The Impact of Learning Mathematical Vocabulary of Functions using the Frayer Model on Conceptual Understanding and Mathematical Performance of Grade 11 Learners

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ABSTRACT

This study investigated the impact of integrating the learning of mathematical vocabulary and the learning of mathematical content by Grade 11 learners in secondary schools in Gauteng province, South Africa. The main research question of the study is: what affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?

A cohort of Grade 11 learners (n=157) took part in this quasi-experimental study with control (n=83) and experimental (n=74). The experimental group was exposed to explicit learning of mathematical vocabulary using the Frayer model, while the control group used any other method preferred by their teachers. During the posttest, learners from the experimental group outperformed their counterparts in associating mathematical vocabulary with a mathematical graph. The study showed that the Frayer model is an effective strategy for learning mathematical vocabulary. When learners learnt mathematical vocabulary using the Frayer model, they mastered more vocabulary than their peers in the control group and this translated into improved conceptual understanding and performance in mathematics. There is a positive moderate correlation ($r = 0.61$) between the quantity of correct mathematical vocabulary that learners know and the marks those learners obtain in a mathematics test. The study further showed that it is possible to adopt Content and Language Integrated Learning (CLIL) in South African Schools.

DEDICATION

This study is dedicated to my family for the unwavering support that each and every one of them gave me throughout my research journey. I dedicate this study to my wife Chemedzai, my daughters Evernice, Eunice and Ethel and my sons Evidence and Elias.

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DECLARATION

I, the undersigned, declare that the thesis which I hereby submit for the degree Doctor of Philosophy to University of the Witwatersrand contains my own, independent work and has not previously been submitted by me for any degree at this or any other tertiary institution.

Signature:

Name: Edwin Madzore

Date: September 22, 2023

CHAPTER 1 : CONCEPTUALISATION OF THE STUDY

This chapter serves as an introduction to a research study that encompassed a well-defined intervention. The study focused on assessing the influence of the Frayer model as a tool for learning mathematical vocabulary related to functions. Conducted in South Africa's Gauteng province, the study aimed to investigate how this intervention impacted students' conceptual understanding, procedural proficiency, and overall mathematical performance.

The study's intervention comprised two distinct phases: an initial information phase and a subsequent implementation phase. These phases were carefully structured to provide learners with a systematic approach to acquiring proficiency in mathematical vocabulary within the context of functions.

Throughout the research, a thorough pretest and post-test assessment were employed to detect significant shifts in participants' understanding of mathematical concepts, their procedural skills, and their overall mathematical performance. The study sought to ascertain whether the focused instruction on mathematical vocabulary using the Frayer model yielded noticeable improvements in these critical aspects of learning in the domain of mathematical functions.

Conceptual understanding is the knowledge of mathematical ideas, operations, and relations such that learners can recognise, and use learnt mathematical ideas in new situations. The knowledge comprises a full understanding of fundamental and underpinning concepts behind the algorithms executed in mathematics (Zeeuw et al., 2013). Learners with conceptual understanding can interconnect mathematical concepts across disciplines. They can critically think about the content and can flexibly communicate key components of mathematics. Conceptual understanding allows learners to make choices and apply their understanding through active engagement. Students with conceptual understanding, can successfully reason in new settings by carefully applying learnt concept definitions and relations or representations of the concepts. In mathematics, such understanding manifests when students manage to recognise and label mathematical objects, sketch graphs, and draw diagrams, identify and recognise when to use a particular theorem, prove theorems, interrelate concepts, and know where to apply mathematical signs and symbols. Students with conceptual understanding know what instructions entail, for example they know the difference between solve and simplify, they can differentiate well between an expression and an equation etc. Conceptual understanding also manifests when students know when to integrate concepts within and across mathematics (Al-Mutawah et al., 2019). Students who have acquired conceptual understanding know how to apply facts and definitions of the mathematical concepts. This implies that a learner who has attained conceptual understanding knows more than isolated facts and methods.

As for procedural proficiency, students demonstrate it in mathematics when they can choose the correct and most appropriate procedure to solve a problem. In addition to choice of procedure, students with procedural proficiency know how to verify and justify the chosen procedure or even tweak the procedure in order to deal with novel situations. (Al-Mutawah et al., 2019). According to Cho and Nagle (2017), procedural proficiency extends to an ability to read and interpret graphs and figures like pie charts, histograms, line graphs, scatter plots etc. In addition to reading and interpreting the graphs, it is also part of procedural proficiency to be able to make geometric constructions, perform calculations and rounding off to a required degree of accuracy. Procedural proficiency is a goal-directed action sequence that points to knowing how to do something (Graven & Stott, 2012). It involves memory strategies and mathematical algorithms. To this end, procedural proficiency entails following a particular sequence of well-defined actions to produce the desired outcome. In the case of mathematics, the desired outcome is the correct answer to the problem. This means that an individual using a procedure is not obliged to reflect on the meanings of the elements being implemented (Hallett et al., 2010). Though conceptual understanding and procedural proficiency are different constructs, they complement each other in their contribution to the success of learners. For this reason, in this study, they have not been viewed as different, but as two sides of the same coin.

In this study, mathematical vocabulary refers to the words/terms that denote mathematical objects, e.g., equation, expression, parabola, and line of symmetry. It also encompasses words used to describe behaviour, such as increasing; features of an object, such as a turning point; and mathematical instructions, including solve, simplify, and so on. Wells (2013) indicated that these are words used by mathematicians and learners of mathematics, either as written or spoken language, to communicate about mathematics. These words include special technical words, ordinary words, phrases, and linguistic constructions with special meanings that sometimes differ from their meanings in everyday language (Wells, 2013), and thus, become a mathematical language of their own. This language is needed to communicate effectively and comprehend various constructs in mathematics classes (Zwiers, 2004). In this research, the phrase “mathematical language” is used to refer to the presence and use of the mathematical vocabulary required for the learning and comprehension of mathematical concepts relating to functions at Grade 11 Level.

The subsequent sections of the chapter present: a background to this study (in Section 1.1), aims (Section 1.2) and objectives (Section 1.3) as well as discussions on the rationale of the study (Section 1.4), research question (Section 1.5), statement of the problem (Section 1.6), and the significance of the study (Section 1.7). Finally, Section 1.8 presents the conclusion to the chapter.

1.1 Background to the study

In South Africa, National Senior Certificate (NCS) mathematics results have been poor for more than two decades (van Jaarsveld & Ameen, 2017). Research has attributed this to a wide range of socio-economic factors (Spaull, 2013a; Taylor, 2019) for example stress and home environment, family income, parental education, etc. Further afield, Duncan and Murnane (2016) noted that, in the United States of America, disadvantaged pupils do not perform academically as well as their more privileged counterparts. This is consistent with research by Okado et al. (2014), where an exploration of the effect of poverty on achievement among African American learners revealed that learners' achievements were strongly negatively correlated to their poverty levels. Though socio-economic factors do affect learners' mathematics performance, they are not the only factors at play.

Given the existing research that examines the role of language and vocabulary in mathematics teaching and learning, it warrants further investigation and its impact on learning mathematics needs to be given thoughtful consideration. A lack of emphasis on and attention to mathematical vocabulary as a part of the teaching-learning process leads to a lack of conceptual understanding, and thus to learners failing to apply the correct computational skills (DBE, 2016). In many cases, learners fail to communicate the mathematical ideas that would prove their understanding, and this results in them appearing less intelligent than they really are (Truxaw & Rojas, 2014). Morgan (2013) holds the view that poor language development results in learners developing misconceptions, and these become a major cause of the obstacles encountered by learners in learning mathematics. In trying to address the mathematical language problem, van Jaarsveld (2016) defined a set of language repertoires for teachers to use in the execution of their duties. He developed literal, algebraic, graphical, Cartesian, and procedural or algorithmic repertoires for teachers but, in the view of this study, this repertoire is also beneficial to mathematics learners as they are the ones who need to understand mathematics and its language.

The problem of learning mathematics in South Africa is reflected in the low national pass rate. This is illustrated by the statistical averages of 55,2% of candidates attaining 30% and above, while 36,1% achieved 40% and above in the South African national senior certificate examinations over the past five years (DBE, 2022).

Table 1: South African National Senior Certificate examination performance over five years.

	2018	2019	2020	2021	2022	Average
% achieved at 30% and above	56	53.6	53.8	57.6	55	55.2
% achieved at 40% and above	36.3	35	35.6	37.6	36	36.1

The problem of poor performance in mathematics is not uniquely South African. In research on the teaching of numeracy in pre-schools and early grades in low-income countries, Atweh et al. (2014) noted that low performance was common in many African, Asian, and Latin American countries. For instance, this problem of low performance in mathematics was noted among secondary-school learners in Zimbabwe, Kenya and Nigeria (Chirume & Chikasha, 2014; Gitaari et al., 2013; Sa'ad et al., 2014). Low performance in mathematics is therefore an international problem. This research study is thus deliberately focused on exploring whether or not learning mathematical vocabulary, as opposed to just drilling basic procedural skills, can improve poor pass rates. The idea that it is backed by Groves' (2012) assertion that successful mathematical learning comprises much more than just knowledge of skills and procedures.

Different studies found that the learning of language within content subjects, such as mathematics, is not an easy exercise, as language and content have been taught and learnt separately for centuries without producing optimal results (Mehisto et al., 2008; Morton, 2018). Fortunately, Mehisto et al. (2008) realised that some language and content teachers have attempted to move the subjects closer together. They attribute this achievement to the Content and Language Integrated Learning (CLIL). David Marsh developed CLIL in 1994: a methodology that provides substantial value for language learning by fusing the subject content and the terminology specific to the content of the subject being taught with the language of the medium of instruction, secondary mathematics, and English (Pérez et al., 2018), in the case of this study. By doing so, it "encourages both independent and cooperative learning, while building common purpose and forums for lifelong development" (Mehisto et al., 2008, p. 7).

Due to the dual focus envisioned in CLIL, equal attention is given to both subject language and subject content. As an educational approach, CLIL allows "curricular content to be taught through the medium of a language that is not a mother tongue of the learners, typically to learners participating in some form of mainstream education at the primary, secondary, or tertiary level" (Dalton-Puffer, 2011, p. 2). The European Commission adopted CLIL as a way of providing learners exposure to the subject language without requiring spending extra time within the curriculum. However, little attention has been given to CLIL in the Global South despite its applicability in South African State schools where about 90% of learners are taught mathematics in English, which is not their native language (Taylor, 2014).

Despite the known affordances of bi/multilingualism in learning mathematics, such as giving learners cognitive flexibility, better problem solving, higher-order thinking skills, and linguistic resources for managing the demands of group mathematics discussions (Anhalt & Pérez, 2013; Hakuta & Diaz, 2014; Zahner & Moschkovich, 2011), learning mathematics in a second language is not a straightforward task. According to Truxaw and Rojas (2014), mathematical language is challenging as compared to informal language. This is because mathematical language is more abstract, contextualised, specific, and culturally determined. Even trying to understand basic academic instructions in a second language is challenging and exhausting (Truxaw & Rojas, 2014). In their study, Truxaw and Rojas (2014) found that learners choose not to openly participate when learning mathematics in a second language, because asking or answering questions in a second language is frightening and tough. Furthermore, if the language and vocabulary are not internalised, learners will not be able to formulate questions. This suggests that learners may not have a conceptual understanding of the mathematics in question.

Nevertheless, research (Ferguson, 2013; Manyonyi et al., 2016; van Jaarsveld, 2011) has shown that learning mathematics using learners' mother languages (especially African languages) has challenges of its own, as most of those languages do not have a fully developed register for this purpose. Chitera et al. (2016) reported efforts being made in some African countries to develop a mathematical register using local languages including Edo and Yoruba in Nigeria, Kiswahili in Tanzania, and Chichewa in Malawi. The strategies used in these countries were either using words from the local languages to develop the register or borrowing terms from mathematical English (Chitera et.al., 2016). Though notable strides in these endeavours were made, the problems associated with the exercise discouraged full-scale implementation and use of the registers in these countries.

Drawing from the insights of Truxaw and Rojas (2014), it becomes evident that the utilization of a second language as a medium for learning mathematics can have a discernible impact on learners' confidence and their levels of active participation. The cause of this may be two-fold. On the one hand, learners might not participate because the language hindered their understanding of the mathematical concepts to be learnt as alluded to in the previous paragraph. Because the concept was not understood, participation was withdrawn. On the other hand, it is possible that conceptual understanding may have been achieved but due to an absence of adequate mathematical language, the learner may not be able to express understanding in a discussion. For these reasons, learners must be encouraged to learn mathematical vocabulary and mathematical language to empower them to engage in mathematical discourses.

In addition to the above, MacSwan and Faltis (2019) state that switching between languages negatively affects the response time for arithmetic operations. In addition, language use and comprehension (or lack of) may sway learners' attitudes and "appreciation of mathematics" (Lin & Tai, 2016) and, in turn, their self-efficacy. Self-efficacy refers to individuals' beliefs about their ability to carry out prescribed tasks (Arslan, 2012; Henk et al., 2012). As learners are taught mathematics in the classroom, enough effort ought to be put into arousing their belief in their abilities and positively influencing how they perceive themselves as doers of the subject. According to Abdullah (2019), individuals' self-perceptions of their abilities are influential in successful goal realisation and the control of the process of attaining these goals. Arslan (2012) further concluded that levels of self-efficacy affect learners' effort, persistence, and resilience. From the above arguments, it can be deduced that strategies that inculcate self-belief and positive perception in learners, not just focus on the manipulation of numbers in mathematics, are necessary. In this research, I suggest that learners ought to be explicitly taught the correct mathematical vocabulary (using the Frayer model) and language as tools for confidently doing mathematics to promote conceptual understanding, procedural proficiency, and enhanced performance.

1.2 Aim of the study

The aim of this study is to assess whether employing the Frayer model as a pedagogical tool for focused mathematical vocabulary acquisition can effectively foster conceptual understanding among mathematics students and consequently lead to notable improvements in their performance within the field of mathematics.

1.3 Research objective

The research endeavour aimed to systematically investigate the impact of targeted mathematical vocabulary acquisition on the overall mathematical vocabulary proficiency of learners and its subsequent influence on their comprehension of mathematical concepts. This inquiry was conducted by rigorously assessing the degree to which facilitating the acquisition of mathematical vocabulary enriched learners' conceptual grasp and subsequently catalysed discernible improvements in their academic performance within the domain of mathematics.

Focused vocabulary instruction was facilitated by the Frayer model. The Frayer model is a graphic organiser. A graphic organiser is an illustration that gives a visual representation of concepts, ideas, or facts. I chose a graphic organiser for the learning of mathematical vocabulary in this research because literature has proved that graphic organisers offer their users several benefits. To start with, Kiewra (2012) claims that they are powerful, easy to use and allow teachers to examine learners' thinking and learning of a specific topic or concept. As visual representations of concepts, they assist learners in summarising, relating, and making meaning of those concepts in a text (Singleton & Filce, 2015). They also disclose students' prior knowledge and promote their involvement in comprehension-facilitation discussions (Kirylo & Millet, 2000, Singleton & Filce, 2015). Lastly, teachers can use graphic organisers for formative assessment purposes (Praveen & Rajan, 2013).

There is a wide range of graphic organisers, e.g. concept maps, mind maps, sequence charts, compare/contrast diagrams, cause-and-effect diagrams, main idea and details charts, attribute charts, etc. In this research, I chose to use the Frayer model, as the use of it allows students to think more deeply about the terms they are learning about (Kustati & Prisillia, 2018). According to Buehl (2017), the purpose of the Frayer model is to identify and define unfamiliar concepts and vocabulary. Learners define a target vocabulary, describe the essential characteristics of the vocabulary, and give examples and non-examples as habits to reinforce their understanding. This information is placed on a chart that is divided into four sections to offer a graphic illustration for students (Frayer, 1969). The Frayer model allows both concept imaging and definition to happen concomitantly, while also functioning as a note-taking instrument (Jiang et al., 2018). Research by Valley (2019) has shown that the Frayer model is an effective strategy for learning vocabulary in language subjects. The Frayer model was also seen to be useful in content subjects, including science and geography (Sacapaño & de Castro, 2022; Rampersad et al., 2020). Wanjiru and O-Connor (2015) also concluded that the model works for learning mathematical vocabulary.

1.4 Rationale of the study

Some mathematics education research (Bhuvaneswari & Umamaheswari, 2020; Kung et al., 2019; Sithole, 2013; Turan & De Smelt, 2022) has shown an interdependence between language ability and the understanding of mathematics by learners. Sithole (2013) alluded to the fact that learning mathematics is dependent on language, as it requires reading, listening, writing, and discussing. In addition, mathematics teachers use language to convey concepts and procedures, read texts, and solve problems (Essien, 2018). This follows, as teachers provide exemplifications and explanatory talks during classroom instruction (Adler & Ronda, 2016).

Due to the role that mathematical language plays, learning mathematical vocabulary and language becomes a paramount task for mathematics learners. Although a considerable amount of research has been done on language in South Africa (Adler, 1998; Berger, 2015; Essien 2010, 2018; Setati, 1996, 1998, 2002, 2008; van Jaarsveld, 2007, 2011, 2016, 2017, 2018) more can still be done on learning mathematical vocabulary and language. Despite a lot of language research, much of the existing research focuses on language as a cooperative tool for teachers and learners. More attention also needs to be given to the pedagogical and conceptual value of the use of precise mathematical language by teachers and learners. Little knowledge of the terms and phrases of the subject language can incapacitate a learner’s understanding of the subject (Zwiers, 2013), as routine proficiency is related to pronounced foci (van Jaarsveld, 2017). To enhance learners' understanding of mathematical concepts and their procedural proficiency, enough focus must be put on their learning of the associated mathematical vocabulary and language.

Similarly, Wilkinson (2019) stated that language could be a major pedagogical and learning tool. With language, learners understand how things are said, and move on to understanding what is being said. Following this understanding, learners will also come to realise why those things are said. Nevertheless, research focusing on factors that affect mathematics achievement (Kiwanuka et al., 2015; Makgato & Mji, 2006; Visser et al., 2015) examined other factors aside from the learning of mathematical vocabulary. Without downplaying the researched foci, this current research explored the extent to which increasing the learning of mathematical vocabulary in the domain of mathematical functions would enhance learners' conceptual understanding and performance in the subject.

While conducting this research, awareness arose regarding arguments against the celebration of mathematical vocabulary acquisition as synonymous with mastering mathematics itself. For instance, (Moschkovich, 1999, 2002, 2007a, 2007b, 2007d, 2013) posits that educators should address more than just vocabulary, emphasizing the need to support learners' engagement in mathematical discussions during English language acquisition. The study conducted by Moschkovich and Cruz (2012) indicated that the primary objective of mathematics instruction should revolve around fostering the active participation of all students, regardless of their English proficiency, in discussions that centre on vital mathematical concepts and logical reasoning, rather than concentrating solely on pronunciation, vocabulary, or rudimentary linguistic skills (Moschkovich & Cruz, 2012, p. 2).

The discovery made by Moschkovich and Cruz, demonstrating the necessity of aiding learners in participating effectively in mathematical discussions, underscores the importance of not underestimating the role of precise mathematical vocabulary. Assisting learners in acquiring and comprehending mathematical terminology serves as a means to enable their enhanced engagement in mathematical discourse. This particular study did not delve into issues related to pronunciation or basic linguistic skills, but rather placed emphasis on vocabulary as a conduit for comprehending essential mathematical principles and fostering a deeper understanding of the associated reasoning processes.

Moschkovich and Cruz (2012, p. 3) concluded that "instruction should provide opportunities for students to actively use mathematical language to communicate about and negotiate meaning for mathematical situations". This conclusion gives yet another justification for learning mathematical vocabulary. Learning the vocabulary includes learning the whole concept image that accompanies a word that, in turn, signifies an object. In the view that for learners to communicate and negotiate effectively, individuals need to know the vocabulary that combines to form the language for that communication. If, for instance, a learner is required to find the sum of two consecutive even numbers; this task will be impossible if the learner does not know what a sum is, what even numbers are, and what consecutive means. The concept resides within the words: sum, even, and consecutive. These meanings necessitate initial unpacking for learners to comprehend and perform the required tasks. Such instances supported by the researcher's teaching experience underscore that fundamental mathematical concepts, both overarching and those particular to mathematical functions, are inherently intertwined with subject-specific vocabulary. Although mastering mathematical vocabulary alone may not provide a comprehensive solution to the challenges encountered in the teaching and learning of mathematics, this research contends that the acquisition and application of precise mathematical terminology can significantly contribute to addressing these issues.

1.5 Research questions

The research explores the link between learners' mathematical vocabulary, their learning of mathematics, and their subsequent performance in mathematics. As an approach, learning vocabulary will be facilitated using the Frayer model.

The main question that guides this research is: **What affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?**

In an endeavour to navigate around this question, some sub-questions emerged, and answers to these sub-questions will help to respond to the main question.

- **Sub-Question 1: How competent are Grade 11 mathematics learners in associating mathematical vocabulary with a given mathematical object?**

Mathematical concepts are abstract, but they are represented by words, symbols, or visuals. The number of words that learners would associate with any given mathematical object related to the concept of the mathematical function had to be seen. The knowledge of the vocabulary would be explicated by way of defining the terms, characterising them, and exemplifying them.

It is one thing to associate a term with an object and another thing to associate and expatiate the term. All of these cognitive skills had to be surveyed in this research.

- **Sub-Question 2: What are the approaches that learners employ to acquire mathematical vocabulary during their mathematics lessons?**

Vocabulary and language envelope mathematical concepts. The mathematical language takes the form of verbal, symbolic, and visual semiotic structures. In this research, the verbal structure was the focal point. As such, it was imperative to investigate what learners had in their academic artillery for learning mathematical vocabulary to ensure improved mathematical language. According to Asyiah (2017), a vocabulary-learning strategy is knowledge about the mechanisms required to learn vocabulary, coupled with the steps/actions that the students take to find the meanings of unknown words, retain these meanings in their long-term memories, recall these meanings at will, and use them in either the oral or written mode. Though it cannot be generalised that students cannot, on their own, come up with such strategies, it is safe to say that teachers need to guide learners in this regard.

- ***Sub-Question 3: In what ways are learners exposed to* the *mathematical vocabulary of functions during lessons as they learn about mathematical functions?***

Classroom discussions present an opportunity for learning and understanding. Aside from being used as a platform for asking questions, classroom discussions serve the purpose of demonstrating understanding (or lack thereof) among learners. Learners were observed as they engaged with mathematical concepts during lessons. During those discussions, I noted the various sources from which the mathematical vocabulary would be imparted to the learners. Their discussion provided an opportunity to assess their understanding of the mathematical vocabulary and concepts.

- **Sub-Question 4: How effective is the Frayer model as an approach for learning the mathematical vocabulary of functions?**

The proposed approach for this research is a specialised graphic organiser – the Frayer model. It then had to be seen if by exposing the experimental group to this instrument and paying special attention to the learning of mathematical vocabulary using this technique, there would be an increase in the vocabulary that the learners retained in their memories and what learners knew about mathematical concepts.

- **Sub-Question 5: To what extent does competency in mathematical vocabulary link to learners' performance in mathematics?**

Several mathematical tasks invoke learners' procedural and conceptual understanding. These manipulative skills are themselves not devoid of connections with other mathematical skills. In this research, there was a desire to establish the extent to which the said manipulative skills were linked to learners' knowledge of mathematical vocabulary, with the latter presumed to be a precursor to the former.

1.6 Statement of the problem

Mulwa (2014) found that in some countries, such as Kenya, the role of mathematical language in the learning of mathematics has not yet been dealt with in practice. In countries where such research was done, it was shown that the language of mathematics challenges learners (Belhiah & Lamallam, 2020; van Jaarsveld, 2016).

Usually, the mention of mathematics is mostly met with dejected bodies and downcast eyes (Capuno et al., 2019) and, when doing mathematics, learners generally regarded the subject to be only the manipulation of numbers and considered language and vocabulary to belong to other disciplines (Capuno et al., 2019; Ramsey, 2013; Seah et al., 2016). In view of this study, this fixation on numerical manipulation may form part of the explanation for the dismal performances in the subject that have persisted over the years. As a result, learners do not put equal effort and/or allocate time to the learning of all the elements of the language of mathematics: words, visual mediators, narratives, and routines, as coined by Sfard (2008). The least considered of these elements are words and narratives (Ramsey, 2013). This is so, despite the dependence of the verbal articulation of one's conceptual understanding on the strength of one's language development (Vygotsky, 1986). Frequently, mathematics learners fail to express themselves, and instead they mumble, are mute, or produce incoherent or unintelligible statements such as "In absolute values you have two answers for x except in the inverse", which was produced by a student trying to describe what an absolute value equation is (van Jaarsveld, 2007, p. 191). This is one dimension of the problem. The correct mathematical vocabulary to interpret an answer may be lacking, despite a routine procedure having been mastered. At times, the procedure is known, but inappropriate vocabulary is used, for example referring to an expression such as $\frac{x^2-x}{x}$ as an equation (van Jaarsveld & Ameen, 2017). Incorrect vocabulary could result in using an incorrect procedure[1], such as simplifying instead of solving.

In similar research with university students, van Jaarsveld (2016) noticed that students were able to simplify a given expression correctly to obtain 2x - 1, however, they continued to equate the simplified expression to zero and forced a solution: $x = \frac{1}{2}$. The error or misconception of forcing a solution in an algebraic expression could have been avoided if students had a better command of their mathematical vocabulary. The ability to distinguish between equations and expressions implies knowledge of what each term means. It also implies the nature of the answers expected from each of them.

When learners with undeveloped mathematical vocabularies are confronted with situations where they are required to justify, explain, or interpret their solutions, they stumble, as reported in the DBE diagnostic reports (DBE, 2021). It is also common for learners not to attempt to answer certain questions because of their unfamiliarity with the vocabulary. This affirms Vygotsky's (1986) assertion that conceptual understanding is impossible without word development, and thinking in concepts only happens within the confines of verbal thinking. Understanding mathematical vocabulary might give learners the basis for thinking and articulating mathematical content. Learners cannot participate proficiently and engage with mathematical content if they are deficient in the language of mathematics.

In this study, the investigation centred on how mathematics learners circumnavigate the issues of mathematical vocabulary and language as learners acquire proficiency in mathematical functions. This research, therefore, explored learners' knowledge of the selected mathematical vocabulary of functions. Irrespective of the way they learn mathematical vocabulary, learners need to have mastery of it to use it to do their mathematics.

While conducting the research, the focus was on comprehending learners' techniques for acquiring vocabulary and the utilization of mathematical vocabulary within mathematics classrooms for questioning, explanation, justification, and demonstration of comprehension. Additionally, a vocabulary-learning tool, namely the **Frayer model**, was introduced to the learners. Subsequently, an examination was conducted on the learners' vocabulary proficiency and their ability to express this vocabulary (mathematical language). The investigation also prioritized exploring the correlation between the acquisition of mathematical vocabulary

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[1] An equation is solved, while an expression is simplified.

(knowledge, vocabulary proficiency, and vocabulary expression) and proficiency in mathematical procedures, as well as conceptual understanding and performance.

1.7 Significance of the study

This study is significant in the sense that it investigates the possibility of increasing learners' mathematical vocabulary knowledge by introducing them to the learning of mathematical vocabulary using the Frayer model within a CLIL model. By doing so, the study assesses whether a CLIL model has value for implementation in South African classrooms.

The research could contribute to the field of education by introducing the Frayer Model as an effective pedagogical approach for teaching mathematical vocabulary. This model focuses on not just memorization but also deep understanding, which could inform teaching strategies for other mathematical concepts as well. The findings of this research might provide insights into the effectiveness of vocabulary-focused teaching strategies in general. Since the Frayer had proved to be successful, this research could encourage educators to explore similar methods in other subjects or at different educational levels. Understanding mathematical concepts involves cognitive development. The research could shed light on how the use of specific instructional techniques, like the Frayer Model, can contribute to cognitive growth, particularly in terms of critical thinking and analytical skills. In the same vein, the positive impact of the Frayer Model could inform curriculum designers and educational policymakers on integrating such vocabulary-building strategies into the official curriculum. This could lead to more comprehensive and effective mathematics education. Additionally, this research could lead to professional development opportunities for teachers to learn about and incorporate this method into their teaching practices, ultimately benefiting their students by making mathematical learning more interactive and enjoyable.

1.8 Conclusion

The chapter highlighted the existence of challenges to the learning of mathematics, both in general, and in South Africa in particular. These challenges manifest in a low pass rate at Grade 12 (the high school exit point). The average national pass rate of achieving 40% and above in the subject has been 36,1% over the last decade. Causes for this low national pass rate had been attributed to socioeconomic factors, overcrowding in classrooms, shortage of qualified mathematics teachers, and so on. The chapter highlighted and explained the importance of knowing subject matter, vocabulary, and the mathematical language (in the form of verbal, symbolic, and visual semiotic structures) in the teaching and learning of mathematics. Generally, language is a cooperative tool between teachers and learners, and humans and text, and is a vehicle for cognitive development. Mathematical vocabulary and mathematical language are the interfaces between the human elements and the subject matter, and they influence the extent of thinking, understanding, and communication in the subject. High-school mathematical content is full of technical language that can either enhance or impede learners' understanding. For this reason, the chapter presented the main research question: **"What affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?"** From this research question, five sub-questions will be used in an endeavour to answer the research question. The chapter also elaborated on the problem statement by highlighting that a lack of understanding of mathematical vocabulary and mathematical language is a possible barrier to the learning of mathematics. Furthermore, the purported contribution of the research was explained under the headings of the rationale and significance of the study.

In the subsequent sections, a literature review, theoretical frameworks, conceptual framework, and the methods of data collection and analysis will be discussed. These discussions will be followed by the presentation, analysis and interpretations of the results obtained during the study. Lastly, conclusions and recommendations will be presented.

CHAPTER 2 : LITERATURE REVIEW

In this chapter, a review of the literature pertaining to the present research is presented. The research conducted explored the impact of employing the Frayer model in focused vocabulary instruction on learners' mathematical vocabulary knowledge and subsequent conceptual understanding and performance in mathematics. The literature review involves an analysis of various studies conducted in the domain of mathematical language and vocabulary. This chapter situates the work within the field and clarifies the specific area addressed by this research, which investigates students' acquisition of both mathematical vocabulary and the concept of mathematical functions. The current study aims to elucidate the affordances that the integration of focused vocabulary learning using the Frayer model and mathematical content offers for conceptual understanding and performance in mathematics. The inaugural section of the literature review, Section 2.1, provides an in-depth exploration of the fundamental concept of mathematical functions. Following this, Section 2.2 delves into the crucial topic of conceptual understanding. Section 2.3 conducts a thorough examination of the relevant literature pertaining to the research sub-questions, offering valuable insights. Lastly, Section 2.4 references existing research in the field of mathematics education that has grappled with the overarching challenge of enhancing the acquisition of mathematical vocabulary, further enriching the research's contextual understanding.

The literature review for Section 2.4 is presented in accordance with the five research sub-questions as follows:

- Competency of mathematics learners in associating mathematical vocabulary with a given mathematical object.
- Vocabulary-learning strategies.
- Ways that learners are exposed to mathematical vocabulary during the learning of mathematics in the classroom.
- The Frayer model as a mathematical-vocabulary-learning strategy.
- The link between competency in mathematical vocabulary and learners' performance in mathematics.

2.1 The concept of a mathematical function

The concept of a function occupies a central position in the mathematics curriculum and is central to learners' ability to "describe relationships of change between variables, explain parameter changes, and interpret and analyse graphs" (Dubinsky & Wilson, 2013). In support of the centrality of the concept of functions in schools, Brahier (2020) advocates for the instruction of functions from pre-kindergarten through Grade 12. However, research has shown that despite its importance, the concept of a function is one of the most difficult concepts for secondary-school and college students to understand (Dubinsky & Wilson, 2013; McCulloch et al., 2020; Muzaffer, 2013). According to Ayalon and Wilkie (2019), learners face numerous problems when attempting to understand the concept of a function, and when they learn to use the various symbols associated with the concept. Additionally, Ayalon and Wilkie (2019) postulated that a function can be regarded in two ways: structurally as an object and operationally as a process. This dual nature of the concept also poses a challenge to learners (Hatisaru, 2020). As an object, literature states that the function is a set of ordered pairs, while, operationally, it is a well-defined means of getting from one system to another (Hatisaru, 2020)

Over the years, the concept of a function has undergone an evolution, right up until it reached modern-day recognition as a univalent correspondence between two sets. In many countries, functions appear in the school curricula with the hope that at the end of high-school, learners will understand the concept of a function in general and be familiar with specific types of functions (Denbel, 2015). In most cases, the learners are expected to know linear, quadratic, general polynomial, exponential, logarithmic, trigonometric, and piecewise functions.

The goals for including functions in curricula vary from one country to another, and from one grade to the next. However, despite the differences in goals, the concept of a function may be included in curricula as an intrinsic part of mathematics. In this regard, some learners learn functions because of the possibility of them becoming relevant in the learners' future studies. On the other hand, functions are learnt as a unifying concept across the entire mathematical curriculum, and they permeate virtually all areas of the discipline, such as algebraic operations on numbers, transformations on points in the plane or space, intersection, the union of pairs of sets, and so forth (Alsina & Salgado, 2022). These concepts can be learnt and investigated, as functions relate to a univalent correspondence between elements of sets.

Functions are used as a vehicle for clarifying mathematical thinking and reasoning, and as a tool for proving statements (Denbel, 2015). They also provide a means for extra-mathematical ends. This manifests in real-life situations that need to be described in the context of correspondence, such as the functional relationship between the speed of a car and the distance it travels.

Teaching experience has also played a role in the selection of the domain of functions as the research focus. The concept is perceived as fundamental within the mathematics curriculum, necessitating a thorough understanding by learners, as it greatly contributes to their performance across various mathematics domains. In the examination of mathematics topics for Grades 10–12, the curriculum and assessment policy statement (CAPS) emphasizes that teachers and learners should "work with relationships between variables in terms of numerical, graphical, verbal representations, and symbolic representations of functions and convert flexibly between these representations (tables, graphs, verbal descriptions, and formulae)" (DBE, 2011, p. 12). The seamless transition between these proposed modes is only feasible if there is an adequate vocabulary to support it. Therefore, the incorporation of van Jaarsveld's (2016) language repertoires offers a facilitative, structured, and focused system of vocabulary associated with each representation.

2.2 Conceptual Understanding and Procedural Proficiency

More than the aspect of simply "knowing that", conceptual understanding can be described as "relational representations," showing that "two or more entities are mentally linked through some sort of a relation" (Österman & Bråting, 2019; Smith et al., 2018). As such, conceptual understanding can be regarded as knowledge-rich in relationships (Hiebert & Lefevre, 1986). Kieren (1993) defined it as "the interweaving of the intuitive and formal knowledge on a personal basis" (p. 49). These ideas point to conceptual understanding as being more than memorisation of separate chunks of material, but rather as the ability to see interconnections between knowledge.

On the other hand, procedural proficiency is a goal-directed action sequence that points towards knowing how to do something (Graven & Stott, 2012). In this manner, it involves memory strategies and mathematical algorithms. It entails following a particular sequence of well-defined actions to produce the desired outcome. In the case of mathematics, the desired outcome is the correct answer to a problem. This means that an individual using a procedure is not obliged to reflect on the meanings of the elements being implemented (Hallett et al., 2010)

Research has shown that conceptual understanding and procedural proficiency have a bidirectional relationship. In this, it implies that conceptual understanding might be thought to come first, before procedural proficiency, and, in turn, procedural proficiency might be thought to come first instead. However, the focus of this research is not on this relationship, so there will be no further discussion of it. The primary concern in this research pertains to the impact that a concerted effort on learning mathematical vocabulary will exert on both aspects, and ultimately, on the learners' performance.

Indicators of conceptual understanding in mathematics of functions

Kasmer and Kim (2011) and Yurekli, Stein, Correnti, and Kisa (2020) have thoroughly highlighted critical indicators of conceptual understanding within the domain of mathematical functions. Their work underscores that true comprehension surpasses mere rote memorization and procedural adherence. Gardee and Brodie's (2021) study emphasizes the limitations inherent in relying solely on rote memorization in education. This conventional approach, characterized by memorizing facts and formulas without meaningful connections to prior experiences or knowledge, often results in a superficial grasp of the subject. Information acquired through rote learning lacks integration with an individual's cognitive framework, making it difficult to establish lasting memory.

To address the shortcomings of rote memorization, educators are increasingly embracing pedagogical strategies that foster meaningful learning (Rugh et al., 2023). One such approach is the utilization of the Frayer Model, a structured framework that facilitates connections between existing knowledge and new concepts. By guiding students through exploring a term's definition, characteristics, examples, and non-examples, the Frayer Model prompts learners to relate new information to their pre-existing mental schema. This cognitive interweaving not only deepens comprehension but also enhances the retention of new knowledge.

The Frayer Model provides a concrete methodology for nurturing authentic understanding. This cognitive scaffold not only enriches the learning experience but also equips students to actively construct knowledge, moving beyond passive accumulation of isolated facts. Embracing this approach empowers educators to transform students into active participants in their learning journey, turning the educational process into a voyage of discovery that cultivates genuine conceptual understanding.

As proposed by Yurekli et al. (2020), several indicators illustrate the manifestation of conceptual understanding:

· Connections between Representations: Students fluently translate between algebraic, graphical, and numerical representations of functions, comprehending how changes in one relate to others.

· Graph Interpretation: Students accurately deduce a function's behaviour from its graph, grasping concepts like slope, intercepts, and transformations.

· Domain and Range Mastery: Students grasp domain and range concepts, identifying limitations on input and output values and explaining inclusions/exclusions.

· Function Composition: Students confidently compose and decompose functions, evaluating them and understanding properties.

· Inverse Functions: Students explain inverse functions and their relationship, finding inverses algebraically and understanding reflections across $y = x$.

· Functional Relationships: Students expound on how parameter changes affect function behaviour, relating shifts and transformations.

· Function Behaviour and Limits: Students predict function behaviour near critical points and at infinity, applying limits.

· Real-World Applications: Students apply functions to real scenarios, modelling and interpreting using functions.

· Function Families: Students recognize patterns in different function families, understanding parameter impacts.

· Problem Solving and Creativity: Students solve complex problems, creatively applying conceptual understanding.

· Effective Communication: Students articulate reasoning, justifying solutions using precise math language.

· Multiple Representations: Students convey concepts through various representations, signifying holistic understanding.

These indicators reveal that conceptual understanding of functions encompasses profound insight into their workings, properties, relationships, and applications. It transcends procedural knowledge to include analysis, explanation, interpretation, and application across contexts.

2.3 Addressing the research sub-questions using literature

As previously mentioned, the initial segment of the literature review aligns with the five research sub-questions proposed for this study. Presented here is a reflection and connection to the existing literature in the field of mathematics education, structured around each question.

2.3.1 Competency of mathematics learners in associating mathematical vocabulary with a given mathematical object

The first research sub-question is answered by assessing the amount of correct and applicable mathematical vocabulary that learners associate with a given mathematical object in the form of a graph. Learners were given a graph and were then asked to list the vocabulary that they would associate with the graph. In the given question, learners were not constrained in terms of the specific vocabulary they could use—whether it pertained to instructions, properties, or features of the mathematical object. Learners could choose to give just one word or a term, but it was mostly preferred for them to provide detailed explanations. This preference stemmed from the recognition that comprehensive responses would greatly aid the researcher in gauging the participants' proficiency in vocabulary. Notably, during both the pretest and post-test phases, learners were explicitly informed that their responses could extend beyond mere words and include sentences and elaborations to enhance their answers. In order to explore this aspect, a comprehensive literature review was conducted, aligning with the focus on learners' grasp of vocabulary pertinent to mathematical concepts and objects.. In the context of this research, "vocabulary proficiency" takes on the specific meaning of the ability to accurately use mathematical words to convey precise facts, provide illustrative examples, and delineate essential characteristics associated with mathematical vocabulary and concepts. Mathematical language can be expressed in words, with graphic representations, and with symbols (Solano-Flores et al., 2013). It cannot be said with an exactitude that one of these forms is easier to understand than the rest. However, learners are encouraged to migrate from one form to another (DBE, 2015; Mofolo-Mbokane, 2012). In the research test, learners were given a graphical representation of a mathematical concept and were asked to write down mathematical vocabulary that they would associate with the visual object. Visuals can make it easy to remember concepts, at least superficially, for some people. A visual representation also holds the potential to scaffold students with strong mathematical ability but weak language skills (Nguyen & Cortes 2013). It was anticipated that, when presented with a visual object, learners would retrieve from memory all associated vocabulary without any prompting, which would then be transcribed. Since the mere recollection of vocabulary does not ensure vocabulary proficiency, learners were urged to expand upon the provided vocabulary.

Despite the recent surge in research focused on mathematical vocabulary, a thorough exploration spanning the years 2012 to 2022, conducted through search engines like Google, failed to yield any discernible evidence of studies explicitly investigating the proficiency of mathematics learners in accurately correlating appropriate mathematical terminology with corresponding mathematical entities. The most common research on graphical competence relates to "levels in the critical understanding of graphs that vary from a complete inability to make sense of the graph, through reading isolated elements or being able to compare elements, to the ability to predict or extrapolate data that are not included in the graph" (Arteaga et al., 2020). In Arteaga et al. (2020), 745 Chilean Grade 6 –7 children were used in a study that aimed to describe the errors conveyed and reading levels reached by these primary-school children when working with line graphs. The study was premised on the fact that, in recent years, reading elementary statistical graphs is regarded as an element of statistical literacy, and is necessary to function in society due to the abundance of data representations presented in the media (Engel, 2019; Ridgway, 2016). In the study, children were asked to do tasks that involved: identifying the title of the graph, identifying variables on the graph, reading data from the graph, and doing an inverse reading of the graph. The study concluded that few of the students achieved the highest reading level and the majority made occasional errors in the reading of the graphs. It shows that graphical literacy was very low among these children. All the information that these children needed for answering the questions was given on the graphs, but they failed to demonstrate graphical literacy. In this current study, participants were provided with a graph, and the task was to associate mathematical vocabulary with it. Differing from typical studies on graphical literacy (Batanero et al., 2018; Díaz-Levicoy et al., 2021; Muñoz et al., 2020) that focused on statistical graphs, the graph used in this study was generated from mathematical functions.

Díaz-Levicoy et al. (2021) concluded that teachers commit errors when defining statistical graphs, identifying concepts linked to bar graphs, and making calculations and predictions based on the information represented on graphs. The study was based on content analysis and was conducted with 102 early-childhood education preservice teachers from Chile. Batanero et al. (2018) also conducted a content analysis of Spanish textbooks to investigate the nature of questions presented to six-to-eight-year-old children related to statistical graphs. They observed that the inclusion of pictograms beginning in Grade 1 was helpful to children, as the icons used to represent the data in these graphs helped children to understand what was being represented. Their study concluded that the development of graphical competence in the Spanish curricular guidelines is reflected and supported by the textbooks. This was also reflected in external assessments, such as assignments, projects, and examinations. Although the textbook supports the learning of graphs, the learning itself is based on identifying and naming graphs, interpreting graphs, drawing graphs, making calculations based on graphs, as well as interpolation and extrapolation with these graphs. None of the exercises and tasks in either Díaz-Levicoy et al. 's (2021) or Batanero et al.'s (2018) studies required learners to associate correct mathematical vocabulary with a mathematical graph, which is the case in this study. Instead, their studies discovered that some textbooks put too much emphasis on computation with graph data when compared to any other skill. The current study regards associating mathematical vocabulary with a graph to be one of the fundamental skills that was lacking in the Spanish textbooks analysed by these previous studies.

Another study conducted with statistical graphs is that of Muñoz et al. (2020), in which 240 university students studying a BA in Elementary Education at a teacher education school in Mexico were given a test in which they had to analyse a statistical table and a graph. After taking the test, a sample of nine students was interviewed to determine their level of comprehension and the difficulties they encountered. The results showed that the students had varying levels of conception of the graph and the table, as well as the basic statistical concepts;

suggesting that the students had not thoroughly worked with statistical graphs and tables before. Like most studies on graphs in statistics, this study did not address the issue of associating mathematical vocabulary with a graph.

Jojo (2019) investigated students' conceptual understanding of cubic functions in differential calculus. In the study, 42 Grade 12 learners from a school in Mount Ayliff in the Eastern Cape (South Africa) were exposed to instructional design using activities aimed at equipping them with a conceptual understanding of cubic functions. The participants then completed a questionnaire, where the researcher wanted to find out the participants' existing conceptual understanding and their general interpretation of cubic functions. Following the questionnaire was an interview with three learners who were purposefully selected from the 42 learners. These three learners had provided satisfactory responses to the questionnaire. During the interview, learners' responses were examined for their "ability to label the intercepts, points of inflection, the minima and maxima and the shape of the drawing produced for the cubic functions" (Jojo, 2019, p. 5). The interview was a "verification and conjecture-making activity" during which the interviewees described the steps they followed when drawing the cubic functions. During the interview, the learners explained that drawing a cubic function was a multi-step process. A conclusion reached by the study revealed that the multiple procedural steps followed when drawing cubic functions requires students to have a manifold understanding of the concept.

Jojo's (2019) study is one of the unique studies that exists in mathematics education, where learners were asked to relate a graph to its correct mathematical vocabulary. This was done by way of asking three students to identify and give meanings of features of a mathematical graph (cubic function). The features that they had to identify and give meanings for were x -intercepts, y-intercepts, stationary points, and shape of the graph. These features together with procedural steps, such as factorisation, differentiation, and the fusion of these concepts, were used to graphically represent the function. Unfortunately, the study did not conclude whether the students were competent or not in identifying and giving meanings of the technical vocabulary in question.

In a study to analyse mathematical vocabulary knowledge, Powell et al. (2017) sampled 65 Grade 3 and 128 Grade 5 children in one school district in a state in the Southwest USA. Three measures were assessed, namely general vocabulary, mathematical vocabulary, and mathematics computation. Lastly, an assessment was done to determine the influence of mathematics computation on mathematical vocabulary. Knowledge of general vocabulary was discovered to positively influence knowledge of mathematical vocabulary. The researchers assessed knowledge of mathematical vocabulary by way of underlining key words in a statement, to indicate to learners that they should demonstrate their knowledge of that vocabulary as they answered a mathematics question, for example "Draw a right angle, Write a proper fraction" (Powell et al., 2017). In this instance, mathematical vocabulary was linked to mathematical computation or geometric construction. This differs from this current study in which learners were tasked with associating mathematical vocabulary with a provided mathematical object.

The study by Powell et al. (2017) concluded that most children did not understand mathematical vocabulary; including vocabulary terms introduced in kindergarten, Grade 1, and Grade 2, for example, proper fraction. To this end, the study recommended that it is necessary to implement focused vocabulary instruction in the same way as in science or history. As such, teachers should learn effective strategies for teaching mathematical vocabulary proactively (Riccomini et al., 2015), and they should also know when to replace informal vocabulary with precise formal vocabulary (e.g. replacing bottom number with denominator).

The study also concluded that because an understanding of mathematical vocabulary aids in mathematics communication, assessing the mathematical vocabulary that students understand or do not understand is helpful to teachers. Another conclusion reached by the study was that there is a positive correlation between mathematics computation and knowledge of mathematical vocabulary. This relationship is such that an improvement in mathematics computation positively influences knowledge of mathematical vocabulary (Powell et al., 2017).

In the prior discussions, the absence of literature suggesting any prior studies evaluating learners' competence in associating mathematical vocabulary with a provided mathematical object was emphasized. This observation holds true even though Bates et al. (2021) pointed out that learners encounter cognitive obstacles when faced with graphs. There is a need for research to examine whether learning mathematical vocabulary can alleviate these cognitive obstacles. Given this context, this research represents an exploration of uncharted territories. In learning areas such as life sciences where learners are asked to label graphs, they are, at times, given a list of words to choose from for the purpose of labelling the graph. In instances where the learners are not given a list of vocabulary to choose from, they do so based on remembering a similar graph that they would have come across in their textbooks previously. In mathematics, where they seldom come across labelled graphs of functions, the task of associating mathematical vocabulary with a graph is both challenging and exciting. That being the case, the research question in this study is a unique question and, as such, it helps the study to address a gap that exists in the literature.

2.3.2 Approaches used by learners to acquire mathematical vocabulary.

Mathematical vocabulary is an integral part of understanding mathematical concepts. Learners must develop the mathematical vocabulary to be able to explicitly communicate their mathematical reasoning with peers (Bicer et al., 2015). Often, if learners are deficient in mathematical vocabulary, they might be unable to process and make sense of what they are trying to learn. In addition, learning mathematics is a building-block process. Concepts and skills build one on top of the other. If learners fail to understand a concept, there is a chance that they will fail to understand the next skill too. In addition, the learners might develop misconceptions. As such, learners must acquire the foundational building blocks first to advance to higher levels. The research posits that a strong proficiency in comprehending mathematical vocabulary and mathematical language plays a pivotal role in acquiring the fundamental building blocks for learning mathematics. In this context, the teaching of mathematical vocabulary throughout the year is viewed as an excellent approach.

Teachers ought to teach their learners mathematical vocabulary as a way of equipping them with this learning resource. Two popularised instructional models for this purpose are contextual learning methods and direct methods. Within these models, vocabulary can be taught either overtly or covertly. Overt strategies are those where vocabulary is explicitly taught to learners, while covert strategies are those where vocabulary is embedded in some tasks and the learners are not made aware of the fact that they are learning the vocabulary.

Contextualised instructional methods allow learners to create mental images by way of enabling them to witness the processes and actions behind the vocabulary (Bicer et al., 2015). Learners are allowed to observe how mathematical vocabulary is used in mathematical contexts. They develop images of the word and a profound understanding of the vocabulary. Within this school of thought, learners are believed to experience a great deal of improvement in vocabulary growth by exposing them to meaningful input such as drama, games, quizzes, and task-based instruction (Willis & Willis, 2013). Cohen and Johnson (2011) and Cohen (2012) also confirmed that imagery-based interventions help learners to form mental images of concepts being taught. This makes it easy for meanings of associated vocabulary to be retained and recalled by learners.

On the contrary, some studies have shown that without direct vocabulary instruction, learners might not make progress in learning mathematical vocabulary (Riccomini et al., 2015). For example, a word such as range needs to be explicitly taught for learners to know how to express a range in functions, as opposed to how to express it in statistics. This approach focuses more on in-class activities. Studies have shown that important vocabulary (mostly technical vocabulary) must be taught using direct instruction, for example using word lists, graphic organisers, word games, etc. (Carrier, 2011). The research advocates for the direct instruction of mathematical vocabulary. It has been demonstrated that direct vocabulary instruction helps learners acquire a greater number of words and has a positive impact on their vocabulary development (Ghapanchi et al., 2012; Soureshjani, 2011).

Some general strategies to carry out direct vocabulary instruction are:

- Asking learners to have mathematical vocabulary notebooks.
- Displaying a mathematics words wall with visuals.
- Encourage learners to use graphics organisers.
- Continuous review of mathematical vocabulary during whole-class or small-group discussions.
- Having learners turn in quick video recordings explaining mathematical vocabulary.

Mahmud et al. (2020) investigated how oral questioning can enhance mathematical vocabulary learning in primary-school children in Malaysia. Six primary-school teachers drawn from six different primary schools were interviewed, their lessons observed, and their field notes examined for the purpose of the study. The study found out that there were six strategies that teachers employed within oral questioning to enhance the mathematical vocabulary of their learners. The identified strategies within oral questioning were: "Asking students to pronounce the information in the question correctly, *Ask students about mathematical terms and keywords,* restate in their own words, *Asking students using verbal-close questions*, Ask questions repeatedly and Using seeking-clarification questions" (Mahmud et al., 2020).

The study did not establish which of the six strategies within oral questioning was more effective than the others. In spite of this, the study did establish that oral questioning enhances students' thinking, and also helps students to improve their mastery of mathematical language. During the oral questioning in the study, the teachers, rather than their learners, initiated the discussions. When the teachers asked their learners about mathematical terms and keywords, they required the learners to give definitions of the terms and solicited them to demonstrate their understanding of the vocabulary, for example: "What do we mean by volume?" (Mahmud et.al., 2020, p. 6). The definitions were constructed during the oral questioning by correcting learners' own definitions if they were incorrect or reinforcing them if they were correct. Though key mathematical vocabulary was discussed, no notes of the definitions were written by the teachers, nor by the learners. In specific instances, participants completed sentences using the correct vocabulary. For instance, "A typical square has…?" (Mahmud et al., 2020, p. 7). Despite the acquisition of mathematical vocabulary and language through oral questioning, there appeared to be no established strategy for vocabulary retention or future recall. In this research, a specific graphic organizer, the Frayer model, will be utilized as an instructional approach for teaching and learning mathematical vocabulary related to functions in Grade 11.

2.3.3 Ways that learners are exposed to mathematical vocabulary during the learning of mathematics in the classroom.

Though this research is not on classroom discourse, but on learning mathematical vocabulary, it was imperative that the research assessed learners' engagement in classroom discussions. Classroom discussions serve as a good measure of learners' proficiency in the mathematical language for learning, explaining, and discursive purposes. Explaining and discussing in a classroom discourse can serve as an indication of intellectual courage. This courage provides an opportunity to demonstrate conceptual understanding, or lack thereof.

According to Krummheuer (2011) and Sfard (2008), there are two perspectives on learners' classroom involvement and learning. The first is a participationist perspective, which refers to all theoretical approaches that conceptualise learning as participation in classroom discourses. This perspective holds the view that learners learn by first participating in a collective, before moving on to perform single-handedly. This view regards mathematical learning as a process of enculturation into mathematical practices (Vygotsky, 1978). On the contrary, the second perspective, an acquisition perspective, considers individual development to be drawn from personal acquisition (epistemic processes), before participation in collective activities. However, Sfard (2008) and Erath et al. (2018) consider interaction and students' epistemic processes to be strongly interwoven.

Irrespective of the perspective, this research considered learning mathematical vocabulary to be vital for both acquisition and participation perspectives. In their study, Erath et al. (2018) investigated the duality of language, i.e. the learning medium and learning goal for the discourse practice of explaining in Grade 5 whole-class discussions. When a mathematical vocabulary word like "function" is presented and discussed in classroom discourse, the word itself serves as a medium for communicating a relation of correspondence between elements of two sets. Simultaneously, understanding the concept of a "function" becomes a learning objective in its own right. The analysis conducted by Erath et al. (2018) revealed that explaining constituted the most prevalent discourse practice, wherein students' dynamic participation exhibits diversity. Moreover, their analysis demonstrated that discourse competence plays a significant role in the acquisition of mathematical knowledge. In this research, the underlying supposition was that discourse competence experiences substantial influence from mathematical vocabulary and proficiency in mathematical language. Consequently, the intended intervention sought to impact learners' mathematical vocabulary and mathematical language skills through direct instruction using the Frayer model.

During classroom discourse and interactions, learners encounter mathematical vocabulary in two main ways. The first way is through written text (the textbook and question papers). According to Kuger (2016), there are three levels to the school curriculum: "the *intende*d – what a system intends students to learn; *the implemented* – what is taught in classrooms; and *the attained* – what students are able to demonstrate". Textbooks convey the curriculum and so they play a critical role in education across school subjects (Fan et al., 2013; Sherman et al., 2016). Learners' textbooks are intended to be used by both teachers and learners. As such, these textbooks take the intended curriculum into the implemented curriculum. Textbooks translate the intended curriculum into a sequence of contents; thus, they influence the implemented curriculum. That being the case, the learners' learning experiences can be limited by what textbooks offer. Hadar (2017) conducted a study to explore the relationship between the quality of learning opportunities provided by mathematics textbooks and children's achievements. To conduct the study, Hadar used two Grade 8 mathematics textbooks commonly used by learners in the Arab community in Israel. Hadar (2017) concluded that in addition to the already known predictors of learners' mathematics achievement such as socioeconomic status, teachers' qualifications and experience, and so on, the textbook also plays an important role. The textbooks provide learners with the opportunities to learn mathematics. The study illuminated that learners who use textbooks that provide highly demanding tasks of understanding are likely to obtain higher scores in standardised tasks. As a result, the findings raised new questions about "how teachers use textbooks and about the role of textbooks in promoting access and equity in mathematics education" (Hadar, 2017).

From Hadar (2017), if the textbook provides an opportunity for learning in the form of exercises, the current research adds to the argument by suggesting that those exercises should also promote the learning of mathematical vocabulary. Research has proved that understanding of mathematical vocabulary aids in conceptual understanding (Riccomini et al., 2015), and it is through conceptual understanding that learners are able to deal with tasks that require higher-order thinking. Hadar's findings did not manage to establish the extent to which the textbooks exposed learners to mathematical vocabulary. The research only focused on comparing the cognitive demands in the exercises and associating them with learners' levels of achievement.

The second manner through which learners are exposed to mathematical vocabulary during lessons is through the teacher. According to Adler and Ronda (2016), teachers introduce concepts followed by explanatory talk. The explanatory talk involves naming and legitimating the concepts. Teachers also give examples of the concept they have introduced. Examples normally take the form of showing the learners how to arrive at the solution to a mathematical problem. This takes the form of a sequence of procedural steps for the learner to follow towards the resolution of a mathematical problem. In some isolated instances, the teachers would explain mathematical words, but would seldom explicitly teach that vocabulary. The teachers were reluctant to use formal mathematical language as it is "abstract and it puts learners off" (Adler & Ronda, 2016). In their work, Adler, and Ronda (2016) used the Mathematical Discourse in Instruction as an analytical framework, and investigated and described the opportunities presented to learners in a mathematics class where the objective was to solve quadratic inequalities. In the study, the teacher provided the learning opportunities. Without any doubt, the teacher took the content from the textbook, but the researchers focused on how the teacher used that content to provide learners with an opportunity to gain experience.

Adler and Ronda (2016) discovered that, during the classroom discourse, the examples provided by the textbook were adequate and provided learners with a good opportunity to gain experience. However, the opportunity was thwarted by the teacher's explanatory talk, how the teacher legitimated the solution strategy, and how he named the concept. In most cases, the teacher in question used colloquial language. For example, the teacher used the term "smiling parabola" to denote a parabola with a minimum value. Use of colloquial language deprives the learners of an opportunity to learn the precise mathematical vocabulary which they need for conceptual understanding.

The preceding discussion reveals that learners receive an opportunity to learn mathematical vocabulary from both their textbooks and their teachers. These two sources (textbook and teacher) play an essential role in the implementation of the intended curriculum. The teachers' explanatory talk should use precise mathematical vocabulary so that the learners can learn how to use the vocabulary in a mathematical discourse.

2.3.4 The Frayer model as a mathematical-vocabulary-learning strategy

In this research, I chose the Frayer model from the variety of methods such as mind maps, concept maps, etc., because it has the elements of an effective language intervention (Aveyard, 2018). The Frayer model has a visual aspect to it, which gives users the opportunity to master the meaning of words and concomitantly making connections and determining relationships in a way that motivates them to embrace the learning process (Sacapaño & de Castro, 2022). Additionally, the Frayer model encourages classroom inquiry among learners. It also facilitates critical thinking, as the learners complete and fill in the Frayer chart sections to identify "characteristics, examples and non-examples, as well as providing a definition in their own way that makes sense to them" (Dunston & Tyminski, 2013; Wati & Alimin, 2022). For instance, if learners are asked to define a function, they would give the definition that they best understand, rather than try to memorise a definition given to them by their teachers. Learners consolidate this definition by way of giving the characteristics of a function, examples of a function, and what it is not (non-examples). Learners can also easily adapt the Frayer model to include some visual pictures, rather than written characteristics, depending on the type of learners they are.

In this study, the model is intended to effectively address the other research objectives: fostering a deeper conceptual understanding of mathematics and enhancing overall performance in the subject. The Frayer model, originally designed for language subjects, might indicate applicability to mathematics, as theoretical literature suggests. Nevertheless, limited empirical research supports this assumption, often lacking relevance to the specific context. Existing research primarily concerns primary-school children and, in certain instances, takes an outdated or unrealistic teaching approach, as demonstrated by the works of Wanjiru and O-Connor (2015) and Heemsoth and Heinze (2014).

In modern research, Wanjiru and O-Connor (2015) investigated the effect of mathematical vocabulary instruction using an ICT-integrated Frayer model on students' achievements in mathematics. This non-equivalent, control group, pretest-post-test, quasi-experimental study was conducted with 216 Form 2 learners in two secondary schools in the Kahuro District in Murang'a County in Kenya. 108 boys and 108 girls took part in this study. The boys, all of whom were drawn from an all-boys secondary school were divided into one experimental group and one control group. The same applied to the girls, who were also drawn from an all-girls secondary school. The two experimental groups were exposed to mathematical vocabulary instruction using the Frayer model with ICT integration. On the contrary, the control groups were taught mathematical vocabulary using the definition-only method. The teaching of mathematical vocabulary lasted 10 weeks for both groups. The study concluded that the teaching of mathematical vocabulary using the Frayer model integrated with ICT caused an improvement in students' achievements in mathematics.

This study bears a close resemblance to a study conducted by Wanjiru and O-Connor (2015). The study and the study by Wanjiru et al., share commonalities in the utilization of graphic organizers, particularly the Frayer model. However, Wanjiru and O-Connor integrated the Frayer model with ICT. While there is agreement with the notion of ICT taking over in the 21st century and beyond, the view is that, presently, not all schools where mathematics is taught possess access to ICT facilities for learning mathematical vocabulary. Advocating for the use of ICT for mathematical vocabulary learning in all South African schools, and most African schools, could be exclusionary. For example, Long and Graven (2017) noted high inequality, high levels of unemployment, and widespread poverty as characteristic contextual features of most schools in South Africa. In addition, Spaull and Kotze (2015) pointed out that schools serving poor communities are characterized by low resource levels and growing learning deficits.

On its own, the Frayer model is cost-effective and accessible to all learners, regardless of their backgrounds and school per capita. All learners, including those in schools without computer laboratories or tablets, can utilize it. This underscores the model's value, particularly as students from underprivileged social backgrounds encounter more obstacles to participation in classroom discourse than their counterparts (Westwood, 2018). Through its cost-effectiveness, the non-ICT-integrated Frayer model provides indiscriminate access to opportunities for learning mathematical vocabulary.

Wanjiru et al.’s (2015) focus was on the utilization of the Frayer model integrated with ICT as a strategy for learning mathematical vocabulary. The study concluded that the Frayer model integrated with ICT offered a valuable opportunity for learners to comprehend the interactions within mathematical content. Consequently, a well-executed strategy for learning mathematical vocabulary can enhance students’ achievements in the subject. Furthermore, the research concluded that the Frayer model integrated with ICT serves as an effective approach for mathematical vocabulary instruction.

The results obtained in Wanjiru et al.’s (2015) study demonstrated that, regardless of the strategy employed, teaching mathematical vocabulary led to improved mathematics achievement, although the Frayer model yielded superior results compared to the definition-only method. While it was concluded that the use of the Frayer model resulted in greater improvements in mathematical results, it did not delve precisely into the connections between learning vocabulary and conceptual understanding, learning vocabulary and procedural proficiency, conceptual and procedural understanding, and mathematics achievement. Neither did it explore the intermediary processes between learning vocabulary and mathematics achievement. This argument does not diminish the significance of their research but raises concerns about the necessity to engage with the other processes inherent in mathematics learning. Focusing solely on achievement results represents a simplified approach to addressing the complexities of mathematics education. Although achievement holds importance, it alone cannot serve as a measure of success in the teaching and learning of mathematics in schools. However, poor mathematics achievement does indicate deficiencies in the teaching and learning process.

Another distinction between this study and that of Wanjiru and O-Connor (2015) is the vocabulary under investigation. Wanjiru and O-Connor (2015) explored vocabulary drawn from various sections of the mathematics syllabus, including terms such as product, factors, chord, polygon, degree, scale, power, area, integer, multiples, perimeter, capacity, ratio, angle, and percentage. In this research, the focus was solely on vocabulary drawn from the domain of mathematical functions, specifically asymptotes, intercept, domain, range, and coordinates. The intention was to investigate vocabulary learning within a specific curriculum domain, recognizing that each section of the curriculum contributes uniquely to the development of mathematics learners.

Ulusoy (2021) conducted a multi-case study involving 126 first year early-childhood pre-service teachers enrolled in a four-year teacher education program at a public university in Turkey. The study aimed to examine concept images and concept definitions for triangles among the participants. The study's results revealed that pre-service teachers encountered difficulties with both the concept image and the concept definition of triangles. Many prospective teachers provided inappropriate definitions of a triangle, often using necessary but not sufficient conditions (a shape with three angles) or conditions that were neither necessary nor sufficient (geometric solid with three vertices). They also frequently used inaccurate terminology in their definitions (using "shape" instead of "polygon"). Furthermore, most participants struggled to generate correct examples and non-examples of triangles. These results underscore the need for a sustainable and effective approach to learning mathematical vocabulary to facilitate retention for future use.

Such a strategy as the use of the Frayer model has already been alluded to by Cunningham and Roberts (2010), when they assessed 23 female elementary pre-service teachers at the College of New Jersey's ability to answer questions involving geometric concepts. Cunningham and Roberts (2010) used a one-group pretest-post-test design. The participants were first exposed to a "passive" or traditional learning environment, where the participants were explicitly given the definitions of words such as parallel and perpendicular lines, altitude, diagonal, etc. When they were tested (pretest) for conceptual understanding, they faltered. An intervention was introduced by way of exposing the participants to the Frayer model for geometry vocabulary learning. The ensuing post-test (same words as in the pretest) results revealed an improvement in the number of pre-service teachers' correct responses. However, for other participants, the use of the Frayer model failed to improve their understanding of the geometric concepts. Though it could not be ascertained with precision why those participants failed to improve, the researchers attributed this to a lack of experience on the part of the participants in use of the Frayer model and the limited number of experimental lessons used in the study.

Sacapaño and de Castro (2022) conducted quasi-experimental research with 60 Grade 9 science children in the Philippines. The 60 children were split into two, heterogeneous groups (control and experiment), and then a pretest was administered to them. After the pretest, the experimental group was exposed to the Frayer model to learn science vocabulary for two weeks, while the control group followed the conventional approach of not putting emphasis on vocabulary. At the end of the two weeks, a post-test was administered to both groups to check if there was any significant difference between the performances of the two groups after implementation of the Frayer model in the experimental group. The study found that the implementation of the Frayer model effectively improved the science achievement of the children to whom it was implemented. Additionally, it was noted that the scientific abilities of the learners in the experimental group increased, and their critical thinking skills, analyses of problems, and thinking of possible solutions improved as a result of using the Frayer model.

The research was conducted within the realm of science, a subject area in which Google searches indicate that the practical application of the Frayer model is relatively infrequent and not widely documented. Most studies that have experimented with the Frayer model did so in language subjects because the model itself was designed for use in languages.

Rampersad et al. (2020), conducted another content subject study with Form 1 geography learners in a secondary school in Trinidad and Tobago. They intended to find out if low academic performance and poor attitude towards "content-oriented reading and writing" improved through use of specific literacy strategies. The literacy strategies adopted by the researchers were the Frayer model, audience-centred teaching, and learning logs. One group of learners took part in the study. A pre-diagnostic test was administered to the learners at the start of teaching a chapter: "The World Around Us". Results of the pre-diagnostic test showed that the average frustration levels of learners were very high when they were given geographic words, their reading skills were poor, and they had a negative attitude towards geographic technical words. For five weeks, the students were exposed to the literacy learning strategies, and their performance was tracked over the period.

When the post-diagnostic test was administered, it was discovered that the literacy strategies had resulted in a positive impact on learners' cognitive development. The findings of the study were that there was an improvement in the learners' academic performance and that their attitude towards reading for meaning in geography had improved drastically. In particular, the Frayer model helped the learners to overcome the fear that they had towards geographic terms.

They developed confidence in reading, writing, and using the geographic words (Rampersad et al., 2020).

Sacapaño and de Castro (2022) and Rampersad et al. (2020) conducted studies with the Frayer model in content subjects, rather than the usual language subjects. These studies attest to the fact that the Frayer model can be used across the school curriculum. The Rampersad et al. (2020) study further demonstrated that, like the Wanjiru and O-Connor (2015) study, the Frayer model can be integrated with other strategies during teaching and learning. Though the two studies proved the possibility of integration, the view is that integration with cost-effective and non-complex strategies such as audience-centred teaching and learning logs is much more plausible than ICT integration. Regardless of the benefits that ICT integration might give, not all schools can afford to provide learners with computer laboratories and ICT gadgets as in the Wanjiru and O-Connor (2015) study. This, however, does not mean that schools that can afford to do so should be discouraged from doing so. It is in the context of advocating for an all-encompassing strategy that this assertion is made.

Rahmadani (2018) conducted research in Indonesia aimed at comparing the Frayer model and conventional strategies in learning English language vocabulary. The research sought to assess and compare students' mastery of English language vocabulary when taught using these two strategies. It was established that there was a significant effect of using the Frayer model on the student's vocabulary mastery. Though the research was conducted for English language vocabulary learning, it is important to know that the usefulness of the Frayer model as a vocabulary-learning strategy was confirmed. In the case of this research, the vocabulary of interest is the mathematical and technical vocabulary of functions.

Kenyon (2016) explored the connections between self-efficacy, vocabulary development, and student academic achievement. It was concluded that self-efficacy, mathematical vocabulary, and academic achievement develop cyclically. A deficiency of mathematical vocabulary precludes students from developing mathematical skills, dispositions, and knowledge. In addition, how students feel about themselves as mathematics learners could hugely influence their academic success in mathematics. Self-efficacy was enhanced by way of direct vocabulary teaching using the Frayer model. According to Kenyon's (2016) findings, the Frayer model has more success in low-ability students, as compared to a definition-only approach.

Despite the successes of the Frayer model in mathematical vocabulary instruction, Kenyon (2016) observed that when learners used the Frayer model to define mathematical vocabulary that also has everyday meaning, e.g. "product", they related it to items that they would buy from a shop. This manifested both prior and post use of the Frayer model for learning mathematical vocabulary. This confirms that vocabulary that represents both scientific and everyday concepts cause problems for some students. To this effect, the Frayer model might not prove to be the right tool to use for such vocabulary. Furthermore, the study concluded that the Frayer model does not suit all students, and thus it should be personalised as much as is feasibly possible. Kenyon (2016) suggested that further investigation might be needed in the future to examine these phenomena more deeply.

In this study, mathematical vocabulary encompassing technical terms (such as asymptote and intercept) and those spanning both scientific and everyday contexts (like domain and range) was incorporated. The term "range" is prevalent in both statistics and functions within the realm of mathematics, but it also holds a place as a common word in the English language. Through the utilization of the Frayer model, the research aims to either validate or challenge Kenyon's (2016) findings regarding the efficacy of the Frayer model in the acquisition of such vocabulary.

Promoting and assessing conceptual understanding using the Frayer model.

Kirsten (2019) indicated that teachers need to be supported in integrating literacy teaching with their ordinary subject teaching in a way that emphasises the objectives of the school subjects. In mathematics teaching, the Frayer model is one such way and it promotes conceptual understanding by providing a structured and comprehensive approach to learning new vocabulary terms. It helps students move beyond rote memorization and fosters deeper comprehension through the following mechanisms:

- In the Frayer model, students are required to provide a concise and clear definition of the mathematical term. This step encourages students to think critically about the essence of the concept and articulate it in their own words, reinforcing their understanding of the term's meaning.

-Learners are prompted to identify the key characteristics and features associated with the term. This process allows students to explore the essential attributes of the concept, enhancing their ability to recognize and apply these features in different mathematical contexts.

-Providing examples and non-examples of the term requires students to distinguish between instances where the concept applies and where it does not. This helps learners develop a deeper understanding of the concept's boundaries and clarifies its usage in specific situations.

- By organizing the information in the Frayer model, students can visualize the interconnections and relationships between the mathematical term and related concepts. This promotes a holistic understanding of how different mathematical ideas are interconnected, contributing to a more comprehensive grasp of the subject matter.

As highlighted by Kenyon in 2016, when utilizing the Frayer model, it is crucial to identify key indicators of conceptual understanding within learners' work. These indicators encompass:

1. Accuracy of Definitions: A well-articulated definition indicates a deeper understanding of the term.

2. Identifying Key Characteristics: This demonstrates their ability to recognize the fundamental attributes of the term.

3. Appropriate Examples and Non-examples: Correctly applying the concept and correctly identifying situations where the concept does not apply is a sign that a learner has understood the concept.

4. Interconnections and Relationships: Demonstration of an understanding of the relationships between the concept and related mathematical ideas is evidence of how learners connect the term to other concepts they have learnt.

5. Application in Problem-solving: A strong conceptual understanding allows students to use the term effectively in various mathematical scenarios.

6. Depth of Understanding: A higher level of understanding goes beyond surface-level explanations and delves into the intricacies and nuances of the concept.

By assessing these indicators in learners' work, educators can gain valuable insights into the students' conceptual understanding and identify areas that may require further support and reinforcement. The Frayer model serves as a valuable tool in facilitating this assessment process and nurturing a deeper conceptual understanding of mathematics among learners.

2.3.5 The link between competency in mathematical vocabulary and learners' performance in mathematics'

Right from the onset of schooling, some huge individual differences in children's performances emerge. Research has identified procedural skills, conceptual understanding, and working memory as the three most prominent cognitive skills that independently predict mathematics performance. Gilmore et al. (2017) worked with 75 UK children aged between five and six years. The researchers investigated the children's procedural skills, conceptual understanding, and working memory and those children's mathematics achievements. They found out that these cognitive skills were indeed independently associated with children's achievements. Apart from independent associations, the skills demonstrated significant associations. The relationship between procedural skills and achievement was moderated by conceptual understanding and working memory. The study aimed to influence conceptual understanding through focused vocabulary learning utilizing the Frayer model. Individuals with strong conceptual understanding tend to make adaptive strategy choices effortlessly. Consequently, as conceptual understanding is acquired, improved procedural skills are often achieved (Rittle-Johnson & Schneider, 2015). Enhanced procedural skills, in turn, have connections with both current and future achievements in mathematics (LeFevre et al., 2010).

As a vocabulary-learning strategy, the Frayer model has proved that it is effective (Nelson et al., 2022; Rahmadani, 2018; Wanjiru et al., 2015). However, it had to be seen whether vocabulary gain translated into a conceptual understanding that was established to be a precursor of procedural skills and achievements or not (Gilmore et al., 2017). Conceptual understanding is important for success in mathematics, as it encompasses the principles and relationships that underlie a domain. When children attain conceptual understanding, they can easily make adaptive strategy choices when working out mathematics problems (Rittle-Johnson & Schneider, 2015). Children with an adaptive ability can quickly see easier and shorter ways of solving a problem. This adaptive ability is made possible by developing procedural skills, which are, themselves, a result of acquiring conceptual understanding.

Empirical study has shown that there is a possibility of the existence of a positive relation between competency in mathematical vocabulary and mathematics performance.

Mathematical vocabulary is thought to mediate the relation between working memory and word problems (Peng & Fuchs, 2017). That being the case, there is a need for intervention studies that investigate the "causal relation between mathematics vocabulary and mathematics performance" (Peng & Fuchs, 2017). To this effect, Peng and Lin (2019) explored how various kinds of mathematical vocabulary found in the curriculum relate to different mathematics outcomes among Grade 4 children in China. They collected data on mathematical vocabulary, general vocabulary (words with usage in everyday life), cognitive skills (IQ and processing skills), and mathematics performance among 237 Grade 4 children. When the researchers controlled general vocabulary and cognitive skills, the study showed that mathematical vocabulary made unique contributions to mathematics performance. The vocabulary contributed as much to word problems as it did to calculations. However, the effects of mathematical vocabulary on performance varied according to types of mathematics outcomes and mathematical vocabulary in question. For instance, mathematical vocabulary related to measurement and geometry had more impact on word problems than vocabulary related to numerical operations. The researchers also concluded that mathematical vocabulary mediates the link between language skills, cognitive skills, and mathematics skills. One question found in the study was: "If the *perimeter* of a *rectangular* garden is 68 meters and the *length* is 14 more meters than the *width*, what is the *length*?" Understanding the mathematical vocabulary is key to answering this question. The vocabulary in the question is from measurement and geometry, and it triggered visuospatial processing skills, because geometry requires "mental and concrete visual manipulations" most (Peng & Lin, 2019).

In a study that explored the relationship between mathematical vocabulary and mathematics achievement, Ünal et al. (2021) conducted research with Grade 8 children drawn from the USA (n = 89) and Turkey (n = 188). The study drew vocabulary from four domains of mathematics: number and operations (e.g. rational numbers), algebra (e.g. equation, function), geometry (e.g. rotation) and data and measurement (e.g. scatter plot). A mathematical vocabulary research instrument was used, where the participants were required to: choose a term that fits a definition, choose a definition that fits a term, or choose an option that fills a space.

Ünal et al. (2021) reached two important findings. The first conclusion was that "mathematics vocabulary predicted mathematics achievement for the US and Turkish higher-achieving children above and beyond their general vocabulary knowledge". For the low-achieving Turkish children, general vocabulary was more significant in predicting their mathematics performance. The second conclusion was that "mathematics vocabulary mediated the relation between general vocabulary and mathematics achievement for US and Turkish higher-achieving groups" (Ünal et al., 2021, p. 7). This conclusion was reached after a formal mediation analysis was carried out and it proved that there is a "significant indirect effect" of general vocabulary on mathematics achievement.

This study is quite significant in the sense that it incorporated both technical vocabulary and general vocabulary. Furthermore, the researchers managed to make a comparison between children from two countries with different demographic situations. More than just this inter-country comparison, the study assessed the results from the high-achieving and low-achieving children to see if the effect of vocabulary was the same for both.

Despite the highlighted scope, the study did not go as far as assessing if the different domains of vocabulary had the same effect on mathematics performance or not. Although this research will not do such a comparison, an assessment will be done to find out if vocabulary drawn from the domain of functions predicts mathematics achievements in functions. This is a narrower scope, but this is intended to give this research a much more focused approach and depth in assessing the presumed relationship.

Lastly, in this regard, an assessment of the findings and implications of the meta-analysis conducted by Lin et al. (2021) would help to provide a more comprehensive summary of the discussion on the relationship between mathematical vocabulary and mathematics performance. In this meta-analysis, Lin et al. (2021) examined 40 studies with 7 988 participants drawn from 55 independent samples. The researchers surfed the internet to search for studies that examined the relationship between mathematical vocabulary and mathematics performance. The search was guided by phrases such as "mathematics vocabulary, mathematics term, mathematics words, mathematics achievement, and so on". When the studies were found, the meta-analysis was conducted and this was the first analysis to "directly investigate whether mathematical vocabulary is more critical for certain types of mathematics skills across development" (Lin et al., 2021, p. 13).

The study found out that, generally, there is a positive, significant relation ($r = 0.49$) between mathematical vocabulary and mathematics performance. In this general relationship, word problems show the strongest relationship with mathematical vocabulary ($r = 0.58$). Other mathematical areas and skills show relatively weak relations with mathematical vocabulary, e.g. fractions ($r = 0.31$), algebra ($r = 0.44$) and number knowledge ($r = 0.47$). The meta-analysis also concluded that the ages and grades of the learners did not significantly influence the relation between mathematical vocabulary and the different types of mathematics. That being the case, it is expected that in this study the results will not be affected by the ages of the participants (Grade 11 learners with an average age of 16 years).

2.4 Research on mathematical language

Mercer and Sams (2006) investigated the use of language as a tool for reasoning among Year 5 students in Southeast England. Their research involved an intervention known as Thinking Together, aimed at developing children's use of language as a tool for reasoning in mathematics. Seven teachers and their classes totalling 109 learners participated as target classes, and another seven teachers, and their 121 learners as control classes. The target classes were exposed to the Thinking Together programme to develop children's abilities to use language as an individual, as well as a group-thinking tool.

Videotape and audio recordings were used for capturing episodes of classroom activities and interviews. In addition, a pretest and a post-test were administered to evaluate the effect of the intervention. Target classes significantly improved their achievement in mathematics. The results demonstrated that explicit reinforcement of language use for collective reasoning resulted in improved thinking patterns for both groups and individuals. Mercer and Sams conducted research involving primary-school children. In contrast, the present study involves an intervention with high-school learners to investigate whether equipping them with precise mathematical vocabulary will lead to changes in their conceptual understanding and mathematical proficiency.

In another study, Powell, and Driver (2015) investigated the influence of mathematical vocabulary that was embedded within tutoring Grade 1 learners who had difficulty with mathematics. With 110 students assigned to three groups for a pretest-intervention-post-test quasi-experiment, they tutored and assessed 19 mathematics terms related to counting, addition, and subtraction to the experimental group with 39 students. Results of the investigation indicated that in comparison to the other students, students in the experimental group demonstrated remarkable improvement in their performance in mathematics. They attributed the improvement to the tutoring of vocabulary. This underscores the indispensability of robust vocabulary in facilitating the comprehension of intricate mathematical ideas.

However, a discernible gap remains in terms of fully integrating vocabulary within broader teaching methodologies, warranting further attention.

In the current research conducted with Grade 11 learners, pre-test and post-test results of an experiment and a control group were compared. The analysis of test results assisted in evaluating the effect of the intended intervention. Powell and Driver (2015) integrated mathematical vocabulary instruction within tutoring. In the current study, overt vocabulary instruction and learning were utilized as the intervention method. Overt instruction was feasible as the research was conducted with high-school learners. High-school learners, as active participants, were expected to appreciate a metacognitive approach.

Van Jaarsveld (2016) highlighted the need for aspiring teachers to be taught exact mathematical language. In research with university students, he advocated for exact mathematical language to be a rigorous aspect in teacher training programmes. He conducted action research with students who were studying to become secondary-school mathematics teachers. Throughout the course of their study, the students' description of mathematical objects was observed to be poor. Four language repertoires, to be described in the next chapter, were developed by the researcher to assist the students in communicating about mathematical objects. His repertoires served as a cognitive resource in the form of an inventory of words (van Jaarsveld, 2017) for his work. Working with the language repertoires, students were encouraged to do small-group discussions and oral evaluations as a way of teaching them exact mathematical language as a pedagogical tool. Acquisition of correct mathematical vocabulary increased students' confidence and gave them the ability to reason mathematically (van Jaarsveld, 2016).

In another study, van Jaarsveld and Ameen (2017) explored the extent to which Inservice teachers used exact mathematical language during teaching in a sample South African classroom. It was discovered that teachers had a store of mathematical vocabulary, but they could not distinguish between the four language repertoires, and used mathematical vocabulary arbitrarily, for example, they referred to "solving expressions". Though they believed in the importance of mathematical language for teaching, the teachers' vocabulary was a hindrance to their descriptions of concepts.

In both studies, van Jaarsveld worked with post-high-school participants. First, with teaching students and then practising teachers. Both sets were expected to work with secondary-school learners using exact mathematical language for conceptual understanding. However, one of his recommendations was that research on teaching and learning an exact mathematical language needs to be done in secondary schools, where mathematical concepts are intertwined with complex vocabulary (van Jaarsveld, 2017). Unlike his action research with adults, this research will involve secondary-school learners.

When van Jaarsveld developed the language repertoires, its intended use was for teachers. In addition to its use by teachers, I propose that learners can also use it. However, for them to competently make use of the repertoires, learners may need to be exposed to learning strategies that help them to learn mathematical vocabulary. One such strategy is the use of the Frayer model, as explained in subsequent sections later in the research.

