

Instrument 1: Pretest

Instructions:

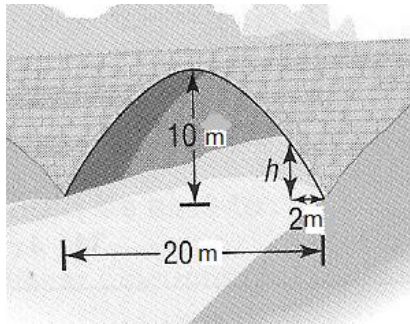
Name: _____ Date: _____

- Complete the problems given in the first column. Use lined paper to complete the problems. Be sure to show all working.
- As you are working, think about the strategies you are using to work on the problem. Complete the next three columns as you are working.

	What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy and/or how do you know when to use the strategy?															
<p>1. The table below contains the mean, median, standard deviation and range of the exam results of Paper 1 and Paper 2 in Mathematics for a group of students.</p> <table border="1" data-bbox="165 767 658 874"> <thead> <tr> <th>Exam</th> <th>Mean</th> <th>Med</th> <th>SD</th> <th>Range</th> </tr> </thead> <tbody> <tr> <td>P 1</td> <td>46.1</td> <td>46</td> <td>15.2</td> <td>91</td> </tr> <tr> <td>P 2</td> <td>60.3</td> <td>60</td> <td>11.6</td> <td>70</td> </tr> </tbody> </table> <p>1.1 The teacher added 4 marks to each of the students' marks for Paper 1. Write down the mean, the median, standard deviation and the range for Paper 1. Explain your reasoning.</p> <p>1.2 There were 50 students in the group and the marks of the top 10% of the students were reduced by 3 marks.</p> <p>Write down the mean, the median and the range for Paper 2.</p>	Exam	Mean	Med	SD	Range	P 1	46.1	46	15.2	91	P 2	60.3	60	11.6	70			
Exam	Mean	Med	SD	Range														
P 1	46.1	46	15.2	91														
P 2	60.3	60	11.6	70														

<p>2. Lesedi is keen to extend her rectangular patio. She has been told that if the length and breadth are each increased by 1 m, the area will be increased by 20 m^2. If the length and breadth are each increased by 2 m, then the area will be doubled. Find the dimensions of the patio (correct to one d.p.).</p>			
<p>3. Study the sequence below and answer the question that follows: $\tan 2^\circ = \tan 2^\circ$ $\tan 22^\circ = \tan 22^\circ$ $\tan 222^\circ = \tan 42^\circ$ $\tan 2222^\circ = \tan 62^\circ$ $\tan 22222^\circ = \tan 82^\circ$ $\tan 222222^\circ = \tan 102^\circ = -\tan 78^\circ$ $\tan 2222222^\circ = \tan 122^\circ = -\tan 58^\circ$</p> <p>Determine the 100th term of the sequence.</p>			

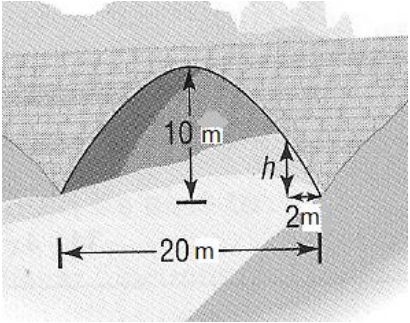
4. A horizontal bridge is in the shape of a parabolic arch. Given the information shown in the figure, what is the height h of the arch 2 m from the shore?



Instrument 1: Memorandum, possible strategy use, and strands of mathematical proficiency required

Problem	Suggested memorandum	Possible strategy used	Strands of mathematical proficiency required to solve problem															
<p>1. The table below contains the mean, median, standard deviation and range of the exam results of Paper 1 and Paper 2 in Mathematics for a group of students.</p> <table border="1" data-bbox="165 547 658 651"> <thead> <tr> <th>Exam</th> <th>Mean</th> <th>Med</th> <th>SD</th> <th>Range</th> </tr> </thead> <tbody> <tr> <td>P 1</td> <td>46.1</td> <td>46</td> <td>15.2</td> <td>91</td> </tr> <tr> <td>P 2</td> <td>60.3</td> <td>60</td> <td>11.6</td> <td>70</td> </tr> </tbody> </table> <p>1.1 The teacher added 4 marks to each of the students' marks for Paper 1. Write down the mean, the median, standard deviation and the range for Paper 1. (4)</p> <p>1.2 There were 50 students in the group and the marks of the top 10% of the students were reduced by 3 marks.</p> <p>Write down the mean, the median and the range for Paper 2.</p> <p>(Buchanan, M., Luis, J. & Reichert, R. (n.d.) <i>Gr. 12 Mathematics: Papers and Answers</i>. Cape Town: The Answer Series. p.2.5.)</p>	Exam	Mean	Med	SD	Range	P 1	46.1	46	15.2	91	P 2	60.3	60	11.6	70	<ul style="list-style-type: none"> • Mean = 50.1 • Median = 50 • Standard deviation=15.2 • Range = 91 <ul style="list-style-type: none"> • Mean = $\frac{60,3 \times 50 - 5 \times 3}{50} = 60$ • Median = 60 • Range = 67 	<ul style="list-style-type: none"> • <i>Working backwards</i> – students know how to calculate the mean, but will have to work backwards to original calculation to find the new mean 	<ul style="list-style-type: none"> • <i>Conceptual understanding</i> is required for this entire problem. There is only one simple calculation required, but learners need to understand how the mean is derived and how the median, standard deviation and range are affected by the increase in scores both overall and at the upper end of the data values. • <i>Strategic competence</i> is needed to set up a meaningful equation for Q. 1.2.
Exam	Mean	Med	SD	Range														
P 1	46.1	46	15.2	91														
P 2	60.3	60	11.6	70														

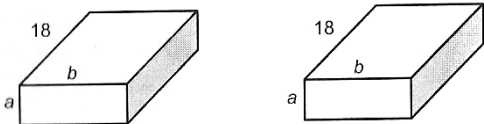
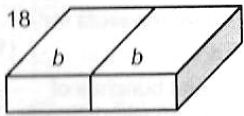
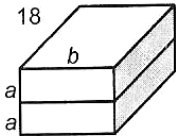
<p>2. Lesedi is keen to extend her rectangular patio. She has been told that if the length and breadth are each increased by 1 m, the area will be increased by 20 m². If the length and breadth are each increased by 2 m, then the area will be doubled. Find the dimensions of the patio (correct to one d.p.).</p> <p>(Source: Buchanan, M., Luis, J. & Reichert, R. (n.d.) <i>Gr. 12 Mathematics: Papers and Answers</i>. Cape Town: The Answer Series. p.1.7.)</p>	$(x+1)(y+1) = xy + 20$ $xy + x + y + 1 = xy + 20$ $x + y = 19 \dots\dots \text{Eq.1}$ $(x+2)(y+2) = 2xy$ $xy + 2x + 2y + 4 = 2xy$ $xy - 2x - 2y - 4 = 0 \dots\dots \text{Eq.2}$ $\text{Eq.1: } y = 19 - x$ $\text{Eq.2: } x(19 - x) - 2x - 2(19 - x) - 4 = 0$ $19 - x^2 - 2x - 38 + 2x - 4 = 0$ $x^2 - 19x + 42 = 0$ $x = \frac{19 \pm \sqrt{19^2 - 4 \times 42}}{2}$ $x = 16,4462\dots \text{ or } x = 2,5537\dots$ $\approx 16,4 \quad \text{or} \quad \approx 2,6$ <p>\therefore Patio is presently 16,4 m \times 2,6 m</p>	<ul style="list-style-type: none"> • <i>Auxiliary elements</i> <ul style="list-style-type: none"> ▪ Draw a figure ▪ Auxiliary problem (simultaneous equation/quadratic equation) 	<ul style="list-style-type: none"> • <i>Strategic competence</i> is needed to set up a meaningful equation • <i>Conceptual understanding</i> is required for the understanding of area and how the increase in one dimension affects the two-dimensional area. • <i>Procedural fluency</i> is required for correctly and efficiently working through all the algebraic steps
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<p>3. Study the sequence below and answer the question that follows: $\tan 2^\circ = \tan 2^\circ$ $\tan 22^\circ = \tan 22^\circ$ $\tan 222^\circ = \tan 42^\circ$ $\tan 2222^\circ = \tan 62^\circ$ $\tan 22222^\circ = \tan 82^\circ$ $\tan 222222^\circ = \tan 102^\circ = -\tan 78^\circ$ $\tan 2222222^\circ = \tan 122^\circ = -\tan 58^\circ$ Determine the 100th term of the sequence. <small>(Source: Buchanan, M., Luis, J. & Reichert, R. (n.d.) <i>Gr. 12 Mathematics: Papers and Answers</i>. Cape Town: The Answer Series. pp.2.10 – 2.11.)</small></p>	$\tan 2^\circ = \tan(0 \times 20 + 2)$ $\tan 22^\circ = \tan(1 \times 20 + 2)$ $\tan 222^\circ = \tan(2 \times 20 + 2)$ $\tan 2222^\circ = \tan(3 \times 20 + 2)$ $\therefore \tan(222\dots 2) = \tan(99 \times 20 + 2)$ $\tan[100 \text{ 2's}] = \tan(1980 + 2)$ $= \tan 182^\circ$ $= \tan 2^\circ$	<ul style="list-style-type: none"> • <i>Did you use all the data?</i> Learners will need to see the pattern in the LHS and RHS and the connection between them. • <i>Generalization</i> Learners will need to generalize the pattern. 	<ul style="list-style-type: none"> • <i>Adaptive reasoning</i> is needed to reflect on the problem, see the pattern and generalize up to the 100th term. • <i>Strategic competence</i> is needed to efficiently set up the RHS • <i>Conceptual understanding</i> is needed of reduction formulae for tan ratios
<p>4. A horizontal bridge is in the shape of a parabolic arch. Given the information shown in the figure, what is the height h of the arch 2 m from the shore?</p>  <p><small>(Source: Sullivan, M. (2008) (8th ed). <i>Precalculus</i>. New Jersey: Pearson Prentice Hall. p. 160.)</small></p>	<p>One of several possible ways to solve depending on how the quadratic equation is set up:</p> $y = a(x - 10)(x + 10)$ $\therefore 10 = a(0 - 10)(0 + 10)$ $\therefore 10 = a(-100)$ $\therefore a = -\frac{1}{10}$ $\therefore y = -\frac{1}{10}(x^2 - 100)$ $\therefore y = -\frac{1}{10}(8^2 - 100)$ $\therefore y = 3,6 \text{ m}$	<ul style="list-style-type: none"> • <i>Working backwards</i> Depending on how the learners conceptualise the arch, the learners have the x-intercepts and need to work back to the equation. • <i>Auxiliary elements</i> Redrawing the picture on a Cartesian plane would be very helpful 	<ul style="list-style-type: none"> • <i>Strategic competence</i> is needed to set up a meaningful equation • <i>Procedural fluency</i> is required for correctly and efficiently working through all the algebraic steps • <i>Conceptual understanding</i> is needed to translate the picture into a graph on the Cartesian plane using the knowledge of the properties of a parabola.

Instrument 2: Post-test

Instructions:

- Complete the problems given in the first column. Use lined paper to complete the problems. Be sure to show all working.
- As you are working, think about the strategies you are using to work on the problem. Complete the next three columns as you are working.

	What strategy(ies) did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
<p>1. Consider the set of numbers: 3; x; y; 10; 12. The mean of the set is 8 and the variance is 10. Determine the numbers x and y.</p>			
<p>2. The dimensions of each of two identical rectangular blocks are a, b, and 18 cm (for height, width and length) as shown below: The two blocks are joined in two different</p>  <p>ways to form designs 1 and 2:</p>  <p>Design 1</p>  <p>Design 2</p> <p>The sum of the height, width and length of Design 1 is 35 units, the sum of the dimensions of Design 2 is 31 units.</p> <p>Calculate the surface area for each design.</p>			

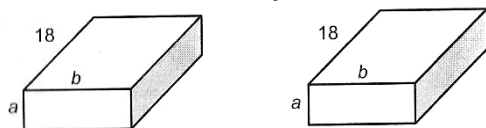
<p>3. The first term of an arithmetic sequence is 3 while T_3, T_6 and T_{10} form consecutive terms of a geometric sequence.</p> <p>Find the value of the tenth term of the arithmetic sequence.</p>			
<p>4. A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 metres apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the centre of the bridge.</p> <p>Find the height of the cables at a point 100 metres from the centre.</p>			

Instrument 2: Memorandum, possible strategy use, and strands of mathematical proficiency required

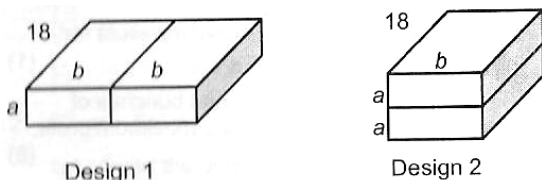
Problem	Suggested memorandum	Possible strategy used	Strands of mathematical proficiency required to solve problem
<p>1. Consider the set of numbers: 3; x; y; 10; 12.</p> <p>The mean of the set is 8 and the variance is 10.</p> <p>Determine the numbers x and y.</p> <p>(Source: IEB National Senior Certificate Examination . (2013). Mathematics: Paper II. Question 8.)</p>	$\frac{3+x+y+12+10}{5} = 8$ $x+y = 15 \dots\dots\dots A$ $\frac{(-5)^2 + (x-8)^2 + (y-8)^2 + 2^2 + 4^2}{5} = 10$ $(x-8)^2 + (y-8)^2 = 5 \dots\dots\dots B$ <p>Substituting A into B:</p> $(x-8)^2 + (15-x-8)^2 = 5$ $x^2 - 16x + 64 + x^2 - 14x + 49 = 5$ $2x^2 - 30x + 108 = 0$ $x^2 - 15x + 54 = 0$ $(x-6)(x-9) = 0$ $x = 6 \text{ or } x = 9$ $y = 9 \quad y = 6$	<ul style="list-style-type: none"> • <i>Working backwards</i> – students will have to work backwards from the variance calculation to the original data values 	<ul style="list-style-type: none"> • <i>Conceptual understanding</i> of how variance is calculated. • <i>Strategic competence</i> is needed to set up a meaningful equation simultaneous equation. • <i>Procedural fluency</i> is needed to work efficiently with the simultaneous equations.

2. The dimensions of each of two identical rectangular blocks are a , b , and 18 cm (for height, width and length) as shown below:

The two blocks are joined in two



different ways to form designs 1 and 2:



The sum of the height, width and length of Design 1 is 35 units, the sum of the dimensions of Design 2 is 31 units.

Calculate the surface area for each design.

(Source: Buchanan, M., Luis, J. & Reichert, R. (n.d.) *Gr. 12 Mathematics: Papers and Answers*. Cape Town: The Answer Series. p.1.8.)

$$\begin{aligned} a + 2b + 18 &= 35 \\ 2a + b + 18 &= 31 \\ a &= 17 - 2b \\ 2a &= 13 - b \\ 2(17 - 2b) &= 13 - b \\ 34 - 4b &= 13 - b \\ -3b &= -21 \\ \therefore b &= 7 \\ a &= 17 - 2 \times 7 \\ \therefore a &= 3 \end{aligned}$$

Surface area :

Design 1:

$$\begin{aligned} SA &= 2(2ab + a \times 18 + 2b \times 18) \\ &= 2(2 \times 3 \times 7 + 3 \times 18 + 2 \times 7 \times 18) \\ &= 696 \end{aligned}$$

Design 2:

$$\begin{aligned} SA &= 2(2ab + b \times 18 + 2a \times 18) \\ &= 2(2 \times 3 \times 7 + 7 \times 18 + 2 \times 3 \times 18) \\ &= 552 \end{aligned}$$

- *Working backwards* – In order to find the surface area, dimensions need to be found. Dimensions can be found from a simultaneous equation relating a , b and 18 to the sum of the dimensions.
- *Auxiliary elements* – auxiliary problem needs to be introduced to find dimensions before finding surface area
- *Strategic competence* is needed to set up a meaningful equation
- *Conceptual understanding* is required for the understanding of surface area
- *Procedural fluency* is required for correctly and efficiently working through all the algebraic steps

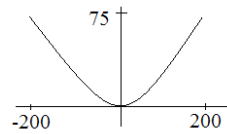
<p>3. The first term of an arithmetic sequence is 3 while T_3; T_6 and T_{10} form consecutive terms of a geometric sequence.</p> <p>Find the value of the tenth term of the arithmetic sequence.</p> <p>(Source: Buchanan, M., Luis, J. & Reichert, R. (n.d.) <i>Gr. 12 Mathematics: Papers and Answers</i>. Cape Town: The Answer Series. pp.1.1).</p>	$T_1 = 3$ $\therefore \frac{T_6}{T_3} = \frac{T_{10}}{T_6}$ $\therefore \frac{3+5d}{3+2d} = \frac{3+9d}{3+5d}$ $(3+5d)^2 = (3+9d)(3+2d)$ $9+30d+25d^2 = 9+33d+18d^2$ $7d^2 - 3d = 0$ $d(7d-3) = 0$ $d = 0 \text{ or } d = \frac{3}{7}$ <p>N.V.</p> $\therefore T_{10} = 3 + 9 \times \frac{3}{7}$ $= \frac{48}{7}$	<ul style="list-style-type: none"> • <i>Did you use all the data?</i> This question has very little information and learners need to understand how successive terms are connected by the common difference or common ratio • <i>Auxiliary element</i> – finding the common difference by setting up the ratio between terms 	<ul style="list-style-type: none"> • <i>Strategic competence</i> is needed to efficiently set up an equation relating the information given • <i>Conceptual understanding</i> is needed of how the terms of arithmetic and geometric sequences are connected by their respective differences and ratios
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4. A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 metres apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the centre of the bridge.

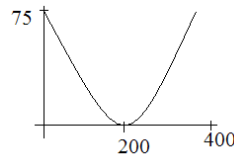
Find the height of the cables at a point 100 metres from the centre.

(Source: Sullivan, M. (2008) (8th ed). *Precalculus*. New Jersey: Pearson Prentice Hall. p. 151.)

One of several possible ways to solve depending on how the quadratic equation is set up:



or



$$y = ax^2$$

$$\therefore 75 = a(200)^2$$

$$\therefore 72 = 40000a$$

$$\therefore a = 0,00187$$

$$\therefore y = 0,00187x^2$$

$$\therefore y = 0,00187(100)^2$$

$$\therefore y = 18,75m$$

- *Working backwards*
Depending on how the learners conceptualise the bridge, the learners have the turning point and need to work back to the equation.
- *Auxiliary elements*
Redrawing the picture on a Cartesian plane would be very helpful

- *Strategic competence* is needed to set up a meaningful equation
- *Procedural fluency* is required for correctly and efficiently working through all the algebraic steps
- *Conceptual understanding* is needed to translate the picture into a graph on the Cartesian plane using the knowledge of the properties of a parabola.

Metacognitive Questioning: Comprehension Questions

HOW TO SOLVE IT: A METACOGNITIVE APPROACH

UNDERSTANDING THE PROBLEM: COMPREHENSION QUESTIONS

<p>FIRST. <i>You have to understand the problem</i></p>	<ul style="list-style-type: none">• WHAT IS THE UNKNOWN?• WHAT ARE THE DATA?• WHAT IS THE CONDITION? ➤ DRAW A FIGURE➤ INTRODUCE SUITABLE NOTATION
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Metacognitive questioning: Connection Questions

CONNECTION QUESTIONS

SECOND.

Find the *connection* between the data & the unknown. You may need to make connections to other areas of mathematics or previously solved problems.

- **DO I KNOW A RELATED PROBLEM?**
- **DID I USE ALL THE DATA?**
- **WHAT IS THIS PROBLEM RELATED TO?**

STRATEGY QUESTIONS

THIRD.

Think of appropriate strategies to tackle the problem.

You may have to consider an *auxiliary problem*.

Obtain a *plan* of the solution.

• **WHAT STRATEGIES CAN YOU USE?**

➤ **TOPIC-SPECIFIC STRATEGIES**

• **HEURISTICS?**

➤ **INTRODUCE AUXILIARY ELEMENTS**

➤ **INTRODUCE AUXILIARY PROBLEM**

➤ **DRAW A FIGURE**

➤ **DECOMPOSE AND RECOMBINE**

➤ **GENERALISATION**

➤ **SPECIALISATION**

➤ **WORK BACKWARDS**

➤ **VARIATION OF THE PROBLEM**

2-D/3-D Trigonometry revision assignment

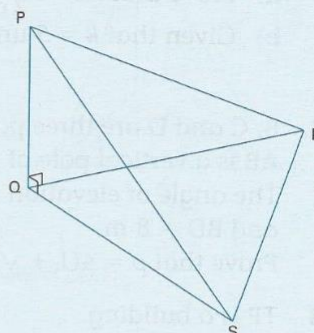
Check your skills

Exercise 6.5

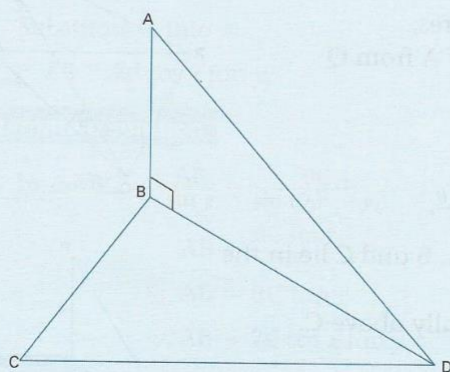
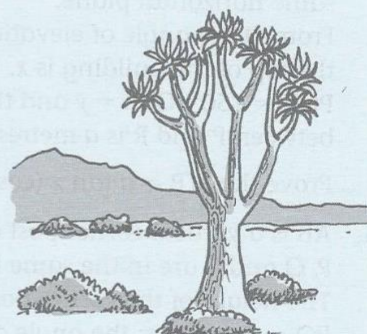
Answers should be calculated correct to one decimal place unless otherwise specified.

