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Pillay, Vasen

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AN INVESTIGATION INTO MATHEMATICS FOR TEACHING; THE KIND OF MATHEMATICAL PROBLEM–SOLVING A TEACHER DOES AS HE/SHE GOES ABOUT HIS/HER WORK.

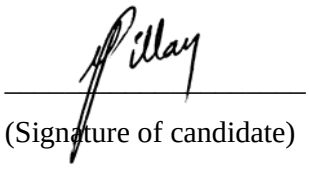
Vasen Pillay

A research report submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science.

Johannesburg, 2006

DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.



(Signature of candidate)

20th day of June 2006

ABSTRACT

This study investigates mathematics for teaching, specifically in the case of functions at the grade 10 level. One teacher was studied to gain insights into the mathematical problem-solving a teacher does as he/she goes about his/her work.

The analysis of data shows that the mathematical problems that this particular teacher confronts as he goes about his work of teaching can be classified as defining, explaining, representing and questioning. The resources that he draws on to sustain and drive this practice can be described as coming from aspects of mathematics, his own teaching experience and the curriculum with which he works. Of interest in this study are those features of mathematical problem-solving in teaching as intimated by other studies, particularly restructuring tasks and working with learners' ideas; which are largely absent in this practice. This report argues that these latter aspects of mathematical problem-solving in teaching are aligned to a practice informed by the wider notion of mathematical proficiency.

The report concludes with a discussion of why and how external intervention is needed to assist with shifting practices if mathematical proficiency is a desired outcome, as well as with reflections on the study and its methodology.

Keywords:

Functions
Mathematical problem-solving of teaching
Mathematics
Mathematics for teaching
Pedagogic content knowledge
Teaching

In memory of my beloved sister-in-law

Dr. Nevashini G. Govender

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Chapter One

"Those who can, do. Those who understand, teach." (Shulman, 1986:14)

Introduction

1.1 Aim

> "**A Chinese teacher on how a profound understanding of fundamental mathematics is attained**:
> One thing is to study whom you are teaching, the other thing is to study the knowledge you are teaching. If you can interweave the two things together nicely, you will succeed … Believe me, it seems to be simple when I talk about it, but when you really do it, it is very complicated, subtle, and takes a lot of time. It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher."
>
> (Ma as cited in Kilpatrick, Swafford and Findell, 2001:370)

Although Ma's study analysed teachers' problem-solving strategies outside of real practice, the words of Ma as quoted above allude to the notion that indeed there is a difference in being an elementary school teacher and being a *good* elementary school teacher. Take the notion of a variable for example, without a range of knowledge resources at the teacher's disposal, including explanations appropriate to different levels of learners, an elementary school teacher might be restricted to stating only that 'a variable is a letter used to represent a number.' To what then can one attribute elements of *good elementary mathematics teaching*?

I am not intending to provide a comprehensive answer to this question through this study. However, it is now more widely accepted that the knowledge that good mathematics teachers need consists of more than knowing mathematics well or understanding how children think at particular developmental stages. It comes from knowing how to apply mathematical knowledge, quickly, in ways that make

sense to learners. In other words, teachers need and use a specialised kind of mathematical knowledge. For example, when a learner uses an unorthodox method of determining if a relationship between two variables will be a function or not and still obtains the correct answer, an effective teacher must determine in a split second whether that learner's approach is a genuine method that generalises to all other relations or whether it is pure luck.

Ma calls this specialised mathematics for teaching Profound Understanding of Fundamental Mathematics (PUFM). The complexity that Ma alludes to in the above extract and the notion of PUFM finds resonance with Shulman's concept of pedagogical content knowledge (PCK) which he defines as "a particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986:9). A level of expertise in both pedagogy and content knowledge is needed for teachers to know, for example, "representations of particular topics and how learners tend to interpret and use them" (Ball and Bass, 2000:87). Furthermore, when you compare teaching in two classrooms, two schools, or two countries, instruction can look quite different since teachers' beliefs, attitudes and knowledge affect instruction, and instruction affects learners' learning.

I have attempted to briefly introduce the problem area that this study addresses. There is increasing agreement that there is a specificity to the mathematical subject knowledge that teachers need to know and know how to use. But what is this 'specificity', and how do we find this out? There is ongoing debate internationally with respect to what exactly is appropriate subject matter knowledge for teaching as well as the relationship of this knowledge to pedagogy (Shulman, 1986; 1987; Ball and Bass, 2000; Ball, Lubienski, and Mewborn, 2001; Adler, 2005). In the next section I will make explicit my research question and the critical questions that underpin this study.

1.2 Statement of the Problem and Critical Questions

From the discussion of the preceding section it is evident that having mathematical subject knowledge is an extremely important but insufficient condition for effective teaching to take place. As Kilpatrick *et al.* (2001:370) argue:

> "A teacher must interpret students' written work, analyze their reasoning, and respond to the different methods they might use in solving a problem. Teaching requires the ability to see mathematical possibilities in a task, sizing it up and adapting it for a specific group of students. Familiarity with the trajectories along which fundamental mathematical ideas develop is crucial if a teacher is to promote students' movement along those trajectories. In short, teachers need to master and deploy a wide range of resources to support the acquisition of mathematical proficiency."

In using the term mathematical proficiency, Kilpatrick *et al.* (op. cit.) point to the complexity of the mathematics itself that learners need to acquire, which in turn has implications for what, mathematically, teachers need to know. They argue that there is no term that "captures completely all aspects of expertise, competence, knowledge, and facility in mathematics" (Kilpatrick *et al*., 2001:116). The notion of mathematical proficiency is used to encapsulate these aspects and to describe that which is necessary for anyone to learn mathematics successfully. Kilpatrick *et al*. (op. cit.) perceive mathematical proficiency as having five strands or components, namely:

- *conceptual understanding* – comprehension of mathematical concepts, operations, and relations;
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently and appropriately;
- *strategic competence* – ability to formulate, represent, and solve mathematical problems;
- *adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification; and

- *productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

They argue further that these strands of proficiency are intertwined like the parts of a rope; they are interdependent and need to be developed simultaneously since mathematical proficiency is "not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands" (Kilpatrick *et al.*, 2001:116).

My study is located in this broad problem arena - of mathematical knowledge for teaching for mathematical proficiency. Ball, Bass and Hill (2004) argue that it is productive to think about the kind of mathematical work that teachers do as a special kind of mathematical problem-solving enacted in the practice of teaching. The route then to finding out more about this kind of mathematical work involves studying teachers in practice.

To this end, this study can be encapsulated as follows:

> *An investigation into mathematics for teaching; the kind of mathematical problem-solving a teacher does as he/she goes about his/her work.*

The questions that would focus my study i.e. the critical/focus questions are:

1. What mathematical 'problems' does the teacher confront as he/she teaches the section on functions in grade 10?
2. What knowledge and experience (resource pool) does the teacher draw on as he/she solves these problems of teaching?
3. Why does the teacher use the knowledge and experiences he/she draws on in the way he/she does?
4. How does this resource pool relate to mathematical proficiency or the potential to promote mathematical proficiency in his/her learners?

To summarise, in this introductory section I am arguing that there is a specificity to maths for teaching (MfT). Secondly, that mathematical proficiency is a desired practice which is accompanied by its own MfT. Therefore the focus questions attempt to elucidate what MfT looks like in a particular practice; how this particular MfT relates to mathematical proficiency; and finally how we may explain all of this. Thus, focus questions 1, 2 and 3 *describe* and *explain* MfT in this particular case, whilst focus question 4 grapples with what it means for mathematical proficiency as a desired or privileged practice i.e. as a *prescription*. In the section that follows I will articulate my reasons for undertaking this study.

1.3 Rationale

> "… today, an understanding of science, mathematics, and technology is very important in the workplace. As routine mechanical and clerical tasks become computerized, more and more jobs require high level skills that involve critical thinking, problem-solving, communicating ideas and collaborating effectively. Many of these jobs build on skills developed through high-quality science, mathematics and technology education. Our nation is unlikely to remain the world leader without a better educated workforce."
>
> (National Research Council, 1997:1)

To develop the skills as alluded to in the extract above it becomes imperative for teachers to develop mathematical proficiency in their learners. For their learners to develop mathematical proficiency, educators:

> "… must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching. They need to know the mathematics they teach as well as the horizons of that mathematics – where it can lead and where their students are headed with it …"
>
> (Kilpatrick *et al.*, 2001:369).

The significance of this study is that it could inform mathematics courses in teacher education and training curricula so as to ensure that we prepare a mathematics teaching workforce that would be skilled in teaching for

mathematical proficiency. In order to do so, we first need to be able to describe and understand what practices exist, hence questions 1, 2 and 3.

A challenge and further possible contribution of this study is that many of the theoretical bases that inform debate in mathematics education are theories of learning, whereas the focus of this study lies in the practices of teaching, which is an emerging field of study. This study forms part of a larger project that hopes to contribute to the growing studies of teaching and knowledge for teaching[1]. Given that this study is a particular case located at a higher level (grade 10), in a 'traditional[2]' classroom, focusing on a key curriculum topic viz. functions, it will provide an important and specific window into mathematical knowledge in and for practice, hence focus question 4. Furthermore, my reason for working with a 'traditional' teacher is that it will help me to identify if there are gaps, what these gaps are and what they may mean – what is the knowledge base that is being drawn on and how this may need to shift.

I have been teaching for the past 14 years, yet only recently have I come across the phrase pedagogic content knowledge (PCK). PCK, in very basic terms, can be seen as the intersection of content knowledge and pedagogic knowledge. Taking this into account, I began to question my teaching and its success since I have consciously focused on content knowledge; the pedagogy that characterised my lessons was a reflection of the teaching methods that I was exposed to in the mathematics classrooms whilst I was at school. This could be perceived as follows: for me the pedagogy was generic, irrespective of the levels of the learners being taught. This therefore sparked an interest in me since I became curious to know how subject knowledge is transformed from the knowledge of the teacher into content of instruction and how this impacts on learners being taught. In

[1] This study forms part of the QUANTUM project – a larger study on mathematics for teaching in teacher education and school classrooms. See Adler & Davis, in press.

[2] Refers to a pedagogical approach which can typically be described as one where the teacher does the explanations from the chalkboard at the front of the class. Thereafter the learners are required to work through questions from the textbook as identified by the teacher. Whilst the teacher speaks the learners sit quietly and listen, watch the chalkboard and write down what they are told to.

essence, what knowledge do educators need to know and know how to use in order to teach for mathematical proficiency?

Reform in education in South Africa gave rise to the Revised National Curriculum Statement for Mathematics, which aims to develop, inter alia, a learner who has the ability to participate in society as a critical and active citizen. The kind of teacher that it encourages includes, inter alia, a teacher who is professionally competent and in touch with current developments, especially in his/her area of expertise. In light of this, this study is particularly important since although it is a single case it can to a degree illuminate on what to include in the curriculum to provide the prospective teacher with the necessary skills to teach for mathematical proficiency within the realm of functions particularly. The study could also inform subject advisors of intervention strategies that they could use to assist teachers in their attempts to teach for mathematical proficiency, specifically with respect to the domain of functions in mathematics.

The domain of functions has been identified to contextualise this study. The reason for the selection of functions and the importance of functions in the mathematics curriculum is elaborated on in the next section.

1.4 The importance of the domain of mathematics in which this study is located

To determine the domain of mathematics that will form the backdrop to this study I asked the teacher participating in this study to choose the section along one of two 'lines': (i) a section that he[3] thinks he teaches well or (ii) a section that he feels he knows well since he is capable of solving any problem within that domain but has difficulty in teaching it to his learners. I adopted this approach so that the teacher would feel comfortable with the domain of mathematics that forms the backdrop to the study. Using the notion of a section that he thinks he teaches well, the teacher has identified the domain of functions as the area of interest.

[3] I am using 'he' to refer to the teacher who participated in this study since the teacher is a male.

The area of functions is particularly important in the curriculum since it is 'all around us', though learners do not always realise this. For example, a functional relationship is at play when we are paying for petrol by the litre or fruit by the gram/kilogram. Algebraic tools allow us to express these functional relationships very efficiently: find the value of one thing (such as the petrol price) when we know the value of the other amount (number of litres), and display a relationship visually in a way that allows us to quickly grasp the direction, magnitude, and rate of change in one variable over the range of values of the other. For 'simple' problems such as determining the petrol price, learners' existing knowledge of multiplication will usually allow them to calculate the cost for a specific amount of petrol once they know the price per litre (say R5). Learners will know that 2 litres will cost R10 and 4 litres will cost R20 and so on. While we can continue listing each set of values in this fashion, it will be efficient to say that for all values in litres (which we call x, by convention), the total cost (which we call y, by convention), is equal to 5x. Writing $y = 5x$ is a simple way of saying a great deal. Furthermore, the concept of function allows one the opportunity to represent the same 'thing' in different representations. Flexibility in moving from one representation to another allows one the opportunity to see rich relationships, to develop a better conceptual understanding and to strengthen one's ability to solve problems.

Within mathematics education the function concept has come to have a broader interpretation that refers not only to the formal definition, but also to the multiple ways in which functions can be written and described (Goldenberg, 1995; Leinhardt, Zaslavsky and Stein, 1990; and Romberg, Fennema and Carpenter, 1993). Common ways of describing functions include tables, graphs, algebraic symbols, words and problem situations. Each of these representations describes how the value of one variable is determined by the value of another. For example, in a verbal problem situation such as 'you get a two rand for every kilometre walked in a walkathon', the amount of money earned depends on, or is determined by, or is a function of the distance walked. Teaching for mathematical proficiency implies that learners need to understand that there are different ways of describing

the same relationship. This does not only mean developing learners' capacity to perform various procedures such as finding the value of y given the x value or creating a graph given an equation, but should also include assisting learners in developing a conceptual understanding of the function concept. This means the ability to represent a function in a variety of ways, and fluency in moving among multiple representations of functions for example, the slope of the line as represented in an equation should have a 'meaning' in the verbal description of the relationship between the variables as well as a visual representation on a graph.

In this section, I have highlighted the importance of functions as a topic in the school mathematics curriculum and why the domain of functions was given prominence in this study. In section 1.3, I provided my rationale for undertaking this study and in doing so I engaged in a discussion that illustrates the value of this study to teacher education. I believe that the value of this study is not just about mathematics for teaching and its possible contribution(s) to teacher education but also about possibly contributing to the literature around the teaching of functions particularly. In the next chapter I will review some of the literature relevant to the teaching of functions as well as articulate the theoretical and analytical framework that underpins this study.

Chapter Two

Literature Review and Theoretical Framework

In this chapter I will formulate a reasonably accurate picture of the prevailing discussions with respect to two central aspects of this study viz.

i. Functions as a teaching domain.
ii. The knowledge that teachers draw on in order to teach for mathematical proficiency.

Furthermore, in this chapter I will provide the 'lens' through which I will view and analyse any data obtained during this study.

2.1 Some perspectives on the teaching of functions from literature

Tall (1992) argues that a major focus in mathematics education is for us as teachers to introduce learners, through our teaching, to the world of the professional mathematician. To accomplish this task it does not imply that as teachers, we initiate learners only in terms of the rigour required, we also provide the experience on which the concepts are founded. The move to advanced mathematical thinking as argued by Tall involves a difficult transition from a "position where concepts have an intuitive basis founded on experience, to one where they are specified by formal definitions and their properties re-constructed through logical deductions" (Tall, 1992:495). Tall further argues that during this transition (and even long after) "there will exist simultaneously in the mind (of the learner) earlier experiences and their properties, together with the growing body of deductive knowledge" (Tall, 1992:495) (brackets own emphasis). This therefore contributes to the wide variety of what Tall (op. cit.) refers to as 'cognitive conflict', which in turn can act as a catalyst in hindering the learning process. This 'cognitive conflict' could be further compounded by the "nature of our own (the teachers') perceptions of mathematical concepts, for even those of

professional mathematicians contain idiosyncrasies dependent on personal experience" (Tall, 1992:495) (brackets own emphasis).

In view of this, a useful way of characterising a person's thinking about functions is in terms of a framework modelled by Vinner (Vinner, 1983; Vinner and Hershkowitz, 1980). This model describes three main mental representations associated with a mathematical concept: (i) the concept definition[4]; (ii) the learners' concept definition[5]; and (iii) the learners' concept image[6]. According to Vinner's theory, when a concept is evoked in the mind, we activate a subset of the mental images or properties associated with the concept. This set of elements that is activated is the learners' activated concept image of the mathematical concept. The concept image may include both relevant and irrelevant features of the mathematical concept and learners may also activate different sets of features under different situations. Therefore, the learners' activated concept image for a mathematical concept may or may not coincide with his/her concept image or his/her concept definition or the mathematical concept definition. So, one of the challenges of teaching is to help learners' concept images and concept definition and the mathematical concept definition to develop into a single entity. So, what would be an appropriate route then to follow in order to accomplish this task? The challenge that this exercise presents seems to be like searching for the Holy Grail and making an attempt at finding a possible route to follow is not the focus of this study.

However, in her elucidation on understanding the notion of function Sierpinska (1992) suggests that in teaching, functions should first appear as models of relationships since this is how they came into being in history. The assumption that Sierpinska is making here is that "the meaning of a concept lies in the problems and questions that gave birth to it, and we wish that our students grasp

[4] The concept definition of a mathematical idea is the minimal set of essential properties or critical attributes that characterise the concept.
[5] The learners' concept definition of a mathematical concept is the definition that learners verbalise when they are asked to provide the definition of the mathematical concept.
[6] The learners' concept image about the mathematical concept is the set of mental images, visual representations, or properties associated or related to a concept in a learner's mind.

the meaning of the notion of function" (Sierpinska, 1992:32). Then what was said earlier seems to be a quite reasonable claim to make. Sierpinska further develops this argument in her epistemological remarks on functions when she mentions:

> "The most fundamental conception of a function is that of a relationship between variable magnitudes. If this is not developed, representations such as equations and graphs lose their meaning and become isolated from one another. … Introducing functions to young students by their elaborate modern definition is a didactical error – an antididactical inversion."
>
> (Sierpinska, 1988:572)

The above argument put forth by Sierpinska (1988) resonates to a certain degree with the obstacle that Tall (1992) highlighted with respect to the utilisation of definitions in mathematics. Tall states that the problem with definitions is that they "are both subtle and generative, while the experiences of students are based on the evident and particular, with the result that the generative quality of the definitions is obscured by the students' specific concept images" (Tall, 1992: 497). Sierpinska (1992), however, does highlight prerequisites concerning the introduction of the general definition of function. To this end she states that the "introduction of the general definition does not make sense before a certain mathematical culture is developed in students, in particular before they are aware of the role and place of definitions in mathematics" (Sierpinska, 1992:57). According to Sierpinska (1992) from a didactical and epistemological point of view the introduction of the concept of function through a definition as a particular kind of relation could to a certain degree be justified. So, informal definitions resembling that of the Dirichlet–Bourbaki[7] concept maybe sufficient at the secondary school level although it does not discriminate between the roles and meanings of the concepts of *relation* and *function* in mathematics. In addition these informal definitions are not sufficient enough to bring out the arbitrariness of functions – where the two sets (x and y) do not have to be sets of numbers it could be rotations, reflections or translations of points in space.

[7] The Dirichlet–Bourbaki concept emphasises the correspondence between two nonempty sets that assigns to every element in the first set (domain) exactly one element in the second set (the co-domain).

In order to facilitate the construction of rich concept images that are consistent with, and highlights for learners, key features of definitions, teachers must use and encourage learners to use images and definitions dialectically (Vinner, 1992). But what is an appropriate formal definition of a function for the classroom? The idea that 'each x-value has a unique y-value, where the set of x-values is called the domain and the set of y-values is called the range' is a typical 'informal' definition resembling that of the Dirichlet–Bourbaki concept of a function. This is the kind of definition of a function that most learners in secondary schools in South Africa will experience as their first encounter of the notion of function. The reason for this stems from the fact that currently the mathematics syllabus is structured in a way that the concept of function is introduced through a 'formal' definition. Here is an example from a Grade 10 textbook used across many schools in South Africa:

> "A function is a rule by means of which each element of a first set (called the DOMAIN) is associated with ONLY ONE ELEMENT of a second set. The set of actual images obtained is known as the RANGE."
>
> (Laridon *et al.*, 1991:96)

From the above discussion it becomes evident that when learners are first confronted with mathematical definitions, and in this case the definition of a function, they will experience a restricted range of possibilities, thus resulting in a limited 'concept image', which in turn will cause future 'cognitive conflicts'. A contradiction comes to light here i.e. between mathematics didactics (i.e. the curriculum) and mathematics practice (i.e. scholarly knowledge); hence in Bernstein's (1990, 1996) terms 'recontextualisation' or what the French refer to as 'transposition[8]' is inevitable i.e. a transposition or recontextualisation from

[8] The French distinguish between scientific knowledge i.e. knowledge which is accessible through books for example and knowledge that is generally accepted by the research community *(savoir savant)* and the taught knowledge i.e. the knowledge that is proposed to learners for example in the form of textbooks or worksheets *(savoir enseigné)*. The notion of transposition can be explained as the process from the *savoir savant* to the *savoir enseigné* (Chevellard (1985) as cited in Pepin and Haggarty, 2001).

curriculum to practice. This has a direct bearing on the notion of PCK and therefore an implication for this study. The quandary that we as educators are now placed in is – what is the starting point to introduce the concept of function to our learners at school, bearing in mind that what we teach is governed by a curriculum. In other words, how do educators bring the notion of function into *'existence*[9]*'* for their learners, and in so doing to provide appropriate *'first encounters*[9]*'* for them, that will facilitate the construction of rich concept images and concept definitions in their minds? In addition, at what level should we as educators strive to get our learners to conceive a function as action, process or object? A brief explanation of these three concepts follows.

Thompson (1994) describes an *action conception* of a function to be when learners come to think of an expression as producing a result of calculating. They see the function as a recipe to apply to numbers and this recipe remains the same across numbers, but they must actually apply it to some number before the recipe produces anything. Thompson (op. cit.) describes the *process conception* of a function to be when learners build an image of 'self-evaluating' expressions. They do not feel compelled to imagine actually evaluating an expression in order to think of the result of its evaluation. Thompson (op. cit.) indicates that when learners perceive a function as a correspondence between two sets – a set of possible inputs to the process and a set of possible outputs from the process then they (the learners) can reason about functions as if they were *objects*.

In light of the above, the desired level at which to perceive a function would be what Thompson (op. cit.) refers to as 'function as an object'. How do we achieve this? Tall (1992) suggests that rather than dealing with formal definitions as the first encounter for learners, it is preferable to search for an approach that builds on concepts that have a dual role or purpose: (i) it is familiar to the learners; and (ii) it provides the basis for future mathematical development. Such an approach is what Tall (op. cit.) refers to as a 'cognitive root'. A 'cognitive root' is different from a 'mathematical foundation' since "a mathematical foundation is an

[9] The terms 'existence' and 'first encounters' are drawn from the theoretical framework which is discussed later in this chapter. See section 2.3 page 31.

appropriate starting point for a logical development of the subject, a cognitive root is more appropriate for curriculum development" (Tall, 1992:497).

By introducing functions through a definition, learners would be able to deal with the more abstract entities without regard for their grounding in everyday experience. Constructing and analysing tables, computing numerical values, developing a quantitative sense, acquiring a notion for what are acceptable and unacceptable approximations, are important aspects of the mathematical competence that may only be attained if one can currently and easily deal with concrete numbers, if possible, coming from real life situations. Approaching functions in this manner would result in learners developing conceptual understanding as well as procedural fluency; however, the other strands of mathematical proficiency as described by Kilpatrick *et al.* (2001) are not likely to develop. Ideas related to variation (increase, decrease, constancy, maximum, minimum), and with variation in variation (fast and slow variation, rate of change, smoothness, continuity, and discontinuity), might be better grasped from graphical representations. Therefore, to be mathematically proficient means to be able to use these concepts to make predictions, interpolate, and extrapolate; to be able to establish relationships among different functions by superimposing graphs; and also, to be able to construct regression curves that approximate relationships for empirically obtained data and have an idea of the degree of association between two variables. Exposure to this kind of approach in dealing with functions will be more likely to develop in learners all five strands of mathematical proficiency with respect to functions. Depending on how the teacher brings the notion of function into 'existence' for his learners and thereby providing them with their 'first encounter', it would be interesting to examine the opportunities that arise for learners to engage with and so reflect on the notion of a function and develop it into something more substantial. As stated previously the terms 'existence ' and 'first encounter' will be explained in the section dealing with the theoretical framework (section 2.3.1, page 32)

In attempting to describe and explain MfT in this particular case and to explore what it means for mathematical proficiency in a particular practice, it becomes imperative to factor in the teacher. In so doing, an intervening factor with respect to the teaching of functions emerges in the form of human resources. This intervening factor in the form of the teacher could be enabling, or an impediment or a mixture between the two. This intervening factor can be construed as an impediment especially when the teacher's conception of functions is incomplete, since, according to Even, it will be "problematic and may contribute to the cycle of discrepancies between concept definition and concept image of functions in students" (Even, 1990:530). Therefore it is essential that teachers update their views on functions to be able to teach for mathematical proficiency. With respect to the subject matter knowledge needed by teachers for the teaching of functions, Even (op. cit.) suggests seven useful aspects that will be highlighted later in this chapter.

If teachers at the onset explicitly expose learners to the definition of a function it will certainly pose a problem to the learners and teachers to, at a later stage, 'panel beat' their learners' behaviour towards functions since it may be different from what the teacher expects. Vinner and Dreyfus (1989) indicate that learners don't necessarily use the definition when deciding whether a given mathematical object is an example or non-example of the concept. They further argue that "in most cases, he or she decides on the basis of a *concept image*, that is, the set of all the mental pictures associated in the student's mind with the concept name, together with all the properties characterising them" (Vinner and Dreyfus, 1989:356).

With respect to this study, it is likely that the teacher (in his own school learning and in his teacher education courses) would have experienced functions formally via the introduction of a definition of a function. This claim is substantiated by the fact that traditionally at schools this is typically how the section on functions was introduced. Textbooks introduce functions in this fashion and they tend to guide the content that teachers teach. What can then be learned from a teacher

who might teach functions in this way, producing, in Sierpinska's terms, a didactical error, and limited possibilities for mathematical proficiency?

