# Mathematical identities and trajectories of secondary school learners

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# Declaration

I, Aarifah Gardee declare that this thesis is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination at any other university.



Aarifah Gardee

24th day of January 2022

# Dedication

For my loving parents:

Yoosuf and Soraya Gardee.

*May the Almighty show mercy upon you, as you have been merciful to me.*

And in memory of my beloved grandfathers:

Moulana Abdul Rahman Gardee and Mr Ahmed Essop.

*May the Almighty be please with you and elevate your status among those who are guided.*

*Aameen.*

# Acknowledgment

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# Abstract

Research on learners' mathematical identities has become increasingly prominent over the last two decades. Despite this interest, various reviews of research on learners' mathematical identities argue that there is a need for the concept of identity to be more clearly defined and conceptualised in theoretically coherent ways that contribute to the concept being empirically accessible. I propose the use of critical realism as a theoretical framework, which has not been used before in this field, to define and research learners' mathematical identities as they are constructed in classrooms, in a non-reductionist manner. Mathematical identity is defined as a social phenomenon, existing in the real world, which emerges from relationships between learner agency, personal identity and social identity, both offered and constructed, but are not reduced to any of these three mechanisms.

Building on the theoretical framework, two explanatory frameworks are developed to support the application of the theoretical framework to data. The first explanatory framework relates teachers' practices with learners' mathematical identities, and the second explanatory framework relates peer interactions with learners' mathematical identities. Most of this study focuses on the first explanatory framework to further our understandings of how learners and teachers contribute to the development of learners' mathematical identities. A small part of the study shows how the second explanatory framework can be used to understand how and why learners construct certain identities in peer interactions.

The methodological approach of this study is a qualitative research approach. Data for this study were collected over two years in a technical high school in Johannesburg over two years in the third school term in both years (August - September), for two weeks in 2015 (Grade 9) and four weeks in 2016 (Grade 10). In total, nineteen learners participated in the study over both years, eight in the first year, with an additional eleven in the second year, along with their five mathematics teachers, two in the first year and three in the second year. Data were collected in the form of videotaped lessons, field notes, photographed learner notebooks and audiotaped interviews with participants.

Using the first explanatory framework, my analysis of the data points to several important findings, which are: the teachers' narratives and actual practices, which incorporate teachers' pedagogical approaches and social relationships with learners are related, and jointly influence the social identities offered to learners; the social identities offered by the teachers, which were informed by how the teachers spoke about errors, the learners and learner

thinking, strongly informed, but did not determine the mathematical identities constructed by learners as learners also played a role in the construction of their mathematical identities; learners were more willing to construct social identities of affiliation with their classroom communities when they felt that their teachers cared about them and others in the classroom; and that learners' mathematical identities changed, some negligibly and other substantially, over the two-year period in response to changes in teachers. It is recommended that teachers remain conscious of the kinds of opportunities they present to learners to learn mathematics, the kinds of narratives they tell about learners and the types of pedagogical approaches they utilise in the classroom, as these narratives and teacher practices inform the kind of opportunities that teachers provide learners to learn mathematics.

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# Chapter 1: Introduction

## 1.1. Background to this study

In South Africa, as in other parts of the world, many learners give up studying mathematics when presented with the opportunity to do so, which is unfortunate for various reasons (Andersson, Valero & Meaney, 2015; Darragh, 2013; Umugiraneza, Bansilal, & North, 2017). In our country, mathematics is an important subject to gain access to tertiary education, especially in fields of engineering, accountancy, health sciences and other mathematically orientated disciplines. So mathematics becomes necessary for access to further education and employment opportunities. Dropping mathematics also has epistemological consequences since being proficient in mathematics enables learners to understand the usefulness of mathematics in their daily lives, by developing their reasoning and critical thinking capacities (Brodie, 2010).

Treating the study of mathematics as a 'gate-keeper' to further education and offering epistemological benefits to learners are somewhat instrumental, viewing the study of mathematics in schools as a means to an end. Perceiving mathematics as instrumental denies learners the opportunity to be "motivated by aspirations of understanding mathematics" (Skovsmose, 2012, p. 5), which involves appreciating mathematics as a body of knowledge, influenced by its own culture and history. In this sense, mathematics is associated with its own ways of working and thinking and can be perceived as being intellectual, sensible and reasonable (Brodie, 2010). The aesthetic appeal of mathematics can influence who we are as learners and doers of mathematics, as well as thinkers more generally.

Our current Curriculum and Assessment Policy (CAPS) for Grades 7-9 reflects each of the above-mentioned uses of mathematics. In terms of mathematics being a gate-keeper to tertiary education, CAPS states that a specific aim of mathematics is to "prepare the learners for future education and training as well as the world of work" (DBE, 2011, p. 8). In terms of the epistemological benefit of learning mathematics, CAPS states that mathematics is essential to "enhance logical and critical thinking skills, accuracy and problem solving, that will contribute in decision making" (DBE, 2011, p.8). CAPS also highlights the aesthetic appeal of learning mathematics by stating that an aim of teaching and learning mathematics is to develop "an appreciation for the beauty and elegance of mathematics" and "a spirit of curiosity and a love for mathematics" (DBE, 2011, p.8).

Despite the many ways in which mathematics is important as stressed in our curriculum policy, the results of our Grade 9 learners' performance in the Annual National Assessments (ANA) and the Trends in International Mathematics and Science Study (TIMMS) in mathematics are disheartening. In 2014, the average national percentage for Grade 9 learners who wrote the ANA mathematics assessment was 11% (DBE, 2014). The 2014 results for Grade 9 were worse than those in 2012 and 2013, 13% and 14% respectively. Worse still, the percentage of learners who achieved more than 50% in these assessments in 2014 was 3% (DBE, 2014). An analysis of these results suggested that learners did not possess adequate knowledge regarding the concepts and vocabulary of algebra and geometry (DBE, 2014). The Annual National Assessments have been discontinued since 2015.

A more recent TIMMS study in 2019 has shown some improvement in the results of Grade 9 learners in comparison to previous years, yet South Africa is still one of the five poorest performing countries in the world (DBE, 2019). The report states that 59% of all Grade 9 learners who participated in the study have not acquired basic mathematics knowledge (DBE, 2019). Aside from the difficulties experienced with learning mathematics in Grade 9, the 2021-2022 Annual Performance Plan report showed that the highest repetition rate with learners repeating the same grade during the 2018 academic year involved Grade 10 learners (approximately 22% of boys and 16,8% of girls in Grade 10) (DBE, 2021).

Research in South Africa illustrates that learners' socio-economic conditions and a lack of resources influence poor performance in schools (Bayaga et al., 2010; Sailors, Hoffman & Matthee, 2007). However, Graven and Heyd-Metzuyanim (2014) argue that research does not explain why South Africa performs worse in mathematics in international tests such as the TIMSS in comparison to its neighbouring countries which are less wealthy. Success in learning mathematics is not solely dependent on learners' abilities and socio-economic factors. Learners' emotions, interest, agency, relationships with their teachers and peers, and how they position themselves as members of their mathematics classroom communities also contribute to how they learn mathematics. To understand learning in its complexity, Darragh (2013) argues that it is necessary to focus on the "whole person and how they become a learner of mathematics" (p.216). To obtain a comprehensive understanding of learning, it is necessary to consider learners' mathematical identities.

## 1.2. My journey

My journey into understanding learners' mathematical identities began with my Honours degree in 2014, where I investigated how a teacher dealt with errors in her mathematics classroom (Gardee & Brodie, 2015). The theoretical framework informing my study was constructivism, which enabled me to understand how individuals actively construct knowledge, and thereby, construct misconceptions and make errors. When analysing my findings, I realised that merely attributing the cause of learner errors to the individual construction of knowledge was very narrow. The lack of complexity in my research made me question how other mechanisms, such as learners' social environments, pedagogical experiences, agency and future projections influence how learners make sense of mathematics.

For these reasons, I thought that the study of learners' mathematical identities was most suitable to enable a researcher to understand the learner as a 'whole' human being (Darragh, 2013). After all, complex problems, like understanding learning and identity, require complex solutions (Goldacre, 2009). I then embarked upon researching learners' mathematical identities for my Masters in Education degree in 2015. Since most of the articles that I read were based on situated theory, I decided that a sociological perspective was most suitable to study identities. My Masters study was focused on understanding the identities and learning trajectories of eight Grade 9 learners in two mathematics teachers' classrooms. Data were collected as videotaped lessons and audiotaped interviews with participants, and I found that there were six dominant mechanisms which influenced learners' learning and identities: learners' past and current mathematics teachers, their experiences of marks in mathematics, agency, family support, the transition between primary and secondary school, and learners' future projections. Among these six mechanisms, the most important mechanism for learners was their current mathematics teachers, involving their pedagogical approaches and the kinds of social relationships shared with learners.

When reflecting on the Masters study after the write up of my dissertation, I realised that I was unsure of what 'identity' meant. In the Masters study, I used Wenger's (1998) and Sfard and Prusak's (2005) definitions of identity as participation and narrative, yet something seemed to be missing in their definitions. Could identity be something so simple as our participation in communities? Or our socially shaped narratives? Or both? It seemed too simplistic to reduce identity to either and I struggled to find an alternative definition in the

literature which made sense to me. I also struggled to understand the gravity of my findings, and I even questioned my theoretical framework. I was unsure how all the mechanisms that I found worked together to produce identity. I struggled to understand learner agency, since my current sociological theoretical framework limited my understanding of how learners could exercise agency as motivated by individual forces, rather than social forces. I felt that I was sitting with a huge jigsaw puzzle, staring at a bunch of pieces of the puzzle, but I could not create the picture based on my current understanding of the literature on identity in mathematics education. The process was frustrating! I made multiple attempts to represent the relationships between mechanisms, all unsuccessfully. What was missing, I later realised, was a clear and perhaps realistic (to me at least) philosophical paradigm.

During the interval between completing the write up of my Masters dissertation and beginning my doctorate degree, I embarked on what my supervisor would call a 'me' search, rather than 'research'. I asked myself: Am I merely a product of my social environment? Are others able to influence me to learn or not learn so easily? Where is the 'me' in all the social discourse? Where is my agency? What about my emotions? My feelings? My motivations? Surely then, learners as individuals must have their own motivations, emotions and aspirations, which would inform how they agentively participate and construct their identities?

Despite all my reservations regarding my Masters research, I was pleased when I found out that I passed my Masters dissertation and was provided with the option to upgrade the Masters to a PhD, which I took. Once I began working on the PhD, all of my past concerns regarding my work motivated me to answer all the questions that I had in mind. I decided that the first step was to define identity.

## 1.3. The problem called "identity"

The last two decades have seen what Chronaki (2013) calls the "identity turn" in mathematics educational research, with the construct of identity being increasingly researched and recognized as being influential to understand learners' learning of mathematics (p. 2). The concept of identity offers valuable insights into how learners engage with mathematics in different classroom contexts (Andersson, et al., 2015; Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009), their relationships with other significant people in relation to learning mathematics (Heyd-Metzuyanim, 2013, 2015; Sfard & Prusak, 2005), the role of discourse in

relation to learners' learning of mathematics (Bishop, 2012; Heyd-Metzuaynim, 2015; Heyd-Metzuyanim & Sfard, 2012; Wood & Kalinec, 2012; Wood, 2013) and the role of certain social categories, such as race, gender and class, in relation to learners' learning of mathematics (Nasir & Cobb, 2007; Gholson & Martin, 2019). Yet, the question of what identity is and how the concept can be used in research remains a topic of concern (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Radovic, Black, William & Salas, 2018).

Over the past few years, there have been several reviews of the concept of mathematical identity aiming to develop suitable definitions of identity (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Radovic et al., 2018). Darragh (2016) analysed 188 articles on identity within mathematics education with a focus on their theoretical underpinnings, research methods and definitions of identity. She found that socio-cultural frames underpinned much identity research, with most studies employing qualitative methods and that there were very few theoretical articles on identity in mathematics education. She identified two distinct frames informing identity research: psychological and sociological frames. For psychological frames, identity is regarded as an acquisition, or something inherent to people. Psychological frames consider the area of affect, involving beliefs, goals motivations and attitudes. Darragh (2016) argues that even though the area of affect makes a valuable contribution to knowledge, a strong research tradition concerning the affective domain already exists in mathematics education. She argues that the affective "domain does not need to be re-branded as identity. Doing so muddies waters already filled with a variety of definitions" (p. 10). Instead, she argues that researchers should employ sociological frames, which consider identity as an action, enabling the understanding of social context, positioning and people's experiences of mathematics teaching and learning.

Polarising the psychological and sociological frames from each other may provide an incoherent definition of identity, as suggested by Radovic et al. (2018) in their review of 69 articles on learners' mathematical identities. Radovic et al. (2018) identified three dimensions for researchers to consider when defining identity, which are the social/subjective dimension, the enacted/representational dimension and the change/stability dimension. For the first dimension, they argue that researchers should consider both the social aspect of identity, understood best using sociological frames, and the subjective/individual aspect of identity, understood best using psychological frames. They argue that ignoring the subjective aspect, by only focusing on the social aspect, is limiting, by ignoring the formation of the self and the important role of individual agency in relation to identity. Conversely, focusing exclusively

on the subjective aspect, by only considering the role of the subjective/individual on the construction of her/his identity, leads to decontextualized understandings of identity, without accounting for the important role of social influences (Radovic et al., 2018). Instead, Radovic et al. (2018) argue that researchers should consider both social and subjective aspects since they both contribute to the emergence of learners' mathematical identities.

For the enacted/ representational dimension, they argue that some researchers view identity primarily as actions, while others view identity as primarily representational. They argue that there is an "overemphasis" on the representational aspect, as research on identity predominantly considers identity as narratives, and collects data in the form of narratives (Radovic et al., 2018, p. 34). Ignoring the enacted aspect of identity is limiting, by neglecting the mechanisms related to the emergence of identity in real contextual practices. However, ignoring the representational aspect of identity is also limiting, by neglecting the role of social discourses on the production of identity. Again, Radovic et al (2018) suggest that researchers consider both enacted and representational aspects of identity to provide operational definitions of identity. For the change/stability dimension, Radovic et al. (2018) showed that most research suggests that identity is malleable, rather than fixed, which is currently a widely accepted view of identity in mathematics educational research (see Bishop, 2012, Graven & Heyd-Metzuyanim, 2019).

Graven and Heyd-Metzuyanim (2019) conducted a more recent review of 47 articles on identity in mathematics education. Like Darragh (2016), they found that there was a very small number of theoretical articles on identity in mathematics education and that most articles utilised qualitative methods and some form of socio-cultural framework. Like Radovic et al. (2018), Graven and Heyd-Metzuyanim (2019) also found that "an overwhelming majority" of studies consider identity as narrative (p. 368). Some of their other important findings are summarised below:

- Only a small number of studies followed learners through their transition from one phase of schooling to another.
- Researchers did not often differentiate between the identity they authored about a participant and the identity authored by the participant him/herself.
- There seems to be a widening gap between studies which consider the area of affect and studies on identity.

- Most importantly, a weakness in mathematics education lies in stating what precisely identity is and how to empirically access and study identity.

## 1.4. Purpose of this study and research questions

Based on the reviews discussed, there is a need for the concept of identity to be clearly defined, theorised and conceptualised in a way that contributes to the concept being empirically accessible (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Langer-Osuna & Esmonde, 2017; Radovic et al., 2018). Further, we do not know much about South African secondary school learners' mathematical identities, and how these identities are related to learners' learning of mathematics in their mathematics classrooms (Brodie, 2017). The purpose of this thesis is to define, theorise and empirically apply the concept of mathematical identity to understand learners' mathematical identities and trajectories in a secondary school in South Africa. Therefore, the main research questions that guided the study are:

1. What is mathematical identity and how can the concept be used in researching secondary school learners' mathematical identities?

2. How do learners and their mathematics teachers, contribute to the development of learners' mathematical identities?

After much discussion with my supervisor, we agreed that I would complete my PhD that includes publications. Like pieces of a puzzle, five interrelating papers were written to answer the two main research questions, to form the big picture. Since this PhD is an upgrade of my Masters study, I had already analysed eight Grade 9 learners' mathematical identities in a school in Johannesburg. My supervisor and I agreed that it would be in the best interest of my study to return to the same school the following year to follow the trajectory of these learners, and to analyse their mathematical identities in their Grade 10 year. In doing so, I aimed to address the gap in empirical research identified by Graven and Heyd-Metzuyanim (2019), by following learners as they transitioned from Grades 9 to 10.

For most secondary school learners, learning mathematics and developing mathematical identities occur in the classroom (Anderson, 2007). Based on the most important finding in the Masters study, i.e., learners' mathematical identities were strongly informed by their mathematics teachers' practices, involving their pedagogical approaches and the kinds of social relationships shared with learners, I decided to focus my research on how the

relationships between teachers and learners contribute to the development of learners' mathematical identities. To do this, I wrote four papers that primarily focused on the relationships between teachers, learners and learners' mathematical identities.

### 1.4.1. The papers

The first paper is a theoretical paper and was written as an extension of a conference paper presented at the International Congress on Mathematical Education (ICME) conference, which focused on answering the first main research question.

**Paper 1:**

1. What is mathematical identity?
2. How can secondary school learners' mathematical identities be researched?

The next three papers focused on applying the framework developed in the first paper to empirical data, with a specific focus on answering the second main research question. These papers focused on how learners and their teachers contributed to the development of learners' mathematical identities (papers 2-4):

**Paper 2:**

a. How are teachers' actual practices and their narratives of their practices related?
b. How do different teacher practices, both actual and narrated, influence how learners construct their mathematical identities?

**Paper 3:**

a. What influences teachers who teach using similar traditional pedagogies to develop social relationships equitably or inequitably with learners?
b. How do social relationships influence how learners construct their mathematical identities?

**Paper 4:**

a. How do teachers speak about and interact with learner errors in their mathematics classrooms?
b. What influences how teachers speak about and interact with learner errors?

c. How do teachers' approaches to errors influence learners' mathematical identities?

As I was analysing the data collected in 2016, I realised that learners' interactions with their peers were also important for the development of their mathematical identities, which led me to revisit the data from the Masters study. Initially, I struggled to find an analytical framework to analyse the data on learner interactions with their peers, and after publishing the first three papers (Gardee, 2019, 2021; Gardee & Brodie, 2022), I came across the work of Wood (2013). According to Wood (2013), learners' mathematical identities can be examined on a micro-scale as micro-identity, or a macro-scale as macro-identity. Micro-identities refer to the moments when identity is constructed and captured during interactions and these identities can shift as learners interact with others in a lesson (Wood, 2013). Macro-identities are the identities that emerge over long periods and these identities are enduring, in the sense that they are persistent, stable and long-term constructions of who a person is (Wood, 2013). I realised that for all of my published papers, I analysed learners' mathematical identities on a macro-scale, as macro-identity. So for the final paper, I modified the existing framework developed in paper 1 to understand how learners' mathematical identities emerge during peer interactions.

**Paper 5:**

1. How do learners construct and offer micro- and macro-identities during peer interactions in mathematics classrooms?
2. How do learners' personal and social identities inform the micro- and macro-identities that they construct and offer during peer interactions?

A conference paper presented at the International Group for the Psychology of Mathematics Education conference (IGPME) was written as an overview of the study and is included in the thesis to support the reading of the empirical papers.

The order of the papers presented above is not the order of publication. I worked on multiple papers at the same time and some of them got published before others. For the full list of publications, please see section 1.7, and for more about my experiences of publishing papers, please see chapter 8. I will now describe the theoretical orientation of the thesis, followed by a discussion of the research design and methodology.

## 1.5. An outline of the theoretical orientation of the thesis

Critical realism is the theoretical underpinning of this thesis and is used to define, theorise, and conceptualise identity, without reducing identity to psychological frames, sociological frames, representational or enacted aspects of identity. To date, critical realism has not been used previously to define and conceptualise learners' mathematical identities, meaning that the use of this theory offers a novel contribution to conceptualising identity in mathematics education.

Critical realism was initially developed by Roy Bhaskar (1978), and he describes critical realism as a philosophy with an under-labouring agenda: the philosophy seeks to clear the philosophical path for the clear expression of science. Bhaskar (1978) developed critical realism as a response to two existing and popular theoretical paradigms: empiricism, which postulates the existence of an external reality corresponding to observations of reality; and interpretivism, which postulates that the world does not exist independently of thoughts, subjective meanings and the language describing it. Researchers usually consider critical realism to be the "middle way" between empiricism and interpretivism, as critical realism maintains the advantages of empiricism, focusing on the existence of an external reality, and the advantages of interpretivism, such as the theory-laden nature of observations, while avoiding potential drawbacks such as relativism, and reducing individuals to discourse or observation (Tikly, 2015, p. 243).

There are several phases to Bhaskar's work, but his initial work, also called basic critical realism, has been most influential in the social sciences (Elder-Vass, 2021). This thesis draws on Bhaskar's (1978) work on basic critical realism. There are three initial claims made by critical realists: 1. there is a difference between the transitive world of knowing and the intransitive world of being; 2. the social world is systematically open; and 3. that researchers should be focusing on the ontological depth of social reality (Scott, 2010). I provide a brief account of these three claims here, and a more detailed account in paper 1, which is presented in chapter 2. For the first claim Bhaskar (1978) distinguishes between the intransitive ontology dimension and transitive epistemology dimension. The intransitive dimension, which refers to objects in the world, involving both natural and social structures and mechanisms, is distinct from and cannot be conflated with the transitive dimension, which refers to knowledge about the world, which is subject to change.

For the second claim, Bhaskar (1978) distinguishes between open and closed systems. He claims that the world is an open system where "no constant conjunctions of events prevail" (p. 2), while closed systems, like laboratories, are those "systems where a constant conjunction of events occurs" (p. 3). In laboratories, scientists exclude certain mechanisms that could interfere with the regularities they wish to investigate. In open systems, there are multiple causal mechanisms that operate and give rise to events, while in closed systems, one or two mechanisms are isolated to understand the effects of the isolated mechanisms (Tikly, 2015).

For the third claim, Bhaskar (1978) argues for depth realism, meaning that the purpose of science is to discover the underlying structures and mechanisms that are responsible for occurrences. Bhaskar (1978) defines generative mechanisms as "nothing other than a way of acting of a thing" (p.42), which endures and is exercised under the appropriate circumstances. In the real world, certain combinations of generative mechanisms give rise to a phenomenon, which is known as the process of emergence (Tikly, 2015). The role of science is to identify and describe these mechanisms, by means of the transitive dimension.

In the context of this thesis, identity is defined as a social phenomenon existing in the real world, emerging from relationships between three generative mechanisms: personal identity, social identity and agency. In paper 1, which is presented in chapter 2, I show how I used the theory of critical realism to define and develop a framework to research learners' mathematical identities.

## 1.6. Research design and methodology

While each paper briefly describes the research design and methodology used to collect and analyse data, details of the research design, participants, collection methods, methods to enhance rigour and ethical considerations, which are not discussed in the papers, are explained here. The methods of data analysis are discussed separately in each paper, since these methods differ based on the focus of each paper.

### 1.6.1. Research design

From a critical realist perspective, qualitative approaches, also called intensive approaches, are important to provide explanations of why certain objects or events occur (Danermark, Ekström & Karlsson, 2019). According to Bhaskar (2014), critical realism is primarily

interested in developing rich explanations of mechanisms that are responsible for the emergence of objects and events. Qualitative/intensive approaches serve this purpose, by providing deep causal explanations of why certain objects or events occur. In other words, qualitative/intensive approaches involve discovering generative mechanisms and investigating how these mechanisms manifest themselves and interact with other mechanisms in contexts (Danermark et al., 2019). The process of discovering mechanisms responsible for the emergence of learners' mathematical identities and understanding the interactions between these mechanisms assisted me in answering the first main research question, which involved defining identity and understanding how the concept of identity can be used to research secondary school learners' mathematical identities.

Qualitative/ intensive approaches were also necessary to assist me in answering my second research question, which involved understanding how learners and their mathematics teachers contributed to the development of learners' mathematical identities, by considering teachers' and learners' experiences and interpretations of the learners' mathematical identities. The importance of understanding how people experience and interpret events requires the qualitative/intensive approach (Danermark et al., 2019). Further, qualitative/intensive research may provide diverse perspectives and a clear understanding of the complexities involved in a particular situation (Creswell, 2012). From a critical realist perspective, it is important to seek a number of different sources of information to develop a context-specific analysis of identity (Marks & O'mahoney, 2014).

### 1.6.2. Research site and participants

*The school*

This study took place in a selective, technical high school in Johannesburg over two years, in the third school term in both years (August - September), for two weeks in 2015 (Grade 9) and four weeks in 2016 (Grade 10). The school focuses on mathematics and physical science and offers courses aimed at enabling learners to gain entrance into university. Being a technical school, the school does not offer mathematical literacy. The school is selective, and learners write an aptitude test to ensure that they have sufficient mathematical and physical science background prior to accepting them. The learners in the school come from both working and middle-class families. The end-of-year mathematics results in this school was low. For example, at the end of the 2016 academic year, the Grade 10 grade average for mathematics was 22,78%. So even though this school may not be representative of all schools

in South Africa, in terms of this school being a technical school, having learners from both working and middle-class families and being selective of the learners who accessed the school, the school faced similar challenges as other schools in South Africa, in terms of learners' poor performances in mathematics (see Spaull, 2019).

There were two reasons for choosing to conduct my research in this school. The first reason was that the school is accessible from the University of the Witwatersrand. The second reason was that the school was part of a project run by my supervisor, Professor Brodie, which involved the creation of a 'mathematics club', aimed at developing learners' proficiency in mathematics, supporting learners' mathematical agency and developing learners' mathematical identities (see Lampen & Brodie, 2020).

*The teachers*

Five mathematics teachers participated in the study: two in the first year, Mr Moyo and Mr Molefe (Grade 9), and three in the second year, Mr Sithole, Mr Ndlovu and Mr Ncube (Grade 10) (all pseudonyms). All of the teachers were qualified to teach mathematics. Mr Moyo was newly qualified and had joined the school at the end of the second term (May), Mr Molefe had five years of experience teaching mathematics, Mr Sithole had thirty-two years, Mr Ndlovu, eight years and Mr Ncube, five years. The teachers' choice of pedagogy and style of interacting with the learners were different, which made them suitable participants for this study. Working with a range of teachers provided me with a stronger understanding of how different pedagogies and teacher-learner social relationships contributed to learners' mathematical identities.

*The learners*

I initially selected Grade 9 learners as research participants for this study because according to the TIMMS and ANA results, Grade 9 learners performed extremely poorly, indicating that Grade 9 learners face serious challenges in secondary school when learning mathematics. In selecting the learner participants, a "purposeful sampling" method was used (McMillan & Schumacher, 2010, p. 138), which requires the researcher to purposefully select individuals who will provide information about the matter being researched (Creswell, 2012). Since I was not familiar with any of the learners in the classroom, I requested each of the two Grade 9 teachers to choose two pairs of learners in their mathematics classroom. Asking the teachers to select learners also gave me an indication of how the teachers positioned these

learners in the classroom. The Grade 9 learners were selected using the following criteria: one learner who is a relatively low achiever in mathematics and someone who sits next to her/him and one learner who is a relatively high achiever in mathematics and someone who sits next to her/him. Studying the identities of learners who displayed a range of achievements in assessments provided me with possibilities to understand how different relationships between generative mechanisms contributed to the development of learners' mathematical identities.

In Grade 10, two of the initial Grade 9 participants transferred to another school and I continued to observe the remaining six learners from Grade 9 to follow their trajectories. I also selected an additional eleven learners, who were seated next to the initial participants. I asked the Grade 10 teachers whether each learner was a low, average or high achieving learner, which enabled me to understand how the teachers' ideas about learner performance influenced teacher and learner interactions. In Grade 10, learners sat in rows of twos, threes or fours and all learners in the same row as an initial participant were invited to participate in the study. Table 1 provides detail of the sample. There are three exceptions to the selection method (symbolised with an * in the Table), and the rationale is explained after the Table.

| **Grade** | **Pseudonym of teacher** | **Pseudonym of learner** | **Mathematics performance** |
|---|---|---|---|
| 9 | Mr Moyo | Senzo | Low achiever |
| | | Lane (sat next to Senzo) | Average achiever |
| | | Shane | High achiever |
| | | Jack (sat next to Shane) | High achiever |
| 9 | Mr Molefe | Tess | Low achiever |
| | | Sunny (sat next to Tess) | Inconsistent |
| | | Sindiswa | High achiever |
| | | Jennifer (sat next to Sindiswa) | Low achiever |
| 10 | Mr Ncube | **Tess (participant from Grade 9)** | Low achiever |
| | | Thandi (sat on the right of Tess) | High achiever |
| | | Amy (sat on the left of Tess) | Average achiever |
| | | Thabo (sat on the left of Amy) | Average achiever |
| | | Mike* (sat behind Tess) | Low achiever |
| | | Jimmy (sat next to Mike) | Low achiever |

| | | | |
|---|---|---|---|
| 10 | Mr Sithole | **Senzo (participant from Grade 9)** | Low achiever |
| | | Lebo (sat between Senzo and Shane) | High achiever |
| | | **Shane (participant from Grade 9)** | High achiever |
| | | **Lane (participant from Grade 9)** | Inconsistent |
| | | Tommy (sat next to Lane) | Low achiever |
| | | **Jack (participant from Grade 9)** | Inconsistent |
| | | Karabo (sat next to Jack) | High achiever |
| | | Phillip* | High achiever |
| 10 | Mr Ndlovu | **Sunny (participant from Grade 9)** | Low achiever |
| | | Fred* (sat in front of Sunny) | High achiever |
| | | Edward (sat next to Fred) | High achiever |

**Table 1.1: Selected learners**

Prior to data collection in 2016, Tess (in Grade 10) was unsure as to whether she would like to continue participating in the study. When she initially agreed to participate, I asked the learners who sat in the same row as her to participate, and they all agreed. She then decided not to participate. Her teacher, Mr Ncube, suggested that I ask Mike and Jimmy instead, who like Tess, were both low achievers, since the other learners who agreed were high and average performers. Both learners agreed to participate. Tess changed her mind again and decided to participate, so all six learners were included in the study.

Like Tess, Karabo, who sat next to Jack in Mr Sithole's classroom, was initially hesitant to participate. Mr Sithole suggested that I ask Phillip, who sometimes worked with Jack, to participate in the study. Karabo then agreed to participate, so I included both him and Philip in the study. In Mr Ndlovu's classroom, the learner who sat next to Sunny did not agree to participate in the study. Mr Ndlovu suggested that I ask Fred and Edward, who sat in front of Sunny, since both learners were high achieving learners, which would enable me to study the identities of learners with different achievements.

### 1.6.3. Methods of data collection

In both years, data were collected in the form of videotaped and audiotaped lessons, audiotaped semi-structured interviews with learner and teacher participants, field notes and photographed learner notebooks (in Grade 10).

I videotaped approximately 8 hours of lessons for each teacher. The video recorder was placed in the front of the classroom, which enabled me to focus it at times on the learners who were of interest to my research, and at other times on the teacher and the board. Videotapes are important to capture nonverbal forms of communication like facial expressions and gestures. McMillian and Schumacher (2010) argue that researchers "can trust participants' responses more if their body language is congruent with their verbal statements", enhancing the validity of my findings (p. 363). Observing the interactions in the classroom using the videotapes enabled me to "see things that might have been unconsciously missed, to discover things that participants might not freely talk about in interview situations" (Cohen, Manion, & Morrison, 2000, p. 305). In a previous research study, I noted the difficulty of capturing learner interactions with a video recorder. To eliminate the issues of poor sound that may pose a threat to capturing data using videotapes, I placed audiotapes on the desks of the learner participants to ensure that all interactions were captured as clearly as possible.

All teacher and learner participants were interviewed individually after the videotaped lessons were collected, and these interviews were captured on an audiotape. According to Silverman (1993), interviews are useful in research as they provide the justification, motives, emotions and reflections of the interviewee. These interviews were semi-structured, which allows for flexibility of discussion and for the interviewer to probe deeper and pose other questions to understand the interviewee's perspective (Freebody, 2003). It was necessary to capture all this detail on an audiotape as it would have been very difficult to document all the details in notes while participating in the discussion.

In both years of data collection, each teacher was interviewed once. In the first year of data collection, learners were interviewed once, and in the second year, twice. The reason that the learners were interviewed twice in the second year was because the interviews in the first year took a lot of time, so I decided to split the interview into two parts, the first focusing on their current experiences of learning mathematics in Grade 10 and the second on their past experiences of learning mathematics in Grade 9. The second interview in the second year provided additional validity to the data collected in the first year. See Appendix A and B for the learner interviews and Appendix C for the teacher interviews.

I took field notes in both years to supplement the data from the videotapes, by noting down my thoughts on the lessons, learner participation and important events that occurred in and

even out of the classroom, which may have not been captured on the video camera. The notes informed both the interviews and the data analysis. I also took photographs of the Grade 10 learners' notebooks to supplement the data from the videotaped lessons. I did not take photographs of the Grade 9 learners' notebooks, as I did not have permission to take the pictures, since they were supposed to write the ANAs immediately after data collection.

### 1.6.4. Rigour

Ensuring the accuracy of one's findings is crucial for results from research to be useful and meaningful. I piloted the study, which enabled me to improve the "reliability, validity and practicality" of my coding system and research tools (Cohen et al., 2000, p.260), and is discussed further below. Other strategies used to enhance the validity of the analysis, such as peer debriefing and triangulation, are discussed in the articles.

*Piloting the study*

To enhance the reliability and validity of my study, I piloted the study over a one-week period in 2015 with Grade 9 learners in a different school, which is similar demographically and is located near the school in which I carried out my final study. The reason I piloted the data collection instruments of my study was so that I could establish whether the instruments provided me with data which were useful for my study, whether the questions were ambiguous, how long it would take me to interview the teachers and learners and whether observations would yield useful data (Cohen et al., 2000).

Piloting my study also provided me with the opportunity to test my coding system and to refine it. I observed two lessons of two different Grade 9 mathematics teachers in the pilot school. In conjunction with observing the lessons, I audiotaped two pairs of learner interactions, one pair in each teacher's class, who were selected by each of their teachers. I practised coding the data using an open coding system and checked my interpretations with my supervisor.

Prior to piloting my study, I had selected a mixed method research approach for this study. I intended to use a sequential explanatory mixed method design which involved analysing quantitative data first followed by analysing qualitative data (McMillan & Schumacher, 2010). Applying the quantitative approach first, I intended to collect data from many learners through questionnaires to enable me to identify how the different mechanisms contributed to the development of learners' mathematical identities. Thereafter, through observing

classrooms and conducting semi-structured interviews, I would seek to obtain explanations about some of the trends found. Unfortunately, the results from the questionnaires in the pilot study were invalid because learners responded extremely positively to almost all of the statements (such as they enjoy mathematics, the way their teacher teaches, perform well in assessment and think they are good in assessment). The response bias could be a result of the learners assuming that they were providing me with answers that I wanted. Graven, Hewana & Stott (2013) encountered a similar problem in their study whereby learners answered questions based on what they thought was the correct expected response. Therefore, I changed my research design to qualitative/intensive research.

Interestingly, the data from the interviews did not reflect the same situation as the questionnaire. It could have been that these learners who were interviewed were comfortable with me because I spoke to them individually prior to collecting data. Piloting the interviews gave me feedback as to whether the types of questions I used were valid. Based on the findings from the pilot interviews, I modified the interviews.

### 1.6.5. Ethical considerations

I obtained ethical clearance from the University of Witwatersrand School of Education Ethics Committee and my protocol numbers are 2015ECE023M and 2016ECE015D. Permission was also obtained from the Gauteng Department of Education to do research in the school for both years, and in the pilot school. Informed consent was requested from the principal, the teachers, and the parents of the learners as well as the learners themselves. The teachers and the learners were told that participation is voluntary, and they could withdraw their participation at any time.

## 1.7. An overview of the thesis

This thesis is divided into eight chapters. In this chapter (chapter one), I provided the background to the study, my personal rationale for conducting the study, the problem statement, purpose, research questions, theoretical orientation, research design and methodology for the study. The next seven chapters consist of interrelated manuscripts that form the central body of the thesis.

For the completion of the thesis, it is required that I publish a minimum of four papers in reputable, peer-reviewed journals. Of the four papers required by the University, one of the

manuscripts is required to be published, one in press, and two accepted for publication. I have four published papers (chapters 2 and 4-6). In addition, I have included a conference paper and a fifth paper that is under review. While these are not required for the completion of the PhD, the inclusion of both papers are purposeful. The IGPME conference paper provides an overview of all the participating learners' mathematical identities, which can support the reading of the remaining empirical papers. The fifth paper uses the framework developed in the first paper and applies it to understanding learners' mathematical identities in peer interaction. Below is a list of the papers, their statuses, and references (Table 2).

| **Chapter** | **Paper** | **Name of paper** | **Status** | **Reference** |
|---|---|---|---|---|
| **2** | Journal paper 1 | A framework for learners' mathematical identities: A critical realist perspective | Published | Gardee, A., & Brodie, K. (2021). A framework for learners' mathematical identities: A critical realist perspective. *Didactica Mathematica, 43,* 5-29. |
| **3** | Conference paper | The influence of teachers on learners' mathematical identities | Published | Gardee, A., & Brodie, K. (2019). The influence of teachers on learners' mathematical identities. In M. Graven, H. Venkat, A. A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education, Vol 2* (pp. 248-255). Pretoria, South Africa: PME. |
| **4** | Journal paper 2 | Relationships between teachers' practices and learners' mathematical identities | Published | Gardee, A. (2021). Relationships between teachers' practices and learners' mathematical identities. *International Journal of Mathematical Education in Science and Technology, 52*(3), 1-27. |
| **5** | Journal paper 3 | Social Relationships between Teachers and Learners, Learners' Mathematical Identities and Equity | Published | Gardee, A. (2019). Social relationships between teachers and learners, learners' mathematical identities, and equity. *African Journal of Research in Mathematics, Science and Technology Education, 23*(2), 233–243. |
| **6** | Journal paper 4 | Relationships between teachers' interactions with learner errors and learners' | Published | Gardee, A., & Brodie, K. (2022). Relationships Between Teachers' Interactions with Learner Errors and Learners' Mathematical Identities. *International Journal of Science and* |

| | | | | |
|---|---|---|---|---|
| | | mathematical identities | | *Mathematics Education, 20*(1), 193-214. |
| **7** | Journal paper 5 | A framework for analysing learners' mathematical identities in peer interactions: accounting for personal and social identities | Under review | Educational studies in Mathematics |

**Table 1.2: Details of manuscripts that make up the chapters of this thesis.**

In chapter eight, I summarise the main findings and reflect on the study and make recommendations for further research. It is my hope that this study can contribute to the research focused on learners' mathematical identities in our country, and internationally.

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# Chapter 2: A framework for learners' mathematical identities: A critical realist perspective

## 2.1. Introduction

This chapter serves to answer the first main research question: What is mathematical identity and how can the concept be used in researching secondary school learners' mathematical identities? In this chapter, I draw on critical realism as a theoretical perspective to define learners' mathematical identities and show how identities can be empirically researched. Using the work of key critical realists, I identified three higher-level mechanisms: personal identity, social identity and agency; and developed a non-reductionist model to understand how the relationships between the mechanisms are responsible for the emergence of learners' mathematical identities. Using these ideas and the ideas of other theorists in the field of mathematics education, I developed an explanatory framework for the relationships between learners' mathematical identities and the social identities offered by teachers in classrooms. This explanatory framework relates two kinds of social identities offered: affiliation with, or marginalisation from, classroom communities; with four kinds of mathematical identities constructed: affiliation with, compliance with, resistance to or marginalisation from classroom communities. The explanatory framework is unpacked in more detail in chapter 4.

This chapter also briefly addresses the second main research question: How do learners and their mathematics teachers contribute to the development of learners' mathematical identities? Following the development of the explanatory framework in this chapter, I then applied the framework to understand the mathematical identities constructed by four learners as they interacted with their teachers in their mathematics classrooms. Each of the four learners participated in different ways and constructed different mathematical identities from each other, which assisted me in understanding how the combination of the three higher-level mechanisms contributed to the emergence of different mathematical identities. I show that to research learners' mathematical identities, the relationships between all three mechanisms are important and should be considered concurrently with each other in a non-reductionist way.

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# A framework for learners' mathematical identities: A critical realist perspective

Abstract: In recent years, many researchers have turned to the concept of identity to better understand learning in social contexts. Despite the increase in research on the concept of identity, there is a need to define identity more clearly, by drawing on established theoretical frameworks. We propose the use of critical realism as a theoretical framework to define and research learners' mathematical identities as they are constructed in classrooms. By employing critical realism, identity is defined as a social phenomenon, which exists in the real world and emerges from combinations of various generative mechanisms. We distinguish between three generative mechanisms: personal identity, social identity and agency, and we illustrate how these three generative mechanisms encompass subjective and social influences, which are responsible for the emergence of learners' mathematical identities. We propose a framework and explore its explanatory power to understand how four secondary school learners constructed their mathematical identities through interactions with their mathematics teachers.

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## Introduction

The last two decades have seen a significant increase in research on identity in mathematics education, with the construct of identity being recognized as being important to understand mathematics learning (Darragh, 2016; Radovic, Williams, Black, Salas, 2018; Graven, Heyd-Metzuyanim, 2019). The concept of identity has offered researchers opportunities to understand learner interests, persistence, and engagement with or disengagement from learning mathematics, without reducing explanations of learning to cognitive deficit discourses (Cobb, Gresalfi, Hodge, 2009; Graven, Heyd-Metzuyanim, 2019; Gresalfi, Hand, 2019).

