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PREPARING PRE-SERVICE MATHEMATICS TEACHERS TO TEACH IN MULTILINGUAL CLASSROOMS: A COMMUNITY OF PRACTICE PERSPECTIVE

A Thesis submitted to the Faculty of Humanities, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Doctor of Philosophy

By

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FEBRUARY, 2013

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DECLARATION

I declare that this research report is my own work, except as indicated in the acknowledgements, the text and the references. It is being submitted in fulfilment of the requirement for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other institution.



16th May, 2013

Signed

Date

Anthony Anietie Essien

DEDICATION

For my wife, Thandekile; and child, Avuzwa

and

My parents, Ete Ambrose Essien who is late, and Anthonia Atim Essien

ABSTRACT

This study takes a particular look at mathematics teacher education communities of practice (CoPs) in order to provide rich descriptions of the CoPs and make claims about its relation/in relation to teacher preparation and particularly the preparation of pre-service teachers for teaching mathematics in multilingual classrooms. The three dimensions of communities of practice proposed by Wenger (mutual engagement, shared repertoire and joint enterprise) were used in conjunction with Mortimer and Scott's notion of meaning making as a dialogic process as a theoretical lens to gain an entry into the nature of communities of practice in pre-service mathematics teacher education classrooms. Data was collected through pre-observation interviews of 12 teacher educators at four Universities in one Province in South Africa in Phase One of the study. A methodological approach based on Wenger's CoP theory and Mortimer and Scott's dialogic process was developed and used to analyse classroom observation videos of four of these teacher educators' classroom communities of practice in two universities in Phase Two of the study. Using the privileged practices in the CoPs as points of departure and how these practices shaped and were shaped by other dynamics in the CoPs, the findings emerging from the study indicate that within the multiply layers of teacher education, there is an overarching emphasis given to the acquisition of mathematical content. Nevertheless, the communicative approaches and patterns of discourse used by the different teacher educators opened up different possibilities as far as preparing pre-service teachers for teaching (in multilingual classrooms) is concerned.

Wenger's community of practice theory has found applications in different spheres of life and in different organisational and educational settings. Its use to understand and describe mathematics pre-service classrooms is, however, still largely unexplored. A theoretical contribution that this study makes lies in the extension of Wenger's CoP theory to include dialogic processes. A methodological contribution lies in the development of an organisational language (based on Wenger's three dimensions of CoP) to characterise pre-service teacher education classrooms.

Keywords: communities of practice, teacher education, multilingual mathematics classrooms, discourse pattern, communicative approach, identities, teacher educators, pre-service teachers; organisational language, joint enterprise, mutual engagement, shared repertoire

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Essien, A. (2010). What teacher educators consider as best practices in preparing pre-service teachers for teaching Mathematics in multilingual classrooms. *Perspectives in Education*, 28(4), Pp 32-42.

Journal article from Chapter Five will be published as:

Essien, A., Adler, J. & Setati, M. (Under review). Using Wenger's Community of Practice Theory to understand pre-service teacher education mathematics classrooms. *Mathematics Education Research Journal*.

Articles from Chapters 7-9 will be published as:

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Essien, A. (forthcoming). Examining opportunities for the development of interacting identities within pre-service teacher education mathematics classrooms. *Perspectives in Education.*

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LIST OF ACRONYMS/ABBREVIATIONS

CoP (plural CoPs)	Community of practice
PST (plural PSTs)	Pre-service teacher
STL	Social theory of learning
TE (plural TEs)	Teacher Educator
DP	Dialogic process
BLMC	Becoming learners of mathematics content
BLMP	Becoming learners of mathematical processes
BTM	Becoming teachers of mathematics
BTMMC	Becoming learners of mathematics in multilingual contexts
LoLT	Language of learning and teaching
OL	Organisational language
TIMMS	Trends in International Mathematics and Science Study
ME	Mutual engagement
SR	Shared repertoire
JE	Joint enterprise
TEIA	University A
TEIB	University B

CHAPTER ONE

CONCEPTUALISING THE STUDY

1.1 Background to the research study: My journey into teacher education

Following the abysmal performance of South African learners in the mathematics component of the Trends in International Mathematics and Science Study (TIMSS) of 1999 and 2003, many researchers, notably Howie (2002, 2003, 2004; Howie & Pietersen, 2001), who analysed the secondary data from the Study suggested that the main factor responsible for the poor performance by South African learners was that of language. Through her analysis, Howie concluded that the poor performance can be attributed to the low English language proficiency level of learners. She then went on to argue that the solution to improving second language learners' performance in mathematics is to develop their English language proficiency. This recommendation sparked off a debate as to whether or not what is fundamental in developing learners' mathematical proficiency is the development of learners' English language proficiency (Also see Cele, 2001); or whether it is the harnessing of the home languages which learners (most of whom are multilingual learners and are still learning the LoLT - English) bring to class.

In my previous research, motivated by the need to inform the above debate, I undertook a quasi-experimental study to investigate whether/how improvement of learners' English language proficiency enables or constrains the development of mathematical proficiency. The research was conducted in a school which was running an English enrichment programme using the ASTRALAB English computer software as an intervention to improve the language proficiency level of the learners. The intervention had an incorporated strategy for testing the language improvement of learners in the programme and the test showed that there was improvement in the English language proficiency of the target class.

My investigation employed both quantitative and qualitative approaches. The quantitative approach was used to investigate the question: How does the improvement of English language proficiency enable or constrain students' mathematical achievement? To do this,

learners in both the control and experimental groups were given a mathematics pre-test and a post-test consisting of questions drawn from the TIMSS of 2003. Statistical methods were used to analyse this pre- and post-tests in order to compare the performance of the learners who underwent the ASTRALAB programme with learners from another class (who served as the control group). The qualitative aspect of the study dealt with communication in the classroom and investigated the question: To what extent (if any) does improving learners' English language proficiency enable or constrain mathematical communication in a mathematics classroom? The classroom communication of the experimental class was analysed in order to ascertain whether the nature of communication changed or remained the same after the intervention.

In defining mathematical proficiency, I had used the understanding of this term as defined by Kilpatrick, Swafford & Findell (2001). Kilpatrick *et al* (2001) defines mathematical proficiency by five interwoven strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Thus, in the study, mathematical proficiency was seen not only as competence in completing procedures, but also as ability to present mathematical arguments and capability of participating in meaningful mathematics discussions, hence the two research questions – one dealing with mathematics achievement (in the sense of scores obtained in mathematics tests) and the other dealing with communication in the mathematics classroom.

Data from the study produced conflicting results: even though the overall mathematics achievement in both tests was low, the experimental group showed statistically significant higher gains from pre-test to post-test; communication in the class, however, showed no difference in the interactive patterns in both the pre- and the post-intervention lessons. In other words, while analysis of test results indicated that improving learners' English proficiency level led to improvement in mathematics performance in the tests, analysis of classroom communication indicated that this improvement of language proficiency did not enable mathematical communication in the classroom. In both lessons, much of the teacher talk was short procedural questions requiring the learners to produce short procedural (and in most cases, one-word) answers. In fact, in general, the interaction in both the pre-intervention lesson and the post-intervention lesson was reminiscent of what Young (1984, in Edwards & Westgate, 1987, p. 143) terms a tendency for learners to be "obliged to respond within the teacher's frame of reference and at the teacher's bidding". The learners were, thus, in both lessons, given little opportunity for using the language of learning and teaching (LoLT – in

this case English) "in more creative ways – such as experimenting with new types of language constructions" (Mercer, 2001, p. 245). And even though both teacher and learners were multilingual, learners were not encouraged to use their home languages and the teacher used learners' home language only as a regulatory tool (For a full report on this study, see: Essien, 2011).

I had, therefore, concluded from the study that even though proficiency in the language of instruction (English in this case) is an important index in improving mathematics achievement, improvement of learners' language proficiency, is not sufficient to impact on mathematical communication in the multilingual classroom. I went on to argue that since in any classroom, the teacher plays a key role in the management of the interaction in the classroom (Edwards & Westgate, 1987), and has the professional responsibility of using language to share and create knowledge and understanding (in the bid to help learners understand a specific body of knowledge) (Mercer, 2000), the teacher's ability to draw on learners' linguistic resources – one of which is structuring questions to allow learners to sufficiently express their thinking – is also of critical importance. Based on the above conclusion, I made the recommendation that since English is the language of instruction and examination in most South African schools, all mathematics activities should be closely linked with improved English language education, and also that necessary mechanisms must be put in place to improve the English language proficiency level of mathematics teachers. Finally, I went on to suggest that mathematics teachers need to be appropriately trained not only in mathematics teaching, but also trained on how to use English language fluently to teach mathematics.

The foregoing discussion foregrounds the fact that teaching and learning are inextricably linked together so much so that investigating one means (implicitly or explicitly) investigating the other. In my case, even though I had originally set out to investigate how the improvement of *learners'* proficiency in the LoLT (English) enabled or constrained mathematical proficiency, through my qualitative analysis of classroom observations, the issue became how the *teacher* can help facilitate the effective use of the LoLT to promote mathematical proficiency in learners, hence the recommendation that teachers need to be proficient in English.

From the account above, two issues emerged that became the *leitmotif* for the present study: the first was the complex nature of multilingualism in South Africa. As I explain below, given that the teacher was teaching in such a multilingual context, representative of the

pedagogic setting in most South African mathematics classrooms, my recommendation above is one-dimensional and thus limited. But it is not enough to put a spotlight on the teachers' teaching. The question is (and this was the second issue): How are teachers trained to deal with teaching mathematics in a multilingual context such as that of South Africa which has been shown by research (Adler, 1995, 2001; Barwell, 2009; Halai, 2009; Heugh, 2002; Setati, 2002, 2005b, 2008; Setati & Adler, 2000; Setati, Adler, Reed, & Bapoo, 2002) to be a complex endeavour? This question prompted me to delve into teacher education and more specifically, to focus on teaching and learning in pre-service mathematics teacher education settings. But in order to understand why research shows that teaching mathematics in multilingual classrooms is complex, I now turn to the South African multilingual context and describe the historical backgrounds and the context of schooling regarding language and learning.

1.2 The South African multilingual context

As Broeder, Extra & Maartens (2002) rightly pointed out, South Africa presents a complex and interesting picture of multilingualism. This is due not only to its political history of apartheid, but also to the distinctive nature of its multilingualism. Of the 11 (official) languages in South Africa, nine are indigenous African languages. These African languages can be grouped into two major groups based on their linguistic distance[1]: The Sotho languages and the Nguni languages. The Sotho languages comprise of South Sesotho, Sepedi, Setswana, while the Nguni languages are isiZulu, isiXhosa, siSwati and isiNdebele. There is mutual intelligibility between the languages in each of these groups (this is not the case with the remaining two African languages: Tshivenda and Tsonga). Because of this mutual intelligibility of languages, other indigenous languages are easily learned. It must be noted that this is not the case with most multilingual settings in other African countries. The example of Nigeria is a case in point. Even though there are over 250 languages in Nigeria, most of them are autonomous in the sense that they share very little (if at all any) common

[1] Linguistic distance is taken as the extent to which two or more languages differ from each other/one another with regards to, amongst myriad other characteristics, vocabulary, grammar, written form, structure and semantic aspects of the language and their status.

vocabularies[2]. In most classrooms in most cities in Nigeria, there might be as many as 10 completely different languages present in the classroom, so much so that if the teacher uses his/her language to teach, a good number of learners would not understand and would thus, be disadvantaged.

In her comparison of the South African linguistic context to that of Nigeria, Cele (2001) ignores both the fact that the Nigerian indigenous languages are mostly autonomous and mostly linguistically distant from one another, and the fact of the existence of the Pidgin language. This has serious implications for her recommendation that "the South African education language policy should be modelled after countries like Nigeria that in spite of many indigenous languages existing in their cultural fabric, English is used as an official unifying factor" (Cele, 2001, p. 192). The nature of multilingualism and, therefore, any attempt at suggesting what the best practices for teaching and learning are for a country, as I have argued previously (Essien, 2010a), depends not only on the nature of the languages (indigenous and otherwise) present in the country in question but also on the colonial legacies or the historical contexts of language development in a country. In South Africa where mother-tongue education was used as a tool for oppression – a tool for "institutionalised racism of apartheid" (Pluddermann, 2002, p. 47) - and where English was synonymous with superiority, power and whiteness, and fluency in English was perceived as an "emblem of educatedness", the issue of creating or building trust between (especially monolingual, namely only English speaking, and bilingual, namely only English and Afrikaans speaking) teacher educators (TEs) and their pre-service teachers, and of creating an environment where pre-service teachers feel comfortable to speak English without fear of ridicule or criticism, becomes essential in multilingual classrooms.

In what follows, I discuss the multilingual situation in South Africa with specific reference to the Apartheid and post-Apartheid eras and the context of schooling regarding language.

1.2.1 The Apartheid Era

English and Afrikaans were the only two official languages during the period of apartheid (1948-1994) despite the presence of African languages. Learners were required to take both English and Afrikaans as subjects for the first 12 years of schooling. But the English that was

[2] There is however the Pidgin language which is spoken across the country and understood by both the literate and the not-so-literate.

taught was "presented as a complexly structured and grammar oriented subject, only taught for the purposes of tracking black people towards low-skills forms of labour" (Cele, 2001, p. 188). Nevertheless, during this period of segregation, mother-tongue schooling for Africans was favoured for quite different reasons than those advanced by educationists and researchers like Cummins (1979a) or Clarkson (1991, 1992). It (mother-tongue education) was used as a tool of oppression and marginalisation of the blacks and this contributed in no small measure to giving mother-tongue education (and to a large extent, the use of African languages) a negative connotation because of the tendency to equate it with the "ravages Bantu education" (Alexander, 2012, p. 5). This was compounded by the fact that during the apartheid and pre-apartheid periods, English competence was perceived to be analogical to superiority, power and whiteness. Fluency in English was perceived as an "emblem of educatedness that socially positioned those with English efficiency as far better than those with limited or no efficiency at all" (Cele, 2001, p. 182).

The language policy designed for separate development of Whites and Blacks coupled with segregated education, unequal resources and a cognitively impoverished curriculum for Blacks resulted in and gave rise to the massive under-education of the majority of the population (Heugh, 2002). The apartheid legacy also affected teacher education in that pre-service teachers whose first language is not the LoLT were inadequately prepared for the provision of emancipatory education and teachers with low proficiency in English often taught in the local languages (Cele, 2001).

1.2.2 The Post-Apartheid Era

The unbanning of liberation movements in the early 90's saw full recognition of the rich multilingual nature of South Africa and the adoption in 1996 of 11 official languages (see Setati, 2002). The apartheid language-in-education policy with its unequal language proficiency demands for learners was replaced in 1997 with a new language-in-education policy based on non-discriminatory language use (Heugh, 2002; Setati, 2002). But as Setati (2002, pp. 7-8) notes,

> While the new language policy in South Africa is intended to address the overvaluing of English and Afrikaans and the undervaluing of African languages, in practice English continues to dominate. Even though English is a main language of a minority, it is both the language of power and the language of educational and socio-economic

> advancement, thus it is a dominant symbolic resource in the linguistic market [ ] in South Africa.

Hence, even though the language-in-education policy and the Constitution of South Africa give provision for learners to learn in any of the 11 official languages of their choice, research has shown that due to economic, political and ideological factors, most learners prefer to learn mathematics in English which for most, is not their first or home language (see Setati, 2008).

1.2.3 The context of schooling regarding language and learning

Broeder, Extra & Maartens (2002, p. 9) argue that the Department of Education encourages developing multilingualism within the framework of additive bilingualism – "while schools are not compelled to offer more than one LoLT, they are encouraged to pursue a policy based on the principles of maintaining home language(s) while providing access to the effective acquisition of additional language(s)".

As indicated by Mati (2004) as far as the foundation phase (Grade 0-3) is concerned, what has been unofficially adopted by most (public) schools is the use of mother-tongue as the LoLT. In some schools in areas where there is a high demand for English by stakeholders, learners are gradually introduced to English as early as Grade One. From Grade Three, officially, at least two languages must be learned as subjects. Hence, from Grade 3 onwards, there is a gradual shift in most schools from using learners' dominant language as the LoLT to using English. The dominant language of the learners and Afrikaans are then taught as subjects. From Grade 10-12, two languages must be passed (see Mati, 2004).

As noted by Mati (2004), in most South African schools with predominantly second language English speakers, learners come in contact with English mainly at school and thus, English learning for these learners is almost entirely a classroom experience. This is so because, in light of what was said above about the mutual intelligibility between languages, more often than not, a learner would be able to understand and/or speak the language(s) of the neighbours around him/her even if the language(s) is/are different from his/her first language. As such, there is no need to switch to a "unifying" language to be able to communicate.

Given the above elucidation of the multilingual context of South Africa and the context of schooling in South Africa, I now turn to the "problem" as far as mathematics teacher education (in such a context) is concerned.

1.3 Defining the problem

In recent years, researchers and educationists have paid increased attention to multilingualism as a phenomenon which relates positively to cognitive development, flexibility and the promotion of academic achievement in learners (Adler, 2001; Agnihotri, 1995; Conteh, 2000; Cummins, 1979a; Gorgorio & Planas, 2001; Halai, 2004,2009; Moschkovich, 1999; Setati, 2002, 2005b; Setati & Adler, 2000). Most mathematics teachers teach in multilingual classrooms in South Africa, and research in South Africa (Adler, 2001; du Plessis & Elsie, 2003; Setati, 2002, 2005b; Setati & Adler, 2000) has shown that learning and teaching mathematics to multilingual learners is complex and that teachers grapple with dealing with this complexity. Adler (1995, p. 265) expresses this complexity in these words:

> …the dynamics of teaching and learning mathematics in multilingual classrooms is not simply about proficiency in the language of learning; nor is it only about access to the (English) mathematics register; nor should it be reduced to social diversity and social relations in the classrooms. The three, while analytically separable, are in constant interplay in the cultural processes that constitute school mathematics learning

Hence, teaching mathematics in multilingual classrooms involves the teacher being confronted by situations constituted by the above triple interplay (Adler, 1995). Re-echoing the same sentiment, Barwell (2009) also argues that in multilingual classrooms of learners whose home language is not the language of learning and teaching (LoLT) and who are not yet proficient in the LoLT, teachers are faced with the triple challenge of striking a balance between attention to mathematics, attention to English (LoLT) and attention to mathematical language. As a result of the intricate relationship between language and mathematics and the complexity of teaching in multilingual classrooms (see Adler, 2001; Barwell, Barton, & Setati, 2007; Conteh, 2000; Gutiérrez, 2002; Setati, 2005b; Setati & Adler, 2000; Young, 1995), it is becoming more and more crucial that teachers are equipped with the understanding and the skills they need to be able to deal with, and support multilingual learners.

Teachers in multilingual classrooms have an additional task of dealing with the language needs of learners whose first language is not the language of teaching and learning. To suggest, as I did in my previous research, that improving the English language proficiency of second language teachers (or/and that of learners) was key in dealing with the challenges of teaching multilingual learners whose dominant language is not the language of teaching and learning, is a one-dimensional approach to the issue. A more holistic approach is therefore needed to address multilingual learners' educational needs. And such an approach can only be possible if the teacher, first and foremost, recognises and embraces multilingualism as a resource rather than as an obstacle to the teaching and learning of mathematics (Agnihotri, 1995). But how do teachers get to know to deal with the challenge of teaching mathematics in multilingual classrooms in a multi-dimensional way? How do teacher educators prepare pre-service teachers to engage with the challenges of teaching mathematics in multilingual classrooms? At the centre of this study is the concern about how pre-service teachers are trained to deal with the complexity of teaching mathematics in multilingual classrooms[3]. I do this by investigating the nature of pre-service teacher education communities of practice mathematics classrooms. The present study is thus about teacher education practice and teachers learning in that community of practice. I therefore turn to the anthropological situative theory which I elaborate in Chapter Two to locate the practice. Crotty (1998, p. 10) argues that every theoretical perspective "embodies a certain way of understanding *what is* (ontology) as well as a certain way of understanding *what it means* to know (epistemology)" (*italics in original*). The epistemological stance taken in this study is that humans are not isolated beings but social beings who interact, and that knowledge is socially constructed/reconstructed. This motivated the choice for a social theory to frame this study. And because this study looks at the negotiated process of knowledge construction in practice and meaning making, the community of practice (CoP) theory becomes useful in achieving that. Of course there are issues with Wenger's (1998) CoP theory when used in teacher education settings. I elaborate on these issues in Chapters Two and Chapter Four and engage with how I dealt with them in my study.

[3]There are different types of multilingual classrooms depending of the sociolinguistic/socio-political settings. In the South African setting, a multilingual classroom is a classroom where the LoLT is an additional language, and where there is a presence of (home) language(s), all or most of which are in use or present a potential to be used in the classroom pedagogic process. A multilingual class in this study is therefore a class where learners and the teachers come to class with different proficiencies in at least two languages overall.

1.4 Purpose of the study

This study uses Wenger's (1998) Communities of Practice (CoPs) theory to explore the nature of the CoPs in contexts involving the preparation of multilingual pre-service mathematics teachers and the implications thereof for teacher development. CoPs as a theoretical stance provides a useful lens with which to explore ways in which pre-service teacher education classroom communities of practice[4] collaborate in the process of knowledge (re)creation and ways of thinking about common concerns and problems (Mitra, 2008). Research thus far has focused on what happens in practice in mathematics classrooms at primary and secondary levels; and research has highlighted that teaching mathematics in multilingual classrooms is fraught with difficulties. The present study focuses on how teacher educators prepare high school teachers to deal with the complexity of teaching mathematics in multilingual classrooms. Hence, the purpose of this study is to understand what happens in pre-service teacher education classrooms of multilingual pre-service teachers who themselves are being prepared to teach mathematics in multilingual contexts. The present study is therefore organised to explore the following questions:

1. What is the nature[5] of the communities of practice of pre-service mathematics teacher education classrooms in one Province in South Africa?

2. What are the implications resulting from the above with regards to preparing pre-service teachers for teaching mathematics in multilingual classrooms?

I revisit and elaborate these questions as I flesh out the theoretical framework in the next chapters, then again in Chapter Five when I deal with the organisational language that informs my methodological approach.

As with most collective bodies, there is one group or individual who is empowered to assess the operating situation and map out a line of action in a community of practice. In the case of

[4] In Chapter Two, I delve deeper into the CoPs theory by Wenger (1998) providing justification for its use, etc.; and in Chapter Four, I argue that the appellation *Communities of Practice* is suited for use in characterising the pre-service teacher education classrooms (as individual social configurations) involved in this study.

[5] I used the three dimensions of Communities of Practice (CoP) as described by Wenger (1998) to gain an entry into the *nature* of CoP of different pre-service teacher education classrooms. Thus, for each CoP, an analysis of the three dimensions of communities of practice – Mutual Engagement, Joint Enterprise and the Shared Repertoire permitted me to access the nature of these communities of practice. I will return to this in subsequent chapters.

a pre-service mathematics classroom, it is the teacher educator who has the pivotal role of steering and managing classroom discourse, norms, practices, etc that are necessary for the smooth running of the community. Hence, how the teacher educator played this role constituted an important index in the overall study.

1.5 Operational definitions

The following terms are used throughout this research study and are defined specifically for this study:

DOMINANT LANGUAGE: Also called first language in this study. It is the main language, usually the home language of students/teachers in which they are most fluent, and speak more often than any other language.

LANGUAGE OF INSTRUCTION: The official language used in teaching and learning which may or may not be the dominant language of students/teachers.

LANGUAGE INFRASTRUCTURE: My use of the term 'language infrastructure' of a classroom resonates with the use of this term by Setati *et al* (2002, p. 129) to denote language teaching and learning contexts in South African classrooms or schools

LANGUAGE PROFICIENCY: Different linguists define language proficiency in different ways. In this study, the use of the term *English language proficiency* resonates with how it is used by Peregoy and Boyle (2000) to refer to an individual's knowledge of English, and the degree to which the individual exhibits control over the use of English, including vocabulary, grammar, and discourse conventions, which may be called upon during any instance of listening (and understanding), reading, oral and written language use.

LANGUAGE OF LEARNING AND TEACHING (LoLT): Simply called language of instruction. The Language-in-Education policy advocates for the adoption of one of the eleven official languages as the language of teaching and learning of a school. LoLT is, thus, the language which a particular school has adopted as the pedagogic language.

SECOND LANGUAGE LEARNERS: Learners who do mathematics in a language other than their home or first language.

MULTILINGUAL(ISM): Since in South Africa, the majority of the population is more accurately described as multilingual rather than bilingual, the term multilingualism would be used instead of bilingualism. Baker (1988) notes that given the number of language skills (reading, writing, listening, speaking, etc) in all languages and the context where the language can or cannot be used, categorisation of who is or who is not bi/multilingual becomes fraught with difficulties. For example, a learner may have greater skills in reading and speaking Zulu but a poor skill in writing this language. This same learner may have a well developed skill in writing English and Xhosa but a poor skill in speaking both these languages. Is this learner Multilingual? Are bi/multilinguals those with well-developed and equal skills (reading, writing, listening, speaking) in two/more than two languages? In this study, multilingualism refers to competence in two or more languages; that is, knowledge of, and relative ease in the use of more than one African language and English language. A multilingual class in this study is therefore a class where learners (and the teachers) come to class with different proficiencies in two or more languages overall and where mathematics is learned in a language other than the first or home language of the majority of students. This understanding of multilingualism encompasses bilingualism.

DISCOURSE: The term 'discourse' is a complex and multifaceted term which has been defined differently by different authors in mathematics and other disciplines (Ryve, 2011). Monaghan (2009, p. 15 drawing on Morgan, 2007), defines discourse as the "patterned uses of language and other forms of communication whose deployment identifies the user as belonging to a particular community at a particular time in a particular setting". This definition is the understanding of the term 'discourse' as it is used in this study. Simply put, it is language-in-use.

PRACTICES: taken-as-shared ways of doing and communicating mathematics which can be idiosyncratic of a person or shared by persons in the same problem situation

1.6 Conclusion

This Chapter has given a global picture of what the present study sets out to investigate. In doing so, it has dealt with the guiding questions, the purpose of the study and imbedded the study in the context of South Africa.

1.7 Structure of the thesis

Chapter Two locates the study within the situative perspective framework and more specifically, within the Wenger's (1998) communities of practice framework. Wenger's communities of practice framework is elaborated and complemented with Mortimer and Scott's (2003) construct of meaning making as a dialogic process. The suitability of the overall framework for the present study is justified.

In **Chapter Three**, I deal with the review of literature concerning the dynamics of teaching and learning in multilingual classroom where mathematics is learned in a language other than learners'/students' first language. I also engage with literature on the teacher educator's role in creating communities of practice and the role identity plays in pre-service teacher education.

The research design, research contexts and the method of data collection used in the study are described in **Chapter Four** where I discuss the population, sample, research instruments, validity and reliability. Justifications are also provided as to why the pre-service teacher education classrooms in my study can be regarded as communities of practice.

In **Chapter Five** I engage with the organisational language that were developed both from theoretical and empirical aspects of the research study in the course of analysis and describe the processes through which this organisational language developed. In so doing, I provide a description of how the shared repertoire, mutual engagement and joint enterprise were analysed in the study.

Chapter Six provides analysis and finding from pre-observation interviews in Phase One of the study and also highlights the issues arising from the data collection in Phase two of the study.

Chapters Seven and Eight present the findings (for University A and University B respectively) which emerged from Phase 2 of the study as far as the shared repertoire and

mutual engagement are concerned, which **Chapter Nine** deals with what can be inferred about the joint enterprise in the four pre-service teacher education classrooms at both universities. I also draw conclusions; make recommendations for teacher educators and teacher education programmes in South Africa.

Finally, Chapter Ten provides a personal account of my PhD journey with regards to the theoretical and empirical fields of my study and provides directions for future research. In this chapter, I also deal with limitations to the study.

CHAPTER TWO
Theoretical and conceptual underpinnings of study

2.1 Introduction

This chapter provides the theoretical stance that informs this study. In this study, elements of the situative perspective are used as a theoretical lens with which to explore the nature of pre-service teacher mathematics classroom communities with the hope of gaining an understanding into how teacher educators prepare pre-service teachers to teach mathematics in multilingual classrooms. More specifically, Wenger's (1998) communities of practice theory is used to frame the study. I complement Wenger's communities of practice theory with Mortimer and Scott's (2003) construct of meaning making as a dialogic process. The chapter concludes with a re-expression of the guiding questions in the language of the theory that frames the study.

2.2 Theoretical background/constructs

The situative perspective focuses on how, in context, different discourses give rise to different kinds or forms of knowing (Putnam & Borko, 2000). Reaffirming this, Brilliant-Mills (1994) argues that the pattern of discourse used by teachers and students within and about a particular content area (mathematics, in the case of this study) would determine the nature of enculturation into the discipline and invariably, would lead to the internalisation of the ability to engage in discursive mathematical practices in particular kinds of ways[6]. In this regard, I argue that a pre-service mathematics classroom which comprises pre-service teachers (PSTs) and teacher educators (TEs) who are monolingual and who share the same

[6] It is pertinent to note that the present study is made more complex by the fact that in any pre-service mathematical classroom, the teacher educator enculturates pre-service teachers into at least two discourses at the same time: mathematics discourse and teaching discourse.

language would "enculturate" the students into a different way of engaging with particular types of practices.

Barab and Duffy (2000) argue that there is a range of perspectives within the situative model: while the psychological perspective highlights cognition and meaning through situated activities in practice fields[7] resembling real life situations, the anthropological perspective on the other hand, focuses on issues around communities of practice and what it means to learn as a member of a community. It focuses on active involvement and the use of concepts and tools to build a rich understanding of the world (Wenger, 1998). The present study is informed by the anthropological perspective of situativity rather than the psychological perspective. This is because, rather than focus on the cognitive apprenticeship of the different pre-service teacher education classrooms involved in the study, it explores mathematics pre-service teacher education classroom *communities* as entities bounded by three dimensions – mutual engagement, joint enterprise and a shared repertoire (Wenger, 1998).

Lo, Wheatley and Smith (1994, in Borko et al., 2000) note that as far as the situative perspective is concerned, it is through participation in classroom pedagogical discourse that students become enculturated into communities of practice. But even though learning, for theorists of anthropological situativity, is an individual process of coming to understand how to participate in the discourse and practices of a particular community, it is at the same time a community process of refining norms and practices through the ideas and ways of thinking that individual members bring to the discourse (Lave & Wenger, 1991). As indicated in Chapter One, as with many collective bodies, there is one group or individual who is empowered to assess the operating situation and map out a line of action. In the case of a pre-service mathematics classroom, it is the teacher educator who does this. In a pre-service mathematics classroom (as in many other settings), the teacher educator has a pivotal role in facilitating this process and in determining how discourse manifests itself in the classroom (Casa & DeFranco, 2005). Hence, the knowledge, skills, and pedagogical practices of the teacher educator in bringing this to bear, of creating an enabling environment of challenging yet supportive discourse in the community of practice, cannot be underestimated[8].

In this study, Wenger's (1998) social theory of learning, and in particular the notion of learning in *communities of practice* was used to frame the study theoretically, conceptually

[7] Practice fields are "contexts in which learners, as opposed to *legitimate participants*, can practice the kinds of activities they will encounter outside school" (Barab & Duffy, 2000, p. 30. Italics in original)

[8] I will engage specifically with the role of the teacher (educator) in Chapter Three.

and methodologically. Wenger's (1998) social theory of learning (STL) constitutes a vital alternative to the then dominant cognitivist view of learning as knowledge acquisition in instruction. STL is premised on the fact that learning is a social enterprise and that learning within communities of practices "shape not only what we do but also who we are and how we interpret what we do" and construct identities in relation to these communities (Wenger, 1998, p. 4). Before delving into Wenger's notion of community of practice, it is expedient to elaborate on Wenger's social learning theory first.

2.3 Wenger's social learning theory

Four assumptions about what matters about learning and about the nature of knowledge, knowing and knowers undergird Wenger's social learning theory (Wenger, 1998, p. 4):

- Humans are social beings (and this, according to Wenger, is the central aspect of learning).
- Knowledge is a matter of competence with respect to valued enterprises.
- Knowledge is a matter of participating in the pursuit of such enterprises, that is, of active engagement in the world.
- Meaning – our ability to experience the world and our engagement with it as meaningful – is ultimately what learning is to produce.

Wenger argues that the primary focus of the STL based on the above assumptions is learning as social participation. He defines such participation as

> "not just to local events of engagement in certain activities with certain people, but [to] a more encompassing process of being active participants in the practices of social communities and constructing identities in relation to these communities" (Wenger, 1998, p. 4).

These four assumptions translate into an integration of four components necessary to characterise social participation: *Meaning* (learning as experience), *practice* (learning as doing), *community* (learning as belonging) and *identity* (learning as becoming) as shown in the diagram below:

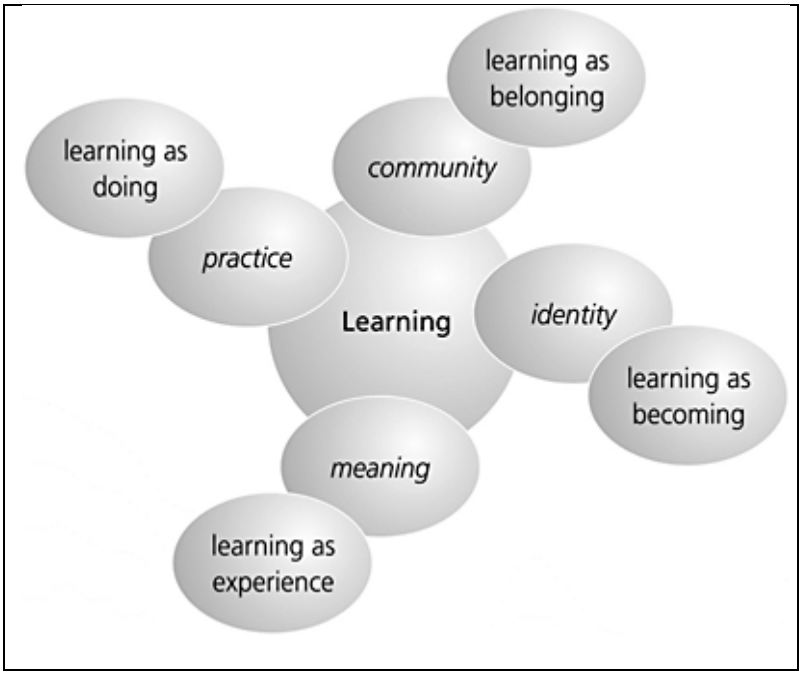


Figure 2.1: Wenger's Components of a social theory of learning (Wenger, 1998, p. 5)

These four components are deeply interconnected and mutually defining according to Wenger.

2.3.1 The concept of meaning

Wenger (1998, p. 5) simply defines meaning as "a way of talking about our (changing) ability – individually and collectively – to experience our life and the world as meaningful". For Wenger, meaning making is integral to all human activities. Wenger holds that although situations may appear to be familiar, by having different interpretations, conversations, etc, meanings are constantly renegotiated anew (see p. 53). This renegotiation of meaning emerges from the dialectic process (tension or duality) of participation and reification.

2.3.1.1 Participation

Wenger (pp. 55-56) defines participation as

> …a process of taking part and also [to] the relations with others that reflect this process. [It is] the social experience of living in the world in terms of membership in social communities and active involvement in social enterprises. Participation in this case is both personal and social.

It is clear that for Wenger, participation shapes not only the individual, but also the collective – the community. Participation defines the way one belongs to the community in which one engages in some enterprises, and through which an identity is developed. Wenger (p. 56) also suggests that a defining characteristic of participation is its ability to bring about the "possibility of mutual recognition" and the mutual ability to negotiate meaning, but does not necessarily entail equality or respect or even collaboration. Hence, for Wenger, participation does not necessarily refer to harmonious relations, but also to conflictual relationships, intimate as well as political, competitive as well as cooperative relations. Finally, for Wenger, participation is broader than direct engagement in practices with specific people and it is not something we turn on and off – it is profoundly social. An example given by Wenger to illustrate this important point about participation is that of someone who is alone in a hotel room preparing slides for presentation the next morning. Even though he/she is alone, Wenger argues that not only is the audience there with him/her as he/she attempts to make his/her points understandable to the audience, but also, his/her colleagues are also there, "looking over your shoulder, as it were, representing for you your sense of accountability to the professional standards of your community" (p. 57). Wenger argues further that even drastic isolation like monastic seclusion or writing is given meaning through social participation and hence, the concept of participation for Wenger, captures this "profoundly social character of our experience of life" (p. 57). Regarding this profound social character, Tusting (2005, p. 38) notes that participation according to Wenger, "draws attention to the 'social-ness' of all sorts of activities which arise from particular identity and community affiliations, even when these activities may not appear in themselves social or participatory".

Participation shapes not only the individual but also the collective. In a multilingual class where teachers are faced with the triple challenge of paying attention to mathematics, attention to English (LoLT) and attention to the mathematical language (as discussed in the previous chapter), the nature of participation can potentially develop both students' spoken language and their mathematical language while at the same time developing mathematical meanings. And in teacher education classrooms, how participation is organised and where authority stems from as members of the community are involved in relations of participation can shape what practices are valued and what practices PSTs are enculturated into.

2.3.1.2 Reification

Reification is 'the process of giving form to our experience by producing objects that congeal this experience into "thingness"' (Wenger, 1998, p. 58). It is treating "an abstraction as substantially existing or as a concrete material object" (p. 58). Reification occurs when metaphors (for example, *war on terror*) or conceptions (like *democracy*, *culture*, etc) are used to capture a situation. By reification, the practices are "congealed" into fixed form, opening up the practice for (re)negotiation of meaning. For Wenger, reification is both the *process* of reification and the reified *products* that emerge from the process of reification.

Although reification is a powerful tool to stabilise practice, Wenger asserts that it is a double-edged sword in the sense that as an evocative shortcut, the process of reification can be very powerful but at the same time its succinctness, its portability can be a danger. As an example of this, Wenger notes that a procedure can reify a concept so that its application is automatic, and a formula can express in a few terms a regularity that pervades the universe; but at the same time, procedures can hide broader meanings in blind sequences of operations, and the knowledge of a formula can lead to the illusion that one fully understands the processes it describes (p.61). This is particularly true in mathematics.

The negotiation of meaning is captured in the interplay of reification and participation. Tusting's (2005, p. 39) articulation of this interplay elucidates better the meaning of reification according to Wenger:

> A book (reification) is not involved in a process of negotiation of meaning until a person reads it (participation). An insurance claims form (reification) is not involved in negotiation of meaning until someone fills it in and processes it (participation). You cannot have a conversation (participation) without drawing on words, linguistic structures and ways of using language (reification). Participation in meaning making always implies reifications and vice versa.

Although reification of a best practice is good, it requires participation to actualise it and make sense of it. Wenger (1998) argues that even when reifications involved in certain practices come from outside the community of practice, "reifications must be re-appropriated into a local process in order to become meaningful" (p. 60). As members participate, new uses or meanings are uncovered and a new reification would occur, influencing participation anew, and vice versa. An example would be useful here: consider the concept of revoicing. As a term, revoicing is abstract. It becomes concretised in practice or through some form of records of practice where revoicing as a pedagogic practice is used. In a study which investigated systematic differences in the ways that revoicing is used in a multilingual

classroom, Enyedy *et al*., (2008) observed that revoicing took on an added meaning when in the second semester, there was a change in the class composition. In the first semester, when the class was bilingual, consisting of solely of native Spanish speakers in an English medium school in the United States, revoicing was used as a tool to transport ideas presented in one language (Spanish) into the dominant language (English) of the classroom. When, in the second semester, African American students who did not speak Spanish but spoke African American Vernacular English were added to the class, revoicing served, among others, as a tool for modelling the academic discourse of mathematics and for positioning the students in relation to the mathematical task at hand. Thus, as members participated in the classroom activities, new meanings and new reifications of revoicing as a practice occurred. In the first instance, revoicing was seen as a go-between practice between Spanish and English; and in the second instance, revoicing was perceived as both a semiotic and a positioning tool.

Wenger argue thus that through negotiation of meaning, participation and reification, we engage in practice and ultimately form communities of practice.

2.3.2 The concept of practice according to Wenger

Wenger (1998, p.5) defines practice as "a way of talking about the shared historical and social resources, frameworks, and perspectives that can sustain mutual engagement in action". Wenger in his first Vignette (pp. 45-46) argues that claims processors have developed a practice, and are involved in a practice when they:

> provide resolutions to institutionally generated conflicts; support a communal memory that allows individuals to do their work without needing to know everything; help newcomers join the community by participating in its practice; generate specific perspectives and terms to enable accomplishing what needs to be done; make the job habitable by creating an atmosphere in which the monotonous and meaningless aspects of the job are woven into the rituals, customs, stories, events dramas and rhythms of community life (p. 46).

Wenger argues that the concept of practice connotes doing, but more than just doing in and of itself, it is doing in a historical and social context that gives structure and meaning to what we do and this always makes practice a social practice (p. 47). Different enterprises give rise to different variations of practice, but "pursuing them always involves the same kind of embodied, delicate, active, social, negotiated, complex process of participation" (p. 49).

Wenger argues that practice does not exist in the abstract – in books or in tools or in a structure that precedes it, even "though it does not start in a historical vacuum" (p. 73), –

practice "resides in a community of people and the relations of mutual engagement" (p. 73). As Tusting (2005, pp. 36-37) notes,

> This concept of 'social practice' offers a way of analysing human activity which brings together the cognitive and the social aspects of human existence. Rather than focussing only on local activity, only on structures and thought or only on broader social structures, it offers us a way of conceptualising the socially situated nature of human activity.

For Wenger, practice is central to meaning making processes and thus the negotiation of meaning. From Wenger's conception of practice, it goes without say that practice varies from one community of practice to the next, and from one historical context to the next[9]. As Grossman *et al*. (2009) rightly point out, practice may vary from context to context and the nature of practices has consequences for what members of a community are able to see and learn about practice.

In pre-service mathematics classroom communities such as the ones in this study, such practice would take on the meaning of *mathematical practices*. I use the term "mathematical practices" to encompass both discursive and pedagogic practices within mathematics classrooms. These three terms – mathematical practices, discursive practices and pedagogic practices – are often used/defined differently or used interchangeably by different scholars. In this study, pedagogic practices are practices that relate to teaching and learning in general; discursive practices are practices that are related to how language and other semiotic devices are used in the negotiation of (mathematical) meaning in class (in this sense, discursive practices would be a subset of pedagogic practices), and mathematical practices is taken as those pedagogic practices that relate specifically to the teaching and learning of mathematics.

My use of the term "mathematical practice" resonates with the way it is used by Godino, Batanero and Font (2007, p. 3) to refer to "any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the solution to other people, so as to validate and generalize that solution to other contexts and problems". They elaborate on this definition thus:

> The practices can be idiosyncratic of a person or shared within an institution. An institution is constituted by the people involved in the same class of problem-situations,

[9] The composition of the community is critical to the practices that are found within it. For example, the community of practice which has a particular TE this year may have different practices to the community of practice of subsequent years even if the incumbent pre-service teachers are still taught by the same teacher educator. Nevertheless, these variations in practices have something of a wider value to contribute to teaching and learning in the South African multilingual context.

> whose solution implies the carrying out of certain shared practices and the common use of particular instruments and tools (Godino et al., 2007, p. 3).

Given this definition of mathematical practices, practices in this study is defined as taken-as-shared ways of doing and communicating mathematics which can be idiosyncratic of a person or shared by persons in the same problem situation. This definition of (mathematical) practices is consistent with Wenger's conception of practice[10] and shared repertoire in that it acknowledges the fact that practices are shared (jointly owned by a community) and are common resources for the negotiation of meaning within communities.

In the present study, an important focus was on what practices are privileged in different teacher education classrooms and how these practices shape and are shaped by these classrooms.

2.3.3 Identity in practice

According to Collanus, Kairavuori and Rusanen (2012), there are two major approaches or views to teacher identity: the essentialist view holds that identity is stable and fixed while the currently dominant view, on the contrary, holds that teacher identity is discursively constructed and constantly changing. Wenger's notion of identity falls categorically into this latter view. The notion of identity is defined by Wenger (p. 5) as a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities. Wenger describes identity as a "constant becoming", as trajectories which are not necessarily linear, and which has no fixed destination. For Wenger, identity is acquired and shaped in the engagement in practices of the community. In fact, for Wenger, participation and reification are dimensions of both practice and identity. Clarke (2008, p. 32) captures this interrelationship between reification and participation which results in identity as follows:

> The dual aspects of meaning, 'reification' and 'participation' are part of Wenger's overall model linking the ongoing (re)creation of a community with the negotiations of meaning entailed by the evolution of its practices, necessitating ongoing learning and resulting in identity (trans)formation

[10] Of course, Godino *et al*. (2007, p. 4) defined the term 'mathematical practice' within the onto-semiotic framework which holds that "teaching involves the participation of students in the community of practices sharing the institutional meaning, and learning is conceived as the students' appropriation of these meanings".

As Hodges and Cady (2012, p. 113) point out, for Wenger, the process of learning is intrinsically linked to identity through "new ways of belonging and being within a community of practice". This is so because for Wenger (1998, p. 215), learning transforms "who we are and what we can do […]. It is not just an accumulation of skills and information, but a process of becoming a certain person". This line of thinking resonates with the later work of Mayer (1999, p. 5) on the importance of teacher identity development:

> Learning to teach for the preservice teachers involve[s] [the] interplay of teaching role and teaching identity. […] Learning to teach can be learning the skills and knowledge to perform the functions of a teacher or it can be developing a sense of self as teacher. In the former, one is 'being the teacher', whereas in the latter, one is 'becoming a teacher'. As has been shown, allowing an emphasis on 'being the teacher' (i.e. the functions of the teaching role) can result in knowledge of content and strategies becoming goals in their own rights. Preservice teacher education becomes 'irrelevant' and survival also becomes a goal in its own right. These distinctions have significant implications for the design and conduct of preservice teacher education programs.

Wenger argues that identity in practice is social not merely because it is reified in social discourse of the self (self-image) and of social categories (what others think or say about us), but "also because it is produced as a lived experience of participation in *specific* communities" (p. 151, my emphasis). Hence, engagement in practice provides for members of the community, certain experience of participation and Wenger argues that what each community pays attention to reifies members of that community as participants.

In exploring identity within the Wenger framework, several aspects of identity can become a focal point. In the present study, I restrict my investigation of identity to how the community's engagement in practice provide a window for what the pre-service mathematics teacher education communities pay attention to, and to how pre-service teachers are positioned in the communities of practice. I would hence talk about identity within pre-service teacher education communities as involving five interacting identities (ways of becoming): becoming a teacher of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics, becoming learners of mathematical practices and becoming proficient English users for the purpose of teaching/learning mathematics. In engaging with the extent to which there is evidence in support of the interacting identities, I explore how members of the different communities in the present study are reified as teachers of mathematics (in multilingual classrooms). Since it is not the aim of this study to explore how the label 'mathematics teacher' is given form in classroom settings in schools (by the PSTs), the study does not investigate whether or not

pre-service teachers have formed an identity, but explores what opportunities exist for the development of the five interacting identities mentioned above. This is why I do not explicitly use Wenger's (1998) three modes of belonging (engagement, imagination and alignment) to understand identity in my study.

2.4 Wenger's conception of Communities of Practice

Wenger (1998, p. 5) defines community as "a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognizable as competence". Cox (2005, p. 532) notes that Wenger's use of the term "community" to describe the emergent relationship around a practice is antithetical to the general/historic usage of and assumptions about the term:

Historic/general usage	Wenger's usage
A rather large, helpful and friendly, bounded group	A community is not necessarily friendly or harmonious (indicator 1)
Communities are usually seen as unpurposive	Communities have a purpose – is it circumscribed by the joint enterprise
Communities tend to imply sameness	Communities as consisting of people who differ, have different skills and knowledge and a mutually defining identity
Communities as rather static	Communities evolve over time, is a creative force

Table 2.1: General vs Wenger's usage of the term community (adapted from Cox (2005, p. 532))

For Wenger, thus, a community of practice is an emergent relationship between people who have come together around a joint enterprise, and is characterised by the existence of mutual engagement in the social practices, and in the process developing a shared repertoire of practices, understandings, routines, activities, common stories, and ways of speaking and acting (Wenger, 1998). I will return to this definition later.

In his earlier work with anthropologist, Jean Lave and Wenger (1991), in contrast to the then dominant cognitive view, focused on situated learning and how through legitimate peripheral participation, newcomers (apprentices) are socialised into a rather static practice community by the more knowledgeable others (masters). As noted by Cox (2005), Li et al. (2009) and Kanes and Lerman (2008), while the work of Lave and Wenger (1991) shares some important

common ground with the later work of Wenger (1998) in that meaning is viewed as locally and socially constructed and identity is central to learning, there are significant divergences in their most basic conceptualisation of community, learning, change, power, and diversity (see table below).

Cox (2005: p. 537) notes the following differences between Lave and Wenger (1991) and Wenger (1998):

	Lave & Wenger (1991)	**Wenger (1998)**
View of learning	Central, and seen as occurring through becoming a member – mostly the socialization of new members by legitimate peripheral participation; learning occurs when there is evidence of increased participation in the community – when there is a movement from the periphery to the core.	An individual learning history is identification with different communities of practice and trajectories through communities; learning is perceived as acquiring competence in the community's joint enterprise(s)
Power and conflict	Between generations, between master, journeymen and novice	Conflict is mostly internal conflict within identity, caused by multi-membership
Formality/informality	Could be in the setting of a formal system of apprenticeship, but sees most learning as informal, that is, unstructured, unplanned, not taught	Authentic engagement around an enterprise, therefore beyond formality. May have a shape and purpose unexpectedly by the designer of the formal system
Diversity	Masters/journeymen/novices – but the practice itself does not have a high division of labour	Includes everyone working on the collective enterprise, mutually defining identities – so could be very diverse with some sort of division of labour – members play different roles
Concept of community	A group of people involved in a coherent craft or practice, example, butchers. Or, not a neat group at all.	A set of social relations and meanings that grow up around a work process when it is appropriated by participants. ('innovative' or problem solving settings)

Table 2.2: Some conceptual differences between Lave and Wenger (1991) and Wenger (1998) concerning the notion of CoP (Adapted from Cox, 2005, p. 537; Li, et al., 2009).

As Cox (2005, p. 536) argues, the above divergences in the conceptualisations of learning, diversity, community, etc, "outweigh the common ground found in the stress on situated negotiation of meaning and the importance of identity in learning". Hence, these divergences have very direct implications for the notion of community of practice that is portrayed by Lave and Wenger (1991) and Wenger (1998). To start with, as Wenger (1998) notes, the work of Lave and Wenger (1991) does not give a clear definition of a community of practice:

> …Our purpose [in the book by Lave and Wenger, 1991] was to articulate what it was about apprenticeship that seemed so compelling as a learning process. Toward this end, we used the concept of *legitimate peripheral participation* to characterize learning. We wanted to broaden the traditional connotations of the concept of apprenticeship – from a master/student or mentor/mentee relationship to one of changing participation and identity transformation in a community of practice. The concept of identity and community of practice were thus important to our argument, but they were not given the spotlight and were left largely unanalyzed *(Wenger, 1998, pp. 11-12: italics in the original)*

More specifically, Kanes and Lerman (2008) call the Community of Practice in Wenger (1998) *Community of Practice Type 1* (CPT1) and in Lave and Wenger (1991) *Community of Practice Type 2* (CPT2)[11]. They argue among others that 1) while CPT1 is concerned with eliciting expositive theoretical structures of learning, CPT2 is concerned rather with eliciting more discursive narratives and trajectories of learning; 2) in terms of the representations of learning that they offer, while for CPT1, to know "is to belong *within* and know that one so belongs", for CPT2, "learning is represented as a linear trajectory, one that transits the periphery of social practices" (Kanes & Lerman, 2008, p. 325); 3) while practice exists as an abstraction from the social world for CPT1, for CPT2, 'practice is concrete and is the world in its socially organised form' (p. 325).

The educational implications for the above differences in both conceptualisations of communities of practice is that while for CPT1 the direction of analysis is towards the abstract/representation, in CPT2, it moves towards the concrete. That said, Kanes and Lerman (2008, p. 326) argue that

> Studies which need or wish to be engaged in theory building around learning or aspects of pedagogy or teacher education and development or other curriculum work may find CPT1 of particular interest… In contrast, studies that take an ethnographic line may be better supported by CPT2, [for example studies involving] the beginning researcher traversing a trajectory peripheral to the practice of mathematics education research.

[11] There are other conceptualisations of community of practice theory which are beyond the scope of this thesis to explore.

The present research is informed by Wenger's (1998) conceptualisation of Community of practices (CPT1) rather than CPT2. The choice of Wenger's (1998) notion of community of practice over Lave and Wenger's (1991) notion of community of practice was because of my conceptualisation of the pre-service multilingual classrooms as a non-homogeneous community where different members play different roles, have varying level of knowledge, confidence and commitment, and fundamentally, where every member is in a learning position as far as the dynamics of the community is concerned[12]. Furthermore, instead of explaining communities of practice using the apprenticeship model of learning in the workplace (which deals with interaction between the newcomers and the more knowledgeable others – the experts, and how newcomers create a professional identity), Wenger (1998) rather describes a community of practice as an entity bounded by three interrelated dimensions – mutual engagement, joint enterprise and a shared repertoire. For Wenger, communities of practice are, as Aguilar and Krasny (2011, p. 219) note, "a place of learning where practice is developed and pursued, meaning and enterprise are negotiated among members, and membership roles are developed through various forms of engagement and participation".

For Wenger (1998), therefore, a community of practice has three components: Joint enterprise (what is it about), mutual engagement (how does it function) and shared repertoire (what capability is produced). As Johnson (2007, p. 281) puts it, "a community of practice is defined in Wenger 1998's text as a community of people with a mutual engagement in a joint enterprise and with a shared repertoire of resources at their disposal".

[12] I will deal with this in a subsequent section

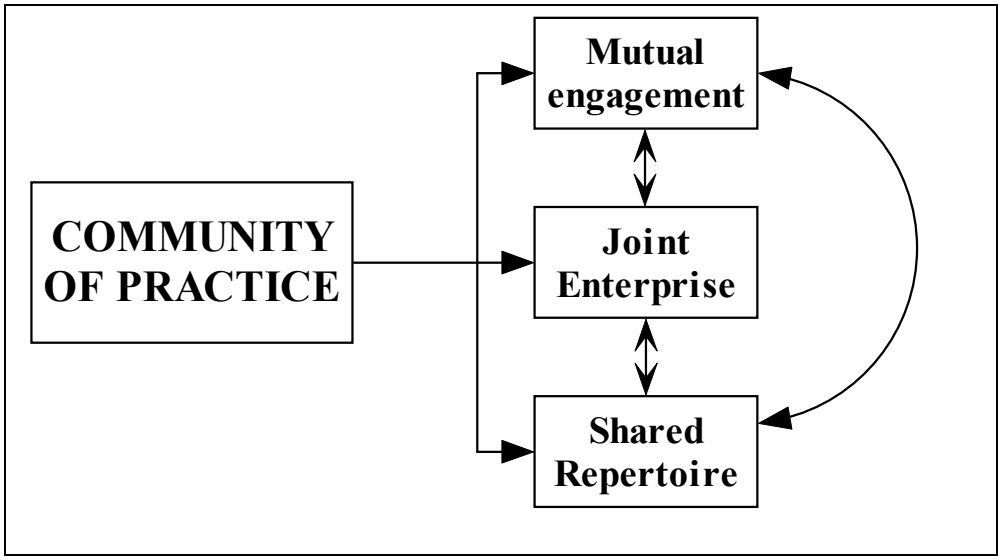


Fig 2.2: Dimensions of communities of practice

Clarke (2008, p. 30) argues that since communities of practice (Wenger, 1998) theory is at once a theory of learning, of identity, of meaning, of community and a theory of practice, CoP "offers considerable potential for thinking about a community of students whose common enterprise is to learn the practices of teaching". As I would explain later, in a multilingual pre-service mathematics classroom, these practices are multifaceted: they include practices specific to being mathematical, practices specific to the teaching of mathematics; practices specific to becoming a teacher and practices specific to becoming a teacher of mathematics in multilingual classrooms[13].

2.4.1 Mutual Engagement

Practice "exists because people are engaged in actions whose meanings they negotiate with one another" (Wenger, 1998, p. 73), and "the relations that constitute practice are *primarily* defined by learning" (Wenger, 1998, p. 131, emphasis added). Practice does not exist in the abstract but resides in a community of people and the relations of mutual engagement by which they can do whatever they do, hence, membership in a community of practice is a matter of mutual engagement (Wenger, 1998, p. 73). Mutual engagement according to Wenger can, thus, be defined as does Clarke (2008, p. 30) as "participation in an endeavour or practice whose meanings are negotiated among participants". A community of practice is fundamentally characterised by mutual engagement of members around the joint enterprise, and this implies, among others, that there is interaction among members (Wenger, 1998).

[13] In a later section, I will engage with the teacher educator's role in these communities of practice.

Wenger argues that it is important to be included in what is 'being engaged' in a community's practice and such engagement is what defines belonging, and so, it is essential for a community of practice to do whatever it takes to enable mutual engagement. Hence, whatever helps the group's dynamics in enabling engagement of its members is welcome – this may include discourse specific talks as well as talks that are geared towards the promotion of an atmosphere of friendliness; it may also include activities by members or a member that are undervalued or unrecognised, but which help in making the daily work/activity of the community more bearable like a constant supply of tea or snacks. For claims processors, Wenger argues that in order "to be a full participant, it may just be as important to know and understand the latest gossip as it is to know and understand the latest memo" (p. 74).

Engagement does not entail homogeneity. Wenger argues that in fact, what makes engagement possible "is much a matter of diversity as it is a matter of homogeneity" (p. 75). Wenger (1998, p. 77) adds that:

> In real life, mutual relations among participants are complex mixtures of power and dependence, pleasure and pain, expertise and helplessness, success and failure, achievement and deprivation, alliance and competition, ease and struggle, authority and collegiality, anger and tenderness, attraction and repugnance, fun and boredom, trust and suspicion, friendship and hatred. Communities of practice have it all.

In fact, the fact that there is heterogeneity within a community may in some instances be a good thing, since as Goos and Bennison (2008, p. 42) note, within a community of practice,

> "productive relationships arise from diversity and these may involve tensions, disagreements and conflicts. However, participants are connected by their negotiation of an enterprise linked to the larger social system in which their community is nested".

Members of a community are different from one another and may have different aspirations and the focus activity of the group may take on unique significance for individual members, but their "responses to dilemmas and aspirations are connected to the relations they create through mutual engagement – they work together, […] they talk with each other all the time, exchange information and opinion…" (p.75). Hence, mutual engagement among members of a community of practice has the potential of helping members overcome differences and diversity.

