THE VIABILITY OF USING MARKOWITZ PORTFOLIO THEORY AS A PASSIVE INVESTMENT STRATEGY ON THE JSE

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Declaration

I declare that this research report is my own unaided work. It is being submitted to the Degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

Signature of Candidate: ……………………………………………………............................

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Abstract

Markowitz Portfolio Theory (MPT) and related research was studied. Objectives were then formulated around whether an MPT model could outperform the returns of the Johannesburg Securities Exchange (JSE) and other financial instruments such as unit trusts. An MPT model was then created in Matlab using the information learnt from the theory and other appropriate sources. The model was used to generate a range of results depending on different inputs into the model. The model outputs were further analysed in Excel and results in the form of tables and graphs were created. It was found that the MPT model considerably outperforms the JSE ALSI and JSE Top 40. There were many positive Sharpe Ratios for various different inputs and model parameters. The JSE ALSI had a 1 year return of 17.13% and 3 year annualised return of 12.83%. The MPT model had 1 year returns of between 17.07% and 37.81%. The MPT model had 3 year annualised returns of between 11.81% and 26.24%. The MPT model outperformed the JSE ALSI with 5 out of 6 portfolios created. The JSE Top 40 had a 1 year return of 18.37% and 3 year annualised return of 13.02%. The MPT model had 1 year returns of 21.49% and 24.24% and 3 year annualised returns of 18.53% and 20.72%. The MPT model for Top 40 data thus outperformed the JSE Top 40 over 1 year and 3 years annualised. The MPT model had two out of its eight portfolios in the top four of the best performing unit trusts over 3 years of total returns. Over a 1 year return, two of the MPT portfolios were the top two performers compared to other unit trusts. This research has thus shown that an MPT model using historical data can outperform the JSE and can perform competitively with other unit trusts.

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Nomenclature

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CF_t	$cash\ flows\ during\ period\ t$
P_E	$price\ at\ the\ end\ of\ period\ t\ (sale\ price)$
P_B	$purchase\ price\ of\ the\ asset$
PC	$price\ change\ during\ the\ period\ (P_E - P_B)$
σ^2	$variance\ of\ a\ set\ of\ values$
X	$each\ value\ in\ the\ set$
$\bar{X}$	$mean\ of\ the\ observations$
n	$number\ of\ returns\ in\ the\ sample$
σ	$standard\ deviation$
$E(R)$	$expected\ rate\ of\ return$
R_i	$ith\ possible\ return$
pr_i	$probability\ of\ the\ ith\ return\ R_i$
m	$number\ of\ possible\ returns\ /\ number\ of\ likely\ outcomes$
$E(R_p)$	$expected\ return\ on\ the\ portfolio$
w_i	$portfolio\ weight\ for\ ith\ security$
$E(R_i)$	$expected\ return\ on\ ith\ security$
n	$number\ of\ different\ securities\ in\ the\ portfolio$
σ_{AB}	$covariance\ between\ securities\ A\ and\ B$
R_A	$one\ possible\ return\ on\ security\ A$
$E(R_A)$	$expected\ value\ of\ return\ on\ security\ A$

ρ_{AB}	$correlation\ coefficent\ between\ stock\ A\ and\ B$
σ_A	$standard\ deviation\ (risk)\ of\ stock\ A$
σ_B	$standard\ deviation\ (risk\ )\ of\ stock\ B$
σ_p^2	$variance\ on\ the\ return\ on\ the\ portfolio$
σ_i^2	$variance\ of\ return\ for\ security\ i$
σ_{ij}	$covariance\ between\ returns\ for\ securities\ i\ and\ j$
R_i	$return\ on\ security\ i$
R_M	$return\ on\ the\ market\ index$
$\bar{r}_m$	$expected\ market\ return$
a_i	$part\ of\ security\ \ i's\ \ return\ independent\ of\ market\ performance$
β_i	$measuring\ expected\ change\ in\ \ variable\ R_i\ given\ a\ change\ in\ \ variable\ R_M$
e_i	$random\ residual\ error$
$E(r_p)$	$expected\ return\ of\ the\ portfolio$
r_f	$risk\ free\ rate$
A	$coefficient\ of\ risk\ aversion$
σ_p^2	$variance\ (standard\ deviation\ squared) of\ the\ portfolio$
y	$optimal\ position\ in\ the\ risky\ asset$
β_a	$beta\ of\ the\ security$

Abbreviations

MPT	Markowitz/Modern Portfolio Theory
JSE	Johannesburg Securities Exchange
RF	Risk Free Rate
TR	Total Return
ALSI	All Share Index
CAPM	Capital Asset Pricing Model

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1 Introduction

1.1 Background

Markowitz Theory, also known as modern portfolio theory (MPT) was developed by Harry Markowitz and published as "Portfolio Selection" in the 1952 *Journal of Finance* [1].

MPT states that it is not good enough to look at the expected risk and return of one single stock, but that by investing in more than one stock the investor can achieve benefits of diversification, the main one being a reduction in the overall risk of the portfolio [1].

1.2 Motivation

Investors look for ways to increase their returns while keeping their risk to a minimum. Using Markowitz theory it is possible to define the risks and returns so that these decisions can be modelled for investors.

With large amounts of data available to many ordinary people investing has become simpler and allowed people who put the time and effort to studying investing, to invest themselves. Tools have become more easily accessible to the average investor. This has allowed more people to handle their own investments without having to spend large sums of money on brokers and advisors. Diversification of portfolio holdings has always been the recommended way to invest more safely by spreading or diversifying ones risk. The principles behind Markowitz Theory, if designed into a simple model can allow ordinary people who have some knowledge of investments to be able to make their own investment decisions. The research problem thus looks at how it can be possible to simplify the theory behind Markowitz Theory and create a practical model which can be used by the average personal investor.

This research project will examine Markowitz Theory and determine how well it performs when compared to general market returns such as those of the Johannesburg Securities Exchange (JSE) index. It will assess whether it is possible to create a Markowitz Theory model which can be more easily understood and used by investors who are not professionals and want more control of making their own investment decisions.

This research will also look at whether investors (over the years since the theory was developed) have been achieving returns which are satisfactory. In addition the theory will be examined to see whether it is currently applicable.

2 Objectives

2.1 Objectives

- Determine whether a Markowitz Theory model outperforms top performing unit trusts and index trackers.
- Examine Markowitz Theory and determine why its application could provide above average returns on the JSE.
- Design and apply a Markowitz Theory model and determine whether it provides above average returns on the JSE.

2.2 Hypothesis 1

Historical data can be used in a Markowitz model to predict future returns by providing above average returns on the JSE.

2.3 Hypothesis 2

Markowitz Theory provides above average risk-returns on the JSE, compared to the overall JSE index return.

2.4 Hypothesis 3

Markowitz Theory does not outperform actively managed equity funds and high risk unit trusts.

3 Literature Review

3.1 Basis of Investment Decisions

3.1.1 Return

People invest because they want to earn a return on the money that they have invested. People also invest to earn a return in inflationary environments, due to the purchasing power of cash diminishing with high rates of inflation [1].

3.1.1.1 Expected return vs. realised return

The expected return is the anticipated return for some future period. The realised return is the actual return over some past period. It is of critical importance to distinguish between these two returns because of the reason that investors invest for the future, i.e. for the returns they expect to earn once the chosen period of investment is over, so that they are left with their realised returns [1].

3.1.2 Risk

Investors seek to maximise their investing returns subject to the risks they are willing to incur. The objective of return is subject to various constraints such as taxes and transaction costs as well as risk. These are seen as the major constraints [1].

Risk can be defined as the uncertainty about the actual return which will be earned on the investment [1].

Investors are risk averse and will not assume a given level of risk unless there is expectation of adequate compensation for taking on the additional risk [1]. Investors can thus not expect to earn larger returns without assuming larger risks.

Investors deal with risk by choosing their risk tolerance (investor's willingness to accept risk when investing). Therefore, in general, investors do not necessarily choose to minimise their risks, but choose risk depending on the returns they want to achieve as well as considering the effect other constraints such as taxes will have on their investment [1].

Investors thus decide on their risk tolerance and how they can maximise their return depending on the chosen risk tolerance and other constraints (taxes and transaction costs) [1].

3.2 Expected Risk-Return Trade-off

Investors can achieve virtually any position on the risk-return graph as shown in Figure 3.1 within the area of financial assets. As the type of assets held by the investor becomes more risky the graph slope increases, achieving a greater return for the increase in risk. The risk free (RF) rate shown in Figure 3.1 is the rate the investor can expect to earn on risk-free assets such as Government Bonds. Depending on the risk appetite of the investor and the amount of return he expects from the investment, the appropriate type of asset investment will be found higher up on the risk-return function line [1].

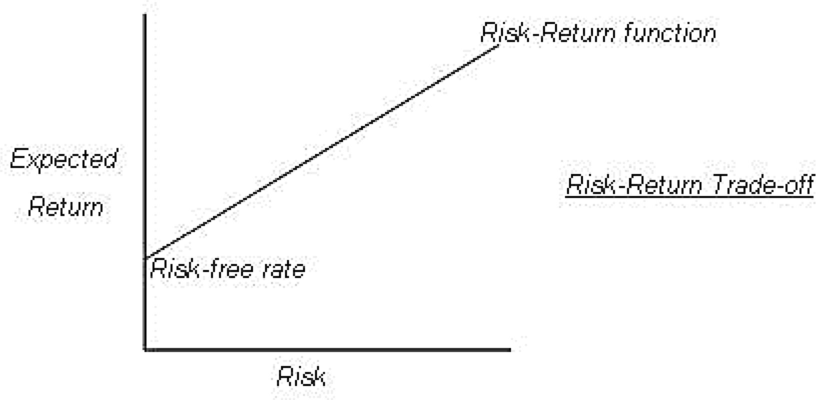

Figure 3.1 Expected risk-return trade-off [1]

3.3 Structuring the Investment Decision Process

3.3.1 Outline

Investors have traditionally analysed and managed securities using a broad two-step process, security analysis and portfolio management [1].

3.3.2 Security analysis

The initial phase of the investment decision process involves the analysis and valuation of individual securities and is known as security analysis. Security valuation is a difficult and time consuming job which looks at firstly, understanding the characteristics of various securities, and secondly the factors that affect these securities. A valuation model is then applied to estimate the value or price of the security. Value is a function of expected future returns and risk attached to the security. These two parameters need to be estimated and then brought together in the model [1].

Usually some type of security analysis is done by the investors who are serious about their portfolios despite the difficulties in performing these valuations. This analysis is necessary unless the investor wants to rely on hunches or suggestions from other people such as friends or brokers, which could be detrimental [1].

Security analysis can be divided into three main areas namely [2]:

- Macroeconomic and industry analysis
- Equity valuation models
- Financial statement analysis

3.3.3 Portfolio management

Portfolio management is the second major step of the decision making process for investors. Once the securities are evaluated a portfolio should be constructed. Much of the work done in portfolio management is in the form of mathematical and statistical models. Once the portfolio is constructed the investor must consider when and how to revise the holdings within their portfolio [1].

The investment process of portfolio construction can be further broken down into four basic steps, namely [2]:

1. Security valuation
2. Asset allocation
3. Portfolio optimisation
4. Performance measurement

These four steps will be discussed briefly below.

3.3.4 Security valuation

Security analysis is used by analysts to forecast dividends and earnings which can be expected from a company [2]. This information is required to eventually calculate which securities should be purchased depending on the security analysis and the share price. Security analysis will allow for the creation of input data for the Markowitz model.

3.3.5 Asset allocation

Asset or capital allocation decision making is the decision on how much of an overall portfolio will be placed in safe, low return money market securities, and how much will be placed in risky, higher return securities such as stocks. The fraction of each type of asset which is allocated is thus known as the asset allocation decision. Once the asset allocation fractions are determined the next step is that of security selection and portfolio optimisation [2].

Table 3.1 shows the optimal asset allocation weights for a South African investor between 1981 and 2006, as shown by Busetti [3].

Table 3.1 Optimal portfolios (1981-2006) [3]

Asset	Portfolio average weight (%)	
	Aggressive	Conservative
SA Equities	33	3
SA Bonds	24	59
SA Cash	4	16
Offshore Equities	39	22
Total	100	100

3.3.6 Portfolio optimisation

Portfolio optimisation follows on from the asset allocation decision. Portfolio optimisation is the decision of which securities should be selected to create the optimal portfolio of risky assets [2].

3.3.7 Performance measurement

Performance measurement is used to evaluate the performance of a portfolio manager on areas such as total portfolio return with the risk involved. Techniques such as Sharpe's measure (discussed in Section 3.20) can be used to measure risk-adjusted performance of a portfolio [2].

3.4 Return

There is no guarantee that the future will be like the past, but knowledge of historical risk-return relationships is a necessary initial step for investors to make investment decisions in the future. There is no reason to assume that relationships will differ significantly in the future. For example if stocks have returned more than bonds over a large period of time there is reason to assume this will continue over a future long-run. It is thus important for investors to understand what has happened in the past [1].

3.4.1 The two components of return

Total returns can be divided into two components, namely yield and capital gain/loss.

Yield – Periodic cash flows (income) on the investment, either interest or dividends received, is the basic component which investors think of when discussing returns on investments. The issuer makes cash payments to the holders of the asset. Yield measures relate these cash flows to a price for the security [1].

Capital gain (loss) – the appreciation or depreciation (price change) of the asset. It is the difference between the purchase price of the security and the amount which it could be sold at. This could lead to a gain or loss [1].

Algebraically adding yield and capital gain/loss, the equation for total return (TR) is formed [1]:

$$Total\ Return\ (TR) = Yield + Price\ Change \tag{1}$$

Where yield can be 0 or +, and price change can be 0, + or –.

3.4.2 Measuring returns

The total return (TR) for a given holding period of a security is a number relating all cash flows received by the investor during the designated time period to the purchase price of the asset [1].

Total return can also be defined as a ratio [1]:

$$TR = \frac{Any\ cash\ payments\ received + Price\ changes\ over\ the\ period}{Price\ at\ which\ the\ asset\ is\ purchased} \tag{2}$$

Where:

$$Price\ change\ over\ period\ =\ Difference\ between\ purchase\ price\ and\ sale\ price$$

$$TR = \frac{CF_t + (P_E - P_B)}{P_B} = \frac{CF_t + PC}{P_B} \tag{3}$$

Where [1]:

$$CF_t = cash\ flows\ during\ period\ t$$

$$P_E = price\ at\ the\ end\ of\ period\ t\ (sale\ price)$$

$$P_B = purchase\ price\ of\ the\ asset$$

$$PC = price\ change\ during\ the\ period\ (P_E - P_B)$$

The cash flows for stocks are dividends and the cash flows for bonds are interest payments received.

3.5 Sources and Types of Risk

3.5.1 Interest rate risk

Interest rate risk is the variability in a securities' return resulting from changes in the level of interest rates. Interest rate risks affect cash and bonds more directly than stocks but is an important consideration for most investors. Security prices move inversely to interest rate with other things being equal [1].

3.5.2 Market risk

Fluctuations in the overall market resulting in variability in returns is known as market risk. Common stocks are primarily affected but all securities are exposed to market risk. Factors involved in influencing market risk include wars, recessions, structural economic changes and consumer preferences [1].

3.5.3 Inflation risk

Also known as purchasing power risk, this is the risk that the purchasing power of invested currency (Rands, Dollars etc.) will decline. The real (inflation adjusted) return involves risk even if the nominal return is safe (government bonds), with uncertain inflation [1].

3.5.4 Business risk

Business risk is the risk of doing business in a particular environment or industry [1].

3.5.5 Financial risk

It is the risk associated with the use of debt financing by companies, i.e. the larger proportion of assets which are financed by debt instead of equity, the larger the variability in returns when other things are equal [1].

3.5.6 Liquidity risk

Risk associated with the secondary market in which a security trades. Liquid investments are those which can be bought and sold quickly and without significant price concession. The higher the uncertainty is with price concession and with time, the greater the liquidity risk [1].

3.5.7 Exchange rate risk

Due to the current global investment arena, investors who invest internationally are at risk of having uncertain returns when converting foreign gains back into their own currency. Exchange rate risk is the variability in returns on securities caused by currency fluctuations [1].

3.5.8 Country risk

Also known as political risk, this is important due to the global market increasingly being involved in the investors' potential decisions. It is the risk associated with doing business within a certain country or the indirect effect the country could have on a business [1].

3.5.9 Systematic risk

Systematic risk, or non-diversifiable risk is the risk which cannot be reduced through diversification. It is the market risk which remains after extensive diversification [2].

3.5.10 Non-systematic risk

Non-systematic risk is the risk that can be eliminated through diversification. It is also known as diversifiable risk. Non-systematic risk is reduced through diversification of portfolios by having a range of different companies in a portfolio which are not always correlated [2].

3.6 Measuring Risk

3.6.1 Variance and standard deviation

Financial asset risks can be measured with an absolute measure of dispersion or variability of returns called variance.

The standard deviation (square root of variance) measures the deviation of each observation from the arithmetic mean of the observations. It is also a measure of the total risk of an asset or portfolio. It captures the total variability in the asset or portfolio return, whatever the source of that variability.

The variance is [1]:

$$\sigma^2 = \frac{\sum_{i=1}^{n}(X - \bar{X})^2}{n - 1} \quad (4)$$

Where:

$$\sigma^2 = \text{ } variance\ of\ a\ set\ of\ values$$

$$X = each\ value\ in\ the\ set$$

$$\bar{X} = mean\ of\ the\ observations$$

$$n = number\ of\ returns\ in\ the\ sample$$

$$\sigma = \sqrt{\sigma^2} = standard\ deviation$$

Therefore the standard deviation of return measures the total risk of one security or of a portfolio of securities.

Using TR's for a specified period of time, the historical standard deviation can be calculated.

Combining the standard deviation with the normal distribution can lead to some useful information about the dispersion or variation in returns.

3.7 Portfolio Theory

3.7.1 Probabilities

A random variable (random values but of a known statistical distribution) can be used to describe the experienced rate of return from a stock. This is used to deal with the uncertainty of returns by dealing with the possibility of more than one outcome. It assesses the probability that each outcome is likely to occur. This is expressed as a fraction or decimal [1].

3.7.2 Probability distributions

Probabilities for possible outcomes are obtained using subjective estimates as well as past occurrences (frequencies) to estimate probabilities. Past frequencies need to be modified for any potential changes expected in the future [1].

Probability distributions can either be discrete or continuous [1]:

- Discrete probability distribution – A probability is assigned to each possible outcome (Figure 3.2).

- Continuous probability distribution – An infinite number of possible outcomes exist. Due to the area under the curve being measured the emphasis is that the probability of a particular outcome is within a specified range as can be seen in Figure 3.3.

The sum of the set of probabilities in a distribution must sum up to 1.0 or 100% because of the requirement to completely describe all possible occurrences [1].

Figure 3.2 Discrete probability distribution [1]

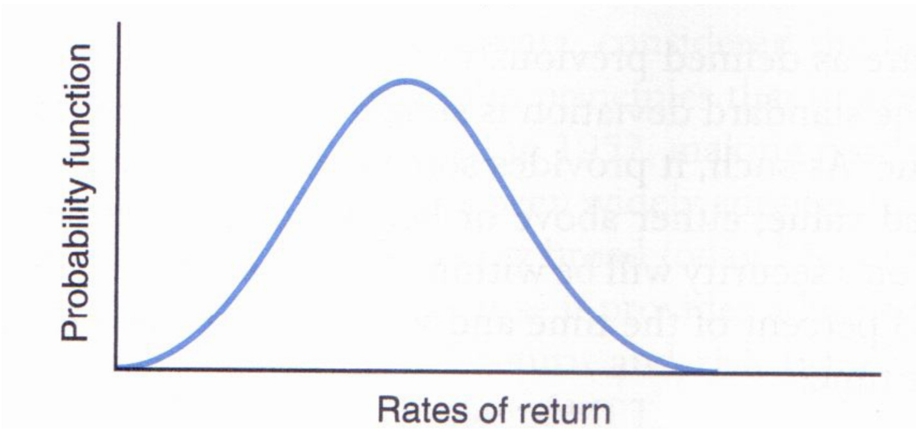

Figure 3.3 Continuous probability distribution [1]

3.7.3 Calculating expected return for a security

The expected value is the weighted average of all possible outcomes. Each outcome is weighted by its probability of occurrence.

The expected return for any security is [1]:

$$E(R) = \sum_{i=1}^{m} R_i pr_i \tag{5}$$

Where:

$$E(R) = expected\ rate\ of\ return$$

$$R_i = ith\ possible\ return$$

$$pr_i = probability\ of\ the\ ith\ return\ R_i$$

$$m = number\ of\ possible\ returns$$

3.7.4 Calculating risk for a security

As discussed earlier, the variance and standard deviation measure the dispersion of a random variable around the mean. A larger dispersion leads to larger variances or standard deviations.

The tighter the probability distribution of the expected returns, the smaller the risk of a security.

Lower standard deviation leads to a tighter probability distribution and hence a smaller risk.

The expected return of the distribution must be calculated first so that the variance or standard deviation can be calculated.

The variance of returns is thus [1]:

$$\sigma^2 = \sum_{i=1}^{m} [R_i - E(R)]^2 pr_i \tag{6}$$

And the standard deviation of returns is:

$$\sigma = \sqrt{\sigma^2} \tag{7}$$

With a normal probability distribution the actual return on a security will be within +- 1 standard deviation of the expected return about 68% of the time and within +- 2 standard deviations of expected return, 95% of the time.

Subjective estimates are required for returns and probabilities when calculating standard deviations using probability distributions. These estimates are required because future returns are uncertain. Prices of securities are based on investors' expectations of the future.

For a well-diversified portfolio, standard deviations are reasonably steady across time. Thus using historical data can be useful in projecting the future [1].

3.8 Markowitz (Modern) Portfolio Theory (MPT)

Markowitz states that there are two stages in selecting a portfolio. The first starts with observing and ends with beliefs of future performance of the securities. The second stage starts with the beliefs of the future performances and ends with choosing a portfolio. Markowitz's research concerns the second stage; creating a portfolio from the inputs determined from the first stage. [4]

Markowitz was the first to develop a measure of portfolio risk and to derive expected return and risk for a portfolio based on covariance relationships [1].

Markowitz showed that portfolio risk is not simply a weighted average of individual security risks within the portfolio, but that the inter relationships between security returns needs to be accounted for to calculate the portfolio risk and to reduce the risk to a minimum for a given level of return [1].

According to Markowitz, a portfolio with maximum expected returns is not necessarily the portfolio with minimum variance. The investor can gain expected returns by taking on variance or reduce variance by giving up expected returns. [4]

3.8.1 Portfolio expected returns

3.8.1.1 Portfolio weights

Portfolio weights are the percentages of each asset which is in the portfolio as can be seen from the following two equations below [1]:

$$w_1 + w_2 + \cdots + w_n = \sum_{i=1}^{n} w_i = 1.0 \tag{8}$$

The expected return on a portfolio *p* can be calculated as:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{9}$$

Where:

$$E(R_p) = expected\ return\ on\ the\ portfolio$$

$$w_i = portfolio\ weight\ for\ ith\ security$$

$$\sum w_i = 1.0$$

$$E(R_i) = expected\ return\ on\ ith\ security$$

$$n = number\ of\ different\ securities\ in\ the\ portfolio$$

The expected return for a portfolio is thus always a weighted average of the expected returns for individual assets in the portfolio.

3.8.2 Portfolio risk

Portfolio risk is measured by the variance of the portfolios return. This leads to the basis of MPT which is described by Jones [1] as follows:

Although the expected return of a portfolio is a weighted average of its expected returns, portfolio risk (as measured by the variance or standard deviation) is typically not a weighted average of the risk of the individual securities in the portfolio.

In equation form the above definition translates as follows, as in Equation 9 [1]:

$$E(R_P) = \sum_{i=1}^{n} w_i\, E(R_i) \tag{10}$$

But

$$\sigma_p^2 \neq \sum_{i=1}^{n} w_i \sigma_i^2 \tag{11}$$

The risk of a portfolio will almost always be less than the weighted averages of risks of individual securities in the portfolio.

3.8.3 The correlation coefficient

The correlation coefficient is a statistical measure of relative movements between returns of securities. It is a measure of the extent of which returns of two securities are related [1].

$$p_{ij} = +1.0 = perfect\ positive\ correlation$$

$$p_{ij} = -1.0 = perfect\ negative(inverse)\ correlation$$

$$p_{ij} = 0.0 = zero\ correlation$$

For a perfect positive correlation, returns have a direct linear relationship. This means that knowing the return on one security will allow a perfect forecast of what the other will do. The risk of a portfolio for perfect positively correlated returns will be a weighted average of the individual risks of the securities [1].

For a perfect negative correlation the securities returns have a perfect inverse linear relationship to each other. When the return of one security is lower, the return for the other security is higher [1].

Zero correlation means there is no linear relationship between returns on two securities.

Having a combination of two of these securities with zero correlation with each other reduces the risk of the portfolio. More securities with zero correlation in a portfolio leads to significant risk reduction [1].

3.8.4 Covariance

The covariance is the extent to which two random variables move together over time. This is required to measure the amount of co-movement among security returns to incorporate it into a measure of portfolio risk.

According to Markowitz, it is not enough to just invest in many securities. It is necessary to avoid investing in securities with high covariances amongst themselves (same or similar industries or companies). Companies in different industries have lower covariances than companies within similar industries. [4]

The formula for calculating covariance on an expected basis is [1]:

$$\sigma_{AB} = \sum_{i=1}^{m} [R_{A,i} - E(R_A)][R_{B,i} - E(R_B)]pr_i \qquad (12)$$

Where:

$$\sigma_{AB} = covariance\ between\ securities\ A\ and\ B$$

$$R_A = one\ possible\ return\ on\ security\ A$$

$$E(R_A) = expected\ value\ of\ return\ on\ security\ A$$

$$m = number\ of\ likely\ outcomes\ for\ a\ security\ for\ the\ period$$

Covariance and the correlation coefficient can be related as follows:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$$

Where:

$$\rho_{AB} = correlation\ coefficent\ between\ stock\ A\ and\ B$$

$$\sigma_A = standard\ deviation\ (risk)\ of\ stock\ A$$

$$\sigma_B = standard\ deviation\ (risk\ )\ of\ stock\ B$$

The covariance can thus be written as:

$$\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B \qquad (13)$$

3.8.5 Portfolio risk calculation

For a two security portfolio the risk can be calculated as follows [1]:

$$\sigma_p = [w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2(w_1)(w_2)(\rho_{1,2})\sigma_1\sigma_2]^{0.5} \tag{14}$$

Analysing the equation it can be seen that three factors determine portfolio risk, namely:

- Variance of each security ($\sigma_1^2 and\ \sigma_2^2$)
- Covariance between securities $(\sigma_1, \sigma_2 and\ \rho_{1,2})$
- Portfolio weights for each security (w's)

For a portfolio with more than 2 securities (n-security case), the portfolio risk is as follows [1]:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}\ \ (i \neq j) \tag{15}$$

Where:

$$\sigma_p^2 = variance\ on\ the\ return\ on\ the\ portfolio$$

$$\sigma_i^2 = variance\ of\ return\ for\ security\ i$$

$$\sigma_{ij} = covariance\ between\ returns\ for\ securities\ i\ and\ j$$

$$w_i = portfolio\ weights\ of\ investable\ funds\ invested\ in\ security\ i$$

3.8.6 Identifying optimal risk-return combinations

Portfolio theory is based on the following assumptions [1]:

1. A single investment period.
2. Liquidity of positions (e.g. no transaction costs).
3. Investor preferences are only based on a portfolio's expected return and risk (measured by variance and standard deviation).

3.8.7 Efficient portfolios

An efficient portfolio is a portfolio with the smallest portfolio risk for a given level of expected return and a given level of risk [1].

The minimum variance frontier can be drawn for minimum variance portfolios as shown in Figure 3.4. Each point on Figure 3.4 represents a single security. The efficient frontier contains portfolios which consist of combinations of securities.

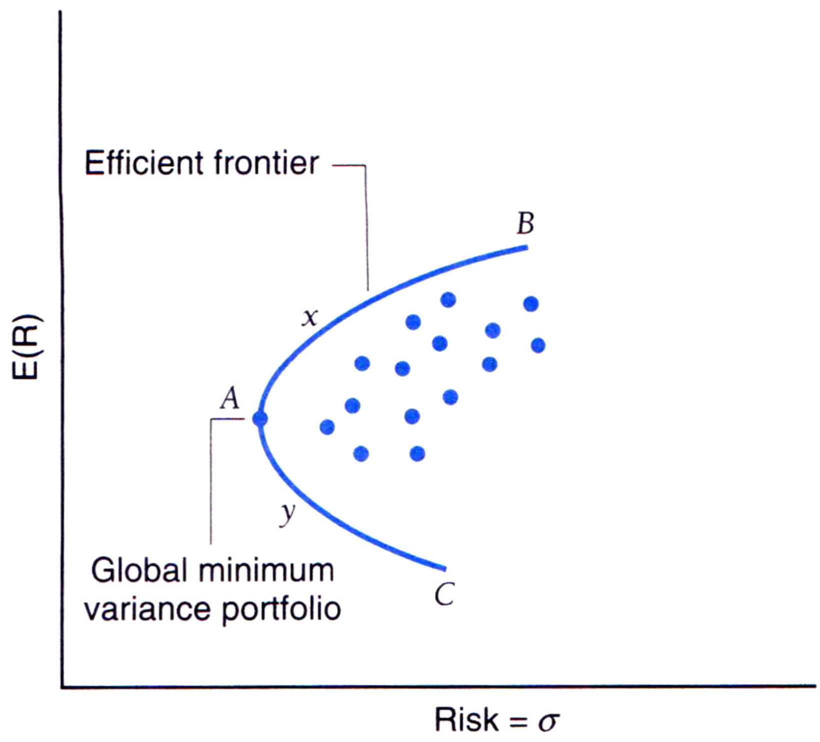

Figure 3.4 The minimum variance frontier [1]

For a given level of risk, portfolio x has a greater expected return than portfolio y, and will thus be chosen as the better portfolio.

The efficient frontier are the portfolios which are located on the curve AB (Figure 3.4) and offer the best risk-return combinations.

The efficient set could be either the portfolios which offer a better return for a given level of risk or the same return for lower risk.

Once the efficient set of portfolios are determined by the Markowitz model it is necessary for investors to select the most appropriate model for themselves. This could be lower or higher risk models with corresponding higher returns which are on the efficient frontier (which makes all of these portfolios optimal) [1].

Utility is an economic term which refers to the subjective satisfaction which an individual obtains from taking certain actions or being subjected to certain circumstances [1]. People with different utility thus might take on more or less risk depending on their own personal subjective needs and thoughts.

To select the expected return-risk combination which will satisfy investors, indifference curves are used. Figure 3.5 shows an example of indifference curves.

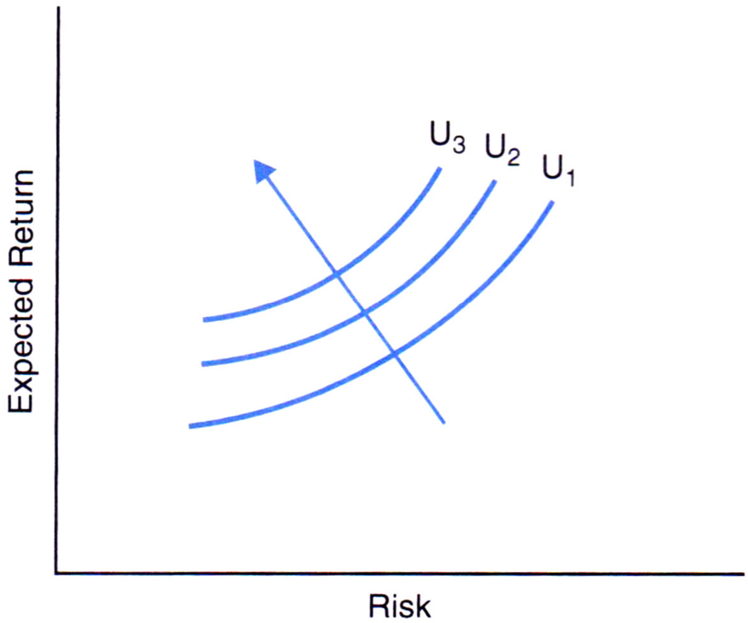

Figure 3.5 Indifference curves [1]

Figure 3.5 shows curves for a risk averse investor. Each of the curves (U_i) represents combinations of expected return and risk which are equally desirable to that particular investor (i.e. they provide the same level of utility) [1].

A few points about indifference curves [1]:

- They cannot intercept because they represent different levels of desirability.

- For risk-averse investors the curve will be upward sloping.
- The curve shapes can vary depending on risk preferences.
- The greater the curve slopes the higher the risk aversion.
- The utility is greater the further the curves are from the horizontal axis (indicated by the arrow in Figure 3.5).

3.8.8 Optimal portfolio selection

The optimal portfolio for a risk averse investor is where the efficient frontier is tangent to the indifference curve. Point 0 in Figure 3.6 is where this occurs and is the point where investor utility is maximised [1].

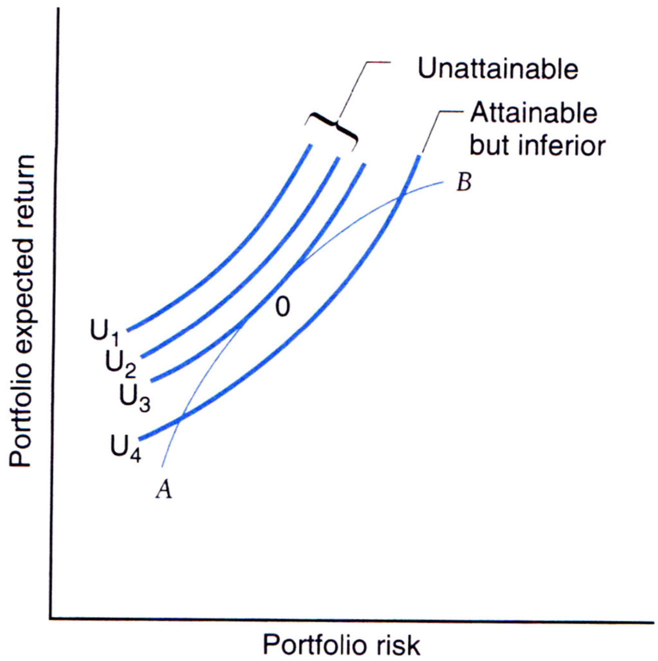

Figure 3.6 Optimal portfolio selection [1]

From Figure 3.6, the indifference curve U_4 is attainable but inferior because it offers less return for the same risk as U_3.

3.8.9 The single index model

To reduce the amount of calculations in the Markowitz model the single index model can be used. It relates returns on each security to the returns on a common index.

The single index model can be described as follows [1]:

$$R_i = a_i + \beta_i R_M + e_i \tag{16}$$

Where:

$$R_i = return\ on\ security\ i$$

$$R_M = return\ on\ the\ market\ index$$

$$a_i = part\ of\ security\ \ i's\ \ return\ independent\ of\ market\ performance$$

$$\beta_i = a\ constant\ measuring\ expected\ change\ in\ \ variable\ R_i\ given\ a\ change\ in\ \ variable\ R_M$$

$$e_i = random\ residual\ error$$

Money and Affleck-Graves found that the Markowitz approach produces results that are significantly superior to the Sharpe Index model, if the investor wants to use a risk-return approach for portfolio selection. [5]

3.9 Utility and the Risky Asset

The optimal position for a risk adverse investor in a risky asset “y” is as follows [2]:

$$y = \frac{E(r_p) - r_f}{0.01A\sigma_p^2} \tag{17}$$

Where:

$$E(r_p) = expected\ return\ of\ the\ portfolio$$

$$r_f = risk\ free\ rate$$

$$A = coefficient\ of\ risk\ aversion$$

$$\sigma_p^2 = variance\ (standard\ deviation\ squared)\ of\ the\ portfolio$$

$$y = optimal\ position\ in\ the\ risky\ asset$$

Utility [2]:

$$U = E(r) - 0.005A\sigma^2 \tag{18}$$

Where:

$$U = utility$$

$$0.05 = scale\ factor$$

A = 0 for risk neutral investors [2].

These formulas are used in the Financial Toolbox in Matlab. Risk aversion coefficients are normally between 2 and 4 [6].

3.10 MPT Assumptions

A number of assumptions are made in the use of MPT. Some of these are: [7]

- All investors are rational and would prefer less risk rather than more risk for any given rate of return.
- All investors have equal and full access to all available information. This results in investors all having similar expectations.
- There are no transaction costs.
- There is no taxation.
- Returns are normally distributed.
- Risks of an asset are known in advance.
- Risk is measured as variability of expected returns.
- Investors want to maximise returns for a given level of risk or minimise risk for a given level of return.
- Investors base decisions solely on expected return and risk.
- Market liquidity is infinite.
- Investors control risk only through the diversification of their portfolio holdings.
- Investors are not large enough players to affect the price of the market.
- Infinite amounts of money can be borrowed at the risk free rate.

3.11 Benefits of Diversification

Fabozzi and Markowitz [8] explain that diversification has been a central theme in finance for a long period of time. The idea of correlation and covariance comes from the fact that generally if one company's share price performs in a certain way, other similar companies will perform in a similar way. Diversification thus helps to mitigate the risk by only investing in certain industries or groups of companies. [8]

Fabozzi and Markowitz [8] explain that numerous innovations around finance are based around the concept of diversification or new methods of obtaining improved estimates of variances and covariances, leading to better estimates of risk. [8]

From Figure 3.7 it can be seen that a relatively small number of shares within a portfolio can lead to a large benefit in total reduced risk. For a holding of between 30-35 shares the total risk cannot be decreased much further. For approximately 20 shares held and more, the marginal benefit of further diversification is considerably reduced [7].

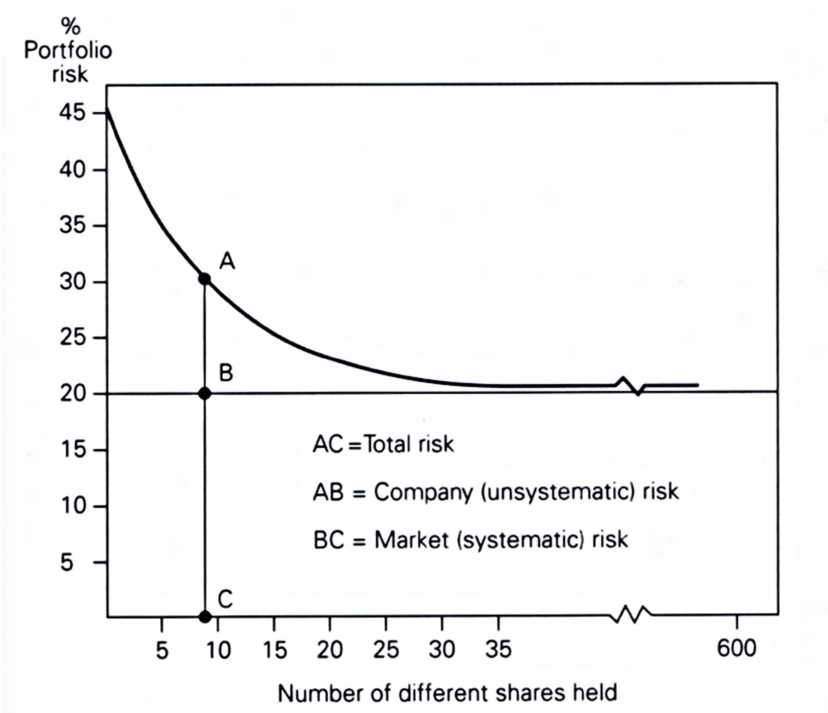

Figure 3.7 Effect of diversification on portfolio risk [7]

According to Busetti [3], the number of stocks required to achieve the lowest possible overall risk in the South African market can be seen in Figure 3.8 [3].

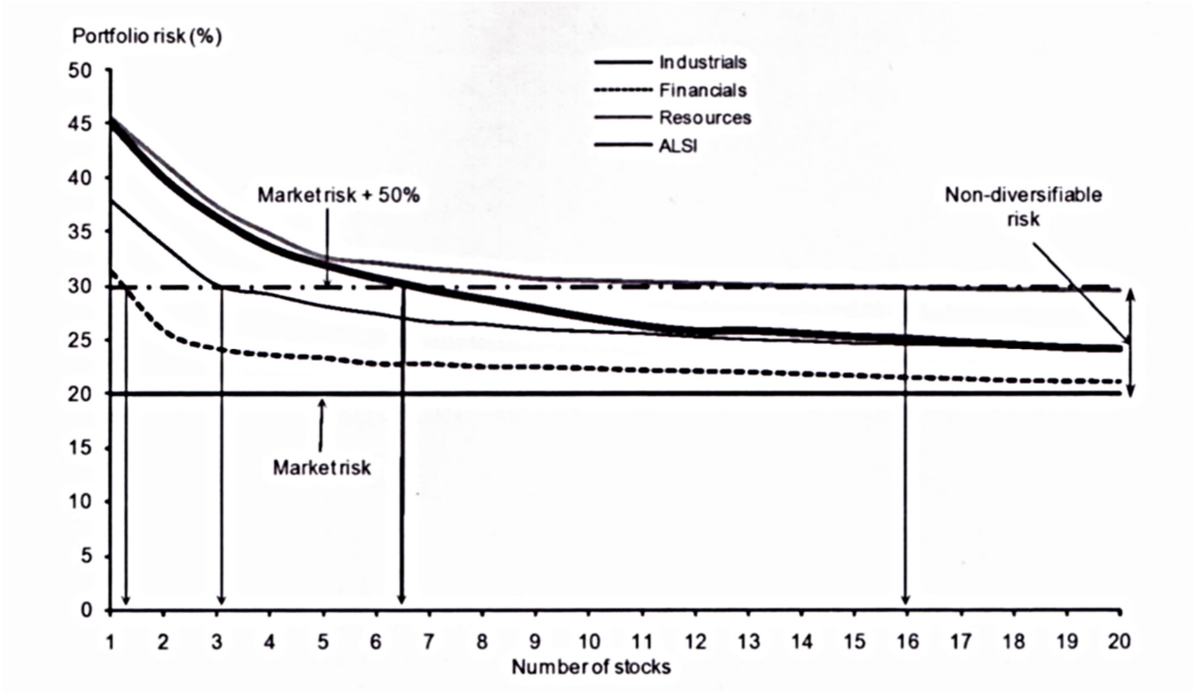

Figure 3.8 Diversification in the South African market [3]

At approximately 20 securities the amount of further diversification adds no significant further risk reduction in the South African market. Further research by Busetti [3] shows that to diversify away the same amount of non-systematic risk, the number of stocks held in different JSE share portfolios shown in Table 3.2 are required.

Table 3.2 The required number of stocks for diversification

Portfolio holding type	Number of stocks required
Financials	1
Industrials	3
ALSI	6
Resources	16

Table 3.2 shows that to diversify away the same amount of non-systematic risk in a portfolio, 16 resources stocks, or the top 6 All Share Index (ALSI) stocks or 3 industrial stocks or 1 financial stock is required [3].

This shows that with the addition of a financial stock in the South African market into a portfolio, the volatility of the portfolio is reduced. With financials it has been shown that risk is diversified away extremely rapidly. This can also be seen in Figure 3.8 (dotted line).

Table 3.3 below shows the number of stocks required to remove the non-systematic risk of the ALSI. These number of stocks include any of those on the ALSI. For example, having three stocks from the ALSI in a portfolio will remove 35% of non-systematic risk [3].

Table 3.3 Number of stocks required to diversify away non-systematic risk [3]

Number of stocks required	Proportion of non-systematic risk (%)
1	0
2	20
3	35
4	46
5	52
10	72
15	79
20	83

Busetti [3] has shown that in 1993 only 6 stocks were required to bring excess risk down to 4%, while later (year 2000) 22 stocks were required. Three reasons were given for this, namely [3]:

- A more highly concentrated market has led to fewer stocks accounting for a larger portion of the index, which in turn dilutes the benefits of diversification.
- Similar types of companies (resource stocks) have become top constituents of the ALSI. Correlation between resource stocks has on average been higher than between resources and financials and industrial stocks. This has created a higher average correlation amongst the top stocks which in turn reduces the benefit of diversification and increases the volatility of the ALSI.
- Top stocks have become more volatile relative to smaller Top 40 stocks. This offsets the risk reduction of the lower average volatility.

Therefore substantially more stocks are required in a portfolio than previously to maintain an equivalent level of risk [3].

3.12 Measurement Period

Correlations between stocks and risk need to be measured over a period of time. The appropriate time period is dependent on the initial investment horizon and the current position in the market [3].

A long term investor would thus use correlations measured over a long historical period which will include at least a full market cycle. Using a short horizon (short term investor), correlations will be measured over the past two to three years [3].

Ulucan [9] found that MPT efficient portfolios have generally better performance in longer-term investment horizons. He found that the best performance was for the 9-month holding period. Ulucan also found that MPT efficient portfolios performed significantly better than index returns on the ISE-100 (Istanbul) and ASE FTSE-40. [9]

3.13 Number of Companies to Consider

Ward and Muller [10] showed that even though there are approximately 350 companies listed on the JSE at any period of time over the last 10 years, only the largest 160 companies which represent 99% of the total market capitalisation are in the All Share Index (ALSI). In their study they only considered the Top 160 companies by market capitalisation in their constructed portfolios [10].

3.14 Use of Historical Values

According to Ringgenberg [11], historical returns can be used as inputs into a Markowitz model, but are not ideal.

Covariances from the rates of return of securities which are to be analysed are usually estimated from historical data [2].

According to Bodie [2], creating the Markowitz optimisation model is not complicated, but the area of competition and where portfolio managers compete, is in sophisticated security analysis to generate input data forecasts for the model.

Fabozzi and Markowitz [8] explain that in the use of historical data, users of MPT use constraints of maximum exposure either in certain areas (industries, markets, etc.) or maximum allocation percentage to compensate for the effect that using historical data could have in predicting future performance. Additional research by an analyst could lead to him placing these constraints so that exposure to certain companies and industries are limited. [8]

Fabozzi and Markowitz [8] explain that historical returns can be used to estimate inputs if the underlying economies giving rise to these outcomes of returns are strong and stable. Strong and stable refers to political stability and consistency in economic policies. [8]

3.15 Portfolio Optimisation

An optimal portfolio needs to be found by taking into account required return and risk, and other considerations such as minimum and maximum weightings, maximum number of shares in a portfolio and possibly omitting certain groups of companies. However the more constraints that are added the less room there is available for optimisation [3].

Becker, Gurtler and Hibbeln [12] found that the impact of constraints was found to be significantly larger than the choice of optimisation procedure (Markowitz vs. Michaud) [12].

As discussed by Busetti [3], setting floor constraints (minimum weights) can be particularly damaging for portfolio optimisation. An example of this is that if a minimum of 2% is held in each JSE Top 40 share, 2% x 40 = 80% of the portfolio will automatically be allocated leaving only 20% available for optimisation [3]. An example of the effect of minimum and maximum constraints can be seen in Figure 3.9 [3].

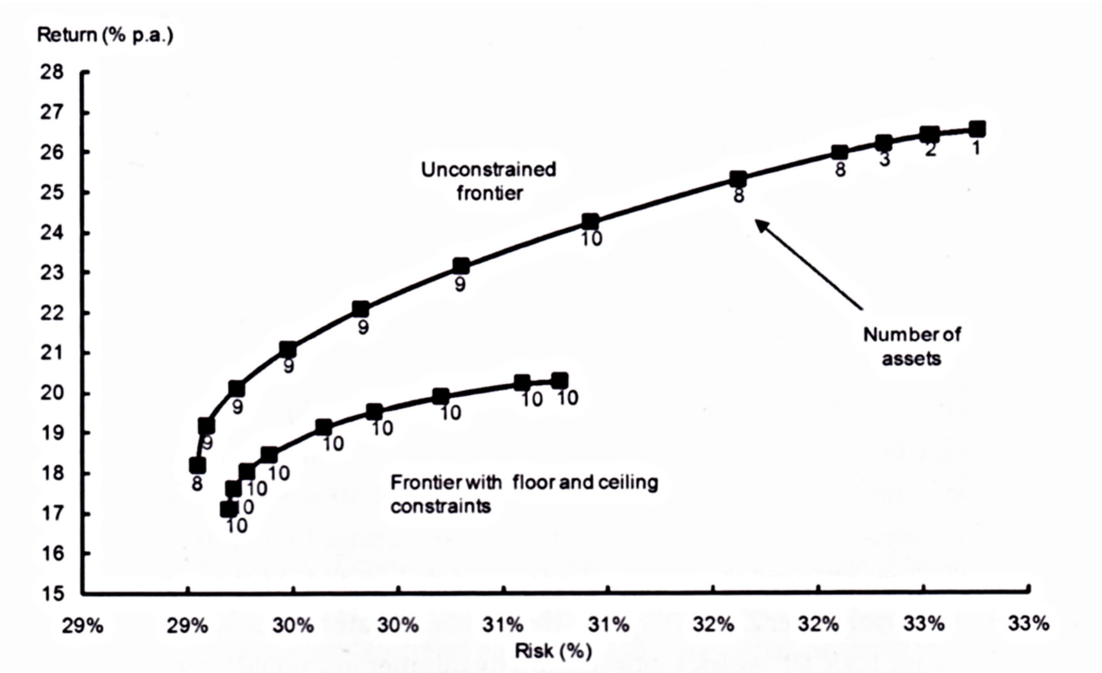

Figure 3.9 Efficient frontier changes with constraints [3]

Minimum and maximum constraints thus lower the efficient frontier and lead to lower returns for a given level of risk. [3]

According to Money and Affleck-Graves [5] varying the upper bound will logically have a much greater effect than varying a lower bound. Decreasing the upper bound requires more securities to be added into a portfolio. The inclusion of an upper bound that is not too high can enforce diversification. If large amounts of money are invested into a portfolio the use of an upper bound increases the practical feasibility of the model because it ensures the proportion invested in any one security is not too large for practicality and can be more realistic. [5]

According to Markowitz [4], statistical techniques and the judgement of practical men need to be combined. Once portfolios are built using a statistical model, and expected returns and risks are found, judgement can be used to increase or decrease these holdings depending on factors not taken into account by the statistical model. The individual investor can thus select the portfolio or weights of securities he finds necessary depending on his experience. For example, if he wants to avoid certain companies or industries for other reasons (political or market risk, personal preferences, other risks). [4]

3.16 Introducing a Risk Free Asset

The concept of a risk free asset makes the assumption that investors can borrow or lend at the risk free rate [7].

According to Correia [7], the effect of introducing a risk free asset such as a Government Bond is as shown in Figure 3.10.

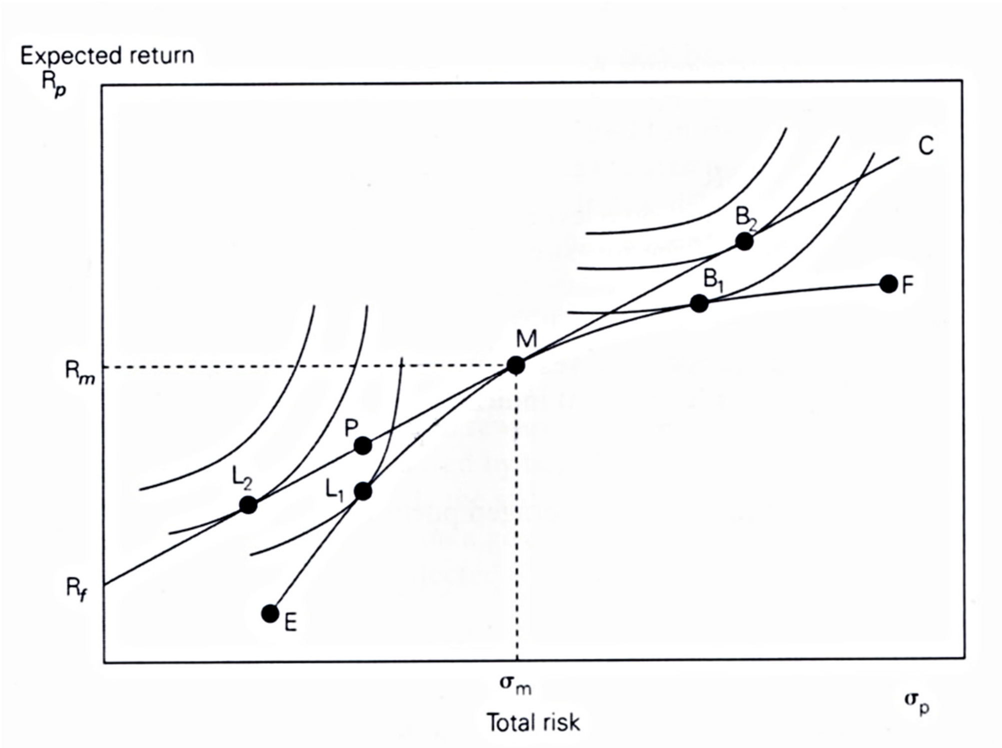

Figure 3.10 The capital market line [7]

Where [7]:

R_f = risk free rate

$R_f MC$ = capital market line (CML)

M = market portfolio

L_1 = Investor L with no risk free assets available

L_2 = Investor L investing personally in the market portfolio and investing (lending) the rest at the risk free rate. This is based on indifference curves and achieving higher utility as a result of being exposed to lower risk than L_1.

B_1 = Less risk averse investor B with no risk free assets available

B_2 = Investor B using borrowed funds at the risk free rate and investing together with personal funds into market portfolio M. Investor B has thus levered his portfolio by borrowing at the risk free rate and paying interest on the borrowed funds but investing them together with personal funds to achieve a net return.

The proportion of the investment in market portfolio M and the risk free asset R is dependent on the point of the tangent with the line R_fMC. The closer to the point R_f of the tangent the higher the proportion which will be invested in the risk free asset [7].

3.17 Ex-ante and Ex-post

Ex-ante refers to future events, such as future returns. An example of ex-ante is to estimate future returns and then compare these returns to the actual stock returns [13].

