). So, based on Schleppegrell's idea, one can view the teacher and learners' math talk, inscriptions made on the board or on paper and actions on materials used in class as all forming part of the 'multi-semiotic' representational system. In this view, mathematical discourse is a form of communication that works across different modes of representation which requires children (and other users of mathematics) to map different elements of this system onto each other (NCTM, 2000). Words are thus employed as an intermediary to do the mapping or to make the links between different elements of this system explicit. From the perspective of this study, the power in using this view of mathematical discourse was the awareness created that each concept or idea in focus could be shared in multiple semiotic formats.

Within mathematics education literature, similar distinctions are made between 'colloquial everyday discourse' and 'scientific discourse' (Scribner, 1983, 1997) and 'everyday discourse' and 'literate mathematical discourse' (Sfard, 2001; Sfard & Cole, 2002). Sfard and Cole argue that the main difference between these two distinct discourses is the way each is mediated: everyday discourses are mediated visually (usually with concrete material) while

mathematical/scientific discourses are mediated symbolically. Sfard further argues that everyday discourses (which occur spontaneously during people's everyday lives) do not naturally morph into mathematical discourses because the latter is mediated by symbolic artefacts designed to communicate specific conceptual understandings relevant only to the field of mathematics. Sfard's (2001) argument thus underscores the need for children to receive explicit instruction in how to use language to reason mathematically as a means of helping them bridge the divide between informal and formal mathematics.

Another reason why discourse forms a crucial part of mathematics classroom interaction is because of 'conceptual compression' (Ball & Bass, 2003) linked to the symbolic notation that commonly forms part of a mathematics lesson. 'Conceptual compression' often masks the underlying reasoning or numerical relationships that exist in the production of solutions. One way in which 'compressed' mathematical ideas can be 'unpacked' so that the coherent reasoning underlying the operations can be made explicit (Ball & Bass, 2003; Adler & Davis, 2006) is through the use of 'math talk'. The nature of teachers' math talk is important because it affects children's opportunity to learn, but more importantly, the math talk learners engage in as they explain their own strategies and make comparisons to those used by others is even more important for the development of mathematical thinking. Reasoning skills are developed as children explain their solution strategies and justify their answers to their peers. Further, flexible thinking is enhanced when children talk about the differences and similarities between various strategies and different mathematical ideas (Anghileri, 2006).

Research shows that student interaction while learning mathematics can improve both their conceptual understanding and problem-solving ability – without loss of computational mastery (Boaler, 1998). This holds true for those considered 'low-attainers' and 'high-attainers' engaged in collaborative work – which goes against the commonly held belief that only low-attainers will gain anything out of paired- or group-work (Howe & Mercer, 2007). If engaging in 'math talk' is so important for developing children's number learning, then we need to know how oral discourse can be encouraged in mathematics classrooms.

2.9.1 A classroom culture that supports oral discourse

Children's acquisition of early number knowledge is highly personalised and influenced by the context in which mathematics evolves (Shumway, 2011). In every classroom, the interaction (which includes words, actions and inscriptions used) between the teacher and learners, and

between learners themselves, forms part of this context which is referred to as the 'classroom culture' or the 'classroom ethos' (Yackel & Cobb, 1996; Askew, 2016). Literature tells us that any opportunity for conversation where children are encouraged to express their thinking and ask or answer questions as they work together is an opportunity for them to develop their mathematical thinking (Anghileri, 2006). But, there are certain classroom cultures that can cause this shared interaction in a group to blossom while others can stifle it.

One way in which a teacher can create a shared classroom culture that guides the nature of interactions is by coordinating the academic tasks given with a social participation structure (Erickson, 1982). 'Revoicing' is a discourse strategy that enables the use of 'linguistic structure' (patterns of discursive interactions between members of a particular group) within a group (O'Connor & Michaels, 1993). By revoicing group members' offers, the teacher structures the pattern of social participation within the group as members talk about elements of the academic task. In my study I encouraged learners to not only share their solutions to tasks but to also share their reasoning using 'because...'. Alternatively, learners could explain to the group how they solved the task. Linguistic structure, or a shared way of talking, is not the only way in which social participation within a group can be structured – the establishment of classroom norms can also structure participation within a group.

Opinions are divided with regard to whether informal language should be used in mathematics classrooms because on the one hand we know that everyday language will not automatically evolve into mathematical language (Sfard, 2001). But, on the other hand researchers like Barwell and Kaiser (2005), Setati (2005) and Schleppegrell (2010) believe that if learners are encouraged to talk about mathematics using 'the language they bring to school' there is more chance that they will be able to develop formal mathematical discourse.

There is broad consensus in the field of primary mathematics education that children whose home language differs from that of the language of instruction must be given extra support in order for them to actively participate in 'math talk' communities. There are differing views as to what this support might look like on the ground. Cummings (2011), drawing on the work of Özçalışkan and Goldin-Meadow (2005), advocates for the increased use of gestures specifically at pre-school level while Setati (2005) and Moshkovich (2007) advocate for the use of 'code-switching' where the teacher and learners move between their home language and the LoLT so that the mathematics is made available to learn. Carey (2004) believes that

through 'bootstrapping' (where children using partial understanding of certain words as 'placeholders') ESL learners can gain greater access to the formal mathematics while Gutiérrez et al. (2010) believes that the dichotomy between registers can be lessened when teachers view everyday discourse and mathematical discourse relationally. What *is* agreed upon is that ESL learners need explicit instruction on ways to reason mathematically using more formal language as a means of bridging the divide between informal and formal understandings of mathematics.

2.9.2 Building a 'math talk' community in any classroom

Talk is an important part of classroom interaction and Askew (2016) suggests that certain kinds of classroom talk supports a collective 'coming to know'. Firstly, greater emphasis must be placed on the skill of listening, which is the 'other side of the coin' with regard to math talk. When group members fail to listen carefully to the ideas of another member, they lose the opportunity to build on that idea and in turn scale-up everybody's mathematical thinking. Secondly, 'math talk' that can be described as classroom *dialogue* enhances interaction more than talk likened to classroom *discussions*. The difference between these two formats of talk is that in the latter people usually have a predetermined position which they are not willing to let go of while the former refers to an exchange of ideas. Finally, multiple opportunities must be provided for children to engage in reasoning and explaining as this type of 'math talk' allows for an extension of their thinking. Often learners realise their own mistakes midway through their explanation of how they solved a problem, which is a more powerful form of learning ('mindful learning') than if someone else were to point out that same mistake. Taken together, these ideas suggest that creating (and maintaining) an environment where meaningful interactional discourse takes place requires hard work. Therefore I agree with Askew's (2016) view that "working cooperatively is a skill that needs to be honed over time" (p. 89) by teachers and learners.

For children to share their mathematical ideas with peers involves a measure of risk – thus a supportive classroom culture is needed for interaction to take place. In light hereof, Askew (2016) draws on different bodies of research to present the following characteristics needed for effective interaction:

- an acceptance that learning happens over time and that applying time pressure can actually reduce learning,

- listening to other's talk about maths is just as important as talking about your own mathematical ideas; this listening can be described as 'deep listening' or 'active listening' that goes further than merely keeping quiet while others speak,
- group/class members build on one another's ideas, thus a safe space is created for members to share 'unpolished' or tentative thoughts,
- meaning becomes clear at the end when children have had experiences to which explanations can be applied, and
- meaning emerges from children's activity and their sharing about the results of that activity so that a collective understanding can emerge from the bottom up (p108-110).

Whilst diagnostic teaching and the use of resources (settings) feature within their intervention programme, the developers of Mathematics Recovery place less emphasis on discourse *per se*. Thus, the attention to oral discourse marks one of the ways in which this study departs from the original Mathematics Recovery programme and from prior South African interventions based on the MR approach. In the following section I outline the MR programme and draw links between it and the literature on early number learning discussed thus far.

2.10 The Mathematics Recovery Programme

The Mathematics Recovery (MR) programme was developed as a 'systematic response to the problem of chronic failure in school mathematics' (Wright et al., 2006, p3). As an individualised intervention programme aimed at advancing low-attainers' strategies for working with early number, the Mathematics Recovery (MR) programme has enjoyed significant success in many schools, across different countries over the last number of years in terms of improved learner attainment. This programme was initially aimed at developing children's early number learning during the second year of schooling, but has evolved over time and is now also used as a classroom numeracy programme, addressing the needs of all ages and levels of ability. The 'Count Me in Too' programme implemented in New South Wales and the 'Numeracy Development Project' implemented in New Zealand are examples of national and state-wide mainstream mathematics programmes that ascribe their origins to the work of Professor Robert Wright and the MR programme.

According to (Wright, 2003, p.6) the original MR programme had the following characteristics:

- a programme of intensive number learning focused mainly on learners in the second year of schooling (6 – 7 year olds)
- intensive, individualised teaching on a daily basis for teaching cycles of 12-15 weeks
- an approach to early number learning that integrates interview-based assessment, documenting learners' current knowledge, and administering instruction

A key element in the MR programme is its framework for learning and assessment, called the Learning Framework in Number (LFIN). The LFIN was developed as a guide to teaching and assessment in early number. This comprehensive framework sets out stages and levels of children's knowledge of number and indicates the likely progression in their learning of number, which correlates with the trajectory for developing counting skills discussed earlier, (based on the work of Carpenter & Moser, 1984) and incorporates Steffe et al.'s (1988) distinction between 'counting' and 'saying a number word sequence' also noted earlier. The MR programme provides detailed assessment tasks that relate closely to the levels of progression identified in the LFIN as well as descriptions of children's likely responses to the different tasks set (Pearn, 2005). In addition, this programme provides exemplary instructional activities and explicit teaching procedures specifically targeted for children at each stage or level of progression within the framework (Wright, 2003).

How is learning described in Mathematics Recovery?

The developers of the Mathematics Recovery programme state that "it is useful to view the aim of students' learning over a series of weeks as the progressive mathematization of their thinking about arithmetic" (Wright et al., 2012, p. 15). For these authors *progressive mathematization* means the 'development of mathematical sophistication over time' (p. 15) which is evident, for example, when a learner who initially needed a concrete representation (like counters) to solve an addition problem is now able to use a more abstract representation like a number line. In later writings emanating from the MR programme (Wright et al., 2012) certain strands of early number learning are grouped together under what the authors refer to as 'domains' of knowledge:

- *Number Words and Numerals* encapsulates knowledge of basic number word sequences and numerals,
- *Structuring Numbers 1 – 20* focuses on facility with non-by-by-one mental strategies for addition and subtraction using combinations and partitions of number (1-20),

- *Conceptual Place Value* involves flexibly incrementing and decrementing numbers by ones, tens and hundreds,
- *Addition and Subtraction to 100* involves facility with mental computation for addition and subtraction in the range to 100, and beyond,
- *Multiplication and Division* focuses on learners' facility with mental computation for multiplication and division in the range to 100, and beyond, and
- *Written Computation* involves the development of written methods for the four operations, from informal jotting methods to formal algorithms.

The trajectories within each of these domains of early number learning can be understood in terms of 'progressive mathematization', which is characterized by Wright et al. (2012) using the following themes: structuring numbers; extending the range of numbers; decimalizing; unitizing; distancing the setting; notating; formalizing; and generalizing. The developers of MR state that these themes – which are the 'mathematically significant developments in arithmetic knowledge' towards which instruction in MR is aimed – elaborate on how the domains develop and interweave, providing a medium-term view of the instructional approach. Therefore, instruction that is based on principles of Mathematics Recovery emphasises 'mathematical thinking' rather than the rote learning of numerical facts and procedures (p. 15).

In this study I structured intervention around the domains of *Number Words and Numerals*, *Structuring Number 1-20* and *Conceptual Place Value* whilst – in line with curriculum goals for Grade 2 learners – my 'eye was on the mathematical horizon' (Ball, 1993) of developing learners' facility with *Addition and Subtraction to 100*. Themes of progressive mathematization like structuring numbers, extending the number range, distancing the setting, decimalizing and unitizing were also employed during intervention to develop learners' sophistication with early number concepts.

2.9.1 The Learning Framework in Number (LFIN)

The LFIN enables summary profiling of children's number knowledge which is an important first step in intervention, because this forms the basis for instruction that is targeted at the cutting edge of the child's current number knowledge (Bobis et al., 2005). Although the LFIN provides a learning trajectory along which children's learning can be tracked, not all points on the learning trajectory are discrete or necessarily passed by all children, and the extent of the

overlap is likely to vary widely across young children (Bobis et al., 2005). The LFIN is made up of eleven interrelated aspects of early number which are organised into four parts of which the Stages of Early Arithmetical Learning (SEAL) is the most important. These parts are:

- Part A – SEAL and Base-Ten Arithmetical Strategies,
- Part B – Forward Number Word Sequences, Backward Number Word Sequences and Numeral Identification,
- Part C – Structuring Number 1-20, and
- Part D – Early Multiplication and Division.

The LFIN contains 11 aspects of early number (listed below) that are described in terms of a progression of 'stages' or 'levels' (the difference between these is explained in Chapter 3, p.112) and form the basis of learners' profiling. These are described by (Wright, 2003) as:

- strategies for counting and solving simple addition and subtraction tasks
- the ability to reason in tens and ones linked to very early place value knowledge
- facility with forward number word sequences
- facility with backward number word sequences
- facility with identifying numerals
- knowledge of early multiplication and division.

The other five aspects of early number that make up the LFIN (under Part C), relate to:

- combining and partitioning of small numbers without counting,
- using five and ten as reference points in numerical reasoning,
- use of finger patterns in numerical contexts,
- relating number to spatial patterns and
- relating number to temporal sequences.

The LFIN aspects that make up Part C do not have a progression of levels but are considered as interrelated with the other aspects of the framework and are needed for developing a clear picture of the child's counting-based strategies (Pearn, 2005). Table 2.1 provides an overview of the LFIN framework (Wright et al., 2006, p. 20).

The Strategies for Early Arithmetical Learning (SEAL) is the only aspect where progression is described in terms of *stages*. Progression in every other early number aspect of the LFIN (except Part C) is described in terms of *levels*. In the Mathematics Recovery programme every

new stage is likened to a 'plateau' and is characterised by 'a qualitative advancement in knowledge' while a level is regarded as 'a point on a continuum' (Wright et al., 2006, p.190). So a stage is qualitatively different to a level, thus progressing from one stage to another involves a bigger shift in conceptual understanding than progressing from one level to another.

Children considered to be at the highest SEAL stage are those who solve 2-digit by 2-digit number tasks using 'non-count-by-one' strategies like known facts and derived facts without the support of a tens-based setting. Literature tells us that children's understanding of ten as a collection of ten ones and as a single unit of ten is crucial for solving 2-digit by 2-digit number problems (Steffe et al., 1988). So, to achieve the highest SEAL stage on the LFIN, learners need an understanding of base-ten, that is, seeing ten as a unit and as a collection. This means that learners' facility with using 'base-ten thinking', as opposed to counting-by-ones, is fundamental to their progress along the LFIN SEAL trajectory.

Table 2.1 The Learning Framework in Number (LFIN)

Part A	Part B	Part C	Part D
Stages of Early Arithmetical Learning (SEAL) Base-Ten Arithmetical Strategies	FNWSs and Number Word After BNWSs and Number Word Before Numeral identification	Structuring Number 1 – 20	Early Multiplication and Division
Early Arithmetical Strategies Stages: 0 – Emergent Counting 1 – Perceptual Counting 2 – Figurative Counting 3 – Initial Number Sequence 4 – Intermediate Number Sequence 5 – Facile Number Sequence Base-Ten Arithmetical Strategies Levels : 1 – Initial Concept of Ten 2 – Intermediate Concept of Ten 3 – Facile Concept of Ten	FNWSs and Number Word After Levels: 0 – Emergent FNWS 1 – Initial FNWS up to '10' 2 – Intermediate FNWS up to '10' 3 – Facile with FNWSs up to 'ten' 4 – Facile with FNWSs up to 'thirty' 5 – Facile with FNWSs up to 'one hundred' BNWSs and Number Word Before Levels: 0 – Emergent BNWS 1 – Initial BNWS up to 'ten' 2 – Intermediate BNWS up to 'ten' 3 – Facile with BNWSs up to 'ten' 4 – Facile with BNWSs up to 'thirty' 5 – Facile with BNWSs up to 'one hundred' Numeral Identification Levels: 0 – Emergent Numeral Identification 1 – Numerals to '10' 2 – Numerals to '20' 3 – Numerals to '100' 4 – Numerals to '1000'	Combining and Partitioning Spatial Patterns and Subitizing Temporal Sequences Finger Patterns Five-based (Quinary - based) Strategies	Levels: 1. Initial Grouping 2. Perceptual Counting in Multiples 3. Figurative Composite Grouping 4. Repeated Abstract Composite Grouping 5. Multiplication and Division as Operations

Discussion of the Stages of Early Arithmetical Learning (SEAL)

Developers of the MR programme regard the SEAL stages to be greatest determiner of learners' facility with early number. Therefore, I delineate each of these stages below by giving an overview of the counting and calculation strategies typical of a learner at each stage based on the MR programme. I also draw links between the SEAL stages and literature on early number learning reviewed earlier. Progression along the trajectory for the SEAL stages involves learners using more advanced or sophisticated counting and calculation strategies to solve additive tasks (Wright et al., 2006).

SEAL Stage 0 – The Emergent Child

The learner at this stage cannot count a visible collection of counters. This learner does not know the 'count list' (LeCorre & Carey, 2008) or how to do 'acoustic counting' (van den Heuvel-Panhuizen, 2008). The 'emergent' learner has not yet mastered the *stable order* and/or *one-to-one principles* (Gelman & Gallistel, 1978). Common errors made by a learner at this SEAL stage include: making an error in reciting the counting sequence; assigning more than one tag to an item or skipping an item that should have been counted (Beckmann, 2014).

SEAL Stage 1 – The Perceptual Child

The 'perceptual' learner can count items that are perceived (seen, heard or felt) and can solve additive tasks that involves one or two collections that are displayed. Here the *one-to-one*, *stable order* and *cardinality principles* are in place (Gelman & Gallistel, 1978). The learner cannot solve tasks involving screened (concealed) items – so at this stage learners are operating on the lowest 'rung' of Steffe et al.'s (1983) hierarchy of countable items, that is, *perceptual items*. The 'count all' strategy, that is counting from 'one' (Carpenter & Moser, 1984), is used to solve additive relations tasks – this count-by-one strategy can also be described as *calculation-by-counting* (van den Heuvel-Panhuizen, 2008).

SEAL Stage 2 – The Figurative Child

The 'figurative' learner can solve additive tasks involving one or two screened collections using the 'count all' strategy (Carpenter & Moser, 1984). From this stage onwards an additional counting principle is foregrounded: the *abstraction principle* (Gelman & Gallistel, 1978) – as the learner now knows that unseen (screened) or imagined items can also be counted. This learner is now able to count what Steffe et al. (1983) refer to as *figural items*, which are more abstract than the perceptual items counted in the previous stage. The learner cannot make

sense of 'missing addend' ($6 + \square = 9$), 'missing subtrahend' (with a difference of four or less, e.g. $10 - \square = 6$) and 'comparison' tasks. Although the 'count all' strategy is carried out on a more abstract level here, it is still a count-by-one strategy and thus involves *calculation-by-counting* (van den Heuvel-Panhuizen, 2008).

SEAL Stage 3 – The Initial Number Sequence Child

Through the 'reification' of some counting processes the learner at SEAL Stage 3 can work with numbers as objects (Sfard, 2008) and thus counting strategies enacted here are curtailed. The learner solves addition tasks using 'count on' (and later 'count-on from the larger number') which is more advanced than 'count all' (Carpenter & Moser, 1984). She also uses 'count-up-to' to solve missing addend tasks: for $6 + \square = 9$ the learner keeps 6 in her head and counts on "seven, eight, nine", the number of counts made is her answer, 3. The learner may also use 'count-down-from' to solve tasks like $15 - 3 = \square$ by saying: "fourteen, thirteen, twelve" (keeping track of the number of backward counts made) and the answer is the number word spoken on the third backward count: "twelve". Here the *order irrelevance principle* (Gelman & Gallistel, 1978) and the commutativity property are what underlays the learner's understanding of addition because the learner knows she can count from the larger number for efficiency and still get the same result. Here the learner works with increasingly abstract countable items (Steffe et al., 1983) which include *figural*, *motor* and *verbal items*. The learner will not use 'count-down-to' which is considered a more advanced strategy than 'count-down-from' as the former relies on the idea of difference while the latter (which is the predominant notion of subtraction taught to young learners) on a take away conception of subtraction. The strategies described here are 'advanced counting strategies' that are more sophisticated than count-by-one strategies used at SEAL Stage 2 but these still fall under the *calculation-by-counting* category (van den Heuvel-Panhuizen, 2008).

SEAL Stage 4 – The Intermediate Number Sequence Child

The learner at this SEAL stage can use both the difference and take away conceptions of subtraction. She can enact 'count-down-to' to solve missing subtrahend tasks like $18 - \square = 15$ by saying: "seventeen, sixteen, fifteen" – the solution is the number of counts made: 3. The learner who grasps the property of inversion may solve the same task by 'counting-up-from' the smaller to the larger number saying: "sixteen, seventeen, eighteen" – here again the answer is the number of counts made: 3. The learner at SEAL Stage 4 is also able to choose the more efficient strategy between 'count-down-from' and 'count-down-to'. She will not use

'count-down-from' to solve $18-16=\square$ because that would require 16 backward counts whereas 'count-down-to' would only require 2. The general rule for efficiency here is that when two numbers in a subtraction task are close together – whether posed as a 'removed items' task ($26-24=\square$) or as a missing subtrahend task ($26-\square=24$) – then 'count-down-to' is more efficient than 'count-down-from'. The opposite is true when the numbers in a subtraction task are not close together, e.g. $26-4=\square$ or $26-\square=4$. The only difference between learners at SEAL Stage 4 and those at SEAL Stage 3 is that the former are able to use the 'count-down-to' strategy efficiently, otherwise all else remains the same.

SEAL Stage 5 – The Facile Child

The learner at this highest SEAL stage uses a range of 'non-count-by-one' strategies. Here learners solve additive tasks using *calculation-by-structuring* (doubles, near doubles, compensation, 1010, N10 and bridging-through-ten) and *formal calculation* strategies (mental strategies that do not rely on structured settings, known facts and derived facts). Here learners are facile with the most advanced counting strategies (Carpenter & Moser, 1984), they show facility when working with *abstract* countable items like numerical symbols (Steffe et al., 1983) and they have mastered all the counting principles outlined by Gelman and Gallistel (1978).

2.9.2 Representational settings used in MR

In MR instruction, structured materials such as bundling sticks and ten-frames are used as settings for posing tasks. A well-designed setting is considered to be one that is amenable to students' initial informal approaches while at the same time, provides a context for learners to reflect on and further organize their thinking. MR makes use of different colours to further structure their materials (like using counters in two different colours when representing a missing addend task); as well as flashing (showing the materials for 1-2 seconds) and screening (covering) as part of the progression in 'distancing the setting' – challenging students to increasingly rely on visualization and mental strategies (Gravemeijer, 1997; Ellemor-Collins et al., 2007).

Developers of the MR programme view structured settings as mechanisms that support learners' progression along the trajectory in early number learning. Mathematics Recovery makes use of structured representations of number and thus promotes the use of ten-frames

and the arithmetic rack in their instructional tasks. These structured settings enculturate children into using base-five and base-ten thinking because both 'five-' and 'ten-ness' are embodied in the structure of these representations. Analogous to points raised previously in the literature reviewed, settings used for initial context-dependent thinking in MR tasks act as a bridge to more independent thinking – thus the *model of* later becomes a *model for* learners' mathematical reasoning which links to the view held by RME (Gravemeijer, 1999; Treffers, 1991b; Beishuizen, 1993) and the Vygotskian notion that actions and talk that first appear on the interpersonal plane later becomes visible on the intrapersonal plane.

The Mathematics Recovery programme distinguishes its inquiry-based use of materials as settings for posing and solving problems from other uses of classroom materials. We know that teachers use materials (or teaching and learning resources) for different purposes: to demonstrate a way of thinking about a task and/or to teach students particular procedures with the materials. However, developers of the MR programme believe that training learners to execute procedures using materials does not bring them any closer to sophisticated mental strategies. These kinds of instructional approaches assume a correspondence between the materials and the mathematical thinking, with an expectation that learners will see mathematical relationships based on the relationships in the materials. However, the developers of the MR programme argue that learners have rarely constructed such a correspondence. Children are unlikely to see the mathematical relationships in the materials; rather, they will see the materials from the perspective of their own informal reasoning (Wright, 1992; Cobb, 1991; Gravemeijer, 1991).

One way in which the MR programme encourages learners to move from context-dependent reasoning to independent reasoning is through gradual concealment of the setting being used. Developers of Mathematics Recovery call this process 'distancing the setting' – where a particular setting is shown and then screened after learners have used it successfully for a while. The use of a screen allows for learners to easily check whether their response is correct or not and also allows for the setting to be made available again should the learner get stuck and need to refer to the setting. More importantly, having the setting there and screening it helps learners to more easily construct a mental picture of the representation used previously. This gradual 'fading' of the setting acts as a bridge between children physically acting on the setting to find a solution and then 'reifying' that activity, using it as a way to reason mathematically in solving similar tasks (Wright et al., 2006).

2.9.3 Key features of the Mathematics Recovery Programme

The key features of the MR programme according to (Wright et al., 2006) are *early intervention, assessment, teaching* and *professional development*. The first three aspects figure within my study and are discussed briefly. Broader professional development was not included in my study and thus this section is not overviewed here.

Early Intervention

Although intervention can take place at any time, proponents of the MR programme believe that this should take place very early in the child's schooling career given the wide range of abilities among children starting school and the increase in this gap as they progress through primary school – a research-based idea noted earlier. While Mathematics Recovery advocates for early intervention, the programme is not started in the child's first year of school because children in the reception year need time to adjust to formal schooling. Thus MR is usually implemented with children in Grade 1 (6-7 year olds).

Assessment

Assessment in Mathematics Recovery involves a distinctive approach to assessing young children's arithmetical knowledge and computation strategies using specific diagnostic tools that do not involve traditional paper-and-pencil tests. MR assessment uses a one-on-one interview, in which the learner is presented with groups of tasks – each group relating to a particular aspect of early number learning. The interviewer follows an assessment schedule (although this is a set procedure, a flexible approach is advised) with the aim of determining the learner's most advanced numerical knowledge and strategies. Assessment within the MR programme produces an individual profile of each learner specifying the child's stage and levels in terms of the LFIN. This profile feeds into specifically developed instructional activities which can be used for one-on-one, small group or class interventions. This type of assessment is useful for documenting the child's progress over time.

The MR programme provides a total of six assessment interview schedules as well as guidance for the administration and analysis thereof. The following MR assessments are administered individually to learners and are routinely videotaped:

- Assessments 1.1 and 1.2 – address the relative sophistication of the child's strategies for adding and subtracting; facility with number words and number word sequences both forward and backward; and identifying, recognizing and sequencing numerals.
- Assessments 2.1 and 2.2 – address the child's knowledge of the tens and ones structure of the numeration system; and the relative sophistication and range of strategies other than counting-by-ones to solve addition and subtraction tasks.

Teaching

In the MR programme assessment and teaching are fully integrated – thus assessment informs teaching, and through the teaching process additional assessment information is gathered about the child by way of careful observation. During MR teaching sessions, which are inquiry-based, the child is presented with tasks which are carefully chosen to provoke the child's numerical thinking. A learner's SEAL Stage (and to a lesser extent her broader LFIN profile) is what forms the key diagnostic determinants of tasks to be set in intervention lessons. The developers of Mathematics Recovery do not consider it important to focus teaching for learners at Seal Stage 3 – the SEAL Stage at which 7 of the 10 intervention learners were placed at after their pre-test interviews – on the development of the counting-down-to strategy (which is typical of SEAL Stage 4). Wright and colleagues believe that when a learner can use counting-on and counting-back strategies, then teaching should focus on advancing such a learner to the Facile Stage (SEAL Stage 5). Planning for MR teaching involves the use of the Instructional Framework for Early Number (IFEN) which is informed by and closely linked to the LFIN. The instructional procedures contained in the IFEN include explicit descriptions of instructional settings (which materials and instructional resources to use), notes on purpose, teaching method, activities, and typical learners' responses to different tasks. This provides guidance for the teacher and ensures that teaching is at the cutting edge of the child's current knowledge and strategies.

I next provide a sample of the tasks used in MR assessment schedules to illustrate the nature of early number skills assessed during pre- and post-test interviews (full copies of assessment schedules are provided in Appendix A). The tasks I present here show how a range of different early number skills are assessed in the MR programme. As noted previously, my research focus was on additive relations thus assessment schedules 3.1 and 3.2 (focused on multiplicative reasoning) were not used in this study.

2.9.5 Examples of task items used in MR Assessment Schedules

All assessment tasks presented here – one from each Mathematics Recovery assessment schedule used in my study – are taken from Wright et al., (2006).

Assessment 1.1 Q8 – Additive Tasks (p. 161,162)

Introductory task: screened, use counters of two colours

$$\boxed{3} + \boxed{1}$$

Interviewer's actions	Interviewer's words
- Flash and screen 3 red counters - Flash and screen 1 yellow counter - Point to both screened collections	<i>There are three red counters under here,</i> <i>and one yellow counter under here.</i> <i>How many counters are there altogether?</i>

If the learner solves the introductory task correctly by counting-on from three or with a known fact (no counting) then the interviewer goes on to pose similar tasks (both addends screened) in a higher number range: Q8a 5+4 and 9+6. Here the first task falls within the finger range and the second out of the finger range. If the learner uses a count-all strategy to answer the introductory task then the interviewer can pose less challenging tasks, where only the first addend is screened: Q8b 5+2, 7+3, 9+4. If the tasks in question 8b are still too difficult then tasks from question 8c are posed – these are the same task items as for question 8b except here both addends are unscreened.

Assessment 1.2 Q1 – Tens and Ones Tasks (p. 166)

These tasks test the learner's facility with incrementing by ten on the decuple using a structured/base-ten setting.

a) Counting by tens with 'strips' – informal familiarization with material	
i.	Put down a ten strip. Ask: <i>How many do we have?</i> If the child says 'one', ask: <i>How many dots are there?</i>
ii.	Put down one strip at a time to 8 strips. Ask: <i>How many altogether?</i> 10 20 30 40 50 60 70 80
iii.	Pick up all the strips. Ask: <i>How many dots do we have?</i> and <i>How many strips are there?</i>

Assessment 2.1 Q3 – Finger Patterns 6 to 10 (p. 170)

Tasks from this question are purely oral and are aimed at determining whether learners can reason more sophisticatedly about finger patterns, that is, not count from one. What is of interest is whether the learner can raise the required number of fingers simultaneously, or raise five fingers on one hand simultaneously and 'count-on' to raise the rest or if the child raises all ten fingers and then closes them sequentially until she has the correct number left.

- a) *Show me 6 on your fingers.*
- b) *Show me 6 a different way.*
- c) *Show me 9 on your fingers.*
- d) *Show me 10 on your fingers.*
- e) *Show me 8 on your fingers.*
- f) *Show me 8 a different way.*

Assessment 2.2 Q2 – Near Doubles (p. 173)

These tasks, aimed at getting children to use their knowledge of doubles, are posed as number sentences.

- | | | |
|------------|------------|------------|
| a) $5 + 6$ | b) $4 + 3$ | c) $7 + 6$ |
| d) $9 + 8$ | e) $8 + 7$ | f) $3 + 2$ |

- Show the learner the number task. Ask: *Can you work this problem out?*
- If you were unable to discern the learner's strategy, ask: *How did you do that?*
- If the learner gives a correct response, check if she has another way to work it out.

Next I delineate my reasons for wanting to explore the efficacy of using Mathematics Recovery principles in developing the early number learning of a sample of South African Grade 2 learners by using a 'wide-angle lens'. With this broad view, I wish to simultaneously take into account the contextual elements on the ground, key literature-based ideas emanating from the field of early number learning and characteristics of the MR programme itself and show how all of these factors have collectively provided the impetus for my research focus.

2.9 6 Why Mathematics Recovery was considered a viable option for developing early number learning in a South African school

As discussed previously, pervasive counting-from-one has been widely described as one of the main reasons for many South African learners' poor number skills development – which is ultimately linked to their poor grasp of 'base-ten thinking' and/or other ways of structuring number (Schollar, 2008; Hoadley, 2008; Ensor et al., 2009). Children who do not know how to structure number using base-five, base-ten, doubles, and so forth have difficulty solving 2-digit by 2-digit additive tasks with efficiency. In the same vein, children who do not know how to add ten to any number as a recalled fact is not considered ready for solving 2-digit by 2-digit additive tasks. These issues can be problematic for the South African Grade 2 learner who is expected to work in the 1-100 number range by the end of the year, which includes working with 2-digit by 2-digit additive tasks. National studies have shown that many Foundation Phase teachers favour un-structured, concrete resources over structured resources during numeracy work and if structured resources are used, then these are used in ways that negate the tens-structure embodied therein. The ubiquitous use of counting strategies and concrete, unstructured resources precludes learners from developing more sophisticated and efficient mental strategies, which literature shows is vital for the development of early number skills. Research also shows that the use of structured materials when learning early number skills can act as a bridge to move children from calculation-by-counting to calculation-by-structuring. All these contextual factors that contribute to the skewed mathematics performance of South African learners' compared to their regional and international contemporaries feature within the early grades. This gap in achievement is shown to grow wider over time.

Much of what is advocated for in literature regarding the development of children's early number learning has been accounted for (and in some cases, extended) within the design of the Mathematics Recovery programme. Mathematics Recovery promotes early intervention which is based on individual, diagnostic assessment of interrelated early number skills which informs teaching in a reflexive way. The Learning Framework in Number (LFIN) provides a research-based trajectory of children's early number learning and details the number skills displayed by children at different stages and levels on the framework. Assessment tasks focused on different early number aspects, and instructions on how to interpret learner responses to these tasks form part of this programme which ultimately allows one to develop a summary profile of the individual child's early number skills. Further, research-based

instructional activities within the MR programme (differentiated for learners at each stage along the trajectory) promote the 'hybrid' orientation to early number learning, which has a dual focus on developing advanced counting strategies (like *count-down-to* to solve $34 - \square = 28$ and *count-up-from* for $48 - 46 = \square$) as well as strategies based on the structuring of number (like the N10 and 1010 strategies). To this end, the MR programme promotes the use of structured representational settings in instructional activities while at the same time encouraging the gradual 'fading' of settings which can help learners enact mental calculations.

I have also drawn attention to the dearth of learner-talk (and classroom discussion in general) that takes place during mathematics lessons which could be linked to various factors such as: teacher-led pedagogy, multi-lingual classrooms and the 'communal' format of teacher-learner interaction. Taken together these South African pedagogic practices overlook the benefits of interaction (including oral discourse) which research has shown to have positive effects on early number learning across the attainment range. Even though Mathematics Recovery does not focus on oral discourse *per se*, I chose to include this aspect in my study because of the research-based evidence that shows the benefits thereof for learning and the lack of 'math talk' that South African learners engage in. The underpinning theoretical assumptions of the MR programme are addressed in Chapter Three.

Parts of the Mathematics Recovery programme have been adapted and used in five MEd studies and one Doctoral study in South Africa. With the exception of one MEd study (Young, 2016) that was based on empirical data collected in Zambia, the rest of these studies were based solely on South African data (Weitz, 2012; Ndongeni, 2013; Mofu, 2013; Wasserman, 2015; Stott, 2015). Thus the body of research based on adaptations of the MR programme in South (and Southern) Africa is growing.

Weitz's (2012) study investigated the use of the LFIN as a tool for assessing the number knowledge of a sample of Grade 2 learners. Weitz found that the LFIN assessments were a better predictor of Grade 2 learners' later mathematics achievement than the national ANA assessments. The two MEd studies completed in 2013 both focused on the development of learners' multiplicative reasoning with a sample of Grade 4 learners. Ndongeni's (2013) study paired the LFIN framework with Kilpatrick, Swafford and Findell's (2011) five strands of mathematical proficiency while Mofu's (2013) investigation was focused on progression of multiplicative reasoning in a group context. Wasserman (2015) explored the effectiveness of

teaching and assessing a group of Grade 4 learners (with a special focus on SEAL and CPV) while Stott's (2015) study investigated the usefulness of an adapted MR programme on Grade 3 learners' progression in number learning in the context of afterschool maths clubs.

My study differs from other South African-based studies that have used parts of the Mathematics Recovery programme in that my study is the only empirical study focused on the development a sample of Grade 2 learners' early number learning based on a withdrawal method of intervention. My study did not use selected items from MR assessment schedules but rather administered individual interviews using all items on the schedules, which provided a broad, rich data set comprising of 20 pre- and post-test interviews and more than 70 intervention lessons across the two groups and two individuals based on Mathematics Recovery principles. My interest in investigating the usefulness of MR for grouped intervention is similar to other studies (Wasserman, Stott) who also took the scale of the problem in the South African context into consideration. The results and recommendations emanating from my study are unique in that it speaks directly to the withdrawal intervention model used in full-service schools⁶ in South Africa that provide different forms of development while it is also of value to class teachers for developing early number skills use during numeracy lessons. Another important aspect of my study was the different layers of comparative analyses that were included, namely, a comparison between learning gains made by an intervention group and a 'matched' control group, and well as comparison between gains made by learners who participated in a grouped format of intervention and individual intervention.

2.11 Concluding Remarks

There is broad consensus in national and international literature on many aspects of early number learning like:

- the importance thereof for further mathematics learning,
- the need for and benefits of early intervention in number learning,

⁶ "Full-service/inclusive schools ... are first and foremost mainstream education institutions that provide quality education to all learners by supplying the full range of learning needs in an equitable manner" (DBE, 2009, p.7)

- the role of advanced counting, structuring number (especially using base-ten) and knowledge of informal place value in developing learners' additive relations strategies (even though there is disagreement on how much emphasis to place on each and at which junctures this should be done in children's learning journeys),
- the role of representational settings and the importance of building flexibility in learners' use of different settings, and
- the importance of oral discourse in developing early number skills.

Other than the added focus on learners' oral discourse during interaction and the push for representational flexibility (which included prominent use of the hundred chart during intervention) all other tenets of the Mathematics Recovery programme were followed in my attempt at developing the early number learning of a sample of Grade 2 learners in a South African primary school. In my study, various factors (like my social constructivist epistemology) caused me to veer away from the original theoretical underpinnings of the MR programme which is discussed in the next chapter.

Chapter Three – Theoretical and Analytical Frameworks

3.1 Introduction to the chapter

Following on from the literature reviewed, this chapter takes a closer look at the theoretical and analytic concepts that informed and supported my research into the development of children's early number learning. The interpretive framework within social constructivist theory that underpins my study was developed in the context of investigations into young children's number learning (Cobb & Bauersfeld, 1995). This theoretical frame used in my study (which draws on cognitive and sociological perspectives) differs from the radical constructivist underpinnings of the original Mathematics Recovery programme (Steffe et al., 1983; Wright et al., 2006) and these differences are explained in this chapter. I also detail my view of learning – as a 'continuous cyclic process' of gradual appropriation that takes place through joint activity and shared meaning-making (Newman et al., 1989; Rogoff, 1990) – and explain why I believe that my epistemological view and the social constructivist theory were a good fit for the purpose of my study. I draw on both cognitive and sociological perspectives in my analysis and explain the aim of these complementary analytical foci and how each was carried out. In this chapter I also delineate the constructs used in my analytical framework – some which were developed inductively from the empirical data in a 'grounded' manner and others that were drawn from the literature base on early number learning, (a large chunk of which was the body of work produced by Wright and colleagues).

3.2 Theoretical Framework

The theoretical framework guiding this study is based on the work of (Cobb & Yackel, 1996) who have developed an interpretive framework within social constructivism which they refer to as an 'Emergent Approach'. In this approach the psychological constructivist analysis of an individual learner's activity is coordinated with an interactionist analysis of classroom interaction and discourse. In a collaborative classroom-based researched study, American researchers (Cobb, Yackel and Wood) who came from a psychological background and German researchers (Bauersfeld, Krummheuer and Voigt) who valued the sociological perspective, found that their seemingly polarised perspectives each only told 'half of a good

story' (Cobb & Bauersfeld, 1995). In a pragmatic move, these researchers decided that their research needed a combined approach that took each individual learner's mathematical interpretations seriously while also seeing their activity as necessarily socially situated. Proponents of emergent theory believe that it augments the psychological constructivist perspective by making it possible to locate analyses of individual learners' constructive activities in social context, while it also strengthens the sociocultural perspective in that characterisations of individual learners' activities, the classroom community, and the broader community are accounted for (Cobb & Bauersfeld, 1995).

According to Cobb and Bauersfeld (1995), the coordination of interactionism and psychological constructivism is the primary defining characteristic of this version of social constructivism which they refer to as the Emergent Approach. Thus, von Glasersfeld's (1987) characterisation of learners as active creators of their mathematical learning and the interactionists' view that learning involves 'the negotiation of meaning in social interaction' (Bauersfeld, Krummheuer, & Voigt, 1988, p. 39) are both drawn upon in this approach. Cobb and Bauersfeld (1995) contend that the Emergent Approach does not merely bolt a social component onto an otherwise unchanged psychological perspective, but rather views the relation between the social and psychological aspects as 'reflexive'. This means they acknowledge that the activities in which children engage in as they construct meaning individually in the mathematics classroom contributes to the microculture of that learning space. They also believe that the microculture established in the classroom has an influence on learners' individual and collective constructions of meaning.

3.2.1 Theoretical underpinnings of Mathematics Recovery

The theoretical underpinnings of the MR programme flow largely from the work of Leslie Steffe, who focused much of his research on developing models of early number learning that could explain and predict children's learning and development. Around the mid-1970s Steffe developed and used a research methodology called 'constructivist teaching experiments' which involved the detailed observation of a small number of children's learning in the context of extensive task-based teaching (Pearn, 2005).

Steffe's particular interest in this approach was the strategies that learners used in situations that were problematic to them, and how those strategies developed over the course of an extended teaching cycle. Changes in learners' strategies were observed mainly in teaching

sessions, but also across pre- and post- interview-based assessments. Steffe's research and the MR programme are both grounded in von Glasersfeld's radical constructivist theory, which is a theory about learning, or how we come to know, and not an approach to teaching *per se* (Steffe et al., 1983). Mathematics Recovery went on to use Steffe's findings to explore the kinds of instruction that would support early number learning.

Thus the theoretical framework guiding this study differs from the framework upon which the MR programme is based, that is, radical constructivism (Steffe et al., 1983; von Glasersfeld, 1987), in that learning in my study is characterised as an individual constructive process that occurs while participating in and contributing to the practices of the local community (Cobb & Yackel, 1996). The differences in the theoretical framework underpinning the use of Mathematics Recovery in my study led me to adapt certain aspects when it came to implementing the programme. The main aspect that stayed unchanged was that all learners were interviewed individually using MR assessment schedules prior to and straight after the intervention period that spanned about 18 lessons. Further, the LFIN was used to construct a summary profile for individual learners prior to and after intervention. The format and content of the intervention lessons were taken from the MR programme, together with the suggested representations (or settings) to be used for each activity. The biggest change was that I used MR principles to develop the early number learning of two groups of learners as I was interested in how various aspects of the group's interaction (their oral discourse and actions on representations) could be used to provide insight into their learning journeys. This is a substantial change as elements of group interaction (like oral discourse) were not used as markers of appropriation in the original design of the MR programme. Alongside this I also used MR principles for individual development of two Grade 2 learners to build in possibilities for comparing any learning gains made by learners who received grouped development and those who received one-on-one development. Other aspects of how the Mathematics Recovery programme is usually administered – like video recording learner interviews – remained the same.

3.3 Epistemological View

Based on the social constructivist view of learning, I believe that learning mathematics is both an individual activity as well as a social activity. Learning opportunities arise for individual children as they participate in classroom social interactions (Cobb, Yackel & Wood, 1992). In

my view children learn early number concepts in the classroom by engaging with meaningful tasks and related subsequent activities while acting on physical materials and sharing solution ideas with peers and teachers. From this perspective, learning can be described as 'appropriation' that takes place through joint activity and shared meaning-making (Rogoff, 1990). Appropriation does not simply involve the observation and imitation of others' behaviour (actions or words) but involves taking what was produced by someone else during collective activity and using it oneself in subsequent activities in a way that makes sense to you – which usually includes transformation of what was appropriated (Rogoff, 1990). I believe that before such 'transformation' takes place, language helps children to 'bootstrap' their understanding of number – where children's partial understandings of number words/concepts which serve as 'place-holders' are later filled with richer meanings (Carey, 2004). The process of appropriation also includes the notion of the expert giving the novice feedback on her actions based on the former's expert understanding of the content (Newman et al., 1989). This feedback given to the novice is usually accompanied by a follow-up task which is also based on the expert's knowledge and experience and is intended to improve the process of appropriation.

My view of learning draws on evidence from many studies involving the use of learning support materials (like ten-structured settings) which show that children do not automatically see 'concepts' in the materials and that their construction of meaning is a prolonged process that is mediated by guided activities and discussion (Moshkovich, 2004; Fuson et al., 1997). The idea of gradual appropriation can be linked to Wright et al.'s notion of 'distancing the setting' where the setting (or representation) originally used to introduce learners to a particular strategy is gradually removed. This 'fading' takes place as follows:

- the setting is first made visible to the child so that the child can enact the strategy (physically or mentally) on the setting
- next the setting is flashed and screened so that the child is now forced to use mental actions to solve a similar task – the benefit of this step is that the setting can be unscreened if the child gets stuck or to check whether the child's solution is correct or not
- lastly, the setting is no longer presented (although the teacher may refer to it) and the mental strategy the child previously performed with support is now carried out without any support.

Further, my view of learning as a 'continuous cyclic process' draws on the Vygotskian idea that an object of children's learning can later become a tool for further learning. So for example, the concept of incrementing and decrementing by ten on and off the decuple could initially be the *object of* children's number learning and later, incrementing and decrementing by ten could be the *tool for* learning, which children use to add and subtract multi-digit numbers. This idea is echoed in Sfard's (1991, 2008) notion that mathematics builds up through discursive layers that depend on learners using reified discursive objects in new operational processes.

3.4 Analytical Framework

Some researchers' analyses of children's mathematical learning are based solely on a sociological perspective – which stems from the belief that mathematical activity is a purely social or discursive practice (Walkerdine, 1988; Solomon & O'Neill, 1998). Such perspectives discount the notion that cognitive models of learning are useful in characterizing children's mathematical learning and are thus equally important as they provide rich insight into such learning (Cobb, Yackel, Wood & McNeal, 1992). Other researchers have placed great importance on the cognitive analyses of children's mathematical learning and to this end have developed cognitive models of learning that delineate children's construction of increasingly sophisticated number concepts (Steffe, et al., 1988) while the social context in which such learning develops is largely ignored. Flowing from the broad theory that underpins this study, the Emergent Approach (Cobb & Yackel, 1996), I draw on both cognitive and sociological perspectives in my analysis of participants' early number learning.

Cobb notes findings from several previous studies which indicate that collaborative learning in small-groups can give rise to individual learning opportunities that would not necessarily have been made available to learners if the format of interaction was whole-class teaching (Barnes & Todd, 1977; Davidson, 1990; Smith & Confrey, 1991; Yackel, Cobb & Wood, 1991 in Cobb, 1995). I agree with Cobb and colleagues (1992) that cognitive and sociological perspectives of learning are complementary in that each can shed light on the other. Findings from this literature, which suggests the usefulness of small group settings, leads to design implications that were taken up in my study through the use of grouped intervention for eight participants. From the sociological perspective, the microculture of a mathematics classroom has the ability to shape certain aspects of individual and collective learning – thus the

sociological analyses, which focusses on group members' talk about their solution strategies and actions on manipulative materials used in the learning environment, are equally important even if the primary focus is on children's individual cognition (Cobb & Bauersfeld, 1995; Cobb & Yackel, 1996). There is thus also a second implication that could be drawn from this literature-base with regard to viable tools for analysing early number learning which have also been taken up in my study through the inclusion of learners' math talk and actions on settings as additional markers of appropriation.

From the cognitive perspective I used Wright et al.'s (2006) LFIN framework to analyse any shifts in learners' strategies for solving early number problems – focussing on their *individual construction* of early number concepts. Based on the sociological perspective, I focused my interest on elements of interaction during intervention that shed light on participants' 'base-ten thinking' as this aspect was the hinge that allowed participants' access to performing calculations beyond twenty – which is the goal for learners in Grade 2. My interactionist analysis thus focussed on children's solution *strategies* which they enacted within certain *representational settings* and the nature of their *math talk* (Anghileri, Heinze et al, Ensor et al). Using the concept of 'appropriation' to describe learning, I believed that learners' talk about their strategies and their actions on representational settings would provide evidence of their facility (or lack thereof) with 'base-ten thinking'. Thus the interactionist-analyses gave me a more nuanced picture of the individual cognition foregrounded by the LFIN. Figure 3.1 (on the next page) outlines these complementary foci.

In the sections that follow I explain the aim of both types of analyses used in my study in greater detail and how these were carried out.

