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CHAPTER II

SOME FUNDAMENTAL CONSIDERATIONS OF THE WATER AND AIR PHASES IN A PARTLY SATURATED SOIL

2.1 Introduction

In chapter I some of the most important work on effective stresses in fully saturated soils was described. Recently attempts have been made to deal with partly saturated soils in terms of effective stresses. However this work on partly saturated soils cannot be considered until some of the properties of the water and air phases of the soil are understood. This chapter is devoted to an examination of the most important properties of the water and air phases in a partly saturated soil.

2.2 Surface Tension

Surface Tension is due to molecular forces which, at a liquid-gas interface, have a resultant attraction towards the denser medium. The best known manifestation of surface tension is the rise of liquids in fine bored tubes. The phenomenon is known as capillarity.

Water in a capillary tube having one end immersed in free water will rise to a height 'h' defined by :

$$h = \frac{2T.\cos\alpha}{r.g.\rho_W} \quad \ldots\ldots\ldots\ldots \quad (2.1)$$

where : T = surface tension of water
α = contact angle of water to the walls of the tube
r = radius of the tube
g = acceleration due to gravity
ρ_W= density of water

when $\alpha = 0$ equation (2.1) reduces to

$$h = \frac{2\ T}{r.g.\rho_w} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2.2)$$

The pressure difference across a curved liquid-gas interface is therefore

$$p_c = g.\ \rho_w.\ h = \frac{2T}{r} \ . \ . \ . \ . \ . \ . \ . \ . \quad (2.3)$$

2.3 Evaporation from an air-water interface.

If the relative humidity of the air above a free water surface is less than 1·0 the water will tend to evaporate until the relative humidity becomes 1·0. If the water is in a state of tension, as in the case of a meniscus of radius r, it ceases to evaporate at a lower value of relative humidity. This lower value is known as the relative vapour pressure of the liquid.

The relationship between relative vapour pressure and the radius of curvature of an air-water interface is given by the well known Kelvin equation :

$$\ln\ p/p_o = -\frac{2\ T.M}{r.g.\ \rho_w.R.\theta.} \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2.4)$$

where : p = the vapour pressure over the curved surface.

p_o = the vapour pressure of free water at temperature θ.

T = the surface tension of water.

M = the molecular weight of water.

ρ_w = the density of water.

g = the acceleration due to gravity.

R = the gas constant

θ = the absolute temperature.

From equations (2.3) and (2.4) the pressure difference across a curved air-water interface can be related to the relative vapour pressure of the water :

$$p_c = - \frac{R.\theta.}{M} g. \rho_w. \ln p/p_o \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2.5)$$

It can be seen that when p/p_o is equal to one, p_c is zero. As p/p_o decreases p_c increases rapidly.

When a curved air-water interface is in contact with an atmosphere having a relative humidity less than the relative vapour pressure of the water, as given by equation (2.4), evaporation will take place. Similarly if the relative humidity of the atmosphere is greater than the relative vapour pressure of the water condensation will take place.

If the air-water interface occurs as menisci between soil grains and if the soil is placed in an atmosphere having a relative humidity less than the relative vapour pressure of the soil water the soil will give up some of its water to the atmosphere. As the soil dries the air-water interfaces retreat into the soil pores and their radii of curvature decrease. Hence from equation (2.4) it can be seen that the relative vapour pressure of the soil water will decrease until it becomes equal to the relative humidity of the atmosphere. At this point evaporation of the soil moisture will cease. It is clear from equation (2.3) that the increase in menisci curvature resulting from the evaporation of the soil moisture gives rise to an increase in the capillary pressure p_c.

2.4 The concept of pressure deficiency in a soil

(a) Development of the concept. The phenomenon of soil suction has been recognised for some time. Briggs (1897) recognised that energy was required to extract water from soil, and that the lower the moisture content of a given soil the greater was the energy required to extract the moisture. Buckingham (1907) introduced what he called 'the capillary potential function'. He defined 'capillary potential' as the work that would have to be done against the 'capillary field force' in transferring a unit mass of water from the soil to free water at zero hydrostatic pressure.

Gardner (1922) related the capillary potential function to measurements obtainable in moist soil with porous cups and vacuum gauges. Joffe and McLean (1925) measured what they called the 'suction force' of soils. The name 'absorbing power' was also used to describe this phenomenon by some workers.

It has now become quite common for workers to describe the tendency for a soil to imbibe water as 'soil moisture tension' or 'soil moisture suction'. These two terms convey the impression that the water in the soil is actually in a state of tension. There has been some doubt as to whether this is, in fact, the case for very dry soils. However, there can be no doubt that the energy of the water in the soil is less than the energy of free water. The potential energy of the water in the soil can be raised to that of free water by applying a pressure to it by means of air or some other gas. The pressure that is required to be applied to the soil water to bring it to the same potential energy as free water may be defined as the 'pressure deficiency' of the soil water and is denoted as p". Strictly speaking it is necessary to stipulate that the temperature be held constant so that the process can be considered as one involving measurements of free energy only. Thus the pressure deficiency p" is really a measure of the free energy of the soil water and there are no implications regarding the physical state of this water. The term 'pressure deficiency' is thought to be less confusing than most other phrases describing the energy of soil water and its use will be adopted for the purposes of this thesis.

(b) The pF scale of measuring p" : p" can reach very high values in a soil. For example when the relative humidity is 93% the equilibrium pressure deficiency is 100kg/sq.cm. and when the relative humidity is 48% p" becomes 1000kg/squ. cm.