But even though Wenger (1998) holds that interaction or engagement is the *sine qua non* condition for the negotiation of meaning[14], and even though Wenger asserts that "interrelations arise out of engagement in practice and not out of an idealised view of what a community of practice should look like" (p. 77), Wenger (1998) as some scholars (Barton & Hamilton, 2005; Creese, 2005; Schegloff, 2006; Tusting, 2005) have pointed out, offers little insight as to how meanings are made and interpreted. Tusting (2005, pp. 39-40) puts it more elaborately as follows:

> Despite the centrality of negotiation of meaning to the communities of practice model, and the key role of language [discourse] within processes of participation and reification, Wenger does not draw out ideals about the relationship between language and meaning making more generally, beyond stating that meaning making cannot be reduced to language alone. However, while Wenger is careful to make clear that he is not just talking about language when talking about meaning, language is clearly central to much of the experience of negotiation of meaning we encounter in communities of practice…[…] language has a privileged place in human communication. […]. Language is one of the principal means by which meaning is reified.

In this study, in order to address this shortcoming, the linguistic tools provided by Mortimer and Scott (Mortimer & Scott, 2003) will be incorporated into Wenger's CoP theory. I will return to this point later in this chapter.

2.4.2 Joint Enterprise

A joint enterprise is the result of mutual engagement, and "refers to the focus of activity that links members of a community of practice" (Clarke, 2008, p. 31). Wenger (1998) explains that an enterprise is joint not in the sense that everyone believes in the same thing or agrees with everything, but "in that it is communually negotiated". This means that members of a community must "find ways to live together", and in some cases, "living with their differences and coordinating their respective aspirations is part of the process" (p. 79) in cultivating/sharing a joint enterprise. Put differently, community members' enterprise and its effects in their lives need not be uniform for it to be a collective product (Wenger, 1998). In short, there are three important points about the enterprise that keeps the community together:

- It is the result of a collective process of negotiation that reflects the full complexity of mutual engagement

[14] And, *ipso facto*, a negotiated joint enterprise and the formation of a community's share repertoire (all of which makes a social configuration a community of practice)

- It is defined by the participants in the very process of pursuing it. It is their negotiated response to their situation and thus belongs to them in a profound sense, in spite of all the forces and influences that may be beyond their control
- It is not just a stated goal, but creates among participants relations of mutual accountability that become an integral part of the practice (Wenger, 1998, pp. 77-78)

The above elaboration of joint enterprise suggests that the joint enterprise is never fully imposed on the group or predetermined, but rather, it is the negotiated result of participants of a community of practice to deal with the situation as they experience it. In a community of practice, members of a community may undertake different tasks, play different roles, have different levels of knowledge, confidence and commitment but they all contribute in some way to a joint enterprise which defines the community. Wenger (1998, pp. 78-79) elaborates on this with reference to claims processors:

> To say that some claims processors share an enterprise is not merely to say that they share working conditions, that they have dilemmas in common, or that they create similar responses. Their individual situations and responses vary, from one person to the next and from one day to the next. But their responses to their conditions – similar or dissimilar – are interconnected because they are engaged together in the joint enterprise of making claims processing real and livable. They must find a way to do that together, and even living with their differences and coordinating their respective aspirations is part of the process. *Their understanding of their enterprise and its effects in their lives need not be uniform for it to be a collective product (*my emphasis*)*.

What this presupposes is that there is normally an overarching broad common enterprise which was responsible for bringing members of a community together in the first place. Wenger (p. 79) holds that communities of practices are not "self-contained entities" – but develop within "larger contexts – historical, social, cultural, institutional – with specific resources and constraints" – that is, it is situated within a broader system (institution) that limits and influences it. But while there is a broad enterprise to which the community owes its (initial) existence, this broad enterprise is appropriated and given meaning within the community of practice in ways that are not determined by the institution:

> Yet even when the practice of a community is profoundly shaped by conditions outside the control of its members, as it always is in some respects, its day-to-day reality is nevertheless produced by participants within the resources and constraints of their situation. It is their response to their conditions, and therefore *their* enterprise […]. In sum, it is only as negotiated by the community that conditions, resources, and demands shape the practice. The enterprise is never fully determined by an outside mandate, by a prescription, or by any individual participation. Even when a community of practice arises in response to some outside mandate, the practice evolves into the community's own response to that mandate. Even when specific members have more power than others, the practice evolves into a communual response to that situation […]. Because members produce a practice to deal with what they understand to be their enterprise, their

> practice as it unfolds belongs to their community (Wenger, 1998, pp. 79-80). *(Italics in original)*

Negotiating the joint enterprise leads to the realisation of relations of mutual accountability, or commitment towards the shared objective among members. Wenger (1998, p. 81) argues that

> these relations of accountability include what matters and what does not, what is important and why it is important, what to do and not to do, what to pay attention to and what to ignore, what to talk about and what to leave unsaid, what to justify and what to take for granted, what to display and what to withhold, when actions and artefacts are good enough and when they need improvement or refinement.

According to Wenger, accountability is central to defining the circumstances under which as a community and as individuals within the community, members feel concerned or unconcerned about what they are doing and what is happening to them and around them (p. 81). Van Zoest and Bohl (2005, p. 337) note that accountability to an enterprise can be regarded as "the set of communal standards (both explicit and implicit) for performance and interaction that develops within and is sustained by a community regarding its work". Some aspects of accountability may be reified into standards and policies, for instance. Accountability can also denote sensitivity to appropriate behaviour within the community.

Because Wenger's construct of joint enterprise is usually the most misunderstood of the three dimensions of CoP, it is expedient to summarise the key features:

- There is usually an initial broad joint enterprise (common purpose) to which the community owes its (initial) existence. This broad enterprise is appropriated and given meaning within the community of practice and as such, the enterprise thus formed is a negotiated result/response of participants of the CoP to deal with their own specific situation.
- Because mutual engagement does not require homogeneity, members in a CoP need not have the same beliefs or agree with everything for the joint enterprise to be joint.
- Members of a CoP may have different tasks/roles/commitment and confidence, but all contribute in a way to a joint enterprise which defines the community.
- The enterprise of a community may affect members' lives differently.
- Negotiating the joint enterprise gives rise to relations of mutual accountability among members.
- To this list, I would add that the negotiation of a joint enterprise or joint enterprises relies heavily on language – on linguistic communication or interaction among members of a community of practice.

In the light of the above conceptualisation of the joint enterprise of a community, what possible variations of joint enterprise are likely to exist in a school mathematics classroom community? Herzig (2005, p. 255) notes that the joint enterprise generally has to do with learning to produce solutions to specific range of problems in algebra, geometry, etc., as students interact with each other and with their teachers to acquire mathematical knowledge. So, in a way, the acquisition of mathematical knowledge is the joint enterprise.

Boylan (2005) argues that the enterprises in school mathematics are different for teachers and students in classroom mathematics contexts, and this from the perspectives both of the formally constituted or recognised enterprises (of students and teachers) and those "that may be inferred from the manner in which students and teachers engage in practices and reports they give of their motivations to engage with the practices in particular ways" (Boylan, 2005, p. 6). Boylan further argues that while the enterprise of the students is, ostensibly, to become proficient in the practices of the school mathematics, the enterprise for the teacher is for students to become proficient in these practices. While acknowledging overlap between these different enterprises, Boylan contends that the nature of the two enterprises is clearly different from those that occur in the situations of community of practice "where the enterprise of the community is extrinsic to the learning of new comers to the community" (p. 6). My contention is that this is a misunderstanding of the Wengerian joint enterprise. In making the above argument, Boylan is working with the assumption that there is only one joint enterprise in a community of practice which must be the same for everyone and affect everyone in the same way. As noted above with Wenger's example of claims processors, individuals in a CoP may respond differently to their conditions and their understanding of their enterprise and its effects in their lives need not be uniform. Also, even though the broad enterprise in a school mathematics community may be the acquisition of mathematical knowledge, how this plays out differs in different mathematics classrooms and hence a class's response to their specific situation would vary from one context to the next.

In the pre-service mathematics communities involved in this study, I would argue that the broad joint enterprise for each of the communities is the teaching and learning of mathematics. Both TEs and Pre-service teachers share one goal – the common desire of developing/improving mathematics proficiency level in schools. Both TEs and pre-service teachers are desirous of a more mathematically literate South Africa. This broad joint enterprise, which is the initial *raison d'être* of pre-service mathematics communities is

negotiated differently across the pre-service teacher education classroom communities in both institutions involved in the present study[15].

2.4.3 Shared Repertoire

Wenger (1998, p. 83) defines a 'repertoire' as "a community's set of shared resources", thereby emphasising both the 'rehearsed character' and the 'availability for further engagement in practice' of a community's repertoire. Put differently, shared repertoire "refers to the common resources for creating meaning that result from engagement in joint enterprise" (Clarke, 2008, p. 31). A shared repertoire is a source of community coherence that is created over time in the process of negotiation of meaning within the joint enterprise of the community (Wenger, 1998). The repertoire of a community combines both reificative and participative aspects (artefacts, ways of talking and being, etc). Hence, the repertoire of a community combines two characteristics that allow it to become a resource for the negotiation of meaning: 1) it reflects a history of mutual engagement; 2) it remains inherently ambiguous (p. 83). Wenger argues that well-established interpretations can take on specific usage in different communities:

> Histories of interpretation create shared points of reference, but they do not impose meaning. Things like words, artefacts, gestures, and routines are useful not only because they are recognizable in their relations to a history of mutual engagement, but also because they can be re-engaged in new situations. This is true of linguistic and non-linguistic elements, of words as well as chairs, ways of walking, claim forms, or laughter. All have well-established interpretations, which can be utilized to new effects, whether these new effects simply continue an established trajectory of interpretation or take it in unexpected directions (Wenger, 1998, p. 83).

Wenger gives examples of repertoire of a community of practice: routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of the practice. In line with the critique of the backgrounding of language in Wenger's community of practice stated above, Tusting (2005, p. 40) notes that these elements are 'partly or entirely linguistic in nature'.

In school mathematics contexts, Herzig (2005, p. 225) contends that a shared repertoire would include 'tacit knowledge', the unspoken norms and practices which form part of the *modus operandi* of the classroom community. These would include practices such as

[15] I will engage more on pre-service mathematics classrooms as communities of practice in a later section.

studying for courses, learning acceptable norms for answering particular types of questions, completing homework assignments, etc. It must be noted that for Wenger, repertoire applies as much to discourse and ways of being in the community as it does to material objects (Johnson, 2007).

For the present study, the shared repertoire consist principally of the practices-in-use in the teacher education classrooms and the norms that enable these practices.

2.5 The three dimensions of CoP as an interdependent whole

Wenger asserts that "learning is the engine of practice, and practice is the history of that learning" (p. 96). Learning in practice or learning as a social practice "implies a mutual engagement in the search of a joint enterprise with a shared repertoire" (Gomez, 2007, p. 20) and according to Wenger (1998, p. 95), includes the following *processes* for the communities involved[16]:

- *Evolving forms of mutual engagement:* discovering how to engage, what helps and what hinders; developing mutual relationships; defining identities, establishing who is who, who is good at what, who knows what, who is easy or hard to get along with.
- *Understanding and tuning the enterprise:* aligning the engagement with the enterprise, learning to become and hold each other accountable to the enterprise; struggling to define the enterprise and reconciling conflicting interpretations of what the enterprise is about.
- *Developing the repertoire, styles, and discourses:* renegotiating the meaning of various elements; producing and adopting tools, artefacts, representations; recording and recalling events; inventing new terms and redefining or abandoning old ones; telling and retelling stories; creating and breaking routines.

Wenger (1998) argues that in a community of practice, mutual engagement, a carefully understood enterprise, and a well-honed repertoire are all investments that make sense with respect to each other. This means that the three dimensions of learning are "interdependent and interlocked into a tight system" (p. 96). For Wenger, it is essential that the three dimensions of a community of practice are present to a substantial and meaningful degree. Wenger later (p. 152) describes these three dimensions of CoP as dimensions of competence since full members of a community who are in familiar territory can handle themselves competently in relation to their mutuality of engagement, their accountability to an enterprise and the negotiability of a repertoire. Conversely, members who have only come in contact

[16] In the next chapter, the three dimensions of communities of practice and their associated processes would provide the backbone for the development of the analytical framework for the present study.

with the CoP experience at that stage a lack of competence in how to interact with others, the subtleties of the enterprise as defined by the community and also lack shared references that participants use.

But can any group of people working or playing together or any social configuration of individuals and activities be regarded as a community of practice?

> Is claims processing in general a community of practice? Should any work group be considered a community of practice? What about a whole company? What about an academic department or a classroom? What about a single individual or a family? What about a hitchhiker and a motorist who share a ride? (Wenger, 1998, p. 122).

Wenger asserts that while calling any social configuration a community of practice would render the concept meaningless, at the same time, too many restrictions in the definition of what qualifies as a community of practice would render the concept less useful. Wenger accounts for the fact that groups of people and their associated activities may form a social configuration without qualifying as communities of practice by engaging with issues around locality of practice, proximity of practices and constellation of practices. What is worth noting, is that for Wenger, the three dimensions of CoP must be present to a substantial degree, and that the relation that constitutes the community's practice must primarily be defined by *learning* (See page 131).

Wenger (1998, pp. 125-126) lists 14 indicators that a community of practice has formed, and these are closely tied to the above-mentioned three dimensions of a community of practice and ensure that the three dimensions are present to a substantial degree:

1. Sustained mutual relationship – harmonious or conflictual
2. Shared ways of engaging in doing things together
3. The rapid flow of information and propagation of innovation
4. Absence of introductory preambles, as if conversations and interactions were merely the continuation of an ongoing process
5. Very quick setup of a problem to be discussed
6. Substantial overlap in participants' descriptions of who belongs
7. Knowing what others know, what they can do, and how they can contribute to an enterprise
8. Mutually defining identities
9. The ability to assess the appropriateness of actions and products
10. Specific tools, representations, and other artefacts
11. Local lore, shared stories, inside jokes, knowing laughter
12. Jargon and shortcuts to communication as well as the ease of producing new ones
13. Certain styles recognised as displaying membership
14. A shared discourse reflecting a certain perspective on the world (pp. 125-126)

Wenger is quick to add that other likely but not necessary conditions include that 1) all participants interact intensely with everyone else or know each other very well, 2) that everything participants do be accountable to a joint enterprise and that 3) much of the repertoire has been locally produced. In Chapter Four, I argue that the teacher education classrooms in my study fulfil the requirement of the appellation "communities of practice" and can therefore be considered as such.

2.6 Complementing Wenger's CoP theory

With regards to the three dimensions that define a community of practice, Herzig (2005, p. 255) notes this with regards to mathematics classrooms:

> Both the joint enterprise and the shared repertoire are constructed and negotiated by participants as they mutually engage in the activities of their classroom communities. These three dimensions of the community of practice of school mathematics entail students' appropriation of mathematical knowledge (entering and constructing the joint enterprise), practices (entering and constructing the shared repertoire), and sense of belonging within the discipline (engaging in mutual ways with the other community members). […] Each of these dimensions is affected by students' interactions with other members of the community – the students and the teachers. These three dimensions of students' learning of mathematics – acquiring knowledge, practices, and a sense of belonging – are all critical components of students' mathematical learning.

What comes out forcefully in the above excerpt is the defining role that interaction plays in the construction and sustenance of communities of practice. To this, I now turn.

2.7 The dialogic process

In the foregoing discussion, I have highlighted the fact that despite the importance accorded to shared repertoire and mutual engagement as dimensions of communities of practice, CoP theory lacks a coherent theory of language in use; and despite the emphasis on a jointly negotiated enterprise and on the negotiation of meaning, little insight is given into how meanings are made and interpreted (Creese, 2005). Coupled with this is the fact that Wenger provides no clear analytic tool. In the present study, this gap is addressed by using Mortimer and Scott's (2003) theoretical constructs of meaning making as a dialogic process. It is my contention that the dialogic process (DP) as constructed by Mortimer and Scott is consistent with the overall framework that guides this study – the anthropological perspective of situativity – in that understanding the social practice and the nature of communities of

practice is tightly bound up with, and indeed is dependent on interaction. It is also my contention that Mortimer and Scott's dialogic process is compatible with Wenger's (1998) CoP theory for a number of reasons: 1) just like CoP theory, DP acknowledges the centrality of purposeful discourse[17] between the teacher and the students in the classroom or learning environment as Mortimer and Scott (2003, p. 3. emphasis in original) argue that "talk is central to *meaning making* process and thus central to *learning*". Hence, both Wenger (1998) and Mortimer and Scott (2003) acknowledge the centrality of discourse (interaction/engagement) in meaning making processes; 2) (and related to the first) both theories are rooted in the premise that learning takes place in social situations where there is social exchange among members of a particular social configuration.

But unlike Wenger, Mortimer and Scott go beyond mere stating the central importance of interaction in the classroom to setting out a multi-level analytic tools for characterising both teacher-student and student-student interactions in the classroom. In what follows, I explore some of the theoretical constructs in dialogic meaning-making process according to Mortimer and Scott. In Chapter Five, I discuss how it (Mortimer and Scott's analytical framework) was used to complement Wenger's CoP theory in providing insights into the teacher education classroom discourse in my study.

2.7.1 Meaning making as a dialogic process

For Mortimer and Scott (2003, emphasis in original), "meaning making can be seen to be a fundamental *dialogic* process, where different ideas are brought together and worked upon". They argue that the dialogic process makes a "shift in focus away from studies of students' alternative conceptions, and towards the ways meanings are developed through language in the […] classroom" (p. 4). This process of meaning making, they contend, happens both on the social and the individual planes:

> On some occasions the dialogic nature of meaning making is obvious, as ideas are, for example, talked through between teacher and student in a social context. Here, an actual dialogue is played out in which there is exploration of ideas between two people. At other times the student may be silent in class, listening to the talk that surrounds them, trying to make sense of what is being said. Here the student is equally engaged in the dialogic process of coming to an understanding, as they struggle to bring together different ideas,

[17] To reiterate, discourse in this study is taken as language and other forms of communication that are in use within a community and define members of such a community (Monaghan, 2009). Discourse is used interchangeably with interaction in this study.

> silent but none the less engaged in dialogic meaning making (Mortimer & Scott, 2003, p. 12).

Mortimer and Scott go on to develop an analytical framework that relates five different, but interconnected aspects of interactions in the classroom with a specific focus on "analysing and characterising the various ways in which the teacher acts to orchestrate […] talk […] in order to support student learning" (p. 24). These 5 aspects of the framework are 1) teaching purposes; 2) content; 3) communicative approach; 4) patterns of discourse; and 5) teacher interventions.

	ASPECT OF ANALYSIS	
FOCUS	Teaching purposes	Content
APPROACH	Communicative approach	
ACTION	Patterns of discourse	Teacher interventions

Table 2.3: Mortimer & Scott's (2003, p. 25) framework for analysing meaning making

As I will explain later, the dominant role played by the teacher in classroom-related discourses does not go against the tenets of Wenger's CoP theory since a community of practice is made up of members who play different roles and with varying competences.

It must be noted that even though Mortimer and Scott's work was in the domain of the sciences, the framework can readily find application in mathematics classrooms. In the present study, two aspects of the framework – communicative approach and patterns of discourse – are used in conjunction with Wenger's CoP theory. The choice of these two stems from the fact that they deal more categorically with issues around communication/interaction in the classroom. Moreover, the other three – teaching purposes, content and teacher interventions – are dealt with in my elaboration of Wenger's CoP theory for use in pre-service teacher education classrooms in Chapter Five[18].

[18] For example, content is dealt when examining the *evaluating mathematics validity practices* under shared repertoire; the teaching purposes are examined through the *interacting identities* under mutual engagement and also form part of the joint enterprise of the classroom communities.

2.7.2 The Communicative Approach

Mortimer and Scott argue that a communicative approach (CA) is at the heart of their framework. CA focuses on "how the teacher works with students to develop ideas in the classroom" (p. 33). Mortimer and Scott (2003, p. 33) explain further:

> This aspect of the framework focuses on questions such as whether or not the teacher interacts with students (taking turns in the discourse), and whether the teacher takes account of students' ideas as the lesson proceeds

They then went on to develop four classes of a communicative approach along two dimensions: *dialogic–authoritative* and *interactive–non-interactive*.

2.7.2.1 The dialogic-authoritative dimension

The dialogic-authoritative dimension has its roots in Wertsch's distinction between authoritative and dialogic functions and also on the notions of authoritative and internally persuasive discourse by Bakhtin (see Scott, Mortimer, & Aguiar, 2006). Mortimer and Scott conceive of the meaning making process as a continuum in which at one extreme there is the dialogic communication approach and at the other extreme, there is the authoritative communication approach. In the dialogic approach, the "teacher hears what the student has to say from the student's point of view", and in the authoritative, "the teacher hears what the student has to say only from the school [mathematics] point of view" (Mortimer & Scott, 2003, p. 33). Hence, "the teacher's purpose is to focus the students' full attention on just one meaning" (Scott, et al., 2006, p. 610). In dialogic discourse, attention is paid to more than one point of view and to more than one voice, and where there is an exploration or, in the Bakhtinian term, where there is the *interanimation* of ideas (Mortimer & Scott, 2003; Scott, et al., 2006). In dialogic discourse, Scott et al. (2006) argue that "the teacher recognizes and attempts to take into account a range of students' and others' ideas" (p. 610) . Scott et al's (2006) use of the term "authoritative" must not be confused with the word "authoritarian". Authoritative is simply used to characterise a situation whereby there is no 'bringing together and exploration of ideas' (p. 610), where the teacher focuses attention on the school content point of view:

> Authoritative discourse is closed to the points of view of others, with its direction having been set in advance by the teacher. More than one voice may be heard, through the contributions of different students, but there is no exploration of different perspectives, and no explicit interanimation of ideas, since the student contributions are not taken into

> account by the teacher unless they are consistent with the developing school [mathematics] account (Scott, et al., 2006, p. 611).

"Dialogic", on the other hand, means that attempts are made to take into account different points of views, different perspectives, and others' ideas. In dialogic discourse "the teacher attends to the students' points of view as well as to the school [mathematics] view" (Scott et al. 2006, p. 610). Scott *et al*. (2006) have engaged decisively with the tension that is engendered by attending to different points of view.

With regards to dialogic discourse, Scott et al. (2006) hold that there are different levels of interanimation of ideas:

> At one extreme the teacher might simply ask for the students' points of view and list them on the board. Here the discourse is open to different points of views, but there is no attempt to work on those views through comparing and contrasting. The teacher's approach involves a low level of interanimation of ideas. On the other hand, the teacher might adopt an approach which involves trying to establish how the ideas relate to one another (*John thinks that this might be the case, but Susan seems to be suggesting something different. Nancy what do you think?*). Both of these approaches are dialogic in the sense of allowing the space for different ideas to be represented, but the second approach clearly involves a higher level of interanimation (Scott, et al., 2006, p. 610. Italics in original).

Scott *et al*. (2006) call the former a low level of interanimation of ideas and the latter which involves comparing, contrasting and developing, they call a high level of interanimation of ideas[19]. Scott et al (2006) argue that for meaningful learning to take place, there has to be movement between dialogic and authoritative discourses as the two are dynamically linked in practice – and therein lies the tension. The tension develops as a result of, for example, a dialogic exploration of a concept which in the long run requires "resolution through authoritative guidance by the teacher". Scott *et al*. (2006, p. 623) conclude that "both dialogicity and authoritativeness contain the seed of their opposite pole […], [and are therefore] tensioned and dialectic, rather than […] an exclusive dichotomy". In the mathematics class, even if there is high interanimation of ideas, there is some kind of authority that is guiding the interaction. This authority is the mathematics itself, of which the teacher (more often than not) has more access at that particular point in time. The tension, therefore, develops between developing the dialogic approach by taking learners views into

[19] An "authority" (mathematics in this case) implicitly or explicitly guides this dialogue/interanimation of ideas. More often than not, the teacher educator has more access to this (authority) than the pre-service teachers. This points to the non-homogeneity of members of the community of practice as discussed earlier.

consideration on the one hand, and on the other hand, developing what is acceptable mathematically.

2.7.2.2 The interactive–non-interactive dimension

Mortimer and Scott (2003) argue that the dialogic nature or the authoritative nature of talk is independent of whether or not it is uttered individually or between people: "what makes talk functionally dialogic is the fact that more than one point of view is represented, and ideas are explored and developed, rather than it being produced by a group of people or by a solitary individual (Mortimer & Scott, 2003, p. 34; also see Scott et al., 2006, p. 611). That said, Mortimer and Scott (2003) argue that talk can be *interactive* which means it is structured to allow for the participation of other people, or *non-interactive* when it excludes the participation of other people. And when the two dimensions of the communicative approach are combined, the upshot for Mortimer and Scott are four fundamental classes of communicative approach:

	INTERACTIVE ⟷ NON-INTERACTIVE	
DIALOGIC	Interactive/Dialogic	Non-interactive/Dialogic
AUTHORITATIVE	Interactive/Authoritative	Non-interactive/Authoritative

Table 2.4: The dialogic-authoritative dimensions of discourse on an interactive–non-interactive continuum (adapted from Mortimer & Scott, 2003, p. 35).

In Scott et al. (2006, pp. 611-612), these four classes of communication approach are summarised as follows:

- *Interactive/dialogic*: Teacher and students consider a range of ideas. If the level of interanimation is high, they pose genuine questions as they explore and work on different points of view. If the level of interanimation is low, the different ideas are simply made available. In interactive/dialogic approach, the teacher seeks to elicit and explore different ideas about a particular issue or concept through 'genuine' questions which probe students' points of view.
- *Noninteractive/dialogic*: Teacher revisits and summarizes different points of view, either simply listing them (low interanimation) or exploring similarities and

differences (high interanimation). This oxymoronic appellation by Mortimer and Scott describes a classroom situation in which the teacher is involved in presenting a specific (mathematics) point of view in a presentational mode (*non-interactive*), but at the same time, explicitly considering and drawing attention to different points of views (*dialogic*), be they students ideas or the difference between everyday language and mathematical language, for example. The key feature of this approach is that even though the teacher makes statements 'that addresses the students', or others' point of views', at the same time, the teacher 'does not call for any turn-taking interaction with the students' (Mortimer & Scott, 2003, p. 38).

- *Interactive/authoritative*: Teacher focuses on one specific point of view and leads students through a question and answer routine with the aim of establishing and consolidating that point of view. Hence, here, the class could be highly interactive, yet authoritative because the teacher pays little or no attention to the students' ideas.
- *Noninteractive/authoritative*: Teacher presents a specific point of view. It involves the teacher presenting a specific mathematics point of view or concept in a formal lecture mode.

Mortimer and Scott (2003) argue that each communicative approach is put into action through specific patterns of interaction (discourse) used by the teacher to which I now turn.

2.7.3 Patterns of discourse

Mortimer and Scott (2003) offer an alternative to the traditional triadic discourse (first proposed by Mehan in the late 70's) of Initiation, Response, Evaluation (I-R-E), where instead of making an evaluation of the student's response, the teacher rather gives feedback, or elaborates on the student's response so that the student is supported in developing his/her own point of view. They called this alternative pattern of discourse, the I-R-F (Initiation, Response, Feedback) pattern of interaction in their early writing (Mortimer and Scott, 2003) and then I-R-P- pattern of interaction in their later writing (Aguiar, Mortimer, & Scott, 2010; Scott, et al., 2006), where *p* stands for prompt. In this pattern of interaction, the "teacher feeds-back the response to the student, in order to *prompt* further elaboration of their point of view [ ] and thereby to sustain the interaction" (Scott et al, 2006, p. 612. Italics in original). They argue that this pattern of discourse can also occur either as a closed chain of interaction. In the closed chain interaction, the pattern takes the I-R-P-R-P-R-…E form, where the

prompt (P) by the teacher is followed by a further response from the student (R), and so on until the chain is closed by an evaluation (E) by the teacher. In the open chain, there is no final evaluation by the teachers, and so the interaction chain takes the I-R-P-R-P-R-P-R- form (Aguiar, et al., 2010; Scott, et al., 2006).

Scott *et al.* (2006) note that this nontriadic pattern of discourse may take a different format if students (rather than teachers) are the initiators of the question in the above chain. They contend that the pattern would also take a different format if different students respond to a particular (initiating) question from the teacher. In that case, the pattern would be I-Rs_1-Rs_2-Rs_3- (where S_1 would be student 1, S_2 student 2, etc). They note that the response from S_2 or S_3 may not necessarily address the initial question posed by the teacher, but may for example deal with issues in the response provided by the previous student (Aguiar, et al., 2010; Scott, et al., 2006).

Mortimer and Scott argue that this pattern of discourse can be used to support dialogic interaction while most authoritative interactions are played out through the I-R-E pattern.

2.8 Guiding questions re-expressed in the language of the theory

In Chapter One, I indicated that my study would be guided by two questions. After an elaboration of the overall theory that frames my study, and more specifically the use of Wenger's communities of practice theory complemented with Mortimer and Scotts' construct of the dialogic process, I re-express below the guiding questions in the form of analysis questions.

Guiding Questions	***Analysis Questions***
1. What is the nature of the communities of practice of pre-service mathematics teacher education classrooms in one Province in South Africa? 2. What are the implications	1. What mathematical practices are in use in the negotiation of meaning in the mathematics community? 2. What norms of practice are in use in multilingual mathematics classrooms of pre-service teachers and how do these norms co-construct the mathematics PST education communities? 3. What are the common discursive

resulting from the above with regards to preparing pre-service teachers for teaching mathematics in multilingual classrooms?	repertoires within the communities, and how do they co-construct these communities? 4. What communicative approaches and patterns of discourse are prevalent in the mathematics PST education classrooms? And where does authority stem from? 5. How does the environment enable or hinder engagement? 6. How does classroom engagement support the different interacting identities within pre-service teacher education classrooms? 7. What can be inferred (as opposed to what conclusion can be drawn for sure) as the joint enterprise(s) in each of the communities that has/have been jointly negotiated or which can be considered as their negotiated response to their specific conditions[20]?

These analysis questions will be further elaborated in Chapter Five where I deal with the emergence of an organisational language for transacting between my theoretical and empirical fields.

[20] In Chapter Five, I argue that in my study, I take joint enterprise as an outcome of the analysis of mutual engagement in the community's set of shared resources (Shared repertoire) used in the negotiation of meaning. This is so because engagement is directed towards a negotiated joint enterprise AND the shared repertoire of a community as described by Wenger, and hence "can be seen as the tangible expression of mutual engagement and the key means of carrying forth a joint enterprise" (Levinson & Brantmeier, 2006, p. 331). Hence, both mutual engagement and the shared repertoire dimensions served as a window through which I gain entry into the communities' joint enterprise.

2.9 Conclusion

In this chapter, I have elaborated on the theoretical perspectives that undergird this study. In so doing, I have discussed Wenger's (1998) community of practice theory and complemented it with Mortimer and Scott's (2003) construct of meaning as a dialogic process. It is important to acknowledge and to deal with the fact that Wenger's theory is a theory of learning developed for workplace settings, thus relevant to informal learning while the present study explores teacher education communities of practice for which the teacher educator plays a central role. Graven and Lerman (2003) argue that in order to use Wenger's theory of learning in formal education settings, much work needs to be done to translate his theory to learning in more formal education contexts (such as those in this study) where teachers play a central role in ensuring successful learning. In the present study, in Chapter Five, I operationalise Wenger's theory for use in teacher education contexts. Before doing this, I engage first with my review of literature in the next chapter.

CHAPTER THREE
Situating the study in the literature

3.1 Introduction

In Chapter One, the context of the present study was explained – that of classrooms of pre-service multilingual teacher educator who themselves are most likely to teach mathematics in multilingual classrooms. Since my study aims to explore the nature of communities of practice in such a context, in this chapter, I locate my study in the nexus of research dealing generally with language issues in the teaching and learning of mathematics, then more specifically, with language issues in the South African curriculum. I also explore some practices in (multilingual) mathematics classrooms. Furthermore, since the concept of identity is an important index in Wenger's (1998) notion of CoPs and also important as far as this study is concerned, I also engage with literature on the role identity plays in pre-service teacher education classrooms. Finally, theoretical and empirical work that addresses the role of the teacher (educator) in creating communities of practice mathematics classrooms is discussed.

3.2 The Relationship of Language and Mathematics and implications for Teaching and Learning

After decades of relative under-emphasis of the importance of language in mathematics teaching and learning, the past three decades has witnessed the mathematical education community across several countries grappling to redefine the role of language in school mathematics (Pimm & Keynes, 1994). Mathematics textbooks and pedagogical practices have, in past and present, led learners and educators alike to portray mathematics as the acquisition of ready-made algorithms and proofs through memorisation and proofs (Siegel & Borasi, 1994) and as an activity done in isolation. More and more, new approaches in the teaching and learning of mathematics are supplanting this traditional approach to mathematics and mathematics learning. With the new approaches, the role of language is increasingly foregrounded (see for example, Gutiérrez, 2002) and more attention is paid to the relationship between language and mathematics (Brown, 1997). Research now not only

recognises the vital role that language plays in mathematics achievement, but also acknowledges its equally important role in classroom discourse and in the acquisition of mathematical concepts and skills (Cuevas, 1984).

Various mathematics education research (Clarkson, 1991, 1992; Cummins, 1979a; Pimm, 1981, 1987; Pimm & Keynes, 1994) into the interplay between language and mathematics point to the intricate link between language competence and mathematical aptitude. Any teaching or learning of mathematics involves activities of reading, writing, listening and discussing (Pimm, 1994). Mathematics is carried in semiotic form and therefore accessed through some form of language. For any teacher, language provides a medium by which concepts and procedures are introduced and conveyed, through which texts and problems are read, solved and by which mathematics achievement is measured[21]. From the notion of mathematics as dependent on competence in language, mathematics educators now have the role of promoting and encouraging written and oral fluency in the language of mathematics (Pimm, 1981). Pimm (1994) highlights four different contexts in a mathematics classroom through which the relationship between language and mathematics are made manifest:

- The spoken language of the mathematics in classroom (including both teacher and student talk)
- The specific uses of language for mathematical purposes (often referred to as the *mathematics register*)
- The language of texts (conventional word problems or textbooks as a whole, including graphic material and other modes of representations).
- The language of written symbolic forms (p. 159).

Although some researchers and scholars (example Goldhaber, 2006) have argued that mathematics is a language in its own right, a more acceptable argument is that mathematics is not a language in the sense that natural/ordinary languages like English, French, German, IsiZulu are languages (Pimm, 1981, 1991; Pimm & Keynes, 1994; Setati, 2002). It is a specialized language[22] and as such, mathematics is read, and written and spoken through the

[21] In Chapter Two, I argued that the pattern of language used by teachers and pre-service teachers in the classroom plays a role in how the pre-service teacher is enculturated into the discursive mathematical practices around the topic at hand.

[22] Halliday (1978) and Pimm (1991) refer to this as mathematics register.

use of ordinary languages. Pimm (1981, 1987) argues that the interaction between ordinary language (for example English) and mathematical language can pose difficulty for mathematics teaching and learning. As Setati (2002) notes, this difficulty is compounded by the fact of teaching and learning in multilingual classrooms where communicating mathematically in the class involves, amongst others: a) managing the interaction between ordinary language and mathematical language; b) managing the interaction between procedural and conceptual discourses; and finally, c) managing the interaction between learners' main language and the language of learning and teaching (where the LoLT is different from learners' main language). This set of demands made of the teacher often give rise to what Adler (2001) has described in terms of three inter-related dilemmas which I engage with in a subsequent section.

In Chapter One, I indicated that most classrooms in South Africa are multilingual, and as such pre-service teachers are most likely to teach in multilingual classrooms after their qualification. I now turn to what curriculum documents in South Africa say about multilingualism and what are the minimum requirements for teacher qualification as far as proficiency in the different languages in South Africa is concerned.

3.3 Language-in-Education policy and the Language-for-teaching in South African curriculum

The South African constitution recognizes eleven official languages and the Language in Education Policy calls for the promotion of multilingualism in the education sector through the use of these languages (DoE, 2002). The language in education policy advocates that learners can choose their LoLT and that School Governing Bodies need to stipulate how the school would promote multilingualism in the school. The policy thus recognizes the multilingual nature of South African classrooms and the importance of using language, which was once a tool for control, marginalization and segregation, as a tool for learners' attainment of their full potential to participate and contribute to, amongst others, the intellectual growth at school and in the society at large (DoE, 2003, pp. 29-30)[23].

The role of language in mathematics is also emphasised in the Revised National Curriculum Statement (RNCS) and the National Curriculum Statement (NCS). The RNCS proposes that

[23] Research by Setati (2008) has, however, shown that given the hegemony of English, the freedom given by the language in education policy for schools to choose their LoLT is indeed a chimera as it does not exist in reality.

in addition to knowledge, skills and values development in learners, some other skills that mathematics ought to promote in learners should be that of reasoning, problem solving and communication. In the *Teacher's Guide for the Development of Learning* Programmes (DoE, 2003, pp. 29-30) the RNCS elaborates on communication in the mathematics class thus:

> Communication is one of the critical skills to be developed throughout the GET phase. Learning Programmes need to ensure ample opportunities for learners to practice communicating. Within the Mathematics Learning Area learners need to develop the ability to:
>
> - Talk, read and write about Mathematics with understanding
> - Listen to and interpret discussions about and involving Mathematics
> - Use mathematical code such as symbols and language that is peculiar to Mathematics
> - Recognise the use of communication techniques (especially mathematical techniques) to misrepresent situations both mathematical situations and others.

More recently, to qualify as a teacher, the *Minimum Requirements for Teacher Education Qualification* (DHET, 2011, pp. 15-16) stipulates that all teachers should

> "be proficient in the use of *at least one* official language as a language of learning and teaching (LoLT), and partially proficient [that is, have conversational proficiency] in *at least one* other official language (including South African Sign Language)."

What is striking in all the above recognition of the intertwinement of language and mathematics is that, how educators need to harness or draw on learners' linguistic resources is not addressed. Also problematic is how teachers are to be trained in order for them to effectively exploit the linguistic knowledge of learners (for effective learning of mathematics to take place) in multilingual settings. In this regard, the present study uses the communities of practice theory propounded by Wenger (1998) as indicated in Chapter Two, to investigate the practices within pre-service multilingual teacher education classrooms and to delineate implications thereof for pre-service teacher training.

3.4 Language and discourse in the mathematics classroom: The centrality of communication

As I indicated in Chapter Two, even though discourse is central to Wenger's community of practice theory, Wenger's work does not highlight the importance of language as indicated in

the previous section, but Mortimer and Scott's work does, and so it is expedient to explore what the literature says with regards to language and discourse generally, and more specifically, language and discourse in the mathematics classrooms.

As I noted in a previous work (Essien, 2011, pp. 27-28), Gorgias' (483-375 B.C.) book entitled *On Not-being or On Nature* sparked off a debate about language and communication that would last centuries amongst philosophers, psychologists and linguists. In his book, Gorgais sets out to prove that "first, nothing exists; second, that even if it does, it is incomprehensible by men; and third, that even if it is comprehensible, it is certainly *not expressible and cannot be communicated to another*" (Borgmann, 1974, p. 17. My emphasis). Even though this position was rejected by many philosophers after him (e.g. Socrates, Plato, Aristotle, etc), it (Gorgias' work) opened up avenues for the recognition and investigation of language as a philosophical problem. A very notable work on language was done by Wittgenstein. The importance of language is a view that Wittgenstein stresses through most of his work. The crucial point for Wittgenstein's philosophy is that language is a crucial part of our ability to conceptualise the world. The meaning of our thoughts and expressions do not exist independently of language. For Wittgenstein language is always practical. It is intended to do something. He holds that "without language we cannot influence other people in such-and-such ways; cannot build roads and machines, etc…" (Wittgenstein, 1972, p. 491). Language is a tool for Wittgenstein. As such, language is a large toolbox with many instruments at our disposal and these instruments have various uses.

It is now a generally accepted proposition that language is a key communication tool necessary for mathematics discourse between the teacher and the learners and between learners (Bednarz, 1996; Ernest, 1994; Reynolds & Wheatley, 1996; Setati, 2005a). It is the very nature of mathematics itself to be communicated through language. Through communication in the mathematics class, learners (along with the teacher) participate in "the dialectical process of criticism and warranting of others' mathematical knowledge claims" (Ernest, 1994, p. 44).

Past research (example Moore & Kearsley, 1996) has shown that there is a direct relationship between learners' active engagement and learning outcomes. This active engagement of learners comes through discourse patterns facilitated by the teacher. Thus, the role of the

teacher in fostering discourse (in fostering induction of learners into the mathematics community and into doing mathematics) is of crucial importance[24].

Mathematics teaching and learning must therefore be seen as a social activity (Bednarz, 1996; Moschkovich, 1999, 2002). As a social process, mathematics is learnt through discussion with others and the sharing of ideas to help develop understanding of concepts and by so doing, create mathematical knowledge in the mathematics community. In the process leading to the formation of mathematical concepts, teacher-learner interaction, learner-learner interaction and learner-content interaction is of crucial importance, and language plays a key role in this process. Moschkovich (2002, p. 192) elaborates on this point in this manner:

> Mathematics learning is […] seen not only as developing competence in completing procedures, solving word problems, and using mathematical reasoning but also as developing sociomathematical norms (Cobb *et al*., 1993), presenting mathematical arguments […], and participating in mathematical discussions […]. In general, learning to communicate mathematically is now seen as a central aspect of what it means to learn mathematics.

As I argued in Chapter Two and to reiterate, even though engagement is central in the creation of communities of practice for Wenger (1998), a coherent theory of discourse is lacking. Given the importance of communicating mathematically and of discussion in mathematics classroom discussed above, it was important to fill this void in my study. For this, I turn to the notion of the dialogic process (Mortimer & Scott) according to which meaning making is seen fundamentally as a dialogic process.

But of course in South Africa, dialogic processes occur in multilingual settings of learners who have English as their second language. I now turn to research on bi/multilingualism and mathematics understanding.

3.5 Bi/Multilingualism and Mathematics Understanding

The disadvantages or advantages of bi/multilingualism for achievement especially in mathematics (and Science) education have been divisive over the centuries. At one end of the spectrum, a camp holds that facility in two (or more) languages leads to less room for mathematical skills. Those who uphold this thesis believe that full attainment in mathematics

[24] I will engage with the role of the teacher educator in creating and maintaining an atmosphere suitable for learning and the development of a community of mathematics practitioners in a later section.

(and the Sciences) may be at risk in bi/multilingual children because of the demands of learning in a second language or through two languages (Baker, 1988). MacNamara's famous and much criticised research in the early 1960s on the mathematics achievement of learners from English speaking homes when taught arithmetic in the medium of Irish falls into this category. MacNamara's sample consisted of 1,084 learners from 119 schools in Ireland. Each of the learners was given tests in problem arithmetic and mechanical arithmetic. The conclusion he drew from his study was that Irish bilingual education has a negative consequence (Baker, 1988).

Later researchers, notably Cummins, disputed the above findings arguing that achievement in problem arithmetic involves language as well as arithmetic skills (Cummins, in Baker, 1988). Using the threshold theory, he attempted to explain why some earlier studies reported bilingual learners as cognitively disadvantaged compared to monolingual learners while others reported bilingual learners as more cognitively advantaged than their monolingual counterpart (Lyon, 1996). The threshold theory stipulates that "those aspects of bilingualism which might positively influence cognitive growth are unlikely to come into effect until the child has attained a certain minimum or threshold level of competence in his second language" (Cummins, 1978, in Lyon, 1996, p. 57).

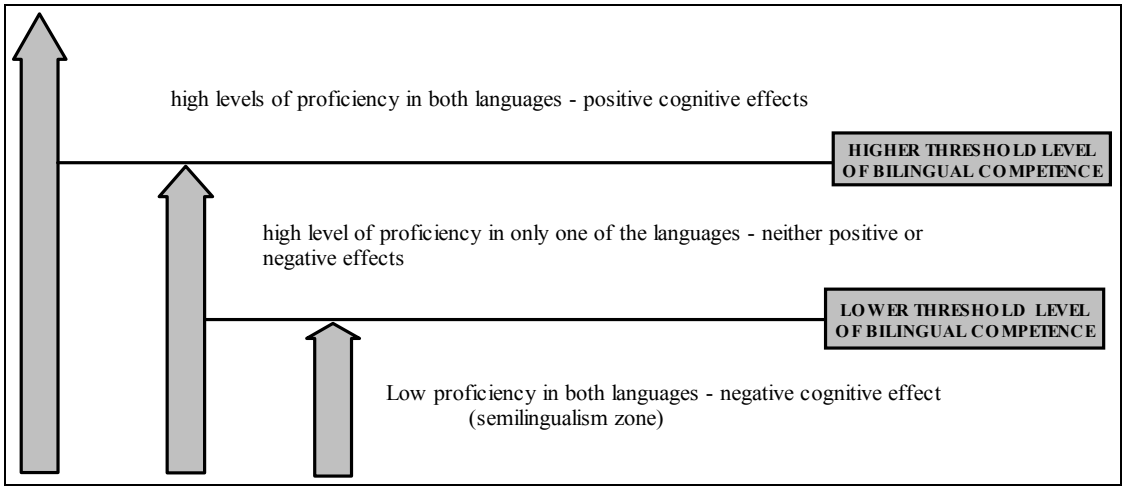


Figure 3.1: Bilingualism, cognitive functioning and the thresholds theory (Adapted from Cummins, 1979a)

Distinguishing two levels of threshold, Cummins (1978) postulates that the lower threshold is sufficient to avoid the negative cognitive and academic effects of bilingualism (in the semilingualism zone) but the higher threshold is necessary to reap the positive benefits of bilingualism (Cummins, 1979a; Cummins, 1979b). This means that once a learner has attained a certain level of linguistic competence in his/her second or third language, positive

cognitive results can occur (Baker, 1988), and the further the child progresses towards proficient bilingualism, "the greater the probability of cognitive advantages (Baker, 1988, p. 175). Combining the developmental interdependence hypothesis and the threshold hypothesis, Cummins (1979a) also suggested that cognitively beneficial bilingualism can be achieved only when the learners' first language is adequately developed. Clarkson's (1992) findings resonate with Cummins' model above. Clarkson (1992) investigated the effect of bilingualism (children's own language and English language) on their capacity for learning in school. The study was prompted by the need to provide research evidence that could inform the debate according to which the use of own language by learners impeded or enhanced mathematical understanding. The research findings revealed that bilingual students with proficiency in both mother tongue and English outperformed students who were proficient in only one of either mother tongue or English, and bilingual students with low competence in both languages performed very poorly.

3.6 Teaching/Learning Mathematics in LoLT as second language

Although the South African constitution gives provision for learners to learn in any of the eleven official languages of their choice, most learners learn mathematics in English, a language which for most, is not their first or home language. Learners who come from homes where the language of learning and teaching is the only language spoken at home are familiar with the linguistic structures they encounter in the mathematics classroom (Barwell, 2002; Cuevas, 1984)[25]. Research (Adler, 2001; Barwell, 1998; Barwell, et al., 2007; Clarkson, 1991; Gorgorio & Planas, 2001; Halai, 2004; Setati, 2002) has shown that this is not the case with learners whose home language is not the language of teaching and learning. English learners in mathematics classrooms do not have to deal with the additional constraint of not being fluent in the LoLT – they are already familiar with the English language as well as its linguistic structure. Of course they have to deal with the structure of the mathematical language (see Pimm, 1987) but additional language learners have to deal with all these simultaneously. These learners need to deal not only with learning the mathematical concepts, but also the language in which these concepts are embedded (Barwell, et al., 2007). In fact, underachievement in Matric examinations (mathematics) in South Africa (and in the

[25] Mercer (2000) is quick to add that even learners studying in their first language have much to learn about how that language is used in the classroom as an educational medium.

TIMMS referred to in Chapter One) has been found to be more prevalent amongst learners who use English language less frequently at home (Simkins in Taylor, Muller, & Vinjevold, 2003) and in areas where English is less frequently used at home. Mathematics (teacher) educators dealing with learners whose first language is not the language of instruction thus need to be conscious of the complex process not only of learning a second language (Cuevas, 1984), but also the even more complex process of learning (mathematics) *in* a second language. As Fillmore (1982, in Cuevas, 1984) notes, second language learning becomes an even more difficult process when parts of the language first encountered is that in textbooks and the classroom. Fillmore (in Cuevas, 1984: 135) elaborates on this:

> [The language of textbooks and instruction] frequently calls for a high degree of familiarity with words, grammatical patterns, and styles of presentation and arguments that are wholly alien to ordinary informal talk.

Hence, second-language learners may find it especially difficult to follow some of the academic language used in the pedagogic process in the mathematics class.

A second and related challenge for the educator is the fact that learners come to class with varied competence in the language of learning and teaching (LoLT), and with different exposure to the LoLT outside the classroom. The educator is faced with the challenge of carrying along all these learners, irrespective of their level of second language development or competence. In a multilingual class, there is also the challenge that stems from not only the interaction of ordinary language and mathematical language, but much akin to that, the challenge of dealing with how the different languages in the class interact with one another and are used in mathematics (Barwell, 1998).

3.7 Multilingual learners/Teaching Mathematics in Multilingual Classrooms

Mercer (2000, 2001) notes that in any situation where the language of instruction is different from the home or dominant language of learners, teachers may have to deal with the multiple task of teaching the language, teaching the educational "ground rules" for using the LoLT in the classroom and finally, teaching the subject content. Using an example from a study conducted in Botswana where the teacher insisted on the simple ground rule of learners responding in full sentences, Mercer (2001) agreed with the researcher that in such bilingual contexts where the language of teaching and learning is different from learners' home

language, the demands of communication are complicated because the teacher is faced with the dual task of getting learners to focus on both the medium (LoLT) and the message (mathematics).

The dynamics of teaching and learning in multilingual contexts takes on an even more added complexity to the situation described above (Halai, 2004). A growing body of research studies on multilingual education have moved away from deficit theories of multilingualism to address issues on how the linguistic resources multilingual learners bring to class can be adequately harnessed to provide high quality mathematics education to such learners (Adler, 2001; Conteh, 2000; Cuevas, 1984; Gorgorio & Planas, 2001; Halia, 2004; Mercer, 2000, 2001; Setati, 2002, 2005a, 2005b; Setati & Adler, 2000). In other words, recently, more and more, research in multilingual classrooms now focus on multilingualism as a resource rather than learners' lack of or limited fluency in the LoLT (Wallace & Goodman, 1989). Most of these researchers, however, are quick to highlight the complexity of teaching and learning in multilingual classrooms where learners are still learning English as a language and simultaneously learning mathematics both as a discipline of knowledge and as a language. Adler (2001, p. 4), talks about this complexity in terms of dilemmas: the dilemma of code-switching (relating to whether or not to switch between the language of teaching and learning and other languages that are present in the class during discussions); the dilemma of mediation (keeping a balance between formal and informal spoken language); and finally, the dilemma of transparency when there is a "tension between implicit or explicit teaching of the mathematics language". Research conducted by Halai (2004) in a different multilingual context to that of Adler (2001) produced similar dilemmas. In her study of two multilingual classrooms in Pakistan, Halai (2004) concluded that in a multilingual classroom the teacher needs to pay particular attention to a number of issues which include: how learners make sense of the mathematics is determined by how they understand the particular usage and structure of the language; how the use of everyday language shape mathematics learning; how learners express mathematical thinking in their own language; and finally, how language is used in the textbooks in contrast to how the teacher uses language.

For Wenger (1998), practices emerge from relations of mutual engagement around a joint enterprise. Hence, a community exists because it engages in practices that are used in the (re)negotiation of meaning. I now turn to practices in (multilingual) mathematics classrooms.

3.8 Practices in (multilingual) mathematics classrooms[26]

In Chapter Two, I dealt more critically with my working understanding of practices. Suffice it to say by way of reiteration, that mathematical practices in my study is defined as taken-as-shared ways of doing and communicating mathematics which can be idiosyncratic of a person or shared by persons in the same problem situation.

One way of looking at what practices pre-service teachers are enculturated into entails looking at the kinds of practices that teacher educators use, why they use these practices, how these practices unfold and how the teacher educator, through these practices, project what it means to teach in a multilingual setting. What follows is a discussion of some of the practices in (multilingual) classrooms that are advocated by literature.

3.8.1 Code switching[27]

Ayeomoni (2006, p. 91) defines code as "a verbal component that can be as small as a morpheme or as comprehensive and complex as the entire system of language". Given this definition, a single morpheme in the IsiZulu language, for example, can be regarded as a code, so also is Zulu language itself. Code switching has been defined by many researchers and scholars. In this study, I take code switching as a term which covers the phenomena of alternating between two or more languages within the same conversation. It is a language practice of using two or more languages present in the classroom by either the teacher or the learners or both.

As noted, in recent years, scholars in bi/multilingual education have moved away from a deficit view of code switching as semilingualism[28] to a view of code-switching as a powerful teaching and learning practice used to promote communication, conceptual understanding and increase participation. Nevertheless, research by Chitera (2009) on the discourse practices of pre-service mathematics teacher educators in Malawi shows that the view of code switching as semilingualism is still persistent in some mathematics classrooms. In her study

[26] My discussion of practices in multilingual mathematics classroom is by no means exhaustive.

[27] Sometimes spelt *code-switching* or *codeswitching*. My personal preference is to write it as two separate words. However, when quoting or paraphrasing scholars who use an alternate form, their original spelling would be preserved.

[28] Semilingualism is the popular belief that bilingual speakers who code switch do so because of their lack of linguistic competence in their repertoire (see Chung, 2006).

of the language practices in multilingual mathematics teacher education classrooms in Malawi, she found that code switching as a practice in multilingual classrooms was seen by the teacher educators as a failure to express one's ideas adequately in the LoLT (English) – as a display of lack of proficiency in English. This led the teacher educators into believing that only the students need such a tool in multilingual classrooms since they (teacher educators) are competent in English.

Research by Setati and Adler (2000) on language practices in multilingual mathematics classrooms in South Africa showed that code switching is a linguistic resource from which educators can choose to draw in order to encourage learners to participate more freely in the class and as a means of enabling learners to harness their main (home) language as a learning resource. Setati and Adler (2000) described and analysed two projects in South Africa on language practices in primary multilingual classrooms where both teachers and learners are multilingual and have English as a second language. The particular focus of their study was on the importance of code-switching in fostering mathematical understanding. Their findings revealed that code-switching was a means used by the teacher to encourage learners to talk more freely in the class and therefore a means of enabling learners to use their main language as a learning resource. Setati and Adler (2000) also argued that even though code switching is of pedagogical and political importance, its use in the multilingual class poses a number of challenges which cannot be overlooked. One of these challenges was evident in the work of Halai (2009) where a mathematical task was set in English while the students code switched to Urdu during group work and classroom discussion. Code switching became problematic in this case because of the specifically different ways of showing degrees of comparison in Urdu compared to English. Halai (2009) concluded that both the understanding of the mathematical intention and the understanding of the language of LoLT are necessary in the discourse of mathematics classrooms where the mathematics texts employ everyday phrases in the language of teaching and learning.

In the present study, of interest, therefore, is how code switching as a practice is used (if at all) by the teacher educators and the pre-service teachers and how the use of code switching shapes the dynamics of the classroom communities.

3.8.2 Revoicing

In any classroom and much more so in multilingual classrooms where learners are still learning the LoLT, it is an observed fact that deep thinking and powerful reasoning by learners do not always translate to or correlate with clear verbal expression on the part of the learners (Chapin, O'Connor, & Anderson, 2003). Revoicing as a pedagogical practice involves repeating what the learner or student has said, most often, using the appropriate mathematical language. Revoicing is "a particular kind of re-utterance (oral or written) of one's contribution by another participant in a discussion" (Ohtani, 2003, p. 3). Research into the use of revoicing (see, for example, Chapin, et al., 2003; Moschkovich, 1999) has shown that it can be used as a pedagogic resource to promote students' participation in, especially whole class discussions on a mathematical activity. Revoicing as a practice in the multilingual class can be used 1) when the teacher understands what the learner is trying to say but not sure if the other members of the community of practice understand; 2) when the learner has not used the correct mathematical language due to linguistic limitations; 3) when the teacher wants to capitalise on a learner's contribution to address the whole class so as to promote collective reflection; 4) when the teacher wants learner(s) to revise his/her/their thinking; etc. In general, revoicing can be used by the educator to support full access to participation. The apprenticeship as far as revoicing is concerned is into the discourse of mathematics.

Enyedy, Rubel, Casterllon, Esmonde and Secada (2008) explored the wider range of epistemic functions revoicing can play within a classroom. In addition to the use of revoicing as a tool for juxtaposing student's ideas, they found that the teacher in their study used revoicing more effectively as a tool to "construct a coherent narrative and to position the students as having co-constructed the narrative with him (p. 149). In my study (this chapter and in the coding scheme), I distinguish between revoicing and *reiterating*. I take revoicing to simply refer to instances of repeating what has been said using the correct mathematical language or repeating an utterance in a different way to aid understanding. Reiteration on the other hand would be when the teacher educator or pre-service teachers repeat their understanding of what another member of the community has said either to check their own understanding or to make sure everyone in the community is on the same page. In addition to this, for the purpose of this study, reiteration was also taken as a situation where community members – PSTs and/or TEs repeat what was said/discussed in a previous class (in order to refresh the memory of the community) to aid their current activity or discussion.

3.8.3 Questioning

I define questioning as a pedagogical practice whereby an educator checks for understanding, requests an explanation of students' thinking, requests justification and so on. Questions, whether asked by learners or by the teacher can aid learners in connecting concepts, making inferences, encouraging creative and imaginative thought, aiding critical thinking processes, and generally helping learners explore deeper levels of knowing, thinking, and understanding (Wilson, 1997). The nature of the questions an educator ask in class, and how the educator structures the questions is critical in promoting critical thinking in learners, and in allowing them to sufficiently express their thinking. Short procedural questions by the educator would prompt short procedural answers from learners and would induce what Young (1984, in Edwards & Westgate, 1987, p. 143) terms a tendency for learners to be "obliged to respond within the teacher's frame of reference and at the teacher's bidding". In a multilingual classroom, *why* questions and generally questions that promote relational understanding (Skemp, 1976) can help improve the language proficiency of the learners while at the same time developing their cognitive ability and promoting conceptual understanding. Such questions can readily provoke extended dialogue in the classroom thereby creating opportunity for more meaningful participation by learners.

3.8.4 Extended dialogue[29]

Dooley (2002) argues that the traditional three part exchange of question-response-feedback, typical of most classrooms closes off the opportunity for extended dialogue through which learners may deepen their conceptual knowledge and refine their language. Research (see Dansie, 2001, Dooley, 2001 in Dooley, 2002; Garcia, 1997 in; Khisty & Morales, 2002) has shown that in a bi/multilingual classrooms where extended dialogue is used as a pedagogical strategy (where learners are still learning the language of learning and teaching), communication between the educator and the learners have, over time, changed from one- or two-word response to learners beginning to initiate questions and taking longer turns. Learners in multilingual classrooms who do mathematics in a language which is not their first

[29] By extended dialogue, I make allusion to situations whereby answers to questions (on a particular concept under consideration) give rise to new questions – a situation where questions, answers, feedback (which may consist of more questions) build progressively into coherent and expanding chain of enquiry and understanding on a concept amongst participants in the community of practice. Extended dialogue so conceived is more about relational reasoning than instrumental reasoning.

or home language need opportunities to engage in extended discussions in the classroom that allow them to use the second language (LoLT), to manipulate it, and to hear from others how the discourse in mathematics is used (Krashen, 1982). By so doing, the learners become enculturated into the values, manner of thinking, and the discourse style that define mathematical proficiency (Pimm, 1987).