As an experienced teacher (this is the situation in this study), it is likely that through practice, and so working with solving problems of teaching (in this case having his learners understand and be able to apply functions), he will bring other resources to bear on his teaching. What are these? How and in what way(s) do they relate to teaching for mathematical proficiency, and to approaches such as introducing functions through investigative tasks as these are posited as more appropriate for learners to begin to develop an understanding of functions as though they are objects? In beginning to think about these questions Boaler's (1997) study highlights that in order for learners to develop more flexible forms of knowledge[10] it is imperative that, what we teach should be connected to the learners' meanings i.e. they should be able to relate to it in some way. If we use examples such as a roller-coaster cart where we get the learners to sketch the graph of the speed of the roller coaster cart against the distance along the track, we will get learners to experience what Thompson (1994) refers to as 'function as covariation'. In terms of mathematical proficiency, the kind of approach as highlighted by Boaler (op. cit.), would lead to the development of strategic competence amongst the learners. By engaging with functions in this fashion teachers would be in a position to assist learners to perceive what Thompson (op. cit.) refers to as 'function as an object'. What is meant by this is that "at the point where the students have solidified a process conception of function so that a representation of the process is sufficient to support their reasoning about it, they can begin to reason formally about functions – they can begin to reason about functions as if they were objects" (Thompson, 1994:8).

[10] Flexible forms of knowledge refers to the kind of knowledge that learners' develop which allows them to use it in a variety of different situations, including the formal school leaving examination and the real world. This is accomplished when mathematics is: i) connected to real world experiences; ii) accompanied by a learning environment that encourages learners' to take responsibility for their learning and; iii) learnt through work on open-ended projects in mixed ability groups. This is opposed to the closed procedural approach to mathematics, which results in learners' developing inert forms of knowledge, that they find difficult using in anything other than a textbook.

What is interesting and reinforces the potential importance of this study and its wider project, is the question of where a practicing teacher would learn about aspects of mathematical teaching like teaching functions as objects? One obvious place is through exposure to the field of mathematics education, in in-service courses or programmes. Another possibility is through the practice of teaching itself, i.e. that these kinds of knowledge are developed in and through practice. So the interesting question for this study is what this teacher does (and could) draw on to teach functions in his class, and why?

Before moving on to the underlying theoretical orientation and analytical framework of this study, it is important first to review relevant literature in the field of mathematical knowledge for teaching. In the introduction I have already discussed some of this literature and pointed to the seminal work of Shulman (1986, 1987) as he identified and named PCK as a specific aspect of knowledge needed for teaching. There is a range of research that builds on Shulman's (op. cit.) work, and focuses on mathematics, some of which has been located in practice.

2.2 Some perspectives from literature on the 'aspects of knowledge' that teachers' need to draw on, in order to teach for mathematical proficiency

What is the difference between the way a mathematics teacher and a mathematician use mathematical knowledge, or for that matter the difference between a science teacher and a scientist? The difference is not simply in the quantity or quality of their subject matter knowledge, but in how the knowledge is organised and used. An experienced mathematics/ science teacher's knowledge is organised from a teaching perspective and is used as a basis for helping learners to understand specific concepts, whilst, the mathematician's/ scientist's knowledge is organised from a research perspective and is used as a basis for developing new knowledge in the respective fields. So, from the perspective of teaching and learning functions in school, knowing the formal definition and its use in

mathematics teaching is likely to be insufficient when teaching for mathematical proficiency.

Nearly two decades ago, Lee Shulman coined a phrase for what he saw as an important aspect of a teachers' knowledge base viz. pedagogical content knowledge (PCK). He did not simply call for the inclusion of "both knowledge of general pedagogy and knowledge of subject matter as equally important yet separately engaged components of a teacher's knowledge base. Rather, he advocated the need to explore the inherent relationship between the two through what he termed 'pedagogic content knowledge'" (Segall, 2004:489). Shulman (1986) further distinguishes between two other important categories of teachers' knowledge base which includes subject matter content knowledge (SMK) and curricular knowledge (CK). In the next three sections I will illuminate aspects of the literature that has an impact on this study.

2.2.1 Pedagogical Content Knowledge (PCK) – What is it?

The distinction between pedagogy and content is a long standing debate with respect to the importance of content (knowledge of the subject) or pedagogy (Shulman, 1986; Ball and Bass, 2000). Considering the meaning of the words and the grammar of the phrase, PCK can be seen as a type of content knowledge particular to teaching. Figure 1 below is intended to provide a basic model of Shulman's notion of PCK.

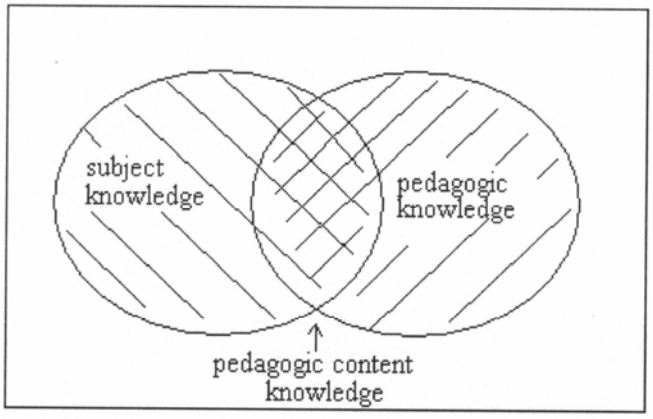


Fig. 1: A Basic Model of Shulman's Notion of PCK

PCK must not simply be taken to be the fusion of subject knowledge and pedagogic knowledge but, according to Shulman, "the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986:9). Included in PCK, according to Shulman (1986) are:

> "… the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstration – in a word, the ways of representing and formulating the subject that make it comprehensible to others… Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons."
>
> (Shulman 1986:9)

PCK is grounded in the beliefs and practices of the teacher. It includes conceptual and procedural knowledge, a repertoire of varied techniques or activities, knowledge of techniques for assessing and evaluating, and knowledge of a variety of resources which can easily be accessed for use in the classroom. PCK "represents a class of knowledge that is central to teachers' work and that would not typically be held by nonteaching subject matter experts or by teachers who know little of that subject" (Marks, 1992:9).

Central to the notion of PCK as argued by Shulman (1986, 1987) is the role of metaphors in teaching since metaphors are pervasive in our daily life. We make use of metaphors to conceptualise, to represent and to communicate many of our thoughts and actions (Lakoff and Johnson, 1980). According to Lakoff and Johnson (1980), a metaphor is a mental construction that helps us to structure our experience and to develop our imagination and reasoning. Moreover, according to Johnson (1987) metaphors are constructed through an 'embodied schema' or an 'image schema'. An embodied schema is defined as "structures of an activity by which we organize our experience in ways that we can comprehend. They are primary means by which we construct or constitute order and not mere passive

receptacles into which experience is poured" (Johnson, 1987:29-30). This means we construct metaphors to link our bodily experience of something to our more abstract thinking, and to "give shape, structure, and meaning to our imagination" (Sfard, 1994:47). This suggests that in fact, the whole conceptual system of how we think and act may be fundamentally metaphorical in nature.

The use of metaphors for pedagogical purposes according to Nolder (1991) is an attempt by the teacher to offer the learner something concrete and familiar to help them understand an unfamiliar or an abstract idea. This is accomplished by making links between the new idea and the past experiences of the learner.

Although Shulman (1986, 1987) provides us with a useful construct known as PCK, it is not devoid of complexities. The advent of PCK foregrounds:

> "questions about the content and nature of teachers' subject matter understanding in ways that the previous focus on teachers' course taking had not. It also led to the crucial insight that even expert personal knowledge of mathematics often could be surprisingly inadequate for teaching … It also requires a unique understanding that intertwines aspects of teaching and learning with content."
>
> (Ball *et al.*, 2001:448)

What a teacher teaches and how the teacher teaches it, is dependent on the individual teacher's own knowledge of the subject. The problem is that "the prevalent conceptualizations and organization of teachers' learning tends to splinter practice, and leave to individual teachers the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work" Ball and Bass (2000:86). Specifically with respect to PCK, Ball and Bass (2000) and Ball *et al.* (2001) echo their concerns regarding the conception of PCK as described by Shulman (1986, 1987). The construct PCK is described by them as:

> " … representations of particular topics and how students tend to interpret and use them, for example, or ideas or procedures with which students often have difficulty - unique subject-specific body of pedagogical knowledge that

> highlights the close interweaving of subject matter and pedagogy in teaching. Bundles of such knowledge are built up by teachers over time …"
>
> (Ball and Bass, 2000:87)

Ball and Bass (2000:88) indicate that although the construct PCK provides "a certain anticipatory resource for teachers", it can be limiting since it is not possible to anticipate in advance all the complexities of practice in the classroom. In light of this, when teachers find themselves in novel classroom situations, they need to reason, and in doing this they need to take into account knowledge from the various domains: content, learners, pedagogy and learning – thus their thinking is dependent on their "capacity to call into play different kinds of knowledge, from different domains" (Ball and Bass, 2000:88). These various classroom situations should be perceived according to Ball and Bass (2000:88) "… as mathematical problems to be solved in practice – entails an ongoing use of mathematical knowledge. It is what it takes *mathematically* to manage these routine and nonroutine problems …".

Brodie (2004) sheds more light on this argument and suggests that thinking about mathematical knowledge for teachers cannot be done in a vacuum detached from the notions of practice[11]. She argues that "mathematical knowledge and teaching practices are mutually constitutive and that the notion of thinking practices draws the two together in a more useful conception of both" (Brodie, 2004:65). The notion of a thinking practice "allows us to look both at what teachers do in the classroom, and how their ongoing thinking about what they do both informs and is informed by their practice, and by the social and institutional constraints of schooling" (Brodie, 2004:73). Brodie's (op. cit.) notion of a thinking practice can be seen as *situated reasoning* which is similar to what Ball and Bass (2000) are referring to.

What becomes evident in this literature is the difficulty in naming as well as describing the specificity of mathematical knowledge for teaching. It is not my

[11] Brodie (2004:73) uses the term practice to refer "to certain activities that people (in this case teachers and mathematicians) engage in on regular basis."

intention here to attempt to resolve this issue, but rather to investigate the application of this notion in practice, and to understand what knowledge and experience a particular teacher in South Africa draws on as he goes about his teaching, and how this relates to the field of *knowledge for teaching* as it is developing.

There is a wealth of literature trying to disentangle and categorise PCK, SMK and CK but this is not the focus of this study, instead, I am interested in the specificity of MfT and will therefore approach this differently. I will confine my discussions in the next two sections to aspects that have some bearing on ways in which concepts could come into existence in the practice of teaching. I will also confine my discussion to aspects that could possibly provide the platform for reflection to take place in the learners' attempt to make the notion of function a more substantial concept. I purposefully choose to do this since these are the kinds of aspects that would, to some degree, influence the analytical framework through the underlining theoretical framework.

2.2.2 Subject Matter Knowledge for Teaching

In a review of the work of others McNamara (1991) suggested the following arguments for SMK:

- If the aim of teaching is to enhance children's understanding then teachers themselves must have a *flexible and sophisticated understanding* of subject matter knowledge in order to achieve this purpose in the classroom.
- At the heart of teaching is the notion of *forms of representation* and to a significant degree teaching entails knowing about and understanding ways of representing and formulating subject matter knowledge so that it can be understood by children. This in turn requires teachers to have a sophisticated understanding of a subject and its *interaction with other subjects*.
- Teachers' subject matter knowledge influences the way in which they teach and teachers who know more about a subject will be more interesting and adventurous in the ways in which they teach and more effective. Teachers with

only a *limited knowledge* of a subject may avoid teaching difficult or complex aspects of it and teach in a didactic manner which avoids pupil participation and questioning and fails to draw upon children's experience.

- Knowledge of subject content is necessary to enable the teacher to *evaluate* text books, computer software and other teaching aids and mediums of instruction.
- During their own education student teachers will have acquired knowledge of subjects in both school and during their higher education courses. They may therefore have *developed attitudes* towards the way in which a subject is studied and *misunderstandings* which need rectifying if they are to teach their subject successfully in school.

(McNamara, 1991:114-115) (italics own emphasis)

The above arguments, as posited by McNamara (op. cit.), highlight aspects of subject matter knowledge that teachers should have. Included is the development of attitudes towards the subject, which resonates with the notion of developing a positive disposition which Kilpatrick *et al.* (2001) have highlighted as one of the strands of mathematical proficiency. McNamara (op. cit.), through his review of work of others avers that SMK should include disposition and attitudes and in Shulman's (1996) terms a whole lot of 'knowing that'. This is not sufficient since Shulman (op. cit.) argues that teachers need to have two kinds of understanding of SMK – 'knowing that' and 'knowing why':

> "We expect that the subject matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject matter major. The teacher need not only understand *that* something is so; the teacher must further understand *why* it is so …"
>
> (Shulman, 1986:9)

This is especially important as teaching for mathematical proficiency, as elucidated by Kilpatrick *et al.* (2001) comes under the spotlight in this study. The kind of knowledge that is being emphasised is the kind of knowledge as described by Ma (1999) cited in Ball and Bass (2000:97) through the simile that "teachers' knowledge of mathematics for teaching must be like an experienced taxi driver's knowledge of a city, whereby one can get to significant places in a wide variety of

ways, flexibly and adaptively." Ball and Bass (2000:97) describe what Ma calls 'profound understanding of fundamental mathematics' in terms of depth, breadth and thoroughness of knowledge that teachers need:

> "'Depth,' according to Ma, refers to the ability to connect ideas to the large and powerful ideas of the domain, whilst 'breadth' has to do with connections among ideas of similar conceptual power. 'Thoroughness' is essential in order to weave ideas into a coherent whole."

All this draws attention to the idea that the teacher's role is to assist his/her learners to achieve understanding of the subject matter being presented. If the aim is to motivate learners to develop mathematical proficiency and meaningful understanding then it is important for the teacher to possess strong concept images and concept definitions to ensure that learners have a solid knowledge of the subject matter to be taught. Even (1990) approaches the question of teachers' knowledge about mathematical topics and provides a framework for subject matter knowledge for teaching a specific topic in mathematics. Even (op. cit.) goes into more detail in her discussion about SMK and includes both the 'knowing that' and the 'knowing why' in the framework that she puts up. She identifies seven aspects to form the main components of teachers' SMK about a specific mathematical topic. What follows is a brief discussion of these seven components:

2.2.2.1 *Even's Components of SMK*

i. *Essential Features*

As alluded to earlier, if the aim of teaching is to foster mathematical proficiency amongst learners, then the teachers' concept definition as well as his/her concept image must have a good match with the specific concept definition. This would ensure that the teacher will be "able to judge whether an instance belongs to a concept family by using an analytical judgement as opposed to a mere use of a prototypical judgement" (Even, 1990:523). Analytical judgements refer to judgements based on the essential features of the concept whilst a prototypical

judgement is based on taking the features of the prototype and imposing it on other examples of the concept. With respect to functions the essential features are the arbitrariness and univalence. Definitions resembling that of the Dirichlet–Bourbaki concept, explicitly highlight the univalence feature of functions i.e. each element in the domain (set x) is associated to one and only one element in the range (set y); whereas the arbitrariness as an essential feature of functions is implicit.

ii. *Different Representations*

Taking the essential features of functions into account, illustrates that the different classes of functions[12] constitute in itself various forms of representation. A further illustration of this idea of different representations prevails in the current school curriculum where we have functions represented in the form of ordered pairs, tables, equations, or graphs. Even (1990) argues that knowing the concept in one representation does not necessarily mean that one understands the concept in another representation. Therefore teachers need to understand the concept in the various representations and also be able to move between the various forms of representation. The importance of various representations of a concept is that it gives "different insights which allow a better, deeper, more powerful and more complete understanding of a concept" (Even, 1990:524).

Under this aspect of different representations I will go beyond Even's (op. cit.) categorises and include analogies, illustrations, examples, explanations and demonstrations as discussed by Shulman (1986). It is the manner in which the teacher represents the subject in order to make it comprehensible to the learners. Drawing from the various domains of knowledge, as alluded to previously, the teacher will identify an appropriate form of representation to present the ideas to the learners. If this form of representation does not aid in the learners

[12] Examples of different classes of functions include inter alia: algebraic operations, trigonometric functions, the exponential function and its inverse, functions of points in a plane or in space and so forth.

understanding of the idea being presented, the teacher needs to draw on other forms of representation. Shulman captures this succinctly:

> "Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice."
>
> (Shulman, 1986:9)

iii. *Alternate Ways of Approaching*

Consider sketching the graphs of: i) $y = 2x^3+3x^2-12x-9$; ii) $y = x+1$; and iii) $y = \sin(60^0+2x)$. In the second example it would be easy enough to use point by point plotting and correctly sketch the graph, but would this approach (i.e. point by point plotting) be effective in correctly sketching the graphs in examples (i) and (iii)? This example is intended to illuminate what Even (1990) implies by 'alternate ways of approaching'. By using an alternate approach, like using the features of the graphs, it would be less cumbersome to sketch these graphs. As Even (op. cit.) indicates that the alternative ways are different from each other and none of them will be suitable for all situations. In order for teachers to teach for proficiency and to help their learners to identify appropriate approaches to functions, the teachers themselves need to posses this kind of knowledge and understanding.

iv. *The Strength of the Concept*

If $f(x) = x+1$ and $g(x) = x^2$; finding $g^{-1}(x)$ or for that matter $g(f(x))$ or even $f(x) + g(x)$ exposes us to 'new' functions (like inverses, composition of functions, substituting functions into each other and so on). "The success of a concept in the discipline of mathematics is rooted in the new opportunities it opens" (Even, 1990:525). Once again, teaching for mathematical proficiency then also implies that teachers should have a good understanding of kinds of characteristics as illustrated by the example above. As Even argues "understanding such sub-topics or sub-concepts requires knowing the general meaning which captures the essence

of the definition as well as a more sophisticated formal mathematical knowledge" (Even, 1990:525).

v. *Basic Repertoire*

The school curriculum for mathematics in both apartheid as well as in post-apartheid South Africa at the secondary school level deals with linear, quadratic and general polynomial, exponential and logarithmic, trigonometric and rational functions. These are the familiar types of functions that learners will experience in their secondary school years, thus these particular functions should constitute the basic repertoire for knowing functions. Therefore secondary school mathematics teachers should have a solid understanding of these types of functions since they will be required to teach it to their learners. Even (1990) suggests that the basic repertoire should include "powerful examples that illustrate important principles, properties, theorems, etc" (Even, 1990:525). The basic repertoire as argued by Even (op. cit.) fulfils the purpose of acting as a reference when having to deal with complex situations.

vi. *Knowledge and Understanding of a Concept*

Under this aspect Even (1990) looks at the importance of both procedural and conceptual knowledge and the relationship between them; and argues that mathematical knowledge in general, should include both kinds of knowledge as well as the relationship between them. Hence, competence in mathematics implies that neither of the two kinds of knowledge should be lacking in any way or if they have been acquired they should not exist in the mind of the acquirer as separate entities. The importance of this is highlighted by some examples which Even (op. cit.) uses to illustrate that correct procedural knowledge can sometimes assist in instances where conceptual knowledge is not fully developed. Thus, "when concepts and procedures are not connected, people may have a good intuitive feel for mathematics but not be able to solve problems, or they may generate answers but not understand what they are doing" (Even, 1990:527).

Although Even (op. cit.) highlights the importance of knowledge and understanding of a concept, it is also important to understand that in order to teach for mathematical proficiency these are not enough. The remaining strands of proficiency viz. strategic competence, adaptive reasoning and productive disposition are just as important.

vii. *Knowledge about Mathematics*

Knowledge and understanding of a concept is not sufficient enough to be classified as knowledge about mathematics. Knowledge about mathematics according to Even (1990) includes knowledge about the nature of mathematics. This resonates with Shulman's (1986) argument, that knowledge about mathematics is not only about 'knowing that' it is also about 'knowing why'. This means that it should include "ways, means and processes by which truths are established as well as the relative centrality of different ideas" (Even, 1990:527).

From the foregoing discussion we see Shulman (1986, 1987), McNamara (1991) and Even (1990) elaborating on their ideas about SMK. Now, what is interesting in these maths focused elaborations is that first, they correlate and second, that while there is a goal to separate SMK from PCK, this boundary is clearly blurred. The boundary is clearly blurred when it comes to aspects such as 'basic repertoire', 'alternate ways of representing' and 'different representations'. In their practice-based notion, Ball, Bass and Hill (2004), list eight elements of MfT which also reinforce the above but they focus on the act of teaching and they do not attempt to categorise further.

In attempting to articulate further the more practiced-based notion of mathematics for teaching, Ball, Bass and Hill (2004:59) list the following:

- Design mathematically accurate explanations that are comprehensible and useful for students;
- Use mathematically appropriate and comprehensible definitions;
- Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process;

- Interpret and make mathematical and pedagogical judgements about students' questions, solutions, problems, and insights (both predictable and unusual);
- Be able to respond productively to students' mathematical questions and curiosities;
- Make judgements about that mathematical quality of instructional materials and modify as necessary;
- Be able to pose good mathematical questions and problems that are productive for students' learning;
- Assess students' mathematics learning and take the next steps.

The above activities describe some of the work that a teacher will find him/herself engaging with. These are the kinds of activities that can possibly provide the platform for reflection to take place as learners attempt to make the notion of function a more substantial concept. Thus, this articulation by Ball, Bass and Hill (op. cit.) could then be perceived as encompassing the kinds of problem-solving teachers confront as they go about their work of teaching. However, this practice-based notion of MfT is lacking in two ways. Firstly, that which does not make its way through from SMK for teaching, is the importance of the first encounter (the introduction of the concept to learners), which Even (op. cit.) and others have talked about. The second aspect, which does not come through from PCK, is the importance of the role of metaphors. It is my view that MfT should comprise of Ball, Bass and Hill's (op. cit.) eight elements as well as the importance of the role of metaphors and the first encounter. This is the kind of composite set of elements of knowing MfT, and here I am grouping them, I am not trying to categorise and distinguish between PCK and SMK.

All of the above research and conceptualisation of specificity of mathematics for teaching is important to this study. This will assist with identifying the mathematical 'problems' the teacher in this study confronts as he goes about his work of teaching. It will also assist with the analysis of what the teacher actually draws on in his teaching. How this relates to the literature above, and why this teacher draws on the knowledge and experiences in the way that he does, will be an important point of reflection for this study.

The discussion centering on aspects of knowledge that teachers need to draw on, in order to teach for mathematical proficiency, illuminates the difficulty in trying to name it and to be clear on the categorisation. From this discussion we can see that researchers are dealing with similar things e.g. learner thinking, representations, attitudes and so forth. The focus of this study is not to resolve this issue, but to answer my research questions I will draw on these aspects. In the next section, I will engage in discussion that will elucidate the theoretical framework that underpins this study.

2.3 Theoretical Framework

This study focuses on mathematics for teaching; the kinds of mathematical problem-solving a teacher does as he goes about his work. Many of the theoretical bases that inform debate in mathematics education are theories of learning whereas the focus of this study lies in the practices of teaching, a less developed field of study. In this section I will make explicit the 'lens' through which I will view the data collected during this study. The purpose for a theory as captured by the words of Olivier is to act:

> "… like a lens through which one views the facts; it influences what one sees and what one does not see. 'Facts' can only be interpreted in terms of some theory. Without an appropriate theory one cannot even state what the 'facts' are."
>
> (Olivier, 1989:10)

The theoretical lens that informs this study is drawn from the QUANTUM project mentioned previously. The methodological question is: How might one 'see' the knowledge resources and experiences the teacher calls on (appeals to) as he goes about teaching the notion of functions to his learners? Starting with the premise that teaching always involves teaching something to someone, the question becomes: How, through his teaching, does the teacher present the notion of a function and in so doing provide opportunities for learners to construct the notion of function, gain experiences of a function, as well as criteria by which to

recognise a function and be able to respond appropriately to tasks involving functions?

Davis, Adler, Parker and Long (2003) turn to Hegel's theory[13] of judgement – an abstract theory of how notions come to be acquired. My concern here is to use this theory but also to extend it into an educational practice i.e. my concern here is not just how notions come to be acquired, but how they come to be acquired in a pedagogic practice. For Hegel, acquiring a notion is bound up with experiences of judgement. In the context of the school, whether explicitly or implicitly, teachers continually exercise judgement as they engage with their learners in relation to the object they wish them to acquire. In other words, as teachers exercise judgement, so learners are afforded opportunities to clarify what it is they are learning i.e. they are offered criteria both implicit and explicit.

According to Hegel's theory the process of judgement splits the notion[14] into subject and one or more predicates that serve to fill out the notion. The four moments of judgement in Hegel's theory are the judgements of: i) existence, ii) reflection, iii) necessity and, iv) notion. In the next four sections I will unpack these moments of judgements and relate them to my study.

2.3.1 The Judgement of Existence:

The essential feature in this judgement is that the judgement of existence has the form of *immediacy*, this is to say that "an initial encounter with a notion is one of *immediacy*; it is simply a 'that', an empty signifier: a verbal or written mark, or gesture." (Davis *et al.*, op. cit.:7). In other words, there is an absence of adequate predication. Davis *et al.* (2003) indicate that to demonstrate that we understand a notion we need to show a series of predicates which is different from the signifier for the notion itself. To illustrate this they (Davis *et al.*, 2003) make use of the

[13] While I am drawing on Hegel's theory it is important to note that I am drawing on its interpretation by Davis *et al* (2003) and how it is being re-interpreted in education and for education.
[14] Notion could be seen as an idea or concept.

following example: if a child was asked 'what is a square?' the response 'a square is a square' is unacceptable, in order to establish 'legitimate' responses one has then to identify the notion with something other than itself like 'it is a floor tile'. It is important to note that the notion is still in its *immediacy*, simply a 'that': a sound or a word or a diagram and hence the relationship between subject and predicate is impossible. It is provocative to think about the formal definition of a function being used to introduce this notion and so operating at the level of judgement of immediacy. It is thus a form of immediacy, simply a 'that'.

Hegel is not talking about or referring to schooling, he is talking about the 'everyday' in general - how do we get to understand that 'that' (an object) is what it is (e.g. an IPOD). So what happens when we take Hegel's construct of 'existence' and apply it to pedagogy, where in pedagogy the first encounter is purposefully designed but for Hegel it is not designed – it is occurring in life? The construct 'first encounter' is taken from the work of the French theory of didactic situation, and is described as the first moment of the didactic process or process of study. It is explained thus:

> "The *first moment* of study is that of the *first encounter* with the organisation *O* at stake. Such an encounter can take place in several ways, although one kind of encounter or 're-encounter', that is inevitable unless one remains on the surface of *O*, consists of meeting *O* through at least one of the types of tasks T_i that constitutes it."
>
> (Barbé, Bosch, Espinoza and Gascón, 2005:238)

We know that in teaching we have to bring a concept into being in the classroom and what Hegel tells us that it is merely a 'that', what the literature (i.e. related to the teaching of functions and on PCK) tells us is that the 'that' actually matters in pedagogy. The literature for the teaching of functions shows that if we are to teach for mathematical proficiency we need to ensure that the first encounter for the learners is a task that will facilitate the construction of rich concept images and concept definitions so as to avoid making a 'didactical error'. What the literature illustrates is what that 'that' could be or what the first encounter could be. For Hegel the first encounter is not an issue since he is only concerned that it

is a 'that', however, this is key in a pedagogic practice. All of this will therefore be of interest to me as I study the teacher and the way notions come into being.