Valley (2019) conducted research with a lower-level English Language Learners (ELL) class at an elementary school in northern New England. The study was conducted in a school with an enrolment of 300 learners, 25% of whom were ELL learners. ELLs are students who come from non-English-speaking backgrounds, are unable to communicate fluently, or learn effectively in English. These students certainly need dedicated instruction in both the English language, and in their academic content – in this case, mathematics. The study was aimed at investigating what impact the teaching of daily mathematics word problems in a Grade 3–6 ELL class would have on mathematical vocabulary use, mathematics word problems, and place value comprehension. In the study, mathematical vocabulary was taught by way of word problems. A pretest and a post-test on word problems were used together with a frequency tally to track mathematical vocabulary used by the learners. Results obtained showed a small improvement in comprehension of word problems and a large improvement in mathematical vocabulary frequency. Furthermore, the study ascertained that daily implementation of mathematics word problems in the classroom significantly increases the use and understanding of English mathematics. Despite these valuable findings, a research gap remains concerning the potential variability in the effectiveness of this approach across different age groups, academic levels, and linguistic backgrounds of ELL students. Further investigation into these nuanced factors could provide a more comprehensive understanding of the strategy's broader applicability and optimization.

In their work, Jannah and Nusantara (2019) observed how the concept of functions was taught by a lecturer and learnt by three university students. In a test that they administered, they asked students to define a function based on their understanding. This required the students to use their own words in their definitions, as an illustration of their understanding. The students were also asked to elaborate on their definitions using examples and non-examples of a function.

One of the students gave the following definition for a function:

> The explanation: $f: A \to B$ is a function mapping from A to B with A as domain and B as codomain. Each member in A must be mapped once to a member in B. Since there is only single $b \in B$ then $\forall a \in A$ will be mapped to b $(R(f)) = B$ with $A \neq \emptyset$, $B \neq \emptyset$. So the range of the function is just b, meaning $b = b'$.

From the student's definition, the student demonstrated difficulties in understanding the formal definition of a function. The researchers made two major findings, the first of which is that the experience that students have from high school influences their understanding in tertiary education. For the students, words such as set, ordered pair, Cartesian product, and relation are the prior knowledge used to form formal definitions of a function. In this case, this student failed to articulate these words in producing their definition for a function. Jannah and Nusantara asked the students to elaborate on their definitions to demonstrate their conceptual understanding. They did so by asking the students to give examples and non-examples of the concept of a function. The students also faltered in their examples and non-examples.

Jannah and Nusantara (2019) demonstrated a belief that the mere definition would not suffice as a demonstration of conceptual understanding. Despite its insufficiency, the definition should stand as a necessary condition for conceptual understanding. In this research, emphasis was placed on definitions, characteristics, examples, and non-examples during the learning and engagement with vocabulary.

Jannah and Nusantara's (2019) other finding were that the three students involved in the research lacked an understanding of the networks of a formal definition of a function, namely sets, ordered pairs, Cartesian product, and relation, and this led to them having cognitive, didactical, and epistemological obstacles in their learning. Cognitive obstacles are difficulties associated with the actual learning process. These obstacles come from the students' previous experiences because their previous experiences influence their internal processing of new experiences. However, when the students are faced with new and more complex situations, where their previous experience is inadequate or difficult to adapt, that experience becomes a hindrance to further learning.

A cognitive obstacle is an ancient idea which first appeared in science, and then later in mathematics education. Bachelard (1938) and Brousseau (1976) concurred in their descriptions that a cognitive obstacle is a piece of knowledge that is anchored in the mind. For a considerable amount of time, that piece of knowledge would have been satisfactory for handling experiences. A time comes, however, when that knowledge becomes inadequate. Students will try to use or adapt their inadequate knowledge, and in doing so, they become confused or develop a misconception. A case in point might be when students learn multiplication with positive whole numbers, where they realise and master that multiplication produces a product that is larger than the multiplicands. They anchor this knowledge in their minds, but as they move on to subsequent sections of the curriculum, this knowledge becomes inadequate. For instance, when they are required to multiply fractions between 0 and 1, or multiply two whole numbers where one is negative and the other positive. The knowledge that multiplication produces larger numbers here becomes a cognitive obstacle.

The other type of obstacle discovered in the Jannah and Nusantara (2019) research was didactical. This occurs because of the nature or method of teaching employed by teachers. As the more knowledgeable other in the classroom, a teacher's actions have a significant and enduring impact on how learners grasp concepts. In the research conducted, investigation into the methods and strategies employed by educators to teach mathematical vocabulary was necessary. This approach focused on zeroing in on and addressing the issue of the didactical obstacle. Additionally, by exposing learners to the Frayer model as a vocabulary-learning strategy deeply rooted in active learning principles, efforts were made to reduce the impact of didactics on mathematical learning.

The last of the obstacles were of an epistemological nature. These hindrances arise due to the inherent nature of the mathematical concept itself. The concept of function holds broad applications in academic and real-life contexts. The associated vocabulary is technical, underscoring the necessity of a thorough understanding of the vocabulary for conceptual understanding. Consequently, the research advocated for the explicit teaching of this vocabulary to explore the influence of such an exercise on conceptual understanding.

While the work touched on the notion that a definition alone might not be sufficient to demonstrate conceptual understanding, it did not delve extensively into alternative pedagogical strategies that could enhance understanding. There is a research gap in terms of exploring innovative teaching methods that can help students not only understand definitions but also grasp the underlying concepts more holistically. This could involve interactive activities, visual aids, or real-world applications that connect the abstract concept of functions to tangible experiences.

Three of the studies were done with primary-school pupils, two with tertiary students and one with practising teachers. The first three studies allowed participants to use mathematical vocabulary while learning mathematics. These studies appreciated the active role of learners in their learning, as opposed to the depository role in teacher-centred approaches. A common finding of these studies was that teaching learners mathematical vocabulary as they learn mathematics brings about improvement in their performance in the subject.

By considering language as a tool for reasoning, Mercer and Sam (2006) conform to the existing understanding that thinking in concepts exists within the confines of verbal thinking (Vygotsky, 1986). As such, when they allowed their primary-school participants to engage in discussions using mathematical vocabulary, they allowed for active participation and encouraged the development of the vocabulary. As seen in their findings, the learners from the experimental group improved in their mathematics because of the "thinking together" practice. This suggests that as the participants gained mathematical vocabulary, they became better thinkers, and hence, better doers of mathematics. Vocabulary gain amplified the thinking horizon, as suggested by Vygotsky. The same applied to Powell and Driver (2015) who, by teaching their participants (experimental group) mathematical vocabulary (though covertly), amplified their thinking horizon and achieved an improvement in performance from the participants. The reported improvement in performance was a consequence of improved reading of texts and enhanced comprehension and learning of content. This can also be said for the participants in van Jaarsveld's (2016) study, who, after engaging with language repertoires, gained confidence in their talk and doing of mathematics. The participants could converse mathematically, and their reasoning improved because their thinking horizons had stretched in response to vocabulary gain.

In her study, Mulwa (2015) found that students have difficulties in using mathematical vocabulary and the associated concepts. She concluded that teaching and learning mathematics requires effective communication, and that when the teacher and the learner do not share a common language, it is difficult to converse mathematically. In this regard, I consider the subject language to be the obligatory common language that must be shared by teachers and learners. Mother languages for the teacher and the learners might differ, but if they all acquire some level of proficiency in the subject language, mathematical conversations can occur. This is supported by the assertion that the function of mathematical language is to enable both the teacher and the learner to communicate mathematics knowledge with precision (Mulwa, 2015).

A noteworthy qualitative case study conducted by Mahmud, Yunus, Ayyub, and Sulaiman (2020) centred on the strategic deployment of mathematical language by teachers during oral questioning. By employing a combination of methods such as observations, interviews, and meticulous field notes, the researchers delved into the ramifications of varied oral questioning techniques on students' fluency in mathematical language and their overall comprehension of mathematical concepts. The findings spotlighted the potency of techniques such as precise pronunciation emphasis and iterative questioning focused on mathematical terminology. These strategies, in turn, resulted in palpable enhancements in both students' mathematical language proficiency and their grasp of complex conceptual nuances.

In another qualitative case study by Moleko and Mosimege (2020), attention was directed towards the intricate challenges intertwined with mathematics word problems. The study elucidated that hurdles such as limited English proficiency, deficient mathematical vocabulary, and misinterpretation of contextual cues impeded the effective instruction and absorption of mathematical concepts embedded within word problems. The findings underscored the imperative of contextual comprehension in formulating meaningful solutions, thereby accentuating the significance of contextualizing challenges to fortify students' comprehension of word problems.

In their 2020 study, Hughes, Powell and Lee conducted insightful research, culminating in a concise yet impactful psychometric report. Within this report, they adeptly crafted and assessed a novel measure targeting mathematics vocabulary proficiency among late middle-school students (specifically, Grades 7 and 8). The primary objectives encompassed gauging the measure's reliability, as well as delving into students' approaches to answering questions concerning mathematics vocabulary terms. Notably, the selected vocabulary terms mirrored those deemed indispensable by middle-school educators for mastering mathematical language within this academic stage. The meticulous analysis undertaken underscored the measure's commendable levels of reliability and validity. However, a compelling revelation emerged—highlighting a prevailing challenge: most students grappled with accurately articulating mathematical vocabulary, illuminating a crucial area for pedagogical attention. This foundational work was subsequently built upon by Erath, Ingram, Moschkovich, and Prediger

(2021), who distilled a set of design principles aimed at amplifying language integration within the domain of mathematics education. Nonetheless, the extant body of work also reveals persistent research gaps, underscoring the need for continued exploration and investigation.

The empirical study conducted by Vula, Avdyli, Berisha, Saqipi, and Elezi (2017) delved into the effects of metacognitive strategies on learners' achievement in solving mathematics word problems. Their findings underscored the pivotal role of metacognitive approaches in influencing students' success, underscoring the significance of active learning strategies in empowering students' learning journey. However, there exists an opportunity to seamlessly integrate these strategies into a coherent teaching and learning framework to achieve a more comprehensive and proactive approach. A qualitative exploration by Jannah and Nusantara (2019) delved into the obstacles encountered by university students in comprehending mathematical functions. The study revealed that cognitive, didactical, and epistemological barriers arose due to insufficient definitions and prior experiences, emphasizing the need for a holistic conceptual understanding to overcome these challenges.

In conclusion, the amalgamation of insights drawn from these diverse studies points to the centrality of mathematical vocabulary and language in enhancing learning outcomes. The convergence of effective vocabulary instruction, active learning strategies, and educator engagement emerges as a pivotal triad in augmenting students' mathematical comprehension and reasoning prowess. Furthermore, acknowledging and addressing cognitive, didactical, and epistemological challenges assumes paramount importance in fostering a nuanced grasp of mathematical concepts. Building on these collective insights, this current research investigates the impact of explicit vocabulary instruction on high school learners' conceptual understanding and mathematical proficiency. Drawing from the tenets of active learning theory and cognitive learning theory, this proposed study advocates for the integration of the Frayer model within content and language integrated learning (CLIL) as a multifaceted approach to enhancing both linguistic competence and mathematical comprehension. By embarking on this trajectory, educators and researchers can collectively contribute to an enriched landscape of mathematics education that prioritizes comprehensive language integration for holistic student development.

2.5 Conclusion

The purpose of this chapter was to situate the research in the field of mathematics education. This was accomplished by examining existing literature and research within the field. The chapter provided explanations regarding the scopes and findings of the examined research. Additionally, an exploration of the likely methodological similarities and differences between the existing research and the current study was conducted. Despite the fact that the discussed research primarily focused on mathematical language and vocabulary, the present research diverges from them in the sense that it proposes the utilization of a vocabulary-learning strategy for acquiring technical vocabulary in the domain of mathematical functions, specifically designed for high-school learners in the South African context.

The discussion presented in the chapter has clarified that there is no existing empirical evidence to indicate whether learners can or cannot associate precise mathematical vocabulary with a given mathematical object. The absence of such evidence implies that the current study will provide insights into learners' competency in associating vocabulary with visual mathematical objects. Learners' abilities to engage in mathematical discourse can be influenced by their proficiency in mathematical vocabulary. Therefore, it is essential to consider how learners acquire this vocabulary. In this context, the chapter conducted an assessment of the vocabulary-learning strategies available to learners in schools. It was also imperative to evaluate the relationship between mathematical vocabulary and mathematics achievement. Emphasizing the learning of mathematical vocabulary would be futile if it did not significantly impact learners' mathematics performance. Research has demonstrated a relatively moderate positive relationship between the two. Some studies have indicated that mathematical vocabulary plays a role in influencing performance, serving as a mediator between general vocabulary and mathematics performance.

Given that the research proposed the use of the Frayer model as a learning strategy to enhance conceptual understanding, it was necessary to review research that has applied this model, considering the specific learning areas, target groups of learners, and the outcomes achieved. Initially designed for language subjects, there exists empirical evidence supporting its effectiveness in non-language subjects, including mathematics, science, and geography. While it has been used generally in mathematics, the current study will employ it specifically for teaching functions.

In the subsequent sections, discussions will be presented regarding theoretical frameworks, the conceptual framework, as well as methods of data collection and analysis. These discussions will precede the presentation of results, their analysis, and interpretations, all of which contribute to addressing the research question at hand. Ultimately, the thesis will conclude with the presentation of conclusions and recommendations.

CHAPTER 3 : THEORETICAL FRAMEWORKS AND A CONCEPTUAL FRAMEWORK

In this chapter, the description of theoretical perspectives that informed the research is provided. The perspectives are a set of preexisting theories and ideas considered relevant to the work. The perspectives presented here include the Active Learning Theory (ALT), detailed in Section 3.2, and the Cognitive Load Theory (CLT), discussed in Section 3.3.

The research participants operated within the framework of these two theoretical foundations within the pedagogy known as Content and Language Integrated Learning (CLIL), which will be examined in Section 3.4. During their engagement in CLIL structured lessons, they utilized the Frayer model, which will be elaborated upon in Section 3.5. In addition to the separate descriptions of the two theoretical frameworks, the pedagogy, and the artefact, integration was carried out, combining the theoretical frameworks, pedagogy, and artefact with literature and teaching experience to develop a conceptual framework referred to as the dyadic framework for Content and Language Integrated Learning using the Frayer model, to be presented in Section 3.6 of this chapter. Sections 3.7, 3.8, and 3.9 are dedicated to the exploration of mathematical vocabulary and language, the examination of language repertoires, and the conclusive remarks concluding this chapter. However, preceding the presentation of ALT, CLT, CLIL, the Frayer model, and the conceptual framework, a brief overview of relevant literature will be provided to introduce and contextualize this chapter.

3.1 Preamble literature

Students' mastery of mathematical language demonstrates a positive correlation with their comprehension of mathematical content (Korhonen et al., 2012). With this objective in mind, the focus shifts towards enhancing mathematical vocabulary among students, which, in turn, can facilitate their engagement with numerical concepts and operations (Dunston & Tyminski, 2013). The present study aimed to introduce learners to the Frayer model as a tool for learning mathematical vocabulary, with a subsequent assessment of the impact of this intervention on their conceptual understanding and performance in mathematics. According to Rossouw et al. (2000), the primary purpose of teaching is to provide learners with opportunities to acquire knowledge. Therefore, this work provided learners with the opportunity to utilize the Frayer model, an educational resource, for the purpose of learning mathematical vocabulary. Through the utilization of the Frayer model, learners had the chance to acquire mathematical vocabulary, with the ultimate goal of enhancing their conceptual understanding and, consequently, improving their performance in mathematics.

When learners use the Frayer model, they actively and intentionally acquire mathematical vocabulary and content by incorporating new experiences into their existing mental structures. These structures are then reorganized by the learner to allow for the handling of more challenging experiences (Kilpatrick et al., 2001. This opposes the view that regards learning as stocking up of knowledge, and teaching as the deposition of knowledge into empty receptacles – a banking education system (Freire, 1970). The banking education system places great emphasis on the teacher as a narrator, while learners do not actively engage with the mathematical language and content. As a result, the development of learners' mathematical language acquisition skills is impeded, which most likely, in turn, negatively affects their mathematical learning performance and conceptual understanding.

This study is located within a constructivist understanding of learning, where knowledge is not passively received from authoritative sources, but is constructed during a sense-making process. Asking learners to use the Frayer model to understand mathematical vocabulary is a way of facilitating sense-making by the learners.

Piaget's (1954) notions of assimilation and accommodation will come into play in this research. Assimilation entails incorporating new experiences into existing mental structures (Piaget, 1954). Students' prior knowledge serves as the basis for further learning. The prior knowledge might be in the form of previously learnt scientific concepts/vocabulary but may also be students' everyday knowledge. Accommodation is the process of reorganizing these structures to become able to handle new experiences. Emphasis is on facilitating learners' engagement with tasks, and graphic organisers will encourage learners to engage more with their tasks. However, constructivism does not espouse any specific teaching practice, so long as learners become active participants. In conducting investigations and managing concrete objects (Rossouw et al., 2000), their practise becomes constructivist in nature.

An Important consideration in this research is cognitively engaging learners to enable them to think through and articulate their learning in a way which makes it useful. Goos et al. (2020) proposed that for learning to be retained and available for use, learners must be encouraged to make their own construction of knowledge and must learn to manage their learning. However, to achieve this level of competency, it is crucial to ensure that learners have the language needed to do so (Coyle et al., 2010). Students must be able to learn mathematical content through vehicular language. They also need to learn the language in which the content is taught (Coyle et al., 2010).

As illustrated in the preceding discussion, there is a need to consider a pedagogy that encourages a fusion of mathematical content learning, and mathematical vocabulary and language learning. The CLIL model serves this purpose. Pinner (2013) claims that academic knowledge and skills could not be developed without access to the language in which the knowledge and skills are embedded, constructed, and evaluated. Likewise, academic language cannot be acquired in a context devoid of content. These two, content and language, must be intertwined in a way that benefits the learners.

This research utilized a dyadic framework, acknowledging the insufficiency of a single framework for the task at hand. In the following section, Active Learning Theory is discussed, given CLIL's presumption that content, and language integration occurs within the realm of active learning. After exploring ALT, the discussion proceeds to Cognitive Load Theory in the context of CLIL, and finally, an examination of the integration of the Frayer Model.

3.2 The Active Learning Theory (ALT)

Active Learning Theory (ALT), rooted in constructivist learning principles, places a profound emphasis on individuals actively constructing their understanding. This process involves connecting new ideas and experiences to existing knowledge, fostering a deeper comprehension of the subject matter (Fazio, 2020). ALT consists of five fundamental elements: materials, manipulations, choice, language, and adult support. These components collectively create an environment where learners actively engage with their education.

Materials are a cornerstone of active learning, providing learners with tools to interact with as they navigate their learning journey. From hands-on equipment to digital resources, the availability of diverse materials is crucial for fostering engagement and exploration. However, learners themselves remain at the core of active learning. The role of adult support is indispensable in guiding and facilitating this active engagement. In active learning environments, students cease to be passive recipients of prepackaged information and instead become active contributors to their own learning experiences.

Effective communication, facilitated through language that encourages both teaching and learning, is another key facet of active learning. This dynamic interaction creates a rich learning environment where students can actively participate and express their thoughts and ideas. The landscape of active learning is marked by diverse scholarly interpretations and definitions, contributing to its multifaceted nature.

Fazio (2020) aptly characterizes active learning as a pedagogical approach that propels students into practical and cognitive activities, forging a deeper connection between doing and understanding. This approach prioritizes critical thinking skills over the mere transmission of information. Similarly, Theobald et al. (2014) emphasize the importance of active involvement through hands-on activities and class discussions, promoting higher-order thinking skills and collaborative group work.

In 2015, Carr et al. introduced a definition of active learning that underscores students' proactive role in shaping their own knowledge construction. These various definitions converge on the idea that active learning strategies aim to develop students' skills while reducing the emphasis on the passive transfer of knowledge. Active learning principles hinge on student activity, encompassing activities such as reading, discussing, writing, questioning, note-taking, and peer interactions, all of which demand higher-order cognitive processes. Additionally, these paradigms encourage students to explore and reflect on their attitudes and values, nurturing a profound understanding of the subject matter.

A common thread among these definitions is the active role students play in constructing knowledge and comprehending concepts. While not explicitly articulated, metacognition, or the ability to reflect on one's own thought processes, emerges as a pivotal component bridging the gap between activity and learning. Integrating metacognitive resources into the framework of ALT could further enhance its applicability and impact within the realm of education.

Empirical research overwhelmingly supports pedagogical approaches centred on active learning. Such approaches consistently yield superior results compared to traditional transmission-based methods. Freeman et al. (2014) conducted a comprehensive meta-analysis across science, technology, and mathematics education, revealing three key findings:

Firstly, students exposed to traditional lectures were 1.5 times more likely to struggle academically than those engaged in active learning methodologies. This disparity in failure rates underscores the effectiveness of active learning, with active learning participants enjoying an 80% likelihood of passing, compared to a 65% pass rate for their transmission-based counterparts.

Secondly, the meta-analysis revealed that active learning's efficacy transcended subject boundaries, spanning an array of academic disciplines. This robustness further substantiates its effectiveness in enhancing learning outcomes.

Lastly, the evidence supporting the benefits of active learning was unequivocally strong. Active learning consistently outperformed transmission-based approaches across various subjects, highlighting its widespread applicability and value in education.

These meta-analysis results align with prior research by Hake (1998), Prince (2004), and Ruiz-Primo et al. (2011), all emphasizing the advantages of active learning. The incorporation of active learning strategies consistently improved student outcomes (Ruiz-Primo et al., 2011). Despite some of these studies dating back to the 1990s, the superiority of active learning over transmission-based methods remains steadfast and relevant.

In the context of this research, Active Learning Theory (ALT) emerges as a vital framework for evaluating the proposed intervention, highlighting the importance of fostering active engagement and metacognition to enhance education outcomes.

3.3 The Cognitive Load Theory (CLT)

Theories of cognition delve into the intricate mechanisms of how humans acquire, organize, retain, and later retrieve knowledge, thereby shaping their thinking and learning processes (Anderson, 1983; Sweller, 2011). One prominent theory, the Cognitive Load Theory (CLT), elucidates how cognitive resources are allocated during learning and problem-solving endeavours. This instructional theory has gained considerable traction over the past three decades, offering valuable insights into strategies that facilitate learning (Ayres & Paas, 2012). Notable applications of CLT include the use of worked examples and the exploration of the modality effect—a phenomenon wherein learner performance is influenced by the mode of presentation of the study material (Plass et al., 2013).

Central to CLT is the understanding of human cognitive architecture, which comprises a limited working memory and an essentially boundless long-term memory (Sweller et al., 2019). Within this framework, CLT can be succinctly defined as the "development of instructional methods that efficiently harness individuals' limited cognitive processing capacity to enhance their ability to apply acquired knowledge and skills to novel situations" (Paas et al., 2003; Sweller et al., 2019).

CLT advocates that when teaching and automating complex knowledge, instructors must provide guidance that optimally matches the cognitive load capacity of students, avoiding overwhelming them (Gilchrist & Cowan, 2011). This concept emphasizes that task complexity should remain within the bounds of working memory capacity, which research by Gilchrist and Cowan (2011) suggests can be as low as two elements. Moreover, CLT prompts educators to consider how the structure of long-term memory either facilitates or hampers learning (Sweller et al., 2019).

Effective instructional materials, in line with CLT principles, are designed to deliver information at a pace and difficulty level conducive to learners' cognitive capacity (Kalyuga, 2011). In essence, such materials channel cognitive resources toward activities crucial for learning, rather than squandering them on preliminary or irrelevant aspects of the subject matter.

CLT, with its focus on sensory perception, reasoning, problem-solving, language use, learning, memory, and action (Medin et al., 2001), is particularly salient in mathematics education. Given the continuous influx of new mathematical concepts and information in the curriculum, CLT plays a pivotal role in this domain compared to other subjects in the school curriculum.

This theory is firmly grounded in research findings concerning memory processes, especially those that transpire between working memory and long-term memory. As the name implies, CLT places significant emphasis on the cognitive load experienced by learners when tackling problems. Cognitive overload can severely impede learning because insufficient working memory resources are available for the requisite learning processes (Sweller, 1988).

Key tenets underpin CLT:

1. Working memory has limited capacity.

2. Long-term memory has virtually limitless capacity.

3. Learning processes necessitate the involvement of working memory in constructing schema.

4. Exceeding the limits of working memory capacity curtails effective learning (Sweller, 1988).

In the context of CLT, the quality of instructional design hinges on the recognition of learners' working memory capabilities and constraints. The working memory, as described by Reid (2020), serves as both a "holding and thinking area" where new information is processed and interacts with previously stored long-term memories. It consists of two primary components: the visual loop and the auditory loop, both orchestrated by a processing centre (Coombs, 2017). Working memory empowers individuals to engage in both creative and logical thinking during problem-solving activities.

Throughout the learning process, a significant portion of cognitive resources is allocated to the sorting process within working memory. This process is instrumental in organizing information through schema formation, thereby reducing the cognitive load on working memory and facilitating storage in long-term memory. However, instructional formats that overload the working memory, such as the use of unfamiliar mathematical vocabulary, inhibit schema formation (Coombs, 2017), hampering comprehension. Hence, the Frayer model is proposed as an effective tool for focused mathematical vocabulary learning, designed to prevent cognitive overload and comprehension impediments.

The cognitive load, a central concept in CLT, arises from sensory input and is placed upon the working memory during information processing (Sweller et al., 2011). It encompasses the intrinsic load, which relates to the complexity of the learning material itself; the germane load, which pertains to the cognitive effort invested in learning; and the extraneous load, associated with irrelevant or unnecessary information (Figure 3.1). Proper management of cognitive load is essential to facilitate effective learning, and the use of the Frayer model is seen as a valuable strategy to mitigate difficulties associated with any of these three types of cognitive load.

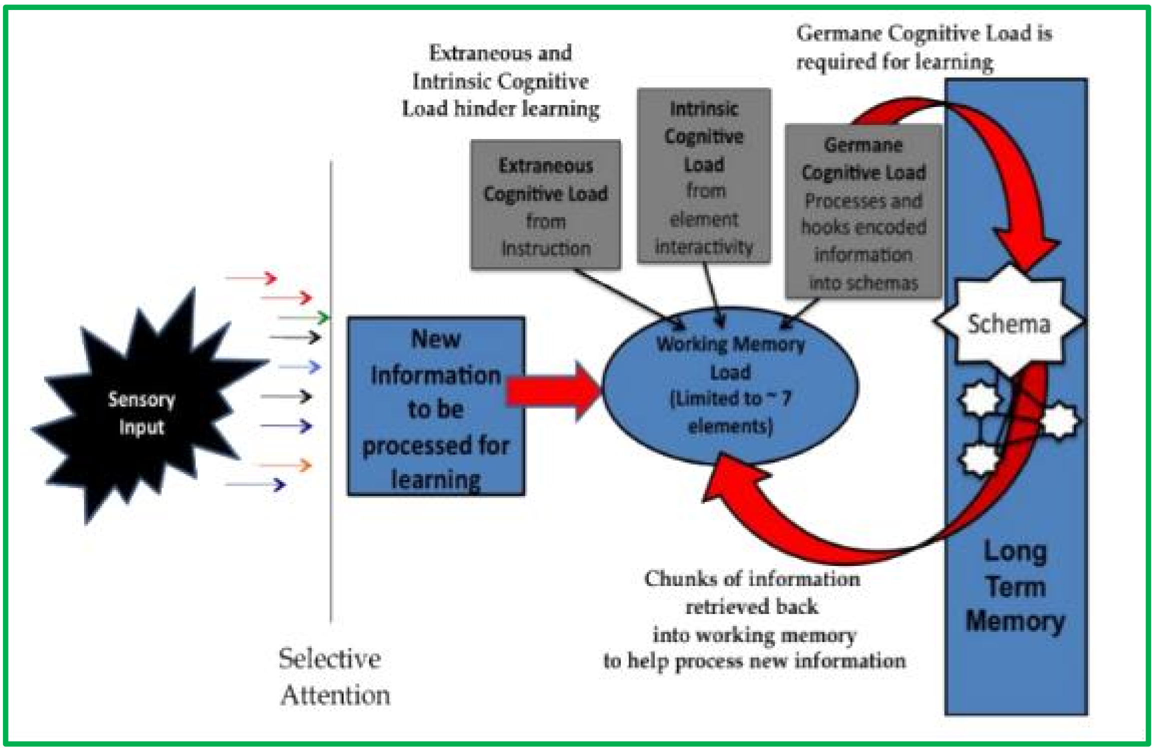


Figure 3.1: The Cognitive Load Theory (CLT) schema (Adapted from Tricot et al. (2020)).

In this comprehensive study, the intricate relationship between cognitive load, mathematical learning, and instructional design is explored. Cognitive load, a pivotal factor in the learning process, manifests itself in the form of both mathematical vocabulary associated with mathematical functions and the inherent complexity of mathematical content. The imperative lies in ensuring that cognitive load remains within the confines of working memory capacity for effective processing. This premise aligns with the findings of Cognitive Load Theory (CLT), as expounded by Sweller et al. in 2011, which posit that students exhibit enhanced learning outcomes when engaging with task information that can be comfortably accommodated within working memory.

However, when confronted with tasks of high complexity and limited knowledge stored in long-term memory, learners face the peril of cognitive overload, leading to a decline in performance. Essentially, any cognitive load that surpasses the threshold of working memory's capacity is destined to be forgotten. This overload not only disrupts the learning process but also induces stress, which, paradoxically, further diminishes the working memory's capacity. Yet, intriguingly, an appropriate level of cognitive load can have a beneficial effect by stimulating thinking and facilitating the consolidation of knowledge into long-term memory.

Considering these insights, it becomes evident that the design of instructional materials holds profound implications for effective learning. The crux of this matter lies in the deliberate cultivation of "beneficial stress" to optimize the learning process. One effective strategy to achieve this is the avoidance of split-source information, which tends to burden learners with unnecessary cognitive load. Split-source information occurs when learners are needlessly required to mentally integrate various sources of mutually referring information, such as separate text and diagrams.

To mitigate the adverse effects of split-source information and foster more effective mathematical vocabulary acquisition, the proposal is made to incorporate the Frayer model into instructional design. The Frayer model represents a pedagogical tool that ingeniously amalgamates a term's definition, its key characteristics, illustrative examples, and non-examples into a cohesive and comprehensive framework. By doing so, the Frayer model serves the dual purpose of bridging the conceptual gap between definition and image and reducing the undue cognitive stress experienced during the learning process. This holistic approach to vocabulary acquisition not only enhances comprehension but also contributes to the optimization of cognitive load, facilitating more effective and stress-free learning experiences for students.

The intricate interplay between cognitive load, instructional design, and learning outcomes underscores the critical importance of thoughtful pedagogical approaches. Strategically managing cognitive load and harnessing beneficial stress through methods like the Frayer model empower educators to guide students through the complex world of mathematics with greater ease and effectiveness, fostering a more conducive learning environment.

3.4 Content and Language Integrated Learning (CLIL)

CLIL is an educational approach in which a language, foreign to the learners, is used as the language for learning and teaching non-language subjects (Marsh, et al. 2012). According to Marsh et al., CLIL suits the times, needs and aspirations of today’s learners and schools, where classrooms across the globe are multilingual because of population movement and migratory patterns. I would like to make clear that this research does not concern linguistic relativity[2], but CLIL's call for a dual focus (learning language and content in the same space) is highly pragmatic.

Consequently, the focus is not on the broader aspect of language learning, but rather on learning vocabulary pertinent to the understanding of mathematical content. In this study, the vocabulary learning will be supervised by mathematics teachers who themselves are not native speakers of the language of instruction and lessons will be timetabled as regular lessons within the school curriculum (Dalton-Puffer & Smit, 2013). The mathematics teachers/researcher will serve as language teachers and, at the same time, as teachers of specific subject matter (Coyle et al., 2010). Language teaching and learning will be emphasised, but it will be fused with the mathematical vocabulary and the teaching and learning of mathematical content.

CLIL encourages learning by construction, rather than by instruction (Marsh et al., 2012) and is influenced by constructivist claims, thus ratifying active, cooperative learning (Slavin, 2011). As a result, in CLIL structured programmes, the focal point is the interrelation between cognitive and thinking skills and communication (Gil, 2010). Equally, in an earlier section of this document, I referred to the DBE's (2016) suggestion on the necessity of emphasizing the correct use of mathematical language to ameliorate learners' cognitive and communicative weaknesses.

CLIL learners are encouraged to oversee their learning process by adopting the learning and problem-solving strategies that best suit their circumstances. In this regard, teachers are encouraged to create environments that cognitively engage the learners. It is through such engagement that effective content learning can be achieved (Coyle et al., 2010).

In the dual endeavour to improve both the learners' knowledge and skills in a subject, and their language skills in the language of teaching and instruction; subject content and language play different roles. Language serves as the medium through which subject content is learnt, and subject content is the resource for learning the language (Coyle et al., 2010). This interplay is summarized by Coyle (2007) in the "four C's" framework: communication (in the language of instruction), content (the knowledge and skills of the subject), culture (intercultural knowledge and understanding), and cognition (thinking skills).

[2] Linguistic relativity claims that cultural concepts that are inherent in different languages affect the perception of the world we experience and cause different language speakers to think and behave differently.

Content determines the thinking skills, language, and aspects of cultural understanding that need to be taught and learnt. As such, Coyle's four C's framework places content at the centre of the model. The framework is shown in Figure 3.2.

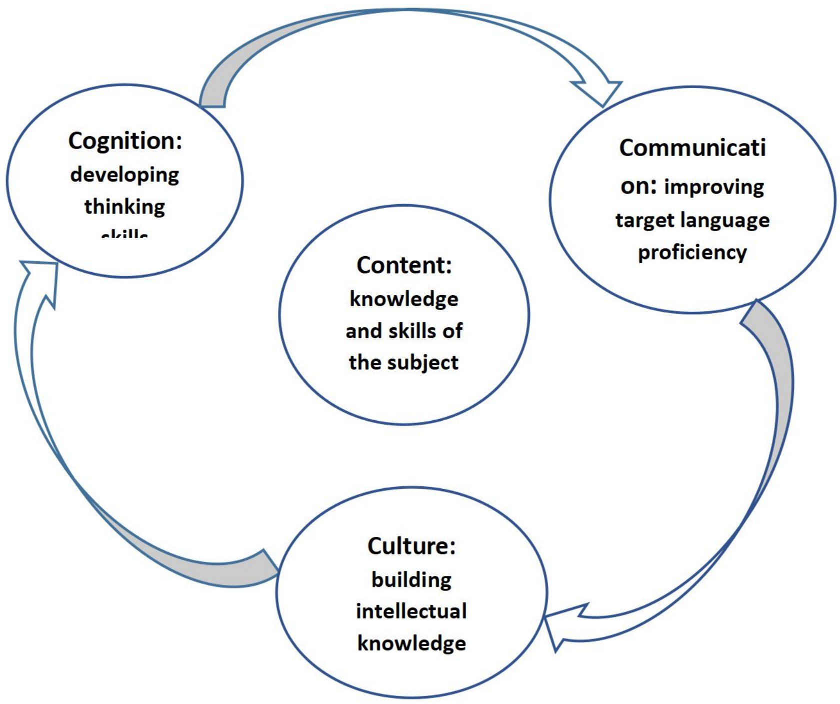


Figure 3.2: The four C's framework (Coyle, 2007).

The target language is derived from the subject content and is thus dominated by subject related vocabulary. In this research, it is dominated by mathematical vocabulary. The vocabulary becomes the focus of the language learning for the purpose of understanding mathematical concepts. The target language is used for exploring, discussing, and writing about the subject matter. Additionally, it is used for employing cognitive skills, such as giving reasons for one's opinion, evaluating, concluding, justifying, and so on, as well as for using learning skills such as interpreting and classifying information.

Vukovic and Lesaux (2013) explored mathematics as a specialized form of language, and they supported the idea that intertwining rich language with rich mathematics is a vital component for effective mathematics learning. In agreement with this, the New Zealand Numeracy Development Project (NZNDP) developed and implemented a learning framework that allows and exposes learners to contexts where they simultaneously develop both concepts and language. The NZNDP framework was built on a constructivist approach, appropriate for settings where learners are engaged with problem solving/investigations (Tweed, 2014).

The NZNDP framework was built on the notion that mathematical language ought to be developed by individual learners, because it is unique to each learner. Learners develop the language as a response to the demands placed on them during their involvement in mathematical tasks. The framework also assumes that one of the best ways to develop mathematical language is through the interaction between learners and teachers as they engage in mathematical activities. In this interaction, learners may develop materials and diagrams as a platform for developing language appropriate to the concepts being studied. In this research, the Frayer model serves to assist with encouraging the learning of mathematical vocabulary.

Teachers play a significant role in both the concept and language development of their learners. Their knowledge and experiences are useful for creating mathematical experience for learners, and for providing meta-information, including examples, titles, or origin of concepts. In addition, teachers ought to place clear demands on learners, such that they develop and use correct language. This assists learners in solving problems and communicating mathematically (DBE, 2017; Tweed, 2014).

For teachers to achieve the above, they must be able to develop learning materials that facilitate content-language integration, but without putting excessive cognitive load on the learners, which would inhibit their learning. The preceding section of this work explained a theory, the Cognitive Load Theory (CLT), that sheds light on the tenets of good instructional material. The CLT implies that the design of instructional material must keep the cognitive load on learners at minimal levels during learning if they are to be effective.

This work proposes the use of the Frayer model as vocabulary-instructional material, as it has the capacity to exclude or minimise external load that inhibits learners' comprehension. For instance, the Frayer model integrates mutually referring information about a concept by putting together diagrams and text about the attributes of the concept in one place. In the next section, I present the Frayer model and discuss it for the sake of highlighting how it integrates with the concepts of the theoretical framework discussed so far.

3.5 The Frayer model

The Frayer model is a graphic organiser used for vocabulary acquisition. Vocabulary acquisition is the process of learning the words of a language (Sunubi, 2018). Vocabulary acquisition is important for learners, as it relates to proficiency and fluency in the language of content subjects, as well as to the language of teaching and learning. It helps the learners to use the skills of understanding, reading, writing, and speaking necessary for participating in the communication of the content subject. The Frayer model helps learners to understand concepts. It can be used as a basis for writing for all, including the youngest, of learners. Due to its layout, the Frayer model allows learners to see what a concept is and what it is not. It has four quadrants, which give a learner the chance to define a concept, give pertinent characteristics of the concept, and give examples and non-examples of the concept (İlter, 2020). The Frayer model can be used for notetaking. Note-taking helps learners to remember what is important about a concept (Lew et al., 2016). If they do not take notes, learners are likely to recall information less than five percent of the time (Wong & Lim, 2021). Figure 3.3 presents a template of the Frayer model.

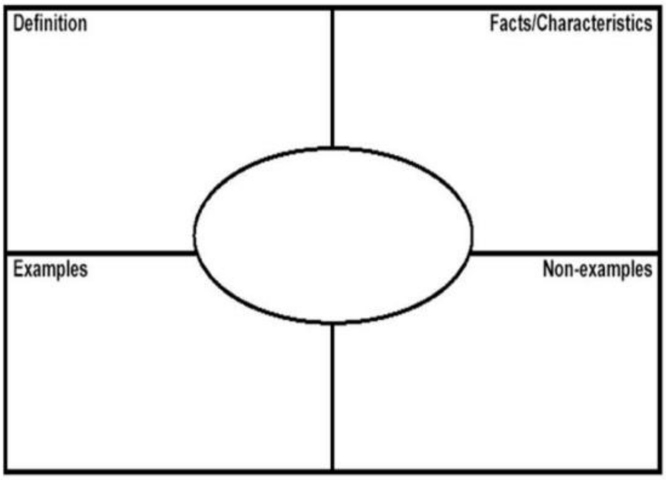


Figure 3.3: Template of the Frayer model for vocabulary acquisition.

The four quadrants of the Frayer model juxtapose the essential attributes of a vocabulary word. More about the Frayer model has already been discussed in Section 2.1.4 of this thesis.

3.6 Dyadic Framework for Content and Language Integrated Learning (DF/CLIL)

It is crucial to emphasize that this study's scope brings together the active learning theory, cognitive load theory, and content and language integrated learning (CLIL) into a unified framework, which serves as the foundation for analysing the proposed pedagogical intervention. The synthesis of these theories illuminates the comprehensive nature of the research, as each theory contributes distinct facets that collectively underscore the effectiveness and significance of the Frayer Model in enhancing the comprehension and mastery of mathematical vocabulary and concepts.

The active learning theory (ALT), rooted in constructivism, forms the guiding principle of this research. As previously mentioned, CLIL operates not as a theory itself but as a pedagogical approach that seamlessly integrates into the ALT framework. Additionally, the Cognitive Load Theory (CLT) is another pivotal theory discussed, emphasizing the need to design and deliver educational content while considering the limitations of human working memory capacity. In the context of this research, these theoretical foundations, the pedagogical approach, and the research artifact have been interwoven into a cohesive framework.

In this integrated framework, the ALT assumes the role of the central guiding theory, which is put into practice through the CLIL pedagogy employing the Frayer Model as a mediation tool. While the ALT, CLIL, and Frayer Model collectively steer the external learning processes and can be observed outwardly, the CLT primarily delves into the internal cognitive mechanisms of learning.

The roles of each component in this framework harmoniously intertwine. Active learning strategies are meticulously crafted based on cognitive principles, tapping into learners' cognitive processes to elevate the overall learning experience. CLIL takes this a step further by not only engaging learners actively but also intertwining content and language acquisition, fostering a comprehensive understanding while refining language skills. The Frayer Model, with its focus on in-depth exploration, aligns seamlessly with cognitive learning theory by prompting students to construct a robust mental framework around each concept.

Ultimately, in this research, the Dyadic framework (figure 3.4) emerges as a comprehensive educational approach, bringing together active learning theory, cognitive learning theory, CLIL, and the Frayer Model in perfect synergy. This approach propels students beyond superficial learning, facilitating active engagement, cognitive growth, language proficiency, and profound comprehension. As students participate actively, engage in cognitive processing of information, delve into subjects using language, and employ strategic tools like the Frayer Model, they emerge as well-rounded learners equipped to navigate the intricacies of the contemporary world.

In Figure 3.4, the dark rectangle encompasses elements of active learning. Within active learning, the participants grapple with a cognitive load that originates from the mathematical content and the mathematical vocabulary. These two units (material which learners manipulate) constitute the first and second elements of active learning. The third element of active learning is adult support. Adult support is provided for the learning of both mathematical content (functions) and mathematical vocabulary. In this study, the subject teachers planned and delivered the content in their normal lessons as per the school timetable. The pink lines in Figure 3.4 show where something is happening, e.g. adult support happens during the learning of content, as well as the learning of mathematical vocabulary. The fourth element of active learning, language, takes the form of language repertoires. The repertoires and the mathematical vocabulary influence one another, hence the double arrow between them. Understanding mathematical vocabulary influences fluency in the repertoires and vice versa.

Double arrows show a dual relationship between phenomena, while single arrows show that the influence is one-directional, e.g. learning mathematical vocabulary affects conceptual understanding and procedural proficiency, but the converse is not true.

The dyadic framework shows that in the active learning process, there is an integration of the learning of both mathematical content and vocabulary. This integration is explained by the pedagogy – Content and Language Integrated Learning (CLIL). As the learners learn mathematical vocabulary, they use the Frayer model, which the study has proposed as the intervention for learning mathematical vocabulary.

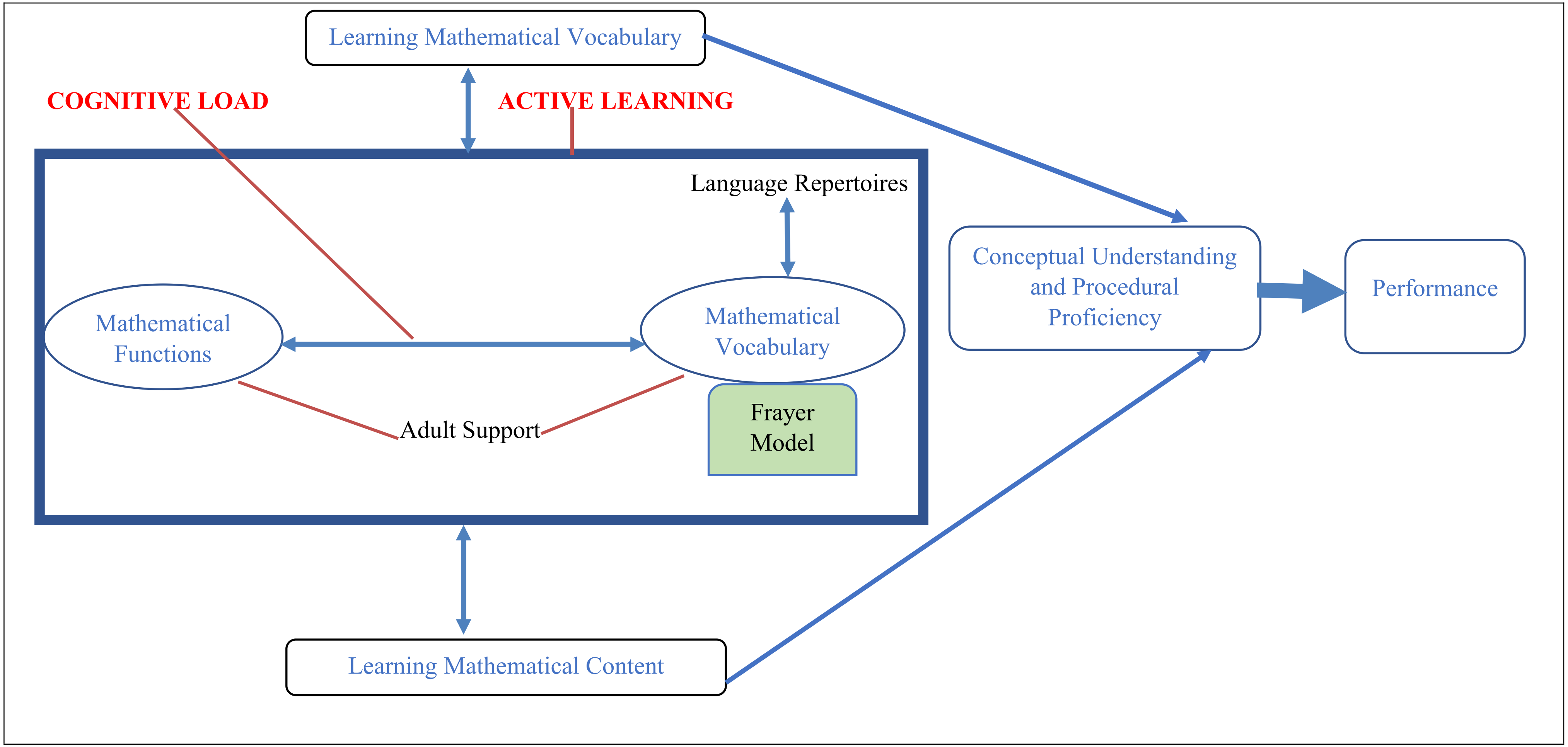


Figure 3.4: Dyadic framework for Content and Language Integrated Learning using the Frayer model.

Learning mathematical vocabulary is deemed a key issue in this research. To this effect, the vocabulary-learning strategies available to the learners were investigated before the Frayer model was used as an approach in this regard. The research sub-questions were aligned to the focus areas of the research: mathematical vocabulary, the Frayer model and conceptual understanding and performance.

Table 3.1 presents the association between the research questions, focus areas, and the respective theoretical frameworks. In the study, the Active Learning Theory was used as a lens for four research sub-questions and the Cognitive Load Theory was used for one sub-question. Details of this link are shown in the subsequent table.

Table 3.1: Linking of research questions to focus areas.

<table>
<tr><td rowspan="10">MAIN RESEARCH QUESTION</td><td rowspan="10">What affordances does the integration of mathematical vocabulary and content learning provide for conceptual understanding and performance in mathematics?</td><th colspan="4">FOCUS AREAS</th></tr>
<tr><th colspan="2">Mathematical Vocabulary</th><th>Frayer Model</th><th>Conceptual Understanding</th></tr>
<tr><td rowspan="2">Knowledge and Proficiency</td><td>Research sub-question 1</td><td>Research sub-question 4</td><td>Research sub-question 5</td></tr>
<tr><td>How competent are Grade 11 mathematics learners in associating mathematical vocabulary with a given mathematical object?</td><td>How effective is the Frayer model as an approach for learning the mathematical vocabulary of functions?</td><td>To what extent does competency in mathematical vocabulary link to learners’ performance in mathematics?</td></tr>
<tr><td rowspan="2">Learning Strategies</td><td>Research sub-question 2</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>What are the approaches that learners employ to acquire mathematical vocabulary during their mathematics lessons?</td></tr>
<tr><td rowspan="2">Language Proficiency</td><td>Research sub-question 3</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>In what ways do learners become exposed to and engage with mathematical vocabulary of functions during lessons as they learn mathematical functions?</td></tr>
<tr><td colspan="3">Active Learning Theory</td><td>Cognitive Load Theory</td></tr>
<tr><td colspan="4">The other two focus areas, i.e., mathematical functions and language repertoires do not have questions that are directly related to them. This is because language repertoires relate to the mathematical vocabulary and mathematical functions through the content from which the vocabulary is derived – in which conceptual understanding and performance is measured. Consequently, these focus areas are necessarily already addressed in the questions asked.</td></tr>
</table>

3.7 Mathematical vocabulary and mathematical language

Mathematical vocabulary is useful for reading texts (Meaney et al., 2012) and is a predictor of competency in learners' comprehension and learning of content (Dunston & Tyminski, 2013). A lack of its knowledge negatively affects learning content (Fisher & Frey, 2014). According to Dennis et al. (2016), the language of mathematics is "critical in helping children develop the means to acquire new concepts" (Dennis et al., 2016), and this further stresses that cognitive barriers that constrain conceptual understanding are removed by understanding mathematical language. This seems to suggest that the learning of mathematical language might serve as a prerequisite for developing understanding of mathematical concepts. This confirms Vygotsky (1986), who had claimed that concepts are only possible with access to words, and that thinking in concepts exists within the confines of verbal thinking. This, therefore, makes mathematical language critical for the teaching and learning of mathematics for all learners (Bowie, 2016).

In South Africa, the Department of Basic Education has, on numerous occasions, through its national diagnostic reports reiterated the necessity to emphasise the correct use of mathematical language by both teachers and learners (DBE, 2016). Riccomini et al. (2015) also recognised that learners need to exert a large amount of effort to use, understand, and apply mathematical words, symbols, and diagrams properly throughout their learning. Similarly, Choat (1974) stressed the close interdependence between language and conceptual understanding by stating that the verbal element is necessary, both as a means of communication and as an instrument of individual representation.

Irrespective of its role, many learners regard mathematics to be a foreign language (Addae & Agyei, 2018), and some educators also view it as a second language (Mbewe & Nkhata, 2019). Consequently, this poses a challenge, as the learners would like to leave mathematical language learning to disciplines other than mathematics. However, according to Dunston and Tyminski (2013), proper use of mathematical vocabulary is a critical tool for improving students' mathematical understanding and their level of achievement in the subject. Riccomini et al. (2015) demonstrated that the development of correct mathematical vocabulary needs to be a point of emphasis that teachers must address in their practise, as the language of mathematics is necessary for the communication of higher-order mathematical reasoning, and as a sign of conceptual understanding.

Despite the affordances of language in mathematics education, the language of mathematics poses a great many challenges to its users (Addae & Agyei, 2018). These challenges come from the fact that mathematical language is context dependent; related terms or pairs of words (e.g. hundred and hundredths, factor and multiple, solve and simplify) are used to describe different concepts (Peng & Lin, 2019; Thompson & Rubenstein, 2014) and some mathematical words have multiple meanings. Furthermore, students have difficulty with the precision of mathematical language (Swan, 2018). This is the case with terms such as average, reflection, even, edge, etc., that are used in everyday life but also appear in mathematics with a similar but more precise meaning. However, learners ought to master the mathematical language and use it when participating in mathematics discourses, as the language is an indispensable component of students' academic achievements (Pearson et al., 2020). Research has also established that in mathematics, there is a positive relationship between vocabulary and conception, such that vocabulary understanding should be regarded as a key component in the understanding of mathematics (Elder & Paul, 2020).

3.8 Language repertoires

Van Jaarsveld (2016) proposed that language repertoires could serve as a resource for teachers as they transition between the various modes of representation of mathematical objects. In this research, the perspective is that akin to their teachers, learners, when utilizing language repertoires, have the potential to adeptly navigate between these representations. Furthermore, each repertoire can function as a tool for enhancing the acquisition of mathematical vocabulary to foster conceptual understanding. The four categories of repertoire are delineated below.

Literal: The literary category demands that learners should be able to use the correct language to read a mathematical object. The object must also be read in the correct sequence. The interval: $0 < x \leq 5$, should be read as "all the real x-values that are greater than zero and at the same time less than or equal to five" (van Jaarsveld, 2016). The way an object is read impacts how it is understood and/or visualised. Improper reading of objects may result in errors and misconceptions. Learners should, in this repertoire, also identify the mathematical object as an expression, equation, inequality, identity, or relationship, and be familiar with the instruction associated with it and that instruction's meaning. For example, equations and inequalities are solved, expressions are simplified, identities are proved, and relationships are drawn or sketched.

Algebraic: This repertoire seeks to address the misunderstandings associated with the operations that make up an object, for example $x + y < 8$, $x, y \in Z+$. The object can be described as "the sum of two different positive integers is less than eight" (van Jaarsveld, 2016).

Graphical or Cartesian: This repertoire of vocabulary focuses on the object as it is depicted in the Cartesian plane. Questions that fall within this category usually pose challenges to learners (DBE, 2016), for example, concerning the sketch below: "Determine the value(s) of x, for which f(x) >g(x)".

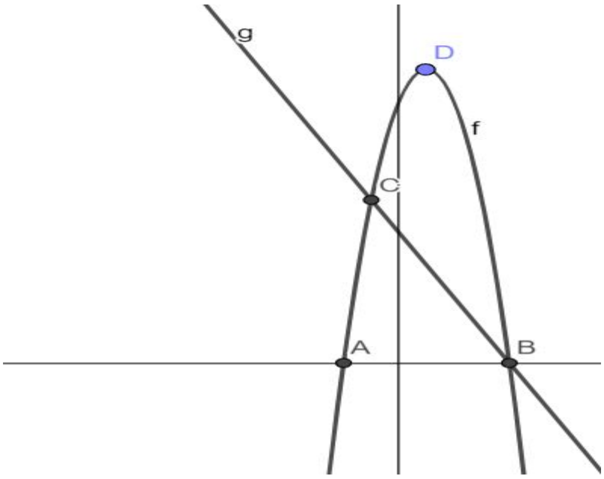


Figure 3.5: Visual images of two functions.

This question (Figure 3.5) seeks the real values of *x* on the *x-axis* for which (where) the maximum value of the quadratic function $y = -2x^2 + 4x + 16$ lies above the decreasing linear function:

$y = -2x + 8.$

Procedural or algorithmic: This repertoire entails the language used in explaining how to "do" mathematics. Taken from van Jaarsveld's work, "the explanation for obtaining a solution to 4x = 32 would be first to see that the equation seeks a value for x for which the left- and right-hand sides are equal in value. Express each term in the equation as a power with a prime base of two because 4 and 32 can both be written as a power of 2. Because we are working in an equation and the bases are equal, the exponents must be equal to, to maintain the equality. So $2^{2x} = 25$ and hence, $2x = 5$. 2 is multiplied by x on the left-hand side, to isolate x, and thus we divide throughout by 2 because the division is the reverse operation of multiplication. In this way, we obtain $x = 2, 5$ which when substituted for x in the original equation will give a true statement" (van Jaarsveld, 2016, p. 13).

In this research exposing learners to the Frayer model, which requires them to be actively involved in learning mathematical vocabulary as a way of knowledge construction and learning, the fundamental tenets of active learning are fully embraced.

3.9 Conclusion

In this chapter, the Active Learning Theory and the Cognitive Load Theory were presented as the guiding theories for this research. Additionally, Content and Language Integrated Learning (CLIL) was outlined as the chosen pedagogy for learning mathematical vocabulary using the Frayer model. The chapter also delved into the synergy between these theories (ALT and CLT), pedagogy (CLIL), and the Frayer model in facilitating the acquisition of mathematical vocabulary and subject content knowledge. The study's content, drawn from the domain of mathematical functions, was highlighted as the source of mathematical vocabulary integrated into the subject matter. The primary aim is to engage learners actively in using the Frayer model for learning mathematical vocabulary within CLII-structured lessons.

Furthermore, the chapter introduced the conceptual framework named the Dyadic Framework for Content and Language Integrated Learning (DF/CLIL), constructed by interconnecting these theoretical perspectives. The focus areas of the study were identified: mathematical vocabulary and language repertoires. The DF/CLIL conceptual framework served as the foundation for formulating the research questions, with a table illustrating the alignment between these focus areas and the research questions provided in this chapter.

The chapter elaborated on the correlation between ALT and CLT in the study. Active learning engages learners cognitively and surpasses traditional exposition methods, which place more emphasis on teachers than on learners. Active learning positions learners at the centre of the learning process, with teachers facilitating and guiding them to take ownership of their learning. In this context, the use of the Frayer model in a CLIL structured environment was discussed, where learners integrated the learning of mathematical vocabulary with the subject content of mathematical functions.

The use of the Frayer model effectively regulated cognitive load for learners, mitigating the effects of split-source information. This model enabled learners to comprehend the definition of a concept, its characteristics, relevant examples, and non-examples simultaneously. Furthermore, the model could be applied individually, in small groups, or by the entire class, consistently regulating cognitive load and facilitating active learning. The study's choice of a dyadic theoretical framework is rooted in the concurrent interaction between active learning and cognitive load regulation through the use of the Frayer model.

The chapter also elucidated the connection between the five research sub-questions and the dyadic theoretical framework. While these constructs are interrelated, the first four sub-questions primarily align with ALT, while the fifth sub-question is more closely related to CLT. This categorization was presented in Table 3.1.

The following sections of this thesis will cover the study's design and methodology, presentation of the results (both pilot and main study), interpretation of the results, and the study's conclusions and recommendations.

CHAPTER 4 : RESEARCH DESIGN AND METHODOLOGY

In this chapter, the research design and methodology are presented concerning the conduct of the research. The research integrated mathematical vocabulary learning with the acquisition of mathematical content knowledge by employing the Frayer model as a vocabulary-learning strategy to facilitate the learning of mathematical vocabulary related to the concept of functions. The research strategy is discussed in Section 4.1, commencing with an exploration of the research method employed and the subsequent discussion of research paradigms (4.1.1), culminating in the selection of the preferred paradigm. Additionally, this section provides insights into the piloting and sampling procedures utilized during the research (4.1.2).

In Section 4.2, the chapter delves into the procedures for data collection, encompassing both the pilot study and the main study. Besides detailing the data collection procedures for the main study, Section 4.2 also elucidates the study's intervention. Given that this is a quasi-experimental study, Section 4.2 offers insights into the constitution of the control and experimental groups. Furthermore, the chapter provides elaboration on methodological rules, reliability, validity, generalization, and ethical considerations in Section 4.3, Section 4.4, Section 4.5, and Section 4.6, respectively.

The organization of the study is addressed in Section 4.7, while the study's limitations are discussed in Section 4.8. Section 4.9 presents the research test for data collection, and Section 4.10 and Section 4.11 respectively provide an overview of the additional data collection methods employed during the research, namely, lesson observations and interviews. Section 4.12 outlines the development of the coding framework, concluding with a summary of the chapter in Section 4.13.

4.1 Research method

This study is centred around a comprehensive exploration of the influence of targeted mathematical vocabulary instruction on students' grasp of mathematical terminology and its correlation with their overall comprehension of mathematical concepts. The investigation employs a combination of quantitative assessment outcomes and qualitative insights drawn from observations and interviews. The chosen research approach employs a mixed method design, which merges quantitative and qualitative research methodologies within a single study, fostering a more profound comprehension of the research subject at hand. The realm of mixed methods research offers several design possibilities, such as convergent design, explanatory sequential design, embedded design, transformative design, multi phase design, sequential transformative design, and triangulation design, which researchers can select based on their research objectives and the nature of their inquiries.

For this research, an Embedded Design with Qualitative-Enhanced Triangulation has been adopted. In this design, emphasis is placed on either quantitative or qualitative data as the primary investigative tool, while the complementary data type serves to augment insights, offer context, or validate findings. This approach is particularly advantageous when researchers seek a deeper understanding of specific facets of their study or aim to unravel the intricacies underlying their primary data outcomes. In this context, the primary focus is on quantitative data collected from research tests, while supplemental insights are derived from lesson observations and interviews. The inclusion of triangulation, involving the utilization of diverse sources, methodologies, or data forms, bolsters the verification and substantiation of research conclusions, thereby bolstering the overall credibility and dependability of the results.

The core of this research centres on the collection and analysis of quantitative data, specifically test results pertinent to students' comprehension of mathematical vocabulary and their performance in mathematics. The quantitative research test is administered both prior to and after the study intervention, facilitating an evaluation of the impact of targeted mathematical vocabulary instruction on students' mathematical comprehension and performance. The study integrates a qualitative facet into the data collection process, encompassing observations and interviews with students. This qualitative dimension aims to capture the students' experiences with the focused vocabulary instruction and their perceptions of its influence on their understanding of mathematical concepts. Analysis of the qualitative data amassed from observations and interviews unveils insights into the students' viewpoints, challenges encountered, and the mechanisms through which focused vocabulary learning shapes their conceptual grasp of mathematics.

Through the fusion of qualitative and quantitative data, the study delves beyond mere statistical outcomes, offering a contextual depth that enriches the findings. By comparing students' personal experiences and perceptions against their quantifiable performance, the research attains a heightened level of credibility and depth. This distinctive approach enables the researcher to attain a comprehensive perspective on the ramifications of concentrated vocabulary learning on students' mathematical comprehension and performance, with qualitative insights lending nuance and context to the quantitative revelations.

4.1.1 Research paradigms

A paradigm is a broad perspective that defines how research could be affected and guided by the researcher’s way of viewing reality and truth (Khaldi, 2017). Choosing the correct paradigm is crucial for any researcher because a paradigm provides viewpoints and guidelines that impact what topics should be investigated, the methodology to employ, and the manner in which study findings should be construed (Kivunja & Kuyini, 2017). As such, a paradigm does more than just locate research ontologically (nature of existence or reality) and epistemologically (what counts as knowledge), but also serves as the researcher’s philosophical orientation that guides the choice of methodology and methods to use in research.

Research paradigms are commonly placed into three taxonomies: positivist, interpretivist and critical paradigms. However, other researchers such as Kivunja and Kuyini (2017) regard the pragmatic paradigm to be a fourth category. The positivist paradigm defines a worldview of research that is based on a scientific method of investigation. It focuses on providing explanations and predictions based on measurable outcomes (Ryan, 2018). Research that utilises this paradigm relies on deductive logic, making of hypotheses, testing those hypotheses, operationalizing definitions and mathematical equations, calculations, extrapolations and expressions, to derive conclusions' (Kivunja & Kuyini, 2017). This paradigm favours quantitative research, where the epistemological standpoint is that of objectivism. This implies that through research, humans gain knowledge which helps them to become more objective in understanding the world around them (Kivunja & Kuyini, 2017; Majeed , 2019). This paradigm favours research where there is a search for cause-and-effect relationships. Manipulation of one variable is expected to produce changes in another (Ling, 2017).

The main competing paradigm is the interpretivist paradigm, whose central endeavour is to understand the subjective world of human experiences (Guba & Lincoln, 1989; Potrac et al., 2014). Interpretivism tries to understand individuals and the way in which those individuals understand the world around them. The major tenet of this paradigm is that reality is socially constructed (Thanh & Thanh, 2015). As such, the researcher engages with the research participants to socially construct knowledge, because of his/her personal experiences of real life, within the natural settings in which the investigations are conducted (Pham, 2018). Because this paradigm favours a subjectivist epistemology, it is suitable for qualitative research. A subjectivist epistemology entails making meaning from data through the researcher's thinking and cognitive processing of data; informed by interactions with research participants.

This research adopted the pragmatic paradigm in response to anticipated limitations in adopting a mono-paradigmatic research orientation. The main argument of this paradigm is that it is not possible to access the truth about the real world using a single scientific method (positivism), nor is it possible to determine social reality constructed in an interpretive way. Proponents of this paradigm (Alise & Teddlie, 2010; Biesta, 2010; Teddlie & Tashakkori, 2012) argue that there is a need for a worldview that provides research methods that are appropriate for studying the phenomena at hand. This comes about by examining approaches that are more practical and pluralistic in nature. As a result, the pragmatic paradigm allows a combination of methods as a way of trying to shed light on the beliefs of the participants, their real behaviours, and the consequences of those behaviours. By combining tenets of both the positivist and the interpretive paradigms, the pragmatic paradigm generates a mixed-method approach to doing research.

According to Kivunja and Kuyini (2017), research using this paradigm is done in such a way that the researcher addresses the questions and issues being investigated without having to worry about whether the methods used are quantitative or qualitative in nature. The main concern regarding this paradigm is its workability. Overall, it provides a worldview that allows for the research design and methodologies that best suit the study and that help in knowledge discovery.

Conducting a quasi-experimental research, the focus was not confined to a single perspective regarding the design, methods, approaches, and procedures for collecting and analysing data. Therefore, a pragmatic paradigm was chosen to enable the utilization of a mixed-method methodology.

4.1.2 Piloting and sampling procedures

While the sampling strategy employed for this research leans towards Convenience Sampling, it also incorporates elements of Stratified Sampling to mirror the diverse demographic composition of South African society.

Convenience Sampling: The choice of four high schools in the Gauteng province, South Africa, can be categorized as Convenience Sampling. These schools were deliberately selected due to their accessibility and willingness to collaborate in the study. It is important to note that the selection was not conducted through random sampling of all potential schools in the province, indicating a convenience-driven approach.

This combined approach aimed to strike a balance between practicality and representation, acknowledging the distinctive characteristics of the chosen schools while ensuring a broader reflection of the local demographic landscape.

Demographic and Socioeconomic Characteristics of the sampled participants

- **Control group**

The control group, consisting of 99 learners from two high schools in the Gauteng province, encompasses a diverse range of demographic backgrounds. Gender classification was not a primary focus of the study.

Demographics: The majority of learners in this group, approximately 82.3%, are of black African origin. Around 8.1% of students have Indian heritage, and the remaining 9.1% represent white and coloured ethnic backgrounds.

Socioeconomic Status: Both schools operate on a fee-paying model, primarily serving families with low to mid-level incomes.

Residential Proximity: Most learners in the control group live within walking distance of their respective schools. A smaller percentage relies on scholar transport services, buses, or parental transportation.

- **Experimental group**

The experimental group, comprising 95 learners across three classes, shares similar demographic and socioeconomic characteristics with the control group. Gender identification was not conducted in this group either.

Demographics: The experimental group closely mirrors the control group, with the majority of learners being of black African descent (approximately 89.4%). Around 6.2% of learners have Indian backgrounds, while the remaining 5.2% encompass white and coloured ethnic backgrounds.

Socioeconomic Status: Schools attended by the experimental group also follow a fee-paying model, serving families with incomes ranging from low to mid-level.

Residential Proximity: Similar to the control group, most learners in the experimental group reside within walking distance of their respective schools. Some use scholar transport services, buses, or parental transportation.

This comprehensive assessment affirmed that the control group and the experimental group shared similar characteristics, setting a solid foundation for the study's subsequent analysis and conclusions.

Throughout the research process, one of the four participating high schools served as the initial research site for the pilot phase and subsequently in the main study. Nevertheless, it is important to note that distinct cohorts of learners participated in these distinct phases. To ensure anonymity and confidentiality, letters of the alphabet were employed to designate the research sites. The designated names for identification were School A, School B, School C, and School D, following the sequence in which the research test was administered. School A underwent the initial research test, while School D was the last in line. The enrolments in these schools were as follows:

School A: Contributed two classes with 24 and 28 learners, respectively.

School B: Contributed one class with 47 learners.

School C: Contributed two classes with 33 and 31 learners, respectively.

School D: Contributed one class with 31 learners.

During the first term of 2020, a pilot study was conducted at School D, which had a single grade 11 class. The group chosen for the study consisted of 38 learners and was intentionally selected due to the proximity of this school to the researcher's place of work. The purpose of doing this was to check the feasibility and usability of the research test before it was used in the main study. During the pilot stage, modifications and adjustments were made to make the research test reliable and credible.

In 2021, a main study was conducted, employing a quasi-experimental design to assess the impact of an intervention. A non-equivalent group design (NEGD) was utilized, wherein preexisting groups considered similar were designated as the experiment and control groups (Gribbons et al., 1997). A convenience sampling technique was employed to select the schools for the research. This non-probability sampling technique focused on the accessibility and proximity of the research participants to the researcher. Selection criteria included schools located within a 15-kilometre radius of the workplace.

Effects of covid-19 on the research

The impact of COVID-19 restrictions on this educational research study was profound, leading to a multitude of challenges. Firstly, the researcher faced limitations in collaboration, as permission was granted for only six classes across four schools due to pandemic-related safety measures. This constrained collaboration resulted in reduced diversity and representativeness among study participants, potentially affecting the generalizability of research findings.

Secondly, the necessity to categorize learners into control and experimental groups was influenced by varying access policies among the schools. While some schools strictly prohibited access to outsiders, including researchers, others were more permissive, introducing complexities in the study's design.

Additionally, the pandemic forced significant alterations in school environments, with mask-wearing and physical distancing becoming the new norm. These changes had far-reaching implications for the overall learning experience, introducing previously unaccounted variables that required thoughtful consideration in the research methodology.

Furthermore, the shift to remote or hybrid learning exacerbated existing disparities in students' access to technology and the internet. The pandemic-induced disruptions, such as school closures and shifts between in-person and remote learning, disrupted students' regular schedules, potentially affecting their readiness and engagement in the study.

Lastly, heightened health and well-being concerns for students and staff, driven by the pandemic, significantly impacted the logistical aspects of the research. Participants' willingness to engage in research activities was also influenced by these health-related concerns, adding complexity to the research process.

These challenges underscore the profound influence of COVID-19 on educational research and emphasize the need for researchers to adapt methodologies while maintaining integrity and relevance. To address these challenges, the researcher maintained an ongoing dialogue with the supervisor, provincial education authorities, and school administrators, demonstrating a collaborative and adaptive approach crucial for mitigating pandemic-related challenges while safeguarding research quality.

Despite these challenges, it is important to highlight that the research method and paradigm chosen worked well, even in the face of the COVID-19 difficulties. The pragmatic research approach, which focuses on practicality and solving real-world problems, was a good match for the quasi-experimental group design selected. This design allowed the researcher to observe students in their regular classroom settings, which closely mirrored real-life situations. It recognized the challenges of conducting tightly controlled experiments and instead focused on making meaningful comparisons in settings similar to real life. This approach effectively bridged the gap between controlled lab studies and the complex dynamics of actual schools, aligning perfectly with the principles of the pragmatic research paradigm.

4.2 Data collection and analysis

4.2.1 Phase 1: The pilot study (March 2020)

- **Data collection process**

The data for the pilot study were collected in March 2020 at School D in the form of a written test. The test comprised five questions, totalling 15 test items, all related to the concept of mathematical functions. The test administration involved explaining its contents to the learners to address any areas needing clarification. A 10-minute reading period was allotted to allow learners to review the test questions and seek clarification on unclear instructions.

Following this, learners commenced answering the questions individually in a classroom setting, with supervision and assistance provided as needed.

The pilot study's purpose was to evaluate the potential for errors in the research test (Leon et al., 2011) before its use in the main study. Additionally, it aimed to assess the clarity and unambiguity of the questions and determine the appropriateness of the proposed test duration. Furthermore, the pilot study facilitated the refinement and enhancement of the coding framework.

In summary, the pilot study aimed to assess the suitability of the proposed research test. Following the pilot study, a thorough discussion with the supervisor led to several adjustments. These adjustments included revising the wording of some questions to enhance clarity and extending the test duration from 1 hour to 1 hour 30 minutes.

In the following section, a comparison is presented between parts of the Pilot questions that underwent modifications (left) and the final questions for the main data collection (right). The complete research test is included as an appendix to this thesis (Appendix 6).

Pilot	**Main Study**	**Reason for Change**
<u>Question 1 instruction</u> Study the mathematical object. below and come up with a list of mathematical vocabulary that can be associated with the object.	<u>Question 1 instruction</u> Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object. Write down the *mathematical vocabulary* used and *identify it* on the graph (explaining its meaning fully). Two examples are given in the answering spaces below.	The instruction during the pilot did not solicit learners to demonstrate knowledge of the vocabulary beyond mentioning it.
<u>Question 2.3</u> Use an algebraic **<u>sentence</u>** (mathematical symbols), to describe the value/s of x *for which* $f(x) < g(x)$	<u>Question 2.3</u> Write down the value(s) of x for which $f(x) < g(x)$	Presence of the phrase during the pilot confused the learners as they thought they needed to write a grammatical sentence.

- **Data analysis**

The learners' written responses underwent marking and coding based on the coding framework displayed in Table 4.9. A coding framework was developed for this purpose, as not all learners' responses received marks. The research test included questions where marks could be assigned, while in other questions, only codes were applied to the learners' responses. After marking and coding all the scripts, a second opinion was sought to ensure reliability.

A peer was tasked with selecting a sample of five of the pilot scripts and coding them using the coding framework (Table 4.9) to assess the coding framework's reliability. Reliability, in this context, refers to the degree to which study results can be replicated using the same methodology (Patton, 2014). To evaluate the coding process's reliability, the peer's coding results were compared to this researcher's results (Rose et al., 2015). Achieving reliability in the coding process is a common methodological requirement achieved through the duplication of said process (Krippendorff, 2011). In this research, reliability was assessed by comparing the peer's use of the designated coding framework to re-code the already coded scripts and arrive at results similar to those obtained by the researcher during coding.

The pilot's results are presented through tables, descriptions, bar graphs, and pie charts. Responses are sometimes depicted as raw data, percentages, or a combination of both. While illustrating specific points in the results presentation, excerpts from the learners' scripts are also provided.

4.2.2 Phase 2: The main study (March 2021 to August 2021)

The main study was conducted in four research sites, all of which were high schools in the Gauteng province of South Africa. The initial plan was to conduct the main study in the 2020 academic year, but this could not happen due to COVID-19 restrictions put in place by the national government. The main study consisted of three stages, namely administration of a pretest (Appendix 6), the intervention, and administration of a post-test (Appendix 6). The pretest and the post-test had similar questions. I wanted to investigate if the intervention would make an impact on the learners. For that reason I opted for a similar test to enable me to sufficiently measure the changes in outcome.

Due to high cases of COVID-19 and the associated disaster management regulations, the administration of the pretest had to be postponed to the first term of the 2021 academic year.

The research test consisted of five questions. All the questions were based on the concept of mathematical functions. The questions were meant to test learners' knowledge of mathematical vocabulary, proficiency in use of the mathematical vocabulary, proficiency in articulating the vocabulary, conceptual and procedural proficiency, and performance in mathematics.

From the four Gauteng high schools that took part in the research at this stage, 194 learners sat for the pretest. School A had 52 learners taken from two classes of sizes 24 learners and 28 learners. School B had one class of 47 learners and School C had a total of 64 learners split into two classes of 33 learners and 31 learners. Lastly, School D had 31 learners that took part in the research.

(a) Administration of the pretest

- **Data collection**

The test could not be administered to all schools in the same week. Instead, it was administered over the first three weeks of the first term of the 2021 academic year that commenced on 15 February. Infrastructural limitations compelled schools to use rotational timetables to avoid overcrowding in schools during the COVID-19 period. With the rotational timetable, it meant that the Grade 11 learners were not in school every day of the week. Schools A, B, and C used week-in-week-out rotations, meaning there would be a full week where the Grade 11 learners would not be in school. School D used a day-in-day-out rotation system, where the Grade 11 learners would for one week attend on Monday, Wednesday, and Friday, and the subsequent week attend only Tuesday and Thursday. For School D, the timetable was structured in such a way that the learners would not necessarily have mathematics lessons on every day that they would be in school. Such timetabling challenges in schools due to the COVID-19 pandemic meant that learners lost a considerable amount of learning time. Despite the challenges, together with others that will be discussed later in this thesis, the pre-test was successfully conducted.

The research test was administered on agreed-upon days between the researcher and the relevant teachers. However, due to COVID-19 protocols and restrictions, complete invigilation in Schools A, B, and C was not feasible. School authorities permitted the researcher to explain the test instructions to the learners and guide them through the test to ensure comprehensive understanding before writing commenced. Following this, the researcher had to leave the subject teachers to oversee the invigilation, waiting for the scripts in the school staff room. At the conclusion of the sessions, subject teachers handed the scripts to the researcher, who then departed from the school. In the case of School D, the situation differed as the school granted permission for full invigilation. The results of the pretests are presented and discussed in Chapter 6.

- **Data analysis**

All 194 scripts were marked, and the results were recorded in a spreadsheet under the heading pre-test results (Appendix 9). The four schools were split into two groups of two schools each. The first group consisted of 99 learners and the second consisted of 95 learners. The 99 learners made up the control group and the 95 learners made up the experimental group. Learners from School A and School B made up the control group while learners from School C and School D made up the experimental group. The recorded results were processed and presented in the form of tables, pie charts, bar graphs, scatter plots, and descriptions in Chapter 6.

The intervention

Throughout the intervention, the Frayer model was utilized alongside the language repertoires as the primary research tool. These language repertoires aided in the identification of vocabulary suitable for the research and later served as an analytical framework. Vocabulary selected for examination was drawn from the four language repertoires (literal, algebraic, graphical, and procedural) as discussed in Section 3.8. As previously discussed, the repertoires delineated the appropriate vocabulary and language for mathematics instruction and learning.

The intervention entailed the integration of mathematical vocabulary from the four language repertoires with the mathematical content of functions. This integration occurred following the administration of a pretest assessing learners' vocabulary understanding and mathematical proficiency. Both a pretest and a subsequent post-test of the same format were administered to participating learners. The choice of tests was made due to their suitability in experimental research (Bertram & Christiansen, 2014), allowing for easy comparison and assessment of the intervention's impact. Following the post-test, learners' conceptual understanding of the same test questions was explored through semi-structured interviews.

During the intervention, learners employed the Frayer model as a tool for learning mathematical vocabulary, which also served as a data collection instrument. After instructing learners to utilize the Frayer model for vocabulary learning, they were tasked with summarizing the vocabulary they had acquired using the model's templates. Learners completed these templates as they learned various concepts. The collected templates were then used as data sources to evaluate learners' conceptual understanding (Miller, 2015). In conjunction with pretest and post-test results, these completed Frayer model templates were analysed by the researcher to assess vocabulary comprehension, conceptual understanding, and mathematical proficiency.

The intervention comprised two phases: an information phase and a practice phase. During the information phase, participants were educated by the researcher about the research's nature, their rights during the research, and the use of the Frayer model as a vocabulary-learning tool. This phase spanned one week, from Monday to Friday, with a maximum duration of 3 hours per day. Training sessions for learners were conducted in typical classroom settings to minimize participant anxiety. Participating teachers received coaching in their offices or base classrooms during school breaks. Inclusion of teachers was necessary, as they would later assist in facilitating the learners' use of the Frayer model.

After the informative phase, in which learners were provided with details regarding the research intervention and informed about their rights in the study, they were actively encouraged to incorporate the Frayer model as a prominent note-taking instrument during their mathematics lessons. They were urged to pay special attention to key words and vocabulary and then utilize the Frayer model for learning and comprehension. The use of the Frayer model was permitted throughout the teaching of functions, as per the work schedule, prior to the administration of the post-test. During this implementation phase, collaboration with participating teachers at the research sites was essential to facilitate the Frayer model's use in lessons.

Figure 4.1 provides a schematic representation of the intervention. The first part involved providing information to both teachers and learners who became research participants. The second part, referred to as the practice/implementation phase, also involved teachers and learners from the experimental group. During this second phase, learners were encouraged to integrate language and mathematical content through Content and Language Integrated Learning (CLIL).

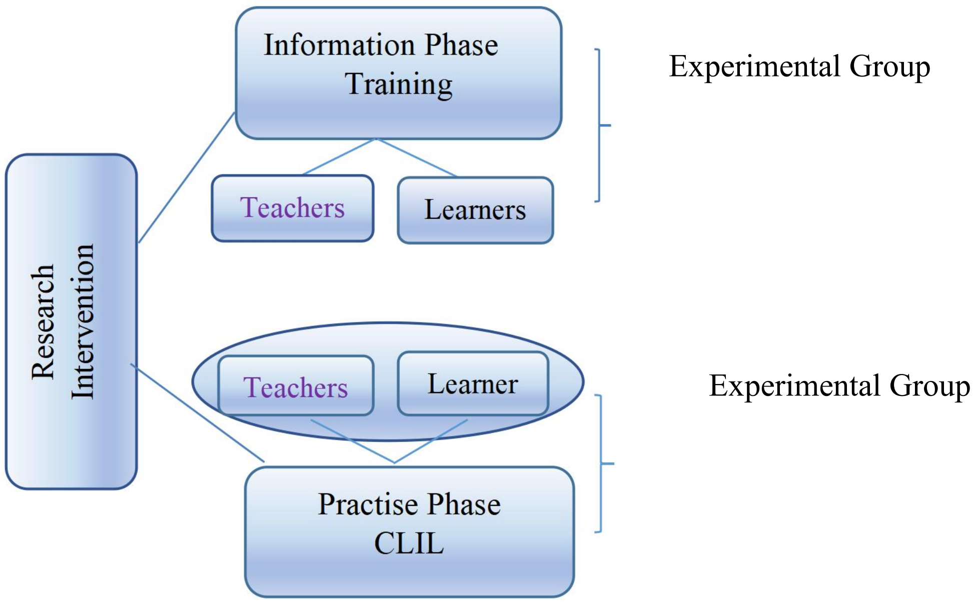


Figure 4.1: The research intervention schema.

- ***Identification of the vocabulary for the intervention.***

In a research study involving in-service university students, van Jaarsveld (2016) advocated for the teaching and learning of mathematics using precise vocabulary. As previously explained, the repertoires offer vocabulary related to representations that, when acquired by learners, can enhance their comprehension of the concept of function. In this study, the proposal was to integrate the learning of accurate mathematical vocabulary with the learning of mathematical content among Grade 11 learners in selected public schools.

The four language repertoires encompass three semiotic resources within mathematical language: verbal structures, mathematical symbolism, and visual images. These resources provide learners with various means to construct mathematical meanings (O'Halloran, 2011). Verbal resources are utilized by learners and teachers to provide concept definitions, explanations, instructions, justifications, and object descriptions, among other functions. In contrast, mathematical symbolism offers "topological descriptions of continuous patterns of covariation" (Malviya, 2019). Furthermore, mathematical calculations and solutions to problems are primarily expressed through symbols. The third semiotic resource is visual representation, which mathematicians view as a source of intuition and a means to connect mathematics with the real world. Visual representations come in the form of graphs, diagrams, and tables. Due to their concrete nature, visual representations provide immediate information and facilitate a better understanding of concepts, contrasting with the abstract algebraic symbolism.

Throughout the intervention phase, learners successfully gained proficiency in mathematical vocabulary within each of the four distinct language repertoires, with a specific emphasis on the domain of mathematical functions. As previously mentioned, these language repertoires encompass the literal, algebraic, graphical, and procedural perspectives. Learners were instructed to employ the correct mathematical vocabulary when interpreting mathematical objects, even if those objects were presented symbolically, requiring the use of verbal semiotic resources. This emphasizes the constant interplay between semiotic codes, generally enhancing learners' understanding. However, it's essential to manage these movements effectively to prevent syntactic ambiguities (O'Halloran, 2011). For example, while the symbolic statement "$r\tan\beta - \tan\alpha = 10$" has no ambiguity, verbalizing it as "r tan of beta take away tan of alpha equals ten" could potentially be understood as "$r(\tan\beta - \tan\alpha) = 10$" (O'Halloran, 2000, p. 381). Proper management of verbalism, symbolism, and visual representations is crucial to avoid significant issues for learners.

