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A statistical approach to some mine valuation and allied problems on the Witwatersrand

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Thus "w" in No. (61) above can be regarded as an indirect measure of the cumulative frequency on the lognormal curve, and the relation between these two quantities can be established directly from any set of tables of areas of the Normal Curve.*

Thus w = function of cumulative frequency (which, from No.(60))

$$= a\sqrt{2}\log x - a\sqrt{2}\log m + \frac{\sqrt{2}}{4a} \qquad \ldots\ldots (63)$$

$$\text{or } \frac{w}{\sqrt{2}} = a\log x - a\log m + \frac{1}{4a} \qquad \ldots\ldots (64)$$

This equation is a straight line formula of the type

$$Y = KX + C$$

where $Y = \frac{w}{\sqrt{2}}$, $X = \log x$,

K = "a," determines the slope of the line

and $C = (\frac{1}{4a} - a\log m)$, is a constant = intercept on Y axis.

As will be shown this leads naturally to a simple graphical straight line fit for the lognormal curve.

2. Logarithmic Probability Paper.

Table 5 reflects the values for cumulative frequencies corresponding to "$\frac{w}{\sqrt{2}}$" values computed from tables of areas of the normal curve,** and from Nos. (61) and (62) above.

Thus if the "$\frac{w}{\sqrt{2}}$" values are plotted as aoscissae on ordinary graph paper, the corresponding values of the cumulative frequency percent can be marked off on this "$\frac{w}{\sqrt{2}}$" scale as illustrated in Diagram No. 3.*** Now, by taking a logarithmic scale for the ordinates as indicated, the logarithms of the gold values corresponding to the various cumulative frequencies as observed, can be plotted, and should for a true lognormal distribution result in the straight line or

equation/...

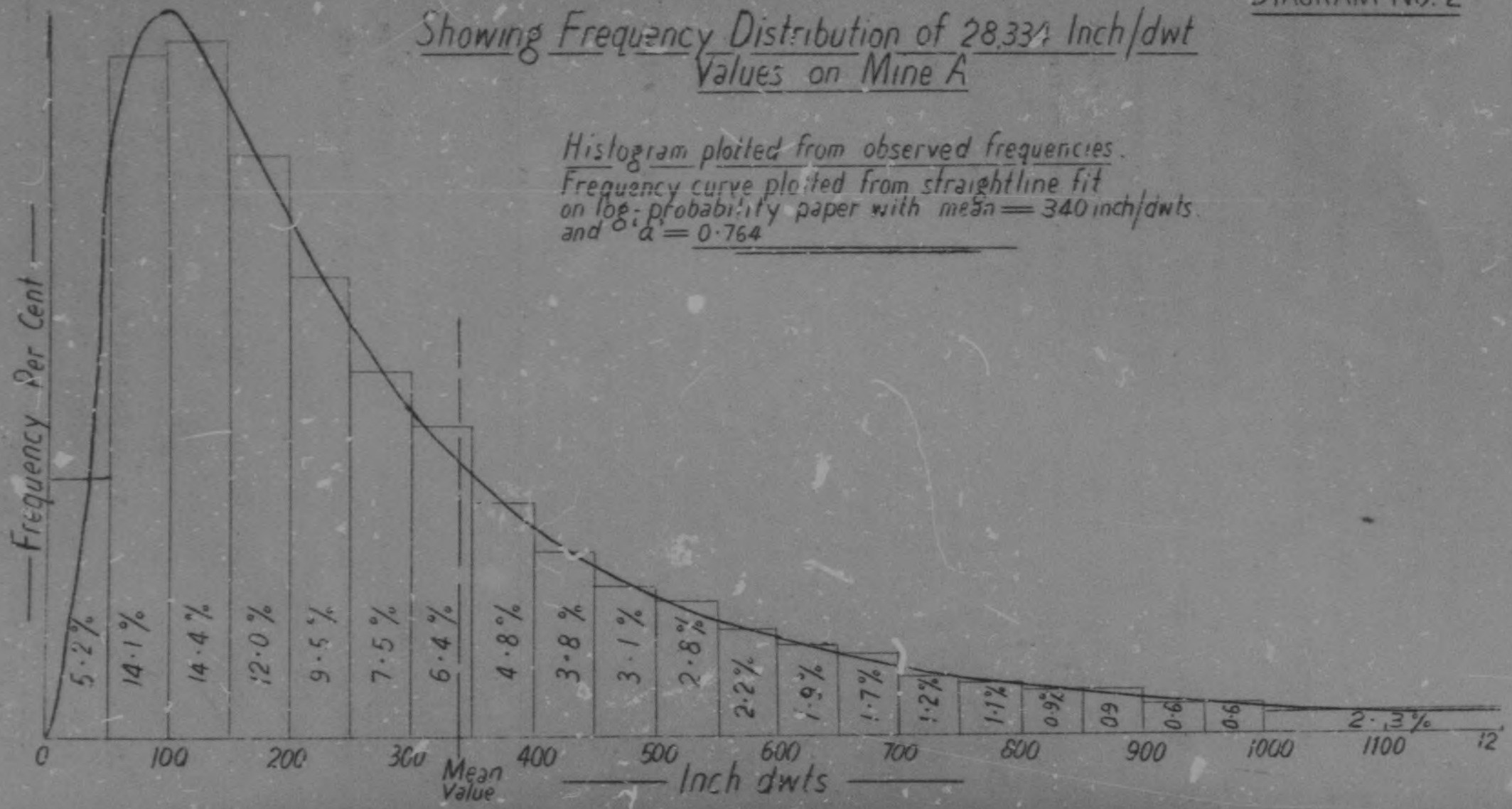
*E.g., Ref. 4, p.237 & Ref. 24. **Ref. 24.
***See map pocket.

.F. %	"$\frac{w}{\sqrt{2}}$"
7	0.5225
8	0.5461
9	0.5702
0	0.5951
1	0.6208
2	0.6472
3	0.6747
4	0.7032
5	0.7329
6	0.7641
7	0.7965
8	0.8308
9	0.8673
0	0.9062
1	0.9481
2	0.9936
93	1.0436
94	1.0994
95	1.1631
96	1.2379
97	1.3299
98	1.4522
99	1.6450
99.5	1.8214
00	$+\infty$

DIAGRAM No. 2

Showing Frequency Distribution of 28,334 Inch/dwt Values on Mine A

Histogram plotted from observed frequencies.
Frequency curve plotted from straightline fit on log: probability paper with mean = 340 inch/dwts and $\sigma' = 0.764$



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3. Graphical D

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*Ref. 10 & Ref.

equation No. (64). The extent to which deviations from a straight line can be expected will be discussed in a subsequent paragraph.

A practical example is provided by the fitting of a lognormal curve to the observed frequency distribution of the inch-dwt values of 28,334 development sampling sections from an area of about 42,000,000 sq. ft., (14,000 ft. on strike x 3,000 ft. on dip), on Mine A. This observed distribution, when plotted as a frequency histogram, yields the step diagram shown on Diagram No. 2 (see opposite), and when plotted on log probability paper, results in the series of points shown by small circles on Diagram No. 3. The straight line AB on the latter diagram provides an excellent fit for these plotted points and converts back into the corresponding lognormal frequency curve shown on Diagram No. 2.

Referring to Diagram No. 3, the following examples of plotted points are self-evident:-

Up to a value of 150 inch-dwts the number of values counted were 9,558 out of 28,334

= 33.7% cumulative frequency.

Up to a value of 400 inch-dwts the number of values counted were 20,939 out of 28,334.

= 73.9% cumulative frequency.

The graph paper as constructed for Diagram No. 3 is known as logarithmic probability paper* and has been used in many branches of applied statistics.

3. Graphical Determination of the Parameter "a."

Reference to No. (64) above reveals that the slope of the plotted line of "$\frac{w}{\sqrt{2}}$" against "log x" is a measure of the parameter "a" of the distribution. A convenient measure of the slope of the line would be the difference between the "log x"/...

*Ref. 10 & Ref. 11, p. 189.

"log x" values corresponding to a fixed horizontal interval, say, that between the cumulative frequencies of 10% and 90%.

At these two frequencies the "$\frac{w}{\sqrt{2}}$" values are -0.9062 and +0.9062, respectively. Thus, from No (64)

$$-0.9062 = a \log x_{10} - a \log m + \frac{1}{4a} \quad \ldots\ldots (65)$$

$$\text{and } +0.9062 = a \log x_{90} - a \log m + \frac{1}{4a} \quad \ldots\ldots (66)$$

where x_{10} = x value corresponding to 10% cumulative frequency

and x_{90} = x value corresponding to 90% cumulative frequency

From which

$$a = \frac{1.8124}{\log x_{90} - \log x_{10}} \quad \ldots\ldots (67)$$

But as the logarithmic scale on the diagram has been constructed from common logs, i.e. logs to the base 10, and since

$$\log_e x = 2.302585 \log_{10} x$$

$$a = \frac{1.8124}{2.302585(\log_{10} x_{90} - \log_{10} x_{10})}$$

$$= \frac{.787115}{\log_{10} x_{90} - \log_{10} x_{10}} \quad \ldots\ldots (68)$$

The table of values of "a" corresponding to values of $(\log_{10} x_{90} - \log_{10} x_{10})$, shown on the right-hand side of Diagram No. 3, has been calculated from equation No. (68), and enables the value of "a" to be interpolated when $(\log_{10} x_{90} - \log_{10} x_{10})$ has been read off on the scale of "log x" (on the right-hand side of the graph).

