and $l_2 = 0.0456 \times 0.16 = 0.0073$ cms.

$\therefore \quad l_g = 0.00456 + 0.00730 + 0.004560 = 0.05766$ cms.

The winding used was a narrow phase spread winding, housed in 2 slots per pole per phase; there being 4 poles.

The number of turns per coil = 12.

Thus, the total number of turns = 12 x 24 = 288.

$$T_1 = \text{turns per phase} = \frac{288}{3} = 96$$

Now, we have that $q = p(6K_5 + 1)$ and $p = 2$

Therefore, the following table can be drawn up.

K_5	- 3	- 2	- 1	0	1	2	3
q	- 34	- 22	- 10	2	14	26	38

where q is the value of the harmonic associated with each value of K_5

The slot angle δ in mechanical degrees $= \frac{360}{24} = 15$

$$\text{Now} \quad Kmq = \frac{\text{Sin. } g\frac{q\delta}{2}}{g \text{ Sin } \frac{q\delta}{2}}$$

Substituting for the above values of q

q	Kmq
2	0.965
-10	0.255
14	-0.255
-22	-0.965
26	-0.965
-34	-0.255
38	+0.255

Also, $Keq = \text{Sin } q \frac{\beta}{2}$

For a full pitched winding, $\beta = \frac{\Pi}{P}$ mechanical degrees

Therefore $Keq = \text{Sin } \frac{q\Pi}{2p} = \text{Sin } \frac{q\Pi}{4}$

CONCLUSIONS

1. MEASURING APPARATUS

The inverted induction motor and associate apparatus was so designed as to facilitate the measurement of various electrical phenomena, such as speed and torque.

By re-designing the apparatus for use with the multi-channel, ultra-violet light recorder, more phenomena could be recorded, the system became considerably more sensitive and everything was now operated at an extremely low voltage. Furthermore, when calibrating the new apparatus, it was found to be completely linear over the working range, whereas the original apparatus was not.

By comparing "old" and "new" oscillogram traces, it was discovered that the damping system used was not ideal; this being due to a change in design of the induction motor. However, by increasing the transducer tension this small inadequacy was compensated for.

Using the new apparatus a number of tests were run in order to try and measure the induction motor "residual torque". From the results obtained one may only conclude that this torque is either non existant, or was of too small a value to be measured by this apparatus.

A large number of readings and records were taken using the re-designed apparatus, and on studying these it was felt that the results were quite satisfactory, the apparatus adequate and a considerable improvement on the original.

2. FLUX INVESTIGATION

From the results obtained in this investigation, a number of conclusions can be drawn. In making these conclusions, however, the various assumptions made throughout the investigation must be borne in mind.

Firstly, the reluctance variation. Assuming the gap to vary as a cosine law, and hence a Fourier series for the effect of both primary and secondary slots. The number of terms considered in the Binomial expansion for the flux density. The assumption that all the flux linked a search coil on a rotor tooth obviously conflicts with the reluctance variation. The "iron path" was ignored in the calculation of the flux.

Remembering the above, it may be concluded from the results obtained that the method employed gives good correlation only in the case of the fundamental wave, even when further terms in the Fourier series and Binomial expansion are considered.

However, a further conclusion that must be drawn is that the method of practical flux measurement employed was far from satisfactory. Due to the fact that the theoretical results give far more frequency components than the practical results, it is felt that the measurements must be recorded while the machine is extremely accurately held at each definite speed; small variations in speed giving erroneous results.

It is also felt that the flux should be measured directly by the use of a device such as a "Hall-generator", which is now commercially obtainable. These paper thin probes may be conveniently situated in the motor in order to measure the various fluxes and hence compute the exact values of flux in the gap, tooth, etc..

Finally, therefore, it may be concluded that the results of this investigation do not prove the methods of flux analysis used to be incorrect, but indicate that more refined methods of practical measurement should be employed.

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INTRODUCTION.

Although the following appendices are not connected with the main body of the thesis, they have been included as they were developed during the flux investigation.

Appendix D is a series of photographs showing the apparatus used.

APPENDIX A.

STATOR ELECTRO MOTIVE FORCE

The electro motive force in the stator is produced by those components of the flux density which move with respect to the stator, i.e.

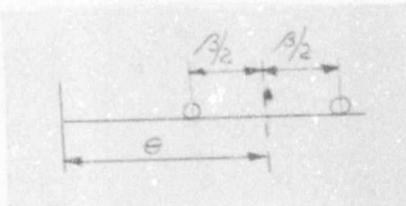
$$\frac{A\ dc\ q\ l_2}{\pi q \quad lg^2} \quad \cos Q_2 \ (\theta_1 - w_s t) \cos q\theta_1$$

$$\theta_1 = \theta_2 + w_s t \quad \text{where} \quad w_s - 2\pi n_s$$

Therefore the flux density with respect to the rotor is,

$$B = \frac{A\ dcq\ l_2}{\pi q\ lg^2} \ \cos Q_2\ (\theta_2 + w_s t - w_s t) \cos q\ (\theta_2 + w_s t)$$

$$= \frac{A\ dcq\ l_2}{\pi q\ lg^2} \ \cos Q_2\theta_2 \ \cos q\ (\theta_2 + w_s t)$$



Coil span = β , therefore flux linking coil

$$\phi = \left\{ \frac{1}{\beta} \int_{\theta_2 - \beta/2}^{\theta_2 + \beta/2} M \cos Q_2\theta_2 \cos q\ (\theta_2 + w_s t)\ d\theta_2 \right\} \frac{\beta}{2\pi} \cdot 2\pi R\ l$$

$$= \frac{1}{\beta}\ lR\beta\ \ M \int_{\theta_2 - \beta/2}^{\theta_2 + \beta/2} \cos Q_2\theta_2 \cos q\ (\theta_2 + w_s t)\ d\theta_2$$

$$= \ lRM \int_{\theta_2 - \beta/2}^{\theta_2 + \beta/2} \cos Q_2\theta_2 \cos q\ (\theta_2 + w_s t)\ d\theta_2 \qquad \underline{\quad\quad} 1$$

Integration only.

$$\int_{\theta_2 - \beta/2}^{\theta_2 + \beta/2} \cos Q_2\theta_2 \cos q\ (\theta_2 + w_s t)\ d\theta_2 \qquad \text{-------} \int AB\ dx$$

$$= \int_{\theta_2 - \beta/2}^{\theta_2 - \beta/2} \frac{1}{2} \left\{ \cos\ (Q_2\theta_2 + q\ \theta_2 + q\ w_s t) + \cos\ (\ Q_2\theta_2 - q\ \theta_2 - q\ w_s t) \right\} d\ \theta_2$$

$$= \frac{1}{2} \int_{\theta_2 - \beta/2}^{\theta_2 + \beta/2} \cos \left[\left\{ (Q_2 + q)\ \theta_2 + q\ w_s t \right\} + \cos\ \left\{ (Q_2 - q)\ \theta_2 - q\ w_s t \right\} \right] d_{\theta 2}$$

$$= \frac{1}{2}\,\frac{1}{Q_2+q}\,\sin\left[(Q_2+q)\theta_2 + qw_s t\right] + \frac{1}{Q_2-q}\,\sin\left[(Q_2-q)\theta_2 - qw_s t\right]\Bigg]_{\theta_2-\beta/2}^{\theta_2+\beta/2}$$

$$= \frac{1}{2}\left[\frac{1}{Q_2+q}\left\{\sin\left[(Q_2+q)(\theta_2+\frac{\beta}{2}) + qw_s t\right] - \sin\left[(Q_2-q)(\theta_2-\frac{\beta}{2}) - qw_s t\right]\right\}\right.$$

$$\left. + \frac{1}{Q+q}\left\{\sin\left[(Q_2-q)(\theta_2+\frac{\beta}{2}) - qw_s t\right] - \left[\sin\,(Q_2-q)(\theta_2-\frac{\beta}{2}) - qw_s t\right]\right\}\right]$$

$$= \frac{1}{2}\left\{\frac{1}{Q+q}\left[\left[\sin\left[(Q_2+q)\,\theta_2 + qw_s t + (Q_2+q)\frac{\beta}{2}\right] - \left[\sin\,(Q_2+q)\theta_2 + qw_s t\right.\right.\right.\right.$$

$$\left.\left.\left. - (Q_2+q)\frac{\beta}{2}\right]\right]\right] + \frac{1}{Q-q}\left[\left[\sin\left[(Q_2-q)\theta_2 - qw_s t + (Q_2-q)\frac{\beta}{2}\right]\right.\right.$$

$$\left.\left.\left. - \sin\left[(Q_2-q)\,\theta_2 - qw_s t - (Q_2-q)\frac{\beta}{2}\right]\right]\right]\right\}$$

$$= \frac{1}{2}\left\{\frac{2}{Q_2+p}\cos\left[(Q_2+q)\,\theta_2 + qw_s t\right]\,\sin\,(Q_2+q)\frac{\beta}{2} + \frac{2}{Q_2-q}\cos\left[(Q_2-q)\theta_2 - qw_s t\right]\,\sin\,(Q_2-q)\frac{\beta}{2}\right\}$$

$$= \frac{1}{Q_2+q}\sin\,(Q_2+q)\frac{\beta}{2}\,\cos\left[(Q_2+q)\,\theta_2 + qw_s t\right] + \frac{1}{Q_2-q}\sin\,(Q_2-q)\frac{\beta}{2}\,\cos\left[(Q_2+q)\,\theta_2 - qw_s t\right]$$