2.3.2 The Judgement of Reflection:

In the judgement of existence the notion is merely an immediate, or abstract notion. Davis *et al.* (2003) indicate that the notion at the level of *immediacy* will generate *reflection* which is an attempt to place some *predication* on the notion so as to transform it from a mere 'that' to something more substantial.

In other words, when the notion comes into existence it can only be abstract in some ways or an experience and therefore it must generate reflection. Reflection is an attempt at trying to transform the notion into something more substantial so that it becomes increasingly comprehensible. When there is no reflection learners can only imitate – all they can do is to reproduce the 'that'. The literature on functions suggests that if learners grapple with aspects such as multiple representations, different approaches to functions and so forth they begin to reflect on the notion thereby transforming it to something more substantial. From a teaching perspective of functions Ball, Bass and Hill (2004) provide a more practice-based notion of mathematics for teaching, which indicates that from a teacher's point of view, actions such as defining, representing, explaining and so forth would begin to drive the process of reflection.

It will be interesting to study how learners are provided with opportunities to elaborate the notion and shift it away from being a mere 'that' to being something more substantial and how it relates to mathematical proficiency. In this study I will be observing and trying to understand that as the teacher works with the notion, what opportunities are provided for reflection.

2.3.3 The Judgement of Necessity:

Further in their interpretation of Hegel, Davis *et al.* (2003) indicate that at some point or the other predication needs to be stopped and when this happens there is a shift in judgement from *reflection* to *necessity*. Taking the example of the square Davis *et al.* (op. cit.) explain that it is the particular features of a square that allow us to differentiate it (a square) from other plane figures and it is these features that are essential to the definition of a square. Therefore, "a *necessary* relation between subject and predicate(s) is established, and the notion no longer collapses into a mere 'that'" (Davis *et al.*, 2003: 8). Judgement of necessity involves moves to fix meaning, so that the notion to be acquired is a full notion. It is interesting to think in the case of a function, how the passage from immediacy to necessity, through reflection pans out in the classroom, what kinds of judgements of immediacy, reflection and necessity teachers enact, and what possibilities there are for learners to grasp the notion of a function. For the purposes of this study the judgement of necessity is used to describe or categorise the appeals the teacher makes in order to fix meanings or to legitimate meanings in the classroom.

2.3.4 The Judgement of the Notion:

According to Davis *et al.* (2003) in the judgement of the notion the predicate is a description of the relationship of object to notion, therefore the concern here is not with filling out of the notion but the adequacy of the notion itself. They (Davis *et al.*, 2003) illustrate this by using the examples: is the object 'good' or 'bad', 'elegant' or 'clumsy' and so forth. The judgement of the notion is related to contingency since "the actuality of the notion depends on the occurrence of an event that is itself irreducibly contingent" (Davis, 2001:10). In a pedagogic practice the contingency is schooling. Contingency features in both production and realisation (reproduction) of the notion. With respect to its production "the notion, in its arrival, retroactively transcodes a series of contingent events into its necessary conditions and every realisation of the notion only comes to be by way of a contingent event that is its index" (Davis, 2001:10). The actuality of the

notion depends on the context in which it is being reflected on and the contingencies here are constrained by the school environment.

Davis *et al.* (2003) elucidate further that the relation between contingency and necessity can be perceived in terms of the idea of a *gap*. The *gap* refers to the space between an object (event) and its notion or as Davis (2001) more appropriately describes the gap as being correlative to the pedagogic subject (eg. the learner). In other words, "the pedagogic subject is the breach between the object and its notion, a point of resistance acting against the self-realisation of the notion" (Davis, 2001:10). So, in the context of schooling the gap comes into existence because of what the learners can manage. The notion that comes into being in a mathematics classroom is an institutionalised notion[15] and as such is a function of learners having to learn. Thus, they become the obstacle in that sense. This inevitably creates the gap between the scholarly notion and the institutionalised notion. In other words, the gap is between the institutionalised notion of mathematics in the school and what mathematicians might regard as viable, and these are not necessarily the same thing. I am more interested in what this might look like particularly from the perspective of mathematical proficiency.

The above rather abstract discussion is an attempt to illuminate the idea that in practice, teachers exercise judgement, ranging from existence to reflection and necessity with the overarching judgement of the notion. In the context of this study, these judgements will entail calling on or appealing to various knowledge resources, and teachers will be doing this in the schooling context. 'Seeing' a

[15] According to the Anthropological Theory of Didactics a teacher's praxeological problem consists of "creating, through a *didactic process*, a specific mathematical organisation in a particular educational institution" (Chevallard as cited in Barbé *et al.,* 2005). In order to solve this problem, "the teacher has some 'given data', such as curricular documentation, textbooks, assessment tasks, national tests, etc., where some components of a mathematical organisation, as well as some pedagogic elements and indications on how to conduct the study can be found. This is how the educational institution 'informs' the teacher about what mathematics to teach and how to do so." (Barbé *et al.*, 2005).

teacher's judgements in action (and so to what they call on in their practice) thus involves understanding pedagogy. It is this last statement that provides the link to the more overarching theory in which this study fits – Basil Bernstein's theory of pedagogy, and specifically his elaboration of the 'pedagogic device'.

Bernstein (1990:180) describes the pedagogic device as a "symbolic ruler of consciousness". It acts to mediate specialised consciousnesses to be formed through pedagogical practice. The pedagogic device has an intrinsic grammar (grammar in a metaphoric sense) that is in turn mediated through three interrelated rules viz. distributive rules, recontextualising rules, and rules of evaluation. Davis *et al.* (2003) elaborate further, but for my purposes and for the purpose of this study, I will focus on the evaluative rules of pedagogic discourse (as it is here that links with Hegel's theory of judgement become evident) since the rules of evaluation emerge through pedagogic discourse i.e. in practice.

Adler, Davis, Kazima, Parker and Webb (2005) argue, drawing on Bernstein, that the "distribution of knowledge and the rules for the transformation of knowledge into pedagogic communication is condensed in evaluation". Evaluation attempts to control the transmission or acquisition of the available potential meaning. In other words, evaluative rules construct the pedagogic practice by providing the criteria to be transmitted and acquired. This means that the possibilities for meaning are condensed in and through moments of evaluation, and in Bernstein's terms, it is these that will function to specialise consciousness (in this case, knowledge of mathematics specifically functions).

In Bernstein's (1996, 1990) terms any pedagogy transmits evaluation rules, this is to say, that in any pedagogic practice, teachers transmit criteria to learners of what it is they are to come to know. In other words, at various points in time the teacher needs to legitimate aspects of the pedagogic discourse (in relation to what it is he wants learners to know), and in order for the teacher to do this he will have to exercise some form of judgement.

For Bernstein, the transmission of criteria to learners functions at two levels – or what he calls two rules of acquisition: recognition and realisation. In the first instance, the learner must recognise what it is he/she is to attend to as well as the specialised language entailed. But recognition (knowing what it is you are meant to know or do) is not sufficient. In Bernstein's terms, recognition needs to translate into realisation, where the learner is able to produce the legitimate text i.e. the kind of response required by the teacher.

For the purposes of this study I am interested in the criteria a teacher transmits through evaluation, and specifically, the knowledge and experience he draws on (appeal to) as he does so. What this suggests methodologically is that it is in evaluative moments in pedagogic practice, what Davis *et al.* (2003) have called evaluative events, that criteria for what is to be acquired become visible. It is through the visibility of the criteria that teachers transmit, that we will be able to 'see' the knowledge and experience they draw on as they go about their work of teaching mathematics i.e. we can trace this through from the first encounter (immediacy) through reflection to necessity. So, in this study I will be interested in observing what knowledge resources the teacher draws on as evaluative moments of judgement unfold over time.

At various instances I have highlighted what it is that I will be interested in observing in this study, but in order to do that I need an analytical framework. In the next section I will expound on this framework.

2.4 Analytical Framework

> "Data analysis is the process of making sense out of the data. And making sense out of data involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read – it is the process of making meaning."
>
> (Merriam, 1998:178)

In my attempt to make sense of the data collected and at the same time to find answers to the focus questions that underpin this study I had to actually chunk the data into units for analysis and the unit is what I call the evaluative event or an episode (see appendix D). An evaluative event or episode is defined when the teacher makes moves to legitimate meaning or to fix meaning. Upon the identification of an evaluative event I established whether the notion in question was conceptual or procedural in nature and whether it had the potential to promote strategic competence amongst the learners. As discussed in the theoretical framework the first level of judgement is at the level of existence (i.e. the first encounter) so with respect to this, three categories were identified viz. verbal; written and activity. It is important to note that these categories are a function of the data i.e. it emerged from the data. Verbal, as the category suggests, the notion comes into existence through someone (teacher or learner) saying something. The next two categories were further subdivided and I draw your attention to figure 2 (page 41) for these refinements.

According to Hegel's theory the next level of judgement is at the level of reflection, this resonates with Ball, Bass and Hill's (2004) practiced-based notions of mathematics for teaching as listed in section 2.2.2 (page 23). The link here is that the activities as described by Ball *et al.* (2004) are teacher driven activities intended to ascribe some form of 'predication' on the notion so as to transform the notion from a mere 'that' to something more substantial. For this analytical framework I have condensed the eight activities as elucidated by Ball *et al.* (2004) into six categories which portray the mathematical work of teaching which are: i) defining; ii) explaining; iii) representing; iv) questioning; v) working with learners' ideas and vi) restructuring tasks. These are what teachers are doing to provide opportunities for reflection, reflection itself should be by the learners – it is the opportunities that are they are given to reflect. Reflect on what? It might well be that they reflect on the definition or the representation and so forth. I refer you to chapter 4 (section 4.3, page 64) for a discussion on the indicators that will exemplify these categories.

As alluded to in the theoretical framework the idea of 'predication' as per Hegel's theory needs to be stopped and when this happens there will be a move to the judgement of necessity. This involves moves by the teacher to fix meaning or to legitimate meaning. In this framework the categories identified as knowledge domains that the teacher might appeal to in order to 'authorise' or legitimate knowledge in class and so ground the learners' ideas is adapted from Adler *et al.* (2005), who in turn developed these from a literature review of the field and the empirical data in the wider study whilst other categories emerged from the data collected. The appeals that characterise this aspect of the framework are:

- Mathematics (M) – principles of mathematics reasoning, defining, representations and so forth. This was further refined to include the following sub-categories: i) empirical (through observation you can see why it is the case); ii) definitions (the teacher's attempt to define notions based on mathematical definitions and rules in mathematics); and iii) rules (conventions in mathematics).
- Experience (E) – the teacher draws on his own experience (personal and professional) and the experience of his learners (you can see it works by metaphorically relating the notion to the everyday). The teacher's professional experience and the everyday formed the two sub-categories that further refined this category.
- Curriculum (C) – teacher legitimates knowledge for example by telling learners 'this is what it says in the textbook or whatever materials are being used or this is what is expected of you in a test or exam'. Two sub-categories were identified viz. textbooks and tests or examinations. This category reconciles with the idea of authority, where authority is seen in the form of curriculum.

To summarise, I present a model of the theoretical framework that underpins this study (see next page).

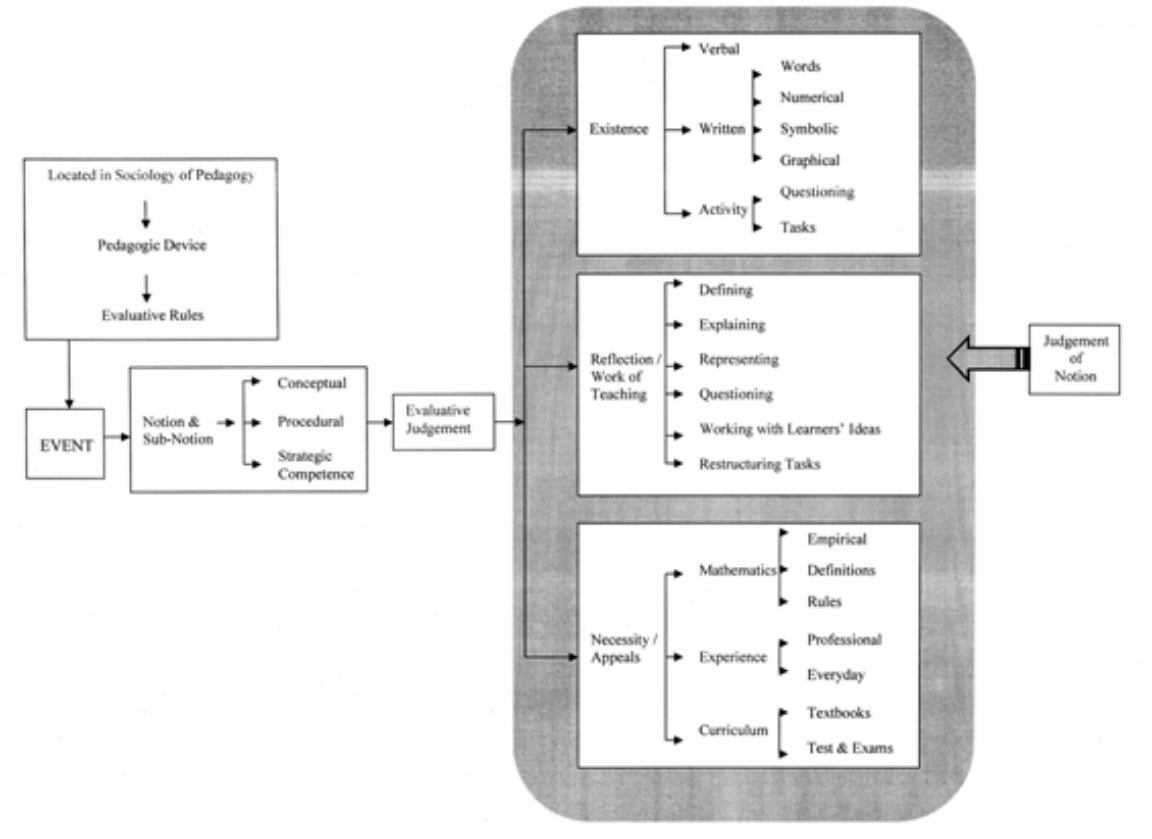


Fig. 2: A Conceptual Map of the Theoretical Framework

Although the model (figure 2), representing the theoretical and analytical frameworks that underpin this study, assists with the analysis, it obscures that in time the judgement of the notion is not a linear process but more cyclical in nature and that it is an overarching notion. Hence, the shaded background and arrow are used to represent the cyclical nature as well as the overarching characteristic of the judgement of the notion.

2.5 Summary

In this chapter I have outlined the theoretical and analytical framework of the study and interrogated the literature pertaining to functions and the teaching of functions. I have also elaborated on Even's (1990) work pertaining to mathematics for teaching.

The literature illuminates that from a teaching point of view the introduction of a concept is very important. From a functions point of view there are a whole lot of questions we need to ask, for instance: what is the best way to introduce the notion of function so that it contributes to developing strong concept images and concept definitions in the mind of the learners? It is interesting for me to see how the teacher, in this study, provides the first encounter as well as opportunities for learners to reflect on the notion in their attempts to transform it into something more substantial. It will also be interesting to observe what the teacher appeals to in his attempt to fix meaning. The next chapter will examine the methodology of this research.

Chapter Three

Methodology

In this chapter I will discuss the research methods that I have adopted for this study and the data collection techniques that I have employed. In addition, I will engage in discussion about the sample used for the study and the ethical issues that I have considered.

3.1 Methodological Approach

Investigation into the issues inherent in my research problem together with the focus questions warrants that my study be located within an interpretive paradigm. The notion of an interpretive paradigm is illuminated to an extent when compared to a normative paradigm:

> "The normative paradigm (or model) contains two major orientating ideas: first, that human behaviour is essentially rule-governed; and secondly, that it should be investigated by the methods of natural science. The interpretive paradigm, in contrast to its normative counterpart, is characterized by a concern for the individual."
>
> (Cohen, Manion and Morrison, 2002:22)

The primary objective of this study is to gain insights into mathematics for teaching – the kinds of knowledge that a teacher draws on in order to teach mathematics successfully. Consequently, a qualitative[16] research approach was most appropriate for this study since this form of research is:

> "an effort to understand situations in their uniqueness as part of a particular context and the interactions there. This understanding is an end in itself, so that it is not attempting to predict what may happen in the future necessarily, but to understand the nature of that setting – what it means for participants to be in that setting, what their lives are like, what's going on for them, what their meanings are, what the world looks like in that particular setting – and in the analysis to be

[16] A qualitative approach does not exclude quantification, but what it does exclude is prediction and causation

> able to communicate that faithfully to others who are interested in that setting … The analysis strives for depth of understanding."
>
> (Patton, 1985 as cited in Merriam, 1998:6)

In light of the above, an appropriate research method for this study would be a case study approach. Cohen *et al.* (2002:182) state that a case study would "strive to portray 'what it is like' to be in a particular situation, to catch the close-up reality and 'thick description' of participants' lived experiences of, thoughts about and feelings for, a situation." This is further amplified by Opie (2004:74) "… a case study can be viewed as an in-depth study of interactions of a single instance in an enclosed system." The 'case' according to Merriam (1998) is common sense obviousness which could include for example: an individual teacher, a single school, or perhaps an innovative programme. Taking my research problem with the focus questions, it immediately becomes evident that the teacher and his class of learners would be the case in my study. A detailed examination of one teacher will provide sufficient evidence for the effectiveness of the research method and sufficient data to respond validly to the research questions.

Since a case study involves mainly the collection of qualitative data, it is capable of "providing a much richer and more detailed description of human behaviour and experience than can be obtained from the collection of quantitative information" (Dyer, 1995:53). The data collected in this study is qualitative data, however, I do quantify some of the data for it is only through this kind of analysis that I can get to see the 'big picture'. The strength of a case study is also found in the fact that it is strong on reality; hence it may hold key features to gaining insights into a situation that may otherwise be lost in research methods that require large scale data e.g. surveys. A case study approach according to Merriam (1998) provides insights into other, similar situations and cases and thereby assisting interpretation of other similar cases, in addition, it can embrace and build in unanticipated events and uncontrolled variables. On the other hand, it is also important for me to take cognisance of the weaknesses that plague this approach since these weaknesses may manifest as limitations to my study. Firstly, the

results of a case study may not be generalisable except where other readers/researchers see their applications. Merriam (1998) indicates that case studies are not easily open to cross-checking; hence they may be selective, biased, personal and subjective.

This study can be considered to be *pure research* as it concerns itself with enriching "the thinking and discourse of educators … by the refinement of prudence through the systematic and reflective documentation of experience" (Stenhouse, 1988:50).

3.2 Data Collection Strategies

This study investigated mathematics for teaching; the kind of mathematical problem-solving a teacher does as he goes about his work. To engage in this type of investigation I had to observe the teacher in practice since it gave me "the opportunity to look at what is taking place *in situ* rather than at second hand." (Patton as cited in Cohen *et al.*, 2002:305), in other words it provided me with the opportunity to record 'behaviour' as it was happening. One may pose the question, then why not interview the teacher and during the interviews provide mathematical problems which the teacher is to answer? Whilst answering, the teacher can be questioned to establish the kind of mathematical problem-solving he does as he goes about his work. For me this seemed 'artificial' in a sense, since analysing the teacher outside the classroom makes the assumption that what the teacher expresses is actually what takes place in the classroom, when in actual fact what takes place in the classroom is not just determined by the teacher. What takes place in the classroom is the amalgamation of the interaction of the teacher, the learners as well as the environment. This argument is amplified by the following extract:

> "Despite the fact that the work on teachers' knowledge has developed innovative means of probing teachers' knowledge of mathematics for teaching – through scenarios and situated examples – responding to such grounded situations is not fully equivalent to the on-line work of teaching"
>
> (Ball *et al.*, 2001:450)

Although I allude to the fact that observations were the key strategy for data collection in this study, it is important to take note that the kinds of observations available "lie on a continuum from unstructured to structured, responsive to pre-ordinate" (Cohen *et al.,* 2002:305). In view of this, the observation strategy that I employed in this study could be categorised as structured observation (pre-ordinate), hence I knew in advance what I was looking for and the observation categories were developed beforehand. Although the categories were developed beforehand, I was also open to the idea that other categories could emerge from the data collected. Hence an observation schedule was designed which was used to capture my observations (see appendix A).

Merriam (1998) indicates that a researcher can assume one of several stances while collecting data as an observer – the stances range from being a full participant to being a spectator. Opie (2004) described these roles as participatory and non-participatory roles. For the purposes of this study my role as observer could be classified according to what Opie (2004:126) described as non-participant "where the researcher has no interaction with the subjects during data collection" which resonates with the notion of structured observation.

In addition to making use of a structured observation schedule to record my observations I also video recorded all the lessons. My use of video recordings stems from what Cohen *et al.*, (2002:313) had to say, that the use of audio-visual recordings "can overcome the tendency towards only recording the frequently occurring events." Furthermore, I felt that this form of data gathering provided me with the opportunity of keeping a more 'permanent' track of the lessons since I could replay it as many times as I wished. This therefore allowed me to focus on events more closely and in greater depth than when recording on the spot. In addition, during lessons much went on and it was highly possible that as an observer I could miss some of the important things that went on, so the use of video recordings assisted me to overcome this obstacle. The video recordings also

allowed me to look at the verbal and non-verbal responses. I made a transcript[17] of each of the lessons recorded and displayed the time intervals between episodes. When reference is made to any of these transcripts the time intervals will serve the purpose of an evidence trail.

I also made use of field notes as another data collection strategy. The purpose of field notes or more specifically *field jottings* as described by Fraenkel and Wallen (1990:381) are "quick notes about something the researcher wants to write more about later. They provide the stimulus to help researchers recall a lot of details they do not have time to write down during an observation or interview."

Interviews augmented the data collection strategies that I have already discussed (see appendix B for the interview schedule). I only interviewed the teacher since the focus of this study is on teaching. To this end I conducted three interviews with the teacher. The first interview was conducted before the period of data collection as the purpose of this interview was merely to obtain biographical information. The second interview was conducted after a week of teaching and the third interview was conducted after all the lessons were taught. All three interviews were transcribed and numbers were inserted on the left hand side of the transcripts[18] to reflect 'turns of talk'. This will serve as part of the evidence trail.

My reasons for conducting interviews were firstly, that it provided me with a mechanism of checking my interpretations of what I had observed in the lesson. Secondly, it provided me with the opportunity to probe for reasons why things were done in the way that they were. In other words, it allowed me the opportunity to probe particular issues in depth. Opie (2004) described interviewing styles along a continuum ranging from structured to unstructured. The interviewing style that best suited my needs in this study can be classified according to what Opie (2004:118) describes as semi-structured:

> "These are a more flexible version of the structured interview which will allow for a depth of feeling to be ascertained by providing opportunities to probe and

[17] The full transcript of each lesson is bound and kept separately.
[18] The full transcript of each interview is bound and kept separately

> expand the interviewee's responses. It also allows for deviation from a prearranged text and to change the wording of questions or the order in which they are asked."

My reason for using this kind of interviewing style is embedded in the notion that this technique allows for more flexibility, with the result I was allowed to probe on certain responses if I felt that such probes enriched my understanding and the analysis of the data collected.

I tape recorded all interviews since I felt that the tape recordings provided me with a more accurate and economical way of capturing what was being said. I also made use of field jottings as alluded to already, since this complemented the tape recordings and provided me with a more holistic account of what transpired during the interview sessions.

3.3 Piloting

Piloting in this study was very difficult. Piloting the study in the true sense of piloting would imply that I would have had to do the study. All that I could do was to pilot aspects of the study. Such piloting was done by viewing pre-recorded lessons that formed part of other studies unknown to me. The recordings of these lessons are kept in archive by my supervisor. In the discussion that follows I will highlight what the piloting process enabled.

I observed four lessons delivered by different teachers employing different teaching methodology ranging from the traditional 'chalk and talk' approach to a more progressive learner-centred approach. The mathematics content that was being taught by these teachers was of no relevance since the focus of this study is to investigate the kind of mathematical problem-solving teachers do as they go about their work. Despite having a preliminary observation schedule, I found that I was not clear on what to focus on whilst watching the lessons. I found myself looking at anything and everything that seemed interesting. I was not sure if I correctly identified episodes or evaluative events and at the same time could

identify the appeal that punctuated the episode so that indeed it was an episode that I had captured. I found myself frequently rewinding the cassettes to make sense of what transpired in the lesson.

During the piloting process two things emerged. Firstly, being able to identify the unit of analysis and secondly, being able to define an event. In addition, it was through the process of piloting that I realised my tools for 'seeing' were inadequate. I realised it was of paramount importance that when viewing lessons I needed to make a concerted effort to 'wear the lens' as prescribed by the analytical framework that underpins this study. The piloting process provided me with the opportunity to practice this. I found that transcribing the data ameliorated this process and it was by watching the lessons again and reading the transcripts that I was able to make sense of the problem-solving that the teacher engaged in and the resources that were drawn on in order to solve these problems. The piloting process contributed to the refinement of my analytical framework in terms of me developing the categories of how a notion could possibly come into existence i.e. the first encounter. It was also as a result of piloting that I was more capable of seeing relationships between some of the aspects of the more practiced-based notion of mathematics for teaching as elucidated by Ball, Bass and Hill (2004). Through a process of grouping I managed to reduce the eight aspects as described by Ball, Bass and Hill (op. cit.) into six categories and renamed them: defining; explaining; representing; questioning; working with learners' ideas; and restructuring tasks.

The piloting process assisted me in ensuring that the observation schedule was 'workable' in the sense that the categories identified were indeed observable in the lessons, and that the categories were comprehensive and discrete i.e. no overlapping between the categories. The piloting also assisted me to gain proficiency and consistency with the capturing of data (observed behaviours) into the various categories. The piloting process also allowed me the opportunity to reflect on what I was observing in the lessons. These reflections in turn assisted me in thinking about the kinds of questions to include in the interview schedule.

Since the piloting was not done by making use of live data it did not provide me with the opportunity of practicing some of the practical skills. I therefore relied on my theoretical knowledge of these skills and had to apply them and refine them whilst in the process of collecting data for the main study, for example: the use of high quality microphones so that when learners who sit next to the video camera start talking you can still hear what the teacher is saying. Also, I did not have the opportunity to interview any of the teachers during the piloting process, so my first experience of interviewing arose whilst in the process of collecting the actual data for this study. I felt that I was prepared theoretically and I heeded the numerous cautions about the difficulties of interviewing (Thompson, 1978; Bell, 1987). I was fortunate enough that the teacher, who participated in this study, was more than willing to reflect on his practice and this contributed to the interviews becoming more conversational in nature, which in turn contributed to the creation of a less stressful environment for both participant and me.