While the concept of identity shows promise in furthering understandings of mathematics learning in schools, there are some challenges associated with the use of the concept in research. The main critique raised by researchers concerns the operationalization of identity, with the concept being poorly defined and at times, theoretically incoherent (Darragh, 2016; Graven, Heyd-Metzuyanim, 2019; Langer-Osuna, Esmonde, 2017). There is a need for researchers to clearly communicate their theoretical frames and analytical methods, as different theoretical frameworks emphasize different aspects of identity (Darragh, Radovic, 2018; Langer-Osuna, Esmonde, 2017). For example, research drawing on sociocultural theories tends to focus on how narratives, positioning and participation, which are socially produced in classroom communities, contribute to learner identity, while underplaying the importance of the individual, their agency, emotions and interests, which also inform learners' actions and identities.

Another important critique concerns the strong focus on social influences on identity, with a limited focus on the role of subjective (or individual) influences on identity (Graven, Heyd-Metzuyanim, 2019; Lutovac, Kaasila, 2018). Currently, much research on learners' mathematical identities in mathematics classrooms shows how social influences, such as what is valued in local communities and the interactions between learners, teachers and peers in their classrooms, contribute to the development of learners' mathematical identities (e.g., Boaler, Greeno, 2000; Bishop, 2012; Heyd-Metzuyanim, Sfard, 2012; Wood, 2013). Lutovac and Kaasila (2018) argue that while social contexts shape people's identities, underplaying the important role of the individual undermines the core concept of identity and limits understandings of how the notion of self and agency are produced and related to learning (Radovic et al., 2018).

To address these concerns, the aim of this paper is to develop a framework to define and research learners' mathematical identity in classroom contexts,

underpinned by a coherent theoretical framework, with a focus on both the subjective and social aspects of identity. Our research is guided by the following research questions:

1. What is mathematical identity?
2. How can secondary school learners' mathematical identities be researched?

To answer the research questions, we present critical realism as a novel theoretical perspective to define and research learners' mathematical identities in classroom contexts. Critical realism focuses on both the subjective and social aspects of identity, as personal and social identity, without reducing identity to either personal or social influences. Most current research looking at identities in mathematics education does not take a critical realist stance, and within the field of critical realism, there is little work on mathematical identities. We propose a framework to understand learners' mathematical identities at a micro-level, as they are constructed in classrooms through interactions with teachers[1]. To illustrate the explanatory power of the framework, we provide an analysis of how four secondary school learners constructed their mathematical identities through interactions with their mathematics teachers, by focusing on both subjective and social aspects of learners' mathematical identities.

## Critical Realism

Critical realism originated with the work of Roy Bhaskar (1978), whose philosophy of science is grounded in three claims: the world as it exists, i.e., intransitive ontology dimension, is different from knowledge of the world, i.e., transitive epistemology dimension; the world is an open system; and science should be committed to uncovering the ontological depth of reality (see Scott, 2010). For the first claim, Bhaskar (1978) argues that ontology is non-reducible to epistemology, as theoretical knowledge about the world in the transitive dimensions is not reducible to the world as it exists in the intransitive dimension. So the transitive world of knowing is different from the intransitive world of being (Scott, 2010). Any inquiry in the social sciences involves studying social objects, such as learners' mathematical identities, which exist in the real world.

[1]We acknowledge that learning emerges from relationships among all the other levels of reality, i.e., the five subsystems: individual; microsystem; mesosystem, ecosystem; and macrosystem. (Bhaskar, 2010; Tikly, 2015), but there is not enough space in one paper to do justice to all, so we focus in-depth on one, the classroom microsystem because for most learners, much of learning mathematics happens in the classroom.

These social objects can change to such an extent that the object may barely be recognisable (Scott, 2010).

The distinction between ontology and epistemology is key to avoiding reductionism, by reducing the world to interpretations or observations, as knowledge of social objects can be fallible (Bhaskar, 1978). Changes in knowledge, i.e., epistemology, do not presuppose a change in the world, i.e., ontology, as theories are subject to change in light of new knowledge about the world (Tikly, 2015). Despite the fallibility of knowledge, the goal of critical realism is to better understand the world, by utilising and constructing theories to create knowledge that approximates reality in better ways.

For the second claim, Bhaskar (1978) argues that the social world is an open system. Open systems are distinguished from closed systems, such as experiments done in laboratory settings, which are controlled so that regular sequences of events occur (Bhaskar, 1978). In closed systems, it is possible to isolate what Bhaskar (1978, p. 38) calls a generative mechanism, which is "nothing other than a way of acting of a thing", which generates certain patterns of events. In contrast to closed systems, the real world is an open system, which is complex, so isolating a single generative mechanism to predict its effects is rare or impossible (Bhaskar, 1978; Tikly, 2015). In open systems, multiple generative mechanisms operate to give rise to phenomena. The purpose of any inquiry into learners' mathematical identities in the transitive dimension needs to acknowledge the openness of the world, so that we can identify and explain how the different mechanisms that exist in the intransitive dimension interact with each other to give rise to learners' mathematical identities. The process where interactions between mechanisms give rise to a phenomenon is called emergence. Identifying the mechanisms that are responsible for the emergence of the social object is called retroduction and explaining the interactions between these mechanisms using available theories is called retrodiction (Danermark, Ekström, Karlsson, 2019).

Building on the second claim, the third claim involves a commitment to the ontological depth of social reality (Scott, 2010). Critical realism offers mathematics education researchers an alternative way to think about identity, by moving beyond questions of what mathematical identities emerge to why and how mathematical identities emerge, by describing the deep nature of social reality in terms of interactions between generative mechanisms. Collier (1994) distinguishes between higher-level and basic mechanisms. Higher-level mechanisms emerge from basic mechanisms, yet higher-level mechanisms need to be discovered first, so they can be explained using basic mechanisms, but are not reduced to them (Collier, 1994). The purpose of science is to describe these mechanisms to better understand the depth of social reality. There can be

many different interpretations of reality, as critical realism accepts epistemological relativism, yet these interpretations are subject to judgmental rationality, which is a process that draws on reasoning to judge the credibility of theory and observations (Bhaskar, 1978; Scott, 2010).

## Critical Realism and Identities

From a critical realist perspective, a learner's mathematical identity is defined as a social phenomenon, which exists in the real world and emerges from various combinations of generative mechanisms. Archer (2002) and Marks and O'mahoney (2014) describe three important higher-level generative mechanisms, whose interactions are responsible for the emergence of identity, i.e., personal identity, social identity and agency. Using their ideas, we developed a model to explain the relationship between mechanisms (Fig. 1).

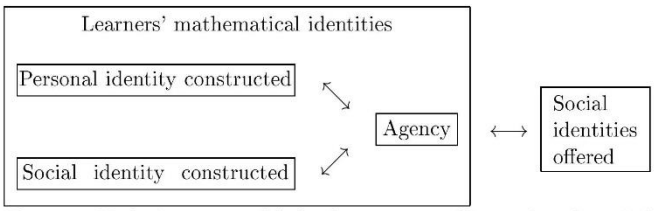



**Figure 1.** Mechanisms responsible for the emergence of learners' mathematical identities.

These higher-level mechanisms in combination are responsible for the emergence of learners' mathematical identities and it is important that we do not reduce identity to any one of the mechanisms, or we will be left with an incomplete, and perhaps incorrect understanding of learners' mathematical identities. Each of these higher-level generative mechanisms emerges from combinations of other basic mechanisms, which are explained as follows.

Personal identity is the subjective dimension of identity (Hartwig, 2007; Radovic et al., 2018). Archer (2002, p. 11, 15) defines personal identity as "a matter of what we care about in the world", since "who we are is a matter of what we care about most". Personal identity is defined as an understanding of self, which emerges from the embodied, reflexive self and is constructed through an individual's emotions and interests[2] (Archer, 2002; Marks,

[2]Other basic mechanisms that can contribute towards the emergence of personal identity are memories, imagination, conscious thought, reflection and personal preferences (see Marks, O'mahoney, 2014). Danermark et al. (2019) argue that explanations can never in-

O'mahoney, 2014). In terms of emotions, Archer (2002) shows how negative and positive emotions are related to participation and practice. She suggests that positive emotions can foster a continuation of practice, whereas negative emotions can lead to a cessation of practice, so success and failure can vindicate or damage self-worth (Archer, 2002). Bhaskar (1993) argues that people can act in accordance with their interests, and these interests are related to what people care about in the world (Archer, 2002). Emotions and interests inform human action and identity, however, it is important that we do not reduce human action and identity to interests and emotions, as the interactions between all three mechanisms are important for the emergence of identity.

Social identity is the social dimension of identity (Hartwig, 2007; Radovic et al., 2018), which emerges through social relations, and is a "navigated position between personal identities and the way in which people believe they should be perceived in a social setting" (Marks, O'mahoney, 2014, p. 72). Social identity involves learners choosing to take up certain roles and responsibilities generated by social structures, such as mathematics classrooms (Archer, 2002; Marks, O'mahoney, 2014). Learners construct their own social identities and these social identities are informed by what they "care about in social roles" (Archer, 2002, p. 17). Social identities are also offered to people, as social structures make available certain social identities for people to take up (Marks, O'mahoney, 2014). In classrooms, learners are offered social identities by others, as teachers and peers provide learners with opportunities to become certain kinds of members of the classroom community. So, it is important to distinguish between the social identities constructed or taken up by learners and the social identities offered to learners by others[3].

Personal and social identity can inform certain kinds of action, yet action is not reducible to either personal or social identity. Action is related to agency, as actions are the outcomes of exercising agency (Hartwig, 2007). According to Bhaskar (1978, p. 37), agency is a praxis and "saying, making and doing" are "characteristic modalities of human agency". Agency involves learners making active choices to participate in certain ways, as influenced by social contexts (Eteläpelto, Vähäsantanen, Hökkä, Paloniemi, 2013). Further, human beings can exercise agency by transforming or reproducing social contexts or struc-

clude all mechanisms and researchers should strive to take the most significant mechanisms into account. In our study, interests and emotions were the key mechanisms that emerged from our data analysis, so we discuss these.

[3]The social identities offered and constructed can also be informed by what Langer-Osuna, Esmonde (2017) call membership identities, which are learners' social memberships related to race, gender, ethnicity and language. Some classroom communities can make it difficult for learners to identify with mathematics because of these social categories and memberships (see Darragh and Radovic, 2018).

tures, so human beings can influence social contexts, rather than merely react to them (Bhaskar, 1998; Mercer, 2011). Human agency and social structures exist "in a relationship of ongoing reciprocal causality in which the emphasis is on the complex dynamic interaction between the two elements" (Mercer, 2011, p. 428). Put simply, there is no human agency without social structures, as these structures are necessary for the possibility of agency, and there are no social structures without human agency, as social structures require human agency to reproduce or transform the structures (Norrie, 2010).

Reasons are central to human agency and according to Bhaskar (1993; 1998), intentional human behaviour is caused by reasons, irrespective of whether agents are aware of the reasons for their intentional behaviours. It is important to understand people's reasons for actions, as reasons can be causes for action. Yet, it is also important for researchers to observe and research the causes of actions, as sometimes, people's accounts of their agency are limited by unacknowledged conditions and unconscious motivations (Norrie, 2010). How learners exercise agency is influenced by the social identities offered to them and their personal and social identities. Concomitantly, as learners exercise agency, they develop their personal and social identities.

Since social objects can change, learners' mathematical identities can change as they interact with their teachers in the classroom over time. Learners can reproduce the same mathematical identities as previously constructed, or transform their identities over time, as influenced by the social identities offered to them by different mathematics teachers, highlighting the contingent nature of identities. Further, as learners enact their identities in classrooms, teachers can continue offering learners the same social identities as previously offered or they can offer learners different social identities from those previously offered.

## Learners' Mathematical Identities

In their review of sixty-nine empirical papers on learners' mathematical identities, Radovic et al. (2018) identified three dimensions for researchers to consider when defining identity. These dimensions are the social/subjective dimension, enacted/representational dimension and change/stability dimension. For the first dimension, they argue that it is important to consider both, the social and subjective aspects of identity, to understand the individual, her agency and her notion of self in social contexts. For the second dimension, they argue that it is also important to consider both, the enacted and representational aspects of identity, to understand how participation and semiotically mediated meanings inform learner identity. For the last category, they contend that

identity is malleable, rather than static.

Building on this work, Darragh and Radovic (2018, p. 582-583) define learner identity as "a socially produced way of being, as enacted and recognized in relation to learning mathematics", which is malleable, "fluid and a constantly changing process". Learner identity involves "stories, discourses, and actions, decisions and affiliations that people draw upon to construct who they are in relation to mathematics", so both the enacted and representational dimension is accounted for in their definition (Darragh, Radovic, 2018, p. 582). They continue by explaining that their definition incorporates how people interact with and position others, how local practices are defined, the kinds of social discourses used to understand mathematics and people's understandings of themselves.

Darragh and Radovic (2018) further explain that their definition considers the importance of the individual's sense of being, which includes self-understandings in relation to mathematics and embodied feelings of who one is, giving the individual a sense of continuity and belonging, which is expressed and recognized in social contexts. The individual's sense of being is in constant negotiation with the social context, so both the social and subjective dimension is accounted for in their definition (Darragh, Radovic, 2018). Further, they argue that there are diverse theories used to operationalize identity, which focus on specific parts of identity, so identity research would require multiple methodologies, data and data analysis methods.

Darragh and Radovic's (2018) definition is compatible with a critical realist perspective because their definition accounts for the importance of both the individual and the social context, and that identity can change. Using critical realism, mathematical identity is defined as a social phenomenon, existing in an open system, and learners' mathematical identities can change over time. A critical realist would build on their definition by explaining that there is a distinction between learners' mathematical identities, which exist in the real world, and our knowledge of identities, which can be fallible. To avoid reductionism, it is important that we do not reduce identities as they exist in reality to knowledge of identities, as theories can change in light of new knowledge. Critical realism can also contribute to their definition by showing how the different components of identity that they discuss are basic mechanisms, that contribute to the development of the higher-level mechanisms, i.e., agency, personal and social identity, whose interactions are responsible for the emergence of learners' mathematical identities. Further, critical realism embraces epistemological relativism, by acknowledging the importance of using different theories and ideas to understand identity as Darragh and Radovic (2018) do, but cautions researchers against ontological relativism, by encouraging the use

of judgmental rationality to judge which theories can explain identity better than others.

We have drawn our understanding of critical realism, Darragh and Radovic's (2018) definition and other key research to develop the following explanatory framework to understand learners' mathematical identities in relation to the social identities offered by teachers in classroom contexts (Table 1) (also see Gardee, 2021).

| **Social identities offered** | **Mathematical identities constructed** |
|---|---|
| Affiliation with the classroom community | Affiliation with, compliance with or resistance to the classroom community |
| Marginalisation from the classroom community | Marginalisation from, compliance with or resistance to the classroom community |

**Table 1.** Identities offered to and constructed by learners in classrooms.

Research in mathematics education shows how teachers can include learners in their classrooms, by providing learners with full access to the tools and resources of the classroom community (e.g., Boaler, Staples, 2010; Cobb, et al., 2009; Turner, Dominguez, Maldonado, Empson, 2013). In doing so, teachers offer learners a social identity of affiliation, by providing them with opportunities to become full flourishing members of their classroom community. Research also shows how teachers can marginalise learners, by constraining learner access to the tools and resources of the classroom community (e.g., Bartholomew, 2000; Black, 2004; Martin, 2006). In these classrooms, teachers offer learners social identities of marginalisation, by alienating them from the classroom community.

Research in mathematics education has also shown how learners can relate to their teachers and tasks, by affiliating with, by complying with, or by resisting teachers and tasks (Boaler, Greeno, 2000; Cobb et al., 2009). Affiliating, complying or resisting teachers and tasks can be understood using Wenger's (1998) distinction between three kinds of participation: full participation, peripherality and marginality. Peripherality is distinct from full participation because learners engage peripherally, rather than fully, with the tools and resources of the community, while marginality is a "form of non-participation [that] prevents full participation" and engagement with the tools and resources of the community (Wenger, 1998, p. 166). Full participation can lead to full membership, peripheral participation can lead to peripheral membership, and in some situations, to full membership over a period of time; and marginal participation can lead to non-membership or marginal positions in communities (Wenger, 1998).

While teachers offer social identities, learners exercise agency and can develop their identities in different ways, as influenced by the social identities offered and their own personal and social identities. When learners are offered social identities of affiliation, they can construct their identities in affiliation, by being invested in becoming full members of the classroom community and by participating fully. Similarly, when learners are offered social identities of marginalisation, they can construct their identities in marginalisation, by marginalising themselves from the classroom community and participating marginally.

Sometimes, learners may choose to develop their identities differently from what is offered, by constructing identities in compliance with or resistance to the social identities offered. Compliance can be seen as a form of passive resistance, with learners becoming peripheral members by participating peripherally, when offered a social identity of affiliation or marginalisation. Resistance is a more active form of non-compliance, with learners marginalising themselves from the community by participating marginally when offered social identities of affiliation or attempting to become a full member by participating fully when offered social identities of marginalisation.

From a critical realist perspective, it is important to observe the teachers' interactions with learners and understand the teachers' reasons for offering certain identities and learners' reasons for constructing certain identities in relation to the mechanisms, as reasons can be causes for actions. Such an analysis would also require an understanding of learner actions and narratives, embracing both the enacted and representational aspects of identity.

## Methods

Data for this paper come from a larger study that began in 2015 (see Gardee 2019, 2021; Gardee, Brodie, 2022). Inspired by the work of Boaler, Greeno, (2000), Cobb et al., (2009), Wenger (1998) and other research on learners' mathematical identities, the focus of the larger study was to investigate the mathematical identities and trajectories of secondary school learners. As we began our quest to research learners' mathematical identities, the difficulty of finding a suitable non-reductionist definition, underpinned by a coherent theoretical framework to research identity became a challenge. Our search to find a suitable definition and method for empirically researching learners' mathematical identities informed the purpose of this paper.

## Participants

This study was conducted in a secondary school in Johannesburg, South Africa. In total, nineteen learners participated in the study over both years, eight in the first year, with an additional eleven in the second year, along with their five mathematics teachers, two in the first year and three in the second year. In 2015, four learners were selected by each of the two Grade 9 mathematics teachers, using the following criteria: one high-performing learner, one low-performing learner and the person sitting next to each of them. In 2016, we observed the same learner participants in Grade 10 and selected an additional eleven learners who were seated next to the initial participants. The Grade 10 mathematics teachers were asked to specify which learners were high-performing, average-performing or low-performing. The purpose of understanding how the teachers identified the learners was so that we could observe how teachers offered social identities to learners who displayed different levels of achievement and how these learners constructed their mathematical identities in relation to the offered social identities.

In this paper, we discuss the mathematical identities constructed by four Grade 10 learners, two in Mr Sithole's (all names are pseudonyms) classroom and two in Mr Ncube's classroom, in relation to the social identities offered by their teachers. Both teachers were qualified to teach mathematics; Mr Sithole had thirty-two years of experience teaching mathematics, while Mr Ncube had five years. In Mr Sithole's classroom, the two learners were Lane, whose marks were inconsistent, and Tommy, who was identified as low-achieving. In Mr Ncube's Grade 10 classroom, the two learners were Thandi, identified as high-achieving, and Thabo, identified as average-achieving. We selected these learners in these teachers' classrooms for discussion for two reasons. First, the teachers interacted with learners and offered learners social identities differently from each other. Second, each of these four learners participated in different ways and constructed different mathematical identities from each other. Our intention is to show how the combination of the mechanisms contributed to the emergence of different mathematical identities.

## Data collection

Data for the study were collected in the 2015 and 2016 academic years during the third school term (August-September), for two weeks in 2015 (Grade 9) and four weeks in 2016 (Grade 10). Data were collected as videotaped lessons, semi-structured interviews with learner and teacher participants, field notes and photographed learner notebooks. We placed the video camera in front of the classrooms to focus on learners to capture their whole-class participation

and how their teachers taught and interacted with them. We collected approximately 8 hours of lessons for each teacher. Audio recorders were placed on learners' desks to capture their conversations with each other and their teachers, as these conversations were sometimes difficult to hear on the video camera. Field notes and photographs of learner notebooks were taken to provide context and to note important events that were not captured in the recordings. These different methods of collecting data are important for triangulation and for understanding the complexity of phenomena.

We conducted semi-structured interviews with the teachers, which were aimed at understanding the kinds of social identities that the teachers offered to the different learners and their reasons for offering certain social identities. Some of the questions asked during the teacher interviews were: how do you see [name of a learner participant] as a learner in your classroom?; how does [name of a learner participant] participate in the classroom?; and do you think that [name of a learner participant] is good at doing mathematics? We also conducted multiple semi-structured interviews with each learner, aimed at understanding how and why learners constructed certain mathematical identities. Some of the questions asked to the learner participants were: how do you see yourself as a mathematics learner?; how do you experience learning mathematics this year?; can you please describe to me what your mathematics lessons are like; how does your mathematics teacher make you feel when you are sitting in your mathematics classroom?; how do you participate in your mathematics classroom?; and is mathematics necessary for your future career or sense of being? All of these questions were followed by further probing questions, aimed at supporting the participants to give detailed explanations.

### Data analysis

All data were analysed qualitatively. From a critical realist perspective, qualitative research, which is also known as intensive research, is needed to provide explanations of why certain objects or events occur (Danermark et al., 2019). Intensive approaches are necessary to apply the processes of retroduction and retrodiction, by identifying mechanisms, and then describing how they interact with other mechanisms in contexts, which is not possible when using extensive quantitative methods.

All videotaped lessons and interviews were transcribed, coded and analysed thematically. When analysing the social identities offered to learners by teachers from the videotaped lessons, we focused on how the teachers interacted with learner participants, in terms of whether the learners were provided with assistance and support in learning mathematics, and whether they were

treated with respect by their mathematics teachers. We coded the social identities offered as affiliation or marginalisation based on whether learners were included in or excluded from the classroom community by their teachers, by being provided with full or marginal access to the tools and resources of the classroom community.

We also focused on how the learners participated in the classroom and who they participated with to learn mathematics in the classroom to understand how they exercised agency. For example, we noted whether learners asked their teachers questions, answered their teachers' questions, discussed their solutions with their peers, demonstrated their answers on the board when provided with the opportunity to do so, assisted other learners in the classroom and completed their tasks. Learners' participation enabled us to understand how the mechanism of agency contributed to the development of their mathematical identities.

The teacher interviews were coded for social identities offered to learners by focusing on how the teachers described each learner participant and their reasons for interacting with learners in certain ways. The learner interviews were coded for personal identities, in terms of their interests or emotions relating to learning mathematics. We coded learners' social identities, in terms of how the learners positioned themselves in their classroom communities and whether they were comfortable to participate as a member of the community. We also coded the learners' experiences of the social identities offered to them by their teachers, in terms of how they felt when participating (or not) in their classrooms. For agency, we coded how learners described their participation to us, in terms of how and with whom they participated with to learn mathematics in their classroom.

We then compared the data analysed from all three sources, i.e., our observations, the teacher interviews and the learner interviews, to understand how and why certain identities were offered and constructed. For the social identities offered to learners, the data from the videotaped lessons provided understandings of how the teachers offered social identities, while the data from the teacher interviews provided understandings of the teachers' interpretations of and reasons for offering certain social identities. The learner interviews provided further understandings of how the social identities offered by the teachers were experienced by the learners. Using our framework, we then coded whether learners were offered social identities of affiliation (A) or marginalisation (M) by their teachers.

Further, our framework assisted us in relating the social identities offered to the kinds of mathematical identities that the learners constructed. Learners' personal and social identities were accessed from their interviews, as it was not

always possible to gauge these two mechanisms from observations. For learners' agency, the data from the videotaped lessons provided understandings of how learners participated in the community, while the data from the learner interviews provided understandings of the learners' interpretations of and reasons for participating in certain ways. Using our framework, we then coded whether learners constructed their identities as affiliation (A), compliance (C), resistance (R) or marginalisation (M) in relation to the social identities offered by the teachers.

By analysing the data using our framework, we assessed the applicability of our framework to abide by the principle of judgmental rationality, by identifying whether our framework can explain learners' mathematical identities comprehensively. To enhance the credibility of our findings, we used a process called peer debriefing, where the first author discussed her preliminary analysis with the second author (McMillian, Schumacher, 2010). Peer debriefing is an iterative process, so after each discussion, the first author re-examined the data and revised the claims made in the paper until both authors were convinced by the analytical claims made.

## Findings

From our observations of Mr Sithole's and Mr Ncube's lessons, we noted that Mr Sithole usually assisted the higher achievers, but not the lower achievers, for example, by telling them "I don't have time for you". In some lessons, he was seen insulting lower achievers and while some learners seemed to find his insults amusing, others seemed upset. In contrast, Mr Ncube invited all learners to participate in his classroom and was willing to assist all learners. He shared a professional relationship with learners and treated all learners and their contributions with respect.

When asking the teachers about how and why they interacted with learners in certain ways, the main reason that the teachers offered was based on their narratives of who could do mathematics. Mr Sithole stated that only certain learners could learn mathematics.

> Mr Sithole: I always think that first of all, there should be a gift into it and then hard work.

He explained that the "gift" to do mathematics was from "God" and this gift informed whether some learners had the "aptitude" to learn mathematics, while others did not. He explained that he did "nothing" to help the weaker learners because some learners do not understand and "we have to accept that there are people like that".

Unlike Mr Sithole, Mr Ncube said that he was willing to assist all learners and that he encouraged learners to participate in his lessons. He explained that all learners could do mathematics and that hard work, effort and practice were responsible for learners' success.

> Mr Ncube: Mathematics is like any other subject meant for everyone to do, but it requires for some to go an extra mile than some.

He explained that some learners needed additional practice and assistance, so he offered all learners extra lessons after school hours for free. He also said that some weaker learners came to him during his free time in school and that he assisted them.

The criteria for learner success in mathematics described by the teachers translated into practice as the social identities offered to learners. Mr Ncube offered all learners identities of affiliation with his classroom community, while Mr Sithole offered learners different social identities depending on whom he thought was able to learn mathematics. Table 2 provides detail of the social identities offered (SIO) by each teacher and how the learners experienced the social identities offered by their teacher.

In Mr Sithole's classroom, Lane and Tommy experienced the social identity of marginalisation offered to them negatively. Both learners were aware that Mr Sithole was not willing to assist them and both learners expressed that he did not care about them. In Mr Ncube's classroom, Thandi and Thabo experienced the social identity of affiliation offered to them positively. When describing Mr Ncube, both learners spoke about how he treated learners equally in the classroom and was concerned about all of them. He was described as being motivating and inspiring and both learners enjoyed his lessons.

A key advantage of considering critical realism as a theoretical framework is that critical realist perspectives do not conflate learners' mathematical identities with the social identities offered by teachers. Even though these learners may have experienced their teachers negatively or positively, not all learners identified with the social identity offered. Table 3 provides detail of the mathematical identities constructed (MID) by the learners, in terms of their personal identities, social identities and agency in relation to the social identities offered (SIO).

| Teacher | Learners | SIO | Teachers' reasons for the SIO (from interviews) | Learners' experiences of the SIO (from interviews) |
|---|---|---|---|---|
| Mr Sithole | Lane (marks inconsistent) | M | Lane did not "grasp... fast enough" and needed more time to understand mathematics. Sometimes, he did well in assessments and other times, he performed "dismally". Mr Sithole did not know what Lane did when he did not understand the concepts. | Mr Sithole "just doesn't care Mam... whether you understand it or not... cos Mam, I've asked him many times to help me. He said no". Mr Sithole "chase(d)" him away when he asked him for help. |
| | Tommy (low achiever) | M | With Tommy, "there is nothing there". He performed "dismally" in tests and Mr Sithole did not think he could ever do well in assessments. "If education is a gift, I think he is gifted somewhere else". | Mr Sithole "doesn't care" when he did not understand. When he went to ask him for help, Mr Sithole would get "angry" and "frustrated" with him and would tell him to sit down or chase him out of the class. |
| Mr Ncube | Thandi (high achiever) | A | Thandi was "a good learner in maths and she is a hard worker". She had an "interest" in mathematics and was "quick to understand the concept(s)". She could work on her own and her solutions were correct. | Mr Ncube "cares a lot" about learners and assisted them in the classroom when they did not understand. He gave all of them attention "equally" and "he focuses on everyone as a class. Makes sure that everyone understands in the class". She enjoyed his lessons and said that he was encouraging. |
| | Thabo (average achiever) | A | Thabo was a "hard worker" but was an "average" performer. He participated often and "benefits a lot from participating in class". He was willing to learn from his mistakes. | He liked Mr Ncube and enjoyed his lessons. He said Mr Ncube made him feel "good about being a maths learner" by "inspiring" the learners "day by day" that mathematics was not difficult. Mr Ncube would not move on until "everyone in the class says they understand". |

**Table 2.** The social identities offered to learners.

| **Teacher** | **Learners** | **SIO** | **MID** | **Personal identity (from interviews)** | **Social identity (from interviews)** | **Agency (from observations and interviews)** |
|---|---|---|---|---|---|---|
| Mr Sithole | Lane (marks inconsistent) | M | C | Mathematics was a "little difficult". He was interested in pursuing mathematics in the future and said that he was trying hard to pass mathematics. | He did not feel comfortable in the classroom with his teacher. He said, in "Mr Sithole's class, you are not comfortable. You are always scared" as Mr Sithole insulted the learners. He was failing mathematics. | He did not participate often with the teacher in the classroom, but he completed his work and asked other learners for help when he did not understand. |
| | Tommy (low achiever) | M | M | He "love(d)" a certain part of mathematics but generally found mathematics "challenging". He says "I don't see myself getting anywhere" with mathematics because of the teacher. He was interested in pursuing mathematics in the future. | He did not feel he could participate "freely" in his classroom. He said that the teacher was "demotivating" and insulting. He was failing mathematics. | Tommy did not participate often with the teacher or others. He did not always complete his tasks, and sometimes, he did not even attempt the task. |
| Mr Ncube | Thandi (high achiever) | A | A | Mathematics was her favourite subject and she liked how "everything links together" in mathematics. She was interested in pursuing mathematics in the future. | She felt like she was "part" of her classroom and felt "good" in her classroom. She was one of the best learners of mathematics in her classroom. | She participated with her teacher when she needed assistance and she assisted others. She usually completed her tasks on her own. |

| | | | | | | |
|---|---|---|---|---|---|---|
| | Thabo (average achiever) | A | C | Thabo enjoyed learning mathematics, but he found mathematics challenging by "holding him down" because of the time required to understand mathematics. However, he was interested in learning mathematics because he wanted to pursue mathematics in the future and he wanted to pass mathematics. | He identified himself as a "slow learner" in comparison to others. He said that his marks were improving, but that his mathematics marks were currently "below average". He was not comfortable asking Mr Ncube for help because Mr Ncube made the solutions seem easy, while he found the problem challenging, making him feel "stupid". | He participated in the classroom with the teacher and others. He usually completed his tasks. |

**Table 3.** Identities constructed by learners.

While Tommy and Thandi constructed their mathematical identities by identifying with the social identities offered by the teachers, Lane and Thabo did not adopt the social identities offered, but rather constructed their identities in compliance with what was offered (Table 3). It is important to consider learners' personal identities, social identities and agency to understand the mathematical identities constructed by learners.

In terms of learners' personal identities, all of the learners were interested to pursue mathematics in the future. The learners generally described their personal identities positively, by being interested in learning mathematics for various reasons, even though some of them found mathematics challenging. For Thandi, mathematics itself was enjoyable, while Thabo enjoyed learning mathematics currently, but like Lane and Tommy, he found learning mathematics challenging. For Thabo and Lane, passing mathematics to pursue their future careers was most important for them, while even though Tommy wanted to pursue a career in mathematics, he said that he did not see himself getting anywhere with mathematics because of his teacher.

Considering personal identity alone to understand learners' mathematical identities is insufficient. For example, Tommy enjoyed learning certain aspects

of mathematics, even though he found mathematics challenging, and he was interested in pursuing mathematics in the future, yet he marginalised himself from the classroom community because of his negative experiences of interacting with the teacher (Table 3). From a critical realist perspective, mathematical identity cannot be reduced to personal influences - learners' personal identities are only one of the generative mechanisms responsible for the emergence of their mathematical identities. If we are to reduce learners' mathematical identities to personal identity, we will develop a decontextualized understanding of identity (Radovic et al., 2018).

In terms of the learners' social identities, Thandi positioned herself as part of her classroom, who felt "good" in her teacher's classroom, and as a learner who was performing well. While Thabo liked Mr Ncube, he was not entirely comfortable asking Mr Ncube for help since Mr Ncube tended to attenuate the difficulty of mathematical problems that he found challenging, which made him feel "stupid". Lane and Tommy were also not comfortable to participate with their teacher in the classroom because of how Mr Sithole treated them, by insulting them and making them feel scared.

From a critical realist perspective, mathematical identity cannot be reduced to social influences alone, as learners' personal identities play an important role regarding whether or not they choose to become members of their classroom communities. For example, Lane experienced Mr Sithole negatively and was not comfortable to participate with him. However, he still maintained a peripheral membership by exercising his agency and doing his work and asking others to help him so that he could pass mathematics, as passing mathematics was important for his personal identity. Reducing learners' mathematical identities to social identity neglects the important role of the individual and their agency in the construction of her/his identity (Radovic et al., 2018).

From this analysis, personal identity and social identity is shown to influence learners' agency, in terms of learner participation in their classroom communities, yet agency is not reduced to either personal or social identity. For example, Thabo exercised agency by completing his work and participating in his lessons with his teacher and others. He said that he asked his teacher questions and answered his teacher's questions. Yet, accessing learner participation alone is not enough to understand how learners construct their identities. While Thabo participated actively with his teacher during the lessons, he developed his identity in compliance and became a peripheral member of the community, because while he currently enjoyed mathematics, he felt that mathematics was "holding him down" and challenging. He was primarily interested in participating so that he could pass mathematics to pursue mathematics in the future. Further, he said that he was not entirely comfortable asking his teacher for

assistance, so he was not comfortable to participate fully in the classroom.

Unlike Thabo, Thandi did not participate as actively with the teacher in the classroom, yet she developed her mathematical identity as a full member of the classroom community. She enjoyed learning mathematics, was comfortable to exercise agency by participating with the teacher if she needed assistance and she assisted others and tried to make sense of mathematics on her own. She said that she wanted to give other learners an opportunity to participate and answer questions, so she only asked her teacher for assistance when she did not understand. It is important to consider the relationships between all mechanisms to understand learners' mathematical identities in a non-reductionist way. Without analyzing learners' personal, social identities and agency, we may not understand why Thandi and Thabo who were offered social identities of affiliation constructed different mathematical identities, one in affiliation and the other in marginalisation.

Similarly, without an analysis of all three mechanisms, we may not understand why Lane and Tommy, who were offered social identities of marginalisation constructed different mathematical identities, one in compliance and the other in marginalisation. Lane developed his identity in compliance with the social identity of marginalisation offered because he was interested in passing mathematics, even though he was marginalised by his teacher. He exercised agency by participating peripherally and was usually seen working on his own or asking others for help. While Tommy was interested in pursuing mathematics in the future and even stated that he enjoyed certain parts of mathematics, his experiences of being marginalised by his teacher informed the identity of marginalisation that he constructed. He exercised agency by participating marginally, as he seldomly interacted with his mathematical tasks, his teacher or others. These findings highlight the importance of considering the interrelated relationship between personal identities, social identities, agency and the social identities offered by teachers, which are all responsible for the emergence of learners' mathematical identities.

## Discussion and Conclusion

The purpose of this paper has been to develop a framework to define and research learners' mathematical identities in classroom contexts, underpinned by critical realism, with a focus on both the personal and social aspects of identity. Although researchers have been researching learners' mathematical identities for some time now, the concept is still not sufficiently understood (Darragh, 2016; Graven, Heyd-Metzuyanim, 2019; Langer-Osuna, Esmonde,

2017). To our knowledge, critical realism has not been used as yet to theorise learners' mathematical identities, and our study points to a number of benefits of using critical realism to sufficiently understand, define and research learners' mathematical identities in a detailed, non-reductionist way.

To answer the first research question, which required a definition of mathematical identity, we showed how we used critical realism to define identity as a social phenomenon existing in the real world, emerging from relationships between generative mechanisms. We drew on research to develop a conceptual framework to describe three higher-level generative mechanisms, which are responsible for the emergence of identity: personal identity, social identity - offered and constructed - and agency. We also accounted for change, by explaining how the identities offered and constructed can be reproduced or transformed over time. Drawing from our understanding of critical realism, Darragh and Radovic's (2018) definition of identity and the works of other key researchers like Boaler, Staples (2010), Cobb, et al. (2009) and Wenger (1998), we then developed a framework for learners' mathematical identity in classroom contexts. Our framework illustrated how learners construct their mathematical identities, in affiliation with, compliance with, resistance to or marginalisation from classroom communities when offered social identities of affiliation with or marginalisation from classroom communities by teachers.

To answer the second research question, which involved showing how secondary school learners' mathematical identities can be researched, we explored the power of the framework and examined how four secondary school learners constructed their mathematical identities through interacting with their mathematics teachers. Two of the learners, Lane and Tommy, were offered social identities of marginalisation and the other two, Thandi and Thabo, were offered social identities of affiliation. We showed how the teachers' reasons for identifying and interacting with learners informed the social identities offered. From these four learners, Tommy and Thandi identified with the social identities offered, while Lane and Thabo complied with the social identities offered. Each learner constructed their identities differently from the other, Lane in compliance with the social identity of marginalisation, Tommy as marginalisation with the social identity of marginalisation, Thandi as affiliation with the social identity of affiliation and Thabo in compliance with the social identity of affiliation. None of the learners resisted the social identities offered in this sample, and across the wider sample, and we have argued elsewhere, that these learners' mathematical identities were informed by their teachers' practices (see Gardee, 2019, 2021; Gardee, Brodie, 2022 for more findings).

While the interactions between all three generative mechanisms contribute to the emergence of learners' mathematical identities, it is important to note

that in each case, some mechanisms seem to contribute more than others. For Tommy, the social identity offered by the teacher contributed largely to the mathematical identity that he constructed. Even though he was interested in pursuing mathematics in the future, and claimed to enjoy certain parts of mathematics, he exercised agency by giving up and not engaging in the classroom. For Lane, his personal identity contributed largely to his compliance, as even though he was marginalised and uncomfortable to participate in his classroom, he exercised agency by trying to learn mathematics by asking others for help and doing his work. For Thabo, his teacher offered him opportunities to affiliate and he exercised agency by engaging with his teacher and others in the classroom, but his personal and social identity largely contributed to his compliance, as he said that mathematics was holding him down, and he was not entirely comfortable to ask his teacher for assistance. While for Thandi, all the mechanisms contributed positively to her identity, supporting her affiliation. From a critical realist perspective, this finding is explained by Collier (1996, p. 42), who argues that "the proportion in which different mechanisms contribute to the course of events will vary from case to case", however, it is still important to explain all higher-level mechanisms as they operate relationally.

As part of our commitment to avoid reductionism, we also showed that reducing mathematical identity to any of the generative mechanisms provides an incomplete, and sometimes incorrect understanding of learners' mathematical identities. Some researchers tend to reduce learners' mathematical identities to the opportunities provided by classroom communities to become competent and effective learners in that community (Cobb et al., 2009), while other researchers tend to reduce identity to personal or social influences (Radovic et al., 2018). Our framework highlights the importance of all three generative mechanisms, i.e., personal identity, social identity (both offered to and constructed by learners), and agency, which need to be considered concurrently with each other in a non-reductionist way. Reducing mathematical identity to any of the mechanisms responsible for its emergence provides an inaccurate understanding of identity.

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## Ramy teoretyczne dla matematycznych tożsamości uczniów: perspektywa krytycznego realizmu

S t r e s z c z e n i e

W ostatnich latach wielu badaczy sięgnęło do koncepcji tożsamości, aby lepiej rozumieć uczenie się w kontekstach społecznych. Pomimo rozwoju badań nad pojęciem tożsamości, istnieje potrzeba dokładniejszego jej zdefiniowania, w oparciu o istniejące opracowania teoretyczne. W artykule tym, jako teoretyczne ramy do zdefiniowania i badania tożsamości matematycznych uczniów, konstruowanych w klasach, zaproponowano wykorzystanie krytycznego realizmu. Stosując realizm krytyczny, tożsamość definiowana jest jako zjawisko społeczne, które istnieje w realnym świecie i wyłania się z kombinacji różnych mechanizmów generatywnych. Rozróżniono trzy mechanizmy generatywne: tożsamość osobistą, tożsamość społeczną i sprawczość oraz zilustrowano, w jaki sposób te trzy mechanizmy generatywne obejmują subiektywne i społeczne wpływy, które są odpowiedzialne za pojawienie się tożsamości matematycznych uczniów.
W niniejszym artykule zaproponowano pewną teorię oraz zastosowano ją, aby zrozumieć, w jaki sposób czterech uczniów szkoły średniej skonstruowało swoją matematyczną tożsamość poprzez interakcje z nauczycielami matematyki.

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# Chapter 3: The influence of teachers on learners' mathematical identities

## 3.1. Introduction

In this chapter, I present a conference paper presented at the International Group for the Psychology of Mathematics Education conference in 2019. This chapter shows how the explanatory framework developed in chapter 2 is in line with Radovic et al.'s (2018) three dimensions discussed in chapters 1 and 2. These dimensions are social/subjective dimension, enacted/representational dimension and change/stability dimension. This chapter also focuses on applying the explanatory framework developed in chapter 2 to all of the teacher and learner participants, by analysing the social identity offered to each learner by their mathematics teacher in relation to the mathematical identity constructed by the learner.

