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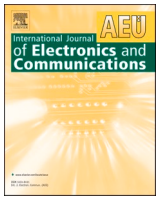
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Regular paper

Enhanced resource optimization with the unevenness coefficients use in regular and irregular graph-based networks

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ABSTRACT

The paper presents a simulation-based study of the impact of transmission resource reallocation on the performance of a communication network. The study considers the unevenness coefficients use to control network resources allocation. The results show that adjusting the transmission resource allocation based on the above-mentioned parameter can significantly improve network performance, leading to a equalization of network characteristics, and all of this without increasing the resources pool. The study's findings provide valuable insights for network designers and operators seeking to optimize network performance in communication systems.

1. Introduction

In modern times, the development of communication networks is one of the most vital tasks that the scientific community has to face. Without communication networks, it would not be possible to develop any distributed technologies. Therefore, it is still worth trying to create network topologies that maximize the efficiency of the use of transmission resources. The performance of a communication network depends on how its connection topology is designed. These networks can be modelled using graphs, so graph theory plays a significant role in this field. Many topologies have been studied, including ring, mesh, torus, and hypercube, each with specific properties.

One topology suitable for telecommunication networks is the chordal ring, proposed by Arden and Lee in 1981 [1]. A chordal ring is a homogeneous, undirected cyclic graph that combines the properties of both ring and full graph topologies. Due to the larger number of paths connecting the source and destination nodes, chordal rings are more resilient than rings while maintaining a smaller diameter and average path length.

Since this proposal, researchers have focused on improving and proposing new versions of the chordal ring topology due to its favorable properties such as low latency, fault tolerance, and symmetry. One way to improve the topology was to increase the degree of classical topologies, such as the fourth [2], fifth [3], and even sixth degree [4]. Another way was to change the ways in which the chordal ring was interconnected, resulting in different types of nodes in the classes e.g. odd-radix chordal rings [14], degree six 3-modified chordal rings [15]. The properties of chordal ring topologies, such as symmetry and Hamiltonicity [5,6], have been studied over the years. Researchers are also interested in topics related to the application of these topologies in compact routing, optimal free-table routing, and broadcasting. The constructs find applications in increasingly modern telecommunication technologies, including wireless [8] and sensor networks [7]. Additionally, the topologies are used in graph algorithms, which are becoming a powerful tool for describing complex systems in the real world, such as those related to social, technological [9], and computer engineering [10].

Therefore, the chordal ring topology has become a subject of extensive research in the field of network topology design, which is of particular interest in the development of telecommunication networks.

The subject of this article is to demonstrate the possibility of equalizing the transmission properties of networks described by regular and irregular graphs with the same number of nodes and the same number of nodes of a given degree, but with different basic parameters, i.e., diameter and average path length, while minimizing the network resources used for information transmission.

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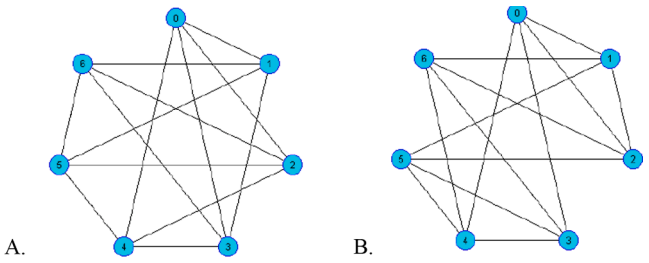


Fig. 1. Analyzed Reference Graphs.

It was assumed that this equalization can be achieved by changing the transmission resources assigned to specific edges of the graph (corresponding to connections) through which the network nodes communicate with each other. The concept of transmission resource, measured in arbitrary units, denotes, for example, the number of time slots or the link bandwidth used for information transmission between network nodes.

As demonstrated in work [11], the transmission properties of networks described by graphs depend not only on their diameter and average path length but also on the use of individual edges of these structures for data transmission, as best illustrated by the Reference Graphs [12], which, despite having the same basic parameters, may differ in the aforementioned properties.

2. Methodology and simulation results

Reference Graphs are the regular structures with a predetermined number of nodes in which the diameter values and the average path lengths from any source node reach the same, theoretically calculated lower size limits.