R.K. Schofield (1935) introduced what he called the pF scale of measuring p". He defined pF as the logarithm of the height in cms. of the water column needed to give the suction in question. p represents the logarithmic character while F is the free energy of the soil water measured on a gravity scale. The relationship between relative humidity, radius of curvature of the meniscus, pressure deficiency p" and pF are tabulated in Table 2.1.

Table 2.1

The relationship between relative vapour pressure, radius of curvature of meniscus, p" and pF.

p/p_o per cent	Radius of curvature r cms	p" psi	pF
99.999	14.7x10	1.42×10^{-1}	1
99.99	14.7	1.42	2
99.9	14.7×10^{-1}	1.42x10	3
99.3	14.7×10^{-2}	1.42×10^{2}	4
93.0	14.7×10^{-3}	1.42×10^{3}	5
49.0	14.7×10^{-4}	1.42×10^{4}	6
0.08	14.7×10^{-5}	1.42×10^{5}	7

2.5 The Measurement of pressure deficiency

Up till about 1920 the only method used for measuring pressure deficiency in a soil was by means of 'tension plates'. In this method a saturated porous cup or plate is placed in contact with the soil. A suction is applied to the water in the porous plate and water passes between the sample and the plate until hydrostatic pressure equilibrium is established. If the suction of the water in the porous plate is greater than the pressure deficiency of the water in the sample flow will take place from the sample through the porous plate until the pressure deficiency in the sample is equal to the suction of the water in the porous plate.

Workers soon found that the pressure deficiencies in soils could greatly exceed one atmosphere. However, the tension plate method cannot be used to measure such high pressure deficiencies since, with practicable apparatus sizes, it is not possible to maintain a suction greater than one atmosphere. Other methods for measuring pressure deficiency had to be devised. Indirect methods such as the freezing point depression, the vacuum desiccator and the sorption balance were developed. These methods are based on the relationship between the pressure deficiency and the free energy of the water in the soil. For example, the temperature at which ice crystals form in a soil is less than 0° C because the free energy of the water in the soil is less than that of water in bulk under atmospheric pressure. For the same reason the pressure of water vapour over a moist soil is less than that over free water at the same temperature. Detailed descriptions of these indirect methods are given by Croney et al (1952).

R.K. Schofield (1935) seems to have been the first to have suggested using air pressures to obtain higher tension values. It can be seen from equation (2.3) that the pressure difference across an air-water interface is independent of the pressure in the air phase. Thus, if the surrounding air pressure where to increase the pressure in the water would increase by the same amount thereby maintaining a constant pressure difference. The independence of meniscus curvature on ambient air pressure was verified experimentally by Hilf (1956). This important concept is employed in the pressure plate method of measuring pressure deficiency. In this method the soil sample is placed in contact with a saturated porous disc in which the water is maintained at atmospheric pressure. Instead of increasing the tension of the water in the porous disc (as in the tension plate method) the pressure of the air surrounding the soil sample is increased until equilibrium is reached. The range of pressure deficiency measurement using this method is limited only by the size of the pores in the porous disc since it is clear that these pores must in themselves remain full of water under the pressure differences which are imposed by the test.

Richards (1941) was the first to use a membrane instead of a plate to measure pressure deficiency. Since the pores of a cellulose membrane are almost molecular in size the magnitude of the pressure deficiency which can be measured employing this method is theoretically very large indeed - far in excess of any value likely to be encountered in normal engineering practice. J.D. Coleman (1959) describes two types of pressure membrane apparatus. One of them is used for routine testing of samples within the range pF2 to pF5. The second type is suitable for very high pF ranges. Results show that this apparatus can be used over the range pF2.5 to pF6.2. The pressure membrane method will be discussed in greater detail in chapter V.

It is worth mentioning one further method employed for measuring pressure deficiencies. This method consists of applying an increased gravitational field to the soil by means of a centrifuge. From equation (2.3) the pressure defference across a curved air-water interface is

$$p'' = g.\ \rho.\ h = \frac{2T}{r}$$

Subjecting the sample to an acceleration has the effect of increasing g. Thus if the soil is permitted to drain freely while in a state of applied acceleration it will eventually come to equilibrium at a pressure deficiency which is governed by this acceleration.

The methods discussed above are listed in Table 2.2 together with the range of pressure deficiencies for which they can be used. These figures are based on values given by Croney and Coleman (1960).

Table 2.2

Methods of measuring pressure deficiency

METHOD	RANGE (pF Units)
Suction plate	0 to 3.0
Pressure plate	0 to 3.3
Pressure membrane	0 to 6.2
Centrifuge	3.0 to 4.5
Freezing point depression	3.0 to 4.0
Vacuum desiccator	5.0 to 7.0
Sorption balance	5.0 to 7.0

2.6 The relationship between pressure deficiency and water content

Early workers like Gardner (1922) and Joffe and McLean (1925) came to recognise that the moisture content of a soil will come to equilibrium if the soil is placed in contact with a porous plate across which a constant pressure reduction is maintained. Thus the concept of a definite moisture content at a given pressure deficiency developed.

Haines (1925) (1927) studied capillary effects in ideal granular materials. He was able to explain the concept of soil suction by making use of the known laws of surface tension. Haines further deduced that the water content was uniquely related to the soil suction. This latter theory was seriously criticised by Fisher (1926) who observed that a wet soil when placed in contact with a dry sample of the same soil does not give up more than a small fraction of its water to the dry one. This observation could not be accounted for in Haine's original theory. As a result of this criticism, Haines (1930) performed tests on granular materials and plotted pressure deficiency against moisture content for wetting and drying cycles. He found that the curves did not fall on a single line but that they formed a hysteresis loop as shown in figure 2.0. Haines was able to explain this hysteresis effect by considering the drainage and refilling of a single soil pore. He showed that a given soil pore which is initially saturated will empty suddenly when the tension in the soil water is just sufficient to 'suck in' the air water interface through its largest entry port. To refill the pore the tension must be reduced until the surface tension can pull the interface past the largest internal cross section of the pore at which point the refilling process will again occur suddenly. Using very large particles Haines actually observed this emptying and filling of pores by 'jerks.' With finer media the individual jerks will not be observed macroscopically but their smoothed out effect will still be present as hysteresis.