3.4 The Sample

> "The quality of a piece of research not only stands or falls by the appropriateness of methodology and instrumentation but also by the suitability of the sampling strategy that has been adopted."
>
> (Cohen *et al.*, 2002:92)

The sample for this study constituted a teacher of mathematics who is mathematically qualified. By this it is meant that Nash[19] has a higher diploma in education majoring in mathematics (content[20] and didactics) and computer science. The teaching experience of Nash equates to twelve years of secondary school experience, teaching mathematics ranging from grade 8 to 12. Nash is an Indian male who is a first language English speaker. Nash teaches at a public school that services learners coming from a range of socio-economic backgrounds

[19] For the purposes of this study I will use the pseudonym Nash to refer to the teacher.
[20] Content is used to describe pure mathematics courses at the level of first and second year university level.

and the language of teaching and learning at the school is English. The selection of my sample can be best described as purposive or purposeful sampling since in this study my intention is to gain insights (Merriam, 1998; Cohen *et al.*, 2002) into aspects of knowledge that Nash draws on when teaching for mathematical proficiency. In addition, the sample was also opportunistic since I knew the principal of the school and this gave me access to the school. Qualification was the criteria for selecting the teacher so as to ensure a reasonable foundation of mathematical knowledge and exposure to epistemological concepts associated with pedagogy.

As a school mathematics teacher, Nash is relatively well resourced. He has access to the relevant curriculum documents issued by the National Department of Education (DoE) i.e. both the Revised National Curriculum Statement (DoE, 2001) as well as the Nated 550[21] (DoE, 1997). He also has access mathematics textbooks, basic teaching aids such as the chalkboard and an overhead projector. In addition, the head of department for mathematics at the school had recently completed a Bachelor of Science with Honours degree in mathematics education which serves as another resource for the teacher. The classroom in which the observations were conducted comprised of 35 learners, 17 female and 18 male.

I visited Nash's class during the third term of the school calendar, more specifically between 15 August 2005 and 22 August 2005. Prior to the first lesson I conducted an interview with Nash in an attempt to collect some biographical data, in this discussion I learnt that Nash serves a very useful and important role at the school. The following extract from the interview illuminates this:

21 VP: How would you describe your role in this school?

22 Nash: Well, at the moment my role is more of a, I'll say multi-purpose. Like if it is a sports team - I will be the utility player because my

[21] Nated (National Education) 550 also known as Report 550 refers to the 'historical' curriculum document that was in place during apartheid South Africa – this is the curriculum currently being phased out of South African schools. The new curriculum has already been phased in from grade 1 to 9. The phasing out of the 'historical' curriculum for the remaining grades are as follows: grade 10 in 2006, grade 11 in 2007 and grade 12 in 2008.

> experiences cover a broad background, I've been in humanities, I am a maths specialist, a computer science specialist. I've taught mathematics and science like I said throughout the entire grades so we are in a situation where we lost seasoned teachers and senior teachers as well, they couldn't just be replaced. The replacements they got were junior teachers or inexperienced teachers as such. So I had to actually replace the senior teacher. For the past 18 months that I have been here, I've been the person shuffled and basically helping out. So during these 18 months I taught from economics, business economics right to physical science and senior mathematics.
>
> (Interview 1, turns 21 – 22)

The following extract highlights that Nash works predominantly in the mathematics department at the school, which he sees as being well organised and effective.

> 24 Nash: Well one of the plus factors is that we have a very organised maths department. We have a subtle blend of experience and 'new blood' not in terms of inexperience but in terms of new ideas coming in all the time and because we share classes amongst each other, like no one is completely responsible for a particular grade – we got three way splits or two way splits and within the department itself we have smaller groups where we have to work and this leads to micro team management and management on a larger side which I find very effective in the long run.
>
> (Interview 1, turn 24)

The significance of these extracts is that they point out that Nash is an active and involved teacher and that he is in a supportive environment. This is my knowledge of the school and of Nash and he experiences it himself and reflects it back to me. This is important for me and this is why I chose him to be a subject for this study.

3.5 Ethical Considerations

> "… a matter of principled sensitivity to the rights of others. Being ethical limits the choices we can make in the pursuit of truth. Ethics say that while truth is good, respect for human dignity is better, even if, in the extreme case, the respect of human nature leaves one ignorant of human nature.'
>
> (Cavan, 1977 as cited in Cohen *et al.* (2002:56)

In this study there were ethical issues surrounding the integrity of the school as well as "informed consent, guarantees of confidentiality, beneficence and non-maleficence" (Cohen *et al.*, 2002:279) of the teacher and his learners that participated in this study.

With respect to the participants of this study I obtained what Cohen *et al.* (2002:51) refer to as informed consent – "the procedures in which individuals choose whether to participate in an investigation after being informed of facts that would be likely to influence their decisions." To this end, I have obtained permission from the Gauteng Department of Education (GDE) to conduct this study in one of their schools (see appendix E). Furthermore, I have also obtained permission from the principal to conduct the study in his school as well as permission from the teacher to be the subject in this study. A consent form was issued and duly completed by the parents of all the learners as well as by all the learners that participated in this study. The following issues were addressed in the consent form:

- A brief outline of the research topic and aims.
- An assurance that the research and findings are in no way any reflection of individual learners, their families or their school, and that participants will be entreated to approach the study in this light.
- A guarantee of autonomy – participation in the study is purely voluntary and that anonymity will be maintained, especially in the reporting of the findings.

- An explanation as to the need for the video recording of lessons as a data collection strategy and seeking of permission to use such a strategy since the learners will appear on the video.
- An explanation of how the research findings are likely to be used.
- A set of tick-boxes in which consent can be indicated for transcripts and recordings to be used by myself, in publications and by other researchers.

(See appendix C). In addition to the above, I have also obtained ethics clearance from the University of the Witwatersrand (see appendix F).

Since this study made use of observations and interviews as data collecting techniques, I will now engage in discussion related to the ethical issues that were considered when making use of such strategies. With respect to interviews it should be noted that it is only the teacher that was interviewed. The ethical dimensions related to interviews concern "interpersonal interaction and produce information about the human condition. One can identify three main areas of ethical issues here – informed consent, confidentiality, and the consequences of the interviews" (Kvale, 1996 as cited in Cohen *et al.*, 2002:292). The issue of confidentiality and informed consent has already been dealt with. I explained to the teacher that the consequences of the interview was that it provides a form of triangulation, this notion will be elaborated on in the next section, as well as to provide insights into that which was observed. To ensure that the interviews were conducted in an appropriate, non-stressful and non-threatening manner I made concerted efforts to ensure gentleness, sensitivity and openness on my part – as already discussed. Furthermore, I took heed and put to practice what Patton (1990) had to say with respect to interviews – the task of the interviewer "is first and foremost to gather data, not to change people. The interviewer is neither judge nor therapist nor a cold slab of granite - unresponsive to the human issues, including great suffering and pain, that may unfold during an interview" (Patton, 1990 as cited in Merriam, 1998:214).

Lesson observations were another means of data collection that I employed. Merriam (1998) indicates that this technique of data collection has its own ethical

pitfalls depending on the researcher's involvement in the activity. As alluded to previously I sought formal permission from the teacher and the learners to observe them as they went about their work, in my capacity as a non-participant observer. The question that arose for me was, what was I going to do if I found myself in a situation where I witnessed utterly ineffective, perhaps potentially damaging teacher behaviour? Merriam (1998) indicates that knowing when and how to intervene is perhaps the most perplexing dilemma facing qualitative researchers:

> "Blanket injunctions such as 'never intervene' offer no practical aid. In the reciprocal relationship that arises between fieldworker and hosts, it seems immoral – and perhaps it is – to stand back and let those who have helped you be menaced by danger, exploitation, and death."
>
> (Cassell, 1982 as cited in Merriam, 1998:215)

In response to this I sought solace in what Taylor and Bogdan (1984) as cited in Merriam (1998:215) have to say: "the literature on research ethics generally supports a non-interventionist position in fieldwork, failure not to act is itself an ethical and political choice."

Finally, as part of my ethical considerations I ensured that the data collection process caused the least possible disruption to the on-going life of the participants as well as to the normal functioning of the school. I also ensured that I gave written and verbal feedback to all interested parties and acknowledged all those who had helped – not at the expense of revealing the identities of the participants.

3.6 Considerations Concerning Rigour

Reliability in research, according to Opie (2004:65) is an "important consideration, in that it may be useful as an indicator of 'goodness' or quality in research." Authors like Bell (1999) and Wellington (2000) as cited in Opie (2004) describe reliability as the extent to which a test, method or instrument gives unswerving results across a range of settings used by a range of researchers. Opie (2004:66) regards reliability as "a property of the whole process of data

gathering, rather than a property solely of the results." Validity on the other hand is described as "the degree to which a method, a test or a research tool actually measures what it is supposed to measure" (Wellington, 2000 as cited in Opie, 2004:68). Cohen *et al.* (2002:119) cite LeCompte and Preissle (1993) as suggesting that "the canons of reliability for quantitative research may be simply unworkable for qualitative research." Thus, in this study I will be using confirmability, credibility and transferability.

Since I am working in a qualitative framework, and it is well known that issues of reliability, validity and generalisability are not appropriate. I will concentrate on confirmability as opposed to reliability, credibility instead of validity, and transferability instead of generalisability.

Credibility can best be described by the following question: "Do the data sources find the inquirer's analysis, formulation, and interpretations to be credible (believable)?" (Guba and Lincoln, 1983:326). I verified my interpretations of what I have observed in the lessons by conducting interviews with Nash. During the interview process I made explicit my assumptions and interpretations to Nash with the understanding that there were strong possibilities that after the interviews my understanding of what I have observed in the lessons may continue to emerge. Cohen *et al.* (2002) warns us about the notion of bias that could crop up in an interview and thereby rendering it less reliable. To this end, I made use of a semi-structured interview schedule, hence in the design of this schedule I made a concerted effort to avoid using leading questions – questions which made assumptions about the interviewee or 'puts words into their mouths' since these kinds of questions influence the answers perhaps in an illegitimate manner. The notion of power[22] could be another source of bias during the interview process. Cohen *et al.* (2002:123) state that "power is fluid and is discursively constructed through the interview rather than being the province of either party." Therefore, to reduce the element of bias from this source according to Cohen *et al.* (2002), I

[22] I am looking at the notion of power in the sense that the teacher involved in the study is aware of the fact that I am a representative from the education department.

made conscious attempts to be i) *gentle* – enable the teacher to say everything he wants to say, in his own time and way; ii) *sensitive* - being empathic, taking into account non-verbal communication and how something is said and iii) *open* – being sensitive to which aspects of the interview are significant for the teacher.

Confirmability is the process whereby the interpretation of data can be confirmed. The data collection methods that I employed in this study could be largely characterised as non-participative observations and interviews. Cohen *et al.* (2002:112) define triangulation "as the use of two or more methods of data collection in the study of some aspect of human behaviour." Hence, there is evidence of triangulation (in a qualitative sense) in this study as the interviews, in addition to augmenting my interpretations and analysis; it also operates as a check against my interpretations of Nash's practice. I am not insinuating that this process pinpoints some sort of 'exactness', but rather it approaches the data from two perspectives in the hope that patterns would emerge that would assist in identifying commonness. At this juncture it is important to note that the observations were conducted by making use of a structured observation schedule which was piloted to ensure that the "observational categories are appropriate, exhaustive, discrete, unambiguous and effectively operationalize the purposes of the research" (Cohen *et al.*, 2002:129).

One should also take cognisance that a limitation of using a case study as a research approach is the issue of generalisability. Guba and Lincoln (1983:326) indicate that "… some degree of transferability is possible if enough 'thick description' is available about both sending and receiving contexts to make a reasoned judgement possible." Although generalisability can be seen as a limitation of this study one should bear in mind that "the study of single events is a more profitable form of research (judged by the criterion of usefulness to teachers), than searches for generalisations" (Bassey, 1984 as cited in Opie, 2004:5). Opie (2004:5) further argues, by drawing on the work of Bessey (op. cit.), that the value of any educational research is "the extent to which the details are sufficient and appropriate for a teacher working in a similar situation to relate

his (or her) decision making to that described." So, although I may not be able to generalise the findings of this study, what is of more importance is the relatability of the study. In view of this, I hope that it opens up other areas for further investigation.

3.7 Summary

In this chapter I have highlighted and elaborated on the research methods that I have employed in this study. The study is a qualitative study; however I do quantify some aspects of the data in order to see the 'big picture'. I have engaged in a discussion that justifies the quantification of data. I have also written about the piloting process and what it enabled for me. I went on to elaborate on the ethical issues that were considered in undertaking this study. With respect to issues of rigour, I explained that for the purposes of this study, I will only concern myself with issues of confirmability, credibility and transferability.

In the next chapter, I will provide a discussion related to the background of the lessons observed and a brief overview of what transpired during each of these lessons - by merely highlighting aspects of the lessons that stood out for me. I will also provide a description and the indicators of the data collected. Furthermore, I will also engage in discussions related to the test results and will provide a quantitative analysis of each lesson which will be guided by the analytical framework. I will then grapple with providing answers to the first two critical questions that guide this study: viz. the mathematical 'problems' that Nash confronts as he goes about the teaching of functions to a grade 10 class and the resource pool that he draws on as he solves these problems of teaching.

Chapter Four

Analysis and Interpretation of Data

4.1 Background to the Lessons Observed

In this section I provide a brief commentary about the setup in Nash's class and a succinct overview of the lessons that I observed during the data collection process. I observed Nash as he went about his work of teaching the section on functions to one of his grade 10 classes over a two-week period. This totalled 16 periods which in turn translates to a total of 8 hours since the duration of a period at the school is 30 minutes. The class that Nash identified to form part of the sample was timetabled to be with him as follows: Monday and Tuesday a single period, Wednesday and Thursday a double period and on Friday a double period split by a lunch break.

During my first interview with Nash it was brought to my attention that there were three teachers (including Nash) who were responsible for teaching grade 10 mathematics at the school and that there was collaboration among them to a very large extent – ranging from preparation of lessons to the setting of assessment tasks. As Nash explained:

26 Nash: We have three teachers, this is our micro team for the grade 10's. We have split between them – higher grade, standard grade and functional mathematics. But at the moment for the functional maths we are running the standard grade syllabus and from grade 11 and 12 they will branch out separately into functional maths itself and there will be standard grade maths and higher grade maths. So, although we work as a team, we use common worksheets but when it comes to evaluation we evaluate separately that means I'm responsible for the standard grades so I'll set the standard grade tasks whereas the higher grade teacher will set the higher grade task and the person doing the functional maths will set the functional maths tasks.

27 VP: What about the preparation of the lessons?

28 Nash: There is common preparation of the lessons because we use common worksheets and we try and run it on a common timeline and we have common tasks as well, like when it comes to groupwork tasks or open assessments, those are actually common tasks.

(Interview 1, turns 26 – 28)

Here again, Nash reinforces the idea that he is an active and an involved teacher who finds himself not only in a supportive environment but that they actually plan together and work together. So again it is reflected that this is a collaborative staff in a well-functioning school.

The seating arrangement in Nash's class for the duration of the two weeks was very 'traditional' i.e. desks were aligned neatly in rows with seating space for two learners per desk. For written communication with the learners, Nash had at his disposal a standard chalkboard, a small white board as well as an overhead projector with the appropriate materials that went with those items. During the two week period Nash covered three types of functions viz. the linear function, the quadratic function and the hyperbola. Nash utilised the 16 periods as follows: the first 10 periods was devoted to the teaching of the linear function, periods 11 and 12 were reserved for a class test[23], periods 13, 14 and a part of period 15 were utilised for the teaching of the quadratic function, the remainder of period 15 and period 16 were used for the teaching of the hyperbola.

This gross imbalance with respect to the 'time weighting' that Nash and his colleagues have appropriated to the teaching of the three types of functions is a derivative of the structure of 'traditional[24]' mathematics textbooks and the curriculum currently being phased out in South Africa i.e. the Nated 550 curriculum (DoE, 1997). Browsing through the contents page of a 'traditional'

[23] A copy of the class test is bound and kept separately

[24] Here traditional is used in a sense to refer to textbooks that emphasises rules or conventions in mathematics, it is heavy laden with drill and practice type of exercises that are designed to reinforce the 'mathematical rules'. Connecting the mathematics to the learners' everyday life is completely absent from these textbooks and investigative type of tasks are lacking or even non-existent. However, these textbooks served the curriculum demands of the day more than adequately.

mathematics textbook for grade 10, reveals that if you are to follow the textbook slavishly then more time is certainly required for the teaching of linear functions then is needed to teach the quadratic function and the hyperbola. For example, a typical chapter on linear functions at the grade 10 level will include: drawing graphs from tables; it would then move on to determining the gradient of a straight line and the intercepts; the chapter could then develop by dealing with the dual-intercept method of drawing straight line graphs, thereafter it will deal with the concept of parallel and perpendicular lines; the chapter could be developed further by requiring one to determine the equation of straight lines and finally the application of linear graphs including solving simultaneous equations graphically. On the other hand, a typical chapter dealing with the quadratic function at this level will include drawing graphs of the form $y = ax^2+c$, and grappling with the effect a and c will have on the graph. Thereafter, solving problems based on the graph of the type $y = ax^2+c$. The chapter dealing with the graph of the hyperbola would include basically sketching the graph of $y=\frac{k}{x}$ by using the table method and then solving problems based on this type of graph. This is the same trend that Nash and his colleagues followed in the design of the exercise sheet[25] that was used for teaching the section on functions.

The imbalance of time as discussed earlier is heavily reinforced in textbooks. Here we see what, in much of the French research on mathematics education, is referred to as 'transposition' from mathematics into the intended curriculum (Chevellard, 1985 as cited in Pepin and Haggarty, 2001) or what Bernstein (1990) refers to as recontextualisation. So we know there is a transposition from mathematics into the intended curriculum and that is affected by the formal curriculum. Therefore, that which Nash and his colleagues emphasise is heavily influenced by the formal curriculum and its interpretation into a textbook than by the relationship of these mathematical concepts to each other and the wider field of mathematics. In other words, we can see that Nash and his colleagues are being influenced by i) the syllabus, which is broad; ii) what is in the textbook i.e. how the textbook has interpreted the syllabus, this pins the syllabus down for

[25] A copy of the exercise sheet is bound and kept separately

Nash and his colleagues; and iii) the examinations, which further pins the syllabus down for them. These are very powerful influences[26] which resonate with what the French refer to as 'transposition' from mathematics into the curriculum i.e. the institutionalisation of mathematics. Because of this imbalance and the significance given to the teaching of linear equations in the current practice I will concentrate only on Nash's teaching of the section on linear functions i.e. up to and including lesson 7. The importance of linear functions is that it provides many learners with their first experience of working with two related variables and this is a significant point of transition in their mathematical development. Some of this will constrain the judgement of the notion. In the next section I will provide a brief description of what transpires in each of these lessons.

4.2 Overview of the Lessons Observed

Table 1 (next page) provides a succinct overview of what Nash did during the seven lessons observed. In Nash's practice it is the main ideas (concepts, skills and formulae) that are dominantly present; this is captured under the heading 'main ideas' in table 1. What also stands out for me is that which is marked by an absence. In other words, where opportunities arose for Nash to engage with the learners' ideas or delve into some kind of activity in these lessons. This is laconically captured as comments in table 1.

[26] The 2005 Senior Certificate Examination - mathematics standard grade paper two, set by the Department of Education in South Africa, set question 1 to a total of 17 marks based on the section of linear functions as one of the questions in analytical geometry. This question contributes 40% towards the analytical geometry section and 11% towards the entire paper. The work done by Nash and his colleagues serves as an excellent platform for the learners to grapple with these concepts again in grade 12 using analytical methods which is an important aspect in the National Examinations.

Date	Duration	Topic	Main Ideas Discussed (concepts, skills, formulae)	Comment
Mon 15/08/05	30 min	Introduction – Linear Function	Dependent & independent variables; relationship; function; gradient; y-intercept; table method	Nash uses metaphors to illuminate the notions of relationship & function. He runs into problems with the use of these metaphors e.g. "That means the y depends on the x, it's like your mother and your father – your mother is dependent on your father in the same way your father is dependent on your (learners chorus mother) mother." He is unaware of the problem inherent to this metaphor and the misconceptions that it could cultivate.
Tue 16/08/05	30 min	Gradient & y-intercept method	Function; gradient; $\frac{\Delta y}{\Delta x}$; y-intercept; application of m & c	Nash recapped some of the concepts from lesson 1. Associated the calculation of m & c to the kinds of questionnaires found in magazines or internet sites. Continued with this idea and engaged in talk about a sales person selling soccer memorabilia. I cannot identify with the relevance of this metaphor – attributed to an inappropriate choice of metaphor. Explained the gradient & y-intercept method by working out examples.
Wed 17/08/05	1 hour	The dual intercept method	y-form; dual intercept; x-intercept; y-intercept; gradient	Lesson commences by Nash reviewing the work done in the previous two lessons. Lesson concluded with Nash asking learners to sketch the graphs of x-2y+2=0 and x-2y-2=0 and to determine what is so special about them.
Thurs 18/08/05	1 hour	Parallel and Perpendicular lines	Parallel; perpendicular; $m_1 = m_2$; $m_1 \times m_2 = -1$	The conclusion of the previous lesson provided the ideal platform for the introduction of this lesson to be driven by an activity based task. However, Nash commenced this lesson by performing the necessary calculations and then drew the two graphs; some learners copied the graphs from the board (they did not do it as homework). Uses only one example of perpendicular lines and expects learners to make the generalisation: $m_1 \times m_2 = -1$
Fri 19/08/05	1 hour	Determining equations of straight lines	Gradient; y-intercept; substitution into $m = \frac{y_2 - y_1}{x_2 - x_1}$; $y-y_1=m(x-x_1)$.	Nash reviewed the various methods of sketching linear functions. Explanations are flawed in some instances – the notion of slope related to going up & down a hill. Introduced learners to the equation $y-y_1=m(x-x_1)$.
Mon 22/08/05	30 min	Determining equations of a straight line from the graph	x & y intercepts; determining gradient from the graph; $m = \frac{y_2 - y_1}{x_2 - x_1}$	Reviewed main ideas from previous lessons. Explained how to determine the equation of a line from its graph by working out problems from the exercise sheet. Problems were selected so that different possibilities were taken into account.
Tue 23/08/05	30 min	Application of linear functions	Functional notation; solving simultaneous equations graphically	Nash drew the graph y=-2x+4 and explained the notion of functional notation by showing learners how to find f(1) for example. Introduced learners to the graphical approach to solving simultaneous equations.
Wed 24/08/05	1 hour	CLASS TEST		

Table 1: Overview of Lessons Observed

4.3 Description of Data and Indicators

As an appropriate point of departure here, as well as for purposes of recapping, I again present the following model representing the analytical framework which is a subset of the model that summarises the theoretical framework that underpins this study.

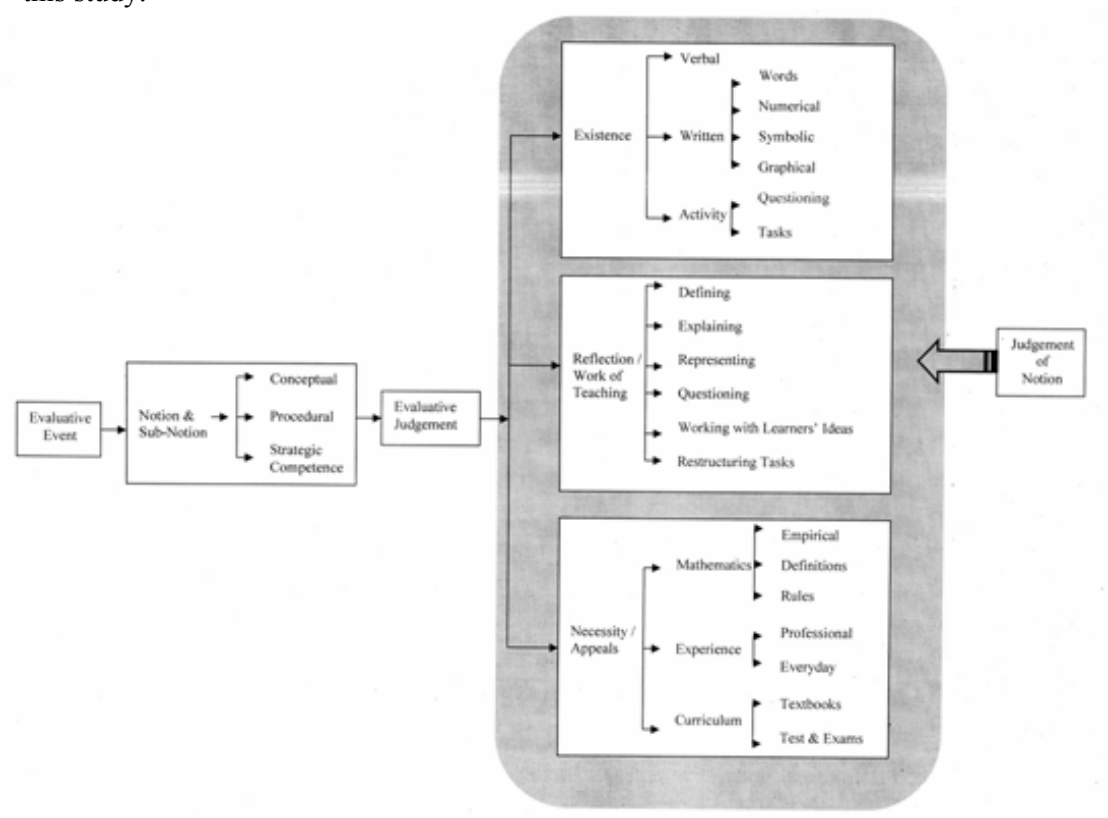


Fig. 3: Summary of the Analytical Framework

To provide the indicators of how I categorised the data it would be gainful to look at the analytical framework in its entirety (fig. 3) in order to discuss all aspects of it i.e. what the notion and its sub-notions are, how it comes into being, what problem-solving (work of teaching) is happening and how it is legitimated (appeals). These aspects will be discussed in this section. In order to accomplish this I am going to discuss an extract which shows a notion and a set of sub-notions and how these notions come into being, the problem-solving that the teacher does and then eventually how they are legitimated.

I will be making use of examples from the actual data that I have collected. The following is an extract from the transcript of lesson 1, this extract is the beginning of lesson 1 and lasts for 7 minutes and 44 seconds – note that time is reflected as 1:20 meaning 1 minute and 20 seconds after the lesson commenced.