Learners were also explicitly exposed to the vocabulary of the algebraic repertoire during the intervention. Similar to the literal category, learners encountered mathematical objects in their symbolic form and were required to interpret the operations within these objects to resolve any misunderstandings related to those operations.

The graphical repertoire pertains to mathematical objects presented as visual images. Learners were encouraged to transition between visual representation and verbal communication, emphasizing the use of accurate mathematical vocabulary to describe the visually represented objects. Lastly, participating learners were taught the vocabulary associated with mathematical procedures. Solving mathematical objects represented either symbolically or visually necessitates procedural knowledge of how to arrive at a solution. This procedure needs to be explained during mathematical discussions and explanations, requiring learners to possess the correct vocabulary to elucidate the procedure for resolving mathematical problems. As with the other three repertoires, the procedural repertoire also entails a continuous movement between the three semiotic modes.

While the focus of this work was on verbal resources, it is essential to acknowledge the role of other semiotic resources, including gestures and body movements. Consequently, during the research, if a learner utilized any resource other than verbal communication, their response was analysed comprehensively without dismissal. This approach allowed for the utilization of the advantages that multiple mathematical representations offer learners in comprehending mathematical concepts for factual, procedural, and conceptual understanding (Ibrahim & Rebello, 2013).

1. *Information phase*

This was done in two schools, here identified as School C and School D. These 2 schools make up the experimental group of the study. In these schools, teachers and learners were oriented on the study.

- **Orientation of the teachers**

The central focus of this research was the utilization of the Frayer model as a vocabulary-learning strategy. Consequently, it was crucial to provide training to both teachers and learners in the experimental group on how to employ the model effectively. A training session was conducted with the three teachers who would be working with the learners from the experimental group.

Two of these teachers were assigned to one research site (School C), allowing for simultaneous training on the same day and at the same time. We convened at their workplace during the last week of the second term, specifically on a Monday. The training took place in the respective head of the department's office during a 30-minute break. The head of department expressed her interest in participating in the training, as the research had a significant impact on her department. It was revealed during the training that these two teachers, along with the head of department, were unfamiliar with the Frayer model, marking their first encounter with it. Participants were provided with copies of model templates and were given explanations on how the templates functioned. Collaboratively, one template for the word 'function' was completed, and the finished template is presented in Figure 4.2

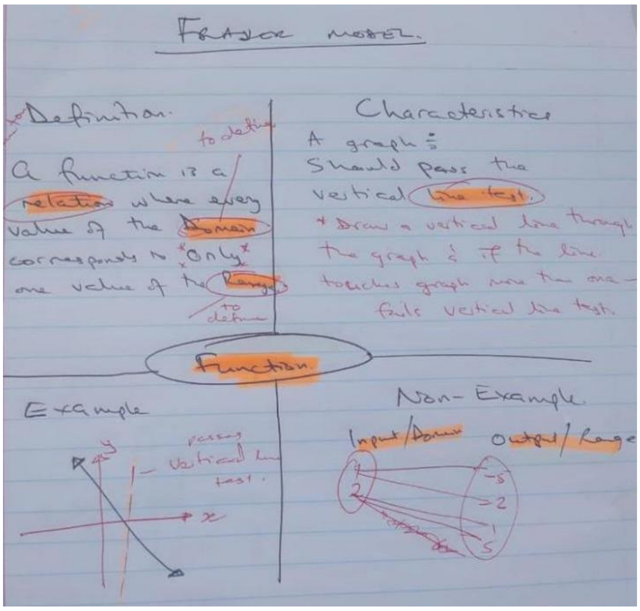


Figure 4.2: Vocabulary "function" as an example Frayer model at School C.

The three participants acknowledged the potential utility of the Frayer model as a strategy for teaching mathematical vocabulary. In addition to agreeing to supervise the learners' use of the Frayer model during the study of functions, the teachers expressed their intention to incorporate it as a teaching strategy in subsequent chapters. Given that this was training for professionals, it was conducted on a single day and lasted for 30 minutes.

In School D, the teacher was trained during break on a Tuesday during the last week of the second term. The training occurred in her base classroom. She candidly admitted to her lack of familiarity with the Frayer model, marking her first exposure to it. During the session, various aspects of the model were discussed and one template for the term "gradient," was completed, the results of which are presented in Figure 4.3.

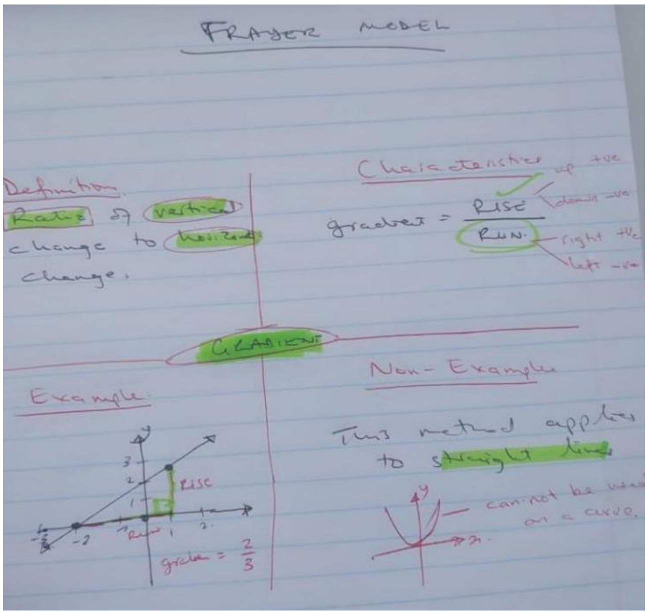


Figure 4.3: Vocabulary "gradient" as an example Frayer model at School D.

Due to the one-on-one nature of this session, it was relatively brief, lasting only 27 minutes. The teacher expressed gratitude for the exposure to the Frayer model and pledged to provide assistance during the research, a promise that was warmly acknowledged.

- **Orientation of the learners: Intervention in School C**

Grade 11 mathematics learners in this school comprised the initial group to undergo intervention training during the first week of Term 3. The purpose of the research and the rights of participants during this study were elaborated upon. Emphasis was placed on comprehending mathematical concepts through the acquisition of mathematical vocabulary. Additionally, the utilization of the Frayer model as a valuable tool for mastering mathematical vocabulary was introduced.

At this school, training sessions were scheduled after school hours (from 14:00 to 15:30), as conducting them during breaks proved impractical. The school's schedule consisted of two 20-minute break sessions, as indicated in Table 4.1.

Table 4.1: School timetable for School C.

Period	Times
Registration	**07:45-08:00**
1	08:00-08:40
2	08:40-09:20
3	09:20-10:00
4	10:00-10:40
Break	**10:40-11:00**
5	11:00-11:40
6	11:40-12:20
Break	**12:20-12:40**
7	12:40-13:20
8	13:20-14:00
Registration	**14:00-14:15**

These break times were too short to conduct reasonable training. After school, training was possible due to where the learners resided. Most of the learners resided in the school's neighbourhood, and some resided in nearby suburbs. Fortunately, these learners needed no scholar transport to travel between home and school as they could walk. As such, these sessions after school did not inconvenience this group of learners. Following this arrangement, the first session was conducted on a Tuesday afternoon.

However, there were seven learners who used scholar transport to come to school, so they excused themselves from the training. This was even though they, together with their parents/guardians had consented to their being part of the research. Additionally, four learners who had taken part in the pretest had changed subjects from mathematics to mathematical literacy. Consequently, this left me with 53 learners eligible to take part in the training, which was a 17,18% drop in enrolment (64 to 53) from the learners who took part in the pretest.

When the pretest was administered during the second week of the first term, the expectation was that functions would be taught to the learners during the second term. Due to rotational timetabling in this school (one week in, one week out), teaching time was seriously impacted to the extent that functions were only taught in the third week of the third term. That being the case, the training was conducted during the first week of the third term.

The vocabulary used for the training did not come from the unit of functions, to avoid giving advantage to the experimental group over the control group. On the first day of training, the intervention essentials were discussed as follows:

- An understanding of mathematical vocabulary is essential for the learning of mathematics.
- Learners use mathematical vocabulary to read mathematical objects.
- Learners use mathematical vocabulary to correctly interpret mathematical operations.
- Learners use mathematical vocabulary to describe and understand mathematical procedures.
- Learners use mathematical vocabulary to interpret Cartesian plane figures and migrate between the different representations used in mathematics, i.e. words, visuals, and symbols.
- Learners were going to be exposed to the Frayer model as a vocabulary-learning strategy.

Two training sessions were conducted during the same week on two different days: Tuesday and Thursday.

- **Orientation of the learners: Intervention in School D**

During the intervention at School D, assistance was provided to learners in comprehending the utilization of the Frayer model. The emphasis was placed on the importance of acquiring mathematical vocabulary as part of their mathematics education. A total of 21 learners participated in this training. At the time of the pretest, the school's enrolment stood at 31 students, but it decreased by 32% between the pretest and the commencement of the training. This drop in enrolment was attributed to students switching to mathematical literacy due to their perception of difficulties in mathematics.

The pretest was administered during the second week of the first term, while the curriculum aimed to cover the chapter on functions during the second term. However, timetabling challenges at this school necessitated a change in plans. In the first term, Grade 11 learners attended school for a maximum of three days per week due to social distancing regulations.

Consequently, the school had to implement a rotational attendance system due to space constraints.

Additionally, the school had a program that grouped subjects into compulsory and optional categories, resulting in students attending different subjects on different days. Mathematics, languages, and life orientation were compulsory subjects, while optional subjects like accounting, economics, and geography were scheduled on separate days. This rotation severely impacted teaching time, leading to the decision to delay teaching functions until the second week of the third term. Accordingly, the intervention training took place in the first week of Term 3.

The first session at this school occurred on a Tuesday during the first week of the third term, involving all 21 learners in a classroom during the break period. Following the break, there was a visual arts examination session for Grade 11 students, which was a continuation of mid-year examinations disrupted during the second term due to school closures related to COVID-19. None of the learners in the mathematics class were enrolled in visual arts, allowing the session to extend into the examination time, providing ample opportunity to cover the training content. The school's schedule is outlined in Table 4.2.

Table 4.2: School timetable for School D.

Period	**Times**
Registration	**07:45-08:00**
1	08:00-08:45
2	08:45-09:30
3	09:30-10:15
4	10:15-11:00
Break	**11:00-11:45**
5	11:45-12:30
6	12:30-13:15
7	13:15-14:00
8	14:00-14:45
Registration	**14:45-15:00**

To commence the training, learners were oriented on the research objectives and their rights during the research. Emphasis was placed on the training's purpose, which was to introduce the use of the Frayer model as a strategy for learning mathematical vocabulary. Alongside explaining the vocabulary benefits of the Frayer model, a diagram of the model was drawn on the chalkboard, accompanied by an explanation of its mechanics. Following the orientation, the concept of an equation was introduced, and learners were tasked with sketching the Frayer model in their notebooks, with the concept/term "equation" in the oval. Learners independently formulated a definition for the term, and a class discussion ensued, refining the definition until a consensus was reached. Learners then generated examples and non-examples of the concept individually. Once the lists were complete, learners collaborated in pairs to identify and record the essential characteristics of the concept. Detailed results from these discussions are presented in Section 6.2.1. In this school, three training sessions occurred on Tuesday, Wednesday, and Thursday, with a total training time matching that of School C (3 hours).

a. *Implementation phase*

The implementation phase signifies the actual use of the Frayer model by learners during their study of functions, which took place in the third term of the 2021 academic year. This study focused on conventional functions, specifically linear functions, quadratic functions, hyperbolas, and exponential functions. Following the completion of this chapter on functions, the intervention was concluded.

Throughout this phase, a total of nine lesson observations were meticulously carried out in accordance with the structured observation schedule outlined in Table 4.3. As there were three teachers, each overseeing a group within the experimental group, observations were conducted for each teacher (three per teacher). Grade 11 mathematics classes at School C were conducted simultaneously for both mathematics groups, making it necessary to alternate observation days. In the third term, Grade 11 learners attended school daily, allowing for more frequent lesson observations. The observation schedule was collaboratively designed with the teachers involved, as shown in Table 4.7. Results from these lesson observations are detailed in Section 6.3.1, following the chronological order of observations as presented in Table 4.3.

Table 4.3: Lesson observation schedule.

Class	C1	C2	D
Monday 26 July 2021	10:00-10:40		
Wednesday 28 July 2021		08:40-09:20	
Friday 30 July 2021			13:15-14:00
Tuesday 3 August 2021			09:30-10:15
Thursday 5 August		1100:11:40	
Friday 6 August	08:00-08:40		
Tuesday 10 August 2021		11:40-12:20	
Wednesday 11 August 2021	13:20-14:00		08:45-09:30

The class names used in this research do not correspond to the names they were known by at the schools. Pseudonyms were employed to safeguard the identities of the participants. Class C1 belonged to School C and constituted the initial class observed during the research. Similarly, Class C2 was also affiliated with School C. The final class, D, was situated at School D, hence its designation as D.

While School C adopted 40-minute periods and School D employed 45-minute periods, the duration of these periods was not within the study's purview. Regardless of this divergence in lesson length, the scheduled lesson observations proceeded smoothly. The outcomes of these observations are detailed in Chapter 6.3 of this thesis document.

Administration of the post-test

- **Data collection**

When the chapter concerning the concept of mathematical functions reached its completion, following the Revised Annual Teaching Plan, the post-test was administered across all four research sites. The test administration took place within the same week. An arrangement was made with the respective teachers, and a mutually agreed-upon schedule was implemented, as depicted in Table 4.4.

Table 4.4: Schedule for administration of the post-test.

Date	**Group**	**Time**
Monday 16 August 2021	Experimental (School D)	14:00–15:30
Tuesday 17 August 2021	Experimental (School C)	13:20–14:50
Wednesday 18 August 2021	Control (School A)	14:00–15:30
Thursday 19 August 2021	Control (School B)	13:00–14:30

In all four schools, invigilation was allowed for the entire duration of the test. At the start of the session, an explanation was given to the learners regarding the test's purpose and what was expected of them in each question. Once clarity on the test was ensured, learners were permitted to begin answering the questions in the provided answering spaces. A class-test style seating arrangement was utilized to promote individuality in test writing. As the test was being written, the researcher carried out invigilation by moving around. However, when clarification was needed on a particular question, the researcher provided an explanation to the learner concerned.

4.3 Methodological rules

The lens through which the participating learners and data are viewed in this study encompasses the Active Learning Theory (ALT) and the Cognitive Load Theory (CLT). In this study, only four of the five elements of ALT will be utilized to assess the presence of active learning. These elements are listed in Table 4.5. The fifth element, which was omitted, pertains to choice. In active learning, learners make content selection, pacing, and task choices. However, in this study, learners were not provided with the opportunity to select content; instead, the content was prescribed to them in accordance with the Annual Teaching Plan.

Table 4.5: Adaptation of four of the five elements of the Active Learning Theory (ALT) in the study.

Elements	Definition	Descriptors
Materials	Artefacts used during the learning of vocabulary of functions.	Frayer model, textbooks, chalkboards and whiteboards, exercise books, notebooks.
Manipulations	Ways of thinking that promote higher-level processing of the content of the mathematics of functions.	Defining mathematical vocabulary, doing mathematical calculations, speaking during lessons, discussing with peers, responding to interview questions.
Mathematical Vocabulary/Language	Verbal component of the mathematical register relating to the mathematical function.	Technical vocabulary of functions: features of objects, instructions, descriptions of objects, definitions.
Adult Support	The assistance that the teachers or the researcher gives to the learners as they learn mathematical functions.	Explanations, instructions, questions, examples, classwork, requests to present work on the board, challenging learners to work.

4.4 Reliability and validity

Reliability encompasses the entire process of data collection and the subsequent findings. According to Mohajan (2017), it pertains to whether results obtained, or conclusions drawn could still be reached if the research were repeated multiple times. It involves consistency in results across various similar settings and repetitions (Bush, 2012; Wellington, 2015). Given that this was a classroom-based research study, replicating it is challenging. Nevertheless, credibility was bolstered throughout the research by posing challenging questions and engaging in self-examination (Phelps-Gregory & Spitzer, 2018). Additionally, journal notes were used and integrated into the final report for added credibility.

Creswell and Poth (2016) viewed validity as an indicator of whether the research has effectively measured its intended variables. To ensure this, bias was mitigated by critically scrutinizing all procedures and decisions. Theory and previous research were employed to critically analyse the methods and decisions made. Furthermore, data triangulation was utilized as a cross-validation technique, involving research tests, semi-structured interviews, naturalistic and non-participant lesson observations, and notes (graphic organizers). Periodic consultations with the research supervisor were also conducted to minimize confounding factors that could affect both internal and external validity. The research adhered to a pretest-post-test design format. During the test administration, every effort was made to maintain the integrity of the process. Tests were personally administered, and in cases where COVID-19 regulations prevented personal administration, proximity to the venue was ensured.

The research employed a pretest-post-test design involving non-equivalent groups (experimental and control) that were not randomly assigned. These groups were not randomly assigned as they consisted of normal Grade 11 classes constructed based on each individual school's criteria. Both the experimental and control groups underwent a pretest, after which the experimental group received an intervention, while the non-equivalent control group did not receive the intervention. Subsequently, a post-test was administered to both groups. The chronology is illustrated in the following schema (Figure 4.4).

O_n Observation by means of a test

X Intervention with the use of the Frayer model

Assign into	Experiment	O_1	X	O_2
Assign into	Control	O_3		O_4

Figure 4.4: Chronology of the pretest, intervention, and post-test

Regarding issues of history and maturation as potential threats to validity, measures were taken to ensure that the research did not span a period longer than a full academic year.

Nevertheless, the identical test instrument was employed for all tests administered throughout the research. While this setup presented a potential challenge to internal validity, steps were taken to mitigate it. Participants (both teachers and learners) were never informed that they would undergo reassessment using the same test instrument at a later stage of the research to prevent any bias. Additionally, to further enhance reliability throughout the research, a pilot study was conducted before commencing the main study.

4.5 Generalisation

Generalization involves making a claim that what is true in one place or time will hold in other places, contexts, and times. The success of everyday social life depends on individuals accomplishing this (Parker and Northcott, 2016). In the case of this study, the results cannot be extended to other settings due to the pilot study involving only 38 learners from a single school, and the main study data being collected from just four high schools with an initial enrolment of 194 learners for the pretest. The enrolment decreased to 157 learners for the post-test, representing a 19.1% decline (37 learners). The decline in the participant numbers can be attributed to several factors. Specifically, 34 of the learners who initially took the pretest made the decision to switch from mathematics to mathematical literacy. Mathematical literacy is an alternative subject chosen by those who opt not to pursue mathematics. Additionally, three learners withdrew from school, further contributing to the reduction in participant numbers.

Despite the inability to generalize the study's results to other settings, they can still be applicable and transferable. Applicability and transferability, closely related terms, have distinct meanings. Applicability concerns the feasibility of implementing an intervention in a new, specific setting (Burchett et al., 2013). It is believed that the intervention used in this research can be applied in different settings, contexts, and times. Transferability, on the other hand, pertains to the likelihood that a study's findings can be replicated in a new, specific setting while maintaining their effectiveness (Burchett et al., 2013).

4.6 Ethical issues

Before the research instruments were administered, research approval requests were sought and granted from the Gauteng Department of Education (2019-180AA) and from the Ethics Committee of the University of the Witwatersrand (H19/09/22). School principals, parents, and learners were made aware of the study's purposes, data confidentiality, and relevance of participation in research of this nature. Formal voluntary consent was sought from all parties concerned. This is in line with the university ethics guidelines on research that involve human participants.

Ethics relate to the rights and the interests of the participants in the research. For the purposes of ethical considerations, teachers and learners were asked to complete consent forms to show their willingness to participate in the study. The learners who took part in the research were minors, so consent was also sought from their parents. The consent also ensured that the results would be treated with the utmost confidentiality. This meant that there was assurance given to the participants that neither their names, nor the names of their institutions, would be revealed anywhere and at any stage of the research, including publication (Novak, 2014). Not anywhere will there be any form of identification of the participants of the research. All participants will remain anonymous and untraceable unless they were to give consent to the contrary (Novak, 2014).

4.7 Delineation

The main study was conducted in four high schools in the Gauteng province of South Africa. However, the intervention was implemented in only two of the four high schools. The high schools sampled for this study were accessible to the researcher. The populations of these four high schools were predominantly black learners. Though the research was conducted in high schools, it only focused on Grade 11 learners who do mathematics. Furthermore, the research did not cover the whole spectrum of the mathematics syllabus. Instead, it focused on the concept of mathematical functions.

4.8 Limitations

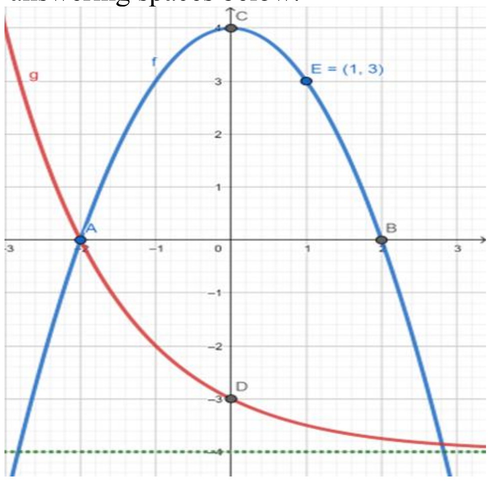
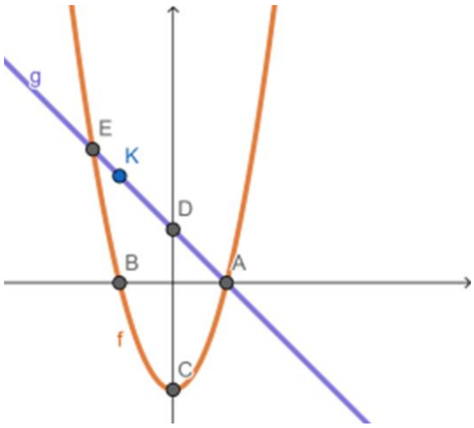
The results of the study could not be generalised due to the smallness of the sample size used. Only Grade 11 mathematics learners from four high schools with a predominantly black population took part in the study. This sample is not representative of the rainbow nation that South Africa is in terms of cultures, values, races, and social classes.

Furthermore, the research was conducted during the COVID-19 pandemic. During the pandemic, teaching and learning was seriously impacted and a lot of the contact time was lost as schools could not open on time for the academic year. When the schools eventually opened, learners were not allowed to come to school on a full-time basis due to COVID-19 regulations. The regulations stated that schools could not operate at full capacity so that social distancing could be observed. In addition to the loss of contact time, both teachers and learners were anxious participants due to the havoc that COVID-19 had caused in communities. Teachers and learners were all directly or indirectly affected by the virus. Such an environment was not fully conducive to learning.

4.9 Research test, data collection method

The research data was collected using a research test, lesson observations and interviews. Table 4.6 presents the research test. The left column shows the category of mathematical vocabulary that each question of the test assessed, and the right column presents the research questions.

Table 4.6: Categorisation of questions of the research test.

Category	Question
Vocabulary Knowledge **(VK)**	1.Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object. Write down the *mathematical vocabulary* used and *identify it* on the graph (explaining its meaning fully). Two examples are given in the answering spaces below. 
1) Marks Obtained **(MO)** 2) Conceptual Understanding and Procedural Proficiency **(CUPP)**	2. The graphs of $\}\{x) = x^2 - 4$ and $g(x =- x + 2$ are sketched below. A and B are the x-intercepts of C and D are the y-intercepts of f and g respectively. K is a point on g such that BK / / . intersect at A (2; 0) and E (-3; 5).  2.1. Write down the coordinates of C. 2.2. Write down the range of f 2.3 Write down the value(s) of x-for which $f(x) < g(x)$ 2.4 Use correct mathematical vocabulary in a sentence to describe the (answer) algebraic notation given in question 2.3.

Language Proficiency **(LP)**	3. Define or describe the following Mathematical Vocabulary: 3.1 *Asymptote* 3.2 *Intercept* *3.3* *Range* 3.4 *Domain*

Vocabulary Proficiency **(VP)**

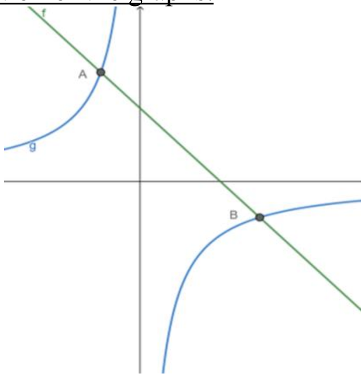
4. Table 1 below has a Mathematical Vocabulary for a Concept, an example of how the concept is expressed symbolically and a characteristic of the concept. Complete table 2 by rearranging and matching the concept in Column A to an example of the concept in Column B and a characteristic of the concept in column C.

Table 1

Column A: Vocabulary	Column B: Example	Column C: Characteristic
4.1 Asymptote	$y = 2^x$	Only the letter representing the independent variable is present
4. 2 Intercept	$2 < x \leq 5$	Linear equation
4.3 Range	$y > 2$	The exponent is a variable.
4.4 Domain	$(2;0)$	Only the letter representing the dependent variable is present
4.5 Exponential equation	$x = 3$	At least one coordinate is equal to zero

Table 2

Column A: Vocabulary	Column B: Example	Column C: Characteristic
4.1 Asymptote		
4.2 Intercept		
4.3 Range		
4.4 Domain		
4.5 Exponential equation		

1) Marks Obtained **(MO)** 2) Conceptual Understanding and procedural Proficiency **(CUPP)**	5. The graphs of $f(x) = -x + 2$ and $g(x) = -\frac{3}{x}$ are sketched below. <u>A and B are the points of intersection of the graphs.</u>  Calculate the coordinates of A and B. Show all steps and <u>next to each step give a reason</u> for doing it (The reasons you give should show that you understand why each step and procedure is done).

Assessment of conceptual understanding using the research test

While each research test question focuses on specific aspects about mathematical vocabulary, their collective significance lies in their contribution to evaluating the extent of conceptual understanding achieved. The subsequent section provides an in-depth elucidation of how each individual question uniquely aids in the assessment of this conceptual understanding.

<u>Question 1</u>

The question was designed to assess conceptual understanding in the context of mathematical vocabulary and its application to a graphical representation. The first step involves recognizing and listing mathematical terms that are relevant to the given mathematical object. This requires students to have a strong grasp of the terminology associated with the specific topic being assessed. Students are then required to identify and label the mathematical vocabulary on the graph itself. This assesses whether students can correctly apply the identified terms to a visual representation. The question further requires students to explain the meaning of each identified mathematical term fully. This aspect of the question assesses whether students understand not only the terms in isolation but also their significance and relevance within the context of the graph. To successfully complete the task, students need to engage in critical thinking by connecting the mathematical terms to the graphical representation. They must consider how each term relates to the graph and its overall meaning.

By combining all the above steps, students are demonstrating their ability to synthesize their knowledge of mathematical vocabulary, graph interpretation, and conceptual understanding.

In summary, this question assesses conceptual understanding by evaluating students' ability to recognize, apply, and explain mathematical vocabulary within the context of a graph. It requires them to think critically, make connections, and provide meaningful explanations, which are key indicators of a deeper understanding of the mathematical concepts involved.

Question 2

The question involves multiple key points on the graphs of functions f(x) and g(x), such as x-intercepts A and B, y-intercepts C and D, and the point E where the two functions intersect. Students need to identify and work with these points, demonstrating their understanding of intercepts, coordinates, and intersections of functions. Students are presented with graphs of two functions. Interpreting and analysing graphs are fundamental skills in mathematics. This question requires students to extract information from the graph, understand the behaviour of the functions, and make connections between the graphical representation and algebraic expressions.

The statement "BK // y-axis" indicates that line BK is parallel to the y-axis. Students are expected to understand the concept of parallel lines and apply it to the given scenario. Furthermore, the question involves coordinate points and their properties. Students need to manipulate coordinates and understand the relationships between points on the graph, including their distances, slopes, and orientations. Students need to know the behaviour of the functions based on their equations. For example, understanding that $f(x) = x^2 - 4$ is a parabolic function that opens upward and that $g(x) =- x + 2$ is a linear function with a negative slope is crucial. The question states that functions f and g intersect at points A (2, 0) and E (-3, 5). Students must understand the concept of function intersection and how to find solutions to systems of equations. While not explicitly mentioned, students may need to recognize that the graph of $f(x) = x^2 - 4$ is a parabola shifted downward by 4 units compared to the basic x^2 graph. Overall, the question requires students to understand the relationships between various mathematical concepts, such as intercepts, parallel lines, function behaviour, and graphical representation. They need to synthesize this knowledge to answer questions about the given scenario accurately.

In summary, the question assesses students' conceptual understanding by testing their ability to interpret graphs, apply geometric and algebraic concepts, recognize mathematical relationships, and analyse the properties of functions. It encourages critical thinking and the integration of multiple mathematical ideas to solve the problem effectively.

Question 3 and 4

Question 3 entailed learners to provide precise definitions for given terms, whereas in question 4, learners were tasked with associating the terms, as defined in question 3, with their corresponding attributes and illustrative examples. The terms in question were asymptote, intercept, domain, and range. For each of these terms, learners need to have certain understanding as illustrated:

1. Asymptote:

- The notion of a curve approaching a line without crossing it.
- The behaviour of functions as they approach infinity or negative infinity.
- The distinction between horizontal and vertical asymptotes and their significance for different types of functions.

2. Intercept:

- The relationship between coordinates and the axis on which the intercept occurs.
- How to find intercepts algebraically by setting one of the variables (x or y) to zero.
- The interpretation of intercepts in real-world contexts and graphical representations.

3. Domain:

- The allowed inputs for a given function.
- Any restrictions or limitations on the values that can be plugged into the function.
- Notions of excluded values, such as denominators that cannot be zero or square roots of negative numbers.

4. Range:

- The possible values that the function can output based on its behaviour and definition.

- How the shape of the graph or the properties of the function impact its range.

- The concept of a function's vertical spread on the coordinate plane.

In summary, question 3 assesses conceptual understanding by requiring students to provide accurate definitions or descriptions of key mathematical vocabulary terms. For each term, students need to demonstrate their understanding of the underlying principles, their ability to relate the term to mathematical concepts, and their capacity to explain these concepts in their own words. On the other hand, question 4 consolidated the understanding demonstrated in question 3.

Question 5

Students need to interpret the given graphs of functions $f(x) =- x + 2$ and $g(x) =-\frac{3}{x}$. This involves recognizing the shape, direction, and behaviour of the graphs. Understanding the graphs is essential for identifying points of intersection (A and B). The first step involves finding the points where the two graphs intersect (A and B). This step requires recognizing that points of intersection occur when the y-values of both functions are equal. Students set the two functions equal: $-x + 2 =-\frac{3}{x}$. This is done to find the x-values where the graphs intersect. The reason for doing this is to solve for the x-values that correspond to the points of intersection. By solving the equation for x students find the x-coordinates of points A and B. This step involves algebraic manipulation. The reason for solving for x is to determine the x-values at which the graphs intersect. The calculated x-values are then substituted into one of the original functions (e.g., $f(x) =- x + 2$ or $g(x) =-\frac{3}{x}$. to find the corresponding y-values. The reason for doing this is to find the complete coordinates (x, y) of points A and B. At each step, students are required to provide a reason or explanation for why they are performing that step. This demonstrates their conceptual understanding by showing that they can relate each step to the underlying mathematical concepts.

In summary, this question assesses conceptual understanding by testing students' ability to interpret graphs, identify points of intersection, set up and solve equations, apply algebraic manipulation, and provide logical justifications for each step taken. Students need to understand the relationships between equations, graphs, and geometric concepts to accurately calculate and explain the coordinates of points A and B where the two functions intersect.

- **Data Analysis**

In this research, a mixed-methods approach was followed, conducting data analysis both quantitatively and qualitatively. The quantitative analysis of test data revealed observable trends in both the control and experimental groups, particularly regarding learners' mathematical proficiency, encompassing their conceptual understanding and procedural proficiency. To categorize the data collected through various research instruments, including research tests, interviews, lesson observations, and learners' notes, a coding framework was developed. These codes facilitated the observation and tracking of potential shifts in learners' conceptual understanding in response to their acquisition of mathematical vocabulary.

Quantitative data were presented using tables, scatter plots, multiple bar graphs, pie charts, and written descriptions. Microsoft Office Excel was employed as a statistical software package for processing research test data. In the case of qualitative data, primarily descriptive techniques were utilized for data presentation. On certain occasions, qualitative responses were quantified, and the results were presented using tables and graphs.

4.10 Lesson Observations in two schools

In the course of the research, the researcher opted for an observation method known as naturalistic and non-participant observation. This decision was made based on several significant factors and considerations.

First and foremost, the research aimed to provide an authentic portrayal of classroom dynamics within genuine educational settings. The naturalistic observation approach was well-suited for this purpose as it allowed for the unobtrusive observation of teaching and learning activities as they naturally occurred (albeit covid-19 induced modifications). This minimized the potential for artificial influences or disruptions, ensuring that the observations closely reflected the real-world classroom experiences.

Furthermore, the study encompassed two distinct intervention research sites, denoted as School C and School D. Each of these sites had its unique characteristics, teaching methodologies, and classroom environments. By employing naturalistic observation, the researcher could delve deeply into the specific contexts of these sites, shedding light on the diverse dynamics at play.

At School C, two teachers, referred to as Teacher C1 and Teacher C2, were observed, each instructing their respective classes, namely Class C1 and Class C2. This differentiation allowed for the exploration of variations in teaching approaches (though both used the Frayer model) and classroom atmospheres, enriching the research findings.

To facilitate clarity and organization in the observation records, a systematic naming convention was implemented. This included designations such as Class C1, Class C2, Teacher C1, and Teacher C2. These labels proved invaluable in distinguishing between different classes and teachers during the subsequent data analysis phase.

In total, the research encompassed the observation of nine lessons, with three lessons documented for each class. This comprehensive approach ensured a thorough and in-depth examination of teaching practices, student interactions, and the overall educational context.

The results obtained from these observations were primarily presented in Chapter 6, Section 6.3.1 of the thesis. This section offered detailed descriptions and valuable insights derived from the observed lessons, contributing significantly to the comprehensive understanding of the research context and its findings.

4.11 Interview with two learners

The original research plan initially outlined two sets of interviews to be conducted with selected participants from the experimental group. The first set was intended to take place after the pretest, and the second set was planned for after the post-test. However, the stringent restrictions imposed on the South African education sector due to lockdown regulations rendered adherence to the original plan unfeasible.

As a result, semi-structured interviews were only carried out with two learners from the experimental group following the post-test. These two learners, identified by the pseudonyms Dean and Dris, were both affiliated with School D, the only school where conducting learner interviews was possible. School C authorities, citing concerns related to COVID-19 regulations, were unable to grant permission for interviews with their learners.

Semi-structured interviews, characterized by a blend of open-ended and closed-ended questions, were employed in this study. While the researcher had a predetermined set of questions, there was flexibility to ask follow-up questions and delve into specific topics in more detail based on the participants' responses. This approach provided a balance between structure and flexibility, making it suitable for gathering both quantitative and qualitative data. It enabled the researcher to delve deeper into the experiences and perspectives of the participants.

The findings resulting from these interviews are comprehensively presented in Chapter 7 of the research report.

4.12 Development of the coding framework

Within the scope of this study, the Frayer model served a dual purpose, functioning both as a valuable instrument for vocabulary acquisition and as an effective tool for collecting research data. In the former use, the Frayer model facilitates active learning. This is supported by research that has shown that when graphic organisers used to present new ideas to learners, learners, in turn, use them to present what they have learnt (Dixon & Lammi, 2014; Tseng, 2020; Yelich et al., 2016). As a data collection instrument, the Frayer model functions as a specialised open-ended questionnaire (Di Paola et al., 2016).

An open-ended-response approach is useful in revealing students' conceptual understanding. In a quest to answer the main research question (What affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?), I developed a coding system for the purpose of data analysis. Coding enabled me to organise data in a way that allowed easy identification and retrieval of content relevant to the research questions (Chowdhury, 2015). It also facilitated conversion of the data into a numerical format for easy statistical processing using computer software (Rose et al., 2015). I used this framework to analyse data obtained through written tests.

Data collected during the study was expected to show trends and the presence/absence of shifts in learners' knowledge of correct mathematical vocabulary and conceptual understanding. That being the case, some categories based on learners' responses were expected to emerge. As the research explored shifts in learners' conceptual understanding and their overall performance, emerging categories from learners' responses were a continuum of their Vocabulary Knowledge, Vocabulary Proficiency, Language Proficiency, and Conceptual

Understanding and Procedural Proficiency. Learners' overall mathematical performance was also measured.

There are five questions in the research test, and what each question is intended to assess is presented in the following Table 4.7.

Table 4.7: Research questions and the category of vocabulary that they assess.

Question	What the question assesses
Question 1	Vocabulary Knowledge and Vocabulary Proficiency.
Question 2	Conceptual Understanding and Procedural Proficiency, Language Proficiency and Marks Obtained.
Question 3	Language Proficiency.
Question 4	Vocabulary Proficiency.
Question 5	Conceptual Understanding and Procedural Proficiency and Marks Obtained.

The subsequent conceptualization pertains to the categories and their associated taxonomy within the coding framework, as illustrated in the table below, featuring five categories and their corresponding codes.

1. Vocabulary Knowledge (VK) - Assesses learners' quantity of vocabulary related to a given mathematical concept in the research test.

2. Vocabulary Proficiency (VP) - Evaluates learners' proficiency in mathematical vocabulary.

3. Language Proficiency (LP) - Focuses on learners' articulation of the language of mathematics as they describe mathematical concepts.

4. Conceptual Understanding and Procedural Proficiency (CUPP) - Assesses learners' accuracy in solving mathematical problems.

5. Marks Obtained (MO) - Represents the marks obtained by learners in questions requiring mathematical solutions, along with their respective mark allocations.

To facilitate coding, numerical codes were assigned to the first four categories. These codes consist of stems 1, 2, 3, and 4, each having leaves 0, 1, 2, and 3 to represent different aspects.

Table 4.8: Key to the research codes.

Category	Abbreviation	Stem
Vocabulary Knowledge	VK	1
Vocabulary Proficiency	VP	2
Language Proficiency	LP	3
Conceptual Understanding and Procedural Proficiency	CUPP	4

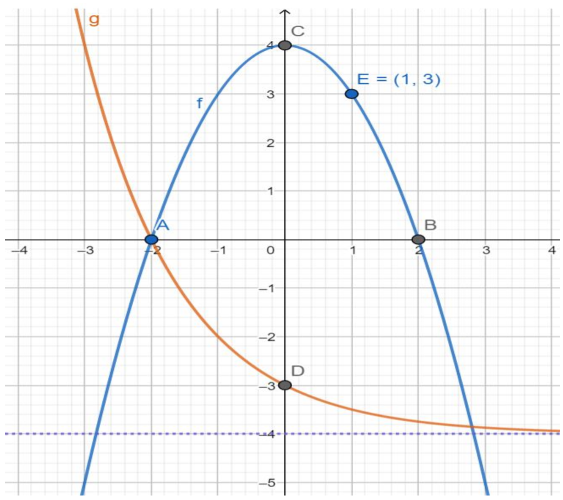
From this (Table 4.8), it follows that any code with stem 1 belongs to Vocabulary Knowledge, any code with stem 2 is of the vocabulary proficiency category, and so forth. The leaves are described in the subsequent conceptualisation.

4.12.1 Conceptualisation of Vocabulary Knowledge

This category refers to learners' knowledge of mathematical vocabulary associated with a given mathematical object. The first question of the research test solicited such knowledge. The intention was to check if learners were aware of the pragmatic areas to which they would apply the mental web of mathematical vocabulary they possess.

Question 1

Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object.



Four codes emerged for this category:

10: Unclassified Knowledge (UK)

11: Little Knowledge (LK)

12: Moderate Knowledge (MK)

13: Satisfactory Knowledge (SK)

For this question, it was possible to come up with 40 or more correct mathematical terms/vocabulary associated with the given mathematical object. The object was a maximum value parabola (f) and a decreasing exponential graph (g) that were drawn on the same set of axes. An asymptote to the exponential graph was visible. Points A, B, C, D and E were marked on the graph. None of the points (except Point E, which was on the parabola), had their coordinates given. The two graphs were labelled, and the intersection of the y-axis and x-axis was marked O. In addition, both axes were labelled.

Where a learner did not attempt to answer the question, the learner's answer (blank) was coded 10. A learner who only managed to come up with up to 10 correct terms was coded 11. A code of 12 was assigned when a learner came up with between 10 and 20 correct words. Lastly, a learner who managed more than 20 correct terms received Code 13.

As a way of scaffolding the instruction of the question, two examples were given to the learners. The examples were the terms coordinates and decreasing. The vocabulary that the learners were expected to write could have been about features of the mathematical object (e.g. coordinates), behaviour of the mathematical object (e.g. decreasing), or instructions associated with the mathematical object (e.g. shift). Though instruction related vocabulary was acceptable as correct responses, no examples of such vocabulary were given, as learners tended rather to follow instructions than give instructions themselves.

4.12.2 Conceptualisation of Vocabulary Proficiency

This category refers to the correctness of facts or characteristics about given mathematical vocabulary. Question 4 of the test involved matching mathematical vocabulary to an example about the vocabulary, as well as a defining characteristic about the same term/vocabulary. The mathematical vocabulary words presented in the question were asymptote, intercept, domain, range, and exponential function. These vocabulary words were presented to the research subjects in a table with three columns as follows:

Question 4

The subsequent (table 1) has a Mathematical vocabulary, an example of how the concept represented by the vocabulary is expressed symbolically and a characteristic of the concept.

Complete Table 2 by rearranging and matching the concept in Column A to an example of the concept in Column B and a characteristic of the concept in column C.

Table 1

Column A: Mathematical vocabulary		Column B: Example	Column C: Characteristic
4.1	Asymptote	$y = 2^x$	Only the letter representing the independent variable is present
4.2	Intercept	$2 < x \leq 5$	Linear equation
4.3	Range	$y > 2$	The exponent is a variable.
4.4	Domain	(2; 0)	Only the letter representing the dependent variable is present
4.5	Exponential equation	$x = 3$	At least one coordinate is equal to zero

Table 2

Column A: Vocabulary		Column B: Example	Column C: Characteristic
4.1	Asymptote		
4.2	Intercept		
4.3	Range		
4.4	Domain		
4.5	Exponential equation		

Column A has the vocabulary, Column B exemplification of the vocabulary, and Column C characterisation of the vocabulary. The vocabulary, examples, and characteristics were mismatched in the table and the subjects were required to correct the matching. An example of correct matching would be *Asymptote,* $x = 3$ *, a linear equation*, denoting that an asymptote can be a vertical line, such as in the case of a hyperbola (National School Certificate Syllabus).

As for the coding for this question, four codes were developed: 20 (Unclassified VP), 21 (Non-Existing VP), 22 (Emerging VP) and 23 (Achieved VP). An Unclassified VP was where a learner did not respond to a particular question. In the event of a response that showed that neither the example, nor the characteristic matched the vocabulary, it was deemed that such a learner had Non-Existing VP. Where either the example or the characteristic (but not both) about the concept was correct, then the learner was considered to have Emerging VP, as there was at least something known about the vocabulary. In the case where both the example and the characteristic of a vocabulary was correct, then VP would have been Achieved. The vocabulary for this question was chosen from commonly used mathematical terms, based on the frequency with which the NSC examinations ask questions related to them, e.g.

- Write down the equation of the asymptotes of f.
- Write down the range of f.
- Write down the value(s) of x for which f is an increasing function.
- Calculate the coordinates of the x and y intercepts of f
- Name the function produced when f (logarithmic function or vice versa) is reflected in the line $y = x$

Vocabulary Proficiency was also assessed in Question 1 of the research test. In this question, participants were requested to write mathematical vocabulary associated with a mathematical object. In addition to writing the vocabulary, they were asked to identify the vocabulary on the mathematical object. For instance, if a participant wrote a vocabulary, such as turning point, he/she would identify the turning point on the graph. Identifying the vocabulary on the graph was a way of checking the participants' knowledge of realistic areas in which they would apply the mathematical vocabulary.

4.12.3 Conceptualisation of Language Proficiency

As learners talk about mathematical objects or procedures, they articulate specific mathematical vocabulary. This articulation results in mathematical language, and, in this research, it is this articulation that is referred to as Language Proficiency. This was assessed as learners answered Questions 2.4 and 3 of the research test, and as they engaged in classroom mathematical talk. Language proficiency is defined as an individual's skill in language use for a specific purpose, and not merely students' conversational fluency in English (Gharbavi & Mousavi, 2012). What matters the most in this case, is how learners engage with precise mathematical vocabulary of functions, and not their general conversational fluency in English.

Haag et al. (2013) indicated that attention to mathematical language gaps is currently on the rise in the field of mathematics education. In addition, Prediger et al. (2018) revealed that most students with low language proficiency face enormous learning obstacles when required to perform tasks with high conceptual demands. Aside from playing a communicative role, mathematical language serves as a medium of thinking (epistemic role) used by learners to construct knowledge (Heller & Morek, 2015; Vygotsky, 1978). Understanding of a mathematical concept requires the acquisition of words and phrases, construction of meaning of the concept (semantic area), and knowledge of the problems and purposes for which it is applied (pragmatic area). Collectively, these are the features of conceptual understanding (Prediger, 2008; Steinbring, 2005), in contrast to the methodical and procedural skills that can be executed without any deeper understanding.

The following questions taken from the research test served the purpose of assessing mathematical language proficiency.

Question 2.4

Use correct mathematical vocabulary in a sentence to describe the answer (algebraic notation) given in Question 2.3.

Question 3

Define or describe the following terms:

3.1 Asymptote

3.2 Intercept

3.3 Range

3.4 Domain

In Question 3, learners were asked to define a given mathematical vocabulary: asymptote, intercept, range and domain. Question 2.4 required learners to describe the answer (algebraic symbol) that they had given in Question 2.3. Asking learners to describe and define terms is in line with a conclusion reached by Moschkovich's (2013) assertion that designing of classroom activities should take cognisance of the fact that instructions should support students' learning of vocabulary as they engage in mathematical reasoning. This was further elaborated on by O' Halloran (2015): "difficulty of scientific language forms the greater theoretical framework within which technical terms are defined" (p. 65).

Language Proficiency (LP) is a continuum ranging from Unclassified, Non-Existing, Emerging, and through to Achieved LP. Language Proficiency is Unclassified (30) if a learner does not answer a question. Where a learner uses incorrect vocabulary to define a concept or describe an algebraic statement, Language Proficiency is Non-Existing (31). Language Proficiency is also Non-Existing if an incorrect definition or description is given for the mathematical vocabulary. Where LP is deemed to be Emerging (32), a learner would have attempted to explain/define a term, but the response shows incomplete/inconsistent constructs. The definition or description given would not be precise but have emerging correct parts. Where well-presented explanations with precise vocabulary are given, with a learner's definition or description matching both the concept definition and concept image, such a learner would be considered to have Achieved (33) LP.

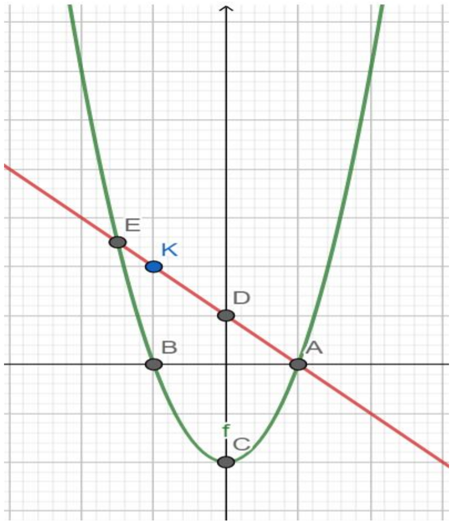
4.12.4 Conceptualisation of Conceptual Understanding and Procedural Proficiency

Despite the interrelationship between conceptual understanding and procedural knowledge, Al-Mutawah et al. (2019) concluded that, for most students, their procedural understanding is at a higher level than their conceptual understanding. While procedural fluency is assessed by checking the accuracy of procedures used in solving problems, conceptual understanding is measured by asking learners to provide definitions, explanations, and reasons (Al-Mutawah et al., 2019). In this research, learners were asked to solve a mathematical problem and, at the same time, provide reasons for executing the steps that they did. As they solved the problem, they engaged in both procedural and conceptual undertakings.

Like the other measures of proficiency, the CUPP category has Unclassified (40), Non-Existing (41), Emerging (42), and Achieved (43) codes. This category appears in Questions 2.1, 2.2, 2.3, and 5 of the research test, where it measured learners' accuracy in doing mathematics**.** Question 2 comprises a Cartesian plane on which a parabola and a straight line are drawn. Equations for both graphs are given. Points A and B are the x-intercepts of the parabola and Points C, and D are the y-intercepts of the parabola and the straight line respectively. Points E and A, the intersection points of the graphs, are also shown. In addition, the coordinates of Points A and E are given.

Question 2

The graphs of $f(x) = x^2 - 4$ and $g(x) =- x + 2$ are sketched. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK $\parallel$ $y - axis$. The graphs of f and g intersect at A (2; 0) and E (-3; 5).



2.1. Write down the coordinates of C. (2)

2.2. Write down the range of f (1)

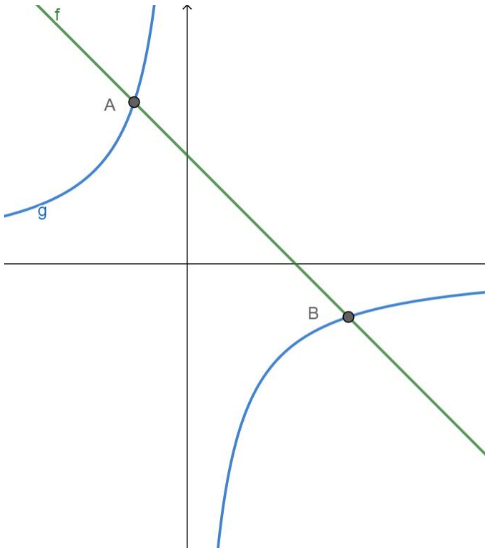
2.3. Write down the value/s of x *for which* *f(x) < g(x).* *(2)*

In Sub-Question 2.1, learners were asked to write down the coordinates of Point C, which is the turning point of the parabola, before they were asked to write down the range of the parabola in 2.2. Answers for Question 2.2 could be presented in interval or set notation. Question 2.3 required learners to use algebraic symbols to describe the values of x for which the y-values of the parabola were smaller than the y-values of the straight line.

Question 5 also measured both conceptual and procedural proficiency. It showed a negative hyperbola and a decreasing straight line that intersect at points P and Q. The equations of both graphs were provided in the question's narration. The research subjects were asked to calculate the coordinates of P and Q.

Question 5

The graphs of $f(x) = -x + 2$ and $g(x) = \frac{-3}{x}$ are sketched below. P and Q are the points of intersection of the graphs.



Calculate the coordinates of P and Q. Show all steps and next to each step give a reason for doing it. (10)

All five questions of this test had an answering space. The answering space for Question 5 had separate spaces for showing calculations/working and for providing reasons for the steps/calculations executed. Explanations were required so as to check on participants' conceptual understanding over and above their procedural proficiency.

As already indicated, the question required learners to calculate the coordinates of P and Q, which are the intersection points of the hyperbola and the straight line. While finding the coordinates, learners were expected to show their working. The workings demonstrated the skills that they had in solving the mathematical problem, and these skills were coded as the responses were marked.

Code 40 indicates that a learner did not answer the question. Where a learner gave incorrect workings/calculations, Code 41 was given. In a case where a learner gave an incomplete/incorrect construct, but the working showed that they had presented emerging facts about the concept, Code 42 was assigned. Code 42 also applied if a learner's workings/reasons showed correct facts but was flawed with imprecision. This also applied to situations where a learner gave correct working with incorrect reasons. Lastly, for this category, a learner Achieved Conceptual Understanding and Procedural Proficiency (43) if he/she showed an accurate understanding of the concept, used correct procedure, used correct terminology to explain the procedure, and gave correct solutions.

4.12.5 Conceptualisation of Marks Obtained

In this research, Marks Obtained (MO) were measured by considering the marks obtained by learners in the test. The parts of the test to which marks were allocated were Questions 2.1, 2.2, 2.3, and 5 (questions already presented under CUPP). On one hand, mark allocation for Questions 2.1, 2.2, and 2.3 was informed by the NSC standard for such types of questions. On the other hand, mark allocation for Question 5 was informed by the steps/skills required in solving the problem. In this regard, the test had 15 possible marks. The following table shows the above-described conceptualisations.

The coding framework presented in Table 4.9 contains the codes that I used to assess learners' responses in the research test. All five categories: Vocabulary Knowledge (VK), Vocabulary Proficiency (VP), Language Proficiency (LP), Conceptual Understanding and Procedural Proficiency (CUPP), and Marks Obtained (MO), with their respective codes, are outlined in Table 4.9. The category, the question that relates to that category, the definition of the category, the respective codes, and the description of each code are also all shown in the table.

Table 4.9: The coding framework.

Questions Taxonomy [VK 1; VP 1 & 4; LP 2.4& 3; CUPP 2.1, 2.2, 2.3, & 5; P 2.1,2.2,2.3 & 5] Vocabulary Knowledge (VK), Vocabulary Proficiency (VP), Language Proficiency (LP), Conceptual and Procedural Proficiency (CUPP) and Performance (P).			
Categories and Questions	**Category definition**	**Codes**	**Code description and Examples of codes applied to learner responses**
Vocabulary Knowledge (VK) **Q1**	**Refers to learners' awareness of mathematical vocabulary associated with a mathematical object.**	10: Unclassified VK	• Question not answered.
		11: Little VK	• Up to 10 correct terms known for the given mathematical object.
		12: Moderate VK	• More than 10 and up to 20 correct mathematical terms known about a mathematical object.
		13: Satisfactory VK	• More than 20 correct mathematical terms are known about a mathematical object.
Vocabulary Proficiency (VP) **Q1 & 4**	**Refers to giving of correct facts/examples/characteristics associated with mathematical vocabulary.**	20: Unclassified VP	• Question not answered.
		21: Non-Existing VP	• Correct vocabulary given but incorrectly or not identified on graph. • Neither the example nor the characteristic about the concept is correct.
		22: Emerging VP	• Correct vocabulary given and partially identified on the graph. • Either the example or the characteristic (but not both) about the concept is correct.
		23: Achieved VP	• Correct vocabulary given and correctly identified on graph. • Both examples and characteristics match the concept.

Language Proficiency (LP) **Q2.4 & 3**	**Refers to articulation of mathematical vocabulary in describing mathematical concepts.**	30: Unclassified LP	• Question not answered
		31: Non-Existing LP	• Incorrect vocabulary used to define mathematical words/terminology/vocabulary/mathematical object. /Incorrect description about a given vocabulary or relationship.
		32: Emerging LP	• An attempt to define/explain a term./Mathematical object is made but the response is not comprehensive.
		33: Achieved LP	• Well-presented explanations about a mathematical object using precise vocabulary. /Correct definition about a term is given using correct mathematical vocabulary.
Conceptual Understanding and Procedural Proficiency (CUPP) **Q2.1, 2.2, 2.3, & 5**	**Refers to learners' accuracy in mathematical procedures and explanations for them.**	40: Unclassified CUPP	• Question not answered.
		41: Non-Existing CUPP	• Concept is not known. / Procedure is incorrect or non-existing.
		42: Emerging CUPP	• There is an Emerging idea about the concept, but procedure is incorrect or incomplete. /Emerging correct explanation about incomplete procedure./ Correct procedure but no explanation./ Correct procedure but given explanation not justifying steps.
		43: Achieved CUPP	• Correct procedure used, correct terminology used to explain procedure and correct solutions given.
Marks Obtained (MO) **Q2.1,2.2,2.3, & 5**	**Refers to learners' achievements based on the marks obtained for Questions 2.1, 2.2, 2.3, and 5, where marks were allocated.**	0: Unclassified MO	• Question not answered.
		1: Poor MO	• $0 \leq \text{Mark} < 5$
		2: Moderate MO	• $5 \leq \text{Mark} < 10$
		3: Satisfactory MO	• $10 \leq \text{Mark}$

The subsequent Table 4.10 outlines the main activities that were undertaken during the research that culminated with the production of this thesis.

Table 4.10: List of the major research activities.

Activity	Purpose and description	Duration
Development of the Coding Framework	To come up with a framework to use for evaluating learners' responses to the research test during pilot, pretest and post-test.	Up to coding of pilot results
Pilot Study	To assess suitability of the research test.	1 hour
Pretest	To gain insight into learners' understanding of the vocabulary used and their mathematical proficiency before a CLIL intervention.	1,5 hours maximum
Intervention Training	To give an orientation to the experimental group and train these participants on how to use the Frayer model.	One week/ maximum 3 hours
CLIL Intervention	Learning of functions and the vocabulary of functions using the Frayer model.	Six weeks or duration of chapter as per ATP
Post-test	To gain insight into learners' understanding of the vocabulary used and their mathematical proficiency after the CLIL intervention.	1,5 hours maximum
Interview	To assess conceptual development and understanding of concepts tested in the post-test.	5 minutes/ participant /session
Data Analysis and Write up	For quantitative and qualitative comparisons of data sets for vocabulary and proficiency shifts.	At most 12 months

4.13 Conclusion

In this chapter, discussions revolved around the positivist, interpretivist, critical, and pragmatic research paradigms. Among these, the pragmatic paradigm was selected for the research, owing to its suitability for a mixed-method approach, which characterizes this study. Details pertaining to the research methodology were also presented.

Data collection predominantly encompassed research tests, lesson observations, and semi-structured interviews. Marking of learners' responses in the research tests involved the application of a coding framework developed by the researcher, with the development process outlined in this chapter. Additionally, considerations of reliability and validity in data collection and analysis were addressed. Furthermore, the chapter delved into the feasibility of applicability and transferability over generalization, given the study's limited participant pool of only four schools in the Gauteng province.

Despite the inherent limitation regarding generalization, the study's findings hold applicability and transferability. Similarly, the results of the pilot study, although not generalizable, contributed to identifying weaknesses in the wording of test questions. Consequently, adjustments were made to the research test in terms of both wording and duration.

Ethical considerations were outlined, particularly in relation to maintaining participants' anonymity and confidentiality. The chapter's delineation provided clarity on the study's focus, while limitations were associated with the small sample size, the exclusion of other racial groups, and the impact of the COVID-19 pandemic on contact time, as well as the associated anxiety and stress experienced by participants.

CHAPTER 5 : THE PILOT STUDY

This chapter presents the results of the pilot study which was an indispensable first phase of this research before the main study. The pilot study only answered the research Sub-Question 1: How competent are Grade 11 mathematics learners in associating mathematical vocabulary with a given mathematical object? The other four research Sub-question were answered in the main study.

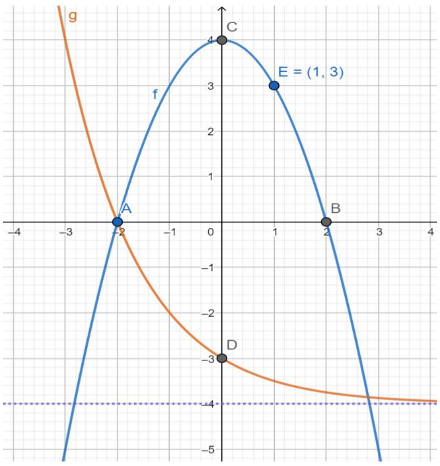
Results of the pilot study are presented in the subsequent sections of the chapter in the form of descriptions, tables, excerpts of learners' work, and bar graphs. These results constitute the first set of results to be presented in this research study. The coding framework presented in Chapter 4 was the one that I used to help assess learners' responses, as presented in this chapter. A question-by-question results presentation is done in this chapter, where Section 5.1.1 presents the results for Question 1. Section 5.1.2 presents the results for Question 2. Results for Question 3 are presented in Section 5.1.3. Sections 5.1.4 and 5.1.5 present results for Questions 4 and 5 respectively. In Section 5.2 I present the demonstrated relationship between Vocabulary Knowledge and Vocabulary Proficiency. Lastly, in Section 5.3 I present a conclusion of the chapter.

5.1 Results of the five-question test used during piloting.

5.1.1 Responses to Question 1

This question was used to measure participants' Vocabulary Knowledge, which is their awareness of mathematical vocabulary associated with a given mathematical object. The question prompted learners to write down vocabulary that they knew about the given mathematical object. Any words that were not related to the object were not considered. It was expected that learners could come up with 32 words or more. As already explained in the preceding table (4.4), Codes 10, 11, 12, and 13 were used to classify learners' responses. The results showed that 26,3 % (10/38) of the learners had Little VK of correct terms associated with the given mathematical object. The results further showed that 47,4% (18/38) of the learners had Moderate VK of the mathematical vocabulary associated with the mathematical object presented to them, and 26,3% (10/38) had Satisfactory VK. The results are illustrated in Table 5.1 and Figure 5.1.

Table 5.1: Results on learners' VK in Question 1 of the pilot study.

1. Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object. Give clear answers and few explanations, where necessary. Two examples are given in the answering space. 	Unclassified Knowledge	Little Knowledge	Moderate Knowledge	Satisfactory Knowledge
	0	10	18	10
	0%	26,3%	47,4%	26,3%

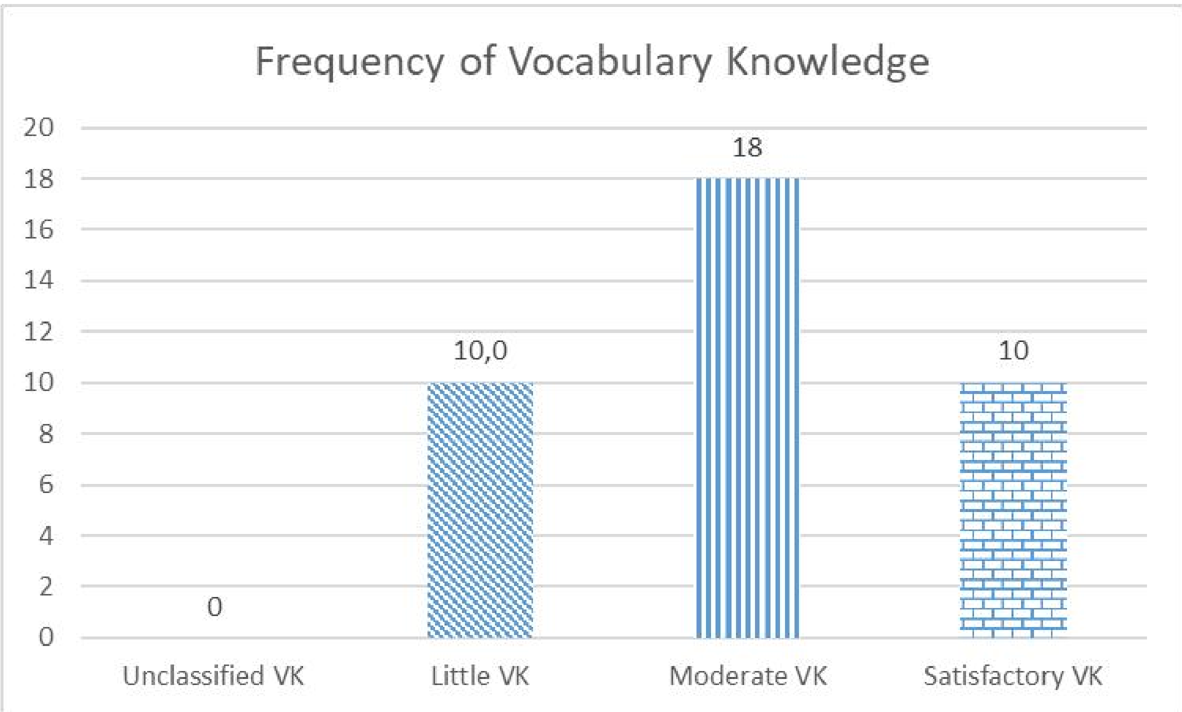


Figure 5.1: Results on learners' VK in Question 1 of the pilot study.

All the learners answered the question without showing any confusion as to what the question required of them. From an assessment of the learner's scripts, those who wrote terms that were not related to the given object did so not because they did not know what the question wanted, but instead because they did not know the correct mathematical vocabulary to give. There were instances where some correct mathematical vocabulary words were provided but were not related to the mathematical object given in the question. In such cases, these words were not considered as correct. The words given by the learners needed to be associated with the mathematical object that was presented to them.

Examples of vocabulary words that were given by learners that this research deemed inappropriate for the question were: hyperbola, letters, numbers, two squares, Euclidean geometry, hill, table, mountain, pineapple, triangle, batton, bisepts, area, and volume (Figure 5.2).

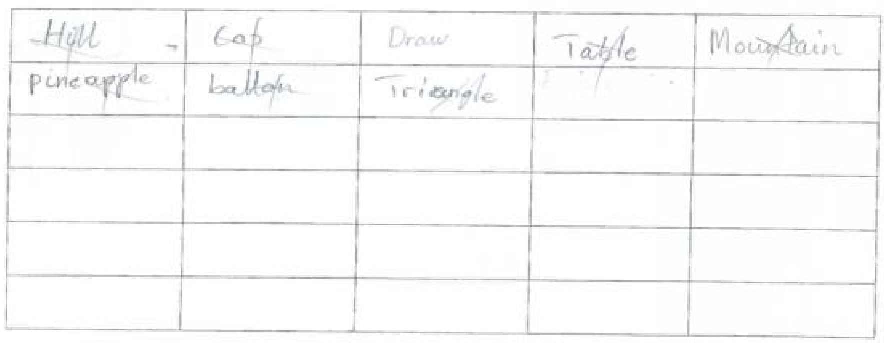


Figure 5.2: Example of learner's vocabulary that the research disregards.

The word triangle is a mathematical vocabulary, no doubt about that, but in this instance, it was not applicable to the given mathematical object. The object presented to the learners had nothing to do with a triangle. Likewise, the vocabulary hyperbola is correct vocabulary for functions, but, in the given mathematical object, there was no hyperbola.

The instruction of the question read:

> Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object.

There seems to be no ambiguity in the question as it precisely requested for vocabulary associated with the presented object. The object consisted of a Cartesian plane, a parabola, an exponential graph, and an asymptote to the exponential graph. Despite this lack of ambiguity, 20 of the 38 participants (52,3%) wrote the vocabulary hyperbola.

This is an indication that the learners did not have a clear understanding of the distinction between a hyperbola and an exponential graph. In essence, the presence of only one asymptote should have guided the learners towards the fact that the graph was not a hyperbola. A hyperbola has two asymptotes (horizontal and vertical), while an exponential graph has only one asymptote (horizontal). After examining all the scripts where the vocabulary hyperbola appeared, it was noticed that three learners who wrote the vocabulary hyperbola did not write an exponential graph/function. For these learners, it can be deduced that they really believed that the graph was a hyperbola.

Though the underlying characteristic of a hyperbola is the presence of an asymptote, four learners who wrote the term hyperbola did not write the vocabulary asymptote. This shows that there was no association between the hyperbola and what characterises it, which is being asymptotic. Likewise, two learners wrote both the terms hyperbola and exponential graph/function but did not write the term asymptote.

The other group comprised 11 learners, who wrote the vocabulary hyperbola as well as the vocabulary asymptote. Though the graph was not a hyperbola, these learners at least seemed to know one fundamental characteristic of a hyperbola. Their deficiency was in recognising the number of asymptotes that a hyperbola has. Lastly, six learners wrote the three vocabulary words, hyperbola, exponential graph/function and asymptote. The assumption is that they associated the asymptote, which is a characteristic, to both the hyperbola and exponential graph. However, for them to refer to the same graph as a hyperbola and an exponential graph is a sign that there is a mix up in their understanding of these graphs. In the view of this research, adopting a learning strategy that emphasises characterising these graphs/functions might help to clear such a mix up.

As for the other words written by learners, had it been with other curricula and not National Senior Certificate (NSC), they could have been correct but in this case, they were not applicable. Words in this category included words such as volume and area. The word area is used in situations where an enclosure between graphs is considered for study. When a selected strip on the graph is rotated about one of the axes then the resultant three-dimensional figure, solid of revolution, results in the concept of volume. Given that the NSC curriculum does not require learners to make those calculations, inclusion of these words (area and volume) was not expected and therefore not accepted. Despite the disqualification of these terms in the pilot, if such words or others come up during the main research, the interview that came after administration of the test would help clarify learner thinking.

Overall, the fact that 18 of the 38 participants (47,4%) and 10 of the 38 participants (26,3%) demonstrated Moderate and Satisfactory VK of the required vocabulary respectively, made me view the question as an understandable one.

However, after consultations with the research supervisor, modifications were made to this question. The question that was taken to the main research task reflects these modifications. The original question only solicited the listing of mathematical vocabulary associated with the given mathematical object. In the modified version, participants were required to list the vocabulary and in addition to listing, they had to identify what it represented on the graph and/or explain the vocabulary. Two examples were given on the question paper for learners to see what was expected of them by the question.

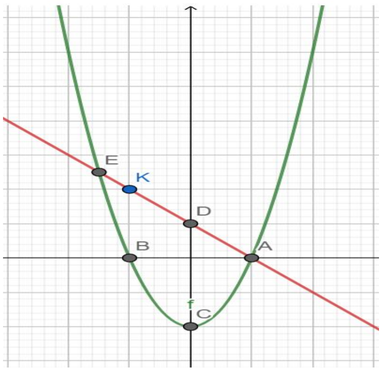
The basis for the modifications was to probe the extent to which participants knew the pragmatic applications of the vocabulary. Such knowledge is necessary as it demonstrates participants' Vocabulary Proficiency, in addition to Vocabulary Knowledge (listing). This is because mere listing of vocabulary without knowledge of the usage of that vocabulary is too superficial a measure.

5.1.2 Responses to Question 2

This question served three purposes in this research. Sub-Questions 2.1, 2.2, and 2.3 were intended to measure participants' conceptual understanding and procedural proficiency. In addition, these sub-questions also measured participants' mathematical performance by allocating marks to the learners' work. On the other hand, Sub-Question 2.4 measured participants' language proficiency.

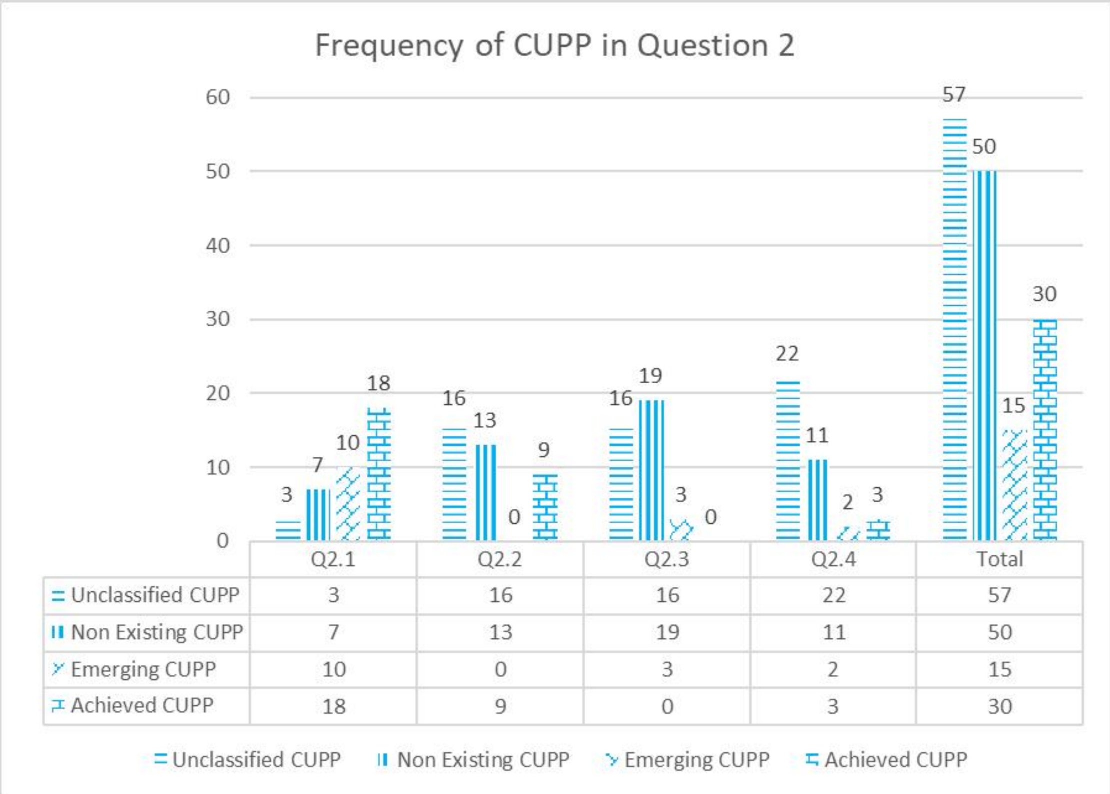
Where Question 2 served as a performance question, learners' achievements were measured based on the marks obtained (out of 5) for Sub-Questions 2.1, 2.2, and 2.3. Possible marks for the sub-questions were 2, 1, and 2 respectively. Sub-Question 2.2 was marked out of 1, so in the ensuing table the nine learners who got this question correct were recorded under full marks and not under 1 mark and the 29 who either did not attempt it or wrote incorrect answers are indicated under 0 marks.

Table 5.2: Results on CUPP in Question 2 of the pilot study.

2. The graphs of $f(x) = x^2 - 4$ and $g(x) = -x + 2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK$\parallel$ $y - axis$. f *and* g intersect at A (2; 0) and E (-3; 5).  Write down the coordinates of C. Write down the range of f Use an algebraic sentence (mathematical symbols), to describe the value/s of *x for which f(x) < g(x)*	***Question***	***0 marks***	***1 mark***	***Full marks***
	2.1	***11***	***10***	***17***
	2.2	***29***	***-***	***9***
	2.3	***35***	***3***	***0***
	Total	***75***	***22***	***26***
		65,8%	***11,4%***	***22,8%***
2.4 Use correct mathematical vocabulary in a sentence to describe the answer (algebraic notation) given in question 2.3.	***Unclassified CUPP***	***Non-Existing CUPP***	***Emerging CUPP***	***Achieved CUPP***
	22	***11***	***2***	***3***
Totals for CUPP in Question 2.	***57***	***50***	***15***	***30***

From Table 5.2, in Question 2, out of the possible 114 responses (38 learners times three sub-questions where marks were awarded), 75 of the responses were awarded 0 marks. This means 65,8% of the learners' responses were 0 marks. 11,4% (13 out of 114) of responses were allocated 1-mark (where the 1 mark was not the full mark for the question) and 22,8% (26 out of 114) responses were allocated full marks.

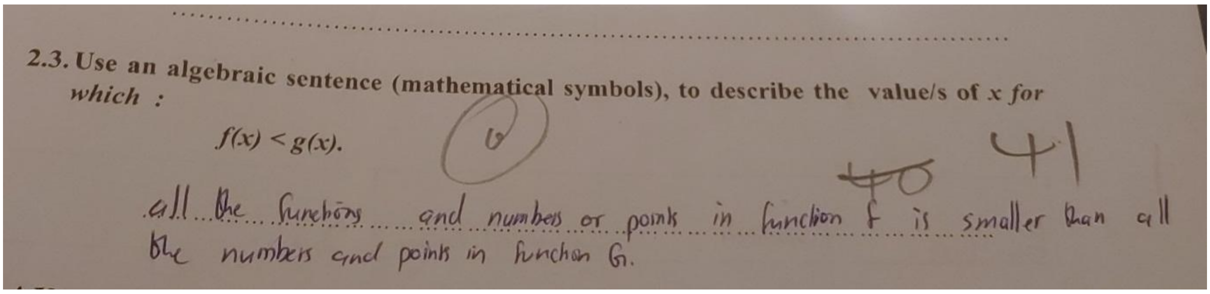
As already alluded to, this question also measured conceptual understanding and procedural proficiency. In this regard, the learners' responses were coded 40 (Unclassified CUPP), 41 (Non -Existing CUPP), 42 (Emerging CUPP), and 43 (Achieved CUPP). There was a total of 57 incidences of unclassified CUPP, 50 incidences of Non-Existing CUPP, 15 incidences of Emerging CUPP, compared to only 30 incidences of Achieved CUPP in Question 2. These incidents arose as totals for the codes across the sub-questions.



	Q2.1	Q2.2	Q2.3	Q2.4	Total
Unclassified CUPP	3	16	16	22	57
Non Existing CUPP	7	13	19	11	50
Emerging CUPP	10	0	3	2	15
Achieved CUPP	18	9	0	3	30

Figure 5.3: Results on CUPP in Question 2 of the pilot study.

Regardless of the poor performance (only 22,3% full marks) and lack of Conceptual Understanding and Procedural Proficiency (only 30 incidences of Achieved CUPP), most learners answered the three sub-questions, with 26 responses awarded full marks (Figure 5.3). This was considered as an indication that Sub-Questions 2.1 and 2.2 were clear and understandable. The poor performance and lack of CUPP in these two sub-questions was mainly caused by factors other than the lack of clarity of the questions themselves. Nonetheless, none of the 38 learners got full marks for Sub-Question 2.3. This could have been because the question was phrased unusually, as it was not worded in the way questions of this nature would normally be in a standard NSC question paper. I discovered that the presence of the word “sentence” in the question posed problems to learners. Learners wrote a full “linguistic sentence” instead of an “algebraic sentence”. An illustration of this error is shown below in the following learner’s script (Figure 5.4).


2.3. Use an algebraic sentence (mathematical symbols), to describe the value/s of x for which : $f(x) < g(x)$.

0 40 41

all the functions and numbers or points in function f is smaller than all the numbers and points in function G.

Figure 5.4 Example of learner response due to poor wording of the question during pilot study.

The learner attempted to explain his understanding of what it means to say $f(x)$ is smaller than $g(x)$. Despite the attempt, this work is evaluated only as Non-Existing CUPP (Code 41). This is so because the correct response is $-2 < x < 2$, which is an algebraic sentence and not a linguistic sentence as given by the learner. The question sought learners’ understanding of writing an inequality, which is more of a procedural skill than grammatical articulation.

Due to the said challenge, Sub-Question 2.3 was rephrased from:

> Use an algebraic sentence (mathematical symbols), to describe the value/s of x *for which* $f(x) < g(x)$

to:

> Write down the value(s) of x for which $f(x) < g(x)$.

As for the Sub-Questions 2.1 and 2.2, no changes were made.

Question 2.4 asked for learners’ articulation of mathematical vocabulary in describing mathematical concepts. Learners were required to use correct mathematical vocabulary to describe their result in the previous sub-question. The results show that none of the 38 learners managed to demonstrate Achieved LP, even though during orientation before the start of the test and after the test they all said they understood what was required of them by the question.

Question 2.4 refers to Question 2.3 where the phrase “value(s) of x for which” and the notation “$f(x) < g(x)$” are used. This is familiar vocabulary and notation for the learners. Though this was an unusual way of asking a question in mathematics, the presence of this familiar vocabulary and notation was expected to scaffold the answering of Question 2.4.

During teaching and learning, asking for and discussing domains of functions is a common exercise. Domains are also talked of orally, but rarely are the learners asked to write a verbal and non-symbolic sentence to describe those domains. Despite this, the question was neither adjusted nor removed because it tests proficiency in the literal repertoire (van Jaarsveld, 2016) of the mathematical register. Such proficiency tends more to conceptual understanding than to procedural competence.

Below is an extract of a response given by one of the learners (Figure 5.5). His response in Question 2.3 was: $-2 < x < 2$

Figure 5.5: Example of an incorrect learner's response to Question 2.4 of the pilot study.

The correct interval was supposed to be $-3 < x < 2$. If the learner made a slip of writing -2 instead of -3, the expected response here was:

> All real x-values that are greater than negative 2, and, at the same time, less than positive 2.

It was imperative that the learner identified that x must be real numbers, as this is the set of numbers that they work with in their syllabus. This is in line with the standard notations for answers normally given for examination questions, e.g. $x \in R$, meaning that the numbers referred to are real numbers. The norm (literal repertoire) is that this should be read from left to right, meaning that x is compared to the value on its left before it is compared to the value on its right. As such, this learner's response was deemed only to show emerging language proficiency. This is because though it seems as if the learner managed to give correct relationships between the variable and the numerical values, the word "and" should have been used in place of "<u>but</u>". The preference towards "and" is because the required values of x fall within an intersection. The learner also left out mentioning the set of numbers from which the required values are taken. In the response, there are correct phrases for an inequality: "less than" and "greater than", but these were not properly articulated in sound mathematical language.

Nonetheless, the question remained the same in the test as a way of checking if learners really understand and can explain their answers from the previous sub-question.

5.1.3 Responses to Question 3

This question tested learners' articulation of mathematical vocabulary in defining or describing given mathematical concepts. As shown in the following Table 5.3, most learners showed that they could not adequately define the given mathematical concepts. This is an unusual question for a mathematics task. During mathematical tasks, learners are seldom asked to define mathematical terms, except during lessons. As this was a test, learners could not have anticipated that they would be asked to give definitions of mathematical vocabulary. Despite this result, the question was not adjusted as I deemed it straightforward and without any ambiguity. Table 5.3 and Figure 5.6 illustrate the results to Question 3.

Question 3 had four sub-questions, all of which required the learners to define a given mathematical vocabulary word. The vocabulary words that needed to be defined were asymptote, intercept, range and domain. The learners' results are presented under the headings Unclassified LP, Non-Existing LP, Emerging LP, and Achieved LP (Table 4.4). Results in both Table 5.3 and Figure 5.6 show that five learners did not define the word asymptote, one did not define intercept, five did not define range, and 10 did not define the word domain. This gives a total of 21 Unclassified LP results. In the results, there were more responses that showed Non-Existing LP than any other category. This shows that on 73 occasions, learners attempted to define the vocabulary words but gave incorrect definitions.

Table 5.3: Results on learners' LP in Question 3 of the pilot study.

Questions		Unclassified	Non-Existing	Emerging	Achieved
3. Define or describe the following terms:	3.1	5	20	12	1
	3.2	1	26	6	5
	3.3	5	18	11	4
	3.4	10	9	17	2
3.1 Asymptote 3.2 Intercept 3.3 Range 3.4 Domain	Total	21	73	46	12

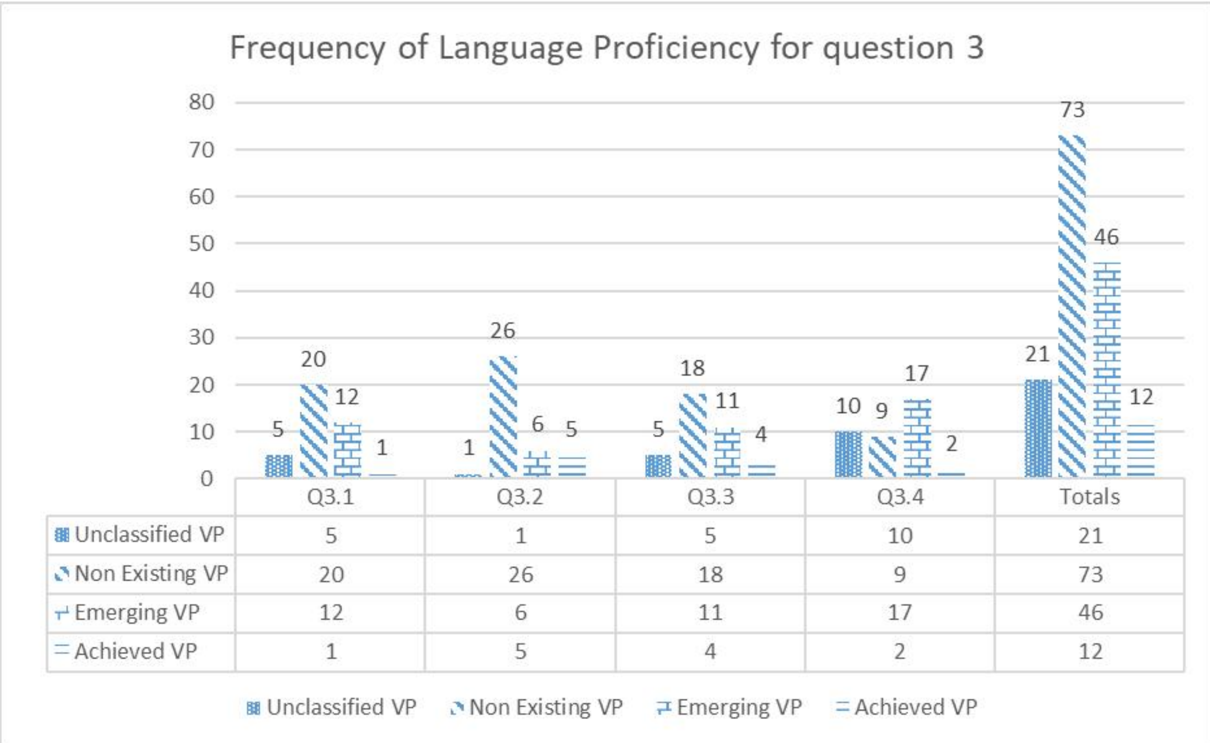


Figure 5.6: Results on learners' LP in Question 3 of the pilot study.