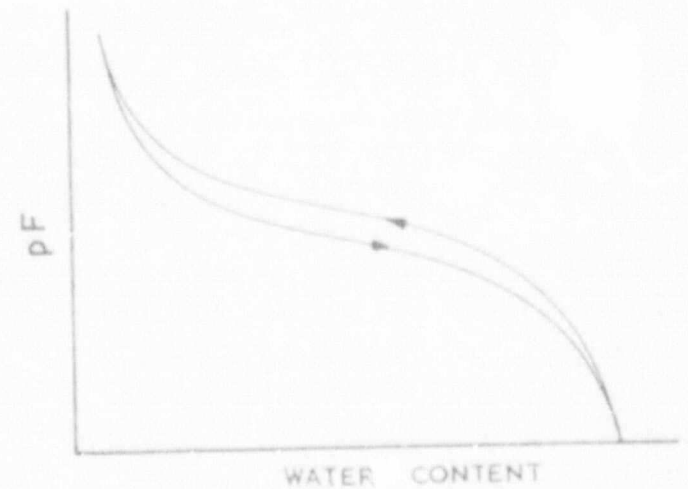


Fig. 2.0

pF : Moisture content curve for a granular material

In the time since Haines presented his theory to account for hysteresis in the pF:moisture content curve many other theories have been advanced. Carman (1953) discussed these theories in detail and concluded that the explanation offered by Haines is the most acceptable even in very fine grained porous materials. Carman attempted to separate the effects of capillary forces and adsorption forces. His results indicate that, insofar as the characteristics of the pF:moisture content curve is concerned, the two effects are synonymous even at high pF values. Aitchison (1960) demonstrated the validity of adopting a simple capillary model approach, similar to that used by Haines, for explaining moisture content : moisture stress relationships.

2.7 The physical condition of the air phase in a partly saturated soil

Free air may exist in two distinct conditions in a partly saturated soil; (i) as occulted bubbles and (ii) as continuous voids open to the atmosphere. Air filled voids can arise in a number of ways, the principle ones being :

(i) Due to a reduction in the pressure in the pore water. This can give rise to both conditions mentioned above. A reduction of the pressure in the pore water will cause the dissolved air to come out of solution and form occulted bubbles in the water. At a limiting soil moisture tension, determined by the dimensions of the soil pores, drainage occurs and the water is replaced by air at atmospheric pressure. The process just described can be represented diagrammatically as in figures 2.1(a), (b) and (c).

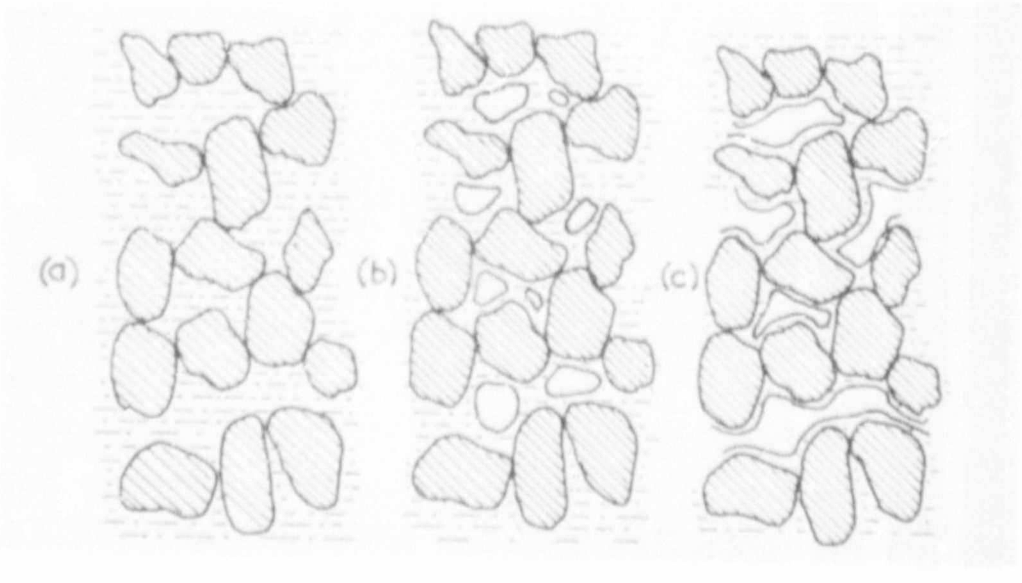


Fig. 2.1

Air voids arising from the reduction in the pressure in the pore water

(ii) Entrapment of air during compaction. This air can be in either of the above two conditions.

(iii) Occulted air bubbles can form as a result of wetting up of the soil either due to an external supply of water or due to a reduction in the volume of voids during compression.

That air can exist in the two conditions described above cannot be doubted. The question is whether air can exist in the form of occulted bubbles for an indefinite period. The importance of this question lies in the fact that occulted bubbles must be in pressure equilibrium with the surrounding water whereas continuous air filled voids remain at atmospheric pressure regardless of changes in pressure in the pore water.

2.8 A review of the literature on the air phase.

Comparatively little work has been done on the physics of the air phase in soil and what little work has been done is mainly in connection with agricultural soils.

