CHAPTER VI

CHANGES OF GRADING DUE TO PHYSICAL AND CHEMICAL WEATHERING.

INTRODUCTION:

This section will describe a short programme of work designed to investigate the effects of chemical and physical weathering on the grading of artificial filter sands, i.e. crusher sands. It includes a description of an investigation of the failure of a practical sewage purification filter due to rapid chemical weathering of the filter sand. It also describes a short series of physical weathering tests carried out over a period of 3 months on a number of different crusher sands.

CAUSES OF THE FURTHER BREAKDOWN OF THE PARTICLES OF A CRUSHER SAND SUBSEQUENT TO CRUSHING.

(i) The Effects of Crushing

Slow motion moving pictures of the action of roll and jaw crushers show that roughly cubical blocks of stone break in two ways: (a) by transverse cracks through the blocks going from roll to roll and yielding more or less tabular slabs, and (b) by radiating cracks from contact points at the roll faces. From observation of the pictures the proportions of grains made by the two crushing actions appear to be of the order of 80% and 20% respectively. The blocks of stone fracture along planes of least resistance. The particles of the resulting crushed aggregate contain innumerable cracks, fissures and surfaces of incipient failure which form an extension in the individual particles of the fracture pattern in the original block of stone.

These small defects in the chunks of crushed aggregate, although invisible to the naked eye, render the apparently sound crushed rock more vulnerable to the action of agents of weathering than the sound rock.

There is an additional source of weakness in rock used for crusher sands which has been mined at great depth, e.g. the waste rock extensively used on the Witwatersrand to provide concrete aggregates and crusher sands. This hard, brittle quartzite invariably must contain numerous hair cracks arising from the tremendous tectonic shearing stresses to which the rock has been subjected through the ages.

(ii) Chemical Weathering

The chemical weathering of rocks takes place by the chemical decomposition of some or all of the component minerals. Combined chemical and mechanical weathering takes place owing to the fact that certain rock minerals, e.g. the felspars give rise on decomposition to an increased volume of products. The expansion of the decomposing minerals may set up sufficiently large stresses to cause cracking of the particles.

(iii) Physical Weathering

The primary mechanical agent which could cause the disintegration of the particles of filter sands is that of temperature variation.

The disintegration of rocks due to temperature changes may result from: (a) Unequal expansion and contraction of component minerals having different coefficients of thermal expansion. In this case the result of fluctuating temperatures is the formation of small cracks into which water may percolate and either set up chemical action or (b) disrupt the rock by freezing. In passing from the liquid to the solid state water expands with a force which varies with the temperature and at a temperature of -22°C may exert a pressure of up to 15T/in.^{2}

Since ice contracts on cooling, a cavity filled at 0°C would not be filled at -10°C. If now, additional moisture freezes in the cavity, then as the temperature rises, disruption of the rock will be caused by the ice expanding up to the melting point.

These then, are the main causes of the breakdown of the particles of artificial crushed filter sands. The effect of crushing and relief of geological stress is probably the initiating cause with chemical and physical weathering playing an important secondary role.

THE FAILURE OF A SEWAGE PURIFICATION FILTER DUE TO RAPID CHEMICAL WEATHERING OF THE FILTER SAND

The filter in this case was a slow sand filter of the "Hamlin" type.

The filtering medium is a layer of fairly coarse sand 6" deep. Below this are three layers of stones of increasing size which allow easy flow of the liquid to the collecting channels below.

The effluent is spilled over the surface of the sand from the distributing channels and passes through the sand and stone to the collecting channels below. A blanket of humus develops on the surface and the interstices of the sand bed gradually become clogged, decreasing the permeability.

The clogged filter is renewed by means of a cleaning operation which is normally carried out once every 24 hours.

The cleaning operation is as follows: Flow is passed in a reverse direction through the lower central channels and up through the filter into the central distributing channel. The upward flow is under a head of only a few inches so that the flow is very gentle and the sand bed itself is hardly agitated. As a result there is very little abrasion between the grains. A travelling screed is moved up and down the filter along the sand surface at a brisk pace to create a wave which agitates the top surface of the sand and helps free the humus. The operation is continued for between 10 and 15 minutes.

It is found that normally the sand loss in such a filter is very little and one such filter has been in operation continuously for more than 30 years, using the original sand without any trouble. In South Africa the life of the filter bed is generally taken to be indefinite.

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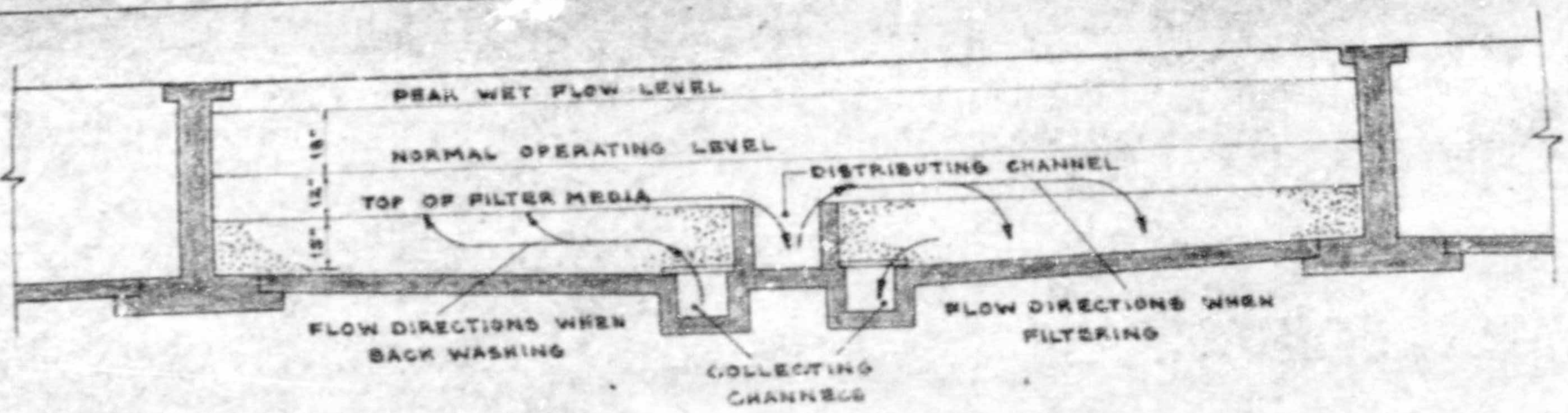

HAMLIN SLOW SAND FILTER.

A TYPICAL SECTION THROUGH A HAMLIN FILTER SHOWING DIRECTIONS OF FLOW DURING FILTERING & BACK WASHING & DISTRIBUTING & COLLECTING CHANNELS.

BASIC CRUSHING ACTIONS.

(i) FRACTURE BY TRANSVERSAL CRACKS GOING FROM JAW TO JAW AND YIELDING TABULAR SLABS. (ii) FRACTURE BY RADIATING CRACKS FROM CONTACT POINTS ON JAW FACES. BOTH ACTIONS WEAKEN THE PARTICLES FOR SUBSEQUENT WEATHERING