Generally, learners failed to give proper definitions for all four vocabulary words, with the worst being asymptote, which was only correctly defined by one learner. The word domain was only correctly defined by two learners. Despite the poor performance in this question, no modifications were made to it.

Examples of learners' response are hereby presented:

- **Emerging Language Proficiency**

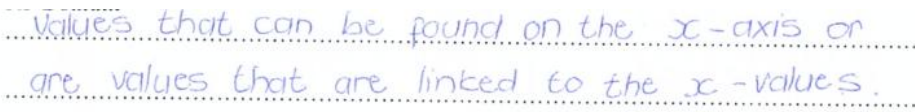

Values that can be found on the x-axis or are values that are linked to the x-values.

Figure 5.7: An example of Emerging LP.

In this instance (Figure 5.7), the learner was defining the term domain. The learner realised that the domain is connected to the x-values but could not explain this clearly. As such, this response was evaluated as Emerging LP. I looked at the same learner's response in Question 4.4 to check for consistency. In Question 4.4, learners were asked to match a vocabulary word in Column A to its example in Column B and its correct characteristic in Column C. It was evident from the learner's script that the vocabulary domain challenged him. In Question 4.4 he gave an example of the domain as $x = 3$ and its characteristic as being that "only the dependent variable is present". The dependent variable on the Cartesian plane is "y", but in answering Question 3.3 the learner referred to "x" values. It was evident from his answers in 3.3 and 4.4 that he did not know what the vocabulary domain represented.

- **Achieved Language Proficiency**


All of the Y-values produced by a graph

Figure 5.8: An example of Achieved LP.

A learner who defined the vocabulary "range" demonstrating Achieved LP gave the following response:

In this response (Figure 5.8), the learner showed good understanding of the vocabulary. There was awareness that the values are "y" values, which are output values as denoted by "produced by a graph". Though the learner used "produced by a graph" in place of "produced by a function", this was not punishable as the terms graph and function were not regarded as the main parts of the answer.

5.1.4 Responses to Question 4

This question was not adjusted, as most learners understood it and it was a question devoid of ambiguity. There were 60, 56, and 70 incidences of Non-Existing, Emerging, and Achieved VP respectively, with only four incidences of Unclassified VP. The following Table 5.4 and Figure 5.9 show the performance of the learners in this question.

Table 5.4: Results on learners' LP in Question 4 of the pilot study.

Question	Unclassified LP	Non-Existing LP	Emerging LP	Achieved LP
4.1: Asymptote	1	16	13	8
4.2: Intercept	2	6	2	28
4.3: Range	0	16	19	3
4.4: Domain	1	17	18	2
4.5: Exponential Function	0	5	4	29

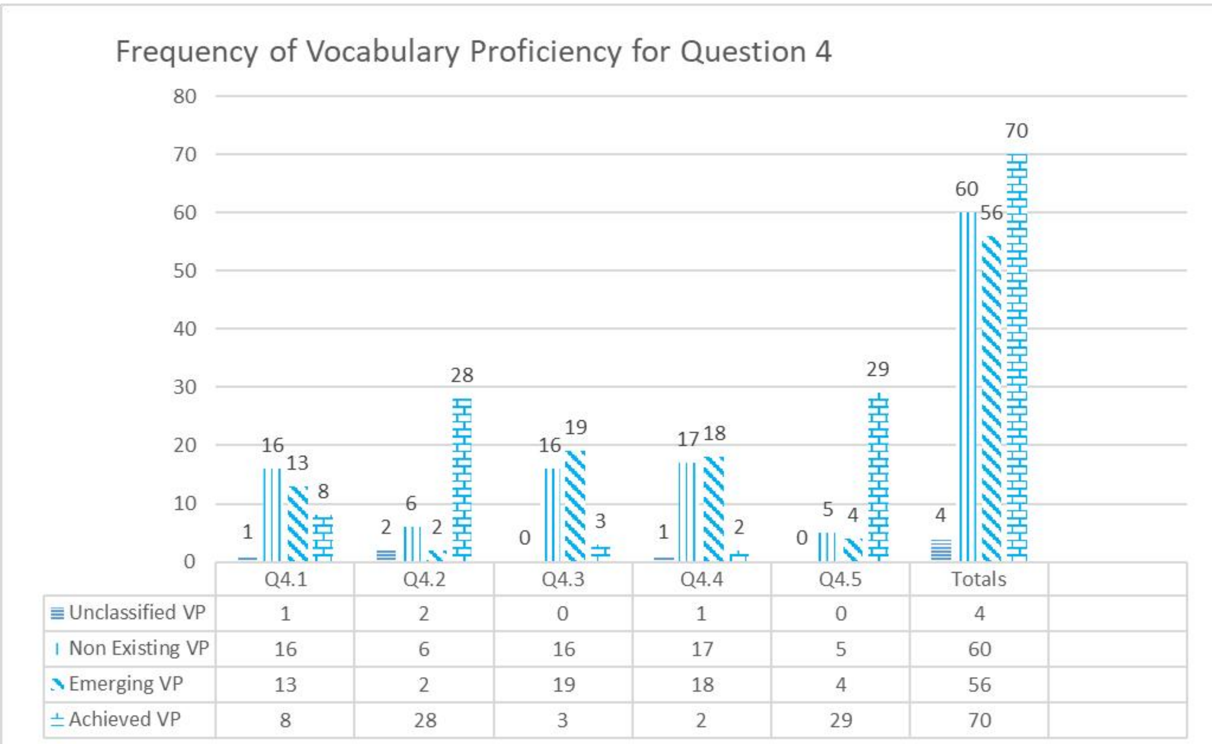


Figure 5.9: Results on learners' VP in Question 4 of the pilot study.

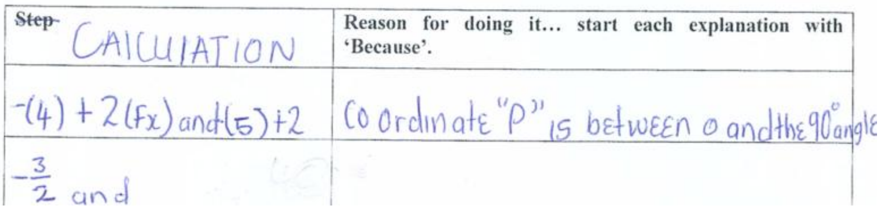
Though the question was generally well answered, it was noted that learners struggled with the vocabulary “range” and “domain”. Only three out of 38 learners (7,9%) managed to get a proper match of the example and characteristic of the vocabulary “range”. 42% of the learners (16/38) neither got the example nor the characteristic of a range correct. Such poor performance was also evident with the vocabulary “domain”, where only two out of the 38 learners (5,3%) showed Achieved VP. 44,7% (17/38) of the learners got neither the example nor the characteristic of domain correct.

5.1.5 Responses to Question 5

This is both an MO and a CUPP question, where learners were asked to answer a typical NSC question appropriate for the grade. 22 learners (57,8%) did not answer the question at all, and six of those who did answer did not manage to finish answering. Only seven learners (18,4%) received the full 10 marks for this question. Consequently, five of the learners were interviewed to ascertain the difficulties they had. Two of the interviewed learners came from the 22 who did not attempt the question, while the other two came from the group of those who started but did not finish. The fifth was a learner who both answered and finished. Those who did not answer and those who attempted but did not finish all had one reason: that the time allocated to the task was insufficient. The learner who did finish the task said that the time allocated was adequate. The learner went on to claim that he completed the task and had excess time left to revise his work. Even though few participants answered this question, it was not adjusted. The conclusion reached was that it was not answered because it was the last question of the test, and the time was up before it could be answered by many learners. For the sake of getting as much data as possible from the participants in the main research, the duration of the test was increased from 1 hour to 1 hour 30 minutes.

An analysis of the learners’ scripts (those who answered) showed that they were seriously challenged by this question. The question required the learners to calculate the coordinates of P and Q, which are intersection points of the functions f and g. This is a question that requires the routine procedure of equating $f(x)$ to $g(x)$ and then solving for the variable x.

- **Non-existing CUPP**

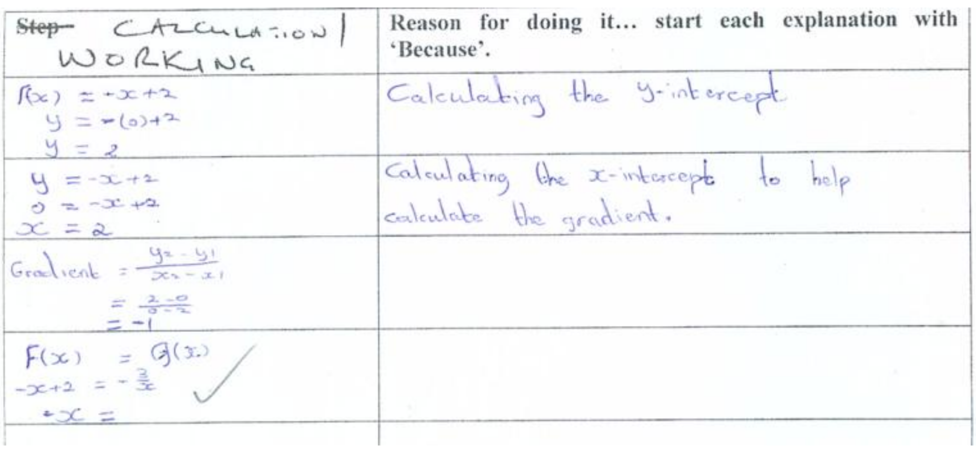


~~Step~~ CALCULATION	Reason for doing it… start each explanation with 'Because'.
-(4) + 2(Fx) and (5)+2	Coordinate "P" is between 0 and the 90° angle
$-\frac{3}{2}$ and	

Figure 5.10: An example of Non-Existing CUPP, learner 1.

This learner (Figure 5.10) did not equate the two functions, so it was no surprise that he could not get the required coordinates. Point P lies in the second quadrant, but the learner describes it as being between 0 and 90° which is a point in the first quadrant. Identifying quadrants was not going to help him get the coordinates of point P. This is a case of a complete procedural and conceptual breakdown.

Another learner who was also regarded as having Non- Existing CUPP presented the following work (Figure 5.11).



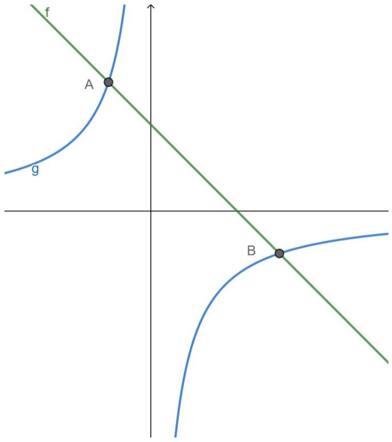
~~Step~~ CALCULATION / WORKING	Reason for doing it… start each explanation with 'Because'.
$f(x) = -x+2$ $y = -(0)+2$ $y = 2$	Calculating the y-intercept
$y = -x+2$ $0 = -x+2$ $x = 2$	Calculating the x-intercept to help calculate the gradient.
Gradient $= \frac{y_2-y_1}{x_2-x_1}$ $= \frac{2-0}{0-2}$ $= -1$	
$F(x) = G(x)$ $-x+2 = -\frac{3}{x}$ $+x =$	

Figure 5.11: An example of Non-Existing CUPP, learner 2.

As this is a routine procedure question, it is imperative that the first step should be to equate the equations of the functions. This learner decided to calculate the y-intercept as his first step. This intercept is nowhere near helping him get the required coordinates. There is also no reason given as to why he calculated this intercept. The second step should have been anything going towards getting the standard quadratic form. In the case of the learner in question, he calculated the x-intercept. After these two intercepts, he calculated a gradient of the line between the intercepts. It is only after three unnecessary steps that he equated *f(x)* to *g(x)* but without any reason as to why he equated them. His first three steps are an indication of both a conceptual and a procedural breakdown. For this reason, he displayed Non-Existing CUPP. However, though there was no reason given for equating the equations, a mark was allocated to this step.

The ensuing table and figure present the overall MO and CUPP categories respectively. Table 5.5 summarises the marks obtained by the learners in Question 5. The question was marked out of 10. From the table it is shown that 25 learners got 0 out of the possible 10 marks. This is 65,8% of the learners. Of these 25 learners, 22 did not give reasons to justify their steps as required by the question. These learners (22 learners) could not justify the steps because to start with, they did not answer the question. Three learners from the 25 who got 0 marks gave incorrect reasons to the steps that they took in answering the question. On the other extreme of the spectrum, only seven learners (18,4%) managed to get the full 10 marks. The full mark distribution is shown in Table 5.5.

Table 5.5: Results on MO by learners in Question 5 of the pilot study.

Question	Marks out of 10										
	0	1	2	3	4	5	6	7	8	9	10
5. The graphs of $f(x) = -x + 2$ and $g(x) = \frac{-3}{x}$ are sketched below. <u>P and Q are the points of intersection of the graphs.</u>  **Calculate the coordinates of P and Q. Show all steps and <u>next to each step give a reason</u> for doing it.**	25	3	1	0	0	0	0	1	1	0	7

In Figure 5.12, I present the results of how the learners performed in Question 5 with respect to CUPP. The results show that 22 learners did not give any reasons for the steps that they took in answering the question. In this case, these 22 learners did not answer the question at all. 11 learners gave reasons for their steps, but their reasons were incorrect. Two learners gave partially correct reasons. These two learners demonstrated Emerging CUPP, while only three learners demonstrated full CUPP, here recorded as Achieved CUPP.

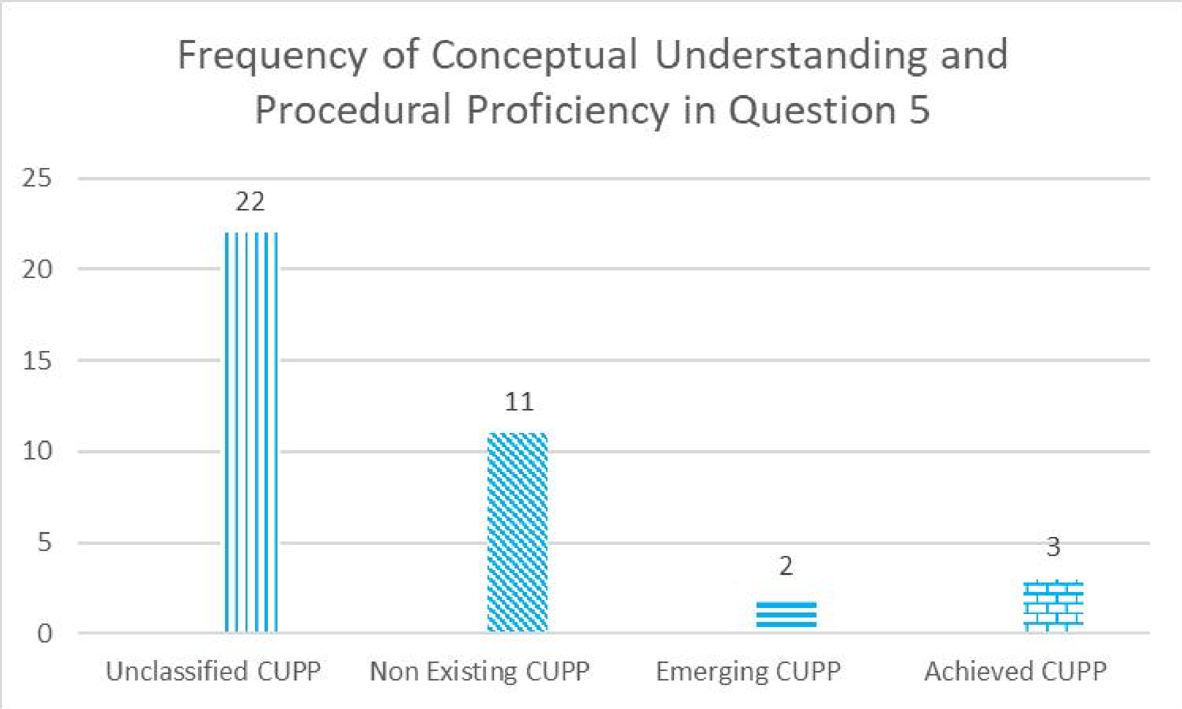


Figure 5.12: Results on learners' CUPP in Question 5 of the pilot study.

From the results in Figure 5.12, it can be concluded that learners showed little understanding of what was required of them when answering this question. They could not explain their steps. This could be an indication that learners memorised steps, but without fully understanding why these steps are undertaken.

5.2 Discussion of the relationship between Vocabulary Knowledge and Vocabulary Proficiency

In this study, Vocabulary Knowledge refers to learners' awareness of the mathematical vocabulary associated with a mathematical object. Learners were asked to identify vocabulary that they would associate with a given mathematical object. This was tested in Question 1 of the research test. In Question 3, they were asked to define the vocabulary, and, in Question 4, they were asked to exemplify and characterise the vocabulary. The vocabulary asymptote, intercept, domain, and range appeared in both Questions 3 and 4. It was discovered that the vocabulary words asymptote, **range**, and domain presented problems to the majority of the learners in Questions 3 and 4.

However, this same vocabulary was listed by many learners in Question 1. Figure 5.13 makes a comparison between the awareness of this vocabulary in Question 1 and proficiency in the same vocabulary in Questions 3 and 4.

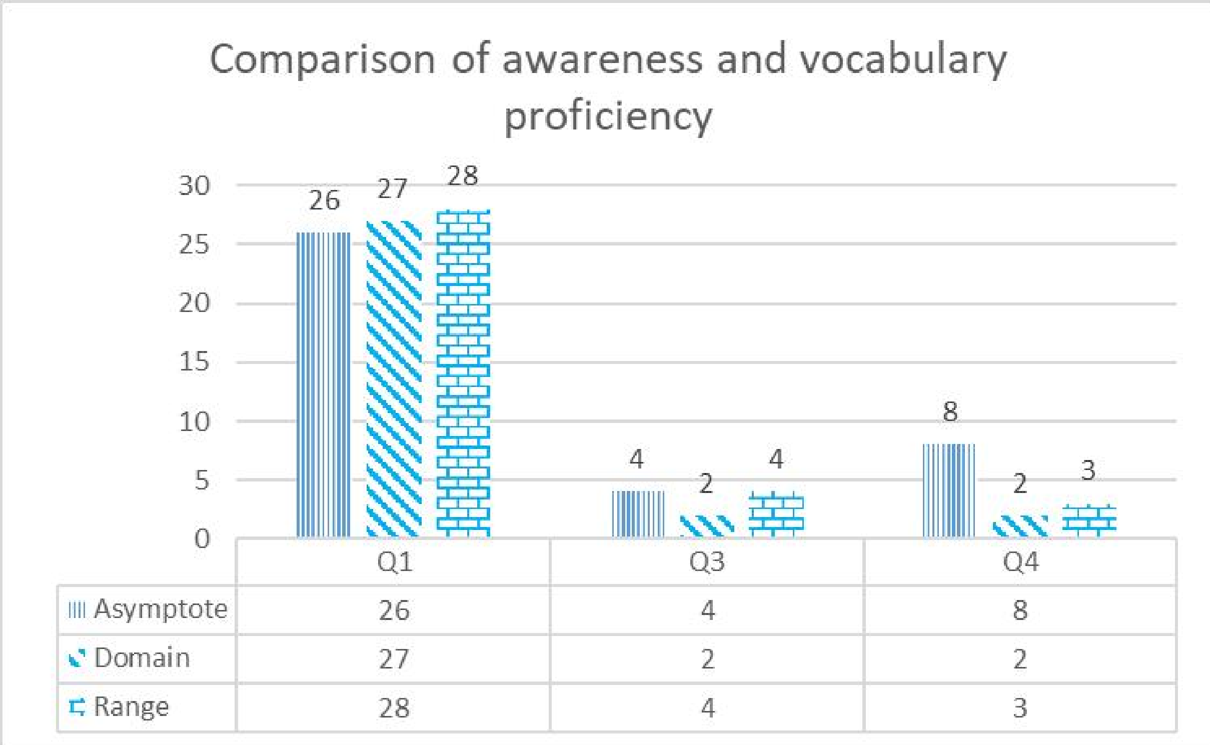


Figure 5.13: Comparison between vocabulary awareness and proficiency in Question 1 and Questions 3 & 4 in the pilot study.

As shown in Figure 5.13, 68,4%, 71,1% and 73,7% of the learners wrote the vocabulary asymptote, domain, and range respectively in Question 1. However, when it came to Questions 3 and 4, few of these learners Achieved LP and VP with these same words. This shows that remembering vocabulary for a mathematical object is not an indicator of a deeper understanding of the vocabulary. Learners remembered the words, but they showed that they lacked knowledge of the meanings of the words, and that they could neither exemplify, nor characterise those words.

Table 5.6 provides an overall summary of the results of the pilot study.

	Categories																			
	Vocabulary Knowledge				Vocabulary Proficiency				Language Proficiency				Conceptual Understanding & Procedural Proficiency				Marks Obtained			
Codes Questions	10	11	12	13	20	21	22	23	30	31	32	33	40	41	42	43	0	1	2	3
Q1	0	10	18	10																
Q2.1													3	7	10	18				
Q2.2													16	13	0	9				
Q2.3													16	19	3	0				
Q2.4									22	11	2	3								
Q3.1									5	20	12	1								
Q3.2									1	26	6	5								
Q3.3									5	18	11	4								
Q3.4									10	9	17	2								
Q4.1					1	16	13	8												
Q4.2					2	6	2	28												
Q4.3					0	16	19	3												
Q4.4					1	17	18	2												
Q4.5					0	5	4	29												
Q5													22	11	2	3				
Q2.1, Q2.2, Q2.3 &Q5																	0	29	1	8
Codes Description	10: Unclassified VK 11: Little VK 12: Moderate VK 13: Satisfactory VK				20: Unclassified VP 21: Non-Existing VP 22: Emerging VP 23: Achieved VP				30: Unclassified LP 31: Non-Existing LP 32: Emerging LP 33: Achieved LP				40: Unclassified CUPP 41: Non-Existing CUPP 42: Emerging CUPP 43: Achieved CUPP				0: Unclassified 1: 0 ≤ Mark <5 2: 5≤ Mark <10 3: 10 ≤ Mark			

5.3 Conclusion

The pilot study aimed to assess the clarity of the research test and refine the coding framework. Participants' responses were coded, with adjustments made to questions where necessary. The test duration was extended from 1 hour to 1.5 hours.

From the pilot study, it was found that 47.4% of participants demonstrated Moderate Vocabulary Knowledge (VK), and 26.3% showed Satisfactory VK in associating mathematical vocabulary with objects. While questions were clear, only 21.1% of learners scored over 30% in marked items, indicating a general challenge with the test, unrelated to question clarity.

The pilot study results led to refinements in question wording for the main study, resulting in clearer questions and richer data collection. Chapter 6 presents the main study results.

CHAPTER 6 : PRESENTATION OF RESEARCH RESULTS

This chapter reports the main study's results, conducted with learners from four schools in two districts of Gauteng province, South Africa. In this phase, a pretest was administered to all four schools, followed by an intervention in two of them, and concluded with a post-test for all four. The two schools receiving the intervention form the experimental group, while the other two constitute the control group.

The presented results address four of the study's five research questions. First, they assess learners' ability to associate mathematical vocabulary with objects (RQ1) by analysing responses to the first research test question. Results from the intervention address how learners are exposed to mathematical vocabulary during learning (RQ3) and the effectiveness of the Frayer model in mathematics learning (RQ4). Additionally, the results explore the link between mathematical vocabulary and learners' mathematics performance (RQ5).

The results are divided into five sections:

1. Section 6.1: Pretest results administered at the research's outset to both control and experimental groups.

2. Section 6.2: Results of the information phase of the intervention.

3. Section 6.3: Results of the practice phase of the intervention.

4. Section 6.4: Post-test results administered to both control and experimental groups after the intervention.

5. Section 6.5: Presentation of statistical tests assessing the significance of observed differences in the results. Results are conveyed through descriptions, tables, pie charts, single bar graphs, and multiple bar graphs across these sections.

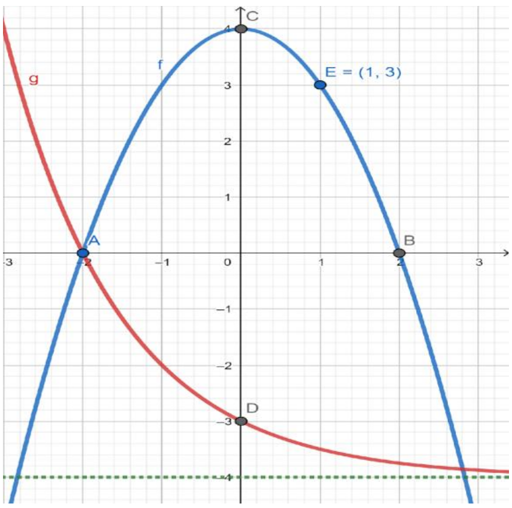
6.1 Pretest results

The pretest involved administering a research test to learners in both the control and experimental groups. This research test comprised five questions with a total of 15 items, all related to mathematical vocabulary concerning functions. Before conducting the pretest at the research sites, confirmation was sought from the subject teachers regarding the completion of the functions chapter by Grade 10 learners in 2020, following the Annual Teaching Plan. All subject teachers at the four research sites confirmed the chapter's completion, enabling the safe administration of the test in these schools. Sections 6.1.1 to 6.1.4 present a question-by-question breakdown of the pretest results.

6.1.1 Discussion of the results of Question 1

The question consisted of a parabola and an exponential graph that were drawn on the same set of axes. An asymptote to the exponential graph was also presented on the graph. The question was as follows:

1. *Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object. Write down the mathematical vocabulary used and identify it on the graph (explaining its meaning fully). Two examples are given in the answering spaces below.*



Learners were asked to write as much mathematical vocabulary that they could associate with the given mathematical object as possible. There was no limit as to the nature of vocabulary that they had to write, as long as it was connected to the presented mathematical object. Such vocabulary could be features of the object, instructions related to the object, descriptions of the object, and so on.

In addition to writing the vocabulary, the learners were asked to identify the use of that vocabulary on the object. Two examples were given to the learners as a way of scaffolding both the instruction and expectation. Apart from the examples, to ensure clarity, I verbally explained the requirements of the question to the learners before they started writing.

Depending on the number of correct vocabulary that learners managed to write, their results were classified as Unclassified Vocabulary Knowledge (Unclassified VK), Little Vocabulary Knowledge (Little VK), Moderate Vocabulary Knowledge (Moderate VK), and Satisfactory Vocabulary Knowledge (Satisfactory VK). These categories were given numerical codes to make it easier to work with the results in a spreadsheet.

The numerical categorisation is as follows: Unclassified VK (10), Little VK (11), Moderate VK (12), and Satisfactory VK (13). In this research, No VK denotes that a learner did not answer the question. When a learner managed to write up to 10 correct mathematical vocabulary for the question, such a learner's result was Little VK. For a learner who attained more than 10 correct vocabulary but did not exceed 20 correct vocabulary, the result was classified as Moderate VK. A result that is regarded to be Satisfactory VK is one where a learner surpassed 20 correct mathematical vocabulary.

Figure 6.1.1 presents a combined result for the learners who sat for the pretest.

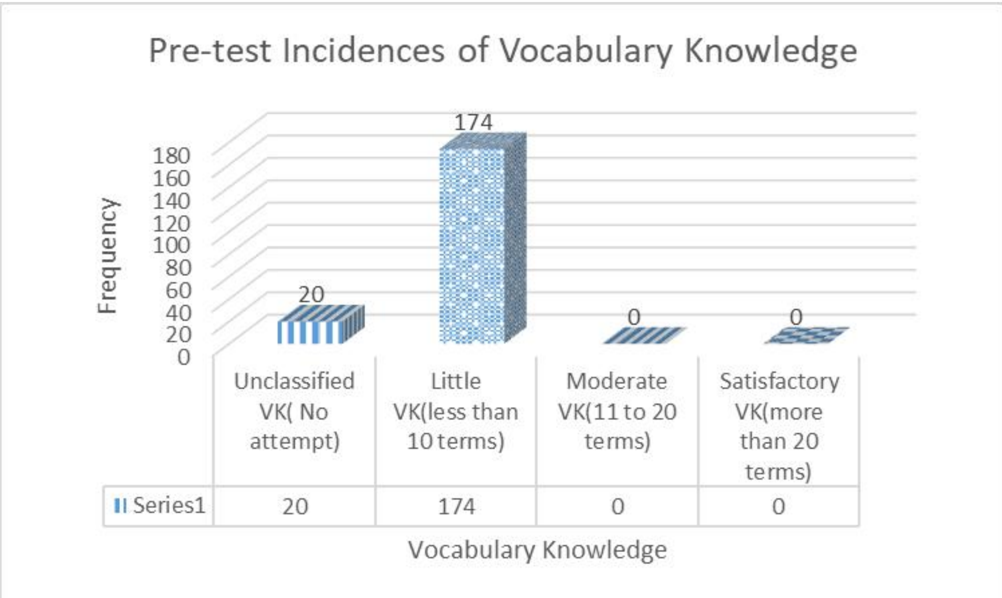


Figure 6.1.1: Results on VK during the pretest.

The results show that of the 194 learners who sat for the pretest, none managed to write more than 10 correct mathematical vocabulary associated with the mathematical object in the question. A total of 20 learners did not even attempt to answer the question. 174 learners only managed to come up with a maximum of 10 vocabulary words. This performance is classified as Little VK, and a breakdown of this result is shown in the subsequent figure.

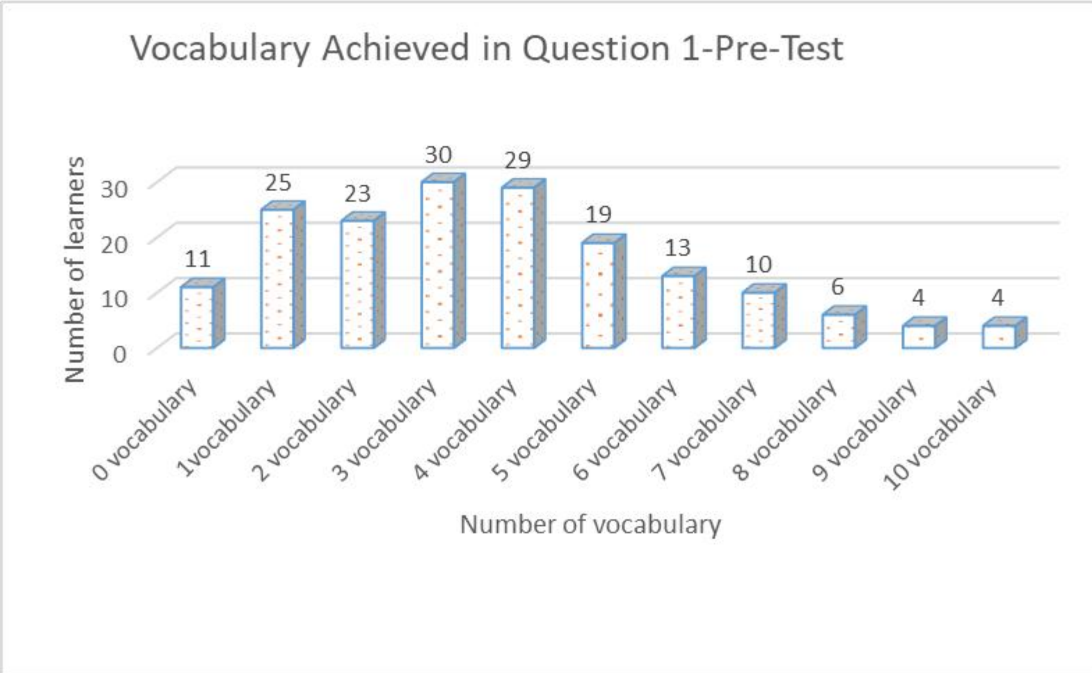


Figure 6.1.2: Breakdown of the VK performance for Question 1 of the pretest.

As shown in Figure 6.1.2, 11 learners wrote vocabulary which could not be accepted as correct vocabulary for the presented mathematical object and only four learners wrote 10 correct vocabulary. Three of the 11 learners with 0 accepted vocabulary repeated the two-vocabulary given as examples in the question, i.e. coordinates and decreasing. What could be seen in their responses, is that they simply gave more examples of the vocabulary "coordinates", for example $A\ (-2;\ 0), B\ (2;\ 0)$, and $E\ (1;\ 3)$. In the following excerpt, this learner could not write correct coordinates for Points C and D. The learner started with the y-values instead of the x-values. C and D are y-intercepts, so the correct coordinates are $C\ (0;\ 4)\ and\ D\ (0; -3)$.

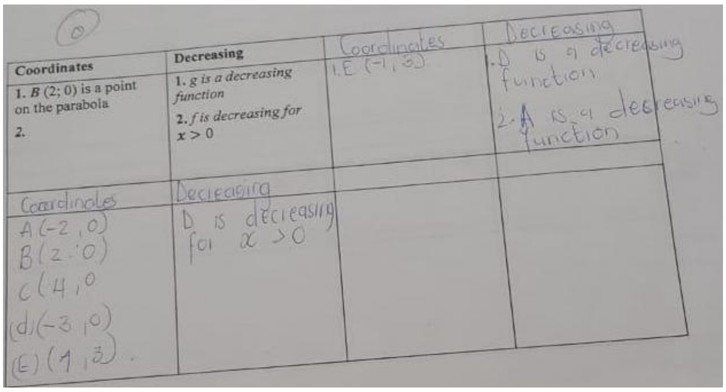


Figure 6.1.3: Example of incorrect coordinates for Points C and D.

For this learner (Figure 6.1.3), the only point whose coordinates were correctly given was $A\,(-2;\,0)$. Points B and E were given in the example and graph respectively, so the assumption is that the learner simply copied these coordinates. Where the learner had to write their own coordinates, without anywhere to copy from, the *x* and *y*-values were interchanged. Interchanging the values in 2 distinct points, C and D, cannot be taken to be a slip but must be a conceptual breakdown. In the case of this learner, interchanging values in Point C was repeated in the next step, D, which shows that the learner did not slip when writing the coordinates of C.

One of the vocabulary that was given in the test as an example was the term “decreasing”. Some of the learners chose to elaborate on this term. However, their explanations showed that they had little understanding of what they were faced with. In the preceding excerpt, the learner wrote that D is a decreasing function. The same learner also wrote that A is a decreasing function. In the object, A and D are points on function *g*. To say that A and D are functions is a sign that the learner could not separate the concepts of a function and a point on a function. I also noted that the learner went on to write that D decreases for $x > 0$. This clearly shows how the learner mixed up a function and a point. There is no valid way that Point D could be found to increase for $x > 0$.

In the following extract (Figure 6.1.4), the learner merely duplicated the example that *g* is a decreasing function. The learner went on to write that *f* is a decreasing function. The function *f* does not decrease for all values of x, but only for $x > 0$, as already stated in the example. Writing that *f* is a decreasing function might be an indication of a knowledge deficiency in respect to the concept of a decreasing function on the part of the learner. Furthermore, the *y*-value of Point D is incorrect. On the graph, it is clearly shown that at D, the 3 is a negative value. Assuming that it was not visible, a conversant learner should have written the correct sign, because D is a point below the x-axis. On the Cartesian plane, points below the x-axis have negative y-values.

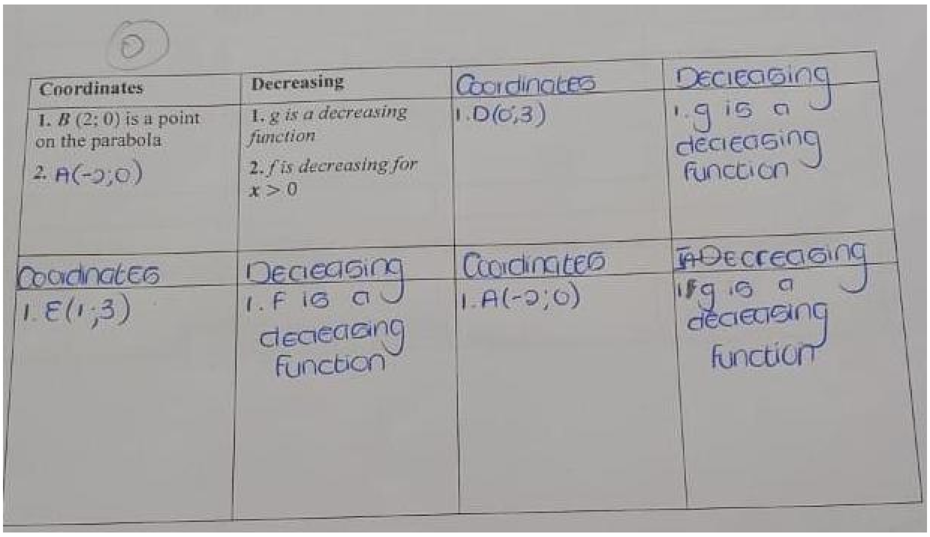


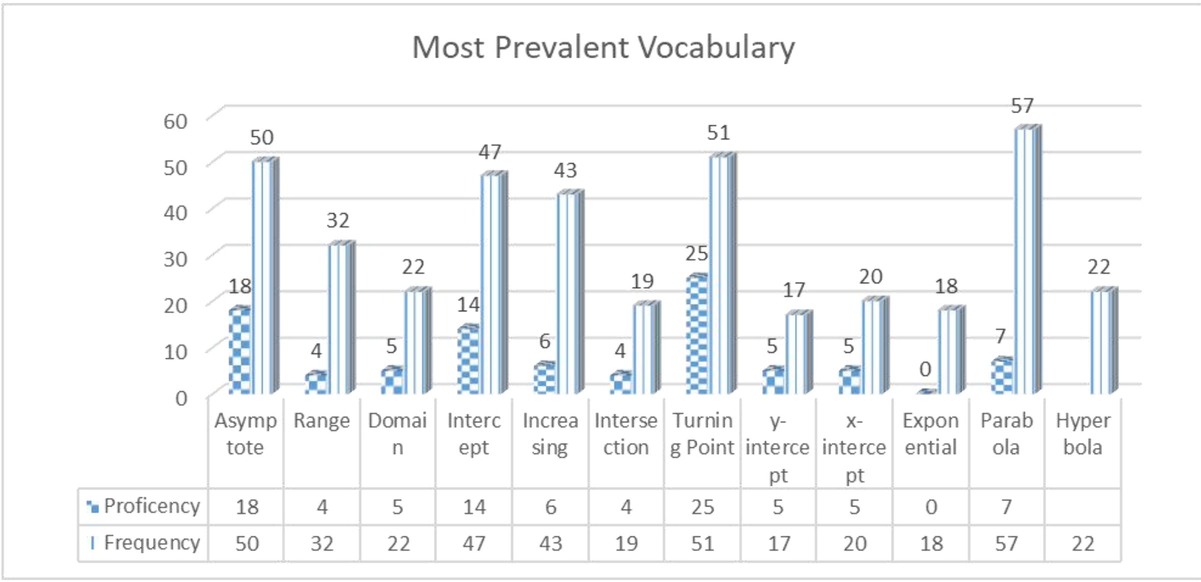
Figure 6.1.4: Example of incorrect sign for the y-value of Point D, below the x-axis.

In all these instances, these learners were regarded as not displaying VK, because they did not go beyond what was already given as examples. In addition to not going beyond the examples, the learners also displayed fundamental conceptual deficiencies.

This test question solicited writing vocabulary and illustrating pragmatic understanding of the vocabulary by means of identifying it on the graph. As such, I did a word-by-word analysis to examine the extent to which the written vocabulary was understood by the learners. A total of 28 words were written by the learners. These words were:

Asymptote, range, domain, intercept, increasing, intersection, coordinates, y-intercept, x-intercept, exponential, parabola, hyperbola, equation, graph, point, tangent, maximum, minimum, period, vertical, horizontal, concave, shift, quadratic, distance, table, line, parallel.

I then split the 28 terms into two groups. The first group is made up of the 12 most prevalent terms found in learners' responses. These 12 terms are presented in Figure 6.1.5.



	Asymptote	Range	Domain	Intercept	Increasing	Intersection	Turning Point	y-intercept	x-intercept	Exponential	Parabola	Hyperbola
Proficency	18	4	5	14	6	4	25	5	5	0	7	
Frequency	50	32	22	47	43	19	51	17	20	18	57	22

Figure 6.1.5: Frequency and proficiency of the most prevalent vocabulary in Question 1 of the pretest.

Three terms: asymptote, parabola and turning point, were written more than 50 times, with parabola being the most prevalent. Even though 57 learners wrote the term parabola, only seven of them identified an aspect of this word on the graph. None of the seven defined, gave an equation, or characterised the term parabola. All they did was to mention that either Points A, B, or C were on the parabola. Although this is a correct fact, it did not prove learners' proficiency beyond what was already given in the example. In the example using the term "coordinate", it was explained that $B\ (2;\ 0)$ is a point on the parabola. Here the term parabola was already given to them. The expectation was that when they wrote the term parabola, they would do more than writing that a given point was a feature of the parabola. However, because there was no restriction as to what the learners could write, their responses were considered to be proficient.

The second most frequent vocabulary was turning point. This appeared 51 times, with a proficiency incidence of 25 times. Of the 51 learners who wrote the vocabulary turning point, 25 of them managed to identify point $C\ (0;\ 4)$ as the turning point of the parabola. This is a maximum turning point. Even though the 25 learners identified C as the turning point, none of them explained that it was a maximum turning point. The term maximum was only written three times and it was written by learners who did not write the term turning point. This shows that the learners did not manage to show full understanding of the concept of a turning point. Characterisation of a turning point of a parabola is that it can either be a maximum or a minimum turning point. It was, however, noted that the three learners who wrote the term maximum also wrote the term minimum, though they did not explain either term.

Upon assessing the vocabulary written by the learners, terms appeared to co-occur frequently in their minds. These terms go as pairs, for example range and domain, x-intercept and y-intercept, and maximum and minimum. Of the 20 learners who wrote the term y-intercept, 17 of them also wrote x-intercept. These 17 learners wrote the terms one after the other, and in most of the responses, the x-intercept was written first. This seems to suggest that when these terms were learnt, this was the order in which they were learnt. To some extent, this seems to suggest that the learning process was based on how the values taken from these axes are related. As independent variables, the x-values derived from the x-axis influence the y-values (dependent variables) on the y-axis. Likewise, domain and range were written following one the other, though range was more prevalent than domain. This seems to suggest that some mathematical words are learnt in pairs.

An important observation from the most prevalent vocabulary is how the terms hyperbola, asymptote, and exponential graph were related – or unrelated for that matter. The term hyperbola appeared 22 times. In those 22 times where it appeared, the term asymptote only appeared twice. This is a challenging scenario, because a hyperbola is a graph that is characterised by the presence of two asymptotes. For this reason, for learners to write the term hyperbola without writing asymptote is an illustration of conceptual deficiency. In the view of this research, the terms hyperbola and asymptote should go as a pair. Additionally, in the mathematical object presented in the question, there was no hyperbola. By writing this term, the learners showed that they lacked knowledge about this function. The two learners who wrote the term hyperbola as well as asymptote cannot be spared from criticism either. A hyperbola has two asymptotes so for them to see only one asymptote and conclude that the curve that they see is a hyperbola is also an illustration of conceptual deficiency.

Another observation is how the terms asymptote and exponential graph were presented. Asymptote was written by 50 learners, while the term exponential graph was written by 18 learners. However, of the 50 learners who wrote the term asymptote only seven wrote both the term asymptote and exponential graph. Those seven learners managed to relate the concept of exponential graph to asymptote because this type of graph has an asymptote. As an asymptote is a fundamental feature of an exponential graph it shows that 43 learners wrote the term asymptote, but without relating it to any specific graph between either f or g. This is a sign of a lack of the ability to relate concepts, as a line only becomes an asymptote with respect to a known graph or function. In this case it was an asymptote to the exponential graph. Just as it was the case with the x-axis and the y-axis, the learners missed an opportunity to write the terms horizontal and vertical. This further demonstrates that the learners had fragmented thought processes. They did not manage to direct their thinking in a way that would put together related terms at one point. After writing the term asymptote, it should have been much easier to classify the asymptote as either horizontal or vertical.

Figure 6.1.6 presents results of the second group of terms, hereby referred to as the less prevalent vocabulary. These are terms which were not common responses from the learners, with nine of them appearing only once.

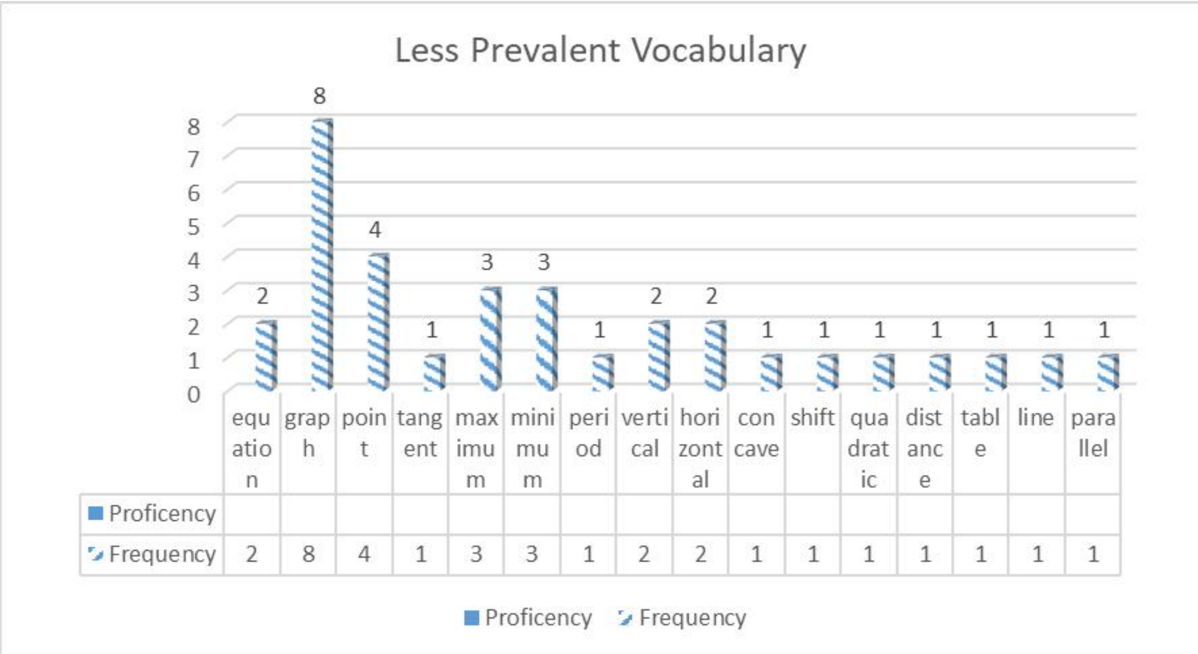


Figure 6.1.6: Frequency and proficiency of the less prevalent vocabulary in Question 1 of the pretest.

From the subset of less commonly used vocabulary, there appeared terms such as "tangent" and "period”. None of the graphs presented in the question were periodic graphs. The terms tangent and period are applicable to trigonometric functions and graphs. Tangent is an acceptable term for functions of this nature, but in the object in question, there was no tangent given. Furthermore, it was difficult to imagine that the concept of a tangent to a curve could have been learnt by these learners in Grade 10, as it is a concept learnt in Grade 12 when they learn calculus. These two terms were accepted as correct mathematical vocabulary of functions on the basis that the general instruction of the test did not exclude vocabulary of trigonometric functions.

Summary and conclusion

The results show that learners displayed little knowledge of the mathematical vocabulary associated with the mathematical object that was presented to them. In addition to this little knowledge, the learners also showed that they lacked proficiency in the vocabulary that they wrote. In this research, VK was measured by the quantity of vocabulary that learners managed to write about the object in question. According to the test memorandum, it was possible to come up with 40 terms that could be associated with the object. I was open to accepting more vocabulary than that which was in the memorandum, so it was possible for the learners to come up with more than 40 terms. However, the maximum that the learners managed was only 10 terms, which was achieved by only 2,1% of the learners (4 out of 194).

Results were also poor for VP, which refers to the ability to give correct facts, examples, or characteristics associated with the mathematical vocabulary. For an example, of the 51 learners who wrote the vocabulary turning point, only 49% of them managed to identify the turning point $C\ (0;\ 4)$ on the graph. Proficiency for the other vocabulary was very low. This showed that despite writing a term, learners could not demonstrate their understanding of the term.

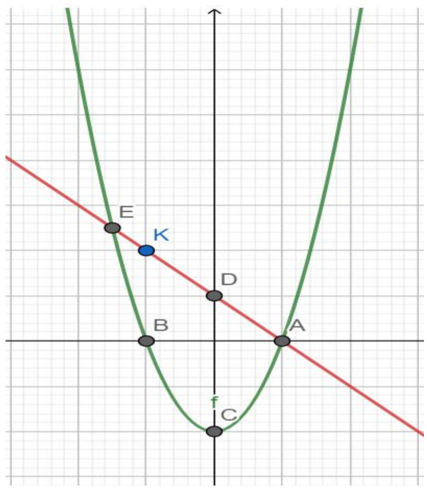
6.1.2 Discussion of the results of Question 2

Question 2 of the research test had three parts (2.1, 2.2 and 2.3) that tested learners' CUPP and one part (2.4) that tested their LP. Parts 2.1, 2.2, and 2.3 also tested mathematical correctness (performance) of learners' answers as examined in formal assessments. In this research, CUPP refers to learners' accuracy in mathematical procedures and explanations for them. This manifests in the ability to produce correct mathematical objects, e.g. writing coordinates of a point, correctly executing systematic calculations, and explaining those steps using precise mathematical language.

In Question 2.1, learners were required to write down the coordinates of Point C, the turning point of $f(x) = x^2 - 4$. Here, learners were tested on their understanding of a vertical shift of the parabola $h(x) = x^2$. $f(x) = x^2 - 4$ is the result of a vertical shift of 4 units downwards on h(x). Given that the coordinates of the turning point of h(x) are $(0;\ 0)$, a vertical shift will not cause any change in the *x*-coordinates but will translate the *y*-coordinate to -4. Hence, the coordinates of the turning point of $f(x) = x^2 - 4$ are $C\ (0; -4)$.

It is evident that these coordinates could be deduced from the equation $f(x) = x^2 - 4$. On inspection, the equation is in the form $f(x) = ax^2 - q$, which is a vertical shift, and the coordinates of the turning point are $TP\ (0;\ q)$. In the case of the function in question, $q = -4$, hence $TP\ (0; -4)$.

In the question, the graph did not have any numbers on either axis:



Hence, the probability of learners employing any method to ascertain the coordinates of point C was diminished, rendering knowledge and a deep understanding of the pertinent concept the primary avenue for such determination. When the learners' answers were assessed, it was imperative to have a correct *x*-value, as this is an indispensable component for a point on the *y*-axis. The correct format/procedure for writing coordinates of a point on the Cartesian plane was also crucial, i.e. $(x;\ y)$, as shown in the following learner's work (Figure 6.1.7).

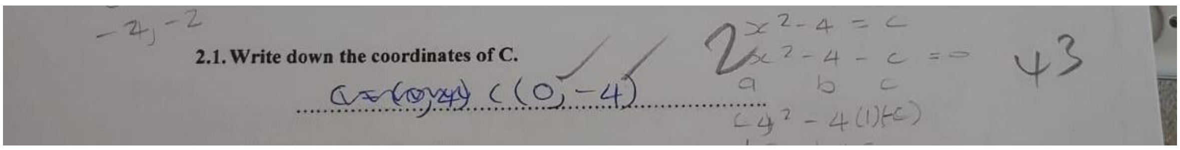


Figure 6.1.7: Correct coordinates of the turning point, C (x; y), in Question 2.1 of the pretest.

Where a learner managed to identify both $x = 0$ and $y = -4$, but wrote the coordinate as $(y;\ x)$, this was regarded as a breakdown and was awarded zero marks (Figure 6.1.8).

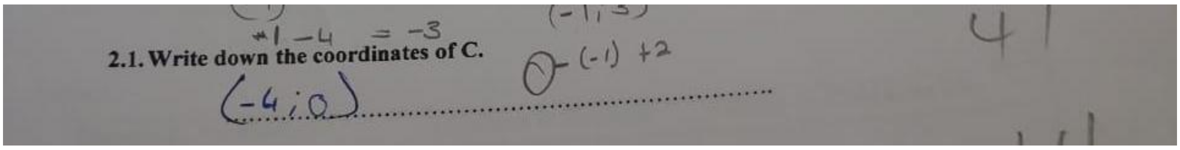


Figure 6.1.8: Breakdown in presenting a coordinate as (y; x) in Question 2.1 of the pretest.

Of the 194 learners who sat for the pretest, 32,2% (76 out of 194) got this question correct, while 29,4% (57 out of 194) had partially correct responses. Responses became partially correct, as in Figure 6.1.9, if only one of the values, either $x = 0$ or $y =- 4$ was correct, and not vice versa with writing the coordinates, i.e. it was imperative to write 0 as the x-value in its correct place.

Figure 6.1.9: Partially correct response to Question 2.1 of the pretest, because the y-value is incorrect.

In Figure 6.1.10, I present the results of the performance of the 194 learners in Sub-Questions 2.1, 2.2, and 2.3. There are four codes to these questions: Unclassified CUPP, Non-Existing CUPP, Emerging CUPP, and Achieved CUPP. The results for Question 2.1 are such that 32,2 % (76 out of 194) and 29,4 % (57 out of 194) of the learners attained an Achieved CUPP and an Emerging CUPP respectively. 14,4% (28 out of 194) and 17% (33 out of 194) attained an Unclassified CUPP and a Non-Existing CUPP respectively in this question. These results, as well as those for Sub-Questions 2.2 and 2.3 are shown in Figure 6.1.10.

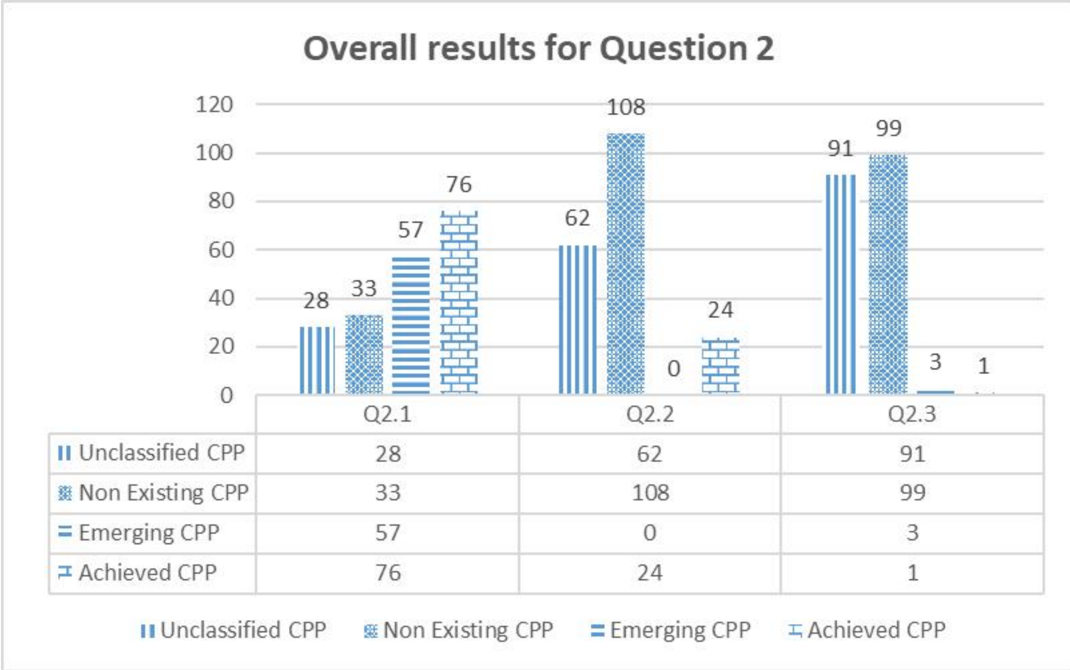


Figure 6.1.10: Overall CUPP results for Question 2 of the pretest.

Sub-Question 2.2 required learners to write down the range of $f(x) = x^2 - 4$. To be able to answer this question, it was crucial for the learners to know the important aspects of the concept of a range as a part of mathematical functions. The learners needed to remember that a range is a set of all possible resulting values of the dependent variable. This knowledge would have assisted them in associating their answer with the variable *y*. Doing so would have led them to focus on the behaviour of the graph of the function with respect to the *y*-axis. That being the case, the learners would have realised that their response to this question should only contain one variable, y, the dependable variable. Responses could have been given in inequality or interval notation as follows: $y \geq -4 \quad OR \quad -4 \leq y < \infty \; OR$.

Regardless of the notation used, it is indispensable to include -4 as part of the values of the range, because the graph turns at $y = -4$. The sign of the inequality was informed by the type of turning point that the graph had. $f(x) = x^2 - 4$ has a minimum turning point at $y = -4$. For this reason, all values of the function lie above $y = -4$. An example of a correct response from a learner is shown in the following extract.

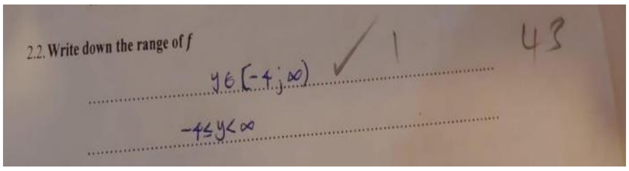


Figure 6.1.11: Example of a correct presentation of the range of f in Question 2.2 of the pretest.

In this response (Figure 6.1.11), the learner used both set notation and inequality notation. The response demonstrates full understanding of the concept of range. In the set notation response, it is unclear as to whether the first bracket includes or excludes -4. However, the inequality notation response clarifies the doubt as it clearly shows that -4 is included in the solution set. Such a response was coded 43, inferring that the learner achieved both conceptual understanding and procedural proficiency. This is despite the learner's use of a semicolon in the set notation response instead of a coma. Use of the semicolon could have been a slip; however, it could also have been a misconception. At this point, this delusion was not penalised because some Grade 10 textbooks, for example *Classroom Mathematics* (p. 153), use this type of notation. In addition to the Code 43 for CUPP, the learner also earned 1 mark for his correct mathematical result (MO).

Learner performance in this question was poor, as 62 out of 194 learners (32%) did not answer the question, and 108 out of 194 (55.7%) gave incorrect responses. Some of the answers were incorrect because the learners used the variable x instead of y, as in the following excerpt.

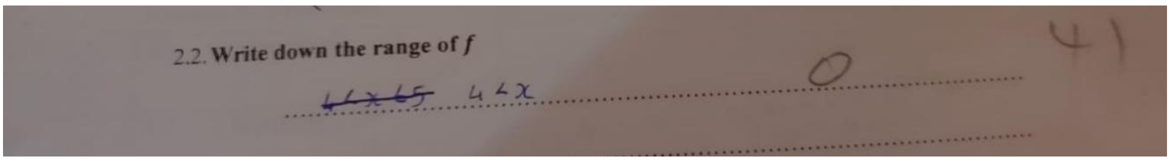


Figure 6.1.12: Example of the incorrect variable used for the domain in Sub-Question 2.2 of the pretest.

In addition to their use of the independent variable instead of the dependent variable, this learner (Figure 6.1.12) also made a mistake in the sign of the value at the turning point. In addition, the inequality is not correct, as it excludes the value at the turning point. As well as this, the learner's first attempt, which he/she scratched out, does not help the situation, because it has the independent variable, and it restricts the values to be between 4 and 5. The learner was coded 41 to indicate that he/she attempted the question, but got a completely incorrect answer.

Another example of an incorrect response is shown in the following learner attempt (Figure 6.1.13).

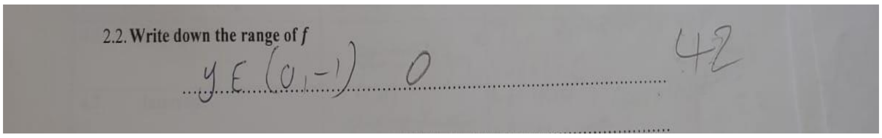


Figure 6.1.13: Example of the correct variable used for range, but the incorrect values used inside the bracket in Sub-Question 2.2 of the pretest.

Though the correct variable, *y*, was used, and set notation was chosen, the values in the set are incorrect. To say that the graph lies between 0 and -1 is incorrect. Furthermore, the learner erred on the algebraic repertoire. The first value in the set should be the smaller of the two values in the set. As for this learner, to have the values placed in that order presupposes that $0 < -1$, which is an incorrect mathematical conclusion.

The third example of an incorrect response is shown in Figure 6.1.14.

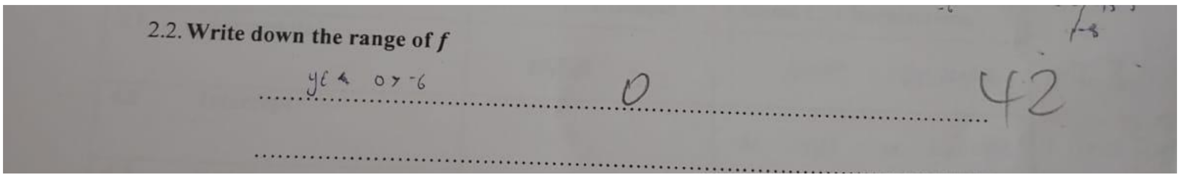


Figure 6.1.14: Example of an incorrect response with both set and inequality notation used in the same response for Sub-Question 2.2 of the pretest.

In this response, the learner used the symbols of set notation, but at the same time also used the symbols of inequality notation. This response is not easy to interpret, as it is a mix up of notations. By using both set notation and inequality notation, the learner showed that he/she did not know what sort of response needed to be given. Of course, the learner identified the correct variable, *y*, for the range, but could not relate it well with the correct values on the graph. The above three incorrect answers were coded 41, 42, and 42 respectively. Code 41 for the first response signifies that the response has nothing correct in it. As for the other two responses, the presence of the dependent variable suggests that there is an Emerging idea about the concept of a range. However, despite knowing that the correct variable for the range was y, nothing beyond that identification was correct in the responses.

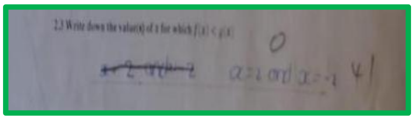
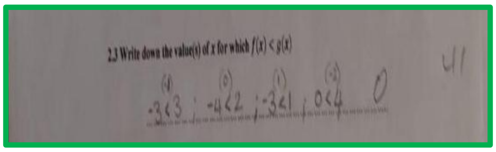
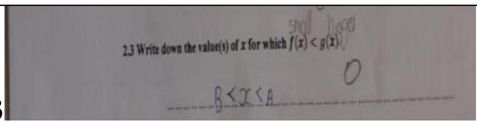
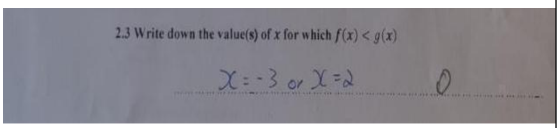
As already alluded to, Question 2.3 assessed both CUPP and performance. Of the three parts of Question 2, 2.3 had the worst responses. Only one of the 194 learners managed to get this question correct, and only three got a partially correct result. In this question, learners were required to write down the value/s of x for which $f(x) < g(x)$. The graph showed the intersection points of *f(x)* and $g(x)$ as $A\,(2;\ 0)$ and $E\,(-3;\ 5)$. Here the learners were expected to focus on the x-values at the intersection points, and then put these values as the extremes of their inequality. In locating the values in the inequality, the smaller of the two values, -3 would need to be placed on the left and the larger, 2, on the right, with variable x in between. This is so, because the required region, $-3 < x < 2$, lies between the intersection points.

91 learners (46,9%) did not answer the question at all, while of those who attempted it, 99 learners (51%) got it completely incorrect. This means that the question was done poorly overall. Only one learner (0,5%) managed to answer it completely correctly, with three learners (1.5%) giving only a partially correct answer.

An answer was deemed to be partially correct if only one of the x-values of the inequality was correct, irrespective of its position in the inequality. As the values were not labelled on the graph, but given in the narration of the question, the mere ability of relating the graph to the narration was a sign of an emerging understanding of the task at hand. That is why the presence of -3 or 2 in the inequality qualifies as Emerging CUPP. If they both appeared but in reversed order, such an answer did not earn full marks but was still regarded as Emerging CUPP.

There was quite a range of incorrect answers that were given by the learners as can be seen in the examples in Table 6.1.

Table 6.1: A set of other incorrect responses to Question 2.3 of the pretest for various reasons.

1. 	$x\ = 2$ is present, but it was not taken from the points of intersection of the graphs. Instead, it was taken from the x intercepts of f. Furthermore, the result given here is not an inequality, but two linear equations
2 	Here, the numerical values were obtained by substituting values of x into the equations of f and g. The shortcoming in this answer is that it has restricted the answers to the set of integers only. The expected response consists of only one inequality, and not the four inequalities given by the learner.
3 	There is an inequality, but it compares the incorrect entities. A and B are x intercepts of f and this inequality does not help answer the question.
4 	Both values, $x\ =\ -3$ and $x\ =\ 2$, are given, but they are in two linear equations and not an inequality. At these values, f and g are equal, so this learner demonstrated a deficiency in their understanding of an inequality. It appears as though this learner failed to differentiate between an equation and an inequality.

The fourth and last part of Question 2 was distinct from the first three parts in the sense that it examined mathematical language proficiency. The literal repertoire of precise mathematical vocabulary requires that learners must be able to read a mathematical object correctly. This is what was expected in this question. Learners were supposed to interpret and describe their responses from Question 2.3. As this question was based on the result in Question 2.3, the principle of consistency accuracy was applied. This means that a learner’s result was marked based on the answer obtained in Question 2.3, provided it was an inequality or a set. Learners were asked to use correct mathematical vocabulary in a sentence to describe the answer (algebraic notation) given in Question 2.3. Figure 6.1.15 shows performance in this question.

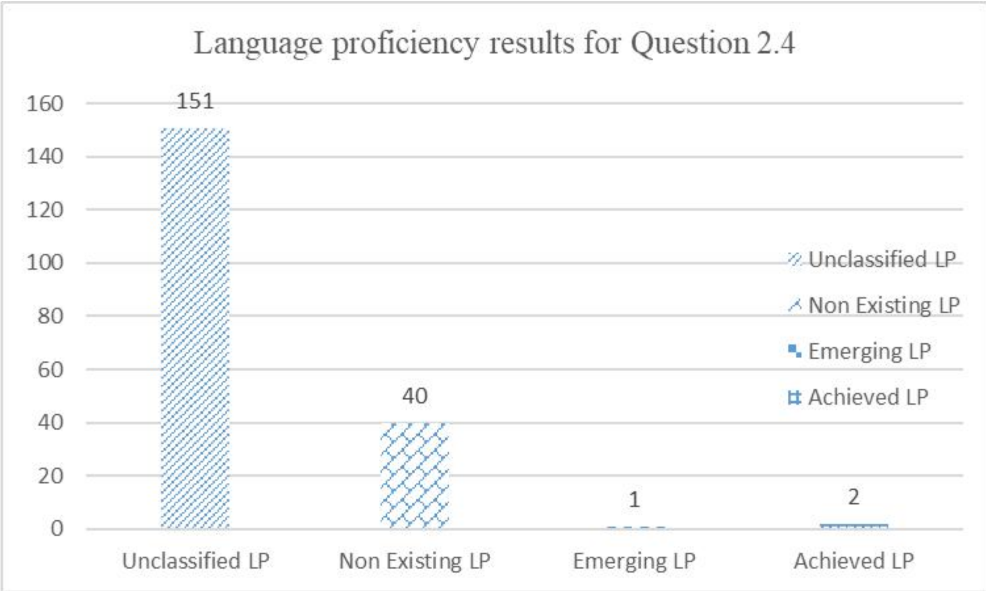


Figure 6.1.15: Overall performance in LP in Sub-Question 2.4 of the pretest.

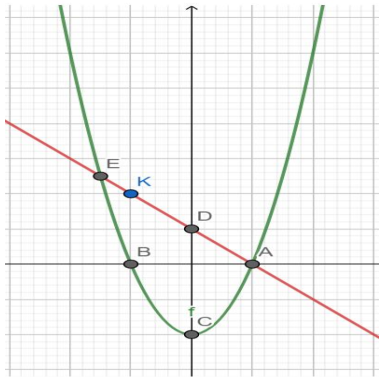
77,8% (151 out of 194) of the learners did not answer the question at all, while only 22,2% (43 out of 194) attempted it. Of those who attempted it, only one learner showed Emerging LP and two learners answered it correctly, thus receiving Achieved LP. The other 40 learners answered it incorrectly. The expected answer for this question was: "All real x-values that are greater than negative 3 and at the same time less than 2."

Summary and Conclusion

Question 2 assessed three aspects: Conceptual Understanding and Procedural Proficiency (CUPP), mathematical performance, and Language Proficiency (LP). A summary of the learners' overall performance in the four sub-questions of the question is presented in Table 6.2. The top part of the table contains the question that the learners had to answer. The sub-questions to Question 2 are in the left section of the table. In the middle of the table are the four codes to the question and the response incidences per code. In the far right of the section of the table are the frequencies of marks obtained by learners in Sub-Questions 2.1, 2.2, and 2.3, out of 2, 1, and 2 marks respectively.

Table 6.2: Summary of performance for the four parts of Question 2 of the pretest.

The graphs of $f(x) = x^2 - 4$ and $g(x) =- x + 2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK || $y - axis$. f and g intersect at $A\ (2;\ 0)$ and $E\ (-3;\ 5)$.



	Unclassified CUPP	Non-Existing CUPP	Emerging CUPP	Achieved CUPP	0	1	2
Q2.1. Write down the coordinates of C.	28	33	57	76	58	60	76
Q2.2. Write down the range of f	62	108	0	24	170	24	
Q2.3. Write down the value/s of x *for which* $f(x) < g(x)$.	91	99	3	1	190	3	1
	Unclassified LP	Non-Existing LP	Emerging LP	Achieved LP			
Q2.4. Use correct mathematical vocabulary in a sentence to describe the answer (algebraic notation) given in question 2.3.	151	40	1	2			
Total Responses per Code	332	280	61	103			

Generally, this question was poorly answered. It is of concern that 170 (87,6%) learners received zero marks for giving the range of the function. This is despite the fact that the concept of a range of a function is one the fundamental aspects that they should know about a function. The 170 learners comprised 62 who did not answer Sub-Question 2.2 at all and the 108 who answered it incorrectly. Only 24 learners got this question correct. These 24 learners are only 12,3% of the learners who sat for the pretest.

Sub-Question 2.3 related to the two functions that were drawn on the same Cartesian plane. As shown in the table, out of a possible 2 marks for this question, 190 learners received 0 marks. These 190 learners comprise 91 learners who did not answer the question and 99 learners who answered it incorrectly. Only one learner managed to answer this question correctly, while three learners had partially correct responses.

Given that 194 learners sat for the pretest, the total number of expected responses was 776 (194×4), because the question has four sub-questions. As testimony to how poorly this question was answered, 332 responses were blank, which constitutes 42,8% of the total responses. 36,1% (280 out of 776) responses were incorrect, 7,9% (61 out of 776) were partially correct and 13,3% (103 out of 776) were totally correct. The results are shown in Figure 6.1.16.

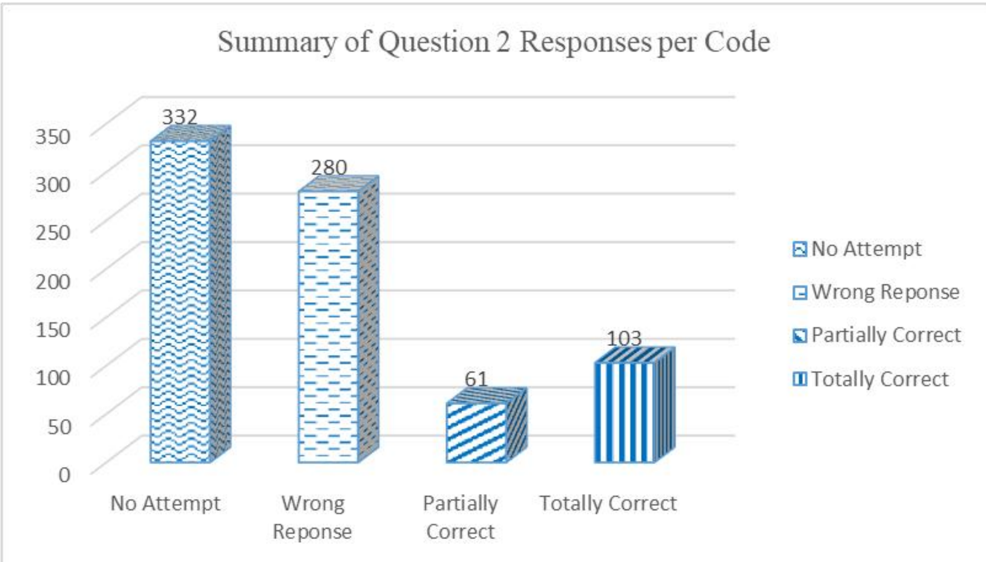


Figure 6.1.16: Code-by-code summary of performance in Question 2 of the pretest.

As shown in Figure 6.1.16, there were 103 totally correct responses. These responses comprise 76 learners who got all 2 marks in Sub-Question 2.1, 24 learners who got the full 1 mark in Sub-Question 2.2, one learner who got the full 2 marks in Sub-Question 2.3 and two learners who got Sub-Question 2.4 correct. The bulk of the totally correct responses, 73,8% (76 out the 103 totally correct responses) came from Sub-Question 2.1, where learners were required to write down the coordinates of a turning point of a parabola. Though it is still too early to conclude, there seems to be a relation between the incidence of the vocabulary turning point in Question 1 and the percentage of totally correct responses in Question 2.1.

The strength of this purported relationship – that the more a term is remembered and conceptualised, the better it is dealt with in assessments – must be tested further in this research.

6.1.3 Discussion of the results of Questions 3 and 4

Questions 3 and 4 had a lot of commonality. This is despite the fact that they assessed different aspects of this research. Question 3 assessed learners' LP, while Question 4 assessed their VP. For the sake of this research, LP refers to articulation of mathematical vocabulary in describing mathematical concepts. In this case, the articulation was done by means of defining some mathematical vocabulary: asymptote, intercept, range and domain. On the other hand, VP refers to giving correct facts/examples/characteristics associated with mathematical vocabulary. The vocabulary that was given to the learners in Question 4 was the same as in Question 3 except for "exponential equation".

The choice and the order of these terms in both Questions 3 and 4 was very deliberate. An asymptote and an intercept have contrasting attributes. An asymptote avoids intersection whereas an intercept is a special point of intersection. Having them follow on from one another in both questions was intended to test if learners would be able to view them in this regard. However, the difference between these two questions was that Question 3 was an open-ended question, where the learners were required to give their own definitions. Their definitions would then be compared to existing concept definitions and coded accordingly.

With Question 4, the learners were given a three-column table in which they were required to match a vocabulary word in Column A to its example in Column B, and to its attribute in Column C. These two questions presented a situation where, in one scenario, learners were expected to think and formulate a definition by retrieval from memory and, in the other scenario, they had to make sense of the facts presented to them in the columns. It had to be seen if there was any association between their responses in the two questions.

In the subsequent figures and analysis, this association is going to be examined. The set of bar graphs examine learners' VP in Question 4, given that they did not give a definition of that term in Question 3 (LP). In the case of the first bar, nine learners who did not define the term asymptote in Question 3 did not answer the question on asymptote in Question 4.

The ensuing figures present the results for corresponding parts of Questions 3 and 4.

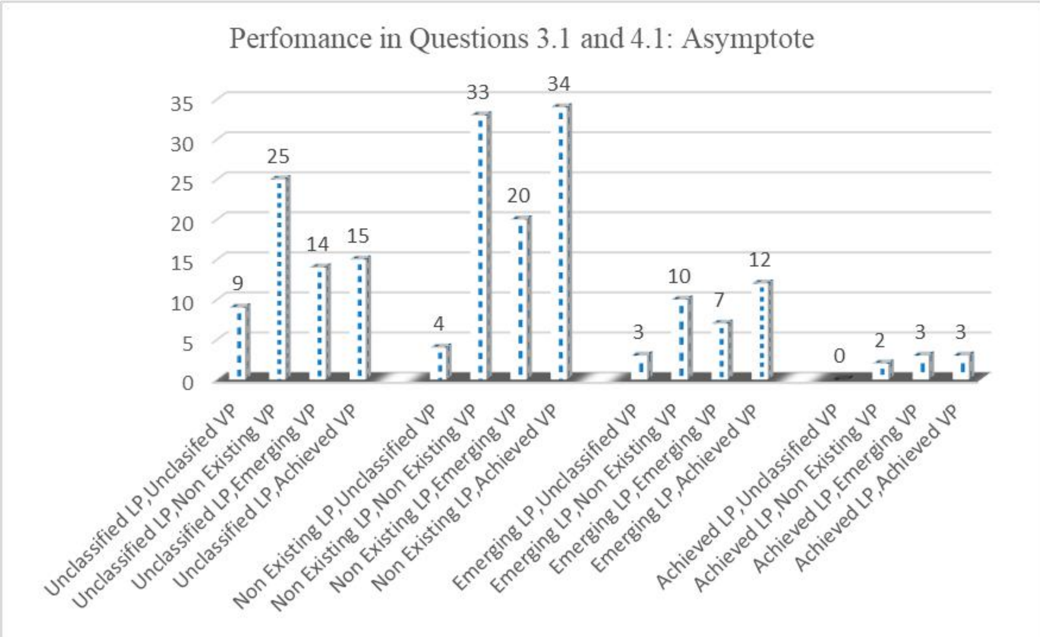


Figure 6.1.17: Relating VP and LP for the term asymptote in Questions 3.1 & 4.1 of the pretest.

Figure 6.1.17 shows the relatedness of LP and VP with respect to the term asymptote. In Question 3.1, learners were asked to define the term asymptote; the same term which they were asked to work with in Question 4.1. As the learners dealt with the same term in two different scenarios, it was necessary to compare and try to establish the link between learners' performances in those cases.

If any Unclassified or Non-Existing result is regarded as poor, and any Emerging or Achieved result is regarded as good, the results in Figure 6.1.17 show that of the learners who received an Unclassified LP in Question 3.1, more received a bad VP than a good VP in Question 4.1. However, when learners attempt to define the term asymptote, irrespective of their responses, attempting to define it is more associated with good VP.

The next term that the learners dealt with was the term intercept. The following figure presents the learners' results in the same format as those for the term asymptote.

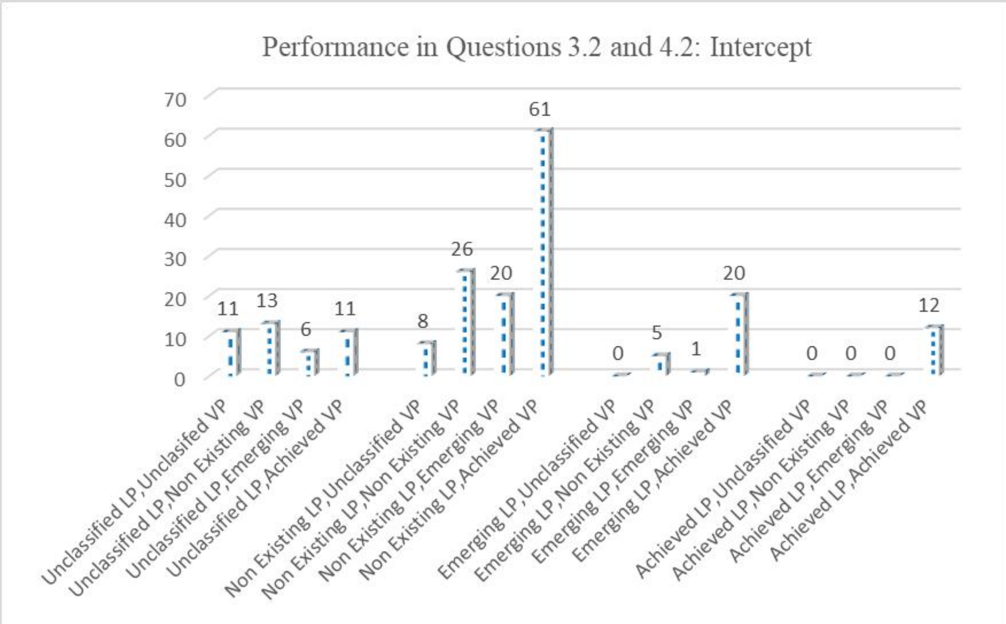


Figure 6.1.18: Relating VP and LP for the term intercept in Questions 3.2 & 4.2 of the pretest.

It is evident from Figure 6.1.18 that not attempting to define the term intercept is associated with poor VP. On the contrary, attempting to define the term resulted in a good VP.

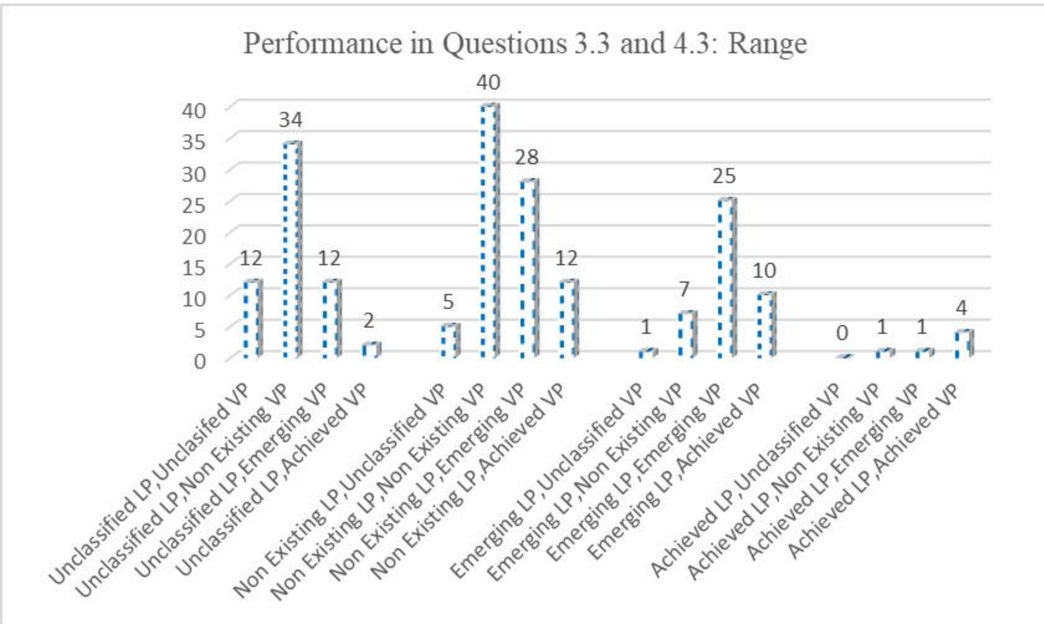


Figure 6.1.19: Relating VP and LP for the term range in Questions 3.3 & 4.3 of the pretest.