1. PQ is a building 150 m high.
The foot of the building Q is in the same horizontal plane as R and S.
The angles of elevation of the top of the building P from the points R and S are 67° and 51° respectively.
 $\widehat{RPS} = 42^\circ$.
Calculate:

- PR
- PS
- RS.

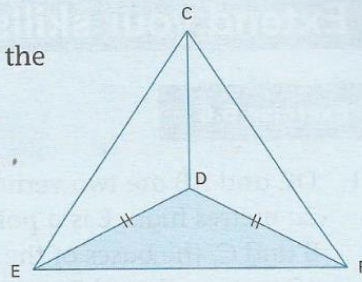


2. AB is a quiver tree in the Kalahari.
The foot of the tree B and the two points C and D are in the same horizontal plane.
C and D are 40 m apart and C is 68 m from the tree trunk.
 $\widehat{BCD} = 132^\circ$
The angle of elevation of the top of the tree from D is 72°

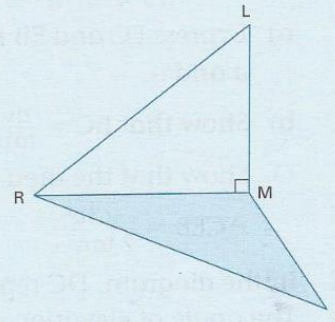


Calculate the height of the quiver tree.

3. CD is a vertical tower 280 m high.
E and F are two observation points equidistant from the tower and in the same horizontal plane as D, the foot of the tower. From both the observation points, the angle of elevation of the top of the tower is 65° and $\widehat{EDF} = 115^\circ$.
Calculate the distance between the observation points E and F.

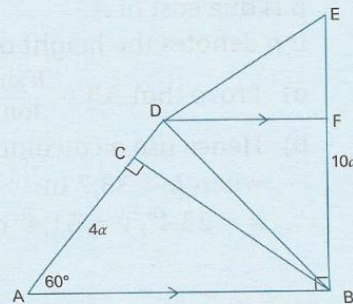


4. LM is perpendicular to the horizontal plane RMS.
The angle of elevation of L from R is ϕ .
 $\widehat{MRS} = \alpha$ and $\widehat{MSR} = \beta$.
The height of the tower is h .
Prove that $RS = \frac{h \sin(\alpha + \beta)}{\sin \beta \tan \phi}$



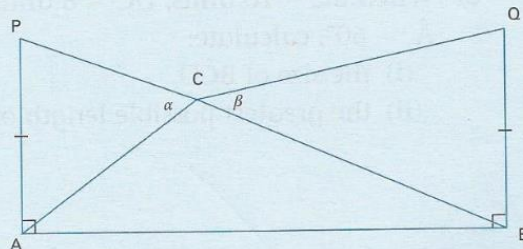
5. In the diagram $\widehat{BAC} = 60^\circ$:
 $AC = 4a$ and $BE = 10a$
 $BD = DE$ and $\widehat{ACB} = \widehat{BDE} = 90^\circ$
 $EB \perp AB$ and $DF \parallel AB$.

- a) Determine:
(i) \widehat{ABC}
(ii) \widehat{DBE}
(iii) \widehat{ADB} .
- b) Calculate, without using a calculator, the length of each of the following in terms of a :
(i) BC
(ii) BD.



6. AP and QB are two identical vertical towers whose bases are situated in the same horizontal plane through A and B. C is a point in this horizontal plane such that $\widehat{CAB} = 90^\circ$. The angle of elevation of P at C is α , and the angle of elevation of Q at C is β .

- a) Show that $\cos \widehat{ACB} = \frac{\tan \beta}{\tan \alpha}$
- b) If $AB = 42$ m; $\alpha = 31,4^\circ$ and $\beta = 17,8^\circ$, determine
(i) \widehat{ACB}
(ii) the height of the towers.



Memo: 2-D/3-D Trigonometry revision assignment

Due: 17/4/2015

Gr. 12

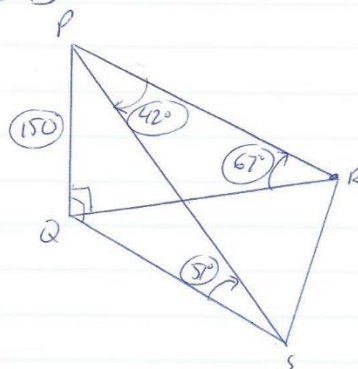
Ex. 6.5

Trig in 3-D

Start of weekly
Revision

Strategy Use (1)

1. (5) Draw a figure
- Need to redraw
w/ ~~given~~ amounts
(circled quantities
in words, not give
in diagram)



Calculate

- a) PR (hypotenuse of $\triangle PQR$)

$$\therefore \sin 67^\circ = \frac{PQ}{PR}$$

$$\begin{aligned} \therefore PR &= \frac{PQ}{\sin 67^\circ} \\ &= \frac{150}{\sin 67^\circ} \\ &= 162,95 \text{ m} \end{aligned}$$

- (5) Decompose + recombine
(5) Work backwards

Note

- highly scaffolded
problem
(?) - Children w/
visual reasoning diff.
will have probs w/draw.

- b) PS (hypotenuse of $\triangle PQS$)

$$\sin 51^\circ = \frac{PQ}{PS}$$

$$\begin{aligned} \therefore PS &= \frac{PQ}{\sin 51^\circ} \\ &= \frac{150}{\sin 51^\circ} \\ &= 193,01 \text{ m} \end{aligned}$$

- c) RS (straight forward cos rule)

$$\begin{aligned} RS &= \sqrt{(193,01)^2 + (162,95)^2 - 2(193,01)(162,95) \cos 42^\circ} \\ &= \cancel{201,08} \text{ m} \quad (?) \\ &= 130,61 \text{ m} \end{aligned}$$

(1)

2. (S) Decompose + Recombine
 (S) Work backwards

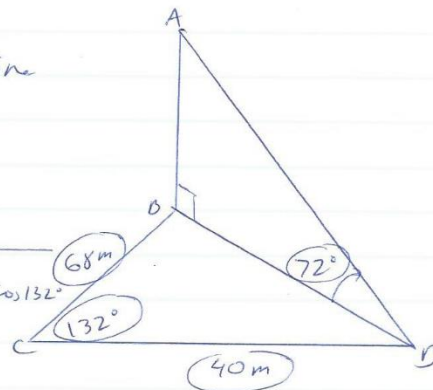
$$AB = BD \tan 72^\circ$$

$$BD = \sqrt{68^2 + 40^2 - 2(68 \times 40) \cos 132^\circ}$$

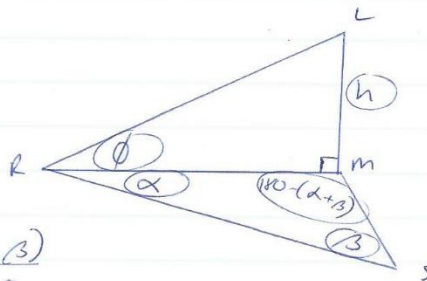
$$= 99,318 \dots$$

$$\therefore AB = 99,318 \times \tan 72^\circ$$

$$= 305,7 \text{ m}$$



4. (S) D + R
 (S) WB



$$\text{RTP: } RS = \frac{h \sin(\alpha + \beta)}{\sin \beta \tan \theta}$$

$$\frac{RS}{\sin(180 - (\alpha + \beta))} = \frac{Rm}{\sin \beta}$$

$$\therefore RS = \frac{Rm \cdot \sin(\alpha + \beta)}{\sin \beta}$$

$$Rm \tan \theta = \frac{h}{Rm}$$

$$\therefore Rm = \frac{h}{\tan \theta}$$

$$\therefore RS = \frac{h \cdot \sin(\alpha + \beta)}{\tan \theta \cdot \sin \beta}$$

(2)

Grade 11/12 Trigonometry Revision

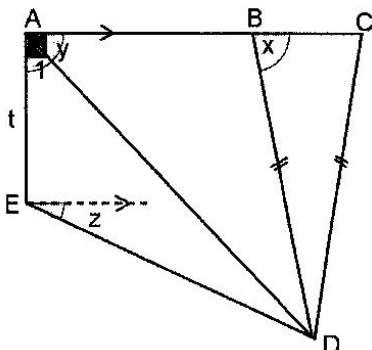
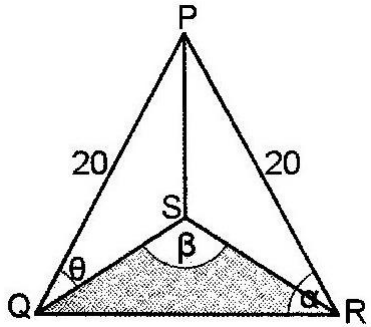
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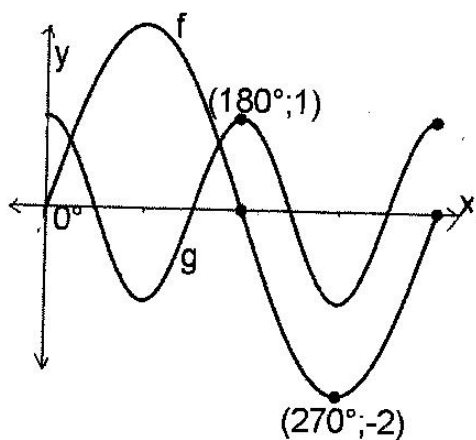
On this paper, you will find Gr. 11 style questions alongside Gr. 12 style questions. Determine the required quantities for each question. Answer the metacognitive question prompts **as you are working on the GRADE 12** question. Do all working out on this page (attach other sheets if necessary).

Grade 11 Content	Grade 12 Content	Metacognitive Questions
<p>1. “Definition” questions:</p> <p>If $\cos A = \frac{4}{7}$ and $A + B = 90^\circ$, use a sketch to find the value of $\frac{\tan A \cdot \sin A}{\cos B}$.</p>	<p>If $\sin A = \frac{12}{13}$, \hat{A} is obtuse and $\tan B = \frac{4}{3}$, $\hat{B} > 90^\circ$, determine $\sin(A - B)$ without calculating the values for the angles.</p>	<ol style="list-style-type: none"> 1) What’s in the question? 2) What are the differences between the problem you are working on and the previous problem 3) What is the strategy/tactic/principle appropriate for solving the problem?
<p>2. “P” Questions:</p> <p>If $\tan 28^\circ = p$, write down the following in terms of p.</p> <p>2.1 $\tan 152^\circ$</p> <p>2.2 $\cos(-28^\circ)$</p> <p>2.3 $\sin 62^\circ$</p>	<p>If $\tan x = p$ and x is acute, write down the following in terms of p.</p> <p>2.1 $\cos 2x$</p> <p>2.2 $\sin 2x$</p> <p>2.3 $\frac{\cos(90^\circ - 2x)}{\cos(90^\circ - x)}$</p>	<ol style="list-style-type: none"> 1) What’s in the question? 2) What are the differences between the problem you are working on and the previous problem 3) What is the strategy/tactic/principle appropriate for solving the problem?

<p>3. Simplifying questions: Simplify: $\frac{2 \cos(-180^\circ - x) \cdot \sin(-x)}{\sin(x - 180^\circ) \cdot \cos(x - 360^\circ)}$</p>	<p>Simplify and evaluate (no calculators!): 3.1 $\cos^2 15^\circ - \cos^2 75^\circ$ 3.2 $\frac{\cos 10^\circ \cdot \cos 340^\circ - \sin 190^\circ \cdot \sin 200^\circ}{\sin 10^\circ \cdot \sin 70^\circ + \sin 100^\circ \cdot \sin 20^\circ}$</p>	<ol style="list-style-type: none"> 1) What's in the question? 2) What are the differences between the problem you are working on and the previous problem 3) What is the strategy/tactic/principle appropriate for solving the problem?
<p>4. Proofs Prove the identity: $\cos \theta \left(\tan \theta + \frac{1}{\tan \theta} \right) = \frac{1}{\sin \theta}$</p>	<p>Prove the identity: $\frac{\sin x + \sin 2x}{1 + \sin x - \cos 2x} = \frac{1 + 2 \cos x}{1 + 2 \sin x}$</p>	<ol style="list-style-type: none"> 1) What's in the question? 2) What are the differences between the problem you are working on and the previous problem 3) What is the strategy/tactic/principle appropriate for solving the problem?

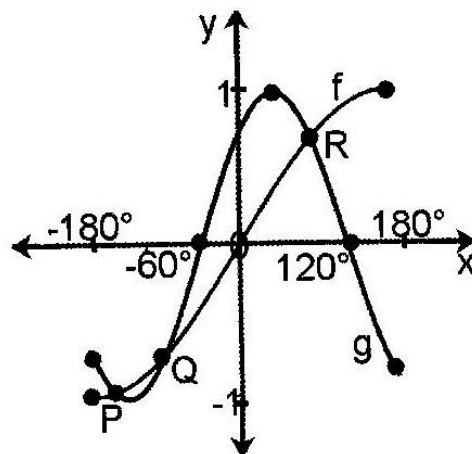
<p>5. Equations Determine the general solution of: $3\cos\theta = 2\sin^2\theta - 3$</p>	<p>Determine the general solution of: $3\sin 2x - 3\sin x = 4\cos^2 x - 2\cos x$</p>	<ol style="list-style-type: none"> 1) What's in the question? 2) What are the differences between the problem you are working on and the previous problem 3) What is the strategy/tactic/principle appropriate for solving the problem?
<p>6. Sine, Cosine and Area rules</p>  <p>Show that $DC = \frac{t \cdot \sin y \cdot \cos z}{\sin x \cdot \sin(y-z)}$</p>	 <p>Given $\hat{SQP} = \theta$, $\hat{PRQ} = \alpha$ and $\hat{RSQ} = \beta$</p> <ol style="list-style-type: none"> 1. Show that the area of $\Delta R\hat{Q}S = 200 \sin \beta \cdot \cos^2 \alpha$ 2. Show that $QR = 20\sqrt{2 + 2\cos 2\alpha}$ 	<ol style="list-style-type: none"> 1) What's in the question? 2) What are the differences between the problem you are working on and the previous problem 3) What is the strategy/tactic/principle appropriate for solving the problem?

7. Functions



The graphs of $f(x) = a \sin x$ and $g(x) = \cos bx$ are given for $x \in [0^\circ; 360^\circ]$.

1. Determine the values of a and b .
2. Hence, determine the amplitude of f and the period of g .
3. Use the graphs to answer the following questions. For which values of x is:
 - 3.1 $f(x) \leq 0$
 - 3.2 $f(x) - g(x) = 3$
 - 3.3 $f(x) \cdot g(x) \geq 0$



The diagram shows the graphs of $f(x) = \sin \frac{x}{2}$ and $g(x) = \cos(x - 30^\circ)$, $x \in [-180^\circ; 180^\circ]$.

1. Determine P , Q and R , the points where f and g intersect.
2. Determine the value(s) of x if:

$$\sin \frac{x}{2} > \cos x \cdot \cos 30^\circ + \sin x \cdot \sin 30^\circ$$
3. Determine the y-intercept of h if

$$h(x) = g(x) - \frac{\sqrt{3}}{2}$$

- 1) What's in the question?
- 2) What are the differences between the problem you are working on and the previous problem
- 3) What is the strategy/tactic/principle appropriate for solving the problem?

Grade 11/12 Trigonometry Revision Memo

Gr. 12 Trig Revision
Memo

Gr. 11

1. $\cos A = \frac{4}{7}$; $A + B = 90^\circ$

$$4^2 + y^2 = 7^2$$

$$y^2 = 49 - 16$$

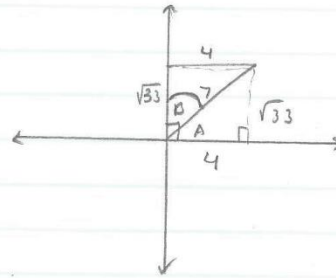
$$\therefore y = \sqrt{33}$$

$$\therefore \tan A, \sin A$$

$$\frac{\cos B}{\cos A} = \frac{\sqrt{33}}{4} \cdot \frac{\sqrt{33}}{7}$$

$$\frac{\sqrt{33}}{4}$$

$$= \frac{\sqrt{33}}{7}$$



Strategy: Draw a figure

Gr. 12 $\sin A = \frac{12}{13}$ $\tan B = \frac{4}{3}$

$$x^2 + 12^2 = 13^2$$

$$\therefore x^2 = 169 - 144 \leftarrow \text{Pythag}$$

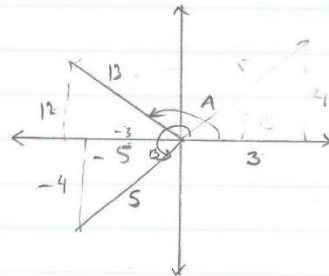
$$\therefore x = 5$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right) - \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right) \leftarrow \text{subs}$$

$$= \frac{-36}{65} - \frac{20}{65}$$

$$= \frac{-56}{65}$$

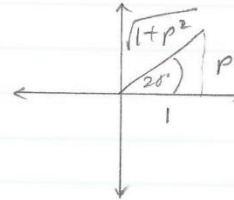


Strategy: Draw a figure

Qr11 2. $\tan 28^\circ = p$

$$1^2 + p^2 = h^2$$

$$\therefore h = \sqrt{1+p^2}$$



2.1 $\tan 152^\circ$

$= \tan (180 - 28^\circ)$ ← reduction to acute

$= -\tan 28^\circ$

$= -p$

Strategy: Draw figure

2.2 $\cos (-28^\circ)$ ← neg. \angle s

$= \cos 28^\circ$

$= \frac{1}{\sqrt{1+p^2}}$ ← figure

← Pythag

2.3 $\sin 62^\circ$

$= \cos 28^\circ$ ← co function

$= \frac{1}{\sqrt{1+p^2}}$

Qr12

2.1 $\cos 2x$

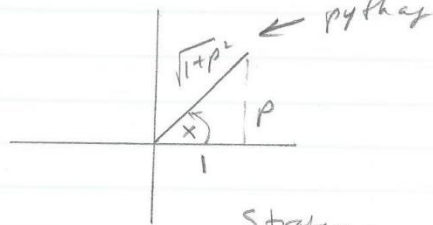
$= 2\cos^2 x - 1$ ← identity

$= 2\left(\frac{1}{\sqrt{1+p^2}}\right)^2 - 1$

$= \frac{2}{1+p^2} - 1$

$= \frac{2 - (1+p^2)}{1+p^2}$

$= \frac{1-p^2}{1+p^2}$



← pythag
Strategy - draw figure

2.2 $\frac{\cos(90-2x)}{\cos(90-x)}$

$= \frac{\sin 2x}{\sin x}$ or $\frac{2\sin x \cos x}{\sin x}$

$= \frac{2p}{1+p^2} \div \frac{p}{\sqrt{1+p^2}} = 2\cos x$

$= \frac{2\sqrt{1+p^2}}{1+p^2} = \frac{2(1)}{\sqrt{1+p^2}}$

2.2. $\sin 2x = 2\sin x \cos x$

$= 2 \cdot \frac{p}{\sqrt{1+p^2}} \cdot \frac{1}{\sqrt{1+p^2}} = \frac{2p}{1+p^2}$

Ex 11.3. $\frac{2 \cos(-180-x) \cdot \sin(-x)}{\sin(x-180^\circ) \cos(x-360^\circ)}$ in the negs
 $= \frac{2 \cos[-(180+x)], \sin(-x)}{\sin[-(180-x), \cos[-(360-x)]}$ ← factor out negs
 $= \frac{2 \cos(180+x) \cdot -\sin x}{-\sin(180-x) \cdot \cos(360-x)}$ ← neg. \angle 's
 $= \frac{-2 \cos x \cdot \sin x}{-\sin x \cdot \cos x}$ ← reduction
 $= -2$ ← simplify

Ex 12.3.1 $\cos^2 15^\circ - \cos^2 75^\circ$
 $= \cos^2 15^\circ - \sin^2 15^\circ$ ← cofunction
 $= \cos 2(15^\circ)$ ← double \angle identity
 $= \cos 30^\circ$
 $= \frac{\sqrt{3}}{2}$ ← special \angle

