A comparison of Real Options and Discounted Cash Flow valuation techniques on a gold mining project

Amtenge Penda Shivute

A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the Degree of Master of Science in Engineering

Johannesburg, 2019

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the Degree of Master of Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

____

Amtenge Penda Shivute

This ________ day of ________________ year _______

ABSTRACT

Valuation of mineral assets is done for several purposes including for investment-decision making. There are three main valuation approaches used in mineral projects valuation including the income approach. Within the income approach, the static discounted cash flow (DCF) is the widely used technique. However, it has several limitations, the main one being that it is deterministic. To try and overcome this limitation Monte Carlo simulation is done on the input parameters to result in a Monte Carlo DCF (MCDCF) technique. Nonetheless, the MCDCF also has limitations, the main one being that it neglects the value of flexibility that management has due to those uncertainties during the life of mine (LoM). All these limitations may lead to undervaluing or overvaluing mineral projects, resulting in unrealistic projects values. Real Options Analysis (ROA) technique has been recognised as a better mineral projects valuation technique that considers uncertainty and managerial flexibility.

The research study applied and compared the results of the static DCF, MCDCF and ROA techniques on an underground mining project with two possible extraction scenarios. The static DCF was used as the basis for the economic value of the mining project, while the MCDCF incorporated uncertainty in its input variables and quantified the risk associated with the project using @RISK software. The Binomial-Lattice was used to value managerial flexibility to defer the start of the project.

The research study found that the static DCF resulted in an NPV of US$9.60 million and an IRR of 8.67% for Scenario 1 while Scenario 2 has an NPV of US$11.35 million and an IRR of 8.95%. These results show that the project is viable since the NPV is positive and the IRR is greater than the 5% discount rate. However, the MCDCF found that there is a chance that the project is unviable which could change the decision made based on the static DCF results from the MCDCF analysis, there is a probability of 0.21 that the project’s NPV and IRR will be negative and less than the discount rate, respectively for Scenario 1. For Scenario 2 there is a probability of 0.17 that the project’s NPV and IRR will be negative and less than the discount rate, respectively. The Binomial-Lattice model resulted in a higher

NPV for both scenarios; US$17.34 million and US$21.52 million for Scenario 1 and Scenario 2, respectively. This is because the Binomial-Lattice uses a risk-free rate to discount cash flows, whereas the static DCF and MCDCF use risk-adjust discount rate. From these findings it can be concluded that the project is viable and that there is more value created when mining projects have the flexibility to delay the start of a project. Therefore, the company should proceed to a full feasibility study. Also, mineral projects valuators must use the MCDCF and ROA to quantify risk in mineral projects and to valuate flexibility in the face of uncertainty.

DEDICATION

This research report is dedicated to my late mother Veronica Nghishidimbwa, may her soul Rest in Peace, to my father Nakathingo Shivute, and my sister Shanyenga Panduleni Shivute.

ACKNOWLEDGEMENTS

Firstly, I give thanks to the creator, the Almighty God for giving me the wisdom, strength and guidance to pursue and complete this degree. Special acknowledgment also goes to the following institutions and individuals:

- My Sponsors, Deutscher Akademischer Austauschdienst (DAAD) In-land Scholarship (Namibia), Namibian Student Financial Aid Fund (NSFAF), Wits Postgraduate Masters Merit Award (PMA) and the University of Namibia (UNAM) for funding my studies and assisting me financially;
- My Supervisor, Mr. T. Tholana for his guidance, knowledge and constant encouragement to complete this degree;
- The University of the Witwatersrand School of Mining Engineering staff members and fellow postgraduate colleagues for their assistance and support during my studies; and
- My family and friends for their words of encouragement and support.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

1.1 Chapter overview

This chapter briefly introduces the concept of value and value creation, and the commonly used mineral valuation approaches used in the mining industry. The application of these valuation approaches to different development stages of mineral properties is also discussed. The problem statement, aim and objectives are also discussed in the chapter. Lastly, the chapter justifies the reason for carrying out this research study and how this research is structured.

1.2 Background information

All mining companies aim to create value from their projects or operations for the benefit of their stakeholders. There are different types of value which include social, intrinsic and economic value. The focus of this research study is on economic value. Economic value can be created through increased production of quality material that may require large capital investment or through cost cutting strategies. The mining industry is unique to other industries in that it is liable to high risks and uncertainties. It is therefore imperative that investment decisions in mineral projects are informed by the appropriate valuation technique(s) that incorporate risk and uncertainties.

The International Valuation Standards Council (2016) has recognised three valuation approaches, namely; the market, cost and income approaches. These approaches are also applicable to mineral asset valuation. Each valuation approach has different techniques that are shown in Table 1.1.

Table 1.1: Classification of valuation techniques for different stages of mineral properties development

(Adopted from: Baurens, 2010)

Valuation approach	Description	Valuation techniques	Exploration properties	Development properties	Production properties
Income approach	Relies on the "value-in-use" principle and requires determination of the present value of future cash flows over the useful life of the Mineral Property	Discounted Cash Flow (DCF)	Not generally used	Widely used	Widely used
		Real Options	Less widely used	Quite widely used	Quite widely used
		Monte Carlo Analysis	Less widely used	Less widely used	Less widely used
		Probabilistic Methods	Not widely used	Not widely used	Not widely used
Cost approach	Relies on historical and/or future amounts spent on the Mineral Asset	Appraised Value	Quite widely used	Not widely used	Not generally used
		Geoscience Factor	Not widely used	Not widely used	Not generally used
		Multiple of exploration expenditure	Quite widely used	Quite widely used	Widely used
Market approach	Relies on the principle of substitution. The Mineral Property being valued is compared with the transaction value of similar Mineral Properties, transacted on an open market	Comparable Transactions	Widely used	Widely used	Widely used
		Options Agreement Terms	Widely used	Widely used	Quite widely used
		Gross "in-Situ" Metal Value	Not acceptable		
		Net Metal Value per unit of metal	Widely used rule of thumb		
		Value per Unit Area	Widely used	Not widely used	Not widely used
		Market Capitalisation	More applicable to single property asset junior companies		

Table 1.1 shows the level to which different valuation techniques are applied to different stages of mineral properties developments. Usually, the valuation technique or techniques applied are dictated by the nature of the valuation, stage of

development on the mineral asset, availability of data, the investors needs and the valuators' professional judgement (SAMVAL Code, 2016; Australasian Code for Public Reporting of Technical Assessments and Valuation of Mineral Assets (VALMIN Code, 2015). However, the most imperative factor that dictates which techniques to apply is the competency and judgment of the valuator (Lawrence, 2001). Valuators must recognise the purpose of the valuation and to whom the valuation is to be done for, and different methods for specific needs. It is also vital that they recognise the different decision criteria for each technique, and the advantages and disadvantages of each technique in order to interpret the results correctly (Lilford & Minnitt, 2005).

1.3 Problem statement

Although there is a wide range of mineral asset valuation techniques as shown in Table 1.1, the static DCF is the most widely used technique for mineral project valuation (Macfarlane, 2001). However, the technique has several limitations which include, it results in a static NPV that neglects the high level of uncertainties associated with input parameters used to calculate the project value. It also neglects the value embedded in the flexibility that management has due to those uncertainties (Bernnan and Schwartz, 1985). The fact that the static DCF technique ignores the ability of management to react and change in the face of uncertainty during the LoM may undervalue or overvalue mining projects resulting in unrealistic projects values (Stojanovic, 2013). This subsequently leads to major losses and eventually premature closure of mining operations as the revenues generated will be unable to sustain costs when the project is overvalued. On the other hand, undervaluing mining projects lead to the missed opportunity of gaining maximum value from the mineral asset. To address the limitations of the static DCF technique, better valuation techniques must be used such as ROA. ROA technique has been recognised as a valuation technique that takes into consideration uncertainty as well as managerial flexibility and can be applied to valuing mineral projects (Haque, *et al.*, 2016).

1.4 Research aim and objectives

The aim of this research is to compare the DCF and ROA valuation techniques on a gold mining project for investment decision-making purposes. The objectives of the research are to:

- Conduct a literature review on mineral project valuation and the application of the DCF and ROA techniques on mining projects;
- Collect the relevant economic and technical data for the selected case study;
- Model selected input parameters using the Monte Carlo simulation technique;
- Compare the valuation results from the two techniques;
- Recommend to the company the best scenario between the two scenarios that are being considered by the company.

1.5 Significance of the study

The global mining sector just like any other sector is liable to risks and uncertainties. This was evident in early 2009 just after the global financial crisis (GFC) occurred in 2008, in which mineral commodities prices dropped by nearly 50% (Deloitte & Touche, 2010). Most mining companies had not anticipated this crisis that negatively impacted their operations. The GFC resulted in smaller and less competitive operations shutting down, while the larger and more competitive ones downsizing their operations in order to survive the aftermath of the crisis. The GFC also left investors all over the world more cautious of where they will invest their money, especially when it comes to investing in mining projects. This is due to the high level of uncertainty caused by the ever-fluctuating mineral commodities prices that are linked to global events, politics, recessions and microeconomics. Coupled with these uncertainties are geotechnical, economic, environmental and social risks. These risks and uncertainties are what makes mining projects riskier to invest in when compared to other investment options.

Investment in the mining industry is capital intensive and are associated with risks and uncertainties. Therefore, it is imperative that investment decisions are informed by the appropriate valuation technique(s) before any capital is committed. It is also important that the valuation technique(s) used to value mineral projects accurately portray its real value. This is to avoid missing out on investment opportunities and having major losses that eventually lead to premature closure of mining operations. Valuation techniques must take into consideration risks and uncertainty to ensure that mining companies make informed investments decisions at the right time to ensure survival during volatile economic periods. Traditional techniques like the static DCF do not take into consideration uncertainty and the flexibility mining project have in its valuation and are unable to quantify risk. This research study will apply selected valuation techniques that incorporate uncertainty and flexibility in the valuation process such as the MCDCF and ROA, respectively. This will provide the company with the risk associated with the project over the LoM and the value embedded in flexibility. It will be able to provide the company with a range of possible outcomes indicating what could likely happen during volatile economic periods. From these outcomes, the company can then make an informed investment decision about where and when to invest capital to remain competitive.

1.6 Research scope and description of the case study

This research study used a gold mine as a case study and due to corporate confidentiality issues, the name of the mine and, company is anonymous throughout this report. Current production at the mine comes from two conventional open pit mines (Pit A and Pit B) that have been in operation since 2014, and as of 2018 together have an expected LoM of nine years. The mine plan assumed seven years of active mining and the remaining two years for processing stockpiles. Pit B has a newly discovered Mineral Resource of 1.49 million tonnes of gold ore that extends deep underground. The mine plans to exploit this newly discovered Mineral Resource using an underground mining method from 2026 before its mining lease expires in 2032. The focus of this research study is to compare the economic value of extracting the newly discovered Mineral Resource using the selected valuation techniques. Figure 1.1 shows the planned ultimate pit layout and the newly discovered underground Mineral Resource below Pit B.

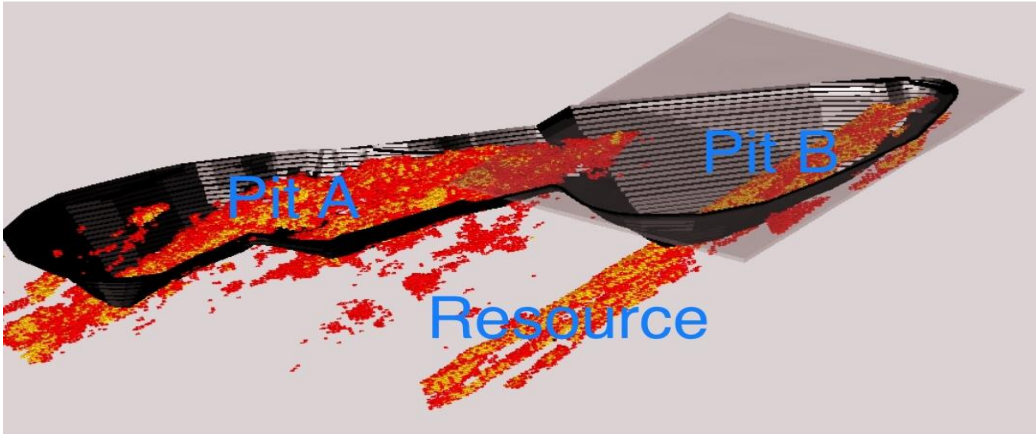


Figure 1.1. Ultimate pit layout and underground resource

(Source: Anonymous, 2017)

According to the preliminary feasibility study (PFS) conducted by the company, the underground deposit will be accessed through a decline that will be developed from Pit B. The PFS assumed that the deposit is likely to be exploited using the bench and fill mining method without pillars in order to maximise Mineral Resource recovery. This mining method will entail mining primary stopes and filling them with pastefill containing 6% cement to support the walls, while the secondary stopes are being mined out. The secondary stopes will be supported by a weaker pastefill that contains 2% cement. Waste from development may be placed in secondary stopes to act as support.

1.7 Chapter summary and structure of the report

The report is structured in five chapters. Chapter 1 has provided a background about the global mining industry and the business environment it operates in. It further introduced the different approaches commonly used in mineral asset valuation. It also defined the problem that this research study is based on and, lastly the chapter listed the research aim and objectives, discussed the significance and scope of the research.

Chapter 2 reviews literature on valuation and valuation approaches with their respective advantages and disadvantages as applied in the minerals industry. The literature on risk and uncertainty is also reviewed, as well as on flexibility. Studies that have been done on the DCF and ROA techniques are also reviewed in this chapter illustrating their application in the minerals industry.

Chapter 3 describes the research methodology used in this study. It describes how the static DCF analysis was applied to the two scenarios. It outlined all assumptions required for the analysis. It also describes how the MCDCF and a Binomial-Lattice in ROA were conducted, outlining the respective assumptions made.

Chapter 4 presents and analyses results from the research. Lastly, Chapter 5 concludes the research and provides recommendations. The systematic workflow of the research is illustrated in Figure 1.2.

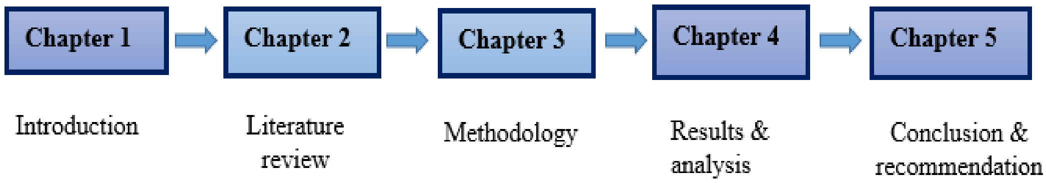


Figure 1.2: Report structure

2 LITERATURE REVIEW

2.1 Chapter overview

This chapter defines economic value and valuation. It further discusses valuation approaches and their respective techniques, reviewing their strengths and limitations. The literature on the DCF and ROA techniques is also reviewed in this chapter. Furthermore, the chapter reviews the importance of flexibility within a mine plan in the presence of risks and uncertainties. Lastly, the chapter reviews the literature on the application of the DCF and ROA techniques in the minerals industry.

2.2 Economic value and valuation

As stated in Section 1.2, there are different types of value. The focus of this research study is on economic value. Njowa (2017 p. 6) defined economic value as a "*dimension of measurement on the worthiness or performance of an entity as determined by an individual's or organisation's preferences, and the trade-offs they choose to make in an open market given their limited resources*". Economic value is created through capital investments from investors to increase production, resulting in increased future cash flows at a rate that is greater than that of the rate of return (Koller, *et al*., 2010). Furthermore, Koller, *et al.* (2010) stated that more value is created when businesses are able to increase their revenues and reinvest their profits in projects with low cost of capital.

Measuring value is a vital process of a business's performance because it considers its long-term welfare and those of all its stakeholder (Koller, *et al.*, 2010). The process of measuring a project's value is known as valuation. Investopedia (2018a) defines valuation as a procedure of determining the intrinsic present value and benefits of investing in a particular business at a specific point in time. In the perspective of mineral properties, The SAMVAL Code (2016 p. 4) defines valuation as "*the practice of defining the monetary value of a mineral asset as a whole at a specified date, under a set of assumptions*". Mineral asset valuation is challenging due to its technical and economic aspects (Lilford and Minnitt, 2005). This is because of the uncertainty in accurately predicting mineral commodities prices, and production profiles. Kramna (2014) noted that valuing mineral properties require the merging of knowledge from a variety of different disciplines such as economics, finance, geology and mining. This also makes valuation a complex process and thus requires a competent valuator to conduct it as stipulated in the relevant mineral asset valuation codes. The valuation of mineral properties involves making a number of assumptions and applying valuation approaches and their principles. The result is a single value that is justifiable based on the available information and assumptions made. It is, therefore, possible for another valuator to estimate different value for the same property (Njowa, 2017).

Mineral asset valuation is done for several reasons at different phases through the mine value chain. According to Cawood (2004), these reasons are categorised into two broad categories, namely; those reasons required by the minerals industry and those controlled by government policies. However, Frimpong (1992) gave four other reasons for valuing mineral properties and these are to:

- outline to potential investors the worth, feasibility, risk, and uncertainty of a mineral project;
- equip organisations with practical, operational and economical guidelines to effectively exploit mineral assets;
- create a basis on which investment decisions such as mergers and acquisitions, project expansions and taxation considerations are made. This is done to provide organisations with the right information and outcomes essential to compare and measure the economic gains of completing projects; and
- enhance the quality of standard operating procedures and control operating adjustments by giving organisations some flexibility.

2.3 Valuation approaches and techniques

Over the years a lot of research has been done in the area of mineral asset valuation. According to the International Valuation Standards Council (2016), there are three valuation approaches used to determine the value of mineral assets as show in Table 1.1. They are classified into the following approaches:

- Market approach;
- Cost approach; and

- Income approach.

2.3.1 Market approach

According to The South African Code for the Reporting of Mineral Asset Valuation (SAMVAL Code 2016 p. 9), the market approach *"relies on the 'willing buyer, willing seller' principle and requires that the monetary value obtainable from the sale of the Mineral Asset is determined as if in an arm's-length transaction".* It also offers a sign of value by comparing the saleable asset with similar assets whose values are known and are available (International Valuation Standards Council, 2016). The approach consists of six techniques: comparable transaction, option agreement, the net metal value per unit, value per unit area, market capitalisation and gross "in situ" metal (SAMVAL Code, 2016).

The market approach has been widely used in the valuation of mineral projects (Guj, 2013). Despite its wide application, it has the following shortcomings:

- the lack of sufficient market data of similar mining projects transactions in order to estimate the value of a mining project, resulting in an unrealistic value of a mining project (Lilford and Minnitt, 2005);
- the real value of mining projects transactions are sometimes not entirely captured in the market because of the market's limitations and policies (Domingo and Lopez-Dee, 2007). This also results in an unrealistic value of the mining project; and
- it does not incorporate market uncertainties and project risks and thus neglects the value in future options (Ampofo, 2017).

Although the market approach has been widely used to value mineral properties at their respective stages of development, it has not been considered in this research study because it does not incorporate uncertainties in its valuation techniques.

2.3.2 Cost approach

The cost approach was adopted from valuing real estate property, hence it can be applied to almost any physical asset. It is based on a principle of a buyer who is not willing to pay extra for an asset than what it will cost them to buy or construct a replica asset of equivalent value. This principle does not take into consideration excessive time stretch and the risk of constructing or buying the replica (International Valuation Standards Council, 2016). The approach calculates the value of substituting or replicating the asset and incorporates depreciation and any significant sort of obsolescence (International Valuation Standards Council, 2016). Techniques within this approach are the appraised value, multiple of exploration expenditure, and the geoscience factor techniques.

The cost approach is one of the simplest valuation approaches, but it sometimes neglects a number of key variables such as commodity prices and exchange rates essential to determining the economic value of mining projects (Lilford and Minnitt, 2005). Therefore, the cost approach is not very suitable to accurately determine the economic value of developmental and operational mineral projects. It is for this reason that it was not used in this research study.

2.3.3 Income approach

The income approach takes into consideration the cash inflow and outflow of business as income and expenses, respectively. The income and expenses are generated by the asset over a projected period of time, normally the life of the asset. The basis of this approach is on the anticipation of future cash flow (International Valuation Standards Council, 2016). There are two techniques that are mainly used under the income approach to determine the economic value of minerals projects; the DCF (static DCF and MCDCF) and ROA techniques (Lilford and Minnitt, 2005). The techniques under the income approach are based on the DCF technique, from which simulations and sensitivity analyses can be conducted.

The major advantages of this approach are that it uses both economic and technical data essential to determining the economic value of minerals projects. It also considers and quantifies the risk associated with mineral projects. Also, uncertainty in inputs variables is accounted for by simulation techniques in order to determine the robustness of mineral projects. Lastly, it is able to incorporate the flexibility that management has in the face uncertainty. It is for these reasons that this approach was used in this research study. The income approach techniques used to determine the economic value of the mining project, taking into consideration uncertainty and quantifying the risk associated with perusing it are discussed in the following section.

2.4 Discounted cash flow analysis

The DCF technique was developed by Irving Fisher in 1930 when it was used in economics to calculate wealth and it validated the optimisation of present value of businesses (Fisher, 1930). When it is applied in the minerals industry the technique has the following four fundamental aspects (Baurens, 2010):

- The orebody is the main asset for any mineral development;
- Revenue is mainly driven by mineral commodity prices, which is calculated by the quantity of valuable mineral sold at a specific market price;
- All costs are calculated by the quantity of valuable mineral mined and processed while; and
- The resultant profits are more liable to changes in revenues as to changes in costs. This is because of the high operating cost nature of mineral operations.

The generic principle of the DCF technique is to estimate the annual cash flowing into the business through its assets from assumed sales and invested capital minus the annual cash flowing out of the business due to taxes, royalties, operating costs and reclamation sinking funds. This results in the net cash flow the business has made during that year which can be calculated using Equation 2.1 (Brigham and Gapenski, 1997):

$$\text{Net cash flow (NCF)} = \text{Revenue} - \text{Costs (including taxes)} - \text{Capital} \qquad 2.1$$

The expected cash flows are projected through the life of the business and according to the risks and uncertainties associated with the business, an appropriate discount rate is applied to discount the annual cash flows. The sum of the discounted cash flows is the NPV of the project. This technique is based on the 'time value of money' concept, which states that cash flows in business have more value now than the same amount received in future due to risks and opportunity cost. Therefore, a discount rate is applied to future cash flows (Pintaric and Kravanja, 2017). The DCF technique uses three main decision criteria to measure the economic viability of a project, which are (Lohmann and Baksh, 1993):

- The Net Present Value (NPV);
- Internal Rate of Return (IRR); and
- The Payback Period (PP).

2.4.1 Net Present Value

The NPV is defined as the sum of discounted annual net incomes after taxes and expenses generated over the life of a project (Stone, 1988). The discount rate applied to the future cash flow estimates takes into account the risk associated with the business. Most businesses use NPV to measure the economic value of a project. The value is measured under definite assumptions to calculate projected cash flows and discount them using an appropriate discount rate. It is used to aid the decision-making process of possible investments. A project is considered viable if the NPV is positive and vice versa. According to Brigham and Gapenski (1997), the NPV of future cash flows is determined using Equation 2.2:

$$NPV = C_0 + \sum_{t=1}^{t=n} \frac{NCF_t}{(1+r)^t} \qquad 2.2$$

Where;

C_0 is the initial capital invested in the project for its development. For mineral projects, this usually includes exploration, prospecting studies, pre-feasibility and feasibility studies, and purchasing production and processing equipment. This investment depends on the magnitude of the project; NCF_t is the yearly projected future cash flows, which are costs including taxes less revenues; r is the discount rate which represents the risk associated with the project; n is the life of the project, and t is the cash flow period.

The discount rate is also referred to as the required rate of return, cost of capital, cost of debt, or alternative cost (Kvalevag, 2009). It can be defined as the percentage rate of return that a business needs to accumulate funds through its operations to reimburse investors for their initial investment and the risk associated with taking such an investment (Park and Matunhire, 2011a). Mun (2006) mentioned that a discount rate can be derived from models such as the Weighted Average Cost of Capital (WACC), Capital Asset Pricing Model (CAPM) and Asset-Pricing Model (APM).

The WACC model is based on a ratio between the cost of capital and debt in a business at a specific point in time. This model is widely used in corporate finance as it takes into consideration the global risks of a company that are associated with the investment-decision making process (Guarnera and Martin, 2011). Therefore, a company's discount rate that is calculated using the WACC model is adjusted for global risks of different operations and projects into one rate (Torries, 1998). Guarnera and Martin (2011) noted that this is one of the model's significant advantages. The resultant discount rate that is calculated from the WACC model can be applied to all the company's different operations, due to the fact that the discount rate incorporates all risks associated with the company's assets (Guarnera and Martin, 2011). However, the major limitation of the WACC model is that it assumes a single discount rate for projects or operations irrespective of geographical locations. According to Steiger (2008), the WACC is computed by finding the product of the weighted streams of capital in alignment to the company's economic structure and their costs. The formula for determining the WACC is given in Equation 2.3 (Steiger, 2008):

$$WACC = \frac{Equity}{Debt + Equity} R_{Equity} + \frac{Debt}{Debt + Equity} R_{Debt} \qquad 2.3$$

Where;

R_{Debt} is the cost of debt, which is the interest rate companies pay for borrowing money; R_{Equity} is the cost of equity and is defined as the rate of return required by a company's shareholders in order for an individual investor to tolerate the risk that comes with holding that company's shares (Financial Times, 2018). This includes capital gains in the company and dividend payments that are compensation methods for undertaking the risk of holding that company's share. The required return is known as the return on equity and can be calculated using the CAPM illustrated in Equation 2.4 (Brealey, et al., 2011):

$$R_{Equity} = R_f + \beta_{Equity}(R_{Market} - R_f) \qquad 2.4$$

Where;

R_f is the risk-free interest rate, β_{Equity} is the systematic risk of the equity which measures the volatility of a company's share price compared to the volatility of the entire market, and R_{Market} is the expected return of the market portfolio.

The CAPM is used to value share prices of companies that are openly available to trade on the stock market. It takes into consideration the fundamentals of an investment, which comprises of the risk-free rate of return, and the volatility of a company's share price versus its average share price as calculated by beta (Guarnera and Martin, 2011).

According to Gardener (2015), the greatest difficulty valuators have to deal with is integrating risk and uncertainty into valuation techniques. Uncertainty affects the process of valuation in different ways. French and Gabbrielli (2005) identified two ways in which uncertainty affects the valuation process. They mentioned that uncertainty causes:

- cash flows from investments to become unpredictable and indefinite; and
- the subsequent value of a project to be uncertain.

The most common way that the static DCF deals with risks and uncertainties is by adjusting the discount factor. Increasing the discount factor under high uncertainty in input variables makes future projected cash flows less significant than earlier cash flow and thus, lowers the NPV. This ultimately demands an increase in the projects' requirements of quality and profitability in order for it to be undertaken (Kvalevag, 2009). In addition to the discount rate, there are other techniques that are used to account for risk and uncertainty in the DCF technique. The most common one is sensitivity analysis which involves adjusting projected cash flows by repeatedly varying input parameters of the DCF (French and Gabbrielli, 2005). Sensitivity analysis is used to observe how significant independent variables impact the value of a business by altering them repetitively and observing their effects on the resultant value generated. For mineral projects, the key variables in a sensitivity analysis are mineral commodity price, production rates, operating costs, operational expenditure and capital expenditure (Kvalevag, 2009; Guj, 2013). There may be other key variables that will arise as more knowledge about the project is discovered as the project progresses. The variables are predicted as two states, either optimistic or pessimistic. The predictions are done on each variable one at a time in order to see how their changes affect the business' value (Kvalevag, 2009). There are two types of techniques used to carry out sensitivity analysis, namely the modelling and simulation techniques, which are discussed in the following sections. However, one major limitation of sensitivity analysis its inability to quantify the risk associated with projects (Bloem and Wees, 2003). Heuberger (2005) valued a small underground mine using the DCF analysis and took into account the impact of economic and geological uncertainties using Monte Carlo simulation. It was concluded that simulation techniques can help to aid management strategies to avoid unwanted outcomes that arise due to uncertainties.

2.4.2 Internal Rate of Return

The IRR is defined as the rate at which a business recovers the initial capital invested and is the discount rate which gives an NPV equal to zero (Salehi and Tavakkoli-Moghaddam, 2008). Van Zyl (2015) mentioned that IRR measures how well an entity effectively uses its capital. It is usually used before any capital is invested in a business. The IRR for a life of the project of t years is determined using Equation 2.5 (Maric, *et al*., 2011):

$$\sum_{n=0}^{t} \frac{NPV}{(1+IRR)^n} = 0 \qquad 2.5$$

The greater the IRR, the faster the capital invested is retrieved making the business more lucrative and desirable. Using this criteria, a project is considered viable if the IRR is greater than the discount rate and vice versa.

2.4.3 Payback period

In addition to NPV and IRR, the payback period (PP) is also used to aid investment decision making. The PP is defined as the duration a business takes to fully recover the initial capital invested in the business. Its measure of economic value in the project valuation is that the longer the resultant payback period, the more undesirable a project becomes. Hence, a shorter payback period is desired because a shorter PP makes the project more liquid. According to Park and Nelson (2013), PP has a minor effect on the overall investment decision when compared to NPV and IRR. The PP can only be calculated once the yearly forecasted cash flows are known. It is the cumulative yearly cash flows until the initial investment has been recovered making the cash flow positive (Runge, 1998). PP can the calculated using Equation 2.6 (Gotze, *et al.*, 2008);

$$PP \approx t* + \frac{NPV_{t*}}{NPV_{t*} - NPV_{t*+1}} \qquad 2.6$$

Where;

t^* is the number of years a business takes before fully recovering the initial capital investment after production has started; NPV_{t^*} is the net present value before fully recovering the initial capital investment, and NPV_{t^*+1} is the net present value after fully recovering the initial capital investment.

2.4.4 Advantages and disadvantages of the DCF technique

The main advantage of the DCF technique, in general, is its simplicity in use and interpretation by valuators, investors, and other stakeholders. This is the main reason for its widespread application in valuing projects and businesses for investment decision making. In addition to this advantage, Mun (2006) stated the following advantages of the DCF technique:

- Its decision criteria (NPV, IRR and PP) are straight forward and consistent for any project valuations;
- Time value of money and basic risk is taken into consideration; and
- It has an acceptable level of precision and economic rationality.

Despite its wide application in a number of industries including the minerals industry, the DCF technique has several limitations. Torries (1998) and Kelly (1998) both stated that it is difficult to determine the most suitable discount rate to accurately calculate the NPV. This is because in reality the determination of the discount rate is dependent on an individual’s experience making the calculation of the discount rate subjective (Park and Matunhire, 2011b). Furthermore, the discount rate in the DCF technique accounts for all the risk in the project over its life. Conversely, the risks of a project may change during the projects life (Mun, 2006). The other limitation of the DCF technique is that it does not integrate the flexibility of management to make decisions when trying to adjust themselves to survive turbulent economic times (Zettl, 2002; Drieza, *et al.*, 2002; Gilbert, 2004; Lilford and Minnit, 2005; Mayer and Kazakidis, 2007; Kvalevag, 2009; Silitonga, 2015).

In addition to the limitations of the DCF technique in general, the static DCF in particular, assumes that all input variables are deterministic and known with certainty throughout the life of the project on the day of valuation and the investment decision is made without considering other factors. In reality, input variables are uncertain, stochastic in nature and can vary in the future which can affect the outcome of the decision criteria of the static DCF. This makes the cash flows uncertain and can ultimately change the investment decision because a profitable project can become unprofitable due to a change of input variables in future (Mun, 2006). This also results in management lacking flexibility in making decisions. Spencer-Young and Durand (2004) also mentioned that the DCF technique does not mirror the value of deferring the start of the project up until favourable conditions prevail and input variables are known with a higher degree of confidence.

The other limitation of the static DCF it that it assumes that once a project commences it is passively managed, not taking into consideration possible decision options, budget constraints and milestones over the projects lifetime. However, in reality, investment decision options and budget constraints are common and management has the flexibility to make informed decisions when the business environment changes (Mun, 2006).

2.5 Risk and uncertainty in the mining industry

Risk and uncertainty in the mining industry have major impacts on the valuation of projects and can make a potentially viable project unviable. Risk, in general, is defined as the measurement of the possibility of an outcome occurring and its impact when it occurs. In financial terms, it is the probability that the actual return in a business will deviate from the projected returns (Investopedia, 2018b). Risks can be measured on input variables and quantified on output variables by using probabilistic models (Ajak, *et al*., 2018). Generally, uncertainty is defined as anything that is not known with accuracy. In financial terms it is defined as information about input variables that are not accurately known at the time of the valuation and the outcome cannot be measured with accuracy (French and Gabbrielli, 2005; Ajak, *et al.,* 2018). The value of a mining project is highly affected by the uncertainties that are associated with the mining industry. The fact that input variables are uncertain makes it possible for the risk of a project to be quantified.