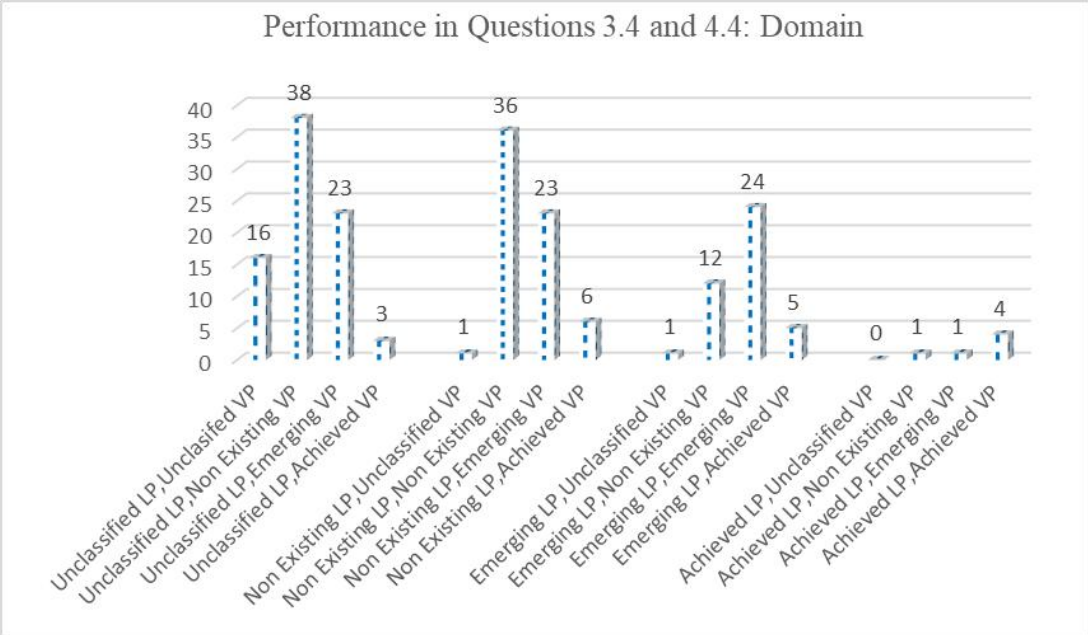


Figure 6.1.20: Relating VP and LP for the term domain in Questions 3.4 & 4.4 of the pretest.

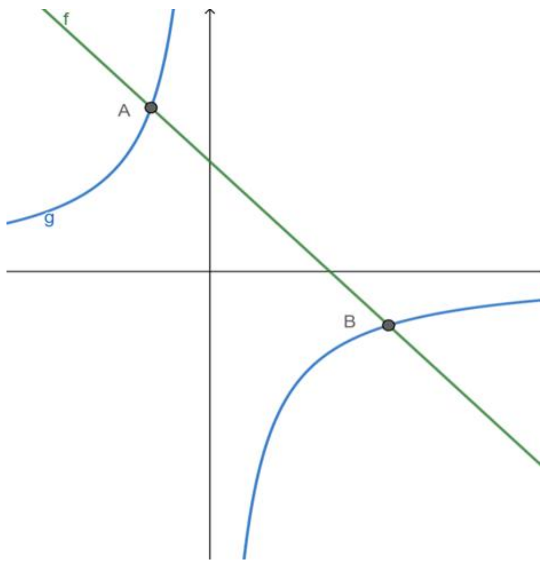
Figure 6.1.17 shows that only three learners (extreme right bar) who managed to give a correct definition for the term asymptote also managed to come up with the correct matching in Question 4. Such poor performance is also evident with the other terms: intercept, range, and domain (shown in Figures 6.1.18, 6.1.19, and 6.1.20 respectively), where 12, four and four learners respectively managed to both give a correct definition and get a correct matching. These low numbers demonstrate that the learners had a challenge in dealing with this mathematical vocabulary. Preliminarily. It can be seen that in Question 2.2: "Write down the range of f", only 12,4% (24 learners) got this question correct. This seems to suggest a link between VP, LP, and mathematical performance. In this case, low VP seems to have negatively impacted upon mathematical performance.

One would wonder where the other numbers come from, as I have indicated that only four learners managed both LP and VP, yet when it came to getting the range of f correctly, there were 24 learners. Of these learners, 10 are those who received Achieved VP, but with Emerging LP, and the other 10 are from the 12 who received Achieved VP but with Non-Existing VP. In all these instances, these learners had Achieved VP. Where these learners faltered is in giving correct definitions by themselves. Possibly, giving the definition in a second language could have been a factor here. The fact that they managed to Achieve VP by correctly matching might be an indication that they knew the concept but lacked the articulation capabilities to define it in Question 3.3. That being the case, VP seems to have come out on top in influencing performance in this regard.

The other term that was asked In Questions 3.4 and 4.4, which was indirectly asked in Question 2.3, is "domain". In Question 2.3, learners were asked to "Write down the value/s of x for which $f\ x) < g(x)$". Here, learners were expected to determine the independent values (domain) that satisfy the given condition. As shown in the previous figure (6.1.20), only four learners managed to achieve both Language and Vocabulary Proficiency. Coincidentally, these are also the only learners who got part or full marks in Question 2.3. This further affirms the preliminary notion that VP tends to impact on performance.

6.1.4 Discussion of the results of Question 5

The graphs of *f* and *g* are sketched below. <u>A and B are the points of intersection of the graphs.</u>



Calculate the coordinates of A and B. Show all steps and <u>next to each step give a reason</u> for doing it (The reasons you give should show that you understand why each step and procedure is done).

This was a poorly answered question where 137 (70,6%) of the learners did not answer the question, 44 (22,6%) got it completely incorrect, despite attempting it, 12 (6,2%) showed signs of Emerging CUPP, and a paltry 1 (0,52%) demonstrated Achieved CUPP. The question consisted of a Cartesian plane on which were a hyperbola and a straight line that intersect at two points. Learners were required to calculate the coordinates of the points of intersection. In addition to the step-by-step calculations, there was an instruction to the learners to explain the rationale of each undertaken step. The required steps in solving the problem enabled allocating marks to the learners. There were 10 identifiable steps that needed to be followed, hence the question was marked out of 10. Explaining the rationale for the steps was necessary as a way of proving conceptual understanding.

That being the case, two aspects were assessed in this question. The first one was performance, which was measured using the marks obtained by the learners. The second aspect was CUPP, which was measured using the learners' ability to justify the rationale of each step in the calculation.

Figure 6.1.21 presents result for CUPP in the question.

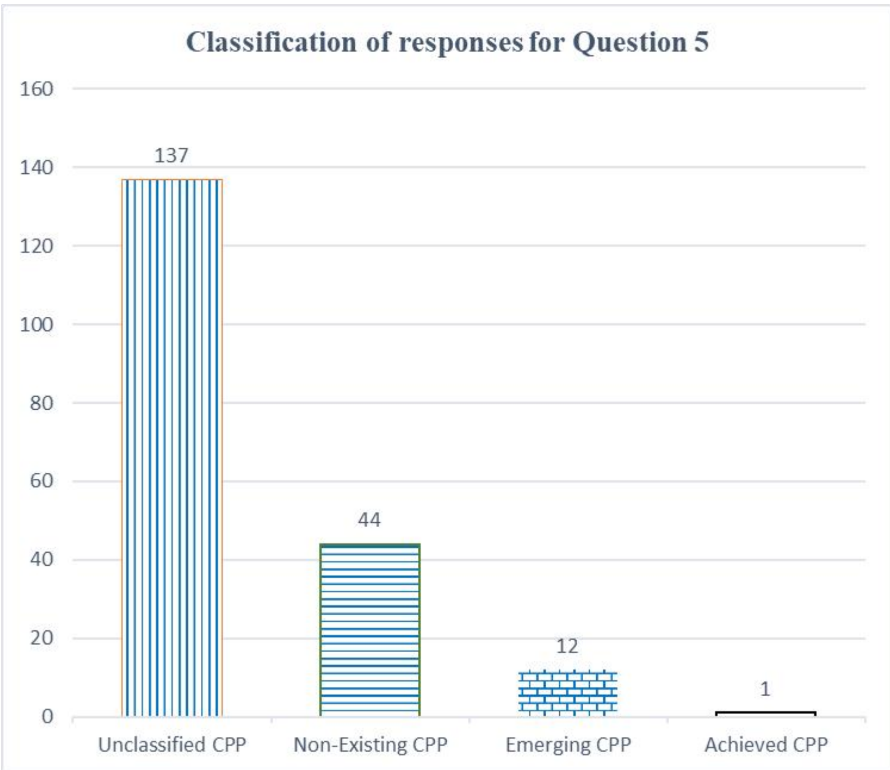


Figure 6.1.21: Overall CUPP results for Question 5 of the pretest.

Figure 6.1.22 presents the performance of the learners in this question. In this question, performance was measured by the number of marks that a learner got. Figure 6.1.22 shows that only 17 (8,7%) of the learners managed to get at least 1 mark in this question. The possible marks for this question were 10. This is a concerning performance, as 137 learners out of the 194 who sat for the pretest did not respond to this question. Additionally, 40 learners attempted to answer the question but received no marks. In total, 177 learners (137+40) received no marks, as depicted in the following figure.

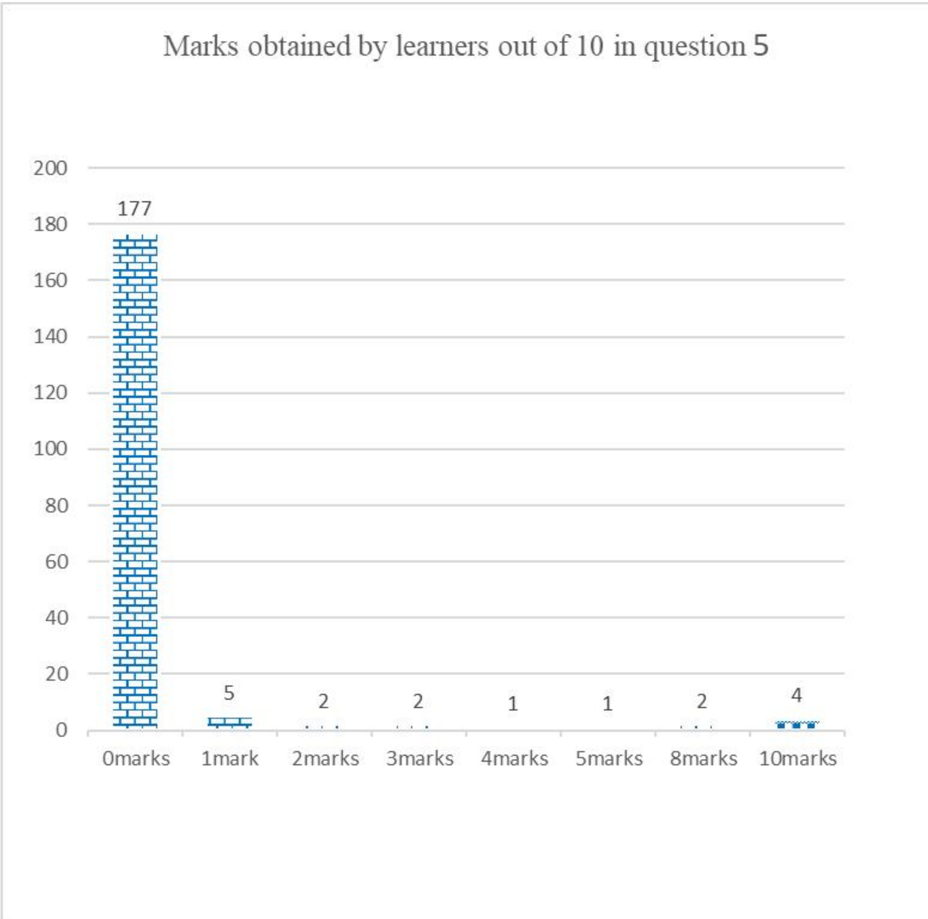


Figure 6.1.22: Mark distribution for Question 5 of the pretest.

The bulk of the learners did not attempt this question (137 out of 194). Without making too much speculation on this, it is highly probable that this is because it was the last question in the test. It being the last question, the possibility was that the time was up before they could attempt it. A follow up interview revealed that the learners ran out of time to answer the question. Furthermore, the learners knew what the question expected of them, but they did not possess the answers to the question. They claimed that they could not remember the solution strategy.

6.1.5 Comparing results of the control group to the results of the experimental group

Though these parameters (VK, VP, and LP) are of a qualitative nature, it is, however, still possible to determine if the difference between them was of any statistical significance. The following figures make a question-by-question comparison of performance between the two groups.

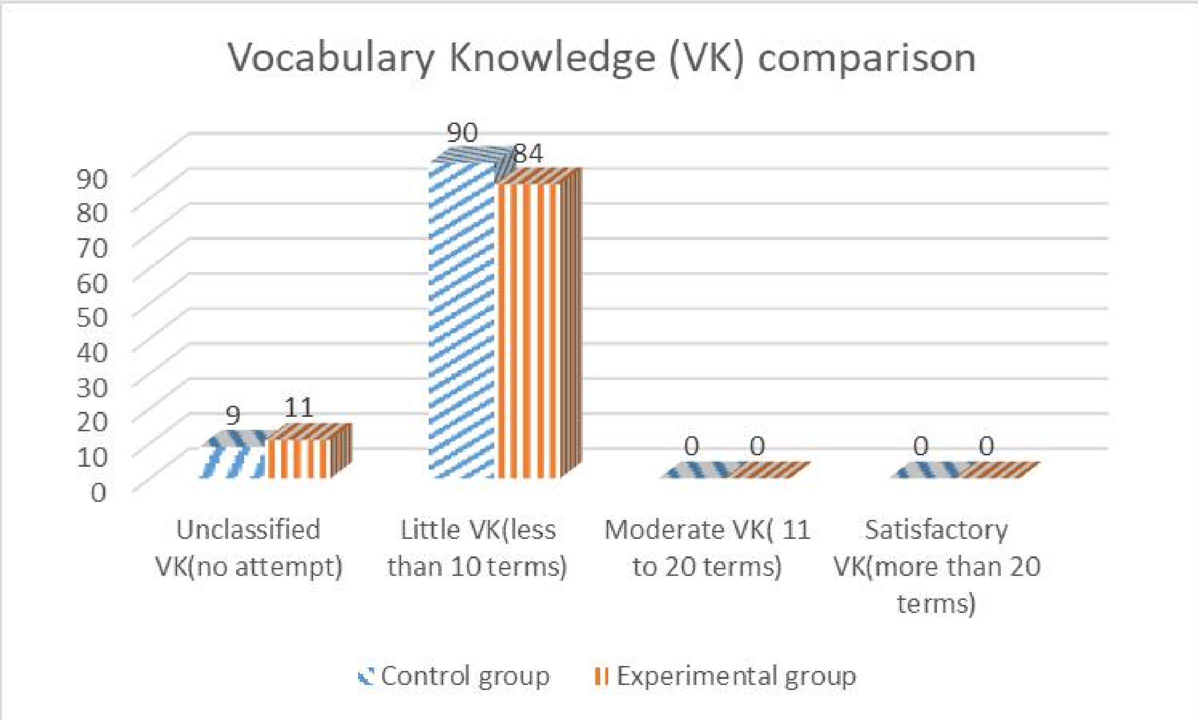


Figure 6.1.23: Comparison of overall VK between the control and experimental groups in the pretest.

The statistics show that of the 20 learners who did not answer Question 1 of the research test (Figure 6.1.23), nine were from the control group and 11 were from the experimental group. Of those who answered it and only managed to come up with up to 10 correct terms, 51,7% (90 out of 174) were from the control group and 48,3 % (84) were from the experimental group. As for the actual numbers of vocabulary achieved, the results were not different either (Figure 6.1.24).

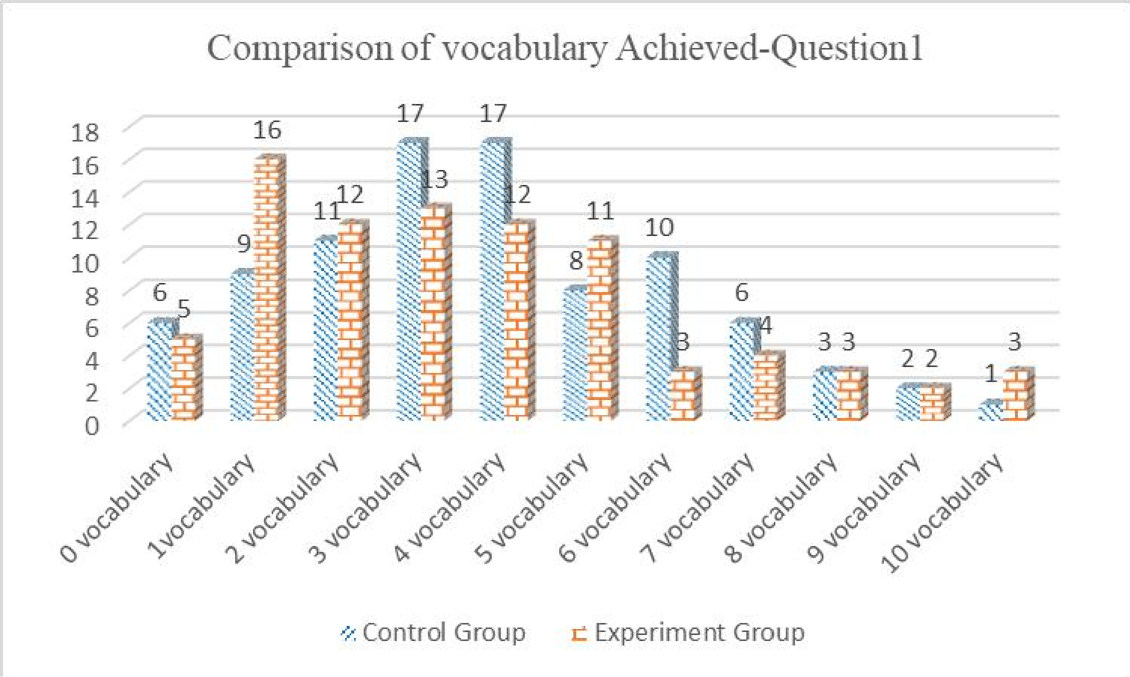


Figure 6.1.24: Comparison of number of vocabulary achieved by learners from the control group and experimental group in the pretest.

The average number of vocabulary achieved by the control group was 3,4747475 (*344 ÷ 99)* compared to that of the experimental group, which was 3.157894737 (*300 ÷ 95)*. This difference is not large enough to warrant suspicion that the two groups are heterogeneous. However, I did a statistical analysis of the means of the vocabulary attained by the two groups during the pretest. The purpose of the test was to ascertain if the difference between the means of these two groups was statistically significant.

To test the difference between the means of the pretest of control and experimental groups, I carried out a two-independent-sample t-test. The two-independent-sample t-test is a method used to test whether the unknown population means of two groups are equal or not. In this test, the null hypothesis (H_0) was that there was no difference between the two means and the alternative hypothesis (H_1) was that there was a difference between the two means. This test was carried out at a 5% significance level, so, H_0 would be rejected if the p-value was less than 0,05. The results are shown in Table 6.3.

Table 6.3: Differences in the vocabulary in the pretest, between the control group and the experimental group.

Group	Sample Size	Mean ± Standard Deviation	t-statistic	P-value
Control	99	3,50 ± 2,444	1,9724	0,2345
Experimental	95	3,16 ± 2,641		

Table 6.3 shows the differences between the vocabulary in the control and experimental groups in the pretest. The slight difference of 0,34 in the pretest, between the control and experimental groups, with the standard deviation of 2,444 and 2,641, respectively, indicates that the result of the pretest on vocabulary, on average, is similar between the two groups. The p-value of 0,2345 shows that the difference of 0,34 is not statistically significant at the 5% level. This means that this slightly observed difference of 0,34 is due to chance alone. Consequently, I failed to reject the null hypothesis (H_0), that there is no difference between the two means.

It was also imperative to compare the incidences of the most prevalent vocabulary. The following figure compares these incidents.

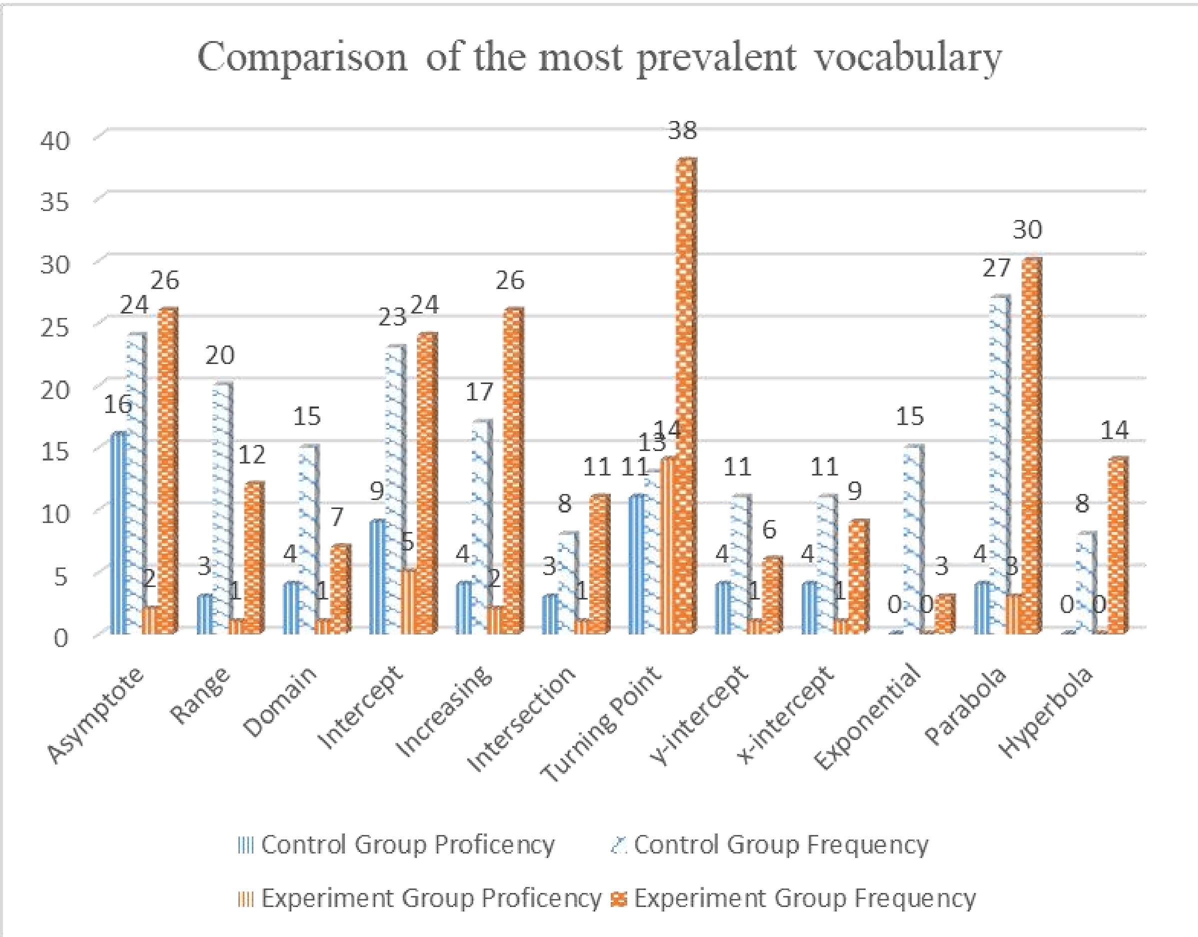


Figure 6.1.25: Comparison of control group and experimental group with regards to the most prevalent vocabulary in the pretest.

For most of the 12 terms that I deemed to be the most prevalent, the word-to-word comparison shows that there was no noticeable difference in the frequency of appearance of these terms between the two groups (Figure 6.1.25). The only striking difference was with the term turning point, which was more common in the experimental group than the control group. However, despite this difference in frequency, there was no significant difference in the proficiency of this term between the two groups. Consequently, it could not be established with certainty that there was any significant difference between these two groups in this respect.

Furthermore, I compared how the two groups performed in Questions 3 and 4. Question 3 assessed learners' Language Proficiency (LP). This was done by asking learners to define the terms: asymptote, intercept, range and domain. The same terms were brought back in Question 4, but the task was slightly different. In Question 4, the learners were assessed on their VP. VP entails the ability to exemplify and characterise the term.

In the ensuing comparison, only Emerging and Achieved LP and VP were compared. As illustrated in the subsequent figure, I attempted to establish the effect of having an Emerging LP about a concept on VP about that same concept. I recognised that there was not much difference in the numbers between the control and the experimental groups in terms of asymptote and intercept. However, with the terms range and domain, 20 learners had both Emerging LP and VP in the control group. On the other hand, there were only five and four learners in the experimental group with Emerging LP and VP for range and domain respectively. This shows that there were more learners (20,2%) in the control group who had some idea about range than there were in the experimental group (4,2%). Also 20,2% of learners in the control group had Emerging ideas about the term domain compared to 3,2% in the experimental group.

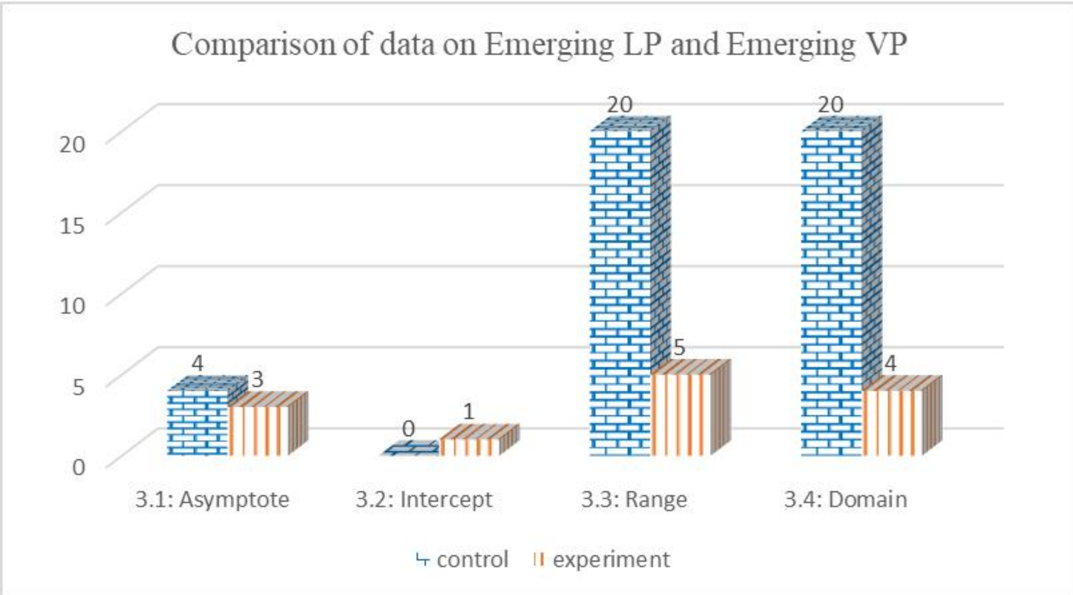


Figure 6.1.26: Comparison of the control and experimental groups with regards to Emerging LP and VP in the pretest.

The preceding differences (Figure 6.1.26) seemed to be great, so I had to refine the criterion of comparison. Next, I compared numbers using Achieved LP and Achieved VP. These are learners who got their definitions, as well as their matching, correct. The difference here was insignificant, as illustrated in Figure 6.1.27. This reveals that learners who had managed both LP and VP in the two groups were equally few for all four terms.

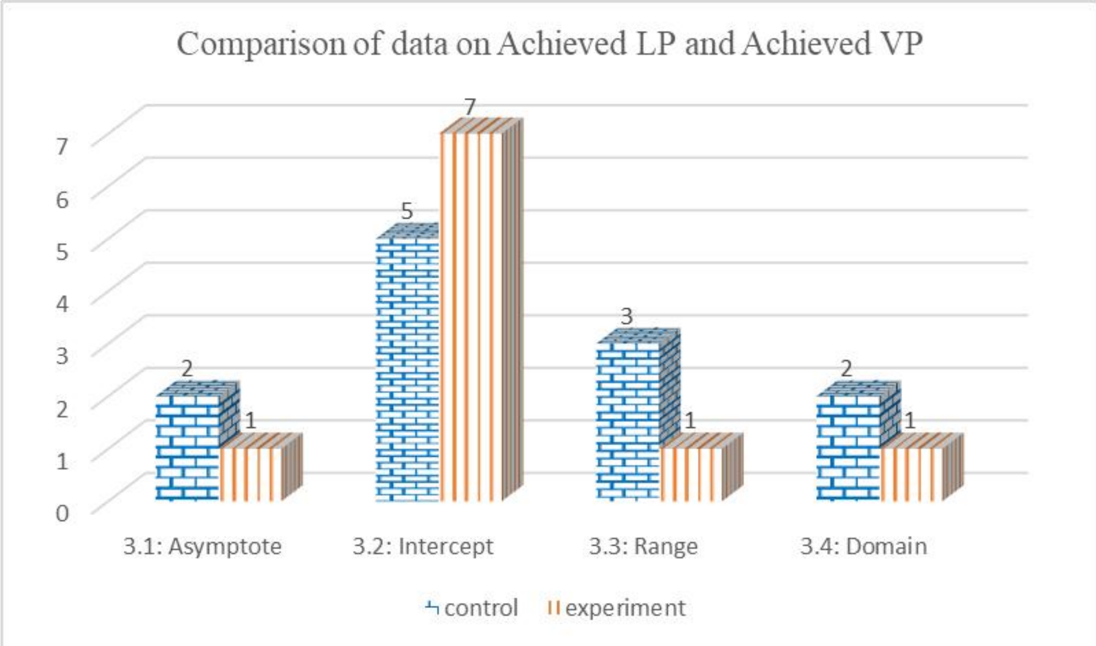


Figure 6.1.27: Comparison of control and experimental groups concerning Achieved LP and Achieved VP in the pretest.

The foregoing comparison was based on VK, LP, and VP. Next, I will compare these two groups based on the actual marks obtained by learners in questions where marks were allocated. Marks were allocated to learners in Questions 2.1, 2.2, 2.3, and 5. A total of 15 possible marks were available to the learners. The marks were very low for the two groups with corresponding means of 1.762677 and 1.522842 for the control and experimental groups respectively. The resulting scatter plots show the mark distributions in the two groups.

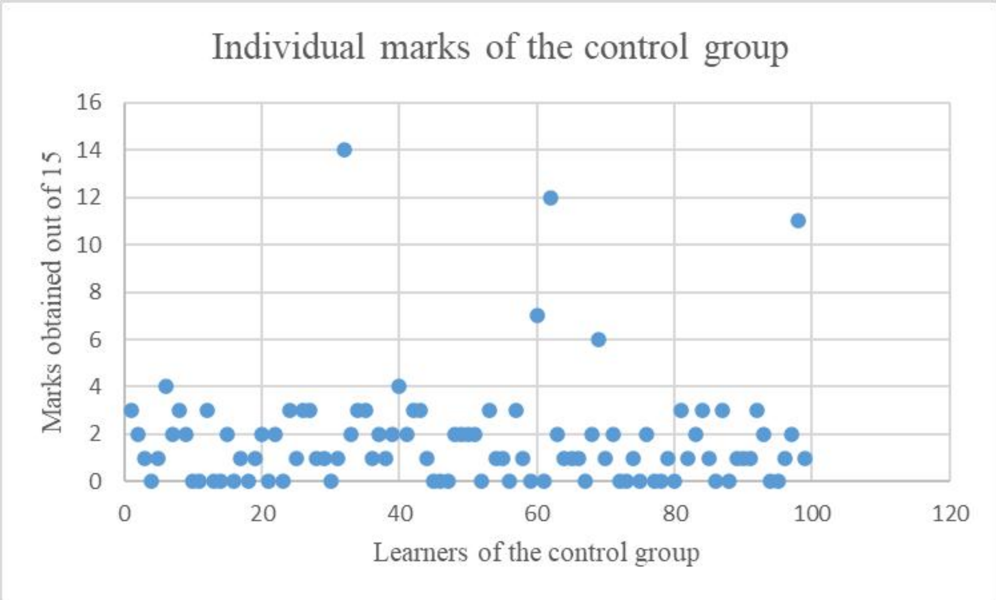


Figure 6.1.28: Marks obtained by learners in the control group in the pretest.

The results depicted in the preceding scatter plot (Figure 6.1.28) show that 28 of the 99 learners from the control group received 0 marks out of the possible 15 marks. Only 2 learners managed to get more than 10 marks. In addition, if I were to use the National Senior Certificate (NSC) benchmark of regarding 30% as a pass mark; only 5,05% (5 out of 99) of the learners passed this assessment. 92 learners got 3 marks and below out of the possible 15 marks.

In the following scatter plot (Figure 6.1.29), I present the results of the experimental group.

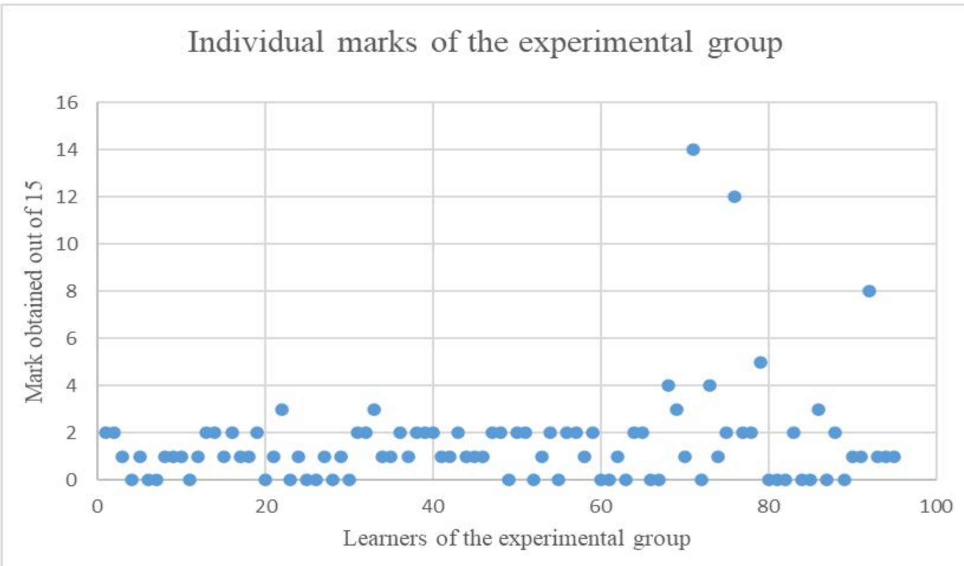


Figure 6.1.29: Marks obtained by learners in the experimental group in the pretest.

The performance of the experimental group also shows that a huge number of the learners (27 out of 99) got 0 marks out of the possible 15 marks. Most of the learners received 2 marks and below. Using the NSC yardstick, only four out of the 95 learners from the experimental group passed the test. This translates to a 4,21% pass rate.

The two scatter plots show that the mark distributions of the two groups have no significant difference. This means that these two groups are analogous in terms of their performance. No group is better than the other. Generally, the overall performance of the two groups in the performance questions (2.1, 2.2, 2.3, and 5) was equally poor for both groups.

In addition to the comparison of the marks using the scatter plots, I also did a statistical analysis to test if the means of the two groups were statistically different. To test the difference between the pretest means of the control and experimental groups, I carried out a two-independent-sample t-test. In this test, the null hypothesis (H_0) was that there was no difference between the two means and the alternative hypothesis (H_1) was that there was a difference between the two means. This test was carried out at a 5% significance level, so H0 was rejected if the p-value was less than 0,05. The results are presented in Table 6.4

Table 6.4: Comparison of the means of the control and experimental groups in the pretest.

Group	**Sample Size**	**Mean ± Standard Deviation**	**t-statistic**	**P-value**
Control	99	1,76 ± 2,317	0,7592	0,448623
Experimental	95	1,52 ± 2,108		

The means of the pretest for the control experimental groups were 1,76 and 1,52 respectively. It can be seen that there is a slight difference of 0,24. The standard deviations of 2,317 and 2,108 for the control and experimental groups respectively, indicate that the scores of the pretest within each group were, on average, similar. That is, in the control group, scores differ from their mean by plus or minus 2,317, while, in the experimental group, they differ from their mean by plus or minus 2,108. The p-value of 0,4486 shows that there is no significant difference, at 5% significance level, between the control and experimental groups in the scores of the pretest. This means that the null hypothesis, that there is no difference between the two means, is accepted and the alternative hypothesis, that there is a difference between the two means, is rejected.

6.2 Information phase results

This represents the research phase considered as the initial stage of the intervention. The intervention encompassed the exposure of research participants to the Frayer model. The exposure consisted of training participants in the utilization of the Frayer model, followed by their application of the model during mathematics lessons. Throughout the information phase, learners within the experimental group had the opportunity to collaborate with the researcher, who introduced them to the Frayer model. Additionally, orientation was provided to four in-service teachers, with three of these teachers subsequently working with learners from the experimental group. The duration of the information phase was 3 hours for learners and 30 minutes for the teachers. The specifics of this phase's proceedings are presented in the following sections.

6.2.1 Teaching how to use the Frayer model.

In this research phase, training was conducted for four in-service teachers on the utilization of the Frayer model. These educators were selected from two different schools, namely, School C and School D. Within this group of four teachers, one held the position of head of department. The head of the department expressed interest in participating in the training, although she did not extend her involvement beyond the training phase. Among these teachers, two were affiliated with School C, while one was associated with School D. The learners who received the training were drawn from both School C and School D.

Training learners from the experimental group on how to use the Frayer model.

The cohort of learners that were trained on how to use the Frayer model comprise the experimental group of this research. These learners were drawn from two schools. In total, 74 learners took part in the training.

6.2.1.1(a) Intervention in School C

- **Report on the first day of the training.**

According to the arrangement, all 53 learners attended the training session after school on a Tuesday afternoon. Due to the large group size and the necessity to adhere to social distancing regulations, the school hall was utilized instead of a regular classroom. This arrangement didn't cause any anxiety among the learners as the hall was regularly used for language lessons due to the large class sizes.

The first task of the day, following orientation, involved working with the vocabulary in the oval of the Frayer model. Learners were instructed to individually complete all four sections of the Frayer model for the term "equation." Excerpts from the work of two learners are presented in Figure 6.2.1(a) and 6.2.1(b).

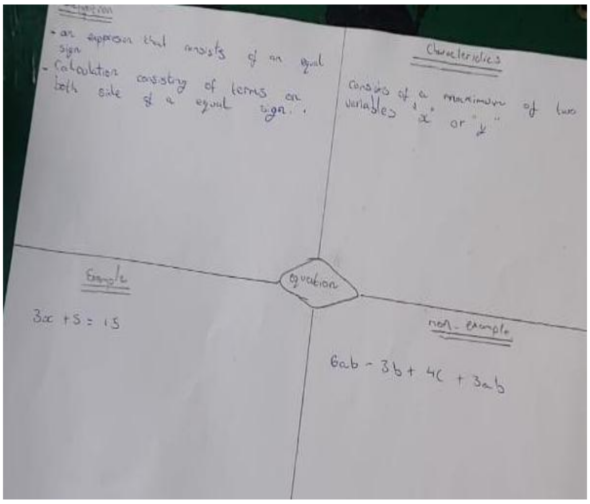


Figure 6.2.1(a): Frayer model for the vocabulary "equation" by Learner C1.

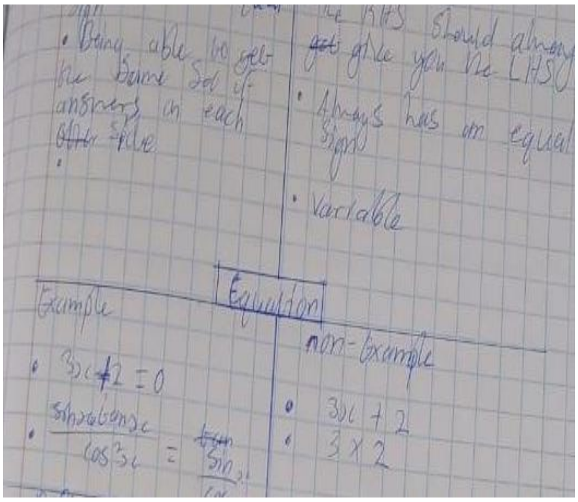


Figure 6.2.1(b): Frayer model for the vocabulary "equation" by Learner C2.

From Learner C1's first definition, it is evident that the learner considers an equation to be an expression. Confusing an equation and an expression is a common practice that most learners do. This is an error, because these two concepts are not the same. Furthermore, the learner gave a characteristic which illustrates more deficiencies. Despite the said weakness, the learner managed to come up with a correct example and a correct non-example.

Learner C2's definition brings to the forefront one of the most common language flaws made by learners in their mathematical talk, regarding an equation as a sum. It was also considered by them to be a process of getting the same answers on both sides of the equation. By stating the phrase "both sides", it might indicate that at the back of the learner's mind, there was the presence of an equal sign. However, what matters is what the learner presented on paper.

We conducted a class discussion as a way of coming up with a common agreed answer for the vocabulary "equation". It was an open discussion, whereas many learners as possible were encouraged to put forward their thoughts. The 10-minute class discussion culminated in the response as presented on the white board by one of the girls in the class.

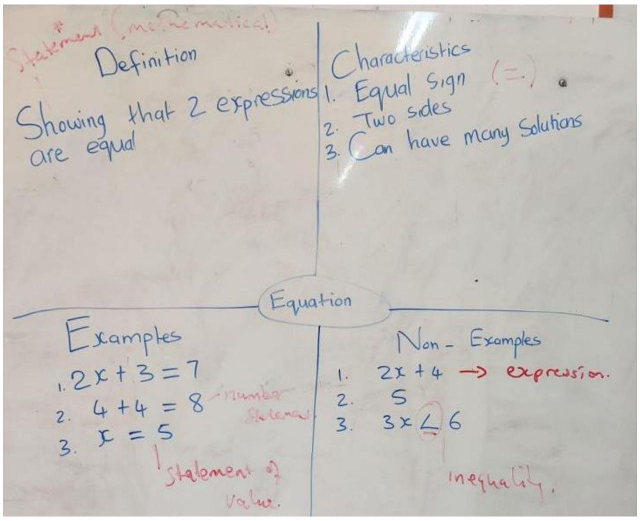


Figure 6.2.2: Frayer model for the vocabulary "equation" by the whole class in School C.

After the learners had assisted each other in creating the work, as shown in Figure 6.2.2, the importance of specifying that an equation is a mathematical statement was emphasized. Additional clarifications (in red) were inserted into the work.

In the final 45 minutes of the session, blank templates of the Frayer model were distributed to each learner, who was then asked to select vocabulary they wanted to work with. Numerous vocabulary terms were discussed, including but not limited to algebraic equation, number patterns, quadratic formula, Pythagoras theorem, multiples, factors, sine, cosine, tangent, and more.

At this point, two learners' Frayer model templates are presented.

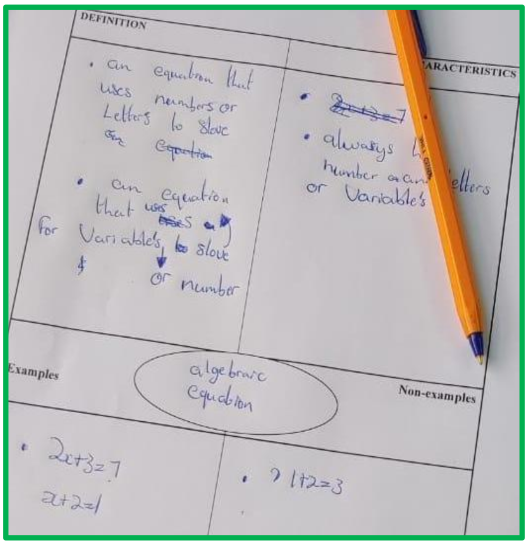


Figure 6.2.3(a): Frayer model for the vocabulary "algebraic equation" by Learner C3.

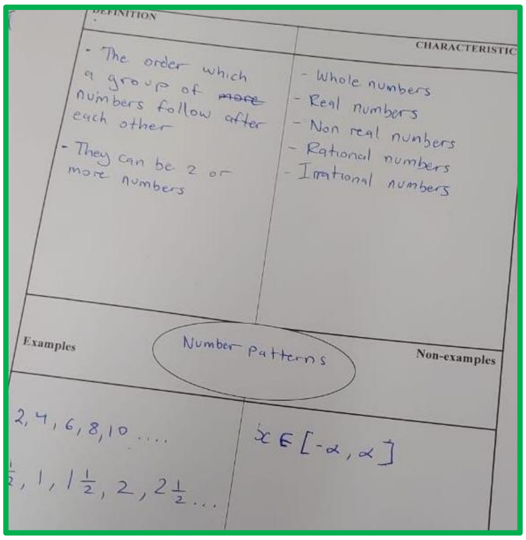


Figure 6.2.3(b): Frayer model for the vocabulary "number patterns" by Learner C4.

Figures 6.2.3(a) and 6.2.3(b) show that these two learners worked with two different concepts. Learner C3 chose to work with an algebraic equation. By adding an adjective, "algebraic", to the vocabulary that we had dealt with as a class, the learner thought that it was a new concept. If the learner had known that the concept remained that of an equation, two possibilities would have been eminent. To start with, the learner would not have written the vocabulary in the oval of the model, as it was already discussed. Alternatively, the learner would have simply copied everything from the model that was still displayed on the chalkboard. The fact that Learner C3 wrote the vocabulary "algebraic equation" is an indication that the learner was not aware of said facts. A look at the learner's work shows that only the examples were given correctly, and the rest of the work was incorrect.

On the contrary, Learner C4 chose the vocabulary "number patterns", which had not been discussed in the previous discussion. Though there were clear signs of deficiencies in the learner's work, especially the definition and characteristics, this learner demonstrated that he had grasped how to use the Frayer model. Both examples given were of arithmetic patterns and the given non-example was also acceptable, as this was a set of numbers which did not necessarily represent a number pattern.

On the first day, when learners were introduced to mathematical vocabulary, their performance was not very satisfactory. The session concluded at 15:30, and the learners were dismissed, with the expectation of meeting again on Thursday of the same week. Meeting on the following day, Wednesday, was not possible due to the school's scheduled assembly session, which coincided with the time the researcher intended to start the training session at 14:00.

- **Report on the second day of the training**

On the second day of the training, Thursday, we started at 14:00, as scheduled. All 53 learners were present, and the training went well. The learners were asked to work with the vocabulary of their choice, as was the case with the last part of the first session. Given that the purpose of the training was to expose learners to using the Frayer model to learn mathematical vocabulary by asking them to choose their own vocabulary, the research wanted them to be the owners of their own learning.

Presented here is a sample of two learners' work. These two learners both dealt with formulas. C5 dealt with the quadratic formula (Figure 6.2.4(a)), while C6 dealt with the Pythagoras theorem (Figure 6.2.4(b)).

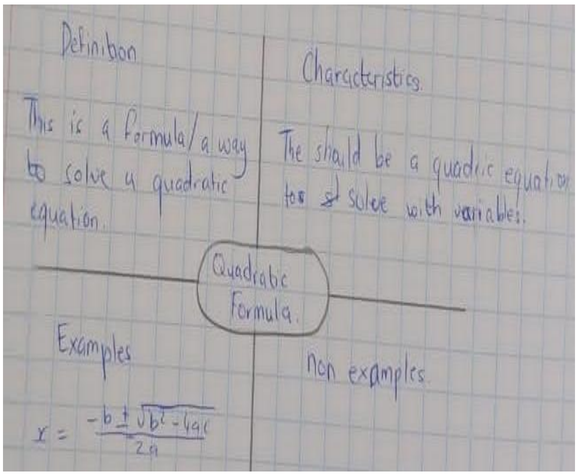


Figure 6.2.4(a): Frayer model for the vocabulary "quadratic formula" by Learner C5.

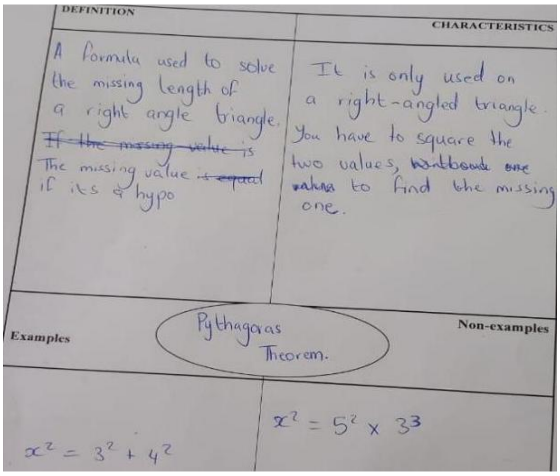


Figure 6.2.4(b): Frayer model for the vocabulary "Pythagoras theorem" by Learner C6.

Both learners precisely gave the definitions and characteristics, with Learner C6 also giving a good example and a good non-example. Learner C5 gave the formula in place of an example. Knowing the formula is one thing, but working with it is another thing. It would have been more accurate if the learner had plugged some figures into the formula. However, as this was a learning exercise, the expectation was that, with time, more precision would be found in the learners' subsequent work.

Following an assessment of most learners' work and individual discussions with them, the nature of the exercise was modified. Learners were provided with Frayer model templates with completed quadrants, and they were instructed to fill in the oval section. This task was assigned when there was less than 30 minutes remaining in the session. A class discussion ensued, and then the training session was concluded.

As per schedule, the training took place over the maximum duration of 3 hours. In this school, the 3 hours were completed in two days: Tuesday and Thursday of the same week.

6.2.1.1(b) Intervention in School D

- **Report on the first day of the training**

Examples of learners' work is presented here:

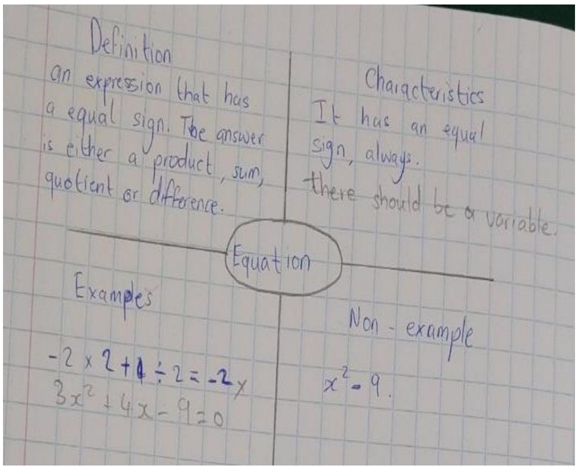


Figure 6.2.5(a): Frayer model for the vocabulary "equation" by Learner D1.

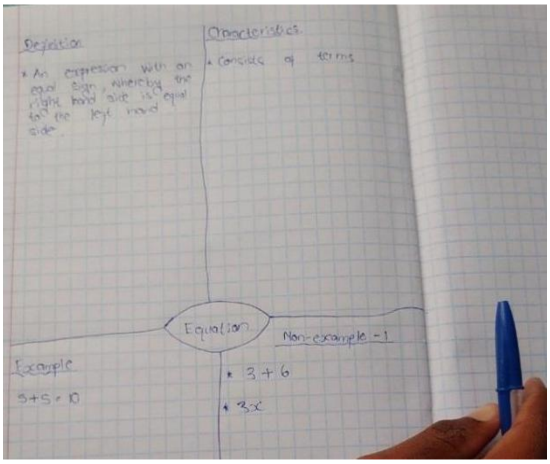


Figure 6.2.5(b): Frayer model for the vocabulary "equation" by Learner D2.

In the preceding learners' excerpts (Figures 6.2.5(a) and 6.2.5(b)), the definitions given allude to the fact that an equation is an expression. Already here, these learners confused the concept of an equation and an expression. Learner D1 went on to suggest the type of answer that an equation has. This suggestion needed not to be in the definition, because it does not help to clarify what an equation is. When it comes to the characteristics, both learners made some errors. Learner D1 wrote that an equation should have a variable. Though this is true for some equations, such characterisation excludes numeric equations. On the other hand, Learner D2 stated that one characteristic of an equation is that it has terms. This is true for all equations, but the most defining characteristic is the presence of an equal sign. Terms without an equals sign represent an expression and not an equation. Both learners gave limited categories of equations. They both gave numeric equations, but Learner D1 went further and give an example of a quadratic equation.

After a 20-minute class discussion, where ideas were taken from various learners, the ensuing Frayer model was completed on the board. During the discussion, one learner from the group volunteered to write what the discussion came up with as shown in Figure 6.2.6.

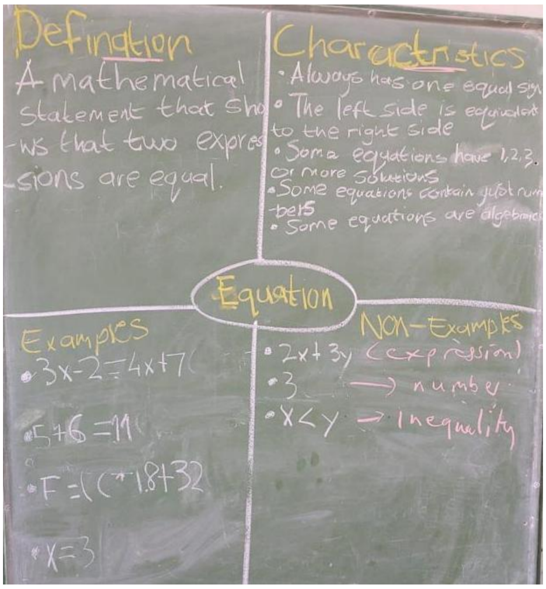


Figure 6.2.6: Frayer model for the vocabulary "equation" by the whole class in School D.

Although not exhaustive, the Frayer model resulting from the class discussion covers enough essentials about an equation to make learners understand the concept. The session concluded after 1 hour 20 minutes.

- **Report on the second day of the training**

On the second day of training, Wednesday, the session was again held during break in the same classroom as the first session. The enrolment for the day was 20 learners. One learner was excused from the training, because he needed to sit for the history examination after break. All of the other 20 learners who remained were not enrolled for history.

During this session, learners were asked to complete templates of the Frayer model for the mathematical vocabulary of their choice. Learners completed models with a wide range of vocabulary. The vocabulary written by learners included: factors, quadratic formula, quadratic equation, roots, product, corollaries, number pattern, intercepts, Pythagoras theorem, axis of symmetry, function, and domain.

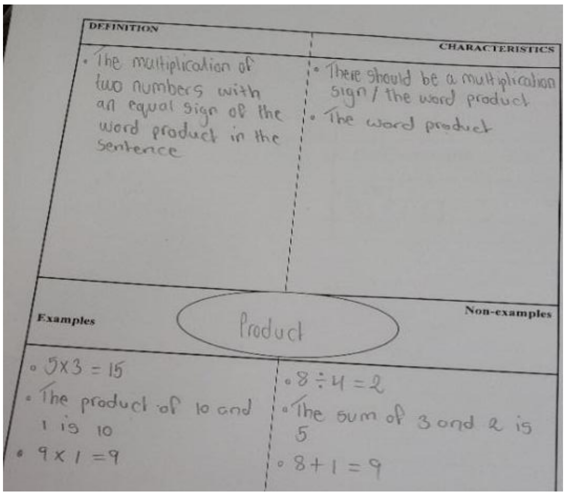


Figure 6.2.7(a): Frayer model for the vocabulary "product" by Learner D3.

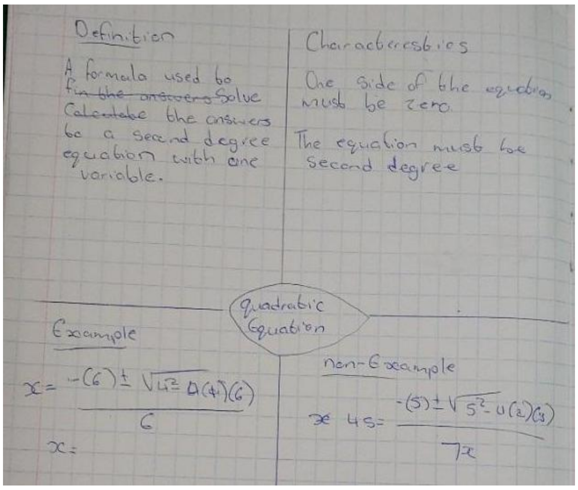


Figure 6.2.7(b): Frayer model for the vocabulary "quadratic equation" by Learner D4.

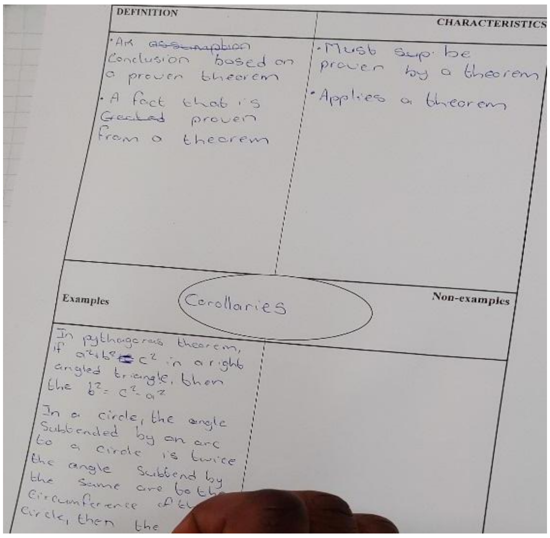


Figure 6.2.8(a): Frayer model for the vocabulary "corollaries" by Learner D5.

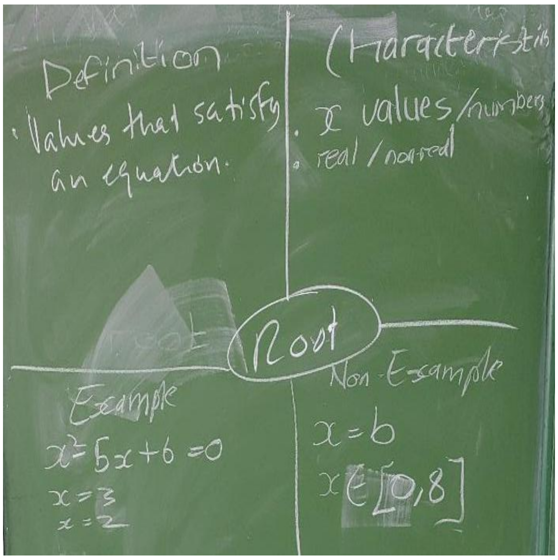


Figure 6.2.8(b): Frayer model for the vocabulary "root" by Learner D6.

Because the learners were working with the vocabulary of their choice, diverse work excerpts were available for selection during the presentation. The work of four learners with vocabulary terms including product, quadratic equation, corollaries, and roots has been presented here. Learner D3 and Learner D4 (Figures 6.2.7(a) and 6.2.7(b)) presented definitions indicating challenges with the selected vocabulary. In the case of Learner D3, a product was defined as a process rather than the result of multiplication. When referred to their definition, Learner D3 acknowledged the omission and emphasized the intention to describe it as "a result of the multiplication of terms." Aside from that omission, Learner D3's work was correct.

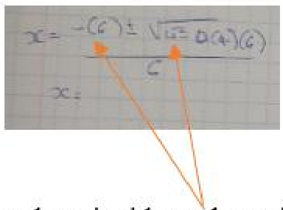
As for Learner D4, who opted to work with "quadratic equations", his work had several errors, starting with the definition. In the definition, the learner considered a quadratic equation to be a formula for solving equations of the second degree. However, an equation of the second degree is a quadratic equation. Therefore, this means that the learner considered a quadratic equation to be a formula for solving a quadratic equation. What the learner wrote as the definition, example, and non-example point to the "quadratic formula", rather than to a "quadratic equation". This contradicts what is in the oval and the characteristics which point to a quadratic equation.

On further inspection of the example that was given by the learner, it is evident that this could not be accepted as a good example of how to use either a quadratic equation or a quadratic formula. Assuming that there was a slip in the oval, meaning that the learner intended to write "quadratic formula", there are still flaws in the values used by the learner.

The quadratic formula is hereby compared to the learner's response:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

"b" inside and outside the radical



different values inside and outside the radical

Figure 6.2.9: Learner D4's incorrect substitution into the quadratic formula.

The fact that the learner used both 6 and 4 in place of "b" is a sign that there were deficiencies in his knowledge of the quadratic formula (Figure 6.2.9).

Learner D5 and Learner D6 (Figures 6.2.8(a) and 6.2.8(b)) demonstrated that they had grasped how to use the Frayer model correctly. Chatting with them revealed that they were fascinated by the Frayer model during the previous training session, so they practised using it at home. Consequently, they managed to produce good pieces of work, as presented in Figures 6.2.8(a) and 6.2.8(b). I asked Learner D6 to put his work on the chalkboard for others to see.

At the time the photo was taken, Learner D5's work was not yet complete, but it shows that the learner was on the right track. He chose unfamiliar vocabulary, but he articulated it very well. Learner D6 worked with the term root which he defined, characterised, and exemplified properly. This was an indication that we were on the right track with the use of the Frayer model. The session lasted 1 hour 20 minutes, and I dismissed the learners and asked them to prepare for the next session the following day.

- **Report on the third day of the training**

On the third and final day of training, which was Thursday, the instructor assigned the learners the task of repeating their work with the Frayer model, just as they had done on the previous day. This exercise served as an opportunity for consolidating understanding of the Frayer model and acquiring mathematical vocabulary explicitly.

After allocating a 20-minute work period to the learners, templates of the Frayer model were distributed, with the quadrants pre-filled. The learners were tasked with identifying the vocabulary in question and recording it in the oval. This marked the last assignment of the day and the entire training program. The session's duration was limited to 40 minutes, which was sufficient to meet the 3-hour training requirement. It is worth noting that, unlike School C, where two sessions of 1 hour 30 minutes each were held, School D conducted three sessions. The first two sessions spanned 1 hour 20 minutes each, while the third session lasted 40 minutes.

6.3 Implementation phase results

The implementation phase encompasses the lesson observations conducted in the two research sites, where learners from the experimental group were located. These learners were categorized into three classes: two classes in School C and one in School D. The aim of the observations was to assess the utilization of the Frayer model by learners in their study of functions. Additionally, there was an interest in gauging the level of exposure to mathematical vocabulary during lessons. A total of nine lessons were observed, and the results are outlined in the subsequent section.

6.3.1 Lesson observations

In this section, C1 denotes Class 1 in School C, and C2 represents Class 2 in School C. School C hosted two Grade 11 mathematics classes, both of which were included in the experimental group for this research. In School D, there was a single mathematics class referred to as Class D. These three classes, C1, C2, and D, constitute the experimental group for the research. Three lesson observations were conducted with each of these classes, and the results of these observations are presented in this section.

In the presentation of the lesson observation results, pseudonyms are utilized for both the learners and the three participating teachers. The teachers are identified as Teacher C1, Teacher C2, and Teacher D. Teacher C1 was responsible for Class C1, Teacher C2 oversaw Class C2, and Teacher D was in charge of Class D.

The pseudonyms assigned to the learners were constructed as follows:

- Names for the learners in Class C1 start with a capital letter C followed by a vowel.
- Names of learners in Class C2 start with a capital letter C followed by a consonant.
- Names of learners in Class D start with a capital letter D.

Class C1: Lesson 1

This lesson was conducted on Monday 26 July 2021, during the fourth period of the school timetable. The class had an enrolment of 26 learners, and they were all present for this lesson. This was an introductory lesson to the chapter of functions, so the teacher started by reminding the learners that they were starting a new chapter. The teacher wrote the name of the chapter on the board as “Functions and Graphs”.

The teacher then immediately reminded the learners that this was a concept that they did in the previous grade (Grade 10), but this time around, they were expected to broaden the scope of the same concept.

> Teacher C1: …remember last year we did functions and drew some graphs associated with the functions that we studied. I also told you that the same functions would be studied in Grade 11 with the only difference being that we will introduce more parameters to the functions. Am I right?

Class: Yes, ma'am.

Teacher C1: Now tell me the names of the functions that we studied in Grade 10 that you still remember.

Following this request, learners named what they remembered, and the teacher wrote the following terms on the board: parabola, line, exponential, hyperbola, Sine, Cosine, and Tangent.

The teacher then pointed out to the learners that these terms would later be sorted into the names of functions, and the names of graphs associated with the functions. Following this, the teacher then asked the learners to define a function. One of the learners, Cathy, asked if the teacher expected them to do so by using what they were taught by "Sir" (me).

Teacher C1: …I like that but, ummm, but for now, I am asking for a definition so that I see if you know what a function is.

Carlos: It is a relationship that we use to draw a graph like so (drawing a curve in the air) but it can also be straight.

Teacher C1: I agree that it is a relation, but ummmm, Carlos, your definition will not make us understand the nature of that relation. Is there anyone who wants to elaborate on what Carlos said?

Cameron: …. the relation is between elements of one set and the other so that they can match each other on the graph.

Teacher C1… (Underlining the topic) its functions and graphs…then the function can be represented on a graph …but what is it then?

Several attempts were made to come up with the definition of a function. When the teacher realised that the learners were struggling with the definition, she turned to Cathy and acknowledged that they were resorting to *that* technique, so that they could use it to dissect and understand the concept. She asked if Cathy or any other learner could go to the board and draw the diagram on which to do the work. Cathy volunteered and she copied the format of the Frayer model from her book on the board and the teacher completed it for the class. She said she had to complete it so that the learners would take down the notes. Figure 6.3.1 is the completed Frayer model by the teacher.

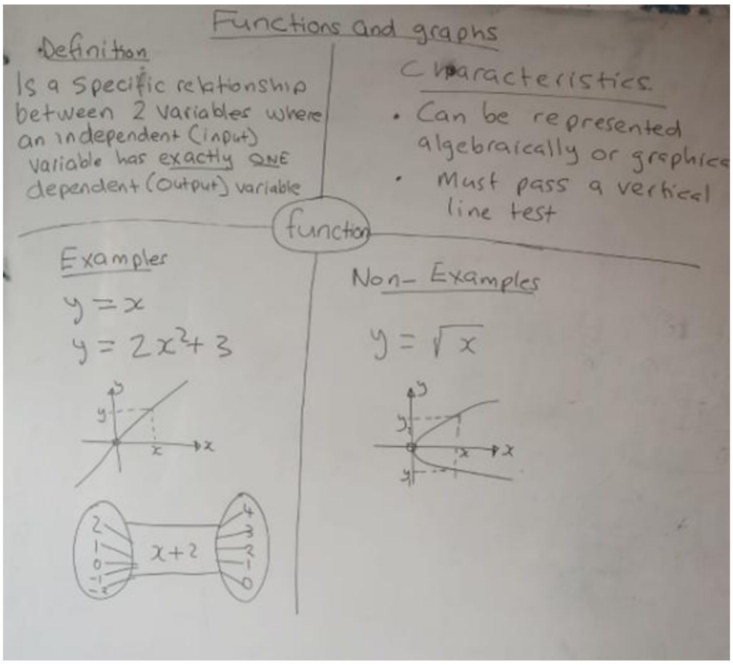


Figure 6.3.1: Frayer model to define the term "function" by Teacher C1 during a lesson observation.

Following the notes and the ensuing explanation, the teacher went straight into the concept of a linear function. She used an example and a flow diagram to illustrate the idea of "correspondence" and "*only* one value" in the second set (Figure 6.3.1). In the beginning, she did not give names to the two sets, but after the flow diagram was completed, she named the sets "Domain" and "Range". She highlighted the terms, and then defined them for the learners, but emphasised that the Frayer model can also be used to define and explain them.

> Teacher C1: The domain is the set from which we get the input values, okay?
>
> Class: Yes ma'am.
>
> Teacher C1: …they are the values of, x, which is the independent variable, neeeh…. and these ones, (pointing to the second set), the, y, values are the output values…dependent variable.
>
> Teacher C1…Go and complete the Frayer model for these words as part of your homework.

The teacher then asked learners to complete Frayer models for these terms as homework. The function that was discussed was of the form $y = mx + c$. This is also referred to as the form $ax + by + c = 0$. For this, the teacher explained the parameters *m* and *c,* with *m* representing the gradient and *c* representing the y-intercept. In her explanation, the other words that emerged were "gradient" and "*y*-intercept".

Teacher C1: In this form (point to $y = mx + c$), m is the gradient and c is the y-intercept, okay?

As the lesson progressed, the teacher and the learners used the dual-intercept method to sketch a graph of a function (already given to). This method entails calculating the coordinates of the x-intercept and the y-intercept. These coordinates are plotted on the Cartesian plane and joined with a straight line. This straight line is the line graph that represents the linear function in question. Throughout the discussion, more terms for functions emerged and the teacher referred learners to them and instructed that they should use the Frayer model for the terms. as part of the homework that she would give them later.

The teacher then gave her learners classwork that involved sketching graphs, stating the domain and range of the functions, writing down the coordinates of the x-intercept and the y-intercept, and describing whether the given functions were increasing or decreasing. The classwork was as follows:

For each of the following linear functions

(i) Calculate the coordinates of the x-intercept.

(ii) Calculate the coordinates of the y-intercept.

(iii) Sketch the graph of the function.

(iv) Write down the domain of the function.

(v) Write down the range of the function.

(vi) State whether the function is increasing or decreasing.

1. $f(x) =- 2x + 6$
2. $g(x) = x + 2$
3. $h(x) = x - 4$
4. $p(x) = \frac{1}{2}x - 1$

As the learners worked through the classwork, with the permission of the teacher, I moved around to check their work. I noticed that some learners had scribbled some sketches of the Frayer model to refer to as they did their work. However, the lesson ended with most of the learners still not having finished the classwork. They were asked to carry over the work as homework that would be discussed the following day.

Class C2: Lesson 1

This lesson was delivered on Wednesday 28 July 2021. It was conducted by the teacher with a class with an enrolment of 27 learners. It was the second period on the timetable, 08:40–09:20. However, this was not an introductory lesson to functions, as the class had started doing functions on Monday 26 July. The teacher started by checking if the learners had done their homework. This checking was done at the door as the learners entered the classroom. As the teacher sanitised the learners' hands, she also asked them to open their books where the homework was written. This was not a check to see the correctness of the work, but simply to check if the homework was done. The homework comprised sketching a parabola. Learners were given the equation of the function and had to sketch the graph and indicate the coordinates of the intercepts and turning point. Part of the homework was to complete Frayer models for key vocabulary. The vocabulary was: turning point, x-intercept, y-intercept, graph, domain, range, and axis of symmetry. They were also asked to consider other pertinent vocabulary in addition to the ones given by the teacher.

Once the learners were settled in the lesson, the teacher asked one of the learners, Choga, to present his work on the whiteboard. This was done to revise the homework and to allow the learners to mark their work. The teacher indicated to me that except for formal assessments, as a COVID-19 protocol, it was encouraged that educators and learners should not exchange books. For this reason, learners had to mark their own books. Choga presented the work on the whiteboard.

When Choga finished presenting the work on the board, I went to see his book and the work in the book was as shown in Figure 6.3.2.

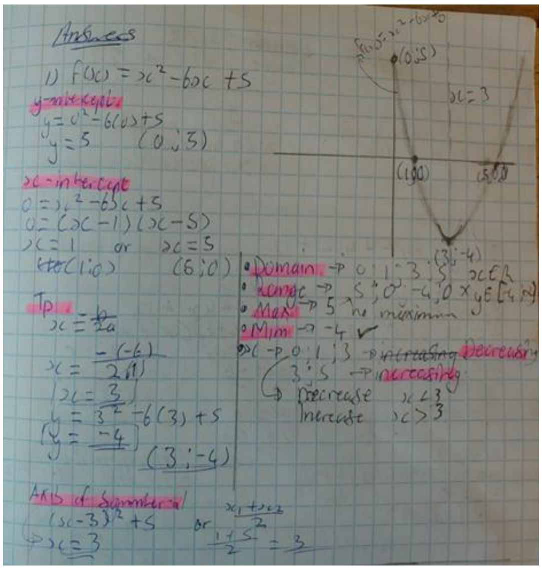


Figure 6.3.2: Excerpt of Choga's homework.

As I looked through the work, I realised that Choga had highlighted some words that appeared in his work. The highlighted words were *y*-intercept, *x*-intercept, TP (for turning point), domain, range, max (maximum), min (minimum), decreasing, increasing, and axis of symmetry. I enquired with the learner as to why the words were highlighted, and his response was that it is because they constituted the key vocabulary for the question. During a discussion with him, he referred to his other notebook where he had sketched some Frayer models and completed them for these words. The learner showed me his models for the terms domain and range. These models are shown in the following figures.

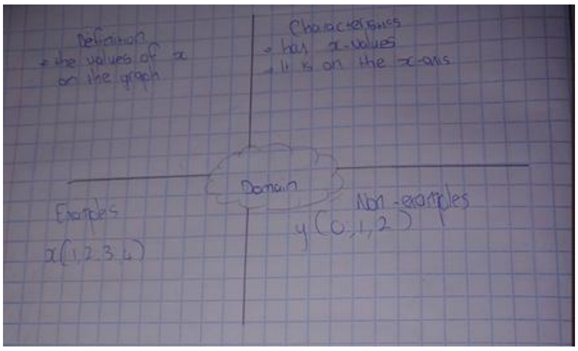


Figure 6.3.3(a): Choga's notes for the term "domain".

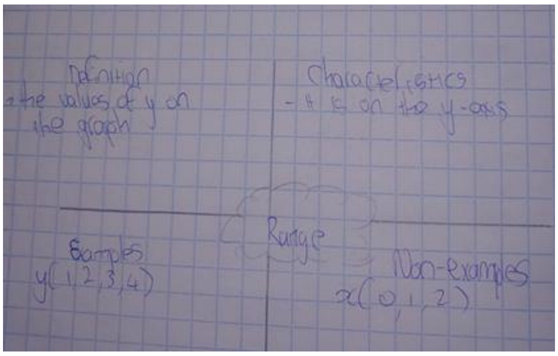


Figure 6.3.3(b): Choga's notes for the term "range".

Throughout the examination of these Frayer models, shortcomings became evident in the treatment of domain, range, decreasing, and increasing. The learner defined the domain as the "values of x on the graph" and the range as the "values of y on the graph." These definitions had deficiencies that resulted in incorrect answers in the learner's homework. As indicated in his answers written in blue (Figure 6.3.2), the domain was specified as particular numerical values (0, 1, 3, 5), and the range similarly consisted of values (5, 0, -4, 0). These answers closely resembled the examples he had already provided in his Frayer models (Figures 6.3.3(a) and 6.3.3(b)).

The learner was instructed to substitute specific x-values into the function, including -3, -1, 5, and 6. Upon doing so, the learner recognized that his graph needed to indicate that the function extended to more x-values, consequently expanding the range as well. He promptly acknowledged that he had constrained his domain and range due to his inadequate definitions. During the subsequent class discussion, when the class had to evaluate the domain and range answers, the teacher pointed out that the ones on the board were incorrect and required correction. Despite accurate calculations and a correctly shaped diagram, the domain and range answers were found to be incorrect. Subsequently, corrections were made to the work during the class discussion, and Choga incorporated these corrections into his work using black ink, as depicted in Figure 6.3.2. Furthermore, the learner was tasked with revising his definitions in his Frayer models.

On completing the revision of the homework, the teacher explained to the class the concept of a quadratic function in the form $f(x) = a(x - p)^2 + q$. In this case, two new terms came into the fold explicitly: vertical shift and horizontal shift. The teacher emphasised that unlike in the form $f(x) = ax^2 + bx + c$, where the coordinates of the turning point are TP$(\frac{-b}{2a}, f(\frac{-b}{2a}))$, in this case the coordinates are TP (p, q) . The teacher explained that parameters p and q represent horizontal and vertical shifts to the parabola respectively. After an illustration on the board, the teacher gave the learners the following classwork.

Given that $f(x) = -(x - 1)^2 + 9$

Calculate the coordinates of the turning point of *f*. (4)

Determine the y-intercept of *f*. (1)

Determine the x-intercepts of *f*. (4)

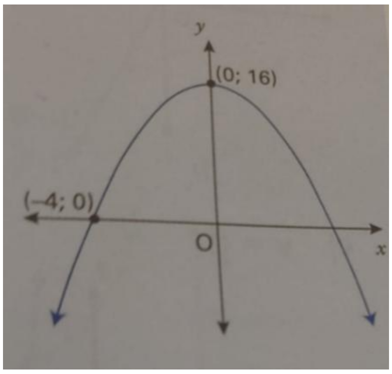
Sketch the graph of *f* clearly showing all intercepts with the axes and turning point. (3)

The lesson ended before the class could discuss this work. To those who had not finished the classwork, this had to be continued as homework. The teacher also gave additional exercises to the learners as homework.

Class D: Lesson 1

This lesson was conducted during Period 7, on Friday 30 July 2021. The lesson started at 13:15 and it ended at 14:00. The chapter on functions had started on Monday 26 July, and there was a double period on Wednesday 28 July. This meant that a lesson that was observed was the sixth period on functions. In the past lessons, the class, with an enrolment of 21 learners, covered linear functions and quadratic functions. During this lesson, learners were practising predominantly with the quadratic function. The exercise of the day required learners to determine the equation of a function from the graph, starting with the following vertically shifted parabola.

Determine the equation of the following parabola:



When the teacher explained the concept to the learners, she emphasised the importance of remembering the vocabulary associated with the given information in the question. In her explanation, the teacher stressed that for any given situation, learners should ask themselves questions related to the question at hand. Answers to these questions become building blocks to the solution. The following conversation is part of the lesson discussion on the day:

Teacher D: What information is given?

Davies: Ma'am.

Teacher D: Yes, Davies (name called in full by the teacher).

Davies: We are given, ummm, points.

Teacher D: Okay. You mean what points?

Davies: Coordinates.

Teacher D: Yes, they are coordinates, but can you say more about the coordinates?

Davies: (0, 16) and (-4,0).

Teacher D: Anything more about the coordinates?

Davies: Ma'am, they are on the graph.

Teacher D: Okay. Can someone add more to what Davies said?

David: They are on the lines, so 0 is there.

Teacher D: Okay, but what more can we say about them?

Dudu: Intercepts Ma'am.

Dennis: That one is not an intercept (laughing).

Teacher D: Which one, Dennis?

Dennis: Ummm never, mind.

Teacher D: Okay. I think that they are both intercepts.

Class: Yes ma'am.

Teacher D: However, one of the intercepts doubles as a turning point. Which one is that Dadi?

Dadi: I think it's (0, 16), because the graph is curving there.

Teacher D: The graph turns and not curves at (0, 16). Anyhow, such a discussion is the one that you should engage yourself in, so that you make it easy for yourself to get answers to the question. Right?

Class: Yes ma'am.

Teacher D: Since the turning point is on the y-axis, the general equation of the function is $f(x) = ax^2 + q$

From the class discussion, it was clear that the teacher was guiding the class towards identifying pertinent vocabulary that they could deduce from the question and use as they sought to answer the question. Considering the teacher's question, "What information is given?" shows that the teacher wanted her learners to say aloud the concepts embedded in the visuals/graph. These concepts needed to be verbalised by the learners, in order for a discussion to be held. Knowing and understanding the said vocabulary became an indispensable component of the teaching and learning that needed to happen in the classroom. These terms/vocabulary were already defined and characterised in the learners' books using the Frayer model. The vocabulary that they brought up in discussion was points, intercepts, y-axis, turning point, graph, equation, function, line, curve, and coordinates.

The teacher then went on to answer the question with the class on the board as shown in Figure 6.3.4.

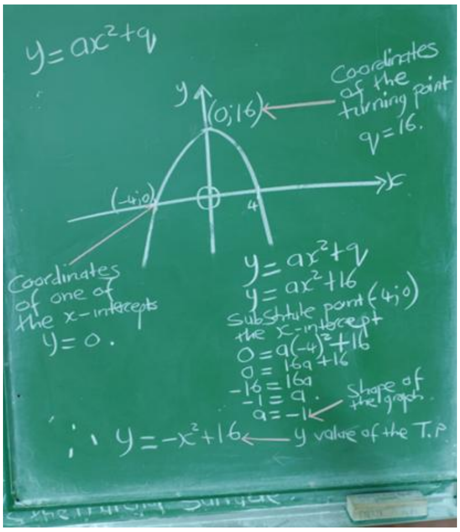
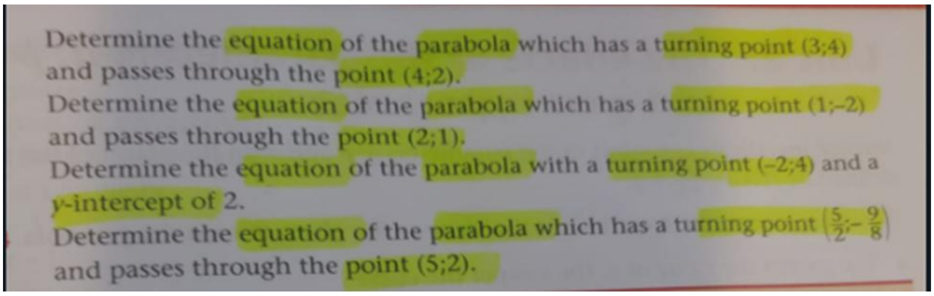


Figure 6.3.4: Example from Class D of finding the equation of a function.

From the working, the equation of the function was deduced to be $f(x) = -x^2 + 16$.

Following this, the teacher explained to the class that in some cases, a question might not provide a graph of the function whose equation must be deduced. Instead, the question might give points through which the graph passes, such as questions taken from their textbook: Platinum Mathematics, Grade 11, page 91, and Question 2.



Determine the equation of the parabola which has a turning point (3;4) and passes through the point (4;2).
Determine the equation of the parabola which has a turning point (1;–2) and passes through the point (2;1).
Determine the equation of the parabola with a turning point (–2;4) and a y-intercept of 2.
Determine the equation of the parabola which has a turning point $\left(\frac{5}{2};-\frac{9}{8}\right)$ and passes through the point (5;2).

Figure 6.3.5: Classwork given to the learners by Teacher D, taken from the textbook.

In all the subsections of Question 2, the coordinates of the turning point were given. In addition, coordinates of another point on the graph were also given. Learners were asked to answer the questions in their textbooks as classwork. As the learners were working, I asked one of the learners to indicate to me what he regarded to be the key mathematical vocabulary in the question. The learner highlighted the words, equation, parabola, turning point, *y*-intercept, and point. The terms were highlighted in the personal copy of the textbook brought to the lesson (Figure 6.3.5). These words had already been recorded in the learner's book as notes using the Frayer model. Upon inquiry, it was revealed that the use of the Frayer model as a note-taking tool had facilitated the learner's improved understanding of mathematical vocabulary.

Class D: Lesson 2

This lesson was conducted on Tuesday 3 August. It was the third period of the school timetable, starting at 09:30. During this lesson, the class was doing Exercise 7 in the Platinum Mathematics textbook, page 97. The exercise required learners to sketch some hyperbola whose equations were given. In addition to these graphs, learners were also asked to answer some questions with respect to the sketched graphs. From Exercise 7, learners were asked to respond to Questions 1, 4, 7, and 8. The questions were as follows:

Sketch each graph on a separate set of axes. In each case:

- Determine the equation of the asymptotes.
- Determine the coordinates of the x-intercept.
- Determine the coordinates of the y-intercept.
- Clearly indicate the intercepts with the axes, the asymptotes, and the symmetry lines.