3.2 $\frac{\cos 10^\circ \cdot \cos 340^\circ - \sin 190^\circ \cdot \sin 200^\circ}{\sin 10^\circ \cdot \sin 70^\circ + \sin 100^\circ \cdot \sin 20^\circ}$
 $= \frac{\cos 10^\circ \cdot \cos 20^\circ - (-\sin 10^\circ)(-\sin 20^\circ)}{\sin 10^\circ \cdot \sin 70^\circ + (\sin 80^\circ)(\sin 20^\circ)}$ ← reduce to acute \angle
 $= \frac{\cos 10^\circ \cdot \cos 20^\circ - \sin 10^\circ \cdot \sin 20^\circ}{\sin 10^\circ \cdot \cos 20^\circ + \cos 10^\circ \cdot \sin 20^\circ}$ ← match \angle 's
 $= \frac{\cos(10^\circ + 20^\circ)}{\sin(10^\circ + 20^\circ)}$ ← cofunction
← compound \angle ident.
 $= \frac{\cos 30^\circ}{\sin 30^\circ}$
 $= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ ← special \angle 's
 $= \sqrt{3}$ ← simplify

$$\text{Qr. 11 4} \quad \cos \theta \left(\tan \theta + \frac{1}{\tan \theta} \right) = \frac{1}{\sin \theta}$$

$$\text{LHS: } \cos \theta \left(\tan \theta + \frac{1}{\tan \theta} \right) \quad \leftarrow \text{start w/ complicated side}$$

$$= \cos \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \quad \leftarrow \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \sin \theta + \frac{\cos^2 \theta}{\sin \theta} \quad \leftarrow \text{distribute}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} \quad \leftarrow \text{LCD (add fractions)}$$

$$= \frac{1}{\sin \theta} = \text{RHS} \quad \leftarrow \text{squares identity}$$

$$\text{Qr. 12 4. } \frac{\sin x + \sin 2x}{1 + \sin x - \cos 2x} = \frac{1 + 2 \cos x}{1 + 2 \sin x}$$

$$\text{LHS: } \frac{\sin x + \sin 2x}{1 + \sin x + \cos 2x} \quad \leftarrow \text{start w/ complicated side}$$

$$= \frac{\sin x + 2 \sin x \cos x}{1 + \sin x - (1 - 2 \sin^2 x)} \quad \leftarrow \text{double \& identities}$$

$$= \frac{\sin x (1 + 2 \cos x)}{\sin x + 2 \sin^2 x} \quad \leftarrow \text{will make factorisable quadratic}$$

$$= \frac{\sin x (1 + 2 \cos x)}{\sin x (1 + 2 \sin x)} \quad \leftarrow \text{factorise}$$

$$= \frac{1 + 2 \cos x}{1 + 2 \sin x} = \text{RHS} \quad \leftarrow \text{factorise}$$

$$\quad \leftarrow \text{simplify}$$

Q.11 5 $3 \cos \theta = 2 \sin^2 \theta - 3$

$3 \cos \theta = 2(1 - \cos^2 \theta) - 3$ ← squares
 ← identity into cos only

$3 \cos \theta = 2 - 2 \cos^2 \theta - 3$ ← distribute

$2 \cos^2 \theta + 3 \cos \theta + 1 = 0$ ← quad. structure ⇒

$(2 \cos \theta + 1)(\cos \theta + 1) = 0$ ← std form

∴ $\cos \theta = -\frac{1}{2}$ or $\cos \theta = -1$ ← factorise

∴ $\theta = \pm 120^\circ + k \cdot 360^\circ$ ← zero-product property

or $\theta = 180^\circ + k \cdot 360^\circ$ ← $\cos^{-1}(\text{ratio}) \Rightarrow 2$ solutions

include multiples of period

↑
 min value
 ∴ only 1 solution per period

Q.12 5 $3 \sin 2x - 3 \sin x = 4 \cos^2 x - 2 \cos x$

$3(2 \sin x \cos x) - 3 \sin x - 4 \cos^2 x + 2 \cos x = 0$ ← double & id.

$6 \sin x \cos x - 3 \sin x - 4 \cos^2 x + 2 \cos x = 0$ ← quad. ∴ std form

$3 \sin x (2 \cos x - 1) - 2 \cos x (2 \cos x - 1) = 0$ ← 4 terms ∴

$(2 \cos x - 1)(3 \sin x - 2 \cos x) = 0$ (look for common bracket)

∴ $2 \cos x = \frac{1}{2}$ or $3 \sin x = 2 \cos x$ ← zero-product property

∴ $\tan x = \frac{2}{3}$ ← can divide by $\cos x$ because still included in tan

∴ $x = \pm 33^\circ + k \cdot 360$

or $x = 33.7^\circ + k \cdot 180$

↑
 2 solutions per period
 inc. period

↑
 1 solution per period; include period.

Q. 11 G. RTP: $DC = \frac{t \cdot \sin y \cdot \cos z}{\sin x \cdot \sin(y-z)}$

$\hat{A}BC = 180^\circ - x$ (\angle s on str. line)

$\hat{B}DA = 180 - (180 - x + y)$ (sum \angle s in Δ)
 $= x - y$

$\hat{A}ED = 90^\circ + z$

$\hat{A}DE = 180 - (90 - y + 90 + z)$ Find all \angle s
 $= y - z$

$DC = BD$ (given)

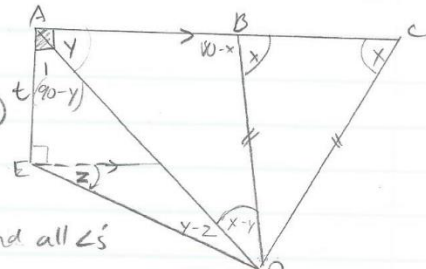
$BD = AD$

$\frac{\sin y}{\sin x} = \frac{\sin(180-x)}{\sin(y-z)}$ ← sine rule
 $\therefore BD = \frac{AD \sin y}{\sin x}$ ← reduction

$\frac{AD}{\sin(90+z)} = \frac{t}{\sin(y-z)}$

$\therefore AD = \frac{t \cdot \cos z}{\sin(y-z)}$ ← cofunction

$\therefore DC = BD = \frac{t \cdot \cos z \cdot \sin y}{\sin(y-z) \sin x}$ ← subs



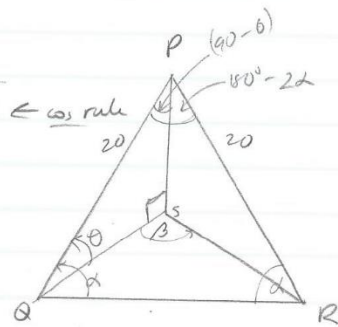
Strategy: Draw figure,

• fill in info.

• Decompose + Recombine

• move from 1 Δ to another by using common sides

Q. 12 G. $2QR = \sqrt{20^2 + 20^2 - 2(20)(20)\cos(180-2\alpha)}$ ← cos rule
 $= \sqrt{800 - 800(-\cos 2\alpha)}$
 $= \sqrt{800(1 + \cos 2\alpha)}$
 $= 20\sqrt{2(1 + \cos 2\alpha)}$
 $= 20\sqrt{2 + 2\cos 2\alpha}$



6.1 RTP: $\Delta RQS = 200 \sin \beta \cos^2 \theta$

$\cos \theta = \frac{QS}{20}$

Strategy: Draw figure

$\therefore QS = 20 \cos \theta$

$\Delta PSR \equiv \Delta PSQ$ (RHS)

$\therefore SR = 20 \cos \theta$

$\therefore \text{Area } \Delta QSR = \frac{1}{2} (20 \cos \theta)^2 \sin \beta$
 $= 200 \cos^2 \theta \cdot \sin \beta$

Functions

Gr. 11

$$1. \begin{aligned} f(x) &= a \sin x \\ g(x) &= \cos bx \\ \therefore a &= 2 \\ b &= 2 \end{aligned}$$

$$2. \begin{aligned} \text{amplitude of } f &= 2 \\ \text{period of } g &= 180^\circ \end{aligned}$$

$$3.1 \quad f(x) < 0: 180^\circ \leq x \leq 360^\circ$$

$$3.2 \quad f(x) - g(x) = 3 \quad \text{when } x = 90^\circ$$

(because $2 - (-1) = 3$)

$$3.3 \quad f(x) \cdot g(x) \geq 0$$

$x \in [0^\circ; 45^\circ]$ (both graphs +)
 $\cup x \in [225^\circ; 315^\circ]$ (both graphs -)

Gr. 12 1. $\sin \frac{x}{2} = \cos(x - 30^\circ)$

$$\cos(90 - \frac{x}{2}) = \cos(x - 30^\circ)$$

$$\therefore 90 - \frac{x}{2} = \pm(x - 30^\circ) + k \cdot 360^\circ$$

$$\therefore 90 - \frac{x}{2} = x - 30^\circ + k \cdot 360^\circ$$

$$\therefore 180 - x = 2x - 60^\circ + k \cdot 720^\circ$$

$$\therefore -3x = -240^\circ + k \cdot 720^\circ$$

$$\therefore x = 80^\circ + k \cdot 240^\circ$$

$$\text{or } 90 - \frac{x}{2} = -x + 30^\circ + k \cdot 360^\circ$$

$$\therefore 180 - x = -2x + 60^\circ + k \cdot 720^\circ$$

$$x = -120^\circ + k \cdot 720^\circ$$

$$\therefore x = 80^\circ \text{ or } -120^\circ \text{ or } -160^\circ$$

$$\therefore R(80^\circ; 0, 64), O(-120^\circ; -0, 87) P(-160^\circ; -0, 98)$$

2. $\cos x \cos 30^\circ + \sin x \sin 30^\circ$

$$= \cos(x - 30^\circ)$$

$$\therefore f(x) > g(x)$$

$$\text{for } 80^\circ < x < 180^\circ \text{ or}$$

$$-160^\circ < x < -120^\circ$$

3. $h(x) = g(x) - \frac{\sqrt{3}}{2}$

$$y = g(x) - \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

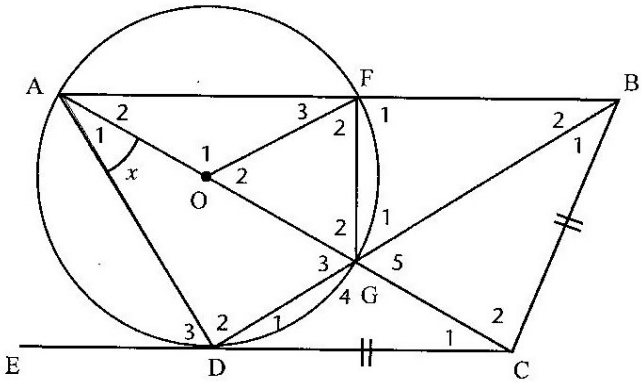
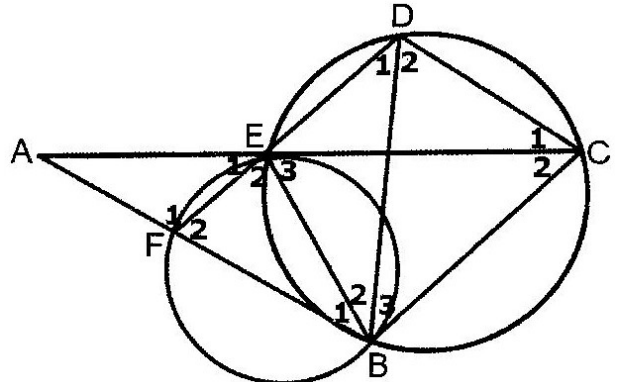
$$= 0$$

Grade 11/12 Euclidean Geometry Revision

Name: _____

Date: _____

On this paper, you will find Gr. 11 style questions alongside Gr. 12 style questions. Determine the required quantities for each question. Answer the metacognitive question prompts **as you are working on the GRADE 12** question. Use this page as a diagram sheet, but attach all working-out pages.

Grade 11 Content	Grade 12 Content	Metacognitive Questions
<p>Circle AFGD with centre O is given. Diameter AG is produced to C. EC is a tangent to the circle at D. DC = BC. AFB is a straight line and $\hat{A}_1 = x$.</p>  <ol style="list-style-type: none"> 1. Prove that ADCB is a cyclic quad. 2. Prove that AC bisects \hat{BAD}. 3. Prove that $DG = FG$. 4. Determine, with reasons, the size of \hat{C}_2. 5. Prove that $\hat{G}_1 = \hat{O}_2$. 6. Prove that FBCG is a cyclic quad. 7. For what value of x will DC be a tangent to the circle through F, B, C and G? 	 <p>AB is a tangent to circle BCDE. $DF \parallel BC$.</p> <ol style="list-style-type: none"> 1. Prove that: <ol style="list-style-type: none"> 1.1 $\triangle BCD \parallel \triangle BEF$ 1.2 AE is a tangent to circle FEB 1.3 $\triangle ABE \parallel \triangle AEF$ 2. If $4FB = 3AB$, show that $AB^2 = 4AE^2$. 	<ol style="list-style-type: none"> 1) What's in the question? 2) What are the differences between the problem you are working on and the previous problem? 3) What is the strategy/tactic/principle appropriate for solving the problem?

Memo: Grade 11/12 Euclidean Geometry Revision

Memo

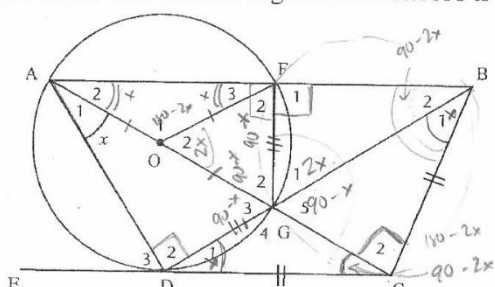
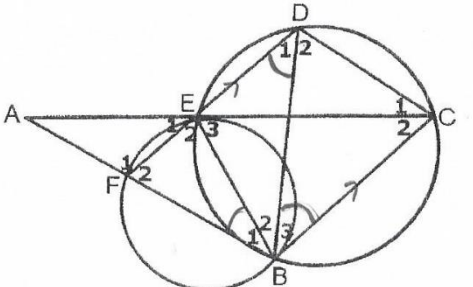
Parktown High School for Girls

Grade 11/12 Euclidean Geometry Revision

Name: Memo

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Geo Revision

Cor. 11:

1. $\hat{D}_1 = \hat{A}_1 = x$ (tan chord)
 $\hat{D}_1 = \hat{B}_1$ (LS opp = sides)
 $\therefore \hat{A}_1 = \hat{B}_1$
 $\therefore ADCB$ is cyclic quad (convex LS in same seg)
2. $\hat{D}_1 = \hat{A}_2 = x$ LS in same seg
 $\therefore \hat{A}_1 = \hat{A}_2$
 $\therefore AZ$ bisects \hat{BAD}
3. $\hat{A}_1 = \hat{A}_2$ proven
 $\therefore DG = FG$ equal chords, equal LS

⑥ In cyclic quad $ADCB$

$$\hat{A}_2 = 90^\circ \quad (\angle \text{ in semi circle})$$

$$\hat{D}_1 = x \quad \text{given}$$

$$\therefore \hat{D}_1 + \hat{D}_2 = 90 + x$$

$$\therefore \hat{B}_1 + \hat{B}_2 = 90 - x \quad (\text{opp LS cyclic quad})$$

In $\triangle OFG$

$$\hat{O}_2 = 2x \quad (\angle \text{ at centre} = 2 \times \angle \text{ at circum})$$

$$\therefore \hat{G}_2 = \frac{180 - 2x}{2} \quad (\text{LS opp = sides = radii})$$

$$= 90 - x$$

$\therefore GFBC$ is a cyclic quad. (convex ext \angle cyclic quad)

$$\therefore \hat{G}_1 = \hat{G}_2$$

⑤ $\hat{G}_1 = \hat{A}_1 = \hat{A}_2$ (ext. \angle of cyclic quad)

$$= 2x$$

$$\hat{O}_2 = 2x$$

$$\therefore \hat{G}_1 = \hat{O}_2$$

(\angle @ centre = $2 \times \angle$ at circum)

$$4. \hat{D}_2 = 90^\circ \quad (\angle \text{ in a semicircle})$$

$$\hat{O}_2 = \hat{C}_2 \quad (\angle \text{ in the same seg})$$

$$\therefore \hat{C}_2 = 90^\circ$$

$$5. \hat{G}_1 = \hat{A}_1 + \hat{A}_2 \quad (\text{ext. } \angle \text{ of cyclic quad})$$

$$= 2x$$

$$\hat{O}_2 = 2x \quad (\angle \text{ @ centre} = 2x \angle \text{ @ circum})$$

$$\therefore \hat{G}_1 = \hat{O}_2$$

$$6. \hat{F}_2 = 90^\circ \quad (\angle \text{ in semicircle})$$

$$\hat{C}_2 = 90^\circ \quad \text{proven}$$

$$\therefore \text{FBCE is a cyclic quad (converse ext. } \angle \text{ of cyclic quad)}$$

$$7. \hat{C}_1 = 90^\circ - 2x \quad (\text{sum } \angle \text{ s in } \triangle OBC)$$

$$\therefore \text{for } OC \text{ to be tangent,}$$

$$\hat{B}_1 = \hat{C}_1$$

$$\therefore 90^\circ - 2x = x$$

$$90^\circ = 3x$$

$$\therefore x = 30^\circ$$

Cr. 12

$$\begin{aligned}
 1.1 \quad \hat{B}_3 &= \hat{D}_1 \quad (\text{alt } \angle s; DF \parallel BC) \\
 \hat{B}_1 &= \hat{D}_1 \quad (\text{tan chord theorem}) \\
 \therefore \hat{B}_3 &= \hat{B}_1 \\
 \hat{D}_1 \hat{C} B &= \hat{E}_2 \quad (\text{ext } \angle \text{ cyclic quad}) \\
 \hat{D}_2 &= \hat{F}_2 \quad (3^{\text{rd}} \angle) \\
 \therefore \triangle BCD &\parallel \triangle BEF
 \end{aligned}$$

$$\begin{aligned}
 1.2. \quad \hat{E}_1 &= \hat{C}_2 \quad (\text{corr } \angle s; FD \parallel BC) \\
 \hat{D}_1 &= \hat{C}_2 \quad (\angle s \text{ in the same seg}) \\
 \therefore \hat{E}_1 &= \hat{D}_1 \\
 \text{but } \hat{D}_1 &= \hat{B}_1 \quad (\text{tan chord theorem}) \\
 \therefore \hat{E}_1 &= \hat{B}_1 \\
 \therefore AE &\text{ is tang. } \quad (\text{converse tan chord theorem})
 \end{aligned}$$

$$\begin{aligned}
 1.3 \quad \hat{A}_1 &= \hat{A}_2 \quad \text{common} \\
 \hat{B}_1 &= \hat{E}_1 \quad (\text{tan chord theorem}) \\
 \hat{E}_1 + \hat{E}_2 &= \hat{F}_1 \quad (3^{\text{rd}} \angle) \\
 \therefore \triangle ABE &\parallel \triangle AEF \quad (\text{AAA})
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{AB}{AE} &= \frac{AE}{AF} = \left(\frac{BE}{EF} \right) \quad | \quad \triangle ABE \parallel \triangle AEF \\
 \therefore AB \cdot AF &= AE \cdot AE \\
 \text{but } 4FB &= 3AB \\
 \therefore FB &= \frac{3}{4} AB \\
 \therefore AF &= \frac{1}{4} AB \\
 \therefore \frac{AB \cdot AB}{4} &= AE^2 \\
 \therefore AB^2 &= 4AE^2
 \end{aligned}$$

Grade 12 Paper 2 Challenge

Gr. 12 Paper 2
CHALLENGE

PARKTOWN HIGH SCHOOL FOR GIRLS
TRIG EQUATIONS WORKSHEET
APRIL 2015

GRADE 12

Solve for x in each of the following:

<p>1. $\cos x = -0,651$ for $x \in [-270^\circ; 90^\circ]$</p> <p>2. $\cos 2x = 0,772$</p> <p>3. $\tan 2x = 1,421$ for $x \in [0^\circ; 180^\circ]$</p> <p>4. $3\sin x = -\cos x$</p> <p>5. $\cos(3x-5) = \cos(x+5)$</p> <p>6. $\tan\left(\frac{x}{2} + 45^\circ\right) = \tan x$</p> <p>7. $\cos(2x-54^\circ) = \sin(x+4^\circ)$</p> <p>8. $\sin(3x-15^\circ) = \sin(4x+20^\circ)$</p> <p>9. $\cos 4x - \sin 4x = 0$ for $x \in [-60^\circ; 120^\circ]$</p> <p>10. $\sin(x+30^\circ) = \sin 2x$</p> <p>11. $2\cos^2 x + 2\sin x \cos x = 0$</p> <p>12. $2\sin^2 x + \sin x - 1 = 0$</p> <p>① 13. $2\cos^2 x - \cos x = 0$</p> <p>14. $\sin 5x = \cos 4x$</p> <p>15. $\cos^2 x = -\cos x + \sin^2 x$</p> <p>16. $\sin 4x + \sin 2x = 0$</p> <p>17. $6\tan x = -\frac{\sqrt{3}}{2}$</p> <p>18. $2\cos^2 x + 3\cos x + 1 = 0$</p> <p>19. $3\cos x = 2\sin^2 x - 1$</p> <p>20. $\cos^2(2x+10^\circ) + 3\cos(2x+10^\circ) + 2 = 0$</p> <p>21. $\frac{5}{\cos^2 x} = 7 - 9\tan x$</p>	<p>22. $\sqrt{3}\cos x + \sin x = 0$</p> <p>23. $-\cos^2 x + 4\sin x + 2 = 0$</p> <p>② 24. $\cos(x+60^\circ) = \sin 2x$ for $x \in [-360^\circ; 0^\circ]$</p> <p>25. $\sin x \cdot \cos 34^\circ + \cos x \cdot \sin 34^\circ = \sin 75^\circ$</p> <p>26. $\sin^2 x + 2\sin x - 2\cos x - \cos x \cdot \sin x = 0$</p> <p>27. $\sin(2x+15^\circ) = \frac{1}{\sqrt{2}}$</p> <p>28. $10\sin x - 6\cos x = 3\cos^2 x - 5\sin x \cdot \cos x$</p> <p>29. $\frac{\cos 2x}{\sin 4x} = 1$</p> <p>30. $2\sin^3 x + 4\cos^2 x - \sin x - 2 = 0$</p> <p>31. $\tan 2x + 2\sin x = 0$</p> <p>32. $\sin 2x = 5\cos x$</p> <p>33. $\sin x + \tan x = 3 + \frac{3}{\cos x}$</p> <p>34. $\frac{2\cos x}{\sqrt{3}} + \frac{4}{\sqrt{3}\tan x} + 1 + \frac{2}{\sin x} = 0$</p> <p>35. $\tan^2 x + 3 = \frac{2}{\cos^2 x}$</p> <p>③ 36. $\cos 2x + 4\sin^2 x - 5\sin x + 1 = 0$</p> <p>37. $2\cos x = \sin(x+30^\circ)$</p> <p>38. $\cos 30^\circ \cdot \sin x = \sin 30^\circ \cdot \cos x$</p> <p>39. $\cos x \cdot \sin(3x+10^\circ) + \cos(3x+10^\circ) \cdot \sin x = 0,5$</p> <p>40. $2\cos x - \frac{4}{\tan x} + \frac{2}{\sin x} = 1$</p>
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Grade 12 Paper 2 Challenge: Memorandum