Practices are closely related to the norms that govern it and to the mutual engagement dimension of community of practice since they influence the patterns of discourse and communicative approach that are used in the classroom. In what follows, I engage with norms of practice in literature and its relevance to the present study.

3.9 Norms of practice

While mathematical practices deal with what mathematical/discursive/pedagogic practices are made available in the community of practice and how this impact on the community, norms of practice are concerned with the patterns and rules of engagement that contribute to the stability of the mathematics discourse and the community of practice. Put differently, mathematical practices, it can be argued, are concerned with the dynamics of the learning process while the norms of practice are concerned with the dynamics of the discourse process. Two constructs pertaining to norms of practice are usually distinguished in mathematical classrooms. These are: social norms and sociomathematical norms (McClain & Cobb, 2001; Voigt, 1995; Yackel & Cobb, 1996)[30].

3.9.1 Social and sociomathematical norms

Voigt (1985, in Yackel, Cobb & Wood) asserts that norms come into being through the expectations that members of a classroom community have of each other and the largely implicit obligations that they have for themselves in specific situations. In a project in which Yackel, Cobb and Wood (1991) investigated the mathematical learning of second-grade learners as they attempted to complete educational learning activities, a variety of social norms emerged. The norms for social cooperation included: that students cooperate to solve

[30] To these two, Sullivan *et al.* (2005) added mathematical community norms. I will engage with this in a later section.

problems, that meaningful activity is valued over correct answers, that persistence on a personally challenging problem is more important than completing a large number of activities, and that partners should reach consensus as they work on the activities. Classroom norms or individual activity included: that students figure out solutions that are meaningful to them, that they explain their solution methods to their partner, and that they try to make sense of their partner's problem-solving attempts. Yackel, Cobb and Wood (1991, p. 398) note that 'individual accountability is defined in terms of operating in accordance with the norms of the classroom community'.

The term *sociomathematical norms* was first used by Voigt (1995, p. 196) to describe "a criteria of values with regard to mathematical activities" that "facilitate the students attempts to direct their activities in an environment providing relative freedom for interpreting and solving mathematical problems". Yackel and Cobb (1996, p. 458) argue that general social classroom norms (as described above) are distinct from sociomathematical norms in the sense that the latter are "specific to the mathematical aspects of students' activity". Perhaps, the most concise distinction between social norms and sociomathematical norms is provided by Yackel (2000, p. 11):

> Social norms [ ] are generally classroom norms that could apply to any subject matter area. They are not unique to mathematics. […]. The distinction between social norms and sociomathematical norms is a subtle one. For example, the understanding *that* students are expected to explain their solutions and ways of thinking is a social norm, whereas the understanding of *what* counts as an acceptable mathematical explanation is a sociomathematical norm *(italics in original)*.

In a study which investigated how social and sociomathematical norms are established during the interactions of pre-service teachers who solve mathematical problems, Tatsis and Koleza (2008, pp. 96-97) identified three social norms (a-c below) and six sociomathematical norms (d-i below):

a) Collaboration norm: the participants are expected to reach a mutual agreement on the solution process and its features, i.e. the concepts and procedures included. This is expressed through the first person plural of the verbs and the questions about the partner's opinion before implementing a method.

b) Justification norm: one has to justify one's opinion, especially when one expresses disagreement with one's partner. This is expressed through words such as 'because' or 'that's why' in a sentence.

c) Avoidance of threat norm: one is expected not to impose a threat towards her partner, i.e. not to insult her/him.

d) Non-ambiguity norm: mathematical expressions are expected to be clear and unambiguous. This is expressed through prompts for rephrasing.

e) Third person comprehension norm: mathematical expressions are expected to be explicit enough so they can be understood by a third person that reads them. This norm is related to the non-ambiguity norm and is expressed through prompts for rephrasing, enhanced with references to the third person.

f) Mathematical justification norm: mathematical methods need some sort of justification before their implementation; there need to be a rationale to support their use. This is mainly expressed through questions beginning with 'why', e.g. 'Why should we use that method?'

g) Mathematical differentiation norm: mathematical areas such as algebra and geometry are distinct, non-overlapping areas; there is also a differentiation between mathematical and everyday practices. This norm is expressed by the students' understandings of what counts as a 'mathematical' and a 'practical' solution to a problem.

h) Validation norm: mathematical methods need to be validated before and/or after they are implemented; this norm is related to the justification norm according to the following scheme: introduction of a method-justification or method-validation of a method. A method may be validated by its difficulty, its time duration or even its result. In some cases the method may be validated by its 'identity', i.e. whether it is a 'pure mathematical' or a 'practical' method (see the differentiation norm above). The validation norm is expressed through queries for information on the above matters, or through relevant quotes, such as: 'Forget about it, it'll take us ages to make that'

i) Relevance norm: the outcome of a method is expected to be relevant to the problem's conditions, In other words, the result has to make sense. This is related to the validation norm, since an irrelevant result may probably lead to the withdrawal of a method.

In providing the organisational language[31] for characterising the norms in my study, the knowledge of these norms, coupled with a careful attention to details in the video recordings and the field notes during the classroom observation enabled me to delineate the norms that

[31] I engage with this in detail in Chapter Five.

were present/being constituted by the TE and the PSTs in each pre-service teacher education classroom.

3.9.2 The case of Mathematical community norms

Sullivan, Zevenbergen and Mousley (2005), in their quest to find out ways in which teachers can improve mathematical learning for students at different stages of readiness, extended the notions of mathematical norms and sociomathematical norms and use the term "*mathematical community norms* to encompass not only 'classroom actions and interactions that are specifically mathematical' (Cobb & McClain, 2001, p. 219) but also norms of practice and other factors that affect learning in mathematics classrooms" (Sullivan, et al., 2005, pp. 249-250). In particular, their conceptualisation includes elements such as routines, social group, language comprehension and usage, and classroom organisation as they relate to the teaching and learning of mathematics, usual practices, organisational routines, and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, and responsibilities of individual learners within the community, all of which helps in community building. The mathematical community norms, however, were not further researched by Sullivan *et al.* (2005) and apart from the paper in the conference proceedings, there are no further publications to elaborate on these norms. Therefore, even though the mathematical community norms would have been of particular significance for the present study which focuses on the community of pre-service mathematics classes in multilingual contexts, I did not explore it further and therefore did not use this conceptualisation.

But communities of practice theory (Wenger, 1998) is not only a theory of learning, of practice and of community, but also a theory of identity in practice (Clarke, 2008). Since my empirical field of research is pre-service teacher education classrooms, I now turn to the role of identity formation in pre-service teacher education.

3.10 The role of identity in pre-service teacher education

Although different authors define identity in different ways drawing on definitions of identity used in the social sciences and philosophy (Beijaard, Meijer, & Verloop, 2004), most research into identity and identity formation in teacher education acknowledge the important

role identity formation plays in learning to be/become a teacher[32]. Within the situative perspective, evidence of a shift in identity occurs when there is a shift in the way members of a community (or an individual in the community) participate with other members of the community in the (mathematical) practices and meaning making processes in which the community is involved (Hodges & Cady, 2012).

Teacher identity, as many authors have argued, is complex. Dinkelman (2011) for example, argues that teacher identities "are multiple, fluid, always developing, shaped by a broad range of sociocultural power relationships, strongly influenced by [a] number of relevant contexts". A review of research into identity with regards to teacher and teacher education conducted by Akkerman and Meijer (2011) reveals three common threads within the different characterisations of teacher identity: the multiplicity of identity, the discontinuity of identity and the social nature of identity. They note that all three characterisations stress that identity "is not a fixed and stable entity, but rather shifts with time and context. This line of reasoning resonates with Wenger's conception of identity as a constant becoming. In Chapter Two, I engaged with Wenger's identity in practice. In Chapter Five, I will deal with how this identity in practice translates to pre-service teacher education settings and in so doing, develop an approach for characterising identity in teacher education.

According to Wenger, within a community, different members play different role towards the realisation of their common goal (joint enterprise). In what follows, I engage with the role of the teacher educator within classroom communities. By highlighting the teacher's role in the creation of communities of practice mathematics classrooms, I by no means wish to disregard or undermine the contributions of pre-service teachers in the constitution of a community of practice. This is so because the role of the teacher educator highlights the non-homogeneity of classrooms communities in teacher education – an important feature of Wenger's concept of the CoP – whereby one person's role is the determinant of whether or not there is a classroom environment conducive for effective mutual engagement in the joint enterprise.

[32] For example, in a longitudinal study exploring the relationship between novice teachers' identities and their learning during their pre-service training, Horn, Nolen, Ward and Campbell (2008) observed that the pre-service teachers' identities as learners and as future teachers influenced their engagement in coursework and their orientation towards learning.

3.11 Teacher Educator's role in the creation of Community of Practice Mathematics Classrooms

Concerns on improving the quality of teacher education have followed quite an interesting trajectory over the past centuries. It can be argued that the paradigm shifts in what constitutes quality mathematics teaching have aligned themselves very intricately to the shift in the perception of the nature of mathematics, what constitutes mathematical knowledge and what it means to engage with mathematical activity (Wood, 1996). Over the years, from the notion of mathematics as "a set of rules and formalisms invented by experts, in which everyone else is to memorise and use to obtain unique correct answers" (Romberg, 1992, p. 453), most psychologists and educationists now conceptualise mathematics as involving the understanding of mathematical concepts as well as the ability to communicate mathematically. Moschkovich (2002), for example, holds that in addition to algorithmic competence, solving word problems and using mathematical reasoning, communication in the mathematics class is also important in the teaching and learning of mathematics. Hence, the argument advanced by both research (Adler, 2001; Moschkovich, 1999, 2002; Pimm, 1987, 1991; Setati, 2005a; Sfard, Nesher, Streefland, Cobb, & Mason, 1998) and curriculum (DoE, 1997, 2002) is that learning to communicate mathematically is one of the central aspects of what it means to learn school mathematics. From this new understanding arise not only questions regarding mathematics knowledge for teaching, but, fundamentally, the nature of classroom discourse and how the teacher positions himself/herself with regards to the teaching and learning process. Students (pre-service teachers) are more likely to see themselves as co-participants in a joint enterprise if the teacher espouses teaching strategies that are not suggestive of mathematics as only an already predetermined body of knowledge and a set of procedures which students need to master. Goos *et al.* (1999) argue that in most 'contemporary classrooms', teachers perceive their job in class to be that of "presenting the subject matter in small, easily manageable pieces and demonstrating the correct procedure or algorithm, after which students work individually on practice exercises". As Povey and Burton (1999, p. 233) and Goos *et al* (1999, p. 37) argue, in such a mathematics classroom this is predicated on an "epistemology of *external authority*" *(italics in original),* students are silenced and "dependent on authoritative others for validation of their knowing". Povey and Burton (1999, p. 37) continue thus:

> "We believe that such a view ignores what is known about the process of coming to know, which, far from being one of cultural transmission, is necessarily one of interpretation and

meaning negotiation in the context of current personal 'knowing' as well as knowledge situated in the community"

What Povey and Burton call external authority has to do with participants in the practice and not the elements of the practice like the discourse itself which in the context of mathematics must relegate some authority. Hence, in mathematics, the voice of mathematics is the external authority, and the teacher has more access to this authority[33]. But how the teacher dispenses of this authority in class can vary in the teacher's *communicative approach* used or *patterns of discourse* (Mortimer & Scott, 2003) that is privileged in the class. The teacher, thus, plays a pivotal role in creating such a learning environment where generally, learners explore deeper levels of knowing, thinking, and understanding (Wilson, 1997) that would lead to the communual construction of mathematical knowledge. In any classroom, the teacher plays a key role in the management of the communication in the classroom (Edwards & Westgate, 1987) and in communual negotiation of meaning in the community of practice, in creating a context where concepts are connected, inferences are made, where creative and imaginative thought are encouraged and critical thinking processes are promoted. As noted by Olson, Knott, & Currie (2009, p. 44), "Social interactions, class structure, and curriculum combine to create opportunities for the unique discourse that may develop in a mathematics classroom. It is however, the type of discursive practices of the teacher [I would say communicative approach and patterns of discourse] that dictate the culture of the classroom through implicit and explicit ways". In the present study, the types of communicative approaches and discourse patterns used in each classroom were indicative of this culture.

3.12 Conclusion

Chapters Two and Three have dealt with the theoretical field (Dowling & Brown, 2010) in which this study is embedded. The figure 3.2 below encapsulates this field:

[33] Of course the teacher educators in pre-service teacher education classrooms also assess their PSTs raising issues about power relations in such classrooms. I contend that this role of assessor does not make the pre-service teacher education classroom less of a community because for Wenger, members of a community need not have the same roles and the same power. Moreover, for Wenger (1998, p. 189), "power is not construed exclusively in terms of conflict or domination, but primarily as the ability to act in line with the enterprises we pursue". Assessment of PSTs and how this assessment is done are part and parcel of how the enterprise of a teacher education community may be carried forth.

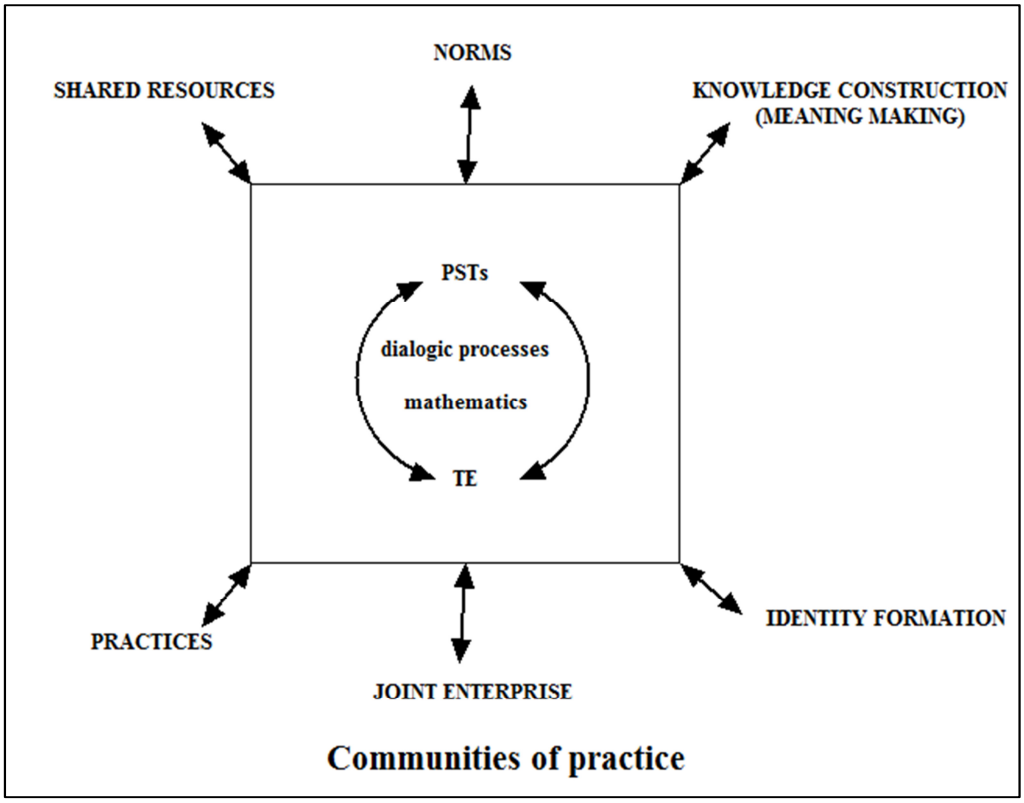


Figure 3.2: Summary of the theoretical field described in Chapters Two and Three

As depicted in the diagram above, communities of practice in this study consist of teacher educators and pre-service teachers who are involved in dialogic processes around mathematics. The result of these processes are practices, identity formation, knowledge construction, norms, shared language which in turn also shape the communities' discourse on the mathematics that is taught and learned.

Research study in mathematics classrooms has pointed to the intricate relationship between mathematics and language and research in multilingual mathematics classrooms points to the complexity of teaching mathematics in multilingual classrooms. To deal with such complexity, various researchers and scholars have alluded to different practices that can help mediate knowledge in the multilingual classroom. In this study, in Chapters 6-9, these practices serve as the point of departure for the analysis of the empirical data collected from pre-service teacher education multilingual classrooms. The next chapter deals with the empirical context of my research.

CHAPTER FOUR
Research design and methodology

4.1 Introduction

As indicated in Chapter One, the aim of this study was to explore the nature of pre-service multilingual teacher education classrooms. This chapter describes the research design and method of data collection used in achieving this aim. In doing this, I rely heavily on the research methodology of Dowling and Brown (2010) and their notion of empirical and theoretical fields. The population, sample, the research instruments, validity and reliability are discussed. I provide grounds for not interviewing all the teacher educators in each institution (university) in which I conducted my research (I call these institutions my 'research institutions') and also include a discussion on, and grounds for the selection of the four teacher educators who took part in Phase two of the study. The chapter concludes with a discussion around the PhD research process and some methodological and ethical issues which emerged from data collection and analysis of data.

Wenger (1998) argues that not all social configurations can be called communities of practice. In that light, before dealing with the above methodological issues, I first provide reasons why the pre-service teacher education classrooms in my study can be regarded as communities of practice in the Wengerian sense.

4.2 The Pre-service mathematics Classroom Communities as Communities of Practice

Up till now, I have worked with an important assumption – that the teacher education classrooms in my study can be considered as communities of practice in the Wengerian sense of the term. The question can be asked: given the characterisation or indicators of a community of practice discussed in Chapter Three, can (mathematics) teacher education classrooms be regarded as communities of practice? If yes, what is the community and what is/are the practice(s) that undergird(s) the community? Is it mathematics or the learning to teach mathematics (from the perspective of the pre-service teachers) or the teaching of mathematics or the pedagogic practice of teaching mathematics in multilingual contexts

(from the perspective of the teacher educators)? Do both teacher educators and pre-service teachers see themselves as engaging in the same practice(s)? What is the joint enterprise? Is it the same for both teacher educators and pre-service teachers in the class? In all these questions, what is at stake is what Wenger's notion of CoP means in a pedagogic context like a classroom or in the case of the present study, how it translates to a classroom setting of pre-service teacher education.

Even though researchers and educationists are in agreement as to the important contribution of community of practice theory in the development of a valuable framework to understand situated learning, many are in disagreement as to whether or not formal education settings can be considered as communities of practice in the Wengerian sense. At one end of the spectrum, Boylan (2005, p. 1), for example, contends that even though the community of practice theory offers important insights into the nature of classroom interaction and learning, school mathematics classrooms generally are not usually communities of practice. He further argues that school mathematics classrooms, and by extension school classrooms, are at best understood as "ecologies of practice within which there is a possibility of developing some of the features found in communities of practice". At the other end, McClain & Cobb (2004), Goos, Galbraith & Renshaw (1999), Graven and Lerman (2003), and Graven (2004) have all argued that the construct of 'community of practice' can be used to characterise the teaching and learning process that occurs in most mathematics classrooms. Graven (2003, 2004) for example, whose research focused on in-service teacher education contends that Wenger's framework is useful in understanding in-service teacher education as a social practice. She however argues that CoP theory in the Wengerian sense, does not deal comprehensively with all primary aspects of learning, and introduces confidence as an overarching fifth component. Like meaning, practice, identity and community, confidence according to Graven, requires discussion and analysis in its own right; it "has its specific features" and should not therefore "be subsumed within the other components" (p. 36). Graven (2004) also engaged with the issue of teaching in CoP theory which I have dealt with in Chapter Two.

Boylan's (2005) argument is grounded in school (mathematics) classroom practices and the nature of classroom practices in such settings and does make sense within school mathematics classrooms. It is my contention that school classrooms are, however, different from pre-service classrooms in a number of ways and hence, most of Boylan's argument against the understanding of mathematics classrooms as communities of practice would not be applicable to the present study which deals rather with pre-service mathematics teacher

education classroom. For a start, in schools, the teacher is not necessarily training students/learners who would take up the teaching of mathematics as a profession. This has implications for identity formation in schools mathematics classrooms as opposed to pre-service mathematics classrooms. Secondly, Boylan argues that in a school mathematics "community", students 'experience isolation and separation from each other rather than a sense of shared or collaborative engagement' (p. 7). This was not the case in the teacher education classes in the present study because of group work and the group dynamics (working with students who are in the same language or linguistically intelligible group) that permeated most of the pre-service classrooms in this study.

With specific regard to the present study, I will begin to unravel the questions posed in the earlier part of this section through the understanding of Wenger's (1998) notion of community of practice as what emerges when there is a sustained mutual engagement around a joint (appropriated) enterprise and as a consequence of such engagement, a shared repertoire of practices, understandings, routines, actions, and artifacts are developed or appropriated. Given this understanding of community of practice, the question that needs to be asked is whether there are aspects of classroom situations that militate against the appropriation of the enterprise by a group in sustained mutual engagement. In this regard, Eraut (2002) and Cox (2005, p. 533) suggest conditions that may limit the appropriation of the enterprise which I have recontextualised below for the classroom environment:

1. Frequent reorganisation, so that engagement between individuals is not sustained
2. Employment of temporary or part-time staff, so that people come and go.
3. A context or situation where there is relationships build up and the individual does not commit to the task (example, when consumption/leisure activities, and not work, is seen as the primary form of identity creation)
4. A situation in which due to tight management, external forces in the school system attempts to define how to do work, therefore limiting the scope for the task to become appropriated and defined locally (by classroom activities in the case of a school).
5. Individualised work, so that there is no collective engagement, only relations between an individual (learner/student) and their teacher/lecturer.
6. Very competitive environments, inhibiting collaboration
7. Time-pressured environment so that there is no time for the development of collective understanding

8. Heavily mediated activities, e.g. by computers, so that interaction is to a large extent less immediate and intense.

Given Wenger's notion of locality, it is not my contention that *any* pre-service mathematics classroom can be regarded as a community of practice. I would, rather, argue that the teacher education classrooms that I observed while collecting data for this study, were structured so that the above conditions are either non-existent or very minimal therefore fulfilling the requirement of communities of practice.

In Chapter Two, I had argued that the choice of Wenger's (1998) notion of community of practice was because Wenger conceptualises communities of practice as a non-homogeneous social configuration where different members play different roles, have varying levels of knowledge, confidence and commitment, and fundamentally, where every member is in a learning position as far as the dynamics of the community is concerned. An important step towards the understanding of teacher education classrooms as a community of practice is to conceptualise teacher education as a process of learning both for the teacher educators and for the pre-service teachers. In this regard, even though the teacher educators involved in this study were experienced teachers, during my interviews with them, they all admitted to being in a learning position when it came to teaching mathematics (in multilingual classrooms). In the extract below where I asked the teacher educators what others can learn from them regarding teaching in the teacher educators context of practice, I use three different teacher educators' responses across the different universities to illustrate the fact that the TEs see themselves as learners within their practice[34]:

R	Given your own experience as a teacher educator in a multilingual context, given your own practice, you have extensive knowledge about preparing pre-service teachers. When it comes to teaching in a context like yours – a multilingual context, what can we learn from you about what it means to teach in multilingual classrooms?
TE1	Elm, teaching is a very difficult task. I always tell my students that…elm…, even myself, up till today, I am still learning how to teach. This is because you are dealing with human beings who come from different background, different contexts and so on. And you are trying to bring them together to understand a particular concept which is not easy.

[34] Even though it is normally not conventional to draw on data in the methodology chapter, I feel it is expedient to do so in my case because it is fundamental that I provide justification as why the classroom communities where my study was conducted can indeed be regarded as communities of practice.

TE2	First, I can say that it is a challenge. Because sometimes, these students what they do…as I hinted in the beginning, you find out that they have a problem in phrasing questions, if they are to ask questions in English. Sometimes they struggle. So, what they do, sometimes they ask me a question in their language. It happens from time to time. The question is a challenge to me if I don't understand the language. Sometimes, it creates communication problems. They have got a problem, but they don't know how to phrase it such that as a lecturer, I understand the question. So, I do experience those kinds of situation from time to time.
TE3	This is difficult because I am still learning. I mean, I am honestly still learning. I have got other lecturers in the classroom who co-teach with me, and they point to me sometimes and say 'the students don't get it because of this or …they whisper to me 'draw a picture or show on the board what you are saying'. I sometimes forget who my audience is. So, I'm still learning to be sensitive to who my audience is.

Table 4.1: TEs as learners in their own practice

The utterances by the three teacher educators suggest that the attribute 'lifelong-learners' who reflect on their practice and change it where appropriate, does not only apply to pre-service teachers and teachers, but also to teacher educators. Hence, the contention of Brown and Borko (1992, in Leikin, Berman, & Zaslavsky, 2000, p. 18) that, "Mathematics teachers often deepen their understanding of a mathematical content as they teach it, and they come to understand both content and possible teaching methods better through this process", is applicable not only to teachers, but to teacher educators as well. As Smith (2006, p. 621) asserts, the teaching of mathematics is "a continual and dynamic process of reconciling who we are as teachers, what we think we know about teaching, and discerning what we think we know to become effective teachers". This view of mathematics teacher educators' growth as learning-through-teaching process (Llinares & Krainer, 2006) suggests that a pre-service classroom situation can be conceptualised as consisting of a community of *learners* of different competences, skills and knowledge, but who have a common goal (joint enterprise) – the becoming of a (better) mathematics teacher in a multilingual context. Such a community is a complex one as it involves subject-specific (mathematics) knowledge, the teaching and learning of mathematics, the language used in providing epistemological access to the mathematical content, and the context in which teaching and learning occurs.

Similarities and differences between each of the two teacher education institutions (involved in this study), and between individual classrooms within an institution imply that each class can be considered as a discrete community of practice. What this means for the present research is that each of the teacher education classrooms is considered as a community of practice. There were four classroom communities of practice – two communities of practice in one university (TEIA) and two from a second university (TEIB). While all these communities of practice have for their overall goal the learning of teaching and learning of

mathematics, the mechanisms for achieving this goal – the practices that each community engage in for achieving this goal - varies from community to community. It is my contention that these practices are shaped by the immediate educational contexts in which the communities of practice (at the universities) are situated. Such contexts include the context of teacher education (the ideas and values of each pre-service teacher education programme in the different universities (see Collanus, et al., 2012)); the context of the individual universities involved in the study; the language infrastructure of the class; and who the teacher educator is (that is, whether the TE is monolingual, bilingual/multilingual and whether or not she shares a common home language with the pre-service teachers[35]).

4.3 Research design

In order to address the guiding questions which this research sought to explore, a qualitative approach was adopted. Qualitative research design is a "systematic approach to understanding qualities, or the essential nature, of a phenomenon within a particular research" (Brantlinger, Jimenez, Klingner, Pugach, & Richardson, 2005, p. 195). Qualitative research provides evidence on a particular phenomenon based on the exploration of specific contexts and particular individuals (Brantlinger, et al., 2005).

I have chosen a qualitative approach mainly for the fact that a qualitative research design involves an in-depth study to understand a phenomenon which makes it a suitable choice in investigating multilingual pre-service mathematics teacher education classrooms.

As indicated in Chapter Two, Wenger's (1998) social theory of learning, and in particular the notion of learning in *communities of practice* was used to frame the study theoretically, conceptually and analytically (methodologically). The methodological aspect of the study would be further understood in the context of Dowling's (2009) notion of 'methologising' and 'organising' of text. I will return to this methodological aspect later in this chapter. But suffice it at this stage to indicate that for Dowling and Brown (2010), what is key in qualitative research is the bringing together of the theoretical and empirical fields – in the

[35] For example, as Staats (2009, p. 32) argues: "A teacher who does not share a home language with her students cannot, for example, use the informality of the home language to develop rapport with students. While the teacher may encourage students to discuss task scenarios in their home language, she cannot codeswitch to emphasize transitions between different pedagogical purposes such as explanation and modeling formal mathematical speech".

dialogue between these two fields through the use of an organisational language[36] that gives meaning to the data[37]. In fact, for Dowling and Brown (2010, p. 108), analysis is judged on four criteria: 1) the internal explicitness and coherence of the theory; 2) the relational completeness of the theory; 3) the integrity of the concept-indicator links; and 4) the direct relationship of the organisational language to the system about which claims are to be made.

What holds these four criteria together is the willingness to look both at the theoretical field and the empirical field. In what follows, I discuss my empirical field.

4.4 Population and research contexts

The population of the study was pre-service multilingual teacher education classrooms in a Province in South Africa, totalling four universities. The decision to focus on this particular Province was motivated by the following reasons:

- *first*, because of its accessibility;
- *second* and more importantly, because of the language infrastructure of this Province. The Province is cosmopolitan and multilingual unlike some of the Provinces in South Africa and thus the challenges of teaching in multilingual classrooms are much more pronounced than in other areas in the country where there is an inclination towards a common African language;
- *third*, the Province is endowed with institutions with well-established pre-service training programs for mathematics teachers.

One obvious limitation of focusing on this province is that teacher education classrooms where there is a pervasive dominant home language between pre-service teacher educators were not included in my study.

4.5 Sample and data collection

Identifying the relevant individuals, institutions, etc (in short the sample) for the study was crucial. Four universities (out of a total of six universities in the Province) were chosen in total in the Province of South Africa selected to conduct my study. The four universities have

[36] I define and engage with this term in section 4.7 of this chapter.

[37] Dowling and Brown (2010) refrain from epistemological classifications of qualitative research as interactionist, critical, post-structuralist, etc. For them, what is important is the articulation of the design of the empirical component with the theoretical framework in which the research occurs.

well established pre-service teacher education programmes. Private universities and universities that do not offer pre-service mathematics teacher education programmes were not included in the sample. Of the four, three are located in urban settings, with diverse students who include both pre-service teachers and teacher educators whose main language is the LoLT and multilingual pre-service teachers and teacher educators. The fourth university is located in a non-urban setting and all the TEs and PSTs were multilingual from previously disadvantaged backgrounds. Three of the universities had a system of pre-service training in which both mathematics content and pedagogy were taught in the education faculty and by faculty staff. In one of the universities, mathematics content was separated from the pedagogy with the former being taught by the Science Faculty and the latter by the Education Faculty. So, the PSTs at this university do mathematics content with those at the university who do mathematics majors (such as Engineering students, etc).

The present study used a two-phased approach – pre-observation interview phase and classroom observation phase – to collect data in order to provide answers to the guiding questions. In what follows, I describe each phase, indicating the data collection procedures, the sample, and providing justifications for my choice of sample.

4.5.1 PHASE 1: Pre-Observation interviews

The first phase of the study was pre-observation interviews of teacher educators at the four universities which were selected for the study. Interviews "facilitate the personal engagement of the researcher in the collection of data" (Brown & Dowling, 1998). They also, according to Denzin & Lincoln (2000, p. 633), "produce […] situated understandings grounded in specific interactional episodes". The aim of the interview was two-fold: *first*, it served as a window into what the teacher educators *know about*/how they talk about the complexities of teaching mathematics in multilingual classrooms and what they understand to be doing to prepare their students to teach in such contexts; *second*, it served as a tool for selecting teacher educators for a more in-depth study in the second phase of my study.

The interview was semi-structured (see appendix A for interview questions) and focused, amongst others, on:

- Teacher educators' awareness of the complexities of teaching in multilingual classrooms,

- What they do in their pre-service programmes (courses/modules/their pedagogic practices, etc) to prepare pre-service teachers to deal with the complexities of teaching in multilingual mathematics classrooms.
- Why they do what they do in the way that they do.
- What teacher educators consider to be the best practice for preparing pre-service teachers for teaching mathematics in multilingual classrooms.
- What they would do differently if they were teaching monolingual students with whom they share a common language.

The interview questions were first piloted with three TEs in a different province and then refined. As a new researcher, the piloting process not only helped me test the adequacy of the interview research instruments, but was also key to my developing the art of interviewing in a semi-structured situation. With the help of my supervisors, I was able to see why the participants answered certain questions the way they did (and hence reformulated the questions so that they were clearer), and what other questions I would have asked which I did not ask in the pilot phase.

4.5.1.1 Selection of sample in Phase 1

In Chapter One, I had argued that South Africa presents a complex and interesting context of multilingualism like many other countries but at the same time has its own distinctive features compared to most other countries in Africa and the rest of the world. On the basis of the distinctive multilingual nature of South Africa as described in Chapter One, therefore, all the teacher educators who were newly employed in the universities where the study was conducted, but who had some experience of teacher education from other countries were systematically excluded from the study. Teacher educators who were newly employed and had no previous experience of teacher education were also excluded from the study since the interviews took place at the beginning of the academic year. Those who were newly employed in the universities, but had experience of teacher education in other teacher education higher institutions in South Africa were, however, considered for interviews in this phase. Furthermore, teacher educators teaching either of (or both of) mathematics methods and mathematics content courses were included in the study. Those who declined to participate in the study and those who belonged to the same research group as me (and were thus too familiar with my study) were also excluded from the interview process. The table

below presents a list of the teacher educators in the institutions where the research was conducted and shows how many were interviewed:

Institution	Total no. of T.E[38]	No. of T.E interviewed
A	4	2
B	9	5
C	2	2
D	3	3
TOTAL	**18**	**12**

Table 4.2: Number of teacher educators in the institutions of research

12 teacher educators in total were interviewed. For four of these 12 TEs, the language of teaching and learning is not their first language while 5 of the TEs have Afrikaans as a first/home language. One of the TEs has English as the first language and the rest have a language other than one of the 11 official languages of South Africa as a first language. In Chapter Six, I will show that the language(s) in which a particular teacher educator is fluent has a direct bearing on what linguistic challenges the teacher educator experiences in class and what classroom practices the TE uses to deal with these challenges. In short, I will argue that the language background of the teacher educator plays an important role in the dynamics of teaching and learning in multilingual classrooms.

4.5.2 PHASE 2: Classroom observation

The second phase of the study involved classroom observation of pre-service teacher education classroom communities. These observations took place from midway through the academic year (2008) until early November. Dowling and Brown (2010, p. 49) argue that through direct experience with the empirical field, the researcher collects information about the setting that enables him/her understand the setting and that "the development of this understanding involves the immersion of the researcher in the practices in the empirical setting". Observation, therefore, was an important method of data collection since the present study set out to understand and describe teacher education settings.

[38] These are the teacher educators involved with pre-service training in mathematics and by no means the total number of teacher educators in the institutions.

4.5.2.1 Selecting the sample in Phase 2

The teacher educators who during the interviews indicated that they paid some attention to providing pre-service teachers with not only the mathematical and pedagogical knowledge, but also knowledge/skills/practices for teaching in multilingual mathematics classrooms, were selected. Four teacher educators teaching in four different classrooms were selected. Two of the teachers were from University A (TEIA) and the other two were from University B (TEIB)[39]. In addition to the reasons elaborated below for the choice of the four teacher educators, the two universities were chosen for the second phase of this study because they present contrasting contexts of pre-service teacher education. TEIA is frequented by PSTs and TEs for whom English (LoLT) is an additional language. TEIB is frequented by PSTs of different linguistic backgrounds, taught by a good number of teacher educators whose first language is the language of teaching and learning.

4.5.2.2 Motivation for the choice of the four teacher educators

The specific criteria which were used in the selection in addition to the general criterion stated above were:

- Willingness to take part in Phase 2 of the data collection.
- Teaching in multilingual classrooms: All classes were multilingual as attested to by all the teacher educators.
- The teacher educators' articulation of their awareness of the complexities of teaching mathematics in multilingual classrooms and also what they understand to be doing to prepare their students to teach in such contexts. This awareness involve some or all of the following as discernible from the teacher educators response to the interview questions:
    - Awareness of the difficulty involve in teaching in such multilingual contexts – that first and foremost, the TE is teaching students who are still learning the LoLT (English).
    - Awareness of the students' language needs and attending to those needs through the practices the TE uses in teaching.

[39] For ethical reasons which I would explain later in this chapter, I do not expound on the empirical context of these two universities beyond their linguistic demographics.

- Awareness of the linguistic distance between African languages and English and awareness of "language interference"
- Impressing the above on students who would most likely be teaching in a similar multilingual contexts.
- The teacher educators selected indicated during the interview that they would have taught differently if they were teaching a monolingual class, therefore, acknowledging the distinctive demands involved in teaching multilingual classrooms.

For TEIA, one teacher educator (TEIA-S) taught the second year mathematics while the second teacher educator (TEIA-M) taught the third and fourth years. Both are (multilingual) South Africans whose main language is not English and who said they were aware of the language needs of the students and tried to attend to these needs in the class. The PSTs were from all over South Africa and with multilingual backgrounds too. The English language proficiency of the PSTs was between low and average as perceived by the teacher educators.

The first teacher educator for TEIB (TEIB-E) has Afrikaans as a home/main language, hence bilingual; the second (TEIB-L) has English as the home language. The PSTs in TEIB were from diverse linguistic backgrounds and able to speak two or more languages. The teacher educators perceived the overall language proficiency to be average.

Both TEIB teacher educators and TEIA teacher educators present a number of interesting similarities and differences in terms of: 1) their familiarity with and understanding of language issues in the teaching and learning of mathematics; 2) their PSTs' English language proficiency level; 3) and their privileged practices used for teaching multilingual PSTs whose first language is not the language of teaching and learning. At this stage, I will expound on some of these similarities and differences as far as the interviews were concerned because these (contextual similarities and differences) were instrumental in understanding the nature of communities of practice in each teacher education classroom.

4.5.3 Similarities and differences between teacher educators in TEIB and TEIA

All four Teacher Educators in TEIB and TEIA teach mathematics content (as distinct from methodology) to pre-service teachers for whom English is a second language, and whom they (teacher educators) believe are not fluent in the language of learning and teaching (English).

Teacher educators at both institutions are of the opinion that, given their experiences, pre-service teachers need to specifically be trained to teach mathematics in multilingual classrooms of learners who learn mathematics in a language other than their first or home language. This is in sharp contrast to the teacher educators at one of the other four institutions who either saw no need for a specific training, or just thought the training was unnecessary and that adjusting to the needs of second language students should be a spontaneous/natural thing. Furthermore, TEs at the two selected institutions indicated that attention should be paid to the fact that the pre-service teachers would, at the end of their qualification, be most likely teaching in multilingual mathematics classrooms. Like the other teacher educators who were interviewed, none of them said they code switched in the class, but they said that they (TEs) encourage the use of code switching during PSTs teaching practice. The four teacher educators also indicated that they encourage the use of different languages during group discussions when the students are working on mathematical tasks or problems.

It must be noted that the present study is by no means a comparative analysis of the different CoPs involved in the study. Neither was it aimed at dichotomising the different research institutions nor in particular the two research institutions involved in this study. Communities of Practice can be viewed in different ways, and this study takes a particular look at teacher education CoPs in order to be able to provide rich descriptions of the CoPs and make claims about its relation/in relation to teacher preparation and particularly the preparation of pre-service teachers for teaching mathematics in multilingual classrooms.

4.5.4 Data collection methods

Data in Phase two was collected through classroom observations. The teacher educators were video-recorded teaching a mathematics concept from start to finish. The focus of the data was on the practices of the teacher education classroom communities. Both TEs and PSTs' voices were part of the classroom observation because both teaching and learning – in short – the communities of practice were in focus in the study. In total, five lessons of three hours each were observed for each classroom in University A and nine lessons of two hours each were observed for each classroom in University B.

It must be noted that the aim of the study was not to explore the extent to which PSTs have acquired the practices that are made available in the teacher education communities of practice, but to explore the shared repertoire, the nature of mutuality of engagement and the

joint enterprise of the community for which exploring whether there was an opportunity made available in the classroom community of practice for the communual construction of meaning/knowledge in mathematics given their context was crucial.

4.6 Data collection challenges

A major constraint in data collection was the cancellation of classes at University TEIA. University TEIA is a 2-hour drive in heavy morning traffic. On four occasions, pre-arranged observation lessons were cancelled when I had already arrived at the university due to strike actions by either the PSTs or the TEs or due to the fact that a good number of the PSTs were on excursion. In one instance, an observation lesson was cancelled because it was Summer Day (September 1) and students engaged in water pouring rituals rather than attend lectures. Another challenge I experienced in data collection was that after interviewing a particular teacher educator and making arrangements for video recordings with her, she was moved to a science class and replaced by a teacher who did not meet the criteria for inclusion in my study.

These data collection challenges did not, however, adversely affect the quality of the data collected. Despite the cancellations, I managed to video record TEIA teacher educators' lessons sequentially without skipping any lessons in-between.

4.7 Data analysis

Yin (2003, p. 109) holds that data analysis "consists of examining, categorizing, tabulating, testing, or otherwise recombining both quantitative and qualitative evidence to address the initial propositions of a study". It is the 'systematic search for meaning' – a way of processing data by means of organising and interrogating data in such a way that "allow researchers to see patterns, identify themes, discover relationships, develop explanations, make interpretations, mount critiques, or generate theories (Hatch, 2002, p. 148).

Brown and Dowling (1998) assert that the bringing to bear of the theoretical framework on the empirical setting enables both theoretical and empirical claims to be made. In describing educational research, they (Brown and Dowling) make a distinction between the theoretical field and the empirical field. The theoretical field is the "broad area of academic and/or professional knowledge, research and debates which contains [the] general area of interest"

and the empirical field is the "general area of practice or activity or experience about which you intend to make claims" (Brown & Dowling, 1998, p. 18). Brown and Dowling (1998) conceive of data analysis as a dialogic process involving the movement between the empirical and theoretical fields. In between the two fields in this dialogic relationship is the language of description which serves as tools for the analysis of the empirical data (Dowling, 1998). In his later work, Dowling (2009), replaces the term 'language of description' with 'organisational language' which according to him is more attuned to sociology as method than the former term. The organisational language is usually generated through dialogue between the considerations of theoretical issues and the empirical context and provides a way of transacting between the theoretical field and the empirical field. As Dowling and Brown (2010, p. 86) assert, "the [organisational] language consist of the categories that in general, have been developed during the process of analysis".

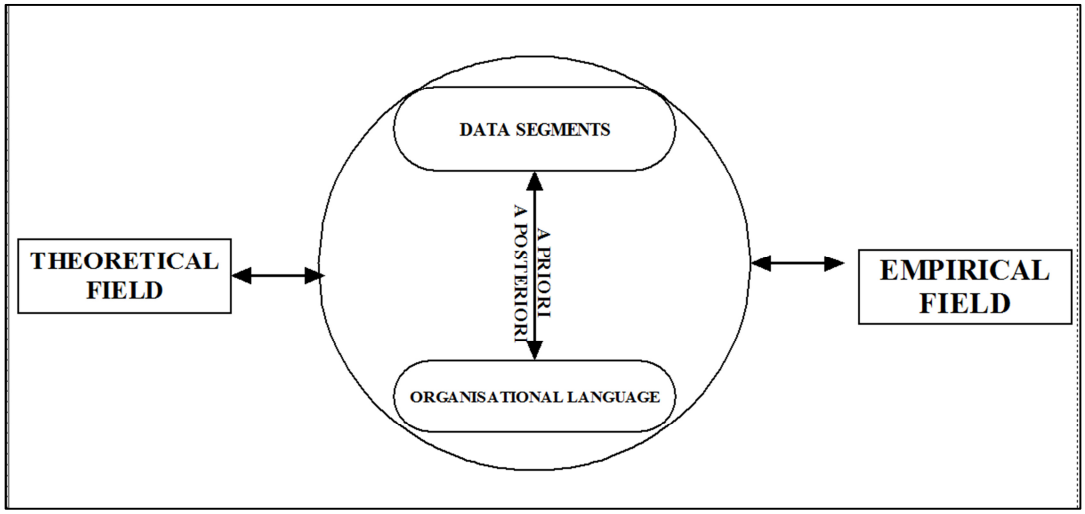


FIGURE 4.1: Analytic strategy used in the study

In my study, the theoretical field is Wenger's Communities of Practice theory and the literature that deals with research and theories of teaching and learning in multilingual settings and in mathematics education. My empirical fields were the pre-service multilingual teacher education classrooms. In Chapter Five which I have entitled *The emergence of an organisational language*, I provide a more detailed description of my organisational language.

The classroom observations (just like the pre-observation interviews) were transcribed entirely and coded. The transcripts were coded and analysed according to segments. A

segment covered the whole section of the data where one mathematical idea or concept was discussed and/or tackled in the class. The table below shows the total number of segments per teacher education classroom community of practice.

Classroom CoP	TOTAL SEGMENTS ANALYSED
TEIA-M	11
TEIA-S	11
TEIB-E	10
TEIB-L	14

Table 4.3: Total segments analysed

It must be noted that for various reasons (some of which have been narrated above in the data collection challenges section), the researcher did not spend an equal length of time in all the TE classroom communities. But one of the advantages of dividing the transcript into segments was that it afforded the researcher the opportunity to describe the kinds of privileged practices in each community of practice, and by so doing, to delineate some patterns within each community.

I start my analysis chapters (Chapters 6-9) by dealing with the analysis of the pre-observation interviews. In Chapter Nine, I pull together both interview analysis and observation analysis in an integrated way. The choice of an integrated analysis was necessary in order to, at once, understand/analyse the classroom activities in the light of the teacher educators' understanding of their contexts because, this understanding had an influence with regards to what was foregrounded or backgrounded in the TEs' respective teacher education classrooms.

4.8 Unit of analysis

Dowling and Brown (2010, p. 28) define *unit of analysis* as "the object that is to be described in terms of the research variables". According to Wenger (1998, p. i),

> the primary unit of analysis [for communities of practice] is neither the individual nor the social institution but rather the informal 'communities of practice' that people form as they pursue shared enterprises over time

Hence, the unit of analysis in this study is neither the teacher educators who are involved in the study nor the pre-service mathematics teacher, but rather the communities of practice – the practices, the norms, the pool of shared language, the mutuality of engagement (all of which contribute in defining their joint enterprise) of the communities of practice involved in the study. But in analysing the segments in the data for the communities' shared repertoire (SR), mutual engagement (ME) and joint enterprise (JE), the utterances which were sub-divided into segments, formed the basic unit of analysis because it is from the utterances that stem the community's SR, ME and subsequently, the community's JE. In order words, since a community, according to Wenger, exists because of participation in certain practices and since such participation are visible through classroom discourse, the utterances of the community (divided into segments) provide a starting point for analysis of the communities' SR, ME and JE.

4.9 Validity and reliability

Validity is the "accuracy and trustworthiness of instruments, data, and findings in research" (Bernard, 2000, p. 47) and deals with, as Dowling and Brown (2010, p. 24) argue, the relationship between the 'theoretical concept variables' and the 'empirical indicator variables'. According to Maxwell (1992, p. 283), validity "pertains to [the] relationship between an account and something outside of that account, whether this something is construed objective reality, the constructions of actors, or a variety of other possible interpretations". Maxwell (1992) makes a distinction between descriptive validity, interpretive validity, theoretical validity, generalizability and evaluative validity.

Descriptive validity is concerned with the factual accuracy of the account (observation, interviews, etc.) made by the researcher (Maxwell, 1992). It is "the extent to which the researcher describes what, in fact, the study set out to do and describe and whether this description was accurate and authentic" (Hitchcock & Hughes, 1995, in Griffiths, 1998). Descriptive validity also concerns issues related to omissions as well as commission, and also concerns accurate accounts of statistical and quasi-statistical descriptions of events or phenomenon (Huberman & Miles, 2002; Maxwell, 1992). In my study, descriptive validity was ensured by video-recording the classroom observations and audio-recording the interviews with the teacher educators. This ensured that classroom interaction was captured more accurately than with the use of only field notes. It also enabled a more accurate

transcription of the data. Furthermore, during coding, accuracy of the transcription, especially as it pertains to non-semiotic events, was ensured by using both video and transcript (hence, mapping the video to the transcript).

Interpretive validity is concerned with a common understanding of the language used in the interpretation of data so that the account is grounded in the language of the participant "and rely as much as possible on their own words and concepts", that is, the interpretation of the data is based on the participants' perspective rather on the researcher's perspective (Maxwell, 1992, p. 289). The piloting phase of the pre-observation interviews proved to be a very valuable exercise as it allowed me to clarify certain terms that are used more commonly in the language and mathematics circle. For example, at the pilot phase, I had used the term language proficiency which I changed to language fluency in the final version of the interview questions because it is a term more easily understood by the TEs. Also, at the pilot phase, I had asked the question: *Given your own practice (context) what would you perceive to be the ideal in teaching in a situation/context such as yours?* I spent about 5 minutes of the interview trying to clarify with the pilot interviewees what I meant by the term "*ideal*" as they wanted to be sure that they clearly understood my understanding of the term. Because of the ambiguity of this term, in the final version of the interview questions, I rephrased the question and used "best practices" rather than what they perceived as "ideal" given their context of practice.

Theoretical validity, sometimes referred to as construct validity, refers to the validity of the concepts (in my case, code switching, revoicing, reiterating, practice, etc) as they apply to the phenomenon under study, and the validity of the relationships between these concepts (Maxwell, 1992). Hence, theoretical validity refers to the extent to which the operational definitions of variables reflect the actual theoretical meanings of the concepts. In Chapter One, I explained my working definitions of the terms used in this study, and in Chapter Five, I developed a methodological approach for understanding and interpreting my data, taking particular care to define the terms used. The framework uses my theoretical underpinnings – Wenger's Community of Practice theory – as the core backbone to understand the different communities of pre-service mathematics teacher education classrooms. In doing this, I developed codes, define them, and provide examples of the codes with regards to how they are used in my study.

The use of multiple sources of evidence (triangulation) has been noted as an effective way to enhance validity. Triangulation involves using multiple perspectives to clarify meaning,

verifying the repeatability of observations and interpretation of data generated (Denzin & Lincoln, 2000). In the present study, data (interviews and classroom observations) were collected from different sites and from different teacher educators.

As far as the coding is concerned, reliability refers to the "measure of the consistency of a coding process when carried out on different occasions and/or by different researchers" (Dowling & Brown, 2010, p. 24). My codes were developed, to a large extent, *a posteriori*[40]. To ensure reliability in my coding, I first started by coding one lesson for each of the TE classrooms (rather than code one classroom community from beginning to the end). After that, I gave my code and coding principles to two colleagues (and of course my supervisors) and then compared their coding with mine. Where there were differences, we looked again at the video, the transcript and the code identification rules and deliberated until a consensus was reached. This was the first phase of the coding process. After comments from my supervisors, I re-organised the data adding extra columns (example, the 'specific activity' column and the 'example form transcript' column) and re-started the coding afresh. In this second coding phase, I started by coding the TE classroom which was the last to be coded in the first phase of coding. In all of these, I guarded against the codes being developed through the lens of only one teacher education classroom community. I will elaborate more on the codes and the coding principles in my next chapter.

Generalizability "refers to the extent to which one can extend the account of a particular situation or population to other persons, times, or settings than those directly studied" (Maxwell, 1992, p. 293). The present study was aimed at investigating teacher education multilingual classrooms to gain insight into the shared repertoire, mutual engagement and joint enterprise, and *ipso facto*, delineate implications with regards to preparing pre-service teachers for teaching mathematics in multilingual classrooms. It was not the aim of this study to generalise findings.

Maxwell (1992, p. 295) argues that unlike descriptive, interpretive, and theoretical validity, evaluative validity is not central to qualitative research as "many researchers make no claim to evaluate the things they study". That said, the aim of this study was not to make any judgement of value with regards to the teacher education classrooms (or institutions) involved in my study; neither was it aimed at a comparison of the different communities of practice to ascertain which had exemplary practices that are worthy of adoption.

[40] I will elaborate on this in Chapter Five.

4.10 Ethical considerations

Access to the universities was negotiated with the heads of the school (faculty) of Education of each university and the teacher educators were asked for a written consent to participate in the research. The teacher educators (and research institutions) were informed by the researcher that their anonymity would be protected. Those who decline to participate in the study were systematically excluded from the videoing during the videoing process by being asked to sit in a position which was not easily captured by the camera. Moreover, the video of class observations was only used for the purpose of data analysis.

It is my contention that the community of teacher educators is different from the community of teachers. The former is much smaller, more academically and research inclined, and more conversant with one another's institutional and historical contexts. This makes research in teacher education an ethical mine-field. It was really difficult to protect, at all times, the anonymity of the research institutions where my research was conducted. In the first place, a description of the context of research could have infringed on anonymity especially if a mention was made of the province involved. A way out was to indicate that the research was carried out in *one* province in South Africa (without necessarily mentioning the name of the province). Second, even though one university responded to all criteria for inclusion in my study, I could not include it due to the nature of its programmes, but nonetheless, delineating the features of this particular institution would have led to easy identification of the province in which I undertook my study.

In this light, in this PhD thesis and in all the publications resulting from this study, I have refrained from both describing the teacher educators involved in the study and the context of the institutions as doing either or both of these would put the anonymity of the teacher educators or the research institutions in jeopardy. Following from this also, the pronoun, 'she' is used for all the teacher educators in this study to protect the anonymity of the TEs. Acronyms are also used for the institutions involved in the study.

The issue of feedback to the research community, to the teacher educators who participated in the study and to the universities is mainly about responsibility and trust (Setati, 2000b). At the end of my study, I would first discuss the findings of my research with all the teacher educators who participated in the study, before giving each of the universities a copy of the thesis for their libraries. Some of the teacher educators involved in this study have participated in conference presentations on the findings of this research already, and I

particularly invite them to my presentation when I see them in the conferences in which I am also a participant.

4.11 Conclusion

In this chapter, I have provided a description of my empirical field, given reasons for the choices I made in the population and sample of my study. A qualitative approach was adopted and a two-phased approach was implemented in order to address the guiding questions. Issues of ethics, validity and reliability were also dealt with. In the next chapter, I provide a description of my organisational language.

CHAPTER FIVE
The emergence of an organisational language

5.1 Introduction

In Chapter Four, I indicated that the organisational language deals with categories that have been developed both from theoretical and empirical aspects of the research study in the course of analysis (Dowling & Brown, 2010). In this chapter, I describe the organisational language (OL) developed for analysis of data, and the processes through which this OL emerged.

As indicated in Chapter Two, Wenger (1998, p. 95) holds that learning in practice includes the following three *processes* for the community involved:

- *Evolving forms of mutual engagement:* discovering how to engage, what helps and what hinders; developing mutual relationships; defining identities, establishing who is who, who is good at what, who knows what, who is easy or hard to get along with.
- *Understanding and tuning the enterprise:* aligning the engagement with the enterprise, learning to become and hold each other accountable to the enterprise; struggling to define the enterprise and reconciling conflicting interpretations of what the enterprise is about.
- *Developing the repertoire, styles, and discourses:* renegotiating meaning; producing and adopting tools, artefacts, representations; recording and recalling events; inventing new terms and redefining or abandoning old ones; telling and retelling stories; creating and breaking routines.

These three dimensions of communities of practice and their associated processes provided the backbone for the development of the organisational language for the present study. As indicated in the previous chapters, Wenger's theory has a number of limitations. First, Wenger is not a mathematician nor a mathematics educationist and was not theorising specifically for the mathematics classroom. As such, Wenger's theory has limitations in terms of providing tools for analysing the (nature of) mathematics classroom communities of practice; Second, as discussed in Chapter Two, Wenger provides no ready-made set of tools

for analysing discourse within a community of practice. Therefore, Wenger's CoP theory is limiting in providing tools for gaining an entry into the negotiation of meaning process occurring in the Classroom CoPs. Moreover, mathematical aspects/perspectives of practice are not dealt with by Wenger. Hence, the challenge for me as a researcher using Wenger's notion of CoP was to draw on CoP theory as a theoretical framework, and then using the teacher education classroom communities of practices in this study, to develop a methodological approach that would capture the three dimensions of CoP in the mathematics pre-service classrooms which were investigated. Doing this meant identifying complementary theories or constructs to fill the gaps mentioned above. Hence, in providing the organisational language for this study, limitations to Wenger's CoP as it applies to the mathematics education context of my study were dealt with by introducing the work of Mortimer and Scott (Mortimer & Scott, 2003) into the mutual engagement process of CoP because of the ability of Mortimer and Scott's (2003) framework to characterise different kinds of discourse patterns and communicative approach. Also, to deal with mathematical aspects of practices in the shared repertoire dimension of CoP, the works by several authors (McClain & Cobb, 2001; Sullivan, et al., 2005; Tatsis & Koleza, 2008; Voigt, 1995; Yackel, 2000; Yackel & Cobb, 1996) were drawn upon. I also drew upon the work of Gomez (2007, 2009) in the elaboration of some key elements of the framework. In drawing on these theoretical sources, I adapted and modified ideas to suit my purposes based on the data collected in pre-service TE mathematics Classroom CoPs, thereby recontextualising them into the wider CoP framework.

In the framework, each *dimension* of CoP is subdivided into *categories*. The categories are then subdivided into *sub-categories.* Codes are used for each of the sub-categories and *code identification rules* are provided for each code. While the dimensions and categories were developed *a priori* by using Wenger's CoP theory and the other literature mentioned above, much of the sub-categories/codes and the code identification rules were developed *a posteriori* from working with data obtained from teacher education classrooms[41].

Research conducted in mathematics multilingual classrooms has sometimes been accused of: 1) being skewed towards analysis of language use and language practices, and 2) of being devoid of the content itself which engenders the talk. It is my contention that the methodological approach described here provides an approach that includes the examination of mathematics practice, mathematics content, the interactional context and the discourses in

[41] I will return to this point at a later stage.

multilingual pre-service teacher education multilingual classrooms in an integrated manner. It offers a more comprehensive approach to characterising the mutual engagement which results in the joint negotiation of the enterprise and the joint negotiation of meaning, and hence, the development of a shared repertoire, all of which are at the heart of Wenger's CoP theory.

5.2 Shared repertoire

As indicated in the previous chapter, the shared repertoire of a community is the common practices, common styles and common representations of a community of practice. In elucidating the OL of the shared repertoire dimension of the different communities of practice in my study, I use particular concepts/constructs within Wenger's notion of shared repertoire alongside sub-categories emerging from my data. In so doing, I use three categories of analysis (and associated sub-categories in each of the categories): mathematical practices (SRMP), norms of practice (SRNP) and pool of shared language and shared representations (SRPSL) that reflect and shape a joint understanding of the community's shared repertoire. I also draw on the work that has been done in these three areas to characterise the shared repertoire of the different communities of practice in my study. These three categories, in my estimation, are representative of the common or shared resources (of a community such as the ones in this study) for the negotiation of meaning.

5.2.1 Mathematical Practices (SRMP)

Wenger (1998, p. 6) argues that "social practice is the key to grasping the actual complexity of human thought as it takes place in real life". As indicated in the previous chapter, Wenger defines practice as "doing, but not just doing in and of itself. It is doing in a historical and social context that gives structure and meaning to what we do" (Wenger, 1998, p. 47). In line with the definition of mathematical practices in Chapter Two, within the shared repertoire of the communities and under the category of 'mathematical practice', the analytical task was to expound on the different practices that were in use in the negotiation of meaning in each community. As indicated in the diagram, to analyse the SRMP of each community, the following questions are used: What mathematical practices are in used in the negotiation of meaning? The questions that enabled the analysis of the norms of practice: What social and norms of practice are in use in the communities of practice? How do they co-construct the CoP? The questions that guided the exploration of the pool of shared language category of

each community were: 1) What are the common discursive repertoires within the communities, and 2) how do they co-construct the community and reflect the different modes of mathematical practices?