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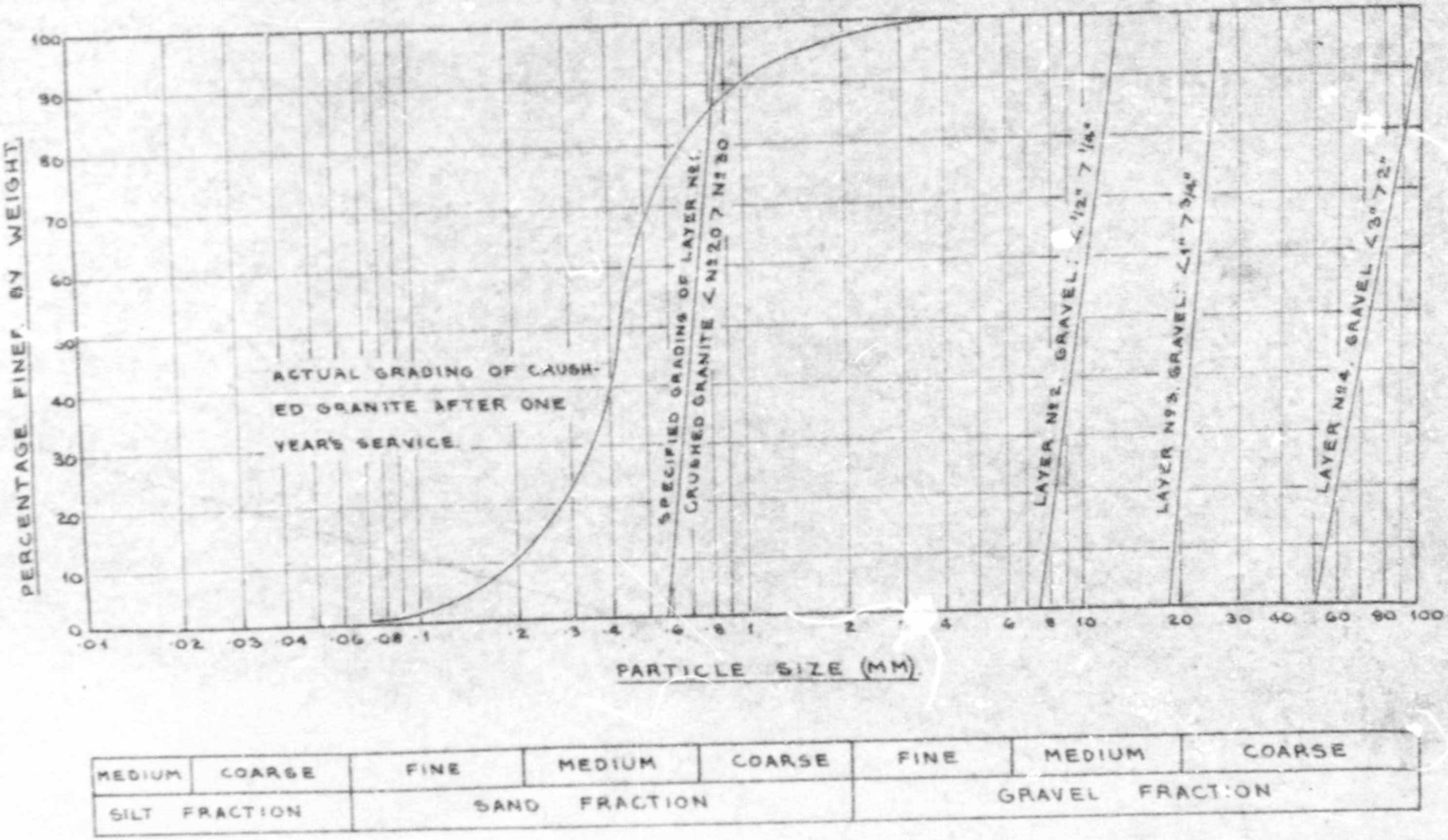
GRADINGS FOR HAMLIN SLOW SAND SEWAGE FILTER.

THE SEWAGE FILTER WHOSE GRADINGS ARE SHOWN HERE FAILED AFTER ONE YEAR'S SERVICE DUE TO CHEMICAL DECOMPOSITION OF THE LAYER OF CRUSHED GRANITE SAND.



In the case considered, the sand was a crushed granite and the filter bed was specified to be laid in the following layers:

6" layer at bottom	-3" to +2" stone
3" layer	-1" to +$\frac{3}{4}$" stone
3" layer	-$\frac{1}{2}$" to +$\frac{1}{4}$" stone
6" layer at top	Sand: -20 mesh to +30 mesh B.S. Sieve

It was the top 6" layer which showed signs of decomposition.

Within a year, it was found that the sand was washing into the stone layers beneath and on examination it was found that the sand contained a large proportion of fine material. The grains were no longer hard and discrete but could be crushed between the fingers and ground to dust.

It was obvious that the felspars of the crushed granite sand had been almost completely decomposed by the organic acids contained in the sewage. The chemical decomposition was almost certainly accelerated in this case by the hair cracks caused in the crushing process.

Comparing the actual grading of the sand after one year's service with the specified grading, it will be seen that the decomposed sand is considerably finer than the specified grading even allowing for quite large variations in the freshly crushed filter sand from the specification.

A standard filter test was carried out on the decomposed sand and it was found that the sand behaved as a filter within itself. The grading curves show that there was hardly any difference between the top and bottom halves of the sample after the test.

It should be pointed out that the top two layers of this filter do not satisfy either the Terzaghi or the U.S.B.R. filter design criteria.

Working on specified particle sizes:

For the Terzaghi criterion:

$$\frac{D_{15}\ \text{FILTER}}{D_{85}\ \text{FOUNDATION}} = 10.0$$

Whereas the maximum permissible value of this ratio is 5.

For the U.S.B.R. criterion:

$$\frac{D_{50}\ \text{FILTER}}{D_{50}\ \text{FOUNDATION}} = 13.8$$

Whereas this ratio has a maximum permissible value of 10.

Nevertheless, according to the engineers responsible for the filter design, the loss of sand from the filter was negligible until decomposition had occurred.

It can safely be concluded that the failure of this filter was caused by a change in the grading of the filter sand brought about by chemical decomposition of its constituent minerals.

PHYSICAL WEATHERING TESTS ON CRUSHER SANDS

This series of four tests was carried out:

(a) to get some idea of the rapidity with which mechanical weathering can affect the grading of a crusher sand;

(b) to establish the possible magnitude of such changes; and

(c) to discover the differing effects of physical weathering on crusher sands derived from varying rock sources.

The samples on which the tests were carried out were as follows:

Sample No.	Rock	Description
1	Pretoria Norite	Sound, hard, unweathered, medium crystalline dark grey norite.
2	Beaufort Sandstone	Very fine grained, uniform, blue-grey sandstone medium to hard in consistency.
3	Transvaal Granite	Pink and white, medium crystalline sound and unweathered granite.
4	Witwatersrand Quartzite	Hard, unweathered, finely grained grey quartzite

Crushed aggregates of all these rocks are commonly used in concrete and road making in the Union and the Norite, Granite and Quartzite are often incorporated in filters in the form of crusher sands.

This selection of samples covers the main range of geological rock types, that is, igneous, sedimentary and metamorphic rocks are all represented.

The tests were carried out in the following manner:

(i) Each rock sample was finely crushed using a jaw crusher. When a sufficiently large quantity of crusher sand had been obtained, the sample was thoroughly mixed and a particle size analysis was carried out on about one-third of the total using wet sieving. This provided the initial grading curve for the crusher sand.

(ii) The weathering test was carried out on the remainder as follows:

The crusher sand was placed in a large galvanised iron basin and heated to a temperature of 110°C in a drying oven. The sample was removed from the oven and doused with a stream of cold water. The wet crusher sand was then frozen to a temperature of -10°C after which the heating and dousing was repeated. Each sample was taken through a range of temperature of 120°C once every 24 hours.