There is a sense of trajectory in this chapter, as I show how the initial six learner participants' mathematical identities changed substantially or negligibly over the two years, as informed by their interactions with different mathematics teachers. I show that the social identities offered to learners were informed by whether the teachers thought that ability or effort were important to learn mathematics. This chapter also points to other important findings, that are further discussed in chapters 4, 5 and 6. These findings are: the teachers' narratives of who could do mathematics influenced how they interacted with learners and the kinds of social identities offered to learners; learners identified teacher practices as the most important factor that influenced their learning of and identification with mathematics; and that none of the learners resisted the offered social identities. This chapter provides some answers to the two main research questions, by showing, although briefly, how the concept of identity can be used in researching secondary school learners' mathematical identities and how learners and their mathematics teachers contribute to the development of learners' mathematical identities.

# THE INFLUENCE OF TEACHERS ON LEARNERS' MATHEMATICAL IDENTITIES

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*In this paper, we develop a framework to define and research learners' mathematical identities, as both constructed by learners and influenced by their teachers. We show how five teachers offered learners different social identities through the kinds of access they provided to learning mathematics. Two teachers offered all learners opportunities to become full members of their classroom communities, and three teachers marginalised those whom they thought could not do mathematics. The offered social identities strongly influenced how learners constructed their mathematical identities and how they learned mathematics.*

## INTRODUCTION

Over the last two decades, the concept of identity has received significant attention in the field of mathematics education and is useful in theorizing learning (Radovic, Black, Williams & Salas, 2018; Sfard & Prusak, 2005). In recent years, researchers are developing operationalized definitions of identity and Radovic et al. (2018) describe three ways in which identity is defined in research: social or subjective; enacted or representational; and transformative or stable. In some research, identity is conceptualised as a social becoming, being constructed in social contexts, while in other research, identity is conceptualised as subjective, emphasising the individual self and personal experiences. Some researchers define identity as enacted and related to action, while others define identity as representational, mediated and constructed through discourse. Most researchers emphasise the malleable and transformative nature of identity, while few view identity as static. In this paper, we begin by presenting a framework to define identity, which moves between the social, subjective, enacted, representational and transformative features of mathematical identity. We use our definition to understand how five secondary school mathematics teachers influenced learners' mathematical identities.

## THEORETICAL FRAMEWORK

Drawing on the work of Marks and O'Mahoney (2014), we define learners' mathematical identities as being constituted by relationships between learners' personal identities, social identities and agency. Personal identities are the subjective dimension of identity, involving learners' understandings of self, as informed by their interests, aspirations, experiences and motivations. Social identities are the social dimension of identity, involving the ways learners position themselves and would like to be perceived by others in their classroom communities. Agency refers to learners' choices in participating in mathematics and developing their identities. Learners'

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mathematical identities are influenced by the social identities offered to them by their parents, teachers and peers, which influence the opportunities provided to learners to participate, learn and become members of the classroom community (Fig 1).

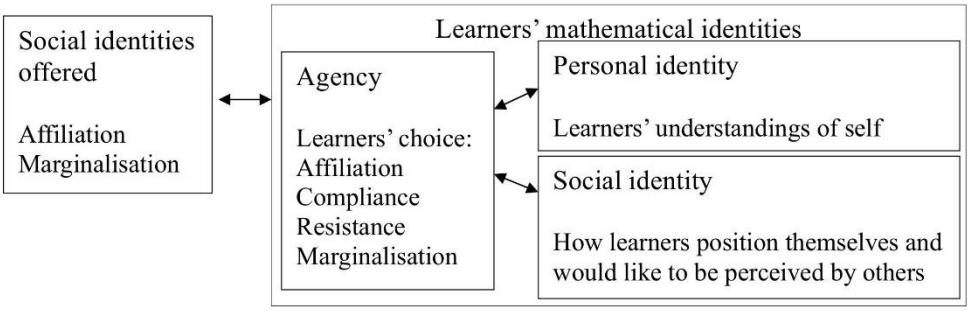



Figure 1: Identities offered and constructed

Teachers can offer learners identities of affiliation, by supporting them to learn mathematics and become full members of their classroom community. Some teachers offer learners identities of partial affiliation, by inviting participation and supporting learners to some extent. Teachers can offer learners identities of marginalisation by limiting learner opportunities to learn mathematics and become full members of the classroom community.

While teachers offer learners certain social identities, learners exercise agency and construct their identities in different ways, influenced by their own personal and social identities. Learners can construct identities of affiliation when offered opportunities to affiliate, or identities of marginalisation when marginalised by teachers. Learners can also exercise agency by developing their identities differently from what is offered, in compliance with or resistance to the offered social identity. Compliant learners often aim to achieve peripheral membership in the classroom by participating peripherally rather than fully when offered identities of affiliation, or may participate with others and not with their teachers when offered identities of marginalisation. Resistant learners may not want to be identified as members of the classroom community when offered identities of affiliation, or may want to achieve full membership when offered identities of marginalisation.

Experiences of exercising agency concomitantly influence learners' personal identities, as they learn more about themselves in terms of their interests, aspirations and motivations, and their social identities, as they evaluate their membership in the community. This means that learners' identities are constantly being developed as they learn and participate (or not) in their classroom communities. As learners enact their identities through interacting with teachers, teachers can continue offering them the same social identities as previously offered or may offer learners different social identities from those offered in the past. Different teachers may also offer different identities to the same learners. Learners may enact and reproduce the same identities

constructed previously or may adopt different identities, as influenced by their experiences with new teachers in different classrooms, highlighting the malleable nature of identity.

Different mathematical identities can be enacted as learners participate with others in the classroom (Radovic et al., 2018). As Wenger (1998) argues, identity is related to learning because as learners participate, they are able to further affiliate themselves with their classroom community. Identity is also representational and constructed through learner narratives (Radovic et al., 2018). Narratives, according to Sfard and Prusak (2005), are a collection of socially shaped stories, which influence action when being reifying, endorsable and significant. Reifying narratives contain verbs and adverbs that stress the repetition of actions. Narratives are endorsable when the storyteller declares the authenticity of narrative, and narratives are significant when a change in story is likely to lead to a change in feelings about an identified person. Observing classroom interactions and accessing narratives enable understandings of the identities offered to and constructed by learners. We now show how our framework is useful to research learners' mathematical identities.

## METHODS

This study took place in a well-resourced, technical school in Johannesburg over a period of two years in the third school term (August - September): two weeks in 2015 (Grade 9) and four weeks in 2016 (Grade 10). Five mathematics teachers participated in the study, two in the first year: Mr Moyo and Mr Molefe (Gr 9), and three in the second year: Mr Sithole, Mr Ndlovu and Mr Ncube (Gr 10) (all names in this paper are pseudonyms). Nineteen learners participated in the study, eight in the first year, with an additional eleven in the second year. From the initial eight participants in the first year, six continued to participate in the second year, while two transferred to another school.

All of the teachers who participated in the study had academic degrees to teach mathematics. Mr Moyo was newly qualified and had joined the school at the end of the second term (May), Mr Molefe had five years of experience teaching mathematics, Mr Sithole had thirty-two years, Mr Ndlovu, seven years and Mr Ncube, five years. The eight learners who initially participated in the study were selected by their grade 9 teachers using the following criteria: one high performing learner, one low performing learner and the person sitting next to them, which enabled us to observe teacher and learner interactions with learners displaying different levels of achievement. In grade 10, we continued to observe the six learners from Grade 9 and selected an additional eleven learners, who were seated next to the initial participants. We asked the Grade 10 teachers whether each learner was a low, average or high achieving learner, which enabled us to understand how teacher ideas about learner performance influenced teacher and learner interactions.

Data were collected in the form of videotaped lessons, field notes, photographed learner notebooks and multiple audiotaped semi-structured interviews with learner and

teacher participants. A total of 480 minutes of lessons were analysed for each teacher. All video and audio recordings were transcribed, and data were analysed qualitatively. From the videotaped lessons, we coded the kinds of social identities offered: affiliation (A), partial affiliation (PA) or marginalisation (M), by considering the interactions between teachers and learners. Observing classroom interactions also enabled us to understand how learners exercised agency and enacted their identities as affiliation (A), compliance (C), resistance (R) or marginalisation (M). We analysed the semi-structured interviews by identifying narrative excerpts to understand the representational component of identity, using Sfard and Prusak's (2005) criteria of narratives needing to be reifying, endorsable and significant.

## FINDINGS

### An overview of our findings

Our analysis showed that two teachers offered all learners identities of affiliation with their classroom communities, while the other three offered learners differential identities depending on learners' achievement in assessments. We found that learners developed their identities in affiliation with, compliance with or marginalisation from the social identities offered. None of the learners resisted the offered social identities, highlighting that learners' mathematical identities are strongly influenced by the social identities offered by teachers. Table 1 shows the social identities offered to learners by the teachers and the identities constructed by learners.

Two teachers, Mr Moyo (Gr 9) and Mr Ncube (Gr 10), offered all learners social identities of affiliation. They engaged with and assisted all learners, including struggling learners, and treated learners and their contributions respectfully. All four learners in Mr Moyo's Grade 9 classroom developed their identities in affiliation with the social identity offered. In Mr Ncube's classroom, three learners developed their identities in affiliation and three developed their identities in compliance with the social identity of affiliation offered.

The other three teachers, Mr Molefe (Gr 9), Mr Sithole (Gr10) and Mr Ndlovu (Gr10) offered learners differential social identities based on learners' achievement. High achieving learners were offered social identities of affiliation - the teachers engaged with them, treated them respectfully and were willing to help them. All the high achieving learners besides Fred, developed their identities in affiliation with the offered identity of affiliation. Low achieving learners were offered social identities of marginalisation - the teachers often ignored them, provided them with limited assistance and in some situations, treated them disrespectfully. Some of the marginalised learners developed their identities in compliance with the offered social identity by participating peripherally, while others developed their identities in marginalisation from the classroom community.

Two learners, Sunny and Jack in Mr Molefe and Mr Sithole's classrooms, whose marks were inconsistent, were offered social identities of partial affiliation - the teachers were willing to assist them to a certain extent and ignored them at other times. Sunny was

identified by Mr Molefe as a "naughty" learner, who was at times the highest achiever in the classroom, and at other times, did not perform as well. Jack was identified as a learner who had "mathematical intelligence", and whose marks were sometimes average and at other times lower. Both learners enacted identities of compliance. Lane, who was identified as a quiet and "slow" learner, whose marks were sometimes average and at other times very low, was marginalised by Mr Sithole and responded with compliance.

| Grade | Teachers | Learners' achievement | Learners | Identity offered | Identity constructed |
|---|---|---|---|---|---|
| 9 | Mr Moyo | High achievers | Shane & Jack | A | A |
| | | Average achiever | Lane | A | A |
| | | Low achiever | Senzo | A | A |
| 10 | Mr Ncube | High achiever | Thandi | A | A |
| | | Average achiever | Amy | A | A |
| | | Average achiever | Thabo | A | C |
| | | Low achiever | Mike | A | A |
| | | Low achievers | Jimmy & Tess | A | C |
| 9 | Mr Molefe | High achiever | Sindiswa | A | A |
| | | Inconsistent | Sunny | PA | C |
| | | Low achievers | Jennifer & Tess | M | C |
| 10 | Mr Sithole | High achievers | Phillip, Shane, Lebo & Karabo | A | A |
| | | Inconsistent | Jack | PA | C |
| | | Inconsistent | Lane | M | C |
| | | Low achievers | Tommy & Senzo | M | M |
| 10 | Mr Ndlovu | High achiever | Edward | A | A |
| | | High achiever | Fred | A | C |
| | | Low achiever | Sunny | M | M |

Table 1: Identities offered and constructed

Over the two-year period, learners' mathematical identities changed, some significantly, others negligibly. Jack, Lane, Senzo and Sunny enacted different identities and provided different narratives of their identities in grade 9 and 10, with their teachers usually being the main reason why they no longer enjoyed mathematics in grade 10. Shane and Tess enacted similar identities in both grade 9 and 10 and both learners' stories of themselves and ways of participating changed negligibly.

**The social identities offered**

The teachers' narratives of who could do mathematics influenced how they interacted with learners and the kinds of social identities offered to learners. For Mr Moyo and Mr Ncube, all learners could learn mathematics. Mr Moyo said that he envisioned a classroom where all learners participated. He believed that struggling learners could learn mathematics and said that he tried to motivate them. Mr Ncube also thought that all learners could learn mathematics. He said, "mathematics is like any other subject meant for everyone to do, but it requires for some to go an extra mile than some", meaning that some learners had to work harder than others to understand concepts. He said he encouraged learners to participate and that he was willing to help struggling learners.

While providing all learners access to learning mathematics, both teachers still saw ability as a characteristic, which enabled some learners to do better than others. For Mr Moyo, some learners achieved well because they were "fast thinkers" because they practised mathematics and had "high IQs". He said that slower learners could also succeed when given more time and that "someone who is just intelligent but who does not practise, that person does not become good in maths". For Mr Ncube, high achieving learners were "gifted" and understood concepts by doing few examples, whereas others may have to practise more to achieve the same level of understanding. Even though both teachers discussed ability, they both stressed the importance of effort in enabling all learners to learn mathematics.

For Mr Molefe, Mr Sithole and Mr Ndlovu, ability was key to learning mathematics. These teachers did not think that all learners could learn mathematics, did not assist all learners and offered learners differential identities. For Mr Molefe, learners who did not understand mathematics had "deeper psychological problems". He said, "yes there are incompetencies in maths but those incompetencies are a result or consequence of these psychological factors". He said he was not "competent" to treat psychological problems and he usually referred these learners to the school counsellor. For Mr Sithole, competence in mathematics was a "gift" from "God" and there was "nothing" he could do for struggling learners so he concentrated on high achieving learners. Mr Ndlovu also attributed mathematical competence to how "God" created people.

> Maths was not made for everybody…because when God probably created people, we know in science there are right people and there are left people. There are people that are good in numbers. There are people who are not good in numbers.

He said that he did not help struggling learners and would advise them to get help from another teacher or drop mathematics as a subject in school.

**Learners' mathematical identities**

Learners identified teacher practices as the most important factor that influenced their learning of and identification with mathematics. Learners who affiliated when offered social identities of affiliation said they enjoyed learning mathematics and identified themselves as successful or growing learners of mathematics, who participated as full

members of the classroom community. For example, Senzo explained why he affiliated with the social identity offered by Mr Moyo in grade 9.

> I am starting to enjoy maths because of Mr Moyo and I realised that it is important for me and my life, like my future. And because at home I revise everything every day, every after school and morning before I come to school.

However, in grade 10, Senzo's identity changed substantively. Senzo, like other learners who developed their identities in marginalisation, said that he did not enjoy learning mathematics and participated in limited ways, because of Mr Sithole.

> I am demotivated because Mr Sithole is demotivating... He gives you a sum, you try it then he comes and checks and tells you that is rubbish. You know that is demotivating. You know, he doesn't really care.

Learners who complied with the social identities offered participated peripherally. Fred, like other compliant learners, was not entirely comfortable to participate. He said that he did not always participate even though he enjoyed learning mathematics. He explained "well, maybe I am scared that I will get such a comment which will… sometimes teachers talk a lot of things that hurt you inside". He said that Mr Ndlovu did not give learners many opportunities to practice mathematics and that Mr Ndlovu was "full of anger". Even though Mr Ndlovu provided opportunities for Fred to affiliate, Fred chose to become a peripheral member because of his discomfort, indicating that learners' experiences of their teachers' practices shaped how they developed their identities.

## DISCUSSION AND CONCLUSION

In this paper, we proposed a framework to define and research learners' mathematical identities. Drawing on Rodovic et all's. (2018) model of the three ways in which identity is defined in research, our framework shows how identity construction is social, as learners' social identities informed how they positioned themselves and were positioned by others in the classroom; and subjective, as learners' personal identities informed how they identified themselves as learners of mathematics.

We showed how teachers' narratives of who could learn mathematics influenced the social identities offered to learners. Two teachers, who thought that all learners could learn mathematics, offered all learners social identities of affiliation, by emphasising effort. While these two teachers stressed the importance of learner effort, they also attributed success to ability. Research points to the dangers of attributing learner success to ability, with some researchers showing how such teacher views result in inequitable teacher practices and exclusion of learners (e.g. Hodgen & Marks, 2009). The dangers of attributing success to ability is highlighted further in our findings, as three teachers offered learners differential social identities based on their ability to perform well in assessments. Sfard (2009) cautions against the use of marks as labels for competence, as these labels may become an obstacle for learners to construct their identities positively.

We showed how learners' personal and social identities were enacted, as learners agentively participated in the classroom. We also showed how identities were representational and constructed through learner narratives, in terms of how learners viewed themselves personally as learners, and socially, as members of their classroom community (Rodovic et al., 2018). We showed that all learners either identified or complied with the social identities offered, with the teacher being the most important reason for learners' identification with mathematics. Learners who affiliated with the offered identities of affiliation were comfortable to participate in the classroom because of their teachers' practices. Their teachers treated them respectfully and provided them with opportunities to affiliate. Learners who complied with or developed their identities in marginalisation from the offered social identities were not comfortable to participate in and become full members of their classroom community, as influenced by their experiences of their teachers' practices. None of the learners resisted any of the social identities offered. Based on these findings, we conclude that learners' mathematical identities are strongly influenced by their experiences of the social identities offered to them by their teachers.

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# Chapter 4: Relationships between teachers' practices and learners' mathematical identities

## 4.1. Introduction

This chapter builds on the explanatory framework developed in chapter 2, which is briefly expanded on in relation to Radovic et al.'s (2018) three dimensions discussed in chapter 3. This paper describes in detail what each of the social identities offered by teachers entail, and how the three higher-level mechanisms work together for the emergence of learners' mathematical identities as affiliation, compliance, resistance or marginalisation. In this chapter, I discuss two teachers whose pedagogical practices and social relationships with learners were different from each other. This chapter provides answers to the first main research question, in terms of what mathematical identity is and by showing how the concept of identity defined in chapter 2 can be used in researching secondary school learners' mathematical identities.

This chapter further explores the key findings discussed in chapters 2 and 3, relating to the importance of teacher practices in a more detailed manner, thereby providing some answers to the second main research question: how learners and their mathematics teachers contribute to the development of learners' mathematical identities. First, I explain that there are two important dimensions of teacher practice found in mathematics education research, which are distinct from each other and are usually discussed independently from each other. The first is between teachers' actual practices and narrated practices, and the second is between teachers' pedagogical practices and social relationships with learners. This chapter considers both dimensions, the actual and narrated teacher practices and their pedagogical approaches and social relationships with learners.

Second, I relate these dimensions of teacher practice with the explanatory framework to analyse the data. I show how teachers' actual and narrated practices, involving both their pedagogies and social relationships are related to each other. I argue that teachers, through their narrated pedagogical approaches, specify *what* is needed from learners to become affiliating members of their classroom communities, while through their narratives of their social relationships with learners, teachers specify *who* can learn mathematics and become full members of their classroom community. These narratives in turn shape teachers' actual

practices, involving both their actual pedagogies and social relationships with learners, and their actual practices further justifies the teachers' narratives of their practices.

I also show how these teacher practices inform the social identities offered to learners and thereby inform learners' mathematical identities. I provide a detailed analysis of the mathematical identities of seven learners and show that the social identities offered by the teachers informed but did not determine the mathematical identities constructed by learners. Some of the other findings in this paper are: the ways in which the teachers dealt with learner errors influenced learner participation; the teachers predominantly asked type 1 gathering information questions and rhetorical questions; and the learners in this sample did not resist any the social identities offered, possibly because learners are strongly influenced by teacher pedagogies and social relationships. One of the key recommendations made in this chapter is that more research is needed on teacher-learner social relationships, as I have shown that these relationships have important implications for learners to access mathematical knowledge.



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# Relationships between teachers' practices and learners' mathematical identities

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**ABSTRACT**

The field of mathematics education has seen an increase in research focused on the concept of identity, with many researchers focusing on the role of teacher pedagogies in shaping learners' identities and their participation in and engagement with mathematics. In this paper, the actual and narrated practices of two teachers, which incorporate teachers' pedagogical approaches and social relationships with learners, are analysed in relation to how they provided opportunities for the construction of learner identities. The paper draws on data that were collected over two years in the form of videotaped lessons, field notes and audiotaped semi-structured interviews with teachers and learners. Data were analysed using qualitative data analysis techniques. The analysis suggests a relationship between the teachers' actual and narrated practices, with the teachers' practices influencing the kinds of identities constructed by learners in their classrooms, in terms of learners' enjoyment of learning mathematics and their willingness to participate in and become fully fledged members of their classroom community.

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Teachers' actual practices; teachers' narrated practices; learners' mathematical identities; pedagogical approaches; social relationships

## 1. Introduction

Many learners experience learning mathematics in schools negatively, consequently giving up on mathematics when presented with opportunities to do so (Andersson, Valero, & Meaney, 2015; Bartholomew, Darragh, Ell, & Saunders, 2011; Black, Mendick, & Solomon, 2009; Darragh, 2013). While research has provided many explanations for limited engagement with mathematics, learning mathematics is still a challenge for many learners, internationally and in South Africa (Adler, 2017; Black et al., 2009; Grootenboer & Zevenbergen, 2008). In recent years, researchers in mathematics education have turned to the concept of 'identity' to better understand engagement in mathematics classrooms, perhaps as a consequence of what Lerman (2000) calls the 'social turn' in educational research (Bartholomew et al., 2011; Cobb, Gresalfi, & Hodge, 2009; Darragh, 2016). The popularity of the concept of 'identity' stems from its capacity to encompass cognitive aspects such as mathematical reasoning, affective dimensions such as attitudes and dispositions and

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social processes such as social relationships in the classroom (Grootenboer & Zevenbergen, 2008).

While researching identity in mathematics education has become widespread, there are some important difficulties associated with the use of the concept in research. First, the absence of clear definitions of identity is an important concern for many researchers (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019). Without clear definitions, the notion of identity cannot be appropriately applied in research, nor enable researchers to engage in productive conversations to further our understandings of learning (Darragh, 2016). Second, when the term identity is used by researchers, they do not often refer to how learners describe themselves (Cobb et al., 2009), but rather, the identities that learners would need to construct to become members of their classroom communities. Third, most researchers focus on one aspect of teacher practice in relation to learners' mathematical identities, either teachers' pedagogical practices, or teachers' social relationships with learners. There is not much research that relates both actual and narrated teacher practices, involving both their pedagogical approaches and social relationships with learners, to learners' mathematical identities.

To address these difficulties, this paper develops a framework to understand learners' mathematical identities as they interact with their teachers in classrooms, and a model which explicates the relationships between teachers' actual practices, their narratives of practices and learners' mathematical identities. I focus on two mathematics teachers, one in grade 9 and one in grade 10 with seven learners across their two classrooms and I examine the relationships between teachers' actual practices, their narratives of their practices[1] and learners' mathematical identities.

I address the following research questions:

(1) How are teachers' actual practices and their narratives of their practices related?
(2) How do different teacher practices, both actual and narrated, influence how learners construct their mathematical identities?

## 2. Theorizing identities

There are many perspectives on identity in research in mathematics education (Darragh, 2016; Radovic, Black, Williams, & Salas, 2018). For this paper, I draw on the works of Wenger (1998) and Sfard and Prusak (2005) because research shows that their definitions of identity are most commonly used to contribute to the discussion and development of definitions of identity[2] (Darragh, 2016). Wenger (1998) and Sfard and Prusak (2005) define identity differently from each other, but both contend that identity is a negotiated process between individuals and communities. For Wenger (1998), identity is enacted and related to practice, whereas for Sfard and Prusak (2005), identity is representational, and defined as narrative. Both the enacted and the representational dimensions of identity are important as Radovic et al. (2018) argue that a focus on only one of the two dimensions of identity is limiting, so I consider both of their works in this paper.

For Wenger (1998), identity construction is social and related to learning. As a person learns, s/he becomes a different person and is able to identify her/himself more or less with the community of practice in which s/he is learning (Wenger, 1998). The process of identity construction is negotiated between persons and their community and is reified

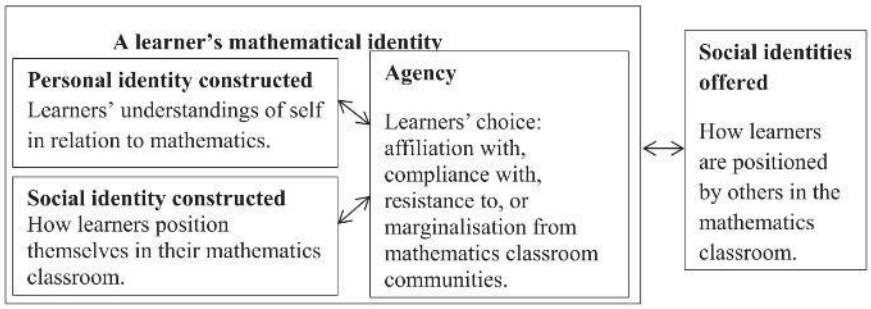



**Figure 1.** Constituents of a learner's mathematical identity in their classroom community.

through practice. As a person engages in practice, they define themselves through the ways in which they experience themselves through participation. The person is also defined by others, which influences the kinds of opportunities presented to the person to learn and become a fully fledged member of the community (Wenger, 1998).

Sfard and Prusak (2005) define identity as a collection of socially shaped stories or narratives about a person, which are reifying, endorsable and significant. Reifying stories involve people narrating who they are, rather than what they can do. Stories are endorsable when narrators declare the authenticity of narratives, while the significance of stories can be determined when changes in stories involve changes in feelings by the storyteller about the person who the story is told about. Narratives can be told about oneself to others and can also be told by significant others, such as stories told by teachers, parents or peers about learners. Significant narratives, which are stories told by significant others, can influence learners' actions and identities (Sfard & Prusak, 2005). While stories may be individually told, they are collectively shaped, meaning that both the individual and the social community shape narratives (Sfard & Prusak, 2005).

For Wenger (1998) and Sfard and Prusak (2005), people position themselves and are positioned by others in communities through engagement with practice (Wenger, 1998) and discourse (Sfard & Prusak, 2005). However, both perspectives overlook the importance of individual agency, which is key to the construction of learners' identities. Drawing on the work of Marks and O'mahoney (2014), I define mathematical identity as being constituted by relationships between personal identity, social identity and agency (Figure 1) in relation to mathematics.

According to Archer (2002), personal identity 'is a matter of what we care about in the world' (p. 15). Personal identity emerges from the embodied and reflexive self, which is forged by individual interests and personal preferences (Marks & O'mahoney, 2014). I extend Archer (2002) and Marks and O'mahoney's (2014) definitions and define personal identity as an understanding of self in relation to mathematics, emerging as individuals engage in mathematical practice and discourse. Learners' beliefs about the nature of mathematics (see Tossavainen, Rensaa, & Johanssona, 2019), their interests, emotions, motivations and future projections of learning mathematics inform their personal identities. Concomitantly, learners' personal identities inform how they learn mathematics, as Archer (2002) argues that personal identities inform what people prioritize and care about most.

Learners also construct social identities, which gives rise to social selves, as they position themselves in their mathematics classroom communities. Marks and O'mahoney (2014) define social identity as being concerned with the embodiment of roles that are made available to people by social structures. Social identity emerges from social relations, involving the kinds of relationships that learners share with others in their mathematics classrooms through practice and discourse. Social identity involves how learners position themselves or would like to be perceived within their mathematics classroom communities, as influenced by their personal identity and by the ways in which they are positioned by others in the classroom community.

In mathematics classrooms, learners are offered certain social identities by their teachers, which may be different from the social identities constructed by learners. Teachers can offer learners social identities of affiliation, by providing them with access to the tools and resources of mathematics to become full members of the classroom community. Teachers can also offer some learners social identities of marginalization, by limiting learner access to the tools and resources of mathematics to become marginal members of their classroom community. Classrooms where all learners are offered opportunities for affiliation with their mathematics classroom communities are those where teachers respect learners, have high regard for learners and believe that all learners can succeed at learning mathematics (Gresalfi & Cobb, 2006). In these classrooms, teachers emphasize that effort, rather than ability, is needed to be successful at learning mathematics (Boaler & Staples, 2008). Classrooms where some learners are marginalized by their teachers are those where teachers attribute success to ability, creating the illusion that not all learners can be successful learners of mathematics (Hodgen & Marks, 2009). Success in these classroom communities are attributed to requiring some form of ability, 'specialness' or 'gift', rather than hard work (Hodgen & Marks, 2009; Mendick, Moreau, & Epstein, 2009).

While learners may be offered certain social identities and opportunities to participate in different ways, learners always have the choice of participating, influenced by their personal and social identities. The notion of agency illuminates how individuals can construct their identities differently from the opportunities offered to them in communities. Exercising agency involves choosing between different forms of intentional action or practice, with social structures enabling or constraining opportunities for people to exercise agency (Eteläpelto, Vähäsantanen, Hökkä, & Paloniemi, 2013; Gresalfi, Martin, Hand, & Greeno, 2009; Norén, 2015). Agency is always exercised and may differ depending on social contexts (Gresalfi et al., 2009). Agency is socially shaped, since how learners participate in their classroom communities, who they participate with and how they learn mathematics, depends on learners' personal and social circumstances.

Experiences of exercising agency concomitantly influence learners' personal identities, as they learn more about themselves, their beliefs, interests, aspirations and motivations; and their social identities, as they evaluate their membership in the community (Figure 1). This means that learners' identities are constantly being developed as they learn and participate in their classroom communities. As learners enact their identities by interacting with teachers, teachers can continue offering them the same social identities as previously offered or may offer learners different social identities from those offered in the past. In what follows, I discuss how I use the concepts of personal identity, social identity – both offered and constructed – and agency, in order to understand the different kinds of

mathematical identities that learners can construct as they interact with their teachers in their mathematics classrooms.

### 2.1. A framework for mathematical identities

Wenger (1998) describes identity as a nexus of multimembership. As learners engage with mathematics in different contexts, either on their own or with teachers, peers or significant others, they develop different kinds of membership in the different communities, which can be reconciled into a single identity. In other words, learners' engagements in different communities of practice may vary, since the repertoires, regimes of accountability, tools and resources of one community may differ from other communities of practice. Together, all the different forms of participation and membership in different communities can be reconciled into one nexus (Wenger, 1998). For this paper, I use the term mathematical identity to refer to a specific kind of identity that learners construct as they interact with their mathematics teachers when learning mathematics in their mathematics classroom communities.

In the field of mathematics education, research has identified three ways in which learners can identify with teachers and activities: by affiliating with teachers and activities; by complying and merely cooperating with teachers; or by resisting teachers and activities (Boaler & Greeno, 2000; Cobb et al., 2009). How learners identify with teachers and activities can be associated with the ways in which learners participate in their classroom communities. There are different kinds of participation in communities – full participation, peripheral participation and marginality – with the different kinds of participation being related to the different identities produced (Wenger, 1998). Full participation is distinguished from peripheral and marginal participation, since the latter two forms of participation involve different degrees of non-participation (Wenger, 1998). Peripheral participation involves a degree of non-participation, 'to enable a kind of participation that is less than full', which can potentially lead to the development of peripheral identities, and in some situations, full membership over time (Wenger, 1998, p. 165). Marginality 'prevents full participation', which may potentially lead to the development of non-membership or marginal identities (Wenger, 1998, p. 166).

To build a framework for mathematical identities, I have drawn on Wenger's (1998) work on participation, Boaler and Greeno's (2000) and Cobb et al.'s (2009) work on how learners identify with teachers and activities, and Marks and O'mahoney's (2014) work on personal identity, social identity and agency. I show how different mathematical identities can be constructed by learners, as influenced by the different social identities offered to them by teachers. The different kinds of mathematical identities constructed by learners are also influenced by learners' relationships with mathematics, in terms of how learners view mathematics as a body of knowledge and how they engage with mathematics in communities (Tossavainen et al., 2019). In this paper, learners' relationships with mathematics are discussed in relation to agency, i.e. how learners participate with the tools and resources of mathematics which are made available to them by their teachers in their mathematics classroom communities.

Table 1 illustrates the framework developed to understand learners' identities, by illustrating the different kinds of mathematical identities which can be constructed by learners

**Table 1.** Offered and constructed identities.

| Social identities offered | Learners' mathematical identities |
|---|---|
| Affiliation with the classroom community | Affiliation with the classroom community<br>Compliance with the classroom community<br>Resistance to the classroom community |
| Marginalization from the classroom community | Marginalization from the classroom community<br>Compliance with the classroom community<br>Resistance to the classroom community |

when they are offered social identities of affiliation with or marginalization from their classroom communities by their teachers.

When learners are offered social identities of affiliation with their classroom community, they may construct their mathematical identities in affiliation with, compliance with or resistance to the social identity offered to them by their teachers (Table 1). When learners meaningfully engage with the tools and resources of the mathematics classroom community, they are constructing their mathematical identities in affiliation with their classroom community. Affiliating learners tend to be emotionally invested and interested in learning mathematics, for their personal identities, and these learners develop their social identities as full members of their classroom community.

When learners engage peripherally with the tools and resources of the mathematics classroom community, by cooperating with teachers without any meaningful engagement, they are constructing their mathematical identities in compliance with their classroom community. Compliant learners are not invested in developing their social identities as full members of their classroom community and may not be personally interested in learning mathematics. When learners choose not to engage with the tools and resources of the mathematics classroom community, or choose an alternative way of participating which is not in compliance with the culture of the mathematics classroom community, they are constructing their mathematical identities in resistance to their classroom community. Resistant learners may not see the use of mathematics for their lives and futures, and these learners do not develop their social identities as members of their classroom community.

When learners are offered social identities of marginalization from their classroom community, they can construct their mathematical identities in marginalization from, compliance with or resistance to the social identity offered to them by their teachers (Table 1). Learners can construct their mathematical identities in marginalization from their classroom community by choosing not to participate with the tools and resources of mathematics in their classroom as a full participant. These learners may no longer see any personal use in learning mathematics and may not wish to develop their social identities as members of their classroom community.

Sometimes, learners who are offered social identities of marginalization by their teachers may still want to develop their social identities as members of the community. Such learners can construct their mathematical identities in compliance with or resistance to the social identity of marginalization offered. Constructing mathematical identities in compliance with the classroom community when offered a social identity of marginalization can happen by learners choosing not to participate much with the teacher, but these learners may try to access assistance from other sources. Compliant learners develop their social identities as peripheral members of their classroom community, perhaps because they wish

to pass mathematics or pursue mathematics in the future, for their personal identities. Learners can also construct their mathematical identities in resistance to the classroom community when offered a social identity of marginalization, by attempting to use the tools and resources of mathematics to develop their social identities as full members of the community. These learners may be emotionally invested and interested in learning mathematics, for their personal identities, and may feel the need to be a member of their classroom community.

Resisting offered social identities of affiliation with classroom communities does not mean that learners are marginalized by others. Concomitantly, resisting offered social identities of marginalization from classroom communities does not mean that learners are affiliating with an offered social identity of marginalization. Learners who resist an offered social identity of affiliation *marginalize themselves* from the classroom community but are not marginalized by the community if their teachers provide them with access and support to learn mathematics. Likewise, learners who resist an offered identity of marginalization cannot become full participants and affiliating members of the classroom community, unless they are offered a social identity of affiliation with the classroom community.

## 3. The influence of teacher practices on learners' mathematical identities

### 3.1. Teachers' actual practices and learners' mathematical identities

What learners learn and how they develop their mathematical identities are closely linked to learners' experiences in the mathematics classroom (Boaler & Greeno, 2000; Boaler & Staples, 2008; Cobb et al., 2009). Different teacher pedagogies have been shown to provide learners with access to learning mathematics in different ways. Boaler and Greeno's (2000) ground-breaking analysis illustrates that different kinds of mathematical identities were developed by advanced placement (AP) calculus students as a consequence of their experiences of reform and traditional pedagogies. Reform pedagogies are characterized by learners engaging collaboratively with others through open-ended problem-solving activities and have been shown to provide learners with better opportunities to participate, develop their reasoning, engage in meaning-making and envision mathematically orientated future projections, thereby offering learners opportunities to construct their mathematical identities in affiliation with mathematical communities of practice (Boaler & Greeno, 2000; Boaler & Staples, 2008; Cobb et al., 2009; Norén, 2015).

While reform pedagogies are the aim of many educational reform projects, traditional pedagogies still dominate classroom instruction, both in South Africa and internationally (Brodie, Jina, & Modau, 2009; Heyd-Metzuyanim, Tabach, & Nachlieli, 2015; Lloyd, 2018; Norén, 2015; Umugiraneza, Bansilal, & North, 2017). Traditional pedagogies are characterized by textbook-based learning with teacher-dominated practices, which are argued to limit the development of learner thinking and meaning-making, with learners often feeling like passive recipients, marginalized, anxious, bored and frustrated, since knowledge is presented in decontextualized ways (Andersson et al., 2015; Boaler & Greeno, 2000; Solomon, 2007).

There is a tendency in research to define traditional pedagogy as a single approach, which negatively influences learning and identity. However, there are different kinds of traditional pedagogies, some of which may encourage learner understanding,

meaning-making and thinking to different extents. Boaler and Brodie (2004) developed a framework for analysing teacher questions, which can be used to determine the extent to which traditional pedagogies encourage learner thinking in whole-class interactions. The framework is composed of nine questioning categories, derived from an analysis of practice from seven teachers in three schools, who taught either using traditional or reform pedagogies. These nine categories are: gathering information and leading students through a method; inserting terminology; exploring mathematical meanings and/or relationships; probing and getting students to explain their thinking; generating discussion; linking and applying; extending thinking; orienting and focusing; and establishing context. Gathering information and leading students through a method involves less learner thinking than the other eight categories, which can require deeper thinking from learners.

In recent years, research considering the importance of supportive social relationships as part of teacher's actual practices in classrooms has grown, with many studies illustrating the importance of supportive social relationships for learning (Bibby, 2009; Cornelius-White, 2007; Gardee, 2019; Hodgen & Marks, 2009; Rytivaara & Frelin, 2017). Listening to learners, placing value on their contributions and creating environments of belonging are shown to enable learners to participate in and construct their mathematical identities in affiliation with their classroom community (Bibby, 2009; Darragh, 2013). For example, Bibby (2009) showed how some learners felt valued by their mathematics teacher, who provided them with opportunities to affiliate with his classroom community, in comparison to a student on teaching practice who marginalized learners and did not value their contributions. Developing supportive classroom environments through strong social relationships between teachers and learners can provide learners with opportunities to construct their mathematical identities positively. These kinds of relationships can occur in classrooms where learners experience traditional or reform pedagogies.

### 3.2. Teachers' narrated practices and learners' mathematical identities

With the exception of Heyd-Metzuyanim's (2013) study, research on teacher narratives predominantly focuses on teachers' identities, rather than accounting for how teachers' narrated practices influence learners' mathematical identities. In her study, Heyd-Metzuyanim (2013) showed how her narrative of labelling a struggling learner as 'clueless' shaped the kinds of opportunities that the learner, called Dana, experienced to learn mathematics and develop her identity as a member of the classroom community. Heyd-Metzuyanim's (2013) narrative relating to Dana contributed to her actual pedagogical practices, as she employed a traditional pedagogical approach only for Dana, which was different from the discussion-based pedagogy that she employed for the rest of the class and which contributed to the development of what the author called Dana's 'identity of failure'. While Heyd-Metzuyanim's (2013) narrative of Dana shaped her interactions with her, her pedagogical practices further contributed to Dana's difficulty to learn mathematics, since she limited Dana's opportunities to engage as a full participant in her classroom.

In Heyd-Metzuyanim's (2013) study, Dana was marginalized from the rest of the classroom, as influenced by her teacher's pedagogical practices, but it is important to note that Dana did not identify herself as a marginalized learner. In her narrative, Dana did not identify herself as a successful learner of mathematics, but she said that she could understand mathematics when provided with assistance from others. She exercised agency and resisted

the identification offered by the teacher during interactions, by participating actively, even though her answers were incorrect, and by pretending to understand the teacher's explanations, even though she might have not understood them. For Dana, it seemed that belonging to and participating in the classroom community as a full participant was important for her identity (Heyd-Metzuyanim, 2013), even though she was offered an identity of marginalization. It is therefore important to consider the role of agency for the development of learners' mathematical identities, as Dana might have not identified herself in the same way as her teacher did.

The research discussed in sections 3.1 and 3.2 show how teachers' actual and narrated practices can provide learners with opportunities to construct their mathematical identities in affiliation with or marginalization from their classroom communities. Most of the research discussed focuses on teachers' actual practices or their narratives of their practices in relation to the social identities offered to learners. There is not much research that relates both actual and narrated teacher practices, involving both their pedagogical approaches and social relationships with learners, to the mathematical identities constructed by learners.

## 4. Methods

### 4.1. Participants

Data for this paper are taken from a study that focuses on understanding learners' mathematical identities as they transitioned from grade 9–10 in a well-resourced secondary school in South Africa. The school population is reflective of the demography of the country and is diverse. In total, 19 learners participated in the study with their five mathematics teachers. This small group of learners was selected to gain in-depth understandings of how they constructed their mathematical identities in their classrooms.

For this paper, Mr Moyo (all names are pseudonyms) who taught grade 9 with four learners in his classroom and Mr Ndlovu who taught grade 10 with three learners in his classroom, are discussed. Both teachers displayed different teacher practices from each other and provided different reasons through their narratives to account for why they thought that their practices were most suitable for promoting learning in their classrooms. A total of 480 min of lessons were analysed for each teacher. Both teachers had academic degrees to teach mathematics. Mr Moyo was recently qualified at the time of data collection, and Mr Ndlovu had eight years of experience teaching mathematics.