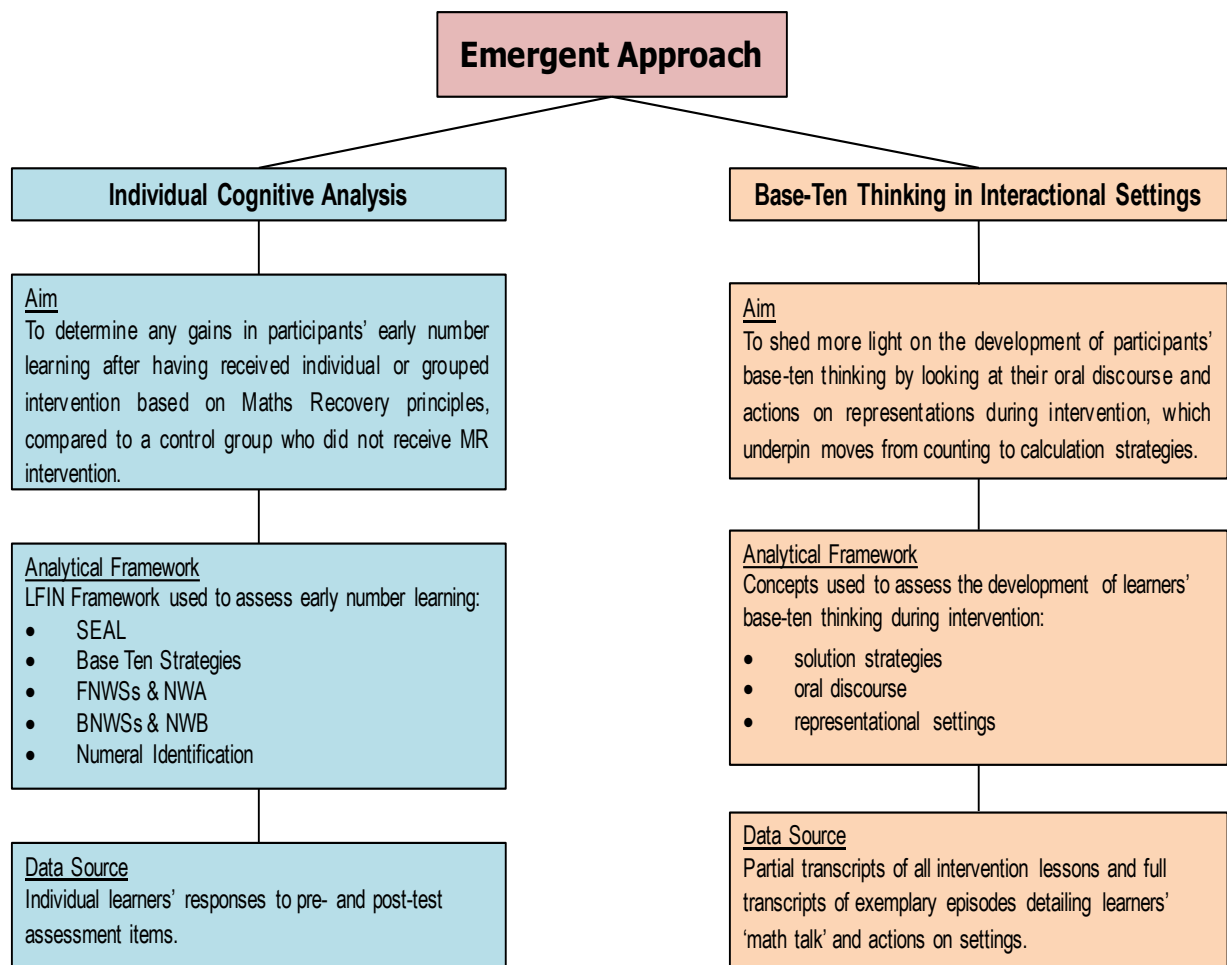


Figure 3.1

3.4.1 Individual Cognitive Analysis

The aim of doing individual cognitive analyses of participants' early number knowledge before and just after intervention was to determine if any shifts were made during the intervention period, and if so, what the nature of these gains were in relation to the LFIN framework. By analysing the control groups' pre- and post-test results I was also able to compare the gains made by learners who received intervention based on Mathematics Recovery principles and their 'matched' peers who did not receive such intervention. These analyses also provided me with a view into the nature of learning gains made by 'singletons' (learners who received one-on-one intervention) compared to gains made by learners who received grouped intervention.

To this end all participants' (intervention and control groups) pre- and post-test interviews were analysed using Wright et al.'s (2006) Learning Framework in Number (LFIN). This analysis provided me with a way of answering the first three research questions:

1. What was the profile of the sample of Grade 2 learners' early number understanding prior to and immediately after receiving intervention based on Mathematics Recovery principles?
2. Following the intervention pathway for learners who made up the intervention group, what learning gains (if any) were produced at the end of the intervention, and how does this compare to learning gains made by the control group?
3. What were the differences and similarities in the nature of learning gains achieved by 'singletons' compared to gains made by learners who received grouped intervention?

3.4.1 a) Learner profiles

As stated previously, five of the six aspects of the LFIN that describe learner progression (using 'stages' and 'levels') across different aspects of early number learning were used for individual learner profiling in my study. These were:

- *Stages of Early Arithmetical Learning (SEAL)* which deals with learners' counting and calculation strategies used for solving simple addition and subtraction tasks,
- *Base Ten Arithmetical Strategies* which speaks to the learner's ability to reason in tens and ones linked to knowledge of structuring number using base-ten,
- *Forward Number Word Sequences (FNWSs)* which deals with the learner's facility with saying forward number word sequences in increasingly higher number ranges,
- *Backward Number Word Sequences (BNWSs)* which is similar to the FNWSs except that here the learner's facility with saying backward number word sequences is in focus, and
- *Numerical Identification* which deals with learners' facility with identifying 1-, 2- and 3-digit numerals.

Table 3.1 Learner LFIN profile	Stages/Levels		Highest stage or level	Highest possible score
LFIN Aspects	Pre-Test	Post-Test		
1. Stages of Early Arithmetical Learning	stage ^{x2}	stage ^{x2}	5	5 ^{x2}
2. Base Ten Arithmetical Strategies	level↓	level↑	3	3
3. Forward Number Word Sequences and Number Word After	level	level	5	5
4. Backward Number Word Sequences and Number Word Before	level	level	5	5
5. Numerical Identification	level	level	4	4
Totals				
Gains				27

In the LFIN profile used in my study (Table 3.1) the relevant LFIN aspects are placed in the extreme left column with the heading: LFIN Aspects. In the two columns immediately to the right I noted the stage or level the learner achieved for each aspect in the pre- and post-test interviews based on the indicators for each LFIN aspect specified by the developers of MR (see Table 3.2 p.114). These stages and levels also acted as scores that were aggregated in the second last row of the profile labelled 'Totals'. Whilst levels attained on the LFIN were translated to scores as is, the SEAL stage achieved was doubled for use as a score (hence the superscript ^{x2}) because of the qualitative difference between stages and levels in the LFIN framework discussed earlier (Wright et al., 2006). I also made use of downward and upward arrows (seen here alongside levels for Base Ten Arithmetical Strategies) to indicate 'high' or 'low' attainment when learners' performances placed them in-between stages or levels. When a learner fulfilled the criteria for being placed at a certain stage or level on the LFIN framework in a fragile⁷ (or weak) manner then I considered her to be at the 'low' end of that LFIN stage or level. However, when the learner exceeded the requirements of a particular stage or level on the LFIN by sporadically exhibiting the criteria for the higher stage or level, then she was placed at the 'high' end of that particular stage or level.

The second last column labelled 'Highest stage or level' indicates the highest stage or level a learner can attain on each LFIN aspect which is helpful for a quick look at where the learner has been placed on that particular trajectory. In Table 3.1 I have included a final column 'Highest possible score' (not included in the learner profiles in Chapter 5) to show how scores relate to the stage or level achieved on the LFIN and the highest total score that could be achieved. In order to develop these profiles I first watched recorded interviews and noted individual learner's answers to each question posed (what they said, actions or gestures made,

⁷ 'Fragile' here refers to the opposite of the robust appropriation of a number skill.

and time taken to answer) on a recording sheet I produced: Record of Response per Task Item (RRTI). RRTI's for each assessment schedule are included as Appendix B. Once all RRTI's were completed for each participant I used indicators in Table 3.2, specified by Wright et al. (2006, p. 21–24), to allocate a stage or level to every individual learner for the five LFIN aspects that made up the profile. When needed, I also made use of another tool that I developed from Wright et al.'s work called Maths Recovery Assessment Purposes (MRAP) to aid the placement of learners at particular stages or levels on the LFIN. This tool sets out the items for each assessment schedule as in the RRTI but here the purpose of each task according to Wright et al. (2006) is included. MRAPs for all assessment schedules are included as Appendix C.

Once individual profiles were developed based on participants' pre- and post-test performances I grouped these profiles according to intervention and control groups for ease of comparison; and within the intervention group I further split learner profiles into 'singletons' and groups. I went through a few iterations of analysis with a different focus each time to answer the different research questions that would help me answer the overarching research question guiding my study. I used all 20 learners' pre- and post-test profiles to determine if any shifts were made. I then looked across the nature of gains made by both intervention and control groups to ascertain if any differences existed between the gains made by each group. I was also interested to see whether there was any difference between gains made by 'singletons' and learners who received grouped intervention. In Chapter 5 I discuss intervention learners' individual early number learning journeys by unpacking their pre- and post-test performances on each of the LFIN aspects that made up their profiles before giving an overview of pre- and post-test results across all participants for comparison across intervention and control groups.

Table 3.2 LFIN Indicators

(based on Wright et al., 2006)

SEAL Stages		Score
Stage 0	Learner cannot count visible items	0
Stage 1	Learner can only count items that are seen, heard or felt	1
Stage 2	Learner can count items in a screened collection using 'count from one'	2
Stage 3	Learner uses 'count on' to solve addition or missing addend tasks, and 'count-down-from' to solve removed items tasks but not 'count-down-to' to solve missing subtrahend tasks	3
Stage 4	Learner uses 'count-down-to' to solve missing subtrahend tasks, and can choose the more efficient strategy between 'count-down-from' and 'count-down-to'	4
Stage 5	Learner uses a number of 'non-count-by-ones' strategies which may involve procedures other than 'count-by-ones' but may also involve some 'count-by-ones'	5

Base-Ten Arithmetical Strategies		Score
Level 1	Learner does not see ten as a unit, so when solving additive tasks involving tens she counts forward or backward by ones. Learner needs to be at SEAL Stage 3 to be placed at this level.	1
Level 2	Learner sees ten as a unit composed of ten ones, but needs a representation like a tens strip to solve written additive problems involving tens and ones.	2
Level 3	Learner can solve additive problems involving tens and ones without using materials or reps.	3

Forward Number Word Sequences and Number Word After (FNWS & NWA)		Score
Level 0	Learner cannot produce the FNWS from 'one' to 'ten'	0
Level 1	Learner can produce the FNWS from 'one' to 'ten' but not the number word just after a number	1
Level 2	Learner can produce the FNWS from 'one' to 'ten' and the number word just after a given number word, but does so by dropping back to 'one'	2
Level 3	Learner can produce the FNWS from 'one' to 'ten' and the number word just after a given number word without dropping back to 'one'	3
Level 4	Learner can produce the FNWS from 'one' to 'thirty' and the number word just after a given number word without dropping back to 'one'	4
Level 5	Learner can produce the FNWS from 'one' to 'one hundred' and the number word just after a given number word without dropping back to 'one'.	5

Backward Number Word Sequences and Number Word Before (BNWS & NWB)		Score
Level 0	Learner cannot produce the BNWS from 'ten' to 'one'	0
Level 1	Learner can produce the BNWS from 'ten' to 'one' but cannot produce the number word just before a given number word	1
Level 2	Learner can produce the BNWS from 'ten' to 'one' and the number word just before a given number word, but drops back to 'one'	2
Level 3	Learner can produce the BNWS from 'ten' to 'one' and the number word just before a given number word without dropping back to 'one'	3
Level 4	Learner can produce the BNWS from 'thirty' to 'one' and the number word just before the given number word without dropping back to 'one'	4
Level 5	Learner can produce the BNWS in the range 'one hundred' to 'one' and the number word just before a given number word without dropping back	5

Numeral Identification		Score
Level 0	Learner cannot count some or all numerals in the range '1' to '10'	0
Level 1	Learner can identify numerals in the range '1' to '10'	1
Level 2	Learner can identify numerals in the range '1' to '20'	2
Level 3	Learner can identify one- and two-digit numerals	3
Level 4	Learner can identify one- two- and three-digit numerals	4

3.4.1 b) How the LFIN was used to develop learner profiles

Table 3.2 details the LFIN indicators used to assign learners to various stages and levels based on their observed strategies for solving early number tasks (Wright et al., 2006). In this section I exemplify how the LFIN indicators for the SEAL Stages and Base-Ten Levels were used in my study to construct individual learner profiles (all learner names in these exemplifications and across the study are pseudonyms). Indicators for other aspects of the LFIN are quite straightforward to apply and are thus not exemplified here.

Stages of Early Arithmetical Learning (SEAL)

Placing a learner at a particular SEAL stage involved determining the most advanced strategy available to the learner at that particular time (Wright et al., 2006). None of the participants were placed at SEAL Stages 0 and 1 so I do not include these stages in my exemplification.

SEAL Stage 2

To find the solution to the task $5+4$ (both addends screened) from Assessment 1.1 Q8a Sibü counted from 'one' to 'five' on one hand, then counted from 'one' to 'four' on the other and finally counted all his raised fingers from 'one' to 'nine'. He answered most additive tasks in his pre-test assessment using this strategy – called 'count all'. This placed Sibü (Group 1) at SEAL Stage 2 because he counted from 'one'.

SEAL Stage 3

Khosi (Group 2) solved additive task $9+6$ (both addends screened) from Assessment 1.1 Q8a by keeping 9 in her head and counting-on "ten, eleven, twelve, thirteen, fourteen, fifteen" keeping track of the number of counts made by raising a finger in co-ordination with each number word spoken. The number word spoken on her sixth count (fifteen) was her answer – which is the 'count on' strategy. She solved 'missing addend' task $12+\square=15$ (Q8f) by keeping 12 in her head and saying "thirteen, fourteen, fifteen" keeping track of her counts with her fingers. The number of fingers she had raised when she said "fifteen" was her answer – the 'count-up-to' strategy. The use of the 'count on' and/or the 'count-up-to' strategies are indicative of a learner at SEAL Stage 3 – also called the 'counting on' stage.

SEAL Stage 4

In her pre-test assessment Julie (Group 2) solved 'missing subtrahend' task $15 - \square = 11$ (Assessment 1.1 Q9b) using a 'count-down-to' strategy and solved two 'removed items' or take away tasks with subtrahends smaller than 5, that is, $15 - 3 = \square$ and $27 - 4 = \square$ (Q9c) by 'counting-down-from'. According to the SEAL LFIN indicators a learner who uses the 'count-down-to' and 'count-down-from' strategies judiciously (that is, the former for 'missing subtrahend' and the latter for 'removed items' tasks in the MR assessments) is at SEAL Stage 4.

SEAL Stage 5

Bongi (Group 1) used a range of non-count-by-one (NCB1) strategies to solve additive tasks during his post-test. He gave immediate⁸ answers (within 3 secs) to most 1-digit additive tasks and used the 1010 and N10 strategies to solve 2-digit by 2-digit written tasks like $42 + 23$ and $39 + 53$ (Assessment 1.2 Q2a), respectively. Bongi used 'compensation for addition' to provide immediate answers to the additive string '6+6, 7+5, 8+3, 9+5' (Q3b) and used the property of inverse operations to solve $18 - 3 = \square$ (Q3d) immediately after initially being presented with the number fact: $15 + 3 = 18$.

Base-ten Arithmetical Strategies

To place learners at particular base-ten levels required determining whether they could see and use ten as a unit in a tens-based setting (like ten dot-strips) and in a setting that did not embody the tens structure like number sentences. Within the MR programme a pre-requisite for being at Base-Ten Level 1 is that a learner has to at least be able to use the 'count on' strategy that is indicative of SEAL Stage 3 (Wright et al., 2006, p.22).

Base-Ten Level 1

Josh (Individual 2) enacted a unit count during his pre-test when tasked with incrementing by ten on the decuple (10, 20, 30... 80) and off the decuple (4, 14, 24... 84) within a tens-based setting (Assessment 1.2 Q1a). He did not use the 'count on' strategy to solve additive relations tasks which would have placed him at SEAL Stage 3 – thus he could not be placed at any level on the LFIN model for Base-Ten Arithmetical strategies.

⁸ Taken from Cowan et al. (2011) inference that learners' correct answers given within 3 seconds indicated that they used 'retrieval' of known answers.

Base-Ten Level 2

To answer pre-test Assessment 1.2 Q1a and b, Khanye (Individual 1) incremented by ten on and off the decuple in the setting of ten strips without attempting to count in ones. She also incremented separately by tens and ones (Q1c, Board 1) although her long pause before giving the last answer could indicate a count-on in ones. She further correctly incremented by tens and ones combined (Q1c, Board 2) up to the third uncovering which would have placed her firmly at Level 2 (Wright et al., 2006, p. 98). However, the halting manner in which she answered Q1c on Board 2 did not convey robust knowledge of incrementing by ten and ones combined, so I placed her at a 'low' Base Ten Level 2: Intermediate Concept of Ten.

Base-Ten Level 3

Julie (Group 2) incremented by ten on and off the decuple in the setting of ten-dot strips and also incremented by tens and ones (separate and combined) when answering questions to post-test Assessment 1.2 Q1a and b – which placed her firmly at Base-Ten Level 2. She further used ten as a unit when she solved 2-digit by 2-digit written additive tasks like $42+23=\square$ and $73-48=\square$ (Assessment 1.2 Q2a) using the 1010 and N10 strategies, respectively, which put her at Base-Ten Level 3: Facile Concept of Ten. Although she had mixed success with these written tasks during her post-test, Julie was still awarded Base Ten Level 3 because all multi-digit tasks do not have to be answered correctly to be awarded Level 3 (Wright et al., 2006, p. 98).

Next I discuss how I analysed elements of social interaction that occurred during intervention lessons and what the aim of this analysis was.

3.4.2 Base-Ten Thinking in Interactional Settings

As mentioned previously, local evidence from the broader South African terrain shows that counting-by-ones is endemic in primary classes well into the Intermediate Phase (Schollar, 2008) and in the pre-test for this study the prevalence of counting-by-ones was seen in the strategies participants used to solve number problems (e.g. tallies and counting-in-ones on their fingers). To shed more light on participants' gains made on the LFIN SEAL trajectory across pre- and post-tests, I took a closer look at learners' strategies and their 'math talk' during intervention (against the backdrop of representational settings used) to gain insight into the nature and progression in their base-ten thinking over the intervention period.

In this part of my analysis I direct my focus to the interaction that took place within the social context of learning, that is, in the immediate, face-to-face interactions between group members (De Abreu, 2000). During intervention I encouraged learners to explain their solution strategies in a way that their peers could understand and to compare someone else's way of working to their own. I stressed the importance of using clear, precise 'math talk' that illuminated the mathematical operation involved. I often modelled (or drew attention to) explanations that did not meet the above criteria and contrasted this with an explanation that did. By doing so learners did not only know what type of explanations they should not give but they also had examples of what acceptable explanations were. My specific focus on: the sophistication/efficiency of participants' strategies for solving 2-digit by 2-digit tasks across different representational settings and the expansions in their 'math talk' while explaining their solution strategies evolved in a 'grounded' way based on my experience of teacher-learner and learner-learner interactions during intervention. I often thought that learners had progressed in base-ten thinking based on their enactment of a ten-based strategy or their explanation of how they solved a task, only to find the same learners reverting back to counting-in-ones when a similar task was posed using a different representational setting. There are links here with South African and international evidence on competences that are 'localized' in the tasks and representations in which they are presented (Venkat & Askew, 2017). In South Africa, this is compounded by the evidence at the pedagogical level of highly 'separate' treatment of examples (Venkat & Naidoo, 2012) and limited evidence of connected moves in the Foundation Phase between more concrete and more abstract representational settings. Connecting between more concrete and more abstract settings for base-ten thinking was therefore a central feature in the intervention lessons.

This analysis was aimed at answering my 4th specific research question:

4. What was the nature of learners' progress (if any) in base-ten thinking during intervention as evidenced by the strategies they used and the nature of their oral discourse across different representations or settings as they worked on additive tasks?

Literature on children's early number learning refers to different markers of appropriation that are visible during interaction, namely, children's choice of solution strategies; the nature of their talk or oral discourse when explaining or justifying these strategies; and their facility with

working flexibly across different representations. Taken together, these markers of appropriation formed the broad framework of my interactional analysis of intervention lessons, which was: to understand (and illuminate) the nuanced shifts in participants' move from counting-by-ones to base-ten thinking during intervention as evidenced by their appropriation of 'calculation-by-structuring' strategies and expanded 'math talk' about these strategies across different representational settings as they solved number tasks.

Following on from the literature detailed in Chapter Two the main constructs that formed the analytical framework for this part of my analysis were learners' *solution strategies* and *oral discourse* used when solving 2-digit by 2-digit additive relations tasks across different *representational settings*. I first describe the constructs that were used in my analysis and thereafter show the relationship between these in diagrammatic form. An explanation of how this analysis was carried out is detailed in Chapter 4.

3.4.2 a) Constructs used in interactionist analysis

Representational Settings

Representations used to pose (and answer) tasks were analyzed using Ensor et al.'s (2009) hierarchy discussed in the previous chapter and represented below in Table 3.3. Here *physical* or *concrete* representations like bundling sticks were at the concrete end and *symbolic-syntactic* representations like number sentences at the other, more abstract, end of the framework. I also drew on Askew's (1998) distinction between *structured* materials like an arithmetic rack that embodies the five- and ten-structure of twenty beads and *unstructured* materials like counters that have no inherent number structure. So, in my analysis certain settings were considered *concrete* but a further distinction was made between those *concrete settings that were structured* like bundling sticks and those *concrete settings that were unstructured* like counters. In my analysis of progression or localization in learners' base-ten strategies, I focused on the setting in which tasks were posed because this gave me insight into the level of abstraction at which learners could access/make sense of the task. I took note of the representational setting used to solve tasks (to a lesser degree) because learners were mostly expected to solve tasks in the setting in which they were set, although 100 charts were always available to be used whenever learners were stuck. So, my first focus was on the setting used to pose or present tasks because I classified tasks according to these settings. When I needed another 'layer' to discern progression or localization in learners' use of settings

(when subsequent tasks were posed in the same setting) then I took note of which representation learners used to solve tasks or those they used for support.

Table 3.3 Framework for Representational Settings based on Ensor et al., (2009)

abstract  concrete	Mode of Representation	Explanation
	No representation used	includes mental arithmetic and formal mathematics
	Symbolic – syntactical	entails the deciphering and production of mathematical statements
	Symbolic – number based	- numerals are used to represent numbers - apparatus include number lines (structured and semi-structured), number charts, etc.
	Indexical	- generic (not realistic) depictions of everyday contexts - apparatus include drawings of tallies, sticks or circles to represent everyday objects
	Iconic	- realistic depictions of everyday context - apparatus include cartoons or drawings of realistic objects like worms or sweets
	Concrete	- manipulation of physical objects - apparatus includes fingers, money, counters, bead string

How representational settings were referred to in analysis

Similar to Hufferd-Ackles et al. (2004), learners in my study displayed fluctuations in the strategies they could enact when the representational setting changed. This led to my interest in finding out when (and how frequently) participants could enact base-ten strategies *within the same settings* and *across different settings* (aspects of progression and reversion are discussed in Chapter 6).

In lesson 5, Kyle (Group 2) solved +11 and +12 tasks posed with bundling sticks – a concrete (Ensor et al., 2009) structured setting (Askew, 1998) – by enacting the count-on strategy and then later in the same lesson he solved similar tasks posed in the same setting using the N10 strategy. This instance was coded as strategy progression *within the same setting* because the setting remained invariant.

In lesson 5, Khosi (Group 2) solved -10 tasks posed orally with bundling sticks for support and then in lesson 8 solved similar tasks mentally without any artefact support. This instance was coded as strategy progression *across different settings* because, although the setting used to pose the tasks remained the same, the setting the learner used to get to an answer changed from a concrete, structured setting (Ensor et al., 2009; Askew, 1998) to the use of no setting.

A reversion in strategy was evident *across different settings* in the way Mali (Group 1) solved +10⁹ tasks during lesson 2. Earlier in the lesson Mali solved +10 oral tasks by making one jump on the 100 chart (i.e. incrementing by 10) but later in the same lesson she solved +10 written tasks by counting-on in ones using her fingers. Here the task remained invariant but the settings in which tasks were posed varied (although not included in Ensor et al.'s hierarchy, oral settings differ to written settings) and the settings used to solve the tasks also varied: from a structured, symbolic-number based setting to a structured, concrete setting. Thus this was coded as strategy reversion *across different settings*.

Oral discourse

My study used learners' discourse or 'math talk' relating to their actions on settings (or strategies used) as a marker of appropriation because literature informs us that children's understanding of mathematics progresses from practical experiences like the physical manipulations of objects to them talking about these experiences. This learner-talk first appears as 'informal language' which later becomes more 'formal language' as they interact with others in their learning environment (Anghileri, 2006, p.8-9). Some researchers make a distinction between 'everyday discourse' and 'mathematical/scientific discourse' (Scribner, 1983, 1997; Sfard and colleagues, 2001, 2003) while Barwell (2005) uses terms like 'informal talk' and 'precise language of mathematics' to describe the progression that can be seen in learners' math talk over time.

Some indicators used to code the nature of children's oral discourse in this analysis were taken from the literature-base discussed previously while others were developed in a 'grounded' way, that is, these emerged from the data itself. Table 3.4 includes the following indicators: *informal language* and *formal language* taken from Anghileri's work noted above,

⁹ +10 tasks are tasks that involve incrementing the first addend by ten, like $\square + 10$ written tasks

Table 3.4 Framework used for analysis of learner's Oral Discourse

Moving from the left to the right was regarded as... Progression 	
no response/cannot explain/gesture offered as response	verbal response/able to explain/one-word response
mathematically imprecise language	mathematically precise language
informal language	formal language
everyday language	mathematical language
mathematical operation not articulated	mathematical operation articulated
explanation offered did not foreground a mathematical reason	explanation offered highlighted clear mathematical reasoning
localized talk	generalized talk
explanation incongruous to solution strategy evidenced by learner's actions	explanation offered matches solution strategy evidenced by learner's actions
 Regression occurred when learners' talk moved from the right to the left	

Some learner responses could be tagged with more than one descriptor from either side of the table. By way of example, a response in which a learner pointed on the 100 chart without offering any oral explanation when asked to explain her reasoning could be described as *gesture offered as response* as well as *mathematical operation not articulated*. Thus I considered any movement from the left to the right of the table as a progression in discourse while the opposite was true for any movement from right to left.

Exemplification of learners' oral discourse in my study

In lesson 8 I wrote a different number on each learner's blank 100 chart and asked them to write the number in the block above and below the given number without counting. Learners completed this task fairly quickly. Sibü (Group 1) wrote 13 in the block above 23 and 33 in the block below. I asked him to tell the group how he knew that these numbers were supposed

to go in the respective blocks and he closed his face with his hands, offering no reply. This was labelled as *no response*. At the end of intervention (L18) Sibü was asked to explain how he knew that the number above 34 on his 'crazy square' was 24 and he replied, "because we are minusing ten less" – labelled as *mathematical operation articulated*. This was coded as one instance of *discourse progression*.

When tasked with finding numbers on a blank 100 chart in lesson 10, Mali (Group 2) used the tens structure of the 100 chart to solve the task and when asked how she worked out where the numbers were to be written she said, "I did think in my head". A few weeks later (L12) Mali solved similar tasks in the same setting by enacting the same strategy, except now she explained her strategy by saying, "I count in ten from 10 to 40 (points to blocks) and then I count from 41 to 45 (points)". Mali's talk in lesson 10 was labelled as *mathematical operation not articulated* and her talk in lesson 12 was described as *mathematical operation articulated*. Thus she showed *discourse progression*.

During lesson 10 I asked Sibü (Group 1) to explain to the group how he found the block for 35 on the blank 100 chart. Pointing to the fifth block in the first row Sibü said, "Five, fifteen, twenty-five, thirty-five" while moving his finger one block down with each count. But, video evidence clearly shows that Sibü did not use the strategy he described. What really occurred was that Sibü found block 35 by pointing to block 12 (previously filled in on the blank 100 chart), he moved 2 blocks down (to block 32) and then moved his finger 3 blocks to the right and wrote 35 in that block – thus he incremented by ten and ones from a known position on the 100 chart. This explanation was thus labelled as *incongruous to the solution strategy* and coded as *discourse 'mismatch'*.

Next I provide a framework for learner solution strategies that I developed from the literature reviewed. Tables 3.3 to 3.5 provide a hierarchy of possible solution strategies to additive tasks. The literature base that these were derived from (Steffe et al., 1983; Carpenter & Moser, 1984; Wright et al., 2006; van den Heuvel-Panhuizen, 2008; Anghileri, 2006) and the settings that could be used for the enactment of each are included. This is followed by an explanation and examples of how this was used to analyse learners' strategies in my study.

Additive Strategies

The ways in which children solve mathematical problems/tasks are often referred to as strategies or methods. In my study I use these terms interchangeably following Wright et al.'s sentiment that the term strategy is a "...generic label for a method by which a child solves a task" (Wright et al., 2006, p.191). The framework for additive solution strategies used in my analysis has been developed from the literature reviewed in Chapter 2 and presented in a hierarchical, tabular form (Tables 3.3, 3.4 and 3.5) for ease of comparison. The first group of strategies (count-from-one) is considered less sophisticated than the next group (advanced counting strategies) and so on. So, if a learner initially used count-from-one to solve an additive task and then later used count-on for a similar task, then this was labelled as **strategy progression** because the former is a less-advanced counting strategy than the latter. If both tasks were posed using the same setting then the learner's strategy progression was labelled as strategy progression *within the same representational setting*. If the tasks were posed in different settings then this progression was labelled: strategy progression *across different representational settings*.

The criteria used for **strategy reversion** in my study were similar to those of strategy progression except here the shift would have been from a more sophisticated/efficient strategy to one that was less sophisticated/efficient based on my framework of learner solution strategies (Tables 3.5 to 3.7). Similar criteria applied for strategy reversion *within the same setting* and *across different representational settings*.

Another phenomenon observed within learners' enactment of solution strategies in my study and described in the literature reviewed (Ch.2) was **representational flexibility** – where learners initially enacted a base-ten strategy in a more concrete, rudimentary setting and then later enacted the same strategy in a more abstract setting (or without any setting). The nature of representational flexibility is that it is evident as learners work across different settings.

Table 3.5 Calculation-by-Counting					
Count-from-One or Count-All Strategies					
Operation	Strategy	Literature base	Example	Settings	Classification using Ensor et al. (2009)
Addition	count-from-one or count all	Carpenter & Moser (1984); Wright et al. (2006)	solves 15+23 by counting out 15 items, then counting out 23 items, then counts all items together from ‘one’ (Wright et al make a distinction here between <i>perceptual counters</i> who need to perceive the two addends and <i>figurative counters</i> who do not)	counters	concrete unstructured ¹⁰ reps or iconic / indexical reps
Subtraction	count out	Carpenter & Moser (1984); Anghileri (2006)	solves 26-14 by drawing 26 items, crossing out 14 of these and then counting what is left from ‘one’ (also called a triple count or count-from-one)	tallies, drawings	
Advanced Counting Strategies					
Operation	Strategy	Literature base	Example	Settings	Classification using Ensor et al. (2009)
Addition	count-on including count-up-from & count-up-to	Carpenter & Moser (1984); Wright et al. (2006)	solves 15+23 by keeping 15 in her head and counting on 23 keeping track of 23 counts using fingers, 100 chart or number line (Wright et al. distinguish between <i>counting-up-from</i> which is typically used to solve additive tasks with two screened collections and <i>counting-up-to</i> when counting-on is used to solve missing addend tasks with a small unknown addend: 18+□=21. Wright et al. regard counting-on and figurative counting as more advanced than perceptual counting)	fingers or number lines, ENL, 100 charts	concrete, structured reps or symbolic number-based reps:
		count-on-from-larger	Carpenter & Moser (1984)	solves 15+23 by keeping 23 in her head and counting on 15 keeping track of 15 counts using fingers, 100 chart or number line; linked to commutative property	
Subtraction	count-down-from	Wright et al. (2006)	solves removed items tasks like 17-3 by counting back from the minuend three counts: sixteen, fifteen, <i>fourteen</i> ... the solution is the number word spoken on the third count, i.e. fourteen	fingers or number lines, ENL, 100 charts	concrete, structured reps or symbolic number-based reps:
	count-down-to	Carpenter & Moser (1984); Wright et al. (2006)	solves missing subtrahend tasks like 23–□=19 by counting down from 23 to 19 (twenty-two, twenty-one, twenty, nineteen) raising a finger with each number word uttered, the solution is the nr of fingers raised on ‘nineteen’, that is, 4 (called counting-back-to by Carpenter & Moser)		
		count-up-from	Wright et al. (2006)		

¹⁰ A distinction between structured and unstructured settings do not form part of Ensor et al.'s (2009) classification. This distinction is taken from Askew (1998) who sees counters as 'unstructured material' because they have no inherent number-structure while fingers are 'structured' because of the 5 and 10 structure inherent in fingers.

Table 3.6						Calculation-by-Structuring					
Base-Ten Strategies											
Op.	Strategy	Literature base	Example			Settings	Classification using Ensor et al. (2009)				
Addition and Subtraction	1010 splitting method	Wright et al. (2006); Anghileri (2006)	to add the learner splits each addend into place value chunks, combines similar place value chunks and finally adds intermediary sums together, e.g. 13+25 as 10+20=30, 3+5=8 and 30+8=38			bundling sticks, MAB blocks or number sentences	base-ten reps ¹¹ or symbolic syntactical reps				
			to subtract the learner splits the minuend and subtrahend into place value chunks, subtracts similar pace value chunks and finally adds the intermediary differences, e.g. 57-34 as 50-30=20, 7-4=3 and 30+3=33.								
	N10 stringing, sequence method or jump strategy	van den Heuvel-Panhuizen (2008); Wright et al. (2006); Anghileri (2006)	to add the learner keeps the first (or biggest) addend whole and then adds the second addend in place value chunks, e.g. 13+25 as 13+20 (or +10+10) = 33 and 33+5=38			bundling sticks, MAB blocks; or number lines; or number sentences	base-ten reps; or symbolic number-based reps; or symbolic syntactical reps				
			to subtract the learner keeps the minuend whole and subtracts the subtrahend in place value chunks, e.g. 57-34 as 57-30 (or -10 -10 -10) = 27 and 27-4=23								
	Bridging-through-ten	Wright et al. (2006); Anghileri (2006)	to add 16+7 the learner considers (without counting) the number to be added to 16 to get to the next decuple (4), she partitions 7 into 4 and 3, then adds 4 to 16 to get to 20 and adds 3 to 20 (without counting) to make 23; this strategy is also called adding/bridging-to-ten and adding-through-ten			number lines, 100 chart or number sentences	symbolic number-based reps or symbolic syntactical reps				
			to subtract 25-9 the learner considers (without counting) what number needs to be subtracted from 25 to get to the previous decuple (5), she partitions 9 into 5 and 4, then subtracts 5 from 25 to get to 20 and subtracts 4 from 20 to get the final answer: 16 (without counting); this strategy is also called subtracting/bridging-to-ten and subtracting-through-ten								
Other Structuring Strategies											
Op.	Strategy	Literature base	Example			Settings	Classification using Ensor et al. (2009)				
Addition and Subtraction	near doubles	Anghileri (2006); Askew & Brown (2003)	learners use their knowledge of doubles facts to easily calculate, e.g. if learners know that double 8 is 16 then 8+9 is 16 and 1 more (17) or 25+24 is 50 and 1 less (49)			number lines, 100 chart or number sentences	symbolic number-based reps or symbolic syntactical reps				
	compensation or varying	van den Heuvel-Panhuizen (2008)	learners alter numbers (typically those that end in 1 and 2 or 8 and 9) to make a calculation simpler; an addition example: 26+39 is calculated as 26+40=66 and then compensate by subtracting the one added: 66-1=65; subtraction: 47-29 as 47-30=17 and compensate by adding 1 because one more than 29 was taken away: 17+1=18								
	column calculation	van den Heuvel-Panhuizen (2008)	learners set out additive calculations using columns and perform the operation using whole-number-based calculation			written/column calculation	symbolic syntactical reps				

¹¹ Ensor et al. (2009) do not include this in their hierarchy of representations; this classification is taken from the RME literature base

Table 3.7 Formal Calculation				
Calculation Strategies				
Operation	Strategy	Literature	Example	No representational settings used
Addition and Subtraction	known facts	Carpenter & Moser (1984); Askew & Brown (2003)	here the learner answers 'out of her head' without any counting or calculating (usually in under 3 sec)	
	derived facts	Carpenter & Moser (1984); Askew & Brown (2003)	the learner uses what she already knows – this varies from one individual to the next – to find out what she does not know, e.g. if a learner knows her 4 times-table well but not her 8 times-table, then she can solve '5x8' by mentally doubling the solution to '5x4'	
	1010 split tens method	Wright et al. (2006); Anghileri (2006)	the learner adds mentally by splitting each addend into place value chunks, then combines similar place value chunks and mentally adds intermediary sums together	
			the learner subtracts mentally by splitting the minuend and subtrahend into place value chunks, then subtracts similar place value chunks and mentally adds the intermediary differences	
	N10 sequence method or jump strategy	Wright et al. (2006); Anghileri (2006)	the learner adds mentally by keeping the first/biggest addend whole and then adds the second addend in place value chunks	
			the learner subtracts mentally by keeping the minuend whole and subtracting the subtrahend in place value chunks	
	bridging-through-ten	Wright et al. (2006); Anghileri (2006)	the learner mentally adds on from the first/bigger addend to get to the next decuple (by splitting the second/smaller addend), then the rest of the second addend is added on mentally, e.g. $16+7 \rightarrow 16+4=20 \rightarrow 20+3=23$	
			the learner mentally subtracts from the minuend to get to the previous decuple (by splitting the subtrahend), then the rest of the subtrahend is subtracted mentally, e.g. $42-8 \rightarrow 42-2=40 \rightarrow 40-6=34$	
	algorithms ¹²	van den Heuvel-Panhuizen (2008)	learners use the column method to do digit-based addition or subtraction	

¹² van den Heuvel-Panhuizen (2008) makes a distinction between digit-based algorithmic calculation and whole-number-based calculation which can both be set out using columns.

Exemplification of learners' enacted strategies in my study

During lesson 2 Bongi (Group 1) solved ± 10 tasks posed with screened bundling sticks by enacting the N10 strategy on the 100 chart and in lesson 9 he solved similar tasks posed in the same setting mentally (without any counting or calculating). In this instance the tasks, and the setting in which the tasks were posed remained invariant but the setting Bongi used to get the answer changed from a structured, symbolic-number based setting to no setting and the enacted strategy changed from calculation-by-structuring to a formal calculation. This was thus coded as *strategy progression within the same setting*.

In lesson 4, I posed 2-digit + 11 oral tasks to Group 2. Kyle enacted the N10 strategy on the 100 chart (structured, symbolic-number based setting) to solve $24+11$ by placing his finger on 24, then he moved one block down to 34 (i.e. adding 10) and from there he moved one block to the right to 35 (i.e. adding 1). Later in the same lesson Kyle tried to solve $43+11$ on the board by counting in ones on the ENL (unstructured, symbolic-number based setting) – he made a hash mark for the first addend (and labelled it) and then made many small jumps on the line with his finger saying the FNWS from 'one' to 'eleven'. Oral tasks are not included on Ensor et al.'s (2009) hierarchy of representations so in this case I considered the setting in which Kyle solved each task. Here the task remained the same; the setting remained at the same level of abstraction (Ensor et al., 2009) although it changed from a *structured* to an *unstructured* setting (Askew, 1998); and the enacted strategy changed from a base-ten strategy (calculation-by-structuring) to an advanced counting strategy (calculation-by-counting). This was thus coded as *strategy reversion* because Kyle solved a similar task using a less sophisticated strategy *across different settings*.

In lesson 5, I posed 2-digit by 2-digit additive tasks to Group 2 using bundling sticks (structured, concrete setting) and later in the same lesson I posed similar tasks using number sentences (symbolic-syntactical setting). To solve these tasks Khosi first enacted the 1010 strategy in the concrete setting and later enacted the 1010 strategy in the symbolic-syntactical setting. Here the task and learner's base-ten strategy were invariant while the setting changed from less to more abstract. This was thus considered an instance of *representational flexibility* (all instances of representational flexibility occurred *across different settings*).

3.5 Concluding Remarks

This chapter explains and justifies the theoretical adaptation of the study, that is, my use of Mathematics Recovery within an Emergent rather than a Radical Constructivist frame. The Emergent Approach allowed me to tell a good story from two distinct angles, as if I were capturing the same subject/content using two different photographic lenses: a telescopic lens to capture minute details of individual learning, and a wide-angle lens to capture individual learning against a larger in-focus background/setting.

As an analytical framework the LFIN enabled me to construct an individual diagnostic picture of each participant's learning journey, while interactionist constructs like oral discourse and actions on settings enabled me to look at how elements of learner interaction 'scaffolded' the early number learning of individual children.

In this chapter I was able to detail the frameworks used in my study to analyse the various data sets in a way that answered the specific research questions. My discussion makes a clear case for the twin foci in my analysis, that is, on cognition and aspects of interaction. In the next chapter (Ch.4) I provide greater detail of: how I gathered the empirical data (detailing the setting, participants and data gathering techniques); the different data sets (primary and secondary data) that were gathered or generated; how this data was organised and analysed; how I dealt with methodological tensions while simultaneously fulfilling the roles of intervention teacher and researcher, and finally, how I dealt with aspects of rigour that speak to the quality of my study.

Chapter 4 – Methodology

4.1 Introduction to the chapter

Educational research has been described as ‘the collection and analysis of information on the world of education so as to understand and explain it better’ (Opie, 2004, p.33) and as a systematic study of a topic of interest using data gathering techniques aimed at answering questions related to that topic (Leedy & Ormrod, 2010). In this chapter I first highlight the broad aim and specific research questions that guided this study before discussing the overall qualitative design of my study which centered around a teaching experiment approach. I then describe the setting or context of the study and provide a detailed account of why and how the purposive sample for this study was selected. Following that I explain my data collection techniques and the sources of information used, as well as the way I organised the information and data collected. Further, I explain how I analysed the data using both ‘cognitive constructivist’ and ‘social interactionist’ lenses that constitute the Emergent Theory which frames my study. Finally, I delineate the steps I took to ensure the rigour of my study, and explain how I dealt with ethical issues related to my research. Next I discuss each of these methodological aspects in detail.

4.1.1 Aim and Research Questions

The aim set out at the start of this study was to understand the extent to which a short-term intervention based on Wright et al.’s (2006) Mathematics Recovery programme proved effective in developing a sample of South African learners’ understanding of early number. Based on this broad aim I developed four specific research questions that guided my sample selection, data gathering techniques and data analysis. These were:

1. What was the profile of the sample of Grade 2 learners’ early number understanding prior to and immediately after receiving intervention based on Mathematics Recovery principles?
2. Following the intervention pathway for learners who made up the intervention group, what learning gains (if any) were produced at the end of the intervention, and how does this compare to learning gains made by the control group?
3. What were the differences and similarities in the nature of learning gains achieved by ‘singletons’ compared to gains made by learners who received grouped intervention?

4. What was the nature of learners' progress (if any) in base-ten thinking during intervention as evidenced by the strategies used and the nature of oral discourse across different representations or settings across work on additive tasks?

4.2 Research Design

The qualitative methodology employed in this educational research is underpinned by an interpretivist epistemology and a constructivist ontology. Thus, my belief that the world is constructed and interpreted by people as they make sense of their reality through social processes (Neuman, 2003) governed my quest to better understand the early number learning of a sample of Grade 2 learners. Whilst the overall frame of my research study was qualitative, quantitative elements relating to the proportions of learners in the intervention and control groups achieving shifts in various aspects of their LFIN profiles were also used.

Qualitative research is an active process wherein those who are studied teach the researcher about themselves and their experiences in a context that is not contrived, but natural (Blaxter, Hughes & Tight, 1996). This type of research often relies on personal contact over a period of time between the researcher and the group being studied (Ulin, Robinson, & Tolley, 2005). Qualitative research often involves collecting large amounts of data on a relatively small purposive sample using data collection strategies like in-depth interviews, participant observation or focus groups. Data collection strategies within a qualitative study are usually flexible and sensitive to the particular social context or setting of the study because the aim is to examine how people within a particular setting give meaning to their daily lives (Hox & Boeij, 2005).

I was personally involved with the early number learning of my sample of Grade 2 learners in their natural learning space – as both teacher of the intervention lessons and researcher of these learning experiences – over an extended period of time to gain deeper insight into their lived experiences, which I believed would add richness and depth to my data. I chose to answer my research questions using a qualitative design because I was interested in the finer shifts in children's early number learning and this design allowed me to engage in deep, textured analysis with a relatively small sample size. Moreover, qualitative research has been widely used in the field of education for more than 30 years (Merriam, 2001). The intervention

approach, which fits well within the research paradigm guiding this research activity, is discussed next.

4.3 Intervention Approach

The intervention approach used in my study was a ‘teaching experiment’. This research approach has been used to explore children’s mathematical learning and explain the progress they make over time in the context of interactive teaching (Steffe & Thompson, 2000). Whilst my motive for using a teaching experiment was to gain a deeper understanding of learners’ mathematical thinking, this approach has also been used in other studies as an inquiry tool into the teacher-researcher’s role in classroom interaction (Czarnocha & Maj, 2008).

All teaching experiments take place within a sequence of teaching episodes (Steffe et al., 1983) and involve a “teaching agent, one or more students, a witness of the teaching episodes, and a method of recording what transpires during the episodes” (Steffe & Thompson, 2000, p. 273). Table 4.1 shows how the elements of this approach, which are germane to all teaching experiments, were accounted for in the design of my study.

Table 4.1 Aspects of a teaching experiment

Aspects of a Teaching Experiment detailed in literature	How aspects of a Teaching Experiment were accounted for in my study
a teaching agent	I planned and presented all lessons
one or more students	10 learners (intervention group) split into two groups of 4 and 2 singletons and 10 learners in the control group
a witness of the teaching episodes	I witnessed all video-recorded lessons
a method of recording what transpired during episodes	all lessons were video recorded; detailed notes were made of each lesson, and full transcriptions were made of exemplary segments

During intervention lessons, I took on the role of the ‘teaching agent’ (or teacher-researcher) and twenty middle-attaining Grade 2 learners from a Johannesburg government school made up the ‘students’ (contextual detail of selected school follows in section 4.4). I was the primary ‘witness of the teaching episodes’ as I viewed video-recorded lessons to observe what took place and plan for the next lesson, with my supervisor as a secondary observer where needed. Detailed notes of each lesson and full transcriptions of exemplary episodes (those episodes that foregrounded a particular aspect that I thought was pertinent to my study) provided a

written record of what transpired during each lesson. These were used for preparing subsequent lessons and conducting retrospective analysis of the teaching experiment (Steffe & Thompson, 2000). The lessons that made up this teaching experiment were based on Wright et al.'s (2006) LFIN and IFEN, and were 'transformative' as the detail of each episode was open to slight changes to match the needs of participants (Confrey & Lachance, 2000). Nevertheless, the format of this teaching-experiment followed that outlined by the Mathematics Recovery programme itself which is explained next.

4.3.1 Intervention Model based on Mathematics Recovery

The intervention followed the 'test-teach-test' model as delineated below:

- Initial individual task-based interviews with twenty learners across the five Grade 2 classes in the selected school (ten learners each for intervention and control groups) using tasks selected from Mathematics Recovery (MR) Assessment Schedules 1.1, 1.2, 2.1 and 2.2 served as the pre-test. These were used to determine each learner's level and stage on the LFIN prior to intervention.
- Implementation of intervention lessons (about forty minutes long) based on Mathematics Recovery principles which involved withdrawal of the ten intervention learners (two groups of four learners and two singletons) for two sessions per week, over a ten-week period, were planned for. There would have been 20 lessons in total for each group and singleton over the course of intervention if absenteeism and clashes in time-table between intervention lessons and school activities were not a factor.
- Final individual task-based interviews with the twenty learners making up the intervention and control groups, using MR Assessment Schedules 1.1, 1.2, 2.1 and 2.2. This served as the post-test used to analyse gains in early number learning.

4.3.2 Adaptations to key features of MR in my study

Early Intervention

In my study, the MR programme was implemented with learners who were in Grade 2, rather than Grade 1. This was because Grade R was not compulsory at the time of my data collection (Gov Gazette, 1988, vol. 400) so many South African children did not have access to formal schooling in the reception year (Grade R). Thus, many South African learners only experienced their second year of schooling in the second grade during 2014.

Assessment

The initial and final learner assessments took the same format (individual task-based interviews) and only made use of the first four MR assessment schedules.

Teaching

In my study, teaching was mostly based on the MR teaching principles previously outlined except that the format of my sessions included individual *and* grouped teaching. The duration of the teaching cycle was 10 weeks, which is shorter than the 12-15 week teaching cycle normally used in Mathematics Recovery, and learners only received intervention twice a week rather than every day. Another difference was the extensive use of the 100 chart during intervention – a decision that was fueled by current practices on the ground. My decision to use the group teaching format and a shorter teaching cycle was motivated by pragmatic concerns: the large number of learners who require intervention in early number learning makes both one-on-one, and longer term intervention more impractical in the South African context where high teacher-pupil ratios and under-resourced schools are common. This format and shorter teaching cycle necessitated decisions about the selection of tasks within the intervention sessions that worked at the cutting edge of learning for different individuals at different points. The provision made for analysis of individual learning in interactional settings which also aimed at opening opportunities for ‘scaffolding’ learning through peer interactions. The study therefore aimed to explore the feasibility and efficacy of this less intensive, adapted MR model on individual learning of early number – in order to gauge the possibility of its broader usefulness.

Discourse, Strategies and Representational Settings

As stated already, my study differs from the original MR programme with regard to the focus on interactional aspects in the public learning space created, that is, on enacted strategies, oral discourse or ‘math talk’ and representations used. The assessment of these social aspects at play during the learning process is not a focal point in the original MR programme but emerged out of my concern about some of the contradictions between claims that I could make based only on learners pre- and post-test LFIN profiles and what I had witnessed during intervention lessons.

The setting of any qualitative study is very important because this is the participants' natural world that they actively give meaning to through various social processes. The setting or context of this study is discussed next.

4.4 Setting

The setting for my research was a local government school selected from the ten primary schools (5 township schools and 5 suburban schools) that participated in WMC-P project's first phase of research and development in the area of primary mathematics. The school is situated on the East Rand in Johannesburg and is called an 'ex-model C' school. These are schools that were formally reserved for the white minority population under the previous apartheid government. As such, the physical infrastructure of the school resembles what is now called a 'private school' – with spacious brick classrooms, a school hall, structured playgrounds, tennis courts, a pool, etc. This school was one of the 5 suburban schools that were selected by the district on the basis of what was seen as 'underperformance' in mathematics relative to their quintile status. This falls in line with broader concerns about historically better-off schools struggling to maintain levels of performance in the context of a changing demographic, and pressures to take on large numbers of learners, with provincial education departments using national systemic evaluation results to guide their decisions on underperformance (Spaull, van der Berg, Wills, Gustafsson, & Kotze, 2016).

The language of learning and teaching (LoLT) at the school is English even though the majority of learners and teachers at the school do not speak English as their first language. As the country moved into democracy, the demographics of the school (and other 'ex-Model C' schools) gradually changed in that the learner population and teaching staff became mostly Black African. The school is a fee-paying government school and at the time of my data collection the school's fees were relatively high compared to the five schools in WMC-P project's suburban cluster¹³. Most of the learner cohort do not live in the more affluent surrounding suburbs and thus travel to school either by public or private transport. Even though the school could be considered to be more privileged than other government schools in township areas, it has a feeding scheme that serves meals to learners from poorer families. I became acquainted with the principal and staff at the school as a Masters Fellow and team

¹³ The annual school fees at the research school for a Grade 2 learner in 2014 was R7000. At the other four suburban project schools the annual fees for a Grade 2 learner in 2014 ranged between R4200 and R5000.

member of the WMC-P project during 2012. Thus, gaining access to the school was fairly unproblematic because I had already built a relationship of trust and camaraderie with a few members of staff prior to my doctoral study. The selection of the setting or context of my study was thus also pragmatic.