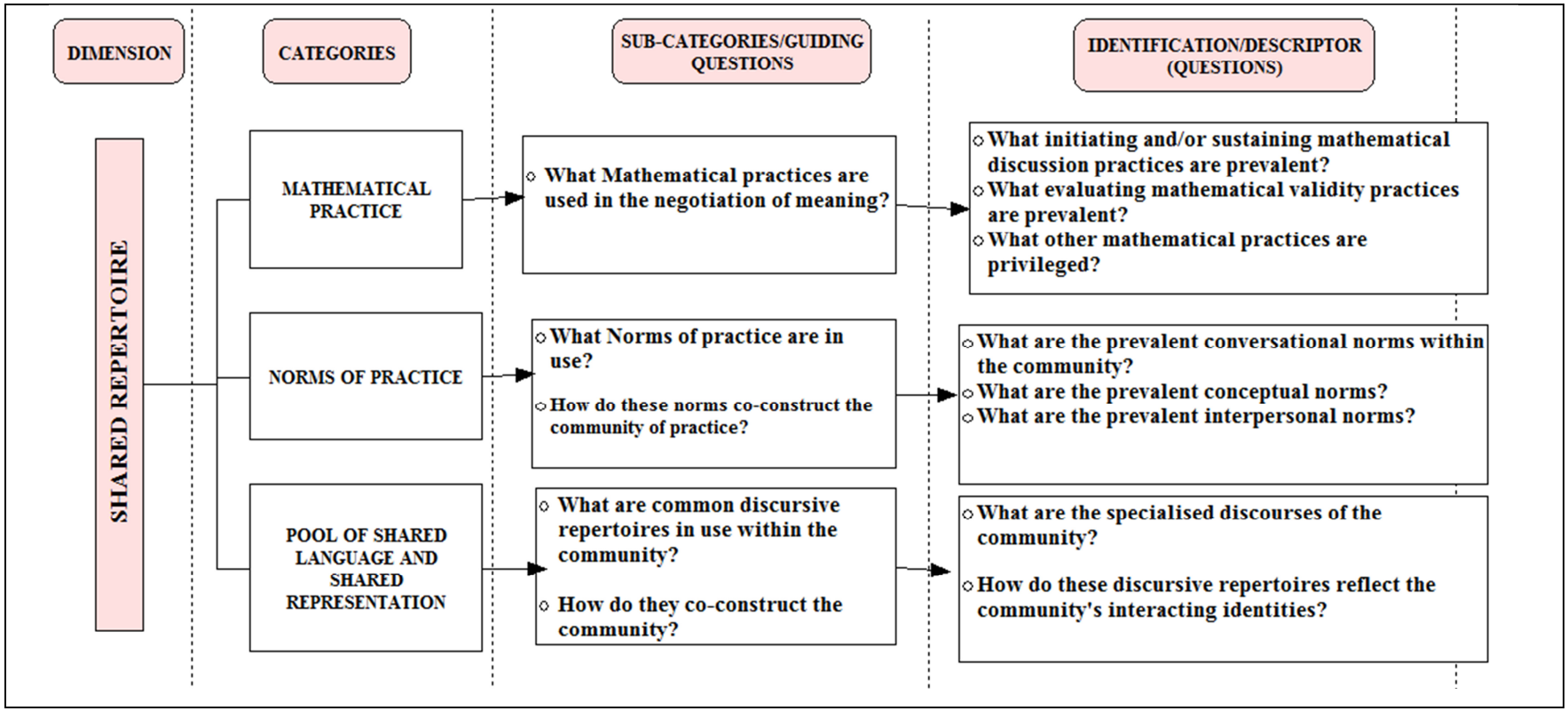


Figure 5.1: Categories of analysis for shared repertoire and associated questions

5.2.1.1 Mathematical Practices (SRMP) Codes and Coding process

A number of practices emerged from the teacher education classrooms. These practices have been categorised into three major headings according to the nature of the practices and the purposes of the practices. There are: 1) *initiating and/or sustaining mathematical discussion* practices; 2) *evaluating mathematical validity* practices; and 3) other mathematical p*ractices*.

- **Initiating and/or sustaining mathematical discussion practices**

These are practices that enable, what some authors have referred to as, productive mathematical discussions (e.g., Stein, Engle, smith, & Hughes, 2008) in the class, and others as productive disciplinary engagement (e.g., Engle & Conant, 2002). These practices are essential in fostering mathematics conversation in the class, which in turn leads to a gradual improvement in the quality of talk/arguments and the development of mathematical ideas and arguments in the class.

- **Evaluating mathematical validity practices**

These are practices that dealt with judgments about what is mathematically legitimate or not. They are the authorising practices.

- **Other mathematical practices**

These are practices which constitute the sort of things one would do in the mathematics class. Practices that neither belonged to the initiating mathematical discussion practices nor the evaluating mathematical validity practices were put in this category.

In the table below, I present a selection of these practices that emerged from our study, the coding scheme and the code identification rule(s) (descriptors). These practices were some of the dominant practices that emerged from the four teacher education classroom communities of practice in my study. The full list is found in appendix D.

Category: Mathematical practices	Mathematical practices-in-use (sub-category)	Code	Code identification rule(s)
Initiating and/or sustaining mathematical discussion practices	Explaining mathematically	MP-EM	When 'what' is used in a question by a community member. Or when the intonation used by the TE or any community member indicates a call for further explanation. Also, the use of the phrase/sentence: -'anything else', eg: anything else you want to add to that? -'no? why not?' -'this is what I mean…' -'what does it mean?' -'Do you understand what you have to do?' COMMENTS: MP-EM could also be a call for someone to shed more light on what has been said. Eg, 'what do you mean by …' MP-EM need not necessarily start in the form of a question. It could also be the explanation of a particular concept or an explanation of another PST's reasoning or solution to a mathematics problem.
	Defining Mathematically	MP-DM	When there is a formal or informal definition of a mathematical concept by either the teacher or the PSTs
	Communicating mathematically	MP-CM	When particular attention is paid to the mathematics register
	Reiterating (As indicated in Chapter Two, for the purpose of this study, I distinguish between reiterating and revoicing.)	MP-Rt	When the TE/PST repeats their understanding of what another member of the community has said either to check their own understanding of what has been said, or to make sure everyone is on the same page (ie, everyone understood the same thing). This is different from revoicing which involves repeating what has been said using the correct mathematical language. Eg: I want to repeat what he has said… "am I right" (when checking to make sure one has correctly captured what the other person has said. "is that what you are saying?" MP-Rt could also be a situation where TE/PST repeats what was said/discussed in previous class (in order to refresh the memory of the community) to aid their current activity or discussion. This is what Andrews

			(2009) calls 'exercising prior knowledge'
	Exemplifying (Providing examples)	MP-PE	When the PST/TE provides an example to demonstrate a mathematics method (eg, example of an application of a mathematics procedure) and in concept development to indicate a mathematics relations (eg, examples of a concept like triangle, etc) (Bills et al., 2006). It could also be when a community member demonstrates how something is done in mathematics, eg, how to draw a frequency table Close to MP-EM. An explanation can be made through the provision of an example. Use of words like: "like…", "example". It can also be a call by a community member for someone to give examples.
Evaluating mathematical validity practices	Critiquing (other's) conjecture	MP-CC	Member of the community challenges other members' conjecture. It could also be a call to critique/validate someone's opinion or to validate one's opinion. Phrases indicating MP-CC: -Do you agree -Is that an assumption you would agree with? -does that make sense? -Anyone wants to challenge that? -Anyone else on this? -Who wants to talk for or against that -is that logical/reasonable? -Is it possible that…? -So, do you mean that… -Am I right?
	Providing Justification	MP-PJ	Close to MP-EM and MP-PE. The "how" question indicates MP-EM while the "why" question would indicate MP-PJ. Instances where a PST/TE is asked to explain the procedures or steps leading to the solution of a maths problem would indicate MP-EM while a call to justify the procedure would be MP-PJ. For example: "who can tell me why the positive sign becomes negative when taken to the other side of the equation?" would be providing justification. The sentence: 'what is your evidence', could indicate either MP-EM or MP-PE or MP-PJ depending on the context of use.

	Critiquing solution	MP-CS	Involves critiquing the solution of a problem proffered by a community member. Different from MP-PJ and MP-CC. Here, a community member critiques his/her or other peoples' solution to a mathematical problem. In MP-CC, postulates are critiqued while MP-PJ involves justification for a conjecture or for the solution to any of the processes involved in the solution of a question. It can also be a call by any community member for other members to critically consider his/her solution to a mathematical problem or the processes involved in finding such solution. Eg: "what did you do wrong", "think carefully why you would make that decision"
Other mathematical practices	Moving between worlds	MP-MBW	When the TE/PST moves between the mathematics world and the real world. In Dowling's (1998) terms, when there is a movement from the esoteric domain to the public domain.
	Writing Mathematically	MP-WM	Any opportunity given to PST to write mathematically. Eg. 'I want you to write…' It could also be showing students the correct way to write/represent, say a frequency table. It could also be writing down a mathematical definition
	Proceduralising	MP-Pc	When the TE or the PST deals with the procedure/steps for solving a particular problem. For instance, if the TE or PST talks about taking a variable to the other side of the equal sign and changing the sign, that would be categorised as MP-Pc. But if a member of the community states why this procedure works, then it was categorised MP-PJ. Could also be a call for a particular procedure or aspects of the procedure to be used in solving a mathematical task: example: "Where do we start?" (which calls for the first thing that needs to be done by way of procedures) "What do we do next?"

Table 5.1: Mathematical practices in use and descriptors

In coding the transcripts, where there were questions followed by an answer, the coding referred to both the question and the answer(s), provided that the answer(s) was/were direct response(s) to the question asked. For example, the question: "what do you mean by…" was coded as a call for an explanation (MP-EM). The response provided to this question was not recoded as MP-EM. So, the question and the answer constituted one code rather than two codes of MP-EM each. Also, where a particular utterance which has already been coded (as writing mathematically for example) was repeated[42] on the same task or sub-task, the utterance was not recoded as writing mathematically but as reiterating. But where there was a different emphasis on the same issue (for example, to a particular member of the community/group), then it was given the same code (in this case, MP-WM).

5.2.2 Norms of practice (SRNP) and codes

As I have already indicated, norms are regularities that guide social interactions. They are expectations/obligations (implicit or explicit) that community members have of one another (Yackel, et al., 1991). Yackel, Cobb and Wood (1991) argue that it is through the interlocking obligations in the mutual construction of classroom norms that make it possible for participants to act appropriately in specific situations giving rise to observable interaction patterns. Hence, norms of practice are tightly related to the mutual engagement dimension of community of practice. The norms of a classroom community, whether implicit or explicit, influence the discourse pattern of the classroom by way of who speaks when and for what reasons, and whether the interaction is dialogic or authoritative or a combination of both.

For the purpose of the present study, I use three categorisations to describe emergent norms in the pre-service teacher education classroom communities of practice: conversational norms, conceptual norms and interpersonal norms which I discuss in the next section. These three categorisations were recontextualised from literature dealing with norms of practice and at the same time, guided by my data. Hence the norms of practice in use were also developed both *a priori* and *a posteriori*.

[42] For example if the teacher educator repeatedly shows the PSTs the correct way to write/represent a mathematical concept

- **Conversational norms**: Norms that guide interaction in the class and do not relate directly to the content of the mathematics at stake. Example: taking turns to speak; speak-out norm.
- **Conceptual norms**: Relates directly to the mathematical object under discussion: Eg. Justification norm; mathematics justification norm; consensus norm; non-ambiguity norm.
- **Interpersonal norms**: related to conversational norms, but in this particular case, these are norms that guide the interpersonal relations in the class. Example: the avoidance of threat norm where one is expected not to ridicule the answer of another community member.

Table 5.2 below presents these norms and their descriptors:

Category: Norms of practices	NP in use (sub-category)	Code	Code identification rule(s)
Conversational Norms	Participation by all norm	[NP-PA]	The expectation that all member of the community participate in the classroom activity. This is evident, when for example -the teacher calls to find out if some less active students are following the lesson -the TE calls out specifically for members who have not given input in the discussion
	Speak-Out norm	[NP-SO]	The expectation that members of the community should speak loud enough for everyone to hear. Phrases like 'louder', 'speak up', etc would indicate the speak-out norm.
Conceptual Norms[43]	Mathematically Sensible norm	[NP-MS]	The expectation that a community members solution or solution strategy makes sense to others or that a community member's explanation of a maths concept makes sense to others. Words like, 'does that make sense to you', anyone wants to challenge that' and 'do you agree' may depict such expectation
	Consensus norm	[NP-CS]	Group members are expected to reach an agreement on the solution to a maths question or explanation of a maths concept.
	Non-ambiguity norm	[NP-NA]	Expectation that mathematical expressions are clear and unambiguous, expressed through prompts for rephrasing. Example: T: What is the formula we use to calculate the distance between 2 points? S: we use the same formula [laughter] T: what is that the same formula? What is that the same formula? Yes sir.
	Justification Norm	[NP-JN]	The expectation that a community member has to justify her/his opinion(s). Expressed through words such as "because", "that is why", "would you explain why…?"
Interpersonal Norms	No Ridicule norm	[NP-NR]	The expectation that no member of the community may be derided if he/she makes a mathematically or grammatically incorrect statement.
	Collaboration norm	[NP-CB]	Relates to group work. The expectation that all members of the group must work together to solve a mathematical problem

Table 5.2: Norms of practice and descriptors

[43] There is obviously a blurred boundary between conceptual norm and mathematical practices because they are both mathematical in a sense. But because consensus norm, non-ambiguity norm, justification norm, etc were more normative (that is, taken as regularities that guided the classroom discourse), they are talked about as norms

In developing conjectures about the emergent norms of practice present/being constituted in each of the mathematics communities in this study, as with McClain and Cobb (2001, p. 241), I looked for instances, regularities and patterns (or breaches to the norms) in the way the pre-service teacher education classroom communities acted and interacted as they engaged with classroom mathematical activities. For example: prompts for rephrasing/reiteration would indicate the non-ambiguity norm; also, words such as 'why' expressed through questions or the use of because would indicate a justification norm. In the justification norm for example (as with other norms), I not only looked for instances where a member of the community is reminded to justify his/her answer or assertion (by for example being asked a 'why' question), but also instances where a member of the community provides justification for his/her answer or conjecture without being asked/reminded directly to do so.

Suffice it to say that for a norm to be considered such, only one instance of, for example prompts for rephrasing, was not sufficient. "Regularities" used in the definition of norms implies that there is some form of consistent reoccurrence of a particular 'instance' of a norm.

It was not the aim of this study to delve into how the norms were communally constituted. To start with, this would entail that the researcher studied the community from its inception which was not the case in my study. Most of the norms were already constituted when the researcher started the classroom observations. The main aim in delineating the norms of practice in this study was to make sense of how certain characteristics of the TE classroom CoPs and regularities in classroom activities are influenced by the social context of the community and how, in turn, they influence the dynamics of teaching and learning in multilingual pre-service teacher education classrooms.

5.2.3 Pool of shared language and shared representation

A community's shared repertoire sometimes derives from the common knowledge base which is reminiscent of the common purpose of the existence of such a community and which is more often than not, unfamiliar to those outside of the community. The specialised discourse used in a community, as discussed earlier may indicate some form of reification, but importantly, this specialised form of language may reflect the different mathematical practices, that is, in the context of the present study, whether there is a balance in or a bias toward (all) any of the interacting identities (that would be discussed in details under mutual engagement) within a teacher education community of practice.

In dealing with the pool of shared language and shared representations, the main questions that guided analysis were:

- What are the common discursive repertoires or specialised discourses used in the community of practice?
- How do these common discursive repertoires co-construct the community

A question arises: what about a situation where, for example, the teacher (or community) uses scaffolding methods, but the term is not mentioned directly by any member of the community? Should scaffolding in such a case not be considered as a shared language? The simple answer to that is that this third category (pool of shared language and shared representation) deals mainly with the shared discursive resources used in the community. Furthermore, such a situation as described above would have been represented already as a practice of the community. This is why an integrated (rather than an isolated) discussion that focuses especially on both the shared repertoire and the mutual engagement dimension of CoP is important in characterising the nature of communities of practice.

In analysing the pool of shared language of the communities of practice, I look therefore at shared references that participants use as they negotiate meaning and how this influenced the practices that are in use and shape the interacting identities of the teacher education community.

5.3 Mutual engagement

In delineating the organisational language for the mutual engagement dimension of the CoPs, two categories were used: pattern of discourse and building of identities (MEBI). As with other dimensions of CoP, these three categories relate to Wenger's process for the community with regards to mutual engagement:

> *Evolving forms of mutual engagement:* discovering how to engage, what helps and what hinders; developing mutual relationships; defining identities, establishing who is who, who is good at what, who knows what, who is easy or hard to get along with (Wenger, 1998, p. 95).

As indicated in Chapter Two, the work of Mortimer & Scott (Mortimer & Scott, 2003; Scott, et al., 2006) was instrumental in providing the analytical structure of engagement in the community in general and of the pattern of discourse category in particular.

Esmonde (2009) argues that in analysing mathematics classroom interactions, it is essential to focus not only on the content of mathematical talk, but also on the interactional context[44] in which talk occurs. To these two (mathematics content and interactional context), I would add that the nature of talk itself (that is, whether it is procedural, dialogic, authoritarian, etc.) is also crucial. To this end, while Wenger's theory provided the backbone for developing the mutual engagement dimension, the three aspects of classroom mathematics interaction (content of mathematical talk, interactional context and nature of talk) provided the guiding principles. While the content of mathematical talk would be dealt with by *who makes substantive contribution*, the interactional context in which talk occurs was taken care of by analysing how participation is organised. Finally, the nature of talk would be analysed through the communicative approach and patterns of discourse aspects of the framework.

[44] Research by Esmonde (2009) on explanation as a discursive practice shows that explanation can be situational – explanation in the presence of a teacher could be different from explanation made to other members of the group and that explanations may vary depending of the students' perceptions of the task.

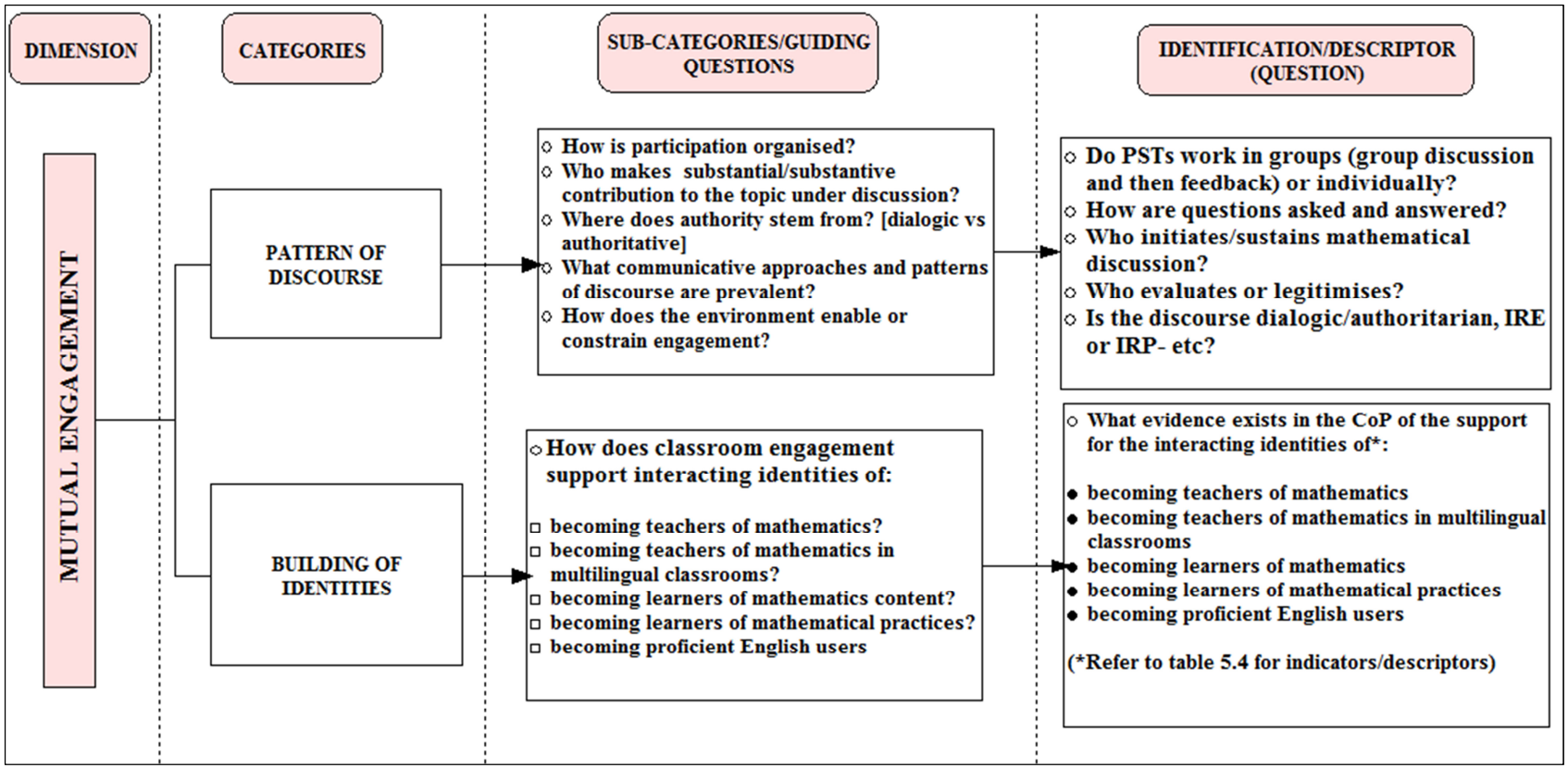


Figure 5.2: Categories of analysis for mutual engagement and associated questions

5.3.1 Pattern of discourse (MEPD)

In mathematics education research, there are many conceptualisations of discourse (see Ryve, 2011) and alongside the different conceptualisations are different methodological approaches for analysing discourse in classrooms. Many of these methodological approaches on discourse have used sociolinguistic methods/approaches to classroom discourse (such as discourse analysis or semantic analysis) and studied the classroom linguistic processes. As I explained in Chapter Two, I have opted to use Mortimer and Scott's (Mortimer & Scott, 2003; Scott, et al., 2006) framework because of its alignment with the overall theoretical lens that guided this study and also because I found it particularly useful in analysing classroom discourse (language-in-use in the classroom) in this study.

The term "pattern" in *pattern of discourse* is used in a broader sense that comprises how participation is organised, who makes substantive contributions, where authority stems from and what communicative approach is prevalent. Hence, in pattern of discourse, I would focus on to what extent there is a *pattern* 1) in how participation is organised; 2) in who makes substantive contributions to the topic under discussion; 3) in where authority stems from; and 4) in which communicative approaches and discourse patterns are prevalent in individual CoPs. Hence, the sub-categories/guiding questions that would help in the analysis of MEPD are:

By substantive contribution, I refer to subject-matter content talk/discourse that contributes to mathematical advancement in terms of knowledge and understanding of the mathematical content at hand, or in the teaching and learning of such content.

In the diagram below, I present these sub-categories and the descriptors, with some examples from classroom observation transcripts. It must be noted that the identification questions were developed *a posteriori* as while analysing the transcript, I added new questions, deleted some and corrected others in the process.

Sub-categories/guiding questions	**Identification/descriptor (questions)**	**Codes (where applicable)**
How is participation organised?	Do PSTs work in groups (group discussion and then feedback) or individually?	
	How are questions asked and answered?	QAS (Question-answer sequences led by the teacher)
		PED (Questions that provoke extended dialogue)
What communicative approaches and patterns of discourse are prevalent in the CoP?	Is the discourse • Dialogic? • Authoritarian? Is the pattern of interaction: • I-R-E? • I-R-P-…? • I-R_s-R_s-R_s-…?	
Who makes substantive contribution?	Who initiates/sustains mathematical discussion?	TEQ (TE asks a question or statement initiating a dialogue – the question ***must*** initiate a dialogue)
		PSTQ (PST asks a question or statement initiating dialogue)
	Who evaluates or legitimises?	TEEv (TE evaluates)
		PSTEv (PST evaluates)
Where does authority stem from?	Is the discourse dialogic or authoritative?	

Table 5.3: Descriptors for analysing patterns of discourse

It must be noted that when the teacher education question (TEQ) involved the steps/procedures in solving a particular task, I coded it as a single TEQ rather than as many TEQs. As an example, the following was coded as a single TEQ:

1. TE: What are the scores that you have on the left hand side? You have 10, that's the highest, right?
2. Some students: Yes
3. T: And then I want to arrange this in descending order. What would be the next number noted?
4. Class: 9
5. T: *[writes 9]*
6. And then followed by?
7. Class: 8

Hence, turn 3 was not re-coded as TEQ, neither was turn 6.

Also, where the TE or PSTs asked a question for clarification purposes, this was also coded as pre-service teacher question (PSTQ) or TEQ.

In exploring the questions about who makes substantive contribution, who initiates/sustains mathematical discussions, who evaluates or from where authority stems, I would in fact be engaging with the key questions: *where does legitimation lie* in the community of practice.

5.3.2 Building of identities

Wenger (1998) notes that identity is in part a trajectory of where members of a community (as a collective and as individuals) have been, where they currently are, and where they are going. Examining this three-tiered trajectory of identity would entail following pre-service teachers as students, as student teachers and then as novice teachers. It was not the aim of this study to do all these. And given that data was only collected during the time interval in which a mathematics topic/concept was addressed in class, the study only focused on the second part of Wenger's identity trajectory – where members *are currently*, while bearing in mind *where they are going*. As Hodges and Cady (2012) note, for Wenger, identity is in part how individuals come "to participate within a community in conjunction with how […] individual[s] talk[…] about and make[…] sense of that participation". This means that access to where member *are currently* is possible through the observation of classroom practices in communities of practice.

A number of authors have argued for the integration of mathematics and language development in multilingual classrooms (Adler, 1995; Barwell, et al., 2007; Smit & van Eerde, 2011)]. These authors have argued against the avoidance of linguistic aspects of teaching and learning mathematics and for attention to be paid to the language needs of

learners/students in multilingual classrooms. This is why as part of the identities involved in teaching and learning mathematics in multilingual pre-service classrooms, it was critical for this study to also examine how each community pays attention to the language needed for mathematical learning. To this effect, under the mutual engagement dimension of CoP, the present study also analysed evidence present in the different CoPs in support of the interacting identities of becoming a teacher of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics, becoming learners of mathematical practices and becoming proficient English users for the purpose of teaching/learning mathematics. These interacting identities were pivotal in the analysis of identities. The table below provides more details about what codes were used and examples from classroom observation transcripts.

Sub-categories/guiding questions	Identification questions	Code and descriptors	Example from transcript (indicators)
How does classroom engagement support interacting identities?	*What evidence exists in support of interacting identities of:*		
	Becoming teachers of mathematics?	[BTM] [Allusion is made to the teaching of mathematics]	TE: And this word 'probable' *[writes:* probable*]* you know if a child had to hear this word for the first time, how could you *as a teacher explain* what probable [in relation to probability] means?
	Becoming teachers of mathematics in multilingual classrooms?	BTMMC [When attention is paid to teaching and learning in multilingual contexts]	TE: I want you to discuss this concept [probability] in your home languages in your group. After that, one member of the group will tell us what it means in a language of your choice and the direct translation
	Becoming learners of mathematics content?	BLMC [when the mathematics concept is the main focus of attention]	TE: An activity that is taking place or will take place is called an Event.
	Becoming learners of mathematical practices?	BLMP [if PSTs are taught, for example, the formal definition of a maths concept, this is BLMC, but if PSTs are 'taught' the importance of defining in the teaching and learning of mathematics, this is BLMP]	TE: I am telling you know, you need to know how to define fractions correctly. Definitions are important in mathematics. I repeat, correct definitions are an important part of mathematics
	Becoming proficient English users	BPEU [General English usage]	TE: And when we speak in English instead of we say… instead of using the word 'probability' what can I say? S: Chance T: What are the chances of getting a …

Table 5.4: Codes and descriptors for the building of identities category of ME

The emphasis on "becoming" rather than "being" must be noted. This is in line with Wenger's notion of learning and identity. Learning, according to Wenger, transforms who we are and what we can do, and as such it is an experience of identity. Also, learning for Wenger is not simply the accumulation of skills and information, "but a process of *becoming* – to become a certain person or, conversely, to avoid becoming a certain person" (p. 215. *My emphasis*).

There are subtle differences between these interacting identities. In what follows, I describe these differences which were evident in my study with the hope that it would help anyone desirous of using this framework to analyse classroom data from pre-service multilingual classrooms:

- **Difference between BTM and BLMC**

Becoming teachers of mathematics (BTM) is about teaching and the TE sees herself as developing this identity in the PSTs and the PSTs see themselves as imbibing this identity. By the same token, in becoming learners of mathematical content (BLMC), the TE sees herself as responsible for teaching the PSTs the mathematics content and the PSTs see themselves as learners of mathematical content. In becoming learners of mathematics content, unlike in becoming teachers of mathematics, what is different is that becoming a learner of mathematics entails knowing, for example, how to move from the ordinary language (OL) to the mathematical language (ML). In the BTM example above, the PST's ability to use her knowledge to teach is called upon (which is not the case with the BLMC example), hence while one is about teaching, for the other, the attention is more towards learning. Put differently, if BLMC entails knowing how to move from the ordinary language to mathematical language, BTM entails being self-conscious/aware of the movement between both. For the learner of mathematics, even though he/she needs to be able to make the moves, the movement does not have to be an explicit part of the learner's practice. To use exemplification (MP-PE) as an example, as a teacher one needs to know what examples to choose in order to cross the boundary between the OL and the ML. As a learner, one does not need to pay attention to what is/is not a good example (and then be able to choose).

Becoming a learner of mathematics content also entails knowing the definitions of mathematical terms, that is, being conversant with mathematical registers. Therefore,

utterances such as "*OK, let's look at what's frequency distribution – the definition part of it*" was coded as BLMC.

Also, being a learner of mathematics content presupposes understanding the demands of the question. So, questions/statements that dealt with understanding the demands of the question were also coded BLMC

When students were asked to go to the board to solve a problem, this is coded as BTM. Incidents in which the TEs asked students to think about what other students were thinking when they solved a problem on the board was also coded at BTM.

- **Difference between BLMC and BLMP**

BLMC is more about becoming knowledgeable about content as opposed to becoming knowledgeable about mathematical processes (BLMP) like the processes of coming to define/prove, etc. (hence it is more about visibility of practice). In the former, the content is the object of attention, and in the latter, the practice becomes the object of attention. For example, understanding probability as a mathematics topic involves being able to distinguish between *certain* events, *likely/unlikely* events, etc, and hence, where there was explicit teaching/attention to the mathematics register in probability, this was categorised as BLMC. This is so because the language of probability forms part of the content of probability. The same would be applicable to other domains in mathematics (example, Statistics and Analytical Geometry, as is the case in this study) – learning the mathematics register in a content domain is part of learning the content of the domain. This is different from the process of coming to define the term probability. If the TE is explaining a mathematical idea so that the students understand the content, this is categorised as BLMC; but if the TE makes explaining/defining as a mathematical practice visible to PST, then this is categorised as BLMP. And in the case where this visibility is such that there is explicit allusion to its importance/use in teaching, this is categorised as BTM.

- **Difference between BTM and BTMMC**

In BTM, there is an emphasis on the PSTs' present/future practice as teachers (such as in the phrase: *how could you as a teacher explain what probable means*?). In BTMMC, there is something specific about teaching in multilingual contexts and so, attention is not only paid

the fact that the PSTs would become teachers, but that they would become teachers in multilingual contexts.

5.4 Joint enterprise

Analysis of the joint enterprise is informed by those dimensions of the community of practice that lend to the appropriation of mathematical knowledge and the associated *processes* of *understanding and tuning the enterprise* (Wenger, 1998). In delineating the categories for joint enterprise, I draw on the work of Gomez (2007, 2009) whose research focused on the development of the three dimensions of community of practice within mathematics pre-service teacher education classrooms. Gomez identifies three categories for the analysis of the joint enterprise of a community of practice: external conditions; content (discourse); and, enterprise and responsibilities. Under joint enterprise, I do not focus on discourse as this is dealt with under mutual engagement.

As indicated in Chapter Two, there is an overarching broad joint enterprise which brought members together in the first place. The way in which the pre-service teachers and the teacher educator (in the individual communities of practice) negotiated different aspects of the joint enterprise of teaching and learning to teach mathematics, and, therefore, how they tune this initial enterprise would be analysed through: 1) the external conditions that constrain and/or enable a particular joint enterprise and how the community adapts or responds to these conditions; 2) how practices in use reflect what is valued by the community and can be perceived as the joint enterprise; 3) how responsibility is defined in the communities of practice. In the diagram below, I present the organisational language used in the analysis of JE. In what follows, I provide a more detailed description and justification of the external conditions and relations of accountability/responsibility that are key in understanding aspects of the joint enterprise. It must be noted that the guiding questions were developed *a priori*.

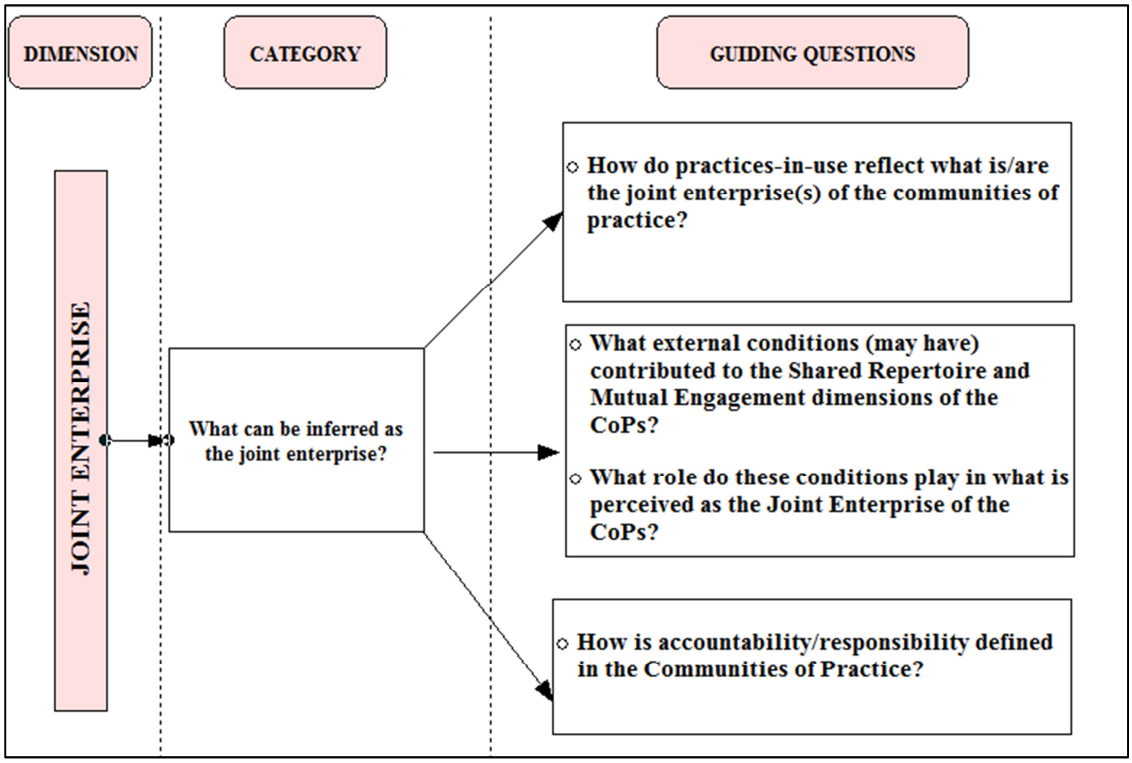


Figure 5.3: Categories of analysis for joint enterprise and associated questions

In analysing the joint enterprise of each of the pre-service teacher education CoPs in my study, I take joint enterprise as an outcome of the analysis of mutual engagement in the community's set of shared resources (shared repertoire) used in the negotiation of meaning. I do this because for Wenger (1998), mutual engagement is fundamentally defining of CoPs and such mutual engagement is directed towards a negotiated joint enterprise. In addition to this, the shared repertoire of a community as described by Wenger, "can be seen as the tangible expression of mutual engagement and the key means of carrying forth a joint enterprise" (Levinson & Brantmeier, 2006, p. 331). Hence, both mutual engagement and the shared repertoire dimensions serve as a window through which I gain entry into the communities' joint enterprise(s).

Since joint enterprise is anchored in mutual engagement and shared repertoire, I do not use codes to analyse it (JE) but provide a general discussion on what can be considered as their negotiated response to their specific conditions. In all these, the question that is in focus was: What can one infer (as opposed to conclude for sure) as the joint enterprise(s) in each of the communities that have been jointly negotiated or which can be considered as their negotiated response to their specific conditions?

5.4.1 External conditions

As indicated in Chapter Three, Wenger (1998, p. 79) argues that communities of practice are often "profoundly shaped by conditions outside the control of its members" (even though invariably, the joint enterprise belongs to the community since it is their collective response to their situation). External conditions is concerned with factors (entities, events, etc.) around a community of practice that may (have) influence(d) its *modus vivendi*. In analysing the external conditions of each of the communities of practice, I not only look at what was the founding external mandate for the community, but also the institutional context of the different university that may have had an impact on the observed nature of mutual engagement and shared repertoire. The analytical task, thus, was to engage with what external conditions (may have) contributed to the CoP's shared repertoire and mutual engagement dimensions of the Communities of practice and what role these conditions play in what is perceived as the negotiated joint enterprise of each community of practice.

5.4.2 Accountability and responsibility

Wenger (1998, p. 81) argues that "relations of accountability include what matters and what does not, […] what to do and what not to do, what to pay attention to and what to ignore, […] when actions and artefacts are good enough and when they need improvement or refinement". Thus put, defining or negotiation of the joint enterprise "produces relations of accountability that are more than norms" (Rock, 2005, p. 85; also see Wenger, 1998;). As Rock (2008, p. 85) argues, "these relations are manifested not [necessarily] as conformity, but as the ability to negotiate actions as accountable to an enterprise". It is with the above in mind that for accountability and responsibility the analysis questions was: how is accountability/responsibility defined in the CoP? In engaging with this question, I look at how members of a community are responsive to the views of others during the mutuality of engagement and to what others are doing in the classroom as well as how members are responsive to the classroom community's norms of practice.

5.5 Conclusion

Providing an organisational language developed through the interplay between the theoretical field and the empirical field was the main purpose of this chapter. The theoretical fields that

enabled the construction of the organisational language were mainly Wenger's theory of communities of practice and Mortimer and Scott's notion of meaning making as a dialogic process. I also indicated in this chapter how my methodological approach helped me in my data analysis and how it was used in my data analysis. In Chapter Six, I present my analysis and findings in Phase One of my study.

CHAPTER SIX
Findings from phase one of study

6.1 Introduction

This study set out to investigate the nature of communities of practice in pre-service teacher education classroom communities in one Province in South Africa and the implications thereof for pre-service teacher education in a context such as that of South Africa. The guiding and analysis questions that informed the study were formulated thus:

Guiding Questions	*Analysis Questions*
1 What is the nature of the communities of practice of pre-service mathematics teacher education classrooms in one Province in South Africa? 2 What are the implications resulting from the above with regards to preparing pre-service teachers for teaching mathematics in multilingual classrooms?	1. What mathematical practices are in use in the negotiation of meaning in the mathematics community? 2. What norms of practice are in use in multilingual mathematics classrooms of pre-service teachers and how do these norms co-construct the mathematics PST education communities? 3. What are the common discursive repertoires within the communities, and how do they co-construct these communities? 4. What communicative approaches and patterns of discourse are prevalent in the mathematics PST education classrooms? And where does authority stem from? 5. How does the environment enable or hinder engagement? 6. How does classroom engagement support the different interacting identities within pre-service teacher education classrooms? 7. What can we infer (as opposed to conclude for sure) as the joint

	enterprise(s) in each of the communities that has/have been jointly negotiated or which can be considered as their negotiated response to their specific conditions?

I used Wenger's three dimensions of communities of practice – shared repertoire, mutual engagement and joint enterprise – as my organisational language that permitted me to gain entry into the nature of the communities of practice at two universities which I have called University A and University B. In this chapter which deals predominantly with findings from the pre-observation interviews in Phase One of the study, I begin to unravel these questions by engaging with how teacher educators project themselves as members of the community of practice they belong to and how they describe the community for which they are members.

Even though the original intention for Phase One was to enable the selection of teacher educators for an in-depth study in Phase Two, I find it useful to provide a discussion on some findings that are pertinent to the discussion of the shared repertoire, joint enterprise and mutual engagement aspects of the communities of practice in both University A and University B, and which ultimately is key in understanding some of the conclusions that were drawn from the study. More specifically, the analysis of the interviews reported in this chapter relates to the analysis question 1 about mathematical practices-in-use and analysis question 5 which deals with how the environment enables or constrains mutuality in communities of practice.

As indicated in Chapter Two, participation according to Wenger is broader than direct engagement in practice (Wenger, 1998) and "draws attention to the 'social-ness' of all sorts of activities which arise from particular identity and community affiliation" (Tusting, 2005, p. 38). The interviews can be viewed as one of those activities as it gives a sense of the teacher educators' discourse when they talk *about* their practice[45]. Three themes which emerged from TEs' projection on their communities of practice through the interviews which helped frame some of the analysis that comes later (and had an impact on the nature of CoP in the pre-service teacher education classrooms) are discussed below.

[45] This, of course, may not necessarily be the same as the discourse they use when they talk within the community.

6.2 Findings emerging from Phase One

I discuss first the extent to which teacher educators are aware of their multilingual context of teaching of pre-service teachers who are being prepared to teach mathematics to multilingual learners and how this awareness inform their practice. This is the first theme and I use solely the pre-observation interviews to engage with this theme. The second theme is about how the teacher education institutions attend to the teaching of mathematics in multilingual contexts. The third theme deals with code switching in the teacher education classroom communities of practice. The first and second themes concern the analysis question 7 above, while the third issue concerns analysis question 1. As a reminder, even though in this chapter I focus mainly on Universities A and B, it must be noted that these issues emerged from four universities involving 12 teacher educators.

6.2.1 Teacher educators' awareness of the context of practice

What value is there in gaining an understanding into teacher educators' awareness of the complexity of teaching mathematics to multilingual second language English pre-service teachers who would at the end of their training teach multilingual learners who may still be learning the LoLT? Because mathematics is abstract science, it is carried in semiotic form and therefore accessed through some form of language. Thus, the awareness of and attention to language use is critical in any classroom and/or community of practice as far as the mutual engagement dimension of CoP is concerned. This awareness becomes even more critical in multilingual contexts where learners (and sometimes teachers) learn/teach mathematics in a language other than their first or home language, and are not yet proficient in the language of learning and teaching. In this study, the use of the construct 'multilingual language awareness' resonates with how it is used by the Association for Language Awareness when they define language awareness as the "explicit knowledge *about* language, and conscious perception and sensitivity in language learning, language teaching and language use" (Garcia, 2008, p. 386 my emphasis). Even though knowledge of the languages present in the class is an added advantage, awareness of one's multilingual context of teaching and learning does not necessarily necessitate the knowledge *of* each and every language present in multilingual settings, but rather, essentially some knowledge *about* the language. As indicated in Chapter One, awareness of the teacher's or teacher educator's multilingual context of mathematics teaching and learning involves, first and foremost, that the teacher or teacher educator

recognise multilingualism as an asset rather than a liability in his/her class; second, that the teacher/teacher educator actively draws on students' multilingualism by creatively tapping into and exploiting the different languages available in the multilingual classroom; third, that the teacher or teacher educator possesses knowledge of how to use the learners' home languages coupled with the knowledge of how these languages may interact with each other and with one another in conveying messages; fourth, that the teacher is attentive to how learners use language as they ask questions and engage with other mathematical activities like making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, and challenging; and fifth, that the teacher is able to use appropriate mathematical language to respond to learners.

In multilingual mathematics contexts, in addition to the above, language awareness include 1) the recognition of multilingualism as a (potential) resource, rather than as hindrance; 2) attention to the linguistic structure in (mathematical/everyday) English as compared to the language structures in the home language(s) of the learners.

Reflecting on the importance of creating language awareness in multilingual contexts, Cummins (2007, p. 229) notes:

> …there are also compelling arguments to be made for creating a shared or interdependent space for the promotion of language awareness and cross-language cognitive processing. The reality is that students are making cross-linguistic connections throughout the course of their learning in a bilingual or immersion program, so why not nurture this learning strategy and help students to apply it more efficiently…If students in bilingual/immersion programs spontaneously focus on similarities and differences in their two or three languages, then they are likely to benefit from systematic encouragement by the teacher to focus on language and develop their language awareness.

Re-echoing the above sentiment, Wagner (2007) asserts that creating a sense of such language awareness as described by Cummins equips students to use language powerfully and also teaches them to problematize language in such a way that they (students) come to the realisation that language problems are inherent in mathematics classroom discourse. Awareness of language so conceptualised, is a fundamental aspect of CoP because it plays an important role in the type of engagement that occurs in the practice and also in the type of practices that are privileged in the community of practice.

The teacher educator's awareness or lack of awareness was evident all through the pre-observation interviews, but more evident in responses to some of the interview questions than in others[46]. Through questions such as those regarding the language background of the pre-

[46] See Appendix A for interview questions

service teachers, and what the TEs think others teaching in the same context can learn from them, the teacher educators' awareness, or lack thereof, of the fact of teaching in multilingual contexts and what this implies came to light. Also, in deciphering teacher educator's awareness of his/her context of teaching, what the TE thought needed to be done specifically to enculturate pre-service teachers into the intricacies of teaching mathematics in multilingual classrooms also provided insight. Of the 12 teacher educators that I interviewed, 10 of them displayed a sense of awareness of teaching in a multilingual context of second language English students who are not yet proficient in the language of instruction. All teacher educators from both University A and University B (which were selected for further study) were amongst the 10 teacher educators. In what follows, I present and discuss excerpts from some of the teacher educators with a particular focus in my selection on the teacher educators who were selected for classroom observations in Phase Two. Even though the teacher educators were aware of the context of their practice, the teacher educators did not talk about their awareness in the same way. For some, awareness is about experience and gained through experience; for others, awareness is about knowledge of the field and knowledge of language issues in the teaching and learning of mathematics. In the excerpt below from University A, the teacher educator's awareness comes through as knowledge about the relationship between language and mathematics gained through acquaintance with research in teaching mathematics in multilingual contexts:

Excerpt 1[47]

1	R		Ok. Do you think that we need to do anything specific to train student teachers to know what it means to teach maths in multilingual classrooms?
2	TE	1	I think in a way yes. This is because the linguistic demands of students are
		2	different. Meaning we have try to meet them half way in terms of using the
		3	language that they understand. Because if they do not understand the LOLT and
		4	the language which the textbook is written and the language of test, then they
		5	would not understand the maths concepts. But if one makes use of their
		6	language, that can enhance their maths understanding and understanding of
		7	concepts and terminologies.
3	R		Ok, you think something needs to be done, but now the question is…
4	TE	8	What is that that needs to be done…
		9	Ok, I think we need to include concepts of teaching our subjects in multilingual
		10	classrooms, so that we look at strategies or approaches of using different
		11	languages in our teaching…
5	R		Thank you. Last but not the least: What would be your advice to someone who is teaching in the same context as you?
6	TE	12	Elm, the fact is that at least the person needs to consult. There have been people

[47] This is TEIA-S described in Chapter Seven.

13 before who have taught in the same context. There are people who have research
14 on the teaching and learning of maths in the same context. So, my advice is for
15 the person to consult, go through some articles of research of people who have
16 gone through the same experience. That can help us develop strategies to
17 approach in the teaching.

In excerpt 1, it can be inferred that the TE is aware of the demands of teaching and learning mathematics in multilingual contexts such as hers. This comes to the fore, firstly, in her affirmation that pre-service teachers do need to be specifically trained to deal with teaching mathematics to multilingual students (turn 2, line 1). Her awareness is also foregrounded by her understanding of the implications of the fact that the PSTs come to class with varying proficiencies in the LoLT (turn 2, lines 3-7). She also goes further to point out that one of the things universities and colleges of Education needs to do is to enculturate pre-service teachers into teaching strategies and approaches that specifically deal with and aim at creating awareness of the relationship between mathematics and language, especially with regards to multilingual students (turn 4). She urges anyone teaching mathematics in multilingual class to consult with more experienced teacher(s) (educators) and literature which deals with teaching mathematics in multilingual contexts (turn 6) in order for them (teachers) to acquaint themselves with the intricacies of teaching mathematics in multilingual contexts. Hence, this teacher educator's awareness comes through as knowledge of the language issues in the teaching and learning of mathematics in contexts where the LoLT is different from the home languages of the PSTs.

The TE in excerpt 2 from University B shows awareness (albeit, differently from the TE in excerpt 1) of teaching mathematics to multilingual students who themselves are still learning the language of teaching and learning. This awareness comes to the fore in the decisive way in which, in her first encounter with students in her class (turn 2, lines 1-6), she impresses upon them (PSTs) that she is aware that, for most of them, English is an additional language and that she understands the implications that stems from this fact for their learning.

Excerpt 2[48]

1	R		…Let's start with students in xxx course. What is their English language proficiency like?
2	TE	1	…Elm, I actually told them from the beginning that I am aware that English is their
		2	second language for most of them, that they need to put away this emotion of
		3	being shy and actually feel free to say to me: please repeat yourself or I didn't

[48] This is TEIB-L described in Chapter Eight

understand what you have just said. I encourage them to do that and I have noticed that, elm, over the last few weeks, more of them are starting to feel comfortable with me, and, to be able to say, 'could you please repeat that again'. Now, I come from a background of working for many years at xxx college of Education where I gained a lot of knowledge on how to speak to students whose second language is English… But what I became aware of is that, I cannot expect a concept to develop within their minds and grow unless I actually said in one way and then say it in another way. That ran instinctively through my work with second language people. It has just become second nature to me that I never explain something, particularly a new maths concept, in one way, in a way an English person would understand first time around. I am aware of the fact that there is mathematical understanding and a language understanding. And so, in my [ ] classes, I think they become aware or they see that I just don't assume that whatever I say is understood by them. I think they have already seen that. […] So, I try to encourage things like…you know, I'm always asking them: did you just understand what I have just said, and I never assume that silence means yes. So, sometimes, I jump the gun, if you like, and I just repeat it to make sure that there is some understanding and usually the repetition is said in another way in English.

For the teacher educator in excerpt 2 above, awareness is gained through experience of teaching second language English learners/speakers. The TE admits (turn 2, lines 7-9) that her background of having taught English second language pre-service teachers has sharpened her awareness of teaching multilingual pre-service teachers who are still learning the LoLT. The awareness of this TE also comes to the fore later in the interview (not captured in excerpt 2) where she says that she pays particular attention to how pre-service teachers use language as they teach during their practical teaching. According to her, she does this because she is "*serious about using language in the classroom and using it correctly, and helping their English meaning develop of the maths word so that it does not hold back their conceptual understanding"***.**

What is interesting in the TE's narrative is that she projects herself as one who knows and understands English second language students because of her many years of experience of teaching such pre-service teachers. This raises questions as to how TEs learn to train teachers for multilingual classrooms. This TE seems to suggest that it is learned from experience. But does this mean that everyone who has taught second language English pre-service teachers knows and understands them and their pedagogic/linguistic needs?

For some other teacher educators like the one in Excerpt 3 from University B, awareness is something gained from both research and from experience.

Excerpt 3:

1	R		Given your vast experience of teaching both in-service and pre-service teachers, what can we learn from you in terms of what it means to teach in multilingual pre-service classrooms of pre-service teachers preparing also to teach in multilingual classrooms?
2	TE	1	I think, elm, … I had in the past researched into the multilingual situations, so I am
		2	very sensitised to the whole importance of the total human being's immersion in
		3	his world. I don't look at a student as someone who is sitting and doing
		4	mathematics. I'm looking at him and trying to find out very early on what is
		5	happening with this student, where does he come from, etc. In short, what I am
		6	trying to say is that it is important that we acknowledge the fact that our students
		7	are coming to our classrooms with a multitude of background information that we
		8	are not familiar with given the multicultural and multilingual situation of our
		9	country. And not only acknowledging it, but making it apparent from the word go,
		10	that you respect them, and that you think that they can make a contribution; that
		11	the learning process is a 2-way street. For me, that has worked over the years,
		12	because that also sets the tone for respect and for trust. Because of trust, the
		13	students knows he/she would not be ridiculed if he/she makes a mathematically
		14	incorrect statement

The TE (from University B) in excerpt 3 is aware of teaching multilingual students and understands what teaching in such a context entails first through her research (turn 2, lines 1-2), then through her practice (turn 2, lines 2-5). This awareness runs through the whole of the interview with her. Later in the interview (not captured in excerpt 3 above), the TE cites an example with PSTs whose home language is also the LoLT, and who do not know the African linguistic structures and do not think there is a big problem in the class since they believe that what is key in teaching and learning is the PSTs' proficiency in English – the LoLT. This also foregrounds the TE awareness of her context of teaching and the PSTs' lack of awareness of the important role language plays in multilingual contexts of teaching.

But while the PSTs who share the same language as the LoLT for the teacher in excerpt 3 are unaware of the African linguistic structures and how they impact on the teaching and learning of mathematics, the TEs also view the PSTs who have a different language to the LoLT (as reported by other TEs as evident in the excerpt below), as unaware of their future role as mathematics teachers in multilingual class:

Excerpt 4

3	R	…you have just told me an interesting story about how you wanted to introduce the multilingual mathematics dictionary in your class. Can you please repeat it?
4	T.E	Yeah, yeah, actually what happened was that I was part of a project…and, elm, I don't know which organisation created this mathematics dictionary

> that include all the official languages. And I thought…Waoh, I am going to include the languages, elm, this dictionary in my study guide here, especially some terminologies to exploit for example, the concept of circle in all the languages. And the students said, no, they don't want all the languages because they are going to teach in English. So, they want English. But the Afrikaan's students, they preferred the dictionary their class would be mainly in English, and they would be teaching in Afrikaans.

In the excerpt above from one TE, the PSTs whose home languages are different from the LoLT were not, according to the teacher educator, keen to use the multilingual dictionary as they felt that this would further disadvantage them in their quest to become more proficient in the language of instruction (English). This resonates with Setati (2008) who argues that learners want access to English because they are concerned with access to social goods and are positioned by the social and economic power of English. Yet, in my observation of these teacher education classrooms in Phase Two, these pre-service teachers code switched to other African languages during group work. The teacher educator also indicated that during teaching practice, the PSTs do code switch. What can be inferred from this is that the TEs perceive the PSTs as having limited awareness of their future role as teachers in multilingual classrooms. They (PSTs) content themselves with, first and foremost, overcoming their own language difficulties/inadequacies and do not foresee that their learners in the class would have similar challenges as they are currently having. As Wagner (2007) argues, students need to be able to problematise language in such a way that they come to the realisation that language problems are inherent in mathematics classroom discourse. In a later section, I discuss what the implications of all these interview findings mean for the nature of communities of practice. For now, I turn to another finding during the interview phase of the study.

6.2.2 How Teacher Training institutions attend to teaching mathematics in multilingual classrooms

Even though most of the teacher educators involved in this study were aware of the context of their practice – that they are teaching mathematics to pre-service teachers who would, at the end of their training teach mathematics to multilingual learners – they (TEs) admit that at the institutional level, they do not receive enough support by way of courses aimed at enculturating pre-service teachers into the dynamics of teaching mathematics in multilingual classrooms as evident in excerpt 5 below:

Excerpt 5

R How do you prepare your students to teach mathematics in multilingual contexts?

TE We don't prepare them in any structured way, we prepare them by taking up suggestion that may arise during practical teaching and talk about those during the method class. And we take it up by making them more explicitly aware during their method class to be aware that people don't necessarily understand their English, but nothing structured.

Excerpt 5 from University B is reminiscent of the sentiments of other TEs in the study as also evident in excerpt 6 from University A below where a teacher educator was asked if he thought there was a need to specifically train pre-service teachers to know what it means to teach mathematics in multilingual classrooms:

Except 6

TE Yes. I think we need to include concepts of teaching in multilingual classrooms, so that we look at strategies or approaching of using different languages in our teaching. Also, it is necessary to have modules which focus on language and mathematics and language and communication. This means we are not preparing our student teachers to teach maths in multilingual classrooms.

The TE in excerpt 6 points out that one of the things universities and colleges of Education needs to do is to introduce modules that specifically deal with and aim at creating awareness of the relationship between mathematics and language, especially with regards to multilingual students. Hence, one of the noteworthy findings in this study was that at the institutional level, there are no explicitly structured courses aimed at creating pre-service teacher awareness of what it entails to teach mathematics in multilingual contexts. I will elaborate more on this finding in Chapter Ten. Suffice it to indicate that for the teacher educators, this constitutes an environmental factor that reflects the type of discourse they feel pertinent to use in their classrooms to make up for this lack (see excerpt 5).

6.2.3 The case of code switching as a practice

Another noteworthy finding in this study was that code switching as a practice was constrained irrespective of the classroom community – irrespective of whether the teacher educator shared the same home language with the PSTs or the TEs' first language is the LoLT and so, they do not share the same home language with the PSTs. All twelve teacher

educators said they did not code switch in class and this was evident during observation of the four teacher educators in Phase Two of the study. Two constraining factors emerged from the interviews as far as code switching is concerned. The first limiting factor concerns the TEs who shared the same home language with the PSTs but did not code switch because of the language infrastructure of the class in which they teach; the second constraining factor concerns TEs who did not share the same home languages as most of the PSTs and hence could not code switch. Excerpt 7, involving three TEs[49] captures both instances:

Excerpt 7

R Do you sometimes code switch?

TE1 Elm, for me, it is difficult because of the background. Because the students we have here are from all over South Africa. You find students as far as Venda. In fact, even in the class, we have Vendas, Spedis, Shangans, we've got Zulus who are in the majority, we've got Xhosa and Ndebele. So, for me as a lecturer, when to code switch is going to be a problem, because some would not understand. It would disadvantage others when I try to…

TE2 ..for me the best practice would be if I know the languages to such an extent that I can use the metaphor of the languages. If I knew these languages, I would still teach in, say, a shared language, English for instance. But if I were aware of imageries evoked by the different languages [present in the class], I would make a point of bringing that in. I am talking about something like, if you think of the concept of multiplication. There is a lovely Zulu word for multiplication called *phinda phinda*, right? When they don't understand and you say 'it's like *phinda phinda'*, ...they say '*ohhhhh, repeat repeatedly*'. And they understand it. So if I know more of those, I would use them and make explicit in teaching. If I knew them, I would bring them in to enrich mathematical discussions. And I think it can help English speaking people to generate more interesting cases of mathematical applications.

TE3 …usually and I say to them, if you can't talk about it in English, I'm going to let you code switch amongst one another if one can help the other better and scaffold the other in their own language. I do allow code switching if they are working informally together and I'm just facilitating and walking around... I don't ban their mother tongue from their classrooms. But their mother tongue is used for scaffolding purposes from their peers who know better than the weaker ones. Sometimes, yes, I feel very sorry that my own education and the times in which I grew up, I wasn't allowed to learn another language which would have enabled me to code switch. But I realise that yes, I think sometimes it is a problem. And it is a problem because I feel disadvantaged, because, sometimes I like to code-switch myself to do the scaffolding, and I can't. So, it is

[49] This is TEIA-S from University A, while TE2 and TE3 are TEIB-E and TEIB-L respectively from University B

30 a problem from that point of view.

