The Prediction of Bankruptcy: An Investigation into The Time and Industry Sensitivity of Predictive Models

Master of Commerce (50% Research) in Finance

Tamara Govender [992077]



Supervisor: Dr. Yudhvir Seetharam School of Economics and Finance

Declaration

Name of Candidate:	Tamara Govender				
Person Number:	992077				
Degree:	Master of Commerce				
Supervisor:	Dr. Yudhvir Seetharam				
School:	School of Economics and Finance				
Title of Dissertation: The Prediction of Bankruptcy: An Investigation into The Time and Industry Sensitivity of Predictive Models					
i. I hereby submit my Masters dissertation for examination (circle applicable).					
ii. I confirm that my signed declaration in terms of Rule G9.7 is included in each copy of the thesis/ dissertation.					
iii. I have checked all copies of my thesis/ dissertation and declare that no pages are missing or poorly reproduced.					
iv. I hereby submit a CD/USB containing a PDF version of my submission for examination and nil bound copies.					
Candidate's Signature: T. Govender					
Date: 23 rd April 2021					

The Prediction of Bankruptcy: An Investigation into The Time and Industry Sensitivity of Predictive Models

Acknowledgements

For Ma.

Abstract

The overall objective of this research is to evaluate the predictive ability of the Altman (1968), Ohlson (1980) and Zmijewski (1984) models when evaluating firms for bankruptcy. An industryadjusted and re-estimated version of each of the three models will be assessed to determine if the re-estimation of the models, using a data from South African firms from 1990 to 2020, results in an increased predictive ability as compared to the original models. The results suggest two main challenges faced when using predictive models in industry: the issue of time sensitivity, as well as industry sensitivity. The issue of time sensitivity was resolved post re-estimating each of the original models using the new sample. The re-estimation of each of the three models resulted in significant improvements in predictive ability across industries. The challenge of industry sensitivity was addressed by re-estimating the models using industry-specific samples of data. The findings showcase near perfect predictive accuracy up to five years prior to bankruptcy. The intended contribution of this research is the practical application of the methods and findings which would serve as a guide for risk assessment by lending institutions, and performance benchmarking for firms.

Keywords: Bankruptcy models; Predictive models; Default prediction; Credit risk; Performance benchmarking

Table of	Contents	
DECLAR	ATION	I
ACKNOV	VLEDGEMENTS	II
ABSTRA	СТ	.III
	TABLES	
LIST OF	FIGURES	VII
1. INT	RODUCTION	1
1.1.	BACKGROUND AND OVERVIEW	2
1.2.	PROBLEM STATEMENT	4
1.3.	RESEARCH OBJECTIVES	4
1.4.	FEASIBILITY OF THE STUDY	4
1.5.	PRIMARY HYPOTHESIS I	5
1.6.	SECONDARY HYPOTHESIS I	5
1.7.	TERTIARY HYPOTHESIS I	5
2. LIT	ERATURE REVIEW	6
2.1.	ACCOUNTING-BASED CREDIT SCORING SYSTEMS	7
2.2.	INDUSTRY-ADJUSTED CREDIT SCORING SYSTEMS	8
2.3.	THE RE-ESTIMATION OF CREDIT-SCORING SYSTEMS	9
2.4.	Models used in industry	9
2.5.	DEVELOPING ECONOMIES AND THE USE OF PREDICTIVE MODELS	10
2.6.	SUMMARY OF LITERATURE REVIEW	10
3. DES	CRIPTION OF DATA AND RESEARCH METHODOLOGY	11
3.1.	DETAILED HYPOTHESIS AND THEORETICAL FRAMEWORK	11
3.2.	DATA	11
3.3.	THE ALTMAN MODEL	12
3.4.	THE OHLSON MODEL	13
3.5.	THE ZMIJEWSKI MODEL	14
3.6.	THE INDUSTRY COMPONENT	14
3.7.	THE TIME COMPONENT	16
3.7.1	Statistical assumptions: Multivariate regression	16
3.7.2	P. Statistical assumptions: Logit and Probit regressions	17
3.8.	LIMITATIONS	17
4. RES	ULTS	18

4	.1.	SAMPLE SELECTION	
4	.2.	ORIGINAL MODEL PERFORMANCE	
	4.2.1	. The original Altman model	
	4.2.2	. The original Ohlson model	
	4.2.3	. The original Zmijewski model	
4	.3.	ANALYSIS OF ORIGINAL MODEL PERFORMANCE	23
4	.4.	THE TIME COMPONENT	24
	4.4.1	. The re-estimated Altman model	
	4.4.2	. The re-estimated Ohlson model	
	4.4.3	. The re-estimated Zmijewski model	
4	.5.	ANALYSIS OF RE-ESTIMATED MODEL PERFORMANCE	
	4.5.1	. The re-estimated Altman model	
	4.5.2	. The re-estimated Ohlson model	27
	4.5.3	. The re-estimated Zmijewski model	
	4.5.4	. Performance comparison	
4	.6.	THE INDUSTRY FACTOR	
	4.6.1	. The industry-adjusted Altman model	
	4.6.2	. The industry-adjusted Ohlson model	
	4.6.3	. The industry-adjusted Zmijewski model	
4	.7.	ANALYSIS OF INDUSTRY-ADJUSTED MODEL PERFORMANCE	35
	4.7.1	. Performance comparison	
4	.8.	INDUSTRY SENSITIVITY: IS RE-ESTIMATION PER SECTOR THE SOLUTION?	
	4.8.1	. The re-estimated models per sector	
	4.8.1	.1. Re-estimation using primary sector data	
	4.8.1	.2. Re-estimation using secondary sector data	
	4.8.1	.3. Re-estimation using tertiary sector data	41
	4.8.2	. Analysis of primary sector re-estimations	
	4.8.3	. Performance comparison	
5.	DISC	CUSSION	44
6.	CON	ICLUSION	45
RE	FEREN	NCES	47
API	PENDI	X	50

List of tables

Table 1: Initial sample 19
Table 2: The results derived when the original Altman model is applied to the initial sample 20
Table 3: The results derived when the original Ohlson model is applied to the initial sample 21
Table 4: The results derived when the original Zmijewski model is applied to the initial sample22
Table 5: The results derived when the re-estimated Altman is applied to the initial sample 27
Table 6: The results derived when the re-estimated Ohlson is applied to the initial sample 28
Table 7: The results derived when the re-estimated Zmijewski is applied to the initial sample 28
Table 8: The adjusted-Altman performance after the inclusion of an industry factor
Table 9: The adjusted-Ohlson performance after the inclusion of an industry factor
Table 10: The adjusted-Zmijewski performance after the inclusion of an industry factor
Table 11: Comparison of model performance after re-estimation using data from each sector 42

List of figures

5	
Figure 1: The predictive accuracy per model 1 year prior to bankruptcy	
Figure 2: The predictive accuracy of the Altman model 1 year prior to bankruptcy	
Figure 3: The predictive accuracy of the Ohlson model 1 year prior to bankruptcy	
Figure 4: The predictive accuracy of the Zmijewski model 1 year prior to bankruptcy	
Figure 5: The predictive accuracy per model 1 year prior to bankruptcy	
Figure 6: The predictive accuracy of the Altman model 1 year prior to bankruptcy	
Figure 7: The predictive accuracy per model 2 years prior to bankruptcy	
Figure 8: The predictive accuracy per model 3 years prior to bankruptcy	
Figure 9: The predictive accuracy per model 4 years prior to bankruptcy	
Figure 10: The predictive accuracy per model 5 years prior to bankruptcy	
Figure 11: The predictive accuracy of the Altman model 2 years prior to bankruptcy	
Figure 12: The predictive accuracy of the Altman model 3 years prior to bankruptcy	
Figure 13: The predictive accuracy of the Altman model 4 years prior to bankruptcy	
Figure 14: The predictive accuracy of the Altman model 5 years prior to bankruptcy 53	
Figure 15: The predictive accuracy of the Ohlson model 2 years prior to bankruptcy	
Figure 16: The predictive accuracy of the Ohlson model 3 years prior to bankruptcy	
Figure 17: The predictive accuracy of the Ohlson model 4 years prior to bankruptcy 55	
Figure 18: The predictive accuracy of the Ohlson model 5 years prior to bankruptcy	
Figure 19: The predictive accuracy of the Zmijewski model 2 years prior to bankruptcy	
Figure 20: The predictive accuracy of the Zmijewski model 3 years prior to bankruptcy	
Figure 21: The predictive accuracy of the Zmijewski model 4 years prior to bankruptcy	
Figure 22: The predictive accuracy of the Zmijewski model 5 years prior to bankruptcy 57	
Figure 23: The predictive accuracy per model 2 years prior to bankruptcy	
Figure 24: The predictive accuracy per model 3 years prior to bankruptcy	
Figure 25: The predictive accuracy per model 4 years prior to bankruptcy	
Figure 26: The predictive accuracy per model 5 years prior to bankruptcy	
Figure 27: The predictive accuracy of the Altman model 2 years prior to bankruptcy	
Figure 28: The predictive accuracy of the Altman model 3 years prior to bankruptcy	
Figure 29: The predictive accuracy of the Altman model 4 years prior to bankruptcy	
Figure 30: The predictive accuracy of the Altman model 5 years prior to bankruptcy	

1. Introduction

Perhaps Black Swan¹ævents are not as rare as we believe. When reviewing only the past two decades, eight economically influential Black Swan events can be observed. At present, we are currently descending into yet another financial crisis which was spurred by the emergence of the COVID-19 virus at the close of 2019. Despite the commonality of the above-mentioned financial turbulence, individuals and companies alike must uphold their responsibilities as borrowers. Thus, it is imperative that an uninvolved technique of measuring credit risk be accessible to lenders. In 1968, Edward Altman questioned if a gap between traditional ratio analysis and more rigorous statistical techniques could be bridged in the assessment of firm performance. More than half a century later, the question remains relevant.

This study compares the predictive ability of three accounting-based models, which each employ different methodologies, in order to establish which model is more accurate in the prediction of bankruptcy. The Altman (1968) discriminant-based model, the Ohlson (1980) logit-based model and the Zmijewski (1984) probit-based model will be compared. In addition, this research aims to simplify the application of these models by rendering a more accurate re-estimated, industry-adjusted version of each model that is applicable across industries and eras.

One of the foremost advantages of considering an accounting-based model is that accounting figures are readily accessible from the annual financial reports of firms. Although data in this realm is plentiful, bankruptcy data is not so. This resulted in this study compiling data from publicly traded firms in South Africa across 33 different industries from 1990 to 2020 which are split into three sectors for testing purposes: primary, secondary and tertiary sectors. This study employs data from the South African market in an attempt to fathom whether the above-mentioned three accounting-based models are as accurately applied to a developing country as they were in the results of the original studies which focused exclusively on data from developed countries. The intended contribution of this research is the practical application of the methods and findings which

 $^{^1}$ A Black Swan event is an unpredictable or unforeseen event, typically one with extreme consequences.

would serve as a guide for risk assessment by lending institutions, and performance benchmarking for firms.

1.1. Background and overview

When securing finance for a business entity, the financier typically requests signed financial statements, management accounts and bank statements. At this point, the financier must determine whether the entity can handle the additional financial strain given but a few pieces of information. The credit review process enables the financier to quantify the probability of default of each entity before entering into or maintaining a lending relationship. The three broad categories of bankruptcy prediction methods include accounting, market, and economic-based models. This research focuses on accounting-based models as a tool for bankruptcy prediction, based on the accessibility of annual financial reports.

Given the exponential advancement of technology over the years, it is curious that lenders still place such a substantial reliance on simple ratio analysis during the credit review process. Altman (1968) analyses the quality of ratio analysis and develops a discriminant ratio model which proves to be accurate in predicting bankruptcy among publicly listed firms. Unfortunately, the model is not without fault. One of the major disadvantages of the model is that it can only forecast the likelihood of failure if the company is comparable to its dataset. The author limited his dataset to firms specific to the manufacturing industry, which severely constrained the application of the model across industries. In an attempt to overcome this constraint, this research aims to explore an industry-adjusted version of Altman's model which can be applied to firms across industries. In addition to the industry-adjustment, the following study considers the effect of re-estimation on the predictive accuracy of the model. Singh and Mishra (2016) compare original and re-estimated models to explore the sensitivity of these models towards the change in time periods and financial conditions. In their study, the authors re-estimate accounting-based bankruptcy models and find that the overall predictive accuracy of the models improved. Therefore, the re-estimation of bankruptcy predication models may prove to be imperative in the case of changing economic environments over the eras.

There are at least four methodological approaches to developing multivariate credit-scoring systems: (i) the linear probability model, (ii) the logit model, (iii) the probit model, and (iv) the discriminant analysis model. As such, this study aims to compare the predictive ability of three accounting-based models, which each employ different methodologies, in order to establish which model is more accurate in the prediction of bankruptcy. The Altman (1968) discriminant-based model, the Ohlson (1980) logit-based model and the Zmijewski (1984) probit-based model will be compared. In addition, this research aims to simplify the application of these models by rendering a more accurate re-estimated, industry-adjusted version of each model that is applicable across industries and eras.

The extensive failure of borrowers to meet their debt obligations ultimately gives rise to the breakdown of the lender. So then, how do financial institutions such as banks and other various lenders predict that a substantial group of their borrowers will be unable to meet their debt obligations? How do these lenders better assess the probability of default in order to mitigate the consequences of this risk becoming an actuality? Is it possible to more accurately measure the probability of default using information that is readily available? This research seeks to explore these real-world questions by avoiding the consideration of unrealistic, complex methods that pose a threat to the availability of the data necessary to carry out the credit assessment.