It should be noted that as the slope of the line decreases and approaches the horizontal, the value of $(\log_{10} x_{90} - \log_{10} x_{10})$ will approach zero and "a" will consequently approach infinity. A very steep line is, therefore, indicative/...

indicative of a very skew and sharply peaked curve with a large coefficient of variation, whereas a very flat line represents a distribution approaching the "normal." (See Chapter III, paragraphs 5 and 6.)

Example: For the distribution represented by the line AB on Diagram No. 3

$\log_{10}x_{90} = 2.86$

$\log_{10}x_{10} = \underline{1.83}$

$\log_{10}x_{90} - \log_{10}x_{10} = \underline{1.03}$

∴ "a" in Column 1 of the table on Diagram No. 3 corresponding to this value of $(\log_{10}x_{90} - \log_{10}x_{10})$ is interpolated as 0.764.

4. Graphical Determination of the Arithmetic Mean.

From No. (64)

$$\frac{w}{a\sqrt{2}} = \log x - \log m + \frac{1}{4a^2}$$

therefore, when $x = m$, $\log x = \log m$

$$\frac{w}{a\sqrt{2}} = \frac{1}{4a^2}, \quad \text{and} \quad \frac{w}{\sqrt{2}} = \frac{1}{4a} \qquad \ldots\ldots (69)$$

The position of the mean on the straight line on Diagram No. 3, therefore, corresponds to a value for $"\frac{w}{\sqrt{2}}"$ of $"\frac{1}{4a}"$ and thus to a cumulative frequency dependent only on the value of the parameter "a." Having determined "a" graphically as outlined in paragraph 3, therefore, this value of "a" can be substituted in equation No. (69) in order to determine the cumulative frequency corresponding to the mean value. To facilitate determinations and to allow direct interpolation of the relevant cumulative frequency values, the further table of these cumulative frequencies against various values for "a" is also reproduced on the diagram.

Determination of the mean value, therefore, merely involves the interpolation of the cumulative frequency (corresponding to the mean value) from the already known value

of/...

of "a" (paragraph 3), and the reading off of the value of "x" on the line at this cumulative frequency.

It is evident on examination of No. (69) that for large values of "a," "$\frac{w}{\sqrt{2}}$" corresponding to the mean value will approach zero, (i.e. a cumulative frequency of 50%), and that for small values of "a," i.e. for very skew and peaked curves, the mean value will correspond to a relatively high cumulative frequency,

e.g. when "a" = 0.5

the cumulative frequency at mean value = 76%

In this case, therefore, more than three-quarters of the total number of values in the population will lie below the mean value.

Example: For the distribution represented by line AB on Diagram No. 3

"a" = 0.764 (see paragraph 3)

∴ Cumulative Frequency at mean value = 67.8% (see Column 3 in table on diagram)

Value on line AB corresponding to this cumulative frequency percentage = 340 inch-dwts = mean value.

5. Graphical Observation of Non-homogeneity of a Distribution.

A detailed discussion of the effect of combining two lognormal populations and the resultant bimodal curves, falls outside the scope of this thesis, and the specific examples on Diagram No. 3 are given merely as illustrations of the types of curves which will be obtained on logarithmic probability paper when the sample values are not representative of a truly homogeneous lognormal population.

Example 1: The combination of the two lognormal populations with the same parameter "a" as represented by the two straight lines CD and EF will result in a type of S curve, shown dotted as GHI.

Example/...

Example 2: The combination of two lognormal populations with different "a's" as represented by the two straight lines JK and LM will yield a curve of the type shown dotted as NOP on the diagram.

6. Graphical Observation of the Absence of an Entire Category of the Lowest Values, e.g. when only the ore reserves above the pay limit are plotted.

Line QRS on Diagram No. 4* represents the complete population (with mean = 7 dwts/ton and "a" = .7) and the curves TUV and WXY represent the position when only the values above "x" values of 2 and 3 dwts/ton, respectively, are plotted as if they are complete populations. These two curves, therefore, indicate the type of curve resulting from the plot of a Payable Ore Reserve tonnage distribution without any allowance for the tonnages of unpay ore below the pay limit.

7. Confidence Zones.

The question now naturally arises whether a limited number of values drawn from a lognormal population will give a straight line plot on the log probability paper, and if not, to what extent the plotted points can deviate from a straight line due solely to the operation of chance in drawing the relevant values.

For this purpose a brief reference to the theory of probability, and more particularly the binominal theory as applied to probability, is required.

The following theorem** can be directly applied to our problem:-

Let p = probability that an event will happen in a single trial

and q = probability that this event will fail in a single trial

= 1 - p

Then/...

*See map pocket. **Ref. 6, p. 7.

Then the probability that the event will happen "x" times in "s" trials

$$= C(s,x)p^x q^{s-x}$$

where $C(s,x) = \frac{s(s-1)(s-2)\ldots(s-x+1)}{x(x-1)(x-2)\ldots 1}$

$$= \frac{s!}{x!(s-x)!}$$

Assume a lognormal population with parameter "a" and mean value such that, say, 20% of the values lie below a certain value, say, 100, and 80% above it

Then in the above formulae

p = .2 (i.e. the probability of striking a value less than 100)

q = .8 (i.e. the probability of striking a value in excess of 100)

Drawing, say, 5 samples at random from this population, i.e. s = 5, the probability that of these 5 values, 2 will lie below 100 and 3 above, is determined as follows:-

s = 5 x = 2

$$\text{Probability} = \frac{5!}{2!(5-2)!}(.2)^2(.8)^{5-2}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1}(.2)^2(.8)^3$$

$$= \frac{20}{2}(.04)(.512)$$

$$= \underline{.2048}$$

i.e., there is approximately a 20%, or 2 in 10, or 1 in 5 chance of this happening.

If this known population is represented by a straight line on log probability paper, a cumulative frequency of 20% would correspond with an "x" value of 100 (as stipulated above). If, however, only 5 samples are drawn from this population, there is a 1 in 5 chance that the plotted data would indicate a 40% cumulative frequency of these 5 samples,

(i.e./...

TABLE 6

Cumulative Frequencies

No. of Samples	Cumulative Frequencies of Parent Population: 5	10	20	30	40	50	60	70	80	90	95
	Outer limits of cumulative frequencies between which any one observed cumulative frequency should lie to cover a 95% probability										
	%	%	%	%	%	%	%	%	%	%	%
10	* 28	38	53	66	77	87	94	100	100	100	100
	ø 0	0	0	0	6	13	23	34	47	62	72
15	* 23	32	46	58	69	79	87	95	100	100	100
	ø 0	0	0	5	13	21	31	42	54	68	77
20	* 19	28	42	53	65	74½	83	92	98	100	100
	ø 0	0	2	8	17	25½	35	47	58	72	81
30	* 16	24	37	48	60	69½	79	87	95	100	100
	ø 0	0	5	13	21	30½	40	52	63	76	84
50	* 13	20	33	44	55	65	74	83	92	98	100
	ø 0	2	8	17	26	35	45	56	67	80	87
100	* 10	17	28½	40	50	60½	70½	79½	88	96	99
	ø 1	4	12	20½	29½	39½	50	60	71½	83	90
250	* 8	14	25½	36½	46½	57	67	76½	85	94	97½
	ø 2½	6	15	23½	33	43	53½	63½	74½	86	92
1,000	* 6½	12	22½	33	43	53	63	73	82½	92	96½
	ø 3½	8	17½	27	37	47	57	67	77½	86	93½

*Upper Limit
øLower Limit

Example: If 20 samples are drawn from a known population there is a 95% chance of the value which, in fact, corresponds to a 30% cumulative frequency in this known population, being found to correspond with a cumulative frequency lying between 8% and 53% in the distribution of the 20 samples.

(i.e., 2 samples out of 5) up to an "x" value of 100.

In this fashion, the probability of any stipulated deviation from the straight line (representing a known population) occurring for any number of samples drawn from the population can be calculated.

When the number of samples is small, the calculation involved is fairly simple, but as the number increases, the calculation becomes more and more laborious and a reasonable approximation is then permitted by the application of the normal law of probability.*

For the purpose of a quick graphical check of whether the plotted points on log probability paper can be regarded as permissible deviations from a straight line representing the theoretical parent population, a confidence coefficient of, say, 0.95 can be adopted, i.e. a probability of 5 in 100 or 1 in 20 of deviations exceeding those calculated. Table 6 has been prepared on this basis.**

A graphical illustration of these 95% confidence zones is given by dotted lines on Diagram No. 4 for an assumed population with "a" = 0.65, and mean = 477 (line AB). From this diagram it is evident that considerable deviations from a straight line can occur when the number of samples is limited, and that even when a 1,000 samples are available a dead straight line cannot be expected.

Practical examples of the distributions obtained in sets of sample values from an observed lognormal parent population corresponding approximately to the line AB on Diagram No. 4, are shown on this diagram by means of the lines CDE and FGH corresponding to the observed distributions of 72 and 360 sample values, respectively, drawn at random from the parent population.

It/...

*Ref. 6, p. 21. **Compiled from graph in Ref.12 - see opposite.

pulation		
80	90	95
tween which any e to cover a 95%		
%	%	%
100	100	100
47	62	72
100	100	100
54	68	77
98	100	100
58	72	81
95	100	100
63	76	84
92	98	100
67	80	87
88	96	99
71½	83	90
85	94	97½
74½	86	92
82½	92	96½
77½	83	93½

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een 8% and 53%

It should be stressed that if an observed distribution yields a plotted line falling entirely within the relevant confidence zone, the evidence is by no means conclusive* that the parent population from which the set of samples was drawn is lognormal and represented by the straight line on which the confidence zone is based. The confidence zones do, however, provide some means of sensing the extent of deviations from a straight line which can be met in practice.