The sources of uncertainty in the mining industry are in input variables of the valuation process and are broadly categorized into internal (endogenous) and external (exogenous) elements (Kazakidis and Scoble, 2003). Internal uncertainties are those uncertainties that are dictated by the orebody and the mining method. External uncertainties are those uncertainties that are governed by factors outside of the project that the business has no direct control over, such as the market in which they operate (Mayer and Kazakidis, 2007). External uncertainties are managed at a strategic level for example by diversification of portfolios and hedging of mineral commodity price. Internal uncertainties are managed at the source where operations take place. This includes stochastic mine planning adjusting production schedules, changing mining and processing equipment, and adding additional capacity. Furthermore, Figure 2.1 gives examples of internal and external uncertainties.

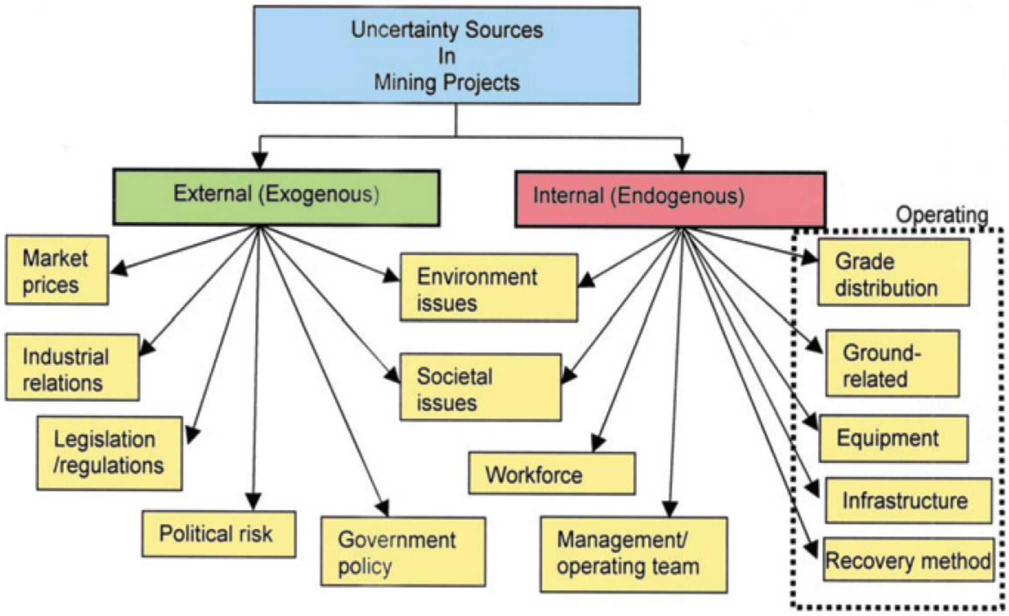


Figure 2.1: Sources of uncertainty in the mining industry

(Source: Kazakidis and Scoble, 2003)

Dehghani and Ataee-pour (2012) also stated that from the establishment of mineral projects the sources of uncertainties can be further classified into three major groups: exploration, engineering and economic uncertainties. Exploration uncertainties are those uncertainties that are based on the orebody characteristics through the different evaluation phases. These uncertainties entail geological knowledge and confidence which are influenced by geological data for ore definition, its interpretation and modelling. These are the key inputs that lead to the mine design and planning process of a project from which investment decisions are made (Morley, *et al.*, 1999; Dehghani and Ataee-pour, 2012). The exploration uncertainty that has the most significant impact on cash flows and net present value is the grade distribution and estimation of the orebody (Morley, *et al.*, 1999). Therefore, it is imperative that the orebody block model be as accurate as possible in order to avoid overestimating or underestimating the value of a mining project.

Engineering uncertainties are those uncertainties that are based on the technical design aspect of a mine that influences the ultimate pit slope angle, stope boundaries and the mine scheduling interval. They take into consideration mainly the mining and processing equipment used to exploit the orebody in order to attain maximum value from it. According to Dehghani and Ataee-pour (2012), engineering uncertainties comprise of defining the mining method, bench or stope geometry, designed grade control, dilution factors, hydrogeological and geotechnical constraints, mining and processing recoveries.

Economic uncertainties are the most significant sources of uncertainty that have a direct consequence on mineral projects' valuations. The two economic parameters with significant uncertainty are the forecasted mineral commodity prices and operating costs (Dehghani and Ataee-pour, 2012). Mineral commodity prices are mostly influenced by supply and demand, global events, politics, recessions and shareholder payouts. Dehghani and Ataee-pour (2012) further stated that uncertainty in mineral commodity prices is caused by two main reasons:

- insufficient information on the factors influencing the increase or decrease in mineral commodity supply and demand; and
- the response which mining companies and their clients have towards influential speculative and political intentions.

The cost estimation aspect of mineral projects is also an important process in valuating mineral projects. The uncertainty in cost estimation arises from the fact that mining operations never know with confidence what incidents could happen at the operation and how much it would cost them. For any project, the operating and capital costs are key inputs into the valuation process. The ambiguity in these costs comes from the early stages of the project when there is insufficient engineering and economic data about the project (Dehghani and Ataee-pour, 2012). The ability that mining operations have to adjust their mine plans in the face of uncertainty is where value is derived from. In response to uncertainty, it would be beneficial for mine planners to be able to adjust their mine plans in order to be competitive and survive volatile economic conditions.

The discount rate discussed in Sub-section 2.4.1 is one of the ways used to address the risks and uncertainties in a business. It is used to illustrate to potential investors the risks and uncertainties affecting a project's expected cash flows. This is done by taking into consideration the technical, economic and political risk factors that influence the determination of the discount rate (Park and Matunhire, 2011b). Creating flexibility in a mine plan is another way of risk management in mining operations.

2.6 Flexibility in mining projects

Uncertainty is intrinsic to the mining industry and it is vital that mining projects cater for it by creating flexibility in their long-term and short-term plans (Elkingto, *et al*., 2006). According to Musingwini *et al.* (2007), there are two types of flexibility; operational (tactical) flexibility and managerial (strategic) flexibility. Operational flexibility is defined as the capability of mining operations to adjust or adapt their operations in response to operational uncertainty (Musingwini, *et al*., 2007). For example, the ability to change the production rate or the quality of material delivered to the processing plant. Managerial flexibility was defined by Musingwini *et. al.* (2007) as the capability of mining operations to achieve the essential return on investment desired by shareholders within a tolerable risk level. It deals with a company's basket of operations in order to disinvest from unprofitable operations. It takes into consideration the timing and diversification of the company's basket of operations.

The flexibility within the mine plan is where the value lies, because the greater the level of uncertainty, the more valuable flexibility becomes (Mayer and Kazakidis, 2007). Flexibility is usually easier to create at the early stages of a project during the feasibility stage as nothing has been set up yet. It is created to counter-act the low confidence level of information at this stage (Mayer and Kazakidis, 2007). It is difficult to create flexibility in older operations because infrastructure and equipment would have already been established (Mayer and Kazakidis, 2007). Mining operations with minimal or no flexibility find it difficult to adjust and survive during volatile economic conditions and also have minimal chance of profit during prospering times (Musingwini, *et al*., 2007). Kazakidis and Scoble (2003) proposed an index to measure flexibility and is shown in Equation 2.7:

$$\text{Flexibility index, F(\%)} = \frac{\text{Option Value, OV}}{\text{NPV passive}} \times 100 \qquad 2.7$$

The option value is the additional cash flow gained from exercising an option over the base case of a project known as the passive NPV, which is shown by Equation 2.8:

$$\text{Option Value} = \text{NPV}_{\text{option}} - \text{NPV}_{\text{passive}} > 0 \qquad 2.8$$

The value of managerial flexibility within a mining project can be determined using simulated techniques and ROA. To improve on the limitations of the static DCF, simulation can be done to incorporate uncertainty.

2.7 Monte Carlo simulated DCF analysis

Monte Carlo simulation, alternatively known as probabilistic simulation, is a statistical problem-solving technique adopted to understand the effect of risk and uncertainty in compound situations (Investopedia, 2018c). The technique uses different probability distributions such as normal, log-normal, uniform and triangular distributions to model the uncertainty of input variables and compute all the possible outcomes (Harrison, 2010). Different simulations are run to produce possible outcomes of the problem. This simulation technique is one of the most used methods for modelling uncertain variables and was used in this research study to model uncertain variables in the MCDCF also known as the probabilistic DCF. It has been applied in a wide range of fields such as corporate finance, engineering and science. In financial markets it is used to simulate stock prices, derivatives and mineral commodity prices by predicting possible trends from historical data that these financial instruments may follow in future (Reddy and Clinton, 2016). Decision makers, investors and valuators use it to aid them in decision making, due to the fact that it quantifies the risk of possible outcomes by giving the probability of each outcome occurring (Van Zyl, 2015). Despite having abundant information available for simulations, it is still very challenging to accurately forecast the future in any industry, especially in unpredictable industries like the mining industry. Monte Carlo simulation makes it possible to view likely outcomes under certain circumstances.

Other common stochastic models used for modelling uncertain variables such as mineral commodity prices and stock prices are the Geometric Brownian Motion

(GBM) and Mean-Reverting Process (MRP). The manner in which the value of a stock moves over time is believed to follow the Brownian motion path (Dmouj, 2006). According to Dimitrakopoulos and Abel (2007), the GBM is ideal to define the unpredictable movement of non-reverting prices such as stock prices and the price of precious metal like gold. Hence, it is one of the most widely used models in financial modelling (Dmouj, 2006).

The Geometric Brownian motion (GBM) was first introduced by a plant biologist Robert Brown who studied the movement of floating pollen particles in a transportation medium of water under a microscope in the 1900s. He noted that the particles moved in a jumpy manner and established that they were ‘alive’ (Reddy and Clinton, 2016). Albert Einstein further studied the water molecules containing pollen under certain conditions and noticed that they moved randomly. This is one of the main assumptions of stock price action on financial markets (Reddy and Clinton, 2016). The idea is that stock price movement follows an unpredictable path known as the random walk which makes it difficult to predict a path that outperforms the market. The GBM integrates the theory of random walk in price movement model by including a component that caters for uncertainty and a component that deals with the trend that the stock price action follows over time (Reddy and Clinton, 2016). According to Brewer *et al.*, (2012), the uncertainty component in the GBM model integrates the time and volatility aspect of the stock price action. According to Dixit and Pindyck (1994), the GBM can be represented by Equation 2.9:

$$\frac{dS}{S} = \alpha dt + \sigma dz \qquad 2.9$$

Where,

S is the metal commodity price; α is the expected trend; σ is the standard deviation; dz is an increment in a standard Weiner process; and dt is an increment of time.

The GBM is based on the following assumptions when projecting stock prices (Dmouj, 2006; Reddy and Clinton, 2016):

- Stock prices are continuous in intervals and value;

- Current historic stock prices are applicable for forecasting imminent stock prices. This progression of prices is known as the Markov process;
- The relative yield of a stock follows a log-normal distribution; and
- The compounding action of returns of stocks follows a normal distribution.

The basis on which these assumptions were made is to incorporate the rules and regulations that govern the financial market into the model (Dmouj, 2006). A MCDCF analysis is very similar to a static DCF in that it projects annual cash flows and applies an appropriate discount rate to discount cash flows. The discounted cash flows are summed to get the project's NPV. The only difference is that a MCDCF caters for uncertainty in its model by fitting probability distributions to uncertain input variables. It is used to simulate uncertain variables in the cash flow such as mineral commodity prices, grade, operating costs and metallurgical recoveries. MCDCF can be used to calculate the flexibility within an operation. Mayer and Kazakidis (2007) calculated the value of flexibility within an underground mine production system by finding the difference between the NPV of a MCDCF and a static DCF. Although the MCDCF technique goes further than the static DCF by incorporating uncertainty and quantifying the risk associated with the projects' cash flow, it has limitations. Its major limitation is that it does not incorporate the flexibility that management has to adjust the project in the face of uncertainty over its life. To incorporate uncertainty and managerial flexibility Real Options has been recognised as a valuation technique that takes into consideration uncertainty as well as managerial flexibility and can be applied to valuing mineral projects (Haque, *et al.*, 2016).

2.8 Real Options analysis

Mun (2006) stated that ROA is a systematic methodology of financial options theory that uses financial economics, statistics, decision sciences and management sciences to aid in valuing real physical assets under uncertainty. A business' ability to adjust its operation due to uncertainty is the source of value that an option holds (Eduardo, 2013). An option is a financial contract between parties giving the holder the right but not the obligation to buy or sell a specified quantity of an underlying asset at a fixed price. It is also known as the strike price or exercise price

(Damodaran, 2000). The contract is subject to specific conditions and has a specified time limit on which the option will expire called the expiration date. The purchaser of this right has the choice of not exercising the right and thus allowing it to lapse. There are two forms of options; a 'call' option and a 'put' option (Palm, *et al.*, 1986). A 'call' is also known as an option to buy, while a 'put' is referred to as a sell option of an underlying asset. Call options can only be utilised or exercised when the price of the underlying asset is greater than the strike price, inversely put options can only be utilised when the price of the underlying asset is less than the strike price. Furthermore, these gains from holding and exercising an option is known as being "in the money" (Damodaran, 2000). However, when determining if the option is in money the cost of the contract is regarded as a sunk cost and is only useful when determining if the purchaser of the contract has really gained from exercising the option in terms of the money invested (Zdravlje, 2011).

Black and Scholes (1973) categorised the call and put options according to a specific timeframe when an option could be exercised. These are the American option and the European option. An American option is an option that can be exercised from the day the option was purchased until the day it expires. A European option is an option that can be exercised on a single agreed date in the future (Black and Scholes, 1973). Bailey, *et al.*, (2004) noted that there are two vital traits that options possess. Firstly, option holders are shielded from loses and have the chance to score high gains. Secondly, options have greater value when there is high uncertainty and risk involved.

To have a better understanding of ROA there is a need to differentiate between financial options and real options. Despite their close similarities, ROA uses the rationale of financial options to valuate real assets by exercising different options at distinct times in a projects life. Financial options are tools used to aid investors in conjecture and hedging on stocks, bonds, exchange rate and commodities (Kvalevag, 2009). According to Zdravlje (2011), the theory of financial options can be applied to value mineral projects, as they both have price uncertainty that is linked to the market. ROA is based on two core assumptions that make the mathematics in financial options techniques applicable to ROA, these assumptions are (Arnold and Shockly, 2010):

- The financial markets have no arbitrage opportunities; and
- Investments must have replicating portfolios.

The first assumption assumes that financial markets have no price differences to take advantage of, which in essences means that arbitrage offers riskless profit. The second assumption means that a series of future cash flows from a project can be mimicked by a given number of shares of the underlying security (Arnold and Shockly, 2010). The foremost justification for conducting ROA in projects is to value the uncertainty and managerial flexibility that is ignored by the traditional DCF analysis (Zdravlje, 2011). ROA allows decision makers of mining projects the flexibility to select and implement different options over the life of the mine as different situations arise due to uncertainties (Haque, *et al.*, 2014). By selecting a suitable option to adjust to different situations such as declining mineral commodity prices, mining operations can create optimum value (Bernnan and Schwartz, 1985). There are different types of options available in mineral project investment.

2.8.1 Types of options available for mining projects

There are four main types of options available in mineral projects, which are the option to expand, contract, abandon and defer or delay the start of a project. The option to expand arises when mineral commodity prices increase remarkably for a prolonged period and when management has the ability to increase its production output which then helps them to increase project value. This option is similar to that of a call option, as expanding production may require an additional capital investment. This increases the overall capital investment but decreases unit operating costs because of economies of scale. Economies of scale is defined as the decrease in unit operation costs as a result of increased production. This is because the fixed costs are spread over a larger output. Mining operations may have mine plans according to the prevailing mineral commodity prices, but if the mineral commodity prices rise significantly, management may decide to create an expansion option to reap the benefits of the price increase (Haque, *et al.*, 2014).

On the other hand, the option to contract presents itself when mineral commodity prices decrease considerably, giving management the option to scale down operations by decreasing mining rates. When exercising this option, it entails decreasing the production scale of the operation, and thus decreasing total operating costs, assuming that all the costs are directly related to the level of production. This will result in lower returns and lower project value, and thus increasing the life of mine depending on the cut-off grade (Haque, *et al*., 2014).

The option to abandon arises when low mineral commodity prices continue to prevail for a prolonged period resulting in continued negative net cash flows. This then forces management to exercise the abandonment option and salvage its physical assets. Management also have an alternative of temporarily suspending operations and placing the operation under care and maintenance, until a time when mineral commodity prices rise (Haque, *et al*., 2014). According to Copeland and Antikarov (2001), the abandonment option is used during mergers and acquisitions which is similar to that of a put option. The option can be exercised only if the value of the underlying asset falls below the exercise price. This signals that there is greater value in disinvesting than remaining invested in the project (Mokenela, 2006).

The option to delay arises when mineral commodity prices decrease and other economic factors are not favourable and if management has the flexibility to defer the start of a new project. The option to delay a project can be exercised until economic circumstances improves, and only when the conditions are optimal to invest (Mokenela, 2006). The duration of the delay differs according to the circumstances on ground. In the minerals industry, the option to delay a project is dependent on the length of a company's mining lease (Mokenela, 2006). The start of a project can be delayed before the expiration of a mining lease, by adhering to the applicable government policies. This option is similar to a call option, as both options give the holder the right but not the obligation to start the project, and can only be exercised when the option value is greater than the exercise price (investment cost) (Bowe and Lee, 2004). According to Zdravlje (2011), the option to defer the start of a project is unique from other options in that it does not require a new cash flow model that looks at different scenarios. Therefore it can be implemented directly to the traditional DCF inputs. Ultimately this permits the straightforward comparison of results from the conventional static DCF and ROA techniques.

The management of the mining project being considered in this research study only has the option or flexibility to delay the start of the project before the mining lease expires in December 2032. In order for the company to exercise this option, capital investment must be acquired on or before the day the mining lease expires. The option to proceed with the development of the project will only be exercised if the expected NPV of the project exceeds the initial capital cost to exercise the option which includes development costs, equipment and infrastructure costs (Kelly, 1998). Brosch (2001) mentioned that the options to expand, contract and abandon are categorised as operating options because they can only be applied to assets that are already established. It is for this reason that the research study only focuses on valuating the option to delay the start of the underground project.

There are four main ROA techniques; the Black-Scholes model, Binomial-Lattice model, Finite Difference technique and Stochastic Monte Carlo technique (Gilbert, 2004; Ampofo, 2017). The most commonly used techniques are the Black-Scholes, the Binomial-Lattice models (Gilbert, 2004). The Finite Difference technique is not commonly used because it becomes more complex with the number of uncertainties in real projects applications, and with compound problems like those in the minerals industry, the techniques become almost impossible and unfeasible to use (Dimitrakopoulos and Abdel, 2007). Additionally, the technique is incapable of incorporating the uncertainty in mineral grade and operating costs over the life of a mine (Dimitrakopoulos and Abdel, 2007). The Stochastic Monte Carlo technique only values European options and in the mining industry projects are complex and are mostly American options (Gilbert, 2004). It is for these reasons that the Finite Difference and Stochastic Monte Carlo techniques were not used in this research study.

2.8.2 Black-Scholes model

The Black-Scholes model is the most popular option pricing model developed by Black and Scholes (1973). The model calculates the value of a call or a put option, which makes it possible for the valuator to divide an option’s lifespan into minor boundless periods. The Black-Scholes model’s is expressed by Equation 2.10 (Black and Scholes, 1973).

$$\text{Value of Call Option} = [N(d_1) \times S] - [N(d_2) \times PV(X)] \qquad 2.10$$

The formula is divided into two components, the first component. $N(d_1)\times S$ is the estimated value from owning the stock, where $N(d_1)$ is the future value of the stock if, and only if, the stock price is above the strike price on the expiry date; and the second component $N(d_2)\times PV(X)$ is the estimated value for exercising the option on the expiry date which is continuously discounted, where $N(d_2)$ is the probability that the stock price will be equal to or greater than the strike price when the option expires. S is the value of the underlying asset which is the NPV of asset excluding the initial capital investment.

Where;

$$d_1 = \frac{\log \frac{P}{PV(X)}}{\sigma \times \sqrt{t}} + \frac{\sigma \times \sqrt{t}}{2} \qquad 2.11$$

$$d_2 = d_1 - (\sigma \times \sqrt{t}) \qquad 2.12$$

$N(d_i)$ is the cumulative standard normal probability distribution function, X is the exercise or strike price of an option (initial capital investment), PV(X) is X discounted by the risk-free interest rate to get the present value of the exercise price, t is the time until the option exercise date, P is the current stock price, σ is the standard deviation of rate of return on stock price, d_1 and d_2 are the standard normal stochastic variables.

The model makes the following assumptions (Black and Scholes, 1973):

- All the options are European and may only be exercised on the expiry date;
- The risk-free rate and volatility of the underlying asset are known and constant;
- The price and returns of the underlying asset follow a lognormal distribution;
- No dividends are paid out during the life of the option; and
- When purchasing the option no transaction cost is charged.

According to Kvalevag (2009), due to the flexibility of the Black-Scholes model, it can be applied to value complex options in mining projects, bonds and foreign exchange. However, it assumes that all options are European, while realistically most options are American in nature (Gilbert, 2004). Furthermore, when calculating the value of an underlying asset for long-term options, unexpected events may occur resulting in sharp upward or downward turns in value which are difficult to incorporate in the Black-Scholes model (Anderloni, 2011). The model also assumes that no dividend payments are made throughout the life of the option, while in reality dividend payments are made as a negative cash outflow that affects the value of the underlying asset (Anderloni, 2011). Therefore, the Black-Scholes model is not suitable for valuing the option to delay mining projects and was not considered for this research study. To address the shortcomings of the Black-Scholes model, the Binomial-Lattice model was developed.

2.8.3 Binomial-Lattice model

The Binomial-Lattice model was developed by Cox *et al.*, (1979) and was used to value stocks in both the European and American options (Gilbert, 2004). It is based on a basic concept of constructing a probability tree in which the value of an option is computed from an expected discrete state of prices throughout the life of the option. The model assumes that the interest rate is constant and the stock price has a multiplicative binomial relationship over each period resulting in two likely values: an upside (*u*) outcome with probability *p* or a downside (*d*) outcome with probability *1-p*. The Binomial-Lattice model is expressed by Equations 2.13-2.17 (Smith, *et al*., 2017):

$$C_{j,i} = \frac{p \times C_{j+1,i} + (1 - p) \times C_{j+1,i+1}}{e^{r_f \times dt}} \qquad 2.13$$

$$dt = \frac{T}{N} \qquad 2.14$$

$$p = \frac{e^{(r_f \times dt)} - d}{u - d} \qquad 2.15$$

$$u = e^{\sigma \times \sqrt{dt}} \qquad 2.16$$

$$d = e^{-(\sigma \times \sqrt{dt})} = \frac{1}{u} \qquad 2.17$$

Where;

C is the option value; r_f is the risk-free rate; σ is the volatility of the stock price or the underlying asset; dt is the time incremental step when major decisions are made in a company; T is the life of the option of the project; N is the number of periods selected; p is the risk-neutral probability; j is time index; i is state at time index; u is >1 and d is < 1.

An example of a multiplicative Binomial-Lattice of an underlying asset (*S)* with its generic calculated call option value (*C),* is illustrated in Figures 2.2 and 2.3, respectively.

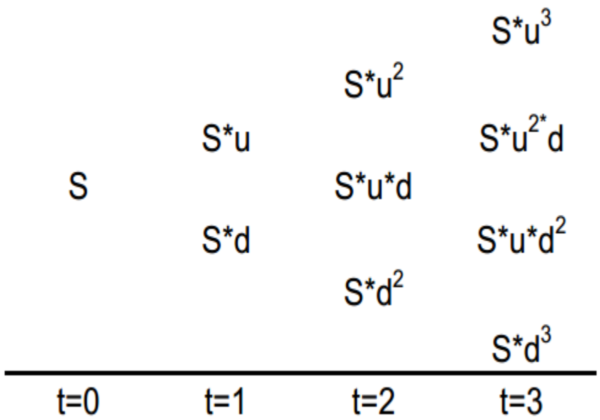


Figure 2.2: Underlying asset value (S) of the Binomial-Lattice

(Source: Kvalevag, 2009)

In Figure 2.2, the lattice represents the value of the underlying asset *S*. The initial value of the asset is entered in the far left node of the lattice and is denoted by *S*.

This value is the NPV of the asset excluding the initial capital investment. The value of the underlying asset can either increase by a factor *u* or decrease by a factor *d*. These factors that influence the value of the underlying asset are influenced by the price volatility of the asset and the length of time *t*. The nodes of the second time step are expanded from the nodes of the first time step. The value of each node increases or decreases by a factor of *u* or *d*, respectively, resulting in the expansion of the lattice until the life of the underlying asset. Each time step between the nodes represents one simulation in the lattice. The exponents in the lattice represent the number of increases or decreases in the assets' value. The increase and decrease by the factors *u* and *d* respectively are attributed to the random walk theory (Smith, *et al., 2017)*. Each node in the lattice signifies a potential decision point at the end of the time increment and the asset values in the final time represent a probability distribution of possible asset value outcomes which follows a log-normal distribution at the time of expiration (Bailey, *et al*., 2004).

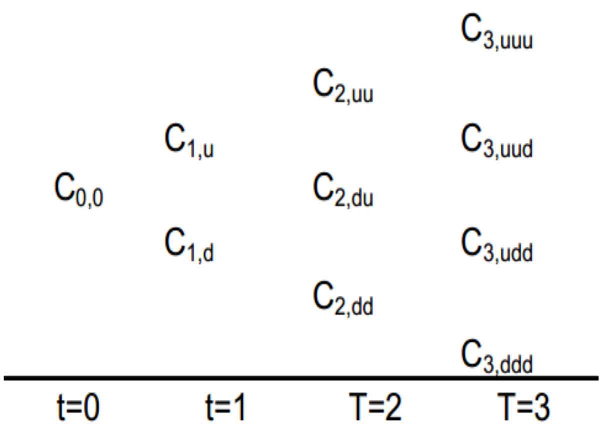


Figure 2.3: Option value (C) of the Binomial-Lattice

(Source: Kvalevag, 2009)

The lattice on the right in Figure 2.3 represents the value of a call option when exercised at different time steps denoted by *C*. The construction of this lattice starts from the right and ends at the left and is calculated using Equation 2.13. First, an option value for the last time steps in the lattice is calculated by subtracting the option cost (initial investment) known as the exercise price *X* from each of the values of the underlying asset *S* obtained in the asset value lattice. This calculation can be summarised by Equation 2.18 (Bailey, *et al*., 2004):

$$C_{j,i} = \max\{S_{j,i} - X; 0\} \quad 2.18$$

Where;

$S_{j,i}$ *is the value of the underlying asset at different time steps and X is the exercise price of the option.*

The option value of the preceding time step is equivalent to the weighted average value of the future option values based on the risk-neutral probability *p* of the asset values in the occurring nodes multiplied by the risk-free discount rate for one time step as shown in Equation 2.13. Option values are computed from the right side of the lattice to the left side of the lattice, resulting in the initial asset value when the option is exercised (Smith, *et al.*, 2017). The initial asset value in the lattice is then compared to the criteria of value from the traditional DCF analysis. The increase in asset value in the ROA is not the real option value, but rather the discrepancy in asset value generated by the ROA value and the NPV of the DCF analysis (Zdravlje, 2011). The probability of each node occurring is based on the size of the lattice and number of paths leading to the final time increment. The number of paths resulting in each outcome in the lattice results in its formation and is simplified by Pascal's triangle (Zdravlje, 2011). According to Weisstein (2003) "*Pascal's triangle is a number triangle with numbers arranged in staggered rows"* and is represented by Equation 2.19:

$$a_{n\,r} \equiv \frac{n!}{r!\,(n-r)!} \equiv \binom{n}{r} \quad 2.19$$

Where,

$\binom{n}{r}$ *is a binomial coefficient; n and r are integers.*

An example of the Pascal's triangle is shown in Figure 2.4 starting with n=0.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Figure 2.4: Pascal's Triangle

(Source: Weisstein, 2003)

Equation 2.19 shows that each succeeding row is calculated by summing up the two entries diagonally above (Weisstein, 2003). The probability of each call option value in the lattice is influenced by the number of paths that leads to its outcome. The resultant distribution of the lattice is identical to that of Pascal's triangle (see Appendix 7.1), which is a normal distribution (Maistrov, 1974). The probability of a call option value of different time states occurring in the lattice is calculated by Equation 2.20 (Maistrov, 1974):

$$p_{c_{ji}} = \frac{X}{2^n} \qquad 2.20$$

Where,

X is the number that appears in a node in Pascal's triangle; and n in number of branches leading to a node in a specific column.

The different options in the Binomial-Lattice model each has slightly modified equations from the generic option value in Equation 2.18. However, the option value to defer a project has the same generic equation. The option value to contract an operation by a factor *x* is calculated by Equation 2.21 (Bjarnadottir, 2013):

$$C = \max\{X - xS; 0\} \qquad 2.21$$

The option value to expand an operation by a factor *x* is calculated by Equation 2.22 (Bjarnadottir, 2013):

$$C = \max\{xS - X - S; 0\} \qquad 2.22$$

The option value to abandon an operation with salvage value s is calculated by Equation 2.23 (Bjarnadottir, 2013):

$$C = \max\{S - s; 0\} \qquad 2.23$$

The Binomial-Lattice model was used in this research study to determine the value of the option to delay the start of the underground project. This is because the option to delay the start of a project can be exercised at any time before the expiration date of the mining licence which makes it an American option. The Black-Scholes model only values European options, while in actual sense the options present in mining projects can be American or European.

2.8.4 Advantages of ROA

According to Kvalevag (2009), the main advantage of ROA in valuation is its ability to measure the value of flexibility within a business. Unlike the DCF valuation technique, ROA gives a business the flexibility to adjust at different times in the project's life as better knowledge of the business environment is gained. As stated in Sub-section 2.8.1 there is value in options and it is vital to integrate it into investment decision making. The other advantage is that discounting of cash flows using ROA is done at a risk-free rate, therefore there is no need to go through the trouble of determining a risk adjusted discount rate as in the case of DCF valuation. Using a risk-free rate, the potential business shareholders risk preference are not taken into consideration (Zettl, 2002). Also, the DCF valuation technique assesses investments on a now basis by either accepting or rejecting an investment. ROA does it in a more realistic and dynamic way, which helps decision makers to comprehend the significance and consequences of new information and timing of investments especially in the minerals industry (Silitonga, 2015).

2.8.5 Limitations of ROA

When it comes to valuing mining operations, the level of mathematical complexity involved in ROA is challenging, because ROA models can be custom fitted for different operations with different risks and uncertainties (Ampofo, 2017). Additionally, ROA uses a risk-free discount rate to discount cash flows, whereas the static DCF and MCDCF use risk-adjust discount rate which incorporates risks in the mining industry. This is the main reason why ROA is not commonly applied in the minerals industry as a valuation technique. Kvalevag (2009) and Ampofo (2017) both mentioned that mining operations are too complex to model, valuators make simplified assumptions to be able to model an adequate amount of payoffs to make an appropriate valuation. This usually causes a model to deviate from reality. Bowman and Moskowitz (2001) stated that the value of an option obtained are based on the inputs discussed in Sub-sections 2.8.2 and 2.8.3 which could be incorrectly estimated resulting in an incorrect option value.

2.9 Comparing the application of DCF and ROA in mineral project valuation

The first application of ROA in the minerals industry was by Bernnan and Schwartz (1985) who applied the Black-Scholes model to valuate a hypothetical gold and copper mine. They argued that the disadvantage of the DCF valuation technique is that it could not take into consideration the volatility of mineral commodity prices and the value embedded in flexibility that management had to these prices (Bernnan and Schwartz, 1985), hence the need to use ROA. The ROA model applies the standard no-arbitrage methodology of option pricing. ROA and DCF models were further extended to valuate a lease of an offshore petroleum company by Paddock, *et al.*, (1988). The evaluation contemplated the option to defer development into the future in order to shield the company from bankruptcy.

Mardones (1993) used financial options and the DCF model to valuate a copper mine with particular application of contingent claim analysis (CCA) and derivative asset valuation (DAV). The evaluation measured managerial flexibility by modifying the cut-off grade in response to copper prices and in doing so adding value (Mardones, 1993). Laughton (1996) compared option pricing and the DCF model on a copper mine with a 20 year LoM to analyse its financial structure. It was concluded that the DCF tends to undervalue long life mines as compared to option pricing models (Loughton, 1996). Furthermore, Samis and Poulin (1996, 1998) used the DCF model and Binomial-Lattice model on two different case studies to valuate a gold and copper mine, respectively. They concluded that despite the DCF model being unable to account for some uncertainties, it is still extensively applied to valuate mining projects. They also concluded that because ROA models account for flexibility they are more appropriate to valuate mining projects as compared to the DCF model (Samis and Poulin, 1996, 1998). Samis and Graham (2003) used a simulated model of the DCF and ROA model on a gold mine in order to value and assess the flexibility of the mine design. They concluded that simulated DCF and ROA models add operating and design flexibility that result in greater anticipated cash flow catering for risk and uncertainty. Smith *et. al.* (2017) also came to the same conclusion when they used the Binomial- Lattice to select and plan capital mining projects.

According to Shafiee, *et al.*, (2009) and Dimitrakopoulos and Abedl (2007), there is a lot of research that applied modified versions of the DCF and Binomial-Lattice models to mining projects between the years 1985 to 2008. Haque, *et al.*, (2014, 2016) applied ROA to valuate a hypothetical gold mine taking into consideration commodity price uncertainty and hedging strategies. They modelled commodity price uncertainty using the mean reversion stochastic model. They used partial differential equations to model ROA and solved the equations using the Matlab software to determine the value of the hypothetical gold mine. The majority of the research conducted on applying ROA to valuate mining projects focused on managerial flexibility and its value in an operation.