Although a plethora studies have been published since, these studies largely consist of the exploration of accounting-based bankruptcy prediction models in the context of developed economies. One of the earliest applications of Altman's model in the context of developing economies can be found in the studies of Lubawa and Louangrath (2016) whom applied the model to a Tanzanian dataset, and Sajjan (2016) who applied the model to an Indian dataset. With the rarity of Black Swan events in question and the current COVID-19 pandemic in play, it is imperative that an uninvolved technique of measuring credit risk be accessible to lenders. Thus, ratios as predictors of impending bankruptcy remain a candidate for further study.

1.2. Problem statement

Will the re-estimation of accounting-based predictive models result in an increase of predictive accuracy?

1.3. Research objectives

- To evaluate the predictive ability of the Altman (1968), Ohlson (1980) and Zmijewski (1984) models when evaluating firms for bankruptcy.
- To determine if the re-estimation of the models results in an increased predictive ability.
- To establish if an industry-adjusted version of each of the three models is able to predict bankruptcy more accurately than the *original* models.
- To ascertain which re-estimated and industry-adjusted model is able to best predict bankruptcy.

1.4. Feasibility of the study

This study compares the predictive ability of three accounting-based models, which each employ different methodologies, in order to establish which model is more accurate in the prediction of bankruptcy. The Altman (1968) discriminant-based model, the Ohlson (1980) logit-based model and the Zmijewski (1984) probit-based model is compared. In addition, this research simplifies the application of bankruptcy models by rendering a more accurate re-estimated, industry-adjusted version that is applicable across industries and eras.

One of the foremost advantages of considering an accounting-based model is that accounting figures are readily accessible from the annual financial reports of firms. Although data in this realm is plentiful, bankruptcy data is not so. This resulted in this study compiling data from publicly traded firms in South Africa across 33 different industries from 1990 to 2020 which are split into three sectors for testing purposes: primary, secondary and tertiary sectors. This study employs data from the South African market in an attempt to fathom whether the above-mentioned three accounting-based models are as accurately applied to a developing country as they were in the results of the original studies which focused exclusively on data from developed countries. The intended contribution of this research is the practical application of the methods and findings which would serve as a guide for risk assessment by lending institutions, and performance benchmarking for firms.

1.5. Primary hypothesis I

Objective: To evaluate the predictive ability of the Altman (1968), Ohlson (1980) and Zmijewski (1984) models when evaluating firms for bankruptcy.

 $H_{0,A}$ = Each of the above models does not provide a statistically accurate predictor of future bankruptcy.

 $H_{1,A}$: Each of the above models provides a statistically accurate predictor of future bankruptcy.

1.6. Secondary hypothesis I

Objective: To establish if an industry-adjusted version of each of the three models is able to predict bankruptcy more accurately than the original models.

 $H_{0,A}$: The industry-adjusted version of each model does not provide a statistically accurate predictor of future bankruptcy.

 $H_{1,A}$: The industry-adjusted version of each model provides a statistically accurate predictor of future bankruptcy.

1.7. Tertiary hypothesis I

Objective: To determine if the re-estimation of the models results in an increased predictive ability as compared to the original models. After establishing the re-estimated models with new coefficients per time period, the performance of the newly re-estimated models will be compared to that of the original models. In order to test this over a long span of time, we will be employing a time-varying approach.

 $H_{0,A}$: Each of the re-estimated models does not provide a statistically superior predictor of future bankruptcy.

 $H_{1,A}$ = Each of the re-estimated models provides a statistically superior predictor of future bankruptcy.

Altman and Saunders (1998) propose some forces which highlight the importance and benefits of studies on the topic of credit risk. These forces include a global increase in the number of bankruptcies and the more competitive margins on loans. Further to these forces, this research simplifies the development of internal credit rating systems. Waagepetersen (2010) describes the increased use of internal credit rating systems in small- and medium- sized banks. No clear consensus has been reached regarding the best model to use during the development of an internal credit rating system. Although quantitative, qualitative or a hybrid combination of the two methods can be used when developing such a model, this research aims to consider an environment in which the qualitative contributions of experts are not readily available.

The opinions of credit experts who assign weights to characteristics that are believed to be influential in an entity's (or individual's) credit rating are key in the understanding and management of credit risk. However, Taffler and Sommerville (1995) question whether this element of human judgement may be prone to bias and potentially affect the consistency of ratings across an institution. This research does not aim to contribute to this hypothesis. Rather, the focus of this paper is aligned strictly to the quantitative aspect of credit risk modelling which can be explored using easily accessible information such as the fundamental characteristics of firms.

The remaining sections of this research are as follows: Section 2 reviews the literature surrounding the prediction of bankruptcy, Section 3 provides an overview of the data and methodology, and lastly, Section 4 presents the results and Section 5 the conclusion of the paper.

2. Literature review

Following the first two decades post the publishing of Altman's seminal article on the predication of corporate bankruptcy in 1968, the evolution of credit risk measurement accelerated rapidly. The following literature review offers a timeline of the development of the accounting-based credit-scoring systems pertinent to this research. In addition, the literature which spurred the inclusion of the industry-adjustment and re-estimation elements are explained.

2.1. Accounting-based credit scoring systems

In univariate accounting-based credit scoring systems, the financial institution decision-maker compares various key accounting ratios of potential borrowers with industry or group norms (Altman & Saunders, 1998). When using multivariate models, the key accounting variables are combined and weighted to produce either a credit risk score or a probability of default measure. If the credit risk score, or probability, attains a value above a critical benchmark, a loan applicant is either rejected or subjected to increased scrutiny. There are at least four methodological approaches to developing multivariate credit-scoring systems: (i) the linear probability model, (ii) the logit model, (iii) the probit model, and (iv) the discriminant analysis model. This research considers three models which employ three different methodological approaches in order to establish which model has the potential to predict bankruptcy most accurately.

The first model which this research considers is the seminal article written by Altman in 1986. He developed the first multivariate discriminant model for default prediction for U.S. companies. The model makes use of five financial ratios to predict the bankruptcy of firms. The author found that the model was able to predict bankruptcy one year prior, with a rate of accuracy of 95%. More recent studies have further confirmed the predictive accuracy of the model (Hayes, Hodge & Hughes, 2010). Despite the success of the model, it is not without fault. One of the major disadvantages of the model is that it can only forecast the likelihood of failure if the company is comparable to its dataset. Altman (1986) limited his dataset to firms specific to the manufacturing industry, which severely constrains the application of the model across industries. To overcome this constraint, this research explores an industry-adjusted version of Altman's model which can be applied to firms across industries.

Later, Martin (1977) presented the first application of logit analysis to the bank early warning² problem. The author found that his model tended to overpredict failure in periods of prosperity and lower loan loss experience. In attempt to refine both Altman's multivariate offering as well as

² Barbu, Dardac and Boitan (2009) define early warning systems as the mechanisms that analyse and transform information held by financial institutions into signals concerning the possibility of an imminent banking crisis.

Martin's initial rendition of the logit approach, Ohlson (1980) sought to employ a logit technique with less restrictive assumptions than those taken in the multivariate discriminant approach to model bankruptcy. Ohlson (1980)'s model uses nine predictive variables which measures firms' size, leverage, liquidity, and performance. Although Ohlson's model produces results more accurate than Altman's original model, the former was highly sensitive to both endogenous and exogenous factors which impacted the accuracy of its predictions.

Lastly, the third model of interest in this research is Zmijewski's (1984) probit approach. The author believed that bankruptcy scoring models oversampled distressed firms and favoured situations with more complete data. He uses financial ratios measuring firm performance, leverage and liquidity. The ratios were selected on the basis of their performance in previous studies, with emphasis largely placed on the leverage factor.

This study compares the predictive ability of the above mentioned three accounting-based models in order to establish which model is more accurate in the prediction of bankruptcy.

2.2. Industry-adjusted credit scoring systems

Up until this point, the accounting-based models mentioned in the previous section were applicable exclusively to the industry with which the study was conducted. However, it was found that industry-relative accounting ratios, rather than firm-specific accounting ratios, are better predicators of corporate bankruptcy (Platt & Platt, 1991; Izan, 1984).

If extreme values of financials ratios are industry specific, then bankruptcy prediction models should not compare unadjusted ratios across companies unless the industry is held constant (Platt & Platt, 1991, p. 1184).

The potential for industry-relative financial ratios to improve predictive ability motivated the introduction of the industry-adjusted component of this research. Lev (1969) provided an industry-relative framework which argues that target financial ratios exist. This principle was employed in research conducted by Platt and Platt (1991). The authors suggest that target ratios are assumed to equal the average ratio value across companies in an industry. The industry-relative modification

then adjusts company ratios by dividing them by industry average ratios. Following the industryrelative modification, it is asserted that ratios can be compared across companies without industryspecific bias. Thus, by adjusting the ratios according to their industry, the following results are intended:

- a) more stable financial ratios,
- b) more stable coefficient estimates over tie, and
- c) less disparity between *ex ante* and *ex post* forecast results.

2.3. The re-estimation of credit-scoring systems

Recently there have been several studies which question the effect of re-estimating models using more contemporaneous data. There has been consensus in that the re-estimation of predictive models results in the overall predictive accuracy of the models improving (Timmermans, 2014; Singh & Mishra, 2016; Karas & Reznakova, 2017). The re-estimation of the models accounts for changing economic environments over the eras. Singh and Mishra (2016) find that the coefficients of the models are sensitive to both time periods and financial condition. The predictive accuracy of the models increases when more recent data is used in the estimation samples. The changes in the financial environment leads to changes in the relation between financial distress and financial ratios. This also alters the comparative ability of the ratios to predict default. In order to contribute to the body of literature which highlights the importance of model re-estimation, this research aims to quantify the extent to which this improvement exists.

2.4. Models used in industry

Presently, a wide variety of models are used to quantify credit risk in industry. In addition to the Altman (1968) model, the models proposed by Jarrow and Turnbull (1995) and Merton (1974) are among those models commonly used today. These additional models are reduced-form and structural-form models respectively, which focus on the modelling of the probability of default of an entity or investment using market-related information. While market-based models such as these exist, no consensus has been reached on whether they are able to outperform accounting-based models. Although, it is true that the information needed to compile a market-based model would be more onerous to source than the information needed to compile an accounting-based model. With one of the core focuses of this research being on the accessibility of information used

to derive the model, and the current disagreement on whether market-based or accounting-based models should be used, only accounting-based models will be considered.

2.5. Developing economies and the use of predictive models

Developing countries possess traits which can be observed in their higher economic growth rates and lower gross domestic product (GDP) per capita than developed countries (Prabhakar, Kaur & Erokhin, 2019). The Altman (1968), Ohlson (1980) and Zmijewski (1984) models were all originated using financial data from American firms, a country which has largely been classified as developed. Considering the differing traits of emerging and developed economies, one would expect the performance of predictive models to vary depending on the level of sophistication of each economy.

Cassim and Swanepoel (2021) test an adjusted-Altman model which was modified to increase its versatility across industries and economies. The authors test the emerging market score (EMS) model using financial data from South Africa firms and conclude that the EMS model is better adept to predicting financial distress in the South African emerging market.

2.6. Summary of literature review

Literature supports the choice to compare the predictive ability of accounting-based models in order to establish which model is more accurate in the prediction of bankruptcy. Studies including Platt and Platt (1991) and Izan (1984) have explored the enhancement of accounting-based models using an industry-adjusted component, having asserted that the enhancement leads to the creation of a more stable model. Several studies including those of Timmermans (2014), Singh and Mishra (2016) and Karas and Reznakova (2017) agree that the re-estimation of predictive models results in the overall predictive accuracy of the models improving. Although research on this topic has been conducted in developing countries in studies such as Lubawa and Louangrath (2016) whom applied the model to a Tanzanian dataset, and Sajjan (2016) who applied the model to an Indian dataset, no such research has been conducted in a South African context.

3. Description of data and research methodology

3.1. Detailed hypothesis and theoretical framework

The overall objective of this research is to evaluate the predictive ability of the Altman (1968), Ohlson (1980) and Zmijewski (1984) models when evaluating firms for bankruptcy. An industryadjusted and re-estimated version of each of the three models will be assessed to determine if the re-estimation of the models results in an increased predictive ability as compared to the original models. After establishing the re-estimated models with new coefficients per time period, the performance of the newly re-estimated models will be compared to that of the original models. It is hypothesised that the re-estimated, industry-adjusted models will predict bankruptcy more accurately than the original models.

3.2. Data

The data that will be used in this study consist of annual figures used in the calculation of the following accounting ratios:

- Working capital
- Total assets
- Retained earnings
- Earnings before interest and tax
- Market value of equity
- Total liabilities
- Sales
- Cash flow
- Net fixed assets
- Total debt
- Short-term debt or current liabilities
- Sales growth (the sales figure in year in the previous year is required)
- Current assets
- Net income
- Funds from operations

- Industry output (this figure will be derived from data retrieved from Bloomberg per country)
- Gross National Product price index level (this figure will be derived from data retrieved from Bloomberg per country)

Data for the abovementioned figures will be retrieved from the Bloomberg terminal for companies trading publicly in South Africa, per industry. The dataset will compromise of both firms which have been declared bankrupt and firms which remain a going-concern. Data will be collected from 1990 to 2020.

3.3. The Altman model

The Altman Z-score is based on five financial ratios that can be calculated using data found on a company's annual financial report. It uses profitability, leverage, liquidity, solvency, and activity ratios to make the predication. The following is a breakdown of the *original* model published in 1968.

$$z_score = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$
(1)

where:

 X_1 = working capital to total assets X_2 = retained earnings to total assets X_3 = earnings before interest and tax to total assets X_4 = market value of equity to total liabilities X_5 = sales to total assets

Z-scores exist in zones of discrimination which indicate the likelihood of a firm going bankrupt. A z-score lower than 1.8 indicates that bankruptcy is likely, while scores greater than 3.0 indicate bankruptcy is unlikely to occur in the next two years. Companies that have a z-score between 1.8 and 3.0 are in the *grey zone*, and bankruptcy is as likely as not.