Learners in the study were initially selected by their grade 9 teachers using the following criteria: one high-performing learner, one low-performing learner and the person seated next to them. The purpose of selecting learners according to these criteria was so that I could observe teacher and learner interactions with learners displaying different levels of

**Table 2.** Selected learners.

| Teachers' pseudonyms | Mr Moyo (grade 9) | Mr Ndlovu (grade 10) |
|---|---|---|
| High-performing learner | Shane | Edward |
| The person sitting next to the high performing learner | Jack (high-performing) | Fred (high-performing) |
| Low-performing learner | Senzo | Sunny |
| The person sitting next to the low-performing learner | Lane (average-performing) | No partner |

achievement. The same learners were observed when they were in grade 10 and additional learners, such as learners who were seated next to the initial participants, were selected to observe teacher and learner interactions. Table 2 provides the pseudonyms of learners in each teacher's classroom and their mathematical achievement, according to their teachers.[3]

### 4.2. Data collection

Data were collected in the third school term (August–September) in 2015 and 2016 – for two weeks in 2015 (grade 9) and four weeks in 2016 (grade 10). For the purpose of understanding teachers' practices as actualized in the classroom, learner participation and the kinds of social identities offered to learners, data were collected in the form of videotaped lessons and field notes. During the lessons, the video camera was placed in the front of the classroom and focused primarily on learners to capture their participation and how the teachers interacted with learners. Audio recorders were placed on learners' desks to capture their interactions, which was often difficult to hear on the video camera. Field notes were taken to supplement the recordings by giving context and noting important incidents that were not captured by the recordings, and the audio from the video cameras enabled understandings of how the teachers structured the lessons and the kinds of questions that they asked.

To understand the teachers' reasons for their practices and the social identities offered to learners from their perspectives, I conducted audiotaped semi-structured interviews with teacher participants. I asked the following types of questions during the teacher interviews: how would you describe your pedagogical approach?; how do you see [name of a learner participant] as a learner in your classroom?; how does [name of a learner participant] participate in the classroom?; and do you think that [name of a learner participant] is good at doing mathematics?

To understand learners' mathematical identities in relation to the social identities offered by their teachers, I conducted audiotaped semi-structured interviews with learners. I asked the following types of questions during the learner interviews: how do you see yourself as a mathematics learner?; how do you experience learning mathematics this year?; can you please describe to me what your mathematics lessons are like; how does your mathematics teacher make you feel when you are sitting in your mathematics classroom?; how do you participate in your mathematics classroom?; is mathematics necessary for your future career or sense of being?; and how does your teacher see you as a learner of mathematics? All questions asked during the interviews were followed by further probing questions, aimed at supporting interviewees to give more detailed explanations.

### 4.3. Data analysis

Qualitative approaches were used to analyse data. Data from lessons and interviews were transcribed, coded and analysed thematically using open coding (McMillan & Schumacher, 2010). Data from lessons were coded according to the extent to which the teachers' pedagogical and social relationships in the classroom provided learners with access to learning mathematics. For teachers' actual pedagogical relationships, I coded how the teachers' structured their lessons, in terms of the time allocated to classroom tasks and the teachers' questions, utterances and prompts in whole-class situations. To categorize the

**Table 3.** Number and type of questions asked by teachers.

| Question type | Number of questions asked by Mr Moyo | Number of questions asked by Mr Ndlovu | Examples |
|---|---|---|---|
| 1. Gathering information and leading students through a method | 227 | 334 | Mr Ndlovu: Look at the mean. The mean is?Mr Ndlovu: What is the average?Mr Ndlovu: What is our maximum? |
| 2. Exploring mathematical meanings and/or relationships | 4 | 0 | Mr Moyo: Now this is how we calculated intercepts using the table. How do we write it in co-ordinate form? |
| 3. Probing, getting students to explain their thinking | 24 | 19 | Mr Moyo: Why are we saying three times zero plus six? Why? |
| 4. Generating discussion | 7 | 42 | Mr Ndlovu: Information and data. Somebody else? What is statistics? What do you understand when you talk about statistics? |
| 5. Establishing context | 0 | 3 | *When explaining to learners the link between statistics in mathematics and planning for an Olympics*.Mr Ndlovu: Where were the Olympics? They still on neh?Mr Ndlovu: Where are they?Mr Ndlovu: In Rio. Where is Rio? |
| 6. Rhetorical question | 56 | 144 | Mr Ndlovu: Today is Wednesday. Listen can you say that what am I given? The condition is today is Wednesday.Mr Ndlovu: Probability that tomorrow is Thursday? It's definite. It's certain. |
| Total | 318 | 542 | |

extent to which teacher questions encouraged learner thinking, I transcribed teacher questions, including utterances and prompts that served the purpose of questions, using Boaler and Brodie's (2004) framework (see Table 3). After coding each question, I counted each question type and created tables to compare the frequency across teacher lessons. For the teachers' actual social relationships, I coded the interactions between teacher and learner participants. I focused on whether learners were provided opportunities to participate and how each teacher interacted with each of the learner participants, in terms of whether the teacher ignored the learner, treated the learner with respect and assisted the learner when the learner required help, enabling me to identify whether learners were offered identities of affiliation with or marginalization from their classroom communities.

Observing learner participation in the classrooms also enabled me to understand how learners constructed their mathematical identities, in relation to the social identities offered to them by their teachers. I coded learner participation by noting whether learners participated with their mathematics teacher and other learners by asking questions, answering questions, discussing their solutions with others, demonstrating their answers on the board when provided with the opportunity to do so, assisting other learners in the classroom and completing their tasks.

The data from the semi-structured interviews were used to further explain the findings from the lesson observations, in terms of why the teachers chose to offer learners certain identities and why learners constructed their mathematical identities in certain ways. Data from the semi-structured interviews with teachers were used to understand teachers' narrated practices. For teachers' narrated pedagogical approaches, I coded how the

teachers described their pedagogical approaches and why they thought that their pedagogies promoted learning in their classrooms. For the teachers' narrated social relationships with learners, I coded how the teachers identified each learner participant and the reasons provided by the teachers for interacting with different learners in certain ways.

Interviews with learners were used to understand their personal identities, social identities, their experiences of the social identity offered to them and how they exercised agency. For learners' personal identities, I coded how learners identified themselves as mathematics learners, how they discussed their emotions relating to their learning of mathematics and the use (or not) of mathematics for their lives and futures. For the social identities constructed by learners, I coded how learners positioned themselves in their classrooms, in terms of their achievements in assessments. For learners' experiences of the social identities offered to them by their teachers, I coded how they experienced their teachers' actual practices. I coded their narrated experiences of participation in terms of how and with whom they participated in the classroom to learn mathematics. I then used the framework discussed earlier in the paper to understand how learners constructed their mathematical identities in relation to the social identities offered to them by their teachers.

To provide credibility in my findings, I continuously compared data from lesson observations, teacher interviews and learner interviews. I also re-watched the videotaped lessons multiple times and created tables to match each claim made in the findings section with excerpts from the interviews and videotaped lessons.

## 5. Findings

In this section, I begin by showing the number and types of questions asked by Mr Moyo and Mr Ndlovu to provide the reader with an understanding of the kind of thinking required from learners through teacher questions in the classroom. I then describe Mr Moyo and Mr Ndlovu's actual practices and narrated practices and I show that there is a relationship between the teachers' actual practices and narrated practices, with the teachers' actual practices influencing the kinds of identities constructed by learners in their classrooms.

### *5.1. Teacher questions*

From Boaler and Brodie's (2004) nine categories used to classify teacher questions, I found that five of the nine categories were evident in the data. These five categories were: gathering information and leading students through a method; exploring mathematical meanings and/or relationships; probing and getting students to explain their thinking; generating discussion; and establishing context. Even though the latter four question types in the framework can require somewhat deeper thinking from learners, I found that the teachers in my study used these question types in ways that did not require much learner thinking, a finding representative of the larger sample. A new code, which I called rhetorical questions emerged from the data, and I define rhetorical questions as non-interactive questions, where teachers answer the questions they ask, rather than allowing learners to respond to the questions. The reason why I coded rhetorical questions as a question, rather than as a teacher explanation was because these non-interactive questions were in the form of interactive questions and had the potential to elicit contributions from learners had the teacher

not answered the questions. Table 3 illustrates the number and types of questions asked in a whole-class situation by each teacher, with examples from the data.

The total number of questions asked by each teacher can be attributed to the amount of time each teacher spent interacting with learners in relation to the time provided to learners to complete tasks on their own or in pairs. Mr Ndlovu did not usually allocate much time to classroom tasks, which is why he asked so many whole-class questions. Mr Moyo allocated a lot of time in his lessons for classroom tasks, which is why he asked fewer whole-class questions than Mr Ndlovu. Table 3 further illustrates that the main types of questions asked by both teachers are of type 1 and 6, constraining learner opportunities to develop their thinking, a finding representative of the larger sample. The other types of questions illustrated in Table 3 were used by the two teachers sparingly and did not often require deeper thinking from learners, a finding which is also representative of the larger sample.

### 5.2. Mr Moyo's actual and narrated teacher practices

Mr Moyo's pedagogical approach was characterized by explanations involving sequences of interactions, typically characterized by type 1 and 6 questions. Thereafter, learners were provided with time to complete classroom tasks from their textbooks, either individually or with the person sitting next to them, a choice Mr Moyo gave to learners. Mr Moyo invited participation in the classroom from all learners, encouraged learners to ask questions if they did not understand, was accommodating of learner errors and was willing to help them at any time during the lesson. At the end of each lesson, Mr Moyo provided learners with the correct answers to the classroom tasks and he often offered different learners opportunities to demonstrate their solutions on the board.

In his narrative, Mr Moyo described his pedagogical approach as one where he provided learners with many opportunities to practice mathematics using classroom tasks.

> Mr Moyo: For their learning, I think it is very important because maths is a practical subject and with a practical subject, the more you are involved the more the logic, the concepts get into them. So let's have them practice more.

He said that he noticed that the more learners practiced, the better they understood the work. Classroom tasks assisted him to identify the challenges learners experienced so that he could help them. He said that he encouraged learners to think and solve problems for themselves because when learners face obstacles and challenges, their minds expand.

> Mr Moyo: What I think is it is very crucial to give learners an opportunity to work out the problems themselves. To find out what is problematic and to try and find solutions. By that, I introduce a concept and I give them examples to work out. Even today when I gave them questions, some of them were saying no Sir I think you must explain to us first before work because it's difficult. But then I was saying for me what is important is for you to see the challenge and your mind expands if you face an obstacle.

Assessment was also important to him to measure the success of his pedagogical approach, as he said 'we teach to assess'.

Mr Moyo provided learners with some opportunities to think for themselves, rather than being dependent on him for answers. However, opportunities to develop thinking through classroom interactions were limited, since Mr Moyo mainly asked type 1 and 6 questions, as illustrated in Table 3.

As for social relationships with learners in the classroom, Mr Moyo helped all learners in the classroom and often encouraged learners to complete their tasks. For Mr Moyo, all learners could succeed in learning mathematics if they practiced and were motivated. Mr Moyo explained:

Mr Moyo: I think generally grade 8 and 9, they just need so much motivation that maths is not difficult and it's a subject that is interesting that opens doors in life.

When discussing why he thought it was important to motivate struggling learners, he said:

Mr Moyo: I think those learners ... what I think is they are not really incompetent but they may just not have motivation, not have really caught up with the subject, what it is about. The fact that to a point where they get that interest, you may actually find out that they are not incompetent but they never got really properly orientated with the subject.

In some lessons, during the time allocated to classroom tasks, Mr Moyo walked to every learner's desk to see if they needed help and would go to learners' desks if they called him. Mr Moyo said that if he noticed that some learners were still not succeeding, he encouraged them to come for help during breaks, after school or to the extra classes that he offered them. Through Mr Moyo's teaching practices, he provided all learners with the opportunity to participate in the classroom and construct their mathematical identities in affiliation with his classroom community.

All four learners chose to construct their mathematical identities in affiliation with the social identity offered by Mr Moyo by participating and developing their social identities as full members of the classroom since learning mathematics was enjoyable and meaningful to them. Shane, a high-achieving learner participated in the classroom by asking questions, answering questions, discussing his solutions, demonstrating his answers on the board, assisting other learners in the classroom and completing his tasks. He described his personal identity by explaining that he viewed himself as a good learner of mathematics, with mathematics being his favourite subject. He was motivated to pursue mathematics in the future and he viewed mathematics as a 'hobby'.

Shane: My favourite subject is maths definitely ... doing maths ... ah let's see, doing maths ... I am doing something enjoyable.

When describing his social identity, Shane said that he was performing well in assessments and that he wanted to be the best mathematics learner in his classroom. Shane experienced learning mathematics in Mr Moyo's classroom positively, as he said that Mr Moyo

allowed learners to approach problems using different strategies and was always willing to help them understand mathematics. When narrating his experiences of the social identity offered to him, Shane described Mr Moyo as a caring teacher who saw potential in him and who would make sure that learners understood what was taught before moving on with the syllabus.

> Shane: It makes me feel nice because it makes me feel good because I see this teacher, yeah he just wants to get his children to pass. He cares about us.

Like Shane, Jack, a high-achieving learner also enjoyed learning mathematics, said he 'love(d)' mathematics and was willing to pursue mathematics in the future, as indicative of his personal identity. He participated actively with Mr Moyo and other learners in the classroom by asking questions, answering questions, discussing his solutions, demonstrating his answers on the board, assisting other learners in the classroom and completing his tasks. He described his social identity by saying that he was one of the most successful learners of mathematics in the grade and was competitive. He found some mathematical topics challenging to learn, but he felt that Mr Moyo was helpful in enabling him to understand because he provided learners with time to grasp concepts, used different teaching strategies and allowed learners to demonstrate their work on the board. When narrating his experiences of the social identity offered to him, Jack described Mr Moyo as a teacher who was a 'soft person', who made him feel 'welcome and comfortable' in the classroom, who cared about them and who would explain for all learners to understand.

> Jack: Cos like if you say you never understood or asked a question, he will like explain more in detail for everyone to understand.

Lane, an average-achieving learner described his personal identity by saying that he 'loves' mathematics, was excited to learn mathematics and was willing to pursue mathematics in the future.

> Lane: I am excited, Mam. When I . . . at first when it was like maths period, the children would say yoh, that maths teacher, she's boring. But now they excited about going to the maths period and things. You know that Mr Moyo cares about them if like you want to . . . if you want help, he helps Mam.

He described his social identity by identifying himself as 'more than an average learner', who was achieving good marks in his classroom. He participated in his classroom community by doing his classroom tasks, working with other learners in the classroom and would sometimes shout out answers, chorus answers and volunteer to answer questions. When narrating his experiences of the social identity offered to him, Lane described Mr Moyo as the 'best teacher' that he ever had for mathematics and as an approachable teacher who explained well, helped them understand and allowed them to make mistakes. Mr Moyo was seen as a teacher who saw potential in him, was motivating and polite and who worked through the syllabus slowly to ensure that they understood the work.

Senzo, a low-achieving learner, described himself as a learner who was 'focused and serious' about learning mathematics in his classroom, and as a learner who saw the relevance of mathematics for his life, as indicative of his personal identity.

> Senzo: Now maths is for me, like I think of it as a subject that I have to do. You can't escape from maths in your life.

He said that he was beginning to enjoy learning mathematics and was motivated to pursue mathematics in the future.

> Senzo: I am starting to enjoy maths because of Mr Moyo and I realised that it is important for me and my life, like my future.

When describing his social identity, Senzo said that he was not good at learning mathematics, but his marks were beginning to improve because he participated in the classroom and was revising mathematics every day at home. He participated in the classroom community by asking questions, answering questions, discussing his solutions, demonstrating his answers on the board, assisting other learners and completing tasks. His enjoyment of learning mathematics was linked to the social identity offered by Mr Moyo, who he described as the 'best teacher' he had. He experienced Mr Moyo as a helpful teacher, who was caring, motivating, concerned about him, approachable and who treated learners equally.

> Senzo: He (Mr Moyo) makes me feel like I am great. I can achieve more, like I can do better.

He said that he enjoyed 'everything' about the way Mr Moyo taught them in the classroom.

### 5.3. Mr Ndlovu's actual and narrated teacher practices

Mr Ndlovu's pedagogical approach involved sequences of interactions with learners, through type 1 questions, followed by lengthy explanations involving no interactions with learners. Mr Ndlovu did not usually provide learners with opportunities to engage with classroom tasks. When starting new topics, he usually invited participation in the classroom by asking learners questions about what they knew about the topic through type 4 questions, without engaging with their ideas. When explaining concepts, he lectured learners, using type 6 questions and by giving multiple examples without inviting much learner interaction. Thereafter, he spent the rest of the lesson doing examples on the board, through the use of type 1 questions. For Mr Ndlovu, answers were either right or wrong and he did not encourage learners to use other methods but the ones he taught. He was not accommodating of learner errors and would sometimes get angry when learners made errors, since he said that he thought that errors cost lives in the workplace.

In his narrative, Mr Ndlovu described himself as a teacher who did not assume that his learners 'understand anything'. He described his pedagogical approach as follows:

Mr Ndlovu: We enforce things to learners to say you have to master this. So you make sure the basics everyone understand, number one. You move on, you make sure they have written formulas. Number two, you make sure that learners recite, that they can recite to a friend. Look at your friend, recite what you have done, recite, recite, recite. That is a method of them sort of memorising and mastering it, they are mastering it. So we do that, I take my time, I don't rush them because I know it's a very important concept. So I might take the whole lesson cultivating on the formula, on where to use the formula and where to use the formula, the difference of the formula so that they have mastered it. Then I can move to the next lesson.

Mastering formulas was central to Mr Ndlovu's pedagogical approach. He further explained why he thought that mastery was important to learning mathematics:

Mr Ndlovu: Maths is all about mastering concepts. Maths is like law, if you don't know concept of laws, in the kind of the law you are doing, definitely you understand certain things. If it is maths principles, you just need to know them because maths principles are the ones that govern you. There is nothing new and everything has been discovered. We use what has been discovered to work our maths like laws of exponents, like Pythagoras theorems, like Euclidean theorems. A learner had to know and master a concept first before he manipulates and applies the concept.

Teaching to the test so that learners were equipped with knowledge to answer questions and mastering formulae were central themes defining how Mr Ndlovu taught. He said that he exposed his learners to different question papers and his high-achieving learners often did very well in their tests. Mr Ndlovu did not encourage much learner thinking in his classroom and believed that the purpose of learning mathematics was to master the concepts and formulae of mathematics, which have already been discovered.

As for Mr Ndlovu's social relationships with learners, Mr Ndlovu treated high-achieving learners like Edward and Fred differently from low-achieving learners like Sunny. He said that 'maths was not made for everybody'. When asked to elaborate, he said:

Mr Ndlovu: Because when God probably created people, we know in science there are right people and there are left people. There are people that are good in numbers. There are people who are not good in numbers. There are people who are good in figures. There are people who are not good in figures. That is the way it is.

He said that he did not help struggling learners.

Mr Ndlovu: You can see consecutively that this learner is not improving so you just leave a learner like that.

He said that he advised struggling learners to ask other teachers for help or to drop mathematics as a subject in school.

In his class, high-achieving learners like Edward and Fred were provided opportunities to construct their mathematical identities in affiliation with the classroom community, whereas low-achieving learners like Sunny, were offered social identities of marginalization from the classroom community. Edward developed his mathematical identity in affiliation with his classroom community and was willing to participate in the classroom. He participated in the classroom with Mr Ndlovu by asking questions, answering questions, demonstrating his answers on the board and completing his tasks. He described himself as a 'good' learner who enjoyed learning mathematics, since mathematics was his favourite subject and he wished to pursue a mathematically orientated degree in university, as indicative of his personal identity.

> Edward: I enjoy maths, like in the class. Even though many people don't enjoy maths like it's a challenge that they have to solve so it's interesting and also I don't have to put a lot of effort since I listen in class and yeah, unlike other people that have to put a lot of effort. Like sometimes I don't go for class, like extra class and yeah. It's quite easy in a way.

He stated that he did not have to put as much 'effort' into learning mathematics in comparison to other learners because he had a 'good memory'. He described his social identity by saying that he was competitive, was performing very well in assessments in his classroom, and felt 'good' about participating in the classroom. Edward said that he enjoyed Mr Ndlovu's lessons because Mr Ndlovu 'maintained discipline' and interacted with him in the classroom, which enabled him to get good marks. The only concern he had with Mr Ndlovu's choice of pedagogy was that Mr Ndlovu did not allow the use of different methods and would penalize them in tests if they tried to approach a problem differently. When narrating his experiences of the social identity offered to him, Edward explained that he enjoyed the kind of social relationship he shared with Mr Ndlovu because Mr Ndlovu corrected him when he made mistakes.

Like Edward, Fred was also identified by Mr Ndlovu as a high-achieving learner and was provided with opportunities to participate in the classroom. However, Fred completed his tasks but only participated with Mr Ndlovu when called by name. In his narrative, Fred described his personal identity by saying that he saw himself as a successful learner of mathematics because he understood mathematics. He was motivated to learn mathematics and pursue mathematics in the future and said that he was trying to be the 'best' that he could be at learning mathematics. When narrating his experiences of the social identity offered to him, Fred explained that Mr Ndlovu displayed much interest in him and his performance in assessments. However, Fred did not develop his social identity as a full member of his classroom as he had less favourable experiences of learning mathematics as a consequence of how Mr Ndlovu taught and treated learners in the classroom when they did not understand. Fred said that Mr Ndlovu did not explain as well as his grade 9 teacher or provide them with much time to engage with classroom tasks. When comparing his grade 9 teacher to Mr Ndlovu, he said:

> Fred: Mr Ndlovu can't explain like how Mr [name of grade 9 teacher] used to. Well [name of grade 9 teacher], Mam, he used to emphasise more on practising and

doing it most of the time and if you didn't understand you could come to him and tell him I don't understand this and he would try to help you.

He further described Mr Ndlovu as a teacher who would get angry with learners if they did not understand:

Fred: He's getting angry when you don't understand ... it makes me feel as if he doesn't understand that sometimes, things take time to sink in.

As a consequence, Fred said that he asked his parents, who were both good at mathematics to help him. Even though Fred was provided with the opportunity to construct his mathematical identity in affiliation with his classroom community, Fred chose to construct his mathematical identity in compliance with his classroom community by participating only when called by name, thereby developing peripheral membership in the classroom community. Fred constructed his mathematical identity differently from Edward since understanding mathematics was more important to him than memorizing formulas. He said that he participated with mathematics out of school by practising at home to understand mathematics.

Unlike Edward and Fred, Sunny was identified as a low-achieving learner by Mr Ndlovu and was offered a social identity of marginalization from the classroom community. Sunny identified himself as a learner who did not see himself as a mathematics learner and who wanted to become a mechanic in the future, as indicative of his personal identity.

Sunny: I don't see myself as a mathematics learner at all.

He did not complete his tasks and sometimes doodled instead of listening to Mr Ndlovu's explanations. He also did not attend some of his mathematics lessons, even though he was present in school. In his narrative, Sunny said that he did not participate in the classroom because he did not 'feel like participating in class', even though I observed him participating, by answering few questions in some lessons. He described his social identity by saying that he was performing 'poorly' in his classroom since he was getting 'very low marks for maths'. He experienced Mr Ndlovu as a teacher who did not use many strategies to teach mathematics or allow them the opportunity to approach problems using different formulae. When narrating his experiences of the social identity offered to him, he described Mr Ndlovu as an 'impatient' and 'aggressive' teacher, who did not assist him when he did not understand what was taught. He said that Mr Ndlovu saw him as a 'pathetic' learner. When discussing how Mr Ndlovu treated him, Sunny said:

Sunny: He treats me in a way that ... he told me stuff like some people like me don't belong in his class and stuff like that because he only teaches excellent learners.

He also said that he cannot learn in his classroom environment, which he described as a 'negative environment'. As a consequence, Sunny constructed his mathematical identity in marginalization from his classroom community and said that he did not enjoy learning mathematics or want to be a member of his classroom community.

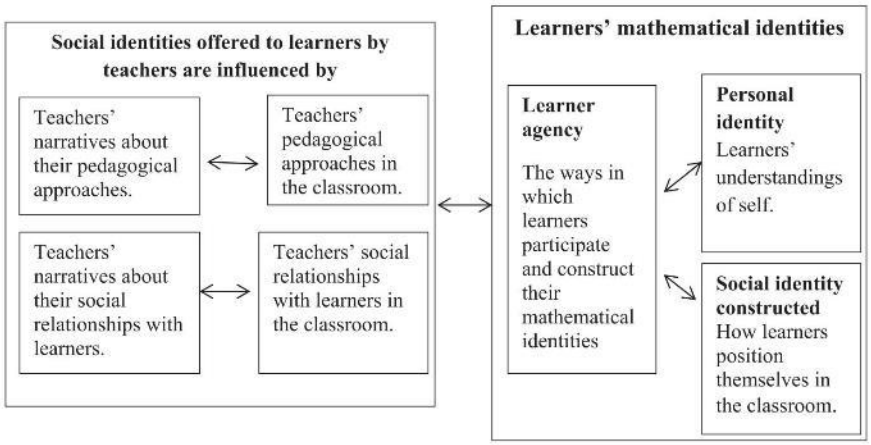



**Figure 2.** Teachers' actual and narrated practices and learners' mathematical identities.

## 6. Discussion

The purpose of this paper has been to identify the relationships between teachers' actual practices, their narratives of their practices and learners' mathematical identities. Teachers, through their actual practices, offer learners opportunities to construct their mathematical identities in affiliation with or marginalization from their classroom communities. I have shown how Mr Moyo and Mr Ndlovu offered learners social identities, through their actual pedagogies and social relationships with learners, with the teachers' actual practices being related to their narratives about their pedagogies and social relationships with learners, as explained further below. I have also shown how the social identities offered to learners influenced how learners constructed their own mathematical identities, as informed by their (learners') agency, their personal identities – their understandings of self, and their social identities – how they positioned themselves in the classroom (Figure 1).

My analysis of the findings discussed in this paper, which are representative of the larger sample, has led me to develop a model of the relations between the social identities offered to learners through their teachers' practices – both actual and narrated – and learners' mathematical identities, as follows (Figure 2).

There are three sets of important relationships illustrated in Figure 2. The first set characterizes the relationship between teachers' narratives about their pedagogy and their actual pedagogical approaches in classrooms. The teachers' narratives of their pedagogy, in terms of how they thought mathematics was best learned, informed how they taught mathematics. Mr Moyo narrated that mathematics was best learned through practice, thereby providing learners with many opportunities to work on classroom tasks, whereas Mr Ndlovu narrated that mathematics was best learned through mastering concepts and formulae, thereby providing learners with opportunities to become proficient in mastering concepts and formulae. Both narrations provide indications of how learners are expected to participate to develop their identities as full members of their classroom communities. For Mr Moyo, learners were primarily required to participate by practising mathematics, whereas for Mr Ndlovu, learners were primarily required to participate by memorizing,

reciting and mastering concepts and formulae. Both narrations of how mathematics was best learned were based on the teachers' experiences of the effectiveness of their pedagogy, as measured through assessments rather than the quality of learner thinking and understanding in the classroom, a finding representative of the larger sample. Their learners' achievements in assessment, as influenced by their pedagogical approaches, reinforced their narratives, as to why they thought that their pedagogies were effective.

In terms of the kinds of questions asked in the classroom as part of the teachers' pedagogical approaches, both teachers predominantly asked type 1 questions, which seems to be one of the most common types of questions asked by mathematics teachers, in my sample, as well as more broadly in South Africa and internationally (Boaler & Brodie, 2004; Boaler & Staples, 2008; Brodie et al., 2009). Given that both teachers asked similar questions, I did not find any relationship between learners' mathematical identities and the kinds of questions asked by the teachers. Rather, the difference in learner identities is explainable in relation to the teachers' broader pedagogical approach, which includes the ways the teachers taught, responded to learner errors and provided time for classroom tasks.

Mr Moyo allocated time in his lessons to classroom tasks, thereby asking fewer questions in comparison to Mr Ndlovu, who asked many questions but did not allocate much time to classroom tasks. So both pedagogies are traditional, but of different kinds, with each kind of traditional pedagogy having different implications for the development of learners' mathematical identities. Mr Moyo primarily focused on providing learners with opportunities to practice mathematics through classroom tasks. He was accommodating of errors and he encouraged learner participation. His pedagogical approach was experienced favourably by his learners, who attributed their enjoyment of and interest in learning mathematics to his teaching approach. Conversely, Mr Ndlovu's pedagogical approach was focused on memorizing rules and formulae. He was not accommodating of learner errors and did not provide learners with many opportunities to practice mathematics or participate with others in the classroom. His pedagogy was only experienced favourably by Edward, who enjoyed learning mathematics because he said that he had a good memory and was corrected by Mr Ndlovu when he made errors. Fred and Sunny did not enjoy learning mathematics in Mr Ndlovu's classroom, since Mr Ndlovu did not encourage practice or the use of different formulae.

The second set of relationships in Figure 2 is between teachers' social relationships with learners and the teachers' narratives of their social relationships, influenced by the teachers' narratives as to who could learn mathematics. Mr Moyo provided all learners with opportunities to participate in the classroom because he said that all learners could succeed at learning mathematics. When teachers believe that all learners can succeed in their classrooms, they are more likely to offer all learners opportunities to construct their mathematical identities in affiliation with the classroom community. Learners in Mr Moyo's classroom participated, were willing to ask him for assistance and were performing well in assessments, influencing his narrative about his social and pedagogical relationships with learners.

In contrast to Mr Moyo, Mr Ndlovu treated learners differentially by providing different opportunities for learners to participate in his classroom because he said that ability was needed to achieve good marks in mathematics. When success in mathematics is reduced to requiring a special ability, rather than effort, some learners who are apparently 'born with the ability' to succeed at mathematics will be offered opportunities to affiliate with

the classroom community, while others will be marginalized (Mendick et al., 2009). In line with Mendick et al.'s (2009) argument, Edward and Fred were able to achieve better in assessments in comparison to Sunny, whose marks were reducing steadily, reinforcing Mr Ndlovu's view that only top achievers can succeed at learning mathematics and become members of his classroom community. When marks are used as labels for competence, not all learners can construct their identities positively (Sfard, 2009).

The third set of relationships is between the social identities offered to learners by their teachers – through teachers' actual and narrated pedagogical practices and social relationships with learners – and learners' mathematical identities – involving learner agency and their personal and social identities (Figure 2). Teachers' narrations of their pedagogies and social relationships with learners inform the social identities that they offer to learners, by demarcating what is needed from learners to become affiliating members of their classroom communities and who they think can learn mathematics and become members of their classroom community. Teachers' narratives of their practices are not always accessible to learners, unless learners are provided with explanations as to why they are taught using a particular approach and why they are treated by their teachers in particular ways. In this study, I found that teacher narratives were related to learners' mathematical identities indirectly, through their actual practices, since the learners were exposed to teachers' actual practices, rather than their narratives.

Through teachers' actual pedagogies and social relationships with learners, teachers offer learners social identities through the different opportunities they offer to learners to participate in the classroom and become members of their classroom communities. Even though teachers, through their actual practices, provide opportunities for learners to construct their mathematical identities in affiliation with or marginalization from their classroom communities, learners construct their mathematical identities, which may be different from the offered identities, as influenced by: their personal and social identities, and how they exercise agency through participating in the classroom.

In Mr Moyo's classroom, all four learners constructed their mathematical identities in affiliation with their classroom community because they enjoyed Mr Moyo's pedagogical approach and they felt comfortable and valued by Mr Moyo in his classroom. All four learners enjoyed learning mathematics, were hopeful to pursue mathematics in the future, indicative of their personal identities, saw themselves as successful learners in their classroom community, indicative of their social identities, and agentively participated as full members in the classroom, thereby developing their mathematical identities as full members of their classroom community.

In Mr Ndlovu's classroom, Edward constructed his mathematical identity in affiliation with his classroom community. He was offered a social identity of affiliation by Mr Ndlovu and he enjoyed Mr Ndlovu's pedagogical approach and shared a positive social relationship with him. He enjoyed learning mathematics and was motivated to pursue mathematics in the future, indicative of his personal identity. Edward agentively participated as a full participant and positioned himself as a successful learner in his classroom community, indicative of his social identity, thereby developing his identity as a full member of his classroom community. Sunny constructed his mathematical identity in marginalization from the classroom community and did not construct his social identity as a learner of mathematics in his classroom. Sunny did not personally enjoy learning mathematics and was not motivated to learn mathematics, since he did not enjoy how Mr Ndlovu taught and

treated him in the classroom. He chose not to develop his identity as a full member of the classroom and did not fully participate in the classroom.

Among the learners discussed in this paper, Fred was the only learner who constructed his mathematical identity differently from the identity offered to him by his teacher.[4] Fred was provided with the opportunity to construct his mathematical identity in affiliation with his classroom community, but he chose to participate peripherally to develop his social identity as a peripheral member of the classroom, thereby constructing a mathematical identity of compliance. He did not enjoy the way in which Mr Ndlovu taught and treated learners in the classroom. However, he was still able to succeed because he enjoyed learning mathematics and was motivated to pursue mathematics in the future, indicative of his personal identity. Fred also had family support and assistance, unlike many other learners who participated in this study. Furthermore, he thought that understanding and practising mathematics was important as the rewards of learning and understanding mathematics were necessary for him so that he could become a better learner of mathematics.

## 7. Conclusion, recommendations and further research

In this paper, I provided a framework to understand learners' mathematical identities as they interacted with their teachers in classrooms, by distinguishing between two kinds of social identities offered by teachers – identities of affiliation with and marginalization from classroom communities – and between different kinds of mathematical identities constructed by learners – identities of affiliation with, compliance with, resistance to and marginalization from classroom communities – as influenced by the social identities offered by teachers, agency and learners' personal and social identities. I showed how two teachers' actual practices and their narratives of their practices had the capacity to influence learners' mathematical identities.

Teacher practices, as I have argued, involve both teachers' pedagogical approaches and social relationships with learners. Teachers, through their narrated pedagogical approaches, specify the criteria for affiliation in their classrooms, based on how learners are expected to participate to become full members of the classroom community. Through their narratives of their social relationships with learners, teachers specify who can learn mathematics and become full members of their classroom community. The ways teachers think and speak about their practices shape how they teach and relate to learners in the classroom. Conversely, how they teach and relate to learners further reifies and justifies why they think that their practices are suitable for teaching mathematics.

Teachers' actual practices informed learners' mathematical identities in important ways, through the kinds of social identities offered by teachers to learners to become full members of their classroom communities. Most of the learners in the sample developed their identities in affiliation with, compliance with or marginalization from the social identities offered to them by their teachers. As each of the learner participants enacted their identities by interacting with teachers, their teachers continued offering them the same social identities as previously offered. A limitation of my analysis is that I did not find the same teacher changing the social identities that they offered to any of the learner participants.[5] Further research is needed to show how often teachers change the social identities offered to learners, the circumstances that would cause such changes, and the relationship between

the different kinds of social identities that can be offered by the same teacher to a learner, in relation to a learner's mathematical identity.

Another limitation of my analysis is that I did not find any learners resisting the social identities offered in the wider sample. Research shows that some learners who come from marginalized communities tend to resist the offered identities of marginalization, with African-American learners persevering and becoming competent learners of mathematics (see Nasir & Shah, 2011). A possible reason for learners not resisting the identities offered can be that learners are strongly influenced by teacher pedagogies and social relationships, and hence the identities offered to them by their teachers[6] (Grootenboer & Zevenbergen, 2008). However, more data are needed to establish why some learners choose not to resist the identities offered to them by their teachers, given that many of these learners belong to marginalized communities in South Africa.

While there is much research drawing upon the seminal work of Shulman (1986) which is focused on improving teachers' mathematical content knowledge and pedagogical content knowledge (see Silverman & Thompson, 2008; Stylianides & Ball, 2008; Thames & Ball, 2010), less attention has been given to the kind of teacher knowledge needed to build and sustain positive social relationships with learners. Further research is needed to show how the importance of building and maintaining positive social relationships can be built into a model of teacher knowledge, as social relationships have important implications for learners to access knowledge. Teacher education programmes can also focus on empowering teachers with the necessary knowledge and skills needed to build and sustain positive social relationships with learners (Cornelius-White, 2007; Gardee, 2019; Rytivaara & Frelin, 2017).

Finally, I encourage teachers to be conscious of the kinds of social identities that they offer learners. I have shown that learners' mathematical identities are influenced by the kinds of social identities offered to them by their teachers, shaping learner participation in and access to learning mathematics. Offering all learners opportunities to construct their mathematical identities in affiliation with classroom communities can be the goal of every teacher so that all learners are provided with opportunities to become successful learners of mathematics. Offering learners the opportunity to construct their mathematical identities in affiliation with their classrooms and developing their thinking and understanding in classroom discussion will be worthwhile for teachers and possibly life-changing for their learners.

## Notes

1. By narratives of teacher practice, I mean stories of teacher practices. Narratives of practice should not be confused with narratives of identity, which are two separate concepts. Narratives of identity are defined later in the paper.
2. According to Darragh (2016), the greatest theoretical influence in the field of identity in mathematics education research is Wenger (41%), which is followed by the work of Sfard and Prusak (21%).
3. Most of the learners who participated in the study were male learners. In the larger study, only six female learners participated, as there were few female learners in each of the classrooms.
4. Within the larger sample, there were other learners who did not adopt the social identities offered to them by their teachers.
5. The influence of different social identities being offered by different teachers to the same learner will be explored in another paper.

6. It is also possible that since the learners were selected by their teachers, the teachers may have ensured that the learners selected were not resistant. However, it was not possible for me to select the learners using any other method since the teachers knew the learners, whereas I did not.

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# Chapter 5: Social Relationships between Teachers and Learners, Learners' Mathematical Identities and Equity

## 5.1. Introduction

This chapter builds on the important concept of social relationships in relation to learners' mathematical identities, as discussed in chapter 4, taking a slightly different approach. In chapter 4, I discussed two teachers whose pedagogical practices and social relationships with learners were different from each other. In this chapter, I focus on two teachers who taught similarly but shared different social relationships with learners, highlighting that while both pedagogical practices and social relationships are key aspects of teacher practices, it is important that we do not conflate these two aspects of teacher practice. Another key difference from the previous chapter is that I discuss the importance of equity, as I noted that research tends to relate pedagogical practices with learners' mathematical identities and equity, and that less attention is given to the relationship between teacher-learner social relationships, learners' mathematical identities and equity.

This chapter looks closely at the identities and trajectories of four learner participants over the two-year period and is focused on answering the second part of the first main research question: how the concept of identity can be used in researching secondary school learners' mathematical identities; and the second main research question: how learners and their mathematics teachers contribute to the development of learners' mathematical identities. This chapter, like the previous two, stresses the importance of teacher narratives of their social relationships with learners, which informed how they offered social identities, either equitably or inequitably, in their classroom communities. Yet this chapter differs from the previous two, by showing that the core characteristic of social relationships that contributed to the kinds of identities constructed by these learners was the notion of care. Care was discussed by these learners, not only in relation to themselves, but also in relation to whether other learners in the classroom were cared for by their mathematics teachers, highlighting the importance of equitable social relationships for the construction of learners' mathematical identities. This chapter concludes by stressing the importance of researching teachers' social relationships with learners, and not only their pedagogical practices, when researching learners' mathematical identities, as I showed that even though these learners experienced similar pedagogical practices over the two years, their identities changed substantially and

were related to the differences in social relationships experienced in their Grades 9 and 10 teachers' classrooms.



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# Social Relationships between Teachers and Learners, Learners' Mathematical Identities and Equity

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**In recent years, the notion of equity has become increasingly prominent in mathematics education, in relation to pedagogy and the opportunities provided for learners to identify themselves as successful mathematics learners. Different pedagogies have been shown to promote equity and access to mathematics in different ways. However, teacher practices are not characterised only by pedagogy, but also by social relationships between teachers and learners, which can be key in supporting all learners to identify with mathematics. Data are drawn from a study involving two teachers who taught using similar traditional pedagogies but had different social relationships with learners in a South African school. Data were collected over two years, in the form of videotaped lessons, field notes and audiotaped interviews with participants, and were analysed qualitatively. The relationship between different teacher–learner social relationships and learners' mathematical identities suggests that different social relationships have implications for equity, shaping access to knowledge and learner identities in important ways.**

**Keywords:** *Pedagogy; social relationships; mathematical identity; equity*

## Introduction

Over the last two decades, there has been a growing interest in the construct of identity to understand learning within social contexts (Graven & Heyd-Metzuyanim, 2019; Radovic, Black, Williams, & Salas, 2018). The popularity of the concept stems from its capacity to capture learner engagement and disengagement (Graven & Heyd-Metzuyanim, 2019), by considering cognitive, affective and social dimensions of learning mathematics.

Recently, research in mathematics education has also focused on the role of pedagogy in providing learners with equitable opportunities to develop their identities as successful learners of mathematics (Boaler & Greeno, 2000; Boaler & Staples, 2008). Equity is concerned with a fair distribution of opportunities for learners to engage with mathematics in their classrooms, influencing their willingness to learn mathematics (Cobb & Hodge, 2011). It has been argued that inquiry-based pedagogies enable all learners to develop deeper conceptual knowledge and positive mathematical identities in an equitable manner, in contrast to traditional pedagogies, which are argued to develop largely procedural knowledge, limiting opportunities for all learners to equitably access and become successful learners of mathematics (Boaler & Staples, 2008; Umugiraneza, Bansilal, & North, 2017). Traditional pedagogies are used widely in South Africa, and are characterised by textbook-based learning and teacher-dominated practices, with teachers holding most of the authority for the knowledge produced (Cobb, Gresalfi, & Hodge, 2009).