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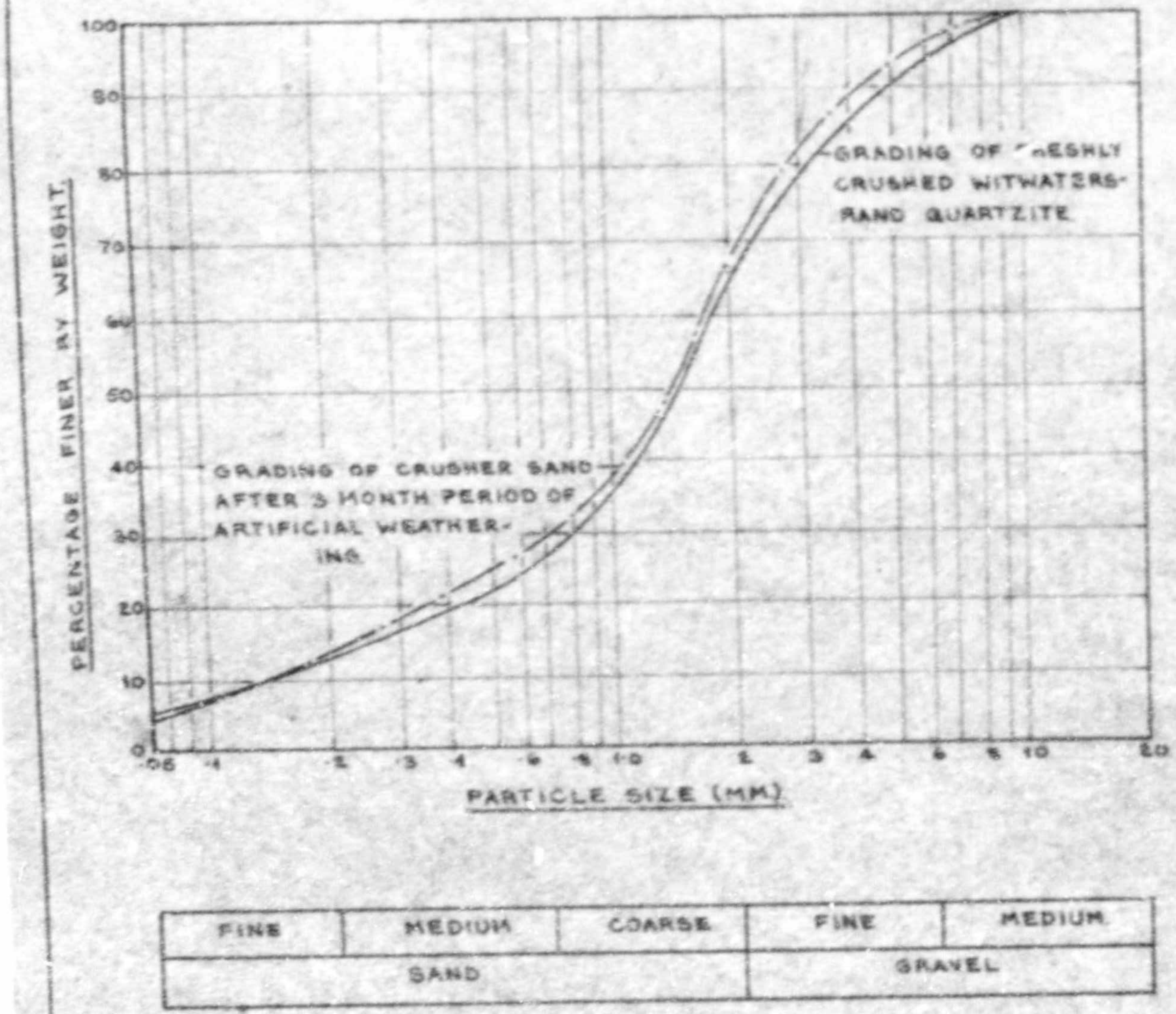
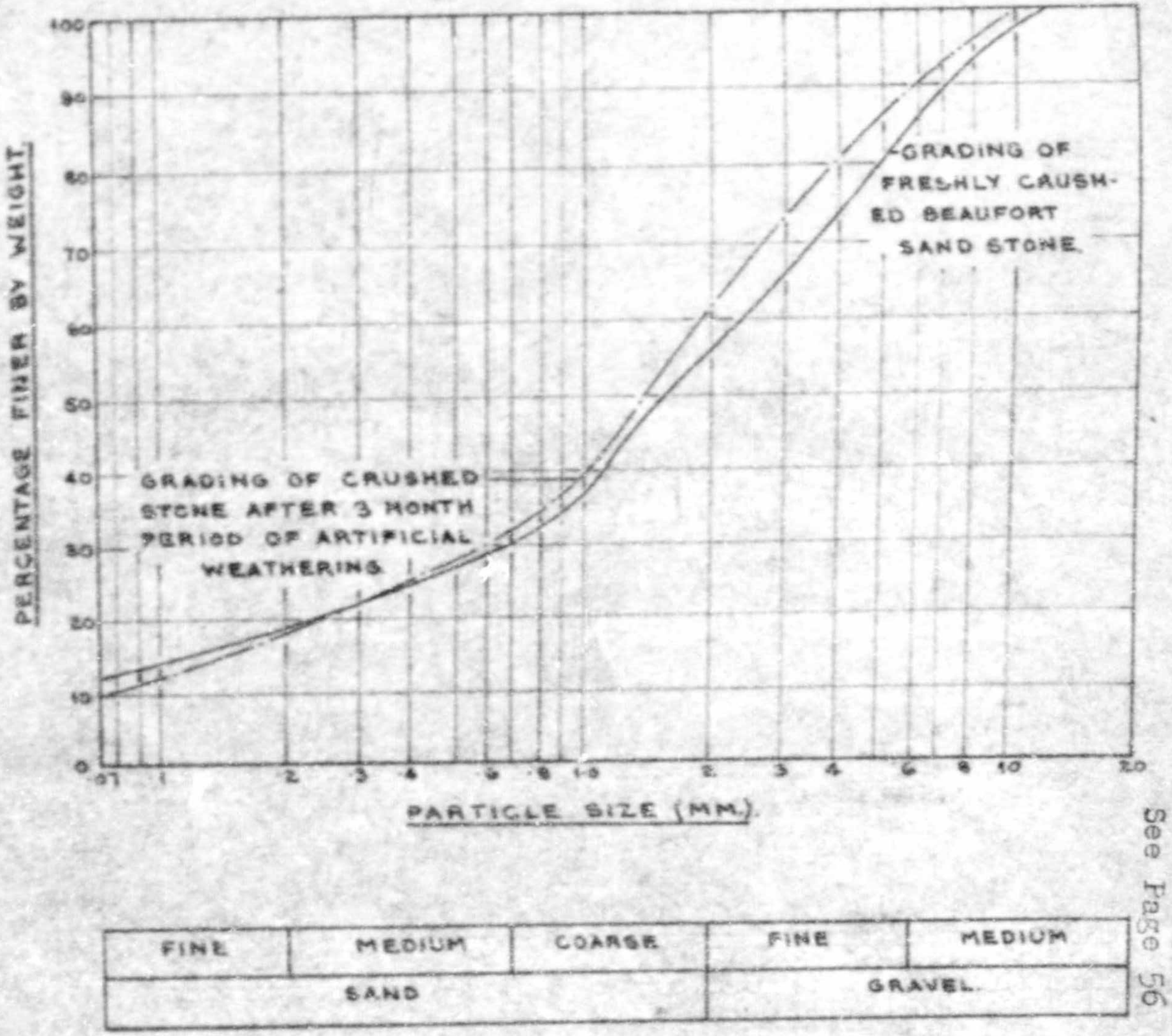
THE EFFECT OF PHYSICAL WEATHERING ON THE GRADING OF A SAND.

THESE TWO SANDS — ONE A CRUSHED QUARTZITE, THE OTHER A CRUSHED SAND STONE BECAME ONLY SLIGHTLY FINER AFTER REPEATED CYCLES OF ALTERNATE HEATING AND FREEZING.





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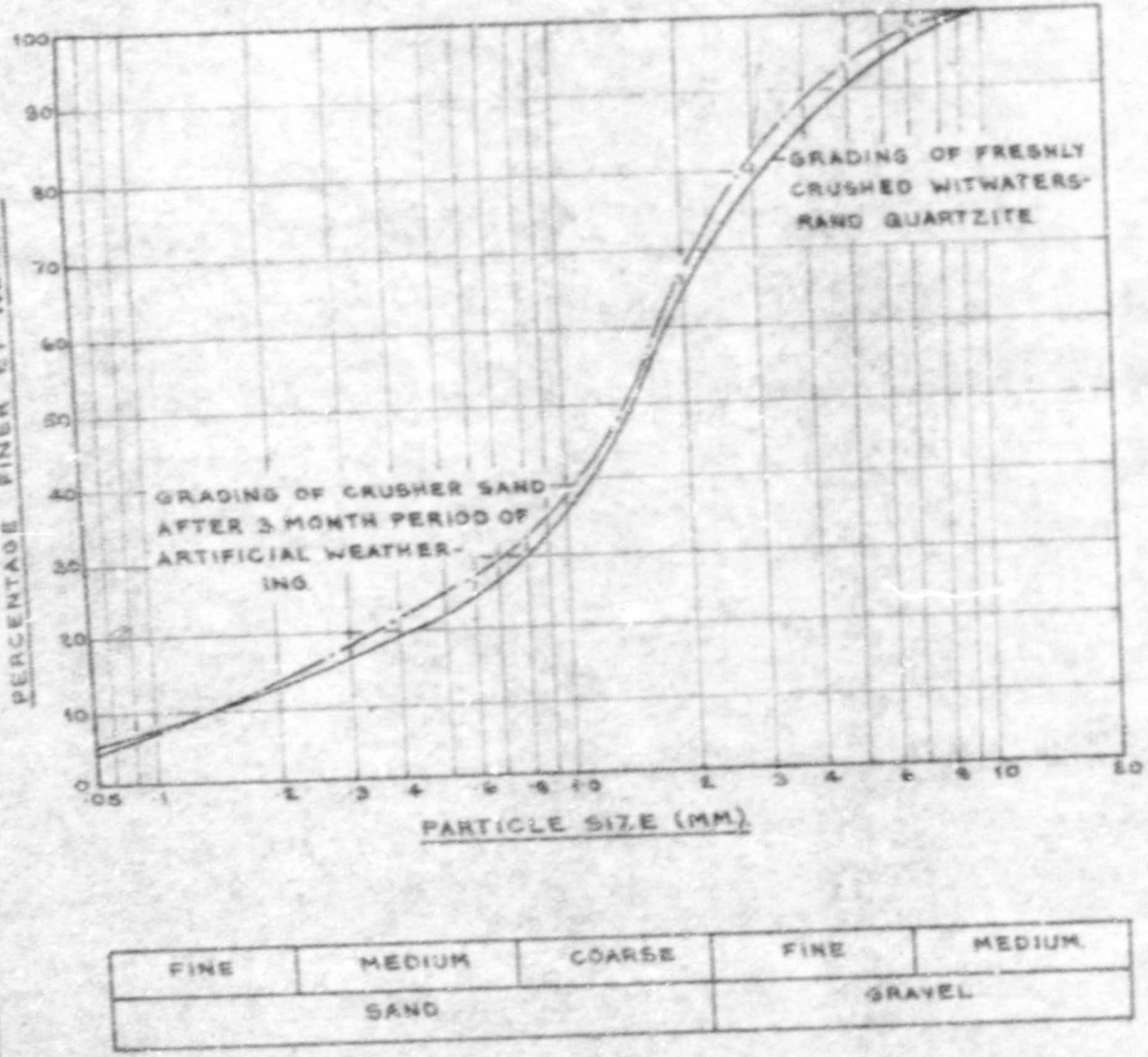