Fig. 1 shows examples of such degree-four graphs created by seven nodes, with diameters of 2 and average path lengths of 1.333.

As stated in [13], the factor causing differences in the transmission properties of networks described by these graphs is the uneven use of individual edges of these structures, which is measured by a parameter called unevenness coefficient. This parameter – w_{spi} is determining the number of uses of a given edge in sets of parallel paths (routes of the same length created by various configurations of these edges) connecting vertices of a graph. Individual edges can be a part of multiple paths, even those that connect the same nodes.

An unevenness coefficient is described by the formula:

$$w_{spi} = \sum_{i=1}^{D(G)} u_{io} \tag{1}$$

where $D(G)$ is the diameter of the graph and u_{io} values are calculated by the formula:

$$u_{io} = \frac{u_k}{k} \tag{2}$$

u_k means the number of uses of a particular edge in the sets of parallel paths of count k [16].

Proposed in [13], the principle of network resource control is implemented according to the values of calculated unevenness coefficients, which allows for a more rational management of global transmission resources in the analyzed network. More resources can be allocated to edges that are used more often to transmit information at the expense of limiting these resources to edges carrying less traffic. The quantity of network resources allocated to each edge is calculated using the rule:

$$RES_{ic} = d_i \bullet RES_g where d_i = \frac{w_{spi}}{\sum w_{spi}} \tag{3}$$

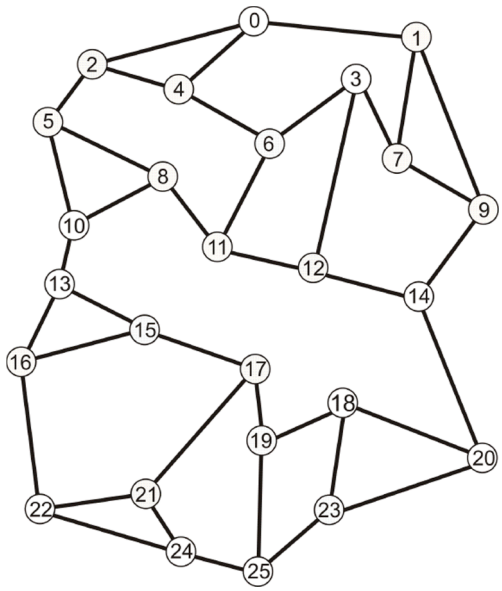


Fig. 2. The studied network structure described by a regular third-degree graph – Graph A.

where RES_{ic} is the resources used by the i-th edge, RES_g is the total network resources, and $\sum w_{spi}$ is a sum of the coefficients calculated for all edges.

The obtained results [13] allowed for the conclusion that by the introduction of network resource control enabled the equalization of characteristics of reference graphs with the same number and degree of nodes.

The above considerations were extended to equalize the characteristics of networks described by graphs whose basic parameters differ from each other. It was assumed that to equalize the transmission characteristics of the tested graphs, it is sufficient to use the ratio of their average path lengths (d_{avA}, d_{avB}) and to multiply the resources corresponding to the edges of the longer graph by a coefficient called Ref_{dav}, calculated using the formula:

$$Ref_{dav} = \frac{d_{avA}}{d_{avB}}. \tag{4}$$

Using the formula (5) given below, the corrected values of resources were calculated, which should enable to obtain similar transmission properties of both networks described by the above-mentioned graphs.

$$RES_{icA} = Ref_{dav} \bullet RES_{iA} \tag{5}$$

Next, an auxiliary parameter called Ref was introduced, which is determined by the formula:

$$Ref = \frac{\sum_{G1wspi}}{\sum_{G2wspi}} \tag{6}$$

where $\sum_{Gwspi}$ represents the total value of the unevenness coefficients for a given graph.

Using formula (7), the corrected resource values were calculated, which according to the adopted assumptions, should enable the achievement of similar transmission properties for both networks described by the aforementioned graphs.

$$RES_{icc} = Ref \bullet RES_{ic} \tag{7}$$

The obtained results [13] confirm the correctness of the thesis that modifying the network resources assigned to the edges of the tested graph can improve its transmission properties and approximate them to the properties of the reference graph with a minimum increase in transmission resources, and can be applied to regular ring graphs.