0:00

Nash: Good morning class

L's: Good morning Sir

Nash: Sit

Nash writes down $y = 2x + 1$ on the board

Nash: Now, on the board there I got an equation, now we saw equations like this before when we were working out simultaneous equations when – then (turns to the board) we said right $y = 2x + 1$. That means the value of the y (points to y in the equation) depends on the value of x (points to the x in the equation). To get the value for y, whatever value we have for x you have to double it (points to the 2 in the equation) and add 1 (points to the +1 in the equation). In other words when you looking at this we say that the y and the x there's some relation – they like cousins. That means the y depends on the x, it's like your mother and your father – your mother is dependent on your father in the same way your father is dependent on your (learners chorus mother) mother. Now to see what's the relationship between them we draw a table.

2:22

(Nash proceeds to construct a table on the board)

Nash: I've got a simple table, let me put my x values on top and my y values at the bottom. Then I'll take x values at random – at random (inaudible) I'll use (Nash selects integer values $-2 \leq x \leq 2$). Why I am taking those values is so that I can see for negative values what happens to the y, I can see for positive values what happens to the y and for zero as well. I don't have to take those values but then these are the most convenient because they cover a broad range. Now I'm going to see (points to the x values in the table) for every x value what happens to the y value. So where I see a x (points to the x in the equation) – for the first one lets say minus two – y will be equal to two then I must multiply it to minus two and I must add 1, then I know (interrupted by someone from outside) I'm gonna get minus four plus 1 which is gonna give me -3. So that value comes here (writes -3 for y under the x value of -2). (The steps are written on the board as:

$y = 2(-2)+1$

= -4 + 1

= -3

Then in the same way I can substitute -1 (Nash proceeds to determine the corresponding y values verbally and then writes these values in the table).

(The following table is produced by Nash)

X	-2	-1	0	1	2
y	-3	-1	1	3	5

3:59

Nash: Now, I got x values I got y values what can I do with it? (pauses for a short while – no learner responds).

4:04

Nash: We say right lets create our axis, we used these before to draw graphs. Going vertically is our y-axis, going horizontally is our x-axis. (Nash draws a set of axes on the board free hand – labels both the x & y axes). So, lets put in values – equal values – we have values for x – negative values and we taking positive values – lets label them, we have 0 and 0 in the middle (Nash calibrates the axes in units of 1).

5:03

Nash: Now, if these are in a relation (points to the x and y values in the table) that means they go together. We say we plot these points – I look for -2 on the x and I look for -3 on the y (interrupted by someone from outside) – (Nash reads the co-ordinates from the table and plots them on the Cartesian plane himself).

5:55

Nash: You notice now if I have to join these points (Nash joins the points freehand), more or less I get a (quick pause) (one learner responds – a straight line).

Nash: A straight line – and let's label our straight line – this straight line has got a particular name – what's it name? This equation (Nash proceeds to label the straight line y = 2x+1).

6:17

Nash: Now all that I did – the straight line is called the linear function. Now, linear because it makes a line.

Nash: Now why function? - all the time we've been saying there's some kind of relationship, now we saying there's simply a function – from a relation it means they were husband and wife – now they having a function – are they getting married now or are they getting married before – a function just basically means

that for every x value (point to the table on the board) I've got a unique y value (interruption by an announcement via the intercom system) (Nash repeats) for every x value there's a unique y value – one x value doesn't have two different y values (Nash pauses due to continued interruption by an announcement via the intercom system). So every husband (points to the x value on the table) has got one unique wife (points to the y value on the table) – so that's why we say linear and it is a function.

7:16

Nash: Now, what I want you to do first take this down (points to the table and the calculation of the co-ordinate (-2:-3)) – then lets see there is some unique features about this (points to the straight line graph) – every straight line – besides just having x values and y values – it's got some unique properties, we are going to try an analyse what are these properties. (interrupted by someone looking for a pupil in the class).

(Lesson 1, time interval 0:00 to 7:16)

This extract was chosen purposefully since firstly, it covers a wide range of categories and will therefore serve to illustrate how I have categorised the data for those categories. It will also provide the relevant indicators for these categories. Secondly, this extract illustrates that a sub-notion is not necessarily singularly located and that an event can comprise of sub-notions. Thirdly, it demonstrates that in a notion or sub-notion there could be more than one problem-solving process (the work of teaching) that Nash grapples with and that there could be more than a single appeal used to legitimate the notion. This illustrates the idea that the process from existence to necessity through reflection is not a linear process. It is the function of the judgement of the notion to create opportunities for contingency to exist at each of these levels, thus making the entire process more cyclical in nature.

Table 2 below is an illustration of how the classification of data was recorded; specifically it shows the chunking of data (the above extract) into episodes where evaluative judgements were made by Nash in an attempt to fix meaning. The table illustrates the timing of events and the identification of a notion and its sub-notions. It also depicts how notions came into existence and the problem-solving

that Nash grappled with in each of these instances. The table further portrays the appeals that Nash makes in each instance in order to legitimate the sub-notion. For purposes of space some of the categories in the table were abbreviated and table 3 provides the key for these abbreviations. The curved arrows above the table symbolise the manner in which the judgement of the notion works i.e. its cyclical nature.

Judgement of the Notion

EVENT	TIMING	Concept						Existence							Problem Solving /Mathematical Work							Necessity / Appeal						
		Notion	Sub - Notions		Conceptual	Procedural	Strategic Competence	VERBAL	WRITTEN				ACTIVITY		Defining	Explaining	Representing	Questioning	Working with learner ideas	Restructuring Tasks	Mathematics			Experience		Curriculum	
									W	N	S	G	Q	T							Em	D	R	P	E	Tb	Ex
Lesson 1 (30 min)																												
1	00:00 - 2:22	Function	1.1	Relationship - using an equation	✓			✓			✓					✓							✓		✓			
	2:22 - 3:59		1.2	Relationship - using a table		✓				✓	✓					✓	✓						✓					
	3:59 - 6:17		1.2	Relationship - drawing a graph		✓						✓					✓						✓					
	6:17 - 7:16		1.4	Definition of a function	✓			✓							✓	✓		✓				✓	✓		✓	✓		

Table 2: Event 1 from Lesson 1

KEY	
W – Words	Em – Empirical
N – Numeric	D – Definitions
S – Symbolic	R – Rules / Conventions
G – Graphical	P – Profession
Q – Question	E – Everyday
T – Task	Tb – Textbook
	Ex – Exams

Table 3: Key for Table 2

The above extract was identified as event one of the lesson with the overarching **notion** being that of a *function* with various sub-notions at play. I will now commence to illustrate how identification and classification of each of these sub-notions were done. Between the time interval 0:00 to 2:22 I have identified the **sub-notion** to be that of *'relationship - using an equation'*. This sub-notion, for me, was *conceptual* in nature and it comes into **existence** in a *written* format, specifically in a *symbolic* fashion. This is evident by the fact that Nash started

out, after greeting his learners, by writing down the equation y = 2x + 1 on the board (the use of symbols); and it is through reference to this equation that the discussion of a relationship commences. The discussion is completely one-sided and controlled by Nash. I classified this discussion as the **mathematical work or problem-solving** that Nash engaged with; and more appropriately as an *explanation* as opposed to the other 5 categories. In Nash's attempt to fix meaning, I classified his **appeal** to be both *mathematics* and *experience*. The mere fact that Nash states, "the value of the y (points to y in the equation) depends on the value of x (points to the x in the equation). To get the value for y, whatever value we have for x you have to double it (points to the 2 in the equation) and add 1 (points to the +1 in the equation)", is an indication for me that Nash is appealing to a *mathematical rule or convention*, thus an appeal to mathematics. Nash also draws on the idea of cousins, mother and father to illustrate the notion of a relationship, therefore giving me the indication that his appeal is also to experience, more specifically that of the *everyday*.

Between the time interval 2:22 and 3:59, I identified another **sub-notion** – *'relationship – using a table'*. I classified this sub-notion as being *procedural* since the main focus here was selecting x-values, substituting them one by one into the equation and calculating the corresponding y-value. The sub-notion in this instance came into **existence**, *symbolically*, since the equation y = 2x + 1 was key to this sub-notion; and because specific x-values were used viz. -2;-1;0;1;2, I have classified this sub-notion as coming into **existence** in a *numeric* fashion as well. The **problem-solving** that Nash was engaged with in this sub-event was clearly that of *representing* since the use of a table with x-values and its corresponding y-values is another way in which a relationship could be represented. In this case I classified Nash as **appealing** to the *rules of mathematics*.

The next time interval, 3:59 to 6:17, was identified as another **sub-notion** viz. *'relationship – drawing a graph'*. This sub-notion was classified as being *procedural* in nature since it dealt with the plotting of coordinates on the

Cartesian plane. It highlights that the notion came into **existence** in a *graphical* format and the **problem-solving** is once again, that of *representing* and the **appeal** is again to the *conventions or rules in mathematics*.

The time interval 6:17 to 7:16 marks the last **sub-notion** for the notion of function which is described as *'definition of a function'*. This sub-notion was categorised as being *conceptual* in nature due to that fact that an attempt is being made to formalise the notion of functions in general by means of a definition. It came into **existence** *verbally* by means of Nash posing questions such as 'now why function?' This underscores the fact that part of the **problem-solving** that Nash engaged with was *questioning*. Since the definition of a function is central in this sub-notion, Nash, at the end would need to come up with a definition, hence *defining* was another **problem-solving** that Nash contended with. During this time we find Nash is also *explaining*, another **problem-solving or the mathematical work of teaching** that he is engaged with, the concept of relationships by associating it with the notion of marriage. This illustrates that Nash is **appealing** to the *everyday* in an attempt to fix meaning. In addition, Nash formalises the definition of a function to be 'for every x value there's a unique y value' this exemplifies the fact that Nash is **appealing** to a *mathematical definition* which will be listed in a mathematics textbook, thus also illustrating that Nash also **appeals** to the *curriculum*, in the sense of a *textbook* in his attempts to fix meaning.

In the above example I have only managed to demonstrate how I classified notions and their sub-notions as being either procedural or conceptual in nature. In terms of the object coming into **existence**, the above example illustrated most of the categories here with the exception of 'words', a sub-category of 'written' and through an activity. To select the sub-category '*words'*, Nash will have to introduce the notion or sub-notion by *writing* it on the board. For example, if the notion was the definition of a function and Nash wrote down the definition on the board, I will then classify it as coming into existence by means of words. With respect to the problem-solving or mathematical work that Nash has to do, the

above example illustrates the idea of defining, explaining, representing and questioning. As far as the appeals go, the above example illustrated the idea of an appeal being to definitions or rules in mathematics, experience with respect to the everyday and the curriculum with respect to the textbook or any other materials that could have been used.

The above example was not sufficient to illustrate all the possible classifications e.g. a notion coming into existence via an activity. Activity is a multifarious word since it could mean various things to different people in various situations. What I mean by it is anything that is not verbal i.e. it is not a statement. So from my perspective the activity can manifest through questioning or through an actual task. The example that I can show is where an activity arises through questioning. I cannot, from the data collected, show a task because Nash does not introduce any of the notions by making use of a task, hence they are largely absent. In a different pedagogy one would see a lot more tasks and a lot less explanations. I will pick up on this again in section 4.5 (this chapter). In the next example I will attempt to address the gap that I have alluded to.

8:59

Nash: What is a straight line? – If they say define a straight line. We doing linear functions, so it's a straight line – What is a straight line?

9:08

Nash: In science when we talked about – we say light travels in a straight line, so what does that mean?
(Nash pauses – giving learners some time to think)

9:19

Nash: If someone asks you – you tell someone you have to go to shop – so the person says what's the quickest way to get to the shop? – What will be the quickest way to get to any place? (Some learners chorus (faintly) straight line)

Nash: (Repeats) A straight line – So if I got a straight line – If here's place A and I want to get to place B (puts two points on the board and labels them A and B), I can walk to there, come back here, come back there, come back there (joins points A and B with a zigzag line) – that's one way of going – that's the way our roads in

South Africa are designed – (some learners laugh) – the easy way or the shortest way will be just to go from A to B (joins points A and B with a straight line).

9:59

Nash: So, the definition of any straight line will be what? (Brief pause)

10:01

Nash: The shortest distance between two points.

(Lesson 1, time interval 8:59 to 10:01)

The above extract, also from lesson 1, illustrates that the notion, defining a straight line, comes into **existence** through an *activity* where Nash resorts to asking a *question* "What is a straight line? – If they say define a straight line. We doing linear functions, so it's a straight line – What is a straight line?" it also illustrates that Nash introduced this notion *verbally*. In an attempt to **legitimate** meaning it is evident that Nash is making an *empirical* appeal where he draws a zigzag line between two points as well as a straight line between the same two points and learners are then required, through a process of observations, to provide an answer to the question – what is a straight line.

If Nash introduced the object / **notion** through some form of investigations for example: on the same system of axis sketch the graphs of $y = 2x + 1; y = 2x - 1; y = 2x + 2; y = 2x - 2; y = 2x - \frac{1}{2}$ and $y = 2x + \frac{1}{2}$ and thereafter asked the learners to discuss in their groups what they observed about the lines and what deductions they could make about the lines and the coefficient of x. I would then classify this notion as coming into existence through an *activity*, specifically a *task*. In instances where Nash legitimates the notion for his learners by drawing on knowledge that he has gained through his training and years of experience, the **appeal** is classified as *experience*, in particular *professional experience*. For example, when a learner asks a question related to the order in which x and y values are selected from two co-ordinates, for substitution into the gradient formula, Nash gets the learners to try both situations out so that they could observe for themselves. An **appeal** to *tests or exams* (i.e. the curriculum) is when Nash specifically tells his learners that they have to do certain things since in the

exams they will get marks for doing so or 'that is what will be required in the exam/test' e.g. the labelling of the axis in a Cartesian plane and the labelling of the graphs drawn by writing down the equation defining the particular graph. In a sense the message that is given to learners in terms of what and how mathematics is constituted, is to a large degree, a set of rules to follow as described by the curriculum.

In the next section, I will quantify the number of times each of the categories is called into play by Nash, lesson by lesson and then taking all the lessons into account.

4.4 A Quantitative Analysis

In order to study how all of this happens over time, what the extent of the appeals are, it was necessary for me to saturate all of the data. By tracing notions and reflections one begins to see the spread of appeals that Nash makes over time in attempting to fix meaning for his learners. This requires being able to quantify some of the data otherwise it becomes too cumbersome and in a sense unreadable to accomplish. Therefore, it lends itself to tallying occurrences, thereby obtaining a picture of presence or absence and frequency. In this sense quantification is used to structure an overview of the data analysis.

4.4.1 The Picture Lesson by Lesson

Table 4, on the next page, is an attempt to quantify, per lesson, the nature of each of the notions identified, how they came into existence, the problem-solving that Nash had to engage with in each case and finally what Nash appealed to in order to fix meaning. The table makes provision, in the last column, for displaying the average of the percentage of occurrence of each item. An explanation regarding the values that appear in the percentage occurred column is provided in the next section.

		Total Occurrences							% Occurred							Averages
Lesson Number		***1***	***2***	***3***	***4***	***5***	***6***	***7***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	
# Notions (including sub-notions)		**12**	**13**	**11**	**7**	**11**	**6**	**5**								
Notion / Object																
Conceptual		6	5	6	3	9	3	4	50	38	55	43	82	50	80	57
Procedural		6	7	5	4	3	3	1	50	54	45	57	27	50	20	43
Strategic Competence		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Existence																
Verbal		6	10	7	5	8	2	4	50	77	64	71	73	33	80	64
Written	Words	0	0	1	1	0	0	0	0	0	9	14	0	0	0	3
	Numeric	1	2	2	1	3	2	2	8	15	18	14	27	33	40	22
	Symbolic	6	5	3	4	5	2	3	50	38	27	57	45	33	60	44
	Graphical	4	5	2	1	1	3	2	33	38	18	14	9	50	40	29
Activity	Question	2	0	0	0	0	0	0	17	0	0	0	0	0	0	2
	Task	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Problem-Solving																
Defining		2	2	0	2	0	0	1	17	15	0	29	0	0	20	12
Explaining		10	11	8	5	9	4	5	83	85	73	71	82	67	100	80
Representing		6	12	6	3	8	3	3	50	92	55	43	73	50	60	60
Questioning		7	3	0	3	1	2	1	58	23	0	43	9	33	20	27
Working with learners' ideas		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Restructuring tasks		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Appeals																
Mathematics	Empirical	5	5	2	3	4	2	3	42	38	18	43	36	33	60	39
	Definitions	2	3	0	3	1	0	0	17	23	0	43	9	0	0	13
	Rules	8	7	6	4	4	1	4	67	54	55	57	36	17	80	52
Experience	Profession	2	7	0	0	6	3	0	17	54	0	0	55	50	0	25
	Everyday	6	4	0	0	3	1	0	50	31	0	0	27	17	0	18
Curriculum	Textbook	3	1	1	0	1	0	0	25	8	9	0	9	0	0	7
	Exam / Tests	2	1	2	0	0	2	0	17	8	18	0	0	33	0	11

Table 4: Quantitative Results per Lesson

4.4.2 The Composite Picture of the Lessons

Table 5 (which follows) provides a composite picture, taking all lessons into account, of how notions came into existence, the problem-solving that took place (i.e. providing learners with opportunities for reflection) and the appeals that were made in an attempt to authorise the notions.

As discussed previously, in section 4.3 (page 64), it is important to remember that the items in each of the categories do not necessarily occur uniquely. For example, something can come into existence in two ways or Nash might appeal to more than one category in his attempt to fix meaning for his learners. Therefore, with reference to table 5 (on the next page), there is a total of 65 events (inclusive of notions and its sub-notions), total occurrences represent the frequency that each of the categories could be identified, taking the above reminder into account i.e. within an evaluative event the categories do not necessarily occur uniquely and Nash could appeal to more than one category. The percentage occurred merely represents a percentage of the total occurrences out of the total of 65.

		Total Occurrences	% Occurred
# Events (including notions & sub-notions)		***65***	
Notion / Object			
Conceptual		36	55
Procedural		29	45
Strategic Competence		0	0
Existence			
Verbal		42	65
Written	Words	2	3
	Numeric	13	20
	Symbolic	28	43
	Graphical	18	28
Activity	Question	2	3
	Task	0	0
Problem-Solving			
Defining		7	11
Explaining		52	80
Representing		41	63
Questioning		17	26
Working with learners' ideas		0	0
Restructuring tasks		0	0
Appeals			
Mathematics	Empirical	24	37
	Definitions	9	14
	Rules	34	52
Experience	Profession	18	28
	Everyday	14	22
Curriculum	Textbook	6	9
	Exam / Tests	7	11

Table 5: Composite Quantitative Results

4.4.3 Results of the Class Test

Table 6 (below) provides some statistics from the test that Nash's class wrote.

Analysis	
# Wrote	35
# Passed	33
# Failed	2
Highest Mark	100
Lowest Mark	28
Average	65
# A's	12
# B's	3
# C's	5
# D's	2
# E's	11
# F's	1
# G's	1

Key - Symbol Distribution	
80 - 100	A
70 - 79	B
60 - 69	C
50 - 59	D
40 - 49	E
30 - 39	F
20 - 29	G
0 - 19	H

Table 6: Test Analysis

4.4.4 A Discussion of the Test Results

As discussed previously, the section on functions formed the backdrop to this study based on the notion that Nash felt he teaches this section well. So, what is the indicator for Nash that tells him his teaching is successful? I have not explicitly asked Nash this question; however during my observations of Nash's lessons and interviews with him I learnt that Nash places a great deal of importance on test and examination results, for example: in the second interview, I asked Nash why he constantly referred to tests and examinations during his teaching, to which he responded:

18 Nash: "…you ask the question from grade 1 right till grade 12 – why are we learning this? And one good reason for why we are learning this is to apply it in an examination. There are other valid reasons, yes, if we talk about careers and talk about skill development and things, but a good valid reason is you have to learn this so that you can apply it and people want to see your ability to apply what you have learnt – that's what life is

all about - applying what you have learnt and one way to apply this is within a test or an exam situation …"

(Interview 2, turn 18)

The results of the test conducted by Nash and his colleagues reveal that Nash's class obtained a 94% pass rate, with the highest mark being 100%. In addition, 34% of the learners in his class obtained an A symbol for this test and the class average was 65%. Taking into account that Nash places a high value on tests and examinations coupled with the fact that his learners generally performed well in the test, confirms that Nash was indeed successful in his teaching of the linear function. However, from these results one cannot deduce that these learners display levels of conceptual understanding, since the questions in the test were a replica of the kinds of questions that appeared in the exercise sheet. To illustrate this point consider the following questions which are taken from the exercise sheet: (i) sketch the graph of $y = \frac{-3}{4}x + 1$; (ii) find the equation of a line passing through (-3;2) and (1;-2). Now compare them to question 2.1 and question 3.2, which are taken from the class test: (2.1) sketch the graph of 2x - 4y = 12; (3.2) determine the equation of the line between points (2;3) and (1;1). How are these sets of questions different? What is the class test really testing? Is it perhaps only testing the recall and application of rules? If this is the case then what conceptions of mathematics are learners' likely to internalise – merely a bag of rules that one has to apply?

These types of questions are aligned to procedural fluency which in fact is extremely important. We know from the discussion of mathematical proficiency that this is not enough, this need to intertwine with conceptual understanding and the other strands of mathematical proficiency. The discussion of this in relation to Nash is further dealt with in chapter 5. The questions in the class test could promote learning of rules without understanding or justification. Interestingly, this relates to the earlier work by Skemp (1978) who distinguishes between two

kinds of understanding in mathematics: relational and instrumental[27]. Skemp (1978) posits these as almost antagonistic. In the strand procedural fluency, what I am trying to see is how the one supports the other. So the kinds of knowledge that these learners are getting access to can be associated with instrumental understanding - not that this is a problem, but if this is the only diet it will not be healthy. Applying their knowledge gained to questions related to investigations or questions that are contextualised in real life situations may pose a challenge to Nash's learners as he indicated:

121 Nash: "Challenges that they may face – again grappling with abstract ideas. // Basically correlating in investigations because lots of the – from my understanding of how the new curriculum presents content, its in a subtle indirect way."

122 VP: What do you mean by that?

123 Nash: "For example, lets take an investigation – you and I as mathematics teachers we know what the goal of the investigation is for example to highlight the effect of c the y-intercept but are our learners actually geared to seeing this because I know especially in science – in my previous teaching of science children might go through the entire experiment and go through the entire investigation but they are unable to correlate everything and come to a decisive conclusion as such; and this is going to be one of the limitations of our learners. That what teachers will ultimately do is perform the entire or facilitate the learners' right through the entire investigation, get the results and than at the end of the day merely just tell them what it is that they should have discovered."

(Interview 3, turns 121-123)

This illustrates that Nash finds himself limited by the pedagogical and philosophical models that have been entrenched in our schooling system as a result of the ideals inherent in the Nated 550 curriculum (DoE, 1997) which is currently being phased out. Nash's style of teaching does not allow him to venture into investigative teaching as he finds himself in a situation of time versus

[27] Relational understanding is described as knowing both what to do and why, whereas instrumental understanding entails "rules without reasons" (Skemp, 1978:20)

syllabus completion which he reflects on when asked what prompted him to move on during a lesson:

22 Nash: "… time is very important because you must take into consideration I'm teaching alongside two other people and we have a standard lesson procedure to follow and we need to complete the syllabus …"

(Interview, turn 22)

The data in table 6 reflects that there is a wide spread of marks ranging from 28% to 100% and the symbol distribution is a bimodal distribution peaking at the A and E symbol range. These are the results of only one class and from just one test, so whether or not it is significant I cannot say, since delving into reasons for this bimodal distribution goes beyond the scope of this study. However, Bernstein (1996:32) indicates that "recognition rules regulate what meanings are relevant and realization rules regulate how the meanings are to be put together to create the legitimate text." So, through a process of speculation and using Bernstein's (1996) notion of 'recognition and realisation rules' a possible reason for this bimodal distribution could be as follows: The learners that performed well recognised that they had to ignore the everyday context that Nash brought to the lesson through the use of metaphors and only learn the mathematical skills that he presented. On the other hand, the learners that did not perform well could possibly be the ones that did not possess the appropriate 'recognition and realisation rules' with the result they were caught up with the everyday as portrayed by the metaphors and were sidetracked. Therefore, in his attempt to get the learners to understand he possibly alienated them further in the sense that the metaphors made it harder for them to get access to the actual mathematics. When Nash was asked if the common errors that learners' made in the test were similar to the problems that they encountered during the lessons he replied:

59 Nash: "No, why I say no is because during the teaching we know that the - I explained, the problems in the test were not related to the section on functions itself but related to previous deficiencies or misunderstandings of concepts; and within the teaching of the lessons basically you are

> guiding them through, they are being made aware of these integers, these concepts. So in class by doing the application as such you realise these common problems don't surface because they are being made aware of looking at those negative numbers, looking at those fractions. In the test situation they resort to their rule making again."
>
> (Interview 3, turn 59)

What Nash is alluding to in his response is that each learner brings a different 'bag' of previous knowledge to the class and this knowledge is embellished with misconceptions. He is unable to fix all the gaps, given that he has to complete a syllabus within a limited timeframe. So, another speculation as to the bimodal distribution could be found in the idea that the level of the test is brought down slightly to ensure that more learners pass resulting in a peak of the E symbol distribution. The learners who are generally performing well now start to score even higher marks, thus the peak on the A symbol distribution. It is of paramount importance that I reiterate that the reasons for this bimodal distribution that I alluded to are merely speculative in nature. It is interesting that there exits a bimodal distribution in this case since one would normally associate test results with a normal distribution, however, I have not analysed the learners answers in detail and further discussion on this is beyond the scope of this study.

Thus far, I have looked at the test results for the purposes of illustrating that as per the perceptions that Nash has with respect to the measure of his success in teaching, his teaching of the section of linear functions was successful. I will now attempt to answer the critical questions that guide this study; an appropriate starting point for tackling this task is to revisit these questions once again. The critical questions that focus this study are:

1. What mathematical 'problems' does the teacher confront as he teaches the section on functions in grade 10?
2. What knowledge and experience (resource pool) does the teacher draw on as he solves these problems of teaching?

3. Why does the teacher use the knowledge and experiences he draws on in the way he does?
4. How does this resource pool relate to mathematical proficiency or the potential to promote mathematical proficiency in his learners?

In the next section I will illuminate the kinds of mathematical 'problems' Nash confronts as he teaches the section on functions to his grade 10 class, as well as the knowledge and experience that he draws on as he solves these problems of teaching.

