A COMPARATIVE ANALYSIS OF MATHEMATICAL PROFICIENCY IN FRACTION WORD PROBLEMS IN GRADE 6 TEXTBOOKS

A Research Report

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By

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ABSTRACT

Fractions are an important aspect of learning mathematics. Despite learners' frequent textbook use as learning materials, learners still struggle with fraction word problems. This study compared three Grade 6 mathematics textbooks—Premier, Headstart and Platinum, and analysed their potential to promote mathematical proficiency. To achieve this objective, the presentation of worked-out examples and word problems exercise examples were analysed according to Kilpatrick et al. strands of mathematical proficiency and categorised accordingly. Findings highlighted a significant focus on the promotion of procedural fluency and productive disposition, creating gaps in the promotion of mathematical proficiency as the other strands were under-represented across all three textbooks. Additionally, the limited presentation of fraction worked-out examples suggests a limited exposure to support a range of word problems in the textbook. The study concludes that while the three selected Grade 6 mathematics textbooks are intended to promote the aims of the curriculum, their fraction content presentation on worked-out examples and fraction word problems hinders an all-encompassing development of proficiency because of the overemphasis on routine fraction problems. The study's findings address a notable practical gap in the three selected Grade 6 textbooks, specifically in their presentation of fraction word problems, The study also makes a methodological contribution by operationalising Kilpatrick et al. five strands of mathematical proficiency for use in analysis of examples (in textbooks).

DEDICATION

With gratitude and love, I dedicate this research to the loving memory of my late mother, and my great grandmother whose presence in my life is a treasured gift. Thank you for everything.

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PLAGIARISM DECLARATION

I, Duduzile Sibiya, with a student number 1106600 hereby declare that this Research Report is original, unaided work except where due acknowledgements have been made. It is my submission for the Degree of Master of Education at the University of Witwatersrand and it has not been submitted for any degree or qualification at any other University.

I acknowledge that I am aware that plagiarism is wrong. I have read and understood the University's policy on academic deception and plagiarism.

I understand that plagiarism is a serious academic offence and the University of Witwatersrand has the authority to take disciplinary actions against me if I am found submitting work that is not entirely mine.

<u>D. E. SIBIYA</u>

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Duduzile Esther Sibiya 17 March 2025

PUBLICATIONS ARISING FROM THIS STUDY

Sibiya, D. & Essien, A. (forthcoming). Affordances for developing mathematical proficiency in fraction word problems in Grade 6 textbooks in South Africa. *South African Journal of Childhood Education*. To be published in 2025.

CHAPTER 1: GENERAL INTRODUCTION

1.0 General introduction to the study

There is a common challenge with primary school learners struggling with the understanding of the concept of fractions (Ubah, 2021; Dogan & Tertemiz, 2020). The challenge learners face regarding fraction comprehension is largely attributed to the abstract nature of fractions (Wilensky, 1991). It has been identified that the challenge learners face concerning fractions in primary school persist into high school mathematics; this deficit poses an even greater hurdle as fractions play a crucial role for progression in high school mathematics (Bruce, Chang, & Flynn, 2013). Cady, Collins, & Hodges (2015) identified concepts like proportionality, algebra, and probability as key concepts in high school mathematics that rely on a good understanding and foundation of fractions. Bruce et al. (2013) further points out that the challenges learners face with fractions persist beyond school education into adulthood.

In addition to learners' struggle with fractions, solving fraction word problem exercise examples has been observed to be an equal challenge to learners. Research, (Pearce, Bruun, Skinner, & Lopez-Mohler, 2013) has recorded that student performance on solving fraction word problems is closely related to learners' reading comprehension skills, devising a plan, computation and higher-order thinking. Additionally, Pearce et al. (2013) also identified that learners' textbooks generally provide procedures and recommendations as worked-out examples on how learners should solve word problems. This relationship between worked- out examples and their influence in problem-solving has motivated the necessity to study the aspects of textbooks that are challenging to learners. As a result, analysing a sample of Grade 6 South African mathematics textbooks will provide both an understanding and an insight on the struggles of fractions and word problems simultaneously.

As a Grade 6 mathematics teacher, I have consistently noticed that a substantial number of my learners demonstrate difficulties when solving fraction word problem exercise examples from their textbooks. When my Grade 6 learners are presented with fraction word problem exercise examples, their attempts to solve the problems reveal a persisting challenge which serves as a good indicator of insufficient mathematical competency. This gap in the application of fractions indicates that learners' fraction proficiency is limited for an effective engagement with solving fraction word problems. Learners have been documented to struggle with word problems in general across grades, but there is very limited research conducted on learners' engagement with fraction word problems specifically in Grade 6 and, even worse, in South Africa. The general mathematics performance of South African learners in the intermediate and senior phase was documented to have been below the target of 60% according to the 2014 Annual National Assessment results. In addition to these findings from ANA, Gauteng's Grade 6 mathematics performance below 40% was recorded to be sitting on 29% in 2014 (DBE, 2014). The results that confirm my classroom experiences are documented in the Annual National Assessment 2014 diagnostic report for intermediate and senior phase, where Grade 6 learners' average performance in numbers, operations and relationships is 41%, one of the skills assessed was the ability to solve problems involving common fractions, decimal fractions and percentages. It was fascinating that the Grade 6 results were documented to illustrate strengths in areas concerning fractions but in Grade 9, areas concerning fractions were listed under areas of weakness. This begs the question of whether learners have mastered their fraction concepts and problem solving skills in Grade 6. If they really did, would not their performance indicate otherwise in Grade 9?

It is paradoxical that while problem solving is intended to be motivational and spark enthusiasm for learners to learn mathematics (Brehmer, Ryve, & Van Steenbrugge, 2015), learners in Grade 6 still encounter challenges with word problems, especially those that involve fractions. Another contributing factor to learners' struggle with fraction word problems is their immediate act to make use of the mathematical operation and numbers given in the word problem without understanding the context and applying the correct reasoning to solve the problem (Pongsakdi, Laine, Veermans, Hannula-Sormunen, & Lehtinen, 2016). This indicates a habit learners have from their constant exposure to routine problems as exercise examples, which limits their development of critical thinking and making use of problem solving strategies. However, there is a suggestion (Alreshidi, 2021; Booth, McGinn, Young & Barbieri, 2015; Trafton, & Reiser, 1993) that worked-out examples have the potential to enhance learners' word problem solving skills, as these examples offer guidance because they illustrate the rules, calculation techniques and procedures necessary to successfully solve word problem exercises.

When the curriculum starts to focus on the fraction word problem unit, learners would exhibit a significant inability to engage with fraction word problem exercise examples that are presented at the end of the chapter in textbooks. According to Steward (2005), learners' exposure to fractions is mostly halves, quarters, and the rest of the ideas of fractions do not form part of learners' livelihood, making learning fractions even more abstract. Their attempts to solve any given fraction word problem often indicates lack of skills to apply mathematical thinking that would be of assistance to solve given fraction word problems. Learners seem not to have the skillset to relate a given fraction word problems with their reality as indicated by Steward (2005). As a result, they avoid using their real life reasoning when it comes to solving word problems. This is very concerning since the concept of common fractions in the South African curriculum is covered in two terms in each grade in the intermediate phase (Grades 4, 5, and 6). The assessment of fractions always includes the requirement of solving word problems. Why do learners still find fraction word problem exercise examples challenging despite utilising textbook examples to guide their understanding regardless of the assistance of a teacher? Bruce et. al (2013), suggest that to identify and understand the fundamental reasons for the poor performance in fraction skills, to address the issue of poor performance, educational mathematics research communities need to prioritise their research on this challenge to develop effective solutions.

My experience has also revealed that during probing and discussions, learners' reasoning is primarily supported by textbooks' worked-out examples and how they understood the examples. This signifies the amount of reliance learners have on worked-out examples in their provided textbooks to solve fraction word problem exercises. Learners depend on textbooks as resources to mitigate their struggles when they are instructed to complete fraction word problem exercise examples they do not have sufficient proficiency in. Due to learner mathematical incompetence, learners find themselves relying on worked-out examples' procedures to make sense of how they should go about solving fraction word problem exercise examples presented to them to solve. Valverde (2002, p.2) furthermore clarifies that "they (textbooks) mediate between the intent of curricular policy and the instruction that occurs in classrooms. This suggests that textbooks have a strong impact on learning that occurs in classrooms". This makes it important that textbooks are studied to ensure that they fulfil their intended purpose.

Kilpatrick, Swafford, and Findell (2001) suggest that textbooks should thoroughly support the development of all the five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition to help learners have an understanding of mathematics. This means that the development of fractions would essentially involve learners' development in their conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Therefore, the purpose of this study is to investigate the affordances of fraction worked-out examples and fraction word problem exercise examples in the three sampled Grade 6, Curriculum Assessment Policy Statement (CAPS) approved textbooks. The topic of fractions appears in Term 1 and Term 4 in all three textbooks. The focus of this study is to analyse both Term 1 and Term 4's fraction content because that will highlight how the promotion of the five strands of mathematical proficiency results in learners completing an academic year without attaining fraction proficiency. The findings of this study are meant to be informative towards instructional practice, textbook use, and classroom practice.

1. 1 Significance of my study

Textbooks, along with their analysis are recognised as a key component of the curriculum, as they function as the mediators of the curriculum and classroom practices. They are therefore, a great influence on teaching and learning mathematics (Tárraga-Mínguez, Tarín-Ibáñez, & Lacruz-Pérez, 2021; Aineamani & Naicker, 2014). Having identified that the learners I teach in Grade 6 are not fully competent or equipped with mathematical skills for solving fraction-related word problem exercise examples presented in textbooks; this comparative textbook study aims to provide insight and understanding of the learning opportunities afforded to learners through their independent use of textbooks in the classroom and how mathematical proficiency is promoted.

Valverde, Bianchi, Wolfe, and Schmidt (2020) suggest that textbooks are essential for understanding learning opportunities provided to learners. In essence, textbooks are analysed with the aim of understanding how they mediate teaching and learning of fractions and their application in a form of word problem exercise examples, guided by the curriculum. In addition to this, research concerning textbook analysis is deemed crucial in ensuring that it helps educational researchers, teachers and research enthusiasts understand what is required in terms of their pedagogical choices and presentation for the sake of the curriculum development (Cheng, Chang, & Silalahi, 2017). This study is of great importance as it aims to investigate the affordances the three Grade 6 textbooks offer by analysing their worked-out examples and fraction word problem exercise examples using Kilpatrick et al. (2001) notion of five strands of mathematical proficiency (2001). The notion of five strands of mathematical proficiency is intended to contribute towards the investigation and understanding of how the different textbooks promote mathematical competence in their worked-out examples and fraction word problem exercise examples.

This study takes into account how important fractions are for learners in primary school. Therefore, the analysis of the textbooks that are mostly used by learners provides an understanding of how learners experience the textbook's content of worked-out examples and fraction word problem exercise examples. In addition to this, Neagoy (2017) has provided three important reasons why the teaching and learning of fractions should be investigated and this is because:

> *1) Fractions play a key role in students' feelings about mathematics*. For many students, fractions present a first mathematical stumbling block. Students begin disliking mathematics when they must surrender their sense making and yield to sense-*less* memorization. 2) *Fractions are fundamental to school math and daily life*. Although fractions underpin many complex mathematical topics, including ratios, rates, percentages, proportions, proportionality, linearity, and slope, their importance is not limited to mathematical study. As the quote on the previous page indicates, fluency with fractions is also required for many activities of daily life: following recipes, calculating discounts, comparing rates, converting measuring units, reading maps, investing money, and more. 3) *Fractions are foundational to success in algebra*. In its final report, *Foundations for Success*, the National Mathematics Advisory Panel (2008) concluded that (1) algebra is the gateway to success in high school and college, and (2) the main reason for U.S. students' failure in algebra is their poor proficiency with fractions. The worthy goal of "algebra for all" is not possible unless "fractions for all" is a reality. And in our present educational system, a solid grounding in algebra is foundational to a STEM career. (p. 1-2)

1.2 Purpose

The purpose of this study is to analyse the affordances fraction worked-out examples and fraction word problem exercise examples offer to learners in the three CAPS approved textbooks in Grade 6: Headstart (Facer et al., 2012), Platinum (Bowie et al., 2012) and Shuters Premier (Tiaden et al., 2013).

This study is informed by the following research questions:

- How is mathematical proficiency using Kilpatrick et al. (2001) five strands promoted in three selected Grade 6 textbooks: Headstart, Premier Mathematics and Platinum textbooks?

- How are fractions worked-out examples and fraction word problem exercise examples presented in the selected Grade 6 textbooks?

- What affordances for learning about fractions are offered in these Grade 6 textbooks?

In response to the above research questions, I engage with Kilpatrick et al. (2001) strands of mathematical proficiency to analyse how mathematical competence is promoted in the fraction worked-out examples and fraction word problems exercise examples under the concept of common fractions in each textbook.

1.3 Conclusion

In this chapter, I have discussed my research rationale. Fraction worked-out examples and fraction word problem-solving under the topic of common fractions have been selected as this research topic of interest in Premier Mathematics, Platinum and Headstart textbooks. This focus is driven by the continuous struggle learners face during teaching and learning when they are expected to demonstrate a satisfactory level of fraction and problem solving proficiency. Additionally, I have discussed the purpose of the study being associated with informing teacher instruction, pedagogical practices and understanding how the three textbooks assist in developing learners' mathematical proficiency. Lastly, I presented the research questions that informed my research paper and how I intend to engage with them using Kilpatrick et al. (2001) notion of mathematical proficiency. The findings from this analysis will inform my teaching instructions towards the use of textbooks with the goal of promoting mathematical proficiency.

The following chapters of this research are structured as follows to communicate and explain how the study was conducted:

- In Chapter 2 , I review existing literature related to this study, I engage with research on the teaching and learning of fractions, the significance of textbooks in the learning of mathematics. A special attention has been devoted to reviewing literature on worked-out examples and fraction word problems. I also describe the study's conceptual framework: Kilpatrick et al. (2001) notion of mathematical proficiency, with the aim of explaining its relevance to this research as it focuses on fractions.
- In Chapter 3, I engaged with the study's research methodology, sample and how data is analysed and presented. I also discuss how Kilpatrick et al (2001) notion of the strands of mathematical proficiency is adapted for this study. Finally, I consider the ethical consideration concerning this study and its validity and reliability.
- In Chapter 4, I present an analysis of fraction worked-out examples and fraction word problem exercise examples for each textbook. The analysis is achieved through categorising each unit of analysis in a table using Kilpatrick et al. (2001) notion of five strands of mathematical proficiency that has been adapted for the analysis of textbooks.
- In Chapter 5, I conclude the study with a summary of findings and reflect on the limitations and implications of the study.

CHAPTER 2: LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.0 Introduction

This chapter presents an overview of a selection of relevant literature that frames the theoretical perspective used for this research's analysis of the textbooks being studied. This study is framed by an adapted version of Kilpatrick et al. (2001) five strands of mathematical proficiency. This chapter begins with a review of literature on teaching and learning of fractions, highlighting the challenges and intricacies related to fractions. This is subsequently followed by a discussion on how the relevant strands of mathematical proficiency are employed as a relevant theoretical perspective to investigate the three selected textbooks.

2.1 Literature Review

In this literature review, I have engaged with relevant research on the teaching and learning of fractions, the importance of textbooks in the teaching and learning of mathematics, the importance of fraction worked-out examples and fraction word problems exercises in mathematics. Lastly, I reviewed scholarly works on the significance of textbooks in the teaching and learning of mathematics and the importance of word problems in mathematics.

2.1.1 Research on the teaching and learning of fractions

The teaching and learning of fractions encompasses a large variety of interrelated mathematical concepts and knowledge that extends to the following grades (Bruin-Muurling, 2010). The demand for fraction proficiency in higher grade mathematics is documented to be an essential element for mathematics performance in topics such as proportionality, probability and algebra (Baiduri, 2020; Bruin-Muurling, 2010; Cady et al., 2015). In addition to this, Hurrell and Day (2014), state that the comprehension of fractions supports other aspects of mathematical thinking and understanding such as geometry and measurements and learners who do not master this thinking may struggle in university.

