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User Pairing and Power Allocation in Underlay Cognitive NOMA Networks

By

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1914265

Submitted in the fulfilment of the academic requirements for the degree of Master of Science in Engineering (MSc. Eng.) in the School of Electrical and Information Engineering in the Faculty of Engineering and the Built Environment, University of the Witwatersrand.

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October 2020

Author's Declaration

I, **Leutsoa Moteka** hereby declare that:

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……..

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Abstract

The unique structure of Non-Orthogonal Multiple Access (NOMA), a candidate for multiple access techniques for fifth-generation mobile networks, poses formidable design challenges when the number of users in the network rises. Fifth generation networks, however, demand hyper connected societies with phenomenal number of users. In a multi-channel NOMA system with a large number of users, the literature has shown that the best performance is achieved if no more than three users share a channel. Most research typically allocates two users to a channel. One of the key issues then is user pairing, which must be done in order to maximize the network capacity. A number of user pairing schemes have been derived in the literature, but these have been done mostly for non-cognitive radio networks. A need for user pairing schemes in cognitive networks has therefore risen. Developed schemes need to take into account a plethora of complications such as energy consumption and an increase in interference raised in the cognitive environment.

The main focus of this dissertation is to mathematically model a framework to optimize power allocation and user pairing in a cognitive NOMA network. In particular, we determine various power allocation schemes that can cope with the severe energy constraints of an underlay cognitive network and employ these schemes for use in different user pairing schemes. First, we employ an underlay random pairing algorithm and an underlay channel state sorting pairing algorithm, for use in a large-scale network. Because of the low complexity of these algorithms, we use their performance to study and compare with other pairing algorithms. Then we propose a near-optimal preference list matching algorithm (PLMA) based on matching theory to perform user pairing. Performance evaluation of the proposed schemes is presented through simulations. Results show how that the preference list matching algorithm effectively outperforms other pairing algorithms and can also perform better that the Hungarian algorithm.

Keywords — Non-orthogonal multiple access; user pairing; underlay cognitive networks.

List of Publications

- Leutsoa Moteka, Fambirai Takawira and Chabalala Chabalala, "User Pairing and Power Allocation in Underlay Cognitive NOMA Networks." *Proceedings of IEEE AFRICON Conference* (IEEE AFRICON 2019).

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List of Acronyms

Abbreviation	Description
5G	Fifth Generation
VR	Virtual Reality
UHD	Ultra-High Definition
OMA	Orthogonal Multiple Access
NOMA	Non-Orthogonal Multiple Access
CR NOMA	Cognitive Radio Non-Orthogonal Multiple Access
CRN	Cognitive Radio Network
TDMA	Time Division Multiple Access
OFDMA	Orthogonal Frequency-Division Multiple Access
LTA	Long-Term Evolution
LTA-A	Long-Term Evolution Advanced
SE	Spectral Efficiency
CDMA	Code Division Multiple Access
LDS-CDMA	Low Density Spreading CDMA
LDS-OFDMA	Low-Density Spreading-Based OFDMA
SCMA	Sparse Code Multiple Access
PDMA	Pattern Division Multiple Access
SDMA	Spatial Division Multiple Access
SC	Superposition Coding
MUD	Multiuser Detection
SIC	Successive Interference Cancellation
AWGN	Additive White Gaussian Noise
CIRs	Channel Impulse Responses
F-NOMA	Fixed Power Allocation NOMA
MIMO	Multiple-Input Multiple Output
CSS-PA	Channel State Sorting Pairing Algorithm
MINLP	Mixed Integer Non-Linear Programming Problem

UE	User Equipment
EE	Energy Efficiency
QoS	Quality of Service
CU	Cognitive User
PU	Primary User
SU	Secondary User
PN	Primary Network
SN	Secondary Network
SNR	Signal-to-noise power ratio
BS	Base Station
RPA	Random Pairing Algorithm
CSS-PA	Channel State Sorting-Pairing Algorithm
D2D	Device-to-Device
IoT	Internet of Things
HetNets	Heterogeneous Networks
SWIPT	Simultaneous Wireless Information and Power Transfer
SM	Spatial Modulation
AS	Antenna Selection
CSI	Channel State Information
JPCAP	Joint Power with Channel Allocation Method
KKT	Karush Kuhn Tucker
UDSA	User Difference Selecting Algorithm
AP	Assignment Problems
3AP	Axial Three-Dimensional Assignment Problem
AMBS	Approximate Muscle Guided Beam Search for Three-Index Assignment Problem
FTPC	Fractional Transmit Power Control
PLMA	Preference list-based matching algorithm
LWF	Linear Water-Filling algorithm

List of Symbols

SU_k – Secondary User *K*

SU_l – Secondary User *L*

$\alpha_{k,f}$ – Power allocation factor (User *k* on frequency *f*)

$\alpha_{l,f}$ – Power allocation factor (User *l* on frequency *f*)

$y_{k,f}$ –Received signal for User *k* in NOMA

$y_{l,f}$ – Received signal for User *l* in NOMA

$\eta_{k,f}$ – Gaussian noise term of User *k* at frequency *f*

$\eta_{l,f}$ – Gaussian noise term of User *l* at frequency *f*

$|g_{m,f}|^2$ – Channel gain from the secondary BS to a PU on frequency *f*

$\left|h_{k,f}\right|^2$ – Channel from the secondary BS to the k^{th} SU on frequency *f*

$\left|h_{l,f}\right|^2$ – Channel from the secondary BS to the l^{th} SU on frequency *f*

I_{PU} – Maximum permissible interference power at the nearest *PU*

I_o – Margin threshold constant

$P_{k,l,f}$ – Transmit power of the secondary *BS*

$R_{k,f}$ – Achievable rate of SU_k on frequency *f*

$R_{l,f}$ – Achievable rate of SU_l on frequency *f*

$R_{k,req}$ – Minimum rate requirements by SU_k

$R_{l,req}$ – Minimum rate requirements by SU_l

γ^k – Signal-to-noise ratio of SU_k

γ^l – Signal-to-noise ratio of SU_l

B – System Bandwidth

Chapter 1

Introduction

1.1 Background

The fifth generation (5G) networks are expected to face phenomenal connectivity and capacity demands due to a huge number of devices expected in the next generation networks [1]. This is seen through the emergence of new modern internet technologies such as the internet of things (IoT), virtual reality (VR) ultra-high definition (UHD) 4K videos, real-time interactive services and cloud computing which have led to an exponential growth of global mobile data traffic [2]. With the limited available spectrum, a plethora of problems many of which are inherent to spectral scarcity characteristic associated with wireless transmission media face the scientists and engineers developing the next generation mobile systems. It has therefore become a challenging but a crucial task to meet this phenomenal increase in live mobile traffic. Non-Orthogonal Multiple Access (NOMA) has been proposed as one of the key technologies for the future 5G mobile systems. NOMA promises to usher in a new era of fast internet speeds which will allow wireless communications to thrive in massive connectivity networks [1]-[4]. Due to its popularity, NOMA has attracted a great deal of interest in the research community over the past years. In contrast to conventional orthogonal schemes, NOMA allows one frequency to be used by more than one user at a time resulting in high spectral efficiency [5]-[8]. It does this by superposing the user signals on the same resource block at the transmitter side and then applying algorithms which separate the signals and cancel out the noise generated on the receiver side [9].

Dynamic spectrum access techniques such as the cognitive radios (CR) have also been envisaged for the 5G mobile systems. Cognitive radios aim to let unlicensed users have access to the licensed spectrum through heterogenous architectures, enhancing spectral efficiency significantly [10]. However, introducing unlicensed users into the licensed spectrum comes at a cost of interference being introduced, and therefore controlling mechanisms would be required for smooth operation [11]. A great deal of researchers have also been attracted to incorporating NOMA into cognitive network to yield extra spectrum efficient technologies.

NOMA enables the power domain to be used to serve multiple users simultaneously, and this nature of NOMA can yield high spectrum efficient technologies when incorporated with other technologies. The combination of NOMA and cognitive networks into cognitive NOMA (CR NOMA) therefore constructively combines the advantages of each technology to further improve spectrum utilization in 5G networks [12].

User pairing has also received a lot of attention recently from researchers as one of the ways in which spectrum efficiency can be enhanced [13]. User pairing aims to pair users in NOMA, in order for the successive interference cancellation (SIC) process at the receivers to be performed with low complexities [14], [15]. In NOMA systems, user pairing and power allocation are critical to achieve high performance gains. This is mainly because NOMA allows multiple users to share the same resources band, and then exploiting the power domain. It is obvious that the performance of NOMA also depends on how the power is distributed amongst the multiplexed users. Existing literature usually has two users paired for each subcarrier and a power allocation scheme employed for the NOMA users. It has also been shown in recent literature that users with large channel gain difference should be paired with each other to achieve high performance in NOMA [16]. Varying both factors to develop schemes which enable high performance in NOMA systems is crucial in the development of algorithms which operate in cognitive NOMA systems. Several algorithms have been developed in this context; however, most do not consider user pairing and power allocation in different paradigms of cognitive networks as this results in high complexity algorithms.

The authors in [17] derive a user pairing scheme in a downlink cognitive NOMA network and employ matching theory to perform user pairing. In their system, the primary users negotiate power allocation for transmission with the cognitive users, guaranteeing a satisfaction of both primary and cognitive user sum rates. Results show that the proposed solution outperforms conventional orthogonal multiple access (OMA) schemes and has a low complexity. A joint user pairing with power allocation scheme is presented in [18], for a full duplex cognitive network. The authors put focus on both uplink and downlink transmissions in an underlay mode. They start with formulating their user problem into an optimization problem and then continue to solve it using a low complexity algorithm and a high complexity algorithm. Their simulation results show that the high-complexity algorithm is superior to low-complexity algorithm in terms of sum rates, for all the power allocation methods used. A channel gains

sorting approach to user pairing is considered in [19]. The authors consider a hierarchical power allocation process. Their simulation results show that the system can scale well and encompass many users while outperforming most recent approaches in sum rates. Moreover, in [20], a novel system with a system model that employs NOMA between three users is considered. Basically, a cell-edge user's performance is boosted by two users who are very close to the base station using NOMA. Simulation results show superior sum rate performance when this pairing scheme is compared to orthogonal schemes.

1.2 Motivation and Problem Statement

Existing literature in user pairing reveals the evolution of different schemes that have been proposed to provide solutions for users pairing in NOMA. Nevertheless, with this literature at hand, most of the user pairing done in NOMA systems is in either downlink or uplink. Not much work has been done in CR NOMA scenarios, let alone the different paradigms of CR NOMA networks. Works presented in different paradigms of CR NOMA, either underlay, overlay or interweave, do not involve user pairing in a secondary network which considers many users while also keeping an eye on individual primary user services. Most schemes consider employing the Hungarian algorithm [21] to perform user pairing which is exhaustive and has a high complexity. Based on our observations, this dissertation provides faster and less complex user pairing schemes which not only consider a system consisting of many users while protecting primary user services, but also a system which operates in underlay cognitive NOMA networks.

The research question for this dissertation is therefore:

Given an underlay CR NOMA network, consisting of a primary network and a secondary network, how can the secondary users that opportunistically access primary user capacity be paired, to form a NOMA system, without compromising the primary user connectivity?

1.3 Research Objectives

The main objectives for the research presented in this dissertation are:

1. To modify existing random pairing techniques such that they work in an underlay cognitive cellular network, operating under strict capacity constraints of the users.

2. To modify existing channel state sorting user pairing techniques by developing a new sorting scheme in an underlay cognitive cellular environment under strict capacity constraints of the users.
3. To study and analyse different power allocation schemes for use in underlay CR networks.
4. To develop a new user pairing algorithm which fits the proposed system model for use in an underlay CR NOMA network.

1.4 Relevance and Research Contributions

Having considered these different user pairing schemes, in different NOMA architectures, it is evident that none of these systems consider secondary user communication with the primary user in underlay mode. 5G on the other hand is expected to have full communication between the secondary network and the primary network. This means that both networks will coexist and operate simultaneously. The proposed technologies, which would undergo standardization under the aegis of 5G network development, include heterogeneous networks (HetNets), machine-to-machine communications, device-to-device (D2D) networks, and internet of things (IoT) among others. These technologies adopt layered architectures thus communication between the secondary network and the primary network in these technologies is essential [22]-[24].

These observations motivate our work to consider an underlay cognitive NOMA network. This will help with enabling the concurrent transmissions between the primary network and the secondary network while protecting primary user services. Moreover, the user pairing schemes discussed in the literature are heuristic and hence their performance can be improved. This research therefore further aims to investigate user pairing schemes in NOMA and then develop better heuristic approaches to the user pairing problem as well as optimal approaches that are less complex.

The research contribution of this dissertation are as follows:

1. User pairing schemes in NOMA are developed for a new heterogeneous network model consisting of a primary cellular network and a secondary cellular network in which transmissions in the secondary network are in underlay mode. Both secondary users

(SUs) and primary users (PUs) can transmit simultaneously, and the transmit power at the secondary BS is constrained according to underlay CR network interference constraint property. This work differs from the studies in the literature in that the individual user capacities in the secondary network are considered before secondary transmissions occur, while at the same time ensuring that the primary network performance is not affected by the secondary user transmissions.

2. This work develops an underlay channel state sorting pairing algorithm (CSS-PA) that ensures that both individual user capacity constraints are met even when operating under the underlay power constraints. This differs from the work in the literature in that, the authors in [25] consider a channel state sorting algorithm but without considering the underlay network interference constraint property. The authors also do not cater for the individual secondary user capacity constraints in the algorithm, this work considers both. The channel sorting scheme is compared with an underlay random user pairing (RPA) scheme which is enhanced in order to include the underlay power constraints and also consider the individual secondary user capacity constraints.
3. A preference list matching algorithm (PLMA) which solves the user pairing problem using matching theory is also presented. Unlike the centralized algorithms, the preference list matching algorithm uses matching theory which makes use of Gale Sharpley theorem to obtain a solution to the matching problem. Like the conventional distributed matching algorithms [25], the preference list matching algorithm needs to know the full channel state information of the users before any user pairing or matching can be performed. This reduces the system complexity. The performance of the preference list matching scheme is shown to closely match the performance of the exhaustive Hungarian method.
4. The effect of power splitting amongst the paired secondary users in NOMA is also investigated in this work. Two power splitting mechanisms, namely LaGrange based power splitting and also a bound based power splitting are considered. From the simulation results, we illustrate that the underlay RPA and the underlay CSS-PA algorithms seem to have a higher user pairing performance when using the bound based power splitting. The results also show that the PLMA algorithm performs better with the bound based power allocation when compared with the Lagrange based power splitting.

5. Development of theorems 2, 5 and 6 in a CR-NOMA network which aid in the solution to the user pairing problem.

1.5 Methodology

The main objective of this research is to find out how can the secondary users that opportunistically access primary user capacity, be paired to form a NOMA system without compromising the primary user connectivity. This is done by implementing modifications to existing algorithms and also development of theorems. To meet the objectives, the research is done by:

(a) Gathering enough comprehensive literature and knowledge on the different interpretations of NOMA, CR networks, user pairing, channel allocation and power allocation in NOMA.

(b) Performing derivations of closed form expressions and theorems for the achievable sum-rates under:

i. transmit power constraint at the secondary nodes;
ii. interference constraint at the primary base station;
iii. the primary interference in the secondary network.

(c) Transforming the system models and the algorithms to pseudo code and implementing them on MATLAB.

(d) Comparing of both analytical and simulation results.

(e) Report Writing

- Journal and/or Conference Papers.
- Dissertation writing.

1.6 Dissertation Organization

The following provides the roadmap for remainder of this dissertation:

Chapter 2 Presents a brief literature survey on NOMA, Cognitive Radios, Cognitive NOMA networks. The principles and features of these components are introduced and discussed in this chapter.

Chapter 3 Explores different research efforts that have been put forward to provide an insight into power allocation and user pairing in NOMA. Furthermore, existing literature on assignment problems and their structure is also discussed.

Chapter 4 Gives the system model and the formulation of the user pairing problem. Performance analysis of the user pairing problem obtained is also given here.

Chapter 5 Proposes modification of the algorithms, random pairing algorithm (RPA), channel state sorting algorithm (CSS-PA) and gives modified algorithms in an underlay cognitive NOMA network. A new preference list matching algorithm is also presented. Simulation results and a performance analysis of the algorithms are also presented.

Chapter 6 Presents a summary of the research done in this dissertation. The main points of the work presented in this dissertation are highlighted. A brief discussion which highlights possible investigations and directions for future research work in this field is presented.

Chapter 2

NOMA and Cognitive Radio Networks: Literature Survey

For decades, the conventional multiple access techniques have been dominant in a variety of wireless networks. Research has been focused on these technologies for almost the past two decades. This includes the well-known and well researched orthogonal multiple access technique which gave birth to wireless communication from 1st generation (1G) to the 4th generation (4G) communications. However, the introduction of NOMA, has caused a major shift in research fields as the race to the 5th generation wireless technologies which can offer high data rates and effective spectrum saving presented itself. Since then, NOMA has been the go-to multiple access technique for majority of researchers and has been envisioned to be a crucial component of the evolving 5th generation mobile networks [26]. From the introduction of this research, it is evident that NOMA when used with different technologies like cognitive radios promises even more advantages. These include improved spectral efficiency, higher cell-edge throughput, relaxed channel feedback, and low transmission latency.

This chapter aims to identify, evaluate and synthesise the relevant literature within NOMA as well as the different paradigms of cognitive radios. Detailed information from current research for each technology is also scrutinized here. This will make it easy to illuminate how the knowledge has evolved within both technologies, in particular shed light on what is emerging, what has already been done, and what is has been accepted.

2.1 Orthogonal Multiple Access (OMA)

Multiple access technologies have changed throughout each generation of wireless mobile communication systems. To this day, there exists only two major approaches, namely OMA and NOMA. All OMA schemes separate user signals in orthogonal resources (frequency, time, code domain), creating the possibility of a perfect receiver which can separate wanted from

unwanted signals [26]. This means that during transmission, signals are orthogonal to each other and only a single user can be served per resource block.

Examples of schemes which employ OMA approaches are frequency-division multiple access (FDMA), Time division multiple access (TDMA), code-division multiple access (CDMA) and orthogonal frequency-division multiple access (OFDMA). These schemes are fully orthogonal. In FDMA the frequency spectrum is divided amongst the number of available users for transmission. Obviously, this is not spectrum efficient. For TDMA, all the users in the system have to share the same frequency band being used for transmission, through taking turns in time. This means that the users will utilize the communication channel one after the other in succession. Each user needs to be given a specific slot for them to use the available frequency [26], [27]. FDMA was predominantly used in the first-generation cellular networks while TDMA was used in the second generation (2G) cellular networks. Similarly, in third generation (3G) cellular networks, CDMA and OFDMA are the main technologies being used currently. Users are assigned resources depending on their code in CDMA, while in OFDMA on the other hand, users employ subcarriers in a frequency band, for transmission. The subcarriers need to be orthogonal to one another before transmission. Fourth generation (4G) wireless network mainly use CDMA and OFDMA multiple access schemes. This approach is used even in practical communication systems, including Long-Term Evolution (LTE) and Long-Term Evolution Advanced (LTE-A) [27]. Although these schemes are being used this much and for different technologies these days, they have major drawbacks that come with using OMA. They can only support a limited number of users in accordance to the available resources and also have major scheduling complications which arise when the number of users becomes enormous.

2.2 Non-Orthogonal Multiple Access (NOMA)

The requirements of 5G are demanding and need a multiple access scheme which offers, high energy efficiency, low latency and high spectral efficiency and tremendous connectivity [27], [28]. This is because the upcoming generation will be a more connected and a hyper related society, demanding 5G technology to deliver technologies which can live up to their expectations of speeds up to 20Gps transmissions. With more businesses going to the cloud, all client and trade aspects will also need extra speeds and connectivity as businesses grow.

The internet of thing (IoT) is also anticipated to launch worldwide and would require high spectral efficient schemes for 5G. This includes schemes which can support more than one million users communicating per square kilometre and still offer gigabit speeds. OMA cannot meet these requirements due to its known limitations [27], [28]. Moreover, it is impossible to meet these requirements using any 4G technology as they put restrictions on the available users per frequency spectrum available. To mitigate these challenges upon wireless communications, NOMA has been proposed as a promising candidate for 5G.

In contrast to OMA, NOMA allows one frequency to be used by more than one user at a time. It does this by superposing the user signals on the same resource block at the transmitter side and then applying algorithms which separate the signals and cancel out the interference generated on the receiver side. NOMA does this by classifying the signals into high level and low level. The high-level signal is isolated and cancelled out using a detection algorithm, and then the low-level signal is decoded. One of the detection algorithms used by NOMA is successive interreference cancellation which we will discuss later in the chapter. By working this way, NOMA not only exploits the path loss differences between the high-level signal and the low-level signal, but it also achieves massive spectrum efficiency [28], [29]. However, this happens at a cost of additional processing power in the receiver, however. This mode of operation has significant spectral efficiency gains when compared to OMA. In the past decade, a number of different NOMA schemes have been proposed, and they allow multiple users to utilize a frequency band then exploit the power domain, whereas the conventional mobile networks have been solely relying on the frequency/time/code domain exploitation [29].

In the recent years several NOMA schemes have been proposed. These can be divided into two main categories, namely, power-domain and code-domain NOMA [30]. Power-domain NOMA focuses on multiplexing users in power domain and then employing a power sharing algorithm to distribute power in the domain. Unlike Power-domain NOMA, Code-domain NOMA symbolises CDMA in some way. Code-domain NOMA uses user-specific spreading codes/sequences which will help in identifying the different users at either side of the communication channel, while power-domain NOMA uses the same code for the users. In this work, however we focus on power domain NOMA only. Moreover, there exists additional multiple access techniques which have been proposed in recent years and have a close relationship with NOMA. These are pattern division multiple access (PDMA) and spatial

division multiple access (SDMA). PDMA designs non-orthogonal patterns by maximizing user diversity and minimizing the overlaps amongst users in the system. SDMA takes some inspiration off CDMA systems. SDMA systems differentiate users in the system through usage of channel impulse responses (CIRs). Each user has its own user-specific channel impulse responses. SDMA is used when the number of receiving base station antennas is lower than users in the uplink [30].

2.2.1 Superposition Coding (SC)

Superposition Coding (SC) technique simultaneously communicates information to multiple users by a single source [31]. In NOMA the base station deals with a problem of having to communicate with multiple users at the same time. Conventional approaches like OMA use orthogonal channels for each user by dividing them in time/frequency/code-division. However, this approach does not achieve maximum possible sum rates although it can eliminate any possible interference.

Superposition Coding (SC) is proven to achieve high sum rates in a multiuser Gaussian system. Authors from [32] also argue that SC performs better, and conserves power compared to conventional multiuser systems without SC. Their argument implies that SC consumes lesser transmission power. Because users in any system will be having different channel gains which can be ordered, superposition coding exploits this channel ordering to superimpose users optimally [32]. In simpler terms, it can manipulate the different distances each user has towards the base station and transmit information to the users based on this information. This feature makes it an important addition in the way NOMA is structured. Because NOMA serves multiple users in the same domain at the same time, employing superposition coding strategy is vital in its operation. To make SC practical, we show that in a two-user system SC can easily be used to transmit information to the users.

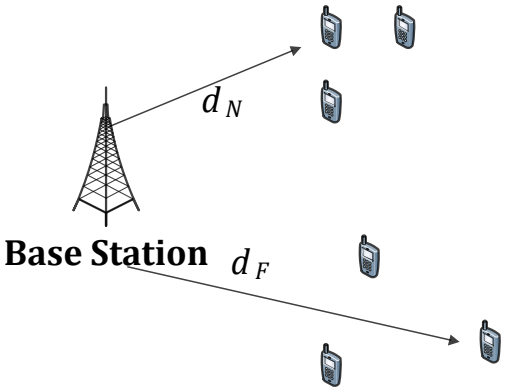


Figure 2. 1 Illustration of two-user superposition coding.

In order to show how SC is performed, we motivate using a two-user scenario in downlink system consisting of several users placed at different distances from the base station. In Figure 2.1, it is always possible to pick two users at different distances d_N and d_F, and refer to them as the near user and the far user, respectively. An observation which can be made is that, geographically, the near user at d_N has a stronger channel as they are closer to the base station. Similarly, the far user d_F has a weaker channel as they are placed far from the base station. This means that any packet to the far user can be decoded at the near user vicinity as well. Thus, the BS can superimpose the information to both the users at different power levels using NOMA in the process. Since the closer user has a stronger link to the base station, they can regenerate and cancel the signal meant for the far user and then decode their own desired signal. This process is called successive interference cancellation (SIC) and we will take a look at it in detail in the sub-section below.

2.2.2 Successive Interference Cancellation (SIC)

When the superposed signal is sent by the transmitter in NOMA, there is a need for an efficient decoding scheme at the receiver which can be employed to retrieve signals. Successive interference cancellation is one such method. SIC is able to consider the difference in signal level of the incoming superimposed signal and then decode the arriving packets simultaneously [33]. The exploitation of the difference in signal level is one main reason why SIC works perfectly with NOMA. In a SIC process with various users, the user with the highest channel gain towards the base station decodes the stronger signal first in the superimposed signal and then subtracts it from the superimposed signal, and then the next higher channel gain user does

the same successively, until the remaining signal is decoded by the weakest user. To make this process possible, users need to be ordered according to their channel gains first [33]. In a NOMA system, successive interference cancellation is employed at the receiver.

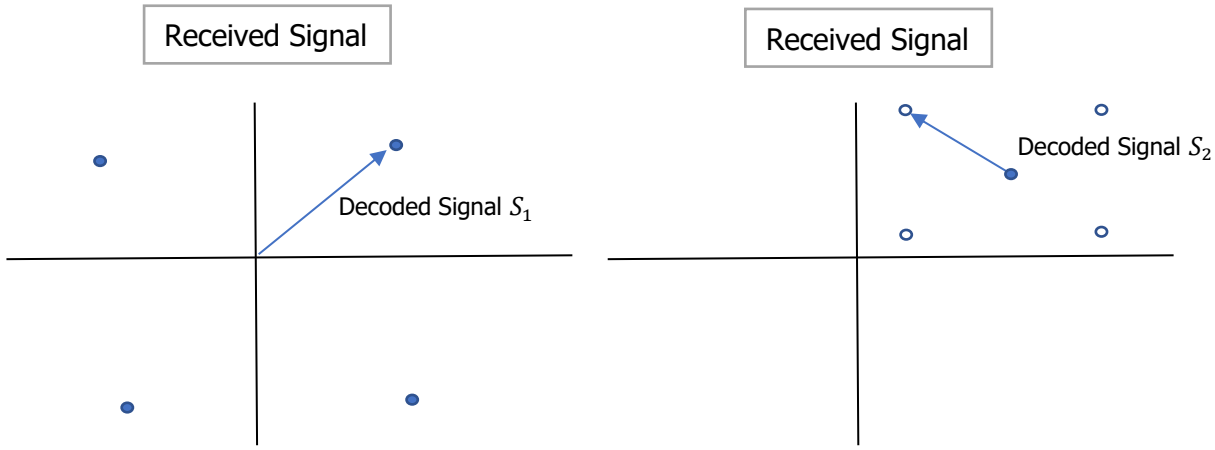


Figure 2. 2 NOMA receiver end decoding scheme.

(a) User 1 (b) User 2

In Figure 2.2, it is assumed that User 1 has a stronger channel gain than User 2. A demonstration of a decoding method for the superimposed signal of the two users is shown. On the left, the constellation diagram shows the point for User 1 is decoded first since its channel gain is the strongest compared to User 2, then after subtracting it from the combined signal the constellation diagram on the right shows the point of User 2 is decoded last. The SIC technique enables the decoding of the weaker user signal efficiently which is a process that would otherwise be difficult.

2.2.3 Downlink NOMA

In downlink NOMA the base station transmits n different signals to n users, by combining them first through superposition coding. In turn, the users will receive their desired messages as a combined signal which comes with interferences from the other users in the combined signal. For these users to get their desired messages only, they first have to employ SIC which will decode dominant interferences from the combined signal and remove them accordingly.