2.10 Chapter summary

This chapter revealed that there are a lot of valuation techniques, which are classified into three valuation approaches; the market, cost and income approach. However, not all approaches are ideal for determining the economic value of mineral projects under risk and uncertainty. The market approach does not incorporate market risks and uncertainties in its valuation techniques. The cost approach neglects key variables such as commodity prices and exchange rates essential to determining the economic value of mineral projects. There are three valuation techniques that are mainly used under the income approach to determine the economic value of mineral projects, which are the DCF (static DCF and MCDCF) and ROA techniques.

The DCF technique uses three criteria to measure the economic viability of a project; the NPV, IRR and PP of a mineral project. However, the static DCF does not take into consideration uncertainty associated with economic and input parameters used to calculate NPV. Simulation techniques like the MCDCF have been developed to incorporate uncertainty and quantify the risk associated with NPV. However, the MCDCF does not incorporate the flexibility that management has to adjust the project in the face of uncertainty over its life. To incorporate uncertainty and managerial flexibility ROA is recognised as a valuation technique that takes into consideration uncertainty as well as managerial flexibility and can be applied to valuing mineral projects. There are two commonly used ROA valuation models used to value flexibility; the Black-Scholes model and the Binomial-Lattice model. Additionally, there are four types of options available in mineral project investment; the option to expand, contract, abandon and delay mineral projects. As justified in this chapter an option to delay a gold mining project was valued in this research study using the Binomial-Lattice model as described in the next chapter.

3 RESEARCH METHODOLOGY

3.1 Chapter overview

This chapter describes the methods used to conduct the research study. It also states the economic and technical input data collected to conduct DCF, MCDCF and ROA to evaluate the case study project. The chapter also describes how the results were analysed.

3.2 Input data and its sources

The economic and technical input data used to conduct the static DCF, MCDCF and ROA was obtained from the companies' published annual reports which are accessible on the public domain. Furthermore, most of the technical and economic data were obtained from the projects' confidential technical report. The historical gold prices were obtained from the World Gold Council (2019). The following is the economic data collected to carry out the static DCF analysis:

- Gold price (US$ / ounce);
- Mining and processing operating costs (US$ / tonne mined or processed);
- General and administrative costs (US$ / tonne processed);
- Capital and operational expenditure (US$); and
- Discount rate, tax rate and royalties (%).

The following technical data was collected as inputs to the static DCF:

- Production rate (tonnes/annum) over the life of the project;
- Grade (g/t); and
- Metallurgical recoveries (%).

In addition to the data collected to carry out the static DCF analysis the following data was collected to conduct a ROA using the Binomial-Lattice model:

- Current asset value (US$) which is the economic value (discounted net cash flow) of the project exclusive of the initial capital invested obtained from the static DCF analysis;
- Price volatility of gold (%) which was calculated from the historical gold prices;

- The risk-free rate of the project (%) which was provided by the company;
- The amount invested to exercise the option which is known as the initial capital expenditure (US$) which was provided in a technical report; and
- Time to the expiration of the option and the option available (LoM-years) obtained from the company's lease agreement and the life of the project.

3.3 Data analysis methods

This research study determined the economic value of a gold mining project that considers the following two scenarios as contained in the company's technical report:

- Scenario 1: The development of the preproduction phase will be conducted by a contractor, thereafter the company will proceed with production; alternatively
- Scenario 2: The company will develop the mine and operate throughout.

The detailed economic and technical differences between the two scenarios are explained in Chapter 4. Although a technical report was provided by the company, no cash flows for the two scenarios were provided in the report. Therefore, the two scenarios were first valued using the economic and technical data mentioned in Section 3.2 to generate a static DCF. The static DCF analysis was used as the basis for the comparison with the MCDCF and ROA results. The analysis further incorporated uncertainty in the valuation process of the static DCF in @RISK software to generate a MCDCF analysis that quantified the risk associated with the project. Finally, the analysis determined the value of the two scenarios with an option to delay the start of the project using the Binomial-Lattice ROA model. The results from the three methods were then compared and analysed as demonstrated in the following Sub-sections.

3.3.1 Static DCF analysis

As discussed in Chapter 2, the static DCF is used to measure the economic value of a project. The key decision criteria of the DCF technique are; the NPV, IRR and PP of the project. The two scenarios of the project were valued and analysed using the static DCF. A simplified hypothetical example of a summarised static DCF analysis of a gold mining project with a life of 5 years is shown in Table 3.1 to demonstrate how the results were analysed in Chapter 4.

Table 3.1: Summarised static DCF analysis results of the hypothetical gold project

Description	Units	1	2	3	4	5	6
Year		2019	2020	2021	2022	2023	2024
Gold recovered	Ounces	-	12 000	13 550	14 350	11 250	9 300
Gold price	US$/oz	-	1 250	1 250	1 250	1 250	1 250
Revenue	**US$**	**-**	**15 000 000**	**16 937 500**	**17 937 500**	**14 062 500**	**11 625 000**
Total operating costs	US$	-	-900 000	-1 100 000	-1 180 000	-899 500	-880 100
Operating profit	US$	-	14 100 000	15 837 500	16 757 500	13 163 000	10 744 900
Total capital cost	**US$**	**-20 130 000**	**-17 790 000**	**-**	**-**	**-**	**-**
Unredeemed capex	US$	-20 130 000	-23 820 000	-7 982 500	-	-	-
Royalties @4%	US$	-	-600 000	-677 500	-717 500	-562 500	-465 000
Net operating profit	**US$**	**-**	**13 500 000**	**15 160 000**	**16 040 000**	**12 600 500**	**10 279 900**
Taxable income	US$	-	-	-	8 775 000	13 163 000	10 744 900
Corporate Tax @20%	US$	-	-	-	-3 071 250	-4 607 050	-3 760 715
Net cash flow	**US$**	**-20 130 000**	**-4 290 000**	**15 160 000**	**12 968 750**	**7 993 450**	**6 519 185**
Discounted Net cash flow @5%	**US$**	**-20 130 000**	**-4 085 714**	**13 750 567**	**11 202 894**	**6 576 231**	**5 107 952**
NPV (US$ million)	**12.42**						
IRR	**21%**						

The static DCF analysis of the hypothetical gold mine in Table 3.1 resulted in an NPV of US$12.42 million at a 5% discount rate. Since the NPV for this example is positive, then based on the NPV decision criteria the project is viable. Figure 3.1 shows the graphical plot of the discounted net cash flows of the hypothetical gold mine generated over the LoM.

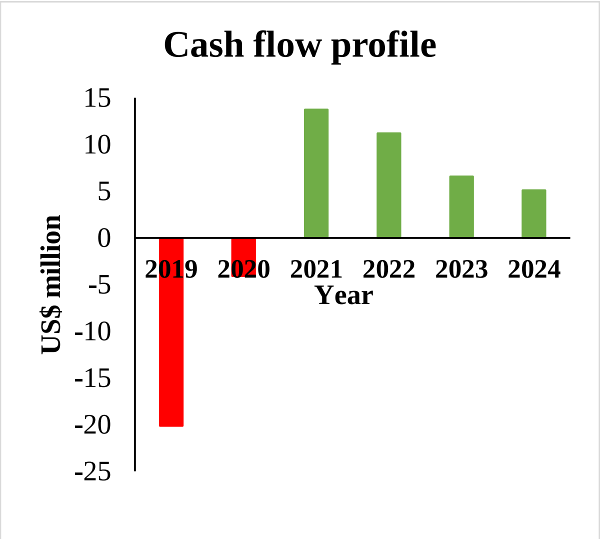


Figure 3.1: Life of mine cash flow profile for the hypothetical gold project

Figure 3.1 shows that after 2021 the project's cash flow started to decline, and the following are the possible reasons for the declining trend:

- Total operating costs decreased as a result of declining production;
- A lower gold grade was mined towards the end of the project's life; and
- Other expenses such as royalties and taxes also decrease because of lower production.

A graphical plot to determine the IRR of the gold mine example is illustrated in Figure 3.2.

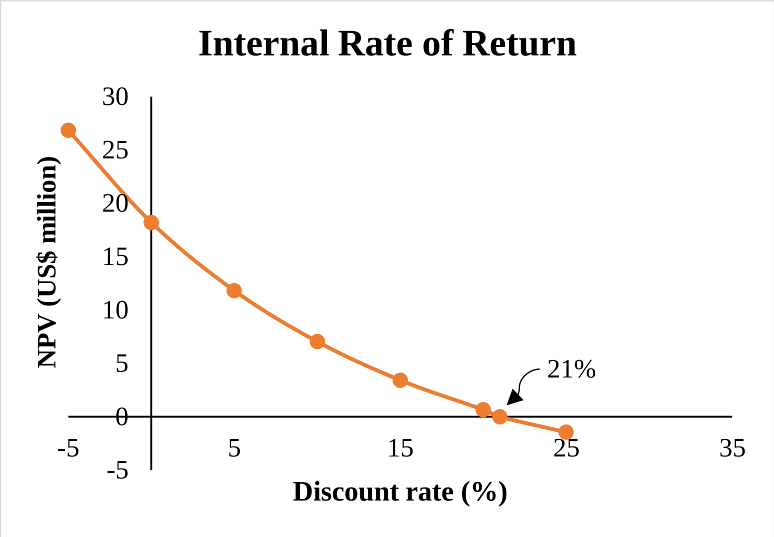


Figure 3.2: Internal Rate of Return curve for the hypothetical gold project

As mentioned in Chapter 2 that IRR is the discount rate where NPV is zero, Figure 3.2 shows that the project has an IRR of 21%. Since the IRR of the project is greater than the 5% discount rate of the project, then based on the IRR decision criteria, the project is viable. Lastly, the PP for the above example was calculated using Equation 2.6 and is shown as follows:

$$PP \approx t * + \frac{NPV_{t*}}{NPV_{t*} - NPV_{t*+1}}$$

NPV_{t*} was calculated by summing each years' discounted net cash flow from year one until the year before the NPV becomes positive or before the initial capital investment has been fully recovered. NPV_{t*+1} was calculated by summing each years' discounted net cash flow from year one until the year the NPV becomes positive or after the initial capital investment has been fully recovered. Table 3.2 demonstrates the calculation of NPV_{t*} and NPV_{t*+1}.

Table 3.2: NPV calculations for PP of the hypothetical gold project

Description	Units						
Number		**1**	**2**	**3**	**4**	**5**	**6**
Year		**2019**	**2020**	**2021**	**2022**	**2023**	**2024**
Discounted Net cash flow @5%	US$ million	-20.13	-4.09	13.75	11.20	6.58	5.11
NPV	**US$ million**	**-20.13**	**-24.22**	**-10.47**	**0.74**	**7.31**	**12.42**

From the cash flow in Table 3.1 and the NPV calculations for the PP in Table 3.2, production starts in 2020 and that is the year in which the PP is counted as year one. The resultant calculation of PP of the project is shown as follows:

$$PP = 2 + \frac{-10.47}{-10.47 - 0.74}$$

$$\therefore PP = 2.9 \text{ years}$$

The project resulted in a payback period of 2.9 years. This means it will take the project 2.9 years to fully recover all the capital invested in the project. The viability of the project based on the PP decision criteria is subjective because it is dependent on how fast the investors want their initial investment back, but a shorter payback period is desired. For a mining project with a 5 year LoM, a payback period of 2.9 years would likely make this project viable.

3.3.2 Monte Carlo simulated DCF analysis

In order to quantify the risk associated with the NPV and IRR, a MCDCF was generated using Palisade's @RISK software. The MCDCF is similar to the static DCF. The only difference is that a MCDCF incorporates uncertainty into its model by fitting probability distributions into the input parameters. Uncertain variables such as gold price, grade, production rate and unit operating costs that influence the NPV of a project were modelled for uncertainty in the research study. However, metallurgical gold recoveries were not modelled for uncertainty. As stated in Chapter 1, there is a currently operating surface mine that has a hydrometallurgical processing plant fitted with a carbon-in-pulp circuit resulting in a high metallurgical recovery (Laxen, *et al.*, 1979). This processing plant will also be used to process ore from the underground mine and due to its high efficiency, the metallurgical gold recovery was assumed to be constant throughout the life of the project.

The GBM and MRP models for forecasting the gold price were not used because the other input parameters were simulated using MCDCF. Additionally, to have a uniform simulation of uncertain variables, gold price was also simulated using Monte Carlo simulation in @RISK software. Furthermore, if the gold price was forecasted using any of the two models it will result in a single value of gold price which will be used as an input in the MCDCF, which will make the gold price static in the MCDCF simulation. Historical data of nominal gold price was used to fit the appropriate probability distribution of the gold price, which resulted in a log-normal distribution. The probability distribution of the gold grade was determined from the historical grades of the current surface operation. A normal distribution was assumed for unit operating costs and production rate. This is because of the lack of historical data for these two parameters. In a similar study conducted by Giovanni, *et al.* (2017) where technical parameters lacked historical data, they assumed the normal distributions for those parameters.

The number of iterations that @RISK software is capable of running in one simulation is nearly unlimited. A greater number of iterations in a simulation is ideal for more accurate results. However, as the number of iterations increase, the distribution outputs in the software converge making the difference in output insignificant and creating a smoother normal distribution curve (Palisade Corporation, 2016). Simulations were run to model and summarise the behaviour of NPV and IRR for the two scenarios of the project. The resultant outcome of the @RISK software is a probability distribution that quantifies the risk associated with each scenario in the project. Figure 3.3 shows the possible range of NPV for the hypothetical gold mine project.

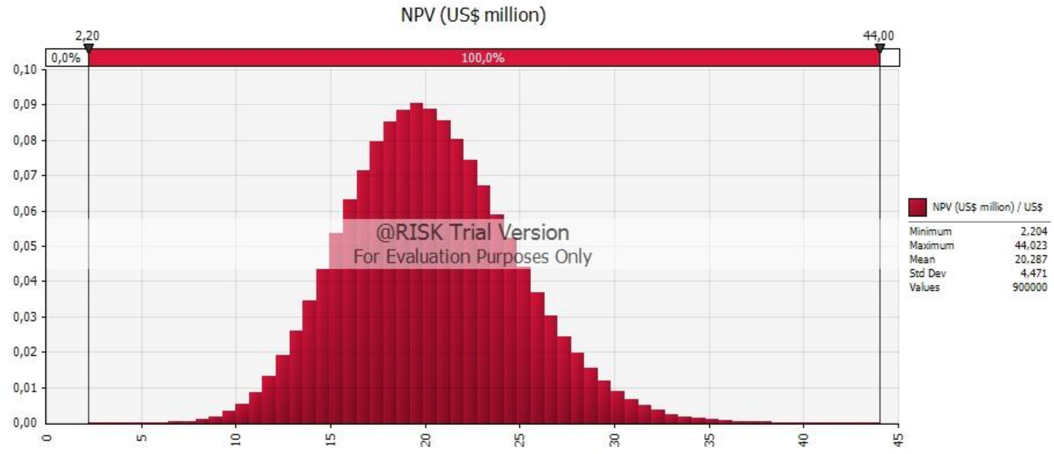


Figure 3.3: Possible range of NPV

Unlike the static DCF results in Table 3.1, Figure 3.3 shows that the NPV for the hypothetical project is likely to range from a minimum of US$2.2 million to a maximum of US$44 million, resulting in a range of U$41.80 million. The wider the range in NPV the greater the risk in NPV and vice versa. Since the minimum value is positive, the project has a probability of 1 to be viable. Knowing the possible range of NPVs, assist investors to make informed investment decisions depending on their risk appetite. For informed decision making it is also important to quantify the risk of the project. Figure 3.4 shows the probability of the project having the NPV obtained from the static DCF.

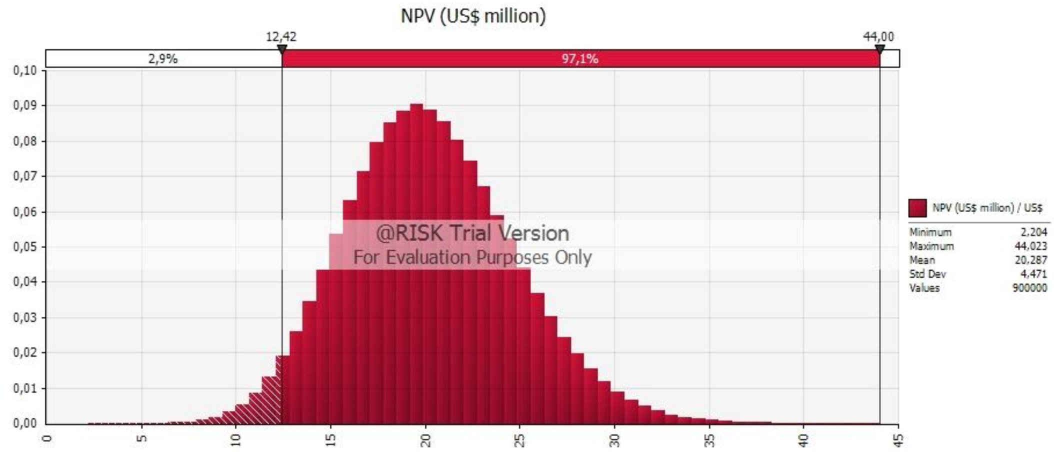


Figure 3.4: Probability distribution of an NPV less and greater than US$12.42 million

Figure 3.4 shows that the project has a probability of 0.97 of having an NPV of US$12.42 million and a probability of 0.029 of them being less than the NPV obtained from the static DCF. Despite the project still being viable based on the DCFs results, the quantified risk of 0.029 can have an influence on the investment decision. The quantification of risk helps investors realise the level of risk associated with the project. This ultimately enables informed investment decision making depending on one’s risk appetite. Figure 3.5 shows the possible range of the IRR for the hypothetical gold mine.

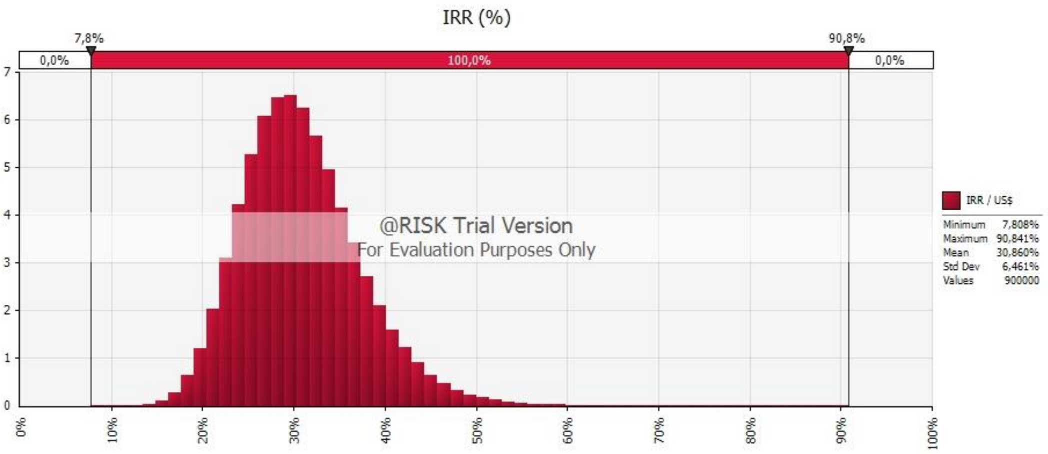


Figure 3.5: Probability distribution of an IRR

The range of possible IRR values ranges from a minimum of 7.8% to a maximum of 90.8%, resulting in a range of 83%. The range indicates how risky the project is, because the wider the range in IRR the greater the risk in IRR and vice versa. Since the minimum IRR is greater than 5% (discount rate), based on the IRR results, the project has a probability of one to be viable. Figure 3.6 shows the probability of the project has an IRR of less than 21% (the one obtained from the static DCF analysis).

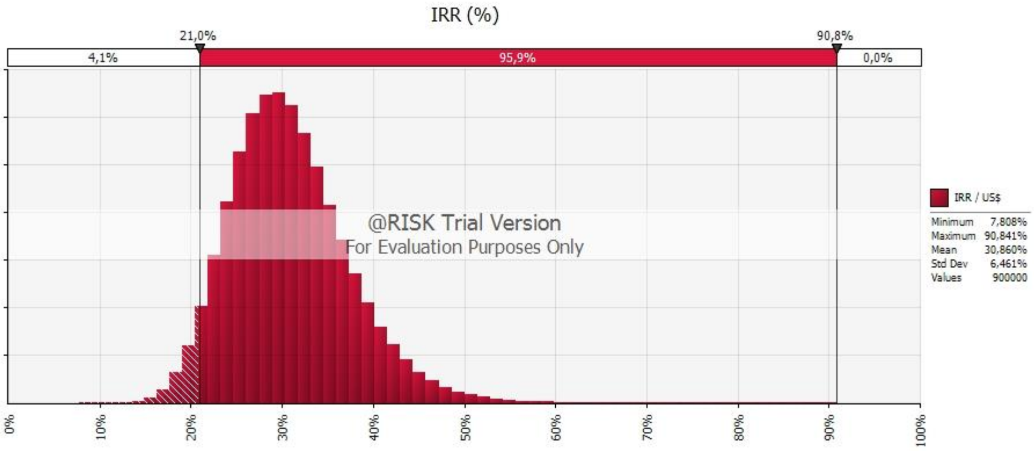


Figure 3.6: Probability distribution of an IRR less and greater than 21%

The project has a probability of 0.96 of having an IRR above 21% and a probability of 0.04 of IRR being less than 21%. According to the IRR results, the project is viable. This project is very lucrative, because it has a high probability of having a positive NPV and also a high probability of having an IRR greater than the 5% discount factor used to discount the cash flows. Knowing these probabilities, an investor would most likely invest in this project with better confidence. Although the MCDCF quantifies risk in its valuation, it does not incorporate the flexibility that management has to adjust the project in the face of uncertainty. Hence, the development of the Binomial-Lattice model.

3.3.3 Binomial-Lattice model

Determining the value of an option to delay the start of the project exercised in both scenarios using the Binomial-Lattice model was done in two steps. In Step 1 the lattice for each scenario that determines the value of the mineral asset was calculated. Each lattice contains decision points that are called nodes, and each node calculated the value of the mineral asset at a specific time in the minerals asset's life in different states. The nodes in the lattice have an incremental time step (*dt*) which is influenced by a company's decision-making process on investments and the lifetime of the project (Smith, *et al*., 2017). Ideally, these decisions are large investment decisions which are made once a year depending on the company's decision-making process. The NPVs excluding the initial capital expenditure obtained from the static DCF were entered into the first node on the left of each lattice as the mineral asset values. The initial mineral asset value for the hypothetical gold mine project is illustrated in Table 3.3.

Table 3.3: Asset value calculation of the hypothetical gold project

Description	Units							Total
Number		1	2	3	4	5	6	
Year		2019	2020	2021	2022	2023	2024	
Net operating profit	US$ million	-	13.50	15.16	12.97	7.99	6.52	56.14
Capex	US$ million	-20.13	-17.79	-	-	-	-	-37.92
Discounted Net cash flow @5%	US$ million	-20.13	-4.09	13.75	11.20	6.58	5.11	12.42
Net DCF excluding capex (Ass	**US$ million**	**0**	**12.86**	**13.75**	**11.20**	**6.57**	**5.11**	**49.49**

The mineral asset value for the gold project to be entered in the first node of the lattice is US$49.49 million. Each node increases and decreases from the first left node by a factor of *u* or *d*, respectively. This results in the expansion with incremental time steps between the nodes to the right until the lattice reaches the expiry date of the option which is the LoM. The upward and downward factors were calculated using Equations 2.14 to 2.17.

The volatility (σ) of the underlying asset used to calculate *u* and *d*, in this case, the present value of the annual cash flows of the project is influenced by key input variables such as production rate, gold prices, grade and operating costs. Lattanzi (2011) stated that the mineral commodity price is the key driver of revenues from which profits are derived. Therefore, the volatility of the underlying asset which is the projects' cash flow was assumed to be that of the historical gold price. This assumed that the hypothetical gold project had an option to be delayed by three years. The input parameters to generate the underlying asset lattice for the hypothetical gold project are shown in Table 3.4 and the resultant lattice is shown in Figure 3.7.

Table 3.4: Input parameters for Binomial-Lattice gold mine example

Description	Value
Asset Value (US$ million)	49.49
Exercise Price (US$ million)	37.92
Option life (years)	3
Volatility	0.082934
Incremental time step	0.3
Up factor	1.046472331
Downward factor	0.955591439
Risk-neutral probability	0.554864684
Risk-free rate	0.02

Underlying Asset Value

No. of Node	0	1	2	3	4	5	6	7	8	9	10
Years	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
	49.49	51.79	54.20	56.72	59.35	62.11	65.00	68.02	71.18	74.48	77.95
		47.29	49.49	51.79	54.20	56.72	59.35	62.11	65.00	68.02	71.18
			45.19	47.29	49.49	51.79	54.20	56.72	59.35	62.11	65.00
				43.19	45.19	47.29	49.49	51.79	54.20	56.72	59.35
					41.27	43.19	45.19	47.29	49.49	51.79	54.20
						39.43	41.27	43.19	45.19	47.29	49.49
							37.68	39.43	41.27	43.19	45.19
								36.01	37.68	39.43	41.27
									34.41	36.01	37.68
										32.88	34.41
											31.42

Figure 3.7: Real Options Binomial-Lattice model example of project value

According to Pascal's triangle (Appendix 7.1), the gold project has a probability of one to having an asset value of US$49.49 million. The probability was calculated using Equation 2.20. For example, the probabilities of the asset values of different states in node 3 are calculated as follows:

$$state_1 = \frac{1}{2^3}$$

$$= 0.13$$

$$state_2 = \frac{3}{2^3}$$

$$= 0.38$$

$$state_3 = \frac{3}{2^3}$$

$$= 0.38$$

$$state_4 = \frac{1}{2^3}$$

$$= 0.13$$

From the above calculation, it can be noted that all nodes have a summed probability of 1 split into different states. In the second step, a lattice for each scenario that determines the value of an option to delay (C), at different time intervals was calculated. Unlike the mineral asset value lattice, the option lattice moves from the right to the left, and thus calculates the present value of the option from future values. Smith *et al.* (2017) noted that this process is referred to as the backward induction. The option values in the last time interval were calculated using Equation 2.18 and the rest of the nodes moving to the left were calculated using Equation 2.13. The resultant lattice is similar to the one in Figure 3.7, but with the values of the option at different time intervals and their respective probabilities. Knowing the probabilities of different states in different nodes, companies can make informed decisions on options available to them. This is because they will now be able to quantify the risk involved.

3.4 Chapter summary

This chapter stated the data collected, the sources of data collected and described the methods used to determine the economic value of the gold project under study. It also illustrated how to carry out a static DCF and MCDCF analysis using a hypothetical project. Lastly, it illustrated how to value an option to delay the start of a project with ROA using the Binomial-Lattice model. The analysis and results of the case study are presented in the next chapter.

4 VALUATION RESULTS AND ANALYSIS

4.1 Chapter overview

This chapter discusses and justifies the input data used in this research study. Furthermore, valuation results from the static DCF, MCDCF and ROA are presented. Finally, a comparison of the results obtained from the different techniques is done.

4.2 Static Discounted Cash flow analysis

The input parameters of the static DCF were provided by the company in the PFS conducted in 2017. These parameters are discussed in the following Sub-section.

4.2.1 DCF input parameters

As mentioned in Chapter 1, the project has a Mineral Resource of 1.4 million tonnes with an average grade of 5.52 g/t of gold (Au). The mine uses a cut-off grade of 2.63 g/t of Au. The average annual production rate for Scenario 1 and Scenario 2 is approximately 200 000 to 240 000 tonnes/annum of ore, respectively. This resulted in a six year LoM for Scenario 1 and a seven year LoM for Scenario 2. This is because in Scenario 1, development of the preproduction phase will be conducted by a contractor, and after year 2 the owner will take over the production phase. In Scenario 2 the owner will develop and operate the mine resulting in late completion of development, commencing production in year 3 quarter 2 of the project. This is because Scenario 2 lacks the same amount of manpower to conduct the preproduction phase and thus prolonging the LoM

The initial capital expenditure for the commencement of the project is US$93.83 million and US$84.92 million for Scenario 1 and 2, respectively. The difference in initial capital expenditure is because of the contractors' extra labours costs for administration and standby. This added 10% and 15% mark-up to the owner's rates for administration and standby, respectively. The initial capital costs for both scenarios are sub-divided into the following categories:

- Capital development cost: that includes the development of the decline and infrastructure to access the deposit during the pre-production phase;

- Mobile equipment cost: which includes the purchasing of equipment and spares for the equipment. A 20% deposit of the purchase price of the equipment is required, while the remaining 80% must be repaid as leasing costs in the operating cost; and
- Operating costs during the preproduction phase.

Furthermore, the sustaining capex of the project for Scenario 1 and 2 is US$7.09 million and US$9.67 million, respectively. Again this is due to the longer LoM of Scenario 2.

As mentioned in Chapter 3, the current processing plant used at the mine is a hydrometallurgical plant fitted with a carbon-in-pulp circuit with a recovery of 98.1% that will also be used for the underground project. However, the mine assumed a conservative long-term recovery of 97% recovery from the processing plant to cater for any losses in the plant. The applicable royalties and corporate tax rate charged on precious/base and rare metal in the country is 4% of the market value of the mineral extracted and 37.5% of taxable income, respectively. The discount rate that is applied to the project by the mining company is 5%. Fluctuations in the exchange rate affect profit margins of mining companies if operating costs are estimated in the local currency. A weaker local exchange rate decreases operating costs in US dollars, which increases a company's profit margins. Inversely, a stronger local exchange rate increases operating costs in US dollars which reduces profit margins (Keeton, 2016). However, all economic data was given by the company in the US dollar and therefore, the effects of the exchange rate on the companies operating costs and profits were not considered because of its short LoM. In their PFS, the company assumed a constant gold price of US$ 1 300/oz over the LoM to determine the economic value of the project. Therefore, this price was also used for the static DCF in this research study. The fixed costs provided by the company for the project which are the site general and corporate costs were considered by the company to vary with the number of tonnes milled in US dollars. Tables 4.1 and 4.2 show a summary of the input parameters used to conduct a static DCF analysis for both scenarios.

Table 4.1: DCF input parameters for Scenario 1

Description	Unit	1	2	3	4	5	6	7	8	Total/average
Year		2026	2027	2028	2029	2030	2031	2032	2033	
Ore milled	tonnes	-	-	164 269	273 750	273 760	273 699	234 748	207 608	1 427 834
Grade	g/t	-	-	6.13	5.93	5.77	5.26	4.38	5.79	5.52
Price	US$/oz	-	-	1300	1300	1300	1300	1300	1300	1300
Recovery	%	-	-	97	97	97	97	97	97	97
Unit variable costs	US$/tonne milled	-	-	114.92	131.35	133.25	135.68	127.13	99.98	123.72
Initial capex	US$ million	27.31	47.62	18.90	-	-	-	-	-	93.83
Sustaining capex	US$ million	-	-	5.65	0.41	0.69	0.35	-	-	7.10

Royalty	4%
Tax rate	37.50%
Discount factor	5%

Table 4.2: DCF input parameters for Scenario 2

Description	Unit	1	2	3	4	5	6	7	8	9	Total/average
Year		2026	2027	2028	2029	2030	2031	2032	2033	2034	
Ore milled	tonnes	-	-	18 899	239 148	273 699	273 749	273 751	234 852	113 735	1 427 834
Grade	g/t	-	-	3.85	5.80	5.89	5.86	5.00	5.11	5.57	6
Price	US$/oz	-	-	1 300	1 300	1 300	1 300	1 300	1 300	1 300	1 300
Recovery	%	-	-	97	97	97	97	97	97		97
Unit variable costs	US$/tonne milled	-	-	-	147.76	144.31	117.36	115.32	112.61	87.13	106.2
Initial capex	US$ million	21.48	25.04	38.4	-	-	-	-	-	-	84.92
Sustaining capex	US$ million	-	-	-	8.99	0.47	0.10	0.09	0.02	-	9.67

Royalty	4%
Tax rate	37.50%
Discount factor	5%

The difference in terms of input parameters between the two scenarios resulted from the difference in LoM and the extra cost of hiring a contractor for the preproduction phase. The extended LoM for Scenario 2 resulted in a different production profile which in turn resulted in a higher sustaining capital expenditure. Also, as explained in Chapter 2 higher production output lead to a decrease in unit operation costs, which is evident in the unit variable costs in Tables 4.1 and 4.2.

Preproduction operating costs that incurred before the end of year 3 in Scenario 2 were transferred to the initial capital expenditure by the company. Appendices 7.2 to 7.4 show the detailed breakdown of the following additional input parameters:

- The Mineral Resource grade and tonnage distribution in primary and secondary stopes;
- Operating costs; and

- Initial capital cost and sustaining costs.

Using these technical and economic data provided by the company static DCF analysis for the two scenarios were done to determine the economic value of the project. The results of the static DCF are presented and discussed in the subsequent Sub-section.