An extensive amount of research has documented that learners continuously demonstrate a limited understanding of fraction concepts (Lortie-Forgues, Tian & Siegler, 2015; Stigler et al, 2010; Behr et al., 1983). Fractions are difficult to teach and learn because the transition from natural numbers to the introduction of positive rational numbers, commonly called fractions in the curriculum, is often complex. Despite intense instructions (Lortie-Forgues et al., 2015) and multiple interventions (Lovemore, Robertson & Graven, 2021; Shin & Bryant, 2015; Greer, 2008) implemented to combat the challenge of fraction learning, the challenges are still persistent. Failure to make a successful transition, learners work with fractions the same way they would when they are working with properties of whole numbers. This leads to what Ni and Zhou (2005) called "whole number bias". Whole number bias poses an interference between the learners' prior knowledge and the construction of new knowledge with regards to learning the concept of fractions. Gabriel, Coché, Szucs, Carette, Rey, and Content, (2013) have documented that there are few core differences between fractions and whole numbers and when they are not addressed thoroughly they have the potential to lead to whole number bias and make the overall learning experience of fractions challenging. A few of these core differences are that there are infinite fractions between two fractions, whereas there are finite whole numbers between two whole numbers. Simply put, there are only three whole number between 4 and 7 but there are infinite fractions between $\frac{1}{4}$ and $\frac{1}{2}$.

The understanding and recognition that there are fractions between $\frac{1}{4}$ and $\frac{1}{2}$ helps eradicate the treatment of fractions as whole numbers in a sense that one cannot simply subtract $\frac{1}{2}$ and $\frac{1}{4}$ the same way they would 7 and 4. However, according to Empson, Levi and Carpenter (2011) there are fundamental connections between properties of whole numbers and how they can be integrated in the learning of fractions. Learners' struggle with learning fractions is that they are often taught in isolation, away from the teaching and learning of whole numbers. The pedagogical decision to teach fractions and whole numbers as two completely different entities leads to learning mathematics in fragments with no connection between mathematical ideas. Learning fractions encompasses "drawing on and reinforcing the fundamental properties that govern reasoning about both whole-number and fraction quantities and operations" (Empson, Levi & Carpenter (2011, p. 41). This is called *relational thinking*, this kind of thinking is meant to help learners create new fraction knowledge connections to their already existing knowledge and properties of whole numbers. Upon studying fraction errors (Kalra, Hubbard & Matthews, 2020), it was suggested that when learners fail at relational thinking and reasoning they often treat fractions as separate quantities resulting in whole number bias and errors.

According to Neagoy (2017), the introduction of fractions brings about a shift in how learners think about the concept of fractions. Neagoy (2017) suggests two ways the shift could be pedagogically addressed. The first one is teacher awareness and sensitivity on the transition from additive to multiplicative thinking, and the second transition is from whole numbers to rational numbers. Additive thinking refers to how learners are used to thinking in the additive path because of how their early experiences of numbers are rooted in adding and subtracting. However, for learners to successfully gauge their way through percent, ratio, and proportion, "Multiplicative thinking is a prerequisite for working with these powerful and necessary ideas. Students cannot be expected to understand and use rational number ideas and representations with any confidence if their understanding of multiplication (and division) is restricted to a 'groups of' model with small whole numbers (Siemon, Breed & Virgona, 2005, p. 1". This means learners should have the cognitive capacity to recognise word problems that require the use of multiplication and division, they should have the skillset to work flexible and efficiently with a range of numbers that involve decimals, ratio, percentages, and percentages and apply appropriate algorithms and strategies. A successful start to introducing multiplication thinking is through the development of the idea of *grouping, sharing,* and/or one-to-many counting.

Neagoy (2017), advocates for the awareness of the shift that has to take place from the "natural way" of learners' thinking about numbers, which is either adding or subtracting, to a multiplicative way. The multiplicative way of thinking of numbers provides a realistic way of thinking about potential real-world situations: "if, say, Plant A grew from 2 to 3 feet, and Plant B grew from 6 to 8 feet, then we would say that Plant A grew $\frac{1}{2}$f its original height, whereas Plant B only grew $\frac{1}{3}$of its original height. Such reasoning exemplifies multiplicative thinking and necessarily involves rational numbers" (Neagoy, 2017, p. 8). The importance of this multiplicative thinking in the teaching and learning of fractions is that it underpins the mathematical understanding that is required beyond primary school mathematics.

Multiplicative thinking captures the essence of fractions as continuous quantities such as the measurement of area, length of time. These quantities cannot be measured using only additive thinking. Hurst and Linsell (2020) defined the concept of multiplicative thinking as linked ideas varying from partition, equal grouping, inverse relationships between multiplication and division, and proportional reasoning.

In as much as the concept of fractions is challenging for learners, and because it requires a great deal of multiplication and division knowledge, it makes it even more challenging for learners because of the degree of multiplicative thinking it requires. According to Hurst and Linsell (2020), multiplicative thinking promotes flexibility when it comes to working with whole numbers, fractions, decimals, percentages, and ratios. In addition to flexibility, multiplicative thinking also promotes the development of conceptual knowledge in a form of understanding the relationship between concepts and their application in different situations. Multiplicative thinking gives learners access to knowing when to divide and when to multiply, which are very important ideas in the teaching and learning of fractions. The development of multiplicative thinking is best related to manipulatives; to comprehend fractions, learners need to completely understand how fractions are represented, how are quantities represented in terms of fractions and the relationship fractions may be representing (Way, 2011). Fractions are introduced to learners with their prior experiences of sharing objects or quantities equally in early grades (Bruin-Muurling, 2010).

The challenges often encountered by learners include the comprehension of the concept of numerator and denominator, insufficient comprehension of adding, subtracting, multiplying, and dividing fractions (Braithwaite & Siegler, 2021; Mark, 1995), and the overall conceptualisation of fractions (Moyo & Machaba, 2001). At the introductory phase, learners are not required to justify how they choose to go about partitioning the objects. Representation is crucial for learners' understanding; representation plays an important role in contextualising the concept of fractions for learners and for the development of conceptual understanding. Way (2011), suggest that children learning fractions need to be provided with the opportunity to explore fractions in different contexts, representations, and engage with manipulatives such as cutting out paper strips, drawing shapes, and sharing items (Moyo & Machaba, 2021).

To understand the development of fraction proficiency in Grades 4 through 9, Bruin-Muurling (2010) begins by defining one of the elements of fraction proficiency as learners' ability to know and apply arithmetic rules and definitions in tasks. According to Siegler, Fazio, Bailey and Zhou (2013), becoming proficient with fractions requires learners to develop solid conceptual and procedural knowledge, which encompasses the ability to apply procedures efficiently. In addition to the two types of knowledge that are expected to take place in the development of fraction proficiency, Siegler et al. (2013) suggest that the development of symbolic and non-symbolic knowledge is just as important, as it enables learners to examine fractional concepts using different representations. In a study conducted by Peck and Jencks (1981) with Grade 6 learners, it was indicated that learners who had gaps in their fraction knowledge often struggled with applying previously learned rules in varying contexts correctly. The study revealed that learners who struggled with the application of rules had knowledge gaps that affected their reasoning, resulting in their dependency on their teachers' validation to determine the accuracy of their answers.

Way (2011) builds on Siegler et al.'s (2013) suggestion that learners need opportunities to explore fractions with different representations and in different contexts to attain conceptual and procedural knowledge. Research (Peck & Jencks, 1981) reports that learners with adequate conceptual and procedural knowledge demonstrate the ability to extend their understanding to more complex concepts of fractions and exude more confidence in their solutions and their reasoning. A segment of the interview revealed that learners with access to various representations could rely on these resource stations rather than external resources for reasoning, as illustrated by a student's response, "Do you want the picture or the rule?" (Peck & Jencks, 1981, p. 346). For fractions, Bruce et al. (2016) suggests that linear visual representations such as number lines, ribbons, strings, and rectangles, are more helpful in introducing fractions because they develop the understanding of fraction size and equivalence.

These are called semiotic interpretations, and they were first proposed by Kieren (1980) as the five sub-constructs of fractions: measurement, part-whole, quotient, ratio, and operator. Understanding the sub-constructs of fractions can help learners make sense of fraction procedures (Braithwaite & Sprague, 2021). This discussion has highlighted three very important types of knowledge crucial for mastering the basics of fractions and decimals: conceptual knowledge, procedural knowledge, and fraction interpretation that are very crucial in the teaching and learning of fractions. The teaching and learning of fractions in Grade 4 concerning sub-constructs involves the understanding of the part-whole interpretation. The part-whole relationship involves recognising that a fraction is one or more equal parts of a single object (e.g., two of eight equal parts of a candy bar) or a subset of a group of objects (e.g., two of five candy bars) (Fuchs et al., 2016, p. 626). Part-whole interpretation is documented to be dominant in schools, particularly American schools, as it formalises children's experiences of sharing. The teaching and learning of the interpretation of measuring is often represented in number lines (Fuchs et al., 2016).

Braithwaite and Sprague (2021), suggests that the conceptual knowledge of fractions encompasses magnitude knowledge, fraction interpretation, place value knowledge, and cross notation knowledge. *Magnitude knowledge* of fraction is knowing that fractions and decimals have magnitudes that can be ordered, compared, and place on a number line just like whole numbers. Magnitude knowledge lessens the chances of learners making errors. For example, a learner should be able to identify an error in $\frac{1}{2}+\frac{1}{3}=\frac{2}{6}$ because the sum cannot be equal or less than one of the addends. A learner with magnitude knowledge can recognise this error before getting into the technicalities of how the procedure of adding fractions is different from that of adding whole numbers by just focusing on the magnitude of the addends and the final sum.

Place value knowledge of decimals refers to understanding that the place value of decimal digits depends on their position relative to the decimal point, which determines the digit's value. This knowledge contributes to decimal arithmetic procedures. For instance, $1 + 0{,}2 = 1{,}2$ is a challenge for learners and results in the most common error related to place value made by learners. When adding the digits, the learner disregards their place value, resulting in an answer of 3 or 0,3.

The last form of knowledge is *cross-notational knowledge* which refers to the relationship of fractions and decimals, and how fractions can be translated to decimals through equivalency. The authors (Braithwaite & Sprague, 2021) assert that cross notation knowledge is very useful in empowering learners to translate between notations in order to successfully problem solve in instances where the problem is represented in mixed notations. For example, a learner presented with the following to add, should know that $5 + \frac{3}{6}$ can also be represented as 5 + 0,5 or $\frac{33}{6} + \frac{3}{6}$ and that the answer 5,5 is the equivalent of $\frac{33}{6}$ relatively.

2.1.2 The importance of textbooks in the teaching and learning of mathematics

Mathematics textbooks are defined by Kilpatrick (2014) as books that are used for the teaching and learning of mathematics (Kilpatrick, 2014; Okeeffe, 2012). Teachers use textbooks to plan and design lessons aligned with the curriculum, making them an essential element in the teaching and learning of mathematics (Cheng, Chang, & Silalahi, 2017). In the classroom, textbooks are used as sources of mathematical content explanations (Weinberg & Wiesner, 2010). As curriculum materials, textbooks are an essential element in the teaching and learning of mathematics because they provide support for the content that will be taught and learnt for the academic year and influence the enacted curriculum (van Zanten & van den Heuvel-Panhuizen, 2018; Gracin, 2018).

Weinberg and Wiesner (2010) suggest that an effective way of using textbooks is for the instructor to acknowledge that the meaning learners generate from the textbooks may not necessarily advance learners' thinking and understanding of mathematics unless the instructor acts as a mediator between the textbook and the students. This implies that teachers' role extends beyond selecting valuable contents from textbooks, transforming knowledge, sharing, implementing and revising the resources through a process called documentation (Gueudet, Pepin, & Luc Trouche, 2013). Research (Weinberg & Wiesner, 2010) has shown that instructor feedback can lead to "significant improvement in students' ability to ask questions rooted in scientific inquiry and to use the textbook as a meaning-generating tool" (p. 61).

Textbooks have been considered as materials and resources that have been employed and enlisted as problem solving materials for self-instructions for learners (Kilpatrick, 2014; Hadaar, 2017). As such, textbooks the closest materials learners have to learn from the curriculum (Okeeffe, 2012). According to Rezat, Fan and Pepin (2018) the relationship between curriculum resources – in this case, textbooks have the potential to afford or constrain the activities they are designed to facilitate. Given the significant influence of textbooks on teaching and learning that greatly influences learners' performance outcomes, research has been inclined to critically examine and analyse textbook content and structure (Cady, Hodges, & Collins, 2015).

Textbooks are considered a resourceful element in the teaching and learning of mathematics, a commonly found in feature in mathematics classrooms (Okeeffe, 2013). Literature indicates that learners use mathematics textbooks independently without any guidance from their teachers, making them an equally influential and important element of the mediator (Rezat, Fan, & Pepin, 2021; Kristanto & Santoso, 2020). According to Weinberg and Wiesner (2010), instructors (teachers) use textbooks with the objective of helping students develop their understanding of mathematical ideas because we (teachers) understand the content in the textbooks but research reveals that learners often struggle with reading textbooks productively. Textbooks are structured and presented in a manner that is helpful towards the teaching and learning of mathematics. Textbooks are designed to have exercises for learners to solve to develop their problem solving skills. Mathematics textbooks consist of various elements that aid towards the learning of mathematics such as explanatory texts, worked- out examples, problem solving tasks. Teachers are therefore responsible for the transformation and the orchestration of how learners should be using textbooks. Textbook use will be different for different mathematics classrooms. Textbooks are an integral part of teachers' daily classroom practice from lesson structuring, planning and often reflect the teachers intended curriculum (Lepik, Grevholm & Viholainen, 2015).

Research by Kristanto and Santoso (2020) found that learners use textbooks to deepen their understanding of concepts they are taught and find solutions to problems. A textbook study (Rezat, 2011) illustrating how a Grade 6 teacher uses a textbook in the classroom showed how influential textbooks they are during teaching and learning. In the textbook study, the teacher uses the textbook to direct learners to tasks or problems for assistance. The author (Rezat, 2011) regards this use of textbooks as a typical use of textbooks, where they are used for tasks and problems, including homework.

2.1.3 The importance of worked-out examples in mathematics

Examples are an essential element in the teaching and learning of mathematics. Research (Bills et al., 2006) suggests that special attention needs to be granted towards understanding their usefulness. Examples serve as essential tools commonly used to communicate mathematical concepts between teachers and learners (Bills et al., 2006). In addition to the communicative relationship of mathematical concepts between teachers and learners, I would like to incorporate textbooks into this relationship, given their role as facilitators of mathematical concept communication between teachers and learners.

Worked-out examples encompass a problem statement and a step-by-step procedure of how to solve the problem from an "expert's model" (Atkinson, Derry, Renkl & Wortham, 2000). Good worked-out examples are expected to contain and demonstrate sufficient information to allow learners to learn through them without the need for additional instructional information (Zhu & Simon, 1987). In mathematics, worked-out examples are regarded as effective modes of learning and if they are designed and sequenced carefully they can induce a skill acquisition (Renkl, 1999). From a work-out example, a learner should be able to emulate studying, comprehend, and emulate the procedure. In other words, learners should have the ability to extend their newly acquired knowledge into other scenarios through the type of generalisation that unifies and subsumes a variety of cases instead of just one (Bills & Rowland, 2009). Internalised concepts and procedures from the worked-out example should be resources that can be applied to new problems. Research (Renkl, 1999) also shows that worked-out examples facilitate learning in a sense that well-conceptualised mathematical ideas can be explained by learners. Additional research (Zhu & Simon, 1987) has concluded that learners who had to learn from worked-out examples without lectures or any form of additional instructions outperformed or performed at the same level of success as those who were learning conventionally, and the duration of learning was shorter. This is because they were able to recognise the rules from the worked-out examples and apply them when they were able to search for and apply the underlying commonalities; moreover, when preschool learners are taught to produce explanations of concepts being instances of examples (Brown & Kane, 1988).