Going back to (1) and letting there be Tc turns

$$\phi = \frac{lRAdcq\; l_2\; Tc}{\pi q\; lg^2}\left\{\frac{1}{(Q_2+q)}\sin\,(Q_2+q)\frac{\beta}{2}\cos\left[(Q_2+q)\theta_2+qw_s t\right] + \frac{1}{Q_2-q}\sin\,(Q_2-q)\frac{\beta}{2}\cos\left[(Q_2-q)\,\theta_2 - qw_s t\right]\right\}$$

$$\text{Now}\quad e = -Tc\,\frac{d\phi}{dt}$$

$$= -K\left\{-\frac{qw_s}{Q_2+q}\sin\,(Q_2+q)\frac{\beta}{2}\sin\left[(Q_2+q)\theta_2 + qw_s t\right] + \frac{qw_s}{Q_2-q}\sin\left[(Q_2-q)\frac{\beta}{2}\cdot\sin\,(Q_2-q)\,\theta_2 - qw_s t\right]\right\}$$

$$e = \frac{Adcq}{\pi q}\,2\pi n_s Rl\; Tc\,\frac{l_2}{lg^2}\left[\left[\frac{q}{Q_2+q}\sin\,(Q_2+q)\,\frac{\beta}{2}\sin\left[(Q_2+q)\,\theta_2 - qw_s t\right]\right] - \frac{q}{Q_2-q}\sin\,(Q_2-q)\,\frac{\beta}{2}\sin\left[(Q_2-q)\,\theta_2 - qw_s t\right]\right]$$

This can be broken up into two vectors as shown below.

1. $H\,\frac{q}{Q_2+q}\,\sin\,(Q_2+q)\,\frac{\beta}{2})\}\underline{(Q_2+q)\theta_2}$ ____ Equation 2.

2. $-H\,\frac{q}{Q_2-q}\,.\,\sin\,(Q_2-q)\,\frac{\beta}{2}\;\}\underline{-(Q_2+q)\theta_2}$ ____ Equation 3.

Where $H = \frac{Adcq}{\pi q}\cdot\pi 2\,n_s Rl\; Tc\,\frac{l_2}{lg^2}$

A P P E N D I X   B .

STATOR ELECTRO MOTIVE FORCE

(Method 2.)

The flux density is given by

$$\hat{B} = \frac{Adcq}{\pi q} \cdot \frac{1_2}{21g^2}$$

Now, the flux per ripple pole for the first wave is

$$\phi_r = \hat{B}\ \frac{2}{\pi}\ 1\ \frac{2\pi R}{2(Q_2+q)} \qquad \text{R is the radius}$$

$$= \frac{Adcq}{\pi q} \cdot \frac{1_2}{1g^2} \cdot \frac{1R}{(Q_2+q)}$$

and for the other wave the flux would be

$$\phi_r = \frac{Adcq}{\pi q} \cdot \frac{1_2}{1g^2} \cdot \frac{1R}{(Q_2-q)}$$

Electro Motive Force from 1st wave

$$Eq_1 = \frac{2\pi}{\sqrt{2}}\ \phi_r\ f_r\ T\ km_1\ Ke_1 \underline{\left|\ -(Q_2+q)\theta_1\right.}$$

The reference is the electro motive force in the coil whose centre line is at zero.

$$\text{Now } f_r = \frac{Q_2 ws}{2\pi}$$

$\therefore$ Above equation is

$$\sqrt{2}\ Eq_1 = 2\pi\ \frac{Adcq}{\pi q} \cdot \frac{1_2}{1g^2} \cdot \frac{1R}{(Q_2+q)} \cdot \frac{Q_2\ 2\Pi N_s}{2\pi}\ T\ Km_1\ Ke_1 \underline{\left|\ -(Q_2+q)\theta_1\right.}$$

$$= \frac{Adcq\ 1_2}{\pi q\ 1g^2}\ \frac{Q_2 1}{(Q_2+q)}\quad 2\Pi R n_s\ T\ Ke_1\ Km_1 \underline{\left|\ -(Q_2+q)\theta_1\right.}$$

$$Ke = \text{Sin}\ (Q_2+q)\beta/2 \qquad \beta = \text{Coil span, mechanical radians}$$

Similarly

$$\sqrt{2}\ Eq_2 = \frac{Adcq\ 1_2}{\pi q\ 1g^2}\ \text{Sin}\ (Q_2-q)\frac{\beta}{2} \cdot \frac{Q_2 1}{Q_2-q}\ 2\Pi R n_s\ T\ Km_2 \underline{\left|\ -(Q_2-q)\theta_1\right.}$$

Stator slot angle as far as first wave is concerned is $\frac{(Q_2+q)2\pi}{Q_1}$

and as far as second wave is concerned $\frac{(Q_2-q)2\pi}{Q_1}$

Thus, distribution factors

$$Km_1 = \frac{\text{Sin}\left[\frac{g\ (Q_2+q)}{Q_1}.\Pi\right]}{g\ \text{Sin}\ \frac{(Q_2+q)}{Q_1}.\ \Pi} \qquad \text{for } 60^\circ \text{ phase spread}$$

$$= \frac{\operatorname{Sin}\left[\frac{2g\,(Q_2 + q)}{Q_1} \cdot \pi\right]}{2g\ \operatorname{Sin}\left[\frac{Q_2 + q}{Q_1} \cdot \pi\right]}$$

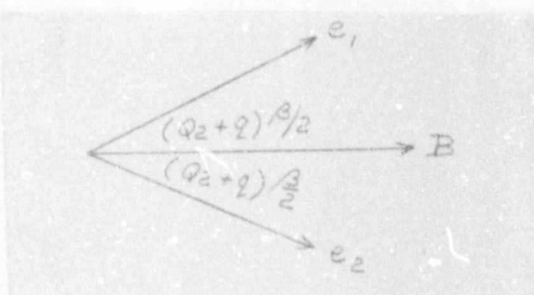
for 120° phase spread

$$Km_2 = \frac{\operatorname{Sin}\left[\frac{2g\,(Q_2 - q)}{Q_1} \cdot \pi\right]}{2g\ \operatorname{Sin}\left[\frac{Q_2 - q}{Q_1} \cdot \pi\right]}$$

Both waves for all values of q produce the same frequency i.e. $\frac{Q_2 w_s}{2}$

the total electro motive force is the vectorial sum of all components.

i.e. $\sum \sqrt{2}\,\dot{E}q_1 + \sqrt{2}\,\dot{E}q_2$



The rotor electro motiv- forces are produced by 5 waves of flux density B.

$$B_1 = \frac{Adcq}{\pi q}\ \frac{1}{lg}\ \operatorname{Cos}\ (q\theta_2 + q w_s t) \qquad f_r = \frac{q w_s}{2\pi} = q\,n_s$$

$$B_2 = \frac{Adcq}{\pi q}\ \frac{l_1}{2lg^2}\ \operatorname{Cos}\left\{ (Q_1+q)\ \theta 2 + (Q_1 + q)\ W_s t \right\} \quad f_r = (Q_1+q)\ n_s$$

$$B_3 = \frac{Adcq}{\pi q}\ \frac{l_1}{2lg^2}\ \operatorname{Cos}\left\{ (Q_1-q)\ \theta_2 + (Q_1 - q)\ w_s t \right\} \quad f_r = (Q_1-q)\ n_s$$

$$B_4 = \frac{Adcq}{\pi q}\ \frac{l_2}{2lg^2}\ \operatorname{Cos}\left\{ (Q_2+q)\ \theta_2 + q\ w_s t \right\} \qquad f_r = q\ n_s$$

$$B_5 = \frac{Adcq}{\pi q}\ \frac{l_2}{2lg^2}\ \operatorname{Cos}\left\{ (Q_2-q)\ \theta_2 - q\ w_s t \right\} \qquad f_r = q\ n_s$$

Thus the electro motive force in a coil of Tc turns with span β mechanical radians is

$$- \frac{Adcq}{\pi q}\ \frac{2}{lg}\ .\ l\ .\ 2\pi R n_s\ \operatorname{Sin}\ q\,\frac{\beta}{2}\ Tc\ \underline{\vert + q\,\theta_2}$$

Angle is displacement of electro motive force from that generated in a coil situated at rotor origin. $\theta_2 = 0$.