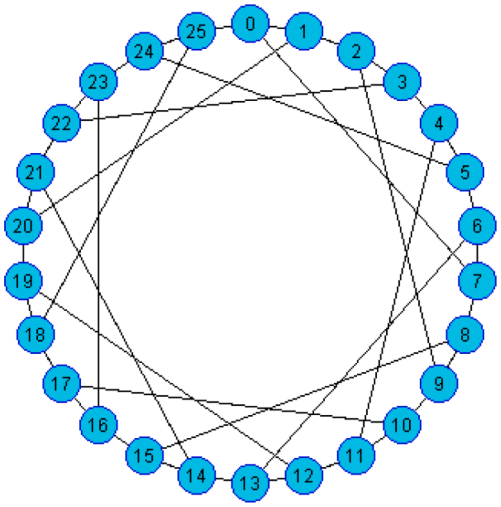


Fig. 3. Third degree, 26 nodes reference graph – graph B.

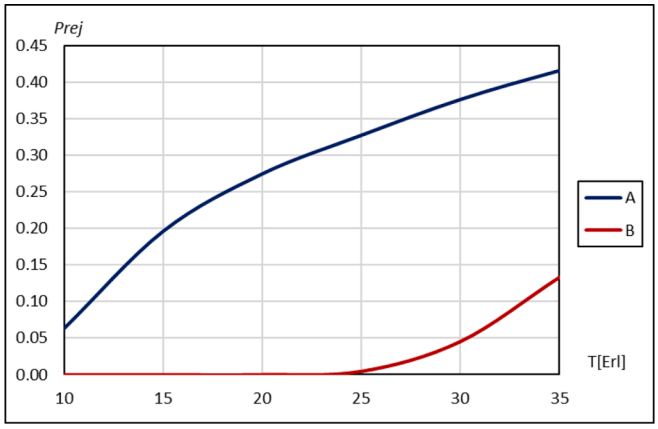


Fig. 4. The results of simulation tests for the networks described by the graphs shown in Fig. 2 (A) and Fig. 3 (B). T – traffic intensity generated in nodes measured in Erlangs, *Prej* – the probability of a service request rejection.

3. Algorithm for equalizing transmission characteristics of networks described by regular and irregular graphs

To generalize the previous findings and verify their accuracy, this section of the article describes the results of a study on structures more closely resembling real-world conditions in terms of their topology (while maintaining the regularity condition).

In Fig. 2, an example of a virtual network described by a third-degree graph with 26 nodes is depicted.

The values of the basic parameters of the graph shown in Fig. 2 are as follows: the diameter is 7, and the average path length is 3.6954. As a reference structure, a network described by a graph shown in Fig. 3 was selected, with a diameter of 5 and an average path length of $d_{av} = 2.84$.

Simulation tests were conducted with own software (C# code developed with Visual Studio 2022) use [3]. Calculation results, saved in csv format, were converted into charts with Excel software use. The results are presented in Fig. 4. It was assumed that to each of the edges (there are 39 of them) were assigned 64 units of transmission resources, meaning that the global transmission resources of both networks amount to 2496 units.

The calculated values of the w_{spi} for the graph labeled with the letter A are provided in the Table 1. All the w_{spi} parameter values of the graph labelled with letter B are equal and amount to 47.333.

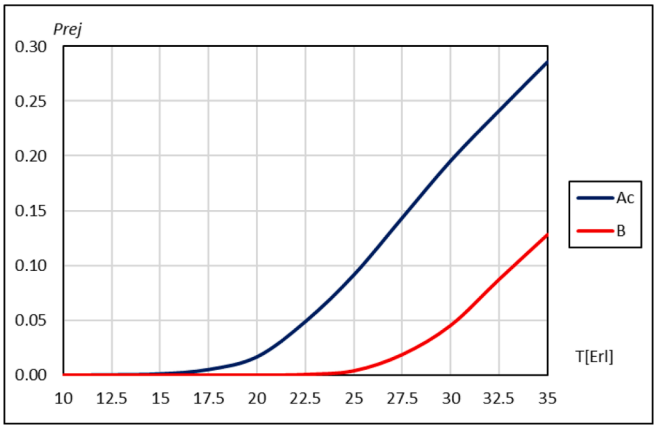


Fig. 5. The simulation results after egdes resources modifications. (A_c). T – traffic intensity generated in nodes measured in Erlangs, *Prej* – the probability of a service request rejection.

