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COLLABORATIVE LEARNING AND KNOWLEDGE BUILDING IN A MATHEMATICS TEACHER EDUCATOR PROFESSIONAL LEARNING COMMUNITY IN GHANA

A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy

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ABSTRACT

This study investigates how collaborative professional learning experiences support the development of Mathematics Teacher Educators' (MTEs') knowledge and practices for teaching mathematics in pre-service teacher education in Ghana. Despite the critical role MTEs play in shaping future mathematics teachers, their own professional development needs and the specific knowledge required for their work remain underexplored, particularly in developing educational contexts.

This research utilised a qualitative case study approach with data collection occurring in two phases: March to May 2022 and February to April 2023, following ethical clearance. Phase One examined collaborative practices within a Professional Learning Community (PLC) of MTEs from one teacher education institution in Ghana, while Phase Two explored the role of critical friendships in supporting MTEs' professional growth. Grounded in sociocultural learning theory, the study integrated Desimone's framework for effective continuous professional development, Wenger's dimensions of practice, and McCrory et al.'s knowledge of algebra for teaching framework as lenses through which data was analysed in the two phases.

Data collection methods included PLC observations, focus group interviews with MTEs, individual interviews with MTEs and their critical friends, and classroom observations of MTEs' teaching practice. The analysis involved both deductive and inductive thematic approaches, employing the theoretical frameworks to examine collaborative learning dynamics and knowledge development among MTEs.

Key findings reveal that content-focused and domain-specific professional learning are crucial for MTEs' development, supporting findings from prior research. The study identified three core knowledge domains vital for MTEs: Pedagogical content knowledge, subject-matter knowledge, and knowledge of educational contexts. Collaborative learning within PLCs and critical friendships were found to enhance MTEs' professional knowledge and practices substantially, fostering reciprocal expertise development and catalysing pedagogical innovation. The study also highlighted the complex, specialised nature of knowledge required for teaching teachers, extending beyond what is typically considered for school-level mathematics teaching.

However, challenges were noted in translating collaborative learning into individual classroom practices, including time constraints and varying levels of participation and commitment of MTEs in the PLC activities, which led to uneven professional growth.

This research contributes to the field by providing an integrated theoretical framework for understanding MTE professional development, offering methodological innovations in studying critical friendships, and identifying specific knowledge domains crucial for MTEs. The findings have substantial implications for policy and practice in mathematics teacher education, suggesting the need for sustained, collaborative professional development models and curriculum reforms in MTE preparation programs.

While contextualised in Ghana, the study offers insights relevant to the broader international discourse on mathematics teacher education, particularly in developing educational contexts. It opens avenues for future research on the longitudinal impacts of MTE professional development, cross-cultural comparisons, and the scaling of successful collaborative learning models in diverse educational settings.

Keywords: Mathematics teacher education, professional learning, professional learning community, mathematical knowledge for teaching teachers, pre-service teachers, critical friendships, collaborative learning, professional practices**.**

DECLARATION

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.



Frank Kwarteng Nkrumah

23rd day of January in the year 2025

DEDICATION

To God Almighty, the source of all wisdom and strength. Your grace has guided me through every challenge and triumph of this journey.

To my beloved family, whose unconditional love, sacrifices, and constant encouragement have been my anchor. Your unwavering support has made this achievement possible.

To my esteemed mentor, Prof Craig Pournara, whose guidance, expertise, and belief in my potential have been instrumental in shaping this work and my growth as a researcher. Your wisdom and patience have been invaluable.

Your collective support has been the foundation of this accomplishment. This thesis stands as a testament to your love, guidance, and inspiration. Thank you.

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LIST OF ABBREVIATIONS

MTE, Mathematics teacher Educator

PST, Pre-service Teacher

NTS, National Teachers Standards

CoE, College of Education

MoE, Ministry of Education

NTECF, National Teacher Education Curriculum Framework

T-TEL, Transforming Teacher Education and Learning

CMPTER, Cabinet Memorandum on Policy on Teacher Education Reform

MKTT, Mathematical Knowledge for Teaching Teachers

WMC-S Wits Maths Connect – Secondary

PLC, Professional Learning Community

CoP, Community of Practice

NCTM, National Council of Teachers of Mathematics

MKT, Mathematical Knowledge for Teaching

PCK, Pedagogical Content Knowledge

SMK, Subject Matter Knowledge

ITE, Initial Teacher Education

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CHAPTER 1

NAVIGATING THE CONCEPTUAL TERRAIN OF THE STUDY

1.0 Introduction

In pre-service and in-service teacher education, the pivotal role of Mathematics Teacher Educators (MTEs) cannot be overstated. These teacher educators are responsible for inculcating pedagogical knowledge and skills, together with essential subject matter, to prepare effective teachers of mathematics (Leikin, 2021). However, the knowledge base necessary for MTEs to excel in their profession remains underexplored (Castro Superfine & Li, 2014; Lloyd et al., 2020). Findings from the few studies undertaken indicate significant variations in this knowledge base due to differences in MTE preparation and different teaching contexts (Dick et al., 2022), together with insufficient support for mathematics teacher educators in this crucial educational domain (Olanoff et al, 2021).

The ability to develop the mathematical potential of pre-service teachers is greatly influenced by the quality of MTE's professional practice. For Robutti et al. (2016), and also Jaworski et al (2017), the development of this expertise is linked to MTEs' collaborative professional learning and depth of professional knowledge. Such professional knowledge can enhance MTEs' grasp of the mathematics they teach, enabling them to navigate the complexities of classroom challenges (Epperson & Rhoads, 2015). It is argued that as teacher educators become learners of their teaching content, their teaching efficacy improves significantly (Loucks-Horsley et al, 2003).

According to Castro Superfine and Pitvorec (2021), a more comprehensive understanding is required to establish how MTEs can strengthen the knowledge they need to teach pre-service teachers effectively. Existing research underlines the need for concerted efforts in theory building to explain the professional practices, foundational knowledge prerequisites, and the impacts of collaborative inquiry that are unique to MTEs (Even, 2014; Triantafillou et al., 2021). Currently, professional learning for teacher educators is negatively affected by the lack of opportunities for collaborative professional learning (Brodie & Chimhande, 2020; DeVries et al, 2014; Garet et al, 2016) in a professional learning community which can provide MTEs with regular, systematic, and sustained learning cycles to enhance their professional capacity (Hairon et al., 2017).

Stoll et al. (2006, p. 5) define a Professional Learning Community (PLC) as "an inclusive group of people motivated by a shared learning vision, supporting and working with each other, finding ways inside and outside their direct community, examining their practice, and together learning new and better approaches that will enhance all pupils' learning." By collaborating to analyse their own practice and pre-service teachers' work samples and instructional plans, MTEs in PLCs can also better calibrate their evaluation of teacher readiness (Bjuland & Mosvold, 2015). However, further research is necessary to develop optimised PLC models and practices tailored to MTEs' specialised needs and knowledge domains in the differing contexts in which teaching and learning are located.

This study explores collaborative practices within a PLC of mathematics teacher educators whose focus is on teaching mathematics in pre-service teacher education in Ghana. It aims to investigate how collaborative efforts, such as critical friendships within a PLC support the development of professional practices and mathematical knowledge for teaching. This exploration aims to assist in bridging the gap between theoretical perspectives on MTE development and the practical realities of classroom instruction, and also to contribute to the ongoing elaboration of theories on specialised knowledge for teaching (Ball & Bass, 2003).

The central premise of the study is that MTEs' capacity to create meaningful teaching and learning experiences for pre-service teachers is fundamentally grounded in their own mathematical knowledge for teaching and that this knowledge can be strengthened through participation in collaborative professional learning environments. Examining the practices of this particular PLC for mathematics instruction in teacher education is thus a timely endeavour, as it aims to offer empirical insights into how collaborative opportunities practically support MTEs' professional growth.

1.1 Statement of the problem

Despite ongoing efforts to improve Ghanaian mathematics teacher education, challenges remain regarding beginning teachers' generic and subject-specific competencies in teaching school mathematics. Mereku (2019) and Buabeng et al. (2020) have raised concerns about the professional knowledge of MTEs involved in preparing pre-service teachers (PSTs) and about PSTs' performance after graduation, as their success in the classroom is considered a benchmark for the quality of teacher education in Ghana (Mereku, 2019).

Questions have been raised about the overall quality of pre-service teacher education in Ghana and the adequacy of preparation provided by teacher educators, particularly in subjects like

mathematics (Akyeampong et al. 2013; Lewin & Staut, 2003), a concern shared by scholars worldwide in the field of mathematics teacher education who argue that pre-service teachers are not receiving sufficient education and training to teach mathematics effectively (Li & Castro Superfine, 2018; Van Es & Conroy, 2009). Interestingly, while considerable attention has been paid to mathematical knowledge for teaching (MKT) school mathematics (e.g., Ball, Thames & Phelps, 2008; Ma, 1999; McCrory et al., 2012; Kazima & Adler, 2006), Jankvist et al. (2020) maintain that there has been far less focus on the knowledge required for teaching in teacher education and how to support its quality. As a result, the professional knowledge and practices unique to MTE remain poorly understood (Even, 2014; Jankvist et al., 2020).

Zaslavsky and Leikin (2004) identified a significant absence of research into becoming an MTE and also found a lack of structured education and training initiatives. They noted that most teacher educators evolve through what Zaslavsky terms 'self-made' efforts (Zaslavsky, 2008, p. 94). More recent studies (e.g., Beswick & Goos, 2018; Masingila et al., 2018) have investigated the mathematical knowledge required for instructing mathematics teachers and have explored practices of MTEs that enable PSTs to develop their pedagogical content knowledge (Appova & Taylor, 2019; Kuzle & Biehler, 2015). Despite these recent efforts, there is still a noticeable absence of research addressing MTE's professional knowledge and collaborative practices. Thus, this study explores how MTEs can collectively develop the ability to extend and critically evaluate their instructional practices.

I posit that though this study is conducted in a Ghanaian context, it can contribute to filling a gap in existing knowledge about the preparation of MTEs, the mathematical practices of MTEs worldwide, their knowledge for teaching in pre-service teacher education and the affordances of collaborative learning for informing their professional practice.

1.2 Algebra community of inquiry: A context for exploration

The mathematical content focus of the study is algebra but this is mostly a context for exploring MTEs' professional learning within a specific community of inquiry. While algebra presents inherent educational challenges, particularly in helping students move from arithmetic to symbolic representations, its main role in this study is to provide a shared pedagogical environment for exploring teacher knowledge development.

Recognising the challenges and fragilities in PSTs' algebraic competencies and the challenges resulting from limited competence, the MTE community came together to re-examine their instructional approaches and professional practices. Internationally, re-learning school algebra

has emerged as a crucial aspect of pre-service mathematics teacher education, which aims to strengthen PSTs' capacity to teach algebra concepts effectively (Warren et al, 2016; Pournara & Adler, 2021). An insight drawn from Pournara and Adler's (2021) South African research points to the fact that limited depth of mathematics knowledge is prevalent among PSTs. The situation in Ghana is similar, so an initial algebra course was made compulsory for PSTs to provide an opportunity for them to revisit and enhance their own understanding of school algebra during their bachelor's degree programme.

Teaching algebra within the context of mathematics teacher education serves multiple critical purposes. While not all PSTs may teach mathematics after graduation, for those who will, having a solid foundation in algebra is essential for understanding and explaining various mathematical concepts to their future students (Huang & Li, 2014). Moreover, mastering algebra places PSTs in a versatile position for teaching in environments where there might not be a specialised mathematics teacher available, thus adding to their teaching repertoire (Castro Superfine et al., 2020).

1.3 Purpose of the study

This study explores collaborative professional learning as a potential means for developing MTEs' professional practices and knowledge for teaching mathematics in pre-service teacher education. By focusing on a PLC of Ghanaian MTEs, I aim to explore the affordances of sustained peer collaboration to enrich both the pedagogical approaches and conceptual expertise needed to train PSTs.

The exploration is informed by a sociocultural perspective on educator learning (Vygotsky, 1978) while also drawing upon approaches to learning proposed by Wenger (1998) and Desimone (2009). Through an in-depth qualitative case study of this MTE PLC, I seek to interpret the constitution and outcomes of collaborations among MTEs as they develop specialised mathematics knowledge for teaching. While acknowledging contextual specificities, it is hoped that the findings contribute to an international understanding of optimal structures for supporting teacher-educator development, professional knowledge, and practice.

1.4 Research questions

The study is guided by the following questions:

1. To what extent does working collaboratively in a professional learning community support the development of mathematical knowledge for teaching in pre-service teacher education?

2. What knowledge do MTEs consider important to focus on in their collaborative work in pre-service teacher education?
3. What counts as evidence of the integration of PLC learnings into MTEs' pre-service teacher education practices, and is there evidence of such integration?
4. To what extent does critical friendship support changes in MTEs' professional knowledge and practice?

1.5 Significance of the study

Globally, the potential for MTE collaboration and reflective practice to enrich expertise is reflected across initiatives in teacher education (e.g., Jaworski, 2006; Joubert & Sutherland, 2010; Robutti et al., 2016; Schuck & Brandenburg, 2020). However, further research is needed to explore how these practices inform MTEs' professional practices and knowledge. This study is positioned to contribute to the knowledge and professional practice of MTEs in the context of collaborative endeavours in mathematics teacher education in Ghana and beyond.

The findings of this study mainly expose the complexities of collaborative endeavours such as critical friendships, the importance of understanding the supportive mechanisms that facilitate MTEs' learning and professional development, and why and how these teacher educators learn. These are important for improving professional learning in Ghana and beyond. For instance, the cyclic approach to critical friendships employed in phase two of this study can serve as a model for other teacher education communities aiming to enhance their professional development through collaborative efforts.

Ultimately, this study aims to contribute to the broader body of research on teacher education and professional learning by providing insights into the nuances of developing specialised knowledge for teaching algebra. It is hoped that these insights may be relevant to researchers exploring similar themes in other domains of education. In summary, this study's significance lies in its potential to inform educational practices, policies, and research in the field of algebra education within pre-service teacher preparation and possibly in other fields. By investigating and reporting on the specific needs and challenges MTEs face, the study aims to contribute to enhancing mathematics education for student teachers.

1.6 Definitions and explanation of terms

This section explains the keywords and concepts used throughout the study.

Professional knowledge is “a body of knowledge and skills needed to function successfully in a particular profession” (Chauraya & Brodie, 2018, p. 223).

Teacher educators commonly refer to both those who educate pre-service teachers and those who educate practising teachers, that is, those who initiate, guide and support teacher learning across the lifespan (Even, 2008).

Mathematics teacher educator: The term refers to individuals who have undergone initial mathematics education training, have the requisite postgraduate degrees and years of experience in mathematics education and provide mentoring and course facilitation for pre-service teachers.

Pre-service teachers (PSTs): Refer to students admitted into initial teacher education programmes in a college of education (CoE) or university faculty.

Mathematical knowledge for teaching teachers: A distinct domain of mathematical knowledge is needed for teaching in teacher education (Zopf, 2010, p. 199).

Continuous professional development: Craft (1996, p. 6) states that “continuous professional development is the type of professional learning educators undertake beyond their initial training”. Therefore, for this study, CPD describes ongoing education and training for practising MTEs to assist them in developing and enhancing their knowledge, methods of instruction and skills.

Educator learning: Refers to the “processes that, whether intuitive or deliberate, individual or social, result in specific changes in the professional knowledge, skills, attitudes, beliefs or actions of teachers” (Fraser, Kennedy, Reid, & McKinney, 2007, p. 157).

Professional learning: Refers to the ongoing learning and development process undertaken by educators to enrich their professional knowledge, competencies, and effectiveness (Avalos, 2011).

Critical friend: Refers to a trusted person who asks provocative questions, provides information to be examined through another lens, and offers a critique of a person’s work as a friend (Costa and Kallick, 1993).

1.7 Structure of the thesis

This thesis is organised into two parts. While Part One explores whether, and if so, how the framing of the PLC supports MTEs' professional learning, practices and knowledge for teaching teachers, Part Two focuses on the role of critical friendships in the collaborative work of the PLC. The thesis is structured as follows:

Chapter 1 introduces the thesis, provides background on the research topic, and outlines the study's aims, research question, and significance. Chapter 2 describes the historical development of education and teacher training in Ghana, examines current policies, and explores the role of MTEs in their own professional growth. It also provides insights into the nature of MTEs' professional learning communities.

Chapter 3 reviews relevant literature by tracing the mathematical knowledge models that have shaped what we know about knowledge for teaching and makes a connection with the development of knowledge for teaching in teacher education. This chapter also connects knowledge to practice and practice to collaboration and critical friendships. Chapter 4 explains the study's theoretical underpinnings, which include Vygotsky's (1978) socio-cultural theory and Desimone's (2009) and Wenger's (1998) approaches to learning and analysis of learning, which are used for various purposes in the study. The chapter also demonstrates how these theories and approaches are synthesised to inform the analysis and interpretation of the study's findings.

Chapter 5 discusses the research methodology used in the study. I describe the research paradigm and approach used in this qualitative case study. I also describe and discuss the research design, issues of trustworthiness of the work, details of the participants, data collection methods and ethical issues that were considered during the study. Chapter 6 presents an account of the data analysis process undertaken in this study. The analysis involved two distinct yet complementary phases, each employing a rigorous combination of deductive and inductive thematic analysis approaches.

Chapters 7 and 9 present and discuss the findings for Part One. Chapters 8 and 10 present discuss findings drawn from both parts. Chapter 11 explores how critical friendships inform MTEs' professional knowledge and practice in mathematics teacher education.

CHAPTER 2

CONTEXT OF THE STUDY

2.0 Introduction

This chapter contextualises teacher education in Ghana and outlines recent major curricula, instructional approaches, and reforms of professional standards. Details on the structure and knowledge pillars of the new Bachelor of Education program are presented. The pivotal role of mathematics teacher educators in supporting revised policies and continuous professional development is discussed. Finally, the chapter describes the local professional learning community of mathematics educators participating in this case study. The background information presented situates the research within a transformative national environment while laying the foundations for interpreting opportunities and challenges related to improved mathematics education in teachers' colleges and school classrooms.

2.1 Historical development of education and teacher education in Ghana

The origins of formal schooling in Ghana can be traced back to Portuguese traders in the 15th century, who established so-called "castle schools" to provide basic education for select young interpreters who facilitated commerce. Initially, access remained limited to a few privileged children of merchants, local elite family members, or mixed-race offspring of European settlers (Owusu-Agyarko et al., 1993). However, successive waves of missionaries from countries such as Britain, Germany and Denmark were interested in mass schooling, especially for conversion purposes. Though still serving the primary goal of religious indoctrination, mission-run boarding elementary schools provided some Ghanaian children with formal education that went beyond inculcating ethnic oral traditions.

As church-run and later government-supported schools expanded, teacher preparation programmes were needed. The Basel Mission established Ghana's first teacher training college in 1848 at Akropong, initiating a long period of instructing local assistants who later facilitated wide-scale classroom teaching (Owusu-Agyarko et al., 1993). By 1909, additional religious seminaries and the first state-sponsored teaching college aimed to meet the ballooning demand for education (Asare-Danso, 2014). By 1957, at the time of Ghana's independence from colonial rule, there were 30 teacher training institutions nationwide. In subsequent decades, various reform efforts sought to expand both access to and quality in teacher education and schooling throughout the country, including lengthening certificate stages and broadening practical curriculum exposure. However, legacy facsimiles of those found across many former

British colonies characterised education in Ghana for more than a generation. Only recently have there been transformational attempts to reshape the education system around national developmental priorities and local realities facing contemporary Ghanaian communities.

While historically, progress in education was a response to colonial interests, today's educational leaders are working to reclaim teaching as an anchor for autonomous advancement and growth. Reforms aim to combine indigenous culture with emerging best practices in education locally and internationally to serve all learners' holistic learning needs within their societal contexts.

2.2 Teacher education reform in Ghana 2012-2023

According to Buabeng et al. (2020), pre-service teacher education in Ghana has long envisaged its graduates as competent, thoughtful professionals who can provide Ghanaian children with an excellent education. For the past two decades, this vision and other educational goals, such as improving access and quality education, have underpinned various reforms within the teacher education system. These reforms have included changes to curricula and the reorganisation of teacher training institutions responsible for training teachers at various educational levels from kindergarten to senior high school. Notwithstanding these reforms, the quality of teaching and learning remains unsatisfactory. One reason for this situation is the mismatch between the expectations outlined in official policy documents and the practical constraints faced within schools and colleges of education. These constraints include limited resources, overcrowded classrooms and inadequate infrastructure. As a result, there is a disconnect between the idealised vision outlined in policy, and realities on the ground, which hinders the effective implementation of educational reforms. (Buabeng et al., 2020; Wilburn et al., 2021).

In 2014, the Ghanaian government, recognising the need for significant change in its education system, launched the Transforming Teacher Education and Learning (T-TEL) programme. Backed by UK Aid and led by Cambridge Education, this four-year initiative undertook to breathe new life into teacher education and revolutionise the learning experiences of Ghanaian students by radically overhauling existing practices, equipping teacher educators with modern, practical skills, boosting PST's critical thinking and problem-solving capabilities, and ultimately delivering improved learning outcomes at colleges across the nation.

Alongside the T-TEL programme, two significant documents paved the way for teacher education reform in Ghana. The Colleges of Education Act 847 of 2012 served as the

foundation, transforming Colleges of Education into autonomous degree-granting institutions with upgraded curricula, resources, and governance. In 2018, the Cabinet Memorandum on Policy on Teacher Education Reform (CMPTER) presented a comprehensive national strategy focused on attracting and retaining quality teachers, strengthening school leadership, and ensuring equity in education. For these reforms to succeed, teacher educators must be well-equipped for their multi-faceted roles. Continuing professional learning for teacher educators to enhance their professional practices and knowledge for effective teaching in teacher education became mandatory in 2014. In 2018, National Teachers Standards (NTS) were introduced, encompassing all teachers from kindergarten to teacher education, along with the National Teacher Education Curriculum Framework (NTECF), both of which were designed to drive educational reforms.

By the 2018/19 academic year, all 46 Colleges of Education (CoEs) in Ghana had been converted into University Colleges and had introduced a new Bachelor of Education curriculum designed to improve the quality of new teachers for the pre-tertiary education levels presented in Table 2.1.