Worked-out examples are an intended part of textbook elements used to introduce a concept and a procedure to learners with or without extra instructions (Zhu & Simon, 1987), the idea is that learners internalise and conceptualise the presentation of the worked-examples enough to exploit them for meaning. Worked-out examples elicit self-explanation, which is essential in developing of problem-solving skills, as it supports knowledge acquisition. Learners who can best explain examples to themselves have been documented to perform successfully because focus on learning outcomes, compared to those who are aware of their learning difficulties and keep monitoring them (*metacognitive monitoring*) (Renkl, 1999). Renkl (2017) mentions that it is important to note that superficially reading examples is not the same as self-explanation, these should not be regarded as the same thing, as superficial reading has been observed as a common habit among learners.

According to Atkinson, Derry, Renkl and Wortham (2000), one of the instructional design principles for effective worked-out examples is that they should utilise multiple representations and follows a labelled and well-sectioned conceptual structure, enabling learners to make sense of them. Brown and Kane (1988) strongly recommend that learning from worked-out examples is most effective when learners are encouraged and taught how to elaborate on and explain worked-out examples as they engage with them, fostering the transfer of ideas and organisation of knowledge. Renkl (2017) asserts that worked-out examples are more effective as a pedagogical method for introducing a new concept compared to the conventional method of using problem solving. Renkl (2017), furthermore mentions that working with worked-out examples pre-empts learners from using shallow strategies, such as copying and pasting a strategy from problem solving.

2.1.4 Word problems in mathematics

Word problems are defined as verbal descriptions of real-life problems represented in a scholastic setting and require the use of mathematical tools, such as the application of various operations to be solved (Verschaffel, L., Greer, B., & De Corte, E., 2000). Word problem solving in mathematics has been evaluated as a cognitively demanding task for learners because of the complexities of language and the comprehension of the texts used for bilingual learners (Madzorera & Essien, 2018; Tong & Loc, 2017; Ahmad, Siti, Salim, & Zainuddin, 2008; Bernardo, 1999). According to Clarkson (2009), learners who utilise their home languages or switched between languages in mathematics classrooms give better mathematical performance in multilingual mathematics classrooms. In addition to the issue of language, word problem solving demands learners to apply their conceptual understanding by assembling and integrating both concepts and procedures to solve a problem (Bernardo, 1999). Simply put, for learners to be well equipped to engage with word problems, they need to be mathematically proficient.

It is notable that, according to Kaiser (2017), word problems or mathematical modelling problems in mathematics do not necessarily have to contain a valid real-world problem but instead focus on the solution to the problem in the mathematics context. In the real-world, fractions have been documented to be concepts that pose challenges to secondary school learners and some young adults, as documented by Elizabeth Green (2014) in an article where a restaurant was selling a burger that had a third-pound of beef in comparison to the famous Quarter-pounder from McDonald's at the same price, but the customers felt that they were being overcharged because, to them, "The '4' in '$\frac{1}{4}$' is larger than the '3' in '$\frac{1}{3}$,' which led them astray" (Hurrell & Day, 2014). Hurrell and Day (2014) propose that since it has been widely recognised that the majority of students enter and leave secondary school without sufficient knowledge of fractions to apply in their everyday use, the challenging big ideas should be identified, and the needed pedagogies should be put in place to help learners. Word problems have been identified to be a difficult part of mathematics for learners (Verschaffel et al., 2020).

The exercise examples in textbooks are regarded as opportunities for learners to practice and test their acquired skills (Kristanto & Santoso, 2020). Topics and exercises in textbooks are sequenced in a manner directed by the objectives of the enacted curriculum (Shield & Dole, 2012). To achieve these objectives and develop conceptual understanding, which is part of a broader picture of mathematical proficiency, Bell (1993) suggest that learners should be given multiple opportunities to develop their understanding and apply their knowledge in various contexts. This implies that learners should be afforded with multiple opportunities to attempt problem solving to enhance and develop their problem solving skills. Problem solving is an important element in mathematics because it allows individuals to make sense and cope with their environments (Kablan & Uğur, 2020).

Marchis (2012) suggest that to develop mathematical proficiency in problem solving, learners need to be competent in problem solving, paying special attention to the emphasis of the development of the five strands of mathematics. In addition to the development of the five strands, authors Kablan and Uğur (2020) further suggest that to develop learners' problem solving skills, appropriate problem types that promote higher-order thinking should be employed. Literature has classified lower-order thinking and higher-order thinking as referring to routine and nonroutine problem types (Kablan & Uğur, 2020).

Nonroutine exercises in mathematics are related to the development of problem solving skills, as they require higher-order thinking skills to be solved, thereby exposing learners to higher degrees of reasoning and new situations to apply their conceptual understanding (Kablan & Uğur, 2020), with the use of a variety of strategies to some up with a solution. On the contrary, solving routine problems does not expose learners to different situations; however, it develops their flexibility, speed, and accuracy in solving problems, making them equally important. Routine problems are problems that learners have encountered and have been taught to solve before and practiced with the appropriate procedures, so that they are familiar to learners.

According to Loewenberg Ball et al. (2005), learning mathematics through the use of real-world problems is both motivational and effective for the introduction of concepts; however, this approach should not be the general principle that every topic in mathematics is taught using real-world problems. Instead, real-world problem solving should be carefully selected to develop the intended mathematical ideas.

2.1.5 Fraction word problems in mathematics

Fraction word problems are mathematical representations of real-world situations. The real-world situations represented in fraction word problems are typically situations that involve money, measuring ingredients, sharing food, and working with distance. Fraction word problems use a pseudo realistic-narrative that is not necessarily applicable to a particular real world situation (Alfonso, García & Gabarda, 2016).

According to Sans and Gómezn (2018), there is extensive research on the classification of word problems, including classification by mathematical structure, phrase structure, length of the problem, number of words, and verb tense. However, Sans and Gómezn (2018) argue that there is no overall acceptance in mathematics education research of the classification of fraction problems. Nicolaou and Pitta-Pantazi (2015), suggest that there are seven fraction abilities that make up fraction understanding in primary schools and these abilities are: (a) fraction recognition, (b) definitions and mathematical explanations for fractions, (c) argumentations and justifications about fractions, (d) relative magnitude of fractions, (e) representations of fractions, (f) connections of fractions with decimals, percentages and division, and (g) reflection during the solution of fraction problems (p. 306). To aid in the development of fraction word problem solving and the seven abilities that constitute fraction understanding, Elbers (2003) asserts that learners benefit significantly when given realistic problems to work with.

One of the contributing factors to learners' struggle with fraction word problems is the language used in the problems, it can be regarded as unfamiliar to learners. Second-language speakers are expected to have a substantial amount of language proficiency in both their languages

According to Chen and Li (2008), there is an association between language proficiency, a robust vocabulary and learners ability to solve fraction word problems. Learners are expected to have an appropriate and adequate mathematics vocabulary to thrive in mathematics classrooms (Xu, Lafay, Douglas, Di Lonardo Burr, LeFevre, Osana, ... & Maloney, 2022; Thompson & Rubenstein, 2000). However, the vocabulary required for engaging with fraction word problems extends beyond technical mathematics vocabulary, such as: numerator, denominator, unit fraction, equivalent, decimal etc. Learners are also required to be proficient in contextual language and this should not be confused with everyday language use. For instance, the word "*share"* holds a different meaning in fraction learning compared to everyday use. In everyday language, *sharing* has a vague meaning and it could imply giving. "Mbalenhle shared her book with her sister" - in this context, *sharing* the book means she gave the book to her sister, and they will likely take turns using the book. Mathematics language is more precise and definite. Therefore, *sharing* in technical terms means dividing a quantity into equal parts or groups. The importance of language in solving fraction word problems involves comprehending the given problem and making use of appropriate operations (Chen & Li, 2008).

The use of manipulatives in the early stages of learning about fractions has been documented to be a beneficial and effective learning approach (Gaetano, 2014). Manipulatives are handson, concrete materials that make abstract mathematical concepts visual and offer a sensory experience that assists learners in communicating and developing their mathematical ideas (Bartolini & Martignone, 2020; Boggan, Harper & Whitmire, 2010; Gaetano, 2014). Fractions are a challenging mathematical concept, and because of their abstract nature, manipulatives are intended to help learners visualise concepts such as equivalence and comparing fractions. Manipulatives in fraction word problems help learners develop a better understanding of mathematical concepts that are encompassed in fraction word problems (Yosuf & Lusin, 2013).

While fraction manipulatives such as fraction strips, fraction circles, and pie charts, are effective in helping learners develop their understanding and problem-solve, this is a potential negative aspect on the use of manipulatives. Overdependence and reliance on manipulatives often result in learners demonstrating a sound ability to solve word problems within a limited scope. However, as fraction word problems become more abstract and require learners to reflect on their problem-solving and thinking process, this may pose a challenge for learners, as they may be required to think more critically in order to transfer their fundamental ideas obtained from the use manipulatives into more abstract scenarios (Belenky & Nokes, 2009). While manipulatives are intended to help learners learn abstract fraction ideas, there is some form of dependency that develops. When they are not gradually removed to assist learners in thinking more abstractly when problem-solving, they can pose a problem that creates knowledge gaps.

2.2 Conceptual Framework

This study was framed by Kilpatrick et al. (2001) five strands of mathematical proficiency.

Kilpatrick et al (2001) notion of the five strands of mathematical proficiency is relevant to my study because it was documented by the authors and suggest that it applies in various domains of mathematics. The framework emphasises that competence, in this case, fraction competence develops over time. For my textbook analysis, a textbook's objective to promote fraction competence should demonstrate development, promotion, and presentation of all five strands, not just a few, over time with the use of the textbooks.

The analysis will be the investigation of the development, promotion, and presentation of procedural fluency, adaptive reasoning, strategic competence, and productive disposition. Mathematical competence strands by Kilpatrick et al. (2001) are regarded as cognitive changes that should be developed over time for successful learning.

2.2.1 Kilpatrick et al. (2001) five strands of mathematical proficiency

This study is analytically framed by the five strands of mathematics proficiency and has been adapted for the analysis of the textbooks used in this study. The five strands of mathematical proficiency were originally developed by authors who have had extensive research in cognitive psychology, mathematics education, and research to provide an overview of a successful, composite, and comprehensive learning process in mathematics (Kilpatrick et al., 2001). The authors' view of what successful learning of mathematics encompasses is explained by their decision to term it "mathematical proficiency". The authors describe mathematical proficiency in these words:

> "Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen *mathematical proficiency* to capture what we believe is necessary for anyone to learn mathematics successfully." (Kilpatrick et al., 2001, p. 116).

Having already discussed the five strands of mathematical proficiency in the reviewed literature, the authors assert that conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition as the five strands of mathematical proficiency are not independent and represent a complex whole (Kilpatrick et al., 2001). This implies that no mathematical strand can be developed first without the others for mathematics to be successfully learned. For example, the development of adaptive reasoning relies on conceptual understanding, for mathematics to be deemed as being worthwhile and for self-efficacy to develop, conceptual understanding and strategic competence need to be part of the process. "Mathematical proficiency is not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands" (Kilpatrick et al., 2001, p. 116).

Kilpatrick et al. (2001) argue that for mathematical proficiency to be an attainable educational goal, instructional programmes need to advocate and address the development of all the strands.

Kilpatrick et al. (2001) strongly advocate that for the development of mathematical proficiency to occur, programmes of instructions, in the case of this study, textbooks should address all the five strands. Fraction proficiency development should provide learners with the skills to cope with everyday mathematics problems and pursue studying mathematics further. The framework suggest that learners possessing full knowledge of fractions, and not only the ability to memorise ideas but to be able to recognise, organise, and represent them leads to facilitated learning. The five strands of mathematical proficiency are:

Conceptual understanding is the comprehension of mathematical concepts and how they are connected and related to one another (Kilpatrick et al., 2001). The possession of conceptual understanding is demonstrated by having knowledge that extends beyond knowing isolated facts but it includes applications and mathematical use. In relation to my study, conceptual understanding will be denoted by the use and promotion of meaningful representations. Representations can be internal and/or external (Samsuddin & Retnawati, 2018), for this analysis, I focused on the external representations in textbooks. External representations are pictures, diagrams, symbols, manipulatives, real-life contexts, and verbal representations and are regarded as meaningful representations once they convey the intended conceptual meaning, facilitate learning, and manage thought processes (Mundy, 1986; Samsuddin & Retnawati, 2018). Learners tend to struggle with the development of representational competence (Tsai & Li, 2016), which is directly linked to what Kilpatrick et al. (2001) would relate to conceptual understanding.

Procedural fluency is the knowledge of how to execute procedures accurately, appropriately and efficiently (Kilpatrick et al., 2001). "It also supports the analysis of similarities and differences between methods of calculating" (Kilpatrick's et al., 2001, p.121). A good comprehension of mathematical concepts also assists with the ability to work out an estimate of what the results could be for the problem. The purpose of the development of this strand is to develop learners' skill that will enable them to see procedures as general algorithms that can be further developed to solve other problems. According to Kilpatrick et al. (2001), this means the general procedures can be beneficial for solving routine tasks, resulting in efficiency. Beyond just being efficient, procedural fluency also requires learners to be flexible because they need to recognise that mathematical situations are not uniform and often necessitate the use of different computational tools. Comprehensive understanding of concepts is considered the basis for procedural fluency is therefore regarded as the skill gained from understanding a concept in depth, leading to fewer mistakes.

Strategic competence is defined as "the ability to formulate mathematical problems, represent them, and solve them" (Kilpatrick et al., 2001, p. 124). The strategic competence strand caters for the fact that learners encounter problems outside the classroom that will require mathematics to be used. In schools, strategic competence is developed through problem-solving with the first step being mathematically formulating the problem, followed by representation and finally problem-solving. Strategic competence development is closely related to non-routine problems as they require productive thinking over reproductive thinking.

Adaptive reasoning is defined as the ability and skill to logically provide clarification and justification. Adaptive reasoning should be correct and be a result of knowledge of concepts, procedures, and methods. This strand develops when learners have enough knowledge about the task, have a good understanding of what is required from them and the context of the task is familiar to them. Adaptive reasoning also encompasses the capacity and ability to provide justification for one's work.

Productive disposition is the continuity of the habit of seeing mathematics as being sensible and useful, and one's self-belief in their potential. This strand develops from giving learners opportunities to experience mathematics and apply their knowledge, giving them the opportunity to see and make their own sensible meaning of mathematics and its usefulness. The advancement of productive disposition stems from progressing from working with easy-to-solve problems to more challenging problems, proving to oneself that they are capable of learning mathematics as they seek higher-order problems for themselves to learn from and see growth.

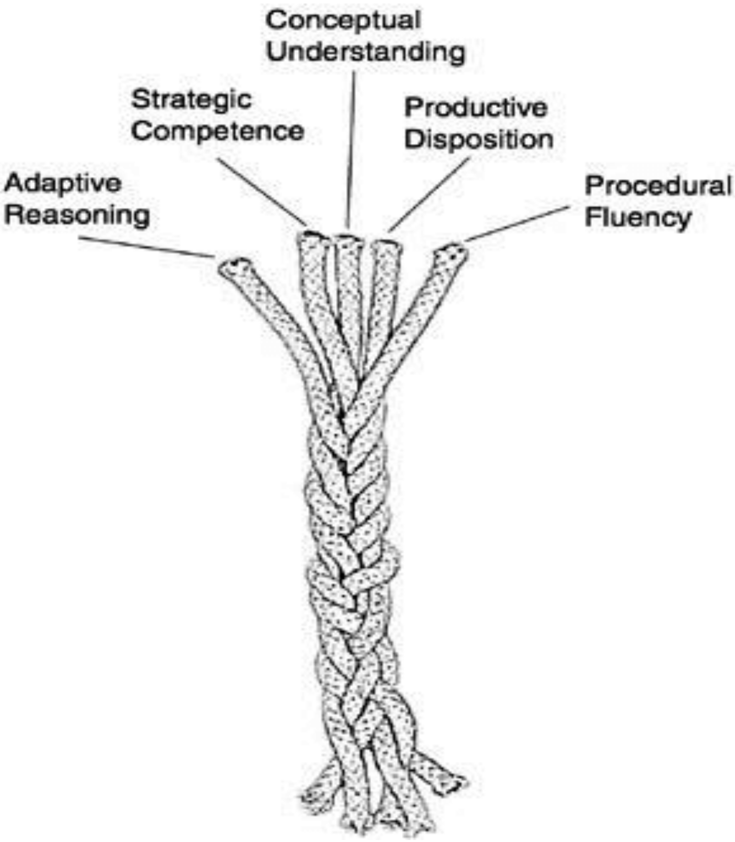


Figure 1A: Kilpatrick et al. (2001) Five strands of Mathematical Proficiency

Figure 1A highlights the key concepts of the notion of the strands of mathematical proficiency is that they are not independent but they are interwoven (Kilpatrick et al., 2001); one strand requires the other strands for its development to take place. The five strands are, and need to be considered as interdependent for the development of mathematical proficiency. To achieve the objective of this study, the main defining features of each strand will be utilised to analyse worked-out examples and word problem exercises. **Table 1** will be used to identify the strand that is promoted by each worked-out example and word problem exercise in the textbook.