Table 1
Unevenness coefficients values for graph A.

Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12
w_{spi}	68.00	57.67	18.33	18.00	59.00	29.50	87.17	39.67	26.50	50.17	33.17	24.00	85.83
Edge	e13	e14	e15	e16	e17	e18	e19	e20	e21	e22	e23	e24	e25
w_{spi}	52.17	26.50	82.00	99.33	98.50	160.8	84.50	105.0	76.67	78.17	175.1	13.17	78.83
Edge	e26	e27	e28	e29	e30	e31	e32	e33	e34	e35	e36	e37	e38
w_{spi}	65.67	79.50	30.67	89.00	8.83	78.50	75.67	20.17	80.17	16.67	16.00	48.33	65.00

Table 2
Network resources allocated to graph A individual edges.

Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12
d_i	0.028	0.024	0.007	0.007	0.024	0.012	0.036	0.016	0.011	0.020	0.013	0.010	0.035
RES_i	70.66	59.92	19.05	18.70	61.31	30.65	90.58	41.22	27.54	52.13	34.46	24.94	89.19
Edge	e13	e14	e15	e16	e17	e18	e19	e20	e21	e22	e23	e24	e25
d_i	0.021	0.011	0.034	0.041	0.041	0.067	0.035	0.043	0.031	0.032	0.072	0.005	0.032
RES_i	54.21	27.54	85.21	103.2	102.3	167.1	87.81	109.1	79.67	81.23	182.0	13.68	81.92
Edge	e26	e27	e28	e29	e30	e31	e32	e33	e34	e35	e36	e37	e38
d_i	0.027	0.033	0.012	0.037	0.003	0.032	0.031	0.008	0.033	0.006	0.006	0.020	0.027
RES_i	68.24	82.61	31.87	92.48	9.18	81.57	78.63	20.96	83.30	17.32	16.63	50.22	67.54

Table 3
Network resources allocated to graph A individual edges after taking into account the *Ref* parameter.

Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12
RES_{icc}	91.94	77.97	24.79	24.34	79.77	39.89	117.8	53.63	35.83	67.83	44.85	32.45	116.0
Edge	e13	e14	e15	e16	e17	e18	e19	e20	e21	e22	e23	e24	e25
RES_{icc}	70.54	35.83	110.8	134.3	133.1	217.4	114.2	141.9	103.6	105.6	236.8	17.80	106.5
Edge	e26	e27	e28	e29	e30	e31	e32	e33	e34	e35	e36	e37	e38
RES_{icc}	88.79	107.4	41.46	120.3	11.94	106.1	102.3	27.27	108.3	22.54	21.63	65.35	87.89

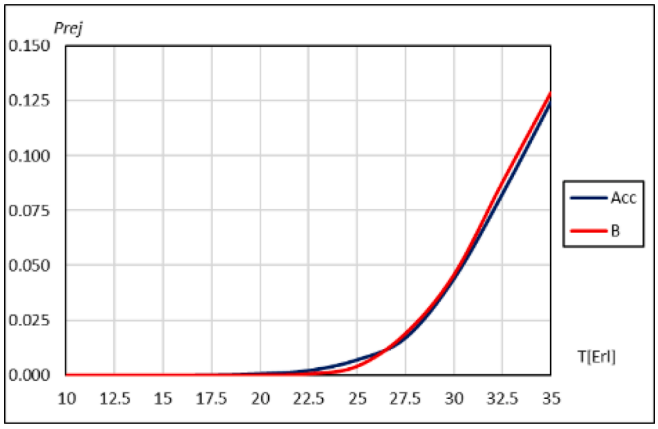


Fig. 6. The simulation results after introducing resource modifications, taking into account the parameter *Ref* (A_{cc}). T – traffic intensity generated in nodes measured in Erlangs, *Prej* – the probability of a service request rejection.