4.5 Participants

I used purposive sampling (Patton, 2002) for my study as opposed to other types of sampling linked to qualitative research. Marshall (1996) argues that through purposive sampling a researcher “actively selects the most productive sample to answer the research question(s)”. These participants are those the researcher considers “richer” than others and are the most likely to “provide insight and understanding for the researcher” (Marshall, 1996, p. 523).

After receiving consent from the principal and Grade 2 teachers at the research school (detailed under Ethical Considerations) I embarked on the participant selection process with the teachers’ help. My selection of participants was based on particular criteria as I wanted the ‘most productive sample’. These criteria and the rationale for each is discussed next.

4.5.1 Criteria for selection

I selected participants for the planned intervention study from the Grade 2 cohort of learners because the Mathematics Recovery programme is usually implemented with children in their second year of schooling. By selecting participants from the second grade in the South African context, the mean age of the sample for this study was higher than for other (international) intervention studies also based on Mathematics Recovery. Nevertheless, I still thought it best not to use children from the Grade 1 cohort as I intended using a withdrawal method of intervention which could potentially cause some interruptions in a Grade 1 child’s school day. Also, children who had already experienced a year of schooling would more likely be able to slip back into their usual school programme after being withdrawn from class twice a week for ten weeks. Moreover, my qualitative study was oriented towards gaining a better understanding of the research problem in its unique context and less concerned with generalizability (Farzanfar, 2005; Ulin et al., 2005). Participants for my study had to be middle-attaining in the school context because these learners were more likely in the country’s skewed international performance to gain from a short-term structured intervention than low-attainers. Participants also had to be relatively proficient in English so that a language barrier

would not be a variable in the research process as I do not speak a vernacular language. A record of regular school attendance was also a criterion, because the intervention involved two withdrawal-teaching episodes across ten weeks which had to be attended in order to judge the efficacy of the programme. Finally, none of the participants had to be repeating the grade because they would then have been in their third year of formal schooling.

I used data from the Grade 1 English and Numeracy ANAs (2013) and an adapted version of the Leverhulme study tests (Brown et al., 2008) conducted by the WMC-P project with the Grade 1 cohort at the end of 2013 to select seven middle-performing learners (justification below) from each of the five Grade 2 classes at the start of the 2014 school year (that was thirty-five learners in total). I first selected learners who fell within the upper quartile for the academic assessments mentioned and who were still enrolled at the school. I then looked at school attendance registers (for the Grade 1 cohort of 2013 and the Grade 2 cohort of 2014) and discussed potential participants' school attendance with their respective teachers to exclude children I had previously selected based on their academic performance but who had a history of irregular school attendance.

After having received informed consent from thirty-two learners and their parents I conducted a one-on-one assessment interview with these learners based on a shortened version of the MR Assessment Schedule 1.1 (about 20min each). Using my assessment of these learners' early number skills during the diagnostic interviews, twenty middle-attaining learners across the five classes were selected to participate in this study – ten learners from the sample made up the intervention group and the other ten 'matched' learners formed the control group (Appendix D sets out the matched learners for each group). Even though I judiciously sought to select Grade 2 participants who were comfortable communicating in English, one intervention learner proved to be reticent when it came to engaging in 'math talk' during intervention (explained in Chapter 6).

Learners who made up the intervention group (two groups of 4 learners and two singletons) were withdrawn from normal teaching for 40 minutes, twice a week, for ten weeks to receive grouped and individual intervention, respectively. The learners forming the control group remained in their classes and followed the usual curriculum as taught by their teachers without receiving intervention. Information about the participants who made up the intervention and control groups are set out in Tables 4.2 and 4.3 respectively.

4.5.2 Participant Information

Table 4.2 **Intervention Group**

Name ¹⁴	Group/ Single	B/G	Age ¹⁵	Class	Pre-Test Date	Post-Test Date
Khanye	Single1	G	7,6	2-1	25.04.2014	09.09.2014
Josh	Single2	B	7	2-5	24.04.2014	12.09.2014
Sibu	Group1	B	6,11	2-3	25.04.2014	12.09.2014
Bongi	Group1	B	7,8	2-2	16.04.2014	15.09.2014
Mali	Group1	G	7,10	2-2	16.04.2014	11.09.2014
Jenna	Group1	G	8,2	2-2	16.04.2014	08.09.2014
Kgomo	Group2	B	6,11	2-4	23.04.2014	09.09.2014
Julie	Group2	G	7,6	2-4	24.04.2014	10.09.2014
Khosi	Group2	G	7,9	2-5	24.04.2014	11.09.2014
Kyle	Group2	B	8,5	2-4	23.04.2014	09.09.2014

Table 4.3 **Control Group**

Name ¹¹	B/G	Age ⁷	Class	Pre-Test Date	Post-Test Date
Thulu	G	6,10	2-5	24.04.2014	12.09.2014
Ntlako	G	7	2-4	23.04.2014	22.09.2014
Rhaka	B	7	2-2	16.04.2014	11.09.2014
Olsen	B	7,3	2-2	16.04.2014	11.09.2014
Sipo	G	7,3	2-4	23.04.2014	26.09.2014
Rila	G	7,4	2-1	25.04.2014	09.09.2014
Yazu	B	7,8	2-4	24.04.2014	22.09.2014
Nozi	G	7,8	2-4	23.04.2014	22.09.2014
Bonang	B	8,4	2-2	15.04.2014	08.09.2014
Abby	G	8,8	2-2	15.04.2014	08.09.2014

¹⁴ All learner names are anonymized

¹⁵ Learners' age in years and months at the time of the pre-test. So, in the intervention group Khanye's age (7,6) means that she was 7 years and 6 months old at the time of the pre-test.

Intervention Group

The ten participants who made up the intervention group are listed in Table 4.2 according to the intervention format they received. This arrangement makes it easy to see that Groups 1 and 2 had the same number of boys and girls (2 each) and the 2 'singletons' were also evenly represented across the gender divide. South African evidence shows no gender disparities in performance at primary school level, so gender was not a focal factor in my study, nevertheless I tried to keep this background factor equitable in my selections. The age range for Group 1 (in years and months) was 6,11 to 8,2 years old with 7,3 as the mean age. Similarly, the age range for Group 2 was 6,11 to 8,5 and the mean age for this group was 7,5. The two 'singletons' were aged 7 and 7,6 and their mean age was 7,3 years old. The average ages of participants in Groups 1 and 2 and the 'singletons' were almost the same (7,3; 7,5 and 7,3 respectively) which adds to the sense of equity amongst groups and 'singletons' although this was not a planned feature. Group 1 had three participants from the same class (class 2-2) and the fourth participant was from the class next door to theirs (class 2-3). The same was true for Group 2: three participants were from class 2-4 and the fourth was from the class next door to theirs (class 2-5). This feature *was* planned. I ideally would have liked to have had all four group members from the same class as this would have made the withdrawal process smoother and interaction during group work may have been easier because the learners would have already been acquainted with each other and all group members would have received the same teaching during their usual class time. Following the criteria for participants mentioned previously, I went for the highest number of learners from one class and then pragmatically selected the fourth from the class next door. Three intervention learners were younger than the 'grade number+6' guideline in Government Gazette vol.400 of 1998 entitled "Age Requirements for Admission to an Ordinary Public school" and 1 learner was older. This is not an uncommon occurrence in public schools in South Africa (as some children start school earlier than the recommended age and others start later due to a variety of factors) thus age was not a factor in my study, rather that participants were in their second year of formal schooling.

Control Group

From Table 4.3 it is evident that five learners from the control group were below the recommended age for this group of Grade 2 learners and two were older (learners in Grade 2 in 2014 should have been born in 2006). The age range was 6,10 to 8,8 for the control group with a mean age of 7,5 which is comparable to that of the intervention group: 7,4. The

control group consisted of 6 girls and 4 boys; 4 learners from each from Grades 2-2 and 2-4; 1 learner each from Grades 2-1 and 2-5 and no learners from Grade 2-3. Looking across both tables it is evident that learners from the same class mostly had their pre- and post-tests done either on the same day or on consecutive days. Where a learner in the class group was absent, then her interview date was slightly different to that of her class group.

4.6 Data Collection

Collecting data for an intervention study based on withdrawing learners from their regular classes can be challenging as a school is a fluid space where certain 'disruptions' like a trip to the Zoo or a Sports Day are not uncommon. However, my familiarity with the school and collegial relationship with members of staff made navigating these potential disruptions to my data collection process less problematic. For example, even though my scheduled data-collection days were every Tuesday and Thursday for the intervention period, when the Grade 2 learners went on a trip to the Zoo (which was on a Thursday) the teachers, HOD and I agreed to change data collection days to the Monday and Wednesday for that week. Other disruptions to planned data collection were navigated in a similar way.

4.6.1 Data collection strategies

Interpretive researchers place strong emphasis on gaining a better understanding of their participants' world through firsthand experience (Merriam, 1998). To this end they employ data gathering methods that are sensitive to context (Neuman, 2003) which enable rich and detailed (or thick) description of social phenomena. I used the following data collection strategies to collect the kinds of data that I believed would help me answer my research questions:

- conducting individual (video-recorded) pre- and post-test interviews using Mathematics Recovery's Assessment Schedules 1.1, 1.2, 2.1 and 2.2 with all twenty participants
- observing video-recorded interviews and completing a Record of Response per Task Item
- using completed RRTI's to analyze every learner's performance on assessment items using the LFIN indicators in order to compose individual profiles of learners' early number knowledge prior to and immediately following intervention
- planning and teaching of grouped and individual intervention lessons (video-recorded) using Mathematics Recovery principles, tasks and representational settings

- observing video-recorded intervention lessons to make detailed notes of learners' actions/strategies, gestures, 'math talk' and settings used as they solved 2-digit by 2-digit additive tasks in order to capture/understand changes (if any) in participants' base-ten thinking during intervention
- document analysis of intervention learners' written work produced while participating in intervention lessons

These data gathering techniques allowed me to gather primary and secondary data. By primary data I refer to the initial 'raw' data generated by the individual task-based interviews and intervention lessons conducted with participants which are mostly in the form of video-recordings. A primary data source used in my study that was not a video recording was learners' written work produced during intervention. Literature describes primary research data as information that is gathered 'first-hand' by the researcher and is oriented towards a specific research purpose, which include interviews and video-recordings (Anderson & Arsenault, 2005).

Sometimes secondary data sources are defined as information that has been collected by someone other than the researcher. Here government documents like CAPS (DBE, 2011a) or large scale international data like TIMSS (2008) which are often used in quantitative studies, are cited as examples of secondary data (Anderson & Arsenault, 2005). However, in my study secondary data sources refer to documents like the pre- and post-test 'Record of Response per Task Item' (RRTI) recording sheets, LFIN profiles, partial transcripts of all lessons and full transcripts of exemplary episodes that I produced from the 'raw' data. Thus, the disadvantage normally associated with secondary data (that is, uncertainty about the degree to which secondary data truthfully reflects the primary source) was not relevant to my study as I was the one who produced the secondary data and could thus refer back to the original data source whenever a secondary source needed to be sharpened, verified or revised. Secondary data sources in my study can thus be described as those that summarize and process primary data sources (based on principles that are discussed below) in a way that made it easy for me to access and analyse the data in a manner that answered the research questions. The tension created by the twin roles I played as the advocate of intervention and as researcher of this intervention was dealt with using Joseph Schwab's idea of 'reflective practice' (discussed further in section 4.8.4).

Table 4.4 (which follows) shows how each research question aligns with other aspects of my methodology. What follows after the table is a concise literature-based overview of each data gathering technique and a description of how each was used in my study.

Research questions	Data gathering technique	Primary Data source	Data processing technique/ Analysis	Secondary Data source	Further Analysis	Theoretical Framework	Analytical Framework
1. What was the profile of the sample of Grade 2 learners' early number understanding prior to the intervention?	* conduct individual, video-recorded interviews using MR assessments prior to intervention with all 20 participants	* individual, video-recorded pre-test interviews of all 20 participants	* complete RRTI's using video data; analyse learner responses during pre-test interviews using LFIN descriptors	* Pre-test LFIN profiles of all 20 participants		* individual constructivist aspect of the Emergent Approach	LFIN descriptors for each stage and level on the framework
2. Following intervention... what learning gains (if any) were produced at the end... and how does this compare to learning gains made by the control group?	* conduct individual, video-recorded interviews using MR assessment schedules immediately after intervention with all 20 participants	* individual, video-recorded post-test interviews with all 20 participants	* complete RRTI's using video data; analyse learner responses to post-test interviews using LFIN descriptors	* Post-test LFIN profiles of all 20 participants	* compare all pre- and post - test LFIN profiles for gains * compare intervention and control groups' gains	* individual constructivist aspect of the Emergent Approach	LFIN descriptors for each stage and level on the framework
3. What were the differences and similarities in the nature of learning gains achieved by 'singletons' compared to gains made by learners who received grouped intervention?		* Intervention groups' pre- and post-test LFIN profiles	* split intervention groups' pre- and post-test LFIN profiles into groups and singletons and collate	* Pre- and post-test LFIN profiles of 10 intervention learners split into Group 1, Group 2 and Singletons	* compare gains made by groups with gains made by singletons	* individual constructivist aspect of the Emergent Approach	LFIN descriptors for each stage and level on the framework
4. What was the nature of learners' progress (if any) in base-ten thinking during intervention as evidenced by the strategies they used and the nature of their oral discourse across different representational settings as they worked on additive tasks?	* plan, present and record grouped and individual intervention lessons based on MR principles	* video-recorded intervention lessons with Group 1, Group 2 and two singletons	* watch video-recorded lessons and make note of: tasks, and activities, example sequences, learner solution strategies, 'math talk' and representational settings used to enact strategies	* quasi-transcripts of whole lessons and complete transcripts of exemplary episodes	* interactional aspects: actions/ strategies, 'math talk' and settings used	* social interactionist aspect of the Emergent Approach * Vygotskian sociocultural concepts	*representational flexibility (Heinze et al) & progression (Ensor et al) * additive strategies (Wright et al, Carpenter & Moser, v den Heuvel-Panhuizen, G & Gallistel) * discourse/'math talk' (Anghileri, Sfard, Barwell, Schleppegrell, Scribner) * 'object of learning' → 'tool for learning' (Vygotsky) * talk/actions on interpersonal plane moving to intrapersonal (Vygotsky) * reification of counting processes (Sfard)

Table 4.4

4.6.1 a) Interviews

As a data collection technique, interviews are regarded as one of the most respected tools of inquiry that a researcher can use (Guba & Lincoln, 1981). In a broad sense interviews are thought of as 'conversations with a purpose' (Dexter, 2006) that are conducted either face-to-face or electronically in a structured, semi-structured or open-ended manner, depending on the nature of the data the researcher hopes to elicit. The interviews used in my doctoral study were highly structured in that they consisted of carefully selected and sequenced numeracy tasks that were posed to each respondent using the same words and materials.

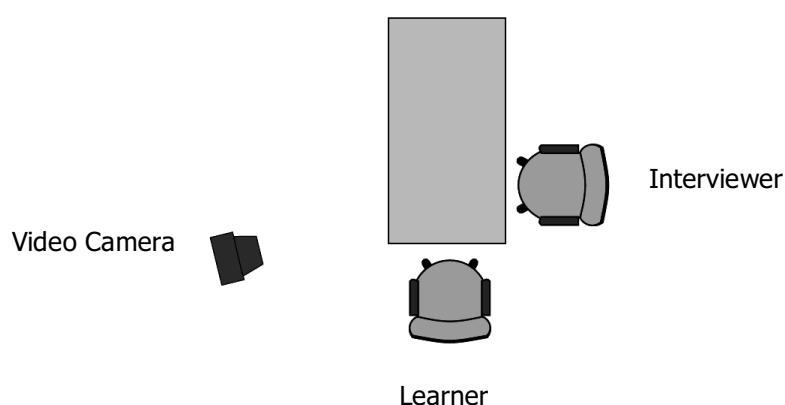
The individual task-based interviews used in my study are derived from Piaget's clinical interview. These interviews were used to help me gain insight into what participants' most sophisticated solution strategies were when working with number problems (Steffe et al., 1983). These interviews followed the MR Assessment Schedules 1.1, 1.2, 2.1 and 2.2 (Appendix A). Unlike conventional clinical interviews that are focused only on determining which level children are at, the task-based interviews being reported on here were further directed towards understanding what progress, if any, children made over a period of time.

Prior to collecting data for my study, I practised using the LFIN assessment schedules with the accompanying materials as each task had to be posed in a particular manner and I had to be careful not to use non-verbal cues that could influence learners' responses. During data collection, I took on the role of the interviewer and posed tasks to participants detailed in the MR Assessment Schedules previously mentioned. I made use of 'talking to the camera' when I observed certain gestures learners' made that I needed to take note of later during transcription. Whilst assessment interviews were very structured my approach during these interviews was relatively flexible as most questions or tasks are structured in such a way that the follow-up question could be 'scaled up' if the child answered the initial question correctly, or 'scaled-down' if the initial answer was incorrect. I tried to establish a relationship of trust with the Grade 2 cohort at the research school prior to the interviews. I ensured that the interview surroundings were familiar and friendly and I was mindful of setting each participant at ease by using age appropriate language. Video recording of all interviews was essential because learners' gestures, as well as their 'math talk', were important for analysis of their solution strategy or counting behaviour. Given that my focus was on observing and responding

to the learner as she completed tasks or answered questions, I did not take additional notes during these interviews, but annotated key reflections after each interview (Steffe et al., 1983; Wright et al., 2006).

The video-recorded interviews were aimed at eliciting participants' most advanced strategies for solving additive relations tasks across various settings. The initial interview data consisted of video recordings of 20 individual task-based interviews that made up the pre-test. These were conducted from the 15th of April 2014 to the 25th of April 2014. The post-test interviews followed the same format and were conducted from the 8th of September 2014 to the 26th of September 2014. During interviews the camera was focused on capturing learners' answers to tasks, their 'math talk' (and sub-vocal talk) and their gestures while responding to tasks. The approximate duration of interviews was: Ass 1.1 – 25 min; Ass 1.2 – 25 min; Ass 2.1 – 8 min and Ass 2.2 – 11 min. These times varied between learners and between pre- and post-test interviews of the same learners. The 'raw' data, that is, the video recording of each participants' task-based interview, was downloaded onto the WMC server and also onto my external hard drive. I watched each recorded interview in turn taking careful note of participants' answers, actions and talk which were all recorded on forms that I designed for use with the Maths Recovery assessment tasks called 'Record of Response per Task Item' or RRTI forms (see Appendix B). Exemplification of how each set of data was analysed is presented in Chapter 3.

Figure 4.1 Set-up of venue where task-based interviews were conducted



4.6.1 b) Intervention lessons

Individual 1 (Khanye) had 19 intervention lessons and Individual 2 (Josh) had 18; Group 1 had 19 intervention lessons and Group 2 had 18 in total (the pilot lesson for each is counted as lesson 1). Within groups there was some variation in the number of lessons members of the group received based on learner absenteeism. When only one learner of the group was absent I carried on with the planned intervention but when two or more learners were absent from the same group I tried to accommodate the group on another day or I extended their subsequent intervention lessons to make up for lost time. One camera (operated by a camera-man) was used to record intervention lessons and this was focused on capturing learners' talk, gestures and settings used. Initial intervention data consisted of a total of 74 video recorded lessons conducted between the 21st of May and the 4th of September 2014.

Following MR principles, the focus of intervention teaching in my study was determined primarily by the learners' SEAL Stage (Wright et al., 2006). For group-intervention lessons I focused primarily on key topics tailored to the three participants who were at the same (higher) SEAL Stage (Stage 3), while my knowledge of tasks aimed the lower SEAL Stage were brought to bear with the fourth learner in each group who was at SEAL Stage 2.

The 'key topics' (related to additive reasoning) that make up the next step for intervention learners at SEAL Stage 3 according to Wright et al. (2006, p.70) were:

- Number word sequences by 2s, 10s, 5s, 3s and 4s
- Numerals from 1 – 1 000
- Incrementing by tens and ones
- Adding and subtracting to and from decade/decuple numbers
- Addition and subtraction to 20, using 5 and 10

The 'key topics' for intervention learners at SEAL Stage 2 in my study were:

- Number word Sequences from 1 to 100
- Numerals from 1 – 100
- Counting-on and counting-back
- Combining and partitioning involving five and ten
- Partitioning and combining numbers in the range 1 to 10

Outline of Intervention Lessons

Intervention lessons with singletons and groups were all modelled after the following lesson outline provided by Wright et al. (2006, p.61, 62)

Step One – Warm Ups [about 5 min]

This time was used to either consolidate skills previously learnt or to develop learners' automaticity with number facts.

Step Two – Problem Solving [about 15 min]

Here the lesson for the day was introduced and new skills were taught.

Step Three – Specific Activity Session [about 15 min]

During this time the learners worked independently, in pairs or groups on similar tasks using the specific skills taught during Step Two.

Step four – Plenary [about 5 min]

The final five minutes of the session was an opportunity for the learners to ask questions and talk about the activities they did during the session. I also used this time to recap, discuss and connect several main ideas relevant to the session.

The venue used for intervention lessons was the school's library which was pleasantly decorated. I usually set the room up as shown in Figure 4.2. However, there were times when I asked learners to sit on the mat – this was usually when learners' squabbles were difficult to control (e.g. kicking each other under the desk) or when we worked with material that required each child to have more space. An anonymized photograph showing learners from Group 1 working on the mat during lesson 3 is included on the next page. Table 4.5 sets out the topics covered for 'Singleton' 1 and Groups 1 & 2 during intervention; and Table 4.6 does the same for 'Singleton' 2. The topics covered during my intervention study were different as these were in keeping with the different foci set out for learners at SEAL Stages 2 and 3 as per the MR programme (Wright et al., 2006).

Figure 4.2 Set up of venue/library where intervention lessons were conducted

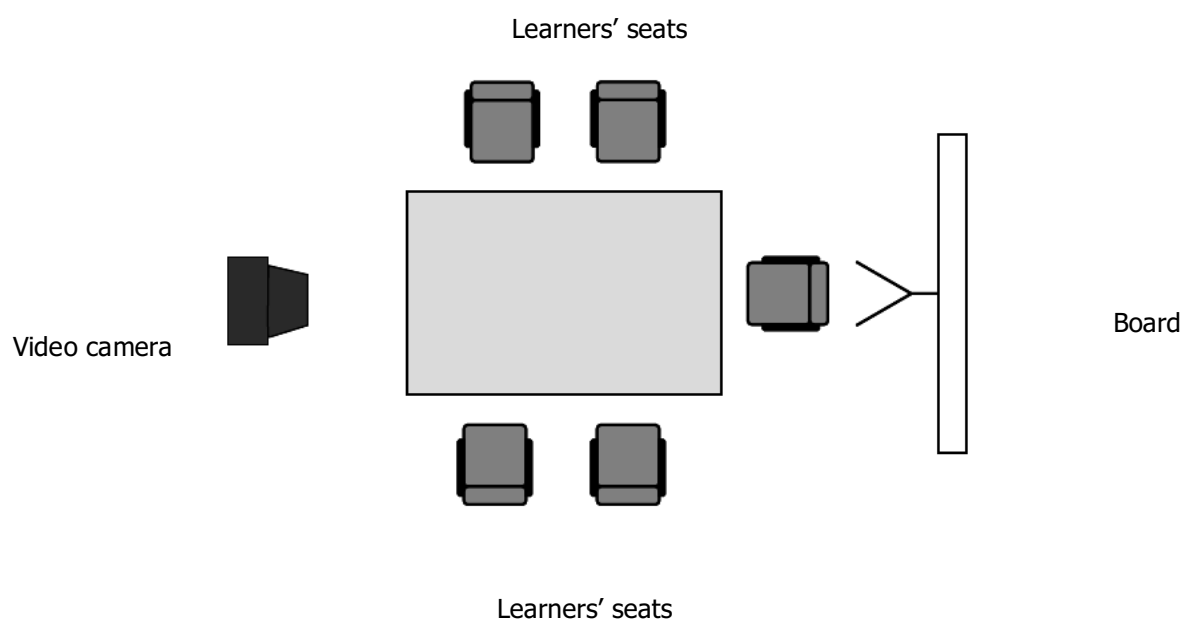


Photo 4.1 Intervention lesson 3 on 2nd June 2014 with Group 1 (Kyle absent)

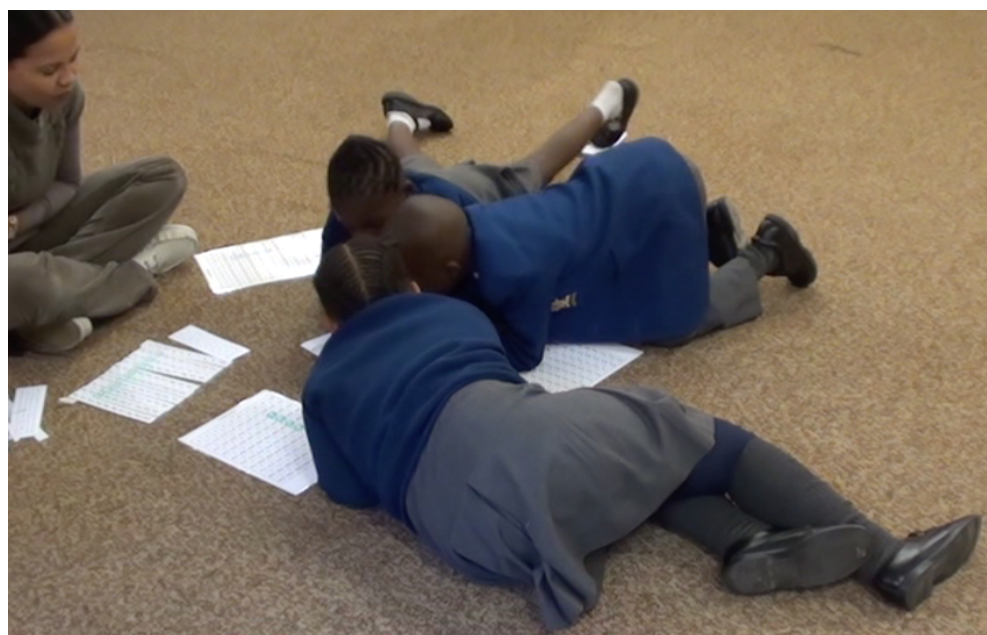


Table 4.5 Overview of topics covered for participants at SEAL Stage 3

Date (2014)	Lesson Number	Topics	Settings
21 May	1	NWS by 2 to 20; identify and sequence 100s numeral cards (100-1000); number before/after on 100s nr track; incrementing by 10 on and off the decuple; base-5	two-dot cards; bundling sticks; numeral cards; 100s number track; 100 chart
29 May	2	F/BNWSs; sequencing numerals; incrementing & decrementing by 10 off the decuple; add from a decuple; subtract to a decuple	100 chart, numeral cards, bundling sticks, ten frames, number sentences (WNS)
2 June	3	F/BNWSs; incrementing & decrementing by ten off the decuple	100 chart; word problems, crazy squares
5 June	4	F/BNWSs; incrementing by 10s and 1s	100 chart, ENL
9 June	5	3-digit numerals; incrementing by 10s and 1s	flard cards, bundling sticks, 100 chart
12 June	6	3-digit numerals; add to a decuple and subtract from a decuple	flard cards, ten frames, WNS
19 June	7	3-digit numerals; add to a decuple; subtract from a decuple	flard cards, ten frames, WNS
22 July	8	F/BNWSs & NWA/B; 3-digit numerals; bonds of ten; incrementing/ decrementing by 10 off the decuple	100 chart; flard cards, ten frames, crazy squares
24 July	9	incrementing & decrementing by 10 off decuple; incrementing by 10 and 1s; add from a decuple; add to a decuple; subtract from a decuple	bundling sticks; ENL; ten frames; WNS
29 July	10	F/BNWSs & NWA/B; 'ten more than' game; subtract from a decuple; filling in numbers on a blank 100ch; incrementing by 10s and 1s	blank 100 chart; ENL; physical and iconic bundling sticks
1 Aug	11	F/BNWSs & NWA/B; 'ten more than' game; filling in number on a blank 100 chart; decrementing by 10 and small multiples of 10; decrementing by 10s and 1s	blank 100 chart; bundling sticks, ENL, 100 chart
4 Aug	12	F/BNWSs & NWA/B; 'ten more than' game; filling in numbers on a blank 100ch; decrementing by 10s and 1s	blank 100 chart; bundling sticks; ENL; 100 chart
7 Aug	13	NWS in 5s; 'ten more/less than' game; base-five; combinations and partitions to 10; commutativity	5-dot cards; ten frames; triads; WNS
19 Aug	14	F/BNWSs & NWA/B; 'ten more/less than' game; identify and expanded notation of 3-digit numbers; combinations and partitions to 20	ten frames, WNS
21 Aug	15	F/BNWSs & NWA/B; 'ten more/less than' game; identify Big Bus flashes; bridging-through-ten (addition)	20-seater Big Bus, red & yellow counters, bus cards; WNS
25 Aug	16	F/BNWSs & NWA/B; 'ten more/less than' game; expanded notation of 3-digit numbers; bridging-through-ten (addition)	flard cards; WNS; Big Bus; red & yellow counters; ENL
29 Aug	17	F/BNWSs & NWA/B; expanded notation of 3-digit numbers; bridging-through-ten (addition with 2-digit as 1 st addend)	Big Bus, red & yellow counters; bead board (rek-n-rek), WNS
1 Sep	18	F/BNWSs & NWA/B; bridging-through-ten (addition & subtraction)	Big Bus, red counters, bead board, WNS, structured nr line
4 Sep	19	NWS by 10s & 20s; structuring strategies: base-ten thinking, doubles & near doubles, compensation; relational thinking using example strings, number triples	number sentences (WNS)

Table 4.6 Overview of topics covered for participant at SEAL Stage 2

Date (2014)	Lesson Number	Topics	Settings
21 May	1	short F/BNWSs & NWA/B; sequencing numerals; number patters; count-on using concrete objects (first addend screened); count-on/ back using worksheets	100 chart; numeral cards; bundling sticks; worksheets
29 May	2	short F/BNWSs & NWA/B; sequencing numerals; count-on using concrete objects (both addends screened); count-on using screened items on worksheets; count-on from the larger number on worksheets	100 chart; numeral cards; bundling sticks; worksheets
2 June	3	short F/BNWSs & NWA/B; 2-digit numeral identification; sequencing numerals; finger patterns (6-9); word problems using count-on from larger	100 chart; numeral cards; word problems; toy cars; pictures of objects
5 June	4	F/BNWSs & NWA/B; 2-digit numeral identification; sequencing numerals; finger patterns using base-5; missing addends tasks with screened concrete objects→ visualization of objects→ written number sentences	100 chart; numeral cards; bundling sticks; WNS
9 June	5	F/BNWSs & NWA/B; 2-digit numeral identification; sequencing numerals; finger patterns 1-5 (on one & two hands); base-5; identifying screened numerals on 100 chart; missing addend tasks	100 chart; numeral cards; bundling sticks; word problems; WNS
12 June	6	F/BNWSs & NWA/B; identifying screened numerals on 100 chart; subitizing regular and irregular (2-8) arrangements; bonds of 10; bonds of 5	100 chart; dot cards; 10 bead string (5 red & 5 yellow); bond house; 5-frame; WNS
19 June	7	F/BNWSs & NWA/B; identifying screened numerals on the 100 chart; bonds of five	100 chart; 2 buckets (orange & lilac); counters; triad diagrams; WNS
22 July	8	F/BNWSs & NWA/B; identify & sequence 2-digit numerals; identifying screened numerals on the 100 chart; recap bonds of 5 and bonds of 10	100 chart; blank 100 chart; numeral cards; 2 buckets & marbles; 10 bead string; WNS
24 July	9	Recap bonds of 10, 5 & commutativity; incrementing and decrementing by 10 on the decuple then off the decuple in increasingly abstract settings; symbolic number patterns	100 chart (some numbers screened); bundling sticks; number patterns; WNS
29 July	10	FNWS; identifying & sequencing 2-digit and decuple numerals; NWS by 10 on decuple forward/bkwd; identifying screened numerals on 100 chart; NWA & two numbers after; NWS by 10 off decuple (forward); numeral sequences	100 chart (some numbers screened); numeral cards; 'numeral ladders'; WNS
1 Aug	11	F/BNWSs & NWA/B; two more than; NWS by 10 on decuple forward/ backward; identify and sequence decuple numeral cards; base-five; bonds of ten; commutativity; doubles and near doubles (1-10 range)	100 chart(some numbers screened); numeral cards; 5 & pair-wise ten frames; WNS
4 Aug	12	F/BNWSs & NWA/B; NWS by 10 off decuple forward/backward; bonds of ten (in increasingly abstract settings)	100 chart (some numbers screened); buckets & marbles; 10 bead string; disappearing number roll; WNS
7 Aug	13	F/BNWSs & NWA/B; number patterns; bonds of 10; commutativity	100 chart (some numbers screened); 5-wise ten frames in two colours; numeral ladder; 10 bead string; WNS
19 Aug	14	F/BNWSs & NWA/B; identifying 2-digit numbers; bonds of 10; commutativity	100 chart (some numbers screened); numeral cards; 10 bead string; 5-w ten frames in two colours; WNS
21 Aug	15	F/BNWSs & NWA/B; number patterns; identify and expanded notation with 2-digit numerals; bonds of ten	100 chart (some numbers screened); numeral ladder; flard cards; 10 bead string; 5-w ten frames in two colours; bonds of 10 worksheet; WNS
25 Aug	16	F/BNWSs & NWA/B; number patterns; identify and expanded notation with 2-digit numerals; identifying 3-digit numerals; bonds of ten	100 chart (some numbers screened); numeral ladder; flard cards; numeral cards; 10 bead string; ten frames; triad diagram; WNS
1 Sep	17	F/BNWSs & NWA/B; number pattern; identify and expanded notation with 2-digit numerals; bonds of 10; base-five	100 chart (some numbers screened); numeral ladder; flard cards; toy bus, picture of 10 seater bus; counters in 2 colours; WNS
4 Sep	18	F/BNWSs & NWA/B; number pattern; identify & expanded notation of 3-digit numerals; NWS by 10; incrementing by 10 and small multiples of 10 using count-on; add-from-a-decuple; decuple + 2-digit number	numeral ladder; flard cards; bundling sticks; 100 chart; WNS

4.6.1 c) Observation

Observation is a commonly used data collection strategy that has the unique strength of allowing the researcher to “gather live data from naturally occurring social situations” (Cohen, Manion & Morrison, 2011, p.456). Through observation the researcher gets first-hand experience of what is taking place *in situ* which can potentially yield more authentic data than second-hand accounts. One caveat with the use this data collection strategy is that what counts as evidence can become cloudy during observation and blur the lines between observation and interpretation. To mitigate against this potential disadvantage researchers should be clear beforehand about what counts as evidence. Observation is also a very useful way of recording participants’ non-verbal behaviour which was important in my study as this gave me another window into learners’ solution strategies which they may have struggled to articulate.

The role an observer plays in a research setting can vary ranging on a continuum from *complete participant* to *complete observer*. During most of my research I took on the role of a *participant-observer* which is considered ‘the most subtly intrusive form of observation’ (Merriam, 1998). Cohen and colleagues (2007) believe that participant observation may prove useful when studying small groups as the researcher assuming this role would spend an extended period of time with participants. Inherent in this role was the potential advantage that I would gain ‘insider knowledge’ of the group being studied but it also had the potential disadvantage of subjectivity in my observations (Cohen & Manion, 1999; Fraenkel, Wallen & Hyun, 2012). Between the two opposite roles of *complete participant* and *complete observer* are the roles of ‘participant-as-observer’ and ‘observer-as-participant’ – the former is described as having comparative involvement to the role of ‘complete participant’ and the latter having a degree of detachment, objectivity and distance that is comparative to the role of a ‘complete observer’ (Cohen et al., 2007). While engaged in data collection activities like conducting interviews and teaching intervention lessons I took on the role of a ‘participant-as-observer’ within the social context, displaying a degree of subjectivity and sympathy with participants. But, while engaged in the analysis of data, I adopted the role of an ‘observer-as-participant’ that was more detached from the events that I was observing. How I managed the tension between these two roles is discussed in section 4.8.4.

"Observation is more than just looking" (Cohen et al., 2011, p.456) thus when I observed video-recorded interviews and intervention lessons my particular purpose was to take note of the solution strategies learners used during task-based interviews and intervention sessions, as well as the 'math talk' and actions on settings that took place during collaborative group work. The analytical framing used for observational data gathered in my study is discussed under section 4.7. Through observation I was able to produce 'secondary data' like transcripts of lessons that were then used in another form of analysis which I discuss next, that is, document analysis.

4.6.1 d) Document Analysis

Literature describes document analysis as a systematic procedure for reviewing or evaluating both printed and electronic documents (Bowen, 2009). This strategy for data collecting is in some instances more narrowly defined as only referring to documents that contain texts and images that have been recorded without a researcher's intervention (Bowen, 2009). However, the broader view: that no written resource can be ruled out of document analysis (Cohen et al., 2007) is writ large here. Organizational texts and institutional documents are usually included in the former account of document analysis while field notes, transcripts and diary entries are included in the latter.

I used document analysis in my study for varying purposes. Documents that can be considered primary data sources, like the written work produced by participants during intervention lessons, were used to gain insight into learners' solution strategies. Other documents examined in my study – previously referred to as secondary data – include: written record of participants' responses (i.e. answers, actions and talk) to each pre- and post-test task item posed (captured on RRTI or 'Record of Response per Task Item' forms); participants' LFIN profiles; partial transcripts of intervention lessons; and full transcripts of exemplary episodes. These data sources were used to track participants' progression on the LFIN for five aspects of early number learning (SEAL, Base Ten, FNWSs & NWA, BNWSs & NWB and Numeral Identification) and give insight into the social interaction during intervention lessons.

I have taken into consideration the critiques levelled against the use of transcripts (and other forms of secondary data) in document analysis as these documents can be seen as containing

'already interpreted data' that "can become an opaque screen between the researcher and the original live... situation" (Kvale, 1996, p.167 in Cohen et al., 2007). However, I believe that because I was not only present but actively involved in the 'original live situations' – that is, as the interviewer during the individual task-based interviews and as the teacher during intervention lessons – this mitigates against the transcripts being an 'opaque screen' between me and the phenomenon that I was researching. Moreover, I did all the transcriptions, not someone who was absent at the time of the 'live' event.

Document analysis was a suitable data-gathering strategy for my study because it made further analyses easy, provided 'contextual richness' that I think may have been overlooked if primary data sources remained in video-format, and because it could be used to corroborate evidence from other sources (Bowen, 2009).

4.7 Data Analysis

Next I explain how different parts of my data set were analysed using the two complementary foci that constitute the Emergent Approach (Cobb and Yackel, 1996), namely, the constructivist analysis of participants' individual early number learning as well as the interpretivist analysis of their actions on settings and oral discourse during intervention. The Emergent Approach acknowledges the importance of both cognitive and social processes in children's learning without subordinating one to the other (Cobb, 1995). Thus, the order in which these aspects are discussed here are not an indication that one aspect is considered more important than the other. I start by describing the analysis process for each data set (collected chronologically) and the analytical frames that were used to code different types of data.

4.7.1 Analysis of Pre- and Post-Test Interviews

Learners' responses to each task posed during individual interviews were recorded on RRTIs for each assessment schedule. Learners' verbal and non-verbal offers were noted down as well as the time within which offers were made: a response offered within 3 seconds was an immediate response and a response offered within 7 seconds was coded as a quick response.

These were then analysed according to Wright et al.'s (2006) Learning Framework in Number (LFIN) which provides a detailed outline of how to assess a range of aspects pertaining to early number knowledge. My summary profile of learners' early number knowledge was based on the following five 'graded' aspects of the LFIN: The Stages of Early Arithmetical Learning (SEAL); Forward Number Word Sequences (FNWSs) and Number Word After (NWA); Backward Number Word Sequences (BNWSs) and Number Word Before (NWB); Numeral Identification and Base-Ten Arithmetical Strategies. The indicators specified by Wright et al. (2006), outlined and discussed in Chapter 3, were used to allocate a stage or level (and correlated score) to every individual learner for each LFIN aspect.

I started analyzing interview data soon after I started data collection, so these processes happened concurrently with the latter taking place at a quicker pace than the former. My first round of data collection and analysis was focused on learners' video-recorded pre-tests which took the form of individual task-based interviews. Immediately after having conducted each interview I noted down anything that stood out for me (like a learner's disposition or a possible speech impediment like stuttering) as I did not record anything during the interview because my focus was on observing and responding to the learner as she completed tasks. While watching the video-recorded interview of every learner I noted the following: the learner's answer to every task and whether this was correct or not; the speed with which answers were offered; the strategies used to produce the answer; and learners' 'math-talk' and gestures used while finding or providing an answer. Learner interviews were conducted in English which allowed me to start analyzing the data as soon as I had collected it. So, each participant's pre-test interview was analysed within two weeks of it having been recorded. Learners' interviews were analysed in the order in which they were conducted – which meant that participants from the same class had their pre-test interviews analysed one after the other.

A similar process was followed for analyzing post-test interviews – these results were recorded in a column to the right of pre-test stages/levels on learners' LFIN profiles. Thus, my analysis of each learner's pre- and post-test interviews produced an individual summary profile of their early number knowledge together with additional notes related to the other interrelated aspects of the LFIN that are not graded, e.g. Spatial Patterns and Finger Patterns. A completed LFIN summary profile is included for each intervention learner in Chapter 5.

4.7.2 Analysis of Intervention Lessons

I planned and taught intervention lessons as described earlier and watched video-recordings of each lesson immediately after returning to campus so that everything that transpired during the lessons were still fresh in my mind. While watching each lesson, I made partial transcripts thereof taking careful note of: the tasks posed to each learner, pair or group; the representational settings used to pose these tasks; learners' verbal responses, response times and gestures; and the strategies and settings learners used when working out solutions to tasks. Making partial transcripts of individual intervention lessons was a bit more straightforward than making notes of grouped-intervention. The strategy I used with the latter was to watch an episode of video data (episodes were based on tasks posed) only focusing on and making notes about one learner in the group and then I did likewise with the same segment of video data for every other participant in the group before moving on the next episode. I also annotated the time-intervals on partial transcripts so that I could easily go back to the exact segment that I wanted to make a full transcription of later. Even though this was time-consuming, it was my way of ensuring that I did not miss anything that group members said or did.

While I observed recorded lessons, and made partial transcriptions of each, I also referred to each learners' written work produced during that particular lesson so that more precise notes could be made of what each learner wrote. I considered learners' actions on settings and 'math talk' that showed them using ten as a unit (or *base-ten thinking*) when solving additive relations tasks as 'critical events' (Wragg, 1994). These were those events that appeared to me to be more of interest than others and thus these were transcribed fully. As mentioned previously, this focus was motivated by the widespread use of counting-based solution strategies to additive and multiplicative tasks well into the intermediate phase in many South African schools which was echoed in learners' solution strategies during their pre-test interviews.

4.7.2 a) Analysis of strategy usage

When a learner first used a less sophisticated/efficient strategy to solve a certain task and then later solved a similar task using a more sophisticated/efficient strategy, I regarded this as a progression in strategy. *What counted as an instance of strategy progression?*

- a visible shift/change in learners' strategy used to solve similar tasks or tasks that require similar number skills (from less to more efficient/sophisticated)
- this change could be visible while learners solved similar tasks within the same setting or across different settings
- this shift/change could occur either within a single lesson or over multiple lessons

When a learner's move from a less efficient to a more efficient strategy (e.g. from 'count all' to 'count on from larger') was already counted as an instance of strategy progression, then that learner's further enactment of 'count on from larger' for a similar task was not counted as another instance of strategy progression. Here the move from 'count all' to 'count on from larger' would already have been accounted for. The same criteria used for strategy progression applied to *strategy reversion* in my study, except that in an instance of strategy reversion the shift/change would have been from a more efficient/sophisticated strategy to one that was less efficient/sophisticated. Representational flexibility occurred when a learner initially enacted an efficient/sophisticated strategy to solve a particular task posed in a more concrete setting and later enacted the same strategy to solve a similar task posed in a more abstract setting. If the learner continued using the more sophisticated strategy to solve other similar tasks, then this was not counted as additional instances of representational flexibility. Exemplification of strategy progressions and reversions as well as representational flexibility (linked to learners' strategy usage) in my study have been included in Chapter 3.

4.7.2 b) Analysis of Oral Discourse

I regarded certain shifts in learners' discourse as *progression in discourse*. For example, when a learner first offered no explanation when asked to explain her solution strategy for a particular task and then later attempted to explain her solution strategy for a similar task even if the language used was not precise; when a learner initially explained her solution strategy to a certain task using imprecise language and then later explained her solution strategy for a similar task using more mathematically precise language; when a learner initially described

her solution strategy to a task by explaining her actions on the setting without illuminating the mathematics involved and then later did the same for a similar task in a way that illuminated the underlying mathematical operation; and finally, when a learner described her physical action (and the underlying mathematics involved) on the setting and then later solves a similar task but this time described her mental action on the setting (and the underlying mathematics involved). So, what counted as an *instance of discourse progression* was a shift/change in the 'math talk' learners used to explain their working, for example, from using everyday/colloquial language to using formal/mathematical language. Shifts in learners' discourse that went in the opposite direction, e.g. from formal/mathematical language to everyday/colloquial language, were described as *reversions in discourse*. When a learner's explanation did not match the strategy she enacted on a setting, then this was labelled as a '*mismatch*' between her discourse and enacted strategy. This category evolved in a 'grounded' way when I discovered (while watching recorded intervention lessons and later *in situ*) that participants would often solve tasks by enacting a base-ten strategy (like N10) on a setting (e.g. 100 chart) and then when asked to explain to the group how they got their answer then they would count in ones on the 100 chart. Sometimes, learners would explain a more efficient strategy than the one they actually used. In both cases their talk was incongruous to the strategy they enacted which I have called a '*mismatch*' between discourse and strategy. My analyses of learners' discourse and strategies used during the intervention period are analyzed against the backdrop of the representational settings in which progressions and/or reversions took place.

4.8 Rigour

Criteria that are often used to judge the quality of any study include validity (also referred to as internal validity), reliability, and generalizability (or external validity). Drawing on the work of Marshall and Rossman (2010) these criteria can be broadly described as follows: validity refers to the truthfulness of a study; reliability speaks of the repeatability of a study; and generalizability speaks of how applicable the study is to other similar participants or settings. Aspects related to rigour speak of the quality of a study and while this was also important in my doctoral study I nevertheless had to remain true to the traditions of the qualitative paradigm framing my research (Cohen et al., 2011). Concurring with this notion, Lincoln and

Guba (1989), posit that although research rigour is a requisite in qualitative studies just as in quantitative studies, *how* this rigour is determined in qualitative studies differs simply because the two research paradigms are based on different ontological and epistemological assumptions.

Aspects of validity within the interpretivist, qualitative paradigm are different to those in the positivist, quantitative paradigm (Feldman, 2003; Maxwell, 1992; Scaife, 2004; Sin, 2010). The trustworthiness of quantitative studies is judged by their degree of internal validity, reliability and external validity, while the criteria for determining the validity or trustworthiness of a qualitative study are the degree to which the findings emanating from the study are credible, relatable/transferable and dependable (Lincoln & Guba, 1985). I agree wholeheartedly with this statement that "...validity is the touchstone of all types of educational research" (Cohen et al., 2011, p.180) so I now discuss how the aspects of validity that pertain to a qualitative study were accounted for in my study.

4.8.1 Credibility of the findings

Credible educational research is described in the literature as research wherein the explanation of a particular event, issue or set of data can actually be sustained by the data (Cohen et al., 2011). Thus, the credibility of a qualitative research study actually speaks to the accuracy of its findings (Creswell, 2012).

Whilst literature in the field of educational research suggests many ways in which to ensure the accuracy of findings in qualitative research (Merriam, 1985; Creswell, 2012; Cohen & Manion, 1999; Cohen et al., 2011), I employed the following strategies to ensure the credibility of my study:

- *submersion in the research situation* – I spent 5 months in the research school gathering data for my doctoral study (i.e. from mid-April 2014 to mid-September 2014) and some more time doing research and development work at the same school with the WMC-P project team prior to and following the data collection period.
- *piloting the task-based interview and practicing my interview skills* – I piloted the task-based interview with a similar group of Grade 2 learners to see whether the tasks were

pitched at an appropriate level for the intended participants and also to practice my interview skills using the LFIN assessment schedules and accompanying materials

- *context-boundedness and thick description* – My data collection took place within participants' natural school setting and a rich description is provided of *how* data was collected, *what* kinds of data was collected, and *why* (or for what purpose) specific data was collected; I also based all interpretive accounts of participants' views using their own words as much as possible
- *the researcher, as part of the researched world, is the primary tool for data collection* – As a teacher-researcher I formed part of the 'researched world' and I was the primary 'tool' for data collection because I collected all the data for my study
- *methodological triangulation* – I looked for consistency in the data I gathered (e.g. video data of learners' explanations and solution strategies and learners' written work produced during intervention) using different data-gathering strategies with the aim of noting recurring patterns and/or inconsistencies
- *investigator triangulation* – Another member of the WMC-P team, who is acquainted with Mathematics Recovery's LFIN, was often involved in observation and analysis of raw data to deal with researcher bias
- *using mechanical means to record, store and retrieve data* – I video-recorded all individual learner interviews and all intervention lessons and stored these on the WMC server and an external hard drive which made it easily retrievable

4.8.2 Relatability of the study

The generalizability or the external validity of any study refers to the extent to which findings from the study can be applied to a wider population or to other situations (Cohen et al., 2011). It is often noted that the greatest stumbling block to claims of value or 'goodness' in qualitative studies is its lack of generalizability (Merriam, 1995). As mentioned previously, the aim of my study was not to make broad generalizations but rather to gain insight into the extent to which the Maths Recovery programme could prove useful in developing a sample of Grade 2 learners' early number learning in a South African context. Therefore, I base the 'goodness' of my study on the relatability thereof (Bassey, 1981), i.e. the degree to which Grade 2 teachers in a similar social context to that being reported on in my study can use the outcomes thereof to

inform their everyday practice or be useful in their attempts at developing the early number learning of a group of children that they teach.

For one school of thought, generalizability through stripping out contextual variables is fundamental, whilst, for another, generalizations that say little about the context have little that is useful to say about human behavior (Schofield, 1990 in Cohen et al., 2011, p.186).