While significant attention has been given to the role of pedagogy in relation to identity and equity, less attention has been given to the role of social relationships between teachers and learners, which are also important in providing learners with equitable access to mathematics. Social relationships and

teacher pedagogy are distinct from each other, with the former focusing on interpersonal relationships (Rytivaara & Frelin, 2017), which involve respecting and valuing learners, and the latter on content matter, lesson structure and classroom management. While there may be some overlap between pedagogy and social relationships, the distinction is important in understanding how teachers who teach using similar pedagogies can have different social relationships with learners, with different implications for equity. Rytivaara and Frelin (2017) argue that social relationships between teachers and learners require more attention throughout the education system, since these relationships are important for the well-being of learners and teachers.

This research was guided by the following research questions:

(1) What influences teachers who teach using similar traditional pedagogies to develop social relationships equitably or inequitably with learners?
(2) How do social relationships influence how learners construct their mathematical identities?

This paper explores the changes in four learners' mathematical identities as they transitioned from Grade 9 to 10 to show how social relationships with teachers influenced how learners constructed their mathematical identities. The practices of two teachers, who taught using similar traditional pedagogies but shared different social relationships, one being equitable and the other inequitable, with the four learners are explored.

## Theorising Mathematical Identity

Wenger (1998) and Sfard and Prusak (2005) contend that identity construction involves negotiation between people and their social communities. For Wenger (1998), identity is related to learning because learning involves becoming a different person and a better participant within a community of practice, as identified and negotiated by the person and other members of the community. Sfard and Prusak (2005) define identity as a collection of socially shaped narratives, which are reifying, endorsable and significant. Reifying narratives are concerned with 'being' certain people, stressing the repetition of events to reinforce identities. Endorsable narratives require the identity-builder to claim the authenticity of the narrative, while significant narratives involve emotional responses from storytellers at the change of story. Narratives told about oneself to others, such as stories told by learners to others, and narratives told by significant narrators, such as stories told by teachers about learners, can influence learners' actions and identities (Sfard & Prusak, 2005).

Wenger's (1998) perspective on identity is related to what Radovic et al. (2018) call the enacted dimension of identity, i.e. identity as a way of participating, which is not necessarily mediated by discourse. Sfard and Prusak's (2005) perspective on identity is related to what Radovic et al. (2018) call the representational dimension of identity, i.e. identity as narrative, which is mediated by discourse. Both Wenger (1998) and Sfard and Prusak (2005) argue that people position themselves and are positioned by others as members of communities in certain ways as they engage in practice and discourse. Both definitions tend to place a stronger emphasis on the social construction of identity, and a weaker emphasis on the subjective construction of identity. According to Radovic et al. (2018), a strong focus on the social construction of identity tends to limit understandings of the notion of self and individual agency.

To bring together the social and subjective, and enacted and representational aspects of identity (Radovic et al., 2018), Marks and O'mahoney's (2014) work is drawn on. Mathematical identity is defined as being constructed through a relationship between personal identity, social identity and agency (Marks & O'mahoney, 2014), in relation to learning mathematics. Personal identity is the subjective component of identity, which is defined as an understanding of self and is developed as individuals participate in practice and discourse. Personal identity emerges from learners' emotions, their experiences of learning mathematics, their motivations, interests and future projections. The influence of the social world in shaping personal identity creates social identity, distinct yet interacting with personal identity. Social identity is the social component of identity, which emerges from social relations with others through practice and discourse and involves the ways one positions oneself within a social

setting (Marks & O'mahoney, 2014). Belonging to specific communities, such as mathematics classrooms, affiliating to the norms and standards of the community, engaging with others by pursuing common goals, utilising the language, tools and culture of the mathematics community are constituents of social identity (Wenger, 1998).

Social identities are also offered to learners, as others position individuals as members through the opportunities presented by communities for individuals to participate (Marks & O'mahoney, 2014). The social identities offered to learners by their teachers are not reducible to the kinds of mathematical identities constructed by learners. While learners may be offered certain social identities by their teachers, learners can participate and construct their mathematical identities in different ways. This choice to intentionally participate in certain ways is referred to as agency, which is always exercised and influenced by social contexts (Eteläpelto, Vähäsantanen, Hökkä, & Paloniemi, 2013). How agency is exercised is key to understanding what it means to achieve success in learning mathematics, considering that learner agency influences whether (or not) learners participate in the classroom and adopt the social identity offered to them by their teachers, as influenced by their personal and social identities.

## A Framework for Mathematical Identities

Wenger (1998) describes different kinds of participation, i.e. full participation, peripheral participation and marginality, which have implications for the development of identity. Peripheral participation is distinguished from full participation, as peripheral participation requires a degree of non-participation to 'enable a kind of participation that is less than full' (Wenger, 1998, p. 165), whereas marginality involves non-participation. Different kinds of participation in classroom communities can be associated with how learners relate to their teachers and tasks. Boaler and Greeno (2000) and Cobb et al. (2009) describe three ways in which learners can relate to their teachers and tasks: by affiliating with teachers and tasks; by cooperating with teachers; or by resisting teachers and tasks. Building on Marks and O'mahoney (2014), Wenger (1998), Boaler and Greeno (2000) and Cobb et al's. (2009) work, the following framework was developed to understand the identities offered to and constructed by learners (Table 1) (see also Gardee, in press).

During social interactions between teachers and learners in classrooms, learners are offered certain social identities by their teachers. Teachers can offer learners social identities of affiliation, by inviting learner participation and providing learners with the tools and resources of mathematics. Some learners can be offered social identities of partial affiliation, where teachers invite participation to some extent, by providing learners with some access to the tools and resources of mathematics. Teachers can also offer learners social identities of marginalisation, by limiting learner participation and by constraining learner access to the tools and resources of mathematics. How and to whom teachers offer social identities of affiliation, partial affiliation and marginalisation supports equitable or inequitable access to mathematics for learners.

To determine the kinds of mathematical identities constructed by learners, learners' personal identities, social identities and agency need to be considered concurrently with each other. When learners are offered social identities of affiliation or partial affiliation with their classroom communities by their teachers, they can construct their mathematical identities in affiliation with, compliance with or

**Table 1.** Identities offered and constructed

| Social identities offered by teachers | Mathematical identities constructed by learners |
|---|---|
| Affiliation or partial affiliation with the classroom community | Affiliation with the classroom community |
| | Compliance with the classroom community |
| | Resistance to the classroom community |
| Marginalisation from the classroom community | Marginalisation from the classroom community |
| | Compliance with the classroom community |
| | Resistance to the classroom community |

resistance to the classroom community. Affiliating with a community involves learners identifying with the norms and standards of the community and utilising the language, tools and culture of the community (Cobb et al., 2009; Wenger, 1998). Affiliating learners tend to be motivated and emotionally invested in learning mathematics for their personal identities, and these learners develop their social identities as full members of the community. Compliant learners engage peripherally with the tools and resources of mathematics, by cooperating with their teachers and completing their tasks diligently (Cobb et al., 2009), but they are not invested in becoming full members of their classroom community, for their social identities. These learners may not be emotionally invested, motivated or fully interested in learning mathematics for their personal identities, and may not be comfortable to participate as full members of the community. Learners can also resist the offered social identities of affiliation, by constructing a mathematical identity of resistance. Resistant learners engage with the tools and resources of mathematics in limited ways or not at all because they may not want to develop their social identities as members of their classroom community, as they may not see the use of learning mathematics for their personal identities.

When learners are offered social identities of marginalisation from their classroom communities by their teachers, they can construct their mathematical identities in marginalisation from, compliance with or resistance to the classroom community. Learners who develop identities of marginalisation may participate in limited ways or not at all, perhaps because they are not personally motivated to learn mathematics, or wish to become members of the classroom community, for their social identities. Sometimes, learners who are offered social identities of marginalisation by their teachers may want to develop their social identities as members of the community, by constructing their mathematical identities in compliance with or resistance to the classroom community. Compliant learners may not always participate with their teachers in the classroom, but may attempt their classroom tasks, perhaps because they are motivated to pass mathematics or pursue mathematics in the future, for their personal identities. Learners who are resistant to the offered identity of marginalisation may try to participate in the classroom, by trying to utilise the tools and resources of mathematics despite being marginalised from the community. Resistant learners are invested in developing their social identities as full members of their classroom community and are emotionally invested and interested in learning mathematics for their personal identities.

The above framework supports analyses of identities as they are offered and constructed in classrooms. In what follows, an overview of the research that illustrates the implications of the different kinds of teacher–learner social relationships on learners' identities, equity and learning mathematics is provided.

## Literature Review

Research demonstrates that what learners learn and how they construct their identities are linked to their experiences of classroom practice (Boaler & Greeno, 2000; Boaler & Staples, 2008; Cobb et al., 2009). In his meta-analysis of research, Cornelius-White (2007) found that positive teacher–learner relationships are associated with optimal learning. Supportive environments are key for the achievement of optimal learning in classrooms because for learners to learn and become successful learners of mathematics they need to feel a sense of belonging, being heard, seen and valued in their classrooms (Bibby, 2009; Darragh, 2013). Bibby (2009) showed how some learners felt marginalised from learning mathematics when taught by a student on teaching practice, who did not value their contributions, in comparison with their mathematics teacher, who supported them by valuing their participation.

Other studies show how teachers' expectations of learner performances are related to learner achievement (Jamil, Larsen, & Hamre, 2018; Turner, Rubie-Davie, & Webber, 2015) and to the kinds of interactions and relationships between teachers and learners (Black, 2004; Hodgen & Marks, 2009). Turner et al. (2015) report that learner achievement in assessments inform teacher expectations of learners, as teachers tend to have higher expectations of higher-performing learners and lower expectations of lower-performing learners. Different teacher expectations have implications

for equity as research shows that teachers who had high expectations of learners treated them differently from others, by valuing their contributions and overlooking their violations of classroom norms, enabling these learners to maintain their status as high-achieving learners (Black, 2004; Hodgen & Marks, 2009). Conversely, teachers who had lower expectations of learners are shown to marginalise such learners, by not interacting with their ideas or treating them in the same manner as the higher-achieving learners, resulting in inequitable teacher practices. Differential treatment of learners tends to create the illusion that natural ability, rather than effort, produces success and that not everyone can do mathematics, having negative implications for learners' mathematical identities (Mendick, Moreau, & Epstein, 2009).

## Methods

This study utilises data collected from a well-resourced secondary school in Johannesburg. Four learners, Senzo, Lane, Jack and Shane (all names are pseudonyms), and their Grade 9 and 10 mathematics teachers, Mr Moyo and Mr Sithole, are discussed. Both teachers displayed similar pedagogical styles but shared different social relationships with different learners. At the time of data collection, Mr Moyo was recently qualified with an academic degree to teach mathematics and Mr Sithole had a teaching diploma with 32 years of experience teaching mathematics. The four learners were selected by Mr Moyo, their Grade 9 teacher, using the following criteria: one high-performing learner, one low-performing learner and the person sitting next to them. In Grade 9, Senzo was a low-performing learner, sitting next to Lane, an average-achiever. Jack was a high-performing learner, sitting next to Shane, another high-performing learner. All four learners identified themselves as successful learners in Mr Moyo's Grade 9 classroom. However, in Mr Sithole's Grade 10 classroom, their perceptions of themselves as learners of mathematics changed substantially, with three of the learners no longer seeing themselves as successful.

Data were collected in the third school term (August–September) in 2015 and 2016: for two weeks in 2015 (Grade 9) and four weeks in 2016 (Grade 10) in the form of videotaped lessons, field notes and audiotaped semi-structured interviews with participants. A total of 480 min of lessons were analysed for each teacher. Questions in the teacher interviews aimed at understanding the teachers' social relationships with learner participants, in terms of the kinds of social identities offered to learners. Questions in the learner interviews aimed at understanding learners' mathematical identities in Grades 9 and 10.

Qualitative approaches were used to analyse the data, which were transcribed, coded and analysed thematically. The social identities offered to learners were coded based on how the teachers interacted with learners in the classroom, how they described the learner participants in their interviews and how learners experienced the social identities offered, enabling understandings of whether the social identities offered were equitable or inequitable. Learners' personal identities were coded based on how learners described themselves, in terms of their emotions relating to their learning of mathematics and the use of mathematics for their lives and futures. Learners' social identities were coded based on how they described their membership in their mathematics classrooms, in terms of their achievements in assessments and whether they felt comfortable participating in their mathematics classrooms. Agency was coded based on how learners participated in the mathematics classroom, as observed in the lessons and narrated by learners in their interviews. To enhance the credibility of the findings, data from lesson observations, field notes and interviews were compared.

## Findings

Mr Moyo and Mr Sithole usually began their lessons by explaining concepts and asking learners questions that required lower-order thinking skills. The rest of the lesson was dedicated to classroom tasks, where learners worked through problems in their textbooks, either individually or with the learner seated next to them.

***Mr Moyo***

Mr Moyo invited participation from all learners in his classroom to answer questions, demonstrate answers on the board and ask him questions when they needed to during his lessons. During the time allocated to classroom tasks, Mr Moyo approached learners who called him for help, and even learners who did not call him, to see if they needed help. He explained:

> In a classroom situation when I am moving around and checking, when I have given homework and I see this is the mistakes that they are having, I can correct them because some homework that I give and we will revise them as a class and there are some homework that I give and I say if you have any problems, then you can raise your hand or if I move around, then I can explain it individually. Some don't really get me when I am speaking on the board there. They need a lot of individual attention so as I move around, I can explain it.

Mr Moyo said that he encouraged learners to ask him questions if they did not understand and that he walked around the classroom to see if learners required any individual attention. He was willing to help lower-achieving learners and thought that all learners could succeed in learning mathematics if they practised and were motivated.

> I think generally Grade 8 and 9, they just need so much motivation that maths is not difficult and it's a subject that is interesting that opens doors in life.

In some lessons, he was seen encouraging all learners to try and complete their tasks. For example, he would tell learners 'you must not be afraid to participate, go and try'. He explained why he thought it was important to motivate struggling learners:

> I think those learners … what I think is they are not really incompetent but they may just not have motivation, not have really caught up with the subject, what it is about. The fact that to a point where they get that interest, you may actually find out that they are not incompetent but they never got really properly orientated with the subject.

If Mr Moyo could not help all learners during the lesson, he said that he encouraged them to come during break time, after school or to the extra classes he offered.

Mr Moyo treated all learners equitably and provided them with opportunities to construct their identities in affiliation with his classroom community through his social relationships with them. All four learners took up the opportunities provided by Mr Moyo and constructed their mathematical identities in affiliation with their classroom community. When describing their personal identities, Shane, Jack, Lane and Senzo said that they enjoyed learning mathematics, and were motivated to pursue mathematics in the future.

| | |
|---|---|
| Shane: | My favourite subject is maths definitely … doing maths … ah let's see, doing maths … I am doing something enjoyable. |
| Jack: | Mam in maths like, I do love maths Mam. |
| Lane: | I do love maths. |
| Senzo: | I am starting to enjoy maths because of Mr Moyo and I realised that it is important for me and my life, like my future. |

All four learners were willing to work with Mr Moyo and with their peers, by assisting others and working together to complete their tasks. When narrating their experiences of the social identity offered, the learners described Mr Moyo as a 'kind, friendly' teacher and 'the best teacher' who saw potential in all learners. All four learners experienced Mr Moyo as a caring teacher who cared about all learners, treated all learners equally and was there for all learners in the classroom, indicating the employment of equitable practices in the classroom.

| | |
|---|---|
| Shane: | I see this teacher, yeah he just wants to get his children to pass. He cares about us. |
| Jack: | Cos like if you say you never understood or asked a question, he (Mr Moyo) will like explain more in detail for everyone to understand. |

Lane: Mam, he cares for us, not like the other teachers. He cares, he talks slow until you understand the … ah the question like factorisation, exponents, everything, algebra, fractions.

Senzo: I think he does care about me … one thing I've seen from Mr Moyo is that at least when we pass his papers, he gets happy that we understand him.

Mr Moyo was seen to be concerned about learners and willing to help them, even when they approached him during his free periods. When describing their social identities, all four learners saw themselves as successful learners of mathematics, who were comfortable to participate in the classroom and whose marks were improving. Mr Moyo was seen to make them feel 'welcome and comfortable' in his classroom, was approachable, motivating and had no favourites. As a consequence of the all learners being treated equitably through the positive social relationships shared in the classroom, all four learners saw themselves as learners who enjoyed learning mathematics and who were willing to participate in and improve their learning of mathematics as full members of their classroom community.

***Mr Sithole***

Mr Sithole taught using a similar kind of pedagogical approach as Mr Moyo, but was experienced negatively by some learners because of his social relationships with learners. Mr Sithole categorised learners according to their marks and usually assisted high-achieving learners, but was not always willing to help struggling learners. He said that he divided the classroom into two parts according to 'noise'. He explained that the disruptive learners were lower-achieving learners sitting on the one side and the quiet learners were the higher-achieving learners on the other:

> Noise ones, I put them this side and try and give some working space to these who are quiet. So anyway, these are the people who get the lowest marks in the class cos the highest marks are this side.

Mr Sithole believed that not all learners had the 'aptitude' for mathematics, which was a 'gift from God'. In some lessons, Mr Sithole ignored struggling learners and refused to help them, thereby offering those learners identities of marginalisation from his classroom community, indicating the employment of inequitable practices in the classroom. In one lesson, he told a learner who called him for help 'I don't have time for you'. In another lesson, he told a learner who attempted a problem 'it needs a mastermind, not you'. Mr Sithole explained that there was 'nothing' he could do for struggling learners so he would rather focus on achieving learners. When discussing struggling learners, he said:

> He can't just understand. You explain to him an equation using $x$. Maybe you think he understands. Now take out $x$, put $y$ in place of $x$, exactly the same things. It is a new and strange thing. I think we have to accept that there are people like that.

From the four learners who participated, Mr Sithole identified Shane as a high-achieving learner and the other three as weaker learners, with Senzo being the weakest. Shane was described by Mr Sithole as being born with a talent to do mathematics.

> He has a talent. Can a talent be influenced by anything? It is something you are born with.

Mr Sithole was willing to assist and build a positive social relationship with Shane, but was not always willing to assist or build positive social relationships with the other three learners, thereby treating learners inequitably. Shane was offered a social identity of affiliation with the classroom community by Mr Sithole. Shane constructed his mathematical identity in affiliation with the classroom community, by participating often with Mr Sithole and other learners, by asking and answering questions and working together with others on classroom tasks. When describing his personal identity, Shane said that learning mathematics was 'good' and that he was hoping to pursue a mathematically oriented degree in the future. He identified himself as a successful learner; however, he felt that he was not performing as well as he would have liked to in assessments because of laziness. When describing his experiences of the social identity offered to him, Shane felt that Mr Sithole's approach to him

changed a little when his marks dropped, by no longer showing the same kind of interest in him as when he was achieving high marks. However, he said that Mr Sithole's approach to him did not influence his learning of mathematics. He was comfortable to participate and he described himself as one of the 'smart guys' in his classroom, indicative of his social identity.

> This year, I am very comfortable, yes. I am in the smart guys.

Jack, Lane and Senzo said that they did not participate much in the classroom. When describing their personal identities, they said that they no longer enjoyed learning mathematics because of Mr Sithole. When narrating their experiences of the social identities offered to them, the three learners described Mr Sithole as a teacher who treated learners inequitably, since he had favourites, ignored learners, was not willing to help or explain to all of them when they asked him for help and was uncaring.

| | |
|---|---|
| Jack: | Mam but like you can't have like all of his jokes because sometimes you will see that okay he has made a joke that will hurt someone … let's say Mam, you tried doing your work and then like you get something wrong. That is when he will tell you like what does he say again? Hm … oh he tells you sometimes, he tells you what you wrote here is the same thing as writing rubbish. |
| Lane: | He just doesn't care Mam … whether you understand it or not … cos Mam, I've asked him many times to help me. He said no. |
| Senzo: | He gives you a sum, you try it then he comes and checks and tells you that that is rubbish. You know that is demotivating. You know he doesn't really care. Mr Sithole … he favours a lot of learners. |

Among the three learners, Jack was offered the opportunity to construct his identity in partial affiliation with the classroom community. Mr Sithole saw him as a learner who had 'mathematical intelligence' but did not put any effort into learning mathematics and was likely to fail. Mr Sithole later explained that learners need to be gifted before hard work can produce success.

> I always think that first of all, there should be a gift into it first, and then hard work.

During the lessons, Mr Sithole sometimes acknowledged Jack's contributions and ignored him at other times. Jack constructed his identity in compliance with the social identity offered, as he participated in some lessons by answering some questions and attempting his classwork, and would sometimes ask other learners for help. When describing his social identity, Jack said that he was not comfortable to ask Mr Sithole questions and that he was failing mathematics. He felt that he was expected to pass mathematics, however, he no longer enjoyed mathematics as he did in Grade 9 or wished to pursue the subject in the future, for his personal identity.

Senzo and Lane were offered social identities of marginalisation by Mr Sithole, who did not engage much with them in the classroom. When describing his social identity, Lane identified himself as a learner who was failing mathematics and who was not comfortable in Mr Sithole's classroom, saying 'Mr Sithole's class, you are not comfortable'. He did not participate much with Mr Sithole in the classroom, but he completed his work, asked other learners for help when he did not understand and he sometimes chorused answers to Mr Sithole's questions. He constructed his mathematical identity in compliance, even though he was marginalised by Mr Sithole, because he was motivated to pass mathematics and pursue mathematics in the future, for his personal identity.

Senzo constructed his identity in marginalisation from the classroom community and rarely participated in the classroom. He identified himself as a learner who was failing mathematics because he was demotivated to learn mathematics owing to his poor relationship with Mr Sithole, as indicative of his social identity. He said that Mr Sithole 'ignored (his) presence' in the classroom and that he was afraid to ask Mr Sithole for help.

> I feel bad but knowing that I have to ask him (for) correction(s) you know so I wouldn't go anywhere. I would just leave that mistake even though I don't know how to correct that mistake cos of being afraid of asking him.

When describing his personal identity, he said that he was no longer motivated to learn mathematics or willing to pursue a mathematically orientated degree in the future.

## Discussion and Conclusion

The purpose of this paper has been to identify the relationships between teacher–learner social relationships, learners' mathematical identities and equity. To answer the first research question, the social identities offered by teachers were influenced by the teachers' narratives of who could do mathematics, and concomitantly, teachers' expectations of learners, which influenced their willingness to support learners. Mr Sithole narrated that ability produced success and used learners' marks as benchmarks to delineate who could become successful learners of mathematics. In concert with current research (Black, 2004; Hodgen & Marks, 2009; Turner et al., 2015), Mr Sithole had high expectations of high-achieving learners and low expectations of low-achieving learners, thereby offering learners social identities inequitably. Mr Moyo narrated that all learners could succeed if they were motivated and made the effort to learn by practising mathematics. Mr Moyo had high expectations of all learners and treated them equitably, by valuing their contributions and offering all learners social identities of affiliation with his classroom community. High regard for learners accompanied by the belief that all learners can succeed are characteristics of equitable teacher practices responsible for learners' increased inclination to learn mathematics (Gresalfi & Cobb, 2006). Concomitantly, the kinds of social identities offered to learners shaped their learning and achievement in assessments, as learners in Mr Moyo's classroom were performing well, whereas in Mr Sithole's classroom, the high-achieving learners were performing well, while the low-achieving learners were performing poorly.[1]

To answer the second research question, learners in Mr Moyo's classroom constructed their identities in affiliation with the classroom community, through their experiences of positive social relationships, by being treated equitably by Mr Moyo. In Mr Sithole's classroom, the same four learners constructed different identities from each other, since Mr Sithole offered social identities inequitably. Moreover, three of the four learners' mathematical identities changed substantively from Grade 9 to 10, as influenced by their experiences of being treated equitably by Mr Moyo and inequitably by Mr Sithole. This finding resonates with current research (e.g. Bibby, 2009; Darragh, 2013), where learners' mathematical identities are related to their experiences of their teachers as being approachable, being willing to assist them, seeing potential in them, making them feel comfortable and treating them kindly and equitably. Most notably, 'care' was discussed by learners as being salient in characterising the kinds of teacher–learner social relationships. Care, as discussed by learners, was not only in relation to themselves, but also in relation to others, highlighting the importance of equitable relationships in promoting access to, identification with and the learning of mathematics.

The findings also go beyond prior work (Boaler & Staples, 2008; Cobb et al., 2009), by showing that, while both teachers taught using similar traditional pedagogies, it was their social relationships with learners that influenced who got access to learning mathematics and who could develop their identities positively. It is therefore insufficient to only consider the role of pedagogy in providing learners with equitable access to learning mathematics, as social relationships are also important, in providing equitable opportunities for all learners to learn and become successful learners of mathematics. Understandings of what it means to be successful in mathematics tend to filter down from teacher to learners, thereby having the capacity to shape learners' mathematical identities. Teacher education programmes can support teachers to become conscious about their expectations of different learners to ensure that all learners are given equitable opportunities to learn and become successful learners of mathematics.

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## Note

1. Some may argue that differences in teacher age could result in differences in social relationships with learners. While age may have some influence more broadly on who does and does not treat learners equitably (after all, this is only a small sample so conclusions cannot be drawn about age), the main argument is still valid: that even though both teachers taught using similar pedagogies, learners experienced different social identities offered because of the different social relationships in both classrooms.

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# Chapter 6: Relationships between teachers' interactions with learner errors and learners' mathematical identities

## 6.1. Introduction

This chapter picks up on the importance of the teachers' interactions with learner errors as part of their pedagogical approaches in relation to learners' mathematical identities, which was briefly discussed in chapter 4. This chapter focuses on the perspectives on and approaches to learner errors of two teachers, and how these perspectives and approaches informed learners' mathematical identities. Like in the previous chapters, there is a clear link between the teachers' narratives and their practices, which in the context of this chapter concerns the relations between their narratives of learner errors and their approaches to learner errors. In this chapter, I show that there were two basic mechanisms that informed how the teachers spoke about and interacted with learner errors: 1. the teachers' views on the role of learner reasoning in mathematics and 2. their narratives about learners. I show that learners' mathematical identities were informed by their teachers' approaches to errors, yet as I have argued in chapters 2, 3 and 4, teacher practices do solely not determine the mathematical identities constructed by learners. The learners also played a role in the construction of their mathematical identities, which was informed by how they exercised agency, and their personal and social identities.

All of the learners in this sample wanted to develop their identities as members of the classroom community, so even though they may have not described errors in the same ways that their teachers did, they altered their participation to maintain their membership in the community. In this chapter, I also provide another possible reason why the learners in the study did not resist the social identities offered by their teachers, stemming from an understanding of their personal and social identities. Resisting teacher practices could have a negative impact on learners, as all the learners in this sample, and most of the learners in the wider sample, aspired to pursue mathematics in the future and aimed at achieving some sense of belonging to the classroom community. Resisting teacher practices could possibly lead to the marginalisation of learners who were not previously marginalised, or further alienation of learners who were marginalised from the classroom community by teachers.

Like chapters 3 and 5, this chapter is focused on answering the second part of the first main research question, and the second main research question: how the concept of identity can be

used in researching secondary school learners' mathematical identities, and how learners and their mathematics teachers contribute to the development of learners' mathematical identities.

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# Relationships Between Teachers' Interactions with Learner Errors and Learners' Mathematical Identities

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**Abstract**
Engaging with learner errors in mathematics classrooms is an important aspect of teacher pedagogy which can support learner identification with mathematics. This study examined the opportunities that teachers provided for the construction of learner identities based on how the teachers spoke about and interacted with learner errors in two secondary school mathematics classrooms. Data were collected over two years in the form of videotaped lessons, field notes, photographed learner notebooks and audiotaped interviews with participants, and were analyzed qualitatively. Two ways in which teachers spoke about errors are discussed: errors are contagious and errors are lessons. How the teachers spoke about and interacted with errors were informed by their views of learners and their reasoning in mathematics, which shaped the identities they offered learners. We show that while learners' experiences of their teachers' approaches to errors informed their identification with mathematics to some extent, the learners also played a role in the construction of their mathematical identities.

**Keywords** Learners' mathematical identities · Learner errors · Teachers' interactions

## Introduction

Making errors is a normal and essential part of learning mathematics (Borasi, 1987; Ingram, Baldry & Pitt, 2013). Even experienced mathematicians make errors and may consequently create new knowledge (Borasi, 1987). Errors are systematic, pervasive

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and persistent mistakes occurring across contexts, indicative of deeper conceptual misunderstandings called misconceptions (Hansen, 2011; Nesher, 1987). Errors are distinguished from slips, which are sporadic mistakes, not indicative of conceptual misunderstandings and easily corrected (Olivier, 1989). Errors are seldom taught, and all learners make them (Brodie, 2014).

Learner participation, learning of and identification with mathematics, can be informed by how teachers interact with learner errors. Unlike other teaching artefacts, for example, teacher explanations, how teachers interact with learner errors specifies the criteria for success in classrooms (Brodie, 2014). In some classrooms, becoming a successful mathematics learner involves not making errors, as errors are associated with labelling learners as intelligent or not (Heyd-Metzuyanim, Tabach & Nachlieli, 2015). In these classrooms, feelings of pride are associated with being correct, while feelings of shame, anxiety and self-doubt may be associated with making errors, so learners may dislike and disengage with mathematics since errors are considered failures and reasons to be embarrassed (Heyd-Metzuyanim et al., 2015). In other classrooms, success in mathematics involves learning from errors. In these classes, teachers draw on learners' incorrect thinking as resources to motivate learners to learn from their errors, develop their conceptual knowledge and appreciate the nature of mathematics (Borasi, 1987). So how teachers deal with errors can support the development of learners' mathematical identities, as they learn what mathematics is about and what being a mathematics learner entails.

Over the last two decades, the construct of identity has been increasingly researched in mathematics education (Darragh, 2016; Hall, Towers & Martin, 2018). A focus on identity in mathematics education is useful in foregrounding learners' experiences regarding what 'doing mathematics' means and who can do mathematics (Gresalfi & Hand, 2019). Researching learners' mathematical identities also enables understandings of learner engagement with and disengagement from learning mathematics (Graven & Heyd-Metzuyanim, 2019). However, research illustrating the relationship between teachers' approaches to learner errors and learners' mathematical identities is limited, even though this relationship can shape learner identification with mathematics.

In this paper, we explore how two secondary school teachers, who taught using traditional pedagogies,[1] spoke about and interacted with learner errors in their classrooms and how their approaches informed learner identities. We address the following research questions:

1. How do teachers speak about and interact with learner errors in their mathematics classrooms?
2. What informs how teachers speak about and interact with learner errors?
3. How do teachers' approaches to errors inform learners' mathematical identities?

[1] Traditional pedagogies dominate classroom instruction, both in South Africa and internationally, and are characterized by textbook-based, teacher-dominated practices (Boaler & Greeno, 2000; Umugiraneza, Bansilal, & North, 2017). Traditional pedagogies are often compared with inquiry-based reform pedagogies, which are characterized by learners engaging collaboratively on open-ended problem-solving activities (Boaler, 2002). Inquiry-based pedagogies are argued to promote deep mathematical reasoning, with competence being evaluated on learner reasoning rather than obtaining the correct answers.

## Theoretical Framework

### Theorizing Mathematical Errors

Constructivist and situated theories of learning are commonly used in researching learner errors in mathematics education. Constructivist perspectives provide understandings of how learner thinking develops and how learner errors emerge through engagement with mathematics tasks, while situated theories provide understandings of how learners learn, make errors and develop their identities through participating in social contexts.

Constructivist theories argue that knowledge is actively built on foundations of prior knowledge. Restructuring of schemata and developing conceptual knowledge occur through processes of assimilation and accommodation (Woolfolk, 2010). The amalgamation of existing knowledge into schemata is characteristic of assimilation, whereas reorganization of schemata as consequences of conflicts between new knowledge and existing schemata is characteristic of accommodation (Woolfolk, 2010).

Assimilation, when existing schemata expand to make sense of new knowledge is not as challenging for the learner as accommodation, where existing schemata must change to make sense of new knowledge. When learners assimilate new knowledge instead of accommodating for it, overgeneralisations based on prior correct knowledge can occur in new domains, where they are no longer correct (Hatano, 1996; Smith, DiSessa & Roschelle, 1993). This implies that correct knowledge from one domain is applied incorrectly in another domain (Smith et al., 1993). For example, the following error made by a grade 9 learner: $\frac{5}{10} + \frac{7}{8} = \frac{12}{18}$, can be explained as an overgeneralisation of prior correct knowledge of addition of whole numbers incorrectly in a new domain: fractions. Therefore, from constructivist perspectives, errors are reasonable and make sense to the learner, illuminating both valid and invalid learner reasoning.

While errors may be reasonable and sensible from constructivist perspectives, the importance of the social context in informing teachers' and learners' views of errors is better understood using situated perspectives. Situated theory forms the backbone of much research on identity, where learning and developing an identity involves becoming a better participant in a practice, using tools and working within the culture of a particular community in which disciplinary knowledge is developed (Boaler & Greeno, 2000; Wenger, 1998).

From situated perspectives, learner errors are indicative of learners not using the appropriate tools afforded to them within their learning communities or are consequences of non-engagement within communities (Brodie, 2014). Errors may also be consequences of newcomers (learners) in the practice using different criteria of what counts as correct mathematical knowledge from the criteria used by oldtimers (teachers) (Brodie, 2014). Using the same example as above, situated theorists would explain that the error emerged as the newcomer engaged with a new task while continuing to use previous tools and experiences, since s/he spent many years doing so correctly, and most likely received praise for doing so. So the newcomer will not develop her/his identity as a full member of her/his current mathematics classroom community, until s/he develops the necessary tools to participate appropriately in the community. Discussions are important for both newcomers and oldtimers to negotiate

and create joint criteria to develop shared understandings of what it means to be a successful learner of mathematics in the community (Brodie, 2014; Sfard, 2007).

Both situated and constructivist theories contend that errors are reasonable, normal and a necessary component of learning mathematics, with classroom conversations needed to take account of current errors to develop new knowledge (Brodie, 2014). We take the position that errors arise from conceptual structures, which is explained by constructivism. These errors emerge as learners engage with mathematics and develop mathematical identities in their classroom communities, which is explained by situated theory. In what follows, we define identity and show how the concept can be applied to understanding learners' mathematical identities.

## Theorizing Identity

The relationships between learning and identity are discussed by Wenger (1998) and Sfard and Prusak (2005). For Wenger (1998), identity and learning are intertwined since learning transforms identity, by changing who we are and how we participate in communities of practice, and learning is seen as an experience of identity since learning involves becoming a member (or not) of a community of practice.

For Sfard and Prusak (2005), identity is a collection of stories or narratives about a person, which constitute a person's identity and shape action. While these stories may be individually told, by the identity-builder or others, they are collectively shaped, meaning that narratives are co-constructed between people and social communities (Sfard & Prusak, 2005). Teachers and other significant individuals tell stories about learners, which inform the opportunities offered to learners to learn mathematics (Sfard & Prusak, 2005).

Based on the work of Wenger (1998) and Sfard and Prusak (2005), we are of the view that learners' mathematical identities inform their possibilities to learn and become members of mathematics communities through practice and narratives. However, Wenger's (1998) and Sfard and Prusak's (2005) definitions of identity tend to place a strong emphasis on the social construction of identity, and a weaker emphasis on the role of the individual in the construction of her/his identity. A strong focus on the social construction of identity limits understandings of the role of individual agency and how the notion of self is produced and related to identity (Radovic, Black, Williams & Salas, 2018). To understand how learners' mathematical identities are constructed, by considering both the individual and social aspects of identity, Marks and O'mahoney's (2014) work on personal identity, social identity and agency is discussed in relation to mathematics (Fig. 1).

Marks and O'mahoney (2014) explain that personal identity "emerges from the embodied, reflexive self, in part forged through interests and actions of the individual" (p. 71). Personal identities involve an understanding of self, emerging from learners' emotions, such as their enjoyment of learning mathematics, their motivations and their future projections of learning mathematics and is informed by social contexts as people learn about themselves through practice and discourse.

Social identities, according to Marks and O'mahoney (2014), are a "navigated position between personal identities and the ways in which people believe they should be perceived in a social setting" (p. 72). So personal identities inform how learners develop their social identities, by informing how they position themselves and would

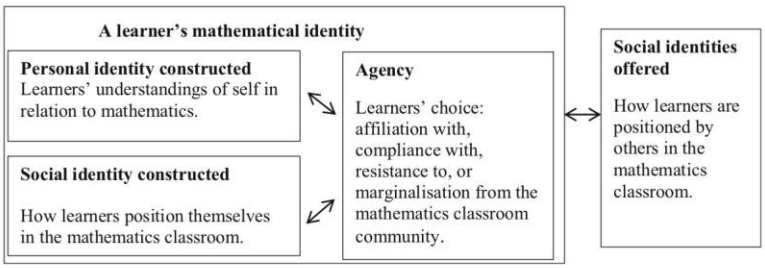



**Fig. 1** Constituents of a learner's mathematical identity

like to be seen by others as members of their classroom communities. Personal and social identities constructed by learners are intimately related to each other and both are related to social contexts, but it is important to keep them analytically distinct (Marks & O'mahoney, 2014).

Social identities are also offered to learners by others, for example teachers, who position learners in classrooms through the stories they tell about learners and the opportunities they provide for learners to participate in and become members of their classroom community, which includes making errors. While teachers offer learners social identities, learners have a choice to construct their identities differently from what is offered. This choice is informed by agency, which involves learners choosing to participate in certain ways, informed by their social contexts (Eteläpelto, Vähäsantanen, Hökkä & Paloniemi, 2013). Learner agency emerges from relationships between individuals and their social contexts and is always exercised (Biesta, Priestley & Robinson, 2017). Learner agency, in terms of how learners participate in the classroom, is informed by the social identities offered to them by their teachers and their personal and social identities. As learners exercise agency, they learn more about themselves, supporting their personal identities, and they assess the importance of their membership in their classroom communities, supporting their social identities.

During classroom interactions, teachers can continue offering learners the same social identity or may offer learners a different social identity from the one previously offered or constructed. Learners can also construct the same mathematical identity as previously, or reconstruct their mathematical identity and enact a different identity from previously, as informed by the social identities offered to them and their personal and social circumstances, highlighting the contingent nature of identities.

## Theorizing Learners' Mathematical Identities as Constructed in Classrooms

For this paper, we use the term 'mathematical identity' to refer to the identities that learners construct as they interact with their mathematics teachers in classrooms. Gardee (2019) has drawn on the work of Wenger (1998), Marks and O'mahoney (2014) and other research in mathematics education to develop a framework for learners' mathematical identities that embraces both individual and social dimensions of identity construction. The framework shows how learners can construct their mathematical identities when offered social identities of affiliation, partial affiliation or marginalization by teachers (Gardee, 2019).

Social identities of affiliation are offered when teachers provide learners with full access to the tools and resources of the community. Social identities of partial affiliation are offered when teachers provide learners with partial access to the tools and resources of the community. Social identities of marginalization are offered when teachers limit learner access to the tools and resources of the mathematics classroom community. Depending on how learners exercise agency and their personal and social identities, learners can identify with, comply with or resist the offered social identities by constructing their mathematical identities in affiliation with, marginalization from, compliance with or resistance to their classroom communities (Table 1) (Gardee, 2019).

Learners can identify with the social identities offered by teachers, by constructing their mathematical identities in affiliation when offered opportunities to affiliate or partially affiliate with the classroom community, or in marginalization when offered social identities of marginalization by teachers. Affiliating learners exercise agency by participating as full members of the community and by utilizing the tools and resources of the mathematics classroom community to develop their social identities as full participants. These learners tend to be emotionally invested in learning mathematics for their lives and futures, as indicative of their personal identities. Concomitantly, learners who develop their identities in marginalization when offered social identities of marginalization by teachers may not be emotionally invested in or motivated to learn mathematics for their lives and futures, as indicative of their personal identities. These learners exercise agency by engaging with the tools and resources of the mathematics classroom community in limited ways or not at all to develop their social identities as marginal members.

Learners who comply with an offered social identity of affiliation, partial affiliation or marginalization tend to exercise agency in similar ways to each other, by participating peripherally with the tools and resources of the mathematics classroom community. Irrespective of the kinds of social identities offered, compliant learners tend to limit their interactions with teachers and usually participate with other learners, so that they may still learn mathematics. These learners may or may not be emotionally invested in learning mathematics. For example, some compliant learners are emotionally invested in learning mathematics, but are not comfortable to participate in and become full members of the classroom community. Other compliant learners are not emotionally invested in learning mathematics and may prefer to participate peripherally to maintain peripheral membership.

**Table 1** Identities offered to and constructed by learners

| | Social identities offered | Mathematical identity constructed |
|---|---|---|
| Learners can identify with: | Affiliation or partial affiliation | Affiliation |
| | Marginalization | Marginalization |
| Learners can comply with: | Affiliation or partial affiliation | Compliance |
| | Marginalization | Compliance |
| Learners can resist: | Affiliation or partial affiliation | Resistance |
| | Marginalization | Resistance |

There may be learners who exercise agency by resisting or opposing the social identities offered to them.[2] Learners who resist the social identities of affiliation or partial affiliation offered by teachers may exercise agency by not participating as members of the community even though they are provided with full access to the tools and resources of the mathematics classroom community. These learners resist the offered social identity by marginalizing themselves from the community, even though they are not marginalized by the community. These learners may not be emotionally invested in learning mathematics or see the use of mathematics for their lives and may not want to develop their social identities as members of their classroom communities.