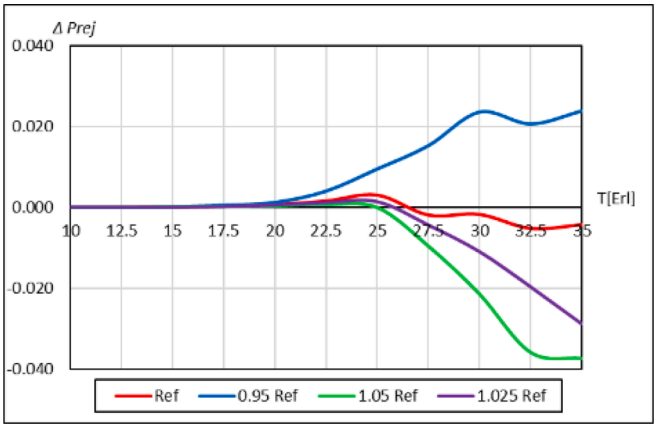


Fig. 7. The differences in the probability of rejecting a service request, denoted as Δ*Prej*, in relation to the characteristic values for graph B, as a function of changes in the generated traffic intensity T measured in Erlangs,

Table 4
The values of *Prej* for changes in the generated traffic intensity in the range of 10 ÷ 35 [Erl] with various values of the *Ref* parameter.

T[Erl]	Graph A				Graph B
	Ref	0.95 *Ref*	1.025 *Ref*	1.05 *Ref*	
10.0	0.000	0.000	0.000	0.000	0.000
12.5	0.000	0.000	0.000	0.000	0.000
15.0	0.000	0.000	0.000	0.000	0.000
17.5	0.000	0.001	0.000	0.000	0.000
20.0	0.001	0.001	0.001	0.000	0.000
22.5	0.002	0.005	0.002	0.001	0.001
25.0	0.007	0.013	0.005	0.004	0.004
27.5	0.017	0.034	0.015	0.009	0.019
30.0	0.044	0.069	0.035	0.024	0.046
32.5	0.083	0.108	0.068	0.052	0.088
35.0	0.124	0.152	0.100	0.091	0.129

To make more rational use of the network's transmission capabilities, based on the results contained in Table 1, the quantities of network resources allocated to individual edges were calculated (Table 2).

The values of resources assigned to the edges of graph B remained unchanged due to the uniform w_{spi}, values, i.e., they amount to 64 units.

Modifications were made to the resources assigned to the edges of graph A, and then simulation tests were conducted again, the results of which are shown in Fig. 5.

In this case a sum of the coefficients calculated for all edges $\sum w_{spiA}$=2402, $\sum w_{spiB}$=1846, so *Ref* ≈ 1.3012.

By applying formula (5), adjusted values of resources were calculated, aiming to achieve similar transmission properties for both networks described by the aforementioned graphs (see Table 3).

The results of the conducted simulations (RES_{icc} values rounded to integers) are presented in Fig. 6.

Based on the obtained results, it can be concluded that by introducing resource control, similar transmission characteristics can be achieved for networks described by graphs with different basic parameters, such as diameter and average path length. This necessitates an increase in overall transmission resources by 724 units.

To demonstrate that the minimum resource value necessary to achieve a rejection probability P_{rej} similar to the reference graph has been reached, additional tests were conducted using network resource adjustments as a function of changes in the *Ref* parameter.

The results of these calculations are presented in Table 4.

Fig. 7 illustrates the differences in the probability of rejecting a service request for graph A, changing values in the range (0.95 to 1.05) of *Ref*, in relation to the characteristic values for graph B.

To summarize the considerations so far, the algorithm for equalizing the transmission properties of networks described by graphs has been specified. This algorithm takes the following two – stages form:

Stage 1. Preliminary Operations:

- Selecting the network with a smaller diameter and average path length from the analyzed networks.
- Conducting preliminary simulations assuming equal network resources for each edge.
- Calculating inequality coefficients for both networks.
- Calculating the values of transmission resources after introducing corrections aimed at their more economical use.
- Conducting simulations after adjusting the resources.