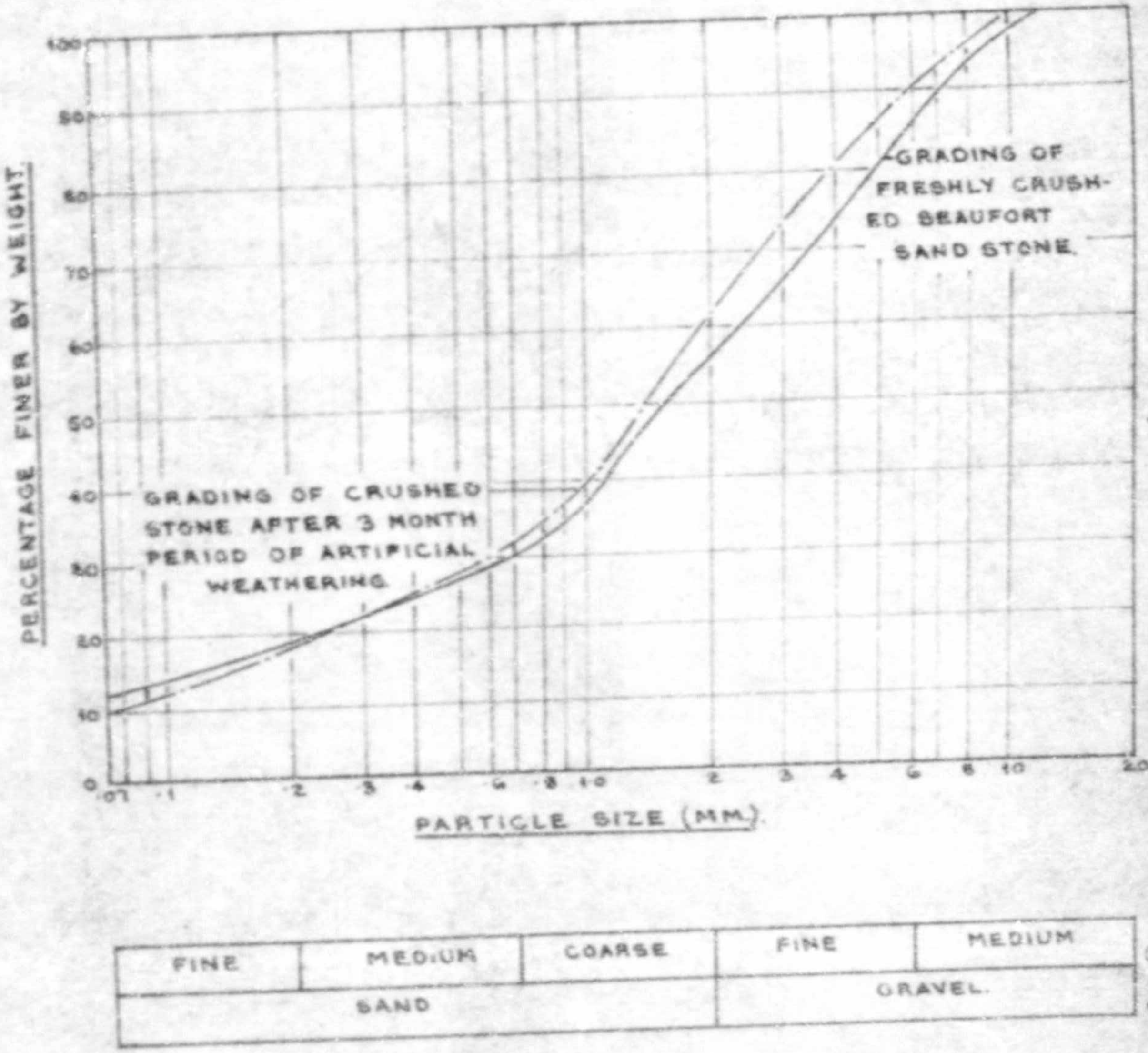

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This process was continued for a period of 3 months, at the end of which period a second particle size analysis was made.

The pairs of grading curves for the four samples accompany this section.

The weathering process to which the sands were subjected was very severe indeed. The most severe weathering took place during the dousing process when the sand fragments of the crystalline rocks could be both heard and seen disintegrating.

This severity would really only be approached in nature under desert conditions. The artificial process is however similar to a natural weathering process except that the time scale is much condensed. The extent of weathering attained in these tests of 3 month duration would probably only be attained under the severest service conditions after a lapse of several years.

The successive grading curves show that all the sands became finer during the course of the test. The fact that the changes were in some cases very small does not invalidate this statement as in this case errors due to natural variation were practically eliminated by the thorough mixing to which the sample was subjected.

The two crusher sands derived from igneous rocks show the greatest changes in grading, the crushed granite being the worse of the two. (This was the same granite which was decomposed so rapidly by sewage acids). This is probably because the crystalline nature of the rock renders it more subject to attack by differential expansion and contraction.

The crushed sandstone was slightly less weathered than the igneous rocks and the quartzite crusher sand proved the most stable of the four sands.

CONCLUSIONS:

It can be concluded that changes in grading due to weathering, both chemical and physical, can be an important factor leading to the breakdown of a filter layer.

If natural sands are used, weathering will have little or no effect. Natural sands are themselves the products of weathering and as such are usually very stable and resistant to the effects of further weathering.

All artificial crushed sands can be affected by physical weathering resulting from sudden temperature fluctuations. The worst types of rock from this aspect are the crystalline igneous rocks.

Furthermore, crusher sands containing unstable minerals such as felspars are unsuitable for use as filtering media for corrosive liquids such as sewage effluent.

CHAPTER VII

A STUDY OF SOME EMPIRICAL RELATIONS FOR THE FLOW OF WATER THROUGH FILTER SANDS.

INTRODUCTION:

It has already been explained in the introduction to this thesis that any investigation of the behaviour of filters and filter sands should enquire not only into the changes of grading undergone by the sands, but also into the hydraulic behaviour of the soil-water flow system as the changes of grading proceed. It should in fact do more, it should attempt to find the link between the variations of grading and hydraulic properties - which are obviously interdependent. That is the object of this chapter.

First of all, the validity of the basic flow law for soils (D'Arcy's Law) will be investigated when applied to flow through a soil whose grading is undergoing change. This is followed up by the development of a semi-empirical general relation for the flow of water through a soil which is undergoing changes of grading. After this the hydraulic behaviour of a soil will be linked to the corresponding changes of grading by considering the phenomenon of the migration of fine soil particles which takes place when water under pressure percolates through a soil.

The variation of the permeability of a freshly placed sand with time immediately following the commencement of flow will also briefly be discussed.