8. The Mathematical Equivalent of the Graphical Fit on Logarithmic Probability Paper.

When a straight line is fitted graphically to an observed distribution of values on log probability paper, it is obvious that the slope of the line will have to conform to the general trend of the plotted points, (corresponding to the observed cumulative frequencies), and that the intersection point of this line on the 50% cumulative frequency ordinate, (i.e. the central ordinate) will have to represent approximately the mean of the log values corresponding to all the plotted points.**

Now, since the slope of the straight line determines the parameter "a" of the theoretical lognormal distribution and the central ordinate, (i.e. log value corresponding to the 50% cumulative frequency line), determines the geometric mean of this distribution, the fitting of this straight line is virtually equivalent to estimating the parameter "a" (and hence the variance $\frac{1}{2a^2}$ of the distribution of the logs of

the/...

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*The best evidence will be provided by the "Chi-square Test" described in most text-books on statistics. There are, however, various practical reasons why this "Test" may, in many cases, indicate a "fit" corresponding to a probability of less than 5%.

**Provided the frequency grouping has been approximately symmetrical on either side of the 50% cumulative frequency.

the observed values), as well as the geometric mean of the observed values (and hence the mean of the logs of the observed values), graphically from these observed values. Referring back to Chapter III, paragraph 13(b), it will be evident, therefore, that the mathematical fitting of the log-normal curve from the calculated variance and mean of the logs of the observed values, is virtually equivalent to the graphical fit discussed in this Chapter.

CHAPTER V/...

CHAPTER V

A PRACTICAL INVESTIGATION INTO CERTAIN BASIC PROPERTIES OF THE DISTRIBUTION OF GOLD VALUES ON WITWATERSRAND MINES

1. Distributions Obtained from Linear and Grid Sampling and the Frequency Weighting of Values.

The following is a brief résumé of the history of the introduction of the idea and methods of frequency weighting on the Witwatersrand.

Based on work done by A. E. Hooper,* Professor Watermeyer** suggested in 1919 that the observed values for a stope face or block should be weighted according to their relative frequencies of occurrence shown by the smooth frequency curve established for the relevant mine, section of mine, or reef, from a larger number of samples. Since high values have relatively low frequencies, and low values, high frequencies in any such curve, (a fact self-evident from a glance at any lognormal curve), such a method of weighting would in general result in lower average values than those obtained from the orthodox arithmetic means of the relevant sets of samples, and could, therefore, possibly be regarded as a "satisfactory" method for improving the so called Mine Call Factor. Such a method with an apparently sound statistical background, would also overcome the anomalies inherent in all the empirical methods of reducing or "cutting" high and apparently anomalous values.

A method of frequency weighting was introduced on the West Rand Consolidated mine by Hooper, and is still

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*Former Chief Surveyor, West Rand Consolidated Mine.
**Refs. 13 & 14.

in use. In a recent paper,* R. J. R. Perkin explained in detail the application of the various master frequency curves for the individual reefs and sections of the mine. A somewhat similar method of frequency weighting was also consistently supported by the late Professor Truscott** even as recently as 1947.***

The basic argument used in favour of these methods has evidently been that a block is only sampled around its periphery, this process yielding what could be termed a "linear" frequency distribution of values, whereas the actual distribution for the whole block would consist of all the possible samples which could be taken throughout the whole extent (or inside) of the block. The contention is, therefore, that by weighting the observed values by their relevant frequencies of occurrence, the unknown missing values in the interior of the block would be compensated for. The question as to whether dwts/ton or inch-dwts should be employed has already been dealt with in Chapter I. It seems, therefore, that if it can be shown that the distribution of inch-dwt values around the periphery of a block is within practical limits, fully representative of the actual distribution of all the possible sample values in the block, the basic argument used in favour of frequency weighting would fall away.

The only practical method of investigating this problem is by examining the distribution of stope sampling values on a grid pattern for an already worked out area, or alternatively, the distribution of development values from a development grid for a large section of a mine.

This method of investigation is based on the assumption that the distribution of values obtained from a regular grid of sampling sections covering an area will be

representative/...

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*Ref. 15, p.145. **Refs. 16 & 17.
***Discussion by Professor Truscott on Ref. 3.

representative of the distribution of all the possible sampling sections for the area. Provided such a grid is reasonably fine in relation to the size of the area involved, there appears to be no sound argument to be advanced against this assumption.

(a) Investigation based on development sampling values: All the development sampling results of the section of Mine A referred to in Chapter IV, paragraph 2, were employed for the purpose of this investigation. The drive-raise grid in this area, which measures some 42,000,000 sq. ft., consists of 9 main drives with 360' backs and 21 lines of raises 600' apart, sampling sections being at 5 foot intervals.

The frequency distribution of all the 28,334 values on this grid was compared with that resulting from the employment of only the peripheral values totally 5,147 along the top and bottom drives and the outer lines of raises. The two distributions obtained were plotted on log probability paper and found to be identical for all practical purposes. A comparison of the percentage cumulative frequencies obtained for the two distributions is given in Table 7 below.

TABLE 7

Inch-dwt value category from zero to:-	75	100	150	200	250	350	500	750
Distribution of grid values	11.7%	19.3%	33.7%	45.7%	55.2%	69.1%	80.8%	90.6%
Distribution of peripheral values	11.0%	18.5%	33.3%	44.5%	54.1%	68.5%	80.2%	90.0%

(b) Investigation based on stope sampling values: A stope sampling plan was prepared for a stoped out section of Mine A measuring some 530 ft. on strike by 1,800 ft. on dip

(960,000/...

(960,000 sq. ft.), and the distribution obtained from 2,448 sample values on a 20' x 20' grid was compared with that of the 310 samples taken along the periphery of the area. (See Table 7A.)

TABLE 7A

Inch-dwt value category up to:-	125	175	225	275	325	425	550	800
Distribution of grid values	14.1%	23.8%	33.3%	42.7%	51.3%	63.8%	75.5%	87.1%
Distribution of peripheral values	11.0%	21.9%	32.6%	40.0%	49.4%	60.6%	72.3%	86.1%

Bearing in mind the relatively small number of values, i.e. 310, constituting the 2nd distribution, the agreement between the observed cumulative frequencies appears reasonable.

(c) Conclusions re frequency weighting of values: It can fairly be concluded from the above results that sampling values obtained in the customary way, i.e. by sampling around the periphery of a block, are reasonably representative of the distribution of all the possible sample values throughout the block and, therefore, that the arithmetic mean of such "peripheral" inch-dwt values provides an unbiassed estimate of the true average (sample) inch-dwt value of the block.* The reliability of such an estimate** will naturally increase with an increase in the number of available sample values.

2. Effect of the Size of a Reef Area on the Relative Shape, i.e. the parameter "a," of the distribution of sample values.

(a) General: As shown previously (Chapter III, paragraph 3), the relative shape of the lognormal curve is determined entirely by the parameter "a," which in turn determines the slope/...

*Provided the locations of raises are not biassed - see Chap.I, par.3.
**Disregarding the question of any bias introduced due to "oversampling"

slope of the corresponding straight line on log probability paper (Chapter IV), as well as the coefficient of variation (No. 10(a)), i.e. the relative variation between the values constituting the lognormal distribution concerned. It is also evident from No. 10(a) that as the relative variation between the sample values constituting a lognormal distribution increases,"a" will decrease in value.

In dealing, for example, with the incomes of the members of a population, it is only natural to expect a larger relative variation between the incomes of members of a town's population than between those of members of the population of a suburb of such a town. Similarly, it can be expected that gold values in a whole mine will be subject to a larger relative variation than those in a portion of the mine, and hence that the relative variation between gold values will tend to increase (and the parameter "a" will tend to decrease) with an increase in the size of the reef area concerned.

This general conclusion regarding the trend in the value of "a" can also be arrived at on the basic assumption that the size of a reef area determines the "a" of the distribution of the sample values within such an area and, therefore, that reef areas of equal size* have identical "a's." Consider a number of reef areas of equal size constituting a large block of ore. If, in addition to identical "a's" such reef areas also have identical mean values, the distributions of the sample values within such areas will be identical (see No. 3), and their combination would yield a further identical distribution for the block of ore as a whole.

It is an observed fact, however, that in general the mean values of reef areas differ, i.e. that their is a variation between such mean values, and it is evident therefore that in combining the distributions of a number of small reef areas

with/...

*A reasonable shape of area is presupposed, i.e. the areas should not be unduly elongated in any direction.

with identical "a's" (or relative variations), a further variation will be introduced, resulting in an overall relative variation for the values within the block of ore, larger than that of each of the constituent small areas. An increase in the size of reef area can, therefore, in general be expected to be accompanied by an increase in the relative variation between the gold values within the area, i.e. by a decrease in the parameter "a."

> The same arguments apply in reverse and the relative variation between values could, therefore, be expected to approach zero as the size of area becomes infinitely small. From a practical point of view it would, for example, be natural to expect the relative variation in respect of a "standard" size reef area, i.e. the area represented by a single sample section, to be zero. There must, however, always remain some variation even within such a small area due to the human factor, e.g. sampling and assay errors, i.e. if it were possible to resample a particular sampling section several times (by replacing the sampled material each time after assaying), differing values would be obtained due to the introduction of these human errors.