Buckingham (1904) studied the diffusion of air and CO_2 through soils and the results of his tests, as reported by Aitchison, show that all the air voids in the soils studied (sands to heavy clays) were in contact with the atmosphere. Zimmerman (1936) has shown that air is entrapped in the pores of soil when the soil is wetted by capillarity from below. Similarly Smith and Browning (1942) have shown that natural soil cores, when wetted under laboratory conditions, are not completely saturated but contain an appreciable amount of air. J.E. Christiansen (1944) has shown that occulted air reduces the permeability of the soil quite considerably. To the Author's knowledge no work has been done by Agriculturalists on the stability of this air.

Rehbinder (1940) dealt with the problem of adhesion of air bubbles to solid surfaces. He showed that if bubbles are present in the adsorbed layers surrounding the surface they are always separated from the solid surface by a thin film of water. Rehbinder did not deal with the stability of bubbles and no conclusions can be drawn from his work regarding the stable size and rate of solution of bubbles in water.

Hilf (1956) considered the stability of single bubbles in relation to the physics of solution of air in water. He deduced that bubbles are inherently unstable in water. However he only considered the air pressure in the bubble and neglected the effects of water vapour pressure. Moreover he gave no indication of the rate of solution of bubbles.

Blight (1961) observed the stability of single bubbles in air-saturated water under a microscope. He found in every case that the bubble radius decreased with time until the bubble finally vanished. Blight also studied the solution of air bubbles in a sealed compressible container. He found that provided the degree of saturation was above some critical value the bubbles dissolved completely. However for degrees of saturation below this critical value the compressibility of the container was too low to allow the bubbles to dissolve completely. Figure 2.2 shows some curves of bubble radius against log time obtained by Blight.

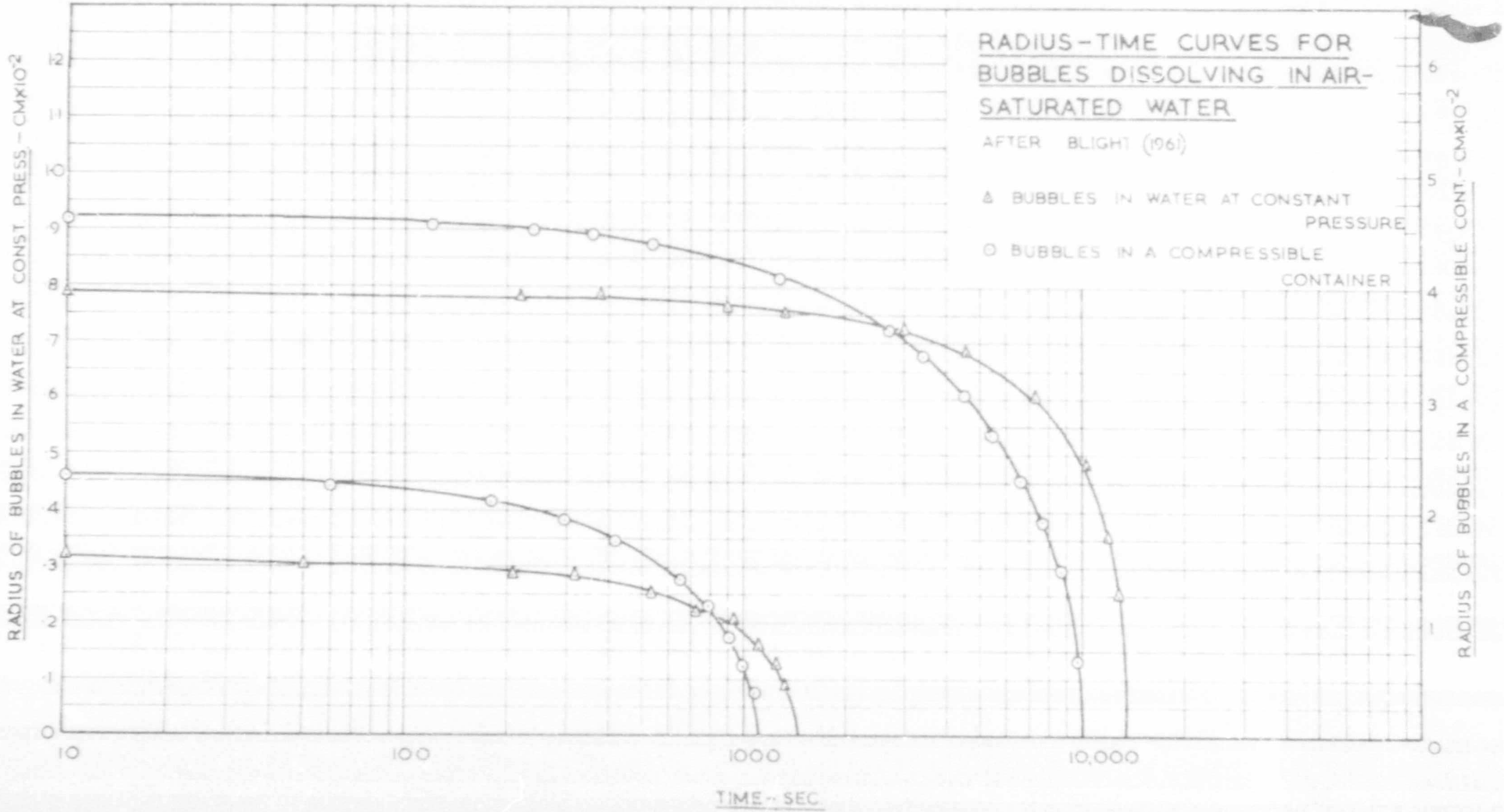
2.9 The physics of gas solution in a liquid

In 1803 Henry formulated a law to account for the solution of gasses in liquids. Henry's law states :

'The mass of gas dissolved by a given volume of liquid at a constant temperature is proportional to the pressure of the gas.'

i.e. $m = K_h . p$ . . . . (2.6)

where m is the mass of gas dissolved in the liquid
p is the pressure of the gas in contact with the liquid.



