

Abstract:

This is an action research study which focuses on a didactical model founded on base ten decomposition as an algorithm for performing division on naturals. Base ten decomposition is used to enhance the algebraic structure of division on naturals in an attempt to cross the cognitive divide that currently exists between arithmetic long division on naturals and algebraic long division on polynomials. The didactical model that is proposed and implemented comprises three different phases and was implemented over five one hour lessons. Learners' work and responses which were monitored over a five-day period is discussed in this report. The structure of the arithmetic long division on naturals formed the conceptual basis from which shorter methods of algebraic long division on polynomials were introduced. These methods were discussed in class and reported on in this study.

Keywords:

Long Division

Base Ten Decomposition

Division Algorithm

Arithmetic

Structure

Polynomials

Cognitive Gap

Variable

Declaration

I declare that this dissertation is my own work. It is submitted for the degree of Master of Science in Mathematics Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination in any other university.

Jacques Desmond du Plessis

2nd day of May, 2007.

To my life partner Mark

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during the writing of this dissertation.*

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Contents

	Page
Abstract.....	i
Declaration.....	ii
Dedications.....	iii
Acknowledgements.....	iv
Contents.....	v
Introduction.....	1
Chapter 1: Division – The jewel in the crown.....	3
Cognitive architecture for algebraic division: Base ten decomposition – an emulation of the ‘real thing’.....	4
.....	4
Division and its importance in the further learning of mathematics	7
Conclusion.....	10
Chapter 2: A search for the cognitive roots that connects learning.....	12
Introduction.....	12
Establishing a cognitive root.....	12
Metacognition: How it assists learning for understanding.....	13
Cognitive schema: The filing of new learning.....	15
Constructivism: The road to teaching for understanding.....	18
Basic skills and conceptual understanding: A balancing act.....	21
The importance of long division and the real division algorithm.....	25
Focus on division problems.....	28
The notation.....	31
Chapter 3: Arithmetic and algebra – The undanced dance.....	34
Historical views of arithmetic and algebra – a dichotomy.....	34
The current debate – a symbiosis.....	36
Defining the cognitive gap: the foci of researchers.....	40
Cognitive architecture: Various approaches.....	42

Structural thinking.....	43
Functional thinking.....	44
The South African context.....	46
Chapter 4: The pilot study	49
The Grade 11 group.....	49
The Grade 9 group.....	53
Step 1: Division that preserves the concept of place value.....	55
Reflection informing the current study.....	59
Step 2: The introduction of the division algorithm.....	60
Reflection informing the current study.....	64
Step 3: The division algorithm and modular arithmetic.....	65
Reflection informing the current study.....	66
Step 4: Division using base ten decomposition.....	67
Reflection informing the current study.....	68
Step 5: Division using a base other than base ten decomposition.....	70
Generalising this idea to polynomial division.....	75
Reflection informing the current study.....	82
In conclusion.....	83
Chapter 5: The didactical model and plan of action	85
Step 1: Focusing on multiple division procedures that enhance the concept of place value.....	86
Step 2: The division algorithm.....	90
Step 3: Base ten decomposition of the divisor and the dividend.....	91
Chapter 6: Implementing the didactical model	97
Introduction.....	97
Lesson 1: Multiple representations.....	98
Lesson 1: Reflection.....	111
Lesson 2: The arithmetic – algebra link.....	125
Lesson 2: Reflection.....	139

Lesson 3: Reinforcing base ten decomposition.....	150
Lesson 3: Reflection.....	155
Lesson 4: Detaching coefficients.....	165
Lesson 4: Reflection.....	176
Lesson 5 and reflection.....	188
Additional work.....	195
Chapter 7: Reflections and future research.....	199
The research focus.....	199
Base ten division as an algorithm.....	200
The division algorithm and the remainder.....	201
The South African curriculum.....	204
Recommendations.....	208
Limitations of the study.....	208
Concluding remarks.....	209
Reference list.....	211