From a practical point of view the only way in which it could be proved that the value of "a" decreases with an increase in the size of the reef area concerned, and that within a particular mine, or section of a mine, reef areas of equal size have identical "a's," would be to obtain a sufficiently large number of sample values in respect of each of a number of reef areas covering a wide range of sizes. The use of existing records on a mine will, therefore, only prove useful for those areas in respect of which relatively large numbers of sample values are generally available. The extent to which such records can, however, serve to provide some measure of practical proof of the general conclusions arrived at above will be shown in subparagraphs (b) to (e) below.

(b) Development values in respect of relatively large areas: All the development sampling values from drives in the

section/...

section of Mine A referred to in paragraph 1 above, were analysed by dividing this section into sets of equal size areas ranging down to areas measuring some 440 ft. x 360 ft. In the latter case only the values from two 440' lengths of driving separated by a 360' length of back were available for each area, but even this limited number of values could in every case be fitted with a lognormal curve. The frequency distributions of the sets of values available in respect of all these individual areas were established and plotted on log probability paper, and the "a" in each case determined graphically as explained in Chapter IV. The results can be tabulated as follows:-

TABLE 8

Area	Size of area Sq. ft.	No.of areas involved	No. of sample values per area	Values determined for "a"	Average value for "a"
1.Whole Section	42,000,000	1	16,934	0.750	0.750
2.4200'x3300'	14,500,000	1	5,960	0.787	0.787
3.1760'x1440'	2,500,000	1	1,600	0.820	0.820
4.1320'x1440'	1,900,000	2	1,200	0.837, 0.829	0.833
5.1320'x1080'	1,400,000	3	960	0.820, 0.842 0.816	0.826
6.1640'x 720'	1,200,000	2	1,440	0.805, 0.865	0.834
7. 880'x 720'	650,000	10	480	.851, .837, .851, .846, .900, .846, .856, .787, .772, .716	0.826
8. 440'x 360'	160,000	28	160	0.645 to 0.984	0.824

(Three mathematical check calculations for "a" were performed by calculating the 2nd moment of the logs of the observed values - see Chapter III, paragraphs 7 and 10(c), and solving for "a" in the equation:

$$\text{2nd moment} = \frac{1}{2a^2}$$

with the following results:-

Area/...

Area: 880' x 720' ("a" graphically = 0.900
("a" calculated = 0.910

("a" graphically = 0.787
("a" calculated = 0.789

440' x 360' ("a" graphically = 0.673
("a" calculated = 0.704

The graphical determinations of the indicated "a's" for the 440' x 360' blocks must be accepted with reserve since the number of samples involved is small, and it would therefore be preferable to calculate the indicated "a's" for these blocks. For all the other size areas, the graphical determinations can, however, be regarded as sufficiently close for all practical purposes.)

(c) Development values in small areas: For the purpose of estimating the "a's" in respect of relative small reef areas, the following assumptions are required:-

(i) that the inch-dwt values of all the possible sample sections even for areas as small as, say, 10' x 10', will be distributed lognormally. (Such an assumption appears justified in the light of the additional evidence furnished by stope values for areas as small as 180' x 180' in subparagraph (d) below), and

(ii) that the "a's" for the distributions within areas of equal size are identical (a question which is dealt with under subparagraph (e) below).

Now, two sampling sections 5 feet apart can reasonably be regarded as "random" sample values drawn from a population of values within an area of, say, 10' x 20'. Similarly, samples taken in a 100 foot stretch of a drive (8' to 10' wide) can be regarded as "random" sample values drawn from an area measuring, say, 100' x 20', and those in a 400 foot stretch as "random" sample values from an area of say, 400' x 20'. In this fashion, sample values in these small "development" areas will represent subpopulations of the parent population comprising all the sample values from the entire development grid.

Since/...

Since the distribution for this grid is known (No. 2 in Table 8), and the distribution of the means of sets of sample values drawn from the subpopulations in respect of, say, the 10' x 20' areas can be determined, formula No. (41) can be applied to estimate the parameter "a" of the distributions of individual values within the subpopulations themselves, i.e. within the 10' x 20' areas.

The distributions of the means of sample values in all the 10', 100' and 400' stretches within the whole section concerned, (i.e. No. 2, Table 8) were determined. The "a's" for the individual 10', 100' and 400' stretches were then calculated by substituting in formula No. (41) with the following results:-

TABLE 9

(Parameter "A" of parent population, No. 2 in Table 8 = 0.787.)

Stretch length	Area	No. of samples per stretch = N	Parameter "ax" of distribution of means	Estimated Parameter "a" for distribution within area concerned
10'	10' x 20'	2	0.95	0.92
100'	100' x 20'	20	1.574	0.87
		10	1.514	0.856
		5	1.312	0.845
		2	0.972	0.867
			Average:	0.861
400'	400' x 20'	80	1.968	0.851

In the case of 100 foot stretches, the application of (41) was checked from a practical angle by taking, firstly all 20 available values per stretch, secondly 10 values only per stretch (every 2nd value only), thirdly 5 values per stretch

(every/...

(every 4th value only) and lastly, 2 values only per stretch (the 1st and the 11th), and determining the "a_x's" and the "a's" for these different values of N. The values obtained for "a" are sufficiently close to accept an average.

(d) Stope sampling values: The stoped out section referred to in paragraph 1(b) above was subdivided into two equal areas with a sampling grid of 12' x 20' in the one case, and 20' x 20' in the other. These two equal areas were further subdivided into sets of equal size subareas down to a size of 180' x 180'. The "a's" for the distributions of the values from all these subareas were determined graphically with the results reflected in Table 10.

TABLE 10

Area	Size	No.of areas involved	No.of samples per area	Values determined for "a"	Average value for "a"
Upper area : 12' x 20' grid	490,000 sq.ft.	1	2,040	0.856	0.848
Lower area : 20' x 20' grid		1	1,224	0.842	
540'x 540' : 12' x 20' grid	292,000 sq.ft.	1	1,21[illegible]	0.846	0.846
20' x 20' grid		1	729	[illegible]	
360'x 360' : 12' x 20' grid	130,000 sq.ft.	2	540	0.854 0.890	0.872
20' x 20' grid		3	324	0.856 0.920 0.915	
180' x 180' : 12' x 20' grid	32,000 sq.ft.	11	135	0.803 to 0.960	0.884
20' x 20' grid		14	81	0.756 to 1.064	0.865

(e) Analysis/...

(e) Analysis of the observed variations in the "a's" for different and equal size areas: Considering the results reflected in Tables Nos. 8 and 9 for development values and those in Table 10 for stope values, separately, the expected tendency of "a" to decrease with an increase in the size of the reef area concerned (subparagraph (a) above), is discernable only as an overall general trend, the "a's" for the range of sizes of ore reserve blocks being approximately constant. Similar but less detailed examinations carried out on two other mines revealed the same general trend. Definite conclusions in respect of a particular mine will, however, only be possible when based on an even more detailed examination than that discussed under (b) to (d) above, and supplemented where necessary with additional underground sampling at close intervals.

The fact that stope values appear to yield larger "a's" than those obtained from development values for equal size reef areas will be discussed in paragraph 4 below.

A further question now naturally arises, i.e. whether the "a's" for equal size areas are identical, i.e. whether the observed variations in the "a's" for equal size areas (in Tables 8 and 10) are due merely to the limited number of samples available in each case. The theoretical variation in "a" due to the latter cause was discussed in Chapter III, paragraph 10(c), and the application of formula No. (38) to the average "a's" in Tables Nos. 8 and 10 reveal the following:-

For Table No. 8, (Development Samples):

The variations in the observed "a's" for areas measuring 1,320' x 1,440', 1,320' x 2,000' and 1,640' x 720', all fall within the limits which can be accounted for by the limited numbers of samples and a probability of 19 in 20.

The variations in the observed "a's" for the 880' x 720' areas also fall within the above limits except for

the/...

the three extreme values of "a," i.e., 0.900, 0.772 and 0.716 The last three determinations of "a," i.e., 0.787, 0.772 and 0.716, are in respect of three areas separated from the other areas by a fault break, and if these are disregarded and a new mean "a" taken for the remaining areas, the variations in the "a's" for these areas all fall within the stipulated limits.

The variations in the "a's" for the 360' x 360' areas cannot all be accounted for solely by the limited number of samples involved. This may be due to the fact that samples from two drive stretches at the top and bottom of a block, i.e. only 2 sides of the block, are not truly representative of the block as a whole and are, therefore, not "random" in a statistical sense.

For Table No. 10, (Stope Samples):

The variations in the "a's" for the 360' x 360' areas all fall within the stipulated confidence limits. It is not possible to be specific for the 180' x 180' areas due to the limited number of samples available and the consequent inability of graphical determinations to give sufficiently close estimates of the relevant "a's." It seems likely, however, that the variation in the "a's" for these areas can also be accounted for (to a very large extent, at any rate) by the limited number of sample values available.

(f) Conclusions: Absolute conclusions cannot be drawn from the above results, but the indications are that in sections of mines, the sizes of which will be dependent on local conditions, specific values for "a" for different size areas* can be determined experimentally in respect of stope and development sampling results. Even where it is established experimentally that the "a's" for a definite size of

reef/...

*Of reasonable shape.

reef area are not identical, the determination of the range of values which is actually covered will serve a very useful purpose in improving the standard of mine valuation.*

It is the writer's opinion that there is considerable scope for fruitful research by interested mine valuators in this specific direction.

The above investigations, as well as similar but less detailed investigations on three other mines, indicate that the range of values of "a" corresponding to the range of sizes of ore reserve blocks on a particular mine, is relatively small, but that the ranges of values covered differ from mine to mine. It also seems that in practice the "a" for values within ore reserve blocks varies from about 0.65 to 0.87,** and for entire mines or large sections of mines, from about 0.55 to 0.75 **

3. The Effect of the Size of the Samples on the Relative Shape of the Frequency Curve for a Specific Area - Distributions for Ore Reserve Blocks.