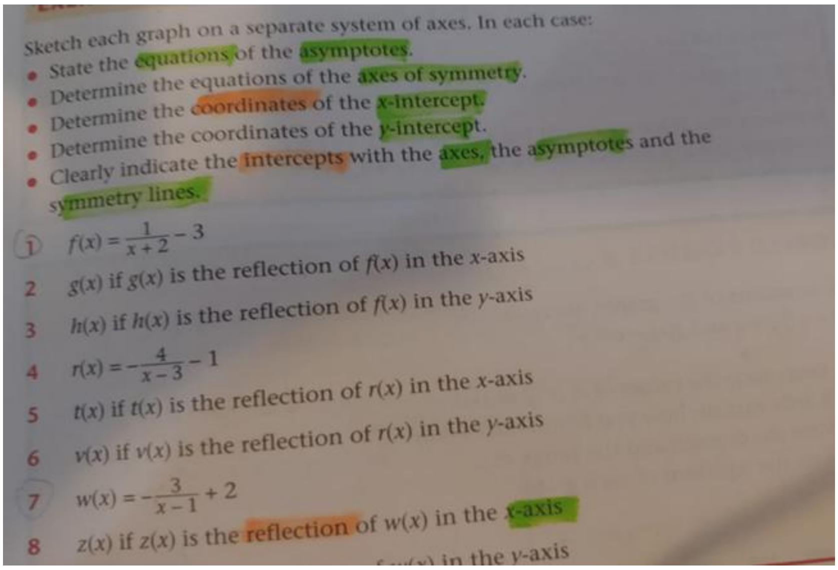
1. $f(x) = \frac{1}{x+2} - 3$

4. $r(x) = \frac{4}{x-3} - 1$

7. $w(x) = -\frac{3}{x-1} + 2$

8. $z(x)$ if $z(x)$ is the reflection of $w(x)$ in the x axis.

As in the previous lesson observation with the same group, I moved around and asked one female learner to identify the key mathematical vocabulary in the classwork. I asked the learner to highlight the vocabulary in the researcher's textbook and she highlighted as follows:


Sketch each graph on a separate system of axes. In each case:

- State the equations of the asymptotes.
- Determine the equations of the axes of symmetry.
- Determine the coordinates of the x-intercept.
- Determine the coordinates of the y-intercept.
- Clearly indicate the intercepts with the axes, the asymptotes and the symmetry lines.

1 $f(x) = \frac{1}{x+2} - 3$

2 $g(x)$ if $g(x)$ is the reflection of $f(x)$ in the x-axis

3 $h(x)$ if $h(x)$ is the reflection of $f(x)$ in the y-axis

4 $r(x) = -\frac{4}{x-3} - 1$

5 $t(x)$ if $t(x)$ is the reflection of $r(x)$ in the x-axis

6 $v(x)$ if $v(x)$ is the reflection of $r(x)$ in the y-axis

7 $w(x) = -\frac{3}{x-1} + 2$

8 $z(x)$ if $z(x)$ is the reflection of $w(x)$ in the x-axis

Figure 6.3.6: Vocabulary highlighted by Deans during classwork.

Nine terms were highlighted using a green highlighter by the learner. Once she had completed highlighting and indicated she was finished, the green highlighter was taken away from her. She was then encouraged to revisit the exercise and attempt to identify additional vocabulary. During this second pass, she requested the highlighter once more, and a pink highlighter was provided. As depicted in Figure 6.3.6, she added three more terms, resulting in a total of 12 highlighted words.

Most of the learners answered Question 1 correctly. There were neither misconceptions nor errors committed by the learners in their work. The following is the work of one of the learners, Doreen (Figure 6.3.7). As seen in this work, the seemingly persistent error most learners made was the types of graphs that they drew. There was a tendency not to draw smooth curves.

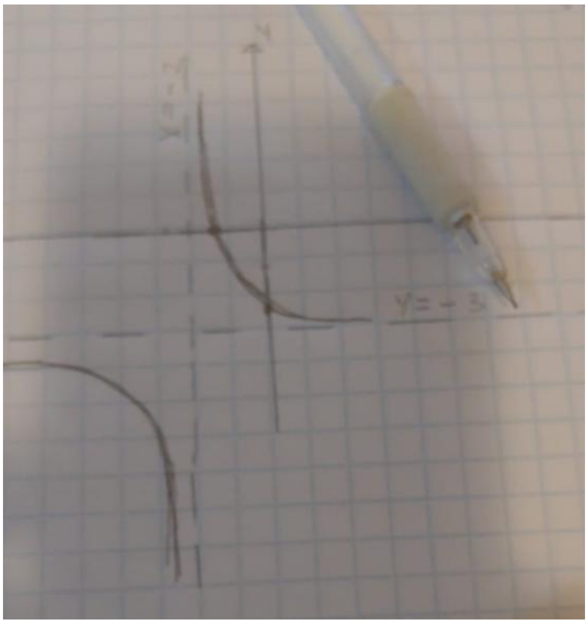


Figure 6.3.7: Doreen's graph and answer to a question from the classwork.

Even though the graph was not yet labelled, including the origin, by looking at the calculations, there was good reason to suggest that the learner was going to label the graph correctly, because her calculations were correct. The teacher allowed the learners to work through the classwork, and they took turns going to the teacher's table to have their work checked and marked. When the lesson came to an end, less than half of the learners had taken their books to the teacher for marking. However, when I moved around to check the learners' work, I realised that the learners had templates of the Frayer model, which they referred to as they worked through the classwork. They confirmed with me that these templates were of vital importance to them, as they served as a vivid reminder of the concepts that they were working with.

Class C2: Lesson 2

The second lesson observation was carried out in C2 during Period 5 on Thursday, 5 August 2021. On that particular day, the class was engaged in the process of understanding how to determine the equation of a hyperbola. This encompassed tasks that involved working with provided points of a hyperbola without the graph and working with a given graph.

The teacher began by asking the learners to give the general equation of a hyperbola.

> Teacher C2: Class! Can I have someone to give us the general equation of the hyperbola?

One female learner volunteered to do so, and wrote the equation on the board as shown in Figure 6.3.8.

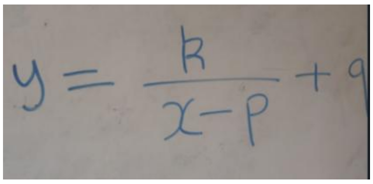


Figure 6.3.8: General equation of a hyperbola as given by a learner in Class C2.

The equation was correct, so the teacher accepted it, and went on to use it for the lesson of the day.

> Teacher C2: From this equation, what do the parameters a, p, and q represent?
>
> Charity: They represent the coordinates of the graph.
>
> Chantel: No, ma'am.
>
> Teacher C2: Can we allow Charity to explain what she means with her answer please? Please be quiet, class. Charity, talk.
>
> Charity: Mara ma'am. I think they are coordinates of the turning point.
>
> Class: (Laughter).
>
> Teacher C2: No, Charity. Remember this is a hyperbola. Class, does a hyperbola have a turning point?
>
> Class: No ma'am.

> Teacher C2: Okay, Chantel, what did you want to say?
>
> Chantel: They are the asymptotes of the hyperbola.
>
> Teacher C2: These are three parameters, so do you mean— (Chulumanco interrupted)
>
> Chulumanco: Sorry ma'am. Only p and q.
>
> Teacher C2: But the asymptotes should be classified, neh? Chipo, classify them for us.
>
> Chipo: Umm. I think p is the flat one (gesturing horizontally). Yaa its horizontal asymptote. So q aaa is the vertical asymptote (gesturing vertically).
>
> Teacher C2: Correct, neeh?
>
> Class: Yes, ma'am.
>
> Teacher C2: So remember, we said that parameter a determines the steepness of the curve, as well as its quadrants.

After this discussion, the teacher went on to explain to the learners how to find the domain and range of a hyperbola. The teacher also taught the learners how to determine the intercepts of hyperbolic functions. Some worked examples were done on the board, and then the teacher gave the learners classwork from their main textbook, Grade 11 Mathematics, Siyavula. The work came from Exercises 5–10 and 5–11 from pages 174 and 175.

What I noted from the class discussion, was the extent to which both the teacher and the learners engaged with the correct mathematical vocabulary for the concepts that they were dealing with. The teacher mentioned 10 terms in her talk: general equation, hyperbola, parameters, steepness, curve, domain, range, x-intercept, y-intercept, and quadrants. On their part, the learners collectively put forward six terms: coordinates, graph, turning point, asymptotes, horizontal, and vertical. This was a rich conversation, which showed that the class community had the correct language at its disposal.

Class C1: Lesson 2

This lesson observation was conducted on Friday 6 August 2021. This was the first period of the day, and, on this day, there were 25 learners in the class. According to the enrolment figures of the class, this meant that two learners were absent on this day. During the lesson, the teacher reminded the learners that, on this day, they were going to determine the equation of a hyperbola when its graph is given. She referred the learners to Question 2 of Exercise 5–14 on page 183 of the Siyavula textbook.

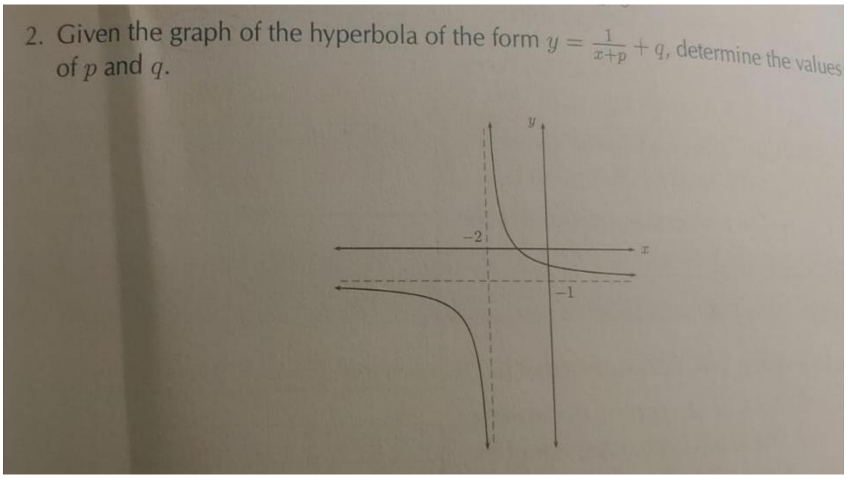


Figure 6.3.9: Question to determine the equation of a hyperbola during classwork in Class C1.

In the question (Figure 6.3.9), the graph of a hyperbola was given. The teacher asked the learners to identify important features of the hyperbola that were shown on the graph.

> Teacher C1: If you look at that graph, you realise that there are features to the hyperbola that are shown on the graph. Remember that I told you that the method that we use to find the equation of a function from its graph is the same for the linear function, parabola, hyperbola. Right?
>
> Class: Yes, ma'am.
>
> Teacher C1: So it means I don't have to go over the procedure again. What I want is to ask one of you to tell us what is evidently given on that graph. Yes, Calvin?
>
> Calvin: The asymptotes.
>
> Teacher C1: Correct. Neeeh?
>
> Class: Yes.
>
> Teacher C1: Okay. Write down those asymptotes in your books. But before you write the asymptotes, draw the graph in your notebooks as well.

The learners started doing what the teacher had asked them to do. I walked around the class, and I noticed that the majority of the learners wrote the asymptotes as $x = -2$ and $y = -1$. However, there was a certain learner who had different asymptotes. His work was as shown in Figure 6.3.10.

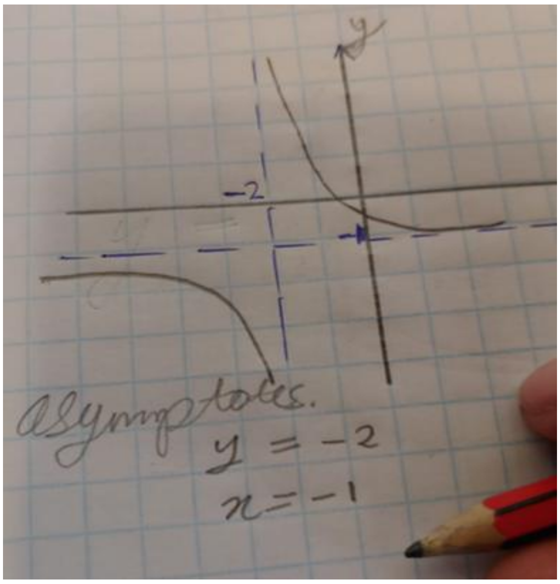


Figure 6.3.10: A learner's swapped equations for the asymptotes of a hyperbola.

Inquiring with the learner was done to understand the rationale behind the learner's answers. The learner provided an explanation, stating that "y goes up and x goes straight," while pointing to the lines he was referring to. This explanation implied that the asymptote with a numerical value of -2 was linked to the y-value because the dotted line was vertical, similar to the y-axis. Likewise, he associated the value -1 with the x-value for the same reason. Consequently, the equation of the hyperbola according to him would be $y = \frac{1}{x+1} - 2$. For the rest of the learners, the equation was $y = \frac{1}{x+2} - 1$.

The discussion continued with reference to the graph once more. When the value -2 appeared on the x-axis, the learner was prompted to provide the coordinates for this point. He correctly identified it as (-2, 0). Subsequently, he was asked to categorize the values -2 and 0. In response, he stated that the x-value at the point was -2, and the y-value was 0. Further exploration led him to deduce that it was, therefore, an x-intercept. During the discussion concerning the vertical line passing through this point (-2, 0), it was concluded that the x-values on this line would consistently remain -2. However, as the line ascends above the x-axis, the y-values increase into positive values, while below the x-axis, they decrease into negative values. Consequently, the only constant aspect was the x-value, and therefore, the line derived its equation from this constant value. This discussion led the learner to realize that the asymptotes should be represented as x=-2 and y=-1.

During the lesson, the teacher spoke very little, as the learners did classwork taken from the Siyavula textbook, Exercise 5-14 Questions 2 and 3 page 183 and Exercise 5-19 Question 8(b), page 196.

Class C2: Lesson 3

The lesson was conducted on Tuesday 10 August 2021, between 11:40 and 12:20. During the lesson, the teacher taught the exponential function. She appeared to be intending to wrap up the chapter on functions. This was picked this up from her statement as follows:

> Teacher C2: …we are supposed to rush through the exponential functions today, so that in tomorrow's lesson, we will be able to finish everything related to the exponential functions…this week we must start the study of Trigonometric functions, okay?

She went ahead with an exposition where she gave the equation of the exponential function, and explained the effects of the parameters a, b,p, and q. This was a more teacher-centred lesson than the ones prior.

The equation that she used for her lesson was in the form $f(x) = ab^{x+p} + q$. In the explanation, the teacher stated that:

> Teacher C2: As you can see here, this function consists of a base (underlining b) that is raised to an exponent, which is a variable (underlining x). This is what separates the exponential function from other functions, okay?

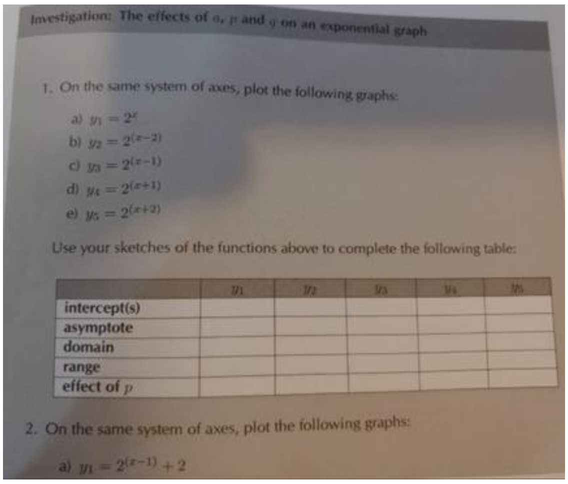
In this explanation, the teacher characterised the exponential functions. She did not define the term exponential function, but by characterising it and giving its general equation, there was an attempt to explain it. Furthermore, the teacher did not ask her learners to use the Frayer model to work with this concept (exponential function). She, however, went on to explain more about the parameters of the exponential function.

> Teacher C2: In this equation, q represents a vertical shift, and it also represents the horizontal asymptote of the exponential function.

She then illustrated on the board that when $q > 0$, there is a vertical upward shift of the function by q units. In the same breath, the teacher explained that when $q < 0$, the function undergoes a vertical downward shift of q units. The teacher also mentioned that the equation of the horizontal asymptote of the exponential function is $y = q$.

Though this was a teacher-centred lesson, after all four parameters were explained, a number of vocabulary were mentioned. This vocabulary was: vertically upward shift, vertically downwards shift, exponential function, horizontal asymptote, units, decreasing, increasing, reflection, direction, shape, and not defined.

Throughout the exposition, the teacher constantly reminded the learners that all of this vocabulary that she was articulating was crucial to the understanding of the exponential function. In some instances, she reminded the learners that they needed to refer to their Frayer model notes in their notebooks. After this exposition, the teacher gave the learners work to do in their exercise books, to help them to investigate the effects of the parameters a, p, and q (Figure 6.3.11).



Investigation: The effects of a, p and q on an exponential graph

1. On the same system of axes, plot the following graphs:

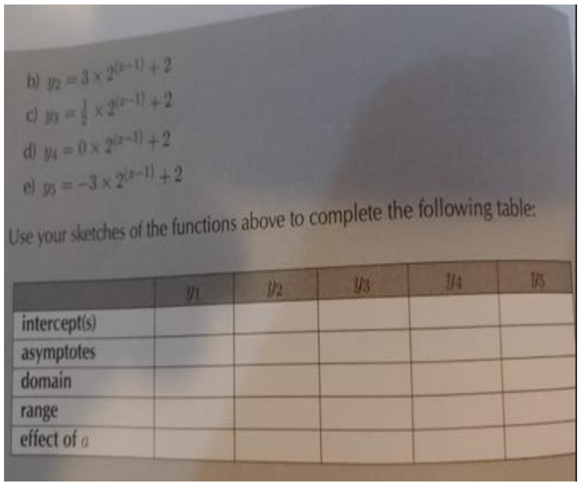
a) $y_1 = 2^x$
b) $y_2 = 2^{(x-2)}$
c) $y_3 = 2^{(x-1)}$
d) $y_4 = 2^{(x+1)}$
e) $y_5 = 2^{(x+2)}$

Use your sketches of the functions above to complete the following table:

	y_1	y_2	y_3	y_4	y_5
intercept(s)					
asymptote					
domain					
range					
effect of p					

2. On the same system of axes, plot the following graphs:

a) $y_1 = 2^{(x-1)} + 2$



b) $y_2 = 3 \times 2^{(x-1)} + 2$
c) $y_3 = \frac{1}{2} \times 2^{(x-1)} + 2$
d) $y_4 = 0 \times 2^{(x-1)} + 2$
e) $y_5 = -3 \times 2^{(x-1)} + 2$

Use your sketches of the functions above to complete the following table:

	y_1	y_2	y_3	y_4	y_5
intercept(s)					
asymptotes					
domain					
range					
effect of a					

Figure 6.3.11: Classwork to investigate the effects of parameters a, p, and q.

It was not possible for me to look at the learners' books during this lesson, because as they were about to start doing the classwork, there was an announcement from the school principal that all Grade 11 and 12 learners were to meet the principal in the school hall immediately.

The teacher asked the learners to take their belongings, and they all left the classroom for the school hall.

Class D: Lesson 3

This lesson was conducted on Wednesday 11 August 2021, from 08:45 to 09:30. During the lesson, the teacher had a quick look at the learners' books, to check if they had done their homework. The homework was an investigation on the effects of parameters a, p, and q, on an exponential function of the form $y = ab^{x+p} + q$. The work was not difficult to check, because the learners had completed a table where, for the given functions, they were required to write down the: shift, asymptote, x-intercept, and y-intercept. (Exercise 13, page 105, Platinum Mathematics). However, what I picked up from the homework, was that it required that the learners understand the said vocabulary, in order for them to be able to do the task.

As for the lesson for the day, the teacher stated that the aim of the lesson was to learn how to find the equation of an exponential graph.

> Teacher D: …as I said earlier on today, we would like to find the equation of an exponential graph. …the method that I said works for this is one. Neeeh. What method is that?
>
> Dickson: Substitution.
>
> Teacher D: Yaa substitution… Right!
>
> Teacher D: Let's turn to Exercise 12 on page 105 and do 1.1 together. (Main textbook, Platinum Mathematics).

The teacher assisted learners in finding the values of variables a, b, and c in the question. As the class worked through the question, the teacher showed the learners how to go about the task. In the process, emphasis was put on the meanings and effects of the parameters, e.g. c is the horizontal asymptote and represents a vertical shift.

> Teacher D: Here, what does c tell us about the function?
>
> Dylan: It's like the asymptote here below.
>
> Teacher D: I agree. So in terms of shifts, what does it tell us about the graph?

Dylan: It's like it moved downwards. Ummm yeah, downwards.

Teacher D: So, let us substitute the given value for c. So, we have done both the shift, and the asymptote here. Right?

Class: Yes, ma'am.

Teacher D: Any suggestions on what to do from here?

Dimpho: Let me try, ma'am. I think we can now put the other values there from the graph, like that one for the intercept on the line.

Teacher D: Which one do you mean? If you look at that graph, you see we have 1, 2, 3 (counting) intercepts, so be specific.

Dimpho: For the red graph…by (0;2).

Teacher D: So, let's substitute it and see the letter that we get.

Substitution was done and a numerical value for a was obtained as $a = 3$.

Teacher D: So, we can substitute the coordinates of the other given point to find the value of b. Correct?

Class: Yes, ma'am.

This was done, and b was determined to be equal to 2. The teacher then asked learners to answer Questions 1.2 and 1.3 in their books as classwork.

As the learners worked through the exercise, I moved around and came across one learner who had done very well. I decided to speak with him to get a better understanding of his work. I asked him to explain the steps that he had followed. From this discussion, we appended a few words to his work, as shown in the following excerpt.

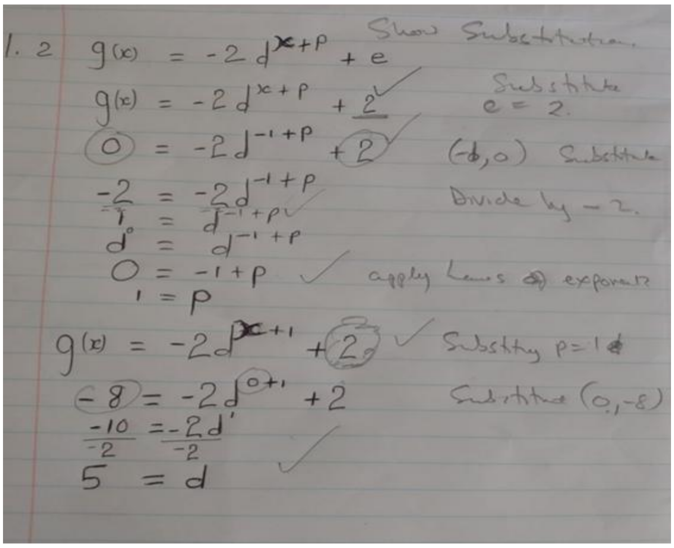


Figure 6.3.12: Excerpt of learner's work finding the equation of an exponential function.

As depicted in the work provided by the learner (Figure 6.3.12), all the calculation steps were mathematically correct. During the discussion with the learner, an inquiry was made to have him explain the relevant steps in his work. In his explanation, the learner circled certain numbers as he worked through the process. After he completed his explanation, the information he provided was added to his work in pencil. Throughout his explanation, the term "substitute" was reiterated four times, underscoring its central role in determining the equation of the exponential function. Additionally, the learner mentioned the terms "divide" and "laws of exponents" once each. Division was employed as a procedural step, while the laws of exponents were introduced as a concept to be incorporated into the procedure.

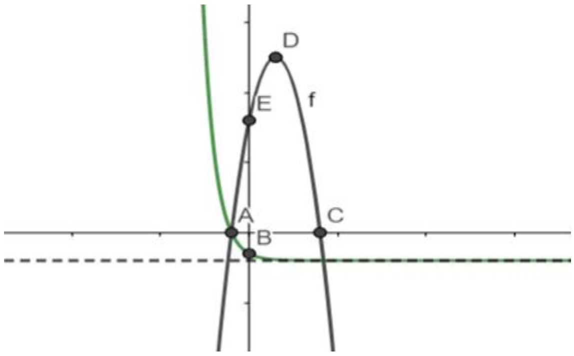
The lesson concluded, and gratitude was expressed to the teacher for granting the opportunity to conduct the research.

Class C1: Lesson 3

This lesson was conducted on Wednesday 11 August 2021, at 13:20. During the lesson, the teacher indicated to the learners that she wanted them to do more practice with a quadratic function combined with an exponential function. She gave the learners a worksheet with the following question that was adapted from the Grade 11 NSC November 2018 Paper 1 question paper:

The diagram below shows the graphs of $f(x) = -(x-3)^2 + 25$ and $g(x) = 2(\frac{1}{2})^{x+1} - 4$. Graph f cuts the x-axis at A and C, the y-axis at E and has a turning point at D. Graph g cuts the x-axis at A and the y-axis at B.

Question 1



1.1 Write down the equation of the asymptote of g. (1)

1.2 Write down the coordinates of D. (2)

1.3 Write down the range of f. (1)

1.4 Calculate the length of EB. (4)

1.5 Determine the values of x for which f is decreasing. (2)

1.6 Calculate the average gradient between points A and B (5)

1.7 Graph t is obtained by reflecting g about the x-axis. Write down the range of t. (2)

1.8 If p(x) = f (x) + 2, write down the coordinates of the turning point of p. (2)

1.9 Determine the value of k for which the straight-line y = 2x+ k will be a tangen**t** to f. (4)

The task was a mixed-question type of exercise that had three functions in the same question: a quadratic function, an exponential function, and a linear function. In the task, there were 18 different terms that have been explicitly stated in the question, in addition to three instruction terms (calculate, write, and determine). In addition to showcasing the integration of concepts, this question underscores the critical significance of teachers and learners comprehending mathematical vocabulary. As the learners worked through the task, it was evident that the task required more than just the manipulation of numbers.

For instance, to answer Question 1.1, there was no need for any calculations. Learners were required to write down the equation of the asymptote of an exponential function. This question required more conceptual understanding than procedural understanding. The following extracts show work by two learners, as they provided answers to Questions 1.1 and 1.2 (Figures 6.3.13(a) and 6.3.13(b)).

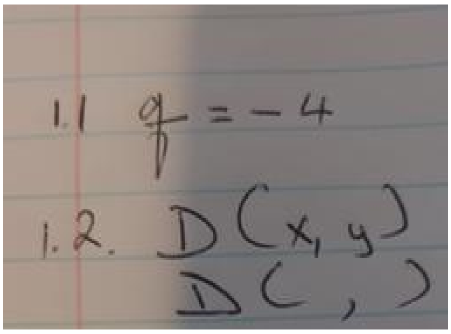


Figure 6.3.13(a): Cathay's answers to Questions 1.1 and 1.2.

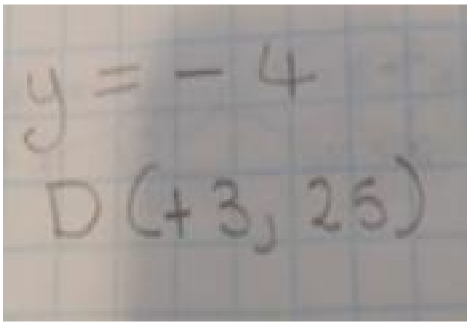


Figure 6.3.13(b): Carol's answers to Questions 1.1 and 1.2.

The two answers for Question 1.1 both have -4 as the constant, but the first learner (Cathay) has an incorrect variable in the equation. During lessons, as they learnt the concept of an asymptote for an exponential function, they were taught that the equation for the asymptote is $y = q$, where, in this case, $q = -4$. Letter q represents both the vertical shift and the numerical value of the constant in the equation for the horizontal asymptote. Cathay did not get this question correct. On the other hand, the second learner (Carol), got both Questions 1.1 and 1.2 correct. The correct answer for 1.1 is $y = -4$. Point D is the turning point of the parabola, and Carol gave the correct coordinates for the turning point in Question 1.2.

I had to engage with Cathay, because her answer to Question 1.1 was incorrect. In this discussion, I reminded her that the equation of the asymptote should have either an independent variable(x) or a dependent variable (y), depending on the axes that the asymptote crosses. She realised that the asymptote crossed the y-axis, so its equation was supposed to be in terms of y and not q. However, I did not discuss with her answer to 1.2, because it was still incomplete. I wanted her to first complete it before I would look at it.

When the lesson ended, most of the learners were on Question 1.4, with just three learners who had started answering Question 1.5. From an assessment of the learners' work, the answers of the learners showed that Question 1.1 was well answered, with 22 of the 26 learners in the class getting the correct answer, $y = -4$. Results for this and other parts of the question are presented in the following table.

Table 6.5: Performance of learners from Class C1 in classwork during a lesson observation.

Question	Correct	Partially correct	Incorrect
1.1	22	0	4
1.2	19	4	3
1.3	23	0	3
1.4	12	13	1
1.5	3	-	-

As can be seen in Table 6.5 and Figure 6.3.14, for the four sub-questions that had been answered by the generality of the learners, most of those learners got correct responses. 19 learners managed to write down the correct equation of the asymptote of the exponential graph in the classwork. This constitutes 76,9%, with only 23,1% completely failing to get this asymptote.

The learners who failed to get the correct answers wrote the following responses per question (Table 6.6).

Table 6.6: The incorrect responses given by learners per question during a classwork in Class C1.

Question 1.1	Question 1.2	Question 1.3	Question 1.4
-4 = -4	D (-3; -25)	25	25
-4	D (x; y)	y is the real number	-
x = -4	D (-3; -11)	x > 3	-
(0; -4)	-	-	-

When assessing the responses, it became apparent where the numbers used as answers by the respective learners had been sourced. In most cases, these numbers were drawn from the questions themselves. However, there was a learner who provided the coordinates of D as D (-3; -11). Nowhere in the entire question was there a figure of -11. Upon a closer examination of the work, it was determined that these coordinates, D (-3; -11), resulted from a calculation error. The learner expanded the expression $-(x-3)^2+25$. She made an error while performing this expansion in the distribution of the negative that is outside the bracket. This distribution error resulted in an incorrect equation, and, hence, incorrect coordinates.

From the above responses for Question 1.1, two of the responses are equations but with the incorrect variables used. The mark allocation for the question was only 1 mark. The norm in mathematics is such that there are no half marks, so in this case, the learner either had to receive 0 marks or 1 mark. For their equations, the learners got 0 marks, so it could not be regarded as partially correct results. The other two responses were not equations at all. One of these responses was simply a numerical value of -4, while the other was a coordinate (0; -4).

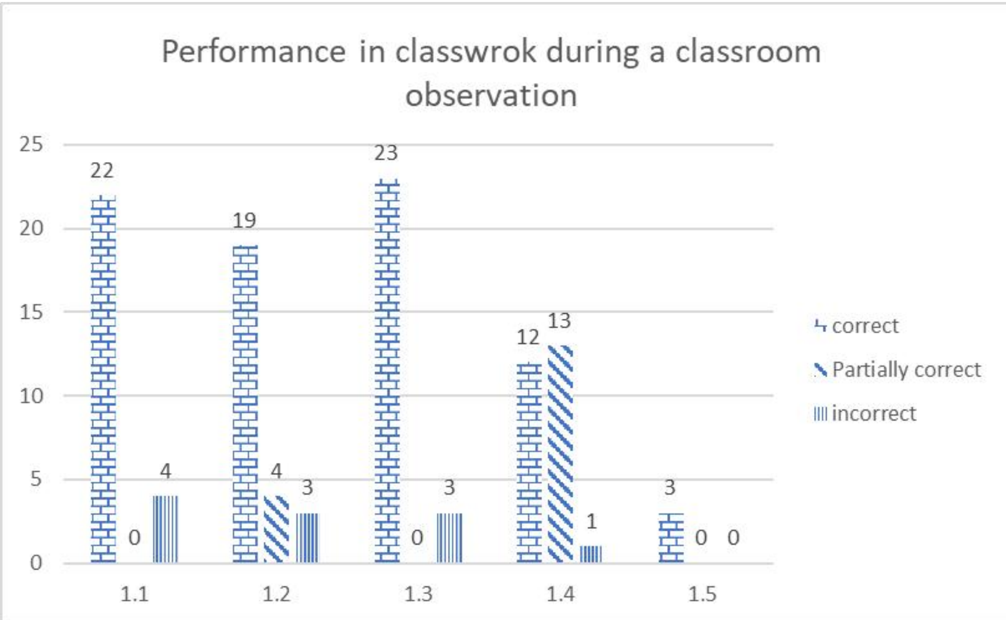


Figure 6.3.14: Performance of learners in class work during a lesson observation of Class C1.

Apart from the responses that were given by these learners, the task that was chosen by the teacher to use for this classwork was a good one. Every question in this task had an explicit mathematical vocabulary word associated with it. Some of the questions, such as 1.1, 1.5, 1.6, 1.7, 1.8, and 1.9, utilised more than one vocabulary. To me, this demonstrated that mathematical tasks within the concept of functions are more than just the manipulation of numbers. It is imperative that learners should have a good conceptual understanding before they can attempt to answer questions.

In the case of the four learners who got incorrect responses for Question 1.1, it was evident from their responses that they blundered and stumbled in their understanding of the concepts of both an equation and an asymptote.

When the lesson ended, only three learners had answered 1.5. These three learners answered it correctly. At the end of the lesson, gratitude was expressed to the teacher for granting the opportunity to collaborate with her and her learners for the research.

6.3.2 Summary of the mathematical vocabulary use during the nine observed lessons.

Class C1

During the first lesson observed for this class, 19 different mathematical terms were discussed. Some of the terms were mentioned just once, but the term function was mentioned 12 times and the term graph was mentioned seven times. In total, the terms were mentioned a combined 41 times. Of these 41 times, the teacher mentioned the terms 23 times, while the learners mentioned them five times. The learners were then asked to do classwork, in which the questions stated the terms 13 times. These terms and their frequencies were as follows: functions (12), graph (7), range (2), y-intercept (2), x-intercept (2), parameter, coordinate (2) straight line, element, set, value, domain (2),gradient, parabola, exponential, hyperbola, sine, cosine, and tangent.

Given that this mathematical vocabulary was mentioned 41 times, it demonstrated that it was imperative for learners to understand these terms. The fact that the teacher and the classwork questions have a combined number of 36 times means that a learner who did not understand this vocabulary would be excluded from the lesson. The only way of bringing the learners to the centre of the class discussion was by ensuring that they had a prior understanding of this mathematical vocabulary.

The second lesson observed included more classwork than class discussion. However, despite this, mathematical vocabulary was mentioned 18 times: 14 times by the teacher, three times by the questions, and once by the learners. The vocabulary in question were graph (7), hyperbola (3), equation, function, linear function, parabola, asymptote (3), and value.

The third and last lesson was entirely classwork. However, in this classwork, learners had to engage with 23 different instances of mathematical vocabulary: diagram, graph, cuts, x-axis (2), y-axis, turning point (2), equation, asymptote, coordinates (2), range (2), length, decreasing, average, gradient, point, reflection, straight line, tangent, and values. All of these terms were mentioned in the questions of the classwork. Neither the teacher, nor the learners mentioned any vocabulary. For this lesson, it was evident that learners needed a good understanding of mathematical vocabulary beforehand in order for them to do the classwork.

A summary of the vocabulary used by the learners, the teacher, and the text during the three lesson observations is given in the following figure.

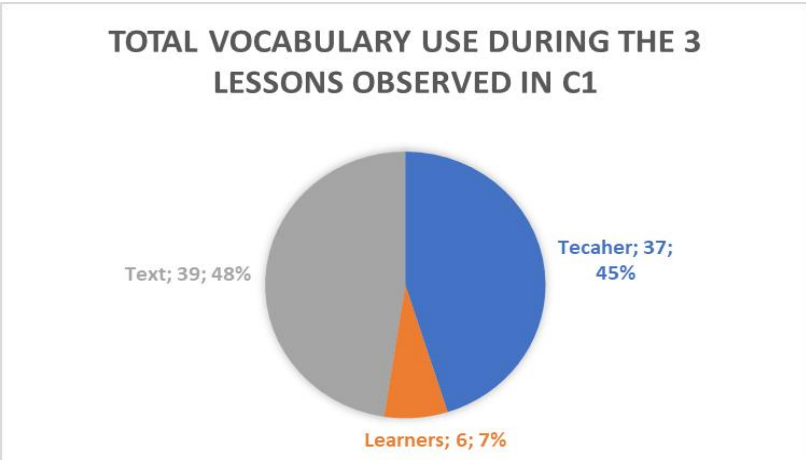


Figure 6.3.15: Vocabulary use during the lesson observations in Class C1.

As shown in Figure 6.3.15, the teacher used the vocabulary the most, at 45%. The text (comprising the textbook and exercise books/homework) make up 48% of the total vocabulary used. The novices in the discussions were the learners/ and it does not come as a surprise that their use of the vocabulary made up only 7% during the entire period of lesson observations.

Class C2

During the first lesson with this class, 18 terms were mentioned and engaged with by the teacher, learners, textbook, and homework. The teacher mentioned the vocabulary 11 times, the homework mentioned it seven times, the learners mentioned it seven times, and the textbook questions mentioned it eight times. This totals 33 instances of mathematical vocabulary use.

The 18 terms that were mentioned and deliberated on were as follows: turning point (4), *x*-intercept (4), *y*-intercept (4), graph (3), domain (2), range (2), axis of symmetry (2), quadratic function, vertical shift, horizontal shift, coordinates (2), parameters, parabola, axes, maximum, minimum, decreasing, and increasing.

As for the second and third lessons, the patterns were not especially different from the first lesson. In the second lesson, 17 terms were used. These terms were used 30 times during the lesson. The teacher used this mathematical vocabulary 17 times, the learners used it eight times, and the questions in the textbook mentioned the vocabulary five times. The 17 terms that were used during the lesson were: general equation (2), hyperbola (5), parameters (3), steepness, curve, domain, range, *x*-intercept, *y*-intercept, quadrants, coordinates, graph, turning point, asymptote, horizontal, and vertical.

During the third observed lesson, the class discussed the following 25 terms: vertical upward shift, vertical downward shift, exponential function (5), horizontal asymptote, units, decreasing, increasing, reflection, direction, shape, not defined,, function, trigonometric functions, base, variables, exponent, equation, function (2), system of axis (2), graph (2), table (2), asymptote (2), range (2), domain (2) and intercepts (2). These terms were used 21 times by the teacher, and 16 times by the mathematical task that the teacher gave to the learners to do as classwork. As this was mainly classwork, learners did not have an opportunity to say any mathematical terms aloud in a general class discussion, except at the micro level that was not possible to capture. Overall, vocabulary use during the three lessons observed with this group was as follows:

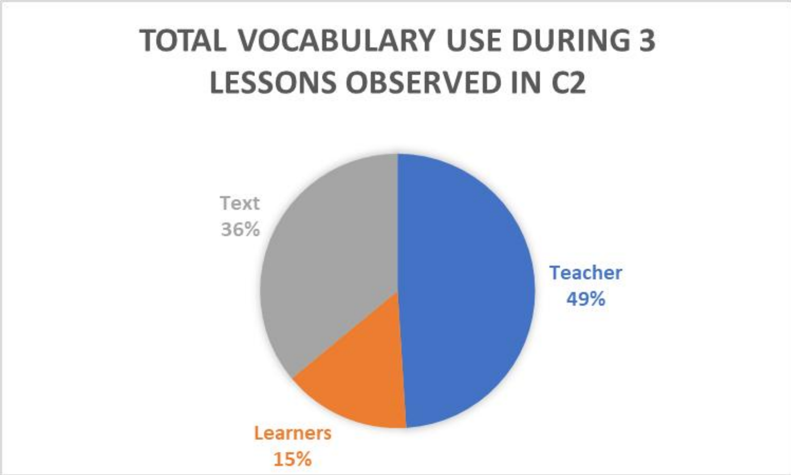


Figure 6.3.16: Vocabulary use during the lesson observations in Class C2.

It is evident from Figure 6.3.16 that the teacher was the main user of mathematical vocabulary. Her vocabulary use makes up 49 % of the discourse, followed by the text, which takes up 36 % of the vocabulary use. The learners make up only 15% of the vocabulary used.

Class D

During the first lesson, 15 different terms were used a total of 49 instances. Of these 49 times, the teacher mentioned the vocabulary 13 times, while the learners had a combined use of the vocabulary seven times. The rest of the use of the vocabulary was in the text, 29 times. In this case, the text included the written words in the textbook, as well as on the chalkboard. The 15 terms used during this lesson were: points (6), intercepts (5), *y*-axis, turning point (9), graph (3), equation (5), function, coordinates (7), parabola (4), *y*-intercept, line, curve (2), *x*-axis (2), shape, and value.

The second lesson that I observed in this class comprised classwork, where the teacher asked the learners to do a task from their textbook. However, before the learners could do the classwork, the teacher demonstrated with an example on the board. During the class deliberations and the task given to the learners, the following 13 terms were used: graph (2), set of axes, equation (4), asymptote (5), coordinates (4), *x*-intercept (4), *y*-intercept (4), intercepts (2), axes, symmetry lines (2), reflection (2), *x*-axis (3), axes of symmetry. These terms were used a total of 35 times during the lesson. The teacher used this mathematical vocabulary 12 times, while the learners did not engage in talk that made use of the mathematical vocabulary. The remaining 23 instances of use of the mathematical vocabulary during this lesson was done by the text.

During the third lesson that was observed for this class, 14 mathematical terms were used a combined 27 times. The distribution of this use was 16 times by the teacher, eight times by the learners, and three times by the text. The terms used were: equation, exponential function, method, function, asymptote (2), shifts (2), graph (4), downwards (2), value (2), intercept (2), line, coordinates, point, and value (6).

The distribution of the use of the terms is summarised in Figure 6.3.17. As shown in the figure, over the entire duration of the lesson observations, the teacher used mathematical vocabulary 37% of the time that vocabulary was used. For this class, the most use of mathematical vocabulary was by the text that accounted for 50% of the times. The learners only used the vocabulary 13% of the times that mathematical vocabulary was used during teaching and learning.

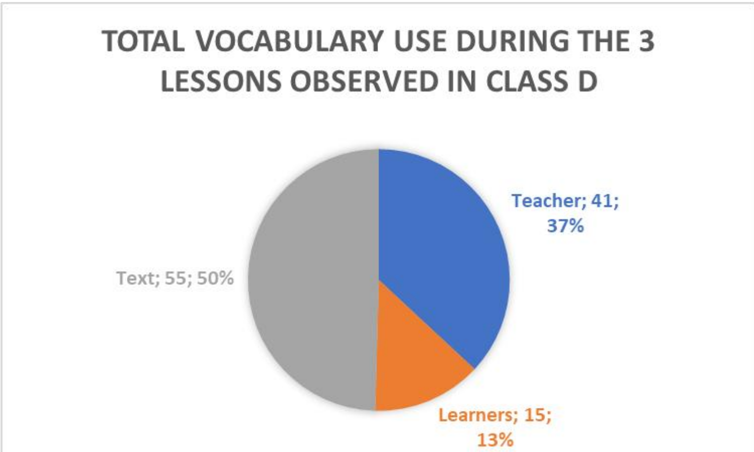


Figure 6.3.17: Vocabulary use during the lesson observations in Class D.

Overview

Over the period of lesson observations in Class C1 (three lessons), correct mathematical vocabulary was used 82 times. The text was the main provider of the mathematical vocabulary (48%); followed by the teacher (45%), and the learners (7%) (Figure 6.3.18(a)). In Class C2, the mathematical vocabulary was mentioned 100 times. The text provided 36%, the teacher 49%, and the learners 15% (Figure 6.3.18(b)). In Class D, where mathematical vocabulary was mentioned 101 times, the text provided 45%, the teacher 41%, and the learners 15% (Figure 6.3.18(c)). Overall, the text contributed 42% of the mathematical vocabulary, the teachers 49%, and the learners 15% (Figure 6.3.18(d)).

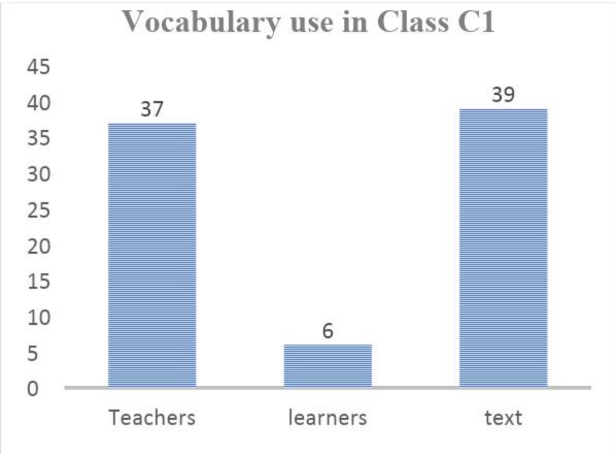


Figure 6.3.18(a): Overall vocabulary use in Class C1.

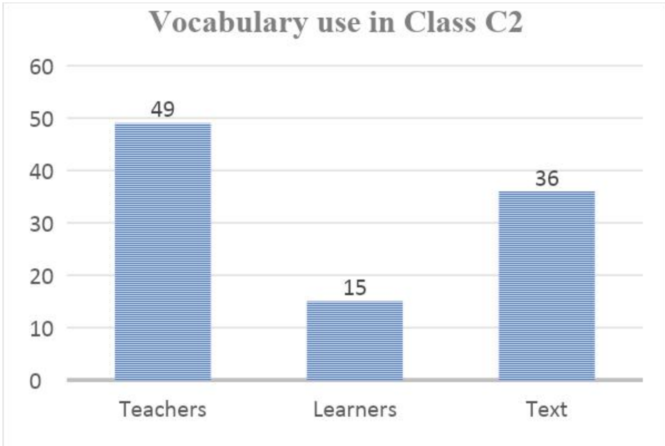


Figure 6.3.18(b): Overall vocabulary use in Class C2.

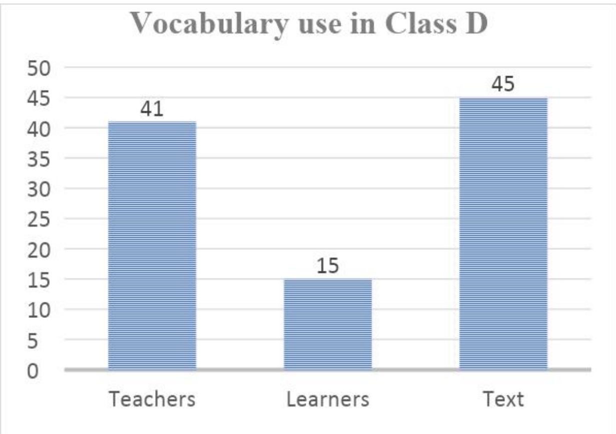


Figure 6.3.18(c): Overall vocabulary use in Class D.

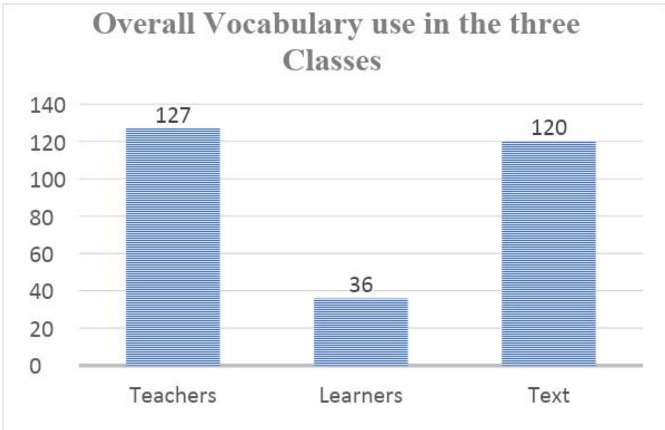


Figure 6.3.18(d): Overall vocabulary use across all three classes.

These results show that the teachers and the text provided far more vocabulary during the observed lessons than the learners.

6.4 Post-test results

6.4.1 Discussion of the results for Question 1

The results show that of the 157 learners who sat for the post-test, 27 managed to write more than 10 correct mathematical vocabulary associated with the mathematical object in the question. Of these 27 learners, 20 wrote between 11 and 20 correct vocabulary and the remaining seven wrote more than 20 correct vocabulary. Nine learners did not attempt to answer the question. 121 learners only managed to come up with a maximum of 10 vocabulary words. The breakdown of this result is shown in the subsequent figure.

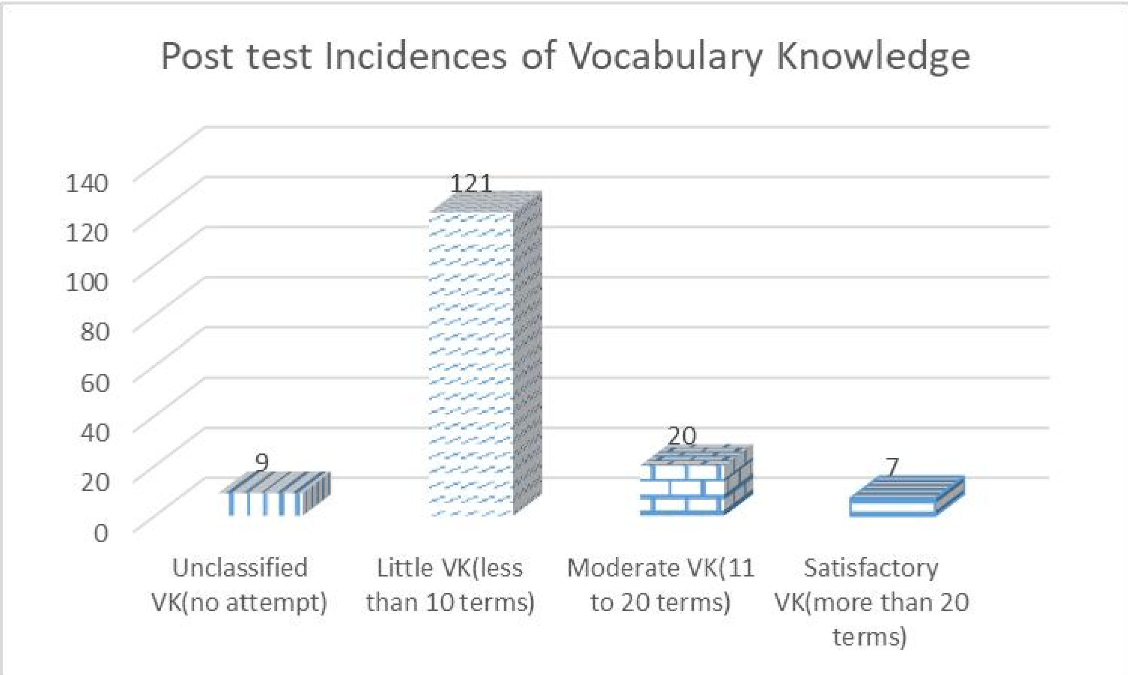


Figure 6.4.1: Overall results on vocabulary knowledge in Question 1 of the post-test.

As shown in Figure 6.4.1, nine learners did not answer the question at all. These are the learners who are recorded under the code Unclassified VK. The remaining 148 learners managed to write various different quantities of vocabulary. The distribution is presented in Figure 6.4.2.

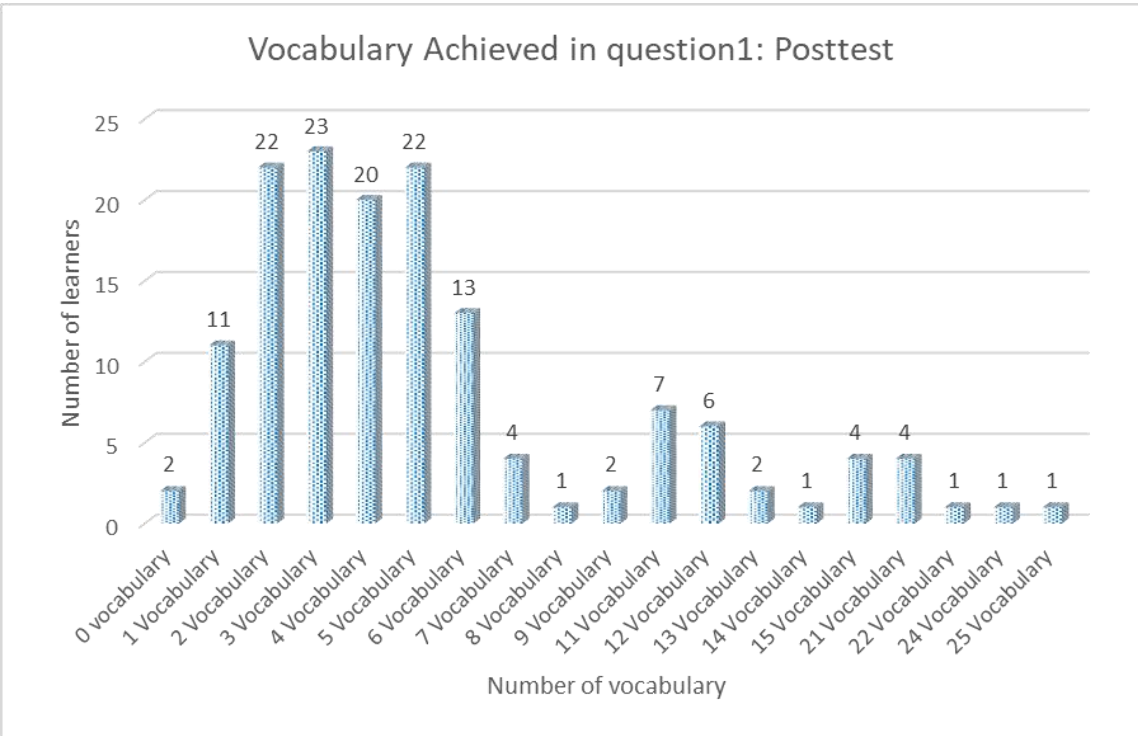


Figure 6.4.2: Breakdown of the VK performance for Question 1 of the post-test.

The distribution shows that 63,9% of the learners managed between two and six correct vocabulary. This is a poor performance, given that the learners had just concluded learning the chapter on functions. Of the 27 learners who managed more than 10 vocabulary, the modal number was 11 terms, with only one apiece for 22, 24, and 25 terms.

The work of the two learners who managed zero terms is presented in the following excerpts (Figures 6.4.3(a) and 6.4.3(b)).

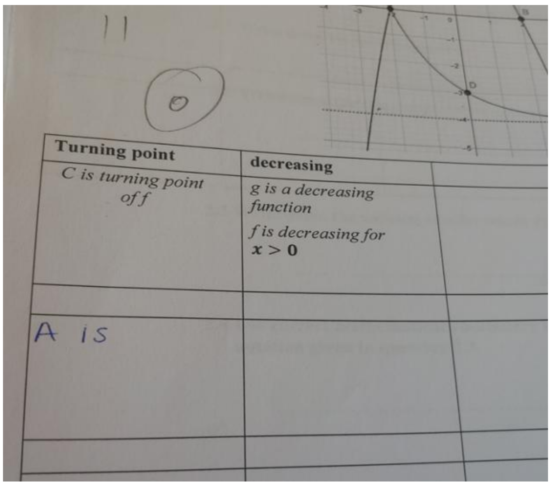


Figure 6.4.3(a): Example of no correct mathematical vocabulary by Learner 1.

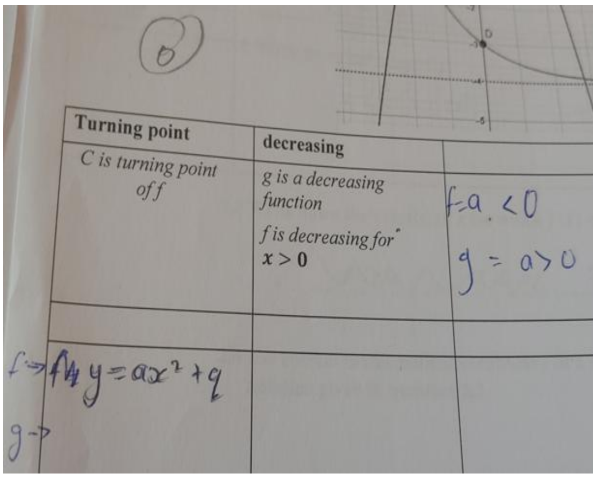


Figure 6.4.3(b): Example of no correct mathematical vocabulary by Learner 2.

Learner 1's work was an incomplete statement, and this could not be accepted as correct mathematical vocabulary. In the case of the second learner, there were four lines in the work: two in the first column and two in the third column. The first sentence in the first column was the general equation of parabola with a vertical shift. This was true for the parabola in question, but the instruction of the question was to write mathematical vocabulary associated with the mathematical object. It would have been acceptable had the learner written the word "equation" (written), instead of writing the equation (symbolic). The second line in the same column was an incomplete sentence, thus it was not accepted as correct vocabulary. The two sentences in the third column were also symbolic notations, rather than as required by the question.

In this test, there were 37 vocabulary words that were written by the learners, and these were: asymptote, **range**, domain, intercept, increasing, intersection, coordinates, y-intercept, x-intercept, exponential, parabola, symmetry, point, maximum, hyperbola, equation, graph, tangent, minimum, period, vertical, horizontal, concave, shift, quadratic, distance, table, line, parallel, value, quadrant, x-axis, y-axis, function, gradient, negative, and positive.

Though some learners remembered some vocabulary associated with the object in question, in many instances, the learners could not demonstrate their understanding of what the vocabulary entailed. As an example, the following excerpt (Figure 6.4.4) presents a case where a learner repeated the terms that were already given as examples in the question. This learner attempted to exemplify further with the given terms and ended up writing incorrect responses. With the term "turning point", this learner stated that D was a turning point of g. This is incorrect, because g is an exponential function. An exponential function does not have a turning point.

The learner further wrote something about the term "decreasing". In the same answering space, the learner mentioned that "g is a decreasing function". Though this is correct, the learner could not be regarded as having gotten it correct, because the learner copied the given example. Stating that "f is decreasing" was also not accepted as a correct answer, because the phrase was copied from the example, but in this case, the learner did not complete the statement. The function f decreases for $x > 0$.

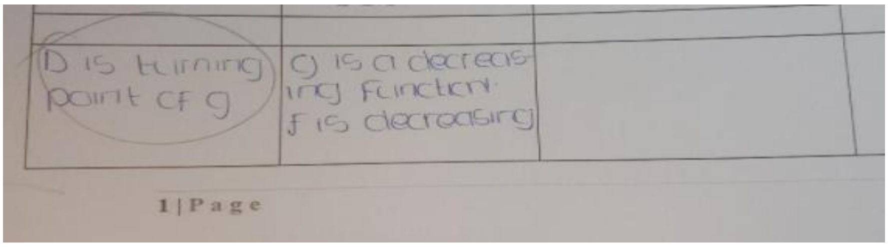


Figure 6.4.4: Example of an incorrect description of the behaviour of a parabola.

In the case of the following learner's work (Figure 6.4.5), though the learner wrote an acceptable term, "parabola", the attempted qualification of the term could not be accepted. The phrase "sad face", as a way of describing the parabola, is not a universally acceptable description. The parabola in question was a maximum value parabola, denoting that it had a maximum turning point.

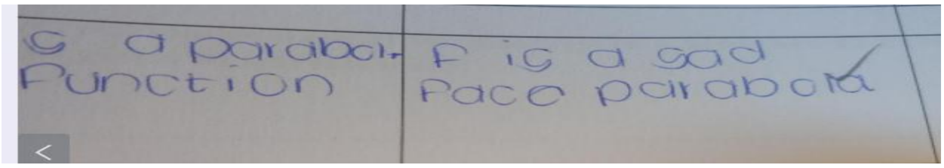


Figure 6.4.5: Example of a description of a parabola that could not be accepted.

Figure 6.4.6 shows the work of a learner who, like many others, wrote the terms "range" and "domain" concomitantly. Though the vocabulary was accepted as correct, the ensuing explanation/elaboration was incorrect. This was incorrect due to the notation used. In the notation used, the learner implied that positive infinite is smaller than negative infinite, which is incorrect.

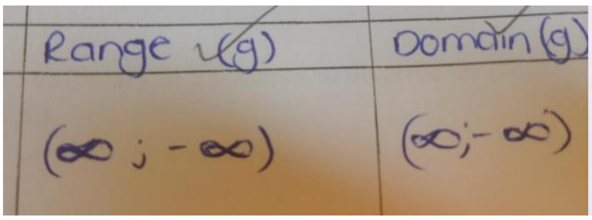


Figure 6.4.6: Example of an incorrect comparison between positive and negative infinite.

Another learner (Figure 6.4.7) who used the notation “infinite”, did so with respect to the term domain. The learner also used incorrect notation, because the domain should have been used together with the independent variable, *x*. Furthermore, the notation should have had a negative infinity on the lower end, and a positive infinity on the upper end, if it were to be correct.

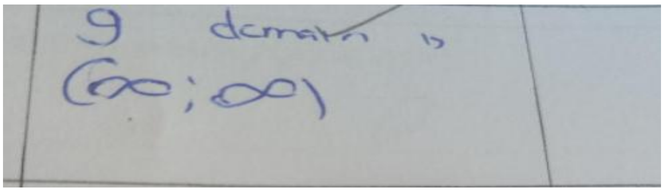


Figure 6.4.7: Example of the incorrect placement of positive infinity on both ends of the notation.

The preceding examples are cases where, though a correct term was written, the learner was not proficient in his/her articulation of the term in question.

With 37 terms recorded by the learners, a division was made into two groups based on their frequency. The terms that appeared more than 15 times were categorized as "prevalent" terms, while the remaining terms were placed in the "less prevalent" group. These terms and their respective frequencies are displayed in Figures 6.4.8 and 6.4.9.

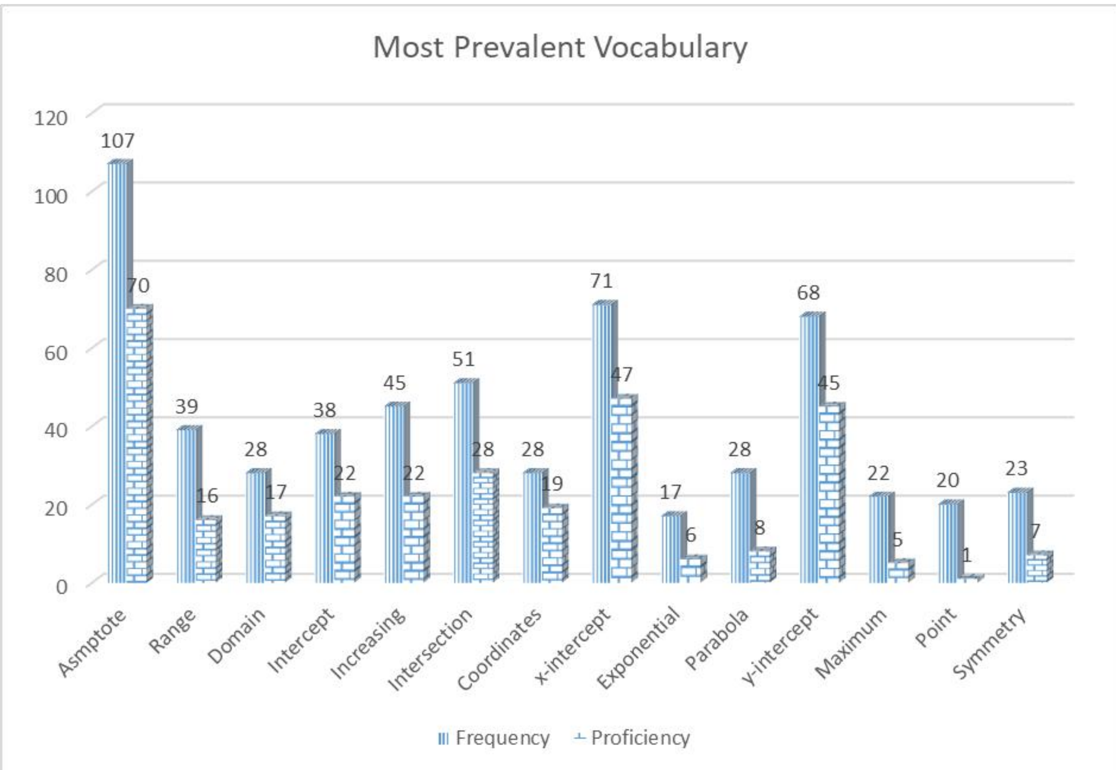


Figure 6.4.8: Frequency and proficiency of most prevalent vocabulary in the post-test.

The prevalent vocabulary from the learners' work, presented in Figure 6.4.8, shows that the most common vocabulary was asymptote. 107 of the 157 learners who took part in the post-test wrote this term. Of the 107 who wrote the vocabulary, 70 of them, constituting 65,45%, managed to articulate this vocabulary by way of identifying the line $y =- 4$ as an asymptote. An asymptote is only an asymptote to a particular graph, however, of the 107 learners who wrote the term asymptote, only 17 wrote the term exponential function/graph. Of the 17 who wrote the term exponential, 13 wrote both the terms asymptote and exponential.

In addition to the term asymptote, two other terms that were common from the learners' responses were x-intercept and y-intercept, with frequencies of 71 and 68 respectively. The terms x-intercept and y-intercept appear concomitantly in both the learners' work and the learners' proficiency to be almost identical. A learner who was proficient with one term was also likely to be proficient with the other.

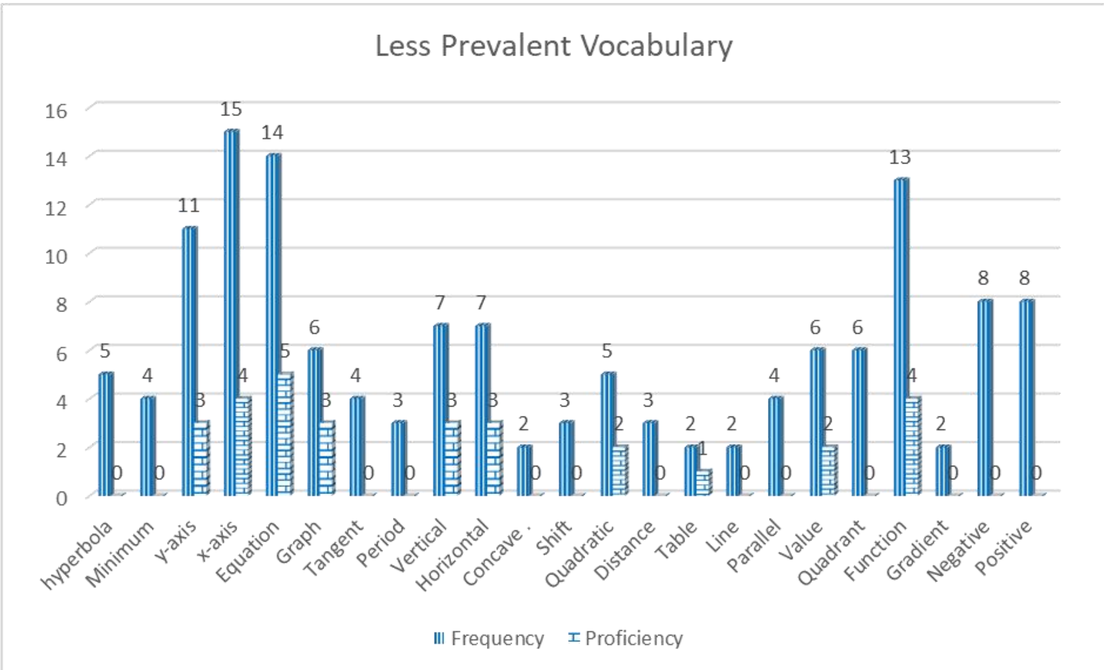


Figure 6.4.9: Frequency and proficiency of the less prevalent vocabulary in the post-test.

From the graph, the terms y-axis and x-axis, equation, and function were the most common in this group of terms. Though the terms positive and negative appear the same number of times, coincidentally no learner managed to explain them or refer to the graph to illustrate where they appear on that graph. Other terms that had a concomitant appearance were vertical and horizontal, which had a frequency of 7 each. Unlike the terms positive and negative, which could not be explained by the learners who wrote them, the terms vertical and horizontal were explained by three of the seven learners who wrote them. Overall, of these 23 less prevalent terms, 13 could not be explained by the learners who wrote them.

6.4.2 Discussion of the results for Question 2

Question 2 had four parts, the first three of which tested learners' CUPP. The results of these first three parts are presented in Figure 6.4.10.

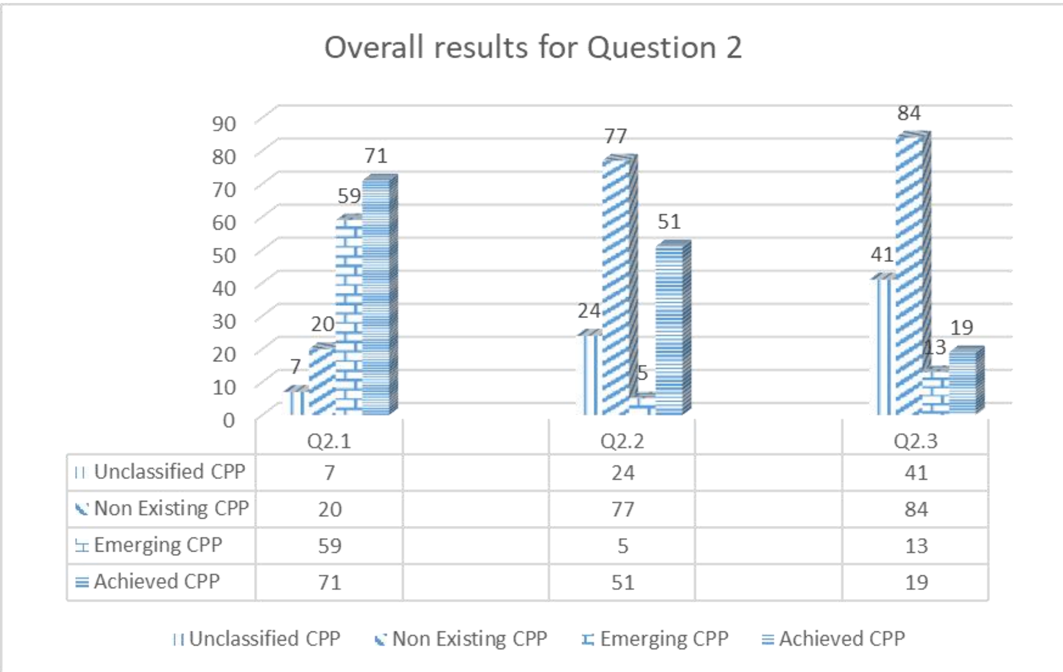


Figure 6.4.10: Overall CUPP results for Question 2 of the post-test.

In Question 2.1, learners were asked to write down the coordinates of Point C, the turning point of a parabola. 71 learners (45,2%) got the coordinates correct, $C\ (0, -4)$. An example of a correct learner response is as shown in Figure 6.4.11.

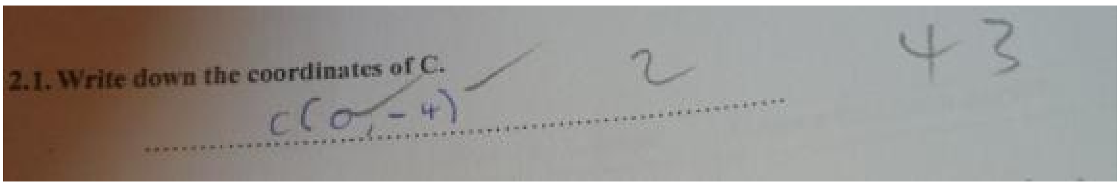


Figure 6.4.11: Correct coordinates of C in Question 2.1 of the post-test.

Because C is a point on the Cartesian plane, the learners were assessed on the x-value, and the y-value separately, though the answer was supposed to be written in coordinate form (x, y). 59 learners got either the x-value or the y-value correct, but not both, such as in Figure 6.4.12.

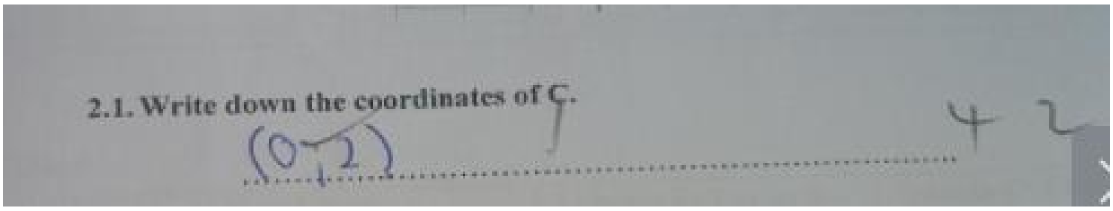


Figure 6.4.12: Partially correct coordinates of C in Question 2.1 of the post-test.

These learners were considered to have Emerging CUPP, and they constituted 36,6% of the total learners.

12,7% wrote completely incorrect responses. Incorrect responses consisted of incorrect values for both x and y, or interchanged correct values of x and y, such as in Figure 6.4.13.

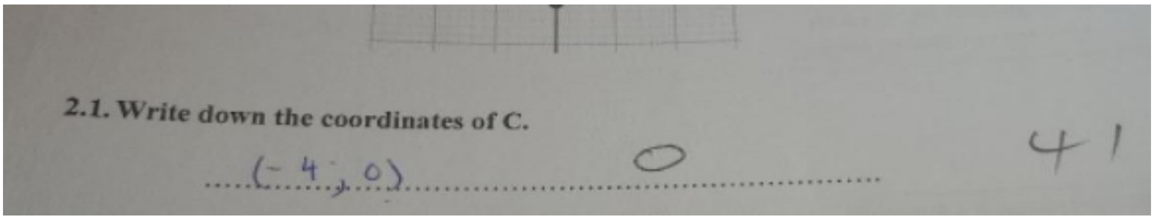


Figure 6.4.13: Incorrectly swapped x and y-values in Question 2.1 of the post-test.

Point $(-4,0)$ is a point on the x-axis. Giving this coordinate as a turning point of the function showed that the learner demonstrated no understanding of what he/she was confronted with, as the correct turning point is on the y-axis.

In the preceding learners' work, I appended some codes to show how the learners' work were rated, for example. 43 to denote Achieved CUPP, 42 denoting Emerging CUPP, and 41 to denote Non-Existing CUPP.

Part 2.2 of Question 2 solicited learners' understanding of the concept of the range of a function. Learners were asked to write down the range of function f, whose graph was shown on the Cartesian plane. The answer to this question was directly related to the answer given in

Question 2.1. The parabola in question was a minimum value parabola, whose turning point was $C(0, -4)$. Learners needed to first realise that the range was associated with the dependent variable, y. Consequently, their responses were supposed to contain this variable. This would be irrespective of the notation used, interval or set notation.

Possible responses expected for this question were: $y \geq -4$ OR $-4 \leq y < \infty$ OR $y \in [-4, \infty)$. From the learners' responses, 24 learners did not respond to this question at all and 77 attempted the question but wrote completely incorrect responses. These responses were incorrect for various reasons, and examples of such responses are presented here.

- **Using x, the independent variable in the range**

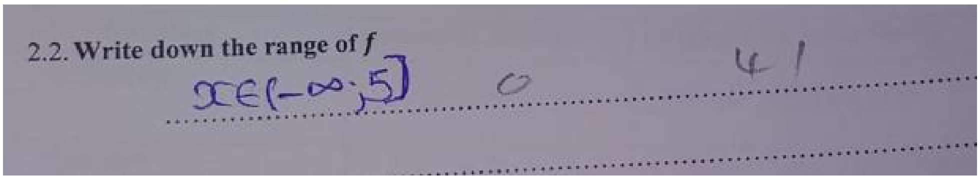


Figure 6.4.14: Example of an incorrect response where x was used in the range in the post-test.

In addition to using an incorrect variable, this response (Figure 6.4.14) does not have any correct element of the expected response. Even if the variable, x was to be replaced with the correct variable, y, this response would still be far from resembling the parabola in question.

- **Not using any variable in the response**

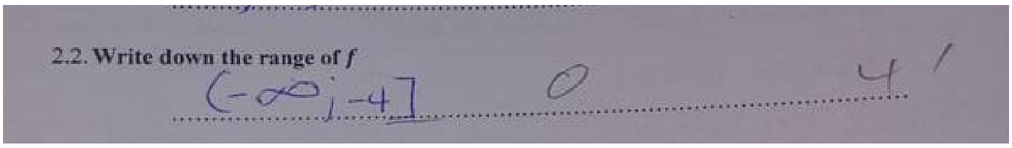


Figure 6.4.15: Example of an incorrect response where no variable was used in the range in the post-test.

In this response (Figure 6.4.15), the learner included one correct aspect of the range in question. The order of the values in the brackets is also correct, but the range itself is incorrect. The order implies that the parabola has a maximum value at $y = -4$. This is incorrect because the parabola in question has a minimum value at $y = -4$.

- **Using the correct component, -4, but without any notation**

Figure 6.4.16: Example of an incorrect response where no notation for range was used in the post-test.

Writing a response such as the one given by this learner (Figure 6.4.16) showed that the learner did not know what a range implied. A range is an aspect of a function that explains the extent of the function along the y-axis. That being the case, writing a range without associating it with a variable is a sign of a lack of knowledge of this concept.

- **Writing the term range itself in the response, and then relating it to a numerical value, -4**

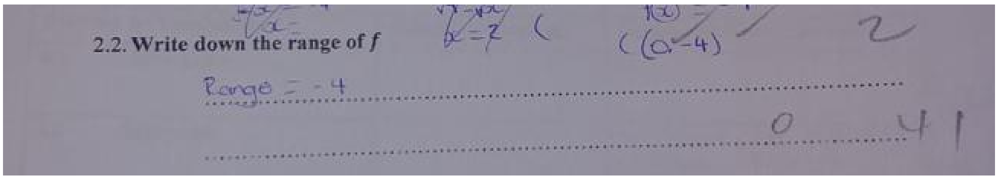


Figure 6.4.17: Example of an incorrect response where the word "range" was used instead of a variable in the post-test.

This (Figure 6.4.17), and the subsequent work (Figure 6.4.18) are illustrations of a lack of understanding of the concept in question. These learners wrote the range as equations, which is completely incorrect. Even if the left-hand side of their equations were to be replaced with the correct variable for a range, their responses would still remain incorrect.

- **Using the notation of the function, f, in writing its range**

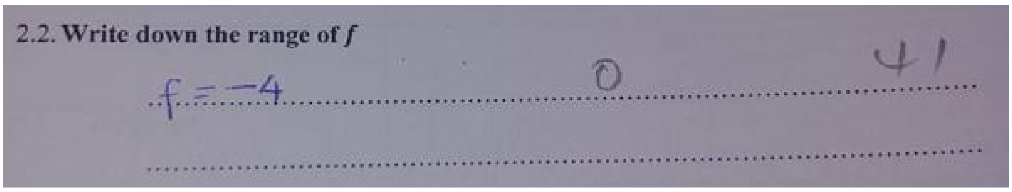


Figure 6.4.18: Example of an incorrect response where the notation "f" was used instead of a variable in the post-test.

- **Making a numerical calculation and writing the range as a numerical value**

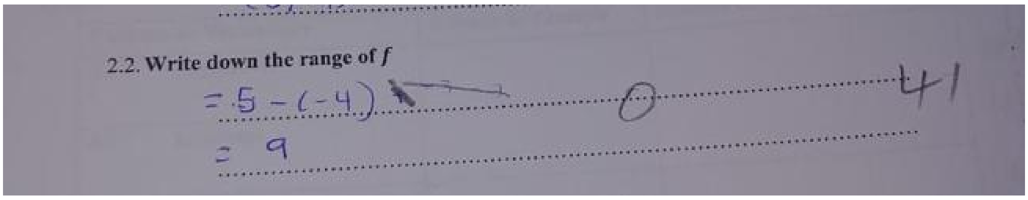


Figure 6.4.19: Example of an incorrect response where a numerical calculation for the range was made in the post-test.

In this case (Figure 6.4.19), the learner decided to make a calculation. In their calculation, the learner plugged in two values, one of which was taken from the turning point of the graph. The other value, 5, was taken from one of the points on the graph, $E(-3{,}5)$. By subtracting these two values, it shows that the learner interpreted the range as the difference between the maximum and the minimum values of a data set. Of the given y-values in the narration of the question, the least was -4 and the highest was 5. This shows that the learner knew that a range is associated with y-values but mixed up the term with how it is used in statistics.

- **Making use of the correct variable, y, but with incorrect values**

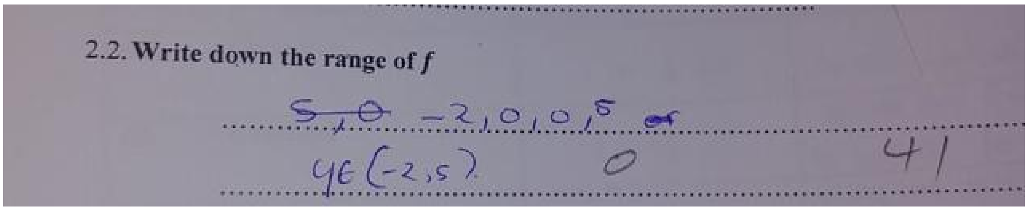


Figure 6.4.20: Example of an incorrect response where the notation "f" was used instead of a variable in the post-test.

This (Figure 6.4.20) is a complicated response because it is not easy to figure out where the learner got the values that were plugged into the interval. The nearest values could have been taken from the Points A (2; 0) and E (-3; 5). If this was the case, then this learner mixed up which values to include in the range, as 2 comes from the set of x-values. while 5 from the y-values.

The other learner who used the correct variable, but incorrect values in the interval has their work presented in Figure 6.4.21. In this learner's work, the parabola has been changed to a maximum value parabola, whose maximum value is $y =- 4$. This is incorrect, thus the code assigned to this work is also 41.

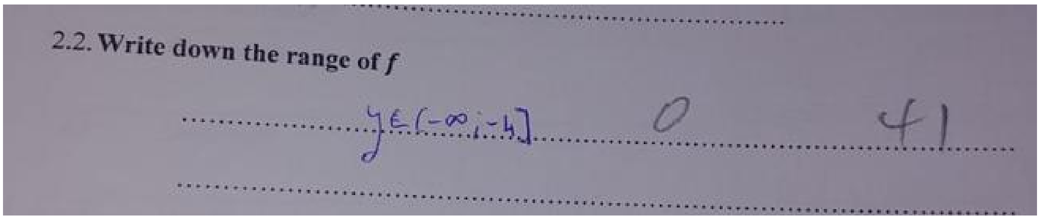


Figure 6.4.21: Example of an incorrect response where the range for a maximum rather than minimum value parabola was written in the post-test.

- **Using both the variables x and y in the response**

Figure 6.4.22 shows one of the rarer responses out of all the learners' responses. The first line, which contains the variable y, shows that the learner considers 5 to be smaller than 0, which is mathematically incorrect. The second part of their response is incorrect, because it contains the variable x.