2.3 Conclusion

This chapter has presented discussions around the five strands of mathematical proficiency that will guide my analysis. Literature on research done on the introduction of fractions, the importance of textbooks in the teaching and learning of mathematics, the importance of worked-out examples in word problems, and word problems in mathematics were reviewed for this chapter.

CHAPTER 3: RESEARCH DESIGN, METHODOLOGY AND ANALYTICAL FRAMEWORK

3.0 Introduction

Chapters 1 and 2 provided an introduction to this study and presented a relevant review of literature and a conceptual framework. In this chapter, I begin by providing a discussion on the implemented research approach for the analysis. This is followed by a discussion on the methodological approach for my analysis. Reasons for the selection of the three textbooks I chose for my analysis are stated and why fraction word problems are the topic of interest for my research are made explicit. This chapter also discusses the analysis of data, reliability, and validity of the study. Lastly, I discuss my studies' ethical considerations.

3.1 Research Design and Methodology

This research study uses a qualitative content analysis research approach. The qualitative aspect of this research aims to provide an in-depth understanding of the textbooks' affordances offered to learners by studying the empirical evidence which comprises of results yielded from the examination of the three Grade 6 sampled textbooks. Kawulich (2004) writes that the qualitative content analysis approach encompasses marking units of analysis, highlighting them, and employing a framework that helps to identify key concepts and themes that offer explanations to findings. This research approach is suitable for this research because it aligns with the study's objective of understanding how worked-out examples and fraction word problems in the selected textbooks contribute to learners' fraction proficiency. Kilpatrick et al. (2001) notion of mathematical proficiency is used to identify the promotion of fraction proficiency in textbooks. The units of analysis for this study comprise worked-out examples and fraction word problem exercise examples from the selected textbooks.

Once again, the objective of this research is to analyse and make sense of the presentation of worked-out examples and fraction word problem exercise examples in the textbooks to inform pedagogical choices of examples, using the strands of mathematical proficiency by Kilpatrick et al. (2001). According to DeJaeghere, Morrow, Richardson, Schowengerdt, Hinton, Muñoz and Boudet (2020), the purpose of qualitative research is to provide a diagnostic of the missing factors and/or gaps between the policy and the practice. However, in this analysis, the data is utilised to provide insight into different affordances the textbooks have to offer as learners engage with the textbooks. This textbook analysis aims for objectivity, with the goal that it is informative to me and other mathematics teachers. "Content analysis assures not only that all units of analysis receive equal treatment, whether they are entered at the beginning or at the end of an analysis, but also that the process is objective in that it does not matter who performs the analysis or where and when" (Krippendorff, 1989, p. 404). To attain objectivity, as suggested by Krippendorff (1989), worked- out examples and fraction word problem exercise examples from the textbooks are treated the same way and were analysed by two other mathematics teachers.

3.2 The Sample

As aforementioned in Chapter 1, this is a Grade 6 mathematics study that compares three textbooks that are CAPS approved and are distributed as per the Learning and Teaching Support Material policy upon the school's requests.

3.2.1 Selection of textbooks

The three selected textbooks for this analytical study are Headstart (Facer et al., 2012), Platinum (Bowie et al., 2012) and Shuters Premier (Tiaden et al., 2013). These textbooks were selected because they are the most commonly utilised textbooks in Grade 6 at the school I teach and they are used by both learners and teachers, establishing them as a major teaching resource. The contents on fractions in all three analysed textbooks is aligned with the curriculum's goal of ensuring that learners are proficient in performing calculations with fractions. The alignment of the textbooks with curriculum makes the comparison legitimate to achieve the purpose of the study as presented in Chapter 1.

3.2.2 Selection of topic (why fractions?)

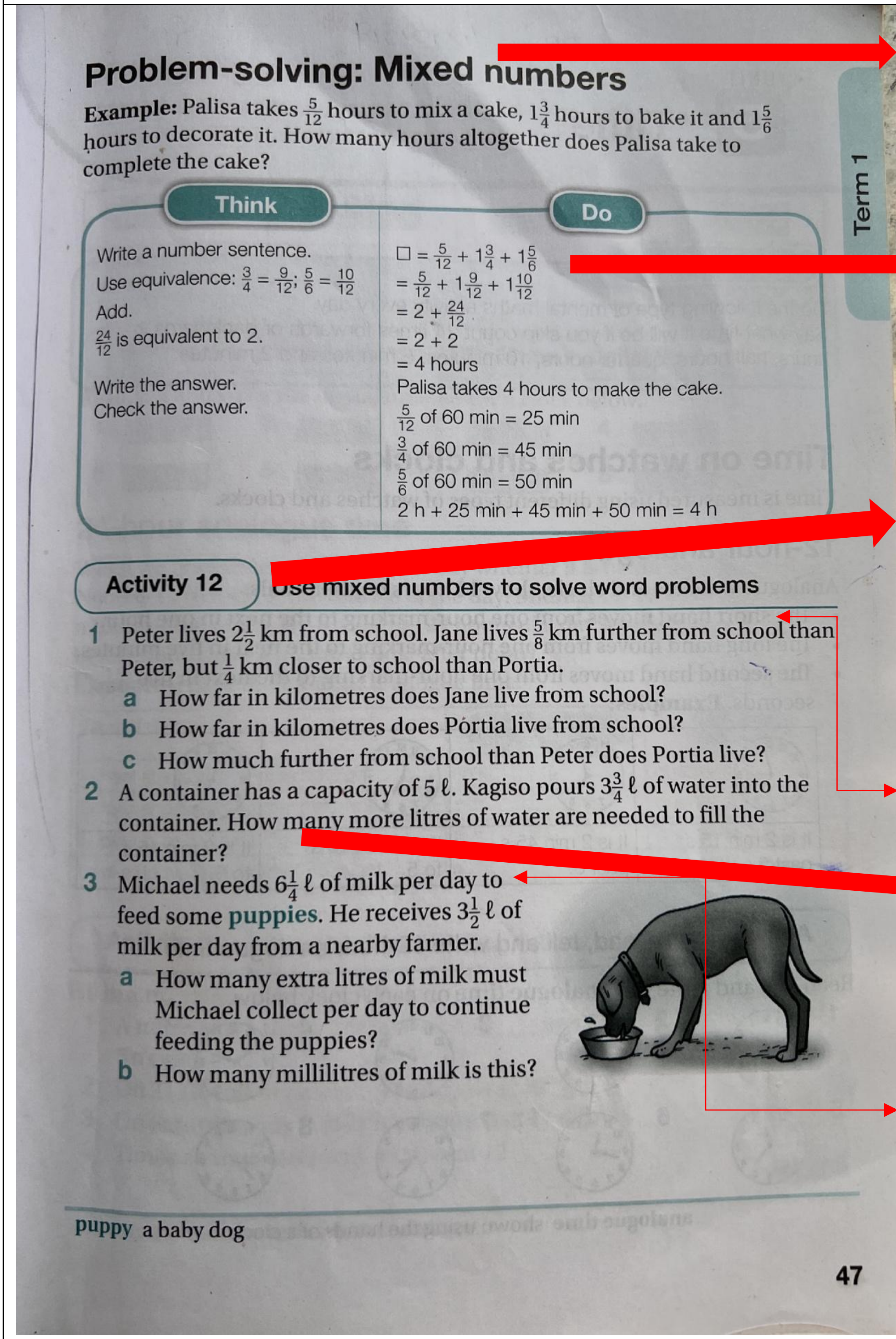
The focus for this study is on fractions because they are one of the most challenging concepts in mathematics for primary school learners (Dogan & Tertemiz, 2020). Additionally, fractions are an important aspect of learning mathematics and they contribute greatly as foundational concepts for more advanced mathematics (Groff, 1994). Therefore, analysing the content of fractions in the selected three Grade 6 textbooks aims to provide an understanding of what could be the potential contributing gaps of in the learning and understanding of fractions.

Worked-out examples were selected for analysis in the three Grade 6 textbooks because they play a crucial role in supporting learners' thinking and the development of their reasoning (Bills et al., 2006). Hence, worked-out examples are linked to the development of reasoning which is an important element in mathematics learning. The analysis of the fraction worked-out examples offer insight on the support offered to learners by the textbooks and what is expected from them as they independently solve fraction word problems. This leads us to the analysis of fraction word problem exercise examples. Word problems are an essential aspect mathematics because they help learners apply and develop their problem-solving skills on how they could be applied in "real life-like situations" (Bates & Wiest, 2004). In addition to this, Chapman (2006) affirms that word problems are important as they can motivate learners to understand the role of mathematics outside the classroom and enhance their interests. I have observed that learners' struggle with fraction word problem exercise examples has resulted in them becoming frustrated and bored. To analyse and investigate fraction word problem exercise examples in the textbooks used by learners will provide insight to some of the observations I have made in the classroom.

3.2.3 Textbook Structure

The three textbooks follow a slightly similar presentation of their content. For the concept of fractions and fraction word problem solving exercise examples, the textbooks follow the structure of one of the pages extracted from the Headstart textbook, **Extract 1**. If there is a worked- out example in the analysed section, it is followed by word problem exercise examples.

Again, at the end of each chapter, there is a series of fraction word problem exercise examples. Some questions have problem statements that can have 2-3 word problem exercise examples, these were analysed as single questions. **Textbook Extract 1,** provides a general overview of what content was considered for analysis for each textbook.

Headstart Textbook Content Overview	**Contents of Analysis**
	Fraction word problem example
	Fraction word problem worked-out example with steps to the solutions
	Exercise examples. Activity 12 has three fraction word problem exercise example
	Fraction word problem exercise example no# 1 has a problem statement and three exercise examples
	No# 2 is a single fraction exercise example.
	Fraction word problem exercise example no# 3 has a problem statement and two fraction word problem exercise examples
Textbook Extract 1: *From Headstart Textbook. Copyright (Oxford University Press Southern Africa). Used with permission.*	

Problem-solving: Mixed numbers

Example: Palisa takes $\frac{5}{12}$ hours to mix a cake, $1\frac{3}{4}$ hours to bake it and $1\frac{5}{6}$ hours to decorate it. How many hours altogether does Palisa take to complete the cake?

Term 1

Think	Do
Write a number sentence.	$\square = \frac{5}{12} + 1\frac{3}{4} + 1\frac{5}{6}$
Use equivalence: $\frac{3}{4} = \frac{9}{12}$; $\frac{5}{6} = \frac{10}{12}$	$= \frac{5}{12} + 1\frac{9}{12} + 1\frac{10}{12}$
Add.	$= 2 + \frac{24}{12}$
$\frac{24}{12}$ is equivalent to 2.	$= 2 + 2$
Write the answer.	$= 4$ hours Palisa takes 4 hours to make the cake.
Check the answer.	$\frac{5}{12}$ of 60 min = 25 min $\frac{3}{4}$ of 60 min = 45 min $\frac{5}{6}$ of 60 min = 50 min 2 h + 25 min + 45 min + 50 min = 4 h

Activity 12 Use mixed numbers to solve word problems

1 Peter lives $2\frac{1}{2}$ km from school. Jane lives $\frac{5}{8}$ km further from school than Peter, but $\frac{1}{4}$ km closer to school than Portia.
- a How far in kilometres does Jane live from school?
- b How far in kilometres does Portia live from school?
- c How much further from school than Peter does Portia live?

2 A container has a capacity of 5 ℓ. Kagiso pours $3\frac{3}{4}$ ℓ of water into the container. How many more litres of water are needed to fill the container?

3 Michael needs $6\frac{1}{4}$ ℓ of milk per day to feed some **puppies**. He receives $3\frac{1}{2}$ ℓ of milk per day from a nearby farmer.
- a How many extra litres of milk must Michael collect per day to continue feeding the puppies?
- b How many millilitres of milk is this?

puppy a baby dog

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3.3 How The Five Strands of Mathematical Proficiency was adapted for this study.

According to Kilpatrick et al. (2001), the five strands of mathematical proficiency are a framework that provide grounds for a discussion on the knowledge, skills, abilities, and beliefs that make up mathematical proficiency (Kilpatrick et al., 2001). As a result, the framework encompasses valuable elements that are important for use in analysing textbooks. The five strands were not originally designed for the analysis of textbook questions but since textbooks are instructional resources, they then have the potential to promote the five strands of mathematical proficiency. To successfully engage with the research questions, the five strands had to be adapted specifically for the task of analysing fraction worked-out examples and fraction word problem exercise examples in textbooks.

Worked-out examples and fraction word problem exercise examples were analysed for their affordances and the promotion of the strands of mathematical proficiency. The first adaptation was to extend the definitions of the five strands of mathematical proficiency and make them applicable for the analysis of fraction worked-out examples and word problem exercises examples. For instance, Kilpatrick et al. (2001) defines strategic competence as "the ability to formulate mathematical problems, represent them, and solve them" (p.118) however, for this study, this definition will be used to investigate how strategic competence is promoted. As a result, strategic competence in textbooks is going to be defined as the promotion of the formulation, representation and solution of mathematical problems. The essence of the original definition is still maintained in the adapted definition. Subsequently, all the definitions of the five strands are adapted to suit their new task of the framework that is to investigate the promotion of mathematical proficiency.

It is important that I acknowledge the argument made by Kilpatrick et al. (2001) that the five strands:

> must be interwoven to be useful reflects the finding that having deep understanding requires that learners connect pieces of knowledge, and that connection is a key factor in whether they use what they know productively in solving problems. Furthermore, cognitive science studies of problem solving have documented the importance of adaptive expertise and of what is called *metacognition:* Knowledge about one's own thinking and ability to monitor one's own understanding and problem-solving activity. This idea contributes to what we call strategic competence and adaptive reasoning. Finally, learning is also influenced by motivation, a component of productive disposition. (Kilpatrick et al., 2001, p. 118)

The issue of Conceptual understanding

According to Verschaffel, Schukajlow, Star, and Van Dooren, (2020) "word problems are thought of as textual problems that can be solved by means of applying a mathematical concept, rule or technique previously learnt at school" (p. 4). Therefore, conceptual understanding can be regarded as a primary strand for the other strands to come into manifestation. Askew (2012), argues that proficiency in mathematics are actions students need to engage in and be experts in when learning and using content. This implies that the strands of mathematical proficiency are action words, verbs. For instance, "procedural fluency is a skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001, p. 116), "to carry out procedure" is to execute and perform the necessary procedures. Therefore, Askew (2012) adds that conceptual understanding cannot be regarded as something learners "do" towards developing mathematical proficiency but it is the results of the other strands. In other words, conceptual understanding cannot be observed as it plays out and "making connections between related ideas, at first glance may look like actions: building, making, describing but what does building 'robust knowledge' or making connections between ideas' look like?" (adapted from Askew, 2012, p. 20).