Notwithstanding the nature of multilingualism in South Africa as discussed in Chapter One, TE1 viewed code switching in her class as a practice that would disadvantage some PSTs (lines 6-7) if used in the negotiation of meaning. Hence, she chose not to code switch while teaching.

For TE2, a good practice would be to be able to code switch in multiple languages to engage with pre-service teachers using different metaphors and imageries around a mathematical concept in the different languages present in her class to enable epistemological access (lines 9-12). For TE2 another related use of code switching for teaching in multilingual class would have been in the ability to use subtle differences for expressing a mathematical idea/word that exist in one language to enrich discussions around the mathematical concept under consideration. She cites an expression in the Zulu language that expresses the concept of multiplication – *phinda phinda* – "repeat repeatedly" and how she uses the few words she knows in some of the pre-service teachers' home languages to teach in class (lines 12-18). Studies (see for example, Arzarello, Robutti, & Bazzini, 2005) have shown that the use of analogical representations (metaphors) can help in the development of mathematical concepts. This, in my opinion, is much more so if in a multilingual class the metaphors are in the home languages of the learners and are used to enrich classroom discussion around a particular mathematical concept.

What also comes to the fore in TE2 and TE3's utterances in excerpt 7 is how they position themselves with regards to code switching. Even though as teacher educators themselves they did not code switch in class, they both agreed that code switching is a good practice which could have added value to their teaching. In fact, all twelve teacher educators who were interviewed said they encourage the pre-service teachers to use code switching in the classes. They do so by asking learners for mathematical expressions in their home languages, by asking learners to interpret/translate to the teacher educators when a pre-service teacher asks questions in another language not familiar to the TE, by encouraging learners to do group discussions in a language they are comfortable in and/or by using metaphors that the different languages in the class potentially provide for use in mathematics. In a way, their mutual engagement in the class and what practices are privileged is shaped to some extent by the compensatory tools that are used in the absence of direct code switching by the teacher

educators themselves. Excerpt 8 is indicative of the sentiments of other TEs with regards to this. TE1 is from University B while TE2 is from University A:

Excerpt 8

1	R		Ok, do you notice any linguistic challenge when you visit them during teaching practice?
2	TE1	1 2 3 4 5	My usual resolve is that if you see that your learners are grappling with the language, break the concept in as many parts as possible. Let the learners bring in some input, maybe *call a learner and say, how would you explain it in Xhosa.* Then the learner would explain in Xhosa and you ask: have you understood? Yes. ...
3	TE2	6 7 8 9 10 11 12	Ok, What I realise is that they use code switching a lot [during teaching practice]. I don't have a problem because the learners would benefit. And I always encourage them in class to code switch. But when on teaching practice, they can only code switch if they realise that the learners have a problem with understanding something. It should not be a matter of making things easy for them [pre-service teachers]. It should be to the advantage of the learners... The medium is English, and from there, they can only code switch where there is a need.

TE1 uses the practice of asking pre-service teachers to explain concepts in their home language in the hope that this would enable epistemological access to concepts which the pre-service teachers are struggling to understand (turn 2, lines 1-4). What is interesting is that TE2 also encourages code switching as a strategy of teaching, but on one condition, namely, that it is for the benefit of the learners in the class and not as a strategy for enabling the pre-service teachers to switch to a more comfortable language for them in order to communicate their message. Hence, code switching is for the learners not for the pre-service teachers. In TE2's practice, she indicated that she tries to get learners who are not proficient in English to ask questions in English and only employs the services of a translator as a last resort. This strategy of persuading pre-service teachers to communicate mathematically in English resonates with the strategy used by other TEs who also said that they encourage their pre-service teachers to present lessons or tasks in English so they can enrich themselves linguistically in the LoLT and become better at communicating mathematically in the LoLT. In this regard, Kasule and Mapolelo (2005, p. 611) note the following concerning the dilemma of being an "African" teacher:

> African teachers know that they must enhance learners' exposure to the English language, must overcome their own sense of inadequacy in that language, and must ensure that

> learners are prepared for higher education and that outside world, so they must not code switch; but they must ensure that learners understand and participate in classroom talk...

This is exactly what TE2 envisages when she discourages her pre-service teachers from code switching to make up for their English language deficiencies, and when she and the other TEs encourage their pre-service teachers to communicate in English despite their pre-service teachers' low proficiency in English. In a study by Setati (2008), it was also found that both teachers and learners who position themselves in relation to English are concerned with access to social goods (higher education, jobs, etc). The conclusion that can be drawn from the above discussion is that even though code switching is an important strategy for access to mathematics in multilingual classrooms, sometimes the deliberate use of English is essential for enculturating pre-service teachers into the (English) mathematics register. In my estimation, the call by de Klerk (2002) for policies and practices which cater for the development of concepts in home languages while at the same time ensuring adequate access to English, is pertinent not only in the early education of learners, but also in pre-service teacher training. The dilemma of code switching, of transparency and of mediation (Adler, 2001) would nonetheless remain a potent reminder of the tension involved in striking a balance between epistemological access, access to language and participation.

6.3 Conclusion

This chapter has provided insight into teacher educators' awareness of the context of their practice and into how teacher education institutions attend to teaching mathematics in multilingual classrooms. It also provided insight into the case of code switching as a practice and why teacher educators say they do not use it in their practice. What these findings in Phase One indicate about the shared repertoire and mutual engagement was also discussed. In the next chapter and in Chapter Eight, I present findings which are predominantly from Phase Two of my study and use some of the findings in Chapter Six to understand data which emerged from classroom observations.

CHAPTER SEVEN
Shared repertoire and mutual engagement at University A

7.1 Introduction

In this chapter, I engage with the shared repertoire and the mutual engagement dimensions within University A[50] using mainly data obtained in phase two. Hence, in this chapter, I provide answers mainly to five analysis questions which relate to the shared repertoire and the mutual engagement dimensions of the communities of practice at University A (TEIA). These questions are:

1. What mathematical practices are in use in the negotiation of meaning in the mathematics community?
2. What norms of practice are in use in multilingual mathematics classrooms of pre-service teachers and how do these norms co-construct the mathematics PST education communities?
3. What are the common discursive repertoires within the communities, and how do they co-construct these communities?
4. What communicative approaches and patterns of discourse are prevalent in the mathematics PST education classrooms? And where does authority stem from?
5. How does classroom engagement support the different interacting identities within pre-service teacher education classrooms?

In answering the above questions, I first discuss TEIA-M's classroom community before engaging with TEIA-S's Classroom CoP. The chapter concludes with a general discussion on University A in relation to the findings relating to the shared repertoire and mutual engagement dimensions of the two communities of practice.

[50] In Chapter Eight, I also deal with the shared repertoire and the mutual engagement at University B, while in Chapter Nine, I draw inferences as to the joint enterprise at these two universities based on the analysis of the ME and the JE of these two universities.

7.2 The Shared repertoire and mutual engagement at University A (TEIA)

The discussion in Chapter Six highlights how teacher educators in my study understand the context of their practice in teacher education classrooms. The teacher educators are aware, albeit differently, of the fact that they are teaching multilingual PSTs who, according to the teacher educators, are unaware of the complexities of teaching mathematics to multilingual learners. The discussion also highlights the fact that at an institutional level, there is no explicit attention paid to the training of pre-service mathematics teachers for teaching in multilingual contexts. Finally, the discussion also highlights the fact that the teacher educators involved in the study do not code switch despite acknowledging the importance of code switching as a practice and encouraging its use in group work and during the PSTs teaching practical in schools. In the next section, I use data mainly from classroom observations to engage with the dominant practices in two teacher education classroom communities of practice at University A. As indicated in Chapter Four, these two classroom communities were taught by two different teacher educators whom I have called TEIA-M and TEIA-S.

In an attempt to analyse the predominant practices in each of the teacher education communities of practice in University A (TEIA) in such a way that goes beyond mere frequency counts of the instances in which a particular practice occurred, I engage (in an integrated manner) with the five analysis questions that concern both the shared repertoire and mutual engagement simultaneously. In doing this, I use the privileged practices as my point of departure, and then reflect on how the practices shaped and are being shaped by other dynamics (such as the norms, the pattern of discourse, the shared language, where authority stems, the projected identities, etc.) in the community of practice. In so doing, I provide answers to the other four analysis questions. Following the "didactic strategy" of Andrews (2009), I also attempt to juxtapose some practices with other practices to gain more insight into the pattern of discourse that is prevalent in the community of practice. For example, where the teacher educator consistently uses examples that gradually lead the discussion into the definitions of concepts embedded in the example, I have tended to juxtapose defining and exemplifying practices in my analysis.

7.2.1 Privileged practices at TEIA-M's CoP

As a background to the series of lessons that were observed in TEIA-M's class, this was a statistics class for third year pre-service teachers. As a product of the old SA curriculum, the pre-service teachers were encountering statistics for the first time since they did not do statistics as a mathematics topic at high school level. English is not the home language for both the teacher educator and the pre-service teachers. The table below is an overview of the predominant mathematical practices and norms of practice in TEIA-M's classroom community.

SHARED REPERTOIRE DIMENSION OF TEIA-M's CoP					
NATURE OF M. PRACTICE	**MP**	**TOTAL**	**NATURE OF N. PRACTICE**	**NP**	**TOTAL**
Initiating and/or sustaining mathematical discussion practices	Explaining mathematically [MP-EM]	48	Conversational norms	Participation by all [NP-PA]	1
	Defining Mathematically [MP-DM]	14		Collaboration norm [NP-CB]	10
	exemplifying [MP-PE]	15		Choral Response	111
Other mathematical practices	Reiterating [MP-Rt]	23		Taking turns to speak norm [NP-TT]	
	Proceduralising [MP-Pc]	66	Conceptual norms	Mathematically sensible norm [NP-MS]	0
	Using Multiple Approaches [MP-MA]	3		Consensus norm [NP-CS]	1
	Writing Mathematically [MP-WM]	22		Non-Ambiguity norm [NP-NA]	0
	Monitoring solution [MP-MS]	8		Justification norm [NP-JN]	0
Evaluating mathematical	Providing Justification [MP-PJ]	8	Interpersonal norms	No sole arbiter of knowledge norm [NP-	0

validity practices				SK]	
	Critiquing solution [MP-CS]	3		No Ridicule norm [NP-NR]	0

Table 7.1: Privileged practices and norms of practice evident in TEIA-M's Classroom CoP

Practices that are privileged[51] in the community are practices that were valued consistently across the data segments of each community of practice and which had a bearing on the overall aim of the study and the overall perceived joint enterprise of the community in question. Table 7.1 above indicates that privileged practices in TEIA-M's classroom community of practice were mostly within the initiating/sustaining mathematical discussion practices and other mathematical practices. Evaluating mathematical validity practices were less prominent in the CoP. As far as the norms of practice are concerned, the table also indicates that collaboration norm and choral response were the most 'visible' norms[52]. To reiterate, in deciding on the frequencies of the norms, I looked at instances of (breaches to) regularities in the way the classroom community acted and interacted. Also, the norms were first developed a priori and a posteriori additions made based on data from TE classrooms.

In what follows, I discuss four of the privileged practices and present a discussion of how each practice was used by the teacher educator and what happens when she uses each practice. In doing this, I bear in mind the analysis questions that this chapter set out to engage with. The four practices are defining mathematically, explaining mathematically, exemplifying and proceduralising. To reiterate, as discussed in Chapter Four, the advantage of using segments in my data coding and analysis was that it enabled me to determine which practices were consistently valued across the segments. The decision as to what practices are prevalent in TEIA-M's community of practice (as with other CoPs) was based on this consistent pattern across the segments.

[51] Sometimes I refer to prominent/dominant/predominant practices. Prevalent practices can be said to be dominant practices which have been privileged by the communities.

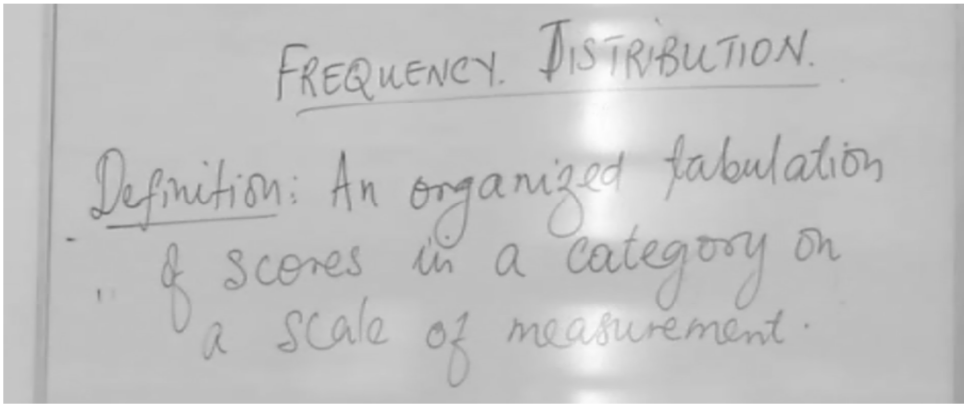
[52] It must be noted that due to the communicative approach and the pattern of discourse used by TEIA-M, it was difficult for me as an outsider to ascertain the presence of other norms.

7.2.1.1 Defining mathematically

One of the prominent practices in TEIA-M's classroom was using defining as a practice. TEIA-M always introduced a new statistical concept by first defining it orally, then writing the definition on the board, and finally reading it out to students. As an example, I will take the introduction of two statistical concepts: frequency distribution and central tendencies. In the definition of frequency distribution in excerpt 9, the teacher educator writes the formal definition on the board:

Excerpt 9

1	TE	Today we look at frequency distribution (writes frequency distribution on the board). What is frequency distribution? Let us look at the definition part of it *(writes "definition" on the board)*. This is nothing else but the organised tabulation of scores in a category on a scale of measurement *(writes this definition on the board)*. [ ] We would start with a few scores and organise them in a table… Let's consider the scores *(writes, 8, 9, 8, 7, 10, 10, 6, 4, 9, 8, 7, 8, 10, 9, 8, 9, 9, 7, 8, 8)*. So, how do we organise this information? *(pause)*. One of the ways of organising this information is using the frequency distribution table. Yes?
2	PST	Yes (in chorus)
3	TE	Right, … From the given scores, what is the smallest?
4	PST	it's 4 (in chorus)
5	TE	The smallest is 4 right. And then, what is the largest?
6	PST	10 (in chorus)
7	TE	10, right. I want you now to arrange these scores in ascending order….What is the frequency of 4?



TEIA-M then goes on to elaborate on key terms like scale of measurement, tabulation, and frequency. This same practice is used in all the definition of terms in the class. In what follows, I present how TEIA-M introduced the concept of central tendency:

Excerpt 10

1	TE	TE: The next organisation of scores is by looking at the concept of expressing the scores in terms of a single score, the concept of central tendency *(writes 'central tendency' on the board)*. What do we mean by this central tendency? If you have a distribution, and then you want to identify a single score that would be representative of the whole population, then we would be speaking of a central tendency. Let me give you an overview: (writes on the board: *Central tendency is the statistical measure that singles out or identifies a single score as a representative of the entire distribution*; then repeats the definition verbally to students and continues)
2	TE	TE: Now, there are 3 main measures of central tendencies *(writes while reading: the main measures of central tendency are the mean, the median, and the mode*). We use the same distribution to look at the 3 measures of central tendencies. Let's start with the mean. How do we calculate the mean from the definition? You add all the scores and then divide by the number of the scores.
3	PST	PST: [PSTs copy from the board]

What comes out clearly in both episodes of defining is that TEIA-M gives the PSTs the conventional or formal definition of the concepts without input from the PSTs. The students are usually not given the opportunity to attempt or question a definition given by the TE. The PSTs seemed to be familiar with this particular practice and so do not offer definitions or suggestions, etc when the TE asks for definitions (for example in turn 1, "*what do we mean by this central tendency?")* because they take it for a rhetorical question, knowing the TE would provide the answer herself. The PSTs preoccupied themselves with copying what the TE had written on the board. In TEIA-M's CoP, the communicative approach that was used in the development of new mathematical ideas was mainly authoritative, and more specifically, interactive/authoritative because there was no interanimation of ideas and the TE focused mainly on one point of view (school mathematics) and even though the class was interactive, the class was authoritative because the TE led the PSTs through a sequence of routine with the aim of entrenching one particular view – that of the formal definition of the mathematical concepts at hand. It is therefore not surprising why there was a low level of evaluating mathematical validity practices in TEIA-M's classroom community.

But even though defining as a mathematical practice was important for TEIA-M, the importance of defining as a mathematical process that is important for teaching and learning in mathematics classrooms was in the background. The practice of defining as used by TEIA-M was anchored in the development of the identity of becoming learners of mathematics content[53] (BLMC). In other words, as far as defining was concerned for TEIA-M's community of practice, the content was the overriding object of attention (and not the process of coming to define)[54]. It can be argued that for TEIA-M, defining as a practice was focused on how the pre-service teachers can use the definition to solve mathematical problems rather than on how the community can construct definitions of concepts through interanimation of ideas that the pre-service teachers bring to class. Defining as a practice thus served a utilitarian purpose of being the window towards mathematical calculations. For example, in excerpt 9 above, after constructing the definition of frequency distribution on her own, the TE then expected the class to apply the concept in solving a mathematical problem. This seems to suggest something about her understanding of the role of definitions in mathematics and in mathematics teaching and learning.

7.2.1.2 Explaining and proceduralising practices

Explaining as a mathematical practice, occurred as part of the process of communicating mathematical strategies, and hence was mostly linked with the practice of proceduralising. In excerpt 11 below, the teacher educator explained how to find the mean of a distribution:

Excerpt 11

1	TE	Right, let's start with the measure of central tendency – the mean. *[writes:* MEAN *].* How do we calculate the mean ? from the definition? You add all the scores. Right. Find the sum of the scores and then you divide them by the number of the scores, so that we've nothing else but the average, né. The average which is the sum of the scores, you divide that by the number of the scores.
2		*[after* MEAN *writes:* (AVERAGE)

[53] In this study, content is taken as the mathematical concept that is the object of attention at a particular time in the classroom discourse.

[54] I will return to this point in the discussion section for University A.

3 TE For that we use capital letter X with a bar above to denote the main score, that will be the average. To determine that, we are going to add all the frequencies. So find the sum of the scores that will be in the frequency… Sorry, the sum of all the scores. In this case it's going to be the frequency multiplied by the score and then you divide this by the sum of the frequencies.

After the definition of the mean, the TE uses the data set that has been used in the teaching of frequency distribution to find the mean of the distribution. The explanations centred on the procedures for finding the mean. Again, the explanation of the mean flows from the definition and is centred on how the mean is calculated.

On some occasions, mathematical explanations were used to explain mathematical concepts (beyond the formal definition of the concept) as in the explanation of the concept of mean and standard deviation in excerpt 12 below. In the discussion about mean below, the class had just determined the mean score from a frequency distribution. On the board was written: $\bar{x} = \frac{162}{20} = 8{,}1$

Excerpt 12

1 TE What, what does average mean? [pause] Look, from the number of scores we can say we have about 20 individuals, né? From that number of scores, the raw scores that we have we can say that because, I mean, the sum of the frequency as it is here it's 20. Right?

2 PSTs Yes (some PSTs)

3 TE So the average means each one of those 20 individuals would have a score of?

4 PST 8,1

5 TE 8,1. When we speak of equal distribution, né? So each one of those individuals will have a score of 8.1. That's what we mean by the mean…

And in another instance where the class was dealing with standard deviations for the distribution, 8, 0, 1, 3:

6 TE Now check from the, those deviations. What is the sum of the deviations?

7 PSTs Zero

8 TE The sum of all is *[writes:* $= \sum (x - \bar{x}) = 0$ *]*

9 PSTs zero

10 TE It's zero. What does it mean?

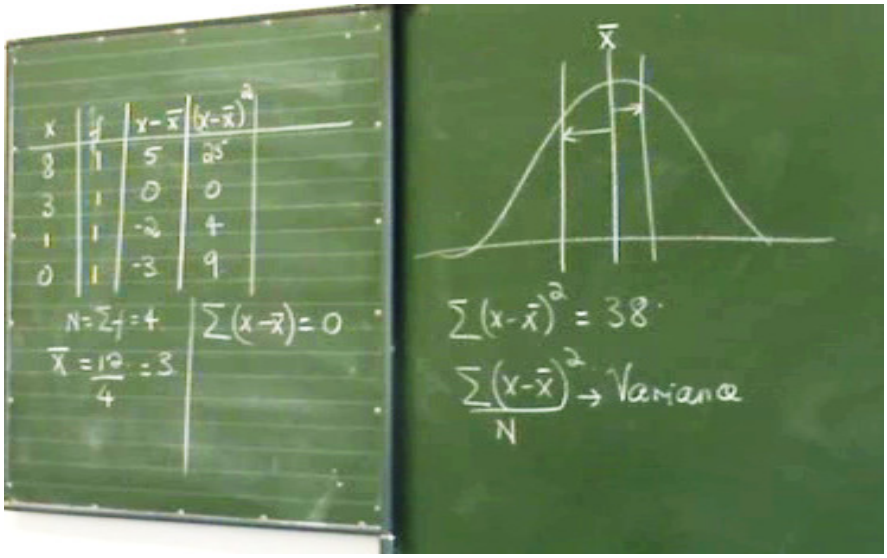
11 PSTs *[silence]*

12 TE Right. Ok. In the normal distribution you have the normal distribution shape like a bell, right

13 *[draws a bell graph]*

14 TE And then we expect all the scores, the mean, to be in the middle. Now the deviations remember that we can have scores to the right of the mean and scores to the left of the mean, right?

15 *[draws this on the graph]*



16 TE So when you take the sum of all the distances of all the scores from the mean either to the right and to the left it should be zero. They should all come back to the mean, right?

17 TE So we are saying that the sum of all the distances of the scores to the right of the mean and the scores to the left of the mean is going to be zero. And that all the scores must converge at the mean. So for some deviations that we have got an indication that the sum of all the scores to the right of the mean and to the left of the mean will be zero. Now looking at this we can now explain the standard deviations using the deviations, right?

The first instance consists of the explanation of the concept of average. In turn 3, the teacher educator explains what 8,1 (which was obtained as the answer to the average of the distribution) mean. In the second instance, the teacher educator also attempts to answer the question as to what $\sum(x - \bar{x}) = 0$ means, even though the question is not fully answered.

Even though proceduralising was used in relation to explaining as a practice, most of the instances of proceduralising, however, were aimed at working with the pre-service teachers towards providing the solution of the mathematical task at hand. In the excerpt below where

the PSTs were asked to find the variance and the standard deviation of a distribution, during whole class feedback, what was evident was the step-by-step interaction by the classroom community as they found the solution to the task:

Excerpt 13

1	TE	Let me give you a question to solve for me. You can use either of the two formulas [teacher writes on the board the following question: Consider the population scores: 1, 9, 5, 8, 7. Compute a) variance, b) standard deviation]
2	TE	*(after 10 minutes)* Can we check those results? Are you through?
3	PSTs	Yes
4	TE	Ok. Let's check, elm *(pause)*. From this *(points to the scores)*, can I deduce the frequency distribution table or go straight to the formula, to using the formula? Right. We can construct the frequency distribution table (draws the table on the board). How many scores do I have in the distribution?
5	PSTs	5
6	TE	Let's start with the highest. What is the highest score?
7	PSTs	9
8	TE	And then?
9	PSTs	8
10	TE	Followed by...
11	PSTs	7
12	TE	And then?
13	PSTs	5, ...and 1
14	TE	Right. 1 is the smallest. Let's check the frequency of each. What is the frequency of 9
15	PSTs	1
16	TE	So each of the frequencies is 1, right *(writes 1 against all the scores in the frequency column)*?
17	PSTs	Yes
18	TE	So, we can calculate the number of the scores. What is the number of scores?
19	PSTs	5
20	TE	From there, what is the average, the mean?
21	PSTs	6
22	TE	30 divided by 5. So the mean is
23	PSTs	6.

[The class continues in a similar manner]

In both excerpts 12 and 13 where explaining and proceduralising were used as practices, much like defining, only the teacher educator was responsible for mathematical explanations during the whole class discussions. The discourse mostly outlined the procedures to be undertaken to solve a mathematical problem and was less designed to explain *why* (providing justification code) or to explain one's/a fellow PST's thinking[55]. Again, like in defining mathematically, what is foregrounded in terms of the interacting identities in TEIA-M's classroom is the identity of becoming learners of mathematical content. Engaging with the importance of how to explain to learners, when to and when not to proceduralise in the class was not a feature of the TEIA-M's discourse and hence, there was no explicit attention paid to becoming learners of mathematical process of proceduralising. Neither was there explicit attention to developing the identity of becoming teachers of mathematics (BTM) beyond the consideration of the teacher educator as modelling what good practice means in terms of these practices.

A feature of TEIA-M's class that is worth mentioning was the choral response norm during explanations and proceduralisation because it permeated the discourse in TEIA-M's class as evident in excerpts 9 and 13 above. Wang (2005) distinguishes four variations of choral responses based on differences on three dimensions – the mathematics content exchanged, the level of discourse formality and the level of participation. The four variations are: a) *choral reading*, in which the teacher asks students to read what has been written on the board, textbook, etc. or to repeat what has just been said by the teacher; b) *co-narrating*, in which the teacher joins students as they respond in chorus; c) *answering simple questions*, in which the teacher asks simple mathematical questions and the students respond in chorus with a short answer; and d) *answering tag questions*, in which the teacher does not expect a formal reply from students and continues even when the students voices could hardly be heard. Wang (2005) argues that of all four variations, the last two are at the lowest level in terms of formality, participation level and the importance of mathematical content. These last two were the two prominent variations of choral response that were prominent in TEIA-M's class. In excerpt 9, the students responded to short simple mathematics questions (answering simple questions) and even when the TE asks for a definition, she did not specifically expect

[55] Esmonde (2009) defines mathematical explanations as utterances that are designed to explain why, explain how and explain one's thinking.

students to respond to the question and provided the definitions herself (answering tag questions). Excerpt 13 shows the same choral response after the students have worked in groups on the problem. In excerpt 13, it is clear that TEIA-M only asked students questions or prompts that require short procedural answers, and also, there are instances in the excerpt where the teacher would pause a bit after asking a question and would eventually answer the question herself (excerpt 12 turns 5 and 14; excerpt 10 turn 1). They were also instances where only a few students responded.

Choral response as a practice has its own advantages. Wang and Murphy (2004) argue that the frequent use of choral responses can reinforce the students' identification with the group and create a sense of solidarity. Wang (2005, p. 50) argues that as a pedagogic strategy, choral responses provide a platform for every student to "get the floor without bidding". But as with choral responses, TEIA-M favoured the group voice over the individual. The implication of this is that pre-service teachers who may have been struggling to understand may have been overlooked, and secondly, value was not placed on speaking mathematically (MP-CM) since the answers were short procedural answers in nature and the group voice and not the individual voice was what TEIA-M perceived as the answer. As Wang (2005, p. 50) puts it, "in Choral Response, the teacher is the only targeted listener, where[as] in individual response, an individual student speaks to both the teacher (primary recipient) and the rest of the class (the secondary recipients). Another drawback of choral response as was evident in TEIA-M's class was that there was no verbal feedback because, most of the times, members of the classroom community realised that the answers are obviously correct due to the nature of the questions posed by the teacher.

7.2.1.3 Exemplifying as a practice in TEIA-M's Classroom CoP

Bills *et al.* (2006) argue that in mathematics, it is not so much the examples in themselves that are important, but what is done with those examples and how they are probed, generalised and perceived. TEIA-M used one or two examples for each statistical concept that she taught. In the excerpts that follow, I use two of such examples to inform my discussion on how TEIA-M uses exemplifying practices to achieve the purpose of her lesson. These examples used by the teacher educator, offer insight as to what the TE perceives as what counts as mathematical knowledge and how it is learned:

Excerpt 14

1	TE	There are three ways of determining the median.*[talks while he writes:* Three ways of determining the median. Right. If you have… the number of the scores is odd you just choose the middle one, number. Right. If N is odd, the middle number is the median. The simple example will be if you take the numbers 1 2 3 4 and 5, the median is just 3.
2		*[next to* (1) *writes:* If N is odd, the middle number is the median. e.g. 1 2 3 4 5 the median is 3. *]*
3	TE	So you could arrange the numbers either in descending order or in ascending order to identify the median. Right. What about if the number of the scores is even? If N is even, right, you take the two middle numbers, you add them and then you divide it by 2. So you take the average of the middle numbers. The average of the two middle numbers is the median.
4		*[writes:* (2) If N is even. The average of the two middle numbers is the median. *]*
5	TE	Right, let's say we take this as the score *[writes:* e.g. 1 2 3 4 5 6 *]* The number of the scores is even. N is even, right? How many scores do they have?
6	PSTs	6
7	TE	6. Is 6 an even number?
8	PSTs	Class: Yes.
9	TE	T: Alright. What are the middle numbers that we have there?
10	PSTs	Class: 3 and 4
11	TE	T: Huh?
12	PSTs	Class: 3 and 4
13	TE	T: So if we add 3 and 4 and divide by 2…
14	PSTs	[Some students] 3.5
15	TE	T: we'll get 3 plus 4, divide by 2. This is the average of this, middle numbers, right? So that is 7 divided by 2, that is 3,5.
16		*[writes:* $\frac{3+4}{2} = \frac{7}{2} = 3{,}5$]
17	TE	You see the median may not necessarily be the number that is part of the distribution. So from this, this indicates that the median may not necessarily be part of the distribution.

Several arguments can be made with regards to the examples provided by the teacher educator in excerpt 14 above which are typical of the type of examples that TEIA-M solved in the class, and excerpt 13 above which is typical of the type of examples the TE gave to

PSTs to solve as a classroom activity. First, there was a mix of what Bills *et al*. (2006, p. 2) call "work(-out) examples" (the examples that the teacher educator performs in class) and "exercise examples" (where tasks are set for pre-service teachers to engage with on their own). The "work(-out) examples" were aimed at both concept development and the application of a mathematical procedure. Bills *et al*. (2006, p. 2) identify three descriptive labels of examples based on the forms and functions of examples and based on how the teacher/learner perceive the mathematical object in question – generic examples, counter-examples and non-examples[56]. A second feature of exemplifying as a mathematical practice in TEIA-M's classroom community was that all the examples were generic examples aimed at serving as a template for pre-service teachers to have tools for solving similar problems involving the concept at hand. As Bills *et al*. (2006, p. 3) note,

> Unfortunately their [generic examples] use in lessons is often reduced to the mere practice of sequences of actions, in contrast to a more investigative approach [ ] in which learners experience the mathematisation of situations as a practice, and with guidance abstract and re-construct general principles themselves.

Exemplifying was used by TEIA-M to explain procedure for solving mathematical problems in the same way definitions were used to explain procedures. What was foregrounded was the application of the definition or example given. Since the pre-service teachers' attention was not drawn to the importance of the choices of examples when working with their future learners, it can be argued that exemplifying as a practice in TEIA-M's class was solely anchored in making the pre-service teachers more knowledgeable in the content of statistics. In other words, the identities of becoming learners of the mathematical processes of exemplifying and becoming teachers of mathematics were backgrounded in the exemplifying practices of the teacher educator. In table 7.2 below, this is evident in the instances of becoming learners of mathematics compared to the other interacting identities.

MUTUAL ENGAGEMENT DIMENSION OF CoP		
CATEGORIES	**CODES**	**TOTAL NUMBER**
PATTERN OF DISCOURSE	Questions that provoke extended dialogue	1

[56] "Generic examples may be examples of concepts or of procedures, or may form the core of a generic 'proof'. Counter-examples need a hypothesis or assertion to counter, but they may do this in the context of a concept, a procedure or even (part of) an attempted proof. Non-examples serve to clarify boundaries and might do so equally for a concept, for a case where a procedure may not be applied or fails to produce the desired result or to demonstrate that the conditions on a theorem are 'sharp'". (Bills et al., 2006, p. 2).

	[PED]	
	TE asks question provoking dialogue [TEQ]	32
	PST asks question provoking dialogue [PSTQ]	5
	TE evaluates [TEEv]	20
	PST(s) evaluate(s) [PSTEv]	0
BUILDING OF IDENTITIES	Becoming teachers of mathematics [BTM]	1
	Becoming teachers of mathematics in multilingual classrooms [BTMMC]	0
	Becoming learners of mathematics [BLMC]	80
	Becoming learners of mathematical practices [BLMP]	0
	Becoming proficient English users [BPEU]	0

Table 7.2: Frequencies of the codes in mutual engagement dimension of CoP for TEIA-M

From the table above, even though there appear to be a lot of instances where the teacher educator asked questions, much of the questions were short procedural questions requiring short procedural answers/responses from the pre-service teachers. There were very limited instances where the questions asked by the teacher educator were aimed at provoking extended dialogue around the issue at hand. More often than not, the teacher educator asked questions that did not initiate dialogue – a typical pattern of engagement in TEIA-M's class. The PSTs seem to be aware of the fact that the TE's questions are rhetorical and as such do not attempt to answer the questions knowing that she would answer them herself. Because this was the case, the PSTs found it difficult to respond to questions that demand justification/extended answers as evident in the excerpt below. The class was working on how to use a simple distribution table of a set of scores (8 9 8 7 10 10 6 4 9 8 7 8 10 9 8 9 9 7 8 8) to work out different statistical problems of which finding the proportion was one. And proportion has been defined as *the score relative to the total number of the scores* which *measures a fraction of the total group associated with each score*.

Excerpt 15

1 TE Now, add these proportions. [on the board is written]

Score (X)	Frequency (f)	P
10	3	0,15
9	5	0,25

8	7	0,35
7	3	0,15
6	1	0,05
4	1	0,05
	∑f =20	

2 PST1 1

3 PSTs [some others] 1

4 TE What is the significance of that? What does it mean? The sum of all these proportions is equal to 1. You've got to add those.

[writes: ∑p = 1 *at the bottom of the* p column *]*

What does it mean?

5 PST1 It's 1

6 PSTs [Some students] It's 1

7 TE What does that mean?

8 *[some laughter]*

9 TE Huh? Anyone? Look, what is the total of the scores that we have here?

10 PST 20

11 TE It's 20. Right? So you are adding these fractions. The total is 20. Right? You are adding the fractions of all these numbers. What is the fraction of, say, 10? That is 3 out of 20

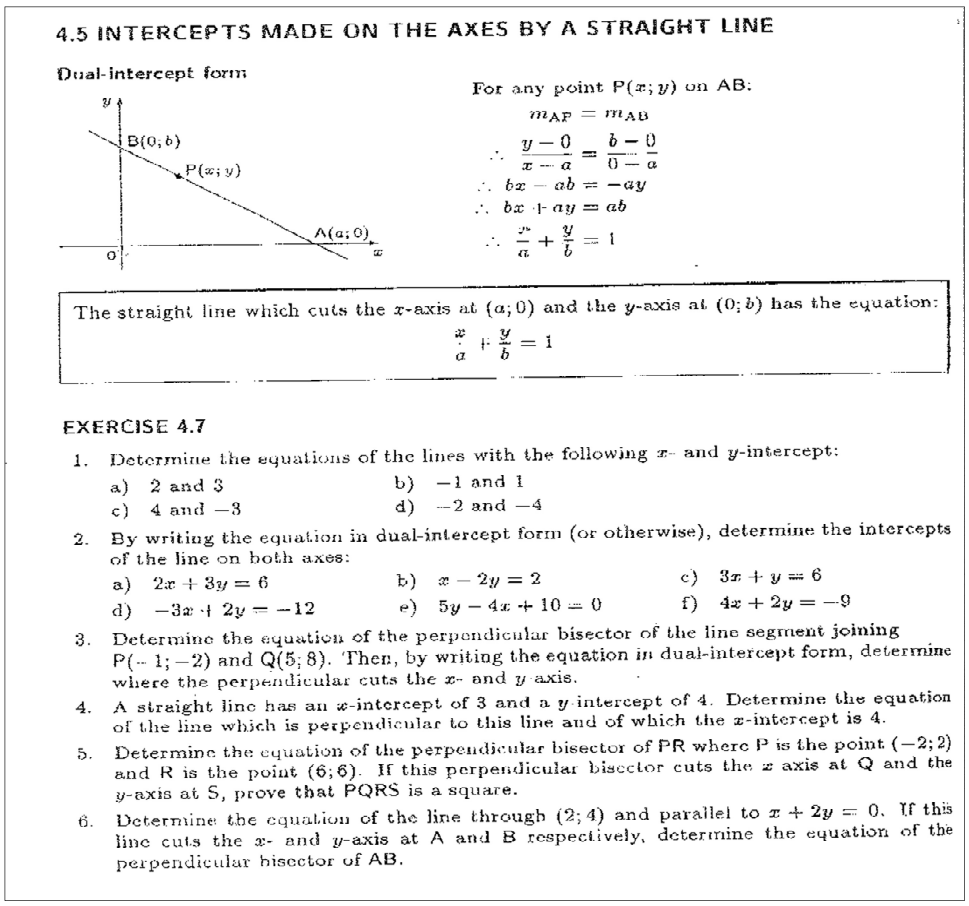
In the excerpt above where the class was discussing the meaning of proportion in relation to the frequency distribution, in turns 4-8, the pre-service teachers struggled to provide an interpretation of the answer which was obtained after the summation of the proportions. This is also the case in excerpt 12 above in turn 1, where the teacher educator asked the PSTs for the meaning of 8,1 which was obtained in answer to the question about the average. After a short pause, the TE answered the question herself. Also, in excerpt 12 turn 6, the teacher educator also asked what $\sum(x\text{-}\bar{x}) = 0$ means and the PSTs still struggled to provide an interpretation. One possible reason for this is that they were not used to being asked justification questions. In general, it can be said that the interaction was reminiscent of what Young (1984, in Edwards & Westgate, 1987, p. 143) terms a tendency for learners to be "obliged to respond within the teacher's frame of reference and at the teacher's bidding". In fact, the pattern of discourse TEIA-M's CoP was mostly in the I-R-E form and from excerpts 9 to 15, it is evident that there was no PST-PST discourse. Much of the validation of

knowledge was done by the teacher educator without input from the PSTs. Due to the manner in which the TE used the I-R-E pattern of discourse (namely in the step-by-step solution to a particular mathematics problem), and coupled with the authoritative communicative approach used in class, it was difficult to delineate the shared references that members of the community used. My data, therefore, does not offer me enough insight into analysis question 3 dealing with common discursive repertoires within CoPs and how these co-construct the community.

I now turn my discussion to the second teacher education classroom in University A–TEIA-S.

7.2.2 Privileged practices at TEIA-S community of practice

Co-ordinate geometry was the mathematical topic which TEIA-S's classroom community was engaged in during classroom lesson observations. The classroom community was comprised of first year pre-service teachers who had been exposed to the topic of co-ordinate geometry previously in high school. To reiterate, English is the additional language for both TE and PSTs in TEIA classroom CoPs. The pre-service teachers were handed-out exercise questions taken from a high school textbook part of which is in the diagram below (see appendix B for the full version):



4.5 INTERCEPTS MADE ON THE AXES BY A STRAIGHT LINE

Dual-intercept form

For any point $P(x;y)$ on AB:

$$m_{AP} = m_{AB}$$
$$\therefore \frac{y-0}{x-a} = \frac{b-0}{0-a}$$
$$\therefore bx - ab = -ay$$
$$\therefore bx + ay = ab$$
$$\therefore \frac{x}{a} + \frac{y}{b} = 1$$

The straight line which cuts the x-axis at $(a;0)$ and the y-axis at $(0;b)$ has the equation:

$$\frac{x}{a} + \frac{y}{b} = 1$$

EXERCISE 4.7

1. Determine the equations of the lines with the following x- and y-intercept:
   a) 2 and 3 b) -1 and 1
   c) 4 and -3 d) -2 and -4
2. By writing the equation in dual-intercept form (or otherwise), determine the intercepts of the line on both axes:
   a) $2x + 3y = 6$ b) $x - 2y = 2$ c) $3x + y = 6$
   d) $-3x + 2y = -12$ e) $5y - 4x + 10 = 0$ f) $4x + 2y = -9$
3. Determine the equation of the perpendicular bisector of the line segment joining $P(-1;-2)$ and $Q(5;8)$. Then, by writing the equation in dual-intercept form, determine where the perpendicular cuts the x- and y-axis.
4. A straight line has an x-intercept of 3 and a y intercept of 4. Determine the equation of the line which is perpendicular to this line and of which the x-intercept is 4.
5. Determine the equation of the perpendicular bisector of PR where P is the point $(-2;2)$ and R is the point $(6;6)$. If this perpendicular bisector cuts the x axis at Q and the y-axis at S, prove that PQRS is a square.
6. Determine the equation of the line through $(2;4)$ and parallel to $x + 2y = 0$. If this line cuts the x- and y-axis at A and B respectively, determine the equation of the perpendicular bisector of AB.

A total of six practices emerged as the dominant practices in TEIA-S's classroom – explaining mathematically, defining, exemplifying, reiterating, proceduralising and providing justification. In what follows, I first provide a table of the frequencies of the shared repertoire codes and then explore in detail these predominant practices within TEIA-S pre-service teacher education community of practice.

SHARED REPERTOIRE DIMENSION OF TEIA-S' CoP					
NATURE OF M. PRACTICE	**MP**	**TOTAL**	**NATURE OF N. PRACTICE**	**NP**	**TOTAL**
Initiating and/or sustaining mathematical discussion practices	Explaining mathematically [MP-EM]	61	Conversational norms	Participation by all [NP-PA]	3
	Defining Mathematically [MP-DM]	12		Collaboration norm [NP-CB]	7
	exemplifying [MP-PE]	15		Speak-Out norm [NP-SO]	0
Other	Reiterating [MP-Rt]	31		Taking turns to speak	0

mathematical practices				norm [NP-TT]	
	Revoicing [MP-Rv]	6	Conceptual norms	Mathematically sensible norm [NP-MS]	5
	Proceduralising [MP-Pc]	40		Consensus norm [NP-CS]	13
	Using Multiple Approaches [MP-MA]	4		Non-Ambiguity norm [NP-NA]	14
	Gestural Symmetry [MP-GS]	2		Justification norm [NP-JN]	18
	Writing Mathematically [MP-WM]	3	Interpersonal norms	No sole arbiter of knowledge norm [NP-SK]	3
	Monitoring solution [MP-MS]	8		Avoidance of threat [NP-AT]	0
Evaluating mathematical validity practices	Providing Justification [MP-PJ]	15		No Ridicule norm [NP-NR]	0
	Critiquing solution [MP-CS]	5			
	Critiquing conjecture	0			

Table 7.3: Privileged practices and norms of practice within TEIA-S's Classroom CoP

From table 7.3 above, it can be inferred that there was a mix of initiating/sustaining mathematical discussion practices, other mathematical practices and evaluating mathematical validity practices. The most prominent evaluating mathematics validity practice was providing justification. As far as the norms are concerned, conceptual norms were the most dominant in TEIA-S's classroom community of practice. In what follows, I provide a more detailed analysis of the shared repertoire in TEIA-S's CoP. To reiterate, privileged practices in the community are practices that were valued consistently across the data segments of each community of practice and which had a bearing on the overall aim of the study and the overall perceived joint enterprise of the community in question. As with TEIA-M's classroom community, I engage with both the shared repertoire and the mutual engagement dimensions of CoP in an integrated manner using the dominant practices as my point of departure and juxtaposing some practices with other practices in an attempt to gain more

insight into the pattern of discourse and communicative approach that are privileged in TEIA-S's classroom community of practice.

7.2.2.1 Explaining mathematically and proceduralising practices

As can be seen in table 7.3, explaining was the most dominant practice in TEIA-S's community of practice. Explaining as a practice in TEIA-S's community of practice occurred mainly as part of the process of communicating mathematical strategies and was, in most cases, intrinsically linked to the practice of proceduralising. In excerpt 16 below where the class was solving the question: *calculate the value of x for which the points M(-2,1) and N(x, -7) are equidistant from P(1,-4)*, after obtaining two values of x, some member of the community thought the positive value of x was the only correct answer because before this, the class had been using the distance formula to find the distance (x) between two points and so thought the x in the problem referred to the distance:

Excerpt 16

1	PST1	why do you take x as equal to 6?
2	TE	yes ma'am, why do we take x = 6? Why don't we take x = -4?
3	PST2	because x is positive
4	TE	ok fine eh calculate the value of x for which M,N are equidistant from this point *[TE repeats the question]*. What do we then do? After finding those 2 values of x ne, after finding those 2 values of x what do we do? We must try and plot and locate, you know, eh the points on the Cartesian plane ok? Who did that? Yes
5	PSTs	[indistinguishable answer]
6	TE	am saying, before or after getting the 2 values right, after getting the 2 values, try and locate a point on the Cartesian plane right, so that you are going to get a value. The person asked the question, the person tried to respond, then I was trying for us to say, after getting the 2 values of x, try and locate a point on the Cartesian plane such that that distance will be equidistant right? So which one is the suitable value of x?
7	PSTs	[indistinguishable chatter]…
8	TE	Let's see if you have drawn that
9		*class works together*

Not realizing that N is in fact a point of which x is the abscissa (and not the distance) and therefore can be either positive or negative, a PST in turn 1 questions why x = -4 and x = 6

are both valid answers. The teacher educator in turns 4 and 6 attempted to explain to the PST and others why both answers were equally valid by giving them a procedure for checking if both answers made sense when plotted on a Cartesian plane. Like TEIA-M, the explanation bordered on procedures, but unlike TEIA-M, the teacher educator in this case demanded of the teacher educators to do the 'checking' themselves.

There were other instances, however, where instructional explanations were not used in the communication of mathematical strategies, but rather for a deeper development of the concept at hand. In such cases, explaining mathematically (MP-EM) was not the same as communicating mathematical strategy or proceduralising (MP-Pc). Such was the case with the instances in excerpt 17 where the class was dealing with the meaning of midpoint:

Excerpt 17

1	TE	Eh fine, do you understand his answer? He is saying the midpoint, according to him, eh, can you explain again? Can we listen to his explanation?
2	PST	The distance from one point to the midpoint is equal to the distance to the another point.
3	TE	That explanation is the right explanation of equidistance. Eh now what are we talking about a midpoint? If we talking about the distance from this corner up to that corner ne, there is somewhere which is the middle of this distance ne, so if we talking about the middle of the distance from that corner to that corner, it must be along the way to the corner ne, right? Do you get that? if we talking about the midpoint from point A, the midpoint of point A to D, it must be somewhere along the distance from point A up to point D eh?
4	PST	murmuring
5	TE	Ok let me explain again; when we talk about a midpoint we are talking about a point which is in the middle, in the middle of what, our way, in the middle of our way to the next point ne? so it must be somewhere along the way right, the midpoint of point A and D, the midpoint of this line segment is a point which is somewhere in the middle of this way from point A to point D; the midpoint of the distance from that wall to that wall will be a point somewhere in the middle of this way; it means the midpoint will be along this way ne? Now let us look at point P. Point P is here, can you see point P?
6	PSTs	Yes

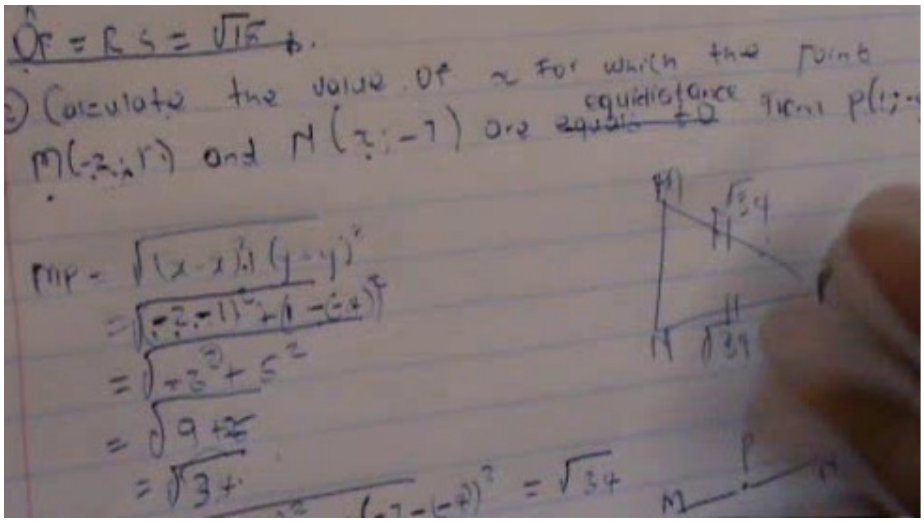
The explanations provided by the TE in turns 3 and 5 does not border on the procedures for finding the midpoint but on the meaning of the term *midpoint*. Put differently, the teacher educator engaged with the *what* rather than the *how* of the concept of midpoint.

A striking phenomenon in TEIA-S's class was that during whole class teaching, explanations were given solely by the teacher educator while during class activities when the TE engages with a particular PST's work as she moved from group to group, explanations were built by both the TE and PSTs through discourse surrounding the task at hand. Leinhardt (2001) argues that when instructional explanations are developed through a process of group discourse, more meaningful classroom talk is promoted. In TEIA-S's classroom community, this was evident in that the communicative approach and pattern of discourse changed depending on whether it was whole-class teaching or a one-on-one with the pre-service teacher. In excerpt 18 below where TEIA-S was engaging with three PSTs (one after the other) on their solution approach to the question in excerpt 16, the communicative approach and pattern of discourse used for one-on-one exchange with PSTs is evident:

Excerpt 18

1	TE	Eh, who got it right? Who thinks got it right? [talking to a student]… eh your PM, why do you have PN^2 why do you have PM^2?
2	PST1	PM =PN
3	TE	Why do you say PM is equal to PN?
4	PST1	Because, it's the line; they say that it's equidistant so
5	TE	and then?
6	PST1	here we take out the square root sign by squaring both sides
7	TE	Eh is this negative y?
8	PST1	negative 4
9	PST1	another question please
10	TE	Eh? Another question? Which other question do you want me to ask?
11	PST1	any other one that will come from your mind concerning these
12	TE	[NOW TALKING TO WHOLE CLASS] Did you get y = -4?
13	PSTs	Yes (some others say no)
14	TE	did you get y -4?
15	PSTs	[multiple chorus responses] x
16	TE	x -4?
17	PSTs	[some students]: Yes. Positive. Yes. No, it's negative…
18	TE	the issue of majority vote does not work in here or else we begin to…[laughter, then discussions and noisy chatting]
19	TE	[with a student] eh why did you use that formula? M1 multiplied by m2 equal to negative 1? What is this m?

20	PST2	It's the gradient
21	TE	It's the gradient so meaning that you started by calculating the gradient. Why did you use this formula?
22	PST2	because they say they are equidistant
23	TE	Equidistant. What does equidistant mean?
24	PST2	They bisect at P
25	TE	They bisect at P; bisecting at P what does it mean? What you think bisecting at P,
26	PST2	they form a 90^o
27	TE	We talked about the concept of bisect. What did we say?
28	PST2	They cut into two equal halves
29	TE	So, according to your understanding, the concept of equidistance means to bisect?
30	PST2	Yes
31	TE	Then what do you understand by $m_1 \times m_2 = -1$? ….
32		….
33	PST3	I want to ask a question: here we're using like this diagram or this diagram, it's like it's equidistant, is it the main point for this diagram? [the diagram below on the right was drawn by this PST]



34	TE	what do you understand by equidistant?
35	PST3	like equidistant, the 2 distance are equal
36	TE	Fine, equal from where to where?
37	PST3	from N to R is equal to from…
38	TE	where do you get the R? [PST3 mistakenly wrote R instead of P]
39	PST3	P
40	TE	I see you have P there, fine come again
41	PST3	like distance from M to P is equal to from N to P meaning that its like isosceles triangle; we are using the midpoint

42	TE	if you have a horizontal line like this, what does it tell you about the x values?
43	PST3	the x values, they are equal

During one-on-one interactions between TE and PSTs (turns 1-11; 19-26 and 33-43), ideas around a concept are probed and developed by both the TE and the PST, instead of simply being produced by the teacher educator. Also, the discussion around the concept or solution process becomes deeper than merely stating of the procedures involved in arriving at the correction answer. It is evident in turns 12 to 18 when the TE moves her attention back to the whole class that what became the object of the discourse was more about the correct solution (and in other cases, the correct procedures for arriving at the correct answer). Immediately when the teacher educator turns her attention back to a specific member of the community, the pattern of discourse also changes from I-R-E to I-R-P-R-P…(Scott, et al., 2006) because the teacher educator prompts for further elaboration of the point that has been made by the pre-service teacher.

Also, most of the justification norm and the non-ambiguity norm became visible during the one-on-one discourse with individual PST because the teacher educator asked for justifications of the PSTs's mathematical procedures (example, in turn 21) and expected that the PSTs' mathematical expression/explanations were unambiguous.

Explanations whether delivered by the teacher or through focused discourse around a task are powerful moments of teaching and can promote active construction of knowledge within the mathematics community (Larreamendy-Joerns & Muñoz, 2010; Leinhardt, 2001, 2010; Wittwer & Renkl, 2008). But explanations not only convey content; in pre-service teacher education settings, effective use of explanations as a practice can help entrench what it takes to be a full participant in a community of practice (Larreamendy-Joerns & Muñoz, 2010; Leinhardt & Steele, 2005). How explanations are modelled in the class can also help develop pre-service teacher educators' identity as teachers of mathematics in such a way that they know *how* to explain, *when* to explain, and *how to integrate* explanations into the learners' on-going cognitive activities. In excerpts 16 to 18, it can be argued that what was the centre of attention for TEIA-S in her explanatory practices was to convey the mathematical content in the form of procedures and concepts. The practice in itself of providing explanation, that is, providing explanation as a mathematical process that is important for teachers of mathematics was not a focal point. The explanatory practices of TEIA-S was therefore

anchored in developing the identity of becoming learners of mathematics content and less of becoming teachers of mathematics.

7.2.2.2 Providing justification

Following from excerpt 18 above and the discussion on explaining mathematically as a prominent practice within TEIA-S's classroom community, it can also be argued that questions that provoked extended dialogue and to a large extent, providing justification [MP-PJ] as a practice happened mainly when the teacher educator was engaged in a one-on-one interaction with PST's work during class activities. With the change in the pattern of discourse discussed above, came more prompts for the PSTs to provide justifications for mathematical claims that they make to the teacher educator, and by so doing, the non-ambiguity norm became more visible during this PST-TE interaction than during whole class interaction or whole class teaching.

7.2.2.3 Reiterating as a practice in TEIA-S's classroom community

In Chapter two, I indicated that reiterating practices can either serve the purpose of checking one's own understanding of a contribution made by a community member, or can be used by either the PSTs or the TE to make sure that everyone is on the same page after a contribution has been made by any of the community members. It can also be used to repeat what was discussed in a previous class to aid the current activity in which the class is engaged in.

Reiterating as a practice in TEIA-S's class was used in conjunction with explaining mathematically (MP-EM) and exemplifying (MP-PE) to repeat (sometimes in the form of questions) what the demands of the example at hand required and/or to explain the demands of the questions:

Excerpt 19[57]

1	TE	now this distance see, still this distance is the same as this distance [indicating on the board] which is the same as this distance ok, meaning that both values stand ne, yes sir
2	PSTs	is P not a midpoint?

[57] Excerpt 19 for which part are in excerpt 17 is one of the instances in which a particular utterance was double coded – in this case, the utterance is both explaining and reiterating.

3	TE	a midpoint of what?
4	PST	[indistinguishable answer]
5	TE	what do you understand by midpoint?
6	PST	distance from one point to the other is the same as from that point to another point
7	TE	eh fine, do you understand his answer? He is saying the midpoint, according to him, eh, can you explain again? Can we listen to his explanation?
8	PST	the distance from one point to the midpoint is equal to the distance to the another point.
9	TE	that explanation is the right explanation of equidistance. Eh now what are we talking about a midpoint? If we talking about the distance from this corner up to that corner ne, there is somewhere which is the middle of this distance ne, so if we talking about the middle of the distance from that corner to that corner, it must be along the way to the corner ne, right? Do you get that? if we talking about the midpoint from point A, the midpoint of point A to D, it must be somewhere along the distance from point A up to point D eh?
10	PSTs	Murmuring
11	TE	Ok let me explain again; when we talk about a midpoint we are talking about a point which is in the middle, in the middle of what, our way, in the middle of our way to the next point ne? so it must be somewhere along the way right, the midpoint of point A and D, the midpoint of this line segment is a point which is somewhere in the middle of this way from point A to point D; the midpoint of the distance from that wall to that wall will be a point somewhere in the middle of this way; it means the midpoint will be along this way ne? Now let us look at point P. point P is here, can you see point P?

In the discussion above, the teacher educator in turn 5 asked the PST to explain his understanding of the concept of midpoint in order to be sure that everyone was on the same page as far as the understanding of the term of midpoint was concerned, before providing an explanation of the term herself. She did not ask what other PSTs think of the explanation of midpoint by their fellow PST but went on to explain the meaning of the term to them. In turn 11, she repeats the explanation of midpoint, again, to make sure that the PSTs all understood the term, before proceeding with the solution to the task at hand.

In a few instances, reiterating was also used by TEIA-S to activate prior knowledge as evident in excerpt 20 below:

Excerpt 20

1	TE	In our previous meeting we ended with the chapter coordinate geometry and we looked at the properties of a line, we looked at midpoint of a line segment, we looked at how to determine the equation of a line. We had on the chalk board the vertices of a rectangle and there was a question which was saying that these are vertices of a rectangle and we drew and we've proven that indeed they are vertices of a triangle, of a rectangle. Fine. This group determines the gradient of one line, this group determines the gradient of another line and when we multiply the two gradients we find the product of the -1 and you therefore concluded to say the two lines are perpendicular. We talked about gradients of parallel lines; what do you know about the gradients of parallel lines?
2	PSTs	[THE ARE EQUAL] the gradients are the same
3	TE	The gradients are the same neh meaning that if 2 lines are perpendicular their gradients will be equal ne, …

The conclusions that can be drawn from TEIA-S's reiterative practices are that, first, reiterating as a practice was mainly used by the teacher educator to make sure everyone was on the same page in their understanding of both the concept at hand and the demands of the task the class was engaged in; secondly, reiterative practices were used by the teacher educator to position the pre-service teachers as learners of mathematics content.

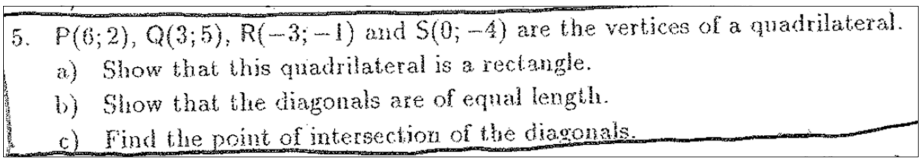
7.2.2.4 Defining and exemplifying practices within TEIA-S's Classroom CoP

TEIA-S's approach to the introduction of a concept in analytical geometry was not to first define the term. The pre-service teachers were first presented with an example of a question that dealt with the concept the TE wanted to introduce. The TE reads out the questions and picks on the concept to be explained and asks PSTs what they understood by the term. Only after the class had understood the concept by way of the definition did the class engage with the mathematics question. In the excerpt below, the question that was to be solved was a question dealing with proving that four given vertices were those of a quadrilateral:

Excerpt 21

1	TE	From the worksheet, on page 118, on page 118, on page 118 let us look at

number 5. There is number 5.


