THE DESIGN AND IMPLEMENTATION OF MANUFACTURING RESOURCE PLANNING AT A PLANT PRODUCING CONTINUOUS SEAM WELDED STEEL TUBING AND A VARIETY OF BATCH PROCESSED TUBE PRODUCTS.

STEFANO CERUSINI

A project report submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 1986

DECLARATION

I declare that this project report is my own, unaided work. It is being submitted for the degree of Master of Science in Engineering at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

This project report is based on work carried out by the author at Woltube Division of Wolhuter Steel, subsequently Woltube Division of Tubemakers of South Africa, in the role of Project Leader for the implementation of Manufacturing Resource Planning (MRP II). All development work described herein was conducted by the author.

(Signature of candidate)

THIRD day of APRIL 1986

ABSTRACT

The implementation of Manufacturing Resource Planning systems in tubemills requires that two major criteria be addressed:

- the production-inventory system consists of a combination of semi-continuous and batch type operations which have to be coordinated, and
- the mills have long changeover times resulting in the need to have cost effective production lot sizes.

The purpose of this report is to describe:

- ways in which standard IBM MAPICS software was used to provide the basic business controls on production and stocks and the special problems encountered,
- the development of a lot sizing technique specifically for the tube milling environment, and
- the influence of the human factor on the implementation tasks.

Simulations carried out on an IBM PC-XT indicate that the lot sizing technique yields potential savings of 18,2% on stocking (replenishment, carrying and shortage) costs. Furthermore, they show that reductions in mill setup times can increase these savings to in excess of 50%.

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1 INTRODUCTION

Increased competitiveness in the tube rolling sector and a shift towards supply side economics prompted top management to provide high customer service levels while at the same time keeping raw material, work in progress and finished goods stocks to a minimum. There was thus an inherent exposure to lost opportunity costs and careful planning and control of manufacturing operations was needed to reconcile and eventually solve this apparent conflict between manufacturing and marketing objectives.

With the emphasis on "return on assets managed" as the key measure of the company's performance the manner in which productive capacity was utilised required intensive investigation. The informal systems and methods in place were not sufficient to respond to the dynamic situation that was being created.

1.1 Key Objectives and Scope of Work

The purpose of this report is to illustrate an approach, partially implemented in practice, to solving the dilemma, and to present the logic and results of research work carried out on the problem of determining production lot sizes for the tube rolling mills at the plant on which this work is based.

The plant is a producer of continuous seam welded steel tubing, some of which undergoes further processing such as flanging, stub-ending, screwing and socketing, galvanising or coating. See APPENDIX A for a more detailed explanation of the milling operation.

1.1.1 Primary Objective

A formal "business case" was drawn up for the project in which a clear overall objective was presented:

"To introduce a workable, simple and effective system of planned production and materials control by 28th February, 1985 so that the necessary management and technical functions of sales/marketing, production and finance are integrated in a common, dynamic manufacturing resource planning system so as to contribute to the growth and profitability of the company."

1.1.2 Primary Performance Measure

The primary performance measure was the projected impact the project would have on RETURN ON ASSETS MANAGED on the basis of EARNINGS BEFORE INTEREST AND TAX ie. EBIT ROAM. The financial definition of ROAM is as follows:

ROAM = ASSET TURNOVER (ATO) X RETURN ON SALES (ROS)

where

ATO = SALES / TOTAL ASSETS, and
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The ROAM model's usefulness as a company performance measure is by virtue of its absoluteness. It focuses on assets only, and not on borrowings or financial structure. Thus, the model *can be put to good use in determining* high return assets (including stock), areas with large potential and areas with the greatest impact on growth and profitability.

The MRP II database was to provide essential information to the model, as well as a "what if" capability to test *alternate strategies. Areas of direct* impact are:

- sales: improved operational efficiency and better customer service,
- stocks: reduced, more balanced and more accurate, and
- EBIT: lower production costs.

The target was set at a ROAM improvement of 13%. This related to a R1 000 000 reduction in stockholding on approximately R15 000 000 and a 3% increase in sales.

1.1.3 Scope of Work

IBM's MAPICS software (Manufacturing, Accounting, Production Information Control System) was s[illegible] as a basis for the implementation. The selection [illegible] rom a shortlist of three standard packages, all of [illegible] an on the existing IBM System /34, later upgraded to a Sy[illegible]m /36.

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The standard modules which were chosen for implementation were:

- *Inventory Management*
- Product Data Management
- Production Control and Costing
- Forecasting
- Master Production Scheduling and Material Requirements Planning
- Capacity Requirements Planning
- *Order Entry and Invoicing*
- Purchasing
- General Ledger
- Debtors
- Creditors

Although standard *business control software* was selected for the task several major peculiarities were identified in the nature of the manufacturing process. The major part of this report deals with discussions of these and the prerequisite development work which they generated.

1.2 An *Industry Perspective*

Steel tubing is, in general, an undifferentiated product in the marketplace. Furthermore, the local market is not large enough to ensure long production runs of a particular size, which are needed for efficient, low cost production. Although the larger producers have found market niches where this is possible these cases are very much the exception.

Some aggressive marketing by most companies has resulted in a substantial export drive which has brought some relief to a sector plagued with excess capacity. However, order quantities for individual sizes have been erratic. A strong industry delegation in the United States has forced the imposition of a "Voluntary Restraint Agreement" limiting and apportioning the steel imports to that country. As a result, even with the generous export incentives that have been created, activity has been curtailed somewhat.

As the standard product market becomes saturated, market segregation is occuring, with the introduction of options (such as special mining products).

In view of the situation, the company has adopted a strategy of accepting a variety of orders, for standard and special products, and for large or small quantities. This has created a very distinct requirement for flexibility on the manufacturing facility. The pressure for diversity contributes substantially to short production runs with associated high setup costs per unit.

The strategy required for establishing a favourable market equilibrium while meeting the demand for special products without overstepping economic constraints requires a suitable manufacturing strategy and a correspondingly suitable planning and control system.

The formulation of these strategies involves careful studies of production costs, quality and customer service, which are all of primary importance in the undifferentiated tube sector. However, from the point of view of our performance measure, ROAM, the productivity focus needs to be on materials since these constitute *85% to 90% of the total cost* of manufacture.

The implications of this statement are vital and far-reaching. Large scrap percentages, large overruns, high stock levels and most *other material related criteria* are of prime concern. These are directly related to the work contained in the remainder of this report.

1.3 The Case History in Retrospect

The severe recession which was the ever present backdrop to all the development work created an unequalled era of turbulence in the company's history. Seven months after the MRP II project was officially approved, the company was taken over by one of two giants in the sector, who, at the time, shared 70% of the steel tube market. The period which followed was one of intense rationalisation and merging of productive capacity and administrative resources. (The author still has doubt as to whether the centralisation strategy was the correct one, in view of the fact that the company was making profits substantially above those of most others in the sector, including the take-over company. The resultant *loss of key personnel* will remain an enduring lesson even although the relative effect of the economies of scale resulting from the combining of plant and resources will will never be fully known. Decisions to combine were taken at a time when there were, and continue to be, strong moves to decentralise, and to reduce the size of business units.)

A year after the project was approved, a further corporate level takeover of the buying company's holding company led to even further uncertainty as to future changes, although it was considered unlikely that these would affect operations.

As a result of these occurences, the only areas into which major in-roads were made were those of regulating the stock debacle which existed, and cleaning up related systems and procedures. The important points are described in chapter 2, entitled "Systems Development". These occured largely in the first 6 months of activity. After the takeover, development work on production planning and control systems was limited to a low key, parallel pilot project, with all accounting sub-systems having being centralised. Important points are likewise covered in chapter 2.

Development work on lot sizing techniques covered in chapter 3 was carried out independently later and although the general operating principles of the tube mill are called on *in developing* the arguments contained therein, none of the findings have been implemented or tested in practice.

The chapter on "The Human Factor" has been included because of the vast insight gained during the various stages of the implementation, into different managerial techniques which were employed by the two camps. This discussion will not be a comparison of the camps themselves, but will extract and present general lessons learned from both, in the hope that a contribution can be made to the theory of the implementation of computerised business systems in general.

1.4 Literature Survey

The theory and practice of the implementation of computerised MRP II systems is abundantly documented. To list all publications which have *had any bearing on the work done* would be onerous. However, all the significantly innovative sections of work done were inspired as a direct result of an intensive search *for answers to specific problems which*, although directly related to tube rolling, were not necessarily unique in nature.

A search through the ANBAR abstracts and indices of the *JOURNAL OF MANAGEMENT* SCIENCES and INTERFACES, the

ENGINEERING INDEX and the APICS BIBLIOGRAPHY yielded little which dealt with steel tube mills in *particular, or steel* rolling mills in general.

Use of the University's "on-line search facility" provided significant leads. Search arguments used were "steel tube mills" and "*production planning*". Publications of interest, found in various local libraries, were as follows:

- *Yamamoto, E; Ohashi, Y;* Kize, K and Matsushita, M. PRODUCTION CONTROL SYSTEM OF STEEL TUBE AND PIPE: Sumitomo Search No. 24 Nov 1980 p154-163, Sumitomo Metal *Industries* Ltd, Kashima, Japan.

- Woodall, A and Saunders, K.H. MODEL BUILDING WITH PARTICULAR REFERENCE TO THE USE OF PRODUCTIVE CAPACITY: Journal of the Iron and Steel Institute, May 1970.

- Konishi, K; Tsukui, T; Seino, H; Kawabata, S; Enomoto, Y and Ida, M: CONSTRUCTION AND OPERATION OF NEW MEDIUM DIAMETER ELECTRIC RESISTANCE WELDED PIPE MILL: Nippon Kokan Technical Report, 1984.

Finally, reference to Peterson, R and Silver, E.A. DECISION SYSTEMS FOR INVENTORY MANAGEMENT AND PRODUCTION PLANNING: John Wiley and Sons, 1970 led to the following two publications on which the major section of this report is based viz. LOT SIZING TECHNIQUES:

- Silver, Edward A., A CONTROL SYSTEM FOR COORDINATED INVENTORY REPLENISHMENT: International Journal of Production Research, 1974, vol.12, no.6, 647-671

- Thompstone, R.M. and Silver E.A., A COORDINATED INVENTORY CONTROL SYSTEM FOR COMPOUND POISSON DEMAND AND ZERO LEAD TIME: International Journal of Production Research, vol.13, no.6, 581-602.

For the sake of continuity the liter[illegible] lot sizing techniques will be discussed in chapt[illegible]

Some of the publications found through the on-line search facility are particularly topical because of their Japanese origin, because of the ever increasing use of Japanese manufacturing techniques worldwide. Predictably, these show a pattern of development which far surpasses that of the local industry.

The following is a summary of the key points of difference:

- The basic items of plant are considerably more automated, with process control computers being used to control the entire operation, from milling to the finishing line. In local plants these form two distinct and separately planned and controlled sections. The Japanese approach results in increased yield, vastly reduced work in progress levels because of the flow line nature of their process, labour savings, focused process control with immediate feedback and action on any problem area and high levels of quality.

- Of critical importance is the fact that the sophistication of the process control extends to changeovers, where a roll change and resetting of the process is achieved in minutes, with very little start-up scrap. The connotations of this to JUST IN TIME techniques are obvious. However, "grouping" of orders with the same production specifications is still carried out to "obtain the largest manageable lots in order to improve work efficiencies at respective processes".

- Because of the process control, Japanese mill outputs are far in excess of comparably sized mills locally. (3000 tonnes/month here compared to 35000 tonnes/month in Japan!).

- Raw steel coil is made at the same facility thus integrating the production requirements. Here again the "JUST IN TIME" nature of the material supply is typically Japanese. The material planning cycle starts with the customer order processing (done by the business control computer) at which point "instruction slips" (KANBAN cards?) are prepared with information on the required billet assortment to produce the necessary tube coils.

- Quality or process control is on-line with sensors monitoring continuously. Operators control the process and material flow by means of shop floor CRT's. Feedback of actual data is continuously compared against the plan. Non-destructive tests are carried out as part of the process and displayed on the CRT's.

- Information on warehouse storage (the final step in the process) is automatically fed to the on-line product shipment system to expedite shipping and packaging operations.

- Overruns are controlled by automatically allocating items to alternative orders. Where this is not possible products are immediately placed under "surplus product management" and are monitored daily for possible allocation or alternatively, they are placed on "inferior product" sheets for immediate discounting and disposal.

Work in progress stocks have been reduced by 44%, they claim, through the improved WIP control and improved surplus product allocation since the installation of their process control computer.

Of significance is FUTURE PROBLEMS AND PLANS. Here, they say: "There still remains much to be examined with respect to stock yard management and work scheduling system which has not been considered this time for the application of a business computer".

2 SYSTEMS DEVELOPMENT

Prior to the commencement of the project some basic computerised systems were in place. Predictably, these were a sophisticated, *standalone General Ledger system* and two modules of IBM's MAPICS, viz. ORDER ENTRY AND INVOICING and INVENTORY MANAGEMENT. The fact that these two basic MAPICS modules were running was one of the main reasons for the decision to use MAPICS as the manufacturing control system. Other reasons were the size of the MAPICS user base internationally and the strength of IBM's support; both were seen as *crucially important criteria*.

Apart from the General Ledger application the computer systems were not well managed. Data was plagued with inaccuracies and no formal systems and procedures were in place to support the information systems.

In particular, stockkeeping is especially difficult in the tubemaking environment because:

- limiting access to stores is virtually impossible with the high production rates of tubes being milled,

- tubes of similar size are often difficult to distinguish,
- the various metal grades are indistinguishable,
- the continuous shop floor activity is difficult to control.

Month end reconcilliations of stock values and quantities were sources of much frustration to the accounting staff. Daily administration of customer orders was done without any reference to computer figures. Informal systems were abundant.

With these conditions prevailing it was clear that the prerequisite to successful manufacturing control systems was to entirely revamp the existing systems to the point where only one well managed and integrated computer system was used to conduct business. For 8 years, various streams of management teams had tried in vain to bring the situation under control. Rapid growth had not made matters easier.

2.1 Administrative Systems

The theories expounded by the American Production and Inventory Control Society, Oliver Wight, Joseph Orlicky, George Plossl and various others were used to good effect. The following paragraphs describe the techniques used to regulate the inventory debacle.

Stock Classification and Production Lot Sizing

At this point the company was plagued with high stocks and *there was much pressure on management to reduce* stockholding. It was thus imperative that a stopgap be devised to limit production runs of low demand items, overruns, scrap and "off-spec" products.

The first step was to produce a Pareto analysis of all end items, the basis for the classification being Rand value of sales over the preceeding 12 months. Of the 3200 items contained in the ITEM MASTER file it was found that only 724 had shown any sales activity during this period. The ABC split is shown in figure 1. The A items, which accounted for 80% of the sales numbered 160 (ie. 22%); the A and B items which accounted for 96,8% of sales amount numbered 362 (ie. 50%). It thus followed that controlling the relatively few A and B items, 362 out of a total of over 3200, would stabilize the situation.

Furthermore, if these end items were to be reduced to their "direct off mill" designations ie. considering only the parameters of diameter, gauge and length, irrespective of finish, then the number of items needing to be controlled would be reduced to under 200.

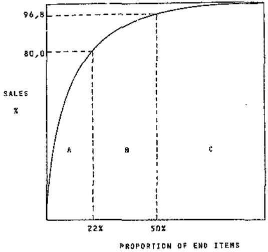


FIGURE 1: ACTUAL PLOT OF PARETO CURVE

The remainder of the exercise led to the establishment of categories of downgraded and slow-moving stock and corresponding lists of quantities on hand (suitably cycle checked). These were the subject of an intensive sales effort to sell off this stock at discounted prices to selected areas of the market where the assurance could be obtained that the products would not be resold as prime tube.

Since the products are of an undifferentiated nature, using sales value as a basis for the PARETO exercise is acceptable. *However, having established the A,B and C categories it was* necessary to review the balance in order to provide full ranges of certain lines of tubing to the marketplace, even though lot sizes would be small. In other cases it was possible to purchase these tubes from other suppliers rather than incurring the penalties associated with short production runs (ie. the idea was not to be a "hardware store").

A variety of daily "watchdog" management reports were used to monitor progress.

Next was the development of a rough and ready lot sizing technique to assist the planners in drawing up mill rolling cycles and determining the corresponding steel requirements (raw material lead times were 6 to 8 weeks).

Since mill cycles (ie the time between successive runs of tubes of the same diameter and guage) were approximately 6 weeks demand figures for eight 6 weekly periods were extracted for each "direct off mill" item. Using this data the average and mean absolute deviation (MAD) was calculated for each item. Using statistical tables (FIG.2) a factor of 2 was used to determine the safety stock required to give a 95% service level. The reorder level was set at:

SAFETY STOCK + AVERAGE DEMAND.

SERVICE LEVEL (% ORDER CYCLES W/O STOCKOUT)	SAFETY FACTOR USING MEAN ABSOLUTE DEVIATION
50,00%	0,00
75,00%	0,84
80,00%	1,05
84,13%	1,25
85,00%	1,30
89,44%	1,56
90,00%	1,60
93,32%	1,88
94,00%	1,95
94,52%	2,00
95,00%	2,06
96,00%	2,19
97,00%	2,35
97,72%	2,50
98,00%	2,56
98,61%	2,75
99,00%	2,91

FIGURE 2: TABLE OF SAFLTY FACTORS FOR NORMAL DISTRIBUTION

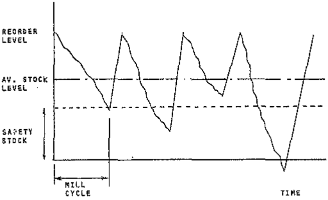


FIGURE 3: GRAPH OF STOCK MOVEMENT WITH TIME FOR AN ITEM

Although this did not take into account variations in cycle length it was decided not to increase reorder levels any further in order to keep stockholding down. (See FIG.3).

Using SAFETY STOCK + 1/2 X AVERAGE DEMAND as the average stock on hand for each item in the report (A and B categories which accounted for 96,8% of sales value) it was found that the cumulative average stock value for these items was approximately 40% of the book value at the time. This gave us a good indication of what was achievable in terms of end item stock reduction. (Figure 4 shows an extract from this report. The value of lot size shown minus the stock on hand was the quantity of tube to be produced).

Documented Procedures, Paperwork and Accountability

The task of developing departmental and functional procedures which are formally documented and adhered to is as critical as it is daunting. All computerised systems have a basic architecture which, if the system has been correctly chosen or designed, will fit the general working patterns of the organisation. However, this is only the first step to a happy union. Latest trends show a distinct movement away from DP departments and DP data capture clerks towards management of computer systems by the various disciplines within a company.

DATE 11/09/84

LOT SIZING AND SAFETY STOCK ANALYSIS

PAGE 1

ITEM CODE	ITEM CLASS	VALUE CLASS	1st CYC	2nd CYC	3rd CYC	4th CYC	5th CYC	6th CYC	7th CYC	8th CYC	MAD	SAFETY STOCK	REORDER LEVEL	AV.STOCK LEVEL	VALUE	TONNES
R0502006400AAA	AF	C	0	0	5	85	0	50	324	205	90	180	263	221	1858	3,24
S0252506400AAA	AF	B	40	380	300	40	0	408	186	185	127	254	446	350	2611	4,04
S0381606400AAA	AF	B	479	235	265	130	40	658	767	335	201	402	768	685	5522	6,59
S0502506400AAA	AF	B	25	25	50	120	327	20	385	75	113	226	354	290	4467	6,96
T0212006400ANB	BJ	A	70	70	30	80	0	150	981	0	201	402	574	488	3589	3,19
T0212007320AAA	AA	B	409	300	395	695	0	200	650	100	193	386	729	557	2660	3,91

FIGURE 4: REPORT SHOWING THE CALCULATION OF REORDER LEVELS AND SAFETY STOCKS

The fear of losing "central control" of the data input and integrity which prevailed in installations in the past has been replaced by a willingness of functional managers and their staff to accept accountability for and ownership of *their data files.*

The company subscribed totally to this end-user participation. However, this brought with it certain prerequisites, viz:

- clear, general understanding of the concepts around which the software was designed and to which the users would have to adhere,
- specific training on the areas of application each person would be involved with,
- understanding of the effects which errors would propagate elsewhere in the system.
- clearly demarcated accountability for all activities.

In order for these prerequisites to be met it was necessary to develop practical procedures, well documented, for each functional area. Further, all paperwork and paper flow had to be reviewed to ensure compatibility with the system.

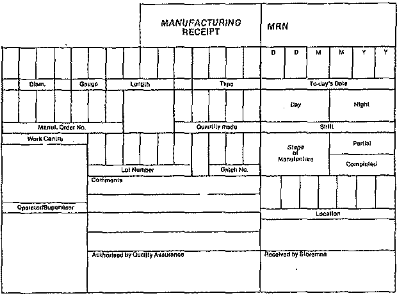
The difficulty arose in balancing the procedures so as to allow a reasonable amount of flexibility to cater for unusual situations and to ensure that business was not impeded in any way. Customer orders, stock transactions, shop activity etc.

was to be entered and verified with the shortest possible delays. Errors would have to be reconciled speedily and the person concerned notified or retrained.

Access to the various parts of the computer system had to be strictly controlled. This was done through the use of passwords.

Figure 5 illustrates the principle of accountability as well as the tight built-in control over bad data creeping into the system. The MANUFACTURING RECEIPT was made out by the operator at a work centre. Production was authorised by means of a computer printed "shop packet" necessitating that manufacturing orders be present in the computer. A quality assurance inspector would be accountable for ensuring that the part number, quantity, quality and label were correct. The storeman would enter the stock location into which the material was transferred, and would be accountable for the correctness of the quantity.

Thus three people were involved in drawing up and verifying the data. Furthermore, the data capture clerk would be averted if no such item, manufacturing order, stock location etc. existed.



MANUFACTURING RECEIPT

MRN

D D M M Y Y

Diam. | Gauge | Length | Type | To-day's Date

Day | Night

Manuf. Order No. | Quantity made | Shift

Work Centre

Stage of Manufacture | Partial | Completed

Lot Number | Batch No.

Comments

Operator/Supervisor

Location

Authorised by Quality Assurance

Received by Storeman

FIGURE 5: SAMPLE MANUFACTURING RECEIPT VOUCHER

This system was very effective in ensuring teamwork at all levels. It forced people to adhere to the rules laid down, to the extent that conflicts periodically arose. In particular, the Q/A inspectors were persecuted for being inflexible in almost any circumstances, since the new procedures had been incorporated into the SABS 0157 procedures submitted to the bureau for approval. The result was a marked decrease in the number of complaints concerning incorrect computer data.

Daily stock reconcilliations checked production against raw materials drawn. Any cases of stock not found in the stated locations would be investigated before leaving work on the same day. Issues and receipts would be in balance daily and *negative stock balances would be eradicated daily.*

Data File Cleanup

Items that had been dormant for over a year were removed to improve the focus on the balance. Only items previously classed as A, B or C end items or semi-finished designations or items not included in the previous categories but for which stock on hand was available were allowed to remain on file. Stricter control on new item entry was set up to prevent unwarranted proliferation.

Stores Layout and Materials Handling Procedures

Notwithstanding the unconventionality of the open storage areas a return to classical storeroom principles was advocated. Tube stock was subdivided into:

- end items: items which could not undergo further processing, and
- components: items which could undergo further processing, even if these were sold before becoming end items.

End items and components housed in clearly demarcated storage areas were under the control of storemen assigned to each area (and were in the stock file). All WIP items not within storage areas were under the production controller's control (and were not in the stock file). These would have works orders against them.

Although it was impractical to fence off stores to limit access each storeman, who was clearly identifiable by virtue of the colour of his overalls, had the simple responsibility of ensuring the integrity of his physical stock by adhering to his set of procedures and filling out paperwork correctly. He would be issued with daily stock lists and cycle counting instructions. The ultimate intention was to allow each storeman to enter his own stock movement transactions into the computer by means of easily accessible shared data capture points housed within shacks on the shop floor. Apart from the automatic software securities and audits, the usual reconcilliations would be carried out by the materials control clerks who would act on inconsistencies promptly.