Coming from an interpretivist qualitative paradigm, my aim was not to disregard the contextual nuances inherent in my research, but rather to provide 'thick description' of all aspects of this study so that readers and users of research can determine to what extent my study (and the findings that emanate from it) might be transferable or relatable to their own context. Literature suggests that naturalistic or qualitative studies cannot be judged on the generalizability of its findings but rather on the degree to which the study is *comparable, transferable or relatable* to similar social situations (Lincoln & Guba, 1985; Cohen et al., 2011). Moreover, some authors argue that it is outside the ambit of the qualitative researcher to prove the relatability of her study. The qualitative researcher's responsibility is to provide 'thick description' of all aspects of her study so that other users of research can decide whether (or the degree to which) the study is relatable to their area of interest or not (Cohen et al., 2011).

4.8.3 Dependability of findings

The suitability of the term 'reliability' in qualitative studies is a contested issue amongst educational researchers (Cohen et al., 2011). In quantitative research reliability is described as: 'The extent to which a test or procedure produces similar results under constant conditions on all occasions' (Bell, 1999, p.103). Scaife (2004) argues that it may be easy to judge scientific experiments according to the reliability of the dataset or data-gathering processes because experimental conditions may be easy to control and thus easy to replicate. But, some researchers propose that this criterion may not be appropriate when judging research conducted in classrooms since we cannot expect 'constant conditions' in such an environment (Feldman, 2003; Merriam, 1995; Scaife, 2004). Thus, the term 'dependability' is used as an indicator of rigour in my study (Lincoln & Guba, 1985).

Some suggestions for how the data-gathering process in educational research could be judged for dependability or consistency include: test – retest, equivalent forms, and split-half procedures (Scaife, 2004); triangulation, peer examination and an audit trail (which involves the researcher providing a thick description of how data was collected and analysed) as proposed by Merriam (1995); using a highly structured interview with the same format and sequence of words and tasks/question for each respondent; and/or inter-rater reliability in the coding of responses and training the interviewer prior to data collection (Cohen et al., 2011).

Using literature-based suggestions I enhanced the dependability of my qualitative study by following methods previously discussed (Merriam, 1995; Scaife, 2004; Cohen et al., 2011):

- triangulation
- an audit trail
- highly structured interviews
- inter-rater reliability in the coding of responses
- training of the interviewer
- ensuring a child-friendly environment for data collection

4.8.4 Dealing with tensions inherent in being a participant observer

As noted earlier, I wore different 'hats' during the course of my study – at times I was more of a subjective participant and at other times I was more of an objective observer. Even though these roles were different, they were inextricably linked. As the intervention teacher, I took on the role of 'participant-as-observer'. I had a vested interest in seeing a positive outcome of the intervention, which was that learners who made up the intervention group improved in their early number learning. But, as the researcher I adopted another role, that of 'observer-as-participant' by taking a step back and putting some 'distance' between myself and the data in order to critically study the change (if any) that I hoped to see. I achieved this degree of distance and objectivity by using literature as a vantage point – I looked at the data gathered with the lens of theory and literature to make sense of it.

During the research process, I relied heavily on the process of 'reflection' (which was usually accompanied by memo writing) as a way to 'think about my thinking' because I was constantly moving in and out of different roles. Joseph Schwab's work emphasizes the idea of 'reflective

practice' which is akin to Schön's notion of a 'reflective practitioner' (1983, 1987) which these authors argue is critical practice for participant observers. I was aware that whatever topics I chose to teach, the sequence of those topics, the choice of settings used, and so forth, all involved my 'informed judgement' that I had gained through the theory and literature that framed the study. I used the knowledge I had and the insight I gained through reflecting on each lesson to inform the following lesson. This 'praxis' – the intersection of action/practice and theory – gave me the ability to use theory to inform the intervention process on one hand and also to take a critical look at the tacit assumptions that guided my practice (Kincheloe, 2003).

4.9 Ethics

Ethical clearance was obtained from the University of the Witwatersrand and the Department of Education through the usual procedures before commencement of my study (Protocol number: 2013ECE104D). I informed the principal, Grade 2 teachers and learners about my study orally and in writing and requested each to complete a consent form if they indeed were willing to participate in my study. Because learners are minors I also received informed consent from their parents/caregivers (My ethics clearance and exemplars of each letter and consent form are included in Appendix D).

When I invited potential participants to take part in my study I explained to them verbally and in writing that their participation would be voluntary, and that if they did choose to participate they had the right to withdraw from the study at any time without penalty. Potential participants were informed that taking part in my study would be neither an advantage nor disadvantage to them; that there were no foreseeable risks in participating; and that they would not be paid for their participation. I further assured all potential participants that their names and identities would be kept confidential at all times and in all academic writing emanating from the study, thus pseudonyms are used in this thesis and in other academic writing. Lastly, I assured potential participants of the safe keeping of video-recorded material and confidential documents and that all data obtained during the course of my study would be destroyed between 3-5 years after completion of the study.

4.10 Concluding Remarks

In this chapter I provided a detailed description of the qualitative research design, the intervention approach, the school setting and the participants involved in achieving the broad aim of this study. I have also described the data collection and analytic procedures followed as well as the rationale for making these selections – which, according to Feldman (2003), adds to the credibility of a qualitative study. The credibility, relatability and dependability of my research findings have also been discussed in detail as well as how ethical considerations were dealt with in my study. I believe the thick description of the methodology provided in this chapter shows the fit between each aspect of the study, the broader qualitative paradigm and the Emergent Approach framing this research. The methodology outlined in this chapter is well-linked to and justified by my aim and specific research questions. It also acts as a 'bridge' between the research questions that guided this study and the findings that emanated from it. In the next chapter I discuss these key findings in detail.

Chapter Five – Analysis

An overview of shifts made using Maths Recovery: The Learning Framework in Number (LFIN)

5.1 Introduction to the chapter

In the previous chapter I presented a detailed description of the research methods employed in this study and explained how the data generated was analyzed within a qualitative frame. My detailed observation of a small number of Grade 2 children's number learning was specifically focused on the counting and calculation strategies they used to solve additive problems – which is seen as fundamental to early number learning. My focus on early number learning was motivated by literature within the international landscape which points to the importance of young children acquiring early number skills (Anghileri, 2006; Askew & Brown, 2003), and the current situation in many South African public primary schools concerning learners' poor early number learning (Spaull, 2013).

In this chapter I present an analysis of data related to intervention and control groups' Mathematics Recovery pre- and post-test results. Whilst the intervention groups' outcomes are discussed in detail (in answer to research question 1) I present control learners' pre- and post-test outcomes in summary form to understand the efficacy of the adapted MR grouped intervention model through including a comparative control (research question 2). As mentioned previously, two learners received individual intervention (called 'singletons') and eight learners (who were split into two groups of four) received 'grouped' intervention in order to provide some comparative insights into the relative outcomes of these two intervention models. Thus, I further compare the LFIN results achieved by 'singletons' to that of learners who received grouped intervention (research question 3).

An interpretive framework within social constructivism, referred to as the Emergent Approach (Cobb & Yackel, 1996), was used to make sense of individual learners' constructive processes that occurred while they participated in and contributed to the practices of the local

community. In this chapter I foreground the constructivist aspect of the Emergent perspective to bring into focus the kinds of shifts visible in each individual learner's construction of more sophisticated strategies for solving additive relations tasks.

I start off by presenting an in-depth analysis of intervention learners' pre- and post-test results using the LFIN framework. This analysis is aimed at shedding light on participants' individual learning journeys and highlights general patterns that have emerged across profiles of individual learning as well as counter-examples. The stages and levels individual learners achieved on the LFIN, based on their pre- and post-test interviews, are presented in a LFIN summary profile. Each learner profile provides a summary of the individual learning that took place over a short period of time. I have opted to showcase the data in this way because my study is essentially a story about individual learning (even though some learners received grouped instruction). Each learner's stages and levels for different LFIN aspects in their pre- and post-test are presented in one profile for ease of comparison. These LFIN aspects are:

- Stages of Early Arithmetical Learning (SEAL),
- Base Ten Arithmetical Strategies,
- Forward Number Word Sequences (FNWSs) and Number Word After (NWA),
- Backward Number Word Sequences (BNWSs) and Number Word Before (NWB), and
- Numeral Identification

Gains made in each aspect of the LFIN are noted and these are then discussed – the SEAL and Base-Ten aspects are discussed separately because they fed into the focal area of my research (which involved moving learners away from solving additive tasks using counting strategies to calculating strategies using base-ten thinking). While the FNWSs, BNWSs and Numeral Identification aspects formed part of participants' early number learning, these were not primary focal points and are thus discussed collectively.

The indicators for achieving each stage or level on the LFIN framework have already been outlined and exemplified (Chapter 3) so what follows here is an unpacking of intervention learners' constructions of early number that resulted in them being placed at certain stages or levels on the framework. I flag up caveats or concerns with regard to learners' profiles as some anomalies existed alongside otherwise coherent displays of early number learning.

Presenting the data in this way forms a comprehensive summary picture of learners' facility with different aspects of early number learning before and after intervention.

Research Question 1

What was the profile of the sample of Grade 2 learners' early number understanding prior to and immediately following intervention based on Mathematics Recovery principles?

To answer my first research question I start by presenting LFIN profiles of the two 'singletons' – the learner with the stronger attainment profile first. By presenting the singletons' data one after the other one can get a nuanced sense of the kinds of gains that were possible to achieve for one of the stronger attainers in the intervention group as a whole, compared to the kinds of gains that were possible to achieve for one of the weaker attainers in the group – both of whom received one-on-one intervention. This also provided insights into the kinds of gains made possible within the original MR individual intervention model.

Next I present the profiles of the learners who received grouped intervention – starting with Group 1 followed by Group 2 – also presented from the strongest pre-test attainer in each group to the weakest. By presenting Group 1 learners' profiles in succession one can get a sense of the kinds of shifts that were made possible for individual learners who participated in the same intervention in a group format, but who started at different points; and this could further be compared to the kinds of gains made possible for 'singletons'. By presenting learners' profiles from the second intervention group in the same way another layer of comparison was made possible (between groups) as children's acquisition of early number knowledge is influenced by the context in which mathematics evolves (Shumway, 2011).

5.2 Khanye (1st Singleton)

Table 5.2 Summary profile for Khanye

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	3 ^{x2}	5 ^{x2}	5
2. Base Ten Arithmetical Strategies	2 ↓ ¹⁶	3 ↓	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5 ↓	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	4	5	5
5. Numerical Identification	3	4	4
Totals	20	27	27
Gains	+7		

Khanye made an overall gain of 7, spread across the following LFIN aspects:

- 2^{x2} SEAL Stages
- 1 Level for Base Ten strategies
- 1 Level for Backward Number Word Sequences
- 1 Level for Numeral Identification

5.2.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Khanye's responses to tasks posed from MR Assessment 1.1 placed her at **SEAL Stage 3**: Initial Number Sequence – also known as the 'counting on' stage. She used the 'count on' strategy to solve an addition task with both addends screened (9+6) and 'count up to' to solve a missing addend task (7+□=10). Khanye answered 3 out of 4 removed items/take away tasks correct (using known facts for the first three and 'count-down-from' for the fourth) which was better than how she solved missing subtrahend tasks, that is, 1 out of 3 correct (the first with a known fact and the next two with sub-vocal counting and finger patterns). Even though she did not get the correct solution when she employed 'count-down-from' to solve Q9c (15-3), Khanye's inclination to use advanced count-by-one strategies rather than count from 'one' is what was more important in my assessment of her early number strategies at that point than the correct execution of the procedure.

¹⁶ The downward arrow is used to indicate an 'early' or 'low' attainment of the particular LFIN stage or level.

Post-Test

Khanye completed most of the additive tasks correctly with immediate responses. She also answered most subtractive tasks correctly: some tasks were answered quickly (within 7 seconds) and for others she used advanced count-by-one strategies like 'count-down-from' and 'count-down-to'. Khanye was placed at **SEAL Stage 5**: Facile Number Sequence.

5.2.2 Base Ten Strategies

Pre-Test

When she completed Assessment 1.2 Q1a and b, Khanye incremented by ten on and off the decuple in the setting of ten strips without attempting to count-on in ones. When tasked with incrementing separately by tens and ones (Q1c, Board 1 shown in Figure 5.1) she answered all eight tasks correctly (but her long pause before giving the last answer could have indicated a count-on in ones).

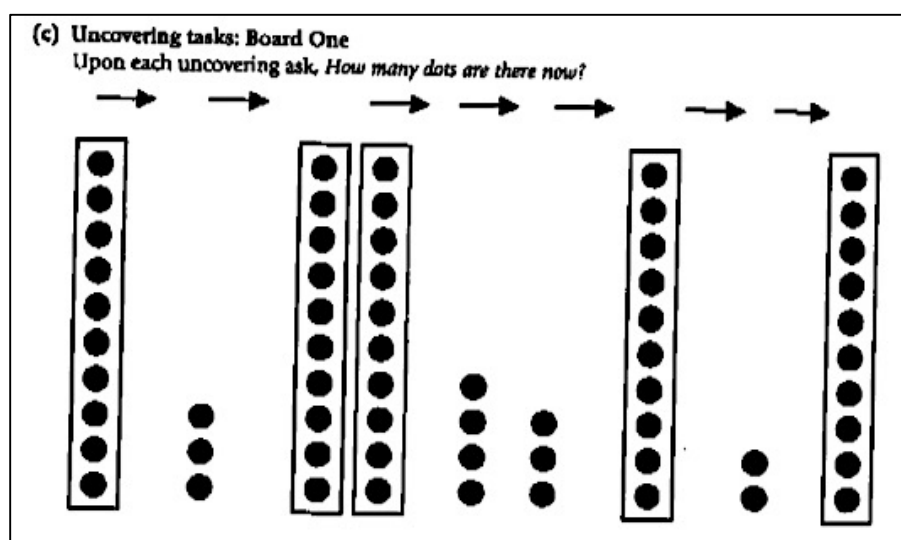


Figure 5.1

When incrementing by tens and ones combined (Q1d, Board 2, Figure 5.2) Khanye answered correctly up to the third uncovering. However, the halting manner in which she answered

Q1d, Board 2 did not convey robust knowledge of incrementing by ten and ones combined, so I placed her at a **'low' Base Ten Level 2: Intermediate Concept of Ten**.

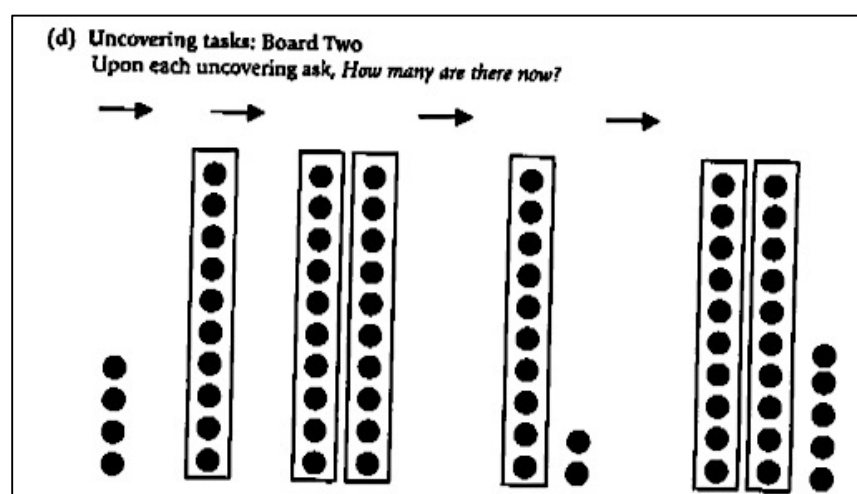


Figure 5.2

Post Test

Khanye incremented fluently by ten on and off the decuple when a tens-based setting was visible and when it was screened. She was also facile when incrementing by ten and ones (separately and combined) when answering uncovering tasks on Boards 1 and 2 – only making an error with the final answer on Board 2. Khanye made use of recalled facts, a mental 1010 strategy (for 2-digit by 2-digit addition with no carrying) and an N10 strategy combined with counting-on to solve 2-digit by 2-digit addition tasks (with carrying) in Assessment 1.2 Q2. Whilst her answers for the four 2-digit by 2-digit additive tasks were all correct, all three of the subtractive 2-digit by 2-digit tasks were incorrect. Although Khanye made some slip-ups with the subtractive tasks (e.g. $56-23$ as $56-20=30...30-3=27$), the strategy she enacted (N10 for all three tasks) showed that she was not thinking in ones. Khanye was thus placed at a **'low' Base Ten Level 3: Facile Concept of Ten**.

5.2.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Khanye was placed at Level 5 for FNWSs and NWA based on her responses to tasks from Assessment 1.1 Q1 and 2. However, she was unable to correctly sequence numeral cards from 46 – 55 in the same assessment (Q7). I thus considered her to be at a **'low' Level 5** because she seemed unable to apply her knowledge of forward number word sequences to another context (numeral cards). I placed Khanye at BNWSs & NWB **Level 4** because she said the BNWSs and the NWB in the range 'thirty' to 'one' without dropping back. Khanye gave the NWB in the range 'one hundred' to 'one' (a greater number range than that indicated for Level 4) but I did not consider this to be enough reason to place her at a higher level for this aspect of the LFIN. Khanye was placed at **Level 3** for Numeral Identification because she correctly identified all the 2-digit numerals shown to her in Assessment 1.1 Q3. She could only identify the '100' of all the 3-digit numerals shown and did not attempt to identify the rest of the 3-digit numerals shown.

Post-Test

Khanye was facile with forward number word sequences from 'one' to 'one hundred and twelve' and gave number words after given numbers in the same range (either quickly or immediately). She also sequenced numerals from 46 – 55 fluently (in ascending order) and showed an awareness of which numeral was next in the sequence she built by often stopping to 'look' for a particular card. Khanye was placed at FNWSs and NWA: **Level 5**. She was facile with backward number word sequences from 'one hundred' to 'one' and swiftly gave the number word before given numbers in the same range. She was thus placed at **Level 5** for this aspect of the LFIN. Khanye correctly identified numerals to 1000; she made an error with '206' but immediately self-corrected. She was placed at **Level 4** for Numeral Identification because she correctly identified all 2- and 3-digit numerals shown.

5.3 Josh (2nd Singleton)

Table 5.3 Summary profile for Josh

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	2 ^{x2}	3 ^{x2}	5
2. Base Ten Arithmetical Strategies		1 [↑] ¹⁷	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	2 ↓	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	2	5	5
5. Numerical Identification	1	3	4
Totals	9	20	27
Gains	+11		

Josh made an overall gain of 11 in the following LFIN aspects:

- 1^{x2} SEAL Stage
- 1 Level for Base Ten strategies
- 3 Levels for Forward Number Word Sequences
- 3 levels for Backward Number Word Sequences
- 2 Levels for Numeral Identification

5.3.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Josh was placed at **SEAL Stage 2**: Figurative Counting after his pre-test interview. He used 'count all' to answer addition tasks (5+4 and 9+6) with both addends screened as well as addition tasks with only the first addend screened (5+2 and 7+3) rather than count on; he only got the first answer correct for each. He offered 'ten' for this missing addend task: 7+□=10 (here his strategy was not visible) and answered two of the three missing subtrahend tasks incorrectly too. He gave the correct answer for the third missing subtrahend task (12-□=9) but the ways in which he connected his strategy to the problem was not clear to me: Josh made a finger pattern for ten, he closed seven fingers sequentially, looked intently at his three raised fingers and said 'three'. He answered two removed items tasks correctly

¹⁷ The upward arrow indicates a 'high' attainment of that particular LFIN stage or level.

('3-1' with an immediate response and '9-4' with a triple count) and the other two ('6-2' and '15-3') incorrectly with sub-vocal counting but no visible strategy.

Post-Test

Josh's most advanced strategy used for solving additive relations tasks during his post-test was to 'count on/back' in ones, usually with the help of his fingers to keep track of his count. After a shaky start with Ass. 1.1 Q8a (addition tasks with both addends screened), Josh went on to correctly complete similar tasks (Q8e) by 'counting on'. He answered both missing addends tasks correctly: the first ($7 + \square = 10$) with a known fact (inferred from his immediate response with no overt counting) and the second ($12 + \square = 15$) with a quick response (unidentifiable strategy). Josh answered three out of five missing subtrahend tasks correctly: the first ($5 - \square = 3$) with an immediate response, the second ($10 - \square = 6$) using 'count-down-from' and the fourth task ($15 - \square = 11$) with an unknown strategy (when I asked him how he got his answer he said he worked it out in his head). Josh was thus placed at **SEAL Stage 3: Initial Number Sequence**, also called the 'counting on' stage. He did however enact a more rudimentary strategy ('count all') when answering one addition and one removed items task but I did not consider this unusual as Wright et al. (2006) note that learners do not always make use of their most sophisticated strategy to answer tasks, and particularly so when these strategies are relatively recent expansions to existing repertoires.

5.3.2 Base Ten Strategies

Pre-Test

Josh enacted a unit count when presented with tasks designed for incrementing in ten on and off the decuple using a tens-based setting (Assessment 1.2). He did not use the 'count on' strategy to solve additive relations tasks which would have placed him at SEAL Stage 3 – thus he could not be placed on any level on the LFIN model for Base Ten.

Post Test

Josh incremented by ten on and off the decuple using ten strips during his post-test interview. He also correctly answered the first four openings of Board 1 (incrementing by ten and ones

separately) which gave me a bit more insight into his emerging appropriation of using ten as a unit. He answered the uncovering tasks as follows: (first opening: ten dots) 'ten', (second opening: three dots) 'thirteen', (third opening: twenty dots) 'thirty-three', (fourth opening: four dots) 'thirty-seven', (fifth opening: three dots) 'thirty', (sixth opening: ten dots) 'forty', (seventh opening: two dots) 'forty-two', and (final opening: ten dots) 'fifty'. Josh really only made two errors – at the fifth opening and the final opening because the other two errors followed through mathematically correctly from his incorrect starting points. Josh demonstrated here that he could add from a decuple ($10+3$ and $40+2$), increment by ten from a decuple ($30+10$) and increment by two groups of ten off the decuple ($13+20$) using what seemed like a base-ten strategy (because of his quick answer $< 7\text{sec}$). Josh was placed at a **'high' Base Ten Level 1: Initial Concept of Ten** because he used ten as a unit to solve a few addition tasks in a tens-based setting but he was not able to do so consistently, thus I considered him to be between Levels 1 and 2. The tasks on Board 2 were not presented to Josh because of the number of errors he made with tasks on Board 1, which were simpler.

5.3.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Josh was placed at a **'low' Level 2** for the FNWSs & NWA aspect because even though he correctly recited the FNWS from 'one' to 'ten' and gave the NWA in the same range he could not correctly sequence numeral cards 1-10 which suggested that his knowledge of the FNWS in this range was 'localized' as he could not use this knowledge in the context of a written number sequence. Josh recited the BNWS from 'ten' to 'one' correctly without omitting any number words and gave the NWA in the same range, but dropped back to 'one' for some answers – he was thus placed at **Level 2: Intermediate BNWS**. Josh fluently identified all 1-digit numerals in his pre-test interview (Assessment 1.1) but could not do the same with 2-digit numerals. He struggled to identify 'teen' numerals (he took very long to identify '15' and made no offer when shown the numeral card for '13') and confused 'teen' and 'ty' numbers, for example, he said 'eighteen' for the numeral '80'. For Numeral Identification Josh was placed at **Level 1: Numerals to '10'**.

Post-Test

Josh was placed at **Level 5**: Facile with FNWSs up to 'one hundred' because he produced the forward number word sequences in the range 'one' to 'one hundred' with only one error (he omitted the sixties in the 48-61 sequence, saying 'fifty-nine, seventy, seventy-one, seventy-two') and he gave the NWA in the same range with one error ('forty-one' offered for NWA 'fourteen'). What was interesting to note was that Josh said the FNWS from 'forty-two' to 'seventy-two' fluently in his attempt at answering ' $42+23$ ' in Assessment 1.2 – so his difficulty with omitting the sixties number words in the 48-61 oral sequence seemed sporadic. Whilst I considered him to be facile with FNWSs and NWA in the range 'one' to 'one hundred' Josh did not sequence numeral cards 46 – 55 correctly (post-test Assessment 1.1 Q7a). Based on my use of corroborating evidence when placing learners at particular LFIN stages/levels, this could have thrown doubt on my assessment of Josh's facility in the FNWSs aspect of the LFIN. However, when I looked closely at how he ordered these numeral cards (51, 52, 53, 54, 55, 46, 47, 48, 49, 50) it looked like he focused on the unit digits without considering the value of the tens digits which is a common error for young children.

Josh produced BNWSs in the range 'one hundred' to 'one' without omitting any number words and gave the NWB given numbers in the same range without dropping back. He still confused the 'teen' and 'ty' numbers, for example, when I asked him to say the 15-10 sequence he said: "fifty, forty-nine..." before I stopped him and clarified that he should start the backward count at fifteen not fifty. He then went on to say the correct sequence fluently. Josh improved by three levels on the LFIN framework to **Level 5**: Facile with BNWSs up to 'one hundred' at the time of his post-test. He was placed at **Level 3**: Numerals to '100' because he correctly identified 1- and 2-digit numerals during his post-test interview. The only 3-digit numeral that he identified correctly was '100' – offering "twelve hundred and three" for '123' and "eighty-two" for '820'. He offered no response for the other 3-digit numerals shown ('206' and '341').

5.3.4 Overview comment for ‘singletons’

Summary profiles of the two learners who received one-on-one intervention show that both learners made gains in the SEAL and Base Ten aspects of the LFIN and that the weaker attainer showed a higher extent of change overall. I return to discuss these interesting outcomes in section 5.12.1.

I next present similar summary profiles for the learners in the two group intervention formats.

5.4 Bongi (Group 1)

Table 5.4 Summary profile for Bongi

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	3 ^{x2} ↑	5 ^{x2}	5
2. Base Ten Arithmetical Strategies	1 ↑	3	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	5	5	5
5. Numerical Identification	3	4	4
Totals	20	27	27
Gains	+7		

Bongi made 7 gains overall which were spread over the following LFIN aspects:

- 2^{x2} SEAL Stages
- 2 Levels for Base Ten strategies
- 1 Level for Numeral Identification

5.4.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Based on how he answered problems posed on Assessment 1.1 Bongi was placed at **SEAL Stage 3**: Initial Number Sequence after his pre-test interview. He answered addition tasks 5+4 and 9+6 (both addends screened) with quick answers and no visible strategy; he answered one missing addend task ($7 + \square = 10$) immediately and the other ($12 + \square = 15$) with no visible strategy. Bongi did not overtly use the 'count on' strategy to solve additive tasks but he counted back in ones to solve both written subtractive tasks posed in Q9a (16-12 and 17-14). For example, to answer 16-12 Bongi put 16 'in his head' and counted back twelve times (fifteen, fourteen, thirteen, twelve, eleven, ten, nine, eight, seven, six, five, four) keeping track of the counts he made on his fingers, with the solution being the number word spoken on the twelfth count. These subtractive tasks were intended to elicit advanced 'counting-by-ones' strategies (Wright et al., 2006) like 'count-down-to' (here the learner would count from the minuend to the subtrahend: fifteen, fourteen, thirteen, twelve – keeping track of the number of counts, i.e. 4, as this would be the answer) or a non-count-by-one strategy

like 'bridging-thru-ten' (here the learner would use ten as a benchmark: $16-6=10$ and $10-6=4$) rather than the more extended 'counting-by-ones' strategy Bongi used which was more prone to errors (Gray, 1991; Wright, 2001). Bongi had mixed success using the 'counting-by-ones' strategy to solve the two written subtractive tasks – the first task was answered correctly and the second incorrectly.

A learner is considered to be at SEAL Stage 4: Intermediate Number Sequence when she uses 'count-down-to' to solve missing subtrahend tasks and when she shows appropriate use of 'count-down-to' and 'count-down-from' strategies (Wright et al., 2006). Even though Bongi correctly answered all four missing subtrahend tasks as well as three out of five removed items tasks – the manner in which he did so did not demonstrate an ability to select the most efficient strategy. Five of the correct solutions to these two sets of tasks were answered immediately with no overt counting or calculating and the other two correct solutions were produced with some sub-vocal counting (SVC) but not any identifiable strategy. One could argue that an immediate response is more sophisticated than any advanced 'counting-by-ones' strategy because the former is most often based on a 'known number fact' – and I will not dispute this. But, I believe that the ultimate aim here was to see whether the learner could choose the most sophisticated (and thus most efficient) strategy for a particular task. Thus, using the evidence I have of Bongi's imprudent use of counting back from 'one' to answer both written subtractive tasks (mentioned earlier) – I believed he had not displayed the ability to select the most efficient or sophisticated strategy for certain tasks. Thus, I placed Bongi at a **'high' SEAL Stage 3**.

Post-Test

After his post-test interview Bongi was placed at **SEAL Stage 5: Facile Number Sequence** – the highest SEAL stage. Bongi answered most additive relations tasks set in Assessment 1.1 with an immediate answer using no overt counting or calculating which suggested that he solved these tasks using known number facts. He also used mental strategies like the 1010 or 'split' strategy to solve subtraction sentence '16-12' and a grouped subtraction strategy to solve '27-4' (as $27-2-2$). Bongi used the 1010 strategy again during Assessment 1.2 to solve a 2-digit by 2-digit addition task ($42+23$) and enacted the N10 or 'jump' strategy on an ENL to solve 2-digit by 2-digit additive relations tasks like '39+53' and '43-15'. In spite of improved

facility with split and jump strategies, Bongi was not always successful with 2-digit by 2-digit additive relations tasks in his post-test – he got two out of three addition tasks correct and one out of three subtraction tasks correct – but SEAL Stage 5 could be applied here due to his fluid recourse to split and jump strategies in the context of these problems.

Additionally, Bongi used other non-count-by-one strategies during his post-test. For example, he 'incremented by one from a constant number' when answering Assessment 1.2 Q3a ($9+3$, $9+4$, $9+5$, $9+6$) fluently and used 'compensation for addition' by answering all but the second item in Q3b ($6+6$, $7+5$, $8+4$, $9+3$) either immediately or quickly. Bongi also used relational thinking like 'using addition to solve subtraction' when answering Q3d ($15+3$, *Can you use what you just done to help you do $18-3$?*) immediately, and used a known subtraction fact to work out an associated subtraction problem in Q3e (*If I tell you $21-16=5$ can you use what I just told you to work out $21-5$?*) by also answering immediately. Further, Bongi could derive two subtraction facts from an addition fact when answering Q3f from the same assessment schedule ($14+4=18$ *There are three numbers here. Can you use these to make a take-away sum? Can you make another one?*). This evidence, which reaffirms Bongi's placement at the highest SEAL stage links to studies on relational working with number conducted by Baroody and colleagues (1983, 1999) which found that children who used advanced addition strategies (based on reasoning or recall of number facts) were more likely to use the complement relation than those who relied on laborious counting strategies to solve addition tasks.

5.4.2 Base Ten Strategies

Pre-Test

During his pre-test (Assessment 1.2 Q1 a, b) Bongi incremented by ten on and off the decuple in the setting of ten strips, without attempting to count on in ones. When incrementing by tens and ones separately with Board 1 (Q1c) Bongi answered five of the eight tasks incorrectly and when incrementing by tens and ones combined (with Board 2) he answered correctly up to the third uncovering. On the surface it seemed as though Bongi fulfilled the requirements of being placed at Base Ten Level 2 (because he answered Board 2 correctly up to the 3rd uncovering) but I did not consider him to be fully at Level 2 because of the five errors he made on Board 1: from 33 dots already added he incremented by four dots offering an

incorrect answer of 24 (12sec) on his first attempt and another incorrect offer of 36 (22sec) on his second; then from 36 (although incorrect) he incremented correctly by three and got 39 (8sec); but from 39 he incorrectly incremented by ten offering 43 (30sec). I considered Bongi to be in-between Base Ten Levels 1 and 2 – at a '**high**' **Level 1**: Initial Concept of Ten.

Post-Test

Bongi was placed at **Base Ten Level 3**: Facile Concept of Ten after his post-test. He was considered to be at this level because he incremented and decremented by tens and ones when solving written number problems without tens-based materials. His responses to tasks set in Q2 of Assessment 1.2 (Table 5.4.1) largely determined whether he would be placed at Base Ten Level 3, but as mentioned earlier, corroborating evidence from other tasks were also considered when placing learners at particular levels for all aspects of the LFIN. However, correctly answering all the tasks set in Q2 of Assessment 1.2 was not a pre-requisite for being placed at Level 3 (Wright et al., 2006, p.167).

Table 5.4.1 MR Assessment Schedule 1.2 Question 2

2. Horizontal Sentences	
(a) Do you have a way to figure out what is $16+10$?	
So, what is $16+9$?	
(b) Do you have a way to figure out what is $42+23$?	
If correct ask, Do you have another way to work it out or check?	
(c) Do you have a way to figure out what is $38+24$?	
If correct ask, Do you have another way to work it out or check?	
(d) Repeat the above questions for: $39+53$	
(e) $56 - 23$	
(f) $43 - 15$	
(g) $73 - 48$	

When answering the Uncovering Boards question in Q1 of Assessment 1.2 (here learners had to increment by ones, tens, and tens and ones in the setting of dot strips) Bongi used ten as a block in various ways: incrementing by ten from a single digit number ($4+10$) and from a

2-digit number ($52+10$); incrementing by two groups of ten off the decuple ($13+20$ and $14+20$) and incrementing by tens and ones off the decuple ($34+12$ and $46+25$). During Assessment 1.2 Q3i Bonggi's immediate answer to '7-5' followed by quick answers to '27-5' and '47-5' led me to infer that he made a connection between these number sentences

From his responses to Q2 of the same assessment, Bonggi showed evidence of incrementing by ten off the decuple when he answered number sentence $16+10$ quickly with no overt counting or calculating. He also worked separately with tens and ones (the 1010 strategy) and incremented by two groups of ten when he solved number sentence $42+23$ as $40+20=60$, $2+3=5$, answer 65. By using a N10 strategy on an ENL to solve 2-digit by 2-digit additive relations tasks in Q2 ($39+53$ and $43-15$) Bonggi again incremented and decremented with groups of ten off the decuple. How he worked during the post-test to answer these written number tasks is shown below.

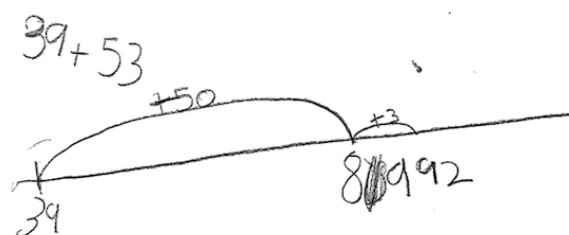


Figure 5.3

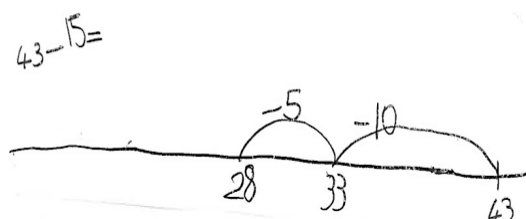


Figure 5.4

5.4.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Bonggi's answers for his pre-test interview (Assessment 1.1 Q1, 2, 5, 6 and 7) placed him at the highest levels for the FNWSs & NWA and BNWSs & NWB aspects of the LFIN – **Level 5**. He produced the forward and backward number word sequences in the range 'one' to 'one hundred' without omitting any number words and without stumbling when crossing decuple boundaries. He also gave the number words before and after given numbers in the same range without dropping back to 'one'. His knowledge of number word sequences was further evidenced when he correctly ordered numeral cards from 46-55 in Q7. Bonggi identified 1- and

2-digit numerals during his pre-test but was not facile with identifying 3-digit numerals (1 out of 5 correct). While trying to identify 3-digit numerals in his pre-test he offered number names such as “twenty-six” when shown the numeral card ‘206’ and “thirty-fourty-one” for ‘341’. Bongi was placed at **Level 3** for Numerical Identification.

Post-Test

In his post-test Bongi showed greater fluency (he took less time to answer) with forward and backward number word sequences (and NWA/B) and was again placed at **Level 5**: Facile with F/BNWSs up to ‘one hundred’ (as there were no higher levels). During his post-test interview Bongi correctly identified all 1-, 2- and 3-digit numerals flashed. This put him at **Level 4**: Numerals to ‘1000’ (the highest level) for Numeral Identification – an increase of 1 level for this LFIN aspect.

5.5 Mali (Group 1)

Table 5.5 Summary profile for Mali

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	3 ^{x2}	4 ^{x2} ↓	5
2. Base Ten Arithmetical Strategies	1 ↑	2	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	3	5	5
5. Numerical Identification	3	4	4
Totals	18	24	27
Gains	+6		

The 6 overall gains can be seen across Mali's pre- and post-test profile in these aspects:

- 1^{x2} SEAL Stage
- 1 Level for Base Ten strategies
- 2 Levels for BNWS & NWB
- 1 Level for Numeral identification

5.5.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Mali solved addition problems $5+4$ and $9+6$ (Q8a, both addends screened) by making a finger pattern for the first addend and then 'counting on' the second addend using a sub-vocal count coordinated with the sequential raising of fingers – which literature shows is a "natural and intuitive scaffold for calculation" (Jordan, et al., 2008, p.662). So, to calculate $5+4$ Mali opened five fingers simultaneously on her left hand saying 'five' sub-vocally and then opened four fingers sequentially on her right hand with a coordinated sub-vocal count-on (a similar strategy was used for $9+4$ except here the first addend was shown on both hands). She solved missing addend tasks ($7+\square=10$ and $12+\square=15$) by using her fingers to 'count up' from the first addend to the sum (with a subtle SVC), with the number of counts/fingers moved as her answer. Mali solved both subtraction sentences ($16-12$ and $17-14$) in the same way. For example, to answer $17-14$ Mali opened seven fingers (subtly) and then closed four of these fingers sequentially; the remaining three fingers were her answer – which showed an emerging ability to work separately with tens and ones. She seemed to use this same strategy,

where she worked separately with tens and ones, to answer removed items tasks (15-3 and 27-4). Mali's explanation for her solution to 27-4 was:

M : I put twenty (*taps left hand once, lightly on the desk*) ... then I put seven (*taps left hand again, slightly to the right of previous tap*) ... then I took four out.

T : Took four out of where? ... Because you were saying you put twenty, then you put seven, and then you took four out.

Mali offers no reply.

T : Did you take the four out of the twenty or out of the seven?

M : Out of the seven.

Mali showed facility with finger patterns to ten and used these readily to solve additive relations problems. She solved missing subtrahend problems $5-\square=3$ and $10-\square=6$ by making a finger pattern for the minuend and simultaneously closing the number of fingers for the difference, with the fingers that remained as her answer. For missing subtrahend problems with minuends bigger than ten ($12-\square=9$ and $15-\square=12$) Mali's strategy was difficult to detect, except for a subtle sub-vocal count for the former. She was not considered to be at SEAL Stage 4: Intermediate Number Sequence because she did not use 'count-down-to' to solve missing subtrahend tasks. Mali was placed at **SEAL Stage 3: Intermediate Number Sequence** because the 'count on' strategy was her most advanced strategy used when solving additive relations tasks.

Post-Test

The strategies employed by Mali during her post-test interview painted a mixed picture of her early number learning up to that point. Her use of particular strategies suggested that she had progressed over the course of intervention but in some cases her choice of strategy suggested otherwise. For example, Mali used a 'count-down-to' strategy to solve missing subtrahend task $12-\square=9$ by counting backwards from the minuend to the difference (e.g. eleven, ten, nine), keeping track of the number of counts on her fingers. The number of counts made (three) was her answer. The use of this strategy for solving a missing subtrahend task was more efficient than a 'count-down-from' or backward count and the judicious use thereof qualified her to be placed at SEAL Stage 4: Intermediate Number Sequence (Wright et al., 2006). However, a learner at SEAL Stage 4 should also have shown appropriate use of the 'count-down-from' strategy (Wright et al., 2006). For example, when solving a removed items task like 15-3 (Q9c) using 'count-down-from' the learner would count back three counts

from fifteen (fourteen, thirteen, twelve) and the last number word spoken would be the answer. In her post-test, Mali used the 'count-down-from' strategy to solve written subtraction sentences 16-12 and 17-14 which was very inefficient. So, Mali seemed to read these subtraction tasks as 'take away' – which research shows to be the predominant understanding related to symbolic subtraction sentences (Fischbein, Deri, Nello, & Marino, 1985) – while using 'count-down-to' to solve missing subtrahend tasks. This suggests a 'direct model' of the situation in both instances in Carpenter et al's (1999) terms, so she did not display the flexibility to choose the most efficient strategy. Developers of the MR programme state that a learner who attempts to solve written subtraction tasks like 16-12 by 'counting-down-from' is not at SEAL Stage 4 (Wright et al., 2006, p74).

Similarly, Mali used rudimentary strategies to solve additive tasks in Q8a where both addends were screened, that is, a finger pattern to solve $5+4$ and a finger pattern and 'counting on' to solve $9+6$. Why Mali chose to use these strategies was not clear to me as she was well able to use either a non-count-by-one strategy like 'bridge-thru-ten' or known number facts as demonstrated during intervention lessons and in other places in the post-test. For example, out of the fifteen additive relations tasks posed in MR Assessment 1.1, Mali offered eight correct answers either immediately or quickly (the speed with which these answers were offered led to the assumption that they were based on known number facts) during her post-test compared to two in her pre-test. Thus, while she definitely improved in her appropriation of early number facts, she lacked consistency in applying these facts during problem solving.

I looked across Mali's responses to tasks in post-test Assessment 2.2 (which is primarily aimed at discerning a learner's facility with structuring numbers 1-20) for evidence that either confirmed or disproved her placement at a 'low' SEAL Stage 4 and found here too that she slipped back to using less efficient strategies at times even though she also showed that she was well able to successfully enact more efficient strategies to solve additive tasks (as seen in post-test Assessment 1.1). Mali was judged to be at a **'low' SEAL Stage 4** at the time of her post-test. This was because she judiciously used 'count-down-to' for missing subtrahend task, $12-\square=9$ (Ass 1.1 Q9b) and enacted other non-count-by-one strategies even though she did not show appropriate use of the 'count-down-from' strategy.

5.5.2 Base Ten Strategies

Pre-Test

Mali had to increment by ten on the decuple (10, 20 ...80) in the setting of tens-based materials in the first task of Assessment 1.2, and she did so correctly. The subsequent task involved incrementing by ten off the decuple (4, 14...74) in the same setting, and she was unable to do so without resorting to 'counting on' in ones. Mali's pre-test interview was stopped here because subsequent tasks were more challenging – requiring learners to increment by tens and ones (separately and combined). Based purely on her responses to pre-test interview 1.2 Mali appeared to be at Base Ten Level 1 because she was unable to increment by ten off the decuple using tens-based materials without 'counting on' in ones. But, as mentioned elsewhere, I used evidence from other MR Assessments to corroborate my findings – and Mali's way of responding to tasks set in Assessment 1.1 provided further insight regarding her ability to reason in tens and ones. For example, when solving subtraction sentences (16-12 and 17-14) and removed items tasks (15-3 and 27-4) Mali showed an emergent appropriation of early place value concepts by mentally organizing numbers into tens and ones for ease of calculation when she solved each of these tasks. Taking this evidence into consideration, Mali was considered to be in-between Base Ten Levels 1 and 2 – at a **'high' Level 1**.

Post-Test

During her post-test interview Mali incremented by ten on and off the decuple (with tens-based materials) without attempting to count on in ones. When incrementing by tens and ones (Boards 1 & 2) she correctly completed the first three of the eight uncovering tasks posed on Board 1, and the first three of the five tasks on Board 2. Developers of the MR programme consider a learner who completes the first 3 tasks on Board 2 to be at Base Ten Level 2, but I had doubts about this when I considered that Mali made five errors on Board 1. Further analysis of her errors on Board 1 showed that when Mali had to increment by four from thirty-three she made an error (answer 'thirty-nine') and from there onwards, although her subsequent increments were correct, all her 'answers' were incorrect. After a closer look at how Mali answered the uncovering tasks on Boards 1 and 2, I concluded that Mali could:

- increment by 10 from a single digit ($4+10$ B2)

- add-from-ten ($10+3$ B1)
- increment by two groups of ten off the decuple ($13+20$ B1 and $14+20$ B2)
- increment by 10 off the decuple ($39+10$ and $42+10$ B1 – even though the starting points here were incorrect).

Further, Mali solved written number problems like $42+23$, $38+24$ and $39+53$ (Ass 1.2 Item 2) using a mental N10 or 'jump' strategy on an ENL. However, she did so with guidance from me, thus I did not consider her to be at Base Ten Level 3, but in-between levels 2 and 3 (a **'high' Base Ten Level 2**). Mali's inability to increment by tens and ones in the setting of tens-based materials (the 3rd and 4th uncovering tasks on Board 2) and her persistent counting on/back when solving tasks that were aimed at eliciting learners' use of knowledge of Base Five and Base Ten in Assessment 2.2, further solidified my assessment of her placement at a 'high' Base Ten Level 2.

5.5.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

During her pre-test interview Mali could produce the FNWSs in the range 'one' to 'one hundred'. She made a small error with the second sequence (48-61) by counting backward rather than forward, but after this was corrected she fluently completed the sequence. Mali also gave the number word after a given number in the same range and was therefore placed at the highest level for this aspect of the LFIN – **Level 5**: Facile with FNWSs up to 'one hundred'. Mali could not give the correct BNWSs for: $23 - 16$, $34 - 27$ and $72 - 67$. With the first sequence mentioned Mali omitted one number word (twenty-one) and in the other two sequences she substituted the decuple number word for that of the previous decuple, for example: "seventy-two, seventy-one, sixty, sixty-nine, sixty-eight" – errors that are commonly made by young children (Wright et al., 2006). Mali had difficulty crossing decuple boundaries in a higher number range but was facile with the BNWS and NWB 'ten' to 'one' and was thus placed at **Level 3**: Facile with BNWSs up to 'ten'. During her pre-test interview Mali correctly identified 1- and 2-digit numerals (which placed her at **Level 3**) with ease but struggled with identifying 3-digit numerals. For example, during the pre-test she identified the numeral card '341' as "thirty hundred and forty-one" and '820' as "eighty hundred and twenty".

Post-Test

Similar to her pre-test interview, Mali produced the FNWSs and NWA in the range 'one' to 'one hundred' almost flawlessly, thus this aspect remained the same (**Level 5**). Although she had difficulty crossing the decuple boundary on the backward count in her pre-test interview, this picture changed by the time of the post-test because Mali was able to correctly produce BNWSs and NWB in the range 'one hundred' to 'one' without any omissions or dropping back. Thus, Mali improved by two levels for the BNWSs and NWB aspect of the LFIN: from Level 3 to **Level 5**. Mali correctly identified all the 3-digit numerals presented to her during her post-test interview. Her skill with identifying numerals improved by one level: from Level 3 to **Level 4**. The errors that she made when identifying 3-digit numerals during the pre-test interview and early intervention lessons were now a thing of the past.

5.6 Jenna (Group 1)

Table 5.6 Summary profile for Jenna

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	3 ^{x2}	4 ^{x2} ↑	5
2. Base Ten Arithmetical Strategies	1	2 ↑	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	4 ↑	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	3	5	5
5. Numerical Identification	3	4	4
Totals	17	24	27
Gains	+7		

Jenna made 7 gains across her pre- and post-tests:

- 1^{x2} SEAL Stage
- 1 Level for Base Ten strategies
- 1 Level for FNWS & NWA
- 2 Levels for BNWS & NWB
- 1 Level for Numeral identification

5.6.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Based on how she responded to tasks posed from MR Assessment 1.1 Jenna was placed at **SEAL Stage 3**: Initial Number Sequence because her most advanced strategy used was 'count on'. Jenna used the 'count on' strategy to solve two addition tasks Q8a 5+4 and 9+6 (both addends screened, only the first task correct). Her strategy for solving missing subtrahend and removed items tasks was impossible to detect even after watching the video footage three times (as she often covered her face with her hands). I also could not discern the strategy she used for her incorrect offer (8) for subtraction sentence '16-12' and when I asked how she got to her answer she replied, "I think in my head". Jenna had mixed success with all types of additive relations tasks posed: 1 out of 3 additive tasks correct (2 tasks with both addends screened and 1 with only the first addend screened), 1 out of 2 missing addends tasks correct, 2 out of 4 missing subtrahend tasks correct, 2 out of 4 removed items tasks correct and the only subtraction sentence posed (as a number sentence) was incorrect.

Post-Test

Jenna's post-test results place her at SEAL Stage 4: Intermediate Counting Phase – which is a gain of 1 SEAL stage. She showed judicious use of the 'count-down-from' and 'count-down-to' strategies when answering tasks in Assessment 1.1 which qualified her to be at SEAL Stage 4 (Wright et al., 2006). For example, to answer the missing subtrahend task $15 - \square = 11$ Jenna said the backward number word sequence: "fourteen, thirteen, twelve, eleven" while raising four fingers sequentially – her answer was the number of fingers raised on the last count which was a 'count-down-to' strategy. She used the 'count-down-from' strategy to answer removed items task $15 - 3$ – here she said the backward number word sequence: "fourteen, thirteen, twelve" keeping track of the number of counts made using her fingers – the third number word spoken (twelve) was her answer. Even though Jenna only showed efficient use of these advanced 'count-by-one' strategies with these two examples she was still considered to be at SEAL Stage 4 because a learner's most advanced strategy was used to determine her stage or level on the LFIN framework (Wright et al., 2006).

A big improvement could be seen between the manner in which Jenna answered tasks in the pre-test compared to the post-test: she used known number facts to answer two tasks during the pre-test but did so for five tasks in the post-test (the learner's correct immediate responses to tasks were indicative of her using a known fact); she used advanced 'count-by-one' strategies like 'count-down-from' and 'count-down-to' appropriately during the post-test while not at all during the pre-test; and she used a mental non-count-by-one strategy (1010 strategy) during the post-test but not in the pre-test.

When considering whether Jenna was at SEAL Stage 5: Facile Number Sequence I looked at how she responded to tasks from the third task group of Assessment 1.2 and task groups 8 and 9 of Assessment 1.1 (Wright et al., 2006). To be awarded SEAL Stage 5 on the LFIN a learner must have used a range of 'non-count-by-one' strategies (like 'bridging-thru-ten', compensation and relational thinking) to solve at least three additive relations tasks across the assessment task groups previously mentioned (Wright et al., 2006, p. 102). The only non-count-by-one strategy Jenna enacted was the 1010 strategy used to answer two subtraction sentences (with mixed success) in Assessment 1.1 and one incident of relational thinking to solve Q3d from Assessment 1.2 (here the learner was asked to solve the number sentence

15+3, and then asked: Can you use what you just done to help you do 18-3?) – which she answered with an immediate correct response. I did not consider this to be enough evidence for awarding SEAL Stage 5 because Jenna used a very limited range of non-count-by-one strategies to solve tasks from the third task group in Assessment 1.2. Other confirming evidence was that she solved ten of the fourteen tasks from this task group using either counting on or back. Based on this evidence Jenna seemed to be between SEAL stages 4 and 5 – a **'high' SEAL Stage 4**.