THE VALIDITY OF D'ARCY'S LAW WHEN APPLIED TO FLOW THROUGH A SOIL WHOSE GRADING IS UNDERGOING CHANGE

The solution of nearly all problems in soil mechanics involving the percolation of water under pressure gradients depends on the validity of D'arcy's Law, viz:

$$q = kiA$$

where: q = flow in unit time through a cross-sectional area A of soil
k = coefficient of permeability
i = hydraulic gradient.

D'arcy's Law, however, applies strictly only to steady state conditions in completely saturated soils. In this series of tests, although the degree of saturation of the soil was always very near to unity as a result of the de-airing precautions which were taken, nevertheless steady state conditions did not prevail because generally the structure of the soil was progressively changing due to internal reorientation of the grains and migration of the finer particles.

Consider now a dimensional expression for the quantity of flow resulting from the seepage of water through a mass of soil which is simultaneously undergoing changes of structure and grading. This will be considered particularly with reference to quantities measurable in a constant head permeability test, viz:

(1) A characteristic particle dimension D which can be chosen from the particle size analysis of the filter material. (In this case the mean particle diameter will be used).

(2) The density of the filter material γ which can be calculated from void ratio measurements and therefore takes into account changes in pore volume within the soil sample.

(3) The length of flow path, l, which is generally taken to be the "direct" flow path or the measured length of a filter sample of constant cross-sectional area.

(4) The rate of flow, q, which can be measured from the total volume of flow Q in time t.

(5) The viscosity of the percolating water at the temperature of the filter cell.

(6) A characteristic mass M_M. which represents the mass of particles of filter material which has migrated since the initiation of flow.

To reduce the number of variables, the effect of the length of flow path l will be assumed to be incorporated along with the effect of varying void ratio and density.

No method has been found of measuring the mass of particles which has migrated after a given time and hence the effect of the variable M_M must be ignored.

The primary variables which have been chosen and their corresponding dimensions are thus summarised as follows:

	Variable	Symbol	Dimension
I	Particle Diameter	D	L
II	Soil Density	γ	ML^{-3}
III	Rate of Flow	q	L^3T^{-1}
IV	Fluid Viscosity	μ	$ML^{-1}T^{-1}$

Assume a dimensional relation for q of the form:

$$q \propto \gamma^a \mu^b D^c$$

Written in dimensional form this becomes:

$$[L^3T^{-1}] \propto [ML^{-3}]^a [ML^{-1}T^{-1}]^b [L]^c$$

By the Principle of Dimensional Homogeneity the indices can be equated to give:

$$\left.\begin{aligned} 3 &= -3a - b + c \\ -1 &= -b \\ 0 &= a + b \end{aligned}\right\} \text{ Whence } \quad \begin{aligned} a &= -1 \\ b &= 1 \\ c &= 1 \end{aligned}$$

i.e. $q \propto \dfrac{\mu D}{\gamma}$

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γ will be the submerged density of a soil of void ratio e and particle specific gravity G, i.e. $\gamma = \frac{G-1}{1+e} . \gamma_w$

Hence the expression for q becomes:

$$q = B \, . \, \frac{\mu D}{\gamma_w} \, . \, \frac{1+e}{G-1}$$

As a result of experience with the percolation of water through unstable soils (see the series of Permeability-Time curves), it has been found that as particle migration and structural change proceed, the rate of flow q will either increase or decrease depending on circumstances. If the internal changes lead to a purely internal redistribution of particles with the concentration of fine particles being retained in the lower portion of the sample, reduced flow passages and a generally closer packing of the grains will result, leading to a reduced rate of flow. If, however, the soil as a whole becomes coarser due to loss of fines from the sample, the rate of flow will increase.

The increase or decrease in the rate of flow thus appears to be mainly a function of the changes in void ratio which occur. It has also been observed that the rate of flow tends to approach a steady value as time proceeds. The expression for q can therefore be modified by introducing an exponential factor.

(i) For a gradually increasing rate of flow:

$$q = B' \, . \, \frac{\mu D}{\gamma_w} \, . \, \frac{(1+e)}{(G-1)} \, . \, \epsilon^{\beta'(e/eo)}$$

Where eo is the initial void ratio.
The index $\beta'(eo/e)$ will ensure that q increases at a diminishing rate until e attains its equilibrium value and a stable grain structure is established.

(ii) For a gradually decreasing rate of flow:

$$q = B'' \, . \, \frac{\mu D}{\gamma_w} \, . \, \frac{(1+e)}{(G-1)} \, . \, \epsilon^{-\beta''(eo/e)}$$

The index $-\beta''(eo/e)$ will ensure a diminution in flow until a steady value of e is reached.

Now compare this general expression

$$q = B \, . \, \frac{\mu D}{\gamma_w} \, . \, \frac{1+e}{G-1} \, . \epsilon^{\pm\beta(eo/e)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

with the D'Arcy expression

$$q = k.i.A. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In (1) $\frac{\mu D}{\gamma_w} . \frac{1+e}{G-1} . \epsilon^{\pm\beta(eo/e)}$ corresponds dimensionally with the quantity k.A in (2) i.e. the coefficient of permeability k is represented by:

$$k \propto . \, \frac{\mu D}{A\gamma_w} \, . \, \frac{1+e}{G-1} \, . \, \epsilon^{\pm\beta(eo/e)}$$

This leaves the hydraulic gradient i to be represented by B (which like i is dimensionless). Thus B is not a constant but can be written as B = B'.i where B' is a constant.

Hence for the case of non-steady conditions D'Arcy's Law can be written as:

$$q = k.i.A$$

Where in this case k is not a constant.

The accompanying illustration shows values of $\frac{\mu D}{\gamma_w} \cdot \frac{1+e}{G-1}$ plotted as ordinates against the corresponding values of the coefficient of permeability k as abscissae for several sands. It can be seen that provided the quantities are plotted on a logarithmic scale, there is a linear relation between k and $\frac{\mu D}{\gamma_w} \cdot \frac{1+e}{G-1}$. The lines generally have a small negative slope.

It must be realised that the values for each sand cover only the small range resulting from internal changes caused by flow under a constant head through a freshly placed soil. It is impossible to say, without further experiments, whether the linear relationship will hold for any practical values of potential head, void ratio, etc. Nevertheless, the conditions under which the tests were made are approximately similar to those which would obtain in a practical soil or sand filter immediately after its construction when water first passes through it.

It can thus be concluded that in any one soil within which changes of grain structure are proceeding due to reorientation of the soil grains and migration of the fine material, a relation exists between the coefficient of permeability and other soil properties which is of the form:

$$\log k = -{}^{1}/_{m} \log \left\{ \frac{\mu D}{\gamma_w} \cdot \frac{1+e}{G-1} \right\} + C'$$

Where m is the (positive) slope of the line and C is the intercept on the Permeability (k) axis.

Also D'Arcy's Law for a soil within which transient flow conditions prevail can be stated as follows:

$$q = k.i.A \qquad \text{OR}$$

$$\log(q) = -{}^{1}/_{m} \log \left\{ \frac{\mu D}{\gamma_w} \cdot \frac{1+e}{G-1} \right\} + \log(iA) + C'$$

The values of C appear to vary in a random fashion from soil to soil and are best determined experimentally.

The graph of the Permeability Function against the Coefficient of Permeability has been plotted disregarding the decimal points of the various values. This was done in order conveniently to show all the curves on the same sheet. To take account of this, put m = m' + p, where p is an integer, positive or negative, introduced to take account of the position of the decimal point in m.

For the five soils tested, the values of m' cluster about a fixed value. This value was estimated statistically in the manner described in section (i) of the appendix to this chapter. The estimated value was:

$$m' = 22.7 \pm 15.4 \quad \text{with } 80\% \text{ confidence.}$$

The range of uncertainty of the value of m' is thus rather large, but it must be remembered that this value was estimated from only five samples. Naturally, the greater the number of samples, the narrower would the limits of uncertainty become.

We thus have the result for the flow of water through a soil which is undergoing changes of grading:

$$\log k = (p - {}^{1}/23) \log \left\{ \frac{\mu D}{\gamma_w} \cdot \frac{1+e}{G-1} \right\} + C'$$

$$k = C \left\{ \frac{\mu D}{\gamma} \right\}^{(p - {}^{1}/23)}$$

The values of p, like those of C vary widely from soil to soil.

A GENERAL RELATION FOR THE FLOW OF WATER THROUGH A SOIL WHICH IS UNDERGOING CHANGES OF GRADING

In this section a general relation is sought between various measurable properties of a soil-water flow system in which a stable structure is in the process of being established. This relation will take into account and illustrate the effects of the variation of these measurable properties from the time flow is first started through the freshly placed sand until steady-state conditions are reached and a stable grain structure has been established within the sand.