A system of "unit loads" was instituted, whereby standard sized bundles of tubing would be made in the production areas prior to the transfer to stores. Thus control of stored pipes became substantially easier and quicker, resulting in vastly improved housekeeping and reduced chaos. Stocktakes, which were frequent, were carried out more accurately and in far less time as a result. The eventual aim was to do away with stock-takes altogether in favour of cycle counting, a practice now accepted by auditing firms.

Customer order procedures were reviewed to ensure fast entry of orders and the accurate reservation of stock. Likewise, invoicing became timeous. The target set for the processing of all data entry was 20 minutes from receipt of the documentation. This target was achieved on several occasions but was difficult to sustain. More work was needed and would have been carried out if events had allowed. The ease with which errors could be reconciled during this period increased significantly, and for the first time in the company's history month-end stock figures were reconciled to 15 tonnes in 8000.

2.2 Production Planning and Control Systems

At this point in the project it was agreed that the prerequisite to the full implementation of MRP II had been successfully completed. Although some refinements were necessary, the solid groundwork had been laid for the next phase. (See FIG.6)

As a result of the takeover mentioned previously a full scale review of operations was requested to determine whether to retain the company as a separate strategic business unit or to rationalise and combine some or all operations to take advantage of certain economies of scale. The project thus lost momentum and had to be rejustified and reviewed in terms of the MIS policies of the takeover company.

A new direction was set for the project at this point. The implementation was to continue as a pilot project incorporating full FORECASTING, MASTER PRODUCTION SCHEDULING, MATERIAL REQUIREMENTS PLANNING, CAPACITY REQUIREMENTS PLANNING and PRODUCTION ACTIVITY CONTROL. Should this prove successful, the project would be expanded and development would continue in the takeover company.

The nature, duration and workload of the rationalisation operations precluded any meaningful progress on the project

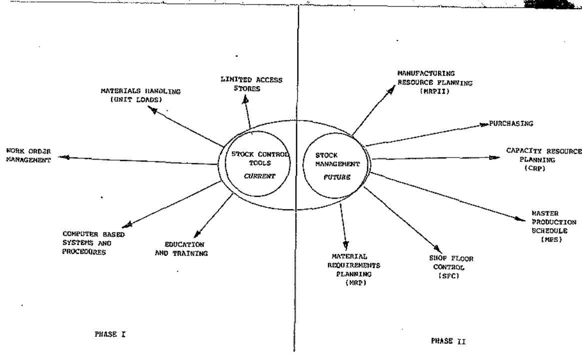


FIGURE 6: A PICTORIAL REPRESENTATION OF THE IMPLEMENTATION PHASES

and development was shelved indefinitely. Thus, all further development described in this report was "extra-mural", and not directly related to the company's operations.

Production Planning of Milling and Processing Operations

The description of operations contained in appendix A demonstrates the split in the nature of the production process. The milling operation consists of the semi-continuous rolling of tube. The output is called "DIRECT OFF MILL" (DOM) tube. This tube can be sold in the DOM state or can undergo one or more operations at which value is added. These operations are carried out in batch mode as in a conventional job shop. It was necessary to effectively address this split in the process.

A further complication was that of long changeover times on the mills. A milling cycle starts with the rolling of a certain diameter of tube. All required gauges (thicknesses) within that tube diameter are rolled sequentially. Changeovers from one gauge to another represent a minor changeover taking about a half hour. Changeovers from one diameter to another requires a complete change of roll tools, an operation taking from 4 to 8 hours, depending on the size range of the mill.

Thus, there was a requirement for long runs at this stage of the process. It was also necessary to forecast demand for the tubes well in advance of the rolling since raw steel delivery lead times ranged from 6 to 8 weeks. Rolling cycle time was approximately 6 weeks.

In order to address these last two issues it was suggested that:

a) raw steel of certain gauges be hedged to increase flexibility and improve response to market requirements. This would shift stockholding from tube to raw steel, however, if a particular gauge of raw steel was in stock, tube of any diameter could be rolled, and

b) an active programme of setup time reduction be pursued. This would imply that smaller rolling runs could be used, and that shorter forecast horisons would be needed. Thus, rollings could relate more closely to actual demand, leading to reduced tube stockholding. (This theory is tested in chapter 3 in detail.)

The question which arose was that of fitting the software to the application described. The textbook concepts of MRP (see bibliography) to which the software adhered, essentially addressed the area of batch production. Thus, the end processing part of the plant was well catered for. But the efficient coordination of milling and end processing needed to be resolved. The following method was tested:

- Demand was created from customer orders in the ORDER ENTRY module and forecasts, entered manually after having been determined from detailed sales forecasting meetings. These meetings were the predecessors of what later became known as MASTER PRODUCTION SCHEDULE meetings. Forecasting took place on an end item level and not on a family level as in the past. Historical sales figures were used to assist in the process. The demand figures included expected sales of both "direct off mill" and end processed tubing.

- Bills of material were developed which caused end processed tubing to generate dependant demand for "direct off mill" tubing. The sum of dependant and independent demands for "direct off mill" tubing were to generate the mill lot sizes.

- Routings were conventional. Virtually all processes were machine governed. Standard processing times were determined by a parametric estimating system which had been developed by us for a standard costing system prior to the MRP II project. By means of activity sampling of milling operations, production and scrap rates for each combination of diameter and gauge of tube were sampled. Three dimensional non-linear regression techniques were used to fit surfaces to the data so that single formulae could be used in programmes to determine production or scrap rates, given the diameter and gauge of the tube.

- The MRP module determined the greater of order demand or forecast in any planning period and generated "planned orders" for this quantity. In addition, orders were created for all the component requirements, including "direct off mill" tube. Discrete order quanities were created for requirements ie. no attempt was made at this stage to calculate economic batch quantities (although MAPICS has a PART PERIOD BALANCING inventory planning facility). One of these order policy codes allowed the entry of user developed code, which was the final destination of the research work to be described in chapter 3.

- Once discrete mill requirements had been calculated as described above, "combine codes" were used to lump common items so as to result in the planning of longer runs. Requirements were combined in 6 weekly buckets as dictated by the mill cycle times. As the setup time reduction programme progressed the intention was to reduce this period accordingly.

- Sequencing of the mill rolling programme had to be done manually at this stage. The planned orders generated by the MRP runs were released to become open orders according to a separately determined schedule which was informally developed according to discussions on priority. Once again the solution to this problem is discussed in chapter 3.

- Once orders were released, the PRODUCTION CONTROL AND COSTING module assumed control. The priority of each order at each work centre was calculated by CRITICAL RATIO, this being the prime prioritising rule provided in the MAPICS software. The accuracy of the time standards available made its use feasible.

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The updating of shop activity in the computer caused all orders to be reprioritised as required, thus ensuring that work was being carried out on the most urgent order at each work centre. The various reports produced by this module enabled us to monitor performance parameters such as standard vs. actual times and costs, work centre efficiencies, and averages and mean absolute deviations of queues (used to balance input to output).

3 THE DEVELOPMENT OF A LOT SIZING TECHNIQUE

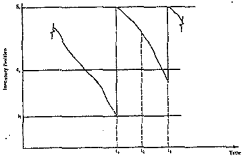
Peterson and Silver (1) in their discussions on stock replenishments at a single echelon, consider the case of items which are interrelated because of the use of a common production facility. Their discussions extend to the case where there is a major setup cost associated with the replenishment of a family of coordinated items and a minor setup cost for each item involved in a particular replenishment. The theory is then extended to the case where items are to be coordinated but the demand is probabilistic.

3.1 A Coordinated Stock Replenishment System

3.1.1 General Description

The system to be considered falls under the generic title of "can-order" systems. It has been specifically developed for the situation where savings in setup costs are of primary concern. It involves continuous review of the stock situation after every transaction and is thus very reactive.

Figure 7 illustrates the principles involved. 'For a given family of items, whenever item i's stock level drops to "s(i)" (called its must-order point) or lower, it triggers a replenishment action to raise item i's level to its order-up-to level "S(i)". At the same time, any other item j within the family with an inventory position at or below its can-order point "c(j)" is included in the replenishment. If item j is included, a quantity is ordered sufficient to raise its level to "S(j)". The idea of having a can-order point is to allow an item j, whose inventory position is low enough (at "c(j)" or lower), to be included in the order triggered by item i, thus eliminating an extra major setup cost that would be likely to occur in the near future due to item j reaching its must-order point. On the other hand, inclusion of item j in the order is not worth while if its inventory position is high enough above its must-order point (ie. above "c(j)")'



At time t_1 this item triggers a replenishment.
At time t_2 some other item in the group triggers a replenishment, but this item is not included because its inventory position is above its can-order point
At time t_3 some other item in the group triggers a replenishment in which this item is now included

FIGURE 7: Behaviour of an Item Under (S,c,s) Control

The (S,c,s) policy does not necessarily minimise the sum of replenishment, carrying and shortage costs. However, it is significantly less complex to achieve good results than a policy that optimises. The cost and practicality of the control system are important criteria from the practitioner's point of view. Peterson and Silver feel that an *(S,c,s) approach achieves a* solution which is close to the best attainable.

In the interest of practicality and usability the author has, for the purposes of this research, developed code from the work of Silver et al, in MICROSOFT BASIC, and has run simulations on an IBM PC-XT. (The original work was done on an IBM 360/75 mainframe).

3.1.2 The Development of Test Code for the IBM PC-XT

The theory of (S,c,s) systems for probabilistic demand has been developed predominantly by Thompson and Silver (2&3). The key criterion was that the system should not involve extensive simulations, but should execute speedily. (previous systems had made use of complex simulations). The reader is referred to references (2) and (3) for detailed study of the theory involved and the results obtained in the extensive tests which were conducted. APPENDIX B contains extracts from these papers.

The object of developing working code on an IBM PC was threefold:

- *the testing of input data specific to the tube sector,*

- the testing of variations of setup times and customer service levels, and the sensitivity of variations in the can-order point,

- the practicality of using the system as a tool in a live environment.

As described previously, a facility is provided within the MAPICS system for the inclusion of a user written lot sizing algorithm, thus making it an integral part of MAPICS.

Conceptual Overview

The code incorporates two separate levels of development in the theory of coordinated replenishment. Firstly, Silver published a paper in 1974 (2) entitled "A control system for coordinated inventory replenishment" in which he describes a practical (no simulations) procedure for determining (S,c,s) limits for the case where demand is Poisson distributed and there is a fixed non-zero replenishment lead time. Although this paper recognises the probabilistic nature of demand, it deals only with unit sized demand transactions.

The case of compound poisson demand becomes significantly more complex. (The use of Poisson demand is discussed under "Limitations of the Model" in section 3.2). In order to keep the system within the bounds of practicality, the data on actual transaction size distribution is converted to a unit sized equivalent in which the mean and variance are the same as those in the original data. Using this technique, values of S and c are calculated, with s set at zero. This involves a one-dimensional search on the discrete variable c to find the value of c which minimises the cost equation (setup, carrying and shortage costs). (In order to reduce processing time on the IBM PC it was necessary to do two search passes, a gross search with stock level increments of 20 and a detailed search with an increment of one around the area of lowest cost. This reduced the search time from 50 to 3 minutes).

At this point Silver's S and c limits are converted back to compound poisson units and these are then increased in increments of one through successive iterations until the chosen customer service level is met. This involves solving a set of simultaneous equations, each of which represents the probability of the item having a certain stock level after a demand transaction, during each iteration. Once the service levels are met, the TOTAL EXPECTED ANNUAL STOCK COSTS , the AVERAGE ORDER QUANTITY PER CYCLE, the EXPECTED STOCK LEVEL, the EXPECTED NUMBER OF REPLENISHMENT OPPORTUNITIES PER YEAR and the CUSTOMER SERVICE LEVEL are reported, along with the resulting (S,c,s) limits.

The developed code was checked using the data from the examples published in (2) and (3). (Results correlated to 4 decimal places.) A full printout of the code is presented in appendix C. The execution time with the test data is 45 minutes.

3.1.3 Published Conclusions

The major effects of (S,c,s) control have been studied through comparisons between the (S,c,s) model and simulation tests involving 64 examples over 4,7 years. The following conclusions have been published (see APPENDIX B):

- For typical values of input data (discussed in the next section) the average cost savings over independent control are in the region of 15 to 20 percent. (Note that in planning rolling cycles for tube mills, comparisons with independent control are irrelevant since rollings are planned on a coordinated basis. This will be discussed in the next section).

- The cost savings increase as the ratio of major to minor setup costs increases.

- The cost savings improve as the number of items in a family increases.

- The percent cost savings tend to diminish as the required service level increases. (A large safety stock dominates either type of control).

- Somewhat surprisingly, for a fixed service level, the cost savings are quite insensitive to the length of replenishment lead time.

- Coordination tends to substantially lower the order-up-to level. This is because under coordination the average setup cost associated with the replenishment of an item tends to be lower, thus the replenishment quantities are lower.

- Coordination, to a lesser extent, lowers the must-order point. Under coordination, an item is often reordered when its inventory level is substantially above s. Hence s can be lowered compared to the case of independent control, while still providing adequate service.

- The major impact of coordination is on the lower usage value items.

- The cost savings increase as the variability of the transaction size distribution decreases.

3.2 (S,c,s) Control System Tests in the Case of Tubemills

A wide ranging series of tests was conducted on the IBM PC-XT, covering various aspects of tube mill lot sizing. An account of these follows.

The tube milling operation requires that steel coils of flat plate be slit into strips suitable in width for the rolling of tubes o the required diameter. These steel strips are then passed through a series of rolls which allow the metal to be formed into circular shape until it gradually resembles a tube. At this point, the opening is fused by a process called induction welding and the tube is cut to the preset length while the mill is in motion, by a travelling saw. (See appendix A for more details).

Loading of slit strip onto the mill is carried out in such a way that the mill is not stopped. This is achieved by means of a device called an accumulator which maintains a buffer length of strip long enough to allow another coil to be loaded and welded to the end of the one being used. Thus, the mill is capable of running continuously as long as required, unless a breakdown occurs.

In order to change the run from tube of one gauge to another, keeping the diameter constant, requires that a strip of material with the new gauge be welded on. This strip would have a width very close to the previous one for a set diameter. All that is now required is to make minor adjustments at various points on the mill, while it is running, and a tube of the same diameter but different gauge will be produced. This is regarded as a "minor" setup.

However, to change over from one diameter tube to another requires all roll tools to be changed, an operation taking from half to a full day in typical plants. This is regarded as a "major" setup. A family would comprise all items within a particular diameter, irrespective of gauge.

The theory developed by Thompson and Silver et al is perfectly suited to this environment. It must be emphasised, however, that comparisons of cost savings must not be related to the case of independent control, since no matter how badly a mill rolling cycle is planned, there is always a significant attempt at coordination.

Under this pretext, once the code had been validated, controlled changes were introduced into the value of the can-order point c to resemble more closely the planning techniques used in the rolling mills. This formed the basis for the first batch of tests.

Typically, a planner would run all items in a family when the cycle called for that family to be run, unless he intuitively knew that stocks of an item were still abnormally high. This could almost be regarded as a crude form of the (S,c,s) control system. This methodology results in an inherent stability in the length of the rolling cycle, notwithstanding any major, sudden change in demand patterns.

To simulate this situation the value of the can-order point c was set equal to the order-up-to level S. This implies that the first item in the family to trigger off a replenishment would cause all items in the family to be replenished as well.

The tests were carried out using the same basic data as used by Thompson and Silver in their examples. (Ideally, if it were possible, live operational data from the tubemill itself would have been used). The basic input information is readily available from computerised administrative systems and is as follows:

- Major and minor setup costs,

- Item standard costs,

- Inventory carrying rate,

- Number of orders taken per year (from customer order history data), for a particular diameter and gauge combination,

- Number of units (tonnes or any relevant convenient measure) in each order (from customer order history data).

The last two are used to calculate the transaction size distribution for each item.

Various intermediate values of c were also tested, and compared to c=S and the unchanged value (designated c(best)). These were based on the AVERAGE ORDER QUANTITY PER CYCLE which is calculated by the model (see FIGURE 8). The significance of this is that an item is replenished only if its stock level is below a predetermined "number of days cover". The AVERAGE ORDER QUANTITY PER CYCLE (AOQPC) relates to the AVERAGE DEMAND PER CYCLE. Thus, if we set c to 3/4 x AOQPC then we replenish if the stock level is below 3/4 of the average demand between now and the next expected rolling of that item.

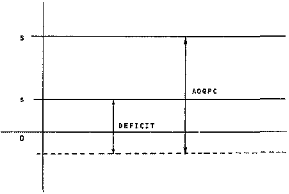


FIGURE 8: AVERAGE ORDER QUANTITY PER CYCLE

The results obtained were as follows (refer to APPENDIX D):

TABLE 1: OVERALL COSTS FOR VARIOUS VALUES OF c.

ITEM NO.	c=s	c=25% AOQPC	c=50% AOQPC	c(best)	c=73% AOQPC	c=100% AOQPC	c=S
1	214	206	203	202	202	202	206
2	36	25	21	20	20	22	32
3	85	72	66	64	64	64	90
4	82	68	62	60	60	61	80
TOTAL	417	371	352	346	346	349	409
% INCR	20,5	7,2	1,6	-	0	0,9	18,2

Note:

- The value of c calculated by the model is optimum in each case.

- The best value of c approximates 73% of the average order quantity per cycle.

- The values display very low sensitivity over a wide range - (50% x AOQPC to 100% x AOQPC)

Based on the results in TABLE 1 it is reasonable to assume that savings of between 0,9% and 18,2% could be expected by using the model. Furthermore, customer service levels would be above the preset level. (In these tests the service level was set at 95%).

Notice that the difference between major and minor setups in tube mills is much larger than the value used. Also, families are typically in the order of 10 items. Both these considerations should improve results.

The fact that the model assumes that replenishment lead time equals zero is significant. In the above case the assumption applied to all tests. Therefore, comparisons were possible. In fact, the system with relatively more replenishments is favoured by having the replenishment lead time equal to zero since there is less probability of stockouts. Therefore, in the comparison between c=c(best) and c=S, the latter is favoured.

Limitations of the Model

The use of mathematical models to represent reality invariably implies the necessity for assumptions in order to reduce the complexity of the model. This practice imposes the need to interpret the output of the model by considering all the aspects which may result in faulty conclusions.

The most severe limitation is the assumption that there is a zero replenishment lead time on the mills. The significance of this is that whenever an item's stock level falls below the must-order point there is an immediate changeover to that family. In practice we cannot just change over, and, having done so, it takes from one day to a week or more to run an entire family. If the data fed to the model is accurate then it is not expected that stock will be depleted before the next family cycle. However, since the model is reactive in nature rather than of the "periodic review" sort, cycle lengths are dynamic.

An important test would be to test the stability of the cycles in a live environment. Large levels of cycle time instability are not expected since all items within a family will have stock levels above the can-order point just after a replenishment run. In general, the next item to trigger a replenishment would probably be the one that was above and nearest the can-order point at the last run.

A further factor which would affect stability is the nature of the demand of each item in the family. The more sporadic the demand, the more cycle instability is expected. This implies more prematurely triggered replenishments and therefore more problems with the assumption of zero replenishment lead time.

Although it is naturally considered that further research be done into this aspect, it is possible to incorporate features into the model to assist in stabilizing the cycles. It is also possible to set rules for interpretation of results, or to set policies for running sporadic demand items on separate facilities.

For ex[illegible] could be possible to stabilize cycles by not trigger[illegible] plenishment unless at least a predetermine[illegible] umber of items are [illegible]e for replenishment at the triggering stage [illegible]his would also depend on the importance of the item or customer for which it is destined).

Notice that items with sporadic demand or for one-off orders can be incorporated by inserting "no. of orders / year" = 1, and an expected order size equal to the expected or actual order quantity.

The important thing is that "what-if" tests could be run to determine the overall effect of accepting one-off orders or of marketing sporadic demand items. If these items are needed to provide a complete range of products, it would be possible to compare the cost of buying the item from another producer to that of producing in-house.

The overriding consideration in the above is that a typical mill can run about 10 families (diameters), each with about 10 items, ie. 100 items in all. These are defined purely by diameter and gauge. However, saleable end items are of various lengths, material grades and end finishes. When the demands for each individual end item are combined into a mill designation (diameter and gauge), the variability of the demand is considerably reduced. The control of the saleable end item production and stocks is left to conventional MRP II systems. Also, the model assumes Poisson demand which approaches reality for many small orders. Since there are numerous small customer orders for saleable end items which make up the mill requirements a Poisson distribution of demand is acceptable.

For the purposes of input data to the model, two key requirements exist. These are moving averages of NUMBER OF ORDERS TAKEN PER YEAR and NUMBER OF UNITS REQUIRED PER ORDER, for each mill specification (diameter and gauge). Although a forecast of this data for the next period or periods is not required, it may prove to be a useful activity.

3.3 Practical Applications of the Model

For a given set of input parameters the model will predict the total relevant stock costs for the year. More importantly, it can be used to test various alternative courses of action:

- Should the major and minor setup costs be reduced, what is the effect on total relevant stock costs?

- What effects arise from altering the required customer service levels?

- If we were to focus our marketing attentions on a product mix with certain ordering frequencies and transaction size distributions (or order sizes), what profitability benefits could we expect?

- The model can be enhanced to assist in setting marketing and business strategies and in the formulation of manufacturing strategy.

Setup Time Reduction

By altering the values of major and minor setup costs and keeping all other input data constant the effects of setup time reduction were studied. The key results are reproduced in TABLE 2.

% REDUCTION IN SETUP COSTS	% REDUCTION IN OVERALL COSTS
20%	10%
50%	28%

These results confirm the previously discussed hypothesis that reductions in setup time holds major advantages for the tube industry.

An analysis of the results presented in appendix D shows that as the setup costs are reduced, there are proportionate decreases in average stock levels - E(I), average order quantities per cycle - AOQPC and overall costs - EC(I), as well as an increase in the number of replenishment opportunities per year - MU(I).

A further factor not considered is the decrease in uncertainty provided by the reduced cycle times, and the resultant increase in flexibility of operations including reduced delivery or manufacturing lead time, especially useful in the case of make-to-order demand.

The above factors point to:

- increased sales.

- reduced operating costs and asset base.

- increased operating flexibility.

These are all vital for achieving improvements in RETURN ON ASSETS MANAGED - our primary measure of performance.

The above results, considered jointly, show that total cost reductions above 40% are possible, which, on an arbitrary stock base of R10 million results in an increase in working capital of R4 million (certainly sufficient to justify a continuation of this research). This would easily justify the purchase of complete sets of duplicate rolls and fixtures and required materials handling equipment as part of a drive to further force down setup times. (This would have to be accompanied by the use of the Japanese tubemill approaches discussed in chapter 1 for the presetting or automation of mill settings to reduce "startup" yield loss (particularly because material costs account for approximately 85% of standard costs).

4 THE HUMAN FACTOR

Because of the takeover which resulted during the project implementation two very diverse forms of management styles became evident. Several useful lessons have emerged. This chapter discusses these and provides recommendations which are considered generally applicable in the implementation of computerised manufacturing systems.

It has become abundantly clear through the writings of Peters, Waterman and Buffa et al that the economies of scale which are theoretically obtainable by centralising operations are, in practice, overshadowed by the increased complexities of managing larger business units. These writers provide ample evidence that the duplication of effort and even product range which could result from decentralising operations is more than offset by the increased flexibility, simplicity and entrepreneurship gained.

In this case, an intensive rationalisation exercise, prompted by the reducing level of economic activity nation-wide, resulted in the decision to centralise all activities of the bought out company, which had been a profitable and expanding unit.

The relative size of the takeover company implied that all systems and procedures would naturally be implemented within the smaller unit. All staff policies had to be made uniform, and management techniques were imposed on the smaller unit at all levels.

