STUDENTS' UNDERSTANDING OF THE LIMIT CONCEPT IN A FIRST YEAR CALCULUS COURSE

by

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DECLARATION

I declare that this research report is my own. It is being submitted in partial fulfillment for the degree of M.S.C at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University, nor has it been prepared under the aegis or with the assistance of any other body or origination or person outside the University of the Witwatersrand, Johannesburg.

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19th day of May 1998

ABSTRACT

This research report focuses on students' understandings of the limit concept in a first year Calculus course. The study was done in two phases. A concept test based on limits in Calculus was given in the first phase to all first year Mathematics students (mostly men) from Medunsa. The students were divided into high, middle and low achievers on the basis of their performance in this test.

In the second phase three-students were randomly selected from each of the three groups. These subjects were individually interviewed. The interview was devised to obtain more information about the conceptions that students held and the reasoning they used. Students were asked about their understanding of the limit concept and the methods they use to solve problems on limits.

Data from the transcripts, capturing essential and relevant bits of student's responses to each question, was collected. The data was analyzed and misconceptions identified. It was found that when solving problems on limits students only think of the manipulative aspect and not of the limit concept per se. They have in mind that the limit concept is about mechanical substitution. Also instead of saying that the function is not defined at a point, they talk of a limit not being defined at a point.

Misconceptions can be remedied if educators are aware of them. The findings of the study are thus important for designing appropriate teaching strategies not only for Medunsa students but more generally.

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Chapter 1

BACKGROUND

1.1 The origins of this research

For generations, mathematics teachers have voiced concern about students' inability to understand certain concepts in mathematics. As a result there is much practical interest in mathematical understanding and as such teachers are urged to emphasize teaching for understanding. Such a view of understanding as a state or complex of states is useful in that it allows us to think of curriculum materials as providing opportunities for students to attain such states or providing criteria against which to assess students' mathematical understanding.

Conference proceedings and the literature generally also show considerable interest in teaching which results in learning mathematics with understanding. What is mathematical understanding? According to the constructivist, understanding is a continuous process of organising one's knowledge structures (von Glasersfeld,1989). For this to be achieved, teachers need to have a sound knowledge of the subject matter. The limit concept is not an exception in this regard.

Mathematics educators today are concerned with the way Mathematics is learned and taught. The call has been for a change to be made in the way teachers teach. The question I often ask myself is whether students understand what is been taught because students construct their own meaning.

The Medunsa first year Mathematics course is divided into two main sections, Calculus and Algebra. The Calculus section is structured in such a way that students do functions, limits, and continuity, differential calculus, special functions, indefinite integrals, applications of integral calculus and plane analytical geometry.

As a lecturer at this institution, I have discovered that there are several problems that students encounter with the limit concept. I found that many students have problems with understanding the definition of the limit and cannot state it correctly. Application of the definition of the limit is most problematic when students solve problems. I have found the meaning attached to the ε, δ definition to be considerably confusing to students.

In this study I will relate the misconceptions found to their possible causes (e.g., language) but this will not be my major research thrust.

The question that have concerned me, relate to what misconceptions students bring to class from their matric studies, as far as the limit concept is concerned? These misconceptions could affect learning in a negative way (Olivier, 1988).

According to Booth (1984b) misconceptions are highly persistent and resistant to change through instruction. They are maintained by their ability to distort or reject incompatible information and by the support from other concepts in the student's conceptual structure. They are further seen as emerging from some interaction between experience and other existing concepts the student has (perhaps themselves misconceptions).

Booth attributes the persistence of these misconceptions to maturation-linked cognitive factors, i.e., that understanding depends on the attainment of a certain general developmental cognitive level.

Where do these misconceptions about limits originate? I am depressed by the poor performance of my students in mathematics more especially their understanding of and performance in limits.

There is a special course in mathematics offered at this institution, but students still have difficulty in understanding the limit concept. The special course is meant for those students who did not get a good symbol in Mathematics (Matric).

This is like a bridging course from Matric to University. There are a small number of students registering for mathematics and from experience the number is reducing by the year (see figure 1.1). The reason might be that students often find Mathematics to be difficult.

This study is thus important because I will be addressing some of the difficulties experienced by students in a crucial area of mathematics for further learning.

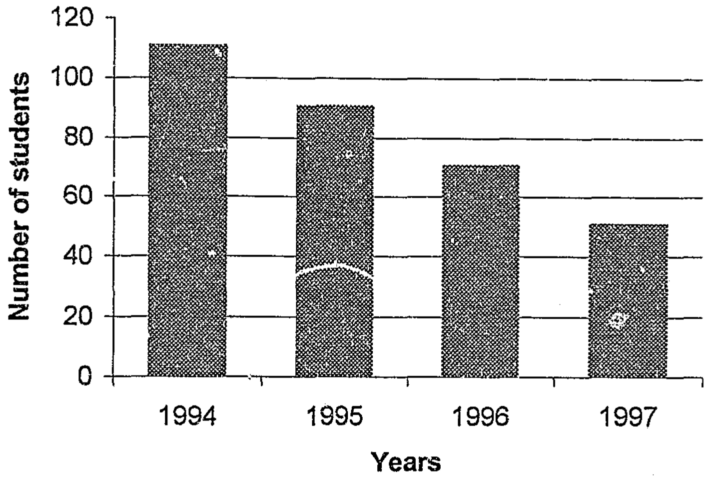

Figure 1.1 Students' registration for Mathematics 1

1.2 Misconceptions

The current study attempts to unearth relevant misconceptions and how these affect understanding of the limit concept in Calculus.

Smith (1992) added that in spite of their flaws, misconceptions indicate a measure of genuine mathematical insight and are thoughtful efforts to make mathematical sense. From this, one can see that it is possible that students can learn new concepts without giving up their misconceived ideas.

Misconceptions are things that "interfere" with learning and students must "give them up" for proper learning to take place (Smith, 1992:22). This is important for my study because if misconceptions are unearthed, a programme to assist students to "give them up" can be developed.

According to Vergnaud(1990) studying students conceptions is very important to achieve two major objectives: on the one hand, to identify difficulties and discontinuities in students reasoning process while approaching some mathematical concepts and on the other hand, to determine in which way students previous conceptions can be used to generate new conceptions. Studying misconceptions is very important in that this will enable us to know what previous conceptions students have and how this will affect future learning.

Clement (1981) refers to misconceptions as "conceptual stumbling blocks", cognitive processes responsible for errors in problem solving in mathematics. According to Perkins and Simmons (1988) misconceptions are conceptions that are in conflict with accepted meanings in science mathematics and programming.

Misconceptions can be a stumbling block to underst nding of scientific concepts. Misconceptions do however form part of students' conceptual structures that will interact with new concepts and influence learning.

1.3 Misconceptions in Calculus

There are general misconceptions in Mathematics more specifically the limit concept in Calculus. According to Monaghan (1986) the limit concept is known to cause difficulty in learning. There are a number of studies that reveal that students conceive the notion of $\lim_{x \to a} f(x) = \ell$ not as a static concept but as a dynamic process of "getting close to a fixed value often with the implication of never reaching the limit" (Cornu, 1992).

The following questions are important as far as misconceptions in Calculus are concerned but more specifically in the limit concept:

1) What kind of misconceptions do students bring to class as far as the limit concept is concerned?

According to Williams (1992) there are three epistemological obstacles regarding misconceptions in the limit concept. Firstly there is confusion over whether a limit is actually reached. This is further supported by Tall (1992) when he talks about the dynamism of a limit. Secondly there is confusion over whether the limit can be surpassed. Students find it difficult to relate continuity with the limit concept and thirdly there is confusion regarding the static character of a limit.