5.6.2 Base Ten Strategies

Pre-Test

During her pre-test interview (Assessment 1.2 Q1 a) Jenna incremented by ten on the decuple in the setting of ten strips, without attempting to count on in ones. But, when this was followed with incrementing by tens off the decuple in the same setting (Q1 b, '4-74') she was unable to do so. Jenna was not posed subsequent tasks that involved incrementing by tens and ones separately (i.e. Q1c Board 1) because she could not successfully increment by ten off the decuple in the previous task. Jenna was thus placed at Base Ten **Level 1** because she struggled to solve additive tasks using ten as a unit in a tens-based setting.

Post-Test

During her post-test Jenna showed facility with incrementing by ten on and off the decuple with tens-based materials (ten strips). She was also facile with incrementing by tens and ones – getting all uncovering tasks correct on Board 1 and up to the third uncovering task correct on Board 2. This qualified Jenna to be placed at Base Ten Level 2. To be placed at Level 3 (the highest Base Ten level) learners had to increment and decrement by ten without tens-based materials when solving 2-digit by 2-digit written number additive relations tasks (Wright et al., 2006) like those in task-group 2 of Assessment 1.2: $42+23$, $38+24$, $39+53$, $56-23$, $43-15$ and $73-48$. Jenna used a N10 strategy to solve these problems but was only successful with the one addition and one subtraction problem that did not require carrying or decomposition ($42+23$ and $56-23$). For example, Jenna solved $42+23$ as $42+10+10+3$ and $56-23$ as $56-10-10-3$. While she successfully incremented and decremented by ten mentally

for some of these tasks she was unable to do so consistently, thus ending up with incorrect answers on most 2-digit by 2-digit additive relations tasks. When I took into account that Jenna answered most tasks from post-test Assessment 2.2 with counting on/ back – bearing in mind that this group of tasks assessed learners’ facility with using Base Five and Base Ten as reference points (Wright et al., 2006) – this confirmed that Jenna was between Base Ten Level 2 and Level 3 for her post-test (a **‘high’ Level 2**).

5.6.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Jenna was sporadically facile with forward number word sequences (FNWSs) up to ‘one hundred’ and was not facile with giving the NWA in the same number range. When posed with NWA tasks Jenna often repeated the number given a few times before answering. Jenna’s responses to FNWSs and NWA tasks made it difficult to place her at a particular level for this aspect of the LFIN, because she did not display robust knowledge of FNWSs and NWA in the range ‘one’ to ‘one hundred’ (Level 5) and yet she was more advanced than Level 4 (facile with FNWSs up to ‘thirty’). Therefore, I placed Jenna at a **‘high’ Level 4** for FNWSs and NWA. Jenna was facile with giving the BNWS from ‘fifteen’ to ‘one’ and the NWB in a higher range (thirty to one). When giving the BNWSs in higher number ranges Jenna made the common error of skipping a decuple when crossing the decuple boundary. For example, on the BNWS for 34-27 Jenna said “twenty” after “thirty-one” instead of “thirty” and skipped the number words “twenty-nine” to “twenty-one”. Based on this she was placed at **Level 3**: Facile with BNWSs up to ‘ten’ for this aspect of the LFIN – even though she exceeded the criteria with regard to giving the NWB. During her pre-test interview Jenna correctly identified 1- and 2-digit numerals. When asked to identify 3-digit numerals she made some errors like saying “one two three” for numeral card ‘123’ and “eighty twenty” for ‘820’. Jenna was thus placed at **Level 3**: Numerals to ‘100’.

Post-Test

Jenna was facile with FNWSs from ‘one’ to ‘one hundred’ and gave the number word just after a given number in the same range without dropping back to one. She thus attained the highest

level for this aspect of the LFIN – **Level 5**: Facile with FNWSs up to 'one hundred'. Jenna was placed at **Level 5** for the BNWSs and NWB aspect of the LFIN – a gain of 2 levels – because she said the BNWSs fluently from 'one hundred' to 'one' and gave the NWB a given number in the same range without dropping back. After her post-test Jenna was placed at **Level 4**: Numerals to '1000' because she correctly identified 1-, 2- and 3-digit numerals (without any hesitation) including 3-digit numerals with zero as a place holder (e.g. 206 and 820) that are often difficult for young children to identify.

5.7 Sibü (Group 1)

Table 5.7 Summary profile for Sibü

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	2 ^{x2}	4 ^{x2}	5
2. Base Ten Arithmetical Strategies		2 ↓	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	3	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	2	4	5
5. Numerical Identification	3 ↓	3 ↑	4
Totals	12	22	27
Gains	+10		

Sibü made 10 overall gains spread across the following LFIN aspects:

- 2^{x2} Seal Stages
- 2 Levels for Base Ten strategies
- 2 Levels for FNWSs & NWA
- 2 Levels for BNWSs & NWB

5.7.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

The manner in which Sibü answered additive tasks in his pre-test Assessment 1.1 placed him at **SEAL Stage 2**. He used a 'count all' strategy (also called a triple count, Anghileri, 2006) to find solutions to additive tasks where both addends were screened in Q8a,e (5+4, 9+6, 8+5 and 9+3) and also in Q8b where only the first/biggest addend was screened (5+2, 7+3 and 9+4). When answering subtraction sentences (Q9a), missing subtrahend tasks (Q9b) and removed items tasks (Q9c) Sibü did a lot of counting by ones, keeping track of counts on his fingers. I could often not discern what strategy he was using because his counting activity would not follow a predictable pattern nor would it produce a correct answer. For example, when solving the missing subtrahend task $10 - \square = 6$, he counted from 'one' to 'six' a few times, and then a few times from 'one' to 'ten', before offering 'one' as his answer. I could not see how he got to 'one' from this counting activity or from his finger-pattern. Another example of this kind of counting strategy was evident in how he answered removed items task: 9-4. Here

he counted on his fingers in ones from 'one' to 'nine' and then from 'one' to 'four' a few times before offering 'eight' as his answer. Again, *how* he got to 'eight' from the counting activity just outlined was unclear to me, and Sibü was unable to offer reasons for his answer.

Post-Test

After his post-test interview Sibü was considered to be at **SEAL Stage 4**: Intermediate Number Sequence. During post-test Assessment 1.1 Sibü answered Q8a $5+4$ (both addends screened) using the 'count on' strategy. With his next task $9+6$ (both addends screened), he used 'count all' – a less efficient strategy. I then encouraged him to rather use 'count on' if he was able to, as one of the principles of MR assessment is that the interviewer should elicit the learner's most sophisticated strategy. Sibü complied and subsequently answered additive tasks $8+5$ and $9+3$ (both addends screened) using the 'count on' strategy. For tasks where only the first addend was screened (Q8b) Sibü initially used a 'count all' strategy but when I asked him to tell me how he found the answer he explained a 'count on' strategy. For the following two tasks in Q8b ($7+3$ and $9+4$) Sibü used the 'count on' strategy. The difference between the strategies Sibü used to answer Q8a and Q8b in his pre- and post-test was evidence that he was at least at Stage 3 (the 'counting on' stage) at the time of his post-test.

Sibü also used 'count-down-from' and 'count-down-to' in his post-test and chose the more efficient of these strategies for particular problems, that is, 'count-down-from' for removed items tasks ($9-4$ and $15-3$) and 'count-down-to' for missing subtrahend tasks ($12-\square=9$ and $15-\square=11$). His use of the more efficient strategy when answering removed items and missing subtrahend tasks was what put him at SEAL Stage 4. Sibü's biggest gain was in the SEAL aspect of the LFIN because a 'stage' in the LFIN is broader than a 'level', based on Wright et al.'s (2006) argument that moves between stages require substantial conceptual reorganization. His jump from SEAL Stage 2 to SEAL Stage 4 was significant because this meant that during the pre-test his most advanced strategy for solving additive relations tasks was 'counting from one' but during his post-test he was able to use advanced counting-by-ones strategies like 'count on' and 'count-down-to' – which pointed to a substantial qualitative change in his early number learning.

5.7.2 Base Ten Strategies

Pre-Test

When tasked with incrementing by ten on the decuple using ten strips (Assessment 1.2) Sibü counted in ones using his fingers. He was unable to increment by ten on or off the decuple using tens-based materials and thus could not be placed at any Base Ten level, as this aspect of the LFIN does not have level 0. Further, to be placed at Base Ten Level 1 a learner must at least have been at SEAL Stage 3 – which Sibü was not at the time of his pre-test interview.

Post-Test

During his post-test Sibü incremented by ten on and off the decuple using tens-based materials. When tasked with incrementing by tens and ones separately (uncovering tasks on Board 1) he answered 7 out of 8 tasks correctly. The next task involved incrementing by tens and ones combined (Board 2) and he answered the first two out of five tasks correctly. His correct offers made during these uncovering tasks show that Sibü used ten as a block when he incremented by ten from a single digit number ($4+10$ B2) and from a 2-digit number ($52+10$ B1) and also when he incremented by two groups of ten off the decuple ($13+20$ B1 and $14+20$ B2). His responses put him very close to meeting the requirements for Base Ten Level 2. I also took into account the fact that Sibü made an attempt to use ten as a reference point when solving written tasks from Assessment 2.2 although his efforts were unsuccessful, thus I placed him at a **'low' Level 2**.

5.7.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Sibü could produce the FNWS 'one' to 'thirty' without omitting any number word but could not give the number word after a given number in the same range even when he dropped back to 'one' (he gave the NWA in a lower number range: 1-10). He was placed at **Level 3**: Facile with FNWSs up to 'ten' for FNWSs and NWA even though he slightly exceeded the requirements for this level on the LFIN framework. I initially struggled to get Sibü to understand the task of counting backwards during his pre-test. His BNWS from 'ten' to 'one'

was a staccato count accompanied by sequential moving of his fingers. His attempt at the backward number word sequence for 15-10 was “fifteen, fifty-nine, fifty-eight” – which could point to an auditory discrimination problem (confusing *fifteen* with *fifty*) and crossing the decuple boundary backwards (if he thought he had to count backwards from fifty). When asked for the NWB single digit numbers he struggled to produce the correct answers – using a lot of sub-vocal counting (SVC). Using the LFIN framework, I placed him at BNWSs & NWA **Level 2**: Intermediate BNWS up to ‘ten’. Although Sibü correctly identified 1- and 2-digit numerals, I noticed that he did a SVC before saying certain numbers. For example, before identifying the numeral ‘80’ Sibü used a sub-vocal forward count in tens (I could not discern his starting point but I heard him count in tens, ending on ‘eighty’) and before identifying the numeral card for ‘12’ he said “ten, eleven, twelve” sub-vocally. He was thus considered to be at a **‘low’ Level 3**: Numerals to ‘100’.

Post-Test

Sibü improved two levels in FNWSs & NWA to **Level 5**: Facile with FNWSs up to ‘one hundred’ – the highest level. During his post-test Sibü could produce FNWSs in the range ‘one’ to ‘one hundred’ without omitting any number words and could also give the NWA a given number in the same range without dropping back to ‘one’. He crossed decuple boundaries fluently, including the boundary of ‘one hundred and ten’. After his post-test interview Sibü was placed at BNWSs & NWB **Level 4**: Facile with BNWSs up to ‘thirty’ because he could produce BNWSs in the range ‘thirty’ to ‘one’ without omitting any number words and also gave the NWB any given number in the same range. Sibü made the following errors when giving the other BNWSs: “thirty-four, thirty-three, thirty-two, thirty-one, twenty, nineteen” and “seventy-two, seventy-one, sixty, sixty-nine, sixty-eight” which Wright and colleagues argue is a common error amongst young learners as they learn to cross decuple boundaries using a backward count (Wright et al., 2006).

Sibü fluently identified all 2-digit numerals in his post-test without any sub-vocal counting, and correctly identified two out of the five 3-digit numerals flashed (these were not even posed in the pre-test). Even though improvement in this LFIN aspect was made, I do not think it was enough to move Sibü to a higher level. He was thus considered to be between levels 3 and 4 for Numeral Identification – a **‘high’ Level 3**. This is the only LFIN aspect for which

Sibu did not move up a level although a shift could be seen *within* the level: from a 'low' to a 'high' Level 3.

5.7.4 Overview comments for Group 1

Across Group 1, all learners showed gains in the SEAL and Base Ten aspects of the LFIN with the greatest extent of gains shown by the weakest pre-test attainer. Thus, Group 1 had similar outcomes to that of the 'singletons'. One caveat that must be noted here is the difficulty I experienced when drawing up the summary profile for Mali: I was torn between placing her at a 'high' SEAL Stage 3 or a 'low' SEAL Stage 4 and after much wrestling I gave her the benefit of the doubt (as learners do not always display the most advanced strategy in their 'arsenal') and settled on the latter. Further discussion of these results follows in section 5.12.1.

5.8 Julie (Group 2)

Table 5.8 Summary profile for Julie

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	4 ^{x2}	5 ^{x2}	5
2. Base Ten Arithmetical Strategies	1 ↑	3	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	5	5	5
5. Numeral Identification	3	4	4
Totals	22	27	27
Gains	+5		

Julie's pre- and post-test results show 5 overall gains spread across these LFIN aspects:

- 1^{x2} SEAL Stage
- 2 Levels for Base Ten strategies
- 1 Level for Numeral identification

5.8.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Based on her pre-test results Julie was placed at **SEAL Stage 4** because she used 'count-down-to' to solve one missing subtrahend task ($15 - \square = 11$) by saying, "fourteen, thirteen, twelve, eleven" with the coordinated raising of four fingers in turn – the number of fingers she raised was her answer. Julie also used the 'count-down-from' strategy to solve two removed items tasks ($15 - 3$ and $27 - 4$). For example, she solved $27 - 4$ by counting backward in ones from 'twenty-seven' for four counts ("twenty-six, twenty-five, twenty-four, twenty-three") while keeping track of the number of counts on her fingers – the number word spoken on her fourth backward count was her answer. She also used the 'counting on' strategy to solve two additive tasks (both addends screened) and answered six of the sixteen additive relations tasks posed during Assessment 1.1 with no overt counting or calculating – five of these answers were given immediately, i.e. within 3 seconds. Placing a learner at a particular SEAL stage involved determining the most advanced strategy available to that child at that time (Wright et al., 2006) – the 'count-down-to' strategy used by Julie is regarded as more sophisticated than the other 'advanced-counting-by-ones' strategies and is thus indicative of

SEAL Stage 4. Further, Julie's judicious use of the 'count-down-from' and 'count-down-to' strategies when solving additive relations tasks confirmed that she was at SEAL Stage 4: Intermediate Number Sequence.

Post-Test

Based on her post-test results Julie was placed at **SEAL Stage 5**: Facile Number Sequence. The awarding of Stage 5 for this aspect of the LFIN was determined by how Julie responded to tasks from the third task group of Assessment 1.2 and task groups 8 and 9 of Assessment 1.1 (Wright et al., 2006). To be awarded SEAL Stage 5 on the LFIN Julie must have used a range of 'non-count-by-one' strategies (like 'bridging-thru-ten', compensation and relational thinking) to solve at least three additive relations tasks across the assessment task groups previously mentioned (Wright et al., 2006, p. 102).

During her post-test (Assessment 1.2) Julie used a known fact to answer $9+3$ (evidenced by no overt counting and an immediate response) and then relational thinking (incrementing by one from a constant number) when solving these subsequent tasks: $9+4$, $9+5$ and $9+6$ (evident by the absence of any overt counting or calculating and the speed at which each answer was given). Julie also used relational thinking when solving other tasks from this task group: $7-5$, $27-5$ and $47-5$ – here she incremented by two groups of ten off the decuple and gave the next example in the sequence ($67-5$) with a correct result. When solving the example string ' $6+6$, $7+5$, $8+4$ and $9+3$ ' Julie seemed to treat the first two examples as separate tasks (an incorrect offer of 15 for the second example that she self-corrected) but her quick responses to the third and fourth examples showed that she noticed the relationship between the addends of each successive example and used compensation to solve these.

When solving multi-digit additive and subtractive tasks like $42+23$ and $56-23$ from a different task group in Assessment 1.2, Julie used base-ten strategies like 1010 and N10. When responding to task groups 8 and 9 in Assessment 1.1, Julie correctly answered twelve out of fifteen additive relations tasks in the range 1 – 20 using a known fact (i.e. with an immediate response and no overt counting or calculating). Her flexible use of known facts within the range 1-10 to solve subtractive and additive tasks in a higher number range was evident in her responses to my question 'How do you know?' For example, after her correct offer of

“four” to the written subtraction task ‘16-12’ Julie responded, “Because four plus two is equal to six”. Similarly, after answering the missing addend task ‘12+□=15’ Julie said, “Because two plus three equals five”. These examples also speak to Julie’s understanding of addition and subtraction as inverse operations as well as her ability to flexibly structure number.

5.8.2 Base Ten Strategies

Pre-Test

During her pre-test Julie incremented by ten on and off the decuple but was more facile doing so with unscreened tens-based materials than with screened materials. For example, she incremented by ten on the decuple (10, 20 30...80) and off the decuple (7, 17, 27...87) with ten strips that were visible, but made some errors when the strips of tens and ones were covered on Board 1 and Board 2. Even though Julie correctly answered the first three items of Board 2 – which qualified her to be at Base Ten Level 2 – I placed her at Level 1 for this aspect of the LFIN because she made four errors with the eight uncovering tasks on Board 1. However, her responses to questions from Boards 1 and 2 showed that she knew how to increment by two groups of ten off the decuple with no overt counting or calculating in the setting of visible and screened dot strips (e.g. 13+20 on Board 1 and 14+20 on Board 2). She also added-from-a-decuple (e.g. 10+3 and 40+2 on Board 1) and incremented by ten from a single digit number (e.g. 4+10 on Board 2). So, I considered Julie’s Base Ten strategies at the time of her pre-test as at a **‘high’ Level 1**.

Post-Test

Based on Julie’s responses to the first two task groups of Assessment 1.2 she was placed at Base Ten Level 3: Facile Concept of Ten for her post-test. Julie incremented by ten, multiples of ten and by tens and ones on and off the decuple using unscreened and screened ten strips – which put her firmly at Level 2. But, she also incremented and decremented by tens and ones using the 1010 and N10 strategies when solving written additive and subtractive multi-digit tasks like 42+23 and 73-48 which put her at Base Ten Level 3. Although she had mixed success with these written tasks during her post-test, Julie was still awarded Base Ten **Level**

3 because all multi-digit tasks did not have to be answered correctly to be awarded Level 3 (Wright et al., 2006, p. 98).

5.8.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Julie was placed at the highest level for the FNWSs & NWA – **Level 5**: Facile with FNWSs up to 'one hundred' for her pre- and post-tests. She produced the forward number word sequences in the range 'one' to 'one hundred' without omitting any number words and also gave the number word after given numbers in the same range without dropping back to one. Julie said backward number word sequences in the range 'one hundred' to 'one' without omitting any number words and gave the number word before given numbers in the same range without dropping back to one. This qualified her to be placed at **Level 5**: Facile with BNWSs up to 'one hundred' for her pre-test. Julie correctly identified 1- and 2-digit numerals during the pre-test but was not facile with identifying 3-digit numerals (two out of five correct). She was thus placed at **Level 3**: Numerals to '100'.

Post-Test

Julie remained on the highest levels for the FNWSs & NWA and BNWSs & NWB aspects after her post-test interview, that is, **Level 5**. She also correctly identified all 1-, 2- and 3-digit numerals shown. This put her clearly at **Level 4**: Numerals to '1000' – the highest level for Numeral Identification on the LFIN framework.

5.9 Khosi (Group 2)

Table 5.9 Summary profile for Khosi

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	3 ^{x2} ↑	5 ^{x2}	5
2. Base Ten Arithmetical Strategies	1	3	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	3	5	5
5. Numeral Identification	3	4	4
Totals	18	27	27
Gains	+9		

Khosi made an overall gain of 9, spread across different LFIN aspects:

- 2^{x2} SEAL Stages
- 2 Levels for Base Ten Strategies
- 2 Levels for BNWSs & NWB
- 1 Level for Numeral identification

5.9.1 Stages of Early Arithmetical Learning (SEAL)

Pre-Test

Based on her pre-test results Khosi was placed at SEAL Stage 3: Initial Number Sequence because her most advanced strategy for solving additive relations problems was 'counting on'. Khosi used 'count on' to solve additive tasks like $9+6$ (both addends screened) and missing addend tasks like $12+\square=15$. She also used 'count up to' to solve a missing subtrahend task: $12-\square=9$. 'Counting up to' is a strategy occasionally used by a learner at SEAL Stage 3 in the context of a missing addend task, but when this strategy is used to solve a missing subtrahend task this is usually indicative of a learner at SEAL Stage 4 (Wright et al., 2006, p.74). However, when placing learners at different SEAL stages I looked for corroborating evidence. For example, the developers of the Mathematics Recovery programme state that a learner who attempts to solve written removed items tasks like $16-12$ by 'counting-down-from' is not at

SEAL Stage 4 (Wright et al., 2006, p.74). During her pre-test Khosi used the 'count-down-from' strategy for written tasks: 16-12 and 17-14. Even though she done so successfully, this was not the most efficient strategy between 'count-down-from' and 'count-down-to'. Another prerequisite for being at SEAL Stage 4 is being able to choose the most efficient between the two strategies (Wright et al., 2006). Thus, Khosi was not placed at SEAL Stage 4 but was identified as being between stages 3 and 4 – a '**high**' **SEAL Stage 3**. Khosi did not only use counting strategies to solve additive and subtractive tasks in the pre-test: she solved seven out of the fifteen additive relations tasks posed in Assessment 1.1 as known facts (indicated by an immediate correct answer).

Post-Test

Khosi's post-test results placed her at **SEAL Stage 5** – showing a gain of 2 SEAL stages. A learner at SEAL Stage 5: Facile Number Sequence uses a range of advanced counting strategies like 'count-down-to' (which may involve some counting by ones) to solve additive and subtractive problems, as well as 'non-count-by-one' (NCB1) strategies such as 'bridging-thru-ten', compensation and subtraction as the inverse of addition.

Khosi used non-count-by-one strategies like N10 and 1010 to solve two-digit by two-digit additive and subtractive problems in Assessment 1.2. For example, she added $42+23$ using a mental 1010 strategy by pointing to the two tens digits and saying "sixty", then she said "five" while pointing to the units digits and gave her final answer as "sixty-five". She also incremented by one from a constant number when presented with an example string like: $9+3$, $9+4$, $9+5$ and $9+6$. After she correctly answered these questions in less than 3 seconds each, I asked her how she got to the answers so fast and she replied, "I just add one more". The speed and accuracy of her responses showed relational thinking in her working with the following tasks from Assessment 1.2: Task 3d) 'What is $15+3$? Can you use this to help you do $18-3$?' and Task 3e) 'If I tell you $21-16=5$ can you use this number sentence to help you to do $21-5$?' Other evidence that confirmed Khosi to be at SEAL Stage 5 was that she used 'count-down-from' to answer two removed items tasks (which is more appropriate than 'count-down-to') and a 'count up to' strategy for a missing subtrahend task ($15-\square=11$), which points to her understanding of addition and subtraction as inverse operations.

Contrary to all other evidence for SEAL, Khosi used 'count-down-from' to solve the subtractive task '16-12' which is not indicative of a learner at SEAL Stage 4 as this is an inefficient strategy for this task. In spite of this, overwhelming evidence of her judicious use of count-down-to/from to solve other subtractive tasks (mentioned previously) and her facility with non-count-by-one strategies pointed to her being placed at SEAL Stage 5 at the time of her post-test.

5.9.2 Base Ten Strategies

Pre-Test

MR Assessment 1.2, which is aimed at assessing children's ability to reason in tens and ones as well as their understanding of early place value, was used to assess Khosi's facility with Base Ten strategies. In task 1 Khosi incremented by ten on the decuple using ten strips (10, 20, 30...80) and off the decuple in the same setting (4, 14, 24...74). She added ten to a multiple of ten without resorting to counting-by-ones and was sporadically facile with incrementing by ten off the decuple using ten strips. However, when tasked with incrementing by tens and ones using ten strips (Board 1) Khosi first used a 1010 strategy and then resorted to counting-by-ones. The last item on Board 1 required learners to increment by ten off the decuple (i.e. adding ten dots to 52 dots already counted) and her strategy here was to count-on in ones with a SVC while keeping track of the count on her fingers. Khosi was not offered Board 2 because she used counting-on by ones with three items on Board 1 and was thus placed at **Level 1** for her use of Base Ten arithmetical strategies (Wright et al., 2006).

Post-Test

Khosi incremented by ten on and off the decuple using ten strips and also incremented by tens and ones in the setting of covered strips. She used more efficient strategies in her post-test compared to her pre-test. For example, during her pre-test Khosi counted-on in ones when adding ten dots to the fifty-two dots already counted (which took 25sec) but in the post-test she answered the same item by incrementing by ten mentally (in 2 seconds) without any overt counting or calculating. When presented with additive and subtractive 2-digit by 2-

digit horizontal number sentences like $38+24$ and $43-15$ she answered all eight tasks correctly using strategies like N10 (mentally and on an ENL). Her ability to solve additive relations problems involving tens and ones without using tens-based settings is what qualified Khosi to be at Base Ten **Level 3**: Facile Concept of Ten.

5.9.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Khosi correctly said the FNWSs from 'one' to 'one hundred' without omitting a number word and gave the number word just after a given number in the same range without dropping back to 'one'. She thus attained **Level 5**: Facile with FNWSs up to 'one hundred' in her pre-test. During her pre-test Khosi could say the BNWS from 'twenty' to 'one' and gave the NWB a given number in a much higher range (to 'one hundred') without dropping back to 'one'. But, following the LFIN framework I placed her at **Level 3** which describes a learner who can produce the BNWS and NWB from 'ten' to 'one' because the next level (Level 4) describes a learner who can produce the BNWSs and NWB in the range 'thirty' to 'one'. Although Khosi exceeded the criteria in terms of giving the NWB in this range, she was not able to say the BNWS from 'thirty' to 'one' without omitting any number words. Khosi correctly identified all 1- and 2-digit numerals in the pre-test and only two of the five 3-digit numerals shown – this placed her at **Level 3**. She struggled to identify 3-digit numerals with zero as a place holder, like 206 and 820, in her pre-test.

Post-Test

Khosi remained at **Level 5** for the FNWSs & NWA aspect of the post-test and shifted 2 levels in BNWSs & NWB to **Level 5**, the highest level for this aspect of the LFIN. During her post-test Khosi produced the BNWSs and NWB in the range 'one hundred' to 'one' without omitting any number words or dropping back to 'one'. I placed Khosi at **Level 4** for Numeral Identification because she correctly identified all 1-, 2- and 3-digit numerals in her post-test – including 3-digit numerals with zero as a place holder for the ones and tens.

5.10 Kyle (Group 2)

Table 5.10 Summary profile for Kyle

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	2 ^{x2} ↑	4 ^{x2}	5
2. Base Ten Arithmetical Strategies		1 ↑	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	5	5	5
5. Numerical Identification	4	4	4
Totals	18	23	27
Gains	+5		

Kyle's pre- and post-test profiles show the following 5 gains overall:

- 2^{x2} SEAL Stages
- 1 Level for Base Ten strategies

5.10.1 Strategies for Early Arithmetical Learning (SEAL)

Pre-Test

Kyle's predominant strategy for solving additive and subtractive tasks during the pre-test interview was 'count from one'. For example, he found the sum of two screened addends (5 and 4) by making a finger pattern for five on one hand and a pattern of four on the other; he then counted all the fingers he had raised by coordinating a spoken number word in the forward number word sequence from 'one' to 'nine' while moving each finger in turn. He used the same strategy for the next additive task posed: 9+6. For subtractive tasks like 16-12 and 9-4 Kyle used a similar counting strategy except here he coordinated the backward number word sequence with the sequential closing of fingers. When answering missing subtrahend questions like $12 - \square = 9$ in his pre-test, Kyle did a lot of counting-by-ones with limited success. I initially deemed Kyle to be at SEAL Stage 2: Perceptual Counting because he used his fingers to solve tasks posed with screened items. However, counting from one was not the only strategy Kyle used – he answered 3 of the 5 removed items tasks (e.g. 6-2) correctly without any overt counting or calculating which points to the use of known number facts. After weighing Kyle's use of 'known facts' against his use of the 'count-from-one' strategy I placed

him at a '**high**' **SEAL Stage 2** because he enacted the latter (inefficient) strategy more than the former.

Post-Test

Kyle used some advanced count-by-one strategies like 'count on' for tasks like $9+6$ and $12+\square=15$; and 'count-down-to' for missing subtrahend tasks like $12-\square=9$ and $15-\square=11$. He was placed at **SEAL Stage 4: Intermediate Number Sequence** because Wright's LFIN descriptors clearly put a learner who uses 'count-down-to' to solve missing subtrahend tasks at this stage. Kyle did not use an overt 'count-down-from' strategy to solve removed items tasks in his post-test. The manner in which he answered two of these tasks led me to believe that he could have worked with tens and ones separately. For example, when solving $15-3$ he immediately answered "two" and then offered "twelve" as his final answer 7 seconds later. After I presented the next removed items task ($27-4$) Kyle paused for a few seconds and then started saying an inaudible number sequence (I could only hear "twenty-six" clearly) before he abandoned that strategy. Thereafter he could be heard saying "one, two, three, four" twice (while doing something with his fingers under the desk) before he gave his final answer: "thirteen". Even though this offer was incorrect, I thought that Kyle worked separately with the tens and ones but only 'put back' ten instead of twenty after he made four backward counts from 'seven'. As a general rule (Wright et al., 2006) a learner had to use at least three instances of Stage 5 strategies across Assessment 1.1 task groups 8 and 9 and Assessment 1.2 task group 3 for her to be placed at SEAL Stage 5 – which Kyle did not do.

5.10.2 Base Ten Strategies

Pre-Test

The LFIN framework regards learners who only count forward or backward in ones to solve additive relations tasks to be at Base Ten Level 1 because they do not work with ten as a unit. Based on his pre-test results, Kyle would have been placed at Level 1 for Base Ten strategies because he counted in ones rather than increment by ten off the decuple using ten-dot strips (4, 14, 24...74) during Assessment 1.2; and because 'counting from one' was his dominant strategy when solving additive relations tasks. Also, the way he solved removed items tasks

(15-3 and 27-4 explained above) led me to believe that he had an emergent understanding of early place value. However, another proviso for being placed at Base Ten Level 1 was the attainment of at least SEAL Stage 3 – which Kyle had not done, thus he was not placed on any level for Base Ten strategies (Wright et al., 2006, p.22).

Post-Test

Kyle's use of Base Ten strategies improved over the course of the intervention period. During his post-test he used advanced 'counting by one' strategies to solve additive tasks like $9+6$ (Q8a) and missing addend tasks like $12+\square=15$ (Q8f) – which clearly put him at Base Ten Level 1. A learner at Base-Ten Level 2 is able to see and work with ten as a unit to solve additive relations tasks, but would only be able to do so using a tens-based setting. During post-test Assessment 1.2 Kyle incremented and decremented by ten on and off the decuple using ten strips, but he was unable to do so fluently – he often dropped back to ten or whatever the starting number was. Tasks that required him to increment by tens and ones separately (Ass. 1.2 Board 1) resulted in Kyle attempting to count-on in ones. I got the impression that Kyle was not serious during his post-test interview and he was thus not showcasing his best maths thinking. Nevertheless, based solely the observable evidence from his post-test I placed him at a **'high' Base Ten Level 1** because he incremented by ten on and off the decuple using ten strips but did not do so consistently.

5.10.3 FNWSs, BNWSs and Numeral Identification

Pre-Test

Kyle attained the highest level for FNWSs & NWA in his pre-test (**Level 5**) because he could correctly produce the FNWSs in the range 'one' to 'one hundred' without omitting any number words and gave the NWA in the same range without dropping back to 'one'. He similarly attained the highest level for the BNWSs & NWB aspect of the LFIN – **Level 5** because he fluently produced the BNWSs and NWB in the range 'one hundred' to 'one'. Kyle attained the highest level (**Level 4: Numerals to '1000'**) in his pre-test as he was facile with identifying 1-, 2- and 3-digit numerals.

Post-Test

In his post-test interview Kyle again attained the highest levels for FNWSs & NWA and BNWSs & NWB, that is, **Level 5** for each. He was fluent with F/BNWSs in the range 'one' to 'one hundred' and gave the NWA/B swiftly in the same range without dropping back to 'one'. He also attained the highest level for Numeral Identification in his post-test – **Level 4**: Numerals to '1000'. His profile thus does not show growth in these three aspects of the LFIN which could be ascribed to a 'ceiling effect'.

5.10.4 Caveat

Kyle seemed to have all the 'building blocks' (like facility with F/BNWSs and identification of numerals) needed for rapid growth in learning early number at the start of intervention but this did not materialize during intervention, which resulted in small gains overall.

5.11 Kgomo (Group 2)

Table 5.11 Summary profile for Kgomo

LFIN Aspects	Stages/Levels		Highest stage or level
	Pre-Test	Post-Test	
1. Stages of Early Arithmetical Learning (SEAL)	3 ^{x2} ↓	3 ^{x2}	5
2. Base Ten Arithmetical Strategies	1	2 ↓	3
3. Forward Number Word Sequences (FNWSs) and Number Word After (NWA)	5	5	5
4. Backward Number Word Sequences (BNWSs) and Number Word Before (NWB)	3	3 ↑	5
5. Numerical Identification	3	4	4
Totals	18	20	27
Gains	+2		

Kgomo's pre- and post-test results show 2 overall gains:

- 1 Level for Base Ten strategies
- 1 Level for Numeral Identification

5.11.1 Strategies for Early Arithmetical Learning (SEAL)

Pre-Test

During his pre-test interview Kgomo used the 'count-on' strategy to answer $9+6$ (Ass.1.1 Q8a, both addends screened) but was uncertain of the solution he obtained and did not offer an answer – even though his solution was correct. We had a small talk about the strategy he was trying to use and Kgomo then used this strategy more confidently to find solutions to additive tasks like $8+5$ and $9+3$ (Ass.1.1 Q8e, both addends screened) which were posed straight afterwards. Kgomo used 'count-up-to' for $12+\square=15$ (Ass.1.1 Q8f) – which is like 'count on' except that this is mostly used to solve missing addend tasks. He initially gave an incorrect response to this task ('five') but changed his answer after explaining to me how he got his answer – which was keeping twelve in his head and saying 'thirteen, fourteen, fifteen' while raising three fingers sequentially in coordination with each number word spoken; his answer was the total number of fingers he raised. Based on his attempts at using the 'count on/ count up to' strategy to answer additive tasks, Kgomo was placed at SEAL Stage 3: Initial Number Sequence. But because he lacked confidence when enacting the 'count on' strategy I considered him to be at a **'low' SEAL Stage 3**.

Post-Test

The next step in the early number progression of a child who can 'count on' (albeit not confidently) would be to use 'count-down-from' when solving removed items tasks (like $15-3$ and $27-4$ in Ass.1.1 Q9c) and 'count-down-to' for missing subtrahend tasks (like $5-\square=3$ and $15-\square=11$ in Ass.1.1 Q9b), in that order (Wright et al., 2006). But, Kgomo answered two of the four removed items tasks in his post-test using tallies; and he answered the missing subtrahend tasks using concrete/inefficient strategies: a finger-pattern for $5-\square=3$, counting-back-in-ones for $10-\square=6$ and tallies for $15-\square=11$ (I was unable to make-out his strategy for $12-\square=9$). It would not be fair to say that Kgomo regressed because he answered some tasks in his post-test using 'count on' confidently and he used known number facts for other tasks (e.g. $3-1$, $6-2$, $9-4$ and $6+6$). For one missing addend task ($7+\square=10$) he offered a voluntary explanation for how he knew the answer: "... because I know that seven plus three is ten". What stood out from his post-test was that Kgomo seemed to resort to more concrete/inefficient strategies when the number range was above 1-10.

An exemplary excerpt from Kgomo's post-test, which shows his reluctance to use a more advanced (and thus more efficient) strategy for solving additive relations tasks is taken from his post-test Assessment 1.2 administered on the 9th of September 2014. Item 2 of Assessment 1.2 consisted of two-digit by two-digit additive relations tasks presented as written number problems. I had presented Kgomo with the first two number problems and he used a 'count-on' strategy for both tasks, keeping track of his counts with his fingers. The first task was answered correctly ($16+10$) but the second was not ($42+23$). Needless to say, Kgomo's chosen strategy was time-consuming. So, when I presented the next task ($38+24$) and he started to 'count-on' in ones using his fingers, I made the following suggestion:

- T : Kgomo, why don't you rather use the empty number line like we did in our group? I think it will be much quicker.
- K : (*Shaking his head and waving his arms*) No, no, no... I don't like using the empty number line!

Kgomo went on to use tallies to solve $38+24$ (a snap shot thereof follows) and the next task: $56-23$. He was the only learner who bluntly refused to use the more advanced strategy I suggested during the post-test interview.



Figure 5.5

Looking across Kgomo's choice of strategies when answering additive tasks in Ass. 1.1 (this assessment is primarily used to determine a child's SEAL Stage 1-4) five tasks were answered using a more sophisticated strategy (mostly known facts) in the post-test when compared to the pre-test – that is $5+4$ (Q8a, both addends screened), $7+\square=10$ (Q8f, missing addend task), $10-\square=6$ (Q9b, missing subtrahend task) and $6-2$ and $9-4$ (Q9c, removed items tasks). Three tasks from Ass.1.1 were answered using a less sophisticated strategy (i.e. counters, fingers or tallies) in the post-test – that is $12+\square=15$ (Q8f, missing addend task), $12-\square=9$ and $15-\square=11$ (Q9b, missing subtrahend tasks). Five tasks from his post-test (Ass. 1.1) were answered using similar strategies as those used in his pre-test.

Thus, Kgomo's choice of strategies when answering additive tasks posed in Assessment 1.1 showed both forward *and* backward moves (while no shift could be seen in some cases), which made it a bit challenging when locating him on the SEAL hierarchy. The forward moves seen in his post-test as discussed above were: he used 'count on' independently and obtained correct answers in both cases, he used known facts four times in the post-test while not doing so at all in the pre-test, and he did not have to 'establish' the numerosity of 'six' in the post-test (by saying the number words 'one, two, three...six' in sequence while raising six fingers in turn) as he did in the pre-test in order to answer the task $10-\square=6$ (Q9b) – in the post-test he looked at his ten fingers and only closed the thumb on his right hand, which suggested that he had the concept of six being five and one more.

The backward moves in Kgomo's post-test were evidenced by his use of more rudimentary strategies like counters and tallies in three instances, which he did not use in the pre-test – a caveat here is that counters and paper for scribbling were not offered to learners during the pre-test. As mentioned previously, Kgomo answered five tasks across his pre- and post-test

using similar strategies – thus showing no ‘movement’. Based on the above evidence Kgomo remained at SEAL Stage 3, although now he could be considered to be more ‘typical’ of a learner at this stage and not at a ‘low’ SEAL Stage 3 level as considered after his pre-test. The main reason for this was that Kgomo’s most advanced strategy displayed in his post-test was the ‘count on’ strategy – the hallmark of a learner at **SEAL Stage 3**.

5.11.2 Base Ten Strategies

Pre-Test

During his pre-test interview Kgomo struggled to increment in ten when solving additive relations tasks in different settings and often resorted to counting-on in ones. For example, during pre-test Assessment 1.2 Kgomo was tasked with incrementing by ten on and off the decuple using ten strips in Q1a, but he was unable to do so. Because he resorted to counting-on in ones when answering the first task, his pre-test ended as the subsequent tasks would have required more of the same but in the more challenging setting of written number problems. Kgomo was placed at **Level 1** for his use of Base Ten arithmetical strategies because he struggled to increment by ten when a tens-based setting was visible (Wright et al., 2006).

Post-Test

During his post-test Kgomo was facile with incrementing by ten on and off the decuple using ten strips (Assessment 1.2 Q1a, b). He also demonstrated facility with incrementing by tens and ones when he correctly completed the eight uncovering tasks on Board 1 and the first three uncovering tasks on Board 2 (Q1c). Kgomo showed facility with incrementing by two groups of ten in the setting of ten strips (3rd uncovering on Board 2) and in the setting of number sentences (Q3c: this included knowledge of the known fact 7-5). He was however not facile with incrementing by tens and ones combined in the setting of ten strips (4th and 5th uncovering on Board 2) nor with the setting of number sentences – this was evident in his incorrect responses to questions like $38+24$ and $56-23$, which he answered using tallies (Q2). Kgomo seemed to have fulfilled the requirements for being placed at Base Ten Level 2 but his persistent use of tallies (Ass. 1.2) and counting on/back to solve tasks from Assessment 2.2

(which were aimed at assessing learners' facility with using Base Five and Base Ten as reference points when solving additive relations tasks) made me consider him to be at a '**low**' **Base Ten Level 2**.

5.11.3 FNWS, BNWS and Numeral Identification

Pre-Test

Kgomo achieved **Level 5**: Facile FNWSs up to 'one hundred' for this aspect of the LFIN for his pre-test because he said FNWSs in the range 'one' to 'one hundred' without omitting a number word and gave the number word just after a given number in the same range without dropping back to 'one'. Based on his pre-test Kgomo was placed at **Level 3** for BNWSs & NWB because he could not say the BNWS from 'thirty' to 'one' without omitting numbers but he could say the BNWS from 'ten' to 'one' and gave NWB in the same range. Even though he gave the NWB numbers up to 'one hundred' without dropping back to 'one' this was not enough to have placed him at a higher level for this LFIN aspect. He correctly identified 1- and 2-digit numeral cards during his pre-test, but was not facile with identifying the 3-digit numerals presented. Kgomo was placed at **Level 3** for Numeral Identification.

Post-Test

Although Kgomo's post-test profile showed him to be ostensibly at the same place in FNWS & NWA as in his pre-test (**Level 5**) – he actually improved in this aspect because he produced the FNWSs in the post-test with less pauses at decuple boundaries and gave NWA with fewer errors. Based on his post-test results, Kgomo was still considered to be at Level 3 for BNWSs and NWB. On the surface his post-test result made it seem as though he made no progress during intervention because he still could not produce the BNWS from 'thirty' to 'one' without omitting any number words. But, there was a distinct difference in the type of errors he made when answering the BNWSs tasks during the pre-test compared to the post-test. Kgomo's incorrect BNWSs in his pre-test were: "34, 33, 32, 31, 20, 19" and "72, 71, 60, 78, 77, 76"; while his incorrect BNWSs in his post-test were: "22, 21, 19, 18, 17, 16"; "34, 33, 32, 31, 29, 28, 27" and "72, 71, 69, 68, 67". Even though he gave more incorrect BNWSs in his post-test, the nature of the errors was different. In his pre-test Kgomo omitted a chunk of the BNWS

34-19 and did so again in the next BNWS, except here he also completely lost his place in the sequence; while in his post-test he only omitted one number word from each sequence (the decuple number), but otherwise finished each BNWS correctly. Thus, I considered him to be at a **'high' Level 3** for BNWSs and NWB. During his post-test Kgomo confidently identified all 2- and 3-digit numerals shown and was placed at **Level 4** (the highest level) for this aspect of the LFIN.

5.11.4 Caveats related to Kgomo's summary profile

My concern regarding Kgomo's ostensible lack of substantial progress across the period of intervention had caused me to pore over all the data generated from his pre- and post-tests numerous times (looking for something that I may have missed) and I am certain that what I have documented here is an accurate account of his early number learning as per the LFIN framework. This closer inspection showed smaller shifts that were not apparent on the LFIN framework. For example, Kgomo did not shift from one SEAL stage to the next but he made a shift *within* SEAL Stage 3: he enacted 'count on' independently during his post-test but had to be encouraged to use this strategy in his pre-test; and when he did use this strategy in the pre-test he was not sure about the answers it produced. In the other LFIN aspects where it seemed like Kgomo did not make any gains (FNWSs and BNWSs) one can actually see small shifts *within* each level (even though a ceiling effect could be the reason for no shift in FNWSs).

5.11.5 Overall comments for Group 2

The greatest gains in Group 2 was made by Khosi – one of the three group members who started with the same LFIN score (18) which was lower than that of Julie (22), the highest pre-test attainer in the group. All four group members improved in the Base Ten aspect of the LFIN but only three of the four improved in SEAL – the weakest post-test attainer did not move to a higher SEAL stage across the pre- and post-test. These outcomes are analyzed further in the following section.

Up to this juncture I focused on intervention learners' individual early number learning journeys across pre- and post-tests which answered the first research question. Next I present

a tabulated overview of intervention and control groups' pre- and post-test LFIN results to get a sense of what the overall gains were for each group and to compare the gains made. Given that the comparison here is between intervention and control group gains, the intervention learners are treated as one group in this first overview analysis.

5.12 An Overview of Intervention and Control Group's LFIN Results

I present an overall view of the pre- and post-test LFIN results achieved by all participants (intervention and control groups) in Tables 5.12 and 5.13, respectively. These tables set out individual pre- and post-test profiles alongside each other (pre-tests are shaded) for ease of comparison. As previously noted, I aggregated the results of pre- and post-test LFIN profiles, giving a double weighting to LFIN stages, which produced a score out of twenty-seven. The last row in each table labeled 'gains' shows the shift made by each learner while the last column labeled 'overall gains' shows the gains made by the whole group for each LFIN aspect. Intervention and control learners have been placed in each table from the strongest attainer (on the left) to the weakest (on the right) according to their pre-test scores. Learners who attained the same overall pre-test score were further ordered according to their post-test scores. Arranging the data in this way allowed me to answer my second research question:

2. Following the intervention pathway for learners who made up the intervention group, what learning gains (if any) were produced at the end of the intervention, and how does this compare to learning gains made by the control group?

Following this, I separated the tabulated data for the intervention learners based on the format of intervention they received. This helped me to compare gains made by 'singletons' and those who received 'grouped' intervention, which in turn allowed me to answer the third research question:

3. What were the differences and similarities in the nature of learning gains made by 'singletons' compared to the gains made by learners who received grouped intervention?

Table 5.12 Overall Performances for Intervention Learners on LFIN Pre- and Post-Tests

	Group 2 Julie		Singleton 1 Khanye		Group 1 1 Bongi		Group 2 Khosi		Group 1 Mali		Group 2 Kyle		Group 2 Kgomo		Group 1 Jenna		Group 1 Sibu		Singleton 2 Josh		Overall gains
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	
SEAL ¹⁸	4x2	5x2	3x2	5x2	3x2	5x2	3x2	5x2	3x2	4x2	2x2	4x2	3x2	3x2	3x2	4x2	2x2	4x2	2x2	3x2	28
BASE 10	1	3	2	3	1	3	1	3	1	2		1	1	2	1	2		2		1	14
FNWS	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	5	3	5	2	5	6
BNWS	5	5	4	5	5	5	3	5	3	5	5	5	3	3	3	5	2	4	2	5	12
Nr Id	3	4	3	4	3	4	3	4	3	4	4	4	3	4	3	4	3	3	1	3	9
Totals	22	27	20	27	20	27	18	27	18	24	18	23	18	20	17	24	12	22	9	20	
Gains	+5		+7		+7		+9		+6		+5		+2		+7		+10		+11		69

Table 5.13 Overall Performances for Control Learners on LFIN Pre- and Post-Tests

	Yazu		Rila		Nozi		Olsen		Ntlako		Sipo		Bonang		Rhaka		Abby		Thulu		Overall gains
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	
SEAL ³	4x2	5x2	4x2	5x2	3x2	4x2	3x2	3x2	3x2	4x2	3x2	4x2	3x2	4x2	3x2	4x2	2x2	4x2	2x2	3x2	20
BASE 10	1	1	2	2	1	1	1	1	1	2	1	2	1	2	1	2		1		1	6
FNWS	5	5	4	5	4	4	4	5	5	5	4	5	4	5	4	5	4	4	4	4	5
BNWS ¹⁹	5	5	4	5	4	5	5	4	3	5	4	4	3	5	3	4	3	4	3	3	7
Nr Id	4	4	3	4	4	4	3	3	3	4	3	4	3	3	3	3	3	4	3	3	4
Totals	23	25	21	26	19	22	19	19	18	24	18	23	17	23	17	22	14	21	14	17	
Gains	+2		+5		+3		+0		+6		+5		+6		+5		+7		+3		42

¹⁸ The score given for the SEAL aspect of the LFIN is double the SEAL stage the learner attained.

¹⁹ There were 8 progressions and 1 reversion so the final score given to this aspect of the LFIN is 7.

Research Question 2

Following the intervention pathway for learners who made up the intervention group, what learning gains (if any) were produced at the end of intervention, and how does this compare to learning gains made by the control group?

Learners from both intervention and control groups showed learning gains across their pre- and post-test LFIN profiles – the intervention group showed a higher number of total gains (69) compared to the total gains made by the control group (42). I next 'unpack' the differences between groups' attainment on the SEAL, Base Ten and FNWSs, BNWS and Numeral Identification aspects of the LFIN which is more useful for getting a nuanced understanding of the differences in gains made by each group.

SEAL Strategies

In both groups the highest number of gains were made in the SEAL aspect of the LFIN: intervention group – 28 and control group – 20 (as the weighting for this aspect was double the stage achieved). Table 5.14 shows that the key difference between the intervention and control groups' pre- and post-test SEAL results was the greater number of intervention learners who improved by two stages, which is substantial (5/10 learners) in comparison to 1/10 learners in the control group. This is particularly of interest when one considers two things: the qualitative change needed in early number understanding to move from one stage to the next; and the short time lapse between the pre- and post-tests (approximately 4 months excluding school holidays).

Table 5.14 Shifts in SEAL Stages

	Intervention Group	Number of SEAL Shifts	Control Group	Number of SEAL Shifts
Nr of learners who made 0 shift in SEAL Stage	1	--	1	--
Nr of learners who made 1 shift in SEAL Stage	4	4X1 → 4	8	8X1 → 8
Nr of learners who made 2 shifts in SEAL Stage	5	5X2 → 10	1	1X2 → 2
Total	10	14 ^{x2}	10	10 ^{x2}

The marked difference in how these groups improved with regard to their strategies for early arithmetical learning is best showcased when one considers what was invariant between both groups at the time of the pre-test and how this had changed at the time of the post-test.

Pre-test profiles across both groups show that they had the same number of learners who achieved SEAL Stage 3 (or the 'counting-on' stage) at the time of the pre-test, that is, 6 learners each. At the time of the post-test there was one similarity across both groups' profiles, that is, that one learner in each group remained at SEAL Stage 3. The biggest difference between learners from the intervention group who were at the 'counting-on' stage during the pre-test, compared to those from the control group, was that half of these intervention learners achieved the highest SEAL Stage (SEAL Stage 5) at the time of post-test while none of the control learners who were at the 'counting-on' stage at the pre-test reached SEAL Stage 5 at the time of the post-test. This point is illustrated below in Figure 5.6.