Concomitantly, learners who resist the social identities of marginalization offered by teachers exercise agency by attempting to participate as full members of the classroom community and by utilizing the tools and resources of the classroom community, despite being marginalized. While these learners resist the offered social identity of marginalization, they cannot fully affiliate to the classroom community since they are being marginalized.[3] These learners may be emotionally invested in and motivated to learn mathematics, as indicative of their personal identities, and may want to develop their social identities as full members of their classroom community.

In what follows, we provide an overview of some of the research on errors in mathematics, which illustrates the implications of learner errors on learners' mathematical identities and learning of mathematics.

## Literature Review

How learner errors are dealt with in classrooms have implications for how learners learn and understand what is needed to be a successful learner of mathematics. Learner errors usually have negative connotations because the use of marks in summative assessments indicates that errors are punishable (Nesher, 1987). There seems to be a tendency for teachers who teach using traditional pedagogical approaches to be unconcerned with the origin of errors, since being correct is the focus of such classrooms, and maintaining a positive identity involves not making errors (Heyd-Metzuyanim et al., 2015). This may not be the situation in all traditional classrooms, as Gardee and Brodie (2015) showed how a teacher who taught using traditional pedagogy used errors in ways that provided her learners with access to mathematics. While this teacher predominantly corrected errors, she also probed errors to access learner reasoning by asking learners for justifications for their errors and she embraced errors, by using learner errors in discussion to develop learner reasoning. Her

[2] While we did not find any learner in our larger study resisting any of the social identities offered to them, we have included the category in our framework for completeness, as other studies have shown that learners can resist the social identities offered to them (see for example, Nasir & Shah, 2011).

[3] In some situations, teachers may continue offering these learners identities of marginalization, as resistance to the offered social identity of marginalization can be seen as a form of resistance to the authority of the teacher. In other situations, teachers may offer learners a new social identity of affiliation when they see learners resisting the offered social identity of marginalization. Only when learners are offered a new social identity of affiliation, instead of marginalization, can they identify with the offered social identity to develop their mathematical identities as affiliating members of the community.

combination of correcting, probing and embracing did support learners' mathematical reasoning (Brodie, 2014; Gardee & Brodie, 2015).

The most common way of teachers interacting with learner errors is by correcting errors, thereby limiting potential opportunities for exploring learner errors in discussion, understanding their conceptual roots and using such errors to promote deeper engagement and reasoning in classrooms (Borasi, 1987; Brodie, 2014). Learners are shown to understand and learn better when errors are discussed in classrooms, rather than being corrected or avoided (Bray, 2011; Gardee & Brodie, 2015; Swan, 2001). Discussions about errors, guided by appropriate norms, can only occur when learners feel safe to make errors (Schleppenbach, Flevares, Sims & Perry, 2007), that is, when learners are offered social identities of affiliation with the classroom community. Avoiding and ignoring errors can potentially offer learners identities of marginalization from the classroom community, since they do not have access to understanding why their errors are incorrect, devaluing their knowledge and further limiting their understanding of mathematics.

Teachers' beliefs and knowledge can inform how teachers deal with learner errors (Bray, 2011; Sun, 2018). Some teachers may ignore or avoid learner errors because they may fear that errors are signs of their own failure (Brodie, 2014), or that errors are contagious and can spread to other learners (Swan, 2001). Teachers may not always be explicit that errors are problematic, but by ignoring and avoiding errors, they imply that errors are problematic and should be avoided (Ingram et al., 2013). Minimizing the opportunities for learners to make errors may inform learners' mathematical identities negatively, by implying that making errors is a reason for embarrassment and that arriving at the correct answer suggests that a learner is 'smart' (Heyd-Metzuyanim et al., 2015). When learners are provided opportunities to make and explore their errors, they construct their identities in ways that enable them to understand that making errors is a normal part of learning mathematics and that mathematical reasoning is more important than obtaining the correct solution.

Learner errors are also learning opportunities for teachers (An & Wu, 2012). Brodie (2013, 2014), Brodie and Shalem (2011), and Chauraya and Brodie (2017) showed how teachers who participated in professional learning communities learned how learner errors are sensible and reasonable from a learner's perspective. These teachers, through their discussions of learner errors, revised their current teaching approaches and developed their mathematical content knowledge. Therefore, engaging with learner errors in the classroom can promote deep mathematical knowledge for both teachers and learners.

## Methods

The data for this article are taken from a larger research study that focused on understanding learners' mathematical identities as they transitioned from grade 9 to 10 in a well-resourced secondary school in South Africa. Nineteen learners participated in the study, 8 in the first year, with an additional 11 in the second year, along with their 5 mathematics teachers, 2 in the first year and 3 in the second year. Two teachers, Mr. Molefe (grade 9) and Mr. Ncube (grade 10)—all names are pseudonyms—with four learners in each classroom, are discussed in this paper. These teachers displayed

different ways of interacting with errors and provided different accounts of how they interacted with learner errors. Both teachers had academic degrees to teach mathematics and each had 5 years of experience teaching mathematics.

Learner participants were initially selected by their grade 9 mathematics teachers using the following criteria: 1 high-performing learner, 1 low-performing learner and the person sitting next to them, which enabled us to observe teacher-learner interactions with learners displaying different levels of achievement. The grade 9 learners were observed again in grade 10, and additional learners, sitting next to the initial participants, were selected.

The four learners in Mr. Molefe's grade 9 class, discussed in this paper, were Sindiswa, selected as high-performing, and Jennifer sitting next to her, and Tess, selected as low-performing, and Sunny sitting next to her. Jennifer was identified as low-performing, while Sunny was identified as a learner whose marks were inconsistent. In grade 10, Sindiswa and Jennifer transferred to another school, while Sunny, who continued to participate in the study, was allocated to another mathematics teacher in the school. Tess was the only learner, from the initial grade 9 participants allocated to Mr. Ncube's grade 10 classroom. Tess sat in a row of four learners, with three other learners, Amy, Thandi and Thabo, who were included in the study. Thandi was a high-performer, Tess continued as a low-performer, while Amy and Thabo were average-performers.

We chose a case study design, which provides a holistic and relational understanding of themes (McMillan & Schumacher, 2010). A case study design was suitable for our study, which focused on the relationships between three themes: teacher practices, learner errors and learners' mathematical identities. The research design informed the data collection methods and analysis, and the findings from the analysis are representative of the larger sample.

Data were collected over 2 years (2015–2016) in the third school term (August–September) in the form of videotaped lessons, audiotaped semi-structured interviews with teachers and learners, field notes and photographed learner notebooks. We placed the video camera in front of the classroom to focus on learners to capture their participation and how their teachers interacted with them and their errors. Approximately 8 h of lessons were analyzed for each teacher. The interviews with the teachers were aimed at understanding how they spoke about learner errors and why they offered learners certain social identities. The interviews with the learners were aimed at understanding how and why they constructed their identities in certain ways and how they spoke about the importance (or not) of their errors.

We analyzed the data using qualitative approaches, by organizing our data into categories, followed by identifying relationships and trends among the categories (McMillan & Schumacher, 2010). We coded the videotaped data by coding the interactions between teacher and learner participants to identify the kinds of identities offered to and constructed by learners. We documented the teachers' pedagogical approaches, in terms of how the teachers structured their lessons, the kinds of opportunities that they provided for learners to participate and their approaches to learner reasoning in the classroom. The videotaped lessons were also analyzed based on how each learner participant participated in the classroom. We documented excerpts of errors that occurred in each lesson, noted which learner made the error and how the

**Table 2** Codes and criteria

| Social identities offered (SIO) | Mathematical identity constructed (MIC) | Personal identity constructed | Social identity constructed | Agency |
|---|---|---|---|---|
| Affiliation (A): Teacher invites participation and supports learners completely. | Affiliation (A) | Learner enjoys learning mathematics and sees the use of mathematics. | Learner is comfortable to be a full member of the community. | Learner participates fully with the tools and resources of the community. |
| Partial affiliation (PA): Teacher invites participation and supports learners to some extent. | Compliance (C) | Learner may or may not enjoy learning mathematics or see the use of mathematics. | Learner wants to maintain membership to some extent in the community. | Learner participates peripherally, usually because s/he is not comfortable to participate with the teacher. |
| | Resistance (R) | Learner does not enjoy learning mathematics or see the use of mathematics. | Learner does not want to become a member of the community. | Learner participates in limited ways, or not at all, with the tools and resources of the community. |
| Marginalization (M): Teacher limits learner opportunities to participate and learn mathematics. | Marginalization (M) | Learner does not enjoy learning mathematics or see the use of mathematics. | Learner does not want to become a member of the community. | Learner participates in limited ways, or not at all, with the tools and resources of the community. |
| | Compliance (C) | Learner may or may not enjoy learning mathematics or see the use of mathematics. | Learner wants to maintain membership to some extent in the community. | Learner participates peripherally, usually because s/he is not comfortable to participate with the teacher. |
| | Resistance (R) | Learner enjoys learning mathematics and sees the use of mathematics. | Learner desires to become a full member of the community. | Learner attempts to participate fully with the tools and resources of the community. |

teacher interacted with the error using Brodie's (2014) framework of correcting, probing and embracing errors.

We analyzed the interviews with a focus on teacher pedagogy, learner errors and learners' mathematical identities. Data from the teacher interviews were used to understand their pedagogical approaches, their understandings of learner errors, their approaches to learner errors and the social identities that they offered to learners. Data from the learner interviews were used to understand their mathematical identities, their understandings of errors and their experiences of their teachers' approaches to errors. We utilized the following criteria to code the identities offered to and constructed by learners (Table 2).

We then coded how learners spoke about errors, in terms of whether errors were important for learning, and we then compared how their understandings of errors were related to their teachers' approaches to errors. Thereafter, we looked for relationships between learners' mathematical identities and their teachers' interactions with errors.

For credibility, we triangulated the findings from all the data collection techniques (McMillan & Schumacher, 2010), that is, our observations of the lessons, the learner interviews and the teacher interviews.

## Results

We begin by discussing each teacher's practices and approaches to learner errors. We then describe and relate the social identities offered to learners by the teachers, the mathematical identities constructed by learners, how learners spoke about errors and learners' experiences of their teachers' approaches to errors.

### Teacher Practices and Approaches to Learner Errors

Teachers, through their pedagogical approaches and interactions with learners, provide messages about what mathematics is about and how it is best learned (Heyd-Metzuyanim et al., 2015). Table 3 shows the differences between the teachers' pedagogical approaches, approaches to learner reasoning, understandings of and approaches to learner errors.

Both teachers' pedagogical approaches were based on what they thought enabled learners to learn mathematics best. For Mr. Molefe, lengthy explanations together with multiple examples enhanced learning, and he discouraged learners from using other methods besides the ones he taught. Whereas Mr. Ncube preferred group discussions where learners devised their own solutions. Both pedagogical approaches were informed by the teachers' approaches to learner reasoning, and ultimately, their understanding of mathematics as a discipline. For Mr. Molefe, mathematics was pre-discovered, and the purpose of learning mathematics was to "make sense" of what was already known. For Mr. Ncube, mathematics was about interrogating solutions and coming to agreements.

These teachers' beliefs and knowledge informed how they understood and dealt with learner errors. For Mr. Molefe, errors had negative connotations, being able to "grow" and "breed". He organized his lessons by limiting learner opportunities to explore and devise their own methods and solutions to problems to avoid the occurrence of errors,

**Table 3** Teacher practices and approaches to learner errors

| | Mr. Molefe | Mr. Ncube |
|---|---|---|
| Pedagogy | Mr. Molefe did not encourage peer work and spent much of his lessons giving lengthy explanations and multiple examples, and asking questions requiring the recall of information with time in some lessons allocated to classroom tasks.<br>Mr. Molefe explained: "I teach by example. So if I have a concept, I try to expand it into five different perspectives. I give an example of this perspective and I give an example of that perspective". | Mr. Ncube asked learners sequences of lower-order questions, followed by providing them with opportunities to work with others on classroom tasks. Learners discussed their classroom tasks with others, while Mr. Ncube walked around and helped learners.<br>Mr. Ncube explained: "When I give them classwork, I'd give them time to discuss about the question before they attempt. Discuss. They discuss, they come to an agreement. If they totally disagree on something, they shout for help. I'll go there then I will assist". |
| Approach to learner reasoning | Mr. Molefe told learners: "If you follow instructions, you will be better than a calculator".<br>He did not encourage learners to use other methods than those that he taught: "I do not want … there are only two ways, this way or that way. There is no other way. Do not try to be creative neh. Please, creativity is good, especially in arts and culture. Not here. Einstein already figured this out. Newton, that's their job. All we do is make sense of it". | Mr. Ncube told learners: "I want you to be in a position to criticize the answer that you get if it does not make sense".<br>He gave learners a chance to work out the solutions for themselves: "After they discuss, each and everyone will write on his or her own, to write the actual writing of the question and the solution, but they have to discuss it first. They assist one another as they are seated before I intervene because I need to give them time". |
| Understandings of learner errors | Errors are contagious. He told learners that he will need to "correct their (learners') brains" if he sees errors. | Errors are lessons. He told learners: "Do not feel bad about being corrected. It is part of learning". |
| Approach to learner errors | Mr. Molefe usually corrected errors by reexplaining concepts; however, he was not accommodating of errors: "I give them strict limitations to say we doing this. My approach is to never really allow them to adventure because if I allow them to explore some of these concepts, I fear that some of the misconceptions come out. But you find that some people say allow them to explore so that they get the feel. It can only work that way if they are doing the right way. But what if they were doing the wrong way? For example, if a learner makes a mistake, you have to correct it as soon as possible before that misconception grows, you breed it". | Mr. Ncube usually corrected errors after probing them. He was accommodating of errors: "They are called learners. They should learn from their mistakes. That is the reason why we call them learners. So I encourage them. They make mistakes. They get corrected then it is easy for them to remember something that they have been corrected because there is no way they can get everything correct. They need to be corrected from their own mistakes. I always, I went down to the extent of telling them that there is a point that I got 2% in maths. But that did not, that did not discourage me from putting more effort". |

and usually corrected errors when they occurred using lengthy explanations. For Mr. Ncube, errors were discussed positively as lessons, and he used errors as a means of encouraging learners to learn from their mistakes. He structured his lessons to support learner reasoning and he often probed learner errors to access their reasoning before correcting errors. For example, he would ask learners to justify their thinking when they made errors, followed by providing learners with the correct answers.

### Identities Offered and Constructed: Learners in Mr. Molefe's Classroom

Mr. Molefe offered learners social identities based on how he identified them as learners in his classroom. Sindiswa was offered a social identity of affiliation since she was identified as a high-performing learner. Sunny was offered a social identity of partial affiliation and was identified by Mr. Molefe as a learner whose marks were inconsistent and as a "naughty learner", who could ask thought-provoking questions followed by doing "something very silly". We did not observe any instances of Sunny displaying unruly behaviour.

Tess and Jennifer were identified as low-performing learners and were offered social identities of marginalization from the classroom community. Mr. Molefe explained that he was not always willing to assist low-achieving learners because he did not think that he had sufficient training to help low performers. He did not interact much with Tess and Jennifer or help them when he saw them struggling or making errors in their books, thereby denying them the opportunity of understanding why their mathematical reasoning was incorrect.

Table 4 illustrates the social identities offered to learners and how learners constructed their identities and discussed the importance of making errors in their classroom (Table 4).

From the four learners in Mr. Molefe's classroom, Sindiswa was the only learner to develop a mathematical identity of affiliation, while the other learners developed identities of compliance (Table 4). Sindiswa was motivated to learn mathematics, indicative of her personal identity, and aspired to develop her social identity as one of the best learners of mathematics in her classroom. Even though Sindiswa developed an identity of affiliation, Mr. Molefe's approach to errors informed her mathematical identity, in terms of how she exercised agency to participate in the classroom. She felt "bad" when making errors even though she thought that errors were important to learn mathematics, so she was sometimes hesitant to participate. However, Sindiswa was able to develop her identity in affiliation and learn from her errors since she was offered a social identity of affiliation and was assisted by Mr. Molefe when she made errors.

Mr. Molefe's approach to errors also informed Jennifer's and Tess's mathematical identities. While Jennifer did not feel "bad" about making errors in her workbook, she did not interact much with Mr. Molefe because she was not comfortable to make her errors public and because he was not always willing to assist her. Tess shared Mr. Molefe's view of errors as problematic and did not interact much with him in the classroom. Even though both learners exercised agency by not participating much with Mr. Molefe, Jennifer and Tess developed their social identities as peripheral members, by asking other learners to assist them because of their personal identities—they saw the use of mathematics for their lives and futures.

Sunny was the only learner who experienced Mr. Molefe's approach to errors differently from the account narrated by Mr. Molefe. He explained that Mr. Molefe's

**Table 4** Identities offered and constructed

| Learners | SIO | MIC | Personal identity constructed | Social identity constructed | Agency | Understanding of errors | Experience of teacher's approach to errors |
|---|---|---|---|---|---|---|---|
| Sindiswa | A | A | She "love(d)" mathematics and was motivated to pursue mathematics in the future. | She was a full member of the classroom community. | She often participated in the classroom but was sometimes hesitant. | Making errors were important to learn. | Mr. Molefe "sometimes becomes shocked", which made her "feel bad"; however, he corrected her errors. |
| Jennifer | M | C | She "love(d) math", wanted to "pass" mathematics and pursue mathematics in the future. | She was not comfortable to be a full member and developed a peripheral membership. | She did not participate with Mr. Molefe. She asked Sindiswa for help. | She did not "feel bad" when making errors in her book. | Mr. Molefe was not always willing to help her and answer her questions. |
| Tess | M | C | She found mathematics "difficult" but she wanted to pass and pursue mathematics in the future. | Even though she did not participate with Mr. Molefe, she maintained a peripheral membership. | She did not participate with Mr. Molefe, but usually asked a friend to help her. | Errors are "bad things" and she did "not feel good at all". | Mr. Molefe discouraged learners from making errors: "He'll tell you do not do mistakes in your book". |
| Sunny | PA | C | He did not "like" mathematics, but he was motivated to pursue mathematics in the future. | He maintained a peripheral membership as he was more concerned about fitting in with his peer group. | He completed his tasks and often attempted to participate in the classroom. | "No one is safe from making mistakes" but "you learn" from errors. | Mr. Molefe responded to his errors by disagreeing with him in a "polite" way and he would ask another learner to help him. |

approach to his errors made him feel "good" and "confident" to engage with mathematics, possibly explaining why he continued to participate in the classroom even though he did not enjoy learning mathematics. However, for Sunny, developing his social identity as a member of his peer community was more important to him than becoming a full member of the mathematics classroom community. He explained that he participated in the classroom community not because he wanted to "excel" in or "fail" mathematics, but because he was a "friendly" person and was concerned with how others would identify him. He was primarily interested in passing mathematics to pursue his future career choice, thereby developing a peripheral membership in the classroom community.

### Identities Offered and Constructed: Learners in Mr. Ncube's Classroom

Mr. Ncube provided all learners in his classroom the opportunity to participate and construct their mathematical identities in affiliation with his classroom community. He encouraged all learners to participate and believed that everyone could do mathematics and that "effort" and "practice" were needed for learners to be successful at learning mathematics. Table 5 illustrates the social identities offered to learners and how learners constructed their identities and discussed the importance of making errors in their classroom (Table 5).

In Mr. Ncube's classroom, Tess and Thabo developed mathematical identities of compliance, while Thandi and Amy developed mathematical identities of affiliation (Table 5). Tess was motivated to pursue mathematics in the future, indicative of her personal identity; however, she did not interact much with Mr. Ncube in the classroom. Rather, she exercised agency by asking others for help, thereby developing a peripheral membership. She explained that she preferred to "listen" to Mr. Ncube teach, rather than to participate with him in the classroom. While Mr. Ncube assisted Tess when she made errors, Tess's past experiences of errors in Mr. Molefe's classroom seemed to have still shaped her participation and perception of errors in Mr. Ncube's classroom. She continued to speak about errors negatively, even though Mr. Ncube made her feel "good" after he corrected her errors.

While Tess experienced Mr. Ncube's approach to errors positively, Thabo did not. Like Tess, Thabo developed his identity in compliance, as he was not comfortable to participate fully with Mr. Ncube. He explained that he was hesitant to ask Mr. Ncube for assistance, because even though Mr. Ncube "smiles" and understood him when he made errors, Mr. Ncube's approach to errors, by making the answers seem easy when he found the problems challenging, made him feel "stupid". In addition, Thabo explained that mathematics was "holding (him) down" due to the time required to understand mathematics, even though he said that he enjoyed learning mathematics, as part of his personal identity. Consequently, Thabo developed his social identity as a peripheral member of the classroom community, by participating peripherally to pass mathematics so that he could pursue his future aspiration.

Like Thabo, Thandi did not experience Mr. Ncube's approach to errors positively. Unlike Thabo, Thandi was comfortable to participate in her classroom community, even though she explained that Mr. Ncube did not seem "happy" when she made errors. Her experiences of Mr. Ncube's approach to errors motivated her to exercise agency by "work(ing) harder" to maintain her full membership in his classroom.

Thandi developed her mathematical identity in affiliation with the classroom community since mathematics was her favourite subject, indicative of her personal identity and that she felt a sense of belonging in her classroom community, as indicative of her social identity.

Like Thandi, Amy developed her mathematical identity as an affiliating member of the community and was comfortable to participate in the classroom and ask Mr. Ncube for assistance. Like Tess, she experienced Mr. Ncube's approach to errors positively and explained that Mr. Ncube would tell her to "think" about her errors before assisting her. She said that she was "grateful" to Mr. Ncube for being accommodating of errors by correcting her errors, which proved to her that he "care(d)" about her. She constructed her social identity as a full member of the community and explained that she enjoyed learning mathematics and was motivated to pursue mathematics in the future, as indicative of her personal identity.

## Discussion

The purpose of this paper has been to identify the relationship between teachers' interactions with learner errors and learners' mathematical identities. To answer our first research question regarding how the teachers spoke about and interacted with learner errors, Mr. Molefe said that errors are contagious, and Mr. Ncube said that errors are lessons and learners can learn from them. These views of errors are common in research as some teachers correct errors for learners to gain access to correct knowledge (Brodie, 2014), whereas others may dislike the occurrence of errors, because they think they are contagious (Swan, 2001).

Both teachers emphasized that errors were mistakes that needed to be corrected, rather than being sensible and reasonable based on some correct mathematical knowledge. Mr. Molefe corrected errors, while Mr. Ncube probed errors before correcting them. Correcting errors is not sufficient in enabling learners to learn from them. From a constructivist perspective, correcting errors does not address underlying misconceptions or lead to the restructuring of knowledge (Borasi, 1987). From a situated perspective, correcting errors may not provide learners with an understanding of why their criteria of what counts as mathematical knowledge were incorrect (Brodie, 2014). Neither teacher embraced learner errors; they did not use errors to generate new knowledge for all learners in the classroom.

The second research question is concerned with what informed how the teachers spoke about and interacted with learner errors. There were two main mechanisms that informed how the teachers spoke and interacted with learner errors: the teachers' views on the role of learner reasoning in mathematics and their narratives about learners. Mr. Molefe was not accommodating of errors because he did not think that learner reasoning was important to learn mathematics, while Mr. Ncube was accommodating of errors, by probing learner errors and encouraging learners to think about their solutions. This finding resonates with current research, which shows how teachers' beliefs and knowledge inform how teachers deal with learner errors (Bray, 2011; Sun, 2018).

How the teachers spoke about learners informed their willingness to help and support learners, and, consequently, the mathematical identities which they offered to learners. For example, Mr. Molefe corrected Sindiswa's and Sunny's errors since he

**Table 5** Identities offered and constructed

| Learners | SIO | MIC | Personal identity constructed | Social identity constructed | Agency | Understanding of errors | Experience of teacher's approach to errors |
|---|---|---|---|---|---|---|---|
| Tess | A | C | Mathematics was difficult. She was motivated to pursue mathematics in the future. | Even though she did not participate with the teacher, she maintained a peripheral membership. | She did not participate with Mr. Ncube unless he called on her to participate. She asked a friend to help her. | Errors were "bad", which made her feel "sad". | Mr Ncube helped her, making her feel "good" after she understood. |
| Thabo | A | C | Mathematics was challenging and "holding (him) down", even though he enjoyed learning mathematics and was motivated to pursue it in the future. | He was not comfortable to participate as a full member and developed a peripheral membership. | He participated with Mr. Ncube but was not comfortable to ask him for help. He participated with other learners. | For him, "all people make mistakes" and he learned from his errors. | Mr. Ncube "smiles" and understood. "He usually says we are all here to learn". However, Mr. Ncube's approach to errors made him feel "stupid", by making the answers that he found challenging seem easy. |
| Thandi | A | A | Mathematics was her favourite subject. She was motivated to pursue mathematics in the future. | She maintained full membership and felt like she was "part" of her classroom community. | She participated with Mr. Ncube when she needed assistance. She assisted other learners. | She made "a lot of mistakes" in mathematics and explained that her teacher assisted her. | Mr. Ncube "does not look very happy. But then, he just explains it till you get it. It makes me feel, work harder. Make him smile". |
| Amy | A | A | Mathematics was her favourite subject. She was motivated to pursue mathematics in the future. | She maintained full membership and felt that she was "part of the class". | She participated with Mr. Ncube and other learners. | She made errors and learned from them because of Mr. Ncube's approach to errors, which made her feel that he "care(d)" about her and "grateful" to him for his assistance. | Mr Ncube "is very helpful". He told her to "think about" the error and suggested how she could correct the error. |

did not see them as low-achieving learners, and they were offered social identities of affiliation and partial affiliation. Tess and Jennifer were described as low-achieving learners by Mr. Molefe, and he was not always willing to assist them and offered them social identities of marginalization. Unlike Mr. Molefe, Mr. Ncube was willing to help all learners and offered all learners the opportunity to participate and construct their mathematical identities in affiliation with the classroom community. All four learners in his classroom explained that he was willing to assist them when they needed help.

The third research question is concerned with how the teachers' approaches to errors informed learners' mathematical identities. In Mr. Molefe's grade 9 classroom, becoming a successful learner of mathematics involved not making errors, while in Mr. Ncube's grade 10 classroom, becoming a successful mathematics learner involved learning from errors. The role of classroom environments in providing learners with opportunities to become members of the community is well established in research (Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009). Different classroom cultures are associated with providing learners with messages about what being a good learner of mathematics means. So, the teachers' approaches to errors, or perceived approaches, had some bearing on how the learners participated in the classroom and constructed their identities. Yet learners too play a role in the construction of their mathematical identities, supported by their personal identities, social identities and agency.

In Mr. Molefe's classroom, Sindiswa and Tess felt guilty for making errors, while Jennifer did not make errors publicly. Sindiswa was hesitant to participate, while Jennifer and Tess did not interact much with Mr. Molefe. Feelings of shame when making errors is common in classrooms where the errors are associated with discourse about people (Heyd-Metzuyanim et al., 2015). For the girls, maintaining a social identity of membership in Mr. Molefe's classroom, where one does not make errors, was more important than learning from errors. Yet Sindiswa's personal and social identity supported her identification with the offered social identity of affiliation, as she was still willing to participate with Mr. Molefe since she was provided with the opportunity to do so and her errors were corrected. Tess and Jennifer, who were both marginalized, were not comfortable to participate with Mr. Molefe, and both learners developed their social identities as peripheral members since they were motivated to pursue mathematics in the future, supporting their compliance.

Unlike the girls, Sunny experienced Mr. Molefe's approach to errors positively, making him feel confident to participate in the classroom. He constructed his identity in compliance, even though he was offered a social identity of partial affiliation, as he was interested in passing mathematics to fit in with his peers. Studies have shown how for some learners, friendship groups may be privileged above the classroom community (Boaler, William & Zevenbergen, 2000). So for Sunny, being a member of his friendship group was more important than becoming a successful learner of mathematics.

In Mr. Ncube's grade 10 classroom, Amy experienced positive emotions when Mr. Ncube corrected her. Her personal and social identity supported her identification with the offered social identity of affiliation and she participated as a full member of the classroom community. Like Amy, Tess too experienced positive emotions when she was corrected by Mr. Ncube, yet she still felt guilty when making errors and was uncomfortable to participate in the classroom. It is likely that her experiences of learning mathematics and making errors in Mr. Molefe's classroom still shaped her

understanding of what being a successful learner of mathematics meant in grade 10, so she continued to enact an identity of compliance.

Thabo and Thandi experienced some level of shame when making errors, based on how they experienced Mr. Ncube's approach to errors. Yet both learners used their experiences differently, with Thandi becoming motivated to work harder to become an affiliating member of the community, which was supported by her personal and social identity, while Thabo was not willing to ask Mr. Ncube for assistance. Like Jennifer and Tess (in both years), Thabo maintained some level of membership in the community so that he could pass mathematics, for his personal and social identity, supporting his compliance.

## Conclusions

Our investigation of the relationship between teachers' interactions with learner errors and learners' mathematical identities revealed that learners' experiences of their teachers' approaches to errors informed learners' mathematical identities to some extent. The learners also played a role in the construction of their mathematical identities, which was informed by how they exercised agency, and their personal and social identities.

The teacher approaches to learner errors were shaped by their narratives regarding the importance of learner reasoning in mathematics and by the kinds of social identities that they offered learners to become members of their classroom communities. Interestingly, the learners discussed in the paper developed their mathematical identities as affiliating or compliant members of their classroom communities.[4] None of the learners resisted the social identities offered to them, or challenged their teachers, even when they did not share the same perspective on errors as their teachers. A possible explanation why none of the learners resisted the offered social identities or challenged their teachers when they did not share the same perspective on errors as their teachers, is that teachers play an important role in shaping learners' mathematical identities in the classroom community (Grootenboer & Zevenbergen, 2008). It is also likely that for some learners, past teachers can play a significant role in informing learners' current mathematical identities, like in the case of Tess who enacted the same mathematical identity in both Mr. Molefe's and Mr. Ncube's classrooms. So while learners' mathematical identities can change as contexts change (Radovic et al., 2018), it is possible that some learners may continue enacting the same identity constructed previously in another teachers' classroom across learning contexts, perhaps because these past experiences were significant to the learner.

Another possible explanation for why none of the learners resisted the offered social identities could stem from understanding the power of learners' personal and social identities. Sfard and Prusak (2005) explain that future aspirations shape actions and identities to a great extent, and Wenger (1998) suggests that these aspirations assist people in understanding whether they belong or not to communities supporting their personal and social identities. These learners were motivated to pursue mathematics in

[4] Within the wider sample, there were learners who developed their identities in marginalization from the community. No learners in the wider study resisted any of the social identities offered. See Gardee (2019).

the future and resisting their teachers' practices could have had negative implications in achieving their future goals and their feelings of belonging to the classroom community. The need for feeling a sense of belonging to the classroom community was key for all these learners, since all of them participated in ways to develop their social identities as full or peripheral members of the classroom community, even though some of them were offered social identities of marginalization. It is therefore important to consider the relationships between personal identity, social identity—both constructed and offered—and agency to understand how learners construct their mathematical identities.

While there have been teacher education programs focused on the importance of learner errors and on ways to create environments for learners to participate in, make and learn from their errors in their mathematics classrooms (see Brodie, 2013, 2014; Chauraya & Brodie, 2017), there is a need for programs to educate teachers about the necessity of offering all learners identities of affiliation. Teacher education programs can assist teachers in becoming aware of the harmful effects of categorizing learners according to their performances in assessments and behaviour, as a means of helping teachers offer all learners equitable opportunities to participate and develop identities of affiliation.

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**Disclaimer** Opinions expressed and conclusions arrived at are those of the authors and not necessarily attributed to the National Research Foundation.

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# Chapter 7: A framework for analysing learners' mathematical identities in peer interactions: accounting for personal and social identities

## 7.1. Introduction

This chapter is different from the previous four chapters, as this chapter focuses specifically on the relationships between learners' mathematical identities and their interaction with their peers. I discuss how two pairs of secondary school learners, who were treated similarly by their teachers and who performed similarly academically, constructed and offered mathematical identities. The reason I chose learners who were treated similarly by their teachers was that there are studies that show how teachers inform the ways learners interact with others and construct identities. My purpose was different, as I wanted to develop a framework to define and research learners' mathematical identities constructed in peer interaction, to show the importance of learners' personal and social identities.

This chapter builds on the non-reductionist model developed in chapter 2 to develop an explanatory framework for the relationships between learners' mathematical identities and the social identities offered by peers during peer interactions in classroom contexts. This chapter provides another perspective to answering the first main research question, in terms of what mathematical identity is and by showing how the concept of identity defined in chapter 2 can be used in researching secondary school learners' mathematical identities constructed during peer interactions. Using the model developed in chapter 2, an explanatory framework is developed to relate the kinds of social identities offered by peers, as higher status, lower status or equal status, to the kinds of mathematical identities that can be constructed during peer interactions, as affiliation with or resistance to the offered social identities. An analytical framework is developed and applied to data to enable understandings of *how* learners construct mathematical identities during peer interactions, and *why* learners construct certain mathematical identities, by drawing on their personal and social identities.

# A framework for analysing the relationships between peer interactions and learners' mathematical identities


## Abstract

Learner interactions have a powerful influence on learners' mathematical identities and their learning of mathematics. Yet the absence of definitions of identity, which consider both subjective/personal and social aspects of identity, is an important concern in mathematics education research. The purpose of this paper is to propose a framework to analyse learners' mathematical identities, by considering personal and social identities. To exemplify the framework, we discuss how two pairs of secondary school learners, who were treated similarly by their teachers and who performed similarly academically, constructed and offered micro-identities during moments of a lesson, and macro-identities across lessons. The two pairs were selected so that we could analyse how they constructed and offered identities differently from each other, independently from the influence of teacher positioning or differences in capabilities, with a focus on their personal and social identities. We show that while learners were offered identities by a peer as higher, lower or equal status, they constructed their identities by affiliating with or resisting the identities offered by their peer. Their identity construction was informed by their personal identities, in terms of their motivations for and enjoyment of learning mathematics, and their social identities, in terms of their choices to occupy certain roles in the classroom community.



Keywords: Learners' mathematical identities, Peer interactions; Participation; Narrations


## 1. Introduction

In recent years, researchers in mathematics education have turned to the concept of identity to understand how learner interactions influence their learning of mathematics (Andersson & Wagner, 2019; Bishop, 2012; Wood, 2013). These researchers have shown how learner interactions inform their mathematical identities, in terms of who they are, who they might become, and how they see others, as learners of mathematics (Andersson & Wagner, 2019; Bishop, 2012; Gresalfi & Hand, 2019). Concomitantly, learners' identities inform their learning, in terms of their engagement in and disengagement with mathematics (Graven & Heyd-Metzuyanim, 2019; Wood, 2013).

Learners' mathematical identities are usually examined on one of two scales: a micro-scale as micro-identity, or a macro-scale as macro-identity (Wood, 2013). Wood (2013) defines a micro-identity as "the position of a person in a moment of time" (p. 780), and these identities can shift in different moments of a lesson as learners interact with others. For example, Wood (2013) showed how in a lesson, a learner shifted his micro-identity from being an explainer of mathematical concepts when positioned as an explainer by his teacher, to being dependent on his peer for knowledge when positioned by his peer as being less mathematically capable than herself, and then back to being an explainer of mathematical

concepts after his teacher positioned him as capable again. So the learner's micro-identity shifted from being mathematically capable to less capable and then back to being capable in different moments of the same lesson, influenced by the different social positions which were offered to him by his teacher and peer (see also Holland & Leander, 2004). So micro-identities are responsive to small changes in contexts, like engaging with different people, mathematical concepts and tasks at different moments in the same lesson (Wood, 2013). Over time, micro-identities can 'thicken', as learners come to identify themselves as certain people, forming their macro-identities (Wood, 2013).

Macro-identities are also constructed through interactions, but unlike micro-identities, these identities emerge over long periods (Wood, 2013). Macro-identities are more persistent, stable and longer-term constructions of who a person is (Wood, 2013). Macro-identities can be accessed through interviews and by observing learner interactions over time. For example, research on macro-identities shows how learners described their identities constructed over a course (e.g., Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009), and how learners' interactions with others contributed to them becoming identified as superior, inferior, smart or bossy across time (Bishop, 2012; Langer-Osuna, 2011). These studies do not show how learners' identities shift from moment to moment, but rather, how learners' identities 'thickened' over time.

Many studies show how learners' interactions with their peers and teachers inform learners' learning of mathematics and their mathematical identities, either in moments during a lesson as micro-identities, or over long periods as macro-identities. Some studies show how learners who collaborated with others on activities learned mathematics and developed their identities positively (DeJarnette & González, 2015; Esmonde, 2009). DeJarnette & González (2015) showed that when learners shared ideas and challenged themselves and others, they deepened their learning and reasoning skills. The learners shared the mathematical authority, so none of the learners sustained a particular micro-identity throughout the interaction sequence (DeJarnette & González, 2015). Esmonde (2009) made a similar finding, as she showed how a group of learners collaborated and did not position each other as being more or less capable than the others, so all learners in the group could make their thinking visible and engage with each other's ideas.

Other studies show how one learner dominated peer discussions and did not consider the ideas of others, especially when that learner was identified by others or identified themself as more capable than others (Bishop, 2012; DeJarnette & González, 2015; Lerman, 2001; Esmonde, 2009). Heyd-Metzuyanim and Sfard (2012) showed how two successful

learners, Dan and Ziv, were offered different micro-identities by other members of their group based on their relationships with the other learners. Dan was well-liked by others and was perceived as being understandable, even though his ideas were incorrect. Ziv was not well-liked by others in the group, as he projected himself as being more capable than the others. His mathematical ideas were correct, but he was excluded from the group and positioned as not being understandable, even though his peers did not listen to or engage with his ideas.

Wortham (2004) showed how a learner was initially identified by her teachers as a good student, and later as disruptive. Over time, the learner's interactions with her teachers and peers changed, which resulted in a 'thickening' of her identity to form her macro-identity, as she became consistently identified as an 'outcast' (Wortham, 2004). Langer-Osuna (2016) found that teacher interactions with learners reinforced authoritative positioning and shaped how learners authoritatively interacted with others. Several other studies have also shown how teachers' interactions with learners informed how learners participated in the classroom and identified themselves and/or others as learners of mathematics (Boaler & Staples, 2008; Hodgen & Marks, 2009; Author, 2019 a, b; Authors, 2021; Author, in preparation; Lerman, 2001; Wood, 2013).

So, the influence of others, such as peers and teachers, on learners' interactions and identities is well explored in research, i.e., most research focuses on the social aspects of identity, rather than the subjective/personal aspects. While there have been attempts to define identity in mathematics education research, the absence of clear definitions of identity with a focus on both, the subjective and social aspects of identity, is an important concern for researchers (Graven & Heyd-Metzuyanim, 2019). The subjective aspect of learners' mathematical identities requires more attention in research, so that the role of the individual in the construction of her/his identity is not neglected (Graven & Heyd-Metzuyanim, 2019; Radovic, Black, Williams & Salas, 2018).

To address these difficulties, the purpose of this paper is to develop a framework to define and research learners' mathematical identities constructed in peer interaction, with a focus on the subjective and social aspects of identity, as personal and social identity. To exemplify the framework, we focus on how two pairs of secondary school learners, who were treated similarly by their teachers and who performed similarly academically, constructed and offered mathematical identities, as micro-identities in moments of a lesson, and macro-identities over time. We selected these two pairs to analyse differences in mathematical identities, as informed by learners' personal and social identities, rather than teacher

positioning or differences in learner capabilities, which is already well-researched (e.g., Bishop, 2012; Hodgen & Marks, 2009; Wood, 2013). Using our framework, we answer the following research questions:

1. How do learners construct and offer micro- and macro-identities during peer interactions in mathematics classrooms?
2. How do learners' personal and social identities inform the micro- and macro-identities that they construct and offer during peer interactions?

Our first research question explores *how* learners construct and offer micro- and macro-identities, while the second research question explores *why* learners construct and offer certain identities, with a focus on their personal and social identities.

## 2. Conceptual and analytic framework

In mathematics education research, the definitions of identity proposed by Wenger (1998) and Sfard and Prusak (2005) are commonly used to understand learners' mathematical identities. For Wenger (1998), identity is enacted, as a negotiated process related to learning, because learning involves becoming a member of a community of practice. As a person learns through participating in the community, s/he defines her/himself and is defined by others as a member of the community. For Sfard and Prusak (2005), identity is discursive, as a collection of socially constructed narratives by and about a person, which shapes their actions. Narratives can be told by people about themselves, or by significant others, such as stories told by teachers or peers about a learner, which can contribute to the learner's actions and identity (Sfard & Prusak, 2005). Both Wenger (1998), and Sfard and Prusak (2005), emphasise the social nature of identity and underplay the subjective, personal aspect of identity, involving the notion of self and agency (Radovic et al., 2018).

There is a need for theory accounting for the social, subjective, enacted and discursive aspects of identity (Radovic et al., 2018). Combining the work of Marks and O'mahoney (2014) and Archer (2002), with Wenger (1998) and Sfard and Prusak (2005) can serve this purpose. Using their ideas, we define learners' mathematical identities as being constituted by relationships between personal identities, social identities and agency, which emerge as learners use discourse to define themselves and participate in communities of practice.