4.5 The mathematical 'problems' / mathematical work of teaching that Nash confronts

To recap, very briefly the theoretical framework shows that firstly a notion comes into being, is reflected on and brought to necessity, keeping in mind that all of this is encased by the judgement of the notion which serves to produce contingency at the various stages. The mathematical work of teaching can be seen as the opportunities that a teacher will afford to his/her learners to reflect on the notion. The purpose for reflection by the learners is to in a sense evolve the notion in their minds from a mere 'that' into something more substantial. In this section I will discuss the mathematical work of teaching that Nash confronts as he teaches the section on functions to his grade 10 class. If we look at event 1 from lesson 1 we see that Nash introduces the notion of a function through an 'informal' definition. Thus, these learners' first encounter of the notion of function is through a definition. This could potentially pave the way to what Sierpinska (1988) refers to as a 'didactical error – an antididactical inversion'. It is interesting here to observe the opportunities that Nash provides to his learners to reflect on the concepts being introduced so that his learners could develop strong concept images and concept definitions. In the next section I will illustrate how I have identified the mathematical 'problems' or the mathematical work of teaching that Nash confronts.

4.5.1 Identifying the Mathematical Work of Teaching

By merely glancing at tables 6 and 7 it is conspicuous that across the lessons when notions come into existence, they frequently come in via *statements* (either verbal or written) as opposed to an *activity*. This has a direct bearing on the mathematical 'problems' or the mathematical work of teaching that Nash confronts. The six problem-solving tasks (defining, explaining, representing, questioning, working with learners' ideas and restructuring tasks) either surface in irregular patterns across the lessons observed or are completely absent across these lessons. The problem-solving tasks of **defining, explaining, representing** and **questioning** are the mathematical problems of teaching that Nash *grapples with* over the series of lessons observed, whereas the mathematical problem-solving tasks (**working with learners' ideas** and **restructuring tasks**) are *completely absent* across these lessons. In an attempt to make sense of this absence I draw your attention to the following: across the lessons when notions come into existence; it is largely absent that it comes into existence via an activity. So what does this imply? A similar study[28] conducted by Kazima and Adler (in press) shows that in a practice which encourages learners to grapple with a set of tasks, components of the mathematical work of teaching (working with learners' ideas and restructuring tasks) become unavoidable. They further argue that this is also the case in the teaching of relatively new topics since it would be difficult to anticipate in advance what ideas learners will bring to the class in terms of what they have to offer and how they will interpret a task. Nash's pedagogy on the other hand was described as 'traditional', which limits an introduction to notions via some form of activity, therefore this absence across the lessons. Kazima and Adler (op. cit.) also reveal that the other aspects of 'problem-solving' viz. defining, explaining, representing and questioning were largely absent in their study; which are central features of Nash's lessons. The mathematical work of teaching can therefore be seen as a function of pedagogy in the sense that the

[28] This study investigated mathematical knowledge for teaching by observing a South African teacher teaching the section on probability. Probability is a relatively new topic in the South African curriculum.

practice influences how a notion comes into existence, and so too the mathematical work that a teacher confronts.

I have identified extracts from different lessons to illustrate the mathematical 'problems' or the mathematical work of teaching that Nash confronts. Throughout the lessons observed, as discussed, Nash only grapples with defining, explaining, representing and questioning as the mathematical work of teaching. My use of extracts from different lessons demonstrates that in every lesson Nash does not necessarily engage with all the categories of the mathematical work of teaching. In some instances some of the aspects of the mathematical work of teaching are minimal or largely absent or alternatively they arise in some combination. In the next 3 sections I will engage in discussion that is intended to elucidate the mathematical work of teaching that Nash confronts.

4.5.2 Explaining

In the quantification of the number of times each of the aspects of reflection or problem-solving viz. defining, explaining, representing and questioning is called upon by Nash, table 4 reveals that the most frequently called upon category which averages 80% is **explaining**. What does this notion of explaining entail? The following extract is intended to shed light on this:

8:14

L: Sir, when you make your brackets like 1 comma 0 or 0 comma 1 – how do you know where's x?

Nash: x goes first – x always comes first – see here, lets take this one (points to the calculation of the x intercept of $x - 2y + 2 = 0$) I start of – I say $x = 0$ - so the first value I know is automatically (point to the abscissa of (0;1)) 0 – and my calculation, I'm only calculating for y – so the y value automatically comes at the back (points to the ordinate of (0;1)). Now look at the second one (references to the calculation of the y intercept) I say $y = 0$ – so that means the second number is automatically 0 (points to the ordinate (-2;0)) – my calculation is for x, so the

first number is -2 (points to the abscissa of (-2;0)). That's how I know which one goes where.

(Lesson 4, time interval 8:14)

In response to the learner's question we find that Nash engages in an explanation in an attempt to create some understanding for the learner. The explanations that Nash offers are not necessarily the best possible ones and may not necessarily be mathematically accurate as the following extract from lesson 5 demonstrates. For purposes of drawing your attention to the specific area of the extract in question, I have highlighted the text by underlining it.

0:41

Nash: Now, before we go on let's recap what we already know – now we almost coming to the last part – so before we go on to determining equations – what do we know? We know that the general form of the function is (states and writes) y=mx+c, right – it's your linear function – also there's a relationship between the y's and the x's – then the m there gives us the gradient (draws an arrow from the m and writes the word gradient) which is either an increasing gradient (draws a line with a positive gradient beneath the word gradient) – like if you with a truck (demonstrates by moving his hand up and away from his body) it's a steep slope – you going uphill. Or it's a decreasing gradient (draws an line with a negative gradient next to the previous line that was drawn) that means you going downhill (demonstrates by moving his hand down and away from his body). And our c value (circle the c in y=mc+c) is our y-intercept, where it cuts the y axis. At this point (draws an arrow from the circle around c and writes x = 0) we learnt later on that x is equal to zero – there's no, no x value – somebody said at that point there is no x value – there is a x value, it's just that the x value is zero – so it's on that y axis, right that's why its called the y-intercept. Then we also learnt there's 3 ways that we can actually plot this graph. The first way was the (states and writes) table method – then what we didn't like about the table method was, there was lots of calculations which you had to do – we had to calculate now for each value, then when you drew it you needed to have every value on the axis itself, so it took a lot of time.

(Lesson 5, time interval 0:41)

This gives me the impression that Nash's conception of slope is pitted with a misconception especially when looking at the notion of slope in terms of some concrete model. This is not a once off occurrence; Nash also repeated this during an interview when I asked him to provide me with an example of how he converted mathematical language into normal English, to which he responded:

69 Nash: "For example gradient, a simple one, I don't even refer to the word gradient in the class or change in y over change in x – we talk about slope, we talk about how steep the graph is – steep means you'll be climbing upwards. The opposite of being steep means you'll be going downwards…"

(Interview 3, turn 69)

It is important to understand and take cognisance of the fact that this study is not intended to assess Nash but rather to investigate the kind of mathematical problem-solving he does as he goes about his work of teaching. In view of this, during the interview I was aware of this error but did not want to question Nash about it. The purpose of highlighting this extract is merely to illustrate that although the quantitative data, as depicted in table 4, shows that the most frequently occurring category of problem-solving that Nash grapples with is explanations; it does not necessarily mean that these explanations are absolutely correct or the best possible ones available, I will pick up on this in the next section.

A distinction can be made with respect to the type of explanations that Nash offers to his learners. Firstly, we see a learner asking a question related to the ordering of numbers that represent the abscissa and ordinate in a coordinate. The nature of questions asked by the learners in this class is typically procedural; hence the explanation follows procedurally in an attempt to answer the question. Secondly, we have Nash providing explanations in the form of recapping. If learners had difficulty, the explanation that Nash offers through the recapping process does not necessarily mean that the learners will automatically understand the concepts or ideas. The recapping process offers Nash the opportunity to consolidate what was

done and in a way to focus his thinking. It also provides a signal to the learners in the sense that after the recapping process, Nash is going to 'move on' with the lesson - a kind of prelude to the next 'chapter' of the lesson.

4.5.3 Representing

The next type of problem-solving confronting Nash is that of **representing**, as per table 4 it is evident that this category is ranked second highest (60%) in terms of frequency of occurrence over the series of lessons observed. There exists a relatively big gap (more than 30%) between the problem-solving categories representing and the remaining two (defining and questioning). Why such a big gap? A possible answer to this question lies in the fact that the section on functions, as described in the syllabus, emphasises the various representations of the linear function, which in turn is interpreted by textbook writers and as discussed previously these are strong influences for Nash and his colleagues. This is clearly evident in the sequencing of Nash's lessons for example: the first encounter of the linear function is through its representation as an equation; thereafter the use of a table to represent a set of co-ordinates (another means in which a function is represented); this is followed by the plotting of the points on the Cartesian plane to yield the graphical representation of a function. This is the manner in which the indicator for the category representing was described earlier and this could be one of the possible contributing factors for the gap in question. Representing, as a category of the mathematical work of teaching, goes beyond just the various ways in which functions can be represented. This is illustrated by the next extract (from lesson 3), and it could be seen as another contributing factor for the gap in question. The highlighted text in the form of underlining is intended to focus your attention on the portion of the extract in discussion.

4:38

Nash: Now, here I got my (points to the 4 equations written on the board - interrupted by someone opening the door) we got our 4 graphs – but then in order for me to draw these graphs I need to have my gradients as fractions because I want to see the change in the y which is on top and the change in the x which is at the bottom. So, one (refers to the first equation) I'll write as 2 over 1 – that's for the

> first one number one. For the second one, its already a fraction – I can see the change in y is 1 and the change in x is 2 – For number three I'll have to make it -3 over 1 – then again the minus doesn't matter, if the minus is on top, the minus is at the bottom or the minus is in front – all the minus and plus will tell us – for minus we just go to the left and for plus we just go to the right. This one is ok (points to the 4th equation on the board) – you can see the change in the y (points to the numerator) you can see the change in the x (points to the denominator).
>
> (Lesson 3, time interval 4:38)

Re-writing the gradient in the form of a fraction could be seen as re-writing the equation $ax + by + c = 0$ to $y = mx + c$. In both instances we certainly have the representation of a function as an equation as described earlier. One may argue that I am contradicting myself by saying that representing as the mathematical work of teaching, goes beyond this i.e. function as an equation, yet I bring it up again. My focus here is to consider that the work that Nash has to confront is not only representing a function as an equation but also the representation of gradient as a fraction to show the change in y over the change in x.

The move between the symbolic and the visual and vice-versa is another form of representing that Nash confronts as part of his mathematical work of teaching. This could be seen as more overarching in nature and each individual lesson is structured in a way to provide the relevant competencies to learners in order to grapple with this type of representation. His learners display proficiency in the move from the symbolic to the visual – the results of the class test bear testimony to this; however the reverse is problematic as he revealed during the interview when asked about the weaknesses of his learners. This problem is directly related to what was emphasised during the lessons since moving from the visual to the symbolic is only dealt with in lesson 6 which was a 30 minute lesson and 2 days away from the day on which the test was written.

4.5.4 Defining and Questioning

Questioning and **defining** are the last two categories, as per the categories that I have identified of problem-solving that Nash confronts during his teaching. These two categories of problem-solving are much more demanding mathematically and as such they do not occur as frequently as compared to the categories already discussed. The following extract (from lesson 1) illustrates Nash engaging with both these 'problems' of teaching. In addition to the time intervals, line numbers are also provided to assist with the discussion and for ease of reference.

	8:42	
1	Nash:	In fact to draw a straight line, how many values do I need? How many points do I need?
2		Here I used five points (points to the graph – do I need always five points?
3		(one learner says no).
	8:47	
4	Nash:	How many points do I need? – 5,4,3,2 – how many points do you need?
	8:55	
		(Nash directs the question to a specific learner by calling his name – learner is unable to answer).
	8:59	
5	Nash:	What is a straight line? – If they say define a straight line. We doing linear
6		functions, so it's a straight line – What is a straight line?
	9:08	
7	Nash:	In science when we talked about – we say light travels in a straight line, so
8		what does that mean? (Nash pauses – giving learners some time to think).
	9:19	
9	Nash:	If someone asks you – you tell someone you have to go to shop – so the
10		person says what's the quickest way to get to the shop? – What will be the
11		quickest way to get to any place? (Some learners chorus (faintly) straight line).
12	Nash:	(Repeats) A straight line – So if I got a straight line – If here's place A
13		and I want to get to place B (puts two points on the board and labels them
14		A and B), I can walk to there, come back here, come back there, come back
15		there (joins points A and B with a zigzag line) – that's one way of going
16		that's the way our roads in South Africa are designed – (some learners
17		laugh) – the easy way or the shortest way will be just to go from A to B (joins points A and B with a straight line).
	9:59	
18	Nash:	So, the definition of any straight line will be what? (Brief pause).
	10:01	
19	Nash:	The shortest distance between two points.

(Lesson 1, time interval 8:40 to 10:01)

Line 1 contains the initial question that Nash poses – the minimum number of points needed to draw a straight line. This question is repeated and undergoes transformation various times in a short space of time – lines 4, 5 (twice), 7 (relating it to the concept of light which was discussed in science), and 9 (relating it to the everyday – going to shop). The time frames captured in the extract are intended to give, inter alia, one a sense of the speed at which the questions come up. Looking at the extract in real time, clearly illustrates that Nash has not given his learners sufficient time to think and to reflect on the questions posed, with the exception of line 8 where Nash pauses for a short while. This does not necessarily mean that if Nash gave his learners more time to think they would come up with the correct answer, they would possibly have come up ideas which Nash could work with in order to steer them to the correct response. This illustrates the opportunity for Nash to grapple with the category working with learners' ideas which he is not aware of. Instead he makes a move to rephrase the question which he refers to as coaxing the learners into getting a correct answer as he explains:

7 Nash: "You see in a classroom situation my or the method that I usually employ, is you actually coax the children into providing particular responses or particular questions and then the development of your lesson depends on these so called questions that you get …"

(Interview 3, turn 7)

The above response by Nash reinforces the notion that the mathematical work of teaching is a function of pedagogy, to the extent that this strategy has been given serious thought and has been consciously factored into his teaching methodology. Finally in lines 9 to 11 we see that Nash has successfully 'coaxed' learners into providing a correct answer to his initial question – the minimum number of points needed to draw a straight line.

Nash continues with his explanation to conclude this event by defining a straight line for his learners (lines 18 and 19). This is the last kind of problem-solving that Nash confronts in the series of lessons observed. So what is the importance of defining? Firstly, according to Ball (2003) defining can be seen as that which successful mathematicians and mathematical users do and thus the act of defining

can be referred to as a mathematical practice. A workable definition according to Ball, Bass and Hill (2004) is one that is mathematically appropriate and usable by learners at a particular level, therefore definitions "must be based on elements that are themselves already defined and understood" (Ball, Bass and Hill, 2004:58). There is a whole lot of mathematics that is being built through the process of definitions because we define what is there and what is not, we also define what constraints are present. So, defining is a significant mathematical practice thus:

> "Knowing mathematical definitions for *teaching*, therefore, requires more than learning mathematically acceptable definitions in courses … Knowing how definitions function, and what they are supposed to do, together with also knowing a well-accepted definition in the discipline, would equip a teacher for the task of developing a definition that has mathematical integrity and is also comprehensible to students."
>
> (Ball, Bass and Hill, 2004:58) (italics in original).

Nash's engagement with the mathematical practice of defining is constrained by his enactment of the practice.

Lines 12 through to 17 show that it is through empirical work that Nash manages to solidify the notion that the shortest distance between two points is a straight line, which allows him to go on to providing a definition. With respect to the defining and explaining the mathematics that the learners get access to, is predominantly – mathematics is a set of rules to follow. Lines 12 to 17 show that Nash tries to back up these rules by doing some empirical work which is probably what teachers would by and large do because it is difficult to get the general mathematical explanation.

Throughout the lessons observed the mathematical 'problems' or the mathematical work of teaching that Nash confronted were explaining, representing, questioning and defining. In this practice Nash relies on the understanding and experience that he 'brings', but this has a limiting effect. The opportunities that Nash provided for his learners to reflect on the notions being introduced, shapes possibilities for them to transform it from a level of immediacy (i.e. a 'that') into something more substantial. What then are the opportunities for

learners to develop strong concept images? Thus far, I have engaged in a discussion aimed at answering focus question one i.e. the mathematical work of teaching that Nash confronts. If these are the elements of problem-solving for Nash, then what resources enable this? In the next section I will answer focus question two which deals with the resources that Nash draws on in order to solve these problems.

4.6 The knowledge and experience that Nash draws on as he solves the problems of teaching that he confronts

As mentioned previously the categories of appeals that have been identified were **mathematics** with the subcategories (*empirical*, *definitions* and *rules*); **experience** with the subcategories (*profession* and *everyday)* and **curriculum** with its subcategories (textbooks and *examinations or tests*). Across the lessons, in an attempt to fix meaning, Nash draws on these resources either in a singular fashion or in some combination, therefore in each event it is not necessarily the case that all these categories are appealed to. Table 4 clearly illustrates this. In the next 2 sections I will illuminate the knowledge and experience that Nash draws on as he solves the problems of teaching that he confronts.

4.6.1 Mathematics

As discussed in the previous section the extract dealing with the ordering of x and y values to form a coordinate resulted in Nash providing an explanation that was procedural in nature. Nash's response "x goes first – x always comes first…' illustrates that he appealed to a *rule* in **mathematics** in an attempt to authorise this notion. His demonstration of the shortest distance between two points, as described in an extract cited in the previous section, by comparing a zigzag line to a straight line between two points illustrates that he is appealing to an *empirical* observation. With the empirical observation the learners' understanding is dependent on the knowledge that they each derive from their experience, particularly from sensory observation, rather than from the application of some

logic. Nash's appeal to rules goes beyond the mere conventions in mathematics but also to instances when he does not have the 'means' to offer a mathematically robust explanation, which in turn fosters instrumental understanding, as he explains:

47 Nash: "… so basically what they're doing is, for application of maths, teachers are basically creating recipes, instead of creating understanding of the mathematical skills required"

49 Nash "… I might be guilty of the same thing, you basically resorting to teaching something the way you were taught. So we just continuing the fallacy as such, because as much as I complain about it / if you use one method and the child does not understand and you use the second method and the child still doesn't understand or doesn't totally grasp the concepts – you find yourself indirectly creating rules or recipes thinking that will be the most efficient way to help the child out."

(Interview 3, turns 47 and 49)

Consider the following extract from lesson 1- I have underlined and italicised text to assist in drawing attention to certain aspects in the discussion that follows:

6:17

Nash: Now why function? - all the time we've been saying there's some kind of relationship, now we saying there's simply a function – <u>from a relation it means they were husband and wife</u> – <u>now they having a function – are they getting married now or are they getting married before</u> – *a function just basically means that for every x value (point to the table on the board) I've got a unique y value* (interruption by an announcement via the intercom system) (Nash repeats) for every x value there's a unique y value – one x value doesn't have two different y values (Nash pauses due to continued interruption by an announcement via the intercom system). <u>So every husband (points to the x value on the table) has got one unique wife (points to the y value on the table) – so that's why we say linear and it is a function.</u>

(Lesson 1, time interval 6:17)

This extract illustrates two things. Firstly, I draw your attention to the underlined text. Here we see Nash engaging in the use of a metaphor to explain the concept of a function. As alluded to in the previous section and it is also evident here that when Nash ventures into the everyday he makes use of metaphors that are not mathematically robust. So what is the problem with his use of metaphors? The use of metaphors is an important aspect of PCK (as discussed previously) and it is not just any metaphor that can be used, the metaphor actually has to have mathematical integrity i.e. the metaphor for the concept must not only make everyday sense it must also make mathematical sense. Nash makes up metaphors by drawing on his own understanding, experiences and assumptions, which were implicit in his communication during an interview:

14 Nash: "Well, with the mother and father the same, I mean in most - well in normal homes, and that's another assumption that I made, I agree that we are going to have one or two children that come from irregular homes, but for the majority there's a father figure and a mother figure, we regard the father figure as the dominant figure and the mother figure as a sub-servant of the father, although they could be earning the same salaries they could be having the same say but in every house we regard the father as the alpha figure in the home as such."

(Interview 2, turn 14)

Nash knows what he is talking about. However, in the extract from the lesson we can see that Nash is having difficulty in manoeuvring the idea of a husband and wife to discuss the notions of relations and functions. This appeal to the *everyday* is inappropriate since the monogamous element connecting husband and wife does not gel well with the mathematical notion of a function. Furthermore, its inappropriateness also surfaces in the sense that in all households it is not always the case that the wife/mother is dependent on the husband/father. It could be the other way around or they could be mutually independent of each other or it could be a single-parent household. Therefore, this metaphor contains the potential to promote misconceptions amongst the learners since the learners come from different backgrounds and their experiences related to marriage relationships are different. Nash is only mapping a part of the everyday domain i.e. the

monogamous element related to the notion of function, to the mathematics domain but this is not made explicit to the learners. The result is that the learners who did not possess the relevant recognition and realisation rules would have mapped a larger portion of their everyday experiences, thus creating the platform for misconceptions to creep in.

I now draw your attention to the italicised text. To me it seems that Nash realises that the metaphor is not working and this metaphor is making it difficult for the function concept to emerge. In an attempt to steer learners on course we see Nash appealing to a mathematics *definition* to legitimate the notion of function. The definition is not a complete version as one would find in a textbook but it certainly stresses the idea of a unique correspondence between x and y (the monogamous element).

4.6.2 Experience and Curriculum

Nash feels that he really knows his learners since he has had eight months exposure to them, with the result, he knows who to question as he explains:

13 Nash: "…I've already had up to eight months exposure to these learners, so I got a clear understanding of who or the different levels or the different backgrounds of each of the learners themselves … I feel I got a good understanding of those. I got a good understanding of which learners will give responses that will progress and develop your lesson, so in the lessons I focus most of my questioning on particular learners because these are the learners that basically span the entire horizon of the class. I know who are my high flayers, who are the weak ones and then through a mixture of questioning of the different students you can basically understand or you get the picture of what's the environment of the class…"

(Interview 3, turn 13)

The above extract illustrates that the central knowledge resource that Nash draws on is his **experience**. This is a kind of tacit knowledge that Nash possesses since

he does not have to think who he is going to question – he knows who to ask. The knowledge of learners refers to the cognitive and affective characteristics of learners. This kind of understanding of learners is a fundamental aspect of PCK in that, as well as facilitating the generation of useful metaphors or representations more broadly, teachers must have a sense of what learners tend to find easy or difficult and what kinds of difficulties they experience.

Ball and Bass (2002) and Ball, Bass and Hill (2004) examine the specialised mathematical problems teachers solve as they go about their work of teaching and highlight an essential feature of knowing mathematics for teaching as 'unpacking' or 'decompression[29]' of ideas. They contrast this with mathematics and "its capacity to *compress* information into abstract and highly usable forms" and posit further that "Mathematicians rely on this compression in their work" (Ball, Bass and Hill, 2004:59, emphases in the original). The unpacking that Ball, Bass and Hill (2204) refer to is unpacking from mathematics. This notion of unpacking resonates with Nash's 'backward chaining' process which came to light during the third interview. In essence Nash describes his backward chaining process as:

> "… the end product – what type of questions do I see in the exam, how does this relate to the matric exams, similar questions that relate to further exams and then work backwards from there."
>
> (Interview 3, turn 89)

Here we see that Nash unpacks from the curriculum and not from mathematics. In the next chapter I grapple with this issue.

Here we can see that Nash 'decompresses' or 'unpacks' from the examinations including the national examinations written at the grade 12 level, as apposed to unpacking from mathematics. I would hasten to add that this is partly due to what one can call institutional pressure. Institutional pressure since schools, specifically secondary schools, in South Africa are ranked and accorded levels of prestige and stature based on their performances in the national examinations. For

[29] Decompression refers to the ability to be able to "deconstruct one's own mathematical knowledge into less polished and final form, where elemental components are accessible and visible" (Ball & Bass, 2000:98).

instance, the following extract taken from lesson 2, demonstrates what Nash appeals to in order to emphasise the importance of labelling:

20:27

Nash: At matric level – when you'll write matric you have to show all this labels (points to the labels on the graphs and the cartesian plane) – they even give you a mark – there's a mark just for having labels (points the label of the y axis), labels (points the label of the x axis) and each graph having a label – because you might have 3 or 4 graphs on the same axes and you don't know which one is which.

(Lesson 2, time interval 20:27)

The above extract illustrates another resource which Nash draws on as he goes about solving the problems of teaching he confronts, is the **curriculum**, specifically in this case an appeal to *examinations*.

The knowledge resources that sustains this practice of Nash is his experience both professional and of the everyday, the experience of his colleagues and the curriculum (textbooks and examinations) as well as his knowledge of mathematics. From the test results, in relation to the expectations in this practice, we can see that this succeeds for most of the learners but not for all.

4.7 Summary

In this chapter I have focused on questions 1 and 2 i.e. the mathematical 'problems' of teaching that Nash confronts and the resource pool that he draws on in solving these problems of teaching. In doing this I have described Nash's practice. The elements of the mathematical work of teaching viz. defining, explaining, representing, questioning, working with learners' ideas and restructuring tasks is key if reflection helps movement from immediacy towards the intended notion. On top of these, the learners master some of it though we might say at a procedural level rather than at a conceptual level. Introducing a notion through an activity is absent in Nash's lessons in the same way are the mathematical work of teaching, viz. working with learners' ideas and restructuring tasks. We know from Kazima and Adler's (in press) study that

when these are present i.e. working with learners' ideas and restructuring tasks, others viz. defining, explaining, representing and questioning become difficult. When this happens some reflective moves are absent and this limits the fullness of the notion. The practice that constitutes all these problem-solving categories (i.e. defining, explaining, representing, questioning, working with learners' ideas and restructuring tasks) is a difficult practice and outside of good models of what it looks like and how to do it there is a difficulty.

In the next chapter I will engage in discussions related to how the resource pool, as illuminated here, relates to mathematical proficiency or the potential to promote mathematical proficiency amongst Nash's learners. In addition, I will also engage in discussion as to why Nash uses the knowledge and experiences he draws on in the way he does.

Chapter Five

Discussion

The discussion engaged with in this chapter is concentrated on answering the last two critical questions that support this study, and they are:

- How does this resource pool relate to mathematical proficiency or the potential to promote mathematical proficiency in his learners?
- Why does the teacher use the knowledge and experiences he draws on in the way he does?

In the previous chapter I have dealt implicitly with these two issues to a certain degree. However, in this chapter I will deal with these issues more explicitly.

5.1 How does the resource pool that Nash draws on relate to mathematical proficiency or the potential to promote mathematical proficiency?

As alluded to in chapter one, the construct mathematical proficiency was drawn from Kilpatrick *et al.* (2001) and the ideas inherent in this construct were expounded on. For purposes of recapping, I will merely list the elements of mathematical proficiency as described by Kilpatrick *et al.* (op. cit.), they are:

- Conceptual understanding;
- Procedural fluency;
- Strategic competence;
- Adaptive reasoning; and
- Productive disposition.