Gr. 12 Paper 2 Challenge
Day 1

Strategy

1. $\cos x = -0,651$ for $x \in [-270^\circ; 90^\circ]$
 $\therefore x = \pm 130,6^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ ← take \cos^{-1} both sides; ①
 $\therefore x \in \{-130,6^\circ; -229,4^\circ\}$ ← 2 solutions per period ②
 • periodicity ③
 $k = -1(360^\circ) + 130,6$ ←
 solution(s) in domain ⑤
2. $\cos 2x = 0,772$
 $\therefore 2x = \pm 39,5^\circ + k \cdot 360^\circ$ ① ② ③
 $x = 19,7^\circ + k \cdot 180^\circ$ ← divide both sides by 2 ④
 or $x = -19,7^\circ + k \cdot 180^\circ$ ← inc. period.
 $k \in \mathbb{Z}$
3. $\tan 2x = 1,421$ for $x \in [0^\circ; 180^\circ]$
 $\therefore 2x = 54,86^\circ + k \cdot 180^\circ$ ← \tan^{-1} of both sides ⑥
 $x = 27,43^\circ + k \cdot 90^\circ$ ④ ← 1 solution/period for tan
 $\therefore x \in \{27,43^\circ; 117,43^\circ\}$ ⑤ ← period = 180°
4. $3 \sin x = -\cos x$ ← divide both sides by $\cos x$ (ok bcz ⑬
 $\tan x = -\frac{1}{3}$ ← inc. in tan) AND both have same ⑩
 ← divide both sides by 3
 $\therefore x = -18,4^\circ + k \cdot 180^\circ$ $k \in \mathbb{Z}$
5. $\cos(3x - 5^\circ) = \cos(x + 5^\circ)$ $k \in \mathbb{Z}$
 $\therefore 3x - 5^\circ = \pm(x + 5^\circ) + k \cdot 360^\circ$ ① ② ③
 $\therefore 3x - 5^\circ = x + 5^\circ + k \cdot 360^\circ$ (NB) if $\cos A = \cos B$
 then $A = \pm B$
 $2x = 10^\circ + k \cdot 360^\circ$
 $x = 5^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ ← solve separately ⑦
 (OR) $3x - 5^\circ = -x - 5^\circ + k \cdot 360^\circ$
 $4x = 0^\circ + k \cdot 360^\circ$
 $x = k \cdot 90^\circ$

Day 1: Cont'd

6. $\tan\left(\frac{x}{2} + 45^\circ\right) = \tan x$
 $\therefore \frac{x}{2} + 45^\circ = x + k \cdot 180^\circ; k \in \mathbb{Z}$ (6) (2) (3)
 $\therefore \frac{x}{2} = x - 45^\circ + k \cdot 180^\circ$
 $\therefore x = 2x - 90^\circ + k \cdot 360^\circ$ (8) multiply both sides by coefficient of x, inc. period
 $-x = -90^\circ + k \cdot 360^\circ$
 $\therefore x = 90^\circ + k \cdot 360^\circ$

7. $\cos(2x - 54^\circ) = \sin(x + 4^\circ)$ (9) Use cofunction - cos is easier
 $\therefore \cos(2x - 54^\circ) = \cos(90^\circ - (x + 4^\circ))$
 $\cos(2x - 54^\circ) = \cos(86^\circ - x)$ (10) SIMPLIFY
 $\therefore 2x - 54^\circ = \pm(86^\circ - x) + k \cdot 360^\circ; k \in \mathbb{Z}$ (1) (2) (3)
 $\therefore 2x - 54^\circ = 86^\circ - x + k \cdot 360^\circ$
 $3x = 140^\circ + k \cdot 360^\circ$
 $x = 46,7^\circ + k \cdot 120^\circ$ (7)
 (OR) $2x - 54^\circ = -86^\circ + x + k \cdot 360^\circ$
 $x = 32^\circ + k \cdot 360^\circ$

8. $\sin(3x - 15^\circ) = \sin(4x + 20^\circ)$
 $\therefore 3x - 15^\circ = 4x + 20^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ (11) (12)
 $\therefore -x = 35^\circ + k \cdot 360^\circ$
 $\therefore x = -35^\circ + k \cdot 360^\circ$
 (OR) $3x - 15^\circ = 180^\circ - (4x + 20^\circ) + k \cdot 360^\circ$
 $3x - 15^\circ = 160^\circ - 4x + k \cdot 360^\circ$
 $7x = 175^\circ + k \cdot 360^\circ$
 $\therefore x = 25^\circ + k \cdot 51,4^\circ$

9. $\cos 4x - \sin 4x = 0$ for $x \in [-60^\circ, 120^\circ]$
 $\therefore \frac{\cos 4x}{\cos 4x} = \frac{\sin 4x}{\cos 4x}$ (12) ← "move" sin 4x to other side
 $\therefore \tan 4x = 1$ (13) ← divide by cos 4x
 $4x = 45^\circ + k \cdot 180^\circ$
 $\therefore x = 11,25^\circ + k \cdot 45^\circ$
 $\therefore x \in \{-33,75^\circ; 11,25^\circ; 56,25^\circ; 101,25^\circ\}$

10. $\sin(x+30^\circ) = \sin 2x$
 $\therefore x+30^\circ = 2x + k \cdot 360^\circ$ (11) or $x+30^\circ = 180^\circ - 2x + k \cdot 360^\circ$ (11)
 $\therefore x = 30^\circ + k \cdot 360^\circ$ or $3x = 150^\circ + k \cdot 360^\circ$

11. $2\cos^2 x + 2\sin x \cos x = 0$ ← quadratic structure
 $\therefore 2\cos x (\cos x + \sin x) = 0$ ← factorise: HCF (14)

(16) $\therefore \cos x = 0$ or $\cos x + \sin x = 0$ ← (NB) Quadratic structure
 $\therefore x = \pm 90^\circ + k \cdot 360^\circ$ or $2\cos x = -\sin x$ ← zero-product property (15)
 $\therefore \tan x = -1$ ← (13)
 $\therefore x = 135^\circ + k \cdot 180^\circ$

12. $2\sin^2 x + \sin x - 1 = 0$ ← quadratic
 $(2\sin x - 1)(\sin x + 1) = 0$ (17) factorise trinomial
 $\therefore \sin x = \frac{1}{2}$ or $\sin x = -1$ (15)

$\therefore x = 30^\circ + k \cdot 360^\circ$ or $x = -90^\circ + k \cdot 360^\circ$ ← (7)
 $\therefore x = 150^\circ - 30^\circ + k \cdot 360^\circ$ or $x = 180^\circ - (-90^\circ) + k \cdot 360^\circ$ ← (7)
 $= 150^\circ + k \cdot 360^\circ$ or $x = 270^\circ + k \cdot 360^\circ$
 (NB) Quadratic structure with 4 solutions (19)

13. $2\cos^2 x - \cos x = 0$ (18) Do not divide by $\cos x$: factorise
 $\cos x (2\cos x - 1) = 0$ (14)
 $\therefore \cos x = 0$ or $\cos x = \frac{1}{2}$ (15)
 $\therefore x = \pm 90^\circ + k \cdot 360^\circ$ or $x = \pm 60^\circ + k \cdot 360^\circ$

List of Content-specific Strategies for Trigonometry

Content-specific strategies for trig

- ① take \cos^{-1} of both sides
- ② 2 solutions per period
- ③ periodicity
- ④ divide both sides by coefficient of x : including period
- ⑤ find solutions in given domain (multiply integer values of k by period)
- ⑥ take \tan^{-1} of both sides
- ⑦ solve 2 trig equations separately
- ⑧ multiply both sides by coeff. of x - inc. period
- ⑨ Use co-function - \cos is easier
- ⑩ SIMPLIFY as you go
- ⑪ take \sin^{-1} ; (θ) ; $(180 - \theta)$
- ⑫ "move" expression to other side
- ⑬ divide by $\cos \theta$ to make $\tan \theta$
- ⑭ Factorise: HCF (Quadratic structure)
- ⑮ Zero-product property
- ⑯ Divide equation in std form through by numerical factor
- ⑰ Factorise trinomial [could use k -sub]
- ⑱ Do not divide by $\cos x$ - one solution will be lost
- ⑲ Quadratic trig equations can have 4 solutions
- ⑳ Put quad. eq. in std form
- ㉑ squares identity
- ㉒ Factorise by grouping (4 terms)
- ㉓ Factorise cubic!

Trig in 2/3 D

- ④ Draw a figure (fill in info)
- ① \angle of elevation = \angle of depression
- ② sine rule
- ③ co-function

Exemplar: Learner Responses with codes (Before coding)

Learner responses with codes	Instrument: 1	Question: 1.1	Class: 3
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LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description								
3.1	<p>Paper 1</p> <table border="1"> <tr> <td>Mean</td> <td>Med</td> <td>SD</td> <td>Range</td> </tr> <tr> <td>46,1</td> <td>46</td> <td>15,2</td> <td>91</td> </tr> </table> <p> $\therefore \text{Mean} = 46,1 \times 4 = 184,4$ $\text{Median} = 46 \times 4 = 184$ $\text{SD} = 15,2 \times 4 = 60,8$ $\text{Range} = 91 \times 4 = 364$ </p>	Mean	Med	SD	Range	46,1	46	15,2	91	None							0	
Mean	Med	SD	Range															
46,1	46	15,2	91															
3.2	<p>Mean: $46,1 + 4 = 50,1$</p> <p>Median: $46 + 4 = 50$</p> <p>SD: $15,2 + 4 = 19,2$</p> <p>Range: $91 + 4 = 95$</p>	<p>What strategy/ies did you use?</p> <p>I added 4 to each value.</p> <p>How did you use the strategy?</p> <p>By adding 4</p>																

Appendix H – 1

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.3	<p>Mean</p> $= \frac{\sum x}{n}$ $= \frac{46 + 15.2 + 91 + 4}{5}$ $= 40.46$ <p>Median</p> $= 32.6$ <p>because $46 + 15.2 + 4$</p> $= 65.2 : 2$ $= 32.6$ <p>Range</p> $= 91 - 15.2 + 4$ $= 79.8$	None								
3.4	<p>mean: $46.1 \times 4 = 184.4$</p> <p>Med: $46 \times 4 = 184$</p> <p>SD</p> <p>Range: $91 - 4 = 22.75$</p>	<p>I just multiplied thought about it.</p> <p>I multiplied everything by 4</p> <p>I did not know what else to do.</p>								

Appendix H – 1

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.5	<p>1.1 Mean = 50.1 med: 52 SD = 21.2</p> <p>Range: 95</p>	none								
3.6	(absent)	(absent)								
3.7	(Note: answered in "How did you use the strategy?" column)	<p>$46.1 \times 4 = 184.4$</p> <p>$46 \times 4 = 184$</p> <p>$15.2 = 60.8$</p> <p>$99 = 364$</p>								
3.8	None	none								
3.9	<p>1.a. Mean: 50.1 (add 4 to each number & divide by no. of values)</p> <p>b. Median: 50</p> <p>c. SD:</p> <p>d. Range: 91 (range remains same as 4 is added to all values)</p>	none								
3.10	<p>Mean Med SD Range</p> <p>48.1 or 46 17.2 95</p> <p>50.1</p>	none								

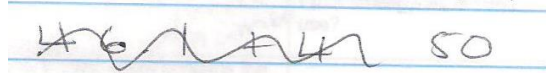
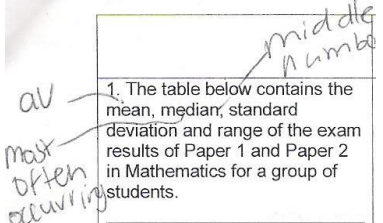
Appendix H – 1

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.11	<p>1.4 Mean - $1 \rightarrow 5$</p> 46.1×0.4 $= 18.44 + 46.1 = 64.54$ 46×0.4 $= 18.4 + 46 = 64.4$ <p>SD</p> 16.2×0.4 $6.08 + 15.2 = 21.28$ <p>R</p> $36.4 + 91 = 127.4$	none								
3.12	none	none								
3.13	(absent)	(absent)								
3.14	<p>Median $\rightarrow 50$</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> Median, middle term, if 4 marks added to each kid - then $46 + 4 = 50$ </div> <div style="border: 1px solid black; padding: 5px;"> General reasoning? </div>								

Appendix H – 1

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.15	<p>Sum of class points no of class points</p> <p>indicator for which is upper or lower median.</p> <p>1.1 Mean Median Standard Range ↳ middle value deviation</p> <p>(original) P1: 46 ; 46 ; 15,2 ; 91</p> <p>D1 = 50,1 ; 50 ; 19,2 ; 94</p> <p>But why? How does this help me.</p> <p>3,8 ← 46 → 61,2 15,2 15,2</p>	None (but working shows learner thinking)								

Learner responses with codes	Instrument: 1	Question: 1.1	Class: 1
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LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
1.1	46,1 + 4 = 50,1% Mean 46,1 + 4 = 50,1% Mean 46 + 4 = 50% Median 15,2 + 4 = 19,2 Range Standard Deviation 91 + 4 = 95% Range	none								
1.2	None	None								
1.3										
1.4	None	none								
1.5	none	none								
1.6	Mean Median Mode.	none								
1.7	None	none								
1.8	none	none								

Appendix H – 1

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
1.9	$\text{mean} = 50.1 \quad \text{Paper 1}$ $= 64.3 \quad \text{Paper 2}$ $\text{median} = 50$ $\text{Standard deviation} : 19.2$	none								
1.10	$\text{Mean} = 46,1 \quad \text{S.D} = 15,2 \times 4 \div 100$ $\text{Med.} = 46 \quad = 60,8 \div 100$ $\text{SD} = 15,2 \quad \text{Range} = 91 \times 4 \div 100$ $\text{Range} = 91 \quad = 364 \div 100$ $\text{Mean} = 46,1 \times 4 \div 100$ $= 184,4 \div 100$ $\text{Median} = 46 \times 4 \div 100$ $= 184 \div 100$	none								
1.11	None	none								
1.12	none	none								

Appendix H – 1

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
1.13	<p> $\mu = 46,1\%$ $= 0,461$ $4\% = 0,04$ $0,461 + 0,04 = 0,0861 \times 100$ $= 8,61\%$ $\therefore 46,1\% + 8,61\%$ $= 54,71\%$ </p> <p> $\text{Median} = 46\% = 0,46$ $4\% = 0,04$ $0,46 + 0,04 = 0,8 \times 100$ $= 50\%$ </p> <p> $\text{Standard deviation} = 15,2\% = 0,152$ $4\% = 0,04$ $0,152 + 0,04 = 0,192 \times 100\%$ $= 19,2\%$ $\therefore 19,2\% + 15,2\%$ $= 34,4\%$ </p> <p> $\text{Range} = 9\% = 0,09$ $4\% = 0,04$ $0,09 + 0,04 = 0,13 \times 100 = 13\%$ $9\% + 18\% = 27\%$ </p>	none								

Instrument 1: Criteria for coding values per question (note: numbers in columns represent codes)

Q.	CU			PF			SC		
	+	+/-	-	+	+/-	-	+	+/-	-
1.1	<p>1. all 3 – 4 answers correct</p> <p>2. used correct formula from memory</p>	<p>1. 2 answers correct</p> <p>2. added 4 to range and SD</p>	<p>1. 0 - 1 answer correct</p> <p>2. Misused numbers given</p> <p>3. Wrong use of operation</p> <p>4. Used wrong value (e.g. 6)</p> <p>5. Gave choice of answer</p> <p>6. little or no progress</p>	<p>1. did correct calculations with correct numbers</p>	<p>1. Continued accuracy: did correct calculations with wrong numbers (i.e. answer was “correct” if values put in calculator gave same result)</p>	<p>1. Incorrect calculations with wrong numbers (i.e. if the values given were put in calculator, calculator would give same result)</p>	<p>N/A</p> <p>This question did not require strategic competence</p>		
1.2	<p>1. all 3 answers correct</p>	<p>1. 2 answers correct</p> <p>2. Correctly found 10% of 50</p> <p>3. Multiplied 5 x 3 but used incorrectly</p>	<p>1. 0 - 1 answer correct</p> <p>2. Misused numbers given</p> <p>3. Wrong use of operation</p> <p>4. Found 10% of another number</p> <p>5. Did not multiply 5 by 3</p> <p>6. subtracted 3 from mean and/or median</p> <p>7. little or no progress</p> <p>8. subtracted 15 from all values</p>	<p>1. did correct calculations with correct numbers</p>	<p>1. Continued accuracy: did correct calculations with wrong numbers (i.e. answer was “correct” if values put in calculator gave same result)</p>	<p>1. Incorrect calculations with wrong numbers (i.e. if the values given were put in calculator, calculator would give same result)</p>			

Appendix I – 1

2	CU			PF			SC		
	+	+/-	-	+	+/-	-	+	+/-	-
	<p>1. $A = l \times b$</p> <p>2. $(l+1)(b+1) = original\ area + 20$</p> <p>3. $(l+2)(b+2) = 2 \times original\ area$</p> <p>4. Retained only positive solution of quadratic equation</p> <p>5. (ca) substituted x to find dimension</p>	<p>1. $(l+1)(b+1) = 20$</p> <p>2. $(l+2)(b+2) = 40$</p> <p>3. correct formulas but with a^2 instead of a</p> <p>4. correct product of dimensions = $dimension + 20/40$</p> <p>5. $(l+2)(b+2) = lb + 40$ and/or $(l+1)(b+1) = lb + 20$</p> <p>6.</p>	<p>1. same variable for both dimensions</p> <p>2. Doubling dimensions instead of area</p> <p>3. guess/assign value for dimension</p> <p>4. confusing dimensions</p> <p>5. equate the areas of the two rectangles</p> <p>6. incorrect product = incorrect area</p> <p>7. (In)correctly solves for x but substitutes back into same equation</p> <p>8. perimeter = area</p>	<p>1. 0 procedural errors (ca)</p> <p>2. correctly solve own equations (possibly one error only) (ca)</p> <p>3. Use quad. formula correctly</p> <p>4. Efficient calculations (necessary and sufficient)</p> <p>5. (ca) substituted x to find dimension</p>	<p>1. 1 – 2 procedural errors only</p> <p>2. Attempt at simul. equations (but incorrect method)</p> <p>3. Sets up quadratic equation but does not/cannot use quadratic formula</p> <p>4. Attempt at simul. equation, but cannot finish</p> <p>5. 2 accurate (ca) equations but cannot continue to solve simultaneously</p> <p>6. (In)correctly solves for x but substitutes back into same equation</p>	<p>1. procedural errors > 3</p> <p>2. Proc errors at low level (e.g. Gr. 8) maths</p> <p>3. product of area does not compute</p> <p>4. Brackets not used</p> <p>5. lengthy, inefficient calculations</p>	<p>1. Assigned separate variables for original length/breadth</p> <p>2. $(x+1)(y+1) = xy + 20$ (i.e. set up equation correctly in terms of own variables)</p> <p>3. $(x+1)(y+1) = 2xy$</p> <p>4. Correctly set up simultaneous equations (possibly one error or ca)</p> <p>5. correctly solves for one variable in terms of another (ca)</p>	<p>1. One side correct for x + 1 equation, but not other side.</p> <p>2. One side correct for x + 2 equation, but not other side.</p> <p>3. Introduce 3 variables</p> <p>4. Use units in equation(s)</p> <p>5. recognises need for simultaneous equations but cannot solve</p> <p>6. sets up simultaneous equation but makes 1 error</p> <p>7. area of 2nd eq. = 2x area of first equation</p>	<p>1. same variable for both dimensions</p> <p>2. not able to set up any equations</p> <p>3. introduce more than 3 variables</p> <p>4. Has two equations but does not set up simultaneous equations</p> <p>5. No variable assigned</p> <p>6. Only has one equation for two unknowns</p>