2) What are the sources of misconceptions in Calculus, in prior knowledge learned from either the classroom or by interaction with the physical and social world. Mamona-Downs(1990) gave Greek and English students the following question to determine the sources of misconceptions. The following question was asked:

Find the following limit $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$?

Answers of the following types were obtained:

$$\lim_{x\to 2}\frac{x^5-2^5}{x-2}=\frac{2^5-2^5}{2-2}=\frac{0}{0}=1;0;\infty.?$$

Such responses were found in both groups. They brought to the surface pupils' difficulties with handling zero.

Once such misconceptions are identified, the knowledge gained can be used as a probe to help in the development of meaningful learning to overcome the misconceptions.

1.4 The problem area

The current study attempts to unearth relevant misconceptions and how these affect the understanding of the limit concept in Calculus. I will further attempt to determine the degree of misconceptions that students have at first year level. Students often find it very difficult to understand the limit concept.

The limit concept is fundamental to understanding the Calculus. Mathematics is like building a house that needs a solid foundation to stand and the limit is such a foundation for Calculus which will enable the students to have an easy passage in Calculus.

1.5 Aims of the research

The research discussed in this report has two underlying aims:

1) to identify misconceptions that Medunsa first year Mathematics students have, after having been taught the limit section.

2) to find out whether particular students have both the correct concept and any related misconception.

1.6 Outline of the research

The purpose of this study was to unearth relevant misconceptions and how these affect the understanding of the limit concept in Calculus amongst Medunsa first year students. The students will be given a test on limits in Calculus. Students will be divided into three groups according to their performance on this test.

Random sampling will be done in each group. Nine students were selected from the three groups. The nine students were all subjected to an interview.

Thus the data will be gathered by means of tests on limits and the transcripts from interviews conducted with selected students. These data will then be analyzed to reveal the underlying conceptions.

CHAPTER 2

LITERATURE REVIEW

2.1 Misconceptions

Misconceptions exist in varying degrees in all disciplines (Brown, 1992). This also includes mathematics and it involves understanding of mathematical concepts and their applications in solving problems. The current study attempts to determine the degree of misconceptions that students have at first year level.

Misconceptions are conceptual difficulties that students experience, which may hinder the development and understanding of scientific concepts e.g., the limit concept in Calculus. Clement (1981) refers to misconceptions as " conceptual stumbling blocks", cognitive processes responsible for errors in problem solving in mathematics.

Misconceptions can be a stumbling block to understanding of scientific concepts but cannot at all times be a stumbling block to learning because they can form part of the student's conceptual structure that will interact with new concepts and influence learning. Olivier(1988) who says that from a constructivist point of view, students' misconceptions are never arbitrary or altogether unreasonable further supports this.

Misconceptions should be viewed as thoughtful efforts to make mathematical sense. They represent students' attempt to cope with new mathematics concepts. According to Perkins and Simmons (1988) misconceptions are conceptions that are in conflict with accepted meanings in science, mathematics and programming. The conflicts arise because of different meanings that pupils have constructed previously,i.e., their own understanding.

Do students construct such knowledge with understanding? If there is construction without meaning, students will create their own "box" in the mind and commit such knowledge to memory (rote learning).

The new ideas that students have will not be linked to the previous knowledge. Rote-learning although good at times, can lead to mistakes, which can result in misconceptions. Students do not easily accommodate new ideas but assimilate the new ideas into existing schemas. The new ideas can be distorted to be "like" the previous ideas.

According to Olivier (1988), from a constructivists point of view, learning leads to changes in our conceptual schemas. Learning is not as for the behaviourist a matter of adding, of stockpiling new concepts to existing ones. The character of existing schemas will determine what the child learns from experience or information and how it is understood.

According to Bodner (1986) learners construct understanding. They do not simply mirror and reflect what they are told or what they read. Learners look for meaning and will try to find regularity and order in the events of the world even in the absence of full or complete information.

The question is, during such construction of new knowledge do students construct with understanding? If there is construction with understanding this will involve the interaction of their existing ideas with new ones.

Misconceptions can results because of learning or instruction in mathematics. What are termed misconceptions relate to many aspect of successful learning. From a constructivist perspective misconceptions are crucially important to learning and teaching because misconceptions form part of students' conceptual structures that will interact with new concepts and influence new learning mostly in a negative way, because misconceptions generate errors. This is further supported by Fischbein et al. (1988) when they say that students can learn the "correct" mathematical concepts without "giving up" their misconceived versions.

Students do construct knowledge from misconceptions and depending how serious the original misconceptions are, the new knowledge will have varying degrees of evidence of continuing misconceptions.

This does not mean that attempts should not be made to unearth misconceptions and remedy the situation. Constructivism should focus on how well the teacher can utilise the misconceptions so that they can be of value.

Prior knowledge that students have, plays a role in students' understanding of concepts. The notion that prior knowledge is important shows that in learning, students build on their previous knowledge. Confrey (1990) further says that the focus on pre-conceptions represents a basic rejection of a tabula-rasa approach to learning.

The existence of misconceptions supports the notion that students are not `empty vessels' as claimed by `behaviourists'. Davies and Vinner (1986) reject the `empty vessel theory' they offer a view of naïve conceptualisation as prior knowledge that may conflict with the new ideas the teacher hopes to teach. They also indicate their epistemological interest by using situations in which a student may hold two competing ideas and will `retrieve' the erroneous response rather than the new one. They studied this by examining students' understanding of sequences after the students had demonstrated competence with the material during the school year.

Students come to class having prior knowledge, which does affect future learning of concepts.

2.1.1 **Sources of Misconceptions:**

The source of misconceptions lies in retrieving the wrong schema and not recognising the retrieval error. For example in my teaching experience I have realized that certain misconceptions acquired by students in earlier mathematical learning do not necessarily get removed by the teaching of later concepts.

Teaching the limit concept in first year does not guarantee a remediation of underlying function concepts. As for remediating misconceptions, Davies (1984) advocates making sure that pupils are aware of both schemas and of the likelihood of incorrect choice. According to Solomon (1983:129) `meanings which are in daily use cannot be obliterated by science lessons, however convincingly presented. Even when the concepts and theories of science have been learnt, the older meanings, and loose explications of the life-world, will still linger on. This implies that our students will acquire, through their instruction in science, a second domain of knowledge, which is radically different from the first but co-existent with it. Under these circumstances we shall want to know if they are aware of these two competing sets of meanings and more importantly, how they decide which one to use during problem-solving exercises'.

Davies and Vinner (1986) argue that students' presentation of an old (and incorrect) idea cannot be taken as evidence that the student does not know the correct idea and that in many cases the students know both but have retrieved the old idea. This clearly shows that if the students have conflicting ideas and do not know which idea to retrieve, the tendency might be for the old idea to be retrieved.

This is an aspect, which I feel is important for a full understanding of student thought patterns.

2.2 Misconceptions in Calculus

The historical development of the limit concept in Calculus can also be a rich source for describing potential misconceptions. Misconceptions can result because of the learning or instruction about the limit concept. Davies and Vinner(1986) suggest that there are seemingly unavoidable misconception stages with the notion of a limit. In that research it was reported that some of the usual misconceptions that students have, result from the influence of language (Tall,1992), since language and thought are closely intertwined. The role of language in the construction and maintenance of misconceptions has received considerable attention in misconceptions research in science education. Pines (1985:108) describe the importance of language as follows: `A word is like a conceptual handle, enabling one to hold on to the concept and manipulate it'.