4.2.2 Static Discounted Cash flow results

Table 4.3 shows a summary of the static DCF analysis results for Scenarios 1. The detailed static DCF results for Scenario 1 are shown in Appendix 7.5.

Table 4.3: Summarised static DCF analysis results for Scenario 1

Description	Number	1	2	3	4	5	6	7	8	
Year		2026	2027	2028	2029	2030	2031	2032	2033	Total
Revenue	US$	-	-	44 790 497.37	72 206 886.40	70 261 206.41	64 036 671.78	45 734 697.25	53 467 816.09	350 497 775.29
Total sales costs	US$	-	-	-109 612.11	-156 113.26	-152 813.18	-142 255.70	-111 213.58	-124 329.78	-796 337.61
Gross profit	US$	-	-	44 680 885.26	72 050 773.13	70 108 393.23	63 894 416.08	45 623 483.67	53 343 486.31	349 701 437.68
Total operating costs	US$	-	-	-19 277 793.48	-36 357 062.50	-36 878 520.00	-37 535 480.32	-30 243 513.24	-21 156 647.84	-181 449 017.38
Operating profit	US$	-	-	25 403 091.78	35 693 710.63	33 229 873.23	26 358 935.76	15 379 970.43	32 186 838.47	168 252 420.30
Total capital cost	US$	-27 313 000.00	-47 618 000.00	-24 539 000.00	-414 000.00	-689 000.00	-352 000.00	-	-	-100 925 000.00
Unredeemed capex	US$	-27 313 000.00	-74 931 000.00	-74 066 908.22	-38 787 197.59	- 6 246 324.36				-221 344 430.18
Operating profit after capex	US$	-	-	-	-	-	19 760 611.40	15 379 970.43	32 186 838.47	165 852 420.30
Royalties @ 4%	US$	-	-	- 1 791 619.89	- 2 888 275.46	- 2 810 448.26	-2 561 466.87	-1 829 387.89	-2 138 712.64	-14 019 911.01
Net operating profit	US$	-	-	23 611 471.88	32 805 435.18	30 419 424.97	23 797 468.89	13 550 582.54	30 048 125.82	154 232 509.29
Taxable Income	US$	-	-	-			19 760 611.40	15 379 970.43	32 186 838.47	67 327 420.30
Corporate Taxes @ 37.5%	US$	-	-	-	-	-	-7 410 229.27	-5 767 488.91	-12 070 064.43	-25 247 782.61
Net Cashflow	US$	-27 313 000.00	-47 618 000.00	-927 527.15	32 391 436.15	29 730 425.94	16 035 240.59	7 783 094.60	17 978 062.37	28 059 732.50
Discounted Net Cashflow @5%	US$	-26 012 380.95	-43 190 929.71	-801 232.82	26 648 514.68	23 294 566.67	11 965 743.41	5 531 300.02	12 168 260.26	9 603 841.57
NPV (US$ million)	9.60									
IRR	8.67%									

As discussed in Chapter 2, the NPV, IRR and PP are the key decision criteria of the DCF technique. The discounted net cash flow of the project resulted in an NPV of US$9.60 million at a 5% discount rate. Since the NPV for Scenario 1 is positive as shown in Table 4.3, then based on the NPV decision criteria the project is viable if Scenario 1 was to be implemented by the company. The graphical plot of the cash flow for Scenario 1 generated over the LoM is illustrated in Figure 4.1

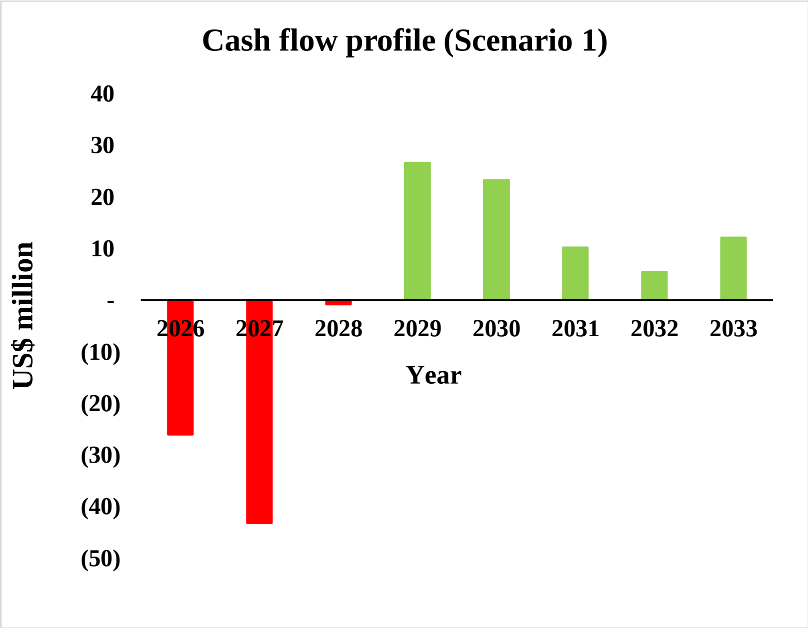


Figure 4.1: Life of mine cash flow profile for Scenario 1

Despite there being a fairly consistent production rate between 2029 and 2033, the net cash flow keeps declining. This is because of the quality of ore processed. Years 2029 and 2030 in which high-grade ore was processed resulted in higher net cash flows and vice versa. Operating costs also had an influence on the net cash flow, as lower production had higher unit operating costs compared to higher production, resulting in higher net cash flows. Overall, the operating costs decreased with the quantity of material processed. This is because more resources and consumables are used to handle and process large amounts of ore. Figure 4.2 shows the IRR for Scenario 1 of the project.

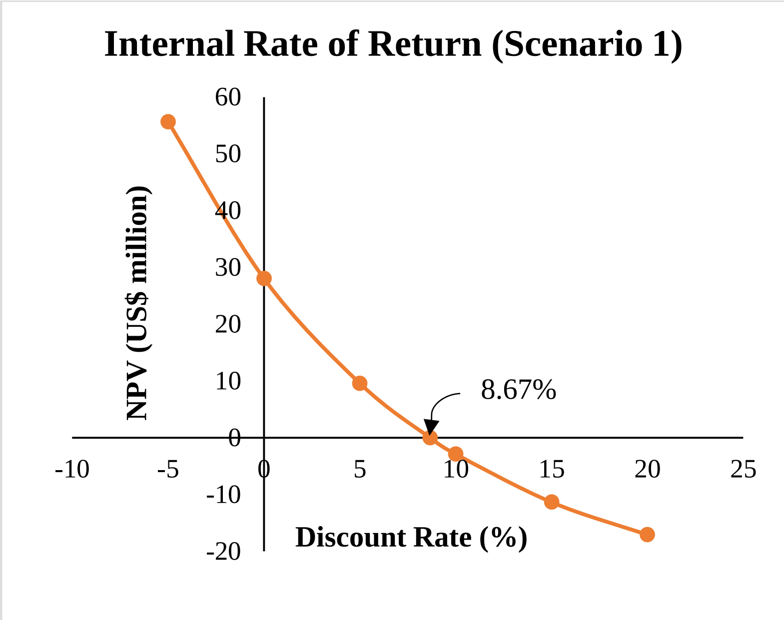


Figure 4.2: Internal Rate of Return curve for Scenario 1

The IRR of the project was found to be 8.67%. Since the IRR of Scenario 1 is greater than the discount rate of the project, based on the IRR decision criteria the project is viable. The PP of Scenario 1 was calculated using Equation 2.6 as described in Chapter 3. Table 4.4 shows the NPVs over the LoM each year that were used to calculate the PP of Scenario 1.

Table 4.4: NPV calculations for PP of Scenario 1

Description	Units	Data							
Number		1	2	3	4	5	6	7	8
Year		2026	2027	2028	2029	2030	2031	2032	2033
Discounted Net cash flow @5%	US$ million	-26.01	-43.19	-0.80	26.65	23.29	11.97	5.53	12.17
NPV	US$ million	-26.01	-69.20	-70.00	-43.36	-20.06	-8.10	-2.56	9.60

From the cash flow in Table 4.3 and the NPV calculations for the PP in Table 4.4, production starts in 2028 and that is the year in which the PP is counted as year one. The resultant calculation of PP of Scenario 1 is shown as follows:

$$PP \approx t* + \frac{NPV_{t*}}{NPV_{t*} - NPV_{t*+1}}$$

$$\approx 5 + \frac{-2.56}{-2.56 - 9.60}$$

$$\approx 5.21\ years$$

Scenario 1 resulted in a PP of 5.21 years, which is approximately the LoM. This means it will take 5.21 years for the project to fully recover all the capital invested in the project. The viability of the project based on the PP decision criteria is dependent on how fast the investors want their initial investment back. A shorter PP is desired in general. For a mining project with a six year LoM, a PP of 5.21 years would likely make the project unviable. However, the major decision criteria of the DCF that have more weight on deciding on the viability of a project are NPV and IRR (Awomewe and Ogundele, 2008).

Table 4.5 shows a summary results of the static DCF analysis for Scenario 2. The detailed results for the static DCF analysis for Scenario 2 are shown in Appendix 7.6.

Table 4.5: Summarised static DCF analysis results for Scenario 2

Description	Number	1	2	3	4	5	6	7	8	9	
Year		2026	2027	2028	2029	2030	2031	2032	2033	2034	Total
Revenue	US$	-	-	3 236 453.35	61 697 070.58	71 706 463.27	71 354 267.89	60 882 925.94	53 380 785.51	28 178 581.32	322 257 966.54
Total sales costs	US$	-	-	-39 131.97	-138 287.48	-155 264.49	-154 667.13	-136 906.60	-124 182.17	-81 436.51	-748 439.85
Gross profit	US$	-	-	3 197 321.38	61 558 783.10	71 551 198.77	71 199 600.76	60 746 019.34	53 256 603.35	28 097 144.81	349 606 671.50
Total operating costs	US$	-	-	-400 000.00	-35 736 508.48	-39 897 502.69	-32 527 182.64	-31 968 965.32	-26 846 683.72	-10 309 730.55	-167 376 842.85
Operating profit	US$	-	-	2 797 321.38	25 822 274.62	31 653 696.08	38 672 418.12	28 777 054.02	26 409 919.63	17 787 414.26	154 132 683.84
Total capital cost	US$	-21 480 000.00	-25 043 000.00	-38 396 000.00	-8 985 000.00	-468 000.00	-101 000.00	-90 000.00	-16 000.00		-94 579 000.00
Unredeemed capex	US$	-21 480 000.00	-46 523 000.00	-82 121 678.62	-65 284 404.00	-34 098 707.92	4 472 710.20				-245 035 080.35
Operating profit after capex	US$	-	-	-	-	-	4 472 710.20	33 159 764.22	26 393 919.63	17 787 414.26	81 813 808.30
Royalties @ 4%	US$	-	-	-129 458.13	-2 467 882.82	-2 868 258.53	-2 854 170.72	-2 435 317.04	-2 135 231.42	-1 127 143.25	-14 017 461.91
Net operating profit	US$	-	-	2 667 863.24	23 354 391.79	28 785 437.55	35 818 247.41	26 341 736.98	24 274 688.21	16 660 271.00	157 902 636.19
Taxable Income	US$	-	-	-	-	-	4 472 710.20	33 159 764.22	26 393 919.63	17 787 414.26	81 813 808.30
Corporate Taxes @ 37.5%	US$	-	-	-	-	-	-1 677 266.33	-12 434 911.58	-9 897 719.86	-6 670 280.35	-24 009 897.77
Net Cashflow	US$	-21 480 000.00	-25 043 000.00	-35 728 135.79	14 369 392.76	28 317 438.52	34 039 982.05	13 816 826.37	14 360 969.32	9 989 991.63	32 643 464.86
Discounted Net Cashflow @5%	US$	-20 457 142.86	-22 714 739.23	-30 863 307.02	11 821 734.99	22 187 454.05	25 401 158.71	9 819 360.54	9 720 069.31	6 439 637.68	11 354 226.17
NPV (US$ million)	11.35										
IRR	8.95%										

The cash flow of Scenario 2 resulted in an NPV of US$11.35 million at a 5% discount rate. Since the NPV for Scenario 2 is positive, then based on the NPV decision criteria the project is viable if Scenario 2 was to be implemented. The graphical presentation of the cash flow profile for Scenario 2 is illustrated in Figure 4.3.

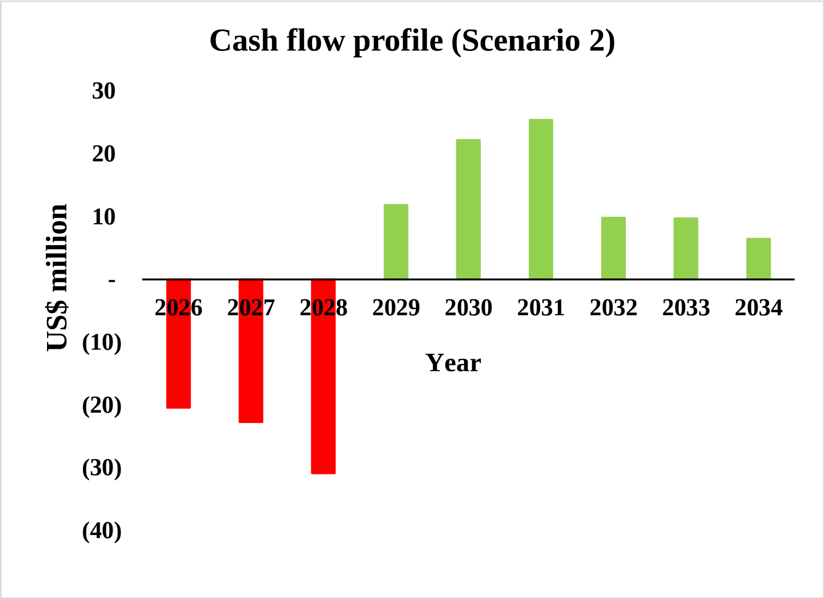


Figure 4.3: Life of mine cash flow profile for Scenario 2

Similar to Scenario 1 there is a fairly consistent production rate between 2030 and 2034, but the net cash flow keeps declining. This was because of the quality of ore processed, years 2030 and 2031 in which high grade ore was processed resulted in higher net cash flows and vice versa. Operating costs also had an influence on the net cash flow, as lower production had higher unit operating costs compared to higher production. Overall the operating costs decreased with the quantity of material processed. This is because more resources and consumables are used to handle and process large amounts of ore. Figure 4.4 shows the IRR for Scenario 2 of the project.

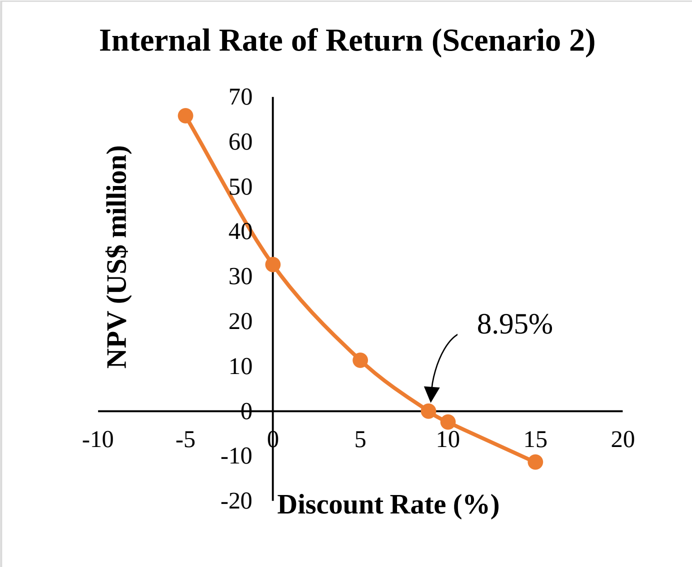


Figure 4.4: Internal Rate of Return curve for Scenario 2

The IRR of the project was found to be 8.95%. Since the IRR of Scenario 2 is greater than the 5% discount rate of the project, then based on the IRR decision criteria the project is viable. The PP of Scenario 2 was also calculated using Equation 2.6. Table 4.6 shows the NPVs over the LoM that were used to calculate the PP of Scenario 2.

Table 4.6: NPV calculations for PP of Scenario 2

Description	Units	Data								
Number		1	2	3	4	5	6	7	8	9
Year		2026	2027	2028	2029	2030	2031	2032	2033	2034
Discounted Net cash flow @5%	US$ million	-20.46	-22.71	-30.86	11.82	22.19	25.40	9.82	9.72	6.44
NPV	US$ million	-20.46	-43.17	-74.03	-62.21	-40.02	-14.62	-4.80	4.92	11.36

From the cash flow in Table 4.5 and the NPV calculations for the PP in Table 4.6, production starts in 2028 and that is the year in which the PP is counted as year one. The resultant calculation of PP of the Scenario 2 is shown as follows:

$$PP \approx t* + \frac{NPV_{t*}}{NPV_{t*} - NPV_{t*+1}}$$

$$\approx 5 + \frac{-4.80}{-4.80 - 4.92}$$

$$\approx 5.49\ years$$

The cash flow of Scenario 2 resulted in a PP of 5.49 years, which is approximately the LoM. The viability of the project based on the PP decision criteria is dependent on how fast the investors want their initial investment back.

4.2.3 A summary of the Static DCF analysis results

Based on the DCF decision criteria it would be in the company's best interest to pursue this project and proceed to conduct a full feasibility study to increase the level of confidence of the project. A summary of the results of the static DCF analysis for the two scenarios is shown in Table 4.7.

Table 4.7: Summary of static DCF analysis results

Measure	Unit	Scenario 1	Scenario 2
NPV	US$	9.60 million	11.35 million
IRR	%	8.67	8.95
PP	years	5.21	5.49

From the NPV and IRR decision criteria, Scenario 2 is the better scenario. This is because it requires lower initial capital investment for hiring a contractor to conduct the preproduction phase of sinking a decline and setting up the necessary infrastructure to access the mineral deposit. Also, Scenario 2 resulted in a higher NPV and higher IRR. However, it resulted in a longer PP of 5.49 years which is 1.51 years before its seven year LoM.

Although the outcomes of the decision criteria of the static DCF in investment decision making show that the project is viable, the static DCF has several limitations that were discussed in Chapter 2. Moreover, the static DCF technique does not present the company with the confidence levels in the NPV and IRR or the risk associated with the economic value of the project. Also, it is possible that based on the three decision criteria a project may be viable at face value, but with a low probability of being viable. To assess the risk and enable properly informed investment decision making, a MCDCF analysis was done and the probabilistic DCF assumptions and results are presented in the following section.

4.3 Probabilistic Discounted cash flow analysis

The basis of the probabilistic DCF are inputs from the static DCF analysis conducted in the previous section. To quantify risk the associated with the project the inputs for the DCF calculation were fitted with probability distributions. Both scenarios were analysed as explained in Chapter 3.

4.3.1 Monte Carlo DCF input parameters

The inputs of the static DCF analysis were simulated using @RISK software to create a MCDCF. The variables that were simulated are:

- Gold price;
- Gold grade;
- Production rate; and
- Unit variable costs.

Other input parameters remained the same as for the static DCF. The gold price used in the static DCF of US$1 300/oz was used in the simulation as the annual average gold price provided by the company. The historical gold price from January 2010 to December 2018 was used to determine the probability distribution of the gold price. The assumption was that the gold price over the LoM will have a similar behaviour with the previous eight years trend. The eight-year historical gold price shows that gold price follows a triangular distribution as shown in Appendix 7.7 which was generated in @RISK. Furthermore, the fitted distribution in the software required statistical parameters of the minimum, maximum and mode of the eight years historical gold price. The parameters were generated by the software from the historical data by a function called distribution fitting. It was found that the data had a minimum, maximum and mode of US$1 038/oz, US$1 898/oz and US$1 198/oz, respectively. However, @RISK does not compute the mode. The mode was computed manually using excel and is not shown in Appendix 7.7.

As explained in Chapter 3, the grade distribution of gold grade was determined from historical grades from the current operating surface mine. The distribution of gold grade was found to follow a positively skewed lognormal distribution as shown in Appendix 7.8. Therefore, the probability distribution selected to simulate the gold grade for this research study is the positively skewed lognormal distribution. Furthermore, Kennedy (1990) noted that the grade of precious metals, including gold grade, tends to follow a lognormal distribution. This type of distribution is usually characterised by low-grade orebodies with inconsistencies in sample values.

Statistical parameters that are required for simulation of a lognormal distribution are the standard deviation and the average gold grade. These were calculated from the stope in-situ Mineral Resource provided by the company and illustrated in Appendix 7.2. The historical gold grades were found to have a standard deviation of 1.32 g/t of Au and an average grade of 5.52 g/t of Au. From the lognormal distribution and its parameters, it was found that the gold grade had a 95% chance of ranging from 3.38 g/t of Au to 8.52 g/t of Au over the project's life as shown in Appendix 7.9 for both scenarios.

As explained in Chapter 3, the study assumed that both production rate and unit variable costs followed a normal probability distribution. The standard deviation and average unit variables costs were calculated from the projections provided by the company used in the static DCF. The resultant average ore milled was calculated to be 237 972 tonnes with a standard deviation of 41 235 tonnes for Scenario 1. Scenario 2 had an average of 203 977 tonnes and a standard deviation of 56 602 tonnes of ore milled. The selected distribution and the inputs for the simulation of the annual production of Scenario 1 had a 95% confidence level of ranging from 157 154 tonnes to 318 791 tonnes as shown in Appendix 7.10. Scenario 2 ranged from 93 048 tonnes to 314 915 tonnes as shown in Appendix 7.11. The input parameters for the simulation of unit variable costs for Scenario 1 and 2 are illustrated in Appendices 7.12 and 7.13. The results of the MCDCF are presented and discussed in the next sub-section.

4.3.2 Monte Carlo DCF results

@RISK software is based on Monte Carlo Simulation. One hundred thousand iterations of each input parameter were conducted in one simulation to create a normal distribution curve as explained in Chapter 3. The outcome of the simulation quantifies the risk associated with the project by generating a probability distribution of the NPV and IRR. Figures 4.5 shows the range of possible NPV for Scenario 1.

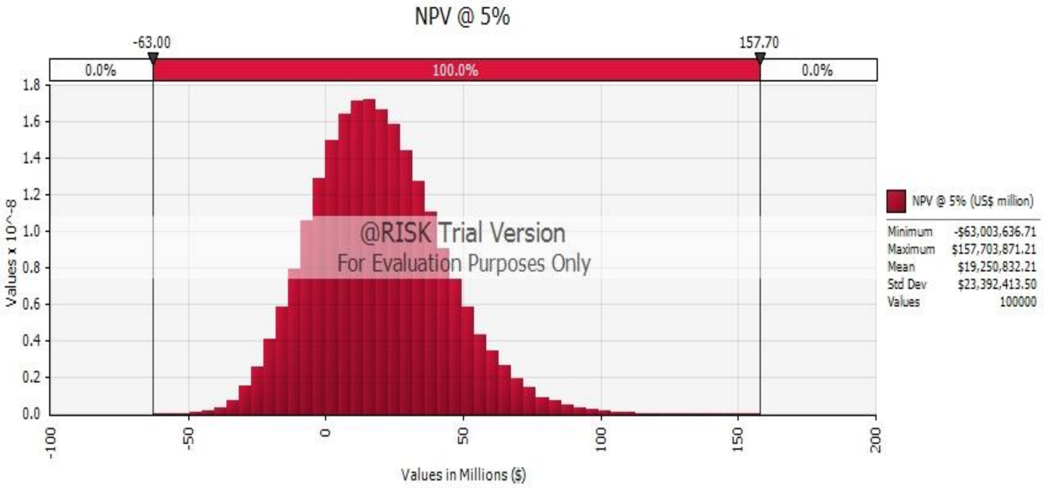


Figure 4.5: Range of possible NPV for Scenario 1

From literature, the static DCF does not show the confidence levels attached to its results. Figure 4.5 shows that the NPV ranges from a minimum of US$-63 million to a maximum of US$157.70 million. From these results, the company is able to see two extreme outcomes of the NPV that Scenario 1 is able to have. Also, from the resultant range of US$220.70 million, the company will have an idea of the risk associated with the project. This is because the wider the range the higher the risk associated with the project and vice versa. For a more comprehensive understanding of the risk, Figure 4.6 shows the probability distribution of the project having a NPV less and greater than the NPV obtained from the static DCF.

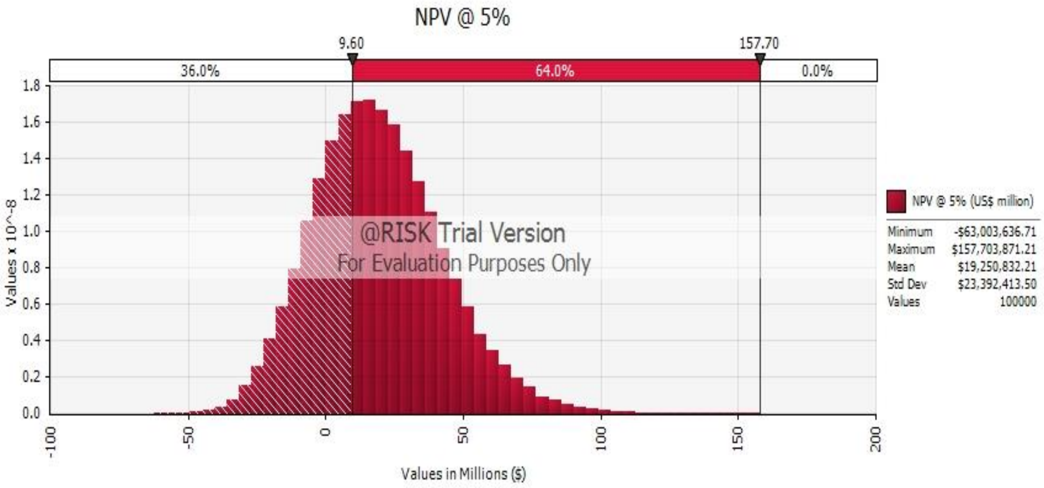


Figure 4.6: Probability distribution of an NPV less and greater than US$9.60 million for Scenario 1

Figure 4.6 shows that there is a probability of 0.64 that the project will have a NPV of US$9.60 million and greater if the company were to implement Scenario 1. The Figure also shows that there is a 0.36 probability that the project will have a NPV less than US$9.60 million. The company is now able to make a decision based on the NPV criteria that are strongly supported by probability. Scenario 1 is still viable, but depending on the risk appetite of the company a different decision can be made. Figure 4.7 shows the risk that the project will not be viable if Scenario 1 will be implemented.

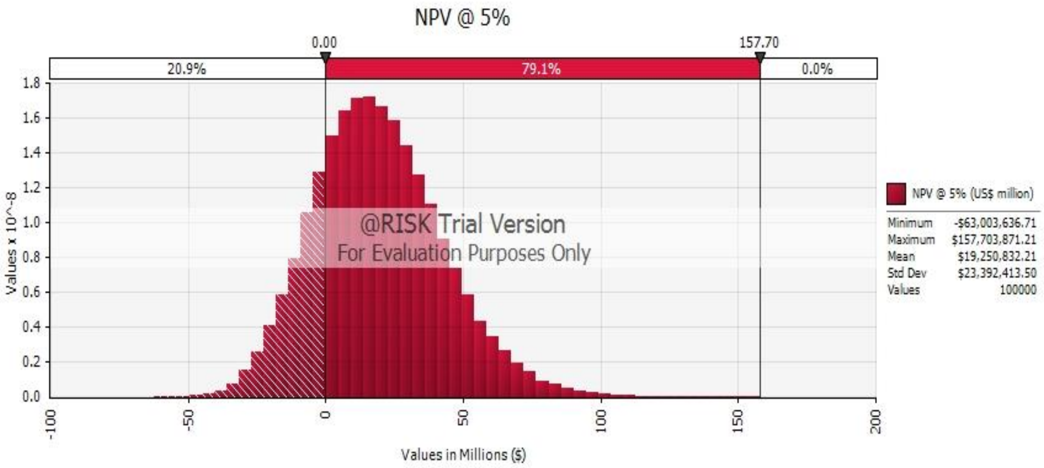


Figure 4.7: Probability distribution of a negative NPV for Scenario 1

Figure 4.7 shows that there is a probability of 0.21 that the project will have a negative NPV for Scenario 1. With this additional information about Scenario 1, the company is able to make informed decisions as compared to the static DCF results based on the risk associated with the NPV. The range of possible IRR for Scenario 1 is shown in Figure 4.8.

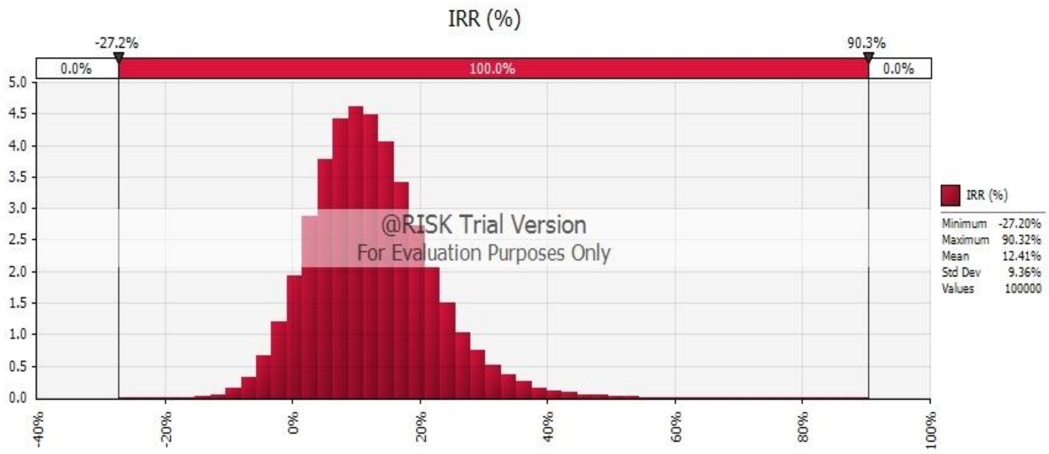


Figure 4.8: Range of possible IRR for Scenario 1

Figure 4.8 shows that the IRR ranges from a minimum of -27.2% to maximum of 90.3%. The resultant range was calculated to be 117.5%. From the resultant range, the company is able to see two extreme IRR in Scenario 1, the worst and best case. As for Scenario 1 IRR, the wider the range the higher the risk associated with the project and vice versa. Figure 4.9 shows the risk that the project will have IRR less and greater than the IRR generated by the static DCF.

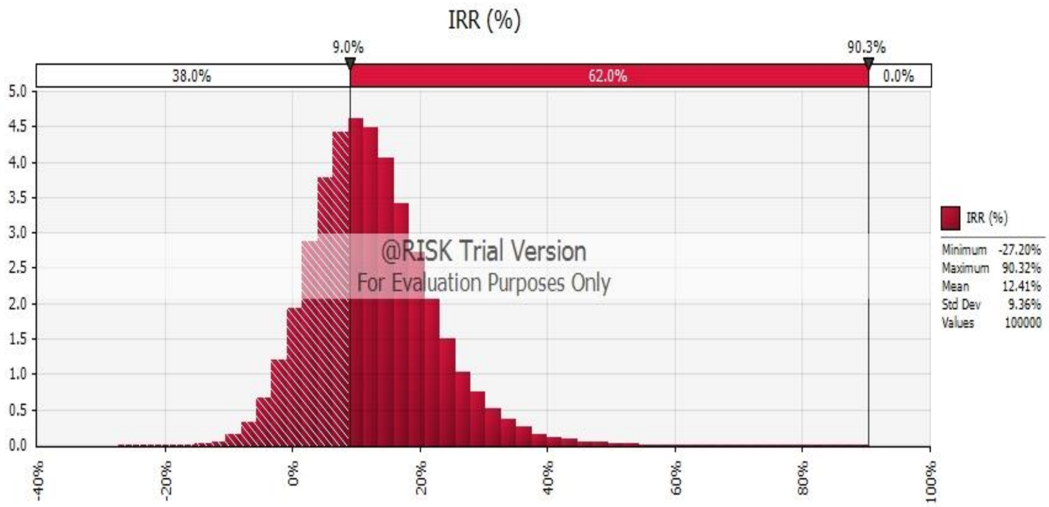


Figure 4.9: Probability distribution of an IRR of less and greater than 9% for Scenario 1

Figure 4.9 shows that there is a probability of 0.62 that the project will have an IRR of 9% and greater, the same IRR generated by the static DCF. The Figure also shows that there is a 0.38 probability that the project will have an IRR less than 9%. Since the risk has now been quantified on the IRR of Scenario 1, the company is able to make an informed decision based on the IRR decision criteria. Figure 4.10 shows the probability of Scenario 1 having a greater and less IRR of 5%.