Appendix I – 1

3	CU			PF			SC		
	+	+/-	-	+	+/-	-	+	+/-	-
	<p>1. correctly reducing $\tan 1982^\circ$ to $\tan 2^\circ$</p> <p>2. correctly recognising linear pattern of degrees of RHS (e.g. by using first differences)</p> <p>3. $T_{100} = \tan 1982^\circ$</p> <p>4. Used correct linear formula (e.g. $T_n = a + (n - 1)d$)</p>	<p>1. find first difference but apply nonlinear formula</p> <p>2. $T_{100} = 1982$ ($^\circ$'s not necessarily specified, but \tan is not mentioned)</p> <p>3. attempt to explore first differences (even if incorrect)</p>	<p>1. confusing \tan of number with number itself (e.g. $\tan 1982 = 1982$)</p> <p>2. Confuses $2 \times n$ and a <i>list</i> of n 2's</p> <p>3. Combined sequences</p> <p>4. finds non-linear pattern</p> <p>5. uses $d = \tan 20^\circ$</p> <p>6. $\tan 20(100) = \tan 2000$</p> <p>7. $\tan 2000 - \tan 18 = \tan (2000 - 18)$</p> <p>8. no conceptual sense</p>	<p>1. substituted correct numbers into formula</p> <p>2. simplified correctly</p> <p>3. Uses correct AP formula</p>	<p>1. correctly substitutes into wrong formula</p> <p>2. correctly simplifies wrong formula</p>	<p>1. substitutes wrong numbers in formula</p> <p>2. Incorrect simplification</p> <p>3. $\tan 20(100) = \tan 2000$ (i.e. incorrect multiplication)</p> <p>4. $\tan 2000 - \tan 18 = \tan (2000 - 18)$ (i.e. combining unlike terms)</p> <p>5. No progress</p> <p>6. Uses incorrect formula</p>	<p>1. applies linear progression formula to degrees on RHS</p> <p>2. Relates number of 2's to n</p> <p>3. Investigates first differences of degrees</p>	<p>1. attempting to find constant difference of degrees on RHS, but not successful</p>	<p>1. found 1st differences of \tan's of RHS</p> <p>2. continued pattern as is</p> <p>3. took 1st differences of degrees of LHS</p> <p>4. Used 1st differences of reduced degrees on RHS</p>

Instrument 2: Criteria for coding values per question (note: numbers in columns represent codes)

Q.	CU			PF			SC		
	+	+/-	-	+	+/-	-	+	+/-	-
1.1	<p>1. Saw numbers as data set</p> <p>2. used mean formula correctly</p> <p>3. used variance formula correctly</p>	<p>1. recognised need for mean formula, but used incorrectly</p> <p>2. recognised need for variance formula, but used incorrectly</p>	<p>1. Saw list of data values as pattern</p> <p>2. tried to find differences between “terms” of “sequence”</p> <p>3. little or no progress</p> <p>4. used quadratic pattern $2a = \dots$ for “2nd difference”</p> <p>5. Assumed <i>linear</i> pattern and found x and y in terms of 1st differences</p> <p>6. ignored the “sum of” aspect of variance equation</p> <p>7. equating unequal quantities</p>	<p>1. able to solve linear mean equation (ca)</p> <p>2. able to solve quadratic (variance) equation (ca)</p> <p>3. able to correctly substitute in to simultaneous equation</p> <p>4. able to correctly solve simultaneous equation</p> <p>5. Successfully used trial-and-improvement method to find x and y</p> <p>6. 0 – 1 procedural errors (ca)</p>	<p>1. used mean formula but with error(s)</p> <p>2. used variance formula but with error(s)</p> <p>3. used simultaneous equation but with error(s)</p> <p>4. Able to simplify quadratic equation to standard form but not solve</p> <p>5. Factorising error</p> <p>6. tried to solve quadratic in form $ax^2 + bx = c$ or $ax^2 = bx + c$</p>	<p>1. significant procedural error(s)</p> <p>2. incorrect substitutions</p>	<p>1. set up mean equation correctly</p> <p>2. set up variance equation correctly</p> <p>3. set up simultaneous equation correctly</p> <p>4. Successfully used trial-and-improvement method</p>	<p>1. set up mean equation but with errors</p> <p>2. set up variance equation but with errors (e.g. ignored sigma)</p>	<p>1. Tried to set up sequence equation</p>

2	CU			PF			SC		
	+	+/-	-	+	+/-	-	+	+/-	-
	<p>1. Understood the need to find values of <u>a and b</u> in order to calculate SA</p> <p>2. Correctly expressed the separate areas of the faces of the <u>both</u> figures</p>	<p>1. Attempted to find an expression for either <u>a or b</u></p> <p>2. correctly expressed the separate areas of the faces of one figure</p>	<p>1. Did not attempt to find an expression for either <u>a or b</u></p> <p>2. did not correctly identify separate areas for either figure</p> <p>3. confused the dimensions of the two figures when calculating SA</p> <p>4. introduced third dimension when calculating SA (therefore, calculating volume, not SA)</p> <p>5. confused <u>sum</u> of dimensions with a single dimension</p> <p>6. added dimensions instead of multiplying</p> <p>7. Very little progress</p> <p>8. used numbers given as <u>as</u> dimensions</p>	<p>1. Wrote down SA formula (possibly from notes) correctly</p> <p>2. correctly solves for <u>a and b</u> in terms of other variable(ca)</p> <p>3. correctly substituted <u>a and b</u> to find SA (ca)</p> <p>4. correctly simplified SA (ca)</p> <p>5. Design 1 SA = $4ab + 36a + 72b$</p> <p>6. Design 2 = $4ab + 72a + 36b$</p>	<p>1. Wrote down SA formula (possibly from notes) but with error(s)</p> <p>2. correctly solves for <u>a or b</u> in terms of the other variable (ca)</p> <p>3. correctly substituted <u>a or b</u> to find SA (ca)</p> <p>4. Simplifies SA with an error</p> <p>5. attempt at simultaneous equation, but cannot continue or mistake</p>	<p>1. Did not use any SA formula</p> <p>2. Incorrectly solves for one variable/or does not solve for variable</p> <p>3. incorrectly substituted <u>a and/or b</u> to find SA (ca)</p> <p>4. substitutes 18 and/or <u>a and b</u> into SA formula</p> <p>5. substitutes any combination of given numbers into SA formula</p> <p>6. multiplied surfaces instead of adding</p> <p>7. “double distribute” 2 outside of bracket to</p>	<p>1. Created two equations to find value of one variable in terms of the other variable correctly</p>	<p>1. Created one equation to find value of one variable in terms of other variable</p>	<p>1. Did not attempt an equation to find a or b</p> <p>2. Introduced different variables for question (e.g. h, l, w)</p> <p>3. Creates 1 or 2 incorrect equations for relationship between variables</p> <p>4. Set up equations but did not solve for one variable in terms of the other</p> <p>5. Set up equation(s) for <u>a and b</u> in terms of original (not stacked) prism</p>

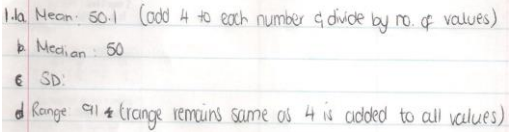
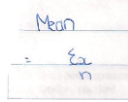
			<p>9. Saw incorrect relationship between variables a, b and 18</p> <p>10. confused SA of unstacked with SA of stacked prisms</p> <p>11. confused (i.e. equated) sum of dimensions with TSA</p> <p>12. Equating unequal quantities</p>			<p>inside bracket</p> <p>8. serious procedural errors</p> <p>9. circular reasoning – solving equation with beginning equation</p>			
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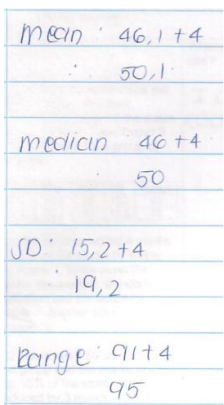
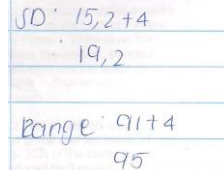
Appendix I – 2

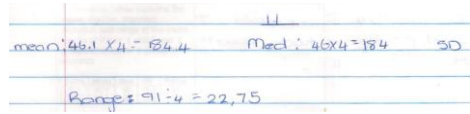
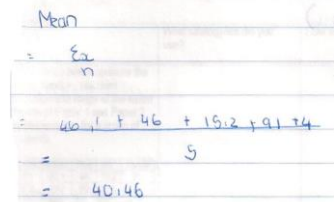
3	CU			PF			SC		
	+	+/-	-	+	+/-	-	+	+/-	-
	<p>1. Expresses all 3 relevant terms i.t.o 3 and multiples of d</p> <p>2. Understands <i>ratio</i> relationship between T_3, T_6, T_{10}</p> <p>3. Expresses 3 terms correctly i.t.o. common ratio</p> <p>4. Saw error and tried a new direction</p>	<p>1. Expresses 1 – 2 terms i.t.o. 3 and multiples of d</p> <p>2. Expresses terms in GP form</p> <p>3. recognises ratio relationship, but not with specific terms</p> <p>4. Found $d = 0$ for first term (i.e. $T_1 = 3$)</p>	<p>1. Incomplete/ incorrect expression of AP terms i.t.o 3 and multiples of d (e.g. does not have value for n)</p> <p>2. Little or no progress</p> <p>3. Found wrong answer for d through misuse of general term.</p>	<p>1. Writes down AP formula</p> <p>2. Correct substitution of values into AP formula for 3 needed terms i.t.o. 3 and multiples of d (formula may not be explicitly stated, but used)</p> <p>3. Correct set up of ratio equation</p> <p>4. Solves ratio equation with no errors</p>	<p>1. Writes down GP formula</p> <p>2. Correct substitution of values into AP formula for 1 - 2 needed terms</p>	<p>1. Writes down AP <i>series</i> formula</p> <p>2. Incorrect/ incomplete substitution of values into AP formula for 1 - 2 needed terms</p> <p>3. Several to many procedural errors</p> <p>4. Incorrect substitution into GP formula</p> <p>5. Cannot solve ratio equation correctly</p> <p>6. Found wrong answer for d through procedural errors working with general term</p>	<p>1. Expresses all AP terms i.t.o. 3 and multiples of d</p> <p>2. Correctly sets up ratio equation using expressions for AP terms i.t.o. 3 and multiples of d</p>	<p>1. Expresses 1 – 2 terms i.t.o. 3 and multiples of d</p> <p>2. Correctly sets up ratio equation but does not use expressions for AP terms i.t.o. 3 and multiples of d</p>	<p>1. Expresses AP terms incorrectly i.t.o. 3 and multiples of d or not at all</p> <p>2. Sets up common difference equation</p> <p>3. Equates T_{10} (and/or T_6) as AP to T_{10} (and/or T_6) as GP</p> <p>4. Found wrong answer for d through misuse of general term and used answer to find value for T_{10}.</p>

Instrument 1: Question 1

Learner examples to determine codes (Exemplar)

Instrument: 1		Question: 1.1
Strand: Conceptual understanding		Rating: +
Code	Description	Learner example
1	all 3 – 4 answers correct	 <p>1.a Mean: 50,1 (add 4 to each number & divide by no. of values) b Median: 50 c SD: d Range: 91 (range remains same as 4 is added to all values)</p>
2	used correct formula from memory	 <p>Mean $= \frac{\sum x}{n}$</p>

Instrument: 1		Question: 1.1
Strand: Conceptual understanding		Rating: +/-
Code	Description	Learner example
1	2 answers correct	 <p>Mean: 46,1 + 4 = 50,1 Median: 46 + 4 = 50 SD: 15,2 + 4 = 19,2 Range: 91 + 4 = 95</p>
2	added 4 to range and SD	 <p>SD: 15,2 + 4 = 19,2 Range: 91 + 4 = 95</p>

Instrument: 1		Question: 1.1
Strand: Conceptual understanding		Rating: -
Code	Description	Learner example
1	0 - 1 answer correct	 <p>mean: $46,1 \times 4 = 184,4$ Med: $46 \times 4 = 184$ SD Range: $91 : 4 = 22,75$</p>
2	Misused numbers given	 <p>Mean $= \frac{\sum x}{n}$ $= \frac{46,1 + 46 + 15,2 + 91 + 4}{5}$ $= 40,46$</p>

3	Wrong use of operation	$\therefore \text{Mean} = 46,1 \times 4$ $= 184,4$ $\text{Median} = 46 \times 4$ $= 184$ $\text{SD} = 15,2 \times 4$ $= 60,8$ $\text{Range} = 91 \times 4$ $= 364$
4	Used wrong value (e.g. 6)	$\text{med: } 52 \quad \text{SD} = 21.2$ <p>(i.e. added 6)</p>
5	Gave choice of answer	<p>Mean</p> <p>48.1 or</p> <p>50.1</p>
6	little or no progress	$46 \times 4 = 184$

Instrument: 1		Question: 1.1
Strand: Procedural fluency		Rating: +
Code	Description	Learner example
1	did correct calculations with correct numbers	$\text{Mean} = 46,1 + 4$ $= 50,1$ $\text{Median} = 46 + 4$ $= 50$

Instrument: 1		Question: 2
Strand: Procedural fluency		Rating: +/-
Code	Description	Learner example
1	Continued accuracy: did correct calculations with wrong numbers (i.e. answer was “correct” if values put in calculator gave same result)	$\therefore \text{Mean} = 46,1 \times 4$ $= 184,4$ $\text{Median} = 46 \times 4$ $= 184$ $\text{SD} = 15,2 \times 4$ $= 60,8$ $\text{Range} = 91 \times 4$ $= 364$

Instrument: 1		Question: 1.1
Strand: Procedural fluency		Rating: -
Code	Description	Learner example
1	Incorrect calculations with wrong numbers (i.e. if the values given were put in calculator, calculator would not give same result)	(None)

Instrument: 1		Question: 1.2
Strand: Conceptual understanding		Rating: +
Code	Description	Learner example
1	all 3 answers correct	(none)

Instrument: 1		Question: 1.2
Strand: Conceptual understanding		Rating: +/-
Code	Description	Learner example
1	2 answers correct	$\therefore \text{Mean} : 57,3$ $\text{Median} : 60$ $\text{Range} : \del{88} 67$
2	Correctly found 10% of 50	$\frac{10}{100} \times 50 = 5$
3	Multiplied 5 x 3 but used incorrectly	$\text{Mean} = 60,3 - (3 \times 5) = 45,3$ $\text{Median} = 60 - (3 \times 5) = 45$ $\text{Range} = 70 - (3 \times 5) = 55$

Instrument: 1		Question: 1.2
Strand: Conceptual understanding		Rating: -
Code	Description	Learner example
1	0 - 1 answer correct	<p>1.2 50 students Marks of top 10% - 3 Median mean range. (original) $P_1 = 60,3, 60, 11,6, 70$ $P_1 = 57,3 ; 57, 8,6, 67$</p>
2	Misused numbers given	$\frac{10}{100} \times 50 = 5$ - reduced by 3 marks: $Q_1 = 1 + 3$ $Q_3 = 1 +$ $\text{mean. med} \quad \text{range } 70 - 0,3 = P_1 - 3$ $60 - 0,3 \quad A_1 = 55 = 59,7 \quad 55 + 0,3$ $= 59,7$
3	Wrong use of operation	$\frac{10}{100} \times 50 = 5$ - reduced by 3 marks: $Q_1 = 1 + 3$ $Q_3 = 1 +$ $\text{mean. med} \quad \text{range } 70 - 0,3 = P_1 - 3$ $60 - 0,3 \quad A_1 = 55 = 59,7 \quad 55 + 0,3$ $= 59,7$

4	Found 10% of another number	$\begin{aligned} &= 10\% \text{ of } 70 \\ &= \frac{10}{100} \times 70 \\ &= 7 \end{aligned}$
5	Did not multiply 5 by 3	$\begin{aligned} &= 10\% \text{ of } 50 \\ &= 5 \\ &= 5 - 3 \\ &= 2 \\ \therefore \text{Mean} \\ &= 60,2 - 2 \\ &= 58,2 \end{aligned}$
6	subtracted 3 from mean	$\therefore \text{Mean} : 57,3$
7	little or no progress	50 students. in top 10% marks : $60 - 3$
8	subtracted 15 from all values	$\begin{aligned} \text{Mean} &= 60,3 - (3 \times 5) = 45,3 \\ \text{Median} &= 60 - (3 \times 5) = 45 \\ \text{Range} &= 70 - (3 \times 5) = 55 \end{aligned}$

Exemplar: Learner Responses with codes (After coding)

Class 12-3 - no absences
127 - 1 absence
No. of respondents: 27

(NB) Need to look at if any improvement in question if context worked on 1st Qn. 12 - i.e. Pattern question

1st attempt at describing strategies

Learner responses with codes Instrument: 2 Question: 1 Class: 3

Note: no other codes used if CU-3 used.

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.1	1. 3, x, y, 10, 12 Find x and y mean = 8 variance = 10 0/11	None	-	3					0	
3.2	3, x, 4, 10, 12 0/11	I need to study a section before I consider the question. I really can't remember what to do and how to do it.	-	3						dismiss. ←
3.3	None	none								
3.4	dismiss Handing 11 diff. found the original expression combined terms substituted set = 0 estimated subbed into original expression set equal to zero solved for y then x from original equation	I used the knowledge of number patterns Simultaneous equations when you have 2 unknown variables to solve for	- - - 7	1 2 7	+ +/-	6 3	-	1		Ex D C S IE Ex P Y " Ex C " "

- conceptual errors:
1) list of numbers separated by semi colon =
2) 1 expression in 2 variables can be solved for

NB: Need to bring in how m.u. affects strands

Page 1 of 12

0.98 = 1,980,000
89.00 = 890,000
= 1210,000

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.5	<p>1. $3; x; y; 10; 12$</p> <p>$3 + x + y + 10 + 12$</p>	<p>* I recall remember how to do the mean problems</p> <p>* I have used the data, sometimes</p> <p>* For some of the students I use their calculator to do much of the work. I can not assure that I am using them correctly</p> <p>* I can do addition, the numbers are small</p> <p>* I can do a simple problem I can do it</p> <p>* I can do some basic I use the same strategy as identifying regular data, working through a problem is asking etc</p> <p>(Note: written on lined paper)</p>	-	3					7	discuss
3.6	<div style="border: 1px solid black; padding: 5px;"> <p>Mean $\frac{25x + 20y}{5} = 8$ ✓</p> <p>$25x + 20y = 40$</p> <p>$x + y = 15$ ✓</p> </div> <p>5/11</p> <div style="border: 1px solid black; padding: 5px;"> <p>Variance $\frac{(x - \bar{x})^2}{n}$ ← formula given but unable to use it</p> <p>$\frac{(x - 8)^2}{5} = 10$ X</p> <p>$(x - 8)(x - 8) = 10$ ✓ CA</p> <p>$x^2 - 16x + 64 = 50$ ✓ CA</p> <p>$x^2 - 16x + 64 - 50 = 0$</p> <p>$x^2 - 16x + 14 = 0$ ✓ CA</p> </div> <p>does not realize applies to all data values.</p>	none	+	1	+	1	+	1	N.	
			+	2	+/-	2	+/-	2		
			+/-	2	+/-	4		✓		
			-	6		✓				
				✓						

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.7	<p><i>discuss</i></p> <p>T_1, T_2, T_3, T_4, T_5 mean = 8 $8, a, y, 10, 12$ variance = 10</p> <p>$x = 6, y = 4$</p> <p>2/11</p> <p>Another case where description does not match what is written down</p> <p>Correct answers from incorrect reasoning (NB)</p>	<p><i>I could not recall how to do this</i></p> <p>1. Consider the set of numbers: $3, x, y, 10, 12$. The mean of the set is 8 and the variance is 10. Determine the numbers x and y.</p> <p>I attempted to find the common difference, but got stuck due to the missing numbers, tried to work it out backwards using the variance and mean.</p> <p>Used the mean formula and worked backwards.</p> <p>I honestly did not know what else to do.</p>	+	-			+	4		<p>Ex O CS PE</p> <p>Ex P H PE</p> <p>Ex C O PE</p> <p>• "Guessed" answers correctly</p>
3.8	none	none								

discuss

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.9	<p><i>discuss</i></p> $\begin{cases} 2x + 3y = 10 & (1) \\ 2x + 4y = 12 & (2) \end{cases}$ <p>for $x = 12 - 2y$</p> $\begin{matrix} (1) & 2x & + & 3y & = & 10 \\ (2) & 2x & + & 4y & = & 12 \\ \hline & & & -y & = & -2 \end{matrix}$ $y = 2$ <p>Substituting $y = 2$ into (1)</p> $2x + 3(2) = 10$ $2x + 6 = 10$ $2x = 4$ $x = 2$ <p>∴ $x = 2$ and $y = 2$</p>	<p>Simultaneous equations. Tables</p> <p>Solve for y, then solve for x by substituting</p>	+	1	+	1	+	1		Ex D CS+H E
			+	2	+	2	+	2		Ex P CS E
			+	3	+	3	+	3		
			✓		+	4	✓			
					+	6				
					✓	5				

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.10	<p>3, x, y, 10, 12.</p> <p>mean = 8 variance = 10</p> <p>3, x, y, 10, 12 \swarrow \searrow \swarrow \searrow $x-3$ $y-x$ $10-y$ 2</p> <p>$2a = y-x - (x-3) - (y-x) - 2 - (10-y)$</p> <p>$2a = y-x - (x-3)$ $2a = y-x + x + 3$ $2a = y - 2x + 3$ $a = \frac{y-2x+3}{2}$</p> <p style="text-align: right;">0/11</p>	none	-	1 2 4 7	+	6	-	1	2	
3.11	<p>Consider the set of numbers</p> <p>3, x, y, 10, 12</p> <p style="text-align: right;">0/11</p>	<p>what type of problem is this</p> <p>used it to think about how to solve the Qs - but didn't get to solve the Q.</p> <p>when I don't know what to do, I think about the types of problems that are, and which is similar to mine</p> <p style="text-align: right;">discuss</p>	-	3						<p>Ex. D. H In</p> <p>Ex P H In</p> <p>Ex C H In</p>

→ equating unequal quantities happened w/ the 'pattern' approach + the 'data' approach

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.12	none	PATTERNS.							N	D C S I E
3.13		none	-	1 2 4 7	-	1	-	1	N	
3.14	<p>3; x; y; 10; 12</p> <p>Mean: $8 = \frac{y}{5}$</p> <p>Variance: $(y - \text{mean})^2$</p> <p>$T_2 - T_1 = x - 3$</p> <p>$T_3 - T_2 = y - x$</p> <p>$T_4 - T_3 = 10 - y$</p> <p>$T_5 - T_4 = 12 - 10 = 2$</p> <p>$\therefore x - 3 = y - x$</p> <p>$\therefore 2x = y + 3$</p> <p>$\therefore 10 - y = 2$</p> <p>$\therefore 10 - 2x - 3 = 2$</p> <p>$\therefore 7 - 2x = 2$</p> <p>$\therefore 5 = 2x$</p> <p>$\therefore x = 2.5$</p> <p>$\therefore y = 7.5$</p> <p>$2 = 10 - y$</p> <p>$y = 8$</p> <p>$10 - y = y - x$</p> <p>$10 = 2y - x$</p> <p>$10 = 2(8) - x$</p> <p>$10 = 16 - x$</p> <p>$x = 6$</p>	<p>Figure out common difference.</p> <p>$T_2 - T_1 = ?$</p> <p>To try figure out if I can do simultaneous equations to figure out x and y</p>	-	1 2 5 7	+	6	-	1		Ex D C S In Ex P C S In Ex C C S In

discuss

linear pattern

Although skill inefficient, are they more articulate!