In downlink NOMA, SIC is performed at the users' receivers. The order in which decoding is done in downlink differs from that of uplink NOMA. We will discuss that of uplink NOMA is the next sub-section.

In downlink NOMA, the decoding order is in the order of increasing channel gains. Once SIC has taken place, users are able to decode their desired signals. This is made possible by the fact that these users are served with different power levels which is crucial in the separation process. It is worth pointing out that each user will be allocated power in accordance with their gains from the base station [34]. Thus, each user will get a power level that is inversely proportional to their gain from the base station. In simpler terms, the user closer to the base station will get a smaller power allocation when compared to the user who is further from the base station. As such, in downlink NOMA the users with high channel gains are allocated lesser power as they incur relatively low interferences.

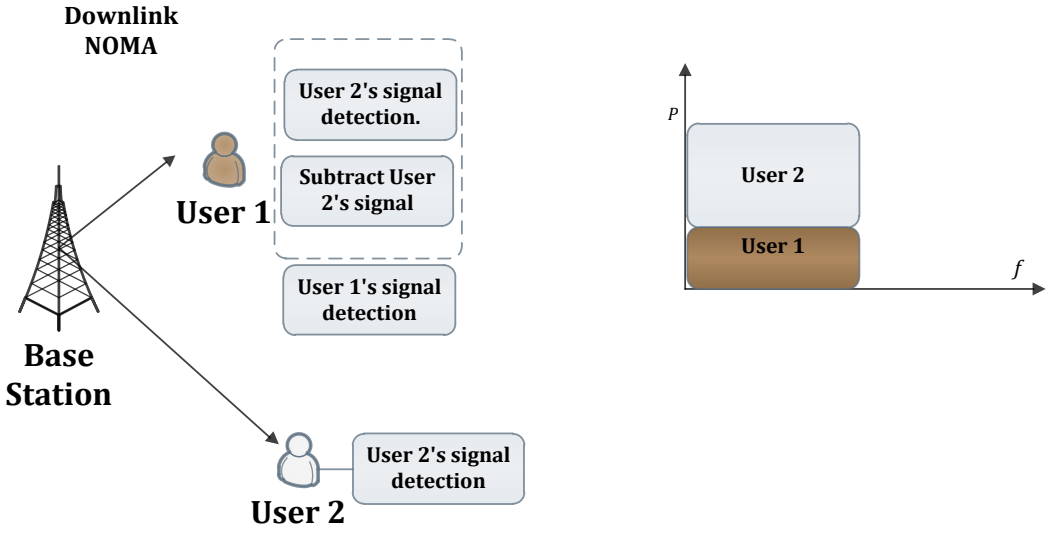


Figure 2. 3 Downlink NOMA with SIC

A two-user system displayed in Figure 2.3 shows that for User 1 to obtain their signal, they first perform SIC to remove any interference caused by User 2. User 2 on the other hand does not have to perform SIC, rather they have to decode their signal using a higher power allocated to their receiver.

2.2.4 Uplink NOMA

The uplink NOMA scenario is different when compared to the downlink one. In uplink, it is a different case because, it is the users who are transmitting to the base station and they interfere with each other. This means that the base station has no control over how much power the users are capable of using for transmission but can only review the received signals from the users. Each of the users solely transmits either using their own maximum transmit power which is proportional to their channel gains from the base station. The challenging task at the base station is that, now all the signals are the desired signals and the base station needs to decode each one of them. The base station then employs successive interference cancellation to decode all the desired signal and cancel out the interreferences. Note that for SIC even in this case, the users would have to be arranged according to their channel gain strengths and the decoding order is in the order of decreasing channel gains. Then, the highest channel gain user get its signal decoded first then the second highest, and so on. This means that the highest channel gain user enjoys interference free transmissions [35].

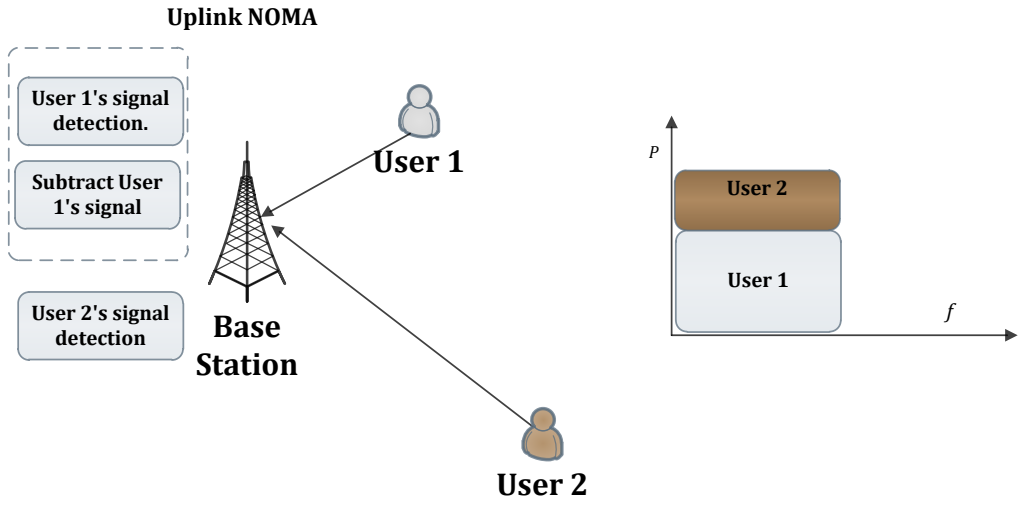


Figure 2. 4 Uplink NOMA with SIC

In Figure 2.4, the base station receiver decodes User 1 signal first and removes its component from the combined signal and then decodes the signal of User 2. Because User 2 is relatively far from the base station his power is low at the base station. However, it should be noted that as the number of users in the system increases, the SIC process becomes more complex [35]. Hence, most research in NOMA typically allocates two users to a channel to avoid this high complexity that SIC generates. However, user pairing can be a solution put in place to

maximize NOMA capacity when two users share a channel for SIC. We will discuss this user pairing issue in detail in the next chapter.

2.3 Cognitive Radios (CR)

In the last two decades, the availability of usable radio frequencies in the wireless frequency spectrum has become severely scarce, leading to a high demand for the spectrum, especially from the new emerging software products and services that require the spectrum to operate. In fact, there is a phenomenal growth in wireless communication devices, systems and services that use the spectrum daily. Studies show that the demand for the spectrum has grown exponentially over the last two decades. This has led to the allocation and buying of spectrum to be a multi-billion industry worldwide. However, it has been proved that, this shortage of spectrum is a consequence of mismanagement of the usable available spectrum as opposed to it being the physical scarcity problem. This means that if efforts are directed to this problem, effective solutions could be developed [33]-[36]. Thus, engineers and researchers have been enthusiastic about developing solutions for this problem, consequently the terms dynamic spectrum access and cognitive radios were born. In this section we look in detail how these technologies changed the spectrum efficiency field and how combining them with latest spectrum saving technologies like NOMA would further be beneficial in the saving the spectrum.

2.3.1 Cognitive Radios and Dynamic Spectrum Access

Dynamic spectrum access refers to the different methods, technologies, theoretical concepts that can introduce flexibility in wireless communications, to let unlicensed users have access to the licensed spectrum [36]. However, introducing these users into the licensed spectrum comes at a cost of interference being introduced, and a controlling mechanism would therefore be required for smooth operation. The existence of interference is a known problem in wireless communication networks, and there has been multiple efforts into leveraging the dynamic spectrum access techniques for the development of software defined technologies and schemes that enable wireless devices to co-exist in the same spectrum while transmitting at the same time. Cognitive radios/software defined radios are a result of such efforts.

Basically, cognitive radios are telecommunications devices that are capable of using advanced sensing mechanisms, to collect information, about the users that are currently using the spectrum, and then intelligently adjust operating parameters in order to begin transmission based on the collected information [36]. These intelligence capabilities ensure that users can coexist and transmit in the same spectrum without degrading each other's performance or interfering with each other. Obviously, this improves spectrum efficiency. These radios are built for software radio platforms and they utilize algorithms which enable them to intelligently detect usable spectrum for unlicensed users. They may as well adapt to the communication environment by adjusting their operating parameters to suit the spectrum they are in. This involves lowering their transmission power or changing the band of operation just so that interference could be avoided. The combination of cognitive networks with NOMA would obviously yield highly spectral efficient communication systems. Multiple research efforts have proved this, and we will look in detail into these efforts later in the chapter.

Based on the type of network, and the type of service being provided by the spectrum, cognitive radios can be classified into three paradigms namely underlay, overlay, and interweave, each which can use sophisticated signal processing to make decisions that ensure the cognitive principle of not degrading system performance is respected.

Underlay Cognitive Networks:

The underlay paradigm adopts a hierarchical structure of operation, while allowing simultaneous non-cognitive and cognitive transmissions [36],[37]. The basic idea of an underlay network is to separate the users into primary users and secondary users. Similar to other paradigms, the spectrum is mainly licensed to be used by the primary users but also open to secondary users for use, as long as during secondary user transmission, the interference perceived by the primary users is below some acceptable threshold. The interference constraint in underlay mode is strict, and all secondary users should be very conservative on their transmission power output, to ensure that their transmit power remains below the prescribed interference threshold. Exceeding the prescribed interference threshold can lead to a dramatic degradation of the primary user services who are the rightful users of the spectrum.

Because of this quite restrictive nature, in underlay mode, secondary users usually perform low power level transmissions and short-range communications which do not go above the

prescribed interference threshold [37]. However, the underlay mode is still the mostly used paradigm of cognitive networks as it encompasses more users into network, meaning that both secondary and primary network transmissions can happen simultaneously. This improves spectral efficiency, while maintaining higher cell-edge throughput at the same time. Moreover, recent literature shows that, many of the interference avoidance and mitigation schemes that have been put forward are proven to be efficient. These include beamforming and short-range communications schemes that restrict the emitted power from the transmitter.

Overlay Cognitive Networks:

Overlay networks were first envisioned by Mitola [38]. In his paper, he referred to overlay cognitive networks as spectrum pooling. In this approach of cognitive networks, secondary users share their user codebooks and their messages with the base station transmitter so that all their information about spectrum usage is known at all times. This can be by directly sending the codebooks to the transmitter or alternatively the secondary users can broadcast all their code books in a periodic manner. The overlay model assumes that prior transmission the codebooks of the secondary users are known. This helps to avoid concurrent transmissions which can result in interference to the primary users and degrading their performance. The base station can then act as a relay and forward the message to the intended receiver when the spectrum holes allow. Alternatively, the primary user can also act as a relay and pass the information to the intended receiver. Knowledge of the secondary user codebooks and message is crucial in overlay networks and may be exploited in various ways with one goal, which is to curb or mitigate interferences during transmissions. Moreover, the primary users may also assist the secondary users by assigning a portion of their power to relay secondary user messages. This is through careful adjustments of the signal-to-noise power ratio (SNR) to enable interference free transmissions. Overlay networks are able to avoid interference because of the codebook knowledge that can easily exploited.

Interweave Cognitive Networks:

The spectrum occupied by the primary users is not in constant use the whole time, and this causes gaps in the spectrum that are referred to as spectrum holes. These spectrum holes change with time and location. To fully utilize this unused spectrum, interweave cognitive networks employ opportunistic communication. This is based on careful and accurate identification of the idle frequencies and then exploiting frequencies that are dynamic without causing any

interference to the licensed users. This is referred to as frequency reuse [36]-[38]. The idle frequencies need to be actively monitored for any change that may occur. A primary user may need to use the spectrum and because they are licensed the secondary users need to give it away. This would then involve careful and intelligent termination of the session that the secondary user was involved in. The only strict policy in interweave networks is that the secondary user may not interfere and interrupt an already existing communication that a primary user is partaking in.

2.3.2 Cognitive NOMA Networks

In the previous sub-section, we looked at two different emerging technologies which target efficient spectrum utilization, namely, NOMA and cognitive radios (CR). Both technologies can be integrated into a holistic system which targets spectral efficiency from different perspectives, resulting in a superior spectrum saving system. In a nutshell, cognitive radios help secondary users by allowing concurrent transmissions between the primary and the secondary networks. This can either be in underlay mode, overlay mode or interweave mode. NOMA on the other hand uses the power domain in order to allow multiple users to transmit on the same spectrum through allocating different power levels to the users. Therefore, a constructive combination of CR and NOMA into cognitive NOMA, can provide tremendous potential for intelligently and efficiently using the spectrum and at the same time increase the number of users in the system. NOMA can be exploited to perform cognitive secondary user transmissions even when the primary users are active. This can be done through one of the paradigms of cognitive radios. This not only increases spectrum efficiency but can also yield high system throughputs and enable massive devices to be connected the network.

Moreover, research also envisions that the combination of the two technologies would result in the mutual interference between the primary and the secondary networks. This means that to reap the benefits of both, algorithms which cater for the involved mutual interference should be developed. Several research efforts into the integration of NOMA with CR have been developed in the past decade. A cognitive non-orthogonal multiple access with cooperative relaying is proposed in [39]. In this work, an integration of NOMA capabilities with CR capabilities is done to achieve a system with intelligent spectrum saving abilities. A careful consideration of different cognitive NOMA architectures is then made. The authors then employ a cooperative relay strategy for reliability enhancement. Their results show that the

proposed strategy significantly achieves low outage probabilities as well as improving reception reliability. In [40], a model which consists of a set of randomly distributed secondary users and primary users, with at least one secondary user closer to the base station is proposed. The authors use theoretical derivations to demonstrate that this model can produce enormous systems throughputs when power allocation is optimally chosen. A similar approach is proposed in [41], in which NOMA is integrated with cognitive radios in an underlay mode of operation. The system model here consists of randomly deployed users. The authors use stochastic tools to come up with important close form expressions which help evaluate the outage performance of the system. They came to realization that NOMA can outperform conventional orthogonal multiple access in underlay CR networks. A sum rate optimization case in cognitive NOMA network is studied by authors in [42]. The authors base their model on an underlay approach. First, they focus on the restrictions which should be maintained in the cognitive network given the underlay scenario. The first one is to keep the interference below the prescribed underlay threshold, and the second is to maintain the SNR of the secondary users above a certain value to ensure that no outage occurs in the secondary network. Thirdly, they derive a low complexity power allocation method which maximizes the sum rate of the secondary users. Their simulation results show that their scheme, in comparison to algorithms which use fixed power allocation, performs better in terms of throughput and outage probability. Their results also show a better performance when compared to TDMA.

Moreover, imperfect successive interference cancellation is one of the burning issues in NOMA. The main reason for this is because SIC in NOMA relies solely on decoding order for each user, and in case of a mismatch the users would need to request for retransmissions which can result in increased processing time. In CR NOMA the issue is the same. The authors in [43] consider how an imperfect successive interference cancellation would perform in an underlay cognitive NOMA network. In their letter, they mainly consider the interference constraint of the underlay network and then derive the outage probability for the secondary users. Their analytic results match with their simulation results when in two cases:

1. when the underlay constraint approaches infinity and,
2. when the secondary base station power goes to infinity.

Furthermore, a dual hop underlay CR NOMA network is considered in [44], whereby the authors aim at evaluating the end-to-end outage probability. Their focus is to compare the conventional multiple access techniques when they employ an amplify and forward (AF) relay to implement the dual hop and also when they implement a decode and forward (DF) approach. Results show that in terms of outage probability, their dual hop underlay model is superior to other conventional techniques. Moreover, a cognitive inspired NOMA network, which employs a technology referred to as, simultaneous wireless information and power transfer (SWIPT) is considered in [45]. The authors here design their framework aiming to cater for simultaneous primary and secondary transmissions. They then propose an adaptive protocol which integrates CR NOMA and SWIPT to exploit relaying possibilities in the network, enhancing the quality of primary user multicast services and conserving power in the secondary network transmissions. Their results show superior energy efficiency and satisfactory outage probability performance when compared to cooperative NOMA schemes.

Although there is not much work on overlay CR NOMA networks, the work in [46], showcases an overlay cognitive radio which utilizes NOMA in the secondary network. Here, the main focus is to address the following question. How would an overlay cognitive radio network perform with NOMA implemented at the secondary users? However, in this work the performance improvement in terms of throughput is only realized for far NOMA users. A higher outage performance is also seen for both near and far users in the secondary network. Fixed power allocation is assumed for the NOMA users. Moreover, one other overlay network scenario which focuses in an overlay cognitive NOMA network is discussed in [47]. But this model is different from most as it employs spatial modulation (SM) and antenna selection (AS). With this in mind, the authors propose a protocol which allows simultaneous secondary and primary transmissions using SM. The reason behind this is to increase the spectral efficiency for both the primary and the secondary network, while at the same time decreasing the complexity of message detection at receivers. This provides high quality transitions and improved spectrum utilization in the primary network without affecting both the secondary user receivers and the primary user receivers.

Furthermore, a study which finds out about the throughput performance and outage performance of a downlink underlay cognitive radio-aware hybrid NOMA network is looked at in detail in [48]. This study also considers both OMA and NOMA scenarios, on the performance of decoding for each user. The main results show that even in the case of imperfect

channel state information (CSI), the NOMA side of the hybrid OMA/NOMA system performed better than the OMA side, for both outage and throughput.

With the existing research efforts in CR NOMA showcasing that the combination of NOMA and CR can yield improved spectrum efficiency, massive connectivity, low latency and better fairness. Most of the work in the literature proves that underlay networks perform better and provide various interference prevention techniques and solutions, protecting the primary user services at all times. Henceforth, this work will be based on involving more technologies in NOMA and improving them, to further better the performance in both the primary and secondary networks.

2.4 Summary

This chapter provides an investigation conducted on the core technologies that this research is built upon. First, we look briefly into the conventional OMA and show its setbacks and why it will not work with the next generations networks. Then we present a brief literature survey on NOMA, cognitive radios and cognitive NOMA networks. For NOMA we introduce the concept, its operation and its network types. We consider both uplink and downlink NOMA systems how they operate differently from each other. We also discuss the main core technologies of NOMA, superposition coding and successive interference cancellation. Moreover, also conduct discussion of cognitive radios and its different paradigms. Furthermore, we discuss how the integration of NOMA and cognitive radios into a holistic system of cognitive NOMA can result in a superior spectrum saving system.

Chapter 3

User Pairing in NOMA: Literature Survey

3.1 Introduction

NOMA represents a paradigm shift which enables the power domain to be used to serve multiple users simultaneously by exploiting the difference in their respective channel gains. This is done through providing different power levels to users sharing a frequency band. Since more users are admitted at the same time to utilize a frequency band, interference is strong in NOMA systems. Consequently, the users received with low power need to employ sophisticated multiuser detection techniques such as successive interference cancellation in their receivers, in order to cancel out any interference caused by multiplexing multiple users in the same spectrum. It has already been shown through the literature in the previous chapter that NOMA results in improved throughputs and is highly spectrum efficient. However, in a multiuser system with a large number of users, the SIC process becomes very complex and is not practical. Up to this day, grouping only two users for NOMA in each resource block has become a common practice.

A promising solution for multi-user systems is to combine both NOMA and multiple access techniques. To put it differently, the users in the network can be divided into various groups with the aim of performing NOMA within each group. Orthogonal bandwidth resources using multiple access would then be offered to each group. As can be seen, the performance of this integrated system depends on how the grouping of the users is done, as the users in a network system are randomly placed. As a result, it is imperative to know which grouping or pairing scheme `degrades the performance and also which one is efficient. In this chapter, we explore different research efforts that have been put forward to provide an insight into user pairing. We also explore some of the recent related topics that assist to explore various techniques and methods used to approach this problem. We first start by exploring and reviewing the literature focused on power allocation in NOMA and then move into user pairing in NOMA. It is imperative to consider power allocation, as it goes hand in hand with NOMA.

3.2 Power Allocation in NOMA

Power allocation considerations in NOMA are crucial and unavoidable. This is mainly because NOMA allows multiple users to share the same resources band, and then exploiting the power domain [49]. Power allocation is basically how the base station allocates power to a group of multiple users who share the same resources band for NOMA. There are a number of considerations that need to be made when looking into power allocation in NOMA. First the sum rate requirements for each user need to be made, then the channel gain of each user needs to be considered as well. In addition, the interference that the user will exert on other users also needs to be put into consideration. It is obvious that the performance of NOMA depends on how the power is distributed amongst the multiplexed users and the user groups. Given a base station that has a total power, P_{max}, and a number of sub carriers, a thorough consideration of how much power is needed for each NOMA user group to utilize the sub carriers is needed. Given also that there are N NOMA groups and N available sub channels, a determination of which frequency band to allocate for the NOMA groups also needs to be considered. Furthermore, within each of the NOMA user groups, a further power split needs to be performed to determine the power fraction each NOMA user gets before NOMA transmission so that the users can perform SIC and achieve higher NOMA capacities. We refer to this process as power splitting. Moreover, before the users start transmitting, each has their own required user rates that must be satisfied first. It is therefore crucial to consider, the minimum power required to meet these rates. The interference also needs to be considered for power allocation in NOMA networks. In this work, we consider both the interferences and the power allocation in underlay mode and aim to develop schemes which achieve all the mentioned aspects. Various works have considered power allocation in NOMA. These have been done in different types of networks. Therefore, this section addresses different power allocation solutions that have been developed so far in the literature for different NOMA systems.

A low complexity power allocation algorithm is proposed by the authors in [49]. To develop their algorithm, the authors first, look into the well-known complex iterative water filling method which is not practical in large systems, and come up with an improved low complexity method for power allocation in sub-carriers. The improved method is based on linear water-filling algorithm (LWF) which is used to allocate power to all the subcarriers at once. In addition to the primary power allocation by LWF, when then users have been distributed power

and are already allocated a subcarrier, they would use a fractional power allocation scheme to reallocate power further. Their model is a downlink NOMA system that aims to allocate power and sub bands to NOMA users. To allocate power to a subcarrier n, the water filling power allocation formula which can allocate power to N groups of NOMA users is given as follows:

$$P_n = \frac{P_{tot}}{N}, \tag{3.1}$$

where P_{tot} is the total power available at the base station and N is the total number of subcarriers. The fractional power allocation scheme is then used to reallocate power further. The fractional power allocation scheme introduces a dynamic power allocation strategy that guarantees fairness of the users with low SNR. On a subcarrier n, the power allocated to a user k is given as:

$$P_{k,n} = \frac{P_n}{\sum_{i \in \Lambda_n} (\psi_{i,n})^{-\alpha_{ftpa}}} \ (\psi_{i,n})^{-\alpha_{ftpa}}, \tag{3.2}$$

where $\Lambda_n$is the superimposed user set on the subcarrier n and ($\psi_{i,n}$) is the SNR of the user k on subcarrier n. Simulation results show relatively low complexity reviewed on the new improved power allocation algorithm and almost equals the water filling method in sum rate.

Moreover, a solution which considers power allocation in an uplink NOMA system is considered in [50]. It is worth pointing out that only a few studies consider power allocation in the uplink scenario. Perhaps, it is because the uplink case is more complicated because of its discrete nature of subchannel allocation, and the power constraint of the users unlike downlink NOMA. The authors in [50] propose an uplink power and channel allocation algorithm, which operates in a system, in which the spectrum is divided into N subchannels with equal bandwidth. In the system the base station is required to schedule subchannels and power across the users optimally so that the capacity of the system is maximized. The authors assign a maximum of K users to a channel. They formulate their problem as:

$$\max_{\{p_{n,m}, a_{n,m}\}} \sum_{n \in N}^{N} \sum_{m \in M}^{M} a_{n,m} \log\left(1 + \frac{p_{n,m} g_{n,m}}{1 + \sum_{i \in M}^{M} p_{i,n} g_{i,n}}\right), \tag{3.3}$$

$$Subject\ to\colon\ (a) \sum_{m \in M}^{M} a_{n,m} \leq K \quad \forall n \in N,$$

$$(b) \sum_{n \in N}^{N} a_{n,m} \leq \infty \quad \forall n \in M,$$

$$(c) \sum_{n \in N}^{N} a_{n,m}\, p_{n,m} \leq p_{max} \;, p_{n,m} \geq 0 \;\; \forall n \in M,$$

$$(d) \qquad a_{n,m} = 1 \;\; or \; 0,$$

where is $a_{n,m}$ a binary variable and the power at each sub band is $p_{n,m}$ with the maximum power of the base station as p_{max}. Given the power constraint of each user, and the set of discrete variables $a_{n,m}$, $p_{n,m}$, the authors found their problem to be NP-hard and non-convex, and the solution intractable. To come up with a solution to the problem, they decompose the problem into two steps.

In the first step, based on the assumption that the maximum power level is subdivided equally among the allocated channels they solve the subchannel-user mapping problem using a many-to-many matching model. They firstly establish that since one user can be assigned either of the multiple subchannels and also one channel can have multiple users, a many-to-many matching model can be developed to solve the first part of the problem. They lay out that, given that a maximum of K users can be multiplexed on a subchannel, M users in set $\boldsymbol{M}$ and N subchannels in set $\boldsymbol{N}$ are two sets of players of this many-to-many matching relation. Therefore, each user m can have infinite number of channels. However, since user m has a constrained maximal power $p_{n,m,}$ this should be subdivided equally among its allocated subchannels. In the second step, they assume that the subchannel user mapping information is known and they employ the iterative water filling method to solve the problem. Results show that the proposed scheme is effective and outperforms existing algorithms in their context. An even improved solution is achieved when the authors apply iterative water filling approach.

Moreover, the authors in [51] propose a power allocation algorithm which considers the dynamic nature of a NOMA network. Because of this nature, it is possible for transmission failures to occur, resulting in a need for retransmissions which require additional power as well. The authors here work in a downlink NOMA system, and propose a packet level scheduling scheme. Although we do not cover packet scheduling in our work, we look into this work focusing on the power allocation side of the problem. In this type of model, the downlink transmitter needs to make dynamic decisions when the packets arrive, whether to use NOMA or OMA. In their system model, the base station is serving multiple users, M, over a single

frequency channel. The packets are scheduled to arrive at the beginning of each time slot t. The authors consider a two-user pairing strategy for NOMA, where two users are allocated different proportions of the total transmit power P. The base station can also transmit a packet using OMA if the channel conditions are not suitable to use NOMA. The authors aimed at developing a power allocation and user scheduling algorithm scheme that maximizes the long-term expected throughput. They formulate their problem as:

$$\max_{t} \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\left(\sum_{m\in M} d_m(t)\right), \tag{3.4}$$

$$Subject\ to: (a)\sum_{n\in N} Q_m < \infty,$$

where $d_m(t)$ is defined as the expected number of packets from a queue. For each considered power P, they formulate a maximization optimization problem and solve the problem by employing Lyapunov optimization toolset to find a solution. Their results from the tool set show significant improvement in throughput when comparing to distance-based approaches and CSI-based approaches.

Moreover, in [52], the authors propose a power allocation algorithm based on particle swarm optimization. They assume the number of users per cell is K and there are B blocks, each with bandwidth W. They denote the power allocated to each block equally as $p_{i,b}$ such that $1 < b < B$. To obtain power for each user in the block, they employ a new addition to evolutionary computation methodologies to search the approximate optimal solutions, particle swarm optimization. The particle swarm optimization utilizes a genetic algorithm-based method for power allocation to maximize the SINR in the block. To get the power of the kth user, the genetic algorithm contain a crossover strategy which calculates the power based on the formulas below:

$$P_{k,b,i} \quad = \alpha_k P_{k,b,i} + \ (1-\alpha_k) P_{k,b,i}\,, \tag{3.5}$$

where $P_{k,b,i}$ represents the power of the kth user in bth block of the ith chromosome according to the genetic algorithm and is the α_k is probability of crossover which is equal to 0~1 randomly. This method leverages the channel state information available for each of the users in each block. The available channel state information is useful for the framework to use the

genetic algorithm. Their simulation results display that the proposed method achieves a better performance in terms of spectral efficiency and fast convergence compared to existing power allocation methods in both OMA and NOMA.