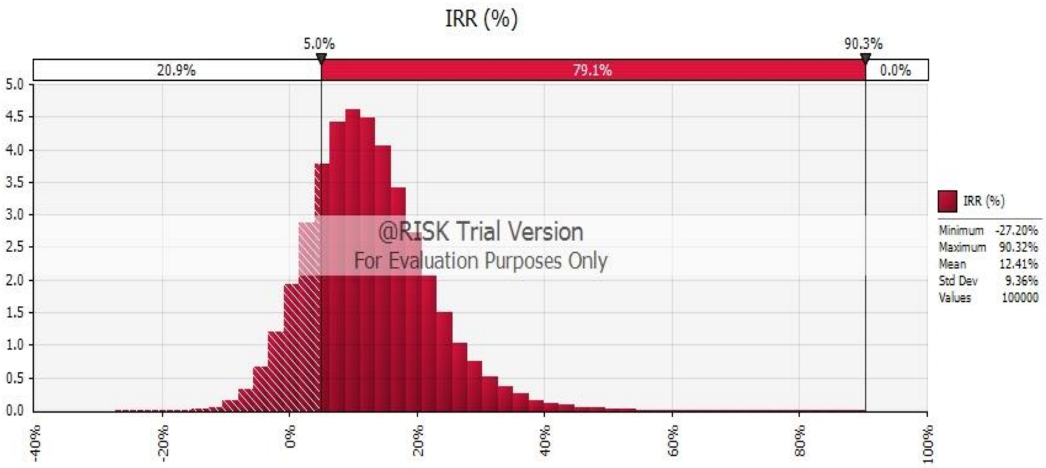


Figure 4.10: Probability distribution of an IRR less and greater than 5% for Scenario 1

Figure 4.10 shows that there is a probability of 0.79 of having an IRR of a 5% discount rate and greater if the company will opt for Scenario 1. The Figure also shows that there is a 0.21 probability that the project will have an IRR of less than 5%. According to the DCF valuation criteria with the respective probabilities of the NPV and IRR, Scenario 1 is still viable. However, depending on the company's risk appetite it may change its investment decision regarding Scenario 1. The following results were obtained when Scenario 2 was analysed in @RISK software. Figure 4.11 shows the range of possible NPV for Scenario 2.

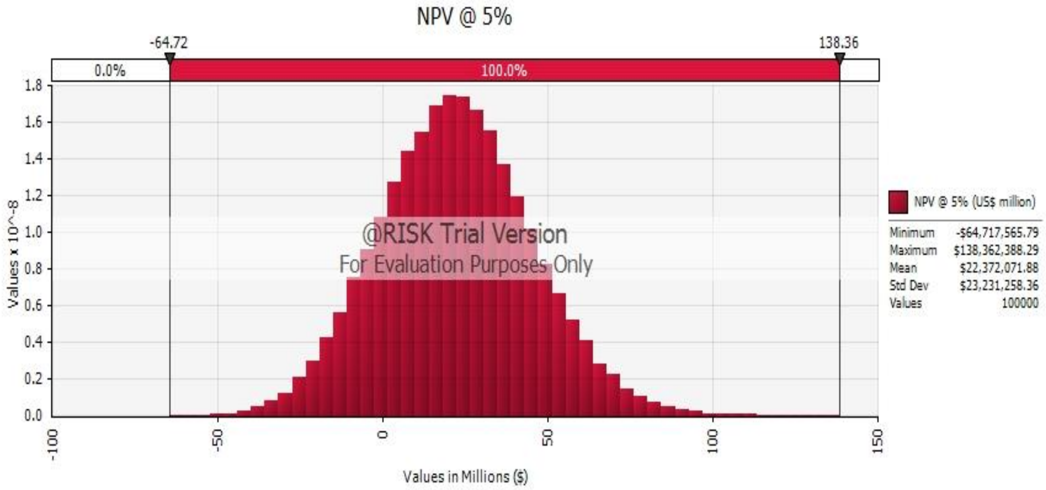


Figure 4.11: Range of possible NPV for Scenario 2

It was found that the NPV ranges from a minimum of US$-64.72 million to a maximum of US$138.36 million. From these results, the company is able to see two extreme outcomes of the NPV that the project is able to have if Scenario 2 was implemented. The range for Scenario 2 was calculated to be U$203.08 million. This is a slightly lower range than that of Scenario 1, which means Scenario 2 is less riskier than Scenario 1. For a more comprehensive understanding of the risk, Figure 4.12 shows the probability distribution of the project having a NPV less and greater than the NPV obtained from the static DCF.

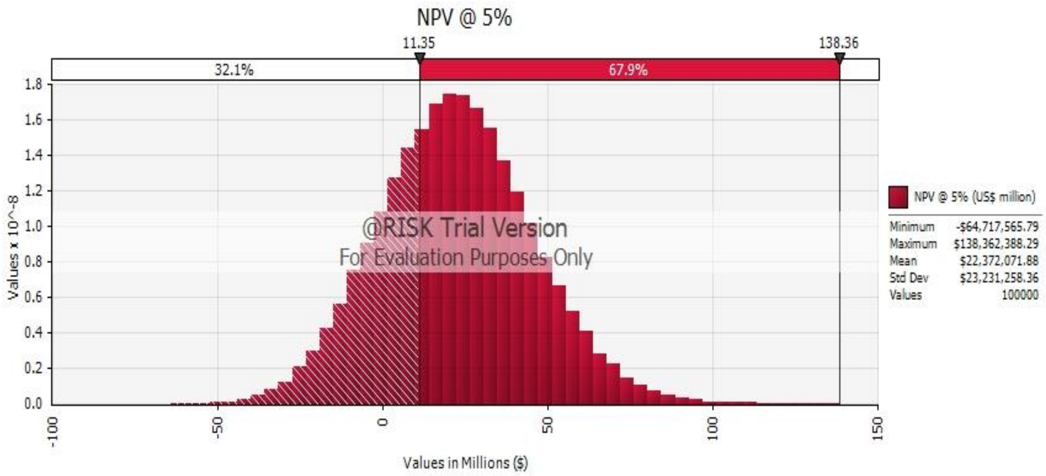


Figure 4.12: Probability distribution of an NPV less and greater than US$11.35 million for Scenario 2

Figure 4.12 shows that there is a probability of 0.68 that the project will have a NPV of US$11.35 million and greater if the company were to implement Scenario 2. The Figure also shows that there is a 0.32 probability that the project will have a NPV less than US$11.35 million. The company is now able to make a decision based on the NPV criteria that are strongly supported by probability. Scenario 2 is still viable, but depending on the risk appetite of the company a different decision can be made. Figure 4.13 shows the risk that the project will not be viable if Scenario 2 will be implemented.

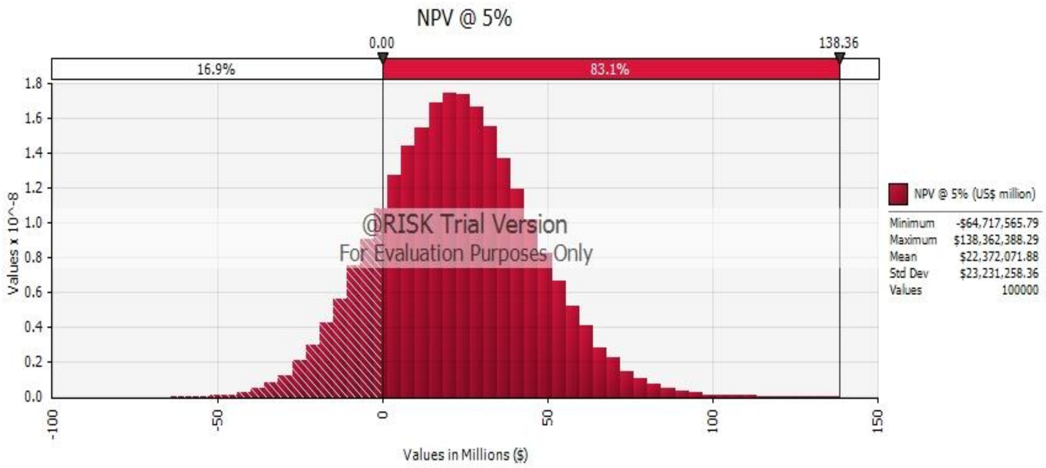


Figure 4.13: Scenario 2 probability distribution of a negative NPV

Figure 4.13 shows that there is a probability of 0.17 the project will have a negative NPV for Scenario 2. The Figure also shows that the project has a probability of 0.83 of having a positive NPV for Scenario 2. With this additional information about Scenario 2, the company is able to make informed decisions as compared to the static DCF results based on the risk associated with the NPV. The range of possible IRR for Scenario 2 is shown in Figure 4.14.

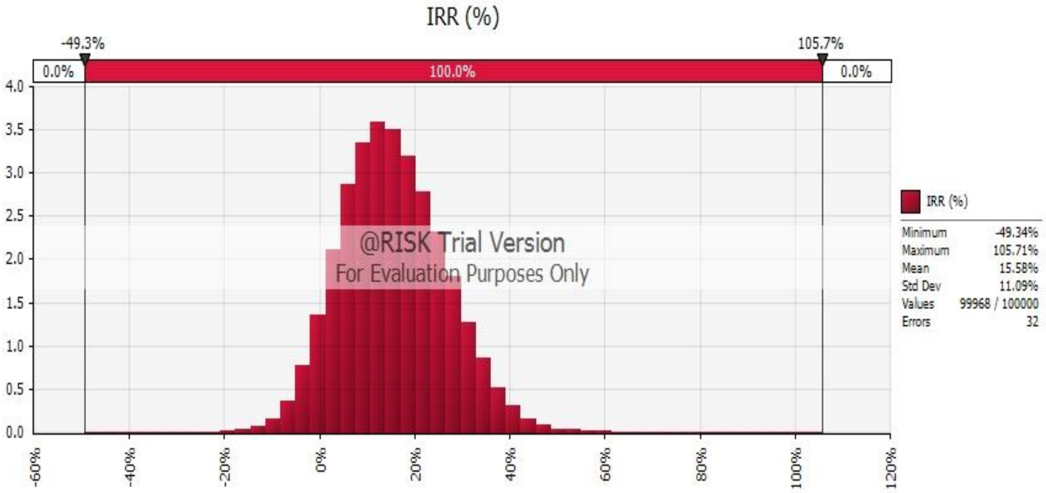


Figure 4.14: Probability distribution for IRR range for Scenario 2

Figure 4.14 shows that the IRR ranges from -49.3%% to 105.7%, with a calculated range of 155%. From these results, the company is able to see two extreme IRR in Scenario 2, the worst and best case. As from previous cases, the wider the range the higher the risk associated with the project. Scenario 2 has a wider range of IRR when compared to Scenario 1, which means it has a greater risk in obtaining its IRR. Figure 4.15 shows the probability distribution of the project has an IRR less than and greater than the one obtained from the static DCF.

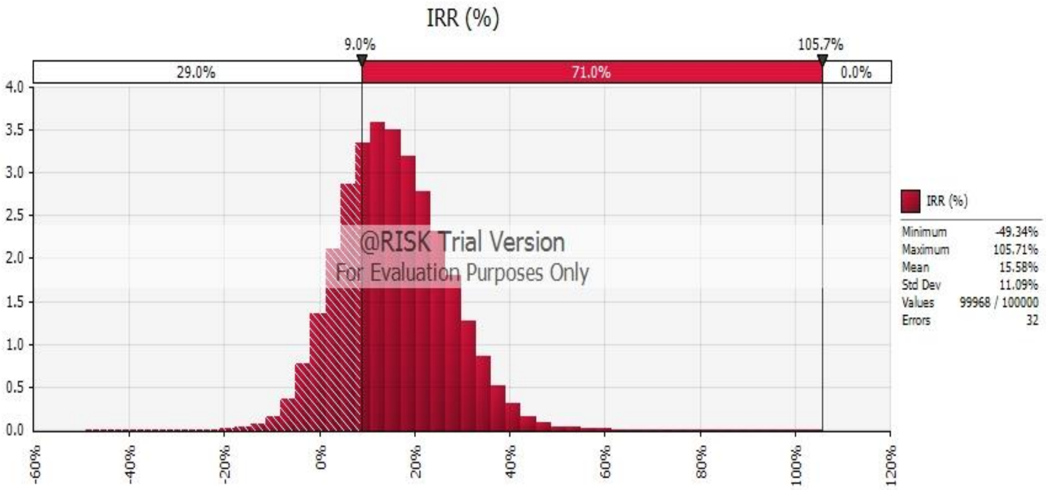


Figure 4.15: Probability distribution of an IRR less and greater than 9% for Scenario 2

Figure 4.15 shows that there is a probability of 0.71 that the project will have an IRR of 9% and greater. The Figure also shows that there is a 0.29 probability that the project will have an IRR of less than 9%. Since the risk has now been quantified on the IRR of Scenario 2, the company is able to make an informed decision based on the IRR. Figure 4.16 shows the probability of Scenario 2 having an IRR less than and greater than 5%.

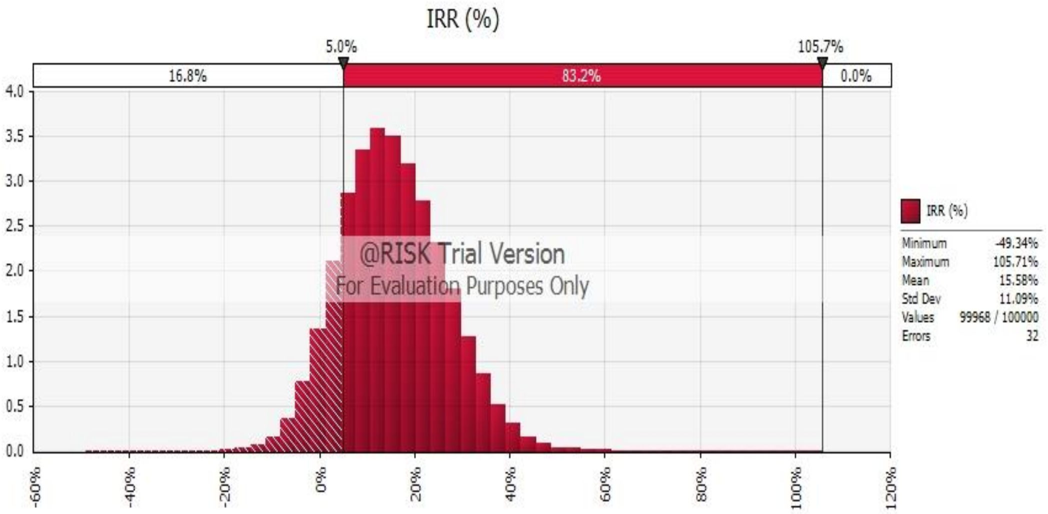


Figure 4.16: Probability distribution of an IRR less and greater than 5% for Scenario 2

Figure 4.16 shows that there is a probability of 0.83 of having an IRR of 5% (the discount rate) and greater if the company will opt for Scenario 2. The Figure also shows that there is a 0.17 probability that the project will have an IRR of less than 5%. According to the DCF valuation criteria with the respective probabilities of the NPV and IRR, Scenario 2 is still viable. However, depending on the company's risk appetite it may change its investment decision regarding Scenario 2.

4.3.3. A summary of the Monte Carlo DCF results

A summary of the results of the MCDCF analysis is shown in Table 4.8.

Table 4.8: Summary of MCDCF analysis results

	Scenario 1		Scenario 2	
Measure	**US$ million**	**Probability**	**US$ million**	**Probability**
NPV	< 9.60	0.36	< 11.35	0.32
NPV	< 0	0.21	< 0	0.17
NPV range	214.78	1	201.83	1
Measure	**%**	**Probability**	**%**	**Probability**
IRR	< 9	0.38	< 9	0.29
IRR	< 5	0.21	< 5	0.17
IRR range	117.50	1	155.00	1

According to the DCF decision criteria of the NPV and IRR, both scenarios are viable. The results from the MCDCF show that Scenario 2 is a better option according to the DCF decision criteria because it has a lower:

- probability and hence lower risk of both NPV and IRR being less than the NPV and IRR obtained from the static DCF;
- risk of NPV being negative;
- risk of IRR is less than the discount rate; and
- range of NPV.

The overall investment decision will depend on the risk appetite of the company. The MCDCF in @RISK software goes further than the static DCF by incorporating uncertainty and quantifying the risk associated with the projects' cash flow. It, however, does not incorporate the flexibility that management has to adjust the project in the face of uncertainty over its life. To incorporate flexibility, ROA was done and the Binomial-Lattice of Real Options assumptions and results are presented in the following Section.

4.4 Binomial-Lattice ROA

As stated in Chapter 2, management of the mining project under study have the option or flexibility to schedule the commencement of the development of the project before the mining lease expires in December 2032. They considered two scenarios which were valued with the static and probabilistic DCF in the previous sections. Among the four possible options, the management only have an option to defer the start of the project in both scenarios as explained in Chapter 2. The assumptions and input parameters for the Binomial-Lattice deferral option are given and discussed in the following Sub-section.

4.4.1 Binomial-Lattice input parameters

In order to calculate the option to defer the start of the project in both scenarios, the initial asset values (S_0) were calculated in the static DCFs analysis in Section 4.2. The initial asset value is equivalent to the present value of the net cash flows without considering the initial capital expenditure of the project (NPV-Total initial capital expenditure). The static DCF analysis valued the project using a fixed gold price of US$1300/oz, and a discount rate of 5%. The price of exercising the option which is also known as the strike price (X) is equivalent to the present value of the initial capital expenditure. The present value of the net cash flow and the initial capital expenditure were calculated using Equation 4.1 (Kvalevag, 2009):

$$PV = \sum_{i=1}^{n} \frac{FV_i}{(1+r)^n} \qquad 4.1$$

Where;

PV is the present value of the cash flow or initial capital expenditure, FV is the future value of the cash flow or initial capital expenditure, r is the discount rate and n is the number of years.

Tables 4.9 and 4.10 show the present values of the net cash flows (asset value) excluding initial capital expenditure for both scenarios, calculated using Equation 4.1.

Table 4.9: Asset value results for Scenario 1

Description	Units									Total
Number		1	2	3	4	5	6	7	8	
Year		2026	2027	2028	2029	2030	2031	2032	2033	
Net operating profit (After tax)	US$ million	-	-	17.98	32.81	30.42	23.80	13.55	30.05	148.61
Capex	US$ million	-27.31	-47.62	-18.90	-	-	-	-	-	-93.83
Discounted Net cash flow @5%	US$ million	-26.01	-43.19	15.53	26.65	23.29	11.96	5.53	12.17	25.93
Net DCF excluding capex (Asset value)	**US$ million**	**0**	**0**	**15.53**	**26.65**	**23.29**	**11.96**	**5.53**	**12.17**	**95.13**

Table 4.10: Asset value results for Scenario 2

Description	Units										Total
Number		1	2	3	4	5	6	7	8	9	
Year		2026	2027	2028	2029	2030	2031	2032	2033	2034	
Net operating profit (After tax)	US$ million	-	-	2.67	14.37	28.32	34.34	13.82	14.36	9.99	117.87
Capex	US$ million	-21.48	-25.04	-38.40	-	-	-	-	-		-84.92
Discounted Net cash flow @5%	US$ million	-20.46	-22.71	-30.86	11.82	22.19	25.40	9.82	9.72	6.44	11.36
Net DCF excluding initial capex (Asset value)	US$ million	0	0	2.30	11.82	22.19	25.40	9.82	9.72	6.44	87.69

Tables 4.9 and 4.10 show that Scenario 1 and 2 have asset values of US$95.13 million and US$87.69 million, respectively. Although both scenarios are valued on the same Mineral Resource, their asset values are different because of the different production schedules that also resulted in different LoM for the project. This makes the schedule with larger cash flows earlier in the LoM to have a greater asset value than one that generates larger cash flow later in the LoM. This is because of the time value of money concept explained in Section 2.4. The next input variable that was calculated is the exercise price of the delay option. Tables 4.11 and 4.12 show the exercise price of the two scenarios which were also calculated using Equation 4.1.

Table 4.11: Initial capital invested for Scenario 1

Initial capital (US$ million)		
Year	**FV**	**PV**
2026	27.31	26.01
2027	47.62	43.19
2028	24.54	21.20
Total	**99.47**	**90.40**

Table 4.12: Initial capital invested for Scenario 2

Initial capital (US$ million)		
Year	**FV**	**PV**
2026	21.48	20.46
2027	25.04	22.71
2028	38.40	33.17
Total	**84.92**	**76.34**

The present value of initial capital resulted in an exercise price of US$90.4 million and US$76.34 million to exercise the option of commencing the project in 2026. The option must be exercised before the company's mining licence expires in 2032, otherwise, the project must be sold. Currently, the company is producing from the surface operation which is expected to close in 2025, therefore the company has the right to exercise the option any time from the year 2026 but before the end of 2032. This means the company has the flexibility to defer the commencement of the project by seven years, otherwise, the deferral option expires without gaining any value from it.

The incremental time step (*dt*) of the lattice is influenced by a company's decision-making process on investments and the lifetime of the project (Smith, *et al*., 2017). In reality, projects require huge amounts of capital and investment decisions are made once or twice a year depending on the company's decision-making process, as such decisions require substantial information. The company under study makes large investment decisions once a year in September, therefore according to Smith, *et al.* (2017) the incremental time step of the lattice can be calculated using Equation 4.2:

$$dt = \frac{1}{project\ life\ time} \qquad 4.2$$

For Scenario 1;

$$dt_{s1} = \frac{1}{6}$$

$$= 0.17$$

For Scenario 2

$$dt_{s2} = \frac{1}{7}$$

$$= 0.14$$

Alternatively, one can select the size of the lattice and according to Kelly (1998), the incremental time step of a lattice can be calculated using Equation 2.14. In this research study, 35-time increments were selected, because this was a reasonable number for presentation purposes. The larger the number of nodes in a lattice the more simulations the lattice does, making the results more accurate and with a larger lattice.

$$dt = \frac{T}{N}$$

Where;

T is the time to maturity of the option or the investment decision horizon in years and N is the selected number of node in the lattice.

The incremental time step for both scenarios resulted in the following:

$$dt = \frac{7}{35}$$

$$= 0.2\ years$$

As stated in Chapter 3 the volatility of the underlying asset (σ) was taken to be the same as the volatility of the historical gold prices. The annualised volatility of gold prices was calculated from the daily gold spot prices for the period from January 2010 to December 2018. The standard deviation of the logarithmic changes in the daily gold price was computed and converted to the volatility of the lattices incremental time step of 0.2 years using Equations 4.4 to 4.7 (Dmouj, 2006)

$$\rho = log\frac{d_n}{d_{n-1}} \qquad 4.4$$

$$\sigma^2{}_n = \frac{1}{m}\sum_{i=1}^{m} \rho_{n-i}{}^2 \qquad 4.5$$

$$d_v = \sqrt{\sigma_n} \qquad 4.6$$

$$\sigma_{dt} = dt\sqrt{d_v} \qquad 4.7$$

Where,

ρ is the daily logarithmic change in gold spot price; n is today's gold price; m is the number of days in the period; σ^2 *is the variance;* d_v *is the standard deviation; dt is the incremental time step in days;* σ_{dt} *is the gold price volatility.*

The resultant gold price volatility from was calculated to be 6.06%. The detailed calculation is shown in Appendix 7.14. From the above input parameters, the upward (u) and downward (d) factors in the lattice were computed using Equations 2.16 and 2.17, respectively:

$$u = e^{\sigma \times \sqrt{dt}}$$

$$= e^{0.06 \times \sqrt{0.2}}$$

$$= 1.03$$

$$d = \frac{1}{u}$$

$$= \frac{1}{1.03}$$

$$= 0.97$$

The risk-free rate (r_f) for the project was provided by the company to be 2.032%. The risk-neutral probability (p) was calculated using Equation 2.15:

$$p = \frac{e^{(r_f \times dt)} - d}{u - d}$$

$$= \frac{e^{(0.02 \times 0.2)} - 0.97}{1.03 - 0.97}$$

$$= 0.57$$

4.4.2 Binomial-Lattice results

The lattices of the underlying asset values were constructed using the upward and downward factors as explained in Chapter 3. The constructed Binomial-Lattice of the underlying asset for Scenarios 1 and 2 are illustrated in Appendices 7.15 and 7.16. With an option life of 7 years the asset values for Scenario 1 ranged from a minimum of US$36.15 million to a maximum of US$241.28 million (the last column of Appendix 7.15), while Scenario 2 ranged from a minimum of US$33.94 million to a maximum of US$226.56 million (the last column of Appendix 7.16). The corresponding probabilities for each node in the lattice were constructed based on the Pascal's Triangle as explained in Chapter 2 using Equation 2.19. A detailed lattice with the probabilities of the corresponding nodes and time states is illustrated in Appendix 7.17. The major advantage of ROA over the DCF is its ability to evaluate flexibility as discussed in the previous chapters. In this research study, the company can determine the value of delaying the start of the project by any duration within the life of the option (seven years). For example, if the company choose to delay the project by 2.6 years, the column (lattice) under node 13 delay period in Appendix 7.15 shows the possible asset value. The probability of those asset value can be read from Appendix 7.17 in the column under node 13. Table 4.13 shows the outcome of possible mineral asset values with their respective probabilities for different states of the 2.6 year delay example.

Table 4.13: Mineral asset values for delaying by 2.6 years with respective probability for Scenario 1

State	Asset value (US$ million)	Probability
1	135.34	0.00012
2	128.20	0.00159
3	121.43	0.00952
4	115.02	0.03491
5	108.94	0.08728
6	103.19	0.15710
7	97.75	0.20947
8	92.58	0.20947
9	87.70	0.15710
10	83.07	0.08728
11	78.68	0.03491
12	74.53	0.00952
13	70.59	0.00159
14	66.87	0.00012
Total		**1**

Each time increment in the lattice has different states with different probabilities. However, in the column of each state there are certain states with the same probabilities. For example states 2 and 13 in Table 4.13 have the same probability. This is because they have the same number in Pascal's triangle and hence from Equation 2.19, they result in the same probabilities. This also results in a time increment having a summation probability of 1. Figure 4.17 shows the relationship between the probability and the asset value at different time states for Scenario 1

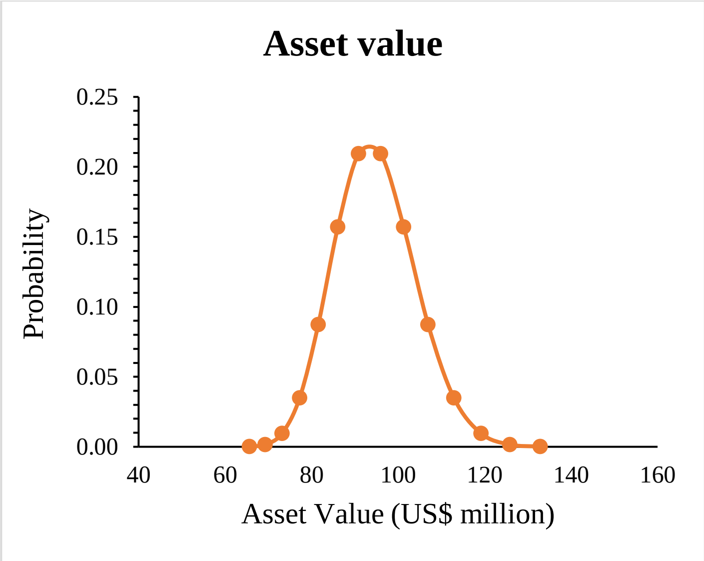


Figure 4.17: Range of mineral asset values for delaying by 2.6 years for Scenario 1

As stated in Chapter 2, the resultant probability distribution for a Binomial-Lattice is a normal distribution. This is shown in Figure 4.17 and this is because the numerical value in each node in the triangle denotes the number of possible paths that results in a specific outcome located in the same position in the lattice (Maistrov, 1974).

The risk-neutral probability, risk-free rate and incremental time step were used to construct the option value lattice using Equation 2.13. The resultant Binomial-Lattice for both scenarios are illustrated in Appendices 7.18 and 7 19. The economic value of the project if exercised in 2028 as planned, for Scenario 1 is US$17.34 million and US$21.52 million for Scenario 2, both having a probability of 1. The graphical representation of the option value at node 0 (initial value without delaying) versus the value generated by the static DCF is illustrated in Figure 4.18.

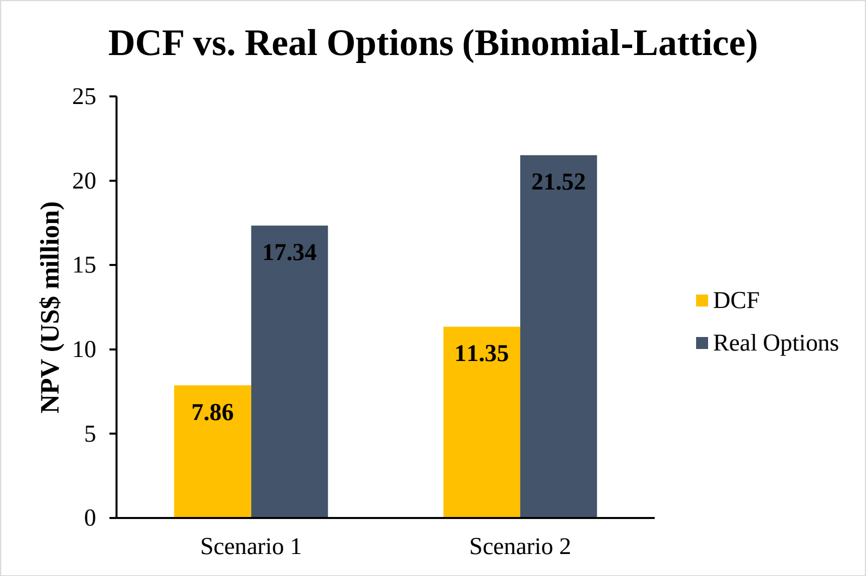


Figure 4.18: Static DCF vs Binomial-Lattice results

As shown in Figure 4.18 the value generated by the Binomial-Lattice is greater than the value generated by the static DCF. This is because the Binomial-Lattice uses a risk-free rate to discount cash flows. The option to defer the start of the project can only be exercised if the value of the underlying asset in each node is greater than the exercise price. Assuming that the project was deferred by 2.6 years, the outcomes of the option value with their respective probabilities for both scenarios were extracted from their respective lattices as shown in Appendices 7.15 to 7.19. Table 4.14 shows the outcome of possible mineral asset values and plan of action with their respective probabilities at the different states.

Table 4.14: The value of delaying the project by 2.6 years of Scenarios 1

Scenario 1				
Exercise price = US$90.4 million				
State	**Asset value**	**Strategy**	**Option Value**	**Probability**
1	135.34	Develop	52.56	0.00012
2	128.20	Develop	45.41	0.00159
3	121.43	Develop	38.65	0.00952
4	115.02	Develop	32.25	0.03491
5	108.94	Develop	26.23	0.08728
6	103.19	Develop	20.61	0.15710
7	97.75	Develop	15.49	0.20947
8	92.58	Develop	10.98	0.20947
9	87.70	Wait	7.23	0.15710
10	83.07	Wait	4.35	0.08728
11	78.68	Wait	2.35	0.03491
12	74.53	Wait	1.11	0.00952
13	70.59	Wait	0.46	0.00159
14	66.87	Wait	0.16	0.00012
Total				**1**

Figure 4.14 ultimately shows the plan of action for Scenario 1 if the project is to be delayed by 2.6 years as an example. This decision is based on the mineral asset value and the exercise price. However, there is a probability attached to the results, which may change the company's decision about delaying the project for 2.6 years. The minimum and maximum possible option value that the company can gain from delaying the project by this period range from US$0.16 million to US$52.56 million. However, the most likely outcome with a probability of 0.21 is US$15.49 million and US$10.98 million. It would be very risky to delay Scenario 1 for this long. The same results for Scenario 2 are shown in Table 4.15.

Table 4.15: The value of delaying the project by 2.6 years of Scenarios 2

Scenario 2				
Exercise price = US$76.34 million				
State	Asset value	Strategy	Option Value	Probability
1	124.76	Develop	54.85	0.00012
2	118.17	Develop	48.26	0.00159
3	111.93	Develop	42.02	0.00952
4	106.02	Develop	36.11	0.03491
5	100.42	Develop	30.52	0.08728
6	95.12	Develop	25.24	0.15710
7	90.10	Develop	20.27	0.20947
8	85.34	Develop	15.67	0.20947
9	80.84	Develop	11.51	0.15710
10	76.57	Develop	7.92	0.08728
11	72.53	Wait	5.02	0.03491
12	68.70	Wait	2.88	0.00952
13	65.07	Wait	1.46	0.00159
14	61.64	Wait	0.64	0.00012
Total				1

The minimum and maximum possible option value that the company can gain from delaying the project by 2.6 years range from US$0.64 million to US$54.85 million. However, the most likely outcome with a probability of 0.21 is US$21.5 million and US$15.54 million. It would also be very risky to delay Scenario 2 for this long. However, it is evident from the range that Scenario 2 has more value in flexibility when delayed because the project generates higher option value in the different states. The company is now able to read the different option values with their respective probabilities from the lattice. The resultant option strategy lattice is illustrated in Appendices 7.20 and 7.21. From the decision criteria of the Binomial-Lattice, the deferral option can be exercised from the initial start dates of the project in 2028. However, in certain nodes for both lattices, the value of the underlying asset is lower than the exercise price, and in these cases, the option should not be exercised. Furthermore, as the value of the option increases the longer one waits to exercise the option, but with lower probabilities. This is because the probability is spread over a wider range of possible outcomes in each column.

4.5 Chapter summary

This chapter applied three valuation techniques to compare the economic value of an underground mining project that considered two extraction scenarios. Scenario

1 considered that development of the preproduction phase will be conducted by a contractor. Alternatively, Scenario 2 considered that the owner will develop and operate the mine.