The results were as follows:

- All "unneeded" staff were retrenched.

- Many of the remaining staff members received cuts in their remuneration.

- Virtually all members of the management team, including the entire board, sought employment elsewhere.

- All incentive schemes were removed.

- All development work was reviewed. (A decision was made to continue with the MRP II project on a pilot basis).

- The sales/marketing and accounting functions, as well as all planning was centralised.

- Stocks were taken over.

The net effect of the decision is not known, however, what is certain is that, generally the takeover was one of assets only, certainly not the total market share, or highly skilled people. All the new ideas and progressive techniques painstakingly developed were lost since very little in depth investigation was done. Virtually all major strengths were lost.

The irony is that any other outcome would have been potentially more destacilising because of the diverse nature of the people and values. Even the retention of the company as a strategic business unit, with all its possible advantages, would have stirred up untold complications at head office level, since even personnel policies would have had to be separate.

The subject of the debate is, given the above circumstances, what would be the best alternative for the group as a whole?

The most fascinating observations on personality traits which emerged daily were almost universally linked to what Peters and Waterman refer to as AMBIGUITY AND PARADOX. When the large picture is overlooked things are very often not what they seem to be.

For example, managers are very quick to install meticulously contrived work centre efficiency measures which they will monitor to the utmost. It seems a superfluous exercise, though, when chaos reigns between centres, with badly planned material flow, extremely high work in progress levels and bad work priorities.

Detailed rationalisation all too often involves paper clip counting exercises at the expense of focus on strategic issues.

At the other extreme, strategic issues are often inadequately considered. For example, long term planning indicates that in order to achieve promised dividends or a preset "bottom line" it is necessary that stockholding be reduced by x million. This becomes policy. Hence, all materials staff cut purchases and production, thereby increasing shortages and lost opportunity, since planning systems are inadequate and have, in any case, not been addressed.

These seemingly irrational occurrances were observed with monotonous regularity at all levels.

Having made these observations it is not easy to state a solution to the problem, which is by no means unique. It is rampant in Western manufacturing circles. A vast "new wave" of theory is emerging, on how to deal with problems of the nature described above. The key universal theme is SIMPLIFICATION. The overriding question is how to disseminate the findings to a largely badly informed management.

On a more mundane level, the importance of people in productivity issues is reinforced by observing the dramatic increase in production levels that can be achieved by introducing large monetary incentives. Although there is intensive ongoing debate about the long term effects of such schemes, the issue is that the increases in performance achieved make a mockery of activity sampling, detailed work study and standards setting. The focus should be on finding successful ways of motivating people and on visibly acknowledging the importance of their role. One factor that emerged very solidly, irrespective of incentive schemes, is that, in general, an over disciplined peoples approach is counterproductive.

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary and Conclusions

The great challenge which manifests itself in the implementation of computer based business control systems in a tube rolling mill is that of achieving lower production costs than competitors for this largely undifferentiated product. Elwood S. Buffa, in his book "Meeting the Competitive Challenge" (8), displays evidence that in order to achieve high operating margins relative to the other companies operating in a particular sector, a company must adopt a manufacturing strategy so as to position itself in one of the following ways:

- high volume, high product availability and low cost production,
- high quality, innovative design and flexible response to the marketplace, or
- meet special needs of a particular market niche or providing lower costs for that segment or both.

Buffa further concludes that a company whose position is unclear in terms of the above has dubious focus in its operations, and is hence likely to suffer the penalty of lower operating margins.

Local tubemills seem to suffer from this lack of focus. They strive for high volume, as expected because of the lengthy changeover times required; they strive for high product availability because of the fierce competitiveness and product similarity; they attempt to provide flexible response to accomodate customers who buy large volumes of certain lines and low volumes of others; they create market segmentation with special mining options and the like.

The argument put forward by the management teams is that such a strategy is necessary in the relatively small local market and highly variable export market. The author is not in a position to test this hypothesis, however, it remains evident that low cost production is imperative in order to maintain adequate margins. Low cost production is not guaranteed by squeezing the last ounce of efficiency from each particular work centre. It is also imperative to consider criteria such as product mix as well as the flow of material between centres, criteria which often do not receive as much attention as personnel productivity audits or "counting paperclips", undertaken by Industrial Engineering departments.

The research work described in this report represents a two pronged approach to lowering operating costs or providing increased profitability. On the one hand, stable, well tested and advanced integrated software systems available "off the shelf" were used to provide the essential information base necessary for assisting in operational decision-making at all levels. On the other, specialised techniques, computerised and manual, were developed in order to provide an increased competitive edge.

5.1.1 Systems development

The selection of a well proven computer system and software, although being a prerequisite for successful business control system implementation, is by no means sufficient. Setting up the company infrastructure required to adequately support the functioning of the software is crucial, and considerably more difficult.

First and foremost, effective operational decision-making requires that all aspects of stock management be done well, this being the nucleus of all operations. The approach adopted in this particularly difficult application proved to be totally successful. The key activities were:

- limiting access to stores,
- handling and storing bundles of tubes in "unit loads",
- redesigning and reprinting company documentation to fit the new systems concepts,
- documenting all systems and procedures,
- creating a well defined accountability structure for all personnel and all activities,
- thoroughly educating and training all involved staff,
- providing strict DP security,

- developing auditing systems and watchdog reports for all computerised data,
- categorising stock into various classifications, each with its own management techniques,
- developing first cut systems to balance stock levels,
- the development of systems to control the movement of work in progress.

The final outcome of all this activity was that month end stock accounts were reconciled to 15 tonnes in 8000 repeatedly, a result which surpassed all expectations.

In the next phase, the anticipated problem of marrying the MRP based software concepts to the mixture of semi-continuous and batch operations was tackled. End item forecasts and customer orders were used to generate demand on the production facility. The greater of the two was used to determine in advance the raw steel requirements and the capacity requirements. Demand on the mills was both dependant (for items requiring value adding activities at other centres), and independent (for items sold in the "direct off mill" form). Requirements for each diameter and guage combination were lumped into time buckets according to a predetermined schedule sequence.

Although these techniqu[illegible] ted on a pilot basis only because of the takeover an[illegible] quent curtailing of the project, indications were that materials and production reporting and control could be significantly improved.

A serious drawback which had to be addressed was that of determining minimum lot sizes to be run on the mills so as not to create severe losses in yield due to frequent setups. Also the drawing up of the schedule sequence was manual, and based on forecasts drawn up informally in sales/master production scheduling meetings. This presented a special challenge whose solution was not to be found in the standard software. It was this inadequacy which led to the major research work on a lot sizing technique, which followed.

5.1.2 The Development of a Lot Sizing Technique

The decision to embark on this research work was instigated by a search, using the University's "on-line search" facility and various other means, for literature on the determination of efficient lot sizes for tube mill rolling cycles. Although literature specificall. related to tube mills did not cover this area, theory developed by PETERSON and SILVER (1) et al was uncovered, which provided a suitable base.

Their work on stock replenishment at a single echelon deals with the case of items which are interrelated because of the use of a common production facility. They consider the case where there is a major setup cost associated with the replenishment of a family of coordinated items, and a minor setup cost for each item involved in a particular replenishment. The work contained herein is based on probablistic demand with arrivals of demand transactions occuring according to POISSON distribution.

The system's generic classification is "continuous review - can order". As each transaction is entered the item's stock level is retrieved. If this is below a "must order point" calculated by a mathematical model which simulates the movement of the overall stocking costs (replenishment, carrying and shortage) then all other items in the same family are reviewed. Those whose stock levels are below a "can order point", also calculated by the model, are marked for replenishment with the transaction item.

As a result, regular, repetitive milling cycles are replaced by reactive and responsive cycles which, although not minimising overall stocking costs, will, with the use of relatively little computational burden and complexity, achieve results very close to optimum.

For the purposes of this research, the model developed by THOMPSTONE and SILVER (3) was coded in Microsoft Basic on an IBM PC-XT. The objectives were:

- to test input data specific to the tube sector,
- to test the effects of varying setup times and customer service levels, and the sensitivity of variations in the "can order point", and,
- to examine the practicality of using the system as a tool in a live environment.

The mutually exclusive results of the simulations carried out are as follows:

- Compared to the conventional techniques used in mill production

planning, savings in stocking costs of up to 18,2% can be expected by using the model.
- For a 20% reduction in setup costs there occurs a 10% reduction in overall stocking costs.
- For a 50% reduction in setup costs there occurs a 28% reduction in overall stocking costs.
- Lowering the customer service level has little effect on the model's results.
- The model's execution time on the IBM PC-XT with the test data was 45 minutes. Since the model would be used to generate "must order", "can order" and "order up to " levels for each item, which would remain reasonably static, it is expected that weekly runs would be sufficient. Systems and procedures would have to be developed to transfer the data to the main database.

Considered jointly, these results indicate that stocking cost savings close to 50% overall are achievable through a coordinated effort.

To place these results in perspective the following factors need to be considered:
- The model assumes a zero replenishment lead time on the mills, ie. when an item falls below its "must order point" there is an immediate changeover to that family. If the data input to the model is sound, stock depletions are not expected until the following cycle.
- sporadic demand items may induce permature triggering of replenishments, which would then be shorter than usual, but still have the effect of destabilising the cycles. Techniques could be

developed to stabilize the cycles.
- The model assumes POISSON demand. This is considered acceptable in view of the large number of small customer orders taken in the sector in general.

The overriding consideration remains that a typical mill can run about 10 families (diameters) each with about 10 items (diameter and gauge combinations). However, saleable end items come in various lengths, finishes and material grades. When demands for each individual end item are combined into a mill designation (diameter and gauge only), the variability of demand is significantly reduced, as is the effect of the above constraints. Ultimately, field tests are required to understand with certainty the true performance characteristics of the model.

Furthermore, Thompstone and Silver (3) state that:
- cost savings increase as the ratio of major to minor setup costs increases,
- cost savings increase as the number of items in a family increases,
- for a fixed service level, the cost savings are quite insensitive to the length of replenishment lead time,
- coordination tends to substantially lower the "order up to level" and the "must order point" compared to classical order point systems,
- the major impact of coordination is on the lower usage items,
- cost savings increase as the variability of the transaction size distribution decreases.

5.2 Recommendations for Future Work

The preceeding comments provide a vast scope for developing the theory and testing it in live environments with expectations of lucrative reward. In addition to this the following ideas are proposed:

- expansion of the theory to coordination across more than one mill,
- expansion of the theory to the coordination of families.

The exciting prospect arises of utilizing this theory to develop and test manufacturing strategies in general, and JUST IN TIME strategies with mixed model production and flexible cellular manufacturing in particular. Given the vast potential for increasing working capital and reducing interest payments on capital tied up in unbalanced and superfluous stocks, further research work along these lines must surely be easy to justify.

APPENDIX A: A DESCRIPTION OF MILLING OPERATIONS AT THE COMPANY

The milling operation consists of the semi-continuous rolling of tube. The output is called "DIRECT OFF MILL" (DOM) tube. This tube can be sold in the DOM state or can undergo one or more operations at which value is added. These operations are carried out in batch mode as in a conventional job shop. There is thus a split in the process into semi-continuous and batch processing. (See FIGURE A1 for a pictorial representation and FIGURE A2 for a schematic diagram of the actual plant layout).

A further complication is that of long changeover times on the mills. A milling cycle starts with the rolling of a certain diameter of tube. All required gauges (thicknesses). within that tube diameter are rolled sequentially. Changeovers from one gauge to another represent a minor changeover taking about a half hour. Changeovers from one diameter to another requires a complete change of roll tools, an operation taking from 4 to 8 hours, depending on the size range of the mill.

Thus, there is a requirement for long runs at this stage of the process. It is also necessary to forecast demand for the tubes well in advance of the rolling since raw steel delivery lead times ranged from 6 to 8 weeks. Rolling cycle time is approximately 6 weeks.

The tube milling operation requires that steel coils of flat plate be slit into strips suitable in width for the rolling of tubes of the required diameter. These steel strips are then passed through

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The tube milling operation requires that steel coils of flat plate be slit into strips suitable in width for the rolling of tubes of the required diameter. These steel strips are then passed through a series of rolls which allow the metal to be formed into circular shape until it gradually resembles a tube. At this point, the opening is fused by a process called induction welding and the tube is cut to the preset length while the mill is in motion, by a travelling saw.

Loading of slit strip onto the mill is carried out in such a way that the mill is not stopped. This is achieved by means of a device called an accumulator which maintains a buffer length of strip long enough to allow another coil to be loaded and welded to the end of the one being used. Thus, the mill is capable of running continuously as long as required, unless a breakdown occurs.

In order to change the run from tube of one gauge to another, keeping the diameter constant, requires that a strip of material with the new gauge be welded on. This strip would have a width very close to the previous one for a set diameter. All that is now required is to make minor adjustments at various points on the mill, while it is running, and a tube of the same diameter but different gauge will be produced. This is regarded as a "minor" setup.

However, to change over from one diameter tube to another requires all roll tools to be changed, an operation taking from half to a full day in typical plants. This is regarded as a "major" setup. A family would comprise all items within a particular diameter, irrespective of gauge.

FIGURE A3 shows a flowchart that was originally used in presentations to describe the Production Planning and Materials Control requirements to management.

FIGURE A4 was used to show the role of Master Production Scheduling in the overall system.

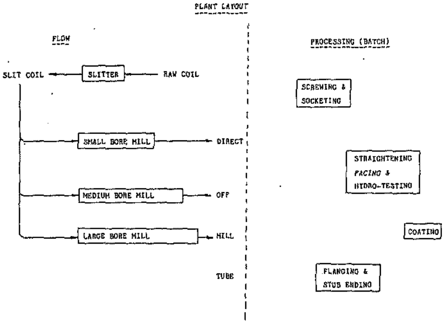
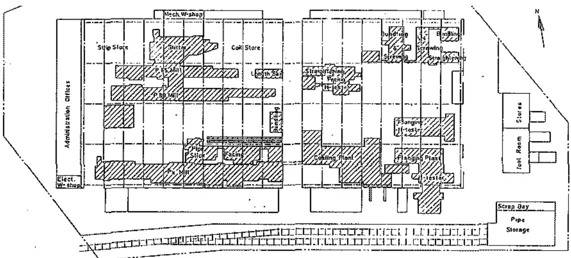
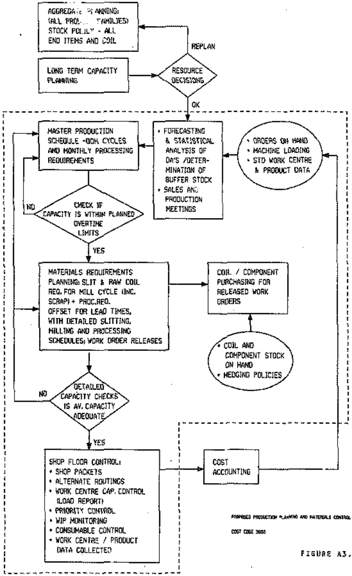


FIGURE A1: PICTORIAL REPRESENTATION OF PLANT LAYOUT

FIGURE A2: SCHEMATIC DIAGRAM OF PLANT LAYOUT



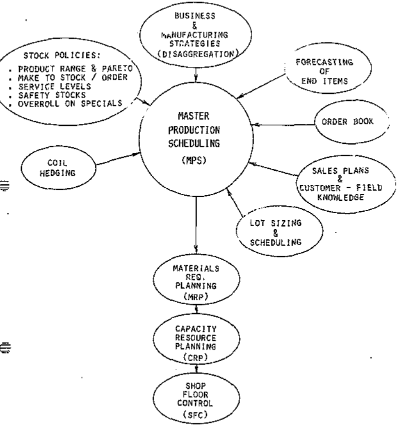
PROPOSED PRODUCTION PLANNING AND MATERIALS CONTROL SYSTEM



COST CODE 3605

FIGURE A3.

MRP II NUCLEUS - MASTER PRODUCTION SCHEDULING



MPS PROVIDES:

- INTEGRATION OF EPARTMENTS (COMPANY GAMEPLAN)
- VISIBILITY INTO THE FUTURE
- ACCOUNTABILITY
- MANAGEMENT OF CHANGE
- "WHAT-IF" SIMULATION

FIGURE A4: THE ROLE OF MASTER PRODUCTION SCHEDULING

APPENDIX B:

PAPERS ON COORDINATED REPLENISHMENT

Coordinated Replenishments at a Single Echelon

In the preceding chapter we discussed control procedures in a multi-echelon (or multi-stage) situation where the replenishment quantities at the different echelons were coordinated. Now we turn to a somewhat different case where coordination is appropriate, namely where all the items involved are stored at the same stocking point and the interrelationship is caused by

i. a common supplier,

ii. a common mode of transportation, or

iii. a common production facility on which the items are produced.

As we shall see in Section 13.1 there are several possible advantages in coordinating the replenishments of such a group of interrelated items. The potential disadvantages will also be discussed. In Section 13.2, for the case of deterministic demand, the economic order quantity analysis of Chapter 5 (which there assumed *independent* control of items) is extended to the situation where there is a major setup cost associated with a replenishment of a family of coordinated items and a minor setup cost for each item involved in the particular replenishment. Section 13.3 extends the quantity discount arguments of Chapter 5 to the case where the discount is based on the magnitude of the total replenishment of a group of items, for example, a freight rate reduction if a carload size replenishment is achieved. Then in Sections 13.4 and 13.5 we turn to the complex situation where items are to be coordinated but demand is probabilistic. Essentially, two approaches are discussed. Section 13.4 deals with so-called "can-order" systems which normally are of a continuous review nature and are not concerned with attaining a specified total replenishment size. In this type of system when the inventory of any item of a coordinated group drops low enough, a replenishment is triggered. Whether or not each other item is included in the replenishment is dictated by how low its inventory level happens to be at that moment. In contrast, the "service-point" approach of Section 13.5 involves periodic review, and the decision of whether or not to place a replenishment order at a particular review is dictated by the group service implications of waiting until the next review. Furthermore, the total replenishment size is normally established to achieve a quantity discount.

It should be noted that in the literature the terminology "joint replenishment" is sometimes used in lieu of "coordinated replenishment."

13.1 *Advantages and Disadvantages of Coordination*

There are a number of reasons for coordinating items when making replenishment decisions. These include:

i. Savings on unit purchase costs: When a group of items is ordered from the same vendor a quantity discount in purchase cost may be realized if the total order is greater than some breakpoint quantity. It may be uneconomical to order this much of a single item, but it could certainly make sense to coordinate several items so as to achieve a total order size as large as the breakpoint. An example of such a situation would be the acquistion by a distributor of a line of steel products from a particular manufacturer. In some cases a vendor-imposed minimum order quantity may dictate the same sort of joint consideration.

ii. Savings on unit transportation costs: The discussion is basically the same as above. Now a grouping of individual item orders may be advisable to achieve a quantity such as a carload. A good example, observed by the authors, is the shipment of cereal products from a supplier to the regional warehouses of a supermarket chain. The MIDAS situation (Case A) provides another example—several items from the parent company in Germany are simultaneously shipped to Canada in the same container.

iii. Savings on ordering costs: In some cases where the fixed (setup) cost of placing a replenishment order is high, it might make sense to put several items on a single order so as to reduce the annual total of these fixed costs. This is likely most relevant where the replenishment is by in-house production. In such a case the major component of the fixed ordering cost is the manufacturing setup cost. An illustration would be in the bottling of beer products. There are major changeover costs in converting the production line from one quality of beer to another. In contrast, the costs of changing from one container type to another are rather minor.

iv. Ease of scheduling: Coordinated handling of a vendor group can facilitate scheduling of buyer time, receiving (and inspection) workload, etc. In fact, we have found that, by and large, managers and purchasing agents alike tend to think and deal in terms of vendors or suppliers rather than individual s.k.u.

On the other hand, there are possible disadvantages of using coordinated replenishment procedures. These include:

i. An increase in the average inventory level: When items are coordinated some will be reordered earlier than if they were treated independently.

ii. An increase in system control costs: By the very nature of the problem, coordinated control is more complex than independent control of individual items. Therefore, under coordinated control review costs, computational costs, etc., are likely to be higher.

iii. Reduced flexibility: Not being able to work with items independently reduces our flexibility in dealing with unusual situations. One possible result is reduced stability of customer service on an individual item basis.

13.4 *Probabilistic Demand: Can-Order Systems*

For the moment we shall discuss two distinct ways of coping with probabilistic demand when coordinating the replenishments of a family of items. The first of these, the so-called "can-order" system, which will be discussed in this section, is specifically geared to the situation where savings in setup costs are of primary concern (for example, where several products are run on the same piece of equipment) as opposed to achieving a specified total replenishment size (for quantity discount purposes). It involves continuous review (transactions recording). The second type of system, to be discussed in Section 13.5, uses the so-called "service-point" approach. It involves periodic review and is particularly suited to the situation where the primary concern is with achieving a specified total replenishment size (for quantity discount purposes). A third intermediate type of system is possible, where each item has a periodic review, order point, order-up-to-level (R, s, S) type of control but with all items of the group having the same review interval. The method of computing appropriate values of R, the s's and S's hinges upon an approximate procedure (Ehrhardt[11] or Naddor[24]) developed for the case of a single item (R, s, S) system, an advanced topic to be covered in Chapter 14. Therefore, we shall return to this third type of system at that time.

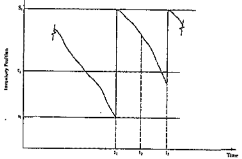
As one would expect from earlier comments, the development of decision rules for coordinated items under probabilistic demand is anything but a simple task—the logic is quite involved. For this reason, in both this section and Section 13.5 we attempt to present only the basic concepts of each system. The reader interested in further details is encouraged to make use of the references provided.

13.4.1 *The Physical Operation of an (S, c, s) System*

Balintfy[3] was the first to propose the use of an (S, c, s) system, a special type of continuous review system for controlling coordinated items. In such a system, whenever item i's inventory position drops to s_i (called its must-order point) or lower, it triggers a replenishment action so as to raise item i's level to its order-up-to-level S_i. At the same time any other item j (within the associated family) with inventory position at or below its can-order point c_j is included in the replenishment. If item j is included, a quantity is ordered sufficient to raise its level to S_j. The idea of having a can-order point is to allow an item j, whose inventory position is low enough (at c_j or lower), to be included in the order triggered by item i, thus eliminating an extra major setup cost that would likely occur in the near future due to item j reaching its must-order point. On the other hand, inclusion of item j in the order is not worthwhile if its inventory position is high enough above its must-order point (that is, above c_j). The behavior of a typical item under such a system of control is shown in Figure 13.2.

Ignall[10] has shown that an (S, c, s) policy does not necessarily minimize the sum of replenishment, inventory carrying, and shortage costs. However, the policy that would minimize these costs would be considerably more

Figure 13.2 Behavior of an Item Under (S, c, s) Control.



At time t_1 this item triggers a replenishment.

At time t_2 some other item in the group triggers a replenishment, but this item is not included because its inventory position is above its can-order point.

At time t_3 some other item in the group triggers a replenishment in which this item is now included complex than the (S, c, s) strategy. Therefore, when one properly takes account of the system control costs, it is felt that an (S, c, s) approach achieves a solution which is close to the best attainable.

13.4.2 The Essence of a Suggested Procedure for Computing Values of the Order-up-to-Levels (S_i's), Can-Order Points (c_i's) and Must-Order Points (s_i's)

One of the authors (Reference 31) has developed a reasonable procedure for ascertaining values of the S's, c's, and s's. The derivation is based on a set of assumptions, the most severe of which are:

> i. The replenishment lead time is of constant known length. Furthermore, its length does not depend upon which subset of the items of the family are involved in the replenishment.
>
> ii. Demand for each item is of a Poisson nature where, of course, each item can have a different demand rate. The Poisson distribution is an approximation to reality, most appropriate for the case of many small customers. The assumption of Poisson arrivals of individual customer transactions is reasonable. However, transactions of greater than unit size would result in what is known as a compound Poisson distribution of demand (an extension of the procedure to handle this more general case, but for the case of a negligible replenishment lead time, is given by Thompstone and Silver[36]).