5. P(6;2), Q(3;5), R(−3;−1) and S(0;−4) are the vertices of a quadrilateral.
a) Show that this quadrilateral is a rectangle.
b) Show that the diagonals are of equal length.
c) Find the point of intersection of the diagonals.

We are given the coordinates of some points there which are vertices of a quadrilateral. Now the question says, if I should start from page 117, which says SHOW THAT these points are the vertices of a quadrilateral. First question, now what is a quadrilateral? What is a quadrilateral? What is a quadrilateral?

2	PST	A cube
3	TE	What is a quadrilateral (pointing to a student)
4	PST	4 sides
5	TE	This is a four-sided figure ne, with how many angles?
6	PSTs	4
7	TE	With four angles ne? It's a four-sided figure made up of four angles. Now to answer the first part of the question what must we do? What must we do? Show that these points P, Q, R, and S are vertices of a quadrilateral. Where do you start?
8	PSTs	[noisy exchange]
9	TE	You draw this. You represent this on a Cartesian plane ne? on the set of axes okay?
10		*CLASS WORKS ON THE QUESTION IN GROUPS FOR FEW MINUTES*

The teacher educator in the excerpt wanted to entrench the concept of quadrilateral before proceeding to solve the question. This strategy of using an example to introduce and define a concept in analytical geometry was used in the introduction of most of the concepts such as the angle of inclination, the distance between two points, line bisector, midpoint, to mention but a few.

While teaching analytical geometry, the teacher educator also used varied examples that catered for working backwards. In the examples below, the TE required, in the first instance for pre-service teachers to find the angle of inclination on the Cartesian plane given the equation of the line, and in the next questions, the task that was to be solved in class was finding the equation of a line given the angle of inclination.

Excerpt 22

1	TE	determine the inclination of the different lines[writes on board] 3x = 2y + 5. Determine the equation, determine the angle of inclination of that line. What do we understand? what do we know about the angle of inclination? What do we know about the angle of inclination? [touches a female student] what do you know about the angle of inclination? Is? Is equal to the gradient ne? Fine, is defined by the gradient of a line. So where do we start? We rearrange ne, we write the equation in standard form ok? So what we got there…
2	PSTs	[chorus] 2y
3	TE	equal to…
4	PSTs	3x - 5
5	TE	[writes] 3x – 5
6	PSTs	[noisy chatter]
7	TE	2y equal to?

And following this example,

8	TE	Okay let's look at number 4. Let's look at number 4, find the equation of the line through a point with an angle of inclination, with a given angle of inclination, which one? Number a, number b,
9	PSTs	Class: a
10	TE	Si: number a. number 4a. Find the equation of a line through 1 and 2 [writes (1,2)] with an angle of inclination of 135^{o}. Where do we start?
11	PSTs	Class: [indistinguishable answer]
12	TE	Si: we must find the gradient right? I agree with you, we must find the gradient. Now the question is how do we find the gradient. We know the equation to determine the gradient neh?
13	PSTs	Class: [chorus] Yes

All the examples discussed in the class were generic examples and work(-out) examples (Bills, et al., 2006) aimed at developing the concept at hand and in entrenching/developing specific techniques for solving the mathematics problem relating to the concept. Since in the discussions, there was no explicit attention paid to the importance of teachers' selection of mathematics examples that can support or impede learning and because there was no attention to the role of examples in the teaching and learning of mathematics, TEIA-S's exemplificative practices were anchored in the identity of becoming learners of mathematics

content. BLMP would entail explicit attention not only to what constitutes a good example, but also attention to what Watson and Mason (2005) refer to as "example space[58]" and how this may be selected/developed bearing in mind the prior knowledge of the learners, the subject-matter knowledge (for example what misconceptions are prevalent in the topic), the language infrastructure of the class, etc. The use of, and knowledge about how to use counter-examples and non-examples is also essential for pre-service teachers. Furthermore, the strategy of asking pre-service teachers to generate or construct their examples (Watson & Mason, 2005) of questions or tasks that can be used to teach a particular mathematics concept would also entail enculturation into the identity of becoming teachers of mathematics or becoming learners of the mathematical process of exemplification.

In general, in TEIA-S's class, as can be seen in the table below, the privileged practices used in the class attended mainly to the development of the identity of becoming learners of mathematics content:

MUTUAL ENGAGEMENT DIMENSION OF TEIA-S' CoP		
CATEGORIES	**CODES**	**TOTAL NUMBER**
PATTERN OF DISCOURSE	Questions that provoke extended dialogue [PED]	12
	TE asks question provoking dialogue [TEQ]	61
	PST asks question provoking dialogue [PSTQ]	10
	TE evaluates [TEEv]	46
	PST(s) evaluate(s) [PSTEv]	2
BUILDING OF IDENTITIES	Becoming teachers of mathematics [BTM]	2
	Becoming teachers of mathematics in multilingual classrooms [BTMMC]	0
	Becoming learners of mathematics content [BLMC]	75
	Becoming learners of mathematical practices [BLMP]	0
	Becoming proficient English users [BPEU]	1

Table 7.4: Mutual engagement code frequencies for TEIA-S' CoP

Of note is the fact that even though TEIA-S's classroom community of practice comprised of multilingual pre-service teachers and a multilingual teacher educator who shares the same home languages as some of the PSTs, attention was not paid to the development of the

[58] A pool of examples that fulfil a specific function

identity of becoming teachers of mathematics in multilingual classrooms. There was also limited positioning of the pre-service teachers as becoming teachers of mathematics and becoming proficient English users as can be seen in table 7.4

Furthermore, even though in general it appears in table 7.4 that the class was dialogic in nature, there was no interanimation of ideas, and the dominant views that were validated in the class were mainly those of school mathematics. In the next section, I expound more on the findings at both TEIA-M's and TEIA-S's communities of practice mathematics classrooms in University A.

7.3 Discussion of University A (TEIA)

Several arguments can be advanced from the above data analysis from the two multilingual pre-service teacher education classrooms. A key one has to do with the discussion pattern in both classrooms. Both teacher educators control and proceduralise the explanation in several steps while eliciting choral responses from the students during whole class interaction or teaching. A key difference, however, between the two classrooms is that while TEIA-M stops at the level of questioning, TEIA-S moves into the domain of extended dialogue when engaging with individual pre-service teacher on a particular task. During such interaction, the pattern of discourse changes from I-R-E to I-R-P-R-P-… . Nonetheless, I-R-E was the prominent interaction pattern in TEIA. In the same vein, there were very limited evaluating mathematical validity practices in TEIA. Much of the evaluating mathematical validity practices were in TEIA-S's class and occurred mainly when the teacher educator was engaged with a particular community member on a task.

Dooley (2002) argues that the traditional three part exchange of question-response-feedback (I-R-E), typical of most classrooms closes off the opportunity for extended dialogue through which learners may deepen their conceptual knowledge and refine their language. Research (see Dansie, 2001, Dooley, 2001 in Dooley, 2002; Garcia, 1997 in; Khisty & Morales, 2002) has shown that in a bi/multilingual classrooms where extended dialogue is used as a pedagogical strategy, communication between the educator and the students have, over time, changed from one- or two-word responses to learners beginning to initiate questions and taking longer turns. Students in multilingual classrooms who do mathematics in a language which is not their first or home language need opportunities to engage in extended interactions in the classroom that allow them to use the second language (LoLT), to

manipulate it, and to hear from others how the discourse, especially the academic language in mathematics, is used (Krashen, 1982). This was not visible during whole class discourse at University A. In a few instances where, especially TEIA-M asked for an explanation/justification from the students, most students who attempted to explain struggled to do so because they have had limited opportunities to express themselves beyond stating steps required to solve mathematical problems. As Nel and Muller's (2010) findings indicate, teachers' English language proficiency influences English second language learners' English language acquisition and academic progress. This is also true for mathematical language. The nature of the discourse patterns used in a community of pre-service multilingual classrooms such as the ones in this study would determine the extent to which the quality of mathematical discourse would evolve. If the pre-service teachers are not themselves fluent mathematically at the end of their training, it is difficult to see how they would help their learners overcome their own mathematico-linguistic challenges when eventually they become teachers.

At University A, defining, exemplifying, explaining and proceduralising were the most privileged practices. But even though these practices were dominant across the two teacher education communities of practice at University A, none of these practices as processes in themselves were the objects of attention at any point in the classroom community. These practices mostly positioned the pre-service teachers as learners of mathematics content. There was limited attention paid to the interacting identities of becoming teachers of mathematics and less still of becoming teachers of mathematics in multilingual classrooms – despite the teacher educators' awareness of the context of their practice and the context in which the PSTs would teach at the end of their qualification as expounded in Chapter Six.

7.4 Conclusion

This chapter has dealt with the analysis of the shared repertoire and mutual engagement dimensions of CoP at University A. The prominent practices in two pre-service mathematics classrooms at the University and what interacting identities they attend to were discussed. Implications for teacher education development were also discussed. In Chapter Eight, I engage with both the shared repertoire and mutual engagement dimensions of CoP at University B.

CHAPTER EIGHT
Shared repertoire and mutual engagement at University B

8.1 Introduction

In Chapter Seven, I dealt with the mutual engagement and the shared repertoire dimensions of CoPs in University A. In this chapter, I engage with these two dimensions of CoP in University B (TEIB). As a reminder, the two teacher educators in University B who were selected for Phase Two of the study did not share the same home language with most of the pre-service teachers. As a reminder also, the aim of this study was not to compare the different practices, norms, pattern of discourse, etc in the communities of practice to ascertain which teacher education classrooms had exemplary practices that are worthy of adoption. Neither was the aim to draw comparisons between the two universities involved in the study. The study aims to explore the nature of CoP in pre-service teacher education classroom communities so as to understand what happens in these classrooms and how pre-service teachers are being prepared to teach mathematics (to multilingual learners). In this chapter, therefore, I attempt to provide answers to the following analysis questions as they concern two mathematics teacher education classroom communities of practice which I have called TEIB-L and TEIB-E:

1. What mathematical practices are in use in the negotiation of meaning?
2. What norms of practice are in use in multilingual mathematics classrooms of pre-service teachers and how do these norms co-construct the mathematics PST education communities?
3. What are the common discursive repertoires within the communities, and how do they co-construct these communities?
4. What communicative approaches and patterns of discourse are prevalent in the mathematics PST education classrooms? And where does authority stem from?
5. How does classroom engagement support the different interacting identities within pre-service teacher education classrooms?

As with Chapter Seven, I attempt to answer the above analysis questions for each teacher educator beginning with TEIB-L. The chapter concludes with a general discussion on the shared repertoire and mutual engagement dimensions of CoP with regards to University B.

8.2 The Shared Repertoire and Mutual Engagement at University B

In analysing the shared repertoire and mutual engagement within University B, I use the privileged practices within each classroom community of practice as a starting point to gain an entry into the community's *modus operandi*. I then use these practices to reflect on other dynamics relating to the shared repertoire and the mutual engagement that are at play in each of the two classroom community of practice.

8.2.1 Shared repertoire and mutual engagement in TEIB-L's CoP

By way of background to the observed lessons in TEIB-L's classroom community, the topic at hand was probability. As a product of the old SA curriculum, the pre-service teachers had not done probability previously and were thus encountering it for the first time. The pre-service teachers were from diverse linguistic background, and for most of them, English was an additional language. English was the home language of the teacher educator.

Table 8.1 below provides a summary of the shared repertoire dimension for TEIB-L's community of practice.

SHARED REPERTOIRE DIMENSION OF TEIB-L's CoP					
NATURE OF M. PRACTICE	**MP**	**TOTAL**	**NATURE OF N. PRACTICE**	**NP**	**TOTAL**
Initiating and/or sustaining mathematical discussion practices	Explaining mathematically [MP-EM]	95	Conversational norms	Participation by all [NP-PA]	4
	Communicating mathematically [MP-CM]	32		Collaboration norm [NP-CB]	1
	Defining Mathematically [MP-DM]	9		Speak-Out norm [NP-SO]	4
	exemplifying [MP-PE]	35		Taking turns to speak norm [NP-TT]	0
	Reiterating [MP-Rt]	38	Conceptual norms	Mathematically sensible norm [NP-MS]	16
	Revoicing [MP-Rv]	16		Consensus norm [NP-	28

				CS]	
Other mathematical practices	Summing up[MP-SU]	4		Non-Ambiguity norm [NP-NA]	2
	Moving between worlds [MP-MBW]	16		Justification norm [NP-JN]	3
	Proceduralising [MP-Pc]	12	Interpersonal norms	No sole arbiter of knowledge norm [NP-SK]	3
	Using Multiple Approaches [MP-MA]	4		Avoidance of threat [NP-AT]	0
	Writing Mathematically [MP-WM]	18		No Ridicule norm [NP-NR]	0
	Monitoring solution [MP-MS]	4			
	Experimenting [MP-Et]	3			
Evaluating mathematical validity practices	Critiquing Conjecture [MP-CC]	4			
	Providing Justification [MP-PJ]	26			
	Critiquing solution [MP-CS]	13			

Table 8.1: Dominant practices and norms present in TEIB-L's Classroom CoP

A number of practices emerged as the dominant practices in TEIB-L's classroom – reiterating, revoicing, exemplifying, moving between worlds, defining, writing mathematically, explaining, communicating mathematically, and evaluating mathematical practices such as providing justification and critiquing solution. It must be noted that these practices are privileged in the sense that they are consistently valued across the different segments of the data obtained from TEIB-L's classroom. Table 8.1 indicates that there was a mix of initiating/sustaining mathematical discussion practices, other mathematical practices and evaluating mathematical validity practices such as providing justification and critiquing solution. As far as the norms are concerned, the table indicates that conceptual norms, notably, consensus norm and mathematical sensibility norm were more 'visible' in this community. How these two norms, in particular, co-construct the community is discussed in this chapter.

In what follows, I elaborate on these practices and as in Chapter Seven, juxtapose some of these practices to provide a more encompassing analysis. I also elaborate on how these practices shape and/or are being shaped by the mutual engagement dimension of CoP.

8.2.1.1 Reiterating and revoicing

In Chapter Four, I drew a distinction between reiterating and revoicing as these terms are understood and used in the present study. Revoicing describes a situation where a community member repeats a contribution using the correct mathematical register while reiterating describes an instance where a community member repeats a contribution either to check their own understanding, or to make sure everyone is on the same page, or to activate prior knowledge. Of all the privileged practices in TEIB-L's class, reiterating was one of the most dominant practices given the number of occurrences in different segments of the data.

In TEIB-L's classroom community of practice, reiterating as a practice was mostly used at the beginning of the class for what Andrews (2009) refers to as activation of prior knowledge to link previous knowledge in the previous class to new knowledge. In excerpt 23 below, to introduce the law of large numbers, the teacher educator first starts by recapping what was done in the previous class on finding the various probabilities of getting the number six when two dice are thrown simultaneously:

Excerpt 23

1	TE	Now … we ended up with a little experiment last time, OK, and what I did was you got dice and what were we doing with the dice? What were we looking for? What was the event? We were looking for?
2	PSTs	For six
3	TE	For Sixes. You were throwing the dice how many times for a game?
4	PSTs	20 times
5	TE	20. Now look, listen to the procedure or the progress, what happened. The development. You threw… You did 5 games, right? And for each of your games (which was 20 throws) you counted the sixes and you got a fraction out of 20. Agreed? So if you'd played 3 games and then I said stop you stopped. But the aim was 5 games, not serious. For each fraction what did I ask you to do?
6	PSTs	[Some students] Find the percentage
7	TE	To find the percentage. That was the Probability of getting a 6, agreed?
8	PSTs	Mmm
9	TE	Then what did we do? Now we found that different people, different little groups got different percentages, isn't that so?
10	PSTs	[Some students] Yes
11	TE	There were a range of percentages. Now that's not going to help me if I come from the street and I walk here and I say, 'Listen, what's the

		probability of throwing a 6 in 20 throws? Because I can't have 10 people telling me different probabilities.' So then what did we do to narrow it down? To get a little bit more cohesion and accuracy?
12		…….
13	TE	What makes it more reliable? That's what I'm asking you. Because this group over here actually might have thrown and made 51%. They saw it with their own eyes, they did the experiment.
14	PST1	Ja, I think inasmuch as you've done all of these experiments in groups so it means that if 18% is considering our differences inasmuch as it is arriving at average.
15	TE	So you're sort of seeing 18% as a sort of average of the, of the, of the class.
16	PST1	Ja
17	TE	It is a way of looking at it, it's getting to a cen… Do you remember with our statistics what did we say 'mean' was in a way? Mean, median and mode, what were they directing us towards?
18	PST2	The central
19	TE	The centre, the central tendencies – agreed? Now 18% is doing that. It's taking us on the number line to the central number that we can say is the chance of throwing a 6 in 20 throws, OK? Now you're still not telling me what's contributing to that , um, to get to that… You're telling me how we're doing it, sure, but there's something very important in Probability. Experiments, that is helping us get 18 as more accurate…

Reiterating practice as used by the teacher educator to activate prior knowledge was aimed at better equipping the PSTs for the next topic. In the case of the excerpt above, the focus of the teacher educator was to engage with the law of large numbers – to engage with arriving at a more accurate experimental probability. In reiterating what was done in the previous class, therefore, the teacher educator enabled continuity between the content learnt and the content to be learnt and also sustained the mathematical discussion around the topic of large numbers.

Revoicing was used in conjunction with reiterating for the development of both the mathematical English and ordinary English. For example, in turn 18 where the PST used the incorrect English term, the TE corrects this by using the correct term. But more often than not, revoicing in TEIA-L's mathematics classroom was used by the teacher educator as a filtering device to enable her choose the correct answer when there is a multitude of responses. Excerpt 24 provides an example of when this was the case. The discussion in the

class was centred around the labelling of the scale ruler using percentages between 0% and 100%. A PST provides an answer of 0,75 instead of 75%...

Excerpt 24

1	TE	T: I could ah… So now do you see what he's mentioning? He's suddenly introduced a decimal number to us because he said 0,75, right? So now what you're learning in probability, that numbers linked to the scale can be said as percentages, which are the most popular, by the way, but they can also be said as decimals. And what do decimal numbers come from?
2	PST1	S: Division
3	PST2	S: Fraction
4	TE	T: From fractions. And so you're learning that there are 3 types of numbers, all linked to each other. Fraction language, OK? So what number goes, what fraction goes here?

In turns 2 and 3, the PSTs produced two different answers to the question posed by the TE. The TE uses revoicing as a practice to point the PSTs to the right answer during choral/multiple response by ignoring the incorrect answer. O'Connor and Michaels (1993) advocate for the use of revoicing in teacher education to give pre-service teachers an expanded voice. This resonates with Mortimer and Scott's (2003) notion of interanimation of ideas. By picking only the correct answers and not probing or engaging with incorrect ones, the teacher educator limits interanimation of ideas that may have uncovered misconceptions or the lack of conceptual understanding of a particular concept.

But even though revoicing was a prominent practice in TEIB-L's classroom, the process of coming to revoice – that is – when to revoice, how to revoice, and the conditions that allow for the successful use of revoicing as a practice was backgrounded in TEIB-L's classroom community of practice. I will return to this point later in this chapter.

8.2.1.2 Exemplifying, defining, communicating mathematically and moving between worlds

Rather than choosing generic examples that are solely aimed at developing the mathematics concept at hand, TEIB-L chose both work(-out) examples and exercise examples that aimed to develop the mathematics concept through relevance to the real world of the pre-service teachers. Hence, the nature of examples chosen by TEIB-L enabled her to move between the mathematics world and the real world.

Examples used by the teacher educator were mainly to clarify the use of probability terms. As such, exemplifying practices in TEIB-L's Classroom CoP were also inextricably linked to communicating mathematically and defining practices. So, rather than defining the term when the term is introduced, the teacher educator first provides an example in real life contexts and asks the PSTs for terms (both in ordinary English and in mathematical English) that describe the story problem. After that, as observed in excerpt 23 (turns 17-19) above, the teacher educator refines the language used by the PSTs to a more mathematical term. Hence, in the exemplifying practices of TEIB-L, practice such as defining, moving between worlds and communicating/writing mathematically were inextricably intertwined. In excerpt 25 below, the teacher educator attempted to differentiate between certain and uncertain events. She starts with a scenario for which the PSTs needed to describe in probability terms:

Excerpt 25

1	TE	Think about me. Did you think… You knew you had Maths today, right? Did you think you were going to see me today? When you woke up this morning how did you, what… Did you think that I would be here or … I know you were probably hoping that I wouldn't be *[Everyone laughs].* OK, so what would you say about me, if your friend said to you this morning before you came here, 'Do you think XXX's going to be in class today?'
2	PSTs	[Some students] No; Other students: Yes
3	TE	So, what, what… Ja, there are different things you thought. What?
4	PST1	100% sure
5	TE	Hmm?
6	PST1	100% sure
7	TE	So you actually said. 'Sure, 100% sure' *[probably during the interruption in the recording T wrote:* unlikely, improbable*, now T writes:* 100% sure*]*
8	PST2	She won't be there
9	TE	*[points to a S)* What did you think?
10	PST2	She won't be there. *[some chuckles]*
11	TE	So you would say… Give me a word for…
12	PST2	[inaudible]
13	TE	OK, but give me a word for, give me a word… Instead of, like you were sure I wouldn't be here, alright, so give me a word that could replace that feeling that she's not going to be here.
14	PST3	It's impossible for her…
15	TE	It's impossible. *[I think at this point T writes: impossible].* What's another word for sure?
16	PSTs	[Some students] Certain

17	TE	Certain. I'm certain that she will be ?. So 'sure' will go with 'certain'. Don't write anything down now. Now what about you people that thought well…she *might* [stresses word] be here?

In turns 13, 15 and 17, the teacher educator insists on the provision of a term/word in English that describes the scenario and then on the mathematics register associated with the ordinary language that has been used by the PSTs to describe the scenario.

Examples used by TEIB-L were not only linked to PSTs everyday life, but also, they were mostly what Zodik and Zaslavsky (2008) have termed, 'spontaneous examples' – examples that the teacher educator generates/constructs in-the-moment to respond to a particular need while teaching. This spontaneous example is seen in turn 1 where the teacher educator attempts to develop the concept of certainty and uncertainty of an event happening. In a subsequent example not captured by excerpt 25, the teacher educator, realising that the example in turn 1 may not be a good example to denote uncertainty (due to the fact that she never had missed lectures in the course of the year) uses another spontaneous example – the chance of snowing – to deal with the situation.

After a brief discussion of a concept using spontaneous examples, TEIB-L dictates to the PSTs the formal definition of concepts and provides more examples to deepen conceptual understanding. While doing this, she pays attention and draws the attention of the PSTs to the probability language that is used and how the language is used, taking care to translate the verbal sentences to mathematical sentences as evident in excerpt 26 below:

Excerpt 26

1	TE	[Dictating] An activity that is taking place or will take place is called an Event. *[writes:* activity Event *]* So underline the word 'event' and underline the word activity. Full stop. The probability of a certain event (no, leave the word 'certain' out) The probability of an event taking place is written in short notation.
2	TE	*[T erases* activity *and* Event*].* So for example *[writes:* eg.*]*…has anyone got a coin here please [gets a coin] Right. We're now about to kick off with the South Africa Bafana Bafana and the Confed Cup, I'm the captain and you're the captain of Iraq. *[points to a student] [everyone laughs].* And the ref comes along. Now before that you know that the coin is tossed. What do you call it in your language?
3	PSTs	*(Students shout out answers)*

4	TE	Anyway, so he comes along *[hands the coin to a student]* You're the ref. Now before he does this activity *[T pushes the door slightly more closed]* Before he does the activity, this is the event, the ref is going to toss the coin. Now whoever gets like a head or a tail, whoever gets it, what happens then?...
5	TE	Now, hang on. Before you toss the coin, listen to me. What are the chances that heads are going to come up?
6	PSTs	Half-half
7	TE	Half-half. What else?
8	PST1	unlikely
9	PST2	Even odds
10	TE	How do we speak?
11	PST	Equally likely
12	TE	Equally likely chance that heads will come up

In excerpt 26, the TE provides the definition of the term 'event' using another spontaneous example (turns 2-4) and taking care to illustrate the words 'activity' and 'event' in her narrative. She then uses this example to reinforce the concept of 'equally likely' event. In turn 6-12, the teacher educator guides the PSTs to the correct mathematical way of describing the outcome of her narrative in probability terms. In using spontaneous examples that PSTs can relate to, the teacher educator was able to increase participation in class. But as I discuss later, due to the teacher educator's receptibility to incorrect answers, only certain PSTs responded to questions posed by the teacher educator. This is reflected also in table 8.1 in the instances in which 'participation by all norm' were 'visible'.

But despite the importance accorded to exemplifying and defining practices in TEIB-L's class, the process of exemplifying (what makes for a good example, etc.), and the process of defining were not in themselves the object of attention. This was also true for the evaluating mathematical validity practices to which I now turn.

8.2.1.3 Explaining, providing justification and critiquing solution practices

Explaining as a practice in TEIB-L's Classroom CoP was more about the explanation of probability terms and aimed at enculturating PSTs to mathematical ways of speaking. In that way, it closely linked to communicating mathematically practice. Esmonde (2009) notes that explanations are utterances that explain *why* (reasoning and proof), *how* (outlining the procedure) and *one's thinking*. Much of the explanation in TEIB-L did not fall into the first two of these categories by Esmonde (2009) as explanations for TEIB-L were designed to

explain the *what* of probability concepts. This is so because the practice of explaining was not so much about explaining the procedures necessary to arrive at the answer, but mainly about developing and deepening conceptual understanding in such a way that goes beyond the procedural or algorithmic. Excerpt 27 below illustrates how the probability concept of *equally likely* was introduced in the class using the probability scale:

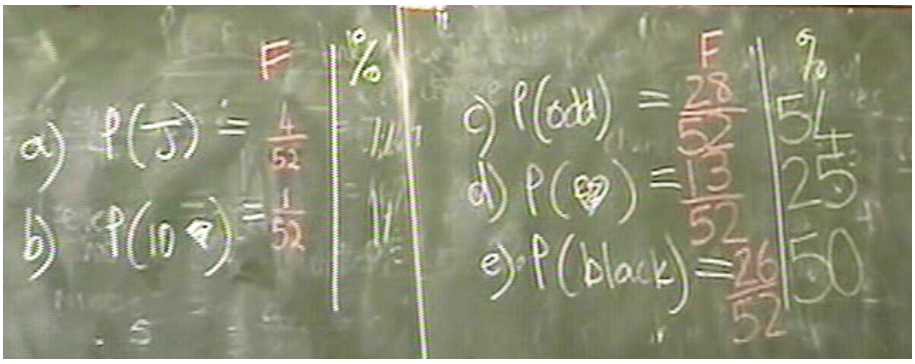
Excerpt 27

1	TE	Yes, we'll see that in our work. Plenty. Plenty. And in fact it was what I was saying just now. I said to him, he wanted to put 'likely' at 50% and I said to him, 'But hang on a minute, isn't that closer to when it was more certain?' Now that's between the two, isn't it? So absolutely. What do we say when it's half-half? Oh, I've just told you!
2	PSTs	Half-half
3	TE	*[laughs]* When it's 50% it will happen, 50% it won't happen, what do we say in English?
4	PSTs	Half-half
5	TE	We often say half-half, agreed?
6	PST	Class: Yes
7	TE	We'd say 'half-half' or we can say – so please write this down – half-half *[in the middle of the board in white chalk writes:* 'half-half'*].* That's how we sometimes speak to mention that 50% chance or we can say it's equally likely *[under 'half-half', in white chalk, writes* equally likely*].* That's language we use as well to represent 'half-half'. OK. Now what number goes half way between 50% and 100%?

The TE starts by describing a situation and asks the PSTs what they would call this situation in ordinary English. From the suggested concept of half-half, the teacher educator then uses the correct mathematical term *equally likely* to describe the situation.

Another feature of TEIB-L's community of practice was that explaining as a practice was done by both the teacher educator and the pre-service teachers both during whole class discussions and when the PSTs went to the board to explain to others. In excerpt 28 below, the teacher educator called on a pre-service teacher to explain the reasoning in the solutions which were proffered by other PSTs after these PSTs had solved the questions on the board. The class was working on finding the probability of picking a jack, a diamond, and a club in a pack of 52 cards.

Excerpt 28

1	TE	Right. Right. Ready. *[looks at her watch]* I'm sorry I'm pushing you. Shh. There is one more little thing I want to do, …um…, but Simon has offered to just volunteer. Now what's going to happen is he's going to go through the thinking – how these people were thinking, see if he agrees with the way they were thinking about the desired outcomes and about the, all the possibilities, OK. And then he's going to look at the fraction, he's going to look, imagine he's a teacher now that's marking this work. So what he wants to do is look at what's going on in the thinking behind these answers, OK. If … let Simon, let him go through all of these 5 first. If there's anything you disagree with we will go back to it. OK? Because one thing you must be clear on, I don't care what phase you are, if ever you are teaching Maths or you're doing a little private lesson at home, or you're helping your little sister, it makes no difference, you've always got to think how they're thinking before you can say 'You are wrong', 'You are right'. And even if they're wrong you want to see what they're thinking about. OK. But let him go through, um, starting with number 1. And I'm going to step aside for a minute and I want you to imagine that you are now looking at their thinking and… carry on. [The solution provided on the board were:]
		
2	PST1	Ja, so for the first one here the thinking is…
3	TE	Well first of all go to the bracket, see what we want.
4	PST1	OK,*[points to (a)]* so in the bracket we have a Jack, so since we know that we have 52 cards all in all, so the Jacks that we have, we have 4 Jacks. So here this fraction tells us that we have 4 Js (Jacks) out of 52 cards. Right?
5	PSTs	[Some students] Mmm
6	PST1	OK, let's go to the second one *[points to (b)]* Since… Since each card is having a dice, a heart, a spade and a…
7	TE	Having a what?
8	PSTs	A dice?
9	PST1	A dice *[draws a diamond]*
10	TE	A diamond, Simon!
11	PST1	Diamond! *[laughs] [laughter from the class]* Simon: *[draws ♠ ♣ ♦ ♥ on the board]* I don't know, what is this? *[points to ♣]*
12	PSTs	A Club

13	PST1	What?
14	PSTs	Clubs
15	PST1	Clubs. So how many 10s? The 10s which… OK, how many 10s? We have which…*[laughs]* Class: *[laughs]*
16	TE	Simon, you're not teaching us. Just look at what's written there and how is the person thinking. Look at the answers.
17	PST1	OK. The person here was thinking that we have one Diamond 10, which is right because you have 4 10s – 1 is this, this, this and this. *[points to* ♠ ♣ ♦ ♥ *on the board]* So here the fraction is 1 over 52, and that is right.
18	PSTs	Yes
19	PST1	Let me go to the 3rd one after the 4th and the 5th.

A number of features of TEIB-L's class are visible in the above excerpt: first is that the class interaction was structured such that both pre-service teachers and teacher educators are able to explain. In turn 1, the pre-service teacher (PST1) was expected to gain an entry into how other PSTs reasoned when they solved the probability problem on the board. The second is that explanatory practices in TEIB-L's CoP were intricately linked with providing justification and critiquing solution practices. Critiquing solution was also done by both the teacher educator and the pre-service teacher and one of the ways in which the teacher educator encouraged the PSTs to critique solutions was to ask them to explain the thinking behind the solutions to classroom activities that have been produced by their fellow pre-service teachers as evident in turns 1 and 16. That said, it can be argued that TEIB-L positioned the PSTs as both becoming learners of mathematics content and becoming teachers of mathematics. This latter comes out forcefully in turns 1 and 16 in excerpt 28 where the TE exhorts the pre-service teachers and PST1 in particular to act like a teacher, and in except 29 below where the class was dealing with the concept of probability:

Except 29

1	TE	Now this word 'Probability' what word does it come from?
2	PSTs	Probable
3	TE	Probable. And this word 'probable' *[writes:* probable*]* you know if a child had to hear this word for the first time, how could you as a teacher explain what probable means? *[points to a student]* Yes?
4	PST2	Is it like the chance or the likelihood of something taking place?
5	TE	Very good answer. It's actually a chance or likelihood that something can happen *[writes:* chance something can happen, likelihood*]*

By asking the PSTs to explain the term 'probable' as they would to a child who was encountering it for the first time, the TE helps the PSTs develop some sort of consciousness about what they want to become in future – teachers. This means that for TEIB-L, the PSTs are becoming teachers of mathematics, and must develop this identity in her classroom community of practice.

But before I delve deeper into what interacting identities were developed in TEIB-L's Classroom CoP, I engage with the use of hedges – a prominent feature in TEIB-L's classroom discourse. The use of hedges by the PSTs featured as a discursive repertoire within the community which impacted on explanatory practices of the PSTs and how they (PSTs) responded to questions posed by the teacher educator.

Contrary to the finding of Rowland (1995) where hedges were used to denote uncertainty, in TEIB-L's Classroom CoP, hedges were used by the PSTs to answer a number of questions posed by the teacher educator because of how the TE received incorrect answers and how she evaluated incorrect answers. In excerpt 30 below, the class was discussing the probability fractions associated with the probability language on a metre scale:

Excerpt 30

1	TE	…And so you're learning that there are 3 types of numbers, all linked to each other. Fraction language, OK? So what number goes, what fraction goes here?
2	PST1	Isn't it three quarters?
3	TE	Three quarters *[under 1 and to the left of 75% T writes in pink chalk: ¾]* And what about this half way mark here? *[T makes a mark half way between 50% and 0%]* What fraction?
4	PST2	S: 1 over 2
5	PSTs	[Some]: 1 over 4
	TE	T: Excuse me?
6	PSTs	[Students shout out] A quarter, one quarter
7	TE	T: Who said 1 over 2? Bow your head in shame!
8		*[laughter]*
9	TE	OK, so it's 1 over 4

In excerpt 29 turn 4, in answer to the question about the meaning of the term 'probable' posed by the TE, rather than respond with an answer which is a statement, the PST responded with a question-like answer. This is also the case with excerpt 30 above in turn 2. The PST's use of the phrases "is it like…" in excerpt 29 and "isn't it…" in except 30 in PSTs's

explanation was a shield against being reprimanded by the teacher educator should the answers not be what the teacher educator was looking for. Esmonde (2009) argues that the use of hedges in explanations provide insight as to the ways in which explanations can position the explainer in relation to others in the classroom and in relation to their mathematical knowledge. The use of hedges by the PSTs positioned them as learners of mathematics content. The table below is an indication of the fact that in TEIB-L's classroom community, the PSTs were positioned not only as becoming learners of mathematics content, but also as becoming teachers of mathematics and as becoming proficient English users. Little attention was paid to becoming teachers of mathematics in multilingual classrooms despite the teacher educator's professed experience and awareness of the context of her practice as described in Chapter Six. The table below provides an indication of what was foregrounded or backgrounded as far as the mutual engagement dimension of CoP was concerned for TEIB-L:

MUTUAL ENGAGEMENT DIMENSION OF TEIB-L's CoP		
CATEGORIES	**CODES**	**TOTAL NUMBER**
PATTERN OF DISCOURSE	Questions that provoke extended dialogue [PED]	19
	TE asks question provoking dialogue [TEQ]	98
	PST asks question provoking dialogue [PSTQ]	14
	TE evaluates [TEEv]	86
	PST(s) evaluate(s) [PSTEv]	14
BUILDING OF IDENTITIES	Becoming teachers of mathematics content [BTMC]	16
	Becoming teachers of mathematics in multilingual classrooms [BTMMC]	3
	Becoming learners of mathematics [BLMC]	111
	Becoming learners of mathematical practices [BLMP]	1
	Becoming proficient English users [BPEU]	23

Table 8.2: Frequencies of the codes in the mutual engagement dimension of TEIB-L's CoP

Also, from table 8.2, it is evident that in the classroom discourse, the teacher educator privileged general explanation and/or justification questions that provoked extended dialogue around probability concepts. Questions that provoked extended dialogue were also posed by the pre-service teachers during classroom discussions. Nevertheless, the communicative approach that was evident in TEIB-L's classroom was interactive/authoritative because the teacher educator used revoicing practices to select the correct answer and incorrect answers were not engaged with as seen in excerpt 24.

It is worth noting that even though explaining, justifying, reiterating, revoicing, exemplifying, moving between worlds, defining, etc. were dominant practices in TEIB-L's mathematics classroom, none of these prevailing practices were in themselves the object of attention. For example, even though the teacher educator used a range of examples both during whole class teaching and during group work, the processes involved in exemplifying (that is, what examples to choose and when, selection of example space (Watson & Mason, 2005), the role of examples in the teaching and learning of mathematics, etc) was not an object of attention. Hence, as with other privileged practices, the pre-service teachers had limited opportunities to develop the identity of becoming learners of the mathematical process of exemplifying.

Having dealt with TEIB-L's mathematics classroom community, I now turn to the second teacher educator in University B – TEIB-E.

8.2.2. Shared repertoire and mutual engagement in TEIB-E's CoP

Before delving into the privileged practices in TEIB-E's classroom community of practice, it is expedient to provide a brief background on the class and the task that the classroom community was engaged in tackling. The observed lessons were statistics lessons with a focus on distribution in two variables. As a product of the old SA curriculum, the pre-service teachers had not done statistics (nor probability) previously and were thus encountering it for the first time. The class was taught by a TE who did not share the same home language as most of the students. The class was engaging with a pre-planned task that was given to them by the TE. The first activity in the task involved determining the most important variables (the most important things to consider) for a transport company whose business is to move stuff (eg, furniture) for people who are relocating. The next activity on the task was to narrow these variables down to the two most important ones so that a scatter plot/line of best fit/regression line can be drawn. The task started as depicted in the diagram below:

THE TASK

Situation 6: Fuel consumption of trucks

Consider a transport company using just one type of truck. Before each transport job, the company has to specify the price for the job. In order to specify a price before a job, the company needs to estimate how much their costs will be for doing the job.

Task 1: Identifying variables and postulating the relationships between them

Discuss: Leaving the overhead costs aside (i.e. salaries, etc.), what are the main costs in this kind of business? How are they related? How can we measure them?

And on page 2, the task continued thus (see appendix B for the full task)

One of the main costs is the cost of fuel, and the main factor influencing the amount of fuel used is the distance. But the load weight also plays a role: the greater the load weight, the higher the fuel consumption. The table below gives information that was recorded for previous transport jobs.

Job number	Distance [km]	Load weight [kg]	Fuel used [litres]
1	1304	5445	879
2	1320	2954	639
3	1151	4705	698
4	1371	4378	787
5	325	3673	176
6	1630	5995	1113
7	1023	5357	600
8	620	4988	382
9	73	1992	35
10	1071	5529	680
11	370	4140	218
12	1423	4062	843
13	394	4068	221
14	1536	1678	682
15	1633	3736	887
16	435	3644	241

Previous transport jobs: distance, load weight and amount of fuel used

This task guided the whole discussion on distribution in two variables and ended with finding the formula for finding the regression line and the formula for the least square regression. Table 8.3 below gives an indication of the dominant practices in TEIB-E's classroom.

SHARED REPERTOIRE DIMENSION OF TEIB-E's CoP					
NATURE OF M. PRACTICE	**MP**	**TOTAL**	**NATURE OF N. PRACTICE**	**NP**	**TOTAL**
Initiating and/or sustaining mathematical discussion practices	Explaining mathematically [MP-EM]	129	Conversational norms	Participation by all [NP-PA]	18
	Making mathematical conjecture [MP-MC]	40		Collaboration norm [NP-CB]	2
	Predicting mathematically [MP-PM]	10		Speak-Out norm [NP-SO]	13
	Communicating mathematically [MP-CM]	6		Taking turns to speak norm [NP-TT]	0
	Defining Mathematically [MP-DM]	0	Conceptual norms	Mathematically sensible norm [NP-MS]	12
	exemplifying [MP-PE]	17		Consensus norm [NP-CS]	19
	Reiterating [MP-Rt]	89		Non-Ambiguity norm [NP-NA]	1
	Revoicing [MP-Rv]	13		Justification norm [NP-JN]	12
Other mathematical practices	Summarising on Board [MP-SB]	11	Interpersonal norms	No sole arbiter of knowledge norm [NP-SK]	13
	Summing up[MP-SU]	11		Avoidance of threat [NP-AT]	3
	Moving between worlds [MP-MBW]	12		No Ridicule norm [NP-NR]	7
	Proceduralising [MP-Pc]	11			
	Using Multiple Approaches [MP-MA]	12			
	Gestural Symmetry [MP-GS]	5			
	Writing Mathematically [MP-WM]	7			
Evaluating mathematical validity practices	Critiquing Conjecture [MP-CC]	70			
	Providing Justification [MP-PJ]	52			
	Critiquing solution [MP-CS]	7			

Table: 8.3 Dominant practices and norms in TEIB-E's Classroom CoP

Table 8.3 indicates that the speak-out norm, participation by all norm, consensus norm and the justification norm were the most 'visible' norm. The table also indicates that the teacher educator valued the co-construction of knowledge by not regarding herself as the sole arbiter of knowledge. This has implications especially for the pattern of discourse dimension of mutual engagement which is discussed later.

Table 8.3 also shows a community that is rich in initiating/sustaining mathematical discussion practices for which explaining, making conjectures and reiterating were the most dominant. The table also depicts a community rich in evaluating mathematical validity practices. In short, six dominant practices emerged in the course of this sub-topic in statistics, viz, explaining mathematically, exemplifying, reiterating, revoicing, conjecturing, critiquing conjecture and providing justification. Of note is the fact that defining mathematics was not privileged as a practice. In what follows, I discuss each of the dominant practices, using them as the point of departure towards elucidating the mutual engagement and shared repertoire of TEIB-E's classroom community of practice. I also juxtapose some of them in order to provide a wider discussion on the practices in TEIB-E's classroom. In engaging with these practices in more detail, I pay particular attention to how other dynamics relating to mutual engagement and share repertoire dimensions in the community shaped or were shaped by these practices.

8.2.2.1 The practice of reiterating in TEIB-E's CoP

Reiterating as a practice in TEIB-E's class was mainly used by the teacher educator to repeat a contribution by a community member in order to ensure that everyone understood and were on the same page as the contributor. Excerpt 31 below (where the class was exploring how best to draw the line of best fit) is an instance in which the teacher educator uses reiterating practice not only to check her own understanding of contributions made by a PST, but also to make sure the other PSTs understood the contribution:

Excerpt 31

1	TE	…That is how our reasoning proceeded right? We can choose 2 points, we connect all the points[drawing] all that, then we have 2 points on that line, to add another sentence, to choose the first point and the last point, I actually thought well that could actually be a line that goes through those sets somewhere. Whether they are the best point is the question
2	PST	Ma'am, I was thinking, maybe cos remember at first we were asking how

		do we get those lines …the first thing we did after checking is to ask yourself is ..and when I was drawing my line, I was thinking of it like this, when am constructing it, I'll choose points whereby if am having the line even though the points won't be equal, the points like below the line and on top of the line must be equal or something
3	TE	Do you follow her? She says she challenges me for choosing the first and the last point. And she says she had a line where she chose the line in such a way that it has an equal number of points above and below. There are 16 so 8 points there and 8 points there. That's how she chose her line. So what did I tell you about error?
4	PST	I didn't think about it
5	TE	You can think about it now?
6	PST	I don't know how to put it.
7	TE	Remember, try. If you try, we can make it better
8	PST	[Inaudible]
9		Loud, loud. You are losing their attention
10	PST	[indistinguishable sounds and voices]
11	TE	You have as many errors there as many errors here, meaning as many there as many here. Okay, so I see you do that, you say it'll balance. Your 8 points there and 8 points there are your strategy to find the line. You have an idea the error should balance up, Isn't it? The positive errors and the negative errors you want them to balance out. Isn't it?
12	PST	Yeah
13	PST2	Ma'am I think that with this line that you put between these two points of course it's a huge error where you underestimate otherwise your company won't survive in six months

As excerpt 31 indicates, reiterating as a practice was used for the benefit of both the teacher educator and the pre-service teachers. In turn 3, the teacher educator reiterates the PST's contribution (in turn 2) for the benefit of the classroom community while in turn 11, reiterating served first and foremost, to benefit the teacher educator herself.

In turn 7, the teacher educator uses a shared language *if you try, we can make it better*. This expression is used by the TE to encourage the PSTs to make contribution even when they struggle to express themselves mathematically. TEIB-E makes them understand that they have to first make an attempt to answer and from there, the classroom community "can make it better". This tolerance towards mathematical and grammar related errors was instrumental in creating a community where the norm of Participation by all and the No ridicule norm were valued in the community. This tolerance also partly explains why there was a high level of interanimation of ideas in TEIB-E's classroom community of practice.

8.2.2.2 Explaining mathematically, conjecturing and evaluating mathematical validity practices

Explanations in TEIB-E's class were jointly built through discourse surrounding the transport company task and involved both the teacher educator and the pre-service teachers working together on the task. After reading the introductory part of the task on page 1, the TE initiated a whole class discussion by calling on the PSTs to explain what the task was about. There was a high level of interanimation of ideas where member of the community tabled their ideas which were then subjected to critique by other members of the community. In excerpt 32 below, after presenting the PSTs with the transport company task and a discussion on what the task was about, the TE asked the PSTs to determines the variables that are necessary to be put into consideration by a transport company:

Excerpt 32

1	TE	…Ok. What do you gather a transport company does? What do you think a transport company does?
2	PST1	Depending on the transport company … Supposed to travel from one place to another
3	TE	So they're moving stuff from one place to another? Right you can see a picture; here is a company with one type of truck, they get phone calls, can you please move my house furniture for me from here to there, what will it cost? Ok, what does this company have to do when they get a call like this?
4	PST2	They have to transport the stuff so that they can…they have to know what they're transporting from which place and to which place so that they can…
5	TE	Yes, do you say km readings are important, they have to go from here to there; km readings are important for you, why?
6	PST2	Because it's like the km readings will determine how much they will charge
7	TE	Do you agree? That how much they charge depends on how far they have to drive? If you disagree, …anything else you want to say that it can depend on?
8	PST3	Think that should be how much fuel they need to go to the place
9	TE	And what will that depend on? And what will that depend on?
10	PSTs	The load
11	TE	Why the load?
12	PST4	It's like, ok the load takes up a lot of petrol if you doing that right, it does like at the end of it the more petrol we use to carry something around and

		since there is, I was gonna say that they could use different transport but we are given that it's only one type of transport so am gonna stick with the petrol that…
13	TE	Is that an assumption you can agree with?
14	PST5	No I was thinking about it depends on the nature of the product I have to transfer, for example you can transport 20 tons of cotton
15	TE	Cotton? What do you mean by cotton?
16	PST5	Yeah for example; they can also transport 20 tons of stones of which the volume of 20 tons of cotton will be much bigger compared to the volume of 20 tons of stone so that one will also determine the price, on how much the customer needs to pay them
17	TE	Ok, so we've got various variables now; that one is an interesting one that I haven't thought of so let's see what sense we can make of it.

Rather than deciding at the onset what the important variables would be given the scenario in the task, the teacher educator called for conjectures as to what those important variables could be. PSTs 1-5 provided different conjectures for what the important variables may be for a transport company, and justifications for why they thought those variables were important facts for that the company needs to consider.

As can be seen in excerpt 32 above and excerpt 33 below, one of the features of explanation in TEIB-E's class was that it was carried out to a large extent by both the teacher educator and the pre-service teachers. Through probing questions that involved the teacher posing *why* questions, the teacher educator developed extended dialogue around the variables that are necessary to consider when determining the cost of moving items from one point to the other. By juxtaposing diverse PSTs' perspectives, TEIB-E investigated mathematical connections among those diverse perspectives and enabled a joint construction of knowledge around the task through meaning negotiation. In engaging with the pre-service teachers' responses, the teacher educator requests for justification for PSTs' conjecture as in turns 5, 9 and 11, and calls on other members of the community to critique the conjectures that have been put forward by a PST.

In engaging with the task, there were a number of discursive repertoires that the community had developed over time in instances that call for further explanation. In Extract 33 below, I provide an example of one of such specialised language in TEIB-E's Classroom CoP and how it was used in developing the content knowledge of the PSTs through explanatory and justificatory practices in which the shared language "does it mean that" was used to get PSTs back on track or to enable them understand the concept at hand. The excerpt is drawn from the discussion on how to draw the line of best fit after plotting the graph:

Excerpt 33

1	TE	Have I lost you?
2	PSTs	[Chorus] Yes
3	TE	Okay then you have to tell me; if I have lost you then help me to find out where I have lost you. You have to tell me *does it mean that*, please can I have a few questions of '*does it mean that*'?
4	PST1	*Does it mean that* we are ignoring these points [points that show fuel deceases as load increases]
5	TE	Does it mean that we are ignoring these points? Yes it is. We ignore those points because we say hey they're unusual. We say it is unusual to get situations in which your fuel will decrease if your load increases. …the logical trend that it should increase the fuel consumption should increase if the weight increases. So it does mean that. Another '*does it mean that*' question?
6	PST2	Yes ma'am, I don't wanna say does it mean we ignore other points but say, *does it mean that* we ignore other factors

In the excerpt above, the TE was attempting to explain to the PSTs that in drawing a trend line, it makes sense to be guided by where the point are clusters than by points which appear to be outliers. Another shared language *does it mean that* was a shared reference that participants used as they negotiated the mathematical knowledge around the concept of trend lines in statistics. The expression positions the teacher educator as having more access to the mathematics knowledge than the pre-service teachers. It also positions the pre-service teachers as attempting to access this knowledge. As indicated previously, in a community of practice, members of the community play different roles and have varying competences, but all contribute to the joint enterprise. In using the expression *does it mean that* as a specialised discourse in her class, the role of the teacher educator, it can be argued, was clearly to enculturate the pre-service teachers into becoming knowledgeable about the content at hand (in this case, the concept of line of best fit).

8.2.2.3 Exemplifying as a practice in TEIB-E's classroom

I have already indicated that TEIB-E used a pre-planned (Zodik & Zaslavsky, 2008) task to introduce and discuss distribution in two variables that extended into the concept of regression line and regression formula. Apart from this major task, spontaneous *empirical* (Goldenberg & Mason, 2008) examples were provided by pre-service teachers to further explain or justify their conjectures. In excerpt 31 turn 14, we see a pre-service teacher

substantiating his claim that the transport company should take into consideration the 'nature of the product' to be transport by giving an example that differentiates between transporting cotton and transporting stones.

But as with TEIB-L, exemplifying as a practice in TEIB-E's classroom was used as a tool for developing PSTs's conceptual understanding of the concept of distribution in two variables. At no point in the discussions was the importance of teachers choices in selecting and using examples in mathematics classrooms attended to.

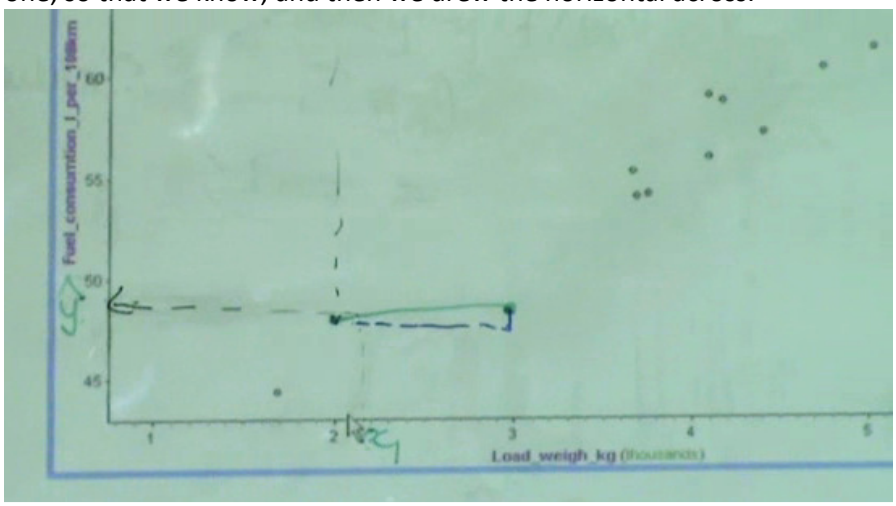
8.2.2.4 Making conjecture and evaluating mathematical validity practices

Conjecturing as a practice was intrinsically linked with the practice of providing justification and critiquing solution. When a conjecture was made, the teacher educator asked questions that required the contributors to justify their thinking. The teacher educator also called for other members of the class to critique the conjectures that have been made. In excerpt 34 below, the discussion was on how to interpolate with the given data set of load versus cost. The class was to devise a strategy for finding the cost of transporting a load of 2000kg and 2500kg – load weights that were not given in the task (See page 2 of the task on page 179). As a reminder, the pre-service teachers were encountering statistics and statistical concepts for the first time.

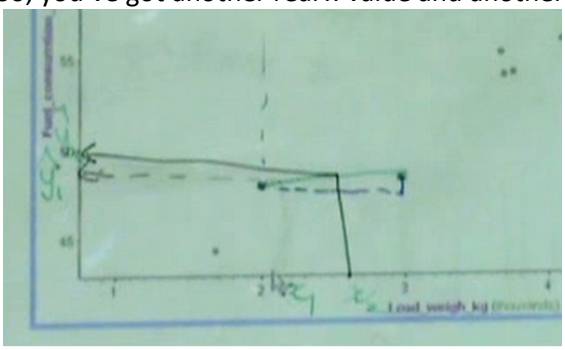
Excerpt 34

1	TE	I might just miss that point so what do I do? What do I do? At 2000, it should be on that point here [pointing to the point], so what will your strategy be in order to predict?
2	PST1	Our strategy was to join the point that is just before 2000 with the next to the point on the scatter plot and then draw a straight line.
3	TE	Okay, what are they saying? They said, let's join the two points on either side of it [either side of 2000]. Why did you draw the line? what's the meaning of this line you drew? [see diagram in turn 6]
4	PST1	It's almost like the difference between the two borders, like the difference in the way fuel consumption grows…[voice fades out, not clear]
5	TE	I am rephrasing what she's saying, I know they're saying slope, I know they're saying difference between 2 consumption rates, okay? As a teacher I am listening for those words because I know the background knowledge is Functions and lines, and line graphs we're drawing functions. They are slopes. She uses that background knowledge, it is valuable for me as a teacher to use that, she gives them to me, I have to take that gift of her thinking and I have to do something with it so [writing on the board], so rephrasing and I am putting my knowledge in. You're saying this line represents for you a change here and a change there [drawing on the board]. Okay, so now how do you go about estimating?

6	PST2	[*PST2 is in the same group as PST1 and so, had worked with PST1 during group work in making a conjecture on how to interpolate*] There we make 2000, and drew a perfectly vertical line up to where it touches the green one, so that we know, and then we drew the horizontal across.



7	TE	And you read it off.
8	PST2	Yes
9	TE	So on my graph yours looks slightly different because of our scales and so it should be something like that [drawing on the board]
10	PST2	and then we did the same with 2500 which is … we drew another vertical line to touch the straight line, and then drew the horizontal line across
11	TE	So, you've got another real x-value and another predicted y-value



12	TE	But I still think of using that line [referring to the line joining two points on the scatter plot]. We will come back to it and challenge it.
13	TE	Anyone on this [referring to the conjecture made by PST1 and PST2]
14		…….
15	PST3	When you check whether that's the green line or those 2 points are going to touch on your fuel consumption line, it's on the same point like it's on 48 for both of them.
16	TE	So you saying on the graph that you see first if I am not saying the wrong thing, that you are also joining 2 points
17	PST3	And we noticed that they're both on 48 there [on the y-axis]
18	TE	So you saying if I read the real y value there [drawing]
19	PST3	Yes
20	TE	And I read the real y value there [drawing]
21	PST3	Yes
22	TE	They seem the same
23	PST3	Ja, and therefore the 2, 2500kg weight will lie on the same y point

24	TE	[writing on the board] so you saying that when you estimating the three and you the values are just the same anyway, the real y value and the predicted y value is the same. Is that a problem? (Writes $\bar{y} = y$)
25	PSTs	[Chorus response] yes

In turns 3,and 24, after conjectures made by PST1 and PST3 respectively on how to interpolate, the TE probes both pre-service teachers for justification and also calls on other members of the community to contribute to the discussion. In so doing, the members of the community are accountable the views of others thereby promoting engagement in the task at hand.

It is worth noting that in TEIB-E's Classroom CoP, mathematical conjectures were mainly made by PSTs, and this was so because of a number of factors: first, the task and the questions posed by the teacher educator were such that engendered making conjectures; second, as outlined previously, the classroom environment promoted the norm of participation by all and norm of no sole arbiter of knowledge.

Another noteworthy feature of TEIB-E's Classroom CoP is the attention paid to the identity of becoming teachers of mathematics as is evident in turn 5 of the except above. In the extracts below in which there was a protracted discussion on finding the trend line, TEIB-E kept using the mathematics that they were doing in class to talk about the teaching of statistics:

Excerpt 35

> TEIB-E: Now before you do that and I'll repeat this, we're sitting here in a situation that is very different from what statisticians do, last week I keep repeating this, statisticians do this, statisticians do that, what we're doing today, no statistician will do that, they won't do it. They go into the computer, they click trend line, it gives them the trend line they want. What we're doing is what teachers do. No statistician will do it but every teacher must do it…

And in another instance,

> TEIB-E: no statisticians do it but teachers do it today. If I ask you to not start with the trend line but background that knowledge of yours, I want you to do it, get the valuable skill of a teacher, background the knowledge that you have and pretend that you know only what the learners in your class know […] How will you predict what the fuel consumption is for 2000kg and for 2500kg?

Mathematical thinking involves, as Stein, Grover & Henningsen (1996, p. 456) put it, "doing what makers and users of mathematics do". Throughout the lesson, TEIB-E kept indicating to

the PSTs what statisticians do and what they do not do and more importantly, what they as mathematics pre-service teachers need to become enculturated into the teaching of mathematics. First, they must be able think like learners who have never been introduced to the concept of a trend line and think of how they would be able to interpolate from a given data; second, they must be able to draw the trend line accurately. Here we see an explicit attention paid to becoming teachers of mathematics through practices such as predicting mathematically, conjecturing, providing justification and critiquing conjectures. TEIB-E is conscious of the fact that she is not teaching mathematics solely for the purpose of content knowledge, but that she is teaching would-be teachers. Table 8.4 below provides an indication of what was privileged in TEIB-E's CoP as far as the interacting identities and pattern of discourse are concerned:

MUTUAL ENGAGEMENT DIMENSION OF TEIB-E's CoP		
CATEGORIES	**CODES**	**TOTAL NUMBER**
PATTERN OF DISCOURSE	Questions that provoke extended dialogue [PED]	51
	TE asks question provoking dialogue [TEQ]	70
	PST asks question provoking dialogue [PSTQ]	33
	TE evaluates [TEEv]	23
	PST(s) evaluate(s) [PSTEv]	20
BUILDING OF IDENTITIES	Becoming teachers of mathematics [BTM]	27
	Becoming teachers of mathematics in multilingual classrooms [BTMMC]	0
	Becoming learners of mathematics [BLMC]	108
	Becoming learners of mathematical practices [BLMP]	7
	Becoming proficient English users [BPEU]	5

Table 8.4: Mutual engagement dimension of TEIB-E's Classroom CoP

Table 8.4 depicts a community where there were a lot of questions that provoked extended dialogue by both the teacher educator and the pre-service teachers, implying that there the class was dialogic rather than authoritative. As can also be noted from the table, little attention was paid to becoming proficient English users and becoming learners of mathematical processes related to the prominent practices that were in use in TEIB-E's classroom. Also, despite the teacher educator's awareness of the context of her practice expressed by the teacher educator in Chapter Six, there was no attention explicitly paid to developing the identity of becoming teachers of mathematics in multilingual context in the Classroom CoP.