3.4. The Ohlson model

The Ohlson O-score is based on a linear factor model. The model uses nine different ratios that can be obtained from the entity's annual financial report. Two of the factors, which are dummy factors, are not entirely necessary as their impact on the formula is typically zero. The following is the formula for the *original* model.

$$o_score = -1.32 - 0.407 \log\left(\frac{TA}{GNP}\right) + 6.03 \frac{TL}{TA} - 1.43 \frac{WC}{TA} + 0.0757 \frac{CL}{CA}$$
(2)
$$-1.72X - 2.37 \frac{NI}{TA} - 1.83 \frac{FFO}{TL} + 0.285Y - 0.521 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$

where:

ΤA = total assets GNP = Gross National Product price index level; this variable is used to adjust total assets for inflationary changes = total liabilities TLWC = working capital CL = current liabilities СА = current assets Χ =1 if TL > TA, 0 otherwise NI = net income FFO = funds from operations Y = 1 if there was a net loss for the last two years. 0 otherwise

Once a value is obtained using equation 3, the O-score needs to be converted into a probability of default.

$$p(failure) = \frac{e^{0-score}}{1+e^{0-score}}$$
(3)

3.5. The Zmijewski model

The Zmijewski model uses a probit analysis to predict an entity's likelihood of bankruptcy within the following two years. The *original* model makes use of three ratios which can be easily obtained from annual financial reports.

$$zmijewski_score = -4.336 - 4.513\frac{NI}{TA} + 5.679\frac{TL}{TA} + 0.004\frac{CA}{CL}$$
(4)

where:

NI	= net income
TA	= total assets
TL	= total liabilities
CA	= current assets
CL	= current liabilities

A Zmijewski-score of less than 0.5 represents a high probability of default.

3.6. The industry component

Platt and Platt (1991) note that the industry-adjusted model includes the additional variable of the average ratio value of the specific industry in question.

Unadjusted model:

$$b_i = f(ratio_i, e_{1a}) \tag{5a}$$

where:

b = a qualitative variable describing the state of bankruptcy or non-bankruptcy
 i = firm (1, ..., n)
 e = a random error term

Adjusted model:

$$b_i = f(ratio_i/ratio_j, e_{1b})$$
(5b)

where:

j = industry in which firm *i* operates

The data will be processed using both the unadjusted model and adjusted model in order to allow for a comparison of the two.

Generalised model:

$$adjusted_z score = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 + 1.0X_6 + 1.0X_7$$
(6)

where:

<i>X</i> ₁	= cash flow to sales
<i>X</i> ₂	= net fixed assets to total assets
<i>X</i> ₃	= total debt to total assets
X_4	= short-term debt to total debt
X_5	= sales growth
X_6	= industry output * cash flow to sales
X_7	= industry output * total debt to total assets

A nonlinear maximum-likelihood estimation procedure will be used to obtain parameter estimates for both specifications of the generalised logit model shown above.

$$P_i = 1/[1 + exp - (b_0 + b_1 X_{i1} + b_2 X_{i2} + \dots + b_n X_{in})]$$
(7)

where:

 P_i = probability of failure of the i^{th} firm X_{ij} $= j^{th}$ ratio of the i^{th} firm b_j = estimated coefficient

3.7. The time component

There has been consensus in that the re-estimation of predictive models results in the overall predictive accuracy of the models improving (Timmermans, 2014; Singh & Mishra, 2016; Karas & Reznakova, 2017). Applying this methodology will allow for the re-estimation of the coefficients using a more contemporary sample. In order to re-estimate the models using a more relevant sample, the data will be processed using SAS software. The following extracts were taken from the SAS/STAT® 14.1 User's Guide and detail the methodology applied.

3.7.1. Statistical assumptions: Multivariate regression

The basic statistical assumption underlying the least squares approach to general linear modeling is that the observed values of each dependent variable can be written as the sum of two parts: a fixed component $x'\beta$, which is a linear function of the independent coefficients, and a random noise, or error, component ϵ :

$$y = x'\beta + \epsilon \tag{7}$$

Further, the errors for different observations are assumed to be uncorrelated with identical variances. Thus, this model can be written

$$E(Y) = X\beta, \quad VAR(Y) = \sigma^2 I$$
 (8)

where Y is the vector of dependent variable values, X is the matrix of independent coefficients, I is the identity matrix, and $\sigma^2 =$ is the common variance for the errors. For multiple dependent variables, the model is similar except that the errors for different dependent variables within the same observation are not assumed to be uncorrelated. This yields a multivariate linear model of the form

$$E(Y) = XB, \quad VAR(vec(Y)) = \sum \otimes I$$
 (9)

where *Y* and *B* are now matrices, with one column for each dependent variable, vec(Y) strings *Y* out by rows, and \otimes indicates the Kronecker matrix product.

Under the assumptions thus far discussed, the least squares approach provides estimates of the linear parameters that are unbiased and have minimum variance among linear estimators. Under

the further assumption that the errors have a normal (or Gaussian) distribution, the least squares estimates are the maximum likelihood estimates and their distribution is known. All of the significance levels ("p values") and confidence limits calculated by the GLM procedure require this assumption of normality in order to be exactly valid, although they are good approximations in many other cases.

3.7.2. Statistical assumptions: Logit and Probit regressions

Logistic regression analysis is often used to investigate the relationship between these discrete responses and a set of explanatory variables. For binary response models, the response, Y, of an individual or an experimental unit can take on one of two possible values, denoted for convenience by 1 and 2. Suppose x is a vector of explanatory variables and $\pi = \Pr(Y = 1|x)$ is the response probability to be modeled. The linear logistic model has the form

$$logit(\pi) = \log\left(\frac{\pi}{1-\pi}\right) = \alpha + \beta' x \tag{10}$$

where α is the intercept parameter and $\beta = (\beta_1, ..., \beta_s)'$ is the vector of s slope parameters. Notice that the LOGISTIC procedure, by default, models the probability of the *lower* response levels.

3.8. Limitations

One evident limitation of this study surrounds the use of accounting-based data. It is true that accounting information may be biased, and accounts may be manipulated. An additional limitation is the scarcity of bankruptcy data. In order to address this constraint, the industries have been segmented according to their sector. The primary sector consists of firms concerned with the extraction of raw materials such as mining or agriculture (Pettinger, 2021). The secondary sector consists of firms concerned with producing finished goods such as firms in the construction and manufacturing industries. Lastly, the tertiary sector consists of firms concerned with offering intangible goods and services to consumers such as retail and banking. Despite the aforementioned limitations, this research aims to contribute a simplified credit risk management tool with a predictive ability which remains largely unaffected by changes in economic climate, which can be applied across industries.

4. Results

4.1. Sample Selection

The initial sample is composed of 80 companies which were split into two groups of 40 firms each, as per the methodology used in Altman (1968). The bankrupt group (1) are composed of 40 firms that can be classified as part of the primary, secondary or tertiary sector. Recognising that this group is not completely homogeneous, due to industry and size differences, a careful selection of non-bankrupt firms was attempted. The going-concern group (2) consists of 40 firms, chosen from 180 firms which were going-concerns, based on the industries and sizes used in group 1. Firms in group 2 were still in existence in 2019 as some firms did not have 2020 data released when the data for this study was collected.

The following industries were excluded from the sample due to data constraints related to the availability of bankruptcy data within each industry: Aerospace and Defense, Automobiles and Parts, Electricity, Fixed Line Telecommunications, Food and Drug Retailers, Health Care Equipment and Services, Household Goods and Home Construction, Life Insurance, Media, Mobile Telecommunications, Nonlife Insurance, Pharmaceuticals and Biotechnology, Technology Hardware and Equipment, and Tobacco.

The following table includes a list of the included industries and describes the segmentation per sector. The total number of bankruptcies and firms per sector are 40 and 220 respectively. Of the 220 firms, 180 firms are going-concerns from which 40 firms were selected based on firm size in order to represent the going-concern sample.

Industry	Number of Firms Per Industry	Sector	Bankruptcies Per Sector	Firms Per Sector	
Food Producers	14				
Industrial Metals & Mining	14	Primary	7	43	
Mining	15				
Chemicals	8				
Construction & Materials	15			32	
Forestry & Paper	3	Secondary	10		
Oil & Gas Producers	5	_ Secondary	10		
Oil Equipment, Services & Distribution	1				
Electronic & Electrical Equipment	4				
General Industrials	8				
Financial Services	23		23	145	
General Retailers	21	Toutions			
Industrial Transportation	5				
Industrial Engineering	3				
Personal Goods	4	Tertiary	25	143	
Real Estate Investment & Services	12				
Real Estate Investment Trusts	24				
Software & Computer Services	8				
Support Services	22				
Travel & Leisure	11				
Total	40	220			

Table 1: Initial sample

4.2. Original model performance

In order to establish a benchmark for the comparison of the accuracy of each model, the three original models are applied to the initial sample. The section is concluded with graphical summary of the results.

4.2.1. The original Altman model

The predictive accuracy is a measure of the mean number of correctly predicted events. Event 1 would be the bankruptcy of a firm, whereas event 2 would be the firm continuing as a going-concern.

As expected, the Altman model, described in equation 1, performs best when applied across the secondary sector because the original model was derived from a sample of manufacturing firms. The reason for including the primary and tertiary industries in this application of the model is to establish a benchmark so that the performance of the model can be analysed after the addition of the industry and re-estimation elements.

$$z_score = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$
(1)

		Predictive	Predictive	Predictive	Predictive	Predictive
Group	Sector	Accuracy 1	Accuracy 2	Accuracy 3	Accuracy 4	Accuracy 5
		Year Prior	Years Prior	Years Prior	Years Prior	Years Prior
	Primary	71.43%	71.43%	57.14%	42.86%	12.50%
1	Secondary	94.44%	88.89%	61.11%	83.33%	87.50%
	Tertiary	58.33%	56.82%	52.50%	47.37%	47.50%
2	Primary	43.75%	50.00%	57.14%	64.29%	50.00%
	Secondary	36.36%	45.45%	54.54%	54.54%	68.18%
	Tertiary	23.81%	45.24%	57.14%	61.90%	65.00%

Table 2: The results derived when the original Altman model is applied to the initial sample

The results indicate the occurrence of both Type 1 and Type 2 errors ³ in both the bankrupt and going-concern groups. The predictive accuracy in the first year prior to the prediction of bankruptcy is the most accurate, as in Altman's original paper.

4.2.2. The original Ohlson model

When applying the Ohlson model, as seen in equation 2, it was found that Type 2 errors were prevalent in both group 1 and 2. Additionally, the model more accurately predicted bankruptcies which occurred in the secondary sector as seen in the previously discussed Altman model results. It can also be noted that the Ohlson model performed better across sectors when compared to the Altman model.

$$o_score = -1.32 - 0.407 \log\left(\frac{TA}{GNP}\right) + 6.03 \frac{TL}{TA} - 1.43 \frac{WC}{TA} + 0.0757 \frac{CL}{CA}$$
(2)
$$- 1.72X - 2.37 \frac{NI}{TA} - 1.83 \frac{FFO}{TL} + 0.285Y - 0.521 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$

		Predictive	Predictive	Predictive	Predictive	Predictive
Group	Sector	Accuracy 1	Accuracy 2	Accuracy 3	Accuracy 4	Accuracy 5
		Year Prior	Years Prior	Years Prior	Years Prior	Years Prior
	Primary	57.14%	42.86%	42.86%	57.14%	25.00%
1	Secondary	100.00%	88.89%	44.44%	88.89%	75.00%
	Tertiary	54.17%	72.72%	70.00%	42.11%	50.00%
2	Primary	87.50%	62.50%	57.14%	57.14%	71.43%
	Secondary	72.72%	54.54%	54.54%	54.54%	63.63%
	Tertiary	76.19%	57.14%	76.19%	90.48%	90.00%

Table 3: The results derived when the original Ohlson model is applied to the initial sample

³ Bhandari (2021) describes Type I statistical errors as a false positive conclusion and Type II errors as a false negative conclusion.

A distinct advantage when using this model as compared to the Altman model is the ease of interpretation of the output of the model. The Ohlson model outputs a probability of default which is easily comparable and interpretable. When using the Altman model, a cutoff score is necessary to determine whether a firm is classified as bankrupt using a point system. If the Altman model were to be re-estimated, establishing a new cutoff score is necessary. An accurate cutoff score is pertinent to the predictive accuracy of the model.

4.2.3. The original Zmijewski model

When considering the results of the original Zmijewski model in equation 4, the increased prevalence of Type 1 and 2 errors is clear. Group 2 presents the most type 2 errors. The model overpredicts bankruptcy which makes the predictive accuracy of the group 1 findings unimpressive when considered in context. Further analysis will follow in the next section in graphical form to more easily illustrate the findings in these sections.

$$zmijewski_score = -4.336 - 4.513\frac{NI}{TA} + 5.679\frac{TL}{TA} + 0.004\frac{CA}{CL}$$
(4)

Group	Sector	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
	Primary	85.71%	71.43%	85.71%	85.71%	100.00%
1	Secondary	55.56%	77.78%	88.89%	55.56%	75.00%
	Tertiary	83.33%	86.36%	90.00%	84.21%	90.00%
2	Primary	0.00%	12.50%	0.00%	0.00%	0.00%
	Secondary	36.36%	9.09%	9.09%	18.18%	0.00%
	Tertiary	23.81%	4.76%	4.76%	9.52%	0.00%

Table 4: The results derived when the original Zmijewski model is applied to the initial sample

4.3. Analysis of original model performance

The strengths and weakness of each model can be observed in the following graph. When the original models are applied to the sample, the obvious flaws of each model are easily detectable. Firstly, the models are extremely sensitive to industry type. The predictive accuracy of each of the three models is vastly improved when applying the model to firms in the secondary, or manufacturing, sector. Secondly, the prevalence of both Type 1 and Type 2 errors is great. Thirdly, the models seem to be sensitive to the time period over which they are being used for the prediction. Considering the above main three difficulties when applying these models, the following sections will attempt to bridge the gap between these issues.