RADIUS-TIME CURVES FOR BUBBLES DISSOLVING IN AIR-SATURATED WATER

AFTER BLIGHT (1961)

FIG. 2.2

K_h is Henry's[*] constant in atmosphere per mol of gas.

If unit volume of liquid is considered, n can be replaced by the molecular fraction or concentration of dissolved gas c.

$$\text{i.e.} \quad c = \frac{\text{no. of molecules of gas}}{\text{no. of molecules of liquid + no. of molecules of gas.}}$$

Hence from equation (2.6)

$$p = c/K_h \quad \dots\dots\dots\dots\dots \quad (2.7)$$

Consider now a mixture of air and water vapour in contact with a water surface. The total pressure in the gas phase is p_t and the partial pressure of the air is p_a. The law of partial pressures states that the pressure exerted by a mixture of gasses (or vapours) is equal to the sum of the pressures which each would separately exert if it alone occupied the space filled by the mixture. If c_a is the molecular fraction or concentration of air in the air-water vapour mixture it can be shown that

$$p_a = c_a.p_t \quad \dots\dots\dots\dots\dots \quad (2.8)$$

If c_{aw} is the concentration of air dissolved in the water then from equations (2.7) and (2.8)

$$c_a.p_t = p_a = c_{aw}/K_h \quad \dots\dots\dots \quad (2.9)$$

This equation governs the relation between the partial pressure of the air, the quantity of air dissolved in water which is in contact with it and the vapour pressure of the liquid.

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[*] K_h is not to be confused with H, the well known Henry's coefficient of solubility. The relationship between K_h and H is derived in appendix C.

2.10 The stability of air bubbles in water

Consider a volume of water V_w in contact with the atmosphere. If the water is saturated with air at atmospheric pressure then, from equation (2.9), the concentration of dissolved air is

$$c_{awi} = p_a \cdot K_h$$

where p_a is the atmospheric pressure.

A bubble of radius R is injected into the water. At time zero the pressure in the bubble is p_b and is given by

$$p_b = p_a + p_w + \frac{2T}{R} \quad \ldots\ldots\ldots\ldots \quad (2.10)$$

where p_w is the pressure in the water surrounding the bubble measured with respect to atmospheric pressure, and T is the surface tension of water.

p_b is composed of two pressures; the partial pressure of the air in the bubble p_{ba} and the partial pressure of the water vapour p_{bw}. Therefore equation (2.10) can be written

$$p_{ba} + p_{bw} = p_a + p_w + \frac{2T}{R} \quad \ldots\ldots\ldots \quad (2.11)$$

The saturated vapour pressure of a liquid is a function of the temperature only. If, therefore, the temperature can be assumed to be constant then p_{bw} is constant.

Consider the stability of the bubble at constant temperature for the following three conditions :

(i) p_{ba} greater than p_a. If p_{ba} is greater than p_a and if the water at the bubble-water interface is assumed to be saturated with air at the pressure in the bubble then the concentration of dissolved air at time zero is $c_{awo} = p_{ba} \cdot K_h$. Since p_{ba} is greater than p_a, c_{awo} is greater than c_{awi} and a concentration gradient is established between the bubble interface and the surrounding water. Air will therefore migrate from the bubble to the surrounding water which in turn becomes super saturated with respect to the free air surface at atmospheric pressure p_a. As the bubble looses air to the water it becomes smaller, i.e. r decreases. Since p_{bw}, p_a and p_w in equ. (2.11) are constant p_{ba} must therefore increase. It can be seen that as the bubble dissolves the concentration gradient increases with time and the bubble must eventually dissolve completely.

(ii) p_{ba} less than p_a. In this case the concentration c_{awo} at the bubble interface is less than c_{awi}. The bubble will therefore take up air from the water. Following the lines of the previous argument, this process will give rise to a concentration gradient, apposite in sign to the previous case, which also increases numerically with time. The ultimate size of the bubble will be controlled by the size of the container holding the water.

(iii) p_{ba} equal to p_a. In this case c_{awo} is equal to c_{awi}. There will therefore be no tendency for air to migrate into or out of the bubble and the bubble size will not change. However, the system is in unstable equilibrium and the slightest fluctuation in atmospheric pressure or temperature will cause the bubble to become unstable.

To summarise it can be seen that when the partial pressure of the air inside the bubble is greater than atmospheric pressure the bubble will dissolve. If, however, the partial pressure of the air is less than atmospheric pressure the bubble will expand until it is held by the boundaries of the container or it 'bursts' into the atmosphere.

It is useful to examine the physical mechanisms by which the conditions of $p_{ab} > p_a$ and $p_{ab} < p_a$ can arise. The case $p_{ab} > p_a$ would arise simply from the process of injecting a bubble into water saturated with air at a pressure p_a. This is because p_{ba} must be greater than $p_a + p_w$ if the bubble is to expand during the injection process (see equ. (2.11)). The case $p_{ab} < p_a$ could arise in the following way. A bubble of radius r is injected into water saturated with air at a pressure p_a. Equation (2.11) defines the pressure conditions within and around the bubble.

$$p_{ba} + p_{bw} = p_a + p_w + \frac{2T}{R}$$

It was shown previously that p_{ba} must be larger than p_a for an injected bubble. p_{bw} and p_a are constant but p_w can be altered by changing the head of water over the bubble. If p_w is suddenly decreased (see fig. 2.3) the bubble will expand and $\frac{2T}{R}$ will decrease. For equilibrium p_{ba} must also decrease and can become less than p_a if p_w is reduced sufficiently.