A dynamic power allocation algorithm which also incorporates user scheduling is considered in [53]. The basic idea here is to develop a stochastic optimization problem and observe its performance when compared to other algorithms. The authors consider a downlink scenario, in which the base station transmits data simultaneously to the users. Power allocation is done at the same time with user scheduling to fully exploit the near-far effect of users and control the decoding complexity. To model the control policy, they define $P(t) = \{p_m(t)\}$ and $A(t) = \{a_m\,(t)\}$ as the power allocation variables. This mean that $p_m(t)$ is the power allocated to the base station to serve user m. Moreover, given the control policy $P(t)$ and $A(t)$, the total power consumption of the whole network for the given slot t is given as:

$$P_{tot} = \sum_{m \in M}^{M} p_m\,(t) + \sum_{m \in M}^{M} p_m\,(t) p_c\,, \tag{3.6}$$

where p_c is the consumed power of any user due to data reception. The problem considers individual user capacity constraints. Before finding the solution, the authors convert the problem into a static optimization problem and then reformulate the problem to adopt the branch and bound structure. Its primal is given as:

$$min\ P_{tot}\ P(t)\ ,\ A(t), \tag{3.7}$$

$$Subject\ to\colon (a) \sum_{m \in M}^{M} p_m\,(t) \leq P_{tot},$$

$$(b) \quad p_{n,m} \geq 0,$$

$$(c) \quad a_m\,(t) = 1\ \ or\ 0.$$

To find the solution the authors exploit the Lyapunov optimization technique and the branch-and-bound method. The simulation results show the proposed algorithm to be superior in optimal control policies while performing with a low complexity as well as in comparison with other schemes.

Furthermore, to improve spectral efficiency of a NOMA system with imperfect channel estimation, [54] proposes two power allocation algorithms, namely, a min-max problem algorithm and a heterogenous user rate-based algorithm. The objective here is to investigate the algorithms in a NOMA system with channel estimation errors and then observe the spectral efficiency as well as the power allocation performance. The authors work with a centralized single antenna base station with K uniformly distributed users. The min-max power allocation problem is described by:

$$\min \max_{k \in K} e_k \ (\boldsymbol{P}), \tag{3.8}$$

$$Subject\ to: (a) \sum_{k \in K}^{K} p_k \leq P_{max},$$

$$(b)\ p_k \geq 0,$$

where e_k is the relative error for user k and the power allocated to the user k is p_k. Their numerical results are able to verify their theoretical assumptions and the algorithms also prove to achieve better overall system performance.

An optimal power allocation scheme for next generation networks is considered in [55]. The authors consider a NOMA system with SIC implemented at the receivers. The proposed power allocation algorithm aims to maximize the overall sum rate capacity using optimum power ratios. The maximization of the sum capacity under a fairness constraint is also put into consideration. For a downlink NOMA system with K users, the fairness index is defined as:

$$F = \frac{\sum_{k \in K}^{K} (R_k)^2}{K \sum_{k \in K}^{K} {R_k}^2}, \tag{3.9}$$

where R_k represents the sum rate capacity for a user k. The power allocation optimization problem is formulated as:

$$\underset{\alpha_k}{\text{maximize}} \sum_{k \in K}^{K} W \log_2 \left(1 + \frac{P_k \, {g_k}^2}{N \ + \sum_{i=1}^{k-1} P_k {g_k}^2} \right), \tag{3.10}$$

$$Subject\ to: (a) \sum_{k=1}^{K} P_k \leq P_T,$$

$$(b)\ \ P_k \geq 0,$$
$$(c) F \geq F',$$

where W is the available transmission bandwidth of the NOMA system, α_k is the power factor each user k and F' is the target fairness index in the network with P_T being the total power of the system. In the equation above g_k is the channel attenuation gain for the link between the BS and the user k. The authors also ensure that fairness is maintained amongst the users in the system. The sum capacity results obtained from the power allocation algorithm come out superior in comparison to all the existing OMA schemes.

Furthermore, the power allocation protocol proposed in [56], deals with joint power allocation algorithms which focus on systematic sum rate optimization rather than individual user required rates. The authors compare their methods with heuristic and ad-hoc methods in the literature. At first, they consider a joint power with channel allocation method (JPCAP) and generate a Lagrangian framework to solve the resulting problem. They aim only to achieve a near optimal solution for a downlink NOMA system with K users, and they describe their joint power and channel allocation problem as:

$$\max_{P} \sum_{k \in K}^{K} w_k \sum_{n \in N}^{N} R_{kn}\, x_{kn}, \tag{3.11}$$

$$Subject\ to: (a) \sum_{k=0}^{K} \sum_{n \in N}^{N} p_{kn} \leq P_{tot},$$

$$(b) \sum_{n \in N}^{N} p_{kn} \leq P_k,$$

$$(c) \sum_{k=1}^{K} x_{kn} \leq M,$$

$$(d)\ \ x_{kn} = 1\ or\ 0,$$

where w_k is the weight coefficient of user k, R_{kn} is the sum rate utility, and p_{kn} is the power allocation variable for a user k utilizing channel n. The authors then employ the Lagrangian dual optimization and dynamic programming to solve the joint power allocation and channel allocation problem. Their proposed solution is able to provide a near-optimal solution as well as bounding the global optimum tightly. Their simulation results also show significant

improvements in fairness and throughput in comparison to existing OFDMA and NOMA power allocation schemes.

With NOMA and power allocation going hand in hand for better throughputs in networks systems, it is equally important to choose the best power allocation scheme that suits any user pairing algorithm to optimally allocate power to the desired user pairs. In the next sub-section, we will review existing user pairing schemes and how they perform in comparison to other schemes.

3.3 User Pairing in NOMA

User pairing refers to the study of how we can group users in a NOMA system so that we can perform SIC with low complexity and achieve good performance for the overall system. This field is maturing with a wealth of well-understood knowledge, methods and algorithms, which have one goal, i.e., to improve the performance of the NOMA system. Few of the earlier works used only the structure of NOMA favouring pairing a far user with a near user to perform user pairing, however later research works show that there are plenty of other ways to approach this problem. Several of the popular recent works are reviewed and presented in this section.

Two sort-based user pairing algorithms studied in [57] perform user pairing for K users divided into M groups, where $(K \geq 2)$. The first pairing scheme pairs the $l-th$ user with the $(KM = 2 + l)$-th user, while the second pairing algorithm differs by pairing the $l-th$ user with the $(KM + 1 - l)$-th user. A power allocation method aimed at maximizing the system spectral efficiency is then applied to the cluster pairs. The overall problem is an optimization problem with M users to be paired and is expressed as:

$$\max_{a_{m,i}} \left(R_{m,1} + R_{m,2}\right), \tag{3.12}$$

$$Subject\ to: (a) R_{m,1} \geq R_{m,1,OMA},$$
$$(b)\ R_{m,1} \geq R_{m,2,OMA},$$

where $R_{m,1}$ and $R_{m,2}$ denote the capacity of the users that are paired and $a_{m,i}$ is the power allocation factor for the users. Simulation results show that both algorithms greatly decrease

error probability while increasing spectral efficiency. This shows that sort-based user pairing is a promising technique for user pairing in NOMA systems.

In [58], the authors present the impact user pairing has in NOMA systems. They consider a NOMA system with fixed power allocation (F-NOMA) towards the users, and also investigate performance in a cognitive inspired NOMA system. In F-NOMA, user pairing is done amongst two NOMA users and a fixed power allocation offered to the users. When the mth user and the nth have been paired together and allocated the fixed power, the probability of the system achieving a lower sum rate than the conventional OMA is found to be:

$$P(R_m + R_n < R'_m + R'_n), \tag{3.13}$$

where R_m and R_n are the OMA sum rates and R'_m and R'_n are the user pairing NOMA rates. For the cognitive NOMA system, they assume the poor channel gain user to be the secondary user and the stronger channel user to be the primary user. Assuming the far user was the mth user, the authors introduce constraints which protect the individual user rates and observe that the CR NOMA system is able to achieve higher throughputs when these constraints are put in place. In addition, a discovery that throughput performance increases when the user with weak channel gain is paired with a user with stronger channel gain is also made by the authors. For the CR-NOMA system, a realization is also made that the channel quality of the secondary user is critical since the interference constraint of CR-NOMA system need to be maintained.

Furthermore, in [59] the authors draw our attention to user pairing in uplink NOMA. In their system the users are divided into disjoint sets resulting into a combinatorial user pairing problem. For a total of M users, and assuming that the paired users, l_{2m-1} and l_{2m} are utilizing the mth sub band, their combinatorial problem is given as:

$$\max_{(l_{2m-1}, l_{2m})} \sum_{m \in M}^{M} (l_{2m-1} + l_{2m}), \tag{3.14}$$

where $m \in \{1, \dots, M\}$ and $l_i \in \{1, \dots, 2M\}$. They then consider two scenarios to get to the solution, one with single antenna base station and the second with frequency flat channels. For the single antenna base stations, when the users are communicating over M sub-channels with a predefined fixed power allocation scheme, the throughput scales with a magnitude of $N \log_2 N$ if the users have small scale fading.

Secondly, for the frequency flat channels the user pairing method analysed shows better results compared to random methods. Their optimal pairing algorithm is implemented using the classic Hungarian algorithm [60] with polynomial complexity. In both approaches, it was observed that NOMA achieves higher power efficiency when compared to orthogonal schemes. In addition, the authors also propose a novel multi-antenna scheme in which transmissions are over a single carrier. When the Hungarian algorithm is employed to improve this approach, significant system performance was achieved. This shows that using the Hungarian algorithm as a reference algorithm for the future is a promising technique for useful comparisons in NOMA systems. Work in [17] puts focus on a downlink NOMA with a primary user (PU) and a cognitive user (CU) in a cognitive NOMA network. The authors employ matching theory to perform user pairing. In their system, the primary users negotiate power allocation for transmission with the cognitive users, guaranteeing a satisfaction of both primary and cognitive user sum rates. For matched users PU and CU, the objective function which maximizes the total sum rate is given by:

$$\underset{\{\alpha_{CU},\alpha_{PU}\}}{\text{Maximize}}\ [R_{PU}(\alpha_{PU}) + R_{CU}(\alpha_{CU})], \tag{3.15}$$

$$\begin{aligned} Subject\ to\ (a) \quad & R_{PU} \geq R_{PU,req}, \\ (b) \quad & R_{PU} \geq R_{PU,req}, \\ (c) \quad & \alpha_{PU} + \alpha_{CU} = 1\,, \end{aligned}$$

where α_{PU} and α_{CU} are the power factors for each of the paired users and R_{PU} and R_{CU} are the paired user NOMA rates. $R_{PU,req}$ and $R_{CU,req}$ are the required rates of the users before user pairing occurs. The authors then employ a distributed matching algorithm to match the primary users with the cognitive user to maximize throughput for both. A Lagrangian based power allocation scheme is employed for power allocation to the user pairs. Simulation results show that the proposed solution outperforms conventional OMA schemes and has a low complexity. The authors in [61], work on user pairing in both downlink and uplink NOMA systems. The authors formulate a mixed integer non-linear programming problem which solves a user pairing problem in a multiuser NOMA system. They consider both uplink and downlink in their problem. For a 3-user cluster, the authors consider a power allocation scheme which is necessary for SIC to take place. Their power allocation scheme is given by:

$$P_i\,\rho_{i-1} \;-\sum_{i=1}^{i-1} P_i\,\rho_{i-1} \;\geq P_T\,, i = 2,3,\dots m, \tag{3.16}$$

where $\rho_i = \frac{h_i}{N_0\,\mathrm{B}}$ and denotes the transmit signal-to-noise ratio (SNR), P_i is the power allocated for a user i and P_T being the total base station power. Putting the user minimum required rates constraint and the power constraints into consideration, the authors formulate the joint user clustering and power allocation problem in uplink NOMA as follows under the constraints of transmission power budget, minimum rate requirements of users, and operation constraints for SIC receivers:

$$\max_{a,\beta,P} \sum_{j=1}^{N/2}\sum_{i=1}^{N} a_j\,\beta_{i,j}\,\log_2\left(1+\frac{P_i\,\rho_i}{\sum_{k=k+1}^{N} P_i\,\rho_i + a_j}\right), \tag{3.17}$$

$$\begin{aligned} Subject\ to\colon\ (a)&\sum_{j=1}^{\frac{N}{2}}\sum_{i=1}^{N}\beta_{i,j}\,P_i \;\leq P_T\,,\\ (b)&\sum_{j=1}^{\frac{N}{2}} a_j\,\beta_{i,j}\,\log_2\left(1+\frac{P_i\,\rho_i}{\sum_{k=k+1}^{N} P_i\,\rho_i + a_j}\right) > R_i,\\ (c)\quad &P_i\,\rho_i\;\beta_{i,j} - \sum_{k=1}^{K} P_i\,\rho_i\;\beta_{i,j} \leq P_{tot},\\ (d)\quad &\beta_{i,j} = 1\ or\ 0, \end{aligned}$$

where $\beta_{i,j}$ is a binary variable and can be either 0 or 1 and a_j is the number of radio channels. The downlink joint user clustering and power allocation problem in uplink NOMA is as follows for a same set of constraints:

$$\max_{P}\ a_j\,\beta_{i,j}\sum_{j=1}^{N/2}\log_2\left(1+\frac{P_i\,\rho_i}{\sum_{k=k+1}^{N} P_i\,\rho_i + a_j}\right). \tag{3.18}$$

Due to the combinatorial nature of their problem, the authors employ a low-complexity sub-optimal user clustering algorithm for both the uplink/downlink NOMA systems. In both of downlink and uplink NOMA systems, the user clusters which had more distinctive channel gains provide superior throughput gains in the NOMA systems in comparison to OMA systems. Numerical results also show that the performance of downlink NOMA deteriorates if the cluster size increases beyond a certain threshold.

A joint user pairing with power allocation scheme is presented in [18], for a full duplex cognitive network. The authors put focus on both uplink and downlink in an underlay mode of operation. They start with formulating the user problem into an optimization problem and then continue to solve it using a low complexity algorithm and a high complexity algorithm. Simulation results show that the high-complexity algorithm is superior to the low-complexity algorithm in terms of sum rates, for all the power allocation methods used. This happens for both uplink and downlink modes of transmission. In [62], a user pairing algorithm in a downlink NOMA network is considered. The authors perform user pairing jointly with power allocation with the aim of optimizing throughputs. The formulation of the problem leads to a mixed integer programming problem. An optimal solution is developed for a NOMA system with two users, then additionally extended to four users. Their results show that the proposed algorithm achieves an optimal solution with better sum rate performance when evaluated against random user pairing schemes and OMA.

The authors in [63] consider user pairing in uplink NOMA. But this research is aimed at exploring how NOMA and user pairing can affect sum capacity of the users individually. Satisfying both individual user rates and observing performance in different scenarios is their goal. Indeed, it is realized in this work that NOMA shows improved capacity performance than OMA schemes in an uplink scenario. In this work, the authors stress out the fact that most researchers are only focused on throughput effect that user pairing has, while other equally important factors such as bit error rate and data reliability can be considered as well, as they have huge impact on the performance of NOMA. A channel gains sorting approach, to user pairing is considered in [63]. The authors consider a frequency selective fading system that pairs users according to their channel gains. They test this sorting approach based on a hierarchical power allocation process. In simpler terms, the proposed power allocation algorithm here focuses on allocating power to only the users in a group of two until all users are allocated. Simulation results show that the system can scale well and encompass a large number of users while outperforming most recent approaches in sum rates. This opened a new door into user pairing approaches based on sorting users according to channel gains.

Moreover, guaranteeing user fairness in user pairing is of equal importance as capacity but however it is a field which is not commonly researched and is still in its infancy. Several studies have proved this. As such, the authors in [25] work on user pairing schemes which focus on

two systems which guarantee user fairness amongst NOMA users. The first algorithm is CSS-PA which sorts users in accordance of their channel gains, and also a user difference selecting algorithm (UDSA) which pairs according the difference in channel gains. Results, both analytical and simulation, show that, apart from user fairness which both algorithms guarantee, CSS-PA offers better throughputs than common user pairing algorithms while UDSA also achieves a $1dB$ performance gain over factor compared to existing random algorithms.

Taking on a different approach, the authors in [64], study the user pairing concept which involves pairing that is done jointly with user scheduling, sub-channel allocation as well as power allocation. Their model is based on a downlink NOMA system. They formulate their problem to determine efficient configurations for capacity efficient scheduling, power allocation and sub-channel allocation. The problem turns into a linear programming problem and solved using the CPLEX optimization tool. Their results show that this kind of joint optimization provides better performance than existing resource allocation schemes in NOMA. In addition, they also make a realization that the energy efficient way of pairing users in downlink is limited by only two users sharing a frequency band.

Moreover, in [20], a novel system with a system model that employs NOMA between three users is considered. Basically, a cell-edge user's performance is boosted by two users who are very close to the base station using NOMA. It is assumed that the base station only serves the three users. The figure below displays how the users are situated in a macro cell for NOMA.

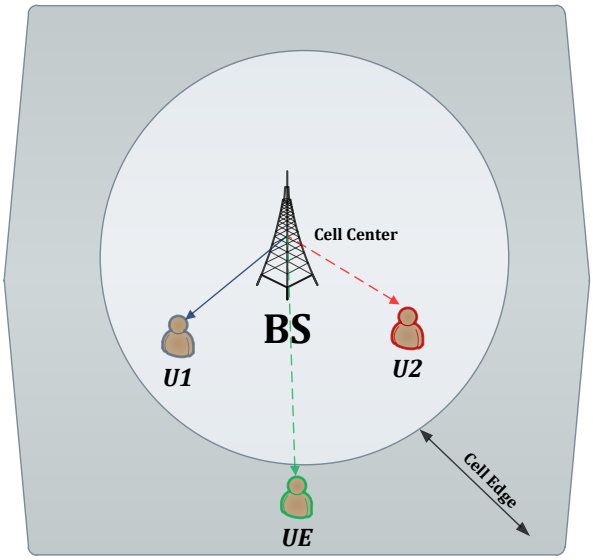


Figure 3. 1 The system model comprised of a base station BS, two cell-centre users $\{\mathbf{U_1}\ \mathbf{U_2}\}$, and a cell-edge user UE.

The cell centre users $\{U_1\ U_2\}$, have direct communication with the base station, while the cell-edge user has a channel gain that is too weak to have access to the base station. For this this reason, the cell-edge user can seek help from the inner users using NOMA so that they are able to communicate. Simulation results show superior sum rate performance when this pairing scheme is compared to orthogonal multiple access schemes. Better fairness is also observed from the results when compared to conventional NOMA and OMA. This is however when the SNR of the system is very low. In [65], the authors consider a downlink NOMA system in which a large number of users are divided into multiple 2-user groups for user pairing. Two user pairing algorithms, namely hill climbing, and simulated annealing are proposed to perform the 2-user user pairing. It is worth pointing out that these two algorithms are independent and do not have to work with a specific power allocation algorithm. In comparison with exhaustive search methods, both methods prove that they can achieve near-optimal solution while having low complexities.

The literature on user pairing reveals the evolution of different schemes that have been proposed to provide solutions for users pairing in NOMA. Nevertheless, with this literature at hand in user pairing, most of the user pairing done in NOMA systems is in either downlink or uplink. Not much work has been done in CR NOMA scenarios. Let alone the different paradigms of CR NOMA. Works presented in different paradigms of CR NOMA do not

involve user pairing in a secondary network which considers a large number of users and while also keeping an eye on individual primary user services. In particular, most schemes consider employing Hungarian algorithm to perform user pairing which is exhaustive. Based on our observations, this dissertation is dedicated to providing faster user pairing schemes which take into account, both a system consisting of a large number of users as well as protecting primary user services. In the next section we will look into different types of problems which can arise when user pairing and power allocation are performed.

3.4 Assignment Problems (AP)

Assignment problems are special members of the linear programming problems. They can be stated as: if there is a number of jobs n in which n employers can work on, find an assignment such that the total cost of assignment is minimum. Recognising that most of the user pairing, channel allocation and power allocation problems involve having to optimally assign two or more sets of elements (users, channels, sub-channels, power) to each other, most of the problems formulated this way take the form of the many variants of the classic assignment problems (AP).

The original version of the assignment problems aims to assign an "agent" to a "task" such that all the agents are optimally satisfied in the assignment [66], [67]. This could be to maximize or minimize a certain factor such as time. Hence, we can have an assignment where agents are assigned tasks. As such, it seems useful to take a look at different forms of the problems generated when we attempt to solve user pairing problems that involve assigning users, channels, sub-channels, power to each other. We can classify these types of problem into, two dimensional and multi-dimensional.

3.4.1 Two-dimensional assignment problems

The classic assignment problem

This is the most basic version of the assignment problems. It aims to minimize a one-to-one matching of users to channels. This can be between any number of n users and n channels. First an n by n cost matrix needs to be developed then the minimization assignment can be obtained. The mathematical representation of the problem can be given as:

$$\max_{x,c} \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}, \tag{3.19}$$

$$Subject\ to: \quad (a)\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1 \ldots n,$$

$$(b)\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1 \ldots n,$$

$$x_{ij} = 1 \ or\ 0,$$

where x_{ij} = 1 if a user has been assigned a channel j,and 0 otherwise, and c_{ij} represents the cost of assigning user i to channel j. Constraint (a) is to ensure that every user is assigned to only one channel, while constraint (b) is to ensure that every channel is assigned to only one user. The maximization case of the problem can be obtained by either:

i. Multiplying the above maximization case by -1,
ii. Instead of calculating c_{ij} , calculate $c_{max} - c_{ij}$.

It should be noted that c_{max} is the maximum of the cost matrix. In this problem, a dummy zero can be added for unbalanced cases and the optimum solution can still be achieved. An algorithm such as the Hungarian algorithm is usually employed to find the solution for the classic maximization problem. In [68], Kuhn and Munkres work on a variation of the Hungarian algorithm, to improve their original problem structure. They present two algorithms, namely, sparsity-based KM (sKM) and parallel KM (pKM). Their numerical results show that for random generated large number of users, both algorithms show improvement in computational performance when compared to existing algorithms. However, for our user pairing problem, to find a solution using the Hungarian algorithm, we would need to consider each frequency one after the next, while the Hungarian is applied in each iteration. This would however affect the complexity, and it would rise. To get to the solution using this algorithm, we would need to employ theorems which can enable us to employ the classic assignment problem and find a solution. We will do this in the next chapter and the results will be presented in Chapter 5.

The k-cardinality assignment problem

The k-cardinality assignment problem is considered in the case where a system model consists of the number of users and the number of resources to be allocated to the users is not equal.

Similar to the classic assignment problem, the k-cardinality assignment problem can be developed to represent a situation in which users need to be allocated to channels but the difference here is that only a subset of the channels is being considered for assignment. In this case, if we have m users and n channels but only k of the users need to be assigned, where $k \leq m$. Gilbert and Hofstra in [69] suggest that this type of an assignment problem arises in wireless communications when time slots of a telecommunications satellite are to transmit information from m earth stations to n different stations on space. Mathematically we can express the k-cardinality assignment problem as:

$$\max_{x,c} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}\,, \tag{3.20}$$

$$Subject\ to\colon\ (a) \sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1 \ldots n,$$

$$(b) \sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1 \ldots n,$$

$$(c) \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} = k,$$

$$(d)\ \ x_{ij} = 1\ or\ 0,$$

where m is the number of users and n is the number of channels but only k of the users need to be assigned, such that $k \leq m$. k represents the limitation on the users to be assigned. There are various efforts in the literature that utilize the k-cardinality assignment problem. In [70], Dell' Amico and Martello present a solution to the problem they describe to have a 2-matroid intersection, which has a framework of the k-cardinality assignment problem. They introduce and showcase how it can be applied to various scenarios. They then develop an efficient processing technique which can find the solution in polynomial time. In addition, the authors add on to solve a slot assignment problem in TDMA, using a pre-processing method which solves the k-cardinality problem. They show that for about 250,000 users, the solution can be reached in a short amount of time. This can be applied to find a user pairing solution in NOMA, when the number of users in the system k is greater than the number of frequencies being allocated.

The assignment problem with side constraints

In most wireless network's problems, the resources are always limited, and constraints need to be considered in the problem formulation. This is where assignment problems with side constraints evolve. These types of problems take care of the budgetary constraints in time, or resources in a network. In these types of assignment problems, for a user to be allocated a resource they must first qualify when the constraints are included. Failure to comply with any of the constraints given ends up in a particular user not being assigned. The mathematical representation of assignments problems with side constraints can be given as:

$$\max_{x,c} \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij} \;\leq\; b_k, \tag{3.21}$$

$$Subject\ to: \quad (a)\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1 \ldots n,$$

$$(b)\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1 \ldots n,$$

$$x_{ij} = 1 \ or\ 0,$$

where b_k can represent the amount of resource k available. There may be additional constraints if necessary, but for each of the users to be paired they must satisfy the constraints. These types of problems are applicable in NOMA networks where budgetary constraints on power and also interference from the secondary network to the primary users' needs to be considered first. For the user pairing problem in this work, the interference from the secondary network to the primary users' needs to be considered first. Several works on the assignment problem with side constraints have been conducted in the literature. Foulds and Wilson, work on an assignment problem with side constraints in [71]. Their problem is for a flexible system which is modelled as a zero-one integer programming model. The problem has restrictions for locating facilities for certain zones. They transform the problem to an assignment problem with side constraints and come up with a branch and bound algorithm which solves the problem. They first calculate the bound of the problem by relaxing the side constraints and then employ the Hungarian algorithm to solve the resulting problem.

3.4.2 Multi-dimensional assignment problems

Assignment problems usually match elements of two or more sets. As explained in the sub-section above, the classical assignment problem and its adversaries are two-dimensional or can equivalently be called two index problems. Extending the two-dimensional assignment problem to any higher dimension produces a multitude of problems which can be modelled as multi-dimensional problems. Unlike the two index problems, which can be solved in polynomial time. Multi-dimensional problems are NP-hard [69]-[71]. Several of such multi-dimensional problems, which have the potential for practical application in cognitive NOMA scenarios are discussed in the section below.

The planar three-dimensional assignment problem

The planar three-dimensional problem results when there are three sets of resources that must be allocated to each other. In this case, we have resources from group a needing to be matched with group b, at the same time with the ones from group c. It is the same case in our cognitive NOMA user pairing problem. A rule in this type of assignment problem is such that each user from group a is allowed to be matched at most one member from each group. The same rule applies to the other two groups b and c. The problem is one of minimizing the total cost of the assignment. A formal mathematical representation of the planar three-dimensional problem can be given as follows. Let A, B, C be different sets with cardinality n, for any triple $(a, b, c) \in A\ x\ B\ x\ C$, let $c_{a,b,c}$ denote the cost and $x_{a,b,c}$ denote the variable of assignment then we can have the problem as:

$$\max_{x,c} \quad \sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c}\,, \tag{3.22}$$

$$\begin{aligned} Subject\ to\text{:} \quad (a)\ & \sum_{a=1}^{n} x_{bc} = 1 \quad for\ each\ b, c\ combination, \\ (b)\ & \sum_{b=1}^{n} x_{ac} \leq 1 \quad for\ each\ a, c\ combination, \\ (c)\ & \sum_{c=1}^{n} x_{bc} \leq 1 \quad for\ each\ a, b\ combination, \\ & x_{a,b,c} = 1\ or\ 0\ \ for\ all\ a, b, c. \end{aligned}$$

In [72], the authors present an algorithm which solves the planar three-dimensional problem using a branch and bound approach. Their algorithm entails a lower and an upper bound procedure which takes advantage of the Latin square to find the solution. A solution for the planar three-dimensional problem is also presented in [73]. Here Gilbert et al, present a polynomial time algorithm which solves a planar three assignment problem that arises in a scheduling application. They develop a solution to the problem by using heuristics. The solution solves the problem in polynomial time, and they prove that it can also solve the travelling salesman problem, which is a variation of three-dimensional problem. The planar three-dimensional assignment problem can be applied directly to our user pairing problem in NOMA to find a solution; however, it is an NP hard problem. Below we have a look at its variants which can also be applied to solve out user pairing problem in CR-NOMA networks.