The static DCF technique determines the economic value of the two scenarios using static inputs. The results from the static DCF analysis show that considering both scenarios the project has a positive NPV with an IRR greater than the 5% discount rate. Scenario 1 has a NPV of US$9.60 million, with an IRR of 8.67% and a PP of 5.21 years. Scenario 2 has an NPV of US$11.35 million, an IRR of 8.95% and a PP of 5.49 years. From the results, Scenario 2 would be a better financial decision for the company. This is because it requires lower initial capital investment for hiring a contractor to conduct the preproduction phase. It also resulted in a higher NPV and IRR than Scenario 1. Even though the static DCF found that the project is viable it did not present the company with the confidence levels in the NPV and IRR or quantifies the risk associated with the economic value of the project. To assess the risk and enable properly informed investment decision making, a MCDCF analysis was done.

The MCDCF incorporated uncertainty in the input parameters, which were gold price, production rate, gold grade and unit variable cost and then quantified the risk associated with each scenario. Scenario 1 has a probability of 0.36 of having an NPV less than US$9.60 million and a probability of 0.38 of having an IRR less than 8.67%. Scenario 2 has a probability of 0.32 of having an NPV less than US$11.35 million and a probability of 0.29 of having an IRR less than 8.95%. The company has the flexibility to defer the start of the project. However, the MCDCF is not capable of introducing such flexibility into its modelling.

The Binomial-Lattice model was used to value the flexibility that management has to schedule the commencement of the project. Management has the option to delay the commencement of the project by 7 years, from 2026 before the mining lease expires at the end of 2032. The value of the project under both scenarios was greater than that of the DCF technique because the cash flows in ROA are discounted at a risk-free rate. The Binomial-Lattice ROA resulted in an option value of US$17.34 million and US$21.52 million for Scenario 1 and Scenario 2, respectively.

5 CONCLUSION AND RECOMMENDATIONS

5.1 Chapter overview

This chapter summarises the findings of the research study. From the findings, conclusions are made. Lastly, the chapter presents recommendations to the company and for future research work.

5.2 Research findings

Despite the wide application of the static DCF technique in mineral project valuation, it has some limitations. The technique assumes that all input variables are deterministic and known with certainty throughout the life of a project at the time of valuation. However, in reality input, variables are uncertain, stochastic in nature and vary in the future which can affect the outcome of the decision made from the static DCF. This makes the cash flows risky and can ultimately change the investment decision. Also, the static DCF technique does not present the company with the confidence levels in the NPV and IRR or the risk associated with the economic value of the project.

To incorporate uncertainty and quantify the risk associated with the case study project, a simulated DCF was used. Simulated cash flow is able to analyse different scenarios by varying uncertain variables in the cash flow such as mineral commodity prices, production rate, grade and unit operating costs. In this research study, simulations were done using @RISK software. This resulted in the project having a probability of 0.64 of having an NPV of US$9.60 million and greater and a probability of 0.36 of a NPV less than that, the same NPV generated by the static DCF, if the company were to invest in Scenario 1. Scenario 1 also resulted in probability 0.62 of an IRR of 8.67% and greater and a probability of 0.38 of an IRR less than that. Alternatively, Scenario 2 had a probability of 0.68 of having an NPV of US$11.35 million and greater, and a probability of 0.32 of a NPV less than that. This was the same NPV generated by the static DCF. Scenario 2 also resulted in a probability of 0.71 of an IRR of 8.95% and greater and a probability of 0.29 of an IRR less than that. Although the simulated DCF goes further than the static DCF by incorporating uncertainty and quantifying risk in the project valuation, it does not incorporate the flexibility that management has to adjust the project in the face of uncertainty over the LoM.

To incorporate flexibility, the Binomial-Lattice model (a model within the ROA techniques) was used to value the flexibility available to management. The real options that are available to management of mining operations are the option to expand, contract, abandon and defer the commencement of a project until a time when more information about uncertainties is available. An option to defer the commencement of the project was the only option valued in this research study because it is the only option available to new projects. From the literature, the value of an option must be greater than the economic value generated by the DCF technique. This is because the cash flows are discounted at a risk-free rate whereas, the static DCF and MCDCF use a risk-adjusted discount rate which is higher than a risk-free rate to discount cash flows. Furthermore, an option can only be exercised if the value of the mineral asset in each node at different states in time is greater than the exercise price. A summary for the results from the static DCF, MCDCF and Binomial-Lattice as shown in Table 5.1

Table 5.1: Summary of DCF, MCDCF and Binomial-Lattice results

	Scenario 1		**Scenario 2**	
Measure	**US$ million**	**Probability**	**US$ million**	**Probability**
NPV	< 9.60	0.36	< 11.35	0.32
NPV	< 0	0.21	< 0	0.17
NPV range	214.78	1	201.83	1
Measure	**%**	**Probability**	**%**	**Probability**
IRR	< 9	0.38	< 9	0.29
IRR	< 5	0.21	< 5	0.17
IRR range	117.50	1	155.00	1
Measure	**US$ million**	**Probability**	**US$ million**	**Probability**
Option value (t=0)	17.34	1	21.52	1
Percentage Increase	120.61%		89.60%	

The research study showed that by applying the Binomial-Lattice model with 35-time increments to the case study, the resultant NPV generated for Scenario 1 and Scenario 2 was US$17.34 million and US$21.52 million, respectively. This was an increase of 120.61 % and 89.60% for Scenario 1 and Scenario 2, respectively. It was noted that the Binomial-Lattice increases in complexity with an increase in the number of nodes because smaller time increments require a greater number of nodes to achieve the required lattice length of the option. The probability of each option value in the lattice is influenced by the number of paths that leads to its outcome. The resultant probability distribution of the lattice is identical to that of Pascal's triangle, which is a normal distribution. Although valuation techniques such as the MCDCF and the Binomial-Lattice of ROA quantify risk, the investment decisions are made based on probability, which makes decision making subjective. This is because different shareholders have different risk appetites and will mostly make their decisions based on past experiences with similar projects.

5.3 Conclusion

Valuation techniques under the income approach are only as accurate as the economic and technical input data available to conduct the valuation of mineral projects. Although the MCDCF quantifies risk and the Binomial-Lattice quantifies risk and flexibility, the ultimate investment decision is based on a company's risk appetite from past experiences, this makes investment decision making subjective. The only option available to new mining project is the delay options because all other options are only applicable to established operations. The Binomial-Lattice illustrated that there is more economic value when mineral projects or operations have flexibility within them. Results from the research study indicated that Scenario 2 is the better option because it has a lower risk of both NPV and IRR being less than the NPV and IRR obtained from the static DCF. Also, it has a higher economic value according to the Binomial-Lattice.

5.4 Recommendations

The results obtained from the research study indicate that the company should implement Scenario 2. This is because it resulted in a higher economic value from the static DCF, MDCF and the Binomial-Lattice. The company should, therefore, invest in increasing the confidence level of the project by conducting a feasibility study that will provide more information to accurately determine the economic value of the project at a greater confidence level. This will take into consideration all the vital risk associated with this project that can render the underground project uneconomical. Different Real Options techniques such as the Black-Scholes, Finite Difference method and Stochastic Monte Carlo methods can also be used to value different options or the same options to see the differences in their outcomes.

6 REFERENCES

Ajak, A. D., Lilford, E. & Topal, E., 2018. Real Option Identification Framework for Mine Operational Decision-Making. *Natural Resources Research,* 1573(8981), pp. 1-22.

Ampofo, K. D., 2017. Reasons why Real Options Analysis is not widely adopted in the mineral industry, *PhD Thesis,* Brisbane: The University of Queensland.

Anderloni, F., 2011. *Project Valuation Using Real Options Analysis.* [Online] Available..at:..http://tesi.cab.unipd.it/27611/1/ANDERLONI_FRANCESCO_578821_-_Project_Valuation_Using_Real_Op.pdf [Accessed 11 October 2018].

Anonymous, 2017. Mineral Resource, *Annual Report,* XYZ Corporation.

Arnold, T. & Shockly, R. L., 2010. Real Options Analysis and the Assumptions of Corporate Finance: A Non-Technical Review. *Multinational Finance Journal,* 14(2), pp. 29-71.

Awomewe, A. & Ogundele, O., 2008. The importance of the Payback method in Capital budgeting decision, *MBA Thesis,* Karlskrona: Blekinge Institute of Technology.

Bailey, W., Bhandari, A., Faiz, S., Srinivansan, S. &Weeds, H., 2004. Unlocking the value of real option. *Oilfields Review,* pp. 4-19.

Baurens, S., 2010. Valuation of metals and mining companies, Zurich: Basinvest.

Bernnan, M. R. & Schwartz, E. S., 1985. Evaluating Natural Resource Investment. *The Journal of Business,* 58(2), pp. 135-157.

Bjarnadottir, E., 2013. Real Options in Corporate Finance, *MSc Thesis,* Reykjavik: Reykjavik University.

Black, F. & Scholes, M., 1973. The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy,* 81(3), pp. 637-654.

Bloem, E. & Wees, J., 2003. Project Evaluation by Sensitivity, Scenario and Risk Analysis. *Australian Institute of Mining and Metallurgy,* pp. 91-93.

Bowe, M. & Lee, D. L., 2004. Project evaluation in the presence of multiple embedded real options. *Journal of Asian Economics,* 15(1), pp. 71-98.

Bowman, E. H. & Moskowitz, G. T., 2001. Real Options Analysis and Strategic Decision Making. *Organisation Science,* 12(6), pp. 772-777.

Brealey, R. A., Myers, S. C. & Allen, F., 2011. Portfolio Theory and the Capital Asset Pricing Model. In: B. Gordon, ed. *Principles of Corporate Finance.* New York: McGraw-Hill, pp. 193-194.

Brewer, K. D., Feng, Y. & Kwan, C. C. Y., 2012. Geometric Brownian motion, option pricing, and simulation: some spreadsheet-based exercises in financial modelling. *Spreadsheets in Education,* 5(4), pp. 1-36.

Brigham, E. F. & Gapenski, L. C., 1997. *Financial Management - Theory and Practice.* 8th ed. Orlando: The Dryden Press.

Brosch, R., 2001. *Portfolio-aspects in real options management,* Frankfurt: Johann Wolfgang Goethe-Universität.

Cawood, F. T., 2004. Mineral property valuation in South Africa: A basket of assets and legal rights. *The Journal of The South African Institute of Mining and Metallurgy,* 104(1), pp. 45-52.

Copeland, T. & Antikarov, V., 2001. *Real Options: A Practitioner's Guide,* New York: Texere.

Cox, J. C., Ross, S. A. & Rubinstein, M., 1979. Option Pricing: A simplified approach. *Journal of Financial Economics,* 7(1), pp. 229-263.

Damodaran, A., 2000. The Promise of Real Options. *Journal of Applied Corporate Finance,* 13(2), pp. 29-44.

Dehghani, H. & Ataee-pour, M., 2012. Determination of the effect of operating cost uncertainty on mining project evaluation. *Resources Policy,* 37(1), pp. 109-117.

Deloitte & Touche, 2010. *Managing Volatility: Risk in mining investment decisions,* Johannesburg: Deloitte & Touche.

Dimitrakopoulos, R. & Abdel, S., 2007. Evaluating mine plans under uncertainty: Can the real options make a difference?. *Resources Policy,* 32(3), pp. 116-125.

Dixit, A. K. & Pindyck, R. S., 1994. *Investment Under Uncertainty,* New Jersey: Princeton University Press.

Dmouj, A., 2006. Stock price modelling: Theory and practice, *MSc Thesis,* Amsterdam: Vrije Universiteit.

Domingo, E. & Lopez-Dee, E., 2007. *Valuation Methods of Mineral Resources.* Pretoria, London Group Meeting.

Drieza, J. A., Kicki, J. & Saluga, P., 2002. Real options in mine project budgeting - Polish mining industry example, *Risk Analysis III,* (ed: C A Brebbia), Southhampton: WIT Press: Southampton.

Eduardo, S., 2013. The Real Options Approach to Valuation: Challenges and Opportunities. *Latin American Journal of Economics,* 50(2), pp. 163-177.

Elkingto, T., Barrett, J. & Elkington, J., 2006. Practical application of real option concept to open pit projects. *International Seminar on Valuation,* Crawley, Australian Centre for Geomechanics.

Financial. Times,. 2018. *Lexicon.* [Online] Available. at: http://lexicon.ft.com/Term?term=cost-of-equity [Accessed 21 June 2018].

Fisher, I., 1930. *The Theory of Interest, as determined by Impatience to Spend Income and Opportunity to Invest it.* 1st ed. New York: Macmillan Publishers.

French, N. & Gabbrielli, 2005. Discounting cash flow: accounting for uncertainty. *Journal of Property Investment & Finance,* 23(1), pp. 75-89.

Frimpong, S., 1992. Evaluation of mineral ventures using modern financial methods, *Thesis,* Edmonton: University of Alberta.

Gardener, N., 2015. A Discounted Cash Flow and Real Option Valuation Approach for Iron Ore mining Company, *MSc Thesis,* Montreal: McGill University.

Gilbert, E., 2004. Investment Basics XLIX. An introduction to real options. *Investment Analysts Journal,* 33(60), pp. 49-52.

Giovanni, F., Carlos, C. & Cindy, P., 2017. NPV risk simulation of an open pit gold mine project under the O'Hara cost model using GAs. *International Journal of Mining Science and Technology,* 27(3), pp. 557-565.

Gotze, U., Northcott, D. & Schuster, P., 2008. *Investment Appraisal.* 1st ed. Berlin: Springer.

Guarnera, B. J. & Martin, M. D., 2011. Valuation of Mineral Properties. In: P. Darling, ed. *SME Mining Engineering Handbook.* Colorado: Society for Mining, Metallurgy and Exploration Inc., pp. 221-226.

Guj, P., 2013. An Introduction to mineral finance. In: P. Maxwell, ed. *Mineral Economics: Australian and Perspective.* Melbourne: The Australian Institute of Mining and Metallurgy, pp. 107-124.

Haque, M. A., Topal, E. & Lilford, E., 2014. A numerical study for mining project using real option valuation under commodity price uncertainty. *Resources Policy,* 39(1), pp. 115-123.

Haque, M. A., Topal, E. & Lilford, E., 2016. Estimation of Mining Project Values Through Real Option Valuation Using a Combination of Hedging Strategy and Mean Reversion Commodity Price. *Natural Resources Research,* 25(4), pp. 459-471.

Harrison, R. L., 2010. Introduction to Monte Carlo simulation. *AIP Conf Proc*, 1204(1), pp. 17-21.

Heuberger, R., 2005. Risk analysis in the mining industry. *The Journal of the South African Institute of Mining and Metallurgy,* 105(2), pp. 75-80.

International Valuation Standards Council, 2016. *IVS 105: Valuation Approaches and Methods,* London: International Valuation Standards Council.

Investopedia, 2018a. *Valuation.* [Online]. Available. at: https://www.investopedia.com/terms/v/valuation.asp [Accessed 13 March 2018].

Investopedia, 2018b. *Risk.* [Online]. Available. at:.https://www.investopedia.com/terms/r/risk.asp [Accessed 15 10 2018].

Investopedia, 2018c. *Monte. Carlo. Simulation.* [Online]. Available. at: https://www.investopedia.com/terms/m/montecarlosimulation.aspb [Accessed 15 June 2018].

Kazakidis, V. & Scoble, M., 2003. Planning for flexibility in underground mine production systems. *Mining Engineering,* 55(8), pp. 33-39.

Keeton, G., 2016. The impact of volatile commodity prices, exchange rates and interest rates: reflections of a former business economist, *SARB Biennial Conference,* Pretoria: South African Reserve Bank.

Kelly, S., 1998. A Binomial Lattice approach for valuing a mining property IPO. *The Quarterly Review of Economics and Finance,* 38(Special Issue), pp. 693-709.

Kennedy, B. A., 1990. *Surface mining.* 2nd ed. Baltimore: Society for Mining, Metallurgy and Exploration.

Koller, T., Goedhart, M. & Wessels, D., 2010. Why value value?. In: *Corporate Valuation: Measuring and Managing the Value of Companies.* New Jersey: John Wiley and Sons Inc., p. 4.

Kramna, E., 2014. Key input factors for discounted cash flow valuations. *WSEAS Transactions on Business and Economics,* 11(1), pp. 454-464.

Kvalevag, T., 2009. How do discounted cash flow analysis and real options differ as basis for decision making about oil and gas field developments?, *MSc Thesis,* Copenhagen: Copenhagen Business School.

Lattanzi, C. R., 2011. *Discounted cash flow analysis input parameters and Sensitivity*. [Online] Available at: http://web.cim.org/mes/pdf/valdaychrislattanzi.pdf [Accessed 3 October 2018].

Lawrence, M. J., 2001. *An outline of market-based approaches for mineral asset valuation best practice,* s.l.: VALMIN.

Laxen, P. A., Becker, S. M. & Rubin, R., 1979. Developments in the application of carbon-in-pulp to the recovery of gold from South African ores. *Journal of the South African Institute of Mining and Metallurgy,* 79(11), pp. 315-326.

Lilford, E. V. & Minnitt, R. C., 2005. A comparative study of Calculation methodologies for mineral developments. *The Journal of the South African Institue of Mining and Metallurgy,* 105(1), pp. 29-42.

Lohmann, J. R. & Baksh, S. N., 1993. The IRR, NPV and Payback Period and their relative performance in common capital budgeting decision procedures for dealing with risk. *The Engineering Economist,* 39(1), pp. 17-47.

Loughton, D. G., 1996. How use of a single corporate hurdle rate can undervalue low cost mines and long life mines and what to do about it: An example of modern asset pricing. Montreal, *Canadian Institute of Mining, Metallurgy and Petroleum.*

Macfarlane, A. S., 2001. A Code for the valuation of mineral properties and projects in South Africa. *The South African Institute of Mining and Metallurgy,* 102(1), pp. 37-48.

Maistrov, L. E., 1974. *Probability Theory: A Historical Sketch.* 1st ed. New York: Academic Press.

Mardones, J. L., 1993. Option valuation of real assets: Application to a copper mine with operating flexibility. *Resources Policy,* 19(1), pp. 51-65.

Maric, B., Bato, K., Radlovacki, V. & Delic, M., 2011. Observing the dependence between dynamic indicators of investment profitability - Relative net present value and internal rate of return. *African Journal of Business Management,* 5(26), pp. 10331-10337.

Mayer, Z. & Kazakidis, V., 2007. Decision making in flexible mine production system design using real options. *Journal of Construction Engineering and Management,* 133(2), pp. 169-180.

Mokenela, L., 2006. Managerial flexibility using ROV: A survey of top 40 JSE listed companies, *MBA Thesis,* Stellenbosch: Stellenbosch University.

Morley, C., Snowden, V. & Day, D., 1999. Financial impact of resource/reserve uncertainty. *The South African Institute of Mining and Metallurgy,* 99(6), pp. 293-302.

Mun, J., 2006. Real Options and Monte Carlo Simulation versus traditional DCF valuation in layman's terms. In: K. Leggio, D. Bodde & M. Taylor, eds. *Managing Enterprise Risk: What the electric industry experience implies for contermporary.* s.l.:Elsevier Global Energy Policy and Economic Series, pp. 75-106.

Musingwini, C., Minnitt, R. & Woodhall, M., 2007. Technical operating flexibility in the analysis of mine layouts and schedules. *The Journal of The Southern African Institute of Mining and Metallurgy,* 107(1), pp. 129-136.

Njowa, G., 2017. A framework to harmonise mineral asset valuation methodologies with existing and emerging financial reporting requirements, *PhD Thesis,* Johannesburg: University of the Witwatersrand.

Paddock, J. L., Siegel, D. R. & Smith, J. L., 1988. Option valuation of claims on real assets: The case of offshore petroleum leases. *The Quarterly Journal of Economics,* 103(3), pp. 479-508.

Palisade Corporation, 2016. *@RISK,* New York: Palisade.

Palm, S., Pearson, N. & Read, J., 1986. Option pricing: a new approach to mine valuation. *CIM Bulletin,* 79(889), pp. 61-66.

Park, H. M. & Nelson, M. G., 2013. *Mining project evaluation process for investment decisions,* s.l.: Mining Engineering magazine.

Park, S. & Matunhire, I., 2011a. Investigation of factors influencing the determination of discount rate in economic evaluation of mineral development projects. *The South African Institute of Mining and Metallurgy,* 111(11), pp. 773-779.

Park, S. & Matunhire, I., 2011b. Investigation of factors influencing the determination of discount rate and the application of quantitative methods for discount rate using risk factors in the minerals industry. *The Southern African Institute on Mining and Metallurgy,* pp. 413-428.

Pintaric, Z. & Kravanja, Z., 2017. The importance of using Discounted Cash Flow methodology in techno-economic analyses of energy and chemical production plants. *Journal of Sustainable Development of Energy, Water and Environment Systems,* 5(2), pp. 163-176.

Reddy, K. & Clinton, V., 2016. Simulating stock prices using Gemometric Brownian Motion: evidence from Australian Companies. *Australasian Accounting, Business and Finance Journal,* 10(3), pp. 23-47.

Runge, I., 1998. *Mining Economic.* Colorado: Society for Mining, Metallurgy and Exploration.

Salehi, M. & Tavakkoli-Moghaddam, R., 2008. Project selection by using a Fuzzy Topsis technique. *International Journal of Industrial and Manufacturing Engineering,* 2(4), pp. 375-380.

Samis, M. & Graham, A., 2003. Using Dynamic DCF and Real Options method to value and assess flexible mine project design. *Canadian Institute of Mining, Metallurgy and Petroleum,* pp. 720-738.

Samis, M. R. & Poulin, R., 1996. *Valuing management flexibility by derivative asset valuation.* Montreal, 98th Annual General Meeting of the Canadian Institute of Mining, Metallurgy and Petroleum.

Samis, M. R. & Poulin, R., 1998. Valuing management flexibility: A basis to compare the standard DCF and MAP valuation frameworks. *The CIM Bulletin*, 90(1), pp. 69-74.

SAMVAL Code, 2016. *The SAMVAL Code - SAMCODES.* [Online] Available at: www.samcode.co.za/codes/category/8-reporting-codes?download=119:samval-code [Accessed 14 March 2017].

Shafiee, S., Topal, E. & Nehring, M., 2009. Adjusted Real Option valuation to maximise mining project value - A Case Study Using Century Mine. Melbourne, *Australian Institute of Mining and Metallurgy,* pp. 125-134.

Silitonga, Y. P., 2015. "Real Options method" vs "Discounted Cash Flow Method" to analyze upstream oil & gas projects. *PM World Journal,* 5(7), pp. 1-26.

Smith, M., Matthews, W. & Driver, R., 2017. Using the real options lattice to select and plan capital projects. *The Business and Management Review,* 9(1), pp. 180-189.

Spencer-Young, J. & Durand, F., 2004. Real option valuation of game lodge concessions. *South African Journal of Business Management,* 35(1), pp. 294-301.

Steiger, F., 2008. The validity of company valuation using discounted cash flow methods, *MBA Thesis,* Cambridge: European Business School.

Stojanovic, C., 2013. Evaluating investment projects in mining industry by combining discount method and real option valuation. *Acta Montanistica Slovaca,* 18(4), pp. 217-225.

Stone, R., 1988. *Management of engineering projects.* London: Macmillan Education.

Torries, T. F., 1998. General description of the evaluation process. In: *Evaluating Mineral Projects: Applications and Misconceptions.* s.l.: Society for Mining, Metallurgy, and Exploration, pp. 3-4.

VALMIN..Code,.2015..*VALMIN*..[Online].Available..at:.http://www.valmin.org/docs/VALMIN_Code_2015_final.pdf [Accessed 23 May 2017].

Van Zyl, C. J., 2015. Mineral asset valuation of Kimberley mine for divestment purposes, *MSc Thesis,* Johannesburg: University of the Witwatersrand.

Weisstein, E. W., 2003. *Wolfram. MathWorld.* [Online].Available..at: http://mathworld.wolfram.com/PascalsTriangle.html [Accessed 15 February 2019].

World. Gold. Council, 2019. *Gold. Prices*. [Online].Available. at: https://www.gold.org/goldhub/data/gold-prices [Accessed 15 January 2019]

Zdravlje, R., 2011. Real Options analysis of mining projects, *MSc Thesis,* Vancouver: University of British Columbia.

Zettl, M., 2002. Valuing exploration and production projects by means of option pricing. *International Journal of Production Economics,* 78(1), pp. 109-116.

7 APPENDICES

Appendix 7.1: Pascal's triangle

(Source: Maistrov, 1974)

Number of Paths	2^n	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	1.64E+04	3.28E+04	6.55E+04	1.31E+05	2.62E+05	5.24E+05	1.05E+06	2.10E+06	4.19E+06	8.39E+06	1.68E+07	3.36E+07	6.71E+07	1.34E+08	2.68E+08	5.37E+08	1.07E+09	2.15E+09	4.29E+09	8.59E+09	1.72E+10	3.44E+10
		1.00E+00	5.00E-01	2.50E-01	1.25E-01	6.25E-02	3.13E-02	1.56E-02	7.81E-03	3.91E-03	1.95E-03	9.77E-04	4.88E-04	2.44E-04	1.22E-04	6.10E-05	3.05E-05	1.53E-05	7.63E-06	3.81E-06	1.91E-06	9.54E-07	4.77E-07	2.38E-07	1.19E-07	5.96E-08	2.98E-08	1.49E-08	7.45E-09	3.73E-09	1.86E-09	9.31E-10	4.66E-10	2.33E-10	1.16E-10	5.82E-11	2.91E-11
			5.00E-01	5.00E-01	3.75E-01	2.50E-01	1.56E-01	9.38E-02	5.47E-02	3.13E-02	1.76E-02	9.77E-03	5.37E-03	2.93E-03	1.59E-03	8.54E-04	4.58E-04	2.44E-04	1.30E-04	6.87E-05	3.62E-05	1.91E-05	1.00E-05	5.25E-06	2.74E-06	1.43E-06	7.45E-07	3.87E-07	2.01E-07	1.04E-07	5.40E-08	2.79E-08	1.44E-08	7.45E-09	3.84E-09	1.98E-09	1.02E-09
				2.50E-01	3.75E-01	3.75E-01	3.13E-01	2.34E-01	1.64E-01	1.09E-01	7.03E-02	4.39E-02	2.69E-02	1.61E-02	9.52E-03	5.55E-03	3.20E-03	1.83E-03	1.04E-03	5.84E-04	3.26E-04	1.81E-04	1.00E-04	5.51E-05	3.02E-05	1.65E-05	8.94E-06	4.84E-06	2.62E-06	1.41E-06	7.56E-07	4.05E-07	2.17E-07	1.15E-07	6.15E-08	3.27E-08	1.73E-08
					1.25E-01	2.50E-01	3.13E-01	3.13E-01	2.73E-01	2.19E-01	1.64E-01	1.17E-01	8.06E-02	5.37E-02	3.49E-02	2.22E-02	1.39E-02	8.54E-03	5.19E-03	3.11E-03	1.85E-03	1.09E-03	6.34E-04	3.67E-04	2.11E-04	1.21E-04	6.85E-05	3.87E-05	2.18E-05	1.22E-05	6.81E-06	3.78E-06	2.09E-06	1.15E-06	6.35E-07	3.48E-07	1.90E-07
						6.25E-02	1.56E-01	2.34E-01	2.73E-01	2.73E-01	2.46E-01	2.05E-01	1.61E-01	1.21E-01	8.73E-02	6.11E-02	4.17E-02	2.78E-02	1.82E-02	1.17E-02	7.39E-03	4.62E-03	2.85E-03	1.74E-03	1.06E-03	6.33E-04	3.77E-04	2.23E-04	1.31E-04	7.63E-05	4.42E-05	2.55E-05	1.47E-05	8.37E-06	4.76E-06	2.70E-06	1.52E-06
							3.13E-02	9.38E-02	1.64E-01	2.19E-01	2.46E-01	2.46E-01	2.26E-01	1.93E-01	1.57E-01	1.22E-01	9.16E-02	6.67E-02	4.72E-02	3.27E-02	2.22E-02	1.48E-02	9.70E-03	6.28E-03	4.01E-03	2.53E-03	1.58E-03	9.80E-04	6.01E-04	3.66E-04	2.21E-04	1.33E-04	7.91E-05	4.69E-05	2.76E-05	1.62E-05	9.45E-06
								1.56E-02	5.47E-02	1.09E-01	1.64E-01	2.05E-01	2.26E-01	2.26E-01	2.09E-01	1.83E-01	1.53E-01	1.22E-01	9.44E-02	7.08E-02	5.18E-02	3.70E-02	2.59E-02	1.78E-02	1.20E-02	8.02E-03	5.28E-03	3.43E-03	2.21E-03	1.40E-03	8.85E-04	5.53E-04	3.43E-04	2.11E-04	1.29E-04	7.83E-05	4.72E-05
									7.81E-03	3.13E-02	7.03E-02	1.17E-01	1.61E-01	1.93E-01	2.09E-01	2.09E-01	1.96E-01	1.75E-01	1.48E-01	1.21E-01	9.61E-02	7.39E-02	5.54E-02	4.07E-02	2.92E-02	2.06E-02	1.43E-02	9.80E-03	6.62E-03	4.41E-03	2.91E-03	1.90E-03	1.22E-03	7.84E-04	4.97E-04	3.13E-04	1.96E-04
										3.91E-03	1.76E-02	4.39E-02	8.06E-02	1.21E-01	1.57E-01	1.83E-01	1.96E-01	1.96E-01	1.85E-01	1.67E-01	1.44E-01	1.20E-01	9.70E-02	7.62E-02	5.84E-02	4.38E-02	3.22E-02	2.33E-02	1.65E-02	1.16E-02	7.99E-03	5.45E-03	3.67E-03	2.45E-03	1.62E-03	1.06E-03	6.85E-04
											1.95E-03	9.77E-03	2.69E-02	5.37E-02	8.73E-02	1.22E-01	1.53E-01	1.75E-01	1.85E-01	1.85E-01	1.76E-01	1.60E-01	1.40E-01	1.19E-01	9.74E-02	7.79E-02	6.09E-02	4.66E-02	3.49E-02	2.57E-02	1.87E-02	1.33E-02	9.39E-03	6.53E-03	4.49E-03	3.05E-03	2.05E-03
												9.77E-04	5.37E-03	1.61E-02	3.49E-02	6.11E-02	9.16E-02	1.22E-01	1.48E-01	1.67E-01	1.76E-01	1.76E-01	1.68E-01	1.54E-01	1.36E-01	1.17E-01	9.74E-02	7.92E-02	6.29E-02	4.89E-02	3.73E-02	2.80E-02	2.07E-02	1.50E-02	1.08E-02	7.63E-03	5.34E-03
													4.88E-04	2.93E-03	9.52E-03	2.22E-02	4.17E-02	6.67E-02	9.44E-02	1.21E-01	1.44E-01	1.60E-01	1.68E-01	1.68E-01	1.61E-01	1.49E-01	1.33E-01	1.15E-01	9.71E-02	8.00E-02	6.44E-02	5.09E-02	3.94E-02	3.00E-02	2.25E-02	1.67E-02	1.21E-02
														2.44E-04	1.59E-03	5.55E-03	1.39E-02	2.78E-02	4.72E-02	7.08E-02	9.61E-02	1.20E-01	1.40E-01	1.54E-01	1.61E-01	1.61E-01	1.55E-01	1.44E-01	1.30E-01	1.13E-01	9.67E-02	8.06E-02	6.57E-02	5.26E-02	4.13E-02	3.19E-02	2.43E-02
															1.22E-04	8.54E-04	3.20E-03	8.54E-03	1.82E-02	3.27E-02	5.18E-02	7.39E-02	9.70E-02	1.19E-01	1.36E-01	1.49E-01	1.55E-01	1.55E-01	1.49E-01	1.39E-01	1.26E-01	1.12E-01	9.60E-02	8.09E-02	6.67E-02	5.40E-02	4.30E-02
																6.10E-05	4.58E-04	1.83E-03	5.19E-03	1.17E-02	2.22E-02	3.70E-02	5.54E-02	7.62E-02	9.74E-02	1.17E-01	1.33E-01	1.44E-01	1.49E-01	1.49E-01	1.44E-01	1.35E-01	1.23E-01	1.10E-01	9.53E-02	8.10E-02	6.75E-02
																	3.05E-05	2.44E-04	1.04E-03	3.11E-03	7.39E-03	1.48E-02	2.59E-02	4.07E-02	5.84E-02	7.79E-02	9.74E-02	1.15E-01	1.30E-01	1.39E-01	1.44E-01	1.44E-01	1.40E-01	1.32E-01	1.21E-01	1.08E-01	9.45E-02
																		1.53E-05	1.30E-04	5.84E-04	1.85E-03	4.62E-03	9.70E-03	1.78E-02	2.92E-02	4.38E-02	6.09E-02	7.92E-02	9.71E-02	1.13E-01	1.26E-01	1.35E-01	1.40E-01	1.40E-01	1.36E-01	1.28E-01	1.18E-01
																			7.63E-06	6.87E-05	3.26E-04	1.09E-03	2.85E-03	6.28E-03	1.20E-02	2.06E-02	3.22E-02	4.66E-02	6.29E-02	8.00E-02	9.67E-02	1.12E-01	1.23E-01	1.32E-01	1.36E-01	1.36E-01	1.32E-01
																				3.81E-06	3.62E-05	1.81E-04	6.34E-04	1.74E-03	4.01E-03	8.02E-03	1.43E-02	2.33E-02	3.49E-02	4.89E-02	6.44E-02	8.06E-02	9.60E-02	1.10E-01	1.21E-01	1.28E-01	1.32E-01
																					1.91E-06	1.91E-05	1.00E-04	3.67E-04	1.06E-03	2.53E-03	5.28E-03	9.80E-03	1.65E-02	2.57E-02	3.73E-02	5.09E-02	6.57E-02	8.09E-02	9.53E-02	1.08E-01	1.18E-01
																						9.54E-07	1.00E-05	5.51E-05	2.11E-04	6.33E-04	1.58E-03	3.43E-03	6.62E-03	1.16E-02	1.87E-02	2.80E-02	3.94E-02	5.26E-02	6.67E-02	8.10E-02	9.45E-02
																							4.77E-07	5.25E-06	3.02E-05	1.21E-04	3.77E-04	9.80E-04	2.21E-03	4.41E-03	7.99E-03	1.33E-02	2.07E-02	3.00E-02	4.13E-02	5.40E-02	6.75E-02
																								2.38E-07	2.74E-06	1.65E-05	6.85E-05	2.23E-04	6.01E-04	1.40E-03	2.91E-03	5.45E-03	9.39E-03	1.50E-02	2.25E-02	3.19E-02	4.30E-02
																									1.19E-07	1.43E-06	8.94E-06	3.87E-05	1.31E-04	3.66E-04	8.85E-04	1.90E-03	3.67E-03	6.53E-03	1.08E-02	1.67E-02	2.43E-02
																										5.96E-08	7.45E-07	4.84E-06	2.18E-05	7.63E-05	2.21E-04	5.53E-04	1.22E-03	2.45E-03	4.49E-03	7.63E-03	1.21E-02
																											2.98E-08	3.87E-07	2.62E-06	1.22E-05	4.42E-05	1.33E-04	3.43E-04	7.84E-04	1.62E-03	3.05E-03	5.34E-03
																												1.49E-08	2.01E-07	1.41E-06	6.81E-06	2.55E-05	7.91E-05	2.11E-04	4.97E-04	1.06E-03	2.05E-03
																													7.45E-09	1.04E-07	7.56E-07	3.78E-06	1.47E-05	4.69E-05	1.29E-04	3.13E-04	6.85E-04
																														3.73E-09	5.40E-08	4.05E-07	2.09E-06	8.37E-06	2.76E-05	7.83E-05	1.96E-04
																															1.86E-09	2.79E-08	2.17E-07	1.15E-06	4.76E-06	1.62E-05	4.72E-05
																																9.31E-10	1.44E-08	1.15E-07	6.35E-07	2.70E-06	9.45E-06
																																	4.66E-10	7.45E-09	6.15E-08	3.48E-07	1.52E-06
																																		2.33E-10	3.84E-09	3.27E-08	1.90E-07
																																			1.16E-10	1.98E-09	1.73E-08
																																				5.82E-11	1.02E-09
																																					2.91E-11