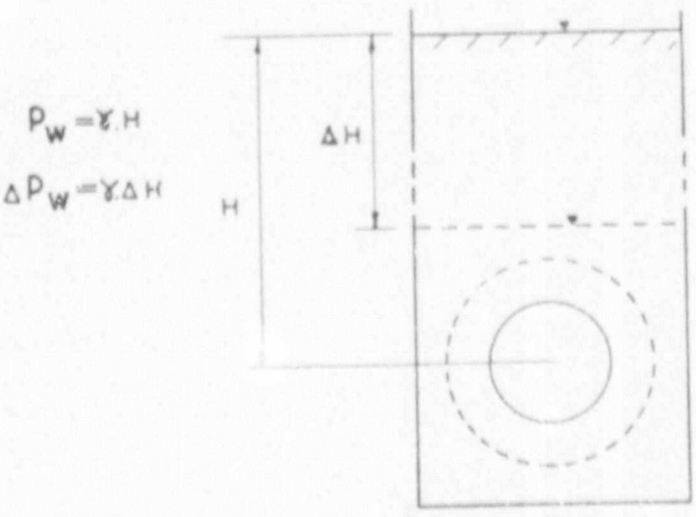

Fig. 2.3

2.11 The concept of a critical bubble size

p_w and p_a in equation (2.11) may be considered constant for conditions of constant temperature, and water with a free surface in contact with the atmosphere. Moreover $\frac{2T}{R}$ can never be negative. Those facts lead to the important conclusion that p_{ba} can only be equal to or less than p_a for certain values of r and p_w. If p_{ba} is set equal to p_a in equation (2.11) limiting condition at which the bubble will just not dissolve is imposed i.e. a condition of zero solution concentration gradient exists at the bubble interface. At 20°C p_{bw} is equal to 23.8 grm/cm^2 and T is equal to 0.075 grm/cm. Hence at the limiting condition of zero concentrati[illegible] adient at the bubble interface equation (2.11) c[illegible] en

$$3. \quad p_w + \frac{0.15}{R_c} \quad \ldots\ldots\ldots\ldots \quad (\;12)$$

where R_c is the critical bubble radius, at any specific water pressure, for the bubble just not to dissolve. For any bubble size less than R_c solution will occur.

Since $\frac{0.15}{R_c}$ can never be negative its limiting value is 0 ($R_c = \infty$). The value of p_w corresponding to $R_c = \infty$ is 23.8 grm/cm^2. For any value of p_w greater than 23.8cm. of water all bubbles must dissolve. For values of p_w less than 23.8 cm. of water only bubbles having radii less than R_c will dissolve. Some values of R_c for decreasing values of p_w are given in table 2.3.

Table 2.3

The relationship between R_c and p_w

P_w	pF	R_c
(cms H_2O)		cms x 10^{-4}
23.8		∞
20.0		3950.0
10.0		1090.0
0.0		630.0
-10.0	1	445.0
-10.0^2	2	121.0
-10.0^3	3	14.6
-10.0^4	4	1.5

2.12 The stability of air bubbles in a soil

From the work in the previous sections it can be deduced that bubbles contained in the pore water in a partly saturated soil will not dissolve if the partial pressure of the air in the bubble is less than or equal to the pressure in the atmosphere. For pressures in the pore water greater than approximately 0.35 p.s.i. this is impossible. It can therefore be stated that in any soil in which the pore pressure is positive all air bubbles present in the soil water must eventually dissolve completely.

Consider now the case of the bubble stability when the pressure in the pore water is less than atmospheric pressure. When p_w is negative the pressure in the pore water can be considered as resulting from the pressure difference across menisci which form at the boundaries of the soil or across pores inside the soil matrix. The case of the stability of a bubble occuring in a meniscus at a grain to grain contact point will be considered first. The mean radius of curvature of the meniscus is r (given by $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ where r_1 and r_2 are the principal radii of curvature). The pressure in the pore water is therefore

$$p_w = -\frac{2T}{r} = -\frac{0.15}{r}$$

Substituting in equation (2.12) gives

$$23.8 = -\frac{0.15}{r} + \frac{0.15}{R_c}$$

$$\text{or} \qquad R_c = \frac{r}{159.r + 1}$$

It can be seen that as r decreases (p" increases), R_c tends to the value of r as shown below

For $r = 0.015$ cms; $p'' = pF1$, $R_c = 0.0044$ cms.
For $r = 0.0015$ cms; $p'' = pF2$, $R_c = 0.0012$ cms.
For $r = 0.00015$ cms; $p'' = pF3$, $R_c = 0.000147$ cms.

Even for values of pressure deficiency as low as pF2, r and R_c are very nearly equal. Reference to the diagram in figure 2.4 will show that it is geometrically impossible for bubbles to form which have radii of curvature greater than the critical radius of curvature R_c. Hence any bubbles occurring in menisci at grain contact points must dissolve.