School Level	Grades	Ages
Kindergarten	Optional	3 – 5
Primary School	1 – 6	6 – 12
Junior High School	7 -9	12 – 15
Senior High School	10 – 12	15 – 18

Table 2.1: Pre-tertiary education schooling system

This shift from a diploma to a degree programme mirrors similar reforms undertaken in South Africa in 2008 (Spaull, 2013). Other African nations like Kenya, Uganda, and Zambia are considering or implementing similar upgrades (Woldegiorgis et al., 2015), suggesting a regional movement towards higher teacher qualification standards.

It is anticipated that in-depth university-level courses will bolster educators' subject knowledge and pedagogical skills, ultimately improving student learning outcomes (Darling-Hammond, 2017). Additionally, obtaining a bachelor's degree can enhance teacher professionalism, potentially leading to increased respect for the profession, better career prospects, and higher morale. This can ultimately contribute to teacher retention (OECD, 2020) and attract more qualified individuals to the teaching profession, creating a stronger talent pool (Schleicher, 2012).

However, challenges must be acknowledged. Implementing degree programmes in teacher education requires significant investments in infrastructure, faculty training, and curriculum development (Momanyi, 2016; Tatto, 2015)). Moreover, the transition might initially lead to a teacher shortage due to more extended training periods, necessitating careful planning and recruitment strategies (Nicolai, Anderson & Hodgkin, 2019). By navigating implementation challenges and ensuring resource allocation, Ghana can position itself as a leader in African education, paving the way for a more qualified and empowered teaching force and ultimately nurturing future generations of successful learners.

2.3 The roles of teacher educators in implementing the reforms

The reform policies prioritise continuous professional learning of teacher educators, offering various capacity-building initiatives, such as practice-based teacher education, mentoring, collaboration, and networking opportunities. The assumption is that by enhancing their professional development, teacher educators can effectively model desired practices in teacher education (MoE, 2017a). As a result, pre-service teachers, upon deployment to basic schools (Grades 1-9), should be better prepared to facilitate their pupils' success in the classroom (T-TEL, 2017).

As stated in the NTECF mandates, teacher educators must understand the NTS and refocus their teaching to support the realisation of the goals of this statement regarding both content and pedagogy. For this reason, all teacher educators, as part of the implementation of the new curriculum framework, have been required to participate in content-focused, domain-specific curriculum-themed weekly professional learning sessions (elaborated later in this chapter), with the assumption that such sessions would be productive for their growth and inform their professional practice and knowledge as teacher educators. Content-focused professional learning is intended to equip teacher educators with subject matter expertise and content-specific pedagogy in domains like mathematics and science. In addition, it is intended to support teacher educators in preparing and guiding pre-service teachers in teaching specific subjects, such as algebra and biology. The underlying assumption is that as a result of the professional learning sessions, MTEs will be able to implement interactive and student-centred instructional approaches, and their pre-service teachers (PSTs) in their classes will learn from their examples and apply similar effective teaching methods during their degree programmes and after they graduate.

While the assumption that PSTs learn from observing their lecturer's use of interactive and student-centred practices holds merit Muldner et al. (2014), I argue that simply observing is

insufficient. Research emphasises the need for meaningful practice and feedback for successful transfer to classroom contexts (Darling-Hammond, 2006; Guskey & Yoon, 2009). Furthermore, fostering critical thinking skills should enable new teachers to adapt these methods to their specific contexts and student needs (Darling-Hammond, 2006). An examination of international examples of teacher education, such as Finland's, where collaboration and autonomy drive successful practices, and Singapore's, where top-down approaches have limitations, highlights the importance of considering cultural context, resource availability, and ongoing teacher support (Ebersöhn & Loots; 2017; Tan, 2019). In my view, avoiding uncritical policy endorsement and embracing a data-driven, adaptable approach to teacher education reforms is crucial for their success.

2.4 Transformations in teacher education programmes: A focus on specialisation

The recently introduced four-year degree programmes in Ghanaian colleges of education (CoEs) offer pre-service teacher education to three distinct groups: early childhood educators, primary school generalists, and junior high school specialists. CoEs in Ghana do not currently train teachers for senior high schools, as this is done by the universities. According to Buabeng et al. (2020), this restructuring of the CoEs is anticipated to offer numerous advantages, including enabling pre-service teachers to develop in-depth knowledge related to their chosen grade band.

In alignment with global trends, these changes aim to enrich pre-service teachers' subject matter knowledge and skill sets through expanded curricula covering education foundations, psychology, instructional methods, and content area Knowledge and skills, as Poole and Russel (2015) highlighted. By raising certification standards, Ghana's degree programmes aim to contribute to the professionalisation of teaching to improve outcomes. However, the successful implementation of best-practice reforms in development contexts requires overcoming political, infrastructural, and scaling challenges. McCulloch and Piron (2019) emphasise the importance of aligning programs with domestic support to generate reform momentum. Thus, as Ghana progresses towards alignment with global standards, it is crucial to strengthen localised support systems to ensure the sustainability and impact of these reforms (Coburn et al., 2012). The overarching goal is to enhance teachers' pedagogical competence across the nation.

2.5 What changed from the old to the new Bachelor of Education curriculum?

The transition from the old teacher education curriculum to the new Bachelor of Education curriculum in Ghana aimed to address several long-standing critiques and challenges. The previous system, which included programs like the Diploma in Basic Education (DBE), faced criticism for its lack of cohesion, generalist approach, compartmentalised subject matter and pedagogy, overreliance on examination-based evaluation, and the prevalence of teacher-centred instructional methods (Wilburn et al., 2021; Abroampa et al., 2020; Asare & Nti, 2014). The new Bachelor of Education curriculum seeks to rectify these issues by introducing several key changes:

1. *Standardization:* The curriculum aims to establish a more cohesive and standardised approach to teacher education across institutions. This is to be achieved by aligning the curriculum with common standards and teaching philosophies, minimising the disparities in training prospective teachers across CoEs.
2. *Specialization:* Instead of producing generalist teachers, the new curriculum introduces specialised programmes tailored to different levels of schooling, such as early grade, upper primary, and junior high school. This approach aims to equip pre-service teachers with more targeted and relevant pedagogical skills for teaching at specific educational levels.
3. *Subject matter and pedagogy integration:* Moving away from the compartmentalised approach of the past, the new curriculum emphasises the integration of subject matter and pedagogical methods. This holistic approach aims to provide pre-service teachers with a more comprehensive understanding of content knowledge and effective teaching strategies.
4. *Competency-based assessment:* The curriculum emphasises competency-based assessment, with clear criteria for evaluating effective teaching practices. This shift aims to remove the over-reliance on examination-based evaluation and promote a more comprehensive assessment of pre-service teachers' skills and competencies.
5. *Student-centred teaching methods:* Unlike previous teacher-centred approaches, the new curriculum promotes student-centred and interactive teaching methods. This aligns with contemporary educational philosophies that prioritise active learning and student engagement.
6. *Tutor qualifications:* To enhance the quality of teacher education, the new curriculum mandates that college tutors meet minimum qualification requirements, such as holding

a research master's degree in a relevant subject area (MoE, 2017; Cambridge Education, 2020).

While the new Bachelor of Education curriculum aims to address the shortcomings of its predecessors, its successful implementation and impact will depend on factors such as the capacity of teacher training institutions, resource availability, and ability to address systemic issues within the broader education system. Ongoing evaluation and continuous improvement will be crucial to ensure that the curriculum effectively prepares pre-service teachers for the practical challenges of real-life school settings (Wilburn et al., 2021).

2.5.1 Ghana's national teacher education curriculum framework (NTECF)

The National Teacher Education Curriculum Framework (NTECF) applies to all teacher training programmes, from early childhood education through senior high school levels. The NTECF proposes four interconnected pillars that encompass the foundational knowledge required for initial teacher certification (MoE, 2017b). These pillars are 1) Subject and Curriculum Knowledge, 2) Literacy Studies in Ghanaian Languages and English, 3) Pedagogical Knowledge, and 4) Supported Teaching in Schools. Rather than treating these as isolated domains, the framework emphasises the integration and interweaving of these knowledge areas. Pillars are "the knowledge mix from which potential broad subject areas and courses will be developed" (MoE, 2017b, p. 17).

The NTECF in Ghana is designed to align with the National Teachers' Standards (NTS) and to be used as a guideline for structuring teacher education programmes. It outlines the expected knowledge and competencies PSTs should acquire across various domains, integrating knowledge, skills, concepts, and abilities (MoE, 2017b).

For example, the framework defines key concepts like "pedagogic content knowledge" (NTECF, p. 29). The NTECF provides recommendations for the curriculum offerings for PSTs within each domain. These include suggestions for specific courses, the balance between theoretical and practical components, and areas of emphasis for assessment (MoE, 2017). Furthermore, the NTECF guides the composition of the overall "knowledge mix" in the Bachelor of Education (B.Ed.) programme by allocating credit hours to different domains, thereby shaping the relative emphasis on various areas (Akyeampong, 2017).

By aligning with the NTS and providing a comprehensive framework, the NTECF aims to ensure that teacher education programmes in Ghana effectively prepare pre-service teachers with the necessary knowledge, skills, and competencies to meet the demands of the teaching

profession. Table 2.2 displays the proposed weightings for these four pillars. Although they are presented as distinct knowledge domains, it is crucial to recognise that these pillars are intended to be interwoven and interconnected, thereby preventing any fragmentation of knowledge, as elucidated by Buabeng et al. (2020).

Table 2.2: Proposed weighting for the four pillars of initial teacher education

Pillar	Proposed % of credit hours.			
	K-P3	P4-6	JHS	SHS
Subject and Curriculum Knowledge	25	25	40	60
Pedagogical Knowledge	25	25	20	18
Literacy Studies: Ghanaian Languages and English	20	20	10	5
Supported Teaching in School	20	30	30	17
Total	100	100	100	100

Source: MoE, 2017b

For students enrolled in either early grade (K-P3) or upper primary (P4-6) programmes, the distribution of course credit hours assigns equal weightings of 25% each to subject/curriculum knowledge and pedagogic knowledge. The most substantial allocation, constituting nearly one-third of the total credit hours (30%), is dedicated to practical teaching or "supported teaching in school" activities, which encompass extended, guided, and evaluated teaching experiences spanning a minimum of 36 weeks (MoE, 2017b, p. 32). This pronounced emphasis on practical teaching experiences is in response, at least in part, to concerns that the previous curriculum under the old diploma programme overly prioritised teachers' subject knowledge, often taught in isolation from pedagogical considerations and disconnected from the realities of the classroom environment (Asare & Nti, 2014; Ntim, 2017). The B.Ed. programme has foregrounded practical experiences on the part of teacher educators to provide PSTs with progressively increasing exposure to real-world teaching contexts, commencing as early as Year One.

The NTECF outlines a comprehensive approach to assessment tailored to each pillar of the curriculum. PSTs undergo evaluation across the three domains of the NTS, aligned with each pillar of the NTECF. This approach aims to ensure that PSTs acquire subject-specific knowledge, pedagogical knowledge and skills, and assessment expertise during their four-year degree which will enable them to effectively teach students of varying ages, abilities, and

aptitudes. A pivotal component of the assessment strategy involves the development of a professional portfolio. This portfolio is designed to encompass two key aspects:

1). *Evidence of learning from the training institution:* assignments, examinations, and lecture notes demonstrating the pre-service teachers' learning during their programme. 2). Evidence of learning from in-school experiences: lesson plans, study materials, teaching resources, assessment records, learner exercise books and evaluations provided by mentors, among other relevant artefacts. The assessment process is structured to be fair and realistic, recognising that pre-service teachers are still in the learning phase of their teaching journey. Emphasis is placed on evaluating student teachers' creativity and innovation and their exemplification of professional values and attitudes.

The National Teaching Council (NTC) has introduced the licensing of the teaching profession. After completing their four-year Bachelor of Education degree, pre-service teachers spend one year teaching in pre-tertiary-level basic schools. Once they successfully complete this period, they receive their teaching licence and attain qualified teacher status. This approach aims to ensure that teachers are well-prepared and qualified to meet the demands of the teaching profession.

2.5.2 The National Teachers Standards (NTS)

The National Teaching Standards (NTS) provide a set of benchmarks applicable to teachers across all levels in Ghana. The three inter-related domains of these standards are outlined as:



Figure 2.1: *The National Teachers' Standards* – Source: MoE (2017a)

For teacher educators, the NTS provides a template for designing curricula and crafting courses to educate and train in-service and pre-service teachers. A notable gap in teacher education in Ghana has been the absence of robust professional development opportunities for teacher educators in pre-service teacher education (MoE, 2017a). Such absence has also been evident in countries like South Africa, Ireland and the Netherlands, in which heavy workloads, undefined career progression pathways and lack of funding constrain professional development access for many university-based teacher educators (Schipper et al., 2022; van der Klink et al., 2017). Critics argue that this lack of formal, systematic development support fails staff and the student-teachers they instruct by allowing outdated, lecture-dominated pedagogical strategies to persist (Koster & Dengerink, 2008).

According to Ghana's Ministry of Education, the ideal Ghanaian teacher educator is pictured as:

> A valued professional in a community of practice…who has secure curricular, subject and pedagogical content knowledge, who plans for and uses different interactive instructional strategies and resources and so engages their learners, who achieve higher learning outcomes for all, more vulnerable learners, those with disabilities, girls and those who need cognitive challenge, and who uses assessment productively in achieving those outcomes (MoE, 2015, p. 10).

In Figure 2.1 above, professional development and community of practice are central to professional values and attitudes. The professional development aspect anticipates that teacher educators will engage in deliberate, collective introspection to enhance their personal and professional practice through lifelong learning and continuous professional learning and demonstrate their leadership attributes within the classroom and the broader educational context (MoE, 2017a). The community of practice component emphasises collaboration and affirming interactions with colleagues and educational systems. While previous policy reforms in Ghana concentrated primarily on enhancing pre-service education, with limited attention to the development of teacher educators (Mereku, 2019), the "Cabinet Memorandum on Policy on Teacher Education Reform" (CMPTER) emphasises the importance of growth in teacher educators' professional practices.

2.6 The role of mathematics teacher educators in the implementation of the national teacher education curriculum

Among the primary responsibilities of mathematics teacher educators is the alignment of teacher education mathematics programmes with the objectives outlined in the new school

mathematics curriculum. Such alignment necessitates a critical assessment of the curriculum's content, adaptation to the changing educational environment and engagement in professional development, particularly in relation to effective pedagogy (Gallagher et al., 2020). Findings from a study by Cardetti and Truxaw (2014) reveal that pre-service teachers who participate in intensive and well-structured mathematics pedagogy courses exhibit heightened confidence and competence in teaching mathematics, underlining the imperative for mathematics teacher education programmes to prioritise effective pedagogy in mathematics.

Fundamentally, effective teaching methods should form the foundation of training for prospective mathematics teachers. As upholders of the principles of the NTECF, MTEs are responsible for empowering PSTs to utilise innovative teaching approaches, such as problem-solving, inquiry-based learning, and technology integration. By demonstrating these methods in their own teaching practices, MTEs can serve as exemplary role models. Through collaborative professional learning efforts, MTEs can contribute substantively to developing novel teaching strategies, curriculum enhancements, and evidence-based policies that elevate mathematics education in Ghana, thus fostering a generation of mathematically proficient learners.

2.7 Continuous professional learning: A standing practice among Ghanaian mathematics teacher educators

While the formal education and training for MTEs is undoubtedly valuable, it may not adequately prepare them to address the challenges they face during their careers. For MTEs in Ghana, continuous professional learning (CPL) has become necessary, in line with Ghanaian NTECF requirements. The main goals of CPL are to assist teacher educators in implementing the new B.Ed. programme and to transition from traditional, teacher-centred methods to a more progressive, learner-centred instructional model (MoE, 2017b; Wilburn et al., 2021).

In Ghana, as in many other countries, pre-service mathematics education has evolved to align with changing curriculum priorities and advances in instructional approaches (Jaworski & Burghes, 2018; Johnston-Wilder & Lee, 2014; Sahlberg, 2011; Toom, 2013). To respond to these changes and to provide quality education to pre-service and in-service teachers through the new curriculum, T-TEL, in partnership with the Ministry of Education (MoE), undertook the development of a comprehensive series of twelve professional development guides. These guides were designed to bolster and enhance classroom practices among college tutors (MoE, 2017a).

The professional development guides cover subject-specific pedagogic content knowledge for teaching B.Ed courses. They are organised by subject, year/level, and semester. Professional Development Coordinators (PDCs) participate in university-based mentoring sessions and share their learning with other MTEs. This domain-specific approach aims to help MTEs improve their professional knowledge and address challenges in teaching specific subjects (Wilburn et al., 2021).

However, despite the availability of this support, uptake by MTEs has been inconsistent, reflecting varying perspectives on the reforms (Wilburn et al., 2021). Research indicates scepticism among some MTEs regarding the feasibility of implementing the new standards due to challenges such as large class sizes, resource constraints, and other systemic barriers (Bjerke & Nolan, 2023). Additionally, those accustomed to conventional teaching styles have resisted shifting their lecturer-centric practices towards the more facilitative methods espoused in guidelines (Westbrook et al., 2013). Challenges in tutor participation in CPL have been reported (Abreh, 2018; Yaqub et al., 2020). Critics have contended that generally, professional learning interventions lack coherence, are disjointed, and fail to address the practical needs of teachers in the classroom, consequently impeding the desired transformation among educators (Abakah, 2023; Abdulrazak, 2020; Opfer et al., 2011; Shriki & Patkin, 2016). Thus, while continuous improvement remains an aspiration, ongoing mentorship, incentive structures, and bottom-up feedback mechanisms are vital for effectively mainstreaming changes in instructional culture.

Despite challenges reported in tutor participation in CPD sessions, T-TEL's annual progress survey indicates a positive trend in teacher educator CPL attendance and understanding of the NTECF since the project's inception in 2015 (Cambridge Education, 2020), which suggests Ghanaian MTEs' commitment to continuous improvement and their recognition of CPD as essential for preparing the next generation of mathematics educators.

The current T-TEL guides for CoE tutors cover subject-specific pedagogic content knowledge for teaching B.Ed. courses. The guides are organised by subject, year/level, and semester (e.g., Algebra, Level 100, 1st semester). The professional development coordinators (PDCs) participate in university-based mentoring workshops (not the focus of this study) at the start of each semester and then return to share what they have learnt using the guides. This suggests that current professional learning activities in Ghanaian pre-service teacher education are domain-specific, with focused and targeted content areas to assist MTEs in extending their professional knowledge (Wilburn et al., 2021). As a result, PLCs for MTEs include small

collaborative groups across teaching faculties within the college who find it useful to learn from and with each other about a particular domain, as in the case of the participants in this study. Guided by the NTECF and NTS, they use each other's practice experience as a learning resource.

The mentoring workshops address challenges from the previous year together with subject-specific teaching methods. MTEs develop their knowledge and skills by discussing how to teach and assess subject-specific topics, challenges they may encounter, and what materials they may need. Some colleges have developed 'resource centres' through T-TEL support and these provide instructional materials for tutors and pre-service teachers. Though studies have reported challenges to tutor participation in CPD sessions in the CoEs (Yaqub et al., 2020; Abreh, 2018), T-TEL's annual survey of progress reported that tutor CPD attendance, understanding of the NTS and use of student-centred methods has improved since the launch of the project in 2015 (Cambridge Education, 2020).

2.8 Overview of the study's professional learning community

The following section provides an examination of the Professional Learning Community (PLC) in mathematics teacher education. It explores the community's foundational structure, including its goals, participants' roles, and the various forms of professional learning activities employed. Here, I provide insights into how the PLC operates, its collaborative approaches, and the ways in which MTEs engage in collective professional development.

The PLC observed in this study is a collaborative network of MTEs in Ghana, designed to facilitate continuous professional development and collective learning. This community operates on a structured forum that has adopted a plan teach-reflect approach. It aims to empower MTEs through shared experiences, knowledge exchange, and collaborative practices to enhance their teaching strategies and elevate the quality of mathematics education.

The number of MTEs meeting for PLC is college-specific and usually ranges between 6 – 10. These MTEs have been teaching algebra for some time. Participants' perspectives, experiences, and expertise contribute to the community's growth. The group meets once a week to plan and reflect on previous lessons.

2.8.1 Goals and objectives of the PLC

The PLC's objectives align with the aims of the new curriculum framework of fostering an environment conducive to growth, innovation, and excellence in mathematics education. Its

primary goal is to encourage a culture of continuous learning and development by offering opportunities for MTEs to engage in reflective practice, discuss emerging educational theories, and apply their insights to refining their teaching methodologies so that these cohere with teaching and learning across school mathematics and impact student teachers' preparation for classroom teaching. The PLC is a space where MTEs can exchange best practices, innovative teaching techniques, and resources.

2.8.2 Participants' roles and engagement within the PLC

Within the PLC, most MTEs play a significant role in the community's collective progress, though engagement varies among participants. MTEs typically participate in rotational leadership or mentoring roles, with many guiding discussions, organising sessions, and offering mentorship to newer or less experienced members. While many share resources, including lesson plans and research findings, and engage in collaborative activities, the level of participation and commitment is not uniform across all members. The community strives to encourage a culture of openness and information exchange, with most participants engaging in reflective practices and providing constructive feedback. However, the PLC's reality is more complex, with some members demonstrating more active involvement than others. Despite these variations, the PLC represents an evolving ecosystem where educators sought to foster collaboration and collective growth in mathematics teacher education in Ghana, recognising that collective progress is an ongoing process with inherent challenges.

2.8.3 Forms of professional learning in the PLC

The Algebra PLC creates a collaborative and supportive environment where MTEs participate in a range of professional learning activities. These activities aim to share knowledge, promote mutual growth, and improve teaching practices in pre-service teacher education. The following sections outline the main forms of professional learning supported by the PLC.

Collaborative lesson planning

MTEs within the Algebra PLC engage in collaborative lesson-planning sessions. During these sessions, they collectively design and structure lesson plans for teaching algebra to PSTs. This collaborative approach allows MTEs to draw from each other's expertise, experiences, and diverse teaching strategies with the intention of producing more comprehensive and effective lesson plans.