LC	Worked solution	Strategy	CU	Codes	PF	Codes	SC	Codes	SU	Description
3.15	<p>mean = $\bar{x} = 8$ Variance = 10</p> <p>3 (x) (y) 10, 12 n = 5.</p> <p>$\rightarrow \frac{\sum (x - \bar{x})^2}{n} = 10$</p> <p>$\frac{\sum (x - 8)^2}{5} = 10 \times$</p> <p>$(x - 8)^2 = 50 \checkmark$ or</p> <p>$(x - 8)(x - 8) = 50$</p> <p>$x^2 - 8x - 8x + 64 = 50 \checkmark$ or</p> <p>$x^2 - 16x + 64 = 50$</p> <p>$x^2 - 16x = -14$</p> <p>$x^2 - 16x + 64 = 50$</p> <p>$(x - 8)^2 = 50$</p> <p>$\therefore x^2 = 16x - 14$ 2/11</p> <p>$x^2 = (8x - 7)^2$</p>	<ul style="list-style-type: none"> • collect data • find a equation to establish a connection • sub in. • factorise <p>(note: done alongside working out)</p>	+	1	✓	2	✓	2		Ex D CS PE
			-	6	✓	6				E P CS PE
			✓	2						

note effect of modelling strategy ← discuss RE: SU

no error

note: while I gave a co mark - this would not happen in real life

Tally of Codes (whole group): Instrument 1

INST 1	Question 1.1	Frequencies							
Strand		CU			Strand		PF		
Rating		+	+/-	-	Rating		+	+/-	-
Codes	Description/Totals	2	8	26	Codes	Description/Totals	5	13	0
1.	3 – 4 answers correct	1			1.	did correct calculations with correct numbers	5		
2.	used correct formula from memory	1			1.	Continued accuracy: did correct calculations with wrong numbers		13	
1.	2 answers correct		4		1.	Incorrect calculations with wrong numbers			0
2.	added 4 to range and SD		4						
1.	0 - 1 answer correct			10					
2.	Misused numbers given			4					
3.	Wrong use of operation			7					
4.	Used wrong value (e.g. 6)			2					
5.	Gave choice of answer			1					
6.	little or no progress			6					
	Question 1.2	Frequencies							
Strand		CU							
Rating		+	+/-	-					
Codes	Description/Totals	0	12	27					
1.	all 3 answers correct	0							
1.	2 answers correct		1						
2.	Correctly found 10% of 50		8						
3	Multiplied 5 x 3 but used incorrectly		3						
1.	0 - 1 answer correct			8					
2.	Misused numbers given			3					
3.	Wrong use of operation			4					
4.	Found 10% of another number			1					
5.	Did not multiply 5 by 3			1					
6.	subtracted 3 from mean and/or median			3					
7.	little or no progress			7					

Appendix L – 1

Instr. 1	Question 2	Frequencies				Strand	Frequencies				Strand	Frequencies			
		CU			Rating		PF			Rating		SC			
		+	+/-	-			+	+/-	-			+	+/-	-	
Codes	Description/Totals	13	18	17	Codes	Description/Totals	15	10	10	Codes	Description/Totals	24	27	15	
1	A = l x b	6			1	0 procedural errors (ca)	7			1	Assigned separate variables for original length/breadth	14			
2	(original length +1)(original breadth +1) = original area + 20	2			2	correctly solve own equations (possibly one error only) (ca)	2			2	sets up (original length +1)(original breadth +1) = original area + 20 equation correctly in terms of own	2			
3	(original length +2)(original breadth +2) = 2 x original area	2			3	Use quad. formula correctly (ca)	3			3	set up (original length +2)(original breadth +2) = 2 x original area equation correctly in terms of own variables	2			
4	Retained only positive solution of quadratic equation	2			4	Efficient calculations (necessary and sufficient)	2			4	Correctly sets up simultaneous equations (possibly one error or ca)	3			
5	(ca) substituted x to find dimension	1			5	(ca) substituted x to find dimension	1			5	correctly solves for one variable in terms of another (ca)	3			
1	(original length + 1)(original breadth +1) = 20		8		1	1–2 procedural errors		4		1	One side correct for x + 1 equation, but not other side.		7		
2	(original length + 2)(original breadth +2) = 40		5		2	Attempt simultaneous equations (but incorrect method)		2		2	One side correct for x + 2 equation, but not other side.		6		
3	correct formulas but with a ² instead of a		1		3	Sets up quadratic equation but does not/cannot use quadratic formula		1		3	Introduce 3 variables		5		
4	correct product of dimensions = one dimension + 20		2		4	Attempt simultaneous equations, but cannot finish		1		4	Use units in equation(s)		3		
5	confused “doubling area” with “increasing area by doubling 20”		2		5	cannot continue to solve simultaneously		1		5	equations but solves by incorrect method		4		
1	same variable for both dimensions			3	6	(ln)correctly solves for x but substitutes back into same equation		1		6	sets up simultaneous equation but makes 1 error		2		
2	Doubling dimensions instead of area			4	1	procedural errors > 3			3	1	same variable for both			3	
3	guess/assign value for dimension			5	2	Proc errors at low level (e.g. Gr. 8) maths			2	2	not able to set up any equations			4	
4	Confusing dimensions			1	3	product of area does not compute			2	3	introduce more than 3 variables			0	
5	equate the areas of the two new patios			1	4	Brackets not used			1	4	Has two equations but does not set up simultaneous equations			1	
6	incorrect product = incorrect area			1	5	Lengthy, inefficient calculations			2	5	No variable(s) assigned			5	
7	(ln)correctly solves for x but substitutes back into same equation			1							Only has one equation for two unknowns			2	
8	perimeter = area			1											

Appendix L – 1

Instr. 1	Question 3	Frequencies					Frequencies					Frequencies		
Strand		CU			Strand		PF			Strand		SC		
Rating		+	+/-	-	Rating		+	+/-	-	Rating		+	+/-	-
Codes	Description/Totals	18	4	16	Codes	Description/Totals	21	0	26	Codes	Description/Totals	15	0	4
1	correctly reducing $\tan 1982^\circ$ to $\tan 2^\circ$	0			1	substituted correct numbers into formula	5			1	applies linear progression formula to degrees on RHS	11		
2	correctly recognising linear pattern of degrees of RHS (e.g. by using first differences)	3			2	simplified correctly	4			2	Relates number of 2's to n	0		
3	$T_{100} = \tan 1982^\circ$	4			3	Uses correct AP formula	12			3	Investigates first differences of degrees	4		
4	Used correct linear formula (e.g. $T_n = a + (n - 1)d$)	11												
1	find first difference but apply nonlinear formula		1		1	correctly substitutes into wrong formula		0		1	attempting to find constant difference of degrees on RHS, but not successful		0	
2	$T_{100} = 1982$ ($^\circ$'s not necessarily specified, but \tan is not mentioned)		2		2	correctly simplifies wrong formula		0						
3	attempt to explore first differences (even if incorrect)		1											
1	confusing \tan of number with number itself (e.g. $\tan 1982 = 1982$)			2	1	substitutes wrong numbers in formula			6	1	found 1 st differences of \tan 's of RHS			1
2	Confuses $2 \times n$ and a list of n 2's			0	2	Incorrect simplification			7	2	continued pattern as is			1
3	Combined sequences			1	3	$\tan 20(100) = \tan 2000$ (i.e. incorrect multiplication)			2	3	took 1 st differences of degrees of LHS			1
4	finds non-linear pattern			2	4	$\tan 2000 - \tan 18 = \tan (2000-18)$ (i.e. combining unlike terms)			2	4	Used 1 st differences of reduced degrees on RHS			1
5	uses $d = \tan 20^\circ$			4	5	No progress			7					
6	$\tan 20(100) = \tan 2000$			2	6	Uses incorrect formula			2					
7	$\tan 2000 - \tan 18 = \tan (2000-18)$			2										
8	no conceptual sense			3										

Tally of Codes (whole group): Instrument 2

Instr. 2	Question 1	Frequencies			Strand	Rating	Frequencies			Strand	Rating	Frequencies		
		CU					PF					SC		
		+	+/-	-			+	+/-	-			+	+/-	-
Codes	Description/Totals	15	6	32	Codes	Description/Totals	14	14	3	Codes	Description/Totals	8	7	4
1	Saw numbers as data set	10			1	able to solve linear mean equation (ca)	5			1	set up mean equation correctly	5		
2	used mean formula correctly	4			2	able to solve quadratic (variance) equation (ca)	1			2	set up variance equation correctly	1		
3	used variance formula correctly	1			3	able to correctly substitute in to simultaneous equation	1			3	set up simultaneous equation correctly	1		
1	recognised need for mean formula, but used incorrectly		1		4	able to correctly solve simultaneous equation	1			4	Successfully used trial-and-improvement method	1		
2	recognised need for variance formula, but used incorrectly		5		5	Successfully used trial-and-improvement method to find x and/or y	1			1	set up mean equation but with errors		2	
1	Saw list of data values as pattern			6	6	0 – 1 procedural errors (ca)	5			2	set up variance equation but with errors (e.g. ignored sigma)		5	
2	tried to find differences between “terms” of “sequence”			5	1	used mean formula but with error(s)		2		1	Tried to set up sequence equation			4
3	little or no progress			9	2	used variance formula but with error(s)		5						
4	used quadratic pattern $2a = \dots$ for “2nd difference”			2	3	used simultaneous equation but with error(s)		3						
5	Assumed <i>linear</i> pattern and found x and y in terms of 1st differences			1	4	Able to simplify quadratic equation to standard form but not solve		2						
6	ignored the “sum of” aspect of variance equation			4	5	Factorising error		1						
7	equating unequal quantities			7	6	tried to solve quadratic in form $ax^2 + bx = c$ or $ax^2 = bx + c$		1						
					1	significant procedural error(s)			2					
					2	incorrect substitutions			1					

Instr 2	Question 2	Frequencies			Strand	Rating	Frequencies			Strand	Rating	Frequencies		
		CU	PF	SC										
Rating		+	+/-	-	Rating	+	+/-	-	Rating	+	+/-	-		
Codes	Description/Totals	10	6	34	Codes	Description/Totals	29	3	14	Codes	Description/Totals	4	3	14
1	Understood the need to find values of <u>a</u> and <u>b</u> in order to calculate SA	6			1	Wrote down SA formula (probably from notes) correctly	16			1	Created two equations to find value of one variable in terms of the other variable correctly	4		
2	Correctly expressed the separate areas of the faces of the <u>both</u> figures	4			2	correctly solves for <u>a</u> and <u>b</u> in terms of other variable(ca)	3			1	Created one equation to find value of one variable in terms of other variable		3	
1	Attempted to find an expression for either <u>a</u> or <u>b</u>		3		3	correctly substituted <u>a</u> and <u>b</u> to find SA (ca)	1			1	Did not attempt an equation to find a or b			9
2	correctly expressed the separate areas of the faces of one figure		3		4	correctly simplified SA (ca)	4			2	Introduced different variables for question (e.g. h, l, w)			1
1	Did not attempt to find an expression for either <u>a</u> or <u>b</u>			10	5	Design 1 SA = 4ab + 36a + 72b	3			3	Creates 1 or 2 incorrect equations for relationship between variables			2
2	did not correctly identify separate areas for either figure			6	6	Design 2 = 4ab + 72a + 36b	2			4	Set up equations but did not solve for one variable in terms of the other			1
3	confused the dimensions of the two figures when calculating SA			1	1	Wrote down SA formula (possibly from notes) but with error(s)		0		5	Set up equation(s) for <u>a</u> and <u>b</u> in terms of original (not stacked) prism			1
4	introduced third dimension when calculating SA (therefore, calculating volume, not SA)			0	2	correctly solves for <u>a</u> or <u>b</u> in terms of the other variable (ca)		1						
5	confused <u>sum</u> of dimensions with a single dimension			1	3	correctly substituted <u>a</u> or <u>b</u> to find SA (ca)		0						
6	added dimensions instead of multiplying			0	4	Simplifies SA with an error		1						
7	Very little progress			5	5	attempt at simultaneous equation, but cannot continue or mistake		1						
8	used numbers given <u>as</u> dimensions			1	1	Did not use any SA formula			3					
9	Saw incorrect relationship between variables <u>a</u> , <u>b</u> and 18			1	2	Incorrectly solves for one variable/or does not solve for variable			0					
10	confused SA of unstacked with SA of stacked prism			4	3	incorrectly substituted <u>a</u> and/or <u>b</u> to find SA (ca)			1					
11	confused (i.e. equated) sum of dimensions with TSA			2	4	substitutes 18 and/or <u>a</u> and <u>b</u> into SA formula			5					
12	Did not attempt to work with areas			3	5	substitutes any combination of given numbers into SA formula			1					
					6	multiplied surfaces instead of adding			1					
					7	"double distribute" 2 outside of bracket to inside bracket			1					
					8	serious procedural errors			1					
					9	circular reasoning – solving equation with beginning equation			1					

Instr 2	Question 3	Frequencies			Strand		Frequencies			Strand		Frequencies		
		CU					PF					SC		
		+	+/-	-			Rating	+	+/-			-	Rating	+
Codes	Description/Totals	11	10	24	Codes	Description/Totals	15	5	10	Codes	Description/Totals	6	3	21
1	Expresses all 3 relevant terms i.t.o 3 and multiples of	5			1	Writes down AP formula	8			1	Expresses all AP terms i.t.o. 3 and multiples of d	4		
2	Understands <i>ratio</i> relationship between T_3 , T_6 , T_{10}	2			2	Correct substitution of values into AP formula for 3 needed terms i.t.o. 3 and multiples of d (formula may not be explicitly stated, but used)	4			2	Correctly sets up ratio equation using expressions for AP terms i.t.o. 3 and multiples of d	2		
3	Expresses 3 terms correctly i.t.o. common ratio	1			3	Correct set up of ratio equation	2			1	Expresses 1 – 2 terms i.t.o. 3 and multiples of d		2	
4	Saw error and tried a new direction	3			4	Solves ratio equation with no errors	1			2	Correctly sets up ratio equation but does not use expressions for AP terms i.t.o. 3 and multiples of d		1	
1	Expresses 1 – 2 terms i.t.o. 3 and multiples of d		5		1	Writes down GP formula		3		1	Expresses AP terms incorrectly i.t.o. 3 and multiples of d or not at all			13
2	Expresses terms in GP form		2		2	Correct substitution of values into AP formula for 1 - 2 needed terms		2		2	Sets up common difference equation			2
3	Recognises ratio relationship, but not with specific terms		1		1	Writes down AP <i>series</i> formula			3	3	Equates T_{10} (and/or T_6) as AP to T_{10} (and/or T_6) as GP			2
4	Found $d = 0$ for first term (i.e. $T_1 = 3$)		2		2	Incorrect/ incomplete substitution of values into AP formula for 1 - 2 needed terms			1	4	Found wrong answer for d through misuse of general term and used answer to find value for T_{10} .			4
1	Incomplete/ incorrect expression of AP terms i.t.o 3 and multiples of d (e.g. does not have value for n)			3	3	Several to many procedural errors			2					
2	Little or no progress			17	4	Incorrect substitution into GP formula			1					
3	Found wrong answer for d through misuse of general term.			4	5	Cannot solve ratio equation correctly			1					
					6	Found wrong answer for d through procedural errors working with general term			2					

Tally of Mathematical Proficiency ratings (per learner): Instrument 1

		Note: Grey blocks = nil response																															
Instrument 1	Question 1.1						Question 1.2						Question 2						Question 3														
	Number of codes in each category/rating						Number of codes in each category/rating						Number of codes in each category/rating						Number of codes in each category/rating														
Strand	CU			PF			CU			PF			CU			PF			SC			CU			PF			SC					
Rating	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)			
Learner																																	
3.1			2			1			2	2					1			1	1	1	5			2	1	1	3			2			
3.2			2		1	1								2				2			1			1				1			1		
3.3		1			3			1			5					2			1				2	1			1		2	1			
3.4					2			1						1	1	1	3		1		1	1	1					1		1			
3.5					2			1			1			1	2			1	1	1	1	1	1	1	2			3		1			
3.7					2			1								3							1	1					1	1			
3.8														2	1				2	1	2												
3.9		1				1				2	1			2				3	1		5	1											
3.10					3			1			1			1	1			2	1		3	1			1	1			1	1		1	
3.11					3			1			3			2			3	3					1										
3.12											1			1									2		1	2	3			1		1	
3.14					1		1							1	1				1		2	2				1			1				
3.15					2						2			1	1			1	1		3	3		3		3	1		4	1			
1.1					2			1	1					2				1	1		1	2		1		1	1		2	1			
1.2														1	1	1	1		1	1	1		2			3			1				
1.3						1					2	2											1			1		1	1				
1.4												1											2	1		3	1		4	1			
1.5														1		2			1	1			1			1		2	1				
1.6						1						1			2				1		1	4			1			1					
1.7														1	1				1		2	2		1									
1.8																																	
1.9					2			1	1						2									1			1		1	1			
1.10						3			1			1						1		1	2							1					
1.11															2	1			1	1		1	1		1		1	2	2	1			
1.12											2	2											1			2			1				
1.13						3			1							3																	
Totals		2	8	26	5	13	0	0	12	27				13	18	17	15	10	10	24	27	15	18	4	16	21	0	26	15	0	4		
Strand	CU			PF			CU			PF			CU			PF			SC			CU			PF			SC					
Rating	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)			

Tally of Mathematical Proficiency ratings (per learner): Instrument 2

Instrument 2	Question 1									Question 2									Question 3									
	Number of codes in each category/rating									Number of codes in each category/rating									Number of codes in each category/rating									
	CU			PF			SC			CU			PF			SC			CU			PF			SC			
Rating	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	
Learner																												
3.1			1								1	1	1	1	1		1				2	1	1	1			1	
3.2			1																									
3.3											1	1									3	1		2			2	
3.4			3	1	1				1		4	2		1			1				1							
3.5			1								1			1							1	1					1	
3.6		2	1	1	1	2		1	1		1		2	1		1	1		2	1		1				1	2	
3.7			1					1			1	1	1				1				2	1	1				1	
3.8										2			4			1				3	1		2	1	1	1	1	
3.9		3			5	1		3			1		2	2		1			1	1	1		1		2	1	1	
3.10			4	1					1		1	2	2		1				1			1					1	
3.11			1							1	1		1		1	1		1										
3.12										1		1	1	1					1	3			4			2	1	
3.13			4			1			1		3	1	2			1					1						1	
3.14			4	1					1	1		1	2		1						1						1	
3.15		1	1	1		2			1		1	1	1				1		1	1	1	1	1			1	1	
1.1		1			1														3			3		2	2			
1.2		1			1	1		1			1	2			2		1			2	1			1			1	
1.3			2								3	1		1				1		1	1						1	
1.4			2								1	1									1						1	
1.5		1		1			1														1						1	
1.7		2	1	1	2	2		1	1			1	1								1	1		1			1	
1.8			1							1		1	1	1	1	1				2	1		1				1	
1.9			1																									
1.10		2	1		1	1		1	1			2	1						1			1					1	
1.11			1								1	3	1				1				1						1	
1.12		1	1			1		1		1		1	3				1		1	1	1		1			3	1	
1.13		1	1	1		3	1		2												1						1	
Totals		15	6	32	14	14	3	8	7	4	10	6	34	29	3	14	4	3	14	11	10	24	15	5	10	6	3	21
Strand	CU			PF			SC			CU			PF			SC			CU			PF			SC			
Rating	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	(+)	(+/-)	(-)	