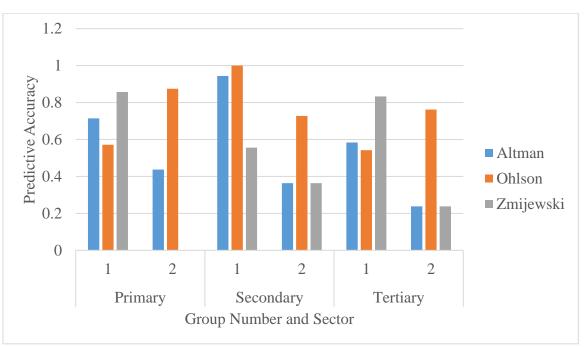


Figure 1: The predictive accuracy per model 1 year prior to bankruptcy

The above figure details the predictive success of the original Altman model across all three sectors. Although the Zmijewski model better predicts bankruptcies in the primary industry in the first year prior to the bankruptcy, the occurrence of Type 1 errors is significant making the predictive accuracy of the bankrupt group less impressive. Again, the same results can be seen in the tertiary group. In conclusion, the original Altman model most accurately distinguishes between bankrupt and non-bankrupt firms across all three sectors in the first year prior to bankruptcy. A similar conclusion is drawn when considering the following figures which detail the predictive accuracy

of each model within the first five years prior to bankruptcy (refer to the appendix for figures 7 to 10 which outline the predictive accuracy per model up to 5 years prior to bankruptcy).

4.4. The time component

In the following sections, the models were regressed according to their original methodology using the new sample with 40 bankrupt firms and 40 going-concern firms across the three industries. The discriminant-based, logit-based and probit-based regressions were performed in SAS. Unfortunately, this resulted in different outputs per model. In some cases, the Somers' D value is used to determine model accuracy, whereas in others the $c value^4$ is used. Both statistics indicate predictive accuracy and can be used in conjunction when determining the fit of the model. The time sensitivity of the models is considered in this section and will be addressed by re-estimating the models using more a more recent data sample in order to establish if this improves predictive accuracy across the three sectors.

4.4.1. The re-estimated Altman model

The re-estimated Altman model yields new coefficients as follows. All variables were statistically significant at the 1% level with the exception of X_1 , the ratio of working capital to total assets, which was statistically insignificant. Despite this, the overall model is statistically significant at the 1% level with an R-squared value of 72.2%. The results of the predictive accuracy will follow in the last sub-section of the time component section.

$$recal_{z}score = 0.0627X_{1} - 0.0590X_{2} + 0.3755X_{3} + 0.0490X_{4} + 0.5122X_{5}$$
(11)

where:

X_1	= working capital to total assets
<i>X</i> ₂	= retained earnings to total assets
<i>X</i> ₃	= earnings before interest and tax to total assets
X_4	= market value of equity to total liabilities
X_5	= sales to total assets

⁴ Values above 0.5 suggest some predictive value, but somewhat weak predictive models may generate c statistics in the 0.75 range (Hermansen, 2008, p. 9).

4.4.2. The re-estimated Ohlson model

The model was re-estimated using the logit methodology as per Ohlson's original paper. Despite the model yielding a Somers' D value of 70.5%, only three of the variables in the model were deemed statistically significant at the 1% level, with the remainder of the variables considered statistically insignificant altogether. It is possible that this outcome was influenced by the quasi-complete separation of data points detected when considering the model convergence status. The model was thus shown based on the last maximum likelihood iteration.

recal_o_score

$$= -9.7352 + 1.0045 \log\left(\frac{TA}{GNP}\right) + 3.6052 \frac{TL}{TA} + 2.7685 \frac{WC}{TA}$$
$$- 0.4637 \frac{CL}{CA} - 12.5161X - 0.4170 \frac{NI}{TA} - 0.9617 \frac{FFO}{TL} + 0.4626Y$$
$$+ 0.1653 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$

(12)

where:

ΤA = total assets GNP = Gross National Product price index level; this variable is used to adjust total assets for inflationary changes TL= total liabilities WC = working capital CL = current liabilities CA = current assets Χ =1 if TL > TA, 0 otherwise NI = net income FFO = funds from operations Y = 1 if there was a net loss for the last two years. 0 otherwise

4.4.3. The re-estimated Zmijewski model

The model was re-estimated based on the probit methodology used in the original paper by Zmijewski. The model convergence status confirms that the convergence criterion were satisfied

prior to the coefficient estimation. Despite this, only the intercept was statistically significant at the 1% level. The ratio of total liabilities to total assets presented a coefficient that was statistically significant at the 5% level, with the remaining variables presenting as statistically insignificant altogether. The Somers' D value for the model was 44.5% indicating the below average predictive ability of the model.

$$recal_{zmijewski_score} = -1.6470 - 0.6498 \frac{NI}{TA} + 0.8192 \frac{TL}{TA} + 0.0604 \frac{CA}{CL}$$
(13)

where:

NI	= net income
TA	= total assets
TL	= total liabilities
CA	= current assets
CL	= current liabilities

The following section outlines the performance of the abovementioned three re-estimated models.

4.5. Analysis of re-estimated model performance

4.5.1. The re-estimated Altman model

As mentioned in a previous section, a new cutoff must be established when re-estimating the Altman model. This is also true of the Zmijewski model. The table below illustrates the increase in predictive accuracy of the newly re-estimated Altman model. The predictive accuracy is also greatly improved in all five years prior to the bankruptcy event. A graphical representation of these results in comparison to the original model performance will follow in the summary section ahead.

Group	Sector	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
1	Primary	100.00%	100.00%	100.00%	83.33%	50.00%
	Secondary	100.00%	100.00%	100.00%	100.00%	100.00%
	Tertiary	72.72%	70.00%	88.89%	82.35%	83.33%
2	Primary	100.00%	100.00%	100.00%	100.00%	N/A
	Secondary	100.00%	100.00%	100.00%	100.00%	100.00%
	Tertiary	100.00%	100.00%	100.00%	100.00%	100.00%

Table 5: The results derived when the re-estimated Altman is applied to the initial sample

*Altman cutoff criteria adjusted: if z_score > 1 then "Bankruptcy Likely"

4.5.2. The re-estimated Ohlson model

The table below illustrates the performance of the newly calibrated Ohlson model. Unlike the overall predictive improvement found in the above Altman model, the Ohlson model illustrates an overall significant decline in predictive accuracy across all three segments in both group 1 and group 2 of the sample. Furthermore, the predictive accuracy over the five years prior to bankruptcy is also severely compromised post the re-estimation.

Group	Sector	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
	Primary	33.33%	66.67%	50.00%	50.00%	75.00%
1	Secondary	0.00%	14.29%	57.14%	14.29%	16.67%
	Tertiary	50.00%	30.00%	27.78%	52.94%	44.44%
	Primary	100.00%	0.00%	100.00%	0.00%	N/A
2	Secondary	0.00%	0.00%	50.00%	0.00%	50.00%
	Tertiary	0.00%	0.00%	50.00%	100.00%	100.00%

Table 6: The results derived when the re-estimated Ohlson is applied to the initial sample

4.5.3. The re-estimated Zmijewski model

Findings similar to those of the newly re-estimated Altman can be seen in the table below. The Zmijewski model illustrates an overall predictive improvement across all three segments for both group 1 and group 2 of the sample. The reduction of Type 1 errors is evident in the below findings, deeming the re-estimation of this model a definite improvement in its predictive ability of both bankrupt and non-bankrupt firms. A comparison of the performance of all three newly re-estimated models follows.

Table 7: The results derived when the re-estimated Zmijewski is applied to the initial sample

Group	Sector	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
1	Primary	100.00%	83.33%	100.00%	83.33%	75.00%
	Secondary	100.00%	85.71%	85.71%	85.71%	100.00%
	Tertiary	68.18%	80.00%	50.00%	58.82%	50.00%
2	Primary	100.00%	100.00%	100.00%	100.00%	N/A
	Secondary	100.00%	100.00%	100.00%	100.00%	100.00%
	Tertiary	100.00%	100.00%	72.22%	50.00%	88.89%

*Zmijewski cutoff criteria adjusted: if z_score > -2.5 then "Bankruptcy Likely"

4.5.4. Performance comparison

Figures 6 and 11 to 14 illustrate the improvement of model performance across all three sectors for the newly re-estimated Altman model (refer to the appendix for figures 11 to 14 which outline the predictive accuracy per model up to 5 years prior to bankruptcy). The reduction of Type 1 errors is evident in the ability of the model to more accurately distinguish between bankrupt, those in group 1 of the sample, and non-bankrupt firms, those in group 2 of the sample. The re-estimation of the model has also illustrated the ability of the model to accurately predict bankruptcy across the different sectors without the aid of an industry factor, although there is still room for improvement.

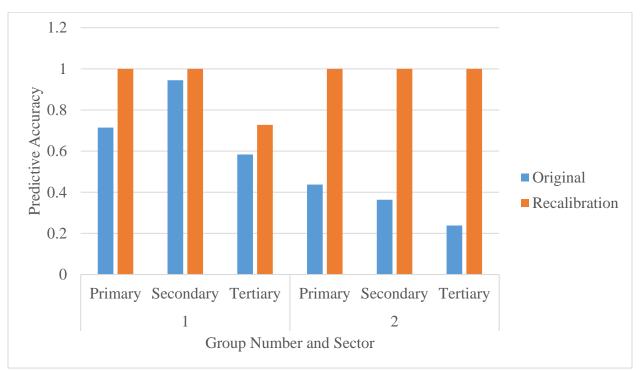


Figure 2: The predictive accuracy of the Altman model 1 year prior to bankruptcy

Figures 11 and 15 to 18 illustrate the performance results of the newly re-estimated Ohlson model (refer to the appendix for figures 15 to 18 which outline the predictive accuracy per model up to 5 years prior to bankruptcy). A decline in predictive accuracy is noted, with an increase in the prevalence of both Type 1 and Type 2 errors across both the bankrupt and non-bankrupt groups. This findings is consistent across all five years prior to bankruptcy and directly contrasts the findings of the above newly calibrated Altman model.

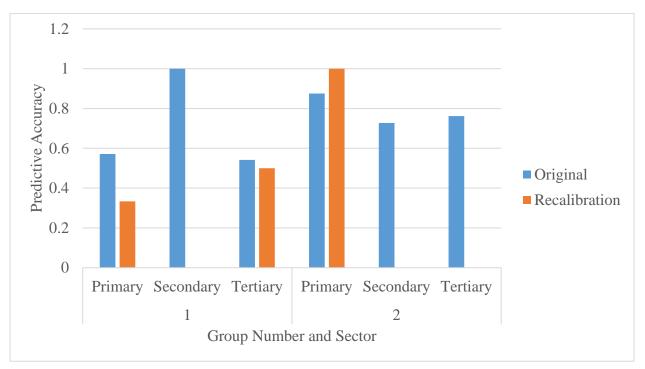


Figure 3: The predictive accuracy of the Ohlson model 1 year prior to bankruptcy

Figures 4 and 19 to 22 detail the performance of the newly re-estimated Zmijewski model (refer to the appendix for figures 19 to 22 which outline the predictive accuracy per model up to 5 years prior to bankruptcy). A significant reduction in the prevalence of Type 1 and Type 2 errors can be observed across all five years prior to bankruptcy. A slight deviation is seen in the tertiary sector of the bankrupt group which shows a slight decline in the predictive accuracy of the bankrupt group across the five years prior to bankruptcy.

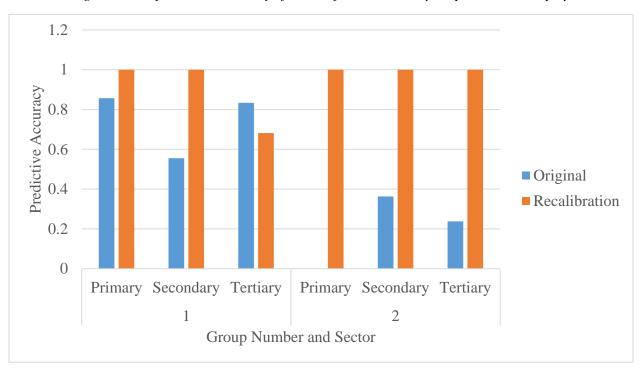


Figure 4: The predictive accuracy of the Zmijewski model 1 year prior to bankruptcy

After addressing the issue of time sensitivity in the models, it is found that two of the three models show a significant improvement in predictive accuracy following a re-estimation using their original methodologies alongside a new data sample. Furthermore, it is found that the predictive accuracy improves across all three sectors (across industries) without the aid of an industry factor, although room for improvement exists.

4.6. The industry factor

The industry sensitivity of each of the three models is considered in this section. The previous section surrounding the time sensitivity of the models found that the time sensitivity can be countered by re-estimating the models. As such, this section combines the re-estimation of the models alongside the inclusion of an industry component. The section is concluded with a visual summary of the performance of each of the models as compared to their original performance.