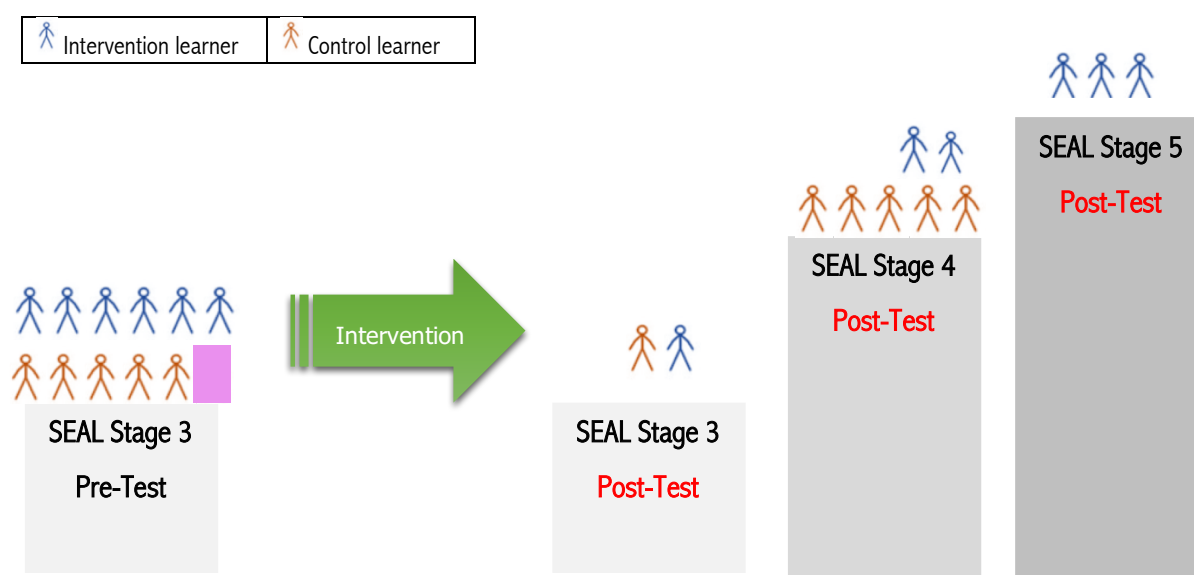


Figure 5.6

Base Ten Strategies

The Base Ten Strategies aspect is of particular interest in this study given that this issue has been flagged as critical in the South African landscape and given its subsequent centrality in

the intervention. Both groups showed gains in the Base Ten aspect of the LFIN across pre- and post-tests, with the intervention group making more than double the number of gains made by the control group, that is, 14 and 6 respectively. This difference in groups' post-test results points to intervention learners' having much more facility with using ten as a unit to solve additive tasks than their peers who made up the control group. Whilst *all* intervention learners improved in the Base Ten aspect of the LFIN, the same was not true for the control group.

Although the Base Ten pre-test results for the four top attainers in each group were: 1, 2, 1 and 1 respectively, the post-test results looked very different (Table 5.15): the four top attainers in the intervention group all achieved the highest Base Ten level for their post-tests but the top four attainers in the control group stayed at the same Base Ten levels across pre- and post-tests.

Table 5.15 Intervention and Control Groups' Top Four Attainers' Base-Ten results

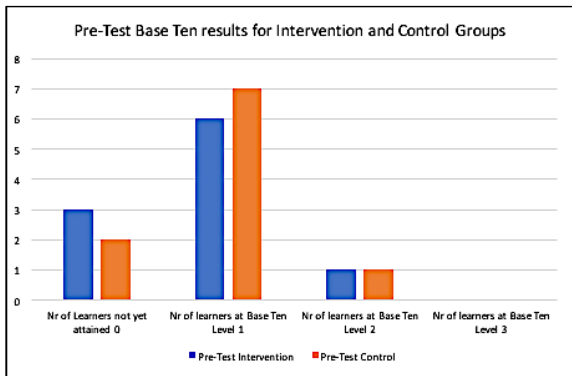
Top Four Intervention Group	Pre-Test	Post-Test		Top Four Control Group	Pre-Test	Post-Test
Julie	1	3		Yazu	1	1
Kamo	2	3		Rila	2	2
Bongi	1	3		Nozi	1	1
Khosi	1	3		Olsen	1	1

Further differences between the two groups' Base Ten results were that: six intervention learners (across the attainment range) improved by one Base Ten level each while only the six lower-attainers in the control group gained one Base Ten level each; four intervention learners improved by two Base Ten levels compared to none from the control group; and six learners from the intervention group attained the highest Base Ten level at the time of the post-test while none in the control group achieved the same.

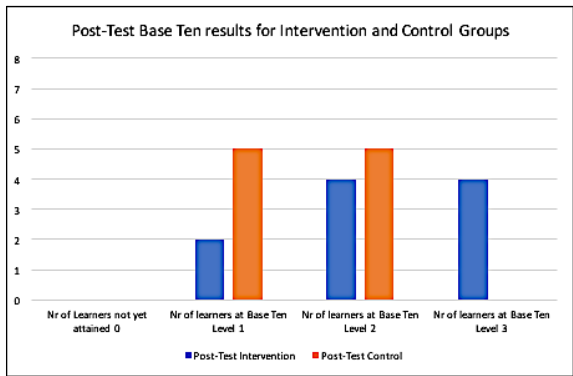
To get an in-depth understanding of the shifts that took place in intervention and control learners' understanding of base-ten over time, I looked once more at what was the same at the time of the pre-test and how this changed (if at all) at the time of the post-test. Both

intervention and control groups had more-or-less²⁰ the same number of learners starting at each level of the LFIN Base Ten model at the time of the pre-test (see Graph 5.1): none at Level 3; 1 each at Level 2; 6 and 7 (respectively) at Level 1; and 3 and 2 that had not yet attained Level 1 (respectively). This low base reflected the kinds of working highlighted in the literature that I flagged in Chapter 2 as perhaps the 'key' problem needing to be addressed in South African early number learning. Whilst the pre-test picture for Base Ten was more-or-less similar between the two groups, the post-test picture looked quite different (see Graph 5.2).

Graph 5.1



Graph 5.2



So, whilst both groups started with more-or-less similar levels of base-ten understanding in the pre-test (with the control group slightly stronger than their intervention counterparts), intervention learners showed much greater understanding/use of ten as a unit at the time of the post-test. Thus intervention learners used the base-ten structure of number more frequently to solve additive tasks (rather than counting in ones) in the post-test, than their counterparts who made up the control group.

FNWSs, BNWSs and Numeral Identification

Table 5.16 shows that both intervention and control groups made gains in these aspects of early number learning, 27 and 16 overall gains, respectively.

²⁰ The control group started from a slightly stronger base at Levels 0 and 1.

Table 5.16 Summary of FNWSs, BNWSs and Numeral identification

	Gains made by Intervention Group	Gains made by Control Group
Forward Number Word Sequences & Number Word After	6	5
Backward Number Word Sequences & Number Word Before	12	7
Numeral Identification	9	4
Total gains made for these 3 LFIN aspects	27	16

The intervention group made greater gains in each of these LFIN aspects – with bigger differences in the BNWSs & NWB and Numeral Identification aspects than the FNWSs & NWA. This comparison suggests that intervention based on MR principles had a greater effect on learners' facility with backward number word sequences and numeral identification than on their facility with forward number word sequences. My expectation was that the weaker attainers in each group would have made the most gains in these LFIN aspects as they are described as being 'narrower' than SEAL stages and because forward/backward counting and numeral identification are basic number skills that children tend to learn early on. Table 5.17 sets out the results of these three LFIN aspects for the two weakest pre-test attainers in each group (all of whom had attained SEAL Stage 2 on their pre-tests).

Table 5.17 Two weakest attainers' LFIN profiles (Intervention and Control)

	Intervention Learners				Control Learners			
	Group 2 Sibu		Singleton 1 Josh		Abby		Thulu	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
FNWSs & NWA	3	5	2	5	4	4	4	4
BNWSs & NWB	2	4	2	5	3	4	3	3
Numeral Identification	3	3	1	3	3	4	3	3
Total	8	12	5	13	10	12	10	10
Total gains made for these 3 LFIN aspects	+4		+8		+2		+0	

From Table 5.17 one can clearly see the big difference between the gains made by the weakest attainer from the intervention and control group, Josh and Thulu. The former, who started off with a much weaker attainment profile than his counterpart, made 8 gains across the pre- and post-tests while his counterpart made none. The second weakest intervention learner (Sibu) made twice as many gains as his counterpart (Abby) in the control group (4 and 2, respectively) despite also starting with a weaker LFIN profile than his peer. What is

also interesting to note (looking across Tables 5.5 and 5.6) is that 5 of the 6 FNWSs gains made by the whole intervention group were made by the two weakest attainers in the group; while none of the 5 FNWSs gains made by the control group as a whole can be attributed to the two weakest attainers in that group. Taken from this, it seems like intervention based on MR principles was very effective in raising the low base of basic number skills for weaker attainers in this school.

5.12.1 A summary of the similarities and differences in gains made between the intervention and control groups

Every research participant made progress over time – those who received intervention based on Mathematics Recovery principles and those who did not. This is not surprising given that participants were more mature at the time of the post-test and they had more learning experiences in the interim too. However, the group of *intervention learners made greater gains overall and in every aspect of the LFIN* than the control learners. This suggests that short-term intervention based on MR principles can be used to accelerate early number learning across the attainment range.

Both groups made a higher number of gains in SEAL – which deals with learners' strategies for solving additive tasks – than any other LFIN aspect. This could be attributed to the fact that additive reasoning is brought sharply into focus in the national curriculum for Grades 1 and 2 (DBE, 2011a). However, the *intervention group made greater gains in the SEAL aspect* than the control group. Three of the six intervention learners who started at the 'counting on' stage in the pre-test achieved the highest SEAL stage at the post-test (compared to none of the six control learners who also started at the same stage). Thus, intervention learners who could 'count-on' appropriated non-count-by-one calculation strategies through intervention based on Mathematics Recovery principles.

The *biggest – and the most significant – difference* between the intervention and control groups was that the former made more than twice the number of gains made by the latter in the Base Ten aspect of the LFIN. I regard this as the most important difference between the two groups given the persistent use of counting strategies across the national terrain

(stretching into the Intermediate phase) that are inefficient and ineffective for solving additive problems as the number range increases across grades. While at one level, this result reflects the attention to base-ten thinking in the intervention lessons, the extent of the gains for the intervention group and the extent of the difference in gains between the intervention and control group represents a very important result for this study. This significant result also speaks to the possibilities of addressing South African concerns about the lack of moves to the use of base-ten structure that underlies the possibilities for moves to calculation strategies.

In the following section I answer the third research question before providing an overview of all insights gleaned from participants' pre- and post-test results.

Research Question 3

What were the differences and similarities in the nature of learning gains made by 'singletons' compared to the gains made by learners who received grouped intervention?

To answer this question, I grouped learners' pre- and post-test LFIN scores according to the format of intervention they received for ease of comparison. To start off, I present the profiles of those learners who received individual intervention side-by-side in Table 5.18.

Table 5.18 Pre- and Post-test LFIN scores for 'Singletons'

	Singleton 1 Khanye		Singleton 2 Josh		Total gains
	Pre	Post	Pre	Post	
SEAL	3 ^{x2}	5 ^{x2}	2 ^{x2}	3 ^{x2}	6
BASE 10	2	3		1	2
FNWS	5	5	2	5	3
BNWS	4	5	2	5	4
Nr Id	3	4	1	3	3
Totals	20	27	9	20	
Gains	+7		+11		+18

The higher gain in the SEAL aspect of the LFIN was made by Khanye, the stronger attainer, that is, an improvement of 2 SEAL stages compared to Josh's improvement of 1 SEAL stage. Both 'singletons' made one gain on the Base Ten aspect of the LFIN but Josh made more gains on the FNWSs, BNWSs and Numeral Identification aspects – 8 and 2 gains (for these 3 aspects), respectively. An interesting outcome for the 'singletons' was that the higher overall gain was made by Josh – who was the weaker of the two. Thus, the attainment gap between these learners, which research indicates grows larger over time (Spaull, 2013), actually became smaller over the course of intervention. Josh, who made much more gains across the FNWSs, BNWSs and Numeral Identification aspects of the LFIN, started from a much lower base in these early number aspects than Khanye – whose gains in these areas may have been stunted by a 'ceiling effect'.

I next present the pre- and post-test LFIN scores of those learners who received grouped intervention. Group 1's scores are presented side-by-side in Table 5.19 and Group 2's in Table 5.20. Learner names in each table have been arranged with the strongest pre-test attainer in the group on the left and the weakest on the right.

Table 5.19 Pre- and Post-test LFIN scores for Group 1

	Bongi		Mali		Jenna		Sibu		Total Gains
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	
SEAL	3 ^{x2}	5 ^{x2}	3 ^{x2}	4 ^{x2}	3 ^{x2}	4 ^{x2}	2 ^{x2}	4 ^{x2}	12
BASE 10	1	3	1	2	1	2		2	6
FNWS	5	5	5	5	4	5	3	5	3
BNWS	5	5	3	5	3	5	2	4	6
Nr Id	3	4	3	4	3	4	3	3	3
Totals	20	27	18	24	17	24	12	22	
Gains	+7		+6		+7		+10		+30

The highest overall gain was made by the weakest attainer in Group 1: Sibu, reflecting the pattern seen across the two singletons above. The highest SEAL and Base Ten gains were made by the weakest *and* strongest members of Group 1, namely, Sibu and Bongi – each improved by 2 SEAL stages (from Stage 2-4 and Stage 3-5, respectively) and 2 Base Ten

levels (from 'not placed' - Level 2 and from Level 1 - 3). As with Josh above, higher gains for the FNWSs, BNWSs and Numeral Identification were skewed towards the weaker learners in the group.

Table 5.20 Pre- and post-test LFIN scores for Group 2

	Julie		Khosi		Kyle		Kgomo		Total gains
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	
SEAL	4 ^{x2}	5 ^{x2}	3 ^{x2}	5 ^{x2}	2 ^{x2}	4 ^{x2}	3 ^{x2}	3 ^{x2}	10
BASE 10	1	3	1	3		1	1	2	6
FNWS	5	5	5	5	5	5	5	5	0
BNWS	5	5	3	5	5	5	3	3	2
Nr Id	3	4	3	4	4	4	3	4	3
Totals	22	27	18	27	18	23	18	20	
Gains	+5		+9		+5		+2		+21

At the start of intervention Julie had stronger number skills than her three group members who all started with the same overall LFIN score: 18. Julie began intervention from a stronger base than Khosi but both girls ended at the highest LFIN levels in the post-test. The highest overall gains in this group were made by Khosi, who ostensibly began intervention from the exact same place on the LFIN framework as Kgomo. The highest SEAL gains (2 stages each) were achieved by Kyle and Khosi: from Stage 2-4 and Stage 3-5, respectively. The two stronger attainers at the time of the post-test (Julie and Khosi) made higher gains in the Base Ten aspect of the LFIN than the two weaker attainers.

5.12.2 Summary of comparison between learners who received individual and grouped intervention

Table 5.21 below sets out the total and mean gains for 'singletons', Group 1 and Group 2 so that an easy comparison can be made (in terms of SEAL, Base Ten, F/BNWSs and Numeral Identification combined, and the overall LFIN scores) between learners who received different formats of intervention. Mean gains for each aspect (and overall score) are highlighted in the

table as these are the gains that allowed me to make more equitable comparisons as there were only two 'singletons' while each group format had four learners each.

Table 5.21 Mean gains for LFIN aspects – Intervention Group

		Singletons	Group 1	Group 2
SEAL	total gains	6	12	10
	mean gains	3	3	2,5
Base Ten	total gains	2	6	6
	mean gains	1	1,5	1,5
FNWSs BNWSs Nr ID	total gains	10	12	5
	mean gains	5	3	1,25
Overall	total gains	18	30	21
	mean gains	9	7,5	5,25

Some similarities and differences were noticeable between mean gains achieved across 'singletons' and the two groups. The aspects where singletons and grouped intervention learners achieved comparable mean scores were in the SEAL and Base-Ten aspects of the LFIN. The mean SEAL scores for singletons and Group 1 were identical (3) while the mean SEAL score for Group 2 was slightly lower: 2,5. The Base Ten mean scores for both groups were the same (1,5) while the Base Ten mean score for singletons was slightly lower: 1. What is not as clear from Table 5.11 is that the SEAL gains made by singletons were skewed towards the stronger learner (noted previously), the gains in Group 1 were evenly spread across the attainment range in the group and no observable pattern could be seen in Group 2's SEAL gains. In the Base-Ten aspect – the singletons and Group 1 had an even spread of gains between stronger and weaker attainers but in Group 2 gains were skewed towards the stronger attainers (also noted previously).

Differences between mean scores were evident in the combined aspects of FNWSs, BNWSs and Numeral Identification as well as the overall gains made by learners who received different formats of intervention. For the three combined aspects of the LFIN, the singletons had the

highest mean score: 5, second was Group 1 with a mean score of 3, and then Group 2 with the lowest mean score of 1,25. Again what is not clear from this table is that gains were skewed towards the weaker singleton, that is, 8/10 compared to 2/10 for Josh and Khanye, respectively. The difference in the overall LFIN mean scores mirrored the differences between the combined LFIN aspects: the highest overall mean score was achieved by singletons (9), the second highest mean score by Group 1 (7,5) and the lowest overall mean score by Group 2 (5,25). These overall gains were also skewed towards the weaker attainer in the individual format of intervention and in Group 1. This indicates that most intervention learners who started from a weaker base in the LFIN with respect to basic number skills (forward and backward acoustic counting and numeral identification), made the most gains in these aspects of early number learning which then translated into the differences seen in the overall gains made between them and their peers who started from a stronger base.

This summary was valuable because it showed that the biggest similarities in mean gains made across different formats of intervention was in the two focal areas pertaining to my research: Strategies for Early Arithmetical Learning (SEAL) and Base-Ten Arithmetical Strategies. A closer look at gains made by two of the weakest pre-test attainers shed more light on this outcome because one of these learners received individual intervention whilst the other participated in grouped intervention. I turn to this next.

5.12.3 A more nuanced comparison between a 'singleton' and one who received grouped intervention

I would like to illustrate the relative similarity in gains made by 'singletons' and 'grouped intervention' learners by looking at the overall gains made by the weakest pre-test attainer (from the intervention group as a whole) who received individual intervention and the second-weakest pre-test attainer who received grouped intervention. My reason for using a 'telescopic lens' to discern the minute differences between these two learners is because they made (the biggest) comparative overall gains on the LFIN framework in my study even though one received individual intervention and the other participated in grouped intervention.

Josh, one of the 'singletons', started with an overall pre-test LFIN score of 9 and ended with a post-test score of 20 – thus making 11 gains overall. Sibü, who received grouped intervention, started with an overall pre-test LFIN score of 12 (a stronger base than Josh) and ended with a post-test score of 22 – thus making 10 gains overall. This is an important result in the context of highly inequitable learning outcomes in South Africa and the scale of the maths crisis on the ground where it is reported that 75-80% of children in public schools are two to four grade-levels below the grade-appropriate levels outlined in the curriculum (Spaull, 2013).

Josh, who was the weakest attainer out of the entire intervention group, made the biggest overall gain (+11). This is significant (even with 'ceiling effects') because the gap between learners with the weakest and strongest attainment learner profiles was closed – the overall pre-test gap between the highest attainer (Julie) and Josh was 13 while the post-test gap was 7. This is contrary to what research tells us: that the gap in early number skills between weaker and stronger attainers grows wider over time (Geary, 2011; Mulligan, 2011; Spaull, 2013).

A more significant outcome of the comparable gains made by the two weakest pre-test attainers in this study is that the second biggest gain (+10) was made by Sibü – the weakest of the 'grouped intervention' learners. He improved by two SEAL stages and two Base Ten levels which is arguably a better improvement than that of Josh whose improvement was by

one SEAL stage and one Base Ten level. Both learners started ostensibly from the same place with regard to SEAL and Base Ten aspects of the LFIN (Sibu was one level better with FNWSs and Numeral Identification in the pre-test) but the learner who received grouped intervention had higher gains in SEAL and Base Ten than the one who received individual intervention. This outcome speaks to the value of interactive learning and it further suggests that grouped intervention is not necessarily less effective than one-on-one intervention which is an encouraging outcome in our current context where the need for intervention is widespread (Schollar, 2008).

5.14 Concluding Remarks

By focusing on the early number learning gains made by individual learners across their pre- and post-tests I foregrounded the constructivist aspect of the Emergent Approach (Cobb & Yackel, 1996) while the interactionist perspective remained largely unarticulated in the background. The differences in SEAL gains between intervention and control learners was significant and I take a closer look at this in my final chapter. The differences between gains made in the more 'basic number aspects' of the LFIN (FNWSs, BNWSs and Numeral Identification) were also interesting and will too be discussed further in Chapter 7.

However, it was the differences in Base-Ten gains made between the intervention and control learners that intrigued me more. The reason for this is twofold: firstly, the persistent use of counting-by-ones amongst Foundation and Intermediate Phase learners in the South African terrain has been widely reported in national and international literature. Children's enactment of time-consuming and error-prone counting strategies have been linked to their poor grasp of structuring number, especially the structure of base-ten. When children cannot structure number, it means that they cannot see and exploit the relationships between numbers and operations when calculating, which precludes them from developing efficient mental strategies when working with numbers bigger than 20. Without an understanding of structuring number children are locked into an unsophisticated and inefficient maze of basic calculation strategies that very rarely produce correct results when working in higher number ranges.

Secondly, empirical evidence shows that South African pedagogic practices do not foreground the base-ten structure of numbers that can act as 'handholds' to assist children in developing efficient mental strategies. This includes (but is not limited to) Foundation Phase teachers' predominant selection of unstructured, concrete representations of number that reinforces ubiquitous counting-by-ones, despite the wellspring of research that structured representations of number (and concomitant actions on these settings that exploit this structure) can help children bridge the divide between calculation-by-counting and calculation-by-structuring. The teacher-led pedagogy prevalent on the ground – where the teacher does most of the talking in a whole-class teaching format and where learners respond with 'chorused' answers – has tended not to make use of the benefits of oral discourse during interactive learning.

Taken together, these key points related to the importance of base-ten thinking and the lack thereof in teaching-and-learning practices in many South African classes, provided the rationale for my emergent focus on elements of interaction during the intervention period (like learners' 'math talk' and actions on representations) that could shed light on the development of individual learners' base-ten thinking. In the next chapter I 'unpack' these focal points that relate to the development of participants' base-ten thinking.

Chapter Six – An Interactionist Analysis

6.1 Introduction

In the previous chapter, I presented detailed analyses of intervention learners' pre- and post-test LFIN profiles that topped and tailed the sequence of about eighteen intervention lessons. I also provided a comparison of intervention and control groups' LFIN scores across the pre- and post-tests and highlighted the differences and similarities in gains made. Across all learners there was evidence in these snapshots, of gains in the profile of skills measured through Wright et al.'s (2006) Learning Framework in Number (LFIN). Whilst the gains made across MR pre- and post-tests were pleasing, assessment literature widely notes the limitations of 'snapshot' assessments: that inevitably they test a limited range of skills, and judgments are made on the basis of a limited range of tasks (Watson, 1998).

Ongoing assessment *for* learning during the course of the intervention provided openings for a more formative type of assessment. In this chapter, I present broader and more nuanced assessment evidence related to learners' base-ten thinking. By 'base-ten thinking' I refer to seeing and using ten as a unit when solving addition or subtraction tasks – which implies moving away from counting-by-ones (Wright et al., 2006). This focus, as noted already, was selected given the widespread South African evidence of counting-by-ones-based approaches to solve arithmetic problems, even in the higher number ranges that render this approach time-consuming and error-prone (Ensor et al., 2009; Schollar, 2008). And from an international perspective, Gervasoni et al. (2011, p.316), drawing on a wide range of literature in the field of early number learning (like Fuson et al., 1997; Cobb & Wheatley, 1988 and Steffe et al., 1988) state that:

Children's success with solving 2-digit by 2-digit problems relies heavily on their understanding of ten as both a collection of ten ones and as a single unit of ten that can be counted, decomposed, traded and exchanged for units of different value.

From the literature review, I noted that a specific aim that emerged during intervention was to move learners towards using what I call 'base-ten strategies', like N10 and 1010, with both

of these strategies resting on seeing ten as a unit as a prerequisite (Anghileri, 2006). 'Bridging-through-ten' is another base-ten strategy that can figure within N10 and 1010 approaches. Learners' appropriation of base-ten thinking, as evidenced by their use of base-ten strategies like N10 and 1010 (and their discourse surrounding this), occurred mostly in the context of solving 2-digit by 2-digit additive relations tasks and this guided the scope of my analysis.

I have already noted that a range of literature in early number emphasizes learning in terms of being able to work across a range of representations (Graham, et al., 2009; Heinze, et al., 2009). Wright et al. (2006) use the notion of moving between 'settings', and stress too the need, over time (in the context of manipulatives and physical resources) to 'distance the setting'. Both of these ideas were important in the context of my work on base-ten thinking during the intervention period. In the literature chapter, I focused on key representational settings that are advocated for the development of learner's base-ten thinking like bundling sticks, the arithmetic rack or bead boards, 100 charts, empty number lines (ENL), and so forth. I also highlighted different ancillary skills required in the context of these different settings – for example, the actions and discourse used when positioning a number on a blank 100 chart are a little different from the actions and discourse that might be needed to position a number on an ENL – a point that Venkat and Askew (2018) attend to within their focus on the coherence and localization of instruction within their 'mediating primary mathematics' framework.

While this body of work tends to emphasize performance in the context of different representations or settings, Anghileri (2006, p. 8-9) and others have also emphasized the idea of 'discourse about actions' as a key marker of increasing proficiency in children's understanding, hence:

Mathematical understanding involves progression from practical experiences to talking about these experiences, first using informal language, and then more formal language. Later, children will learn to use the symbols that characterize the conciseness and precision of a mathematically reasoned argument.

This idea aligns well with Cobb and Yackel's (1996) position on Emergence, and the Vygotskian argument about interpersonal actions moving to the intrapersonal plane, with individual

actions and discourse that follow interactional actions and discourse providing key markers of appropriation (Kozulin, 2003).

Children's early number strategies have been noted as moving in non-linear ways in the shorter term, with reversions to more rudimentary representational settings found to be common. Similarly, appropriation of discourse also tends to be fragmented and partial before moving to more fluid coherence. Thus, the analysis presented in this chapter focuses on the progressions and reversions in participants' base-ten thinking inferred from the **solution strategies** and **oral discourse** they used when solving additive tasks in different **representational settings**²¹. This analysis gives a nuanced and textured account of what underlay the substantial gains in intervention learners' base-ten thinking noted in the previous chapter, as well as providing evidence of non-linear moves in solution strategies and oral discourse across representational settings that characterized the minutiae of the overall forward trajectory.

Whilst teaching the intervention lessons (and when I watched the video-recorded lessons afterwards) I often observed – as the literature indicates – that learners' progress in their base-ten thinking, as displayed in their enacted strategies, was not a linear process. There were many forward *and* backward moves in participants' ability to enact base-ten strategies that seemed to be linked to shifts in task settings. Similarly, my interest in the nature and progression of participants' oral discourse during intervention was also piqued. I had a general sense that learners were better able to explain their thinking over the course of intervention. However, in the midst of these positive changes there were times when learners' explanations did not match their enacted strategies – which I refer to as a 'mismatch' in discourse. My interest was thus in a more fine-grained analysis of changes over time in learners' base-ten thinking, evident in their choice of solution strategies and in their 'math talk' while solving additive tasks in different settings. These experiences during intervention led to me using two focal questions (and sub-questions) as a guide for investigating the nature and progression of learners' base-ten thinking as displayed by their strategy usage and oral discourse, respectively. The first question was:

²¹ Tasks have been grouped according to the mode of representation in which they were presented.

- 1) What was the nature and progress of participants' **solution strategies** related to base-ten thinking:
 - a) *within the same* representational setting? and
 - b) *across various* representational settings over the course of intervention?

The second focus question that guided my enquiry into participants' base-ten thinking was:

- 2) What was the nature of participants' **oral discourse** related to base-ten thinking:
 - a) with regard to progression or expansion in learners' explanations? and
 - b) with regard to 'mismatches' between learners' explanations and their enacted strategies?

These questions also act as an organizing principle for presenting data in this chapter. So, to answer the first focus question, I next discuss the nature of participants' solution strategies.

6.2 The nature/progress of participants' solution strategies

In this analyses I make reference to participants' use of strategies that reflected progression, reversion and flexibility in how they solved similar tasks *within the same settings* and *across different settings*. I considered **strategy progression** to have occurred *within the same setting* when a learner solved an additive task (posed in a particular setting) by initially using a more rudimentary/inefficient strategy and then later solved a similar task (posed in the same setting) using a more sophisticated/efficient (base-ten) strategy. The opposite was true for **strategy reversion** – where the move was from a more sophisticated/efficient strategy to one that was more rudimentary/inefficient. When strategy progression/reversion was evident across similar tasks (e.g. +11 tasks) that were posed in different settings (e.g. bundling sticks and number sentences) then that progression/reversion was considered to have taken place *across various settings*. **Representational flexibility** – which only occurred as learners worked on similar tasks across different settings – was evident when a learner enacted a base-ten strategy in a more concrete setting and was later able to do the same in a more abstract setting. As explained in Chapter 4, what counted as an 'instance' in each case was the shift in strategy, 'math talk' or setting used by learners across similar additive tasks. Learners' continued use of the more efficient strategy, more abstract setting or more formal 'math talk'

(previously counted as an instance) did not count as additional instances of progression/reversion/ flexibility as that particular shift had already been accounted for.

6.2.1 Group 1's strategies within the same settings

Group 1's learner names have been arranged from left to right in Table 6.1 starting with the learner with the strongest LFIN attainment profile (Bongi) to the one with the weakest LFIN profile (Sibu) as in the previous chapter. This learner arrangement is kept the same for all subsequent comparison tables for Group 1 in this chapter. In this section I firstly discuss learners' strategy progressions *within the same settings* followed by strategy reversions. For the purpose of further comparison, I included an aggregation of the total number of instances of strategy progression and reversion for the group in the table.

Table 6.1 Group 1's strategies within the same settings

	Bongi	Mali	Jenna	Sibu	Total
Progression	4	3	4	3	14
Reversion	--	--	1	1	2

There were 14 instances of strategy progression *within the same settings* during intervention for Group 1. Two learners each had 4 instances of strategy progression and two had 3 instances each. Sometimes learners' progression from a less to a more efficient/sophisticated strategy occurred in one lesson and at other times over several lessons as they solved similar tasks posed within the same setting. For example, within the setting of bundling sticks, Mali solved $\square + 11$ tasks in lesson 4 using a count-on strategy: she solved $26 + 11$ by keeping 26 'in her head' and then made 11 counts, keeping track of her counts on her fingers: "twenty-seven, twenty-eight, twenty-nine... thirty-seven". The number-word spoken (thirty-seven) in coordination with the eleventh finger she raised was Mali's answer to $26 + 11$. In lesson 5 Mali solved similar tasks (presented in the same setting) using the N10 strategy: she solved $34 + 11$ by keeping the first addend whole and added the second addend in place-value chunks, i.e. she calculated $34 + 11$ as $34 + 10 + 1$ using the 100 chart for support. In first incrementing by ten and then by 1, Mali solved a $\square + 11$ task in two jumps which is much quicker than doing the same in eleven jumps. Thus Mali progressed from a less efficient (count-on) to a more efficient (N10) strategy when solving $\square + 11$ tasks posed within the same setting over these two lessons.

An example of strategy progression within a single lesson is taken from another member of Group 1, Bongi. In lesson 2 he initially answered 'ten more/less than' oral tasks by making one vertical jump on the 100 chart (i.e. incrementing and decrementing by ten) but later in the same lesson, was able to answer similar oral tasks mentally, without needing the support of the 100 chart. This is an example of progressing from a less to a more sophisticated/efficient strategy as Bongi initially used a tens-based setting to increment and/or decrement by ten but later did so mentally (without the support of a setting). There was a greater incidence of strategy progression over two or more lessons (11 instances) than within single lessons (3 instances) for Group 1.

There was 2 instances of strategy reversion – moving from a more efficient/sophisticated strategy to a less efficient/rudimentary strategy – *within the same setting* during the intervention period for Group 1. This happened with the weakest attainer in the group (Sibu) and one of the middle attainers (Jenna). At the end of lesson 2 Sibu answered 'ten more than' oral tasks with a quick²² or immediate²³ answer but when the same oral tasks were posed in lesson 12 he needed the 100 chart to answer (that is, to find the given number and move one block down). In lesson 3 Jenna incremented by ten mentally (ten more than 46 is 56) and gave her justification as "Because I'm counting in ten" but then reverted to a count-on strategy using her fingers when I reposed the question saying, "So, fifty-six is how much more than forty-six?" In Jenna's case the strategy reversion occurred within the same lesson but in Sibu's case it occurred after many lessons.

6.2.2 Group 1's strategies across various settings

In this section learners' strategy progression *across various settings* is discussed first followed by strategy reversion and representational flexibility. As previously explained: strategy progression entails a shift from a more rudimentary/inefficient strategy to a more sophisticated/efficient one; strategy reversion speaks about a shift in the opposite direction; and representational flexibility involves a learner using a base-ten strategy in a more abstract setting after having used it in a more concrete setting to solve a similar additive task. The

²² A response given within 7 seconds.

²³ A response given within 3 seconds.

total number of instances of strategy progression, reversion and flexibility for the group have been included in Table 6.2.

Table 6.2 Group 1's strategies across various settings

	Bongi	Mali	Jenna	Sibu	Total
Progression	--	--	--	--	0
Reversion	--	2	3	3	8
Flexibility	7	4	4	1	16

Whilst there were **no instances of strategy progression**, Group 1 displayed **8 instances of strategy reversion** across various representational settings during the intervention period. These instances were more-or-less evenly spread across three group members but none were evident for Bongi. Instances of strategy reversion were more evident within single lessons (7 instances) than over two or more lessons (1 instance). An instance of strategy reversion that occurred within one lesson was the way in which Sibu solved 2-digit+11 tasks during lesson 9. Early on during lesson 9 Sibu solved 2-digit+11 tasks posed with *bundling sticks* using the N10 strategy (with the 100 chart for support). So, to solve 42+11 he placed his finger on 42, moved one block down to 52 and one block to the right: 53. But, at the end of lesson 9 when similar tasks were posed as *number sentences* he reverted to a less efficient strategy: he counted-on in ones on the 100 chart. Reversion was also seen in Jenna's strategy for answering +/-10 oral tasks. At the start of lesson 19 Jenna solved +/-10 *oral tasks* mentally but later in the same lesson she solved similar *written tasks* by using the more inefficient unit count strategy. All instances of strategy reversion occurred when tasks moved from being posed in more concrete to more abstract settings, for example, from bundling sticks to number sentences or from oral 'ten more/less than' tasks to 'ten more/less than' on crazy squares (in the former the instruction to increment/decrement by ten is explicit but in the latter, it is implicit).

Sixteen instances of representational flexibility were identified during the intervention period – where learners enacted a base-ten strategy in a more concrete setting and were later able to do the same in a more abstract setting. Nine of these instances occurred within single lessons and 7 occurred across different lessons. In lesson 5 Bongi enacted the N10 strategy to solve +11/+12 tasks posed with *bundling sticks* and later used the same strategy to solve +11/+12 tasks posed as *number sentences*, the latter being a more abstract setting. All the

learners incremented and decremented by ten on/off the decuple with varying degrees of facility across different settings (oral tasks, bundling sticks, word problems, blank 100 chart, number sentences) over two or more lessons. A very distinct gap in representational flexibility is evident between the strongest attainer and the two middle attainers and between the latter and the weakest attainer, i.e. nearly half of these incidents are attributed to Bongi, a quarter of these each to Mali and Jenna and one to Sibü. This suggests – as has been noted in the literature (Baroody & Dowker, 2013) – that this kind of representational flexibility across settings is an important marker of conceptual understanding and higher attainment related to base-ten thinking, and therefore important to build into instruction.

There were a few instances when I was able to intervene just as learners reverted to a more rudimentary strategy. Sometimes I only needed to make the link between the tasks more explicit (even though they were posed in different settings) for a learner to see the connection and continue using the more advanced strategy but at other times this level of support was insufficient. In the latter case I overlapped the more concrete setting with the more abstract setting and gradually ‘distanced’ the former. Sibü’s approach to solving 2-digit-by-2-digit subtraction tasks over two lessons provides an example of this. In lesson 11 Sibü solved 2-digit–11 and 2-digit–12 tasks posed with *bundling sticks* by enacting the N10 strategy on the 100 chart but in the next lesson (L12) he attempted to count-by-ones on the 100 chart to solve similar tasks posed as *number sentences*. I intervened by making the link between the tasks explicit but when this was not enough I overlapped the more concrete setting (*bundling sticks*) with the more abstract setting (*number sentences*) and Sibü was then able to enact the N10 strategy on the 100 chart to solve the written tasks. After two examples I removed the extra layer of support (bundling sticks) and Sibü continued using N10 on the 100 chart to solve written tasks.

6.2.3 Discussion of Group 1’s solution strategies

By juxtaposing the nature of Group 1’s strategies across the same and different settings (Table 6.3), the patterns in how learners appropriated base-ten thinking (evidenced by their more advanced solution strategies and representational flexibility) is made clearer.

Table 6.3 Group 1's strategies within and across settings

		Bongi	Mali	Jenna	Sibu	Total
Progression	within same setting	4	3	4	3	14
Reversion		--	--	1	1	2
Progression	across various settings	--	--	--	--	0
Reversion		--	2	3	3	8
Flexibility		7	4	4	1	16

Firstly, **strategy progression** for Group 1 only occurred when learners solved similar tasks *within the same setting* (14 instances) as no instances of progression were observed when they solved similar tasks *across various settings*. Secondly, there was a higher frequency of **strategy reversion** when learners solved similar tasks *across various settings* (8 instances) than when they solved similar tasks *within one setting* (2 instances). **Strategy progression** *within settings* was more-or-less evenly spread across the attainment range in the group but **strategy reversion** *within and across settings* was skewed toward the weaker attainers in the group.

Further, **strategy progressions** for Group 1 occurred *more across* multiple lessons (11) and *less within* the same lessons (3) while the opposite is true for **strategy reversions** – these occurred *less across* different lessons (2) and *more within* the same lessons (8). So, the progressions in learners' base-ten thinking, as evidenced by their use of more efficient/sophisticated solution strategies, were more stable when learners worked *within the same settings* – and this applied in a greater degree to the weaker attainers. These progressions in their base-ten thinking sometimes fell away over time or when an element of 'breadth' was introduced across settings, which was a predictable result. In addition, progressions in learners' base-ten thinking (as inferred from their solution strategies) were more prevalent when they worked within the same settings *over two or more lessons* rather than in a single lesson.

In Group 1 distinct differences in **representational flexibility** – which only occurs across different settings – were evident across the attainment range during the intervention period. Bongi, the strongest attainer, had the highest rate of representational flexibility while Sibu, the weakest attainer, had the lowest. The two middle attainers (Mali and Jenna) had the same score which was 3 less than Bongi's score and 3 more than Sibu's. The gap between these

learners' representational flexibility was bigger than the gap seen between their strategy progressions and/or reversions. Moreover, and interestingly, ranking the representational flexibility scores of members of Group 1 (Table 6.4) reflected the ranking of their post-test LFIN scores.

Table 6.4 Group 1's Representational flexibility and LFIN post-test score

	Bongi	Mali	Jenna	Sibu
Representational flexibility	7	4	4	1
Overall LFIN post-test score	27	24	24	21

Next I discuss the nature and progress of the solution strategies used by learners from Group 2 and both 'singletons'. Many of the results regarding learners' use of solution strategies that I present next were comparable to that of Group 1, therefore I present Group 2's use of strategies *within the same setting* and *across various settings* in one table. Salient points with regard to similarities and differences between group's strategy usage is highlighted.

6.2.4 Group 2's strategies within the same settings and across various settings

Group 2's learner names have been arranged from left to right in Table 6.5 starting with the strongest pre-test attainer on the left (Julie) and the weakest on the right (Kgomo). The nature of learners' strategy usage *within the same setting* and *across various settings* are presented in one table here for the sake of brevity. The total number of instances of progressions, reversions and representational flexibility for Group 2 have been aggregated in the table for comparative purposes.

Table 6.5 Group 2's strategies within and across settings

		Julie	Khosi	Kyle	Kgomo	Total
Progression	within same setting	1	2	5	5	13
Reversion		--	--	1	4	5
Progression	across different settings	--	1	--	--	1
Reversion		1	1	3	3	8
Flexibility		6	6	2	1	15

The nature of Group 2's strategy usage during the intervention period was similar to that of Group 1: there were *more instances of strategy progression within the same representational setting (13)* than *across various settings (1)* and most strategy progressions occurred across multiple lessons (10) rather than within single lessons (4). *Strategy reversions* (which also followed the same pattern as for Group 1) were more prevalent in Group 2 when learners worked *across various settings* to solve similar additive tasks – 8 reversions *across various settings* compared to 5 *within the same setting*. These reversions were more prevalent within single lessons (10) than across multiple lessons (3) and were skewed towards the weaker attainers in the group.

There were some differences between the two groups' strategy usage: Group 1 did not show any strategy progressions across different settings but Group 2 had one such instance; for Group 1 two out of eight strategy reversions occurred across multiple lessons but for Group 2 all strategy reversions occurred within single lessons. Next I present exemplification of instances coded as strategy progression/reversion and representational flexibility for Group 2.

Khosi showed *strategy progression* in lesson 3 when she initially solved +10 *word problems* by 'counting-on' in ones on the number track but later in the same lesson answered similar +10 *word problems* mentally. So, she progressed from a more rudimentary strategy to a more efficient strategy when answering +10 word problems. In lesson 5 Kyle solved +11/+12 tasks posed with *bundling sticks* by 'counting-on' (with the 100 chart for support) but later in the same lesson he solved similar tasks using a mental N10 strategy – so he also progressed from a more rudimentary to a more efficient strategy. Khosi's shift in strategy for solving +10 word problems can be described as a move from calculation-by-counting to formal calculation, while shifts in Kyle's strategy for +11/+12 tasks can be described as a move from calculating-by-counting to calculation-by-structuring (Van den Heuvel-Panhuizen, 2008).

There were *5 instances of strategy reversion within the same representational setting* for Group 2: none for the two learners with the stronger post-test attainment profiles and the most for Kgomo, the weakest attainer in the group. In lesson 5 Kgomo solved 32(scr)+11 presented with *bundling sticks* (first addend flashed and screened) using the N10 strategy: $32+10=42$ and $42+1=43$, but when his turn came around again and he was presented with

the next task: $45(\text{scr})+11$ he could not enact the same strategy, closing his face with his hands when his peer encouraged him by saying, "Kgomo you just did it now". In lesson 10 Kyle solved a 2-digit+2-digit task presented with indexical drawings of *bundling sticks* using the 1010 strategy: he solved $21+23$ by adding the tens ($20+20=40$) and then the units ($1+3=4$) and finally adding the two interim sums ($40+4=44$) to get the answer but was unable to solve the next two similar tasks ($23+33$ and $32+31$) using the same strategy. In both cases these learners were eventually able to re-enact their respective base-ten strategies with support from me. The number of reversions across and within lessons were relatively close: 3 and 2 instances respectively. Kgomo had 3 reversions across lessons and he and Kyle had 1 reversion each within a lesson.

As in Group 1, a distinction is apparent when one considers the difference in the number of times the two stronger attainers showed *representational flexibility* (6 instances each) compared to the two weaker attainers (2 and 1 respectively). This means that Khosi and Julie were far more facile with enacting base-ten strategies when the original task settings changed from more concrete to more abstract. On the other hand, Kyle and Kgomo could enact base-ten strategies when tasks were presented in more concrete settings but could not do the same when the setting changed to a more abstract one. A similar relationship between the ranking of learners' frequency of representational flexibility was evident in their LFIN post-test scores (see Table 6.6). This relationship was interesting because the association between representational flexibility and LFIN post-test score was more regular than for the progression/reversion counts across learners in relation to LFIN post-test score.

Table 6.6 Group 2's Representational flexibility and LFIN post-test score

	Julie	Khosi	Kyle	Kgomo
Representational flexibility	6	6	2	1
LFIN post-test score	27	27	23	20

Next I highlight the main points regarding Group 2's usage of solution strategies during intervention. For the sake of brevity, I once again relate these to the outcomes for Group 1.

6.2.5 Comparison of Group's solution strategies

A pattern in learners' solution strategies emerged for Group 2 that was similar to that for Group 1: they had more **strategy progressions** when they worked on similar tasks using the *same representational settings* and a higher frequency of **strategy progressions** across two or more lessons. Group 2 also had a greater number of **strategy reversions** when they worked *across different settings* and had a higher frequency of **reversions** within individual lessons. **Strategy reversions** were also skewed towards the weaker attainers in the group; the only difference in this case was that the weaker attainers also had a higher number of strategy progressions. Further, progressions in Group 2's use of base-ten strategies were more widespread when they worked within the same settings *over two or more lessons* rather than in individual lessons – which was the same for Group 1. The big differences between stronger and weaker attainers' facility with **representational flexibility** in both groups were mirrored in their overall LFIN post-test scores; and stronger attainers in both groups had a higher frequency of **representational flexibility** than the weaker.

So, to sum up: **representational flexibility** was skewed towards the stronger attainers in both groups while **strategy reversions** were skewed towards the weaker. The difference between the group's **strategy progressions** was that in Group 1 this was more-or-less evenly spread across the attainment range but was much higher for weaker attainers in Group 2.

Next I look at the nature and progression in singletons' use of solution strategies when solving 2-digit by 2-digit additive problems. The singleton with the stronger attainment profile (Khanye) had similar intervention to that received by learners in Groups 1 and 2 (aimed at learners at SEAL Stage 3) whilst the singleton with the weaker attainment profile (Josh) received intervention aimed at SEAL Stage 2 (as explained earlier). Thus, the number of 2-digit by 2-digit additive tasks posed to (and solved by) Josh was much less than that which Khanye received over the course of their individual intervention pathways. Because of the time spent on appropriating number concepts relevant to SEAL Stage 2, Josh was only posed with 2-digit by 2-digit additive tasks during the last two intervention lessons. Thus, most of the examples related to Josh's solution strategies used during intervention are taken from the tail-end of the intervention period and are more fine-grained. I present my analysis of these learners' strategies alongside each other for brevity and for openings into the shifts related to

base-ten thinking that were made available to a learner with a stronger attainment profile who received one-on-one intervention compared to a learner with a weaker attainment profile. In my exemplification and discussion of singletons strategy usage I compare results to those of the two groups that have already been noted.

6.2.6 Singletons' solution strategies within the same settings and across various settings

Khanye (the singleton who started intervention at SEAL Stage 3) is placed on the left in Table 6.7 below and Josh (the singleton who received intervention for SEAL Stage 2) is on the right. This learner arrangement is kept the same for all subsequent tables for singletons in this chapter.

Table 6.7 Singletons' strategies within and across settings

		Khanye	Josh	Total
Progression	within same setting	3	1	4
Reversion		--	--	0
Progression	across different settings	--	--	0
Reversion		2	1	3
Flexibility		6	2	8

Khanye had more instances of **strategy progression** *within the same settings* over the course of intervention, that is, 3 compared to Josh's 1. This result (where progression was skewed towards the stronger attainer) differs to that for Group 1, where progression was more-or-less evenly spread across the attainment range, and also to Group 2's strategy usage, which was skewed towards the weaker attainers. Josh's one instance of strategy progression could be described as a very 'fine-grained shift' because here he enacted the N10 strategy to solve 'decuple+2-digit' tasks (L17) posed with bundling sticks (screened) with increased independence and speed: he solved '30+12' and '20+22' with a lot of guidance from me in the form of 'cue' questions like, "What will you add first?" and "What do you add next/still need to add?" → then he solved '30+43' with very little guidance in 33 secs → and finally solved '50+27' in 3 seconds (immediately) with no guidance.

There were no instances of **strategy progression** for singletons and Group 2 *across different representational settings*; and only 1 instance for Group 1. The 3 instances of **strategy reversion** for singletons (2 for Khanye and 1 for Josh) showed a small difference between the stronger and weaker attainers, unlike that for both groups, where strategy reversions were skewed towards weaker attainers. Both singletons had an instance of strategy reversion when they initially used a more efficient/sophisticated strategy to solve a task posed with bundling sticks but subsequently used a more rudimentary strategy to solve a similar task when the setting was concealed (or 'distanced'). Khanye (the stronger attainer) had another instance of **strategy reversion** when she enacted a sophisticated strategy in one setting (she incremented and decremented by ten mentally in the setting of bundling sticks) and was then subsequently unable to enact the same strategy to solve similar tasks posed as number sentences (here she incremented and decremented by ten using the 100 chart for support). For both learners, strategy reversion only took place within individual lessons. There were no instances of **strategy reversion** *within the same settings* for singletons and 3 instances *across various settings*, which was comparable to the results for Groups 1 and 2 which had more reversions across settings.

As with both groups, singletons' facility with **representational flexibility** was skewed towards the higher attainer. Khanye had 6 instances of **representational flexibility** during intervention (5 *within individual lessons* and 1 *across different lessons*) and Josh had 2 (both *within individual lessons*). One example of **representational flexibility** across different lessons was evidenced when Khanye incremented and decremented by ten in various settings (oral tasks, 100 chart, bundling sticks, dot strips, ENL and written tasks). An instance of representational flexibility shown in the same lesson occurred in lesson 17 when Josh first solved 'decuple + decuple' tasks (e.g. $40+30$) posed with bundling sticks and then later in the same lesson solved similar tasks posed as number sentences. This is counted as representational flexibility because he used the same strategy (incrementing by a multiple of ten) to solve similar tasks posed first within a more concrete setting and then later within an abstract setting. The pattern of higher representational flexibility being associated with a higher LFIN post-test score is maintained across the singletons as seen in Table 6.8.