The axial three-dimensional assignment problem (AP3)

The first example of the axial three-dimensional assignment problem is provided by Gilbert and Hofstra in [69], [73]. In their work, they develop an assignment problem in which a set A of jobs was being assigned to a set B of workers using a set C of machines where $a \leq b \leq c$. Their objective being to minimize the total time to complete the jobs. This can type of problem can apply to a user pairing problem with three entities that must be allocated to each other. A formal mathematical representation of the planar three-dimensional problem can be given as follows. Let A, B, C be different sets for jobs, workers and machines respectively, where $a \leq b \leq c$. For any triple $(a, b, c) \in (A\ x\ B\ x\ C)$, let $c_{a,b,c}$ denote the cost and $x_{a,b,c}$ denote the variable of assignment. The three axial assignment problem is finding triples amongst all sets of a jobs such that no two triples agree in any coordinate. The mathematical representation of the axial three-dimensional problem can be given as:

$$\max_{x,c} \sum_{a=1}^{n} \sum_{b=1}^{n} \sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c}\,, \tag{3.23}$$

$$Subject\ to\text{:}\quad (a) \qquad \sum_{a=1}^{n} \sum_{b=1}^{n} c_{a,b,c}\ x_{a,b,c} \leq 1\ for\ all\ a, b, c,$$

$$(b) \qquad \sum_{b=1}^{n} \sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c} \leq 1\ for\ all\ a, b, c,$$

$$(c) \quad \sum_{a=1}^{n}\sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c} \leq 1\ f for\ all\ a,b,c,$$

$$(d) \quad x_{a,b,c} = 1\ or\ 0\ for\ all\ a,b,c.$$

Several authors have provided solutions to the axial three-dimensional problem, and these solutions can be modified and constrained to find a solution to the user pairing problem in a CR-NOMA system. The authors in [74], explore the axial three-dimensional assignment problem (3AP) and calculate the lower bounds for the solution. They transform the 3AP to a series of equivalent 3APs which provide the possibility of improving the lower bound. They first employ the projection method which utilizes a Hungarian algorithm to find a solution based on a Lagrangian relaxation approach. Secondly, they employ a transformation scheme which solves the problem iteratively by providing the possibility of using a lower bound to solve the problem. Their overall results show efficient computations in relation to the low bound. Moreover, a variation of the axial three-dimensional assignment problem is presented in [75], and a solution which provides competitive running times is obtained. The authors here use a heuristic method named Approximate Muscle Guided Beam Search (AMBS) for three-index assignment problem. The method combines the approximate muscle with beam search. Their extensive results are able to obtain competitive results which can be employed on a large scale. Their work also comprises of a promising method which aims to improve the efficiency of the beam search algorithm.

A study of new Lagrangian relaxation sub-optimal algorithms which solve multidimensional problems is presented in [76]. The algorithms presented are proven to have capabilities of solving different types of multidimensional assignment problems. These include the AP3, combinatorial optimization problems, and also non-smooth problems. These algorithms achieve solutions in real time and also enable solutions to be performed for systems having large datasets. Furthermore, in [77] a scheduling algorithm is associated with assigning teachers to students in a college, and the formulated problem is a 3-dimensional assignment problem. Similar to [76], a Lagrangian relaxation-based solution is described, and the results they get show that a maximum number of assignments are possible using this structure of solution. Moreover, a Lagrangian based solution which exploits the structure of the three axial dimensional problem is also presented in [78] and incorporates a class of facet inequalities that are solved by modifying a sub gradient procedure for good lower bounds. To find the solution

several variables are fixed to reduce computational complexity. Their results show that problems with up to about 343,000 different variables can be solved.

Fuzzy Three-Dimensional Axial Assignment Problem

When the costs of the assignment problem cost matrix are not deterministic but imprecise ones, such cases can be referred to as fuzzy [79]. Fuzzy dimensional assignment problems are also an extension of the general linear assignment problems. Most real-world problems and applications have costs that are not precise, or deterministic, for example available power. Besides that, vagueness may exist in most problems, hence during the construction of the problem its crucial to also look at the fuzzy case. Many researchers have already studied two-dimensional assignment problems in a fuzzy environment, which gave a need for research in the multi-dimensional problems as well.

The fuzzy three-dimensional axial assignment problem is usually NP-hard. It tries to create a one-to-one correspondence between three sets of assignment, as is the case in other multi-dimensional problems. This problem is the same assignment as when n worker to n machines to perform n jobs. A mathematical model of the fuzzy three-dimensional axial assignment problem can be given as:

$$\max_{x,c} \sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c}\,, \tag{3.24}$$

$$Subject\ to: \quad (a)\ \sum_{a=1}^{n}\sum_{b=1}^{n} c_{a,b,c}\ x_{a,b,c} \le 1\ for\ all\ a,b,c,$$

$$(b)\ \sum_{b=1}^{n}\sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c} \le 1\ for\ all\ a,b,c,$$

$$(c)\sum_{a=1}^{n}\sum_{c=1}^{n} c_{a,b,c}\ x_{a,b,c} \le 1\ for\ all\ a,b,c,$$

$$(d)\quad x_{a,b,c} = 1\ or\ 0\ \ for\ all\ a,b,c,$$

where each of the involved variables a, b, c are from fuzzy sets. Below we will give how the sets of the fuzzy model of assignment problems would differ from the normal sets. It is worth

noting that the fuzzy sets can be considered as an extension of conventional classical sets. To understand how fuzzy sets, operate, the context of set membership should be considered. In a nutshell, fuzzy sets allow partial membership in a set, which means that it contains elements that have varying degrees of membership in the given set. Below we consider some of the important concepts to consider when dealing with fuzzy sets for assignment. Solutions to different types of fuzzy assignment problems have been developed. Lin and Ma formulate a fuzzy dimensional axial assignment problem [80]. In their work, they focus on developing a solution for the fuzzy three-dimensional assignment problem that is focused on efficiency. Their initially constructed fuzzy three-dimensional model results in a mixed nonlinear programming problem. They then convert the model into a special multi-fractional max-min 0-1 programming problem. Two algorithms, branch and bound, and f-g trade off algorithm are introduced to solve the resulting programming problem. Their results show that the two algorithms perform more efficient than any existing algorithm in every way. Another algorithm which solved a fuzzy solid assignment problem is proposed in [81], using a method that is adopted for ranking imprecise data. The solution proposed for the problem is illustrated with a real-life problem first. The resulting problem is turned to a crisp problem then solved by a plane point method. The employed plane point method is able to find a solution. In addition, the authors argued that the method can even be used for various types three dimensional problems, which apply in real life.

3.5 Matching Theory and the Gale Sharpley Theorem

Wireless Networks have been gradually evolved leading to complex multi-tiered heterogeneous wireless architectures. The introduction of cognitive radios in wireless networks increased the complexity of wireless networks. Unlike the conventional cellular mobile systems which employ centralized access management strategies, wireless networks which utilize cognitive radio systems are distributed and cannot use these centralized strategies. Thus, in CR networks it is not possible to use an optimal centralized methodology for either channel allocation, user pairing or access to the spectrum. This nature of CR networks has motivated the need for new mathematical tools that can optimize resource allocation and access management in many emerging wireless systems. The proposed tools included random algorithms, centralized optimization algorithms and game theory.

However, these tools result in increased complexity when they deal with combinatorial integer programming problems in wireless networks which result from either channel allocation or user pairing. The high complexity is basically because the CR networks require global network information and centralized control, hence leading to a significant overhead and increased complexity. For a large number of users, random algorithms are significantly worse than the best centralized strategy. However, since the loss of random channel allocation is unacceptable and the nature of cognitive radios does not allow for centralized algorithms, only methods that consider channel gain or quality from the base station are needed. To cater for these distinctive features of the wireless networks which utilize cognitive radios, a novel, wireless-oriented classification of matching theory is proposed [82]. The matching theory approach not only overcomes the limitations provided by game theory and centralized algorithms, but they are also a promising technique in wireless communications for resource allocation.

3.5.1 Matching Theory

Matching theory essentially constitutes of managing the optimal allocation of resources to users in complex environments. This includes cognitive radio networks, small cell networks and large-scale device-to-device networks. Existing results by authors in [82] show that matching theory effectively improves resource allocation. The classification of matching problems can conventionally be found to be into three groups, namely, one-to-one matching many-to-one matching and many-to-many matching. In one-to-one matching each player can be matched to at most one partner from the other set while in many-to-one matching, we have multiple players from one set that can be matched to at least one member from the other set. Many-to-many matchings involve each of the sets having to match to at least one or more members in the other set [82].

On the other hand, for wireless oriented classification, matching can be classified into three classes namely, canonical matching, matching with externalities, and matching with dynamics. In canonical matchings, the preference of a user being matched to a resource depends solely on the information that is available at the time of the matching. This is usually used in CR networks, which have the secondary users in need of the licensed bands. Matching with externalities consists of matchings in which the preference of a user in matching translates to interdependencies developing between the users being matched. For example, when a user in the network gets matched to a resource, the preference of other users automatically changes.

For matchings with dynamics, each user being matched must first adapt to the matching dynamics of the matching environment. These can be fast fading, mobility and conditions in traffic. Matching theory has been proposed as one of the prominent solutions that could be employed for resource allocation in CR networks. Matching theory is a Nobel Prize winning framework that was proposed to cater for the different features in wireless networks especially in resource management. It has been proved that matching theory can improve the performance of resource allocation in wireless networks. With this in mind, it is easy to see that matching theory can be employed for user pairing also in cognitive networks.

3.5.2 Stable Matchings

Stable matching is a theory from game theory which forms a basis for resource allocation in CR networks, for users who share a set of frequencies. This theory was proposed by the Gale-Shapley as a solution to the stable marriage problem hence resulting in the emergence of the stable matching theorem. The authors in [83] analysed the Gale-Sharpley stable matching algorithm for channel allocation scenario. Their results show that stable matching approach has a performance that is in its worst case one half that of the optimal centralized allocation, which is significantly better than the random allocation.

One major advantage of the stable matching problem is that it can be computed by the Gale-Sharpley theorem, which is decentralized in nature, hence can be used in wireless networks with cognitive networks. The stable matching problem states that a stable allocation always exists. In simpler terms, it states that whenever we have two sets of N men and N women, given that every man and woman has their own preference regarding the opposite sex players, a stable matching can always be found. If we consider a general scenario with two sets of K users and L users, such that the set of users in one set is denoted by $k = \{1, \dots K\}$ and that of the other set is denoted by $l = \{1, \dots L\}$. Assuming that each user l has channel utility functions representing the transmission quality for each user l being matched with. This can be in terms of capacity of each user. Therefore, at any given time the utility functions are almost surely all different. We can then assume that $C_{k,l}$ represents the rate that user k can achieve when matched with user l, assuming that each user is capable of being matched to only one user at a time. From the given constructs we give below some of the important concepts to consider when dealing with matching theory.

Definition 3.1 We define a matching P as a *one-to-one* matching of the set $L \cup K$. We refer to $P(l)$ and $P(k)$ as the partner of l and k, respectively. Then we have $P(l) \in K$ and $P(k) \in L$.

Definition 3.2 A *preference list,* LIST $_k$ or LIST $_l$ can be defined in terms of a utility function that satisfies the QoS requirements through user matching of either user k based on l or user l based on k. The concept of a preference list represents the individual view that one user from one set of users has of the other set, based on local information provided. The preference of any user depends solely on the information available at this user based on the users with which it is seeking to match. This means that any user k trying to be matched would have to collect the information from the opposite set of, user $l's$ and perform a ranking according to its preferences and have an individual view of the preferences from the other side.

Definition 3.3 A matching P of user sets K and L is *stable* if and only if for every $k \in K$ and $l \in L$ satisfying $P(k) \neq l$ either LIST $_k(P(k)) >$ LIST $_k(l)$.

More explicitly a matching is stable, if for any pair k and $l \neq P(k)$ either k prefers $P(k)$ over l or l prefers its own partner over k. The Gale-Shapley theorem states that for any preference functions there is always a stable matching although not necessarily an optimal matching.

Definition 3.4 A matching is deemed to be *blocked* by an individual $k \in K$ or $l \in L$ when the individual $k \in K$ or $l \in L$ would rather prefer not to be matched, instead of being matched to its current partner. This happens when the current partner results in a low revenue for the individual k or l or fails to meet the minimum requirements of the user.

Definition 3.5 A The pair (k, l) is referred to as a *blocking pair* for the matching, if both k and l would prefer to be matched with each other, but they are not matched. Both users would achieve a higher rate if they are matched in comparison with their current matching partners.

3.5.3 Gale-Shapley theorem

The Gale-Shapley theorem solves the stable matching problem. It aims to find a stable matching in any two given sets K and L, of the same size M. For each user $k \in K$ there is a one-to-one function which forms a preference list for the user LIST $_k(l)$ which ranks the preferences of users from set L. Similarly, for the set of users in L there is a one-to-one function

LIST $_l(k)$ which ranks the preferences of each user k from the set K where a higher value means a higher preference. A matching is a one-to-one function from K to L.

Goal. Given a set of preference lists LIST $_k(l)$ and LIST $_l(k)$ among two given sets K and L, find a matching in which all users are satisfied with their matches.

Unstable Matching: Occurs in two scenarios:

- when user K prefers a certain user L apart from their match.
- when user L prefers a certain user K apart from their match.

Stable Matching: Assignment with no unstable pairs form both user groups.

Solution: Gale Sharpley Theorem

---

01. **Input** (Preference lists for both K and L)
02. LIST $_k(l)$ and LIST $_l(k)$
03. **Initialize** matching set P to empty matching.
04. **while** (some user k is unmatched and hasn't proposed to every user l)
05. $l \leftarrow$ first user on k's list to whom k has not yet proposed.
06. **if** (l is unmatched)
07. Add $k-l$ to matching P
08. **else if** (l prefers k to current partner k')
09. Replace $k'-l$ with $k-l$ in matching P.
10. **else**
11. l rejects k.
12. **return** stable matching P.

The Gale Sharpley theorem can therefore be applied to solve a two-dimensional assignment problem given the preference lists. The theorem can also be used to solve a three-dimensional assignment problem, given one dimension is held while solving for the other two variables. Thus, for a given assignment problem above, adding a third dimension f as shown below, we use the Gale Sharpley theorem by holding the dimension f each time and solving for the preference lists of both K and L as inputs at each f.

$$Minimize \sum_{f=1}^{F} \sum_{k=1}^{K} \sum_{l=1}^{L} c_{f,k,l} \; x_{f,k,l} . \tag{3.25}$$

This means that given the preference lists for K and L, the solution to the three-dimensional case can be achieved. We will use the theorem to solve a similar problem developed in the next chapters, that is used to solve an optimization problem given additional constraints.

3.6 Conclusion

In this chapter, we survey different power allocation solutions that have been developed in NOMA. We also introduce the user pairing concept in a NOMA scenario and look deeply into the currently existing frameworks that solve the user pairing problem. Moreover, we look into the concept of assignment problems after recognising that most of the user pairing, channel allocation and power allocation problems take different form of the assignment problems. We then look in detail at both the two-dimensional assignment problems and also multi-dimensional assignment problems. Moreover, we introduce the fuzzy assignment problems which are mainly used in assignment problems in which the costs are not deterministic but imprecise ones and also look at different solutions that have been proposed in the literature for this kind of problems. Lastly, we look at the concept of matching theory and the Gale Sharpley theorem which is utilized by one of our proposed algorithms in the later chapters.

Chapter 4

System Model and Problem Formulation

4.1 Introduction

In this chapter, we develop the system model based in underlay mode of cognitive operation and then we formulate NOMA capacity equations based on the resulting model. We then consider different power allocation schemes in an underlay scenario. Moreover, we formulate the user pairing optimization problem based on the model and outline limiting conditions of the optimization problem in terms of theorems. Both the formulated equations and the power allocation schemes are used to solve our optimization problem in an underlay scenario. Furthermore, we demonstrate that a solution to the optimization problem can be found by the Hungarian algorithm in which its results can be used as reference for new developed algorithms in the next chapter.

4.2 System Model for an Underlay CR-NOMA Network

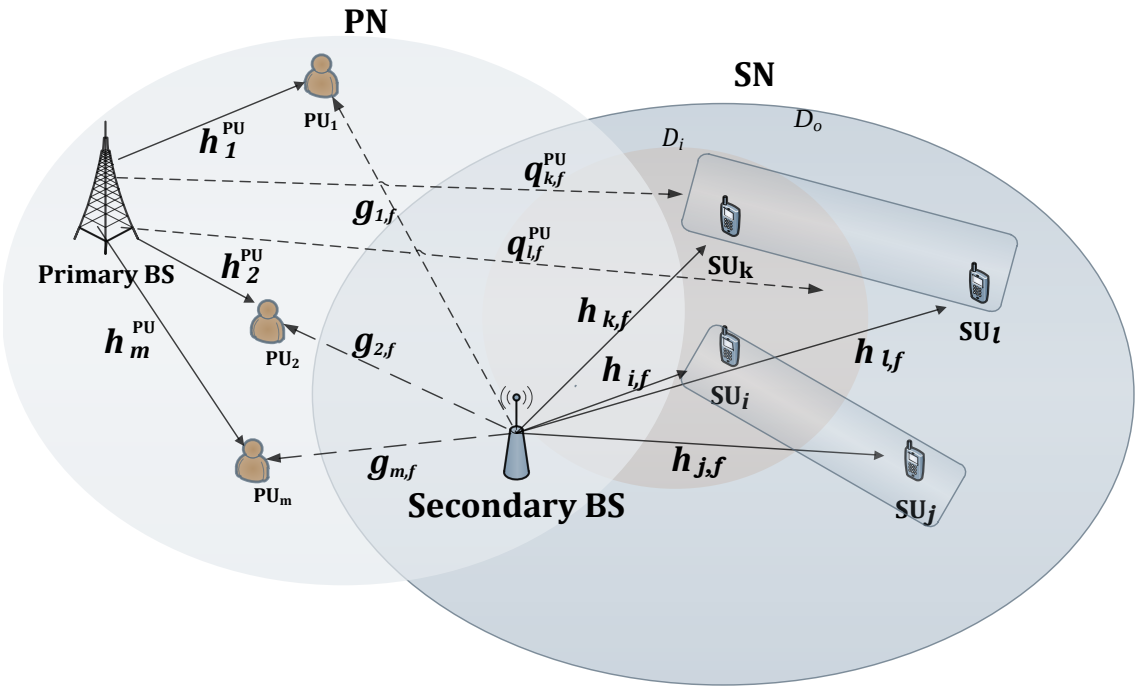


Figure 4. 1 The system model showing users $\mathbf{SU}_k$ and $\mathbf{SU}_l$ in a CR-NOMA system.

We model a wireless communication system which assumes the coexistence of a licensed Primary Network (PN) and a Secondary Network (SN) where secondary user (SU) nodes in the secondary network can access the PN spectrum in an underlay mode. In the PN, we consider a primary Base Station (BS) and a random number of primary users (PUs) distributed in a finite two-dimensional plane. In the SN, we consider uniformly deployed secondary users around the secondary BS, and this is region for NOMA and user pairing. Thus, the secondary BS communicates with the SUs using NOMA. It is crucial to note that the transmission power at the secondary BS is constrained to limit the interference at the PUs. At the same time if the PN transmission power is over a specified threshold, user pairing cannot occur. This is because the SU's have opportunistically accessed the spectrum in use by the PUs and will deteriorate the performance of the PUs if interference is not monitored and also the primary BS's interference' effect on the SUs needs to be monitored. This is the same case in underlay cognitive radio networks. We assume that the SN region is divided into two regions. A region D_i for the SUs which are located near the secondary BS and region D_o for the ones far from the BS. All regions within the SN have SUs randomly distributed in a finite two-dimensional plane. Let M represent all the PUs in the PN such that $0 < m \leq M$, and let f , be one of the channels occupied by a PU such that $0 < f \leq M$. Then, let K be the set of all the SUs in the region near the secondary BS such that $0 < k \leq K$, and let L be the set of all the SUs in the outer region of the SN such that $0 < l \leq L$. This means that for NOMA, user l and user k are paired with each other to occupy a frequency borrowed from the PN. To simplify calculations in further stages, we will also refer to the total number of users in the secondary network as $N = (K + L)$.

Furthermore, the channels in the SN are sorted as $|h_{K,f}|^2 > \cdots > |h_{k,f}|^2 > |h_{1,f}|^2$ and $|h_{L,f}|^2 > \cdots > |h_{l,f}|^2 > |h_{1,f}|^2$. Where $|h_{k,f}|^2$ defines a channel from the secondary BS to the k^{th} SU at a frequency f, and $|h_{l,f}|^2$ defines a channel from the secondary BS to the l^{th} SU at a frequency f. Each of the secondary users k and l, experiences interference from the primary BS through channels $q_{k,f}^{PU}$ and $q_{l,f}^{PU}$, as the channels receiving the interference for the users, respectively. It should be noted that the SU pairs are expected to transmit data on the same spectrum with the PUs. According to underlay CRN, the transmit power at the secondary BS is constrained as follows:

$$P_{k,l,f} \quad < \left[\frac{I_{PU}}{|g_{m,f}|^2} \right], \tag{4.1}$$

where $P_{k,l,f}$ is the transmit power at the secondary BS and must always be less than the total base station power P_T and I_{PU} is the maximum permissible interference power at the PU that is using frequency f, and $|g_{m,f}|^2$ is the overall channel gain from the secondary BS to a PU at frequency f. If this constraint is violated and the PU detects a higher interference than the constraint, user pairing cannot occur. Figure 4.2 on the left below shows that in the beginning any user can be matched with any user if the underlay constraints are satisfied, but the question remains, which user is the perfect match for another user. We would be looking to end up with a pairing match as shown on Figure 4.2 on the right. It displays the form of our problem and how we want to assign a user k to a user l and then assign the pair a frequency f.

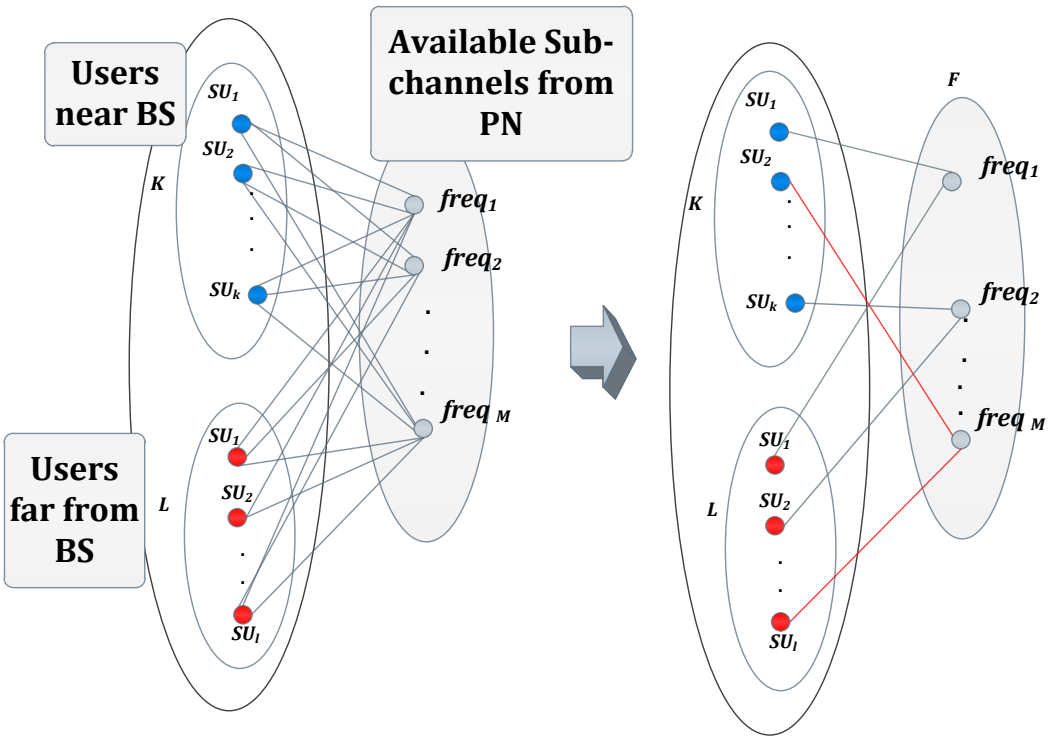


Figure 4. 2 The assignment of a user $\boldsymbol{k}$ to user $\boldsymbol{l}$ to frequency $\boldsymbol{f}$.

In Figure 4.2, we have users in the set K (near users), users in the set L (far users) and also a set of available frequencies from the primary network set F. It has been shown in the literature shows that NOMA works best if we pair a near BS user with a far user. We aim to assign each user from the set K, a partner from the set L then give them a frequency in F for use in underlay mode. This is shown in Figure 4.2 on the right. It should be noted that any user k can be paired

with any user l for any frequency f as seen on the left. This results in a 3 by 3 assignment problem. In essence, we have three entities that have to be assigned to each other, being K users to L users to F frequencies. This is essentially the three-dimensional problem explained in the previous chapter.

4.2.1 Sum Rate Requirements for Paired NOMA Users

Now, let us take a look at how the sum rate equations look like. We consider the NOMA system model in downlink scenario. We assume user l and user k have been paired for transmission on frequency f and have been allocated the power $P_{k,l,f}$ by the secondary base station. According to NOMA, the received signal at the inner k^{th} secondary user $\text{SU}_k$is given by:

$$y_{k,f} = h_{k,f}\left[\sqrt{\alpha_{k,f}\ P_{k,l,f}}\ s_{k,f} + \sqrt{\alpha_{l,f}\ P_{k,l,f}}\ s_{l,f}\right] + q_{k,f}^{PU}\sqrt{P^{PU}}\ s_{PU} + \eta_{k,f}, \qquad (4.2)$$

and the signal received at the outer SU_l at the edge is expressed as:

$$y_{l,f} = h_{l,f}\left[\sqrt{\alpha_{k,f}\ P_{k,l,f}}\ s_{k,f} + \sqrt{\alpha_{l,f}\ P_{k,l,f}}\ s_{l,f}\right] + q_{l,f}^{PU}\sqrt{P^{PU}}\ s_{PU} + \eta_{l,f}, \qquad (4.3)$$

where $P_{k,l,f}$ is the constrained transmit power at the secondary BS and P^{PU} is the constrained transmit power from the primary network. The expression $\left[\sqrt{\alpha_{k,f}\ P_{k,l,f}}\ s_{k,f} + \sqrt{\alpha_{l,f}\ P_{k,l,f}}\ s_{l,f}\right]$ represents the combined signal of SU_k and SU_l as it is emitted from the secondary BS. The expressions $q_{k,f}^{PU}\sqrt{P^{PU}}\ s_{PU}$ and $q_{l,f}^{PU}\sqrt{P^{PU}}\ s_{PU}$ are also signals from the PN. The channels $h_{k,f}$ and $h_{l,f}$ are from the secondary base station to user k and l, respectively and the channels $q_{k,f}^{PU}$and $q_{l,f}^{PU}$are form the primary network. For power allocation in the secondary network, $\alpha_{k,f}$ and $\alpha_{l,f}$ are used to represent the power splitting coefficients of each SU_k and each SU_l such that $\alpha_{k,f} + \alpha_{l,f} = 1$. Similarly, $\eta_{k,\text{f}}$ and $\eta_{l,\text{f}}$ are the Gaussian noise terms, with zero mean and a single sided power spectral density N_0. It is important to note that successive interference cancellation is employed at both secondary users to cancel the strong signal form the primary BS. This is because the power from the primary base station is strong and as it is expected to be the largest compared to the secondary BS, thus needs to be cancelled first.