Team teaching

Another approach employed is team teaching, where two or more MTEs jointly plan and deliver algebra instruction to PSTs. Team teaching allows MTEs to model effective teaching practices and fosters an active learning environment where they can observe and support each other in real-time.

Critical friendships

MTEs often form critical friendships within the PLC. In these pairs or small groups, they visit each other's lessons and provide constructive feedback and reflections. These critical friendships are based on trust and a commitment to mutual improvement, enabling MTEs to enhance their instructional practices through constructive peer evaluation. In the context of this study, critical friendships between MTEs are the focus of chapter 11.

Reflective practice

Reflective practice is a cornerstone of the PLC's approach. MTEs engage in regular self-reflection and group discussions about their teaching experiences, challenges encountered, and successes achieved. This reflection on practice promotes a culture of continuous improvement, where MTEs adapt and refine their teaching methods based on collective insights.

Sharing of resources

MTEs within the PLC actively share resources related to algebra teaching. These resources encompass various materials, from lesson plans and teaching aids to research findings and best practices. This sharing of resources not only supports ongoing professional development but also ensures a rich repository of teaching materials.

Collaborative problem-solving

The PLC serves as a platform for collaborative problem-solving. MTEs come together to address challenges they face when teaching algebra to PSTs. They collectively seek solutions, brainstorm innovative approaches, and share effective problem-solving strategies.

Lesson study

MTEs participate in lesson study activities, collaboratively planning, teaching, and reflecting on lessons. This approach enables them to assess the impact of their teaching methods and refine their strategies for improved learning outcomes among PSTs. These strategies and

methods are part of the PLC's efforts to create a dynamic and supportive environment where MTEs can continuously develop their professional knowledge, share insights, and enhance their teaching practices. This collaborative approach contributes to teaching algebra in pre-service teacher education by being effective, innovative, and responsive to the evolving needs of PSTs.

2.9 Chapter summary

This chapter has provided a contextual overview of the history of formal education in Ghana, starting from the colonial era and progressing through various reforms in teacher preparation and school and teacher education curricula. It highlights recent significant initiatives such as the Transforming Teacher Education and Learning (T-TEL) programme (2014-2018). The chapter argues that past reforms focused on pre-service teacher education and training, with insufficient attention paid to the professional development of practising teachers and teacher educators, thus allowing outdated pedagogical approaches to persist. The chapter contends that the new standards and curriculum frameworks prioritise continuous professional development, recognising it as vital for improving the current practices of teacher educators and teachers in schools.

However, uptake of what these new standards and curriculum frameworks require has been inconsistent due to doubts regarding feasibility and resistance from teacher educators to shifting their embedded practices. Mathematics teacher educators must now align their teaching to school mathematics curricula and standards, necessitating engagement with current best practices in mathematics pedagogy and the modelling of interactive teaching strategies.

While progress has been made, transforming instructional culture requires sustained, coherent efforts that address on-the-ground needs. One aim of professional development opportunities for MTEs is to extend their content and pedagogic knowledge. The next chapter reviews literature relevant to situating this exploration of how collaborative engagement enhances knowledge for teaching in pre-service mathematics teacher education programmes.

CHAPTER 3

SITUATING THE STUDY WITHIN THE RELEVANT LITERATURE

3.0 Introduction

This chapter reviews the literature on models of teacher knowledge that have substantially influenced teacher education. It begins by clearly distinguishing between mathematics teacher knowledge and the knowledge required for mathematics teacher educators. The chapter then provides an overview of the current academic discourse surrounding mathematical knowledge for teaching. Additionally, it discusses studies that establish the relationship between knowledge and practice in pre-service mathematics teacher education. The final section focuses on the extensive literature regarding professional learning initiatives and collaborations for mathematics teacher educators, including approaches such as critical friendships.

3.1 The distinction between mathematics teacher knowledge and mathematics teacher educator knowledge

The knowledge required for mathematics teacher educators (MTEs) is distinctly not the same as that needed by mathematics teachers (MTs), though there are substantial areas of overlap. While MTs need to know how to teach mathematics to students in schools, MTEs must understand how to teach teachers to teach mathematics – a fundamental distinction that Zaslavsky and Leikin (2004) conceptualise as knowledge orders. First-order knowledge represents the direct application of mathematical pedagogical skills within classroom settings, while second-order knowledge encompasses a meta-level understanding of how mathematical teaching and learning occur. This differentiation is not merely hierarchical but represents a fundamental transformation in professional perspective and analytical capacity.

Jaworski's (2008) concept of nested layers of practice provides a robust framework for understanding this distinction. The knowledge base of MTEs can be conceptualised as a multilayered construct that encompasses, yet extends beyond, traditional mathematics teaching knowledge. These layers include foundational mathematical content knowledge, pedagogical content knowledge, and a meta-layer of understanding teacher learning processes.

The distinction becomes particularly clear when considering the practice of teaching school mathematics and the practice of (pre-service) mathematics teacher education. Taking the teaching of fractions as an illustrative example, an MT primarily focuses on identifying and addressing student misconceptions about fractions and developing targeted instructional strategies to enhance student understanding. In contrast, an MTE must operate at a more

advanced level, simultaneously understanding student misconceptions, recognising potential misconceptions held by pre-service teachers, and developing strategies to support teachers' professional growth.

Superfine and Li (2014) capture this distinction through their characterisation of a fundamental shift from "knowledge of teaching mathematics" to "knowledge of teaching about teaching mathematics." This transformation represents more than a simple expansion of knowledge; it requires a fundamental reconfiguration of professional understanding. Mathematics teacher educators must develop the capacity to make implicit teaching practices explicit, facilitate deep professional discussions, and support the ongoing development of teachers' professional identities.

The theoretical framework proposed by Even (2008) introduces the concept of "meta-mathematical pedagogical knowledge," which further elaborates on the unique knowledge base of mathematics teacher educators. This form of knowledge transcends traditional pedagogical content knowledge, requiring MTEs to simultaneously consider multiple levels of mathematical and pedagogical understanding. They must navigate a complex three-way relationship between mathematical content, teachers' learning of this content, and the development of teachers' instructional capabilities (Leikin, 2021).

The relationship between these knowledge bases is not merely additive. MTEs don't simply need more knowledge than MTs; they need qualitatively different knowledge that enables them to support teacher learning effectively. This includes understanding how to make teaching practices explicit, how to facilitate productive discussions about teaching, and how to support the development of teachers' professional identity. These aspects of knowledge are unique to the work of teacher education and represent a distinct professional domain.

Understanding this distinction is crucial for research and practice in mathematics teacher education. It suggests the need for specific attention to how MTEs develop their specialised knowledge base and how this development can be supported through professional learning opportunities that explicitly address the unique demands of teaching teachers.

Table 3.1: A nested model showing the relationship between MT and MTE knowledge

Aspect	Mathematics Teacher	Mathematics Teacher Educator
Primary Focus	Teaching students mathematics	Supporting teachers' learning to teach mathematics
Misconception Handling	Addressing student misconceptions	Understanding and addressing both student and teacher misconceptions
Pedagogical Development	Applying pedagogical strategies	Developing teachers' pedagogical reasoning and reflection
Knowledge Complexity	First-order knowledge	Second-order, meta-level knowledge

3.2 Conceptualising knowledge for teaching: a review of knowledge models that have subsequently evolved into MTE knowledge

The knowledge required for effectively teaching mathematics is distinct from the mathematical knowledge needed in other professions (Ball et al., 2008; Hoover et al., 2016). Researchers characterise this as "mathematical knowledge for teaching" - this specialised mathematical understanding that is intrinsically tied to carrying out the recurring tasks and responsibilities of the mathematics teaching profession (Ball et al., 2008, p. 399). In contrast to the mathematics typically covered in tertiary-level content courses, teachers must develop an integrated grasp of mathematical concepts and pedagogical methods.

Researchers have used the phrase "for teaching" and "practice-based" to explicitly denote that this form of knowledge is inextricably linked to the actual work and practice of teaching mathematics in the classroom setting (Ball et al., 2008; Hoover et al., 2016). Mathematical knowledge for teaching encompasses a fusion of content expertise and an understanding of how to effectively guide student learning within a specific content domain. It represents the specialised mathematics knowledge that allows teachers to responsively employ instructional techniques, assess student thinking, select appropriate representations, and facilitate conceptual discussions - all in the context of actively teaching mathematical concepts (Ball et al., 2008). Delineating this distinct knowledge domain has been foundational for research into the preparation and professional development of mathematics teachers

Castro Superfine et al. (2020) argue that there is widespread consensus within the mathematics teacher education community regarding the concept of mathematical knowledge for teaching teachers (MKTT). The dimensions of MKTT encompass not only fundamental teacher-related knowledge but also include expansions that transcend the conventional scope of teacher knowledge. Researchers have explored the exact characteristics of these MKTT dimensions in varying degrees of detail (Goodwin & Kosnik, 2013; Zaslavsky & Leikin, 2004).

Most extant MKTT research makes a connection between the knowledge domains of MTEs and those of teachers, particularly emphasising the expertise required to assist PSTs in cultivating specialised content knowledge. For instance, Chauvot (2009) self-analysed her role as an MTE and conceptualised diverse knowledge domains of MTE expertise. Her comprehensive knowledge map for MTEs consists of three core categories: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. This domain expands on Shulman's (1987) framework for teacher knowledge. Chauvot also adapts elements from Ball and colleagues' (2008) domains of mathematical knowledge for teaching, tailoring

them to the specific context of MTEs. These adaptations encompass the knowledge and skills needed to foster PSTs' specialised content and pedagogical content knowledge, along with strategies to effectively familiarise PSTs with content in ways aligned with instructional practices.

Significantly, Chauvot's knowledge map (2009) considers contextual factors, echoing Grossman's (1990) insights, that shape the operational landscape of MTEs. These include considering school standards for mathematics and curriculum programmes, thereby extending the spectrum of knowledge that MTEs require. In the next section, I review several models of teacher knowledge that have influenced that conception of what I call mathematical knowledge for teacher education and connect these to current literature on MKTT.

3.3 Teacher knowledge from Shulman's (1986) perspective

Shulman (1986) proposed that educators have a unique kind of knowledge that distinguishes them from subject matter specialists and termed this pedagogical content knowledge (PCK). By coining PCK as a term for the integration of subject-specific content and pedagogy, Shulman (1986) aimed to address a "missing paradigm" in research on teaching and educator knowledge that had until that time, focused predominantly on generic aspects of teaching. The four categories of teacher knowledge suggested by Shulman (1986) are Subject Matter Content Knowledge (SMK), Pedagogical Knowledge (PK), Pedagogical Content Knowledge (PCK) and Curricular Knowledge (CK). SMK "refers to the amount and organisation of knowledge per se in the teacher's mind" (Shulman, 1986, p. 9). Together, these four knowledge domains comprise Shulman's "missing paradigm." Concerning instructing teachers, Shulman proposed distinct categories of knowledge that are conducive to facilitating impactful and efficient instruction. The domains include:

- Content knowledge
- General pedagogical knowledge, which includes broad classroom management and organisational strategies that apply across various subject matters
- Curriculum knowledge, which involves a thorough understanding of teaching materials and programs
- Pedagogical content knowledge, which represents the unique combination of content and pedagogy specific to teachers
- Knowledge of learners and their characteristics

- Knowledge of educational contexts, ranging from classroom dynamics to the governance and financing of educational institutions
- Knowledge of educational goals, purposes, and values, as well as their philosophical and historical foundations (Shulman, 1987, p. 8)

Shulman emphasised that teachers need to know how an algorithm operates, why it functions that way, and the theoretical underpinnings that justify its accuracy. CK encompasses substantive structures, organising basic principles, laws, and concepts within a discipline, and syntactic structures, which refer to the processes for generating and validating new knowledge (Shulman, 1986). PCK, residing at the intersection of curriculum content knowledge and pedagogy, involves making the subject matter comprehensible to students through effective analogies, accurate representations, and efficient instructional strategies. Shulman (1986, p. 9) argued that "If teachers were to be successful, they would have to confront both issues of content and pedagogy simultaneously, by embodying the aspects of content most germane to its teachability." Curricular Knowledge is knowledge of a range of programmes designed to teach specific subjects and includes an awareness of the instructional materials available for those programs and their effective use. A solid grasp of curricular knowledge supports establishing connections between topics, helping students build upon prior knowledge and scaffold future learning.

Shulman (1987) highlighted the significance of PCK among the various types of teacher knowledge, as it represents an integration of content and pedagogy adapted to accommodate learners' diverse needs and abilities. Shulman's work offered useful characteristics of content knowledge for effective teaching in general. His work also shifted researchers' attention from a focus on pedagogy alone to a focus on both CK and PCK. Ball et al. (2001) argue that PCK is an essential component of teacher knowledge for three reasons. Firstly, it fills the gap left when focusing only on the teacher's credentials. Secondly, it improves understanding of the knowledge required for teaching. Through Shulman's framework, educators and researchers realised that teachers' mathematical knowledge differs from that of mathematicians. Thirdly, it has produced new methods of teacher study that have resulted in the generation of rich knowledge (Ball et al., 2001). Ball et al. (2008) acknowledged two contributions of Shulman's framework. Firstly, it has shifted researchers' attention from general teaching aspects to content's role in teaching. Secondly, it presents understanding as a special component of the profession of teaching.

However, Jones et al. (2002) argue that Shulman's model may be too simplistic because it does not make a distinction between different school subjects. They note that teaching Mathematics is complex and different from other subjects, hence the need for a specialised framework. Ball et al. (2001) also observe that the framework might be very difficult to use because it does not provide specific guidelines for teaching Mathematics well. Furthermore, Ball et al. (2005) argue that Shulman's framework might be difficult to operationalise because it does not clarify the difference between CK and PCK. I argue that Shulman (1986) does not present his framework as a final tool for understanding knowledge for teaching any subject but as a lens for guiding further research on knowledge for teaching a particular subject. The framework acts as a springboard for researchers to understand the different dimensions of teacher knowledge for every subject. By suggesting that more research be conducted to ensure a deep understanding of PCK for specific subjects, Shulman acknowledges that the requirements for teaching different subjects are not the same. In this study, Shulman's model of teacher knowledge can be useful in studying how the collaborative endeavours of MTEs inform their mathematical knowledge for teaching by providing a comprehensive framework for examining and enhancing the various domains of knowledge required for effective teacher education through shared insights, critiques, and development of instructional approaches within a collaborative setting.

3.4 Grossman's (1990) model of teacher knowledge

Though Grossman is not a mathematics education specialist, her model of teacher knowledge (Grossman, 1990) demonstrates a comprehensive understanding of the domains that contribute to effective teaching. This model draws from the earlier work of Shulman (1986) by highlighting four distinct areas of teacher knowledge: 1) Subject Matter Knowledge (SMK): The realisation that teachers need to have a thorough comprehension of the subjects they teach is at the heart of Grossman's model. This goes beyond a mere grasp of content; it involves understanding the subject's structure, language and symbols (syntactic structures), and the underlying concepts and principles (substantive structures). This depth of knowledge enables teachers to convey content in ways that resonate with students and to design learning experiences that facilitate meaningful comprehension. 2) General Pedagogical Knowledge: Grossman's (1990) model emphasises the general pedagogical knowledge and skills that are essential for effective teaching across various subjects. This domain encompasses the broader instructional strategies and techniques that are applicable beyond specific disciplines. It involves managing a classroom, designing curricula, engaging students, and selecting

appropriate instructional materials. This foundational pedagogical knowledge forms the basis for creating an environment conducive to learning. 3) Knowledge of context: For Grossman, teaching occurs within dynamic and varied contexts that include factors such as the school environment, the community, student demographics, and prevailing educational standards. Grossman emphasises the importance of teachers being aware of these contextual elements as they shape the educational experience. Teachers attuned to these factors can adapt their teaching methods to align with the needs and expectations of their students and the broader educational landscape.

Grossman's fourth area of teacher knowledge is Pedagogical Content Knowledge (PCK), which represents the integration of subject matter expertise with pedagogical knowledge and skills, just like Shulman (1986). PCK involves understanding how to effectively teach specific content to particular students. It includes recognising how students conceptualise the subject, selecting appropriate instructional strategies, and adapting teaching approaches to suit diverse learning styles, thus expanding Shulman's (1986) idea of PCK. PCK bridges the gap between content knowledge and effective teaching methods. Grossman's model underscores the interconnectedness of these four domains. Effective teaching requires a balance among all areas of knowledge. Mastery of subject matter alone is insufficient; effective educators possess content expertise, pedagogical strategies, awareness of the learning context, and the ability to translate subject matter into comprehensible and engaging instruction. This holistic approach to teacher knowledge guides educators seeking to provide meaningful and impactful learning experiences for their students. For instance, in this study, MTEs emphasised that knowledge of the mathematics teacher education context is important for them to thrive as teacher educators.

3.5 Ma's notion of a profound understanding of fundamental mathematics

Ma's (1999, 2010) work is based on her study of mathematics education in China and the United States. She noticed a stark contrast between how mathematics was taught and learned in these two countries. In China, students seemed to have a deeper and more intuitive grasp of fundamental mathematical concepts than their American counterparts (Ma, 2010). Ma (1999) described the knowledge that the Chinese teachers in her study displayed as involving a "*profound understanding of fundamental mathematics*" (p. 120). Her concept of "profound understanding of fundamental Mathematics" (PUFM) is involves a richly connected web of understandings that gives teachers a deep understanding of the domain and ways to help students learn mathematics. Ma emphasised that Chinese teachers demonstrated greater

mathematical knowledge, which was noticeably absent in the American teachers she interviewed. She states:

> An understanding of the terrain of fundamental mathematics that is deep, broad and thorough. Although the term 'profound' is often considered to mean intellectual depth, its three connotations, deep, vast, and thorough, are interconnected. (Ma, 2010, p. 120).

This profound understanding enables teachers to foresee and address common student misconceptions, present concepts in diverse ways, draw connections between mathematical ideas, and promote a conceptual understanding rather than rote memorisation. Ma (2010) contends that PUFM is composed of well-organised "knowledge packages," a term she uses to describe teachers' subject matter knowledge (SMK).

Ma explains that when introducing a new topic, teachers should recognise its place within the broader mathematical framework. Teachers should be able to identify and connect related topics, which together form the knowledge package for the topic being taught (Ma, 1999, p. 118). These knowledge packages contain "key pieces," essential related topics that help understand the main subject. A teacher can build a cohesive and thorough picture of a mathematical concept using this comprehensive information. Ma asserts that PUFM is based on a well-structured collection of these knowledge packages and suggests ways to cultivate such understanding among teachers.

Ma argues that PUFM is intrinsically mathematical and pedagogical. While the core tenets revolve around mathematical structures, they also focus on meaningful and deep learning. Ma notes that a proficient teacher's knowledge overlaps with that of an accomplished mathematician but with distinct differences. For instance, teachers with PUFM might not know the formal definitions that mathematicians are familiar with, but they possess unique insights into elementary mathematics that are specifically relevant to teaching. Ma's model highlights that teaching in a classroom informed by a teacher's PUFM is characterised by attention to:

Basic Ideas – Central to PUFM is the idea that teachers should have a strong and deep understanding of the foundational concepts in mathematics. This goes beyond surface-level familiarity and requires a grasp of underlying principles and relationships. When teachers have this deep understanding, they are better equipped to explain concepts from various angles, answer students' questions effectively, and anticipate the challenges that students might face.

Connectedness – It is crucial to emphasise the connections between various mathematical concepts. Teachers should help students see the relationships between ideas, preventing a disjointed experience and fostering a unified body of knowledge (Ma, 2010, p. 122).

Multiple Perspectives—Teachers should present mathematical concepts using various representations and problem-solving strategies. This encourages students to develop flexible thinking and adapt their approaches to different problems. Different students learn in different ways. PUFM supports teachers in presenting mathematical concepts using a variety of representations, such as visual diagrams, concrete manipulatives, and symbolic notation. Similarly, teachers with a profound understanding of mathematics can introduce students to various problem-solving strategies, enabling them to develop flexible thinking, adapt their approach to different problems, and consider them critically.

Longitudinal Coherence – "Teachers with PUFM are not limited to the knowledge that should be taught in a certain grade; rather, they have achieved a fundamental understanding of the whole elementary mathematics curriculum." (Ma, 2010, p. 122). PUFM emphasises the importance of seeing how different mathematical ideas connect and relate. Teachers aware of the connections between concepts can assist students in filling in knowledge gaps and seeing the logical flow from one idea to the next. This longitudinal coherence promotes more effective learning by ensuring that students' mathematical knowledge builds upon a strong foundation.

In summary, Ma's (1999) concept of a profound understanding of fundamental mathematics underscores the critical role of teacher knowledge in shaping effective mathematics education. Similarly, I argue that when MTEs understand fundamental algebraic concepts, connections, and pedagogical strategies, they can create more meaningful and enduring learning experiences for their PSTs.

3.6 Teacher education development study in mathematics (TEDS-M)

Tatto et al. (2008) developed the TEDS-M model, which has been used to develop instruments for investigating and evaluating both primary and lower secondary teacher education preparation worldwide. The authors aimed to develop test items that would enable different countries to analyse their education systems and policies and compare their expectations about future mathematics teachers. The TEDS-M theoretical framework of teachers' professional competence regards MKT as comprising of Shulman's (1986) two teacher knowledge categories: mathematical content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) (Tatto et al., 2008). The domains of MCK include the mathematical

content that the teachers are required to teach, as well as mathematics beyond the level that the teachers are required to teach. The domains of MPCK included Mathematics curricular knowledge, knowledge of planning, and knowledge of enacting Mathematics (Tatto et al., 2008). The assumptions underlying the TEDS-M model are as follows:

- Teacher education is understood and implemented differently across national settings and even between institutions within the same country.
- Teacher education and teacher learning are complex, contested processes.
- Knowledge of the content to be taught is a crucial factor influencing the quality of teaching.
- Teacher education requires understanding and addressing how teachers should think about mathematics, teaching, and learning.
- Teacher education embodies a developmental logic for how teachers acquire professional knowledge for teaching mathematics and other subjects.
- Knowledge for teaching involves consideration of the situational contexts where teachers will teach.
- Teacher education is assumed to be linked to student achievement, but this relationship is poorly understood.