4.6.1. The industry-adjusted Altman model

In this section, the model was re-estimated using the multilinear regression methodology used by Altman, with the addition of an industry factor. The industry factor was estimated by calculating the mean of the ratio of total liabilities to total assets across all the included industries. The industry variable was then included in the re-estimation of the model. The following newly re-estimated and industry-adjusted Altman model was derived and presented an R-squared value of 81.5%. All six variables, as well as the overall model, were deemed statistically significant at the 1% level.

$$ind_adj_z_score$$

$$= 0.6473X_1 + 0.0065X_2 + 0.4842X_3 + 0.0356X_4 + 0.0576X_5$$

$$+ 1.0286X_6$$
(14)

where:

<i>X</i> ₁	= working capital to total assets
<i>X</i> ₂	= retained earnings to total assets
<i>X</i> ₃	= earnings before interest and tax to total assets
X_4	= market value of equity to total liabilities
X_5	= sales to total assets
X_6	= industry factor

4.6.2. The industry-adjusted Ohlson model

The re-estimation and addition of an industry factor resulted in the following derivation of the Ohlson model. The logit methodology was used despite the quasi-complete separation of data points detected within the sample. Due to the model convergence status, the maximum likelihood estimate was based on the last maximum likelihood iteration. The industry factor yielded a coefficient of zero as it was comprised of a linear combination of the other variables. Thus, caution is warranted when testing for the validity of this model. Despite this, a moderately strong Somers' D value of 70.5% was yielded indicating the moderate predictive accuracy of the newly defined model. It must be noted that this model is identical to the previously re-estimated model due to the nil coefficient for the industry factor.

ind_adj_o_score

$$= -9.7352 + 1.0045 \log\left(\frac{TA}{GNP}\right) + 3.6052 \frac{TL}{TA} + 2.7685 \frac{WC}{TA}$$
$$- 0.4637 \frac{CL}{CA} - 12.5161X - 0.4170 \frac{NI}{TA} - 0.9617 \frac{FFO}{TL} + 0.4626Y$$
$$+ 0.1653 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|} + 0x_6$$
$$= -9.7352 + 1.0045 \log\left(\frac{TA}{GNP}\right) + 3.6052 \frac{TL}{TA} + 2.7685 \frac{WC}{TA}$$
$$- 0.4637 \frac{CL}{CA} - 12.5161X - 0.4170 \frac{NI}{TA} - 0.9617 \frac{FFO}{TL} + 0.4626Y$$
$$+ 0.1653 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$

(15)

where:

TA = total assets

GNP = Gross National Product price index level; this variable is used to adjust total assets for inflationary changes

TL = total liabilities

WC = working capital

- *CL* = current liabilities
- CA = current assets

$$X = 1$$
 if TL > TA, 0 otherwise

NI = net income

FFO = funds from operations

Y = 1 if there was a net loss for the last two years. 0 otherwise

 X_6 = industry factor

4.6.3. The industry-adjusted Zmijewski model

The model was re-estimated using the probit regression methodology similar to the original study by Zmijewski. The re-estimation and addition of an industry factor resulted in the following derivation of the Zmijewski model. The industry factor yielded a coefficient of zero as it was comprised of a linear combination of the other variables. The Somers' D value, which indicates the predictive ability of the model, carries a value of 44.5% suggesting that the model has a moderately accurate predictive ability. It must be noted that this model is identical to the previously re-estimated model due to the nil coefficient for the industry factor.

$$ind_adj_zmijewski_score$$

$$= -1.6470 - 0.6498 \frac{NI}{TA} + 0.8192 \frac{TL}{TA} + 0.0604 \frac{CA}{CL} + 0X_{6}$$

$$= -1.6470 - 0.6498 \frac{NI}{TA} + 0.8192 \frac{TL}{TA} + 0.0604 \frac{CA}{CL}$$
(16)

where:

NI	= net income		
ΤA	= total assets		

TL = total liabilities

CA = current assets

CL = current liabilities

 X_6 = industry factor

4.7. Analysis of industry-adjusted model performance

The following three tables summarise the performance results of the re-estimated models after the addition of an industry factor. In order to visualise these results, the next section presents figures detailing the model performance as compared to the original model performance.

Group	Sector	Accuracy Accuracy A		AccuracyAccuracyAcc3 Years4 Years5 Y		Predictive Accuracy 5 Years Prior
	Primary	50.00%	50.00%	50.00%	50.00%	25.00%
1	Secondary	100.00%	85.71%	85.71%	71.42%	83.33%
	Tertiary	59.10%	40.00%	38.89%	35.29%	50.00%
2	Primary	0.00%	100.00%	100.00%	100.00%	N/A
	Secondary	100.00%	100.00%	50.00%	50.00%	50.00%
	Tertiary	50.00%	100.00%	50.00%	50.00%	50.00%

Table 8: The adjusted-Altman performance after the inclusion of an industry factor

*Altman cutoff criteria adjusted: if z_score > 0.8 then "Bankruptcy Likely"

Table 9: The adjusted-Ohlson performance after the inclusion of an industry factor

Group	Sector	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
	Primary	33.33%	66.67%	50.00%	50.00%	75.00%
1	Secondary	0.00%	14.29%	57.14%	14.29%	16.67%
	Tertiary	50.00%	30.00%	27.78%	52.94%	44.44%
2	Primary	100.00%	0.00%	100.00%	0.00%	N/A
	Secondary	0.00%	0.00%	50.00%	0.00%	0.00%
	Tertiary	0.00%	0.00%	50.00%	100.00%	100.00%

Group	Sector	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
	Primary	16.67%	16.67%	16.67%	16.67%	0.00%
1	Secondary	57.14%	28.57%	28.57%	42.86%	33.33%
	Tertiary	18.18%	15.00%	16.67%	17.65%	16.67%
	Primary	0.00%	100.00%	0.00%	100.00%	N/A
2	Secondary	100.00%	0.00%	0.00%	50.00%	0.00%
	Tertiary	50.00%	50.00%	0.00%	0.00%	0.00%

Table 10: The adjusted-Zmijewski performance after the inclusion of an industry factor

*Zmijewski cutoff criteria adjusted: if zmijewski_score < 0 then "Bankruptcy Likely"

4.7.1. Performance comparison

Figures 5 and 23 to 26 illustrate a slight improvement in the predictive ability of the Altman model in the secondary and tertiary sectors, but not in the primary sector even after the inclusion of the industry factor (refer to the appendix for figures 23 to 26 which outline the predictive accuracy per model up to 5 years prior to bankruptcy). The Ohlson and Zmijewski models have presented improvements in their predictive abilities across the three sectors, but this is mainly due to their re-estimation as the industry factors for both these models yielded coefficients of zero. Although a slight improvement can be seen after the inclusion of the industry factor in the Altman model, these results do not motivate a significant improvement in predictive ability. Furthermore, the incorporation of an industry factor may not always be possible due to the linear relationships shared between variables in the Ohlson and Zmijewski models. The problem of industry sensitivity still exists after the analysis of these results.

The following section attempts to address these challenges but suggesting the re-estimation of models per sector in order to improve the overall predictive accuracy of the models on an industry basis.

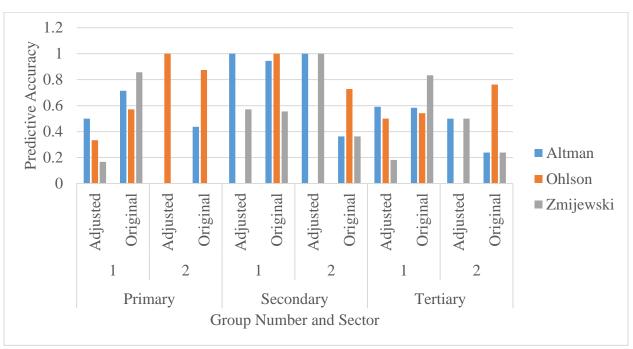


Figure 5: The predictive accuracy per model 1 year prior to bankruptcy

4.8. Industry sensitivity: Is re-estimation per sector the solution?

In this section the industry sensitivity of the models is tested. This is achieved by re-estimating the models using sector level data. In the previous two main sections detailing the results after the reestimation and subsequent addition of an industry factor, several variables presented as statistically insignificant. In an attempt to reduce the Root Mean Square Error (RMSE) per model, a stepwise regression is performed to remove statistically insignificant variables prior to re-estimation.

4.8.1. The re-estimated models per sector

4.8.1.1. Re-estimation using primary sector data

Using only primary sector data to run the stepwise regression, variable X_3 is removed from the model as it is statistically insignificant. The model is then re-estimated using the same linear regression methodology as previously stated and yields the following statistically significant model. The RMSE was reduced from 34.30% to 33.94% after the removal of variable X_3 .

$$recal_primary_z_score = -0.4046X_1 - 0.019X_2 + 0.1898X_4 + 0.4438X_5$$
(17)

A similar approach was used when re-estimating the Ohlson model. The ratio of funds from operations to total liabilities was removed, although no significant change in either the Somers' D value or *c value* was detected. The only coefficient with a significant change from the previously estimated model across all three sectors was the ratio of net income to total assets, indicating the relative influence of this variable when considering the primary sector. The following model was yielded.

$$recal_primary_o_score$$

$$= -9.8958 + 1.0294 \log\left(\frac{TA}{GNP}\right) + 3.4966 \frac{TL}{TA} + 2.7990 \frac{WC}{TA}$$

$$- 0.4683 \frac{CL}{CA} - 12.7694X - 2.1629 \frac{NI}{TA} + 0.4415Y$$

$$+ 0.1790 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$
(18)

Lastly, the Zmijewski model was re-estimated using the same methodology. Following the stepwise procedure, the ratio of net income to total assets was removed from the model. This resulted in the *c value* decreasing from 72.20% to 67.80%, which suggests a slight improvement in the predictive accuracy of the model.

Values above 0.5 suggest some predictive value, but somewhat weak predictive models may generate c statistics in the 0.75 range (Hermansen, 2008, p. 9).

Following the re-estimation using only data from the primary sector, the following model was yielded.

$$recal_primary_zmijewski_score = -1.7514 + 0.9974 \frac{TL}{TA} + 0.0677 \frac{CA}{CL}$$
(19)

4.8.1.2. Re-estimation using secondary sector data

In this section each of the models is re-estimated using the same methodology as described in the last section. A stepwise regression will be performed, and insignificant variables will be removed. This will be followed by the regression in the methodology described in the original articles.

The re-estimated Altman model, although significant, presented with an R-squared value of 48.59% after the removal of variable X_4 . Furthermore, the removal of this variable resulted in a slight increase in the RMSE from 60.08% to 61.40%. The following model was derived.

$$recal_secondary_z_score = 0.3291X_1 - 0.2263X_2 + 0.6080X_3 + 0.4749X_5$$
(20)

The following Ohlson model was derived, with a Somers' D value of 99.60% following the removal of the *X* variable.

$$recal_secondary_o_score$$

$$= 47.7537 + 6.3817 \log\left(\frac{TA}{GNP}\right) + 177.1000 \frac{TL}{TA} - 325.0000 \frac{WC}{TA}$$

$$- 199.5000 \frac{CL}{CA} + 203.5000 \frac{NI}{TA} - 99.2586 \frac{OI}{TL} - 2.2401Y$$

$$- 6.3061 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$
(18)

When re-estimating the Zmijewski model, the removal of the ratio of total liabilities to total assets was suggested by the stepwise methodology, however when further analysis into the outcome of this removal was conducted it was found that the Somers' D value would have shifted from 60.20% to 14.3%. The deduction of the variable was thus decided against, yielding the following model.

(19)

recal_secondary_zmijewski_score

$$= -3.8303 + 3.4929 \frac{NI}{TA} + 4.8527 \frac{TL}{TA} + 0.1478 \frac{CA}{CL}$$

4.8.1.3. Re-estimation using tertiary sector data

Following the same methodology as stated in the previous two sections, this section makes use of tertiary sector data. The re-calibrated Altman yields an R-squared value of 79.28%, with a RMSE of 44.06%. The model is statistically significant at the 1% level.

$$recal_tertiary_z_score$$

$$= -0.0838X_1 - 0.0573X_2 + 0.2258X_3 + 0.0684X_4 + 0.5401X_5$$
(20)

Baring the removal of any variables, the Ohlson model yields a Somers' D value of 91.20%, indicting a significant predictive ability.

recal_tertiary_o_score

$$= -1.5522 - 1.8522 \log\left(\frac{TA}{GNP}\right) - 0.6141 \frac{TL}{TA} + 0.0675 \frac{WC}{TA}$$
$$- 11.9781 \frac{CL}{CA} - 0.3016 \frac{NI}{TA} - 0.1588 \frac{OI}{TL} + 0.7945Y$$
$$- 0.0760 \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$

(18)

(19)

No stepwise effects met the 0.5 level for entry into the Zmijewski model. The original model variables were regressed using the probit methodology and presented a *c value* of 68.40%, indicating a satisfactory predictive ability.

recal_tertiary_zmijewski_score

$$= -1.7618 - 0.8947 \frac{NI}{TA} + 0.0589 \frac{TL}{TA} + 0.1721 \frac{CA}{CL}$$

4.8.2. Analysis of primary sector re-estimations

The following three tables present the results from the performance analysis using the three newly re-estimated per sector models detailed above. Although the results quite plainly illustrate the improvement in predictive accuracy per model across all three sectors, the following section graphically illustrates these findings.