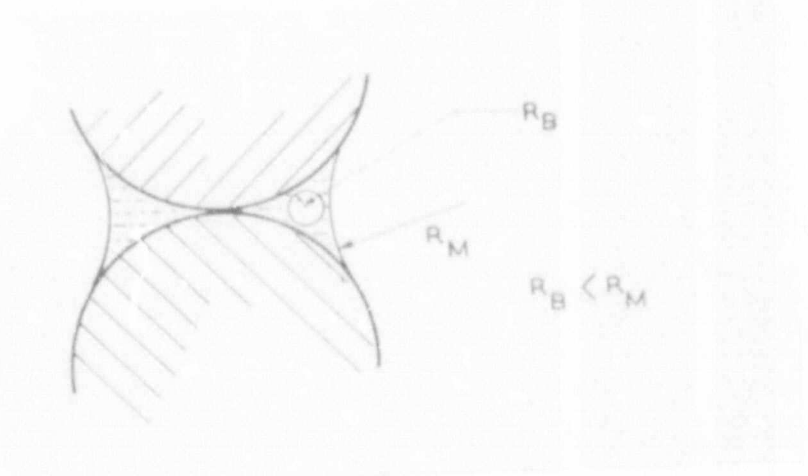

Fig. 2.4

Diagram showing that the radius of a bubble formed in a meniscus must be less than the radius of the meniscus

In cases where r forms at the boundaries of the soil it is possible for a bubble having a radius greater than r (and hence greater than R_c) to form. Figure 2.5 shows a number of soil grains enclosing a large void. Under suitable conditions a bubble having a radius larger than R_c could form in such a void. In this case the bubble would expand until it occupied the whole void. The chances that such a bubble would ultimately join up with a neighbouring air filled void which is in contact with the atmosphere are very good.

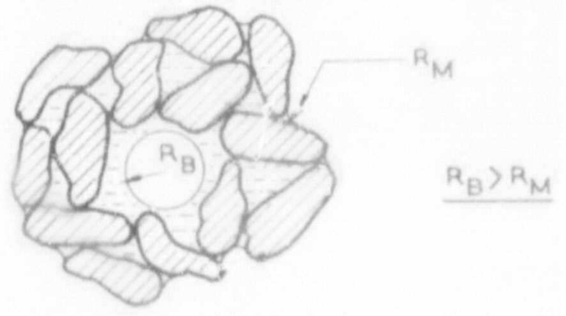

Fig. 2.5

Diagram showing a bubble having a radius which is larger than the radii of menisci formed at the soil boundaries.

In section 2.9 the mechanism whereby a bubble could be formed which would not dissolve was outlined. This mechanism involved the rapid lowering of the water pressure p_w. A condition whereby the pressure in the pore water in a soil is rapidly lowered exists during the sampling operation. The sampling operation would therefore appear to favour the formation of bubbles which do not dissolve. The theoretical discussion presented in this chapter indicates that, with the exception of the case just mentioned, enclosed air bubbles in a soil are unstable and must dissolve or join up with continuous air voids.

2.13 The rate of solution of air bubbles

Up till now the work on the air phase in a soil has been concerned mainly with the problem of whether occulted bubbles will dissolve or not. The rate of solution of these bubbles is another problem requiring investigation. Epstein and Plesset (1950) derived an equation for the solution of a bubble having an initial radius R_o which is placed in a liquid containing gas of concentration c_i.

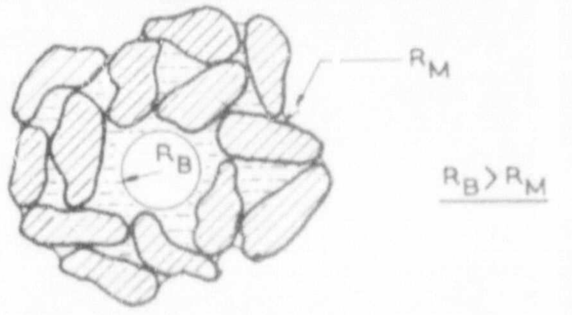

Fig. 2.5

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This equation is :

$$\frac{dR}{dt} = \frac{K(c_i - c_s)}{\rho(\infty) + \frac{2T}{3R}} \left[\frac{1}{R} + \frac{1}{\sqrt{\pi . K . t}} \right] \ldots\ldots \quad (2.13)$$

where R is the bubble radius at time t.

K is the diffusivity of the gas in the liquid.

c_i is the concentration at a distance from the bubble.

c_s is the concentration at the surface of the bubble.

T is the surface tension.

$\rho(\infty)$ is the density of air in the bubble at the same conditions of pressure and temperature with an interface of zero curvature.

For water initially air saturated at atmospheric pressure equation (2.13) was modified by Donald (1961) to the following form :

$$\frac{t}{t_{R=0}} = 1 - \frac{\frac{R_o . P_w}{2T} \left(\frac{R}{R_o}\right)^3 + \left(\frac{R}{R_o}\right)^2}{\left(1 + \frac{R_o . P_w}{2T}\right)} \ldots\ldots \quad (2.14)$$

This equation was shown by Donald to agree fairly well with results obtained by measuring the change of bubble radius with time under a microscope.

2.14 Experimental investigation of the solution of air entrapped in submerged porous materials.

The experimental work of Blight (1961) on the solution of single bubbles in water confirms the theoretical work presented in this chapter. However, there is no experimental evidence on the stability of enclosed air voids in soils and other porous bodies. A detailed experimental investigation of the phenomenon of air bubbles in soil water is outside the scope of this thesis. However, a few experiments have been carried out on rigid porous bodies in order to check, in a preliminary fashion, the validity of the theoretical conclusions reached in the preceding sections.

The experimental procedure was simply to immerse an oven dry porous body in water. The body was allowed to stand in this condition for a long period of time. At frequent intervals the body was weighed (in water). Once it had reached constant weight any air remaining in the system was driven off by boiling and its saturated weight in water was obtained. The difference between the weight of the body in water at a specific time and its final saturated weight in water gives a direct measure of the volume of air in the voids of the porous body. The total volume of voids was obtained by weighing the body in air in an oven dry condition and then in a fully saturated condition. From the above observations it is possible to calculate the volume of air in the system at any time and hence the degree of saturation at any time.