8.3 Discussions on University B

The predominant mathematical practices that were used in meaning negotiation in University B were mainly reiterating, revoicing, exemplifying, moving between worlds, defining, writing mathematically, explaining, communicating mathematically and evaluating mathematical validity practices for TEIB-L; and explaining, exemplifying, reiterating, revoicing, conjecturing, and providing justification for TEIB-E. These practices were used differently by the two teacher educators and they also shaped the mutual engagement dimension of each community of practice differently. For TEIB-E, the movement was from setting up a mathematical task to providing epistemological access to the concept. After giving the PSTs a task, the PSTs offered different answers which the TEIB-E and the pre-service teachers evaluate preliminarily; then if the PSTs' contributions were too divergent, the teacher guides the classroom community towards one definite argument or solution. The community then reflects on the meaning of the concepts at hand.

The communicative approach (CA) used by TEIB-L was mainly the interactive/authoritative approach as the teacher educator focuses on one point of view and excludes other points of view. The use of hedges by the PSTs contributed in shaping the mutual engagement dimension of CoP in TEIB-L's class.

On the other hand, the CA used by TEIB-E was mainly interactive/dialogic because a range of ideas were considered and explored in the class. There was also a high level of interanimation of ideas because not only were different points of view explored, but also the PSTs had the opportunity to engage in turn-taking discourse on, for instance, what they would do to interpolate from the known to the unknown in a given data set. Nevertheless, despite the fact that different points of views were explored, TEIB-E had on page 2 of the task (which she requested PSTs not to turn to during the exploration phase), the two variables which she deemed the most essential in the problem context raising the question of how to deal with tension that is inherent in the use of the dialogic approach as discussed in Chapter Two.

The discourse pattern used by TEIB-L was I-R-E while for TEIB-E, the discourse pattern took the I-R-P-R-P- form, and in many cases, the I-Rs_1-Rs_2-Rs_3-…P- form where pre-service teachers engaged with contributions made by their peers. Through the pattern of discourse and the communicative approach used by TEIB-E, a more complex and dynamic process of argumentation develops that involves new considerations by members of the community. In

TEIB-E's class, during the interaction process, the community negotiates the mathematical meanings and tries to come to an agreement about what arguments could be taken as mathematical solutions and appropriate mathematical explanations respectively. A key feature of TEIB-E's Classroom CoP is that the community does not stop at the level of questioning, but moves into the domain of extended dialogue. Certain classroom norms made the interactive/dialogic CA approach and the I-R-P-R-P- discourse pattern possible in TEIB-E's class: First, participation by all norm whereby all PSTs were expected to contribute to the classroom discussions; second, the no sole arbiter of knowledge norm through which PSTs' contributions were also valued. The discursive repertoire which the community had developed as shared language/reference such as the *does it mean that* and the *if you try, we can make it better* clauses also helped in shaping the discourse pattern of the community.

Perhaps, one of the most significant findings in University B is that attention is paid to the development of the identities of becoming teachers of mathematics and becoming proficient English users alongside the identity of becoming learners of mathematics content. But even though the TEs in University B were aware of their context of teaching attention was not paid to enculturating the PSTs into becoming learners of mathematical processes of, for example, exemplifying or justifying. And the teacher educators did not attend to the development of the identity of becoming teachers of mathematics in multilingual classrooms.

8.4 Conclusion

In this chapter, I dealt with the shared repertoire and mutual engagement dimensions of CoP in University B. In doing this, I dealt with the privileged practices in two teacher education classrooms in University B and how these practices are related to other dynamics of the community such as the communicative approach, the pattern of discourse, the norms of practice and the interacting identities. The privileged practices in both teacher education classrooms ranged from defining mathematically to evaluating mathematical validity practices such as providing justification and critiquing solutions. I have also shown that these practices were anchored, as far as the interacting identities are concerned, in becoming learners of mathematical content and in becoming teachers of mathematics. The identities of becoming learners of mathematics in multilingual context and becoming learners of mathematical processes were largely unattended to.

In Chapter Nine, I deal with issue around the joint enterprise at both University A and University B and attempt to deduce what for them was a key joint enterprise.

CHAPTER NINE
Cross-case synthesis and implications for joint enterprise

9.1 Introduction

In Chapters Seven and Eight, I dealt with the shared repertoire and mutual engagement dimensions of CoP in University A and University B respectively. In doing this, I used the privileged practices as my point of departure. In this chapter, I engage with the joint enterprise dimension of CoP in the two universities involved in my study. As indicated in Chapter Five, according to Wenger (1998), the joint enterprise is the result of the process of negotiation of meaning and not necessarily stated as a goal. Thus, in analysing the joint enterprise of the classroom communities in my study, I make the joint enterprise an outcome of the analysis of the mutual engagement and the shared repertoire dimensions of the communities of practice involved in my study. The analysis question that is in focus in this chapter is:

- What can one infer (as opposed to conclude for sure) as the joint enterprise(s) in each of the communities that have been jointly negotiated or which can be considered as their negotiated response to their specific conditions?

In dealing with this analysis question, and given the understanding of joint enterprise as described above and the fact that the joint enterprise is the common purpose which provides a unifying goal and coherence for actions within a community (Wenger, 1998), I look at how practices-in-use (and what these practices attend to) reflect what is/are the joint enterprise(s) of each of the communities of teacher education classroom involved in my study. The joint enterprise also creates relations of mutual responsibility and accountability among members of the community. Hence, I also delve into issues of accountability and responsibility towards the joint enterprise in these communities.

Because of the conceptualisation of joint enterprise as the result of mutual engagement in shared resources, in this chapter, I also do a cross-case synthesis of findings emerging from

University A and B, while engaging with implications and recommendations for pre-service teacher educators and pre-service teacher education programmes.

9.2 How the joint enterprise is reflected through practices-in-use and relations of mutual accountability and responsibility

In Chapter Two, I indicated that for Wenger (1998), the three dimensions of CoP are inextricably intertwined and mutually defining. This intertwinement in my study was evident in the fact that the practices (shared repertoire) that are privileged in the classroom communities of practice lend themselves to the building of identities and what is foregrounded in the interacting identities is evident of what the community values as the joint enterprise(s). In what follows, I use the interacting identities to access what is privileged as the joint enterprise in the classroom communities of pre-service teacher education.

According to Wenger (1998), one learns when one develops competence with respect to a particular valued enterprise and through participation in such enterprise. And through such participation in the practices that are informed by the enterprise of the community, identities are constructed in relation to the community in which one is a participant. As discussed in Chapter Two, there is a broad joint enterprise which serves as an initial *raison d'être* of pre-service mathematics teacher education classroom communities. I have argued that for the pre-service teacher education classrooms involved in my study, the common goal for which the CoPs came to be can be understood in terms of the common desire to develop/improve mathematics proficiency in schools. Both TEs and PSTs are desirous of a more mathematically literate South Africa and this is achieved through their own teaching and learning of mathematics as a broad joint enterprise. But even though all the CoPs share the same goal or broad joint enterprise which is the *raison d'être* of the pre-service mathematics teacher education communities, this joint enterprise is negotiated differently across the four communities of practice classrooms involved in this study. This difference in the way the teacher education classrooms negotiated their joint enterprise is foregrounded in the nature of participation and what is reified in the communities, the type of practices-in-use and how these practices are used, and finally, in what is privileged in terms of the interacting identities. The importance of creating identities that reflect the multiple layers of teacher education, which has been discussed in previous chapters, cannot be overemphasised. In my study, I have argued that within these multiple layers, the identities that the teacher educator develops or sees herself as developing reflect what enterprise or enterprises the teacher

educator values in her community of practice classroom. In what follows, I engage with what was privileged in the different mathematics teacher education classrooms involved this study.

The table below provides a general indication as to whether the practices were anchored in becoming teachers of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics content, becoming teachers of mathematics in multilingual contexts or becoming proficient English users. It must be noted that what is important about this table is not the number of times each identity occurred. It is more about the pattern and what can be perceived as the privileged joint enterprise(s) in each of the teacher education classrooms:

Evidence in support of:	Number of occurrence –TEIA-M	Number of occurrence-TEIA-S	**UNIVER-SITY A**	Number of occurrence – TEIB-L	Number of occurrence – TEIB-E	**UNIVER-SITY B**	**To-tal**
Becoming teachers of mathematics	1	2	3	16	27	43	**46**
Becoming teachers of mathematics in multilingual classrooms	0	0	0	3	0	3	**3**
Becoming learners of mathematics content	80	75	155	111	108	219	**374**
Becoming learners of mathematical practices	0	0	0	1	7	8	**8**
Becoming proficient English users	0	1	1	23	5	28	**29**

Table 9.1: A cross-sectional view of interacting identities in the four CoP classrooms

I use this table to present a discussion on the joint enterprise at University A and University B and also for the cross-sectional discussion on the joint enterprise at both teacher education institutions.

9.2.1 Joint enterprise in University A and accountability to enterprise

From the table above, because TEIA-M predominantly enculturates the pre-service teachers into the identity of becoming learners of mathematical content, it can be inferred that the overriding/valued joint enterprise for TEIA-M's community of practice classroom was the acquisition of mathematical knowledge. TEIA-M perceived her responsibility in the community to be mainly that of fulfilling this role in the class and the PSTs see themselves as recipients of this knowledge. This is evident in the way she uses practices such as defining, explaining, exemplifying, etc. that were discussed in Chapter Seven to develop the mathematical knowledge of the pre-service teachers through an authoritative communicative approach (Mortimer & Scott, 2003) that uses the I-R-E discourse pattern. These practices in themselves were not the focus of attention, nor were they reified as practices for the development of the pre-service teacher educators' identity as teachers of mathematics.

It is also evident from table 9.1 that the valued enterprise in TEIA-S' community of practice classroom is the development of mathematical knowledge, and the practices used by TEIA-S were designed to fulfil this purpose. The teacher educator also sees herself as fulfilling this role in the class. Of note is the fact that despite both TEIA-S and TEIA-M's awareness of the context of their practice and the future context of practice of the PSTs (as discussed in Chapter Six), they did not focus attention on the development of the identity of becoming teachers of mathematics in multilingual classrooms. Therefore, it can be argued that training PSTs to deal with the challenges of teaching and learning in multilingual contexts was not a privileged enterprise at University A. Neither were the development of English language proficiency and engaging with mathematical process of, say, defining, privileged enterprises as depicted in table 9.1. In short, there was less emphasis in University A's communities of practice on all the other forms of identities except that of becoming learners of mathematics content.

9.2.2 Joint enterprise in University B and accountability within CoP

Table 9.1 is an indication that for TEIB-L, not only was the acquisition of mathematical knowledge an important enterprise, but also the development of the identity of the pre-service teachers as future teachers of mathematics was a valued enterprise. This latter enterprise comes to the fore in the class where the teacher insists that PSTs explain the thinking (as teachers would) behind solutions proffered by fellow PSTs and also required of them to

explain terms as they would to learners who were encountering the terms for the first time (see excerpts 28 and 29 in Chapter Eight). The PSTs see themselves as fulfilling both the roles of developing mathematical knowledge of the concepts and of developing competence as teachers of that content. TEIB-L also sees her role as that of enabling the development of English language proficiency in the PSTs. As can be deduced from table 9.1, attention to the development of the PSTs as proficient users of English was one of the valued enterprises in her community of practice.

The valued enterprises in TEIB-E's class were mainly the acquisition of mathematical knowledge and becoming teachers of mathematics. The teacher educator and the design of the transport task strongly encouraged the pre-service teachers to become stakeholders accountable to the contributions of others in the knowledge construction around the task. The teacher educator takes responsibility for PSTs knowledge construction by both emphasising the mathematics content and providing the PSTs with the opportunity to give their perspective on this content by creating a conducive ambiance for making this knowledge construction possible (Hansson, 2010).

9.3 Cross-case synthesis

A remarkable finding that comes to the fore from table 9.1 above is that even though the TEs in the study were aware of their context of teaching – that they were teaching multilingual pre-service teachers who themselves would teach in multilingual contexts at the end of their qualification – this awareness was not reflected unequivocally in their practice. The practices-in-use in their classrooms were mostly those that inducted the pre-service teachers into becoming learners of mathematics content. There were very limited practices aimed at inducting PSTs into becoming teachers of mathematics and even more limited ones that inducted them into becoming teachers of mathematics in multilingual classrooms. In terms of the multidimensional aspects of teacher education that are reflected in the interacting identities, and discussed earlier, in both University A and University B, the development of the pre-service teachers as becoming mathematics teachers in multilingual classrooms and as proficient learners of mathematical process (of say, revoicing, reiterating, etc) were less privileged in the communities of practice. This research therefore shows that mathematics pre-service teachers were in fact not being prepared adequately to understand and subsequently deal with the challenges involved in teaching mathematics in multilingual

contexts which is the case in South Africa. This has significant implications for teacher training in South Africa where most of the classes are multilingual and where most learners, despite their low English language proficiency, choose to do mathematics in English (Setati, 2008).

But although there is an overarching emphasis given to the development of the identity of becoming learners of mathematics content (that is, the acquisition of mathematical content as a joint enterprise), the four teacher education communities of practice in this study experienced this enterprise differently. What is interesting in this study is that these four CoPs opened up different possibilities for the pre-service teachers as far as preparing for teaching mathematics (in multilingual classrooms) is concerned[59]. In order to disaggregate these differences, I take another look at the shared repertoire and the mutual engagement dimensions of CoP in the four communities. To do this, I reflect on how practices, norms, communicative approaches and discourse patterns reflect the nuances in different communities of practice. I argue that the way in which the teacher educator organised participation and where authority stems from shaped what practices were valued which in turn shaped and was shaped by mutual engagement around these practices.

For University A, both TEIA-M and TEIA-S privileged practices such as defining, exemplifying, explaining and proceduralising. The interactive/authoritative communicative approach to the teaching and learning of mathematical concepts with limited interanimation of ideas around the mathematical concepts meant that the PSTs had limited opportunity to develop how to make use of contributions in class in furthering the mathematical development of concepts. It also meant that the PSTs had limited opportunity in engaging in extended discussions around mathematical concepts. Since the practices of the teacher educators in University A largely focused on procedures for arriving at the correct answer, the acquisition of knowledge of concepts taught by TEIA-M and TEIA-S (during whole class discussions) was mainly algorithmic knowledge and thus, there were limited opportunities for the development of relational understanding and relational reasoning for the PSTs. The I-R-E pattern of discourse used by teacher educators in University A was such that there was limited enculturation of the PSTs into the evaluating mathematical validity practices.

In terms of communicative approach and discourse patterns, the practices that were used by TEIB-L and TEIB-E (University B) in the negotiation of meaning around mathematics

[59] And this despite that three of these classroom communities were engaging in mathematics topics which the PSTs were encountering for the first time.

concepts were different and shaped the mutual engagement differently. But even though TEIB-L used largely the interactive/authoritative approach while TEIB-E used the interactive/dialogic approach in the negotiation of meaning, there was a high level of interanimation of ideas in both teacher education classrooms. In requesting for clarifications or further elaboration, in extending invitations to other students for evaluation, and in reiterating the PSTs' contributions, the teacher educators in University B developed mathematical knowledge in PSTs while at the same time providing them with opportunities for the development of their (PSTs) mathematics discourse. To conclude, although the four teacher education communities of practice in Phase Two of this study all steered toward the acquisition of mathematical knowledge as a joint enterprise, they did not all learn content in the same way in terms of the pattern of discourse and the communicative approaches (that were used by the four teacher educators) which shaped the nature of the content. Different aspects of becoming learners of mathematical content were emphasised across both Universities. What does this mean for communities whose initial joint enterprise was the learning and teaching of mathematics and for PSTs who would teach in multilingual mathematics classrooms at the end of their qualification? In a sense, it can be argued that the pre-service teachers in University B have had some experience of dialogic and interactive processes even though multilingualism was not foregrounded[60]. In TEIB-L's and TEIB-E's classrooms, because there was a high level of interanimation of ideas and extended dialogue around the concepts at hand, PSTs had the opportunity of developing both spoken language and mathematical language while simultaneously developing mathematical meanings. The short procedural questions that required short procedural answers used especially in TEIA-M's classroom limited PSTs' opportunity to engage in extended interactions using both the LoLT and the mathematical language. The result of this were that, first, there was limited evaluating mathematical validity practices in University A and second, on the few occasions where TEIA-M asked for explanations/justifications, the PSTs struggled to do so. It can therefore be argued that unlike the PSTs in University A, the PSTs in University B were more exposed to ways of dealing with the triple challenge of paying attention to mathematics, attention to the LoLT and attention to mathematical language as discussed in Chapters One and Three.

[60] The PSTs in TEIA-S's classroom also have this experience but limited to when the teacher educator engages in a one-on-one with them on a particular mathematics task.

9.4 Further implications and some recommendations

One of the findings of this study indicates that at one University (University A), the valued enterprise is the acquisition of mathematical knowledge. This means that beyond the consideration of the teacher educator as modelling what good practice means in terms of the practices that they use, pre-service teacher educators in University A have limited enculturation into the identities of becoming teachers of mathematics. Research on teachers' mathematics knowledge has underscored the importance of teachers' knowledge base of mathematics (subject-matter knowledge or mathematics content knowledge) (Thompson, 1984) and teachers' pedagogic content knowledge (Shulman, 1986). Shulman (1986) for example argues that teachers need not only know both the substantive and the syntactic structures of a subject[61], but also possess the subject matter knowledge *for teaching* which involves a whole range of skills which amongst others include knowledge of the "ways of representing and formulating the subject that makes it comprehensible to others" (p. 9), and the understanding of what makes a particular topic in the subject difficult or easy for students and the knowledge of what research studies have found on a particular topic. The latter knowledge is what distinguishes a pure mathematician from a teacher of mathematics, and thus, in pre-service teacher education classrooms, one of the enterprises needs to be the development of the pre-service teachers' identity as teachers of mathematics.

A related finding from this study is that even though University B pays attention to the development of the identity of becoming teachers of mathematics, just like University A, little attention is paid to the dimension of becoming teachers of mathematics in multilingual contexts. Conteh (2000) argues that the principles which underpin good practices in multilingual contexts are essentially the same as should operate in all classrooms. In a sense, this is true since multilingual mathematics classrooms are first and foremost mathematics classrooms. Thus what needs to be attended to in these classrooms are not just issues of language and communication but also critical issues of access to mathematical knowledge and pedagogy. But while most practices (revoicing, reiterating, explaining, code switching etc) may be present in all mathematics classrooms, in a multilingual classroom, they are more complex and substantively different and impact differently on the nature of discourse that takes place in the classroom. One of such differences is in the obvious fact that multilingual

[61] This was Shulman's critique of Schwab. Substantive structures are "the variety of ways in which the basic concepts and principles of the discipline are organised to incorporate its facts" and the "syntactic structures of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established"(see Chung, 2006).

classrooms present an opportunity for the use of multiple languages in the setting of and discussions around tasks which is clearly not feasible in a monolingual class. Another difference is in the use of code switching as a practice in the classroom. In any mathematics classroom, there is necessarily switching of code of a sort between registers (for example, between ordinary English and mathematical English). What is distinctive in a multilingual class where learners are doing mathematics in a language other than their home language is that there would be both code switching and register switching, that is, there would be switching between registers and between languages (as opposed to only register switching in a monolingual classroom). In a multilingual classroom, therefore, the switch is not necessarily only between registers, but between registers and languages. The know-how of being able to engage with students using certain practices in multilingual classrooms, therefore, is more complex and differs to a certain degree with non-multilingual settings. Thus, this know-how consists of skills that need to also be in focus in teacher training as such skills are learnt and not necessarily acquired through the pre-service teachers' mere experience of being in a multilingual environment. One way of equipping pre-service teachers with this skill is first and foremost creating awareness of the intertwinement between language and mathematics. The interviews in the study revealed that TEs were aware that PSTs did not understand the demands of teaching in multilingual context of students whose proficiency in the LoLT is still developing. Developing the identity of becoming teachers of mathematics in multilingual classrooms would mean creating opportunities where this awareness is entrenched in PSTs. As previously outlined, this awareness in multilingual classroom involve first, the recognition of multilingualism by pre-service teachers as a (potential) resource rather than as a hindrance to epistemological access; second, attention to the linguistic structure in (mathematical/everyday) English as compared to the language structures in the home language(s) of the learners. Beyond creating awareness of the complexity involved in teaching in multilingual classrooms, the teacher educator needs to actively draw on PSTs' multilingualism by creatively tapping into and exploiting the different languages available in the multilingual classroom. The study by Van Jaarsveld (2011) is a case in point. Through the discussion on the meaning of 'turning-point' in a multilingual classroom, it was discovered that this term has no IsiZulu equivalent because it is an oxymoron. Further discussions in the class prompted the learners to suggest "place of turning" as a "useful descriptor for the path of a locus of points referenced in the left-to-right reading of a graph" (Van Jaarsveld, 2011, p. 412). In excerpt 26 turn 2 (Chapter 8), the teacher educator asks the class what tossing a coin is called in their language, but

immediately continues with the class without giving the PSTs any time to respond to the question. As Van Jaarsfeld (2001) argues, the value of using different languages for particular mathematics discussion lies in the use of learners' home language to facilitate mathematical thinking. Strategic use of PSTs' home language(s) to develop mathematical concepts, and what it entails to do this would go a long way in developing PSTs' identity of becoming teachers of mathematics in multilingual contexts.

It was also found that becoming learners of mathematical practices was not a privileged enterprise at both universities. I will use exemplification as a case in point. As far as exemplifying as a practice is concerned, many of the examples provided by the teacher educators in both University A and University B came from either the teacher educators or textbooks or both and they were designed to develop mathematics knowledge acquisition. The process of coming to exemplify (what makes a good example and example space for dealing with particular concepts in the topic area, etc) was not a focal point in the class discussion. What do teachers need to do to develop the identity of becoming learners of mathematical processes of say, exemplifying? One strategy to do this is the use of pre-service teacher generated examples as described by Watson and Mason (2005) and Zazkis and Leikin (2008). These generated examples may include counter-examples and non-examples.

A finding that came to light in Phase One is that at institutional level, there are no structured courses aimed specifically at enculturating pre-service teachers into the dynamics of teaching and learning in multilingual contexts. Yet, most of the pre-service teachers who are trained at these institutions would, at the end of their programme, teach mathematics to multilingual learners for whom English is an additional language. This concern that teacher education courses do not attend specifically to the challenges of teaching mathematics in multilingual classrooms of mostly second language English learners, is not new to pre-service education in South Africa; neither is it new that in pre-service education, the centrality of the importance of language in the teaching of mathematics is not adequately attended to. In the early 90's, the National Education Policy Investigation Report (NEPI, 1993, p. 181) made the following observations:

> A questionnaire sent out by one of our subgroups to teacher training institutions in South Africa revealed that the development of this understanding [that all teachers need to understand the role of language in the education of their pupils] is not currently a focus area in teacher training. As a result, teachers are often unaware of the complex role of language in cognition, in the construction of knowledge, and in the formation of

> individual and group identity. The questionnaire also revealed that there is no component in the training of primary school teachers or of high school subject teachers that prepares them for the challenges of teaching through the medium of a language other than the pupils' home language.

The NEPI (1993) concluded by recommending that notwithstanding whether the LoLT is the home language or a second language, these gaps in teacher training seriously affect the ability of teachers to use the LoLT in the best interests of their pupils when eventually they go to teach in schools at the end of their qualifications. Two decades after this finding, this recommendation remains valid for teacher training institutions in South Africa. Increased attention (in teacher training) to teaching for teaching of mathematics, teaching for teaching of mathematics in multilingual contexts, teaching for learning of mathematical practices, and teaching for development of proficiency in the language of teaching and learning would go a long way in bridging this gap and in producing teachers who are more prepared to deal with the challenges involved in teaching mathematics in a context such as that of South Africa. At the teacher training level, a course that attends to the complexities of teaching and learning in multilingual classrooms is essential. But a single course is not enough to enculturate pre-service teachers into the intricacies involved in teaching mathematics to multilingual learners. Hence, the enterprise of the development of teachers of mathematics in multilingual contexts and what it entails should be a thread that runs through the entire teacher education (mathematics) curriculum.

9.5 Conclusion

This chapter has focused on what the findings from earlier analysis chapters mean for the joint enterprise. It has hence delineated what the joint enterprise or joint enterprises are for each of the communities of practice in my study. In addition to engaging with the conclusions from my study, I also engaged with what these findings mean for teacher education in a context such as that of South Africa. In Chapter Ten, I discuss the contributions this research has made, and the limitations of the study. I also engage with future research directions.

CHAPTER TEN
Contributions, limitations and future directions

Teacher education classrooms seen through the lens of Wenger's communities of practice theory and in particular the notion of communities as comprising of three interrelating and mutually defining dimensions (shared repertoire, mutual engagement and joint enterprise) enabled me gain an understanding of what happens in pre-service teacher education classroom communities and how pre-service teachers are being prepared to teach mathematics to multilingual learners. Teacher education classrooms are complex communities of practice which require an elaborated framework to disentangle the multiple layers involved in teacher development and how multiple identities are produced.

A gap in teacher education that came to light in my previous study (dealing with how improvement of learners' English language proficiency enables or constrains the development of mathematical proficiency) inspired me to undertake this study. In South Africa where most of the classes are multilingual and given the distinct nature of multilingualism discussed in Chapter One, one of the greatest challenges facing teacher education institutions is how to prepare pre-service teachers to deal with the complexity of teaching effectively in multilingual mathematics classrooms (Young, 1995). There is a dearth of research into how this complexity of teaching multilingual mathematics learners (especially at secondary level) whose first language is not the language of learning and teaching is addressed in pre-service teacher education classrooms. In general, teacher education research on mathematics education, thus far, has rarely focused on multilingual mathematics education and research on multilingual mathematics education has hardly focused on teacher education.

This study is one study focusing on teaching and learning mathematics in multilingual teacher education classrooms of pre-service teachers who are being trained to teach in multilingual classrooms at the end of their qualification. The overall aim of the study was to investigate the nature of communities of practice in pre-service teacher education classrooms in order to understand what happens in these classrooms and how pre-service teachers are

being prepared to teach mathematics (in multilingual classrooms). The questions that guided this investigation were:

1) What is the nature of the communities of practice of pre-service mathematics teacher education classrooms in one Province in South Africa?
2) What are the implications resulting from the above with regards to preparing pre-service teachers for teaching mathematics in multilingual classrooms?

I used seven analysis questions based on the three dimensions of CoP by Wenger (1998) to access the nature of each of the communities of practice in my study. The findings from the study (and what they mean for teacher education in a context such as that of South Africa), recommendations and nuances around the nature of the four teacher education classroom communities of practice involved in this study have been discussed in Chapter Nine and are not repeated in this chapter. What is discussed in this chapter are the strengths and the limitations of this study, and what future directions there are for research on teacher education using the theoretical and methodological approach developed and used in this study. This approach is an empirical and original contribution of the study and hopefully a springboard for further research. As claimed earlier, Wenger's CoP theory was the enabling mechanism that enabled me open up teacher education. In what follows, I provide details as to how Wenger unlocked teacher education for this study and what limitations came with it.

Wenger contends that the three dimensions of communities of practice are interrelated and interlocked into a tight system. They are also mutually defining. This interrelationship between the dimensions of CoP became evident for me in the course of analysis. At the centre of the three dimensions of CoP is the joint enterprise (figure 10.1) to indicate that both the mutual engagement dimension and the shared repertoire dimension fed into and shaped the joint enterprise dimension of CoP in my study.

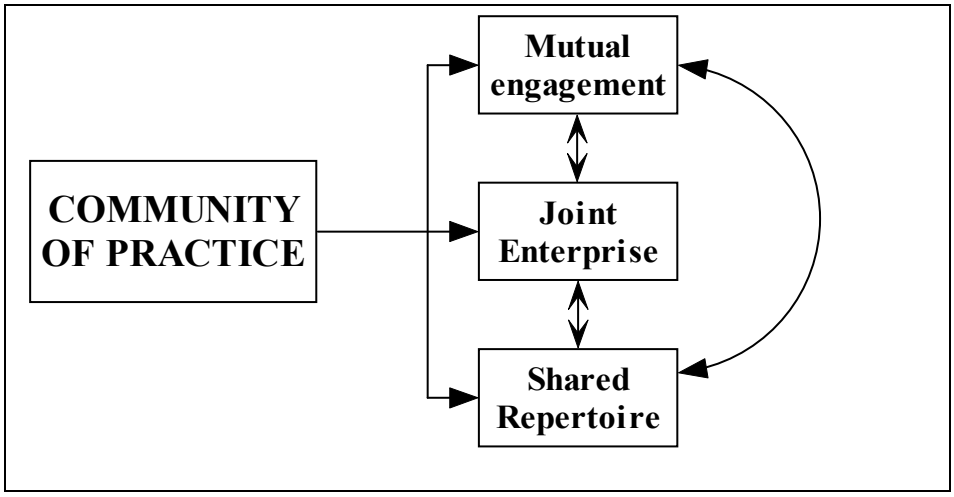


Figure 10.1: The interrelated dimensions of communities of practice

In using the methodological approach to analyse my data, an issue that arose was the fact that the shared repertoire dimension of CoP and the mutual engagement dimension were difficult to analyse separately. For example, in working with the methodological approach, I came to realise that I cannot do an analysis of the data beyond mere description of the practices (and norms) present in the class if I analysed the shared repertoire dimension as an independent entity. For a deeper analysis, I needed to combine the analysis of the different categories within shared repertoire and mutual engagement at the micro level, and between shared repertoire and mutual engagement at a macro level. For example, it was not in the naming of the different practices present in the CoPs that I saw differences between the TE classroom communities but in examining how these practices shape and are shaped by the norms of practice and the mutual engagement dimension of CoP. What classroom norm governs *explaining mathematically* as a mathematical practice? In one community, explanations dealt more with explaining a procedure while in another community, it was more on clarifying a concept. In both cases, the discourse around the concept shaped the nature of the content and provided an indication as to what the pre-service teachers were enculturated into and how their identities were shaped. Thus, both shared repertoire dimension and mutual engagement dimensions analysed together provided a richer description of the classroom communities involved in this study, and *ipso facto*, enabled me to make inferences as to what the joint enterprise(s) of these communities is/are. Of particular significance was the fact that within the organisational language, a methodological contribution of this study (which is developed throughout the thesis) is in the foregrounding of the fact that the negotiation of the joint enterprise relies heavily on the dialogic processes (communicative approach and patterns of discourse used by the teacher educator) that are privileged in the community.

But using Wenger's CoP theory was not without some challenges and limitations. The challenge I needed to deal with using Wenger's notion of CoP was to draw on CoP theory to develop a methodological approach that captures the three dimensions of CoP in the mathematics pre-service classrooms which were investigated. The theoretical contribution the study has made lies in the extension of Wenger's theory to include discourse processes. The resource I brought in to achieve this was the notion of meaning making as a dialogic process proposed by Mortimer and Scott. Akin to this theoretical contribution is a methodological contribution in the development of an organisational language for characterising (multilingual) pre-service teacher education classrooms. The methodological approach developed in this study enabled me to interrogate pre-service teacher education classrooms in an encompassing manner that examines the mathematics content, the interactional context and the discourses in multilingual pre-service teacher education multilingual classrooms in an integrated manner.

Even though the methodological approach is useful in thinking about teacher education communities of practice in terms of mutual engagement, shared repertoire and joint enterprise, the approach however, presents a number of limitations. First, it does not capture the effect of boundary practices of other communities of practice that the pre-service teachers and the teacher educators belong to and how they (boundary practices) impact on the classroom CoPs. Clarke (2008, p. 94) argues rightly that "in conceptualizing the student teachers' community of practice within the wider set of communities of practice that comprise the enterprise of education, the issue of boundaries [in which the students learn to teach through participation in the university and the school communities] must inevitably arise". With regards to this point, one general limitation of this study is that the researcher did not follow the students to their practical teaching and so, cannot analyse PSTs' boundary-crossing practices. Moreover, the methodological approach was not developed to capture and explore the extent of PSTs' enculturation into the practices that are privileged in the CoP or the extent to which the PSTs have formed each of the interacting identities. A different study which aims to explore these would need not only a different methodological approach to the one developed in this study, but also a different set of data.

In terms of the group dynamics (how the group cohere), the methodological approach is limited in capturing how the environment enable and constrain engagement and how members recognise and respond to the needs of others.

A further general limitation of this study is that the data does not permit me to make inferences as to why, for instance the teacher educators in University A focused solely on the development of mathematical knowledge in the communities of practice and backgrounded the development of BTM. Can this be because of the PSTs' level of mathematical knowledge? Or could it be perhaps because of the dominant discourse in the country that it is important to improve mathematics teaching and learning in the country through the improvement of content knowledge? My data does not offer me insight into these questions. If I were to redo this study, rather than a two-phase study, I would make it a three-phase study by including post-observation interviews of teacher educators based on the data collected and particularly based on their privileged practices and dialogic processes. This would enable me to make inference as to why they do what they do in the way they do it. I would also include pre-service teacher interviews in this third phase. This PST interviews would centre on their awareness of the context of their future practice and to what extent they feel adequately prepared to deal with this context. Also, in any teacher education classroom, what is assessed and how assessment practices are carried out provide indicators as to what enterprise is privileged in the community. For example, questions that deal solely with the regurgitation of previously learnt facts and knowledge, and facility in completing procedures leading to the successful execution of a task would indicate a privileging of knowledge acquisition (BLMC). However, assessment practices that include, in addition to the above, questions/tasks that deal with accessing learner thinking through solutions proffered by learners and the PSTs' interpretation of learner solutions indicates a privileging of both BLMC and BTM. In this regard, if I were to repeat this study, I would take assessments and assessment practices into consideration in my data collection and analysis. That said, I turn to future research directions.

How can teacher educators develop the identities of becoming learners of mathematics content (BLMC) and becoming teachers of mathematics (BTM) alongside the development of the identities of becoming learners of mathematical processes (BLMP) and becoming teachers of mathematics in multilingual context (BTMMC)? The identity of becoming learners of mathematical processes is what distinguishes a teacher of mathematics from a pure mathematician and the identity of becoming teachers of mathematics in multilingual context distinguishes teacher prepared for a 'monolingual' class from those prepared for multilingual class. Further study is needed to investigate how PSTs may best acquire the understanding of the complexity of teaching and learning mathematics in multilingual

classrooms, where the challenges of developing PSTs' identity of BLM and BTM are intertwined with the challenges of developing BTMMC, BLMP and becoming proficient English users.

While I understand the need for a teaching methodology course that deals with some of these issues, at the same time, more often than not, methodology courses do not run alongside mathematics content component courses and are rarely taught by the same teacher educators. In fact in University A, as at the time of my research, the methodology course was only done in the last year (fourth year) of pre-service teacher training.

But the journey that led to Wenger is important to reflect on because it was not intuitive. It is important for me to conclude my discussion on my theoretical/methodological approach by indicating that my journey did not start with Wenger's CoP theory as a lens. The initial focus of my study was on exploring what language skills, what language knowledge for teaching mathematics teacher educators need to apprentice pre-service teacher educators into for them to be able to teach mathematics effectively to multilingual learners at the end of their qualification. I had wanted to develop a theory on *Language for Teaching Mathematics* as a framework for mathematics second language teachers teaching in multilingual classrooms. To this end, I used grounded theory to frame my work. I soon realised that developing a theory was not what I really wanted to do. I wanted to rather explore the pedagogic practices that teacher educators use in teaching multilingual pre-service teachers who themselves would teach multilingual learners at the end of their qualification. This led me to situated cognition as a theoretical perspective at the proposal submission stage with a focus on what pedagogic practices teacher educators use for cognitive apprenticeship of pre-service teachers into the dynamics of teaching and learning in multilingual contexts. My choice of situated cognition was motivated by the fact that for situated cognition theorist, learning is an integral part of *generative* (an act of creation or co-creation) *social practice* (it involves partnership) *in the lived-in world* (which denotes that learning takes place in real world settings that make it relevant, useful and transferable) (Brill, 2001; Lave & Wenger, 1991). I soon came to realise that my study was not about cognition as such – that my study was not an investigation into how teacher educators think when, for example, they use a particular

pedagogic practice[62], but on what practices are made available to and the affordances of these practices for pre-service teachers. Hence, I felt a need search for a different theoretical lens.

My search then led me to the use of elements of the situative perspective used in conjunction with Bernstein's notion of pedagogic discourse and Halliday's systemic functional linguistics to provide a theoretical lens with which to explore how teacher educators prepare pre-service teachers to teach mathematics in multilingual classrooms. My central argument and reason for choosing this framework was because of the fact that while most of discursive practices (revoicing, code-switching, etc) are present in all mathematics classrooms, in a multilingual classroom, they are more complex and substantively different and impact differently on the nature of pedagogic discourse that takes place in the classroom. Hence, I argued that while the situative perspective[63] would provide the central theory that framed the study, Bernstein's pedagogic discourse[64] would provide a lens for examining/disentangling the teacher pedagogic discursive talk through which teaching (and learning) takes place and systemic functional linguistics would provide a tool for examining the forms and function of talk and why the teacher educator selects one rather than the other.

After data collection, I soon realised that these theories would not help me unravel what happens in pre-service teacher education classrooms of multilingual pre-service teachers who are being prepared themselves to teach mathematics in multilingual contexts. Systemic functional linguistics focuses analytically on how people use language in social contexts to create meaning and how language is structured or organised to make meaning (Eggins, 2004). Since it was not my intention to do an analysis on how the teacher educator uses language in making ideational, interpersonal and textual meanings, and in language as a semiotic system, I opted for the community of practice aspect of the situative perspective and specifically on Wenger's CoP theory. Changing my theory meant changing, to some extent, the focus of my attention from teacher educators to pre-service teacher education mathematics classroom communities where both teacher educators and pre-service teachers are in focus. This had

[62] At this stage, my study was still mainly focused on pedagogic practices used by in pre-service teacher education classrooms, how the teacher educators implement these practices, how teacher educators model what it means to teach in multilingual classrooms and why teacher educators do what they do in such classroom and why.

[63] Which alludes to specificity to particular context and how such specificity impact on how and what knowledge is produced. In the case of this study, the specificity lies in the presence of multiple languages in the class which present a potential to be used as pedagogic resources.

[64] With a particular focus on the notion of reconceptualisation in relation to pedagogical discourse defined as "the process of moving practice from its original site, where it is effective in one sense, to the pedagogical site where it is used for other reasons"(Lerman, 2000, p. 29).

implications for both the research design and the methodological approach used in data analysis. Research is indeed not a linear process.

As a final remark, this research has made a personal contribution to my development in terms of thinking of my own teaching. This research has sensitised me to the importance of the multidimensional layers involved in teacher education. As a teacher educator involved with preparing PGCE[65] pre-service teachers, I have often tried to model how to teach using different practices in my class. These practices were never themselves the object of attention in our discussions beyond their mere definitions. After conducting this study, I feel the need to deal with, for instance exemplifying and revoicing practices in my class and take my pre-service teachers through not only what these practices are, but more importantly, what makes a good example and example space. PST-generated examples will also form part of my classroom practice from henceforth.

[65] The Post Graduate Certificate in Education (PGCE) is a one-year programme designed for PSTs who have acquired the subject-specific knowledge but not the pedagogy involved in teaching such content.

REFERENCES

Adler, J. (1995). Dilemmas and a paradox--secondary mathematics teachers' knowledge of their teaching in multilingual classrooms. *Teaching and Teacher Education, 11*(3), 263-274.

Adler, J. (2001). *Teaching Mathematics in Multilingual Classrooms*. Dordrecht: Kluwer Academic Publishers.

Agnihotri, R. K. (1995). Multilingualism as a classroom resource. In K. Heugh, A. Siegruhn & P. Pluddemann (Eds.), *Multilingual Education for South Africa*. Johannesburg: Heinemann.

Aguiar, O. G., Mortimer, E. F., & Scott, P. H. (2010). Learning from and responding to students' questions: The authoritative and dialogic tension. *Journal of Research in Science Teaching, 47*(2), 174-193.

Aguilar, O. M., & Krasny, M. E. (2011). Using the communities of practice framework to examine an after-school environmental education program for Hispanic youth. [Article]. *Environmental Education Research, 17*(2), 217-233.

Akkerman, S. F., & Meijer, P. C. (2011). A dialogical approach to conceptualizing teacher identity. *Teaching and Teacher Education*(27), 308-319.

Alexander, N. (2012). The centrality of the language question in postapartheid South Africa: Revisiting a perennial issue. *South African Journal of Science, 108*(9/10), 1-7.

Andrews, P. (2009). Mathematics Teachers' Didactic Strategies: Examining the Comparative Potential of Low Inference Generic Descriptors. *Comparative Education Review, 53*(4), 559-581.

Arzarello, F., Robutti, O., & Bazzini, L. (2005). Acting is learning: focus on the construction of mathematical concepts. *Cambridge Journal of Education, 35*(1), 55-67.

Ayeomoni, M. O. (2006). Code-Switching and Code-Mixing: Style of Language Use in Childhood in Yoruba speech Community. *Nordic Journal of African Studies, 15*(1), 90-99.

Baker, C. (1988). *Key Issues in Bilingualism and Bilingual Education*. Clevedon: Multilingual Matters.

Barab, S., & Duffy, T. (2000). From Practice Fields to Communities of Practice. In D. Jonassen & S. Land (Eds.), *Theoretical Foundations of Learning Environments*. Mahwah, NJ: Lawrence Erlbaum Associates.

Barton, D., & Hamilton, M. (2005). Literacy, reification and the dynamics of social interaction. In D. Barton & K. Tusting (Eds.), *Beyond communities of practice: Language, power and social context* (pp. 14-35). Cambridge: Cambridge University Press.

Barwell, R. (1998). Maths in a Multi-Lingual Environment. *Mathematics Teaching, 165*, 26-27.

Barwell, R. (2002). The development of a discursive psychology approach to investigate the participation of students with English as an additional language (EAL) in writing and solving arithmetic word problems with peers. Unpublished Doctoral Thesis. University of Bristol: Unpublished Doctoral Thesis.

Barwell, R. (2009). Multilingualism in mathematics classrooms: An introductory discussion. In R. Barwell (Ed.), *Multilingualism in Mathematics Classrooms: Global perspectives* (pp. 1-13). Bristol: Multilingual Matters.

Barwell, R., Barton, B., & Setati, M. (2007). Multilingual issues in mathematics education: Introduction. *Educational Studies in Mathematics, 64*, 113-119.

Bednarz, N. (1996). Language Activity, conceptualization and problem Solving: The role played by verbalisation in the development of mathematical thought in Young Children. . In H. Mansfield, N. Pateman & N. Bednarz (Eds.), *Mathematics for Tomorrow's Young Children: International perspectives on Curriculum* (pp. 228-239). Dordrecht: Kluwer.

Beijaard, D., Meijer, P. C., & Verloop, N. (2004). Reconsidering Research on Teachers' Professional Identity. *Teaching & Teacher Education: An International Journal of Research and Studies, 20*(2), 107-128.

Bernard, R. H. (2000). *Social Research Methods: Qualitative and Quantitative approaches*. Thousand Oaks: Sage Publications, Inc.

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in Mathematics Education. In J. Novotna (Ed.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*. Prague, Czech Republic: PME.

Borgmann, A. (1974). *The Philosophy of Language: Historical Foundations and Contemporary Issues*. Hague: Martinus Nijhoff.

Borko, H., Peressini, D., Romagnano, L., Knuth, E., Willis-Yorker, C., Wooley, C., et al. (2000). Teacher Education does matter: A situative view of learning to teach secondary Mathematics. *Educational Psychologist, 35*(3), 193-206.

Boylan, M. (2005). School classrooms: Communities of practice or ecologies of practices?, *Paper presented at the First Socio-Cultural Theory in Educational Research*. Manchester University, UK. http://orgs.man.ac.uk/projects/include/experiment/mark_boylan.pdf.

Brantlinger, E., Jimenez, R., Klingner, J., Pugach, M., & Richardson, V. (2005). Qualitative Studies in Special Education. *Exceptional children, 71*(2), 195-207.

Brill, J., M. (2001). Situated Cognition. In M. Orey (Ed.), *Emerging perspectives on learning, teaching and technology*. : Department of Educational Psychology and Instructional Technology, University of Georgia.

Brilliant-Mills, H. (1994). Becoming a Mathematician: Building a Situated definition of Mathematics. *Linguistics and Education, 5*, 301-334.

Broeder, P., Extra, G., & Maartens, J. (2002). *Multilingualism in South Africa with a focus on KwaZulu-Natal and Metropolitan Durban*. Cape Town: University of Cape Town.

Brown, A., & Dowling, P. (1998). *Doing Research/Reading Research: A mode of interrogation for Education*. London: The Falmer Press.

Brown, T. (1997). *Mathematics Education and Language*. Dordrecht: Kluwer.

Casa, T. M., & DeFranco, T. C. (2005). *Making decisions about discourse: Case studies of three elementary-level teachers with various backgrounds and years of teaching experience.* Paper presented at the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.

Cele, N. (2001). Oppressing the oppressed through language liberation: Repositioning English for meaningful education. *Perspectives in Education, 19*(1).

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2003). Classroom Discussions Using Math Talk in Elementary Classrooms. *Math Solution,* 1-3.

Chitera, N. (2009). Code-switching in a college mathematics classroom. *International Journal of Multilingualism, 6*(4), 426-442.

Chung, H. H. (2006). Code Switching as a Communicative Strategy: A Case Study of Korean–English Bilinguals. *Bilingual Research Journal, 30*(2), 293-307.

Clarke, M. (2008). *Language teacher identities: Co-constructing discourse and community*. Clevedon: Multilingual Matters.

Clarkson, P. (1991). Mathematics in a Multilingual Society. In K. S. Durkin, B. (Ed.), *Language in Mathematical Education: Research and Practice* (pp. 237-246). Philadelphia: Open University Press.

Clarkson, P. (1992). Language and Mathematic: A Comparison of Bilingual and Monolingual Students of Mathematics. *Educational studies in Education, 23*, 417-429.

Cobb, P., & McClain, K. (2001). An approach for supporting teachers' learning in social context. In F.-L. Lin & T. Cooney (Eds.), *Making sense of mathematics teacher education*. Dordrecht, The Netherlands: Kluwer.

Collanus, M., Kairavuori, S., & Rusanen, S. (2012). The identities of an arts educator: Comparing discourses in three teacher education programmes in Finland. *International Journal of Education Through Art, 8*(1), 7-21.

Conteh, J. (2000). Multilingual Classrooms, Standards and Quality: Three Children and a lot of Bouncing Balls. *Language and Education, 14*(1), 1-17.

Cox, A. (2005). What are communities of practice? A comparative review of four seminal works. *Journal of Information Science, 31*(6), 527-540.

Creese, A. (2005). Mediating allegations of racism in a multiethnic London school: What speech communities and communities of practice can tell us about discourse and power. . In D. Barton & K. Tusting (Eds.), *Beyond communities of practice* (pp. 55-76). Cambridge: Cambridge University Press.

Crotty, M. (1998). *The foundations of social science research:meaning and perspective in the research process*. New SouthWales: Allen and Uwin.

Cuevas, G. J. (1984). Mathematics learning in English as a second language. *Journal of research in Mathematics Education, 15*(2), 134-144.

Cummins, J. (1979a). Linguistic Interdependence and the Educational Development of bilingual Children. *Review of Educational Research, 49*, 222-251.

Cummins, J. (1979a). Linguistic Interdependence and the Educational Development of Bilingual Children. *Review of Educational Research, 49*(2), 222-251.

Cummins, J. (1979b). Cognitive/academic language proficiency, linguistic interdependence, the optimum age question and some other matters. *Working Papers on Bilingualism, 19*, 121-129.

Cummins, J. (2007). Rethinking monolingual instructional strategies in multilingual classrooms. *Canadian Journal of Applied Linguistics, 10*, 221-241.

de Klerk, V. (2002). Language issues in our schools: Whose voice counts? Part 1: The parents Speak. *Perspectives in Education, 20*(1-14).

Denzin, N., & Lincoln, Y. (2000). Methods of collecting and analyzing empirical materials. In N. Denzin & Y. Lincoln (Eds.), *Handbook of Qualitative Research*. California: Sage Publications.

DHET. (2011). Minimum Requirements for Teacher Education Qualification. *Government Gazette No. 583*, Pretoria.

Dinkelman, T. (2011). Forming a teacher educator identity: uncertain standards, practice and relationships. *Journal of Education for Teaching, 37*(3), 309-323.

DoE. (1997). *Language-in-Education Policy*. Pretoria: Department of Education.

DoE. (2002). *Language policy for Higher education*. Pretoria: Department of Education.

DoE. (2003). *Teacher's Guide for the Development of Learning Programmes: Mathematics*. Pretoria: Department of Education.

Dooley, K. (2002). Genre-based pedagogy and cooperative learning: Integrating strategies to achieve high quality outcomes for ESL students in mainstream primary school classes. *TESOL in Context, 11*(2).

Dowling, P. (1998). *The Sociology of Mathematics Education: Mathematics Myths/Pedagogic Texts*. London: The Falmer Press.

Dowling, P. (2009). *Sociology as Method: Departures from the Forensics of Culture, Text and Knowledge*. Rotterdam: Sense Publishers.

Dowling, P., & Brown, A. (2010). *Doing Research/Reading Research*. New York: Routledge.

du Plessis, S., & Elsie, N. (2003). Needs of teachers in preschool centres with regard to multilingual learners. *South African Journal of Education, 23*(2), 122-129.

Edwards, A., & Westgate, D. (1987). *Investigating Classroom Talk*. London: The Falmer Press.

Eggins, S. (2004). *An introduction to systemic functional linguistics*. New York: Continuum.

Engle, R. A., & Conant, F. R. (2002). Guiding Principles for Fostering Productive Disciplinary Engagement: Explaining an Emergent Argument in a Community of Learners Classroom. *Cognition and Instruction, 20*(4), 399-483.

Enyedy, N., Rubel, L., Castellon, V., Mukhopadhyay, S., Esmonde, I., & Secada, W. (2008). Revoicing in a Multilingual Classroom. *Mathematics Thinking and Learning, 10*, 134-162.

Eraut, M. (2002). *Conceptual analysis and research questions: Do the concepts of 'learning community' and 'community of practice' provide added value?* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. April 1-5.

Ernest, P. (1994). The Dialogical Nature of Mathematics. In P. Ernest (Ed.), *Mathematics, Education and Philosophy: An international perspective* (pp. 33-38). London: The Falmer Press.

Esmonde, I. (2009). Explanations in mathematics classroom: A discourse analysis. *Canadian Joural of Science, Mathematics and Technology Education, 9*(2), 86-99.

Essien, A. (2010a). Mathematics teacher educators' account of preparing pre-service teachers for teaching mathematics in multilingual classroom: The case of South Africa. *The International Journal of Interdisciplinary Social Sciences, 5*(2), 33-44.

Essien, A. (2011). *Exploring the English Proficiency-Mathematics Proficiency relationship: An investigation using instructional English Computer Software*. Saarbrücken: VDM.

Garcia, O. (2008). Multilingual language awareness and teacher education. In J. Cenoz & N. H. Hornberger (Eds.), *Encyclopedia of Langauge and Education: Knowledge about Language* (2nd ed., Vol. 6, pp. 385-400): Springer Science & Business Media LLC.

Godino, J. D., Batanero, C., & Font, V. (2007). The Onto-semiotic approach to research in mathematics education. *ZDM, 39*(1-2), 127-135.

Goldenberg, P., & Mason, J. (2008). Shedding light on and with example spaces. *Educational Studies in Mathematics, 69*(2), 183-194.

Goldhaber, A. (2006). Math as a Language in its own right. *American Scientist, 94*, 185-186.

Gomez, P. (2007). Learning within communities of practice in preservice secondary school teachers education. *PNA, 2*(1), 17-28.

Gomez, P. (2009). Toward a methodology for exploring mathematics preservice teachers' learning from a sociocultural perspective. *PNA, 3*(4), 157-168.

Goos, M., & Bennison, A. (2008). Developing a communal identity as beginning teachers of mathematics: Emergence of an online community of practice. *Journal of Mathematics Teacher Education, 11*(1), 41-60.

Goos, M., Galbraith, P., & Renshaw, P. (1999). Establishing a community of practice in a secondary mathematics classroom. In L. Burton (Ed.), *Learning Mathematics: From Hierarchies to Networks* (pp. 36-61). London: Falmer Press.

Gorgorio, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics, 47*, 7-33.

Graven, M. (2003). Teacher Learning as Changing Meaning, Practice, Community, Identity and Confidence: the Story of Ivan. *For the Learning of Mathematics, 23*(2), 25-33.

Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of Confidence. *Educational Studies in Mathematics, 57*, 177-211.

Graven, M., & Lerman, S. (2003). Wenger, E. (1998). Communities of practice: Learning, meaning and identity. *Journal of Mathematics Teacher Education, 6*(2), 185-194.

Griffiths, M. (1998). *Educational research for social justice*. Buckingham: Open University Press.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shaha, E., & Williamson, P. W. (2009). Teaching Practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055-2100.

Gutiérrez, R. (2002). Beyond Essentialism: The Complexity of Language in Teaching Mathematics to Latina/o Students. *American Eductional Research Journal, 39*(4), 1047-1088.

Halai, A. (2004). *Teaching Mathematics in Multilingual classrooms*, Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education.

Halai, A. (2009). Politics and practice of learning mathematics in multilingual classrooms: Lessons from Pakistan. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 33-47). Bristol: Multilingual matters.

Halia, A. (2004). *Teaching Mathematics in Multilingual classrooms*, Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education.

Halliday, M. A. K. (1978). *Language as Social Semiotic: The Social Interpretation of Language and Meaning*. London: Arnold.

Hansson, Å. (2010). Instructional responsibility in mathematics education: modelling classroom teaching using Swedish data. *Educational Studies in Mathematics, 75*(2), 171-189.

Hatch, J. A. (2002). *Doing qualitative research in educational settings*. New York: SUNY.

Herzig, A. H. (2005). Goals for achievin diversity in mathematics clasrooms. *Mathematics Teacher, 99*(4), 253-259.

Heugh, K. (2002). The case against bilingual and multilingual education in South Africa: Laying bare the myths. *Perspectives in Education, 20*(1).

Hodges, T. E., & Cady, J. A. (2012). Negotiating Contexts to Construct an Identity as a Mathematics Teacher. *Journal of Educational Research, 105*(2), 112-122.

Horn, I. S., Nolen, S. B., Ward, C., & Campbell, S. S. (2008). Developing Practices in Multiple Worlds: The Role of Identity in Learning to Teach. *Teacher Education Quarterly, 35*(3), 61-72.

Howie, S. (2002). *English Language Proficiency and Contextual Factors Influencing Mathematics Achievement of Secondary School Pupils in South Africa*. Enschede: PrintPartners Ipskamp.

Howie, S. (2003). Language and other background factors affecting secondary pupil's performance in Mathematics in South Africa. *African Journal of Research in Science, Mathematics and Technology Education, 7*, 1-20.

Howie, S. (2004). A national assessment in mathematics within an international comparative assessment. *Perspectives in Education, 22*(2), 149-162.

Howie, S., & Pietersen, J. (2001). Mathematics Literacy of final year students: South African realities. *Studies in Educational Evaluation, 27*, 7-25.

Huberman, M., & Miles, M. B. (2002). *The Qualitative Researcher's Companion*. Califonia: Sage Publications.

Johnson, H. (2007). Communities of practice and internationl development. *Progress in Development Studies, 7*(4), 277-290.

Kanes, C., & Lerman, S. (2008). Analysing Concepts of Community of Practice. In A. Watson & P. Winbourne (Eds.), *New Directions for Situated Cognition in Mathematics Education*. New York: Springer.

Kasule, D., & Mapolelo, D. (2005). Teachers' strategies of teaching primary school matheamtics in a second language: A case of Botswana. *International Journal of Educational Development, 25*, 602-617.

Khisty, L. L., & Morales, H. (2002). Discourse matters: Equity, Access and Latino's learning mathematics. Retrieved January 7, 2008, from http://www.icme-organisers.dk/tsg25/subgroups/khisty.doc

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping Children learn Mathematics*. Washington DC: National academic press.

Krashen, S. D. (1982). *Principles and practice in second language acquisition*. Oxford: Pergamon Press.

Larreamendy-Joerns, J., & Muñoz, T. (2010). Learning, Identity, and Instructional Explanations. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 23-40): Springer US.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.

Leikin, R., Berman, A., & Zaslavsky, G. (2000). Learning Through Teaching: The Case of Symmetry. *Mathematics Education Research Journal, 12*(1), 18-36.

Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and location for contrast. In V. Richardson (Ed.), *Handbook for research on teaching (4 th Ed.)* (pp. 333-357). Washington, DC: American Educational Research Association.

Leinhardt, G. (2010). Introduction: Explaining Instructional Explanations. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 1-5): Springer US.

Leinhardt, G., & Steele, M. D. (2005). Seeing the Complexity of Standing to the Side: Instructional Dialogues. *Cognition and Instruction, 23*(1), 87-163.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning: International perspectives on mathematics education*. Westport: Ablex.

Levinson, B. A. U., & Brantmeier, E. J. (2006). Secondary Schools and Communities of Practice for Democratic Civic Education: Challenges of Authority and Authenticity. *Theory & Research in Social Education, 34*(3), 324-346.

Li, L., Grimshaw, J., Nielsen, C., Judd, M., Coyte, P., & Graham, I. (2009). Evolution of Wenger's concept of community of practice. *Implementation Science, 4*(1), 1-8.

Llinares, S., & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. In A. Gutierez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathemtics Education: Past, Present and Future* (pp. 429-459). Rotterdam: Sense Publishers.

Lyon, J. (1996). *Becoming Bilingual: Language acquisition in a Bilingual Community*. Clevedon: Multilingual Matters.

Mati, X. (2004). *Using code switching as a strategy for bilingual education in the classroom.* Paper presented at the 21st annual AEAA Conference, Cape Town, South Africa.

Maxwell, J. A. (1992). Understanding and Validity in Qualitative Research. *Harvard Educational Review, 62*(3), 279-300.

Mayer, D. (1999). Building teaching identities: Implications for preservice teacher education. In AARE (Ed.), *Global Issues and Local Effects: The Challenge for Educational Research*. Melbourne: Australian Association for Research in Education Conference, 27 Nov - 2 Dec 1999.

McClain, K., & Cobb, P. (2001). An analysis of the development of sociomathematical norms in one First-Grade classroom. *Journal for Research in Mathematics Education, 32*(3), 236-266.