Ex-post refers to being after the fact. An example of ex-post is to use historical returns to forecast future returns [13].

3.18 Markowitz Model on the JSE

Similar previous research was done using a Markowitz model on the JSE by du Plessis and Ward [14].

Their hypothesis was that having a passive investment strategy using Markowitz optimal portfolio theory will not outperform the South African stock market in the medium to long term [14].

In the research, 11 year weekly data from January 1997 to end December 2007 was used. Data was acquired from all the shares listed but only using the JSE Top 40 in the research, thus ignoring any small and medium capitalised companies. Dividends were not included in the return calculations [14].

The model was created in Excel and the Solver add-in was used to solve the optimisation model.

Ex-ante returns were used to produce the expected future returns and covariance matrix. The method assumed "momentum in portfolio returns" which implies that shares which outperformed in a prior period would continue to outperform in the following period. This was stated as being in line with the research found by and Page (2000), who found evidence of a momentum effect on the JSE [14], [15].

A re-balancing period of 26 weeks was used, after which new shares would be selected or dropped according to the model [14].

The research also used various constraints such as short selling, no short selling and -10% and 10% maximum weights per share [14].

The research found that all of the calculated Markowitz portfolios outperformed the JSE Top 40 in terms of return and risk adjusted return measured by the Sharpe ratio [14].

3.19 Relevance of MPT

Fabozzi and Markowitz [8] conclude that MPT has been found to be relevant in modern financial theory and practice, and that MPT will continue to occupy a permanent place in the practice and theory of finance. [8]

Research done by Garaba [16] shows that MPT does not play a significant role in current asset management companies in South Africa. It was found that MPT is not being used in specific ways. This was mainly because of the limitations of public domain data and the uncertainty in forecasting risk and return characteristics of stocks. [16]

It was found that asset managers in South Africa regard fundamental analysis as the most significant method of security evaluation. Technical analysis and econometric models were found to play moderate roles and used to complement fundamental analysis. Behavioural finance was found to play the least role. [16]

Due to the difficulty and time-consuming job in performing fundamental analysis, MPT is one of the best and easiest ways an individual investor can use a mathematical model for their asset allocation. [16]

3.20 Criticisms and Drawbacks of MPT and Portfolio Optimisation

Some of the drawbacks and criticisms about MPT given by various authors include:

- Difficulty of including real world constraints and issues into the model [3].
- Assumptions such as no transaction costs, minimum and maximum constraints [3].
- The assumption of normally distributed returns is not true. Returns have long tails and real world events add shocks which create these long tails. Using the normal distribution assumption thus severely understates the true risk [3].

3.21 Portfolio Performance Evaluation (Sharpe's Measure)

Sharpe's measure can be used to evaluate portfolio performance and is a reward to variability ratio [2]. The formula for Sharpe's measure is as follows [2]:

$$S = \frac{E(r_p) - r_f}{\sigma_p} \tag{19}$$

The ex-post Sharpe measure uses realised portfolio return instead of expected return in the same formula as given in Equation 19 [2].

The objective of a portfolio manager is to maximize the Sharpe measure (ratio).

3.22 Tax implications

Investments may be subject to a number of taxes. These are principally:

- Income taxes.
- Capital gains taxes.
- Withholding taxes on dividends and interest.

Companies and individuals are taxed on both incomes and capital gains, albeit at different tax rates, while withholding taxes are deducted at source regardless of the recipient [17].

3.22.1 Company tax

Any company, such as an asset management firm, portfolio management firm or company that trades, will need to pay income tax at a tax rate of 28% [17]. This will be on net income which is similar to, but not the same as profit – due to certain expenses and deductions not being allowed / allowed. Certain income, such as dividends, is not included in taxable income. However capital profits from the sale on investments (gains) are included in taxable income at a rate 66.6% of the profit amount (i.e two thirds of the profit); (two thirds of losses are allowed as a deduction) [17]. This inclusion rate is higher than that for individual taxpayers.

Thus any investor holding his portfolio in a company, will be subject to taxes at a different rate and on "net income" calculated differently, primarily due to the inclusion rate of capital gains.

Professional portfolio managing firms, who manage funds on behalf of clients, would not pay taxes on the net incomes generated by those funds. They however are required to report various information in respect of individual taxpayers set out in the income tax act regulations from time to time. This includes information such as, interest paid, dividend income as well as capital gains and losses realised (IT3B form).

Taxes paid by individuals in respect of their investment portfolios are summarised below:

3.22.2 Dividend taxes

Dividend tax of 15% will be paid directly by the investment company to SARS on behalf of the individual taxpayer. This percentage is based on the actual value of the dividends paid [17].

3.22.3 Individual income tax

The taxation of profits (net of losses) made from buying and selling shares depends whether there is regular trading or whether the shares are held as investments. Trading profits are included in an individual's net taxable income fully, while net capital gains are included only at a rate of 33.3%. The actual tax rate will depend on the individual taxpayer's tax brackets. These range up to the maximum marginal rate of 41% [17].

3.22.4 Effect of tax on MPT

Even though MPT assumes that no tax is paid, the investor unfortunately will need to take tax into account. The net profit (or losses) that would arise from a rebalancing of the portfolio would at best under the capital gains rules, be subject, at an inclusion of 33.3% of the net profit, to the marginal tax rate of the individual (typically 41% for higher income earners) [17]. Should the rebalancing happen very frequently and appear to be a trading scheme then, at worst, the full net profit would be subject to tax at the marginal tax rate of the individual.

The taxation rules are beyond the scope of this paper. For the purposes of the testing the hypothesis, it is assumed that the tax treatment would be identical for any investment scheme and hence has a neutral impact.

3.23 Bias

3.23.1 Bias overview

Bias in investing is when an investor follows a certain path or forms a perspective based on notions and beliefs which are predetermined. Investors who act on bias do not fully explore issues and can be ignorant of evidence which contradicts their own opinions [18].

Biases should be avoided as far as possible to allow for a more impartial decision to be made by the investor solely based on data which is available [18].

Biases with regards to investing can be divided into numerous categories. These are discussed in further detail below.

3.23.2 Survivorship bias

Survivorship bias results from using sets of data that include only surviving companies over a certain period. Companies which have been delisted or have been taken over or merged, not being included in the data sets creates survivorship bias [19]. Ignoring delisted companies will produce an upward bias in the measurement of the returns on these risky stocks [19].

The main reason for survivorship bias being ignored in financial research is that collecting data for delisted companies can be time consuming and expensive, and might not always readily be available [19].

Gilbert and Strugnell [19] concluded that any financial research which excludes delisted stocks is likely to be affected by survivorship bias. However, the authors stated that survivorship bias might not materially affect the outcomes of a specific study, depending on what the study is trying to achieve [19].

3.23.3 Home country bias

Home country bias is when investors tend to only focus or to mostly focus on domestic or home markets. This type of bias could lead to less diversification of the investor's portfolio because of the lack of exposure to foreign markets. This is especially true if the country where all of the investments are suffers from potential political and economic risks which could only effect that country [18].

3.23.4 Look-ahead bias

Look-ahead bias is when information or data is used in a simulation which would otherwise not have been known during the period which is being analysed [18].

3.23.5 Attribute bias

Attribute bias is the selection of stocks through a quantitative technique which selects stocks with similar fundamental characteristics such as low price earnings ratios, etc. Most models would normally contain parameters which will exclude certain stocks based on the chosen parameters. The danger in having an attribute bias is that stocks could be selected from the same industry because of similar attributes and could thus lead to a less diversified portfolio [18].

3.23.6 Instant history bias

Instant history bias is an inaccuracy which occurs when only successful funds' returns are recorded and unsuccessful fund returns are not factored into an overall performance record [18].

3.23.7 Sample selection bias

Sample selection bias is caused by having non-random data for statistical analysis. A subset of data could be excluded due to it having a particular attribute. Exclusion of subsets of data could produce distorted results and influence the statistical significance of the test. An example of sample selection bias is only looking at companies who have a certain range of data, such as 10 years' worth of price data. This sample selection bias thus excludes companies who are newer (less than 10 year's data) [18].

3.23.8 Outcome bias

Outcome bias is a decision based on the outcome of previous events without taking regard of how past events have developed. This leads to past factors leading to a previous event to be ignored while only overemphasising the outcome. An example of outcome bias is when an investor invests in a stock due to it having a high return over a specific period instead of looking at the factors which led to the success of the stock [18].

3.23.9 Hindsight bias

Hindsight bias is a psychological phenomenon where past events seem more prominent now than they would have been while they were occurring. This could lead a person to think an event was more predictable than it actually was [18].

3.24 Momentum Effect

Jegadeesh and Titman [20] have shown that stock buying strategies which buy stocks that have performed well in the past and selling stocks which have performed poorly, generate significantly positive returns over holding periods less than 12 months. It was also found that part of the returns generated over the first year of selection dissipates in the following two years.

A momentum effect has thus been shown to exist over a period of 3 to 12 months.

Rebalancing a portfolio every 12 months becomes important to limit the effect of the possible dissipation of returns over the following years [20].

3.25 Introduction to Other Investment Theories

3.25.1 Post-modern portfolio theory

This theory uses the downside risk of returns instead of mean variance returns as used by Markowitz. The difference between the models is between each theory's definition of risk and how it influences expected returns. Markowitz theory uses the standard deviation of all returns as a measure of risk, whereas Post-modern theory uses the standard deviation of negative returns. The downside risk is what investors fear, i.e. having negative returns [21].

3.25.2 Mutual fund theorem

The theorem states that investors should hold an identically comprised portfolio of risky assets combined with a certain percentage of risk-free assets or cash. The more risk-averse the investor, the higher percentage of cash they would hold, but will have the same basket of risky securities as an aggressive investor. This model follows similarly to Markowitz in that diversification is assumed to limit portfolio risk [2].

3.25.3 Capital Asset Pricing Model (CAPM)

This model describes the relationship between risk and expected return (similar to Markowitz). It states that investors must be compensated in two ways, namely time value of money and risk. Time value of money is represented by the risk free rate in Equation 20, and is used to compensate investors for placing money in an investment over a period of time. Risk is represented by the rest of the equation and calculates the amount of compensation the investor requires for taking additional risk. This is done by taking a risk measure (beta) that compares returns of the asset to the market over a period of time and to the market premium $(\bar{r}_m - r_f)$ [2]:

$$\bar{r}_a = r_f + \beta_a(\bar{r}_m - r_f) \tag{20}$$

Where:

$$r_f = risk\ free\ rate$$

$$\beta_a = beta\ of\ the\ security$$

$$\bar{r}_m = expected\ market\ return$$

If the expected return does not meet or beat the required return the investment should not be undertaken [2].

3.25.4 Random Walk Theory

In 1973 Burton Malkiel wrote the book "A Random Walk Down Wall Street" which states that the past movement of a stock's price or market cannot be used to predict the future movement of that stock or market. The theory states that price fluctuations of stocks are independent of one another and have the same probability distribution. People who follow the random walk theory believe that it is impossible to outperform the market without assuming additional risk. Malkiel also states in his book that technical and fundamental analysis are a waste of time and are still unproven in outperforming the markets [22].

Following on from this book from Malkiel, a research article by Research Affiliates selected 100 random portfolios containing 30 stocks each selected randomly from a 1000 stock list. This process was repeated for every years' worth of stock returns from 1964 to 2010. On average 98 of the 100 portfolios beat the 1000 stock capitalisation weighted stock list each year. The main reason for this outperformance was that the random selection was from any company (small or large cap) and due to the large capitalisation of the top 30 companies having lower returns the inclusion of smaller cap companies being of the same weighting added extra returns [23].

3.25.5 Behavioural Finance combined with MPT

MPT can be improved by incorporating behavioural finance. MPT assumes that investors behave rationally, but this has been found to be incorrect. Curtis [24] explains that MPT can be improved by firstly designing a normal MPT optimal portfolio and secondly designing a behavioural portfolio. A behavioural portfolio would depend on the specific customer and could include various investment goals such as [24]:

- Liquidity
- Income
- Capital preservation
- Growth

This has been called the Statman/Brunel approach.

This approach is more cautious (liquidity, income, capital preservation are desired) than aggressive (growth is desired). This leads to a sub optimal portfolio that creates comfort over high investment growth. This dilemma can be solved by merging MPT with behavioural portfolio theory to more suit individual/family investors needs. [24]

3.26 Conclusion

3.26.1 Portfolio construction

From the literature review into the development and application of MPT, it was found that the investment process of portfolio construction falls into four areas: security valuation, asset allocation, portfolio optimisation and performance measurement.

Security valuation is found to be a complex task, analysing financial data such as financial statements of each company and doing industry and economic analysis. Equity valuation models are used to gauge what a stock's price should be or could be in the future. It was also found that security analysis is often the arena in which asset managers compete, to forecast the best input data for a portfolio model.

3.26.2 Required number of stocks

As discussed in Section 3.11, the number of stocks required to get the most out of diversification ranges between 20 and 25 in the South African market. This information will thus be used when creating constraints with regards to the MPT model. However 10-15 stocks have been found to reduce diversifiable risk by approximately 70-80% (as shown in Table 3.3), thus adding more stocks will create a less optimal portfolio and not significantly decrease risk. These figures can change over time and will need to be taken into account in future calculations of the MPT model. A range of different numbers of stocks will be chosen and compared to see how the potential returns differ.

3.26.3 Measurement period

The measurement period is discussed in Section 3.12. For a long term investment a longer horizon needs to be used. Therefore data from at least the past 5 years will need to be used in the MPT model.

3.26.4 Stock data range

As shown in Section 3.13 and 3.18, previous research of MPT on the JSE used only the Top 160 and Top 40 companies respectively. This research project will include all the companies available from the available data, and will also use data from only the JSE Top 40 to make easier comparisons to current Top 40 products. This will be done to see whether the possible inclusion of small and medium cap companies can make a difference in the potential amount of return which could be received.

3.26.5 Constraints

As shown in Section 3.15 the addition of floor/minimum weight constraints can be particularly damaging to optimisation. This research will thus not have any floor constraints in the MPT model. However, maximum weight constraints will be used. The reason for this is to limit exposure to only a few companies. Because historical data will be used, a company which has performed exceptionally in the past might be the sole company chosen by the model, even though it is being potentially overvalued now, or its future growth opportunities are not as good as in the past. This is one of the drawbacks of using historical data. Security analysis of a certain company now could show that it is overvalued or its future growth prospects are unfavourable. Historical data can be overly optimistic with respect to a handful of stocks based on past data. Thus using maximum constraints could reduce the influence large gains in past data will have in influencing selection in the model.

3.26.6 Selection of MPT

Markowitz Theory or Modern Portfolio Theory was chosen to be used in this research report because of numerous factors including:

- The theory behind MPT is the forefather to many newer portfolio theories.
- It is the one of the first areas in portfolio management that is discussed in textbooks.
- To test whether MPT is still applicable in this day and age.
- To see whether MPT models can be created more easily because of advances in computing capabilities.
- To test whether MPT can be used during times such as a financial crash (September 2008).

4 Methodology

4.1 Research Methodology Overview

Figure 4.1 shows the general methodology which will be followed to achieve the objectives.

Firstly an understanding needs to be gained of the actual Markowitz portfolio theory as well as other theories and techniques involved in selecting and optimising a portfolio. Then a method of comparison to the Markowitz model should be devised to compare results. Once the choices of models and theories have been made it is necessary to gather the required data which would be used for input to these models. The model will be designed accordingly. Results can be compared to other models and research and to the formulated objectives. Results will be discussed and recommendations made for further future research in this field.

The research methodology that will be used is fully quantitative. Large volumes of price data will be analysed using a model that has been formulated around the theories discussed by Markowitz.

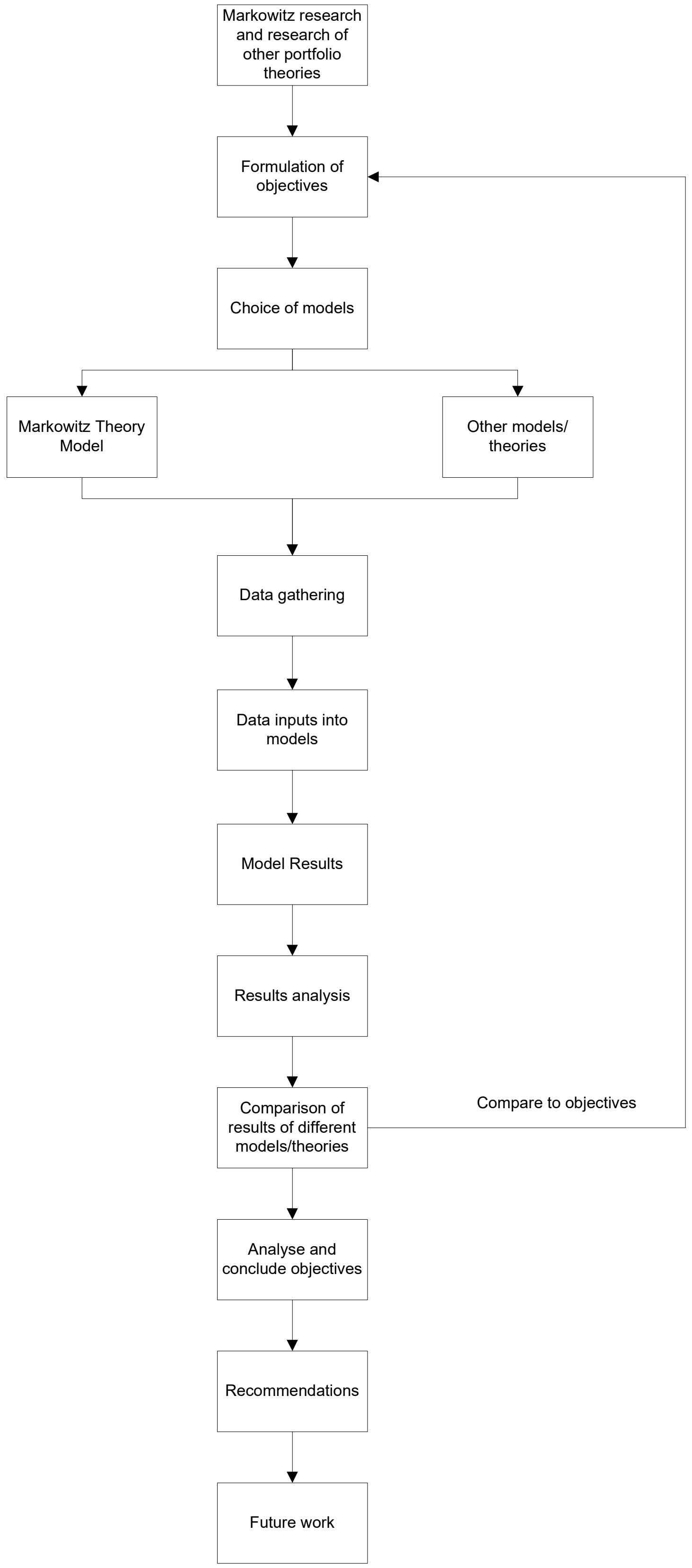

Figure 4.1 General research methodology

4.2 Detailed Methodology

4.2.1 Introduction

There are numerous techniques which can be used when selecting securities. The main one which will be looked at is the Markowitz Portfolio Theory (MPT).

There are numerous choices which can be made when deciding where to invest money. This project will look at and compare the possible returns which could be or would have been achieved if MPT was used. These returns will then be compared to returns from other similar financial instruments (i.e. equity instruments). Some of these include:

- Unit trusts
- Actively managed funds
- Passively managed funds (Satrix)

This research report looks at which method can be used to maximise your return from an equity based investment. The assumption is made that an investor has a choice of whether to invest a certain amount of money in the stocks estimated from the MPT model or to rather invest the money in another equity financial instrument and then hold for a long term period (5-10 years) without interfering or changing any of the holdings in either the portfolio or financial instruments.

4.2.2 Analysis and discussion of initial research findings

Due to the scope of this research, and the fact that large amounts of time and resources are required to do a thorough security analysis to create forecast input data, historical data will be used. As discussed earlier, historical data is not ideal but it has been used before and some authors say it is viable to assume future performance can be predicted using past data, as discussed in Section 3.14.

For the asset allocation decision it is assumed that the potential investor already has advice or already knows what his/her ideal percentage mix of assets are. It is thus assumed that whatever money is to be invested in an MPT model will be the amount which is allocated to the local equities portion of the person's asset allocation. The scope of this research is not how an investor should split their money into various asset classes, but rather which local equities should be selected and optimisation of that portfolio.

Portfolio optimisation is the area of portfolio construction which will be mostly considered in this research report. Portfolio optimisation is involved in the selection of securities and then finding the optimal risk-return for given constraints and investor risk appetite. This research will thus create an MPT model, set realistic constraints determined by research, and use historical data inputs to allow the model to calculate which stocks should be purchased and the percentage of each required for different portfolios.

Performance measurement of the portfolio risk-adjusted performance will be calculated by using the Sharpe ratio. This will allow for a comparison to be made of how well each portfolio would have performed had an actual investment been made into the securities selected by the model.

4.3 Software Selection for Model Creation

4.3.1 Possible Software Choices

Two programs, Microsoft Excel and Matlab are available which have the capabilities to create an MPT model. Excel has been used by South African researchers as discussed in the literature review. Matlab has been used by international researchers. These two programs will be discussed and compared to decide which will be the best to create a model required by this research.

4.3.2 Requirements for the Model

The MPT model requires a range of data which it needs to convert and calculate to create an efficient frontier.

As discussed in Section 3.8, the stock returns, standard deviations (risk), portfolio weights and covariance between stocks needs to be calculated.

The returns and standard deviations can be calculated easily in Excel and Matlab. However the optimal portfolio weights and covariance are more difficult to calculate.

In Excel a N x N (N = number of stocks) matrix needs to be created manually when calculating the covariance as given by Equation 12 and 13 (Section 3.8.4)

Matlab has a function built in to calculate covariance's for all the input values.

Optimal portfolio weights can be calculated in Excel using the Solver add-in. Matlab's financial toolbox add-in has a function where portfolio weights can be calculated.

The portfolio risk needs to be calculated once the optimal portfolio weights and variances (standard deviation squared) are known, as shown in Equation 15 (Section 3.8.5). In Excel this would be straightforward to calculate but could take some time to add in all of the weights. In Matlab the portfolio risk is automatically calculated when using the financial toolbox function and a few lines of code.

The number of portfolios created (different risk-return values) is done automatically in Matlab by adding lines of code which allow for having different points (different risk-return portfolios) on the efficient frontier. The amount of portfolios shown on the efficient frontier can be changed easily using a line of code for the number of portfolios required. In Excel these will have to be calculated manually.

Matlab's financial toolbox code can also automatically create the efficient frontier graph. In Excel creating this graph will be easy once the different portfolio risk-returns have been calculated.

Matlab's financial toolbox allows for adding a risk free rate to borrow at, as well as creating an optimal portfolio for a given risk free rate. Risk aversion factors can also be added in easily.

Constraints can be added into Excel and then the Solver add-in can be used to find a solution. Matlab requires adding extra lines of code using the built in functions of the financial toolbox to create these constraints.

4.3.3 Microsoft Excel

4.3.3.1 Advantages

- There are a large range of built-in functions available.
- Data is easily analysed and organised.
- Different types of graphs and graphical data can be easily created.

4.3.3.2 Disadvantages

- Less data can be analysed than in Matlab.
- Complex calculations can take a long time to calculate.
- More manual operations are required.
- Larger spreadsheets with high volumes of data become more difficult to use and become slow.

- It is more difficult to trace mistakes over large numbers of spreadsheets.
- Excel is not robust

4.3.4 Matlab

4.3.4.1 Advantages

- Matlab can easily handle large arrays.
- Code can be programmed to add what is required.
- The financial toolbox add-in has many built in functions directly involving MPT.
- Calculations and programming can be straight forward if knowledge of the program is known.
- Matlab has code, which is far more compact than looking at multiple spreadsheets.
- Matlab is extremely good at performing matrix calculations.

4.3.4.2 Disadvantages

- Not as simple to use and understand as Excel.
- Some programming skill will be required.
- High cost compared to Excel

4.3.5 Choice of Software

Through analysing the software requirements and the advantages and disadvantages of each program, Matlab was chosen to create the MPT model. The main reason for this is the financial toolbox add-in which significantly reduces the amount of code required due to its built in functions specifically for MPT and creating an efficient frontier.

The financial toolbox is an add-in (toolbox) of Matlab. Matlab is used, amongst other things to analyse large volumes of data that can be represented in matrix form. Matlab is designed to be able to quickly analyse these matrices by performing the necessary matrix calculations.

The financial toolbox enhances Matlab by allowing it to do the required matrix algebra and then further analysing this data to the user's requirements. In this case these requirements are for the returns data to be represented by matrices and then analysed and manipulated by Matlab. The results include covariances and correlation coefficients. Matlab allows this huge volume of matrix data to be quickly analysed and calculated to the required sets of covariance and other data. Built-in functions create a set of optimal portfolios depending on the number of portfolios selected. Other built-in functions allocate weights to securities and create an efficient frontier.

Matlab and the financial toolbox are thus just programmes which do basic mathematical calculations but on large volumes of data.

Excel was chosen to analyse the data created by Matlab so that graphs and tables can more easily be created. Basic calculations using the Matlab output data will be done in Excel due to its ease of use and better organisation of summarised data.

4.4 Experimental Facilities

A PC with Matlab (with the financial toolbox installed) was used to formulate the Markowitz mean variance model and generate the results. Microsoft Excel was used for pre-model data processing. This is the processing of the data which is required before the data is input into Matlab. Post-model data processing was also done using Excel by copying and pasting the results from Matlab into an Excel spreadsheet for further processing.

Matlab was used to formulate the MPT model because of the financial toolbox add-in which greatly eases the task of creating a model to analyse input data.

A list of software and hardware used throughout the research is shown below.

<u>Software:</u>

Matlab version 2013a with the financial toolbox installed

Microsoft Excel 2013 (Office 365 Home Premium)

McGregor BFA Research Domain accessed through the University of the Witwatersrand (Wits) website

<u>Hardware:</u>

Intel Core i5-3230M CPU @ 2.60GHz (4 CPU's)

4.00GB RAM

64-bit Windows 8 Operating System

Intel HD Graphics 4000, 1792 MB Approximate total memory

4.5 Overview of MPT Modelling Process

This section illustrates the MPT modelling process.

1. Obtain share price and dividend data from McGregor BFA [25].
2. Calculate share price returns on your chosen range (e.g. weekly or yearly returns).
3. Read company return data into Matlab MPT model.
4. Change constraints (upper limit, lower limit, risk aversion factor, etc.) to your choosing.
5. Run Matlab MPT model.
6. Export MPT model data (portfolio selections for each company, expected portfolio risk and expected portfolio returns) to Excel.
7. Calculate portfolio returns for each MPT model portfolio in Excel.
8. Calculate the Sharpe Ratio of the expected portfolio returns in Excel.
9. Select the portfolio with the highest Sharpe Ratio of expected portfolio returns as the portfolio to invest in.
10. Calculate the actual return for that portfolio if the actual portfolio was invested into (Excel).
11. Choose a new set/range of data to input into the MPT model.
12. Repeat steps 3 to 11 to rebalance for your chosen number of years.
13. Combine data from different tests together to create comparisons to discuss (Excel).

4.6 Input Data Methodology

The total returns were calculated for each share listed on the JSE so that this data could be used as an input into the MPT model. The following procedure was used for gathering and processing the input returns data to make it ready to be read into the MPT model.

4.6.1 Procedure

1. Share price and dividend pay-out data is obtained using the McGregor BFA Research Domain, through the University of the Witwatersrand website [25].
2. The share price and dividends data is then saved from the research domain and exported to Excel.
3. Once all the price and dividend data has been acquired, the total returns need to be calculated.
4. The returns in percentage can be calculated in Excel using the following formula:

$$x = \frac{(Closing\ price - Opening\ price) + Dividends}{Opening\ price} X\ 100 \qquad (21)$$

5. Once the total returns are calculated for each company the pre-processing phase is complete.

4.6.2 Precautions

- Make sure the dividend is added to the correct time period (i.e. add dividend of 2012 to year 2012). This is important due to many companies only paying dividends intermittently and some companies paying dividends more than once a year.

4.7 Model Input Data and Assumptions

4.7.1 Model Input Data

As discussed in Section 4.6: Input Data Methodology, the data which will be used is that of historical price and dividend data to calculate total returns. This data was gathered through the McGregor BFA research domain [25]. Two sets of data will be used, namely yearly and weekly.

4.7.1.1 Calculating total return

The total return from each year is calculated using the return calculation discussed in Section 4.6.1 (Equation 21). This data is calculated in Excel for each company. The total returns for each company on each specified date are then put in one table, an example of which is shown in Table 4.4.

Table 4.1 shows the price data which was acquired from McGregor BFA for SAB Miller. The table shows semi-annual price data and various other information such as high and low price data which is not used. The closing price data value was used when calculating the returns.

Table 4.1 SAB Miller price data from McGregor BFA [25]

Price History - SABMILLER PLC (SAB)

Date	EY (%)	P/E	DY (%)	High (Cents)	Low (Cents)	Open (Cents)	Close (Cents)	Volume
08-Nov-13	3.29	30.36	1.55	53481	46515	47810	52843	123872337
28-Jun-13	3.63	27.58	1.64	52496	39150	39150	48000	200947807
31-Dec-12	3.95	25.29	1.75	40360	32639	32850	38944	227785652
29-Jun-12	4.08	24.53	2.03	33333	27984	28750	32990	226275299
30-Dec-11	4.2	23.83	2.12	29500	22651	24650	28309	248838660
30-Jun-11	4.4	22.72	2.29	25971	22010	23500	24500	220731554
31-Dec-10	4.23	23.66	2.21	24200	20645	21400	23553	203572565
30-Jun-10	4.62	21.65	2.43	23775	19800	21888	21490	182634499
31-Dec-09	4.94	20.24	2.09	22631	15705	15839	21640	226994309
30-Jun-09	6.73	14.87	3.35	17500	13010	16250	15691	221467662
31-Dec-08	6.11	16.37	3.09	18149	13433	18000	16232	284953972
30-Jun-08	5.33	18.78	2.28	20970	14701	19000	17805	218502805
31-Dec-07	4.53	22.08	1.9	21543	17341	17750	19001	197776971
29-Jun-07	4.39	22.76	1.98	18545	14900	16498	17706	195429580
29-Dec-06	4.59	21.77	2.03	16500	12200	12765	16044	206550737
30-Jun-06	5.4	18.52	2.45	13497	11371	11771	12821	293306139
30-Dec-05	5.35	18.69	2.13	12900	10100	10450	11775	204942168
30-Jun-05	6.17	16.22	2.28	10672	8750	9550	10410	240300374
31-Dec-04	6.25	16.01	2.37	9810	7535	8030	9545	177387886
30-Jun-04	6.78	14.76	2.54	8290	6550	6853	7990	267396015
31-Dec-03	7.16	13.97	2.86	6994	6050	6200	6801	60288149

Table 4.2 shows the dividend data given by McGregor BFA for SAB Miller. The amount paid and the date at which the dividend was declared is required to calculate yearly returns.

Table 4.2 SAB Miller dividend history from McGregor BFA [25]

Dividend History		
Type	**Declared**	**Amount (Cents)**
SAB (March)		
FINAL	23-May-13	634.93
INTERIM	22-Nov-12	181.7
FINAL	24-May-12	500.84
INTERIM	18-Nov-11	176.14
FINAL	19-May-11	424.42
INTERIM	18-Nov-10	136.84
FINAL	20-May-10	384.36
INTERIM	19-Nov-09	126.04
FINAL	15-May-09	327.24
INTERIM	13-Nov-08	167.82
FINAL	15-May-08	334.03
INTERIM	15-Nov-07	106.26
FINAL	18-May-07	255.08
INTERIM	09-Nov-06	100.03
FINAL	18-May-06	226.36
INTERIM	10-Nov-05	87.21
FINAL	20-May-05	164.06
INTERIM	18-Nov-04	73.43
FINAL	20-May-04	152.49
INTERIM	20-Nov-03	50.16
FINAL	22-May-03	144.3

Table 4.3 shows the calculation table for calculating the yearly return without dividends and with dividends. The total return which will be used is that with dividends added in to take into account the total return an investor would have received if they had actually invested in the specific company.

A sample calculation for the total return with dividends for the period 30 December 2011 to 31 December 2012, using the information from Table 4.1 and 4.2, is as follows:

$$x = \frac{(Closing\ price - Opening\ price) + Dividends}{Opening\ price}\ X\ 100$$

$$= \frac{(38944 - 28309) + (181.7 + 500.84)}{28309}\ X\ 100$$

$$= 39.34\%$$

This is then the total return for the year. Total returns for all companies are thus calculated the same way and a sample of the total returns for a selection of companies can be seen in Table 4.4.

Table 4.3 Calculated SAB Miller yearly return

Date	Open (Cents)	Close (Cents)	Dividend date	Dividend (Cents)	Yearly return with no dividend	Total yearly return
04-Nov-13	47810	52843	16-Aug-13	634.93	35.69%	37.32%
28-Jun-13	39150	48000	07-Dec-12	181.7		
31-Dec-12	32850	38944	10-Aug-12	500.84	37.57%	39.34%
29-Jun-12	28750	32990	02-Dec-11	176.14		
30-Dec-11	24650	28309	05-Aug-11	424.42	20.19%	21.99%
30-Jun-11	23500	24500	03-Dec-10	136.84		
31-Dec-10	21400	23553	06-Aug-10	384.36	8.84%	10.62%
30-Jun-10	21888	21490	04-Dec-09	126.04		
31-Dec-09	15839	21640	21-Aug-09	327.24	33.32%	35.33%
30-Jun-09	16250	15691	28-Nov-08	167.82		
31-Dec-08	18000	16232	11-Jul-08	334.03	-14.57%	-12.81%
30-Jun-08	19000	17805	30-Nov-07	106.26		
31-Dec-07	17750	19001	13-Jul-07	255.08	18.43%	20.02%
29-Jun-07	16498	17706	01-Dec-06	100.03		
29-Dec-06	12765	16044	07-Jul-06	226.36	36.25%	38.18%
30-Jun-06	11771	12821	02-Dec-05	87.21		
30-Dec-05	10450	11775	08-Jul-05	164.06	23.36%	25.08%
30-Jun-05	9550	10410	03-Dec-04	73.43		
31-Dec-04	8030	9545	09-Jul-04	152.49	40.35%	42.59%
30-Jun-04	6853	7990	05-Dec-03	50.16		
31-Dec-03	6200	6801	11-Jul-03	144.3		

Table 4.4 shows the total return data over a number of years for a number of companies. Once the total returns for each company are calculated for each of the years/months/weeks under consideration, these values are used as inputs for the MPT model in Matlab.

Table 4.4 Calculated total yearly returns for selected companies

Date	SAB Miller	Sabvest	Sacoil	Sanlam	Santam	Santova	Sappi	Sasfin	Sasol
04-Nov-13	37.32%	32.38%	-10.00%	19.33%	0.95%	20.14%	-7.83%	24.80%	42.58%
31-Dec-12	39.34%	101.50%	-41.18%	59.69%	35.08%	39.71%	29.83%	24.06%	-3.28%
30-Dec-11	21.99%	28.23%	-65.77%	7.45%	13.07%	-15.00%	-30.00%	-26.44%	14.19%
31-Dec-10	10.62%	5.17%	727.78%	27.30%	29.35%	100.00%	-4.23%	2.23%	18.79%
31-Dec-09	35.33%	-16.83%	-76.62%	39.59%	42.24%	-20.00%	-5.59%	61.96%	8.56%
31-Dec-08	-12.81%	-17.74%	-9.41%	-21.19%	-19.81%	-44.44%	-57.40%	-52.79%	-14.64%
31-Dec-07	20.02%	48.00%	750.00%	28.52%	48.98%	-47.06%	-15.40%	59.28%	33.27%
29-Dec-06	38.18%	94.19%	-50.00%	24.75%	12.92%	41.67%	64.68%	25.82%	16.15%
30-Dec-05	25.08%	42.27%	100.00%	20.69%	31.73%	33.33%	-10.59%	37.68%	89.75%
31-Dec-04	42.59%	11.50%	-75.00%	52.27%	42.86%	-18.18%	-6.59%	210.00%	29.84%

4.7.2 Model Data Assumptions

The following is a list of the simplifying assumptions which have been made when designing the model:

- Historical data is acceptable for the MPT model.
- Investing only occurs in local equities. Cash and bonds are assumed to have been invested in separately already.
- The portfolio which will be selected out of the number of portfolios function (*NumberPortfolios*) which Matlab generates will be the one with the highest Sharpe ratio using the expected portfolio returns and risk calculated by Matlab.
- The decision is to invest your money either into what the model suggests or into another financial instrument such as a unit trust or product like the Satrix Top 40. Returns will be compared to make a decision on each instruments' performance.
- Dividends will be added to yearly data but not to weekly data due to the added complexity in adding different dates of dividends to a large range of data.
- Dividends are assumed to be paid within the same year that they are declared in.
- All incomes are assumed to be reinvested
- A risk free rate (r_f) of 6.5% is assumed, as offered by the Government Retails Savings Bonds 2 year fixed interest rate as of December 2013 [26].
- No short selling is allowed.
- It is assumed that the investor will only use their own money and not borrow any money to invest.

- The Top 40 companies are assumed to be the same as those as of 31 December 2013.
- Data for companies which have existed in the past and no longer exist are not included, due to the difficulty in obtaining data of past companies.
- Share buy backs are assumed to have been factored into the share price.

4.8 Design of the Model

4.8.1 Introduction

The basic code was written in Matlab and various literature was referred to [6, 11, 27] to create maximum limits for the assets, to use historical data and selecting types of data to be input into the model. The required code was then created which would allow for the generation of results using the MPT model.

4.8.2 The MPT Model

The actual Matlab code which was created and used to create the required results can be found in Appendix C: Matlab Code.

4.8.3 Explanation of How the Code Works

4.8.3.1 Constraint matrix

A constraint matrix needs to be created for the minimum and maximum allocation per asset. This is done by creating a matrix, the size which depends on the number of assets which is read into the model (*NumberAssets*). The matrix *ConstraintMatrix* is created using the "portcons" function of the Financial Toolbox [6].

4.8.3.2 Mean, covariance and standard deviation

The mean (m) (to calculate the expected return of each asset using historical data), covariances (c) (the extent to which two assets move together) and standard deviations (stdev) (risk) need to be calculated in order for Matlab to create the different portfolios and the efficient frontier. The number of portfolios (*NumberPortfolios*) was chosen as ten. This creates different points on the efficient frontier.

These values will then be included with the constraint matrix in the "portopt" function, in which Matlab creates the portfolio weights (*PortfolioWts*), portfolio returns (*PortfolioRet*), and portfolio risk (*PortfolioRisk*) [6]. The amount of data which will be created will be one column matrix for returns (*PortfolioRet*) and one column matrix for portfolio risk (*PortfolioRisk*). The number of rows would be ten for each of these, with each row representing one of the ten portfolios specified by the number of portfolios (*NumberPortfolios*) function. The portfolio weights (*PortfolioWts*) matrix is a m by n matrix (10 by *NumberAssets*) which is created by the "portopt" function. Each column of the portfolio weights matrix represents one of the assets (companies). Each row is one of the ten created portfolios which increase in risk and return as the row number increases.

4.8.3.3 Optimal risky portfolio

This section explains the creation of an optimal risky portfolio and optimal capital allocation using the "portalloc" function. This function computes the optimal risky portfolio as well as how funds should be allocated between the risky portfolio and the risk-free asset [6]. The function uses inputs such as the riskless rate (the rate at which an investor can invest in risk-free), as well as the borrow rate, which is the interest an investor would pay if they borrow money which they would then invest. A risk aversion factor is chosen – in this case 2 (lowest risk aversion, therefore accepting the highest risk) for which the investor is most comfortable with. In this research report it is assumed that the investor will not borrow any money to invest (therefore *BorrowRate* = NaN). The matrices created include *RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction.*

The Markowitz model can be used to create an optimal portfolio while using borrowed money. In this research it is assumed that the average person who would use a model like the one presented here (focus is on individual investor, not asset management company) would prefer not to borrow money. The model could be modified to deal with investing borrowed money and creating an optimal risky portfolio of stocks and bonds/cash. It is assumed as discussed earlier that the investor already has optimised the portfolio with respect to asset allocation, and that the focus is on security selection for the part of the portfolio that will consist of South African equity.

4.8.3.4 Creation of the efficient frontier graph

The efficient frontier graph needs to be created so that an overall picture of the risks and returns can be seen more easily. This is done by using a basic Matlab function for creating a scatter plot of the portfolio risks and portfolio returns.

4.8.4 Matrices Created by the MPT Model

Once the MPT model runs in Matlab it will create matrices. Each of these matrices contains data which needs to be further analysed to get the final results, as follows:

- The matrices c (covariance), m (mean) and stdev (standard deviation) are created initially using the input company return data.
- *ConstraintMatrix* - (constraint matrix) creates a matrix for the lower and upper limits (percentages) for each asset.
- *PortfolioRet* – The expected return on each portfolio (depending on the number of chosen portfolios *NumberPortfolios*).
- *PortfolioRisk* – The Matlab calculated risk of each of the portfolios.
- *PortfolioWts* – The weights which each stock will have in each of the different portfolios.
- *RiskyFraction* – The fraction of the portfolio, when borrowing at a risk free rate which will be made up of stocks (equities).
- *RiskyReturn* – The return which the optimal portfolio is expected to have when borrowing at the risk free rate.
- *RiskyRisk* – The risk which the optimal portfolio is expected to have when borrowing at the risk free rate.
- *RiskyWts* – The weights given for each stock for the portfolio when borrowing at the risk free rate.

4.9 Model Methodology

4.9.1 Procedure

This is the procedure which needs to be followed to gather data from the MPT model in Matlab:

1. Open Matlab and click on the "Import Data" tab on the top ribbon.
2. Select the file (Excel file) of the price return data that needs to be analysed.
3. The spreadsheet of the selected Excel file will open. Select the range of data that needs to be used (leave out any irrelevant data such as dates and the name of the companies).
4. Select 'matrix' at the top ribbon under the "imported data" tab.
5. Click on "import selection" in the top ribbon to import the selected returns data.
6. A matrix will be created in the workspace. The name of the created matrix will be dependent on the name of the spreadsheet where the data was read from.
7. Inside the m-code (MPT code) the function "data" needs to equal the new input data matrix which has been created. E.g. if the matrix created is called "yeardata", set: data

= (yeardata). This will create a matrix called "data" using the values of the input data file.

8. Another number which must be changed manually in the m-code is that of the number of assets (*NumberAssets*). The number of assets is the amount of companies whose data is read in from the input Excel spreadsheet.
9. Constraints which can be changed depending on the user are *AssetMin, AssetMax, NumberPortfolios, RisklessRate, BorrowRate* and *RiskAversion*. These are all changeable depending on which outcomes the user wishes to measure or which variables they want to change.
10. The rest of the code will run automatically once it is started and the constraints are changed according to the users' requirements.
11. Once all the initial values are selected (*NumberAssets*) and the constraint values are chosen, the "run" button on the top ribbon can be clicked to start the code.
12. The code will run automatically and will take a few seconds to compute all the data, depending on how large the input spreadsheet is. The model will then create an efficient frontier graph, and matrices *OverallReturn, OverallRisk, PortfolioRet, PortfolioRisk, PortfolioWts, RiskyFraction, RiskyReturn, RiskyRisk, RiskyWts,* as well as other matrices which are required to calculate the aforementioned data.
13. The efficient frontier graph can then be manually saved if required. This is done by selecting "File" at the top of the figure and clicking on "Save as". The graph can then be saved in the correct folder and a choice of file type can be made (in this case a jpeg format is chosen).
14. The *RiskyFraction, OverallRisk* and *OverallReturn* are used when money is borrowed at the risk free rate to invest in the *RiskyWts* portfolio. These will not be considered due to the initial assumption that money will not be borrowed to invest. These are thus calculated and shown for the interest of the reader.
15. The most important matrix is that of *PortfolioWts* as discussed earlier. Right click on the *PortfolioWts* matrix in the workspace. Click on "open selection". A spreadsheet will open in Matlab. This spreadsheet contains rows of the number of portfolios selected (*NumberPortfolios*) and the number of columns are the actual companies. Each rows return increases (according to the model) with increasing row number. The top row (top portfolio) should thus be low risk and lower return and the bottom row (bottom portfolio) should have higher returns and thus higher risk. The value in each cell is the corresponding percentage which should be invested in that specific

company (column) for that specific portfolio (row). These values all need to be selected and copied.

16. These values are then pasted into an Excel spreadsheet which has the company names in the order in which the input data was read into Matlab as the top row. Each corresponding value from the pasted cells can then be associated with each company.
17. The actual returns need to be calculated, to see what return an investor would achieve at some future date, if an investor invested at the end of the time period of the input data e.g. end of 2012 to end of 2013.
18. The return over the period under consideration (i.e. the return from end 2012 to end 2013) is calculated by multiplying the weights of each asset by the return of the asset.
19. The total return which could have been realised had the assets chosen by the model been invested into is then calculated by adding the returns of each asset multiplied by its percentage as chosen by the model.
20. The total return for each of the portfolios is calculated the same way.
21. These returns can then be compared to one another.
22. The process can be repeated by altering values such as the minimum or maximum constraints in the Matlab code and then repeating the procedure to find returns for various different portfolios, including for the JSE Top 40.

4.9.2 Precautions

- Select only the price data when importing the data into Matlab. Leave out any irrelevant data such as dates or the company names. The resultant data will be in the same order as the data in the initial spreadsheet.
- Save each set of calculated data under specific names so that when they need to be compared the differences in each is clearly known (i.e. changing maximum asset allocation percentage).

5 Model Data Observations

5.1 Data Output by the Model

This section shows a sample of the data created by the Matlab model. The raw data output of the model is in the spreadsheet which is included on the digital appendix CD (MPTmodel.xlsx).

A point in time had to be selected in which to consider which companies are in the Top 40, because the Top 40 constantly changes depending on changes in each company's share price and thus changes in its market capitalisation. The sample shown below is for weekly price data (dividend's excluded) for the time period 1998 to 2012 (15 years of weekly data). There were companies which fell into the Top 40 at the date of consideration (end December 2013) but who do not have enough data to go back the entire 15 year period. All the companies who did not have the entire range of data were thus excluded and the next highest market cap companies then replaced them. A range of different time periods were calculated including 15 years, 10 years and 5 years for weekly data. Therefore the same companies which were analysed using 15 year data are not all the same companies which were analysed for 5 year data. This project looks at total returns and which instruments deliver better returns. It is thus not necessary to have the same companies in a calculation of the model than is found in an actual Top 40 financial instrument such as the Satrix Top 40. The returns are what is important, not the companies used in each comparison.

The table has been split into four parts for ease of presentation (Table 5.1). Table 5.1 shows the raw data which is output by the Matlab MPT model. Each row is one of the 10 portfolios as specified in the Matlab code. Every portfolio is optimal and are thus all on the efficient frontier. Each column is a separate company which was in the JSE Top 40 as of end December 2013. The values within the table are the percentage which should be selected in each asset, as determined by the model, for each numbered portfolio (1 to 10). Portfolio 1 should be the lowest risk (according to the model calculation) and the risk (and return) increase as each portfolio number increases. Portfolio number 10 should thus be the highest risk and highest return portfolio.

The constraints selected for this sample are a 15% upper asset limit and 0% lower asset limit. The time horizon is over a 15 year period, to 31 December 2012 using weekly price data for the Top 40 companies which had data available for that range of time.

Table 5.1 15 Year, weekly data for the Top 40 JSE companies (part 1)

Portfolio Number	African Rainbow Minerals	Anglo American PLC	Anglo American Plat	Anglo Gold Ashanti	Aspen Pharmacare	Assore	Barclays Africa (ABSA)	BHP Billiton	Bidvest	Compagnie Fin Richemont
1	0	0	0	7,71%	2,05%	12,13%	0	0	0,52%	2,04%
2	0	0	0	7,46%	3,41%	14,01%	0	0	0	2,08%
3	0,39%	0	0	6,96%	4,82%	15,00%	0	0,11%	0	1,97%
4	1,10%	0	0	5,80%	6,46%	15,00%	0	0,86%	0	1,39%
5	1,75%	0	0	4,21%	8,45%	15,00%	0	1,17%	0	0,33%
6	2,41%	0	0	2,32%	10,74%	15,00%	0	1,43%	0	0
7	3,03%	0	0	0,26%	13,07%	15,00%	0	1,60%	0	0
8	3,71%	0	0	0	15,00%	15,00%	0	0,53%	0	0
9	4,58%	0	0	0	15,00%	15,00%	0	0	0	0
10	0	0	0	0	15,00%	15,00%	0	0	0	0

Table 5.1 15 Year, weekly data for the Top 40 JSE companies (part 2)

Portfolio Number	Firstrand	Growthpoint Property	Impala Plat	Mediclinic International	Mr Price	MTN	Naspers	Nedbank	RMB	SAB Miller
1	0	0,83%	0	10,69%	0	0	0	0	0	1,69%
2	0	1,00%	0	12,91%	0	0	0	0	0	0,89%
3	0	1,20%	0,56%	15,00%	0	0	0	0	0	0
4	0	1,46%	1,95%	15,00%	0,54%	0	0,89%	0	0	0
5	0	1,76%	3,17%	15,00%	1,52%	0	2,02%	0	0	0
6	0	2,13%	4,28%	15,00%	2,38%	0	3,07%	0	0	0
7	0	2,50%	5,45%	15,00%	3,03%	0,03%	4,04%	0	0	0
8	0	3,11%	6,75%	15,00%	3,75%	0,47%	6,07%	0	0	0
9	0	4,09%	8,61%	15,00%	5,48%	1,10%	9,56%	0	0	0
10	0	15,00%	10,00%	15,00%	0	0	15,00%	0	0	0

Table 5.1 15 Year, weekly data for the Top 40 JSE companies (part 3)

Portfolio Number	Sasol	Shoprite	Standard Bank	Tiger Brands	Woolworths	MMI	Imperial	Liberty	Netcare	Gold Fields
1	0	5,20%	0	6,77%	3,14%	0	0	7,98%	0,54%	0,71%
2	0	5,79%	0	5,46%	3,07%	0	0	7,62%	0,33%	0,34%
3	0	6,46%	0	3,86%	2,89%	0	0	7,12%	0,06%	0
4	0	7,24%	0	1,67%	2,42%	0	0	6,21%	0	0
5	0	7,90%	0	0	1,72%	0	0	4,90%	0	0
6	0	8,33%	0	0	0,72%	0	0	2,87%	0	0
7	0	8,54%	0	0	0	0	0	0,53%	0	0
8	0	7,23%	0	0	0	0	0	0	0	0
9	0	4,93%	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Table 5.1 15 Year, weekly data for the Top 40 JSE companies (part 4)

Portfolio Number	Lonmin	Distell	Tsogo Sun	Nampak	Barloworld	Brait SE	Investec	Pick n Pay Stores	Santam	The Foschini
1	3,09%	15,00%	4,54%	1,18%	0	4,33%	0	2,97%	6,89%	0
2	2,39%	15,00%	5,00%	0,38%	0	3,48%	0	2,68%	6,70%	0
3	1,43%	15,00%	5,54%	0	0	2,57%	0	2,39%	6,66%	0
4	0	15,00%	6,44%	0	0	1,54%	0	2,12%	6,91%	0
5	0	15,00%	7,37%	0	0	0,20%	0	1,47%	7,05%	0
6	0	13,68%	8,29%	0	0	0	0	0,45%	6,90%	0
7	0	12,13%	9,19%	0	0	0	0	0	6,61%	0
8	0	7,88%	11,23%	0	0	0	0	0	4,27%	0
9	0	1,27%	15,00%	0	0	0	0	0	0,37%	0
10	0	0	15,00%	0	0	0	0	0	0	0

6 Model Data Processing

6.1 Sample Calculation

6.1.1 Return calculations

Raw data needs to be created by the Matlab MPT model for various different constraints (upper asset percentage limits), time data (yearly or monthly price data) and time data ranges (15 year, 5 year data ranges, etc.). The potential returns of each of the calculated portfolios needs to be calculated if the assets chosen by the model (with the asset percentages) had actually been bought. All of the data was analysed using price data up to the end of 2012. The return is then calculated depending on the return which would have been received between 2012 and 2013. For example, the MPT model is run at the end of 2012 (hypothetically). The model shows which assets should be chosen and the percentage of each which should be chosen. An amount is then invested on the last day of 2012 into each of these assets, the amount in each of which depends on the model output. The return is then calculated at the end of 2013, being the return that would have been achieved if the actual investment had taken place. This is then the "realised return" which would have been achieved. This realised return for different constraints, etc., can then be compared to what the return would have been for other financial instruments over the same time period.

The values in Table 6.2 are calculated by multiplying the percentage amounts from Table 5.1 by the total return over the year (return from 2012 to 2013) shown in Table 6.1. This then gives the return which would have been realised for each company in each portfolio.

Sample calculations were done using Mediclinic and Portfolio Number 10. The MPT model calculated that Mediclinic should account for 15% of the portfolio in Portfolio Number 10 (Table 5.1). The total return for Mediclinic between 2012 and 2013 was 0.3899 (38.99%). These two numbers are then multiplied to calculate Mediclinic's return contribution to portfolio number 10.

$$Fractional\ Contribution = \ 0.15\ X\ 0.3899 = 0.0585$$

The total return which Mediclinic would have contributed to portfolio number 10 would thus be 0.0585 (5.85%) (Table 6.2). This process is repeated for all the companies and the total returns for each asset and portfolio is shown in Table 6.2.