Therefore, we set an interference power constraint in which SIC needs to be applied once the signal from the primary network violates this constraint. This means that user pairing cannot occur before the PU signal from the primary BS is cancelled by both SU users. The set constraint is given below:

$$\left|q_{k,f}\right|^2 P^{PU} > I_o \left|h_{k,f}\right|^2 P_{k,l,f}, \tag{4.4}$$

$$\left|q_{l,f}\right|^2 P^{PU} > I_o \left|h_{l,f}\right|^2 P_{k,l,f}, \tag{4.5}$$

where I_o is the margin threshold. For user pairing to occur the previous constraint needs to be respected first and SIC performed accordingly. After this, each SU_k will then also perform SIC within each pair. This is because within each of the NOMA pairs we have $\alpha_{l,f} > \alpha_{k,f}$, therefore according to successive interference cancellation, SU_k decodes the SU_l's message and then removes it, before it decodes its own message in a successive manner [84], while the SU_l the will treat SU_k's message as noise each time. After SIC of the PU and SU_l in the secondary network, the achievable rate of SU_k is then given by:

$$R_{k,f} = \log_2\left(1 + \frac{\alpha_{k,f} P_{k,l,f} \left|h_{k,f}\right|^2}{N_0}\right). \tag{4.6}$$

As a result, the achievable rate of each SU_lafter SIC of the PU signal can be written as:

$$R_{l,f} = \log_2\left(1 + \frac{\alpha_{l,f} P_{k,l,f} \left|h_{l,f}\right|^2}{\alpha_{k,f} P_{k,l,f} \left|h_{l,f}\right|^2 + N_0}\right). \tag{4.7}$$

Specifically, any SU_l is willing to perform NOMA with another only if it can achieve a higher rate in comparison with the conventional OMA transmission. In OMA, we can set the rates of both SU's using conventional OMA as the minimum rate requirements, namely $R_{k,req}, R_{l,req}$ and they can be expressed as:

$$R_{k,req} = \frac{1}{2} \log_2\left(1 + \frac{\alpha_{k,f} P_{k,l,f} \left|h_{k,f}\right|^2}{N_0}\right), \tag{4.8}$$

and

$$R_{l,req} = \frac{1}{2} \log_2\left(1 + \frac{\alpha_{l,f} P_{k,l,f} \left|h_{l,f}\right|^2}{N_0}\right), \tag{4.9}$$

where the factor $\frac{1}{2}$ is due to the fact that the bandwidth resources are split between two users. Moreover, according to our system model, knowing the total power to be shared between the users and also, given the secondary users' predefined constraints and the QoS requirements are met, the NOMA capacity of the system throughput can then be formulated as:

$$C_{k,l,f} = R_{k,f} + R_{l,f},$$

$$C_{k,l,f} = \log_2\left(1 + \frac{\alpha_{k,\mathrm{f}}\, P_{k,l,f}\ |h_{k,f}|^2}{N_0}\right) + \log_2\left(1 + \frac{\alpha_{l,\mathrm{f}}\, P_{k,l,f}\ |h_{l,f}|^2}{\alpha_{k,\mathrm{f}}\, P_{k,l,f}\ |h_{l,f}|^2 + N_0}\right). \qquad (4.10)$$

The capacity in equation 4.10 occurs when a user k has partnered with user l, on a particular frequency f. The secondary users are willing to perform NOMA with one another only if they can achieve a higher rate in comparison with the conventional OMA transmission. We can show the performance gain of the users when they are paired for NOMA, over conventional OMA transmission, by easily illustrating using a high SNR analysis. We will also show the bounds in which the NOMA capacity of the paired users occurs. The theorem below from [26] illustrates the rate comparison given a NOMA system. We have modified the theorem to include an available frequency from the primary network. The bandwidth of the system, the power split coefficients for the two users k and l in NOMA, and the total power to be shared $P_{k,l,f}$ should also be given for the rate comparison. The theorem provides an SNR analysis for NOMA and OMA in our system model at an operating frequency f from the primary network, which provides the NOMA bounds for a chosen frequency.

Theorem 1: For a single placement of users from set K and set L, the NOMA capacity of user k paired with user l on a frequency f, $C_{k,l,f}$ is bounded by:

$$\frac{1}{2}\log_2\left(1 + \frac{\alpha_{k,f}\, P_{k,l,f}\ |h_{k,f}|^2}{N_0}\right) + \frac{1}{2}\log_2\left(1 + \frac{\alpha_{l,f}\, P_{k,l,f}\ |h_{l,f}|^2}{N_0}\right) \le C_{k,l,f} \le$$

$$\log_2\left(1 + \frac{\alpha_{k,f}\, P_{k,l,f}\ |h_{k,f}|^2}{N_0}\right) + \log_2\left(1 + \frac{\alpha_{l,f}\, P_{k,l,f}\ |h_{l,f}|^2}{N_0}\right) \qquad (4.11)$$

Proof:

The proof for this theorem which provides a rate comparison given a NOMA system, is given in Section 3, Lemma 1 in [59], in which the authors show that the bounds above can be proven easily using the inequality of arithmetic and geometry.

■

4.2.2 Total Power Allocation for Paired NOMA Users

Taking into account the requested rates above, the power that is going to be split between the two users that will be using frequency f from the primary network, $P_{k,l,f}$ could be derived. After this power is derived, the power can now be split between the two users in NOMA. The power splitting is such that the splitting coefficients of each SU_k and SU_l pair, follow the equation $\alpha_{k,f} + \alpha_{l,f} = 1$. To calculate each user's power share, a power allocation scheme needs to be employed. We will explore different power allocation schemes in the next section. We will first start with a theorem which obtains the total power that is going to be shared amongst the users in the secondary network. The theorem is outlined below.

Theorem 2*:* In a NOMA system with two users, k and l, the total power, $P_{k,l,f}$ to be shared between the two users using frequency f, in order to meet both user's required rates $R_{k,req}$ and $R_{l,req}$ is found to be:

$$P_{k,l,f} = N_0 \left[\frac{2^{2R_{k,req}} - 1}{\left|h_{k,f}\right|^2} + \frac{2^{2R_{l,req}} - 1}{\left|h_{l,f}\right|^2} \right]. \tag{4.12}$$

Proof:

From equation, (4.8) and (4.9) we have the requested rates of the secondary users, SU_k and SU_l given as shown below, respectively.

$$R_{k,req} = \frac{1}{2}\log_2\left(1 + \frac{\alpha_{k,f}\ P_{k,l,f}\ \left|h_{k,f}\right|^2}{N_0}\right),\ R_{l,req} = \frac{1}{2}\log_2\left(1 + \frac{\alpha_{l,f}\ P_{k,l,f}\ \left|h_{l,f}\right|^2}{N_0}\right).$$

Below we rearrange the two equations to solve for $P_{k,l,f}$ and obtain the total power that is going to be shared amongst the users. Rearranging the two equations we obtain:

$$2^{2R_{k,req}} = \left(1 + \frac{(1-\alpha_{l,f})\, P_{k,l,f}\ |h_{k,f}|^2}{N_0}\right), \text{ and } 2^{2R_{l,req}} = \left(1 + \frac{\alpha_{l,f}\ P_{k,l,f}\ |h_{l,f}|^2}{N_0}\right).$$

Now, solving the simultaneous equations to eliminate $\alpha_{l,f}$, we obtain:

$$\frac{\frac{\alpha_{l,f}\ P_{k,l,f}\ |h_{k,f}|^2}{N_0}}{\frac{\alpha_{l,f}\ P_{k,l,f}\ |h_{l,f}|^2}{N_0}} = \frac{2^{2R_{k,req}} + \frac{P_{k,l,f}\ |h_{k,f}|^2}{N_0}}{2^{2R_{l,req}} - 1}.$$

$$P_{k,l,f} = N_0 \left[\frac{2^{2R_{l,req}} - 1}{|h_{l,f}|^2} + \frac{2^{2R_{k,req}} - 1}{|h_{k,f}|^2}\right]. \tag{4.13}$$

This concludes the proof.

■

4.3 Power Splitting Schemes

It is obvious from the total power equation in the section above that, the power that is going to be split and utilized by the paired NOMA users k and l, not only affects the achievable throughput of user k but also the throughput of user l. This is mainly due to power-domain multi-user multiplexing. Therefore, it is imperative that we consider how the power can be split optimally for the two users and allocated. Multi-user transmit power allocation and user pairing are connected to each other. In our case, optimal power allocation remains computationally complex because, for each candidate user, a set of all possible combinations of power allocations from each side must be considered and at the same time the total power allocated should be within base station budget.

To allocate power to the two paired users k and l, we consider three schemes, namely a Lagrange based power splitting, which aims to maximize the total sum-rate of matched users, a bound based power splitting method which promises to offer near optimal power allocation to NOMA users in user pairing and lastly a fractional transmit power control algorithm which aims to offer users who are far from the base station more power. Both the Lagrange based power splitting and the bound based power splitting algorithms are formulated based on the individual user required rates in equations. All these power splitting mechanisms are

incorporated into our user pairing algorithms in Chapter 5. The fractional transmit power control algorithm is used also for user pairing algorithms which are used as reference.

4.3.1 Lagrange Based Power Splitting

In this method the Lagrange multipliers are employed to formulate a power splitting strategy for NOMA system users within each pair that maximizes the total sum-rate of matched users, k and l. The method was adopted from the literature and the authors employ the individual user required rates obtained in the previous section. A power splitting algorithm was then deduced by using Lagrangian Multipliers and apply the Karush-Kuhn-Tucker (KKT) conditions, to obtain the power allocation strategy. This is mainly to deduce the split coefficients $\alpha_{l,f}$, and $\alpha_{k,f}$ for each pair. For this power splitting scheme, the power allocation coefficient $\alpha_{l,f}$ of the user SU_l who has been paired with SU_k was given as shown below in the theorem.

Theorem 3: Given the total power, $P_{k,l,f}$ for a pair of NOMA users, and the individual user requested rates, $R_{k,req}$ and $R_{l,req}$ at a particular frequency f, the power allocation coefficient $\alpha_{l,f}$ that maximizes the total sum-rate of matched users is given as:

$$\alpha_{l,f} = \begin{cases} \dfrac{\frac{P_{k,l,f}}{N_0}\,|h_{k,f}|^2 + 1 - 2^{R_{k,req}}}{\frac{P_{k,l,f}}{N_0}.\,|h_{k,f}|^2} & \text{if } R_{k,f} < R_{l,f} \\ \dfrac{\frac{P_{k,l,f}}{N_0}\,|h_{l,f}|^2 + 1 - \sqrt{\frac{P_{k,l,f}}{N_0}.\,|h_{l,f}|^2 + 1}}{\frac{P_{k,l,f}}{N_0}.\,|h_{l,f}|^2} & \text{if } R_{k,f} > R_{l,f} \end{cases} \quad (4.14)$$

Proof:

The proof for this theorem is given in Section 3, A of [17], in which the authors decouple the objective function to deduce the power allocation factor using the Lagrange multipliers and apply the Karush-Kuhn-Tucker (KKT) conditions, to obtain the power allocation strategy.

■

4.3.2 Bound Based Power Splitting

For a NOMA system with two users, a power allocation mechanism can be deduced to obtain a near optimal power split for the two NOMA users using a bound based splitting mechanism. This can then be used for each of the borrowed frequency from the PN. From the objective function that maximizes the total sum rate of the paired users:

$$\underset{\{\alpha_{l,f}\}}{\text{Maximize}} \left[R_{k,f}(\alpha_{l,f}) + R_{l,f}(\alpha_{l,f})\right] \tag{4.15}$$

$$\begin{aligned} Subject\ to\ (a) \quad & R_{k,f} \geq R_{k,req}, \\ (b) \quad & R_{l,f} \geq R_{l,req}, \\ (c) \quad & \alpha_{k,f} + \alpha_{l,f} = 1\ , \end{aligned}$$

The power split variable, can be obtained from the differetial of the objective function. This power splitting method was obtained from the literature in [62] and it is given by Theorem 4.

Theorem 4*:* In a NOMA system which has two users, SU_k and SU_l, the power allocation coefficient $\alpha_{l,f}$ based on the objective function above is given by:

$$\alpha_{l,f} \;=\; \frac{\sqrt{1 + \left|h_{l,f}\right|^2 \gamma} - 1}{\left|h_{l,f}\right|^2 \gamma} \tag{4.16}$$

Where $\gamma = \frac{P_{k,l,f}}{N_0}$.

Proof:

The proof for this theorem is given in [62] as Thoerem 1 in which the authors show that obtaining the differential of the objective function and using the constraints can find the optimal value of $\alpha_{l,f}$.

■

4.3.3 Fractional Transmit Power Control (FTPC)

The structure of NOMA enables it to offer better overall throughputs when a user who is further from the based station is allocated more power. This is also the same case when user pairing is performed with users who differ more in channel gains from their base stations. With fractional transmit power control (FTPC), the SNR values of the users are fully considered and then more power is allocated to the weaker user. FTPC power allocation algorithm is adopted from the literature [25]. The FTPC algorithm makes it easy for the weaker users with poor channel gains to obtain more power and guarantees their required rates. The algorithm does this with low complexity. With FTPC, the transmit power of user SU_k and SU_l paired in frequency f is dynamically allocated according to the channel gains of the multiplexed users as follows:

$$P_{k,f} = \frac{P_{k,l,f}\left(\left|h_{k,f}\right|^2\right)^{-\alpha}}{\left[\frac{\left|h_{k,f}\right|^2}{N_0} + \frac{\left|h_{l,f}\right|^2}{N_0}\right]^{-\alpha}} \tag{4.17}$$

$$P_{l,f} = \frac{P_{k,l,f}\left(\left|h_{k,f}\right|^2\right)^{-\alpha}}{\left[\frac{\left|h_{k,f}\right|^2}{N_0} + \frac{\left|h_{l,f}\right|^2}{N_0}\right]^{-\alpha}} \tag{4.18}$$

Where $0 < \alpha \leq 1$ is equal to the individual transmit power allocation among the users. The more α is increased, the more power is allocated to the users.

4.4 Optimization Problem Based on the System Model

It is worthy to note that in the NOMA system, each user from K can only pair with one user from L. To the get the optimization problem for all the users, the total power for the two users is calculated and power allocation coefficients for the users are also calculated. The optimal total sum-rate of all matched secondary users SU_k and SU_l, can then be expressed as:

$$\max_{x \in \{0,1\}} \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{f=1}^{F} x_{k,l,f}\, C_{k,l,f}, \tag{4.19}$$

$$
\begin{aligned}
Subject\ to: \quad & (a) \quad R_{k,f} \geq R_{k,req}, \\
& (b) \quad R_{l,f} \geq R_{l,req}, \\
& (c)\ \alpha_{k,\mathrm{f}} + \alpha_{l,\mathrm{f}} = 1\ , \\
& (d)\ P_{k,l,f} \quad < \left[\frac{I_{PU}}{\left|g_{m,f}\right|^2}\right], \\
& (e) \quad \left|q_{k,f}\right|^2 P^{PU} > I_o \left|h_{k,f}\right|^2 P_{k,l,f}, \\
& (f)\ \left|q_{l,f}\right|^2 P^{PU} > I_o\ \left|h_{k,f}\right|^2 P_{k,l,f}, \\
& (g) \sum_{k=1}^{K} x_{k,l,f} \ \leq\ 1, \\
& (h) \sum_{l=1}^{L} x_{k,l\ ,f} \leq\ \ 1,
\end{aligned}
$$

where $x_{k,l\ ,f}$ can be 0 or 1 depending on the outcome of the user pairing. The objective is to optimize the sum-rate utility. The constraints (a) and (b) are respectively imposed to ensure that, the minimum rate requirements of the SU_k and SU_l can be achieved. The constraint (c) ensures that the total power budget will not be exceeded by the power of the secondary base station and constraint (d) ensures that the maximum permissible interference power at the nearest PU is never violated. The constraints (e) and (f) ensure that the interference from the primary network is cancelled, should it violate the constraints. The conditions (g) and (h) ensure that each SU_k will only be matched to one SU_l.

4.5 Solution by the Hungarian Algorithm

Once a power allocation scheme has been chosen, the optimization problem can be solved. There are many methods of solving this problem, but since this optimization problem takes form of an assignment problem, we can use the conventional Hungarian algorithm to solve it when holding one dimension as this is a 3-dimensional problem. We can then use its results as reference for our new algorithms in Chapter 5. In this section we show that the maximization problem above can be solved using the Hungarian algorithm, if we choose a frequency f and solve the resulting 2-dimensional problem for all the frequencies. At this frequency combined NOMA capacity of the two users has the form $R_{k,f} + R_{l,f}$. The contribution of the users in the

sum rate can be decoupled. $R_{k,f}$ represents the sum rate contribution of user k at frequency f frequency while, $R_{l,f}$ represents the contribution of user l at frequency f.

Moreover, taking note of NOMA rates of the paired users, we can approximate the value of $C_{k,l,f}$ for a single frequency f. Thus, for a chosen frequency f the optimization problem can be solved optimally using the Hungarian Algorithm with a polynomial time complexity. This can be possible when the entries of the Hungarian algorithm are made with respect to the user pairing problem constraints when the capacities of the users are being calculated. Thus, for a chosen frequency f, the optimization problem $\max_{x \in \{0,1\}} \sum_{k=1}^{K} \sum_{l=1}^{L} x_{k,l,f} C_{k,l,f}$ can be solved optimally using the Hungarian algorithm with a polynomial time complexity. We can choose the frequencies starting with the highest performing frequency and compare it to when they are chosen sequentially.

To show that we can use the Hungarian method, we assume the K users are jobs such that $k \in \{1, \dots K\}$, and they need to be matched with L users as workers such that $l \in \{1, \dots L\}$ and that a rating matrix $\boldsymbol{X}$ which has user capacity entries at frequency f of $C_{k,l,f}$, can be given, where the members of the matrix are positive sum rates for all k and l. In matrix $\boldsymbol{X} \in \mathbb{R}^{\mathrm{KxLxF}}$ the $(k,l)th$ entry is equal to capacity $C_{k,l,f}$ for a chosen frequency. To obtain the optimal pairs, we can solve an equivalent problem which is a linear assignment problem with the cost matrix $\boldsymbol{X_c} \triangleq [\boldsymbol{K}\ \boldsymbol{L}]$ for each f. This problem takes the same form as the golden job-worker problem with K workers and L jobs, and we want to assign each job k to a worker l, at each frequency. This is a linear assignment problem that can be solved optimally using the Hungarian algorithm with a polynomial time complexity.

The solution to this is having the objective function to be maximized in each case. The solution considers the problem constraints, that is, any non-negative entry $C_{k,l,f}$ should provide $R_{k,req}$ and $R_{l,req}$ required rates in such that the OMA rates are not violated. This means that no job-worker assignment will be less than initial rating in that job, in the golden assingment problem terms. The maximization problem however cannot be solved by the Hungarian and needs to be converted to a minimization equal. This can be done by (a) multiplying all of the $C_{k,l,f}$ by -1, or (b) replacing each $C_{k,l,f}$ by $C_{max} - C_{k,l,f}$. Thus converting the problem to a minimization case. The minimization problem answers the problem, what is the smallest total matching

$R_{1,f} + \cdots + R_{K,f} + R_{1,f} + \cdots + R_{L,f}$ possible for the budget. The problem dual can now be solved using the Hungarian algorithm to obtain for each f to obtain $C_{k,l,f}$.

$$C_{k,l,f} = \quad R_{k,f} + \ R_{l,f}. \tag{4.20}$$

It is important to note that the Hungarian algorithm solves the minimization case of the problem which can give an equal solution to the maximization case. Once the solution to the minimization problem has been obtained, we can replace each $C_{k,l,f}$ by $C_{max} - C_{k,l,f}$ to get the maximization case solution. Later, the results of the Hungarian problem solution will help in comparison to newly developed schemes that help solve the problem.

For a single placement of users, the total number of user pairs given the number of users available from the secondary network N, where $N = (K + L)$, is $\frac{(N)!}{2^{(N)}}$, can be approximated using Stirling's approximation which is given by $\sqrt{4\pi(N)}(\frac{\sqrt{2}\,(N)}{e})^{2(N)}$. It can be seen from the given approximation theorem that the total number of user pairs will exponentially increase for a large number of placed users in the secondary network. In that case the, optimum solution is not feasible using the Hungarian algorithm. Thus, we need to simplify the problem in order to achieve an optimum solution.

4.6 Limiting Conditions of the Optimization Problem

This section outlines some of the limiting conditions in the form of theorems that facilitate a way to the formulation of a less complex solution to the optimization problem in equation 4.19. As it was shown in the previous section that, employing the Hungarian algorithm can work to find a solution but the algorithm is complex. In this section, we will work towards building assumptions and theorems which lead to a solution for the optimization problem that is less complex.

Theorem 5: For a two user NOMA system with k_1 and l_2 and two available frequecies, f_1 and f_2, with $\left|h_{k_1,f_1}\right|^2 > \left|h_{k_1,f_2}\right|^2 > \left|h_{l_2,f_1}\right|^2 > \left|h_{l_2,f_2}\right|^2$, it is best to pick the highest channel gain user first to get optimal capacities when performing user pairing.

Proof:

The proof for this thereom is presented in Appendix A.

Theorem 6: In a NOMA system with with K users and L users, if the channel coeficients in the NOMA system are arranged in descending order such a way that $|h_{K,f}|^2 > \cdots > |h_{k,f}|^2 > |h_{1,f}|^2$ and $|h_{L,f}|^2 > |h_{l,f}|^2 > |h_{1,f}|^2$, then the optimum user pair P_{opt} which maximizes the sum rate at a frequency f when user k is paired with user l can be shown as:

$$P_{opt} = \{(K - (j-1)), (1 + j))\}, \tag{4.21}$$

where $0 < j < L$.

Proof:

The proof for this thereom is presented in Appendix B.

4.7 Conclusion

In this chapter we develop the system model and then formulate the user pairing problem according to the underlay cognitive network, taking care of the underlay constraints as well. We start by deriving the NOMA capacity expressions based on the underlay environment. We then use the resulting capacity expressions to formulate the user pairing optimization problem based on the model. Furthermore, we consider the power allocation mechanisms that can be used in the system model and also consider the power split needed to allocate the total power to the paired users in an underlay cognitive network. Both the derived expressions and the power allocation equations are used to solve our optimization problem in an underlay scenario. Lastly, we look into the limiting conditions in the form of theorems that facilitate a way to the formulation of a less complex solution to the optimization problem.

Chapter 5

Proposed User Paring Schemes

In this Chapter, we discuss various approaches to solve the user pairing problem formulated in Chapter 4. Most of the existing studies which solve the user pairing problem focus mostly on the capacity of the users. These studies leave out how the capacities would change in cognitive environments, where the primary network transmissions are affected by the interference from the secondary network. In this chapter, we discuss algorithms which cater for both individual required rates and also operate in underlay cognitive environments. We describe Underlay RPA and CSS-PA Underlay and a new PLMA algorithm which we propose to solve the optimization problem in an underlay cognitive NOMA network. For Underlay RPA and CSS-PA Underlay, we adopt the RPA and CSS-PA algorithms and use them as reference. The key in these algorithms which makes them suitable for underlay environments is that they always consider the interference from the primary network before performing user pairing, at the same time they consider the interference inflicted upon the primary users in an underlay scenario. For the PLMA algorithm we employ the concept of preference lists and matching theory in the secondary network to find best pair solutions. We then compare the new PLMA algorithm to exhaustive algorithms such as the Hungarian algorithm. It is worth mentioning that, part of the work presented in this chapter has been presented at IEEE AFRICON conference, held in Accra, Ghana 2019 [85].

5.1 User Pairing Algorithms

1. **Underlay Random Pairing Algorithm, (RPA-Underlay):** The objective of this algorithm is to assign each user k from the set K, where $k = \{1 \dots K\}$, a partner l from the set L , where $L = \{1 \dots L\}$ then give them a frequency in set f, where $f = \{1 \dots F\}$ for use in underlay mode. This method is adapted from Han Zhang et al [40], in which they consider user pairing algorithms in a SIC NOMA based system.

   For our system model we modify the algorithm to operate in an undelay cognitive NOMA network. In our method, we first consider the interference from the primary network and cancel it out if the set constraints are not met. Then we calculate the

secondary base station power using Theorem 2 from Chapter 4. We then ensure that the calculated power respects the secondary base station constraint that protects the primary network users. If satisfactory, we calculate the new user rates in a NOMA scenario and compare the obtained rates to individual user requested rates. If also satisfactory, we perform user pairing. The total capacity for the NOMA system should always be according to Theorem 1 given in Chapter 4. The power allocation schemes used in the user pairing algorithm are obtained from Theorem 3 and 4 from the previous chapter as well.

For all users, this is done by randomly picking a user SU_k and then randomly picking their partner l from the set L. After that, an available frequency from the primary network f is also chosen at random for the pair. Because of the low-complexity performance of the algorithm, we use this algorithm mainly to study and compare with other pairing algorithms. The method is shown below:

Underlay Random Pairing Algorithm

01. **Input:** I_{PU} , P_T , $R_{l,req}$, $R_{k,req}$, I_o, P^{PU}, $Total\ Placements$, P
02. **Output:** $C_{k,l,f}$
03. **Initialize:** $h_{k,f}$, $h_{l,f}$, $g_{m,f}$, $q_{l,f}$, $q_{k,f}$
04. **for** all frequencies in F**,** choose random frequency f, where f is not in P
05. **for** all secondary users in K, choose random user k at a frequency f, where k is not in P
06. **for** all secondary users in L, choose random user l at a frequency f, where k is not in P
07. Compute $P_{k,l,f}$ using equation (4.1)
08. **if** ($|q_{k,f}|^2 P^{PU} > I_o |h_{k,f}|^2 P_{k,l,f}$) or ($|q_{l,f}|^2 P^{PU} > I_o |h_{l,f}|^2 P_{k,l,f}$)
09. Perform SIC to cancel out noise from the PN
10. Calculate $\alpha_{l,f}$, $\alpha_{k,f}$ using a chosen power allocation scheme
11. **if** ($P_{k,l,f} < \left[\frac{I_{PU}}{|g_{m,f}|^2}\right]$) and ($P_{accumulated} < PT$ **)**
12. Calculate NOMA capacity for SU_k using equation (4.4)
13. Calculate NOMA capacity for SU_l using equation (4.5)
14. **if** ($R_{k,req} < R_{k,f}$) and ($R_{l,req} < R_{l,f}$)
15. Compute $C_{k,l,f}$
16. Perform User Pairing of SU_k and SU_l
17. Update the pair SU_k and SU_l in pairing matrix P
18. and go to *line* 05.
19. **end if**
20. **end if**
21. **end if**
22. **end for**
23. move to next frequency
24. **end for**
25. **end for**
26. **return** $C_{k,l,f}$

2. **Underlay Channel State Sorting-Pairing Algorithm, (CSS-PA-Underlay):** Previous Works on NOMA have proved that pairing users with a higher channel gain difference is beneficial in improving both individual user rates and the system throughput. The CSS-PA algorithm adopts this concept. CSS-PA was adopted and modified from [25] in which it was used mainly to increase user fairness using the channel gains considerations of the users.