Appendix 7.2: Stope insitu Mineral Resource

Stope no.	Stope type	High grade		Low grade		Internal waste		Stope Insitu Resource	
	Primary/Secondary	tonnes	g/t of Au	tonnes	g/t of Au	tonnes	g/t of Au	tonnes	g/t of Au
1	S	28 018.00	8.63	13 343.00	0.86	593.00	0.06	41 954.00	6.04
2	P	24 672.00	8.60	13 679.00	0.94	384.00	0.07	38 735.00	5.81
3	S	21 736.00	7.81	12 651.00	0.83	97.00	0.06	34 484.00	5.23
4	P	22 146.00	6.80	15 422.00	0.71	134.00	0.06	37 702.00	4.28
5	S	25 027.00	8.03	16 173.00	0.65	1 243.00	0.05	42 443.00	4.98
6	P	32 227.00	10.13	13 650.00	0.70	3 218.00	0.07	49 095.00	6.85
7	S	37 749.00	10.63	11 770.00	0.76	1 911.00	0.11	51 430.00	7.98
8	P	37 941.00	10.26	17 028.00	0.78	399.00	0.07	55 368.00	7.27
9	S	36 347.00	9.48	16 683.00	0.74	432.00	0.06	53 462.00	6.68
10	P	37 718.00	8.17	17 181.00	0.59	159.00	0.08	55 058.00	5.78
11	S	34 101.00	7.22	18 606.00	0.58	216.00	0.15	52 923.00	4.86
12	P	30 756.00	7.34	19 300.00	0.62	804.00	0.11	50 860.00	4.68
13	S	32 677.00	8.06	19 341.00	0.52	468.00	0.10	52 486.00	5.21
14	P	33 774.00	8.75	20 116.00	0.55	131.00	0.04	54 021.00	5.68
15	S	36 924.00	8.76	19 106.00	0.57	749.00	0.05	56 779.00	5.89
16	P	41 445.00	8.88	23 270.00	0.64	1 059.00	0.06	65 774.00	5.82
17	S	39 144.00	10.32	21 053.00	0.74	697.00	0.06	60 894.00	6.89
18	P	40 235.00	11.26	18 385.00	0.89	280.00	0.06	58 900.00	7.97
19	S	42 787.00	11.79	17 615.00	0.91	126.00	0.07	60 528.00	8.60
20	P	47 889.00	10.95	17 850.00	0.78	294.00	0.04	66 033.00	8.15
21	S	46 558.00	8.29	15 981.00	0.65	236.00	0.06	62 775.00	6.31
22	P	36 300.00	7.22	15 089.00	0.61	53.00	0.07	51 442.00	5.27
23	S	27 422.00	7.70	12 136.00	0.57	0.00	0.00	39 558.00	5.51
24	P	23 719.00	7.23	13 702.00	0.55	197.00	0.06	37 618.00	4.76
25	S	20 025.00	6.81	12 053.00	0.58	117.00	0.05	32 195.00	4.45
26	P	16 734.00	6.84	12 590.00	0.60	0.00	0.00	29 324.00	4.16
27	S	17 223.00	7.98	16 469.00	0.67	13.00	0.06	33 705.00	4.41
28	P	18 384.00	8.44	18 350.00	0.68	38.00	0.03	36 772.00	4.56
29	S	18 828.00	7.74	16 184.00	0.61	129.00	0.05	35 141.00	4.43
30	P	19 931.00	7.04	16 741.00	0.58	274.00	0.06	36 946.00	4.06
31	S	17 231.00	6.66	16 107.00	0.64	286.00	0.03	33 624.00	3.72
32	P	10 895.00	6.15	8 605.00	0.59	0.00	0.00	19 500.00	3.70
Total		**956 563.00**	**8.78**	**516 229.00**	**0.68**	**14 737.00**	**0.07**	**1 487 529.00**	**5.88**

Appendix 7.3: Mineable Resource after applying modifying factors

	Primary		Secondary		Development		Total	
	tonnes	g/t of Au	tonnes	g/t of Au	tonnes	g/t of Au	tonnes	g/t of Au
Insitu resource	597 301.00	6.29	643 714.00	6.23	246 512.00	3.99	1 487 527.00	5.88
West wall	15 303.00	0.65	15 480.00	0.68	-	-	30 783.00	0.62
Stope back	10 742.00	0.62	10 469.00	0.60	-	-	21 211.00	0.61
Back & sidewalls	-	-	-	-	9 561.00	0.61	9 561.00	0.61
Backfill walls	3 762.00	-	28 806.00	-	-	-	32 568.00	-
Geotech pillars	-	-	92 144.00	5.74	-	-	-	-
Total	627 108.00	6.02	606 325.00	5.76	256 073.00	3.86	1 489 506.00	5.56
Recovery	0.95	-	0.95	-	1.00	-	-	-
Mineable Resource	**595 752.60**	**6.02**	**576 008.75**	**5.76**	**256 073.00**	**3.86**	**1 427 834.35**	**5.52**

Appendix 7.4: Capital costs for Scenario 1 and 2

Scenario 1	
Description	**US$ millions**
Initial capital costs	
Capital development	35.143
Operating cost transferred to capital	21.544
Initial mobile equipment	2.440
Mine infrastructure	34.707
Sub-total	**93.834**
Sustaining capital costs	
Capital development	2.710
Mobile equipment	0.943
Mine infrastructure	3.438
Sub-total	**7.09**
Total	**100.92**

Scenario 2	
Description	**US$ millions**
Initial capital costs	
Capital development	31.169
Operating cost transferred to capital	16.091
Initial mobile equipment	2.924
Mine infrastructure	34.737
Sub-total	**84.92**
Sustaining capital costs	
Capital development	5.91
Mobile equipment	0.46
Mine infrastructure	3.292
Sub-total	**9.661**
Total	**94.81**

Appendix 7.5: Detailed static discounted cash flow for Scenario 1

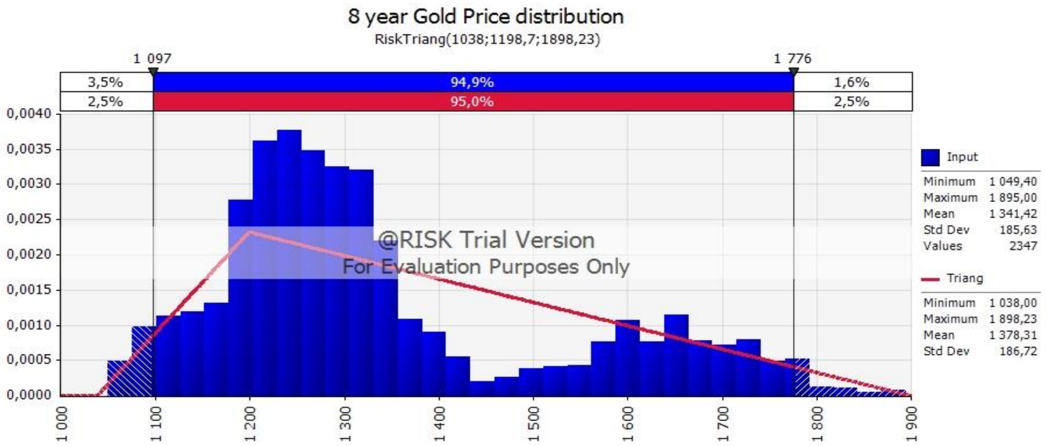
Description	Year	2026	2027	2028	2029	2030	2031	2032	2033	Total
Ore milled	Tonnes	-	-	164 269.00	273 750.00	273 760.00	273 699.00	234 748.00	207 608.00	1 427 834.00
Grade	g/t	-	-	6.13	5.93	5.77	5.26	4.38	5.79	5.52
Metallurgical recovery	%	-	-	97%	97%	97%	97%	97%	97%	73%
Gold recovered	g	-	-	976 759.90	1 574 637.38	1 532 207.34	1 396 467.04	997 350.35	1 165 988.81	7 643 410.82
Gold recovered	Ounces	-	-	34 454.23	55 543.76	54 047.08	49 258.98	35 180.54	41 129.09	269 613.67
Gold price	US$	-	-	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00
Revenue	**US$**	-	-	**44 790 497.37**	**72 206 886.40**	**70 261 206.41**	**64 036 671.78**	**45 734 697.25**	**53 467 816.09**	**350 497 775.29**
Sales costs										
Transportation, security and insurance	US$	-	-	-47 415.92	-55 908.68	-55 305.97	-53 377.80	-47 708.41	-50 103.89	-309 820.66
Refinery treatment, assay, finance charges, payable metal loss	US$	-	-	-55 308.79	-89 101.39	-86 703.20	-79 031.03	-56 472.58	-66 004.18	-432 621.17
Other costs	US$	-	-	-6 887.40	-11 103.20	-10 804.01	-9 846.87	-7 032.59	-8 221.70	-53 895.77
Total sales costs	**US$**	-	-	**-109 612.11**	**-156 113.26**	**-152 813.18**	**-142 255.70**	**-111 213.58**	**-124 329.78**	**-796 337.61**
Gross profit	**US$**	-	-	**44 680 885.26**	**72 050 773.13**	**70 108 393.23**	**63 894 416.08**	**45 623 483.67**	**53 343 486.31**	**349 701 437.68**
										-
Unit variable costs										
Processing cost	US$/tonne milled	-	-	12.72	12.72	12.72	12.72	12.72	12.72	
Site general	US$/tonne milled	-	-	3.50	3.50	3.50	3.50	3.50	3.50	
Corporate costs	US$/tonne milled	-	-	2.15	2.15	2.15	2.15	2.15	2.15	
Development in waste	US$/tonne milled	-	-	15.75	7.47	12.48	14.02	2.72	0.70	
Development in ore	US$/tonne milled	-	-	5.62	10.17	7.43	8.12	10.35	3.79	
Stope grouting	US$/tonne milled	-	-	1.70	2.27	1.93	2.15	2.69	0.71	
Bench stoping & mucking	US$/tonne milled		-	8.94	9.88	10.54	10.44	9.83	11.45	
Truck haulage	US$/tonne milled	-	-	15.98	16.13	19.41	21.59	16.20	14.74	
Pastefill	US$/tonne milled	-	-	8.72	14.34	10.06	10.33	11.94	6.60	
Mine services	US$/tonne milled	-	-	7.79	11.09	11.36	10.96	11.51	10.45	
Mine supervision & technical	US$/tonne milled	-	-	8.74	11.38	11.29	10.46	12.05	11.36	
Contractor indirects	US$/tonne milled	-	-	2.99	-	-	-	-	-	
Leasing	US$/tonne milled	-	-	8.30	11.24	11.26	10.97	11.18	3.79	
Mine power	US$/tonne milled	-	-	12.02	19.01	19.12	18.27	20.29	18.02	
Total unit variable costs	**US$/tonne milled**	-	-	**114.92**	**131.35**	**133.25**	**135.68**	**127.13**	**99.98**	**123.72**
Variable costs										
Processing cost	US$	-	-	-2 089 501.68	-3 482 100.00	-3 482 227.20	-3 481 451.28	-2 985 994.56	-2 640 773.76	
Site general	US$	-	-	-574 941.50	-958 125.00	-958 160.00	-957 946.50	-821 618.00	-726 628.00	
Corporate costs	US$	-	-	-353 178.35	-588 562.50	-588 584.00	-588 452.85	-504 708.20	-446 357.20	
Development in waste	US$	-	-	-2 587 236.75	-2 044 912.50	-3 416 524.80	-3 837 259.98	-638 514.56	-145 325.60	
Development in ore	US$	-	-	-923 191.78	-2 784 037.50	-2 034 036.80	-2 222 435.88	-2 429 641.80	-786 834.32	
Stope grouting	US$	-	-	-279 257.30	-621 412.50	-528 356.80	-588 452.85	-631 472.12	-147 401.68	
Bench stoping & mucking	US$	-	-	-1 468 564.86	-2 704 650.00	-2 885 430.40	-2 857 417.56	-2 307 572.84	-2 377 111.60	
Truck haulage	US$	-	-	-2 625 018.62	-4 415 587.50	-5 313 681.60	-5 909 161.41	-3 802 917.60	-3 060 141.92	
Pastefill	US$	-	-	-1 432 425.68	-3 925 575.00	-2 754 025.60	-2 827 310.67	-2 802 891.12	-1 370 212.80	
Mine services	US$	-	-	-1 279 655.51	-3 035 887.50	-3 109 913.60	-2 999 741.04	-2 701 949.48	-2 169 503.60	
Mine supervision & technical	US$	-	-	-1 435 711.06	-3 115 275.00	-3 090 750.40	-2 862 891.54	-2 828 713.40	-2 358 426.88	
Contractor indirects	US$	-	-	-491 164.31	-	-	-	-	-	
Leasing	US$	-	-	-1 363 432.70	-3 076 950.00	-3 082 537.60	-3 002 478.03	-2 624 482.64	-786 834.32	
Mine power	US$	-	-	-1 974 513.38	-5 203 987.50	-5 234 291.20	-5 000 480.73	-4 763 036.92	-3 741 096.16	
Total variable costs	US$	-	-	-18 877 793.48	-35 957 062.50	-36 478 520.00	-37 135 480.32	-29 843 513.24	-20 756 647.84	
Other costs										
CSR projects & charity	US$	-	-	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-2 400 000.00
Total costs	**US$**	-	-	**-19 277 793.48**	**-36 357 062.50**	**-36 878 520.00**	**-37 535 480.32**	**-30 243 513.24**	**-21 156 647.84**	**-181 449 017.38**
Operating profit	**US$**	-	-	**25 403 091.78**	**35 693 710.63**	**33 229 873.23**	**26 358 935.76**	**15 379 970.43**	**32 186 838.47**	**168 252 420.30**
Capital costs										
Development	US$	-15 136 000.00	-12 415 000.00	-7 592 000.00	-	-	-	-	-	
Operationg costs transferred to capital	US$	0.00	-11 819 000.00	-9 725 000.00	-	-	-	-	-	
Initial mobile equipment	US$	-281 000.00	-882 000.00	-1 278 000.00	-	-	-	-	-	
Mine infrastructure	US$	-11 896 000.00	-22 502 000.00	-309 000.00	-	-	-	-	-	
Sustaining capital cost	US$	-	-	-5 635 000.00	-414 000.00	-689 000.00	-352 000.00	-	-	-7 090 000.00
Total capital cost	**US$**	**-27 313 000.00**	**-47 618 000.00**	**-24 539 000.00**	**-414 000.00**	**-689 000.00**	**-352 000.00**	-	-	**-100 925 000.00**
Unredeemed capex	**US$**	**-27 313 000.00**	**-74 931 000.00**	**-74 066 908.22**	**-38 787 197.59**	**-6 246 324.36**				**-201 583 818.78**
Operating profit after capex	**US$**	-	-	-	-	-	**19 760 611.40**	**35 140 581.83**	**32 186 838.47**	**165 852 420.30**
Royalties @ 4%	**US$**	-	-	-1 791 619.89	-2 888 275.46	-2 810 448.26	-2 561 466.87	-1 829 387.89	-2 138 712.64	-14 019 911.01
Net operating profit	**US$**	-	-	**23 611 471.88**	**32 805 435.18**	**30 419 424.97**	**23 797 468.89**	**13 550 582.54**	**30 048 125.82**	**154 232 509.29**
Taxable Income	**US$**	-	-	-			19 760 611.40	35 140 581.83	32 186 838.47	87 088 031.70
Corporate Taxes @ 37.5%	**US$**	-	-	-	-	-	**-7 410 229.27**	**-13 177 718.19**	**-12 070 064.43**	**-32 658 011.89**
Net Cashflow	**US$**	**-27 313 000.00**	**-47 618 000.00**	**-927 527.15**	**32 391 436.15**	**29 730 425.94**	**16 035 240.59**	**372 865.33**	**17 978 062.37**	**20 649 503.22**
Discounted Net Cashflow @5%	**US$**	**-26 012 380.95**	**-43 190 929.71**	**-801 232.82**	**26 648 514.68**	**23 294 566.67**	**11 965 743.41**	**264 988.43**	**12 168 260.26**	**4 337 529.97**

NPV (US$ million)	**4.34**
IRR	**6.75%**

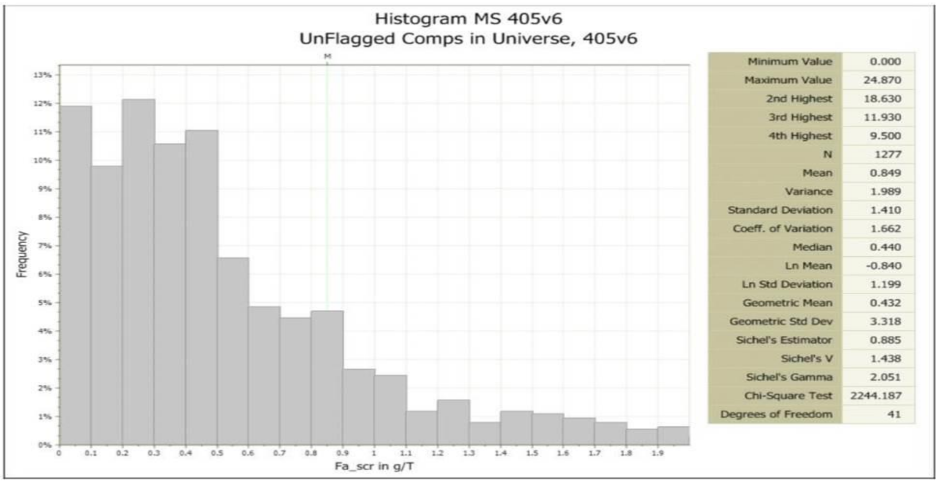
Appendix 7.6: Detailed static discounted cash flow for Scenario 2

Description	Year	2026	2027	2028	2029	2030	2031	2032	2033	2034	Total
Ore milled	Tonnes	-	-	18 899.00	239 148.00	273 699.00	273 749.00	273 751.00	234 852.00	113 735.00	1 427 833.00
Grade	g/t	-	-	3.85	5.80	5.89	5.86	5.00	5.11	5.57	6.00
Metallurgical recovery	%	-	-	97%	97%	97%	97%	97%	97%	97%	97%
Gold recovered	g	-	-	70 578.32	1 345 446.65	1 563 724.50	1 556 044.07	1 327 692.35	1 164 090.91	614 498.83	7 642 075.62
Gold recovered	ounces	-	-	2 489.58	47 459.29	55 158.82	54 887.90	46 833.02	41 062.14	21 675.83	269 566.58
Gold price	US$	-	-	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00	1 300.00
Revenue	**US$**	**-**	**-**	**3 236 453.35**	**61 697 070.58**	**71 706 463.27**	**71 354 267.89**	**60 882 925.94**	**53 380 785.51**	**28 178 581.32**	**350 436 547.86**
Sales costs											
Transportation, security and insurance	US$	-	-	-34 543.75	-52 653.06	-55 753.66	-55 644.56	-52 400.86	-50 076.93	-42 270.06	-343 342.89
Refinery treatment, assay, finance charges, payable metal loss	US$	-	-	-4 090.55	-76 147.31	-88 484.58	-88 050.48	-75 143.82	-65 896.91	-34 833.45	-432 647.11
Other costs	US$	-	-	-497.67	-9 487.11	-11 026.25	-10 972.09	-9 361.92	-8 208.32	-4 333.00	-53 886.36
Total sales costs	**US$**	**-**	**-**	**-39 131.97**	**-138 287.48**	**-155 264.49**	**-154 667.13**	**-136 906.60**	**-124 182.17**	**-81 436.51**	**-748 439.85**
Gross profit	**US$**	**-**	**-**	**3 197 321.38**	**61 558 783.10**	**71 551 198.77**	**71 199 600.76**	**60 746 019.34**	**53 256 603.35**	**28 097 144.81**	**349 606 671.50**
											-
Unit variable costs											
Processing cost	US$/tonne milled	-	-	-	12.72	12.72	12.72	12.72	12.72	12.72	12.72
Site general	US$/tonne milled	-	-	-	3.50	3.50	3.50	3.50	3.50	3.50	3.50
Corporate costs	US$/tonne milled	-	-	-	2.15	2.15	2.15	2.15	2.15	2.15	2.15
Development in waste	US$/tonne milled	-	-	-	22.16	17.56	6.64	4.82	2.45	0.00	8.94
Development in ore	US$/tonne milled	-	-	-	9.99	12.12	6.25	7.80	8.19	1.21	7.59
Stope grouting	US$/tonne milled	-	-	-	2.47	2.04	1.88	2.40	2.00	0.00	1.80
Bench stoping & mucking	US$/tonne milled		-	-	9.93	9.35	10.91	10.53	10.31	12.11	10.52
Truck haulage	US$/tonne milled	-	-	-	20.84	20.14	17.23	17.38	15.85	13.96	17.57
Pastefill	US$/tonne milled	-	-	-	12.33	10.23	11.08	11.18	10.24	8.70	10.63
Mine services	US$/tonne milled	-	-	-	11.94	11.88	10.42	10.31	10.91	7.24	10.45
Mine supervision & technical	US$/tonne milled	-	-	-	11.20	11.35	10.50	10.30	11.74	9.33	10.74
Contractor indirects	US$/tonne milled	-	-	-	-	-	-	-	-	-	-
Leasing	US$/tonne milled	-	-	-	11.62	11.88	5.85	4.71	2.72	0.27	6.18
Mine power	US$/tonne milled	-	-	-	16.91	19.39	18.23	17.52	19.83	15.94	17.97
Total unit variable costs	**US$/tonne milled**	**-**	**-**	**-**	**147.76**	**144.31**	**117.36**	**115.32**	**112.61**	**87.13**	**106.23**
Variable costs											
Processing cost	US$	-	-	-	-3 041 962.56	-3 481 451.28	-3 482 087.28	-3 482 112.72	-2 987 317.44	-1 446 709.20	
Site general	US$	-	-	-	-837 018.00	-957 946.50	-958 121.50	-958 128.50	-821 982.00	-398 072.50	
Corporate costs	US$	-	-	-	-514 168.20	-588 452.85	-588 560.35	-588 564.65	-504 931.80	-244 530.25	
Development in waste	US$	-	-	-	-5 299 519.68	-4 806 154.44	-1 817 693.36	-1 319 479.82	-575 387.40	0.00	
Development in ore	US$	-	-	-	-2 389 088.52	-3 317 231.88	-1 710 931.25	-2 135 257.80	-1 923 437.88	-137 619.35	
Stope grouting	US$	-	-	-	-590 695.56	-558 345.96	-514 648.12	-657 002.40	-469 704.00	0.00	
Bench stoping & mucking	US$	-	-	-	-2 374 739.64	-2 559 085.65	-2 986 601.59	-2 882 598.03	-2 421 324.12	-1 377 330.85	
Truck haulage	US$	-	-	-	-4 983 844.32	-5 512 297.86	-4 716 695.27	-4 757 792.38	-3 722 404.20	-1 587 740.60	
Pastefill	US$	-	-	-	-2 948 694.84	-2 799 940.77	-3 033 138.92	-3 060 536.18	-2 404 884.48	-989 494.50	
Mine services	US$	-	-	-	-2 855 427.12	-3 251 544.12	-2 852 464.58	-2 822 372.81	-2 562 235.32	-823 441.40	
Mine supervision & technical	US$	-	-	-	-2 678 457.60	-3 106 483.65	-2 874 364.50	-2 819 635.30	-2 757 162.48	-1 061 147.55	
Contractor indirects	US$	-	-	-	-	-	-	-	-	-	
Leasing	US$	-	-	-	-2 778 899.76	-3 251 544.12	-1 601 431.65	-1 289 367.21	-638 797.44	-30 708.45	
Mine power	US$	-	-	-	-4 043 992.68	-5 307 023.61	-4 990 444.27	-4 796 117.52	-4 657 115.16	-1 812 935.90	
Total variable costs	US$			-	-35 336 508.48	-39 497 502.69	-32 127 182.64	-31 568 965.32	-26 446 683.72	-9 909 730.55	
Other costs											
CSR projects & charity	US$	-	-	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-400 000.00	-2 800 000.00
Total costs	**US$**	**-**	**-**	**-400 000.00**	**-35 736 508.48**	**-39 897 502.69**	**-32 527 182.64**	**-31 968 965.32**	**-26 846 683.72**	**-10 309 730.55**	**-167 376 842.85**
Operating profit	**US$**	**-**	**-**	**2 797 321.38**	**25 822 274.62**	**31 653 696.08**	**38 672 418.12**	**28 777 054.02**	**26 409 919.63**	**17 787 414.26**	**154 132 683.84**
Capital costs											
Development	US$	-9 976 000.00	-11 660 000.00	-9 533 000.00	-	-	-	-	-	-	
Operationg costs transferred to capital	US$	0.00	-931 000.00	-15 160 000.00	-	-	-	-	-	-	
Initial mobile equipment	US$	-1 892 000.00	-133 000.00	-898 000.00	-	-	-	-	-	-	
Mine infrastructure	US$	-9 612 000.00	-12 319 000.00	-12 805 000.00	-	-	-	-	-	-	
Sustaining capital cost	US$	-	-	-	-8 985 000.00	-468 000.00	-101 000.00	-90 000.00	-16 000.00	-	-9 660 000.00
Total capital cost	**US$**	**- 21 480 000.00**	**-25 043 000.00**	**-38 396 000.00**	**- 8 985 000.00**	**- 468 000.00**	**- 101 000.00**	**- 90 000.00**	**- 16 000.00**		**-94 579 000.00**
Unredeemed capex	**US$**	**- 21 480 000.00**	**-46 523 000.00**	**-82 121 678.62**	**-65 284 404.00**	**-34 098 707.92**					**-245 035 080.35**
Operating profit after capex	**US$**	**-**	**-**	**-**	**-**		**4 472 710.20**	**33 159 764.22**	**26 393 919.63**	**17 787 414.26**	**81 813 808.30**
Royalties @ 4%	**US$**	-	-	-129 458.13	-2 467 882.82	-2 868 258.53	-2 854 170.72	-2 435 317.04	-2 135 231.42	-1 127 143.25	-14 017 461.91
Net operating profit	**US$**	**-**	**-**	**2 667 863.24**	**23 354 391.79**	**28 785 437.55**	**35 818 247.41**	**26 341 736.98**	**24 274 688.21**	**16 660 271.00**	**157 902 636.19**
Taxable Income	**US$**	-	-	-	-	-	4 472 710.20	33 159 764.22	26 393 919.63	17 787 414.26	81 813 808.30
Corporate Taxes @ 37.5%	**US$**	**-**	**-**	**-**	**-**	**-**	**-1 677 266.33**	**-12 434 911.58**	**-9 897 719.86**	**-6 670 280.35**	**-24 009 897.77**
Net Cashflow	**US$**	**-21 480 000.00**	**-25 043 000.00**	**-35 728 135.79**	**14 369 392.76**	**28 317 438.52**	**34 039 982.05**	**13 816 826.37**	**14 360 969.32**	**9 989 991.63**	**32 643 464.86**
Discounted Net Cashflow @5%	**US$**	**-20 457 142.86**	**-22 714 739.23**	**-30 863 307.02**	**11 821 734.99**	**22 187 454.05**	**25 401 158.71**	**9 819 360.54**	**9 720 069.31**	**6 439 637.68**	**11 354 226.17**
NPV (US$ million)	**11.35**										
IRR	**8.95%**										

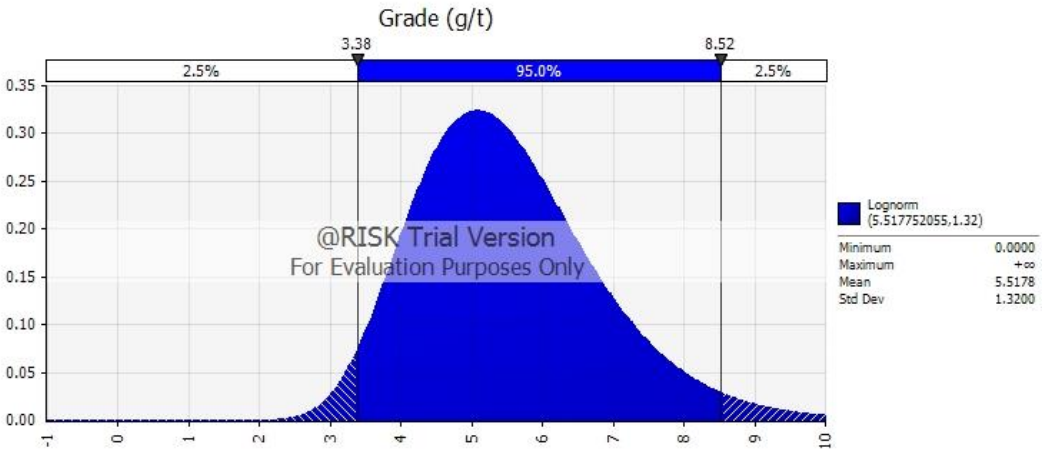
Appendix 7.7: Probability distribution of gold grade



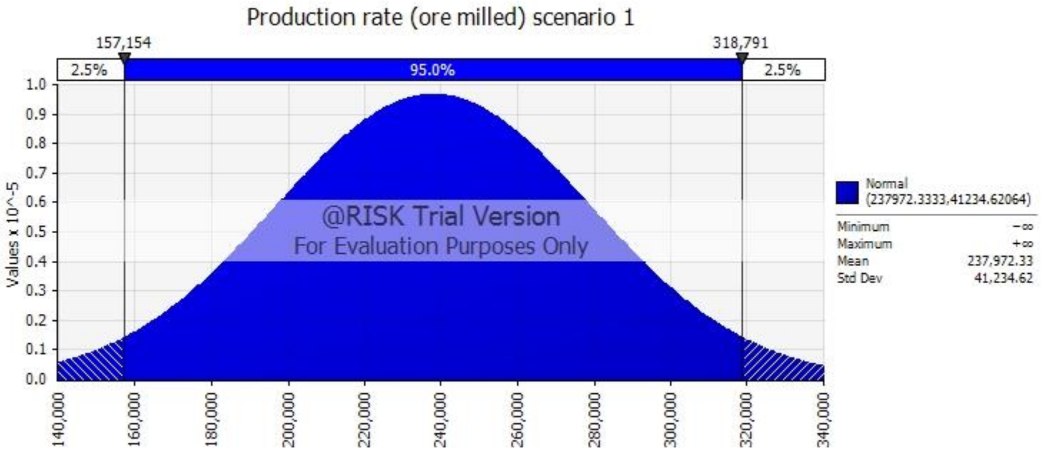
Appendix 7.8: Probability distribution of gold grade from the surface operation