Table 6.1 Top 40 company total returns from 2012 to 2013

Date	African rainbow minerals	Anglo american plc	Anglo american plat	Anglo gold ashanti	Aspen pharmacare	Assore	Barclays Africa (ABSA)	BHP Billiton
31-Dec-13	2.17%	-10.76%	-11.30%	-52.98%	59.16%	-15.66%	-17.53%	12.35%
	Bidvest	Compagnie fin richemont	Firstrand	Growthpoint prop	Impala Plat	Mediclinic international	Mr Price	MTN
31-Dec-13	26.09%	58.32%	18.02%	-0.90%	-26.35%	38.99%	18.57%	24.60%
	Naspers	Nedbank	RMB	SAB Miller	Sasol	Shoprite	Standard bank	Tiger Brands
31-Dec-13	102.37%	13.47%	21.00%	38.43%	44.93%	-18.91%	10.53%	-17.12%
	Woolworths	MMI	Imperial	Liberty	Netcare	Gold fields	Lonmin	Distell
31-Dec-13	6.91%	17.06%	3.82%	11.00%	33.59%	-67.68%	36.93%	41.17%
	Tsogo sun	Nampak	Barloworld	Brait SE	Investec	Pick n Pay stores	Santam	The Foschini
31-Dec-13	13.87%	29.85%	15.23%	43.35%	28.78%	17.16%	-1.00%	-28.92%

Table 6.2 15 Year, weekly processed return data for the Top 40 JSE companies (part 1)

Portfolio Number	African Rainbow Minerals	Anglo American PLC	Anglo American Plat	Anglo Gold Ashanti	Aspen Pharmacare	Assore	Barclays Africa (ABSA)	BHP Billiton	Bidvest	Compagnie Fin Richemont
1	0	0	0	-4,08%	1,21%	-1,90%	0	0	0,14%	1,19%
2	0	0	0	-3,95%	2,02%	-2,19%	0	0	0	1,21%
3	0,01%	0	0	-3,69%	2,85%	-2,35%	0	0,01%	0	1,15%
4	0,02%	0	0	-3,07%	3,82%	-2,35%	0	0,11%	0	0,81%
5	0,04%	0	0	-2,23%	5,00%	-2,35%	0	0,14%	0	0,19%
6	0,05%	0	0	-1,23%	6,36%	-2,35%	0	0,18%	0	0
7	0,07%	0	0	-0,14%	7,73%	-2,35%	0	0,20%	0	0
8	0,08%	0	0	0	8,87%	-2,35%	0	0,07%	0	0
9	0,10%	0	0	0	8,87%	-2,35%	0	0	0	0
10	0	0	0	0	8,87%	-2,35%	0	0	0	0

Table 6.2 15 Year, weekly processed return data for the Top 40 JSE companies (part 2)

Portfolio Number	Firstrand	Growthpoint Property	Impala Plat	Mediclinic International	Mr Price	MTN	Naspers	Nedbank	RMB	SAB Miller
1	0	-0,01%	0	4,17%	0	0	0	0	0	0,65%
2	0	-0,01%	0	5,03%	0	0	0	0	0	0,34%
3	0	-0,01%	-0,15%	5,85%	0	0	0	0	0	0
4	0	-0,01%	-0,51%	5,85%	0,10%	0	0,91%	0	0	0
5	0	-0,02%	-0,84%	5,85%	0,28%	0	2,07%	0	0	0
6	0	-0,02%	-1,13%	5,85%	0,44%	0	3,14%	0	0	0
7	0	-0,02%	-1,43%	5,85%	0,56%	0,01%	4,13%	0	0	0
8	0	-0,03%	-1,78%	5,85%	0,70%	0,12%	6,21%	0	0	0
9	0	-0,04%	-2,27%	5,85%	1,02%	0,27%	9,79%	0	0	0
10	0	-0,13%	-2,64%	5,85%	0	0	15,36%	0	0	0

Table 6.2 15 Year, weekly processed return data for the Top 40 JSE companies (part 3)

Portfolio Number	Sasol	Shoprite	Standard Bank	Tiger Brands	Woolworths	MMI	Imperial	Liberty	Netcare	Gold Fields
1	0	-0,98%	0	-1,16%	0,22%	0	0	0,88%	0,18%	-0,48%
2	0	-1,10%	0	-0,93%	0,21%	0	0	0,84%	0,11%	-0,23%
3	0	-1,22%	0	-0,66%	0,20%	0	0	0,78%	0,02%	0
4	0	-1,37%	0	-0,29%	0,17%	0	0	0,68%	0	0
5	0	-1,49%	0	0	0,12%	0	0	0,54%	0	0
6	0	-1,57%	0	0	0,05%	0	0	0,32%	0	0
7	0	-1,62%	0	0	0	0	0	0,06%	0	0
8	0	-1,37%	0	0	0	0	0	0	0	0
9	0	-0,93%	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Table 6.2 15 Year, weekly processed return data for the Top 40 JSE companies (part 4)

Portfolio Number	Lonmin	Distell	Tsogo Sun	Nampak	Barloworld	Brait SE	Investec	Pick n Pay Stores	Santam	The Foschini
1	1,14%	6,18%	0,63%	0,35%	0	1,88%	0	0,51%	-0,07%	0
2	0,88%	6,18%	0,69%	0,11%	0	1,51%	0	0,46%	-0,07%	0
3	0,53%	6,18%	0,77%	0	0	1,11%	0	0,41%	-0,07%	0
4	0	6,18%	0,89%	0	0	0,67%	0	0,36%	-0,07%	0
5	0	6,18%	1,02%	0	0	0,09%	0	0,25%	-0,07%	0
6	0	5,63%	1,15%	0	0	0	0	0,08%	-0,07%	0
7	0	4,99%	1,28%	0	0	0	0	0	-0,07%	0
8	0	3,25%	1,56%	0	0	0	0	0	-0,04%	0
9	0	0,52%	2,08%	0	0	0	0	0	0,00%	0
10	0	0	2,08%	0	0	0	0	0	0	0

The total returns for each portfolio are then calculated by adding up all of the individual returns for each company. The result for portfolio number 10, with the data being taken from the final row in Table 6.2, are shown in Table 6.3.

Table 6.3 shows each company's calculated return, which is added together to calculate the portfolio (portfolio 10) return for the period under consideration. This value can then be compared to the other portfolios (portfolios 1 to 9) of the same set of data as well as to returns for different constraints and financial instruments.

Table 6.3 Companies selected by MPT model and company returns

Company	Total Return (%)
Aspen Pharmacare	8.87
Assore	-2.35
Growthpoint Properties	-0.13
Impala Platinum	-2.64
Mediclinic International	5.85
Naspers	15.36
Tsogo Sun	2.08
Total:	**27.04**

Table 6.4 shows a summary of various outputs from the Matlab MPT model (portfolio risk or standard deviation, and expected portfolio returns) and calculations of realised returns as discussed above, for each of the 10 previously illustrated portfolios. Also shown in Table 6.4 is the yearly portfolio risk (converting the weekly risk of the output Matlab data into a yearly risk) and the expected yearly return. The yearly risk and expected yearly return are then calculated (converted) from the weekly data. This is done as follows (still using portfolio number 10) [28, 29]:

$$Yearly\ Portfolio\ Risk = Weekly\ Portfolio\ Risk\ X\ \sqrt{52}$$

$$Yearly\ Portfolio\ Risk = 0.0328\ X\ \sqrt{52}$$

$$Yearly\ Portfolio\ Risk = 0.2367$$

$$Expected\ yearly\ return = [(1 + expected\ weekly\ return)^{52}] - 1$$

$$Expected\ yearly\ return = [(1 + 0.0072)^{52}] - 1$$

$$Expected\ yearly\ return = 44.96\%$$

Table 6.4 Top 40, 15 year, weekly price data summary

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.0192	0.0045	0.1382	26.29%	10.63%	6.50%	1.43	0.30
2	0.0193	0.0048	0.1388	28.25%	11.12%		1.57	0.33
3	0.0195	0.0051	0.1407	30.23%	11.72%		1.69	0.37
4	0.0200	0.0054	0.1440	32.24%	12.90%		1.79	0.44
5	0.0207	0.0057	0.1489	34.28%	14.77%		1.87	0.56
6	0.0216	0.0060	0.1558	36.35%	16.87%		1.92	0.67
7	0.0228	0.0063	0.1645	38.45%	19.25%		1.94	0.78
8	0.0243	0.0066	0.1755	40.59%	21.13%		1.94	0.83
9	0.0266	0.0069	0.1917	42.76%	22.92%		1.89	0.86
10	0.0328	0.0072	0.2367	44.96%	27.04%		1.62	0.87

6.1.2 Sharpe ratio calculations

The next step is to calculate the Sharpe Ratio and Ex-post Sharpe Ratio. The Sharpe and Ex-post Sharpe Ratio for this sample calculation can be seen in Table 6.4. Ratios using portfolio number 10 are calculated as follows:

Sharpe Ratio:

$$S = \frac{E(r_p) - r_f}{\sigma_p}$$

$$S = \frac{44.96 - 6.5}{23.67}$$

$$S = 1.62$$

Ex-post Sharpe ratio:

$$S = \frac{r_{realised} - r_f}{\sigma_p}$$

$$S = \frac{27.04 - 6.5}{23.67}$$

$$S = 0.87$$

7 Model Data Results and Discussion

7.1 Introduction

From Table 6.4 (Section 6.1), the next step is to select one of the 10 portfolios created by the model. The only criteria, which can be correctly used to select one of the portfolios is the Sharpe ratio of the expected portfolio returns and the risk calculated by the Matlab MPT model. Thus the portfolio which will be selected is that which has the highest Sharpe ratio. This information will be known before the investor invests, because it will be the Sharpe ratio from which the model estimates or forecasts the return for the next time period (in this case one year ahead).

Table 7.1 illustrates this; the portfolio with the highest Sharpe Ratio, using the expected portfolio return for the year 2006 and the portfolio risk as calculated by the model, is portfolio number 3 with a Sharpe Ratio of 5.23. This portfolio will thus be selected. The annual realised return column shows the actual return which would have been achieved if the stocks selected by the model were actually invested in. This return is for the period between the 31 December 2005 and 31 December 2006. It is thus the return for that year if the data up to the end of 2005 was put into the model and then the selection made according to the stocks chosen by the model.

Table 7.1 also shows that just because a portfolio is expected to have the best Sharpe Ratio it doesn't necessarily achieve the best Ex-post Sharpe Ratio when using the actual realised return. In this case portfolio 3 only had the third best Ex-post Sharpe Ratio of 1.55 after portfolio 1 (3.44) and portfolio 2 (2.89). This is due to the fact that the actual realised return was not as high as predicted by the model. However the Sharpe Ratio achieved by portfolio 3 is still acceptable.

Table 7.1 15 Year Weekly data, rebalanced yearly up to year 2005, measuring the actual return of 2006

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.067	23.93%	29.47%	6.50%	2.61	3.44
2	0.015	0.009	0.106	60.47%	37.03%		5.11	2.89
3	0.027	0.014	0.193	107.53%	36.38%		5.23	1.55
4	0.044	0.019	0.316	168.04%	20.36%		5.11	0.44
5	0.071	0.024	0.509	245.75%	0.18%		4.70	-0.12

In the analysis and discussion below, only the portfolio with the highest Sharpe Ratio from each set of data will be shown and summarised. This is to reduce the amount of data shown and because an investor will have to choose only one of the portfolios to invest in.

The following are the different test cases created by the Matlab MPT model, where weekly return data was used:

Data Names (data sets):

- Weekly 1 (15 year data)
- Weekly 2 (10 year data)
- Weekly 3 (5 year data)
- Weekly Top 40 1 (15 year data)
- Weekly Top 40 2 (10 year data)
- Weekly Top 40 3 (5 year data)

Each of these data sets have different portfolios for the following upper limit stock holding percentage constraints, (the maximum proportion that the model could select of a stock):

- No constraints
- 8%
- 10%
- 15%
- 20%

The abovementioned tests were done by selecting data over the period of years as stated, until the end of 2012. The actual realised return for the end of 2013 was then calculated based on the MPT model selected stocks as at the end of 2012 to be chosen for 2013. This was done as an initial test before other tests which will be discussed later. Further tests were done where the portfolios were rebalanced annually instead of just looking at the returns over one year, namely 2013.

Rebalancing a portfolio is to run the model again at certain time periods (in this case every year over a certain number of years). New portfolios are then created every year through this rebalancing. The portfolios could change significantly, slightly or not at all, depending on how Matlab analyses each new years' worth of data. Rebalancing is important because as discussed in Section 3.22 it allows for the possible dissipation of returns to be limited over the following years (due to share price momentum). A potential investor would probably want to look at rebalancing their portfolios yearly so that their portfolios will stay optimal.

The rebalanced portfolios using weekly data created the following test results:

Data names (data sets):

- Rebalance Weekly (15 year data)
- Rebalance Weekly (10 year data)
- Rebalance Weekly (5 year data)
- Rebalance Weekly Top 40 (15 year data)

These portfolios were rebalanced yearly, using weekly return data for the number of years shown.

The upper limit constraints used for these sets of data were:

- 10%
- 15%

The results of the data sets discussed above will be shown in the sections to follow. These results will then be compared to other financial instruments as well as to the JSE.

Top 40 companies are used to compare to past research in which only the Top 40 companies are used. Creating portfolios which include other companies allows more exposure to smaller companies which could have potential for higher returns, and higher variance. From research done [14] it has been found that most South African researchers only took into account the Top 40 companies when creating an MPT model. Including only Top 40 companies could also be seen as a better way for more risk averse investors to gain exposure to equities of companies which are successful and have a longer track record.

7.2 Weekly Data with No Rebalancing

7.2.1 No rebalancing

Initial tests were done to see how the Matlab model operates, as well as to look at ways in which the volume of data could be reduced before the rebalancing tests were done. These tests were also done to look at whether, and by how much, returns would change depending on changing constraints such as the maximum upper limit (percentage allowable) of a single stock in a portfolio as well as the effect different ranges of data (number of years) could have.

7.2.2 Efficient frontier graphs

Only some of the efficient frontier graphs are shown to provide the reader with an overview of what the model does. The efficient frontier graphs shown are from the Weekly Top 40, 10 year data set for upper limit constraints of no constraint, 8%, 10%, 15% and 20% (Figures 7.1 to 7.5). Efficient frontier graphs for the rest of the tested data sets (no rebalancing) can be found in Appendix B, Figures B1 to B25.

Tables 7.2 to 7.6 show a selection of data as calculated by Matlab and Excel. Each numbered portfolio is that as created by the Matlab model, with the associated portfolio risk and expected returns. These portfolios risks and expected returns are weekly values. These are then converted to yearly portfolio risk and expected return in Excel. The realised annual return is the actual return which would have been realised if the portfolios calculated by the MPT model were actually invested into for a year. The calculated Sharpe and Ex-post Sharpe Ratios are also shown for each portfolio. The shaded blocks are those portfolios which have been selected based on the highest Sharpe ratio. The summaries of data for the other tested data sets (no rebalancing) can be found in Appendix A, Tables A1 to A25.

Figures 7.1 to 7.5 show the efficient frontiers created by the Matlab MPT model. Each point on the graph represents a single stock as well as the stock's expected return and risk (standard deviation). These returns and risks are a weekly percentage, due to weekly data being used. The solid curved line shown on the figures is that of the efficient frontier. The efficient frontier is the line where optimal portfolios are found. Which point on the line to select is dependent on the amount of risk an investor is willing to take. Therefore each point on the efficient frontier is mathematically optimal but the choice of how much risk to take and thus which portfolio on the frontier to choose lies with the investor.

Table 7.2 Summary of Matlab MPT model results and calculated realised returns and Sharpe Ratios for no upper limit constraints and Weekly Top 40 10 year data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.016	0.004	0.112	21.98%	14.41%	6.50%	1.38	0.70
2	0.016	0.004	0.113	24.58%	16.47%		1.60	0.88
3	0.016	0.005	0.116	27.23%	19.38%		1.79	1.11
4	0.017	0.005	0.119	29.93%	22.17%		1.96	1.31
5	0.017	0.005	0.125	32.69%	24.71%		2.10	1.46
6	0.018	0.006	0.131	35.50%	26.35%		2.21	1.51
7	0.019	0.006	0.139	38.38%	28.19%		2.29	1.56
8	0.021	0.007	0.151	41.31%	30.25%		2.31	1.58
9	0.024	0.007	0.171	44.31%	37.07%		2.22	1.79
10	0.048	0.007	0.344	47.37%	-15.66%		1.19	-0.65

Figure 7.1 differs in comparison to the other Figures 7.2 to 7.5, in that the efficient frontier extends horizontally to a specific point, this point being the stock (company) Assore. When no constraints are used in the MPT model it can choose only one share, since it is not bound to choosing more than one. In this case the MPT model calculated that holding only a single share (Assore) would create a portfolio which could be found on the efficient frontier. In this case that portfolio is portfolio number 10 as shown in Table 7.2. The model predicted that this portfolio which contains 100% of the share Assore, would create an expected yearly return of 47.37%. However the portfolio and the share made a loss of 15.66%. This reveals one of the drawbacks of MPT, in that it assumes that returns are normally distributed. However, as described in Section 3.19, in the real world returns are not normally distributed and can have long tails because of shocks in the market. This is the main reason for why the stock (and in this case also the portfolio) of Assore had a return so different to what the model predicted. This is the reason why other upper limit constraints were tested in this research. The constraints are necessary to create optimal portfolios which at least hold a number of shares in other companies to limit this "tail" risk.

The actual data which shows the percentage holding of each share for each portfolio for each different constraint fills a large spreadsheet which cannot be shown in this report. It was demonstrated in Section 5 how Matlab creates the data and how it is further analysed. The spreadsheet will be available in the digital appendix ("MPTmodel.xlsx").

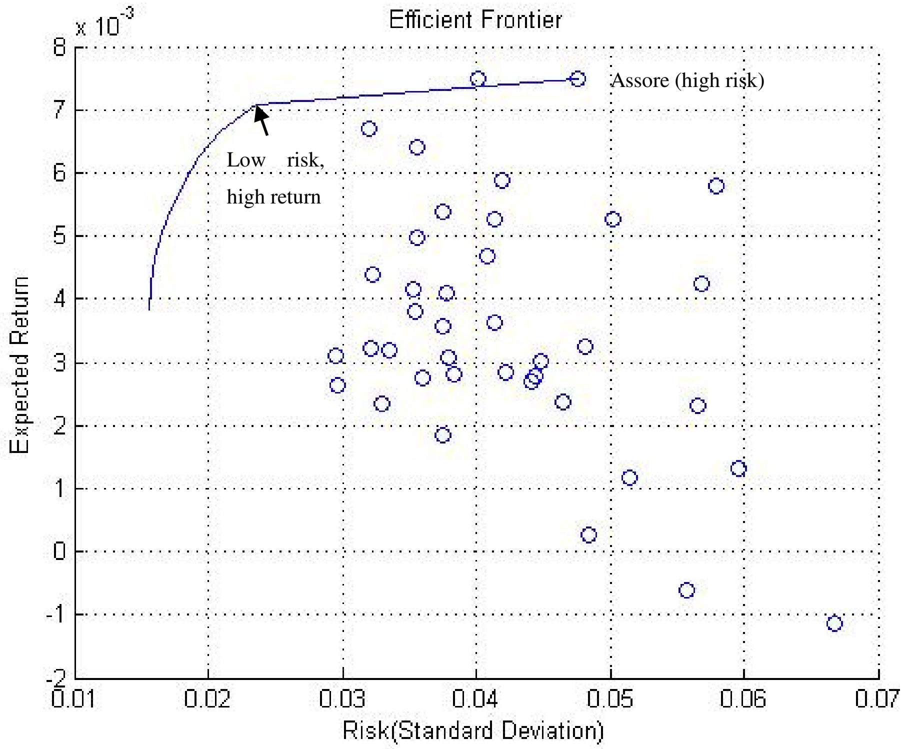

Figure 7.1 Efficient frontier graph for Weekly Top 40 (10 year) data with no upper limit constraint

Table 7.3 Summary of Matlab MPT model results and calculated realised returns and Sharpe Ratios for 8% upper limit constraints and Weekly Top 40 10 year data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.016	0.004	0.116	23.24%	13.02%	6.50%	1.44	0.56
2	0.016	0.004	0.116	24.46%	13.71%		1.54	0.62
3	0.016	0.004	0.117	25.70%	15.69%		1.64	0.78
4	0.016	0.005	0.119	26.95%	19.36%		1.72	1.08
5	0.017	0.005	0.121	28.21%	23.19%		1.80	1.38
6	0.017	0.005	0.124	29.48%	25.97%		1.86	1.57
7	0.018	0.005	0.128	30.77%	26.09%		1.90	1.53
8	0.019	0.005	0.134	32.07%	25.96%		1.91	1.46
9	0.020	0.006	0.141	33.38%	25.73%		1.91	1.36
10	0.022	0.006	0.160	34.70%	22.05%		1.77	0.97

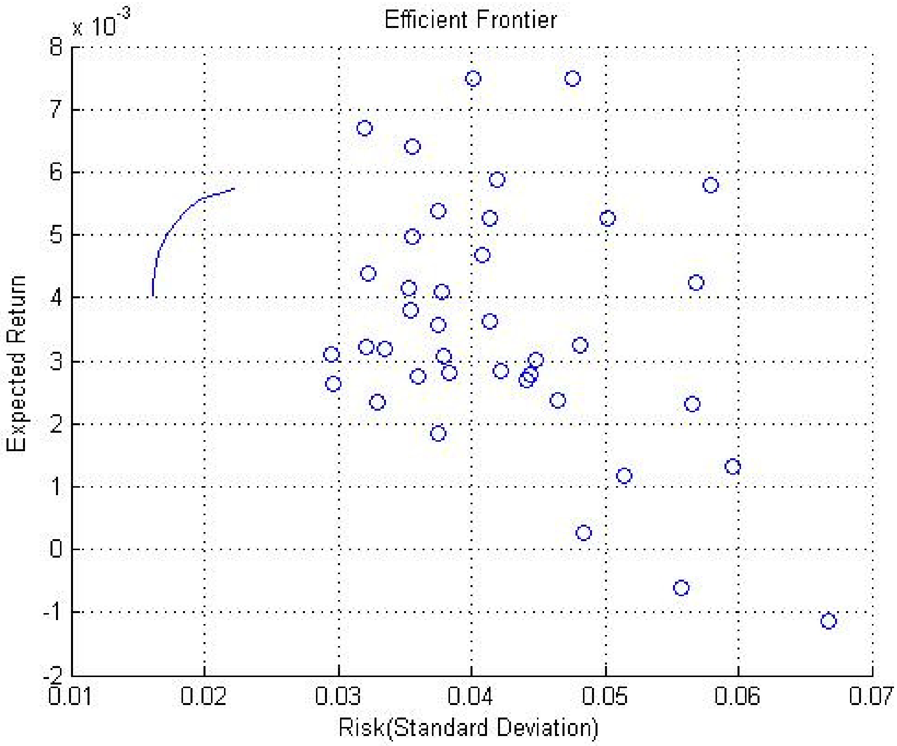

Figure 7.2 Efficient frontier graph for Weekly Top 40 (10 year) data with 8% upper limit constraint

Table 7.4 Summary of Matlab MPT model results and calculated realised returns and Sharpe Ratios for 10% upper limit constraints and Weekly Top 40 10 year data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.016	0.004	0.114	22.86%	13.74%	6.50%	1.44	0.64
2	0.016	0.004	0.114	24.35%	14.83%		1.57	0.73
3	0.016	0.004	0.115	25.86%	16.88%		1.68	0.90
4	0.016	0.005	0.117	27.38%	18.95%		1.78	1.06
5	0.017	0.005	0.120	28.93%	20.91%		1.87	1.20
6	0.017	0.005	0.123	30.49%	24.33%		1.95	1.45
7	0.018	0.005	0.127	32.07%	27.58%		2.01	1.66
8	0.018	0.006	0.133	33.68%	28.13%		2.05	1.63
9	0.019	0.006	0.140	35.30%	27.13%		2.06	1.47
10	0.024	0.006	0.170	36.94%	24.08%		1.79	1.04

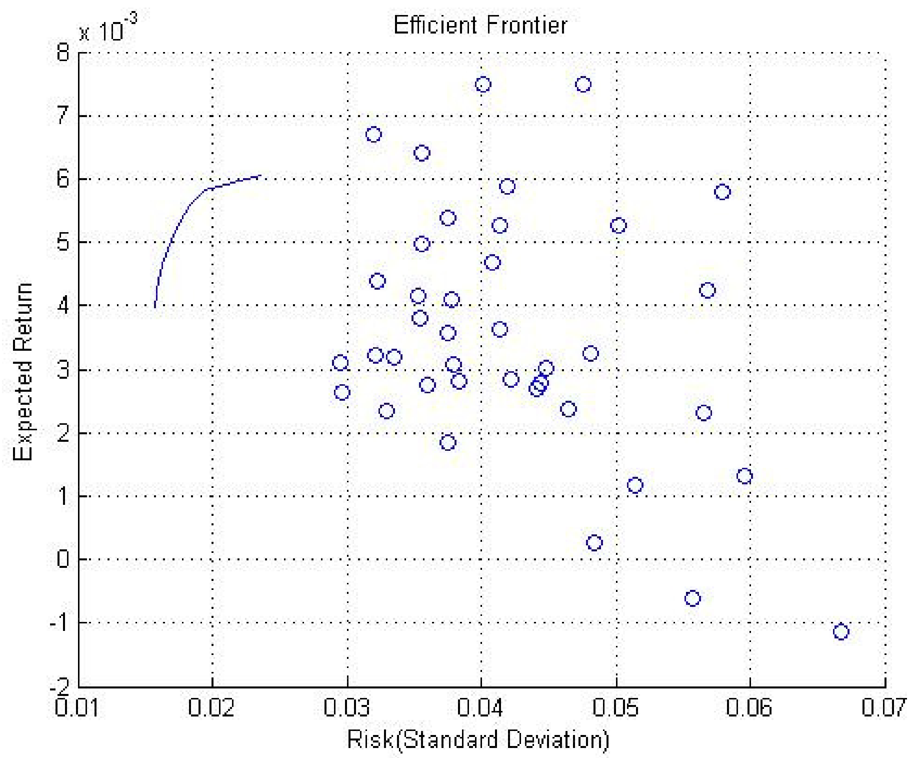

Figure 7.3 Efficient frontier graph for Weekly Top 40 (10 year) data with 10% upper limit constraint

Table 7.5 Summary of Matlab MPT model results and calculated realised returns and Sharpe Ratios for 15% upper limit constraints and Weekly Top 40 10 year data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.016	0.004	0.112	21.98%	14.41%	6.50%	1.38	0.70
2	0.016	0.004	0.113	23.87%	15.92%		1.54	0.83
3	0.016	0.004	0.114	25.79%	17.58%		1.69	0.97
4	0.016	0.005	0.116	27.74%	20.12%		1.83	1.17
5	0.017	0.005	0.119	29.72%	22.13%		1.94	1.31
6	0.017	0.005	0.124	31.72%	24.13%		2.04	1.43
7	0.018	0.006	0.128	33.76%	25.37%		2.12	1.47
8	0.019	0.006	0.134	35.83%	26.80%		2.18	1.51
9	0.020	0.006	0.141	37.93%	29.82%		2.22	1.65
10	0.023	0.007	0.168	40.06%	23.79%		1.99	1.03

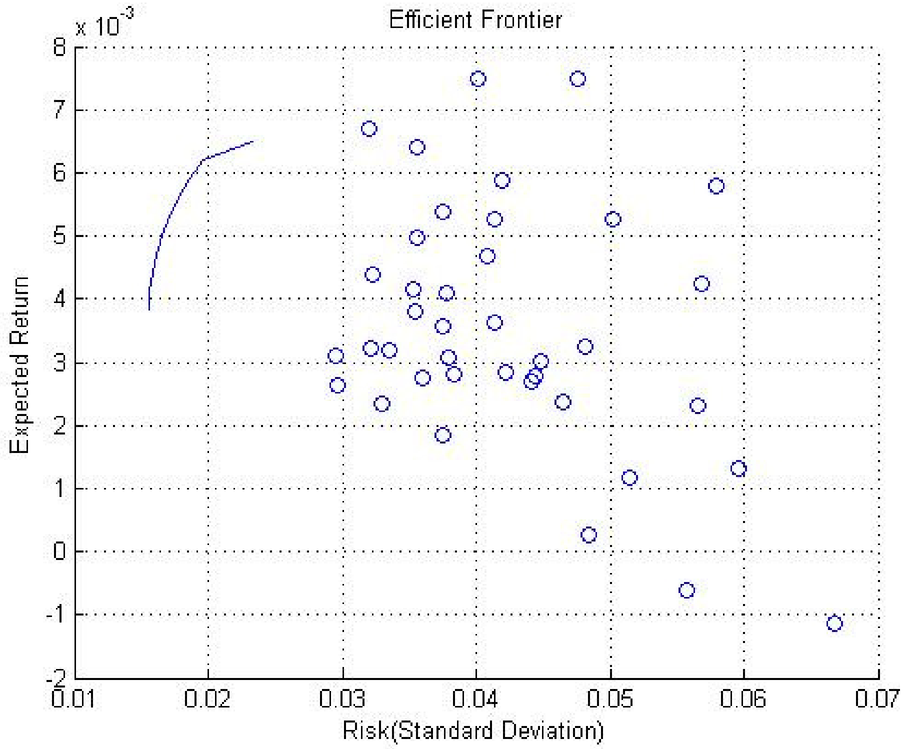

Figure 7.4 Efficient frontier graph for Weekly Top 40 (10 year) data with 15% upper limit constraint

Table 7.6 Summary of Matlab MPT model results and calculated realised returns and Sharpe Ratios for 20% upper limit constraints and Weekly Top 40 10 year data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.016	0.004	0.112	21.98%	14.41%	6.50%	1.38	0.70
2	0.016	0.004	0.113	24.08%	16.08%		1.56	0.85
3	0.016	0.004	0.115	26.21%	18.09%		1.72	1.01
4	0.016	0.005	0.117	28.38%	20.66%		1.87	1.21
5	0.017	0.005	0.120	30.58%	22.80%		2.00	1.35
6	0.017	0.005	0.125	32.83%	24.80%		2.11	1.46
7	0.018	0.006	0.131	35.11%	26.14%		2.19	1.50
8	0.019	0.006	0.137	37.42%	27.70%		2.25	1.54
9	0.020	0.006	0.146	39.78%	28.92%		2.27	1.53
10	0.023	0.007	0.164	42.18%	33.19%		2.18	1.63

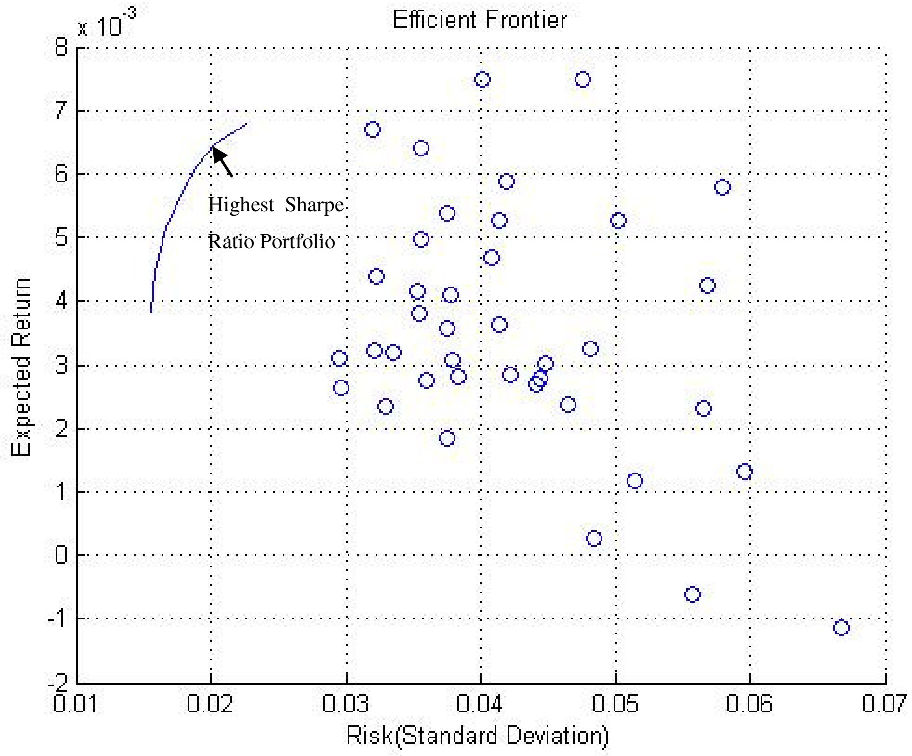

Figure 7.5 Efficient frontier graph for Weekly Top 40 (10 year) data with 20% upper limit constraint

7.2.3 Actual measured returns

Once the portfolios are created the actual measured returns need to be calculated to determine what return an investor could have achieved if the chosen portfolios were actually used.

Table 7.7 shows the summarised actual annual returns for a variety of constraints. These returns are for those portfolios which had the highest Sharpe Ratio.

The data in Table 7.7 is all created using weekly return data but for different constraints. The measured actual return shown in the table are all for the year ending 2013, using weekly return data up to the end of 2012.

The data from Table 7.7 can be easily compared in graph form (Figure 7.6).

Table 7.7 Summarised realised returns for 2013 using different constraints

Data Set	Upper Limit Constraint						
	None	8%	10%	15%	20%	Average Return	Average Sharpe Ratio
Weekly 1 (15 year data)	15.99%	14.88%	15.80%	15.03%	16.60%	15.66%	4.25
Weekly 2 (10 year data)	37.23%	37.57%	33.85%	36.93%	37.80%	36.67%	7.07
Weekly 3 (5 year data)	27.78%	25.94%	29.15%	26.90%	23.73%	26.70%	7.65
Weekly Top 40 (15 year data)	19.71%	17.70%	16.44%	19.25%	19.00%	18.42%	1.91
Weekly Top 40 (10 year data)	30.25%	25.96%	27.13%	29.82%	28.92%	28.42%	2.15
Weekly Top 40 (5 year data)	28.86%	28.09%	25.87%	41.54%	27.05%	30.28%	1.67

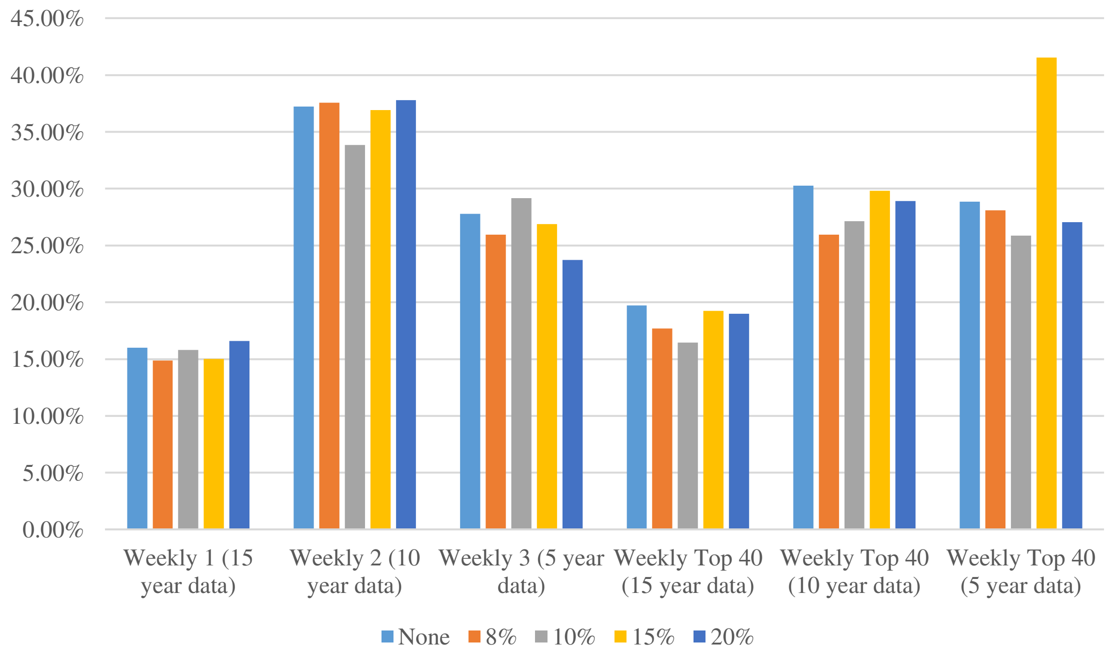

Figure 7.6 Comparison of 2013 actual returns with different upper limits and data ranges

Figure 7.6 shows that other than one outlier (Weekly Top 40, 5 year data) for the 15% upper limit, different upper limit constraints are on average very similar across the entire data set. There is no upper limit constraint which is obviously superior to the others for this set of data. This is an observation for one year's recorded return data (2013), and might not be true for larger sets of data across many different years.

For the following section where data was rebalanced yearly over a number of years, the number of upper limit constraints was reduced from five to two. This was done to reduce the amount of data to analyse and due to the fact that, as shown in Figure 7.6, there is little difference in realised return across the different data sets for the different upper limit constraints. The number of portfolios chosen to be created by the Matlab MPT model was also reduced from 10 to 5 portfolios.

The outlier in Figure 7.6 (Weekly Top 40, 5 Year data, 15% Upper Limit) could be due to a number of reasons. The years under consideration are from 2009-2013. This coincided with the global financial crisis of 2008 when share prices all over the world decreased significantly. In the years since the share prices have continued to recover. The high return for this specific portfolio could be due to the model, at a 15% upper limit, selecting some companies whose performance greatly outperformed those of other companies. This could create a spike in returns if certain companies were selected instead of others. This portfolio ended up being the highest risk portfolio created by the model. This in turn created the portfolio with the highest returns.

The major contributors to this performance of a total return of 41.5% were Aspen contributing 9%, Mondi – 10%, Mediclinic International – 6% and Naspers – 15%. Each of the other chosen portfolios from the other upper limit constraints contained lower percentages of these high return companies. The value of a 15% upper limit allowed higher percentages of these high performing companies.

7.3 Weekly data with yearly rebalancing

7.3.1 Yearly rebalancing

This section looks at the annual returns which could have been achieved if the portfolios were rebalanced once a year over a certain number of years.

Upper limit constraints of 10% and 15% were chosen for these data sets. The tables contain the returns for each year over a number of years, as well as the Ex-post Sharpe Ratios for the portfolios with the best Sharpe Ratios of expected returns. The tables also show the calculated, annualised and total returns so that these can be compared to published data of various indices and other unit trusts.

Table 7.8 shows the number of companies which can be found in each data set. Certain companies which are less than 5 years old were not considered in the MPT model. This was due to the constraint being set that a minimum of 5 years of data be used. The longer the amount of years back considered, the less companies there will be in each set due to new companies being formed and thus not having been around for long.

Table 7.8 Number of companies considered in each data set

Data Set	Number of companies in set
Weekly 1 (15 year data)	140
Weekly 2 (10 year data)	183
Weekly 3 (5 year data)	236
Weekly Top 40 (15 year data)	40

In all of the following data tables, an Ex-post Sharpe Ratio which is positive, is what an investor wants. The more positive the ratio the higher the risk adjusted return over the chosen risk free rate of 6.5%. It is not ideal generating large returns with an equally large amount of risk. The Sharpe Ratio thus shows the excess risk adjusted return which the investor will achieve.

7.3.2 15 year weekly data

Table 7.9 shows the yearly rebalanced results for 15 year weekly data. Only companies who have at least 15 years' worth of data (1998-2012) were considered in the model. Thus 140 companies' worth of return data was used (Table 7.8).

Table 7.9 shows the yearly returns which would have been achieved if the portfolio with the highest Sharpe Ratio was chosen for each year. Also shown in the table are the annualised and total returns for 3, 5 and 8 years. The upper limit constraint of 15% performs slightly worse than the 10% constraints over the annualised and total return periods. The negative Sharpe Ratios show when the return for the year was less than the chosen risk free rate of 6.5%. The effects of the recession of 2008 on actual returns can be clearly seen by the approximately 30% loss, and large negative Sharpe ratios.

The summaries of data for the other tested data sets (rebalancing) can be found in Appendix A, Tables A26 to A81.

Table 7.9 Returns using yearly rebalancing and 15 year weekly data

Rebalance Weekly (15 year data) Returns per year				
Year	Upper Limit Constraint		Ex-post Sharpe Ratio	
	10%	15%	10%	15%
2006	36.38%	37.30%	1.55	2.54
2007	83.11%	69.13%	4.11	5.25
2008	-33.26%	-30.48%	-2.07	-3.02
2009	18.61%	18.08%	0.59	0.45
2010	52.07%	61.03%	2.24	2.11
2011	3.10%	-0.02%	-0.11	-0.16
2012	26.39%	18.82%	1.02	0.50
2013	17.07%	17.67%	0.57	0.48
8 year annualised return	20.97%	19.97%		
5 year annualised return	22.44%	21.59%		
3 year annualised return	15.12%	11.81%		
8 year total return	358.61%	329.11%		
5 year total return	175.17%	165.80%		
3 year total return	52.55%	39.79%		

7.3.3 10 year weekly data

As shown in Table 7.10 the 10% upper limit constraint performs considerably better than the 15% limit on a 5 year annualised and total return basis. The effect of a small increase in annualised return as shown for the 8 year annualised return from 19.18% for a 15% upper limit and 22.86% for a 10% upper limit can be considerable when compounding. This 3.68% difference on an annualised basis can have a huge difference on the 8 year total return. The difference of the total return over 8 years is 112.25%. This shows the effect of compounding and how much even a slight difference in the annualised return can have a large impact on the total return over a period of time.

Table 7.10 Returns using yearly rebalancing and 10 year weekly data

Rebalance Weekly (10 year data) Returns per year				
Year	Upper Limit Constraint		Ex-post Sharpe Ratio	
	10%	15%	10%	15%
2006	42.84%	42.98%	2.87	2.11
2007	76.08%	90.86%	6.05	5.58
2008	-39.09%	-40.52%	-4.19	-3.31
2009	20.58%	-14.96%	1.16	-0.28
2010	40.18%	46.56%	2.92	2.82
2011	19.16%	21.57%	1.16	1.14
2012	23.27%	20.09%	1.60	1.10
2013	36.50%	37.81%	2.97	2.64
8 year annualised return	22.86%	19.18%		
5 year annualised return	27.65%	20.19%		
3 year annualised return	26.10%	26.24%		
8 year total return	419.28%	307.03%		
5 year total return	238.93%	150.77%		
3 year total return	100.51%	101.19%		

7.3.4 5 year weekly data

This section shows the results using 5 year weekly data. The amount of companies considered are considerably larger than the two previous sets of data due to the shorter time period of data being used.

Efficient frontier graphs are shown for 5 year weekly data and thus consist of 236 companies (circles) compared to the 40 companies (circles) shown in the efficient frontier graphs in Section 7.2.2.

7.3.4.1 Efficient frontier graphs

Figures 7.7 to 7.10 show the efficient frontier graphs for weekly 5 year data with a 15% upper limit. These figures are each for a separate year which is rebalanced yearly. Figure 7.7 shows the efficient frontier of the data used up to the end of 2009. Each subsequent figure shows the efficient frontier graph for all the data up until the next year. Efficient frontier graphs for the rest of the tested data sets (rebalancing) can be found in Appendix B, Figures B21 to B76.

As can be seen in Figures 7.7 to 7.10, each year added to the data changes the position of each company on the graph. This is because, with each new year of weekly data added to the previous sets of data, the overall risk and return of each company will change. The efficient frontiers are similar for all of the figures.

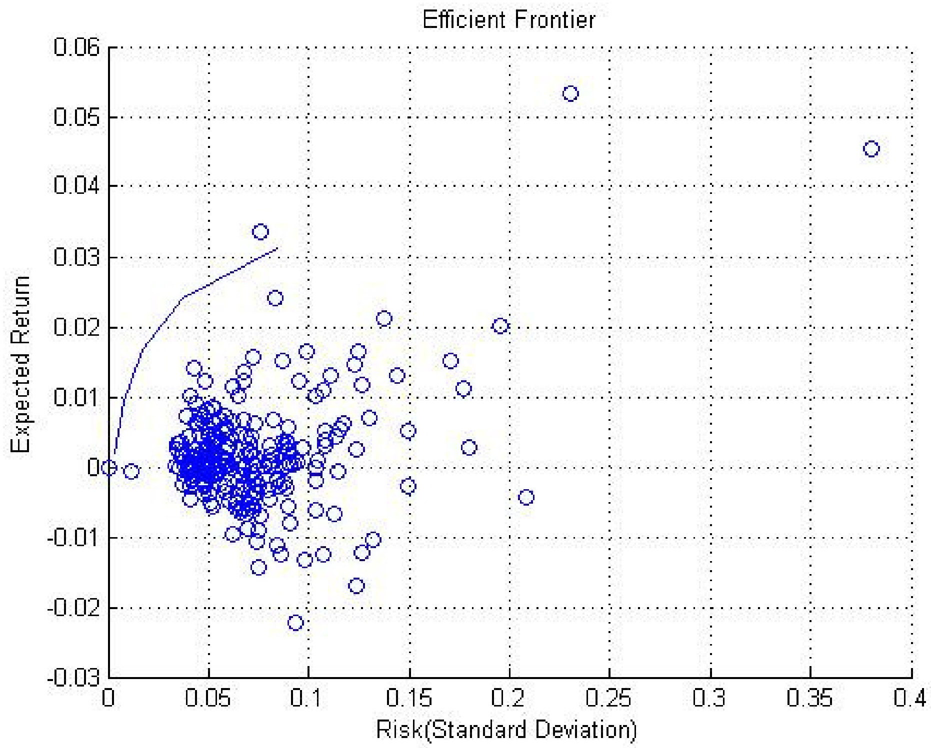

Figure 7.7 Efficient frontier graph for weekly (5 year) data with 15% upper limit constraint with data up till end 2009

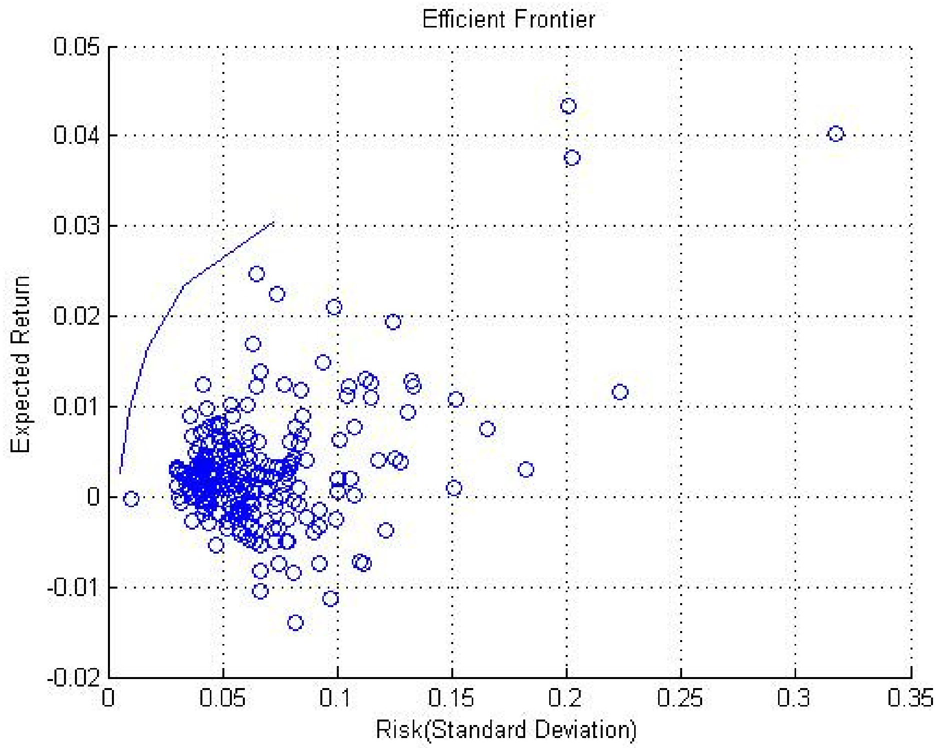

Figure 7.8 Efficient frontier graph for weekly (5 year) data with 15% upper limit constraint with data up till end 2010

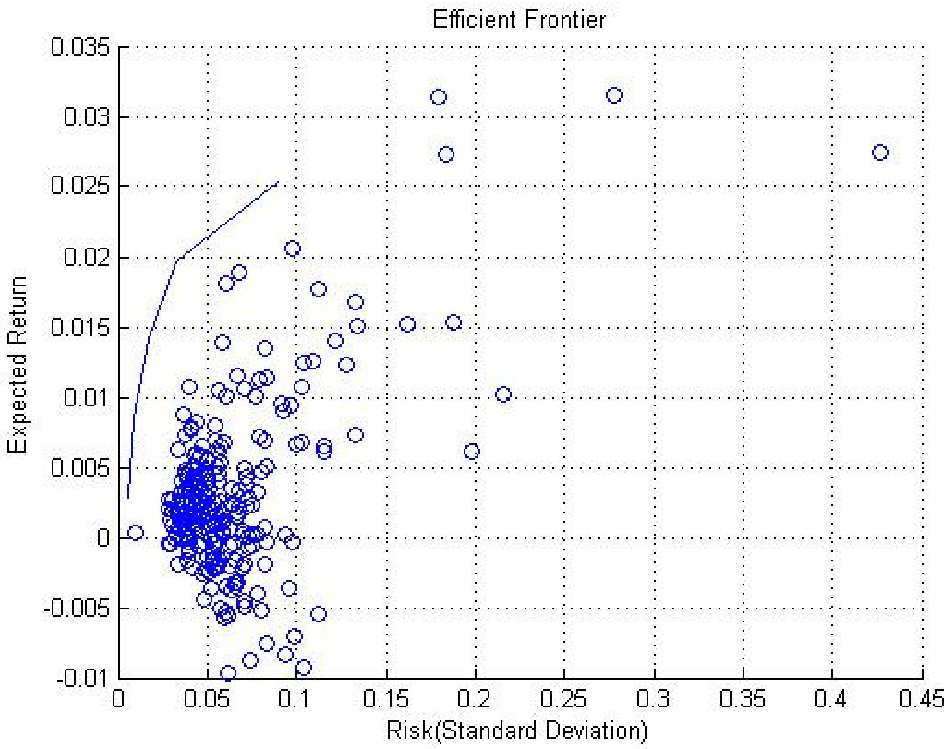

Figure 7.9 Efficient frontier graph for weekly (5 year) data with 15% upper limit constraint with data up till end 2011

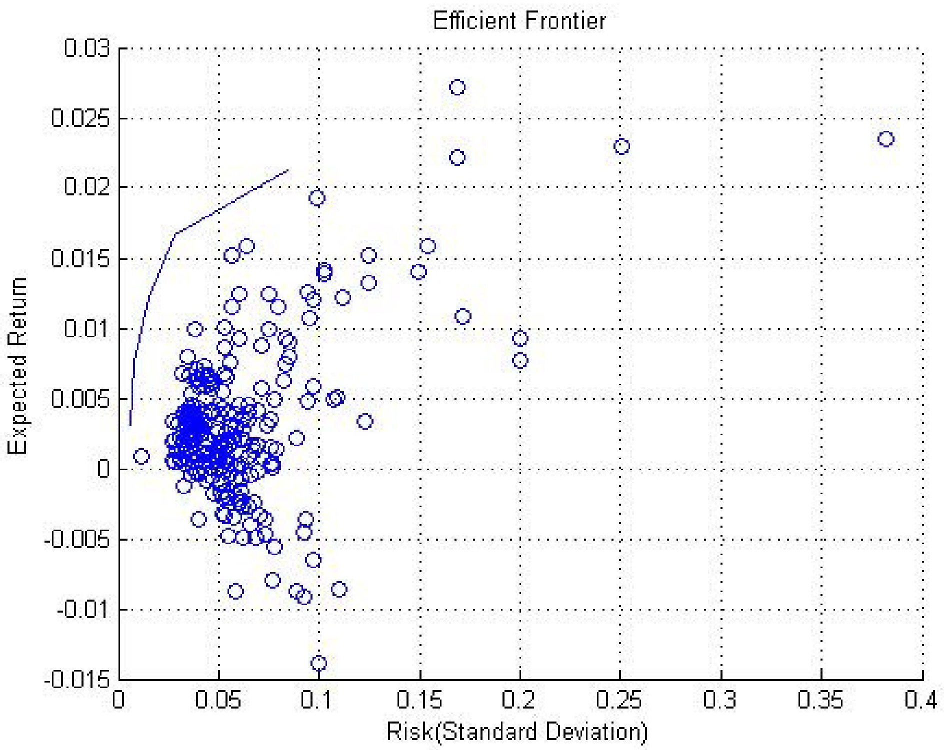

Figure 7.10 Efficient frontier graph for weekly (5 year) data with 15% upper limit constraint with data up till end 2012

Table 7.11 shows a selection of companies whose return data was analysed in the MPT model. The percentage company selection is the percentage of each company which the MPT model selected for portfolio number 3, which had the highest Sharpe Ratio. Each year which is rebalanced is shown as well as the percentage amount in each share which should be invested in. The next large column shows the actual percentage return per company. This is the percentage of each share multiplied by the return percentage of the year under consideration. When these are all added up the total portfolio return which could have been achieved if the investment was made is shown at the bottom of the table. The second last row of the table shows a summation of the percentage selections, and should add up to 100%. The last row of the table shows the number of companies which will be selected in each portfolio for each year. These range between 30-43 companies. This compares well with the theory discussed in Section 3.11 (i.e. that at least 22 companies are required in a portfolio to diversify away excess risk).

The table also shows that from one year to the next, no drastic changes need to be made with regards to the holdings in each portfolio. The only differences are for the percentage holding of each company. Rebalancing every year thus doesn't have much influence on changing the actual holdings in each portfolio, but does involve changing the amount held in each share. This was seen to be true across all of the different sets of data analysed in this report.