   In the CSS-PA method users adopt the same methodology used in RPA underlay but in CSS-PA, the channel gains are sorted in descending order first before user pairing. This benefits the users that are far from the BS. They benefit from the fact that NOMA allocates them more power, to counter them being far from the BS. This also further enhances the system capacity in the secondary network, and simultaneously improves user fairness in the NOMA system. This happens all under strict primary user interference constraints and QoS requirements constraints of the users being paired. When the underlay constraints have been met, user pairing is performed. CSS-PA benefits the correct receiving of poor channel-condition users at the receiver, further enhancing the system fairness while raising system capacity. The method is shown below:

Underlay Channel State Sorting-Pairing Algorithm

01. **Input:** I_{PU} , P_T , $R_{l,req}$, $R_{k,req}$, I_o, P^{PU}, $Total\ Placements$, P

02. **Output:** $C_{k,l,f}$

03. **Initialize:** $h_{k,f}$, $h_{l,f}$, $g_{m,f}$, $q_{l,f}$, $q_{k,f}$

04. **for** all frequencies in F**,** choose a frequency f, where f is not in P

05. Order users in users in K, order in descending order of $h_{k,f}$

06. Order users in users in K, order in descending order of $h_{l,f}$

07. **for** all secondary users in K, choose a user k at a frequency f, where k is not in P

08. **for** all secondary users in L, choose a user l at a frequency f, where l is not in P

09. Compute $P_{k,l,f}$ using equation (4.1)

10. **if** ($\left|q_{k,f}\right|^2 P^{PU} > I_o \left|h_{k,f}\right|^2 P_{k,l,f}$) or ($\left|q_{l,f}\right|^2 P^{PU} > I_o \left|h_{l,f}\right|^2 P_{k,l,f}$)

11. Perform SIC to cancel out noise from the PN

13. Calculate $\alpha_{l,f}$, $\alpha_{k,f}$ using a chosen power allocation scheme

14. **if** ($P_{k,l,f}$ $< \left[\frac{I_{PU}}{|g_{m,f}|^2}\right]$) and ($P_{accumulated} < PT$ **)**

15. Calculate NOMA capacity for SU_k using equation (4.4)

16. Calculate NOMA capacity for SU_l using equation (4.5)

17. **if** ($R_{k,req} < R_{k,f}$) and ($R_{l,req} < R_{l,f}$)

18. Compute $C_{k,l,f}$

19. Perform User Pairing of SU_k and SU_l

20. Update the pair SU_k and SU_l in pairing matrix P

20. and go to *line* 05.

21. **end if**

22. **end if**

22. **end if**

23. **end for**

24. move to next frequency

25. **end for**

26. **end for**

27. **return** $C_{k,l,f}$

3. Existing User Pairing Algorithms

For comparison purposes, we compare our two schemes above with the pairing algorithms below and use them as reference for our improved algorithms above. Insight into how the original algorithms operate is given below.

A. Random Pairing Algorithm, (RPA): RPA is simple, as user' pairs from secondary network are randomly chosen from the candidate users to pair. This means that any user in the set K would just be paired with any user who is far from the BS. RPA is not recommended since the existing information regarding user's channel states is not used which increase complexity and also no user fairness is considered. We chose this algorithm to set a reference for a clear comparison with our modified underlay pairing algorithms above.

Random Pairing Algorithm

01. **Input:** I_{PU} , P_T , $R_{l,req}$, $R_{k,req}$, I_o, P^{PU}, $Total\ Placements, P$
02. **Output:** $C_{k,l,f}$
03. **Initialize:** $h_{k,f}$, $h_{l,f}$, $g_{m,f}$, $q_{l,f}$, $q_{k,f}$
04. **for** all frequency in F**,** choose random frequency f, where f is not in P
05. **for** all secondary users in K, choose random user k at a frequency f, where k is not in P
06. **for** all secondary users in L, choose random user l at a frequency f, where l is not in P
07. Compute $P_{k,l,f}$ using equation (4.1)
08. **if** $(|q_{k,f}|^2 P^{PU} > I_o |h_{k,f}|^2 P_{k,l,f})$ or $(|q_{l,f}|^2 P^{PU} > I_o \, \varepsilon \, |h_{l,f}|^2 P_{k,l,f})$
09. Perform SIC to cancel out noise from the PN
11. Compute $P_{k,f}$, $P_{l,f}$ using FTPC power allocation algorithm
12. **if**$(P_{k,f} + P_{k,f}) < P_T$)
13. Calculate NOMA capacity for SU_k using equation (4.4)
14. Calculate NOMA capacity for SU_l using equation (4.5)
15. Compute $C_{k,l,f}$
16. Perform User Pairing of SU_k and SU_l
17. Update the pair SU_k and SU_l in pairing matrix P
17. and go to *line* 05.
18. **end if**
10. **end if**
19. **end for**
20. move to next frequency
21. **end for**
22. **end for**
23. **return** $C_{k,l,f}$

B. Channel State Sorting-Pairing Algorithm, (CSS-PA): This algorithm modifies the RPA above to improve not only throughput but and also user fairness. Users in each of the sets K and set L are ordered according to their channel gains first before user pairing can occur. This will result with a good channel-condition secondary user from the set K being paired with user with a poor channel-condition user from the set L.

Channel State Sorting-Pairing Algorithm

01. **Input:** I_{PU} , P_T , $R_{l,req}$, $R_{k,req}$, , I_o, P^{PU}, $Total\ Placements, P$

02. **Output:** $C_{k,l,f}$

03. **Initialize:** $h_{k,f}$, $h_{l,f}$, $g_{m,f}$, $q_{l,f}$, $q_{k,f}$

04. **for** all frequency in F**,** choose a frequency f, where f is not in P

05. Order users in users in K, order in descending order of $h_{k,f}$

06. Order users in users in K, order in descending order of $h_{l,f}$

07. **for** all secondary users in K, choose a user k at a frequency f, where k is not in P

08. **for** all secondary users in L, choose a user l at a frequency f, where l is not in P

09. Compute $P_{k,l,f}$ using equation (4.1)

10. **if** $(\left|q_{k,f}\right|^2 P^{PU} > I_o \ \left|h_{k,f}\right|^2 P_{k,l,f})$ or $(\left|q_{l,f}\right|^2 P^{PU} > I_o \ \left|h_{l,f}\right|^2 P_{k,l,f})$

11. Perform SIC to cancel out noise from the PN

12. Compute $P_{k,f}$, $P_{l,f}$ using FTPC power allocation algorithm

13. **if**$(P_{k,f} + P_{k,f}) < P_T)$

14. Calculate NOMA capacity for SU_k using equation (4.4)

15. Calculate NOMA capacity for SU_l using equation (4.5)

16. Compute $C_{k,l,f}$

17. Perform User Pairing of SU_k and SU_l

18. Update the pair SU_k and SU_l in pairing matrix P

19. and go to *line* 05.

20. **end if**

10. **end if**

21. **end for**

22. move to next frequency

23. **end for**

24. **end for**

25. **return** $C_{k,l,f}$

5.1 Simulation Results

This section presents the simulation results for performance evaluation comparing RPA-Underlay and CSS-PA Underlay and their RPA and CSS-PA counterparts which will be used as reference. The results are average capacities and the number of users which can be paired. The interference inflicted on the primary network and also the QoS requirements are also considered.

Table 5. 1 Simulation parameters.

Parameters	**Values**
PN radius, SN radius	5km, 3km
Number of subcarriers	F=M
Number of users, SU_k, SU_l	$K = 20$, L=20
SU_k required rates	$R_{k,req} = 8kbps$
SU_l required rates	$R_{l,req} = 10kbps$
Number of random placements	5000
Path and penetration loss at distance d (km)	$148.1 + 37.6\ log10(d)$ dB
Secondary BS SNR	$200dB - 260dB$
I_o	$1x10^{-5}$
I_{PU}	$1x10^{-8}$
PT	$1x10^{-3}$
P^{PU}	$1x10^{-3}$

In our simulation, we assume a PN radius and the SN radius of 5km, and 3km respectively as shown in the table above. The available number of available channels form the PN is assumed to be 20 as well as the total number of users K and L to be paired. The required transmission rates of the users are assumed as shown in the table for a total of 5000 placements. As seen in Table 5.1, each user SU_k and SU_l has QoS requirement rates of 8kbps and 10kbps. Each

capacity is obtained by dividing the average capacity for each case by 1000 to get our capacity in kbps. In our simulations, user pairing cannot occur when either the channel gains or power allocation resulted in a lower rate than the stipulated rate.

The choice of these parameters is motivated by [86], in which the authors layout their cell structure in a downlink scenario based on the release 9 of the 3GPP physical aspects [87]. Our channels, path and penetration loss at distance are also adopted from [86] as shown in the table. The SNR is obtained by dividing the total base station power P_T by N_0, the noise power and then obtaining the SNR. The choice of the SNR range is high because low SNR scenarios do not produce any user pairs. However, because in downlink NOMA, the base station has sufficient power and can be increased if needed, we consider a high SNR scenario to ensure user pairs.

For comparison purposes, we employ the simple and less complex FTPC power allocation algorithm for use in RPA and CSS-PA. For underlay RPA and underlay CSS-PA we employ the Lagrange based power allocation scheme and a bound based power allocation scheme. We use a power allocation factor of $\alpha = 0.1$ for FTPC, for both RPA and CSS-PA. The value of α is obtained through simulations by considering various values and observing which values give out best performance. For a total of 5000 random placements user pairing performed in the secondary network, ensuring all underlay constraints. The simulation results obtained from the above assumptions are shown below.

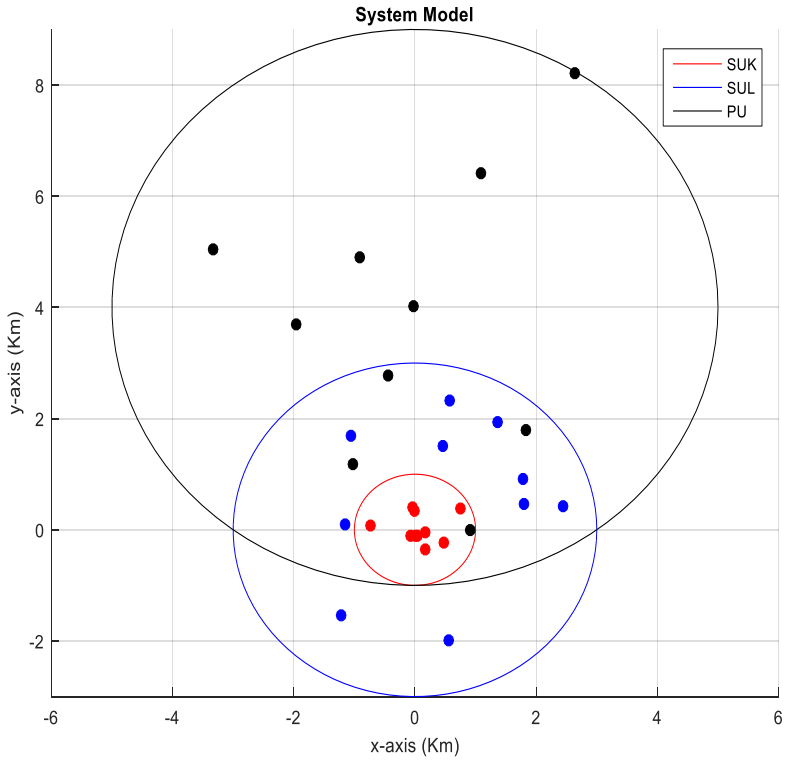


Figure 5. 1 Secondary users actively distributed in the PN and in the SN.

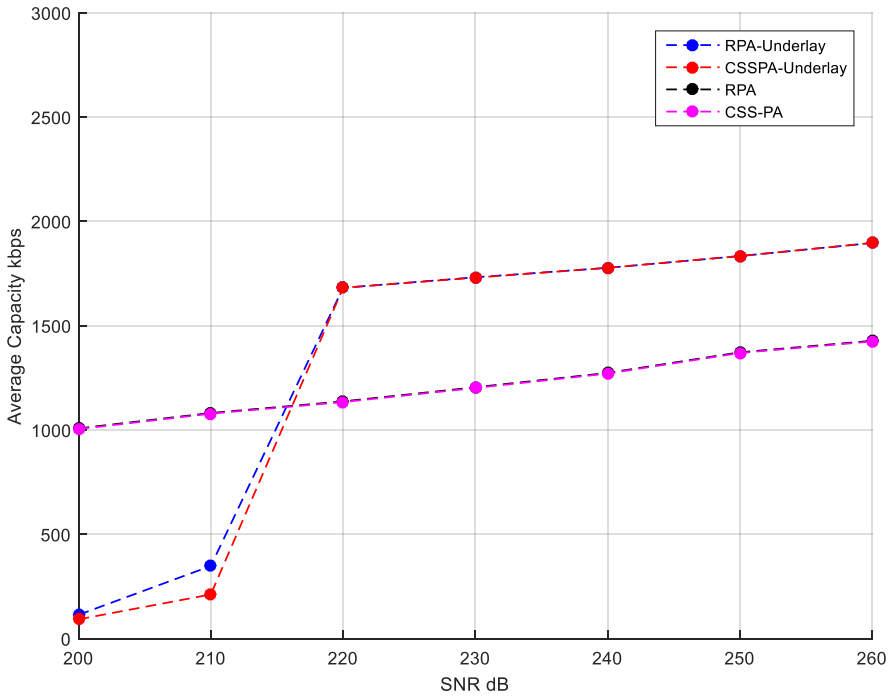


Figure 5. 2 Average Capacities for each pairing algorithm Lagrange based power allocation scheme for Underlay schemes.

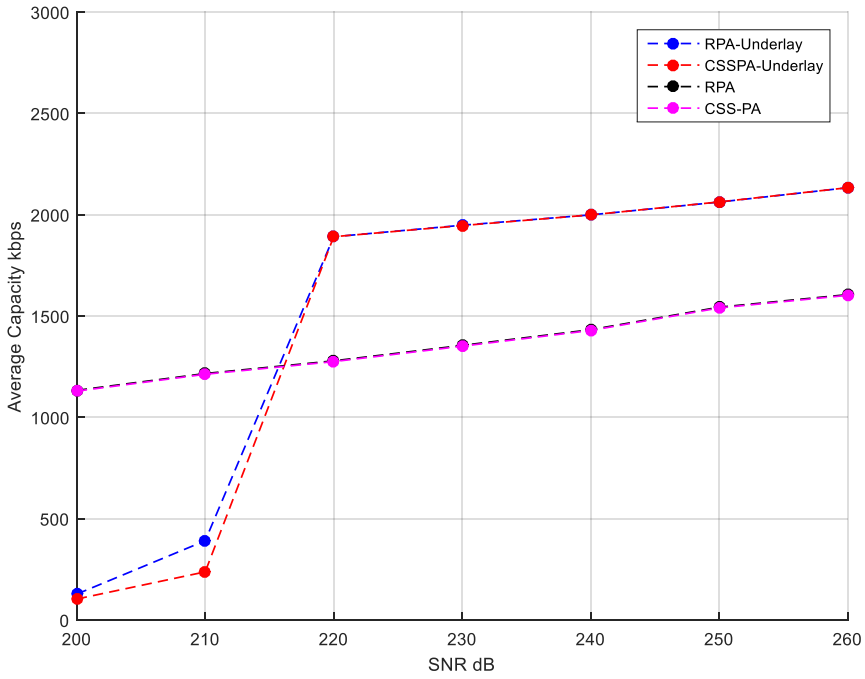


Figure 5. 3 Average Capacities for each pairing algorithm using bound based power allocation for underlay schemes**.**

Figure 5.1 shows, using a cartesian plane a random distribution of secondary users in the SN for one placement. A random placement of the primary users in the PN is also shown. Figure 5.2 shows the average channel capacity achieved for RPA, CSS-PA and also the underlay version of the user pairing algorithms using a Lagrange based power allocation. It can be observed that the modified underlay algorithms achieved higher rates in comparison to the methods adopted from the literature. The proposed user pairing schemes seem to underperform at some SNR range however, this because at low SNR the interference is not being cancelled and user pairs do not occur. The RPA and CSS-PA perform better at this SNR range as they do not consider any interferences. Figure 5.3 shows the average channel capacity achieved for RPA, CSS-PA and also the underlay version of the user pairing algorithms using the bound based power allocation scheme. It can be noticed that the bound based power allocation algorithm shows superior performance compared to the other power allocation schemes. The results show that it is important to consider power allocation schemes when performing user pairing. Failure to critique a power allocation scheme for user pairing can result in low sum rates for a network. The results also showcase the importance of successive interference cancellation in NOMA. It can be observed that low sum rates are expected when SIC is not

being performed. Below we look at the results which show the performance of the user pairs for each of the employed algorithms.

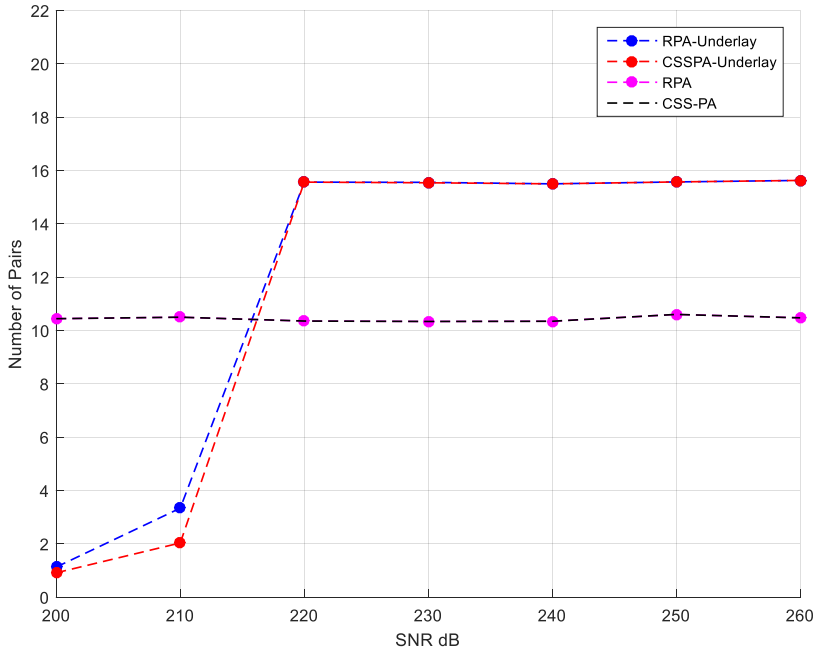


Figure 5. 4 Number of $\mathbf{SU}_{\boldsymbol{k}}$ and $\mathbf{SU}_{\boldsymbol{l}}$ user pairs among different SNR levels using Lagrange based power allocation scheme for Underlay schemes.

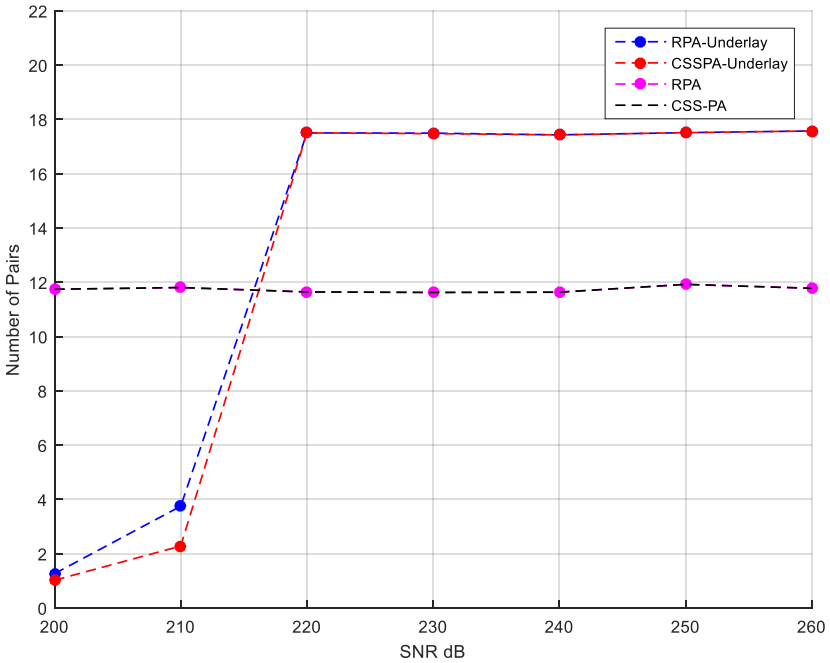


Figure 5. 5 Number of $\mathbf{SU}_{\boldsymbol{k}}$ and $\mathbf{SU}_{\boldsymbol{l}}$ user pairs among different SNR level using the bound based power allocation for underlay schemes.

Figure 5.4 shows that out of 20 users from both user K and user L, close to 16 pairs would be obtained for the underlay pairing algorithms while a low 12 pairs was observed for both RPA and CSS-PA. Again, the modified undelay algorithms show significance performance over the algorithms form the literature. This was using the Lagrange based power allocation scheme. And improvement in performance can be observed for the bound based power allocation scheme in Figure 5.5 as almost 18 pairs of users were obtained when using this power allocation scheme. It can be noticed that the bound based power allocation algorithm shows superior performance compared to Lagrange based power allocation scheme. The results show again that, it is important to consider power allocation schemes when performing user pairing.

In Figure 5.6 and 5.7 below, we look at the results which show the performance of the number of user pairs that are able to pass the secondary network interference test using different power allocation schemes.

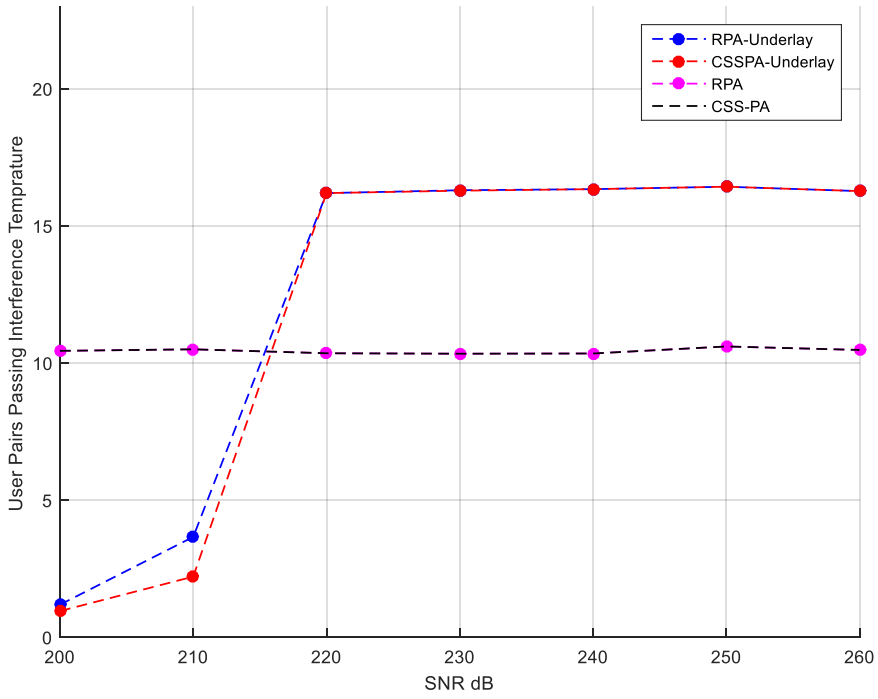


Figure 5. 6 Number of $\mathbf{SU}_{\boldsymbol{k}}$ and $\mathbf{SU}_{\boldsymbol{l}}$ user pairs passing PU interference test using Lagrange based power allocation scheme for Underlay schemes.

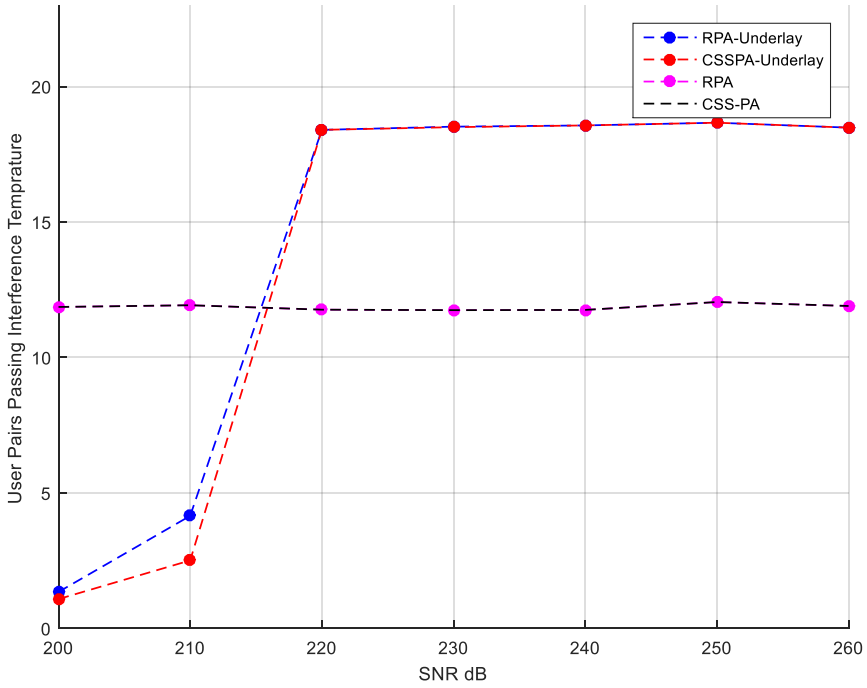


Figure 5. 7 Number of $\mathbf{SU}_{\boldsymbol{k}}$ and $\mathbf{SU}_{\boldsymbol{l}}$ user pairs passing PU interference test using bound based power allocation scheme for Underlay schemes.

In Figure 5.6 it can be observed that, the modified underlay algorithms seemed not be interfering with the PUs, while the random algorithms were mostly found to be interfering with the primary network users. This is because the proposed user pairing algorithms consider the interference being inflicted on the primary network before performing user pairing, enabling them to pass the interference test more than the algorithms which do not. From these results, it can be observed that, an algorithm which does not check the performance, in terms of interference towards the primary network will not perform well towards the primary network. This occurred for both power allocation schemes as observed in Figure 5.7. It show that it is crucial to consider interference in underlay networks.

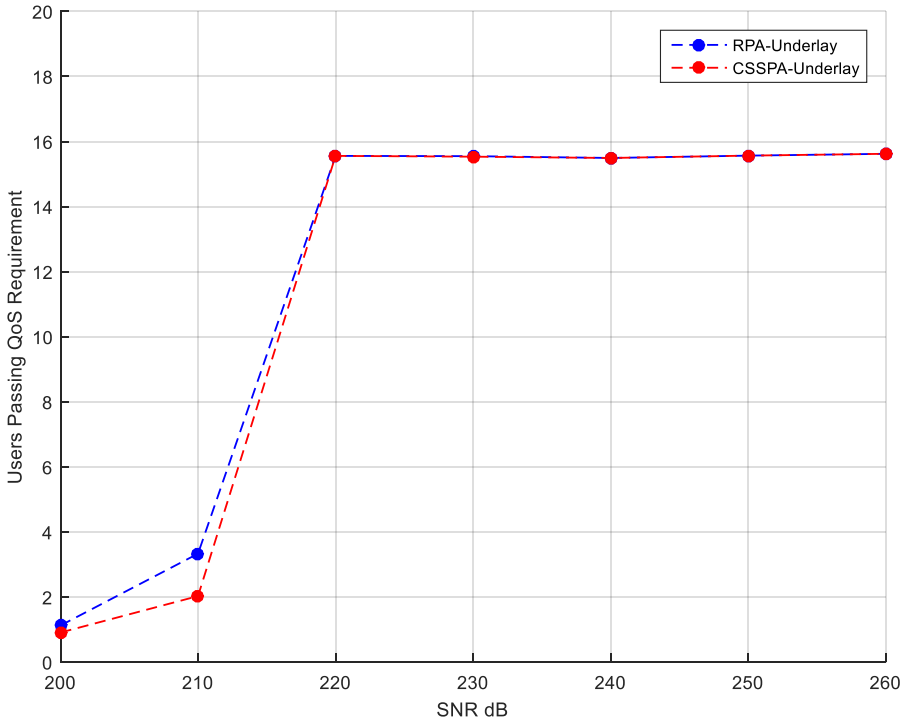


Figure 5. 8 Users that pass the underlay QoS requirements using Lagrange based power allocation scheme for Underlay schemes.