The assumptions underpinning the TEDS-M project relate to the findings of some empirical studies. For instance, the assumption that educators' content knowledge influences the quality of teaching is similar to that of Hill et al. (2005), who argue that teacher practices influence students' achievements in mathematics. The assumption that knowledge for teaching involves consideration of the situational contexts where teachers will teach aligns with the research of Ding et al. (2005), who found that the contexts within which mathematics is taught around the world vary in terms of curriculum specification, student age level, teacher knowledge, and so on.

Lin and Rowland (2016) argue that the TEDS-M framework ignores the role of mathematics classrooms in advancing teachers' professional development by assuming that teacher knowledge can be evaluated only through pencil and paper tests taken outside mathematics classrooms. This means that the framework ignores the usefulness of teacher-student interaction in mathematics education. In my view, the framework also ignores the importance of assessing students' mathematical thinking. Like some other mathematical teacher knowledge frameworks, the PCK categories of the TEDS-M framework describe general knowledge for

both primary and secondary school pre-service teachers. This implies that the framework does not recognise that PCK is strongly influenced by specific subject matter knowledge.

3.7 Mathematical knowledge for teaching

Ball, Thames, and Phelps (2008) emphasised that the issues of teacher knowledge identified by Shulman are important to research on teaching and teacher education. They explicitly identified Mathematical Knowledge for Teaching (MKT) as the mathematical knowledge, skill, and habits of mind entailed by the work of teaching and proposed a further refinement of Shulman's (1986) subject matter and pedagogical content knowledge into six sub-domains as illustrated in Figure 3.1.

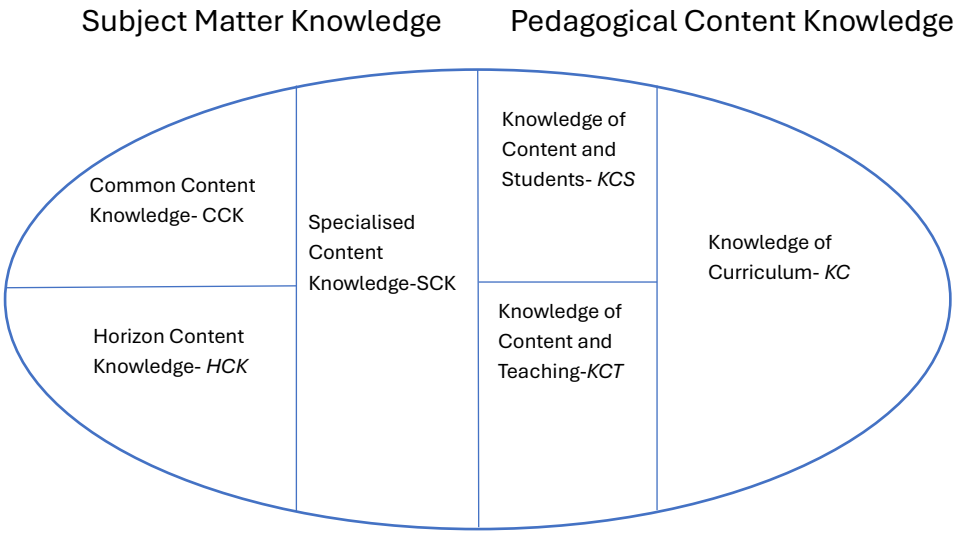


Figure 3.1: Mathematical knowledge for teaching framework by Ball, Thames and Phelps (2008, p. 403).

The MKT framework is divided into two main categories:

Subject Matter Knowledge (SMK):

- Common content knowledge (CCK): Mathematical knowledge used in settings beyond teaching.
- Specialised content knowledge (SCK): Mathematical knowledge and skills unique to teaching.
- Horizon content knowledge (HCK): An understanding of how mathematical topics are connected within the broader mathematical landscape.

Pedagogical Content Knowledge (PCK):

- Knowledge of content and students (KCS): Understanding of both mathematics and student cognition.

- Knowledge of content and teaching (KCT): Knowledge of mathematics intertwined with effective teaching methods.
- Knowledge of curriculum (KC): Understanding of the curriculum's structure and interconnections.

Horizon Content Knowledge (HCK) is particularly noteworthy. It refers to an awareness of how mathematical concepts are situated within the discipline and plays a crucial role in connecting learners with the broader field of mathematics. HCK includes an understanding of core disciplinary orientations, values, and essential structures of mathematics. This conceptualisation suggests that HCK is instrumental in enabling students to make connections across various mathematical concepts and topics, thereby enhancing their learning experience. The concept of HCK aligns with Ball's (1993) initial reference to the "mathematical horizon" in the context of elementary teaching. It involves making pedagogical decisions that maintain mathematical integrity and consistency with future learning. For instance, when introducing negative numbers, teachers must consider representations that will remain valid when students encounter more advanced concepts like absolute value in later years. Because MTEs assume an extensive range of roles in mathematics teacher education (Coles & Helliwell, 2023), it is a challenge to precisely delineate the knowledge imperative for their work (Ping et al., 2020).

In this study, because the PSTs with whom MTEs work come from different backgrounds, I argue that having knowledge of the mathematical horizon could help MTEs bridge the gap between PSTs' existing knowledge of school mathematics and the concepts being taught in teacher education. By being aware of the typical mathematical progression and instructional approaches used at various levels, MTEs could better assess PSTs' prior understanding, identify potential gaps or misconceptions for discussions in the PLC, and design instruction that builds coherently on their backgrounds. Such knowledge would enable MTEs to make meaningful connections, use relevant examples and representations familiar to PSTs, and address their specific needs based on their mathematical experiences

3.8 Positioning mathematical knowledge for teacher education in academic discourse

Building on established models of teacher knowledge, some MTEs have conceptualised and examined their professional knowledge in teaching mathematics to pre-service teachers (PSTs) through self-reflective analysis (Chapman, 2013; Chauvot, 2009; Tzur, 2001). Chauvot's (2009) work is an illustrative example, where she used Shulman's framework to create a knowledge map encompassing subject matter content knowledge, pedagogical content knowledge, curricular knowledge, and contextual knowledge for MTEs. This map guided her

teaching and mentorship roles. Notably, Chauvot emphasised the significance of "knowledge of research" in shaping her teaching approaches.

Zaslavsky and Leikin (2004) also conceptualise mathematical knowledge for teaching teachers (MKTT) to support PSTs in their learning of content. In particular, this MKTT involves understanding techniques for guiding PSTs' learning processes and understanding what subject matter PSTs need to learn. Zaslavsky and Leikin assert that MKTT further entails attuning to PSTs' sensibilities and evaluating their comprehension of mathematical concepts.

Perks and Prestage (2008) conceptualise the domains of MTE knowledge as encompassing opportunities to support PST content learning. They describe these domains as the interrelationships between:

1. Extended knowledge (the knowledge that develops as MTEs progress in their profession).
2. Practical wisdom (the activities and strategies employed during teaching).
3. Professional traditions (the established practices and norms for working with teachers – both in-service/ pre-service, as well as the research on teaching and learning).

In essence, they suggest that MTE knowledge is enriched by connections between the knowledge they acquire over time, their practical teaching strategies, and the professional standards, practices, and research surrounding teaching and learning. Notably, Perks and Prestage (2008) emphasise the significance of the dynamic relationship between the collaborative approaches used with PSTs and the concerted efforts to foster the development of the comprehensive teaching knowledge base expected of MTEs, as these two elements are closely intertwined and mutually influential.

In similar vein, drawing from Ball and colleagues' work, Zopf (2010) explored the knowledge required by MTEs to perform the mathematical tasks of teaching, termed "*mathematical knowledge for teaching teachers*". Zopf breaks the work of teaching MKT into some core tasks, such as:

1. Selecting interpretations and representations used for teaching mathematical knowledge for teaching.
    - *Unpacking details compressed within mathematical concepts and procedures.*
    - *Surveying interpretations and representations that are appropriate for teaching particular concepts in ways that unpack these details.*
2. Selecting examples used for teaching mathematical knowledge for teaching.

- *Sequencing identified examples (to unpack the mathematics layer by layer)*

3. Managing the enactment of mathematical tasks used for teaching mathematical knowledge for teaching (Jankvist et al., 2020, p. 317).

According to Zopf (2010), the execution of pedagogical responsibilities in teacher education necessitates having mathematical knowledge for teaching teachers (MKTT). This expertise involves a nuanced understanding of mathematical knowledge for teaching (MKT) intertwined with a special knowledge of teaching about teaching. Zopf (2010) characterises MKTT as panoramic, connected, fluent, and deliberate. The panoramic quality refers to the need for a broad understanding of the wider mathematical landscape when teaching MKT. This understanding includes knowledge of how fundamental mathematical concepts are developed and how they connect to more advanced mathematics, aligning with Ma's (1999) notion of a profound understanding of fundamental mathematics (PUFM). MKTT is described as connected knowledge because it requires making connections within and across different mathematical domains.

Based on the aforementioned studies, Castro Superfine, Prasad, Welder, Olanoff, and Eubanks-Turner (2020) conceptualised Mathematical Knowledge for Teaching Teachers (MKTT) as an extension of Ball and colleagues (2008) domains of mathematical knowledge for teaching. They proposed that MKTT comprises two main domains: 1) Subject Matter Knowledge for Teaching Teachers (MTE-SMK) and 2) Pedagogical Content Knowledge for Teaching Teachers (MTE-PCK), as shown in Figure 3.2. They assumed that MTEs need knowledge of mathematical content (MTE-SMK) and ways of facilitating PSTs' learning and relearning of content (MTE-PCK). They included Mathematical Knowledge for Teaching (MKT) for classroom teachers as a subdomain of MTE-SMK, referred to as MTE-Knowledge of MKT. They elaborate on these domains and their related sub-domains, as indicated below.

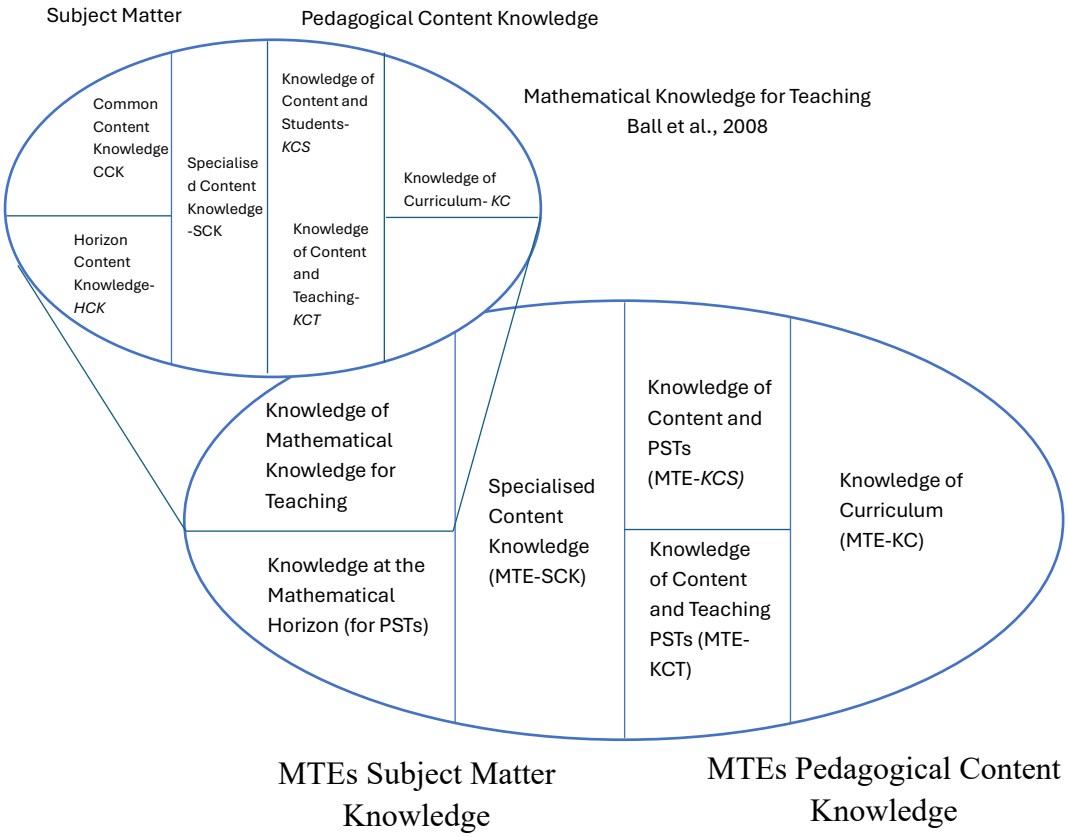


Figure 3.2: Mathematical knowledge for teaching teachers (Castro Superfine et al., 2020, p. 372)

MTE-SMK consists of three subdomains: MTE knowledge of MKT, MTE specialised content knowledge (MTE-SCK), and knowledge at the mathematical horizon for PSTs. PSTs need to learn content in ways specific to teaching (i.e., MKT as defined by Ball et al., 2008), and thus, MTEs must have knowledge of the content PSTs need to know (MTE-Knowledge of MKT). Additionally, MTE-SCK refers to the specialised content knowledge unique to teaching MKT to PSTs, including understanding the mathematical purposes underlying the specialised content PSTs learn. MTEs also require knowledge at the mathematical horizon for PSTs, encompassing the content PSTs will encounter in their subsequent coursework.

MTE-PCK consists of three subdomains: knowledge of content and PSTs (MTE-KCS), knowledge of content and teaching PSTs (MTE-KCT), and knowledge of curriculum (MTE-KC). MTE-KCS includes knowledge of content and PSTs as adult learners, where the content to be learned is MKT. MTE-KCT involves knowledge of how to facilitate PSTs' learning of MKT in relation to teaching school mathematics Additionally, MTE-KC refers to

understanding how to use or develop curricular materials to create learning environments that enable PSTs to unpack and relearn mathematical content.

Castro Superfine et al. (2020) conceptualised MKTT as comprising not only the content PSTs need to know but also specialised knowledge of content unique to teaching PSTs and knowledge of how to facilitate PSTs' relearning mathematics. They considered MKTT to be an elaborated extension of teacher knowledge, including domains distinct from teacher knowledge, stemming from MTEs' work in supporting PSTs' relearning of Mathematics. The authors assert that MTEs' mathematical knowledge for teaching teachers must be fluent, allowing them to unpack and represent mathematical concepts and procedures accessibly and meaningfully to facilitate PSTs' learning.

Through examples in their study, Castro Superfine et al. (2020) highlight the distinctions between designing tasks for students and tasks for PSTs. This distinction illustrates the difference between the knowledge required to teach mathematics to students (MKT) and the knowledge required to teach MKT to PSTs (MKTT). They also present three examples: 1) Supporting Unconventional Thinking, 2) Developing Sense-Making Strategies, and 3) Developing Specialised Content Knowledge. The first example demonstrates a task to help PSTs relearn basic mathematical content by engaging with a student's unconventional thinking. This approach challenges PSTs to reconsider their understanding and adapt their teaching strategies accordingly. The second example describes a task that aids PSTs in relearning fractions by encouraging them to develop sense-making strategies instead of relying solely on procedural methods. This approach foregrounds the importance of understanding rather than reliance on rote procedures.

The example of developing specialised content knowledge involves revising tasks for PSTs to promote the development of specialised content knowledge (SCK). This process of revision also enhances MTEs' understanding of content and PSTs. Castro Superfine et al. (2020) argue that while these examples contribute to the limited research on MTE knowledge, there is a need for more comprehensive research that specifically explores the differences between MKTT and MKT. They suggest that research on teachers' development of MKT can provide insights into understanding the distinctions between MKT and MKTT. Castro Superfine et al. (2020) envision a parallel between how teachers use their MKT to help students learn and how MTEs use their MKTT to aid PSTs in relearning mathematics.

In essence, the MKTT framework aligns with the foundational comprehension of teaching and learning seen in the MKT framework, given that both frameworks perceive teaching as an interactive process between educators and learners centred around specific subject matter (Jankvist et al., 2020). Nonetheless, these frameworks diverge in at least three significant aspects: *content, learners, and objectives.* Firstly, the distinction lies in the content being addressed. In the context of conventional schooling, Mathematics is a subject taught to students. Conversely, MKTT focuses on the educational content within teacher training programmes Jankvist et al. (2020).

Secondly, the frameworks focus on different categories of learners. The MKT domain pertains to teaching school mathematics, while the MKTT is tailored for teacher education mathematics. Learners diverge not only in terms of their ages but also in their existing knowledge of mathematics. Thirdly, the objectives of the frameworks differ markedly. The MKT domain aims to equip school learners with mathematical understanding for personal application. In contrast, the MKTT framework addresses the needs of pre-service and in-service teachers, equipping them to teach mathematics to others (Zopf, 2010) effectively. In agreement with Castro Superfine et al. (2020), my view is that MKTT has components of mathematics and components of teaching mathematics, both of which need explicit study and explicit attention.

Masingila et al. (2018) investigated the development of MKTT among novice teacher educators. After analysing their teaching of PSTs via problem-solving, Masingila and colleagues argue that these novice MTEs developed MKTT through three parts of their work. First, determining mathematical learning goals prompted their development of MKTT. Second, their developing knowledge was stimulated by selecting and using tasks for problem-solving. Third, the researchers discovered that asking questions to facilitate PSTs' learning from problem-solving required a deep and connected understanding on their part as teacher educators.

Castro Superfine and Pitvorec's recent work (2021) has provided insights into developing and articulating a pedagogy for mathematics teacher education. Their study involved creating and analysing lesson plans, focusing on "problems of mathematics teacher education practice." A novice mathematics teacher educator (MTE) developed their knowledge of practice, supported by more experienced colleagues. The aim was not to produce detailed lesson plans for every topic or create *blueprints* for practice but rather to potentially explain components of a knowledge base related to the content of lesson plans through collaborative analysis of

problems of practice. While this process was immediately helpful for the novice MTE, it may also benefit other teacher educators by illustrating approaches they could use to develop their pedagogical content knowledge (PCK) and subject matter knowledge (SMK). This current study contributes to the limited research on teacher educator professional growth and professional knowledge by providing a detailed example of subject-specific pedagogy for teacher education.

Venkat and Askew (2021) focused on subject matter knowledge in a study aimed at building the capacity and confidence of teacher educators (referred to as Math Subject Advisors or MSAs) to support primary school teachers in teaching multiplicative reasoning. The purpose was to build the MSAs' research-informed mathematical knowledge and understanding. The outcomes showed professional growth for the MSAs regarding their subject matter knowledge (SMK) for multiplicative reasoning and their pedagogical content knowledge (PCK) of classroom teaching of this topic. They developed "more skilled pedagogic content knowledge" and "mathematically attuned noticing" through working with teachers in the professional development programme. Additionally, they developed their knowledge of teaching adult learners.

These studies highlight ongoing efforts to understand and develop the specialised knowledge base required for effective mathematics teacher education, including subject matter knowledge, pedagogical content knowledge, and knowledge of practice. They emphasise the importance of collaborative learning, identity relationships with mathematics, and engaging with problems of practice in shaping the professional growth trajectories of MTEs.

In the next section, I focus on issues surrounding the definition and understanding of practice(s) and point out different ways the term has been conceptualised. Next, I turn to the scholarly work surrounding the definition and classification of core or high-leverage practices in mathematics teacher education and link this work with the collaborative practices of MTEs.

3.9 Knowledge-practice relationship in mathematics teacher education

The relationship between knowledge and practice in pre-service mathematics teacher education can be understood from three perspectives: knowledge for practice, knowledge in practice, and knowledge of practice (Cochran-Smith & Lytle, 1999). Knowledge-for-practice refers to the theoretical knowledge derived from academic research and university programmes and includes content knowledge, pedagogical methods, educational theories, teaching strategies, human development concepts, and learning theories (Cochran-Smith & Lytle, 1999).

In contrast, knowledge-in-practice results from the experiential learning MTEs acquire through classroom teaching experiences. This practical knowledge is developed through inquiries, sharing best practices, and reflecting on their own teaching methods. It becomes embedded in how they make decisions, take actions, and exercise judgment based on the unique contexts of their classrooms and school environments (Cochran-Smith & Lytle, 1999; Ozmantar & Agac, 2023).

The third perspective, knowledge-of-practice, involves critically examining externally generated knowledge-for-practice. In this mode, MTEs actively generate knowledge by investigating their teaching contexts and seeking to relate their practices to broader educational issues for a wider audience (Cochran-Smith & Lytle, 1999). By doing this, they aim to bridge the gap between externally sourced knowledge and the practical insights arising from personal experiences in order to develop a more holistic understanding of effective teaching.

Regarding teacher educators, Goodwin et al. (2014) suggest that the same three perspectives can be employed. For example. knowledge-of-practice corresponds to the research endeavours undertaken by teacher educators as active academic community members. While Cochran-Smith and Lytle (1999) originally focused on teacher learning, Goodwin et al. (2014) highlight the value of their three perspectives for linking teacher educators' learning processes with the types of knowledge they acquire.

3.10 Conceptualising practice in mathematics teacher education

Charalambous and Delaney (2019) noted that the term "practice" is rarely explicitly defined. Lampert (2001) provides a comprehensive overview of what practice entails and how its complexity can be managed by conceptualising practice as 1) that which contrasts with theory: "what people do rather than what they think or know" (p. 24). It is the process of actively implementing an idea as opposed to the process of conceiving an idea; 2) doing something repeatedly by way of study to improve its performance in the form of a rehearsal; 3) "the carrying on or exercise of a profession" (p.29); and 4) typically used in the plural sense of instructional practices, routines and specific tasks educators employ "constantly and habitually" (p. 25), in their line of work.

In mathematics teacher education, there has been work on core or high-leverage practices of MTEs, which are considered important for assisting beginning teachers to conceptualise their practice. In this section, I draw on the fourth conception of practice outlined by Lampert (2010) and define practice as the regular and habitual classroom routines engaged in by a community

of MTEs. I refer to these known practices as "core practices". In the following sections, I explain how practice has been conceptualised within mathematics teacher education.

Over time, two different lines of research on practice have developed concerning mathematics teacher education (Charalambos et al., 2019; Snider, 2016). The first focuses on identifying and codifying teaching practices in general (Charalambous & Delaney, 2019), particularly core or high-leverage practices of teacher educators, which influence student learning (Grossman et al., 2018). The second focuses on identifying practice-based pedagogies in pre-service teacher education (Lampert et al., 2013; McDonald et al., 2013) that could constitute teacher preparation. Examples of this research focus are understanding the work involved in equitable Mathematics teaching or inquiry-based learning (Snider, 2016).