Tally of Strategy Use ratings (per learner): Instrument 1

Instrument 1	Note: grey blocks mean = nil response																																						
	Question 1.1					Question 1.2					Question 2					Question 3					Question 4																		
	Explicit			Implied		None	Explicit			Implied		None	Explicit			Implied		None	Explicit			Implied		None															
Awareness	Decl			Proc		Cond	Decl			Proc		Cond	Declarative			Proc		Cond	Declarative			Proc		Cond	Declarative			Proc		Cond									
MC know	H	CS	O				H	CS	O				H	CS	O				H	CS	O				H	CS	O				H	CS	O						
Strategy																																							
Learner																																							
3.1						✓						✓				PE			E							PE						IE							
3.2		PE		CS/PE															IE		CS/IE	CS/IE					IE		CS/IE	CS/IE						✓			
3.3						✓													IE													IE							
3.4				IE	CS/IE	O/IE						✓	PE			H/PE	H/PE																			✓			
3.5						✓						✓																								✓			
3.7					IE							✓	IE			CS/IE	H/IE*					PE			H/PE	H/PE		IE			H/IE								
3.8						✓						✓	IE			CS/IE	CS/IE	IE				IE														✓			
3.9						✓						✓							E																		✓		
3.10						✓						✓							PE																		✓		
3.11						✓						✓							IE																		✓		
3.12						✓										IE																IE							
3.14		E			O/E							✓	PE			CS/PE	CS/PE	PE													IE								
3.15					PE											PE																PE					✓		
1.1						✓						✓							PE																		✓		
1.2						✓						✓	PE			CS/PE	CS/PE	PE				PE			CS/PE			IE			CS/IE								
1.3					IE		PE		O/PE													PE														✓			
1.4						✓						✓							IE							PE			CS/PE									✓	
1.5						✓						✓																											✓
1.6						✓						✓	E			CS/PE	CS/PE	PE				IE														✓			
1.7						✓						✓							PE																		✓		
1.8						✓						✓																											✓
1.9						✓						✓																											✓
1.10						✓						✓							IE																		✓		
1.11						✓						✓							PE																		✓		
1.12						✓						✓																											✓
1.13						✓						✓																											✓

Learner Strategy Use: Instrument 1

1. Question 1

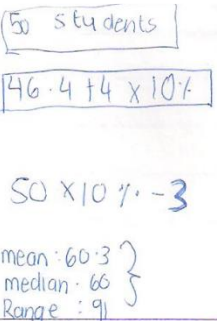
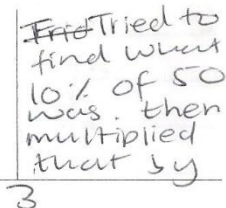
1.1 Question 1.1

For Question 1.1, out of the 26 respondents for this instrument, only three learners attempted to report their strategies. Of those, three of the learners wrote down what strategy they used; two wrote down how they used that strategy and two attempted conditional knowledge. The table below gives the scanned versions of their responses under the specific headings:

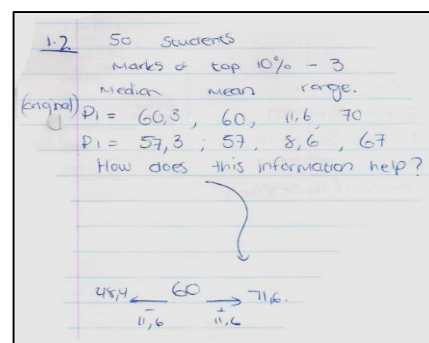
What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
I added 4 to each value.	By adding 4	
I just multiplied thought about it.	I multiplied everything by 4	I did not know what else to do.
Median, middle term, if 4 marks added to each kid - then $46 + 4 = 50$		General reasoning?

1.2 Question 1.2: Instrument 1

Evidence of strategy use was very minimal with Question 1.2. A table with learner responses in the strategy columns is given below (see figure 16). Only two learners wrote something in the “strategy columns” and one of those responses may have simply been the start of procedural steps. One learner did state “tried to find what 10% of 50 was, then multiplied that by 3” which was coded as a declarative, content-specific strategy. However, because she could not complete the problem, it was a partially effective strategy. She also wrote “used a calculator then” under the “how” column, which was coded the same.

What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
	<p>Given</p>	<p>Given.</p>
	<p>Used a calculator then</p>	

One learner reproduced her thinking process while working, as she did in Question 1.1 (see figure alongside). She writes down the given original data values, then adds three to all values and writes down the self-directed question: “How does this information help?” Like before, she draws an arrow to an interval of values one standard deviation around what she has labelled as the mean, but what is actually the median in the question. Although this indicates the learner is reflecting on her solution process, she only wrote down the correct answer for the range; this was coded as “other” and received a partially effective rating.



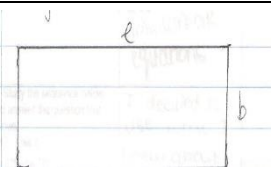
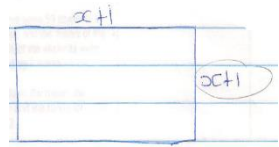
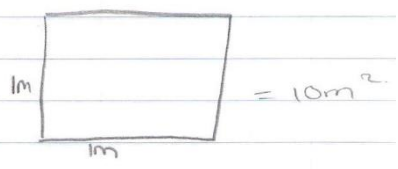
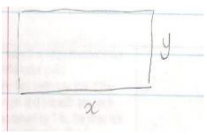
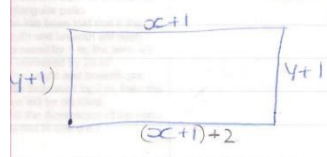
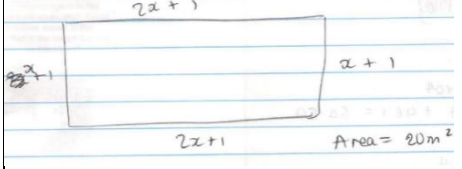
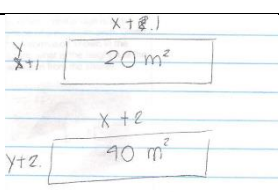
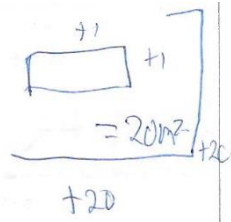
2. Question 2

In Question 3, 10 out of the 26 learners explicitly stated their strategy use in the strategy columns; their responses are reproduced in the table below. A few common trends were apparent. Most frequent was the association of the problem with arithmetic number patterns: in the “what strategy” column, five of the 10 learners specifically mentioned the word “pattern” or “progression,” and one learner mentioned “tried to find the ‘a’ or ‘d’ or ‘r’”. Two learners implied the recognition of patterns; one learner wrote “it goes up in 20’s” and another used the arithmetic progression formula in the strategy columns.

Appendix O – 1

Learner no./ (score)	What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
3.1 (3)	I decided to use what I know about number patterns and sequences in order to solve this question		
3.2 (0)	Tried to find the "color" or "cr" "r"	To find, the first term is equal to d.	To find a general term
3.5 (4)	I asked what pattern is kind of pattern this pattern was like above pattern similar to?	I used this strategy by asking questions	I used it because each time I see a difficult problem I ask "What do I know?"
3.7 (0)	$\tan 2^\circ = \tan 2^\circ$ $\tan 22^\circ = \tan 22^\circ$ (It goes up in 20°)	$\tan 2222222222 =$ $\tan 122 + 20 + 20 + 20$ $= \tan 182$	I could see that the tans to the right of the equal sign increased by 20 units.
3.8 (0)	↳ number patterns		
3.12 (3)		$T_n = a + (n-1)d$ $T_n = 78 + (n-1)(-20)$ $T_n = 78 + (100-1)(-20)$ $= -1902$ $\therefore T_{100} = \tan 1982 - 1902$	Saw a common difference.

1.2 (4)	Arithmetic Patterns	Because the difference is constant	
1.3 (2)	Looks like a arithmetic Progression question Linear		
1.4 (2)	I used a Simultaneous equation method to find the term	I put $T_1 = \tan 2^\circ$ as my a and $T_2 = \tan 22^\circ$ and $T_3 = \tan 42^\circ$. I then subtracted T_3 to T_2 to get the d	
1.6 (0)	Take $\tan 2$ to its hundreth 2 and put \tan^{-1} to your answer.		

Effective	Partially effective	Ineffective
		
		
		

3. Question 3

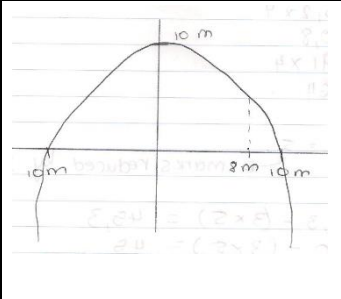
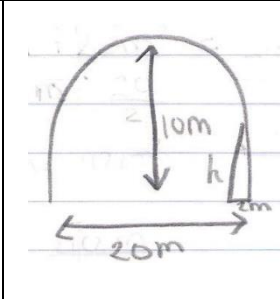
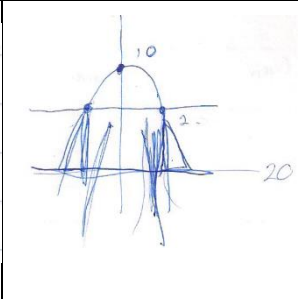
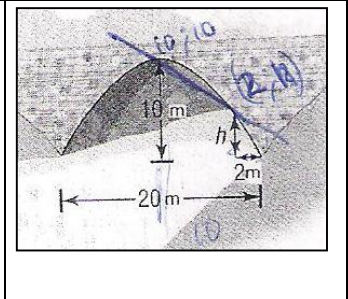
In Question 3, 10 out of the 26 learners explicitly stated their strategy use in the strategy columns; their responses are reproduced in the table below. A few common trends were apparent. Most frequent was the association of the problem with arithmetic number patterns: in the “what strategy” column, five of the 10 learners specifically mentioned the word “pattern” or “progression,” and one learner mentioned “tried to find the ‘a’ or ‘d’ or ‘r’”. Two learners implied the recognition of patterns; one learner wrote “it goes up in 20’s” and another used the arithmetic progression formula in the strategy columns.

Learner no./ (score)	What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
3.1 (3)	I decided to use what I know about number patterns and sequences in order to solve this question		
3.2 (0)	Tried to find the "a" or "d" or "r"	To find the first term is equal to d.	To find a general term
3.5 (4)	I asked what pattern is kind of pattern this pattern was like about pattern similar to?	I used this strategy by asking questions	I used it because each time I see a difficult problem I ask "What do I know?"
3.7 (0)	$\tan 2^\circ = \tan 2^\circ$ $\tan 22^\circ = \tan 22^\circ$ It goes up in 20's.	$\tan 2222222222 =$ $\tan 122 + 20 + 20 + 20$ $= \tan 182$	I could see that the terms to the right of the equals sign increased by 20 units.
3.8 (0)	↳ number patterns		

<p>3.1 2 (3)</p>	<div style="display: flex; justify-content: space-between;"> <div style="border: 1px solid black; padding: 5px; width: 45%;"> $T_n = a + (n-1)d$ $T_n = a + (n-1)d$ $T_n = 2 + (n-1)20$ $T_{100} = 2 + (100-1)20$ $= 3960$ </div> <div style="border: 1px solid black; padding: 5px; width: 45%;"> $T_n = a + (n-1)d$ $T_n = 78 + (n-1)(-20)$ $T_n = 78 + (100-1)(-20)$ $= -1902$ $\therefore T_{100} = \tan 1982 - 1902$ </div> </div>		<p>Saw a common difference.</p>		
<p>1.2 (4)</p>	<p>Arithmetic Patterns</p>	<p>Because the difference is constant</p>			
<p>1.3 (2)</p>	<p>Looks like a arithmetic Progression question</p> <p>Linear</p>				
<p>1.4 (2)</p>	<p>I used a simultaneous equation method to find the term</p>		<p>I put $T_1 = \tan 2^\circ$ as my a and $T_2 = \tan 22^\circ$ and $T_3 = \tan 42^\circ$. I then subtracted T_3 to T_2 to get the d</p>		
<p>1.6 (0)</p>	<p>Take $\tan 2$ to its hundredth 2 and put \tan^{-1} to your answer.</p>				

4. Question 4

Three of the learners tried to use the strategy of drawing a figure and one learner tried to annotate the figure; their attempts are illustrated below. These would be categorised as implied strategies.

			
Learner 3.1	Learner 3.3	Learner 3.12	Learner 3.14

I drew up a parabola and worked from there

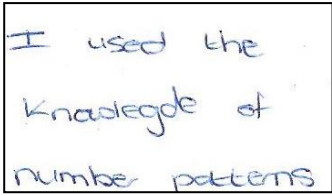
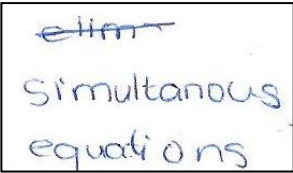
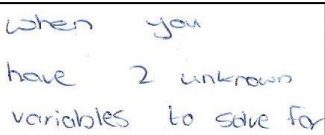
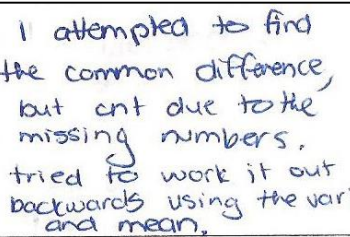
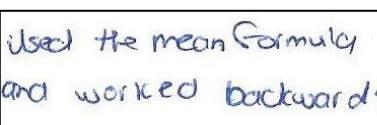
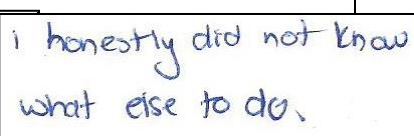
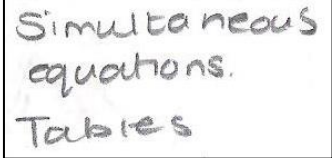
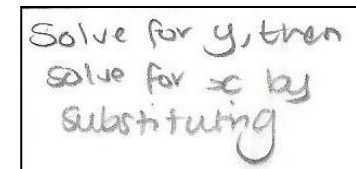
The question mentioned that the arch was in the shape of a parabola so it is bound to have the same characteristics as a parabola.

Because the height is 10 onto the breadth of 20m the ratio is 1:2 so if the breadth is 2 then the height is 1

Strategy Use: Instrument 2 (With commentaries)

1. Question 1

Reproduction of self-reported strategies with commentary

Learner No.	What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
3.4			
<p>The learner is mathematically incorrect, but is self-aware of the content area she is using for this problem and is thus employing metacognitive knowledge. Her strategy was to use her “knowledge of number patterns” and the way she used it was through “simultaneous [sic] equations.” She knows she needs simultaneous equations because there are “2 unknown variables to solve for.” Her self-reported strategy use matches her mathematical working and she (valiantly) tries to use simultaneous equations to relate the variables. She demonstrates declarative, procedural and conditional knowledge even if conceptionally incorrect for the problem.</p>			
3.7			
<p>This learner wrote quite a bit regarding her strategy use, but her actual mathematical working had scant detail. She found the correct values for x and y, but it is unclear from her working how she arrived at those values. There is evidence on her working-out that she did try to use differences between terms, as she stated. However, there is no evidence of her using the mean formula and working backwards.</p>			
3.9			
<p>This learner wrote a brief, but concise explanation of her strategy use. In answer to “what strategy did you use?” she replies: “Simultaneous equations Tables.” She used that strategy by solving for one variable then the other by substituting. Her working-out matches her strategy use exactly, and she was the one learner who came closest to the memorandum. She indicates declarative and procedural knowledge of the strategies she employed.</p>			

3.11	what type of problem is this	Used it to think about how to solve the Qs - but didn't get to actually find solve the Q.	when I don't know what to do, I think about the types of problems they are, and which is similar to
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This learner was quite descriptive in her strategy use, but did not proceed past rewriting the set of given data values. She shows she can reflect on the problem, but her reflection is not aiding her ability to solve the problem. She also describes the heuristic “similar problem” strategy.

3.12	PATTERNS.		
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This learner said she used “patterns” to solve this problem, but showed no further working-out. She could be demonstrating declarative knowledge.

3.14	Figure out common difference.	$T_2 - T_1 = ?$	To try figure out if I can simultaneous equations to figure out x and y
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This learner describes using the content-specific strategy of finding the common difference and employs that strategy by using the second term minus the first term. She also implies in the “why” column that she is using this strategy in order to set up simultaneous equations to “figure out x and y”. Her working-out does match her strategy use and she tries to use the constancy of the first differences to solve for x and y. She demonstrates declarative, procedural and conditional knowledge, even if her mathematical understanding is faulty.

3.15	<ul style="list-style-type: none"> • Collect Data • Find a equation to establish a connection • sub in. • Factorise 		
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Note: written alongside working-out

This learner did not use the three columns, but rather described her content-specific strategies alongside her working-out. This may have been because I often modelled my content-specific strategy use this way when working out problems on the board or on my written memoranda. She did use the variance formula, albeit incorrectly, and her working-out did match the strategy use. She seems to demonstrate declarative knowledge.

1.1	I used trial and error.	I inserted numbers into my calculator in place of x and y until I got to one that equals a mean of 8.	I was given that the mean was 8 so I guessed until I got to an answer that worked.
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This learner used a “trial and error” heuristic and articulately described how she used the strategy and why. Her working-out matched her strategy use and she discovered the correct answers for x and y. However, she did not check if the numbers worked as well in the variance formula.

1.2	- finding first difference		
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This learner stated that “finding first difference” was her strategy, but this does not match her working-out. She does show that the difference between 10 and 12 is two, then starts with a geometric sequence, scratches that out and then (correctly) tries to find an expression for x in terms of y from the mean formula.

1.4	<ul style="list-style-type: none"> • Rewrite pattern. • find first difference • use $T_n = a + (n-1)d$ $\text{so } \frac{T_3}{T_2} = \frac{T_2}{T_1}$	<ul style="list-style-type: none"> • work backwards backwards. 	<ul style="list-style-type: none"> • The x & the y values / numbers are next following each other so it's better to try work from the end / back of the pattern.
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This learner writes responses in all three columns, which could be indicative of declarative, procedural and conditional knowledge. However, what she has written does not match her working-out: she only writes down T_1 T_2 T_6 T_{10} , with a 3 below T_1 and a “?” below the other three terms. In addition, her content-specific strategy use is faulty because she may be confusing the idea of a difference and a ratio. Also, she states that how she will use the strategy is to “work backwards” but in the “why” column, she states that “it’s better to try to work from the end/back of the pattern” which is not necessarily the same as the “work backwards” heuristic. While the learner appears to demonstrate declarative, procedural and conditional knowledge by writing responses in the three columns, I am not too sure if she really understands what she is writing about.

1.5	<ul style="list-style-type: none"> Determine whether the pattern is linear, geometric or Quadratic Then you create a sumat simultaneous equation. 		
<p>This learner clearly articulated what can be construed as declarative knowledge, but again the working-out does not match the strategy-use. Her strategy discusses patterns but the working-out involves an incorrect use of the variance formula.</p>			
1.10	<ul style="list-style-type: none"> I worked backwards to find the sum of the data altogether 	<ul style="list-style-type: none"> I said: mean = $\frac{?}{5} = 8$ $\therefore 5 \times 8 = 40$ I said: deviance = $\frac{?}{5} = 10$ $\therefore 5 \times 10 = 50$ 	
<p>For this learner, the strategy-use clearly matched the working-out. She demonstrates declarative and procedural knowledge and was able to use her strategy to correctly find an expression for x in terms of y.</p>			
1.12	<p>figuring out the mean & variance</p>		
<p>This learner states that her content-specific strategy is “figuring out the mean and variance.” However, the mean and variance were given in the problem so her strategy does not make sense as stated. However, she did attempt (although incorrectly) to form an expression for x in terms of y from the mean formula.</p>			

2. Question 2

Learner No.	What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
3.4	<p>The Formula for the surface area</p>	<p>I placed each number I saw into the formula.</p>	<p>When you hear surface area you know that you need to use the formula for the surface area.</p>

This learner described exactly what she did: she placed each number she saw into the formula. Unfortunately, the numbers she used, 18, 35 and 31, had nothing to do with the dimensions of the figures. While the learner articulates declarative, procedural and conditional content-specific strategies, her methods were rated as ineffective.