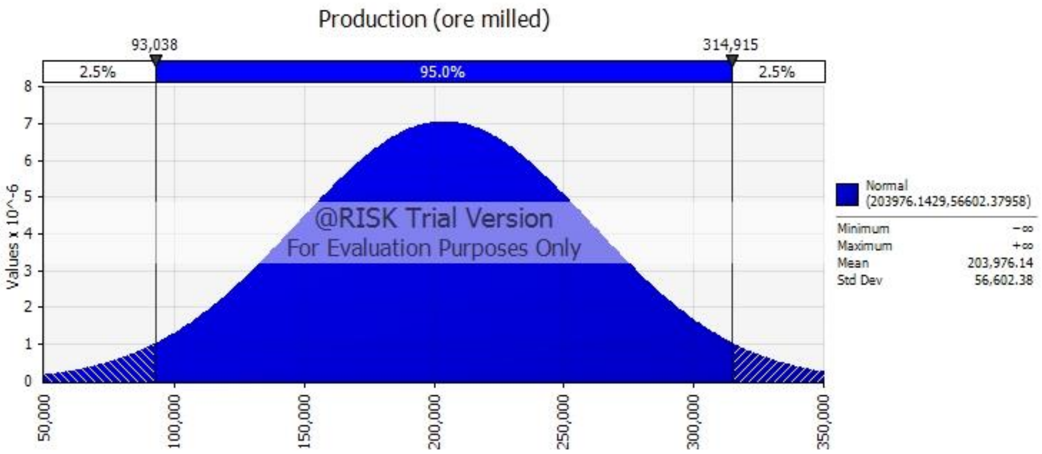
Appendix 7.9: Probability distribution of grade



Appendix 7.10: Probability distribution of production rate for Scenario 1



Appendix 7.11: Probability distribution of production rate for Scenario 2



Appendix 7.12: Unit variable cost inputs for the MCDCF for Scenario 1

Activity	Unit costs (US$/tonne milled)	
	Average	Standard deviation
Development in waste	8.86	3.08
Development in ore	7.58	2.34
Stope grouting	1.91	0.62
Bench stoping & mucking	10.18	0.77
Truck haulage	17.34	2.37
Pastefill	10.33	2.42
Mine services	10.53	1.27
Mine supervision & technical	10.88	1.06
Leasing	9.46	2.74
Mine power	17.79	2.68

Appendix 7.13: Unit variable cost inputs for the MCDCF for Scenario 2

Activity	Unit costs (US$/tonne milled)	
	Average	Standard deviation
Development in waste	10.73	6.51
Development in ore	7.59	3.40
Stope grouting	1.80	0.23
Bench stoping & mucking	10.52	0.86
Truck haulage	17.57	2.36
Pastefill	10.63	1.11
Mine services	10.45	1.57
Mine supervision & technical	10.74	0.80
Leasing	6.18	2.12
Mine power	17.97	1.35

Appendix 7.14: Gold price volatility calculation

(Source: World Gold Council, 2019)

Annual variance	1.92%
Annual volatility	13.84%
Annual standard deviation	14.34%
Daily standard deviation	0.87528%
0.2 years	6.06%

Year	US$/oz	Log change	Variance
2018	1 268.5	0.90%	0.01%
2017	1 257.2	0.51%	0.00%
2016	1 250.8	7.53%	0.57%
2015	1 160.1	-8.77%	0.77%
2014	1 266.4	-10.83%	1.17%
2013	1 411.2	-16.78%	2.81%
2012	1 669.0	6.02%	0.36%
2011	1 571.5	24.95%	6.22%
2010	1 224.5	23.06%	5.32%
2009	972.4		

Appendix 7.15: Binomial-Lattice underlying asset value for Scenario 1

Underlying Asset Value

Nodes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50	182.38	187.40	192.55	197.84	203.28	208.87	214.61	220.51	226.57	232.80	239.20	245.78
		92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50	182.38	187.40	192.55	197.84	203.28	208.87	214.61	220.51	226.57	232.80
			90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50	182.38	187.40	192.55	197.84	203.28	208.87	214.61	220.51
				87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50	182.38	187.40	192.55	197.84	203.28	208.87
					85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50	182.38	187.40	192.55	197.84
						83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50	182.38	187.40
							80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13	172.75	177.50
								78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26	163.63	168.13
									76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85	155.00	159.26
										74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88	146.81	150.85
											72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34	139.06	142.88
												70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20	131.72	135.34
													68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43	124.77	128.20
														66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02	118.18	121.43
															65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94	111.94	115.02
																63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19	106.03	108.94
																	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75	100.43	103.19
																		59.99	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58	95.13	97.75
																			58.39	59.99	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70	90.11	92.58
																				56.82	58.39	59.99	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07	85.35	87.70
																					55.30	56.82	58.39	59.99	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68	80.84	83.07
																						53.82	55.30	56.82	58.39	59.99	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53	76.58	78.68
																							52.38	53.82	55.30	56.82	58.39	59.99	61.64	63.34	65.08	66.87	68.70	70.59	72.53	74.53
																								50.98	52.38	53.82	55.30	56.82	58.39	59.99	61.64	63.34	65.08	66.87	68.70	70.59
																									49.62	50.98	52.38	53.82	55.30	56.82	58.39	59.99	61.64	63.34	65.08	66.87
																										48.29	49.62	50.98	52.38	53.82	55.30	56.82	58.39	59.99	61.64	63.34
																											47.00	48.29	49.62	50.98	52.38	53.82	55.30	56.82	58.39	59.99
																												45.74	47.00	48.29	49.62	50.98	52.38	53.82	55.30	56.82
																													44.52	45.74	47.00	48.29	49.62	50.98	52.38	53.82
																														43.33	44.52	45.74	47.00	48.29	49.62	50.98
																															42.17	43.33	44.52	45.74	47.00	48.29
																																41.04	42.17	43.33	44.52	45.74
																																	39.94	41.04	42.17	43.33
																																		38.87	39.94	41.04
																																			37.83	38.87
																																				36.82

Appendix 7.16: Binomial-Lattice underlying asset value for Scenario 2

Underlying Asset Value

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62	168.12	172.74	177.49	182.37	187.38	192.53	197.83	203.27	208.85	214.59	220.49	226.56
		85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62	168.12	172.74	177.49	182.37	187.38	192.53	197.83	203.27	208.85	214.59
			83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62	168.12	172.74	177.49	182.37	187.38	192.53	197.83	203.27
				80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62	168.12	172.74	177.49	182.37	187.38	192.53
					78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62	168.12	172.74	177.49	182.37
						76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62	168.12	172.74
							74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98	159.24	163.62
								72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80	150.84	154.98
									70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05	142.87	146.80
										68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71	135.33	139.05
											66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76	128.19	131.71
												65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17	121.42	124.76
													63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93	115.01	118.17
														61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02	108.94	111.93
															59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42	103.19	106.02
																58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12	97.74	100.42
																	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10	92.58	95.12
																		55.30	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34	87.69	90.10
																			53.82	55.30	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84	83.06	85.34
																				52.38	53.82	55.30	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57	78.68	80.84
																					50.98	52.38	53.82	55.30	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53	74.52	76.57
																						49.62	50.98	52.38	53.82	55.30	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70	70.59	72.53
																							48.29	49.62	50.98	52.38	53.82	55.30	56.82	58.38	59.99	61.64	63.33	65.07	66.86	68.70
																								47.00	48.29	49.62	50.98	52.38	53.82	55.30	56.82	58.38	59.99	61.64	63.33	65.07
																									45.74	47.00	48.29	49.62	50.98	52.38	53.82	55.30	56.82	58.38	59.99	61.64
																										44.51	45.74	47.00	48.29	49.62	50.98	52.38	53.82	55.30	56.82	58.38
																											43.32	44.51	45.74	47.00	48.29	49.62	50.98	52.38	53.82	55.30
																												42.16	43.32	44.51	45.74	47.00	48.29	49.62	50.98	52.38
																													41.04	42.16	43.32	44.51	45.74	47.00	48.29	49.62
																														39.94	41.04	42.16	43.32	44.51	45.74	47.00
																															38.87	39.94	41.04	42.16	43.32	44.51
																																37.83	38.87	39.94	41.04	42.16
																																	36.82	37.83	38.87	39.94
																																		35.83	36.82	37.83
																																			34.87	35.83
																																				33.94

Appendix 7.17: Probability lattice

(Source: Maistrov, 1974)

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Delay period (Years)	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2	3.4
Number of branches leading to that column (2^n)	1.00E+00	2.00E+00	4.00E+00	8.00E+00	1.60E+01	3.20E+01	6.40E+01	1.28E+02	2.56E+02	5.12E+02	1.02E+03	2.05E+03	4.10E+03	8.19E+03	1.64E+04	3.28E+04	6.55E+04	1.31E+05
	1.00E+00	5.00E-01	2.50E-01	1.25E-01	6.25E-02	3.13E-02	1.56E-02	7.81E-03	3.91E-03	1.95E-03	9.77E-04	4.88E-04	2.44E-04	1.22E-04	6.10E-05	3.05E-05	1.53E-05	7.63E-06
		5.00E-01	5.00E-01	3.75E-01	2.50E-01	1.56E-01	9.38E-02	5.47E-02	3.13E-02	1.76E-02	9.77E-03	5.37E-03	2.93E-03	1.59E-03	8.54E-04	4.58E-04	2.44E-04	1.30E-04
			2.50E-01	3.75E-01	3.75E-01	3.13E-01	2.34E-01	1.64E-01	1.09E-01	7.03E-02	4.39E-02	2.69E-02	1.61E-02	9.52E-03	5.55E-03	3.20E-03	1.83E-03	1.04E-03
				1.25E-01	2.50E-01	3.13E-01	3.13E-01	2.73E-01	2.19E-01	1.64E-01	1.17E-01	8.06E-02	5.37E-02	3.49E-02	2.22E-02	1.39E-02	8.54E-03	5.19E-03
					6.25E-02	1.56E-01	2.34E-01	2.73E-01	2.73E-01	2.46E-01	2.05E-01	1.61E-01	1.21E-01	8.73E-02	6.11E-02	4.17E-02	2.78E-02	1.82E-02
						3.13E-02	9.38E-02	1.64E-01	2.19E-01	2.46E-01	2.46E-01	2.26E-01	1.93E-01	1.57E-01	1.22E-01	9.16E-02	6.67E-02	4.72E-02
							1.56E-02	5.47E-02	1.09E-01	1.64E-01	2.05E-01	2.26E-01	2.26E-01	2.09E-01	1.83E-01	1.53E-01	1.22E-01	9.44E-02
								7.81E-03	3.13E-02	7.03E-02	1.17E-01	1.61E-01	1.93E-01	2.09E-01	2.09E-01	1.96E-01	1.75E-01	1.48E-01
									3.91E-03	1.76E-02	4.39E-02	8.06E-02	1.21E-01	1.57E-01	1.83E-01	1.96E-01	1.96E-01	1.85E-01
										1.95E-03	9.77E-03	2.69E-02	5.37E-02	8.73E-02	1.22E-01	1.53E-01	1.75E-01	1.85E-01
											9.77E-04	5.37E-03	1.61E-02	3.49E-02	6.11E-02	9.16E-02	1.22E-01	1.48E-01
												4.88E-04	2.93E-03	9.52E-03	2.22E-02	4.17E-02	6.67E-02	9.44E-02
													2.44E-04	1.59E-03	5.55E-03	1.39E-02	2.78E-02	4.72E-02
														1.22E-04	8.54E-04	3.20E-03	8.54E-03	1.82E-02
															6.10E-05	4.58E-04	1.83E-03	5.19E-03
																3.05E-05	2.44E-04	1.04E-03
																	1.53E-05	1.30E-04
																		7.63E-06

Node	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	3.6	3.8	4	4.2	4.4	4.6	4.8	5	5.2	5.4	5.6	5.8	6	6.2	6.4	6.6	6.8	7
Number of branches leading to that column (2^n)	2.62E+05	5.24E+05	1.05E+06	2.10E+06	4.19E+06	8.39E+06	1.68E+07	3.36E+07	6.71E+07	1.34E+08	2.68E+08	5.37E+08	1.07E+09	2.15E+09	4.29E+09	8.59E+09	1.72E+10	3.44E+10
	3.81E-06	1.91E-06	9.54E-07	4.77E-07	2.38E-07	1.19E-07	5.96E-08	2.98E-08	1.49E-08	7.45E-09	3.73E-09	1.86E-09	9.31E-10	4.66E-10	2.33E-10	1.16E-10	5.82E-11	2.91E-11
	6.87E-05	3.62E-05	1.91E-05	1.00E-05	5.25E-06	2.74E-06	1.43E-06	7.45E-07	3.87E-07	2.01E-07	1.04E-07	5.40E-08	2.79E-08	1.44E-08	7.45E-09	3.84E-09	1.98E-09	1.02E-09
	5.84E-04	3.26E-04	1.81E-04	1.00E-04	5.51E-05	3.02E-05	1.65E-05	8.94E-06	4.84E-06	2.62E-06	1.41E-06	7.56E-07	4.05E-07	2.17E-07	1.15E-07	6.15E-08	3.27E-08	1.73E-08
	3.11E-03	1.85E-03	1.09E-03	6.34E-04	3.67E-04	2.11E-04	1.21E-04	6.85E-05	3.87E-05	2.18E-05	1.22E-05	6.81E-06	3.78E-06	2.09E-06	1.15E-06	6.35E-07	3.48E-07	1.90E-07
	1.17E-02	7.39E-03	4.62E-03	2.85E-03	1.74E-03	1.06E-03	6.33E-04	3.77E-04	2.23E-04	1.31E-04	7.63E-05	4.42E-05	2.55E-05	1.47E-05	8.37E-06	4.76E-06	2.70E-06	1.52E-06
	3.27E-02	2.22E-02	1.48E-02	9.70E-03	6.28E-03	4.01E-03	2.53E-03	1.58E-03	9.80E-04	6.01E-04	3.66E-04	2.21E-04	1.33E-04	7.91E-05	4.69E-05	2.76E-05	1.62E-05	9.45E-06
	7.08E-02	5.18E-02	3.70E-02	2.59E-02	1.78E-02	1.20E-02	8.02E-03	5.28E-03	3.43E-03	2.21E-03	1.40E-03	8.85E-04	5.53E-04	3.43E-04	2.11E-04	1.29E-04	7.83E-05	4.72E-05
	1.21E-01	9.61E-02	7.39E-02	5.54E-02	4.07E-02	2.92E-02	2.06E-02	1.43E-02	9.80E-03	6.62E-03	4.41E-03	2.91E-03	1.90E-03	1.22E-03	7.84E-04	4.97E-04	3.13E-04	1.96E-04
	1.67E-01	1.44E-01	1.20E-01	9.70E-02	7.62E-02	5.84E-02	4.38E-02	3.22E-02	2.33E-02	1.65E-02	1.16E-02	7.99E-03	5.45E-03	3.67E-03	2.45E-03	1.62E-03	1.06E-03	6.85E-04
	1.85E-01	1.76E-01	1.60E-01	1.40E-01	1.19E-01	9.74E-02	7.79E-02	6.09E-02	4.66E-02	3.49E-02	2.57E-02	1.87E-02	1.33E-02	9.39E-03	6.53E-03	4.49E-03	3.05E-03	2.05E-03
	1.67E-01	1.76E-01	1.76E-01	1.68E-01	1.54E-01	1.36E-01	1.17E-01	9.74E-02	7.92E-02	6.29E-02	4.89E-02	3.73E-02	2.80E-02	2.07E-02	1.50E-02	1.08E-02	7.63E-03	5.34E-03
	1.21E-01	1.44E-01	1.60E-01	1.68E-01	1.68E-01	1.61E-01	1.49E-01	1.33E-01	1.15E-01	9.71E-02	8.00E-02	6.44E-02	5.09E-02	3.94E-02	3.00E-02	2.25E-02	1.67E-02	1.21E-02
	7.08E-02	9.61E-02	1.20E-01	1.40E-01	1.54E-01	1.61E-01	1.61E-01	1.55E-01	1.44E-01	1.30E-01	1.13E-01	9.67E-02	8.06E-02	6.57E-02	5.26E-02	4.13E-02	3.19E-02	2.43E-02
	3.27E-02	5.18E-02	7.39E-02	9.70E-02	1.19E-01	1.36E-01	1.49E-01	1.55E-01	1.55E-01	1.49E-01	1.39E-01	1.26E-01	1.12E-01	9.60E-02	8.09E-02	6.67E-02	5.40E-02	4.30E-02
	1.17E-02	2.22E-02	3.70E-02	5.54E-02	7.62E-02	9.74E-02	1.17E-01	1.33E-01	1.44E-01	1.49E-01	1.49E-01	1.44E-01	1.35E-01	1.23E-01	1.10E-01	9.53E-02	8.10E-02	6.75E-02
	3.11E-03	7.39E-03	1.48E-02	2.59E-02	4.07E-02	5.84E-02	7.79E-02	9.74E-02	1.15E-01	1.30E-01	1.39E-01	1.44E-01	1.44E-01	1.40E-01	1.32E-01	1.21E-01	1.08E-01	9.45E-02
	5.84E-04	1.85E-03	4.62E-03	9.70E-03	1.78E-02	2.92E-02	4.38E-02	6.09E-02	7.92E-02	9.71E-02	1.13E-01	1.26E-01	1.35E-01	1.40E-01	1.40E-01	1.36E-01	1.28E-01	1.18E-01
	6.87E-05	3.26E-04	1.09E-03	2.85E-03	6.28E-03	1.20E-02	2.06E-02	3.22E-02	4.66E-02	6.29E-02	8.00E-02	9.67E-02	1.12E-01	1.23E-01	1.32E-01	1.36E-01	1.36E-01	1.32E-01
	3.81E-06	3.62E-05	1.81E-04	6.34E-04	1.74E-03	4.01E-03	8.02E-03	1.43E-02	2.33E-02	3.49E-02	4.89E-02	6.44E-02	8.06E-02	9.60E-02	1.10E-01	1.21E-01	1.28E-01	1.32E-01
		1.91E-06	1.91E-05	1.00E-04	3.67E-04	1.06E-03	2.53E-03	5.28E-03	9.80E-03	1.65E-02	2.57E-02	3.73E-02	5.09E-02	6.57E-02	8.09E-02	9.53E-02	1.08E-01	1.18E-01
			9.54E-07	1.00E-05	5.51E-05	2.11E-04	6.33E-04	1.58E-03	3.43E-03	6.62E-03	1.16E-02	1.87E-02	2.80E-02	3.94E-02	5.26E-02	6.67E-02	8.10E-02	9.45E-02
				4.77E-07	5.25E-06	3.02E-05	1.21E-04	3.77E-04	9.80E-04	2.21E-03	4.41E-03	7.99E-03	1.33E-02	2.07E-02	3.00E-02	4.13E-02	5.40E-02	6.75E-02
					2.38E-07	2.74E-06	1.65E-05	6.85E-05	2.23E-04	6.01E-04	1.40E-03	2.91E-03	5.45E-03	9.39E-03	1.50E-02	2.25E-02	3.19E-02	4.30E-02
						1.19E-07	1.43E-06	8.94E-06	3.87E-05	1.31E-04	3.66E-04	8.85E-04	1.90E-03	3.67E-03	6.53E-03	1.08E-02	1.67E-02	2.43E-02
							5.96E-08	7.45E-07	4.84E-06	2.18E-05	7.63E-05	2.21E-04	5.53E-04	1.22E-03	2.45E-03	4.49E-03	7.63E-03	1.21E-02
								2.98E-08	3.87E-07	2.62E-06	1.22E-05	4.42E-05	1.33E-04	3.43E-04	7.84E-04	1.62E-03	3.05E-03	5.34E-03
									1.49E-08	2.01E-07	1.41E-06	6.81E-06	2.55E-05	7.91E-05	2.11E-04	4.97E-04	1.06E-03	2.05E-03
										7.45E-09	1.04E-07	7.56E-07	3.78E-06	1.47E-05	4.69E-05	1.29E-04	3.13E-04	6.85E-04
											3.73E-09	5.40E-08	4.05E-07	2.09E-06	8.37E-06	2.76E-05	7.83E-05	1.96E-04
												1.86E-09	2.79E-08	2.17E-07	1.15E-06	4.76E-06	1.62E-05	4.72E-05
													9.31E-10	1.44E-08	1.15E-07	6.35E-07	2.70E-06	9.45E-06
														4.66E-10	7.45E-09	6.15E-08	3.48E-07	1.52E-06
															2.33E-10	3.84E-09	3.27E-08	1.90E-07
																1.16E-10	1.98E-09	1.73E-08
																	5.82E-11	1.02E-09
																		2.91E-11

Appendix 7.18: Binomial-Lattice option value for Scenario 1

Option Value

Nodes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	0	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00	4.20	4.40	4.60	4.80	5.00	5.20	5.40	5.60	5.80	6.00	6.20	6.40	6.60	6.80	7.00
	17.34	19.41	21.60	23.91	26.34	28.87	31.50	34.23	37.05	39.96	42.97	46.07	49.27	52.56	55.95	59.43	63.03	66.73	70.54	74.46	78.50	82.66	86.94	91.34	95.87	100.54	105.35	110.29	115.38	120.61	126.00	131.55	137.25	143.12	149.16	155.38
		14.79	16.71	18.77	20.96	23.27	25.69	28.22	30.85	33.57	36.40	39.31	42.31	45.41	48.60	51.89	55.28	58.76	62.36	66.05	69.86	73.78	77.81	81.97	86.25	90.65	95.18	99.84	104.65	109.59	114.67	119.90	125.29	130.83	136.53	142.40
			12.40	14.17	16.08	18.13	20.31	22.61	25.03	27.56	30.19	32.92	35.74	38.65	41.65	44.75	47.94	51.22	54.60	58.09	61.68	65.37	69.18	73.09	77.13	81.28	85.55	89.95	94.48	99.14	103.94	108.88	113.96	119.19	124.57	130.11
				10.19	11.80	13.55	15.44	17.48	19.66	21.95	24.37	26.90	29.53	32.25	35.07	37.98	40.98	44.08	47.26	50.55	53.93	57.41	60.99	64.69	68.49	72.40	76.43	80.58	84.85	89.25	93.77	98.43	103.23	108.16	113.24	118.47
					8.19	9.61	11.19	12.92	14.80	16.83	18.99	21.29	23.70	26.23	28.86	31.58	34.40	37.31	40.31	43.40	46.58	49.86	53.24	56.72	60.30	63.99	67.79	71.70	75.73	79.88	84.14	88.54	93.06	97.72	102.51	107.44
						6.41	7.64	9.02	10.57	12.28	14.14	16.16	18.32	20.61	23.03	25.55	28.18	30.91	33.73	36.63	39.63	42.72	45.90	49.18	52.55	56.03	59.61	63.30	67.09	71.00	75.02	79.17	83.43	87.82	92.34	97.00
							4.86	5.89	7.08	8.43	9.95	11.63	13.48	15.49	17.64	19.93	22.35	24.87	27.50	30.23	33.05	35.95	38.95	42.03	45.21	48.49	51.86	55.33	58.91	62.59	66.39	70.29	74.31	78.45	82.72	87.10
								3.55	4.39	5.38	6.52	7.83	9.32	10.98	12.81	14.80	16.95	19.24	21.66	24.19	26.82	29.54	32.36	35.26	38.26	41.34	44.52	47.79	51.16	54.63	58.20	61.88	65.67	69.58	73.60	77.73
									2.49	3.14	3.93	4.86	5.96	7.23	8.68	10.31	12.13	14.11	16.26	18.54	20.96	23.50	26.13	28.86	31.67	34.57	37.56	40.64	43.82	47.08	50.45	53.92	57.49	61.17	64.96	68.86
										1.66	2.14	2.74	3.47	4.35	5.40	6.62	8.03	9.64	11.43	13.41	15.55	17.84	20.26	22.80	25.44	28.16	30.98	33.87	36.86	39.94	43.11	46.38	49.74	53.20	56.77	60.45
											1.04	1.38	1.81	2.35	3.02	3.84	4.83	6.01	7.38	8.95	10.72	12.69	14.83	17.13	19.56	22.10	24.74	27.46	30.28	33.17	36.16	39.23	42.40	45.66	49.02	52.48
												0.61	0.83	1.11	1.49	1.97	2.58	3.34	4.27	5.39	6.71	8.25	10.00	11.96	14.10	16.41	18.85	21.40	24.04	26.76	29.57	32.46	35.44	38.52	41.68	44.94
													0.32	0.46	0.63	0.87	1.19	1.61	2.15	2.84	3.70	4.76	6.03	7.53	9.26	11.22	13.37	15.68	18.13	20.69	23.33	26.05	28.86	31.75	34.73	37.80
														0.16	0.23	0.33	0.46	0.66	0.92	1.27	1.74	2.35	3.13	4.12	5.33	6.79	8.50	10.45	12.62	14.95	17.42	19.98	22.62	25.34	28.14	31.03
															0.07	0.10	0.15	0.22	0.32	0.46	0.67	0.95	1.34	1.87	2.57	3.47	4.61	6.02	7.71	9.67	11.86	14.23	16.71	19.27	21.90	24.62
																0.02	0.04	0.06	0.09	0.13	0.20	0.30	0.45	0.67	0.97	1.41	2.00	2.81	3.87	5.22	6.89	8.87	11.11	13.51	15.99	18.54
																	0.01	0.01	0.02	0.03	0.04	0.07	0.11	0.17	0.27	0.42	0.64	0.97	1.45	2.13	3.08	4.36	6.02	8.07	10.39	12.79
																		0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.08	0.13	0.21	0.35	0.56	0.90	1.43	2.23	3.41	5.09	7.35
																			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.07	0.13	0.22	0.39	0.70	1.23	2.18
																				0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																						0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																							0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																								0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																									0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																										0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																												0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																													0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
																														0.00	0.00	0.00	0.00	0.00	0.00	0.00
																															0.00	0.00	0.00	0.00	0.00	0.00
																																0.00	0.00	0.00	0.00	0.00
																																	0.00	0.00	0.00	0.00
																																		0.00	0.00	0.00
																																			0.00	0.00
																																				0.00

Appendix 7.19: Binomial-Lattice option value for Scenario 2

Option Value

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Delay period (Years)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
	21.52	23.59	25.76	28.00	30.33	32.73	35.21	37.77	40.41	43.13	45.93	48.82	51.79	54.85	58.00	61.24	64.58	68.01
		19.00	20.98	23.05	25.21	27.46	29.78	32.18	34.67	37.22	39.86	42.58	45.38	48.26	51.23	54.29	57.43	60.67
			16.58	18.46	20.44	22.51	24.67	26.91	29.23	31.63	34.11	36.67	39.31	42.02	44.82	47.70	50.67	53.72
				14.27	16.04	17.91	19.89	21.96	24.11	26.36	28.68	31.08	33.56	36.11	38.75	41.46	44.25	47.13
					12.09	13.73	15.49	17.36	19.33	21.40	23.56	25.80	28.12	30.52	33.00	35.55	38.18	40.89
						10.05	11.56	13.19	14.94	16.80	18.77	20.84	22.99	25.24	27.56	29.96	32.43	34.99
							8.17	9.53	11.02	12.64	14.38	16.24	18.21	20.27	22.43	24.67	26.99	29.39
								6.47	7.67	9.00	10.47	12.08	13.81	15.67	17.63	19.70	21.86	24.10
									4.97	5.99	7.15	8.46	9.92	11.51	13.24	15.09	17.05	19.12
										3.68	4.52	5.51	6.64	7.92	9.35	10.93	12.65	14.50
											2.60	3.27	4.07	5.02	6.11	7.37	8.78	10.35
												1.75	2.25	2.88	3.63	4.53	5.59	6.81
													1.10	1.46	1.92	2.48	3.19	4.04
														0.64	0.88	1.19	1.59	2.10
															0.34	0.48	0.67	0.93
																0.16	0.24	0.34
																	0.06	0.10
																		0.02

Node	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
	71.55	75.19	78.94	82.80	86.77	90.86	95.07	99.39	103.85	108.43	113.15	118.00	123.00	128.14	133.42	138.86	144.46	150.22
	64.01	67.44	70.98	74.62	78.36	82.22	86.19	90.27	94.48	98.81	103.26	107.84	112.55	117.41	122.40	127.53	132.82	138.25
	56.86	60.10	63.44	66.87	70.40	74.04	77.78	81.64	85.60	89.69	93.89	98.21	102.66	107.24	111.95	116.80	121.79	126.93
	50.10	53.15	56.29	59.53	62.86	66.29	69.82	73.45	77.20	81.05	85.01	89.09	93.29	97.61	102.06	106.64	111.35	116.19
	43.69	46.56	49.52	52.57	55.71	58.95	62.28	65.70	69.23	72.87	76.60	80.45	84.42	88.49	92.69	97.01	101.45	106.03
	37.61	40.32	43.11	45.99	48.95	51.99	55.13	58.36	61.69	65.11	68.64	72.27	76.01	79.85	83.81	87.89	92.08	96.40
	31.86	34.41	37.04	39.75	42.54	45.41	48.36	51.41	54.55	57.77	61.10	64.52	68.05	71.67	75.41	79.25	83.21	87.28
	26.42	28.82	31.29	33.84	36.46	39.17	41.95	44.82	47.78	50.82	53.95	57.18	60.50	63.92	67.44	71.07	74.80	78.64
	21.28	23.52	25.85	28.24	30.71	33.26	35.88	38.58	41.37	44.23	47.19	50.23	53.36	56.58	59.90	63.32	66.84	70.46
	16.47	18.54	20.70	22.95	25.27	27.66	30.13	32.68	35.30	38.00	40.78	43.64	46.59	49.63	52.76	55.98	59.30	62.71
	12.06	13.91	15.88	17.95	20.12	22.36	24.68	27.08	29.54	32.09	34.70	37.40	40.18	43.04	45.99	49.02	52.15	55.37
	8.20	9.75	11.46	13.31	15.28	17.36	19.53	21.78	24.10	26.49	28.95	31.49	34.11	36.80	39.58	42.44	45.38	48.42
	5.05	6.24	7.60	9.15	10.85	12.70	14.68	16.77	18.94	21.19	23.51	25.89	28.36	30.89	33.51	36.20	38.97	41.83
	2.74	3.54	4.51	5.66	7.00	8.52	10.23	12.09	14.08	16.17	18.35	20.59	22.91	25.30	27.76	30.29	32.90	35.59
	1.28	1.73	2.31	3.04	3.95	5.06	6.37	7.89	9.59	11.46	13.47	15.57	17.75	19.99	22.31	24.69	27.15	29.68
	0.49	0.70	0.98	1.37	1.88	2.54	3.39	4.44	5.72	7.23	8.95	10.84	12.86	14.97	17.15	19.39	21.70	24.08
	0.15	0.22	0.33	0.49	0.71	1.02	1.45	2.04	2.81	3.81	5.05	6.55	8.29	10.22	12.26	14.37	16.54	18.78
	0.03	0.05	0.08	0.13	0.20	0.31	0.47	0.70	1.05	1.53	2.21	3.13	4.33	5.84	7.63	9.61	11.65	13.76
	0.01	0.01	0.01	0.02	0.04	0.06	0.10	0.16	0.26	0.42	0.66	1.03	1.58	2.40	3.55	5.11	7.03	9.00
		0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.06	0.11	0.18	0.31	0.53	0.91	1.55	2.64	4.50
			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.07	0.13	0.23
				0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
						0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
							0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
								0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
									0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
										0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
												0.00	0.00	0.00	0.00	0.00	0.00	0.00
													0.00	0.00	0.00	0.00	0.00	0.00
														0.00	0.00	0.00	0.00	0.00
															0.00	0.00	0.00	0.00
																0.00	0.00	0.00
																	0.00	0.00
																		0.00

Appendix 7.20: Binomial-Lattice strategic action outcome for Scenario 1

Option Strategy																																				
Nodes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
		Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
			Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
				Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
					Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
						Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
							Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
								Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
									Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
										Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
											Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
												Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
													Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
														Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
															Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
																Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
																	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop
																		Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop
																			Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																				Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																					Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																						Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																							Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																								Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																									Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																										Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																											Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																												Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																													Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																														Wait	Wait	Wait	Wait	Wait	Wait	Wait
																															Wait	Wait	Wait	Wait	Wait	Wait
																																Wait	Wait	Wait	Wait	Wait
																																	Wait	Wait	Wait	Wait
																																		Wait	Wait	Wait
																																			Wait	Wait
																																				Wait

Appendix 7.21: Binomial-Lattice strategic action outcome for Scenario 2

Option strategy																																				
Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Delay period (Years)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
		Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
			Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
				Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
					Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
						Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
							Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
								Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
									Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
										Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
											Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
												Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
													Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
														Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
															Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
																Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
																	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop	Develop
																		Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop	Develop	Develop
																			Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop	Develop	Develop
																				Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop	Develop	Develop
																					Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Develop
																						Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																							Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																								Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																									Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																										Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																											Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																												Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																													Wait	Wait	Wait	Wait	Wait	Wait	Wait	Wait
																														Wait	Wait	Wait	Wait	Wait	Wait	Wait
																															Wait	Wait	Wait	Wait	Wait	Wait
																																Wait	Wait	Wait	Wait	Wait
																																	Wait	Wait	Wait	Wait
																																		Wait	Wait	Wait
																																			Wait	Wait
																																				Wait