Table 7.11 Percentage company selection and actual returns for portfolio 3 using weekly 5 year data with 15% upper limit

Company	Percentage company selection				Actual percentage return per company			
	2009	2010	2011	2012	2009	2010	2011	2012
Adaptit	0	0	0.018	0.017	0	0	0.008	0.047
Adrenna	0.035	0.029	0.032	0.031	-0.008	-0.002	-0.009	0.005
African Media	0	0.041	0.048	0.032	0	0.014	0.006	0.017
Aspen pharmacare	0.003	0	0	0	0.001	0	0	0
Bauba plat	0.026	0.029	0.030	0.030	-0.005	-0.002	-0.010	-0.017
Bowler Metcalf	0.041	0.010	0	0	0.009	0.001	0	0
Business Connexion	0.023	0	0	0	0.006	0	0	0
Cafca	0.013	0.002	0.008	0.004	0	0	0	0.003
Calgro m3	0	0	0.022	0.031	0	0	0.017	0.004
Clicks	0.139	0.038	0.027	0.032	0.084	0.003	0.011	-0.001
Conduit capital	0.025	0.017	0.023	0.029	-0.001	0.005	0.016	0.009
Consolidated infrastructure	0	0.015	0.023	0.029	0	0.002	0.015	0.011
Control Instruments	0.007	0.016	0.016	0.026	0.001	-0.006	0.010	0.016
Convergenet	0.058	0	0	0.001	-0.035	0	0	-0.001
Crookes brothers	0.091	0.050	0.037	0.006	-0.011	-0.004	0.018	0.001
ELB	0	0	0	0.003	0	0	0	0.001
Ellies	0.036	0.011	0	0.039	0.010	0.000	0	-0.012
EOH	0	0	0	0.007	0	0	0	0.008
Fairvest prop	0.005	0.020	0.009	0.007	0.001	0.004	0.001	0.001
Goliath Gold	0.023	0.062	0.058	0.048	0.016	0.016	-0.007	-0.019
Howden Africa	0	0	0	0.006	0	0	0	0.003
Hwange colliery	0.003	0	0	0	0.001	0	0	0
Kap industrial	0.045	0.051	0.059	0.042	0.004	0.011	0.010	-0.001
Kaydav	0	0	0.008	0.007	0	0	0.012	0.001
Litha healthcare	0	0.117	0.099	0.070	0	0.005	0.024	-0.018
Metair investments	0.030	0.046	0.054	0.060	0.043	0.030	0.037	0.015
Micromega	0.018	0.008	0.024	0.025	-0.005	0.015	-0.002	0.049
Miranda mineral	0	0	0.004	0.024	0	0	-0.001	-0.004
Mix telematics	0.009	0.007	0.004	0.015	0.002	0.002	0.003	0.011
Oando	0.003	0	0	0	-0.001	0	0	0
Oceana	0	0.021	0.001	0.045	0	0.005	0.000	0.008
Orion Real Estate	0	0	0.010	0.019	0	0	0.003	-0.001
Pinnacle tech	0	0.012	0.019	0.030	0	0.008	0.014	0.008
Purple capital	0	0	0.016	0.015	0	0	-0.004	0.002
Putprop	0.032	0	0	0	0.004	0	0	0
Randgold and exploration	0	0.007	0	0	0	-0.003	0	0
Rockwell diamonds	0.009	0.012	0.008	0.004	-0.003	-0.002	-0.004	0.003
Rolfes	0.005	0.005	0.015	0.008	0.006	0.005	0.005	0.003
Sabvest	0	0	0.005	0.023	0	0	0.005	0.006
Sacoil	0	0.032	0.025	0.022	0	-0.021	-0.010	-0.002
Santova	0	0	0.005	0.004	0	0	0.002	0.001
Securedata	0.059	0.053	0.038	0.024	0.017	-0.023	-0.006	0.011
Sekunjalo investments	0	0.004	0.012	0.012	0	0.003	-0.004	0.002
Spanjaard	0.045	0.044	0.032	0.014	-0.001	0.012	0.005	-0.002
Taste	0	0	0.022	0.014	0	0	0.032	-0.002
Tradehold	0.020	0.016	0.011	0.007	-0.002	-0.003	0.004	0.005
Trematon capital	0	0.008	0.042	0.020	0.000	0.004	0.003	0.025
Value group	0.150	0.146	0.100	0.096	0.021	0.011	0.044	-0.001
Verimark	0	0.005	0	0	0	0.000	0	0
Village main reef	0.008	0.011	0.013	0.010	0.010	-0.001	-0.003	-0.007
Wescoal	0.003	0.036	0.025	0.013	0.002	-0.012	-0.003	0.031
Witwatersrand consolidated gold	0.037	0.020	0	0	-0.010	-0.006	0	0
Check/return	100%	100%	100%	100%	15.63%	7.08%	24.06%	22.07%
Number of Companies	30	34	38	43				

7.3.4.2 Actual measured returns

Table 7.12 shows the actual returns which could have been achieved for 2010-2013 for 10% and 15% upper limit constraints. Again the 10% upper limit portfolio outperforms the 15% upper limit portfolio.

Table 7.12 Annual returns using yearly rebalancing and 5 year weekly data

Rebalance Weekly (5 year data) Returns per year				
Year	Upper Limit Constraint		Ex-post Sharpe Ratio	
	10%	15%	10%	15%
2010	16.41%	15.63%	1.05	0.76
2011	8.01%	7.08%	0.15	0.05
2012	26.33%	24.06%	1.94	1.47
2013	26.83%	22.07%	2.18	1.46
3 year annualised return	20.06%	17.48%		
3 year total return	73.07%	62.15%		

7.3.5 15 year weekly data (Top 40)

The simplifying assumption was made to use the Top 40 companies as of 31 December 2013. Even though this is not ideal, the bulk of the companies contained within the Top 40 would have a presence within the Top 40 for a long period of time. Also, due to the 15 year period used in this test, some of the companies which are in the Top 40 as of the end 2013 are not included in the model calculations due to them not having enough data. Therefore the Top 40 in this calculation actually consists of 40 companies from the Top 65 as of end 2013.

Table 7.13 shows the actual returns which could have been achieved for 2006-2013 for 10% and 15% upper limit constraints. For the Top 40 data, unlike for the previous data for all the companies, the 15% upper limit outperforms the 10% upper limit. The outperformance is not large but can be significant over a long period of time.

Table 7.13 Top 40 annual returns using yearly rebalancing and 15 year weekly data

Rebalance Weekly Top 40 (15 year data)				
Year	Upper Limit Constraint		Ex-post Sharpe Ratio	
	10%	15%	10%	15%
2006	41.78%	35.62%	1.92	1.42
2007	45.54%	54.89%	2.13	2.42
2008	-18.26%	-17.01%	-1.40	-1.23
2009	43.65%	47.41%	1.99	2.11
2010	22.55%	21.93%	0.87	0.82
2011	1.49%	2.02%	-0.28	-0.24
2012	49.06%	55.73%	2.42	2.73
2013	18.53%	20.72%	0.71	0.82
8 year annualised return	23.25%	24.37%		
5 year annualised return	25.85%	28.08%		
3 year annualised return	21.49%	24.24%		
8 year total return	432.42%	472.52%		
5 year total return	215.67%	244.72%		
3 year total return	79.31%	91.79%		

7.3.6 Rebalanced data comparison

Table 7.14 shows a comparison between using larger sets of data, with regards to the number of years into the past the model considers. There is a significant difference in the returns between 15 year, 10 year and 5 year data sets. This can more easily be seen in Figures 7.11 and 7.12. The 10 year data set considerably outperforms the 15 year and 5 year data sets. The 5 year data set also outperforms the 15 year data set. The reason for this could be that with having more companies in the 10 year and 5 year data sets than the 15 year one, that newer companies have been outperforming older ones. However when considering Table 7.15 and Figures 7.13 and 7.14, the 10 and 15 year data are much more even when it comes to returns over periods longer than 5 years. It is therefore inconclusive that newer companies have performed better than older companies in the portfolios. For the 3 year annualised and total return data 5, and 10 year data again outperform 15 year data, with 10 year data also outperforming 5 year data. Effects such as the global financial crisis of 2008, and the subsequent recovery in share prices could be a reason why the 10 year data set outperforms the 5 and 15 year data sets.

Table 7.14 Rebalanced weekly annual returns for 10% upper limit on portfolio holdings

Return type and period	15 Year data	10 Year Data	5 Year Data
8 year annualised return	20.97%	22.86%	
5 year annualised return	22.44%	27.65%	
3 year annualised return	15.12%	26.10%	20.06%
8 year total return	358.61%	419.28%	
5 year total return	175.17%	238.93%	
3 year total return	52.55%	100.51%	73.07%

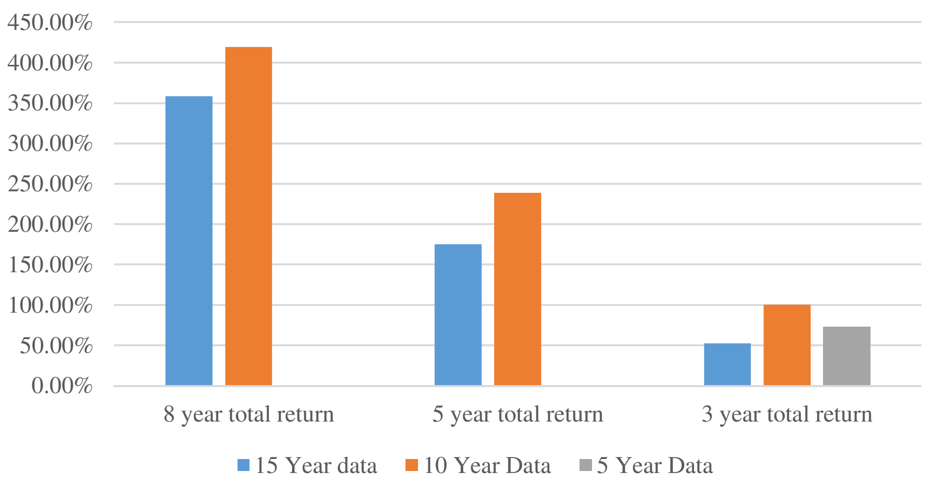

Figure 7.11 Total annual returns for MPT data set compared to different lengths of year data at a 10% UL

Figure 7.12 Annualised returns for MPT data set compared to different lengths of year data at a 10% UL

Table 7.15 Rebalanced weekly annual returns for 15% upper limit on portfolio holdings

Return type and period	15 Year data	10 Year Data	5 Year Data
8 year annualised return	19.97%	19.18%	
5 year annualised return	21.59%	20.19%	
3 year annualised return	11.81%	26.24%	17.48%
8 year total return	329.11%	307.03%	
5 year total return	165.80%	150.77%	
3 year total return	39.79%	101.19%	62.15%

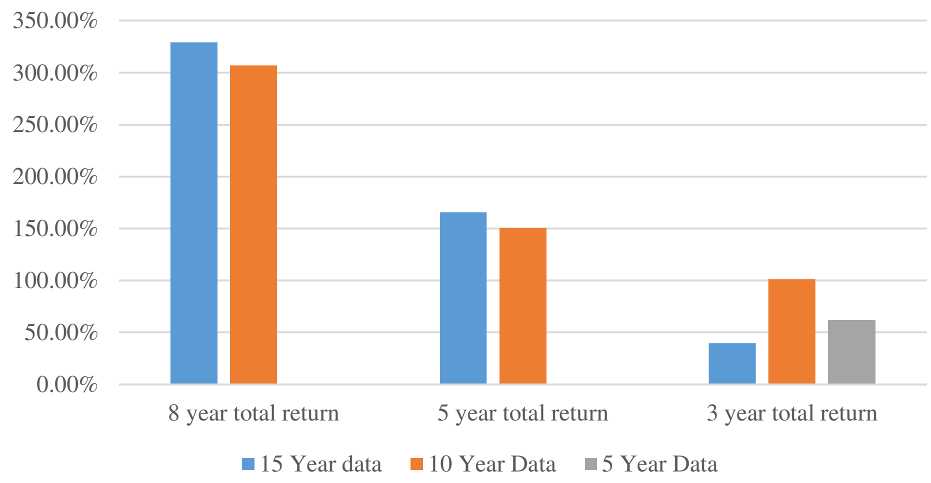

Figure 7.13 Total returns for MPT data set compared to different lengths of year data at a 15% UL

Figure 7.14 Annualised returns for MPT data set compared to different lengths of year data at a 15% UL

7.4 Comparison Data

This section contains graphs and tables of indices and unit trust performances which can be used to compare to the MPT results.

7.4.1 Introduction

As unit trusts are one of the main financial instruments which can be used to compare to the MPT results, a graph (Figure 7.15) below shows how many actively managed general equity unit trusts outperformed the JSE All Share Index [30].

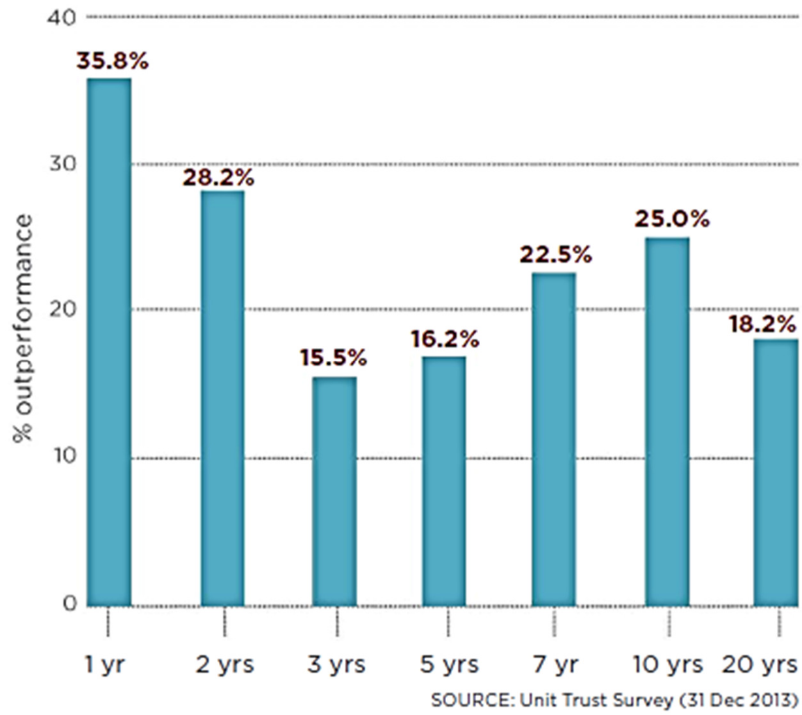

Figure 7.15 Percentage of actively managed general equity unit trusts that outperformed the FTSE/JSE All Share Index [30]

Figure 7.15 shows that only between 15% and 25% of the sampled unit trusts outperform the JSE ALSI over a medium to long period (5-20 years).

7.4.2 JSE Top 40 and ALSI

Tables 7.16 and 7.17 show the price and yearly returns of the JSE Top 40 and JSE ALSI for the last 4 years. Also shown are the three year annualised returns.

Table 7.16 JSE Top 40 return data [31]

JSE Top 40				
Date of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	24996.97			
2010-12-31	28639.4	14.57%		
2011-12-31	28469.81	-0.59%		
2012-12-31	34930.61	22.69%		
2013-12-31	41348.03	18.37%	44.37%	13.02%

Table 7.17 JSE ALSI return data [32]

JSE ALSI				
Date of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	27666.45			
2010-12-31	32118.89	16.09%		
2011-12-31	31985.67	-0.41%		
2012-12-31	39385.04	23.13%		
2013-12-31	46131.29	17.13%	43.63%	12.83%

7.4.3 Index trackers data

Table 7.18 shows the summarised one year return for various indices and index trackers. Further return data and 3 year annualised returns for these index trackers can be seen from Table 7.19 to 7.23.

Table 7.18 Summarised index and index trackers return data

Index/ Index Tracker	One Year Return
JSE Top 40	18.37%
JSE Alsi	17.13%
Satrix Top 40 Index Fund	16.44%
Satrix RAFI	13.84%
Satrix RESI	-4.83%
Satrix INDI	33.23%
Satrix FINI	13.94%

Table 7.19 Satrix Top 40 index fund return data [33]

Satrix Top 40 Index Fund				
Date of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	2500.75			
2010-12-31	2857.5	14.27%		
2011-12-31	2927	2.43%		
2012-12-31	3559	21.59%		
2013-12-31	4144	16.44%	45.02%	13.19%

Table 7.20 Satrix RAFI return data [33]

Satrix RAFI				
Data of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	631.25			
2010-12-31	734.25	16.32%		
2011-12-31	766.5	4.39%		
2012-12-31	939	22.50%		
2013-12-31	1069	13.84%	45.59%	13.34%

Table 7.21 Satrix RESI return data [33]

Satrix RESI				
Data of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	5124.5			
2010-12-31	5639.25	10.04%		
2011-12-31	5346	-5.20%		
2012-12-31	5358	0.22%		
2013-12-31	5099	-4.83%	-9.58%	-3.30%

Table 7.22 Satrix INDI return data [33]

Satrix INDI				
Data of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	2152.25			
2010-12-31	2673.75	24.23%		
2011-12-31	2917	9.10%		
2012-12-31	4090	40.21%		
2013-12-31	5449	33.23%	103.80%	26.78%

Table 7.23 Satrix FINI return data [33]

Satrix FINI				
Data of price	Price	Return	Total return	3 Year Annualised return
2009-12-31	736			
2010-12-31	825	12.09%		
2011-12-31	853.75	3.48%		
2012-12-31	1119	31.07%		
2013-12-31	1275	13.94%	54.55%	15.62%

7.4.4 Unit trust data

Tables 7.24 and 7.25 show a summary of the 12 month and 3 year total returns for the top performing and other selected unit trusts.

Table 7.24 South African unit trust top performers for 12 month return (part 1) [34]

Rank	Fund	Type	12 Month Return
1	PSG Equity Fund (A)	General	31.21%
2	PSG Equity Fund (B)	General	30.36%
3	Investec Value Fund (H)	General	28.09%
4	Investec Value Fund (A)	General	28.08%
5	Investec Value Fund (B)	General	28.08%
6	Investec Value Fund (R)	General	28.00%
7	Momentum Industrial Fund (A)	Industrial	27.00%
8	36ONE MET Equity Fund (A)	General	24.51%
9	Coronation Industrial Fund (A)	Industrial	24.42%
10	Old Mutual Investors' Fund (R)	General	24.30%
11	Old Mutual Investors' Fund (A)	General	23.66%
12	Foord Equity Fund (B2)	General	23.59%
13	Investec Active Quants Fund (R)	General	23.51%
14	SIM General Equity Fund (R)	General	23.27%
15	Old Mutual Industrial Fund (R)	Industrial	23.25%
16	Coronation Equity Fund (R)	General	23.14%
17	Foord Equity Fund (R)	General	23.07%
18	Old Mutual Industrial Fund (A)	Industrial	22.92%
19	Coronation Top 20 Fund (B4)	General	22.77%
20	Coronation Equity Fund (B2)	General	22.74%
21	Investec Active Quants Fund (B)	General	22.56%
22	SIM General Equity Fund (A)	General	22.50%
23	Coronation Equity Fund (A)	General	22.39%
24	Coronation Top 20 Fund (A)	General	22.20%
25	Prudential Equity Fund (B)	General	21.81%
26	PSG Multi-Management Equity Fund of Funds (B)	General	21.59%
27	Prudential Equity Fund (A)	General	21.32%
28	PSG Multi-Management Equity Fund of Funds (A)	General	21.32%
29	Investec Equity Fund (B)	General	21.28%
30	Investec Equity Fund (A)	General	21.28%
31	Investec Equity Fund (R)	General	21.12%
32	Prudential Dividend Maximiser Fund (B)	General	21.00%
33	PSG Wealth Creator Fund of Funds (D)	General	20.68%
34	Kagiso Equity Alpha Fund (A)	General	20.50%
35	Prudential Dividend Maximiser Fund (A)	General	20.44%
36	PSG Wealth Creator Fund of Funds (B)	General	20.09%
37	PSG Wealth Creator Fund of Funds (A)	General	19.91%
38	Investec Equity Fund (H)	General	19.45%
39	Stanlib ALSI 40 Fund (A)	Large cap	19.34%
40	Investec Commodity Fund (R)	Resources	19.15%
41	Momentum Top 40 Index Fund (A)	Large cap	19.00%
42	MET Value Fund (A)	General	18.82%
43	Momentum Best Blend Specialist Equity Fund (A)	General	18.71%
44	SIM Value Fund (A)	General	18.49%
45	Investec Commodity Fund (A)	Resources	18.49%

Table 7.24 South African unit trust top performers for 12 month return (part 2) [34]

Rank	Fund	Type	12 Month Return
46	Investec Commodity Fund (B)	Resources	18.49%
47	MiPlan IP Beta Equity Fund (B2)	General	17.80%
48	Kagiso Islamic Equity Fund (A)	General	17.68%
49	Coronation Financial Fund (A)	Financial	17.54%
50	Imara MET Equity Fund (A)	General	16.68%
51	Nedgroup Investments Rainmaker Fund (A)	General	16.63%
52	Coronation Smaller Companies Fund (R)	Mid and small cap	16.42%
53	Allan Gray Equity Fund (A)	General	16.16%
54	MET General Equity Fund (A)	General	16.03%
55	Old Mutual High Yield Opportunity Fund (A)	General	15.99%
56	Lynx SCI Opportunities Fund of Funds (B1)	General	15.91%
57	Oasis Crescent Equity Fund (A)	General	15.57%
58	Momentum Equity Fund (R)	General	15.25%
59	Momentum Equity Fund (A)	General	14.93%
60	Old Mutual RAFI 40 Tracker Fund (A)	Large cap	14.27%
61	Stanlib Industrial Fund (R)	Industrial	14.24%
62	SIM Small Cap Fund (R)	Mid and small cap	14.14%
63	Stanlib Industrial Fund (A)	Industrial	13.59%
64	SIM Small Cap Fund (A)	Mid and small cap	13.51%
65	Stanlib Value Fund (B1)	General	13.19%
66	Stanlib Value Fund (A)	General	12.55%
67	Investec Emerging Companies Fund (R)	Mid and small cap	12.23%
68	ABSA Select Equity Fund	General	12.17%
69	Old Mutual Small Companies Fund (R)	Mid and small cap	11.83%
70	Stanlib Resources Fund (R)	Resources	11.75%
71	Nedgroup Investments Value Fund (A)	General	11.72%
72	Investec Emerging Companies Fund (B)	Mid and small cap	11.62%
73	Investec Emerging Companies Fund (A)	Mid and small cap	11.62%
74	Marriott Dividend Growth Fund (R)	General	11.55%
75	Old Mutual Small Companies Fund (A)	Mid and small cap	11.23%
76	Prime General Equity Fund (B)	General	11.23%
77	Momentum Resources Fund (A)	Resources	11.01%
78	Stanlib Resources Fund (A)	Resources	11.00%
79	Coronation Resources Fund (A)	Resources	10.97%
80	Cadiz Mastermind Fund (A)	General	10.07%
81	Cannon MET Equity Fund (A)	General	8.23%
82	Old Mutual Mining & Resources Fund (R)	Resources	7.63%
83	Momentum Value Fund (A)	General	7.41%
84	Old Mutual Mining & Resources Fund (A)	Resources	7.31%
85	Momentum Small/Mid-Cap Fund (A)	Mid and small cap	7.05%
86	Investec Growth Fund (B)	General	5.46%
87	Investec Growth Fund (R)	General	5.34%
88	Stanlib Gold and Precious Metals Fund (R)	Resources	-1.26%
89	Stanlib Gold and Precious Metals Fund (A)	Resources	-1.84%
90	SIM Dividend + Index Fund (A1)	General	-4.74%
91	Old Mutual Gold Fund (R)	Resources	-6.12%
92	Old Mutual Gold Fund (A)	Resources	-6.63%

Table 7.25 South African unit trust top performers for 3 year return (part 1) [34]

Rank	Fund	Type	3 Year Return
1	Coronation Industrial Fund (A)	Industrial	112.93%
2	36ONE MET Equity Fund (A)	General	108.88%
3	Stanlib Industrial Fund (R)	Industrial	92.86%
4	Stanlib Industrial Fund (A)	Industrial	89.74%
5	Foord Equity Fund (R)	General	89.67%
6	Investec Emerging Companies Fund (R)	Mid and small cap	86.48%
7	Old Mutual Industrial Fund (R)	Industrial	85.48%
8	Momentum Industrial Fund (A)	Industrial	84.93%
9	Coronation Financial Fund (A)	Financial	84.14%
10	Old Mutual Industrial Fund (A)	Industrial	84.05%
11	Investec Emerging Companies Fund (A)	Mid and small cap	83.60%
12	Investec Emerging Companies Fund (B)	Mid and small cap	83.59%
13	PSG Equity Fund (A)	General	81.29%
14	Investec Active Quants Fund (R)	General	80.71%
15	PSG Equity Fund (B)	General	80.52%
16	Coronation Equity Fund (R)	General	78.67%
17	Coronation Equity Fund (B2)	General	78.12%
18	Coronation Equity Fund (A)	General	76.90%
19	Coronation Top 20 Fund (A)	General	76.32%
20	Imara MET Equity Fund (A)	General	76.04%
21	Investec Active Quants Fund (B)	General	75.77%
22	Prudential Equity Fund (B)	General	75.75%
23	Prudential Equity Fund (A)	General	73.21%
24	SIM General Equity Fund (R)	General	73.13%
25	Old Mutual Small Companies Fund (R)	Mid and small cap	71.93%
26	Prudential Dividend Maximiser Fund (B)	General	71.66%
27	SIM General Equity Fund (A)	General	71.45%
28	Prudential Dividend Maximiser Fund (A)	General	69.56%
29	Old Mutual Small Companies Fund (A)	Mid and small cap	69.24%
30	Investec Equity Fund (B)	General	68.00%
31	Investec Equity Fund (A)	General	68.00%
32	Investec Equity Fund (R)	General	67.95%
33	Old Mutual Investors' Fund (R)	General	67.06%
34	PSG Multi-Management Equity Fund of Funds (B)	General	67.01%
35	Old Mutual Investors' Fund (A)	General	66.69%
36	Allan Gray Equity Fund (A)	General	65.00%
37	PSG Wealth Creator Fund of Funds (D)	General	64.82%
38	PSG Multi-Management Equity Fund of Funds (A)	General	64.71%
39	Marriott Dividend Growth Fund (R)	General	64.15%
40	Momentum Small/Mid-Cap Fund (A)	Mid and small cap	63.59%
41	Momentum Best Blend Specialist Equity Fund (A)	General	63.32%
42	MiPlan IP Beta Equity Fund (B2)	General	62.66%
43	MET General Equity Fund (A)	General	62.66%

Table 7.25 South African unit trust top performers for 3 year return (part 2) [34]

Rank	Fund	Type	3 Year Return
44	Stanlib ALSI 40 Fund (A)	Large cap	62.05%
45	PSG Wealth Creator Fund of Funds (B)	General	61.00%
46	PSG Wealth Creator Fund of Funds (A)	General	60.03%
47	Momentum Top 40 Index Fund (A)	Large cap	59.99%
48	MET Value Fund (A)	General	59.98%
49	SIM Value Fund (A)	General	59.70%
50	Kagiso Equity Alpha Fund (A)	General	58.65%
51	Lynx SCI Opportunities Fund of Funds (B1)	General	56.71%
52	Momentum Equity Fund (R)	General	56.21%
53	Coronation Smaller Companies Fund (R)	Mid and small cap	56.12%
54	Momentum Equity Fund (A)	General	54.97%
55	Investec Growth Fund (B)	General	53.84%
56	Investec Growth Fund (R)	General	53.83%
57	Stanlib Value Fund (B1)	General	53.28%
58	ABSA Select Equity Fund	General	52.58%
59	Old Mutual RAFI 40 Tracker Fund (A)	Large cap	52.02%
60	Stanlib Value Fund (A)	General	50.83%
61	Nedgroup Investments Rainmaker Fund (A)	General	50.43%
62	Oasis Crescent Equity Fund (A)	General	50.22%
63	Nedgroup Investments Value Fund (A)	General	49.09%
64	Prime General Equity Fund (B)	General	48.84%
65	Cadiz Mastermind Fund (A)	General	45.39%
66	Kagiso Islamic Equity Fund (A)	General	44.84%
67	Old Mutual High Yield Opportunity Fund (A)	General	41.91%
68	SIM Small Cap Fund (R)	Mid and small cap	41.73%
69	SIM Small Cap Fund (A)	Mid and small cap	39.49%
70	Investec Value Fund (A)	General	34.83%
71	Investec Value Fund (B)	General	34.82%
72	Investec Value Fund (R)	General	33.87%
73	Momentum Value Fund (A)	General	28.12%
74	Cannon MET Equity Fund (A)	General	27.89%
75	Investec Commodity Fund (R)	Resources	26.06%
76	Investec Commodity Fund (B)	Resources	24.06%
77	Investec Commodity Fund (A)	Resources	24.05%
78	Stanlib Resources Fund (R)	Resources	10.86%
79	Momentum Resources Fund (A)	Resources	9.97%
80	Coronation Resources Fund (A)	Resources	9.82%
81	Stanlib Resources Fund (A)	Resources	8.94%
82	Old Mutual Mining & Resources Fund (R)	Resources	-3.94%
83	Old Mutual Mining & Resources Fund (A)	Resources	-4.76%
84	Stanlib Gold and Precious Metals Fund (R)	Resources	-5.11%
85	Stanlib Gold and Precious Metals Fund (A)	Resources	-6.83%
86	Old Mutual Gold Fund (R)	Resources	-17.62%
87	Old Mutual Gold Fund (A)	Resources	-18.08%

Figures 7.16 to 7.18 show the summarised data from Tables 7.14 to 7.23 in graph form. These bar graphs also show the various data set results from the MPT model calculations. Figure 7.16 shows that the MPT model returns outperform nearly all of the indices and index trackers. Only the JSE Top 40 and Satrix INDI fund outperform the worst of the MPT model returns, namely 15 year 10% upper limit and 15% upper limit. All the other MPT model returns outperform the JSE Top 40 and ALSI. Both 10 year MPT data sets outperform the best index tracker (Satrix INDI). Overall the MPT model returns over one year outperform all the JSE indices and most of the Satrix index trackers.

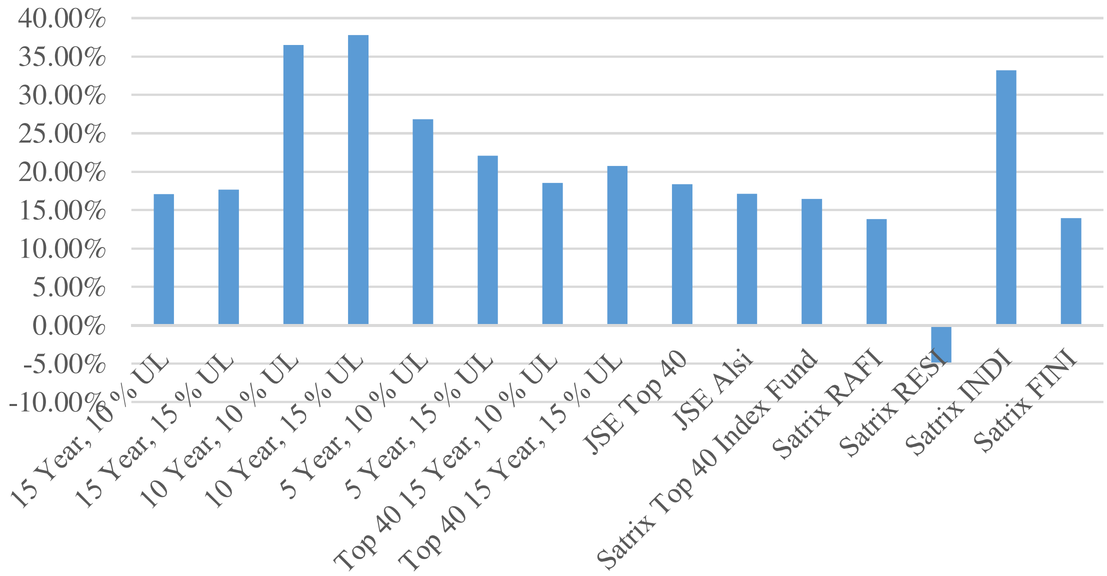

Figure 7.16 One year returns (2013) of MPT model data compared to various indices and index trackers

Figure 7.17 shows the three year total returns (2011-2013) of the MPT model results compared to various indices and index trackers. Again all of the MPT model returns outperform most of the indices and index trackers except for the 15 year data sets. In this case over a 3 year period the Satrix INDI tracker slightly outperforms the two best MPT model data sets (10 year data sets). Overall the MPT model returns outperform all the indices and index trackers except for the Satrix INDI tracker.

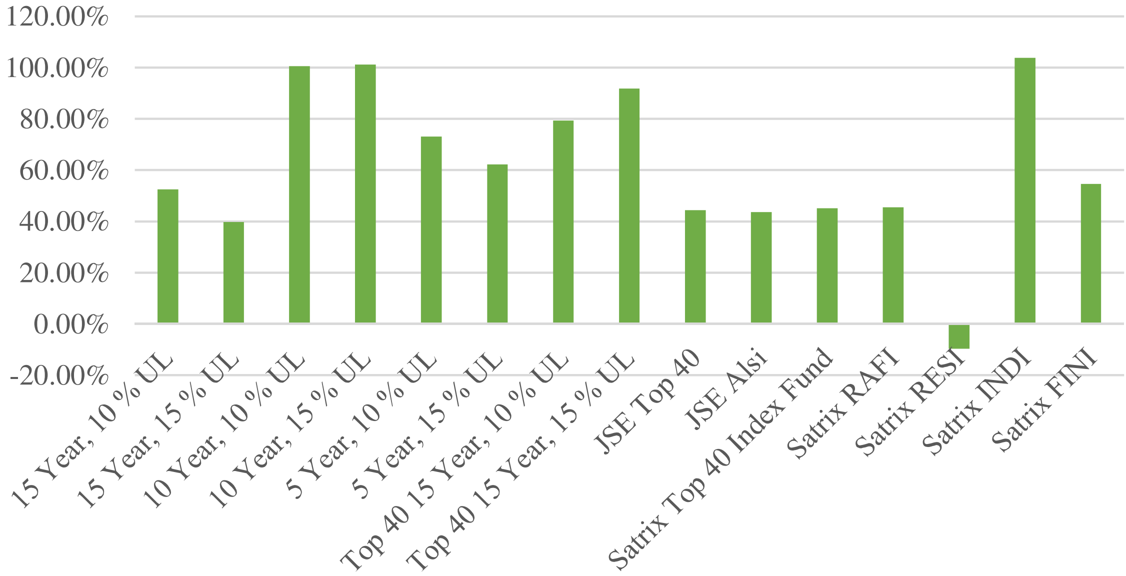

Figure 7.17 Three year total returns (2011-2013) of MPT model data compared to various indices and index trackers

Figure 7.18 shows the three year annualised returns of the MPT model data compared to various indices and index trackers. Using annualised data it is easier to see the scale of outperformance over the three year period. Once again a similar pattern is found in Figure 7.18 as in Figure 7.17. Most of the MPT returns outperform the JSE indices and all the index trackers, except for the Satrix INDI tracker, on an annualised basis. The Satrix INDI fund is again the top performer overall.

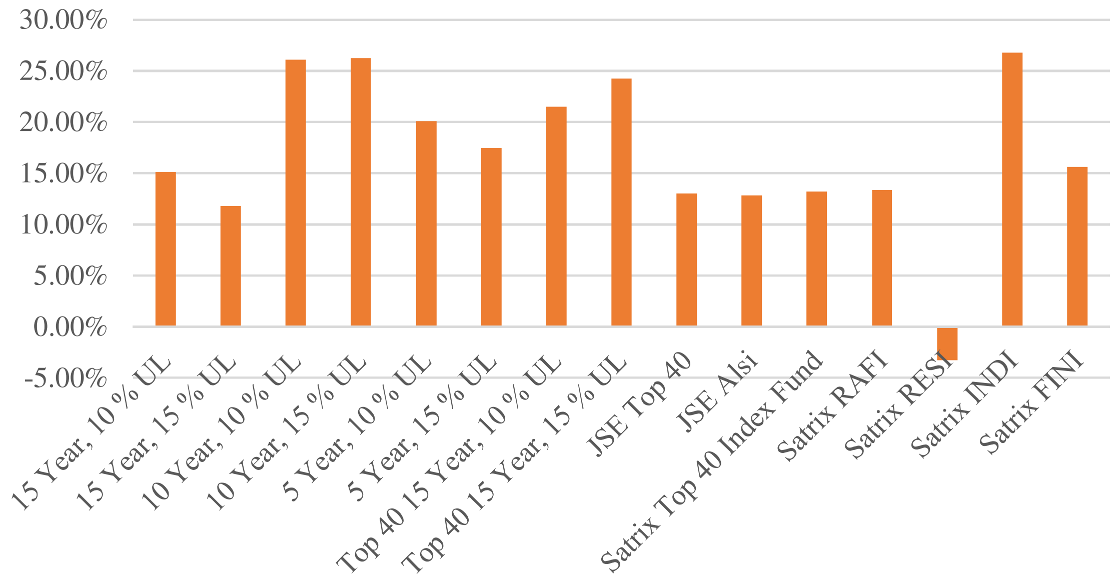

Figure 7.18 Three year annualised returns (2011-2013) of MPT model data compared to various indices and index trackers

Figures 7.19 and 7.20 show the summarised data from the top 10 companies in Tables 7.24 and 7.25, as well as the data sets from the MPT model calculations.

Figure 7.19 shows a comparison of the MPT model returns over one year to the top 10 performing unit trusts of the same year (2013). Over one year the two 10 year MPT data sets outperform all of the top 10 unit trusts. The 5 year 10% upper limit MPT data set would have been rated as the eighth best return at 26.83%.

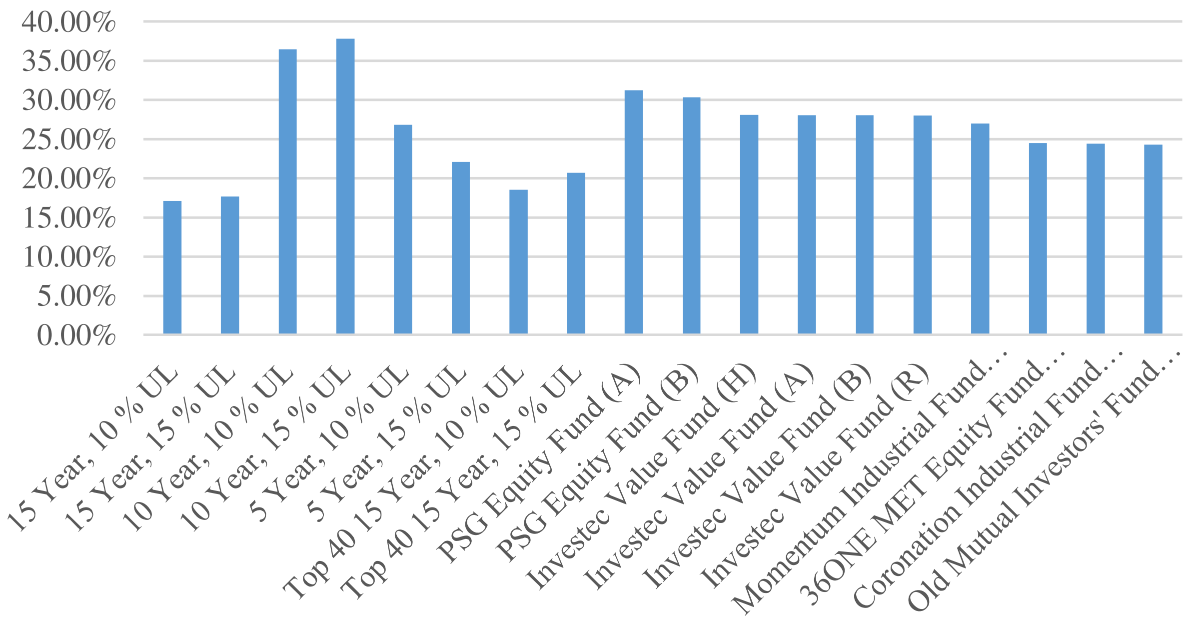

Figure 7.19 One year returns of MPT model data compared to the top 10 performing unit trusts

Figure 7.20 shows the three year total returns of the MPT model data compared to the top 10 performing unit trusts of the past three years. The MPT model data returns would have been ranked third (10 year 15% upper limit), fourth (10 year 10% upper limit) and sixth (Top 40 15 year, 15% upper limit), when compared to the top performing unit trusts.

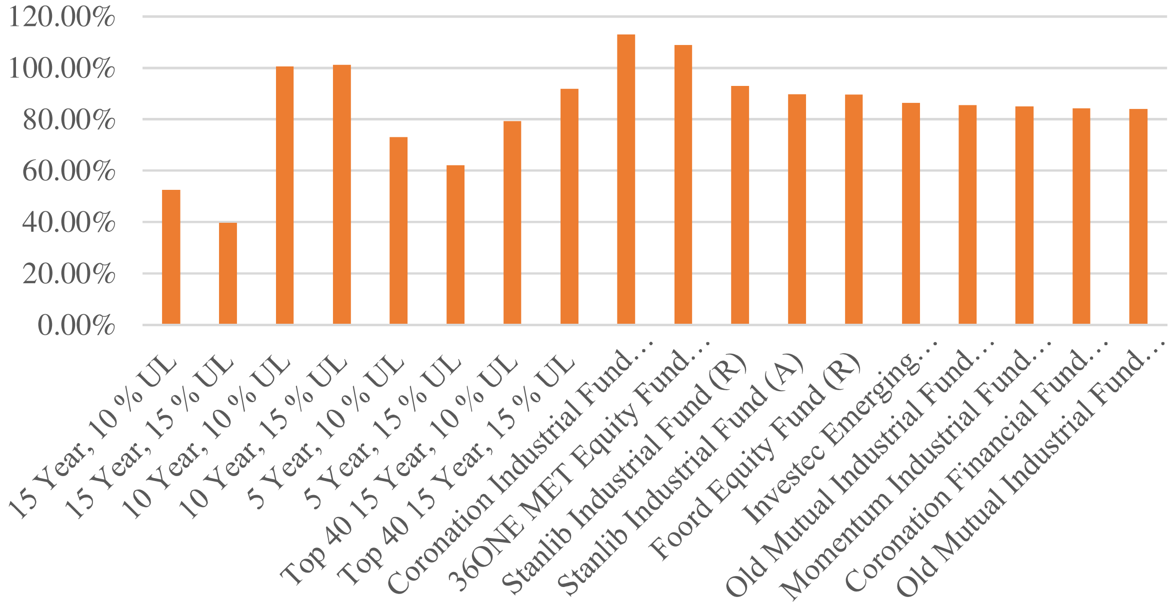

Figure 7.20 Three year total returns (2011-2013) of MPT model data compared to the top 10 performing unit trusts

7.4.5 Comparison to past research (Ward and Du Plessis)

This research follows on from that undertaken by Ward and du Plessis [14]. It expands on the work of Ward and du Plessis on a number of fronts.

These include:

- Analysing data from all listed JSE companies which have sufficient amount of return data available, not just the Top 40.
- Compare returns from the MPT model to unit trust and indices, not just indices.
- Testing not just a 10% upper limit, also 8%, 12%, 15% and 20%.
- Using data of different periods in history

Other differences include:

- Du Plessis and Ward allowed short selling.
- Use of Matlab instead of Excel.
- Rebalanced once a year instead of twice a year.
- Different ranges of data used, 5 year, 10 year and 15 year (not just 11 year).

- This research identifies numerous portfolios on an efficient frontier and selects the portfolio that sits in the optimal position based on the best Sharpe ratio.

Ward and du Plessis [14] found that their Markowitz based portfolios outperformed the Top 40 index in terms of pure returns as well as risk adjusted returns (Sharpe ratio).

They achieved the highest returns with two portfolios that had no upper limit constraints. This research report has found that there is close to no difference (Figure 7.6) between choosing an upper limit constraint and not choosing one. No upper limit constraint was found to be superior to another across the range of different data used. The actual length of time over which data was analysed e.g. 5 year, 10 year or 15 year played a much bigger role in producing larger returns.

This research analysed a larger range of portfolios and data than Ward and du Plessis. Ward and du Plessis' study used data up until 31 December 2007. This was before the financial market crash of September 2008. This research looks further at what effect a crash could have on the performance of an MPT model.

Ward and du Plessis [14] recommended that a longer time period should be used for the study, and more shares should be analysed. Recommendations also included using more constraints and alternative approaches to estimate future returns (analyst consensus).

This research report thus carries out numerous of those recommendations. This research used a larger amount of shares, data over a longer period of time, as well as additional upper limit constraints.

7.5 Effect of Biases on this Research

There is some effect of biases (discussed in Section 3.21) on this research. These biases are mainly attributed to making simplifying assumptions. The effects of the applicable biases are discussed below.

Biases identified in this research include survivorship, home country, and sample selection bias.

The most significant bias in this research would come from survivorship bias. This is due to ignoring companies who have been delisted in the MPT model. As discussed in Section 3.8, MPT looks at the inter relationships between stock returns when portfolio risk is calculated.

This inter relationship between stocks is measured by the correlation coefficient or covariance between stocks. If a stock does not exist anymore (delisted), it cannot have an effect on the MPT because the model only uses data from companies which exist at the period of time when the data is used by the model.

Thus the only effect of survivorship bias is when portfolios are rebalanced from a past period (e.g. year 2006) at the current point in time (end year 2013), with only the current surviving companies data available. It is not ideal to ignore delisted companies, but due to the scope of this research and the difficulty in getting data of delisted companies and exact dates and circumstances behind the delisting (merger, takeover, bankruptcy, going private etc.), it is difficult to factor it into the MPT model.

Sample selection bias was created by only considering companies who had a certain range of data available for each data set (Only 140 companies had enough returns data over a 15 year period). This bias was reduced by selecting a smaller range of price data for certain data sets (i.e. Weekly 3 had all companies who had at least 5 years of data). This can be seen summarised in Table 7.26. Table 7.26 shows the number of companies considered in each data set, as well as the percentages of these companies compared to the number of companies on the JSE and the data available from McGregor BFA. The number of companies who had at least 1 year worth of returns data on McGregor BFA are 293 [25]. The total approximate number of companies listed on the JSE and Altx (according to the JSE) is 400 as of 31 December 2013 [35]. A potential investor would probably require at least 1 years' worth of data to analyse in an MPT model. The final column in Table 7.26 is thus better to make comparison to rather than the current total JSE and Altx listing. This is also due to only the JSE listed companies being considered (not including Altx listed companies). Table 7.26 shows that the percentage of companies in each data set ranges from 48% to 81% of the total JSE listed companies who had at least 1 years' worth of data on McGregor BFA.

The reason for including 10 year and 5 year data was thus to reduce the impact of sample selection bias. However, depending on the type of investor, if he is conservative he might not want to have companies without a proven track record in his portfolio (e.g. companies less than 5 years old).

Table 7.26 Number of companies in chosen data sets compared to JSE and McGregor BFA data

Data Set	Number of companies in set	Percentage of companies in data set compared to JSE	Percentage of companies in data set compared to McGregor
Weekly 1 (15 year data)	140	35%	48%
Weekly 2 (10 year data)	183	46%	62%
Weekly 3 (5 year data)	236	59%	81%
Weekly Top 40 (15 year data)	40	N/A	N/A
Companies with at least 1 year of return data on McGregor BFA	293	73%	
JSE Main Board and Altx Listings (as of 31 December 2013)	400		

The effect of home country bias can also be ignored because this research is looking at creating a local equity portfolio, and not discussing asset allocation amongst a range of different assets (e.g. international equities, bonds, property, cash).

The biases in this research have been reduced by having a few ranges of results as well as being backed up by findings of other researchers in areas such as share price momentum and use of historical data. The simplifying assumptions are acceptable within the scope of this research as well as what is practically useful to private investors who do not have large resources at their disposal.

7.6 High performance of MPT Model

This model is based on the effect of momentum of share prices. As discussed, the momentum effect in share prices have been shown to be found on the JSE [14; 15]. The recovery in share prices since the global financial crisis of 2008 is also a contributing factor to the high performance. Future work to further test the resilience of the model could involve selecting lower rebalancing periods, different limit constraints and testing price data from further back in time and where share prices were not significantly growing over a short period of time. This additional work can give a better understanding of the effect of momentum versus diversification

7.7 Sharpe Ratio of expected returns as a future performance measure

It is necessary to evaluate whether the Sharpe ratio of the expected portfolio returns is a successful measure of a top performing portfolio, and a predictor of actual future returns.

Figure 7.21 shows a summary of the percentage of times the Sharpe ratio chosen portfolio was found in each position (first is highest actual return) ranked according to the other portfolios not selected. None of the portfolios was worse than sixth place in the group of portfolios of actual returns (out of 10 portfolios created). The selected portfolios were the best approximately 10% of the time, second best 16% of the time etc. This shows that it is not guaranteed that the Sharpe ratio of portfolio expected returns will be the best portfolio of actual returns. However it is not a bad method of choosing a portfolio because it never ends up being one of the worst performing ex post Sharpe ratios.

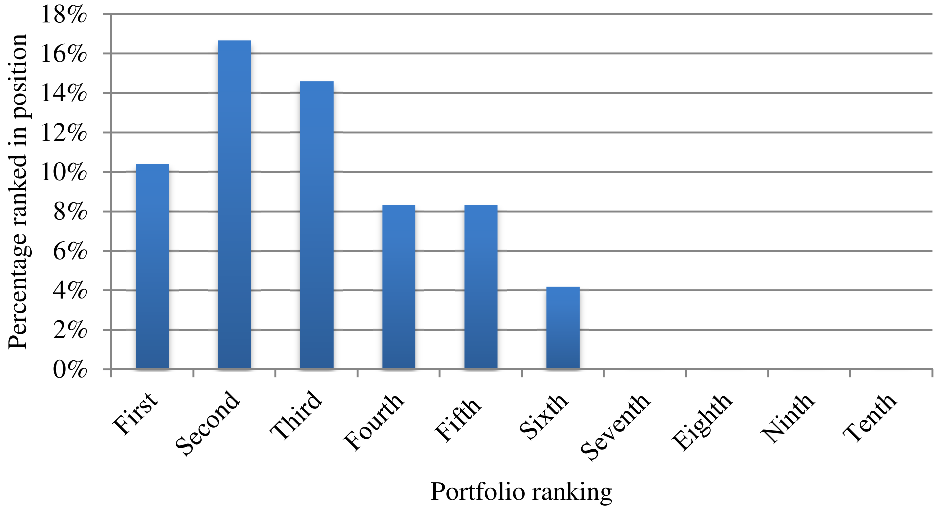

Figure 7.21: Number of times in various positions for each portfolio (no rebalancing)

Similarly, Figure 7.22 shows the number of times each position is achieved for the portfolios where rebalancing takes place. The number of times the portfolio chosen is the best is 23% of the time, second best 20 % of the time, etc. None of the selected portfolios performed worst overall.

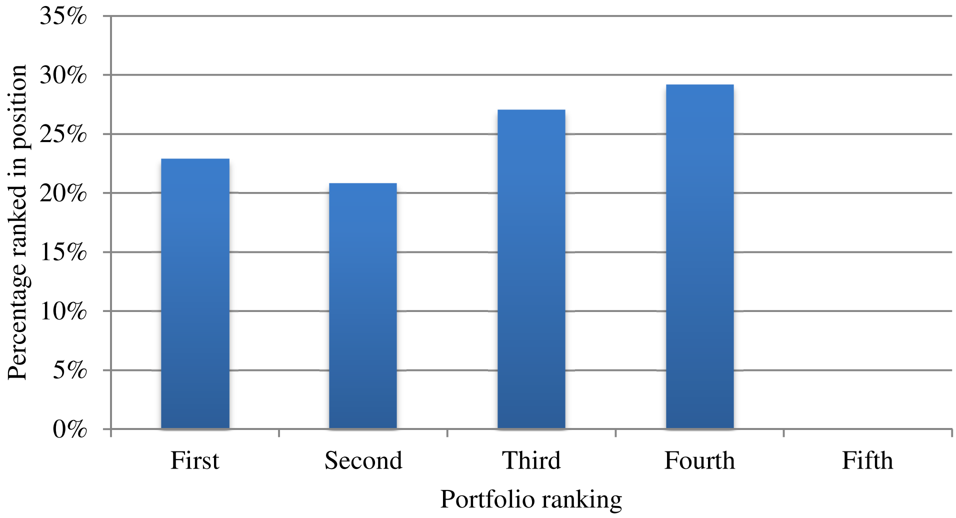

Figure 7.22: Number of times in various positions for each portfolio, with rebalancing

The Sharpe ratio of expected portfolio returns is thus not the ideal way of selecting a portfolio, but is the only way of selecting a portfolio using the information available before seeing the actual returns. The Sharpe ratio of expected portfolio returns performs satisfactorily in that many of the results shown earlier show good performance even though the best portfolio from a Sharpe ratio perspective is not always chosen.

8 Conclusion

- Markowitz Portfolio Theory (MPT) could provide above average returns on the JSE because of the manner in which it combines different shares into an optimal portfolio. Diversification has been proven to reduce risk, and through creating a portfolio with shares of different rates of risk the overall return of the portfolio can be enhanced. These optimal portfolios have been shown to produce above average returns compared to the overall JSE ALSI and JSE Top 40.
- All but one of the MPT model data sets created exceeded the JSE ALSI one year return of 17.13%, three year annualised return of 12.83% and three year total return of 43.63%. Only the 15 year data set at a 15% upper limit performed worse at a three year annualised return of 11.81% and three year total return of 39.79%.
- Both the Top 40 MPT model data sets outperformed the JSE Top 40 which had a one year return of 18.37%, three year annualised return of 13.02% and three year total return of 43.63%. These returns were 20.72% (one year), 24.24% (three year annualised) and 91.79% (total) for a 15% upper limit. The returns for the MPT model with a 10% upper limit were 18.53% (one year), 21.49% (three year annualised) and 79.31% (total).
- Through this research it has been found that the MPT model data sets performs well compared to many unit trusts. Compared to the top ten unit trusts the 10 year data sets outperform all the unit trusts over a one year period. Over a three year period the total returns of three of the MPT model data sets, namely the two 10 year data sets and the Top 40 set (15 year, 15% upper limit) are all three found in the top 10 of the overall comparison between unit trusts. Overall the MPT model results outperform many unit trusts and are even found amongst the top performing unit trusts over the last three years.
- The Sharpe Ratio was used to measure risk adjusted performance and was found to be satisfactory for this purpose.