[illegible]

1.	$\dfrac{A_5}{\pi(-Q_2+p)lg}$	[illegible] $Q_2 +$ [illegible]	$\dfrac{[illegible] (-Q_2+p)w_s}{2\pi}$
2.	$\dfrac{A_5 l_1}{2\pi(-Q_2+p)lg}$	[illegible] $Q_2 - p$	$\dfrac{(Q_1+Q_2-p)w_s +}{2\pi}$ [illegible]
3.	$\dfrac{A_5 l_1}{2\pi(-Q_2+p)lg^2}$	[illegible] $Q_2 + p$	$\dfrac{(Q_1-Q_2+p)w_s -}{2\pi}$ [illegible]
4.	$\dfrac{A_5 l_2}{2\pi(-Q_2+p)lg^2}$	[illegible]	$\dfrac{w - (-Q_2+p)w_s}{2\pi}$
5.	$\dfrac{A_5 l_2}{2\pi(-Q_2+p)lg^2}$		$\dfrac{w - (-Q_2+p)w_s}{2\pi}$

The results can now be tabulated, since column 1 above has already been evaluated (see pp. 65) and is the value k in the basic equation I on pp. 71. The pole pairs are Λ, and hence the following table can be extracted.

k.	Λ.	$\sin \frac{\Lambda\beta}{2}$	$\hat{\phi} = \frac{Dlk}{\Lambda} \sin \frac{\Lambda\beta}{2}$	f.	$E = 4.44\phi$ ft.

It can be seen from the basic equation I (pp.71) that all terms of the same frequency (i.e. the same Ω_2) must add with due respect to the sign of both k and $\sin \frac{\Lambda\beta}{2}$. This sum should correspond to the measured values of the magnitudes at the various frequencies.

Starting with the synchronous wave, i.e. $q = p$ we obtain the following for the various terms in the table.

1. $k = 13.65 \times 10^{-2} \times 2.5 = 0.341.$

2. $\Lambda = p = 2.$

3. $\sin \frac{\Lambda\beta}{2} = \sin \left(\frac{2\times 10}{2}\right) = \sin 10$ [illegible] $0.174.$

4. $\phi = \frac{Dlk}{\Lambda} \sin \frac{\Lambda\beta}{2}.$ $\quad Dl = 217 \times 10^{-4}$ ___ constant.

   $\therefore \phi = \frac{217 \times 0.341}{2} \times 0.174 \times 10^{-4}$ [illegible] 0.645×10^{-3} wb
   (as before)

5. $F = \frac{w - pws}{2\pi} = \frac{w}{2\pi}$ = 50 cps. For speed = 0 rpm

6. $E = 4.44\ \phi\ ft$ $\qquad (= 44.4\ \phi\ f\ )$

$= 4.44 \times 0.645 \times 10^{-3} \times 50 \times 10$

$= 1.43$ volts.

In order to obtain the final values all 25 terms ($q = p$, $q = \pm Q_1 + p$, $q = \pm Q_2 + p$) must be calculated (procedure as above) and the terms of the same frequency added together.

<u>0 rpm.</u>

k.	A.	Sin $A\beta/2$	ϕwb		f.	E.mV.
.341	2	0.174	0.645	$\times 10^{-3}$	50	1430
.0135	22	0.934	0.0124	$\times 10^{-3}$	50	27.3
.0135	26	0.766	0.00865	$\times 10^{-3}$	50	19.2
.0216	34	0.174	0.00239	$\times 10^{-3}$	50	5.3
.0216	38	-0.174	-0.00214	$\times 10^{-3}$	50	-4.75
-.0275	26	0.766	-.0176	$\times 10^{-3}$	50	-39.1
-.00104	2	0.174	-.00197	$\times 10^{-3}$	50	-4.37
-.00104	50	-0.939	+.000563	$\times 10^{-3}$	50	1.25
-.00166	10	0.766	-.00237	$\times 10^{-3}$	50	-5.26
-.00166	62	-0.766	.000287	$\times 10^{-3}$	50	.637
.031	22	0.934	.0286	$\times 10^{-3}$	50	63.5
.00123	46	-0.766	-.00045	$\times 10^{-3}$	50	-1.0
.00123	2	0.174	.00234	$\times 10^{-3}$	50	5.2
.00197	58	-.939	-.00694	$\times 10^{-3}$	50	-15.4
.00197	14	.939	.00287	$\times 10^{-3}$	50	6.37
-.00475	38	-.174	+.000472	$\times 10^{-3}$	50	1.0
-.000187	14	.939	-.000273	$\times 10^{-3}$	50	-.58
-.000187	62	-.766	+.00000502	$\times 10^{-3}$	50	.0106
-.0003	2	.174	-.000568	$\times 10^{-3}$	50	-1.2
-.0003	74	.174	-.000153	$\times 10^{-3}$	50	-.324
-.00612	34	.174	-.00068	$\times 10^{-3}$	50	-1.44
-.000242	58	-.939	.00085	$\times 10^{-3}$	50	1.80
-.000242	10	.766	.000402	$\times 10^{-3}$	50	-.853
-.000388	70	-.174	.0002	$\times 10^{-3}$	50	.044
-.000388	2	.174	-.000735	$\times 10^{-3}$	50	-1.56

Resultant:	f.	E.mV
	50.	1485.7

$$- \frac{q}{Q_2-q} \frac{1_2}{1g^2} \quad \mathrm{Sin} \quad (Q_2-q)\frac{\beta}{2}\Big] \quad \underline{|+ q\ \theta_2}$$

at frequency $q\ n_s$

and $\sqrt{2}\ Ec_2$ at frequency $(Q_1+q)\ n_s$

and $\sqrt{2}\ Ec_3$ at frequency $(Q_1-q)\ n_s$

If the coil spans a slot pitch then $\beta = \frac{2\pi}{Q2}.a$

$$(Q_2+q)\frac{\beta}{2} = (Q_2+q)\frac{\pi a}{Q_2} = \frac{q\pi a}{Q_2} + a\pi$$

$$(Q_2-q)\frac{\beta}{2} = -\frac{q\pi a}{Q_2} + a\pi$$

$$\frac{q\beta}{2} = \frac{q\pi a}{Q_2}$$

The coil span factors will be of the same value but may have different signs depending on the value of a.

The Ec_4 and Ec_5 combinations may be combined as

$$(Q_2+q)\frac{\beta}{2} = q\frac{\pi a}{Q_2} + a\pi \qquad \text{i.e. } \beta = \frac{2\pi}{Q_2}a$$

and $$(Q_2-q)\frac{\beta}{2} = -\frac{q\pi a}{Q_2} + \pi a$$

Then $\mathrm{Sin}\ (Q_2+q)\frac{\beta}{2} = -\mathrm{Sin}\ (Q_2-q)\frac{\beta}{2}$ for all values of a.