(a) Size of samples: In this section "sample" refers to the reef matter per "unit area" along the plane of the reef, and for a mine as a whole the size of "sample" can, therefore, theoretically vary from the minimum or "standard" size as represented by the cross-sectional area of the channel cut in ordinary sampling, i.e. 6 sq. inches,*** to the maximum size equivalent to the mine as a whole, in which case only one sample is theoretically possible.

In/...

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*See Chapter VI, paragraph 5(b).

**In the case of Mine A, check samples are taken at every section, and the average value of each set of two complete sampling sections in the same groove is therefore equivalent to taking samples double the ordinary size. Those upper values for "a" are consequently higher than normal for Mine A.

***See Chapter I, paragraph 1.

In the case of the "standard" sample, the frequency curve for the whole mine can be determined as indicated in the previous paragraph. Similarly, by dividing the mine into blocks of ore of a specific size, the average "a" for standard size samples within such blocks can be determined and will be of a somewhat larger value than the "a" for the mine.* Regarding these blocks of ore now as larger "samples" within the mine, the "a" for the distribution of the true mean values** of these blocks can be determined from No. (40),

$$\text{i.e. } \frac{1}{a_m^2} = \frac{1}{A^2} - \frac{1}{a^2}$$

where a_m = parameter of distribution of mean values of blocks, i.e. of the large "samples"

A = parameter of distribution of values of standard size samples within the mine

and a = parameter of the distributions of values of standard size samples within individual blocks.

Based on the contention that "A" must always be smaller than, or equal to, "a" (since the mine must be larger than, or in the limiting case, equivalent to the block of ore), it follows that "a_m" must always be larger or at least equal to "a." Further, since for a specific mine "A" is fixed, and accepting that "a" decreases with an increase in the size of the block, "a_m" will increase in value with an increase in size of the blocks or "samples," and will range from a value = A (when "a" = ∞, i.e. relative variation within block = o) to ∞ (when "a" = A, i.e. the "sample" size = size of mine).

For/...

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*Where conditions vary materially from section to section in a mine, the sections can be treated individually as "mines."

**"True mean value" implying the mean value of all the theoretically possible sample values which can be obtained by repeated sampling.

For a specific mine, therefore, the parameter "a_m" for the frequency curve of "sample" values will increase with the size of "samples" taken within the mine.* In the present case the actual curve of relationship could not be established over the practical range of sample sizes, since the gap between "samples" of "standard" size and those of the smallest blocks of ore is too wide to allow of direct interpolation for sample sizes of, say, double or treble the standard size.

(b) Ore reserve distribution for Mine A: It will, however, be interesting to check the above conclusions regarding "sample" sizes for "samples" of a size equivalent to that of the average ore reserve block. The "Ore Reserves" on Mine A, including all blocks of ore irrespective of pay limit, were analysed and found to consist of blocks of an average size equivalent to 98,000 sq. ft., (31,380 tons over a stoping width of 46" = 270' on strike x 360' on dip). Further, the tonnages of these blocks segregated into their different inch-dwt categories gave the following frequency distribution:-

TABLE 11/...

*This conclusion is, in a way, analogous to that reached by Ross, (Ref. 2), for varying "stretch lengths" in development sampling, since a specific "stretch length" can be regarded as a "sample" of specific size. He, however, dealt with the means of the available sample values in the various stretches and not with the true mean values of the "stretch lengths," i.e., the mean values which would be obtained from unlimited numbers of sample values in all such "stretches."

TABLE 11

Inch-dwt category	Percentage of total tonnage	Cumulative percentage tonnage up to upper limit of value category
0 to 100	0.6	0.6
100 to 125	0.6	1.2
125 to 150	4.5	5.7
150 to 175	2.3	8.0
175 to 200	9.2	17.2
200 to 225	4.4	21.6
225 to 250	9.4	31.0
250 to 275	5.6	36.6
275 to 300	9.7	46.3
300 to 325	5.6	51.9
325 to 350	6.4	58.3
350 to 375	5.1	63.4
375 to 400	8.3	71.7
400 to 425	4.3	76.0
425 to 450	4.6	80.6
450 to 475	3.2	83.8
475 to 500	2.5	86.3
500 to 550	5.0	91.3
550 to 600	4.5	95.8
600 to 700	2.9	98.7
700 to 800	0.5	99.2
800 to 900	0.8	100.0

The frequency distribution of the tonnages in the blocks of ore, when plotted on log probability paper, can be fitted practically without any latitude, with a straight line representing a lognormal curve having an "a" = 1.657.

Now/...

Now, if the contention regarding the effect of the size of "samples" on the relative shape of the frequency distribution is correct, the above value for "a" should also be found by solving in equation No. (40)* for "a_m" from the knowledge of the "A" of the frequency distribution of the values of "standard" samples for the section of the mine containing the above li d blocks of ore and of the "a" indicated for the values of "standard" samples in an "area" (or block of ore) measuring about 98,000 sq. ft. Accepting the determinations of "A" and of "a" based on grid samplings, (i.e. raises + dr'ves for the section of the mine concerned, and stope values for the blocks of ore), the following is found:-

"A" for the section of the mine concerned = 0.764**

"a" for an area of 98,000 square fee' = 0.872***

Then from No. (40)

$$\frac{1}{a_m^2} = \frac{1}{A^2} - \frac{1}{a^2} = 0.3981$$

$$a_m = 1.585$$

The difference between this value and that obtained directly from the ore reserve distribution above (i.e., 1.657) is not significant and can probably be accounted for by the fact that individual block valuation is based largely on drive and raise samplings which will probably yield an "a" for values within the blocks somewhat below the 0.872 based on stope values and employed above.****

It/...

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*Formula 41(a) should strictly speaking be used, but "N," i.e. the average number of available sample values per block is large (approximately 150), and No. (40) consequently provides a sufficiently close approximation.

**See Chapter IV, par. 3.

***See Table 10.

****A value of 0.858 is required for "a" to yield an "a_m" of 1.657.

It may also be stressed that since the variation in the customary sizes of blocks of ore appears to have only a very slight, if any, affect on the "a's" for the values of standard size samples within these blocks, (see Tables 8 and 10), the average size of block for a mine will within reasonable limits not have an appreciable affect on the relative shape (i.e. of the "a_m's") of the distributions of the mean values of such blocks within the mine. Also, where a mine operates to a pay limit, the percentage payability, i.e. the percentage of the blocked out tonnage above the pay limit, can, from an analysis of formula No. (23), be shown to be entirely dependent on the relative shape of the ore reserve distribution curve, i.e. on the value of "a_m." The conclusion, therefore, that for practical purposes the "a_m" will not be affected by the average size of the ore reserve blocks for a mine, suggests that in practice the percentage payability of a mine will not be affected to any appreciable extent by changes in the average size of the ore reserve blocks.* Detailed examination on mines other than Mine A may, however, indicate that this conclusion cannot be applied in all cases.

Other ore reserve distributions

(c) Other ore reserve distributions: The determination of the theoretically ideal lognormal curve which can be fitted to the ore reserve tonnages on the average Witwatersrand mine is complicated by various factors, the main ones being:-

(i) blocking of ore below the pay limit is not generally done, and the percentage tonnage below the pay limit is therefore unknown;

(ii) even where the blocking is done over the full range of value categories, or alternatively, where the percentage of unpay ore can be determined accurately, the natural/..

*Provided block valuation is based on a sufficient number of samples per block - see Chap. VI, par. 1.

natural distribution of the tonnages of ore has usually been upset due to past mining to a pay limit, and the consequent accumulation of tonnages in the categories below the pay limit;

(iii) the varying sizes of ore reserve blocks which lead to "samples" of varying sizes. This factor is not in itself very serious since the variation in the "a_m's" for ore reserve tonnages is relatively small over the range of block sizes met in practice, but the smaller blocks are usually valued on a relatively small number of samples, which do not always provide a sufficiently close estimate of the true mean value of such a block;

(iv) arbitrary "cutting" of calculated block values to varying extents in the different value categories, thus upsetting the natural distribution of tonnages in such categories.

The ideal distribution for ore reserve tonnages can, therefore, not be expected, even where the "unpay" tonnages have also been blocked, the blocks are roughly of a regular size and no "cutting" has been done. The ideal case will only be met where, in addition, the ore reserves are determined for a more or less intact and fully developed mine, or section of a mine. The theoretical determination of the tonnage and average value of the ore reserves for the average mine (where such ideal conditions are not applicable), based on the knowledge of the relative shapes of the frequency distributions of values in the mine and in the average size ore reserve block can, therefore, only serve as a guide and cannot, in the writer's opinion, replace the orthodox basis of valuing individual/...

individual blocks.*

Ross** approached the problem of the distribution of ore reserve tonnages from the theoretical point of view, viz., that the ideal balancing point in ore reserve blocking would coincide with that type of curve which indicated "maximum selectivity" over the full range of values. He thus concluded that the "a" for the frequency distribution of the values of ore reserve tonnages should be equal to unity. He also indicated that the distribution for a combination of the ore reserve tonnages of 23 lease mines could be fitted with a curve having an "a" = 1, (after having indirectly made allowance for unpay tonnages not blocked out). The latter fit has been confirmed as reasonable by the author using the graphical method on log probability paper, but the examination of the ore reserves of a few selected mines indicate that with normal methods of blocking, the "a" for ore reserve distributions can be as high as 1.66, (Mine A above), and often exceeds unity.