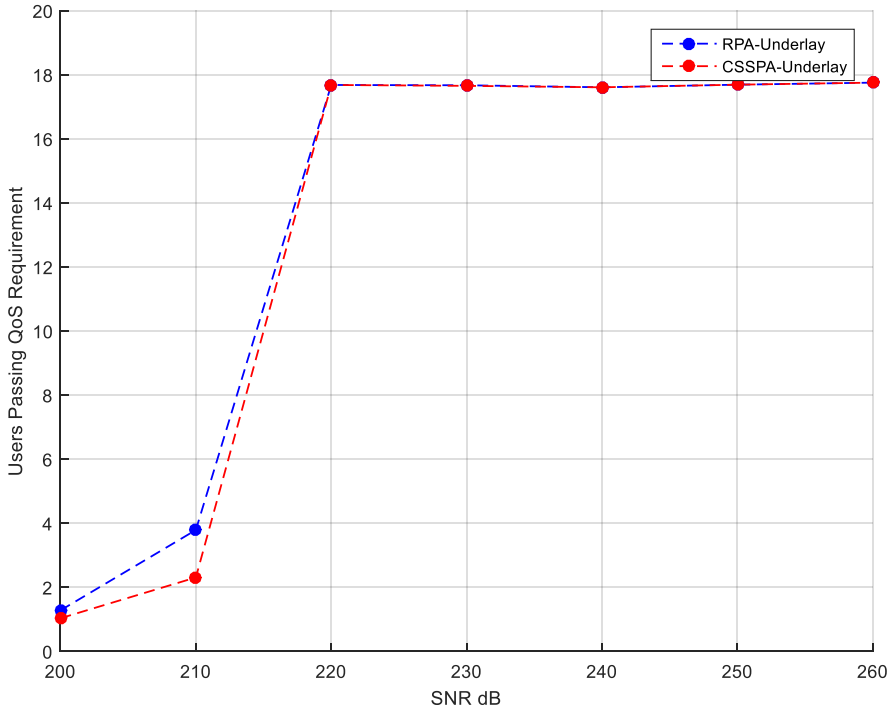


Figure 5. 9 Users that pass the underlay QoS requirements using the bound based power allocation scheme for Underlay schemes.

Figure 5.8 shows the statistics for the underlay algorithms as to, how many, out of 20 users were able to pass QoS requirements before pairing. This is a requirement for user pairing in the proposed algorithms as shown in the previous chapters. The underlay algorithms show that almost 16 pairs were able to pass the QoS for user pairing, while the algorithms from the literature show almost 10 pairs. This means that, the proposed user pairing algorithms are able to meet the individual user sum rate requirements before performing user pairing. The RPA and the CSS-PA algorithms do not have this functionality, which results in them having low number of users passing the individual QoS requirement test. This means that, in user pairing, it is important to add a step which ensures that the individual user, sum rates requirements are met before user pairing. There results in Figure 5.8 are for the Lagrange based power allocation scheme. The bound based power allocation algorithm has a superior performance compared to the Lagrange power allocation scheme and shown in Figure 5.9.

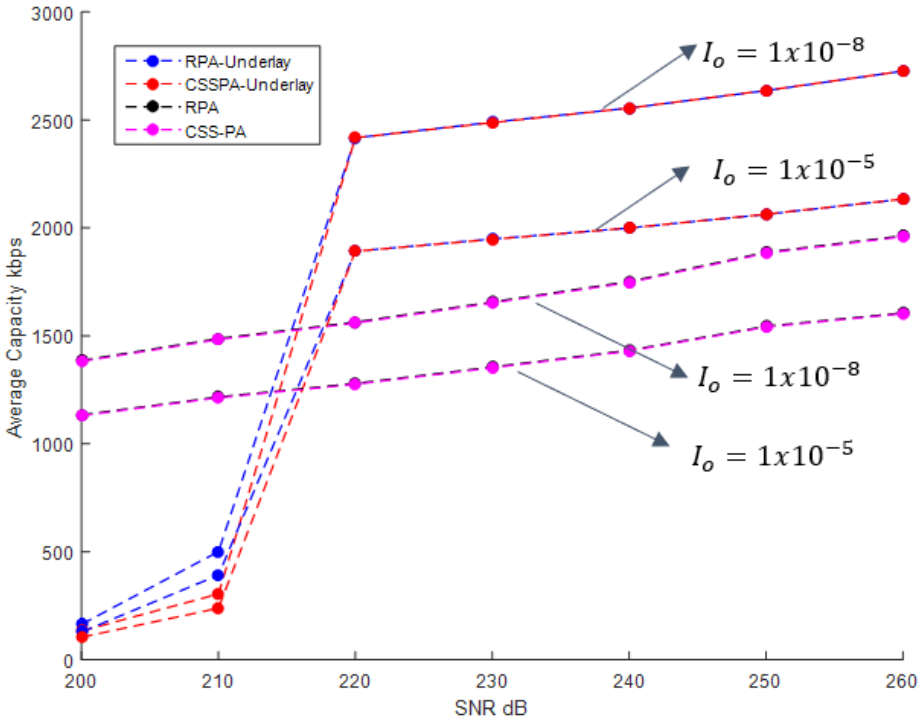


Figure 5. 10 How the change of the constant I_o affects the average user capacity.

In Figure 5.10, the aim is to investigate the effect of changing the constant I_o from the constraint that ensures that the SIC is performed to cancel the interference from the primary network. It can be observed that when the constant is lowered the average capacity of the user pairs increases and when it is increased the average capacity of the paired users

decreases. This means that the average capacity can be controlled using the constant, depending on the interference that is felt in the primary network. This is a useful feature as it can make the system flexible and allow adjustments in power.

5.2 Preference List Matching Algorithm (PLMA)

In this section, we provide the details of the solution employed to solve the user pairing problem. This method unlike the centralized algorithms uses matching theory which makes use of the Gale Sharpley theorem to obtain a solution to the matching problem. In the PLMA algorithm, the BS needs to know the full CSI of the users before any user pairing or matching can be performed. This reduces the system complexity. The algorithm utilizes the channel gains of the users in the SN to build preference lists for each user, that will feed the Gale Sharpley theorem. This makes our algorithm fully distributed and convenient for cognitive environments.

It is worth pointing out that, if the power constraints are not met, the preference lists cannot be constructed and thus user matching will not occur. According to our system model the secondary BS broadcasts the channel gains information to all the users in the SN. Thus, the preference list for any SU_k is given by SUK_LIST and the preference list for any SU_l is given by SUL_LIST. For both preference lists, the following conditions must be satisfied.

$$R_{k,f} \geq R_{k,req} \qquad k \in (1, \dots, K), \tag{5.1}$$

$$R_{l,f} \geq R_{l,req} \qquad l \in (1, \dots, L). \tag{5.2}$$

The total power constraints and interference temperature constraint to the primary network should also be respected when constructing the preference lists. Thus:

$$P_{k,l,f} < \left\lfloor \frac{I_{PU}}{|g_{m,f}|^2} \right\rfloor, \tag{5.3}$$

$$P_{k,l,f} < PT. \tag{5.4}$$

The interference power constraint from the primary network must also be ensured when constructing the preference lists. The constraint are given below:

$$\left|q_{k,f}\right|^2 P^{PU} > I_o \ \left|h_{k,f}\right|^2 P_{k,l,f}, \tag{5.5}$$

$$\left|q_{l,f}\right|^2 P^{PU} > I_o \ \left|h_{k,f}\right|^2 P_{k,l,f}. \tag{5.6}$$

The indexes of the SU_l's are recorded in each of the preference lists, when the above conditions have been satisfied for each k. It is assumed that at the top of the list SUK_LIST would lie the SU_k with the highest rate $R_{k,f}$ when paired. On the other hand, each SU_l 's also constructs their own preference list SUL_LIST and at the top of the list would lie the SU_k with the highest rate $R_{l,f}$ when paired. This means that the elements in the lists are ranked in decreasing order according to the achievable rates.

5.2.1 Preference List Matching Algorithm (PLMA)

This algorithm also ensures that both QoS requirements of the users and importantly the interference to the primary network are respected. The key idea of this algorithm is to start matching by using the users with the highest channel gain at a particular frequency first, then use the Gale Sharpley concept to perform user pairing at a particular frequency. This achieves the near optimal sum rates as shown by Theorem 6.

Each SU_k having the high channel gain makes an offer to match with a SU_l and the SU_l can accept or reject the match based on the NOMA capacity generated when matched with the current SU_k. Additionally, if an SU_k attempts to match with an already matched SU_l the algorithm opts for the SU_k who could provide a higher revenue. The preference list matching algorithm method is shown below. According to Theorem 6, it is imperative to note that an optimum matching will occur when the highest channel gain user is chosen first to pair. The PLMA will be compared to the Hungarian method when the highest channel gain user is chosen.

Preference List Matching Algorithm (PLMA)

01. **Input:** K, L, F
02. **Output:** $PAIRING\ MATRIX, P$
03. Generate Channel Gains $h_{l,f}$, $h_{l,f}$, $g_{m,f}$, $q_{l,f}$, $q_{k,f}$
04. **for** loop from 1 to F
05. Choose SU_k with highest channel gain
06. Note f at this highest channel gain
07. Construct SUK_LIST for all SU_l at f, respecting constraints given
08. Order SUK_LIST according to Capacity
09. Choose Q best performing SU_l's from SUK_LIST
10. **for** all SU_l from 1 to Q
11. Construct SUL_LIST for all SU_k at f, respecting constraints given
12. **if** SU_l is not paired yet and member of SUK_LIST
13. **if** (SU_k is member of SUL_LIST)
14. Match SU_k with SU_l in $PAIRING\ MATRIX$
15. **else if (**SU_l is already matched to SU_k')
16 **if** (Current match Capacity > Previous match Capacity)
16. Match current SU_k with SU_l in $PAIRING\ MATRIX$
17. **endif**
18. **end for**
19. **end for**
20. **Return** $PAIRING\ MATRIX, P$

A flowchart of the preference list matching algorithm is displayed below.

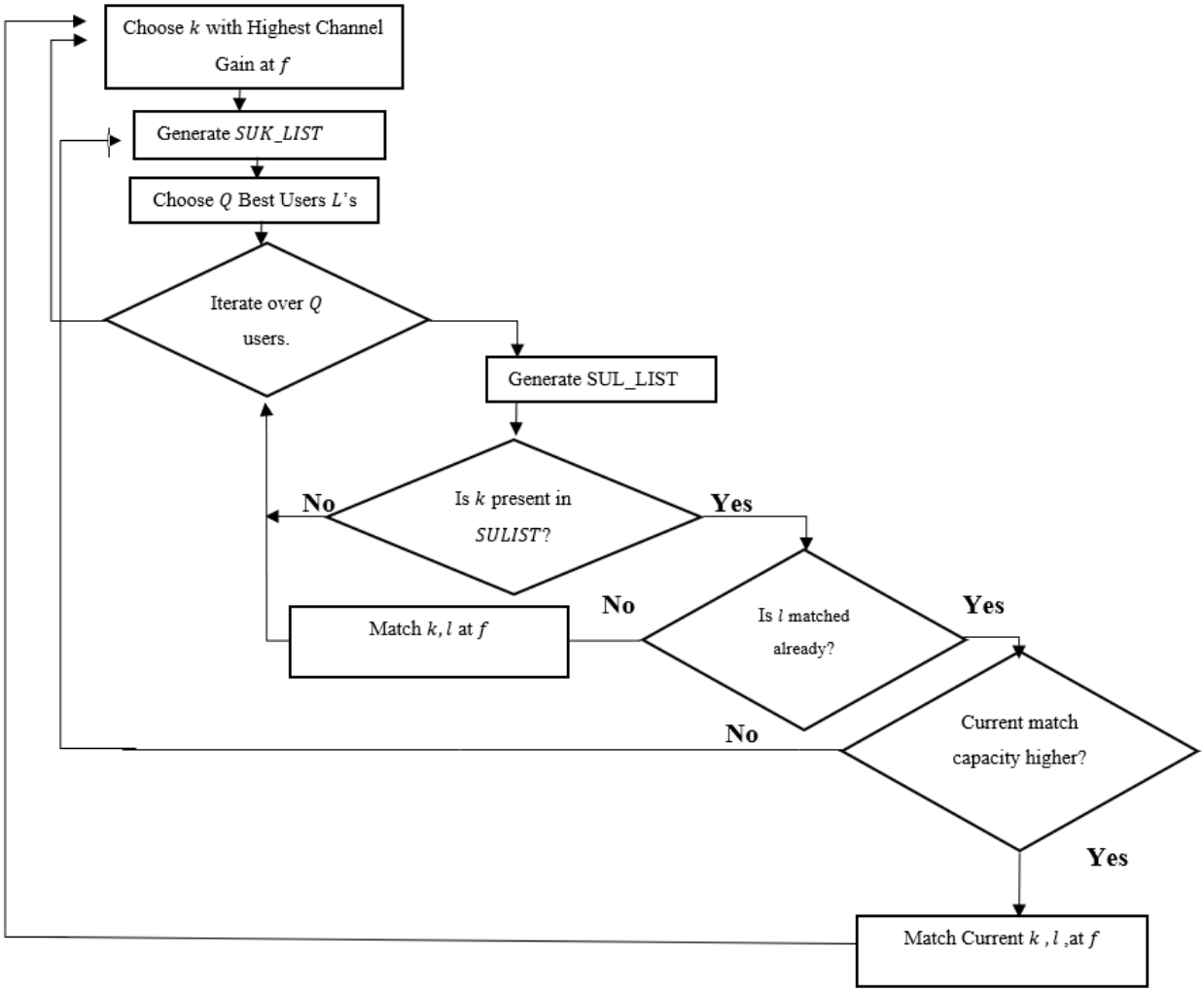


Figure 5. 11 Preference List Matching Algorithm flowchart.

5.2.2 Stability of the PLMA Algorithm

Based on the PLMA algorithms in Section 5.2, we proceed to prove that our algorithm results in a stable matching of the users. We assume that the final matching provided by our algorithm is P.

According to definition 3.4 in Chapter 3, any matching in the above algorithm is deemed to be blocked by a user SU_k , or SU_l where $\mathrm{SU}_k \in K$ and $\mathrm{SU}_l \in L$, when at a chosen frequency $f \in F$:

- The user would rather prefer not to be matched than being matched to its current partner under P, where a low capacity rate for SU_k , or SU_l is obtained.
- The interference to the primary network when being matched in P is above the constraint to be maintained as in underlay cognitive networks. This mathematically implies that:

$$R_{k,f} < R_{k,req} \ or \ R_{l,f} < R_{l,req}, \tag{5.7}$$

$$(P_{k,l,f} \geq \left[\frac{I_{PU}}{|g_{m,f}|^2}\right]) \text{ and } (P_{k,l,f} > PT). \tag{5.8}$$

Now, taking a look at the way our preference lists are constructed, at each picked frequency with highest gain f, they are based on the satisfaction of equations of the sum rate equations from Chapter 4. This means that there exist no blocking users in the final matching P because in each preference list all the users satisfy the corresponding inequalities corresponding to the minimum capacity requirements.

Moreover, any matching is deemed a blocking pair for the matching, if both SU_k and SU_l prefer to be matched with each other but are not matched. If both users are matched, they would achieve a higher rate, in comparison with their current matching partners. This mathematically implies that:

$$R_{k,f} > R^k_{k,\ P(l),f} \quad or, \quad R_{l,f} > R^l_{P(k),l,f}. \tag{5.9}$$

Now, if we consider the way matching is done in our algorithm, let's assume that at the termination of the algorithm, SU_l and SU_q $(q \in K)$ are not matched with each other under the final matching P, but SU_l prefers to be matched with SU_q instead of his current partner under the final matching P, $SU_{(P)l}$ at a particular frequency $f \in F$. This means that at f, $SU_q$is acceptable to be matched with SU_l but meets all the matching constraints. Hence, during the algorithm SU_l must have received an offer to match with SU_q at f before it was matched with its current partner $SU_{(P)l}$,under a different frequency or before being rejected by all its acceptable partners. Since SU_l is not matched with SU_q when the algorithm ends, it implies that SU_l the matching, SU_l - SU_q, must have been rejected because SU_q was in favour of another SU_t $(t \neq l, t \in L)$ at a different frequency e $(e \neq f, e \in F)$, which offered a better capacity

compared to the one offered by SU_l. Therefore, SU_q is matched to its current partner SU_t occupying frequency e, where $t = P(q)$, which is better than SU_l. In conclusion, neither SU_l nor SU_q blocks the matching P hence the preference list matching algorithm is stable.

5.3 Simulation Results

Our simulation results below display the performance of our proposed PLMA algorithm when two different power allocation algorithms are employed, namely the Lagrange based power allocation scheme and also the bound based power allocation scheme. In both runs, the Hungarian algorithm is employed for use as reference as it can achieve an optimal solution but with higher complexity. A modified Hungarian algorithm which performs user pairing starting with the highest frequency first was added for observation as well. The aim was to find out if the algorithm can achieve a performance close to the Hungarian algorithm while doing it with a low complexity as compared to the Hungarian algorithm. This would be in terms of the achieved sum rates and also the number of users which are paired per algorithm. In addition, which power allocation performed better would also be showcased by the simulation results. The results would also be compared to the modified underlay RPA and the modified underlay CSS-PA algorithms proposed in the previous section. The two figures below, Figure 5.13 and 5.13 display the results of the average capacities achieved and also the number of user pairs achieved by the proposed PLMA algorithm when the Lagrange based power allocation scheme is employed.

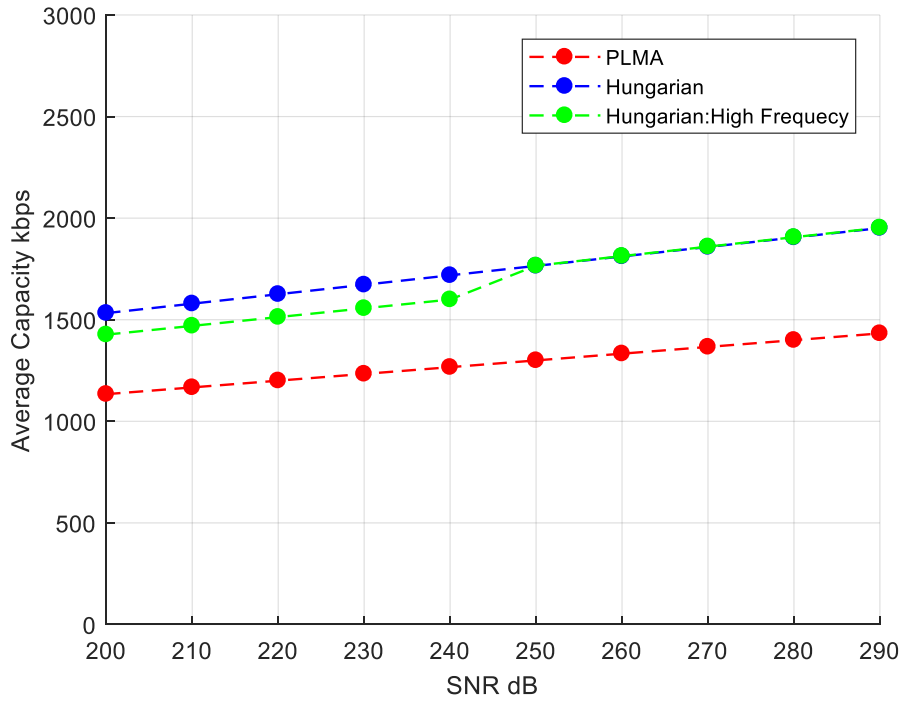


Figure 5. 12 PLMA algorithm performance with Lagrange based power allocation scheme.

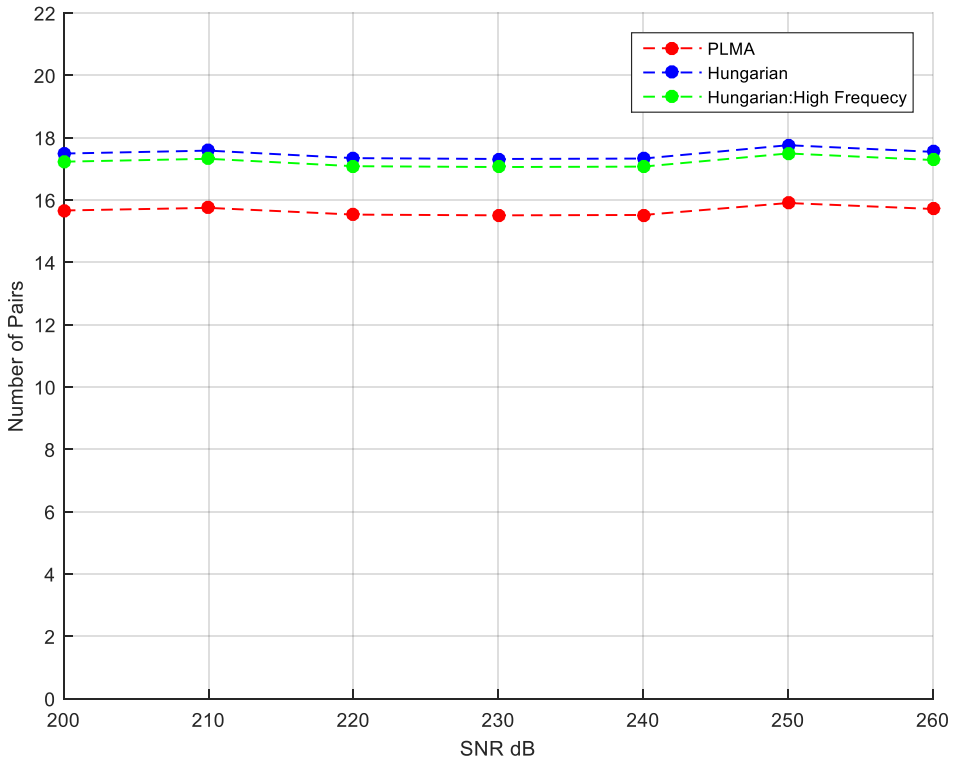


Figure 5. 13 Number of $\mathbf{SU}_{k}$ and $\mathbf{SU}_{l}$ user pairs among different SNR levels using Lagrange based power allocation.

It can be observed in Figure 5.12 that, performance of the proposed PLMA algorithm nears that of the Hungarian when using the for the Lagrange based power allocation. The number of pairs performance is also the same for the algorithms as observed in Figure 5.13. This means that, for a user pairing scheme, a method used in power allocation can deteriorate its performance regardless of how good the algorithm is. It can be seen that, for the chosen LaGrange power allocation method, the performance is lower than the Hungarian algorithm.

The two figures below display the same set of results as in Figure 5.12 and 5.13 but using a different power allocations scheme, a bound based power allocation scheme. Results of the average capacities achieved and also the number of user pairs achieved by the proposed PLMA algorithm are shown below.

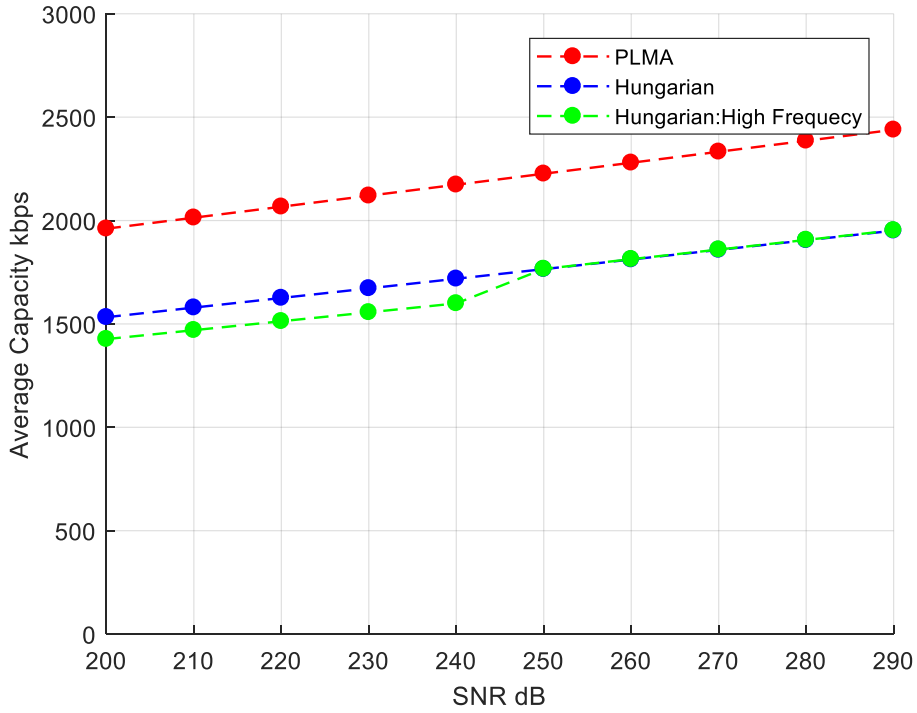


Figure 5. 14 PLMA algorithm performance with the bound based power allocation scheme.

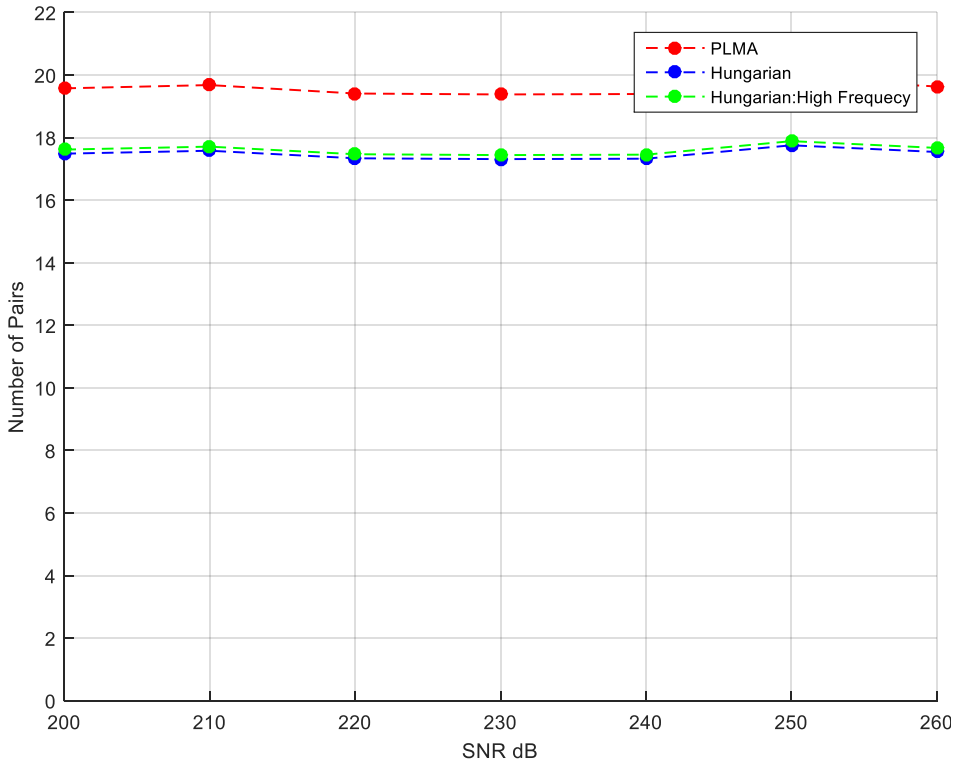


Figure 5. 15 Number of $\mathbf{SU}_{\boldsymbol{k}}$ and $\mathbf{SU}_{\boldsymbol{l}}$ user pairs among different SNR levels using Lagrange based power allocation.

It can be observed in Figure 5.14 that when employing the bound based power allocation scheme the proposed PLMA algorithms is superior to the Hungarian algorithms. This means that the Lagrange based power allocation scheme is outperformed by the bound based power allocation scheme as shown in the figures. Another reason for the high performance of the PLMA algorithm is because, it employs the Gale Sharpley theorem. This is because the Gale Sharpley theorem starts the matching by using the users with the highest channel gains at a particular frequency first, then use the Gale Sharpley concept to perform user pairing at a particular frequency. The theorem also ensures that users with higher capacities are matched first, which results in higher throughputs. This is also observed through the number of user pairs of each algorithm in Figure 5.15.

5.4 Complexity Analysis

The computational complexities of each of the algorithms discussed, is considered in the section below in terms of the number of operations n [88]. We consider the worst-case

scenarios as they require the greatest number of operations. RPA has the complexity equal to finding the nth element in an unordered list which can be given by $O(n)$. For CSS-PA, the complexity involves sorting out of the channel gains before user pairing changing the complexity to $O(n\log(n))$. Moreover, the complexity of the Hungarian algorithm is the same as the complexity for the problem of finding the maximum-weight matchings in bipartite graphs, which is sometimes called the assignment problem and can be given by $O(n^3)$.