All word problem exercises promote the development of a certain mathematical concept, therefore, learners will have to demonstrate their conceptual understanding but it cannot be something they "do" to attain conceptual understanding. Kilpatrick et al. (2001) articulation of this is that when learners do not possess conceptual understanding, methods and procedures to be used will not be recalled incorrectly thus influencing procedural fluency. A good grasp of concepts and how they relate to one another supports the development of procedures being carried out efficiently and accurately. To be able to formulate mathematical problems, represent and solve them will also require an in-depth understanding of the situation at hand the concepts involved to avoid relying on the "number grabbing" methods which indicate lack of understanding. To provide reasons and justifications for mathematical claims, conceptual understanding is needed, and the more self-efficacy and usefulness of mathematics develops, the development of productive disposition takes place. For this study and the reasons aforementioned, conceptual understanding will be required to be demonstrated in all word problem exercise examples.

In as much as the strands are interconnected, a fraction word problem exercise examples can promote procedural fluency and not adaptive reasoning as according to the adapted framework. This is because the framework recognises adaptive reasoning in word problems as possessing the feature of justification while procedural fluency questions will have verbal cues. As a result, a word problem can promote procedural fluency and not adaptive reasoning.

Adaptation of Mathematical Proficiency Strands for this Analysis

A further elaboration of what will be regarded as verbal cues for this analytical study is informed by Jerman and Rees' (1972) conception of verbal cues being words that are a cue or rather a guide for an operation to be used. Jerman and Rees' (1972) notion on verbal cues regarded words such as *and*, *left*, *each*, *average* or *each* as verbal cues for addition, subtraction,

multiplication and division (in that order). However, Jerman and Rees (1972) recognise that "verbal cues" can be distracting. "Distractors" are verbal cues that are not cues for the intended operation that should be used. A very common distractor is "*each*", as it can suggest a multiplication or division pathway. Such distractors do still suggest a pathway for learners to follow. However, they (learners) must use their mathematical thinking to use the correct operation to problem solve. Therefore, only distractors that are in the mathematical language would be considered as promoting procedural fluency. Verbal cues in the everyday English language will also be regarded as verbal cues. It is important to mention that I have also taken into consideration how everyday English language verbal cues may suggest incorrect computation pathways depending on context and learners' interpretation of the word problem. For this study, I used the notion of everyday English and mathematical language verbal cues used in the study by Jerman and Rees (1972) to predict the relative difficulty of verbal arithmetic problems. The verbal cues to be considered for procedural fluency are:
Addition: *added, altogether, gained, total*
Subtraction: *how much less, lost, left*
how much larger… than
how much smaller… than
how much greater… than
how much further…than
Multiplication: *each, times, of* (for fractions of a whole). "*Of*" is a modification made for this study.
Division: *average, each*

To operationalise the concept of **productive disposition** beyond self-efficacy, in fraction word problems, worthwhileness and positivity will be recognised by the everyday context being embedded in the problem. The idea of productive disposition being linked to worthwhileness and positive attitudes towards mathematics will be identified by the potential to lead to efficacy.

To analyse the exercises question and worked- out examples, I begin by investigating the potential promotion of procedural fluency. Procedural fluency in this analysis will be sub-categorised into four categories. The first category is what I will term the 1st degree of procedural fluency. 1st degree procedural fluency will be identified by the use of only **general vocabulary** everyday English language. These are signifiers that are used daily and may be common in learner's everyday speech . These promote **1st degree procedural** fluency when they are applied in a mathematical context. They offer a suggestive pathway to solving a word problem because they are utilised in their everyday speech, making it easy to mathematically translate them to the correct operation when problem solving.

General vocabulary will be considered as offering a suggestive pathway when it is used and interpreted in the right context. For example, the verb "*eat*" used in a word problem could possess various meanings but once its context is taken into consideration, the suggestive pathway will become apparent. "Two friends share a chocolate bar divided into 12 pieces, the first friend eats $\frac{2}{3}$ and the second friend eats $\frac{1}{6}$ of the chocolate". If the first question asks how many pieces did the first friend eat? This is a question of finding a fraction of a whole. However, if the second question asks to calculate how much chocolate did the two friends eat altogether?

Learners will have to find the sum of the two fractions to solve the problem. Another question would be who ate the most chocolate? And this will be a comparison of the two fractions representing the pieces eaten. The point here is that no assumption about a verbal cue should be made, all **general everyday language** cues will be analysed according to their intended context.

Next is the **2nd degree procedural fluency**, unlike the previous degree of procedural fluency, this degree will be identified by a different kind and more mathematical form of vocabulary. This kind of vocabulary is what Mulyran (1984) regarded as mathematical terms, a word that has specific mathematical meaning in the context it is used in they can be both technical or special and the difference between technical and special mathematical terms is that technical terms are individualistic to mathematics as a field of study. On the other hand, special words are used on a daily basis but can convey a different special or mathematical meaning in the context of mathematics, e.g. "*match*", "*set*", "*group*", "*figure*". It is important to bring up the distinction of the 1st degree and 2nd degree of procedural fluency beyond the presented verbal cues, the idea is that the flexibility, efficiency and accuracy procedures become slightly more mathematically developed for the sake of proficiency. In other words, with the conceptual understanding of mathematical vocabulary that is routinely used in mathematics textbooks and word problems, learners get access to mathematical tools that enable them to learn mathematics (Miller, 1993).

Unlike the first two degrees of procedural fluency, the **3rd degree of procedural fluency** is a routine problem that does not contain any obvious form of verbal cues to the solution of the problem. All fraction word problem exercise examples that were once categorised as promoting strategic competence because they are non-routine and only appear only once in the textbook will be analysed as promoting strategic competence. However, if the same kind of fraction word problem exercise example appears for the second time it will be regarded as promoting procedural fluency because the second exposure to the fraction word problem exercise example offers learners the skill sets and cognitive tools to solve the problems more flexibly and more accurately. The fraction word problem exercise examples that fit this category will be recorded as promoting procedural fluency at the **4th degree of procedural fluency** as they also possess qualities of strategic competence and this is what differentiates this level of degree compared to the first three that are actually routine problems. Note that the shift from the promotion of strategic competence to procedural fluency does not take away from the fact that the fraction word problem exercise example being analysed as promoting procedural fluency should equate to easy problem solving.

To investigate the promotion of strategic competence, all fraction word problem exercise examples that are non-routine and are appearing for the first time in the textbooks will be categorised as non-routine because they do not offer any suggestive pathway to solving the problem. There should be various forms of strategies that can be utilised to solve fraction word problems and those strategies should promote the essence of problem formulation and problem representation. To measure the promotion of the development of reasoning, fraction word problem exercise examples that probe for further explanation, justification, reasoning, potentially proof, or what Essien and Adler (2016) refer to as authorising practices, will be analysed as promoting adaptive reasoning. The last strand that will be analysed is the productive disposition strand and it has been divided into three sub-cob-categories, the first being **PD**$^{+,}$ fraction word problem exercise examples s will be regarded as promoting productive disposition at first degree when the information/ data in the word problem exists in the real world. The second degree, **PD**$^{++}$ will be identified by the evidence of real data being used and the event described has taken place or has the potential to take place. The last degree is **PD**$^{+++}$ and the data and the scenario created are exaggerated for the learning of mathematical concepts.

In **Table 1**, I present the analytical framework with adapted definitions of the strands of mathematical proficiency and I explain how each strand will be recognised during the analysis of the fraction worked-out examples and fraction word problem exercises. Each strand of mathematical proficiency has been sub-divided into varying degrees as discuss earlier in the chapter. For each category, an example is illustrated below it. The analytical framework also consist of rules each fraction worked-out example and fraction word problem exercise example has to adhere to for analysis.

Table 1: *Definitions and rules of recognition of the five strands being promoted*

Mathematical strands of proficiency	Definition	Description	Categories of description	Rules	Categorisation Justification
Procedural Fluency	The promotion of flexibility, accuracy and efficiency in the development of knowledge procedures	I will see this when a word problem is a routine problem.	**PF^{+} :** contains general verbal cues Example: Molly saved $\frac{1}{4}$ of the money that she earned. If she earned R100, how much did she spend? **PF^{++}:** contains mathematical verbal cues or both mathematical and general verbal cues Example: At the end of the week, a farmer expects to harvest of his bean crop altogether. If he has already harvested of the crops, what difference in the crop remains to be harvested this week? **PF^{+++}:** contains obvious suggestive pathways Example: In a bouquet of 24 red and white flowers, $\frac{1}{3}$ of the flowers are red.	**PF^{+}:** everyday verbal cues have to be words used in the everyday English language. They should provide a clear and concise pathway to solve the problem in the correct context. **PF^{++}:** mathematical verbal cues have to be mathematical cues that are commonly used in mathematics as described by Jerman (1972, p. 309) "His conception of verbal cues included words such as 'added', 'altogether', 'gained', 'how much less', 'lost', 'left', 'each', 'average" **PF^{+++}:** suggestive pathways are unclear as they cannot be classified as everyday English or mathematical verbal cues but the problem is a routine problem.	"By studying algorithms as "general procedures," students can gain insight into the fact that mathematics is well structured (highly organised, filled with patterns, predictable) and that a carefully developed procedure can be a powerful tool for completing routine tasks" (Kilpatrick et al., 2001). ● Saved, earned, spent and eat are everyday English cues. ● Added, altogether, gained, divided are clear mathematical verbal cues that do not possess any ambiguity in the mathematical context. ● Finding a fraction of a whole is a routine exercise but there is no cue that suggests the exact pathway to be followed.

			How many of the flowers in the bouquet are white? **PF^{++++}:** If a strategic competence question appears for the second time, it becomes a routine problem		
Strategic competence	The formulation, representation and solution of mathematical problems	I will see this when a word problem is a nonroutine question with no straightforward pathway.	**SC:** First time appearance in the textbook. Example: Tom, Sarah and Jack are related. Jack is $\frac{1}{3}$ of Sarah's age and Tom is $\frac{1}{4}$ of Sarah's age. If Tom is five years younger than Jack, how old is Sarah?	**SC^{+}:** problem is less structured, more complex, ambiguous and risky. These are Higher Order Problems. The context of the question is set in an abstract world of mathematics and not limited to real-life contexts. The problem has no clear or prescribed method or approach for the solution.	Robertson argues that if a problem has been solved before using previously taught methods then that task can be considered an "exercise" because of its reappearance (Robertson, 2017), not a problem . Hence, the second time the question is asked, it will be considered as procedural fluency.
Adaptive reasoning	The development of the skill of reasoning with logic, explanation and justification .	I will see this when a word problem requires an explanation or justification.	**AR:** contains the requirement to explain, provide proof or justification. Example: Nkanyezi and Alex are planning a trip from Rustenburg to Johannesburg. Their petrol tank is $\frac{4}{7}$ full, and they know they need $\frac{3}{4}$ of their tank to complete the trip. Do they have enough petrol to complete the trip? Show your calculations and justify your answer.	**AR: The** problem's objective is to develop adaptive reasoning through "explaining, proving or justifying" but this is allowed to be posed in different and creative ways in the problem.	One manifestation of adaptive reasoning is the ability to justify one's work (Kilpatrick et al., 2001).

Productive disposition	The promotion of sensibility, usefulness, worthiness and self-efficacy in mathematics.	I will see this when the everyday life situation embedded in the word problem emulates reality.	**PD^{+}:** Information/ data in the word problem exists in the real world. Example: Thandi baked a dozen muffins. She gave $\frac{1}{3}$ of them to her neighbour. How many muffins did Thandi give to her neighbour? **PD^{++}:** Real data is used and the event has taken place or has the potential to take place. Example: A typical school day in Roodeplaat has 7 hours of instruction. Due to a special school event, classes were shortened, and students only had $\frac{2}{3}$ of a typical school day. How many hours of instruction did the students have on that day? **PD^{+++}:** Data and the scenario created are exaggerated for the learning of mathematical concepts. Example: On the candy planet of Sweetopia, the Great Gumball Tree produces 1 000 000 gumballs every hour. If a swarm of Sugar Snails manages to slurp up $\frac{15}{10\ 000}$ of the hourly gumball production, how many gumballs do these speedy snails devour each hour?	**PD:** Real world word problems and their information should be problems that have been experienced by learners or have the potential to be experienced. This promotes a higher sense of useful mathematics.	'Realistic' word problems have been documented to have the potential to promote learners' willingness and the ability to introduce and make use of realistic responses (Cooper & Harries, 2000). "A higher proportion of the students working with more authentic word problems seem to have engaged more often in thorough task analysis that other students (Palm, 2007, p.53)

3.4 Ethical Considerations

This research project primarily focused as the analysis of three Grade 6 textbooks that are used by both teachers and learners and learners as the school I teach in. I applied for an ethics applications to the Wits University Ethics Committee to ensure that the research project adhered to the standard procedures and respected intellectual property of the textbooks. The ethical clearance certificate was granted by the University with the protocol number: 2022ECE014M.

3.5 Validity and Reliability

Validity and reliability are an important aspect of this study and they were considered throughout the implementation of the study. The approach this study took was a content analytical approach of three Grade 6 mathematics textbooks. According to Fraenkel, Wallen and Hyun (2012) content analysis offers researchers the opportunities to study and analyse communication and the communication can be of any type. To strive for validity, I used Kilpatrick et al (2001) notion of the strands of mathematical proficiency as my categories for the study to analyse the promotion of mathematical proficiency and the affordances the textbook have to offer.

According to Oleinik, Popova, Kirdina and Shatalova (2013), reliability depends on the consistency and the stability of the research framework used for analysis and research findings over time. To analyse the textbooks' contents, reliability is of paramount importance to ensure that the findings are not dependent on a single person's interpretation. To attain reliable research findings, the study used an analytical framework that was adapted specifically for the analysis of fraction word problems in Grade 6 textbooks. Another Grade 6 mathematics teacher was asked to use the same analytical framework to analyse the textbooks to see if the same result can be reproduced.

The textbooks that were sampled for this comparative analysis were sampled on the basis that the results would be a valid representation of the textbooks used in the school I teach by both teachers and learners. The sample size aligns with the purpose of the research study to inform my teaching and use of the studied textbooks. According to Palinkas, Horwitz, Green, Wisdom and Duan and Hoagwood (2015) the approach I have chosen for the Grade 6 textbooks I have sampled is considered purposeful sampling as it is related to a phenomenon of interest and not specifically generalisation.

3.6 Conclusion

This chapter has provided an overview of the research methodology and design used in this study. I discussed the comparative analytical approach this research undertook to support the analysis of the research focus; the focus is the promotion of the five strands of mathematical proficiency promoted by the word problem exercise examples and worked-out examples in each teach textbook. Sampling of the textbook and the textbook was done deliberately to inform my teaches practices and gain insight on the gaps that contribute to incompetent fraction knowledge. Furthermore, this chapter provided justification for the framework and how it was adapted for the study. I engaged with the concepts of reliability and validity by outlining how the two were considered for this research.

CHAPTER 4: DATA ANALYSIS AND DISCUSSION OF RESULTS

4.0 Introduction

This chapter of the study provides a presentation of the analysis of fraction worked-out examples and fraction word problem exercise examples from Premier, Platinum and Headstart textbooks. This analysis begins with a brief description of how the three Grade 6 textbooks' content is structured with the intention of providing a clear context for this analysis. Upon identifying all the worked-out examples and word problem exercises in each textbook, the process of examining each textbook's content follows. The outcome that follows the examination of the individual worked examples and exercise word problems will frame the discourse that will address my research questions on how mathematical proficiency using Kilpatrick et al. (2001) strands is promoted in the three selected Grade 6 textbooks, and how are fractions worked-out examples and word problem exercises presented in Headstart, Premier Mathematics, and Platinum textbooks? To achieve this goal, textbooks' contexts are elucidated.

4.1 Analysis of textbooks

To address the central point of this chapter, which is the analysis of the promotion of the strands of mathematical proficiency in work-out examples and fraction word problem exercise examples, it is crucial to note that the quantity of the number of worked-out examples and the fraction word problem exercise examples presented in each textbook varied. I begin this aspect of the analysis by with a summary of the number of worked-out examples and fraction word problem exercise examples in a graph as they appear in each textbook. This is followed by an analysis of the worked-out examples and fraction word problem exercise examples in a table categorises each fraction worked-out example and fraction word problem exercise example into their appropriate categorisations adapted from Kilpatrick et al. (2001).