Taking the case of Mine A, the "a" required within blocks to result in an "a_m" of unity, being the ideal suggested by Ross for the distribution of ore reserve tonnages, can be calculated from No. (40).

$$\frac{1}{a^2} = \frac{1}{.2} - \frac{1}{a_m^2} = .7132$$

$$a = 1.184$$

An examination of Table 9 will indicate clearly that the theoretical "a" for the distribution of means of blocks even as small as 10' x 20', is estimated at only 0.92, and it seems, therefore, that on this mine at any rate, no method of ore reserve blocking can ever yield an "a" of unity for the distribution of ore reserve tonnages.

In/...

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*Individual block valuations can, however, be improved - see Chap. VI, par. 5.
**Ref. 2.

In cases where a lognormal curve is fitted to the ore reserve tonnages for a mine (after allowance for tonnages below the pay limit), and the resultant parameter "a" is not unity, there is therefore not necessarily any cause for criticising the methods of blocking. Where, however, previous mining operations have not upset the natural balance between the various grade categories of ore, any deviations in the distribution of the mean values of the ore reserve blocks from the theoretical distribution based on No. (41) will in general be indicative of wrong blocking methods, e.g. irregular cutting of block values.

4. The Differences in the Relative Shapes of the Frequency Curves obtained from Dip, Strike and Grid Sampling.

In dealing with the development sampling results of the section of Mine A referred to in previous paragraphs, it was observed that two slightly different curves were obtained in plotting the frequencies of drive values and of raise values separately, and that a third curve with an in-between shape resulted from the combination of these two sets of values. The same tendency was, on examination, found for a portion of the section referred to, the results being the following:-

TABLE 12

Area	Size of area	Samples	No. of samples	Parameter "a"
Whole section	42,000,000 sq.ft.	Drives	16,934	0.750
		Raises	11,399	0.787
		Total:	28,334	0.764
4,200' x 3,300'	4,500,000 sq.ft.	Drives	5,960	0.787
		Raises	3,473	0.833
		Total:	9,433	0.803

Comparison/...

Comparison of the average values for "a" determined from drive values (Table No. 8) with those from stope values on a grid basis (Table No. 10) for similar size areas, reveals that here also the "a's" from grid sampling (comparable to raises + drives above) are consistently larger than those from drive sampling. The only feasible explanation of this tendency appears to be that the distribution of values along the strike of the reef is subject to a greater actual relative variation, (i.e. a smaller "a") than that of values down the dip. This is confirmed by the fact that the distributions obtained for individual drives gave an average "a" of about 0.83 as against an average value of about 0.88 for the individual lines of raises.

It is not known whether a similar tendency will be exhibited on other mines on the Rand, although it is natural to expect that on the Far East Rand the relative variation between values along lines parallel to the general shoot direction, (i.e. lines either wholly inside or outside a shoot) will be less than that between values along lines at right angles to the shoot direction. The general direction of the shoots on the Far East Rand happens to coincide with the direction of dip on Mine A (on the West Rand), and it is possible therefore that the tendency concerned will be found to persist throughout the Witwatersrand field. It is the writer's opinion that if this should prove to be the case, it may disclose new avenues of approach to the question of the origin and mode of deposition or introduction of the gold in the Witwatersrand reefs.

CHAPTER VI/...

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CHAPTER VI/...

CHAPTER VI

SOME PRACTICAL APPLICATIONS OF STATISTICS TO MINE VALUATION AND ALLIED PROBLEMS

1. The Reliability of Individual Face and Block Valuations.

As stressed at the outset in Chapter I, paragraph 1, the individual sample values as computed from assay results have been employed throughout, and no attempt has been made to determine the bias errors which may be introduced in the actual physical acts of sampling and assaying. A useful paper on this aspect was published by Sichel* in 1947. The reference to the reliability of block and face values in this thesis, therefore, merely implies that the reliability of accepting the mean of a limited number of sample values as an estimate of the correct average sample value of the relevant block or face will be examined. The criterion for measuring reliability will, consequently, be the average value of a theoretically infinite number of sample sections in the relevant block or stope face. This average value has been, and will be, referred to as the true mean value of the block and will only equal the actual mean value of the block where on average no over- or under-sampling is carried out.

As concluded in paragraph 2(f) of Chapter V, the frequency curves formed by all the sample values within the average size block, and the same will presumably apply for practical purposes to the average length of stope face, can apparently be fitted with lognormal curves having parameters "a" ranging between 0.65 and 0.87. Accepting 0.7 as a common figure, the following will be the distribution of values within a block corresponding to this value of "a," the values

determining/...

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*Ref. 3.

determining the limits of the value categories being expressed as percentages of the true mean value of the block.

TABLE 13

Value Category - values as percentages of True Mean Value	Percentage of total values in category
0 to 10%	4%
10% to 20%	10%
20% to 30%	11%
30% to 40%	10%
40% to 50%	8%
50% to 75%	16%
75% to 100%	10%
100% to 150%	13%
150% to 200%	6%
200% to 300%	6%
Above 300%	6%

(a) <u>One sample per block or face</u>: If one sample is taken at random from this average block of ore (or stope face), the probabilities of drawing a value falling within these respective value categories will be represented by the percentages of values in the various categories; e.g., the chances of obtaining a value falling within the range of, say, 75% to 100% of the true mean value of the block, i.e. a value corresponding to a negative error of between 0 and 25% will be 10%, i.e. 1 in 10.

Similarly, the probability of an error not exceeding plus or minus 50% of the true mean value is represented by the percentages in the three categories from 50% to 150% of the mean value, i.e. 39%. There will, therefore, only be a chance

of/...

of about 4 in 10, i.e. less than an even chance, of not exceeding an error of 50%. In other words in an average of 6 cases out of every 10, the value obtained will be in error by more than 50% of the true mean value of the block of ore.

As could have been anticipated, one sample value as an estimate of the true block value will therefore serve little practical purpose.

(b) A number of samples per block or face: It is an accepted fact that the reliability of the determination of any quantity increases as the number of individual measurements of the quantity is increased. In the present case, therefore, where the true mean value of the block of ore is required to be "measured," the reliability of the measurement will depend on the number of sections sampled ... $m^2(e^{2a^2} - 1)$* where m = true mean value of block

A method of measuring the reliability of the observed mean of a set of N sample values available in respect of such a block is provided by the conclusions reached in paragraph 10(a) of Chapter III, viz., that the distribution of the observed means of sets of N sample values each drawn from a lognormal population will, for practical purposes, also be lognormally distributed with

$$\text{Variance} = \frac{\text{Variance of parent population}}{N}$$

Since in the present case the variance of the parent population consisting of all the individual sample values in the ore block is defined by

$$m^2(e^{2a^2} - 1)^{\frac{1}{2}}* \quad \text{where m = true mean value of block}$$

and a = 0.7

$$= 1.7743m^2$$

the variance of the observed means of the sets of sample values will

$$= \frac{1.7743m^2}{N} = m^2(e^{2am^2} - 1)^{\frac{1}{2}}$$

where/...

*See Formula No. (9).

where N = number of samples per set

and a_m = parameter of lognormal distribution of observed means

i.e., $e^{\frac{1}{2a_m^2}} = 1 + \frac{1.7743}{N}$ ..... (63)

and with N, say, equal to 10, $a_m = 1.75$

The distribution of the observed means of sets of 10 sample values each will then be as follows:-

TABLE 13A

Value Category - values as percentages of True Mean Value of block	Percentage of total number of observed mean values of sets of 10 samples each within the value category
0 to 50%	7%
50% to 75%	24%
75% to 100%	27%
100% to 150%	31%
150% to 200%	8%
Above 200%	3%

From this table it is evident that if a large number of sets of 10 samples each are taken from this particular block of ore, the chances are that, e.g., 31% of all the observed mean values of these sets of 10 sample values each will be within the range of values from 100% to 150% of the true mean value of the block.

If, therefore, the observed mean value of 10 samples is accepted as the true mean value of the block, then the chances of being in error by not more than 50% of the true mean value will be 82%* or, say, 8 in 10.

Following/...

*24% + 27% + 31%.

Following a similar method of calculation for various numbers of samples per set, the following table of probabilities was prepared:-

TABLE 14

No. of samples per set = N	"a_m" for distribution of observed means	Maximum error either side of true mean value of block*				
		10%	20%	30%	40%	50%
		Probability percent				
1	0.7	7	14	22	30	39
5	1.28	13	27	42	56	71
10	1.75	18	38	55	72	83
15	2.12	23	45	65	79	88
20	2.42	27	51	71	85	92
30	2.95	33	60	80	92	96
50	3.78	41	72	90	97	99
100	5.32	55	87	97	99	100
200	7.50	71	98	99	100	100
500	11.87	92	100	100	100	100

E.g., there will be a 51% probability, say, an even chance, of the observed mean of 20 sample values not being in error by more than 20% either side of the true mean value.

It becomes clear from this table what "chances" are taken by the valuator in the valuation of individual blocks and stope faces. In order to ensure, say, that the observed mean of the sample values for a block or face will not be in error by more than 10% in an average of 9 cases out of every 10, (i.e., a 90% probability or confidence limit), a total of some 500 samples will be necessary. This is far in excess even of

the/...

*These probabilities were obtained graphically and are therefore not necessarily correct to the nearest 1%.

the usual number of sample sections available for an average ore reserve block and appears nearly astronomical when compared with the number of stope sampling sections per face, usually ranging from 10 to 30.

It is also obvious that the customary practice of starting and stopping stope faces, or portions of stope faces, on the evidence of one, or even two or three, stope samplings, must inevitably result in the stoping of a considerable tonnage of unpay ore and in the rejection of a fair percentage of pay ore.