For each item i there are three quantities (S_i, c_i, and s_i) to specify, that is, if the family has 10 items, there are 30 interrelated control variables that must be given values. In Chapters 5, 6, and 7 we advocated first determining Q (or R) and then finding s (or S) conditional on the specified value of Q (or R) rather than attempting to simultaneously select the (s, Q) or (R, S) pair. Again here a sequential approach is used. The S's and c's are found, by an approximate iterative procedure, for the case of a negligible replenishment lead time (each s is zero in this case), then, conditional on these S and c values, one finds the lowest s_i which satisfies a prespecified service constraint for the particular item i (in contrast with the approach to be discussed in Section 13.5, here the desired service level can vary from item to item). As earlier we know that the S's, c's, and s's so obtained will not strictly minimize the sum of replenishment and carrying costs. However, the cost penalty is likely to be low and we are willing to absorb it in order to have a computationally feasible scheme for evaluating the control parameters.

13.4.3 *Effects of (S, c, s) Control*

A Numerical Illustration—To indicate the effects of coordinated control on the control parameter settings and the expected total relevant costs we present the four-item example of Table 13.3. No details of the derivation of the parameter settings are given. (These are available in Reference 31.) Examination of Table 13.3 reveals the following points (typical of all examples tested):

i. Coordination tends to substantially lower the order-up-to-levels (the S's). This is because under coordination the average setup cost associated with the replenishment of an item tends to be lower, thus we need not order as much each time we replenish.
ii. Coordination, to a lesser extent, lowers the must-order points (the s's). Under coordination, an item i is often reordered when its inventory level is substantially above s_i, hence s_i can be lowered (compared with the case of independent control) while still providing adequate service.
iii. The major impact of coordination is on the lower dollar usage items (items 2, 3, and 4 in the example).
iv. The cost saving is substantial (here approximately 15 percent)

Table 13.3 A Numerical Example of Coordinated Control $A = \$50$, $a_i = a = \$10$, $r = 0.2$/yr., $L = 1$ month. Service level, $P_1 = 0.95$ for all items

			Independent Control			Coordinated Control			
Item, i	D_i (units/yr.)	v_i ($/unit)	s_i	S_i	EC_i^a ($/yr.)	s_i	c_i	S_i	EC_i^a ($/yr.)
1	290	6.90	33	192	232.03	32	119	184	221.74
2	41	1.20	7	150	35.34	4	41	100	23.26
3	77	3.90	11	120	68.86	9	52	97	70.03
4	122	2.30	16	194	84.98	13	82	156	67.19
					Total = $441.21/yr				Total = $383.12/yr.

^{a} EC_i—the expected costs (replenishment plus carrying) per year associated with item i

Cost Savings of Several Examples—Based on a number of test examples performed (see References 31 and 36) the following conclusions can be drawn (at least for the cases of Poisson and compound Poisson demand) concerning the cost savings possible through the use of the (S, c, s) procedure discussed in the preceding subsection:

> i. For typical values of the input factors (A, a_i's, D's, σ's, number of items, etc.) the average cost savings over independent control are in the neighborhood of 15 to 20 percent.
>
> ii. The cost savings increase as the a/A ratios decrease, certainly intuitively appealing.
>
> iii. The cost savings improve as n increases; the more items there are in the group, the more attractive coordinated replenishment becomes.
>
> iv. The *percent* cost savings tend to diminish as the required service level increases (a large safety stock dominates either type of control).
>
> v. Somewhat surprisingly, for a fixed level of service, the cost savings are quite insensitive to the length of the replenishment lead time.

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A control system for coordinated inventory replenishment*

EDWARD A. SILVER†

There are frequent situations where the coordination of replenishment orders for selected groups of items can lead to significant savings in the costs of replenishment. This paper is concerned with a proposed procedure for selecting the order-up-to levels, can-order points and must-order points of a particular coordinated control system. Demand is Poisson and a fixed non-zero replenishment lead time is assumed. Cost comparisons with the best independent control strategy indicate that substantial [illegible] [illegible] can be obtained through coordination.

Introduction

One of the fundamental assumptions inherent in the derivation of the basic economic order quantity (or Wilson lot size) is that the items are maintained under independent control. This assumption is likely to be inappropriate under any of the following circumstances:

(1) several items are produced on the same equipment, in which case coordination of run quantities may significantly reduce set-up costs;
(2) several items are purchased from the same supplier, in which case coordination may allow use of 'group' quantity discounts;
(3) several items share the same transportation facility, in which case coordination may result in transportation economies (e.g. full car loads).

A number of authors have discussed methods of coordinating items for replenishment purposes. Brown (1967) grouped items into families and established a coordinated policy for the case of deterministic demand. Again, for the deterministic case, Maxwell (1964) illustrated that, when one takes account of set-up times, the determination of a feasible, let alone an optimal, production schedule for a number of items on a production facility is a non-trivial task.

Recognizing the probabilistic nature of demand, Balintfy (1964) was the first to advocate the use of an (S, c, s) system‡, the topic of this paper.

* Based on an earlier paper presented at the Joint Conference of the Operations Research Society of America, The Institute of Management Sciences and the Systems Engineering Group of the American Institute of Industrial Engineers, Atlantic City, N.J., November 1972.
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† Department of Management Sciences, University of Waterloo, Waterloo, Ontario, Canada.

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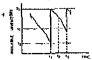
‡ In an (S, c, s) policy we have a continuous review (i.e. transactions reporting) system. Whenever the available inventory level (on hand plus on order minus backorders) of item i hits s_i or lower it triggers a replenishment action so as to raise item i's level to S_i. At the same time any other item j (within the associated family) with available inventory level at or below its can-order point c_j is included in the replenishment. If item j is included, a quantity is ordered sufficient to raise its level to S_j. The behaviour of a typical item under such a system of control is shown in fig. 1.

However, he did not propose a practical means of specifying the values of the control variables, namely the S's, c's and s's. Other references relevant to this type of policy include Silver (1965), Ignall (1969), Davey et al. (1970), and Schaack and Silver (1972). In each of the latter two references a procedure was suggested for obtaining the optimal values of the control parameters. However, a time consuming simulation was an integral component of both of these procedures. In closely related work, Cohen (1973) used deterministic approximations to develop rules for selecting values of the control parameters, but he again required parameter values which must be ascertained by simulation or by observation of the actual behaviour of the system. Finally, the author (1971), in an earlier publication, has developed an algorithm, not involving simulation, for the special case of a replenishment lead time of negligible length. In fact, as we shall see, this zero lead time routine is a key element of the solution of the more general problem.

Fig. 1



Behaviour of an item under (S, c, s) control. At time t_1 this item triggers a replenishment. At time t_2 some other item in the group triggers a replenishment, but this item is not included because its available inventory is above the can-order point. At time t_3 some other item in the group triggers a replenishment in which this item is now included.

Chern (1974) has considered a slightly different control strategy where a fixed order quantity Q_i is used for item i rather than an order-up-to-level S_i. Renberg and Planche (1967) analysed a replenishment policy in which an order is triggered for a group of items as soon as the total stock of the group falls below a certain level. In a somewhat related approach, Hill [illegible] (1966) have presented a strategy where a periodic review is used and a predetermined group order quantity is apportioned among the items. Simmons (1972) has also analysed a periodic review coordinated control procedure.

Ignall (1969) has shown that an (S, c, s) policy does not necessarily minimize the sum of replenishment, inventory carrying and shortage costs. However, the policy that would minimize these costs would be considerably more complex than the (S, c, s) strategy. Therefore, when one properly takes account of the control system costs it is felt that the (S, c, s) approach achieves a solution to the coordinated replenishment problem which is close to the best attainable. Considerably further discussion of this type of approach to complex problems of production planning and inventory control will be available in Peterson and Silver (in preparation).

In the next section we lay out the assumptions of the model and the basic notation to be used in the remainder of the paper. This is followed by a presentation of the cost structure and service constraints implied by the assumptions. Next, the case of independent control, needed later in the paper for comparison purposes, is reviewed. In the fifth section we discuss a single item problem whose solution is a key element in the derivation of the control parameter values of the (S, c, s) strategy. The sixth section represents the heart of the paper, namely the algorithm for finding the S, c and s values. It is followed by a cost and service comparison of coordinated (S, c, s) and independent control for some 107 cases involving 8 different basic data sets (item characteristics and set-up costs). The paper ends with some concluding remarks.

Assumptions and notation

(1) Assumptions

We are dealing with a group of items where the cost of replenishing two or more items at the same time is less than the total cost of replenishing the same number in separate individual replenishments. To be more explicit, we assume that there is a fixed (or header) cost A associated with each replenishment, as well as a variable (or line) cost a associated with each item involved in the replenishment. Therefore, the set-up cost of an order involving k items is $A+ka$ whereas the set-up cost of k replenishments, each involving a single item, is $k(A+a)$. There would be no difficulty in generalizing to the case where a became a_i, i.e. the line cost depended upon the specific item i.

In this paper we restrict attention to the case of Poisson demand where, of course, each item can have a different demand rate. Obviously the Poisson distribution is an approximation to reality, most appropriate for the case of many small customers. The assumption of Poisson arrivals of transactions is reasonable as reported elsewhere by the author (1970). However, transactions of greater than unit size would result in a compound Poisson distribution of demand; in further research we are exploring this extension.

The unit variable cost of an item does not depend on the size of the replenishment, i.e. there are no quantity discounts.

Another assumption that we make is that the replenishment lead time is of constant (finite) length. Furthermore, its length does not depend upon which subset of the items of the family are involved in the replenishment.

Another assumption made, which is common in the modelling of inventory control problems, is that inventory carrying costs are proportional to the average inventory level.

An additional point, which is not really an assumption but rather an accounting convenience, is that we shall say that the entire cost A is assigned to the item that triggers the replenishment.

Rather than explicitly costing shortages we assume instead that the control parameters are selected so as to provide customer service at or above a prescribed level. For each item, service is defined in one of two ways:

(1) Probability of no shortage per replenishment cycle.
(2) Fraction of demand to be satisfied directly from shelf.

This 'servicelevel' approach is widely used in practice.

We further assume that all demands when out of stock are back-ordered. However, the model can easily be adapted to cope with complete or partial lost sales.

One further assumption is made concerning the probabilistic description of the generation of opportunities for any particular item j to be included in a replenishment triggered by another item. However, it is best to postpone discussion of this assumption until use is made of it in the derivation.

(2) Basic notation

[A few other symbols will be defined throughout the paper as they are needed.]

A, fixed or header cost per replenishment (independent of the number of items involved), in \$.

a, (line) cost per item involved in a replenishment, in \$.

c_i, can-order point of item i.

EC_i, expected relevant cost per unit time for item i in \$/yr.†

EC, expected relevant costs per unit time for all the items in the family, in \$/yr.

$ESPRC_i$, expected shortage per replenishment cycle of item i, in pieces.

FDS_i, fraction of demand satisfied directly from shelf for item i.

$\bar{I}_i$, average on hand inventory level of item i, in pieces.

L, replenishment lead time, in years.

n, number of items in the family under consideration (i.e. a replenishment can include up to n different items).

N_i, expected number of replenishments which involve item i per year.

NT_i, expected number of replenishments triggered by item i per year.

$PNSPRC_i$, probability of no shortage per replenishment cycle for item i.

P_1, desired value of $PNSPRC$.

P_2, desired value of FDS.

$\bar{Q}_i$, average order quantity of item i, in pieces.

r, inventory carrying charge, in \$/\$/yr.

s_i, must-order point of item i.

S_i, order-up-to-level of item i.

v_i, standard unit cost of item i, in \$/piece.

w, available inventory level of an item when it is included in an order being placed.

λ_i, Poisson demand rate for item i, in pieces/yr.

$p_w^i(w_0)$, probability for item i that the random variable w takes on the value w_0.

$p_0(x_0|z)=\frac{z^{x_0}\exp(-z)}{x_0!}$, probability that a Poisson variable with parameter (mean) z takes on a value of x_0.

$p_{\le}(x_0|z)$, probability that a Poisson variable with parameter z takes on a value less than or equal to x_0.

† Any other unit time period can be used as long as one is consistent throughout.

Mathematical statement of the decision problem

Using the above assumptions and notation the expected relevant costs per year for item i are given by

$$EC_i=\bar{I}_i v_i r+NT_i A+N_i a \tag{1}$$

The first term represents the inventory carrying costs and the other two the set-up charges allocated to item i. The total expected relevant costs per year of the group of items are given by

$$EC=\sum_{i=1}^{n} EC_i=\sum_{i=1}^{n}(\bar{I}_i v_i r+NT_i A+N_i a). \tag{2}$$

The service constraint takes one of two forms depending upon the measure of service used:

Probability of no shortage per replenishment cycle

For item i, we want‡

$$PNSPRC_i\ge P_1 \quad (i=1,2,\ldots,n). \tag{3}$$

If we let A_i be the event of no shortage of item i in a particular replenishment cycle, then

$$PNSPRC_i=p(A_i)=\sum_{w_0=s_i}^{S_i} p(A_i|w_0)p_w^i(w_0)$$

Now, a shortage occurs if and only if the lead time demand, which is Poisson with parameter $\lambda_i L$, is greater than the available inventory level of the item when the order is placed, i.e.

$$p(A_i|w_0)=p_{\le}(w_0|\lambda_i L).$$

Therefore eqn. (3) becomes

$$\sum_{w_0=s_i}^{S_i} p_{\le}(w_0|\lambda_i L)p_w^i(w_0)\ge P_1 \quad (i=1,2,\ldots,n). \tag{4}$$

Fraction demand satisfied directly from shelf

The expected shortage of item i per replenishment cycle is

$$ESPRC_i=\sum_{w_0=s_i}^{S_i}\left[\sum_{x_0=w_0+1}^{\infty}(x_0-w_0)p_0(x_0|\lambda_i L)\right]p_w^i(w_0). \tag{5}$$

The quantity in square brackets is the expected shortage in a cycle given that the item is ordered when its available stock is at the level w_0.

Now the fraction of demand satisfied directly from shelf is, at least to a first approximation,

$$FDS_i=1-\frac{ESPRC_i}{\bar{Q}_i}. \tag{6}$$

‡ There would be no difficulty in handling the case where the desired probability was P_{1i}, i.e. it depended on the specific item i.

Therefore, with this service measure we want, using eqns. (5) and (6),

$$1-\frac{1}{\bar{Q}_i}\sum_{w_0=s_i}^{S_i}\sum_{x_0=w_0+1}^{\infty}(x_0-w_0)p_0(x_0|\lambda_i L)p_w^i(w_0)\ge P_2 \quad (i=1,2,\ldots,n). \tag{7}$$

We can now state our problem mathematically as follows:

Select the S's, c's and s's (i.e. the values of $3n$ different variables) so as to minimize EC of eqn. (2) subject to the n constraints of either eqn. (4) or eqn. (7). The quantities $\bar{I}_i$, NT_i, N_i, $p_w^i(w_0)$ and $\bar{Q}_i$ depend upon one or more (most often, all) of the S's, c's and s's in a rather complex fashion. It is clear that this is an extremely complicated optimization problem.

Independent control

As mentioned in the introduction, the case of independent control must be analysed for purposes of comparison. Independent control of item i is a special case of (S, c, s) control where we set $c_i=s_i$, i.e. the item is never ordered until it hits its must order point. The order quantity is always of size S_i-s_i. Furthermore, no other item is ever included in the replenishment, hence a set-up cost $(A+a)$ is incurred with each replenishment.

For a single item i there are 2 decision variables, namely S_i and s_i. Recognizing that $c_i=s_i$, eqns. (1), (4) and (7) become

$$\text{Minimize } EC_i=\left(\frac{S_i-s_i+1}{2}+(s_i-\lambda_i L)\right)v_i r+\frac{(A+a)\lambda_i}{S_i-s_i}, \tag{8}$$

subject to

$$p_{\le}(s_i|\lambda_i L)\ge P_1 \tag{9}$$

or

$$1-\frac{1}{S_i-s_i}\sum_{x_0=s_i+1}^{\infty}(x_0-s_i)p_0(x_0|\lambda_i L)\ge P_2. \tag{10}$$

Strictly speaking, the pair (S_i, s_i) which minimizes eqn. (8) while satisfying either eqn. (9) or (10) should be found. However, it has been observed (see, e.g. Silver and Wilson (1972)) that, under usual conditions, a much simpler sequential approach does almost as well from the standpoint of reducing replenishment and carrying costs. In this simple approach we first select the order quantity $Q_i=S_i-s_i$, which minimizes EC_i ignoring the constraint (eqn. (9) or (10)). Then, given this value of S_i-s_i, the constraint is used to find the lowest allowable value of s_i. The first step leads to the usual economic order quantity expression:

$$\hat{Q}_i=S_i-s_i=\sqrt{2(A+a)\lambda_i/v_i r}. \tag{11}$$

In general this will not be an integer. Therefore, we test the two integer values of Q_i around $\hat{Q}_i$. The one giving the lowest value of EC_i in eqn. (8) is the one to use.

Equation (9) shows that, for the service measure being the probability of no shortage per cycle, the s_i value does not depend upon S_i; we solve for s_i using the cumulative distribution of a Poisson variable. For the other service measure, it is shown in Appendix A, that from eqn. (10) it follows that the lowest s_i which satisfies the following inequality should be used:

$$\lambda_i \bar{L}_i - s_i - \lambda_i L p_{po\geq}(s_i - 1 | \lambda_i L_i) + s_i p_{po\geq}(s_i | \lambda_i L_i) \leq Q_i (1 - P_i). \qquad (12)$$

This is a very special case of the type of numerical problem which must be solved in the coordinated control situation. Therefore, we postpone discussion of the solution procedure until a more appropriate time later in the paper.

Two illustrative numerical examples (one for each type of service measure)

Example 1. Probability of no shortage per replenishment cycle.
Consider the following example involving four items:

$A = \$40$
$a = \$10$
$r = 0.2$ \$/\$/yr.
$L = 1$ month (i.e. $\frac{1}{12}$ year)

Item, i	λ_i pieces/yr	v_i \$/piece	P_i (desired $PNSPRC_i$)
1	200	0.60	0.95
2	41	1.20	0.95
3	77	3.00	0.95
4	122	2.50	0.95

Use of eqn. (11) gives the Q_i entries shown in table 1. The tests of neighbouring integers lead to the results in the next column of the table.

Table 1. Results of numerical example 1 under independent control

Item, i	Best non-integer Q_i	Best integer $Q_i = S_i - s_i$	s_i	EC_i
1	159.8	160	33	1232.03
2	143.2	143	7	25.31
3	105.8	106	11	98.84
4	178.4	178	14	81.85
			Total	\$441.21

We illustrate the determination of s_1 using item 1.

$$\lambda_1 \bar{L} = \frac{200}{12} = 16.13.$$

Equation (8) says to select the smallest value of s_1 such that

$$p_{po<}(s_1 | 16.17) \geq 0.95.$$

Using a computer simulation of the Poisson distribution or a look-up of Poisson tables, such as those in Molina (1942), one finds $s_1 = 33$. In a similar fashion the other s_i values shown in table 1 are developed. The values of $S_i - s_i$ and s_i are then substituted into eqn. (8) to obtain the EC_i values shown in the table. The total expected costs per year are seen to be \$441.21.

Example 2. Fraction of demand satisfied directly from shelf

We use the same data as in example 1 except that the service measure is now the fraction of demand satisfied directly from shelf. Suppose that the desired service level is 0.99 for each of the four items.

The best integer Q_i values are unaffected by the service measure. Hence they are the same as in example 1.

The computation of s_i is again illustrated for item 1. Equation (12) becomes

$$24.17 - s_1 - 24.17 p_{po\geq}(s_1 - 1 | 24.17) + s_1 p_{po\geq}(s_1 | 24.17) \leq 1.60.$$

The smallest s_1 which satisfies this inequality is 26. The other s_i values are shown in table 2 as are the EC_i values resulting from substituting the $S_i - s_i$ and s_i values into eqn. (8). We see that the total expected costs per year for the four items are \$423.33.

Table 2. Results of numerical example 2 under independent control

Item, i	Best integer $Q_i = S_i - s_i$	s_i	EC_i
1	160	26	1229.09
2	143	3	24.28
3	106	7	85.74
4	178	10	82.22
		Total	\$423.33

A single item problem related to the case of coordinated control

Before considering the case of coordinated control it will be found helpful to briefly discuss a related single item problem whose solution will be a key element in the algorithm for determining the parameters of the (S, c, s) control system for the case of a zero length lead time.

Consider a typical single item, i, in a group (or family) of coordinated items when the replenishment lead time is of negligible length. The assumption of unit sized (Poisson) demands together with a negligible lead time implies that $s_i = 0$, i.e. there is no point in ordering an item until its inventory level drops to 0.

Now, this item i, from time to time, will be faced with the opportunity of a replenishment at the reduced set-up cost of only a (such an event being caused by another item triggering a replenishment). Occasionally this item itself will trigger an order when its inventory hits the zero level. Under such circumstances we would like to know the (S, c) pair to choose so as to minimize the expected costs per unit time of the item. This question can,

and has been answered, provided one more assumption is employed. This additional assumption is that the opportunities to replenish at the reduced cost (i.e. triggerings by other items) occur probabilistically according to a Poisson process with a rate μ per year, where μ is the expected number of orders triggered per year by all other items in the group. This assumption becomes more and more reasonable as the number of items in the group increases. In any event, the assumption would appear to be justified by the relative mathematical simplicity compared with any non-Poisson case.

The author (1973) has shown by three different methods (one of which had been developed previously by Friend (1960) and generalized by Naddor and Kaminsky (1964)) that the total expected relevant costs per year for item i are given by†

$$EC_i = [S - c + \rho(1 - \rho^c)/(1 - \rho)]^{-1} [(S - c)(S + c + 1)v r/2 \;+\; \rho(\ldots)\,[\text{illegible}] + \lambda \rho^c A + \lambda a], \qquad (13)$$

where

$$\rho = \lambda/(\lambda + \mu), \qquad (14)$$

and all the other symbols are as defined earlier.

A by-product which we shall need for the coordinated control algorithm is NT_i, the expected number of replenishments triggered by item i per year.†

$$NT_i = \lambda \rho^c / [S - c + \rho(1 - \rho^c)/(1 - \rho)]. \qquad (15)$$

Friend (1960) has suggested a procedure for selecting the values of S and c which minimize the EC_i expression of eqn. (13). Without getting into the mathematical details, the steps of a somewhat modified version of Friend's procedure are as follows:

Step 1. We take the partial derivative of EC_i with respect to S and set it equal to zero (this is a necessary condition for the minimum).‡ Solve this equation for S in terms of c and all the other parameters. Call this solution $\hat{S}(c)$ to indicate that it is the best S for the given c value. It can be shown that $\hat{S}(c)$ is given by

$$\hat{S}(c) = c - \rho(1 - \rho^c)/(1 - \rho)$$
$$+ \sqrt{[\ldots]}\;[\text{illegible}] \qquad (16\,a)$$

Now, the expression within the square root in eqn. (16 a) can be negative, i.e. $\hat{S}(c)$ is imaginary. In such a case, it can be shown that the minimum cost for a given c value is achieved by setting

$$\hat{S}(c) = c, \qquad (16\,b)$$

Physically, this means that the can-order point is equal to the order-up-to level, in which case this item will always be included in any order triggered by another item.

† For simplicity the subscripts have been suppressed in λ_i, ρ_i, v_i, S_i and c_i.
‡ The procedure will, in general, produce a non-integer value of S. Strictly speaking, S should be an integer but we neglect this refinement at this stage.