The problem will be approached by the use of dimensional analysis.

As a result of experience arising from the many observations taken during the course of this investigation, the following five variables were selected as having probably the greatest effect on the hydraulic properties of any soil:-

(1) The density of the soil. It is widely recognised that the void ratio of a soil has a fundamental influence on the flow characteristics of that soil. The density is a direct function both of the void ratio and of another fundamental soil property, the particle specific gravity and thus should have an important effect on the flow characteristics of a soil. The density which will be used is the submerged density

$$\gamma = \frac{G-1}{1+e} \cdot \gamma_w$$

(2) A characteristic particle dimension. It has long been recognised that the flow characteristics of a cohesionless soil such as a filter sand are dependent to a large extent on the distribution and magnitudes of the particle sizes in the soil. The characteristic dimension which is used in what follows constitutes the "measure of position" of the grading curves and will here be denoted by d. It is a statistical average particle size.

(3) The permeability of the soil. The permeability of a soil is generally regarded as being the basic measure of the flow characteristics of a soil and as such its inclusion into the analysis needs no further justification. The value here used is D'Arcy's coefficient of permeability k, the constant of proportionality in the simplified flow equation v = ki.

(4) The viscosity of the percolating fluid. Especially in the case of viscous flow through soils, the value of the viscosity must have a fundamental influence on the friction losses during flow and hence on the flow characteristics of the system as a whole.

(5) The seepage force per unit volume. This seepage force makes its influence felt chiefly on the void ratio of the soil and to a lesser extent on the structural rearrangement and migration of the particles of the soil as flow proceeds. This seepage or body force can be shown to be given at any point within the soil by the relation

$$S = i\gamma_w$$

Where i is the hydraulic gradient at the point considered.

Summary of Variables and their Dimensions

	Variable	Symbol	Dimension
I	Density	γ	ML^{-3}
II	Particle Diameter	d	L
III	Permeability	k	LT^{-1}
IV	Viscosity	μ	$ML^{-1}T^{-1}$
V	Seepage Force	S	$ML^{-2}T^{-2}$

No. of variables: $n = 5$.
No. of Primary dimensions: $k = 3$.
Applying Buckingham's 'π' Theorem, the number of independent dimensionless groups which can be formed is: $n-k = 2$.
Denote these dimensionless groups by π_1, and π_2
Assume dimensional relationships of the form

$$\pi_1 = \gamma^{x_1} d^{y_1} k^{z_1} . \mu$$

$$\pi_2 = \gamma^{x_2} d^{y_2} k^{z_2} . S$$

Expressing π_1 in dimensional form:

$$\pi_1 = [ML^{-3}]^{x_1} . [L]^{y_1} . [LT^{-1}]^{z_1} . [ML^{-1}T^{-1}]$$

Equating indices by the principle of dimensional homogeneity:

$$\left.\begin{aligned} 0 &= x_1 + 1 \\ 0 &= -3x_1 + y_1 + z_1 - 1 \\ 0 &= -z_1 - 1 \end{aligned}\right) \text{ Hence: } \begin{aligned} x_1 &= -1 \\ y_1 &= -1 \\ z_1 &= -1 \end{aligned}$$

$$\text{i.e. } \pi_1 = \frac{\mu}{\gamma dk}$$

Expressing π_2 in dimensional form:

$$\pi_2 = [ML^{-3}]^{x_2} . [L]^{y_2} . [LT^{-1}]^{z_2} . [ML^{-2}T^{-2}]$$

Equating indices by the principle of dimensional homogeneity:

$$\left.\begin{aligned} 0 &= x_2 + 1 \\ 0 &= 3x_2 + y_2 + z_2 - 2 \\ 0 &= -z_2 - 2 \end{aligned}\right) \text{ Hence: } \begin{aligned} x_2 &= -1 \\ y_2 &= +1 \\ z_2 &= -2 \end{aligned}$$

$$\text{i.e. } \pi_2 = \frac{d.S}{\gamma k^2}$$

Thus provided that the variables chosen do have the greatest influence on the flow phenomena, the conditions of flow in a soil can be represented by the relation

$$F\{\pi_1, \pi_2\} = 0$$

i.e. $$F\left\{\frac{\gamma dk}{\mu}, \frac{d.S}{\gamma k^2}\right\} = 0$$

Since $\gamma = \frac{G-1}{1+e} . \gamma_w$, the π's can be rewritten as follows:

$$\pi_1 = \frac{G-1}{1+e} . \frac{\gamma_w dk}{\mu} \qquad \pi_2 = \frac{1+e}{G-1} . \frac{d.i.}{k^2}$$

i.e. The flow relationship now becomes:

$$F\left\{\frac{G-1}{1+e} . \frac{\gamma_w.d.k}{\mu}, \frac{1+e}{G-1} . \frac{d.i.}{k^2}\right\} = 0$$

It may be noted that both π_1 and π_2 are analogous to quantities commonly used in studies of pipe flow.

π_1 is analogous to the Reynolds Number for pipe flow: $R_N = \frac{\gamma.V.d}{\mu}$, while π_2 is the counterpart of the Friction Factor $S = \frac{d.\Delta p}{2L.\gamma.V^2}$

The accompanying dimensionless graphs show values of π_1 plotted as ordinates against π_2 as abscissae for various sands. The figures used in the calculations form the results of the standard filter tests carried out as described elsewhere in this report. The values shown cover the period during which the stable grain structure within the sand was being established. All the changes in properties which occurred to give the range of points shown in each curve occurred solely as a result of the influence of the percolating water. No artificial control was imposed over any of the variables, the only controlled factor being the constant head applied to the test cylinders.

The dimensionless graphs show that for a sand in the process of undergoing structural change a linear relationship exists between the logarithms of the dimensionless quantities π_1 and π_2. In other words, the general equation to the flow of water through a sand which is undergoing internal structural rearrangement of the grains including compression of the void spaces and migration of the fine material can be expressed over the range investigated as:

$$\log \pi_1 = -n \log \pi_2 + A$$

OR:

$$\log\left\{\frac{\gamma.d.k}{\mu}\right\} = -n \log\left\{\frac{d.s}{\gamma.k^2}\right\} + A$$

OR:

$$\frac{\gamma.d.k}{\mu} = a\left\{\frac{d.s}{\gamma.k^2}\right\}^{-n}$$

Where A and a are constants.

The values of a in the latter relationship vary widely from soil to soil. The logarithmic graphs of π_1 versus π_2 have been plotted disregarding the position of the decimal point in individual values in order to show all the curves on a single sheet. To take account of this, put $n = n' + p$ where p is an integer, positive or negative, introduced to take account of the absolute value of n.

The values of n' were found to cluster closely about a fixed value. For the five soils tested, this fixed value was estimated statistically at:

$n' = 0.526 \pm 0.075$ with 95% confidence.

Considering that this estimate was made on the basis of 5 samples only, the limits of uncertainty are narrow.

We may therefore conclude that the general equation to the flow of water through a soil which is in the process of undergoing changes of grading is:

$$\frac{\gamma dk}{\mu} = a\left\{\frac{dS}{\gamma k^2}\right\}^{(p-0.53)}$$

A noteworthy point is that as all the curves have a constant value of n', the type of flow (viscous or turbulent) taking place through the sands must have been the same in each case. This was despite the fact that individual values of the coefficient of permeability varied from 0.4×10^{-3}CM/SEC to 16×10^{-3}CM/SEC.

THE PHENOMENON OF THE MIGRATION OF FINE SOIL PARTICLES WHICH TAKES PLACE WHEN WATER UNDER PRESSURE PERCOLATES THROUGH A SOIL

The phenomenon of the migration of fine soil particles through a soil under the action of percolating water has already been described elsewhere in this report. In this section the causes, the mechanism and the effects of the particle migration or diffusion will be studied both from a qualitative and, later, from a quantitative point of view.

The Rate of Diffusion or Migration of Particles

It is clear that the void spaces in a mass of soil subjected to seepage forces will be reduced partly by the elastic deformation of the soil particles and partly by a reorientation and slippage of the soil grains relative to each other. The void spaces will further be altered by the migration of particles which is started by the slipping of fine particles out of their original places in the structure of the soil skeleton and into the voids. If the particles are retained in the voids into which they have slipped, the void space will be further reduced. If the particles are small enough to escape and travel onwards, out of the body of the soil, the ultimate effect will be to increase the void space. The mechanism for the compression of a soil by seepage forces may therefore be considered in three steps:

(a) Compaction by pure slippage of the grains,
(b) Diffusion or migration of fine particles,
(c) Elastic deformation of the grains.