Example: Consider a mine on which the selection of its stope tonnages as pay or unpay, is based in general on the observed means of, say, 30 samples per working face.

Referring now to the above Table No. 14, it is evident that there is little chance of a stope face with a true mean value of double the pay limit value, (i.e. where the pay limit = 50% of true value of face) yielding an observed unpay mean value on 30 sample values (since 96% of all the observed averages of sets of 30 sample values will not be in error by more than 50%). Stope faces with lower true mean payable values will, however, all yield a varying proportion of observed unpay mean values based on which such faces would be stopped as unpay in spite of the fact that they are actually payable. Similarly, stope faces with true unpay mean values will all yield some percentage of observed pay mean values on which such faces would be worked, although actually unpay.

(c) Conclusion re stoping policy: It is evident from the above analysis, that due to the limited number of sample sections per stope face, the practice of basing stoping policy on current stope sampling is in general unsound.

Unless the number of stope sampling sections per face is increased at least tenfold, it therefore seems that stoping policy should be based solely on block valuation, and that this

in/...

in turn should be based on as large a number of sample sections per block, as practically possible. Such block valuations could then be relied upon irrespective of any subsequent stope sampling results based on a lesser number of sections.

(d) The advisability, or otherwise, of continuing with stope sampling: The conclusions reached under (c) above immediately raise the question as to whether stope sampling should not be discontinued. It might be held that stope sampling serves a useful purpose in the following instances:-

(i) Where the reef band stoped is narrow and difficult to follow, stope sampling serves as a check on the stoping of the correct band of reef.

(ii) Where no block valuation is available, or where it has in any case been based only on the raise or a previous stope sampling, e.g. in the case of footwall haulages with practically no driving on reef.

(iii) Where stope sampling serves as an overall control on the grade of ore mined in the mine as a whole.

(iv) Where a sampling of a stope face is required for the purpose of blocking out a new ore reserve block.

In the case of (i), regular inspection by experienced samplers with, perhaps, only an occasional check sample, will serve the purpose even better than a monthly, or quarterly, stope sampling. Such inspections can also cover aspects such as the amount of external waste stoped, etc.

As far as (ii) is concerned, it is a question of policy and judgment whether it would not be preferable to confine stoping operations entirely, if possible, to areas which have been properly blocked out. In blocking out, the ideal

conditions/...

conditions naturally exist where a block has been developed around its entire periphery. Mining policies frequently do not permit of this being done. In such a case where, for example, little or no driving is normally done, it may be preferable from an economic point of view to resort to stope driving, since the additional cost, if any, of developing such drives well ahead of the working faces instead of advancing the stope faces without driving, can in many instances be outweighed by the increased accuracy of selectivity obtained in valuation.

Where practical mining considerations and the general layout do not permit of the proper development of a block before stoping commences, and blocking has to be done off raises or stope faces only, the continuation of stope sampling will be inevitable, but should rather be less frequent with as large a number of individual sample sections per sampling as possible.

In the case of (iii) above, block values will, on average, serve as good or better a purpose as an overall grade control from stoping sources, and stope sampling under (iv) will not be required where the previous block valuation was based on a sufficiently large number of development sections, since it will be preferable to apply the original block valuation until the whole block is worked out, rather than valuing a remaining portion of the block anew on a limited number of values.

It seems therefore, that stope sampling could justifiably either be discontinued, or very much curtailed, and that the labour at present employed for this purpose could be more profitably engaged in increasing the number of development sampling sections per ore reserve block to a maximum.

2. Forecasts/...

2. Forecasts of Mine Grade from Borehole Results.

(a) Confidence to be placed in borehole results.* Since a borehole core section can, for practical purposes, be regarded as a "perfect" sampling section of a somewhat smaller than "standard" size, the distribution of the values of all the theoretically possible boreholes which can be sunk on a mining property should be of the same nature as that of all the possible underground sampling section values for the same mine. Further, as the latter has been shown to be lognormal in form, with parameters "a" covering (in the cases considered) the range 0.55 to 0.75,** a figure of 0.65 being common, it is obvious that the confidence which can on average be placed in the observed mean of a number of borehole values will be rather less than that reflected for specific values of "N" in Table No. 14 (which was based on "standard" size samples and an "a" value of 0.7). For practical purposes this table can, however, also serve to indicate roughly the probability of any specified maximum error in the observed average of the relevant number of borehole values,

e.g., the probability of the observed mean of, say, 10 borehole values not being in error by more than 30% of the true mean value of the mine, is roughly 55%, i.e. there is roughly an even chance of obtaining an observed mean value within 30% either side of the true mean value of the mine.

Reference to Table No. 14 also indicates that, whereas a considerable increase in the confidence to be placed on the observed mean of borehole values can be obtained by increasing the number from 1 to 5, and to a somewhat lesser

extent/...

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*The question of incomplete core recoveries will not be considered and "borehole values" consequently imply only values obtained from complete core recoveries.

**Chapter V, paragraph 2(f).

extent, from 5 to 10, an increase in the number of boreholes beyond 10 requires to be considerable to be of appreciable additional benefit.

(b) Additional confidence gained from deflections: The problem of deflections was approached practically by regarding, e.g., two adjacent development sampling sections (with sampling at 5 ft. intervals) as being equivalent to a borehole section + 1 deflection, and three consecutive sampling sections as being equivalent to a borehole + 2 deflections.

The drive sampling results in a portion of Mine B were employed for this purpose, the area measuring 6,000' on strike x 4,000' on dip. The distribution of the individual sample section values (equivalent to boreholes) was first determined, followed by that for the observed means of all the pairs of adjacent section values (equivalent to boreholes + 1 deflection each), and then by the distribution for the observed means of sets of three consecutive sampling section values (equivalent to boreholes + 2 deflections each). The results were as follows:-

	Value of "a"
Distribution of single sample values:	0.651
Distribution of means of 2 adjacent sample values:	0.716
Distribution of means of 3 consecutive sample values:	0.743

The variances of these three distributions will be:*

(i) Single sample values: 2.2538 m^2 where m = true mean value of the section of Mine B concerned

(ii) Means of sets of 2: 1.652 m^2

(iii) Means of sets of 3: 1.474 m^2

Now, referring to No. (24), the variance of the distribution of the means of sets of N random sample values drawn

from/...

*Formula No. (9).

from No: (i) above can be accepted as

$$\frac{2.2538\ m^2}{N} \qquad \ldots\ldots\ (64)$$

and similarly for N random sample values from (ii) and (iii), the variances will be

$$\frac{1.652\ m^2}{N} \text{ and } \frac{1.474\ m^2}{N}, \text{ respectively} \qquad \ldots\ldots\ (65)$$

Now, since N sample values drawn from (i) can be compared with N borehole values in a mine, N sample values from (ii) with the values of N boreholes, each with one deflection, and N sample values from (iii) with the values of N boreholes, each with two deflections, and as two lognormal distributions with the same mean value and the same variance will be identical, the variances in Nos. (64) and (65) above can be equated in order to determine the relative "N" values for equivalent confidence limits, e.g.,

The means of the values in sets of 20 boreholes + 1 deflection each will be distributed with variance

$$= \frac{1.652\ m^2}{20} = .0826\ m^2$$

and the means of the values of N single boreholes will be distributed with variance $= \frac{2.2538\ m^2}{N}$

For these two variances to be identical

$$.0826\ m^2 = \frac{2.2538\ m^2}{N}$$

$$\text{and } N = 27.3$$

i.e., on average 20 boreholes each with 1 deflection will yield as reliable an indication of the mine's mean value as 27 single boreholes.

The following table was compiled on this basis to indicate the number of boreholes (with and without deflections),

which/...

which will, on average, yield equally reliable results under the conditions stated.

TABLE 15

Boreholes without deflection(s)	Boreholes with 1 deflection each	Boreholes with 2 deflections each
1.36 say 1	1	-
2.73 " 3	2	-
6.82 " 7	5	-
13.6 " 14	10	-
27.3 " 27	20	-
68.2 " 68	50	-
136.4 " 136	100	-
1.53 say 2	-	1
3.1 " 3	-	2
7.6 " 8	-	5
15.3 " 15	-	10
30.6 " 31	-	20
76.4 " 76	-	50
152.9 " 153	-	100

e.g., 10 boreholes each with 1 deflection will yield, on average, as reliable an indication of the true mean value of the mine as 14 single boreholes.

It is evident from the above that even 2 deflections at a borehole are not equivalent in value, for confidence purposes, to an independent separate borehole. This table will obviously only apply where conditions on a mine will be identical to those on Mine B, but a similar investigation for Mine A, where conditions are radically different and somewhat exceptional ("a" for large section of mine = .764, "a" for

pairs/...

pairs of development values = .948)* shows that the general indications in the above table will not vary overmuch from mine to mine, e.g. on Mi . A, 10 theoretical boreholes each with 1 deflection will on average be equivalent to 18 theoretical boreholes without deflections.

(c) The effect of the distance of a deflection from the original intersection: The question naturally arises to what extent borehole + deflection results can be rendered more useful by spacing the deflections further apart than the 5 feet on which the above investigations were based.

Based on the development values of the section of Mine B already referred to, the type of investigation outlined above was repeated, and the variances of the distributions of the observed mean values of pairs of development values 50' and 100' apart, respectively, were determined (on log probability paper), and also for sets of 3 consecutive values at 100' intervals:

Distribution	"a"	Variance
Means of 2 values 50' apart (equivalent to borehole + 1 deflection 50' away)	.743	1.474 m^2
Means of 2 values 100' apart (equivalent to borehole + 1 deflection 100' away)	.772	1.314 m^2
Means of 3 values at 100' intervals (equivalent to borehole + 2 deflections 100' on either side)	.795	1.206 m^2

Adopting the previous line of argument, the theoretical "value" of deflections in terms of the equivalent number of independent boreholes** for a mine where conditions are

approximately/...