Also $\mathrm{Sin}\ (Q_2+q)\frac{\beta}{2} = \mathrm{Sin}\ (\frac{q\pi a}{Q_2} + a\pi)$

Thus

$$\sqrt{2}\ Ec_6 = -\frac{Adcq}{\pi q} 2\Pi Rn_s 1\ Tc \left[\frac{2}{1g}\ \mathrm{Sin}\ \frac{q\pi a}{Q_2} + \frac{1_2}{1g^2}\left\{\frac{q}{Q_2+q} + \frac{q}{Q_2-q}\right\} \times \right.$$

$$\left. \mathrm{Sin}\ (\frac{q\pi a}{Q_2} + a\pi)\right] \underline{|q\ \theta_2}$$

$$= -\frac{Adcq}{\pi q} . 2\Pi Rn_s 1\ Tc \left[\frac{2}{1g} \mathrm{Sin}\ q\frac{\pi a}{Q_2} + \frac{1_2}{1g^2}\frac{2qQ_2}{Q_2^2-q^2}\ \mathrm{Sin}\ (\frac{q\pi a}{Q_2} + a\pi)\right] \underline{|q\ \theta_2}$$

$$\mathrm{Sin}\ \frac{q\pi a}{Q_2} = \pm\ \mathrm{Sin}\ \frac{q\pi a}{Q_2} + a\pi \qquad \begin{matrix} + \text{ for a even} \\ - \text{ for a odd}\end{matrix}$$

Thus

$$\sqrt{2}\ Ec_6 = -\frac{Adcq}{\pi q}\ 2\Pi Rn_s 1\ Tc\ \mathrm{Sin}\ (\frac{q\pi a}{Q_2})\left[\frac{2}{1g} \pm \frac{1_2}{1g^2}\frac{2\ q\ Q_2}{Q_2^2 - q^2}\right] \quad \underline{|q\ \theta_2}$$

+ for a even
- for a odd

From the bracketed term, the 1st term gives an indication due to the "fundamental" wave and the other term, that due to the rotor slots.

The equations for Ec_2 and Ec_3 allow these two electro motive forces to be found separately. These are at frequencies $(Q_1 + q)n_s$ and $(Q_1 - q)n_s$ respectively but it should be noted that for certain conditions these frequencies fit into the series associated with Ec_1, Ec_4 and Ec_5 whose frequencies are $q\,n_s$.

The value of q is given by $q = p\,(6\,K_1 + p)$. Thus if the frequency $(Q_1 + p_1)n_s$ is to be the same as a frequency $q_2\,n_s$ (for different values of q),

$$(Q_1 + q_1) = q_2$$

i.e. $$Q_1 + p\,(6K_1 + p) = p\,(6K_1^1 + 1)$$

therefore $$\frac{Q_1}{p} + 6K_1 + 1 = 6K_1^1 + 1$$

$$\frac{Q_1}{p} + 6K_1 = 6K_1^1$$

i.e. $$\frac{Q_1}{6p} + K_1 = K_1^1$$

Now K_1 and K_1^1 must both be whole numbers which can only occur if $\frac{Q_1}{6p}$ is a whole number.

$$\frac{Q_1}{6p} = \frac{Q_1}{3 \times 2p} = g = \text{number of slots/pole/phase}$$

Thus for an integral slot winding on the stator the frequency of $(Q_1 + q)n_s$ for one value of q is equal to the frequency of $q\,n_s$ for another value of q.

The same can be obtained for frequencies $(Q - q)n$.

APPENDIX C.

DIRECT CURRENT IN A POLYPHASE WINDING

Direct Current in the Polyphase Winding.

A magneto motive force wave similar in shape to that of the A.C. wave can be obtained by putting the correct values of D.C. through the winding. The currents in the m-phases in the A.C. case are given by

$$i_1 = \sqrt{2}\ I \ \text{Sin} \ wt$$

$$i_2 = \sqrt{2}\ I \ \text{Sin} \ \left(wt - \frac{\pi b_1}{m_1}\right)$$

$$i_3 = \sqrt{2}\ I \ \text{Sin} \ \left(wt - \frac{\pi b_1 2}{m_1}\right)$$

If we take the instant of time when the current in phase 1 is a maximum then the instantaneous distribution of currents is,

$i_1 = \sqrt{2}\ I,$

$wt = \Pi/2$

$i_2 = \sqrt{2}\ I \ \text{Sin} \ (\Pi/2 - \Pi b_1/m_1) = \sqrt{2}\ I \ \text{Cos} \ \pi b_1/m_1$

$i_3 = \sqrt{2}\ I \ \text{Sin} \ (\Pi/2 - 2\Pi b_1/m_1) = \sqrt{2}\ I \ \text{Cos} \ 2\pi b_1/m_1$

If we wish to set up this system of currents in any winding by D.C. then if Idc flows in phase 1 the currents in the other phases must have the following magnitudes.

$i_1 = Idc$

$i_2 = Idc \ \text{Cos} \ \pi b_1/m_1$

$i_3 = Idc \ \text{Cos} \ 2\pi b_1/m_1$ etc.

In a narrow phase spread three phase winding $b_1 = 1$ and $m = 3$ but alternative groups have opposite signs when b is odd.

i.e. $i_1 = Idc$

$i_2 = - Idc \ \text{Cos} \ \Pi/3 = - \frac{Idc}{2}$

$i_3 = Idc \ \text{Cos} \ 2\Pi/3 = - \frac{Idc}{2}$

i.e.

The equation for the magneto motive force wave can be obtained from the original expression by putting $wt = \Pi/3$ and $\sqrt{2}\ I = Idc$.

$$AT = \sum \frac{m \ Idc \ T_1}{\pi q} \ Kmq \ Keq \ \text{Sin} \ (\Pi/3 - q\,\theta_1)$$

$$= \sum \frac{m \ Idc \ T_1}{\pi q} \ Kmq \ Keq \ \text{Cos} \ q\,\theta_1$$

$$q = \frac{2 K_5 m_1 p}{b_1} + p$$

The spacial angle between phases being $\frac{\pi b_1}{pm}$ this gives

$$fph_1 = \frac{2\ Idc\ T_1}{\pi q}\ Kmq\ \ Keq\ \ Cos\ \ q\ \ \theta_1$$

$$fph_2 = \frac{2\ Idc\ Cos\ \frac{\pi b_1}{pm}\ \ T_1}{\pi q}\ \ Kmq\ \ Keq\ \ Cos\ \ q\ \ \theta_1$$

etc.

The sum of m phases is

$$AT = \frac{2\ Idc\ T_1}{\pi q}\ Kmq\ Keq\ \Big[Cos\ 0\ Cos\ q\ \theta_1 + Cos\ \frac{\pi b_1}{m}\ Cos\ q\ (\theta_1 - \frac{\pi b_1}{pm}) + Cos\ \frac{2\pi b_1}{m}\ Cos\ q\ (\theta_1 - \frac{2\pi b_1}{pm}) + \ldots \Big]$$

$$= \frac{2\ Idc\ T_1}{\pi q}\ Kmq\ Keq\ \frac{1}{2}\Big[Cos\ q\ \theta_1 + Cos\ (-\ q\ \theta_1) + Cos\ (q\ \theta_1 + \frac{\pi b_1}{m}(1 - q/_p) + Cos\ \Big(-q\ \theta_1 + \frac{\pi b_1}{m}\ .\ (1 + q/_p)\Big)$$

$$+ Cos\ \Big(q\ \theta_1 + \frac{2\pi b_1}{m}(1 - q/_p)\Big) + Cos\ \Big(-\ q\ \theta_1 + \frac{2\pi b_1}{m}(1 + q/_p)\Big) + \ldots\ldots$$

The first group of terms in the bracket will sum to 0 unless $\frac{\pi b_1}{m}(1 - q/_p) = 2\pi K_5$ when the sum will be m and the second group will sum to 0 unless $\frac{2\pi b_1}{m}(1 + q/_p) = 2\pi K_5$ when sum will be m.