The way a student communicates about a mathematical concept and his understanding of it are mutually interdependent. Language is a tool and a sign which can play a role in development of thought (Vygotsky,1986). Tools cannot be ignored but can be improved with time.

Some of the limitations as stated by Confrey (1990), are that Vygotsky focused on the privilege of language to the detriment of other forms of interaction and inquiry, e.g., giving of definitions in the teaching of mathematics has proven to be inadequate.

Adler (1995) shows that even if the language spoken is not adequately mastered students can understand the

Mathematics that they are being taught. The type of activities that were used here, were based on triangles and students could derive meaning from these activities even when the language spoken was not their mother-tongue.

Although we need a tool like language to communicate mathematics, students who are not fluent in English can perform much better than those who are fluent in English when English is the language of instruction but not the mother tongue. Numerous studies have shown that even with those students whose first language is English misunderstanding words commonly used in mathematics can be a source of misconceptions.

There are some words that intrude into mathematics, which result in misconceptions. In addition to the words there are ideas that these words conjure up, which have their origins in earlier experiences. In my research I will thus look into the possibility of language as being a cause of any misconception exposed. I will investigate whether students hold both a misconception and a related proper understanding.

According to Vergnaud (1990) studying students' reasoning processes while approaching some mathematical concept helps to determine in which way students' previous conceptions can be used to generate new conceptions.

Students have definite ideas about mathematical concepts prior to classroom instruction (naive conceptions) but their ideas are often different from accepted forms of knowledge.

Tall (1980) describes the word "limit" as having many connotations in everyday life which are at variance with the mathematical idea. An everyday limit is often something that cannot or should not be passed such as a "speed limit". The terminology associated with mathematical limiting processes includes phrases such as"tends to", " approaches" or "gets close to".

Smith (1992) argues that in spite of their flaws, misconceptions indicate a measure of genuine mathematical insight and are thoughtful efforts to make mathematical sense. This does not mean that one should not unearth the misconceptions that students have because these might affect new learning in a negative way (Olivier, 1988). Investigating misconceptions will enable us to know what previous conceptions students have and how this will affect future learning.

Misconceptions arise out of prior knowledge learned either from classroom instruction or the social world and as a result influence learning. Misconceptions can be a stumbling block to understanding of scientific concepts (e.g., the limit concept) but are not necessarily a stumbling block to learning. Misconceptions form part of a student conceptual structure which will interact with new experiences and concepts and so be influenced in the learning process.

There are misconceptions of mathematics generally, and more specifically of the limit concept in Calculus. According to Monaghan (1986), the limit concept is known to cause difficulty in learning. The students that I teach at first year level, normally have difficulties in understanding and handling the limit concept.

There are also a number of studies that reveal that students conceive the notion of $\lim_{x \to a} f(x) = \ell$ not as a static concept but as a dynamic process of "getting close to" a fixed value, often with the implication of "never reaching" the limit (Cornu,1992). I regard the following question as very important in so as far as misconceptions of the limit concept are concerned:

What kind of misconceptions do students bring to class, which are pertinent to the limit concept?

According to Williams (1992) there are three epistemological obstacles regarding misconceptions in the limit concept. Firstly there is confusion over whether a limit is actually reached. This is further supported by Tall (1992) when he talks about the dynamism of a limit. This confusion often prevails in the classroom situation that the researcher handles. Students think of a limit as something that is not reachable.

Secondly there is also confusion over whether the limit can be surpassed. Students find it difficult to relate continuity to the limit concept. There is also confusion regarding the static character of a limit.

Once the misconceptions are identified, they can be used as a probe to help in the development of meaningful learning, which will be a follow up to this study.

Misconceptions represent students' attempt to cope with the development of mathematical concepts on the basis of new experiences. According to Smith(1992) misconceptions provide the main raw material for building constructivist theories of the learning of mathematics. This is further supported by Ausubel(1968) who argues that students construct their own meaning. From this it follows that it will depend on a individual what constructions are made in his mind. According to Bodner(1986) learners construct understanding. They do not simply mirror and reflect what they read.

Learners look for meaning and will try to find regularity and order in the events of the world even in the absence of full or complete information. The construction is influenced by the teacher but the teacher cannot control it because the teacher does not have control over the previous knowledge that students have nor of the construction process in individual student's minds.

Students do not easily accommodate new ideas but assimilate the new ideas into existing schemas. The new ideas should be distorted to be "like" the previous ideas. This is supported by Vygostky (1979) when he says that learning encountered in school always has a previous history.

The limit concept is fundamental to mathematical thinking however it proves to be difficult for many students to use as a basis of their thinking. The limit concept should be approached in such a way that previous knowledge that students have is taken into consideration. Limits occur in many different mathematical contexts including the limit of a series and in connection with functions.

According to Tall and Vinner (1981) the formal definition of the limit $\lim_{x \to a} f(x) = \ell$, is very difficult for students to understand because of the epsilon(ε), delta(δ) approach. This was confirmed when 70 highly competent first year university mathematics students were requested to write down a definition of $\lim_{x \to a} f(x) = \ell$ if they knew one.

The students had just passed A-level examinations and would be expected to have been given a dynamic definition (f(x) gets close to ℓ as x gets close to a although some might have been shown the formal definition. Those who replied did so as shown by the following table:

Students definition of the limit concept

Correct	Incorrect
Formal 4	14
Dynamic 27	4

The majority of those who recalled the (easier) dynamic definition could state it correctly, whereas the majority of those who chose to give the formal definition were not able to recall it in a satisfactory way, mistaking it in various ways such as $|f(x)-c|\le\varepsilon$ for all positive values of ε with x

sufficiently close to a. As $x\to a$, $c-\varepsilon\le f(x)\le c+\varepsilon$

The formal definition of the limit is very difficult for students to state with understanding because of the epsilon(ε), delta(δ) approach. The majority of students when asked to give a formal definition were not able to recall it in a satisfactory way, mistaking it in various ways.

Students in some cases can state the definition correctly, more especially the ε,δ definition of the limit. What is important is whether the students understand the definition or not. The individual's method of thinking about mathematical concepts depends on more than just the form of words used in a definition.

According to Tall (1992) within mathematical activity, mathematical notions are not only used according to their formal definition but also through mental representations, which may differ for different students.

According to Mamona-Downs (1990) the concept limit has a special place in mathematics as being almost symbolic of the first cross-over from naive mathematics to rigour and as a result has attracted a fair amount of attention in educational research.

Mamona-Downs(1990) mentions among others

Sierpinska (1985) who focused her attention on the "epistemological obstacles" relative to the notion of limit. Mamona-Downs (1990) gave groups of Greek and English students the problem: Find the $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$ to determine what approach the students would use. In responding to this problem there were two main conceptual formations. One was exclusively used by the English and the other by the Greeks.

The following are representative answers from each group:

Representative English Answer

a) Let $x=2-\delta x$ then :

$(2-\delta x)^5-2^5 = 2^5-5.2^4\delta x+.....-2^5$

$= 80-80\delta x+....+(\delta x)^4$. as $\delta x \to 0$, the function

$\to 80$

b) Greek Answers

Representative example

$$= \lim_{x\to 2}\frac{[(x-2)(x^4+2x^3+4x^2+8x+16]}{(x-2)}$$

$=\lim_{x\to 2}(x^4+2x^3+4x^2+8x+16)$

$=80$

Substituting x by 2 is permitted here because the function is a polynomial, so continuous.

The following is an exception to the above two responses:

The $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$ is the derivative of the function $y=x^5$ at $x_o=2$

We know that $y'=5x^4$. So the limit is $5x_0^{\;4}=5.2^4=80$

In this study the Greeks gave answers similar to (b) because they have been exposed to the formal treatment. The L'Hospital rule was also used in certain instances.