Figure 2: *An overview of the worked-out examples and exercise word problem in each textbook.*

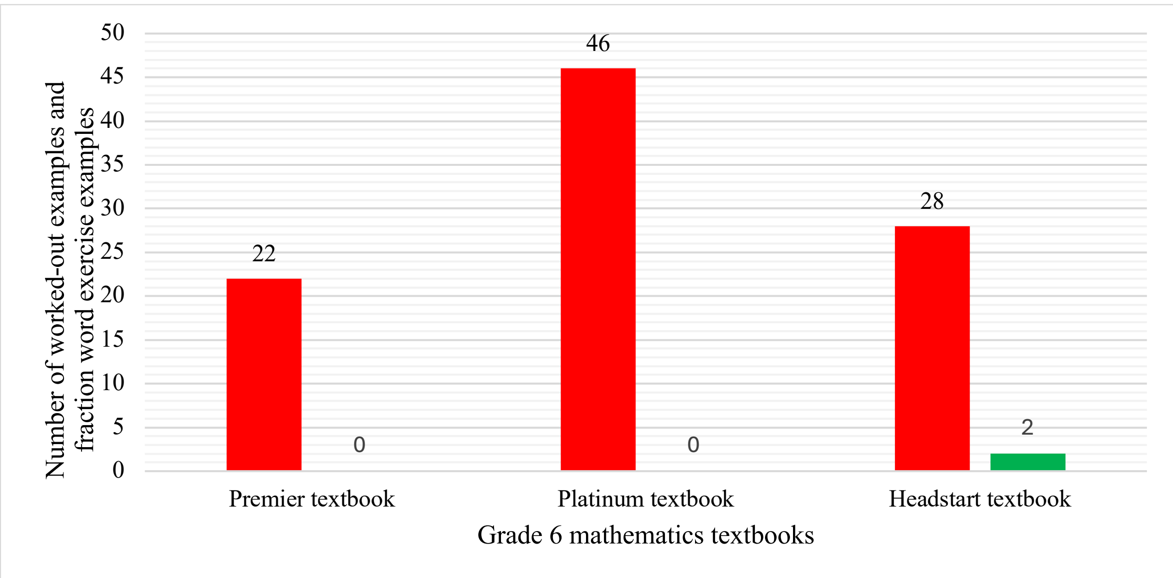

Figure 2 provides a clearer overview of the number of worked-out examples and word problem exercise examples in each textbook. From **Figure 2**, it is evident that the Platinum textbook has more fraction word problem exercise examples compared to the other two textbooks. There are only two word worked out examples appear in the Headstart textbook.

4.1.1 Textbook Contexts: Analysis of textbooks

In this part of the analysis, I analyse each textbook individually. The analysis for each textbook begins with an overview of the number of worked-out examples and word problem exercise examples in each textbook. The examples are then categorised under each strand of mathematical proficiency with percentages for comparative purposes in a summary table.

4.1.1a Content analysis and discussion for the Premier textbook

Premier Textbook: In the Premier textbook, the topic of the concept of fractions word problem solving appears in Term 2 and again in Term 4. The textbook does not have any worked-out examples for both Term 2 and Term 4. There are a total of 22 fraction word problem exercise examples that were analysed in the Premier textbook. Since there were no worked-out examples for word problems, the analysis for this textbook only focuses on word problem exercise examples; this observation will form part of answering the second research question that focuses on the presentation of the worked-out examples. **Table 1a** is a presentation of the categorisation of fraction word problem exercise examples from the Premier textbook. The fraction word problem exercise examples under each category were individually counted after summary and their total was represented in percentages as they will be compared amongst the three textbooks.

Table 1a: *summary of analysis of worked-out examples from the Premier Textbook*

Mathematical proficiency strands	Codes	Word problem exercises examples (Premier textbook)
Procedural fluency	**PF^{+}**	6 (13%)
	PF^{++}	7 (15%)
	PF^{+++}	12 (26%)
	PF^{++++}	0
Strategic Competence	**SC**	0
Adaptive Reasoning	**AR**	0
Productive Disposition	**PD^{+}**	1 (2%)
	PD^{++}	19 (40%)
	PD^{+++}	2 (4%)

From summary **Table 1a**, the first degree of procedural fluency (**PF^{+}**) is accounted for by six fraction word problem exercises, that is 13% of the exercise examples in the textbook. This suggests that six exercise examples provide an in-depth practice of foundational concepts through routine exercises that promote rules, algorithms and the selection of appropriate procedures to solve fraction word problem exercises. Newton, Lange and Booth (2020) argue that the acquisition of fully developed flexibility is rare amongst primary school learners compared to some progressive high school learners.

A closer examination of the six fraction word problem exercise examples that promote **PF**$^{+}$ reveal the textbook's promotion to give access to develop flexibility and practice of accuracy through the use of the everyday verbal cues incorporated within them. 15% (seven exercise examples) of the fraction word problem exercise examples promote the second degree of procedural fluency through their use of technical mathematical verbal cues. Fraction word problems in this category put an emphasis on the shift from everyday language application to a more technical language.

The language transition to a mathematical language is essential in the learning of fractions or any other mathematical concept, as it aids in the familiarity of mathematical terminology and develops precise problem-solving techniques. This transition is not only critical for the development of mathematics terminology but Ilany and Margolin (2010) assert that the transition is also important because mathematical language does not come naturally learners and it requires to be learnt and developed. Therefore, the transition from **PF**$^{+}$ and **PF**$^{++}$ exposes learners to a language that is more discipline specific. Developing a mathematics discipline specific language should be reinforced at the beginning stages of primary school (Colwell, Hutchison & Woodward, 2021) and it should be integrated in instructions (Siffrinn & Lew, 2018).

Additionally, the analysis revealed that 26% of the fraction word problems in the Premier textbook promote procedural fluency to the third degree (**PF**$^{+++}$). All fraction word problem exercise examples in this category (**PF**$^{+++}$) are categorised by their integration of both everyday language verbal cues and mathematical cues. It is recommended by Patkin (2011) that learners be encouraged to move back and forth between everyday language and a mathematical language. The integration of both everyday language and a mathematical language generally promotes mathematical thinking, and increases learners' understanding in the mathematics discipline (Patkin, 2011). In this study, the set of fraction word problem exercise examples in the **PF**$^{+++}$ category promote the understanding of fraction concepts and recognise the incorporation of everyday language and technical language in fraction problem-solving.

The findings of this analysis furthermore revealed that there are no fraction word problem exercise examples that promote procedural fluency to the fourth degree (**PF**$^{++++}$). As previously discussed in Chapter 3, **PF**$^{++++}$ fraction word problem exercise examples are categorised as non-routine problems, however, upon repeated exposure in the textbook they get recategorized into routine problems. Regardless of the recategorisation of non-routine word problems to routine, these types of fraction word problem exercise examples are crucial for fraction proficiency. They help support learners become expert problem solvers who are exposed to a cognitively varying problems, which in turn prompts critical thinking and develops proficiency in higher-order problems.

The absence of $\mathbf{PF}^{++++}$ is a concern because there is an increase in demand for such problem solving skills in the 21st century (Nguyen, Guo, Stamper & McLaren, 2020) and without nonroutine problems the textbook, learners do not get the opportunity to refine their problem solving skills that prepare them for the real world.

For strategic competence and adaptive reasoning, the analysis revealed that there were no fraction word problem exercise examples that promoted these two strands and their absence has a negative impact on the promotion of fraction proficiency for the textbook user. A Romanian study (Marchis, 2012), demonstrated that learners consistently exposed only to routine problems achieved lower marks on nonroutine problems. The Premier textbook only has routine fraction word problem exercise examples that require known algorithms and strategies.

The analysis furthermore revealed that productive disposition was promoted at varying degrees. 2% (one exercise example) of the fraction word problem exercise examples promote productive disposition on the first degree; in this category ($\mathbf{PD}^{+}$), the present exercise example was categorised by the use of data that exist in the real world and is important for fraction proficiency because they help learners understand fraction concepts from an everyday experience perspective instead of fractions being regarded as abstract concepts. A realistic mathematics approach makes learners aware of the uses of mathematics and how it connects to the real world as they know and interact with it (Haji, Yumiati & Zamzaali, 2019. The Premier textbook offers one word problem exercise example that makes use of authentic data and is relatable to learners, making problem solving to be viewed as worthwhile. 40% of the fraction word problem exercise examples (19 exercise examples) promoted productive disposition on the second degree, this is the most prominent degree of productive disposition. The second degree of productive disposition entails the inclusion of data and/ or experiences that have taken place in the real world or has the potential to take place in the real world.

Parallel to the one word problem exercise example from the first degree of productive disposition, the 19 fraction word problem exercise examples in the second degree also provide learners with the opportunity to regard fraction word problems as relatable. However, there is a notable distinction between the first and second degree, there is a gradual expectation from learners to contextualise fraction word problems within a mathematical setting in order to develop mathematical modelling skills. According to (Keijzer, 2003), when the concept of fractions arises in an everyday situation as a problem that needs to be solve, the use of a circle as a form of modelling can become helpful to act as a representation to solve the, "for example, if the sharing of pizzas constitutes a situation that generates fractions, a circle is the context-specific model as it provides for an image of the pizzas in the sharing process" (p.16), visual representations help simplify complex situations for learners. Competency in mathematical modelling enables learners to use models as applications to help them solve real world problems. Lastly, there were two fraction word problem exercise examples (4%) that promoted productive disposition on the third degree. These fraction word problem exercise examples were categorised by the use of exaggerated everyday scenarios for the learning of mathematical concepts.

Ultimately, the analysis of the Premier textbook indicate a skewed distribution of the strands of mathematical proficiency, with 54% of the fraction word problem exercise examples promoting procedural fluency and it being a prominent strand promoted in the Premier textbook. 46% of the fraction word problem exercise examples promoted productive disposition and there were no exercise examples that promoted strategic competence and adaptive reasoning. The implications of the absence of strategic competence and adaptive reasoning will formulate a discussion in Chapter 5.

4. 1.1b Content analysis and discussion for the Platinum textbook

Platinum Textbook: In the Platinum textbook, the topic appears in Term 1 and Term 4. The concept of fractions is under Topic 4 in Term 1, there is only one example that will be regarded as worked-out examples for the purpose of this analysis because of how they model a process or procedure for learners to actually follow. Overall, there are 46 fraction word problem exercise examples in the Platinum textbook. **Table 2a** provides an illustration of the summary of the analysis.

Table 2a: *summary of analysis of worked-out examples worked-out examples from the Platinum Textbook.*

Mathematical proficiency strands	Codes	Word problem exercises examples (Platinum textbook)
Procedural fluency	**PF**$^{+}$	8 (11%)
	PF^{++}	29 (32%)
	PF^{+++}	3 (3%)
	PF^{++++}	1 (1%)
Strategic Competence	**SC**	0
Adaptive Reasoning	**AR**	4 (5%)
Productive Disposition	**PD**$^{+}$	2 (5%)
	PD^{++}	36 (41%)
	PD^{+++}	2 (2%)

From the summary **Table 2a** of the Platinum textbook, 53% of the word exercise examples promote procedural fluency, making it a prominent strand of proficiency in this textbook. Procedural Fluency has been subdivided into four degrees. 13% of the fraction word problem exercise examples promote procedural fluency to the first degree (**PF**$^{+}$**),** the word problem exercise examples in this level are identified by everyday verbal cues and require learners to rely on their memorised procedures and how they are linked to the word fraction exercise example. There are 15% and 26% exercise examples that promote procedural fluency at the second degree (**PF**$^{++}$**)** and third degree (**PF**$^{+++}$) in the Platinum textbook. This indicates a gradual increase in the cognitive demands of the exercise examples and the development of knowledge. According to Hiebert and Lefevre (1986), procedural knowledge is defined as "knowledge that consists of rules, algorithms, or procedures used to solve mathematical tasks" (p. 6). The exposure to the procedural fluency word problem exercise examples with both everyday English and mathematical verbal cues cultivates flexibility, accuracy and efficiency in problem solving

Strategic competence and adaptive reasoning are significant strands of mathematical proficiency and there are no fraction word problem exercise examples of these two strands. Strategic competence consists of mathematizing problems through representation and establishing appropriate strategies to the problem (Kilpatrick et al., 2001). In the context of fraction word problems, Copur-Gencturk and Doleck (2021) suggests that multistep word problems that are non-routine foster strategic competence. The Platinum textbook neglects multistep, non-routine fraction word problem exercise. The absence of strategic competence fraction word problem exercise examples limits learners' exploration of a range of fractional representation that cannot be developed through routine exercises.

Productive disposition was also subdivided into three degrees. There was one fraction exercise example that promoted first degree productive disposition (**PD^{+}**), that is 2%. This is the only worked out example in the entire textbook that contains real-world data and has the potential to be both relevant and relatable to learners' experience with the chance to see the applicability of fractions in the real world. 40% of the fraction exercise examples in the Premier textbook promoted second degree productive disposition (**PD^{++}**), that is 19 of the exercise examples. Given that productive disposition is defined as a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, p. 116), 19 of the fraction word problem exercise examples exposes learners to scenarios that may be deemed as worthwhile and useful because of their realistic approach. Lastly, two fraction word problem exercise examples that promoted productive dispositions to the third degree (**PD^{+++}**), which involves exaggerated data that is not realistic but is intended for mathematical exploration. The two exercise examples have the potential to develop learners' fraction problem solving skills and encourage them to think more critically.

Overall, the findings from the Platinum textbook suggest that the textbooks' primary focus is mainly on the development of procedural fluency and productive disposition.

4.1.1c Content analysis and discussion for the Headstart textbook

Headstart Mathematics: The common fraction exercise word problems also appear in Term 1 and Term 4. There are an overall of 28 word problem exercises and two worked-out examples in the Headstart textbook.

Table 3a: *summary of analysis of worked-out examples from the Headstart Textbook*

Mathematical proficiency strands	Codes	Word problem exercises examples (Headstart textbook)	Worked-out examples (Headstart textbook)
Procedural fluency	**PF^{+}**	7 (13%)	
	PF^{++}	16 (29%)	1 (2%)

	PF^{+++}	4 (7%)	
	PF^{++++}	0	
Strategic Competence	**SC**	0	
Adaptive Reasoning	**AR**	1 (2%)	
Productive Disposition	**PD**$^{+}$	0	
	PD^{++}	27 (49%)	1 (2%)
	PD^{+++}	0	

The Headstart textbook had one fraction worked-out example that promoted procedural fluency in the second degree (**PF**$^{++}$) and productive disposition (**PD**$^{++}$) in the second degree. It is worth pointing out that majority of the fraction word problem exercises are also saturated in these two strands and at the exact same degree. Could this be a deliberate choice to promote certain strands of proficiency over the others? Meanwhile, there is no evident research on what constitutes a sufficient spread of the strands of mathematical proficiency but the complete absence of certain strands limits textbook users from exploring a range of exercise examples that will develop their proficiency.

There were seven fraction word problem exercise examples (13%) that promoted procedural fluency on the first degree (**PF**$^{+}$**).** To reiterate on the categorisation of the exercise examples in this category, all the fraction word problem exercise examples were categorised for their incorporation of everyday verbal cues to offer learners suggest pathways to the appropriate procedures to employ. Similarly to the other previous textbooks, **PF**$^{+}$ promotes the recognition of multiple procedures, identification of efficient procedures and the use of procedures effectively (Newton et al., 2020). Everyday verbal cues in this category (**PF**$^{+}$**)** help support learners' understanding of fraction concepts and help learners see the relevance of fraction in their everyday lives. In the second degree of procedural fluency, the fraction word problem exercise examples include both every day and mathematical verbal cues and there were 16 (29%) exercise examples in this category. There were 4 (7%) fraction word problem exercise examples in the Platinum textbook in the third degree of procedural fluency (**PF**$^{+++}$**)** and the exercise examples in this category have suggestive pathways that are unclear and they cannot be classified as everyday English or mathematical verbal cues but the exercise is a routine exercise.