In all practical cases, the deformation of the particles under seepage forces is negligible.

The variables which appear to have most effect on the rate at which diffusion proceeds are as follows:

(i) The seepage force arising from viscous drag of the percolating fluid on the surfaces of the particles. This seepage force is not only the agent of transport in the phenomenon of migration but also affects the rate of migration and, if large enough, the extent to which migration will continue, due to its effect on the void spaces in the soil.

(ii) The void ratio of the soil. The void ratio is a measure of the void space in a soil. It can be expected that the greater the initial void space in a soil, the more likely is it that diffusion will take place and the greater will be the initial rate of this diffusion.

(iii) The particle size distribution or grading. Soils which contain large percentages of fine particles and soils which cover a large range of particle sizes have been found on the average to be more likely to suffer from the effects of extensive particle migration than are other soils. Migration would be at a minimum in a perfectly uniform soil. In this analysis the particle size distribution will be described by a single characteristic particle dimension d.

Although these variables appear to affect the rate of particle migration, they seem to have little effect on the final grading attained by the sand.

If it is assumed that the slippage forces are negligible as compared with the seepage forces applied to a mass of soil and that the soil is confined within immovable boundaries, dimensional analysis can be used to obtain a reasonably rational expression for the rate of particle migration.

Let the rate of migration be described by the mass per unit volume of particles M_M migrating through unit distance in unit time. Assume the variable void ratio to be adequately described by variations in the submerged density γ of the soil.

The variables and their dimensions are thus:

	Variable	Symbol	Dimension
I	Rate of Migration	M_M	$ML^{-4}T^{-1}$
II	Seepage Force/Unit Vol.	S	$ML^{-2}T^{-2}$
III	Soil Density	γ	ML^{-3}
IV	Particle Diameter	d	L

Assume a dimensional relationship of the form:

$$M_M \propto S^a \gamma^b d^c$$

Expressing this in dimensional form:

$$\left[ML^{-4}T^{-1}\right] \propto \left[ML^{-2}T^{-2}\right]^a \left[ML^{-3}\right]^b \left[L\right]^c$$

Equating indices by the principle of dimensional homogeneity:

$$\left.\begin{aligned} 1 &= a + b \\ -4 &= -2a - 3b + c \\ -1 &= -2a \end{aligned}\right) \quad \text{Whence:} \quad \begin{aligned} a &= 1/2 \\ b &= 1/2 \\ c &= -3/2 \end{aligned}$$

Hence the dimensional relationship is:

$$M_M \propto S^{1/2} \,.\, \gamma^{1/2} \,.\, d^{-3/2}$$

OR: $M_M = A \sqrt{\dfrac{S.\gamma}{d^3}}$ Where A is a constant of proportionality.

If the seepage force increases gradually from zero, it is reasonable to expect that migration would not begin until the seepage force reached a certain critical value S_c and that no further migration would occur once the seepage force had exceeded a certain ultimate value S_u sufficient to bodily transport the soil mass. The increasing seepage force would also lead to an increasing density. The relation between seepage force and density would probably be of the form:

$$\gamma = \gamma_1\, e^{\alpha . S/S_u}$$

Neglecting the variation in d which must occur if diffusion takes place, a likely expression for the rate of migration of the fine particles of a soil under the action of flowing water would be:

$$M_M = A \sqrt{\frac{(S-S_c).\gamma_1}{d^3}} \,.\, \epsilon^{-\alpha . \frac{S}{S_u}}$$

Hence we have the qualitative result that the rate of migration is proportional to the square roots of seepage force and soil density and is inversely proportional to the $^3/_2$-power of the grading characteristic of the soil.

The exact form of this relationship and the values of the constants A, α, S_c and S_u cannot be determined experimentally as there is no way of measuring the rate of migration M_M. It is felt, however, that the expression does in part explain the mechanism of particle migration in soils.

Changes in Specific Surface Area due to the Migration of Particles in Sands.

No quantitative results could be arrived at for the rate at which particle migration takes place. The ultimate degree to which diffusion of particles within a given sand will take place under a given seepage head will now be studied in relation to the effects of this diffusion on the hydraulic properties and grading of the soil.

Earlier in this report, the effects of particle migration on the gradings of filter sands was discussed and it was stated that the area of the loop between the cumulative grading curve for the original sand and the curve for the sand once it had been altered by particle diffusion would give a measure of the change of specific surface area of the sand between initial and final conditions and hence also of the mass of fine particles which had migrated under the influence of the flowing water.

First of all evidence will be provided that the area beneath the cumulative grading curve for a sand is a measure of the specific surface area of that sand.

The specific surface area of a sand between any two particle size limits can easily be calculated if an algebraic expression can be found which will describe the cumulative grading curve of the sand between the chosen particle size limits. The calculation can be done as follows:

Assume, for example, that the cumulative grading curve can be accurately described between the particle size limits x_1 and x_2 by the equation:

$$y = Cx^{m+1}$$

Where: x = size of sieve aperture
y = cumulative percentage by weight of sand passing through sieve aperture x.

The amount of material dz ranging in size from x to $x + dx$ is:

$$dz = y_{(x + dx)} - y_{(x)}$$

$$dz = C(x+dx)^{m+1} - Cx^{m+1}$$

$$dz = Cx^{m+1} + C(m+1)x^{m}.dx + \frac{C(m+1)mx^{m-1}(dx)^2}{2!} + \,..\, -Cx^{m+1}$$

Neglecting infinitesimal quantities of order higher than unity:

$$dz = C(m+1)x^{m} \,.\, dx$$

In a mass of 100 gm. of material, the mass of particles ranging in size from x to $x + dx$ is dz. This mass would contain:

$$dN = \frac{dz}{\rho x^3}$$ cubical particles of edge x.
Where ρ is the solids density.

In the range x_1 to x_2, therefore, the total number of particles is:

$$N_{x_1}^{x_2} = \int_{x_1}^{x_2} \frac{C(m+1)x^{m}}{\rho x^3} dx = \frac{C(m+1)}{\rho} \int_{x_1}^{x_2} x^{m-3} dx$$

$$N_{x_1}^{x_2} = \frac{C}{\rho} \cdot \frac{(m+1)}{(m-2)} (x_2^{m-2} - x_1^{m-2})$$

The area per cubical particle is $6x^2$; for dN cubical particles of side x it is:

$$ds = \frac{6x^2.C(m+1)x^{m}.dx}{\rho x^3} = \frac{6C(m+1)}{\rho}x^{m-1}.dx$$

The total area of the particles ranging in size from x_1 to x_2 is therefore:

$$s_{x_1}^{x_2} = \frac{6C(m+1)}{\rho} \int_{x_1}^{x_2} x^{m-1} dx = \frac{6C}{\rho} \cdot \frac{(m+1)}{m}(x_2^{m} - x_1^{m})$$

The total mass of the particles ranging in size from x_1 to x_2 is:

$$z_{x_1}^{x_2} = C(m+1) \int_{x_1}^{x_2} x^{m} dx = C(x_2^{m+1} - x_1^{m+1})$$

Thus the specific surface area or the surface area per unit mass of the sand ranging in particle size from x_1 to x_2 is:

$$S_{x_1 x_2} = \frac{s_{x_1}^{x_2}}{z_{x_1}^{x_2}} = \frac{6(m+1)}{\rho m} \cdot \frac{x_2^{m} - x_1^{m}}{x_2^{m+1} - x_1^{m+1}}$$

This calculation has been based on the assumption of cubical particles. If s is the surface area on the basis of cubical particles and S is the real surface area then

$$S_x^{x+dx} = r_x s_x^{x+dx}$$

or, assuming r_x to be a constant,

$$S = r.s \text{ for any given sand.}$$

Hence the real specific surface area per unit mass of the sand ranging from x_1 to x_2 is given by:

$$S_{x_1x_2} = \frac{6r}{\rho} \cdot \frac{m+1}{m} \cdot \frac{x_2^m - x_1^m}{x_2^{m+1} - x_1^{m+1}}$$

This expression will be valid and will yield a positive result for all values of m, positive or negative different from zero.