8.1 Hypothesis 1

Hypothesis 1 can be accepted because it has been shown that Markowitz portfolio theory can outperform the JSE Top 40 and other indices.

8.2 Hypothesis 2:

Hypothesis 2 is inconclusive. In some cases the MPT model data sets outperformed the top ranking actively managed equity funds and high risk unit trusts, while some of them did not. Overall the MPT data sets together outperformed many unit trusts and equity funds.

8.3 Hypothesis 3:

Hypothesis 3 can be accepted. It has been shown that within the constraint, and the type and length of data chose, that historical data can be used in an MPT model to provide above average returns.

9 Recommendations and Future Work

- Companies can be grouped in certain industries and those industries can be limited to a certain percentage of the asset allocation depending on industry and economic analysis. This would reduce exposure to certain industries depending on personal views (ethics etc.) or due to other research done by the investor. This can be added to the Matlab code using the built in function in the financial toolbox. However, knowledge of the industry and economic environment is required to fully make or justify this decision.
- Build a more detailed Matlab MPT model that can take into account more constraints such as costs of buying and selling a share.
- Use data such as daily and monthly price data and repeat the MPT model tests to compare whether the type of data makes a difference or not.
- Consider portfolio rebalancing more often, such as every month or twice a year, to see whether it will have an impact on the overall returns in each portfolio.
- Use shorter time periods, such as 6 months or 1 years' worth of data, to use as inputs into the model.
- Loops could be programmed into the Matlab code to make the reading and exporting of data automatic and save time in running the model.

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12 Appendices

12.1 Appendix A: Portfolio Summaries

12.1.1 Weekly 1 (15 year) data sets

Table A1 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for no upper limit constraints and Weekly 1 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.014	0.007	0.100	46.73%	13.49%		4.02	0.70
3	0.023	0.011	0.167	77.52%	15.99%		4.26	0.57
4	0.036	0.015	0.260	114.61%	17.91%		4.16	0.44
5	0.052	0.018	0.375	159.27%	19.91%		4.07	0.36
6	0.071	0.022	0.513	213.01%	10.77%		4.03	0.08
7	0.093	0.026	0.674	277.64%	-5.81%		4.02	-0.18
8	0.126	0.030	0.910	355.30%	1.12%		3.83	-0.06
9	0.181	0.033	1.305	448.57%	13.04%		3.39	0.05
10	0.268	0.037	1.930	560.50%	-10.00%		2.87	-0.09

Table A2 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 8% upper limit constraints and Weekly 1 (15 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.011	0.005	0.076	31.67%	11.92%		3.31	0.71
3	0.013	0.007	0.093	43.03%	13.18%		3.92	0.72
4	0.016	0.009	0.117	55.34%	13.88%		4.16	0.63
5	0.020	0.010	0.146	68.69%	14.88%		4.25	0.57
6	0.025	0.012	0.181	83.16%	17.67%		4.25	0.62
7	0.030	0.013	0.220	98.85%	18.46%		4.21	0.54
8	0.037	0.015	0.265	115.86%	19.55%		4.13	0.49
9	0.044	0.017	0.316	134.29%	21.28%		4.04	0.47
10	0.055	0.018	0.395	154.26%	16.91%		3.74	0.26

Table A3 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Weekly 1 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.011	0.006	0.078	33.16%	12.12%		3.43	0.72
3	0.014	0.007	0.099	46.28%	13.46%		4.01	0.70
4	0.018	0.009	0.129	60.66%	14.05%		4.21	0.59
5	0.023	0.011	0.164	76.42%	15.80%		4.26	0.57
6	0.029	0.013	0.206	93.70%	17.68%		4.22	0.54
7	0.035	0.015	0.255	112.63%	17.81%		4.16	0.44
8	0.043	0.016	0.310	133.38%	21.19%		4.09	0.47
9	0.052	0.018	0.372	156.10%	23.03%		4.02	0.44
10	0.063	0.020	0.456	181.00%	16.11%		3.83	0.21

Table A4 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Weekly 1 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.011	0.006	0.082	36.27%	12.56%		3.63	0.74
3	0.016	0.008	0.113	53.16%	13.91%		4.14	0.66
4	0.021	0.010	0.154	72.11%	15.03%		4.25	0.55
5	0.029	0.013	0.206	93.35%	17.66%		4.22	0.54
6	0.037	0.015	0.267	117.16%	18.04%		4.15	0.43
7	0.047	0.017	0.336	143.83%	19.96%		4.09	0.40
8	0.057	0.020	0.413	173.71%	19.95%		4.05	0.33
9	0.070	0.022	0.502	207.17%	15.30%		4.00	0.18
10	0.087	0.024	0.628	244.63%	-1.71%		3.79	-0.13

Table A5 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 20% upper limit constraints and Weekly 1 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.012	0.006	0.086	38.58%	12.79%		3.75	0.74
3	0.017	0.009	0.124	58.40%	13.88%		4.19	0.60
4	0.024	0.011	0.175	80.99%	16.60%		4.25	0.58
5	0.033	0.014	0.240	106.74%	17.71%		4.18	0.47
6	0.044	0.017	0.316	136.06%	19.67%		4.10	0.42
7	0.056	0.019	0.401	169.45%	19.28%		4.06	0.32
8	0.069	0.022	0.499	207.46%	12.18%		4.03	0.11
9	0.084	0.024	0.609	250.72%	3.71%		4.01	-0.05
10	0.105	0.027	0.756	299.93%	-5.31%		3.88	-0.16

12.1.2 Weekly 2 (10 year) data sets

Table A6 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for no upper limit constraints and Weekly 2 (10 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.015	0.012	0.110	84.52%	37.23%		7.08	2.79
3	0.036	0.019	0.257	168.54%	38.66%		6.30	1.25
4	0.066	0.027	0.478	289.79%	46.24%		5.93	0.83
5	0.122	0.034	0.882	464.27%	53.43%		5.19	0.53
6	0.208	0.041	1.503	714.72%	26.38%		4.71	0.13
7	0.321	0.048	2.313	1073.32%	-7.23%		4.61	-0.06
8	NaN	1.000	#VALUE!	##########	#VALUE!		#VALUE!	#VALUE!
9	NaN	1.000	#VALUE!	##########	#VALUE!		#VALUE!	#VALUE!
10	1.608	0.070	11.597	3351.92%	13.51%		2.88	0.01

Table A7 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 8% upper limit constraints and Weekly 2 (10 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.009	0.007	0.063	44.99%	24.62%		6.10	2.87
3	0.012	0.010	0.085	66.20%	31.19%		6.98	2.89
4	0.017	0.012	0.119	90.44%	37.57%		7.05	2.61
5	0.023	0.015	0.165	118.15%	38.33%		6.75	1.92
6	0.031	0.018	0.223	149.79%	35.27%		6.41	1.29
7	0.041	0.020	0.295	185.93%	29.61%		6.09	0.78
8	0.053	0.023	0.384	227.18%	24.20%		5.75	0.46
9	0.083	0.026	0.596	274.25%	25.96%		4.49	0.33
10	0.146	0.028	1.054	327.95%	21.81%		3.05	0.15

Table A8 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Weekly 2 (10 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.009	0.007	0.065	47.28%	25.17%		6.26	2.87
3	0.013	0.010	0.092	71.47%	33.85%		7.05	2.97
4	0.019	0.013	0.134	99.56%	38.63%		6.97	2.41
5	0.026	0.016	0.191	132.14%	37.95%		6.59	1.65
6	0.036	0.019	0.260	169.92%	37.44%		6.28	1.19
7	0.048	0.022	0.343	213.72%	33.52%		6.04	0.79
8	0.063	0.025	0.452	264.46%	28.12%		5.71	0.48
9	0.104	0.028	0.751	323.23%	19.47%		4.22	0.17
10	0.182	0.031	1.310	391.26%	23.63%		2.94	0.13

Table A9 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Weekly 2 (10 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.010	0.008	0.069	51.62%	26.23%		6.52	2.85
3	0.015	0.012	0.106	81.71%	36.93%		7.09	2.87
4	0.023	0.015	0.164	117.63%	38.50%		6.76	1.95
5	0.034	0.019	0.243	160.48%	37.92%		6.35	1.29
6	0.046	0.022	0.334	211.59%	43.12%		6.14	1.10
7	0.062	0.026	0.449	272.49%	36.09%		5.92	0.66
8	0.088	0.029	0.637	345.02%	27.48%		5.32	0.33
9	0.159	0.033	1.147	431.35%	20.41%		3.70	0.12
10	0.262	0.036	1.889	534.04%	19.91%		2.79	0.07

Table A10 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 20% upper limit constraints and Weekly 2 (10 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.010	0.008	0.073	55.08%	27.17%		6.68	2.84
3	0.016	0.012	0.119	90.06%	37.80%		7.05	2.64
4	0.027	0.016	0.192	132.75%	37.96%		6.59	1.64
5	0.040	0.020	0.287	184.80%	40.26%		6.22	1.18
6	0.055	0.024	0.399	248.23%	45.19%		6.07	0.97
7	0.077	0.028	0.557	325.44%	39.77%		5.72	0.60
8	0.116	0.032	0.838	419.38%	27.90%		4.93	0.26
9	0.211	0.036	1.521	533.57%	27.08%		3.46	0.14
10	0.344	0.040	2.483	672.28%	36.51%		2.68	0.12

12.1.3 Weekly 3 (5 year) data sets

Table A11 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for no upper limit constraints and Weekly 3 (5 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.005	0.003	0.039	15.12%	18.52%	6.50%	2.20	3.07
2	0.006	0.005	0.046	32.52%	22.80%		5.63	3.53
3	0.009	0.008	0.064	52.51%	27.94%		7.20	3.36
4	0.012	0.011	0.090	75.43%	27.78%		7.67	2.37
5	0.018	0.014	0.130	101.73%	13.77%		7.34	0.56
6	0.027	0.016	0.193	131.89%	-2.40%		6.51	-0.46
7	0.042	0.019	0.306	166.45%	-15.25%		5.23	-0.71
8	0.065	0.022	0.472	206.05%	-29.58%		4.23	-0.76
9	0.098	0.024	0.708	251.41%	-37.39%		3.46	-0.62
10	0.169	0.027	1.219	303.34%	-55.17%		2.43	-0.51

Table A12 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 8% upper limit constraints and Weekly 3 (5 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.043	18.15%	20.63%	6.50%	2.74	3.32
2	0.006	0.005	0.045	28.69%	23.25%		4.92	3.71
3	0.007	0.007	0.052	40.16%	25.93%		6.41	3.70
4	0.009	0.008	0.064	52.62%	27.97%		7.21	3.36
5	0.011	0.010	0.078	66.16%	28.90%		7.61	2.86
6	0.013	0.011	0.097	80.88%	25.94%		7.65	2.00
7	0.017	0.013	0.122	96.88%	17.21%		7.39	0.88
8	0.021	0.015	0.154	114.27%	5.30%		7.00	-0.08
9	0.028	0.016	0.204	133.15%	-0.54%		6.19	-0.34
10	0.047	0.018	0.342	153.67%	6.54%		4.30	0.00

Table A13 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Weekly 3 (5 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.042	17.85%	20.42%	6.50%	2.70	3.32
2	0.006	0.005	0.045	29.19%	23.19%		5.04	3.71
3	0.007	0.007	0.054	41.59%	25.82%		6.55	3.61
4	0.009	0.008	0.067	55.16%	28.51%		7.31	3.31
5	0.012	0.010	0.083	70.01%	29.15%		7.65	2.73
6	0.015	0.012	0.105	86.24%	22.65%		7.59	1.54
7	0.019	0.014	0.134	104.00%	12.52%		7.29	0.45
8	0.024	0.016	0.173	123.40%	0.94%		6.77	-0.32
9	0.033	0.017	0.237	144.62%	-2.07%		5.82	-0.36
10	0.057	0.019	0.411	167.81%	-10.69%		3.92	-0.42

Table A14 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Weekly 3 (5 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.041	17.08%	19.86%	6.50%	2.60	3.28
2	0.006	0.005	0.045	30.00%	22.88%		5.25	3.66
3	0.008	0.007	0.056	44.31%	26.04%		6.76	3.49
4	0.010	0.009	0.072	60.16%	28.71%		7.48	3.10
5	0.013	0.011	0.093	77.72%	26.90%		7.67	2.20
6	0.017	0.013	0.122	97.16%	16.39%		7.42	0.81
7	0.022	0.015	0.161	118.68%	2.67%		6.96	-0.24
8	0.031	0.017	0.223	142.51%	-4.60%		6.09	-0.50
9	0.044	0.019	0.320	168.87%	-15.19%		5.07	-0.68
10	0.084	0.021	0.609	198.04%	-23.49%		3.14	-0.49

Table A15 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 20% upper limit constraints and Weekly 3 (5 year) data

Portfolio number	Portflio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.040	16.38%	19.28%	6.50%	2.48	3.21
2	0.006	0.005	0.045	30.64%	22.40%		5.38	3.54
3	0.008	0.007	0.058	46.61%	26.58%		6.90	3.46
4	0.011	0.010	0.076	64.49%	28.77%		7.58	2.91
5	0.014	0.012	0.102	84.50%	23.73%		7.61	1.68
6	0.019	0.014	0.139	106.89%	10.24%		7.25	0.27
7	0.027	0.016	0.193	131.95%	-2.42%		6.51	-0.46
8	0.039	0.019	0.282	159.97%	-11.84%		5.45	-0.65
9	0.058	0.021	0.420	191.31%	-23.34%		4.40	-0.71
10	0.107	0.023	0.770	226.34%	-29.80%		2.86	-0.47

12.1.4 Weekly Top 40 (15 year) data sets

Table A16 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for no upper limit constraints and Weekly Top 40 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.004	0.138	0.261	11.50%	6.50%	1.42	0.36
2	0.020	0.005	0.141	0.303	11.94%		1.69	0.39
3	0.021	0.006	0.148	0.346	13.52%		1.89	0.47
4	0.022	0.006	0.161	0.390	16.60%		2.02	0.63
5	0.025	0.007	0.179	0.436	19.71%		2.07	0.74
6	0.029	0.008	0.206	0.484	22.90%		2.03	0.80
7	0.036	0.008	0.257	0.533	30.15%		1.82	0.92
8	0.046	0.009	0.335	0.583	37.83%		1.55	0.94
9	0.059	0.010	0.426	0.635	45.51%		1.34	0.91
10	0.073	0.010	0.527	0.689	59.16%		1.18	1.00

Table A17 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 8% upper limit constraints and Weekly Top 40 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.004	0.142	0.249	9.48%	6.50%	1.29	0.21
2	0.020	0.005	0.143	0.263	9.49%		1.39	0.21
3	0.020	0.005	0.144	0.278	10.10%		1.47	0.25
4	0.020	0.005	0.147	0.292	10.84%		1.55	0.30
5	0.021	0.005	0.150	0.307	12.00%		1.61	0.37
6	0.021	0.005	0.154	0.321	14.11%		1.66	0.49
7	0.022	0.006	0.160	0.336	16.33%		1.70	0.62
8	0.023	0.006	0.167	0.351	17.70%		1.71	0.67
9	0.025	0.006	0.178	0.366	18.56%		1.69	0.68
10	0.028	0.006	0.200	0.382	15.67%		1.58	0.46

Table A18 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Weekly Top 40 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.004	0.140	0.257	9.54%	6.50%	1.37	0.22
2	0.020	0.005	0.141	0.273	9.91%		1.48	0.24
3	0.020	0.005	0.143	0.290	10.46%		1.57	0.28
4	0.020	0.005	0.146	0.306	11.17%		1.65	0.32
5	0.021	0.005	0.150	0.323	12.87%		1.72	0.43
6	0.022	0.006	0.155	0.340	14.60%		1.77	0.52
7	0.022	0.006	0.162	0.357	16.44%		1.81	0.61
8	0.024	0.006	0.171	0.375	19.72%		1.81	0.77
9	0.026	0.006	0.187	0.392	19.90%		1.75	0.72
10	0.030	0.007	0.213	0.410	21.68%		1.62	0.71

Table A19 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Weekly Top 40 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.004	0.138	26.29%	10.63%	6.50%	1.43	0.30
2	0.019	0.005	0.139	28.25%	11.12%		1.57	0.33
3	0.020	0.005	0.141	30.23%	11.72%		1.69	0.37
4	0.020	0.005	0.144	32.24%	12.90%		1.79	0.44
5	0.021	0.006	0.149	34.28%	14.77%		1.87	0.56
6	0.022	0.006	0.156	36.35%	16.87%		1.92	0.67
7	0.023	0.006	0.164	38.45%	19.25%		1.94	0.78
8	0.024	0.007	0.176	40.59%	21.13%		1.94	0.83
9	0.027	0.007	0.192	42.76%	22.92%		1.89	0.86
10	0.033	0.007	0.237	44.96%	27.04%		1.62	0.87

Table A20 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 20% upper limit constraints and Weekly Top 40 (15 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.004	0.138	0.261	11.50%	6.50%	1.42	0.36
2	0.019	0.005	0.139	0.283	11.77%		1.57	0.38
3	0.020	0.005	0.141	0.306	11.96%		1.71	0.39
4	0.020	0.005	0.145	0.329	12.67%		1.82	0.43
5	0.021	0.006	0.150	0.352	13.71%		1.92	0.48
6	0.022	0.006	0.157	0.376	16.19%		1.99	0.62
7	0.023	0.006	0.166	0.400	19.00%		2.02	0.75
8	0.025	0.007	0.179	0.425	21.31%		2.02	0.83
9	0.027	0.007	0.194	0.450	23.57%		1.98	0.88
10	0.037	0.008	0.269	0.476	19.09%		1.53	0.47

12.1.5 Weekly Top 40 (5 year) data sets

Table A21 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for no upper limit constraints and Weekly Top 40 (5 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.017	0.002	0.122	0.137	5.57%	6.50%	0.59	-0.08
2	0.017	0.003	0.123	0.166	7.42%		0.82	0.07
3	0.017	0.003	0.126	0.195	9.73%		1.04	0.26
4	0.018	0.004	0.129	0.225	13.05%		1.24	0.51
5	0.019	0.004	0.134	0.256	16.24%		1.43	0.73
6	0.019	0.005	0.140	0.288	19.12%		1.59	0.90
7	0.020	0.005	0.147	0.321	21.93%		1.74	1.05
8	0.022	0.006	0.156	0.354	24.76%		1.85	1.17
9	0.023	0.006	0.166	0.388	28.86%		1.95	1.35
10	0.032	0.007	0.230	0.423	38.99%		1.56	1.41

Table A22 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 8% upper limit constraints and Weekly Top 40 (5 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.003	0.133	0.173	8.28%	6.50%	0.81	0.13
2	0.019	0.003	0.134	0.187	10.53%		0.91	0.30
3	0.019	0.004	0.136	0.202	13.65%		1.01	0.53
4	0.019	0.004	0.138	0.218	16.77%		1.11	0.74
5	0.020	0.004	0.141	0.233	20.22%		1.19	0.97
6	0.020	0.004	0.145	0.249	21.78%		1.27	1.05
7	0.021	0.005	0.150	0.265	25.03%		1.33	1.24
8	0.022	0.005	0.156	0.281	27.66%		1.38	1.36
9	0.023	0.005	0.165	0.297	28.09%		1.41	1.31
10	0.025	0.005	0.178	0.313	27.96%		1.39	1.21

Table A23 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Weekly Top 40 (5 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.018	0.003	0.129	0.164	7.19%	6.50%	0.77	0.05
2	0.018	0.003	0.130	0.183	8.58%		0.91	0.16
3	0.018	0.004	0.132	0.202	11.25%		1.04	0.36
4	0.019	0.004	0.135	0.221	15.86%		1.15	0.69
5	0.019	0.004	0.140	0.240	18.79%		1.25	0.88
6	0.020	0.004	0.145	0.260	21.50%		1.35	1.04
7	0.021	0.005	0.150	0.280	24.26%		1.43	1.18
8	0.022	0.005	0.157	0.300	25.87%		1.50	1.23
9	0.023	0.005	0.165	0.321	29.39%		1.55	1.39
10	0.026	0.006	0.185	0.342	27.60%		1.50	1.14

Table A24 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Weekly Top 40 (5 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.017	0.003	0.123	0.147	6.14%	6.50%	0.67	-0.03
2	0.017	0.003	0.125	0.171	7.26%		0.85	0.06
3	0.018	0.003	0.128	0.195	10.24%		1.02	0.29
4	0.018	0.004	0.132	0.220	13.17%		1.18	0.51
5	0.019	0.004	0.137	0.246	15.77%		1.32	0.68
6	0.020	0.005	0.143	0.272	18.22%		1.44	0.82
7	0.021	0.005	0.150	0.298	20.78%		1.55	0.95
8	0.022	0.005	0.159	0.325	23.61%		1.64	1.08
9	0.023	0.006	0.169	0.353	27.34%		1.71	1.24
10	0.025	0.006	0.184	0.381	41.54%		1.72	1.91

Table A25 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 20% upper limit constraints and Weekly Top 40 (5 year) data

Portfolio number	Portfolio risk (std dev)	Portfolio expected returns	Yearly portfolio risk	Expected yearly return	Realised return (annual)	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.017	0.003	0.122	0.140	5.75%	6.50%	0.61	-0.06
2	0.017	0.003	0.123	0.165	7.40%		0.81	0.07
3	0.017	0.003	0.125	0.192	9.33%		1.01	0.23
4	0.018	0.004	0.129	0.219	12.40%		1.19	0.46
5	0.019	0.004	0.134	0.246	15.09%		1.35	0.64
6	0.020	0.005	0.141	0.274	17.81%		1.49	0.80
7	0.021	0.005	0.148	0.303	20.58%		1.61	0.95
8	0.022	0.006	0.157	0.333	23.74%		1.70	1.10
9	0.023	0.006	0.168	0.363	27.05%		1.78	1.23
10	0.026	0.006	0.191	0.394	30.24%		1.72	1.24

12.1.6 Yearly rebalance, Weekly 1 (15 year) data sets

Table A26 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2005)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.067	23.93%	29.47%	6.50%	2.61	3.44
2	0.015	0.009	0.106	60.47%	37.03%		5.11	2.89
3	0.027	0.014	0.193	107.53%	36.38%		5.23	1.55
4	0.044	0.019	0.316	168.04%	20.36%		5.11	0.44
5	0.071	0.024	0.509	245.75%	0.18%		4.70	-0.12

Table A27 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2006)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.067	24.53%	35.41%	6.50%	2.71	4.35
2	0.014	0.009	0.103	59.72%	63.23%		5.17	5.51
3	0.026	0.014	0.187	104.60%	83.11%		5.26	4.11
4	0.043	0.019	0.309	161.78%	123.11%		5.03	3.78
5	0.070	0.023	0.504	234.56%	211.02%		4.52	4.06

Table A28 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2007)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.068	25.69%	-20.11%	6.50%	2.82	-3.92
2	0.015	0.009	0.105	61.75%	-29.94%		5.25	-3.46
3	0.027	0.014	0.192	107.90%	-33.26%		5.27	-2.07
4	0.044	0.019	0.319	166.90%	-37.17%		5.03	-1.37
5	0.073	0.024	0.524	242.24%	-29.82%		4.50	-0.69

Table A29 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2008)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.072	22.27%	15.55%	6.50%	2.20	1.26
2	0.016	0.009	0.115	55.98%	20.34%		4.30	1.20
3	0.028	0.013	0.205	98.77%	18.61%		4.50	0.59
4	0.045	0.018	0.328	153.01%	15.86%		4.47	0.29
5	0.071	0.023	0.512	221.69%	-3.56%		4.20	-0.20

Table A30 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.073	22.87%	21.41%	6.50%	2.26	2.05
2	0.016	0.009	0.114	56.16%	36.11%		4.36	2.60
3	0.028	0.013	0.203	98.25%	52.07%		4.51	2.24
4	0.045	0.018	0.323	151.41%	74.12%		4.48	2.09
5	0.070	0.023	0.503	218.48%	88.98%		4.21	1.64

Table A31 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.071	22.61%	11.50%	6.50%	2.26	0.70
2	0.015	0.008	0.111	55.21%	3.50%		4.38	-0.27
3	0.027	0.013	0.198	96.26%	5.98%		4.53	-0.03
4	0.043	0.018	0.311	147.91%	3.10%		4.54	-0.11
5	0.068	0.022	0.490	212.83%	-2.71%		4.21	-0.19

Table A32 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.071	21.75%	25.39%	6.50%	2.16	2.67
2	0.015	0.008	0.110	52.35%	28.74%		4.15	2.01
3	0.027	0.012	0.194	90.47%	26.39%		4.32	1.02
4	0.042	0.017	0.306	137.89%	10.04%		4.30	0.12
5	0.068	0.021	0.489	196.84%	-6.57%		3.89	-0.27

Table A33 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (15 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.015	0.008	0.106	49.75%	13.71%		4.08	0.68
3	0.026	0.012	0.185	84.86%	17.07%		4.25	0.57
4	0.041	0.016	0.295	128.01%	20.21%		4.11	0.46
5	0.063	0.020	0.456	181.00%	16.11%		3.83	0.21

Table A34 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2005)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.067	23.91%	29.15%	6.50%	2.61	3.40
2	0.017	0.010	0.121	69.61%	37.30%		5.20	2.54
3	0.033	0.016	0.241	131.73%	34.21%		5.19	1.15
4	0.057	0.022	0.414	216.00%	7.55%		5.07	0.03
5	0.097	0.028	0.697	330.13%	-14.00%		4.64	-0.29

Table A35 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2006)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.067	24.53%	35.41%	6.50%	2.71	4.35
2	0.017	0.010	0.119	69.57%	69.13%		5.29	5.25
3	0.033	0.016	0.239	130.47%	93.73%		5.18	3.65
4	0.059	0.022	0.423	212.68%	159.38%		4.88	3.62
5	0.097	0.028	0.696	323.46%	294.13%		4.55	4.13

Table A36 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2007)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.009	0.004	0.068	25.69%	-20.11%	6.50%	2.82	-3.92
2	0.017	0.010	0.122	71.93%	-30.48%		5.34	-3.02
3	0.034	0.017	0.249	134.76%	-36.21%		5.16	-1.72
4	0.060	0.023	0.432	219.95%	-35.31%		4.94	-0.97
5	0.097	0.029	0.700	335.25%	-27.04%		4.69	-0.48

Table A37 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2008)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.072	22.27%	15.55%	6.50%	2.20	1.26
2	0.018	0.010	0.133	65.02%	20.99%		4.41	1.09
3	0.036	0.015	0.258	122.35%	18.08%		4.49	0.45
4	0.060	0.021	0.430	199.08%	7.91%		4.48	0.03
5	0.096	0.027	0.690	301.61%	-14.94%		4.27	-0.31

Table A38 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.073	22.91%	21.37%	6.50%	2.26	2.05
2	0.018	0.010	0.133	65.68%	38.93%		4.44	2.43
3	0.036	0.016	0.259	122.95%	61.03%		4.50	2.11
4	0.060	0.021	0.431	199.52%	93.51%		4.48	2.02
5	0.095	0.027	0.688	301.70%	124.51%		4.29	1.72

Table A39 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.071	22.61%	11.50%	6.50%	2.26	0.70
2	0.018	0.010	0.130	64.49%	3.47%		4.47	-0.23
3	0.035	0.015	0.250	120.30%	5.54%		4.54	-0.04
4	0.057	0.021	0.413	194.58%	-0.02%		4.55	-0.16
5	0.091	0.027	0.660	293.26%	1.23%		4.35	-0.08

Table A40 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.071	21.75%	25.39%	6.50%	2.16	2.67
2	0.018	0.009	0.128	60.90%	29.20%		4.25	1.77
3	0.034	0.015	0.245	112.33%	18.82%		4.32	0.50
4	0.056	0.020	0.404	179.78%	2.12%		4.28	-0.11
5	0.089	0.025	0.644	268.12%	-17.66%		4.06	-0.37

Table A41 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (15 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.010	0.004	0.069	21.20%	10.18%	6.50%	2.12	0.53
2	0.017	0.009	0.122	57.70%	13.85%		4.19	0.60
3	0.033	0.014	0.235	104.92%	17.67%		4.19	0.48
4	0.054	0.019	0.392	165.92%	20.10%		4.06	0.35
5	0.087	0.024	0.628	244.63%	-1.71%		3.79	-0.13

12.1.7 Yearly rebalance, Weekly 2 (10 year) data sets

Table A42 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2005)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.005	0.007	0.035	41.51%	28.99%	6.50%	10.01	6.43
2	0.018	0.020	0.127	183.21%	42.84%		13.95	2.87
3	0.051	0.034	0.367	461.65%	54.98%		12.41	1.32
4	0.135	0.047	0.975	1003.99%	65.80%		10.23	0.61
5	0.311	0.061	2.243	2051.30%	63.19%		9.12	0.25

Table A43 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2006)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.006	0.040	33.69%	28.31%	6.50%	6.76	5.42
2	0.016	0.018	0.115	148.74%	76.08%		12.36	6.05
3	0.043	0.030	0.312	359.45%	148.43%		11.33	4.56
4	0.100	0.042	0.723	742.58%	278.09%		10.18	3.76
5	0.273	0.054	1.970	1434.44%	337.54%		7.25	1.68

Table A44 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2007)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.006	0.045	35.05%	-18.78%	6.50%	6.31	-5.59
2	0.015	0.017	0.109	135.96%	-39.09%		11.89	-4.19
3	0.039	0.027	0.285	309.83%	-41.83%		10.66	-1.70
4	0.081	0.038	0.582	607.68%	-36.07%		10.33	-0.73
5	0.246	0.049	1.773	1115.10%	-35.75%		6.25	-0.24

Table A45 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2008)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.053	28.37%	19.67%	6.50%	4.13	2.49
2	0.017	0.014	0.122	109.32%	20.58%		8.46	1.16
3	0.039	0.024	0.284	239.77%	11.95%		8.21	0.19
4	0.074	0.033	0.537	449.07%	-1.58%		8.24	-0.15
5	0.226	0.043	1.627	783.41%	-2.04%		4.78	-0.05

Table A46 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.008	0.005	0.055	29.17%	20.77%	6.50%	4.11	2.59
2	0.016	0.014	0.115	101.00%	40.18%		8.19	2.92
3	0.036	0.022	0.261	211.63%	56.35%		7.84	1.91
4	0.069	0.031	0.495	381.37%	89.24%		7.57	1.67
5	0.211	0.039	1.519	640.90%	71.78%		4.18	0.43

Table A47 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.008	0.005	0.054	28.67%	12.12%	6.50%	4.08	1.04
2	0.015	0.013	0.109	92.91%	19.16%		7.94	1.16
3	0.034	0.021	0.243	188.30%	29.22%		7.48	0.93
4	0.063	0.028	0.455	329.55%	31.88%		7.10	0.56
5	0.198	0.036	1.428	538.05%	21.62%		3.72	0.11

Table A48 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.008	0.005	0.055	27.34%	24.01%	6.50%	3.80	3.19
2	0.015	0.012	0.105	85.28%	23.27%		7.51	1.60
3	0.032	0.019	0.232	168.87%	9.16%		6.99	0.11
4	0.061	0.026	0.438	289.13%	2.32%		6.45	-0.10
5	0.191	0.034	1.376	461.73%	2.51%		3.31	-0.03

Table A49 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (10 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.014	0.011	0.101	78.11%	36.50%		7.09	2.97
3	0.031	0.018	0.224	150.33%	37.00%		6.42	1.36
4	0.058	0.024	0.419	251.07%	29.96%		5.84	0.56
5	0.182	0.031	1.310	391.26%	23.63%		2.94	0.13

Table A50 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2005)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.005	0.007	0.035	41.34%	28.31%	6.50%	9.99	6.25
2	0.024	0.024	0.173	236.96%	42.98%		13.35	2.11
3	0.074	0.041	0.531	691.89%	58.09%		12.90	0.97
4	0.222	0.058	1.598	1735.53%	58.45%		10.82	0.33
5	0.462	0.075	3.335	4098.05%	39.63%		12.27	0.10

Table A51 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2006)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.006	0.040	33.69%	28.31%	6.50%	6.76	5.42
2	0.021	0.021	0.151	188.36%	90.86%		12.02	5.58
3	0.059	0.036	0.429	515.06%	198.12%		11.85	4.47
4	0.159	0.051	1.143	1197.70%	400.44%		10.42	3.45
5	0.404	0.066	2.916	2609.21%	475.68%		8.93	1.61

Table A52 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2007)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.006	0.045	35.05%	-18.78%	6.50%	6.31	-5.59
2	0.020	0.019	0.142	170.96%	-40.52%		11.57	-3.31
3	0.055	0.033	0.395	438.65%	-38.92%		10.94	-1.15
4	0.123	0.046	0.888	961.24%	-28.79%		10.76	-0.40
5	0.364	0.060	2.623	1972.67%	-30.77%		7.50	-0.14

Table A53 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2008)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.053	28.37%	19.67%	6.50%	4.13	2.49
2	0.021	0.017	0.154	135.63%	17.17%		8.38	0.69
3	0.052	0.028	0.378	329.50%	-4.77%		8.54	-0.30
4	0.108	0.040	0.780	677.53%	-14.96%		8.61	-0.28
5	0.334	0.052	2.406	1298.19%	-20.04%		5.37	-0.11

Table A54 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.008	0.005	0.055	29.17%	20.77%	6.50%	4.11	2.59
2	0.020	0.015	0.142	122.23%	46.56%		8.16	2.82
3	0.049	0.026	0.351	280.21%	59.50%		7.79	1.51
4	0.099	0.037	0.712	546.94%	105.84%		7.59	1.40
5	0.310	0.047	2.239	994.87%	109.72%		4.41	0.46

Table A55 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.008	0.005	0.054	28.67%	12.12%	6.50%	4.08	1.04
2	0.018	0.015	0.132	111.52%	21.57%		7.94	1.14
3	0.045	0.024	0.327	246.07%	37.61%		7.32	0.95
4	0.090	0.034	0.653	463.60%	31.06%		7.00	0.38
5	0.291	0.043	2.102	813.71%	27.69%		3.84	0.10

Table A56 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.008	0.005	0.055	27.26%	24.08%	6.50%	3.78	3.20
2	0.017	0.013	0.124	99.38%	20.09%		7.50	1.10
3	0.042	0.022	0.300	211.16%	6.77%		6.83	0.01
4	0.083	0.031	0.600	383.79%	-6.35%		6.29	-0.21
5	0.276	0.039	1.988	649.42%	-3.33%		3.23	-0.05

Table A57 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (10 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.007	0.005	0.054	26.44%	18.65%	6.50%	3.70	2.25
2	0.016	0.012	0.119	90.10%	37.81%		7.05	2.64
3	0.040	0.020	0.287	184.91%	40.27%		6.22	1.18
4	0.080	0.028	0.574	325.68%	29.32%		5.56	0.40
5	0.262	0.036	1.889	534.04%	19.91%		2.79	0.07

12.1.8 Yearly rebalance, Weekly 3 (5 years) data sets

Table A58 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (5 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.004	0.003	0.029	13.92%	8.79%	6.50%	2.55	0.79
2	0.007	0.008	0.050	54.63%	13.40%		9.62	1.38
3	0.013	0.014	0.095	109.50%	16.41%		10.88	1.05
4	0.027	0.020	0.195	183.34%	38.81%		9.05	1.65
5	0.061	0.026	0.438	282.54%	107.35%		6.30	2.30

Table A59 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (5 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.040	15.47%	20.36%	6.50%	2.26	3.48
2	0.008	0.008	0.058	54.84%	11.79%		8.38	0.92
3	0.014	0.014	0.098	107.28%	8.01%		10.26	0.15
4	0.025	0.020	0.178	177.04%	4.54%		9.57	-0.11
5	0.051	0.025	0.366	269.67%	13.21%		7.20	0.18

Table A60 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (5 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.041	16.69%	22.74%	6.50%	2.46	3.91
2	0.008	0.008	0.060	50.18%	26.41%		7.34	3.34
3	0.014	0.013	0.102	93.05%	26.33%		8.46	1.94
4	0.025	0.018	0.181	147.84%	14.31%		7.81	0.43
5	0.062	0.022	0.449	217.82%	-6.04%		4.71	-0.28

Table A61 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly (5 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.042	17.85%	19.93%	6.50%	2.70	3.20
2	0.008	0.007	0.056	44.87%	26.10%		6.79	3.47
3	0.013	0.011	0.093	77.94%	26.83%		7.66	2.18
4	0.022	0.015	0.161	118.39%	3.09%		6.93	-0.21
5	0.057	0.019	0.411	167.81%	-10.69%		3.92	-0.42

Table A62 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (5 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.004	0.002	0.026	11.54%	7.07%	6.50%	1.93	0.22
2	0.008	0.009	0.056	62.65%	16.93%		10.05	1.87
3	0.017	0.017	0.120	136.55%	15.63%		10.79	0.76
4	0.037	0.024	0.266	243.10%	42.76%		8.88	1.36
5	0.084	0.031	0.604	396.32%	151.46%		6.46	2.40

Table A63 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (5 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.005	0.003	0.038	14.56%	19.85%	6.50%	2.13	3.52
2	0.009	0.010	0.064	63.96%	10.12%		8.95	0.56
3	0.017	0.016	0.123	134.09%	7.08%		10.36	0.05
4	0.033	0.023	0.240	233.39%	3.33%		9.45	-0.13
5	0.072	0.030	0.521	373.70%	4.86%		7.05	-0.03

Table A64 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (5 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.005	0.003	0.040	15.86%	22.34%	6.50%	2.36	4.00
2	0.009	0.008	0.064	55.03%	27.02%		7.60	3.21
3	0.017	0.014	0.119	107.11%	24.06%		8.42	1.47
4	0.032	0.020	0.234	176.24%	8.94%		7.27	0.10
5	0.089	0.025	0.642	267.86%	-4.15%		4.07	-0.17

Table A65 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly (5 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.006	0.003	0.041	17.08%	19.37%	6.50%	2.60	3.16
2	0.008	0.008	0.060	48.12%	26.76%		6.99	3.40
3	0.015	0.012	0.106	87.19%	22.07%		7.58	1.46
4	0.028	0.017	0.205	136.32%	-3.48%		6.34	-0.49
5	0.084	0.021	0.609	198.04%	-23.49%		3.14	-0.49

12.1.9 Yearly rebalance, Weekly Top 40 (15 year) data sets

Table A66 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2005)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.143	30.13%	43.93%	6.50%	1.66	2.62
2	0.020	0.006	0.148	36.94%	43.44%		2.06	2.50
3	0.022	0.007	0.161	44.10%	42.65%		2.33	2.24
4	0.026	0.008	0.184	51.63%	41.78%		2.45	1.92
5	0.036	0.009	0.256	59.55%	52.81%		2.07	1.81

Table A67 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2006)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.142	31.42%	37.75%	6.50%	1.75	2.20
2	0.020	0.006	0.147	37.87%	40.85%		2.13	2.33
3	0.022	0.007	0.161	44.63%	44.47%		2.38	2.37
4	0.025	0.008	0.183	51.72%	45.54%		2.47	2.13
5	0.034	0.009	0.246	59.14%	27.08%		2.14	0.84

Table A68 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2007)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.141	31.03%	-15.27%	6.50%	1.74	-1.54
2	0.020	0.006	0.145	36.98%	-16.12%		2.10	-1.56
3	0.022	0.007	0.157	43.19%	-17.57%		2.34	-1.54
4	0.025	0.008	0.177	49.68%	-18.26%		2.44	-1.40
5	0.031	0.009	0.226	56.46%	-17.20%		2.21	-1.05

Table A69 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2008)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.021	0.005	0.150	27.69%	37.18%	6.50%	1.41	2.04
2	0.021	0.005	0.154	32.50%	38.39%		1.69	2.07
3	0.023	0.006	0.165	37.48%	41.60%		1.88	2.13
4	0.026	0.007	0.187	42.64%	43.65%		1.93	1.99
5	0.033	0.008	0.239	47.99%	47.35%		1.73	1.71

Table A70 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.021	0.005	0.148	27.73%	24.22%	6.50%	1.43	1.19
2	0.021	0.005	0.152	32.45%	22.70%		1.70	1.06
3	0.023	0.006	0.163	37.34%	22.57%		1.89	0.99
4	0.026	0.007	0.184	42.41%	22.55%		1.95	0.87
5	0.033	0.008	0.237	47.66%	19.83%		1.73	0.56

Table A71 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.146	26.35%	10.10%	6.50%	1.36	0.25
2	0.021	0.005	0.149	30.57%	7.56%		1.61	0.07
3	0.022	0.006	0.159	34.93%	4.54%		1.79	-0.12
4	0.025	0.006	0.177	39.43%	1.49%		1.86	-0.28
5	0.032	0.007	0.233	44.07%	-3.50%		1.62	-0.43

Table A72 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.004	0.144	25.17%	42.89%	6.50%	1.30	2.53
2	0.020	0.005	0.148	28.86%	43.84%		1.51	2.53
3	0.022	0.005	0.158	32.66%	45.38%		1.66	2.47
4	0.024	0.006	0.176	36.57%	49.06%		1.71	2.42
5	0.030	0.007	0.219	40.59%	49.45%		1.55	1.96

Table A73 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 10% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.004	0.140	25.71%	9.54%	6.50%	1.37	0.22
2	0.020	0.005	0.144	29.38%	10.61%		1.59	0.29
3	0.021	0.006	0.152	33.14%	13.74%		1.75	0.47
4	0.023	0.006	0.168	37.01%	18.53%		1.81	0.71
5	0.030	0.007	0.213	41.00%	21.68%		1.62	0.71

Table A74 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2005)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.005	0.139	30.09%	40.68%	6.50%	1.70	2.46
2	0.020	0.006	0.147	38.99%	40.08%		2.21	2.28
3	0.023	0.008	0.169	48.49%	38.84%		2.49	1.92
4	0.028	0.009	0.205	58.62%	35.62%		2.55	1.42
5	0.043	0.010	0.310	69.43%	44.90%		2.03	1.24

Table A75 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2006)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.005	0.139	31.09%	47.87%	6.50%	1.77	2.98
2	0.020	0.006	0.146	39.28%	50.47%		2.24	3.01
3	0.023	0.008	0.166	47.98%	52.41%		2.51	2.77
4	0.028	0.009	0.200	57.20%	54.89%		2.53	2.42
5	0.041	0.010	0.299	67.00%	33.04%		2.02	0.89

Table A76 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2007)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.005	0.138	31.67%	-12.70%	6.50%	1.82	-1.39
2	0.020	0.006	0.144	39.01%	-13.75%		2.25	-1.40
3	0.022	0.007	0.161	46.76%	-15.40%		2.50	-1.36
4	0.026	0.008	0.191	54.93%	-17.01%		2.54	-1.23
5	0.041	0.010	0.295	63.54%	-24.37%		1.94	-1.05

Table A77 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2008)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.148	28.46%	37.14%	6.50%	1.49	2.07
2	0.021	0.006	0.152	34.34%	38.08%		1.84	2.08
3	0.023	0.007	0.166	40.49%	42.73%		2.04	2.18
4	0.027	0.007	0.194	46.90%	47.41%		2.08	2.11
5	0.037	0.008	0.264	53.61%	41.57%		1.78	1.33

Table A78 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2009)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.146	28.42%	24.55%	6.50%	1.50	1.23
2	0.021	0.006	0.150	33.99%	22.72%		1.83	1.08
3	0.023	0.006	0.163	39.80%	22.38%		2.04	0.97
4	0.026	0.007	0.187	45.85%	21.93%		2.10	0.82
5	0.036	0.008	0.256	52.17%	24.44%		1.78	0.70

Table A79 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2010)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.005	0.143	27.03%	9.67%	6.50%	1.43	0.22
2	0.020	0.005	0.147	32.09%	7.82%		1.74	0.09
3	0.022	0.006	0.160	37.36%	4.60%		1.93	-0.12
4	0.025	0.007	0.183	42.83%	2.02%		1.98	-0.24
5	0.035	0.008	0.250	48.52%	-0.30%		1.68	-0.27

Table A80 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2011)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.020	0.004	0.142	25.71%	45.07%	6.50%	1.35	2.72
2	0.020	0.005	0.146	30.18%	48.40%		1.63	2.88
3	0.022	0.006	0.158	34.79%	51.67%		1.79	2.86
4	0.025	0.006	0.180	39.57%	55.73%		1.84	2.73
5	0.034	0.007	0.244	44.52%	53.67%		1.56	1.94

Table A81 Summary of Matlab MPT model results and calculated realised returns and Sharpe ratios for 15% upper limit constraints and Rebalance Weekly Top 40 (15 year) data (end year 2012)

Portfolio Number	Weekly portfolio risk (std dev)	Weekly portfolio expected returns	Yearly portfolio risk	Yearly expected portfolio return	Annual realised return	Risk Free Rate	Sharpe Ratio	Ex-post Sharpe Ratio
1	0.019	0.004	0.138	26.29%	10.63%	6.50%	1.43	0.30
2	0.020	0.005	0.141	30.73%	11.94%		1.71	0.38
3	0.021	0.006	0.152	35.31%	15.70%		1.89	0.61
4	0.024	0.006	0.172	40.05%	20.72%		1.95	0.82
5	0.033	0.007	0.237	44.96%	27.04%		1.62	0.87

12.2 Appendix B: Efficient Frontiers

12.2.1 Weekly 1 (15 year data) data sets

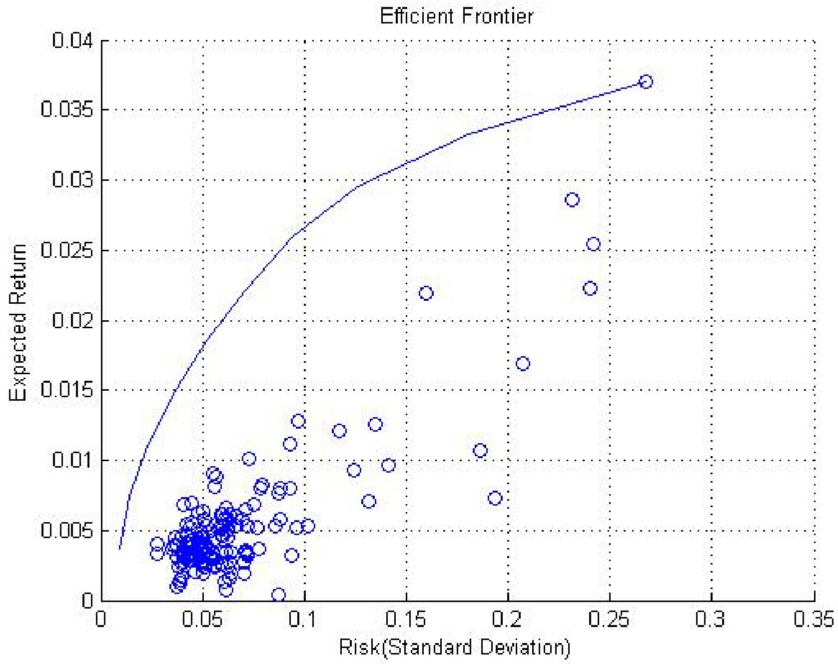

Figure B12.1 Efficient frontier graph for Weekly 1 (15 year) data with no upper limit constraint

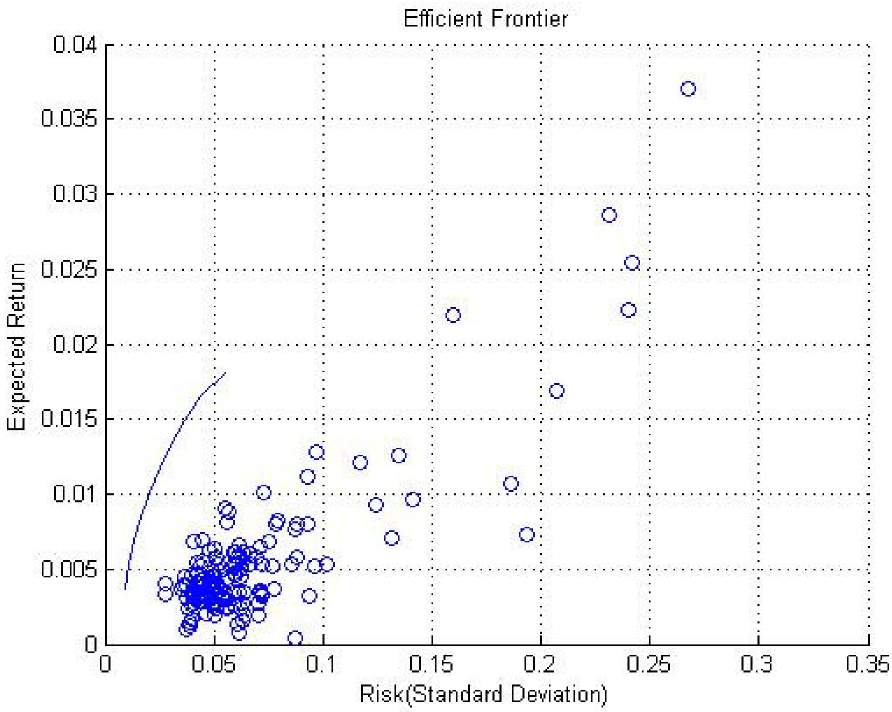

Figure B12.2 Efficient frontier graph for Weekly 1 (15 year) data with 8% upper limit constraint

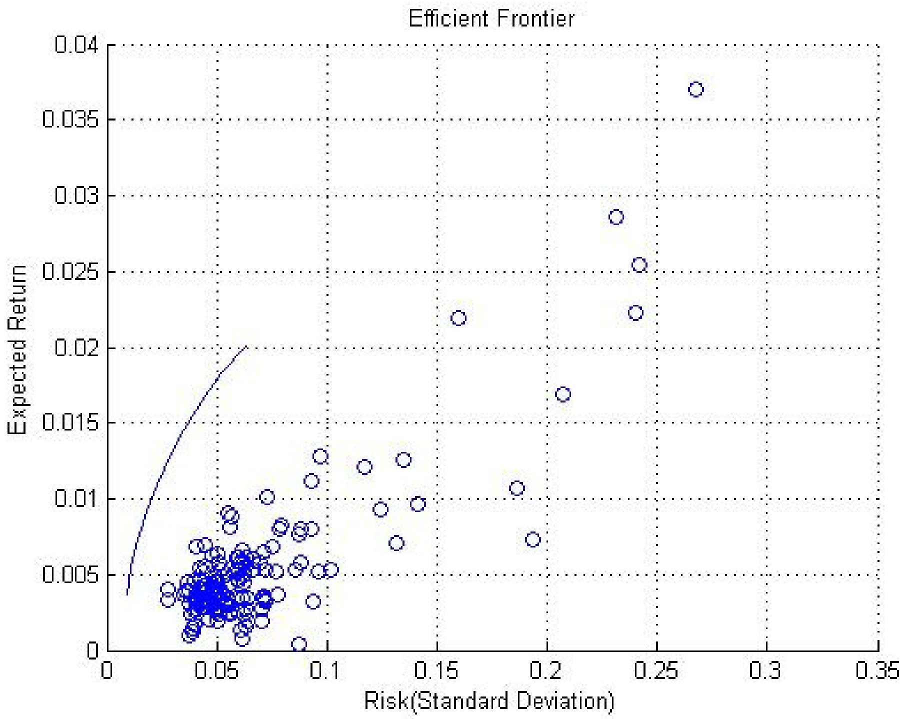

Figure B12.3 Efficient frontier graph for Weekly 1 (15 year) data with 10% upper limit constraint

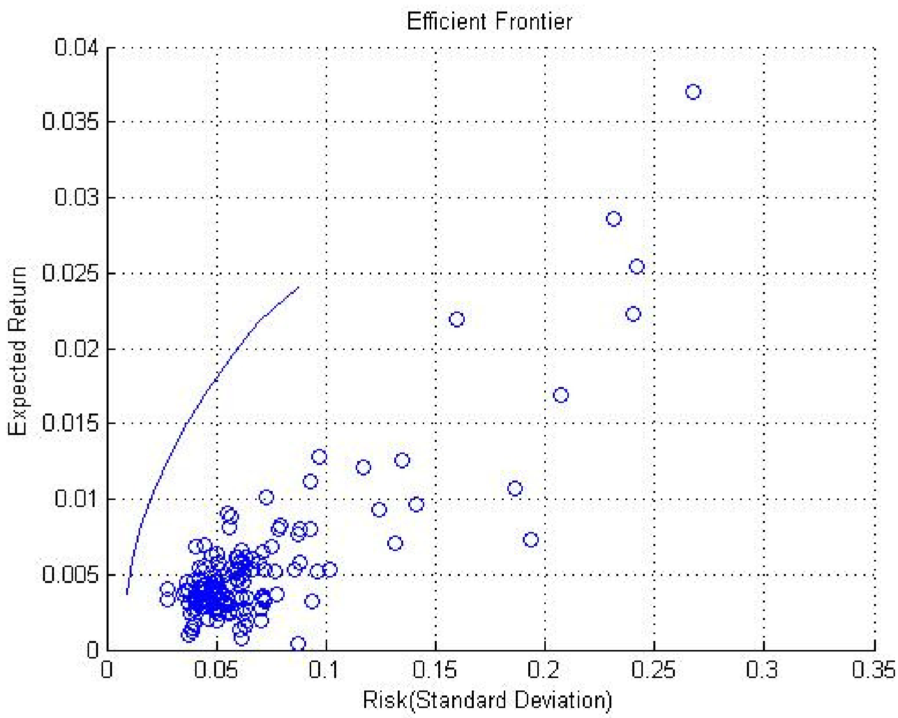

Figure B12.4 Efficient frontier graph for Weekly 1 (15 year) data with 15% upper limit constraint