McClain, K., & Cobb, P. (2004). *The critical role of institutional context in teacher development.* Paper presented at the 28th conference of the International Group for the Psychology of Mathematics Education, Bergen, Norway.

Mercer, N. (2000). How is Language used as a medium for Classroom Education? In B. Moon, M. Ben-Peretz & S. Brown (Eds.), *Routledge International Companion to Education*. London: Routledge.

Mercer, N. (2001). Language for teaching a language. In C. N. Candlin & N. Mercer (Eds.), *English Language Teaching in its social context: a Reader*. London: Routledge.

Mitra, D. L. (2008). Balancing power in communities of practice: An examination of increasing student voice through school-based youth–adult partnerships. [Article]. *Journal of Educational Change, 9*(3), 221-242.

Monaghan, F. (2009). Mapping the mathematical langscape. In R. Barwell (Ed.), *Multilingualism in Mathematics Classrooms: Global perspectives* (pp. 14-31). Bristol: Multilingual Matters.

Moore, M., & Kearsley, G. (1996). *Distance education: A systems view*. New York: Wadsworth.

Mortimer, E. F., & Scott, P. H. (2003). *Meaning making in secondary science classrooms*. Philadelphia: Open University Press.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the learning of mathematics, 19* (1), 11-19.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*, 189-212.

Nel, N., & Muller, H. (2010). The impact of teachers' limited English proficiency on English second language learners in South African Schools. *South African Journal of Education, 30*, 635-650.

NEPI. (1993). *The framework report and final report summaries*. Cape Town: Oxford University Press/NECC.

O'Connor, M. C., & Michaels, S. (1993). Aligning Academic Task and Participation Status through Revoicing: Analysis of a Classroom Discourse Strategy. *Anthropology & Education Quarterly, 24*(4), 318-335.

Ohtani, M. (2003, December 1-3). *Social Formation of Mathematical Activity in a Japanese Mathematics Classroom: "Revoicing" as a Unit of Analysis.* Paper presented at the Learner's Perspective Study, Melbourne.

Olson, J. C., Knott, L., & Currie, G. (2009). Discursive practices in college pre-calculus classess. In L. knott (Ed.), *The Role of mathematics discourse in producing leaders of discourse*. Charlotte, N.C: Information Age Publishing.

Peregoy, F., & Boyle, F. (2000). English learners reading English: What we know, what we need to know. *Theory Into Practice, 39*(4), 237-247.

Pimm, D. (1981). Mathematic? I speak it fluently. In A. Floyd (Ed.), *Developing Mathematical thinking* (pp. 139-150). Addision Wesley: Workingham.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge & Kegan Paul.

Pimm, D. (1991). Communicating mathematically. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education: Research and Practice* (pp. 17-23). Philadelphia: Open University Press.

Pimm, D., & Keynes, M. (1994). Mathematics classroom Language: Form, Function and Force. In Biehler R., Sholz R., Straber R. & Winkelmann B. (Eds.), *Didactics of mathematics as a scientific Discipline* (pp. 159-169). Dordrecht: Kluwer Academic Publishers.

Pluddermann, P. (2002). Action and reflection: Dual-medium primary schooling as language policy realisation. *Perspectives in Education, 20*, 47-64.

Povey, H., & Burton, L. (1999). Learners as authors in the mathematics classroom. In L. Burton (Ed.), *Learning Mathematics: From Hierarchies to Networks* London: Falmer Press.

Putnam, R. T., & Borko, H. (2000). What do new views of Knowledge and Thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4-15.

Reynolds, A., & Wheatley, G. (1996). How do Social Interactions among Children contribute to learning? . In H. Mansfield, N. Pateman & N. Bednarz (Eds.), *Mathematics for Tomorrow's Young Children: International perspectives on Curriculum* (pp. 186-197). Dordrecht: Kluwer.

Romberg, T. (1992). Further thoughts on the Standards: A reaction to Apple. *Journal for Research in Mathematics Education, 23*(5), 432-437.

Rowland, T. (1995). Hedges in mathematics talk: Linguistic pointers to uncertainty. *Educational Studies in Mathematics, 29*(4), 327-353.

Ryve, A. (2011). Discourse research in mathematics education: A critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education, 42*(2), 167-198.

Schegloff, E. A. (2006). Interaction: The infrastructure for social institutions, the natural ecological niche for language, and the arena in which culture is enacted. In N. J. Enfield & S. C. Levinson (Eds.), *Roots of Human Sociality: Culture, cognition and interaction* (pp. 70-96). London: Berg.

Scott, P. H., Mortimer, E. F., & Aguiar, O. G. (2006). The tension between authoritative and dialogic discourse: A fundamental characteristic of meaning making interactions in high school science lessons. *Science Education, 90*(4), 605-631.

Setati, M. (2000b). *Classroom-based research: From with or on teachers to with and on teacher.* Paper presented at the Second International Mathematics Education and Society Conference, Lisbon.

Setati, M. (2002). Researching Mathematics Education and Language in Multilingual South Africa. *The Mathematics Educator, 12*(2), 6-20.

Setati, M. (2005a). Mathematics Education and Language: Policy, Research and Practice in multilingual South Africa. In R. Vithal, J. Adler & C. Keitel (Eds.), *Researching Mathematics Education in South Africa*. Cape Town: HSRC Press.

Setati, M. (2005b). Teaching Mathematics in a Primary Multilingual Classroom. *Journal for Research in Mathematics Education, 36*(10), 1-20.

Setati, M. (2008). Access to mathematics versus access to the language of power: the struggle in multilingual mathematics classrooms. *South African Journal of Education, 28*, 103-116.

Setati, M., & Adler, J. (2000). Between language and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*(3), 243-269.

Setati, M., Adler, J., Reed, Y., & Bapoo, A. (2002). Incomplete Journeys: Codeswitching and Other Language Practices in Mathematics, Science and English Language Classrooms in South Africa. *Language and Education, 16*(2), 128 - 149.

Sfard, A., Nesher, P., Streefland, L., Cobb, P., & Mason, J. (1998). Learning Mathematics through Conversation: Is It as Good as They Say? *For the Learning of Mathematics, 18*(1), 41-51.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Siegel, M., & Borasi, R. (1994). Demystifying Mathematics Education through Inquiry. In P. Ernest (Ed.), *Constructing Mathematical knowledge: Epistemology and Mathematical Education* (pp. 201-214). London: The Falmer Press.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Smit, J., & van Eerde, H. (2011). A teacher's learning process in dual design research: learning to scaffold language in a multilingual mathematics classroom. *ZDM, 43*(6), 889-900.

Smith, T. (2006). Becoming a teacher of Mathematics: Wenger's social theory of learning perspective. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australia)*. Sydney: MERGA.

Staats, S. (2009). Somali Mathematics Terminology: a community exploration of Mathematics and Culture. In R. Barwell (Ed.), *Multilingualism in Mathematics Classrooms: Global Perspectives* (pp. 32-46). Bristol: Multilingual Matters.

Stein, M., Engle, R. A., smith, M., & Hughes, E. (2008). Orchestrating productive mathematical discussions: Five practices for helping teacher move beyond show and tell. *Mathematical Thinking and Learning: An International Journal, 10*(4), 313-340.

Stein, M., Grover, B., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An analysis of Mathematical Tasks Used in Reform Classrooms. *American Eduational Research Journal, 33*(2), 455-488.

Sullivan, P., Zevenbergen, R., & Mousley, J. (2005). *Planning and teaching mathematics lessons as a dynamic, interactive process.* Paper presented at the 29th Conference of the International Group for the Psychology of Mathematics Education, Melbourne.

Tatsis, K., & Koleza, E. (2008). Social and socio-mathematical norms in collaborative problem-solving. *European Journal of Teacher Education, 31*(1), 89-100.

Taylor, N., Muller, J., & Vinjevold, P. (2003). *Getting Schools Working: Research and Systemic School reform in South Africa*. Cape Town: Pearson Education.

Thompson, A. G. (1984). The relationship of Teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Tusting, K. (2005). Language and power in communities of practice. In D. Barton & K. Tusting (Eds.), *Beyond communities of practice: Language, power and social context* (pp. 36-54). Cambridge: Cambridge University Press.

Van Jaarsveld, P. (2011). Do multilingual mathematics classrooms promote or impede teaching and learning? Episodes of communication around the features of a parabola. In M. Setati, T. Nkambule & L. Goosen (Eds.), *Proceedings of the ICMI study 21 conference: Mathematics education and language diversity* (pp. 406-413). Sao Paulo, Brazil. 16 – 20 September.

Van Zoest, L. R., & Bohl, J. V. (2005). Mathematics Teacher Identity: a framework for understanding secondary school mathematics teachers' learning through practice. *Teacher Development, 9*(3), 315-345.

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meanings: Interaction in classroom cultures* (pp. 163-201). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. *Mathematical Thinking and Learning, 9*(1), 31-50.

Wallace, C., & Goodman, Y. (1989). Research Currents: Language and Literacy Development of Multilingual Learners. *Language Arts, 66*(5), 542-551.

Wang, T. (2005). Choral response in two six-grade mathematics classrooms in China: From discourse, pedagogical, and cultural perspectives. Ed.D. dissertation, Harvard University, United States -- Massachusetts: Dissertations & Theses: The Humanities and Social Sciences Collection.(Publication No. AAT 3176361).

Wang, T., & Murphy, J. (2004). An examination of coherence in a Chinese mathematics classroom. In L. Fan (Ed.), *How Chinese learn mathematics: perspectives from insiders* (pp. 107-123). London: World Scientific Publishing Co.

Watson, A., & Mason, J. H. (2005). *Mathematics as a Constructive Activity: Learners Generating Examples*. Mahwah, New Jersey: Erlbaum.

Wenger, E. (1998). *Communities of Practice: Learning, Meaning, and Identity*. Cambridge: Cambridge University Press.

Wilson, L. (1997). New Views of learning: Types of questions. Retrieved June 1, 2005, from http://www.uwsp.edu/education/lwilson/learning/quest2.htm

Wittgenstein, L. (1972). *Philosophical Investigations*. Oxford: Blackwell.

Wittwer, J., & Renkl, A. (2008). Why Instructional Explanations Often Do Not Work: A Framework for Understanding the Effectiveness of Instructional Explanations. *Educational Psychologist, 43*(1), 49-64.

Wood, T. (1996). Events in Learning Mathematics: Insights from Research in Classrooms. *Educational Studies in Mathematics, 30*(1), 85-105.

Yackel, E. (2000). *Creating a mathematics classroom environment that fosters the development of mathematical argumentation.* Paper presented at the Ninth International Congress of Mathematical Education, Tokyo/Makuhari, Japan.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for research in Mathematics Education, 27*(4), 458-477.

Yackel, E., Cobb, P., & Wood, T. (1991). Small-Group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education, 22*(5), 390-408.

Yin, R. K. (2003). *Case Study Research: Design and Methods*. Califonia: Sage Publications.

Young, D. (1995). Preparing Teacher trainees to teach in multilingual classes. In K. Heugh, A. Siegruhn & P. Pluddemann (Eds.), *Multilingual Education for South Africa*. Johannesburg: Heinemann.

Zazkis, R., & Leikin, R. (2008). Exemplifying definitions: a case of a square. *Educational Studies in Mathematics, 69*(2), 131-148.

Zodik, I., & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. [Article]. *Educational Studies in Mathematics, 69*(2), 165-182.

APPENDIX A

INTERVIEW QUESTIONS

(SEMI-STRUCTURED) INTERVIEW QUESTIONS (For Phase 1)

1. STUDENTS' USE OF LANGUAGE IN THEIR LEARNING

A. I want to confirm with you. You said you teach XXX course, is that right?

B. Which group of students (year) do you lecture?

C. What is the language background of your students? Are they mostly first-language English speakers or non-first language English speakers? (In the case where the T.E. teaches a wide spectrum of students, say from year one to year three or four, the question: *How does the English language fluency of students improve over the years*, would be asked at some point in the interview).

D. Tell me about the proficiency of your students in the Language of Learning and teaching (English)

E. There is a growing consensus that we talk to learn, that language enables epistemological access to mathematics, and that learning means being able to talk about what you have learnt. Do you agree?

F. How do you think your students learn to talk and talk to learn in your class?

G. Are there linguistic challenges your students experience while teaching them in the LoLT? If yes, how do you deal with those challenges and why do you do what you do in the way you do it?

2. MODELLING (T.E own use of language when they teach)

A. (Given the background of your students) what kind of schools do you think they will teach in at the end of their pre-service teacher training?

B. How do you prepare your students to teach in such context?

3. **BEST PRACTICES IN OWN CONTEXT (what they construct as best practices)**

A. Given your own practice (context) what would you say constitutes best practices in teaching in a situation/context such as yours? (if there were no constraints (linguistic or otherwise), what would be the best practices in your situation?)/You are multilingual and you are teaching multilingual students, what would be best practices in a context such as yours?

B. Why would …be a best practice for you?

- You are a professional and have experience in teacher education. What can we learn from you about…

- Are there things that you worry about/ What's good about what you do/Why do you think what you do meet the needs in question 1 above?

4. **<u>OWN TEACHING (MATHS/METHODS COURSE)</u>**

A. Are the things we have talked about (example best practices) specific to your context…

   a. Is your best practice specific to your context?

   b. Would it be different if for example you were teaching the Mathematics course/methodology course? (only if lecturer is teaching both)

   c. Would it be different if the students' language background were different?

   d. Would it be different if you were multilingual/monolingual?

HELPFUL NOTES:

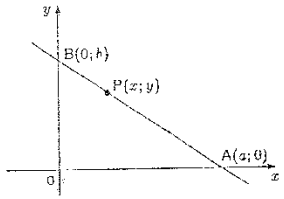
- Remember to ask for difficulties experienced and for scenarios/examples
- …How should we, how do we prepare pre-service teachers for teaching in multilingual classrooms?
- What do you think we can do about teaching in multilingual classrooms?
- What do you think needs to be done with someone in the same situation?
- What do you think: should there be a need to deal with training pre-service teaches for teaching in multilingual contexts? Should that be a concern?

APPENDIX B

CLASS TASKS

4.5 INTERCEPTS MADE ON THE AXES BY A STRAIGHT LINE

Dual-intercept form



For any point $P(x;y)$ on AB:

$$m_{AP} = m_{AB}$$

$$\therefore \frac{y-0}{x-a} = \frac{b-0}{0-a}$$

$$\therefore bx - ab = -ay$$

$$\therefore bx + ay = ab$$

$$\therefore \frac{x}{a} + \frac{y}{b} = 1$$

The straight line which cuts the x-axis at $(a;0)$ and the y-axis at $(0;b)$ has the equation:

$$\frac{x}{a} + \frac{y}{b} = 1$$

EXERCISE 4.7

1. Determine the equations of the lines with the following x- and y-intercept:
   a) 2 and 3 b) -1 and 1
   c) 4 and -3 d) -2 and -4
2. By writing the equation in dual-intercept form (or otherwise), determine the intercepts of the line on both axes:
   a) $2x + 3y = 6$ b) $x - 2y = 2$ c) $3x + y = 6$
   d) $-3x + 2y = -12$ e) $5y - 4x + 10 = 0$ f) $4x + 2y = -9$
3. Determine the equation of the perpendicular bisector of the line segment joining $P(-1;-2)$ and $Q(5;8)$. Then, by writing the equation in dual-intercept form, determine where the perpendicular cuts the x- and y-axis.
4. A straight line has an x-intercept of 3 and a y-intercept of 4. Determine the equation of the line which is perpendicular to this line and of which the x-intercept is 4.
5. Determine the equation of the perpendicular bisector of PR where P is the point $(-2;2)$ and R is the point $(6;6)$. If this perpendicular bisector cuts the x-axis at Q and the y-axis at S, prove that PQRS is a square.
6. Determine the equation of the line through $(2;4)$ and parallel to $x + 2y = 0$. If this line cuts the x- and y-axis at A and B respectively, determine the equation of the perpendicular bisector of AB.

Summary of formulae and facts established in this chapter

1. The distance between $A(x_A;y_A)$ and $B(x_B;y_B)$ is:
   $$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
2. The coordinates of the mid-point of the line segment joining $A(x_A;y_A)$ to $B(x_B;y_B)$ are:
   $$\left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right)$$
3. The gradient of the line PQ joining $P(x_P;y_P)$ to $Q(x_Q;y_Q)$ is given by:
   $$m = \frac{y_P - y_Q}{x_P - x_Q} \text{ if } x_P \neq x_Q$$
4. The inclination, θ, of a line through $A(x_A;y_A)$ and $B(x_B;y_B)$ is given by:
   $$\tan\theta = \frac{y_B - y_A}{x_B - x_A} = m \text{ if } x_B \neq x_A$$
5. Non-vertical parallel lines have equal gradients.
6. a) If l_1 and l_2 are non-vertical perpendicular lines with gradients m_1 and m_2 respectively, then $m_1.m_2 = -1$.
   b) Non-vertical lines with gradients m_1 and m_2 are perpendicular if $m_1.m_2 = -1$.
7. The equation of the line with gradient m through $A(x_A;y_A)$ is given by:
   $$y - y_A = m(x - x_A)$$
8. The equation of the line through $A(x_A;y_A)$ and $B(x_B;y_B)$ is given by:
   $$y - y_A = \left(\frac{y_B - y_A}{x_B - x_A}\right)(x - x_A) \text{ if } x_B \neq x_A$$
9. a) The horizontal line through $A(x_A;y_A)$ has the equation $y = y_A$.
   b) The vertical line through $A(x_A;y_A)$ has the equation $x = x_A$.
10. The straight line which cuts the x-axis at $(a;0)$ and the y-axis at $(0;b)$ has the equation $\frac{x}{a} + \frac{y}{b} = 1$.

In general, it is best to draw a sketch to confirm the algebraic solution to the question.

EXERCISE 4.8 (Mixed examples)

1. Show that:
   a) $P(-1;1)$, $Q(0;4)$, $R(3;5)$ and $S(2;2)$ are the vertices of a rhombus.
   b) $(6;-4)$, $(5;3)$, $(-2;2)$ and $(-1;-5)$ are the vertices of a square.
   c) $A(5;1)$, $B(-2;-6)$, $C(-1;1)$ and $D(6;0)$ lie on a circle with centre $(2;-3)$ [i.e. show that these points are equidistant from $(2;-3)$].
   d) $M(2;-1)$ is the circumcentre of ΔPQR with vertices $P(6;2)$, $Q(-1;3)$ and $R(-1;-5)$.
   e) $(-6;4)$ lies on the perpendicular bisector of the line segment AB where A is the point $(3;6)$ and B is the point $(1;-2)$. (You should be able to see from a sketch that all points which are equidistant from two given points lie on the perpendicular bisector of the line segment joining these points.)

TEIA-S's Classroom Task

2. Show that the lines joining the mid points of the sides of quadrilateral PQRS bisect each other if P(−1; 1), Q(−3; −2), R(7; −3) and S(3, 5) are the vertices of the quadrilateral.

3. Show that the lines joining the mid-points of the opposite sides of any quadrilateral bisect each other.

4. a) Find the fourth vertex of parallelogram ABCD in which the vertices are A(−2; 0), B(2; 1) and C(1; −3) respectively.
   b) Then show that the diagonals of this parallelogram bisect each other at right angles
   c) What kind of parallelogram is ABCD?

5. P(6; 2), Q(3; 5), R(−3; −1) and S(0; −4) are the vertices of a quadrilateral.
   a) Show that this quadrilateral is a rectangle.
   b) Show that the diagonals are of equal length.
   c) Find the point of intersection of the diagonals.

6. A(−3; −1), B(−2; 2), C(1; 3) and D(0; 0) are the vertices of a quadrilateral.
   a) Show that this quadrilateral is a parallelogram.
   b) Show that the diagonals bisect each other at right angles.
   c) What kind of quadrilateral is this?

7. a) Prove that △ABC with vertices A(−5; 3), B(4; 0) and C(6; 6) is right-angled at B.
   b) Determine the coordinates of D of rectangle ABCD if D is in the second quadrant.

8. A(−2; 1), B(2; 5) and C(4; −3) are the vertices of △ABC. X and Y are the mid-points of AB and AC. Show that:
   a) XY ∥ BC b) $XY = \frac{1}{2}BC$

* 9. M(2; 3), N(−4; −1) and K(8; 1) are the vertices of a triangle. Determine the equations of the medians of the triangle and show that they are concurrent.

*10. The vertices of △ABC are A(−7; 3), B(−4; −1) and C(4; 5).
   a) Show in two different ways that $A\hat{B}C = 90°$.
   b) Calculate the coordinates of the circumcentre of △ABC.
   c) Determine the equation of the perpendicular bisector of AB and show that this line passes through the circumcentre found in b).

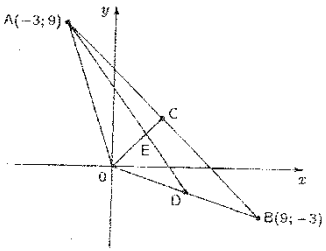
11. P(−3; 10) and Q(3; 4) are vertices of △PQR. If $\hat{Q} = 90°$, determine:
   a) the equation of QR.
   b) the coordinates of the orthocentre of △PQR. (The orthocentre is the point where the altitudes of a triangle meet.)
   c) the possible coordinates of R if $QR = 4\sqrt{2}$.

*12. In △ABC, D(2; −3), E(1; 1) and F(5; −1) are the mid-points of AB, BC and AC respectively. Calculate the coordinates of C.
   [**Hint**: BC ∥ DF and DE ∥ AC. Determine the equations of the lines BC and AC and then find the point of intersection C.]

13. a) Calculate the inclination of:
      (i) the line joining A(4; 3) and B(−2; −6).
      (ii) the line $y + 2x = 3$.
   b) Hence determine the acute angle between these two lines.

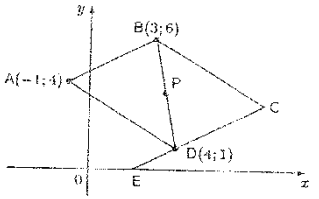
*14. Determine the acute angle between the lines $5y = x - 5$ and $y = 5x - 1$. Show that the gradient of the bisector of this angle is 1 and hence determine the equation of this bisector.

15. A(−3; 9) and B(9; −3) are two given points.
   a) Determine:
      (i) the coordinates of C and D if C is the mid-point of AB and D is the mid-point of OB.
      (ii) the equation of the median OC.
      (iii) the equation of the median AD.
      (iv) the coordinates of E, the point of intersection of OC and AD.



   b) Prove that △OAB is an isosceles triangle.
   c) Show that OC ⊥ AB.

16. ABCD is a parallelogram with A(−1; 4), B(3; 6) and D(4; 1). Determine:
   a) the gradient of AB.
   b) the mid-point P of BD.
   c) the coordinates of C.
   d) the equation of CD.
   e) the coordinates of E if E is the x-intercept of the line CD produced.
   f) the inclination of the line AE.
   g) the size of $A\hat{E}D$.
   h) the length of BC.



17. A(−7; 3) and B(5; 7) are two given points.
   Determine:
   a) the coordinates of M, the mid-point of AB.
   b) the distance from C(−2; −5) to M.
   c) the equation of AB.
   d) the equation of the line perpendicular to AB which passes through M.

18. △PQR is right-angled at P. The points P(2; 6), Q(−1; 2) and R(6; k) are given.
   a) Determine k.
   b) Determine the coordinates of S, such that QPRS is a rectangle.

* Example 4

Determine the equations of the tangents to the circle $x^2 + y^2 = 13$ from the point $P(-1;8)$ outside the circle.

Solution:

From the sketch, A is the point: $(x; \sqrt{13 - x^2})$

Gradient of the normal OA is: $\dfrac{\sqrt{13-x^2}}{x}$

$\therefore$ Gradient of the tangent AP is: $\dfrac{-x}{\sqrt{13-x^2}}$

Also the gradient of the tangent AP is: $\dfrac{\sqrt{13-x^2}-8}{x+1}$

$$\therefore \frac{-x}{\sqrt{13-x^2}} = \frac{\sqrt{13-x^2}-8}{x+1}$$

$$\therefore -x^2 - x = 13 - x^2 - 8\sqrt{13-x^2}$$

$$\therefore x + 13 = 8\sqrt{13-x^2}$$

$$\therefore x^2 + 26x + 169 = 64(13 - x^2)$$

$$\therefore 65x^2 + 26x - 663 = 0$$

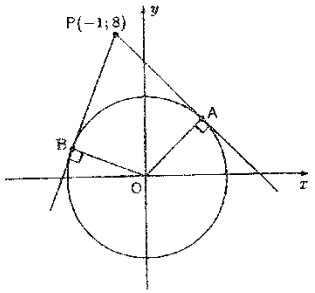
$$\therefore 5x^2 + 2x - 51 = 0$$

$$\therefore (5x + 17)(x - 3) = 0$$

$$\therefore x = -\frac{17}{5} \text{ or } x = 3$$

$\therefore$ At A: $x = 3$ and $y = 2$

At B: $x = -\dfrac{17}{5}$ and $y = \dfrac{6}{5}$ (See sketch.)



Gradient of the tangent PA is:

$$\frac{8-2}{-1-3} = \frac{6}{-4} = -\frac{3}{2}$$

Equation of the tangent PA is:

$$\frac{y-8}{x-(-1)} = -\frac{3}{2}$$

$$\therefore 2y - 16 = -3x - 3$$

$$\therefore 2y + 3x = 13$$

i.e. $y = -\dfrac{3}{2}x + \dfrac{13}{2}$

Gradient of the tangent PB is:

$$\frac{8-\frac{6}{5}}{-1-(-\frac{17}{5})} = \frac{17}{6}$$

Equation of the tangent PB is:

$$\frac{y-8}{x-(-1)} = \frac{17}{6}$$

$$\therefore 6y - 48 = 17x + 17$$

$$\therefore 6y - 17x = 65$$

i.e. $y = \dfrac{17}{6}x + \dfrac{65}{6}$

EXERCISE 5.6

*1. The equations of a straight line and a circle are given. Find the point(s) of intersection of the line and the circle in each case. State whether the line is a tangent or a secant to the circle:

a) $y + 2x = 10$ and $x^2 + y^2 = 25$

b) $3y + x = 5$ and $x^2 + y^2 = 5$

c) $x + y = 4$ and $x^2 + y^2 = 8$

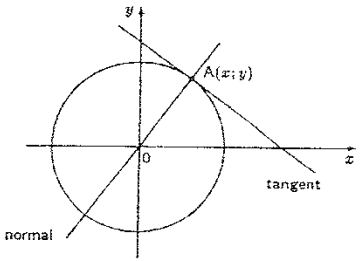
d) $y + 2x = 15$ and $x^2 + y^2 = 50$

e) $y - 2x - 5 = 0$ and $x^2 + y^2 = 5$

*2. Find the equation of the circle, with centre at the origin, passing through A. Find the equation of the tangent and the normal at A to that circle. In each case:

a) A$(-1;2)$ b) A$(1;1)$

c) A$(-3;2)$ d) A$(2;-1)$



3. a) $y = 7x$ cuts the circle $x^2 + y^2 = 50$ at A and B. Find the coordinates of A and B.
   b) Show that C$(5;5)$ lies on this circle.
   c) Show that $A\hat{C}B = 90°$. What can you deduce about AB?
   d) Find the equation of the tangent to the circle at C.

4. Find the equation of the tangent which touches the circle:
   a) $x^2 + y^2 = 25$ at the point $(3;4)$.
   b) $x^2 + y^2 = 5$ at the point $(1;-2)$.
   c) $4x^2 + 4y^2 = 65$ at the point $(-\frac{1}{2};4)$.
   d) with centre at the origin, at A$(2;-2)$, a point on the circle.
   e) with centre at the origin, at B$(-\sqrt{2};1)$, a point on the circle.
   *f) $(x-3)^2 + (y+1)^2 = 20$ at $(5;3)$.
   *g) $(x+3)^2 + (y-4)^2 = 25$ at the origin.
   *h) with centre M$(-1;1)$, at B$(3;2)$, a point on the circle.

*5. Find the equations of the tangents:
   a) to $x^2 + y^2 = 5$ which have a gradient of -2.
   b) to $x^2 + y^2 = 18$ which have an inclination of $135°$.
   c) to $(x+1)^2 + y^2 = 20$ which are parallel to $2y - x = 0$.
   d) to $(x-2)^2 + (y+3)^2 = 16$ which are parallel to the y-axis.

6. a) Find the equation of the circle, with centre at the origin, through A$(-2;4)$.
   b) Find the equation of the tangent to the circle at A.
   c) If this tangent cuts the x-axis at B, find the length of AB.
   *d) Find the equation of the other tangent to the circle from B.

7. M$(2;2)$ is the mid-point of the chord AB in the circle $x^2 + y^2 = 10$.
   a) Find:
      (i) the equation of the secant AB.
      (ii) the coordinates of A and B.
      (iii) the length of the chord AB.
      (iv) the equations of the tangents at A and B.
      (v) the coordinates of C, the point of intersection of these two tangents.
   b) Show that AC = BC.

*8. Find the length of the tangent from:
   a) $(5;4)$ to the circle $x^2 + y^2 = 16$.
   b) $(-3;5)$ to the circle $x^2 + y^2 = 18$.
   c) $(-3;1)$ to the circle $(x-3)^2 + (y+1)^2 = 4$.

9. The tangents to a circle with centre at the origin, at P(5;5) and Q(7;1), intersect at R. Find R and show that RO is perpendicular to PQ, where O is the origin of the circle.

Summary of formulae and facts established in this chapter

1. If P divides the straight line AB in the ratio $m : n$ then: $\frac{AP}{PB} = \frac{m}{n}$
2. **For enrichment:**
   The coordinates of the centroid $D(x_D ; y_D)$ of $\triangle ABC$ with vertices $A(x_A; y_A)$; $B(x_B; y_B)$ and $C(x_C; y_C)$ are:
   $x_D = \frac{x_A + x_B + x_C}{3}$ and $y_D = \frac{y_A + y_B + y_C}{3}$
3. The locus of a point $P(x; y)$ is the set of all possible positions (or locations) of P which satisfy the given conditions.
   The equation that expresses the relationship between x and y is also called the locus of $P(x; y)$. It is usually possible to give a geometric description of this locus.
4. The equation of the circle with centre at the origin and with radius r is:
   $x^2 + y^2 = r^2$
5. The equation of the circle with centre $(a; b)$ and radius r is:
   $(x-a)^2 + (y-b)^2 = r^2$
6. A tangent to a circle touches the circle at one point only. The normal to the circle at that point is perpendicular to the tangent:
   Gradient of the normal $\times$ gradient of the tangent $= -1$
7. In general, it is best to draw a sketch to confirm the algebraic solution to the question.

EXERCISE 5.7 (Mixed problems)

This exercise covers the work from both Chapter 4 and Chapter 5.

1. Points P(−3;−3); Q(1;−5) and R(2;2) are given.
   a) Determine the equation of the perpendicular bisector of PQ.
   b) Test whether R lies on this perpendicular bisector.
   c) Determine the coordinates of S such that PSQR is a rhombus.
   d) Calculate $P\hat{R}Q$ (answer correct to one decimal place).

* 2. The circle with equation $x^2 + y^2 = 50$ is given.
   a) Determine the coordinates of the x-intercepts of the circle. (Leave your answers in surd form).
   b) Calculate the coordinates of the points at which the line $x - y - 6 = 0$ cuts the circle.
   c) Give the equation of the locus of points equidistant from the x-intercepts of the circle.

3. $\triangle ABC$ has vertices A(4;−3); B(8;5) and C(−2;3) respectively.
   a) If BE is the median on AC, prove that the angle between EC and the x-axis is 135°.
   b) Determine the equation of the median BE.
   *c) Determine the coordinates of the centroid D of $\triangle ABC$.
   d) Verify that D lies on the median BE.

4. The distance between A(−3; −4) and B(b; 0) is 5. Calculate all possible values of b.

* 5. Determine the equation of the locus of a point which is always 7 units from the origin.

6. Points P(−1;−3); Q(1;−2) and R(3;−1) are given. Prove that P; Q and R are collinear.

7. A(−2;1) and B(3;4) are two points. Calculate the coordinates of $P(x_P; y_P)$ which divides AB in the ratio 1 : 1.

8. A(−2;−5) and B(6;10) are two points in the Cartesian plane.
   a) Calculate the length of AB.
   b) Calculate the inclination of AB.
   c) Determine the coordinates of M, the mid-point of AB.

* 9. a) Points A(−4;−1) and B(2;−1) are two vertices of equilateral $\triangle ABC$. Find the possible positions of C in coordinate form. (You may leave your answer in surd form if necessary.)
   b) Suppose A(−4;−1) and B(2;−1) are the vertices of isosceles $\triangle ABC$. Find the equation(s) of the locus of all the possible positions of C.

*10. Two circles $x^2 + y^2 - 16x - 8y + 35 = 0$ and $x^2 + y^2 = 5$ are given.
   a) Determine the centre of each circle.
   b) Prove that the circles touch each other.

*11. Determine the equation of the set of points which are twice as far from the straight line $x = -1$ as from the point $(4; -1)$.

*12. Points A(−1;1); B(3;5) and C(1;0) are the vertices of a triangle in the Cartesian plane. Determine:
   a) the angle between AC and the positive x-axis.
   b) the equation of CE, a median of $\triangle ABC$.
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   d) the equation of the locus of P, if the distance from P to C is the same as the distance between P and the straight line with equation $x = -1$.
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   b) Calculate the inclination of AB.
   c) Determine the coordinates of M, the mid-point of AB.

* 9. a) Points A(−4;−1) and B(2;−1) are two vertices of equilateral $\triangle ABC$. Find the possible positions of C in coordinate form. (You may leave your answer in surd form if necessary.)
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   a) the angle between AC and the positive x-axis.
   b) the equation of CE, a median of $\triangle ABC$.
   c) the x-intercept of BA produced.
   d) the equation of the locus of P, if the distance from P to C is the same as the distance between P and the straight line with equation $x = -1$.
   e) the coordinates of F, if ACBF is a parallelogram.

TEIB-E's Classroom Task

Situation 6: Fuel consumption of trucks

Consider a transport company using just one type of truck. Before each transport job, the company has to specify the price for the job. In order to specify a price before a job, the company needs to estimate how much their costs will be for doing the job.

Task 1: Identifying variables and postulating the relationships between them

Discuss: Leaving the overhead costs aside (i.e. salaries, etc.), what are the main costs in this kind of business? How are they related? How can we measure them?

One of the main costs is the cost of fuel, and the main factor influencing the amount of fuel used is the distance. But the load weight also plays a role: the greater the load weight, the higher the fuel consumption. The table below gives information that was recorded for previous transport jobs.

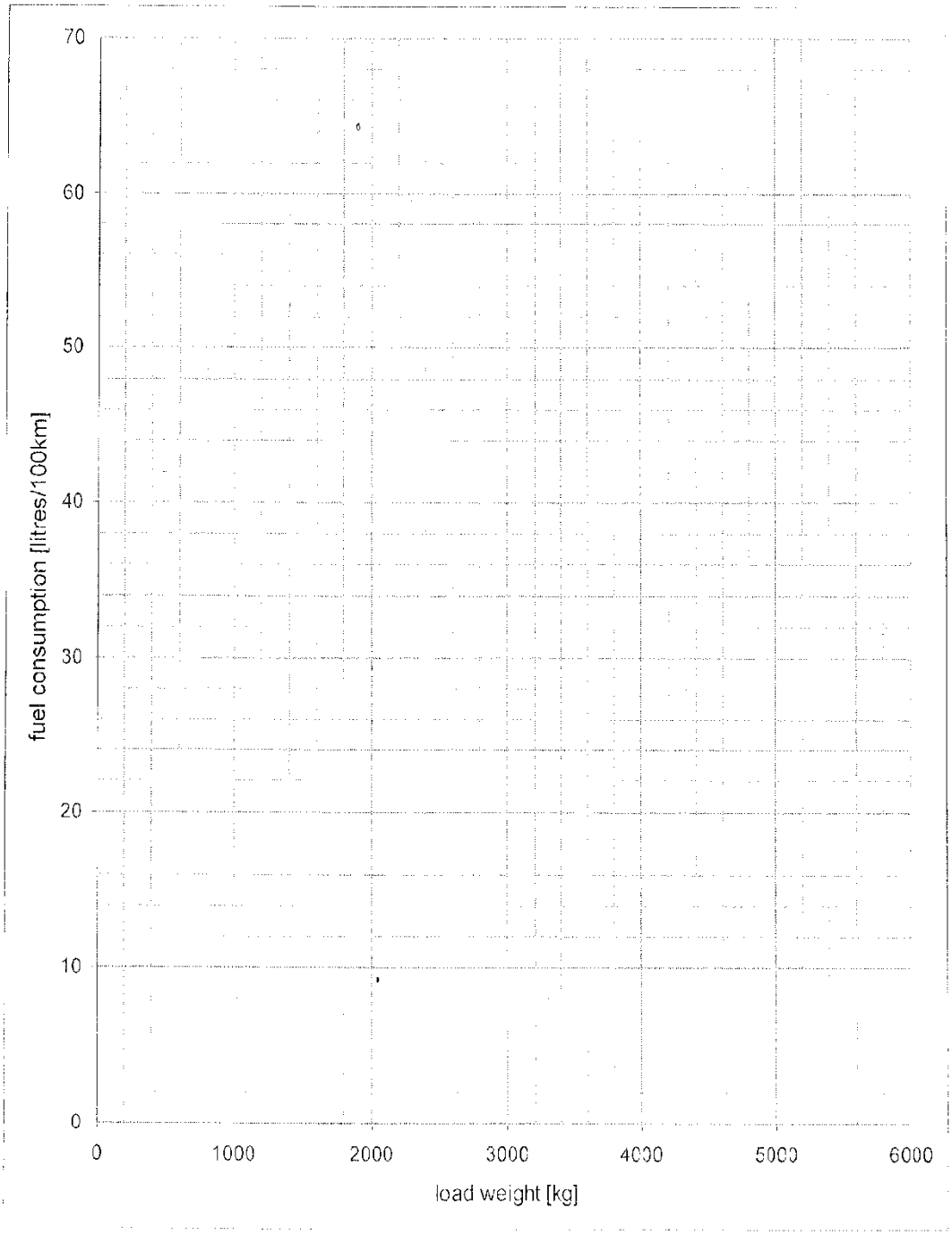
Job number	Distance [km]	Load weight [kg]	Fuel used [litres]
1	1304	5445	879
2	1320	2954	639
3	1151	4705	698
4	1371	4378	787
5	325	3673	176
6	1630	5995	1113
7	1023	5357	600
8	620	4988	382
9	73	1992	35
10	1071	5529	680
11	370	4140	218
12	1423	4062	843
13	394	4068	221
14	1536	1678	682
15	1633	3736	887
16	435	3644	241

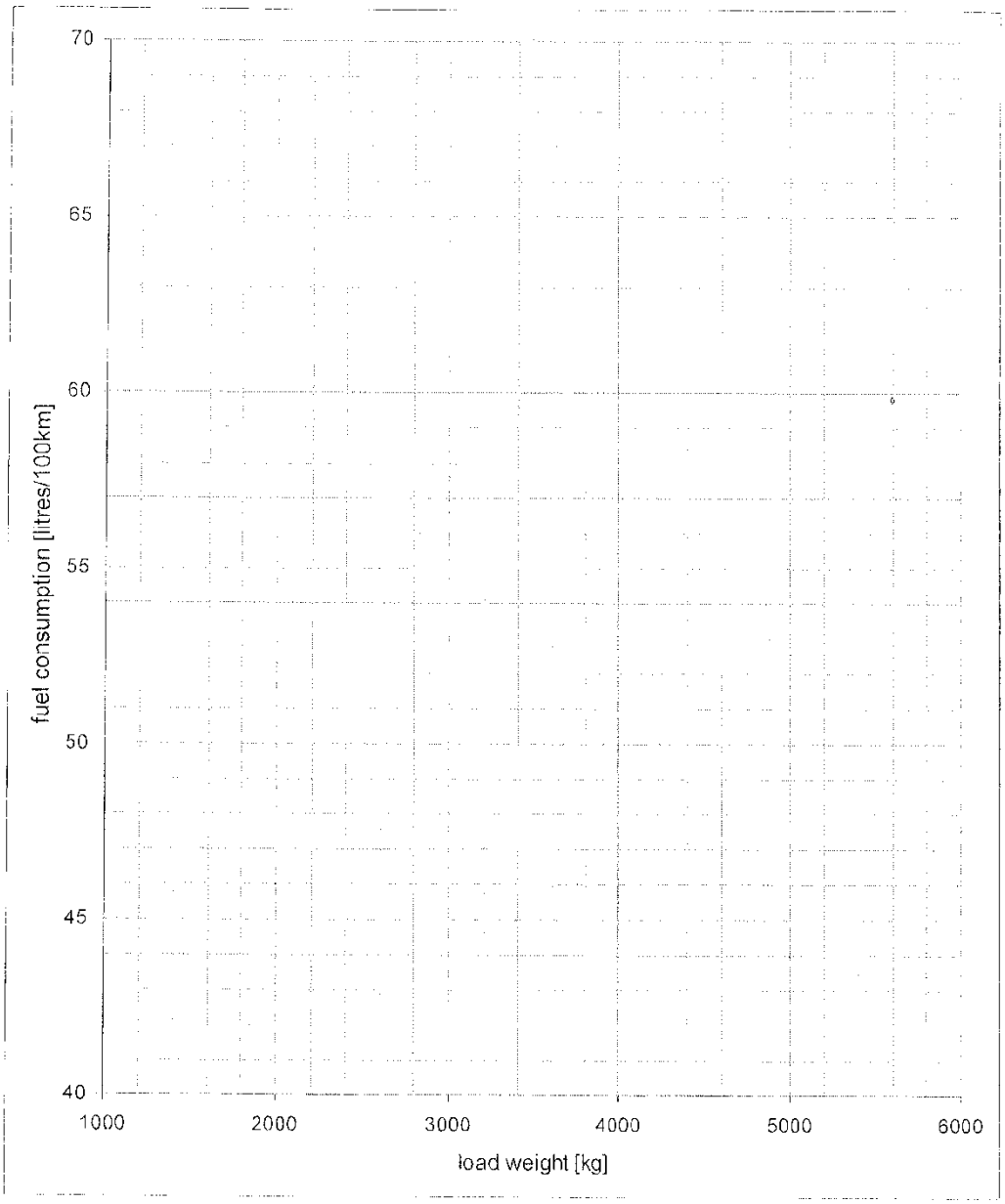
Previous transport jobs: distance, load weight and amount of fuel used

Task 2: Estimating the relationship between fuel consumption and load weight

a) Find a unit of measurement that allows you to work with the relationship between fuel consumption and load weight.

b) Make a graph that can be used to estimate the fuel consumption for any specified load weight. One should be able to look at a certain load weight on the x-axis and then read down the fuel consumption of the y-axis.





c) You have worked with lines to predict values on a scatterplot before.
(i) Explain why a line is a good mathematical model for prediction.
Explain why a line may be a bad model.
(ii) What are the ideal properties of such a line? (i.e. how will you know you are using the best of all possible lines?)

d) Now make a formula to estimate the fuel consumption (litres/100km) depending on the load weight.

e) How much do you trust your formula? Why?

APPENDIX C

CONSENT FORMS AND LETTERS

Wits School of Education

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STUDENT NUMBER: 0302922V
Protocol 2008ECE22

13 August 2008

Mr. Anthony Essien
WSoE
Marang Centre

Dear Mr. A Essien

Application for Ethics Clearance

I have pleasure of advising you that the Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has agreed to approve your application for ethics clearance submitted for your proposal entitled:

Pedagogical practices of teacher educators preparing pre-service teachers for teaching mathematics in multi-lingual classrooms

Recommendation:

Ethics clearance is granted

Yours sincerely

Matsie Mabeta
Wits School of Education

Cc Supervisor: Prof. J Adler (via email)

PERMISSION LETTER FROM HEAD OF SCHOOL

My name is ESSIEN Anthony. I am currently doing my PhD in Mathematics Education at the MARANG Centre of the University of the Witwatersrand. In my study, I am investigating how the complexity of teaching mathematics in multilingual classrooms is attended to in teacher education. More specifically, my research is concerned with what teacher educators do to support student teachers learning to teach mathematics in multilingual classrooms, and how and why they do what they do. I am also interested, given the multilingual character of teacher education classes, in understanding how teacher educators model what it means to teach (mathematics) in a multilingual classroom).

I write this letter to formally request permission to conduct some of my research in your university. The study intends to involve 5 institutions in the Pretoria and Johannesburg region, through a similar process.

The data collection will be done in two phases. In the first phase of the data collection, I will ask teacher educators teaching mathematics to pre-service teachers to participate in an individual interview. The focus of this interview is for me to find out what teacher educators understand to be doing to prepare students to teach mathematics in multilingual classrooms. It will also enable me to find out what courses, programmes, modules, etc are in place (in the institution) for preparing pre-service teachers for multilingual classrooms in your institution. This interview will be between 30-45 minutes long, at the teacher educator's convenience, and preferably in the teacher educator's office.

In the second phase, I will select a few cases in whose classrooms I will explore richer descriptions of what teacher educators do in their classrooms to prepare student teachers for teaching in multilingual mathematics classrooms, I would like to observe selected teacher educators' classes for an agreed-upon time frame. Selection for this phase would depend on which cases I would like to pursue further in my research. If your university is selected for the second phase of data collection, lessons will be video-recorded so that I can ensure that I make an accurate record of what teacher educators say and do and so that I could transcribe the whole lessons and use it to reflect on the different pedagogical strategies used by the teacher educators. I have opted for video-recording rather than audio-recording since the classroom observation would involve writing/data projected information, gestures, etc which cannot be captured by the tape-recorder. Post observation interviews will be conducted with the teacher educators whose classes have been observed. This interview would focus on the observed lessons and on how the teacher educators believe the course/pedagogic practices, etc would help the pre-service teachers deal with the challenges of teaching in multilingual classrooms. I would also like to collect study/course materials where these would help me gain a deeper insight into how multilingualism is attended to in the teacher educator mathematics class.

I hereby guarantee your anonymity and the confidentiality of the University and of responses by the teacher educators to the fullest possible extent. The name of your University and details of teacher educators will be kept in a separate file from any data obtained from the University and will not be used in any publication emerging from this research.

Once the research has been completed, a brief summary of the findings, and subsequently, the whole thesis, will be made available to the University. It is also possible that findings will be presented at academic conferences and published in national and international academic journals.

If you agree that your University participates in this study, please read though and sign the attached institutional consent form for data collection (feel free to modify it as you please).

Should you require any further information do not hesitate to contact me – my telephone numbers are below.

Anthony Essien
(011)7173408 or 0722712570
Email: anthony.essien@wits.ac.za

Supervisors: Prof. Mamokgethi Setati (012)4292851 setatrm@unisa.ac.za Prof. Prof. Jill Adler (011)7173413 jill.adler@wits.ac.za

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Institutional consent for collecting PhD data in the Bachelor of Education programme

As Head of school of Education at the above University, I am happy to give you permission to collect your data for your doctoral study within the School. Based on your letter of request, I understand that the focus is on multilingual mathematics pedagogical practices and what problems and issues teacher educators face as they prepare multilingual students for teaching in multilingual mathematics classrooms, and the following details pertain to data collection for which I give my permission:

- You will conduct a semi-structured interview with the mathematics teacher educators in phase 1 of the study mainly for the purpose of sample selection .
- The teacher educators selected will then be video-recorded as they teach for an agreed-upon time frame negotiated between you and the teacher educator involved
- You may need to collect some study/course materials where these would help you gain a deeper insight into how multilingualism is attended to in the teacher educator mathematics class.
- You will conduct a post-observation interview with the teacher educator whose classes you have observed.

You have also indicated that you may use the data in the future for publication and other research dissemination purposes (e.g. conference presentation).

Yours truly

Signature: _____ ____ Date: ___/_____/____

LETTER OF CONSENT: TEACHER EDUCATOR PARTICIPATION

My name is ESSIEN Anthony. I am currently doing my PhD in Mathematics Education at the MARANG Centre of the University of the Witwatersrand. In my study, I am investigating how the complexity of teaching mathematics in multilingual classrooms is attended to in teacher education. More specifically, my research is concerned with what teacher educators do to support student teachers learning to teach mathematics in multilingual classrooms, and how and why they do what they do. I am also interested, given the multilingual character of teacher education classes, in understanding how teacher educators model what it means to teach (mathematics) in a multilingual classroom).

Your Head of School has given me permission to send you this letter to invite you to participate in this research project. Once you have read the letter you can decide whether you want to take part or not. Should you agree to participate, you would be asked to contribute in two ways. First, in the first phase of the data collection, I will ask you to participate in an individual interview. The focus of this interview is for me to find out what teacher educators understand to be doing to prepare students to teach mathematics in multilingual classrooms. It will also enable me to find out what courses, programmes, modules, etc are in place (in the institution) for preparing pre-service teachers for multilingual classrooms in your institution. This interview will be between 30-45 minutes long, at your convenience, and preferably in your office.

In the second phase, to explore richer descriptions of what teacher educators do in their classrooms to prepare student teachers for teaching in multilingual mathematics classrooms, I would like to observe your classes for an agreed-upon time frame. With your permission, the interview would be tape-recorded and if your university is selected for the second phase of data collection, your lessons will be video-recorded so that I can ensure that I make an accurate record of what you say and do. When the tape has been transcribed, you would be provided with a copy of the transcript, so that you can verify that the information is correct. As part of this second phase, I will like to conduct a post-observation interview with you after I have observed your class. This interview would focus on the observed lessons and on how you as the teacher educators believe the course/pedagogic practices, etc would help the pre-service teachers deal with the challenges of teaching in multilingual classrooms. I would also like to collect study/course materials where these would help me gain a deeper insight into how multilingualism is attended to in your mathematics classroom.

I will ensure your anonymity and the confidentiality of your responses to the fullest possible extent. Your name and contact details will be kept in a separate file from any data that you supply. This will only be able to be linked to your data by me. In any publication emerging from this research, you will be referred to by a pseudonym. I will remove any references to personal information that might allow someone to guess your identity. If, however, for any reason you would like your real name to be used in the publications you will need to make a written request to me.

Once the research has been completed, a brief summary of the findings will be made available to you. It is also possible that findings will be presented at academic conferences and published in national and international academic journals.

Please be advised that your participation in this research project is completely voluntary. Should you wish to withdraw at any stage, or withdraw any unprocessed data you have supplied, you are free to do so without prejudice. Your decision to participate or not, or to withdraw, will be completely independent of your dealings with the University of the Witwatersrand.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form.

Should you require any further information do not hesitate to contact me – my telephone numbers are below.

Anthony Essien
(011)7173408 (work) or 0722712570 (cell)
Email: anthony.essien@wits.ac.za

Supervisors: Prof Jill Adler and Prof Mamokgethi Setati

LETTER OF CONSENT: STUDENT PARTICIPATION

My name is ESSIEN Anthony. I am currently doing my PhD in Mathematics Education at the MARANG Centre of the University of the Witwatersrand. In my study, I am investigating how the complexity of teaching mathematics in multilingual classrooms is attended to in teacher education. More specifically, my research is concerned with what teacher educators do to support student teachers learning to teach mathematics in multilingual classrooms, and how and why they do what they do. I am also interested, given the multilingual character of teacher education classes, in understanding how teacher educators model what it means to teach (mathematics) in a multilingual classroom).

Your Head of School and lecturer have given me permission to send you this letter to invite you to participate in this research project. With your permission, I would like to video record lessons in which you as a student will be participating. These observations will not harm you as a student in any way, neither would they (observations) be disruptive as they will take place only during normal lecture periods and in agreement with your lecturer and the Head of School.

I hereby guarantee your anonymity, commit myself to acting responsibly and will do everything in my power to protect yours rights as participants. In reporting on this research therefore, neither video recordings nor people's names nor institutional names will be used.

Since this research is about how the complexity of teaching in multilingual classrooms is attended to pre-service training, it is anticipated that your participation in this research will help me understand more what pedagogical practices teacher educators use while preparing you to teach in multilingual classrooms which is a reality in most classroom settings in South Africa.

Please be advised that your participation in this research project is completely voluntary. Should you wish to withdraw at any stage, or withdraw any unprocessed data you have supplied, you are free to do so without prejudice. Your decision to participate or not, or to withdraw, will be completely independent of your dealings with the University of the Witwatersrand.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form.

Should you require any further information do not hesitate to contact me – my telephone numbers are below:

Anthony Essien

(011)7173408 (work) or 0722712570 (cell)

Email: anthony.essien@wits.ac.za

Supervisors: Prof Jill Adler and Prof Mamokgethi Setati

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I, _____, hereby acknowledge that:

- The aims and methods of the research have been explained to me.
- I voluntarily and freely give my consent to my participation in the research study.
- I understand that the results will be used for research purposes and may be reported in conferences and academic journals.
- I am free to withdraw my consent at any time during the study.

I hereby give consent for the following:

- To be video recorded during the lesson
  Yes ☐ No ☐

Signature: _____ Date: ___/_____/____

APPENDIX D

FULL DESCRIPTION OF CODES

Category: Mathematical practices (SRMP)	**MP in use (Sub-category)**	**Code**	**Code identification rule/comments**
Initiating and/or sustaining mathematical discussion practices	Explaining mathematically	MP-EM	When 'what' is used in a question by a community member. Or when the intonation used by the TE or any community member indicates a call for further explanation. Also, the use of the phrase/sentence: -'anything else', eg: anything else you want to add to that? -'no? why not?' -'this is what I mean…' -'what does it mean?' -'Do you understand what you have to do?' COMMENTS: MP-EM could also be a call for someone to shed more light on what has been said. Eg, 'what do you mean by …' MP-EM need not necessarily start in the form of a question. It could also be the explanation of a particular concept or an explanation of another PST's reasoning or solution to a mathematics problem.
	Making mathematical conjecture	MP-MC	There is a fine line between MP-MC and MP-PM. The difference is that MP-MC would depict an act of reasoning that goes beyond the stated information by applying some background knowledge while MP-PM depicts an act of reasoning about something in the future either based on past experience or based on some form of mathematical projection. The context of the utterance is important in deciding whether it is MP-PM or MP-MC or both. Eg: 'Isn't 'probable' like the chance or likelihood of something taking place?' would be MP-MC rather than MP-PM. And 'can you come up with a plan and use your plan predict for a weight of 2000kg' would be MP-PM.
	Predicting mathematically	MP-PM	
	Communicating mathematically	MP-CM	When particular attention is paid to the mathematics register

	Defining Mathematically	MP-DM	When there is a formal or informal definition of a mathematical concept
	Exemplifying (providing examples)	MP-PE	When the PST/TE provides an example to demonstrate a mathematics method (eg, example of an application of a mathematics procedure) and in concept development to indicate a mathematics relations (eg, examples of a concept like triangle, etc) (Bills, et al., 2006). It could also be when a community member demonstrates how something is done in mathematics, eg, how to draw a frequency table Close to MP-EM. An explanation can be made through the provision of an example. Use of words like: "like…", "example". It can also be a call by a community member for someone to give examples.
Classroom practices	Summarising on Board	MP-SB	When the TE or another community member summarises what has been discussed on the board (by writing) EG: "I am going to jot down what we have said so far"
	Summing up	MP-SU	Pulling together the arguments or contributions, but unlike MP-SB, this is done orally COMMENT: Both MP-SB and MP-SU are different from reiteration which is done in the course of a discussion/negotiation of meaning
	Reiterating (As indicated in Chapter Two, for the purpose of this study, I distinguish between reiterating and revoicing.)	MP-Rt	When the TE/PST repeats their understanding of what another member of the community has said either to check their own understanding of what has been said, or to make sure everyone is on the same page (ie, everyone understood the same thing). This is different from revoicing which involves repeating what has been said using the correct mathematical language. Eg: I want to repeat what he has said…

			"am I right" (when checking to make sure one has correctly captured what the other person has said. "is that what you are saying?" MP-Rt could also be a situation where TE/PST repeats what was said/discussed in previous class (in order to refresh the memory of the community) to aid their current activity or discussion
	Revoicing	MP-Rv	When a member of the community repeats what another member has said using the correct mathematical language or when the TE/PST repeats what he/she has said or what has been said in a different way to aid understanding. MP-Rv could also be instance where there are more than one responses from students and the TE only repeats the correct answer
	Moving between worlds	MP-MBW	When the TE/PST moves between the mathematics world and the real world. In Dowling's (1998) terms, when there is a movement from the esoteric domain to the public domain.
	Proceduralising	MP-Pc	When the TE or the PST deals with the procedure/steps for solving a particular problem. For instance, if the TE or PST talks about taking a variable to the other side of the equal sign and changing the sign, that would be categorised as MP-Pc. But if a member of the community states why this procedure works, then it was categorised MP-PJ. Could also be a call for a particular procedure or aspects of the procedure to be used in solving a mathematical task: example: "Where do we start?" (which calls for the first thing that needs to be done by way of procedures) "What do we do next?"
	Monitoring Solution	MP-MS	

			-'Let's check what you have done so far'
	Reading Mathematically	MP-RM	A community member asks someone to read a maths question, or a mathematical comment, etc either to themselves or for the benefit of everyone. Eg: "I am giving you 2 minutes to read the question on your own and after that, you tell me what it means".
	Experimenting	MP-Et	When a member of a class performs an experiment like the throwing of a dice. It could also be a call for an experiment by a community member.
	Writing Mathematically	MP-WM	Any opportunity given to PST to write mathematically. Eg. 'I want you to write…' It could also be showing students the correct way to write/represent, say a frequency table. It could also be writing down a mathematical definition
	Using Multiple Approaches	MP-MA	When a member of the community uses different approaches to solve the mathematical task at hand Also when different approaches to solving a mathematics task is encouraged.
	Gestural symmetry	MP-GS	A member of the community uses gestures synchronously with utterance to explain a concept. It could also be a call to do so by the TE or another member of the community
Evaluating mathematical validity practices	Critiquing (other's) conjecture	MP-CC	Member of the community challenges other members' conjecture. It could also be a call to critique/validate someone's opinion or to validate one's opinion. Phrases indicating MP-CC: -Do you agree -Is that an assumption you would agree with? -does that make sense? -Anyone wants to challenge that? -Anyone else on this? -Who wants to talk for or against that -is that logical/reasonable? -Is it possible that…? -So, do you mean that…

			-Am I right?
	Providing Justification	MP-PJ	Close to MP-EM and MP-PE. The "how" question indicates MP-EM while the "why" question would indicate MP-PJ. Instances where a PST/TE is asked to explain the procedures or steps leading to the solution of a maths problem would indicate MP-EM while a call to justify the procedure would be MP-PJ. For example: "who can tell me why the positive sign becomes negative when taken to the other side of the equation?" would be providing justification. The sentence: 'what is your evidence', could indicate either MP-EM or MP-PE or MP-PJ depending on the context of use.
	Critiquing solution	MP-CS	Involves critiquing the solution of a problem proffered by a community member. Different from MP-PJ and MP-CC. Here, a community member critiques his/her or other peoples' solution to a mathematical problem. In MP-CC, postulates are critiqued while MP-PJ involves justification for a conjecture or for the solution to any of the processes involved in the solution of a question. It can also be a call by any community member for other members to critically consider his/her solution to a mathematical problem or the processes involved in finding such solution. Eg: "what did you do wrong", "think carefully why you would make that decision"