Regarding the first line of research, scholars have selected from different possible sets of practices what they consider most important for beginning teachers as they enter their classrooms (e.g., Shaughnessy & Boerst, 2018; Wager, 2012). However, these researchers have conceptualised the practice of teaching in different ways. Several studies refer to best practice (e.g., Eddy et al., 2015; Silver, 2010). For Lampert (2010), teaching is more than acquiring skills or best practices. It is what educators do in common rather than what a particular educator does in isolation. Hence, it includes people who have adopted a teacher's identity and have been acknowledged as such, as well as those who have adopted shared principles, values, language, and teaching resources (Charalambous & Delaney, 2019). In this conceptualisation, teaching becomes the coordinated activities of individuals and groups in doing their real work (Cook & Brown, 1999), informed by a particular organisational context. For example, Grossman (2018) focused on practices that frequently occurred in teaching, enabled important teacher education work, were aimed at helping all pre-service teachers learn, and could be used within various instructional approaches.

The second line of research on practice has focused on practice-based pedagogy in teacher education, emphasising the importance of teacher educators engaging pre-service teachers in practical and hands-on experience during their studies (Hauser & Kavanagh, 2019). In this study, I use practice-based pedagogy in teacher education to describe a cycle of collaborative learning among MTEs for engaging in instructional activity that incorporates and connects opportunities for PSTs to experience and develop their own pedagogy for teaching in practical ways (Grossman et al., 2018; McDonald et al., 2013).

3.11 High-leverage teaching practices in mathematics teacher education

The literature often employs two synonymous terms, "core" and "high leverage," practices to describe fundamental teaching practices in mathematics teacher education (McDonald et al., 2013; Shaughnessy & Boerst, 2018; Ball & Forzani, 2009; Forzani, 2014; Sleep & Boerst, 2012; Forzani & Ball, 2016; Charalambous & Delaney, 2019). While these terms are often used interchangeably, there exists a subtle difference between them (Charalambous & Delaney, 2019). Core practices generally refer to general teaching practices, while high-leverage practices specifically focus on those mathematics teacher education practices meaningful for pre-service teachers (PSTs) to learn, with a deliberate focus on supporting them in learning to teach (Charalambous & Delaney, 2019).

High-leverage practices involve instructional tasks and activities that significantly facilitate learning and form the essential core of effective teaching (Forzani & Ball, 2016). They describe such activities as "high leverage" because they matter for PST's learning and advancement in teaching, playing a vital role in helping PSTs learn important content. The high-leverage practices that were focused on in this study include designing, modifying and selecting mathematical tasks; establishing and revising mathematical goals for PSTs; accessing and using tools and resources for teaching, explaining mathematical ideas and solving mathematical problems Huang (2012). In the context of this study, I also found building and supporting a mathematical community and discourse a fundamental practice in skilful teaching in the context of this study. Each of these is discussed below.

For Grossman, core practices are "identifiable components of teaching to enact to support learning. These include instructional strategies and subcomponents of routines and moves. The core practices include general and content-specific practices" (Grossman, 2018, p. 184). MTEs' core practices may involve asking and responding to questions or presenting the PSTs with exercises that provoke or allow them to share their mathematical thinking to evaluate their understanding, guide instructional decisions, and surface ideas that benefit other students. When engaging in these practices, the MTE may face such challenging tasks as asking "productive mathematical questions" and evaluating "the plausibility of PSTs' claims" (Ball et al., 2008, p. 400). The mathematical teaching tasks included in various core practices may vary depending on the context (teacher education or school mathematics).

3.11.1 Designing, modifying and selecting mathematical tasks

Several studies have contributed to a growing understanding of the use of tasks in mathematics teacher education (Zaslavsky, 2008; Watson & Sullivan, 2008; Zaslavsky & Sullivan, 2011; Epperson & Rhoads, 2015). Epperson and Rhoads' (2015) define a task as "a mathematical problem, prompt, or guided exploration that is posed to learners" (p. 38), while Epperson and Rhoads (2015) and Watson and Sullivan (2008) consider a task to be "a starting point of mathematical activity" (Watson & Sullivan, 2008, p. 109). In this review, I focus on tasks in which PSTs engage as adult learners in a college of education or university setting.

Five elements have emerged from the literature as important in the process of MTEs designing lessons to enhance PSTs' deep understanding of the mathematics they will teach, as opposed to their knowledge of mathematics: 1) identifying the mathematical goal of the lesson; 2) anticipating PSTs' responses and possible misconceptions; 3) identifying specific questions that will assess or advance PSTs' understanding while they work; 4) selecting and sequencing examples, and 5) specifying how to orchestrate a whole-class discussion that builds on PSTs' thinking and makes salient the mathematics of the lesson (Hughes, 2007).

Setting clear mathematical goals is a critical first step when designing lessons involving tasks for PSTs. The process involves the MTE explicitly determining the specific mathematical concepts and understandings they want PSTs to develop during the lesson. It is important to identify these goals with precision, as it guides the overall lesson plan and instructional decisions (Lampert, 2001; Fernandez & Yoshida, 2004; Leinhardt & Steele, 2005).

Rather than focusing on the procedural skills PSTs will practice or tasks they will accomplish, the goals should articulate the conceptual understandings of the mathematics PSTs should gain by the end of the lesson. Well-defined goals clarify the key mathematical ideas, relationships, representations, or ways of thinking that the PSTs need to grasp more deeply. Having this conceptual learning as the clear objective allows the MTE to thoughtfully craft examples, questions, and facilitation plans to build PSTs' understanding of the targeted mathematical concepts.

Orchestrating a meaningful whole-class discussion involves two important aspects: 1) evidence that the discussion was building on students' mathematical thinking and/or work in solving the problem, and 2) identifying specific questions that would enable PSTs to make connections between solutions or make the mathematics of the lesson salient. MTEs could plan a discussion that was built on PSTs' mathematical thinking in several ways: 1) purposefully selecting their

solutions for whole class discussion; 2) determining the sequence in which those solutions should be discussed; or 3) identifying specific questions to ask PSTs that explicitly referred to their thinking or work on the problem (Stigler & Hiebert, 1999; Lampert, 2001; Stein et al.,2001). Hughes (2007) has shown that planning for these practices improves classroom discourse.

While the general principles of task design may be similar, it is essential to recognise that designing tasks for PSTs in teacher education programmes differs from designing tasks for school mathematics in several ways. Based on the distinctions elaborated in the preceding paragraphs, the summary table highlights the key differences:

Aspect	Tasks for pre-service Teachers (PSTs)	Tasks for teaching in Schools
Goals and Purpose	Developing a deep understanding of the mathematical concepts PSTs will teach and knowledge of teaching those concepts effectively (Zaslavsky, 2008; Epperson & Rhoads, 2015).	Develop students' understanding and skills in specific mathematical topics.
Prior Knowledge	As adult learners, build upon PSTs' existing mathematical knowledge, addressing misconceptions or gaps (Zaslavsky & Sullivan, 2011).	It is tailored to students' developmental levels and prerequisite knowledge.
Focus	Emphasise mathematical content and pedagogical aspects, such as representations, examples, explanations, and anticipating student thinking (Lampert, 2001; Fernandez & Yoshida, 2004).	Primarily focus on developing students' mathematical knowledge and skills.
Classroom Discussions	Unpack the mathematics for teaching, connecting solutions and highlighting pedagogical implications (Stigler & Hiebert, 1999; Stein et al., 2001).	Centre on students' mathematical thinking and understanding of the content.
Learner Roles	Position PSTs as both learners of mathematical content and future teachers, bridging their understanding of content and pedagogy (Hughes, 2007).	Engage students primarily as learners of mathematical concepts and skills.
Professional Development	Contribute to PSTs' professional development as future mathematics teachers, preparing them for their teaching roles (Leinhardt & Steele, 2005).	Typically, not designed with a direct focus on students' future professional development.

Table 3.2: Key differences in designing mathematical tasks for PSTs vs. tasks for school mathematics students

While both types of tasks involve mathematical content and problem-solving, tasks for PSTs in teacher education programmes must explicitly address the intersection of content knowledge and pedagogical knowledge necessary for effective mathematics teaching.

3.11.2 Choosing and using representations

Representations are formed by the assembly of certain signs and symbols that obey well-set rules. Algebra, daily language, graphics, tables and computer languages are all referred to as *semiotic registers of representations*. Recent scholarship builds upon Kapu's foundational assertion that representation is the very "heart of mathematical content," revealing increasingly sophisticated ways we construct and communicate mathematical ideas (Kaput, 1987). Contemporary researchers like Duval (2006) and Goldin (2008) have expanded this

perspective, arguing that representations are not just passive containers of mathematical meaning, but active cognitive tools that shape mathematical understanding itself. In the NCTM (2000, p. 67) standards, "the term 'representation' is used to refer both to process and product… to the act of capturing a mathematical concept or relationship in some form and to the form itself."

Emerging research provides evidence on the role of representation in school mathematics teaching showing that students benefit in various ways from multiple representations. When students are provided with opportunities to transfer between different representations or to discuss multiple representations, they make connections between multiple representations (Ainsworth et al., 2002; Brenner, et al, 1997). Similarly, PSTs can improve their mathematical abilities, problem-solving, and reasoning skills when MTEs use multiple representations (Greeno & Hall, 1997).

For mathematics teacher educators, these insights demand a reimagining of pedagogical approaches. Representation is no longer a peripheral skill but a core competency. Research by Bolden et al. (2013) demonstrates that pre-service teachers who develop sophisticated representational thinking become more effective mathematics educators, capable of supporting students' deeper mathematical understanding.

The quest to understand mathematical representations is far from complete. Each new study reveals additional layers of complexity, how representations interact, how they emerge in different cultural contexts, and how digital technologies are transforming representational possibilities. What remains clear is that representations are not mere tools for communicating mathematical ideas, but the very fabric through which mathematical thinking occurs.

3.11.3 Explaining mathematical ideas and solving mathematical problems

Leinhardt et al. (1991) defined instructional explanation as the way in which teachers communicate subject matter to students. This process goes beyond mere explanations to include a deliberate arrangement of experiences aimed at helping students construct a meaningful grasp of concepts and procedures. Leinhardt elaborated on this definition in her later work (2001), describing instructional explanations as educators' expressions, demonstrations, or structuring of experiences. These explanations are designed to effectively convey specific portions of the subject matter to students, focusing on enhancing their understanding of the content. Leinhardt outlined key criteria for explanations, requiring them to be clear, relevant to students' comprehension, and respectful of the inherent nature of the

subject's knowledge. Providing instructional explanations involves the crucial task of transforming mathematical content in academically valid and pedagogically sound ways, making it a fundamental aspect of teaching.

Leinhardt (2010) emphasises that to improve learning, instructional explanations in mathematics must effectively bridge informal, colloquial language with formal disciplinary discourse. This approach requires teachers to coordinate familiar forms of expression with more rigorous mathematical terminology. Crucially, teachers must make implicit assumptions explicit, ensuring that foundational concepts are clear to all learners. They must also provide justifications for connections between mathematical ideas, helping students understand the logical relationships within the subject. Additionally, different representations of mathematical concepts should be explicitly mapped, and their interrelations explained.

Instructional explanations are a network of goals, actions, and knowledge assumptions. At the heart of the goal system of explanations is the posing of a powerful and important question. In an effective instructional explanation, the problem must be carefully unpacked and examined, not just stated. This means identifying problematic features in the query and summoning effective and important examples. The knowledge system that supports framing a significant problem combines deep disciplinary knowledge and solid pedagogical understanding (Chapman, 2013).

Crucial to an instructional explanation are the interconnections within and the completion of a discussion. Connections need to be made to prior knowledge and examples, showing the principles of use and limits of use of the concept or procedure and displaying how errors are connected to the misapplication of principles or actions. Explanation must be grounded in the teacher education setting, as it is important for PSTs to see the explicit connection between the mathematics they learn and what they will teach (Epperson & Rhoads, 2015). From a teaching perspective, this means keeping track of all the little questions and examples that have emerged during a discussion and pulling them back into the activity of explaining.

Another important goal in an instructional explanation is the systematic and careful development of examples and representations. These are the analogical instructional tools. The tools can be developed by the MTEs or with the PSTs. However, the examples and representations (pictures, formalisms and diagrams) must be appropriate to the situation, and they must be unpacked and carefully mapped or flagged to the salient aspects of the answer to

the query. The orchestration of representations and examples to serve an instructional goal is challenging and complex.

In studying the practice of teaching, Leinhardt (2010) argues that many educators design instructional settings that meet some of these goals, but in many cases, some critical goals – such as establishing a central query that the explanation is in the service of – are left out. The new teacher, for example, may over-focus on the activity (the worked example or representation) and forget to have the students grapple with the central question or assume that the example is the explanation or that a representation is self-evident.

3.11.4 Eliciting, interpreting and developing PSTs' thinking

One of the crucial roles of MTEs is to understand and enhance the thinking processes of PSTs. MTEs can employ various strategies, such as posing thought-provoking questions and presenting thought-provoking tasks. Unlike teaching school mathematics, teaching algebra in teacher education requires the MTEs to elucidate the underlying reason behind mathematical concepts. For this reason, developing PSTs' thinking also involves MTEs in analysing and responding to PSTs' errors and comments while managing productive discussions. Research has emphasised the importance of teacher educators recognising and addressing misconceptions in mathematics, especially among those PSTs who lack proficiency in basic mathematical knowledge and skills (Schnepper, 2011). Remediating these misconceptions can lead to rapid new learning and long-term retention (Sayce, 2009; Schnepper, 2011). The analysis of errors and misconceptions serves multiple purposes, such as identifying correct steps taken by students, categorising the errors made (Brown & Skow, 2016) and determining if an error is a one-time mistake or a persistent misunderstanding of a mathematical concept or procedure. Furthermore, this analysis aids MTEs in selecting appropriate instructional approaches to rectify misconceptions and support understanding of correct concepts, strategies, or procedures by PSTs. To develop the ability to elicit PSTs' effective thinking, MTEs must make informed decisions before, during, and after the lesson (Pothen, 2011). Before the lesson, MTEs should anticipate potential errors in PSTs' thinking and plan accordingly. During the lesson, they need to decide how to guide PSTs' thinking in a way that promotes deeper understanding, and after the lesson, they need to determine whether and how to address any remaining confusion.

3.11.5 Articulating ideas through examples and PSTs' solution processes

Carpenter and Lehrer (1999) define articulation as conveying knowledge verbally or through alternative means such as diagrams or models. This process requires reflection, where critical elements of an activity are extracted to communicate its essence. Thus, when teachers articulate ideas from examples, they engage in a reflective process, identifying and describing essential components of the material. Carpenter and Lehrer posit that articulation serves as a public form of reflection.

Johnston-Wilder and Lee (2008) argue that articulating mathematical ideas significantly builds mathematical resilience while learning. Mathematical resilience is characterised as positive adaptation in adversity (Newman, 2004). Articulation of ideas is likely to enhance students' confidence in their learning and competence in using mathematical concepts, ultimately increasing their mathematical resilience (Johnston-Wilder & Lee, 2008). While articulation is not classified as one of the high-leverage practices by Ball and colleagues, teachers need to recognise its fundamental role in students' learning. Promoting their articulation of ideas fosters PSTs' ability to be lifelong users of mathematical skills, thinking, and reasoning, enhances their readiness to face the challenges inherent in understanding mathematics and empowers them to become proficient mathematical educators.

3.12 Conceptualising professional learning as a means of informing MTEs' practice and professional knowledge

The concept of professional learning for educators has been variously interpreted within scholarly discussions. Some scholars use the term synonymously with continuous professional development (CPD) (Borko, 2004; Cochran-Smith & Lytle, 1999; Wilson & Berne, 1999), while others differentiate between the two terms (Corcoran, 1995; Fraser et al., 2007; Webster-Wright, 2009).

Cochran-Smith and Lytle (1999) argue for a symbiotic relationship between educator professional learning and CPD, highlighting how both contribute to educators' acquisition of professional knowledge and enhance teaching practices. Similarly, Cole (2012) characterises professional learning as comprising both formal and informal learning endeavours pursued by educators and school administrators to refine their professional knowledge and skills, ultimately benefiting students' engagement and learning outcomes.

In contrast, Fraser et al. (2007) clearly distinguish between professional development and professional learning. They define professional learning as specific efforts aimed at individual and group learning that result in changes in educators' professional knowledge, skills, attitudes,

beliefs, or actions. On the other hand, professional development refers to broader changes within the teaching profession over a longer period, leading to qualitative shifts in professionalism.

To prevent potential conflict between the usage of CPD and professional learning, Webster-Wright (2009) introduces the term "Continuing Professional Learning" (CPL), which defines the learning undertaken by practising professionals. In the context of this study, CPL encompasses all forms of learning arising from CPD, Professional Development (PD), Continuing Education (CE), or any other relevant activity (Webster-Wright, 2009). Common characteristics are shared by CPD, professional learning and CPL, emphasising a commitment to ongoing learning for educators' growth. Educators acquire knowledge from diverse sources, including structured workshops, seminars, and collaborative interactions, whether formal or informal, that are embedded in their daily practices.

3.13 Professional learning needs of mathematics teacher educators

Teacher educators are responsible for PSTs to be effective teachers of their subject(s). Literature suggests that MTEs often transition from classroom teaching to teacher education without formal education in pedagogy, curriculum design, and instructional strategies (Even et al., 2020; Goos & Beswick, 2021). This lack of formal preparation highlights the need for targeted professional learning to enhance teacher educators' practice.

Within mathematics education, MTEs assume roles as faculty members within education and mathematics departments at both undergraduate and graduate levels (NCTM, 2020). Their core tasks revolve around educating pre-service and in-service teachers, extending their influence across diverse institutions such as colleges of education, universities, and alternative teacher certification programmes. This extensive range of roles undertaken by MTEs (Coles & Helliwell, 2023) makes it challenging to precisely delineate their required professional capabilities and the knowledge imperative for their work (Ping et al. 2020).

Nevertheless, research findings indicate that teacher educators generally display a variety of professional learning needs (Czerniawski et al., 2016; Hadar & Brody, 2017) stemming from their diverse backgrounds and experiences. Those transitioning from senior high school teaching backgrounds may need assistance comprehending adult learning principles and the unique instructional design associated with teacher education (Czerniawski et al., 2016). Conversely, lecturers in Mathematics may seek guidance in translating theoretical knowledge into pragmatic teaching strategies.

Tamir (1991) contended that the expertise teacher educators draw upon to educate pre-service and in-service teachers differs distinctly from that of classroom teachers. Recent investigations (Beswick & Goos, 2018; Castro Superfine & Li, 2014; Zaslavsky, Chapman & Leikin, 2003; Rider & Lynch-Davis, 2006) support this contention by demonstrating that the knowledge of MTEs varies somewhat from that of mathematics teachers.

As a result, scholarship in this domain proposes diverse strategies for addressing the professional learning needs of MTEs. For instance, Olanoff, Prasad and Welder (2020) reviewed several avenues for the professional growth of MTEs. These avenues include learning through graduate preparation and/or professional development activities, engaging in communities of inquiry, conducting reflective self-study, undertaking research, and learning through practical teaching experience. Each of these strategies offers valuable opportunities for MTEs to enhance their professional knowledge and skills, ultimately contributing to their success in educating future teachers.

Notably, a handful of institutions globally, such as the MOFET Institute in Israel which is a national intercollegiate centre for the research and development of curricula and programs in teacher education and teaching in the colleges, and the Norwegian National Research School in Teacher Education (NAFOL), offer formal models of MTE professional development (Smith, 2020). MOFET aims to augment teacher educators' professional development and enrich their subject-specific knowledge, while NAFOL concentrates on enhancing Norwegian teacher educators' research competencies. However, many countries lack formal professional development programmes for MTEs, as highlighted by exploratory international studies (Van der Klink et al., 2016) that emphasise the need for such initiatives.

While there is limited literature describing formal professional development opportunities for MTEs, some studies indicate that MTEs have pursued graduate-level courses (Flores et al., 2017) or engaged in other structured professional development activities (Hauk et al., 2017; Castro Superfine & Li, 2014). Flores and colleagues (2017) suggest that undertaking graduate courses or participating in formal professional development can offer significant opportunities for MTEs to reflect on and expand their own knowledge base.

Meanwhile, in the absence of formal institutional, professional development opportunities, communities of inquiry (COIs) or PLCs have proved effective for MTEs' professional learning and development (Robutti et al., 2016; Jaworski et al., 2017), particularly within mathematics teacher education. Within these communities, MTEs collaborate with shared goals and

interests, collectively pursuing learning. A case in point is the study by Hadar and Brody (2010), where eight teacher educators participated in a year-long professional learning community to implement 'thinking education' into their practices. This three-phase process entailed exploring theories and materials, implementing them in lessons, and reflecting collaboratively on the outcomes. Such collaborative interactions facilitate the exchange of experiences and ideas and cultivate a collective comprehension of effective practices (Ping et al., 2020). Some groups of MTEs (e.g., Appelgate et al., 2020; Olanoff et al., 2021) have formed communities of practice to improve the professional knowledge and practices of all group members and initiate novice MTEs in the field (Castro Superfine & Pitvorec, 2021).

According to Ping et al. (2018), studies in teacher education indirectly point to sources from which teacher educators derive their professional learning. For Zaslavsky and Leikin (2004), MTEs develop professional knowledge in two ways: ''through learning, as facilitated by an MTE or through teaching, when they facilitate pre-service teachers' learning'' (p. 9). Thus, looking at the use and development of MKTT requires examining both. Zopf (2010) and others, such as Jankvist et al. (2020), suggest that MTEs develop professional knowledge on the job by teaching teachers. This learning typically occurs informally within the workplace (van Velzen et al., 2010), further deepening teacher educators' understanding. In teacher education, engagement in the educational project and collaborative reflection within their community of practice have been found to aid in developing mathematical knowledge for teacher education (Masingila et al., 2018).

The professional learning needs of MTEs stem from the unique nature of their roles and responsibilities. Unlike classroom teachers who primarily focus on student learning, MTEs must navigate the complexities of adult learning and effectively translate theoretical knowledge into practical teaching strategies for their pre-service and in-service teacher students (Czerniawski et al., 2016; Hadar & Brody, 2017). Additionally, MTEs are expected to stay abreast of current research, trends, and pedagogical approaches within mathematics and teacher education (Ping et al., 2021) and, in many institutions, to become research active themselves.