Personal identity focuses on internal processes and emerges from the embodied and reflexive self, involving a person's interests, motivations, emotions and practices (Archer, 2002; Marks & O'mahoney, 2014). Personal identity involves an understanding of self and is "a matter of what we care about in the world", which informs what people prioritise (Archer,

2002, p. 15). Archer (2002) explains that engaging with people and tasks inform personal identity in terms of self-worth, since success and failure are reflected emotionally, thereby regulating behaviour. While personal identity is often discussed in relation to personal preferences, the role of the social world is intimately related to personal identity (Archer, 2002; Marks & O'mahoney, 2014). However, neglecting the importance of personal identity in research undermines the importance of the person in constructing her/his identity.

Social identity focuses on what people "care about in social roles" (Archer, 2002, p. 17). Marks and O'mahoney (2014) explain that social identity is a "navigated position between personal identity and the way in which people believe they should be perceived in a social setting" (p. 72). So social identity is distinct from yet related to personal identity, shaping how learners position themselves in communities by choosing to embody roles they care about, which are generated by classroom communities. Society makes social identities available to people, through the roles generated by social structures (Marks & O'mahoney, 2014). We refer to these social identities as the social identities offered to learners. In mathematics classrooms, teachers and peers offer learners social identities through the opportunities they provide for learners to participate in and become members of the classroom community.

While learners may be offered certain social identities, they exercise agency, by appropriating, or not appropriating, the social identities offered to them. Agency involves learners making active choices, which are influenced by social contexts (Eteläpelto, Vähäsantanen, Hökkä & Paloniemi, 2013). These choices can be exercised through compliance or resistance (Gresalfi, Martin, Hand & Greeno, 2008). How learners exercise agency, by participating in classrooms and learning mathematics, is influenced by the social identities offered to them by their peers and teachers, and their personal and social identities. Concomitantly, through exercising agency, learners learn more about themselves, supporting their personal identities, and they can evaluate their membership in the community, supporting their social identities.

Drawing from the ideas of Marks and O'mahoney (2014), Esmonde (2009) and Gresalfi et al. (2008), we developed the following diagram to illustrate the relationship between the social identities offered to learners by their peers and learners' mathematical identities, which can be accessed in moments of a lesson as micro-identities, or across lessons as macro-identities (Fig. 1).

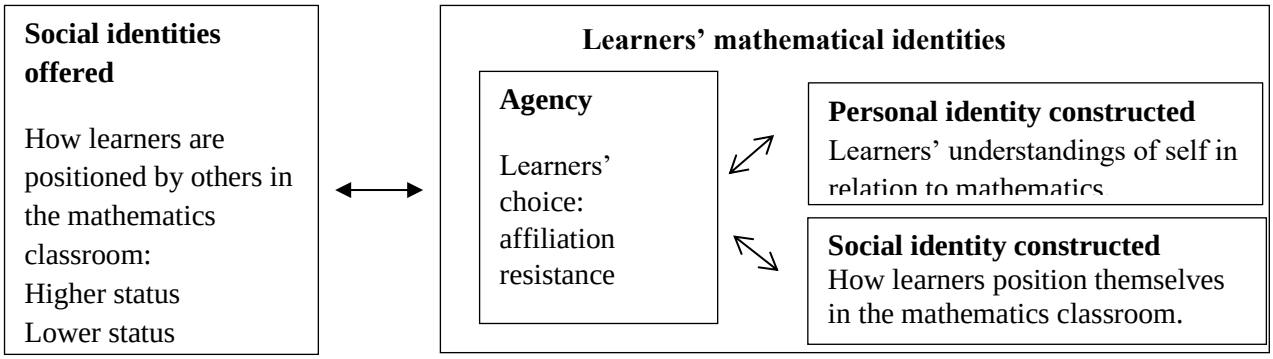



**Fig 1 Constituents of mathematical identity**

Mathematical identity is a social phenomenon and emerges from relationships between learners' personal identities, social identities and agency. Learners' mathematical identities are shaped by the social identities offered to them by the people they interact with, like their peers and teachers [1]. The social identities offered to learners by others can be examined on a micro-scale, as the offered micro-identities at a moment, or a macro-scale, as the offered macro-identities over long periods. The micro-identity offered to a learner can shift in moments, as the learner may be offered different positions by her/his peer in different moments. These offered micro-identities can thicken to form the macro-identities offered, as the peer comes to identify and interact with the learner in certain ways across lessons.

In peer interactions, learners can offer their peers micro- and macro-identities of higher, lower or equal status. Micro- and macro-identities of higher status are offered when learners position their peer as being more mathematically capable than themselves (Esmonde, 2009). Micro- and macro-identities of lower status are offered when learners position their peer as being less mathematically capable than themselves (Esmonde, 2009). Micro- and macro- identities of equal status are offered when learners position their peer as equally knowledgeable.

Depending on learners' personal and social identities and how they exercise agency, they can construct their micro- and macro-identities by affiliating with or resisting the offered micro- or macro-identities. Learners affiliating with an offered micro- or macro-identity of higher status may participate in ways that demonstrate that they are more capable than their peer. These learners may be personally invested in developing their social identities as knowledgeable, thereby constructing their identity as higher status. Learners affiliating with

[1] Elsewhere, we show how learners' mathematical identities are informed by their interactions with teachers (see Author, 2019 a,b; Authors, 2021).

an offered micro- or macro-identity of lower status may take on a more passive role during their interactions with their peer. These learners may be personally invested in developing their social identities as less knowledgeable, thereby constructing their identities as lower status. Learners affiliating with an offered micro- or macro-identity of equal status engages with their peer's ideas, because they are personally interested in learning mathematics with others and developing their social identities as cooperative members.

Depending on learners' personal and social identities, learners resisting an offered micro- or macro-identity of higher status may construct their identities as lower status, by taking on a more passive role during peer interactions, or as equal status, by working cooperatively with their peer. Learners resisting an offered micro- or macro-identity of lower status may construct their identities as higher status, by taking on an authoritative role during peer interactions, or as equal status, by considering the ideas of their peer. Learners resisting an offered micro- or macro-identity of equal status may construct their identities as higher or lower status, by either exerting their authority, or depending on their peer for knowledge.

Guided by our conceptual framework and existing research, we developed an analytical framework to analyse learners' micro- and macro-identities constructed during peer interactions. We follow Davis and Harrè's (1990) and Harrè & Van Langenhove's (1999) theorization of positioning, as the way participants use discourse to position themselves and others in conversation, thereby constructing and offering micro- and macro-identities. We identified five positioning utterances from research that learners use to construct and offer micro- and macro-identities during interactions. These positioning utterances are authoritative, diffident, disputational, cumulative and exploratory utterances.

Bishop's (2012) study describes authoritative utterances as characterised by one learner dominating peer interactions, by using an authoritative voice, not considering the ideas of others and identifying themselves as more mathematically capable than others. We propose a second positioning utterance, which we call a diffident utterance. Diffident utterances are characterised by learners deferring to other learners, by asking others for help, declaring their errors, and identifying themselves as less mathematically capable than others.

Wegerif and Mercer's (1997) framework for analysing peer interactions is used to identify three other positioning utterances: disputational, cumulative and exploratory utterances. Disputational utterances are characterised by disagreements, with learners making decisions on their own and seldom attempting to pool resources, offer constructive criticism or make suggestions (Wegerif & Mercer, 1997). Learners who make use of disputational utterances are likely to identify themselves as more mathematically capable than others.

Cumulative utterances involve learners building positively on what others have said uncritically to create common knowledge (Wegerif & Mercer, 1997). Learners who make cumulative utterances are likely to identify themselves and their peer as equal status.

Exploratory utterances involve learners building knowledge by engaging critically and constructively with each other's ideas, by considering different opinions, accompanied by justifications, with all learners participating in discussion (Wegerif & Mercer, 1997). Learners who frequently make explorative utterances are likely to identify themselves and their peer as equal status. The key difference between cumulative and exploratory utterances is that learners who use exploratory utterances are likely to create more opportunities for mathematical reasoning, than those who predominantly use cumulative utterances.

Positioning is reciprocal because with every utterance, a person simultaneously positions themselves and others in the interaction (Harrè & Van Langenhove, 1999). In doing so, Learner A constructs her/his micro-identity at the moment while offering Learner B a micro-identity in relation to Learner A's micro-identity at that moment (Wood, 2013). In each scenario, Learner B can either resist the offered micro-identity and construct another micro-identity, or affiliate with the offered micro-identity by constructing the same micro-identity as the one offered. Table 1 shows the possibilities, with some examples given below.

| **Learner A's utterance** | **Identity constructed by Learner A** | **Identity offered to Learner B** | **Learner B's response** | **Identity constructed by Learner B** | **Learner B's agency** | **Identity offered to learner A** |
|---|---|---|---|---|---|---|
| Authoritative | Higher status | Lower status | Authoritative | Higher status | **Resistance** to the identity of lower status offered | Lower status |
| | | | Disputational | Higher status | | Lower status |
| | | | Cumulative | Equal status | | Equal status |
| | | | Exploratory | Equal status | | Equal status |
| | | | Diffident | Lower status | **Affiliation** with the identity of lower | Higher status |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | status offered | |
| Disputational | Higher status | Lower status | Authoritative | Higher status | **Resistance** to the identity of lower status offered | Lower status |
| | | | Disputational | Higher status | | Lower status |
| | | | Cumulative | Equal status | | Equal status |
| | | | Exploratory | Equal status | | Equal status |
| | | | Diffident | Lower status | **Affiliation** with the identity of lower status offered | Higher status |
| Diffident | Lower status | Higher status | Diffident | Lower status | **Resistance** to the identity of higher status offered | Higher status |
| | | | Cumulative | Equal status | | Equal status |
| | | | Exploratory | Equal status | | Equal status |
| | | | Authoritative | Higher status | **Affiliation** with the identity of higher status offered | Lower status |
| | | | Disputational | Higher status | | Lower status |
| Cumulative | Equal status | Equal status | Authoritative | Higher status | **Resistance** to the identity of equal | Lower status |
| | | | Disputational | Higher status | | Lower status |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | Diffident | Lower status | status offered | Higher status |
| | | | Cumulative | Equal status | **Affiliation** with the identity of equal status offered | Equal status |
| | | | Exploratory | Equal status | | Equal status |
| Exploratory | Equal status | Equal status | Authoritative | Higher status | **Resistance** to the identity of equal status offered | Lower status |
| | | | Disputational | Higher status | | Lower status |
| | | | Diffident | Lower status | | Higher status |
| | | | Cumulative | Equal status | **Affiliation** with the identity of equal status offered | Equal status |
| | | | Exploratory | Equal status | | Equal status |

**Table 1 Relationship between positioning utterances and learners' micro-identities**

Table 1 shows the relationships between Learner A's utterance, the micro-identity constructed by Learner A and the micro-identity offered to Learner B (Columns 1-3). Table 1 also shows Learner B's possible responses to Learner A's utterance, the micro-identities constructed by Learner B, and the micro-identities offered to Learner A by Learner B (Columns 4-7). For example, if Learner A makes an authoritative utterance at a particular moment by telling Learner B "you should have added the two numbers together", Learner A constructs a micro-identity of higher status and offers Learner B a micro-identity of lower status (Row 1, Table 1). S/he projects her/himself as being more knowledgeable than Learner B, while positioning Learner B as the one who needs assistance.

Learner B can resist the micro-identity of lower status offered, by not constructing her/his micro-identity as lower status, when responding to Learner A with an authoritative, disputational, cumulative or exploratory utterance (Row 1-4, Table 1). If Learner B responds to Learner A by making an authoritative utterance, for example, by saying "do your own

work", Learner B constructs a micro-identity of higher status by telling Learner A what to do, and s/he offers Learner A a micro-identity of lower status. If Learner B responds to Learner A by making a disputational utterance, for example, by saying "you are wrong", s/he constructs a micro-identity of higher status, by projecting her/himself as knowing better than Learner A, so s/he offers Learner A a micro-identity of lower status.

If Learner B responds to Learner A by making a cumulative utterance, for example, by saying "Yes. That is how we do it", Learner B contributes to the discussion by building positively on what Learner A said. S/he displays her/his solidarity with Learner A and offers Learner A a micro-identity of equal status. Learner B can also contribute to the discussion as an equal by responding to Learner A with an exploratory utterance, for example, by asking Learner A "why did you add the two numbers?". S/he is interested in Learner A's thinking by requesting a justification and offers Learner A a micro-identity of equal status.

Learner B can affiliate with the micro-identity of lower status offered, by constructing her/his micro-identity as lower status, when s/he responds to Learner A with a different response (Row 5, Table 1). For example, if Learner B responds to Learner A by saying "what do you do then?", s/he positions Learner A as knowledgeable, so s/he offers Learner A a micro-identity of higher status.

In lessons, learners interact with each other by using different combinations of positional utterances, thereby constructing and offering different micro-identities at different moments. Over time, these micro-identities constructed and offered thicken through the frequent use of particular positioning utterances, to form the macro-identities constructed and offered. Using the same example, if Learner A predominantly makes authoritative utterances across many lessons, s/he constructs a macro-identity as higher status and offers Learner B a macro-identity of lower status. Learner B can resist the offered macro-identity if s/he predominantly responds to Learner A by making authoritative, disputational, cumulative or exploratory utterances across many lessons. If Learner B predominantly makes authoritative or disputational utterances, s/he constructs a macro-identity as higher status and offers Learner A a macro-identity of lower status. If Learner B predominantly responds to Learner A by making cumulative or exploratory utterances, s/he constructs and offers Learner A a macro-identity of equal status. Learner B can affiliate with the offered macro-identity when s/he predominantly responds to Learner A by making diffident utterances, and s/he offers Learner A a macro-identity of higher status.

Sometimes, learners may not engage with only one kind of utterance and may shift their micro-identities, thereby constructing and offering different micro-identities at different

moments. For example, in some moments during a lesson, a pair of learners may make authoritative utterances to show their authority over their peer, while at other moments, the same pair can make diffident, cumulative or exploratory utterances. In these situations, both learners do not sustain a particular type of identification across time, as their micro-identities shift during different moments of a lesson and authority is spread evenly between the pair. These learners take up positions as higher, lower and equal status in different moments, to construct macro-identities as equal status.

So the analytical framework illuminates *how* learners exercise agency, by interacting with others to construct and offer micro and macro-identities. Relating learner agency to their personal and social identities in order to understand learners' mathematical identities as proposed in our conceptual framework, requires an analysis of learners' narrated macro-identities to understand *why* learners exercise agency to construct and offer certain micro- and macro-identities during interactions.

## 3. Methods

This paper comes from a larger study that was conducted over two years (2015-2016) during the third school term (August - September) in a well-resourced secondary school in South Africa. The study involved nineteen learner participants, eight in the first year (Grade 9), with an additional eleven in the second year (Grade 10), along with their five mathematics teachers, two in the first year and three in the second year. In these classrooms, learners usually interacted with the person who sat next to them. Four learner participants were selected by each of the Grade 9 teachers using the following criteria: one high-performing learner, one low-performing learner and the person sitting next to each of them. The purpose of selecting learners according to the achievement criteria was so that we could observe how learners displaying different levels of achievement constructed identities when interacting with their teachers and peers. Since research shows that peers are significant to learners and their mathematical identities, we invited each learner selected by their teacher and the person who sat next to them to participate in the study.

In Grade 10, two of the initial participants transferred to another school. The remaining six learners from Grade 9, who were distributed across the three mathematics teachers' Grade 10 classrooms, continued to participate in the study. We also selected an additional eleven learner participants, who sat next to the initial participants, to analyse their identities and interactions with the initial participants. In the Grade 10 classrooms, learners

sat in rows of twos, threes or fours, so all learners who sat with an initial participant were invited to participate. To preserve the natural setting of the classroom, learners' seating arrangements were not changed.

In this paper, we focus on two pairs, one in Mr Moyo's Grade 9 classroom, and another in Mr Ncube's Grade 10 classroom. Mr Moyo was a newly qualified teacher and Mr Ncube had five years of experience teaching mathematics. The selection of a pair of learners in each of these teachers' mathematics classrooms was purposeful. From the five teachers who participated in the study, only these two teachers offered all learners "identities of affiliation" with the classroom community, meaning that they supported all learners to learn mathematics and become full members of their classroom community (see Author 2019 a, b; Authors, 2021 for a detailed analysis). In Mr Moyo's classroom, the two learners were Shane and Jack, who were good friends and were identified by Mr Moyo as high-performing learners. In Mr Ncube's classroom, the two learners were Amy and Thabo, who were good friends and were identified by Mr Ncube as average-performing learners. We selected these two pairs for two reasons. First, these two pairs interacted with each other more frequently than any of the other pairs in the wider sample, giving us more data to make convincing analytical claims. Second, we were interested in analysing peer identities and interactions, which were not informed by negative peer relationships or differences in teacher positioning or learners' academic achievements.

Data were collected during the third school term (August-September) in each year as videotaped lessons, audiotaped, semi-structured interviews with all participants, field notes and photographed learner notebooks (only Grade 10) [2]. The video camera was placed in front of the classrooms to focus on learners to capture their whole-class participation and audio recorders were placed on learners' desks to capture their conversations with each other. Approximately 8 hours of lessons were collected for each teacher, and we coded all peer interactions that occurred between participating learners during that time. In both classrooms, the teachers taught using traditional, textbook-based pedagogies, but provided learners with opportunities to engage with others to complete classroom tasks in almost every lesson (see Author 2019 a, b; Authors, 2021).

When analysing the data, all video- and audio-recordings of the lessons were transcribed. The interactions of the learner participants in each lesson were analysed as

[2] Fields notes and photographs of learners' notebooks were taken to provide context and supplement the recordings.

follows. We began by dividing the transcripts into excerpts based on the mathematics task under discussion, with each task counted as a single excerpt. All tasks were routine, textbook-based tasks. We coded each turn in each excerpt by identifying learners' positioning utterances and their micro-identities constructed and offered in each moment. We paid particular attention to how the learners' interactions unfolded, in terms of whether they predominantly used a single positioning utterance, or whether their interactions shifted over time, by using different positioning utterances. A single turn could contain more than one positioning utterance, so learners can construct and offer more than one micro-identity through an utterance (see excerpt 3). We calculated the frequency of each kind of positioning utterance, to understand how the micro-identities constructed at different moments during lessons formed learners' macro identities.

Positioning utterances can take on different meanings in different contexts (Herbel-Eisenmann, Wagner, Johnson, Suh & Figueras, 2015). For example, saying "what did you do?" could be considered an authoritative utterance when reprimanding a peer, a diffident utterance when asking for more information or an exploratory utterance when pressing another learner for justifications. When coding learner utterances, we considered the learners' tones of voice and their larger dataset of interactions. Table 2 summarises the criteria used to code learner utterances and their micro-identities.

| **Code** | **Criteria** | **Example from transcripts** | **Micro-identity constructed by learner** | **Micro-identity offered to peer** |
|---|---|---|---|---|
| **Authoritative (A)** | Learner exerts authority over others. | Shane: Yeah, look. What's there? | Higher status | Lower status |
| **Diffident (DIF)** | Learner defers to others. | Thabo: It doesn't work? | Lower status | Higher status |
| **Disputational (DISP)** | Learner disagrees with others. | Jack: You are wrong! | Higher status | Lower status |
| **Cumulative (C)** | Learner builds positively and uncritically on what others said. | Amy: Uh hm.<br>Thabo: Seventeen plus nine. Nine over twenty-six plus seventeen. | Equal status | Equal status |

| | | | | |
|---|---|---|---|---|
| | | Amy: Yeah. | | |
| **Exploratory (E)** | Learner engages critically with other learners' ideas. | Shane: How? | Equal status | Equal status |

**Table 2 Codes and criteria for learner utterances**

Over the eight hours of lessons collected for each teacher, we coded 57 excerpts with 330 utterances for Shane and Jack, and 44 excerpts with 320 utterances for Amy and Thabo. For this paper, we selected three excerpts for discussion, two excerpts for Shane and Jack, which were approximately one minute each, and one excerpt for Amy and Thabo, which was approximately one and a half minutes. All the excerpts were usually short, since interactions around a single task counted as a single excerpt, and it did not take learners long to work on each task. We used our calculations of the frequency of each positioning utterance to make sure the excerpts are representative of each pair's interactions. Since the purpose of this study is concerned with the relationship between peer interactions and learners' mathematical identities while doing mathematics, we did not code off-task utterances, utterances directed to the teacher, other non-participating learners or to the learner her/himself, and utterances that were unclear.

While the data from the videotaped lessons enabled understandings of how learners constructed and offered micro- and macro-identities in the classroom, data from the semi-structured interviews provided an understanding of learners' narrated macro-identities, in terms of why learners constructed and offered certain identities. Each learner was interviewed individually after we observed and videotaped the lessons, and the data from the semi-structured interviews were transcribed. Some questions asked to all learner participants were: how do you see yourself as a mathematics learner?; who do you participate with in the classroom; and how and why do you participate with them? All questions were followed by further probing questions, aimed at supporting participants to give detailed explanations. From the learner interviews, we coded learners' macro-identities by considering their personal identities, in terms of how learners described themselves as learners of mathematics, their emotions and motivations for learning mathematics, and their social identities, in terms of how they identified themselves as members of their mathematics classrooms in relation to their peers. For credibility, we engaged in a process of peer debriefing, where the second author discussed the first author's preliminary analysis (McMillian & Schumacher, 2010).

The first author revisited the data and made revisions until both authors were convinced by the analytical claims made.

## 4. Findings

In this section, we begin by showing the frequency of positioning utterances for each pair, to understand learners' macro-identities constructed across lessons. We then provide excerpts to illustrate how learners' micro-identities constructed during moments of a lesson thickened, to form their macro-identities. We conclude by showing how learners' narrations of their personal and social identities account for the micro- and macro-identities constructed and offered during interactions.

### 4.1. Shane and Jack: Both high status

Shane and Jack's discussions were largely characterised by authoritative and disputational utterances. Shane made more utterances than Jack, because sometimes, Jack got angry and ignored Shane when he used to insist that Jack's solutions were incorrect. Table 3 illustrates the type and frequency of utterances for the pair.

| Learners | Number of coded utterances | A | DIF | DISP | C | E |
|---|---|---|---|---|---|---|
| **Shane** | 174 | 73 | 11 | 72 | 18 | 1 |
| **Jack** | 156 | 85 | 17 | 42 | 12 | 6 |
| **Total** | 330[3] | 158 | 28 | 114 | 30 | 7 |

**Table 3 Shane and Jack's utterances**

Shane and Jack constructed macro-identities of higher status through their frequent use of authoritative utterances (Shane: 73 utterances and Jack: 85 utterances). Neither learner aligned with the offered identity of lower status through their frequent use of disputational utterances (Shane: 72 utterances and Jack: 42 utterances), which illustrates how these two learners exerted their agency by resisting the identities offered to them by the other. There were a few instances when they constructed micro-identities of lower status, by asking each

[3] As a reminder to the reader, the total number of utterances coded is not the total number of positioning utterances made, as a single utterance can contain more than one positioning utterance.

other for help (28 diffident utterances in total). There were also a few instances when they constructed micro-identities of equal status, by agreeing with each other or engaging critically with each other's ideas (30 cumulative utterances and 7 exploratory utterances in total). Excerpt 1 illustrates how Shane often disagreed with Jack.

*Task: Calculate the surface area of a cylinder with a diameter of 25cm and a height of 23cm.*

Jack incorrectly wrote the formula to calculate the volume of a cylinder, instead of surface area and he asked Shane for help.

Excerpt 1: Shane disagreed with Jack

| **Turn** | **Speaker** | **Utterance** | **Code** | **Micro-identity constructed** | **Micro-identity offered** |
|---|---|---|---|---|---|
| 3 | Jack | (*Shows Shane his book*) What sign do you put? Multiplication sign? | DIF | Lower status | Higher status |
| 4 | Shane | No. You are wrong. | DISP | Higher status | Lower status |
| 5 | Jack | Okay I am right. | DISP | Higher status | Lower status |
| 6 | Shane | You wrong. What did you use to calculate? | DISP & E | Higher status and equal status | Lower status and equal status |
| 7 | Jack | That's the $pi$. | A | Higher status | Lower status |
| 8 | Shane | (*Later*) It's not. You wrong. You are forgetting your crime. | DISP | Higher status | Lower status |

Jack began the interaction by constructing a micro-identity of lower status, as he was uncertain about the formula, so he asked Shane for help. Instead of helping him, Shane exercised his authority, by demonstrating his higher status and repeatedly asserting that Jack was wrong (turns 4, 6 and 8). Jack resisted the offered micro-identity of lower status, by making disputational utterances (turn 5), which shows his annoyance with Shane who did not assist him. At one point, Jack authoritatively tried to justify his solution (turn 7), by showing

Shane that he did know some aspect of the formula, after Shane inquired what he used to calculate the surface area (turn 6). Shane jokingly claimed that Jack's suggestion of using the multiplication sign in the formula was a crime, so he constructed his micro-identity as being of a higher status than Jack. Excerpt 2 documents how Jack disagreed with Shane.

*Task: Calculate the gradient of a straight line with points (-4;-6) and (3;-2).*

Jack was arguing with Shane because Shane had substituted the numerical values for the $x$ and $y$ coordinates interchangeably.

Excerpt 2: Jack disagreed with Shane

| **Turn** | **Speaker** | **Utterance** | **Code** | **Micro-identity constructed** | **Micro-identity offered** |
|---|---|---|---|---|---|
| 1 | Shane | Minus six over (mumbles something to himself). | Not coded | | |
| 2 | Jack | You are wrong! | DISP | Higher status | Lower status |
| 3 | Shane | Look negative six… | DISP | Higher status | Lower status |
| 4 | Jack | (Interrupts louder) You are wrong! | DISP | Higher status | Lower status |
| 5 | Shane | How? | E | Equal status | Equal status |
| 6 | Jack | Wena[4] six. | DISP | Higher status | Lower status |
| 7 | Shane | Yeah, look. What's there? | A | Higher status | Lower status |
| 8 | Jack | Variable. It's $x$. $x$ is not minus six. $y$ is minus six. | DISP | Higher status | Lower status |
| 9 | Shane | It is. Look at the formula. $y$ two. | DISP | Higher status | Lower status |

[4] Wena means "you" in IsiZulu.

| | | | | | |
|---|---|---|---|---|---|
| 10 | Jack | Where? | E | Equal status | Equal status |
| 11 | Shane | There. | A | Higher status | Lower status |

Jack's insistence that Shane's answer was wrong, through his tone, indicated Jack's determination to be correct, so he constructed his micro-identity as higher status. He exercised his authority and disputed Shane's solution to show his superiority (turns 2, 4, 6 and 8). Shane resisted the offered micro-identity of lower status, by making disputational utterances and explaining why his answer was correct (turns 3, 7, 9 and 11). Shane challenged Jack, by asking him 'how' (turn 5), when Jack insisted that he was wrong, and Jack challenged Shane, by asking him 'where' (turn 10), when Shane directed his attention to the formula, which shows that the boys were interested in each other's thinking at that moment. These challenging moves required learners to justify their claims and were counted as exploratory utterances. These exploratory moves provided the boys with an opportunity to construct their macro-identities as equal status, if they explained their thinking. However, the pair often lost opportunities to develop their understanding and reasoning by engaging with each other's ideas, because that might suggest that the other learner's answer was correct. The boys both constructed their macro-identities as higher status, and resisted the offered macro-identities of lower status. These identities constructed during interactions can be better understood by considering learners' narratives of their macro-identities (Table 4).

| **Learner** | **Personal identity** | **Social identity** |
|---|---|---|
| **Shane** | Mathematics was his "favourite" subject. He wanted to pursue mathematics in the future. He enjoyed competing with others and said that competition motivated him to enjoy learning mathematics: "I got competition in the class so I enjoy it". | He positioned himself as being one of the best learners of mathematics in his classroom. He explained that "it is a competition between me and Jack". He wanted to become recognised as one of the best learners of mathematics in his classroom: "I want them to think yes, this one is the best in the class". |
| **Jack** | He "love(d)" mathematics and wanted to pursue it in the future. He was motivated to learn by being competitive. He said that competition "ma(kes) me enjoy | He positioned himself as being one of the best learners of mathematics in his classroom. He explained that he competed with Shane: "We compete together to see who is going to get the |

| | | |
|---|---|---|
| | maths because I kn(ow) that everyone want(s) to compete against each other". | best marks". Performing better than Shane made him feel like "the king for that day like you won the maths challenge and you beat everyone else". |

**Table 4 Shane and Jack's narrated macro-identities**

Shane and Jack were motivated to learn mathematics for various reasons. When describing their personal identities, both learners enjoyed mathematics, saw the use of mathematics for their lives and futures and were motivated to learn mathematics because of the competition between them[5]. Mr Moyo discouraged them from competing and in one lesson, he explained to them that they should rather work on convincing each other instead of always competing. The learners explained to him that they were still friends and they "just want to know who is right", and so the boys continued to compete. For their social identities, both learners identified themselves as being among the best mathematics learners in their classroom, and they aspired to perform better than others.

### 4.2. Amy and Thabo: Equal status

Amy and Thabo made a similar number of authoritative, diffident, disputational and cumulative utterances. Table 5 illustrates the type and frequency of utterances for the pair.

| Learners | Number of coded utterances | A | DIF | DISP | C | E |
|---|---|---|---|---|---|---|
| **Amy** | 161 | 59 | 37 | 20 | 52 | 8 |
| **Thabo** | 159 | 59 | 33 | 31 | 41 | 4 |
| **Total** | 320 | 118 | 70 | 51 | 93 | 12 |

**Table 5 Amy and Thabo's utterances**

Amy and Thabo constructed and offered macro-identities of equal status. In some moments, they constructed micro-identities as higher status by exercising their authority over each other by making authoritative utterances (59 utterances each) and disputing each other's ideas by making disputational utterances (Amy: 20 utterances and Thabo: 31 utterances). In

other moments, they constructed micro-identities of lower status by asking each other for help by making diffident utterances (Amy: 37 utterances and Thabo: 33 utterances). They also constructed micro-identities of equal status by agreeing with each other (Amy: 52 cumulative utterances and Thabo: 41 cumulative utterances) and probing each other's thinking (12 exploratory utterances in total). Authority was spread evenly between the pair since both learners made a similar number of each of the positioning utterances. Excerpt 3 documents how Amy and Thabo worked together.

*Task: Calculate the probability of obtaining a letter from the word trapezium from the alphabet.*

Excerpt 3: Thabo and Amy's interactions

| **Turn** | **Speaker** | **Utterance** | **Code** | **Micro-identity constructed** | **Micro-identity offered** |
|---|---|---|---|---|---|
| 1 | Amy | Trapezium. How you going to get a letter in the word trapezium? | DIF | Lower status | Higher status |
| 2 | Thabo | Amy, You not thinking. You not thinking. | A | Higher status | Lower status |
| 3 | Amy | It's the first two periods on Monday. | DIF | Lower status | Higher status |
| 4 | Thabo | Nine minus twenty-six is how much? | A | Higher status | Lower status |
| 5 | Amy | Nine minus twenty-six or twenty-six minus nine? | A | Higher status | Lower status |
| 6 | Thabo | It's fifteen. No seventeen. | A | Higher status | Lower status |
| 7 | Amy | Uh hm. | C | Equal status | Equal status |
| 8 | Thabo | Seventeen plus nine. Nine over twenty-six plus seventeen. | C | Equal status | Equal status |
| 9 | Amy | Yeah. | C | Equal status | Equal status |
| 10 | Amy | (*After a while*) No. | DISP | Higher status | Lower status |

| | | | | | |
|---|---|---|---|---|---|
| 11 | Thabo | It doesn't work? | DIF | Lower status | Higher status |
| 12 | Amy | No. Your answer is wrong. | DISP | Higher status | Lower status |
| 13 | Thabo | You are stupid (l*aughs*). That was just stupid. | A | Higher status | Lower status |

Amy initially constructed a micro-identity of lower status, by positioning herself as less capable than Thabo by asking him for help as she was unsure about the task (turn 1). She later constructed a micro-identity of higher status by correcting his mistake and declaring that his answer was incorrect (turns 5, 10 and 12). Thabo constructed a micro-identity of higher status by showing his superiority, and telling her that she was not thinking, asking her the difference between nine and twenty-six and giving her the answer to his questions (turns 2, 4 and 6). He later constructed a micro-identity of lower status, by being unsure of his solutions through making diffident utterances and asking her if his strategy did not work and then joked that she was stupid, which was followed by berating himself for calling her stupid, by saying that what he said was stupid (turns 11 and 13). There was also a point in their discussion where both learners constructed micro-identities of equal status, by agreeing with each other cumulatively (turns 7-9). These learners constructed macro-identities of equal status, which can be better understood by considering their narratives of their macro-identities (Table 6).

| **Learner** | **Personal identity** | **Social identity** |
|---|---|---|
| Amy | Mathematics was her "favourite" subject and she was motivated to pursue mathematics in the future. She identified herself as a "growing learner", who was "not great" in mathematics, but "good" at mathematics. She was motivated to work with others in the classroom. | She said that she was "part of the class" as her teacher treated the learners equally. She identified her role in the classroom as one who assisted others, so that they could "help one another in the class". She was also open to the "advise other learners" gave her in the classroom. |
| Thabo | Mathematics was challenging, but he enjoyed learning mathematics and was motivated to pursue mathematics in the future. He said that he "prefer(ed) to work with others" and that he learned better when he worked with others. | He explained that he was "a bit below average", as he was not performing as well as others in his classroom, but his marks were improving "slowly but surely". He worked with Amy and others |

| | | |
|---|---|---|
| | | in the classroom by sharing his ideas and listening to their ideas. |

**Table 6 Amy and Thabo's narrated macro-identities**

Amy and Thabo enjoyed learning mathematics in their classroom. When describing their personal identities, both learners spoke about the importance of mathematics for their futures and the importance of working with others to learn mathematics. For their social identities, both learners positioned themselves as learners who assisted others, shared ideas, and listened to others when interacting in the classroom.

## 5. Discussion

Research shows how learners' mathematical identities are informed by the social identities offered by teachers and peers, yet the role of the individual, such as her/his motivations to learn mathematics, is often unaccounted for in identity research (Graven & Heyd-Metzuyanim, 2019; Lutovac & Kaasila, 2018). While there have been attempts to define identity in mathematics education research, there is a need for researchers to develop clear definitions of identity with a focus on both, the subjective and social aspects of identity (Graven & Heyd-Metzuyanim, 2019; Radovic et al., 2018). Our work contributes to research on learners' mathematical identities, as the purpose of this paper was to propose a framework to define and research learners' mathematical identities constructed in peer interaction, by accounting for the importance of both personal and social identity. We organise our discussion by drawing on our framework and the analysis of each pair's interactions and identities to answer our research questions. First, we discuss how we applied the analytical framework to analyse *how* learners constructed and offered micro- and macro-identities during interactions. Second, we discuss how our conceptual framework can relate the micro- and macro-identities constructed and offered during peer interactions with each pair's personal and social identities, to understand *why* learners constructed certain identities.

Using our analytical framework, we found that Shane and Jack predominantly interacted by making authoritative or disputational utterances, constructing micro-identities of higher status and offering micro-identities of lower status in many moments during the lessons. These micro-identities thickened to form the macro-identities of higher status constructed and lower status offered by each boy to the other. There were few moments during some lessons when they shifted their micro-identities, illustrating a change in

positioning when asking each other for help, agreeing with each other or exploring each other's ideas. Unlike the learner in Wood's (2013) study, the shifts in these learners' micro-identities were not significant or sustained during their interactions, as both learners usually reverted to using authoritative and disputational utterances. For example, when Jack asked Shane for help, Shane immediately told him he was wrong without assisting him (excerpt 1). Jack responded to Shane by telling him that he (Jack) was right.

Understanding why peers do not always contribute positively to each other's learning and identities is not sufficiently understood in research (Heyd-Metzuyanim & Sfard, 2012). Using our conceptual framework, we can establish why this pair did not contribute positively to each other's identities, by drawing on their narratives of their personal and social identities. When describing their personal identities, Shane and Jack said that they enjoyed learning mathematics and saw the value of learning mathematics for their futures. Yet, competition was also important for them, and they discussed how competition motivated them to work harder and brought them joy. For their social identities, both learners positioned themselves as being among the best learners of mathematics in their classroom. These learners' narratives illuminated what they thought was important when learning mathematics, i.e., being competitive and being the best learner in their classroom, which informed their interactions and supported their resistance to the micro- and macro-identities of lower status offered.

Wood (2013) argues that learners who have negative macro-identities may have moments of positive micro-identities, which when encouraged, can lead to positive macro-identities. Based on our findings, we think that encouraging such shifts may not always be so easy, as Mr Moyo unsuccessfully tried to shift Shane and Jack's micro-identities. Guided by our conceptual framework, we argue that positive shifts in these learners' micro-identities, and eventually, their macro-identities, may require a shift in both learners' interactions and their narratives. To shift these learners' narrated macro-identities, the learners may need to be convinced that working collaboratively with others on tasks is important for their learning, compared to competing with others. A shift in context can lead to a shift in narrative, so creating more opportunities for learners to engage meaningfully with mathematical content and others in classrooms, where teachers' expectations of what counts as productive forms of peer interactions are clear, can be a starting point to facilitate such change (Boaler & Greeno, 2000; Boaler & Staples, 2008, Cobb et al., 2009; Andersson, Valero & Meaney, 2015).

Amy and Thabo interacted with each other by using authoritative, cumulative, diffident and disputational utterances. Their interactions shifted in different moments so they

constructed micro-identities of higher, lower and equal status. For example, both learners asked each other questions, exerted their authority on each other and agreed with each other (excerpt 3). Neither learner consistently constructed micro-identities of higher or lower status, so authority was distributed evenly between them and both learners constructed and offered macro-identities of equal status. Some learners in DeJarnette and González's (2015) study also shifted their interactions in different moments, yet they argue that more research is needed to understand why those learners interacted as they did.

Using our conceptual framework, we can establish why this pair worked together and constructed macro-identities as equals. When describing their personal identities, this pair said that they enjoyed learning mathematics and were motivated to pursue mathematics in the future. They prioritised listening to each other and assisting one another, as they were motivated to work with others. They did not construct their social identities as being more or less knowledgeable than the other. For them, learning mathematics involved working with others, so both learners affiliated with the micro- and macro-identities offered by the other. So, by using our framework, we can understand how learners construct and offer micro- and macro-identities by analysing their interactions with their peers, and why they construct and offer those identities by analysing their narratives of their personal and social identities.

Learners' mathematical identities are constructed relationally, so our findings cannot be generalised to the interactions and identities of other pairs of learners, or even to the interactions and identities constructed by one of these participating learners when s/he interacts with another learner. The learners reported here were good friends, were treated similarly by their teachers and performed similarly academically. Learners who are not good friends, who are treated differently by their teachers and perform differently academically may interact differently from the learners in our sample, so we encourage researchers to assess the applicability of our framework to other pairs. We did not consider the roles of different mechanisms, such as the cognitive demand of tasks, race, socio-economic status and gender on learner identities and interactions. We need further research to understand the complex relationships between different mechanisms, peer interactions and learners' mathematical identities.

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# Chapter 8: Conclusion

## 8.1. Introduction

The purpose of this thesis was to define, theorise and empirically apply the concept of mathematical identity to understand learners' mathematical identities in a secondary school. This chapter serves to conclude my research. I present a synthesis of the preceding chapters to show how all the chapters are interrelated and provide original contributions to existing knowledge in the field. I then answer the overarching research questions that guided the research, discuss the implications of the findings, reflect on the study, and conclude the research.

