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The transition across the cognitive gap - the case for long division - : Cognitive architecture for division : base ten decomposition as an algorithm for long division

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2008-11-04T12:53:01Z
http://hdl.handle.net/10539/5830

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Abstract:

This is an action research study which focuses on a didactical model founded on base ten decomposition as an algorithm for performing division on naturals. Base ten decomposition is used to enhance the algebraic structure of division on naturals in an attempt to cross the cognitive divide that currently exists between arithmetic long division on naturals and algebraic long division on polynomials. The didactical model that is proposed and implemented comprises three different phases and was implemented over five one hour lessons. Learners' work and responses which were monitored over a five-day period is discussed in this report. The structure of the arithmetic long division on naturals formed the conceptual basis from which shorter methods of algebraic long division on polynomials were introduced. These methods were discussed in class and reported on in this study.

Keywords:
Long Division
Base Ten Decomposition
Division Algorithm
Arithmetic
Structure
Polynomials
Cognitive Gap
Variable

Declaration

I declare that this dissertation is my own work. It is submitted for the degree of Master of Science in Mathematics Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination in any other university.

Jacques Desmond du Plessis

2nd day of May, 2007.

To my life partner Mark

Thanks for the great inspiration, love, support and encouragement during the writing of this dissertation.

To my friend, mentor and colleague, Kgethi

Your friendship and support for my work is a lifeline which is always available when things get difficult.

To Mr Derrick Kruger, Principal of the host school, who passed away during this study

His support of and interest in this research study and the warm friendship he extended to the University of the Witwatersrand, is greatly appreciated.

Acknowledgements

Professor Mamokgethi Setati –

Thank you for supervising my work over the past three years so meticulously and for being so honest in your assessment of the work in progress. Your belief in me and my work, through the productive and fallow times, inspired me and placed me back on track again. The financial support that you have extended to me over the past two years has made it possible to contribute to the debate on division, which would not have been possible otherwise.

Mark Wilmot –

Thank you for reading my dissertation time and time again and for your honesty when ideas were just not coming across the way they should.

Lynne Southey –

Your efforts in proof reading this dissertation at very short notice are much appreciated. The meticulous attention to detail, in language, structure and mathematics, was very beneficial to this dissertation.

Erna Lampen –

You possess a wealth of knowledge, and are generous in your support and suggestions. Thank you for the honest comments and ideas that you recommended while reading my work.

Professor Jill Adler –

Your encouragement and support during your tenure as the Head of the Division Mathematics and Science Education, facilitated this study and is deeply appreciated.

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