As discussed by Kilpatrick *et al.* (op. cit.) these five strands are interwoven and need to be developed simultaneously. Furthermore, proficiency develops over time therefore learners need enough time to engage in the section on functions. In this study, I have only concentrated on lessons conducted over a seven day period, for reasons already discussed. The section on functions was further grappled with by Nash and his learners after the collection of data and they will also encounter the concept of functions again in grades 11 and 12, certainly at increasing levels

of complexity though. Taking this into account then, these learners ought to become increasingly proficient since according to Kilpatrick *et al.* (op. cit.) proficiency develops over time. The mathematical proficiency in question here is not a matter of all or nothing since "students should not be thought of as having proficiency when one or more strands are undeveloped" (Kilpatrick *et al.*, 2001:135).

Researchers (Ball and Bass, 2000; Ball, Lubienski & Mewborn, 2001; Ball, Bass & Hill 2004; Even, 1990; and McNamara, 1991) allude to the notion that knowing mathematics for teaching requires knowing in detail the topics and ideas that are fundamental to the school curriculum and beyond. This detail involves a kind of 'unpacking' or 'decompressing' of mathematical ideas. When Ball, Bass and Hill (2004) talk about unpacking, they talk about unpacking from mathematics in relation to learner thinking. What they (Ball, Bass and Hill, 2004) are saying is that you have to unpack it so that someone else can understand it. In other words, for them it is the relationship between the epistemic and the cognitive root. It is not merely the blending of the two, what is missing is the fact that it is institutionalised; the unpacking is constrained by the context of schooling – particular curricula, particular social practices and so forth. On the other hand, the unpacking associated with Nash's notion of 'backwards chaining' is from the curriculum. For Nash unpacking is a function of schooling and the curriculum and this resonates with what the French refer to as the institutionalised notion of schooling as described by Barbé *et al.* (2005). Thus, what drives the mathematics that Nash thinks about is different; it is not the principles of mathematics or student learning or student thinking, as is the case with Ball, Bass and Hill (op. cit.). Therefore the object between these two ideas is different. Nash articulates his idea of 'backwards chaining' as:

89 Nash: "First and foremost when you looking at the topic / my preferred method is what is referred to as backwards chaining. Backwards chaining means the end product – what type of questions do I see in the exam, how does this relate to the matric exams, similar questions that relate to further exams and then work backwards from there – what leads up to

> completing a complicated question or solving a particular problem and then breaking it down till you come to the most elementary skills that are involved; and then you begin with these particular skills for a period of time till you come to a stage where you're able to incorporate all these skills to solve a problem or the final goal that you had."
>
> (Interview 3, turn 89)

The above extract illustrates that assessment, in the form of examinations and tests, is an important resource that Nash draws on. The examinations and tests that Nash draws on currently for teaching at this level is largely influenced by the principles inherent in the Nated 550 curriculum (DoE, 1997). The doctrines imbued within this curriculum make it difficult or impossible to observe some of the strands of proficiency as described by Kilpatrick *et al.* (2001). So in Nash's practice, examinations and tests have the potential to promote mathematical proficiency amongst his learners provided that the values around proficiency as, captured in the current National assessments, change to include the five strands of proficiency as described by Kilpatrick *et al.* (2001). The implication of this would be that Nash's practice in turn will have to change since his practice is driven by the 'backwards chaining' process from the assessment standards provided. So if the assessments were changed so that it measures all the competencies as described in the five strands of Kilpatrick *et al.* (op. cit.) then Nash's backward chaining process would begin to drive a change in his practice, but then another 'problem' emerges in the sense that he does not have the relevant experience either.

Nash's sense of working with learners' ideas is through his conception of 'coaxing' which he explains:

> 7 Nash: "You see in a classroom situation my or the method that I usually employ, is you actually coax the children into providing particular responses or particular questions and then the development of your lesson depends on these so called questions that you get … you actually learn more from misconceptions and errors in your judgement then you

> do by actually doing the right thing. If you put a sum on the board and everybody gets it right, you realise after a while the sum itself doesn't have any meaning to it, but once they make errors and you make them aware of their errors or aware of their misconceptions – you realise that your lessons progress much more effectively or efficiently by correcting these deficiencies, by correcting these errors and misconceptions."
>
> (Interview 3, turn 7)

The above extract is taken from the last interview with Nash. His talk about coaxing learners shows that he has an interpretation of what it is to work with learners – Nash knows that he has to work with learners' errors. Ball, Bass and Hill (2004) indicate that part of the mathematical knowledge needed for teaching entails more than just the ability to recognise that a learner's answer is wrong, a teacher needs to identify the 'site' and 'source' of the error. The error analysis that Ball *et al.* (2004) allude to is something that Nash consciously factors into his delivery of lessons. In fact it is one of the drivers in his lessons and the resource that he draws on is not a theory of misconceptions but his own experience.

The role of teachers is to assist their learners to achieve understanding of the subject matter that they are teaching, and in order to accomplish this task the teachers themselves need to have a thorough and sound understanding of the subject matter. Many authors, for example Ball and Bass (2000); Ball, Lubienski & Mewborn, (2001); Ball, Bass & Hill (2004); Even (1990); and McNamara (1991) allude to the notion that teachers who have a solid understanding of the subject matter being taught are the kinds of teachers who will be more capable of creating learning environments that will be conducive to promoting mathematical proficiency and thereby moving their learners to a level where they (the learners) achieve meaningful understanding of the subject matter. As discussed in chapter two, subject matter knowledge is only one of the categories of teachers' knowledge base that Shulman (1986) distinguished; nevertheless it is an important one. Even (1990), as discussed in chapter 2, offers us some 'tools' which allow us to get a handle of what is entailed in the notion of teachers' knowledge about mathematical topics. She identifies seven aspects that form the main facets of

teachers' subject matter knowledge, one of which is the strength of the concept. By this she means that teachers should have "a good understanding of the unique powerful characteristics of the concept. The related important sub-topics or sub-concepts, as with any other concept, cannot be fully understood or appreciated if viewed in one simplistic way only" (Even, 1990:525). Consider the following extract taken from the last interview with Nash:

67 Nash: "First, I think I even mentioned this in the previous interview; you've got to have an understanding of where this concept of functions are going to. That means the graphs, your hyperbola, parabola, although for the purposes of the research I focussed on the straight line graph but thereafter your parabola, hyperbola, circle – these concepts are going to be developed further in grade 11 and then in grade 12 when you come to your cubic graphs. So any grade 10 mathematics teacher has to highlight important or must have an understanding of where these concepts are going to, because you don't want a situation where you come to grade 12 and you teaching cubic graphs and you coming back to the concept of turning point, or y-intercept or x-intercept and there are particular methods you have to solve those in matric, not the same methods that you using in grade 10 or grade 11 but then again the concrete concepts of these features of the graph are being lost and you've got to re-teach all of those than. So basically the teacher needs a good understanding of the mathematics, the concepts itself …"

(Interview 3, turn 67)

This extract illustrates that Nash is aware of the importance of what Even (1990) refers to as the strength of the concept. Although the strength of the concept is just one of the seven features of teachers' subject matter knowledge, it is an important contribution to the promotion of mathematical proficiency. Nash's knowledge and understanding that he reflects on, as captured in the above extract, is not drawn from some theory arising from studies in mathematics education but from his own experience. Thus, once again illustrating that Nash's experience (a resource) that he draws on does to a degree facilitate or has the potential to promote mathematical proficiency amongst his learners.

The knowledge and experience that Nash has gained through his years of teaching are resources that have the potential to promote mathematical proficiency amongst his learners. As discussed earlier in this section Nash unpacks from examinations and tests through his backward chaining process, but currently the domain from which unpacking is done does not explicitly reflect all five strands of proficiency e.g. productive disposition. Nonetheless, the extract that follows shows that Nash sees the importance of developing a productive disposition amongst his learners towards mathematics which contributes to the promotion of mathematical proficiency:

19 Nash: "Learners' response to the lessons, where you don't see it immediately but later on when they actually enquire about whether this will be a major section in the exam, that means they are actually looking forward to having something based on functions in the exam, knowing that this is actually an area of maths that they understand that they actually looking forward to it. It's a break away from your normal factorising and solving for x, something that's basically abstract – this is more concrete – you taking equations you turning it into something that you can actually see. The graphs itself is actually something that you can apply in daily life. So by just looking at the responses and the enthusiasm to actually do more, to actually work out more and look forward to actually seeing these types of questions in the exams tells you that there must have been some kind of success in the lessons itself."

(Interview 3, turn 19)

In this section I looked at how the resource pool that Nash draws on relates to mathematical proficiency or the potential to promote mathematical proficiency. The next interesting question that arises is why does Nash use the knowledge and experience that he draws on in the way that he does? In the next section I will engage in a discussion in an attempt to answer this question.

5.2 Why does Nash use the knowledge and experiences that he draws on in the way he does?

From the discussions in section 4.4.4 of chapter 4 we see that Nash's teaching does lead to success but here, as mentioned previously, success is defined by the learners' performance in examinations and tests which to a large degree can be ascribed to the social and institutional constraints of schooling – as alluded to previously, schools are currently ranked according to their performance in the senior certificate examinations (i.e. National examinations) and this is the main resource that Nash uses to 'unpack' from. And as alluded to in section 5.1 (this chapter) this does not necessarily mean that the learners display proficiency in all five strands of mathematical proficiency as described by Kilpatrick *et al.* (2001). Nash realises that there exists a problem, and he reflects on it during the third interview:

49 Nash: …I might be guilty of the same thing, you basically resorting to teaching something the way you were taught. So we just continuing the fallacy as such, because as much as I complain about it / if you use one method and the child does not understand and you use the second method and the child still doesn't understand or doesn't totally grasp the concepts – you find yourself indirectly creating rules or recipes thinking that will be the most efficient way to help the child out.

50 VP: And do you think that this is the most efficient way of helping a child out?

51 Nash: I think that's the worst way of trying to help the child out, but because we were taught that way - like I say / when all else fails you sit back and think how was I taught this, because if I understand it at this point probably that was the best way and you find yourself making rules or recipes.

(Interview 3, turns 49 – 51)

Furthermore, in the same interview when Nash was asked what would he find challenging with the new curriculum he said:

111 Nash: "A challenge will be, we are presented with a new curriculum but then we still, like I am guilty of that – we still have old ideas and old ways of presenting this information. Like I said before if push comes to shove we are going to resort to our abstract ideas and our rule making again so that's going to be one of the challenges. Finding new innovative ways to present this new curriculum."

(Interview 3, turn 111)

The above extracts show that Nash realises that there needs to be a move or shifting of his pedagogy but he is also indicating that without some intervention in mathematics education in terms of what this other pedagogy could look like, he does not know what to do. So there is a missing resource that would need to come from somewhere else. He is certainly not picking it up from is own research using internet sources and from discussion with his colleagues and the substance of what he has to do is not easy for him.

During the third interview I created the scenario where I took a newly written textbook that incorporates the principles inherent in the new curriculum for grade 10 and showed Nash the kinds of questions that appeared in the book specifically looking at the section on functions. Firstly, we looked at an investigative activity that investigated the effect of c in the equation y=mx+c, where the gradient is kept constant and the y-intercept is changed. Secondly, we looked at a question set in the context of the everyday e.g. water. I asked Nash to comment on the challenges that he thinks he might face if he had to engage with these kinds of problems in the classroom. He responded as follows:

117 Nash: "Challenges again // the way that I actually foresee the development of a lesson – like I said I always do backwards chaining with my main goal on developing learners' problem-solving abilities, developing their understanding of mathematics and preparing them for future application of the concepts. Now, like I said problem-solving and future development is not part of the outcomes, so the challenge that I'm gonna

> face is now – actually determining what are the goals and outcomes for this new era of learners because the goals have changed, like I said our understanding is still the same but we've got a completely different breed of learner that they have different goals and different things motivate them as such."
>
> (Interview 3, turn 117)

The above response by Nash illustrates that he recognises that the learners are the 'obstacles' in the teaching and learning process since the learners have changed. The learners have changed in a sense that they cannot be treated as learners were treated in the past, 'this new breed of learners' needs to be stimulated, they need to be more involved in order for them to learn. Nash recognises that there is a new moral order and this resonates with the broader notion of a change in social order in the world which has been discussed at a very general level by Bernstein (1990). The implementation of Curriculum 2005 is a manifestation of this new moral order. Nash recognises this but what does it mean to realise a practice that deals with this effectively? Nash indicates that he does not know what to do, even though in his practice he can see it, yet it does not give him the resources to work with it.

The answer does not only lie in the implementation of some new kind of pedagogy. There also needs to be what Margolinas, Coulange and Bessot (2005) refer to as external interventions – I will discuss the notion of external intervention in the next chapter. What could possibly constitute this external intervention for Nash? Reading some research on the construction of knowledge and learners misconceptions might give Nash a formal understanding of what is happening with learners. But this might still not equip him with the practical skills; it might require something else that would begin to shift his practice. In Nash's case the external intervention that could perhaps achieve this is a change in assessment since Nash's practice is driven by assessment. However, this needs to be followed with additional support around the epistemic and cognitive roots in order to strengthen his ability to unpack the mathematics he needs to teach.

5.3 Summary

In this chapter I provided a discussion exploring how the resource pool that Nash draws on promotes mathematical proficiency or has the potential to promote mathematical proficiency. It is important to note that when I talk about mathematical proficiency I refer to the way in which Kilpatrick *et al.* (2001) describe it. Unpacking, as a concept related to MfT, is epistemic, it is cognitive and it is institutional. Unpacking needs to be done with all three aspects in mind, in Nash's case we see unpacking being done from the curriculum alone, in the form of assessments, through a process which he terms 'backwards chaining'. The backwards chaining process shows that assessments, in Nash's case, has the potential to promote mathematical proficiency. Another revelation was the idea of 'coaxing', which can be construed as Nash's conception about working with learners' ideas. Here again it is not theories in mathematics education, e.g. a theory of misconceptions, that he draws on, but rather his own experiences. We see that it is Nash's knowledge and experience that he has gained over the years that also has the potential to promote mathematical proficiency. The question then is – how do we provide Nash with experiences that would adequately equip him to teach for mathematical proficiency? How do we provide him with the tools that would assist him to approach his teaching from a perspective that would allow him to expose his learners to the world of the professional mathematician? In doing so, he would provide his learners with opportunities to develop what Vinner (1983) refers to as strong concept images and concept definitions.

In terms of providing insights into why Nash uses these resources in the way he does, we see that to a large degree, it can be ascribed to the social and institutional constraints of schooling. This stems from the fact that schools are ranked and given status dependent on their performance in assessments. Secondly, Nash alludes to the idea that the learners are the 'obstacles' in the teaching and learning process since the learners have changed. This implies that learners these days need to be stimulated. They need to be more involved in order for them to learn (thus the concept of learner centred practice). Nash, also realises that his practice

needs to change, but in the absence of what this ‘new’ practice should look like it is a daunting task for him to change.

In the concluding chapter I will reflect back on the study, its theoretical underpinnings and its results.

Chapter Six

Conclusion

6.1 Introduction

This study was concerned with investigating mathematics for teaching; the kinds of mathematical problem-solving a teacher does as he goes about his work. The questions that guided the investigation were:

1. What mathematical 'problems' does the teacher confront as he teaches the section on functions in grade 10?
2. What knowledge and experience (resource pool) does the teacher draw on as he solves these problems of teaching?
3. Why does the teacher use the knowledge and experiences he draws on in the way he does?
4. How does this resource pool relate to mathematical proficiency or the potential to promote mathematical proficiency in his learners?

The domain of functions provided the backdrop to this study as this was the aspect of mathematics identified by the teacher, based on the notion that he feels he teaches this section well. The domain of functions is an important aspect of the curriculum and was elaborated on in chapter one.

In this study I drew on the work of Kilpatrick, Swafford and Findell (2001) to elaborate on the notion of mathematical proficiency. As discussed in chapter 2, many of the theoretical bases that inform debate in mathematics education are theories of learning, whereas this study is located in the practices of teaching – which is a less developed field of study. In view of this, the theoretical lens that informed this study was drawn from the QUANTUM project. It is located in the sociology of pedagogy and Bernstein's (1990) notion of the pedagogic device provided the platform here. Hegel's theory of judgement provided me with the ability to look into a practice and to see what transpired over time. It was the work of Ball, Bass and Hill (2004), specifically their elucidation of the more

practice-based notions of mathematics for teaching, which assisted me in identifying the categories that were used to describe the mathematical work of teaching. In Hegel's terms these are the activities that provide learners with opportunities to reflect on a concept so that they can transform it from the first encounter into something more substantial. I will pick up on these practice-based notions of mathematics for teaching a little later in this chapter. The categories that were used to identify how notions come into existence as well as the appeals Nash made in order to fix meaning emerged from the data that was collected for this study.

After chunking the data into evaluative events it immediately became visible that the mathematical work of teaching (mathematical problems) that Nash encountered were defining, explaining, representing and questioning. The mathematical work of teaching viz. working with learners' ideas and restructuring tasks were absent in Nash's practice. I link this to the manner in which concepts or notions came into being. In Nash's case notions came into being either verbally or through some written activity as depicted in the analytical framework. A similar study conducted by Kazima and Adler (in press) found that when notions come into existence through an activity then the reverse happens i.e. working with learners' ideas and restructuring tasks become inevitable whilst the mathematical work of teaching viz. defining, explaining, representing and questioning are largely absent. Defining, explaining, representing and questioning are key elements within the process of reflection in terms of propelling a notion from the level of immediacy to the level of necessity with respect to what the notion is and what it is not. As alluded to previously, a practice that constitutes all of these 'problem-solving' i.e. defining; explaining; representing; questioning working with learners' ideas and restructuring tasks is a difficult practice and outside of good models of what it looks like and how to do it, there are difficulties with its realisation.

Nash has, in Bernstein's terms, recognition that something should change but does not know what to do, he cannot realise it in any way. It seems that what Nash is

saying is that there should be some intervention; that his own actions, his own work with his learners, his talking with them, and whatever the curriculum has to offer is not sufficient. This resonates with what Margolinas *et al.* (2005) say that in fact the practice is only disturbed when there is an external intervention. Margolinas *et al.* (op. cit.) concur with the community of research in mathematics education on the importance of "enabling teachers to reflect on their practice from a cognitive perspective" (Artzt and Armour-Thomas as cited in Margolinas *et al.*, 2005: 229). But they (Margolinas *et al.*, 2005) make an even bolder claim by saying that the conditions needed to stimulate this kind of reflection will not necessarily be found in the practice of teaching alone. In other words, in attempting to gain some principled understanding, and from their (Margolinas *el al*., 2005) perspective, it would mean gaining a deep grasp of some mathematics education research, and a teacher would need to distance himself/herself from his/her practice. To do this, the teacher would require something to assist with creating the distance from practice and then to aid with reflecting back on the practice. So, they are suggesting that "significant change may be brought about by external influences when teachers interact in groups with the potential for strong internal dynamics" (Ponte *et al.*, 1994 as cited in Margolinas *et al.*, 2005: 229). Therefore, it is unlikely that courses on its own will change Nash's practice. So where should this intervention be? To change practice, in Nash's case, the external intervention needs to come in through a change in the National assessments. This will encourage him to change his practice to comply with the requirements of the curriculum since, he unpacks from the curriculum. Therefore, if assessments embraced and highlighted the strands of mathematical proficiency as described by Kilpatrick *et al.* (op. cit.) and if Nash was given additional support around the epistemic and cognitive root then Nash would begin to teach for mathematical proficiency. A change in assessment alone will not necessarily result in the desired change in Nash's practice. Such change needs to also be supported by providing appropriate scaffolds to Nash in terms of the epistemic and cognitive roots, thereby strengthening his ability to unpack.

The second thing that is very interesting, which is a more theoretical point, is that Nash has some intuitive understanding that there is a change in the moral order. Nash does not know what to do with it, the problem is that the learners have changed, and so the 'obstacle' has changed for him and his practice no longer really works.

6.2 The 'problem' with Ball, Bass and Hill's description of mathematical knowledge for teaching

In studying the practice of teaching and analysing the mathematical demands of the teachers' daily work both in and out of their classrooms, Ball, Bass and Hill (2004) put up aspects (refer to chapter 2 section 2.2.2.1) that illuminate the mathematics that teachers have to do in the course of their work. In essence they are saying that mathematics for teaching is about defining, explaining, representing, and questioning (what Nash does). They are saying that it is also about working with learners' ideas and restructuring tasks. In Nash's practice we do not see these two aspects. Why?

That the activities such as defining, explaining and representing are present does not mean that they are error free and it certainly does not mean that they are mathematically rigorous. Furthermore, it does not mean in Hegel's terms, that moments of reflection and necessity, necessarily lead to a 'full' notion. There is always contingency. In Nash's classroom this is at the level of curriculum for example: this is a definition; this is what is in the textbook; and ultimately when there are some questions from the learners – this is what you need to do; this is in the exams. In a context where results from the current National examinations are prioritised, Nash is thus a successful teacher. He draws on precisely the kinds of knowledge resources he needs to in order to be successful in his practice. He is able to 'backwards chain' from the curriculum, he is able to represent, he is able to explain, and these are some of the key things that are needed to ensure a successful practice.

What is interesting in Nash's practice is the absence of restructuring tasks and working with learners' ideas. Where does this come from? Why am I saying that there is this absence? The answer to these questions lies in the notion that Ball, Bass and Hill (2004) provide a template for looking at MfT but it does not state explicitly that the template is informed by a privileged practice. Therefore, the problem with Ball, Bass and Hill's (op. cit.) practice-based notion of MfT is that they want it to work for any pedagogy but actually in a sense it is not a *description* of MfT but rather a *prescription*. This prescription is not only about the practice-based notion of MfT but is also related to a particular conception of mathematics proficiency, which in turn can also be perceived as some kind of prescription. This is not a conception that Nash has of what constitutes mathematics proficiency hence; it is not surprising that there are absences in his practice.

This study is about the mathematical 'problems' a teacher confronts and what knowledge resources he draws on to solve these problems and why he does what he does. The knowledge resources that Nash draws at the moment are sufficient to sustain his practice. Though we understand that through his practice it is institutionalised notions that emerge. If there is a privileged practice in mind which is somewhat different, and this is where curriculum 2005 seems to be going, then Nash is going to need other knowledge resources. These knowledge resources will then relate particularly to the design of tasks and the restructuring of tasks, to definitions and to working with learners' ideas. Currently, it appears that Nash does not have the tools or knowledge resources to do this. So, what might be done? We can offer Nash a set of courses, to access some of the activities mentioned. But, given his practice and the current institutional demands; if assessment stays as it is then no matter what this goal might be and this privileged practice might be, such courses are unlikely to drive a change in his practice. Another external motivator and force on Nash to change his practice would be if assessments change as they should if there is a new curriculum.

In Nash's practice we see that the knowledge resources that he draws on includes empirical mathematics and we see some limits to that; he draws on his experience and he draws on the curriculum. He does all of these quite effectively. Nash also draws on definitions, but defining in mathematics, as discussed previously, is not merely about knowing formal definitions, it is also about defining as a practice. We can see that even within the activities that he engages with, there are additional mathematical problem-solving resources and knowledge resources that need to be enabled to ensure that Nash gets better access to these activities. These are certainly not emerging from his practice and as Margolinas *et al.* (2005) argue, there are limits to what teachers learn in and from their practice. Engaging effectively with learners thinking (and hence task based activity), in their research, was a function of external intervention.

The above discussion has argued that the elements of mathematics for teaching identified and described by Ball, Bass and Hill (2004) are a function of a particular conception of mathematical practice, an argument derived from a description of MfT in Nash's classroom. Their elaboration, while apt and useful, is for a prescribed practice. In addition, there are elements of MfT identified in this study that Ball, Bass and Hill's (op. cit.) description do not elaborate, particularly the significance of the first encounter, and metaphors for teaching. Both these elements of MfT are identified in the wider literature, and arose as important in this study.

6.3 Reflections

As discussed in the section of piloting (chapter 3, section 3.3), I explained that piloting in the 'true' sense of piloting was impossible since it implied that I would have had to do the study. I therefore prepared myself theoretically but soon discovered, whilst in the process of collecting data, that I was still unprepared. The methodology employed for this study was a non-participatory observation of classroom practice and as such, whilst in the classroom I experienced difficulty in restraining myself from interjecting whilst Nash was teaching – the 'teacher in

me' wanted to surface. It was difficult being in the classroom and merely observing while learners experienced difficulties or being unable to draw Nash's attention to it. To ensure that my role of non-participant observer was not compromised I had to exercise self-restraint which required me to be disciplined as a researcher. Similarly, when analysing data it was hard to avoid being judgemental in terms of how I, as a teacher, would have done something as compared to the way in which Nash had done it - this required discipline in my doing the analysis and in reporting it.

My role as observer during the classroom observations was to remain detached from the participants. Although this was a daunting task, I accomplished it without any compromise; however, my mere presence in the class meant that I was being obtrusive. 'Fortunately' my obtrusiveness impacted positively on the learners to an extent, as commented on by Nash:

> 21 Nash: Again the // we looking at the average being 65%, but if we look at the highest and the lowest itself we still have learners that, we won't say are failing but then lower down we've got a hand full of learners who still weren't influenced by the entire process, because another question people raise – if this was a normal lesson that you were presenting and if it was done the same way as the two other teachers, how come your standard grade class actually performed better than the higher grade class? But I think it's this whole placebo effect of having outsiders in the class and the story of recording, basically there's more interest in there. But then again on the other hand for some of them it was just business as usual.
>
> (Interview 3, turn 21)

The theoretical framework used in this study is Hegel's theory of judgement and as indicated previously I drew on this theory through Davis *et al's.*, (2003) interpretation. Over a period of time I got to appreciate Hegel's judgements of existence, reflection and necessity. However, it is still not crystal clear what the fourth notion is i.e. the judgement of the notion.

One of the aims of this study was to build a model to represent this framework and it was difficult to represent and capture this fourth moment of judgement. In other words, trying to represent what it is and how it works was difficult. This provides opportunity for further research.

In retrospect, when I go back to the initial stages of this study I felt that the more practice-based notion of mathematics for teaching as discussed by Ball *et al.* (2004) would possibly provide the theoretical framework on which to rest this study. However, there was a sense of absence when attempting to use this practice-based notion of mathematics for teaching as the theoretical framework. It is not only absent in Ball, Bass and Hill's (2004) work, it is not found in Bernstein's work either or for that matter in any other work on pedagogic practice. The absence that I talk about is found in the idea of what happens over time, and it is Hegel's theory that provided the lens to be able to see what happens in a pedagogic practice over time. The idea of 'over time' refers to 'real' time - as it is happening in the classroom; as things unfold in 'real' time.

If we look at Shulman's (1986, 1987) work we see categories of PCK, SMK and CK. If we look at Ball *et al.* (2004) we see categories prescribing mathematics for teaching. From the work of Even (1990) we see that she picked up on the first encounter (concept comes into existence). The ability to look at a pedagogic practice and to observe the opportunities that a teacher provides to his learners in terms of bringing a notion into existence was helpful in designing the theoretical framework. Furthermore, to observe the teacher providing his learners with opportunities to reflect on the notion, in order to transform it into something more meaningful, coupled with the appeals the teacher makes in order to fix meaning also assisted in the design of the theoretical framework.