## 8.2. Contributions of each chapter

Each of the seven preceding chapters was aimed at answering one or both main research questions. As a reminder to the reader, some of the chapters responded to question 1 completely, which was concerned with what mathematical identity is and how the concept can be used in researching secondary school learners' mathematical identities, while other chapters only answered the second part of question one, which was how the concept can be used in researching secondary school learners' mathematical identities. Some of the chapters addressed question 2, which was concerned with how learners and their mathematics teachers contribute to the development of learners' mathematical identities, while others did not address this question. Table 1 presents the aim of each chapter, which includes the main research questions that were answered, and key contributions of each chapter.

| **Chapter** | **Aim of chapter** | **Main contributions to research** |
|---|---|---|
| 2. A framework for learners' mathematical identities: A critical realist perspective | This chapter explored the use of critical realism in defining and researching learners' mathematical identities.<br><br>The main research question addressed in this chapter is research **question** | 1. Critical realism is presented as a novel theoretical perspective used to define and analyse learners' mathematical identities.<br>2. A conceptual framework is presented, and three key higher-level generative mechanisms are discussed: personal identity, social identity and agency. All three mechanisms are important to analyse learners' mathematical identities.<br>3. Two kinds of offered social identities are described: affiliation with and marginalisation from communities. For each offered social identity, three kinds of mathematical identities can be constructed, as learners can identify with, comply with or resist the offered social identity. |

| | | |
|---|---|---|
| | **1,** and very briefly, **question 2.** | 4. The relationships between all three higher-level mechanisms need to be considered in order to understand how learners construct their mathematical identities through interactions with their mathematics teachers. |
| 3. The influence of teachers on learners' mathematical identities | This chapter provided an overview of the whole sample, in terms of the social identities offered by teachers and the mathematical identities constructed by learners.<br><br>The main research questions addressed in this chapter are the second part of research **question 1,** and very briefly, research **question 2**. | 1. A social identity of partial affiliation, where teachers support learners to some extent, is introduced.<br>2. Teachers' narratives of who could learn mathematics influenced the social identities offered to learners.<br>3. Two teachers, who thought that all learners could learn mathematics, offered all learners social identities of affiliation, by emphasising effort.<br>4. The other three teachers thought that not all learners can learn mathematics and emphasised ability, so they did not assist all learners and offered learners differential identities.<br>5. None of the learners resisted any of the social identities offered. So, the learners' mathematical identities were strongly influenced by their experiences of the social identities offered to them by their teachers.<br>6. There is evidence of learners' trajectories, as the initial six learner participants' mathematical identities are shown to have changed substantially or negligibly over the two years as informed by their experiences of interacting with different teachers. |
| 4. Relationships between teachers' practices and learners' mathematical identities | This chapter focused on describing the kinds of social identities that can be offered by teachers through their actual and narrated practices, involving their pedagogical approaches and social relationships with learners. These offered social identities are analysed in relation to learners' mathematical identities.<br><br>The main research questions addressed in this chapter are | 1. There is a deeper explanation of the explanatory framework, in terms of the social identities offered and learners' mathematical identities constructed.<br>2. Teachers' narratives of their pedagogical approaches provide detail as to *what* is needed from learners to become affiliating members of their classroom communities.<br>3. Teachers' narratives of their social relationships with learners provide detail as to *who* can learn mathematics.<br>4. Teachers' narratives and actual practices, which incorporate teachers' pedagogical approaches and social relationships with learners are related, and jointly influence the social identities offered to learners.<br>5. The social identities offered by teachers did not determine, but informed learners' mathematical identities. |

| | | |
|---|---|---|
| | research **questions 1 and 2.** | |
| 5. Social Relationships between Teachers and Learners, Learners' Mathematical Identities and Equity | This chapter focused on the different social relationships between teachers and learners, and how these relationships inform learners' mathematical identities.<br><br>The main research questions addressed in this chapter are the second part of research **question 1,** and research **question 2**. | 1. The teachers taught using a similar kind of pedagogy, yet their social relationships with learners informed the identities offered to and constructed by learners.<br>2. The teachers' narratives of their social relationships with learners informed how they offered social identities, either equitably or inequitably, in their classroom communities.<br>3. Learners were more willing to construct social identities of affiliation when they felt that their teachers cared about them and others in the classroom.<br>4. There is evidence of learners' trajectories, as the four learner participants' mathematical identities are shown to have changed over the two years, as informed by their social relationships with their Grades 9 and 10 mathematics teachers. |
| 6. Relationships between teachers' interactions with learner errors and learners' mathematical identities | This chapter examined the relationship between teachers' narratives and interactions with learner errors and learners' mathematical identities.<br><br>The main research questions addressed in this chapter are the second part of research **question 1,** and research **question 2**. | 1. The ways in which the teachers spoke about errors, the learners and learner thinking, informed how they addressed learner errors in the classroom.<br>2. All learners wanted to develop their identities as members of the community, so even though they may have not described errors as their teachers did, they altered their participation to maintain their membership in the community. |
| 7. A framework for analysing the relationships between peer interactions and learners' mathematical identities: accounting for personal and social identities | This chapter focused on presenting a framework to analyse and describe the kinds of identities that can be offered and constructed by learners during peer interactions.<br><br>The main research question addressed | 1. An explanatory framework to understand the relationships between learners' mathematical identities and peer interaction is developed.<br>2. For the framework, three kinds of offered social identities are described: higher status, lower status and equal status. For each offered social identity, two kinds of mathematical identities can be constructed, as learners can affiliate with or resist the offered social identity.<br>3. The framework accounts for learners' micro-identities, which are constructed during moments of a lesson, and their macro-identities, constructed over longer periods of time. |

| | | |
|---|---|---|
| | in this chapter is research **question 1.** | 4. Using the explanatory framework, an analytical framework is developed to relate learners' positioning utterances to their micro- and macro-identities.<br>5. The explanatory and analytical frameworks are used to understand *how* and *why* learners construct certain mathematical identities, by drawing on their personal and social identities. |

**Table 8.1: Summary and contributions of chapters**

I now describe in detail how chapters 2-7 came together like pieces of a puzzle, contributing to answering the research question in a holistic way, and presenting an original contribution to the existing knowledge in the field.

## 8.3. Answering the research questions

### *What is mathematical identity and how can the concept be used in researching secondary school learners' mathematical identities?*

Given that one of the biggest challenges faced by mathematics education researchers involves describing what identity is and how the concept of identity can be empirically accessed and researched in a theoretically coherent way (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Langer-Osuna & Esmonde, 2017; Radovic et al., 2018), the main purpose of chapter 2 was to show how critical realism can be used to define and research learners' mathematical identity in classroom contexts. Critical realism has not been used previously to define and conceptualise learners' mathematical identities, so chapter 2 offers a novel contribution to research, by conceptualising learners' mathematical identities from a critical realist perspective, while the remaining chapters show how this concept can be used in mathematics education research. Chapter 2 also contributes to the theoretical papers on mathematical identities, which are currently few in number (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019).

In chapter 2, mathematical identity is defined as a social phenomenon. There are three central tenets of critical realism, which were discussed in chapters 1 and 2, and were used to enhance this basic definition of identity. These tenets are: 1. The intransitive dimension is different from the transitive dimension, 2. The openness of the social world; and 3. The importance of the ontological depth of social reality (Scott, 2010). For the first tenet, a learner's mathematical identity, being a social phenomenon, exists in the intransitive dimension and is the focus of inquiry. Objects in the intransitive dimension, like learners' mathematical

identities, exist independently of our knowledge of the phenomenon, as our knowledge of phenomena exists in the transitive dimension. Further, a social phenomenon existing within the intransitive dimension can change (Scott, 2010). By applying the first tenet, learners' mathematical identities can be defined as a social phenomenon, which exists in the real world and can change.

Building on the first tenet, chapter 2 explores in detail the second and third tenets, by describing that the real world as an open system, where multiple mechanisms operate to give rise to events. Uncovering the underlying mechanisms that are responsible for the emergence of a phenomenon is needed to achieve an understanding of the ontological depth of social reality (Bhaskar, 1978). Further, these underlying mechanisms need to be understood in a non-reductionist way, firstly, by not reducing the mechanism to our interpretations of it, and secondly, by not reducing a phenomenon to one or two of the mechanisms responsible for the emergence of the phenomenon.

In chapter 2, a non-reductionist model of identity, forming the conceptual framework of this study, is discussed. Three higher-level mechanisms that are responsible for the emergence of learners' mathematical identities are identified from Archer (2002) and Marks and O'mahoney's (2014) work, i.e., agency, personal identity and social identity, and the interactions between all three mechanisms are described in chapters 2 to 7. Figure 8.1 provides the model used in these chapters to explain the interactions between mechanisms. Please see chapter 2 for a detailed explanation of the model.

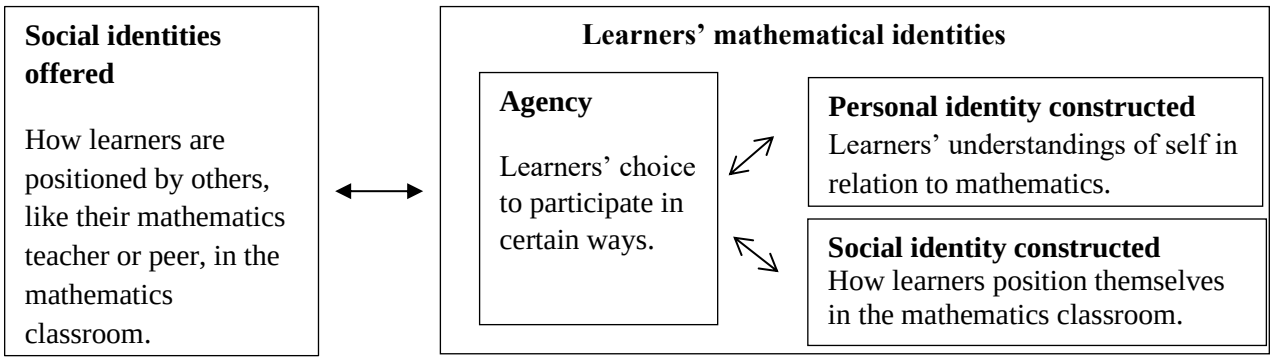



**Figure 8.1 Constituents of mathematical identity**

The conceptual framework developed in chapter 2 considers the importance of all three mechanisms independently from each other and then in relation to each other to show how together, all three mechanisms are responsible for the emergence of learners' mathematical

identities, without reducing identity to any of the three mechanisms. By applying all three tenets of critical realism to the basic definition, mathematical identity is defined as**:**

*a social phenomenon, existing in the real world, which emerges from relationships between learner agency, personal identity and social identity, both offered and constructed, and that these identities can change over time.*

In chapter 2, I've argued that from a critical realist perspective, the contribution of each of the three higher-level mechanisms to learners' mathematical identities differs from case to case, however, it is still important to explain all higher-level mechanisms as they operate relationally (Collier, 1996). Further, I argued that reducing mathematical identity to any of the generative mechanisms provides an incomplete, and sometimes incorrect understanding of learners' mathematical identities.

To apply the concept of identity defined in chapter 2 to empirical research, there was a need to create explanatory frameworks for identity. Chapter 2 provides an explanatory framework to relate the kinds of social identities offered by teachers: affiliation with or marginalisation from classroom communities, with four kinds of mathematical identities constructed: affiliation with, compliance with, resistance to or marginalisation from classroom communities. Chapter 3 introduces a third kind of social identity that can be offered to learners called partial affiliation, where teachers invite participation and support learners to some extent. Chapter 4 elaborates on the explanatory framework in chapter 2, by providing detailed explanations of the social identities offered and the mathematical identities constructed by learners, while chapters 5 and 6 elaborate on the social identity of partial affiliation introduced in chapter 3. Like chapter 2, chapter 6 also offers an explanatory framework, but differs from chapter 2 by relating the kinds of social identities offered by peers, i.e., higher status, lower status or equal status, to the kinds of mathematical identities that can be constructed during peer interactions, i.e., affiliation with or resistance to the offered social identities. So, all of these chapters contribute to understandings of *what* mathematical identity is, *how* learners can construct identities and *why* learners would construct certain identities and not others.

By using the explanatory frameworks, chapters 2 to 7 contribute to answering the second part of this main research question, which involved showing how the concept of identity can be used in researching secondary school learners' mathematical identities. To do this, chapters 2 to 7 show how the concept of identity is: 1. social and subjective by considering personal and

social identity: 2. representational and enacted by accounting for both narratives and enacted practices; 3. transformative rather than stable by accounting for change (see Radovic et al., 2018). The conceptual and explanatory frameworks contribute to research in important ways, by addressing the following concerns raised by researchers, as discussed previously in chapter 1:

- Radovic et al. (2018) encouraged researchers to develop suitable definitions of identity, which account for both social and subjective aspects of identity. My conceptual and explanatory frameworks show how identity is both social and subjective/individual, by focusing on the importance of both personal and social identity, without reducing identity to either personal or social aspects.
- Radovic et al. (2018) and Graven and Heyd-Metzuyanim (2019) were concerned that "an overwhelming majority" of studies consider identity as narrative and that overlooking the importance of the enacted dimension of identity neglects the important role of the contextual practices (p. 368). Chapters 2 to 7 each show how both enacted practices and narratives can contribute to our understanding of learners' mathematical identities.
- Graven and Heyd-Metzuyanim (2019) showed that researchers did not often differentiate between the identity they authored about a participant and the identity authored by the participant him/herself. Critical realism demands that a researcher differentiates between these different sources of data. In chapters 2 to 7, I differentiate between teachers' narratives of the learners, learners' narratives about their identities and my observations of learners' identities.
- Graven and Heyd-Metzuyanim (2019) argued that only a small number of studies followed learners through their transition from one phase of schooling to another. Instead of following learners across phases of schooling, I followed learners through their transition from one grade to another, illustrating the malleable and transformative nature of learners' mathematical identities. Evidence of change in identities is discussed briefly in Chapter 3, where I show the trajectory of six Grade 9 learners' mathematical identities, as they progressed to Grade 10. Chapter 5 describes in detail the change in identity of four of the six Grade 9 learners as they progressed to Grade 10, while chapters 4 and 6 need to be read together to understand, in detail, the identity trajectories of the remaining two Grade 9 learners as they progressed to Grade 10.

- Graven and Heyd-Metzuyanim (2019) were concerned about the widening gap between research on affect and research on mathematical identities. My conceptual and explanatory frameworks include the importance of personal identity, which includes the area of affect in relation to learners' emotions and interests, as one of the emergent mechanisms responsible for learners' mathematical identities.

The application of the conceptual and explanatory frameworks is discussed in relation to the second main research question in the next section.

***How do learners and their mathematics teachers, contribute to the development of learners' mathematical identities?***

Chapters 2 to 6 focused on the important relationships between teachers and learners in relation to the social identities offered to learners by teachers and the kinds of mathematical identities constructed by learners. Taken together, these chapters highlight the importance of teachers' narratives and practices about their pedagogies and social relationships, which inform the social identities that are offered to learners. Narratives about the teachers' choices of pedagogy, the learners in their classrooms, the importance of learners' reasoning and errors were shown to specify *what* is needed from learners to become affiliating members of their classroom communities and *who* could learn mathematics and become an affiliating member of the community. Chapters 2 to 6 also show the relationships between teachers' narratives and their actual practices, as teachers' narratives were shown to inform their practices, while their experiences of their social relationships with learners and the success of their pedagogies reinforced their narratives. Table 8.2 provides a summary of the kinds of social identities offered by the five teachers, which include both their actual practices and narratives of practices.

| **Teachers** | **Chapters** | **Social identities offered** | **What is needed to learn mathematics: Pedagogy** | **Who can learn mathematics: Social relationships** |
|---|---|---|---|---|
| Mr Moyo (Grade 9) | Chapters 3, 4 and 5 | Affiliation to all learners. | Working on tasks was important for Mr Moyo as he described mathematics as a "practical" subject. Errors were seen as opportunities for learning and he was accommodating of errors. | Mr Moyo was seen assisting and interacting with all learners in the classroom. He believed all learners could do mathematics through practice, hard work and motivation. |

| | | | | |
|---|---|---|---|---|
| Mr Ncube (Grade 10) | Chapters 2, 3 and 6 | Affiliation to all learners. | Having learners work on tasks with other learners was important for Mr Ncube. Errors were described as lessons and he was accommodating of errors. | Mr Ncube was seen inviting participation and assisting all learners. He believed all learners could do mathematics, through effort and hard work. |
| Mr Molefe (Grade 9) | Chapters 3 and 6 | Affiliation to high achievers and marginalisation to low achievers. | Peer work was not encouraged in his classroom. Mr Molefe spent much of his lessons giving lengthy explanations. He described errors as being contagious, so he was not keen to have learners explore mathematical ideas. | He was usually seen assisting and interacting with higher achievers and ignoring the lower achievers. He explained that low achievers did not have the aptitude or personality for mathematics. |
| Mr Sithole (Grade 10) | Chapters 2, 3 and 5 | Affiliation to high achievers and marginalisation to low achievers. | Mr Sithole provided learners with opportunities to work individually, or with others on classroom tasks. He described errors as opportunities for learning, yet he responded to errors differently depending on who made the error. | He usually assisted higher achievers and was seen refusing to assist lower achievers. He believed that only learners with "aptitude", "talent" and a "Gift from God" could do mathematics. He said that there was "nothing" he could do for the lower achievers. |
| Mr Ndlovu (Grade 10) | Chapters 3 and 4 | Affiliation to high achievers and marginalisation to low achievers. | Mr Ndlovu did not usually provide learners with opportunities to engage with classroom tasks. He usually gave lengthy explanations, accompanied by asking learners type 1 recall question. He described errors as costing lives in the work environment. He was not accommodating of learner errors and would sometimes get angry when learners made errors. | "Maths was not made for everybody" since "God" created people differently. He did not assist struggling learners and he advised them to ask other teachers for help or to drop mathematics as a subject in school. |

**Table 8.2: The social identities offered to learners by the teachers**

Mr Moyo and Mr Ncube described the importance of learner effort when learning mathematics over ability (Table 8.2). Both teachers were concerned with including all learners and accommodated learner thinking in their lessons to some extent. For Mr Molefe, Mr Sithole and Mr Ncube, ability was important to learn mathematics and all three teachers

were not accommodating of all learners, their thinking and errors (Table 8.2). These practices inform how learners are offered social identities, either equitably or inequitably, in their classroom communities.

The model developed at the end of chapter 4 can be applied to understanding the relationships between these teachers' narratives and their actual practices, which support and justify each other and lead to the continuation of these social identities being offered by the teachers in the classroom to learners. Building on Radovic et al.'s (2018) work, I show that it is important to understand the relationships between the representational and enacted social identities offered by teachers to understand the teachers' reasons for offering social identities and why these social identities that are offered to certain learners are reproduced and offered continuously across lessons.

While these social identities offered are important for the emergence of learners' mathematical identities, chapters 2 to 6 show that the social identities offered by teachers inform, but do not determine, the kinds of mathematical identities constructed by learners. This is an important point, as some researchers use the term identity to explain what learners are required to affiliate with to become competent mathematics learners in the classroom community, rather than how learners view or describe themselves (Cobb et al., 2009).

Learners' mathematical identities emerge through relationships between the three higher-level mechanisms, i.e., personal identity, social identity and agency. Table 8.3 provides a summary of the kinds of mathematical identities constructed by learners as affiliation (A), compliance (C), resistance (R) or marginalisation (M), in relation to the social identities offered by teachers as affiliation (A), partial affiliation (PA) or marginalisation (M).

| Grade | Teachers | Learners | Identity offered | Identity constructed |
|---|---|---|---|---|
| 9 | Mr Moyo | Shane, Jack, Lane & Senzo | A | A |
| 10 | Mr Ncube | Thandi, Amy & Mike | A | A |
| | | Thabo, Jimmy & Tess | A | C |
| 9 | Mr Molefe | Sindiswa | A | A |
| | | Sunny | PA | C |
| | | Jenifer & Tess | M | C |
| 10 | Mr Sithole | Phillip, Shane, Lebo & Karabo | A | A |
| | | Jack | PA | C |
| | | Lane | M | C |

| | | | | |
|---|---|---|---|---|
| | | Tommy & Senzo | M | M |
| 10 | Mr Ndlovu | Edward | A | A |
| | | Fred | A | C |
| | | Sunny | M | M |

**Table 8.3: The mathematical identities constructed by learners**

Across chapters 2 to 6, I showed that when all three higher-level mechanisms contributed positively to the emergence of learners' mathematical identities, learners constructed their mathematical identities in affiliation with the social identities of affiliation offered. For example, in chapters 4 and 5, I showed how four learners in Mr Moyo's classroom became full affiliating members of their classroom community and they were offered social identities of affiliation. These learners enjoyed learning mathematics in his classroom and were appreciative of the way Mr Moyo taught his lessons, approached their errors and interacted with them by showing care. All four learners participated in the lessons to become full members of the classroom community. Yet, three of these four learners' mathematical identities changed substantively in Mr Sithole's Grade 10 classroom. These learners described Mr Sithole as an uncaring teacher, providing evidence of how different teachers can contribute to changes in learners' mathematical identities.

When two or three higher-level mechanisms, especially learner agency, contributed negatively towards the emergence of learners' mathematical identities, learners constructed their mathematical identities as marginalised learners. For example, in chapter 2, Tommy experienced the social identity of marginalisation offered to him by Mr Sithole negatively, as Mr Sithole shared a poor social relationship with him and was not accommodating of learner errors. Even though Tommy was interested in pursuing mathematics in the future, and said that he enjoyed certain parts of mathematics, he was not comfortable to be a member of his classroom community and he exercised agency by not engaging in the classroom, developing his mathematical identity as a marginalised learner.

When learners wanted to maintain some form of peripheral membership to the community, for their social identities and/or personal identities, they constructed their mathematical identities in compliance with their classroom community. For example, in chapters 2 and 6, I showed how Mr Ncube offered Thabo opportunities to affiliate. Thabo exercised agency by engaging with his teacher and others in the classroom, but his personal and social identity largely contributed to his compliance, as he said that mathematics was holding him down,

and he was not entirely comfortable to ask his teacher for assistance. In chapters 2 and 5, I showed another form of compliance, where Lane was offered a social identity of marginalisation by Mr Sithole. Instead of adopting the social identity offered to him, Lane exercised agency by participating peripherally and was usually seen working on his own or asking others for help because he was interested in passing mathematics.

While I did not find any learners resisting the social identities offered by the teachers, I included the category in my explanatory framework for completeness. I provided some possible reasons for not finding resistance in chapters 4 and 6. The first reason discussed in chapter 4 was that these learners may have been strongly influenced by their teachers' practices, and the second reason discussed in chapter 6 was that resisting teacher practices could have a negative impact on learners, which could result in their alienation from the classroom community.

## 8.4. Implications

Most of the chapters in this thesis present the implications of the findings for research and practice based on the chapter's topic of discussion. Instead of repeating those implications, I am going to present some implications that emerged when looking at the thesis as a whole.

### 8.4.1. Implications for research

As I was analysing my data, I wondered if researching the teachers' identities could have provided me with a deeper understanding of why they offered learners certain social identities. An empirical review of research on mathematics-related teacher identities showed that the concept of teacher identity in research faces similar challenges to research on learners' mathematical identities (see Lutovac & Kaasila, 2018). Research on mathematics-related teacher identity is usually informed by socio-cultural orientations, with a strong focus on narrative-driven methodologies and frameworks (Lutovac & Kaasila, 2018). Further, social practices and structures are foregrounded in these papers, while the importance of the individual in the construction of her/his identity is underplayed (Lutovac & Kaasila, 2018). It may be possible to use my conceptual framework to better understand mathematics-related teacher identity, which can perhaps further our understandings of the social identities offered to learners.

I am also interested in understanding whether the mechanisms that I identified in this study can be applied to understanding learner identity in different school subjects beyond mathematics and in different schools. It may be possible for other researchers to apply the framework to understand learners' learning of other subjects to understand learning holistically in terms of learner identities, rather than reducing learning to cognition or social influences.

When compiling my response to research question 1 in the previous section, I realized that there is a need to create a unifying framework of learners' mathematical identities that take into account the different kinds of social identities offered by various significant narrators. One of my initial ideas for modelling the relationships is presented below.

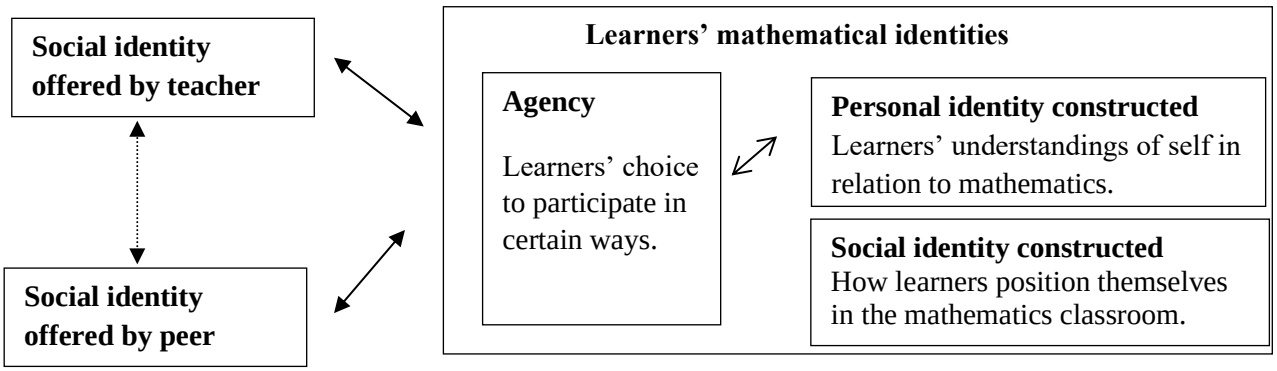



**Figure 8.2: Possible model of mathematical identity**

In chapter 7, I showed that there is much research that relates teacher positioning with the way learners' position and offer identities to each other in classrooms. Yet, this relationship between the social identities offered by significant others may not manifest itself for every learner. For example, Shane and Jack were not convinced by their teacher, who encouraged them to work collaboratively rather than competitively. So even though both learners were identified as high achievers and were provided opportunities to affiliate with the classroom community, their own personal and social identities motivated them to compete to become better than the other. Hence, I have included a dotted line between the social identity offered by the teacher and peer. More research may be needed to assess the applicability of figure 8.2 to data.

After attending and participating in the 2021 International Association for Critical Realism conference, which was focused on the importance of social transformation and emancipation, I became aware of some of the limitations of my PhD work, which could act as a springboard

for further research. First, my PhD work focused closely on the relationships between learners' mathematical identities and the mathematics teaching and learning that happened in classrooms. So other mechanisms, such as experiences of race and gender were not explored, leading to a limited understanding of the concept of identity.

Second, my research was underpinned by a descriptive and explanatory agenda, as one of the main purposes was to define identity and use this definition to understand how the mechanisms responsible for the emergence of identity operate in classrooms, rather than a transformative agenda, by seeking ways to transform inequitable practices that may constrain certain learners from accessing mathematics and becoming successful mathematics knowers and doers. So, there is a need for broader research on learners' mathematical identities, with both explanatory and transformative agendas, to understand how social and economic inequalities also inform learner identities and learning of mathematics in school, and how social and economic inequalities can be transformed for all learners to become successful mathematics learners, doers and thinkers.

### 8.4.2. Implications for practice

Teachers have an ethical responsibility to care for and about their learners, both in a pedagogical sense and in developing social relationships with learners. According to Van Niekerk (2010), caring for others in education requires one to "reflect deeply on living with others who are different, but nevertheless with a specific common humanness expressed in care and responsibility for others" (p. 87). Caring for others is not just what is required to be a teacher, but it is in essence an ethical requirement, which is the hallmark of what characterises us as humans. Care involves both a "way of doing and being" (Peterson & Osman, 2010).

Displaying care through pedagogy involves ensuring that learners are provided with opportunities to learn mathematics well in a safe space. Teaching mathematics learners using the traditional approach has been critiqued extensively through research (Boaler, 1998; Boaler & Greeno, 2000). I found that the teachers predominantly asked lower-order questions which usually required recall and learners were not given many opportunities to explore and use the tools of mathematics to make sense of it. In her inaugural lecture, Brodie (2015) argued that "learners who must accept the rules and concepts of the discipline as unquestionable become unquestioning". Being uncritical has many negative implications for

learners, not only for learning mathematics, but also for becoming a productive citizen in society (Graven, 2015).

Most of the learners described care as being central to the kinds of social relationships shared with the teachers. They spoke of care in relation to teachers being concerned about them and others (or not) in the classroom. For example, in chapter 5, care was spoken about in relation to whether the teacher was concerned about learners, in terms of whether they understood the work and their performances in assessments. Further a caring teacher was described as one who would explain for all learners to understand and who would be happy with their good performance in assessments. I suggest that teachers reflect on their practices and see how they can display care through both pedagogy and the social relationships they share with learners.

Further, I was surprised to find that the narratives that teachers told me about their practices and learners actually reflected the kind of opportunities and the ways in which the teachers taught and interacted with particular learners. According to Bishop (2012), words are very powerful. I only understood the profound effect of words when I noticed how the teachers' narratives of their pedagogies and learners influenced learners' learning and opportunities in the classroom. I suggest that teachers regularly reflect on their reasons for using certain pedagogical practices, and on their perceptions of learners in their classrooms.

## 8.4. Reflections

### 8.4.1. Reflections relating to my study

From a critical realist perspective, researchers are part of their research context and therefore should be be transformed by their research process (Price, 2015). I can see a marked difference between the person, writer and researcher I was at the beginning of the PhD process and the person, writer and researcher that I am now. I was always a person who loved writing a lot of pages, with long and often verbose sentences and paragraphs. From my journey as an undergraduate student to becoming a postgraduate student, I could never meet word or page limits. When my supervisor first introduced me to the idea of doing a PhD by publication, I was very worried. How could I possibly say everything that I would like to say in a journal article which has word limits? At first, I struggled and would often look for journals that did not have low word or page limits. With the expert guidance of my supervisor, I learned how to articulate myself in clear, accessible ways and meet word or

page limits. One of my supervisor's lines, which plays in my head when I proofread my work is to place myself in the shoes of my reader.

The publication process was a lot more difficult than I expected. One of the greatest struggles I faced at the initial stages of the PhD when trying to get my papers published was rejection. I remember my heart sinking the day I received my first rejection, and I was devastated for days thereafter. Over time, I have learned to change my way of thinking when dealing with rejection. When a paper is rejected, I look forward to reading the reasons for rejection from the reviewers so that I can change the paper in such a way that the paper will be less likely to be rejected again. I have learned to change my mindset about the outcome of the peer review process, from being a scary, potentially heart-breaking outcome, to be a way of helping me to create a paper that I can be proud of. I am deeply grateful to every editor and reviewer for their critical comments and suggestions, as I can see a marked difference between the paper that was initially submitted to the journal and the paper that was finally published.

Given that my PhD study took me very long to complete, there were times during the PhD process when I felt demotivated and depressed. Aside from talking to my supervisor, family and friends, one of the greatest support structures was the research weekend hosted by the University four times a year. I loved listening to the celebratory speakers, who usually offered motivation and helpful suggestions on how to cope with the stress of completing the PhD and reconnecting with the work. I also enjoyed presenting my work at the research weekends and at conferences, where I was able to show my research, discuss my thinking with others, receive helpful suggestions on how I could improve the work, watch other people's presentations and learn from their work.

### 8.4.2. Limitations

If I were to repeat this study, I would make changes to the data collection and analysis methods. For collection, I only investigated and reported on findings of data collected in one school for a short period of time. If I were to repeat this study, I would like to spend more time with learners to understand learners' micro-identities in the classrooms, in terms of how they interact with their teachers and peers across different topics in mathematics. Further, I think more insights can be obtained by following learners as they progress from primary to secondary school as well as to find out what career path they have chosen once they finish school to gain a more valid understanding of how different mechanisms contribute to the identities the learners construct over a longer period of time.

In addition, my research is primarily guided by data collected in classrooms, which can be critiqued as I did not engage with mechanisms which were beyond the mathematics classroom because they were not apparent in the data. Indeed, "the most significant potential to influence students' identities exist in the mathematics classroom" (Anderson, 2007, p. 12), even though learners' mathematical identities and trajectories are socially, historically, politically and institutionally mediated. Stenthoft and Valero (2009) argue that ignoring the other mechanisms besides those directly linked with mathematics and education may lead to the fallacy that other mechanisms external to the mathematics classroom are not important. While I acknowledge the importance of the external mechanisms in chapter 2, an explicit focus on these mechanisms is beyond the scope of this study and requires further research.

## 8.5. Final conclusion

Developing learners' mathematical identities in positive ways cannot be done by the teacher or the learner alone. It is a joint effort as identities emerge through relationships between a person and her/his social context. In order for learners to develop positive mathematical identities, learning interactions need to be fundamentally about mathematics and teachers need to care about learners as part of their professional and ethical duty. In return, learners need to exercise their agency in ways that enhance their learning of mathematics to become successful learners of mathematics, with teachers supporting learners and providing them with opportunities to think for themselves and understand mathematics with the aid of higher-order questions and tasks. Teachers need to be conscious of the kinds of opportunities they present to learners to learn through their interactions with learners, the kinds of narratives they tell about learners and the type of pedagogical approach they utilise in the classroom. Yes, there are mechanisms that are beyond the capacity of the teacher to change, such as learners' past experiences of learning mathematics, learners' socio-economic conditions and the kinds of family support they receive. However, the social identities offered by teachers are a key mechanism responsible for the emergence of learners' mathematical identities, and by changing and reflecting on our practices, we can strive to create active, independent, caring, critical learners, citizens and future parents.

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# Appendix A

Questions for a semi-structured interview with learners:

Some general questions so that learners feel comfortable talking to me:

- What is your favourite subject in school?
- What makes your favourite subject so special to you?

**Current experiences**:

How do you experience learning mathematics in grade 9?

Possible probing questions

**What is mathematics about?**

- Is mathematics important to you? Why?
- Do you enjoy learning mathematics? Why?
- What aspects of mathematics do you enjoy most? How do you know? Why?
- What aspects of mathematics do you enjoy least? How do you know? Why?
- What do you think about making mistakes in your mathematics classroom?
- Do you make mistakes in your mathematics classroom?
- How do you feel when you make a mistake? Why do you feel that way?

**How are you taught mathematics? (Pedagogy)**

- Can you please describe to me what your mathematics lessons are like?
- Please share with me what you think about the way your mathematics teacher teaches?
- Do you enjoy the way he teaches?
- What do you enjoy most about the way he teaches mathematics?
- What do you enjoy least about the way he teaches mathematics?
- Is he helpful or not helpful in enabling you to learn mathematics? How do you know? What makes him helpful or unhelpful?
- How does he react when learners make mistakes in the classroom?
- How do you feel about the way he reacts when learners make mistakes?

**What are the social relations like in your classroom with your teacher?**

- How do you feel when you are sitting in your mathematics classroom? Why?
- How does your teacher make you feel? Why?

**How do you participate in your mathematics classroom?**

- Do you participate in your mathematics classroom?
- Do you feel comfortable participating in the classroom? Why?
- How do you participate in your mathematics classroom?
- Why do you participate in the classroom?
- Do you prefer to work on your own or with others in your classroom? Why?
- Who do you participate in the classroom with?
- How do you participate in the classroom with them?
- Do you do your classwork and homework? Why?
- Do you think doing your classwork and homework helps you understand mathematics? How does it help or not help?

**What does it mean to be good at mathematics?**

- Who in your class is good at doing mathematics? How do you know? What makes them good?
- Who in your class is not good at doing mathematics? How do you know? What makes them not good enough?
- Are you good at doing mathematics? How do you know? What makes you good or not good enough?

**Perceptions of self by significant narrators**

- How do your parents see you as a learner of mathematics? How do you know? Why?
- How does the way your parents see you as a learner of mathematics make you feel?
- How does your teacher see you as a learner of mathematics? How do you know? Why
- How does the way your teacher see you as a learner of mathematics make you feel?
- How do your friends see you as a learner of mathematics? How do you know? Why?
- How does the way your friends see you as a mathematics learner make you feel?
- How do you perform in mathematical assessments? Why?

**Primary school**

How did you experience learning mathematics in primary school?

Possible probing questions

- Could you please describe to me what your lessons were like in primary school?
- Did you enjoy learning mathematics in primary school? Why?
- Did you make mistakes in primary school?
- How did you feel when you made mistakes and why did you feel that way?
- How is learning mathematics now different or similar to how you learnt mathematics in high school?
- Do you think the mathematics you learned in primary schools helps you learn mathematics in high school? How?

**How were you taught mathematics? (Pedagogy)**

- Please share with me what you think about the way your primary mathematics teacher taught you?
- Did you enjoy the way s/he taught?
- What did you enjoy most about the way s/he taught mathematics?
- What did you enjoy least about the way s/he taught mathematics?
- Was s/he helpful or not helpful in enabling you to learn mathematics? How do you know? What made her/him helpful or unhelpful?
- How did s/he react when learners made mistakes in the classroom?
- How did you feel about the way s/he reacted when learners made mistakes?

**What were the social relations like in your classroom with your primary school mathematics teacher?**

- How did you feel when you were sitting in your primary mathematics classroom? Why?
- How did your teacher make you feel? Why?

**How did you participate in your mathematics classroom?**

- Did you participate in your primary mathematics classroom?

- Did you feel comfortable participating? What made you feel comfortable or uncomfortable?
- How did you participate in your primary school mathematics classroom?
- Why did you participate in the classroom?
- Did you prefer to work on your own or with others in your classroom? Why?
- Who did you participate in the classroom with?
- How did you participate in the classroom with them?
- Did you do your classwork and homework? Why?

**Future projections**

Is mathematics necessary for your future career or sense of being?

Possible probing questions

- Would you like to be better at mathematics in the future? Why or why not?
- Who would you like to see yourself becoming in the future (maybe in ten years' time)?
- How do you think mathematics can be helpful to you in the future? Why or why not?

# Appendix B

## Interview 1

## Questions for a semi-structured interview with learners:

Some general questions so that learners feel comfortable talking to me:

- What is your favourite subject in school?
- What makes your favourite subject so special to you?

**Current experiences**:

How do you experience learning mathematics in grade 10[6]?

Possible probing questions

**What is mathematics about?**

- Is mathematics important to you? Why?
- Do you enjoy learning mathematics? Why?
- What aspects of mathematics do you enjoy most? How do you know? Why?
- What aspects of mathematics do you enjoy least? How do you know? Why?
- What do you think about making mistakes in your mathematics classroom?
- Do you make mistakes in your mathematics classroom?
- How do you feel when you make a mistake? Why do you feel that way?

**How are you taught mathematics? (Pedagogy)**

- Can you please describe to me what your mathematics lessons are like?
- Please share with me what you think about the way your mathematics teacher teaches?
- Do you enjoy the way he/she teaches?
- What do you enjoy most about the way he/she teaches mathematics?
- What do you enjoy least about the way he/she teaches mathematics?
- Is he/she helpful or not helpful in enabling you to learn mathematics? How do you know? What makes him/her helpful or unhelpful?

[6] Grade 10 in 2016 and grade 11 in 2017

- How does he react when learners make mistakes in the classroom?
- How do you feel about the way he/she reacts when learners make mistakes?

**What are the social relations like in your classroom with your teacher?**

- How do you feel when you are sitting in your mathematics classroom? Why?
- How does your teacher make you feel? Why?

**How do you participate in your mathematics classroom?**

- Do you participate in your mathematics classroom?
- Do you feel comfortable participating in the classroom? Why?
- How do you participate in your mathematics classroom?
- Why do you participate in the classroom?
- Do you prefer to work on your own or with others in your classroom? Why?
- Who do you participate in the classroom with?
- How do you participate in the classroom with them?
- Do you do your classwork and homework? Why?
- Do you think doing your classwork and homework helps you understand mathematics? How does it help or not help?

**What does it mean to be good at mathematics?**

- Who in your class is good at doing mathematics? How do you know? What makes them good?
- Who in your class is not good at doing mathematics? How do you know? What makes them not good enough?
- Are you good at doing mathematics? How do you know? What makes you good or not good enough?

**Perceptions of self by significant narrators**

- How do your parents see you as a learner of mathematics? How do you know? Why?
- How does the way your parents see you as a learner of mathematics make you feel?
- How does your teacher see you as a learner of mathematics? How do you know? Why
- How does the way your teacher see you as a learner of mathematics make you feel?
- How do your friends see you as a learner of mathematics? How do you know? Why?

- How does the way your friends see you as a mathematics learner make you feel?
- How do you perform in mathematical assessments? Why?

## Interview 2

### Past experiences

How did you experience learning mathematics in grade 9[7]?

Possible probing questions

- Could you please describe to me what your lessons were like last year?
- Did you enjoy learning mathematics last year? Why?
- Did you make mistakes last year?
- How did you feel when you made mistakes and why did you feel that way?
- How is learning mathematics now different or similar to how you learnt mathematics last year?
- Do you think the mathematics you learned last year helps you learn mathematics this year? How?

**How were you taught mathematics? (Pedagogy)**

- Please share with me what you think about the way your last year's teacher taught you?
- Did you enjoy the way s/he taught?
- What did you enjoy most about the way s/he taught mathematics?
- What did you enjoy least about the way s/he taught mathematics?
- Was s/he helpful or not helpful in enabling you to learn mathematics? How do you know? What made her/him helpful or unhelpful?
- How did s/he react when learners made mistakes in the classroom?
- How did you feel about the way s/he reacted when learners made mistakes?

**What were the social relations like in your classroom with your last year's mathematics teacher?**

- How did you feel when you were sitting in your mathematics classroom? Why?

[7] Grade 9 for 2015 and 10 for 2016

- How did your teacher make you feel? Why?

**How did you participate in your last year's mathematics classroom?**

- Did you participate in your mathematics classroom?
- Did you feel comfortable participating? What made you feel comfortable or uncomfortable?
- How did you participate in your mathematics classroom?
- Why did you participate in the classroom?
- Did you prefer to work on your own or with others in your classroom? Why?
- Who did you participate in the classroom with?
- How did you participate in the classroom with them?
- Did you do your classwork and homework? Why?

**Future projections**

Is mathematics necessary for your future career or sense of being?

Possible probing questions

- Would you like to be better at mathematics in the future? Why or why not?
- Who would you like to see yourself becoming in the future (maybe in ten years' time)?
- How do you think mathematics can be helpful to you in the future? Why or why not?

# Appendix C

## Semi-structured interview with the teacher:

Each teacher will be asked the same questions for each of the learners who are being researched in their classroom.

Questions for a semi-structured interview with the teacher:

**What characterises a successful mathematics learner in you class?**

Possible probing questions

- In what ways do you think that the way you teach promotes competence?
- How do you recognise competence?
- How do you recognise incompetence?

**How does Learner A participate in the classroom with you, each-other, their peers and their work?**

Possible probing questions

- Does s/he engage with you in the classroom? If so, how?
- Does s/he ask or answer questions?
- Are his /her responses to questions in the classroom usually correct or incorrect?
- Does s/he complete their work and homework timeously and correctly?
- How does s/he perform in mathematical assessments?

**What do you know about Learner A's past experiences of mathematics?**

Possible probing questions

- What do you know about the way s/he was taught?
- Do you think s/he learnt mathematics well last year?
- What do you think influenced him/her to learn/not to learn mathematics well last year?

**Do you think that Learner A is good at doing mathematics? Why or why not.**

Possible probing questions

- What aspects of mathematics is s/he good at?
- What aspects of mathematics is s/he not good at?
- How do you know what s/he is good or not good at in mathematics?

**Do you think Learner A will pursue mathematics in his/her future?**

Possible probing questions

- Do you know what career choice s/he wishes to pursue?
- Do you think s/he will pursue an academic degree? Why or why not?
- Do you think that s/he could pursue an academic degree which is mathematically orientated?