The students were further asked to justify their responses. Those who used factorization and cancellation under the lim-sign gave the following typical explanations:

"We may divide x-2 into x^5-2^5 whenever x is not 2, it's true then, however close x is to 2, and so we can do the same in the limit". There were also answers of this type:

$$\lim_{x\to 2}\frac{x^5-2^2}{x-2}=0?1?0?$$

which were found in both groups and this brought to surface students' difficulties with handling zero.

Mamona-Downs(1990) asserts that the handling of zero has nothing to do with the limit concept. Zero becomes a problem for nearly every student some time in his (or her) mathematical career.

2.3 Remediation of misconceptions

Remediation of misconception is not part of this research project but will be the object of subsequent work.

Misconceptions can be remediated if we are aware of them which makes this study very important. Instruction can be constructed in such a way as to address the misconceptions by helping students to integrate new and previous knowledge. According to Smith (1992) misconceptions can be neutralized by organizing instruction so as to confront students misconceptions with accepted knowledge.

Davies(1984) suggested that separate conflicting "frames" should be created for students to distinguish between two competing frames, the main aim is for the students to recognise the retrieval error because many misconceptions lie in retrieving wrong frames. This will enable the students to be aware of both frames and the likelihood of an incorrect choice will be limited. When conflicting "frames" are created, students can be asked to explain two opposing statements and are required to choose the one most like their own view.

Nesher (1987) argued the importance of the teachers' role in designing learning environments which ensure that common misconceptions are exposed and dealt with. It is very important that the teacher should not view student errors in mathematics as an inability to cope with mathematics due to low intelligence.

Mistakes and errors that students make should be viewed as rational and meaningful efforts to cope with mathematics. If we want to account for students misconceptions we must look at students current schemas and how they interact with instruction and with experience.

Constructivism argues that we should focus on how well we can utilise the misconceptions so that they are of value to us. The million dollar question is "can we remove the misconceptions and replace them with acceptable knowledge?". From a constructivist perspective, misconceptions are crucially important to learning and teaching because they form part of the learners' conceptual structure that will interact with new conceptual structures.

New learning will thus be influenced, mostly in a negative way because misconceptions generate errors. To the constructivists, learning leads to changes in our schemas.

The only way to replace misconceptions is by constructing a new concept that appropriately explains experiences. Group discussions can see to it that misconceptions are exposed and the conflicts resolved by discussion. According to Bell and Bassford(1988) misconceptions need to be exposed and the conflicts resolved by discussion.

The discussions should be structured in such a way that there is mediation during such interactions. Brodie(1995) regards pupil-pupil interaction as important for learning to take place but this does not necessitate the need for a teacher to be removed from a classroom situation. Intervention of the teacher as mediator will be needed in this regard.

Brodie(1995) further asserts that if the teacher is removed from the classroom situation there will be no-one to deal with the misconceptions that students have. The teacher as somebody who is knowledgeable, will be able to address any misconceptions and flaws that exists during pupil-pupil interaction.

In order for accommodation to occur there must be some sense of dissatisfaction with the existing conceptual framework. There must be alternate conceptions which are both intelligible and initially plausible. The alternate conceptions must be seen as fruitful, useful or valuable for learning to take place.

2.4 Conclusion

Students' understanding of the limit concept in Calculus has been a subject of many investigations and this has resulted in some persistently reported misconceptions. From the literature one can see that this study is very important because of the contribution it will make to research done in this field. The study will enable the researcher to further do some remedial work after unearthing the misconceptions.

Chapter 3

METHODOLOGY AND DATA COLLECTION

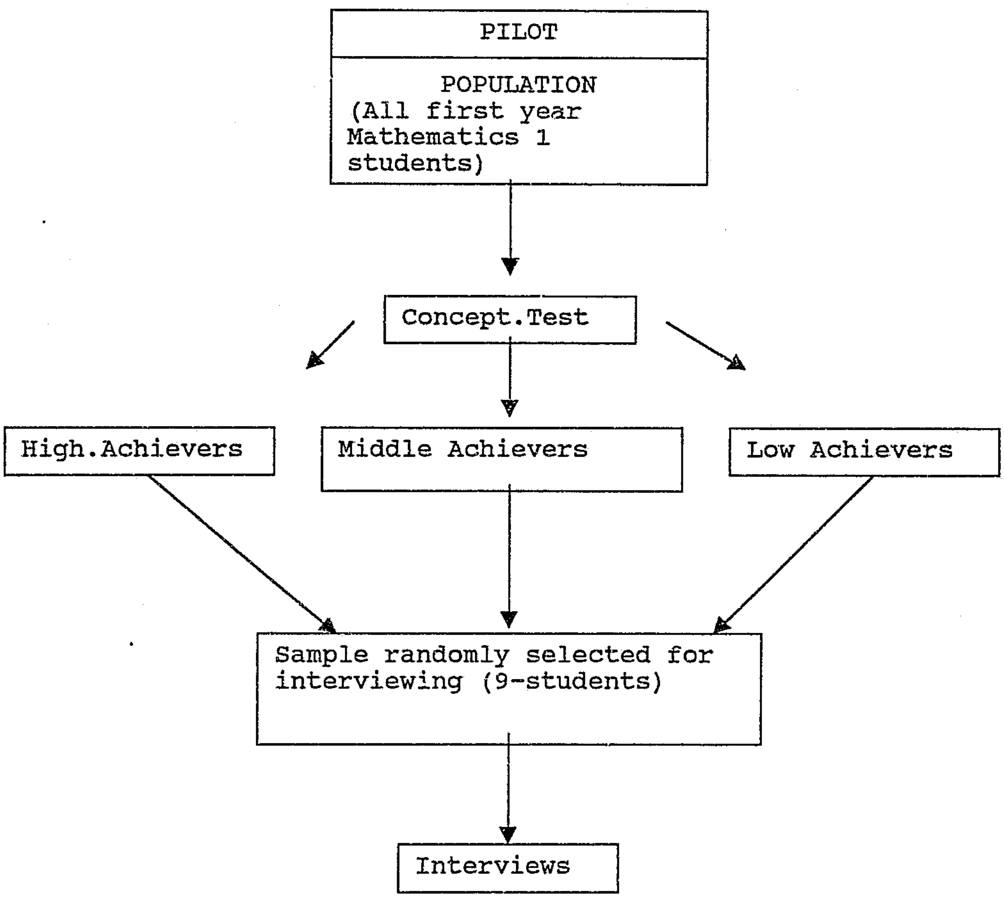

Figure 3.1 Research plan

3.1 Overview of methodology used

3.1.1 Research aims and questions

Aims:

1) To identify misconceptions that Medunsa first year Mathematics students have on limits after they have been taught the limit section.
2) To find out whether students have both the correct concept and a related misconception.

This can be reformulated into the following questions:
1) What misconceptions exist after students have been taught the limit concept?
2) Do students have a correct concept and a related misconception?

3.1.2 Structure of the Research

The structure of the research consists of two phases (see fig. 3.1). A pilot study was conducted among all the first year Mathematics 1 students. The concept test (see appendix A) based on the section dealing with limits in Calculus was given. Data was collected from transcripts of the interview. The data was analysed as seen from chapter 4.

3.1.3 Motivation

What influenced me to divide the research into two phases, was that in the first phase by giving the concept test I was able to divide students into high, middle and low achievers on the basis of their performance in this test. This enabled me to randomly select 9 subjects across all the three groups for interviews.

