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Ndlovu, Williams Chapasuka

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LEARNERS' MATHEMATICAL REASONING WHEN GENERALIZING FROM NUMBER PATTERNS IN THE GENERAL EDUCATION AND TRAINING PHASE

By

WILLIAMS CHAPASUKA NDLOVU

STUDENT NUMBER: 0616576G

SUPERVISOR:

PROFESSOR MARGOT BERGER

A research report submitted to the Wits School of Education, Faculty of Science, University of the Witwatersrand in partial fulfilment of the requirements for the degree of Master of Science by combination of coursework and research.



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ABSTRACT

This study aims to explore GET learners' mathematical (algebraic) reasoning when generalizing from number patterns. Data was collected in a former model C school in greater Johannesburg area by means of a questionnaire based task involving number patterns. The mathematical reasoning of the grade 9 participants when generalizing from number patterns was examined within a commognitive framework. According to this perspective, thinking is a special activity of communication in which a participant of a discourse engages. The participants' responses to questions in the questionnaire based task were classified according to particular aspects of the discourse they used, specifically routines (strategies) and visual mediators. The participants' generalization routines were further classified into one of the three main categories; numeric, figural and pragmatic generalizations. The analysis focused on how the learners' derived rules for the *n*th term and their justifications for their responses.

The results of this study strongly support the notion that students' algebraic reasoning when generalizing in number patterns is intertwined with their choices of routines and mediators. Most learners used recursive routines while a few used explicit routines (classified and categorized as numeric routines) and number-mediators. Also, most participants found it easier to informally verbalize their generalizations. However participants' spoken justifications of their written and spoken responses often did not match their use of routines and visual mediators. As such, an awareness and appreciation (by teachers) of students' diverse use of routines and mediators when generalizing from number patterns could have direct pedagogical implications in the mathematics classrooms.

KEYWORDS

Algebra, Generalization, Commognition, Thinking, Communication and Reasoning

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science at the University of the Witwatersrand, Johannesburg South Africa. It has not been submitted before for any degree or examination at any other University.

_____

(Signature of Candidate)

7th day of June in the year 2011

DEDICATION

In loving memory of my late Parents: Enala & Chapasuka

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ABBREVIATIONS

GET Education and Training

GD Gauteng Department of Education

FET Further Education and Training

OBE Outcome Based Education

RNCS Revised National Curriculum Statement

C2005 Curriculum 2005

NCS National Curriculum Statement

DoE Department of Education

LR Learners' Routines

LVM (LM) Learners' Visual Mediators

QRASS Question Response and Summary Sheet

LD Level Descriptors

TLA Task Level of Attainment

MALATI Mathematics Learning and Teaching Initiative

SO Specific Outcomes

AS Assessment Standards

LO Learning Outcomes

AF Analytical Framework

NCTM National Council for Teachers Mathematics