Thus, it can be shewn

$$AT = \frac{m\ Idc\ T_1\ Kmq\ Keq}{\pi q}\ Cos\ q\ \theta_1$$

$$\text{where } q = \frac{2\ K_5 m_1 p}{b_1} + p$$

For narrow phase spread winding $m = 3$ $\quad b_1 = 1$

$$AT = \frac{3\ Idc\ T_1}{\pi q}\ Kmq\ Keq\ Cos\ q\ \theta_1 \qquad q = p\ (6K_5 + 1)$$

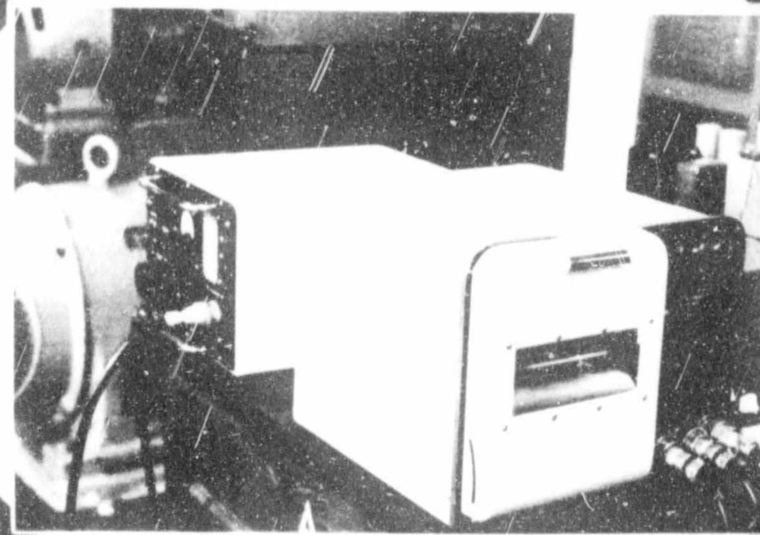
K_5 any intiger.

Similarly to the AC case

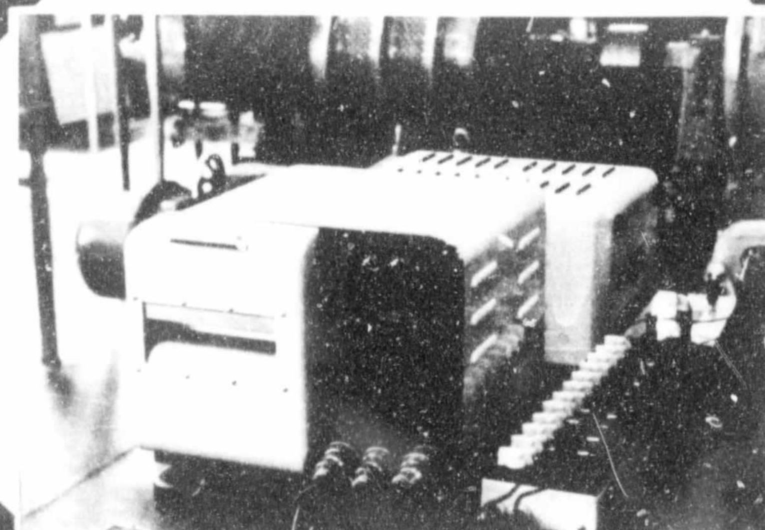
$$B\ dcq = \frac{\mu_o 3\ Idc\ T_1\ Kmq\ Keq\ Cos\ q\ \theta_1}{\pi q\ \{lg - l_1\ Cos\ Q_1\theta_1 - l_2\ Cos\ Q_2\ (\theta_1 - w_s t)}$$

APPENDIX D.

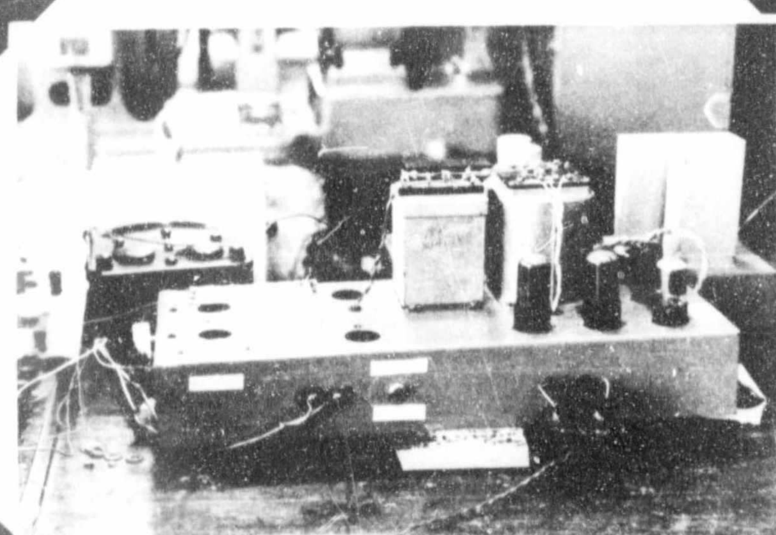
PHOTOGRAPHS



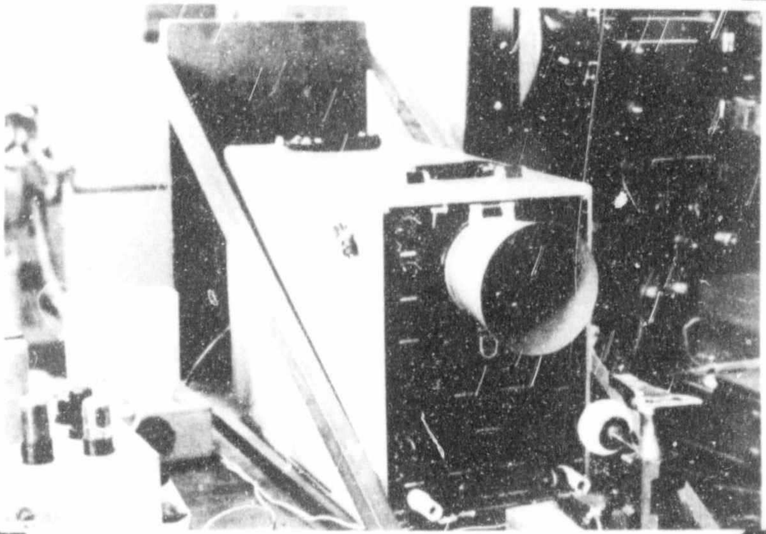
U.V. RECORDER AND POWER PACK



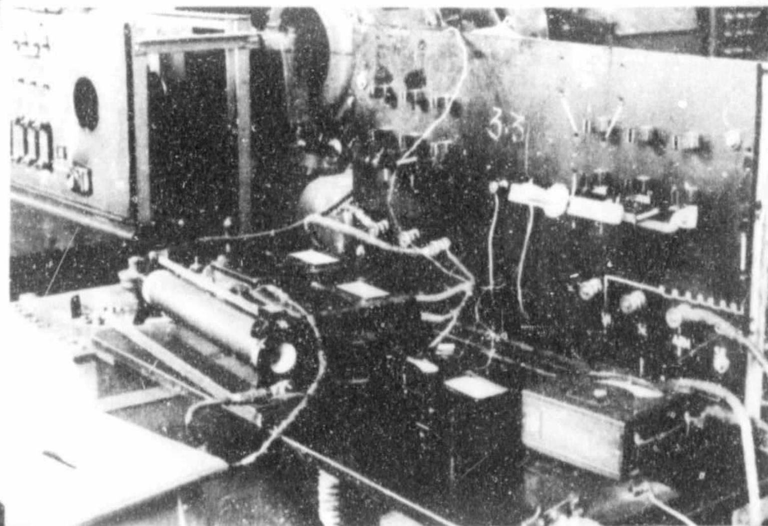
U.V. RECORDER AND "CHANNEL" SWITCHES



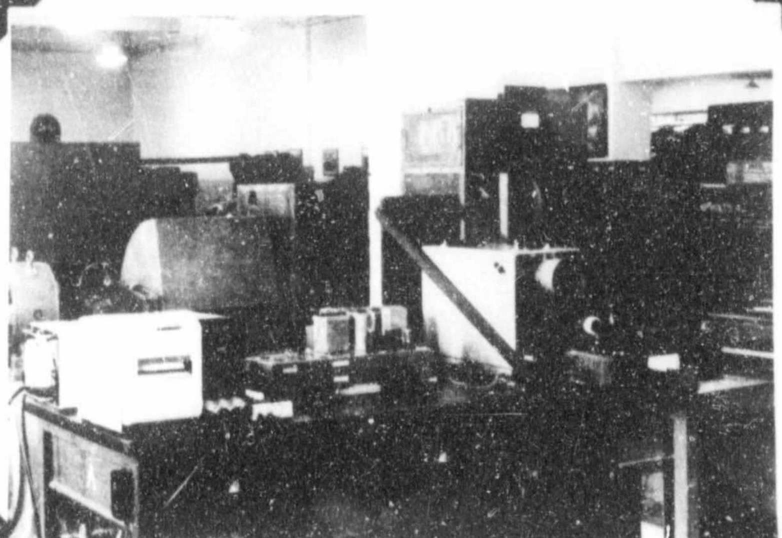
OSCILLATOR, DE-MODULATOR AND FILTER



OSCILLOSCOPE AND CAMERA SUPPORT



EQUIPMENT FOR D.C. DRIVE MOTOR



ASSOCIATED EQUIPMENT

$$= \frac{Adcq \operatorname{Cos} q\,\theta_1}{\pi q \left\{lg - l_1 \operatorname{Cos} q\,\theta_1 - l_2\right.} \operatorname{Cos} Q_2 (\theta_1 - w_s t)$$