Sector	Group	Model	Predictive Accuracy 1 Year Prior	Predictive Accuracy 2 Years Prior	Predictive Accuracy 3 Years Prior	Predictive Accuracy 4 Years Prior	Predictive Accuracy 5 Years Prior
Primary	1	Altman	100.00%	100.00%	100.00%	83.33%	100.00%
1 minur y		Ohlson	100.00%	100.00%	100.00%	100.00%	100.00%
		Zmijewski	100.00%	100.00%	100.00%	100.00%	100.00%
	2	Altman	100.00%	100.00%	100.00%	100.00%	N/A
		Ohlson	100.00%	0.00%	100.00%	100.00%	N/A
		Zmijewski	100.00%	100.00%	100.00%	100.00%	N/A
Secondary	1	Altman	85.71%	100.00%	100.00%	100.00%	100.00%
Secondary		Ohlson	100.00%	100.00%	100.00%	100.00%	100.00%
		Zmijewski	100.00%	100.00%	100.00%	100.00%	100.00%
	2	Altman	100.00%	100.00%	100.00%	100.00%	100.00%
		Ohlson	50.00%	0.00%	0.00%	0.00%	0.00%
		Zmijewski	100.00%	100.00%	100.00%	100.00%	100.00%
Tertiary	1	Altman	100.00%	100.00%	100.00%	100.00%	100.00%
Tertiary		Ohlson	100.00%	100.00%	100.00%	100.00%	100.00%
		Zmijewski	100.00%	100.00%	100.00%	100.00%	100.00%
	2	Altman	100.00%	100.00%	100.00%	100.00%	100.00%
		Ohlson	100.00%	100.00%	100.00%	100.00%	100.00%
		Zmijewski	100.00%	100.00%	100.00%	100.00%	100.00%

Table 11: Comparison of model performance after re-estimation using data from each sector.

*NA: No relevant data for this time period to be considered.

*Altman cutoff adjusted for primary sector dataset: if *z_score* > 1.1 then "Bankruptcy Likely"

*Zmijewski cutoff adjusted for primary sector dataset: if zmijewski_score < -1.5 then "Bankruptcy Likely"

- *Altman cutoff adjusted for secondary sector dataset: if z_score > 1 then "Bankruptcy Likely"
- *Zmijewski cutoff adjusted for secondary sector dataset: if zmijewski_score < -1.5 then "Bankruptcy Likely"
- *Altman cutoff adjusted for tertiary sector dataset: if *z_score* > 1.2 then "Bankruptcy Likely"
- *Zmijewski cutoff criteria adjusted for tertiary sector dataset: if zmijewski_score < -3 then "Bankruptcy Likely"

4.8.3. Performance comparison

Figures 6 and 27 to 30 present the performance results of the re-estimated per sector Altman model (refer to the appendix for figures 27 to 30 which outline the predictive accuracy per model up to 5 years prior to bankruptcy). A significant increase in the predictive ability of the model is evident. Furthermore, a significant decline in the presence of Type 1 and Type 2 errors is evident, improving the overall reliability of the model. Interestingly, the predictive ability of the re-estimated per sector Altman model has drastically improved up to five years prior to bankruptcy.

Similar results can be observed in the figures representing the results from both the Ohlson and Zmijewski models. Figures 31 to 40 (see appendix) which present the results of the Ohlson and Zmijewski models detail the significant decrease in the presence of Type 1 and Type 2 errors from which these models succumb to previously.

An overall significant improvement following the re-estimation per sector can be observed across all three models.

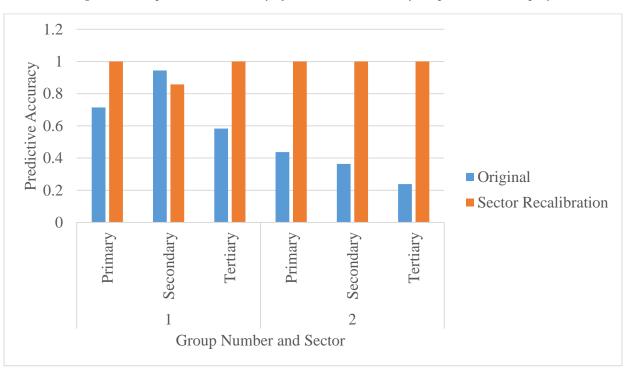


Figure 6: The predictive accuracy of the Altman model 1 year prior to bankruptcy

5. Discussion

In this section the results of the predictive accuracy of the per sector re-estimated models in this research are compared to the results of the predictive accuracy of the models from the original studies.

Altman (1968) reported a predictive accuracy of 94% of his initial sample, with 95% of all firms in the bankrupt and non-bankrupt groups assigned to their actual group classification. Furthermore, Altman's model predicted bankruptcy accurately up to two years prior to actual failure, with the accuracy diminishing rapidly after the second year. Ohlson (1980) conveyed the importance of cutoff points and noted a predictive accuracy of 82.6% in the non-bankrupt sample of firms and 87.6% in the bankrupt sample of firms. Ohlson also notes the prevalence of Type 1 and Type 2 errors when applying bankruptcy prediction models. Zmijewski (1984) achieved a maximum predictive accuracy of 83.5% for his incomplete data group.

This research improves on the predictive accuracy of all three models by unveiling the influence of time and industry sensitivity on accounting-based bankruptcy predictive models used in industry. As and when samples change, so should the coefficients of each model in order to more reliably interpret those samples. The re-estimation of models using per sector, or industry-specific, samples of data proved to improve the predictive accuracy of all three models across both time and industries. These results agree with the finding by Platt and Platt (1991).

If extreme values of financials ratios are industry specific, then bankruptcy prediction models should not compare unadjusted ratios across companies unless the industry is held constant (Platt & Platt, 1991, p. 1184).

Not only do the results suggest this is most reliable way to predict bankruptcy with near perfect predictive accuracy, but the results further indicate that this is true up to five years prior to bankruptcy. A significant decrease in the number of Type 1 and Type 2 errors across all three models is also noted.

6. Conclusion

In 1968, Edward Altman questioned if a gap between traditional ratio analysis and more rigorous statistical techniques could be bridged in the assessment of firm performance. More than half a century later, the question remains relevant. This paper aimed to uncover an unrivaled accounting-based model with the ability to accurately predict bankruptcy several years prior to the event across multiple industries. Although this was not directly achieved, the objective directed this paper to a finding which is just as paramount to our understanding of bankruptcy prediction models. The results suggest two main challenges faced when using predictive models in industry: the issue of time sensitivity, as well as industry sensitivity.

The issue of time sensitivity was resolved following the results yielded post re-estimating each of the original models using the new sample. As and when samples change, so too should the coefficients of each model in order to more reliably interpret those samples. The re-estimation of each of the three models resulted in significant improvements in predictive ability across industries.

The second challenge, being that of industry sensitivity, was initially addressed by suggesting the inclusion of an industry factor. Although the use of an industry factor did improve the predictive accuracy of one of the three models across sectors, a more promising solution to industry sensitivity is the re-estimation of models using per sector, or industry-specific, samples of data. The findings suggest this to be the most reliable way to predict bankruptcy with near perfect predictive accuracy up to five years prior to bankruptcy, as well as a significant decrease in the number of Type 1 and Type 2 errors across all three models.

Following the findings of this study, an interesting future addition to this realm of research would be the investigation into a hybrid accounting-based and market-based model. Nakajima (2011) explains the basic estimation methodology of the time-varying parameter vector autoregression (TVP-VAR) model by reviewing an estimation algorithm for a univariate TVP regression model with stochastic volatility. The application of this methodology could also be seen as an opportunity to further delve into the element of time sensitivity, possibly unearthing a model which considers the elasticity of variables in relation to model predictive accuracy over time. The intended contribution of this research is the practical application of the methods and findings related to the time and industry sensitivity of predictive models. Considering the impact of these elements, this research serves as a guide for risk assessment by lending institutions, and performance benchmarking for firms. By highlighting the sensitivity of time and industry to the application of bankruptcy prediction models, this research lessens the gap between traditional ratio analysis and more rigorous statistical techniques in the assessment of firm performance when evaluating credit risk.

References

- Altman, E. (1968). Financials Ratios, Discriminant Analysis and the Predication of Corporate Bankruptcy. *The Journal of Finance 23(4)*, 589-609.
- Altman , E., & Saunders, A. (1998). Credit Risk Measurement: Developments Over the Last 20 Years . Journal of Banking & Finance 21(1), 1721-1742.
- Barbu, T. C., Dardac, M., & Boitan, I. (2009). The Effects and the Cost of Bank Recapitalisation in the Context of Financial Crises. *Theory of Applied Economics* 12(541), 274-281.
- Bhandari, P. (2021, January 18). *Type I and Type II Errors*. Retrieved from Scribbr: https://www.scribbr.com/statistics/type-i-and-type-ii-errors/
- Cassim, R. J., & Swanepoel, M. J. (2021). The Bankruptcy Prediction Approach: An Empirical Study of Comparison Between the Emerging Market Score Model and Bankruptcy Prediction Indicators Approach in the Johannesburg Stock Exchange. *Journal of Economic* and Financial Sciences 14(1), 1-8.
- Hayes, S. K., Hodge, K. A., & Hughes , L. W. (2010). A Study of the Efficacy of Altman's Z To Predict Bankruptcy of Firms Doing Business During Contemporary Times. *Economics & Business Journal: Inquiries & Perspectives 3(1)*, 130-134.
- Hermansen, S. W. (2008). *Evaluating Predictive Models: Computing and Interpreting the c Statistic.* Maryland: SAS Global Forum 2008.
- Izan, H. (1984). Corporate Distress in Australia. Journal of Banking & Finance 8(1), 303-320.
- Jarrow, R. A., & Turnbull, S. (1995, March). Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance 50(1)*, 53.85.
- Johnson, O. (2002, April). On Internal Ratings, Models, and the Basel Accord Issues for Financial Institutions and Regulators in Measuring and Managing Credit Risk. Retrieved from IMF eLibrary: https://www.elibrary.imf.org/view/IMF072/02402-9781589060128/02402-9781589060128/ch03.xml?language=en&redirect=true
- Karas, M., & Reznakova, M. (2017). Could the Coefficients Re-estimation Solve the Industry or Time Specific Issues? *Journal of Economics and Management Systems 2(1)*, 206-213.
- Lev, B. (1969). Industry Averages as Targets for Financial Ratios . *Journal of Accounting Research* 7(2), 290-299.

- Lubawa, G., & Louangrath, P. (2016). Using Altman Z-Score to Assess the Financial Effects of Multiple Loans on SMEs . *International Journal of Research & Methodology in Social Science 2(1)*, 63-86.
- Martin , D. (1977). Early Warning of Bank Failure: A Logit Regression Approach . Journal of Banking & Finance 1(1) , 249-276.
- Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates . Journal of Finance 29(1), 449-470.
- Nakajima , J. (2011). Time-Varying Parameter VAR Model with Stochastic Volatility: An Overview of Methodology and Empirical Applications. *Monetary and Economic Studies* , 107-142.
- Ohlson, J. A. (1980). Financial Ratios and the Probabilistic Predication of Bankruptcy. *Journal of Accounting Research*, 109-131.
- Pettinger, T. (2021, April 5). *Sectors of the Economy*. Retrieved from Economics Help: https://www.economicshelp.org/blog/12436/concepts/sectors-economy/
- Phillips , P. C., Gao, J., & Li, D. (2017). Kernel-Based Inference in Time-Varying Coefficient Cointegrating Regression. *Research Gate Electronic Journal*, 1-60.
- Platt, H. D., & Platt, M. B. (1991). A Note of the Use of Industry-Relative Ratios in Bankruptcy Prediction . *Journal of Banking and Finance 15(1)*, 1183-1194.
- Prabhakar, A. C., Kaur, G., & Erokhin, V. (2019). *Regional Trade and Development Strategies in the Era of Globalisation*. Pennsylvania: IGI Global .
- Sajjan, R. (2016). Predicating Bankruptcy of Selected Firms by Applying Altman's Z-Score Model . International Journal of Research 4(4), 152-158.
- Singh, B. P., & Mishra, A. K. (2016). Re-estimation and Comparisons of Alternative Accounting-Based Bankruptcy Prediction Models for Indian Companies. *Financial Innovation 2(6)*, 1-28.
- Sommerville, R. A., & Taffler, R. J. (1995). Banker Judgement Versus Formal Forecasting Models: The Case of Country Risk Assessment. *Journal of Banking & Finance 19(2)*, 281-297.
- Timmermans, M. (2014). U.S. Corporate Bankruptcy Predicting Models . Tilburg : Tilburg University .
- Waagepetersen, R. (2010). A Statistical Modeling Approach to Building an Expert Credit Risk Rating System. *The Journal of Credit Risk* 6(2), 81-94.

Zmijewski, M. E. (1984). Methodological Issues Related to the Estimation of Financial Distress Prediction Models . *Journal of Accounting and Research 22(1)*, 59-82.