3.7	Volume, area, perimeter applies here.	Face Attempted to work out the TSA.	The question asks to calculate the surface area, so I assumed we had to use the TSA formula.
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This learner redrew Design 1 in addition to her self-reported strategies and set out one equation to find a and b . She was not able to solve for one variable in terms of another. She also wrote down the surface area formula, but did not proceed further. Although she mentions the idea of volume, she did not use it in her working out. She articulates declarative, procedural and conditional content-specific strategies and was closer to moving down a correct solution pathway than Learner 3.4, but was not able to proceed far at all. Her strategies were rated as ineffective.

3.8	<ul style="list-style-type: none"> → simultaneous equations to solve for a and b → substitution methods → changing the subject of an equation → used the diagram as assistance 		
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This learner was the only one to solve the problem almost correctly and her strategies and solution will be discussed separately.

3.9	Area equation $A = l \times b$	Substitute values into area equation ↳ $\times 2$ for each surface as there are 2 surfaces that are equal in area.	
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This learner’s response was somewhat unique. She writes about using the area equation in the “what strategy” column. This is matched by her working which correctly determines the surface area in terms of multiples of 18, a and b . She also demonstrates sound conceptual knowledge in her description of *how* the surface area formula works by writing “substitute values into area equation → $\times 2$ for each surface as there are 2 surfaces that are equal in

area.” Even though she wrote down the surface area as given in the notes, from her explanation of *how* she worked and her accurate working-out, she may have demonstrated more conceptual understanding than other learners because she seemed to *start with* the separate areas of the prisms.

3.11	<p>Best way to solve this problem? what do I know? Have I even done this before?</p>		<p>You know when to use the strategy when you are stuck. Ask yourself what is the best way you can do it or have you even done it before.</p>
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This learner seems to present variations on the “*did you use all the data*” and “*similar problem*” heuristics. Her solution process is somewhat complicated and warrants a separate discussion.

3.13	<p>Double (a) in design 2 Variant of known rules</p>	<p>Double (b) in design 1, and (a) in design 2</p>	
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This learner may be trying to employ two content-specific strategies, “double (a) in design 2” and doubling may be seen as a “variant of known rules.” She may also have been utilizing the heuristic strategy of “similar problems.” However, her statement of how she worked, “double (b) in design 1, and (a) in design 2” may rather indicate a content-specific strategy. In her working, she made serious procedural errors (replaced the summing of areas with the multiplication of areas). Although ended with the correct multiples of 18, a and b for Design 1, the operations between them was incorrect. Her strategy use matched her working and this response was rated as partially-effective.

3.14	<p>Keep variables</p>		
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This learner’s terse response, “keep variables,” resulted in her finding the correct relationship in multiples of 18, a and b, amongst the areas of the two designs, but procedurally she made the error of “double distributing” the outside factor of 2 to what she saw as two terms in the bracket. This resulted in her doubling the surface area, and also she was not able to simplify her final answer. Because she was able to identify the separate areas in both figures, her response was rated partially effective and her strategy use matched her working.

3.15	<ul style="list-style-type: none"> • collect data and what is asked <ul style="list-style-type: none"> • equations that link. <p>(Note: done next to working out, not in columns)</p>		
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This is another response that requires a separate discussion.

1.2	<p>-drawing a figure</p>		
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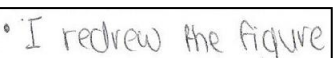
This learner self-reported the heuristic “auxiliary elements - draw a figure”. She only drew the unstacked prism and made the conceptual error of equating the given sum of the dimensions to the total surface area for one, unstacked prism. She then makes other procedural errors (discussed in MP section) and ends with an unsolvable quadratic equation. Her strategy use was rated as ineffective, but her strategy use matched her working.

1.3	<p>Ask myself how they would work like in</p>	
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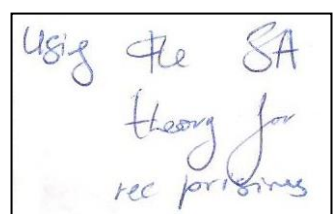
This learner’s strategy is somewhat unclear, except that it seems to involve self-questioning. In her drawing, she also indicates self-questioning. She does not seem able to make sense of the diagram relative and her strategy was rated ineffective. However, she did make use of the surface area formula and tried to substitute the values of 18 and 9 into the formula. As her strategy involved inquiring about the dimensions of the figure and her working out used the surface area formula, her strategy was judged to match the working.

1.4	<ul style="list-style-type: none"> • Redraw rectangular boxes. • colour code the rectangles. • Solve first design • Solve second design. 	<ul style="list-style-type: none"> • Use the surface area formula. 	<p>Understand.</p> <ul style="list-style-type: none"> • Its better to see things in my hand-writing.
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This learner states a heuristic comparable to draw a figure, adds that she uses colour coding and also she states the order of her working. This is all in the “what strategy” column. In answer to “how do you use the strategy?” she states, “use the surface area formula.” This does not seem to follow from the “what strategy?” question; or she be answering the question, “how will you solve this problem.” The response in the third column seems to follow her response in the first column because she is explaining why she redraws figures. Thus, the learner seems to indicate declarative and conditional knowledge, but not necessarily procedural. In her working out, she only managed to redraw the figures and write the surface area formula so the heuristic was rated as ineffective, although her strategy partially matched her working.

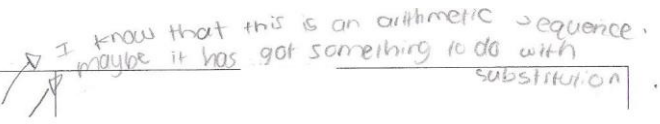
1.10			
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This learner states that she redraw the figure and it was coded as a heuristic strategy. However, in her actual working she did not redraw the figure and only wrote down the surface area formula and its meaning. Her strategy was coded as declarative and rated ineffective, and her strategy did not match her working.

1.12			
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This learner indicates using the “SA theory for rec prisms” as her strategy, which caused it to be coded as a content-specific strategy. As she can state what she is using, her statement indicates declarative knowledge. She was able to simplify the surface areas of the two designs correctly in terms of multiples of 18, a and b, and was thus coded as partially effective and her strategy matched her working.

3. Question 3

Learner No.	What strategy/ies did you use?	How did you use the strategy?	Why did you use the strategy or how do you know when to use the strategy?
3.3			

This learner may be using a variant of “Did you use all the data?” because she writes, “I know that this is an arithmetic sequence” and her arrow is pointing to the arithmetic sequence formula. This was coded as a declarative, heuristic strategy and rated ineffective as she incorrectly found an answer for d. However, her strategy use did match her working.

3.7	Formulas for the arithmetic sequence.	Subbing in the given values (although not much was given)	The question is clearly a patterns question and mentions the arithmetic sequence.
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This learner's strategy was coded as content-specific and rated as ineffective, as she was not able to proceed very far with her working. Her strategy use matched her working out, though, and her response is discussed below in the light of her mathematical working.

3.8	<ul style="list-style-type: none"> → Geometric and arithmetic sequences (number patterns) → Lots of trial and error → Write down everything I know 	<ul style="list-style-type: none"> → Re-arranging / manipulating general formulae, through, substitutions and changing the subject of the formula 	<ul style="list-style-type: none"> → look for key words e.g. arithmetic, geometric sequence → mention of term number
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In brief: she has three declarative strategies, two of which were coded as heuristic and one as content-specific. These were rated as partially effective and effective, respectively. Her procedural knowledge was coded as content-specific and rated as partially effective. The conditional knowledge was coded as "other" and rated as effective. Her working matched her strategy use.

3.9	Use equations for arithmetic & Geometric equations	Equate the two	
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This learner describes her content-specific strategies quite tersely, but are an accurate reflection of her working out. Her mathematical working was discussed in the Mathematical Proficiency section. Her strategies were rated as partially effective.

3.15	<ul style="list-style-type: none"> • Data collect • what is asked. • equations with correct connection <p>(note: done next to working-out, not in columns)</p>		
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As mentioned in Question 2, this learner describes the strategies she employs as she is working instead of in the columns. She seems to know in theory what to do, but is unable to put much in practice. Her declarative and procedural strategy was coded as given a partially effective rating.

1.1	I used the formulas to try and figure out what the answers were.	Inserted numbers and values into the equation.	
<p>This learner's mathematical working was discussed in the Mathematical Proficiency section. She did exactly as she said and her content-specific strategies were partially effective.</p>			
1.4	<ul style="list-style-type: none"> • Rewrite pattern • find T₁₀ 	<ul style="list-style-type: none"> • Use the geometric sequence formula 	<ul style="list-style-type: none"> • It's better to understand questions in my own handwriting.
<p>In this case, the learner's strategies did not match her working: she said she rewrote the pattern and instead wrote down T1 with a "3" below it, then T2, T6 and T10 with "?" below them. She also stated she used the geometric sequence formula, but no formula was displayed in her working-out. Her strategies were coded as "other" and content-specific and rated as ineffective.</p>			
1.5	<ul style="list-style-type: none"> • Redraw the pattern. • Use Geometric Formula. 		
<p>Like Learner 1.4, this learner also wrote down T1, T2, T3, . . . T10 and did not use the geometric formula at all. Her strategies were coded as "other" and content-specific and rated as ineffective.</p>			
1.7	<ul style="list-style-type: none"> • Working backwards to solve for d. 		
<p>This learner's response was a self-reported heuristic strategy but it was difficult to determine if she did "work backwards" or not because she used T1 to incorrectly determine a value for d. Her solution was discussed in the Mathematical Proficiency section. Her strategy use was rated ineffective.</p>			

3.1		None
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This learner has created a cusp at the turning point instead of a smooth curve; or she has created two upside-down parabolas that meet. She has not interpreted the dimensions correctly.

3.3			<div style="border: 1px solid black; padding: 5px; width: fit-content;">used a picture.</div>
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This learner has made the parabola with a sharp v-shape, but has interpreted the height of the towers and the distance between them correctly. She indicates declarative knowledge of her heuristic strategy.

Partially Effective Drawings		
LC	Learner drawing	Reported strategy use
3.11	<p>metres apart. The cables are <u>parabolic</u> in shape and are suspended from the tops of the towers. The cables touch the road surface at the <u>centre</u> of the bridge.</p> <p>Find the height of the cables at a point 100 metres from the centre.</p>	<div style="border: 1px solid black; padding: 5px;"> <p>Important information</p> <p>kind of problem</p> <p>What is this?</p> <p>Have I seen it before</p> <p>which is the best way to go about answering it.</p> </div>
<p>Learner 3.11 seems to be employing a self-questioning strategy in order to make sense of the problem. She also underlines key words in the question and attempts to create a drawing that incorporates that information. Her drawing is parabolic shape and she seems to know it is related to the parent parabolic function because she draws that to the left of her other parabola. She somewhat gets the sense of the height of the towers, but does not draw them, and may be confused about the distance between them.</p>		

1.8		None
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This learner, after two attempts at upside-down parabolas, settles on the correct orientation. She correctly interprets the height of the towers and the distance between them and seems to indicate by a dot that the turning point may be important. She crosses out “(200; y)” at the turning point, indicating she does know about the symmetry of the parabola, but has not interpreted the phrase from the question that the height at that point is zero.

3.6		none
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This learner has a better sense of the shape of the cables and correctly interprets the width between the towers, the height of the towers and the importance of the centre.

3.7		<p>I drew out what the question was asking.</p> <p>I think that you had to apply the parabola theorem somewhere,</p>	
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This learner also correctly interpreted the shape of the cables, width between towers and height of the towers. She adds the additional information that the centre is 200 metres from the first tower. In her self-reported strategy, she states the strategy of drawing out “what the question was asking.” She also thinks that the “parabola theorem” needs to be applied “somewhere,” but does not seem to know how.

1.2		<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>- Draw picture of sum</p> </div>
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This learner demonstrates declarative knowledge of her strategy of drawing a figure. The parabolic shape is evident, as well as the height of the towers and the distance between

them. She does seem confused, though, about what the height at 100 m from the centre means. However, she does bring the idea into her drawing.

1.12	<p>Handwritten diagram showing a parabolic cable between two towers. The towers are 75m high. The distance between them is 400m. The origin (0,0) is marked at the center of the base. The equation $y = a(x-p)^2 + q$ is written, with a note $y = ax^2 : (0:0)$.</p>	none
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This learner shows understanding of the parabolic shape, the height and distance between towers which she models more abstractly than some of the others. She also seems to realise that the origin may be used. She is the only learner to attempt a quadratic equation for the function.

3.13	<p>Handwritten diagram showing a parabolic cable between two towers. The towers are 75m high. The distance between them is 400m. The origin (0,0) is marked at the center of the base. The equation $y = a(x-p)^2 + q$ is written, with a note $y = ax^2 : (0:0)$.</p>	none
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This learner demonstrates the parabolic shape, and the height and distance between the two towers. She tries to interpret the idea of 100 m from the centre, but seems to think 100 m up.

3.10	<p>Handwritten diagram showing a parabolic cable between two towers. The towers are 75m high. The distance between them is 400m. The origin (0,0) is marked at the center of the base. The equation $y = a(x-p)^2 + q$ is written, with a note $y = ax^2 : (0:0)$.</p>	none
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In addition to interpreting the parabolic shape, tower height and distance between towers correctly, this is one of the few learners who correctly interpreted the height of the cables 100 m from the centre. She also made the turning point of her parabola sit at a height of zero metres.

Effective Drawings		
3.14	<p> $TP = (0, 0)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{75 - 0}{400 - 0}$ $\frac{75}{400}$ $\approx \frac{3}{16}$ </p> <p> $y = ax^2 + bx + c$ $y =$ </p>	<p>Tried to bend the equation to how I want it</p> <p>Easier</p> <p>I set my equation based on the turning point being (0; 0)</p> <p>I thought it would work out nicely.</p>

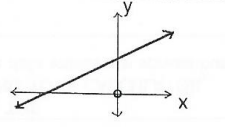
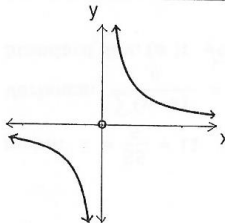
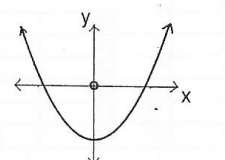
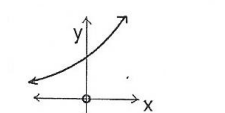
This learner showed declarative, procedural and conditional knowledge of the strategy she employed. She states that she “tried to bend the equation to how I want it” by basing the turning point at (0, 0) because she thought “it would work out nicely.” This was a well-applied strategy and would have been helpful if she had the conceptual understanding to apply a quadratic equation, which she did start with, but made the mistake of employing the linear slope formula. However, she shows an accurate understanding about the height of the towers and the distance between them. She also shows understanding of how the horizontal distance of 100 m from the centre relates to the height value asked for in the question. She also has a sense of directed numbers, which only one other learner demonstrated. She was one of two learners to place the parabola on the Cartesian plane and her strategy was rated as effective.

3.15	<p> Final y-value to x value of 100 But values a, b and c unknown What can I do to find these values? if you could work out 2 values Just plug in to find 1000 But then sub in x-value 100 to find y-value which is answer </p> <ul style="list-style-type: none"> • Data collect • what's been asked? • draw sketch then graph it
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This learner, whose work has been discussed previously, has her “bullet point” list that seems to be a variant of “what are the data, what is the unknown, what is the

condition?” substrategy. As well, she writes down her thinking process: “Find y-value to x value at 100 But values a, b and c unknown What can I do to find those values?” By asking that question of herself, she indicates that she realises she is working with a parabola which in general form would be $ax^2 + bx + c$. She also orients her parabola on the Cartesian plane with the turning point marked at (200,0) and marks the y-value of $x = 100$ as “?”. She knows what she is looking for, but not how. However, she demonstrates understanding of the particulars of the question and her strategy was rated effective.

“Notes” Page for Instrument 2

<p>A. Straight Line</p> <p>$y = ax + q$ or $y = mx + c$</p> <p>a or m • gradient q or c • y-intercept</p>	
<p>B. Hyperbola</p> <p>$y = \frac{a}{x}$</p> <p>$y = \frac{a}{x} + q$</p> <p>$y = \frac{a}{x-p} + q$</p> <p>q • vertical translation p • horizontal translation</p>	
<p>C. Parabola</p> <p>$y = ax^2$ Turning Point (0;0)</p> <p>$y = ax^2 + q$ Turning Point (0;q)</p> <p>$y = ax^2 + bx + c$ Turning Point $(\frac{-b}{2a}, \frac{4ac - b^2}{4a})$</p> <p>$y = a(x - p)^2 + q$ Turning Point (p;q)</p>	
<p>D. Exponential</p> <p>$y = ab^x$</p> <p>$y = ab^x + q$</p> <p>$y = ab^{x-p} + q$</p>	

a – first term
 n – number of terms or
 – position of term
 T_k – value of k th term

d – common difference
 r – common ratio
 l – last term
 T_n – value of n th term

Arithmetic (common difference... d)

$T_3 - T_2 = T_2 - T_1 = d$ Sequence: $a ; a+d ; a+2d ; a+3d...$

Sequence

General term:

$T_k = a + (k-1)d$ or

$T_n = a + (n-1)d$

Series

Sum of:

$S_n = \frac{n}{2}[2a + (n-1)d]$ or

$S_n = \frac{n}{2}(a+l)$

Geometric

(common ratio... r)

$\frac{T_3}{T_2} = \frac{T_2}{T_1} = r$

Sequence: $a ; ar ; ar^2 ; ar^3 ; \dots$

Sequence

General term:

$T_k = ar^{k-1}$ or

$T_n = ar^{n-1}$

$a \neq 0, r \neq 0$

Series

Sum of:

$S_n = \frac{a(r^n - 1)}{r - 1} ; r > 1$

$S_n = \frac{a(1 - r^n)}{1 - r} ; r < 1$

16. **Outliers:** data values less than $[Q_1 - 1,5 \times IQR]$
 data values greater than $[Q_3 + 1,5 \times IQR]$

B. Grouped data

17. Σ : sum of
 n : total number of scores
 x : each score in sample
 x_1 : midpoint of interval
 f : frequency of score

18. **Mean:** $\bar{x} = \frac{\sum f \cdot x_1}{n}$

19. **Variance:** $\frac{\sum (x_1 - \bar{x})^2}{n}$

20. **Standard deviation:** $\sqrt{\text{variance}}$

Example 1

Data	f	cum. f	x_1	$f \cdot x_1$
$10 \leq x < 20$	2	2	15	30
$20 \leq x < 30$	5	7	25	125
$30 \leq x < 40$	18	25	35	630
$40 \leq x < 50$	20	45	45	900
$50 \leq x < 60$	15	60	55	825
				2510

Mean: $\bar{x} = \frac{\sum f \cdot x_1}{n} = \frac{2510}{60} = 41,83$

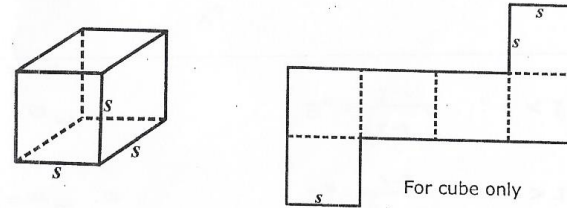
Example 2

Data(x)	$x - \bar{x}$	$(x - \bar{x})^2$
8	-3	9
10	-1	1
12	1	1
14	3	9
11	0	0
55		20

1. **mean:** $\bar{x} = \frac{55}{5} = 11$
 2. **Variance:** $\frac{\sum (x - \bar{x})^2}{n} = \frac{20}{5} = 4$
 3. **Standard dev. (σ):** $\sqrt{4} = 2$

MAKE SURE THAT YOU CAN DETERMINE THE STANDARD DEVIATION SD (σ), BY USING YOUR CALCULATOR.
 A high standard deviation indicates that the data values are spread out and have a large range.

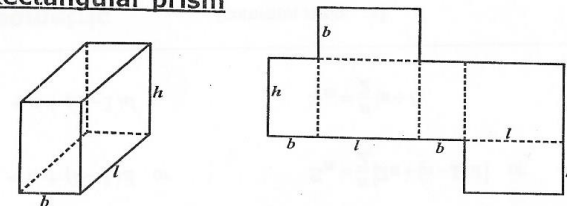
Cube



Surface area = the sum of the areas of all the faces
 $= 6(s \times s)$
 $= 6s^2$

Volume = area of base \times height
 $= (s \times s) \times s$
 $= s^3$

Rectangular prism



Surface area = the sum of the areas of all the faces
 $= 2(\ell \times b) + 2(b \times h) + 2(\ell \times h)$
 $= 2\ell b + 2bh + 2\ell h$
 $= 2(\ell b + bh + \ell h)$

Volume = area of base \times height
 $= (\ell \times b) \times h$
 $= \ell b h$

Title Approval

UNIVERSITY OF THE
WITWATERSRAND,
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09 January 2015
Person No: 579332
PAG

Mrs CS Price
60 Hettie Street
Cyrildene
2198
South Africa

Dear Mrs Price

Master of Science (Dissertation): Approval of Title

We have pleasure in advising that your proposal entitled *Connecting metacognitive knowledge and mathematical proficiency: A case study of South African Matriculants* has been approved. Please note that any amendments to this title have to be endorsed by the Faculty's higher degrees committee and formally approved.

Yours sincerely

A handwritten signature in black ink, appearing to read 'René Vosloo'.

Ms René Vosloo
Faculty Registrar
Faculty of Science