In the preference list matching algorithm, the complexity increases according to number of K users and the number of L users in the system. The worst-case scenario occurs when the user L updates their preference list $(K-1)$ times before finding a match. This happens until the last user left in the preference list making the number of operations equal to the time complexity of $O(n^2)$. It can be observed that the Hungarian algorithm has a significantly higher complexity when compared to other algorithms. The table below summarises the complexities in table form for all the algorithms, starting with the algorithm with least performance and ending with the algorithm with the greatest sum rate performance. The trade-off between performance and complexity for each algorithm can be deduced from the table. It is worth mentioning that, although the Hungarian algorithm can perform better than some of the discussed pairing algorithms, this happens at the expense of the high time complexity.

Table 5. 2 Time complexities of the user pairing algorithms

	User Pairing Algorithm	Time Complexity
1.	Random Pairing Algorithm (RPA)	$O(n)$
2.	Channel State Sorting Algorithm (CSS-PA)	$O(n\log(n))$
3.	Hungarian Algorithm	$O(n^3)$
4.	Preference list matching algorithm (PLMA)	$O(n^2)$

5.5 Conclusion

This chapter presents the simulation results for our proposed algorithms and also the algorithms adopted from the literature. In general, the presented results illustrate that the underlay RPA and the underlay CSS-PA algorithms seem to have a higher user pairing performance than the literature algorithms, RPA and CSS-PA regardless of the underlay system constraints. Moreover, it can be observed that the Lagrange based power allocation scheme is outperformed by the bound based power allocation scheme. It can also be seen that the Lagrange based power allocation provides less pairs in user pairing than the bound based power allocation scheme. This symbolises that with a clever choice of a power allocation algorithm, higher throughputs can be achieved in a user paring algorithm. It can therefore be concluded that the proposed PLMA algorithm performs better when being used with the bound based power allocation scheme as the power allocation algorithm to perform user pairing. This also symbolises that, the use of the Gale Sharpley theorem can result in increased sum rate performances in user pairing algorithms.

Chapter 6

Conclusion and Future Directions

6.1 Conclusion

Non-Orthogonal Multiple Access has recently been proposed as a key enabling solution to meet 5G demands. The fifth-generation networks are expected to face phenomenal connectivity and capacity demands due to a huge number of devices expected in the next generation networks. With the limited available spectrum, a plethora of problems many of which are inherent to spectral scarcity characteristic associated with wireless transmission media face the scientists and engineers developing the next generation mobile systems. Cognitive Radio networks have also been envisioned to provide more bandwidth to mobile users through heterogeneous architectures and dynamic spectrum access techniques. User pairing has also attracted a lot of attention lately as one of the ways in which spectrum efficiency can be enhanced for future wireless technologies. This research therefore aims to incorporate different user pairing techniques and power allocations techniques in cognitive radios NOMA systems to constructively improve the system capacity of secondary users.

In this work, we formulate and implement different user pairing algorithms namely, underlay RPA, underlay CSS-PA and PLMA algorithm. We then demonstrate using sum rates capacities and user pairs, the performances of the different algorithms. We compare these algorithms to other pairing algorithms from the literature. We accomplish this, by exploring different research efforts that have been put forward to provide an insight into power allocation, user pairing and cognitive radios in Chapter 2 and Chapter 3. Then we work on modifying the existing user pairing solutions to work in underlay environments. We then develop a framework of power allocation schemes to go with these user pairing techniques. In Chapter 4, the system model based in underlay mode of cognitive operation is constructed then we formulate NOMA capacity equations based on the resulting model. Moreover, based on the same model, a formulation different power allocation schemes for underlay environments is done. We then propose a new set of user pairing algorithms in an underlay cognitive NOMA

network, namely, an underlay random pairing algorithm, and an underlay channel state sorting algorithm. A new algorithm preference list matching algorithm based on matching theory and the Gale-Sharpley is also presented. Simulation results illustrate that the underlay RPA and the underlay CSS-PA algorithms have a higher user pairing performance than the literature algorithms RPA and CSS-PA regardless of the underlay system constraints. The results also show that the PLMA algorithm performs better with the bound based power allocation when compared with the Lagrange based power allocation, Moreover, we conclude that with a clever choice of the power allocation algorithm a near optimal performance can be achieved by the preference list matching algorithm in an underlay mode.

6.2 Future Directions

The work in this dissertation mainly focused mainly on developing the power allocation schemes and user pairing schemes in an underlay cognitive NOMA network. The problems encountered in this pursuit included different types of assignment problems that are an extension of the linear assignment problems. It would be beneficial to have a detail look at these problems when dealing with user pairing problems. Looking at these problems in a fuzzy environment may also be helpful to future user pairing problems. Although the costs of some of these problems are deterministic, the costs of many assignment real world assignment problems may not be deterministic numbers and in such situations, it is important to study the assignment problems in a fuzzy environment. Extending the work in this dissertation to involve a wide range of fuzzy assignment problems therefore constitutes a future direction of this work.

Furthermore, the work in this dissertation investigates the performance of user pairing algorithms in an underlay environment, extending this work to also cater for both interweave and overlay NOMA network could be a great addition to this work. Finally, the practical implementation of the proposed routing schemes in a real testbed in order to thoroughly investigate and analyse their performance with experimental results is mandatory and could be a great challenge as part of the future research for the work presented in this dissertation.

Appendix A

A1. Theorem 5

In a NOMA system with with K users and L users, if the channel coeficients in the NOMA system are arranged in descending order such a way that $|h_{K,f}|^2 > \cdots > |h_{k,f}|^2 > |h_{1,f}|^2$ and $|h_{L,f}|^2 > |h_{l,f}|^2 > |h_{1,f}|^2$, then the optimum user pair P_{opt} which maximizes the sum rate at a frequency f when user k is paired with user l can be shown as:

$$P_{opt} = \{(K - (j-1)), (1 + j))\}, \quad (A1.1)$$

where $0 < j < L$.

Proof:

We will use a NOMA system with 4 users to prove the theorem above. In such a system, the user channel gains are sorted as $|h_{k_1,f}|^2 > |h_{k_2,f}|^2$ and $|h_{l_1,f}|^2 > |h_{l_2,f}|^2$, which means that in terms of the channel gains the users are sorted as $k_1 > k_2$ and $l_1 > l_2$. In this case the optimal solution pairing for a certain f is expected be found to be:

$$P_{opt} = \{(k_1, l_2)\}. \quad (A1.2)$$

In this NOMA system, there is a posibility of 4 cases that the users can be paired to get an optimal pairing solution.

Case 1: This is when user k_1 has paired with l_2, the resulting sum rates can be obtained as $\{(k_1, l_2)\}$:

$$C_{case1} = R^l_{1,2,f} + R^k_{1,2,f.} \quad (A1.3)$$

Case 2: This is when user k_1 has paired with l_1, the resulting sum rates can be obtained as $\{(k_1, l_1)\}$:

$$C_{case2} = R^l_{1,1,f} + R^k_{1,1,f}. \quad (A1.4)$$

Case 3: This is when user k_2 has paired with l_2, the resulting sum rates can be obtained as $\{(k_2, l_2)\}$:

$$C_{case3} = R^{l}_{2,2,f} + R^{k}_{2,2,f}. \quad (A1.5)$$

Case 4: This is when user k_2 has paired with l_1, the resulting sum rates can be obtained as $\{(k_2, l_1)\}$:

$$C_{case4} = R^{l}_{2,1,f} + R^{k}_{2,1,f}. \quad (A1.6)$$

We then need to show that:

$$C_{case1} \geq (C_{case2}, C_{case3}, C_{case4}\,). \quad (A1.7)$$

With the help of the Lemma below, the inequality above will be proved in order to showcase and compare the sum rates and get which pairing is the highest at f. Now we will consider the Lemma below first.

Lemma 1*:* In a NOMA user pairing system with four users in which the user channel gains are sorted as $\left|h_{k_1,f}\right|^2 > \left|h_{k_2,f}\right|^2$ and $\left|h_{l_1,f}\right|^2 > \left|h_{l_2,f}\right|^2$, the sum rates are such that:

$$C_{case1} \geq (C_{case2}, C_{case3}, C_{case4}\,). \quad (A1.8)$$

Proof:

The NOMA capacity for each case follows the sum rate equation as given in the previous chapter, and can be given as:

$$C_{k,l,f} = \log_2\left(1 + \frac{\alpha_{k,\mathrm{f}}\, P_{k,l,f}\ \left|h_{k,f}\right|^2}{N_0}\right) + \log_2\left(1 + \frac{\alpha_{l,f}\ P_{k,l,f}\ \left|h_{l,f}\right|^2}{\alpha_{k,\mathrm{f}}\, P_{k,l,f}\ \left|h_{l,f}\right|^2 +\ N_0}\right). \quad (A1.9)$$

It is obvious from the capacity equation above that the capacity depends on the value of $\alpha_{l,f}$. Using the power allocation equation obtained from the bound based power allocation scheme in Theorem 4, the value of $\alpha_{l,f}$ is given as $\alpha_{l,f} = \frac{\sqrt{1+\left|h_{l,f}\right|^2\gamma}-1}{\left|h_{l,f}\right|^2\gamma}$.

From that observation we have the value of $\alpha_{l,f}$, for the different cases the users can be paired in the NOMA system being shown below.

When the NOMA users (k_1, l_2) are paired we have the value of $\alpha_{l,f}$ from Case 1 as:

$$\alpha_{1,f}^{(1,2)} = \frac{\sqrt{1 + \left|h_{l_2,f}\right|^2 \gamma} - 1}{\left|h_{l_2,f}\right|^2 \gamma} \triangleq \beta_1. \tag{A1.10}$$

When user the NOMA users (k_1, l_1) are paired we have the value of $\alpha_{l,f}$ from Case 2 as:

$$\alpha_{1,f}^{(1,1)} = \frac{\sqrt{1 + \left|h_{l_1,f}\right|^2 \gamma} - 1}{\left|h_{l_1,f}\right|^2 \gamma} \triangleq \beta_2. \tag{A1.11}$$

When user the NOMA users (k_2, l_2) are paired we have the value of $\alpha_{l,f}$ from Case 3 as:

$$\alpha_{2,f}^{(2,2)} = \frac{\sqrt{1 + \left|h_{l_2,f}\right|^2 \gamma} - 1}{\left|h_{l_2,f}\right|^2 \gamma} \triangleq \beta_3. \tag{A1.12}$$

When user the NOMA users (k_2, l_1) are paired we have the value of $\alpha_{l,f}$ from Case 4 as:

$$\alpha_{1,f}^{(2,1)} = \frac{\sqrt{1 + \left|h_{l_1,f}\right|^2 \gamma} - 1}{\left|h_{l_1,f}\right|^2 \gamma} \triangleq \beta_4. \tag{A1.13}$$

Then from above, if we define $\beta_i\ (x) = \frac{\sqrt{1+x}-1}{x}$ and let $x = \left|h_{i,f}\right|^2 \gamma$, where $i \in \{1,2,3,4\}$. Then, equation 4.33 can also be written as:

$$\beta_i = \frac{(1+x)^{\frac{1}{2}} - 1}{x}. \tag{A1.14}$$

From equation 4.34 above if we multiply the numerator and the denominator by $\left[(1+x)^{\frac{1}{2}} - 1\right]$, then we have:

$$\beta_i = \frac{[(1 + x)^{\frac{1}{2}} - 1]\left[(1+x)^{\frac{1}{2}} - 1\right]}{x\left[(1+x)^{\frac{1}{2}} + 1\right]}$$

$$= \frac{(1+x) - 1}{x\left[(1+x)^{\frac{1}{2}} + 1\right]} = \frac{x}{x\left[(1+x)^{\frac{1}{2}} + 1\right]}$$

$$= \frac{1}{\left[(1+x)^{\frac{1}{2}} + 1\right]}. \qquad (A1.15)$$

Moreover, if we consider any two values of x, x_1 and x_2 where $x_1 > x_2$, then we clearly have:

$$\frac{1}{\left[(1 + x_1)^{\frac{1}{2}} + 1\right]} < \frac{1}{\left[(1 + x_2)^{\frac{1}{2}} + 1\right]}. \qquad (A1.16)$$

For this reason, according to the assumption that $|h_{l_1,f}|^2 > |h_{l_2,f}|^2$, then it can be concluded that:

$$\beta_2 < \beta_1. \qquad (A1.17)$$

$$\beta_3 = \beta_1. \qquad (A1.18)$$

$$\beta_4 < \beta_{1.} \qquad (A1.19)$$

Because of the arragement of the channel gains and the power sharing based equation above we can use the NOMA capacity equations 4.6 and 4.7 to show that for the four cases we have Case 1 as being superior. We will illustrate this by showing that the expressions resulting when the other cases are subtracted from Case 1, the results is greater than one.

$C_{case1} - C_{case2}$

In this case the channel gains are such that $|h_{l_1,f}|^2 > |h_{l_2,f}|^2$, and thus $\beta_2 < \beta_1$. Then we want to show that:

$$R^{l}_{1,2,f} + R^{k}_{1,2,f} - \left(R^{l}_{1,1,f} + R^{k}_{1,1,f}\right) \geq 0. \qquad (A1.20)$$

Then, this means that,

$C_{case1} - C_{case2} = C$

$$C = \log_2\left(\frac{N_0 + (1-\beta_1)\left|h_{k_1,f}\right|^2\gamma}{N_0}\right) + \log_2\left(\frac{\left|h_{l_2,f}\right|^2\gamma + 1}{(1-\beta_1)\left|h_{l_2,f}\right|^2\gamma + 1}\right)$$
$$- \left[\log_2\left(\frac{N_0 + (1-\beta_2)\left|h_{k_1,f}\right|^2\gamma}{N_0}\right) + \log_2\left(\frac{\left|h_{l_1,f}\right|^2\gamma + 1}{(1-\beta_2)\left|h_{l_1,f}\right|^2\gamma + 1}\right)\right]$$

$$C = \log_2\left(\frac{N_0 + (1-\beta_1)\left|h_{k_1,f}\right|^2\gamma}{N_0}\right)\left(\frac{\left|h_{l_2,f}\right|^2\gamma + 1}{(1-\beta_1)\left|h_{l_2,f}\right|^2\gamma + 1}\right)$$
$$- \log_2\left(\frac{N_0 + (1-\beta_2)\left|h_{k_1,f}\right|^2\gamma}{N_0}\right)\left(\frac{\left|h_{l_1,f}\right|^2\gamma + 1}{(1-\beta_2)\left|h_{l_1,f}\right|^2\gamma + 1}\right)$$

$$C = \log_2\left(\frac{N_0 + (1-\beta_1)\left|h_{k_1,f}\right|^2\gamma \quad (\left|h_{l_2,f}\right|^2\gamma + 1)}{N_0\,((1-\beta_1)\left|h_{l_2,f}\right|^2\gamma + 1)} \quad \frac{N_0\,((1-\beta_2)\left|h_{l_1,f}\right|^2\gamma + 1)}{(N_0 + (1-\beta_2)\left|h_{k_1,f}\right|^2\gamma) \quad (\left|h_{l_1,f}\right|^2\gamma + 1)}\right)$$

$$C = \log_2\left(\frac{(N_0 + (1-\beta_1)\left|h_{k_1,f}\right|^2\gamma) \quad (\left|h_{l_2,f}\right|^2\gamma + 1)}{(N_0 + (1-\beta_2)\left|h_{k_1,f}\right|^2\gamma) \quad (\left|h_{l_1,f}\right|^2\gamma + 1)} \quad \frac{N_0\,((1-\beta_2)\left|h_{l_1,f}\right|^2\gamma + 1)}{N_0\,((1-\beta_1)\left|h_{l_2,f}\right|^2\gamma + 1)}\right).$$

From above we can note that we can reduce similar terms in the numerator and denominator. Then noting that $\beta_2 < \beta_1$, from above we can reduce the terms which multiply $\left|h_{k_1,f}\right|^2\gamma$ and then proceed to reduce similar numerator and denominator expressions.

$$C \geq \log_2\left(\frac{(\left|h_{l_2,f}\right|^2\gamma + 1)}{(\left|h_{l_1,f}\right|^2\gamma + 1)} \quad \frac{(\left|h_{l_1,f}\right|^2\gamma - \beta_2\left|h_{l_1,f}\right|^2\gamma + 1)}{(\left|h_{l_2,f}\right|^2\gamma - \beta_1\left|h_{l_2,f}\right|^2\gamma + 1)}\right) \geq \log_2\left(\frac{-\beta_2\left|h_{l_1,f}\right|^2\gamma}{-\beta_1\left|h_{l_2,f}\right|^2\gamma}\right)$$

$$\mathrm{C} \geq \log_2 \left(\frac{-\frac{\sqrt{1+\left|h_{1,f}\right|^2\gamma}-1}{\left|h_{1,f}\right|^2\gamma}\left|h_{l_1,f}\right|^2\gamma}{-\frac{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1}{\left|h_{2,f}\right|^2\gamma}\left|h_{l_2,f}\right|^2\gamma} \right)$$

$$\mathrm{C} \geq \log_2 \left(\frac{\sqrt{1+\left|h_{1,f}\right|^2\gamma}-1}{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1} \right) \geq 0. \qquad (A1.21)$$

Similarly, following the similar approach as above, we can have the following case as:

$$C_{case1} - C_{case3} = C$$

$$= R_{1,2,f}^{l} + R_{1,2,f}^{k} - \left(R_{2,2,f}^{l} + R_{2,2,f}^{k}\right) \qquad (A1.22)$$

$$C \geq \log_2 \left(\frac{-\beta_3\left|h_{l_2,f}\right|^2\gamma}{-\beta_1\left|h_{l_2,f}\right|^2\gamma} \right)$$

$$C \geq \log_2 \left(\frac{-\frac{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1}{\left|h_{2,f}\right|^2\gamma}\left|h_{l_2,f}\right|^2\gamma}{-\frac{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1}{\left|h_{2,f}\right|^2\gamma}\left|h_{l_2,f}\right|^2\gamma} \right) \geq \left(\frac{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1}{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1} \right) \geq 1. \qquad (A1.23)$$

Based on the two previous approaches we can also have the last case being:

$$C_{case1} - C_{case4} = C$$

$$= R_{1,2,f}^{l} + R_{1,2,f}^{k} - \left(R_{2,1,f}^{l} + R_{2,1,f}^{k}\right). \qquad (A1.24)$$

$$C \geq \log_2 \left(\frac{-\beta_2\left|h_{l_1,f}\right|^2\gamma}{-\beta_1\left|h_{l_2,f}\right|^2\gamma} \right)$$

$$C \geq \log_2 \left(\frac{-\frac{\sqrt{1+\left|h_{1,f}\right|^2\gamma}-1}{\left|h_{1,f}\right|^2\gamma}\left|h_{l_1,f}\right|^2\gamma}{\frac{-\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1}{\left|h_{2,f}\right|^2\gamma}\left|h_{l_2,f}\right|^2\gamma} \right)$$

$$C \geq \log_2 \left(\frac{\sqrt{1+\left|h_{1,f}\right|^2\gamma}-1}{\sqrt{1+\left|h_{2,f}\right|^2\gamma}-1} \right) \geq 0. \qquad (A1.25)$$

With the help of the lemma above we can conclude that Case 1 has the grater capacity than Case 2, Case 3 and Case 4, thus is the optimal matching for the four user NOMA scenario. Thus, because of the lemma above, the theorem has been proved.

■

Appendix B

B1. Theorem 6

For a two user NOMA system with k_1 and l_2 and two available frequecies, f_1 and f_2, with $\left|h_{k_1,f_1}\right|^2 > \left|h_{k_1,f_2}\right|^2 > \left|h_{l_2,f_1}\right|^2 > \left|h_{l_2,f_2}\right|^2$, it is best to pick the highest channel gain user first to get optimal capacities when perfoming user pairing.

Proof:

There are two ways the pairing can happen between the two users and the frequencies, but we want to show that if user pairing is perfomed considering the highest channel gain user first, then an optimal pairing occurs. Assume $\left|h_{k_1,f_1}\right|^2$ is the best perfoming channel gain. Then two cases are listed below:

Case 1:

Pair: k_1, f_1 and l_2, f_1.

Case 2:

Pair: k_1, f_2 and l_2, f_2.

We can recall the NOMA capacity as given in the previous chapter, and can be given as:

$$C_{k,l,f} = \log_2\left(1 + \frac{\alpha_{k,\mathrm{f}}\, P_{k,l,f}\ \left|h_{k,f}\right|^2}{N_0}\right) + \log_2\left(1 + \frac{\alpha_{l,\mathrm{f}}\, P_{k,l,f}\ \left|h_{l,f}\right|^2}{\alpha_{k,\mathrm{f}}\, P_{k,l,f}\ \left|h_{l,f}\right|^2 + N_0}\right). \qquad \text{(B1.1)}$$

It is obvious from the capacity equation above that the capacity depends on the value of $\alpha_{l,f}$. Using the power allocation equation obtained from the bound based power allocation scheme in Theorem 4, the value of $\alpha_{l,f}$ is given as $\alpha_{l,f} = \frac{\sqrt{1+\left|h_{l,f}\right|^2\gamma}-1}{\left|h_{l,f}\right|^2\gamma}$.

From the channel gains arrangement, the values of $\alpha_{l,f}$ for f_1 and f_2, can be deduced as:

Case 1:

$$\alpha_{f1}^{(1,2)} = \frac{\sqrt{1 + \left|h_{l_2,f_1}\right|^2\gamma} - 1}{\left|h_{l_2,f_1}\right|^2\gamma} \triangleq \beta_1. \qquad \text{(B1.2)}$$

Case 2:

$$\alpha_{f2}^{(1,2)} = \frac{\sqrt{1 + \left|h_{l_2,f_2}\right|^2\gamma} - 1}{\left|h_{l_2,f_2}\right|^2\gamma} \triangleq \beta_2. \qquad \text{(B1.3)}$$

Then from above, if we define $\beta_i\ (x) = \frac{\sqrt{1+x}-1}{x}$ and let $x = \left|h_{i,f}\right|^2\gamma$, where $i \in \{1,2\}$. Then, the equation above can also be written as:

$$\beta_i = \frac{(1+x)^{\frac{1}{2}} - 1}{x}. \qquad \text{(B1.4)}$$

From the equation above if we multiply the numerator and the denominator by $\left[(1+x)^{\frac{1}{2}}-1\right]$, then we have:

$$\beta_i = \frac{[(1+x)^{\frac{1}{2}} - 1]\left[(1+x)^{\frac{1}{2}}-1\right]}{x\left[(1+x)^{\frac{1}{2}}+1\right]}$$

$$= \frac{(1+x) - 1}{x\left[(1+x)^{\frac{1}{2}}+1\right]} = \frac{x}{x\left[(1+x)^{\frac{1}{2}}+1\right]}$$

$$= \frac{1}{\left[(1+x)^{\frac{1}{2}}+1\right]}. \quad \text{(B1.5)}$$

Moreover, if we consider any two values of x, x_1 and x_2 where $x_1 > x_2$, then we clearly have:

$$\frac{1}{\left[(1+x_1)^{\frac{1}{2}}+1\right]} < \frac{1}{\left[(1+x_2)^{\frac{1}{2}}+1\right]}. \quad \text{(B1.6)}$$

For this reason, according to the assumption that the channnel gains are ordered as: $|h_{k_1,f_1}|^2 > |h_{k_1,f_2}|^2 > |h_{l_2,f_1}|^2 > |h_{l_2,f_2}|^2$.

Then,

$$\beta_1 < \beta_2. \quad \text{(B1.7)}$$

Because of the arragement of the channel gains and the power sharing based equations above we can use the NOMA capacity equations to show that it is best to pick the highest channel gain user first to get optimal capacities when perfoming user pairing. We will illustrate this by showing that the expressions resulting when the Case 2 is subtracted from Case 1, the result is greater than one.

$C_{case1} - C_{case2} = C$

$$= R^l_{1,2,f_1} + R^k_{1,2,f_1} - \left(R^l_{1,2,f_2} + R^k_{1,2,f_2}\right) \quad \text{(B1.8)}$$

$$C \geq \log_2\left(\frac{N_0+(1-\beta_1)\left|h_{k_1,f_1}\right|^2\gamma}{N_0}\right)+\log_2\left(\frac{\left|h_{l_2,f_1}\right|^2\gamma+1}{(1-\beta_1)\left|h_{l_2,f_1}\right|^2\gamma+1}\right)$$
$$-\left[\log_2\left(\frac{N_0+(1-\beta_2)\left|h_{k_1,f_2}\right|^2\gamma}{N_0}\right)+\log_2\left(\frac{\left|h_{l_2,f_2}\right|^2\gamma+1}{(1-\beta_2)\left|h_{l_2,f_2}\right|^2\gamma+1}\right)\right]$$

$$C \geq \log_2\left(\frac{N_0+(1-\beta_1)\left|h_{k_1,f_1}\right|^2\gamma}{N_0}\right)\left(\frac{\left|h_{l_2,f_1}\right|^2\gamma+1}{(1-\beta_1)\left|h_{l_2,f_1}\right|^2\gamma+1}\right)$$
$$-\log_2\left(\frac{N_0+(1-\beta_2)\left|h_{k_1,f_2}\right|^2\gamma}{N_0}\right)\left(\frac{\left|h_{l_2,f_2}\right|^2\gamma+1}{(1-\beta_2)\left|h_{l_2,f_2}\right|^2\gamma+1}\right)$$

$$C \geq \log_2\left(\frac{N_0+(1-\beta_1)\left|h_{k_1,f_1}\right|^2\gamma\ \ (\left|h_{l_2,f_1}\right|^2\gamma+1)}{N_0\,((1-\beta_1)\left|h_{l_2,f_1}\right|^2\gamma+1)}\ \ \frac{N_0\,(\ (1-\beta_2)\left|h_{l_2,f_2}\right|^2\gamma+1)}{(N_0+(1-\beta_2)\left|h_{k_1,f_2}\right|^2\gamma)\ \ (\left|h_{l_2,f_2}\right|^2\gamma+1)}\right)$$

$$C \geq \log_2\left(\frac{(N_0+(1-\beta_1)\left|h_{k_1,f_1}\right|^2\gamma)\ \ (\left|h_{l_2,f_1}\right|^2\gamma+1)}{(N_0+(1-\beta_2)\left|h_{k_1,f_2}\right|^2\gamma)\ \ (\left|h_{l_2,f_2}\right|^2\gamma+1)}\ \ \frac{N_0\,(\ (1-\beta_2)\left|h_{l_2,f_2}\right|^2\gamma+1)}{N_0\,((1-\beta_1)\left|h_{l_2,f_1}\right|^2\gamma+1)}\right).$$

From above we can note that we can reduce similar terms. Noting that $\beta_2 > \beta_1$, and also that $\left|h_{l_2,f_1}\right|^2 > \left|h_{l_2,f_2}\right|^2$ from above we can reduce the terms in the numerator and denominator.

Then we have:

$$C \geq \log_2\left(\frac{-\beta_1\left|h_{l_2,f_1}\right|^2\gamma}{-\beta_2\left|h_{l_2,f_2}\right|^2\gamma}\right)$$

$$C \geq \log_2\left(\frac{-\frac{\sqrt{1+\left|h_{l_2,f_1}\right|^2\gamma}-1}{\left|h_{l_2,f_1}\right|^2\gamma}\ \left|h_{l_1,f_1}\right|^2\gamma}{\frac{-\sqrt{1+\left|h_{l_2,f_2}\right|^2\gamma}-1}{\left|h_{l_2,f_2}\right|^2\gamma}\ \left|h_{l_2,f_2}\right|^2\gamma}\right)$$

$$C \geq \log_2\left(\frac{\sqrt{1+\left|h_{l_2,f_1}\right|^2\gamma}-1}{\sqrt{1+\left|h_{l_2,f_2}\right|^2\gamma}-1}\right) \geq 0. \qquad \text{(B1.9)}$$

Because of the arragement of the channel gains and the power sharing based equations, using the cases above we use the capacity equations to show, it is best to pick this highest channel gain user first to get optimal capacities when perfoming user pairing. Thus, the theorem has been proved.

■

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