Appendix

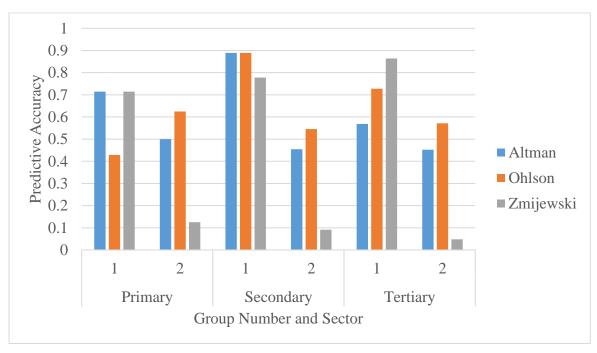
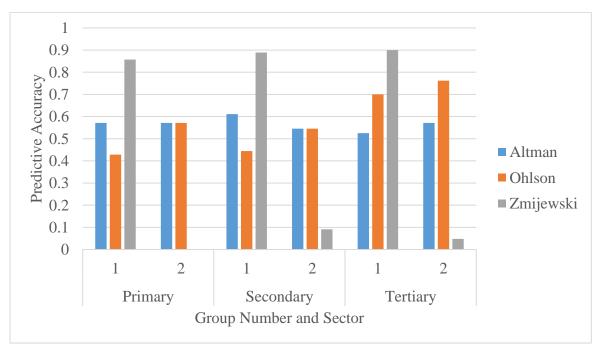


Figure 7: The predictive accuracy per model 2 years prior to bankruptcy

Figure 8: The predictive accuracy per model 3 years prior to bankruptcy



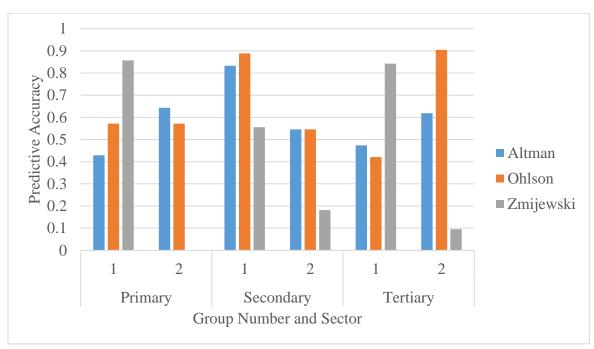
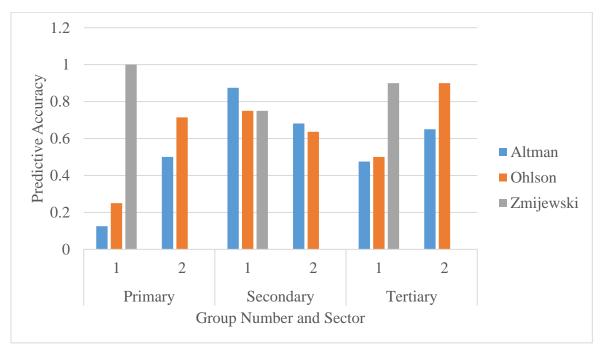


Figure 9: The predictive accuracy per model 4 years prior to bankruptcy

Figure 10: The predictive accuracy per model 5 years prior to bankruptcy



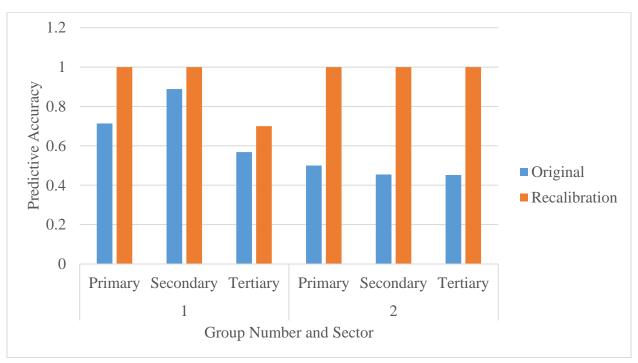
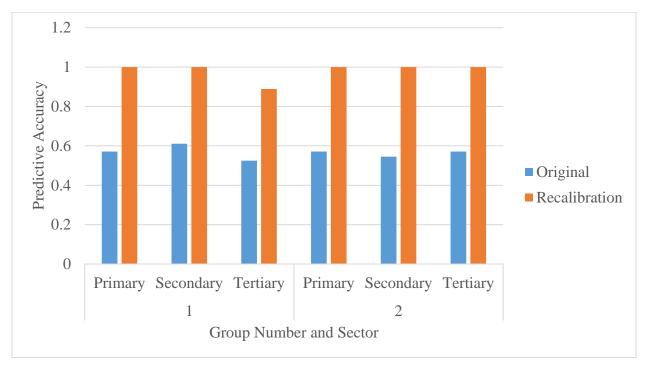


Figure 11: The predictive accuracy of the Altman model 2 years prior to bankruptcy

Figure 12: The predictive accuracy of the Altman model 3 years prior to bankruptcy



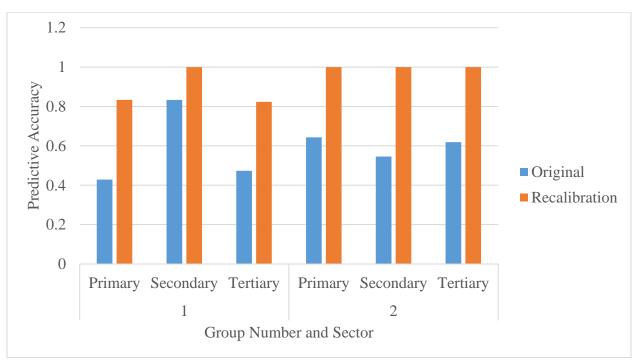
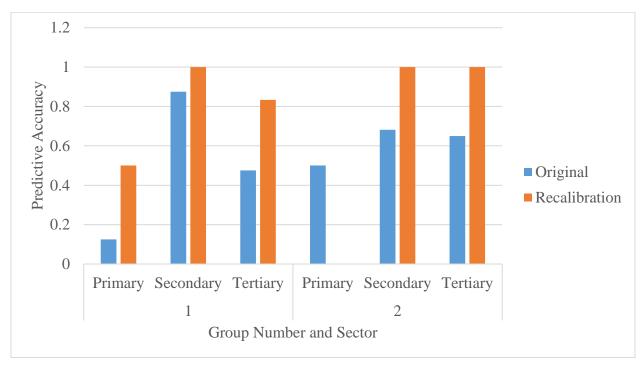


Figure 13: The predictive accuracy of the Altman model 4 years prior to bankruptcy

Figure 14: The predictive accuracy of the Altman model 5 years prior to bankruptcy



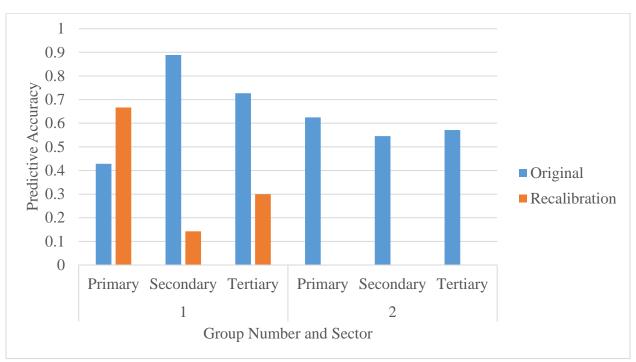
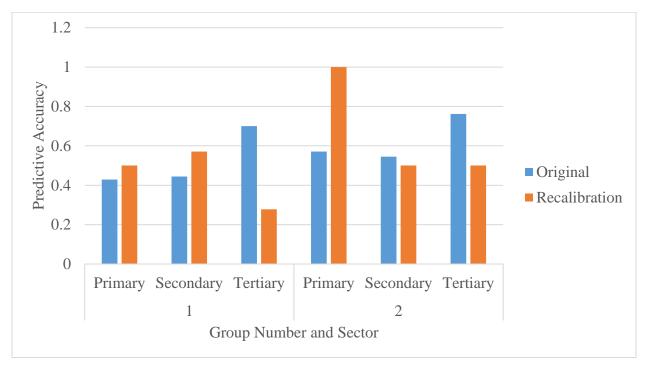


Figure 15: The predictive accuracy of the Ohlson model 2 years prior to bankruptcy

Figure 16: The predictive accuracy of the Ohlson model 3 years prior to bankruptcy



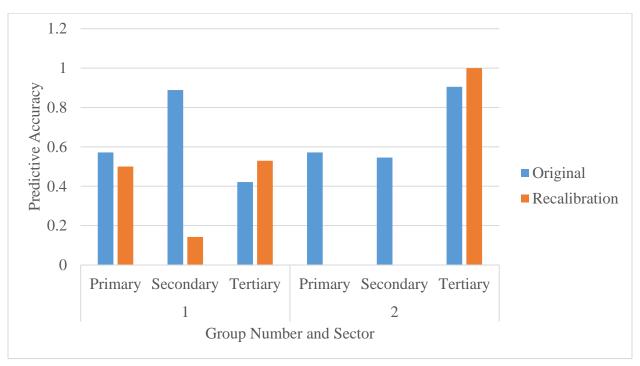
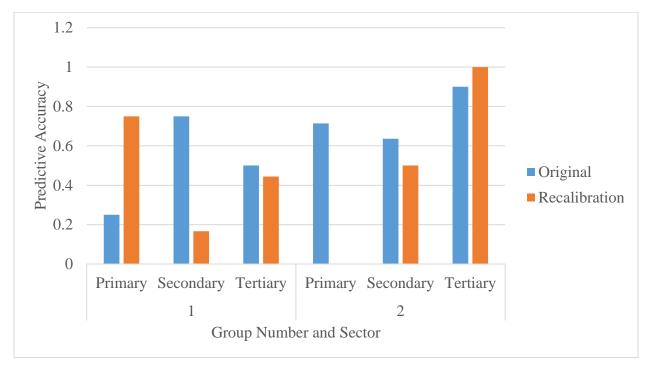


Figure 17: The predictive accuracy of the Ohlson model 4 years prior to bankruptcy

Figure 18: The predictive accuracy of the Ohlson model 5 years prior to bankruptcy



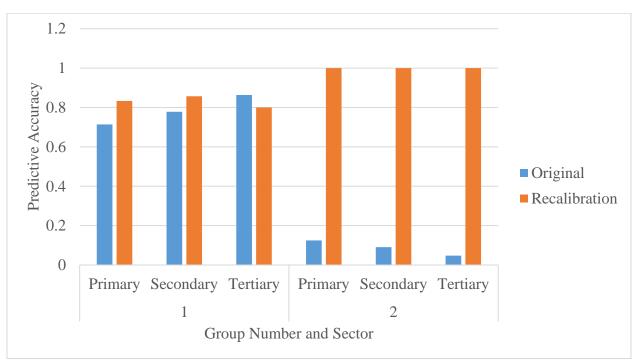
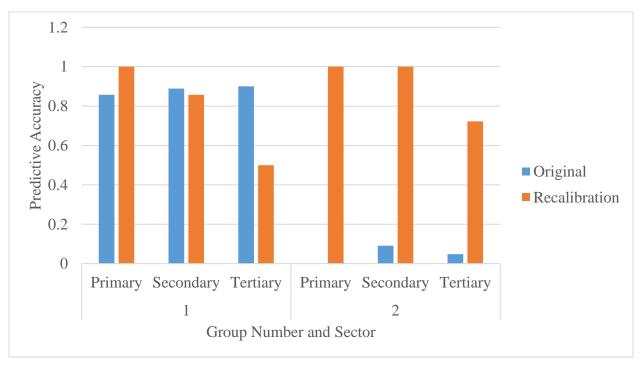


Figure 19: The predictive accuracy of the Zmijewski model 2 years prior to bankruptcy

Figure 20: The predictive accuracy of the Zmijewski model 3 years prior to bankruptcy



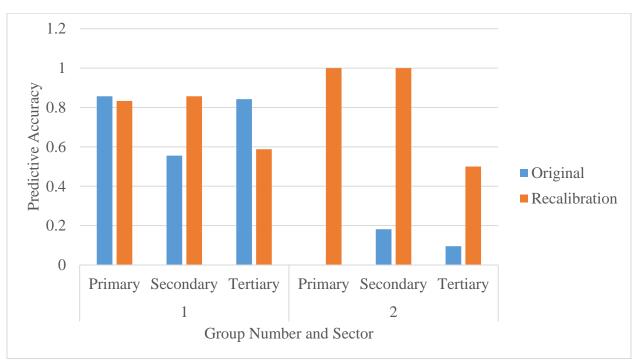
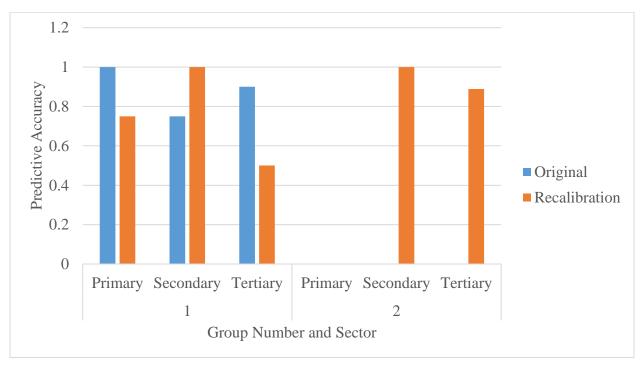


Figure 21: The predictive accuracy of the Zmijewski model 4 years prior to bankruptcy

Figure 22: The predictive accuracy of the Zmijewski model 5 years prior to bankruptcy



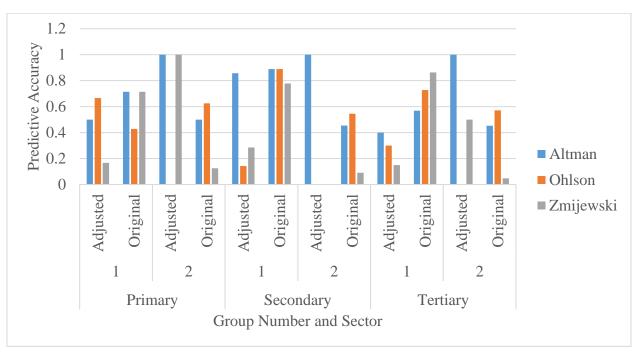
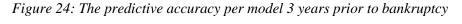
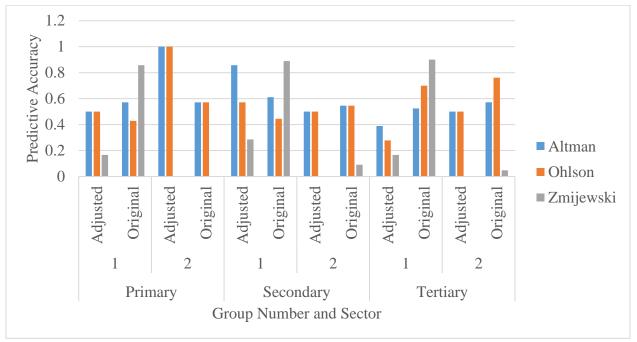