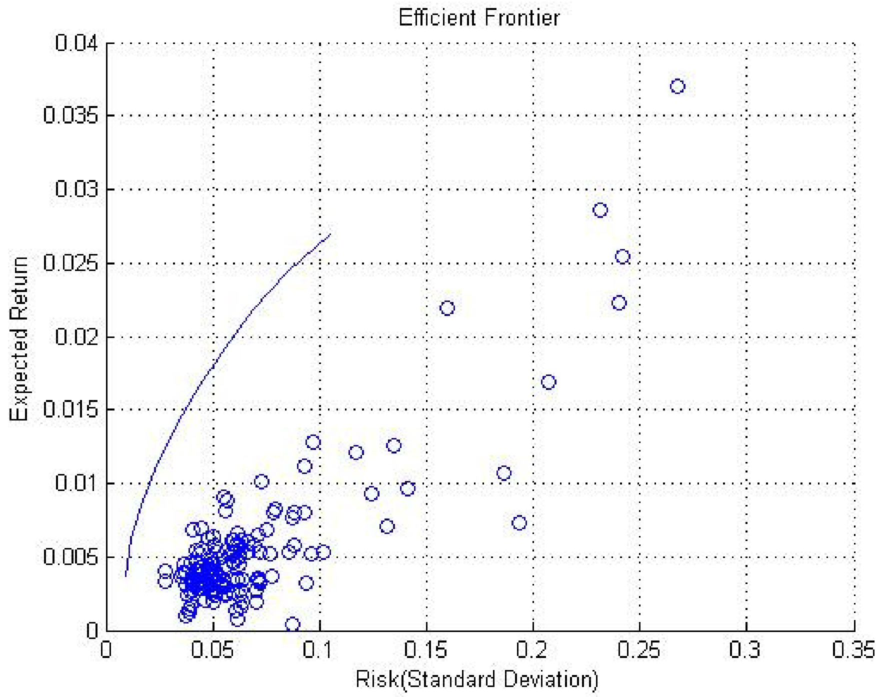

Figure B12.5 Efficient frontier graph for Weekly 1 (15 year) data with 20% upper limit constraint

12.2.2 Weekly 2 (10 year data) data sets

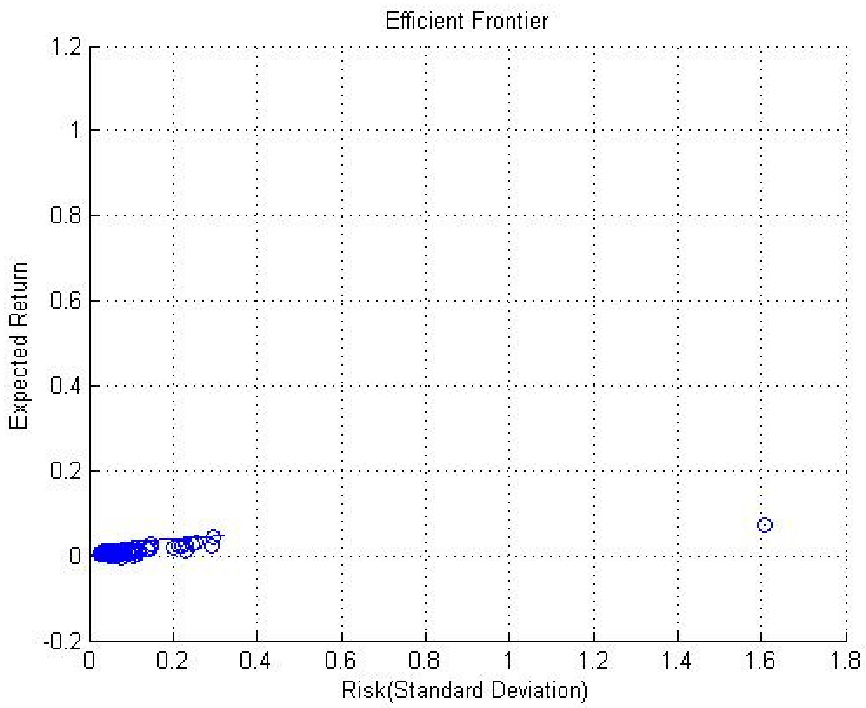

Figure B12.6 Efficient frontier graph for Weekly 2 (10 year) data with no upper limit constraint

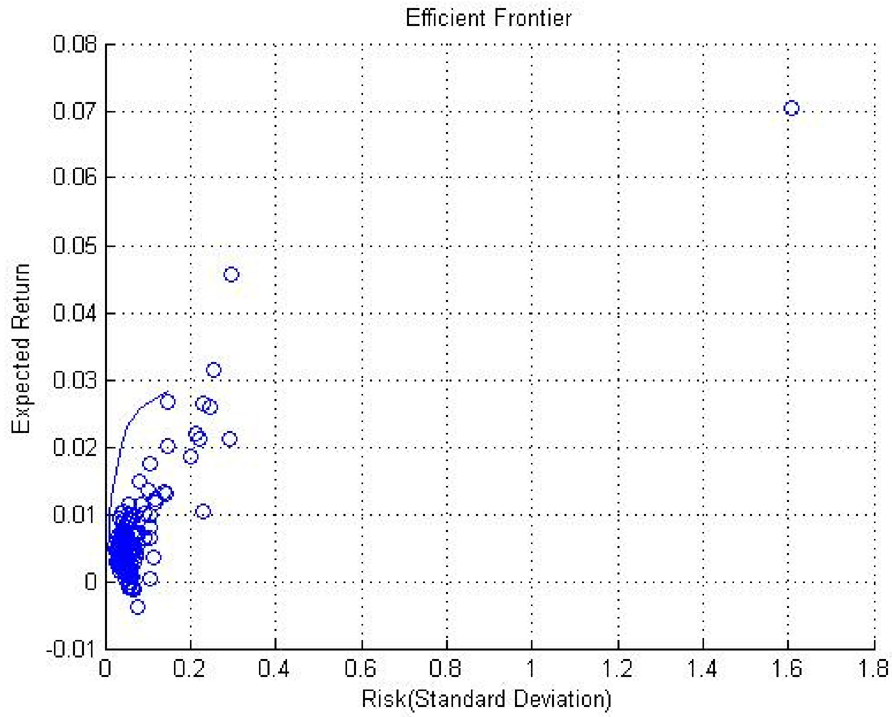

Figure B12.7 Efficient frontier graph for Weekly 2 (10 year) data with 8% upper limit constraint

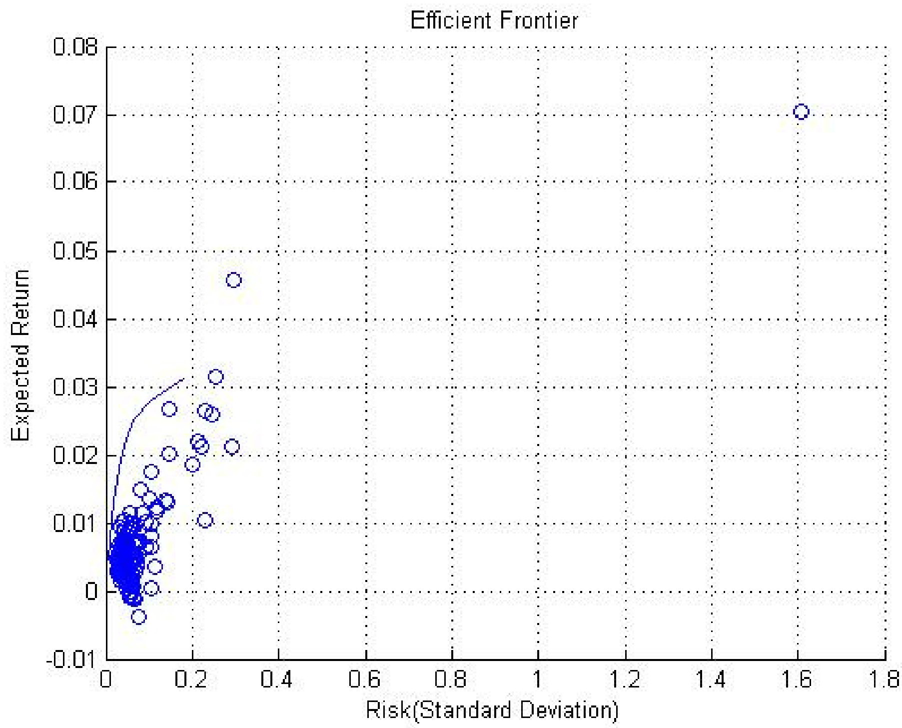

Figure B12.8 Efficient frontier graph for Weekly 2 (10 year) data with 10% upper limit constraint

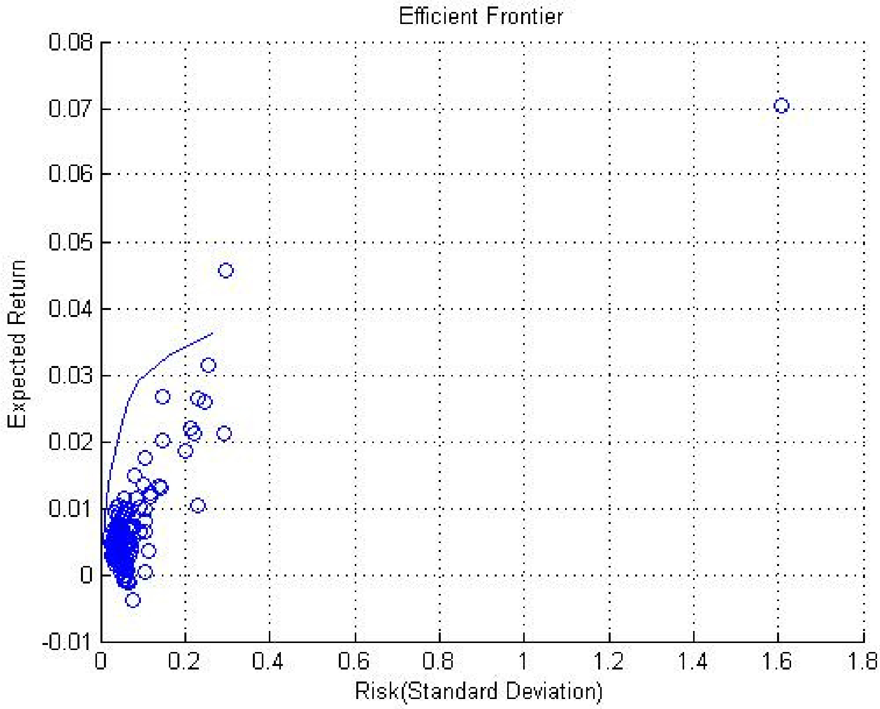

Figure B12.9 Efficient frontier graph for Weekly 2 (10 year) data with 15% upper limit constraint

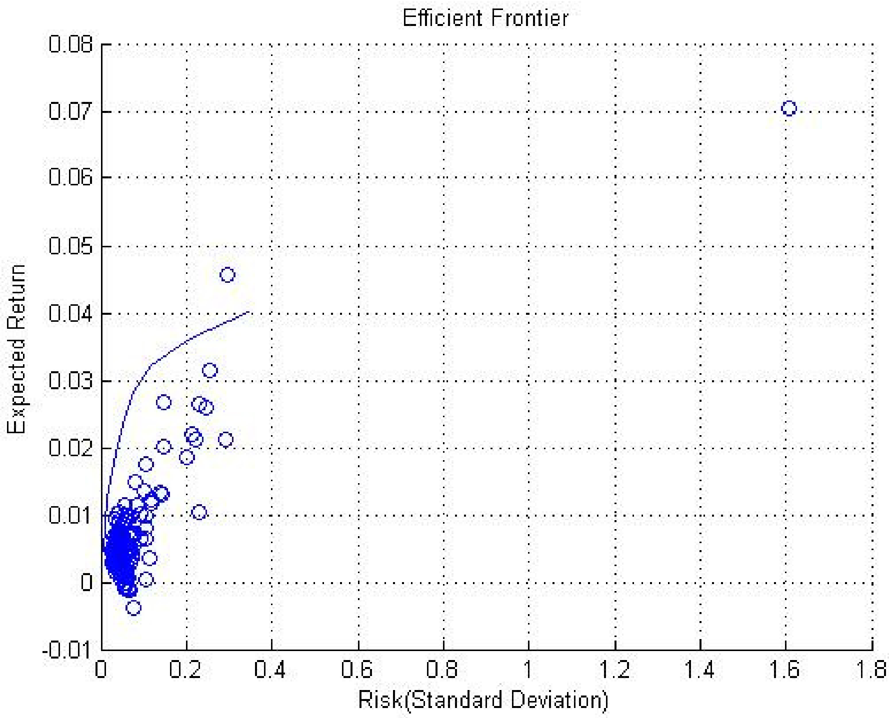

Figure B12.10 Efficient frontier graph for Weekly 2 (10 year) data with 20% upper limit constraint

12.2.3 Weekly 3 (5 year data)

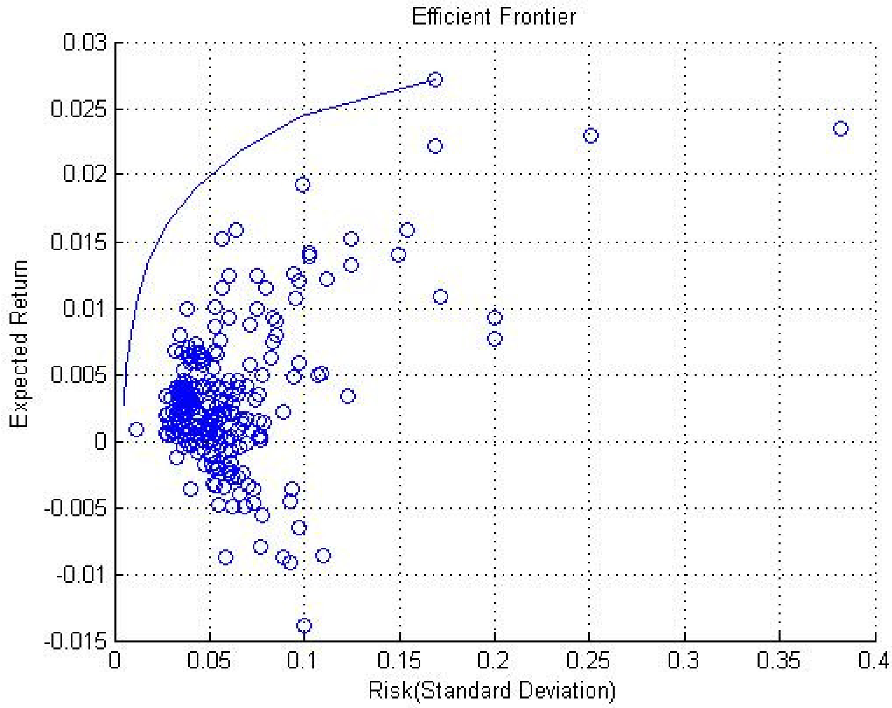

Figure B12.11 Efficient frontier graph for Weekly 3 (5 year) data with no upper limit constraint

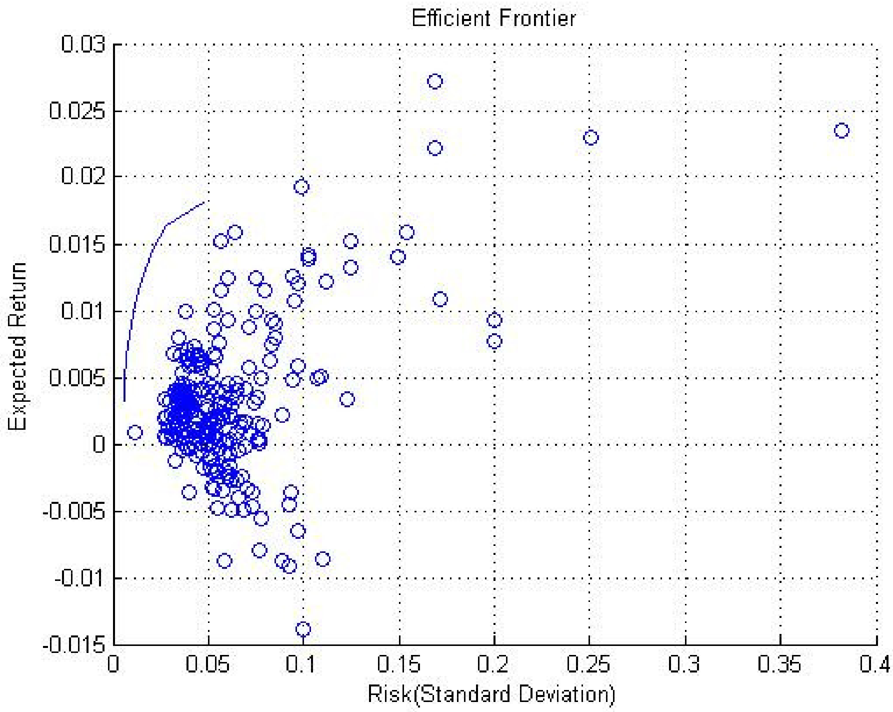

Figure B12.12 Efficient frontier graph for Weekly 3 (5 year) data with 8% upper limit constraint

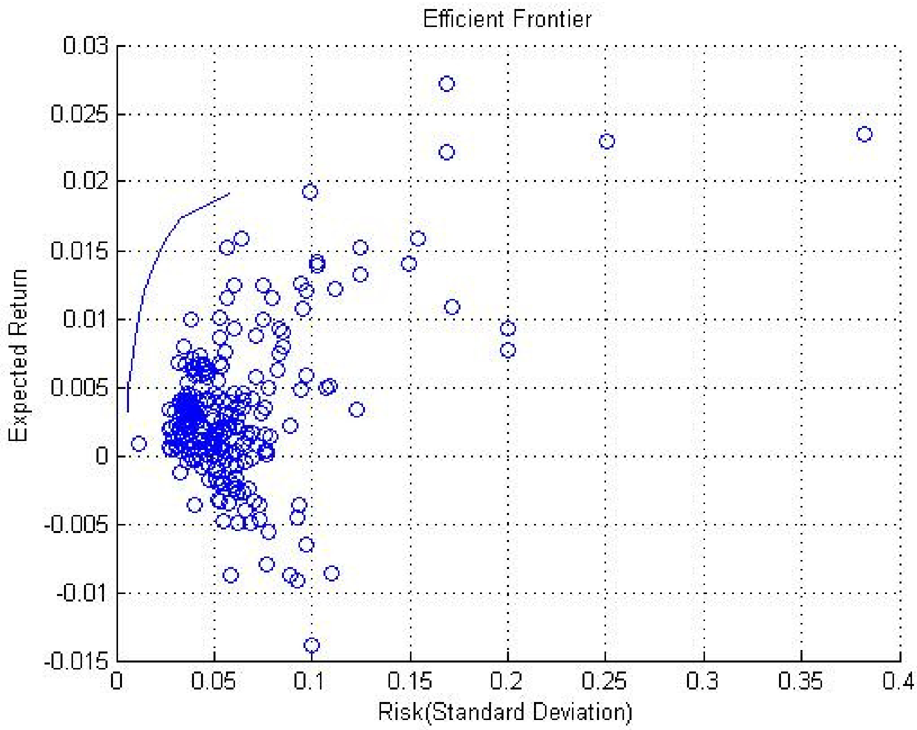

Figure B12.13 Efficient frontier graph for Weekly 3 (5 year) data with 10% upper limit constraint

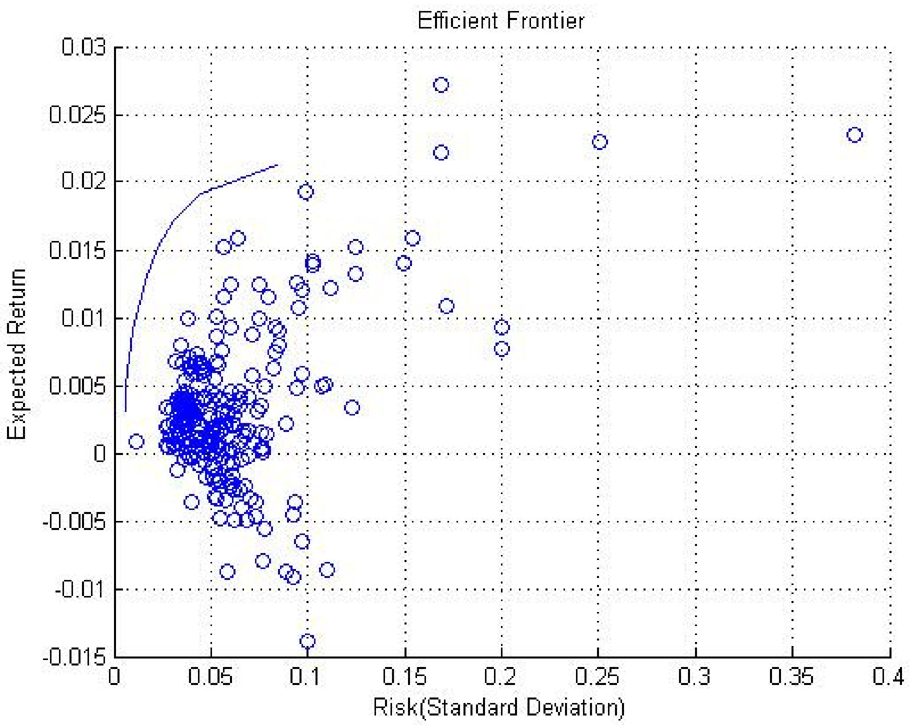

Figure B12.14 Efficient frontier graph for Weekly 3 (5 year) data with 15% upper limit constraint

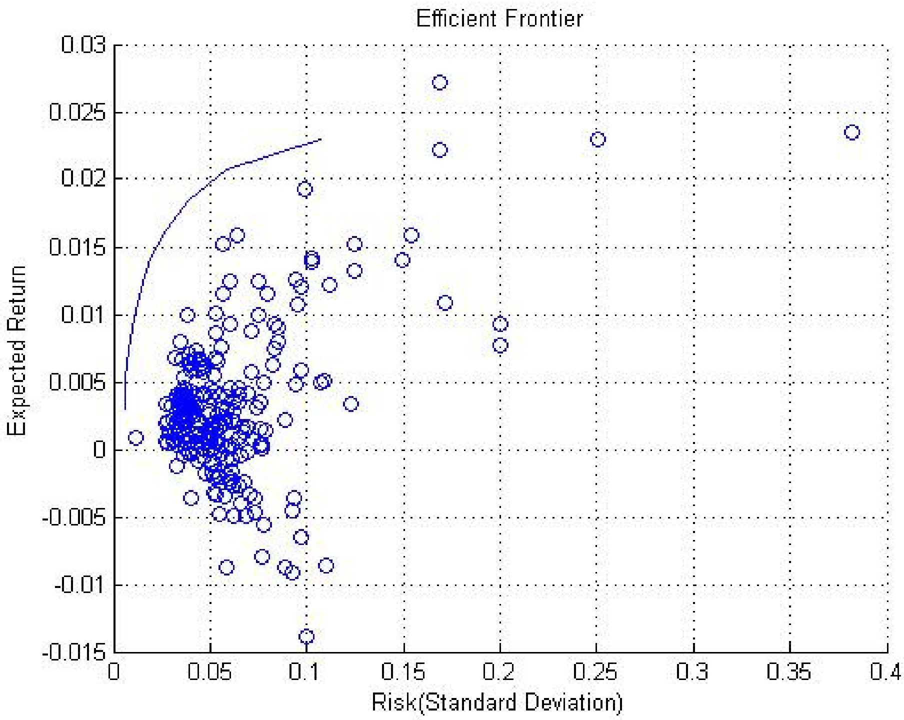

Figure B12.15 Efficient frontier graph for Weekly 3 (5 year) data with 20% upper limit constraint

12.2.4 Weekly Top 40 (15 year data) data sets

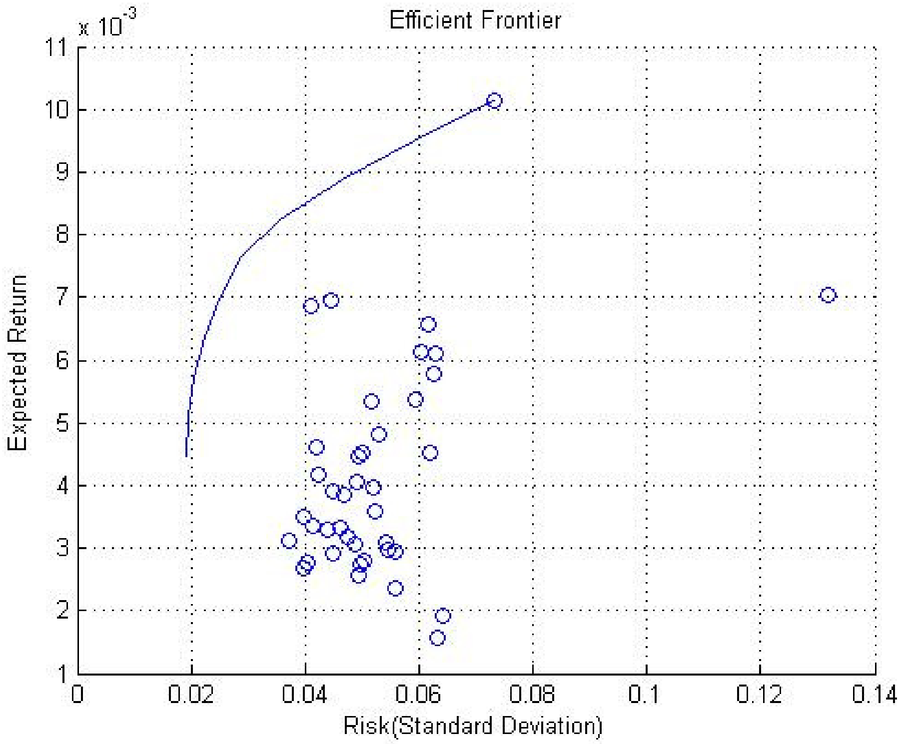

Figure B12.16 Efficient frontier for Weekly Top 40 (15 year) with no upper limit constraint

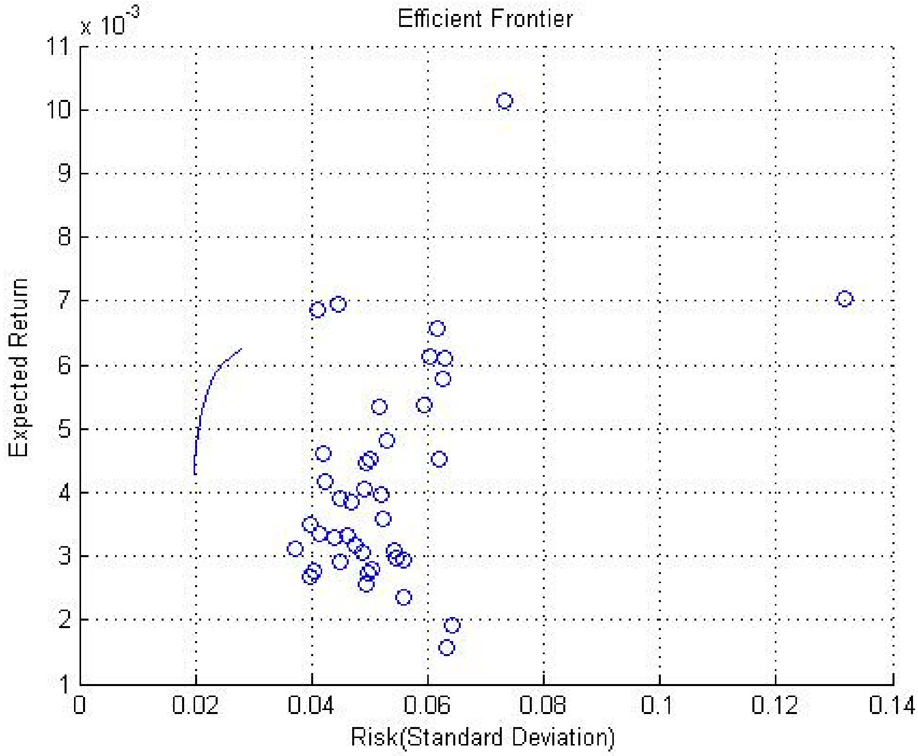

Figure B12.17 Efficient frontier for Weekly Top 40 (15 year) with 8% upper limit constraint

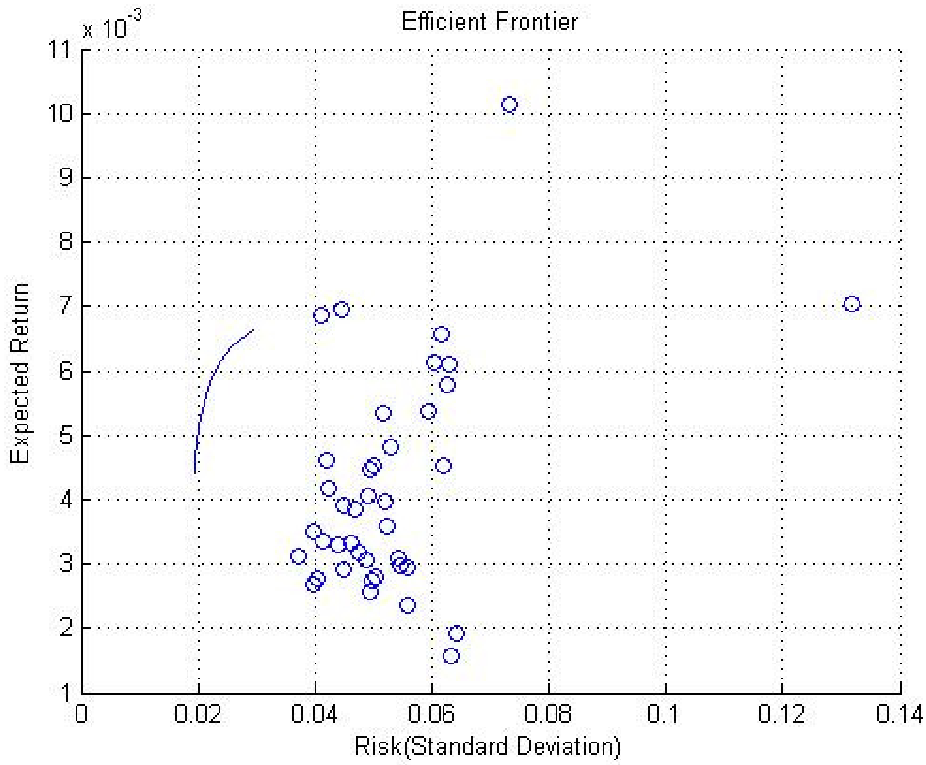

Figure B12.18 Efficient frontier for Weekly Top 40 (15 year) with 10% upper limit constraint

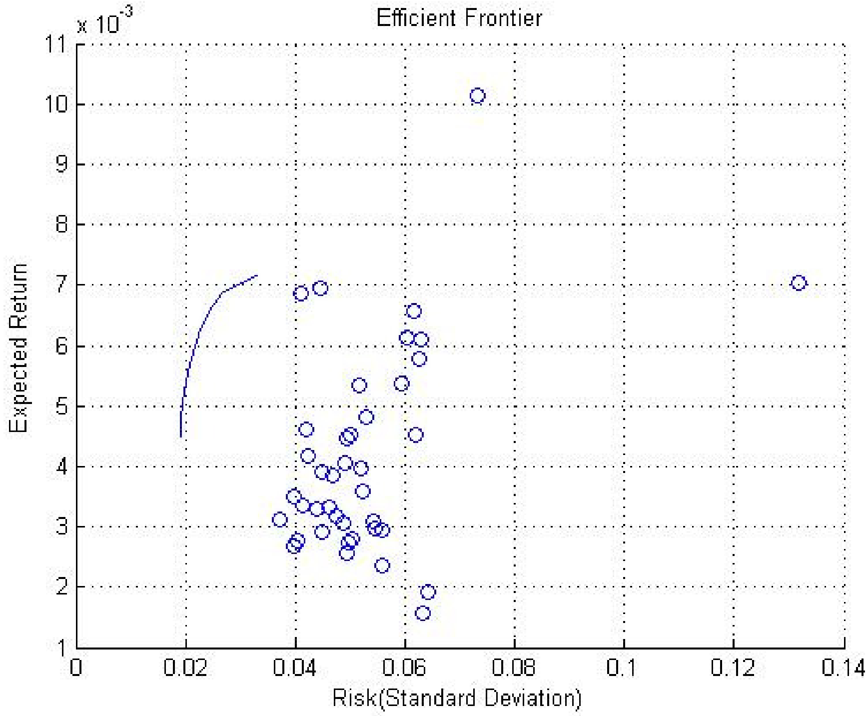

Figure B12.19 Efficient frontier for Weekly Top 40 (15 year) with 15% upper limit constraint

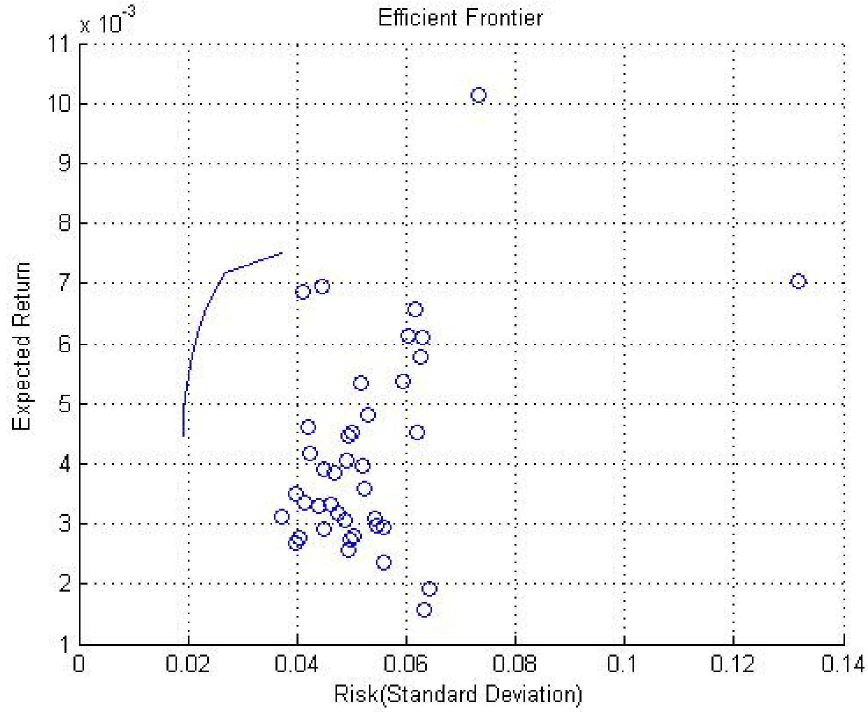

Figure B12.20 Efficient frontier for Weekly Top 40 (15 year) with 20% upper limit constraint

12.2.5 Weekly Top 40 (5 year data) data sets

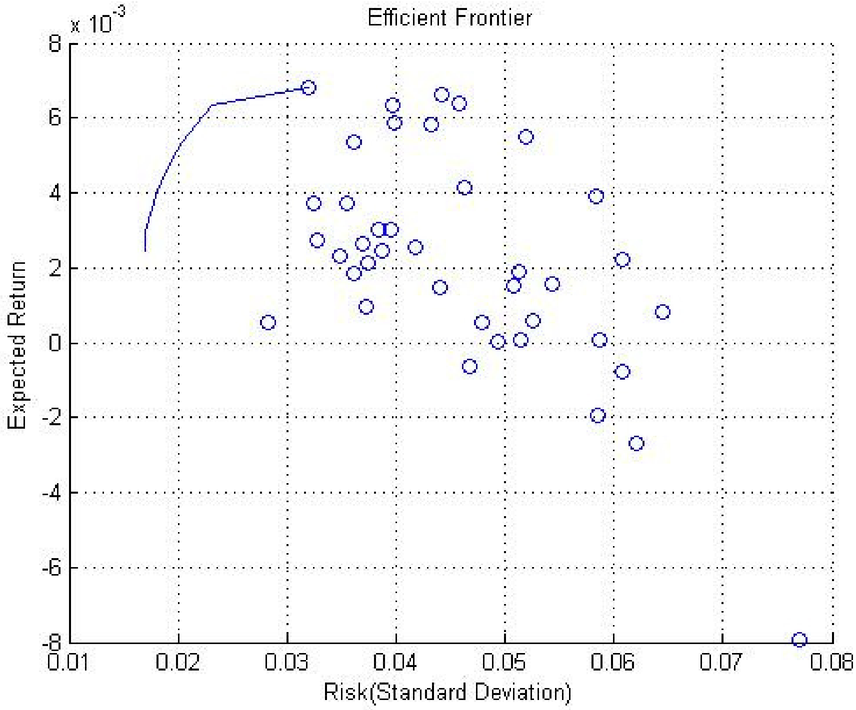

Figure B12.21 Efficient frontier graph for Weekly Top 40 3 (5 year) data with no upper limit constraint

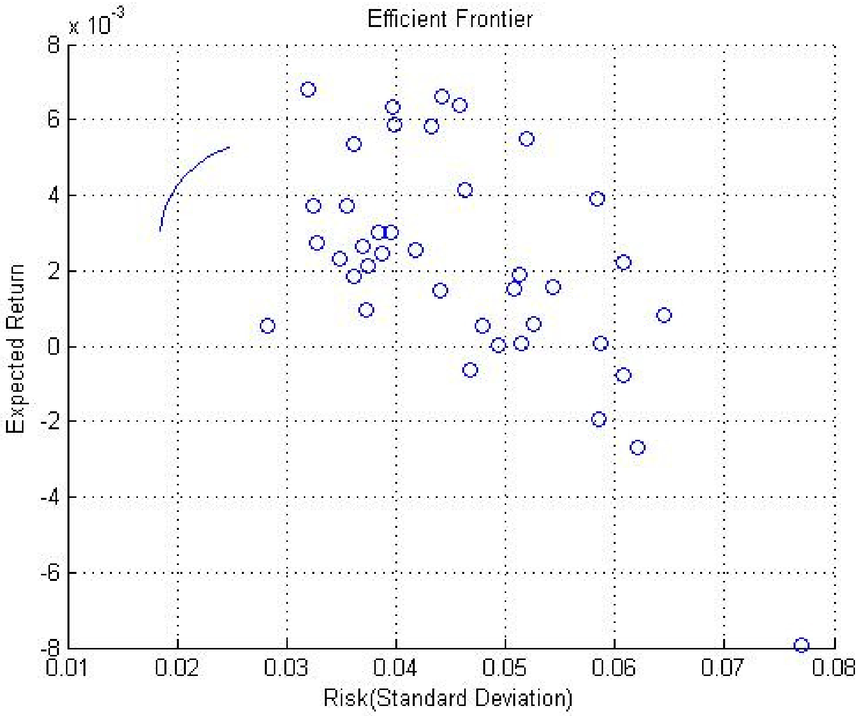

Figure B12.22 Efficient frontier graph for Weekly Top 40 3 (5 year) data with 8% upper limit constraint

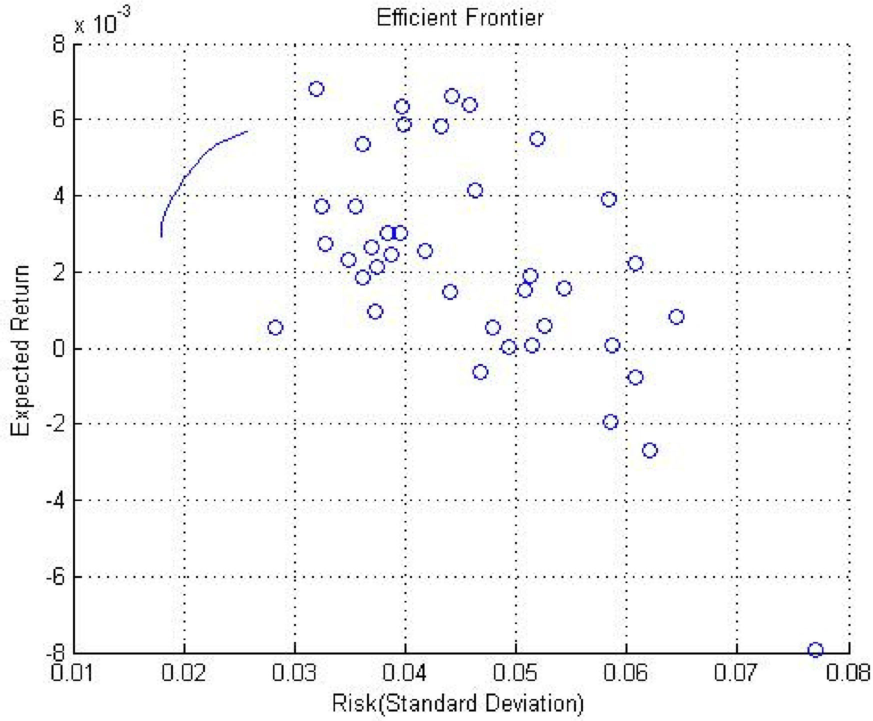

Figure B12.23 Efficient frontier graph for Weekly Top 40 3 (5 year) data with 10% upper limit constraint

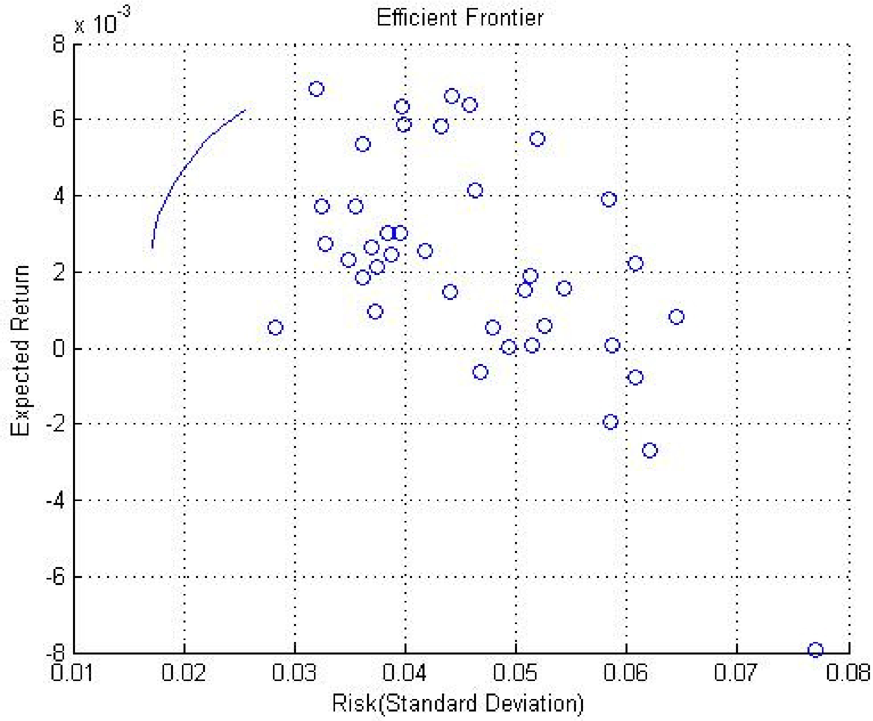

Figure B12.24 Efficient frontier graph for Weekly Top 40 3 (5 year) data with 15% upper limit constraint

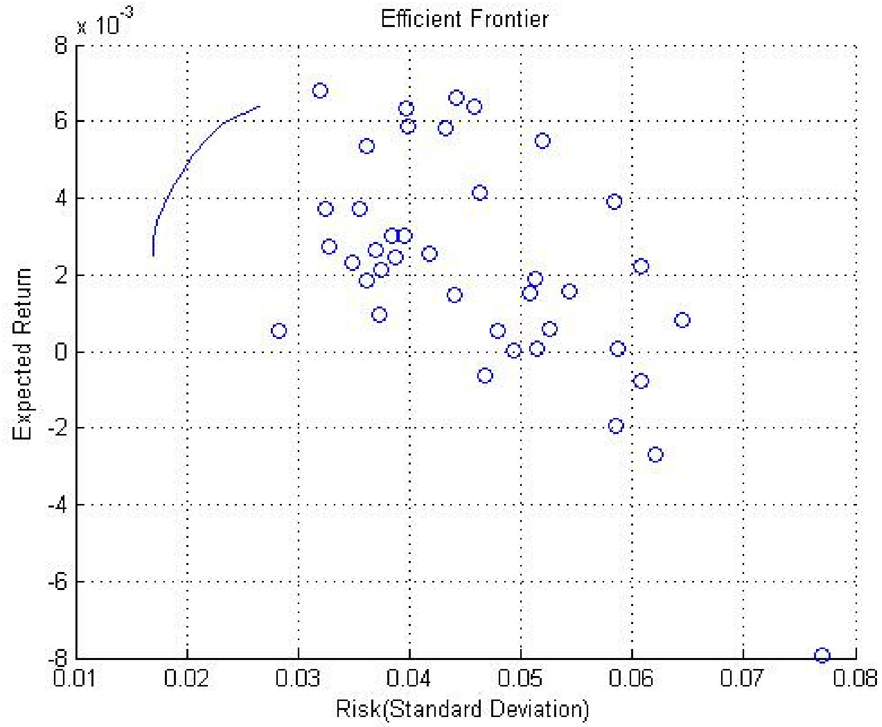

Figure B12.25 Efficient frontier graph for Weekly Top 40 3 (5 year) data with 20% upper limit constraint

12.2.6 Yearly rebalance, Weekly 1 (15 year data) data sets

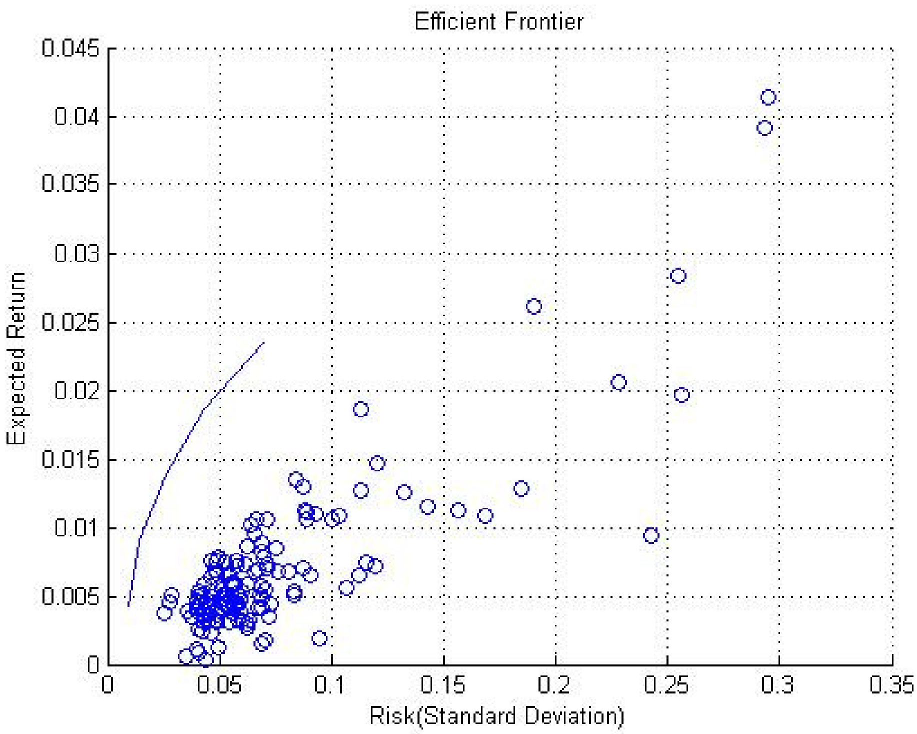

Figure B12.26 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2006)

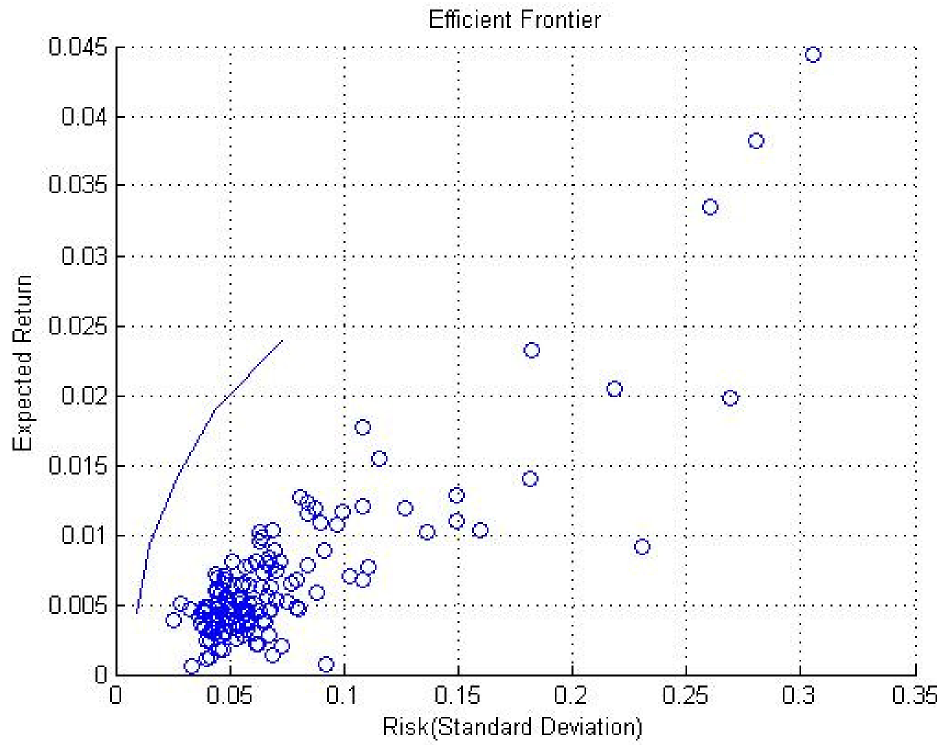

Figure B12.27 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2007)

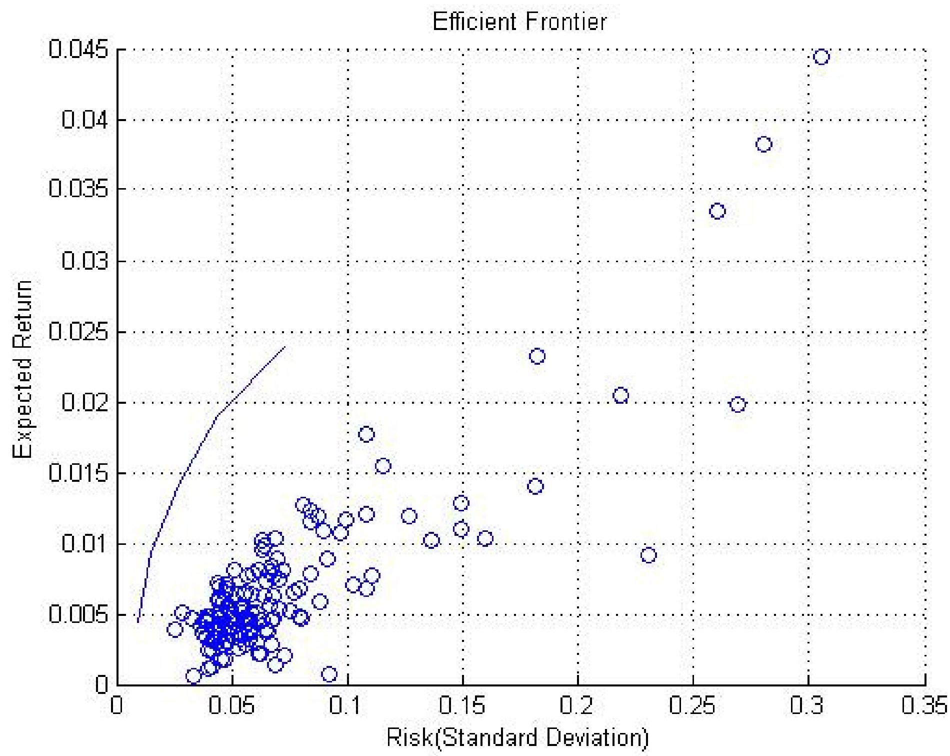

Figure B12.28 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2008)

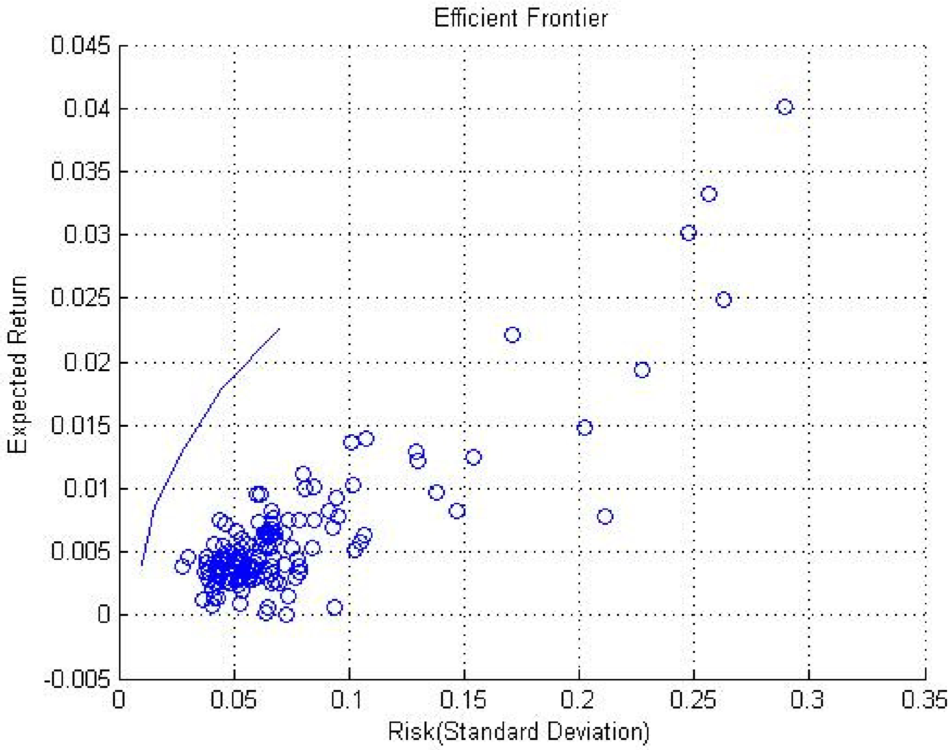

Figure B12.29 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2009)