Step 2. Substitution of eqn. (16 a) back into eqn. (13) gives $\widehat{EC}_j(c)$, the best cost for the given c value. It can be shown that this expression is simply

$$\widehat{EC}_j(c) = w[\hat{S}(c) + 1]. \qquad (17\ a)$$

For the case where eqn. (16 b) is valid, its substitution into eqn. (13) gives

$$\widehat{EC}_j(c) = [\ldots]^{-1}[\ldots][w/(1-\rho) + \lambda\rho^c A + \ldots]. \qquad (17\ b)$$

Step 3. Next a one-dimensional search† is performed on the discrete variable c to find the value c_o which minimizes $\widehat{EC}_j$, using whichever of eqns. (17 a) or (17 b) is appropriate for each c value tested. (Such a search‡ has been programmed in FORTRAN.) Once c_o is found, we have an associated $\hat{S}(c_o)$ which need not be an integer. In such a case, for the two integer values of S around $\hat{S}(c_o)$ we evaluate EC_j using eqn. (13) with $c = c_o$ for the specific S value. Whichever S gives the lower cost is the one to use.

In an actual application only step 3 need be performed. Steps 1 and 2 are mathematical manipulations which have prepared the ground work, once and for all, for step 3.

The algorithm to find the S's, c's and s's

Our approach here is analogous to [illegible] in the independent case, namely we find the S's and c's for the case of a negligible replenishment lead time (each s is zero in this case); then, conditional on these S and c values, we find the lowest s_i which satisfies the service constraint for the particular item i. As in the independent case we know that the S's, c's and s's so obtained

Table 2 Control parameters for $L=0$ and $L>0$

	$L=0$	$L>0$
Independent control		
Order-up-to-level	EOQ	$s + EOQ$
Must-order point	0	s
Coordinated control		
Order-up-to-level	S	$s + S$
Can-order point	c	$s + c$
Must-order point	0	s

† For a one-dimensional search on a variable which has no upper limit (as is the case with c) it is extremely helpful to have an upper bound on the best value of c. It has been shown by the author (1973) that a very simple upper bound is given by $\sqrt{(2\lambda A + \ldots)}$, the economic order quantity when the item is treated independently of all other items.

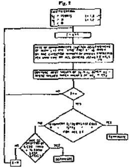
‡ The specific search routine used was the 'Golden Section Rule'; see, for example, Wilde (1964).

will not minimize the sum of replenishment and carrying costs. However, the cost penalty is likely to be low and we are willing to absorb it in order to have a computationally feasible scheme for evaluating the control parameters. A tabular summary of the relationships between the zero and non-zero lead time cases is shown for both independent and coordinated control in table 2.

The same numerical examples as used for the case of independent control will be employed here for both illustrative and comparative purposes.

(A) *The case of $L=0$ (i.e. each $s_i=0$).*

Fig. 2



Algorithm for case of $L=0$.

General remarks

The flow diagram of fig. 2 may be helpful in following the reasoning. The procedure for the case of a negligible lead time is iterative in nature. The key step can be summarized as follows: knowing the current values of all S_i's and c_i's, a new pair (S_j, c_j) are obtained for a particular j by using the one-item optimization routine described in the previous section. The key input needed to do this is an estimate of μ_j, the rate of opportunities for ordered

set up (no) replenishment of item j. But this is given by the expected number of orders triggered per year by all other items, i.e.

$$\mu_j = \sum_{i \neq j} NT_i. \qquad (18)$$

For example, if there were four items in the group and we were considering item 2, we would have

$$\mu_2 = NT_1 + NT_3 + NT_4.$$

Two problems remain, namely initialization and termination. To initialize we assume independent control of the items. From earlier, we know that this implies

$$c_i = 0, \quad S_i = \sqrt{(2(A + a_i)\lambda_i/\nu r)} \quad (i = 1, 2, \ldots, n).$$

A reasonable way of terminating is to wait until the entire set of S's and c's does not change from one iteration to the next (an iteration being a complete pass through all n items). This criterion may be over exacting; in fact, there are cases where a cycling among two or three sets of S's and c's sets in. To counteract this effect we have used an alternative criterion for termination, namely, that the total expected costs have changed by less than a certain percentage from the preceding iteration. In all examples tested the number of iterations required before termination has been quite small.

We now turn to the details of the algorithm. No loss of continuity will result from bypassing this subsection.

The details of the algorithm

Notation:

Let $c_i^{(k)}$ represent the value of c for item i on iteration k,

$S_i^{(k)}$ represent the value of S for item i on iteration k,

$\mu_i^{(k)}$ be the value of μ_i used to evaluate $c_i^{(k)}$ and $S_i^{(k)}$,

and

$\mu_i^{(k)} = \lambda_i/[\ldots]$.

Step 1 (Initialization)

Set $c_i^{(1)} = 0 \quad i = 1, 2, \ldots, n,$

$S_i^{*} = \sqrt{(2(A + a_i)\lambda_i/\nu r)} \quad i = 1, 2, \ldots, n.$

Test the two integer values around S_i^{*} in eqn. (13) (with $c=0$). The one giving the lowest cost is to be taken as $S_i^{(1)}$. Each NT_i is evaluated using eqn. (15).

Set $k = 2$.

Step 2

$$\mu_1^{(k)} = \sum_{i=2}^{n} NT_i$$

Use method of the fifth section to find best $S_1^{(k)}$ and $c_1^{(k)}$. NT_1 and EC_1 are evaluated using eqns. (16) and (13).
Proceeding to the second item

$$\mu_2^{(k)} = \sum_{i \neq 2} NT_i.$$

Use method of the previous section to find best $S_2^{(k)}$ and $c_2^{(k)}$. NT_2 and EC_2 are evaluated using eqns. (15) and (13).
Moving on to item 3 we have

$$\mu_3^{(k)} = \sum_{i \neq 3} NT_i.$$

For item *n* we have:

$$\mu_n^{(k)} = \sum_{i=1}^{n-1} NT_i.$$

Use method of the fifth section to find best $S_n^{(k)}$ and $c_n^{(k)}$. NT_n and EC_n are evaluated using eqns. (15) and (13).

Step 3 (Termination)

Is $c_i^{(k)}$ equal to $c_i^{(k-1)}$ for all i and $S_i^{(k)}$ equal to $S_i^{(k-1)}$ for all i? If the answer is yes, we terminate and the current values of $c_i^{(k)}$ and $S_i^{(k)}$ are the appropriate ones to use. If the answer is no, we compare current value of EC with its value on last iteration. If a decrease of less than x% (where x must be specified by the user) has occurred, we terminate. If not, we set $k = k+1$ and return to the start of step 2.

Numerical examples

The two numerical examples used in the fourth section differed only in terms of the service criterion. The results for the case of $L = 0$ do not depend on the service criterion. Therefore, we have the same results for both examples for the $L = 0$ phase. The detailed output of the algorithm is as follows:

k	i	$S_i^{(k)}$	$c_i^{(k)}$
1	1	180	0
	2	133	0
	3	109	0
	4	178	0
2	1	148	80
	2	67	33
	3	87	42
	4	146	72
3	1	162	81
	2	66	33
	3	89	43
	4	143	69
4	1	Same as previous iteration	
	2		
	3		
	4		

Through the use of eqn. (13) the algorithm also generates the following cost estimates under the selected set of S's and c's:

Item, i	EC_i \$/yr
1	119·03
2	15·12
3	88·91
4	45·80
Total	268·86

These cost estimates are only approximate, because of the assumption (of Poisson approximation for replenishment at reduced cost) underlying the model upon which the algorithm is based.

(ii) *The case of $L > 0$*

As was shown in table 2, for item i we now find s_i such that the must-order point is s_i, the can-order point is $s_i + c_i$ and the order-up-to level is $s_i + S_i$ where c_i and S_i are assumed known at this stage as output from the algorithm for the $L = 0$ case. It should be emphasized that we are able, if desired, to select the s's so as to satisfy different, specified service levels for different items. This is in contrast to the Thill (1965) 'service point' approach where one can only aim for an average service level for the entire group of items.

Because of the way that we are computing the must-order point, the can-order point and the order-up-to level from the S and c for the $L = 0$ case, we can make the following statements:

(1) The NT_i's and N_i's will be the same for $L > 0$ as they were for $L = 0$. Therefore, the expected replenishment costs per unit time will remain the same.

(2) Item i will be ordered on the average when its stock is s_i higher for the $L > 0$ case than for the $L = 0$ case. However, an average quantity $\lambda_i L$ will be depleted before the order arrives. Therefore,

(average inventory of item i for $L > 0$) = (average inventory for $L = 0$) + $(s_i - \lambda_i L)$.

(3) Statements 1 and 2 together imply

$$EC_i \text{ (for } L > 0) = EC_i \text{ (for } L = 0) + (s_i - \lambda_i L) r v_i, \tag{19}$$

with EC_i (for $L = 0$) being given by eqn. (13).

As we shall see, it is advantageous to establish an expression for the service provided in terms of the probability distribution of the available stock of the item when it is included in an order being placed.

Level of available stock (w_0) when item is ordered

Under the assumptions of the model the author (1973) has shown elsewhere that w_0 is a random variable with a truncated geometric distribution, viz.:

$$p_{w_0}(w_0) = \begin{cases} (1-p_i)p_i^{s+c-w_0} & w_0 = s+1, s+2, \ldots, s+c \\ p_i^{c} & w_0 = s, \end{cases} \tag{20}$$

where, as earlier,

$$p_i = \lambda_i/(\lambda_i + \mu_i)$$

with μ_i being an output of the $L = 0$ algorithm.

We now treat the cases of the two different measures of service separately.

Service measure: Probability (P_1) of no shortage per cycle

Equation (4) gives the constraint which s_i must satisfy. We repeat it here†:

$$\sum_{w_0} p_{w_0} \{p_{\leq}(w_0 | \lambda_i L)\} \geq P_1 \tag{21}$$

In Appendix A we indicate how substitution of the truncated geometric distribution of eqn. (20) into eqn. (21) leads to the following condition:

Select the smallest s such that‡

$$\{1/p\}p_{\leq}(s+c|\lambda L) - p_{\leq}(s+1|\lambda L) - p^{c+1}\sum_{w_0=s+1}^{s+c} p_{w_0}(w_0|\lambda L) \geq 1 - \frac{P_1}{p} \quad (c > 0) \tag{22}$$

Again, S_i does not enter into this expression because the service measure is on a per cycle basis.

Numerical example 1

This is the same as example 1 under independent control, i.e. $P_1 = 0.95$ for all four items. The $L = 0$ solution, developed in the previous subsection, gives the S, c and p values shown in table 4. Use of eqn. (22) then leads to the s values shown. Finally, eqn. (16) gives us the model's estimate of the expected cost per unit time for each item. The estimated total expected cost per unit time are seen to be \$383·12, a saving of 13.2% over the independent control result (see table 1).

Table 4. Results of numerical example 1 under coordinated control ($P_1 = 0.95$)

From $L=0$				Must-order point s_i	Can-order point $s_i + c_i$	Order-up-to level $s_i + S_i$	EC_i
Item, i	S_i	c_i	p_i				
1	162	81	0·9971	32	113	184	1221·74
2	56	33	0·9961	4	41	100	22·38
3	88	43	0·9713	0	42	97	79·93
4	137	69	0·9835	13	82	156	61·10
						Total =	1383·12

† In contrast with earlier, now the can-order point is $s_i + c_i$.
‡ For simplicity we have suppressed the i subscript on c_i, s_i, λ_i and p_i.

Service measure: Fraction (P_2) of demand to be satisfied directly from shelf

Equation (7) gives the constraint which s_i must satisfy, namely select the smallest s_i such that

$$\sum \sum (x_0 - s_i) p_{x_0}(x_0 | \lambda_i L) p_u(u_i) \leq \hat{Q}_i (1 - P_2) \quad (23)$$

In Appendix A we show how substitution of the $p_u(u_i)$ of eqn. (20) into eqn. (23) leads to the following rule:

Select the lowest s such that†

$$[illegible]$$

$$+ (1 - p) p^{s+x} \sum [illegible]$$

$$+ [illegible] \geq (S - c + p(1 - p^{s})(1 - p)) (1 - P_2). \quad (24)$$

This equation looks extremely complicated but we have programmed its solution using recursive procedures to relate the left-hand side for $s+1$ to its value for s.

Numerical example 2

We use the same data as in example 2 under independent control, i.e. $P_2 = 0.90$ for all four items. The $L = 0$ coordinated solution, developed earlier, gives the S_i, c and p values shown in table 6. Use of eqn. (24) then leads to the s values shown. Equation (19) gives the model's estimate of EC_i. The total expected costs per unit time are seen to be 1306·39, a saving of 12·5% over the independent control result (see table 2).

Table 6. Results of numerical example 2 under coordinated control ($P_2 = 0.90$)

From $L=0$				$L>0$			
Item, i	S_i	c_i	p	Must-order point, s_i	Can-order point, $s_i + S_i$	Order-up-to-level $s_i + S_i$	EC_i
1	145	87	0·0681	25	112	170	1212·08
2	36	27	0·0487	−1	36	35	32·00
3	46	43	0·0743	6	49	52	67·81
4	145	62	[illegible]	7	74	152	54·43
						Total =	1306·39

Comparison of coordinated (S, c, s) and independent control

In previous work on the $L = 0$ case (Silver (1974)) some 22 different numerical examples were tested. Some of these were selected from U.S.

† Again, the subscripts have been omitted.

Army Inventory Research Office data (Clymer (1973)) while the others were generated from typical inventory populations provided by Brown (1967). Those earlier experiments showed that the ratio s/A was important, not the individual values of s and A. Bearing this fact in mind and recognizing that each basic example selected would have to be run for several combinations of lead time, service measure and service level, we reduced the original set of 22 examples to 8 for the $L>0$ experiments. These 8 examples are shown in table 8.

Table 8. Basic examples tested

Example number	A	s	Number of items	i	λ_i	v_i
	$	$			pieces/yr.	$/piece
1	155	40	10	1	40·00	0·45
				2	4·00	38·11
				3	31·96	12·57
				4	4·24	4·40
				5	1·24	4·06
				6	1·00	38·41
				7	4·24	1·05
				8	24·79	3·02
				9	4·00	38·41
				10	1·00	94·41
2	150	80	5	1	4·00	38·41
				2	4·00	38·41
				3	15·18	3·02
				4	49·02	9·48
				5	29·78	3·02
3	40	8	8	1	200	4·00
				2	41	1·20
				3	71	3·05
				4	122	2·30
				5	40	1·50
				6	161	3·09
				7	87	2·30
				8	20	1·29
4	40	10	8		Same as example 3	
5	40	20	8		Same as example 3	
6	50	5	4	1	200	4·00
				2	41	1·20
				3	71	3·06
				4	122	2·30
7†	40	10	4		Same as example 6	
8	40	25	4		Same as example 6	

† This is the numerical example used earlier in this paper.

For each of these basic examples 13 sub-cases were run as follows:

Sub-case	L-value (months)	Service measure	Service level
i	0	—	—
ii	1	P_1	0·98
iii	1	P_1	0·92
iv	1	P_2	0·98
v	1	P_2	0·90
vi	3	P_1	0·85
vii	3	P_1	0·90
viii	3	P_2	0·95
ix	3	P_2	0·98
x	12	P_2	0·85
xi	12	P_1	0·90
xii	12	P_1	0·95
xiii	12	P_2	0·95

For each of the 104 combinations the following were done:

(a) Determination of independent control parameter values and associated expected costs.

(b) Determination of coordinated control parameter values and expected costs estimated by the model.

(c) As was discussed earlier, the total cost figure predicted by the mathematical model may be in error because of the assumption of Poisson opportunities for replenishment at reduced set-up cost. A similar statement applies to the service level predicted by the model. Therefore, a simulation of the actual behaviour of the (S, c, s) system was performed for each of the 104 examples to obtain more accurate estimates of the total expected costs per year and the service levels provided. (Actually, only 8 simulations were needed, one for the $L = 0$ case of each basic example. The results for $L > 0$ can be deduced from those for $L = 0$ using the concept of 'virtual measures' suggested by Carlson and Ignall (1972)). It should be emphasized that simulations are needed only for testing purposes and not for day-to-day use of the (S, c, s) control procedure.

The detailed results, case by case, are shown in Appendix B. For one example, these are shown graphically in figs. 3 and 4. In summary, the following statements can be made.

(a) In all 104 cases coordinated control results in total costs of replenishment and carrying inventory significantly lower than under independent control. The average cost savings are 18·8%.

(b) In all cases the model is conservative in predicting cost savings, the average prediction being only 14·8%.

(c) The model is also conservative in estimating the service level. In all cases where $L > 0$ the actual average (across items) service is higher than that predicted. Therefore, not only is the (S, c, s) control method achieving an average cost savings of 18·8% but it is also providing better average service than is independent control. In a few isolated cases, the actual service for an individual item is slightly lower than that predicted (this level of detail

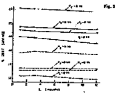


Fig. 2

...al savings of (S, c, s) over independent control for example No. 1. [illegible]: predicted by model; [illegible] actual (by simulation); P_1: probability of no shortage per replenishment cycle; P_2: fraction of demand satisfied directly from shelf.

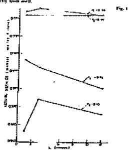


Fig. 3

...al service provided by (S, c, s) system for example No. 1. P_1: target probability of no shortage per replenishment cycle; P_2: target fraction of demand satisfied directly from shelf.

is not shown in Appendix B); when this occurs, the item involved is the one that triggers the majority of the replenishments in the group.

(d) Cost savings of coordinated control increase as a/A decreases, certainly intuitively appealing.

(e) Cost savings improve as n increases; the more items there are in the group, the more beneficial coordinated replenishment becomes.

(f) For a given service measure and lead time the cost savings increase as the desired service level decreases.

(g) For a given service measure and desired level of service the cost savings are quite insensitive to the length of the lead time.

Conclusions

In this paper we have developed a practical procedure for computing the values of the control parameters in an (S, c, s) coordinated replenishment system. The procedure requires no input data beyond that needed for independent control. In numerical examples tested, the average savings in replenishment and carrying costs were 13·8%. The method has not really been streamlined for computational purposes; despite this fact, running the 12 sub-cases of an example for both independent and coordinated control took less than 1 min of IBM 360-75 time.

As mentioned earlier in the paper, an extension being explored is compound Poisson (instead of regular Poisson demand). Other extensions which come to mind are (i) the case where the lead time depends upon the number of items involved in the replenishment and (ii) the situation where the replenishment quantity must be of a particular size (e.g. enough to fill a rail car).

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Appendix A—Derivations

(1) Independent control with service measure: fraction demand satisfied

Equation (18) says that we want the smallest (to keep carrying costs at a minimum) value of s, such that

$$1-\frac{1}{Q_i}\sum_{x_0=s_i+1}^{\infty}(x_0-s_i)p_{po}(x_0|\lambda_i L)\geq P_2. \quad (A\ 1)$$

But Hadley and Whitin (1963) have shown that

$$\sum_{x_0=s_i+1}^{\infty}(x_0-s_i)p_{po}(x_0|\lambda_i L)=\lambda_i L p_{po}(s_i|\lambda_i L)+(\lambda_i L-s_i)\sum_{x_0=s_i+1}^{\infty}p_{po}(x_0|\lambda_i L)$$

$$=\lambda_i L p_{po}(s_i|\lambda_i L)+(\lambda_i L-s_i)[1-P_{po}(s_i)]$$

$$=\lambda_i L-s_i-\lambda_i L P_{po}(s_i-1|\lambda_i L)+s_i P_{po}(s_i|\lambda_i L). \quad (A\ 2)$$

Substitution of this expression into eqn. (A 1) immediately leads to eqn. (17).

(2) Coordinated case with service measure: probability of no shortage per replenishment cycle

According to eqn. (21) we wish to select the smallest (to keep carrying costs as low as possible) value of s_i such that†

$$\sum_{n_i} p_{po}(n_i|\lambda_i L)p_c(n_i)\geq P_1. \quad (A\ 3)$$

where, from eqn. (20),

$$p_c(n_i)=\begin{cases}(1-\rho)\rho^{S_i-c_i-n_i} & n_i=s_i+1, s_i+2, \ldots, S_i\\ \rho^{\ldots} & n_i=s_i.\end{cases} \quad (A\ 4)$$

Substituting eqn. (A 4) into eqn. (A 3) we have

$$P_{po}(s_i|\lambda_i L)\rho^{\ldots}+\sum_{n_i=s_i+1}^{S_i}p_{po}(n_i|\lambda_i L)(1-\rho)\rho^{S_i-n_i}\ldots\geq P_1$$

or

$$P_{po}(s_i|\lambda_i L)+(1-\rho)\rho^{\ldots}\sum_{\ldots}\ldots\sum_{\ldots}P_{po}(\ldots|\lambda_i L)\geq\frac{P_1}{\rho^{\ldots}}.$$

Reversing the order of the double summation and doing considerable simplification, we are led to the result of eqn. (22).

Note: One of the simplifying operations is invalid for $c=0$, but for that case we simply use the independent control result of eqn. (9).

(3) Coordinated case with service measure: fraction demand satisfied

Equation (23) indicates that we want the smallest s satisfying

$$\sum_{n_i}\sum_{x_0}(x_0-n_i)p_{po}(x_0|\lambda_i L)p_c(n_i)\leq \bar{Q}(1-P_2). \quad (A\ 5)$$

We substitute $p_c(n_i)$ from eqn. (A 4), substitute $\bar{Q}=S-s+\rho(1-\rho^{\ldots})/(1-\rho)$ (see Silver (1973)) and use the result of eqn. (A 2). This immediately leads to eqn. (24).

† For simplicity we suppress the letter i denoting the item number.

Appendix B—Detailed numerical results

Example number		s/A	n	L (months)	Service measure	Desired service level	% Cost savings over independent: Predicted by model	% Cost savings over independent: Actual (by simulation)	Actual average† service
(1)	i	0.1	10	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
(2)	i	[illegible]	6	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
(3)	i	[illegible]	8	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
(4)	i	[illegible]	8	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]

Example number		s/A	n	L (months)	Service measure	Desired service level	% Cost savings over independent: Predicted by model	% Cost savings over independent: Actual (by simulation)	Actual average† service
(5)	i	0.5	4	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
(6)	i	[illegible]	6	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
(7)	i	[illegible]	4	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
(8)	i	[illegible]	4	0	—	—	[illegible]	[illegible]	—
	ii			1	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	iii			1	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	iv			1	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	v			1	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	vi			3	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	vii			3	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	viii			3	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	ix			3	[illegible]	0.99	[illegible]	[illegible]	[illegible]
	x			12	[illegible]	0.85	[illegible]	[illegible]	[illegible]
	xi			12	[illegible]	0.90	[illegible]	[illegible]	[illegible]
	xii			12	[illegible]	0.95	[illegible]	[illegible]	[illegible]
	xiii			12	[illegible]	0.99	[illegible]	[illegible]	[illegible]
						Average	14.9	15.8	

† Averaged across all items in the example.

[illegible]

[illegible]

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A coordinated inventory control system for compound Poisson demand and zero lead time†

ROBERT M. THOMPSTONE‡ and EDWARD A. SILVER§

Often coordination of items for replenishment purposes can lead to significant savings in the costs of replenishment. This article presents a reasonable procedure for determining the values of the various control variables of an (S, c, s) control system, namely the order-up-to-levels, the can-order points and the must-order points. It is assumed that demand is compound Poisson and the replenishment lead time is of negligible duration. Comparison with the best independent control strategy indicates that substantial savings (averaging 16·9% over some 64 examples) are possible through coordinating replenishments.

Introduction

Most inventory control policies involve individual control of items. Under certain circumstances coordinating the control of a group of items may result in substantial cost savings. Balintfy (1964) has proposed the (S, c, s) policy as a reasonable strategy for coordination.‖ Silver (1974) has suggested situations where coordination may be advantageous and has outlined the development of control methods for this purpose. He presents an algorithm for selecting the control variables for an (S, c, s) policy for items under unit-sized Poisson demand and replenishment lead times greater than or equal to zero.¶ In this paper we demonstrate how control variables may be selected for this policy when demand is compound Poisson, i.e. not restricted to unit-sized transactions, but the lead time is of negligible length. Unlike previous solutions to this problem (Curry *et al.* 1970, Schaack and Silver 1972, and Cahen 1972) the algorithm does not incorporate time-consuming simulations. Ideas are given for expanding it to cover lead times greater than zero.