3.2 Sampling

3.2.1 The general sample

The subjects were all first year mathematics students (mostly men) from Medunsa. The choice of the subjects was influenced by the following considerations:

1) The university is sufficiently close to the researcher and there is easy accessibility of apparatus and a better place to conduct an interview.

2)The students had all passed Matric mathematics.

3)The subjects were taught by the researcher and always available for the research study.

The students were all enrolled for a BSc course, the duration of which is three years. Their ages range from 15 years to 35 years. The subjects were taught mathematics in English. The Medunsa first year mathematics course is divided into two main sections, Calculus and Algebra.

The two sections of Mathematics 1 course were presented simultaneously for the whole year. Calculus was presented twice in a week and Algebra once a week. The Calculus section is structured in the following sequence: functions; limits and continuity; differential calculus; special functions; indefinite integrals; applications of differential calculus; integration techniques; applications of integral calculus and plane analytical geometry.

All the students learned the same mathematical content using the same study manual and textbook for the same duration of time except for those who were repeating.

The teacher presented the limit concept to the whole class except on a few occasions when some students were not in class.

3.2.2 The interview sample

All the students were given a concept test on limits in Calculus in the first phase (see fig. : 1). The test was discussed with some of my colleagues. The items from the test were from the literature, the prescribed textbook and the study manual that the students use. The test items addressed students' definitions of the limit concept and some examples on limits. The students were divided into high, middle and low achievers on the basis of their performances in the test (see fig. 3.1).

All the nine students agreed to participate in this study. Though this was a sample of convenience the gender composition was 3 woman and 6 men which was representative of Mathematics 1 students population in first year Mathematics 1 at Medunsa. The students were all taught the limit concept by the researcher. All students learned the same mathematical content with the same prescribed textbook for the same duration of time

Teaching mathematics as well as teaching other subjects, involves the use of special technical language. The language used as a medium of instruction was English. This was the second language to most of the students. As a result I will look at how language plays a role in students' understanding of the limit concept but this is not my major field of study

The discussion of mathematical ideas has three qualitatively different functions: the cognitive function for the articulation of one's own thought processes, the communicative function for making one's ideas available to another, and the listening function for attempting to incorporate another's scheme into one's own.

In traditional whole class instruction, the teacher verbalizes the special concept, whereas the students themselves usually only listen to the teacher. The question is, what meaning do students construct when a concept is taught. When students are given instruction, the normal response and general phrases used are, for example: "I don't understand anything" and not specifying the source of their difficulty. Hoyles (1985) indicated that young children recognized the need to explain their solutions but nonetheless had difficulties in articulating their thought processes.

They often therefore tended to resort to simple enumeration and description of the given problem. Older children however, by way of contrast, frequently lost sight of the given problem in their search for a mathematical explanation.

3.3 DATA COLLECTION

3.3.1 Methods used for verification

In order to establish the validity of the procedure and instruments, a pilot study of the interview was done in the year 1995. The questions for the interview were discussed with my supervisor and some of my colleagues who are teaching mathematics at Medunsa so as to establish face validity. The questions were revised according to the recommendations of my colleagues and supervisor. The working suitability of the interview instrument was established by the pilot which also further sharpened the interview skills of the researcher.

Tape recordings were used to check the wording of any the need to explain their solutions but nonetheless had difficulties in articulating their thought processes.

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Tape recordings were used to check the wording of any statement that I wished to quote. Accurate field notes were kept. With the experience I gathered from the pilot interviews, I was able to devise a system of writing up notes immediately or very soon after the interview ended and this made it possible to produce a reasonable record of what was said in the key areas. All this was made easier because I was videotaping and audio-taping all the sessions.

3.3.2 Interview Procedure

Careful preparation for an interview helped me to record responses for all the 9 interviews. Whenever possible, interview transcripts, and particularly statements that are used as direct quotations in this report, were verified with the respondents.

Subjects who agreed to be interviewed were given some consideration and this fitted in well with their plans, however inconvenient it was for me at times. A venue was fixed for all the interviews. I tried to avoid my office because of disturbances that might have destroyed any chance of continuity.

Although difficult to lay down rules for the conduct of an interview, I was able to tell the interviewees to speak very loudly and if there was anything that they did not understand they should feel free to say so. I made it quite clear what I would do with the information.

When I made the appointment with the interviewees I told them how long the interview is likely to last and I tried my best not to exceed the stated time. It is very easy to become so interested in the topic of discussion that time slips by and before you know it, you have exceeded the time limit but besides this I tried to stick to the time frame.

It is the responsibility of the interviewer, not the interviewee to end an interview. If an interview takes two or three times as long as the interviewer said it would, the interviewee will be irritated in retrospect, however enjoyable the experience may have been at the time. This sort of practice breaks one of the ethics of professional social research, which is that the field should not be left more difficult for subsequent investigators to explore by disenchanting respondents with the whole notion of research participation.

Interviewing is not easy and many researchers have found it difficult to strike the balance between complete objectivity and trying to put the interviewee at ease. There are particular difficulties in interviewing senior colleagues, as one first time researcher found when he interviewed his headmaster. It is difficult to know how these difficulties can be overcome.

Honesty about the purpose of the exercise, integrity in the conduct and in the reporting of the interview. I promised to allow interviewees to see the transcripts and\or the draft of the report. Cost and time made it difficult to circulate drafts. I took care to keep whatever promises were made, so I did not promise too much.

From the pilot study I was able to consider where to put the tape recorder and the video recorder. The video-recorder and audio-recorder where not placed close together because of sound interference. I thus guarded against the recording of information strongly influencing the pace of the interview, the nature of the responses and the quality of the analysis.

Several factors had to be weighed up before deciding on the recording method to be used in this study. There was always the danger of bias creeping in to interviews, largely because, as Selltiz, Wrightsman, and Cook (1959) point out, that interviewers are human beings and not machines and their manner may have an effect on the respondents. Where a team of interviewers is employed, serious bias may show up in data analysis, but if one researcher conducts a set of interviews, the bias may be consistent and therefore go unnoticed.

Many factors can influence responses, one way or another. Borg (1981) draws attention to a few of the problems that may contribute to data obtained from the interview which are among others, eagerness of the respondent to please the interviewer, vague antagonism that sometimes arises between interviewer and respondent, or the tendency of the interviewer to seek out the answers that support his preconceived notions. It is easier to acknowledge the fact that bias can creep in than to eliminate it altogether. Gavron (1966) carried out research into the position and opportunities of young mothers, and was very conscious of the dangers inherent in research by solitary interviewers. Gavron further asserts that it is difficult to see how this can be avoided completely, but awareness of the problem plus constant self control can help. If I knew that I held strong views about some aspect of the topic, I was particularly careful about the way I put questions. It is even easier to "lead" in an interview than it is in a questionnaire.

The same question put by two people, but with different emphasis and in a different tone of voice, can produce very different responses. Complete objectivity was my aim.

I ensured, as far as I was able, that this recording method enabled me to record all substantive communication from the interviewee. Some interviewers simply write as much as they can believing that exhaustive recording by the interviewer is the best path. Others prefer to record only the main points of the respondent sacrificing some recording in the interaction. I tried to record everything that the respondent said.

Only a few interviewers rely completely on a tape recorder, mainly in cases where precise quotations will be crucial. In fact, inarticulate respondents and\or noisy background conditions may require interviewers to go back over a tape in order to hear each word slowly and distinctly before attempting an analysis. I tried to use both the audio and video recordings to help identify the respondents and to hear exactly what was said.