Figure 23: The predictive accuracy per model 2 years prior to bankruptcy





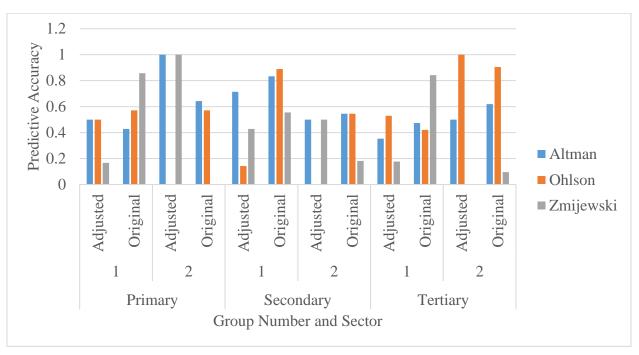
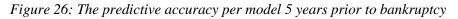
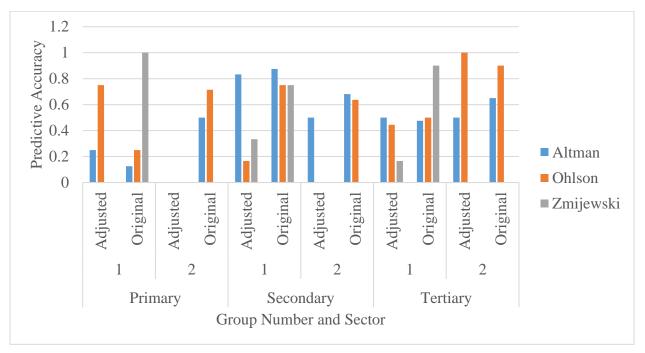


Figure 25: The predictive accuracy per model 4 years prior to bankruptcy





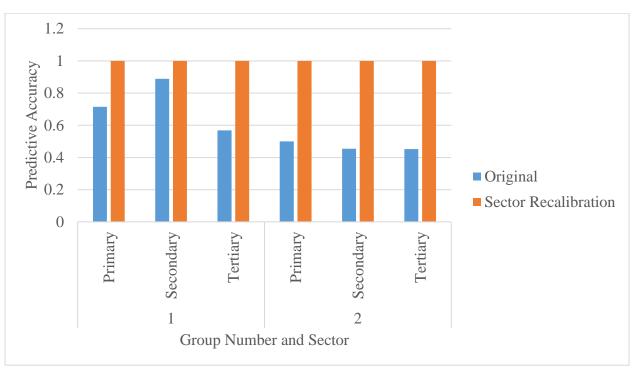
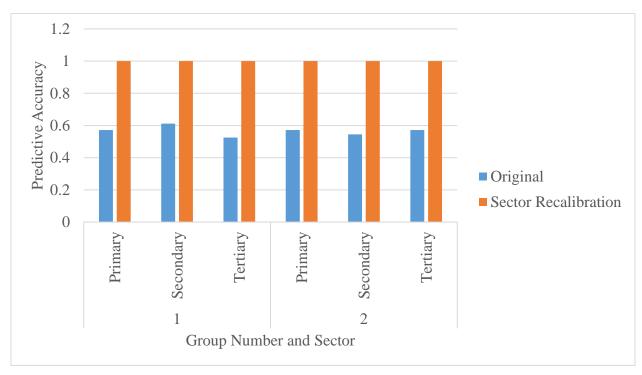


Figure 27: The predictive accuracy of the Altman model 2 years prior to bankruptcy

Figure 28: The predictive accuracy of the Altman model 3 years prior to bankruptcy



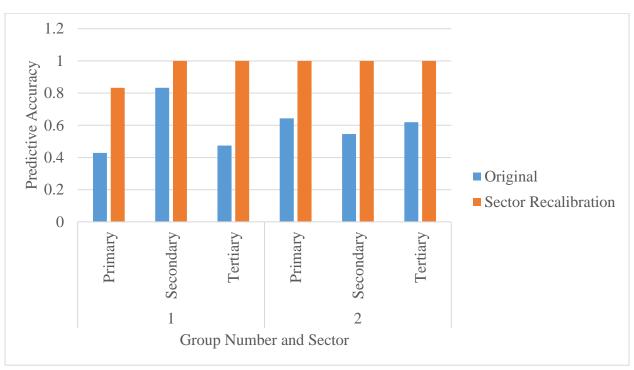
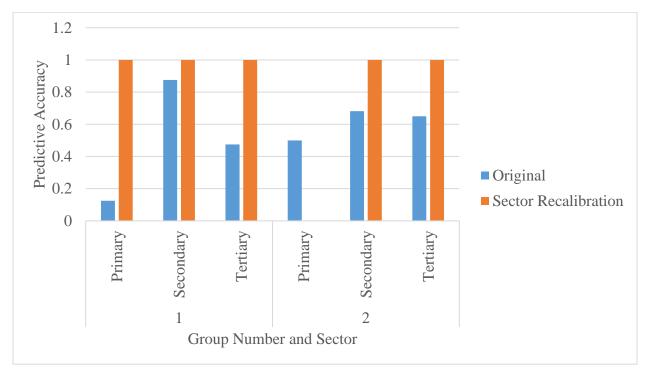


Figure 29: The predictive accuracy of the Altman model 4 years prior to bankruptcy

Figure 30: The predictive accuracy of the Altman model 5 years prior to bankruptcy



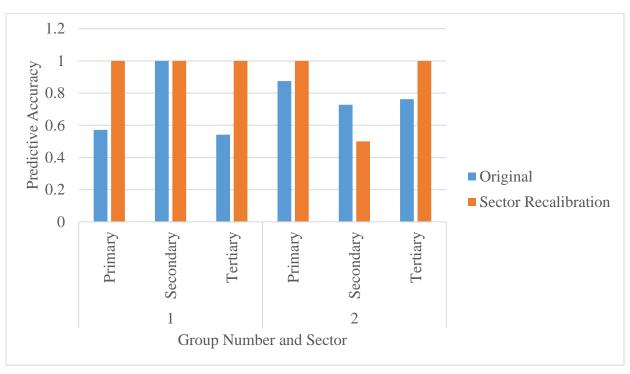
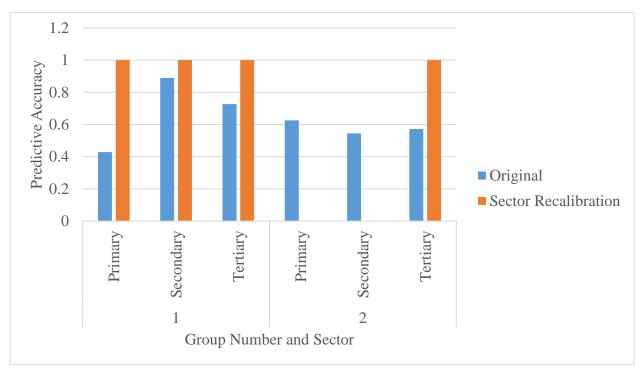


Figure 31: The predictive accuracy of the Ohlson model 1 year prior to bankruptcy

Figure 32: The predictive accuracy of the Ohlson model 2 years prior to bankruptcy



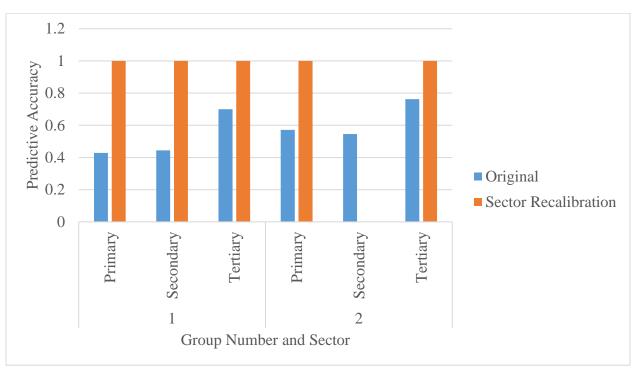
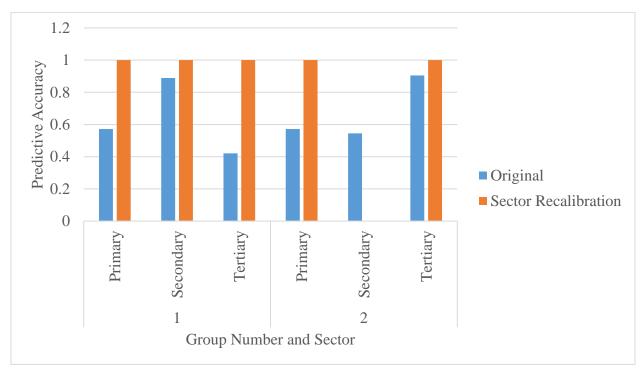


Figure 33: The predictive accuracy of the Ohlson model 3 years prior to bankruptcy

Figure 34: The predictive accuracy of the Ohlson model 4 years prior to bankruptcy



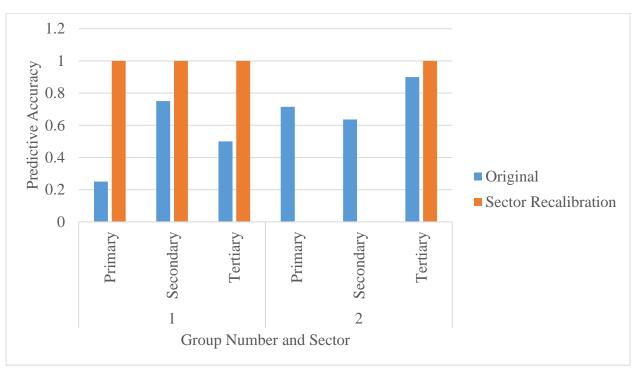
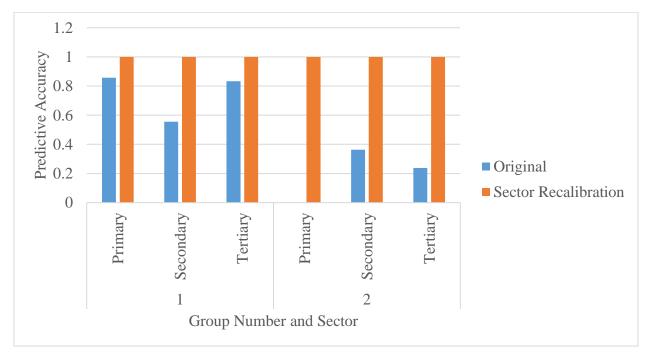


Figure 35: The predictive accuracy of the Ohlson model 5 years prior to bankruptcy

Figure 36: The predictive accuracy of the Zmijewski model 1 year prior to bankruptcy



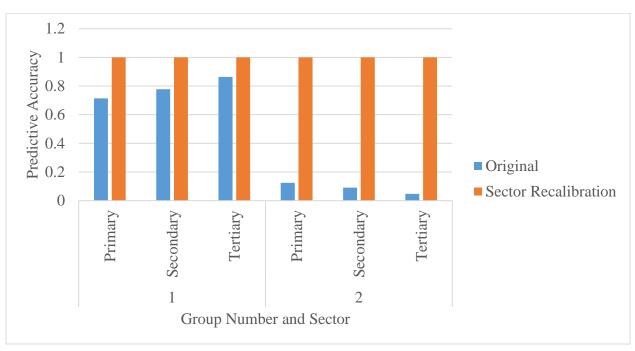
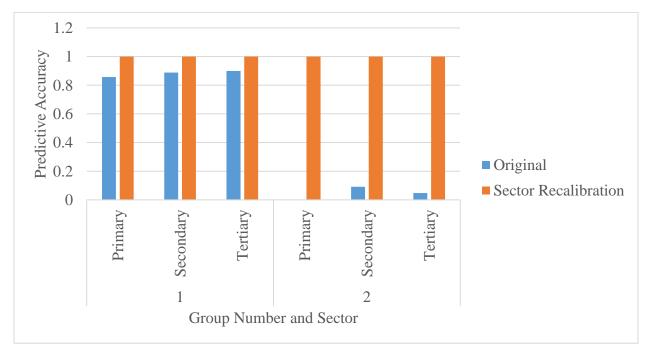


Figure 37: The predictive accuracy of the Zmijewski model 2 years prior to bankruptcy

Figure 38: The predictive accuracy of the Zmijewski model 3 years prior to bankruptcy



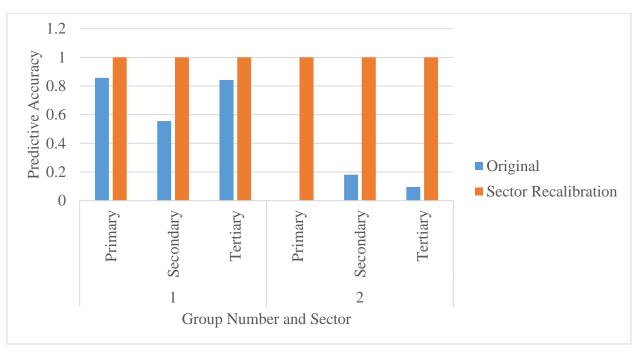


Figure 39: The predictive accuracy of the Zmijewski model 4 years prior to bankruptcy

Figure 40: The predictive accuracy of the Zmijewski model 5 years prior to bankruptcy