The next section gives a more complete problem statement and the basic notation. Independent control under compound Poisson demand is then discussed and numerical examples are presented. We proceed to discuss a single item problem whose solution is a key element in the derivation of the

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‡ Aluminium Company of Canada, Arvida, Quebec, Canada.

§ Department of Management Sciences, University of Waterloo, Waterloo, Ontario, Canada.

‖ Inventory levels are reviewed continuously and whenever item i's available (on-hand plus on-order minus backorders) inventory hits s_i or lower it triggers a replenishment to raise item i's level to S_i. At the same time any other item j (within the associated family) with available inventory at or below its can-order point c_j is included in the replenishment to raise its level to S_j.

¶ To avoid excessive repetition of material, we have used several results from Silver (1974) without reshowing the derivations. Also, we have not repeated the literature review presented in the earlier article.

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control parameter values of the (S, c, s) strategy. An algorithm for coordinated control of a family of items is developed and a simulation test for comparing these results with independent control is described. Numerical examples of coordinated control are given. A set of 64 examples are used to develop a relationship between cost savings of coordinated control over independent control and various characteristics of the family of items. The paper ends with some concluding remarks.

Problem statement and basic notation

Demand characteristics

In deriving realistic inventory control strategies one generally must use a probabilistic description of demand. It is useful to view the total demand in a time period as having two components: (i) the number of transactions during the period; and (ii) the magnitudes of the individual transactions. Each component may have its own probability distribution. Empirical evidence suggests the pattern of arrivals of transactions can be adequately represented by a Poisson process

$$p_x(x_0) = \frac{(\lambda t)^{x_0} \exp(-\lambda t)}{x_0!}, \quad t \geq 0,\ x_0 = 0, 1, 2, \ldots$$

where $p_x(x_0)$ is the probability that there are x_0 arrivals in the time t and λ is the expected number of arrivals per unit time period.

If arrivals of transactions are thus described and the transaction sizes have some specified probability mass function (PMF), the total demand in a time period is described by a compound Poisson PMF. We shall treat the case of a general distribution of transaction sizes, letting

$p_t(t_0)$ = probability that the transaction is of magnitude t_0, $t_0 = 1, 2, \ldots, t_{max}$, where t_{max} is the largest possible transaction size.

The variance of the compound Poisson distribution equals or exceeds its mean, a characteristic often exhibited by demand data.

Cost structure†

Two costs are considered, setup (or ordering) costs and inventory carrying costs.

A group of items is involved for which the cost of replenishing two or more items at the same time is less than the total cost of replenishing the same number in separate individual replenishments. It is assumed that a fixed (or header) cost A is associated with each replenishment and a variable (or line) cost a is associated with each item involved in the replenishment. Thus the setup cost of a single order involving k items is $(A + ka)$.

An important aspect of this paper is the comparison of costs under independent control and coordinated control. The basic cost structure for item i is

$$(\text{expected cost})_i = (\text{setup cost})_i + (\text{carrying cost})_i$$

† The cost structure and service criteria are discussed more fully by Silver (1974).

For independent control this is

$$EC_i(S_i, s_i) = N_i(A + a) + E(I)_i v_i r \quad (1)$$

where :

EC_i = the expected dollar cost of controlling item i per year which is a function of two control variables, S_i and s_i ;

S_i = the order-up-to-level for item i, in pieces ;

s_i = the must-order point for item i, in pieces ;

N_i = the expected number of orders of item i per year ;

$E(I)_i$ = the expected inventory level for item i, in pieces ;

v_i = the dollar value or standard unit cost of item i, in dollars per piece ; and

r = inventory carrying charge, in dollars per dollar per year.

and A and a are as defined earlier ;

Under the coordinated control system

$$EC_i(S_i, c_i, s_i) = NT_i A + N_i a + E(I)_i v_i r \quad (2)$$

where :

EC_i = the expected dollar cost of item i per year which is a function of three control variables, S_i, c_i and s_i ;

c_i = the can order point for item i, in pieces ;

NT_i = the expected number of replenishments *triggered* by item i, per year ;

and all other variables are as previously defined.

The total expected relevant cost of the group of items under either control system is simply

$$EC = \sum_{i=1}^{n} EC_i \quad (3)$$

where n is the number of items in the group or family.

Service criteria

The 'best' control strategy is that which minimizes the total relevant cost and simultaneously satisfies service constraints.

Two service measures considered are :

P1 = the probability that the cycle ends with no backorders (or lost sales) ; and

P2 = the fraction of demand satisfied without backordering, i.e. satisfied directly from on-hand stock.

This research is limited to the zero lead time case, i.e. it is assumed that replenishment orders arrive immediately. Some ideas for the non-zero case are discussed later. Although the lead time may be negligible, it is still meaningful to consider the above service constraints. For example, the fraction of demand satisfied without backordering may be viewed as the fraction of demand satisfied directly from the shelf rather than from some back storeroom.

Independent control under compound Poisson demand

Exact costs and service levels can be computed for an item under compound Poisson demand controlled independently by a given (S, s) strategy (i.e. order point, order-up-to-level) as shown by Silver (1970). We now review some key results from that paper.

For unit-sized transactions under an (S, s) system the available (on-hand plus on-order minus backorders) inventory level equals the must-order point each time a replenishment order is placed. When demand transactions are not unit-sized the transaction that triggers the order may instantaneously drop the inventory level well below the must-order point, possibly into an immediate backorder situation. The amount by which the must-order point is passed by the transaction is called the deficit. It will be denoted here as the random variable z.

The total relevant cost for a single item under independent control has been expressed in eqn. (1). Dropping subscripts this may be re-written as

$$EC(S, s) = \frac{\lambda E(t)}{S - s - E(z)} (A + a) + E(I)vr \tag{4}$$

where

λ = Poisson rate of demand transactions;

$E(t)$ = the expected value of the transaction size which can be calculated directly from the transaction size PMF;

$E(z)$ = the expected value of the deficit z; and

$E(I)$ = the expected value of the on-hand inventory level.

The probability distribution of the available stock I is needed to determine the average on-hand inventory level $E(I)$ and the PMF of the deficit, z. Since the time until the next transaction does not depend upon the current level of the available stock, the probability distribution of the available stock immediately after a transaction (and any ensuing replenishment action) is equivalent to the probability distribution at a random point in time (the latter being what is required).

The available inventory starts a cycle at the order-up-to-level S and, as transactions occur, it moves in jumps downward until a transaction drops it to or below the must-order point s. At that time a replenishment is made and the available inventory instantaneously jumps back up to S, starting a new cycle.

Let

$p_I(I_0)$ = prob {available stock is at level I_0},
$I_0 = s+1, s+2, \ldots, S-1, S$;

and

$p_t(t_0)$ = prob {transaction size is t_0};
$t_0 = 1, 2, 3, \ldots, t_{max}$.

The following set of equations govern the probabilities

$$\left.\begin{aligned}
p_I(S-1) &= p_I(S)p_t(1)\\
p_I(S-2) &= p_I(S)p_t(2) + p_I(S-1)p_t(1)\\
p_I(S-3) &= p_I(S)p_t(3) + p_I(S-1)p_t(2) + p_I(S-2)p_t(1)\\
&\ \,\vdots\\
p_I(s+1) &= p_I(S)p_t(S-s-1) + p_I(S-1)p_t(S-s-2) + \ldots + p_I(s+2) \cdot p_t(1)
\end{aligned}\right\} \tag{5}$$

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The total relevant cost for a single item under independent control has been expressed in eqn. (1). Dropping subscripts this may be re-written as

$$EC(S, s) = \frac{\lambda E(t)}{S - s + E(z)} (A + a) + E(I)vr \tag{4}$$

where

λ = Poisson rate of demand transactions ;

$E(t)$ = the expected value of the transaction size which can be calculated directly from the transaction size PMF ;

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The following set of equations govern the probabilities

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&p_I(S-3) = p_I(S)p_t(3) + p_I(S-1)p_t(2) + p_I(S-2)p_t(1) \\
&\vdots \\
&p_I(s+1) = p_I(S)p_t(S-s-1) + p_I(S-1)p_t(S-s-2) + \ldots + p_I(s+2) \cdot p_t(1)
\end{aligned}\right\} \tag{5}$$

Note that in cases where $S-s-1>t_{max}$ the last equations of this set will accordingly have fewer terms.

Equation set (5) may be solved as follows:

(i) Set $p_I(S)=1{\cdot}0$;

(ii) $p_I(S-1)$ can be calculated from the first equation, then $p_I(S-2)$ from the second, etc.;

(iii) Sum $p_I(I_0)$, $I_0=s+1, s+2, \ldots, S$ and divide each by the sum to normalize.

Once the probability mass function of the available inventory level has been computed, the expected inventory level can be calculated and inserted in the expected cost eqn. (4).

When the available stock level is x above s and a transaction of size $x+z_0$ occurs, the result is a deficit of size z_0. The deficit distribution can be derived using the following equations after performing the above steps (i) and (ii) *but not step* (iii):

$$p_z(z_0)=\sum_{x_0=s+1}^{S} p_I(x_0)\cdot p_t(x_0-s+z_0), \quad z_0=0, 1, 2, \ldots, t_{max}-1 \tag{6}$$

The expected deficit can be computed directly and an exact figure for the expected cost then follows using eqn. (4).

Exact service levels for the given (S, s) values can be calculated using the deficit distribution. The probability of no shortage per replenishment cycle, $P1$, is simply the probability that the deficit is less than or equal to s, i.e.

$$P1=\sum_{z_0=0}^{s} p_z(z_0) \tag{7}$$

To compute the fraction of demand satisfied without backorders, P2, first calculate the expected backorders per cycle (EB).

$$EB=\sum_{z_0=s+1}^{t_{max}-1} (z_0-s)p_z(z_0) \tag{8}$$

The average order quantity per cycle (AOQPC) is

$$\text{AOQPC}=S-s+E(z) \tag{9}$$

and it is straightforward to calculate

$$\text{P2}=1{\cdot}0-\frac{EB}{\text{AOQPC}} \tag{10}$$

This algebra required for calculating exact costs and exact service levels for a given independent (S, s) control strategy demonstrates that for compound Poisson demand the problem of determining *optimum* (S, s) values would be extremely complicated. However, it will be shown that some approximations can be used to estimate reasonable (S, s) values.

The first of these approximations concerns the deficit distribution. By application of renewal theory Karlin (1958) has developed some very useful results that apply to the distribution of z whenever the order quantity is much larger than the average transaction size (i.e. $S - s \gg E(t)$). He shows that

$$p_z(z_0) \simeq \frac{1}{E(t)} \sum_{t_0 = z_0 + 1}^{t_{max}} p_t(t_0), \quad z_0 = 0, 1, 2, \ldots, t_{max} - 1 \tag{11}$$

and

$$E(z) \simeq \frac{1}{2}\left[\frac{E(t^2)}{E(t)} - 1\right] \tag{12}$$

where $E(t)$ is the average transaction size and $E(t^2)$ is the average squared transaction size. The important point to note about these expressions is that they are independent of S and s.

A second approximation concerns the average inventory level. If all transactions were of average size, $E(t)$, there would be $k = (S-s)/E(t)$ transactions each cycle. The inventory level I would on the average spend $1/k$ of the time at $s + (S-s)/k$; another $1/k$ of the time it would be at $s + [2(S-s)]/k$ another $1/k$ at $s + [3(S-s)]/k$, and so on, up to $s + [k(S-s)]/k$. Thus

$$E(I) = \frac{1}{k}\left[s + \frac{(S-s)}{k}\right] + \frac{1}{k}\left[s + \frac{2(S-s)}{k}\right] + \ldots + \frac{1}{k}\left[s + \frac{k(S-s)}{k}\right]$$

$$= s + \frac{k+1}{2k}(S-s)$$

By definition $k = (S-s)/[E(t)]$, so the approximation used is

$$E(I) = s + \frac{S-s}{2} + \frac{E(t)}{2} \tag{13}$$

The cost equation can now be re-written as

$$EC(S, s) = \frac{\lambda E(t)(A+a)}{S - s + E(z)} + \frac{S + s + E(t)}{2} vr \tag{14}$$

where $E(z)$ is approximated as in eqn. (12).

Suppose P1, the probability of no shortage per replenishment cycle, is constrained to be greater than or equal to MP1:

$$\sum_{z_0 = 0}^{s} p_z(z_0) \geq MP1 \tag{15}$$

It is evident that the smallest s that satisfies eqn. (15) also minimizes that cost in eqn. (14). Thus equation set (11) can be used in conjunction with eqn. (15) to evaluate s. Once s has been found, S can be computed by setting

$$\frac{\partial EC(S, s)}{\partial S} = 0$$

The result is

$$S = s - E(z) + \sqrt{\left[\frac{2(A+a)\lambda E(t)}{vr}\right]} \tag{16}$$

In general this S will not be an integer and it will be necessary to test the integers above and below S to find which of these has minimum cost.

Suppose now that P2, the fraction of demand satisfied without backorder, is constrained such that P2 ≥ MP2. The expected backorders per cycle are given by (6) where $p_z(z_0)$ can be calculated by equation set (11). The average order quantity per cycle, $AOQPC = S - s - E(z)$, can be evaluated using eqn. (12). Thus the service constraint may be written as

$$1{\cdot}0 - MP2 \geq \frac{\sum_{z_0 = s+1}^{t_{max}-1} (z_0 - s)p_z(z_0)}{S - s + E(z)}$$

or

$$S \geq \frac{1}{1{\cdot}0 - MP2}\left[\sum_{z_0 = s+1}^{t_{max}-1} (z_0 - s)p_z(z_0)\right] + s - E(z) \qquad (17)$$

This provides a lower bound on S for a given s, but this is not necessarily the S that minimizes costs for given values of s and the service constraint MP2. The value of S that does minimizes costs for given s is as per eqn. (16). Hence for given s, S should be found by eqn. (17) but if this is less than that given by eqn. (16), the value from (16) is preferable. To find the best s value a one-dimensional search can be performed on all s, $0 \leq s \leq t_{max}$, with corresponding S from eqn. (16) or (17), whichever is applicable.

Numerical examples—independent control

Above we have shown that through the use of two reasonable approximations it is rather easy to establish (S, s) values for independent control with constraints on either P1 or P2. Appropriate FORTRAN language computer programmes were written and this was done for a family of items with $A = \$50{\cdot}00$, $a = \$5{\cdot}00$, $r = 0{\cdot}2$ and all having the same transaction size distribution shown in Table 1. The results for the service constraint P1 ≥ 0·95 are given in Table 2, and those for P2 ≥ 0·99 are in Table 3. Once (S, s) values were computed using the approximations, the algebra presented earlier was used to calculate the exact values of the expected costs and service levels. The results suggest that the model, using the approximations, has a tendency to slightly over-estimate costs. Required service levels are met in each case, despite the fact that the model appears to over-estimate service by a very small amount.

t_0	$p_t(t_0)$	t_0	$p_t(t_0)$	t_0	$p_t(t_0)$
1	0·00973	6	0·16356	11	0·02130
2	0·04205	7	0·13570	12	0·01091
3	0·09304	8	0·09904	13	0·00524
4	0·14311	9	0·06487	14	0·00235
5	0·16973	10	0·03872	15	0·00102

$E(t) = 5{\cdot}97$
$\sigma_t = 2{\cdot}38$
Coefficient of variation = 0·398

Table 1 Transaction size distribution.

	(1)	(2)	(3)	(4)
Item :				
λ (transactions/year)	48·4	6·8	12·8	20·4
v ($/piece)	0·90	1·20	3·90	2·80
S (pieces)	156	140	108	175
s (pieces)	7	7	7	7
Model results :				
$E(z)$ (pieces)	2·96	2·96	2·96	2·96
$E(I)$ (pieces)	84·48	70·45	60·48	93·96
EB (pieces)	0·10	0·10	0·10	0·10
AOQPC (pieces)	151·96	135·96	103·96	170·96
EC ($/year)	221·12	34·77	87·58	82·40
P1	0·95015	0·95015	0·95015	0·95015
Exact results :				
$E(z)$ (pieces)	2·96	2·96	2·95	2·96
$E(I)$ (pieces)	83·46	75·40	59·45	92·98
EB (pieces)	0·10	0·10	0·10	0·10
AOQPC (pieces)	151·96	135·90	103·96	170·96
EC ($/year)	219·71	34·53	86·78	81·95
P1	0·95010	0·95011	0·95012	0·95009

Total group cost, model = $425·86/year
Total group cost, exact = $422·95/year

Table 2. Examples of independent control parameters set with constraint on service level P1 (P1 ≥ 0·95).

	(1)	(2)	(3)	(4)
Item :				
λ (transactions/year)	48·4	6·8	12·8	20·4
v ($/piece)	0·90	1·20	3·90	2·80
S (pieces)	151	145	104	170
s (pieces)	2	2	3	2
Model results :				
$E(z)$ (pieces)	2·96	2·96	2·96	2·96
$E(I)$ (pieces)	79·48	76·48	56·48	86·98
EB (pieces)	1·32	1·32	0·38	1·32
AOQPC (pieces)	151·96	146·96	103·96	170·96
EC ($/year)	214·22	33·05	84·47	80·10
P2	0·99135	0·99090	0·99200	0·99231
Exact results :				
$E(z)$ (pieces)	2·96	2·96	2·96	2·96
$E(I)$ (pieces)	78·46	75·48	55·45	87·98
EB (pieces)	1·46	1·46	0·05	1·46
AOQPC (pieces)	151·96	145·96	103·96	170·96
EC ($/year)	212·81	33·40	83·06	79·63
P2	0·99046	0·99001	0·99085	0·99147

Total group cost, model = $412·44/year
Total group cost, exact = $409·51/year

Table 3. Examples of independent control parameters set with constraint on service level P2 (P2 ≥ 0·99).

A single item problem related to the case of coordinated control

Before investigating joint control consider a related single item problem, the solution to which is a key element in determining (S, c, s) values for an entire family. The problem consists of a single item faced with Poisson opportunities at rate μ to replenish at reduced cost a. Opportunities may be viewed as being caused by another item triggering a replenishment and the rate μ is effectively the expected number of orders per year triggered by all other items in the family. Silver (1974) discusses the justification for the … n assumption.

…en (S, c, s) values the expected cost and service levels are to be computed. As for independent control, the probability distribution of the available stock is used to determine the average on-hand inventory level, the probability of no shortage per replenishment cycle, and the fraction of demand satisfied without backordering.

Since opportunities for reduced-cost replenishments and demand transactions occur according to Poisson processes with rates μ and λ, respectively, the probability at any random point in time that the next event is a demand transaction is $\rho = \lambda/(\lambda+\mu)$ and the probability that the next is a replenishment opportunity is $(1-\rho) = \mu/(\lambda+\mu)$.

When the available stock is in the range $c+1, c+2, \ldots, S-1, S$, all opportunities for reduced cost replenishment will be ignored. Thus the following equations govern probabilities of various inventory levels immediately after a transaction.

$$\left.\begin{aligned}
p_I(S-1) &= p_I(S)\,.\,p_t(1)\\
p_I(S-2) &= p_I(S)\,.\,p_t(2)+p_I(S-1)\,.\,p_t(1)\\
p_I(S-3) &= p_I(S)\,.\,p_t(3)+p_I(S-1)\,.\,p_t(2)+p_I(S-2)\,.\,p_t(1)\\
&\;\;\vdots\\
p_I(c+1) &= p_I(S)\,.\,p_t(S-c-1)+p_I(S-1)\,.\,p_t(S-c-2)+\ldots\\
&\qquad +p_I(c+2)p_t(1)\\
p_I(c) &= p_I(S)\,.\,p_t(S-c)+p_I(S-1)\,.\,p_t(S-c-1)+\ldots\\
&\qquad +p_I(c+1)p_t(1)\\
p_I(c-1) &= p_I(S)\,.\,p_t(S-c+1)+p_I(S-1)\,.\,p_t(S-c)+\ldots\\
&\qquad +p_I(c+1)\,.\,p_t(2)+[p_I(c)\,.\,p_t(1)]\,.\,\rho\\
p_I(c-2) &= p_I(S)\,.\,p_t(S-c+2)+p_I(S-1)\,.\,p_t(S-c+1)+\ldots\\
&\qquad +p_I(c+1)\,.\,p_t(3)+[p_I(c)\,.\,p_t(2)+p_I(c-1)\,.\,p_t(1)]\,.\,\rho\\
p_I(c-3) &= p_I(S)\,.\,p_t(S-c+3)+p_I(S-1)\,.\,p_t(S-c+2)+\ldots\\
&\qquad +p_I(c+1)\,.\,p_t(4)+[p_I(c)\,.\,p_t(3)\\
&\qquad +p_I(c-1)\,.\,p_t(2)+p_I(c-2)\,.\,p_t(1)]\,.\,\rho\\
&\;\;\vdots\\
p_I(s+1) &= p_I(S)\,.\,p_t(S-s-1)+p_I(S-1)\,.\,p_t(S-s-2)+\ldots\\
&\qquad +p_I(c+1)p_t(c-s)+[p_I(c)\,.\,p_t(c-s-1)\\
&\qquad +p_I(c-1)\,.\,p_t(c-s-2)+\ldots+p_I(s+2)p_t(1)]\,.\,\rho
\end{aligned}\right\}\qquad (18)$$

To obtain the deficit distribution start by setting $p_I(S)=1{\cdot}0$ and calculate $p_I(S-1)$ from the first equation of (13), $p_I(S-2)$ from the second equation, etc. Then

$$p_z(z_0)=\sum_{I_0=c+1}^{S} p_I(I_0)p_1(I_0-s+z_0)+\sum_{I_0=s+1}^{c} p_I(I_0)p_1(I_0-s+z_0)\cdot\rho \qquad z_0=0,1,2,\ldots,I_{max}-1 \tag{19}$$

P1, the probability of no shortage per replenishment cycle, is

$$\mathrm{P1}=1{\cdot}0-\sum_{z_0=s+1}^{I_{max}-1} p_z(z_0) \tag{20}$$

For P2, the fraction of demand satisfied without backorders, consider first the average order quantity per cycle. Orders can be triggered in the range $s+1, s+2, \ldots, c$ or at or below s. Thus

$$\mathrm{AOQPC}=\sum_{I_0=s+1}^{c} p_I(I_0)[S-I_0](1-\rho)+\sum_{z_0=0}^{I_{max}-1} p_z(z_0)[S-s+z_0] \tag{21}$$

$$EB=\sum_{z_0=s}^{I_{max}-1}(z_0-s)p_z(z_0) \tag{22}$$

and

$$\mathrm{P2}=1{\cdot}0-\frac{EB}{\mathrm{AOQPC}} \tag{23}$$

The probability of a reduced cost replenishment (PRCR) is

$$\mathrm{PRCR}=\sum_{I_0=s+1}^{c} p_I(I_0)(1-\rho) \tag{24}$$

Now, unlike the independent case, the probability distribution of the available stock immediately after a transaction (or any ensuing replenishment action) is not equivalent to the probability distribution at a random point in time. For inventory levels $c+1, c+2, \ldots, S$ the expected time until the next event is the expected time until the next demand transaction, $1/\lambda$. For levels at or below c, the expected time until the next event is the expected time until either the next demand transaction or the next opportunity to replenish at a reduced setup cost, i.e. $1/(\lambda+\mu)$.