The pilot study helped me to decide how much notetaking would be necessary and to supplement notes with tape recordings. I was careful to ensure the smooth flow of the interview was facilitated by the method of recording information that I used. I was able to consider the best order of questioning and manage my schedule to make sure that the form of questioning was clear.

3.4 INSTRUMENTS USED

3.4.1 The concept test

All the students were given a test based on the section dealing with limits in Calculus in the first phase (see fig. 3.1). The test items were from prescribed textbook and the study manual. The test items addressed students' definitions of the limit concept and some examples on limits (see Appendix A).

3.4.2 The structured Interview

The interview was a structured one in that all questions were asked exactly the same way and in the same order for all respondents. All the nine subjects were interviewed for about half an hour each. The interviews were audio recorded and video recorded with the students' agreement.

The students were told that the interview questions were not to evaluate their performances in Mathematics. The interview schedule was devised to obtain more information about the conceptions that students held and the reasoning they used. Each student was encouraged to think aloud while addressing interview questions. Students were asked about their understanding of a limit and asked about methods they use to solve problems on limits.

3.4.3 The interview questions

The items for the interview were the following:

1) Explain the following definition in your own words :

$\lim_{x \to a} f(x) = \ell$ (Tall and Vinner, 1981)

2) Explain the method you will use to find the following limit :

$\lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}$ (Mamona-Downs, 1990)

3.4.4 Reasons for selecting the items

1)

(a) To find out whether students think of a limit as something that is a particular real number. According to Tall (1980) the word "limit" has many connotations in everyday life, which are at variance with the mathematical idea.

(b) To find out whether students think of a limit as something that can or cannot be approached sufficiently close by taking x sufficiently close to a. Cornu (1992) asserts that student conceive the notion of a limit ($\lim_{x \to a} f(x) = \ell'$) not as a static concept but as a dynamic process of "getting close to" a fixed value often with the implication of "never reaching" the limit. According to Williams (1992) there is confusion over whether a limit is actually reached or not. Tall (1992) talks about dynamism of a limit and whether a limit can be surpassed (see Chapter 2).

c) To determine students' understanding of the ε, δ definition. Tall and Vinner (1981) stated that the formal definition of the limit ($\lim_{x\to a} f(x)=\ell$) is very difficult for students to understand because of the epsilon (ε), delta (δ) approach. According to their study students could not recall the formal definition in a satisfactory way (see Chapter 2).

(d) To explore any other misconceptions which may emerge.

2

(a) To find out what approach students use to solve this limit(see section 3.4.3). Mamona-Downs (1990) gave groups of Greek and English students a limit problem to determine what approach the student would use to solve the problem (see section 2.2).

(b) To find out whether students have difficulties in handling zero as a denominator (see section 2.2). Mamona-Downs (1990) gave students the following problem $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$ to solve. This brought into surface students' difficulties with handling zero. Answers of this type were given 0? ∞?.

Mamona-Downs asserts that the handling of zero has nothing to do with the limit concept.

(c) To find out whether students will be able get the limit even though the function concerned is not defined at x=2. In the study by Mamona-Downs students were asked to justify factorization and cancellation under the lim-sign.

3.4.5 **Analysis of transcripts**

The 9-subjects were interviewed. The aim was to collect data from the transcripts capturing essential and relevant bits of students responses to each question. The data collected was analyzed (see chapter4).

The meaning and interpretation of the individual interviewed were bracketed (Cohen and Manion, 1980). These enabled me to understand what the interviewee said rather than what I expected the person to say. The transcripts were read a number of times. Redundancies were eliminated and general meaning reduced to units of meaning relevant to the research questions. A summary of the interview to accurately capture misconceptions was done (see section 4.1.2.1).

3.5 CONCLUSION

The data were collected from the transcripts made from the audio and videotape of all the interviews. The interviews were transcribed verbatim. Analysis of the transcripts is reported on Chapter 4.

CHAPTER 4

ACTUAL FINDINGS

4.1 The results

4.1.1 Results in Phase I

4.1.1.1 Concept test results

All the students were given a concept test(see Appendix A) on limits as discussed on section 3.4.1. The test was marked and subjects arranged from high to low marks. The test was out of sixty marks. The following marks were obtained:

Marks	No. of students
60-40	11
40-20	24
20-0	16

Fig. 4.1 Pilot test marks

Based on this test the students were divided into high, middle and low achievers. Students were divided into three groups i.e. the first top seventeen, the middle seventeen students and the low seventeen students. The students were fifty-one in number. Three students in each group were randomly sampled.

4.1.1.2 Conclusions

Although the results (see fig 4.1) for phase I might lead one to speculate that misconceptions where held only by those students who obtained low marks, the results of the interviews conducted later on, suggested otherwise. Every interviewee, including those who had made only few errors on the test, gave evidence of holding misconceptions (see section 4.1.2.1).

4.1.2 Analysis of Results of Phase II

4.1.2.1 Critical incidents from transcripts of interviews

As will emerge below results of the interviews show that misconceptions persist in all the groups (i.e.: low, middle and high achievers). Some interviewees reported that it is very difficult to state the ε,δ definition of the limit i.e. ($\lim_{x \to a} f(x)=\ell$). Statements such as "I can't recall it .... its difficult" were the order of the day. This difficulty was experienced across the three groups.

According to Tall and Vinner (1981) (see section 2.2) the formal definition of the limit concept is very difficult. According to Tall and Vinner the majority of those who gave the formal definition were not able to recall it in a satisfactory way mistaking it in various ways. This was the case with most of the students that the researcher interviewed (see extract 1).

The following extract from the transcripts shows how the students find it difficult understanding the limit concept.

EXTRACT 1

J=Reseacher (M,N) Students

J : Explain the following definition in your own words:

$\lim_{x \to a} f(x) = \ell$

M : In this context (trying to give ε,δ definition)

.... we deal with $\varepsilon > 0$ such that x-a $\rangle 0, f(x) - \ell > \varepsilon$

N :.. . .I can't recall it.

The students(M,N) are not able to state the ε, δ definition correctly. According to Tall and Vinner(1981) the definition of the limit ($\lim_{x \to a} f(x) = \ell$) is very difficult for students to understand because of the ε,δ approach. In this study students could not recall the definition in a satisfactory manner either.

Students cannot analyse the definition given, to put it in their own words. The students have learnt the idea of a limit in a haphazard manner.

The following extract shows that the student(L), has a vague idea about the definition of the limit.

EXTRACT 2

J=Researcher L=Student

J : Explain the following definition in your own words

$$\lim_{x \to a} f(x) = \ell$$

L: There is a number called **a** in the x-axis and that number as it approaches **x**, there is again another number in the y-axis which is ℓ it also approaches **f(x)**.

The student has a vague idea in that he says **a** approaches **x** and as ℓ approaches **f(x)** instead of as **x** approaches **a**,

f(x) approaches ℓ. The idea of the limit is confused. The student most probably didn't understand the definition when it was taught. The student has learned the concept but he can't put it in his own words. English could be a problem in this instance and this hinders his grasp of the mathematical language and the ability to communicate. In research by Davies and Vinner(1986) it was reported that some of the usual misconceptions that students have, result from the influence of language. Alder(1995) reported that even if the language spoken is not adequately mastered students can understand the mathematics that they are being taught.

The following students also tried to give the dynamic version of the definition, which was the most popular response in all the three groups as can be seen in the following extract:

EXTRACT 3

J=Reseacher; (B,N)Students

J: Explain the following definition in your own words

$$\lim_{x \to a} f(x) = \ell$$

B:.....as x approaches a the limit will be equal to ℓ. (There is confusion regarding the static character of a limit).

N:…f(x) cannot pass the point ℓ when x is approaching that point a.