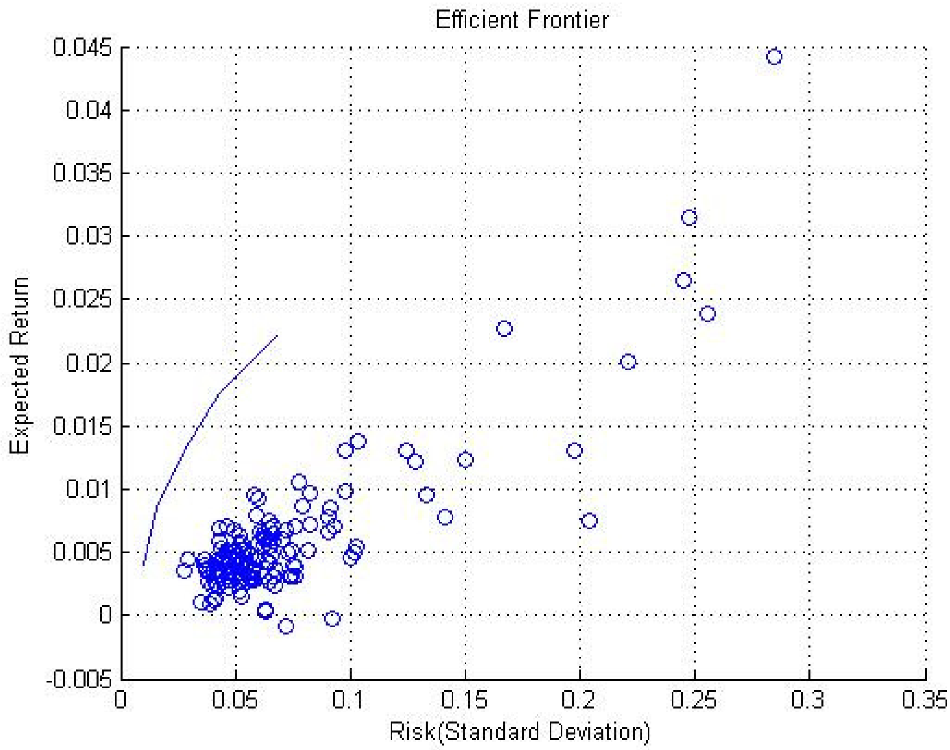

Figure B12.30 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2010)

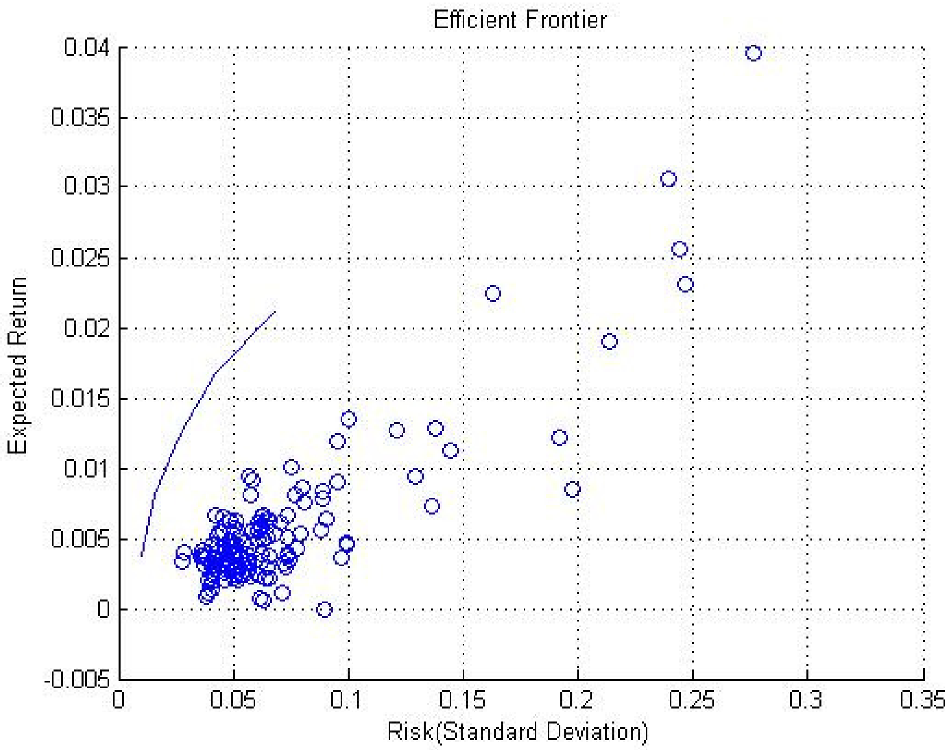

Figure B12.31 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2011)

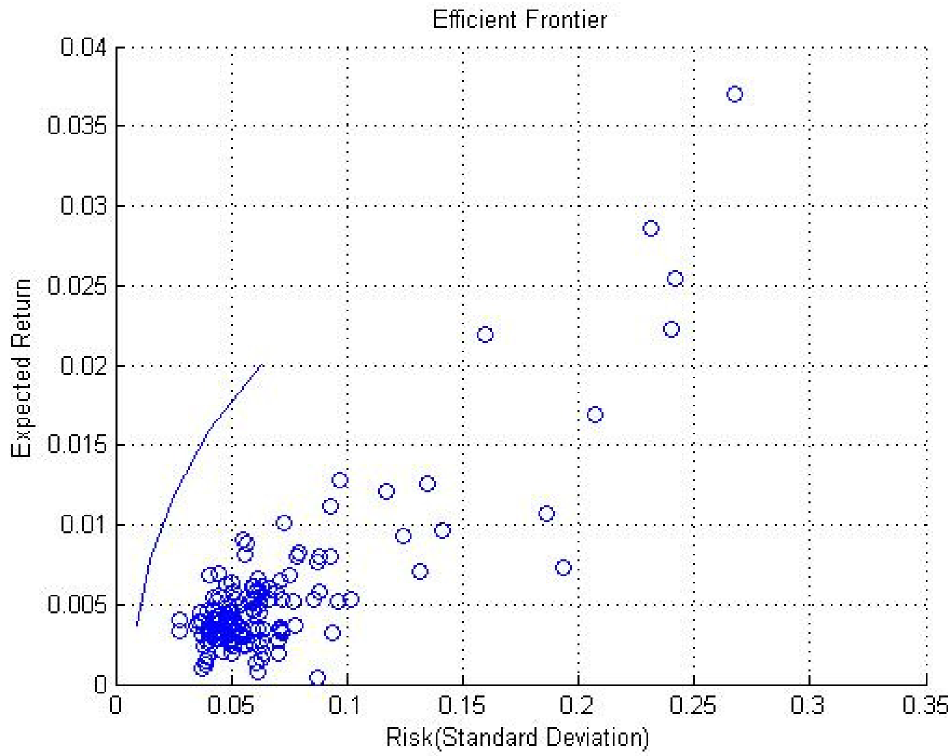

Figure B12.32 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 10% upper limit constraint (end year 2012)

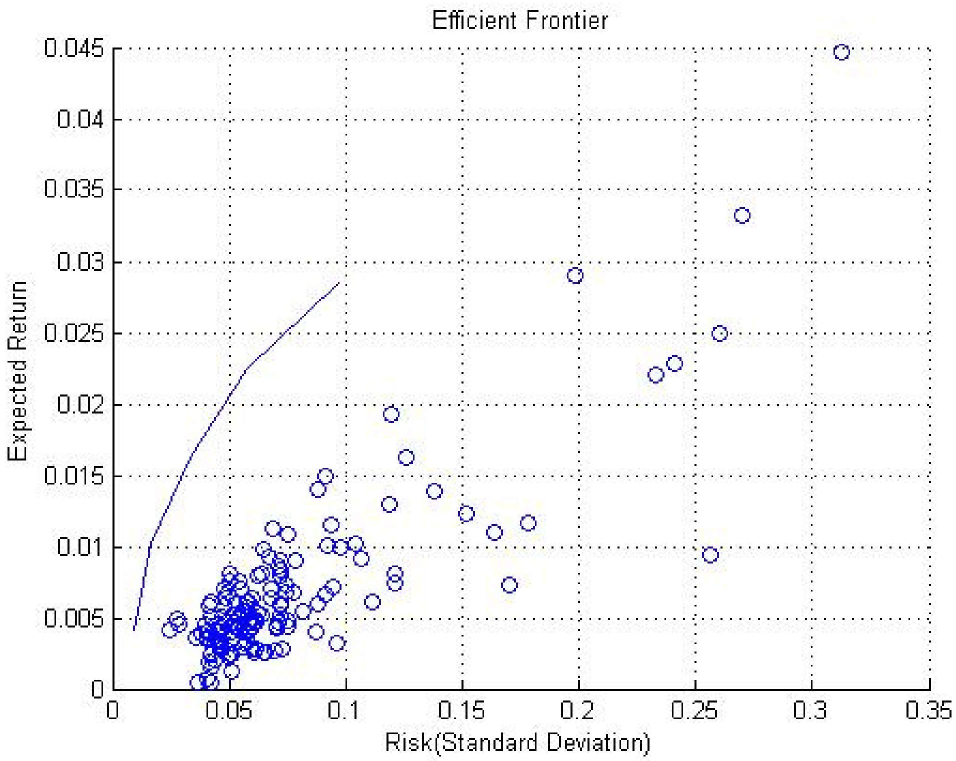

Figure B12.33 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2005)

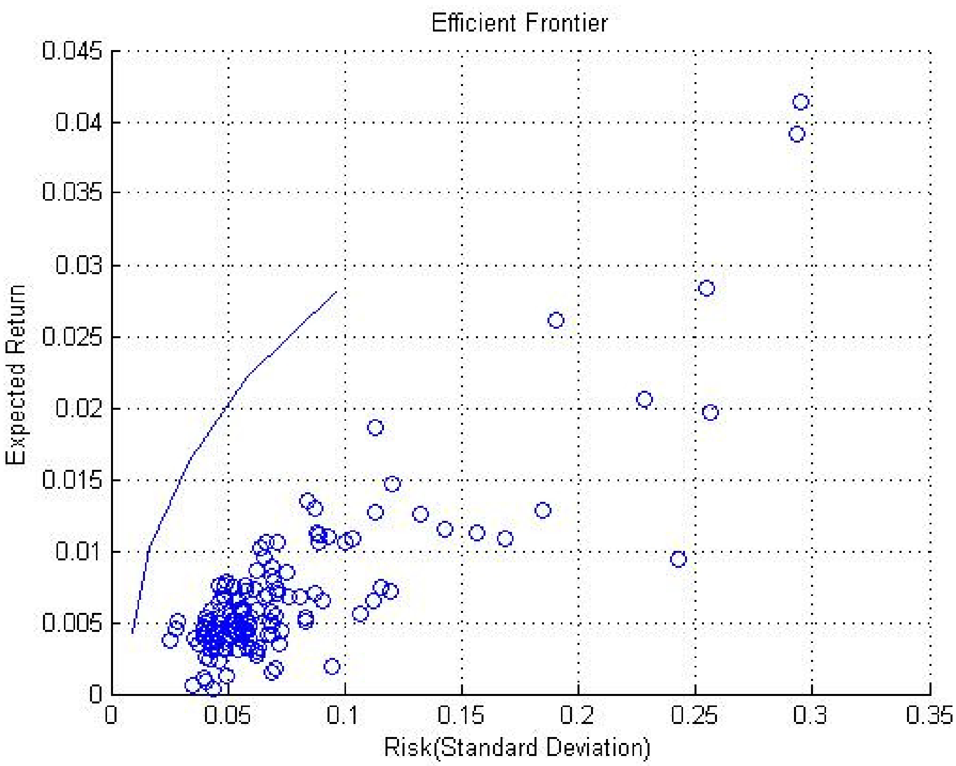

Figure B12.34 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2006)

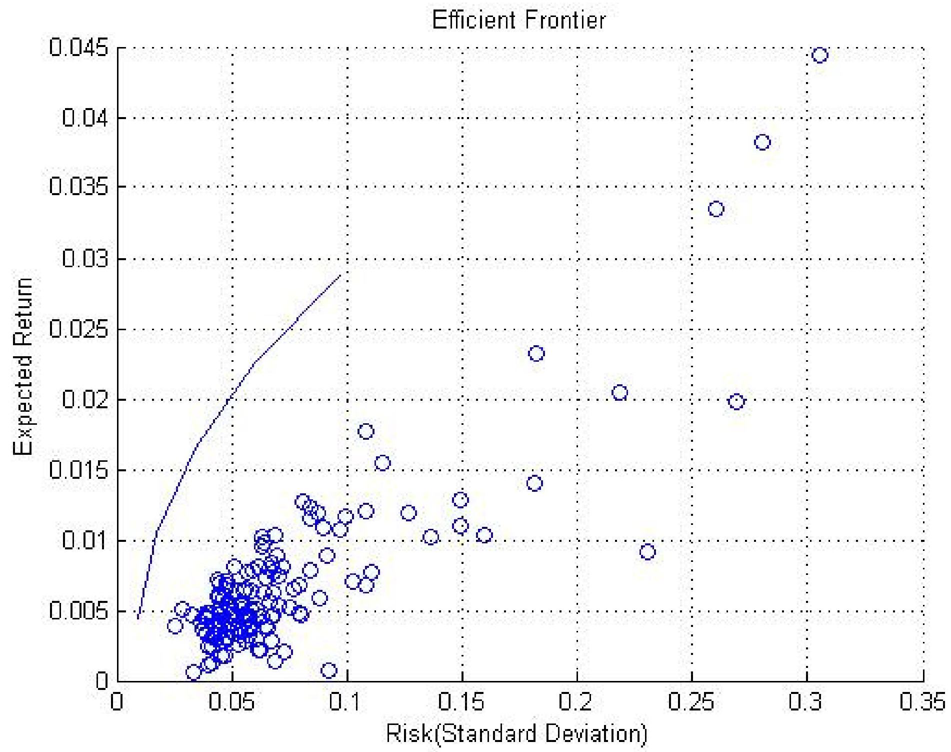

Figure B12.35 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2007)

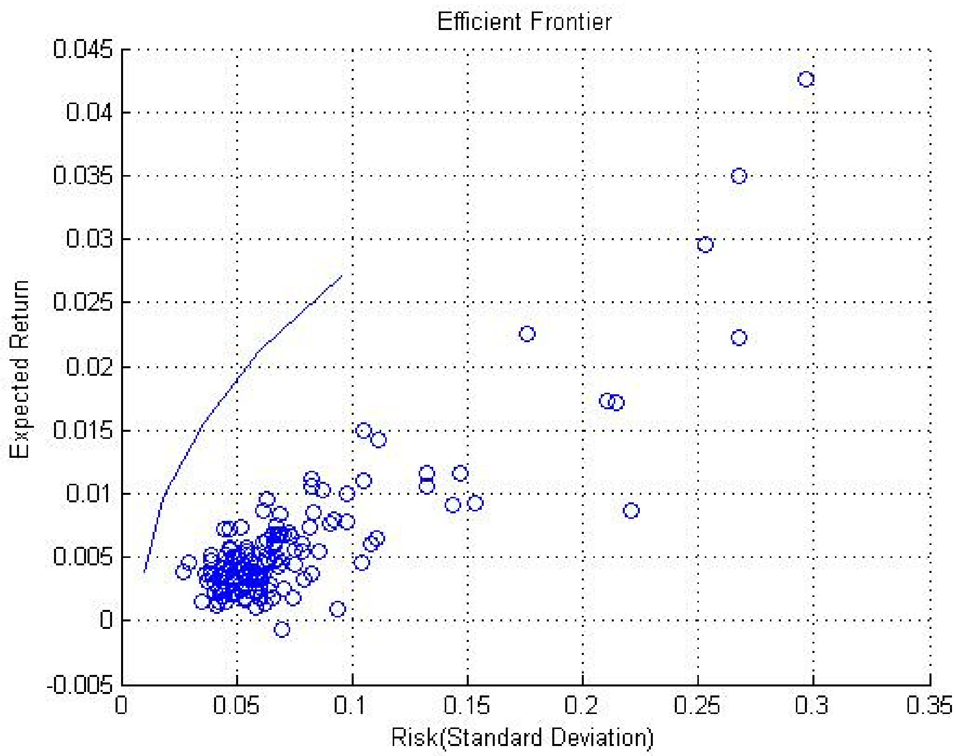

Figure B12.36 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2008)

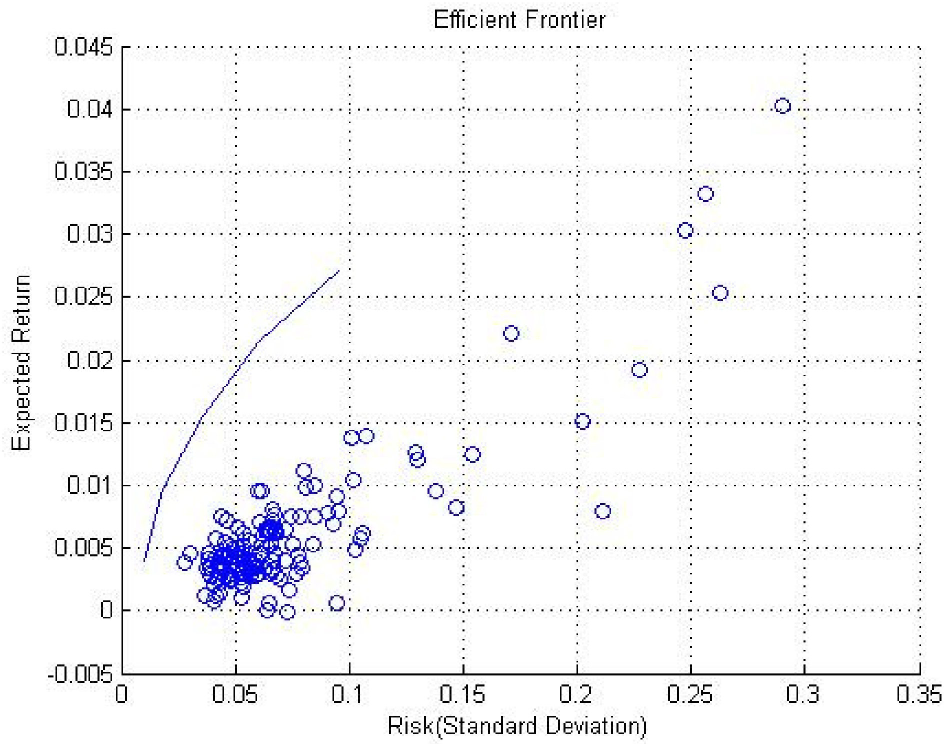

Figure B12.37 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2009)

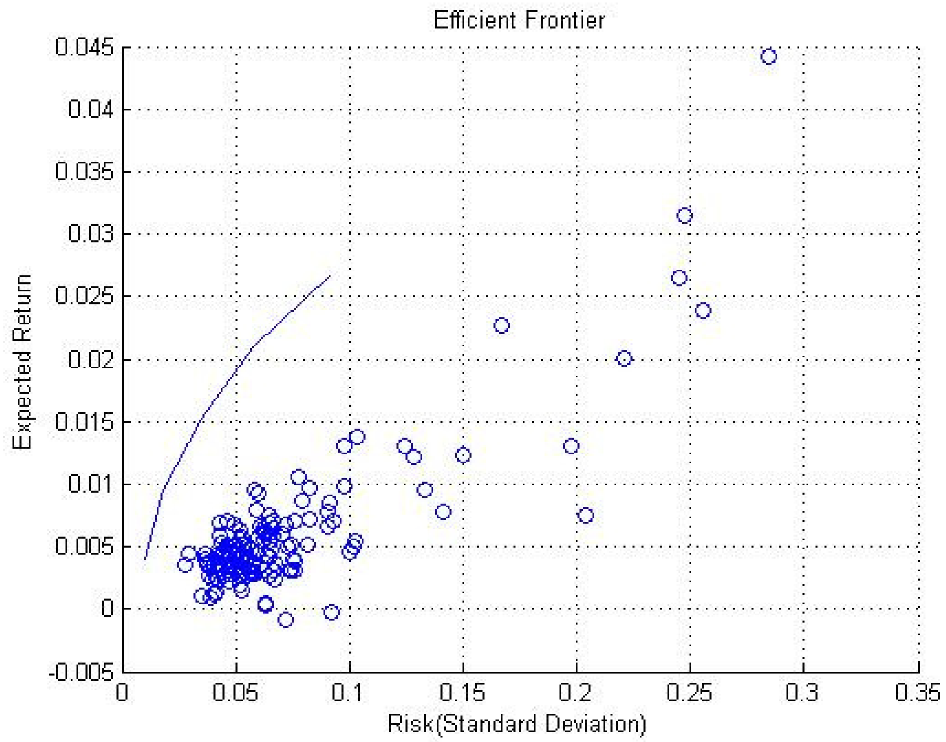

Figure B12.38 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2010)

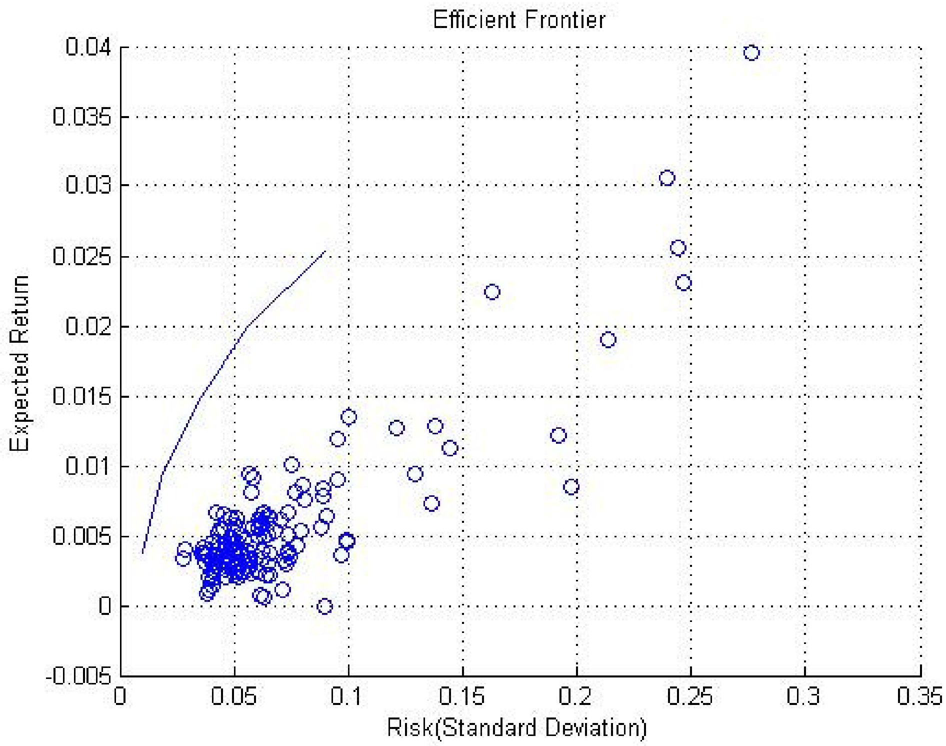

Figure B12.39 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2011)

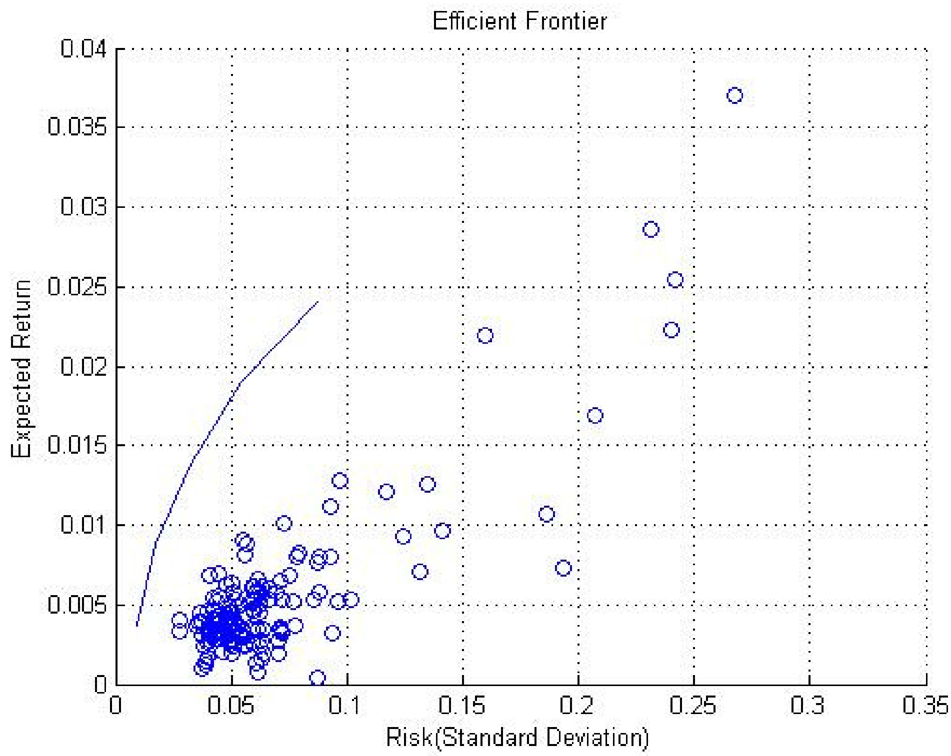

Figure B12.40 Efficient frontier graph for Rebalance Weekly 1 (15 year) data with 15% upper limit constraint (end year 2012)

12.2.7 Yearly rebalance, Weekly 2 (10 year) data sets

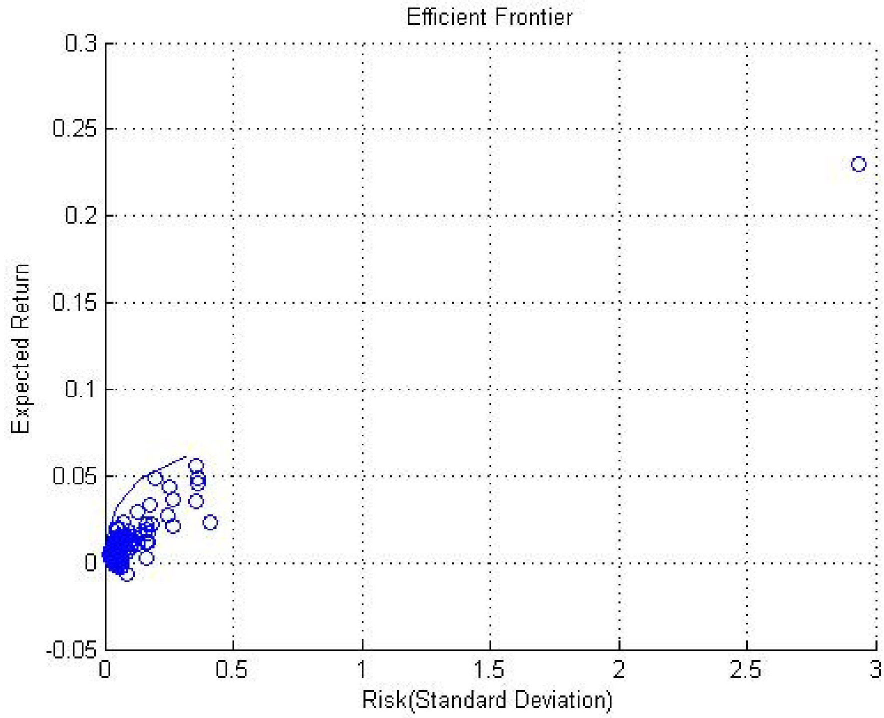

Figure B12.41 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2005)

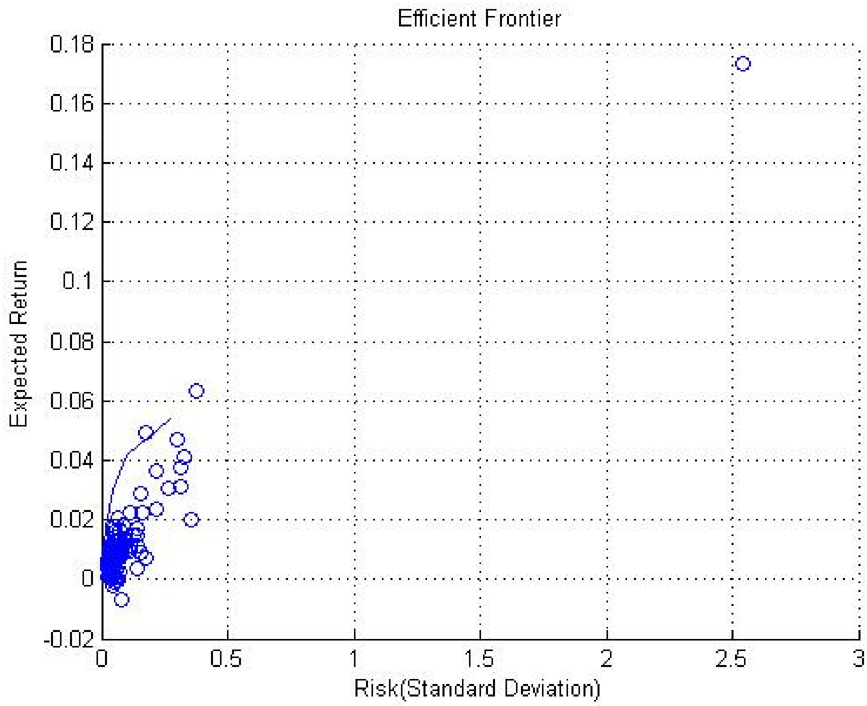

Figure B12.42 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2006)

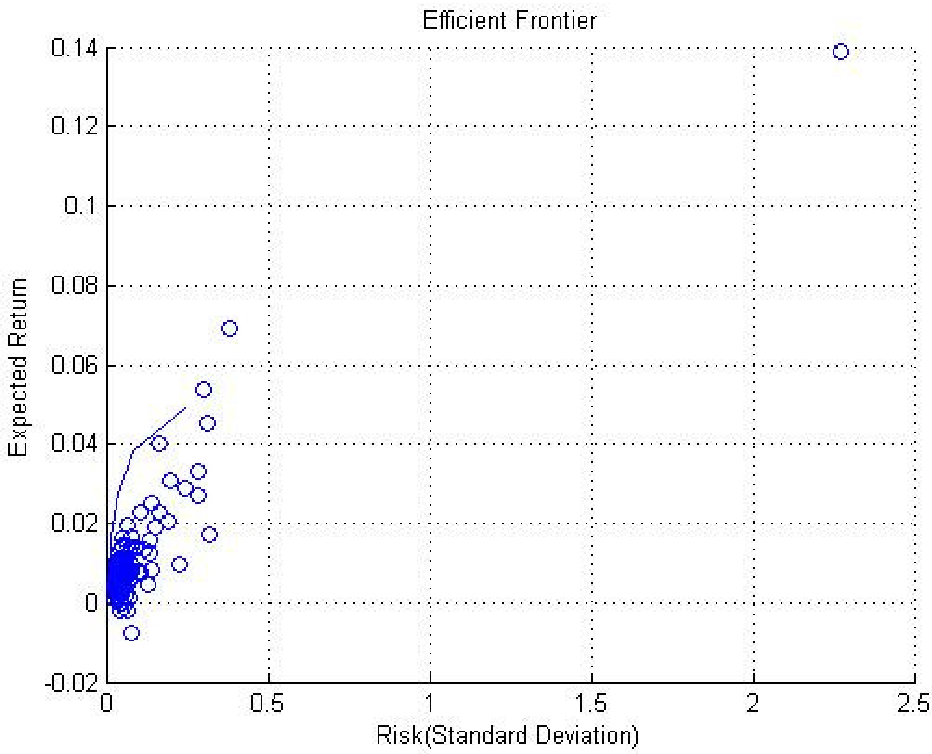

Figure B12.43 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2007)

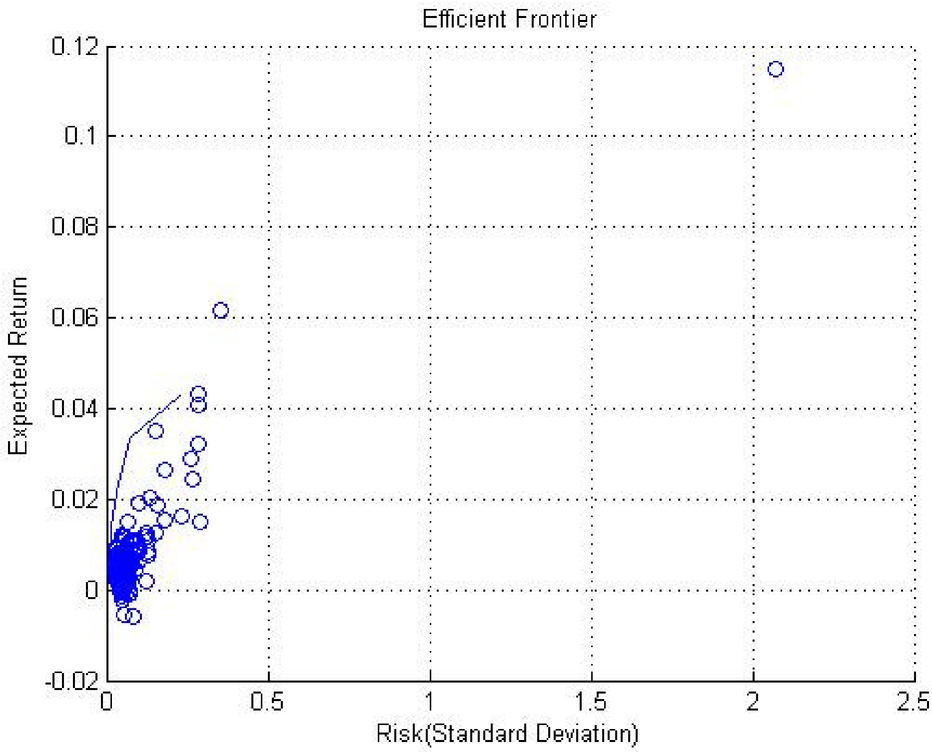

Figure B12.44 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2008)

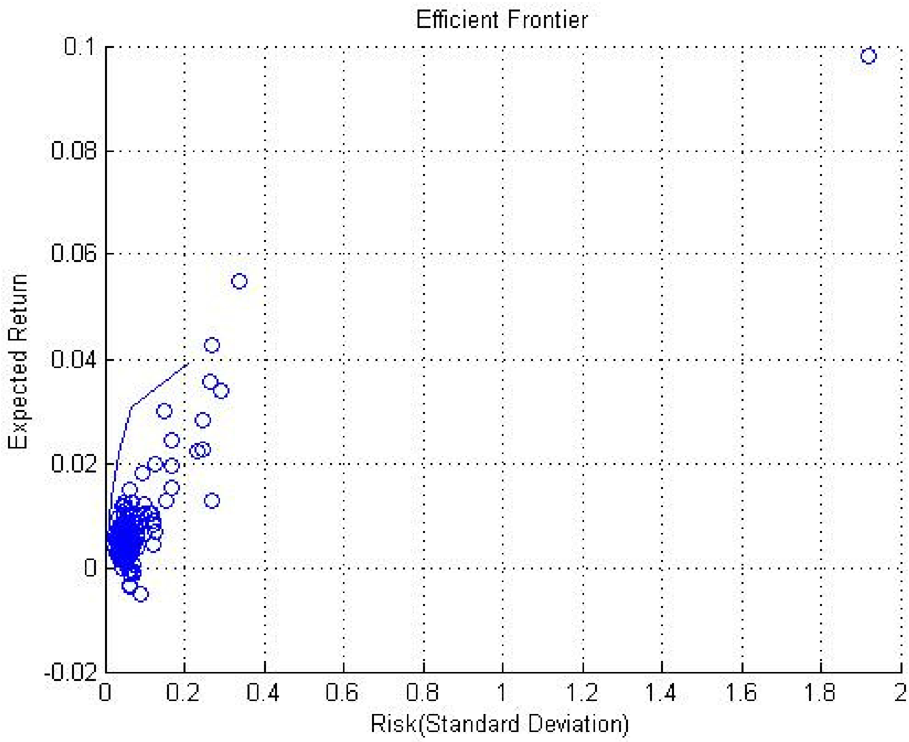

Figure B12.45 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2009)

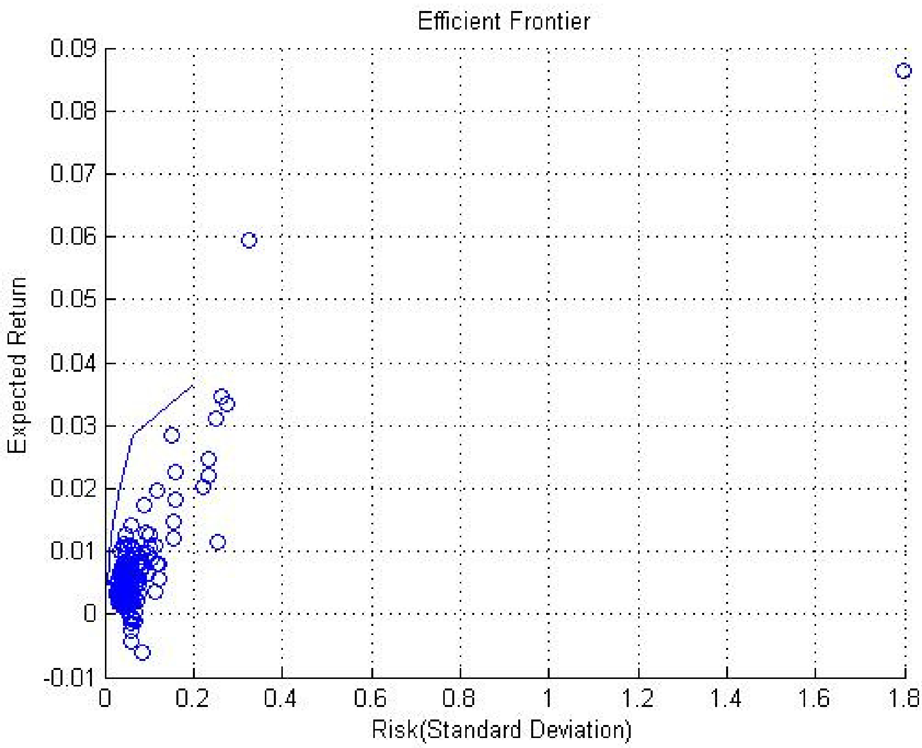

Figure B12.46 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2010)

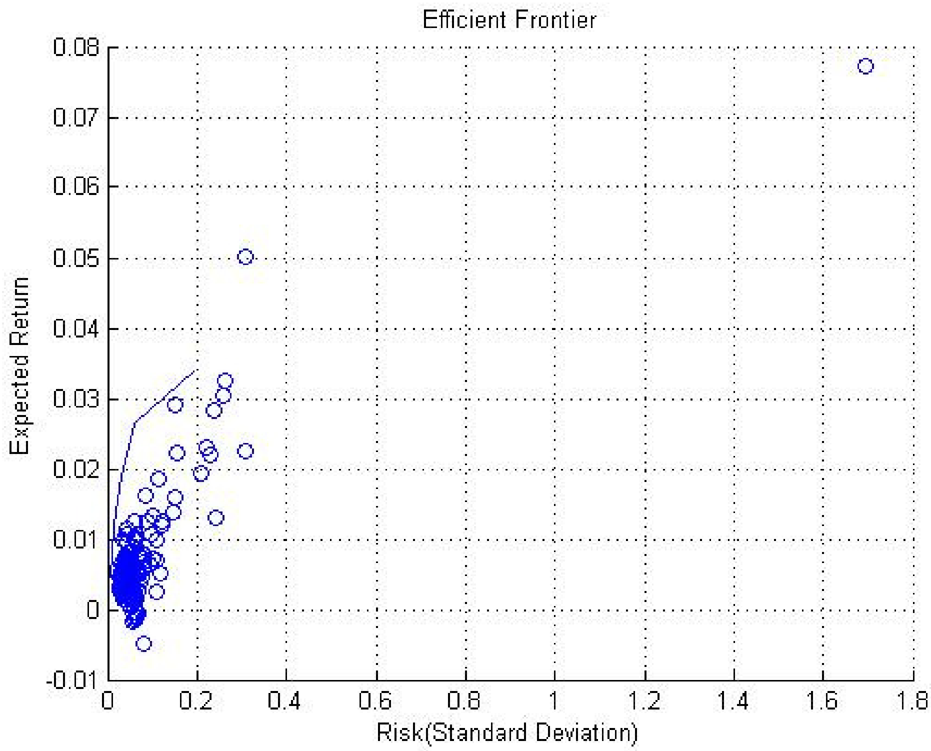

Figure B12.47 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2011)

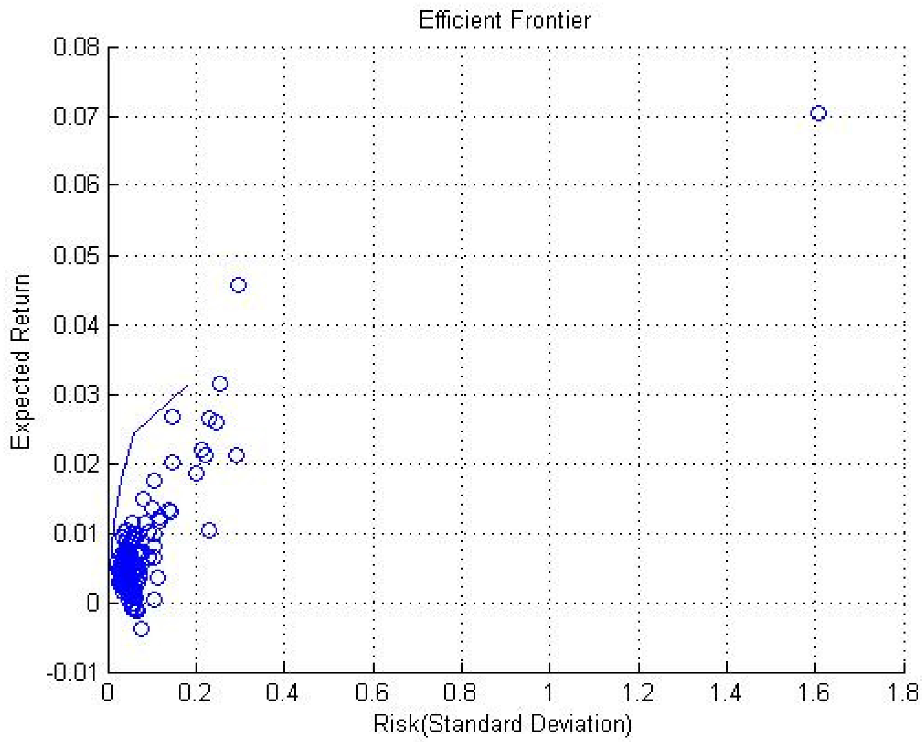

Figure B12.48 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 10% upper limit constraint (end year 2012)

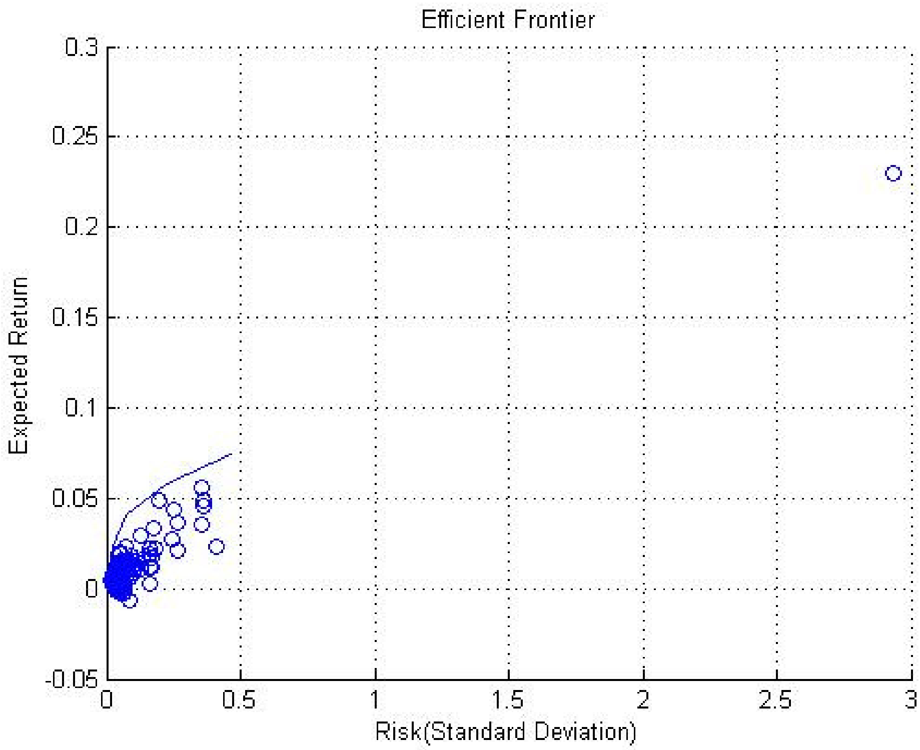

Figure B12.49 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2005)

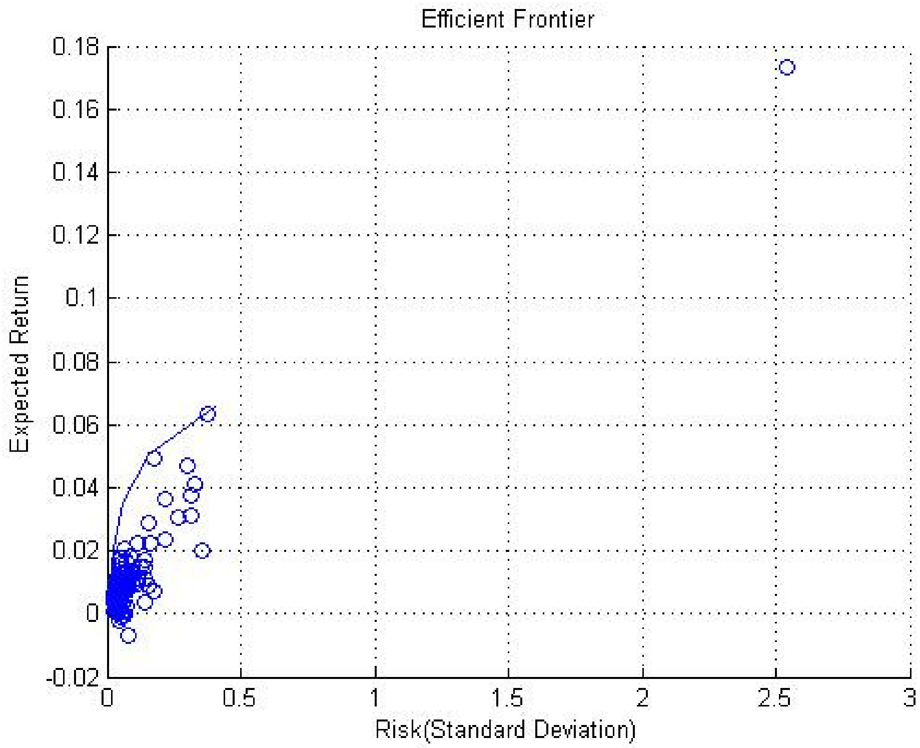

Figure B12.50 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2006)

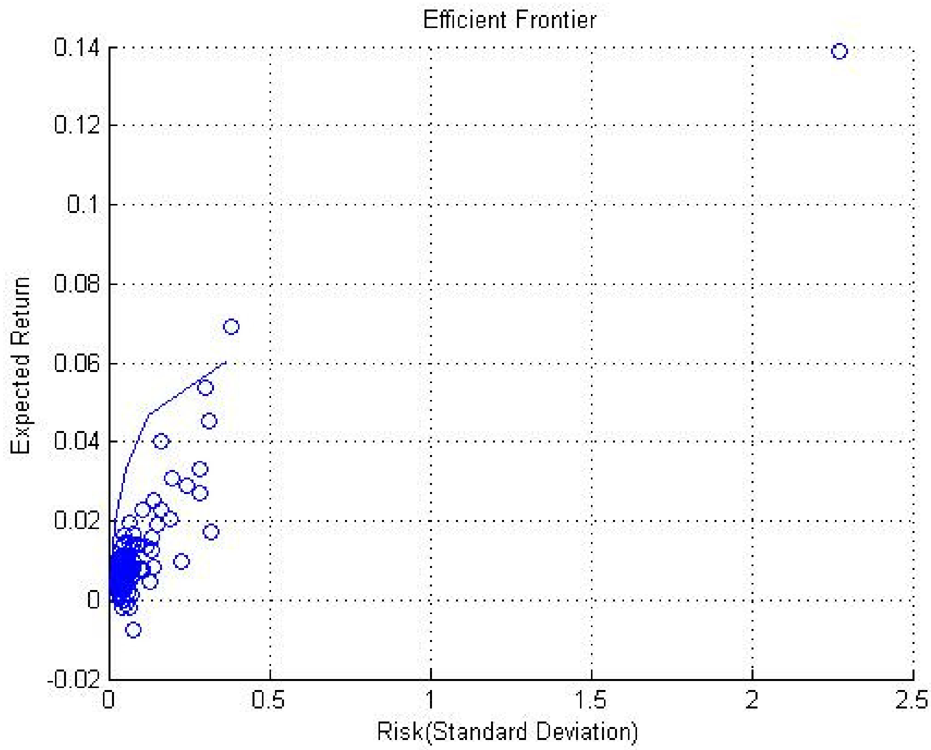

Figure B12.51 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2007)

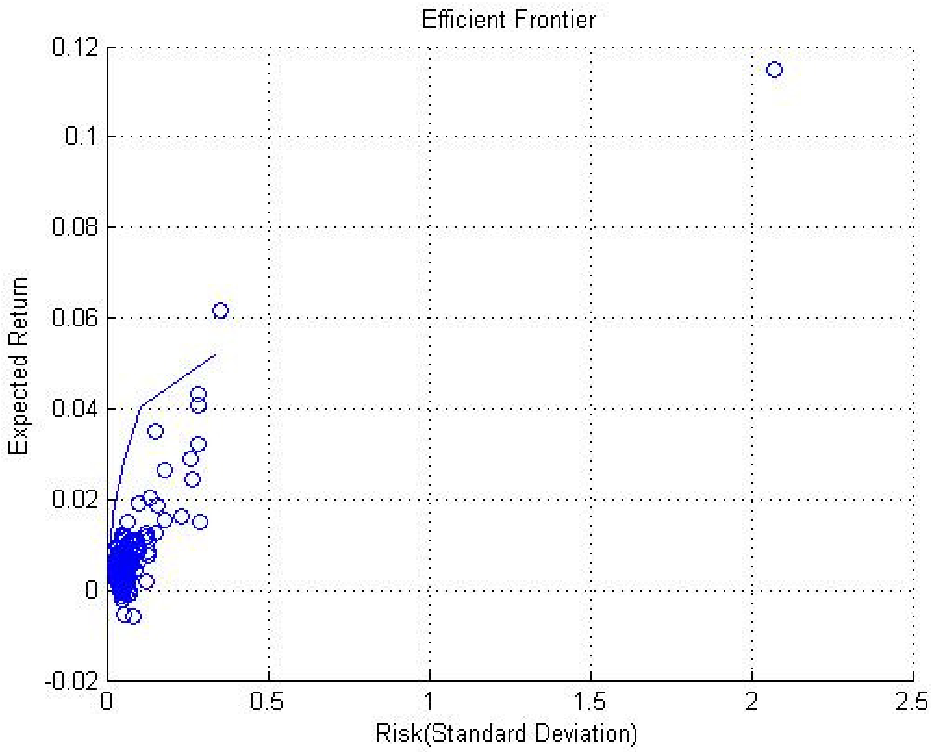

Figure B12.52 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2008)

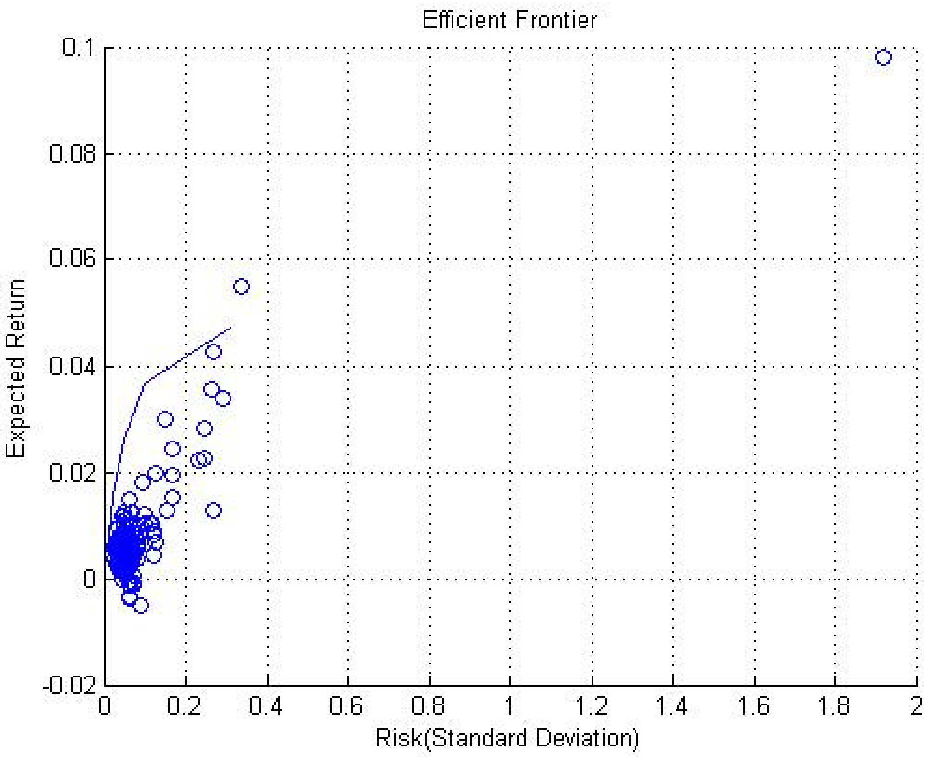

Figure B12.53 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2009)