Stage 2. Equalization of Transmission Characteristics:

- Determining the total value of w_{spi} coefficients.
- Calculating the value of the *Ref* coefficient.
- Adjusting network resources considering the *Ref* value.
- Conducting simulations after making adjustments.
- Determining the costs of implementing modifications (the amount of additional transmission units).

From the foregoing considerations, it can be concluded that, at the expense of increasing transmission resources allocated to individual edges of the network, calculated using the *Ref* parameter, it is possible to achieve similarity in transmission properties for networks described by

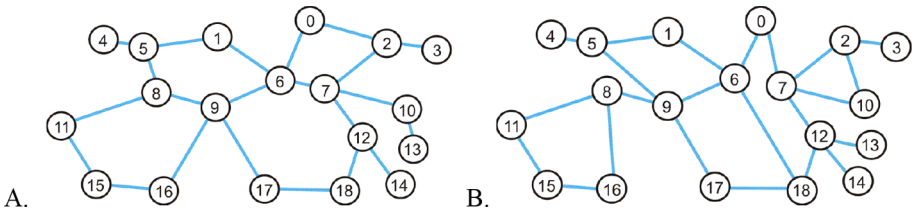


Fig. 8. Analyzed irregular networks.

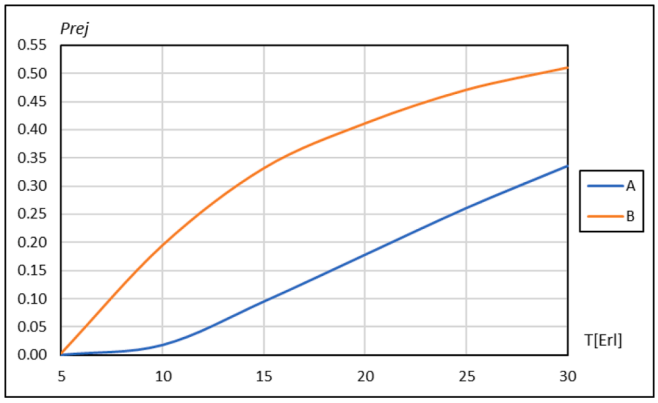


Fig. 9. Simulation results. T – traffic intensity generated in nodes measured in Erlangs, *Prej* – the probability of a service request rejection.

regular graphs of the same degree and the same number of nodes. This achievement is possible without the need to change the topology of inter-node connections and with a minimal increase in network resources used for information transmission.

4. Equalization of transmission characteristics for networks described by irregular graphs with equal node counts of a specific degree, varying diameters, and average path lengths

In the next part of the article, the aim is to investigate whether the use of unevenness coefficients can also be employed to equalize transmission properties in the case of networks described by irregular graphs. However, there is a condition that both the node counts and the counts of nodes with a specific degree are equal.

On Fig. 8, schematic representations of sample networks meeting the specified conditions are presented.

The diameter of the graph describing the A network is 6, and average paths length $d_{av} = 3.1637$, while the graph B diameter is 7 and $d_{av} = 3.6023$. The transmission resources of both networks are 1408 units.

The results of the conducted simulation tests for both networks are shown in Fig. 9.

Following the rules of the algorithm previously provided, the testing process of the mentioned networks was conducted. Table 5 presents the calculated inequality coefficients for both graphs.

The results of the transmission network resources modification are included in Table 6.

The network resources for individual edges, calculated after considering the *Ref* parameter value, equal to 1.18299, are provided in Table 7.

Fig. 10 depicts the plots of the study results for both graphs after modifying the resources of graphs A_c and B_c and considering the Ref parameter value – B_{cc}.

These results were achieved by increasing the resources by 256 units. To ensure that the minimum values were reached, the network was tested by changing the *Ref* parameter, and the obtained results are shown in Fig. 11.