$$= \frac{Adcq}{\pi q\ lg} \operatorname{Cos} q\,\theta_1 + \frac{Adcq\ l_1 \operatorname{Cos} Q_1\,\theta_1 \operatorname{Cos} q\,\theta_1}{\pi q\ lg^2}$$

$$+ \frac{Adcq\ l_2 \operatorname{Cos} Q_2 (\theta_1 - w_s t) \operatorname{Cos} q\,\theta_1}{\pi q\ lg^2}$$

$$= \frac{Adcq}{\pi q} \left[\frac{1}{lg} \operatorname{Cos} q\,\theta_1 + \frac{l_1}{2lg^2} \left\{ \operatorname{Cos} (Q_1 + q)\,\theta_1 + \operatorname{Cos} (Q_1 - q)\,\theta_1 \right\}\right.$$

$$\left. + \frac{l_2}{2\ lg^2} \left\{ \operatorname{Cos} \left[(Q_2 + q)\theta_1 - Q_2 w_s t\right] + \operatorname{Cos} \left[(Q_2 - q)\,\theta_1 - Q_2\,w_s t\right]\right\}\right]$$

The flux density wave with respect to the rotor is obtained by putting $\theta_1 = \theta_2 + w_s t$.

$$B_r\ dcq = \frac{Adcq}{\pi q} \left[\left[\frac{1}{lg} \operatorname{Cos} q\,(Q_2 + w_s t) + \frac{l_1}{2lg^2} \left[\operatorname{Cos} \left\{ (Q_1 + q)\,\theta_2 + \right.\right.\right.\right.$$

$$\left. (Q_1 + q)\,w_s t\right\} + \operatorname{Cos} \left\{ (Q_1 - q)\,\theta_2 + (Q_1 - q)\,w_s t\right\}\Big] + \frac{l_2}{2lg^2} \Big[\operatorname{Cos} \Big\{(Q_2 + q).$$

$$\left.\left.\left. \theta_2 + q\,w_s t\right\} + \operatorname{Cos} \left\{(Q_2 - q)\,\theta_2 - q\,w_s t\right\}\right]\right]\right]$$

This also gives five waves.

Number of poles.	Magnitude	Velocity with respect to stator mech. rads per second.	Velocity with respect to rotor mech. rads per second.
1. $2q$	$\frac{Adcq}{\pi q\ lg}$	0	$- w_s$
2. $2(Q_1+q)$	$\frac{Adcq\ l_1}{\pi q\ lg^2}$	0	$- w_s$
3. $2(Q_1-q)$	$\frac{Adcq\ l_1}{\pi q\ lg^2}$	0	$- w_s$
4. $2(Q_2+q)$	$\frac{Adcq\ l_2}{\pi q\ lg^2}$	$\frac{Q_2\ w_s}{Q_2\ +q}$	$- \frac{q\ w_s}{Q_2\ +q}$
5. $2\ (Q_2-q)$	$\frac{Adcq\ l_2}{\pi q\ lg^2}$	$\frac{Q_2 w_s}{Q_2 - q}$	$+ \frac{q\ w_s}{Q_2\ -q}$

$$+ \frac{l_2^{\,2}}{4lg^3} \left[\text{Sin} \left\{ (2Q_2 - q)\theta_2 + (w - qw_s)t \right\} + 2\,\text{Sin} \left\{ (w - qw_s)t - q\theta_2 \right\} \right.$$

$$\left. - \text{Sin} \left\{ (2Q_2 + q)\theta_2 - (w - qw_s)t \right\} \right] \Bigg] \Bigg]$$

This gives rise to a number of additional waves as follows.

	Magnitude	Pole Pairs	Frequency
1.	$\frac{Aql_1^{\,2}}{4\pi qlg^3}$	$2Q_1 - q$	$\frac{(2Q_1 - q)w_s + w}{2\pi}$
2.	$\frac{Aql_1^{\,2}}{4\pi qlg^3}$	q	$\frac{w - qw_s}{2\pi}$
3.	$\frac{Aql_1 l_2}{2\pi qlg^3}$	$Q_1 + Q_2 - q$	$\frac{(Q_1 - q)w_s + w}{2\pi}$
4.	$\frac{Aql_1 l_2}{2\pi qlg^3}$	$Q_1 - Q_2 + q$	$\frac{(Q_1 - 2Q_2 + q)w_s - w}{2\pi}$
5.	$\frac{Aql_1 l_2}{2\pi qlg^3}$	$Q_1 - Q_2 - q$	$\frac{(Q_1 - q)w_s + w}{2\pi}$
6.	$\frac{Aql_1 l_2}{2\pi qlg^3}$	$Q_1 + Q_2 + q$	$\frac{(Q_1 + 2Q_2 + q)w_s - w}{2\pi}$
7.	$\frac{Aql_2^{\,2}}{4\pi qlg^3}$	$2Q_1 - q$	$\frac{w - qw_s}{2\pi}$
8.	$\frac{Aql_2^{\,2}}{4\pi qlg^3}$	q	$\frac{w - qw_s}{2\pi}$
9.	$\frac{Aql_2^{\,2}}{4\pi qlg^3}$	$2Q_2 + q$	$\frac{w - qw_s}{2\pi}$

As before, we must consider the following harmonics.

Synchronous wave	$q = p$
Stator slot	$q = \pm Q_1 + p$
Rotor	$q = \pm Q_2 + p$

However, before tabulating the various quantities of magnitude, Pole pairs and frequency for the five waves, consider the magnitudes of the nine terms as shown in the above table.

Remembering that $l_1 = 0.00456$ cms, $l_2 = 0.0073$ cms, $l_3 = 0.0456$ cms and $lg = 0.05746$ cms, and comparing the magnitudes to those given on page   we have the following :

1. Similar to (2), but reduced by a factor $\frac{l_1}{2lg}$, i.e. by 0.0396

2.	Similar to (2), but reduced by a factor $\frac{l_1}{2l_g}$, i.e. by 0.0396
3, 4, 5 and 6.	Similar to (2), but reduced by a factor $\frac{l_2}{l_g}$, i.e. by 0.127
7, 8, 9 and 10.	Similar to (4), but reduced by a factor $\frac{l_2}{2l_g}$, i.e. by 0.0635

Thus it is immediately obvious that any results obtained by including a further term in the Binomial Expansion will have a negligibly small effect on the theoretical values already obtained.

CONCLUSIONS

$$\sqrt{2}\ Ec_1 = -\frac{Adcq}{\pi q} \cdot 2\Pi Rn_s\ 1.\ \frac{2}{lg} \text{ Sin } \frac{q\beta}{2} \text{ Tc } \underline{|q\theta_2}$$

$$\sqrt{2}\ Ec_2 = -\frac{Adcq}{\pi q} \cdot \frac{l_1}{lg^2}\ 1.\ 2\Pi Rn_s \text{ Sin } (Q_1+q)\frac{\beta}{2} \text{ Tc } \underline{|(Q_1+q)\theta_2}$$

$$\sqrt{2}\ Ec_3 = -\frac{Adcq}{\Pi q} \cdot 2\Pi Rn_s\ 1.\ \frac{l_1}{lg^2} \cdot \text{ Sin } (Q_1-q)\frac{\beta}{2} \text{ Tc } \underline{|(Q_1-q)\theta_2}$$

$$\sqrt{2}\ Ec_4 = -\frac{Adcq}{\pi q} \cdot 2\Pi Rn_s\ 1\ \frac{l_2}{lg^2}\ \frac{q}{q+Q_2}\ 2\Pi Rn_s \text{ Sin } (Q_2+q)\frac{\beta}{2} \text{ Tc } \underline{|(Q_2+q)\theta_2}$$

$$= -\frac{Adcq}{\pi q}\ 2\Pi Rn_s\ 1\ \frac{l_2}{lg^2}\ \frac{q}{Q_2+q} \text{ Sin } (Q_2+q)\frac{\beta}{2} \text{ Tc } \underline{|(Q_2+q)\theta_2}$$

$$\sqrt{2}\ Ec_5 = +\frac{Adcq}{\pi q}\ 2\Pi Rn_s\ 1.\ \frac{l_2}{lg^2}\ \frac{q}{Q_2-q} \cdot \text{ Sin } (Q_2-q)\frac{\beta}{2} \text{ Tc } \underline{|-(Q_2-q)\theta_2}$$

Phase difference between Ec_4 and Ec_5 is $(Q_2+q)\theta_2 - \{-(Q_2-q)\theta_2\}$

$$= 2\ Q_2\theta_2$$

As coils are placed in slots θ_2 has only certain descrete values for each slot.

i.e. for origin $\theta_2 = 0$

for next slot $\theta_2 = 2\pi/Q_2$

for next slot $\theta_2 = 4\pi/Q_2$ etc.