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*Underground samples are all taken in duplicate and section values are therefore equivalent to those of samples of double the standard size.

**"Independent" boreholes should be "random" samples and should therefore be roughly spaced on a grid pattern for the mine as a whole. It is assumed that only the values for complete recoveries in boreholes or deflections are considered.

approximately similar to those on Mine B, will be as follows:-

Deflection No. - Distance	Number of Equivalent Independent Boreholes
1 - 5' away	.36
2 - 5' away (either side)	.53
1 - 50' away	.53
1 - 100' away	.72
2 - 100' away (either side)	.87

Depending, therefore, on the depth of the reef, and the relative costs of a new borehole and of the type of deflections listed above, the advisability of sinking a new borehole, or of resorting to deflections, can be weighed against one another on relative merit and cost. In doing this, it must, however, also be borne in mind that boreholes are not only required for valuation purposes, but also for examining reef continuity, geological structures, faulting, etc.

(d) General conclusions regarding the under- or over-estimation of a mine's value from borehole results: Referring again to Table No. 13, which can also serve as a rough indication of a typical distribution of borehole values in a mine, it is evident that 69%* of the individual borehole values can, on average, be expected to have a value lower than the true mean value of the mine, and consequently, that if one borehole only is sunk on a property, the chances are 69 against 31 in favour of the borehole value obtained being lower than the true mean value for the mine. In roughly 2 cases out of every 3, therefore, a single borehole value will provide too low an indication of the true mean value of the mine.

Where more than one borehole is drilled, the odds in favour of obtaining too low a value will decrease gradually,

but/...

*i.e., for an "a" = 0.7.

but will only disappear when an infinite number of boreholes are drilled. The following provides a direct indication of the odds in favour of too low an indicated value for a mine based on various numbers of boreholes drilled:-

TABLE 16*

No. of boreholes drilled	Approximate Odds in favour of too low an indicated value
1	2 to 1 (69%)
5	3 to 2 (61%)
10	53 to 47 (53%)
20	52 to 48 (52%)

With an "a" of less than 0.7, which is quite common, the odds will be somewhat greater than those listed above.

It is therefore clear that borehole results will in the majority of cases tend to indicate too low an average value,** but that the odds in favour of this practically disappear when the number of boreholes drilled on a property exceeds 10.

3. Bias Errors Introduced in Mine Valuation due to the Limited Number of Available Sample Sections.

(a) Bias errors in different Ore Reserve value categories: For the purpose of illustrating the extent to which such bias errors can be introduced, the following theoretical case in which conditions as assumed roughly approximate those on Mine C, will be considered.

Consider a developed section of a mine fully blocked out, the true values of the various block tonnages of ore being

distributed/...

*Based on Formula No. (24).
**Apart from any bias due to incomplete core recoveries.

TABLE 17

Distribution of true mean values of all blocks	Row No.	Value category limits - dwts	Freq. %	Mean values	Distribution of means of 60 samples per block: 1	2	3	4	5	6	7	8	9
	Column No.		1A	1B									
	Value category limits - dwts				2.55	2.95	3.95	4.95	5.95	6.95	7.95	8.95	
	1A		% Freq.		12.57	6.32	22.42	19.21	13.46	9.36	6.20	3.66	6.80
	1B		Freq. %	Mean values	2.05	2.76	3.45	4.43	5.42	6.41	7.41	8.41	11.20
	1	2.55	9.55	2.12	8.26	0.48	0.81	-	-	-	-	-	-
	2	2.95	7.10	2.75	2.88	2.02	2.06	0.14	-	-	-	-	-
	3	3.95	22.03	3.46	1.43	3.41	12.67	4.03	0.49	-	-	-	-
	4	4.95	20.74	4.43	-	0.41	5.81	9.44	3.94	0.97	0.17	-	-
	5	5.95	15.35	5.42	-	-	1.00	4.45	5.45	3.15	1.01	0.23	0.06
	6	6.95	10.06	6.41	-	-	0.07	0.99	2.67	3.12	2.06	0.80	0.35
	7	7.95	6.2[illegible]	7.41	-	-	-	0.15	0.77	1.55	1.77	1.12	0.84
	8	8.95	3.69	8.41	-	-	-	0.01	0.14	0.51	0.92	0.89	1.22
	9		5.28	10.88	-	-	-	-	-	0.06	0.27	0.62	4.33
	10	Mean of true values in columns		4.90	2.42	3.19	3.70	4.57	5.40	6.17	6.97	7.89	9.73
	11	Theoretical Block Plan Factor		-	118%	116%	107%	103%	100%	96%	94%	94%	87%
	12	Actual B.P.F. - Mine C		-	-	119%	106%	108%	105%	101%	98%	93%	81%

distributed (lognormally with "a" = 1.648, and mean = 4.90 dwts/ton) in the value categories as shown in column No. 1A of Table No. 17.*

The distribution of sample values within each block of ore (assuming equal "a's") follows a lognormal curve with parameter "a" = .65, and an average of 60 sampling sections are available for estimating the true mean value of each block. The observed means of all the possible selections of such sets of 60 samples from each block can, therefore, in turn be regarded as lognormally distributed with parameter "a_x" obtained in solving the equation

$$\frac{m^2(e^{2(.65)^2} - 1)}{60} = m^2(e^{2a_x^2} - 1)^{**}$$

$$\text{i.e. } a_x = 3.672$$

Consider now the category of blocks having a true mean value of 2.75 dwts (i.e. row No. 2). If these blocks are now sampled one by one to the extent of 60 samples per block, the means of the sets of sample values so obtained will be distributed lognormally with parameter "a" = 3.672 and mean value = 2.75 dwts/ton as shown by row No. 2 in this table, e.g. of the total of 7.10% of the mine tonnage in this value category, only 2.02% will yield <u>indicated</u> values in the category 2.55 to 2.95 dwts/ton, 2.06% will yield <u>indicated</u> values in the category 2.95 to 3.95 dwts/ton, etc. In this way the complete distribution of the block tonnages in the various value categories based on the means of sets of 60 values per block can be determined as indicated by all the rows (Nos. 1 to 9).

A study of Table No. 17 now reveals that on sampling as stipulated, the <u>apparent</u> tonnage in, e.g., the value category 2.95 to 3.95 dwts/ton (column No. 3) will be 22.42% and

will/...

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* See opposite.
** See Chap. III, par. 10.

will actually consist of

0.81%	with a	true	value	of	2.12	dwts/ton	
2.06%	"	"	"	"	"	2.75	"	"
12.67%	"	"	"	"	"	3.46	"	"
5.81%	"	"	"	"	"	4.43	"	"
1.00%	"	"	"	"	"	5.42	"	"
0.07%	"	"	"	"	"	6.41	"	"
Total: 22.42%	with a	true	mean value	of	3.70	"	"

In a similar manner the <u>true</u> values of the blocks falling in the other "indicated" value categories can be determined and are shown in row No. 10. But the indicated average values of the blocks within these categories, as obtained from sampling results, will be the values lying approximately halfway between the respective category limits* as shown in row No. 1B. Comparing now the values in this row with those in row No. 10, it is obvious that <u>block valuation based on a limited number of samples per block will result in the under-valuation of blocks listed in the low grade categories and over-valuation of blocks listed in the high grade categories</u>.

Now, it is a wellknown fact on most mines on the Witwatersrand that when ore reserve blocks are extracted, the results from blocks valued as high grade are on average disappointing, whereas results from blocks valued as low grade on average exceed expectations.** The best measure of those phenomena is provided by the Block Plan Factors observed in the various/..

*The correct mean values do not coincide exactly with the central values of the columns and were determined by fitting a lognormal curve to the distribution in row 1A and further calculation from formula No. (23).

**Applicable only where no cutting of block values is done.

various categories, those for Mine C being listed in row No.12 (Table No. 17). The question naturally arises why current stope sampling can be accepted as a criterion for criticising ore reserve valuations in the various categories, but the explanation can readily be supplied. Take, for example, the blocks as valued in the category 2.55 to 2.95 dwts/ton (column 2, Table 17), and in fact comprising a range of blocks with true values in the categories from 0 to 4.95 dwts. The current face samplings in respect of these blocks (based on an even smaller number of samples per block than in the case of block valuations) will err on either side of the true values in the range 0 to 4.95 dwts, but the mean of all such samplings (particularly over a whole year of operation) will provide a reasonable estimate of the true mean value of the combination of blocks under consideration.*

(b) Block Plan Factors in value categories - actual and theoretical: The theoretical Block Plan Factors in row No. 11, Table No. 17 (i.e. true values in row 10 divided by indicated values in row 1B) can now be compared with the observed Block Plan Factors (in the various value categories) for Mine C (row No. 12), where the conditions assumed in the table are roughly approximated. The agreement is sufficiently close to indicate that the contentions on which the calculation of the theoretical Block Plan Factors was based probably provide the solution to the problem of the variation in the Block Plan Factors as observed in the various value categories.

A shorter and more exact method of arriving at the theoretical Block Plan Factors is provided by the brief investigation into lognormal correlation and lines of regression in Chapter III, paragraph 14, and the application of formula No. 59(b).

The/...

*Provided the total number of samples taken in the category is sufficiently large - see paragraph 1 above.

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