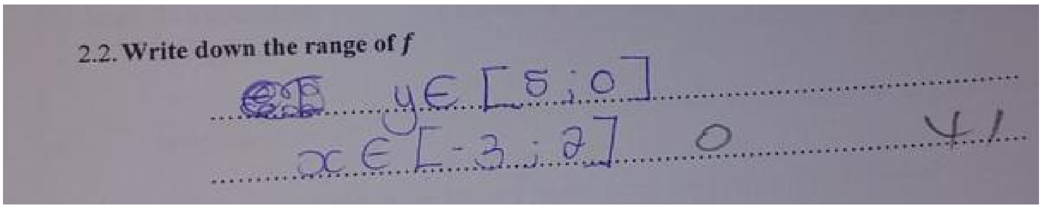


Figure 6.4.22: Example of an incorrect response where values for both x and y were given in the post-test.

Question 2.3 required learners to write down the values of x for which $f(x) < g(x)$. The response could have been given in either set notation or interval notation. As the question explicitly stated that the values solicited were to be values of x only, what was looked for in the responses were x values and either notation. For this reason, a response such as $(-3, 2)$ would be accepted as correct, though the most appropriate way of presenting it would have been $x \in (-3; 2)$ or $-3 < x < 2$.

In the subsequent section of the work, results for Question 2.3 are presented.

A grand total of 125 learners encountered considerable difficulty in obtaining even a single mark on Question 2.3. Among this group, 41 learners opted not to respond to the question altogether, while the remaining 84 made attempts but their responses were entirely incorrect.

Nevertheless, there was a subset of 32 learners who were able to secure at least 1 mark. Within this subset, 13 learners received a score of 1 mark, while an additional 19 impressively attained the highest possible score of 2 marks. This compelling data serves as clear evidence of the overall poor performance of learners in this particular aspect of the assessment. Excerpts from some of the 84 learners who wrote completely incorrect responses are presented in Figure 6.4.23 to show why they were disregarded.

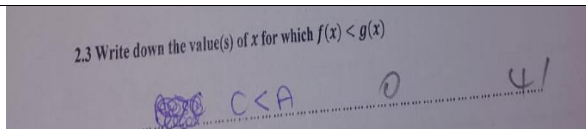
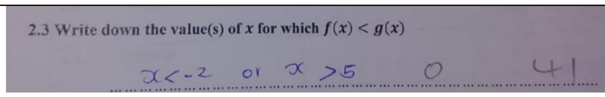
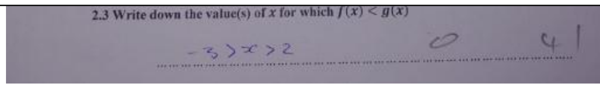
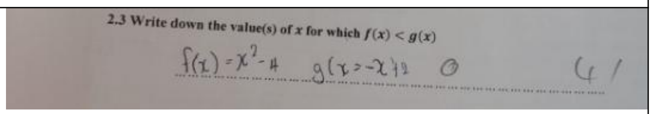
Use of an inequality with letters of points on the graph. A and C are given points on the graph and cannot be used as values of *x*, as required in the question.	
Separate inequalities with the connector OR. The required values of *x* are between two values, so the appropriate connector is AND.	
Correct values -3 and 2 are present in an inequality but are reversed. This now implies that -3 is greater than 2, which is incorrect.	
Repeated the given equations from the question, thus this does not answer the question.	

Figure 6.4.23: Excerpts of incorrect responses to Question 2.3 of the post-test.

The last part of Question 2 (2.4) assessed learners' ability to explain a segment of a graph, where one function was greater than the other in words. In Figure 6.4.24, the learners' results presented for this question show that 56,7% (89 out of 157) of the learners did not attempt to answer the question. 31,8% attempted the question but got it completely incorrect. Of the 157 learners, 10 had partially correct responses and only eight had correct responses.

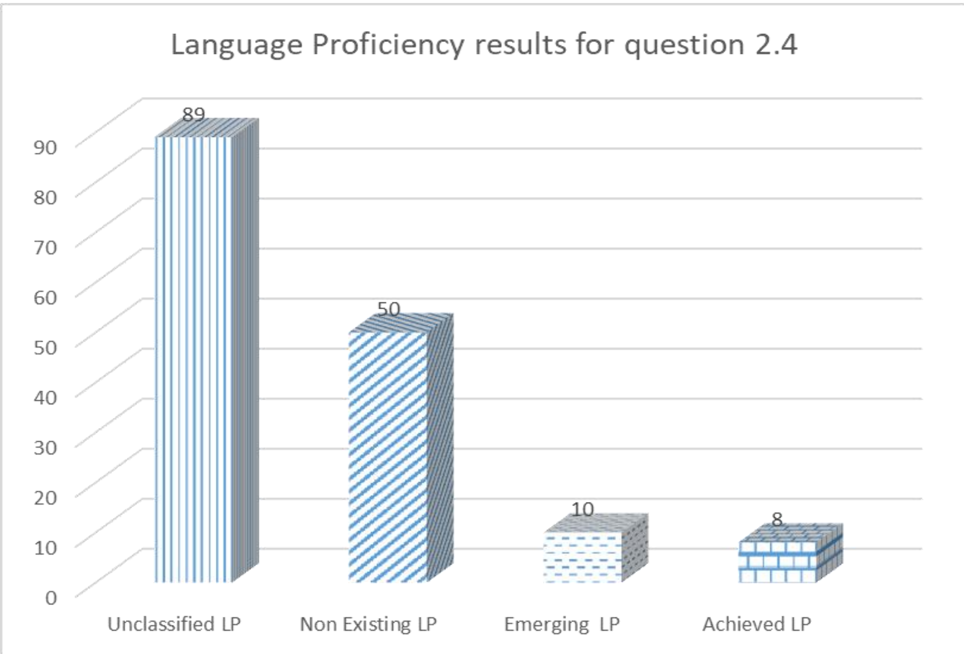
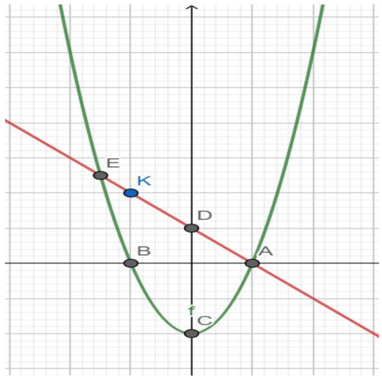


Figure 6.4.24: Overall performance in LP in Question 2.4 of the post-test.

The result presented in Figure 6.4.24 shows that this question was poorly answered by the learners. It is a rare question in mathematics, as it asked learners to explain in words the response that they had given in the previous question. Despite this being a rare question, this does not mean that learners will not come across questions, such as this one, that require them to articulate their responses in mathematics. The following Table 6.7 and Figure 6.4.25 give an overview of the responses given by learners in Question 2.

Table 6.7: Summary of responses per sub-question per code in Question 2 of the post-test.

The graphs of $f(x) = x^2 - 4$ and $g(x) =- x + 2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK || $y - axis$. f and g intersect at A $(2;\ 0)$ and E $(-3;\ 5)$.



	Unclassified CUPP	Non-Existing CUPP	Emerging CUPP	Achieved CUPP	0	1	2
Q2.1. Write down the coordinates of C.	7	20	59	71	27	59	71
Q2.2. Write down the range of f	24	77	5	51	101	56	
Q2.3. Write down the value/s of x *for which* $f(x) < g(x)$.	41	84	13	19	125	13	19
	Unclassified LP	Non-Existing LP	Emerging LP	Achieved LP			
Q2.4. Use correct mathematical vocabulary in a sentence to describe the answer (algebraic notation) given in question 2.3.	89	50	10	8			
Total Responses per Code	161	231	87	149			

The total responses per code in this Question 2 are presented in the following bar graph. Given that 157 learners took part in the post-test, it meant that a total of 628 responses were expected from this question. Of the 628 responses given by the learners, 149 of them showed Achieved CUPP which translated into 23,7%. Emerging CUPP comprised 13,9% of the total responses, while 36,8% and 25,6% were for Non-existing CUPP and Unclassified CUPP respectively.

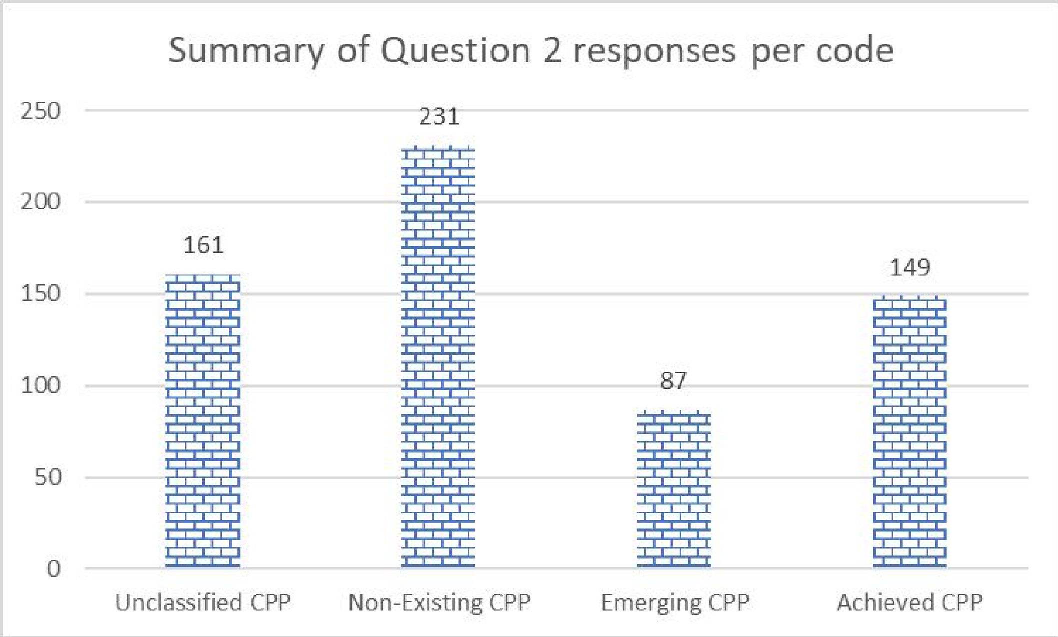


Figure 6.4.25: Summary of the responses to Question 2 of the post-test per code.

The majority, 71 out of 149 (47,7%), of the Achieved CUPP responses came from Question 2.1, where learners were asked to write down the coordinates of the turning point of a parabola. Overall, Question 2.4 had the least number of responses that showed Achieved CUPP. This means that most learners could not explain in words the region of the graph where $f(x) < g(x)$. This result is related to the result in Question 2.3, where the learners also failed to describe the said region algebraically.

6.4.3 Discussion of the results for Questions 3 and 4

Question 3 and Question 4 were closely related. The former required learners to define given mathematical vocabulary associated with functions, while the latter required them to demonstrate their proficiency in working with that same vocabulary that they had defined. There were four terms that were dealt with for this purpose: asymptote, intercept, range, and domain. In the subsequent presentation, results for these two questions are provided and discussed.

The first term to be discussed is "asymptote", which appeared in Questions 3.1 and 4.1. The results are provided in Figure 6.4.26.

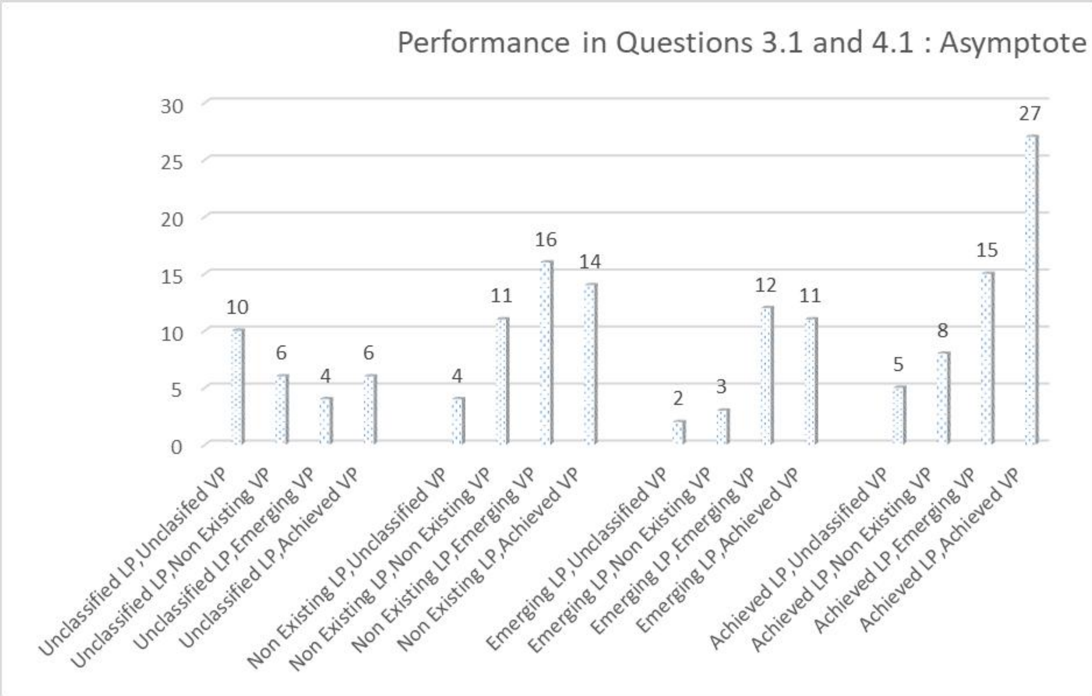


Figure 6.4.26: Relating LP and VP for the term "asymptote" in Questions 3.1 and 4.1 of the post-test.

The result presented here regards learners who attained a particular result in Question 3.1. The results of these learners were then compared to their results in Question 4.1. The reason for doing so was to find out if there was any correlation between the responses in Question 3.1 and performance in Question 4.1. In Question 3.1, learners were asked to define the term asymptote. In Question 4.1 the learners were asked to match the term asymptote (Column A) to its correct example (Column B), and correct characterisation (Column C). It was discovered that 10 learners who did not answer Question 3.1 also did also not answer Question 4.1. However, it was also discovered that six learners who did not define the term asymptote in Question 3.1, managed to answer correctly in Question 4.1.

In this analysis, whenever the code for an aspect was Unclassified or Non-existing, I deemed knowledge of that aspect to be poor. On the other hand, when the code was Emerging or Achieved, knowledge of that aspect was deemed to be good.

From Figure 6.4.1, it is evident that poor LP was associated more with poor VP. Conversely, a good LP tends to be more associated with a good VP. In fact, attempting to define the term Intercept (Non-existing, Emerging or Achieved) tends to be associated with an improved result for VP. For instance, 16 learners who attempted to define the term Asymptote demonstrated emerging proficiency in Question 4.1. In addition, 14 learners who attempted to define it got the characterisation and exemplification right.

In Figure 6.4.26, the tallest bar is that for the Achieved LP, Achieved VP. This points to the fact that when learners managed to define the term asymptote, they were also most likely to manage to exemplify and characterise the term too.

Figure 6.4.27 provides results for Questions 3.2 and 4.2, both of which dealt with the term "intercept".

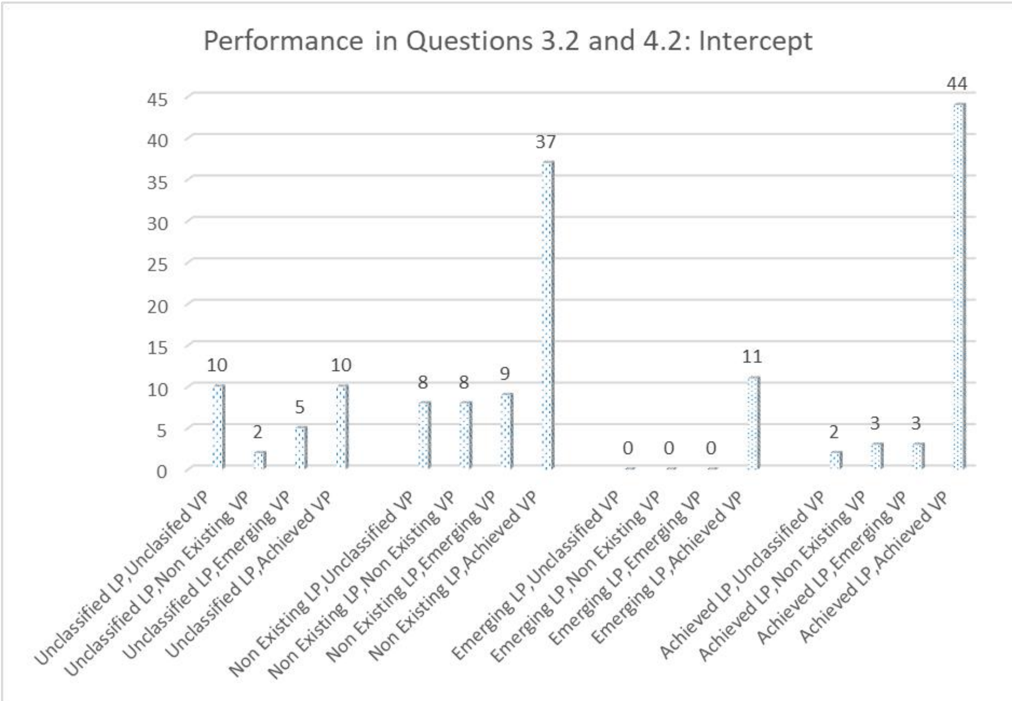


Figure 6.4.27: Relating LP and VP for the term "intercept" in Questions 3.2 and 4.2 of the post-test.

For the term intercept, the trend of poor LP associated with poor VP seems not to hold throughout. This is seen in the totals of the first four bars, where the sum of the first two bars is 12, compared to the sum of the third and fourth bars, which is 15. This implies that despite no attempt to define the term intercept, 12 learners had a poor VP compared to 15 with good VP. However, of the learners who had completely incorrect definitions, 16 had poor VP and 46 had good VP. This indicates that attempting to define the term is associated more with good VP than poor VP. Furthermore, 44 learners who had correct definitions also had correct exemplification and characterisation of the term intercept.

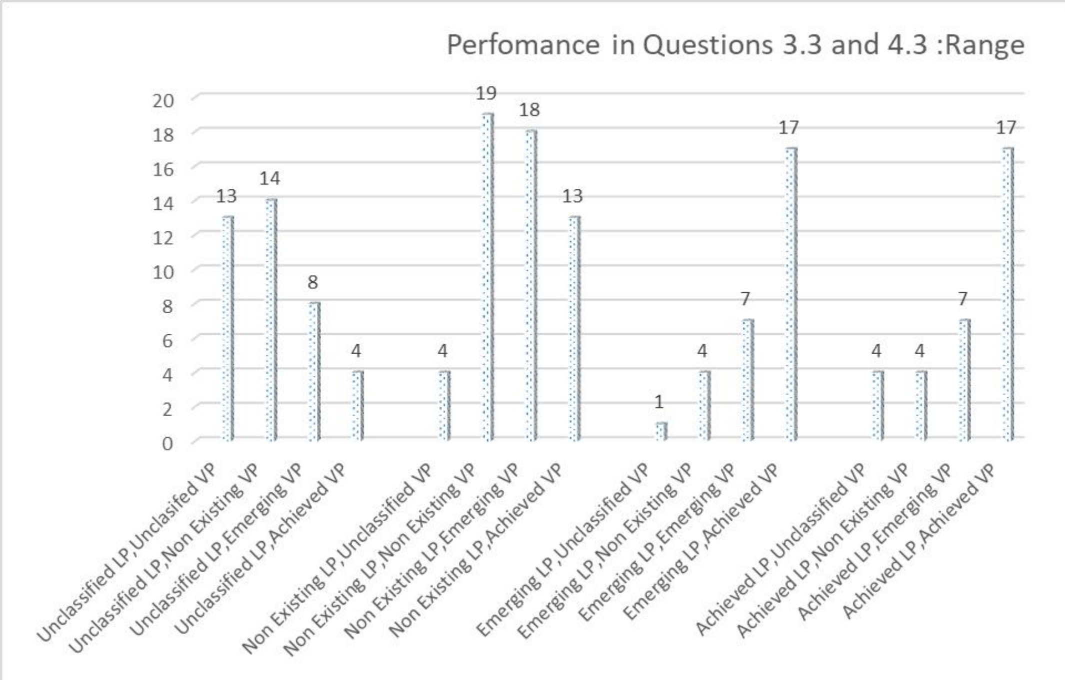


Figure 6.4.28: Relating LP and VP for the term "range" in Questions 3.3 and 4.3 of the post-test.

As depicted in Figure 6.4.28, a noteworthy observation emerges: 13 learners chose not to address either Question 3.3 or 4.3. This indicates that they neither provided definitions nor exemplifications for the term "range." Curiously, a parallel pattern emerges when examining the term "domain" in Questions 3.4 and 4.4, as illustrated in Figure 6.4.29.

The results for both "range" and "domain" exhibit striking similarities. Learners who attempted to define these terms exhibited a commendable level of conceptual understanding. Intriguingly, it was also apparent that those learners who successfully defined the term "range" similarly managed to define "domain." This proficiency extended to their performance in answering Question 4.4, suggesting a strong correlation between the comprehension of these two mathematical concepts and their ability to articulate them effectively.

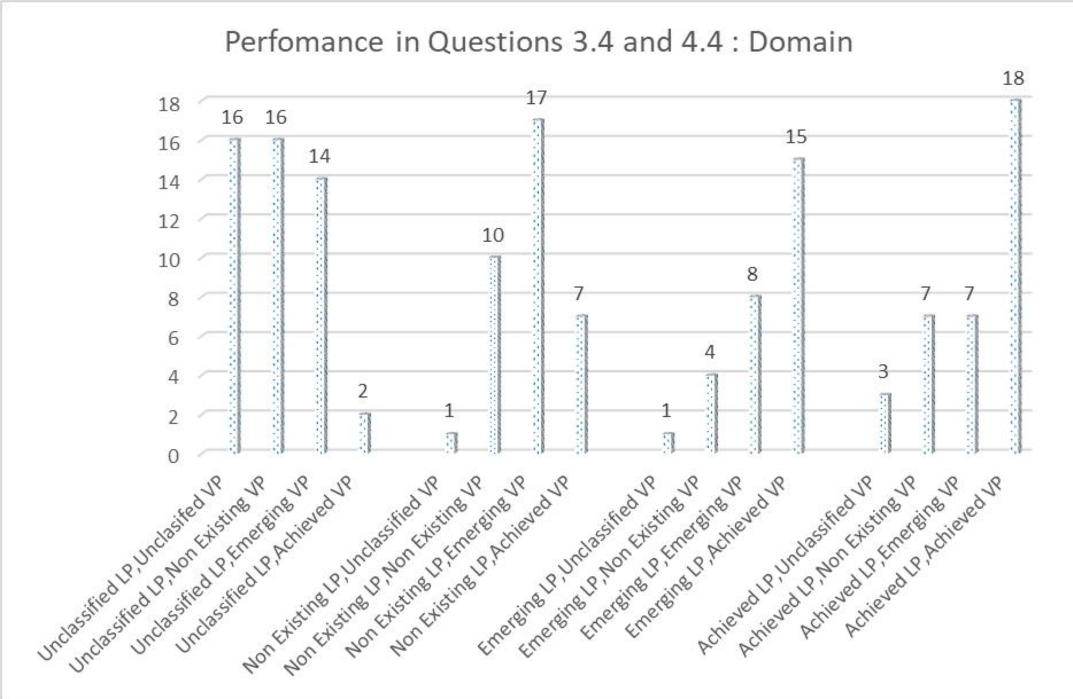


Figure 6.4.29: Relating LP and VP for the term "domain" in Questions 3.4 and 4.4 of the post-test.

6.4.4 Discussion of the results for Question 5

This question required making some mathematical calculations to get the coordinates of the points of intersection of two graphs. In addition to these calculations, learners were required to give a reason for each step that they undertook in their calculation. Figure 6.4.30 gives the results of this, based on the ability of the learners to justify the steps in their calculations. Meanwhile, Figure 6.4.31 gives the results of the marks obtained by the learners in the actual calculations.

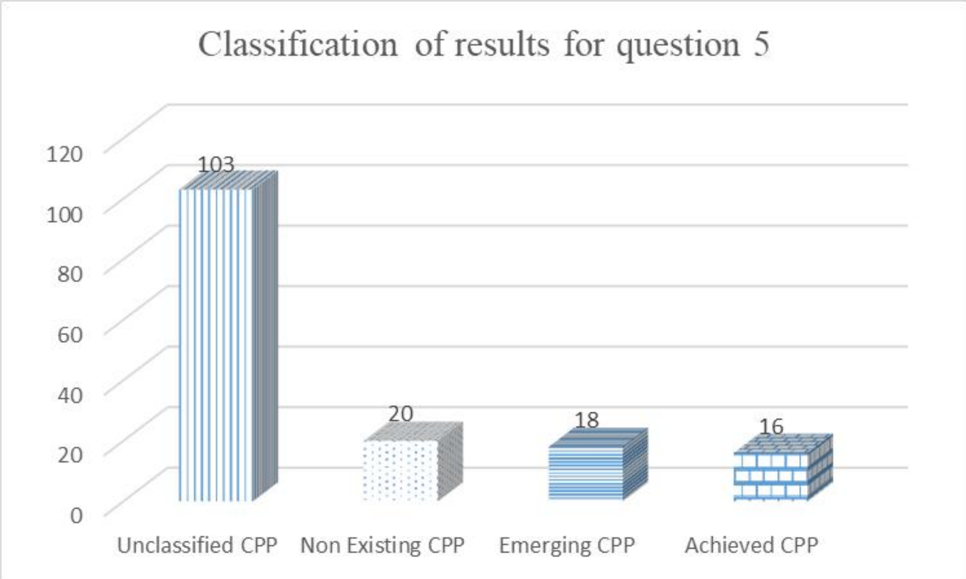


Figure 6.4.30: Overall CUPP results for Question 5 of the post-test.

From Figure 6.4.30, 103 learners did not give reasons for the steps that they undertook. This number comprises 55 who did not answer the question at all and 48 who answered it but did not give reasons for their steps as required by the question. 20 learners attempted to give reasons, but their reasons were not acceptable for the steps. 18 learners had partially correct reasons and 16 had correct reasons.

The mark distribution for this question is presented in Figure 6.4.31.

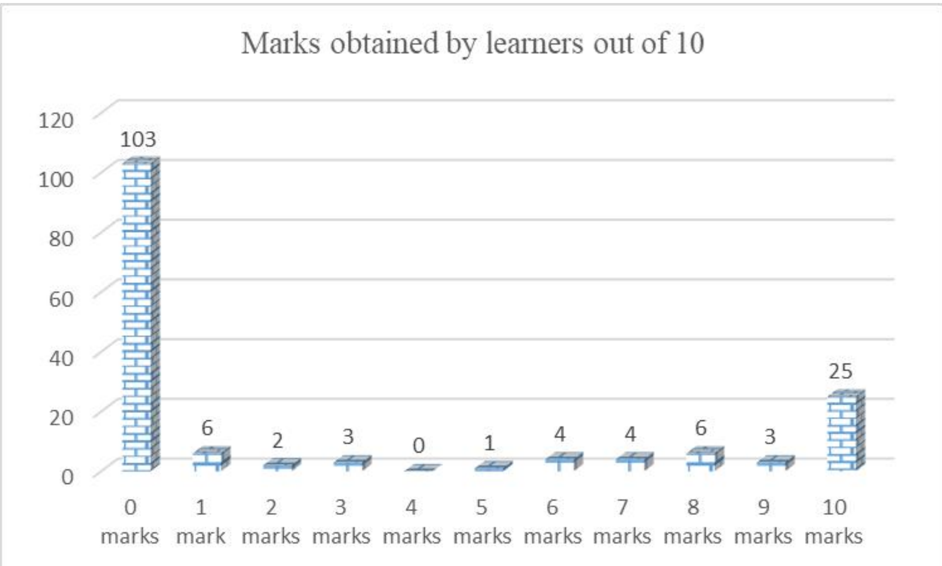


Figure 6.4.31: Mark distribution for Question 5 of the post-test

The performance in this question reveals a clear need for improvement, as a substantial group of 103 learners garnered zero marks for this particular question. Remarkably, only 43 learners managed to secure a minimum of 5 marks out of the attainable 10. These results emphasize the scope for enhancement in learners' performance in this specific area of assessment.

Assessing Conceptual Understanding in Learners' Responses

In addition to conducting a comprehensive evaluation of the learners' overall performance, an in-depth analysis was conducted of select learners' responses to gain insights into the reasoning behind their problem-solving approaches. Among the 43 learners who achieved a minimum of 5 marks for this particular question, two standout scripts were identified for detailed scrutiny. Similar to the other participants in the research, these two learners had encountered challenges in providing justifications for their steps during the pretest. Subsequently, the authors of these scripts were invited for interviews in a later stage to seek clarifications and further understanding of specific aspects. Below, excerpts from these noteworthy scripts are provided, offering a glimpse into the thought processes.

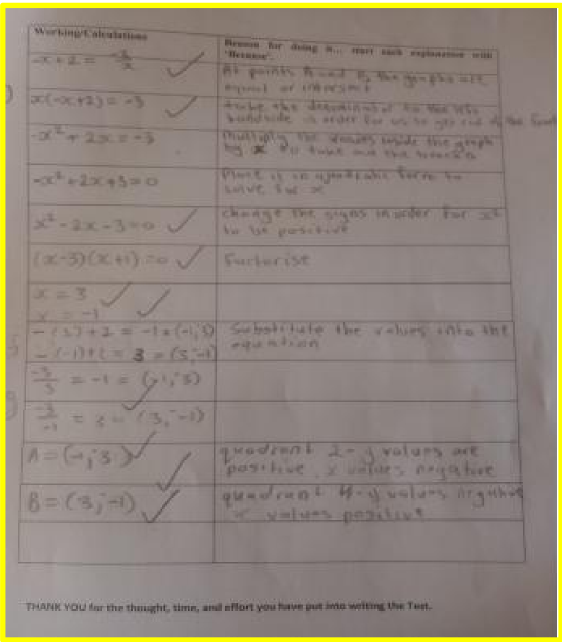


Figure 6.4.32(a): Dris's Comprehensive Step-by-Step Rationale.

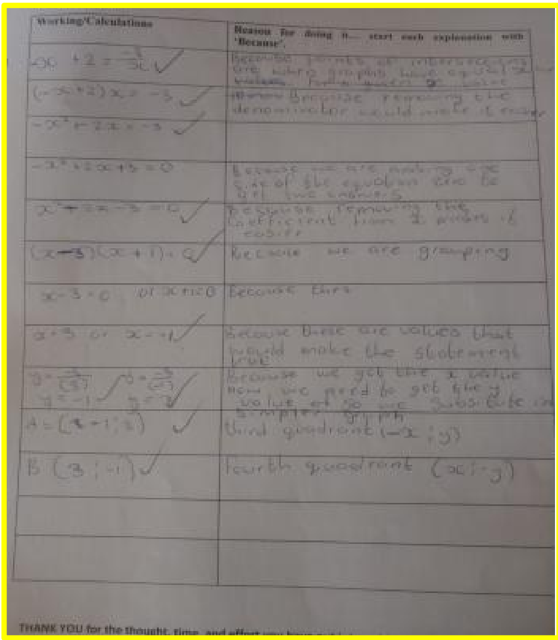


Figure 6.4.32(b): Dean's Comprehensive Step-by-Step Rationale.

These two excerpts clearly demonstrate the learners' adeptness in providing well-justified explanations for the steps they took in arriving at the solution to the given problem. Notably, both learners exhibited a commendable mastery of precise mathematical terminology in their justifications, showcasing their skilful use of vocabulary.

6.4.5 Comparing results of the control group to the results of the experimental group

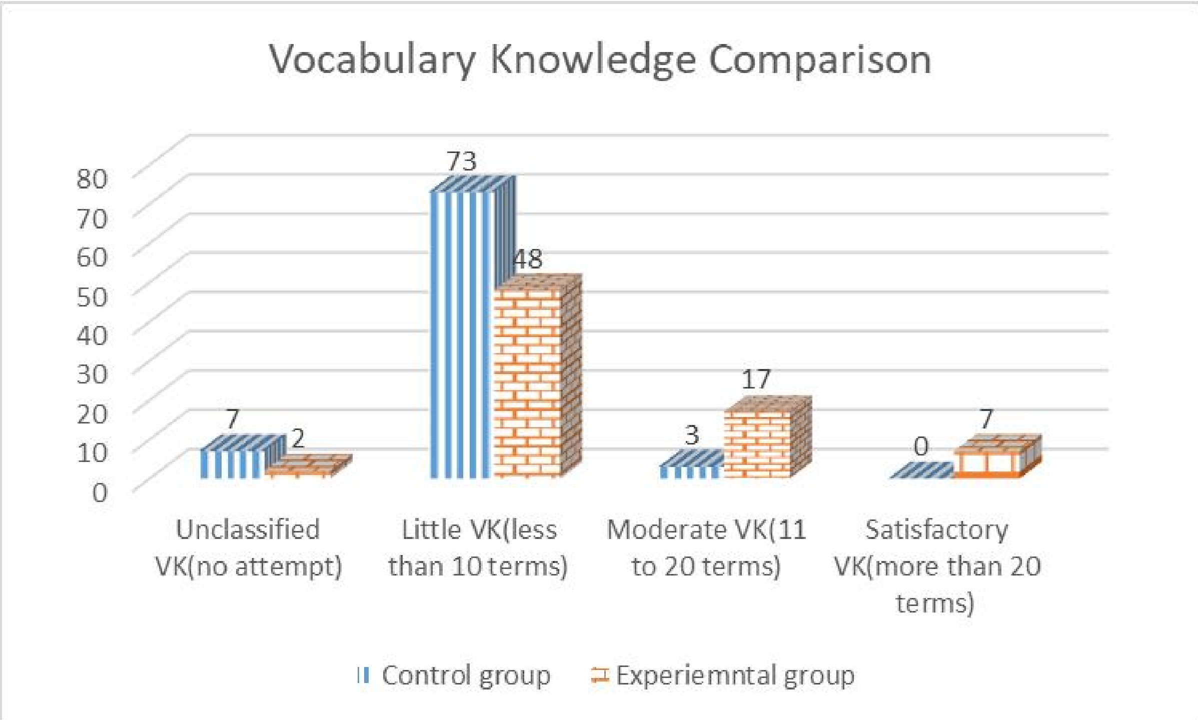


Figure 6.4.33: Comparison of the overall performance of the control and experimental groups on VK in the post-test.

Of the nine learners who did not answer Question 1, seven were from the control group and two were from the experimental group. From the results shown in Figure 6.4.33, it is striking to note that all seven learners whose number of vocabulary was deemed to be Satisfactory came from the experimental group. Also, there were more learners (17) with moderate vocabulary knowledge from the experimental group than the control group (3). This demonstrates that in this test, the performance of the experimental group was better than that of the control group. However, a check as to whether this difference is statistically significant will be done in a later section of this chapter.

In the subsequent Figure 6.4.33, a comparison is made between the number of vocabulary achieved by learners from the control and experimental groups. For attaining five terms and below, the trend is such that there were more learners from the control group than from the experimental group. This trend changes when assessing those that attained six terms and above, where, generally, there were more learners from the experimental group than the control group, e.g. six learners from the experimental group attained 12 terms, compared to 0 learners from the control group. Furthermore, the average number of vocabulary attained by the control group was 3,439638554, compared to the 7,743243243 attained by the experimental group. As to whether this is statistically significant, a two-independent-sample t-test was done, and the results are shown in Table 6.10.

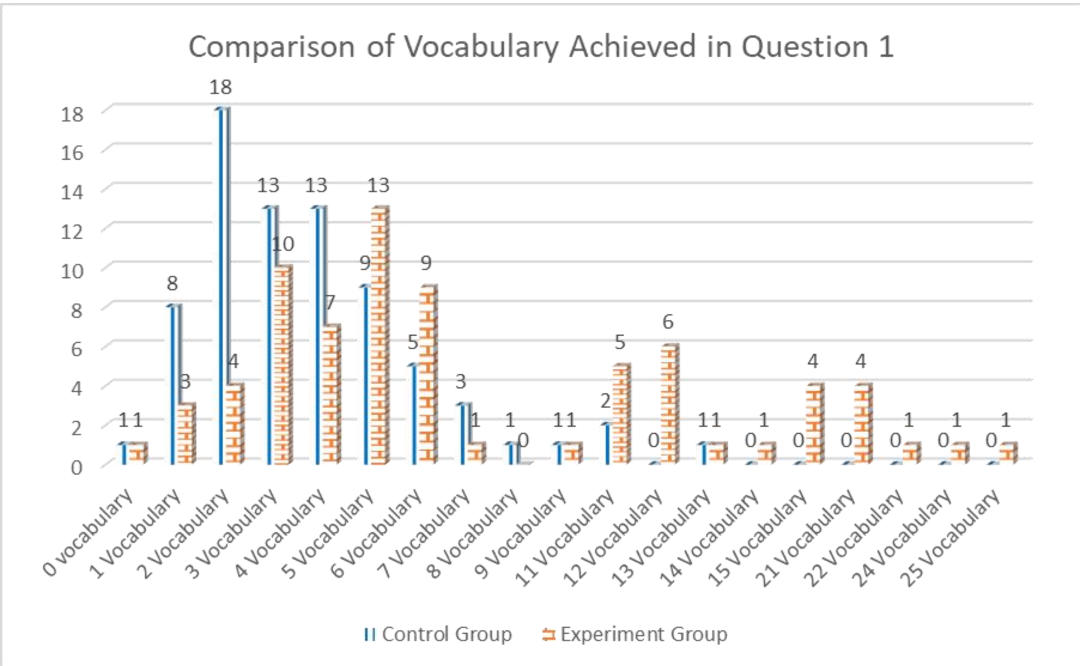


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Figure 6.4.34: Comparison of the number of vocabulary words achieved by learners from the control and experimental groups.

The results presented in Figure 6.4.34 reveal a significant difference in vocabulary acquisition between the experimental group and the control group during the post-test. Learners in the experimental group clearly outperformed their counterparts in the control group by demonstrating a greater ability to generate vocabulary. This suggests that the intervention or treatment provided to the experimental group had a positive impact on their vocabulary development.

This observed difference is not isolated but is corroborated by the findings in Table 6.10. A detailed analysis of the table data further supports the notion that the experimental group excelled in vocabulary acquisition. The data in Table 6.10 provides the mean scores that demonstrate the superiority of the experimental group in terms of vocabulary production compared to the control group.

Figure 6.4.34 extends the discussion by delving into the performance of both groups on the most prevalent vocabulary terms. This comparative analysis gives deeper insights into the nature of the vocabulary acquisition differences. The results in Figure 6.4.34 clearly illustrate that the experimental group surpassed the control group in terms of generating the most prevalent vocabulary terms.

For each of the terms analysed, with the exception of "increasing" and "symmetry," the experimental group consistently outperformed the control group. This finding is of particular importance as it underscores the effectiveness of the experimental group's learning approach in mastering not only overall vocabulary but also specific high-frequency terms. The contrast in performance between the groups on these specific terms suggests that the experimental group's learning intervention may have targeted and facilitated the acquisition of domain-specific or essential terminology.

In summary, the results from both Figure 6.4.34, along with the supplementary data in Table 6.10 and Figure 6.4.34, collectively support the conclusion that the experimental group exhibited superior vocabulary acquisition skills compared to the control group. These findings imply that the instructional approach or treatment applied to the experimental group holds promise for enhancing vocabulary development in similar educational contexts.

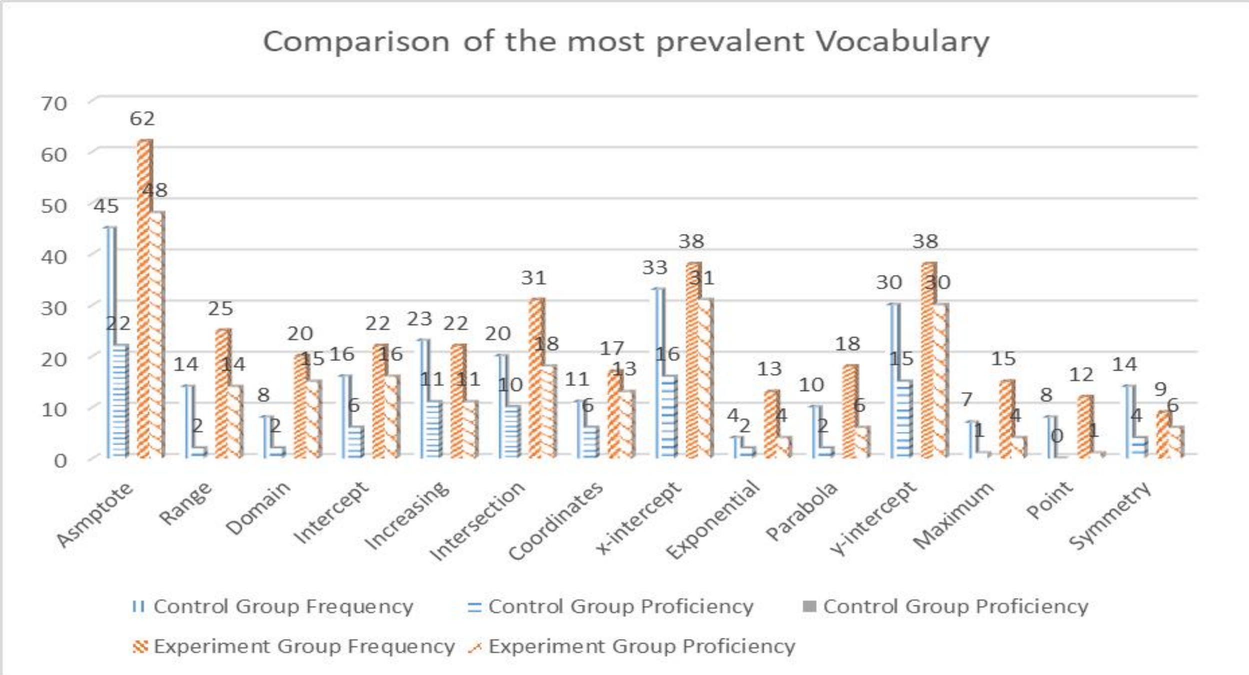

Figure 6.4.35: Comparison of the control and experimental groups concerning the most prevalent vocabulary.

In addition, a comparison was conducted between the performance of the two groups in relation to Emerging and Achieved LP and VP. This involved assessing the number of learners from each group demonstrating emerging constructs of definitions and emerging constructions of exemplifying and characterizing (Figure 6.4.36). Figure 6.4.36 displays a comparison of learners who successfully defined the terms and exemplified and characterized them. The values for this comparison are depicted in the following figures.

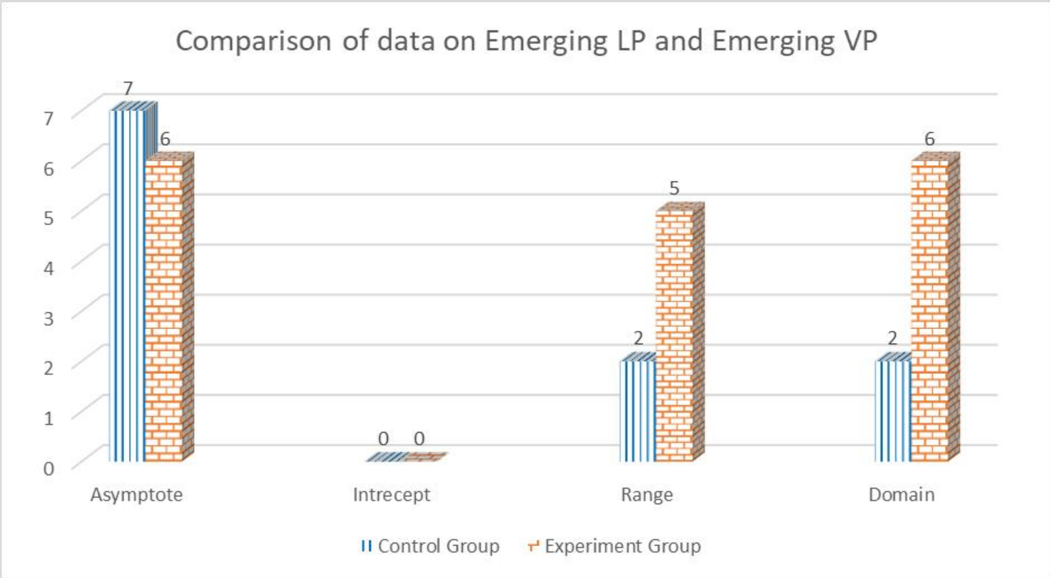

Figure 6.4.36: Comparison of the control and experimental groups concerning Emerging LP and VP.

In Figure 6.4.36, it is shown that for the terms “range” and “domain”, there were more learners in the experimental group than the control group who, when demonstrating Emerging LP, also demonstrated Emerging VP.

When it came to Achieved LP and VP, learners from the experimental group outperformed those from the control group. For instance, with the term “asymptote”, 19 learners from the experimental group answered Question 3.1 well and answered Question 4.1 correctly, compared to only nine in the control group. This is illustrated in the following Figure 6.4.37.

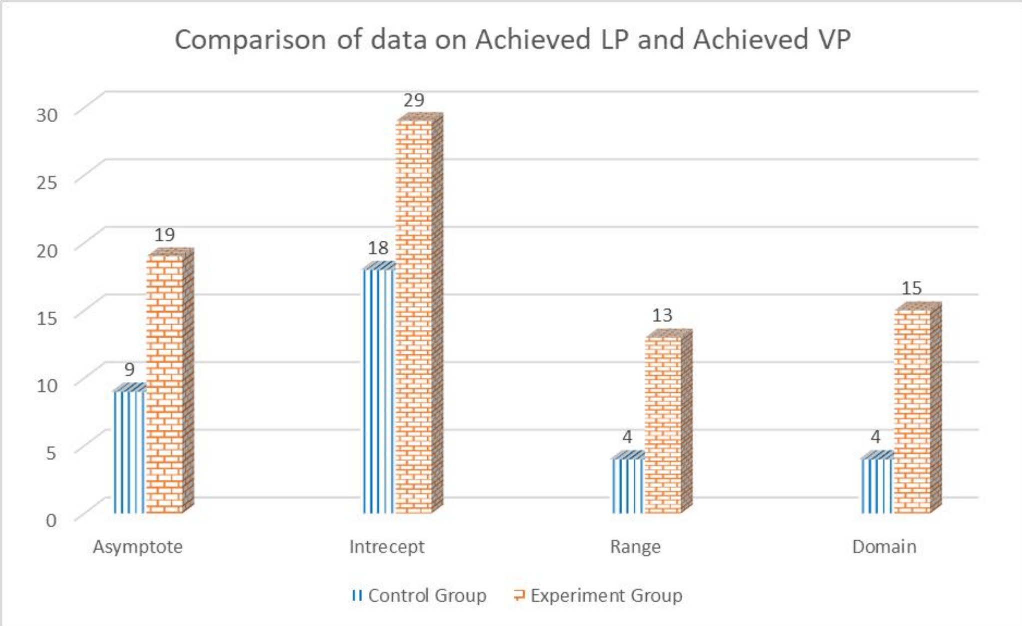

Figure 6.4.37: Comparison of the control and experimental groups concerning Achieved LP and VP.

The following box and whisker diagrams (Figures 6.4.38 and 6.4.39) show the mark distributions of the control group and the experimental group in the performance questions (2.1, 2.2, 2.3, and 5).

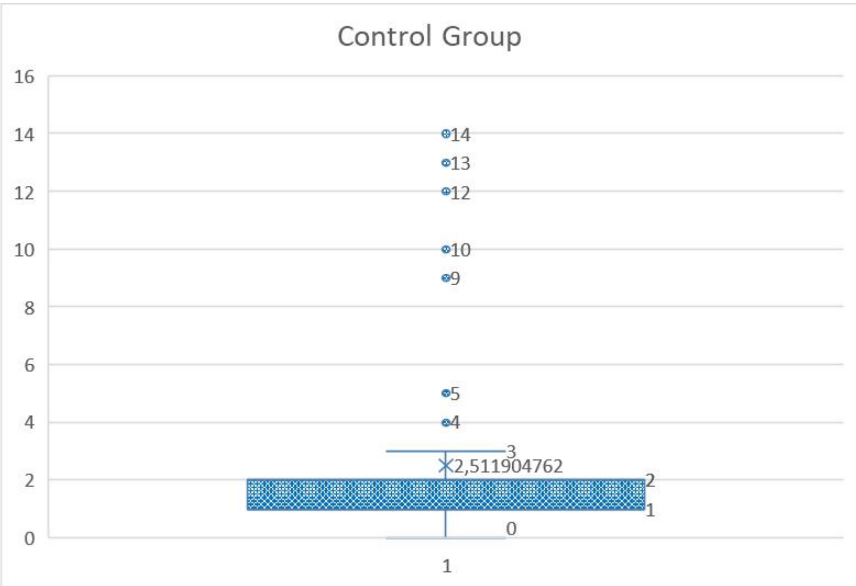

Figure 6.4.38: Performance of the learners in the control group with respect to MO.

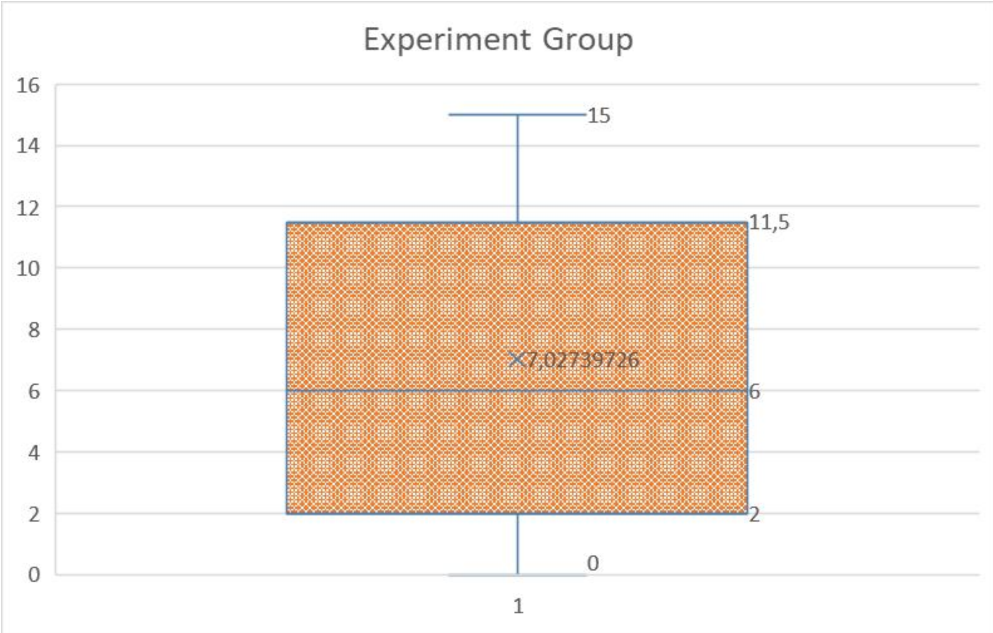

Figure 6.4.39: Performance of the learners in the experimental group with respect to MO.

The statistical data unveils a clear contrast in performance between the control and experimental groups. Specifically, the mean marks for the control and experimental groups were 2.5119 and 7.0274, respectively. These figures highlight a substantial divergence, with the experimental group displaying markedly higher average marks compared to the control group. Furthermore, it's worth noting that both groups exhibited a common range of scores, with minimum marks at 0, signifying the lowest performance level, and the control group's maximum score at 14, compared to the experimental group 15, indicating the highest attainable score.

The discernible difference in performance underscores the potential efficacy of the intervention implemented within the experimental group, suggesting that it holds promise as a favourable approach to learning.

However, it is imperative to emphasize that, as of now, it could not be definitively concluded whether this difference is statistically significant or simply a result of random variation. To ascertain the statistical significance of these findings, a comprehensive statistical test has been conducted, the results of which are detailed in the following section.

In this forthcoming section, the outcomes of the statistical test enabled the research to establish whether the observed distinctions in mean marks between the control and experimental groups hold statistical significance. This analysis takes into account various factors, including sample size and within-group variability, to provide a robust and data-driven foundation for our interpretations. By doing so, this ensures that any conclusions drawn from this study are firmly rooted in empirical evidence, enabling the research to make well-informed and credible claims about the effectiveness of the applied intervention.

6.5 Statistical report of the research test results.

6.5.1 Comparisons of the test means within the same group.

In the main study, a pretest and a post-test were administered to both the control group and the experimental group. Enrolment in both groups changed between the pretest and the post-test. Upon investigation, only two reasons were attributed to this change in enrolment. One reason was learners dropping out of the schooling system or transferring to other schools. The primary reason, however, was learners switching subjects from mathematics to mathematical literacy. Mathematical literacy is an alternative subject taken by learners who are not competent enough to pursue mathematics in the Further Education and Training phase (Grades 10–12).

In addition to dropout, transfers, and subject changes, enrolment between the two tests was affected by some learners not sitting for either of the tests due to absenteeism. To compare the means within the same groups, a paired-sample t-test was employed. This statistical method necessitates the use of results from the same participants for comparison. The paired-sample t-test, also known as the dependent-sample t-test, is used to compare the means of the same group under two separate scenarios and determine whether the difference is equal to 0. The sample size here exceeded 30, allowing for the assumption of normality and thus the use of the paired-sample t-test. Consequently, all learners without results for either the pretest or the post-test were excluded from the analysis, leaving 157 learners who met the criteria (83 control and 74 experimental).

When the pretest was administered to the control group, 99 learners took the test, and their responses were recorded and entered into a spreadsheet. During the post-test administration, 87 learners participated. Among these, four were absent during the pretest, so their results were excluded from this analysis. It was also noted that 12 learners had results for the pretest but did not sit for the post-test. Consequently, only 83 learners were finally considered for the analysis. The paired-sample t-test was conducted on these 83 learners, and the means were obtained and compared, as detailed in Table 6.8.

Table 6.8: Comparison of the means of the pretest and post-test of the control group.

	Sample Size	Mean ± Standard Deviation	Mean Difference	t -statistic	P-value
Pretest	83	1,73 ± 2,264	0,78	-1,8852	0,06295
Post-test	83	2,51 ± 3,013			

The mean difference of 0,78 between the pretest and post-test, in the control group, is little. Although there was an improvement in performance of the learners from the control group in the post-test (mean = 2,51) compared to their performance in the pretest (mean= 1,73), this improvement is not statistically significant. The p-value of 0,06295 indicates that there is no statistical difference between the results of the pretest and the post-test in the control group.

A dependent-sample t-test was also conducted for the experimental group. Table 6.9 presents the results of this paired-sample t-test of the pretest and post-test for the experimental group.

Table 6.9: Comparison of the means of the pretest and post-test of the experimental group.

	Sample Size	Mean ± Standard Deviation	Mean Difference	t-statistic	P-value
Pretest	74	1,53 ± 2,174	5,52	-9,3097	0,00001
Post-test	74	7,05 ± 5,066			

From the results shown in Table 6.9, the pre-test mean, and the post-test mean, show a marked difference. There is a difference of 5,52 between the means of the pretest and the post-test. The difference of 5,52 is statistically significant, at the 5% significance level, as the p-value of 0,00001 is less than 0,05. This indicates that the learners' performance during the post-test was remarkably better than their performance in the pretest.

6.5.2 Comparing the results of the control and experimental groups

- **Comparing the number of mathematical vocabulary words found by the control and experimental groups.**

In order to ascertain the significance of the aforementioned difference in the number of vocabulary words found by both groups during the post-test, an independent-sample t-test was conducted. The results of this test are presented in Table 6.10.

Table 6.10: Differences in the number of vocabulary words found in the post-test by the control and experimental groups.

Group	Sample Size	Mean ± Standard Deviation	t-statistic	P-value
Control	83	3,44 ± 2,585	1,9754	0,00001
Experimental	74	7,74 ± 6,133		

There is a large difference of 4,3 observed between the means in the post-test of the control and experimental groups, which is statistically significant at the 5% level, as the p-value is less than 0,05. The difference in their standard deviations shows that the results of the post-test are not similar between the two groups. This result confirms that during the post-test, the learners from the experimental group outperformed their counterparts in the control group.

- **Comparing the post-test means of the control group and the experimental group.**

A test was conducted to ascertain the statistical difference between the mean of the control group, and that of the experimental group. Table 6.11 presents the results of this statistical test.

Table 6.11: Comparison of the means of the control and experimental groups in the post-test.

Group	Sample Size	Mean ± Standard Deviation	t-statistic	P-value
Control	83	2,51 ± 3,013	-6,9087	0,00001
Experimental	74	7,03 ± 5,037		

The means of post-test for the control and experimental groups were 2,51 and 7,03 respectively. A huge difference of 4,52 was observed between the two groups. The standard deviations of 3,013 and 5,037 for the control and experimental groups respectively, indicate that the scores in the post-test of the experimental group are more spread from the mean than those of the control group. That is, in the experimental group, scores differ from their mean by plus or minus 5, while in the control group, they differ from their mean by plus or minus 3. The p-value of 0,00001 shows that there is a significant difference, at the 5% significance level, between the control and experimental groups, in their scores in the post-test.

From the results of the statistical analysis, it follows that during the post-test, in terms of marks obtained, learners from the experimental group performed better than the learners from the control group.

6.5.3 The relationship between the number of mathematical vocabulary found, and the marks that learners obtained in the post-test.

After comparing the means of the pretest and the post-test within and between the groups, an investigation was also conducted into the relationship between the number of vocabulary words that learners achieved during the post-test and the corresponding marks they obtained in that same test. In this comparison, both groups were combined since it wasn't a comparison between them. Consequently, all 157 learners who participated in the post-test were included in this analysis, and these same learners had previously taken part in the mean comparisons outlined in Table 6.8 and Table 6.9. The results of this comparison and the resulting association are presented in Table 6.12.

Table 6.12: The association between the number of vocabulary and marks obtained by learners.

Variable	Sample Size	Correlation Coefficient	P-value
Number of Vocabulary	157	0,61	0,00001
Performance	157		

The correlation coefficient indicates the strength and direction of the linear relationship between the number of vocabulary and the performance. The correlation coefficient value of 0,61 shows that there is a positive moderate correlation between the two variables. That is, the number of vocabulary and performance change in the same direction. When the number of vocabulary increases/decreases, so does the performance. The p-value of 0,00001 shows that the association between the number of vocabulary and the performance is statistically significant, at the 5% significance level.

6.6 Conclusion

This chapter presented the results of the main study. These results were presented in five main sections. Section 6.1 presented the results of the pretest that was administered to the two groups. The control group consisted of 99 learners, while the experimental group was made up of 95 learners. These 194 learners sat for a research test, and their work was marked according to a predesigned coding framework. On comparing the results of the two groups, it was found that the performances of the groups were homogeneous.

Section 6.2 discussed how the research participants were oriented for the research. The participants who went through the orientation were the teachers and learners from the experimental group. These participants were introduced to the Frayer model and trained on how to use it. This was in preparation for the implementation of the intervention during the research. Following the orientation was the actual implementation, where the participants used the Frayer model during the learning of the mathematical vocabulary of functions. In this chapter, the orientation and the implementation were jointly referred to as the intervention.

After the intervention, a post-test was administered to the learners from both the control group and the experimental group. The work was marked and recorded before it was analysed. A question-by-question presentation of the results was done for both the pretest and the post-test.

The results presented in this chapter demonstrated that, in the post-test, learners from the experimental group performed better than learners from the control group in all aspects. It was proven that the results of the post-test were better than those of the pretest for both groups, but, in the case of the experimental group, the difference was statistically significant in all respects. During the post-test, learners in the experimental group managed to write more correct mathematical vocabulary than their counterparts, they were more proficient in that vocabulary, and they obtained higher marks in the test than those from the control group. In addition to improved acquisition of mathematical vocabulary, the results also showed that the marks obtained by the learners who used the Frayer model were superior to those of their counterparts in the control group.

The later sections of this thesis present results from the interviews conducted with two learners from the experimental group, interpretation of the results, answering the research questions, conclusions, and recommendations.

CHAPTER 7 : THE INTERVIEWS WITH TWO LEARNERS

In this chapter, I delve into the outcomes drawn from insightful interviews conducted with two carefully selected learners hailing from the experimental group. The criteria for the selection of these interviewees were stringent, aimed at ensuring the depth and relevance of the insights garnered. Specifically, these participants met several key prerequisites:

Firstly, they had actively participated in both the pretest and post-test phases of the study, establishing a comprehensive foundation in experiencing the educational intervention under investigation. This criterion was crucial in enabling them to provide informed perspectives on the utility of the Frayer model in mathematics learning. Secondly, the inclusion of these participants was contingent upon the obtaining of consent from multiple stakeholders. Not only did the learners themselves willingly participate, but their schools and parents also granted their informed consent. This rigorous process further authenticated the ethical considerations of the research. Lastly, the selection of these interviewees was guided by the analysis of their test results. Those individuals whose test scores exhibited an indication of valuable insights were chosen. This discerning approach ensured that the interviews would be conducted with participants who not only met the technical requirements but also possessed the potential to contribute meaningfully to the research findings.

It's worth highlighting that the selection process adhered to these stringent criteria to meticulously identify individuals who could provide comprehensive and illuminating insights. Within Section 8.1, a detailed exposition of the interview findings with one of the participants, namely Dris, is presented. Following this, Section 8.2 offers an in-depth exploration of the interview outcomes derived from another participant, referred to as Dean. A comprehensive analysis that dissects the results of both interviews is then presented in Section 8.3.

Furthermore, Section 8.4 undertakes a comparative examination of the pre-test and post-test results as exhibited by the participants. As the chapter progresses, Section 8.5 encapsulates the key points by providing a concise summary. Throughout the ensuing discourse, the primary researcher is denoted as 'R', whereas the interviewees are anonymous using the pseudonyms 'Dris' and 'Dean'. This approach ensures a cohesive and organized presentation of the research outcomes and the subsequent discussion.

7.1 Presentation of the interview results with Dris

The interviewee was welcomed to the interview and asked to explain his experience with the recent research test that he had written. The interview was meant to address the research question on competence in associating mathematical vocabulary with a mathematical object, as well as addressing the research question on usefulness of the Frayer model in the learning mathematical vocabulary. That being the case, the codes Vocabulary Knowledge (VK) and Marks Obtained (MO) will come to the fore.

Dris stated that he found it easy to complete the test, particularly the first question.

> R: I welcome you to this interview session, which is in the section of mathematical functions. We will talk about the research test that you sat for on Tuesday. What was your experience with the research test?
>
> Dris: Yaa Sir, I think my experience was good. I saw that graph and I started writing those words in the spaces.
>
> R: Okay Dris…what do you mean if you say those words?
>
> Dris: I mean the test wanted us to write the words about the graph like …. asymptote.
>
> R: Okay, how easy was it for you to relate the graph to those words that you said you started writing in the spaces?
>
> Dris: It was easy because those words that I wrote there were… some of them…. I mean I did them in my book.

The learner then spoke about how he related the features of the mathematical object in the test to the words that he had learnt using the Frayer model. The learner stated that the use of the Frayer model assisted him in remembering words that were associated with the mathematical object. After using the Frayer model, Dris became competent in associating the mathematical vocabulary with a mathematical object (VK).

> Dris: The graph is not the same, but the words that I can write about the graph are the ones that I have in my book… I mean I write them in the Frayer model.
>
> R: Ooh, the Frayer model?
>
> Dris: Yes sir.

Then, he was asked to explain what the Frayer model was and how it assisted in generating the necessary words for Question 1.

R: What was the Frayer model all about by the way? How did you use it in your learning?

Dris: It was that graph… umm…. diagram, that I used to do the maths words. It is like I would put the word in the centre and then define it and eeeh, give some examples.

R: So how did that model help you to come up with words for the graph in the research test?

Dris: Sir, you see there, I could see the line and how it was going and know that it was an asymptote.

R: What did you see on the line that made you say it was an asymptote?

Dris: It was not touching the graph. As the graph was going, it only got close to the line, but does not touch it.

The learner went on to define the vocabulary word "asymptote". In his second line of the interview, he had already mentioned the word asymptote. The second time he mentioned the word asymptote was in his line number 7 of the interview. After mentioning the word for the second time, the researcher asked him to define the word asymptote. His definition was correct, as it matched the concept definition, as per the memorandum for the research test.

R: So how would you define an asymptote?

Dris: It is a line that a graph gets close but does not touch it forever.

R: Correct.

After the learner had defined the word asymptote, he was probed to find out if he remembered when his first encounter with the word asymptote was. He stated that he had first learnt about the word asymptote in Grade 10. However, he quickly jumped to concluding that, despite this encounter, he had not understood the word. Two suggestions were made to him for possible reasons as to why he did not understand it then. The first reason that was attributed to him was playfulness, and the second one was immaturity. Though he conceded that this could have been the case, he also brought to the fore the usefulness of the Frayer model in this regard. Dris stated that while using the Frayer model, he had worked on the definitions, characteristics, and exemplifications by himself. The teacher played the role of an assessor who would assess and give feedback. In a way, Dris suggested that for him, the Frayer model was an effective tool for learning mathematical vocabulary.

> R: When did you learn about an asymptote?
>
> Dris: I think it was last year, but I did not understand it.
>
> R: Do you now understand? Don't you think that in Grade 10 you did not understand it because you were still playful or young.
>
> Dris: Yaa I think so but this one…. The Frayer model helped me to understand it, because I could do it myself and then the teacher marks it.

The learner confirmed that he did not notice that it was the same test that they sat for in the two sessions. However, the learner was confident that he did better in the second test than in the first one. He attributed this performance to the Frayer model. The learner was confident that in this test, he had managed to come up with many correct mathematical vocabulary words. He even started mentioning some of these words.

> R: What can you say about the research test that you wrote last term, and this one that you wrote on Tuesday?
>
> Dris: Like what Sir?
>
> R: Which one of the tests was more difficult?
>
> Dris: I think they were almost the same, but I think for me this one was much easier… I think I passed it.
>
> R: If someone would say to you that it was the same test, what would you say?
>
> Dris: I don't know, maybe they are similar.
>
> R: Okay, Dris, what do you think could have been the cause of your difference in performance between the tests?
>
> Dris: I think this time I was knowing many words. Yaaaa, there were many because I practised the Frayer model.

R: You keep on talking of the Frayer model, how much did it help you to learn mathematical vocabulary this term?

Dris: A lot, Sir. I now know many words. Like asymptote, parabola, domain, range… a lot of them Sir.

Dris was also asked if he thought that knowing mathematical vocabulary was of any help to his learning of mathematics.

R: Do you think that knowing the vocabulary can be of any help in your understanding of mathematics?

Dris: Yes, Sir.

R: In what ways or how?

Dris: I now understand the questions and can work them out because I know what they want me to do.

Before concluding the interview with him, he was asked about the usability of the Frayer model in other subjects. He mentioned that he uses it in physical sciences and life sciences.

R: So, did you ever try to use the Frayer model in other subjects?

Dris: Yes, I now use it in physics and life sciences.

R: What about in English?

Dris: I never used it in English. Ummmm but I think it can also work. In physics, it helps me a lot….and life sciences also.

The learner was thanked for valuable contributions and then released to conduct an interview with the next learner.

7.2 Presentation of the interview results with Dean

This interviewee did not realise that the post-test was the same as the pretest. However, he thought that the second test was much easier than the first test. He was also confident that he had managed to come up with about 25 correct mathematical vocabulary associated with the given mathematical object in Question 1.

> R: You are welcome to this interview session, Dean. I would like you to share with me your thoughts and experience about the research test that you wrote on Tuesday.
>
> Dean: Okay, Sir.
>
> R: The first question in the test required you to write down vocabulary associated with the graph that was drawn there. What was your experience in answering that question?
>
> Dean: Ummmm, that question was not very difficult, but I could not manage to fill all the spaces that were there.
>
> R: How many words did you write if you can still remember?
>
> Dean: I am not sure Sir but there were many… like 25.
>
> R: From what you saw, what was the difference between this previous test and the other one I gave you last term?
>
> Dean: There are some similarities but ummmm, to me this one was much better than the first one.

The learner insisted that he wrote about 25 correct mathematical vocabulary in Question 1. He indicated that the Frayer model was the reason why he answered Question 1 with ease. According to Dean, the Frayer model made it easy for him to associate the vocabulary words with graphs. He further mentioned that when he worked with the Frayer model, he inserted some small graphs into the model for illustrations. He was also asked to define the word domain and he did so correctly.

> R: You said you wrote about 25 words, right?
>
> Dean: Yes, somewhere around 25
>
> R: 25 is a good number…what made it easy for you to come up with those words and associate them with the graph?

Dean: Since last term when I started using that Frayer model, I can quickly see and match words with diagrams and graphs and I now know them.

R: Okay.

Dean: Yaa because when I do the Frayer model, at times I draw some small graphs inside.

R: So, can you say the Frayer model helped you here?

Dean: Yes, it helped me a lot.

R: So, do you mean that if you were to be asked to define a word like, domain, will be able to define it?

Dean: Yes, I can define it.

R: Define it for me.

Dean: It is a set of all possible values of x that make the function work.

R: Okay, I agree with your definition.

Dean was asked if he thought knowing mathematical vocabulary was of any help to his learning of mathematics.

R: Do you think that knowing mathematical vocabulary can be of any help in your understanding of mathematics?

Dean: Yaaaa, I think it helps a lot.

R: In what ways does it help?

Dean: Mostly when the teacher said things, at times it wasn't easy to get what she said because I didn't know what she was talking about…. But if you practise those words and know them, then it is easy to follow the teacher.

R: So, you want this vocabulary to help you follow what the teacher says?

Dean: Not only that Sir, also to understand questions in the test.

The interviewee was also drawn into a discussion about the applicability of the Frayer model in his other subjects. He mentioned two subjects in which he had used the Frayer model. The two subjects that he mentioned were business studies and economics.

R: Have you ever tried to use the Frayer model in other subjects?

Dean: Yes, I use it in business studies and economics.

R: Is it easy to do so, and does that work?

Dean: Its very helpful. In economics, there are a lot of terms that we are required to define and use in essays. The same also applies to business studies.

R: Okay.

Dean: Yaa because when I want to write an essay, I must first know the terms that are in the essay question, so I can use the Frayer model to explain the terms and use those explanations in the essay.

The learner was thanked for his time and his valuable contribution to the research and ended the session.

7.3 Analysis of the interview results

The two learners interviewed emphasized the importance of precise mathematical vocabulary in mathematics learning. Their conclusions were derived from statements made during the interviews. Dris conveyed, "Understanding the questions and solving them is easier now because I comprehend what they require." Dris highlighted that mathematical vocabulary aids in question comprehension, given that it constitutes the language of mathematics education. Some mathematical questions explicitly rely on the use of this vocabulary, such as "write down the equation of the horizontal asymptote of *f*," necessitating a solid grasp of key mathematical terms like "equation" and "horizontal asymptote."

Dean's response, "It wasn't easy to get what she said because I didn't know what she was talking about," suggests that familiarity with mathematical vocabulary enhances classroom communication. When teachers explain concepts, learners require the appropriate vocabulary to comprehend and engage in discussions effectively. This vocabulary serves as the medium of classroom discourse.

Additionally, during the interviews, both learners praised the effectiveness of the Frayer model in their learning. Dris and Dean both asserted that the Frayer model significantly improved their vocabulary acquisition. Dris remarked, "I believe I knew many words this time because I practiced the Frayer model." He compared his performance in the pretest to that in the post-test, attributing his improvement to the Frayer model. Dean noted that using the Frayer model enabled him to quickly associate words with diagrams and graphs, facilitating enhanced comprehension and recall of concepts.

Furthermore, both interviewees concurred that the Frayer model extends its usefulness beyond mathematics to other school subjects. While it wasn't possible to verify its applicability in mathematics chapters beyond functions due to the research timeline, their statements suggested potential utility across various mathematics topics. Dris mentioned using it in physical sciences and life sciences, while Dean utilized it in business studies and economics. These responses indicated that the Frayer model found application in both science and commerce subjects.

7.4 Comparison of the interviewees' pre-test and post-test results

7.4.1 Dris' pre-test and post-test results compared.

To verify and gain more clarity on the claims made by Dris during the interview, an item-by-item comparison of his two sets of results was conducted. The results are displayed in Table 7.1, where only the relevant codes for the results are presented.

23: Achieved Vocabulary Proficiency.

33: Achieved Language Proficiency.

42: Emerging Conceptual Understanding and Procedural Proficiency.

43: Achieved Conceptual Understanding and Procedural Proficiency.

Table 7.1: Comparison of Dris' results in the pretest and post-test.

	Q1 VK	2.1 CUPP	2.2 CUPP	2.3 CUPP	2.4 LP	3.1 LP	3.2 LP	3.3 LP	3.4 LP	4.1 VP	4.2 VP	4.3 VP	4.4 VP	4.5 VP	Tot MO	Q5 CUPP
Pretest	6	42	43	43	33	33	33	33	33	23	23	23	23	23	9	42
Post-test	25	43	43	43	33	33	33	33	33	23	23	23	23	23	15	43

The results in Table 7.1 show that Dris did well in both the tests. The only notable difference in his results was between the VK and the MO in the two tests. In the pretest, Dris only managed to come up with six correct mathematical vocabulary in Question 1. This is a small number compared to the 25 correct vocabulary that he attained in the post-test. Dris also obtained only 9 marks out of a possible 15 during the pretest, compared to the 15 marks out of 15 that he obtained during the post-test. This result supports the notion that there is a positive moderate correlation between number of vocabulary achieved and marks obtained. An improvement in the number of vocabulary written by Dris during the pretest from six to 25 in the post-test is followed by an improvement in the marks obtained from 9 during the pretest to 15 during the post-test.

7.4.2 Deans' pre-test and post-test results compared.

An item-by-item comparison of Dean's pre-test results with post-test results was also conducted. The results are displayed in Table 7.2.

21: Non-existing Vocabulary Proficiency.

23: Achieved Vocabulary Proficiency

30: Unclassified Language Proficiency.

31: Non-existing Language Proficiency

40: Unclassified Conceptual Understanding and Procedural Proficiency.

41: Non-existing Conceptual Understanding and Procedural Proficiency.

42: Emerging Conceptual Understanding and Procedural Proficiency.

43: Achieved Conceptual Understanding and Procedural Proficiency.

Table 7.2: Comparison of Dean's results in the pretest and post-test.

	Q1 VK	2.1 CUPP	2.2 CUPP	2.3 CUPP	2.4 LP	3.1 LP	3.2 LP	3.3 LP	3.4 LP	4.1 VP	4.2 VP	4.3 VP	4.4 VP	4.5 VP	Tot MO	Q5 CUPP
Pretest	3	43	40	40	30	30	32	31	31	23	23	21	21	23	4	42
Post-test	22	43	43	41	32	33	33	33	33	23	23	23	23	23	13	42

The results in Table 7.2 show a large difference between Dean's results in the two tests. During the pretest, there were four questions that Dean did not attempt to answer. These were Questions 2.2, 2.3, 2.4, and 3.1. For instance, during the pretest, Dean did not answer 2.2, which required him to write down the coordinates of C, the turning point of a parabola. In the post-test, Dean answered the question well. He also did not answer Question 2.3 in the pretest, which required him to write down the value/s of x for which $f(x) < g(x)$. During the post-test, Dean attempted the question, though his response was completely incorrect. As he did not answer 2.3 in the pretest, it was logical that he did not answer 2.4, because 2.4 was derived from the response in 2.3. However, during the post-test, Dean attempted 2.3 and gave an incorrect answer. Following his incorrect response in 2.3, Dean showed Emerging LP in Question 2.4. Lastly, Dean did not define the vocabulary word asymptote during the pretest (Question 3.1). During the post-test, he defined the vocabulary word correctly.

The pattern in the results shows that Dean showed an improvement in all aspects of the test. For example, during the pretest, Dean wrote three correct mathematical vocabulary in Question 1, compared to the 22 that he wrote in the same question during the post-test. In terms of marks obtained, during the pretest, Dean got 4 out of 15 whereas during the post-test, he got 13 out of 15.

7.5 Conclusion

The chapter discussed the results obtained from the interviews done with two learners from the experimental group. The questions put to the participants were meant to solicit the participants' views on their experience when they used the Frayer model. Both interviewees expressed that they found the Frayer model user friendly and helpful in learning mathematical vocabulary.

The interviewees stated that the Frayer model helped them to understand mathematical vocabulary. This understanding of mathematical vocabulary helped them to understand the teacher's explanations, questions, and instructions. The learners mentioned that such understanding helped them to perform better in mathematics. In the chapter, the interviewee's pre-test and post-test results were compared, and it was drawn from the comparison that both interviewees experienced an improvement in their mathematical vocabulary knowledge, as well as their performance in mathematics. The interviewees acknowledged that their improved performance in mathematics was the result of their enhanced understanding of explanations given by the teacher, improved understanding of question instructions, and better understanding of mathematics texts.

The discussions reveal that the interviewed learners were aware that, to do well in mathematics, one needs to be able to understand and correctly use precise mathematical vocabulary. What the interviewed learners emphasised was that without understanding the correct mathematical vocabulary, one will not comprehend concepts taught, and consequently one will falter in mathematics.

CHAPTER 8 : DISCUSSIONS, INTERPRETATIONS AND CONCLUSIONS

In this chapter, the results of the investigation into the impact of learning mathematical vocabulary for functions using the Frayer model are presented. During this study, the integration of mathematical vocabulary learning was implemented alongside the learning of mathematical content related to functions. This integration was carried out within Content and Language Integrated Learning (CLIL) structured lessons in the experimental group. In these CLIL lessons, the mathematical content of functions served as the source of mathematical vocabulary, including terms like domain, range, asymptote, and more, which the learners explicitly learned using the Frayer model. As a result, as learners acquired the mathematical content of functions, including performing calculations and creating graphs, a deliberate effort was made to instruct them in the associated mathematical vocabulary.

The impact of this integration was assessed by examining the Grade 11 learners' conceptual understanding and mathematical performance, as detailed in the conceptual framework presented in Chapter 3. The study's research question, "What affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?" is addressed based on the five research sub-questions outlined in Section 8.2.

Section 8.1 provides an overview of the research, reiterating the research objective, rationale, and the five research sub-questions. The research sub-questions are addressed in Section 8.2, followed by an answer to the main research question in Section 8.3. Recommendations drawn from this study are presented in Section 8.4, followed by a discussion of the study's limitations in Section 8.5.

Analysis and interpretations are conducted to establish correlations between performance in these focal areas. Consequently, a meaningful connection is established between the use of the Frayer model for learning mathematical vocabulary, the gain in mathematical vocabulary, and performance in mathematics. Finally, the conclusions are discussed.

8.1 Overview of this research

This study aimed to find out if focused mathematical vocabulary learning using the Frayer model would assist mathematics learners in gaining conceptual understanding and, eventually, improve their performance in mathematics. In Chapter 1, I highlighted the rationale of the study, which was the interdependence between mathematical language ability and the understanding of mathematics by learners. However, the researcher's teaching experience spanning more than two decades, a background in marking National Senior Certificate examinations for nine years, and extensive literature review led to the belief that one of the contributing factors to inferior performance in mathematics is the insufficient emphasis on learning mathematical vocabulary in the classroom. Given this challenge, the research question posed was:

"What affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?" To tackle the research question effectively, the research embarked on a quest for answers to five distinct research sub-questions, which are outlined in the following section.

- **Sub-Question 1: How competent are Grade 11 mathematics learners in associating mathematical vocabulary with a given mathematical object?**
- **Sub-Question 2: What are the approaches that learners employ to acquire mathematical vocabulary during their mathematics lessons?**
- **Sub-Question 3: In what ways are learners exposed to the mathematical vocabulary of functions during lessons as they learn mathematical functions?**
- **Sub-Question 4: How effective is the Frayer model as an approach for learning the mathematical vocabulary of functions?**
- **Sub-Question 5: To what extent does competency in mathematical vocabulary relate to learners' performance in mathematics?**

In the sections that follow, the main research question in this study is addressed under each of the five sub-questions.

8.2 Addressing the research questions for this study

This study aimed to assess whether employing the Frayer model as a pedagogical tool for focused mathematical vocabulary acquisition can effectively foster conceptual understanding among mathematics students. As already alluded to, the main research question was answered using five research sub-questions.

8.2.1 How competent are Grade 11 mathematics learners in associating mathematical vocabulary with a given mathematical object?

The research assessed how well learners could connect mathematical terminology with graphs. It involved a pilot study and a main study with pretest and post-test phases. In the pilot study, only 26.3% of learners listed more than 20 correct mathematical vocabulary words associated with the given graph. In the main study's pretest, 89.7% listed fewer than 10 correct mathematical terms. Learners struggle with Vocabulary Proficiency (VP), often failing to identify terms on the graph correctly. In the post-test, 77.1% listed fewer than 10 correct vocabulary words, while only 4.5% listed more than 20. Learners faced difficulties linking visual graphs with mathematical vocabulary.

The research found that learners require focused support in understanding and applying mathematical vocabulary, which aligns with Quinnell's (2014) findings that mathematics demands a distinct literacy approach. Bridging the gap between graphical and verbal representations is crucial, as emphasized by Lowrie & Dieman (2011) and Quinnell & Carter (2012), who stressed the importance of interpreting graphics and symbols in mathematical problems. Enhancing literacy skills is vital for better mathematics learning (Åberg-Bengtsson & Ottosson, 2006, p. 56–57). Effective teaching strategies should reinforce conceptual understanding (DBE, 2011, p. 8). In summary, this study highlighted significant difficulties among learners in connecting mathematical concepts to appropriate vocabulary, underscoring the importance of improving literacy skills in mathematics education.

8.2.2 Sub-Question 2: What are the approaches that learners employ to acquire mathematical vocabulary during their mathematics lessons?

This study focused primarily on investigating the instructional strategies employed by teachers in the teaching of mathematical vocabulary. It brought to light a significant gap in specialized methods within this educational domain. Through in-depth conversations with three teachers responsible for instructing the experimental group, a surprising revelation emerged: none of them had prior experience with specific strategies for teaching mathematical vocabulary, and they were unfamiliar with the Frayer model, a tool introduced during the intervention.

One noteworthy example was a head of department at an experimental school who openly admitted her lack of familiarity with explicit instruction in mathematical vocabulary within her teaching practice. Instead, she mentioned relying on a basic "definition-only" approach, wherein she verbally conveyed definitions to students without reinforcing their comprehension through writing or emphasizing engagement. Recognizing the limitations of this approach in stimulating students' enthusiasm for mastering vocabulary, she expressed a willingness to explore new teaching strategies.

The absence of familiarity with the Frayer model among these teachers highlighted its absence in their previous teaching repertoire. Interestingly, one of the teachers had access to mathematical charts supplied by the Department of Education, which adorned the classroom walls. These charts contained formulas, proofs, diagrams, and a few term definitions. However, classroom observations revealed that the teacher seldom made use of these resources.

Upon further discussion, it became evident that these teachers heavily relied on an oral-questioning strategy (Mahmud et al., 2020). They initiated sentences and paused to allow students to complete them in a group format. Unfortunately, this approach proved ineffective in facilitating meaningful acquisition of mathematical vocabulary. The teachers also acknowledged that they rarely allocated dedicated time to teach mathematical vocabulary within their lessons. Consequently, they faced challenges in conceiving effective strategies for this crucial task. This phenomenon resonates with the prevailing perception of mathematics as primarily centred on numerical manipulation and proof-based learning (Seah et al., 2016).

In summary, this study shed light on the lack of structured strategies for teaching mathematical vocabulary among the surveyed teachers. The absence of explicit instruction in this critical area underscores the essential need for teacher training and resources aimed at enhancing mathematical vocabulary acquisition in the classroom.

8.2.3 Sub-Question 3: In what ways are learners exposed to the mathematical vocabulary of functions during lessons as they learn mathematical functions?

In the course of the research, a comprehensive analysis was conducted utilizing a diverse range of data collection methods, including research tests, lesson observations, and interviews. The specific insights pertinent to addressing the sub-question were drawn from meticulous examination of lesson observations. These observations involved the active participation of three educators and their cohorts of mathematics learners across two distinct research sites.