For the Nth slot $\theta_2 = \frac{(n-1)\ 2\pi}{Q_2}$

For this slot the phase displacement between Ec_4 and Ec_5 is

$2Q_2 \frac{(n-1)\ 2\pi}{Q_2} = 2(n-1)2\pi$ i.e. they are in phase.

Also the phase angle for Ec_4 may be written $Q_2\theta_2 + q\ \theta_2$ and for the discrete values given above this angle is

$$\frac{Q_2\ (n-1)2\pi}{Q_2} + \frac{q\ (n-1)2\pi}{Q_2}$$

$$= q/Q_2\ (n-1)2\pi + (n-1)2\pi$$

$$= q/Q_2\ (n-1)2\pi$$

The phase angle for any slot n of electro motive force Ec_1 is

$$q\ \theta_2 = q\ \frac{(n-1)2\pi}{Q_2}$$

i.e. for any particular slot n the electro motive forces Ec_1, Ec_4 and Ec_5 are all in phase and may be added arithmetically.

Thus

$$\sqrt{2}\ Ec_6 = \sqrt{2}\ (Ec_1 + Ec_4 + Ec_5)$$

$$= -\frac{Adcq}{q}\ 2\Pi Rn_s\ 1\ \text{Tc}\Big[\text{ Sin } \frac{q\beta}{2} + \frac{q}{Q_2+q}\ \frac{l_2}{lg^2} \text{ Sin } (Q_2+q)\frac{\beta}{2}$$

From page 29 of "The Performance and Design of Direct Current Machines" by Clayton, we have that

$$Kg_1 = \frac{\text{Reluctance with slotted armature}}{\text{Reluctance with smooth armature}}$$

$$= \frac{l_1 + l_3}{l_3} \quad \text{—————— (1)}$$

and $$Kg_2 = \frac{l_2 + l_3}{l_3} \quad \text{—————— (2)}$$

Considering (1)

$$l_3 Kg_1 = l_1 + l_3$$

Therefore

$$l_1 = l_3 (Kg_1 - 1)$$

and from (2)

$$l_2 = l_3 (Kg_2 - 1)$$

In using the above formulae, it must be remembered that the "slotted armature reluctance" is an average value as well as being an approximate assumption. Furthermore, the determination of l_1 and l_2 is only a possible approximate method.

Kg_1 and Kg_2 can be determined from the normal methods (see the above mentioned book by Clayton). l_1 and l_2 are equivalent lengths on the assumption that the length varies as a cosine law, despite the relative proportions of the slot opening, slot pitch and gap length.

Now, if the ampere-turns at any point are AT, then the flux density at that point (on the stator) is

$$B = \frac{\mu_0 \, AT}{lg - l_1 \cos Q_1 \theta_1 - l_2 \cos Q_2 (\theta_1 - \omega_s t)}$$

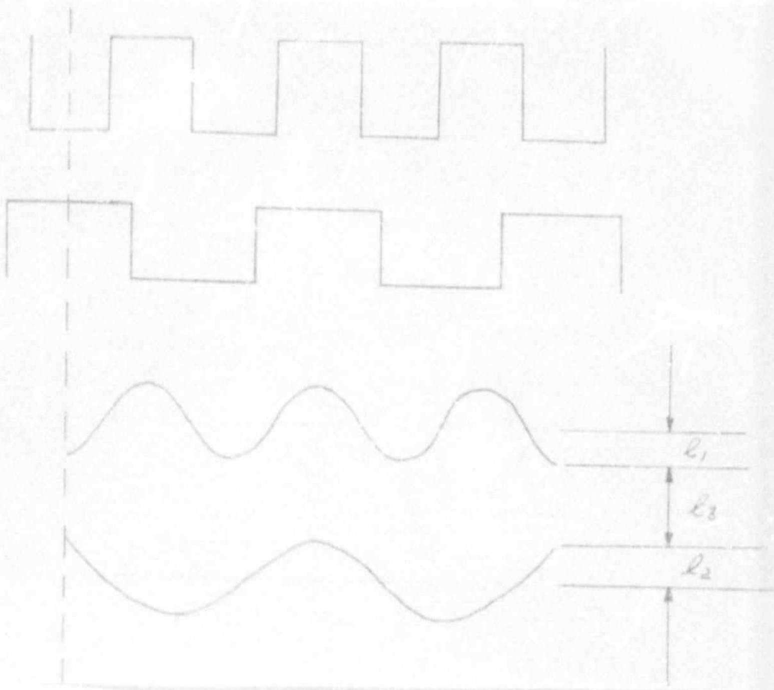
If the ampere turns are produced by an m- phase winding, housed in the stator slots, carrying a current I_1, and having T_1 turns per phase

$$AT = \sum \frac{2^{\frac{1}{2}} m I_1 T_1}{q} \, Kmq \, Keq \, \text{Sin} (wt - q\theta_1)$$

Footnote:

p = number of pole pairs.

b_1 = number of poles after which a fractional slot winding repeats itself.



We will assume the stator to have Q_1 teeth, and the rotor to have Q_2 teeth. Let us assume the stator to be the primary. The length of the airgap between the tops of the teeth is l_3.

Assume that the effect of the primary slots can be considered by increasing the airgap, and that this increase varies as a cosine law from the centre line of one stator tooth. The increased airgap length due to the stator is then $l_1 - l_1 \operatorname{Cos} Q_1 \theta_1$ where θ_1 is measured in mechanical radians relative to the stator. By this assumption, all harmonics caused by the slots are completely neglected.

If the secondary slots, Q_2, are moving with respect to the primary at a speed W_s mechanical radians per second, then the increase in the airgap length due to the secondary slots is

$$l_2 - l_2 \operatorname{Cos.} Q_2 (\theta_1 - \omega_s t)$$

This means that at time $t = 0$ the centre of a particular tooth on the secondary is opposite to the centre of the stator tooth from which measurement is made.

The total length of the airgap is thus:

$$l_3 + l_1 - l_1 \cos Q_1 \theta_1 + l_2 - l_2 \cos Q_2 (\theta_1 - \omega_s t)$$

$$= lg - l_1 \cos Q_1 \theta_1 - l_2 \cos Q_2 (\theta_1 - \omega_s t)$$

where $lg = l_1 + l$ [illegible]