I argue that addressing these professional learning needs through targeted and ongoing professional development opportunities is crucial for enhancing the capability of MTEs. Suppose they are provided with support for engagement with adult learning principles, instructional design for teacher education, and translating research into practice. In that case,

MTEs can better equip themselves to nurture the next generation of highly skilled and knowledgeable mathematics teachers.

3.14 The role of collaboration in MTEs' professional learning communities

The coexistence of mathematicians and mathematics teacher educators in teacher education presents pre-service mathematics teachers with a rich yet sometimes conflicting array of mathematical approaches and pedagogies (Marshman, 2021; Marshman & Goos, 2018). Acknowledging this, Bleiler (2015) underscores the importance of collaboration among mathematics teacher educators (MTEs) to prepare pre-service teachers effectively. Research findings suggest that such collaborations significantly enhance the professional practices and knowledge of MTEs, thereby enhancing the quality of teacher education programmes (Goos & Bennison, 2018).

While collaborative learning within PLCs is gaining traction in mathematics teacher education (Jaworski et al., 2017; Lloyd & Chapman, 2019), it remains an emerging field. It aligns with contemporary socio-constructivist perspectives, built on the work of Vygotsky almost a century ago (Newman & Latifi, 2021). However, the term "collaboration" is often misunderstood, with its complexity highlighted by Robutti et al. (2016), who differentiate it from "cooperation." Collaboration entails working jointly, while cooperation involves individual contributions to various aspects of a task (Robutti et al., 2016). These terms are intertwined in describing collective efforts, as Forte and Flores (2014) noted, who argue that true collaboration involves activities with lasting implications for multiple parties, surpassing mere superficial aid and exchange.

Hargreaves (1994) identifies two distinct cultures of collaboration in educational settings: "collaborative cultures" and "a culture of contrived collegiality." He characterises collaborative cultures as having spontaneous, voluntary, and development-oriented working relationships among teachers. These relationships are pervasive across time and space and tend to be unpredictable in nature. This type of culture fosters organic, teacher-driven collaboration that emerges naturally from the professional community.

Hargreaves contrasts the culture of contrived collegiality with more organic forms of collaboration. He describes contrived collegiality as structured and administratively regulated, involving mandatory participation, an implementation-focused approach, and collaboration scheduled at specific times and locations. This culture tends to be more predictable and controlled. Wang (2015) presents an intriguing perspective on the relationship between these

two cultures, suggesting that contrived collegiality can act as a catalyst for fostering more authentic collaborative cultures. He argues that genuine collegiality can emerge from intentionally organized structures that promote disciplined collaboration, shared responsibility, and ongoing professional learning (Wang, 2015).

In the Ghanaian context, this interplay between contrived collegiality and collaborative culture relates to the transformation of regulated MTE collaboration into self-driven collaborative learning spaces, as evident in the learning community that is the focus of this study. The interconnection between teacher knowledge and collaboration becomes apparent as collaborative activities can potentially enhance professional knowledge (Chauraya & Brodie, 2018).

PLCs have emerged as essential forums for MTEs to engage in sustained learning cycles and draw insights from their teaching experiences (Brodie & Chimhande, 2020; Hairon et al., 2017). Effective PLCs require shared goals, interdependence, congruence with practice, allocated time, leadership, and trust (Olivier et al., 2010; Prenger et al., 2017). However, interdependence requires self and collective examination of practice and discussions on practice improvement (Stoll et al., 2006; Murdaugh, 2017).

For Wenger (1998), belonging to a PLC involves engagement, imagination, and alignment with the goals and norms of the PLC. Learning occurs in the PLC as a process of social participation. Researchers have observed that some educators adopt an inquiry stance within PLCs, challenging the status quo and questioning accepted practices (Goodchild et al., 2013). Inquiry as a stance involves deliberately investigating teaching, schooling, and teacher education (Cochran-Smith, 2003) and the subject matter, in the case of this study, mathematics. It involves generating local knowledge, theorising practice, and critically engaging with theory and research (Cochran-Smith & Lytle, 1999).

Adopting an inquiry stance means inquiring about a practice norm within a PLC (Goodchild et al., 2013). Cochran-Smith (2003) suggests that teacher educators should examine teaching, schooling, and teacher education through an inquiry lens. Teacher educators generate local knowledge in an inquiry community, envision and theorise their practice, and critically interpret theory and research (Cochran-Smith & Lytle, 1999). They treat their work as a site for intentional investigation, using the knowledge and theory produced by others as generative material for reflection and interpretation (Cochran-Smith, 2003).

This approach to inquiry is crucial for examining both one's own work and the work of others. This study used an inquiry stance to explore MTEs' professional practice in teaching algebra in pre-service teacher education. This inquiry stance involved identifying the knowledge considered important for their work and understanding how they use it to support instruction.

3.15 Critical friendships as a collaborative practice within MTEs PLC

While the term "critical friends" may not be widely used in mathematics education research (Rojas et al., 2022), the idea of collaborative, self-study and reflective practices in mathematics teacher education can be found in various studies and initiatives across mathematics teacher education (e.g., Jaworski, 2006; Joubert & Sutherland, 2010; Robutti et al., 2016; Schuck & Brandenburg, 2020). A critical friend is a dependable person who poses challenging questions, provides classroom or practice-based information to be analysed from a different perspective, and criticises a colleague's work out of friendship built on the foundations of trust and openness (Farrell, 2001; Fern Thorgersen, 2014). Instead of using critical friendship for assessment and evaluation, feedback is primarily intended to assist and promote a colleague's work and professional learning (Costa & Kallick, 1993). The work of a critical friend entails knowledge and understanding of the context of a colleague's work and the expected outcome their colleague is working towards (Ragoonaden & Bullock, 2016; Carlson, 2019).

According to Nilsson et al., (2018) combining the two terms "critical" and "friend" can be tricky, as these terms are often perceived as conflicting. Gibbs and Angelides (2008) propose the distinctive use of the terms 'friend', 'acquaintance' and 'companion'. They argue that "A participant is a 'critical friend' when the actual engagement for critique is situated within the acceptance of the duties and obligation of 'perfect' friendship as a moral agency, seeking a just and fair engagement" (p. 221-222). They suggest using the term "critical acquaintance" in the context of explicitly short-term useful and friendly engagements in which there is no development and maintenance of trust. Gibbs and Angelides (2008) describe critical companionship as a friendship based on gains. Although it still has the appearance of friendship, both parties acknowledge that it is a utilitarian friendship. The foundation of critical friendship is a relationship in which the parties mutually engage and critique their practice (Carlson, 2019)

For Baskerville and Goldblatt (2009), adopting criticism protocols can help enhance critical friendships. Critical friends can challenge each other's practice in a secure setting thanks to protocols that outline participants' roles and obligations (Blake & Gibson, 2021). All

participants need to understand that they are engaged in a joint, non-hierarchical process and that critical friendship is a long-term relationship based on trust (Coghlan & Brydon-Miller, 2014). Such friendships enable the giving and receiving of professional advice (Nilsson et al., 2018).

Nilsson and colleagues emphasise that critical friends must learn how to work to foster trust while being critical. Regardless of the level of trust, agreement on boundaries should be reached in the early stages when both parties sit down and have an initial conversation about the whole idea of enlisting the support of a critical friend in practice. Stolle et al. (2019, p. 25) have outlined questions to guide collaboration by critical friends.

Table 3.3: Critical friends guide for quality assurance (Stolle et al., 2019, p. 25) adapted.

Start	Why would I have critical friends?
	What is the purpose of the critical friendships?
	What do I hope to gain?
Throughout	What do critical friends do?
	What should the critical friends reflect on?
End	How did critical friendship inform practice?
	Did the critical friends offer an alternative perspective, lead you to new insights, or help to reframe your thinking?

Critical friendships have been conceptualised in different learning contexts, such as school management (e.g., Swaffield, 2004; Baskerville & Goldblatt, 2009) and professional learning communities focused on action research (e.g., Wennergren, 2016). Critical friendships in professional learning communities must be negotiated by members who have similar needs, difficulties, and concerns rather than being 'forced' by outsiders.

According to Swaffield (2004), critical friends do not always need to be experts in the field of work that the friendship focuses on. She contends that it is feasible for a critical friend to pose legitimately intriguing yet potentially naive questions. While critical friends may not necessarily need a high degree of technical competence, Swaffield (2004) notes that they need to be aware of the context so that their inquiries prompt thinking and do not irritate coworkers. By contrast, Stoll et al. (2006) argue that critical friends should have at least the same level of knowledge as their colleagues to suggest improvements to a problem, initiate meaningful conversations, and give constructive feedback. Ainscow and Southworth (1996) characterise

the critical friend as a fresh set of eyes that may bring new viewpoints to practice and help it be seen from a different angle. In the ongoing debates about critical friendships, Loughran and Brubaker (2015) argue that critical friends can assist teacher educators in looking beyond their own worldviews and broadening their perspectives on the practical situation meaningfully. As a result, the idea of a "critical friendship" can spur significant change by encouraging mathematics teacher educators to engage in reflective practice. According to Schunk and Russell (2005), a critical friend can serve as an investigative sounding board for other educators as they work to improve their teaching skills via reflection.

3.16 Critical friendships: complexity, tensions and limitations

Critical friends may perform varied roles in different contexts and collaborative situations (Blake & Gibson, 2021). Enacting these roles includes being accountable to one another and providing the following: emotional support, diverse perspectives, motivation, and constructive feedback based on honest and well-intentioned critiques that highlight strengths and limitations in each other's work (Noor & Shafee, 2021). More importantly, knowing someone is invested in their success can motivate individuals to push their boundaries and strive for excellence in their endeavours. However, like any relationship, critical friendships are likely to be characterised by complexity, tensions, and drawbacks (Richards & Shiver, 2020; Stolle et al., 2019). One of the key challenges is managing disagreements and tensions that may arise within the relationship of critical friends. According to William Rawlins (1992), critical friendships require individuals to engage in open and honest discussions about their differing opinions or perspectives. This can be difficult as it requires vulnerability and the willingness to have uncomfortable conversations. As Rawlins states, True friends must face the music together, even if it's discordant (Rawlins, 1992, p. 152).

According to Wennergren (2016), one potential limitation of critical friendship is the inability to maximise the learning potential of such friendships. Another is power dynamics (Richards & Shiver, 2020), which can emerge within critical friendships, especially if one person consistently provides critique while the other primarily receives it. A power imbalance can undermine the equality and reciprocity that should ideally exist between critical friends. Both individuals need to have specific roles and opportunities to offer feedback and engage in reflective dialogue (Baskerville & Goldblatt, 2009). For Chang et al. (2016), the power dynamics in critical friendship can be diffused through strong collaboration.

Managing expectations can be a challenge in critical friendships. It is common for individuals to have different expectations regarding the level of involvement or commitment required from each other in these relationships. Felmlee et al. (1990) point out in their study on friendship patterns that unfulfilled expectations can lead to setbacks and strain critical friendships. Clear communication and ongoing dialogue about expectations are necessary to avoid misunderstandings and potential conflicts. It is important to approach such friendships with open-mindedness, a commitment to self-reflection, and a willingness to engage in honest and constructive dialogue (Richard & Shiver, 2020).

In conclusion, critical friendships have great potential for contributing to professional growth but also come with their fair share of challenges. Individuals must navigate disagreements, maintain a balance between constructive criticism and supportiveness, build and maintain trust, and manage expectations. To manage all this, I used the critical friends' groups (CFGs) as a safe space for critical friends to openly discuss their experiences, challenges, expectations, and communication needs. I maintain that individuals can cultivate meaningful and transformative relationships that foster professional development by acknowledging these challenges and employing effective communication strategies.

3.17 Situating critical friendships in a range of educational contexts

The literature review suggests that critical friendships have been explored in various educational contexts, primarily aimed at promoting reflection, collaborative support, and professional development among educators. Several studies have investigated critical friendships in preservice teacher education to support the learning and growth of preservice teachers (PSTs) through collaborative reflection and peer feedback (Smith, 2019; Carlson, 2019; Bognar & Krumes, 2017). Other researchers have examined critical friendships within the context of professional learning communities (PLCs) and how they can facilitate educators' engagement in systematic inquiry, pedagogical innovation, and collegial interactions (Kintz et al., 2015; Wennergren, 2016; Fletcher et al., 2016; Males et al., 2010).

While these studies highlight the potential benefits of critical friendships in various educational settings, there appears to be a gap in the literature regarding the specific application and investigation of critical friendships in the context of mathematics teacher educators. The existing research does not explicitly address how critical friendships can support the professional learning and development of mathematics teacher educators or their ability to facilitate the learning and growth of preservice mathematics teachers. This study addresses this

gap by exploring the role and impact of critical friendships in the context of a PLC for MTEs. It investigates how critical friendships among MTEs can foster collaborative reflection, pedagogical innovation, and professional learning specific to the discipline of mathematics and the preparation of future mathematics teachers.

3.18 Critical friendships in mathematics teacher education

Within mathematics teacher education research, the focus on PLCs has been particularly prominent in recent years, and the importance of collaboration and collegiality in promoting effective classroom practice has been recognised (Jaworski, 2020). According to Kintz et al. (2015), the justification for using PLCs in teacher education is based on two premises. 1). Knowledge resides in teacher educators' experiences in practice and is understood by collaborative reflection. 2). Knowledge growth and improved student learning outcomes are linked to active engagement in the learning process. Males et al. (2010) claim that educators can offer fresh perspectives and innovative teaching techniques by closely examining their own teaching. Collaboration can help MTEs gain a sense of professional sovereignty and take charge of their work with collegiality facilitated by PLCs and critical friendships.

In one of the few examples of research on critical friendships with a group of MTEs, Kastberg and Grant (2020) identified two key transformative traits of critical friends that contributed to recognising themselves as MTEs: 1) significant otherness and 2) conversational residue. Significant otherness trait involves positioning a critical friend so that their expectations or perspectives influence the positioner. In the context of professional identity transformation, significant otherness occurs when one individual seeks to be influenced by the other in relation to their professional thinking, teaching practices, or other actions. It signifies a shift in perspective where the critical friend's insights and feedback lead to explicit professional changes in the individual being influenced.

On the other hand, conversational residue involves taking up an idea or insight from a critical friend's conversation to address a professional dilemma or challenge. It refers to the lingering impact of conversations that evolve in internal dialogue and practice. Conversational residue can serve as a catalyst for reflection, learning, and growth as individuals reflect on past discussions and apply insights gained to navigate professional challenges. By recognising and embracing these transformative traits within their critical friendship, MTEs can navigate challenges, provoke critical reflection, and ultimately transform their professional identities as mathematics teacher educators (Kastberg & Grant,2020).

Similarly, Rojas eta al. (2022) report on a self-study by two MTEs which focused on their critical friendship sessions in preparation for and delivery of a teaching sequence by one of them. They collected data by recording three critical friendship meetings at the end of implementing a sequence of lessons related to quadratic equations. Rojas and colleagues discuss the different types of questions raised by these critical friends that provide opportunities for considering different perspectives (Schuck & Brandenburg, 2020) and that focus on understanding the MTEs' pedagogical reasoning and teaching practice (Rojas et al., 2022). The findings also demonstrated awareness of the personal resources that each MTE provides during a conversation that can be related to each one's teaching experiences and thus help improve their teaching practice. Additionally, Rojas et al. (2022, p.7) argue that "critical friendship is a tool that causes the teacher educators involved to evolve", using their study as an example in which critical friendship evolved from dynamic-based questioning to one in which MTEs shared experiences that allowed them to engage in critical reflection. Given the limited research focus on the role of critical friendships in the professional learning of MTEs, I seek to explore how the support of critical friends may inform MTE's pre-service mathematics teacher education practice. To this end, I conclude this chapter with a short section on protocols for critical friendships.

3.19 Some common protocols of critical friendships

3.19.1 Establishing trust in critical friend relationships

Trust is the overarching principle underpinning critical friend relationships (Farrell, 2001; Swaffield, 2004). It is necessary for creating a safe and supportive environment where CFs can engage in open dialogue, share vulnerabilities, and provide constructive feedback, without which CFs will not thrive in a learning community (Olivier et al., 2010). While the role combines unconditional support with constructive critique (Macbeath & Jardine, 1998), it can be uncomfortable for some individuals (Schuck & Russell, 2004). Therefore, it is essential to be sensitive about the nature of the relationship and establish trust before starting the CF (Farrell, 2001).

Studies have shown that CFs need time to develop deep trust in each other (e.g., Baskerville & Goldblatt, 2009; Golby & Appleby, 1995) and to be in a supportive environment (Farrell, 2001) where the critical friend will "listen well, clarify ideas, encourage specificity in teaching practices, advocate for the success of the work, and not use the relationship for judging" (Costa & Kallick, 1993, p. 50). Providing constructive feedback is a critical aspect of trust-building in critical friendships. Feedback should be given to support growth and improvement (Schuck &

Russell, 2005). Critical friends should strive to balance acknowledging strengths and offering suggestions for improvement. Framing feedback in a non-threatening and respectful manner should enable it to be received and given with an open mind and a willingness to learn (Wennergren, 2014). Collaboration and shared ownership of professional learning contribute to building trust within critical friend relationships (Nilsson et al., 2018). CFs should actively engage in collaborative decision-making processes, establish shared goals, and take collective responsibility for the pair or group's success. This mutual ownership contributes to an inclusive and participatory environment where everyone's input is valued and considered, fostering a sense of trust and commitment among the members

3.19.2 Maintaining a reflective dialogue

According to Berry and Russell (2014), when educators make their work available to others and work as critical friends, it improves the quality of their practice so that it is not limited to their own viewpoint, judgment, or opinion. A reflective dialogue is an interaction in which two or more colleagues reflect together and thoroughly engage each other based on experience, data, and literature (Martin, 2016).

Experience is considered the most important starting point for reflection (Korthagen & Vasalos, 2005). A dilemma (experience) within a concrete situation is often the starting point for CFs to reflect. This dilemma might entail an event that occurred during practice and is still fresh in the educator's mind (e.g., students' misconceptions and errors, the educator's inability to deal with in-the-moment situations in the classroom and other difficulties associated with teaching and learning. Randall and Thornton (2001) note that the educator may not evaluate the experience soon after the dilemma and may need a little space to self-reflect and prepare for a reflective dialogue with a CF. Reflective dialogue should occur in a non-evaluative, safe, and non-judgmental environment where educators feel comfortable sharing their thoughts, experiences, and vulnerabilities regarding classroom practice (Baskerville & Goldblatt, 2009).

Schildkamp et al. (2016) argue that CFs must combine experience with evidence during reflective dialogue. Decisions solely based on intuitive experience do not work all the time. Therefore, data and literature can inform decision-making about one's practice based on evidence, supporting professional learning and student achievement. CFs must be aware of what kind of support they intend to give one another and provide reasons for it (Cosh, 1999) through comments, questions, and suggestions related to the educator's practice. I concur with Farrell (2001) that some engagement rules must be established. For instance, I contend that the

CFs should not enter the reflective dialogue "as an expert in a special field of academic knowledge, who puts right wrongs" (Holden, 1997, p. 444). Instead, the CFs should strive to balance acknowledging strengths, offering suggestions for improvement, and sharing perspectives in a supportive atmosphere that fosters mutual gain and pedagogical insights (Nilsson et al., 2018).

3.19.3 Peer lesson observation

Peer observation is vital to professional learning among critical friends (CFs). The conventional focus on observation as a performance marker (O'Leary, 2022) has minimised the value of observation as a formative tool for professional educator learning. Tilstone (2012, p. 60) uses the term *'partnership observation'* to describe peer observation and argues that such partnerships offer different perspectives to CFs that enable professional learning. She maintains that any successful partnership should be underpinned by 'trust, commitment, and shared understanding that prioritises individual needs. In this non-evaluative professional learning space, CFs work collaboratively to observe each other's lessons and to provide mutual support centred on nurturing pedagogical knowledge, skills, and curriculum development (e.g., Tilstone, 2012; Martin, 2016). Peer observation should not result in a 'product' but should be a catalyst for sharing ideas and stimulating reflective dialogue. For Fielding et al. (2005), if educators are to get the most out of professional learning tools such as observation, they should experiment with new ways of thinking and doing and be ready to reflect freely and honestly on their own and colleagues' practice. The general idea is that working in this way will support their ongoing professional learning and ultimately improve the quality of their teaching and the student's learning experiences.

One of the distinguishing characteristics of peer observation in critical friendships is the notion of reciprocal learning and the expectation that those involved will mutually support each other rather than assess each other's classroom performance (O'Leary & Price, 2016). Lasagabaster and Sierra (2011) advise that the observation should not concentrate on compiling negative feedback. Instead, it should support an agenda for both an observer and the observed to discuss and refine practice (Donnelly, 2007). But for this to function, it needs more than just a commitment to reciprocity. CFs must have equal agency and respect for one another regardless of someone's hierarchical position and rank within the institution (O'Leary & Price, 2016).

3.20 Chapter summary

This chapter has situated the study within existing theoretical literature and empirical studies related to professional knowledge. It has examined the knowledge models shaping modern educational practices in teacher education and has explored mathematical knowledge for teaching teachers, examining its development. The chapter has foregrounded the relationship between professional knowledge and practice in pre-service teacher education. It concluded with an exploration of collaborative professional learning practices in pre-service mathematics teacher education, focusing on critical friendships among mathematics teacher educators (MTEs) which have the potential to impact on teacher education practices.

Overall, the chapter has aimed to provide a foundational understanding of knowledge acquisition, professional practice, collaborative learning and critical friendships among MTEs. The next chapter will explain the theoretical and conceptual frameworks that were used in this study as analytical tools.