During this process, close observations were made, recordings taken, and transcriptions completed for three separate lessons for each teacher. Observations revealed a noteworthy trend - students were intentionally immersed in the world of precise mathematical terminology during their learning experiences. The teachers, who exhibited a high degree of expertise in their field, consistently used mathematical vocabulary when explaining various concepts. Importantly, the learners actively engaged with this specialized lexicon, posing questions and responding using mathematical terminology. Even the instructional materials, including textbooks and question papers, were designed to incorporate this specialized vocabulary when presenting problems. This observation challenged the conventional notion that learners' access to mathematical discourse was solely mediated by instructors, a perspective put forth by Adler and Ronda in their work from 2016. Findings highlighted that learners were active contributors to this discourse, not just passive recipients.

Analysis further revealed that the primary sources of mathematical vocabulary were the instructional texts and the teachers themselves. In contrast, learners used this terminology less frequently, which was in part due to their status as novices in this domain. Teachers and instructional material creators, who were seasoned experts in the subject matter, played pivotal roles in imparting precise mathematical vocabulary to their learners. Given this dynamic, it became evident that novice learners needed specific instruction on acquiring mathematical vocabulary to foster more meaningful engagement with both teachers and instructional materials.

Throughout the lessons, instructional texts provided a substantial 42% of the vocabulary used by the students, encompassing instructions, explanations, and questions. Despite not having an audible voice, texts communicated meaning to the readers, underscoring the importance of students possessing a solid grasp of mathematical vocabulary to effectively interact with these materials. Likewise, educators themselves made significant contributions, accounting for 45% of the vocabulary employed. In all instances, both texts and teachers utilized exemplification and explanatory discourse techniques, aligning with insights drawn from Adler and Ronda's work in 2016.

For example, in classes C1 and D, teachers-initiated lessons with illustrative examples on the chalkboard, accompanied by explanatory discussions to scaffold comprehension. An example was presented as a specific "instance from a broader category that allows for generalization," in line with Zadik and Zaslavsky's definition from 2008. Such examples were commonplace in secondary-school mathematics classes. In the study, an examination was conducted into how learners were exposed to mathematical vocabulary, closely examining these examples to gauge the extent and manner of this exposure. These examples effectively clarified the learning objectives and provided the necessary technical mathematical vocabulary. The accompanying explanatory dialogues featured both colloquial and formal mathematical vocabulary, serving as vital sources of terminology for the learners.

Research findings further illuminated that learners' contributions were primarily confined to simple "yes" or "no" responses, with limited opportunity for ideas to permeate classroom discussions in depth or detail. This was reflected in the modest 13% contribution of learners to the overall vocabulary during the nine observed lessons, consistent with Adler and Ronda's 2016 findings, which suggest that learners do not engage in classroom discourse to the same extent as educators. Notably, while Adler and Ronda indicated that some teachers were hesitant to employ formal mathematical language, considering it "abstract and off-putting," the study found that teachers in the research were committed to utilizing precise mathematical vocabulary without reservation. Furthermore, a commendable coherence was demonstrated between illustrative examples and accompanying explanations, a practice consistent with insights of Venkat and Adler from 2012. The teachers' orientation during the initial phase of the intervention aptly prepared for the task, highlighting readiness to seamlessly incorporate mathematical vocabulary into lessons.

8.2.4 Sub-Question 4: How effective is the Frayer model as an approach for learning the mathematical vocabulary of functions?

The research assessed the effectiveness of the Frayer model in aiding learners' acquisition of mathematical vocabulary to enhance their conceptual understanding. In the experimental group, learners used the Frayer model during their mathematical lessons, focusing specifically on the concept of functions. In contrast, the control group employed different vocabulary learning methods, primarily using a definition-only approach.

The study compared the results of mathematical vocabulary assessments between the two groups. Initially, there was no significant difference in performance during the pretest. However, the post-test revealed a statistically significant improvement in the experimental group's performance after implementing the Frayer model. These findings suggest that the Frayer model had a notable positive impact on learners' comprehension of mathematical vocabulary compared to other strategies.

The research also examined participants' language and vocabulary proficiency. The test required learners to define terms like "asymptote," "intercept," "range," and "domain" and match them to their respective definitions and characteristics. The results showed that learners exposed to the Frayer model outperformed their counterparts in the control group, with the difference being statistically significant at a 95% confidence level. This outcome supports the effectiveness of the Frayer model in improving learners' understanding of mathematical vocabulary, aligning with previous research by Rahmadani (2018) that highlights the Frayer model as a useful strategy for learning vocabulary.

Furthermore, the research found that the Frayer model not only facilitated vocabulary acquisition but also encouraged active engagement with the material. Learners actively defined, characterized, and exemplified vocabulary terms, as suggested by Dunston and Tyminski (2013) and Wati and Alimin (2022). They used completed Frayer models in their notebooks as effective note-taking tools, promoting active learning and deeper understanding and retention of the material, as emphasized by Lew et al. (2016), Bui et al. (2013), and Jiang et al. (2018).

Through observations and interviews, it became evident that learners using the Frayer model not only improved their vocabulary retention but also demonstrated a better grasp of mathematical concepts. They showed improved problem-solving skills and a heightened ability to understand their teachers' explanations. The structured nature of the Frayer model also reduced cognitive load by facilitating a comprehensive understanding of vocabulary terms through characteristics, examples, and non-examples.

Furthermore, the research revealed that the Frayer model had broader applicability beyond mathematics. Learners found it valuable in other subjects such as physical sciences, life sciences, economics, and business studies. This aligns with previous studies by Sacapaño and de Castro (2022) and Rampersad et al. (2020), which demonstrated the model's effectiveness in various content subjects, including geography and science.

In summary, the research findings strongly support the efficacy of the Frayer model in enhancing mathematical vocabulary acquisition, improving language and vocabulary proficiency, fostering active engagement, deepening conceptual understanding, and reducing cognitive load. Its versatility extends to a range of subjects, making it a valuable tool for vocabulary acquisition and concept comprehension across the curriculum.

8.2.5 Sub-Question 5: To what extent does competency in mathematical vocabulary link to learners' performance in mathematics?

This study assessed learners' Vocabulary Knowledge (VK) and the marks obtained by the learners (MO) in the test. The research findings highlight a significant correlation between a strong grasp of mathematical vocabulary and improved performance in the test, as demonstrated by the two learners interviewed during the study.

Before the interview, these two learners struggled to list mathematical vocabulary correctly in the pretest, with Dris listing six and Dean listing three correct mathematical terms. Consequently, their MO for the test was low, with Dris scoring 9 and Dean scoring 4 out of a possible 15 marks. However, in the post-test, both learners improved their performance, listing 25 and 22 correct mathematical terms, respectively. This marked improvement was attributed to the effective use of the Frayer model as a vocabulary-learning strategy, leading to higher MO for Dris (from 9 to 15) and Dean (from 4 to 13).

Beyond these individual cases, an analysis of 157 learners who participated in the post-test revealed a strong positive correlation (r = 0.61) between the number of mathematical vocabulary terms known and their performance. This suggests that there is a direct and moderately strong relationship between the knowledge of mathematical vocabulary and marks obtained in mathematics tests. The more mathematical vocabulary a learner knows, the more likely they are to perform well in mathematics, challenging the findings of Powell & Driver (2015), which claimed the reverse relationship.

The interviewed learners emphasized that knowledge of mathematical vocabulary significantly aided their ability to read and understand mathematics questions, comprehend their teachers' explanations, and articulate and justify their answers effectively. These skills, stemming from an enhanced knowledge of mathematical vocabulary, serve as precursors to improved mathematics performance.

Furthermore, this study suggests that mathematical content presented to learners can be a cognitive load. The extent to which this load affects learners' success in solving problems is influenced by their familiarity with mathematical vocabulary. Increased knowledge of mathematical vocabulary reduces the cognitive load, enhances understanding of teacher explanations, question instructions, and peer discussions, and improves the ability to ask questions and explain answers. This leads to a state of conceptual understanding, which is associated with improved mathematics performance.

This conclusion is supported by the findings of Bhuvaneswari and Umamaheswari (2020), indicating that investing time in learning mathematical vocabulary yields dividends by enabling students to develop a clear understanding of mathematical concepts and actively engage in the process of understanding mathematical language and content.

8.3 Answering the main research question for the study

In the study, the aim was to answer a fundamental research question: **What affordances does the integration of focused vocabulary and mathematical content learning provide for conceptual understanding and performance in mathematics?** The investigation yielded several crucial findings that shed light on the relationship between vocabulary acquisition and mathematical proficiency.

One of the primary observations was that learners encountered significant difficulties in associating the appropriate mathematical vocabulary with graphical representations. This challenge stemmed from the predominant use of the definition-only teaching method employed by participating teachers. This method had several drawbacks: it was teacher-centred, lacked active engagement from learners, and failed to encourage notetaking. Consequently, learners exposed to this approach exhibited limited cognitive engagement with the vocabulary, leading to poor retention and recall. It became evident that these inadequately learned terms were unlikely to be stored in long-term memory for future application.

Additionally, the study revealed that learners struggled to define or characterize technical mathematical vocabulary related to functions, including terms like asymptote, intercept, domain, and range. This deficiency in vocabulary understanding directly impacted their ability to solve mathematical problems related to these concepts. For instance, learners who could not provide a correct definition for "domain" were less likely to correctly determine the domain of a given function, a pattern reflected in their performance on examinations. The Department of Basic Education (DBE) also noted a general lack of understanding of concepts such as asymptote, domain, and range among candidates.

However, a significant turning point in the research was the introduction of the Frayer model as a vocabulary-learning tool. Implementing this technique led to a substantial improvement in learners' knowledge and comprehension of mathematical vocabulary. The study observed a moderately positive correlation ($r = 0.61$) between the number of correctly defined mathematical terms known by a learner and their performance in mathematics tests. This relationship demonstrated that a deeper understanding of mathematical vocabulary translated into enhanced achievement in mathematical tasks.

Based on these findings, the study concluded that it is highly advantageous to encourage learners to explicitly integrate the learning of mathematical vocabulary with subject content.

Mastery of mathematical terminology directly contributes to improved conceptual understanding and performance in the subject. Moreover, the study affirmed that the use of the Frayer model for vocabulary learning was particularly effective in achieving this desired outcome. Integrating focused mathematical vocabulary learning with mathematical content is instrumental in fostering conceptual understanding, a critical factor influencing mathematics performance.

Furthermore, the research demonstrated the feasibility of seamlessly integrating mathematical vocabulary acquisition with subject content, underscoring the potential of Content and Language Integrated Learning (CLIL) in South African mathematics classrooms. Utilizing the Frayer model within a CLIL framework holds substantial promise for enhancing mathematics learning outcomes, as it promotes active learning, reduces cognitive load, fosters vocabulary mastery, and ultimately elevates both conceptual understanding and mathematical performance. The study paves the way for the effective use of CLIL, with the Frayer model as a valuable tool, to positively impact mathematics education in South Africa.

mathematics learning.

8.4 Practical implications of the study

The research carries several practical implications for education. Firstly, it highlights the effectiveness of the Frayer Model in enhancing conceptual understanding and mathematical performance, offering educators a valuable pedagogical tool. Furthermore, the study underscores the significance of vocabulary acquisition in mathematics, emphasizing the need for a structured approach to teaching mathematical terms. Educators can incorporate the Frayer Model to cultivate a deeper understanding of mathematical language, aiding students' comprehension of complex concepts.

Additionally, the findings prompt curriculum designers to consider integrating vocabulary-building strategies into mathematics education. This could lead to more holistic and effective learning experiences for students. Overall, the study's practical implications advocate for innovative strategies that bridge the gap between mathematical language and conceptual comprehension, fostering improved mathematical proficiency among Grade 11 learners.

8.5 Recommendations of the study

The recommendations from this study are intended to benefit a wide range of stakeholders, including in-service mathematics teachers, curriculum content developers, examining bodies, and educational authorities within the Department of Education

8.5.1 The teaching of mathematics

Teachers should pay attention to the mathematical concepts and definitions when teaching functions, as these are related to the mathematical register using precise and standardized language, ensuring clarity and consistency in communication about mathematical ideas. Teachers can do this by spending time discussing the basic concepts of functions. As this study has already suggested, teachers can introduce their learners to the Frayer model, so that the learners can understand the fundamental aspects of this mathematical vocabulary, e.g. all points on the x-axis have a y-coordinate of 0 and all points on the y-axis have a x-coordinate of 0, the domain is always a set of x-values, and the range is always a set of y-values, a graph of a function cannot cut the asymptote of the function. The Frayer model facilitates knowing vocabulary that is mathematical, as well as understanding that vocabulary.

Currently, teachers use the definition-only method for teaching mathematical vocabulary. This method is less effective in the sense that it does not fully engage learners cognitively when learning mathematical vocabulary. In addition, the definition-only method tends not to compel learners to take down notes. As a result, the learners easily forget the definitions, forget where to use the words, and struggle to answer questions associated with the words.

8.5.2 The role of content developers

Content developers play a crucial role in the education system. They are the ones who produce the texts that expose the learners to the curriculum content. This study recommends that content developers should incorporate examples and tasks that push the learners into learning mathematical vocabulary. They should introduce learners to the explicit learning of mathematical vocabulary, for example by incorporating the Frayer model and other graphic organisers (concept maps, flow charts) for learning mathematical vocabulary into their texts. This must be evident in different textbooks, written and prescribed to different schools, for different grades and curricula.

8.5.3 Examining literacy.

In order to enforce literacy in mathematics, examination authorities ought to assess candidates' literacy in all chapters of the syllabus. This can be done by including parts in the examination that compel candidates to understand mathematical vocabulary. Candidates must be compelled to realise that mathematics is not restricted to only the manipulation of numbers. Examination questions should incorporate defining terms, matching concepts to their definitions and characteristics, completing sentences, and labelling mathematical objects such as graphs. Including questions where learners are compelled to give synonyms of certain vocabulary items used in mathematics should be of great importance. The topic of functions is done at many levels of schooling, and it is a foundation of university mathematics, where such questions are possible. This would foster more conceptual understanding for this topic.

8.5.4 The role and responsibilities of the Department of Education

The Department of Education is the custodian and implementer of national policies on education. In any country, education is vital for the country's realisation of economic development. Mathematics education significantly helps in the development of STEM subjects. It is also useful in commerce, business, culture, medicine, government, farming, architecture, as well as natural and social sciences. Due to its usefulness for the overall functionality of a country, the Department of Education needs to explore the integration of the learning of mathematical vocabulary and mathematical content as a way of improving results in mathematics. This can be done by making amendments to the curriculum, so that this integration is reflected and enforced in the classrooms. Most importantly, the department must ensure that the textbooks used in schools adhere to reinforcing mathematical vocabulary learning.

8.6 Limitations of the study and directions for further research

A significant limitation inherent in this study is rooted in its methodology, specifically the constrained sample size and limited scope of data collection. The study was exclusively conducted within the confines of four schools, utilizing a relatively modest sample of learners. This restricted representation could hinder the generalizability of the study's findings to broader educational contexts. The further restriction of lesson observations to merely three teachers and three classes across two schools adds another layer of limitation. This selective sampling raises concerns about the robustness and representativeness of the observed teaching and learning behaviours.

Moreover, the potential for the Hawthorne effect cannot be overlooked. Given that the observations were conducted in a manner that was apparent to the participants, there is the possibility that the awareness of being observed influenced the instructional strategies employed by teachers and the engagement of learners. This effect was palpably demonstrated when a student sought clarification regarding the researcher-taught strategy, illustrating the potential bias introduced by the observation itself.

Adding to the complexity, the diversity in instructional resources and teaching methodologies across different classrooms and schools introduces the challenge of accounting for textbook and teacher effects. The inherent variations in these factors can introduce confounding variables that might compromise the internal validity of the study.

Despite these methodological limitations, it is important to acknowledge the potential transferability of the study's outcomes to analogous educational settings with similar characteristics.

To address these methodological limitations and enhance the robustness of the study's findings, future research should consider adopting a more expansive and comprehensive approach. This could involve expanding the participant pool to a wider range of schools and learners, employing a more diverse set of teaching methods, and employing more rigorous control measures to mitigate the potential influence of observer effects.

Furthermore, extending the duration of the study and incorporating longitudinal elements could provide insights into the long-term effects and sustainability of the observed outcomes.

This longitudinal perspective could also help in understanding how the benefits of the Frayer model unfold over time and whether they persist beyond immediate exposure.

In conclusion, while this study offers valuable insights, it is imperative to acknowledge and critically engage with its methodological limitations. Doing so not only strengthens the credibility of the research but also highlights directions for more robust future investigations.

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APPENDICES

Appendix 1: Information Letter to School Principals

DATE: 18/02/2019

Dear Principal

My name is **Edwin Madzore,** I am a student in the School of Education at the University of the Witwatersrand. I am doing research on the

Use of precise Vocabulary and Language in the Domain of Mathematical Functions

My research involves teaching and training learners on the use of precise vocabulary as a way of equipping them with the correct language for learning Functions. This involves fusing the teaching of the Language of Teaching and Learning (LoTL) with the learning of the subject content as envisioned in Content and Language Integrated Learning (CLIL). CLIL is a learning methodology that provides substantial value for language learning by fusing the content and language of the language of teaching and learning and the language of the content of the subject being taught, English and secondary mathematics in the case of this study. Due to the dual focus envisioned in CLIL equal attention is given to both subject language and subject content. As an educational approach CLIL allows curricular content to be taught through the medium of a language that is not a mother tongue of the learners, typically to learners participating in some form of mainstream education at the primary, secondary, or tertiary level. This study suits the current state of affairs in most schools in the country as 90% of our learners are taught Mathematics in English which happens not to be their mother tongue.

A CLIL intervention will be done with learners and participating teachers after school hours for two weeks during which they will be familiarized with the nature of the research. During this informational phase of the research, participants will be trained on the use of the Frayer model as a vocabulary learning technique. After the information giving phase, learners will be encouraged to use the Frayer model as a note taking tool during the learning of mathematics.

I will conduct the training in a normal classroom and the training does not attract any charge on the part of the school, the students or me. After the two weeks of training, I will allow the students to implement CLIL for the duration of the learning of Mathematical Functions. During the course of the research, participating students will do a pre-test at the beginning of the research and a post- test at the end as a way of checking the effectiveness of the intervention program. I will also interview certain learners (experiment group) at the end of each of the two tests. The interview sessions will take at most 5 minutes per participant. The interviews will take place on school premises in a classroom after school hours so that they do not interfere with lessons.

The reason why I have chosen your school is because I am a mathematics teacher in the same province as your school. It will be beneficial for the learners to undergo the training as in the process they will become equipped with skills that can help them become better doers of mathematics.

I am inviting your school to participate in this research. It is not compulsory that your school takes part but I am kindly requesting your assistance in this regard.

The research participants will not be disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The names of the research participants and identity of the school will be kept confidential and anonymous at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as it is convenient.

Yours sincerely,

Edwin Madzore
ADDRESS: 22 Essex Street, Kensington, Johannesburg, 2094
EMAIL: madzorebest@yahoo.com TELEPHONE NUMBER: 0790579112

Appendix 2: Information Letter to Learners

DATE: 18/02/2019

Dear Learner

My name is Edwin Madzore and I am a student in the School of Education at the University of the Witwatersrand. I am doing research on

Use of precise Vocabulary and Language in the Domain of Mathematical Functions

My investigation involves training students on use of the Frayer model as a vocabulary learning tool. I will train you after school for two weeks in a classroom on the school premises. After the training, you will be expected to make use of the learnt techniques in your mathematics lessons, precisely as you learn the concept of functions. As part of the research, you will write some tests but these tests will not be for marks and they do not have any effect on your term or year mark. The marks obtained in these tests are only for the purpose of the research and not for classifying you in any way.

Would you mind if I ask you to take part in my research? This is not a compulsory exercise but I am kindly inviting you to participate in the research.

I need your help with taking part in the training and then later in the use of the methods learned during training in learning your Mathematical Functions topic. I will also ask you during the research to complete some Frayer model templates as well as answer a few interview questions as a way of helping me to evaluate the effectiveness of the training program.

Remember, all the tasks that you will do during the research are not for marks and it is voluntary which means that you don't have to do them if you decide not to. Also, if you decide halfway through that you prefer to stop, that will be completely your choice and will not affect you negatively in any way.

I will not be using your real name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join me in the study should your parents' consent.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Edwin Madzore

ADDRESS: 22 Essex Street, Kensington, Johannesburg, 2094
EMAIL: madzorebest@yahoo.com: TELEPHONE NUMBER: 0790579112

Appendix 3: Information Letter to Parents

DATE: 18/02/2019

Dear Parent

My name is Edwin Madzore and I am a student in the School of Education at the University of the Witwatersrand. I am doing research on

Use of precise Vocabulary and Language in the Domain of Mathematical Functions

My research involves teaching and training students on the use of the Frayer model as a vocabulary learning tool. The training that I will give them helps them to fuse the learning of the language of teaching and learning with the language of the Mathematics content. I will do the training during school days and there are no costs involved in doing the training. Training sessions will take place after school hours so they do not affect your child's lesson times.

The reason why I have chosen your child's class is because the research is based on Senior Secondary School Mathematical Functions. This targeted topic of Functions is done in grade 11 so the fact that your child is in this grade at the school where the research will be conducted makes your child's class a potential participating group.

Would you mind if I ask you to allow your child to take part in the research? In addition to being trained and observed as he works with the research tools, your child will be asked to write two tests and answer some interview questions. I will interview him/her once at the end of each of the two tests. Both tests are not for marks so they will not affect your child's progress in any way. The interview session will take at most 5 minutes. The interview will take place in a classroom after school so that it does not interfere with lessons.

Your child will not be disadvantaged in any way. He/she will be reassured that s/he can withdraw his/her permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for taking part in this study.

Your child's name and identity will be kept confidential and anonymous at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.
Yours sincerely

Edwin Madzore

ADDRESS: 22 Essex Street, Kensington, Johannesburg
EMAIL: madzorebest@yahoo.com TELEPHONE NUMBER: 0790579112

Appendix 4: Learner Consent Form

Dear Learner

Please fill in the reply slip below if you are willing to participate in my study called:

My name is: ____

Please circle only YES or NO to each of the following questions/statements.

Permission to review/collect documents/artifacts
I agree that my Frayer model templates can be used for this study only. YES NO

Permission to observe you in class
I agree to be observed in class. YES NO

Permission to be audio/ video recorded
I agree to be video recorded for this study YES NO

Permission to be interviewed
I would like to be interviewed for this study. YES NO

I know that I can stop the interview at any time and don't have to answer all the questions asked. YES NO

Permission for questionnaire/test
I agree to write some tests for this study. YES NO

Informed Consent
I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be videotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Signature______________________________ Date__________________________

Appendix 5: Parents Consent Form

Dear Parent(s)

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called:

I, ________________________________ the parent of ______________________

Please circle <u>only</u> YES <u>or</u> NO to each of the following questions/statements.

Permission to review/collect documents/artifacts

I agree that my child's templates can be used for this study only.	YES	NO

Permission to observe my child in class

I agree that my child may be observed in class.	YES	NO

Permission to be audio/ video recorded

I agree that my child can be video recorded for this study	YES	NO

Permission to be interviewed

I agree that my child may be interviewed for this study.	YES	NO
I know that he/she can stop the interview at any time and doesn't have to answer all the questions asked.	YES	NO

Permission for questionnaire/test

I agree that my child may write some tests for this study.	YES	NO

Informed Consent

I understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of the school will not be revealed.
- My child does not have to answer every question and can withdraw from the study at any time.
- My child can ask not to be videotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of the project.

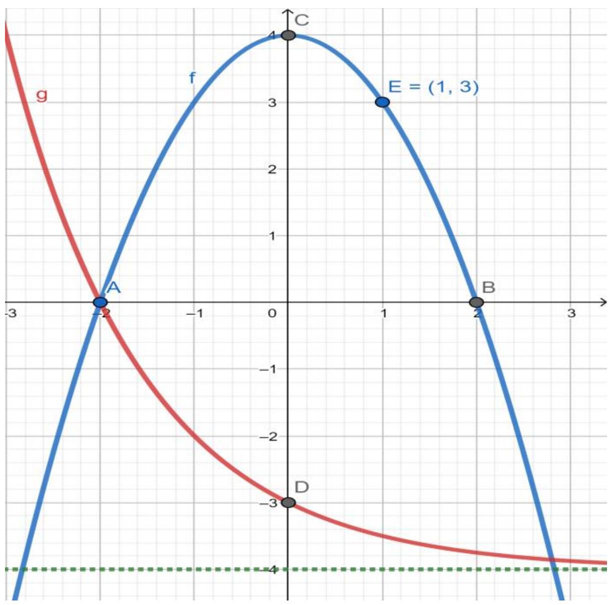
Signature____________________________ Date__________________________

Appendix 6: Research Test

Surname: .... Name: ………………..........................

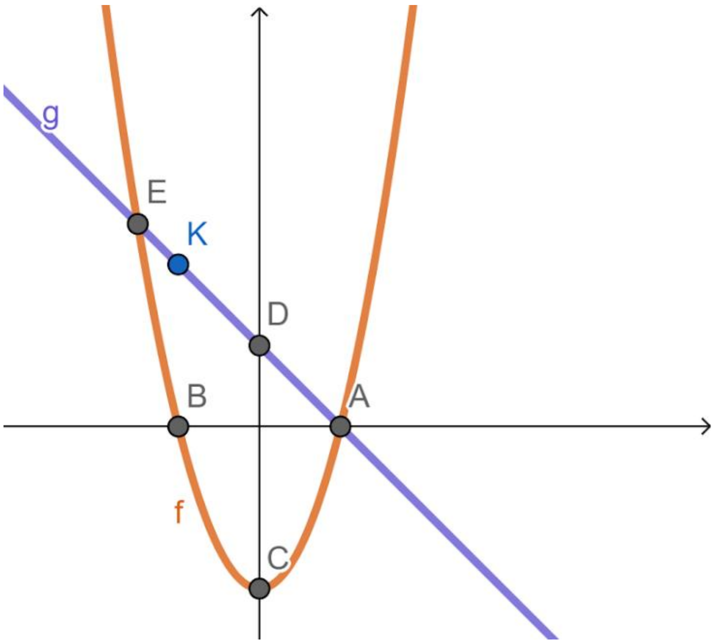
Questions in this task refer to the mathematics language and vocabulary used to communicate and explain mathematical concepts in the domain of FUNCTIONS. *Kindly answer the following questions as honestly as you can knowing that your identity will never be disclosed in this research.*

1 Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object. Write down the *mathematical vocabulary* used and *identify it* on the graph (explaining its meaning fully). Two examples are given in the answering spaces below.



Coordinates	Decreasing		
1. *B* (2; 0) is a point on the parabola *2.*	1. *g is a decreasing function* 2. *f is decreasing for* $x > \mathbf{0}$		

2. The graphs of $f(x) = x^2 - 4$ and $g(x) =- x + 2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK$/ / y - axis. f\ and\ g$ intersect at A (2; 0) and E (-3; 5).



2.1 Write down the coordinates of C.

……………………………………………………………

2.2 Write down the range of f

……………………………………………………………………………………

……………………………………………………………………………………

2.3 Write down the value(s) of x for which $f(x) < g(x)$

…………………………………………………………………………………………….

2.4 Use correct mathematical vocabulary in a sentence to describe the (answer) algebraic notation given in question 2.3.

……………………………………………………………………………..

3 Define or describe the following Mathematical Vocabulary:

3.1 *Asymptote*

………

………

3.2 Intercept

………

3.3 Range

………………………………………………………………………………………………………

………………………………………………………………………………………………………

3.4 Domain

………………………………………………………………………………………………………

………………………………………………………………………………………………………

4 Table 1 below has a Mathematical Vocabulary for a Concept, an example of how the concept is expressed symbolically and a characteristic of the concept.

Complete table 2 by rearranging and matching the concept in Column A to an example of the concept in Column B and a characteristic of the concept in column C.

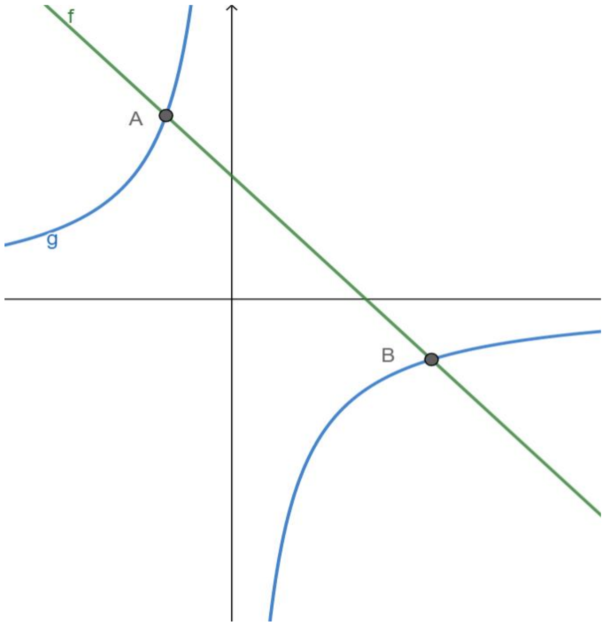
Table 1

Column A: Vocabulary	Column B: Example	Column C: Characteristic
4.1 Asymptote	$y = 2^x$	Only the letter representing the independent variable is present
4. 2 Intercept	$2 < x \leq 5$	Linear equation
4.3 Range	$y > 2$	The exponent is a variable.
4.4 Domain	$(2; 0)$	Only the letter representing the dependent variable is present
4.5 Exponential equation	$x = 3$	At least one coordinate is equal to zero

Table 2

Column A: Vocabulary	Column B: Example	Column C: Characteristic
4.1 Asymptote		
4.2 Intercept		
4.3 Range		
4.4 Domain		
4.5 Exponential equation		

5 The graphs of $f(x) = -x + 2$ and $g(x) = \frac{-3}{x}$ are sketched below. A and B are the points of intersection of the graphs.



Calculate the coordinates of A and B. Show all steps and next to each step give a reason for doing it (The reasons you give should show that you understand why each step and procedure is done).

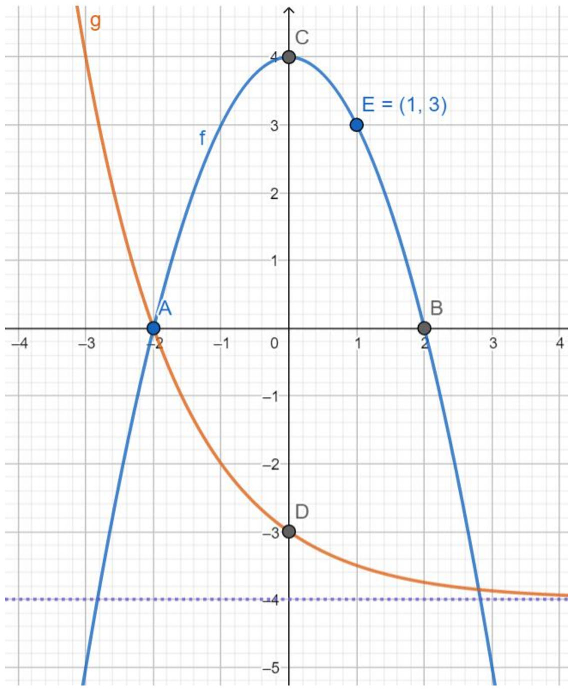
Working/Calculations	Reason for doing it… start each explanation with 'Because'.

THANK YOU for the thought, time, and effort you have put into writing the Test.

Appendix 7: Research Test Memorandum

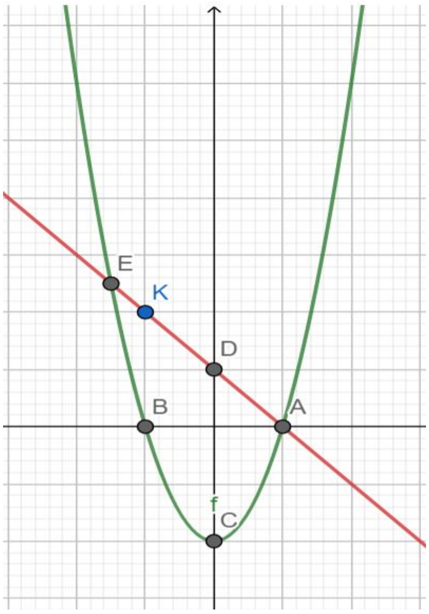
Questions in this task refer to the mathematics language and vocabulary used to communicate and explain mathematical concepts in the domain of FUNCTIONS. *Kindly answer the following questions as honestly as you can knowing that your identity will never be disclosed in this research.*

1. Study the mathematical object below and come up with a list of mathematical vocabulary that can be associated with the object.



Turning point	**increasing**	**decreasing**	**quadrants**	**Cartesian plane**
C is turning point of f	*f is increasing for* $x < 0$	*g is a decreasing function* *f is decreasing for* $x > 0$	*1: (x;y) 2:(-x;y) 3:(-x;-y) and 4:(x;-y)*	*2 perpendicular number lines*
Origin	**asymptote**	**exponential function**	**function**	**graph**
Intersection of the axes (0; 0)	*Never intersects with the graph for the entire range of the graph* *Dotted horizontal line y = -4*	*Exponent is the independent variable* *g is exponential graph*	*Relationship of correspondence between element of domain and range.*	*Visual representation of a function on Cartesian plane*
Parabola	**Quadratic function**	**coordinates**	**intercept**	**x-axis**
Graph of a quadratic function *f is the parabola*	*Highest exponent of the variables is 2*	*Values showing exact position on the Cartesian plane.*	*Point where graph crosses an axis* *A, B ,C and D*	*Horizontal number line of the Cartesian plane*
y-axis	**range**	**domain**	**intersect**	**roots**
Vertical number line of the Cartesian plane	*Output values of a function* $y \leq 4$ *for parabola* $y > -4$ *for exponential graph*	*Input values of a function* $x \epsilon R$ *for both f and g*	*Meet and cross at a point* *f and g intersect at A*	*Solutions/zeros of the function* $x = 2$
line	**vertical shift**	**Horizontal shift**	**maximum**	**curve**
$y = -4$	*Movement along the y-axis*	*Movement along the x-axis*	*No other value above it* $y = 4$ *for the parabola*	*Parabola* *Exponential graph*
y-intercept	**x-intercept**	**Line of symmetry**	**reflection**	**image**
Point where graph crosses y-axis *C and D*	*Point where graph crosses the x-axis* *A and B*	*Line that bisects a figure/graph* $y - axis\ to\ the\ paraola$	*Type of transformation* *y-axis*$(x = 0)$ *is mirror line*	*A is image of B and vice versa after a reflection in* $x = 0$
Distance	**horizontal**	**vertical**	**value**	**Point**
Length of space between points *CD =7*	*At right angles with the vertical* *x-axis*	*At right angles to the horizontal* *y-axis*	*Number* *2*	*Position on the Cartesian plane* *E(1;3)*
Symmetry	**zeros**	**parallel**	**variables**	
Being made of similar parts	*Roots/solution* $x = -2$	*The asymptote is parallel to the x-axis*	*x is independent y is dependent*	

2. The graphs of $f(x) = x^2 - 4$ and $g(x) =- x + 2$ are sketched below. A and B are the x-intercepts of f. C and D are the y-intercepts of f and g respectively. K is a point on g such that BK $//y-axis$. f and g intersect at A (2; 0) and E (-3; 5).



2.1 Write down the coordinates of C.

C(0;-4)

2.2 Write down the range of f

$y \geq -4$ *OR* $-4 \leq y < \infty$ *OR* $y \in [-4, \infty)$

2.3 Write down the value/s of *x for which f(x) < g(x).*

$-3 < x < 2$

2.4 Use correct mathematical vocabulary in a sentence to describe the answer (algebraic notation) given in question 2.3.

All real x values that are greater than negative 3 and at the same time less than 2

3 Define or describe the following terms:

3.1 *Asymptote*

A line that a function keeps getting closer to but never actually touches/ a line that a graph gets closer and closer to as x approaches a specific value or as x approaches infinity.

3.2 Intercept

A point where a graph crosses, or cuts the x or the y axes.

3.3 Range

Complete set of all possible resulting values of the dependent variable, y, after substituting the domain/ set of all possible values that the function takes, or is valid for.

3.4 Domain

Complete set of valid/acceptable/possible values of the independent variable/ Set of all values for which the function is defined/ set of all possible x values that makes the function work.-

4 Table 1 below has a Mathematical Vocabulary for a Concept, an example of how the concept is expressed symbolically and a characteristic of the concept.

Complete table 2 by rearranging and matching the concept in Column A to an example of the concept in Column B and a characteristic of the concept in column C.

Table 1

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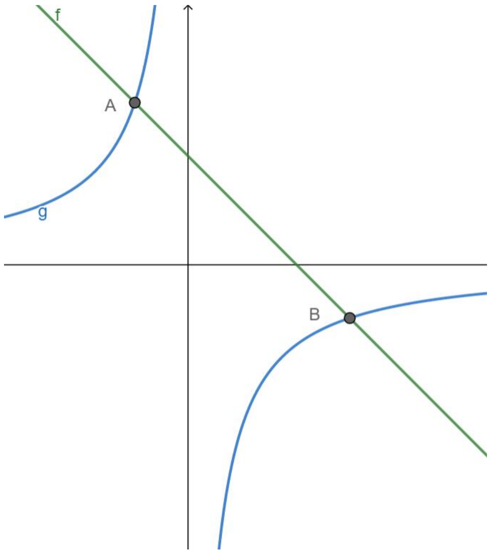
4.4 Domain	(2; 0)	Only the letter representing the dependent variable is present
4.5 Exponential equation	$x = 3$	At least one coordinate is equal to zero

Table 2

Column A: Concept	Column B: Example	Column C: Characteristic
4.1 Asymptote	$x = 3$	*Linear equation*
4.2 Intercept	(2; 0)	*At least one coordinate is equal to zero*
4.3 Range	$y > 2$	*Only the letter representing the dependent variable is present*
4.4 Domain	$2 < x \leq 5$	*Only the letter representing the independent variable is present*
4.5 Exponential equation	$y = 2^x$	*The exponent is a variable.*

5 The graphs of $f(x) =- x + 2$ and $g(x) = \frac{-3}{x}$ are sketched below. A and B are the points of intersection of the graphs.



Calculate the coordinates of A and B. Show all steps and next to each step give a reason for doing it.

Step	Reason for doing it… start each explanation with 'Because'.
$-x+2=\frac{-3}{x}$	*Because A and B are intersection points of the graphs, the graphs intersect where their y values are equal.*
$x(-x+2)=-3$	*Because to eliminate denominators Multiply both sides by LCD, x.*
$-x^2+2x=-3$	*Because to get the equation into quadratic form we need to Expand to remove brackets.*
$-x^2+2x+3=-3+3$	*Because to keep the equation balanced we need to Add 3 both sides to set equation to zero/write the equation in standard quadratic form.*
$-(x^2-2x-3)=0$	*Because factorising is easier when the leading term has a positive coefficient, Factor out leading coefficient -1 to remain with a positive leading coefficient*
$(x^2-2x-3)=0$	*Because -1 is a constant we can Multiply or Divide throughout by it.*
$(x-3)(x+1)=0$	*Factorise the quadratic equation*
Either $x-3=0$ *or* $x+1=0$	*Because If the product of two factors is zero then one of the factors or both factors are equal to zero.*
If $x-3=0$ *then* $x=3$	*Because to isolate x/make x subject of the formula, and keep the equation balanced add 3 to both sides. Equate linear equation to zero to solve for x*

If $x+1=0$ $then$ $x=-1$	*Because to isolate x/make x subject of the formula, and keep the equation balanced add -1 to both sides. Equate other linear equation to zero to solve for x*
Therefore $x=3$ or $x=-1$	*Solutions/roots*
$f(-1)=-(-1)+2$= 3	*Because to find values of y, we substitute values of x in any of the functions/equations*
$f(3)=-(3)+2$= - 1	
A(-1;3) and B (3;- 1)	*Because A and B are points so they have coordinates. (Check relative positions of A and B on the Cartesian plane).*

Appendix 8: Pilot Study Data

Categories	Vocabulary Knowledge	Vocabulary Proficiency (VP)	Vocabulary Proficiency (VP)					Language Proficiency(LP)					Conceptual & Procedural Proficiency(CUN)				Performance (P)				Marks
	Q1		Q4.1	Q4.2	Q4.3	Q4.4	Q4.5	Q 2.4	Q3.1	Q3.2	Q.3.3	Q3.4	Q2.1	Q2.2	Q2.3	Q5	Q2.1	Q2.2	Q2.3	Q5	**Total**
LEARNER																	2	1	2	10	**15**
P1	12		23	23	22	22	23	31	31	31	31	31	40	40	41	40	0	0	0	0	**0**
P2	11		23	23	22	22	23	31	31	31	30	30	42	41	40	41	1	0	0	1	**2**
P3	11		21	21	22	22	23	31	32	31	32	32	43	43	41	43	2	1	0	10	**13**
P4	12		22	21	22	21	23	30	31	32	31	30	43	40	40	41	2	0	0	10	**12**
P5	10		22	23	21	21	23	30	31	31	30	30	40	40	40	40	0	0	0	0	**0**
P6	11		22	23	21	22	23	30	32	31	32	32	42	41	41	40	1	0	0	1	**2**
P7	10		22	21	21	21	23	31	30	31	31	32	41	41	41	41	0	0	0	0	**0**
P8	12		23	23	22	22	23	30	32	31	31	31	43	43	40	40	2	1	0	0	**3**
P9	12		22	23	22	22	23	30	31	31	31	31	43	40	40	40	1	0	0	8	**9**
P10	10		22	23	22	22	23	30	30	30	30	30	43	41	41	41	2	0	0	10	**12**
P11	10		21	23	21	22	22	30	30	31	31	30	41	40	41	40	0	0	0	0	**0**

P12	12		21	21	21	22	21	31	32	31	32	32	41	41	41	41	0	0	0	0	**0**
P13	12		22	23	21	21	23	31	32	32	32	32	43	41	41	43	2	0	0	10	**12**
P14	11		22	23	21	21	23	31	32	31	31	31	43	41	41	40	2	0	0	0	**2**
P15	11		23	23	23	23	23	31	31	31	31	32	42	40	40	40	1	0	0	0	**1**
P16	12		21	22	22	21	23	31	32	31	32	32	43	43	42	43	2	1	1	10	**14**
P17	11		22	23	21	22	23	31	32	31	30	30	42	40	41	40	1	0	0	0	**1**
P18	11		21	23	22	21	23	31	32	33	33	32	43	43	41	41	2	1	0	1	**4**
P19	10		21	22	21	21	21	30	31	31	31	31	42	41	40	40	1	0	0	0	**1**
P20	11		21	23	22	21	23	31	30	31	32	32	43	41	41	41	2	0	0	2	**4**
P21	11		20	20	22	21	21	30	31	33	32	32	42	40	40	40	1	0	0	0	**1**
P22	11		23	23	22	22	23	30	31	33	31	31	43	43	40	40	2	1	0	0	**3**
P23	12		22	21	22	22	21	30	31	31	31	31	41	40	41	40	0	0	0	0	**0**
P24	10		21	21	21	21	23	31	31	31	31	31	41	41	41	40	0	0	0	0	**0**
P25	11		22	23	23	22	23	30	32	31	31	32	42	40	42	40	1	0	1	0	**2**
P26	10		23	23	22	22	23	30	33	33	32	32	43	43	41	42	2	1	0	7	**10**
P27	11		23	23	22	22	23	30	31	32	33	32	43	43	40	41	2	1	0	10	**13**

P28	11		22	23	21	21	23	31	31	31	31	31	43	40	41	40	2	0	0	0	**2**
P29	11		21	23	21	21	23	30	32	33	31	30	41	40	40	40	0	0	0	0	**0**
P30	12		21	23	21	22	23	30	31	32	32	32	43	43	40	40	2	1	0	0	**3**
P31	11		21	23	22	23	23	30	31	31	33	33	40	40	40	40	0	0	0	0	**0**
P32	12		22	23	22	21	22	31	31	32	32	32	43	43	41	41	2	1	0	0	**3**
P33	11		21	23	21	21	21	31	31	31	32	32	43	40	42	41	2	0	1	0	**3**
P34	11		21	23	23	21	23	30	32	32	33	33	43	41	40	42	2	0	0	10	**12**
P35	10		21	23	21	21	23	31	31	31	30	32	41	40	40	40	0	0	0	0	**0**
P36	10		21	23	22	22	22	30	30	31	31	30	42	40	41	41	0	0	0	0	**0**
P37	10		21	20	21	20	22	30	31	31	31	30	42	41	40	40	1	0	0	0	**1**
P38	11		23	23	22	22	23	31	31	31	31	30	42	41	41	40	1	0	0	0	**1**

Appendix 9: Pretest Data

		Vocabulary Knowledge (VK)		Vocabulary Proficiency (VP)					Language Proficiency(LP)					Conceptual & Procedural Proficiency(CUN)				Performance			
		Q1	Q1	Q4. 1	Q4. 2	Q4. 3	Q4. 4	Q4. 5	Q2. 4	Q3. 1	Q3. 2	Q3. 3	Q3. 4	Q2. 1	Q2. 2	Q2. 3	Q5	Q2. 1	Q2. 2	Q2. 3	Q5
	LEARNER																	2	1	2	10
School A	1C	11	6	23	23	21	21	23	30	30	31	31	30	43	43	41	40	2	1	0	0
CONTROL	2C	11	2	22	23	21	21	22	30	31	32	31	31	43	41	41	40	2	0	0	0
	3C	11	4	23	23	22	22	23	30	32	31	31	31	42	41	40	40	1	0	0	0
	4C	11	10	20	20	20	20	20	30	32	31	30	30	40	40	40	40	0	0	0	0
	5C	11	6	22	22	22	22	20	30	31	31	32	32	42	41	41	40	1	0	0	0
	6C	11	9	20	20	20	20	20	30	32	31	30	30	43	43	41	40	2	1	0	1
	7C	11	7	23	23	23	23	23	30	30	30	30	30	43	41	41	40	2	0	0	0
	8C	11	4	23	22	22	21	22	31	31	31	32	32	43	43	41	41	2	1	0	0
	9C	11	6	21	21	22	22	21	31	30	30	30	30	43	41	41	41	2	0	0	0
	10C	10	N	23	22	21	21	23	30	31	31	30	30	40	41	41	40	0	0	0	0

	11C	11	4	22	23	21	22	23	30	30	31	31	30	40	40	40	40	0	0	0	0
	12C	11	3	22	23	22	22	23	30	31	31	31	31	43	43	40	40	2	1	0	0
	13C	11	4	22	23	21	21	23	30	31	31	31	31	41	40	41	40	0	0	0	0
	14C	11	6	21	23	22	22	21	31	32	31	31	31	41	41	41	41	0	0	0	0
	15C	11	2	21	23	21	22	23	30	30	30	30	30	43	41	41	40	2	0	0	0
	16C	11	0	23	23	22	22	23	30	31	31	31	31	41	40	41	40	0	0	0	0
	17C	11	2	22	22	22	21	22	30	32	31	30	30	42	41	40	40	1	0	0	0
	18C	10	N	22	23	21	21	23	30	31	30	30	30	41	40	40	41	0	0	0	0
	19C	11	6	21	21	22	22	23	30	31	31	32	32	42	41	40	40	1	0	0	0
	20C	11	7	22	22	21	21	22	30	30	30	30	30	43	41	41	40	2	0	0	C
	21C	11	4	22	23	22	22	23	30	31	31	31	31	40	40	40	41	0	0	0	0
	22C	11	4	22	23	21	21	22	30	30	30	30	30	43	41	40	40	2	0	0	0
	23C	11	4	23	22	21	21	23	30	31	31	30	30	41	41	40	40	0	0	0	0
	24C	11	5	21	21	21	22	22	30	32	31	31	30	43	43	40	40	2	1	0	0
	25C	11	0	21	21	22	22	23	30	30	30	30	30	42	41	40	41	1	0	0	0
	26C	11	5	22	23	23	22	23	30	30	31	32	32	43	43	41	41	2	1	0	0
	27C	11	8	22	23	22	22	22	31	31	32	32	32	43	43	41	41	2	1	0	0

	28C	11	1	23	23	22	22	23	30	31	32	30	30	42	41	41	40	1	0	0	0
	29C	11	4	21	22	22	22	21	31	31	31	32	32	42	41	41	41	1	0	0	0
	30C	11	1	22	23	21	21	22	30	30	31	31	30	41	40	40	40	0	0	0	0
	31C	11	4	23	23	23	22	23	30	31	31	32	32	42	41	41	40	1	0	0	0
	32C	11	8	22	23	22	22	23	31	31	32	32	32	43	43	42	43	2	1	1	10
	33C	11	4	21	23	22	21	21	30	30	30	30	30	43	41	40	40	2	0	0	0
	34C	11	5	21	23	23	22	21	30	31	32	32	32	43	43	40	41	2	1	0	0
	35C	11	7	23	23	22	22	23	30	32	31	32	32	43	43	41	41	2	1	0	0
	36C	11	2	21	23	22	23	23	31	31	31	32	32	42	41	41	41	1	0	0	0
	37C	11	1	22	23	23	22	23	30	30	31	31	30	43	40	40	40	2	0	0	0
	38C	11	2	22	23	23	23	22	30	32	30	31	31	42	41	41	40	1	0	0	0
	39C	11	3	22	23	21	21	23	30	30	31	30	30	43	41	41	40	2	0	0	0
	40C	11	4	23	23	22	22	23	30	32	33	32	32	43	43	42	40	2	1	1	0
	41C	11	0	23	23	21	21	23	30	31	31	30	31	43	41	41	40	2	0	0	0
	42C	11	4	23	23	22	22	23	30	31	32	32	32	43	43	41	41	2	1	0	0
	43C	11	7	21	23	22	21	23	30	31	31	31	31	43	43	41	40	2	1	0	0
	44C	11	3	22	23	23	21	21	31	32	31	31	31	42	41	41	41	1	0	0	0

	45C	11	4	21	21	23	22	23	30	32	31	31	31	41	41	40	40	0	0	0	0
	46C	11	4	22	21	21	20	22	30	31	31	30	30	40	40	40	40	0	0	0	0
	47C	10	N	22	23	22	22	22	31	30	31	31	31	41	41	41	40	0	0	0	0
	48C	11	3	23	23	22	22	23	31	31	31	31	31	43	41	41	40	2	0	0	0
	49C	11	3	22	23	21	22	22	30	33	33	30	30	43	41	41	40	2	0	0	0
	50C	11	1	21	21	21	22	22	30	32	32	31	33	42	41	40	40	2	0	0	0
	51C	11	9	22	21	21	21	22	30	31	31	31	31	43	41	41	40	2	0	0	0
	52C	11	2	21	21	22	21	21	30	32	32	30	30	41	41	41	40	0	0	0	0
School B	53C	11	6	23	23	22	22	23	30	31	31	32	32	43	41	40	41	2	0	0	1
CONTROL	54C	11	0	23	20	20	20	20	30	30	30	30	30	42	41	40	40	1	0	0	0
	55C	11	2	23	21	21	21	23	30	30	31	31	32	43	41	40	40	1	0	0	0
	56C	11	4	23	22	21	21	22	30	30	30	30	30	40	40	40	40	0	0	0	0
	57C	11	7	23	23	23	23	23	31	31	33	33	33	43	41	41	41	2	0	0	1
	58C	11	6	23	23	21	21	21	30	31	31	31	31	42	41	41	41	1	0	0	0
	59C	11	4	23	22	21	21	23	30	30	31	31	31	41	41	40	41	0	0	0	0
	60C	11	1	23	23	21	21	23	32	30	31	32	32	43	41	41	42	2	0	0	5

	61C	11	4	22	21	21	22	21	30	30	30	30	30	40	40	40	40	0	0	0	0
	62C	11	5	23	23	22	22	23	30	31	31	30	30	43	40	40	42	2	0	0	10
	63C	11	3	23	23	20	20	20	30	30	30	30	30	43	41	41	40	2	0	0	0
	64C	11	6	23	23	23	23	23	31	31	32	33	33	42	41	41	40	1	0	0	0
	65C	11	7	23	22	20	21	23	30	30	31	31	31	42	41	40	40	1	0	0	0
	66C	11	1	23	23	22	22	23	30	31	31	31	31	42	41	40	41	1	0	0	0
	67C	11	2	23	23	23	23	23	30	30	31	30	30	41	41	40	41	0	0	0	0
	68C	11	3	22	22	21	21	21	30	32	31	32	32	42	43	40	40	1	1	0	0
	69C	11	6	23	23	22	22	23	30	31	31	32	32	43	41	41	42	2	0	0	4
	70C	11	3	23	23	21	21	23	30	31	31	31	31	42	40	40	40	1	0	0	0
	71C	10	N	21	23	22	21	21	30	30	31	30	30	43	40	40	40	2	0	0	0
	72C	10	N	20	20	20	20	20	30	30	30	30	30	40	40	40	40	0	0	0	0
	73C	11	3	23	23	23	23	23	30	32	31	33	32	40	40	40	40	0	0	0	0
	74C	10	N	23	23	22	22	23	30	31	31	32	32	41	43	40	40	0	1	0	0
	75C	11	3	23	23	23	23	23	30	31	31	31	31	40	40	40	40	0	0	0	0
	76C	11	3	23	21	21	21	23	30	32	32	32	31	43	40	40	40	2	0	0	0
	77C	11	2	22	23	21	21	21	30	31	32	32	31	41	41	41	40	0	0	0	0

	78C	11	5	23	23	22	22	23	31	31	33	32	32	41	40	41	40	0	0	0	0
	79C	11	5	23	21	23	21	23	30	31	31	32	32	42	41	40	40	1	0	0	0
	80C	11	0	23	23	22	22	22	30	33	31	32	32	40	40	40	40	0	0	0	0
	81C	10	N	23	23	22	21	21	30	31	31	32	32	43	43	40	40	2	1	0	0
	82C	11	1	23	23	22	22	23	30	30	30	31	30	42	41	40	40	1	0	0	0
	83C	11	2	23	20	21	20	23	30	30	30	30	30	43	40	41	40	2	0	0	0
	84C	11	0	23	23	22	22	23	30	32	31	32	32	43	43	40	40	2	1	0	0
	85C	11	2	23	23	22	22	23	30	31	31	31	31	42	41	40	40	1	0	0	0
	86C	11	3	23	23	21	21	23	30	32	31	31	31	41	40	40	40	0	0	0	0
	87C	11	3	23	23	22	22	23	30	31	31	32	32	43	43	41	40	2	1	0	0
	88C	10	N	23	21	23	21	23	30	31	31	31	31	40	40	40	40	0	0	0	0
	89C	11	8	23	23	22	22	23	31	31	31	32	32	42	41	41	40	1	0	0	0
	90C	11	1	23	22	21	22	21	30	31	31	31	31	41	41	41	42	0	0	0	1
	91C	11	3	23	23	22	23	23	30	31	31	31	30	42	41	40	40	1	0	0	0
	92C	11	5	23	23	22	22	23	31	32	31	32	32	43	43	41	40	2	1	0	0
	93C	11	1	23	23	21	22	23	30	31	32	31	30	43	41	41	40	2	0	0	0
	94C	10	N	21	21	21	21	21	30	31	31	30	31	40	40	40	40	0	0	0	0

	95C	11	5	23	23	23	23	23	30	31	31	32	32	41	40	40	40	0	0	0	0
	96C	11	3	23	23	21	21	23	30	32	31	30	30	42	40	41	40	1	0	0	0
	97C	11	3	23	21	21	22	23	30	31	31	31	31	43	41	40	41	2	0	0	0
	98C	11	6	23	23	23	23	23	30	33	33	32	32	43	43	40	42	2	1	0	8
	99C	11	3	23	22	21	21	21	30	31	30	31	30	42	41	41	41	1	0	0	0
School C	1E	11	5	21	23	23	21	23	30	33	33	31	31	43	41	41	40	2	0	0	0
EXPERIMENT	2E	11	1	20	22	21	21	21	31	31	31	31	30	43	41	41	40	2	0	0	0
	3E	11	1	21	23	21	22	23	30	30	31	30	30	42	40	41	40	1	0	0	0
	4E	10	N	21	21	21	21	22	30	31	30	30	30	41	41	40	40	0	0	0	0
	5E	10	N	20	20	20	20	20	30	30	31	31	30	42	40	41	40	1	0	0	0
	6E	10	N	21	20	21	21	22	30	31	30	32	32	41	41	41	41	0	0	0	0
	7E	10	N	20	20	20	20	20	30	30	30	30	30	40	40	40	40	0	0	0	0
	8E	11	1	21	21	22	21	23	31	31	31	31	31	42	41	41	40	1	0	0	0
	9E	10	N	21	21	21	21	21	30	33	32	30	30	41	43	41	41	0	1	0	0
	10E	11	2	21	22	21	21	21	30	31	31	31	31	42	41	41	40	1	0	0	0
	11E	11	0	22	22	22	23	22	30	31	31	31	31	40	40	40	40	0	0	0	0

	12E	11	2	21	21	22	22	22	30	31	31	31	31	42	41	41	40	1	0	0	0
	13E	11	2	20	20	20	20	20	30	30	30	30	30	43	40	41	40	2	0	0	0
	14E	11	3	23	23	21	21	23	30	30	30	30	30	43	40	41	40	2	0	0	0
	15E	10	N	22	23	22	22	23	30	30	30	30	30	42	40	41	41	1	0	0	0
	16E	11	6	21	23	21	22	23	30	30	31	31	30	43	40	40	40	2	0	0	0
	17E	11	5	21	23	21	21	21	30	30	31	31	30	42	40	41	40	1	0	0	0
	18E	11	1	20	20	20	20	20	31	30	30	30	30	42	40	41	41	1	0	0	0
	19E	11	5	21	21	21	21	23	31	31	31	31	31	43	41	41	41	2	0	0	0
	20E	11	4	21	21	21	21	21	30	30	30	30	30	40	40	40	40	0	0	0	0
	21E	11	2	21	21	21	22	23	30	31	30	31	31	42	40	40	40	1	0	0	0
	22E	11	4	21	23	22	22	23	30	32	33	31	31	43	43	40	40	2	1	0	0
	23E	11	1	22	21	21	22	22	30	30	31	31	30	41	40	40	40	0	0	0	0
	24E	11	4	21	22	21	21	23	30	31	31	31	30	42	40	40	40	1	0	0	0
	25E	11	0	21	21	21	22	23	30	30	30	30	30	40	40	40	40	0	0	0	0
	26E	11	0	20	20	20	20	20	30	30	30	30	30	40	40	40	40	0	0	0	0
	27E	11	1	22	23	21	21	22	31	30	31	30	30	42	41	41	40	1	0	0	0
	28E	11	3	20	20	20	20	20	30	31	30	31	30	40	41	41	40	0	0	0	0

	29E	11	2	21	21	22	21	23	30	30	31	31	30	42	41	41	40	1	0	0	0
	30E	11	7	21	23	23	21	23	30	31	31	31	31	40	40	40	41	0	0	0	0
	31E	11	5	20	20	20	20	20	30	32	31	31	30	43	41	41	40	2	0	0	0
	32E	11	3	21	21	21	21	21	30	32	31	33	33	43	41	40	40	2	0	0	0
	33E	10	N	21	22	22	21	22	33	31	32	32	32	42	43	42	40	1	1	1	0
	34E	11	1	21	23	21	22	21	30	30	31	31	31	42	41	40	40	1	0	0	0
	35E	11	0	21	21	23	21	21	30	31	31	31	32	42	40	40	40	1	0	0	0
	36E	11	5	20	20	20	20	21	30	30	31	30	30	43	40	40	40	2	0	0	0
	37E	11	1	21	23	21	21	21	30	30	31	30	30	41	40	40	41	1	0	0	0
	38E	11	5	23	23	22	22	23	30	32	31	31	31	43	41	41	40	2	0	0	0
	39E	11	7	21	20	20	21	23	30	30	31	31	30	43	40	41	40	2	0	0	0
	40E	11	4	20	20	20	20	20	31	30	31	32	31	43	41	41	40	2	0	0	E
	41E	11	4	21	23	21	21	21	31	32	31	32	31	42	41	41	41	1	0	0	0
	42E	11	2	20	22	21	21	23	30	31	31	32	30	41	41	40	40	1	0	0	0
	43E	10	N	21	23	21	21	23	30	31	31	30	30	43	41	40	40	2	0	0	0
	44E	11	2	21	22	22	22	23	31	31	31	31	31	42	41	41	41	1	0	0	0
	45E	10	N	21	20	20	20	22	31	30	30	30	30	42	41	41	40	1	0	0	0

	46E	11	5	21	21	21	21	22	30	31	31	31	31	42	41	41	40	1	0	0	0
	47E	11	9	21	21	21	21	23	30	32	31	31	30	43	41	40	41	2	0	0	0
	48E	11	8	23	23	22	22	23	30	31	31	31	30	43	41	41	40	2	0	0	0
	49E	11	4	21	22	21	21	23	30	30	30	30	30	41	41	41	40	0	0	0	0
	50E	11	5	21	22	21	21	23	30	31	31	31	30	43	41	41	40	2	0	0	0
	51E	11	4	22	21	22	21	23	31	31	31	32	31	43	41	41	41	2	0	0	0
	52E	10	N	23	23	21	21	23	30	32	32	31	31	41	41	41	40	0	0	0	0
	53E	11	1	21	21	21	22	21	30	30	30	30	30	41	41	41	40	1	0	0	0
	54E	11	0	22	21	21	21	22	31	31	31	31	31	43	41	41	41	2	0	0	0
	55E	11	3	22	23	21	21	23	30	30	30	30	30	40	40	40	40	0	0	0	0
	56E	11	4	20	21	21	21	23	31	31	31	31	30	43	41	41	40	2	0	0	0
	57E	11	3	21	21	21	21	22	31	31	31	31	31	43	41	41	40	2	0	0	0
	58E	11	5	22	23	21	21	23	31	32	33	31	31	42	41	41	41	1	0	0	0
	59E	11	2	21	21	21	22	21	30	30	30	30	30	43	40	40	40	2	0	0	0
	60E	11	3	22	23	22	22	23	30	31	31	31	31	40	40	40	41	0	0	0	0
	61E	11	2	22	22	22	21	23	30	31	30	30	30	40	40	41	40	0	0	0	0
	62E	11	2	21	23	21	22	23	31	31	31	31	32	42	41	41	40	1	0	0	0

	63E	11	2	21	22	21	21	22	30	30	30	30	30	41	40	40	40	0	0	0	0
	64E	11	10	22	23	22	23	23	31	31	32	31	31	43	41	41	40	2	0	0	0
School D	65E	11	5	21	23	23	22	23	30	32	32	32	30	43	41	40	40	2	0	0	0
EXPERIMENT	66E	11	9	22	23	21	21	23	31	32	33	30	31	41	41	41	41	0	0	0	0
	67E	11	3	21	23	23	21	23	30	31	32	32	31	41	41	40	41	0	0	0	0
	68E	11	3	23	23	21	21	23	30	30	32	31	31	43	40	40	42	2	0	0	2
	69E	11	4	23	23	22	22	23	30	30	31	31	31	43	41	41	41	2	0	0	1
	70E	11	7	21	21	23	21	23	31	31	31	32	32	42	41	41	40	1	0	0	0
	71E	11	6	23	23	23	23	23	33	33	33	33	33	42	43	43	42	1	1	2	10
	72E	11	4	21	23	21	21	23	31	31	31	31	31	41	40	41	40	0	0	0	0
	73E	11	1	22	23	22	21	21	31	31	32	32	32	42	41	41	42	1	0	0	3
	74E	11	8	22	23	22	22	23	30	32	31	31	31	42	41	40	41	1	0	0	0
	75E	11	1	21	22	21	21	23	30	30	31	30	31	43	40	40	40	2	0	0	0
	76E	11	6	22	23	21	21	23	31	31	32	31	31	43	41	41	42	2	0	0	10
	77E	11	3	21	23	21	21	23	30	31	32	31	31	43	41	41	40	2	0	0	0
	78E	11	4	21	21	21	22	21	30	30	30	30	30	43	41	40	41	2	0	0	0

	79E	11	2	21	23	22	21	22	31	30	31	31	31	43	41	41	42	2	0	0	3
	80E	11	1	21	21	22	21	22	30	30	30	30	30	40	40	40	40	0	0	0	0
	81E	11	1	22	23	22	23	22	30	33	33	31	33	40	40	40	40	0	0	0	E
	82E	11	1	21	21	21	22	23	30	31	31	31	32	40	40	40	40	0	0	0	0
	83E	11	4	21	23	23	23	21	30	31	31	31	31	43	40	40	40	2	0	0	0
	84E	11	5	23	23	22	22	23	30	32	32	30	30	40	40	40	40	0	0	0	0
	85E	11	3	21	21	23	22	23	31	31	31	32	31	41	41	41	41	0	0	0	0
	86E	11	7	21	23	22	22	23	30	31	33	32	32	42	41	40	42	1	0	0	2
	87E	11	3	22	20	23	21	23	31	30	31	31	31	41	41	41	40	0	0	0	0
	88E	11	1	22	22	22	22	21	30	33	31	31	32	43	41	40	40	2	0	0	0
	89E	10	N	21	21	22	21	21	30	30	30	33	32	40	40	40	40	0	0	0	0
	90E	11	1	23	22	21	21	22	31	31	31	30	30	42	41	41	41	1	0	0	0
	91E	11	8	21	21	22	22	23	30	31	32	31	31	42	41	40	40	1	0	0	0
	92E	11	10	21	23	22	23	23	30	31	32	32	32	41	41	41	42	0	0	0	8
	93E	11	3	20	20	20	20	20	30	30	30	30	32	42	40	40	40	1	0	0	0
	94E	11	3	22	23	23	23	23	30	31	32	31	31	42	41	41	41	1	0	0	0
	95E	11	10	21	21	21	21	23	30	30	30	30	30	42	41	40	40	1	0	0	0

Appendix 10: Posttest Data

		Vocabulary Knowledge (VK)		Vocabulary Proficiency (VP)					Language Proficiency(LP)					Conceptual & Procedural Proficiency (CUN)				Performance				Marks
		Q1	Q1	Q4.1	Q4.2	Q4.3	Q4.4	Q4.5	Q2.4	Q3.1	Q3.2	Q3.3	Q3.4	Q2.1	Q2.2	Q2.3	Q5	Q2.1	Q2.2	Q2.3	Q5	Total
	LEARNER																	2	1	2	10	15
School A	1C	11	2	20	20	22	22	21	30	30	31	30	30	43	43	42	40	2	1	1	0	4
Control	2C	11	6	22	23	22	22	23	31	32	33	33	33	43	41	41	41	2	0	0	1	3
	3C	11	5	23	23	22	21	23	30	31	33	31	31	40	40	40	40	0	0	0	0	0
	4C	11	5	22	23	21	22	23	30	31	32	31	31	41	41	40	40	0	0	0	0	0
	5C	11	3	23	23	22	22	23	30	33	31	31	33	43	43	40	40	2	1	0	6	9
	6C	11	2	21	22	22	23	21	30	31	31	33	32	43	41	41	40	2	0	0	0	2
	7C	11	3	22	23	22	23	23	31	33	33	33	33	43	41	41	40	2	0	0	0	2
	8C	11	3	22	22	21	20	20	30	31	33	30	30	43	40	40	40	2	0	0	0	2
	9C	11	3	20	20	20	20	20	30	32	31	31	33	43	41	41	40	2	0	0	0	2
	10C	11	4	23	23	22	22	23	30	31	32	31	33	43	41	40	40	2	0	0	3	5

	11C	11	8	23	23	22	22	23	31	30	30	32	32	42	41	41	40	1	0	0	8	9
	12C	12	13	20	20	20	20	20	30	30	30	30	30	40	40	40	40	0	0	0	0	0
	13C	11	4	21	23	21	22	23	30	31	31	30	30	43	41	41	40	2	0	0	0	2
	14C	11	5	22	20	20	20	20	30	32	31	33	30	42	43	40	40	1	1	0	0	2
	1C5	12	11	22	21	22	21	22	31	33	33	33	33	42	41	41	40	1	0	0	0	1
	16C	11	2	21	21	20	20	20	32	30	31	30	30	41	41	42	40	0	0	1	0	1
	17C	11	5	21	23	21	21	23	31	33	32	31	33	43	43	41	41	2	1	0	6	9
	18C	11	5	23	23	23	23	23	31	33	31	33	33	43	41	41	40	2	0	0	0	2
	19C	11	2	21	22	21	21	22	30	31	31	33	33	43	43	43	40	2	1	2	0	5
	20C	11	2	21	23	21	22	23	31	32	33	33	33	43	41	41	40	2	0	0	0	2
	21C	11	3	23	23	23	23	23	31	30	33	33	33	43	41	42	40	2	0	0	0	2
	22C	11	4	21	23	22	21	22	30	30	33	30	30	40	41	41	40	0	0	0	10	10
	23C	11	3	23	23	23	23	23	31	33	31	32	32	43	43	41	41	2	1	0	1	4
	24C	11	9	23	23	22	23	23	31	31	33	32	32	43	43	41	41	2	1	0	10	13
	25C	11	6	22	23	23	22	23	30	31	33	32	32	41	43	43	41	0	1	2	1	4
	26C	11	6	23	23	21	21	23	30	31	30	31	31	43	41	43	41	2	0	2	8	12
	27C	12	11	23	23	23	23	23	31	33	32	32	32	41	43	43	41	0	1	2	10	13

	28C	11	1	21	23	22	21	21	31	33	33	31	30	41	41	41	41	0	0	0	1	1
	29C	11	3	21	21	23	22	23	30	33	33	30	30	43	41	41	40	2	0	0	0	2
	30C	11	2	22	23	22	22	23	32	32	31	30	30	42	41	43	40	1	0	2	0	3
	31C	11	4	23	23	22	22	23	30	33	33	30	30	41	40	40	40	1	0	0	0	1
	32C	11	5	21	23	21	22	23	30	33	31	31	31	41	41	41	40	1	0	0	0	1
	33C	11	2	21	20	22	21	21	30	32	31	31	31	42	40	41	40	1	0	0	0	1
	34C	10	0	22	23	21	21	23	30	30	30	30	30	40	40	40	40	0	0	0	0	0
	35C	11	1	22	23	21	21	23	30	32	33	31	30	42	41	41	41	1	0	0	0	1
	36C	11	4	21	21	21	21	23	30	31	31	30	30	41	40	42	41	1	0	0	0	1
	37C	10	0	21	21	21	23	23	30	31	31	31	31	42	40	40	40	1	0	0	0	1
	38C	11	3	23	23	23	23	23	30	33	30	31	31	41	40	40	41	0	0	0	0	0
	39C	11	2	20	20	20	20	20	30	30	30	30	30	42	40	40	40	1	0	0	0	1
	40C	11	4	22	23	23	22	23	30	33	33	31	30	41	41	41	40	0	0	0	0	0
	41C	11	2	22	23	21	22	23	30	31	33	31	31	41	41	41	40	0	0	0	0	0
	42C	11	5	20	20	20	20	20	30	30	30	30	30	41	40	40	40	0	0	0	0	0
	43C	11	2	20	20	20	20	20	30	33	31	31	32	43	41	41	40	2	0	0	0	2
	44C	11	4	20	20	20	20	20	31	31	33	30	30	43	41	41	40	2	0	0	0	2

	45C	11	6	21	21	21	21	21	31	32	31	32	32	42	41	41	40	2	0	0	0	2
	46C	11	2	23	23	21	21	23	30	33	32	32	30	42	40	40	40	1	0	0	0	1
School B	47C	11	4	22	23	22	22	23	33	33	33	31	31	42	42	41	40	1	1	0	0	2
Control	48C	11	1	21	23	21	22	23	30	31	31	31	30	42	41	40	40	1	0	0	0	1
	49C	11	3	22	23	21	21	23	30	33	32	31	31	42	40	40	40	1	0	0	0	1
	50C	10	0	21	23	21	22	22	30	30	30	30	30	43	41	40	40	2	0	0	0	2
	51C	10	0	21	23	21	21	21	30	31	31	32	30	42	41	41	40	1	0	0	0	1
	52C	11	1	21	23	23	21	23	30	33	31	33	33	43	41	41	40	2	0	0	0	2
	53C	11	3	22	23	21	21	23	31	32	33	32	31	42	41	41	40	1	0	0	0	1
	54C	11	7	21	23	21	22	23	30	31	31	31	31	41	40	41	40	0	0	0	0	0
	55C	11	2	20	20	20	20	20	31	31	30	30	30	42	41	41	40	1	0	0	0	1
	56C	11	7	20	20	20	20	20	30	33	31	33	33	42	40	40	40	2	0	0	0	2
	57C	11	4	23	23	23	23	23	31	33	33	32	32	42	43	41	43	1	1	0	10	12
	58C	11	2	22	22	21	21	22	30	31	30	30	30	42	41	40	40	1	0	0	0	1
	59C	10	0	23	22	21	21	23	30	30	30	33	33	41	40	40	40	0	0	0	0	0
	60C	11	4	22	23	22	22	22	30	31	31	30	30	43	40	41	40	2	0	0	0	2

	61C	11	3	23	23	22	22	23	30	33	31	33	33	41	40	40	40	0	0	0	0	0
	62C	11	4	22	23	22	22	23	30	33	33	33	33	43	42	41	40	2	0	0	0	2
	63C	11	7	20	20	20	20	20	30	30	30	30	30	43	41	41	40	2	0	0	0	2
	64C	11	2	22	23	21	21	22	31	31	31	31	32	42	41	41	40	1	0	0	0	1
	65C	11	1	20	22	20	20	22	30	31	31	32	30	42	41	40	40	1	0	0	0	1
	66C	11	0	22	23	23	21	23	30	32	31	30	30	43	43	41	40	2	1	0	0	3
	67C	11	4	22	23	21	21	23	30	30	30	30	30	42	40	41	40	1	0	0	0	1
	68C	10	0	23	23	21	21	23	30	30	33	30	30	43	41	41	40	2	0	0	0	2
	69C	11	2	23	23	22	22	23	30	31	31	31	31	43	41	40	40	2	0	0	0	2
	70C	11	5	20	20	20	20	20	30	33	31	33	33	43	41	41	40	2	0	0	0	2
	71C	11	6	23	23	22	22	23	30	31	31	31	31	42	41	41	40	1	0	0	0	1
	72C	10	0	22	23	21	21	23	30	30	30	30	30	43	41	40	40	2	0	0	0	2
	73C	11	4	22	23	23	22	23	30	31	31	30	30	43	43	41	40	2	1	0	0	3
	74C	11	1	23	22	22	23	23	30	31	30	30	30	43	43	41	40	2	1	0	0	3
	75C	11	2	20	20	20	20	20	31	33	30	30	30	43	43	41	40	2	1	0	0	3
	76C	11	3	20	20	20	20	20	30	30	30	30	30	42	41	40	40	1	0	0	0	1
	77C	11	0	20	20	20	20	20	30	30	30	30	30	40	40	40	40	0	0	0	0	0

	78C	11	2	23	23	23	23	23	31	32	31	32	32	43	41	41	40	2	0	0	0	2
	79C	11	1	21	21	21	22	20	31	31	31	31	31	42	41	41	40	1	0	0	0	1
	80C	11	5	22	22	23	23	23	33	32	31	33	33	43	41	43	40	2	0	2	0	4
	81C	11	3	21	21	21	21	21	30	33	31	30	30	42	40	40	40	1	0	0	0	1
	82C	11	2	20	20	20	20	20	30	30	30	30	30	42	41	41	40	1	0	0	0	1
	83C	11	1	21	21	21	21	21	31	31	31	31	31	43	41	41	40	2	0	0	0	2
School C	1E	10	0	20	22	21	21	22	30	31	31	31	31	42	43	40	40	1	1	0	0	2
Experiment	2E	11	3	21	22	21	21	22	30	30	30	30	30	43	41	41	41	2	0	0	0	2
	3E	11	3	22	23	21	22	23	30	31	31	30	30	40	41	40	40	0	0	0	0	0
	4E	11	6	23	23	22	22	23	31	33	31	32	32	42	41	41	41	1	0	0	0	1
	5E	11	3	23	23	23	23	23	31	31	31	31	31	42	41	41	42	2	0	0	8	10
	6E	11	3	22	23	22	23	23	31	32	31	31	31	42	41	41	40	2	0	0	2	4
	7E	12	15	23	23	23	23	23	31	32	33	32	32	42	43	43	43	2	1	2	10	15
	8E	13	21	23	23	23	23	23	31	31	31	31	31	42	43	41	43	2	1	0	10	13
	9E	11	5	23	23	23	22	22	31	33	33	31	31	42	43	41	43	2	1	0	8	11
	10E	11	4	21	21	23	23	22	31	31	31	32	32	42	41	42	43	2	0	0	9	11

	11E	12	12	23	23	23	23	23	30	31	32	32	32	42	40	40	42	2	0	0	10	12
	12E	12	11	22	22	23	22	23	31	33	33	31	31	42	43	41	42	2	1	0	8	11
	13E	11	5	23	23	23	23	23	31	32	32	32	32	42	41	41	42	2	0	0	10	12
	14E	11	5	22	23	23	22	23	30	33	33	31	31	42	43	40	40	2	1	0	10	13
	15E	11	3	22	23	23	22	22	30	33	31	31	31	42	43	40	43	2	1	0	9	12
	16E	11	2	22	23	22	22	23	31	32	31	31	31	42	43	41	41	2	1	0	0	3
	17E	11	4	23	23	23	23	23	31	33	32	33	33	42	43	42	40	2	1	1	7	11
	18E	12	12	23	23	23	23	23	33	32	33	31	31	42	43	43	43	2	1	2	10	15
	19E	12	15	23	23	23	23	23	32	32	33	32	32	42	43	43	43	2	1	2	10	15
	20E	11	5	23	23	22	22	23	30	32	30	30	30	42	43	41	40	2	1	0	6	9
	21E	11	3	21	21	22	21	21	31	30	30	30	30	42	43	42	40	1	1	0	0	2
	22E	11	1	22	23	23	23	22	31	32	33	31	31	43	43	42	40	2	1	0	0	3
	23E	10	0	22	23	22	22	23	30	32	30	33	33	43	42	42	40	2	1	1	0	4
	24E	11	2	22	23	22	22	22	31	32	32	31	32	43	43	42	40	2	1	0	3	6
	25E	11	1	22	22	22	22	22	30	31	31	31	31	42	41	40	40	0	0	0	0	0
	26E	11	2	23	23	23	23	23	32	30	33	32	32	41	43	41	40	0	1	0	0	1
	27E	11	6	23	23	23	23	23	30	32	31	32	32	43	43	41	43	2	1	0	10	13

	28E	11	5	23	23	23	23	23	32	33	33	31	33	43	43	41	40	2	1	0	0	3
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	30E	11	6	23	23	23	23	23	32	33	33	31	33	43	43	41	40	2	1	0	0	3
	31E	11	5	20	20	20	20	20	32	33	33	30	30	43	43	43	40	2	1	2	0	5
	32E	11	1	23	23	21	21	23	30	33	33	31	33	43	41	40	42	2	0	0	7	9
	33E	11	3	22	22	22	22	22	30	32	33	31	33	42	43	42	42	1	1	1	6	9
	34E	11	6	23	23	23	23	23	31	33	33	31	33	43	43	41	42	2	1	0	3	6
	35E	11	5	22	23	23	22	23	30	30	31	32	32	43	42	42	40	2	0	1	0	3
	36E	11	3	20	20	20	20	20	30	30	30	30	30	40	40	40	40	0	0	0	0	0
	37E	11	6	22	23	21	22	21	30	33	33	30	30	42	41	40	40	1	1	0	0	2
	38E	11	3	23	23	22	22	23	32	32	33	31	33	43	43	41	43	1	1	1	0	3
	39E	11	4	23	23	22	22	22	30	33	31	32	32	42	41	41	40	1	0	0	0	1
	40E	11	3	22	22	20	20	22	30	33	30	31	31	42	40	40	40	1	0	0	0	1
	41E	12	12	22	22	22	22	23	33	31	31	32	32	43	42	42	40	2	1	1	0	4
	42E	11	5	23	23	23	23	23	30	33	33	32	32	43	41	43	42	2	0	2	5	9
	43E	11	5	20	23	20	22	23	30	30	33	33	33	43	41	40	40	2	0	0	0	2
	44E	12	11	22	23	21	21	23	30	33	31	31	31	41	41	43	42	0	0	2	9	11

	45E	11	5	23	23	23	23	23	31	31	31	33	33	43	43	41	41	2	1	0	7	10
	46E	12	13	22	22	23	23	23	31	33	31	32	32	42	41	41	40	1	0	0	0	1
	47E	11	2	21	23	21	22	23	30	33	33	31	33	43	43	41	40	2	1	0	0	3
	48E	12	14	23	23	23	23	23	30	30	30	30	30	43	43	43	41	2	1	2	10	15
	49E	11	4	22	23	21	21	23	32	31	33	31	31	42	41	41	40	1	0	0	0	1
	50E	11	5	23	23	23	23	23	31	31	33	33	33	43	43	41	43	2	1	0	10	13
	51E	11	4	20	20	20	20	20	31	32	31	31	33	43	41	41	40	2	0	0	0	2
	52E	11	5	22	23	22	22	23	31	31	33	32	32	43	41	43	41	2	0	2	2	6
	53E	11	6	23	23	22	22	23	31	32	33	31	31	43	43	41	41	2	1	0	7	10
School D	54E	11	9	22	23	22	21	23	30	31	32	32	32	41	41	41	40	0	0	0	0	0
Experiment	55E	12	15	22	23	21	21	23	30	33	33	33	33	42	41	41	41	1	0	0	1	2
	56E	13	22	23	23	23	23	23	32	33	33	33	33	43	43	41	42	2	1	0	10	13
	57E	11	7	22	22	22	22	23	30	33	33	33	33	43	41	40	43	2	0	0	10	12
	58E	12	12	23	23	22	22	23	31	32	31	33	33	42	41	41	43	1	0	0	10	11
	59E	13	25	23	23	23	23	23	33	33	33	33	33	43	43	43	43	2	1	2	10	15
	60E	12	12	22	23	23	21	23	31	31	31	32	32	43	43	41	42	2	1	0	10	13

	61E	11	6	23	23	23	23	23	30	33	31	33	33	43	43	40	40	2	1	0	0	3
	62E	13	21	23	23	23	23	23	33	33	33	33	33	43	43	43	42	2	1	2	10	15
	63E	11	6	22	23	23	22	23	30	33	33	33	33	42	41	41	41	1	0	0	0	1
	64E	12	11	23	23	22	22	23	31	33	31	31	31	42	41	41	43	1	0	0	10	11
	65E	11	4	22	23	22	22	23	30	31	31	33	33	43	41	43	40	2	0	2	1	5
	66E	12	15	23	23	22	22	23	31	33	33	31	31	41	41	41	42	0	0	0	10	10
	67E	12	11	23	23	23	23	23	33	32	31	32	32	43	41	43	42	2	0	2	0	4
	68E	13	24	23	23	23	23	23	33	33	33	33	33	43	43	43	43	2	1	2	10	15
	69E	11	5	21	21	21	21	21	31	30	30	30	30	42	41	40	40	1	0	0	0	1
	70E	11	6	23	23	23	23	23	31	33	32	33	33	43	43	41	42	2	1	0	0	3
	71E	12	11	21	21	23	23	23	30	33	33	33	33	43	41	41	42	2	0	0	8	10
	72E	13	21	23	23	23	23	23	31	31	33	33	33	42	41	41	42	1	0	0	10	11
	73E	12	12	23	23	23	23	23	30	33	33	33	33	43	43	41	42	2	1	0	10	13
	74E	13	21	23	22	21	21	22	30	31	31	31	31	41	41	41	40	0	0	0	0	0