Table 6.8 Singletons' representational flexibility and LFIN post-test score

	Khanye	Josh
Representational flexibility	6	2
LFIN post-test score	27	20

6.2.9 Key similarities across all intervention learners' solution strategies

The more predictable patterns that emerged across singletons and groups in relation to learners' use of strategies during intervention were:

- learners displayed more **strategy progression** when they worked on similar tasks *within the same settings*
- **strategy progressions** were more prevalent over the course of two or more lessons for both groups and Khanye (Josh's one instance of progression occurred in lesson 17)
- no **strategy progression observed** *across various settings* for all intervention learners, except the one instance for Group 2
- learners had more **strategy reversions** when they solved similar tasks *across different settings*
- **strategy reversions** were more prevalent within individual lessons for all intervention learners

The instances of learners' strategy usage that related to representational flexibility – which was evidenced when a learner enacted a base-ten strategy in a more concrete setting and then later solved a similar task by enacting the same strategy in a more abstract setting – revealed the following:

- stronger attainers had a higher incidence of **representational flexibility** than weaker attainers
- there was a higher incidence of **representational flexibility** within single lessons than across two or more lessons
- a patterned association between learners' **representational flexibility** and their overall LFIN score emerged as an interesting finding (as shown in Table 6.9). While the dataset here was too small for correlational analyses, this result – which has backing in the literature – pointed to a finding worthy of further investigation.

Table 6.9 Intervention groups' representational flexibility and LFIN post-test score

	Bongi	Khanye	Julie	Khosi	Mali	Jenna	Kyle	Sibu	Kgomo	Josh
Representational flexibility	7	6	6	6	4	4	2	1	1	2
LFIN post-test score	27	27	27	27	24	24	23	21	20	20

Next I deal with the second question which focuses on participants' discourse:

2. What was the nature of participants' oral discourse related to base-ten thinking:
 - a) with regard to progression or expansion in learners' explanations?
 - b) with regard to 'mismatches' between learners' explanations and their enacted strategies?

To recap, what counted as an instance of *discourse progression* related to base-ten thinking in my study was a shift or change in the 'math talk' learners used to explain their working. When a learner first used colloquial language to explain why she solved a particular task using base-ten and then later used more formal or mathematical language to explain similar reasoning, then this was regarded as an instance of *discourse progression*. Changes in learners' discourse from: mathematical language to colloquial language, or from an explanation that illuminated the mathematical operation to an explanation in which the underlying mathematical operation was not illuminated – were described as *reversions in discourse*. When a learner's explanation did not match the strategy she enacted on a setting, then this was labelled as a '*mismatch*' between her discourse and enacted strategy. The evidence of 'mismatches' provided here is solely based on a comparison between learners' observed physical actions as they solved 2-digit by 2-digit additive tasks (observed either in real time or later via video recordings of the lesson) and their oral explanation of the solution strategy they used, and is therefore limited to episodes that were observed and/or captured on video.

While I judiciously sought to select participants who were comfortable communicating in English, Josh was reticent when it came to engaging in 'math talk' during intervention. His poor number skills – together with having all attention on him in the one-on-one format of

intervention – appeared to cause Josh to feel anxious, and across his responses, he made widespread use of gestures and one-word answers in reply to my questions during the first half of intervention. Also, the type of number tasks Josh solved during most of the second half of intervention (e.g. 4+9) lent themselves to explanations like, “Nine in my head (touches his head with left hand), ten, eleven, twelve, thirteen” (opening four fingers sequentially on his right hand). All these factors (and some I may be unaware of) led to Josh using oral discourse that was different to that of his peers and resulted in me not being able to discuss his oral discourse using the markers that applied to all other intervention learners. This is not to say that Josh showed no expansion or progression in his oral discourse, because he did.

6.3 Participants’ Oral Discourse

To answer the second question guiding this chapter, I first describe the instances where *progression* was evident in participants’ oral discourse followed by a description of instances of ‘*mismatch*’ and *evidence of base-ten thinking* for each group and ‘singleton’ in turn.

6.3.1 Discussion of Group 1’s Oral Discourse

Group 1’s learner names have been arranged from left to right in Table 6.10: Bongi (the strongest attainer in the group) on the left and Sibü (the weakest attainer) on the right. This table sets out the number of observed instances per learner for each marker (i.e. *progression* and ‘*mismatch*’) related to oral discourse. Discourse *reversions* have not been included in Table 6.10 as no instances thereof were observed for Group 1 during the intervention period. For purposes of further comparison, I included an aggregation of the total number of instances for the group per category in each table.

Table 6.10 Group 1’s Oral Discourse

	Bongi	Mali	Jenna	Sibü	Total
Progression	6	3	2	3	14
‘Mismatch’	--	2	1	4	7

There were no instances of discourse reversion for Group 1 and 14 instances of discourse progression over the intervention period. This progression only occurred when learners solved tasks *within the same setting* – no matter what that setting was: bundling sticks, crazy

squares, blank 100 charts or word problems. As with strategies, and once again, predictably, these progressions were more frequent over the course of two or more lessons (10 instances) than within individual lessons (4 instances) and was skewed towards the strongest attainer in the group: Bongi. What follows are two instances of discourse progression for Group 1.

Progression in Bongi's discourse was evident *within the same setting* across lessons 3, 8 and 18 with ± 10 tasks on the *crazy square or blank 100 chart*. In lesson 3 Bongi justified why he said 36 should go above 46 in the crazy square using talk that did not illuminate the mathematical operation underlying his solution like "Because it's lower" but later (L8) he justified why 35 would go in the block above 45 using talk that illuminated the mathematical operation underlying his solution hence, "Because it's ten less". Also, in lesson 18 Bongi justified why 46 should be written in the block above 56 and 66 in the block below 56 using: talk that illuminated the mathematical operation underlying his solution: "Because we are minusing ten" and "Because we are adding ten", respectively.

During lesson 12 there was evidence of discourse progression *within one lesson* when Sibü explained how he subtracted 20 from 2-digit numbers in the setting of *bundling sticks*. Early on in lesson 12 Sibü solved '73-20' by moving his finger on the 100 chart and explained his strategy using discourse that focused on his physical actions on the setting: "I moved two blocks up". Here Sibü's discourse did not illuminate the mathematical operation underlying his physical actions. Later in the same lesson Sibü solved '21-20' in the same way (by moving his finger 2 blocks up on the 100 chart) but here his discourse described the mathematical operation involved in his physical action, hence, "I minused twenty". There were no instances of discourse progression *across different representational settings* and no discourse reversions for Group 1 during the intervention period.

There were 7 instances of 'mismatch' for Group 1 – where learner explanations were incongruous with their actual solution strategy. The 7 instances of 'mismatch' were skewed towards the weakest attainer, Sibü. In 5 out of the 7 cases of 'mismatch' the learners described an inefficient calculation-by-counting strategy rather than the more efficient calculation-by-structuring strategy they actually employed. In the other 2 cases of 'mismatch' learners explained a more efficient strategy than what they had used. Most instances of 'mismatch'

occurred during the second half of intervention: L9, L10, L10, L12, L12, L19 and L19 and in most instances pointed to a 'lag' between learners' actions on settings and the mathematical discourse available to describe and replay these actions. Examples of the kinds of 'mismatch' evident between Group 1 learners' strategies and their discourse are presented next.

In lesson 9 Mali solved $+11$ and $+12$ tasks on the 100 chart by placing her finger on the first addend, moving down 1 block (incrementing by ten) and then moving her finger either 1 or 2 blocks to the right (to increment by 1 or 2, respectively). But, when asked to explain her solution strategy for ' $24+11$ ' to the group, Mali put her finger on the first addend (on the 100 chart) and said "twenty-four". She then made 11 jumps by moving her finger from 24 to 35 on the 100 chart while saying the FNWS "twenty-five, twenty-six... thirty-five" in coordination with the jumps. But, the video recording shows that Mali actually solved $24+11$ by placing her finger on 24, she moved 1 block down to 34 (incrementing by ten) and then moved her finger 1 block to the right, 35 (incrementing by one). Thus, Mali's discourse in lesson 9 belied the efficient strategy she used to solve $+11$ and $+12$ tasks on the 100 chart: she described an inefficient 'count-on' strategy (calculation-by-counting) while she actually enacted the more efficient N10 (calculation-by-structuring) strategy.

Both Mali's instances of discourse 'mismatch' occurred in the context of her solving 2-digit by 2-digit additive relations tasks. The first was an addition task in lesson 9: $24+11$, where she explained a *more inefficient* strategy than the one she used and the second (L19) was a subtraction task where she explained a *more efficient* strategy than what she actually used. The second example was interesting because in that lesson I was trying to get the group to see the value of using the 'adding-up-to' strategy when the two numbers in a subtraction sentence were close: it was much quicker to 'add-up-to' than to count-back-in-ones. After doing a few similar problems as a group using the 'add-up-to' strategy I gave learners the problem ' $17-15$ ' to solve on their own. I first asked Bongi to share his strategy and he explained the N10 strategy ($17-10=7$ and $7-5=2$). Even though this was not what I expected, I accepted it because he was applying a base-ten strategy we had learnt previously. What happened after that was the interesting bit because the rest of the group (including Mali) explained the exact same strategy. I could confidently refute Mali and Sibu's explanations

because I clearly saw how each solved that task on the structured number line. When I refuted Mali's N10 explanation she admitted to having counted back in ones.

When tasked with positioning numbers on a blank 100 chart in lessons 10, 12 and 19 Sibü's answers were all correct but his explanations did not match the strategies he used. In lesson 12 there were two instances of 'mismatch' between Sibü's explanations and his strategies. One of these instances took place when his group was tasked with finding 99 on the blank 100 chart and Sibü was very quick to write 99 in the correct block. When I asked him to explain his strategy Sibü pointed to the 9th empty block in the first row and said, "nine" before moving his finger one block down in coordination with each subsequent number word spoken: "nineteen, twenty-nine... ninety-nine" (a similar explanation was given for finding block 35 in lesson 10 detailed previously). But, video evidence clearly showed that Sibü placed his finger on the last empty block of the blank 100 chart (block 100) and then moved his finger 1 block to the left and wrote 99 in that block. In all 3 instances of 'mismatch' between Sibü's strategies and discourse (in the context of finding numbers on the blank 100 chart), his explanations started from a single digit in the first row and continued with him saying the consecutive 'ten more' numbers in coordination with his finger moving down the respective column until he got to the given number. His discoursing was thus linked to a type of counting sequence that we often practiced in the first five minutes of every lesson, that is, counting in tens off the decuple.

I next discuss the changes evident in Group 2's oral discourse during the intervention period.

6.3.2 Discussion of Group 2's Oral Discourse

In Table 6.11 Group 2's learner names have been arranged from left to right: Julie and Khosi (the learners with the stronger attainment profiles in the group) on the left and Kyle and Kgomo (those with the weaker attainment profiles) on the right. The number of observed instances per learner for each marker related to oral discourse have been included and aggregated in the table. As for Group 1, discourse *reversions* have not been included in Table 6.11 as no instances thereof were observed.

Table 6.11 Group 2's Oral Discourse

	Julie	Khosi	Kyle	Kgomo	Total
Progression	1	4	2	2	9
'Mismatch'	3	--	1	--	4

I observed 9 instances of discourse progression across the intervention period as learners from Group 2 solved similar tasks *within the same representational settings*. Khosi, one of the stronger attainers, had the highest frequency (4) of these discourse progressions and the rest were more-or-less evenly distributed amongst the other three learners in the group. Discourse progressions were slightly more prevalent over the course of two or more lessons (5 instances) than within single lessons (4 instances). Two instances of discourse progression for Group 2 are presented next.

An example of discourse progression within a single lesson occurred during lesson 3. Here learners were faced with word problem situations where Baby Frog (BF) could only jump onto a rock that was 'ten more' than the rock she was on whilst Mother Frog (MF) could only jump onto a rock that was 'ten less' than the rock she was on as they tried to find each other. When Khosi's turn came around to move BF closer to her mother she said that BF must jump from rock number 38 to rock number 48 because "...the number must end in eight". Using the same reasoning, I asked Khosi why BF could not jump to rock number 58 because that rock also had a number that ends with '8' and the following exchange ensued:

- K : If we are at thirty-eight... and you want to jump to fifty-eight, it means you are going to count twenty more.
T : So is fifty-eight twenty more?
K : (*nods her head in agreement*)
T : So forty-eight is how much more?
K : Ten

This shift in discourse was counted as progression/expansion because Khosi's first explanation for the solution she offered did not foreground a mathematical reason which was in contrast to the second explanation she offered, "... it means you are going to count twenty more" which was based on clear mathematical reasoning.

Discourse progression across lessons was evident in Kyle's explanation in lesson 10 compared to that offered in lesson 5. In lesson 10 I asked Kyle how he found block 76 so quickly on his blank 100 chart and he said, "I know that here is nineties (pointing along last row on the 100

chart) and here is eighties (pointing along the second last row) so here must be seventies (pointing to the third row from bottom) and I said one, two, three, four, five, six, (pointing from the 1st to 6th block along the row), seventy-six". Here Kyle's discourse can be described as more 'generalized talk' based on the tens structure of the setting because his explanation used decuple words (nineties, eighties, seventies) to refer to the setting in more broader terms. This type of talk can be contrasted with Kyle's earlier explanation (L5) for how he knew that 17 was in the block above 27 on the crazy square: "Because of seven, seventeen, twenty-seven, thirty-seven that's how I know". The latter, which was a mere recital of a particular number sequence rather than an explanation (and thus narrowly focused), only referred to one example and was thus labelled 'localised talk'. There were no discourse progressions *across different representational settings* for Group 2 and no discourse reversions across the intervention period.

There were 4 instances of 'mismatch' between learners' enacted strategies and their discursive explanations. These instances of 'mismatch' occurred mostly during the first few lessons (2 each in lessons 3 and 4) for Julie and Kyle who started intervention being more comfortable conversing in English than Khosi and Kgomo. In all instances of discourse 'mismatch' learners' talk described a rudimentary strategy which belied their more efficient base-ten strategy used. It is interesting to note that only the two learners who started intervention being more confident to talk were guilty of 'mismatches' in their math talk, and that these 'mismatches' only occurred within the first few lessons. This result could simply be linked to the fact that Julie and Kyle were the two group members who were more communicative at the start of intervention and that they may have misunderstood what I meant when I asked, "How did you work that out?"

During lesson 3 Julie seemed to know the results of Baby Frog's 'ten more' jumps (from 13 to 23 to 33 and so forth). When BF was on rock 53, Julie immediately offered 63 as the number that was ten more and I asked her to explain to her group how she knew the answer. Julie then 'counted-on' in ones on the number chart (reciting the number names 'one' to 'ten' while pointing her finger from 54 to 63) as her way of explaining how she knew the answer, even though she did not use this method. I challenged Julie, saying that I did not see her counting

and she then said that she knew that adding ten to 53 would be 63. A few examples later this exchange ensued:

Julie had just offered "sixty-eight" in reply to "Where can BF jump to from rock fifty-eight?"

T : How do you know, Julie? Why do you say sixty-eight?

J : Because we are adding ten.

T : So, fifty-eight plus ten is...?

J : Sixty-eight

In the next task, Julie said that Mother Frog (who could only jump ten less) had to jump from rock 86 to rock 76 – without having looked at the 100 chart. When she was asked to explain to her group how she knew, Julie again used a unit count, this time backwards on the 100 chart – demonstrating the forwards and backwards nature of learning.

In lesson 4 Kyle enacted the N10 strategy to solve $24+11$ on the 100 chart but 'counted-on' in ones when asked to explain how he got his answer – which was a 'mismatch' between his enacted strategy and discursive explanation. In another episode (L4) I interrupted Kyle's attempt to enact calculation-by-counting and asked him why he counted in ones when he had proven that he can count in tens and he replied, "Because that's how my teacher showed me". I got the distinct impression that Julie and Kyle may have been misinterpreting what I wanted them to do when I asked questions like, "How do you know?" or "Show us how you worked that out". I made it very clear to the group what I meant when I posed such questions and explained that their teachers would also be very happy if learners showed them that they can count in tens. This seemed to have worked because thereafter no more 'mismatches' were evident between learners' actions and their explanations that relayed these actions.

6.3.3 Discussion of Khanye's (Singleton 1) Oral Discourse

In Table 6.12 I present the instances of progression and 'mismatch' observed in Khanye's oral discourse during the intervention period. Immediately following the table I provide details of Khanye's discourse progressions and also example thereof. Thereafter I do the same for 'mismatches' in Khanye's math talk during intervention.

Table 6.12 Khanye's Oral Discourse

Progression	'Mismatch'
8	1

Khanye showed 8 instances of **discourse progression/expansion** when she solved similar tasks *within the same settings* across the intervention period. Six of these instances occurred within individual lessons while 2 occurred across more than one lesson. There were thus three times more shifts that occurred within individual lessons than across two or more lessons.

Khanye showed discourse progression across four lessons in the context of the oral 'ten more than' game we often played from the middle of the intervention period onwards, where we each got a turn to ask the other 'ten more than' questions. I often made 'silly mistakes' but found it difficult to get Khanye to challenge my incorrect offers. In lesson 11 (about mid-way through the intervention period) I made my first 'mistake' by offering 35 in answer to: "What is ten more than 15?". Khanye's mouth dropped open and she had an expression of shock on her face but she did not challenge my incorrect offer. When she wanted to carry on posing 'ten more than' questions to me I asked her if 35 was the correct answer and she shook her head (meaning 'no'). After she told me that 25 was the correct answer I encouraged Khanye to challenge any incorrect offer I made. In lessons 12 and 13 I again made 'silly mistakes' when we played the 'ten more than' game but Khanye was still reluctant to say anything – at first she only covered her mouth and laughed but later on she verbalized her disagreement saying 'No'. These one-word verbal responses, compared to the gestures she made previously, were considered discourse expansions but without foregrounding a mathematical reason. Khanye moved on from simply saying 'No' to just repeating the question when I gave an incorrect answer, but a major shift came about during lesson 15:

The lesson started with Khanye saying the FNWS and BNWS between 116 and 125, NWB and NWA in the same number range and answering my questions in the 'ten more than' game. Thereafter it was her turn to pose the 'ten more than' questions to me:

- K : Ten more than fifteen?
T : Twenty-five
K : Ten more than one hundred and ninety-five?
T : Two hundred and five.
K : Ten more than seventy-nine?
T : Sixty-nine
K : Huh-uh (*accompanied by shaking of her head*), I didn't say ten less!

Khanye's expression: "Huh, uh (meaning no), I didn't say ten less!", which was coded as explanation offered highlighted clear mathematical reasoning, showed two things. Firstly, that she was on top of the concepts: 'ten more' and 'ten less' and secondly, that she was able (and

confident enough) to use these maths terms to express *why* my answer was incorrect. This exchange was thus considered an instance of discourse progression.

I observed 1 instance of 'mismatch' during intervention between Khanye's explanation and her enacted strategy. I think this may be due to the fact that we worked together closely in the one-on-one format of intervention. In lesson 11 Khanye was tasked with filling in numbers on a blank 100 chart and had correctly filled in numbers 98 and 41. I next asked her to fill in number 88 and she did so in 4 seconds. The following exchange then took place:

- T : How did you know?
K : Here is nineties (*points along the last row*) and here is eighties (*points along the second last row*) and here is eighty-eight.
T : But your answer was very quick. Are you sure you didn't look at ninety-eight and minus ten to get to eighty-eight?
K : Yes
T : So, if this is eighty-eight where will seventy-eight be?
Khanye immediately writes 78 in the block above 88.
T : How did you find 78?
K : It's ten less
T : So, where will 58 be?
Khanye quickly writes 58 two blocks above 78.
T : How did you find 58 so quick? That isn't ten less.
K : I did twenty less

I was not convinced that Khanye had used the strategy she previously explained – although it was a perfectly acceptable strategy – and therefore I proposed what I thought her strategy was. However, I had to use the follow-up questions (find blocks 78 and 58) to ensure that she was not merely agreeing with what I had proposed. The manner in which she answered the follow-up questions convinced me that Khanye had used the placement of 98 (and not the long-winded strategy she first explained) to quickly fill in 88 on the blank 100 chart.

Next I present a brief summary of all learners' oral discourse used during intervention before answering research question 4.

6.3.4 Key similarities across all Intervention Learners' Oral Discourse²⁴

- **discourse progression** for all learners only occurred when they solved similar tasks *within the same representational setting*
- most instances of **discourse progression** for the groups occurred across multiple lessons (15 instances) rather than within individual lessons (8) but for Khanye discourse progression was more prevalent within single lessons (6) rather than across multiple lessons (2)
- **discourse progression** was skewed towards the stronger attainers in the intervention group as a whole
- in almost all cases of discourse '**mismatch**', learners explained a more rudimentary strategy (calculation-by-counting) than the more efficient or sophisticated strategy (calculation-by-structuring) that they actually employed
- the prevalence of learners' explanations at odds with enacted strategies points to broader limitations of working with young learners' explanatory discourse – and more so in a context where children are learning in a second or third language as is the case in many South African primary schools. During the intervention period, the combined attention to enacted strategies and discourse pointed to prevalence of a lag between appropriations of actions and the appropriation of a discourse describing these actions.

Taken together, my analysis of intervention learners' solution strategies and oral discourse used across different settings to solve additive tasks during intervention leads me to answering the fourth research question.

Research Question 4

4. What was the nature of learners' progress (if any) in base ten thinking during intervention as evidenced by the strategies used and the nature of oral discourse across different representational settings across work on additive tasks?

²⁴ This section speaks to those intervention learners who have been included in the aforementioned analysis of oral discourse, which excludes Josh.

Whilst considering the changes in intervention learners' base-ten thinking as inferred from their use of solution strategies and oral discourse, I noticed a few definite patterns that emerged. Firstly, a few similarities existed between **progressions** in learners' solution strategies and oral discourse that may be considered to be relatively predictable:

- progression in both aspects were more prevalent *across two or more lessons*,
- progression in both aspects had a higher rate of occurrence when learners worked on similar tasks *within the same representational settings*,
- progression in oral discourse was skewed towards the stronger attainers

The first two bullet points indicate that progression in this sample of Grade 2 learners' base-ten thinking was predicated on them spending sufficient time working within a particular setting to solve similar tasks over more than one lesson. The third point indicates that the nature of intervention learners' base-ten thinking was mirrored in the nature of their 'math talk' because learners with stronger LFIN attainment profiles showed more progress in their 'math talk'.

The converse was true with regard to **reversions** in learners' base-ten thinking inferred from their strategy usage, that is, that strategy reversions were more prevalent when learners solved similar tasks *across various representational settings* (usually when moving from more concrete to more abstract settings) within single lessons. This strengthens the point highlighted above: that learners need to work on similar tasks within the same settings over an extended time in order to fully appropriate the intended base-ten reasoning. This assertion holds true for all learners but more so for learners with weaker attainment profiles.

Intervention learners' **representational flexibility** appeared to correlate well with their overall attainment because learners with stronger LFIN attainment profiles showed greater flexibility than those with weaker attainment profiles. Representational flexibility can thus be regarded as another good marker (together with oral discourse) of overall attainment.

6.4 Concluding Remarks

I think a key outcome of this interactional analysis was that stronger attainers showed greater ease with representational flexibility and progression in oral discourse. This means that they could enact base-ten strategies across different representational settings rather than reverting back to a more rudimentary counting strategy when the setting changed. These learners' ability to replay their physical and mental actions on settings grew alongside their facility with working across different representations.

Another important result of this analysis is the commonality between learners' strategy progression and discourse progression. That is, that in both these aspects progression was far more prevalent as learners worked *within the same representational setting* than when they worked across various settings. Two (almost contradictory) points could be deduced from this result. First and most importantly, the progress that was made in participants' strategy usage and oral discourse was 'narrow' in that most of it took place when learners were working within the same setting. This speaks to the need to purposefully build more 'breadth' into the use of representational settings during the teaching and learning of base-ten thinking so that progressions are not restricted to particular settings. This point is especially important for weaker learners as the greater number of strategy reversions across settings was skewed towards the weaker learners in the group.

Secondly, children need sufficient time to work on similar tasks within the same settings as this affects the degree to which they can appropriate the desired actions on settings as well as the discursive explanations about these actions which are seen as indicators of learning. However – bearing the first point in mind – this does not mean that children should only be encouraged to work in one particular setting but that additional settings should be introduced judiciously.

Discourse progression was skewed towards the stronger attainers while a 'mismatch' between learners' discourse and strategy was skewed towards learners with weaker attainment profiles. The greater degree of 'lag' between weaker attainers' explanatory discourse and enacted strategies (compared to stronger learners) points to the need for more attention to oral discourse as a key part of entrenching the closing of the attainment gap that was shown

in the pre-/post-test results in the previous chapter. Sfard (2008) points to mathematics building up through discursive layers that depend on learners using reified discursive objects in new operational processes. The more limited discursive appropriation for the weaker learners point to potential problems in this regard, and could hinder onward progression for these learners.

In the final chapter I highlight the key findings emanating from my research and make a few recommendations for theory and practice in light of the South African research base. I also consider the limitations of my study and end with a short autobiographical reflection.

Chapter Seven

Findings, Recommendations and Contributions

7.1 Introduction to the chapter

In this final chapter I reflect on the research process and I highlight the key findings emanating from my study by situating these within the current literature base on developing children's early number knowledge. I further relate findings from my study to those of national and international studies also based on Mathematics Recovery and I discuss how these are similar and different. This reflection allows for a careful consideration of the outcomes from the study that I considered particularly useful for teachers and/or policy makers and the theoretical and methodological developments that allowed these findings to emerge. I further note the limitations of the study and offer recommendations for future research relating primarily to the results that I have highlighted as particularly interesting or surprising when considering the existing South African research base and the directions of its policy recommendations.

7.2 An overview of the study

This study was aimed at exploring the efficacy of short-term intervention, based on Mathematics Recovery (MR) principles, in developing the early number learning of a sample of Grade 2 learners in a South African primary school. I considered children's early number learning to be an important research focus because of the weight of research evidence depicting South African learners' poor numeracy levels upon leaving primary school (Schollar, 2008; Ensor et al., 2009; Spaul, 2013) as well as the importance of early number learning for future success in mathematics. At the same time, another body of evidence showing the improvement that is made possible in children's early number learning through implementation of the Mathematics Recovery programme, albeit in developed countries like Australia, England and the USA, sparked my interest (Wright et al., 2006; Pearn, 2005). Further, the importance of early intervention in closing the attainment gap as well as research

showing that children's learning deficits in number can be remediated, provided the impetus for this study (Spaull et al., 2016).

Using a 'teaching experiment', I selected 20 mid-attaining Grade 2 learners (in the school context) and allocated half of these to the 'intervention' group and the other half to the 'control' group. The changes recorded across learners' pre- and post-test LFIN profiles determined the learning gains made. Comparative analysis between gains made by intervention and control groups was aimed at determining whether intervention based on MR principles moved children further along the trajectory for early number learning, and if so, in which aspects of early number this could be seen. In another layer of analysis gains made by the 2 'singletons' across their pre- and post-test LFIN profiles were compared to gains made by 8 learners who received grouped intervention (i.e. two groups of 4 learners each). This added layer of LFIN analysis (between individual and grouped intervention) was aimed at determining the differences (if any) in gains made by learners who participated in different modes of the same type of intervention.

Based on an Emergent Approach (rather than the Radical Constructivist framing of the original MR programme), I switched between cognitive and sociological perspectives in my analysis to understand the individual gains made by participants (Cobb & Yackel, 1996; Bauersfeld, 1994). Analysis based on aspects of interaction – this included learners' oral discourse and their actions/strategies enacted on various representational settings during intervention – added depth to the cognitive aspects highlighted by the LFIN. This analysis, borne out of the sociological perspective, was aimed at shedding light on the kinds of change noticed across intervention learners' pre- and post-test results as well as the antecedent practices used in intervention that may have given rise to these changes.

In this chapter I present a summary of my findings from both sociological and cognitive perspectives and make recommendations for practice based on these. I then present what I believe are the contributions to knowledge made by this work in the field of early number learning. I also discuss limitations to this study as well as the steps I took to mitigate against these. And finally, in agreement with Kolb's view that "an experience that is not reflected upon

is unrealized learning” (Jackson & Caffarella, 1994, p.7), I share some autobiographical reflections.

7.3 Key Findings and Recommendations for Practice

Higher level gains made by the intervention group and no difference between individual and grouped intervention learners’ gains

When I compared the nature and extent of gains made by intervention and control groups I found that the *intervention group made higher level gains than the control group in all aspects* of early number detailed in MR’s Learning Framework in Number (LFIN), that is, in strategies used for solving early number tasks; in using ten as a unit when calculating rather than counting in ones; with saying forward and backward number words sequences in increasing number ranges and with identifying 1-, 2- and 3-digit numerals. This points to the comprehensive nature of the MR programme and the kinds of gains that can be made across various integrated areas of number learning in a context like South Africa where many challenges exist on the ground. Further, there was little difference in the nature and extent of gains made by learners who participated in grouped intervention and those who received individual intervention. While the base in two groups and two individuals does not allow for strong claims that group-based intervention can be recommended, this outcome certainly suggests that a grouped-based model is feasible, with results indicating that Mathematics Recovery principles can be used to develop number learning in a way that takes the scale of the problem on the ground into account. An interesting (and somewhat surprising) result was the 2 stages gain on the SEAL aspect of the LFIN made by five intervention learners across the attainment range and different modes of intervention. This result points out that what is described as ‘significant cognitive reorganization’ by developers of the MR programme was possible in an intervention that provided, in total, about 12 hours of structured teaching. This suggests that a structured teaching approach focused on progression and expansion in early number teaching and learning can make a substantial difference.

Significantly higher level of gains for intervention group in Base-Ten

Whilst the overall gains made were very pleasing, it was the *big differences in gains made by the intervention group in the Base-Ten aspect of the LFIN* compared to the control group that intrigued me most. The intervention group made more than twice the number of gains (14) in Base-Ten than the control group (6), and this too, over the short period of the intervention. This is an important result in light of research evidence that bases South African children's pervasive counting strategies (that precludes them from efficient calculation in higher number ranges) on their poor grasp of base-ten understanding. As highlighted in Chapter 2, children's success with mental calculations depends largely on their facility with *sequence* (N10, bridging-through-ten) and *split-tens* (1010) strategies – which involves learners making 'jumps-of-ten' and/or jumps-via-ten' (Anghileri, 2006) – which are both premised on base-ten thinking. So, conceptual knowledge of base-ten plays an important role in the additive strategies employed by learners in Grade 2 who need to progress from working within the 1-20 to the 1-100 number range (Beishuizen, 1993). The substantial gains made in the Base-Ten aspect of the LFIN by the intervention group speaks to these learners' facility with solving written tasks using 'jumps-in-ten' and 'jumps-via-ten' without the support of a tens-based setting, rather than error-prone counting strategies. The margin of difference in gains between the intervention and control groups in the area of number learning considered to be at the root of South African learners' poor progress in number skills is thus highly significant.

The literature reviewed in Chapter 2 also speaks to the 'bridging' function that structured settings (e.g. arithmetic rack) provide in that they assist learners with enacting calculation-by-structuring (van den Heuvel-Panhuizen, 2008). I therefore expected learners to use the structured settings made available during the post-test to enact calculation-by-structuring. While this was true for some intervention learners, the 4 stronger intervention learners (who attained SEAL Stage 5 in the post-test) enacted strategies based on structuring number (base-ten strategies) in the absence of structured settings. Even though these learners did not always get the correct solutions because of 'slip ups' made during calculation, the fact that they chose to enact strategies based on the structuring of number without using a structured setting (made available during the post-test) spoke to their growing confidence in their structuring number skills at the time of the post-test.

Oral Discourse and Representational Flexibility correlating with attainment

Analysis based on the interactional perspective showed a *link between intervention learners' overall attainment on the LFIN framework and two interactional aspects* related to their base-ten thinking, namely, their *oral discourse* and *representational flexibility*. This outcome is very interesting because it denotes *oral discourse* and *representational flexibility* as reliable markers of early number learning. Progressions or expansions in oral discourse or 'math talk' in my study spoke to learners' ability to relay their physical and mental actions on settings and representational flexibility referred to learners' facility with working across various representational settings.

A patterned association between the ranking of learners' frequency of representational flexibility and their LFIN post-test summary profiles was evident. This relationship is particularly interesting because the association between representational flexibility and LFIN post-test scores was more regular, or stronger, than the association between the strategy progression/reversion counts across learners' LFIN post-test scores. South African research over the last decade has tended to stress the need for strategic progression and representational progression (Ensor et al., 2009) but paid less attention to representational flexibility as a goal. My study results suggest that the latter may be more important to attend to, at least in the shorter term. Clarification notes for Foundation Phase mathematics in our CAPS curriculum (DBE, 2011a) note the different representations that can be used in the teaching and learning of early number concepts (concrete counters, Dienes blocks, number grids and number lines) as well as how these can be used. However, what is lacking is a strong message about the importance of having learners work *across different representations* when solving similar tasks (Heinze et al., 2009). Instead, the curricular advice tends to suggest that teachers can choose whichever representation they feel comfortable with and stick with that. In this presentation, neither flexibility nor progression are clearly marked as important. This gap is problematic in an environment where primary teachers' mathematical knowledge and pedagogical content knowledge have been acknowledged as weak for many (Schollar, 2008; Hoadley, 2012; Spaull et al., 2016).

While structured working with 'building-up and breaking-down' strategies are foregrounded in the Foundation Phase CAPS curriculum, the findings from this study points to potential

limitations with how these are presented in curriculum documents and in the prescribed solution strategies in national assessment tasks. The representation that most often accompanies this strategy in curriculum documents is a symbolic 'splitting' or 1010 strategy. The foregrounding of the 'building-up and breaking-down' strategy with its concomitant symbolic representation negates the importance of representational flexibility (not to mention the fact that research shows the splitting strategy to be problematic for learners when decomposition is required with subtraction). In order to move away from what research depicts as 'localized' ways of working on the ground that leads to a 'narrow' appropriation of number skills (Venkat & Askew, 2012, 2018), findings related to representational flexibility in my study shows that it is both possible and essential to build 'breadth' into the settings used to develop early number skills and to encourage learners to work across these settings.

Literature related to developing learners' oral discourse in classroom settings (discussed in Chapter Two) shows that reasoning skills and flexible thinking are developed as children explain their ways of working and compare their methods to those of their peers (Anghileri, 2006), which seems to be echoed by the correspondence between learners' LFIN post-test scores and progressions in oral discourse. The link between progressions in learners' oral discourse and overall attainment suggests the need for a greater focus on the development of a 'math talk' community in classrooms despite the challenges faced on the ground (like a difference between the content and pedagogical registers and multilingual classes) that seem to militate against such moves. A cursory note regarding learner explanations is made in the CAPS curriculum (DBE, 2011a) but the importance thereof for developing early number skills is lacking. What is also absent in our curriculum documents is an exemplification of progressions and expansions in learners' 'math talk' that can be used as markers of their appropriation of early number concepts. Using the notion of strengthening the 'instructional core' in Foundation Phase classrooms (Spaull et al., 2016) – which refers to the 'actual interactions between teachers, learners and content' – I now take findings from my study to highlight contributions and formulate recommendations which I believe can directly affect the 'learning reality' in many classrooms.

7.4 Theoretical and Methodological contributions

Attention to the nature of and progression in learners' 'math talk' as they explain their thinking (or relay their solution strategies) as well as their facility with working across various representational settings (moving from less to more abstract) while engaged in additive number tasks is the theoretical development my study makes to the implementation of Mathematics Recovery principles for purposes of intervention. The relationship between these two interactional variables and learners' LFIN profiles in my study is the basis for this theoretical insight. My study built in another layer alongside the Base Ten aspect of the LFIN by documenting progressions and reversions in learners' base-ten thinking as displayed by their oral discourse and representational flexibility with regard to their solution strategies.

Progression in oral discourse is evident, for example, when a learner who first explained her solution strategy using everyday/colloquial language next uses formal/mathematical language to relay the same strategy, as illustrated in Table 7.1.

Table 7.1 Indicators for Oral Discourse

Moving from the left to the right is regarded as... Progression 	
Everyday /colloquial language	Formal /mathematical language
Task: 36+23	Task: 45+24
Learner explanation: "I put my finger on thirty-six then I <u>jumped two blocks down</u> to fifty-six then I <u>moved three blocks to the right</u> to fifty-nine"	Learner Explanation: "From forty-five I <u>added twenty</u> then I got to sixty-five then I <u>added four</u> and I got to my answer, sixty-nine"
Discussion: Expressions like 'I jumped two blocks down' and 'I moved three blocks to the right' can be labelled as everyday language	Discussion: 'I added twenty' and 'I added four' are mathematical phrases
Mathematical operation not articulated	Mathematical operation clearly articulated
Task: 36+23	Task: 45+24
Learner explanation: "I said fifty, fifty-six, fifty-seven, fifty-eight, fifty-nine".	Learner explanation: "I said forty plus twenty is sixty, plus five is sixty-five and plus four: sixty-six, sixty-seven, sixty-eight, sixty-nine. My answer is sixty-nine".
Discussion:	Discussion:

The phrase: 'I said fifty' implies that the learner added 30 and 20 but this mathematical operation is not made explicit. The same applies to 'fifty-six...fifty-nine' as this is merely stating the result of a mathematical action but not what the action is.	In 'forty plus twenty is sixty' the learner states what the mathematical operation is as well as the result of that operation. So, the people listening to her explanation do not have to wonder what she did to get sixty because this would have been made clear.
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Representational flexibility in solution strategies is evident when learners can work across various representations when solving similar tasks (Heinze et al., 2009). Using Ensor et al.'s (2009) hierarchy of more concrete versus more abstract representations as well as Askew's (1998) notion of structured and unstructured representations, one can note whether a learner is able to enact base-ten strategies across settings when solving similar tasks. The movement or progression that should be encouraged here is from more concrete to more abstract representational settings. So, representational flexibility is evident when a learner solves, for example, 2-digit+11 tasks first using bundling sticks (concrete, structured representation) and then later solves similar tasks on a number line (symbolic-number based representation), as illustrated in Table 7.2.

Table 7.2 Indicators for Representational Flexibility

Working across concrete to more abstract settings to solve similar tasks is representational flexibility 	
More concrete representation	More abstract representation
Task: 24 (scr) +11	Task: 35+11
Rep Setting: Bundling sticks, first addend screened	Rep Setting: ENL
Action on setting: Learner places the bundle under the screen saying "thirty-four, then she places the stick under the screen saying, "plus one is thirty-five".	Action on setting: Learner makes a mark to the left of the ENL labelling it '35' she then makes a big jump to the right and writes +10 above the jump and 45 where she lands; she then makes a smaller jump to the right labelled +1 and writes 46 where she lands.

The caution, of course, is that the correspondence between learners' overall attainment profiles, representational flexibility and oral discourse in my study cannot be taken as evidence of causation. The nature of my data does not allow me to make any claims about whether a greater degree of representational flexibility and expansions in learners' oral discourse results in higher overall attainment, or vice versa. This is an important avenue for future research,

as in the international literature in mathematics education there is strong support for the importance of this kind of representational flexibility and oral discourse.

On the methodological side, my study involved South African learners in their second year of formal schooling using a withdrawal method of intervention based on Mathematics Recovery principles – which has not been done before. I explicitly built in two layers of comparison based on the Mathematics Recovery LFIN framework, that was, a comparison between learning gains achieved using an individual model of intervention and a grouped model; as well as a comparison between gains made by an intervention group who missed much of their Numeracy lessons on the intervention days (18 or 19 days) and a control group who continued with normal class teaching. These methodological features are unique to my investigation into the efficacy of using MR principles for the development of early number in the South African terrain.

Next I overview the relationship between findings from my research and previous research also based on Mathematics Recovery. I first relate the findings from my study to other national studies before doing likewise with international studies.

7.5 Relationship with Previous Research

Findings from my study backs up the assertion from the prior research base that short-term intervention based on Mathematics Recovery principles can be effective in developing learners' early number skills. One difference was that learning gains displayed over the course of my intervention study were achieved by learners who had only experienced one year of formal schooling, so their base-line knowledge was not yet as well established as the older participants in other South African studies. Also, through participation in my classroom-withdrawal study learners missed much of the normal class-time for numeracy on intervention days. Thus, a greater percentage of learning gains produced across pre- and post-tests in my study could be interpreted as a direct result of intervention, compared to learning gains produced by intervention studies conducted after school, where participants did not miss out on any part of the normal school curriculum, with the comparative control group design offering further support for this view.

The association between intervention learners' attainment profiles and their representational flexibility builds on the work of researchers like Heinze et al. (2009) who found that children who could seamlessly move between different representational settings had deeper conceptual understanding of the concepts being taught than those who could not work across various settings. The further correspondence to progressions in learners' oral discourse tends to back up the notion that children's facility with language is closely tied to their number learning (van den Heuvel-Panhuizen, 2008) and that oral discourse can enhance children's conceptual understanding of number concepts (Dunphy et al., 2014) without loss of computational mastery (Boaler, 2008).

There are three main differences between findings emanating from my intervention study and other international studies also based on MR. Firstly, findings from my study emerged over a shorter intervention period (two 40min lessons over ten weeks) compared to the intensive teaching cycle of Math Recovery programmes in America where 30min lessons, for four or five days per week over 12 to 15 weeks is the norm. This finding may relate to a contextual base in which there is evidence of primary classroom teaching in which conceptual working is largely sidelined (Hoadley, 2012), underpinned by very limited attention to connections and sometimes lacking even in coherence (Venkat & Naidoo, 2013) leading to stronger effects from carefully structured interventions than might be common in environments where these aspects are more commonly part of the classroom fabric.

Secondly, my study's findings were drawn from a cognitive perspective using the LFIN framework *and* a sociological perspective using aspects of interaction. Results from my intervention study are thus more nuanced than studies based on a purely cognitive approach. Lastly, findings emanating from my study with regard to the development of Grade 2 learners' early number skills were based on two layers of comparison: firstly, between a group of learners who received intervention based on MR principles and a control group who did not receive intervention but rather participated in normal classroom activities; and secondly, between learners who participated in 'grouped' intervention based on MR and those who received 'one-on-one' intervention also based on MR principles. These textured findings make the practical relevance thereof more widely applicable.

7.6 Limitations of my study

In this study, as with all research, there were certain aspects that could be viewed as “potential weaknesses or problems with the study that may have affected the results” (Creswell, 2012, p.199). I believe that the main limitation to my study was that I lacked formal training (as the intervention teacher) on how to implement the Mathematics Recovery programme – including aspects of teaching, assessment, use of settings, analysis using LFIN, and so forth. When I joined the WMC-P project as a Masters Fellow, elements of the MR programme were already being used in the broader project and this was how I came to know about Bob Wright’s work. Despite being very interested in attending a Mathematics Recovery training course, resource constraints did not allow for direct access to this training which then led to the decision that I would have to learn all I could about the programme through self-study using the vast number of resources produced by Wright and colleagues. This means that my understanding of the MR programme is, in some ways, different to that of an intervention teacher who received MR training and thus there may have been some things that I could have done differently/better during intervention or there may have been things that I missed during analysis. To mitigate against this limitation, I planned intervention lessons by carefully following guidelines in the Instructional Framework in number (Wright et al., 2006) and worked from these lesson plans as much as possible. I also developed some additional tools to aide my understanding of the MR programme using Wright et al.’s material, like the Record of Response to Task Items and MR Assessment Purposes (Appendix B and C, respectively). These materials acted as scaffolds during the assessment and analysis processes.

Additionally, when reflecting on my own pedagogic approach, my sense was that my style of teaching (steeped as I was in training and teaching in the largely traditional teacher-led format of South Africa) was not wholly ‘inquiry-based’ during intervention. Even though I planned intervention lessons with an inquiry-based approach I found that I sometimes veered from this path. What added to this problem was that most participants in my study displayed behaviour during intervention (like reluctance to tackle a problem unless I first showed them how to do it; reluctance to work in pairs or a group in ways that built on each other’s ideas; laughing at others’ mistakes; and so forth) that indicated that they too had very little experience with ‘inquiry-based’ learning. So, to implement a pedagogical approach that

neither I nor the learners were accustomed to was challenging, with this factor likely writ large in the broader South African landscape. Given this factor, the exemplars of progression and expansions in oral discourse presented above are probably particularly important for setting out markers of what is possible in environments where 'math talk' can be encouraged.

Zooming in on learner responses during the intervention, there were certain aspects of the planned intervention that I knew would be novel for participants in my study, like the use of the empty number line (ENL), but did not anticipate the challenge that it would present to most learners. Even though my introduction and use of the ENL during intervention was done with fine-grained increments, the take-up of this resource was seen to a small extent in two of the top four intervention learners. I introduced the ENL from the second intervention lesson onwards but only used this to recap actions that learners had previously enacted on other settings like the 100 chart and bundling sticks. Learners' initial use of the ENL was also to simply demonstrate an action that they had already performed on another resource. However, during the second half of intervention when I thought that learners could start using the ENL as a 'tool for thinking' I found that most preferred to use the 100 chart. During the post-test, only a few learners made use of the ENL when prompted by me to do so.

Fraenkel et al. (2012, p.458) note that "all researchers have certain biases" and point out that "in qualitative research, much depends on the perspective of the researcher". From the outset, I believed that the MR programme held some promise for developing early number learning but I expected to only see gains in LFIN 'levels' as these were not as broad as 'stages' which developers of the MR programme liken to a qualitative change in number learning. I anticipated that the individual format of intervention would yield more gains than grouped intervention because the individual model would be tailored to the specific needs of the learner (and this was the original format of MR intervention) while the grouped format would not. Thus, the results produced at the end of my intervention study – where most intervention learners shifted in the LFIN levels *and* stages, and the comparable learning gains made by 'singletons' to those made by learners who received grouped intervention – were really unexpected. So, as much as my perspective may have predisposed me to see gains in learning after intervention, the results produced were (pleasantly) contrary to what I expected.

7.7 Autobiographical Reflection

The person I was when I started this doctoral journey is definitely not the person I am today. There were many valuable lessons that I learnt about myself, the topic I chose to study and the field of primary mathematics education on the whole.

I learnt that I have an enduring passion to see primary mathematics teaching and learning improve in our country (and in places facing similar contextual challenges) that is born out of my desire to serve others and improve their lives. I have also discovered that I can be tenacious when I believe that the goal I am pursuing is one that is worthwhile even in the face of hardship and challenges.

Learning about the development of children's early number skills has opened my eyes to the many opportunities teachers have to make a big difference in this all-important foundational skill by making a few changes to their instructional approach. I admit that there is quite a lot that underpins 'a few changes' in pedagogical style but I believe that these changes are possible and worth pursuing. Through this journey I have found that the field of primary mathematics education as a whole is a very exciting research space, and that this doctoral study has been my first real 'taste' of this excitement.

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Appendix A

in a 30-minute lesson the teacher may be working towards addition tasks with two collections, the first of which is covered with numbers in the range beyond 10, for example, $7 + 4$. The lesson could commence with saying short, FNWSs commencing at 5, then 6, 7, 8, and so on. This could be followed by the teacher saying one number and the child saying the next two, three or four number words. For example, Five ... 6, 7, 8; ... Nine 10, 11, 12; Thirteen ... 14, 15, 16; and so on. The activity could shade to saying the number word after a number or even saying two or three numbers after. Similarly activities could involve saying the number word before, two before, or even three before. A numeral track, or covered number line, would enable the same activities to be done in a different setting. Activities involving flashing spatial patterns using a filled Ten Frame and a partially filled Ten Frame with 2, 3, 4, or 5 dots could be used with the child being asked, 'How many altogether?' The Figurative Counting Key Topic involves a variety of settings such as seven animals in one barn and four animals in the field; seven bees in the hive and four flying around; seven letters in the post box and four more being posted, and so on. The key element of the activity is that the first collection is briefly displayed and then screened.

GUIDING PRINCIPLES IN MATHEMATICS RECOVERY TEACHING

The teaching sessions, whether for an individual, group or class, afford an opportunity to provide intensive, high-quality teaching. In order to do this the teacher must have a clear model of the children's current knowledge and strategies in early number, and a clear idea of the progress in learning that is a reasonable goal for them. The origins of Mathematics Recovery teaching lie in large part in research projects which involved longitudinal observation and study of children's developing strategies and learning as they occurred in interactive teaching sessions.

We conclude this chapter on how Judy, Denise and Michael were able to make such good progress by summarizing the teaching approach in the following set of nine guiding principles of Mathematics Recovery teaching.

1. The teaching approach is enquiry-based, that is, problem-based. Children are routinely engaged in thinking hard to solve numerical problems which for them, are quite challenging.
2. Teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching. The latter refers to the teacher's informed understanding of the child's current knowledge and problem-solving strategies, and continual revision of this understanding.
3. Teaching is focused just beyond the 'cutting-edge' of the child's current knowledge.
4. Teachers exercise their professional judgment in selecting from a bank of teaching procedures, each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.
5. The teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies.
6. Teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.
7. Teaching supports and builds on the child's intuitive, verbally based strategies and these are used as a basis for the development of written forms of arithmetic which accord with the child's verbally based strategies.
8. The teacher provides the child with sufficient time to solve a given problem. Consequently, the child is frequently engaged in episodes which involve sustained thinking, reflection on her or his thinking and reflecting on the results of her or his thinking.
9. Children gain intrinsic satisfaction from their problem-solving, their realization that they are making progress, and from the verification methods they develop.

Appendix 1

The Mathematics Recovery Programme

Assessment Interview Schedule 1.1 Early Arithmetical Strategies and Numerical Knowledge

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

	SEAL	FNWS	No. Id.	BNWS
Stage/level				

1 Forward Number Word Sequence

Start counting from ** and I'll tell you when to stop.

- | | |
|----------------|-----------------|
| (a) 1 (to 32) | (b) 48 (to 61) |
| (c) 76 (to 84) | (d) 93 (to 112) |

2 Number Word After

Say the word that comes straight after ***. Example: Say the word that comes straight after one.

(a) Entry task			
14	11	19	12
23	29	20	

(b) Less advanced task				
5	3	9	6	7

(c) More advanced task				
59	70	65	99	32

3 Numeral Identification

Show each card in turn, saying, What number is this?

(a) Entry task									
80	10	12	15	17	99	13	20	21	66

(b) Less advanced task					(c) More advanced task				
8	3	5	7	9	100	123	206		
6	2	4	1		341	820			

4 Numeral Recognition

Arrange the cards from 1 to 10 randomly. Which number is ... ?

6 4 7 9 8

5 Backward Number Word Sequence

Example: Count backwards from 3. Three, two, one.

Now count backwards from ** and keep going until I say stop.

- | | |
|---------------------|---------------------|
| (a) 10 (down to 1) | (b) 15 (down to 10) |
| (c) 23 (down to 16) | (d) 34 (down to 27) |
| (e) 72 (down to 67) | |

6 Number Word Before

Say the number word that comes just before ***. Example: Say the number just before 2.

(a) Entry task			
24	17	20	11
13	21	14	30

(b) Less advanced task				
7	8	10	3	4

(c) More advanced task				
67	50	38	100	
83	41	99		

7 Sequencing Numerals

Show the ten numbered cards face up in random order, asking the child to identify each number as you put it out. Then say, Can you place the cards in order? Start by putting the smallest down here.