The validity of this expression for the specific surface area of a sand depends mainly on the accuracy with which the expression $y = Cx^{m+1}$ describes the cumulative grading curve of the sand between the particle size limits x_1 and x_2. Expressing this relation logarithmically:

$$\log y = \log C + (m+1).\log x$$

which is the equation to a straight line on a log.-log. plot.

The accuracy of the calculated values could naturally be improved if the value of r were known. This could be obtained fairly simply by comparing measured values of the specific surface area with values calculated from the above formula. This would hardly be profitable, however, as there is such a wide variation in particle shape from sand to sand. Values of r would depend on the origin of the sand, its geological history and the constituent minerals of its grains. In all numerical work in this thesis, the value of r is assumed to be unity.

In order to determine suitable limiting particle sizes x_1, x_2 and the constants C and m, for a sand, the cumulative grading curve of the sand was plotted on a log.-log. basis. The particle sizes between which the grading curve was a straight line were chosen as x_1 and x_2. m was evaluated from the slope of the straight line and C from the intercept of the straight line portion with the "Cumulative % Passing" axis. Once these constants had been determined, the calculation of the specific surface between the chosen limits followed simply.

It can now be shown that the area beneath the cumulative grading curve for a sand is a measure of the specific surface area of that sand.

Let S_{ox} be the specific surface of a given sand between the particle size limits 0 and x. Let A_{ox} be the area under the cumulative grading curve between the limits 0 and x.

$$S_{ox} = \frac{6r}{\rho} \cdot \frac{m+1}{m} \cdot \frac{x^m}{x^{m+1}} = \frac{6r}{\rho} \cdot \frac{m+1}{mx}$$

$$A_{ox} = \int_0^x Cx^{m+1}dx = \frac{C}{m+2}\, x^{m+2}$$

$$\frac{A_{ox}}{S_{ox}} = \frac{C\rho}{6r} \cdot \frac{m}{(m+1)(m+2)} \cdot x^{m+3}$$

putting $\frac{C\rho}{6r} \frac{m}{(m+1)(m+2)} = R$

$$\frac{A_{ox}}{S_{ox}} = Rx^{m+3}$$

Taking logarithms:

$$\log A_{ox} = \log S_{ox} + (m+3) \log x + \log R$$

This shows that there is theoretically a linear relationship between the logarithms of Specific Surface Area and the Area beneath the cumulative semi-logarithmic grading curve if this curve can be accurately described by $y = Cx^{m+1}$.

To show that this is a general relationship for any filter sand, the specific surface areas between suitable limits for 9 sands (which underwent extensive changes of grading in the filter tests) were plotted on a log.-log. scale against the corresponding areas under the cumulative grading curves. The correlation obtained is remarkably good considering the extremely simplifying assumptions made in calculating the specific surface areas. The major source of scatter is probably neglect of the true value of the ratio r in the calculations.

The line of best fit for the correlation between the Specific Surface Area and the area under the Grading Curve for a sand was calculated by the Method of Least Squares in the manner described in section (ii) of the Appendix to this chapter. The equation to this line of best fit is:

$$\log_{10} A = -0.465 \log_{10} S + 1.99$$

i.e.

$$A = 98. S^{-0.47}$$

This equation is of a similar, but not identical form to the theoretical equation

$$A = R.x^{m+3} . S$$

which can be rewritten as:

$$A = (Rx^{m+3} . S^{1.47}) S^{-0.47}$$

The experimental results show that the average value of $(Rx^{m+3} . S^{1.47})$ is 98. This suggests that as the supposed constant is in fact a function of the Specific Surface Area, the true relationship between log A and log S is not linear, but of a more complex nature. The linear approximation is probably close enough for most practical purposes.

Thus it will be accepted that the relationship between the Area under a Grading Curve and the corresponding Specific Surface Area is:

$$A = 98. S^{-0.47}$$

The Correlation Coefficient for this data is $r = -0.850$ which indicates an excellent degree of correlation.

The Area of the "Hysteresis" Loop as a Measure of Particle Migration

Consider a sand whose grading is changing due to particle diffusion:-

If fine particles are being washed out of the body of the sand, the sand as a whole is becoming coarser and its grading curve will move from its original position towards the right-hand or 'coarser' side of the grading chart. At the same time its specific surface area will be decreasing. The total decrease will be measured by the area of the loop between the new and the original grading curves.

Similarly, if fine particles are being washed into the sand, the sand becomes finer, its grading curve moves toward the 'finer' side of the grading chart and its specific surface area will increase.

For the samples used in the standard filter tests, in general, the upper portion of the sand grew coarser, its specific surface area decreasing, while the lower portion grew finer, its specific surface area increasing. This process produced a double loop on the grading chart, with one portion on each side of the original grading curve for the sand. The overall change in the specific surface area of the sample was thus measured by the difference in areas between the "coarser" and "finer" hysteresis loops.

(Note that the use of the term "hysteresis" has some justification. The area of the loop represents, to some scale, the work done by the percolating water on the sand in changing its grading).

As it has now been shown that the area beneath a grading curve is a direct measure of specific surface area, the phenomenon of particle migration can now be further investigated by setting up a dimensional relation between specific surface area and other measurable properties of a Soil-Water flow system.

It will be assumed that Particle Migration, measured by changes in specific surface area is a function of:

(a) The quantity of flow q.
(b) The density (and hence the void ratio) of the sand γ
(c) The seepage force per unit volume $p = gi\gamma_w$.

The variables and their dimensions are thus:

	Variable	Symbol	Dimension
I	Specific Surface Area	S	$L^2.M^{-1}$
II	Quantity of Flow	q	$L^3.T^{-1}$
III	Soil Density	γ	$M.L^{-3}$
IV	Seepage Force	p	$ML^{-2}T^{-2}$

Assume a relationship of the form:

$$S \propto q^a . \gamma^b . p^c$$

Expressing this in dimensional form:

$$\left[L^2 M^{-1}\right] \propto \left[L^3 . T^{-1}\right]^a . \left[M . L^{-3}\right]^b . \left[M . L^{-2} . T^{-2}\right]^c$$

Equating indices by the Principle of Dimensional Homogeneity:

$$\left.\begin{aligned} 2 &= 3a - 3b - 2c \\ -1 &= b + c \\ 0 &= -a - 2c \end{aligned}\right\} \text{ Whence: } \quad \begin{aligned} a &= -2/3 \\ b &= -4/3 \\ c &= 1/3 \end{aligned}$$

Hence the relation becomes:

$$S = \beta \,.\, q^{-2/3} \,.\, \gamma^{-4/3} \,.\, p^{1/3}$$

where β is a constant of proportionality.

OR:

$$S = \frac{\beta^{1/3} \,.\, g^{1/3} \,.\, i^{1/3}}{q^{2/3} \,.\, \gamma_w} \left\{\frac{1+e}{G-1}\right\}^{4/3}$$

Since g and γ_w can be regarded as constant under all conditions and for all sands:

putting $\dfrac{\beta^{1/3}\, g^{1/3}}{\gamma_w} = B$ (say)

$$S = B \,.\, \frac{i^{1/3}}{q^{2/3}} \left\{\frac{1+e}{G-1}\right\}^{4/3} = B.\sigma \qquad \text{(say)}$$

The nett change in Specific Surface area due to the flow of water over the duration of the test is represented by the nett area of the hysteresis loop L.

$$\text{i.e.} \quad F(S_i - S_f) = L = B(\sigma_i - \sigma_f)$$

Where F( ) denotes the known relationship between S and L and the subscripts i and f represent, respectively, initial and final conditions in the filter test.

For the 9 sands which exhibited significant changes in grading, L has been plotted against $(\sigma_i - \sigma_f)$ on a log.-log. basis. There is a fair correlation between L and $(\sigma_i - \sigma_f)$ and it seems reasonable to assume that a linear relation exists on a logarithmic basis. This relation can be expressed as:

$$\log L = \mu \log (\sigma_i - \sigma_f) + \log E$$

OR:

$$L = E(\sigma_i - \sigma_f)^{\mu}$$

Where E and μ are constants.

We already have the relationship

$$\frac{A_{ox}}{S_{ox}} = Rx^{m+3}$$

If x is in this case the largest particle size occurring in the sand, then S_{ox} will be the average specific surface area for the entire sample and A_{ox} will be the total area underneath the grading curve. In this case

$L = A_{ox_i} - \overline{A}_{ox_f}$ = the nett change in area between initial and final conditions.

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