The porous materials used in the tests consisted of thin $1\frac{1}{4}$ inch diameter porous stones. Fig. 2.6 shows a set of $S_r : \sqrt{t}$ curves for the sintered glass discs. In Fig. 2.7 the same results are plotted using $\frac{V}{V_o}$ as the ordinate where V is the volume of occulted air at time t and V_o is the volume of occulted air at time zero. The curves show that the volume of air enclosed in the pores of the discs decreases with time. The volume of air dissolved appears to follow a straight line relationship with the square root of time over most of the solution process. At degrees of saturation of about 98% the rate of solution of the air suddenly decreases to almost zero. It would appear that at some very small bubble size solution ceases. This phenomenon is not displayed in the results obtained by Blight for single bubbles (see fig. 2.2). The size of bubble at which solution ceases is probably too small to be seen under a normal microscope but the effect would become apparent when a large number of bubbles are involved as was the case with the porous discs. It is apparent from figure 2.7 that the air enclosed in the discs with fine pores dissolved more rapidly than the air in the coarser discs. This result is in agreement with the results obtained by Blight in which small bubbles were found to dissolve more rapidly than large bubbles. In order to permit easier comparison with results obtained for the porous discs, the curves obtained by Blight (see figure 2.2) have been re-plotted in terms of volume against root time in figure 2.8. It can be seen that the curves for single bubbles are very similar to those obtained for air enclosed in a porous body. In figures 2.9 and 2.10 the curves given in figures 2.7 and 2.8 have been plotted non-dimensionally together with the theoretical relationship derived from equation (2.14). It can be [illegible] the experimental curves agree reasonably [illegible] the theory.

The above experiment was repeated using some ½" thick porous stones thereby enabling lower degrees of saturation to be obtained. Moreover the formation of large irregular shaped air voids (as occur in soils) were thought to be more likely to occur in a thick porous stone than in a thin disc. Curves of degree of saturation against root time for two similar porous stones are given in figure 2.11. The initial steep portion of the curves can be attributed to the relatively rapid solution of air in irregular shaped voids. An irregular shaped void will dissolve more rapidly than a spherical air bubble of the same volume because of its larger surface area. Once a sufficient volume of air has been dissolved the air bubbles become spherical and at this stage the graph of degree of saturation against root time becomes a straight line. As in the case of the sintered glass discs solution of the air ceased at approximately 98% saturation.

The results for the porous stones have been plotted non-dimensionally in figure 2.9 . It can be seen that the experimental curves deviate considerably from the theoretical curve. This deviation can be attributed mainly to the effects of irregular shaped air voids discussed previously.

The three most important conclusions to be drawn from the work in the air-phase described in this chapter are :

(i) Enclosed air bubbles in soil water are inherently unstable and will either dissolve or join up with neighbouring air filled voids which are connected to the atmosphere. A possible exception to this appears to be the case of a soil which has been subjected to a sudden decrease in pore-water pressure due (say) to sampling. In this instance there is a likelihood that air bubbles will form in the pore water and that they will continue to expand until they fill the voids containing them.

(ii) The rate of change of volume of the air voids is proportional to the square root of time. The smaller the initial size of the air voids the more rapid is the rate of solution.

(iii) The solution of air from the pores of a submerged soil appears to cease at degrees of saturation of between 99 and 99.5 percent. Under certain circumstances the effect of such a small quantity of air in the compressibility of the pore fluid might be significant. This would apply particularly to cases in which complete incompressibility of the pore fluid has been assumed.

2.15 Categories of unsaturation in a soil.

This chapter has dealt with some of the properties of the water and air phases in a partly saturated soil. It is appropriate, therefore, to conclude the chapter by defining the categories of unsaturation in a soil.

Aitchison (1956) defines three major categories of partly saturated soil:

(i) Quasi-saturated
(ii) Partially saturated
(iii) Unsaturated.

A description of each of these categories is given below.

(i) Quasi-saturated. In a quasi-saturated soil all the pores are filled with water but the pressure in the pore water is less than atmospheric pressure. In sands the quasi-saturated state can only extend over a small range of p'' while in very compressible clays the range can be very large.

(ii) Partially saturated : A partially saturated soil is one having a degree of saturation less than one but the pore water pressure greater than or equal to atmosphere. The air phase in a partially saturated soil is mainly occulted and hence from the work described in the previous sections such soils can only exist in a transitory state. Ultimately all the air must either dissolve out, in which case the soil becomes fully saturated or quasi-saturated, or the air will form continuous voids in which case the soils become unsaturated.

(iii) Unsaturated : An unsaturated soil is one in which the degree of saturation is less than one and the pressure deficiency is greater than zero. In an unsaturated soil the air can be present in the form of occulted bubbles or free air connected to the atmosphere or both.

Aitchison defines a partly saturated soil as one which - at its specific condition of external stresses - can absorb additional water (if supplied air-free at atmospheric pressure). This definition does not include a partially saturated soil as such a soil will not absorb free water at atmospheric pressure. The definition given by Aitchison has therefore been modified so as to include this category. For the purposes of this thesis the following definition will be used to define a partly saturated soil :

'A soil is defined as partly saturated if it possesses at least one of the following two properties :

(i) The degree of saturation less than one.
(ii) A pressure deficiency greater than zero.

Both these properties being measured under specific conditions of applied stress.'

This definition takes account of the partially saturated soil and also the case of a soil subjected to a raised atmosphere in the laboratory. It is important that the soil should be considered at a specific condition of applied stress as almost any soil can be made to become partly saturated if the external stresses are altered radically enough.

Author Burland J B
Name of thesis The Concept Of Effective Stress In Partly Saturated Soils. 1961

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