CHAPTER 4

THEORETICAL AND CONCEPTUAL FRAMEWORKS

"Through others, we become ourselves." - Lev Vygotsky

4.0 Introduction

This chapter outlines the key theoretical and conceptual frameworks that have guided the analysis and interpretation of the research data. The overarching theory is Vygotsky's (1978) sociocultural theory (SCT) of learning, which emphasises the importance of social and cultural contexts in knowledge construction. The study utilises Wenger's (1998) dimensions of practice framework for understanding the dynamics of collaborative endeavours, such as critical friendships in mathematics teacher education. It also employs Desimone's (2009) framework for effective continuous professional development (CPD) and McCrory et al.'s (2012) knowledge of algebra for teaching (KAT) framework. These provide lenses through which to analyse the professional learning and teaching practices of MTEs. Socio-cultural theory as a main theoretical framework, together with Desimone's and McCrory et al.'s conceptual frameworks, provides analytical tools for examining and interpreting the findings of this study of MTEs' collaborative professional development and learning experiences and how each of these informs instructional practices and knowledge for teaching in pre-service teacher education.

4.1 Theoretical framework

As the study explores how MTES collaborate within a professional learning community to advance their professional practice and build knowledge for teaching algebra, Vygotsky's (1978) sociocultural theory of learning provides a useful framework for conceptualising and implementing the study. Central to this theory is the idea that higher mental functioning originates from engaging with cultural tools and more experienced others (Vygotsky, 1978). Hence, learning is an interpersonal process shaped by participation in social and material contexts rather than solely an individual cognitive act.

Vygotsky's work explains how collaborative engagements with more knowledgeable peers can, under supportive conditions, foster conceptual understanding and capacity building. For this study, a PLC for MTEs was established to enable peer collaborations aimed at refining and extending the specialised pedagogic knowledge and skills of a group of mathematics teacher

educators. Lave and Wenger's (1991) situated learning theory explains how groups jointly immersed in professional work create mutually understood meanings and evolve towards mastery by establishing norms, relationships, dialogues, and identity around practice. Lave and Wenger's (1991) concept of communities of practice is relevant for investigating informal collective learning by MTEs. Although such MTEs may be experienced, engaging with peers provides learning opportunities for tackling novel challenges.

Key concepts in Vygotsky's theory, such as the zone of proximal development (ZPD) and mediation and the concept of scaffolding developed by later scholars in response to his foundational work, are significant for explaining how collaborative environments enable progression from novice to expert. The ZPD refers to the gap between what learners can achieve independently and what they can achieve with guidance from a more knowledgeable other. Scaffolding involves calibrated support from an expert tailored to a learner's ZPD that tapers off and is eventually removed as mastery develops.

Vygotsky emphasised the role of tools and artefacts in shaping cognition. In particular, language is pivotal in mediating learning by enabling communication and thought. Vygotsky argued that speech and thinking are intertwined, forming the bedrock for cognitive growth (Vygotsky, 1978). Within mathematics teacher education, sociocultural theory informs how specialised knowledge is co-constructed through collaborative learning, group discourse, and joint meaning-making. Exploring how collective efforts between educators shape pre-service teachers' mathematical knowledge development requires a theory which foregrounds sociocultural influences on learning.

Although Vygotsky's work focused on children's learning in formal and informal settings, its translation to adult informal learning settings has precedents (Goos, 2004; Jaworski, 2006; Lieberman & Miller, 2008; Goos, 2014). Building on these established applications, this study utilises a sociocultural lens to analyse and interpret data from Ghanaian MTEs' professional development activities in a PLC. Their collaborative engagement in pedagogical improvement endeavours represents socially mediated capacity building. In the next section, I outline some of Vygotsky's key concepts.

4.1.1 Mediation

Vygotsky dismissed the notion of learning as an individualistic pursuit. For him, an interactive, dialogic, and collaborative mediation process is central to learning. He highlighted the pivotal role of social interaction in human cognition, particularly in two key processes of social

mediation: the inter-psychological interactions among individuals and the intra-psychological cognitive development of each individual (Vygotsky, 1978). Mediation also involves the process of internalisation. As individuals engage in social interactions and utilise cultural tools, external knowledge and resources are internalised and become part of their cognitive processes, leading to higher mental functions and greater self-regulation (Johnson & Golombek, 2003).

Internalisation is a term used for the process through which individuals integrate external information, social interactions, and cultural practices into their cognitive structures. This integration empowers them to utilise these internalised resources independently, even without immediate external support (Rogoff & Chavajay, 1995; Johnson & Golombek, 2002). Through internalisation, individuals appropriate cultural tools or artefacts and adapt them according to their understanding, transforming the construction and utilisation of knowledge (John-Steiner & Mahn, 1996). This process also leads to the contextualisation of learning, recognising that the context in which knowledge is acquired and applied significantly influences its meanings (Wenger, 1998).

Contextualisation involves linking the subject matter to the broader context, considering cultural norms, values, historical background, and social dynamics. This concept emphasises that meaning is co-constructed through interactions within the social and cultural environment (Tang & Choi, 2009). Educators can use contextualisation to adapt learning experiences to address specific needs in classroom practice (Tang & Choi, 2009). In the context of mathematics teacher educators, the collaborative support that informs their mathematical knowledge for teacher education depends on various types of expertise.

4.1.2 Cultural tools

Within socio-cultural theory (SCT), cultural tools serve as instrumental elements that mediate learning and development. These tools, whether physical or symbolic, are not mere objects; they are deeply ingrained within specific cultural and social contexts, carrying their communities' meanings, values, and practices (Vygotsky, 1978). According to SCT, the internalisation of knowledge does not occur directly; rather, it is achieved through tools shaped by sociocultural evolution (John-Steiner & Mahn, 1996; Wertsch, 1991). Language, artefacts, conceptual usage, and other educational aids serve as tools in the environment of MTEs to mediate new thought patterns and mental functions.

These tools are not static but evolve, adapting to different contexts and cultures (Kozulin, 2003). Their mediation can range from object-regulated to other-regulated and self-regulated

(Lantolf et al., 2015). In essence, physical and symbolic tools are pivotal in the sociocultural learning theory. They mediate cognitive development by offering individuals resources to engage with their environment, communicate, and gain knowledge.

Cultural tools, such as mathematical language, writing systems, and technology, are deeply embedded in cultural contexts and actively shape learning experiences and cognitive processes. The accessibility of specific teaching and learning materials relevant to teaching algebra is essential for the MTEs in this study as they undertake specific teaching tasks within the context of algebra in teacher education.

4.1.3 Vygotsky's zone of proximal development ZPD

The Zone of Proximal Development (ZPD) is a fundamental construct within sociocultural learning theories. First, the work of Vygotsky (1978) establishes the differentiation between two developmental levels: the actual and the potential. While the actual represents learning outcomes that learners can exhibit independently, the potential refers to learning outcomes that call for support or direction. In Vygotsky's (1978, p. 89) definition, the ZPD is "the distance between the actual development level as determined by independent problem solving and the potential level as determined through problem-solving under adult guidance or in collaboration with more capable peers." It is where learning is most effective and allows learners to bridge the gap between their current abilities and their potential for further development. Learning is viewed as a development that occurs through exchange with others from which shared knowledge is constructed.

Within the ZPD are the mediational approaches that contribute to knowledge construction, such as object-, other-, and self-regulated (Johnson & Golombek, 2003). In object-regulated knowledge construction, teacher educators as learners rely on contextual or cultural artefacts to promote their understanding. When knowledge construction is other-regulated, teacher educators seek assistance from other colleagues, and when it is self-regulated, they take control of and master their learning activities (Johnson & Golombek, 2003). The provision of scaffolding, which denotes support given by a knowledgeable other to a novice individual or a colleague seeking insights, expands the possibility for learning and progress through aided performances (Lantolf et al., 2015). Therefore, the quality of the interactive learning context is likely to influence how well educators learn within the ZPD. The role of the more knowledgeable others, such as experienced MTEs or critical friends, is crucial in providing the necessary support and scaffolding to help the MTEs progress. The support can take various forms, including lesson demonstration, questioning, providing hints, and breaking down

complex tasks into manageable steps. By receiving this targeted assistance, MTEs as learners can achieve objectives that they might not be able to accomplish independently. In effect, the ZPD is created not just within the individual learner but also in the interaction between the learners, co-participants, and available tools during the involvement in a common enterprise (Gan, 2014). I argue that the success of MTEs' learning within the ZPD will depend on their engaged participation in a socially interactive context.

4.1.4 Social interaction as a situated learning and community of practice

Lave and Wenger (1991) introduced the concepts of "situated learning" and "legitimate peripheral participation" to describe the process of learning through engagement in social environments. In situated learning, individuals, over time, negotiate their competence within a social process and form a community of practice (Farnsworth et al., 2016). Wenger (1998, p.79) emphasises that communities of practice (CoPs) are not independent entities but are situated within a broader context, encompassing historical, cultural, and social systems that significantly influence them. CoPs are constituted by a group of individuals with particular expertise in specific areas of cultural practice (Nuthall, 1997). Wenger (1998) defines CoPs as "a kind of community created over time by the sustained pursuit of shared enterprise." He asserts that organisations cultivate a 'shared practice' when group members collectively create and apply knowledge. Learning, for Wenger, is embedded within this shared practice.

Participation within communities of practice connects an individual's experiences to those of others in their social context. Lave and Wenger's (1991) concept of "community of practice" (CoP) has three fundamental features: 1) Domain, 2) Community, and 3) Practice. When learning occurs within a social setting, there is an interplay between the domain or the shared area of interest, the collective engagement of the community, and the practices employed within that community.

4.1.4.1 The concept of domain

The domain represents the shared sphere of interest or common purpose that unites members of a community of practice. Specifically, the domain provides the glue that binds individuals based on mutual professional interests, knowledge, and skills (Wenger, 1998). For instance, the domain may revolve around a particular subject matter expertise, a craft, or an occupation. According to Wenger (1998), the domain gives the community an identity and defines joint objectives, values, and behaviours. In this study, the domain of the mathematics teacher-educators is algebra instruction. I engaged with a professional group focused on strengthening algebraic competence for their work in pre-service teacher preparation. Teaching algebra

effectively poses challenges, as it involves the beginning of advanced mathematical reasoning for many pre-service teachers. Building algebraic knowledge and skills undergirds competence in further areas like geometry and calculus and is critical for mathematics teachers (Star et al., 2015).

For Wenger (1998), the domain is a focal point that attracts individuals with similar interests and expertise, fostering a sense of belonging and shared commitment. It enables members to engage in discussions, share experiences and seek solutions to challenges related to their shared interests. This mutual interest in the domain facilitates the exchange of knowledge, expertise and best practices among the community members, leading to collective learning and improvement in professional knowledge. Additionally, the domain influences the evolution of the community of practice over time. As Wenger et al. (2002) highlight, the domain establishes the boundaries and scope of the community, defining what is considered relevant knowledge and expertise within the group. In this regard, the domain helps maintain the coherence and focus of the community, shaping who its members become and contributing to advancing knowledge within the field.

Considering all the above, I argue that a domain is not static but evolves in response to changes in the external environment and the community's dynamic needs. Wenger (1998) emphasises that members' active participation and engagement in the domain are crucial for its development. Through their interactions and contributions, members can collectively shape and redefine the boundaries and practices of the domain, contributing to its relevance and adaptability. This dynamic relationship between the domain and the community enables the community of practice to remain vibrant and responsive to its members' evolving needs to the challenges they face, and to the broader context. In summary, the domain is the foundation of the community of practice, influencing the community's identity, its practices and knowledge boundaries while evolving in response to its members' challenging needs and contributions.

4.1.4.2 The community dimension

The concept of community centres on social ties and relationships within a collective domain of interest (Lave & Wenger, 1991; Smith et al., 2017). Wenger (1998) stresses that shared identities and a sense of belonging are integral for binding members together. By interacting, collaborating, and engaging in the group, individuals are likely to develop communal experiences, trust, and camaraderie. Members actively participate in discussions and activities and invest in the social fabric, bolstering cohesion and strength. Thus, learning in communities of practice occurs through involvement in collective endeavours rather than individual efforts

(Wenger, 1998). Novices learn from veterans, while experts refine understanding by mentoring and teaching newcomers. As connections strengthen, members feel comfortable seeking feedback and sharing knowledge, thus powering continuous development.

Additionally, communities provide safe spaces for members to take risks, challenge norms, and pioneer innovations within their sphere of work (Wenger et al., 2002). Members co-construct knowledge and practices by cooperating in collective sensemaking, enriching expertise. However, participation levels may vary among members from core to peripheral (Wenger, 1998). Still, even observing can foster gradual integration as individuals gain insights into the community's goals and ways of functioning.

4.2 Mathematical practices and learning communities

Practice encompasses the activities and interactions members engage in within a community or context (Lave & Wenger, 1991). Practice involves actively participating in meaningful tasks such as observing, imitating, collaborating, and serving as needed. For Wenger (1998), a practice involves more than mastering procedures; it includes understanding cultural and social ways of being, such as norms, tools, concepts, codes of conduct, and application of knowledge and skills. Participating in community activities enables PLC members to acquire both explicit and tacit knowledge central to the domain (Wenger et al., 2002). Practice also entails developing specialised vocabulary, symbols, and representations to facilitate knowledge transfer (Wenger, 1998). The shared language or discourse reinforces belongingness while enabling effective communication within the learning community.

Within the discipline of mathematics, practice has been conceptualised in several ways. For Li (2013), mathematical practice refers to the skilled activities and strategies enacted by experts to solve problems, convey solutions, validate methodology, and generalise understanding across situations. In other conceptualisations, practice comprises "shared ways of doing and communicating mathematics" within an institution or group tackling particular problems (Essien & Adler, 2016, p. 178). This social conceptualisation of mathematical practice aligns with the notions of participation and reification proposed in Wenger's framework (1998). According to Wenger (1998), participation refers to active participation in the community's activities and interactions. As members participate, they develop a shared repertoire of experiences, tools, concepts, and methodologies. Reification refers to manifesting these shared practices and constructs in concrete artefacts and representations. Examples include symbols, terminology, models, documents, or protocols encapsulating aspects of the community's

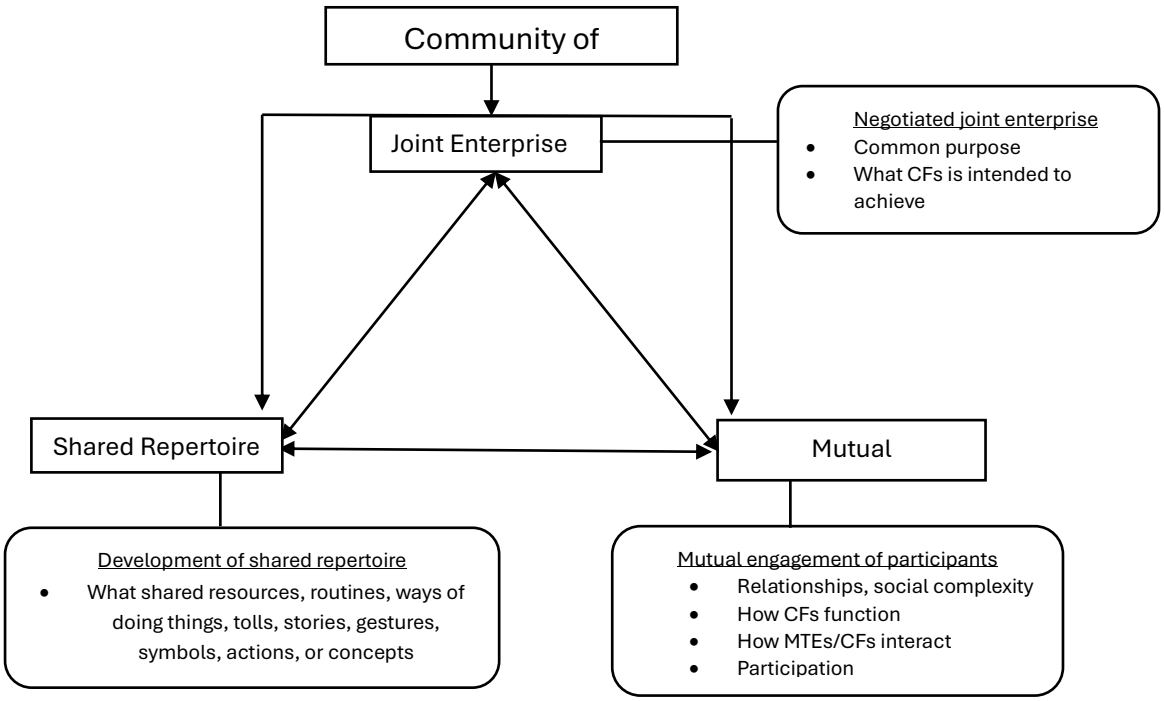
knowledge and behaviours. Through repeated cycles of participation and reification, the community continues to refine and extend its understanding of the domain.

For MTEs engaging in collaborative inquiry, developing mathematical practices would involve maximising the benefit of participation and reification to advance their instructional approaches collectively. This could entail jointly analysing challenges in teaching algebra, co-creating novel pedagogical resources or activities, designing assessments, and formalising successful strategies into sharable workshop materials or teaching guides. Here, the emphasis remains on participatory sensemaking and manifesting the outcomes in tangible artefacts that capture the community's evolving shared knowledge. Maintaining this productive interplay between participation and reification enables growth (Wenger, 1998). As the community crystallises insights into concrete forms, it will likely uncover knowledge and skills gaps that require further investigation. Mathematical practices progress through iterative collaborative construction undergirded by artefacts that embody the community's strengthening practice.

4.3 Wenger's (1998) dimensions of practice

Wenger (1998) distinguishes three interrelated dimensions of practice within a community: mutual engagement of participants, negotiation of joint enterprise and developing a shared repertoire. As Essien and Adler (2016) note, for Wenger, these dimensions of a community of practice must be substantial and interdependent because their interdependency makes them the source of a community's coherence (see Figure 4.1)

Figure 4.1: Dimensions of a community of practice (Researcher's Construct)



4.3.1 Mutual engagement of participants

Engagement in social practice is the fundamental process by which community members learn and become who and what they are (Wenger, 1998). Wenger emphasised that practice does not exist in a vacuum but resides in a community of people and their connection to the mutual engagement by which they act. A community of practice is more than just a social group determined by a network of affiliations. It involves interactions, diversity, and long-lasting interactions focused on the involvement of participants in their work. It implies that they are speakers of a common language with a set of universally understood meanings. A community of practice "can become a very tight node of interpersonal relationships" (Wenger, 1998, p. 76) that exist through engagement in practice.

In the context of MTEs' professional learning and collaborations, mutual engagement involves active participation and teamwork in a joint enterprise, which entails interaction among professional learning community members. MTEs engage in reflective dialogue, share experiences, and examine mathematical practices to promote coherence and consistency in their teaching practice (Wenger, 1998; Kuh, 2016). Through mutual engagement, MTEs can reflect on their instructional strategies, gain new insights into their pedagogical practices, and receive feedback from their critical friends about their teaching practices. Mutual engagement fosters a sense of trust, respect, and a shared commitment to professional growth. Clarke (2008, p.30) defines mutual engagement as "participation in an endeavour or practice whose meanings are negotiated among participants." In mutual engagement, the negotiations result in a joint enterprise, which Clarke refers to as "the focus of an activity that links members of a community of practice" (Clark, 2008, p. 31). In a PLC of MTEs, forms of engagement can include collaborative lesson planning, observation and feedback cycles, analysing student work, and co-authoring papers - all focused on enhancing practice in their sphere of education (Hochberg & Desimone, 2010)

4.3.2 A negotiated joint enterprise

Wenger (1998, pp.77-78) established three key points that distinguish what is and is not a joint enterprise:

> "it is the result of a collective process of negotiation that reflects the full complexity of mutual engagement. It is defined by the participants in the very process of pursuing it. It is their negotiated response to their situation and thus belongs to them in a profound sense, despite all the forces and influences that are beyond their control. It is not just a

> stated goal but creates among participants relations of mutual accountability that become an integral part of the practice."

He explains that a joint enterprise is joint not because every member agrees with every community undertaking but because the joint enterprise is socially negotiated. He emphasises that disagreement, challenges, and competition can be considered productive. This explanation implies that joint enterprise is not imposed on the community but instead comes as a negotiated result of participation in a learning community that engages with dilemmas of practice (Essien, 2016). According to Wenger (1998), members of a CoP may undertake different tasks, assume various roles and exhibit different levels of expertise and commitment, yet all contribute their quota to a joint enterprise that defines the community. The joint enterprise regarding MTEs' professional learning in mathematics teacher education revolves around supporting each other in constructing bridges between teaching and research and developing an inquiry approach to mathematics teaching and learning (Potari, 2013). Potari believes that the intentions of the mathematics educator should be "not to transmit knowledge from the research to the teachers but to form a community of inquiry where the teachers use research as a tool for their inquiry" (Potari, 2013, p. 509).

Clark (2008) holds that the joint enterprise is the focus of activities that connect members to the learning community, which is what the collaborations within the PLCs are intended to achieve. Therefore, in the context of professional learning, collaborating MTEs aim to enhance their teaching practices by tapping into the expertise, skills, and other support membership may afford. They work together to identify areas for improvement, set goals, and develop strategies to address specific challenges or dilemmas in the practice (Wenger, 1998). The joint enterprise provides a sense of direction and purpose, guiding the collaborative efforts between MTEs. Essien (2016) argues that the dimensions of mutual engagement and shared repertoire, as proposed by Wenger's community of practice framework, provide a means to access and understand the joint enterprise. In other words, how MTEs interact and engage with one another and the shared resources and practices they develop can offer insights into their professional learning communities' collective aims and endeavours.

4.3.3 Development of a shared repertoire

For Wenger (1998, p. 83), a shared repertoire can be described as the "routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions or concepts that the community has adopted in the course of its existence." These resources assist in realising the common goal

and the joint enterprise and are the cornerstone for building a learning attitude (Gau, 2016). According to Levinson and Brantmeier (2006, p. 331), the shared repertoire of a community "can be seen as the tangible expression of mutual engagement and the key means of carrying forth a joint enterprise." Motivated by joint enterprise, the members engage each other in everyday, meaningful mathematical practices and gradually merge into a learning culture to shape the different types of shared repertoire (Gau, 2016). A channel for accessing the collaborative endeavour of a mathematics teacher education community is provided by the mutual engagement and shared repertoire dimensions of practice (Essien, 2016). I hold that in mathematics teacher education, developing a shared repertoire encompasses the resources, strategies, and tools that MTEs and critical friends utilise to support learning and professional growth (Wenger, 1998). It includes a collective linguistic resource and disciplinary discourse, lesson plans, assessment tools, and instructional strategies. For example, critical friends contribute to the shared repertoire by sharing their experiences, expertise, and knowledge of effective mathematics teaching practices. This shared repertoire is valuable for MTEs, enabling them to access and apply relevant information and strategies in their *mathematical practices*. Essien (2016) holds that the shared repertoire is the community's way of doing things.