(a) Entry task
Cards from 46 to 55

(b) More advanced task
Cards from 1 to 10

8 Additive Tasks (screened, use counters of two colours)

Introductory task:

There are three red counter under here, and one yellow counter under here. How many counters are there altogether?

3	+	1
---	---	---

(a) Entry tasks (both collections screened)

$$\boxed{5} + \boxed{4} \qquad \boxed{9} + \boxed{6}$$

If one or more incorrect continue below to 8(b). If both correct go to 8(e).

(b) Less advanced task (first collection screened)

$$\boxed{5} + 2 \qquad \boxed{7} + 3 \qquad \boxed{9} + 4$$

If (b) is too difficult go to (c)

(c) Unscreened collections

$$5 + 2 \qquad 7 + 3 \qquad 9 + 4$$

(d) Perceptual counting

Would you count to see how many counters there are altogether in this group?

Place out 13 counters.

Place out 18 counters

(e) Supplementary additive tasks (screened, use counters of two colours)

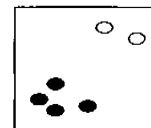
Supplementary tasks to (a) if further clarification is needed. The tasks are presented totally screened.

$$\boxed{8} + \boxed{5} \qquad \boxed{9} + \boxed{3}$$

(f) Missing addend

Here are four red counters. Now look away. While you were looking away I put some more yellow counters under here. Now there are 6 counters altogether. How many yellow counters did I put under here?

Introductory task $4 + \boxed{} = 6$



Tasks $7 + \boxed{} = 10$

$12 + \boxed{} = 15$

9 Subtractive Tasks**(a) Subtraction sentences**

Present the tasks as a written number sentence on card. Say to the child, *What does this say? Do you have a way to work out what the answer is?* [Note: using counters is not an option]

Entry task	Supplementary task
$16 - 12$	$17 - 14$

(b) Missing subtrahend

[Note: use counters of one colour only]

Here are five counters. (Ask the child to look away. Remove and screen two counters.) There were five counters. While you were looking away I took some away. Now there are only three. How many did I take away?

(i) Introductory task	5 to 3	$\{5 - \boxed{} = 3\}$
(ii) Entry tasks	10 to 6	$\{10 - \boxed{} = 6\}$
	12 to 9	$\{12 - \boxed{} = 9\}$
(iii) More advanced task	15 to 11	$\{15 - \boxed{} = 11\}$

(c) Removed items

Here are three counters. (briefly display, then screen). If I take away one, (remove one counter, display briefly, then re-screen) how many are left under here? (Indicate the first screen.)

(i) Introductory task	$3 - 1$		
(ii) Entry tasks	$6 - 2$	$9 - 4$	$15 - 3$
(iii) More advanced task	$27 - 4$		

Mathematics Recovery Programme: 1.1 – pupil profile sheet

Pupil: Age: yrs months		DoB: Assessment date:	Teacher: Assessor:	Class:
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Aspect/Task Item	Level	Comments
FNWS and NWA	1 2 3 4 5	
Numerical Id.	1 2 3 4	
BNWS and NWB	1 2 3 4 5	
Sequencing Nos		
Additive Tasks and Subtractive tasks. Use the information on each section to arrive at the Stage.		
Additive Tasks		
Missing Addends		
Subtractive Tasks		
Missing Subtrahends		
Removed Items		
STAGE on SEAL	1 2 3 4 5	

Appendix 2**The Mathematics Recovery Programme****Assessment Interview Schedule 1.2
Base-Ten and Advanced Arithmetical Strategies**

Child's name:
DoB: Age: years months
Interviewer's name:
Date of interview:
Teacher/class:

Tasks	Comments
Tens and Ones Level 0 1 2 3	
Non-Count-By-Ones strategies	

1 Tens and Ones Tasks

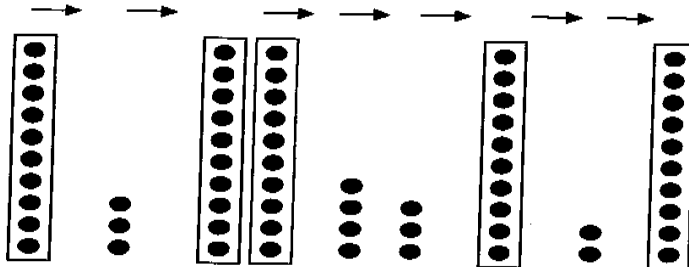
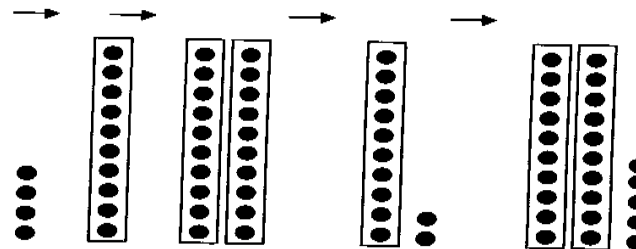
(a) Counting by tens with 'strips' - informal familiarization with material

- (i) Put down a ten strip. *How many do we have? If the child says 'one', ask, How many dots are there?*
- (ii) *'How many altogether?'* Put down one ten strip at a time to 8 strips.
10 20 30 40 50 60 70 80
- (iii) Pick up all the strips.
How many dots do we have?
How many strips are there?

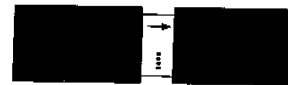
(b) Incrementing by ten

- (i) Place out the 'four dot' strip. *How many dots are there?*
- (ii) Place out a ten strip to the right of the four strip. *How many dots are there altogether?*
- (iii) Continue placing ten strips to the right of the four strip. *How many dots are there altogether?* 24 34 44 54 64 74
- (iv) If necessary, repeat the whole task with either the 3 strip or the 7 strip.

(c) Uncovering tasks: Board One

Upon each uncovering ask, *How many dots are there now?*(d) Uncovering tasks: Board Two
Upon each uncovering ask, *How many are there now?*

Example of Uncovering tasks: Board One, fourth move



2 Horizontal Sentences

- (a) *Do you have a way to figure out what is?* $16 + 10 =$
So what is $16 + 9$?

- (b) *Do you have a way to figure out what is?* $42 + 23$

If correct ask, *Do you have another way to work it out or check?*

- (c) *Do you have a way to figure out what is?* $38 + 24$

If correct ask, *Do you have another way to work it out or check?*

- (d) Repeat the above questions for:

$39 + 53$

- (e) $56 - 23$

- (f) $43 - 15$

- (g) $73 - 48$

3 Tasks to Elicit Non-Count-By-Ones Strategies and Relational Thinking

(a) $9 + 3$ Can you use ... to help you do? $9 + 4$ $9 + 5$ $9 + 6$ and so on if necessary	(b) $6 + 6$ Can you use ... you help you do? $7 + 5$ $8 + 4$ $9 + 3$ and so on if necessary	(c) $7 - 5$ Can you use ... to help you do? $27 - 5$ $47 - 5$ What do you think the next take-away will be?
(d) $15 + 3$ Can you use ... to help you do ...? $18 - 3?$ How do you use ... to help you?	(e) If I tell you $21 - 16 = 5$ Can you use ... to help you do ...? $21 - 5?$	(f) $14 + 4 = 18$ There are three numbers here. Can you use these to make a take-away sum? Can you make another one?

Appendix 3

The Mathematics Recovery Programme

Assessment Interview Schedule 2.1
Early Grouping: Structuring Numbers 1 to 10

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Subitizing and Spatial Patterns	Regular	Irregular	Domino
Finger Patterns	1 to 5		6 to 10
Five Frame Patterns	1 to 5		
Pair-wise Patterns on a Ten Frame	1 to 5		6 to 10
Five-wise Patterns on a Ten Frame	1 to 5		6 to 10
Combining to make Five			
Combining to make Ten			

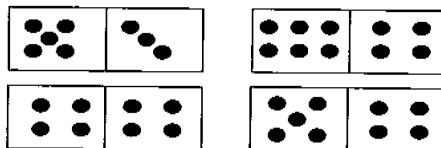
1 Subitizing and Spatial Patterns

Show each card briefly, for about half a second, saying, *I'm going to show you some cards very quickly. Tell me how many spots are on each one.*

- | | | | | | |
|-----------------------|---|---|---|---|---|
| (a) Flashed Regular | 4 | 3 | 2 | 5 | 6 |
| (b) Flashed Irregular | 6 | 7 | 4 | 5 | 8 |
| (c) Domino Cards | | | | | |

Show each card briefly, for about half a second, saying, *I'm going to show you some cards very quickly.*

- (i) What did you see?
 (ii) Tell me how many spots are on one side.
 (iii) Tell me how many spots are on the other side.
 (iv) How many are there altogether?



2 Finger Patterns 1 to 5

- (a) Show me three on your fingers.
 3 2 5 1 4
 (b) Using two hands show me.
 3 2 5 4

3 Finger Patterns 6 to 10

- (a) Show me 6 on your fingers.
 (b) Show me 6 a different way.
 (c) Show me 9 on your fingers.
 (d) Show me 10 on your fingers.
 (e) Show me 8 on your fingers.
 (f) Show me 8 a different way.

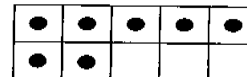
4 Five Frame Patterns



Flash the Five Frame.

- (a) What did you see?
 (b) How many spots did you see?
 3 2 5 1 4

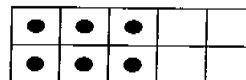
5 Five-wise Patterns on a Ten Frame



Flash five-wise cards 1–10

- (a) What did you see?
 (b) How many spots did you see?
 7 10 8 6 9

6 Pair-wise Patterns on a Ten Frame



Flash pair-wise cards 1–10

- (a) What did you see?
 (b) How many spots did you see?
 4 2 5 1 3
 7 10 8 6 9

7 Combining to make five

I will say a number and you say the number that goes with it to make five.

4 2 1 3 5

8 Combining to make ten

- (a) Tell me two numbers that add up to 10.
 (b) Tell me 2 other numbers that add up to 10.
 (c) Can you tell me another two?
 (d) I have 8 apples, how many more do I need to make 10?
 (e) I have 4, how many more to make 10?
 (f) I have 7, how many more to make 10?

Appendix 4

The Mathematics Recovery Programme

Assessment Interview Schedule 2.2 Advanced Grouping: Structuring Numbers 1 to 20

Child's name:

DoB: Age: years months

Interviewer's name:

Date of interview:

Teacher/class:

Tasks	Comments
Doubles	
Near Doubles	
Addition using five, ten or doubles, and so on	
Subtraction using five, ten or doubles, and so on	
Addition and subtraction with one addend/subtrahend > 10	
Relational Thinking	

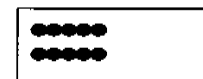
Appendix 4 173

Assessment Interview 2.2 Advanced Grouping: Structuring Numbers 1 to 20

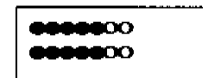
1 Doubles

Show spot cards (two seconds)

- (a) $5 + 5$ (b) $4 + 4$ (c) $2 + 2$
 (i) What do you see?
 (ii) How many on the top row?
 (iii) How many on the bottom row?
 (iv) How many altogether?



- (d) $7 + 7$ (e) $9 + 9$ (f) $6 + 6$ (g) $8 + 8$
 (v) What do you see?
 (vi) How many on the top row?
 (vii) How many on the bottom row?
 (viii) How many altogether?



2 Near Doubles

Show the number problem.
 Can you work this problem out?
 How did you do that?
 Do you have another way to work it out?

- (a) $5 + 6$ (b) $4 + 3$ (c) $7 + 6$
 (d) $9 + 8$ (e) $8 + 7$ (f) $3 + 2$

3 Addition Using Five, Ten or Doubles, and so on

Show the number problem. Do you have a way to work out this problem?
 How did you do that?

- (a) $9 + 3$ (b) $8 + 5$ (c) $9 + 6$
 (d) $8 + 6$ (e) $5 + 7$ (f) $4 + 9$

4 Subtraction using five, ten or doubles, and so on

Show the number problem. Do you have a way to work out this problem?
 How did you do that?

- (a) $17 - 7$ (b) $11 - 4$ (c) $12 - 9$
 (d) $17 - 8$ (e) $13 - 5$ (f) $14 - 8$

5 Addition and Subtraction with One Addend/Subtrahend Greater than Ten

Show the number sentence. Do you have a way to work out this problem?

How did you do that?

- (a) $13 + 5$ (b) $11 + 8$
(c) $19 - 7$ (d) $17 - 15$

6 Relational Thinking

- (a) Present the card
What does this say?
Can you work it out?

$14 + 2$
$2 + 14$

Can you use this number problem to help you work out ...?

- (b) Present the card
What does this say?

$7 + 7$
$8 + 6$
$9 + 5$

Can you use this ... to help you do ...

Can you use this ... to help you do ...

And so on ... if necessary.

- (c) Present the card
What does this say? Do you have a way to work this number problem out?
Can you use this number problem to help you work out this?

$13 + 4$
$17 - 4$

How did you do that?

- (d) Present the card

Can you use this number problem to help you work out this?

$6 + 8 = 14$

- (i)

$8 + 6$

 (ii)

$14 - 6$

(iii) What other number facts would you know from this?

Appendix B

Wits School of Education



27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa
Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website:
www.wits.ac.za

Student Number:
515290
Protocol Number:
2013ECE104D

5 November 2013

Dear Samantha Morrison

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school.

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely


Matsie Mabeta
Wits School of Education

011 717 3416

Cc Supervisor: Prof. H Venkatakrishnan

LETTER TO THE PRINCIPAL

Date: _____

Dear _____

My name is Samantha Morrison and I am a PhD student in the School of Education at the University of the Witwatersrand. I am doing research on the remediation of numeracy with learners at Foundation Phase level. My research study is entitled '**Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school**'.

My study is aimed at exploring a classroom withdrawal-based model for remediating the early number learning of a sample of Grade 2 learners, with a focus on understanding the interactional processes within which shifts towards more sophisticated understandings of number occur. This short-term intervention programme is based on an adaptation of the Maths Recovery programme developed by Wright, Martland and Stafford (2006), which has been used successfully for individual remediation in developed countries such as Australia, Britain, Ireland, the USA and Canada. My study aims to adapt the Maths Recovery programme so that it could be used for remediating a group of learners' number skills, and then exploring the extent to which this adapted programme might be effective in remediating a sample of South African learners' strategies for working with early number.

My reasons for embarking on this study are inspired by the challenges currently facing primary school teachers in many South African schools. Within the South African context, national assessments point to the majority of learners falling behind the levels of the prescribed curriculum by the end of Foundation Phase, rather than the minority – which is the case in the developed world. In the midst of such a great need for intervention on the ground, a model in which intervention is provided on a one-on-one basis is too costly and not practical in public schools that are under-resourced – therefore my study proposes to adapt the one-on-one Maths Recovery programme by using it in a 'group intervention' model where more than one child can receive intervention simultaneously. Additionally, my focus on the learning opportunities that may arise for individual children as they interact during group intervention is intended to provide insight that can feed into the mainstream classroom.

This study will involve selecting 8 'middle-attaining' learners from each of the three Grade 2 classes at the start of the 2014 school year as participants - 24 learners in total. Half of these learners will make up the 'intervention groups' (4 learners from each class – 12 in total) and the other half will make up the 'control group' (4 learners from each class – 12 in total). I will conduct individual task-based interviews with all 24 learners participating in this study at the end of the first term – this would serve as a pre-test. Learners in the three 'intervention groups' will then receive Maths Recovery remediation (in groups of four) for 40minutes, twice a week, over eight weeks during the second school term of 2014, while learners making up the 'control group' will not receive Maths Recovery intervention but will continue receiving Maths tuition as per usual from their class teachers. After the eight-week intervention period, I will conduct a final individual task-based interview with all 24 learners – this will serve as a post-test. The pre- and post-testing of all learners participating in this study will provide comparative data on individual learner's strategies and performance before and after the intervention period; and as well as comparative data across 'intervention' and 'control groups', which will allow me to determine the effectiveness of using the Maths Recovery programme for grouped remediation in the South African context. All individual task-based assessments and group intervention sessions will be video recorded. This is absolutely necessary because, as the interviewer or teacher during these sessions, I will not be able to take detailed notes of learners' strategies when working with number problems during interviews, nor the interactions (which include talk and gestures) between learners in the group sessions while simultaneously conducting these interviews or group intervention sessions.

I am inviting you, and the school, to participate in this research study on remediation in numeracy. My reason for choosing your school as a potential site for my study is because I have already developed a rapport with some of the Foundation Phase teachers at your school over the past two years as a member of the Wits Maths Connect

Primary project team. And, I believe that the feelings of mutual respect and trust that has already been established between me and the teachers at your school will make for the smooth running of this study. Another reason for choosing your school is because the LoLT of your school is English, which means that the Maths Recovery intervention sessions and individual assessments making up my study (which will also be conducted in English) will be conducted in the language of teaching and learning that is not unfamiliar to participating learners.

Participation in this study is voluntary and all participants will be assured that they are free to withdraw consent and discontinue participation in this study at any time without prejudice. Participants will not be paid for taking part in this study and will not be advantaged or disadvantaged in any way through their participation. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing emanating from the study. Your individual privacy will be maintained in all published and written data resulting from the study. All data collected and generated by my study will be treated with the strictest confidentiality and I will ensure the safekeeping of all data related to my research. All research data will be destroyed 5 years after completion of the project.

Please let me know if you require any further information. If you agree to allow your school to participate in this study, kindly complete the consent form attached or reply to me in writing.

I look forward to your response as soon as is convenient.

Yours sincerely,

Contact person:

Samantha Morrison

89 Du Toit Street, Discovery

samantha.morrison@wits.ac.za

011 717 3319 (W)

082 9282 494

Principal's Consent Form

Dear Mr./Mrs./Ms. _____

Please complete and return the consent form below indicating your willingness for your school to participate in my voluntary research study called: Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school.

Permission to participate

***Please delete as appropriate**

I, _____ (principal's name), give / do not give* consent for my school to participate in the above mentioned study.

Kindly circle your response to each of the following statements:

I know that my school, and participants from my school, may withdraw from the study at any time without prejudice or penalty. YES/ NO

I know that the school and participants will not be advantaged or disadvantaged in any way during participation. YES/ NO

I know that Samantha Morrison will first get informed consent from teachers, learners and parents before commencing with the proposed study. YES/ NO

I know that participants do not have to answer every question posed. YES/ NO

I know that the all data collected will be used for this study only. YES/ NO

I know that all data collected will be destroyed 5 years after completion of this project. YES/ NO

I know that Samantha Morrison will keep my information confidential and safe, and that participants' names and the name of our school will not be revealed. YES/ NO

Principal's Signature: _____ Date: _____

TEACHER'S INFORMATION SHEET

Date: _____

Dear _____

My name is Samantha Morrison and I am a PhD student in the School of Education at the University of the Witwatersrand. I am researching the remediating of numeracy with learners at Foundation Phase level. My research study is entitled '**Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school**'.

My study is aimed at exploring a classroom withdrawal-based model for remediating the early number learning of a sample of Grade 2 learners, with a focus on understanding the types of interactions within which shifts towards better understandings of number occur. This short-term intervention programme is based on an adaptation of the Maths Recovery programme developed by Wright, Martland and Stafford (2006), which has been used successfully for individual remediation in developed countries such as Australia, Britain, Ireland, the USA and Canada. My study aims to adapt the Maths Recovery programme so that it could be used for remediating a group of learners' number skills, and exploring the extent to which this adapted programme might be effective in remediating a sample of South African learners' strategies for working with early number.

My reasons for embarking on this study are motivated by the challenges currently facing teachers in many South African primary schools. Within the South African context, national assessments point to the majority of learners falling behind the levels of the prescribed curriculum by the end of Foundation Phase, rather than the minority – which is the case in developed countries. In the midst of such a great need for maths intervention, a model in which intervention is provided on a one-on-one basis is too costly and not practical in public schools that are under-resourced – therefore my study proposes to adapt the one-on-one Maths Recovery programme by using it for 'group intervention' where more than one child can receive remediation at the same time. Additionally, my focus on the learning opportunities that may arise for individual children as they interact during group intervention is intended to provide insight that can feed into the mainstream classroom.

This study will involve selecting 8 'middle-attaining' learners from each of the three Grade 2 classes at the start of the 2014 school year as participants for this study - 24 learners in total. Half of these learners will make up the 'intervention groups' (4 learners from each class – 12 in total) and the other half will make up the 'control group' (4 learners from each class – 12 in total). I will conduct individual task-based interviews with all 24 learners participating in this study at the beginning of the second term to assess the strategies they use when working with number problems and their overall performance. Twelve learners making up the 'intervention groups' will then receive Maths Recovery remediation (in groups of four) for 40 minutes, twice a week, over eight weeks during the second school term of 2014; while learners making up the 'control group' will not receive Maths Recovery remediation but will continue receiving Maths tuition as per usual from their class teachers. After the eight week intervention period, I will conduct a second round of individual task-based interviews with all 24 learners participating in the study. The outcome of the first and final individual interviews with these learners will allow me to compare individual learner's strategies and performance before and after the intervention period. This will also allow me to compare learners' performances across 'intervention' and 'control groups', which will then make it possible for me to determine the effectiveness of using the Maths Recovery programme for grouped remediation in the South African context. All individual task-based interviews and group intervention sessions will be video recorded. This is absolutely necessary because, as the interviewer or teacher, I will not be able to take detailed notes of learners' strategies when working with number problems during interviews, nor the types of interaction (which include talk and gestures) between learners in the group teaching sessions while simultaneously conducting these interviews or teaching sessions.

I am inviting you to participate in this research study on remediation in numeracy. My decision to work with you (and other Grade 2 teachers at your school) is because the Maths Recovery programme is usually implemented

with children in their second year of formal schooling, which are Grade 2 learners in the South African context. Also, you and your colleagues may know me through my work as a member of the Wits Maths Connect team that has been working alongside you in improving primary maths teaching and learning at your school for the past three years – and this relationship of mutual respect and trust would make the research process run smoother. Additionally, the LoLT of your school is English, which means that the Maths Recovery intervention sessions and individual interviews which form part of my study (also to be done in English) will be conducted in a language that is not unfamiliar to learners in your class.

If you agree to participate in my study, I would need you to consent to an interview with me during the second month of the first school term in 2014. During this interview, I will explain the procedures involved in this study and elicit your help in selecting 8 learners from your class to participate in this study. The criteria to be used for the selection of these learners will also be discussed during the interview. Once we have agreed on which 8 learners to select, I will need your help in explaining to learners what the study will entail. Thereafter, I will need you to help me send information letters and consent forms home with learners for them and their parents to read at home and complete if they agree to participate. We would also need to come to an agreement concerning the best times and days for me to conduct the individual interviews with the 8 learners from your class (before and after intervention) and for the withdrawal of the 4 'intervention group' learners. We would also have to talk about how these learners will be assisted in catching up with important work missed.

By participating in my study you will not be advantaged or disadvantaged in any way. Your participation would be voluntary, thus you will be free to withdraw from the study at any time without penalty. There are no foreseeable risks involved and you will not be paid for participating in this study. Your name and identity, as well as the names learners and the school, will be kept confidential at all times and in all academic writing about the study. All data collected and generated by my study will be treated with the strictest confidentiality and I will ensure the safekeeping of data related to my research. All research data will be destroyed 5 years after completion of the project.

Please let me know if you require any further information. If you agree to participate in this study, kindly complete the consent form attached and return the form to me as soon as is convenient, or you may reply to me in writing.

Thank you very much for your help.

Yours sincerely,

Contact person:

Samantha Morrison

89 Du Toit Street, Discovery

samantha.morrison@wits.ac.za

011 717 3319 (W)

082 9282 494

Teacher's Consent Form

Date: _____

Dear Mr./Mrs./Ms. _____

Please fill in the consent form below indicating your willingness to participate in my voluntary research study entitled 'Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school'.

Thank you
Samantha Morrison

Kindly circle your response to each of the following statements:**Permission to participate**

I agree to participate in the above-mentioned study. YES/ NO

I know that I may withdraw from the study at any time, and that I will not be advantaged or disadvantaged in any way. YES/ NO

I know that the all data collected will be used for this study only. YES/ NO

I know that all data collected will be destroyed 5 years after completion of this study. YES/ NO

Permission to be interviewed

I agree to be interviewed. YES/ NO

I know that I do not have to answer every question posed. YES/ NO

I know that Samantha Morrison will keep my information confidential and safe, and that learners' names and the name of our school will not be revealed. YES/ NO

Teacher's Signature: _____

Date: _____

PARENTS' INFORMATION SHEET

Date: _____

Dear Parent/Guardian

My name is Samantha Morrison and I am a PhD student in the School of Education at the University of the Witwatersrand. I am doing research on '**Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school**'.

My study is aimed at exploring a model for remediating the early number learning of a sample of Grade 2 learners, with a focus on understanding the types of interactions within which shifts towards better understandings of number occur. This short-term intervention programme is based on an adaptation of the Maths Recovery programme, which has been used successfully for individual remediation in developed countries such as Australia, Britain, the USA and Canada. My study aims to adapt the Maths Recovery programme so that it could be used for remediating a group of learners' number skills, and exploring the extent to which this adapted programme might be effective in remediating a sample of South African learners' strategies for working with early number.

This study is motivated by the challenges currently faced by many South African parents and teachers of primary school children. Within the South African context, the Annual National Assessments (ANAs) point to the majority of learners falling behind the levels of the prescribed curriculum by the end of Foundation Phase, rather than the minority – which is the case in developed countries. While there is such a great need for maths intervention in schools, providing intervention on a one-on-one basis is too costly and not practical in public schools that are under-resourced – therefore my study proposes to adapt the one-on-one Maths Recovery programme by using it for 'group intervention' where more than one child can receive remediation at the same time. Additionally, my focus on the learning opportunities that may arise for individual children as they interact during group intervention is intended to provide insight that can help teachers and learners in ordinary classrooms as they work with numeracy concepts.

This study will involve selecting 8 'middle-attaining' learners from each of the three Grade 2 classes at the start of the 2014 school year to take part in this study - 24 learners in total. Half of these learners will make up the 'intervention groups' (4 learners from each class) and the other half will make up the 'control group' (4 learners from each class). I will conduct individual task-based interviews with all 24 learners participating in this study at the end of the first term – this will serve as a pre-test. Twelve learners making up the three 'intervention groups' will then receive Maths Recovery remediation (in groups of four) for 40 minutes, twice a week, over eight weeks during the second school term of 2014; while learners making up the 'control group' will not receive Maths Recovery remediation but will continue receiving Maths tuition as per usual from their class teachers. After the eight-week intervention period, I will conduct a second round of individual task-based interviews with all 24 learners participating in the study – this will serve as a post-test. The pre- and post-testing of all 24 learners will allow me to compare individual learner's strategies and performance before and after the intervention period. This will also allow me to compare learners' performances across 'intervention' and 'control groups', which will then make it possible for me to determine the effectiveness of using the Maths Recovery programme for grouped remediation in the South African context. All individual task-based interviews and group intervention sessions will be video recorded. This is absolutely necessary because, as the interviewer or teacher during these sessions, I will not be able to take detailed notes of learners' strategies when working with number problems during interviews, nor the interactions between learners in the group sessions at the same time.

I am inviting you and your child to participate in this research study. My reason for choosing your child's class is because the Maths Recovery programme is usually implemented with children in their second year of schooling, which are Grade 2 learners in the South African context. The language of learning and teaching at your child's

school is English, which means that the Maths Recovery intervention and individual interviews (also done in English) will be conducted in a language that is not unfamiliar to your child.

If you allow your child to participate in my voluntary study, I would need you to give consent for your child to be interviewed by me and for the interviews to be video recorded. I would also need you to agree to your child being either part of the 4 learners from each Grade 2 class which would make up the 'intervention groups' or part of the learners making up the 'control group'. Remember, learners making up the 'intervention groups' will be withdrawn from class for 40 minutes, twice a week for eight weeks, at a time agreed upon by the class teacher. These learners will be given time in class to catch up on any work missed, and they will not get extra homework. Learners making up the 'control group' are those learners who will remain in class for the intervention period and be taught as usual by their respective class teachers.

By participating in my study your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study. Your child's name and identity, as well as the names of her/his teacher and school, will be kept confidential at all times and in all academic writing about the study. All data collected and generated by my study will be treated with the strictest confidentiality and I will ensure the safekeeping of data related to my research. All research data will be destroyed 5 years after completion of the project.

Please let me know if you require any further information. If you agree to allow your child to participate in this study, kindly complete the consent form attached and return the form to your child's class teacher as soon as is convenient, or you may reply to me in writing.

Thank you very much for your help.

Yours sincerely,

Contact person:

Samantha Morrison

89 Du Toit Street, Discovery

samantha.morrison@wits.ac.za

011 717 3319 (W)

0829282494

Parent Consent Form

Date: _____

Dear Parent/Guardian

Please complete this consent form and indicate your willingness to allow your child to take part in my voluntary research project called 'Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school'. Kindly return the completed form to your child's class teacher as soon as is convenient.

Thank you

Samantha Morrison

* Please delete as appropriate

I, _____ the parent/guardian of _____
give/ do not give* consent for my child to take part in the above-mentioned project.

Kindly circle your response to each of these statements:

Permission for participation

I know that I may withdraw my consent allowing my child to participate in the study at any time, and that my child will not be advantaged or disadvantaged in any way.	YES/ NO
I know that my child can choose to withdraw from the study at any time.	YES/ NO
I know that Samantha Morrison will keep my information confidential and safe and that my child's name and the name of his/her school will not be revealed.	YES/ NO

Permission for interviews

I agree for my child to be interviewed.	YES/ NO
I know that my child does not have to answer every question when interviewed.	YES/ NO
I know that data collected from interviews will be kept safe, and this data will be destroyed 5 years after completion of this study.	YES/ NO

Permission for video recordings

I agree for my child to be video recorded.	YES/ NO
I know that the videotapes will be used for this study only.	YES/ NO
I know that all video data collected during the course of this study will be destroyed 5 years after completion of this project.	YES/ NO

Parent/Guardian Signature: _____

Date: _____

Contact person:

Samantha Morrison

89 Du Toit Street, Discovery

samantha.morrison@wits.ac.za

011 717 3319 (W)

082 9282 494

LEARNER'S INFORMATION SHEET

Date: _____

Dear Learner

My name is Samantha Morrison and I am a PhD student in the School of Education at the University of the Witwatersrand. I am doing research on: **Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school.**

Through my study I want to find out if the Maths Recovery programme can be used to help South African children to work better with numbers. I will choose 24 Grade 2 learners, that is, 8 children from every class, and interview them one-by-one to see how they work with numbers. Then I will take a group of 4 learners from each class and teach them how to work with numbers using the Maths Recovery methods. These lessons will take place during school, twice a week for 8 weeks, when your teacher says it will be fine to do so. After the 8 weeks of Maths Recovery teaching, I will again interview all 24 learners one-by-one. When I interview children one-by-one or teach them how to work with numbers in a group, I will use a video recorder to record what we are doing and saying. I will watch the videos afterwards to see what each person did and said while we worked with numbers.

I was wondering whether you would like to take part in my study. I would need your help with taking part in individual interviews and agreeing to be video recorded so that I can see how you work with numbers. I may also need you to be part of the 4 learners who will be taught how to work with numbers using the Maths Recovery methods, if you are one of the 4 learners chosen for this out of your class, or to be part of the 4 learners who will stay in class and be taught by your teacher as usual.

Remember, taking part in this study is not for marks and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you want to stop, this is up to you and you will not be punished in any way. If you agree to take part in this study, your teacher will give you time in class to catch up with any work that you missed, and you will not be given extra homework to do.

I will not be using your own name but I will make up a name so that no-one will be able to identify you. All information about you, your teacher and your school will be kept a secret in all my writing about the study. Also, all collected information will be stored safely and destroyed 5 years after I have finished my study.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join in this study. Kindly complete the attached consent form if you agree to be part of my study.

Please feel free to contact me if you have any questions. I am looking forward to working with you!

Thank you

Contact person:

Samantha Morrison

89 Du Toit Street, Discovery

samantha.morrison@wits.ac.za

011 717 3319 (W)

082 9282 494

LEARNER CONSENT FORM

Date: _____

Dear Learner

Please fill in the consent form below if you agree to participate in my voluntary study called 'Remediating early number learning: Using an adapted version of Maths Recovery in a South African primary school'.

Thank you

Samantha Morrison

Learner's name: _____

Grade: _____

Kindly circle your response to each of the following statements:

Permission to participate

I agree to participate in the study.	YES / NO
I know that I can withdraw from this study at any time.	YES/ NO
I know that Samantha Morrison will keep my name, my teacher's name and the name of my school a secret.	YES / NO
I know that Samantha Morrison will keep all information collected during this study in a safe place.	YES/ NO

Permission to be interviewed

I agree to be interviewed.	YES/ NO
I know that I do not have to answer all the questions.	YES/ NO
I know that taking part in the interviews will not count for marks.	YES/ NO

Permission to be video recorded

I agree to be video-recorded in class.	YES / NO
I know that I can stop the video-recording at any time.	YES / NO
I know that the videotapes will be used for this project only.	YES / NO
I know that the videotapes will be destroyed 5 years after this study is finished.	YES/ NO

Sign: _____

Date: _____

Contact person:

Samantha Morrison
89 Du Toit Street, Discovery
samantha.morrison@wits.ac.za
011 717 3319 (W)
082 9282 494

Appendix C

Maths Recovery Assessment Schedule 1.1

Record of Response per Task Item

(To determine SEAL 1 – 4; F/BNWS & Numerals)

Description	Item		Learner's Response
Q 1. FNWS	a	1 – 32	
	b	48 – 61	
	c	76 – 84	
	d	93 – 112	
Q 2. NWA	a	11 – 30	
	b	1 – 10	
	c	31 – 99	
Q 3. Numeral ID	a	10 – 99	
	b	1 – 9	
	c	100–1000	
Q 4. Num Recog		1 – 10	
Q 5. BNWS	a	10 – 1	
	b	15 – 10	
	c	23 – 16	
	d	34 – 27	
	e	72 – 67	
Q 6. NWB	a	11 – 30	
	b	1 – 10	
	c	31 – 99	
Q 7. Sequencing Numerals (Num ID)	a	46 – 55	
	b	1 – 10	
Q 8. Additive Tasks			
Q 8a. Both addends screened	i	5 + 4	
	ii	9 + 6	
Q 8b. First addend screened	i	5 + 2	
	ii	7 + 3	
	iii	9 + 4	
Q 8c. Both addends unscreened	i	5 + 2	
	ii	7 + 3	
	iii	9 + 4	
Q 8d. Perceptual counting	i	13 counters	
	ii	18 counters	
Q 8 e.	i	8 + 5	

Q 9. Subtractive Tasks

Q 9a. Subtraction sentences	i	$16 - 12$	
	ii	$17 - 14$	
Q 9b. Missing subtrahend	i	$5 - [] = 3$	
	ii	$10 - [] = 6$	
	iii	$12 - [] = 9$	
	iv	$15 - [] = 11$	
Q 9c. Removed items	i	$3 - 1$	
	ii	$6 - 2$	
	iii	$9 - 4$	
	iv	$15 - 3$	
	v	$27 - 4$	

Summary

Maths Recovery Assessment Schedule 1.2
(To determine SEAL 5 & Base Ten)

Record of Response per Task Item

Q 1. Tens and Ones Tasks

Task		Item	Learner's Response	Description
Q 1a. Incrementing by tens on the decade (10s strips)	i	10		
	ii	20 – 80		
	iii	80 dots		
	iv	8 strips		
Q 1b. Incrementing by tens off the decade (with strips)	i	4 14 – 74		
	ii	7 17 – 87		
Q 1c. Incrementing by tens and ones	Board 1 10, 13, 33, 37, 40, 50, 52, 62			
	Board 2 4, 14, 34, 46, 71			

Q 2. Horizontal Number Sentences

Task		Item	Learner's Response	Description
Q 2a. Adding 2digit+2digit	i	16 + 10		
	ii	42 + 23		
	iii	38 + 24		
	iv	39 + 53		
Q 2b. Subtracting 2digit- 2digit	i	56 – 23		
	ii	43 – 15		
	iii	73 – 48		

10min57sec

Q 3. NCB1 Strategies and Relational Thinking

Task		Item	Learner's Response	Description
Q 3a.	i	9 + 3		
	ii	9 + 4		
	iii	9 + 5		
	iv	9 + 6		
Q 3b.	i	6 + 6		
	ii	7 + 5		
	iii	8 + 4		
	iv	9 + 3		
Q 3c.	i	7 – 5	2	
	ii	27 – 5	22	
	iii	47 – 5	42	
Q 3d.	i	15 + 3	18	
	ii	18 – 3	15	
Q 3e.		21 – 5		
Q 3f.		14 + 4 = 18		

Maths Recovery Assessment Schedule 2.1
Record of Response per Task Item

(For Structuring numbers 1 – 10)

Task	Item	Learner's Response	Description
Q 1a. Subitizing Regular Patterns 1 – 10	4		
	3		
	2		
	5		
	6		
Q 1b. Subitizing Irregular 1 – 10	6		
	7		
	4		
	5		
	8		
Q 1c. Subitizing Dominoes	5 – 3 – 8		
	6 – 4 – 10		
	4 – 4 – 8		
	5 – 4 – 9		
Q 2a. Finger Patterns 1-5 (one hand)	3		
	2		
	5		
	1		
	4		
Q 2b. Finger Patterns 1-5 (two hands)	3		
	2		
	5		
	4		
Q 3. Finger Patterns 1 – 10	6		
	9		
	10		
	8		
Q 4. 5 Frame	3		
	2		
	5		
	1		
	4		
Q 5. 5-Wise on Ten Frame	7		
	10		
	8		
	6		
	9		
Q 6. Pair-Wise on Ten Frame	4		
	2		
	5		
	1		
	3		
	7		
	10		
	8		
	6		
	9		
Q 7. Combinations to Five	4		
	2		
	1		
	3		
	5		
Q 8. Combinations to Ten	Learner's own		
	Learner's own		
	Learner's own		

Maths Recovery Assessment Schedule 2.2

Record of Response per Task Item

(For Structuring Numbers 1 – 20)

Q 1. Doubles (shown on bead board)

	Item	Learner's Response	Description
a	5 + 5		
b	4 + 4		
c	2 + 2		
d	7 + 7		
e	9 + 9		
f	6 + 6		
g	8 + 8		

Comments

Q 2. Near doubles

	Item	Learner's Response	Description
a	5 + 6		
b	4 + 3		
c	7 + 6		
d	9 + 8		
e	8 + 7		

Comment on 'near doubles' tasks

Q 3. Addition using 5, 10, doubles, etc.

	Item	Learner's Response	Description
a	9 + 3		
b	8 + 5		
c	9 + 6		

Appendix D

Format	Intervention Group							Control Group							
	Grade	Learner	SEAL	Base Ten	FNWS	BNWS	Num Id.	Grade	Learner	SEAL	Base Ten	FNWS	BNWS	Num Id.	
Single 1	2-1	Khanye (F)	3	2	5	4	3	2-1	Rila (F)	4	2	4	4	3	
Single 2	2-5	Josh (M)	2	--	2	2	1	2-5	Thulu (F)	2	--	4	3	3	
Grouped Intervention 1.	2-2	Mali (F)	3	1	5	3	3	2-2	Bonang (M)	3	1	4	3	3	
	2-2	Jenna (F)	3	1	4	3	3	2-2	Rhaka (M)	3	1	4	3	3	
	2-2	Bongi (M)	3	1	5	5	3	2-2	Abby (F)	2	--	4	3	3	
	2-3	Sibu (M)	2	--	3	2	3	2-2	Olsen (M)	3	1	4	5	3	
Grouped Intervention 2.	2-4	Julie (F)	4	1	5	5	3	2-4	Nozi (M)	3	1	4	4	4	
	2-5	Khosi (F)	3	1	5	3	3	2-4	Yazu (M)	4	1	5	5	4	
	2-4	Kgomo (M)	3	1	5	3	3	2-4	Ntiko (F)	3	1	5	3	3	
	2-4	Kyle (M)	2	--	5	5	4	2-4	Sipho (F)	3	1	4	4	3	
		TOTAL: 172		28 (x2)	8	44	35	29	TOTAL: 180		30 (x2)	9	42	37	32

Appendix E

Assessment Schedule 1.1 Numerals)

(To determine SEAL 1 – 4; NWS &

Task	Item		Purpose
Q 1. FNWS	a	1 – 32	To assess the child's facility with forward number word sequences. You observe whether the child has problems crossing decades and one hundred bound-aries. Also interesting is to see whether she omits any numbers or decades. Also interesting is to see what she says after 'one hundred and ten' and so on. (p36)
	b	48 – 61	
	c	76 – 84	
	d	93 – 112	
Q 2. NWA	a	11 – 30	Can the child give the NWA immediately or soon after being asked? Does the child drop back to one (or cannot answer)? Of particular interest is how the child responds to number words with '9' or '0' in the ones place, e.g. twenty-nine. (p37)
	b	1 – 10	
	c	31 – 99	
Q 3. Numeral ID	a	10 – 99	It is useful to observe whether the child is confident and swift in response. If there is a delayed response, observe whether the child says the FNWS from or subvocally, in order to generate the name of the numeral. Some children say particular numerals like 12 and 21. Listen to the child's articulation of '12' because they often confuse it with '-teen' numbers. (p39)
	b	1 – 9	
	c	100 – 1000	
Q 4. Num Recog	1 – 10		
Q 5. BNWS	a	10 – 1	Similar to FNWS and NWA. Some children can respond to these tasks but slowly and with what appears to be a good deal of mental effort. The observer should observe carefully for signs of distress in the child. Saying the BNWSs is typically more difficult than FNWSs in the teens. Does the child drop back subvocally to determine a particular number word? Does she omit the decade numeral (e.g. 38) or say the decade which is ten less than the correct decade (41, 30, 38)? (p38)
	b	15 – 10	
	c	23 – 16	
	d	34 – 27	
	e	72 – 67	
Q 6. NWB	a	11 – 30	Can the child give the NWB immediately or soon after being asked? Does the child drop back to one (or cannot answer)? Of particular interest is how the child responds to number words with '0' or '1' in the ones place, e.g. sixty and one. (p42)
	b	1 – 10	
	c	31 – 99	
Q 7. Sequencing Numerals	a	46 – 55	Children's solutions to sequencing numerals tasks are likely to draw on their knowledge as well as their numeral identification knowledge. Therefore these tasks provide interesting additional insights into these aspects of children's number knowledge. (p43)
	b	1 – 10	

Q 8. Additive Tasks strategies)

(Overall the aim is to understand the child's solution

a) Both addends screened	i	5 + 4	a i) is within the finger range and ii) is beyond the finger range (p47)
	ii	9 + 6	
b) First addend screened	i	5 + 2	These are Less Advanced Additive tasks because only the first (a) addend is screened) (p47)
	ii	7 + 3	
	iii	9 + 4	

f) Missing addend	i	$7 + [] = 10$	To assess the child's use of the 'counting-up-to' / 'count on' strategy, the child may use subtraction to get the answer. (p. 48)
	ii	$12 + [] = 15$	

Q 9. Subtractive Tasks

Q 9a. Subtraction sentences	i	16 – 12	These are the hardest subtraction tasks. Check if the child uses N strategies or parts thereof. (p. 49)
	ii	17 – 14	
Q 9b. Missing subtrahend	i	5 – [] = 3	These are less difficult than 9a and more difficult than 9c. To elicit strategy of ‘counting-down-to’ (p. 49, 50)
	ii	10 – [] = 6	
	iii	12 – [] = 9	
	iv	15 – [] = 11	
Q 9c. Removed items	i	3 – 1	These are the easiest subtraction tasks in this assessment. To elicit strategy of ‘counting-down-from’. (p49, 50)
	ii	6 – 2	
	iii	9 – 4	
	iv	15 – 3	
	v	27 – 4	

Assessment Schedule 1.2

Base Ten)

(To determine SEAL 5 &

Q 1. Tens and Ones Tasks

Task	Item		Purpose
Q 1a. Incrementing by tens on the decade (10s strips)	i	10	Base-Ten level 1 The purpose of these tasks is to determine where the child can increment by tens on the decade where the strips are displayed rather than screened and where the child can increment by one ten only (2006, p93)
	ii	20 – 80	
	iii	80 dots	
	iv	8 strips	
Q 1b. Incrementing by tens off the decade (with strips)	i	4 14 – 74	Base-Ten level 1 Same purpose as above except this is off the decade (2006, p94)
	ii	7 17 – 87	
Q 1c. Uncovering tasks with tens and ones	Board 1 10, 13, 33, 37, 40, 50, 52, 62		Base-Ten level 2 Board 1 – 10s and 1s separate; Board 2 (4th and 5th) – 10s and 1s combined. To determine whether the child can increment by one ten and one one in increasingly complex ways (2006, p95).
	Board 2 4, 14, 34, 46, 71		

Q 2. Horizontal Number Sentences

Task	Item	Purpose
Adding 2digit+2digit	16 + 10	Base-Ten level 3 Children are judged to be at Base-Ten level 3 if they are able to increment and decrement by tens and ones when solving bare number tasks that do not involve base-ten materials, i.e. horizontal number sentences. It is not necessary for children to answer all tasks correctly. Note whether they can solve addition but not subtraction tasks and whether they have multiples strategies or are dependent on one strategy. (2006, 98)
	42 + 23	
	38 + 24	
	39 + 53	
Subtracting 2digit-2digit	56 – 23	
	43 – 15	
	73 – 48	

Q 3. NCB1 Strategies and Relational Thinking

Task	Item	Description
a)	9 + 3	SEAL Stage 5 To determine whether the child can use the NCB1 strategy: incrementing by one from a constant number
	9 + 4	
	9 + 5	
	9 + 6	

e)	$21-16=5$ $21-5$	SEAL Stage 5 NCB1 strategy: using a known subtraction fact to work out an associated subtraction problem
f)	$14+4=18$ Use these 3 numbers to make a take-away sum	SEAL Stage 5 NCB1 strategy: deriving a subtraction fact from an addition fact

NB! As a general rule, at least three instances of Stage 5 strategies across Assessment 1.1 task groups 8 and 9 and Assessment 1.2 task group 3 (above) must be observed for a child to be placed at SEAL stage 5.

Assessment Schedule 2.1
numbers 1 – 10)

(For Structuring

Task	Item	Purpose
Q 1a. Subitizing Regular Patterns 1 – 10	4 3 2 5 6	To assess the child's facility with immediately recognizing regular spatial patterns and recalling the number that corresponds to a pattern. If the child takes a few seconds she might be visualizing the pattern and attempting to place dots in the visualized image. The delay may be accounted for by head nods or hand movements and/or verbal or non-verbal utterances (p. 105). Subitizing four dots can indicate that the child has a concept of '4', for example (p. 26)
Q 1b. Subitizing Irregular Patterns 1 – 10	6 7 4 5 8	Same as above except that these patterns are irregular. Irregular patterns are more difficult to subitize than those above.
Q 1c. Subitizing Dominoes	5 – 3 – 8 6 – 4 – 10 4 – 4 – 8 5 – 4 – 9	Configurations here involve two domino patterns, making the task more challenging than the above (105). This provides insight into the child's emerging strategies for combining partitions to 10. (p. 106)
Q 2a. Finger Patterns 1-5 (one hand)	3 2 5 1 4	If the child raises fingers sequentially to make these patterns then she does not have a facile finger pattern for that number. By raising fingers sequentially she 'establishes' a pattern for that number. Check whether the child has her fingers in order to make the finger patterns. (p. 2)
Q 2b. Finger Patterns 1-5 (two hands)	3 2 5 4	Same as above. I think using two-hands speaks to children's ability to partition number 1-5, i.e. emerging strategies for combining a partition to 5.
Q 3. Finger Patterns 1 – 10	6 9 10 8	Assess whether the child has a facile finger pattern for 10 (if she raises 5 fingers simultaneously) – the child who can raise 5 fingers simultaneously may now 'count on' from 5. When the question is posed check whether the child uses a systematic approach to producing the new finger pattern or whether the question is seen as a new, unrelated task. Some children solve this in a cumulative/additive sense (as just expected: raise ten fingers and then proceed to reduce the amount to the required total. (p. 107)

Q 5. 5-Wise on Ten Frame	7	To assess knowledge of the number five as a base (p. 111). Offers the child the opportunity to show her ability to understand and partition numbers 6-9 using 5 as a base. [deduct 1 point for “again” on page 112 in purpose for pair-wise ten frame]
	10	
	8	
	6	
	9	
Q 6. Pair-Wise on Ten Frame	4	Pair-wise patterns use two as a base unit and relate even numbers and doubles and near doubles (p. 111). This task will expose to what extent the child has developed conceptually based images for the numbers 1 to 10 in context to the five-wise pattern (p. 112). Offers the child the opportunity to show her ability to understand and partition numbers in range 1 to 10 (p. 112).
	2	
	5	
	1	
	3	
	7	
	10	
	8	
	6	
9		
Q 7. Combinations of Five	4	To assess the child’s knowledge of the combinations that make up the support of equipment to give the corresponding image (p. 112).
	2	
	1	
	3	
	5	
Q 8. Combinations of Ten	learner’s own	Assesses the child’s ability to make various combinations presented as verbal questions – thus extending the range of problems presented without equipment (p. 112).
	learner’s own	
	learner’s own	
	8 + [] = 10 4 + [] = 10 7 + [] = 10	

Assessment Schedule 2.2

Checking child's facility with mental addition and subtraction in the range 1-20 using doubles and base-5 and -10 as reference points (p.113)

Advanced Grouping: Structuring Numbers 1 to 20

Task Groups	Item	Purpose
Task Group 1 Doubles	5+5, 4+4, 2+2 7+7, 9+9, 6+6, 8+8	facility with addition in range 1-20 in the setting of doubles
Task Group 2 Near Doubles	5+6, 4+3, 7+6, 9+8, 8+7, 3+2	facility with addition in range 1-20 using knowledge of doubles as a reference point
Task Group 3 Addition using five, ten or doubles, etc.	9+3, 8+5, 9+6 8+6, 5+7, 4+9	Task groups 3, 4 and 5 assess the child's facility to use number relationships and the properties of operations. Does the child have ways of reorganizing the tasks by using five or ten as a reference point or using their knowledge of doubles? (p115)
Task Group 4 Subtraction using five, ten or doubles, etc.	17 - 7, 11 - 4, 12 - 9 17 - 8, 13 - 5, 14 - 8	
Task Group 5 Addition and Subtraction with addend/subtrahend >10	13+5, 11+8 19 - 7, 17 - 15	
Task Group 6 Relational Thinking	14+2 and 2+14	commutativity
	7+7, 8+6, 9+5	compensation
	13+4 and 17 - 4	addition to solve subtraction
	6+8=14, 8+6, 14 - 6	use the given number sentence to solve an addition problem and a related subtraction problem

These were omitted from my test.