Majority of the word problem exercise examples are concentrated at the first, second and third degree. There are no questions that are promoting procedural fluency at the fourth degree (**PF**$^{++++}$). The absence of the fourth degree of procedural fluency limits the potential for mathematical thinking, development of metacognition and high-level thinking skills that are acquired through **PF**$^{++++}$. **PF**$^{++++}$ consists of word problem exercise examples that were first identified as non-routine but were analysed and categorised as routine because of their second appearance. Despite this reclassification from non-routine to routine, these word problem exercise examples still possess an increased cognitive demand and their absence has the potential to result in overreliance on algorithms presented on the the first, second and third degree set of exercise examples.

There were no fraction word problem exercise examples that promoted procedural fluency to the third degree, strategic competence and productive disposition on the first degree. These findings are consistent with the analysis for the other two textbooks. However, a singular exercise example was analysed to promote adaptive reasoning, this is a distinctive aspect of the textbook in comparison to the other two textbooks. There was a limit of the promotion of productive disposition in the first and third degree, the exercise examples were saturated in the second degree of productive disposition.

A salient point to acknowledge is that even though there are notable gaps in the promotion of the strands of mathematical proficiency in the textbook, this analysis was not intended to evaluate the textbooks against any standardised framework. However, this descriptive approach of the analysis helps provide a nuanced and detailed analysis without the imposition of external expectations.

4.2 Textbook Analysis: A Comparative Overview of the Three Mathematics Textbook

For each textbook, a table was used to conduct a comprehensive analysis of the individual questions and the mathematical strands they promote. This process involves categorising each question according to how it aligns with the specific mathematical proficiency categories **Tables 1a, 2a** and **3a** has provided an illustration of the summary of the analysis. The tables that follow will provide an insight from the above summary. The insight highlights the trends and patterns, strengths and weaknesses of each textbooks' alignment, Kilpatrick et al. (2001) strands of mathematical proficiency. For each textbook, the data is represented in an elaborate table that shows the frequency of each strand, those are totalled. It is important to remember that one exercise can promote more than one strand, therefore the number of exercises analysed will not be the same number of how frequent the strands appear. From the table, a bar graph is employed for the study of trends we get from the strands of mathematical proficiency. From the graphs, there will be a discussion around how the three textbooks differ in their approach on the emphasis of mathematical proficiency.

A detailed analysis for each textbook

1. This section aims to provide a detailed overview of the results as tabulated in **Table 1a,** this is the analysis for the first textbook, Premier textbook. There were a total of 26 questions that were analysed. The same question has the potential to promote more than one mathematical strand because of their interwoven nature. Twenty-five questions promoted procedural fluency at three varying degrees. Six word problems promoted procedural fluency to the first degree (PF^{+}), these word problems contained everyday verbal cues that offer a suggestive pathway to solve the problem. The next degree of procedural fluency (PF^{++}) contained strictly mathematical verbal cues, this still promoted suggestive pathways to be followed and there were seven questions that promoted PF^{++}. There were 12 questions in the third degree of procedural fluency (PF^{+++}), in this degree, the exercise questions contained both every day and mathematical verbal cues. There were no exercise problems that promoted SC and AR. Twenty-two word problem exercises promoted Productive disposition to varying degrees. One exercise question promoted PD^{+} , 19 exercise questions promoted PD^{++}, and two exercise questions promoted PD^{+++}. The question under PD^{+} promotes a foundation for mathematical proficiency by making the mathematical connection to the everyday life context. The next level PD^{++} that has 19 questions enhances the same idea in PD^{+} by establishing a link that encourages critical mathematical applications in the real world. PD^{+++} exercise questions extend beyond realistic contexts by focusing on actual structures of mathematics and relations.

Textbook: Premier Textbook

Table 1a*:* *Analysis of the strand of mathematical proficiency in the Premier textbook*

Unit 4	**Procedural Fluency**				**Strategic Competence**	**Adaptive Reasoning**	**Productive Disposition**		
	PF +	**PF^{+} +**	**PF^{++} +**	**PF^{++} ++**	**SC**	**AR**	**PD^{+}**	**PD^{++}**	**PD^{++} +**
Exercise 2 (pg. 17)									
1		✓						✓	
2		✓						✓	
3		✓						✓	

4		✓						✓	
Exercise 9 (pg. 9)									
1		✓						✓	
2			✓					✓	
3 a)			✓						
b)			✓						
c)			✓						
d)	✓							✓	
4			✓						✓
5	✓							✓	
6			✓					✓	
7	✓						✓		
8			✓					✓	
9			✓					✓	
10			✓					✓	
11	✓							✓	
12	✓							✓	
Exercise 6 (pg. 172)									
1 a)		✓						✓	
b)		✓						✓	
2			✓					✓	
3	✓							✓	
4			✓					✓	
5			✓						✓
6		✓						✓	
TOTAL	**6**	**7**	**12**	**0**	**0**	**0**	**1**	**19**	**2**

2. **Table 1b** is the analysis of the second textbook, Platinum. In the Platinum textbook, a total of 48 fraction word problem exercise examples were analysed. Eight exercise examples were analysed and were recorded to be promoting PF^{+}, 28 fraction word problem exercise examples promote PF^{++} and three fraction word problem exercise examples promote PF^{++++}. Similar to the first textbook analysis, the analysis on the promotion of procedural fluency indicates a gradual increase in the increment on the cognitive demands and mathematical refinement in the word problem exercises. There are no fraction word problem exercise examples that initially promoted PF^{++++}, this implies there were no fraction word problem exercise examples that were initially documented to be nonroutine and their second appearance qualified them to be routine questions. There are no non routine questions in this textbook, therefore SC is not a promoted mathematical proficiency strands. Four fraction word problem exercise examples promoted AR. Forty word problem exercises promoted PD in the same degrees explained in **Table 1.** Majority of the exercise questions that promoted PD, promoted PD in the second degree (PD^{++}). This level promotes the use and application of mathematics in real life contexts.

Textbook: Platinum

Table 1b: *Analysis of the strand of mathematical proficiency in the Platinum textbook.*

Topic 4	**Procedural Fluency**				**Strategic Competence**	**Adaptive Reasoning**	**Productive Disposition**		
	PF +	**PF**$^{+}$ +	**PF**$^{++}$ +	**PF**$^{++}$ ++			**PD**$^{+}$	**PD**$^{+}$ +	**PD**$^{++}$ +
Exercise 4.2 (pg. 19)									
4 a)		✓				✓		✓	
c)		✓				✓		✓	
e)		✓				✓		✓	
Exercise 4.3 (pg. 20)									
1 a)		✓						✓	

b)		✓					✓		
c)		✓					✓		
3		✓						✓	
Exercise 4.5 (pg.21)									
1 **a)**		✓						✓	
b)		✓						✓	
c)		✓						✓	
d)		✓						✓	
2 **a)**			✓					✓	
b)		✓						✓	
c)		✓				✓		✓	
3 **a)**		✓						✓	
b)		✓						✓	
c)		✓						✓	
d)		✓						✓	
4 **a)**		✓						✓	
b)		✓						✓	
c)		✓						✓	
d)		✓						✓	
5		✓						✓	
Exercise 4.11 (pg. 26)									
1 **a)**		✓						✓	
b)		✓						✓	
c)	✓							✓	
d)	✓							✓	
2		✓						✓	

3	✓							✓	
4 **(a)**		✓							✓
(b)		✓						✓	
(c)	✓								✓
5 **(a)**	✓							✓	
(b)	✓							✓	
6 **(a)**	✓							✓	
(b)	✓							✓	
7 **a)**			✓					✓	
b)		✓						✓	
c)			✓					✓	
d)		✓						✓	
TOTAL	**8**	**28**	**3**	**0**	**0**	**4**	**2**	**36**	**2**

3. **Table 1c** is the analysis of the third textbook, Headstart. In the Headstart textbook, a total of 28 fraction word problem exercise examples were analysed. Seven fraction word problem exercise examples were analysed and were recorded to be promoting PF^{+}, 16 word problem exercises promote PF^{++} and four questions promote PF^{++++}. Similar to the Platinum textbook. there are more questions that promote that PF^{++}. There are no fraction word problem exercise examples that initially promoted PF^{++++}, this implies there were no questions that were initially documented to be nonroutine and their second appearance qualified them to be routine questions. There are no non routine questions in this textbook, therefore SC is not a promoted mathematical proficiency strands. Four word problem exercises promoted AR. Forty word problem exercises promoted PD in the same degrees explained in **Table 1a**. Majority of the exercise questions that promoted PD, promoted PD in the second degree (PD^{++}). This level promotes the use and application of mathematics in real life contexts. This is the only textbook that has a worked-out example and the example promotes procedural fluency to the first degree and procedural fluency to the second degree as well. This is interesting because the majority of the exercise questions in the textbook also promote the strands the worked-out example has emphasised.

Textbook: Headstart Mathematics

Table 1c: *Analysis of the strand of mathematical proficiency in the Headstart textbook*

Activity 11, Page 46	**Procedural Fluency**				**Strategic Competence**	**Adaptive Reasoning**	**Productive Disposition**		
	PF +	**PF**$^{+}$ +	**PF**$^{++}$ +	**PF**$^{++}$ ++			**PD**$^{+}$	**PD**$^{++}$	**PD**$^{++}$ +
1 (a)			✓					✓	
(b)		✓						✓	
(c)		✓						✓	
2 (a)	✓					✓		✓	
(b)		✓						✓	
3 (a)			✓					✓	
(b)		✓						✓	
Activity 12, page 47									
	PF +	**PF**$^{+}$ +	**PF**$^{++}$ +	**PF**$^{++}$ ++			**PD**$^{+}$	**PD**$^{++}$	**PD**$^{++}$ +
1 (a)		✓						✓	
(b)		✓						✓	
(c)		✓						✓	
2	✓							✓	
3 (a)	✓							✓	
(b)			✓					✓	

Activity 2. Page 295									
	PF +	**PF^{+} +**	**PF^{++} +**	**PF^{++} ++**			**PD^{+}**	**PD^{++}**	**PD^{++} +**
3 (a)		✓						✓	
(b)		✓						✓	
(c)	✓							✓	
4 (a)		✓						✓	
(b)	✓							✓	
(c)			✓					✓	
5 (a)		✓						✓	
(b)		✓						✓	
(c)		✓						✓	
6 (a) NAFQ									
(b)		✓						✓	
(c)	✓							✓	
(d)		✓						✓	
7 (a)	✓							✓	
(b)		✓						✓	
TOTAL	**7**	**16**	**4**	**0**	**0**	**1**	**0**	**27**	**0**
Worked-out example , page 46									
		✓						✓	

A comparative analysis of the three textbook

The analysis for each textbook begins with an overview of the number of worked-out examples and word problem exercise examples in each textbook. The examples are then categorised under each strand of mathematical proficiency as adapted from Kilpatrick et al. (2001) with percentages for comparative purposes in **Table 2**.

Table 2: *Comparative analysis of mathematical proficiency in each textbook*

		Textbooks					
Mathematical proficiency strands	**Codes**	**Premier Mathematics Gr 6**		**Platinum Mathematics Gr 6**		**Headstart Mathematics Gr 6**	
		Word problem exercises examples	Worked-out examples	Word problem exercises examples	Worked-out examples	Word problem exercises examples	Worked-out examples
Procedural fluency	PF^{+}	6 (13%)	0	8 (11%)	0	7 (13%)	0
	PF^{++}	7 (15%)	0	29 (32%)	0	16 (29%)	1 (2%)
	PF^{+++}	12 (26%)	0	3 (3%)	0	4 (7%)	0
	PF^{++++}	0	0	1 (1%)	0	0	0
Strategic Competence	SC	0	0	0	0	0	0
Adaptive Reasoning	AR	0	0	4 (5%)	0	1 (2%)	0
Productive Disposition	PD^{+}	1 (2%)	0	2 (5%)	0	0	0
	PD^{++}	19 (40%)	0	36 (41%)	0	27 (49%)	1 (2%)
	PD^{+++}	2 (4%)	0	2 (2%)	0	0	0

Table 2 shows that all three textbooks prioritise the promotion of mainly procedural fluency and productive disposition. Both these strands are important for the development of mathematical proficiency, Foster (2013) argues that learner confidence in solving routine problems prepares them for advanced and complex problems. An identifiable issue from this finding, is the absence of the promotion of complex problems. Textbook users are not offered opportunities to apply their basic procedural knowledge to more complex and nonroutine questions because they are not there in all three textbook. According to Foster (2017), problem solving that promotes strategic competence needs to be prioritised in mathematics education because we are in a technological era where there are computers and calculators that can easily solve routine questions. Learners struggle with fractions and have a limited fraction proficiency in primary school (Roesslein, Codding, 2018) and the distribution of the these strands have the potential to reproduce learners that struggle with the concept of fraction word problem solving simply because they do not offer opportunities to explore challenging problem solving. In a study (Turner, Meyer, Midgley & Patrick, 2003) that analysed the influence of teacher discourse and student behaviour in mathematics classrooms revealed that Grade 6 learners who were challenged by their teachers in classrooms showed more adaptive strategies compared to those who voided the challenge completely. In the context of this research, learners equally struggle with challenging problems during teaching and learning from observation because the

textbooks they get to independently use do not challenge them to a degree of proficiency and comfortability solving nonroutine problems. The overall distribution of the strands of mathematical proficiency in each textbook is similar.

Procedural Fluency

Looking at **Table 2**, procedural fluency in Premier mathematics textbook is promoted at 54%, 47% in the Platinum and for Headstart, it is 49%. What insight do these findings from the analysis reveal? These findings suggest that learners using these textbooks are provided with ample opportunities to explore, advance and develop their procedural fluency through engaging with routine fraction word problem exercises. The recurring appearance and engagement of procedural fluency questions in textbooks affords learners the opportunity to practice and refine their thinking, and by so doing there is an opportunity to focused on areas of weaknesses that will result in their (learners') development (Foster, 2013). Conceptual understanding is a fundamental strand for mathematical proficiency and procedural fluency builds foundation that later enables slightly advanced problem solving (Kilpatrick et al., 2001; Mellor et al., 2018). In what follows, we disaggregate the promotion of these findings on procedural fluency into their levels as discussed in **Table 1**.

PF^{+}: To reiterate on the categorisation of examples in this category, all the fraction word problem examples were categorised for their incorporation of everyday cues that offer learners suggestive pathways to the appropriate procedures to employ. In Premier textbook, the first degree of procedural fluency (**PF^{+}**) is accounted for by only six fraction word problem examples (13%). This suggests that only six exercise examples provide an in-depth practice of foundational concepts through routine exercises that promote rules, algorithms and the selection of appropriate procedures to solve fraction word problem exercises. In Platinum, 11% of the fraction word problem exercise examples promote procedural fluency to the first degree (**PF^{+}**). Lastly, Headstart has seven fraction word problem exercise examples (13%) that promoted procedural fluency on the first degree (**PF^{+}**). Newton, Lange and Booth (2020) argue that the acquisition of fully developed flexibility is rare amongst primary school learners compared to some progressive high school learners. All three textbooks provide learners with comparable opportunities to encourage and develop flexibility and accuracy through everyday language within the exercise problems.

PF^{++} : Table 2 also reveals that 15% (seven exercise examples) of the fraction word problem exercise examples in Premier textbook promote procedural fluency to the 2nd degree with the addition of technical mathematical verbal cues. Fraction word problems in this category put an emphasis on the shift from everyday language application to a more technical language. Platinum textbook offers more affordances to learners than the Premier textbook, with 32% exercise examples that promote procedural fluency at the 2nd degree (**PF^{++}**). Learners using the Platinum textbook are exposed to an important aspect of language transitions. The language transition from everyday English to a mathematical language is essential in the learning of fractions or any other mathematical concept, as it aids in the familiarity of mathematical terminology and develops precise problem-solving techniques. Headstart also provides a significant amount of **PF^{++}** coverage, with 16 (29%) exercise examples in this category. The opportunities offered in the transition from **PF^{+}** and **PF^{++}** exposes learners to a language that is more discipline specific. Therefore, Platinum and Headstart, affords learners more opportunities to learn and use mathematics terminology precisely. This transition is not only critical for the development of mathematics terminology but as Ilany and Margolin (2010) assert, transition is also important because mathematical language does not come naturally to learners. It needs to be learnt and developed.