There is also confusion here as to whether the limit can be surpassed. This was also reported by Tall(1992) when he talked about the dynamism of a limit. The student (N) has in mind that the limit is not reachable. In most instances students tried to give the dynamic version which is easier than the formal definition, i.e., the ε, δ definition.

Student (B) thinks of mechanical substitution when we talk of a limit. The following extract shows this:

EXTRACT 4

J=Reseacher; B=student

J:.........What about ℓ?

B:........because there is no x in ℓ we can't find the limit.

The student thinks that we can only get a limit if we are able to substitute for x in the function.

The following student(T) also had the same problem as can be seen from the following extract:

EXTRACT 5

T=Student

T : The $\lim_{x \to a} f(x)$ is the same as f(a)

The student thinks of continuity of a function at point x=a. This is not always the case because it is possible at times that the function is not defined at a whereas $\lim_{x \to a} f(x)$ exists.

The following extracts relate to question 2 of the interview.

EXTRACT 6

J=Reseacher B=Student

J:Explain the method that you will use to find the following limit: $\lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}$

B: This one we can't immediately plug in 2. We have to find a way of factorising the numerator so that it could cancel out with the (x-2).

In this instance again the student thinks of mechanical substitution as in question 1 (see extract 4)

The students know that x^5-2^5 must be simplified but do not know how. Most of them have forgotten the factor theorem; remainder theorem and long division. This was so in all the cases. The following is a typical extract in this regard.

EXTRACT 7

J = Reseacher T=Student

J : Is your function defined at 2 ?

T : Yes, if you get the factor **(x-2)** it will be defined.

Clearly the student confuses the functions defined by $f(x)=\frac{x^5-2^5}{x-2}$ and $f(x)=x^4+2x^3+4x^2+8x+16$. The function is not defined at 2 but we can get the limit at this point. This was the case in most of the students that I interviewed(see extract 8)

EXTRACT 8

J=Reseacher (L,N) Students

L :. . . we have to factorise the top part and be able to cancel out the (x-2)

J : Can you get the limit if you can't factorise the numerator?

L : No....

N : I can't find the limit. (The student is unable to factorise the numerator.)

Most of the subjects had an idea as to what to do with the question. They only had problems with factorisation and didn't notice that they could use the remainder theorem; factor theorem and long division method to get the factors. When the students tried to factorise the numerator and got frustrated they normally said the limit does not exist as can be seen in the following extract.

EXTRACT 9

J=Researcher; B=Student

J : ... meaning this function is not defined at 2, but, can we still get the limit?

B : Oh no ... we can't get the limit. The limit of this function as $x \to 2$ is not defined.

The student has a misconception in that he talks of a limit not being defined at 2 instead of saying that the function is not defined at 2.

EXTRACT 10

J = Reseacher (N, M, R)Student

J : Does the limit exists at 2?.

N : I will only get the limit if I can simplify.

Student(N) is thinking only about the manipulative aspect and not of the concept of a limit.

M : We must try to simplify our numerator first so that the denominator can cancel out.

R : You gonna try to simplify this .... put it in simpler form .... its very difficult to simplify.

The students only thinks mechanically and not about the limit concept.

4.1.2.2 **Summary of identified Misconceptions**

a) Students have in mind that the limit concept is about mechanical substitution.

b) Students talk of a limit not being defined at a point when it is the function that is not defined at the point.

c) Students think only about the manipulative aspects and do not focus on the concept of a limit.

4.2 INTERPRETATION OF THE FINDINGS

4.2.1 Understanding of definition of the limit concept

Throughout the study students were asked to give their own interpretation of the definition of the limit ($\lim_{x \to a} f(x) = \ell$). Students found it very difficult to give the ε, δ definition of the limit concept (see Tall and Vinner 1981 and section 2.2) in their study of 70 highly qualified first year university students who were requested to write down the definition of $\lim_{x \to a} f(x) = \ell$.

Most of the subjects couldn't analyse the definition given, to put it in their own words. I think English is also a problem which hinders the students to grasp the mathematical language. Tall(1992) asserts that misconceptions that students have result from the influence of language.

According to Alder(1995) even if the language spoken is not adequately mastered, students can understand the mathematics that they are being taught. Students often think of mechanical substitution when we talk of a limit (see extract 3; 6 and 10).

There is also a confusion whether a limit can be surpassed and also the static character of a limit. Students talked of a limit as not being defined at a point instead of saying that the function is not defined at a point. Students also think of a manipulative aspects and not of the concept of a limit.

4.2.2. Interpretation of the second question

Most students were aware that, substituting directly by 2 would force them to divide by zero. They knew that X^5-2^5 must be simplified but didn't know how. The problem was with manipulative skills. The subjects cannot relate limits to the remainder theorem, long division and factor theorem. Most of them had in mind that substituting directly by 2 is not allowed (see extract 8). Mamona-Downs (1990) asserts that handling of zero has nothing to do with the limit concept. Zero becomes a problem for nearly every pupil sometime in his mathematical career. As a result if the student find it difficult to factorize, they believe that the limit does not exist.

The student had a misconception in that he talks of a limit not being defined at 2 instead of saying the function is not defined at [illegible]. Students also think of manipulative aspect and not of the concept of a limit.

CHAPTER 5

SUMMARY AND CONCLUSION

5.1 Summary of research approach

The research consists of two phases (see fig 3.1). A concept test based on the section dealing with limits in Calculus was given in the first phase. Based on the marks the students obtained in this test, the students were divided into high, middle and low achievers.

In the second phase 9-subjects were randomly selected across all three groups, 3 from each group. The subjects were individually interviewed. The interviews were audio-recorded and video-recorded.

The data was collected from the transcripts made from the audio and videotape of all the interviews. The qualitative data thus recorded was analysed (see chapter 4).

5.2 **Summary of results**

Although results (see fig 4.1) might lead the reader to speculate that misconceptions where held by only a relatively small portion of the students, the interview results suggest otherwise.

Every interviewee, including those who had made a few errors on the test written gave evidence of holding some misconceptions. In fact all interviewees reported some difficulty as far as the definition of the limit is concerned. Statements such as "I can't state it properly" "Its difficult....." were the order of the day.

Results show that misconceptions persist in all of them. This supports the claim that the limit concept causes some conceptual difficulty in learning.

Students were asked to solve a problem related to situations commonly thought to involve limits. The data show that students supplied responses that they can't substitute the 2 in the expression.

Also of interest is that no student noticed that in question 2 (see section 4.2.2) long division; the factor theorem and remainder theorem can be used.

5.3 Implications for teaching

The findings of this study are particularly relevant in the teaching of the limit concept. The findings provide information about what ideas students have about the limit concept (see section 4.1.2.2).

The research will enable us to alert future graduates to the existence of misconceptions related to limits. This is important in improving the understanding of the limit concept.

5.4 Recommendation for further research

The researcher suggests that research on misconceptions related to limits, be done on more diverse population groups, given the rather select nature of my sample.

5.5 Limitations of the study

The possible limitation of the study might be that the interviews were conducted in English which is a second language to most of these students.

This limitation was addressed by asking students if there was anything that they would wish to have rephrased in the questions so that it would be well-understood. There was no comment from the students as far as rephrasing of questions is concerned.

The 9-students selected is a small sample on which to base generalisation.

5.6 Conclusion

Generally the results of the survey indicate that misconceptions concerning definitions of the limit concept exists. The most essential message of this research is that we should have sympathy for students and the misconceptions that students have in mathematics. Misconceptions that students have are rational and meaningful efforts to cope with the learning of mathematics.

Alerting future teachers to the existence of misconceptions is very important in improving their students' competence in Mathematics.