By reflecting on this, I must also admit that this aspect of the study bordered on becoming a nightmare for me in my valiant attempts to get an understanding of Hegel's theory of judgement – a rather very abstract theory indeed. I would hasten to add, that this study has provided me with *opportunities* to *reflect* on

Hegel's theory. However, I am still in the process of *transforming* it into something more *substantial* with reference to my understanding. The study has not made me an expert on this theory but it has given me a certain degree of access into the theory.

Coming to the end of this study, I sit back and wonder if there was no Hegel on the agenda, how might this study have looked? I came out of this study feeling that without the tools that Hegel's theory offers it would have been like owning and trying to drive a car without wheels.

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Appendix A

University of the Witwatersrand

Classroom Observation Schedule

Research: An investigation into mathematics for teaching: the kinds of mathematical problem-solving teachers do as they go about their work.

Researcher: Vasen Pillay

Observed lesson number: _____ Date of observation: _____

Number of female students: _____ Number of male students: _____

Lesson topic: _____

Duration of lesson: _____ Time of lesson: _____

Video recording of lesson (tick appropriate block):

YES	NO

(Capture observations by placing a tick next to the appropriate category. If the categories provided are insufficient, list the new category that best captures the observation.)

Key: P = Procedural – approaches the solution of a problem in the form of an algorithm

C = Conceptual - comprehension of mathematical concepts, operations, and relations

SC = Strategic Competence - ability to formulate, represent, and solve mathematical problems

Make a rough sketch of the learners seating arrangement:

Event:_____	Mathematical			Everyday	
	P	C	SC	Visual	Verbal
Representation of ideas:	Numerical Verbal Symbolic		Graphical Tabular	Pictorial Metaphorical	Story
Learner Activity	Asks Answers Writing		Comments Groupwork	Asks Answers	Comments
Appeals:	Mathematics Mathematics Education Curriculum			Authority Experience Metaphorical	
Comments:					

Event:_____	Mathematical			Everyday	
	P	C	SC	Visual	Verbal
Representation of ideas:	Numerical Verbal Symbolic		Graphical Tabular	Pictorial Metaphorical	Story
Learner Activity	Asks Answers Writing		Comments Groupwork	Asks Answers	Comments
Appeals:	Mathematics Mathematics Education Curriculum			Authority Experience Metaphorical	
Comments:					

Appendix B

Interview Schedules

Interview schedule – Interview 1:

Biographical Data:

- What qualifications do you have?

- How many years have you been teaching for?

- What subjects have you taught and at which grades have you taught these subjects.

- For how many years have you taught each of these subjects?

- Have you attended any in-service training?
  If yes: What did this training include?
  When did you attend?
  Who were the providers of this training?
  What was the duration of the training?
  How did you benefit from this training?

- How would you describe your role in this school?

- How does the Mathematics Department operate (in terms of meetings, assistance to teachers in the department itself)?

- From your timetable I deduce that there are other teachers also teaching grade10, can you explain how preparation of lessons are done as well as the compilation of worksheets and tasks used?

Interview schedule – Interview 2:

- You have completed five days of teaching; I am drawing you back to the introductory lesson on Monday. In preparation for the introduction of the topic of linear functions on what assumptions, if any, did you make?

- I notice that you introduced the notion of a function by defining a function as: for each x value there is a unique y value. What is it that informed your choice?

- Do you think that there are other ways in which you could have introduced it?

  If ***NO:*** go to last one on this page

If ***YES***: Could you please elaborate and explain your reasons for not using this approach?

If by investigation: What is the value of this approach (textbooks introduce it by making use of a definition)?

If investigations are not mentioned: What is your opinion about the use of investigative tasks, like getting learners to investigate the effect of m in y = mx + c whilst keeping c constant?

- You related the notion of a function to the analogy 'a mother is dependent on a father', what is the importance, if any, of doing this?

- In your opinion, is there any value in relating mathematics to the child's everyday experiences?

- I notice that at various times you refer learners to the exams and how questions could be asked. What impact does this have on your lesson preparations?

- What is it that informed the decision of including the type of questions that are included in the worksheet?
  What resources were used?
  If the learners' ability levels are not mentioned: What about the learners' ability levels?

Interview schedule – Interview 3:

- *Thank you for making this time available to meet with me, before I continue I would like to express my sincerest thanks and appreciation for allowing me to observe you over the two week period as you went about your work, for allowing me access to the materials that were prepared for the teaching of the section on functions as well as to the test, marking memorandum and learners' work also, thank you for allowing me to conduct the interviews with you. I would like you to look at this session as a conversation rather than an interview so please feel free to interject and comment whenever you feel like.*

- As I've just mentioned I have observed you over a two week period and your learners wrote a test as well and we have also engaged in discussions, are there any burning issues for you that you would like to raise?
  If no: **Leave it**
  If yes: (This is an unplanned part - depending on what the teacher says I will have to follow-up and probe further if necessary)

- I'd like to start with the test that was conducted – how would you describe the results for your class?

- What pleased you about the results? Why?

- What disappointed you with these results? Why?

- What do you attribute your disappointments and successes to?

- Did all the grade 10's taking mathematics write this test?
  If no: **Leave it**
  If yes: How did the results of your class compare with the results of the other classes?
  Probe what is meant by response (if better, same or worse) e.g. better in what way? What do you attribute this to?

  Did the other teachers assist in the setting of the test?
  If yes: Explain the process that was involved in the setting of this test?
  Depending on the answer – What was your contribution?
  What was it that informed your choice of questions to include?
  If no: What is it that informed your choice of questions to include in the test that was conducted?

- In the setting of tests where do you get your questions from?

- What were the common problems experienced by learners? Provide Examples?

- What do you attribute these to?

- Could any of these problems be classified as the 'hot potatoes' that you alluded to in the previous interview?
  If no: Did the questions in the test provide opportunity for the possibility of these 'hot potatoes' to emerge?
  If yes: Could you give me an example?
  And they did not emerge – what do you attribute this to?
  If no: Why did you exclude such questions?
  If yes: Could you perhaps provide me with an example?
  What do you attribute this to?

- The common errors that learners' made in the test are they similar to the problems that they encountered during the lessons?
  If yes: Why do you think this is the case?
  If no: How do the errors differ and to what do you attribute this to?

- How do you think these problems could be remediated?

- What were the learners' strengths and to what do you attribute this to?

- What were the learners' weaknesses and to what do you attribute this to?

Thank you - this discussion about the test and results. I would like to talk a little more about some of your ideas about teaching.

- From your experience and point of view, what do you think are the main things a teacher needs to know in order to teach functions in grade 10, as you have done?

  Answers might refer to content, curriculum, the students …
  If content is not referred to: What about the actual mathematics?
  If content is referred to: What do you mean by (phrase in teacher's own words)

- There are interesting views on this – on what teachers need to know. Do you think that if you learned to do mathematics, then you can teach it?

  If yes: So if for example, I have learned how to find the equation of a straight line that is perpendicular or parallel to another line and passes through a given point, then I would be able to teach it to the next person?
  If no: Why not?

- I know you have been teaching for a number of years, and at grade 10, so you must have taught functions many times at this level. Do you always teach linear functions the way that you did this year that which I have observed?
  If yes: Have you ever taught it differently?
  Would you consider teaching it differently?
  If no: **Leave it**
  If yes: What would prompt you to teach it differently?
  How would you?
  What resources would you need to draw on?
  Would the learning be different?

  If no: How else do you teach it?
  When would you do it that way?
  What resources would you need to draw on?
  Would the learning be different, do you think?

We have talked a lot about functions – and this was a topic we agreed on for this project together, and functions is something that you are well familiar with. What about new topics...

- When you plan the teaching of a new topic, how do you start?

- What happens next?
  I would anticipate a response like: "I try to imagine what pupils would find difficult? What kinds of exercises would enable them to negotiate with the concepts?"

  (This is unstructured and possibly I will ask a question like)

- What are the important things you consider?
  And probe – ways of teaching?
  And probe – what about learners and learning?
  And probe – what about the content – where and how is this considered?

- In what ways do you think the teaching of mathematics has strengthened your understanding of mathematics?

- If you wished to enhance your teaching, how would you go about doing this?

- Where do you see resources for the improvement of practice?

- A lot of people feel that the new curriculum for grade 10 mathematics is different from the current curriculum. What do you think?
  If he agrees: How is it different?
  What do you think is interesting and why?
  What do you think will be challenging for you?
  If he disagrees: Why do you think that there is nothing different?

I was looking through a new grade 10 textbook and the section on functions. One of the things I noticed is that there were different examples from those you used in the lessons I observed. So I have brought an example with me…

- What experience or knowledge do you think you will need to draw on in order for you to teach using these types of questions?
  What challenges do you think you may face and why?
  What would you do to overcome these challenges?
  What challenges do you think your learners may face and why?
  How would you do to help them to overcome these challenges?

I have asked lots of questions – thanks for your inputs. I wonder if there is anything that I have not asked about that you would like to share?

Thank you for your willingness to participate in this study, I found it refreshing speaking to you.

Appendix C

Letters Seeking Permission

Letter to the Principal

592 Patel Street
Actonville
Benoni
1501
[Date]

The Principal
[Name of school]
[Address of school]

Dear [Name of principal]

Information for participation in the mathematical knowledge for teaching functions research project and consent form.

Following our conversation at the beginning of this year, I write to formally request your consent to participate in the mathematical knowledge for teaching functions research project. This research project is one of the requirements for the completion of my MSc degree.

The aim of the project is to investigate the mathematical knowledge that teachers need to know and know how to use in order to teach functions well to secondary school students. To accomplish this task I will need to observe the teacher in practice, that is to say the focus will be on classroom teaching of functions in one grade 10 class. The lessons will continue as normal and as scheduled according to the teacher's timetable. To this end I have identified [name of teacher] who has already given me verbal consent. I therefore, humbly request your permission to allow me the opportunity to observe one grade 10 mathematics class as they are being taught the section on functions for two weeks. I further request that I videotape the lessons and also have access to copies of any materials produced by the teacher for teaching the section on functions as well as any materials produced by the learners during this time. In addition, I also intend to interview the teacher at least three times during the two weeks – once in the middle of each of the two weeks and once at the end of the two weeks. The interviews will be conducted after the lesson at a time that is convenient for the teacher so as not to interrupt the smooth functioning of the school. The purpose of conducting interviews is for me to gain insights into what I have observed during the lessons and for providing me with the opportunity to probe particular issues in depth.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports. Video extracts, where anonymity cannot be provided, will only be used with the consent of the participants. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years. The findings of my study will be communicated with you upon completion of my study. If at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Kindly complete the attached form and return it to me at your earliest convenience. I will be happy to answer any questions or queries that you might have. Furthermore, if there is anything, within reason, that I can do in return, please do not hesitate to inform me. Hoping for a favourable response from my earnest and humble entreaty.

Yours sincerely.

_____

Vasen Pillay
Tel: 0824158932
E-Mail: vasenpi@gpg.gov.za

CONSENT FORM (PRINCIPAL):

I, _____
(please print)

Principal of [name of school] give consent to the following:

1. The research related to mathematical knowledge for teaching functions can be conducted at my school.

   YES [ ] NO [ ] please tick

2. Videotaping of lessons on mathematics functions in one grade 10 class.

   YES [ ] NO [ ] please tick

3. Copies made of class notes, tasks and assessment that the teacher and students might produce as part of the lessons assigned to the teaching of functions.

   YES [ ] NO [ ] please tick

4. Interviewing the teacher to probe particular issues in depth.

   YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Letter to the Teacher

592 Patel Street
Benoni
1501
[Date]

[Name of teacher]
[Name & Address of school]

Dear [name of teacher]

Information for participation in the mathematical knowledge for teaching functions research project and consent form.

Following our conversations over the past few months, I write to formally request your consent to participate in the mathematical knowledge for teaching functions research project. This research project is one of the requirements for the completion of my MSc degree.

The aim of the project is to investigate the mathematical knowledge that teachers need to know and know how to use in order to teach functions well to secondary school students. To accomplish this task I will need to observe you in practice, that is to say the focus will be on classroom teaching of functions in one grade 10 class. I therefore, humbly request your permission to allow me the opportunity to observe you as you engage in the teaching of functions to one of your grade 10 classes for approximately two weeks. I further request your permission to allow me to video record the lessons, since this will provide me with a more permanent recording of the lessons so that I could focus on events more closely. To augment my data collection strategies I also intend to interview you with the purpose gaining insights into what I have observed during the lessons as well as for providing me the opportunity to probe particular issues in depth. To keep an accurate account of the interviews I will be tape recording all interviews. I intend to interview you at least three times during the two weeks – once in the middle of each of the two weeks and once at the end of the two weeks. The interviews will be conducted after the lesson at a time that is convenient for you so as not to interrupt the smooth functioning of the school. I further request your permission to have access to copies of any materials produced by you and your learners during teaching the section on functions.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports. Video extracts, where anonymity cannot be provided, will only be used with your and your learners consent. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years. The findings of my study will be communicated with you upon completion of my study. If at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Kindly complete the attached form and return it to me at your earliest convenience. I will be happy to answer any questions or queries that you might have. Furthermore, if there is anything, within reason, that I can do in return, please do not hesitate to inform me.

Hoping for a favourable response from my earnest and humble entreaty.

Yours sincerely.

_____

Vasen Pillay
Tel: 0824158932
E-Mail: vasenpi@gpg.gov.za

CONSENT FORM (TEACHER):

I, _____
(please print)
mathematics teacher at [name of school] give consent to the following:

1. Videotaping of lessons on mathematics functions in which I, the teacher, will appear as part of the videotext.

   YES [ ] NO [ ] please tick

2. Copies made of class notes, tasks for students and assessments that I might produce as part of the lessons assigned to the teaching of functions.

   YES [ ] NO [ ] please tick

3. Conducting interviews with me, the teacher, in order to probe particular issues in depth.

   YES [ ] NO [ ] please tick

4. Tape recording of interviews conducted with me, the teacher, with the purpose of providing an accurate record of the interviews.

   YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Letter to Learners

[Date]

Dear Grade 10 Learner
[Name of School]

Information for participation in the mathematical knowledge for teaching functions research project and consent form.

I am currently studying for a Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. As part of my thesis, I am investigating mathematical knowledge that teachers need to know and know how to use in order to teach well. To this end, I have already obtained permission from the Gauteng Department of Education, the principal of the school as well as the teacher concerned to conduct this study. This letter is to request your consent for your participation in the above mentioned research project.

In this phase of the project the focus will be on classroom teaching of functions in grade 10. I plan to observe lessons that are dedicated to the teaching of functions. I plan to videotape these lessons as well as to have access to copies of some of the materials produced by you during these lessons. Since you are one of the students in these classes, I ask for your consent to appear as part of the videotext and where necessary to have access to copies of materials that you might produce. Lessons will continue as normal and as scheduled, with my presence in the back of the classroom.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports by making use of a pseudonym to refer to the school, teacher and students. Video extracts, where anonymity cannot be provided, will only be used with your consent. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years. The findings of my study will be communicated with you, if you so desire, upon completion of my study.

Please note that if consent is not granted I will respect your decision. Therefore you together with any other students not participating in the project will be seated on one side of the classroom and will not be videotaped. Furthermore, any text that you might produce will not be used in the project. In addition, if at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Please compete the form attached and return it to the teacher at your earliest convenience. I will be happy to answer any questions or queries that you might have.

Looking forward to hearing from you.

_____

Mr V. Pillay
Tel: 0824158932
E-Mail: vasenpi@gpg.gov.za

CONSENT FORM (LEARNER):

I, _____ (please print full name), a student in grade 10, give consent to the following:

1. Videotaping of lessons on mathematics functions in which I might appear as part of the videotext.

   YES [ ] NO [ ] please tick

2. Copies made of class work, homework or assessments that I might produce as part of these lessons.

   YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Letter to Parent/Guardian

[Date]

Dear Parent/Guardian

Information for participation in the mathematical knowledge for teaching functions research project and consent form.

I am currently studying for a Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. As part of my thesis, I am investigating mathematical knowledge that teachers need to know and know how to use in order to teach well. To this end, I have already obtained permission from the Gauteng Department of Education, the principal of the school as well as the teacher concerned to conduct this study. This letter is to request your consent for your child/ward to participate in the above mentioned research project.

In this phase of the project the focus will be on classroom teaching of functions in grade 10. I plan to observe lessons that are dedicated to the teaching of functions. I plan to videotape these lessons as well as to have access to copies of some of the materials produced by your child/ward during these lessons. Since you are the parent/guardian of a student in these classes, I ask for your consent to allow your child/ward to appear as part of the videotext and where necessary to have access to copies of materials that your child/ward might produce. Lessons will continue as normal and as scheduled, with my presence in the back of the classroom.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports by making use of a pseudonym to refer to the school, teacher and students. Video extracts, where anonymity cannot be provided, will only be used with your and your child/wards' consent. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years. The findings of my study will be communicated with you, if you so desire, upon completion of my study.

Please note that if consent is not granted I will respect your decision. Therefore your child/ward together with any other children not participating in the project will be seated on one side of the classroom and will not be videotaped. Furthermore, any text that your child/ward might produce will not be used in the project. In addition, if at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Please compete the form attached and return it to the teacher at your earliest convenience. I will be happy to answer any questions or queries that you might have.

Looking forward to hearing from you.

_____

Mr V. Pillay
Tel: 0824158932
E-Mail: vasenpi@gpg.gov.za

CONSENT FORM (PARENTS/GUARDIANS):

Parent/Guardian's consent

I, _____ (please print fullname), parent/guardian of

_____,
[Full name of child/ward]

give consent to the following:

1. Videotaping of lessons on mathematics functions in which my child/ward might appear as part of the videotext.

   YES [ ] NO [ ] please tick

2. Copies made of classwork, homework or assessments that my child/ward might produce as part of these lessons.

   YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Appendix D

Categorising and Chunking of Lessons into Evaluative Events

| EVENT | TIMING | Concept | | | | Existence | | | | | | | Judgement of the Notion | | | | | | |

3	03:56 - 04:57	Gradient	3.1	Gradient as a fraction	✔			✔			✔					✔	✔	✔					✔	✔			
	04:57 - 08:48		3.2	Formula for gradient		✔					✔					✔	✔	✔				✔	✔		✔			
4	08:48 - 09:25	Gradient & y-intercept method for sketching straight lines	4.1	Using the y-intercept		✔		✔				✔				✔	✔	✔			✔		✔					
	09:25 - 10:39		4.2	Working with the gradient	✔			✔				✔				✔	✔				✔		✔					
	09:59 - 10:39		4.3	m & the relationship between x & y	✔			✔				✔				✔	✔				✔			✔				
5	12:43 - 13:44	Errors in counting when working with the gradient	5.1	Counting from the y-intercept		✔						✔				✔	✔				✔			✔				
	13:44 - 16:25		5.2	Inclusion of (0;0)		✔						✔				✔	✔				✔			✔				
6	16:25 -17:05	Negative sign in front of a fraction	-	-		✔		✔		✔						✔	✔						✔	✔				
7	20:27 - 20:49 and 21:25 - 22:08	Labelling of graphs	-	-		✔		✔									✔							✔		✔	✔	
8	20:49 - 21:25	Ordering of values in a co-ordinate	-	-				✔		✔	✔					✔	✔						✔	✔				
Lesson 3 (1 Hour)																												
1	3:15 - 3:37	Sketching of graphs	1.1	Labelling		✔		✔								✔											✔	
	3:37 - 4:38		1.2	Choice of method		✔		✔								✔											✔	
2	04:38 - 5:46	Gradient	2.1	As a fraction	✔			✔		✔	✔					✔	✔						✔					
			2.2	Sign of fraction	✔												✔						✔					
3	10:34 - 15:07	Writing an equation in y-form	3.1	Making y the subject of the equation		✔		✔			✔					✔	✔						✔					
	15:07 - 16:20		3.2	Commutative property - addition	✔			✔									✔						✔					
4	30:21 - 30:53	Dual Intercept Method	4.1	Meaning	✔				✔							✔							✔					
	31:53 - 33:07		4.1	Determining the coordinates of the intercepts	✔					✔	✔					✔	✔						✔					

	33:07 - 35:06	Method	4.3	Sketching the graph		✔						✔					✔				✔						
5	35:32 - 36:28	Gradient	5.1	Slope of the line		✔		✔				✔				✔					✔						
	40:21 - 43:19		5.2	Does the line have a gradient?	✔			✔								✔										✔	
Lesson 4 (1hour)																											
1	08:14 - 08:54	The order within a coordinate	-	-		✔		✔				✔				✔							✔				
2	11:20 - 11:34	Perpendicular Lines	2.1	Angle between these lines		✔					✔	✔			✔		✔	✔			✔	✔					
	11:34 - 11:54		2.2	The term perpendicular		✔			✔									✔				✔					
	12:09 - 16:46		2.3	Product of m = -1	✔			✔		✔	✔				✔	✔	✔						✔				
3	29:04 - 29:58	Parallel Lines	3.1	Meaning of parallel	✔			✔							✔	✔					✔	✔					
	29:58 - 36:54		3.2	Relationship between gradients	✔			✔			✔					✔	✔				✔		✔				
4	52:26 -52:39	Writing in y-form	-	-		✔		✔								✔							✔				
Lesson 5 (1hour)																											
1	0:41 - 4:42	Review of lesson 1 to 4	-	-	✔	✔		✔								✔								✔			
2	4:42 - 5:44	Determining equations of straight lines	2.1	Relationship to previous work		✔		✔								✔								✔	✔		
	7:01 - 9:00		2.2	The focus	✔			✔		✔	✔					✔	✔				✔						
3	9:47 - 11:59	Gradient	3.1	$m = \frac{\Delta y}{\Delta x}$	✔			✔			✔					✔	✔					✔	✔				
	11:59 - 15:00		3.2	Order of (x1;y1) & (x2;y2)	✔					✔	✔						✔				✔			✔			
	15:00 - 16:33		3.3	Interchanging x & y		✔		✔								✔							✔	✔			
	17:24 - 18:59		3.4	y is always on top	✔			✔									✔									✔	
	42:43 - 48:19		3.5	// and ⊥lines	✔					✔	✔					✔	✔	✔					✔	✔			
4	20:00 - 23:24	y-intercept	-	-	✔			✔			✔					✔	✔						✔	✔	✔		
5	26:05 - 28:02	Negative gradient	-	-	✔							✔				✔	✔				✔				✔		

6	39:26 - 42:43	Interchanging (x1;y1) & (x2;y2)	-	-	✔			✔								✔	✔				✔								
Lesson 6 (30 min)																													
1	6:57 - 8:31	Determining the equation of a linear function from a graph with the intercepts plotted	1.1	Determining what is given in a sketch		✔						✔					✔				✔								
	8:31 - 8:42		1.2	y-intercept		✔						✔					✔	✔			✔								
	9:06 - 11:37		1.3	Gradient	✔					✔	✔	✔					✔	✔						✔				✔	
2	15:12 - 17:11	The use of formulae	-	-		✔		✔			✔						✔							✔				✔	
3	19:01 - 22:35	Application of linear functions	-	-	✔			✔									✔		✔						✔				
4	22:35 - 24:14	Subtracting from a negative number	-	-	✔					✔							✔						✔	✔					
Lesson 7 (30 min)																													
1	10:12 - 12:30	Functional notation	1.1	Meaning of f(1)	✔			✔		✔	✔				✔	✔	✔						✔						
			1.2	Determining f(1) graphically		✔					✔	✔				✔	✔				✔		✔						
	15:30 - 16:04		1.3	Meaning of f(x)	✔			✔								✔							✔						
	17:58 - 19:34		1.4	Meaning of f(a) = value	✔			✔		✔	✔	✔				✔	✔	✔			✔		✔						
2	13:47 - 14:42	Accuracy of readings	-	-	✔			✔								✔					✔								
				Total Occurrences	36	29	0	43	2	13	28	18	2	0	7	52	41	17	0	0	24	9	34	18	14	6	7
	Tot # notions incl sub-noti		**65**																										
				% of Occurrence	55	45	0	66	3	20	43	28	3	0	11	80	63	26	0	0	37	14	52	28	22	9	11

Appendix E

Letter of approval to conduct study issued by the Gauteng Department of Education



UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Date:	**22 June 2005**
Name of Researcher:	**Pillay Vasen**
Address of Researcher:	**592 Patel Street**
	Actonville
	Benoni, 1501
Telephone Number:	**(011) 4225794**
Fax Number:	**(011) 4226123**
Research Topic:	**An investigation into mathematics for teaching: the kind of mathematical problem-solving teachers' do as they go about their work**
Number and type of schools:	**1 Secondary School**
District/s/HO	**Ekurhuleni East**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. ***The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.***
2. ***The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.***
3. ***A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.***

Office of the Senior Manager – Strategic Policy Research & Development
Room 525, 111 Commissioner Street, Johannesburg, 2001 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0488 Fax: (011) 355-0286

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Senior Manager (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Senior Manager: Strategic Policy Development, Management & Research Coordination with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Senior Manager concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

ALBERT CHANEE
ACTING DIVISIONAL MANAGER: OFSTED

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	11-07-2005

Appendix F

Ethics Clearance issued by the

University of the Witwatersrand

University
of the Witwatersrand,
Johannesburg

Humanities: Education

Private Bag 3, Wits 2050, South Africa • Fax: +27 11 717-3219 • Telephone: +27 11 717-3018 • E-mail: chapmanh@hse.wits.ac.za

Mr V Pillay
592 Patel Street
Actonville
1500 Benoni

STUDENT NUMBER 8710172X
Protocol EC03

29 July 2005

Dear Mr V Pillay

Application for Ethics Clearance Master of Science (Science Education)

I am pleased to inform you that the Ethics Committee in Education has approved your application submitted for ethics clearance for the degree of Master of Science (Science Education).

Your attention is drawn to the following comments:

- grammatical errors in the letter to students, parent and guardian. Also, in the letter to students, it is not clear who is giving consent, whether it is the student, parent or guardian. The Committee recommended that there should be separate letters to the students, parent and guardian;
- two words are being confused autonomy and anonymity on pages 3 and 17 of the research proposal;
- the Committee questions in what way the feedback would be given to teachers and principal without breeching confidentiality?;
- if the results were to be reported in the form of a case study, would the name of the school be disguised? (confidentiality issue).

The Committee has suggested that you discuss the above issues with your supervisor.

Yours sincerely

Mathoto Senamela
Faculty Officer for Faculty Registrar
Faculty of Humanities

cc Chair GSEC
Supervisor
Ethics File
HDethics clearance