PF^{+++}: 26% of the fraction word problems in Premier textbook promote procedural fluency to the 3rd degree (**PF^{+++}**) and 7% in the Headstart textbook. Fraction word problem exercise examples in the Platinum textbook are lesser compared to the other two textbooks, with only 3% of the fraction word problem exercise examples under this category. As a result, Premier affords learners more opportunities to understand technical language in fraction problem solving. The high exposure in Premier textbook of exercise examples in this category offers learners the opportunity to move back and forth between everyday language and mathematical language. The integration of both everyday language and a mathematical language generally promotes mathematical thinking and increases learners' understanding in the mathematics discipline (Patkin, 2011). Premier textbook offers a gradual increase in the cognitive demands of the exercise examples and the development of knowledge. Compared to Platinum and Headstart, Premier exposes learners to more opportunities to enhance their procedural fluency through word problem exercise examples with both everyday English and mathematical verbal cues.

PF^{++++}: All non-routine exercise examples that appear for the second time promote the development of procedural fluency to the 4th degree. There were no fraction word problem exercise examples that promoted procedural fluency in the 4th degree in Premier and Headstart textbooks. Compared to Premier and Headstart, Platinum has only one fraction exercise example that promotes procedural fluency to the 4th degree. One exercise example is not adequate exposure to offer learners the skill sets and cognitive tools to solve the problems more flexibly and more accurately. Furthermore, the general absence of PF^{++++} in both fraction word problem work-out examples and exercise examples indicates failure to promote strategic competence, to which we now turn.

Strategic competence

Table 2 shows no evidence of Strategic Competence across all three textbooks. Strategic competence consists of mathematising problems through representation and establishing appropriate strategies to the problem (Kilpatrick et al., 2001). In the context of fraction word problems, Copur-Gencturk and Doleck (2021) suggests that multistep word problems that are non-routine foster strategic competence. The absence of strategic competence fraction word problem exercise examples limits learners' exploration of a range of fractional representation that cannot be developed through routine exercises. These kinds of fraction word problem exercise examples are crucial for fraction proficiency because they help expose learners to cognitively varying problems, prompting critical thinking and developing proficiency on higher order problems.

The absence of examples that promote strategic competence is a concern because there is an increase in demand for such problem-solving skills in the 21st century (Nguyen, Guo, Stamper & McLaren, 2020) and without nonroutine problems in all three textbooks, learners do not get the opportunity to refine their problem-solving skills that prepare them for the real world. The limited exposure and development of strategic competence is a concern as textbooks are used for instructional purposes, and for a subject like mathematics that is hierarchical in nature, learners need to such exposure (Doorman, Drijvers, Dekker, Heuvel-Panhuizen, Lange & Wijers, 2007).

Adaptive reasoning

For Adaptive Reasoning, **Table 2** reveals that there are no fraction word problem exercise examples in Premier textbook in this category. Platinum textbook offers some affordance to learners in this strand, with 5% of the fraction word problem exercise examples promoting adaptive reasoning. According to the National Research Council (2000), learners can be taught strategies that include the ability to explain their thinking and develop their metacognition. Headstart has 2% of fraction word problem exercise examples for this strand. The exposure to adaptive reasoning problems is limited in all three textbooks. The three textbooks do not take the metacognitive approach due to the scarcity of fraction word problems that develop adaptive reasoning.

Productive disposition

Table 2 shows that in Premier textbook, productive disposition is promoted at 46%, 48% in Platinum and 49% in Headstart.

PD^{+}: The analysis revealed that 5% of the fraction word problem exercise examples promote productive disposition in the first degree in Platinum textbook. There is only one fraction exercise example that promotes first degree productive disposition (**PD^{+}**) in Premier textbook. There are no word problem exercise examples in the first degree of productive disposition in Headstart textbook in this category (**PD^{+}**). Both Premier and Platinum offer learners minimal opportunities towards a more realistic mathematics approach which according to Haji, Yumiati and Zamzaali, (2019) makes learners aware of the uses of mathematics and how it connects to the real world as they know and interact with it. Premier textbook offers one word problem exercise example that makes use of authentic data and is relatable to learners, making problem solving to be viewed as worthwhile. However, an exposure to one word problem is not sufficient to help learners understand fraction concepts from an everyday experience perspective.

PD^{++}: In Premier Mathematics, 40% of the fraction word problem exercise examples (19 exercise examples) promote productive disposition to the second degree. Compared to Platinum, 41% of the fraction exercise examples (36 in total) in the Premier textbook promote second degree productive disposition. Headstart textbook offers learners with more opportunities to develop productive disposition at the second degree compared to Premier and Platinum, with 49% of its fraction word problem exercise examples promoting PD^{++}. Given that productive disposition is defined as a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al. p. 116), Headstart exposes learners to more scenarios that may be deemed as worthwhile and useful because of their realistic approach. The high exposure of **PD^{++}** in Headstart affords learners the opportunity to develop competency in mathematical modelling and enables learners to use models as applications to help them solve real world problems.

PD^{+++}: The fraction word problem exercise examples under this category were categorised by the use of exaggerated everyday scenarios for the purpose of learning mathematical concepts.

Table 2 shows that in the Headstart textbook, there were no fraction word problem exercise examples under this category. The absence of this aspect of productive disposition in Headstart limits the potential for mathematical thinking, development of metacognition and high-level thinking skills that are acquired through **PD**$^{++++}$.

4.3 Conclusion

In this chapter, I have presented the three Grade 6 mathematics textbooks using Kilpatrick et al. (2001) strands of mathematical proficiency. The analysis showed a constant trend across the textbooks, the over emphasis of the promotion of procedural fluency and productive disposition. The distribution of the strands is similar across the textbooks. In the next chapter, I provide a summary of the findings and discuss the implications of the findings and the study's limitations .

CHAPTER 5: FINDINGS AND CONCLUSIONS

5.0 Introduction

In engaging in this comparative textbook study, the aim of this investigation was to undertake a comprehensive analysis of the presentation and the affordances of common fractions' worked out examples and word problem exercises presented in Grade 6 Premier, Platinum and Headstart textbooks.

5.1 Focus of the study

The investigation was grounded on the notion of Kilpatrick et al. (2001) strands of mathematical proficiency. The primary focus of this analytical research was to conduct a comparative study of the three Curriculum and Assessment Policy Statements (CAPS) approved textbooks, focusing on their fraction word problems, the worked examples and the exercise examples. Ultimately, the objective of this textbook analysis is to provide me, a mathematics teacher with an in-depth understanding of how mathematical proficiency is promoted from the presentation of the fraction contents of the textbooks that have been sampled for this study. My teaching experience of teaching Grade 6 mathematics has shown that learners struggle with the comprehension and application of the concept fractions in word problems especially when they are encouraged to engage with the fraction word exercise examples in the textbooks. This observation has motivated this research inquiry. Textbooks are widely deemed to be the most prominent and influential learning resources in education settings. As a result, the roles textbooks play can either facilitate learning or hinder it. To gain insight and understanding from examining the presentation of the fraction word problems requires an in-depth study of the textbook contents. This analytical investigation undertook a nuanced exploration of the presentation of fraction word problems with the goal to provide answers to the three research questions that follow:

- How is mathematical proficiency using Kilpatrick et al. (2001) five strands promoted in three selected Grade 6 textbooks: Headstart, Premier Mathematics and Platinum textbooks?

- How are fractions worked-out examples and fraction word problem exercise examples presented in the selected Grade 6 textbooks?

- What affordances for learning about fractions are offered in these three Grade 6 textbooks?

To address the above research questions, a comprehensive analysis was conducted on all the worked out examples and fraction word problem exercise examples in each textbook. This analysis was visually simplified and facilitated by the use of tables that resulted to a more systematic comparison of the three textbooks' fraction contents. The distribution and frequency of Kilpatrick et al. (2021) strands of mathematical proficiency were quantified and expressed in percentages for a quantitative presentation of the research findings.

In this chapter, I will provide and outline the findings that respond to the research questions of this study, reflect on the research methodology, discuss the implications of the study, present recommendations and identify potential avenues for future and further research.

5.2 Summary of results

After a close inspection of the findings in this textbook comparative analysis in Chapter 4, the Premier textbook had a percentage of 54% fraction word problem exercise examples that promoted procedural fluency, 26% of the 54% of the exercise word problems promote procedural fluency at the third degree. This indicates that about 12 questions are routine and they incorporate suggestive pathways to solving the questions that will not be classified as everyday nor mathematical verbal cues. Compared to the Platinum textbook, 48% of the fraction word problem exercise examples promoted procedural fluency, with 28 (33%) fraction word problem exercise examples promoting procedural fluency to the third degree, more routine exercise examples in the Platinum textbook contain mathematical and everyday verbal cues. Unlike Premier, the Platinum textbook does contain questions that promote procedural fluency to the fourth degree, these are questions that were initially documented as nonroutine during the analysis but upon second appearance they were documented as promoting procedural fluency. Similar to Premier, Headstart textbook, does not have questions that promote fourth degree procedural questions. However, majority of the procedural fraction word problem exercise examples in Headstart promote fluency at the second degree. There are 49% exercise questions that promote procedural fluency, and 29% (16 exercise questions) of these exercise promote fluency at second degree.

What insight do these findings from the analysis reveal? From these findings, learners are provided with ample opportunities to explore, advance and develop their procedural fluency through an engagement of routine questions. The recurring appearance and engagement of procedural fluency questions in textbooks affords learners the opportunity to practice refining their thinking and the application of concepts, practice focused on areas of weaknesses will result in their development (Foster, 2013). With conceptual understanding being the fundamental strand of mathematical proficiency, procedural fluency builds foundation for slightly advanced problem solving (Kilpatrick et al. 2001, Mellor et al., 2018).

All three textbooks have shown no record of nonroutine questions that promote strategic competence. Premier had no exercises that promote adaptive reasoning, there were 4 in the Platinum textbook and one in Headstart. From this observation, it is clear that procedural fluency and productive disposition are the most prominent strands promoted across the three analysed textbook. Adaptive reasoning and strategic competence were completely neglected altogether in the textbooks. The textbooks presented a limited presentation of fraction exercise examples that promoted productive disposition. Majority of the questions were centred around the second degree of productive disposition. The insufficient opportunities in all three textbooks limit learners' development of the other strands to develop.

5.3 Limitations of the study

This comparative textbook study only focused on only three Grade 6 textbooks and the focus was limited to worked out examples and fraction word problem exercise examples. The textbooks that were sampled for this analytical study may not be a reflection of the variety of textbooks that are CAPS approved and are widely used in South African classrooms. It is important to note that different textbooks including the ones that were sampled for this analysis have varying educational outcomes and may have varying and diverse levels of prioritising mathematical proficiency development of fractions.

For the scope of my research study, the sampled textbooks were carefully sampled with the objective to not generalise the findings of the research but to provide an in-depth analysis with valuable insights into the research questions of this study. The specific focus and the scope of my master's research guidelines on fraction worked out examples and word problem exercise examples may not capture the three textbooks' overall presentation and promotion of the strands of mathematical proficiency for other concepts. It is worth emphasising that the modified framework by Kilpatrick et al. (2021) was revised with the intention to be applicable to a wide range of mathematical concepts word problems and not just fractions.

5.4 Implications for my findings

This part of my discussion will focus on the implications of my findings. The implications discussed will be for textbook development in line with fraction proficiency and the teaching and learning of fraction word problems through textbooks.

Textbooks are both dominant and influential concerning classroom practice (Mellor et al., 2018). From the analysing, the three textbooks have a similar distribution of the strands of mathematical proficiency. Firstly, given that attaining procedural fluency is essential in mathematics learning and fraction proficiency, there are however implications with the abundance of routine fraction word problem exercise examples and the prioritisation of the development of procedural fluency. However, the overuse of routine questions and lack non-routine exercises will result in pseudo-confidence, where learners think they have fully understood a concept and its application while this is not true simply because of their minimum to no exposure of problem solving and problem formulation that comes from non-routine questions. Routine exercises produce flexibility to help solve nonroutine problems (Kilpatrick et al., 2001), the issue becomes the lack of exposure to actively make use and apply the knowledge obtained through routine problems. The promotion of procedural fluency that supersedes that of strategic competence, disadvantages learners' skill development of not simply following cues and not to simply look similarities in solving problems but to "focus more on the structural relationships within problems, relationships that provide the clues for how problems might be solved" (Kilpatrick et al., 2001). It is important that there is a meaningful distribution of all the strands for the development of mathematical proficiency.

The three textbooks are aligned with the CAPS document, as teaching and learning materials they are intended to prepare learners for assessments. According to specific aim (2.4) number 5 and 6, learners are expected to know how to problem solve and build an awareness on the important role of mathematics in their everyday life situations. According to Section 4 of the CAPS (DBE, 2012, p. 296) document, mathematics assessments should adhere to the four cognitive levels. These levels are: Knowledge (25%), routine procedures (45%), complex procedures (20%) and problem solving (10%). Each one of the five strands of mathematical proficiency addresses a cognitive level that needs to be part of assessment. From the textbook analysis, the five strands are not distributed in a manner that aligns with the Curriculum and

Assessment Policy, even though the books are said to be CAPS compliant. From the analysis, it has been found that the three commonly used textbooks are not compliant with the policy. All three textbooks do not have exercises that develop learner's cognitive levels to engage and solve "higher order questions". As South Africa edges toward a new curriculum that will necessitate the publication of new (mathematics) textbooks, the findings of this study are vital in informing content creation of examples that go into these textbooks.

5.5 Suggestions for further study

Future and further research directions emerging from this study are multidimensional. The first suggestive approach for future research involves broadening the scope of the research. To augment generalisability of the study, the sample size will be increased and diversified to contribute towards capturing a broader range of perspectives. Additionally, to attain more comprehensive and insightful findings from the study, I suggest incorporating learners' and teachers' outlook through interviews and observing their engagement with the textbooks and studying how the presentation of fraction proficiency plays out during teaching and learning. While on the matter of expansion, further investigations could involve investigating a closely related concept to fractions such as decimals and their word problems. The incorporation of decimals in an investigation should provide insight on how fraction proficiency aids with learning decimals being a natural extension of fractions.

5.6 Concluding Remarks

This study has investigated fraction worked-out examples and fraction word problem exercise examples in the Grade 6 textbooks. The findings, underpinned by Kilpatrick et al. (2001) strands of mathematical proficiency have revealed that the textbooks commonly used at the school I teach by both teachers and learners afford learners limited opportunities to develop their fraction proficiency because of the gaps and focus of textbooks. The development of procedural fluency and productive dispositions is essential for mathematical learning.

5.7 Reflection and Implications Regarding my Teaching Practice and Philosophy

My initial motivation to carry out this study was motivated by the struggles I observed from my learners during the segment of fractions, specifically word problems. As a teacher, I was concerned by their level of fraction proficiency simply because the textbooks that we use in class are the same textbooks I use as resources to inform my lessons. The comprehensive analysis of the three textbooks and the engagement with Kilpatrick et al. (2001) strands of mathematical proficiency has started to transform the way I approach the teaching and learning of fraction word problems. The approach is now more holistic and takes into account the shortfalls and gaps in the textbooks' fraction word problems. This study has supported my belief in the importance of providing learners with opportunities to develop their overall proficiency in fraction. I now provide learners with additional material that include nonroutine exercises, I ask them to reflect on their solution to be able to justify their thought processes. This approach has helped my learners and I to bridge the gap on proficiency that contributed to their struggles and I was not initially sure what was the contributing factor. This study has also enriched my knowledge on the subject matter.

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Appendix A: Ethics Clearance Certificate

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2022ECE014M**
PROJECT TITLE	*The promotion of mathematical proficiency of common fractions in worked-out examples and word-problem exercises*
INVESTIGATOR	Duduzile Esther Sibiya
SCHOOL/DEPARTMENT OF INVESTIGATOR	Wits School of Education
DATE CONSIDERED	22 April 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	No
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	**CHAIRPERSON** Mbokane Dr. Batseba Mofolo-Mbokane

cc: Prof. Anthony Essien

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

Signature ____________________ **Date** ______/______/______

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