To calculate $E(I)$, we first weight each level by the expected duration, add and normalize.

$$p_I'(I_0)=\frac{p_I(I_0)\cdot\frac{1}{\lambda}}{\sum_{I_0=c+1}^{S} p_I(I_0)\cdot\frac{1}{\lambda}+\sum_{I_0=s+1}^{c} p_I(I_0)\cdot\frac{1}{\lambda+\mu}} \qquad I_0=c+1, c+2, \ldots, S \tag{25}$$

$$p_I'(I_0) = \frac{p_I(I_0)\cdot\frac{1}{\lambda+\mu}}{\sum_{I_0=c-1}^{s} p_I(I_0)\frac{1}{\lambda} + \sum_{I_0=s+1}^{c} p_I(I_0)\cdot\frac{1}{\lambda+\mu}}$$

$$I_0 = s+1,\ s+2,\ \ldots,\ c$$

Then

$$E(I) = \sum_{I_0=s+1}^{c} p_I'(I_0)\ I_0 \qquad (26)$$

The expected cost is therefore

$$EC(S, c, s) = \frac{\lambda E(t)}{\mathrm{AUQPC}}\left\{\left[1 - \sum_{I_0=s+1}^{c} p_I(I_0)\cdot(1-\rho)\right] A + a\right\} + E(I)vr \qquad (27)$$

Thus it is possible to calculate exact costs and service levels of a single item given (S, c, s) values and a known Poisson rate of opportunities to replenish at a reduced setup cost. However, it is evident that, in general, for compound Poisson demand an optimization of (S, c, s) values for an entire family with constraints on service levels is computationally intractable. (Note that Karlin's approximations of the deficit distribution cannot be applied in this case because of the possibility that cycles end 'prematurely', i.e. by taking an opportunity to replenish at a reduced set-up cost.)

Coordinated control for a family of items

Silver (1974) has developed an algorithm for selecting (S, c) control variables for a family of items under unit Poisson demand with zero lead time. Friend (1961) in related single item work has suggested that the unit Poisson distribution can provide a good fit to demand even when transactions are not of unit size. Thus one may transform the given compound Poisson demand distribution into Friend's 'equivalent unit Poisson' distribution and use Silver's algorithm to calculate $(S^*, c^*, 0)$ control variables for each item. The asterisk indicates that these values are in equivalent units: the must-order point s^* is zero because the lead time is zero. Both S^* and c^* can be converted back into original units S' and c' and the development of the previous section can be used to compute exact service levels for the control variables $(S', c', 0)$, $(S'+1, c'+1, 1)$, $(S'+2, c'+2, 2)$, ..., etc., until some set (S, c, s) meets the service constraint. This will not necessarily be the set of mathematically optimal control variables but rather a reasonable set of values obtained without excessive computations.

It should be noted that the procedure for iterating to meet a given service constraint is not as computationally demanding as it may initially appear to be. The equations that govern the probabilities of various inventory levels at the event prior to that ending the cycle (i.e. before normalizing) represent a significant portion of the computational burden of the exact service levels algorithm, but they do not have to be re-computed on successive iterations as s is increased in increments of 1. When one increases from s to $s+1$ the transformation is that $p_I(I_0)$ for $s+1$ equals $p_I(I_0-1)$ for s, The deficit distribution is recomputed by equation set (18) and service levels are readily calculated for the current (S, c, s) set.

Friend's fitting technique consists of using past data to estimate the mean m and variance σ^2 of demand per unit time, and substituting an 'equivalent' Poisson distribution in which transactions of constant unit size $U = \sigma^2/m$ occur with an average frequency $F = m/U$. The mean $UF = m$ and variance $U^2F = \sigma^2$ are the same as those of the original demand data. In practice the expected transaction size, $E(t)$, the average squared transaction size, $E(t^2)$, the rate of demand transactions, λ, and the value, v, of each item are used to calculate equivalent units, an equivalent transaction rate, and an equivalent unit value as follows:

$$m = \lambda E(t) \tag{28}$$

$$\sigma^2 = \lambda E(t^2) \tag{29}$$

$$U_{equiv} = \sigma^2/m \tag{30}$$

$$\lambda_{equiv} = m/U_{equiv} \tag{31}$$

$$v_{equiv} = U_{equiv} v \tag{32}$$

The preceding was summarized into an algorithm for setting (S, c, s) control variables for all items in a family with constraints on P1 or P2. It was programmed in FORTRAN and is presented in flow chart form in Fig. 1.

A simulation test for coordinated control

These (S, c, s) variables found in the preceding section are not optimal in the strict mathematical sense of the word—the assumption of Poisson opportunities for reduced cost replenishments and the use of Friend's equivalent units introduce some uncertainty regarding the model's estimates of costs and service levels. It is possible to investigate the reasonableness of the control variables it selects by digital computer simulation of the operation of the system. Simulation can be continued or repeated for several time periods and the cost of operating the system observed for each of these. The objective is then to estimate the population mean (actual cost of operating the coordinated control system) from this sample. An estimate of the variance of the sample mean is required to determine how representative the sample mean is of the population mean. For uncorrelated observations, the sample population variance divided by the sample size provides a useful estimate of the variance of the sample mean. However, in simulation experiments, such as discussed here, it is possible that observations are serially correlated. For correlated data, the variance of the sample mean is a function of the correlation between observations and consequently the estimation of this variance becomes more difficult. Fishman (1969) has incorporated an estimator of the variance of the sample mean into a method for estimating and collecting the sample size needed to evaluate the mean of a process with a specified level of statistical accuracy. The estimator is a function of the autoregressive representation of the process under study.

An important aspect of this research is the reliable demonstration of cost savings which will be realized if the coordinated control system were

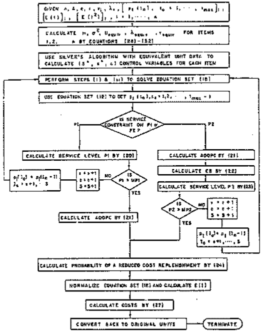

Figure 1. Selecting control variables for coordinated replenishment.

used in preference to independent control for a given family of items. A computer programme was written to use a simulation routine and Fishman's technique to terminate the simulation so as to estimate the cost of coordinated control such that the probability of the exact cost of coordinated control being greater than the estimated cost of coordinated control plus 10% of the absolute difference between coordinated and independent cost was less than 1%, i.e.

$$\text{Prob}\,\{EC_c > \widehat{EC}_c + 0{\cdot}1|EC_{ind} - \widehat{EC}_c|\} < 0{\cdot}01 \qquad (33)$$

where EC is the unknown true cost of coordinated control, EC_{ind} is the known cost of independent control, and $\widehat{EC}_c$ is the estimated (by simulation) cost of coordinated control. This probability statement is illustrated in Fig. 2. Essentially it means that one can be 99% confident of realizing 90% or more of the cost savings reported here.

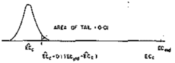

Figure 2 Reliability criterion applied to estimate of expected cost for coordinated control (based on simulation of system).

A further aspect of interest in the simulation was whether or not the selected control variables for the coordinated system would meet the service constraints. In the case of P1, the probability that the cycle ends with a non-negative inventory level, the appropriate statistical test is one concerning proportions as described by Freund (1971, pp. 275–76). Approximating the binomial distribution with the normal distribution leads to treating the statistic

$$w = \frac{x - (n)(MP1)}{\sqrt{\{(n)(MP1)(1 - MP1)\}}} \tag{34}$$

as a value assumed by a random variable having the standard (unit) normal distribution. The variable x is the number of cycles which ended with non-negative inventory levels and n is the total number of cycles, i.e. the number of times the item was ordered. Thus it is possible to test the null hypothesis $P1 = MP1$ against the alternative $P1 < MP1$. The fraction of demand satisfied without backordering is tested similarly since it too involves a proportion.

Additional details of this simulation experiment may be found in Thompstone (1974). It should be emphasized that simulations are needed only for testing purposes and not for day-to-day use of the (S, c, s) control procedure.

Numerical examples—coordinated control

The algorithm shown in Fig. 1 was used to set control variables for the two numerical examples described earlier and the variables were tested in a simulation experiment. The results are summarized in Tables 4 and 5 (corresponding to Tables 2 and 3, respectively). They indicate cost savings of 17·2 and 17·9%, respectively, when coordinated control variables selected by the algorithm presented herein are used in lieu of the best independent control. The desired service levels for the group were exceeded on the average in both cases. Only for one item (No. 1 in Table 4) did the observed

Item	1	2	3	4
Control variables selected by model				
S (pieces)	144	86	86	139
c (pieces)	86	14	53	67
s (pieces)	7	3	5	5
Model (algorithm—Fig. 1)				
$E(z)$ (pieces)	2·31	0·37	0·85	0·75
$E(I)$ (pieces)	80·06	54·93	54·05	65·97
AOQPC (pieces)	129·20	30·58	61·24	97·23
μ (opp./year)	0·707	2·409	2·130	2·140
EC (\$/year)	209·32	21·07	68·76	62·35
P1	0·96114	0·95529	0·95058	0·96154
Simulation (4·7 years per observation, 30 observations)				
Orders/year	2·15	0·74	1·45	1·43
Orders triggered/year	1·88	0·01	0·17	0·09
$E(z)$ (pieces)	2·42	0·01	0·37	0·26
$E(I)$ (pieces)	75·60	61·32	59·82	97·48
AOQPC (pieces)	136·45	54·72	56·28	85·65
μ (opp./year)	0·270	2·142	1·979	2·057
EC (\$/year)	212·35	18·81	62·40	56·59
P1	0·93549	1·00000	0·95438	0·96992
w stat, P1	−1·159	2·340	0·320	1·296

Group cost, independent control (exact) = \$422·95/year
Group cost, coordinated control (model) = \$359·52/year
Group cost, coordinated control (simul.) = \$350·14/year
Percent cost savings = 17·2%
Group mean P1 = 0·96507

Table 4. Examples of coordinated control parameters set with constraints on service level P1 (P1 ≥ 0·95).

mean service level fall below that which was required. Nevertheless, it was not possible to reject, at the 90% level of significance, the hypothesis that the service level of that item is met.

Comparison of coordinated (S, c, s) and independent control

The six factors given in Table 6 were considered in estimating the relationship between the expected cost savings of coordinated control (in comparison with independent control) and various characteristics of the group of items to be controlled. Using two levels for each gives 64 examples.

Factor 6 obviously has only two alternatives. Factors 3 and 5 illustrate values covering a range that one may expect to observe in practice. The transaction rate λ_i was set to keep the demand rate $\lambda_i E(t)_i$ constant as $E(t)$ varied. The v_is used were 0·90, 1·20, 3·90, 2·30, 1·20, 3·90, 2·30 and 1·20 dollars per piece and corresponding demand rates, the $\lambda_i E(t)_i$s, were 290, 41, 77, 122, 59, 154, 87 and 25 pieces per year. Groups of four and eight items

Item	1	2	3	4
Control variables selected by model				
S (pieces)	139	82	81	134
c (pieces)	91	49	48	82
s (pieces)	2	1	0	0
Model (algorithm–Fig. 1)				
$E(z)$ (pieces)	2·31	0·37	0·85	0·75
$E(I)$ (pieces)	75·06	51·93	50·05	81·97
EB (pieces)	1·14	0·37	0·86	0·75
AOQPC (pieces)	129·20	59·58	61·24	97·23
μ (opp./year)	0·787	2·419	2·130	2·140
EC ($/year)	302·42	20·33	63·66	60·05
P2	0·99119	0·99379	0·99009	·99232
Simulation (4·7 years per observation, 30 observations)				
Orders/year	2·13	0·74	1·38	
Orders triggered/year	1·94	0·02	0·11	0·06
$E(z)$ (pieces)	2·66	0·04	0·22	0·14
$E(I)$ (pieces)	73·03	57·85	56·67	92·32
$E(B)$ (pieces)	1·30	0·04	0·15	0·14
AOQPC (pieces)	136·87	64·95	53·67	86·11
μ (opp./year)	0·154	2·106	2·023	2·071
EC ($/year)	208·59	18·67	56·59	52·50
P2	0·99052	0·99940	0·99754	0·99854
w stat, P2	1·048	7·822	7·257	11·909

Group cost, independent control (exact) = $409·51/year
Group cost, coordinated control (model) = $346·48/year
Group cost, coordinated control (simul.) = $336·35/year
Percentage cost savings = 17·9%
Group mean P2 = 0·99650

Table 5. Examples of coordinated control parameters set with constraints on service level P2 (P2 ≥ 0·99).

Factor	Levels
(1) Expected transaction size, $E(t)$	6, 12
(2) Coefficient of variation of transaction size, c.v.	0·4, 0·7
(3) Ratio of line cost to header cost, a/A	0·1, 0·5
(4) Number of items in group, n	4, 8
(5) Service level	0·95, 0·99
(6) Service criterion	P1, P2

Table 6. Factors and levels used to estimate cost savings.

were selected similar to earlier research by Silver (1971, 1974). Examples with four items used the first four listed above.

The first two factors concern the transaction size distribution. Features desired include:

(1) $p_t(t_0)=0$ for $t_0=\ldots-2, -1, 0$.

(2) Parameters which achieve desired combinations of mean and coefficient of variation.

(3) *A skewness to ensure sufficient generality for most applications.*

Unfortunately the more common probability mass functions do not satisfy these requirements. The negative binomial distribution,

$$\left.\begin{aligned} p_y(y_0|r,p)&=\binom{y_0+r-1}{r-1}p^r(1-p)^{y_0} \\ y_0&=0, 1, 2, \ldots \\ r&=1, 2, 3, \ldots \\ 0\cdot0&<p<1\cdot0 \end{aligned}\right\} \quad (35)$$

is not appropriate because of the non-zero probability of a transaction size of 0. Thus a transposed negative binomial distribution was used:

$$\left.\begin{aligned} p_x(x_0|r,p)&=\binom{x_0+r-2}{r-1}p^r(1-p)^{x_0-1} \\ x_0&=1, 2, 3, \ldots \\ r&=1, 2, 3, \ldots \\ 0&<p<1\cdot0 \end{aligned}\right\} \quad (36)$$

$$E(x)=r\left(\frac{1}{p}-1\right)+1 \quad (37)$$

$$\sigma_x^2=\frac{r(1-p)}{p^2} \quad (38)$$

and

$$\begin{aligned} \text{c.v.}&=\frac{\sigma_x}{E(x)} \\ &=\frac{\sqrt{[r(1-p)]}}{r(1-p)+p} \end{aligned} \quad (39)$$

The proposed distribution would imply a maximum transaction size, t_{max}, of infinity. Consequently it was truncated at the lowest point x' such that

$$\sum_{x_0=1}^{x'} p_x(x_0) \geq 0\cdot999$$

The 'remaining' probability, i.e.

$$1\cdot0-\sum_{x_0=1}^{x'} p_x(x_0)$$

was distributed over the values $x_0 = 1, \ldots, x'$ proportionally to their probabilities before truncation.

Values of r and p given in Table 7 were selected such that levels of the factor $E(t)$ would be approximately 6 and 12 and levels of c.v. would be approximately 0·4 and 0·7, these being determined to be among the mutually attainable pairs. Thompstone (1974) describes additional characteristics of the transposed negative binomial distribution.

	r	p	$E(t)$	c.v.
1	33	0·866	5·96743	0·39835
2	2	0·260	5·96972	0·68799
3	10	0·477	11·94397	0·39703
4	2	0·154	11·96701	0·69557

Table 7. Parameters of transaction size distributions.

Reasonable (S, s) values were estimated for independent control for all 64 examples and resulting exact costs and service levels were calculated. Reasonable (S, c, s) control variables were computed for coordinated control (the 64 examples required only 1·2 min of central processing unit execution time on an IBM 360/75 computer system). Simulation was used to estimate percentage cost savings which would actually be realized if coordinated control were used in preference to independent control. The w statistics (eqn. (34)) were calculated to investigate how well service constraints were met. Detailed numerical results are given in the Appendix.

These results were subjected to an analysis of variance and the following models were fit by linear regression.

For service criterion P1,

$$\begin{aligned}\text{Percentage cost savings} = {} & 13{\cdot}5 + 0{\cdot}684 \,.\, E(t) \\ & - 2{\cdot}89 \,.\, [\text{c.v.}] - 11{\cdot}5 \,.\, [a/A] + 2{\cdot}66 \,.\, n - 1{\cdot}12 \,.\, E(t) \,.\, L \\ & - 0{\cdot}258 \,.\, E(t) \,.\, n + 0{\cdot}0825 \,.\, E(t) \,.\, [a/A] \\ & - 2{\cdot}62 \,.\, [a/A] \,.\, n + 0{\cdot}55 \,.\, E(t) \,.\, [\text{c.v.}] \,.\, [a/A] \end{aligned} \tag{40}$$

and for P2,

$$\begin{aligned}\text{Percentage cost savings} = {} & 13{\cdot}3 - 0{\cdot}946 \,.\, E(t) \\ & - 2{\cdot}17 \,.\, [\text{c.v.}] - 12{\cdot}3[a/A] + 2{\cdot}60 \,.\, n + 0{\cdot}717 \,.\, E(t) \,.\, L \\ & - 0{\cdot}0364 \,.\, E(t) \,.\, n + 0{\cdot}428 \,.\, E(t) \,.\, [a/A] \\ & - 2{\cdot}47 \,.\, [a/A] \,.\, n - 0{\cdot}264 \,.\, E(t) \,.\, [\text{c.v.}] \,.\, [a/A] \end{aligned} \tag{41}$$

where L is the required service level for P1 or P2.

Goodness of fit measures were $F_{[illegible]} = 592$ and $F_{[illegible]} = 444$, respectively, both being statistically significant at the 99·9% level of significance. Multiple correlation coefficients were 0·92 and 0·90 and standard errors of estimates were 2·6 and 3·0, respectively. Reasonable cause-effect relationships seen in these equations include cost savings increasing as: (i) the number of items in the group increases; (ii) variability of the transaction size distribution decreases; and (iii) the line cost (a) becomes smaller relative to the header cost (A). Actual relationships may not be strictly linear but these models give a general indication of potential savings. Regression estimates are compared with actual savings in the Appendix.

Analysis of the results indicates:

(a) In all examples coordinated control costs less than independent control, savings averaging 15·9%.

(b) In most cases the model of the system used to set control variables is conservative in predicting cost savings, the average prediction being 14·3%. (Examples for which the model is not conservative are 6, 14, 16, 32, 40, 56 and 64.)

(c) The model is generally conservative in meeting service levels. In 61 of the 64 cases the actual average (across items) service is higher than required and in the other three the service is negligibly lower than required. (Of the three examples, 20, 21 and 22, the worst was lower by $0.99-0.9875=0.0025$, i.e. $\frac{1}{4}$%.)

Thus, not only is the (S, c, s) control method achieving an average cost savings of 15·9%, but it is also providing better average service than is required. In a few isolated cases (examples 10, 13, 29 and 58) for one item in the group the w statistic indicates that at the 99% level of significance one can reject the hypothesis that the actual service level is equal to the predicted level in favour of the alternative that the actual service is lower than the predicted. This indicates 4 out of 388 (or approximately 1% of the) items are below their required levels—random fluctuations could account for this. The item which statistically speaking is worst off (example 58, w statistic $= -4.163$) involves an actual service level of 0·98865 in comparison with a predicted level of 0·99. (To conserve space the w statistics have not been tabulated.)

Previous research into coordinated control strategies with unit Poisson demand and zero lead time with equivalent demand rates and standard unit costs and combinations of a/A and n by Silver (1974) shows average actual cost savings of 18·6%. This is consistent with the earlier conclusion that cost savings increase as the variability of the transaction size decreases (in his case of unit Poisson demand, the coefficient of variation of the transaction size is 0).

Conclusions

This research has developed a practical procedure for setting the control variables of an (S, c, s) coordinated inventory control system for a group of items under compound Poisson demand and zero lead time. Cost savings in comparison with independent control averaged 15·9% over 64 examples. The coordinated procedure requires no input beyond that needed for independent control. Although computations have not been streamlined, the 64 examples required only 1·3 min of execution time on an IBM 360/75 computer system.

An obvious extension of this research would be to relax the assumption of zero lead time. A recurrence relationship developed by Adelson (1966) would be very useful in calculating the lead time demand distribution.

Appendix

Percentage cost savings of coordinated control systems over independent control for 64 examples

Example	$E(t)$	c.v.	a/A	n	Service criterion	Service level (actual average)	Percentage cost savings (actual)	Percentage cost savings (system model)	Percentage cost savings (regression model)
1	6	0·4	0·1	4	P1@0·95	0·95507	17·2	14·5	18·1
2	12	0·4	0·1	4		0·97143	16·7	15·1	15·4
3	6	0·7	0·1	4		0·98106	17·6	14·7	17·3
4	12	0·7	0·1	4		0·95992	15·0	14·8	14·7
5	6	0·4	0·5	4		0·95972	10·7	8·2	10·0
6	12	0·4	0·5	4		0·95807	7·0	8·3	8·1
7	6	0·7	0·5	4		0·96219	9·2	8·2	9·7
8	12	0·7	0·5	4		0·95643	8·8	8·5	8·2
9	6	0·4	0·1	8		0·97766	26·7	23·8	27·1
10	12	0·4	0·1	8		0·96332	24·3	22·4	23·7
11	6	0·7	0·1	8		0·96979	26·7	23·1	26·3
12	12	0·7	0·1	8		0·96844	23·5	22·4	23·1
13	6	0·4	0·5	8		0·95643	15·0	12·4	14·8
14	12	0·4	0·5	8		0·96405	12·5	12·7	12·2
15	6	0·7	0·5	8		0·95849	14·0	12·3	14·1
16	12	0·7	0·5	8		0·95689	12·2	12·8	12·4
17	6	0·4	0·1	4	P1@0·99	0·99192	17·7	15·1	17·8
18	12	0·4	0·1	4		0·99131	15·1	13·9	14·9
19	6	0·7	0·1	4		0·99111	17·5	15·0	17·1
20	12	0·7	0·1	4		0·98826	13·8	13·8	14·2
21	6	0·4	0·5	4		0·98050	9·9	8·0	9·8
22	12	0·4	0·5	4		0·98750	8·1	7·6	7·5
23	6	0·7	0·5	4		0·99007	9·7	8·3	9·4
24	12	0·7	0·5	4		0·99184	7·9	7·8	7·7
25	6	0·4	0·1	8		0·99530	26·0	22·5	26·9
26	12	0·4	0·1	8		0·99421	22·8	21·0	23·2
27	6	0·7	0·1	8		0·99403	26·2	22·6	26·0
28	12	0·7	0·1	8		0·99484	21·1	20·4	22·5
29	6	0·4	0·5	8		0·99020	11·1	12·5	14·5
30	12	0·4	0·5	8		0·99431	12·0	11·7	11·7
31	6	0·7	0·5	8		0·99240	14·5	12·4	14·2
32	12	0·7	0·5	8		0·99316	11·8	11·0	11·8

Appendix (*continued*)

Example	$E(t)$	c.v.	σ/d	n	Service criterion	Service level (actual average)	Percentage cost savings (actual)	Percentage cost savings (system model)	Percentage cost savings (regression model)
33	6	0·4	0·1	4	P2@0·95	0·99137	16·8	14·9	18·3
34	12	0·4	0·1	4		0·99377	16·8	14·6	16·0
35	6	0·7	0·1	4		0·98986	17·8	14·6	17·6
36	12	0·7	0·1	4		0·97661	10·4	14·6	15·3
37	6	0·4	0·5	4		0·99266	10·5	8·1	10·2
38	12	0·4	0·5	4		0·99096	8·9	7·9	8·7
39	6	0·7	0·5	4		0·99040	8·4	7·9	9·3
40	12	0·7	0·5	4		0·97502	7·1	7·8	7·6
41	6	0·4	0·1	8		0·99567	26·8	22·8	26·0
42	12	0·4	0·1	8		0·98713	24·0	21·8	23·7
43	6	0·7	0·1	8		0·99119	26·0	22·1	2[illegible]·2
44	12	0·7	0·1	8		0·98260	22·0	21·2	23·0
45	6	0·4	0·5	8		0·99136	14·3	12·3	14·8
46	12	0·4	0·5	8		0·98560	11·0	11·9	12·6
47	6	0·7	0·5	8		0·99152	13·4	11·8	13·0
48	12	0·7	0·5	8		0·98037	11·4	11·1	11·3
49	6	0·4	0·1	4	P2@0·99	0·99650	17·9	15·6	18·5
50	12	0·4	0·1	4		0·99520	16·0	15·0	16·4
51	6	0·7	0·1	4		0·99657	18·7	15·2	17·8
52	12	0·7	0·1	4		0·99313	14·6	14·0	15·6
53	6	0·4	0·5	4		0·99467	10·4	8·8	10
54	12	0·4	0·5	4		0·99116	8·6	8·2	9·0
55	6	0·7	0·5	4		0·99470	10·4	8·5	9·5
56	12	0·7	0·5	4		0·99292	7·6	8·1	7·9
57	6	0·4	0·1	8		0·99720	27·9	23·2	27·0
58	12	0·4	0·1	8		0·99520	21·3	22·5	24·1
59	6	0·7	0·1	8		0·99603	26·7	23·3	26·3
60	12	0·7	0·1	8		0·99393	22·8	21·4	23·3
61	6	0·4	0·5	8		0·99542	14·7	13·0	15·0
62	12	0·4	0·5	8		0·99402	13·6	12·9	12·8
63	6	0·7	0·5	8		0·99517	14·6	12·7	14·1
64	12	0·7	0·5	8		0·99272	12·0	12·6	11·7

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Il arrive très souvent que la coordination des réapprovisionnements de plusieurs articles d'un même stock se traduise par une diminution significative des coûts du lancement des commandes. Cet article propose un moyen raisonnable pour déterminer les différentes variables d'un système de gestion (*S*, *c*, *s*): niveau de réapprovisionnement, seuil éventuel de commande, seuil de commande. On suppose que la demande suit une loi de Poisson mélangée et que les délais de réapprovisionnement sont fixés à zéro. La comparaison avec un système de gestion article par article montre que la coordination des commandes permet des économies substantielles. (Sur 64 exemples traités, la moyenne des réductions de coût réalisées est de 13·8%.)