FLUX INVESTIGATION

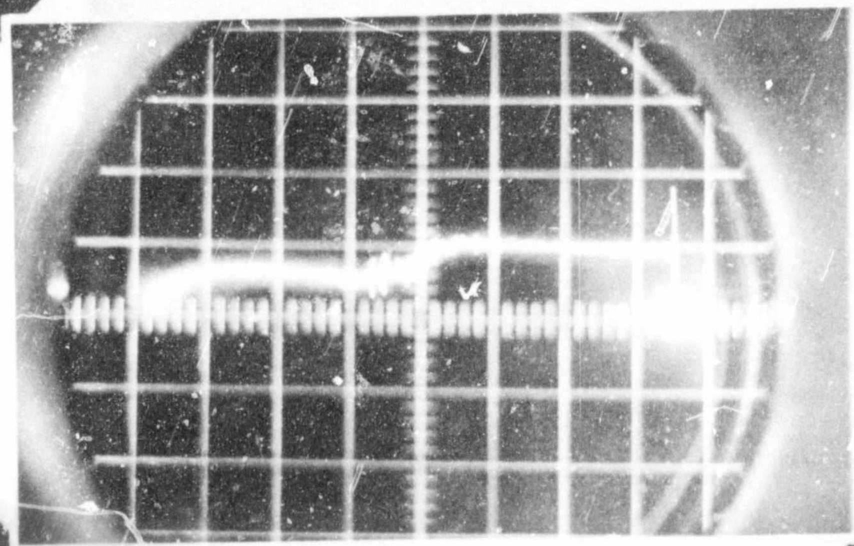


PHOTO XIII

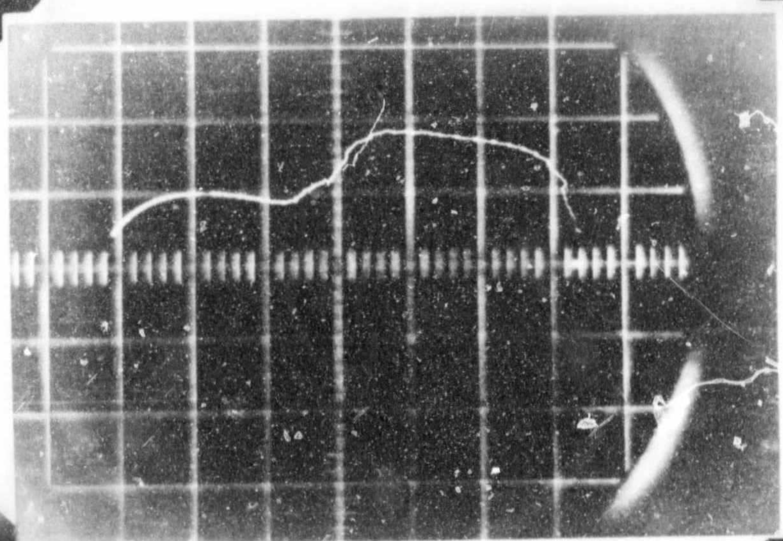


PHOTO XIV

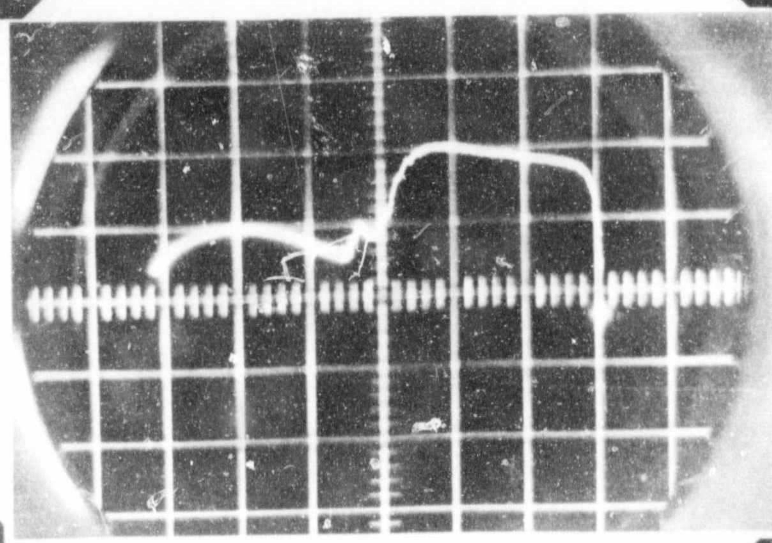


PHOTO XV

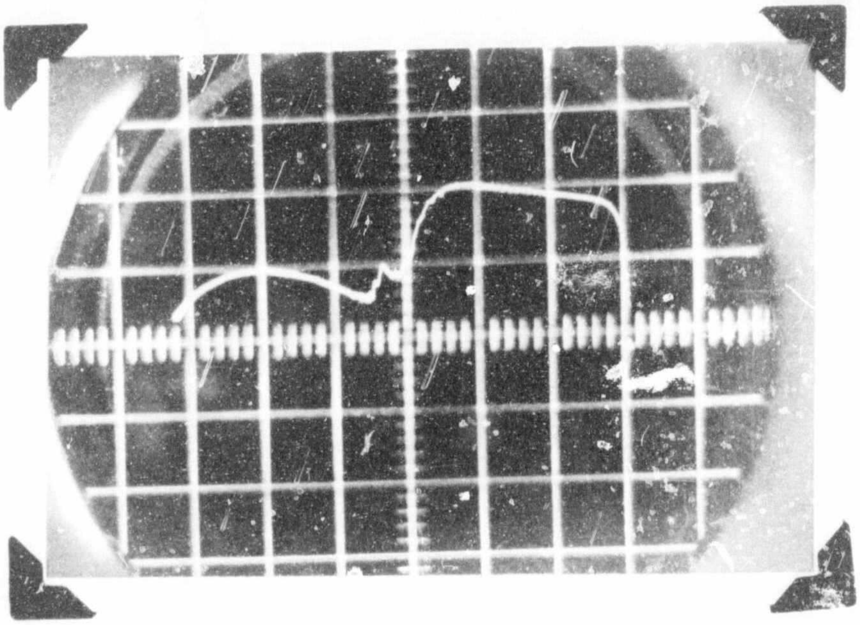


PHOTO X

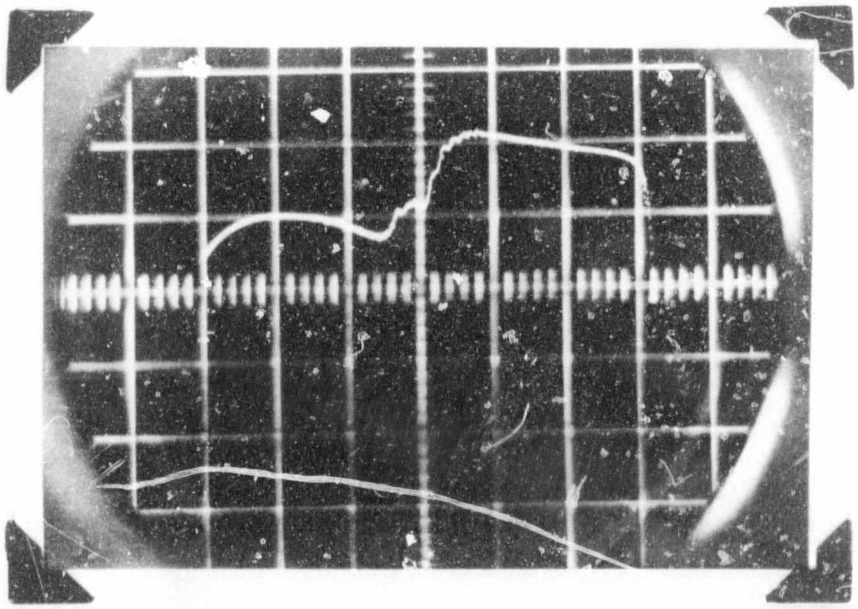


PHOTO XI

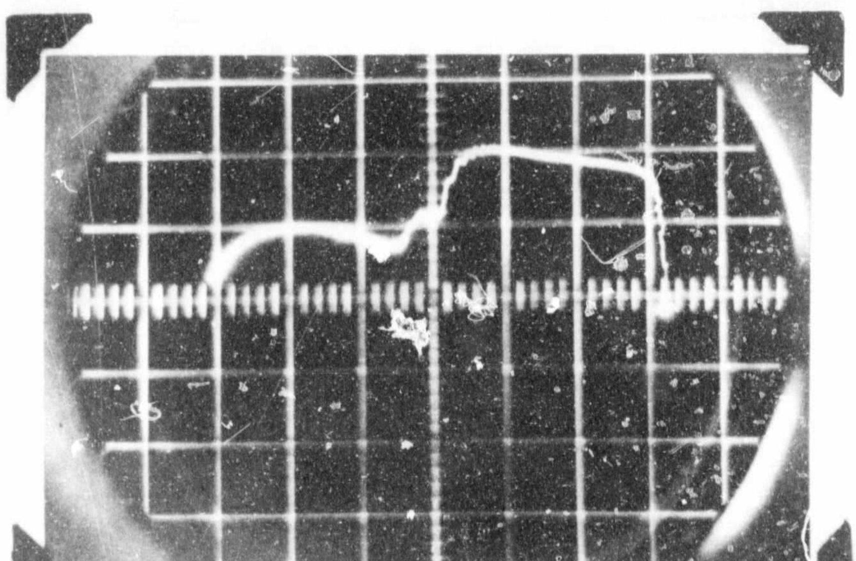


PHOTO XII