For me personally, the study is very important. The research has made me aware of misconceptions arising from my teaching, that I was not aware of before. I will plan my future teaching so as hopefully to avoid these misconceptions arising in my students to be.

APPENDIX A

CONCEPT TEST

QUESTION 1

1 **Complete the following:**

$\lim_{x \to a} f(x) = \ell$ means ________________________________ (3)

2 **Use definition of a limit to show the following:**

$\lim_{x \to 2} (5-2x) = 1$ (5)

QUESTION 2

Determine the following limits if they exist.

a) $\lim_{x \to -1} (2x^3 - 6x)$ (2)

b) $\lim_{x \to 2} \dfrac{x^5 - 2^5}{x - 2}$ (6)

c) $\lim_{x \to 3} \dfrac{\sqrt{x} - \sqrt{3}}{x - 3}$ (5)

d) $\lim_{x \to -1} \dfrac{x^3 + 1}{x + 1}$ (5)

e) $\lim_{x \to 0} 3$ (3)

QUESTION 3

3.1 **Complete the following:**

$\lim_{x \uparrow a} f(x) = \ell$ ________________________________ (3)

3.2 **If** $f(x) = \begin{cases} 1+x & \text{if } x \leq 0 \\ 2-x & \text{if } x<0 \end{cases}$

Is f continuous at 0? (6)

3.2 **Determine the following:**

a) $\lim\limits_{h \to 0} \dfrac{1 - \cosh}{h^2}$ (5)

b) $\lim\limits_{x \to 2} \dfrac{\sin(x^2 - 4)}{x - 2}$ (3)

c) $\lim\limits_{x \to 0} \dfrac{\sin x}{|x|}$ (5)

d) $\lim\limits_{x \to \infty} \dfrac{(x-a)(x-b)}{(ax+b)^2}$ (4)

e) $\lim\limits_{x \uparrow 1} \dfrac{1}{x}$ (2)

f) $\lim\limits_{x \to 1} \dfrac{x}{(x-1)^2}$ (3)

BIBLIOGRAPHY

Adler, J (1995) :Participatory, inquiry pedagogy, communicative competence and mathematical knowledge in a multilingual classroom: A Vignette Paper presented at PME 19- Recife, Brazil July 1985

Ausubel, D P(1968) : Educational psychology : A cognitive view. New York : Holt, Rinehart and Winston

Bell, A and Bassford, D (1988) : A conflict and investigation teaching methods and an individualised learning scheme- A comparative experiment on the teaching of fractions. Shell centre for Mathematical Education,University of the Nottingham

Bodner, G M(1986) : Construction : A theory of knowlegde Journal Chemistry Education, 63 : 873

Booth, L.R.(1984). Misconceptions Leading to Error in Elementary Algebra. Journal of Structural learning 8: 125-138

Borg, W.R (1981) : Applying educational research: A practical guide for teachers: New-York, Longman.

Brodie, K (1995) : Peer interaction and the development of mathematical knowlegde. Paper presented to PME -19, Recife, July 1995

Brown, D E (1992) : Using examples and analogies to remediate misconceptions in physics. Journal of Research in Science Teaching 29 : 17-34

Clement, J (1981) : Analogy generation in scientific problem solving. Proceedings of the Third Annual Meeting of the Cognitive Science Society (Vol.3;137-140). Hillsdale, N.J. : Lawrence Erlbaum

Cohen, L and Manion, L (1980) : Research Methods in Education. Croom Helm Ltd (ed). London and New York

Confrey, J (1990) : A review of the research on student conceptions in Mathematics, Science and programming . In Cazden (ed) Review of Research in Education Vol 16 (pp 3-56)

Cornu, B (1992) : Limits In : Tall, D O (ed) Advance Mathematics Thinking, 153-166 Dordrecht

Davies, R B (1984) : Learning Mathematics . The cognitive science approach to Mathematics Education, London : Croom Helm, Davies, R and Vinner, S (1986). The notion of limit : some seemingly unavoidable misconceptions stages. Journal of Mathematical Behaviour 5 , 281-303

Davies, R B and Vinner, S(1986): The notion of Limit: Some seemingly Unavoidable Misconceptions Stages . Journal of Mathematical Behaviour 5, 281-303

Fischbein, E.,Deri,.M.,Sainati,W.,and Marino,M S(1988): The role of implicit models in solving verbal problems in multiplication and division. Journal for Research in Mathematics Education 16 : 3-18

Gavron, H (1966) : The captive wife : Conflicts of house bound mothers : London, Routledoe.

Hoyles, C (1985) : What is the point of group discussion in mathematics? Educational Studies in Mathematics, 16, 205-214.

Mamona-Downs,J(1990): Pupils' interpretations of the limit concept: A comparison study between Greeks and

English. Proceedings PME-14 Conference, Mexico.

Monaghan, J D(1986): Adolescent's understanding of Limits and Infinity . Unpublished PHD thesis , Warwick University, U.K.

Olivier, A(1988) : The construction of an Algebraic concept through conflict . (In Brabas , A Ed (1988). The Proceedings of PMEXII . Hungary, 511-518

Nesher, P(1987) : Towards an instructional Theory : The role of students' Misconceptions. For the learning of mathematics, 7(3), 33-40.

Perkins, D N and Simmons, R(1988): Patterns of misunderstanding : An integrative model of science, mathematics and programming. Review of Educational Research, 58(3), 303-326

Pines(Eds), A.L. (1985). Towards a taxonomy of conceptual relations. In L.West & A.L. Pines(Eds), Cognitive structure and conceptual change (pp. 101-116). New York :Academic Press.

Selltiz, C, Wrightsman, L S, and Cook, S W(1959) : Research methods in social relations. New-York: Holt, Rinehart and Winston

Sierpinska, A (1985) : `Obstacles Epistemologigues Relatifs a la' in Recherches en Didactique des Mathematiques, 6(1), 5-67

Smith, J P (1992) : Misconceptions and the construction of mathematical knowledge. (In Geslin, W and Graham, Eds (1992). The proceedings of PME XVI, Durham USA, 319-326

Solomon, J.(1983). Messy, contradictory, and obstinately persistent: A study of children's out-of-school ideas about energy. School Science Review, 65(231).

Tall, D O (1980) : Mathematical intuition, with special reference to limiting processes. Proceedings of the fourth International conference for the Psychology of Mathematics Education.

Tall, D O (1992) : The transition to advanced mathematical thinking: functions, infinity and proof (In : Grouws, D A(Ed) : Handbook of Research on Mathematics Teaching and Learning : New York L : Macmillan, 495-511).

Tall, D O and Vinner, S (1981) : Concept image and concept definition in mathematics with particular reference to limits and continuity . Educational Studies in Mathematics , 12 , 151-169

Von Glasersfeld, E (1984) : An introduction to Radical Constructivism" Watziawick, P.(ed) The Invented Reality. NY: WW Norton

Von Glasersfeld, E (1989) : Introduction (In von Glasersfeld, E Ed (1991) Radical Constructivism in Mathematics Education: xiii-xx Kluwer.)

Vergnaud, G (1990) : La theorie des Champs Conceptuals. Recherches en Didactique des Mathematiques. Vol 10 No 23 , 133-170

Vygtosky, L.S. (1986): Thought and Language(A. Kouzulin,ed) Cambridge, MA: Massachusetts Institute of Technology.

Vygtosky, L.S. (1979): Mind in Society: The Development of higher psychology processes.Cambridge, Massachusetts: Harvard University press.

Williams, S.R (1992) : The understanding of limit : Three perspectives. Washington State