Koordinierung von Produkten in der Bestellpolitik kann häufig zu erheblichen Kostenersparnissen führen. Ziel dieser Arbeit ist es, ein vernünftiges Verfahren zu entwickeln, welches 'order-up-to-levels', 'can-order-points' und 'must-order-points' für ein gegebenes Koordinations-system bestimmt. Es ist unterstellt, dass die Nachfrage einer compound Poisson-Verteilung folgt und die Lieferfrist vernachlässigt werden kann. Die beste, nicht koordinierte Bestellpolitik als Basis benutzend kann gezeigt werden, dass Koordination zu erheblichen Kostenersparnissen (im Durchschnitt 13·8% in 64 durchgerechneten Beispielen) führen kann.

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APPENDIX C:

PROGRAMME LISTING

```
1 N=4:NA=50:HI=5:RATE=.2:TMAX=15:TMIN=1:INC=1:C
SL=.95
2 LANDA(0)=40.4:LANDA(1)=6.0:LANDA(2)=12.8:LANDA(3)=20.4:VALUE(0)=6.[illegible]:VALUE(1)
.2:VALUE(2)=3.7:VALUE(3)=2.5
3 DIM TO(2000,1):TO(0,0)=1:TO(1,0)=2:TO(2,0)=3:TO(3,0)=4:TO(4,0)=5:TO(5,0)=6:TO(
6,0)=7:TO(7,0)=8:TO(8,0)=9:TO(9,0)=10:TO(10,0)=11:TO(11,0)=12:TO(12,0)=13:TO(13,
0)=14:TO(14,0)=15
4 TO(0,1)=.00973:TO(1,1)=.04295:TO(2,1)=.09764:TO(3,1)=.1[illegible]:TO(4,1)=.[illegible]:TO(
5,1)=.16354:TO(6,1)=.1[illegible]:TO(7,1)=.09904:TO(8,1)=.06487:TO(9,1)=.0[illegible]:TO(10,1)=
.0213:TO(11,1)=.01071:TO(12,1)=.00524:TO(13,1)=.00238:TO(14,1)=.00[illegible]
5 GOTO 110
10 REM [illegible] LEVELS AND TOTAL EXPECTED [illegible]
20 REM DATA INPUT [illegible]
30 INPUT "NO. OF ITEMS IN FAMILY: ";N
40 INPUT "MAJOR SETUP COST: ";MA
50 INPUT "MINOR SETUP COST: ";MI
60 INPUT "AVERAGE [illegible] CARRYING RATE: ";RATE
70 INPUT "MAX. [illegible]: ";TMAX
80 INPUT "MIN. [illegible]: ";TMIN
90 INPUT "INCREMENT: ";INC
100 INPUT "IS SERVICE [illegible] ON CUSTOMER SERVICE LEVEL [illegible]: ";CSL
110 REM SET UP ARRAYS [illegible]
120 REM [illegible] TO(I,1)
150 DIM HLAD(N,1)
160 DIM STPLV(N,1)
170 DIM BEQUIV(N,1)
180 DIM LEQUIV(N,1)
190 DIM VEQUIV(N,1)
200 DIM OUTL(N,1)
210 DIM GAL(N,1)
220 DIM NN(N,1)
240 DIM NT(N,1):REM EXPECTED NO. OF REPLENISHMENTS TRIGGERED BY THIS ITEM
250 DIM MKN(N,1):REM RATE OF REDUCED COST REPLENISHMENT OPPORTUNITIES
260 DIM RO(N,1):REM PROBABILITY THAT THE NEXT EVENT IS A DEMAND TRANSACTION
270 DIM SOPT(100,N,1)
280 DIM COPT(100,N,1)
290 DIM OPTSC(N,3)
300 DIM FLAG(N)
```

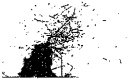
```
310 DIM SCOPT(N)
320 DIM OPTS(N)
330 DIM OPTC(N)
332 DIM P1(500,N-1)
334 DIM P2(N-1,(MAX [illegible]/INC))
336 DIM P3(N-1)
338 DIM C(N-1)
340 DIM ADDC(N-1)
350 DIM PROB(N-1)
360 DIM [illegible](500,N-1)
370 DIM [illegible](N-1)
380 DIM LL(N-1)
385 DIM FRUTM(6,2)
387 DIM COST(100,N-1)
388 GOTO 460
390 REM DATA INPUT [illegible]
392 I=0
393 FOR I=1 TO N
394 PRINT "[illegible] FOR ITEM ",I:INPUT [illegible]
395 PRINT "VALUE FOR ITEM ",I:INPUT VALUE(I)
396 NEXT I
400 C1=0
410 FOR I=0 TO [illegible]
430 PRINT "[illegible] DISTRIBUTION FACTOR ",I:INPUT [illegible]
440 TO(C1,0)=I:TO(C1,1)=[illegible]
445 C1=C1+1
450 NEXT I
460 E=0
470 FOR I=0 TO [illegible]
480 ET=ET+TO(I,0)*TO(I,1)
490 ET2=ET2+(TO(I,0))^2*TO(I,1)
500 NEXT I
501 LPRINT "ET","ET2"
505 LPRINT ET,ET2:LPRINT
510 EZ=.5*((ET2/ET)-1):LPRINT "EXPECTED VALUE OF DEFICIT ",EZ
560 REM EQUIVALENT UNIT POISSON [illegible]
570 I=0
580 FOR I=0 TO N-1
```

```
570 MEAN(I)=LAMDA(I)*LT
600 STDEV(I)=LAMDA(I)*LT^2
610 UEQUIV(I)=STDEV(I)/MEAN(I)
620 LEQUIV(I)=MEAN(I)/UEQUIV(I)
630 VEQUIV(I)=UEQUIV(I)*VALUE(I)
627 LPRINT "MEAN(I)","STDEV(I)","UEQUIV(I)","LEQUIV(I)","VEQUIV(I)"
640 LPRINT MEAN(I),STDEV(I),UEQUIV(I),LEQUIV(I),VEQUIV(I):LPRINT
650 NEXT I
660 REM SILVER'S ALGORITHM*********************************[illegible]
670 I=0
680 FOR I=1 TO N-1
690 COL(I)=0
695 PRINT NG,NT,LEQUIV(I),VEQUIV(I),RATE
700 QUIL(I)=SQR((2*[illegible]*LEQUIV(I))/(VEQUIV(I)*RATE))
710 NT(I)=LEQUIV(I)/QUIL(I)
715 PRINT QUIL(I),NT(I)
720 NEXT I
730 Z=0
740 IF Z=10 THEN [illegible] "NO CONVERGENCE"
745 LPRINT:PRINT
750 [illegible]
760 FOR I=0 TO N-1
770 MU(I)=0
780 FOR J=0 TO N-1
790 MU(I)=NT(J)+MU(I)
800 NEXT J
810 MU(I)=MU(I)-NT(I)
820 RO(I)=LEQUIV(I)/(LEQUIV(I)+MU(I))
825 PRINT "RO,MU",RO(I),MU(I)
830 COL(I)=QUIL(I):L=1
840 SQ=((NA*RO(I)*COL(I)+MU)*2*LEQUIV(I)/(VEQUIV(I)*RATE))+((2*COL(I)*MU(I)*(COL(I)+1))/(1-RO(I)))-((MU(I)*(1-RO(I))*COL(I))*(1+RO(I)*(COL(I)+1)))/(1-RO(I))^2)
850 IF SQ<0 THEN SCOPT(I)=COL(I) ELSE SCOPT(I)=COL(I)-RO(I)*(1-RO(I))*(COL(I))/(1-RO(I))+SQR(SQ)
890 OPTB(I)=SCOPT(I)+OPTC(I)-COL(I)
892 GROUP=RO(I)*(1-RO(I)-COL(I))/(1-RO(I))
895 IF SQ<0 THEN EC(I)=(MU(I)*(COL(I)-GROUP)*VEQUIV(I)*RATE/(1-RO(I))+LEQUIV(I)*(RO(I)*COL(I))*[illegible](I)*MU(I))/GROUP ELSE EC(I)=VEQUIV(I)*RATE*[illegible]
```

```
896 IF L=1 THEN ECMIN(0,0)=EC(I)
897 IF EC(I)<ECMIN(0,0) THEN 898 ELSE 900
898 ECMIN(0,0)=EC(I):ECMIN(0,1)=OPTS(I):ECMIN(0,2)=OPT(I)
900 L=0:IF COUNT>0 THEN COL(I)=COL(I)-1 ELSE COL(I)=COL(I)-20
905 IF COUNT>0 [illegible] COUNT=COUNT+1
906 IF COUNT>40 THEN 920
910 IF COL(I)<0 [illegible] ELSE 840
912 IF COUNT>0 THEN 920
914 COL(I)=ECMIN(0,2)[illegible]COUNT=1:GOTO 840
920 COUNT=0:SOPT(Z,I)=ECMIN(0,1):COPT(Z,I)=ECMIN(0,2)[illegible]
925 PRINT SOPT(Z,I),COPT(Z,I),COST(Z,I):REM [illegible]
927 NT(I)=[illegible]
,1))/(1-[illegible](I)))
928 ECMIN(0,0)=0:ECMIN(0,1)=0:ECMIN(0,2)=0
930 NEXT I
940 IF Z=0 THEN 950 ELSE 960
950 Z=Z+1:GOTO [illegible]
960 J=0
970 FOR I=0 TO N-1
980 IF ABS(SOPT(Z,I)-SOPT(Z-1,I))<1 THEN FLAG(I)=1 ELSE FLAG(I)=0
990 NEXT I
1000 J=0
1010 FOR I=0 TO N-1
1020 IF FLAG(I)=0 THEN 1041
1030 NEXT I
1040 GOTO 1049
1041 FOR I=0 TO N-1
1042 IF (ABS((COST(Z,I)-COST(Z-1,I))/COST(Z,I))*100<1 THEN FLAG(I)=1 ELSE FLAG(
I)=0
1043 NEXT I
1044 FOR I=0 TO N-1
1045 IF FLAG(I)=0 THEN 1046 ELSE 1047
1046 Z=Z+1:GOTO 740
1047 NEXT I
1049 PRINT:PRINT:PRINT:PRINT:PRINT:PRINT "[illegible]"
1060 FOR I=0 TO N-1
1070 FOR J=0 TO 1
1080 IF J=0 THEN [illegible](I,J)=SOPT(Z,I) ELSE [illegible](I,J)=COPT(Z,I)
```

```
1090 NEXT J
1095 LPRINT OPTSC(I,0),OPTSC(I,1)
1100 NEXT I
1110 I=0
1120 FOR I=0 TO N-1
1130 IF OPTSC(I+1,0)>OPTSC(I,0) THEN SMAX=OPTSC(I+1,0)
1140 NEXT I
1150 FOR I=0 TO N-1
1160 OPTSC(I,2)=0
1162 NEXT I
1164 REM CONVERT BACK TO ORIGINAL UNITS************************************
1165 FOR I=0 TO N-1
1166 FOR J=0 TO 2
1167 PRINT I;OPTSC(I,J) OPTSC(I,J)*UEQUIV(I)
1168 NEXT J
1169 OPTSC(I,0)=OPTSC(I,0)\[illegible]
1170 NEXT I
1175 LPRINT "S","C","T"
1176 FOR I=0 TO N-1
1177 LPRINT OPTSC(I,0),OPTSC(I,1),OPTSC(I,2)
1178 NEXT I
1180 REM [illegible]****************************************
1190 OPTSC(I,0)=OPTSC(I,0)\[illegible]
1200 FOR I=0 TO N-1
1210 S=OPTSC(I,0)\[illegible]
1215 IF [illegible] THEN PI(S,I)=1
1220 FOR J=1 TO S-1
1230 FOR K=1 TO J
1240 PI(S-J,I)=PI(S-J,I)+[illegible](S-J+K,I)*TO(K-1,I)
1250 NEXT K
1255 PRINT I,S-J,PI(S-J,I)
1260 NEXT J
1270 X=0
1280 FOR J=S-C+1 TO S-MOP-1
1290 X=X+1
1300 FOR K=1 TO J
1310 PI(S-J,I)=PI(S-J,I)+PI(S-J+K,I)*TO(K-1,I)
1315 IF [illegible] PI(S-J,I)=PI(S-J,I)*RO(I)
```

```
1350 NEXT K
1375 PRINT I,S-J,P1(S-J,I)
1380 NEXT J
1385 PRINT:PRINT
1390 NEXT I
1400 REM DEFICIT DISTRIBUTION****************************
1420 FOR I=1 TO N-1
1430 FOR J=0 TO TMAX-1
1435 P2(I,J)=0
1440 FOR K=OPTSC(I,1)+1 TO OPTSC(I,0)
1450 P2(I,J)=P2(I,J)+P1(K,I)*(Q(K-OPTSC(I,2)+J-1,I))
1460 NEXT K
1470 FOR K=OPTSC(I,2)+1 TO OPTSC(I,1)
1480 P2(I,J)=P2(I,J)+P1(K,I)*(Q(K-OPTSC(I,2)+J-1,I) [illegible]
1490 NEXT K
1495 PRINT I,J,P2(I,J)
1500 NEXT J
1505 P1(I)=0
1507 IF ITER=1 THEN 1560
1510 NEXT I
1520 REM SEPARATE COMPUTATIONS****************************
1530 REM P1=PROBABILITY OF NO SHORTAGE PER REPL. CYCLE
1540 REM P2=FRACTION OF DEMAND SATISFIED WITHOUT BACKORDER
1560 FOR I=1 TO N-1
1570 ITER=0
1575 FOR J=OPTSC(I,2)+1 TO TMAX-1
1580 P1(I)=P2(I,J)*P1(J)
1590 NEXT J
1600 P1(I)=1-P1(I):LPRINT:LPRINT:LPRINT P1(I)
1610 IF P1(I)>CSL THEN 1649
1611 LPRINT OPTSC(I,0),OPTSC(I,1),OPTSC(I,2),"......NEXT ITERATION"
1620 OPTSC(I,0)=OPTSC(I,0)+1
1630 OPTSC(I,1)=OPTSC(I,1)+1
1640 OPTSC(I,2)=OPTSC(I,2)+1
1642 FOR J=OPTSC(I,2)+1 TO OPTSC(I,0)-2
1644 P1(OPTSC(I,0)-J,I)=P1(OPTSC(I,0)-J-1,I)
1646 NEXT J
1648 ITER=1:GOTO 1430
```



```
1649 NEXT I
1690 REM AVERAGE ORDER QUANTITY PER CYCLE*************************
1700 FOR I=0 TO N-1
1710 FOR J=OPTSC(I,2)+1 TO OPTSC(I,1)
1720 AOQPC(I)=AOQPC(I)+P(J,I)*(OPTSC(I,0)-J)*(1-RO(I))
1730 NEXT J
1740 FOR J=0 TO ULR-1
1750 AOQPC(I)=AOQPC(I)+P(J,I)*(OPTSC(I,0)-OPTSC(I,2)+1)
1760 NEXT J
1770 NEXT I
1780 REM PROBABILITY OF A REDUCED COST REPLENISHMENT*****************
1790 FOR I=0 TO N-1
1800 FOR J=OPTSC(I,2)+1 TO OPTSC(I,1)
1810 PRCR(I)=P(J,I)*(1-RO(I))
1820 NEXT J
1830 NEXT I
1840 REM EXPECTED SHORTAGES*************************
1850 FOR I=0 TO N-1
1851 DENOM=0
1852 FOR K=OPTSC(I,1)+1 TO OPTSC(I,0)
1853 DENOM=DENOM+P(K,I)/LAMDA(I)
1854 NEXT K
1855 FOR K=OPTSC(I,2)+1 TO OPTSC(I,1)
1856 DENOM=DENOM+P(K,I)/(LAMDA(I)+MU(I))
1857 NEXT K
1900 FOR J=OPTSC(I,2)+1 TO OPTSC(I,0)
1930 IF J<=OPTSC(I,1) GOTO 1950
1940 PIPRIME(J,I)=(P(J,I)/LAMDA(I))/DENOM:GOTO 1955
1950 PIPRIME(J,I)=(P(J,I)/(LAMDA(I)+MU(I)))/DENOM
1955 NEXT J
1960 NEXT I
1970 FOR I=0 TO N-1
1980 FOR J=OPTSC(I,2)+1 TO OPTSC(I,0)
1990 EI(I)=PIPRIME(J,I)*...(I)
2000 NEXT J
2010 NEXT I
2020 REM EXPECTED ISSUES*************************************
2030 FOR I=0 TO N-1
```

```
2035 EC(I)=0
2040 FOR J=OPTEC(I,2)+1 TO OPTEC(I,1)
2050 EC(I)=EC(I)+E(J,I)*(1-RO(I))
2060 NEXT J
2070 EC(I)=[illegible]LAMDA(I)*L[illegible]
2075 NEXT I
2080 LPRINT "EC(I)","EI(I)","ABSOR(I)","NB(I)","ET(I)"
2090 FOR I=0 TO N-1
2100 LPRINT EC(I),EI(I),ABSOR(I),NB(I),ET(I),
2110 NEXT I
2112 LPRINT:LPRINT "I","EC","S"
2114 FOR I=0 TO N-1
2116 LPRINT OPTEC(I,0),OPTEC(I,1),OPTSE(I,2)
2118 NEXT I
2190 GOTO 2210
2200 PRINT "NO CONVERGENCE"
2210 END
```

APPENDIX D: (S,c,s) Model Test Results

The results tabulated below represent the output from the model for a series of cases tested.

Case 1: Using the Thompson and Silver example with no changes to the input data.

Case 2: c = 73% of AOQPC (average order quantity per cycle)

Case 3: c = s (must-order point)

Case 4: c = S (order-up-to level)

Case 5: c = 50% of AOQPC

Case 6: c = AOQPC

Case 7: c = 25% of AOQPC

Case 8: Customer service level reduced to 80%

Case 9: Customer service level reduced to 50%

Case 10: Major and minor setup costs reduced by 20%

Case 11: Major and minor setup costs reduced by 50%

DEFINITIONS:

EC(I) : The total expected cost (setup + carrying + shortage) for item I per year.

P1(I) : The probability that the cycle ends with no backorders (or lost sales) ie. customer service level, for item I.

AOQPC(I): The average order quantity per cycle = S-s-deficit below s, for item I. (See FIG.8)

NU(I) : The number of replenishment opportunities per year for item I.

EI(I) : The expected inventory level for item I (pieces).

ITEM	EC(I)	P1(I)	AOQPC(I)	MU(I)	EI(I)	S	c	s
CASE 1: NO CHANGE TO INPUT DATA								
1	201,8	0,96	132	0,825	81	151	104	7
2	20,1	0,95	62	2,410	56	89	43	2
3	63,8	0,96	60	2,120	53	87	53	5
4	59,9	0,96	98	2,133	84	137	79	5
TOTAL 345,7								
CASE 2: c = 73% x AOQPC								
1	201,9	0,96	132	0,825	81	151	103	7
2	20,2	0,96	58	2,410	58	89	47	2
3	63,9	0,96	63	2,120	53	87	49	5
4	59,9	0,96	100	2,133	83	137	77	5
TOTAL 345,8								
CASE 3: c = s								
1	213,8	0,95	147	0,825	77	151	7	7
2	36,3	0,95	90	2,410	48	94	7	7
3	85,0	0,95	85	2,120	46	89	7	7
4	82,1	0,95	135	2,133	71	139	7	7
TOTAL 417,2								

ITEM	EC(I)	P1(I)	AOQPC(I)	MU(I)	EI(I)	S	c	s
CASE 4: c = S								
1	206,3	0,97	115	0,825	82	151	151	7
2	32,5	0,99	14	2,410	73	87	87	0
3	90,0	0,96	28	2,120	56	84	84	2
4	80,0	0,96	45	2,133	91	134	134	2
TOTAL 408,9								
CASE 5: c = 50% x AOQPC								
1	202,8	0,96	140	0,825	79	151	73	7
2	21,0	0,96	71	2,410	54	91	35	4
3	66,0	0,96	73	2,120	50	88	36	6
4	61,8	0,97	116	2,133	78	138	55	6
TOTAL 351,6								
CASE 6: c = AOQPC								
1	202,3	0,97	121	0,825	82	151	139	7
2	22,1	0,97	42	2,410	66	87	62	0
3	64,1	0,96	51	2,120	55	86	64	4
4	60,6	0,96	79	2,133	89	136	102	4
TOTAL 349,1								

ITEM	EC(I)	P1(I)	AOQPC(I)	MU(I)	EI(I)	S	c	s
CASE 7: c = 25% x AOQPC								
1	206,3	0,96	145	0,825	77	151	40	7
2	24,8	0,97	83	2,410	51	93	22	6
3	72,2	0,97	81	2,120	47	89	22	7
4	67,9	0,97	129	2,133	73	139	32	7
TOTAL 371,2								
CASE 8: P1 = 80%								
1	200,0	0,82	132	0,825	79	148	101	4
2	20,4	0,92	62	2,410	58	87	41	0
3	63,6	0,84	60	2,120	53	83	49	1
4	59,7	0,84	98	2,133	83	133	75	1
TOTAL 343,3								
CASE 9: P1 = 50%								
1	199,9	0,52	132	0,825	79	145	98	1
2	20,4	0,92	62	2,410	58	87	41	0
3	62,8	0,80	60	2,120	52	82	48	0
4	59,3	0,80	98	2,133	82	132	74	0
TOTAL 342,4								

ITEM	EC(I)	P1(I)	AOQPC(I)	MU(I)	EI(I)	S	c	s
CASE 10: Setup costs reduced by 20%								
1	181,1	0,96	120	0,927	73	136	90	7
2	18,2	0,96	53	2,716	53	81	43	2
3	57,5	0,96	56	2,380	48	79	46	5
4	53,8	0,97	85	2,392	76	123	75	5
TOTAL	310,6							
CASE 11: Setup costs reduced by 50%								
1	144,2	0,96	94	1,178	59	109	76	7
2	14,8	0,96	47	3,423	41	66	31	3
3	46,2	0,96	43	3,043	40	64	40	5
4	43,0	0,97	68	3,041	61	99	60	5
TOTAL	248,2							

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