The tests described above were conducted under the assumption that the traffic intensity generated at each node is uniform. The simulation program developed by the authors of the article allows for the differentiation of these intensities, enabling the examination of network behavior in such situations as well. For illustration, an example was provided that relates to the graphs shown in Fig. 8. It was assumed that the traffic intensity at node 2 would be ten, at node 5 – five, at node 12 –

Table 5
Calculated inequality coefficients.

Graph A	Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
	w_{spi}	29.00	45.67	45.00	63.00	36.00	50.33	36.00	33.00	110.0	99.33	68.00
	Edge	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21
	w_{spi}	67.67	66.00	39.00	57.00	52.33	36.00	11.00	36.00	35.67	29.00	37.00
Graph B	Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
	w_{spi}	176.0	168.0	29.00	45.67	36.00	64.00	4.00	36.00	50.33	129.0	37.33
	Edge	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21
	w_{spi}	32.00	96.00	120.0	49.00	49.00	30.67	19.00	36.00	36.00	19.00	18.00

Table 6
The determined values of resources.

Graph A	Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
	RES_{ic}	38	59	59	82	47	65	47	43	143	129	88
	Edge	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21
	RES_{ic}	88	86	51	74	68	47	14	47	46	38	48
Graph B	Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
	RES_{ic}	194	185	32	50	40	70	4	40	55	142	41
	Edge	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21
	RES_{ic}	35	106	132	54	54	34	21	40	40	21	20

Table 7
The resources assigned to the edges of graph B, taking into account the value of the parameter *Ref*.

Graph B	Edge	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
	RES_{icc}	230	219	38	59	47	83	5	47	65	168	49
	Edge	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21
	RES_{icc}	41	125	156	64	64	40	25	47	47	25	24

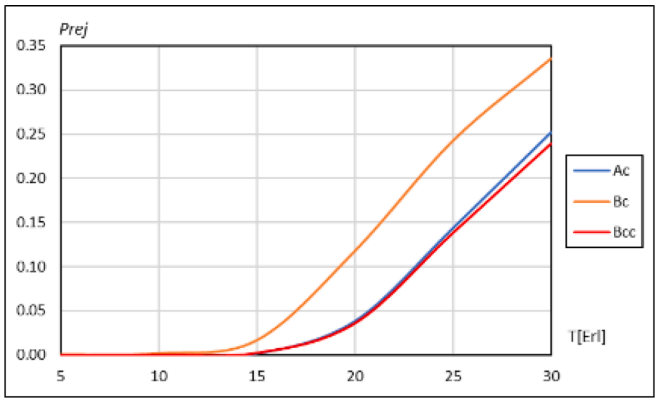


Fig. 10. Simulation results. T – traffic intensity generated in nodes measured in Erlangs, *Prej* – the probability of a service request rejection.