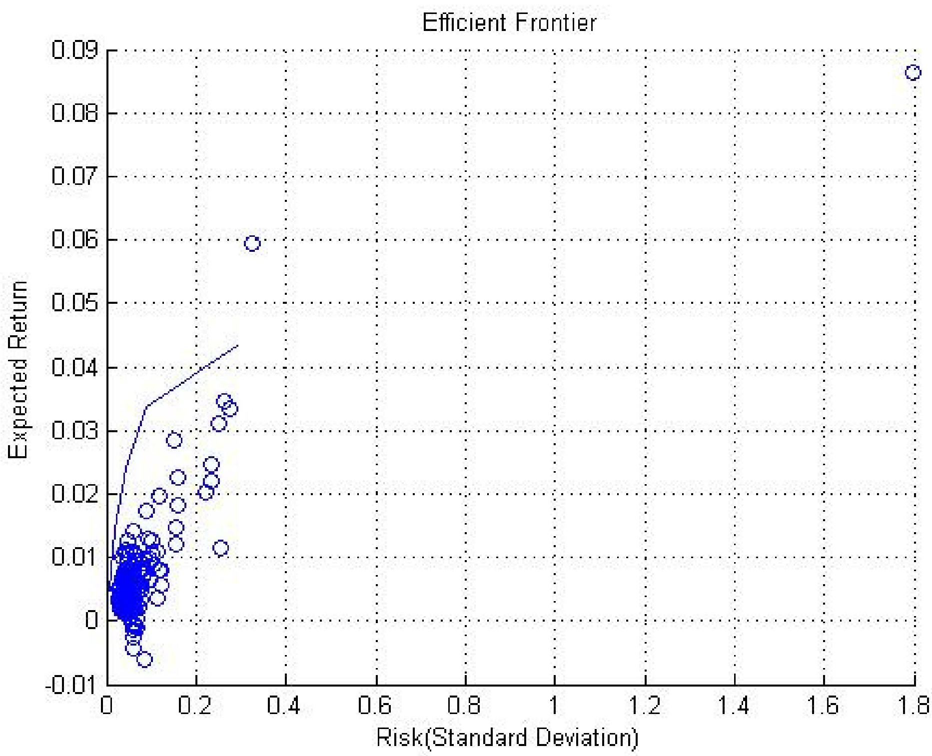

Figure B12.54 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2010)

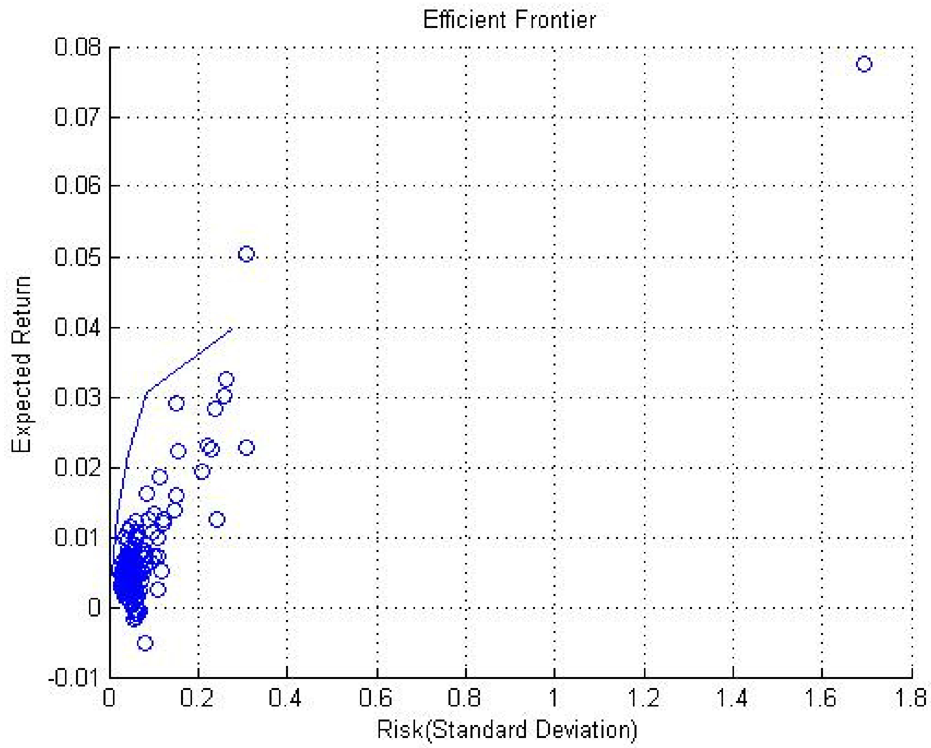

Figure B12.55 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2011)

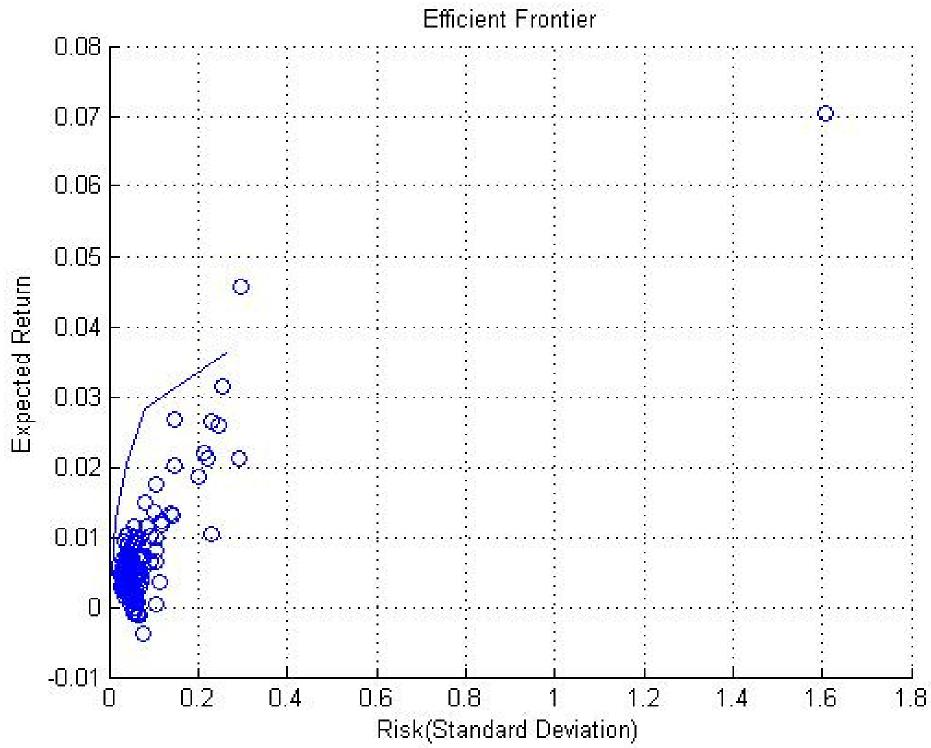

Figure B12.56 Efficient frontier graph for Rebalance Weekly 2 (10 year) data with 15% upper limit constraint (end year 2012)

12.2.8 Yearly rebalance, Weekly 3 (5 year) data sets

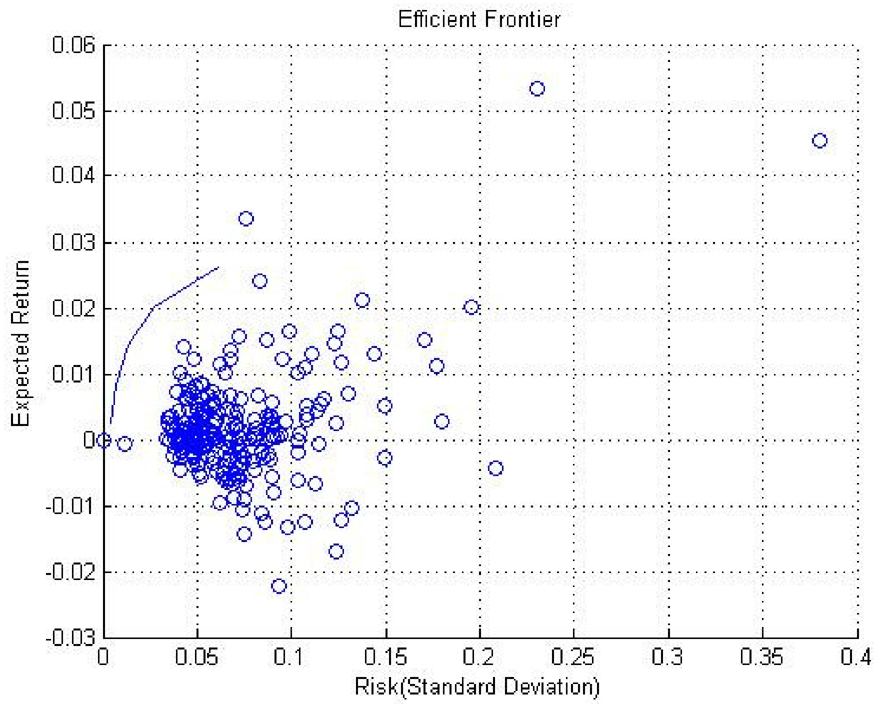

Figure B12.57 Efficient frontier graph for Rebalance Weekly (5 year) data with 10% upper limit constraint (end year 2009)

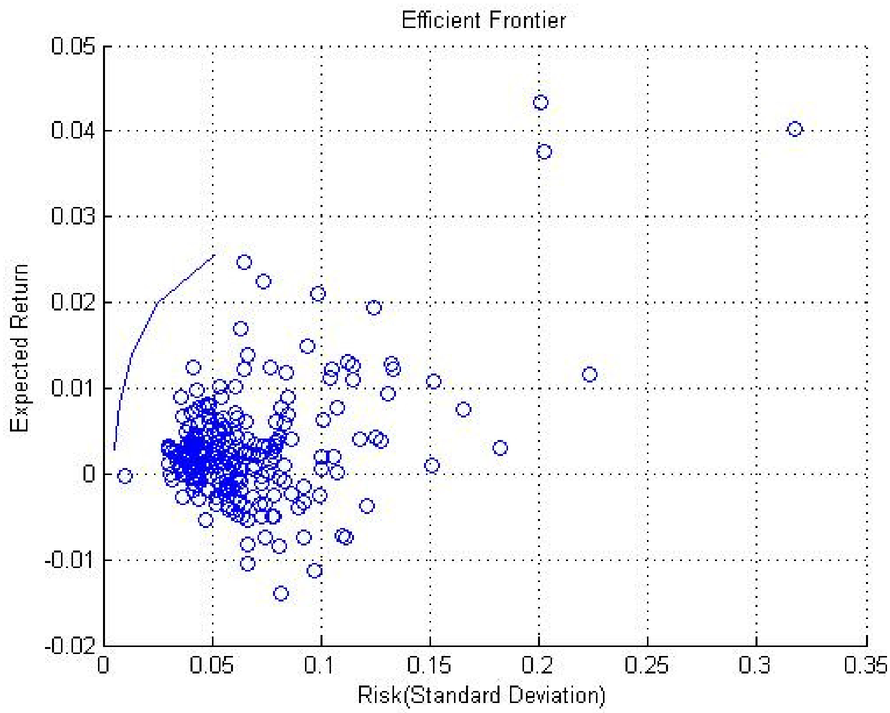

Figure B12.58 Efficient frontier graph for Rebalance Weekly (5 year) data with 10% upper limit constraint (end year 2010)

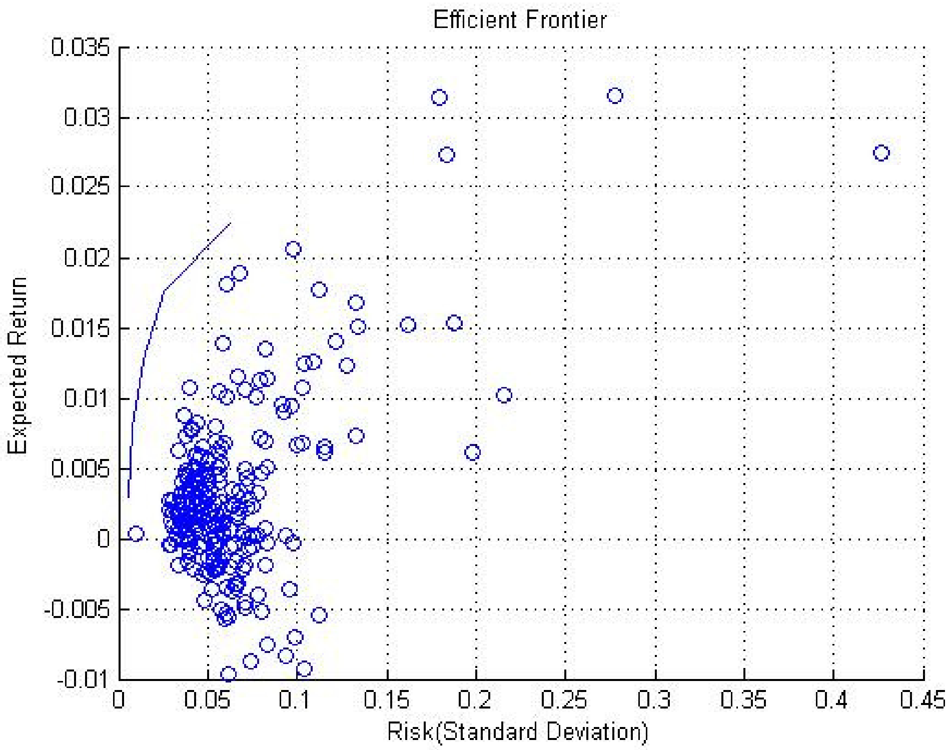

Figure B12.59 Efficient frontier graph for Rebalance Weekly (5 year) data with 10% upper limit constraint (end year 2011)

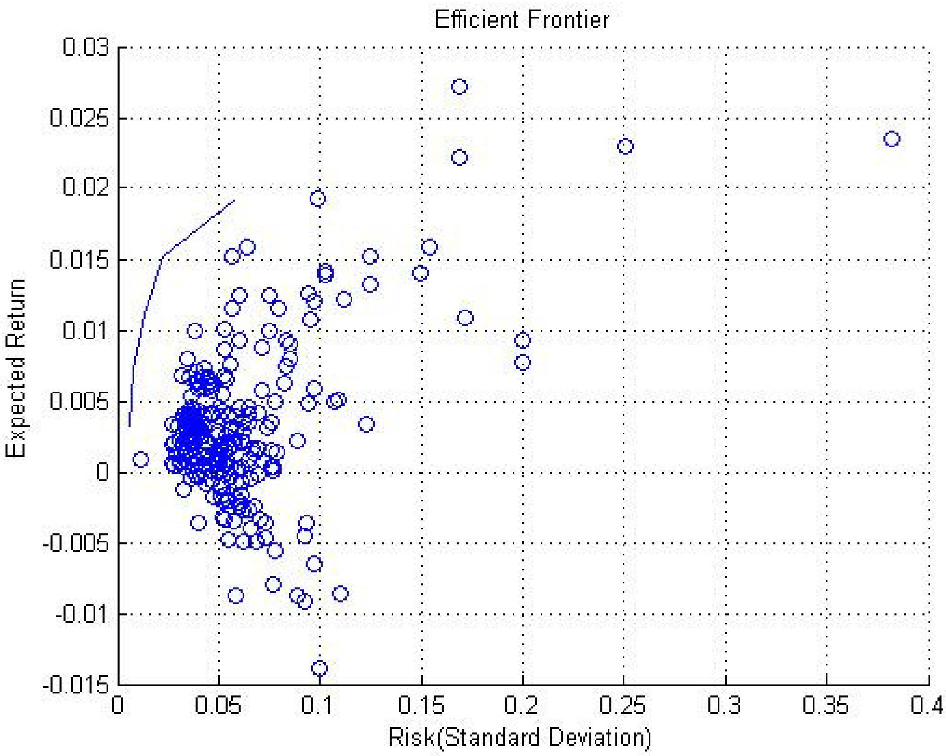

Figure B12.60 Efficient frontier graph for Rebalance Weekly (5 year) data with 10% upper limit constraint (end year 2012)

12.2.9 Yearly rebalance, Weekly Top 40 (15 year) data sets

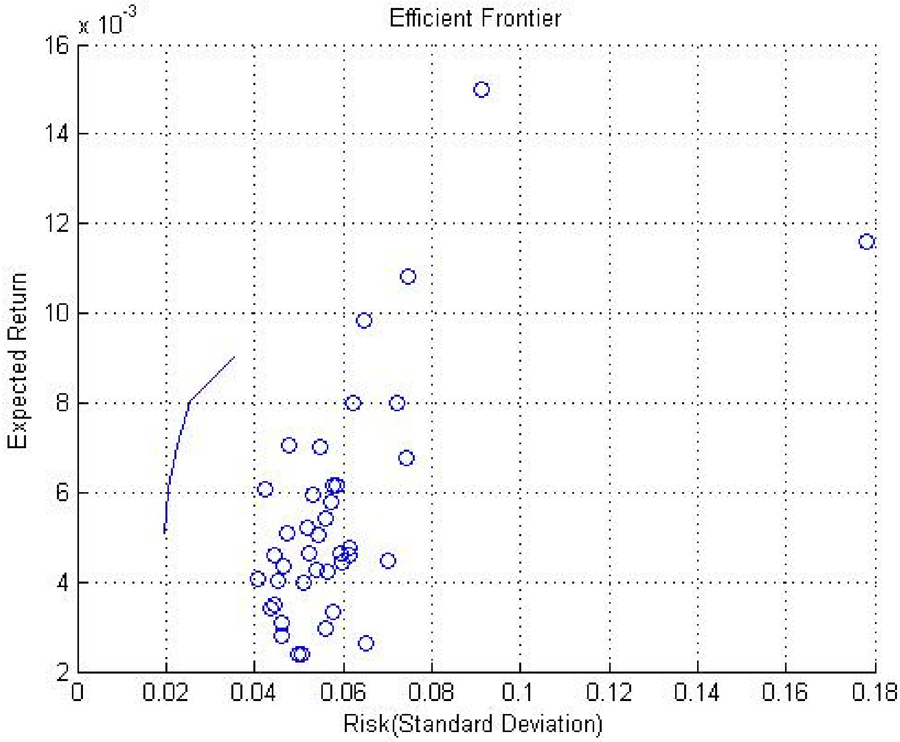

Figure B12.61 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2005)

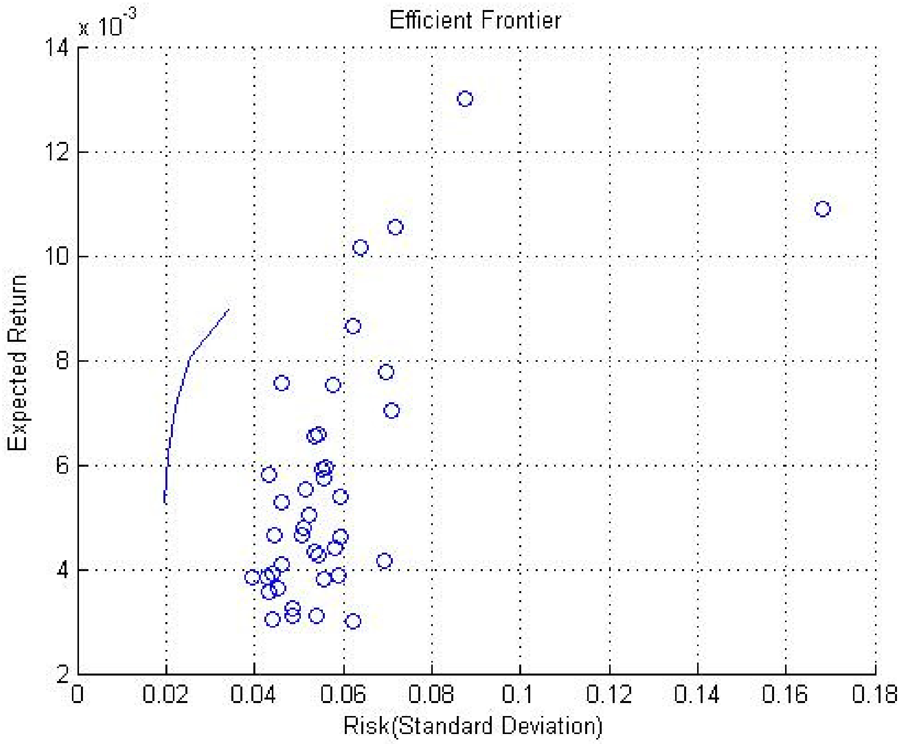

Figure B12.62 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2006)

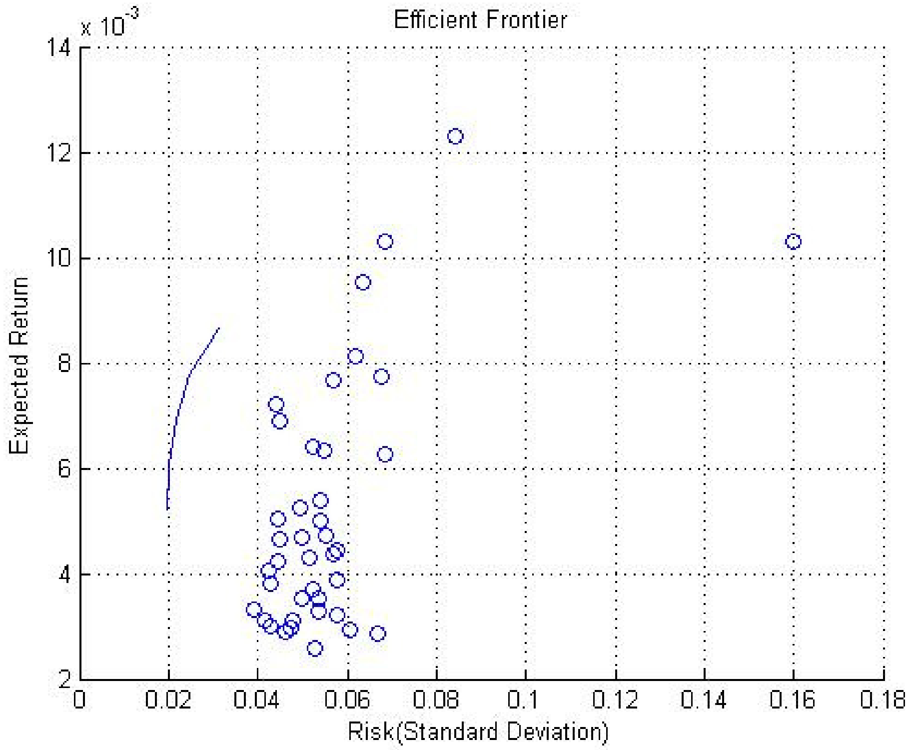

Figure B12.63 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2007)

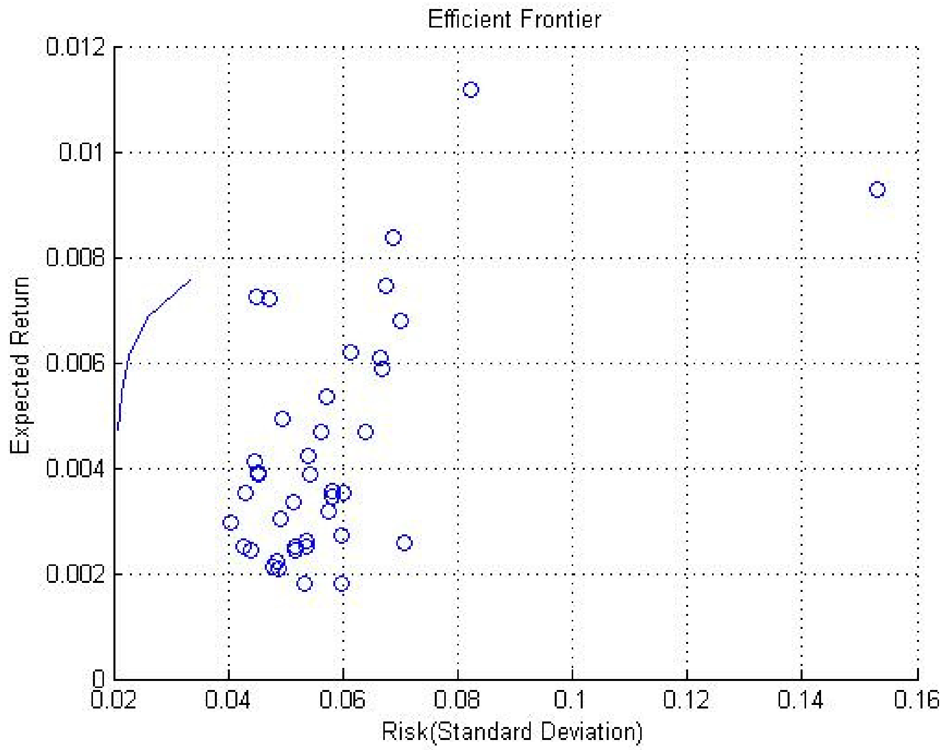

Figure B12.64 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2008)

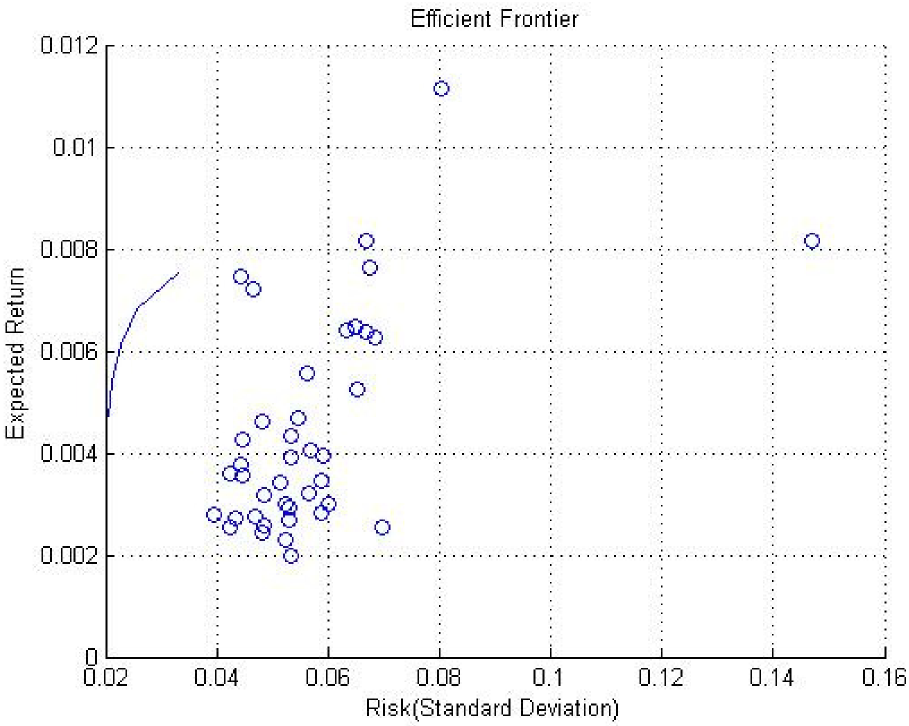

Figure B12.65 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2009)

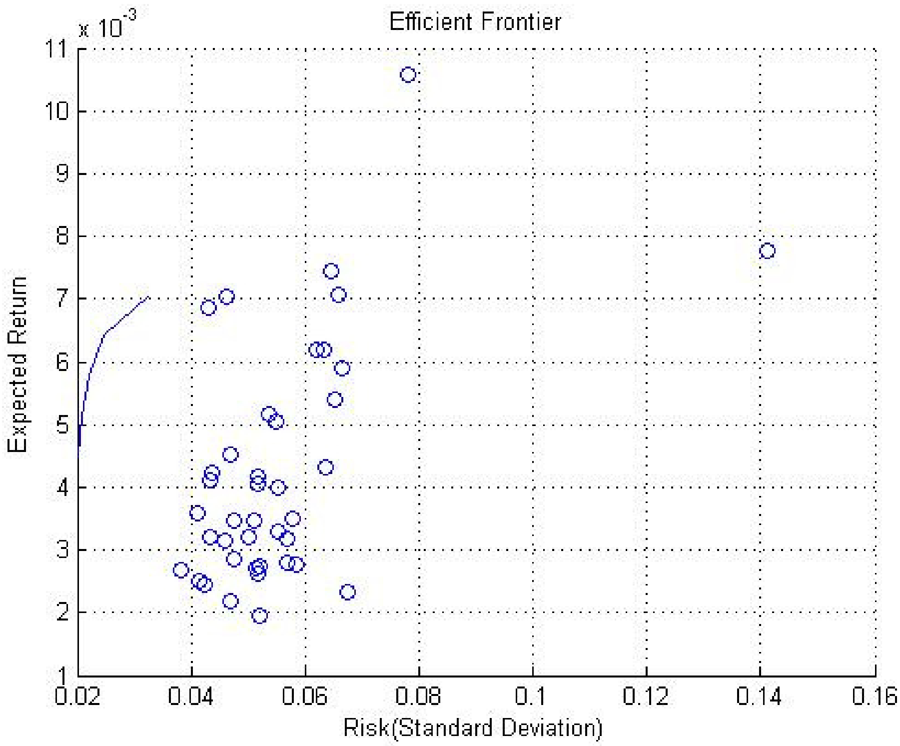

Figure B12.66 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2010)

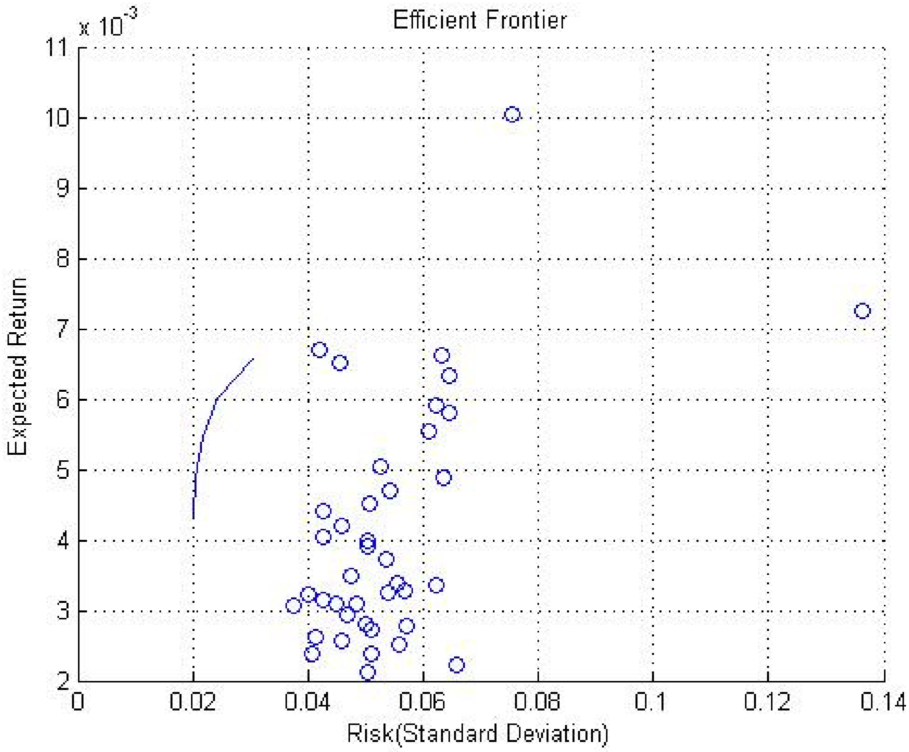

Figure B12.67 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2011)

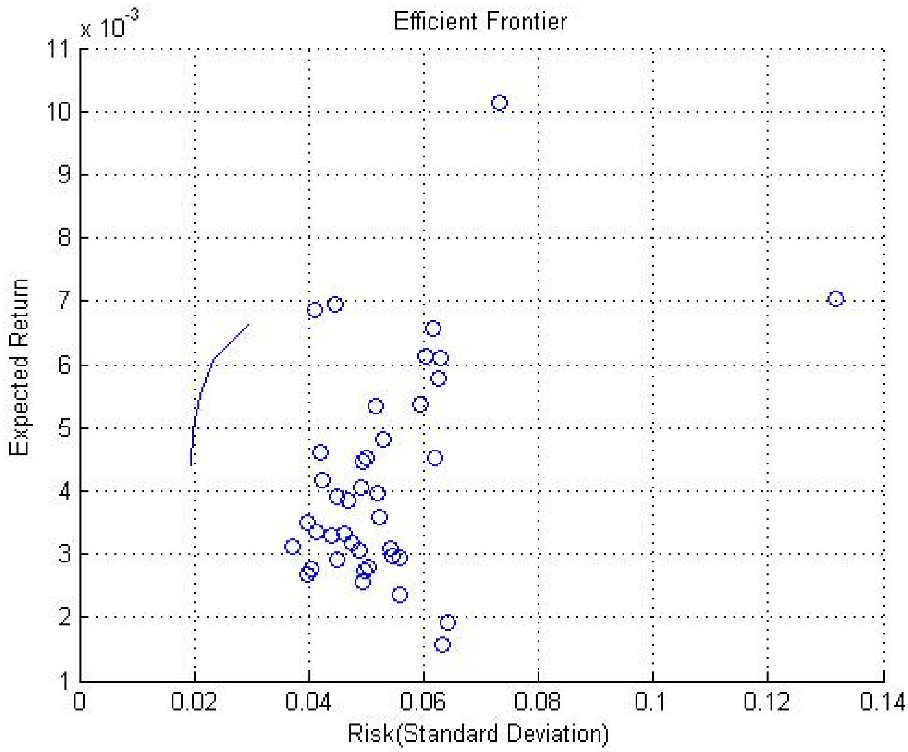

Figure B12.68 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 10% upper limit constraint (end year 2012)

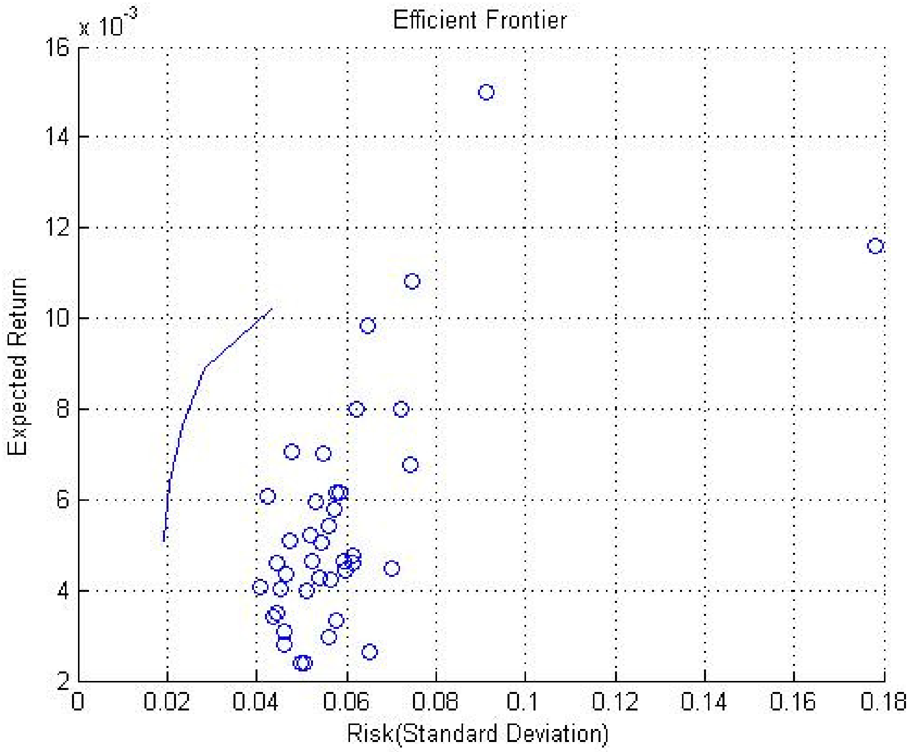

Figure B12.69 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2005)

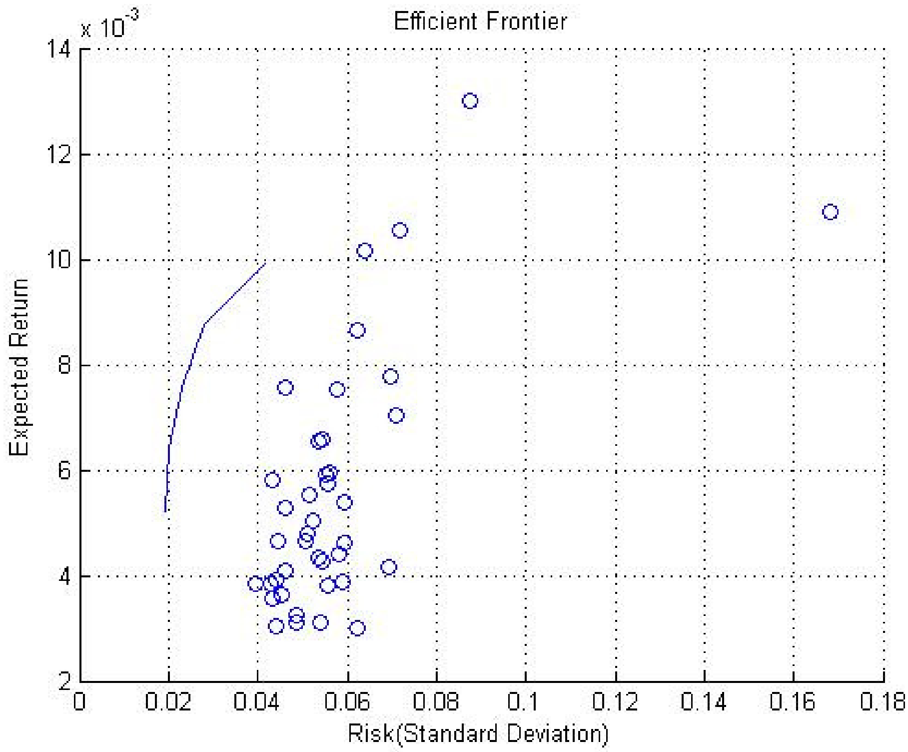

Figure B12.70 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2006)

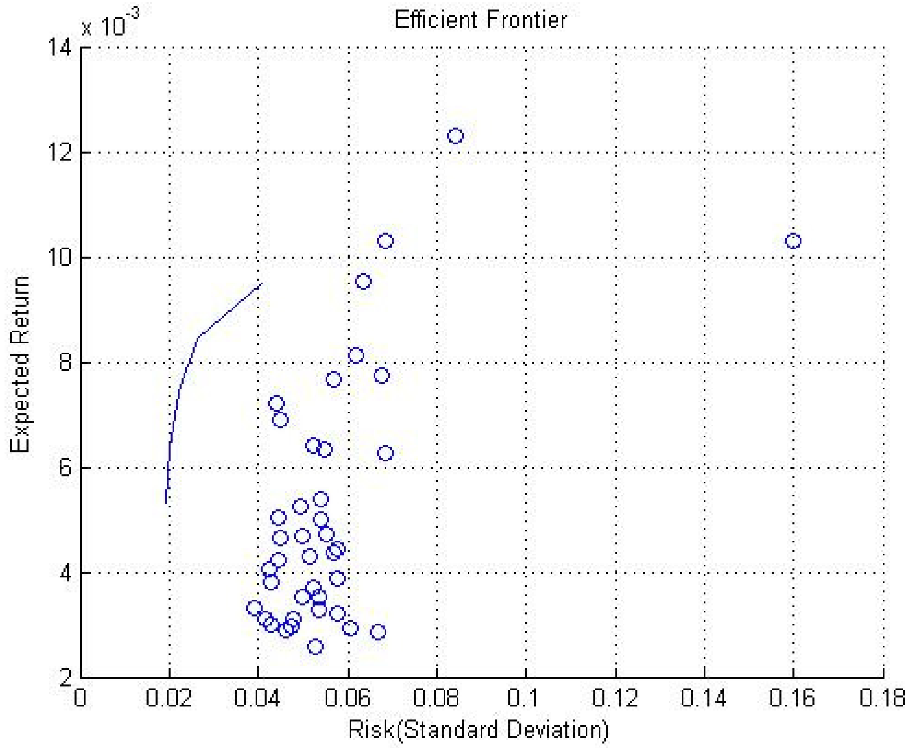

Figure B12.71 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2007)

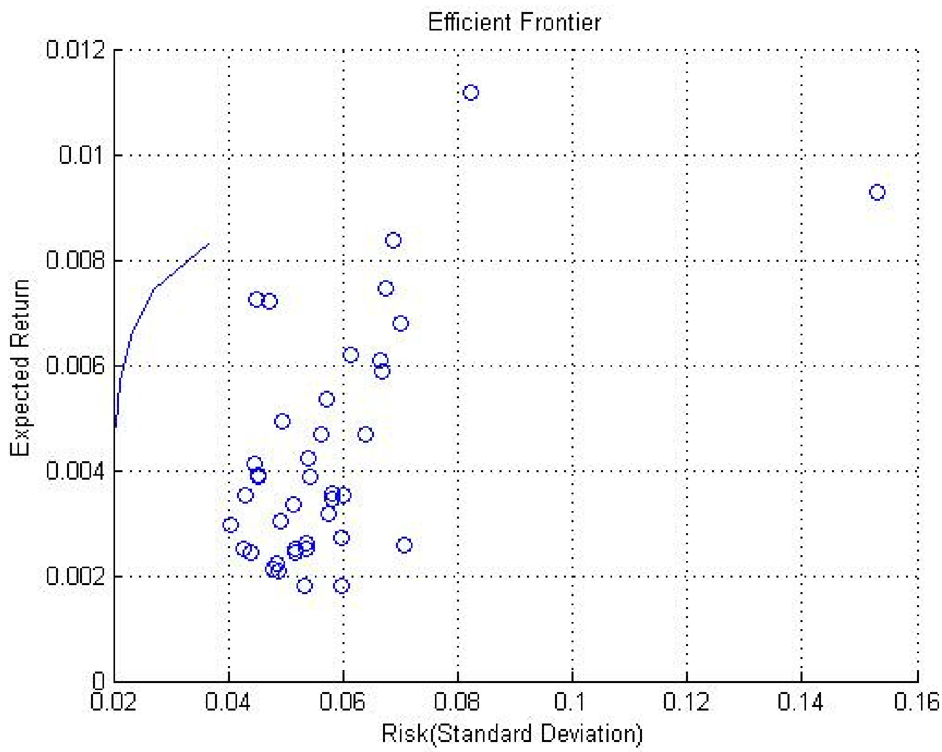

Figure B12.72 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2008)

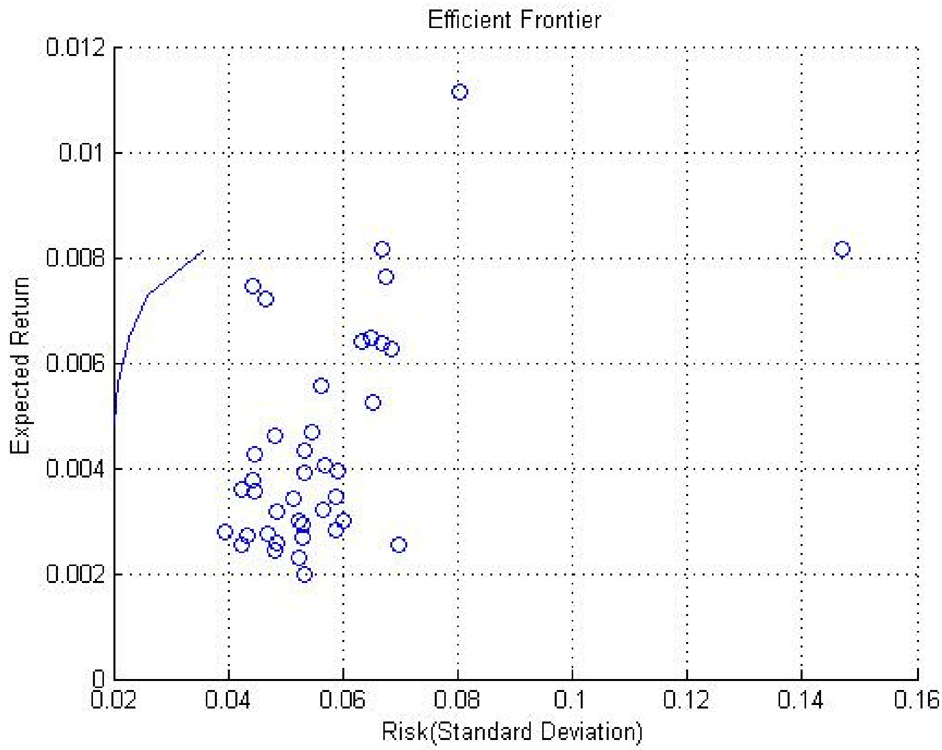

Figure B12.73 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2009)

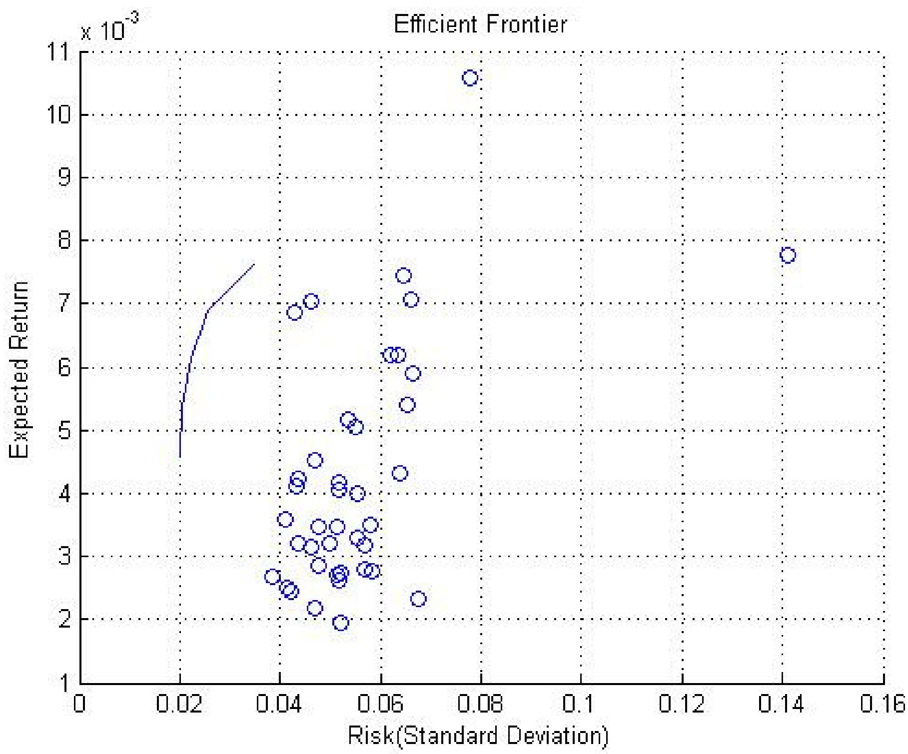

Figure B12.74 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2010)

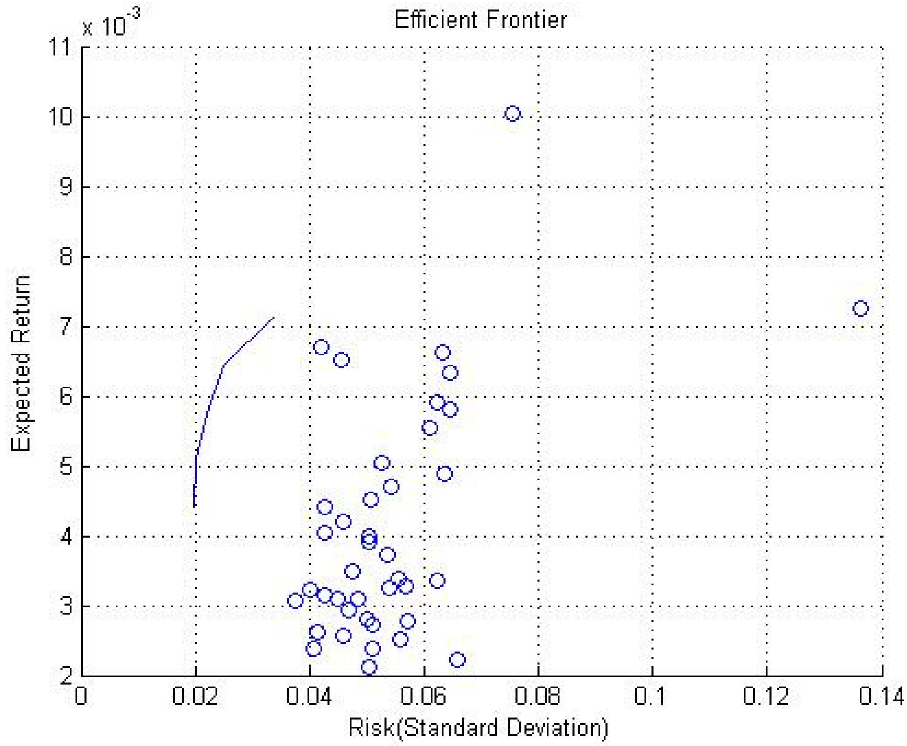

Figure B12.75 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2011)

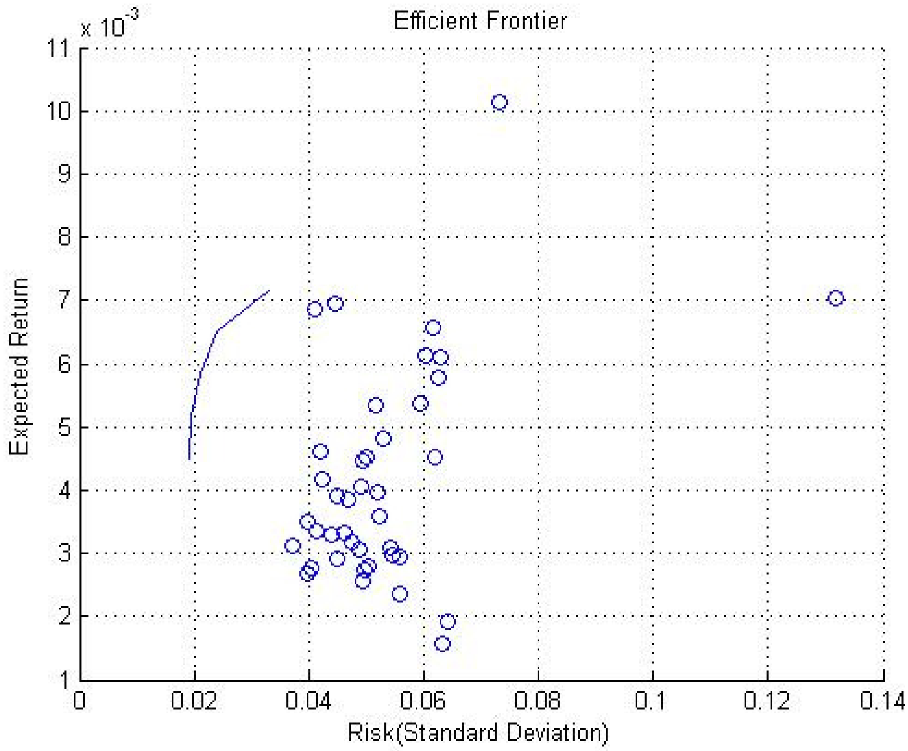

Figure B12.76 Efficient frontier graph for Rebalance Weekly Top 40 (15 year) data with 15% upper limit constraint (end year 2012)

12.3 Appendix C: Matlab Code

```
% Markowitz Portfolio Theory Matlab Code

% Constraints for the number of assets which are to be considered as well
% as the minimum and maximum percentage which is to be allocated to each
% asset in each portfolio

NumberAssets = 40;
AssetMin = 0;
AssetMax = 0.10;

% A matrix needs to be created which contains each of the above mentioned
% constraints

ConstraintMatrix                                                        =
portcons('Default',NumberAssets,'AssetLims',AssetMin,AssetMax,NumberAssets)

% The data which has been imported is then changed to the matrix named data

data = (Top40);

% The mean, covariance and standard deviations need to be calculated in
% order for the portfolios to be created. The number of portfolios to be
% created has arbitrarily been chosen as 10

m = mean(data);
c = cov(data);
stdev = std(data);
NumberPortfolios = 10;

% The portfolio creation tool of the financial toolbox function called
% portopt needs to be created to create the portfolio returns,
% risk and weights of which each asset has in each portfolio. This function
% uses the mean, covariance, constraint matrix and the number of portfolios
% which are to be created

[PortfolioRisk,PortfolioRet,PortfolioWts]                               =
portopt(m,c,NumberPortfolios,[],ConstraintMatrix);

[PortfolioRet,PortfolioRisk]
PortfolioWts

% The section below is for the creation of the optimal risky portfolio
% which includes borrowing at a risk free rate. A risk aversion factor is
% added depending on how risk averse an investor is. This is just shown for
% interest sake, but is not included in the results due to the assumptions
% discussed in the report of not allowing an investor to borrow money to
% invest

RisklessRate = 0.065;
BorrowRate = NaN;
RiskAversion = 2;
[RiskyRisk,RiskyReturn,RiskyWts,RiskyFraction,OverallRisk,OverallReturn]  =
portalloc(PortfolioRisk,PortfolioRet,PortfolioWts,0.65,0.85,2)
```

```
portalloc
(PortfolioRisk,PortfolioRet,PortfolioWts,RisklessRate,BorrowRate,RiskAversi
on);

% The section below is where a graph is created of the efficient frontier
% of the data calculated previously

scatter(stdev,m)

hold on
plot(PortfolioRisk,PortfolioRet,'DisplayName','PortReturn                vs.
PortRisk','XDataSource','PRisk','YDataSource','PRet');figure(gcf)
xlabel('Risk(Standard Deviation)')
ylabel('Expected Return')
title('Efficient Frontier')
grid on
```