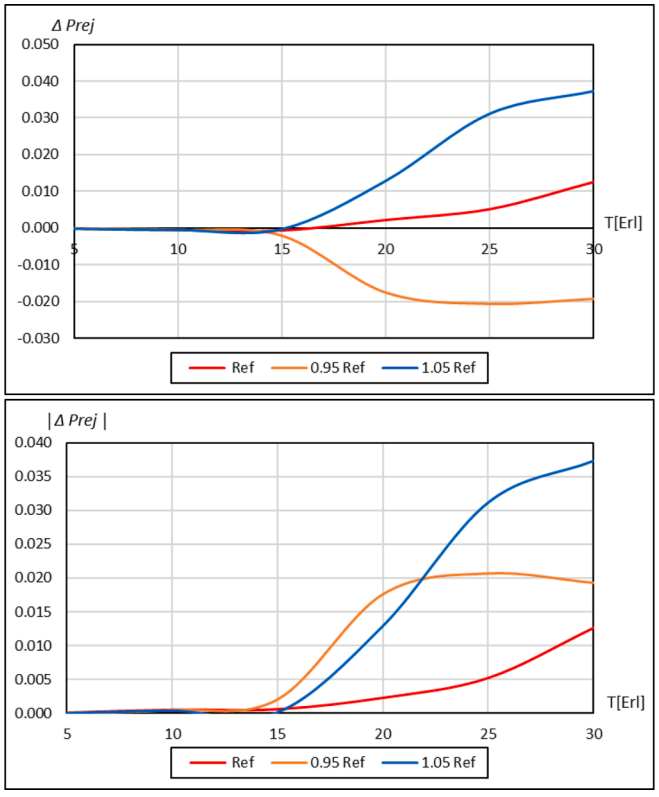


Fig. 11. The results of the probability difference calculations and their absolute values are shown as a function of changes in the intensity of generated traffic based on the parameter *Ref*. T represents the intensity of traffic generated at nodes measured in Erlangs|Δ*Prej*|– represents the difference in the probability of rejecting a call for processing in relation to graph B.

seven, and at node 15 – four times greater than in the remaining nodes. The obtained results are depicted in Fig. 12.

From the presented graph, it can be inferred that in this case, introducing control over transmission resources allows achieving similar

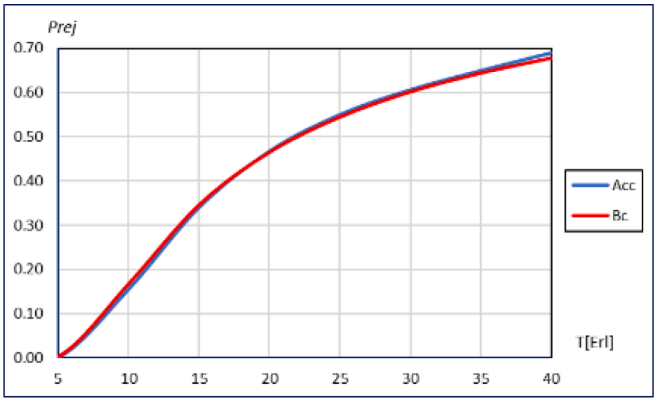


Fig. 12. The results of simulation tests under conditions of uneven traffic intensity generated at nodes. T – traffic intensity generated in nodes measured in Erlangs, *Prej* – the probability of a service request rejection.

transmission properties for the network described by the selected graph in relation to the adopted reference structure as well. In the analyzed examples, the proposed resource reallocation method reduces the probability of connection rejection, resulting in a 10 Erlangs increase in carried traffic using exactly the same amount of resources – see Fig. 10. The modified version of the Bc network – Bcc achieves traffic carrying characteristics similar to the reference network Ac.

5. Summary and conclusions

Most works on network capacity analysis modeled by graphs conclude that it is sufficient to find the network with the shortest average path length. The novelty of the proposed method is that it takes into account not only the average path length but also the resource allocation. As demonstrated in our previous publications, average path length condition is not sufficient because, with the same or even greater average path lengths, the carried traffic can be higher. This depends on the distribution of the occurrence of individual edges in the set of shortest paths. For a balanced distribution, the average path length will indeed be a sufficient parameter, but unfortunately, a significant portion of regular graphs does not meet this condition. Therefore, we attempted to find a solution to this problem with resources reallocation use. When resources are not properly allocated, some edges may have unused resources, while the edges connected to them lack available resources. The solution was to appropriately relocate resources for individual links depending on the network topology. This allowed for the utilization of all network resources.

From the considerations presented, it can be inferred that utilizing unevenness coefficients to control network resources allows for a rational utilization of network resources used for information transmission. This, in turn, improves the transmission properties of networks modeled using graphs. Additionally, under the assumption of the same number of nodes and an equal number of nodes with the same degree in both regular and irregular graphs describing the studied networks, by increasing the size of transmission resources assigned to individual edges (resulting from the value of the Ref parameter), it is possible to achieve similarity in the transmission properties of networks with different diameters and average path lengths. This can be accomplished with minimal increases in network resources, without the need to

change the structure of inter-node connections. The limitation of the proposed method is the necessity of having the capability to reallocate resources, e.g. such as in leased networks.

CRediT authorship contribution statement

Sławomir Bujnowski: Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Zbigniew Lutowski:** Writing – original draft, Software, Conceptualization. **Olutayo Oyeyemi Oyerinde:** Investigation, Conceptualization. **Sebastián García Galán:** Software, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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