4.4 Learning as shared meaning-making

Learning is an intrinsically social process interweaving experiences and interactions to inform collective growth (Wenger, 1998). As a communal act, learning involves negotiating meanings regarding shared situations to construct understanding. Thus, participation in context shapes identity formation over time. Within communities of practice, varied interpretations are inevitable as members approach situations informed by their unique histories (Wenger, 1998). Hence, reconciling alternative viewpoints through collective reflection enables the co-evolution of perspectives. Sustaining affirmative dialogue and balancing perspectives allows "reification", tacit understandings transitioning into commonly owned explicit knowledge (Ewing, 2017; Goos, 2004). Thus, the crystallisation of new shared mental models, practices, and language reflects the CoP's learning trajectory.

For MTEs, learning in the CoP entails ongoing negotiation of pedagogical techniques and theoretical orientations when tackling classroom challenges together (Graven, 2004). Whereas independently reaching a consensus may prove difficult, engaging differences in a collegial spirit is more likely to result in breakthroughs in thinking. The resultant discourse distils collective wisdom from individual insights by reconciling varied best practices interpretations.

Thereby, the community jointly advances fundamental pedagogical knowledge and capabilities.

4.5 Implications of socio-cultural theory for MTEs' professional knowledge and practice

Socio-cultural theory provides a framework for examining the connection between educators as learners during their professional learning (PL) and the broader context in which their professional learning practices occur. Primarily, SCT suggests that learning is not an independent process but occurs through participation and interaction within social practices, implying that learning is a collaborative effort among participants within a particular context. To optimise this collaborative learning, participants should set clear goals for their PL (Brown et al., 2016).

Vygotsky's theorisation of learning as a mediated activity which is facilitated by tools and artefacts, has implications for educators' PL. Tools aiding teachers to extend their ZPD encompass expert guidance, teaching materials, textbooks, technology, and professional documentation (John-Steiner & Mahn, 1996; Eady & Lockyer, 2013). In the context of MTEs' PL, access to and utilisation of these tools can facilitate active learning and enhance classroom performance. Training manuals and relevant artefacts are crucial for mediating learning during and after CPD sessions.

Additionally, the construct of the ZPD helps understand educators' progress and potential for further development (Mansour et al., 2014). Warford (2011) extends the ZPD concept to educator development, defining it as the space between what educators can do independently and the level they can achieve through strategically mediated assistance. Effective PL programs must accurately assess participants' learning needs and goals and guide them to their potential level of development. Involving MTEs in planning their PL programmes is essential for promoting effective learning outcomes. Senior MTEs or critical friends can scaffold and support the learning of novice MTEs.

By emphasising that knowledge construction is not isolated from social, cultural, and historical contexts, SCT underscores the critical role of context in influencing educators' involvement in CPD. Indeed, educator learning and CPD are intricately linked to the broader contexts of teachers' professional practice (Hardy, 2012; Larsen & Allen, 2021; Abakah, 2023). In essence, SCT provides insights into the complexities of educator learning and CPD and explains various aspects of educator growth in diverse sociocultural contexts (van den Bergh et al., 2015). In

essence, SCT contributes to understanding of how educators learn and translate knowledge into practice, underlining the critical role of context in shaping CPD activities. Ultimately, SCT provides valuable insights into the multifaceted nature of educator learning and development in varied sociocultural settings.

4.6 Additional conceptual frameworks

In this section, I describe the CPD frameworks of Desimone (2009) and McCrory et al.'s (2012) practice-based KAT framework as analytical tools useful for informing the discussion and interpretation of the findings of this study. Desimone's framework provides a lens through which to explore interactions, experiences, activities, and resources in a professional learning community or community of practice, while the KAT framework though used sparingly in the study provides a lens through which to explore the classroom practices of MTEs. Both frameworks are used at different stages of the study alongside Wenger's (1998) dimensions of practice.

4.6.1 Desimone's conceptual framework for effective CPD

This section describes Desimone's (2009) conceptual framework which is used as an analytical tool for informing the discussion and interpretation of findings from part one of the study. Her framework consists of three parts: (1) a description of key characteristics defining effective CPD; (2) an operational theory elucidating how CPD influences educator learning and student outcomes; and (3) contextual factors that exert an impact on CPD. For Desimone (2009), the critical features of CPD are content focus, active learning, coherence, duration, and collective participation. Each of these is described in the sub-sections that follow.

4.6.1.1 Content-focused professional learning

Content focus refers to the specific material teachers learn through CPD (Garet et al., 2016). The content selected should aim to augment teachers' subject matter expertise and pedagogical understanding of how students learn that content. In content-focused professional learning, the primary emphasis is on improving educators' understanding of the content they teach. This can include deepening their knowledge of specific subject areas, curriculum standards, instructional materials, and assessment methods that pertain to their teaching assignments (Garet et al., 2016; Ingvarson et al., 2005). Kennedy (2014) also divides content learnt through CPD into two main categories: knowledge of the subject matter or content knowledge (CK) and pedagogical content knowledge (PCK). This study focuses on the content that MTEs consider to be important for discussion in the PLC.

4.6.1.2 Active learning

Professional learning activities should be engaging, participatory, and relevant to MTEs' classroom practice. Active learning approaches can include hands-on activities, collaborative projects, and problem-solving exercises that encourage educators to apply what they have learned (Darling-Hammondet al., 2009). Interactive experiences encouraging active learning, such as observing expert/ experienced teacher educators, participating in model lesson delivery, reviewing students' work, scoring assessments, leading discussions, coaching and mentoring, or participating in a community of practice, contribute to improving teaching (Franke & Kazemi, 2014). It is crucial, for example, that MTEs read about current issues in mathematics education and extend their understanding of teaching and learning. Studies have revealed a positive association between active learning and educators' knowledge (Rotgans, & Schmidt, 2011; Ingvarson et al., 2005).

4.6.1.3 Coherence in professional learning

Coherence refers to aligning professional learning opportunities with educators' expertise, beliefs, and instructional objectives (Desimone, 2011; Kang et al., 2013). Coherent professional learning promotes the collective adoption of pedagogical strategies aligned with the content focus.

In this study, coherence pertains to the degree to which professional learning activities address what MTEs need to know and be able to do in order to enrich PSTs' knowledge and practice. For Roach et al. (2008), an educational system is an integrated entity with alignment among components such as curricula, materials, pedagogy, and assessments. Within communities of practice, sustained engagement should enable coherent understandings to develop through negotiating meaning regarding challenges, goals, and innovations shaping collective work (Wenger, 1998). Alignment is not an end state but an ongoing, collaborative process for reinforcing shared purpose across the professional domain.

4.6.1.4 Duration of professional learning

Duration refers to the total contact hours spent in professional development activities and the time during which the engagement occurs (Hochberg & Desimone, 2010; Kang et al., 2013). Research indicates meaningful intellectual and instructional change requires substantial duration, including overall timespan and hours invested (Cohen & Hill, 2001; Guskey, 2002b). Studies link greater time commitments to enhanced teacher outcomes (Banilower et al., 2007). Sufficient duration allows for in-depth discussion, the practice of new strategies, meaningful collaboration, and ongoing refinement. Protracted activities spread over a semester or year enable educators to iteratively apply emerging ideas in the classroom, analyse effectiveness,

and improve implementation with peer input. Conversely, brief workshops often fail to provide sufficient time for comprehension, experimentation, and mastery. While particular formats and individual needs vary, impactful professional development generally entails extensive engagement, allowing for progression through a series of learning cycles.

4.6.1.5 Collective participation

Collective participation refers to educators from the same institutional context engaging in shared professional learning (Hochberg & Desimone, 2010). This enables interaction and discourse, which nurtures collaborative knowledge development (Banilower & Shimkus, 2004). Research has shown that collective participation positively impacts teaching practices, as common learning experiences combined with collegial support facilitate the integration of new strategies (Penuel et al., 2015). Desimone (2002) found that professional development successfully transforms classrooms when educators from the same schools participate cooperatively. Working in isolation inhibits meaningful feedback and evaluation (Hargreaves & Fullan, 2012). By creating a professional learning community, educators reinforce each other's growth by jointly analysing challenges, brainstorming ideas, refining implementation plans, and examining progress. This collective enterprise, fuelled by a "culture of collaboration", propels positive change (Prenger et al., 2017). As Wenger (1998) notes, shared meaning-making and community participation enable the effective adoption of productive educational approaches. This section concludes with a diagrammatic illustration of a conceptual framework for studying the effects of CPD on MTEs.

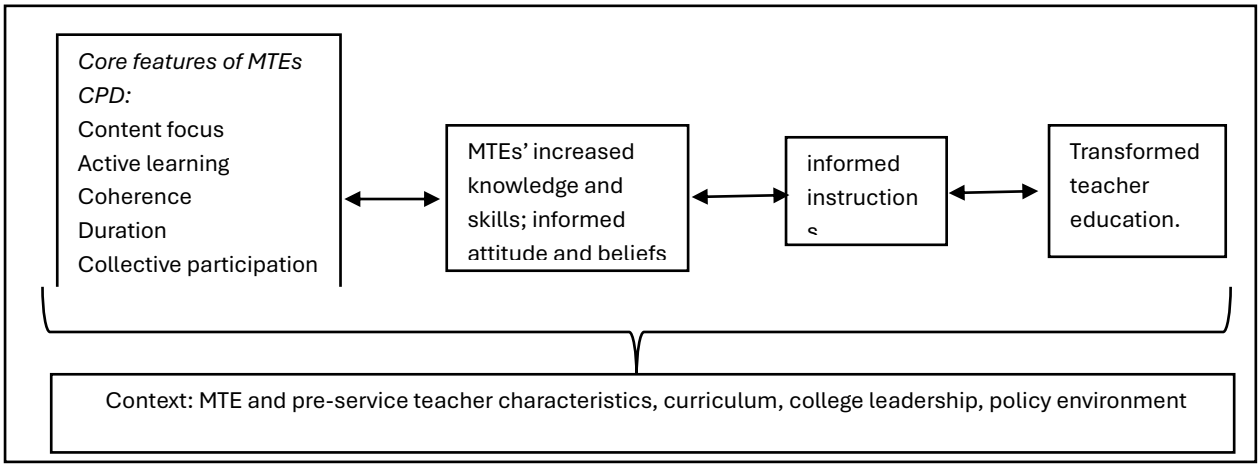


Figure 4.2: Conceptual framework for studying the effect of CPD on MTEs (adapted from Desimone, 2009, p. 185).

Desimone's (2009) framework includes the crucial role of the context in which professional learning occurs. I find the framework useful for this study for the following reasons. First, it allows for exploring transformation in learning and how it influences educators' ability to teach and students' likely achievement, all of which play a vital role in understanding how professional learning works (Wayne et al., 2008). Second, the significance of each component of the iterative pathways involved in the model is consistent with the literature and links well with teacher educators' knowledge (Hill et al., 2008; Phelps & Schilling, 2004) and learning. Third, the model moves professional learning beyond craft-oriented approaches (Fishman et al., 2003).

Guskey (2002a) devised an approach for exploring professional learning that includes five crucial levels of data that need to be collected and analysed: (1) participants' reactions, (2) participants' learning (new knowledge and skills gained), (3) organisation's support and change (organisation's advocacy and support for change if necessary), (4) participants' use of new knowledge and skills (level of implementation), and (5) student learning outcomes. This study focuses on the second, third and fourth levels, that is, participants' learning, support and use of knowledge and skills.

4.7 The knowledge of algebra for teaching (KAT) framework

The KAT framework explains categories of knowledge and practice that support effective algebra instruction (McCrory et al., 2012). Informed by the conceptualisation of mathematical knowledge for teaching (Ball et al., 2008), this framework consists of three categories of knowledge with algebra in focus: *Knowledge of School Algebra* which encompasses educators' grasp of the curriculum content they instruct and is akin to common content knowledge (CCK) and specialised content knowledge (SMK) (Shulman, 1986; Ball et al., 2008). *Advanced Algebra Knowledge* which provides more sophisticated conceptual insights beyond school algebra to enable richer representations and connections. *Teaching Knowledge* which involves pedagogical techniques and knowledge of students central for instruction but often excluded from formal mathematics preparation.

McCrory et al. (2012) identified three additional domains of teaching practices for leveraging algebraic knowledge: *Trimming* which involves simplifying advanced methods for the level of learners without compromising mathematical integrity, reminiscent of Bruner's (1960) concept of intellectually honest translations of complex ideas. *Bridging* which entails linking across mathematical topics, courses, and concepts to illustrate connections, similar to Tyler's (1949) notion of horizontal and vertical curriculum alignment. *Decompressing* which requires

unpacking formalised knowledge into elemental forms where foundational structures are visible (Adler & Davis, 2006), aligning with Ball and Bass's (2009) idea of rendering implicit understanding explicit for instruction. These practices characterise the complex art of translation at the core of skilled algebra instruction in mathematics teacher education.

As McCrory and colleagues argue, evaluating educators' capacities for effectively teaching algebra necessitates examining all combinations of knowledge types and practices. However, in this study, to gain a deeper understanding of how collaboration informs knowledge for mathematics teacher education, I chose to focus on teaching practices for leveraging algebraic knowledge. McCrory et al.'s (2012) framework provides a lens for use in evaluating how MTEs' collaborative engagement shapes algebraic knowledge and pedagogical techniques for PST preparation. For this reason, I find the KAT framework useful as it draws on a practice-based theory of mathematics for teaching. Secondly, the strength of KAT lies in domain-specific and measurable terms. Finally, the KAT framework is aligned with the work of both Shulman (1986) and Ball et al. (2008): teaching mathematics involves doing work that requires specialised knowledge.

4.7 Synthesising the frameworks

This section presents a synthesis of the various frameworks discussed in the chapter. Vygotsky's sociocultural theory offers the metatheoretical framework for conceptualising professional learning as socially mediated knowledge construction, shaped by participation in particular cultural contexts. Wenger's constructs of "community of practice" and "dimensions of practice" enable a focus on how collaborative activity in groups, organised around domain-related work, enables members to extend their expertise and adopt shared repertoires. Desimone's features of effective professional development which encompass content focus, active learning, coherence, duration and collective participation directly map onto the learning dynamics that Wenger theorised as characteristic of communities of practice, providing key indicators of members' deepening engagement. Lastly, McCrory et al.'s (2012) frameworks delineate the multifaceted aspects of mathematical and teaching knowledge pivotal for mathematics instruction, offering a lens through which to evaluate the impact of collaborative engagements on classroom teaching practices.

Together, these frameworks provide a multilayered analysis which situates teacher-educator collaboration within a sociocultural paradigm of social knowledge building, while tracing adaptive shifts in practice repertoires, strategic knowledge, and motivations through markers

of community participation and professional exchange. The frameworks provide theoretical and practical tools for examining, characterising and evaluating collective learning for improved practice.

4.8 Chapter summary

In this chapter, I have explained the theoretical and conceptual frameworks that underpin this study. Socio-cultural theory (SCT) frames the exploration and interpretation of MTEs' learning processes in a professional learning community while Desimone's and McCrory et al.'s conceptual frameworks offer complementary lenses, enriching the examination of MTEs' participation in professional learning community practices, experiences and use of knowledge in their classroom practices. Throughout the study, both the theoretical underpinning of SCT and the conceptual frameworks are referred to during data analysis, interpretation and discussion.

CHAPTER 5

RESEARCH METHODOLOGY

5.0 Introduction

The chapter begins with a discussion of the research paradigm, the research design and the rationale for the choice of a case study. Next, I describe the research procedures used in two phases of data collection. This part of the chapter includes a description and discussion of my role as a researcher and of the methods employed for data collection, selection and analysis. Issues of reliability and validity, ethical considerations and the limitations of the study are discussed in the concluding section.

1. To what extent does working collaboratively in a PCL support the development of mathematical knowledge for teaching in pre-service teacher education?
2. What knowledge do MTEs consider important to focus on in their collaborative work in pre-service teacher education?
3. What counts as evidence of the integration of PLC learnings into MTEs' pre-service teacher education practices, and is there evidence of such integration?
4. To what extent does critical friendship support changes in MTEs' professional knowledge and practice?

5.1 Research paradigm

Denzin and Lincoln (2000) describe paradigms as "human constructs" that indicate where the researcher is coming from when constructing meaning from evidence. Khatri (2020) states that a paradigm is a framework of connected practices and ways of thinking that establish inquiry parameters in relation to ontology, epistemology, and methodology, each of which is explained briefly in the sub-sections below.

5.1.1 Ontological assumptions

In the context of this study, the ontological assumption guiding the research is firmly rooted in the belief that reality is socially constructed (Gergen,2015). Herein lies my role – not as a detached observer but as an active participant, seeking to understand and interpret the subjective experiences and perspectives of the participants. This ontological assumption is underpinned by the recognition that diverse perspectives on the world and individual experiences are not merely idiosyncratic but are deeply influenced by the interplay of social and cultural factors (Al-Ababneh, 2020). In embracing this ontological stance, I explored the socially constructed nature of reality within a qualitative case study.

The research embraces the idea that the nature of reality and existence, particularly within the space of collaborative professional learning in mathematics teacher education, is not an objective, pre-given entity but is actively shaped, negotiated, and constructed through social interactions, cultural contexts, and shared meanings - in this case, by MTEs within a professional learning community who co-construct their understanding of teaching knowledge through collaborative endeavours. One of my assumptions is that the MTEs' experiences, perspectives, and knowledge they generate collectively contribute to shaping their professional identities.

This ontological perspective is consistent with the broader understanding that social interactions, cultural influences, and shared practices within a professional learning community play a fundamental role in shaping the MTEs' perceptions of what constitutes effective teaching knowledge and practice. As Scotland (2012) suggests, ontology is concerned with understanding the fundamental assumptions employed to acknowledge the existence of something or the essence of social phenomena being studied. In the context of this research, another assumption is that the nature of knowledge for teaching is socially constructed through collaborative professional learning practices within the mathematics teacher education community.

Furthermore, this ontological stance aligns with the social constructivist view that foregrounds social interactions and collaboration in the construction of knowledge (Ulbert, 2014). The MTEs' shared experiences within the PLC are seen as integral to the co-construction of knowledge for teaching in mathematics pre-service teacher education. A further assumption is that different perspectives and diverse experiences within the community contribute to the multifaceted and socially negotiated nature of knowledge for teaching.

5.1.2 Epistemological assumptions

Epistemology is concerned with knowledge acquisition and validation (Gall et al., 2003): with understanding how we can learn about the world and what forms the basis of our knowledge (Ormston et al., 2014). In the social sciences, there are two primary epistemological debates. The first revolves around the question of whether knowledge is arrived at inductively (by observing patterns in the world) or deductively (through the testing of logically derived hypotheses against observations). This debate is relevant to the exploration of how MTEs construct their knowledge during collaborative professional learning sessions.

The second critical debate relates to the relationship between the researcher and the subjects being studied, which influences the interaction between 'facts' and 'values.' This study is underpinned by the belief that participants' experiences and knowledge are co-constructed through the researcher-participant interaction. This belief acknowledges my involvement in shaping and interpreting the information gathered through a dialogic process, emphasising that the researcher cannot be entirely neutral (Ritchie et al., 2003; Ormston et al., 2014). Consequently, the collection and interpretation of data are influenced by the interactive relationships between the researcher and the researched community.

5.1.3 Social constructivism

Social constructivists theorise that learning occurs predominantly through social interaction and within cultural contexts, rather than solely within individuals (Schreiber & Valle, 2013). In this study, the learning environment is the PLC of the MTEs. Prawat and Floden (1994) emphasize the key role of consensus-building processes within learning communities, where members collaboratively construct meanings and knowledge.

While critics highlight potential pitfalls of social constructivism, such as over-reliance on group consensus and the risk of prioritising socially accepted "truths" over objectivity (Krahenbuhl, 2016), for this study a social constructivist lens remains valuable for understanding the complex interplay between practitioners, their work, and the broader socio-cultural and political contexts that shape the community of practice in which MTEs' professional knowledge develops (Lave & Wenger, 1991). Specifically, a social constructivist perspective enables explanation of how MTEs, through interactions within their PLC, negotiate and co-construct understandings about effective pedagogies, curriculum design, and their roles as teacher educators. These shared understanding emerge from the MTEs' diverse experiences, dialogue, and efforts to reach consensus, which are inevitably shaped by the norms, discourses, and power dynamics operating within the broader professional and institutional contexts they inhabit.

5.2 Qualitative research approach

Qualitative researchers aim to explore and understand phenomena within their real-world contexts (Denzin & Lincoln, 2023) and thus a qualitative approach is particularly well-suited for studies that aim to capture the rich complexity of social settings, interactions, and meanings from the perspectives of the participants themselves (Creswell & Poth, 2016). In this study,

adopting a qualitative research approach enabled an in-depth examination of the professional learning experiences and practices of MTEs within the context of their PLC.

Bryman (2008) outlines four key characteristics that distinguish qualitative research:

1. *Inductive Approach:* Qualitative research employs an inductive analytical process, where theories and hypotheses emerge from the data rather than being predetermined. This allows for the exploration of unanticipated findings and the development of context-specific understandings.
2. *Focus on Meanings and Interpretations:* Qualitative research prioritises understanding how individuals make sense of their social realities and the meanings they ascribe to their experiences, behaviours, and interactions within particular contexts.
3. *Naturalistic Setting:* Data collection in qualitative research typically occurs in naturalistic settings, allowing researchers to study phenomena as they unfold in their authentic environments, without manipulating or controlling variables.
4. *Flexible and Emergent Design:* Qualitative studies often employ flexible and emergent research designs that can adapt to unanticipated circumstances, evolving insights, and emerging lines of inquiry throughout the research process.

By embracing these characteristics, the qualitative approach taken in this study facilitated an in-depth exploration of the MTEs' professional development journeys within their PLC. It allowed for the collection and analysis of rich, contextual data from a purposefully selected sample of MTEs, shedding light on their lived experiences, interactions, and meaning-making processes.

The qualitative nature of this research enabled the examination of nuanced aspects of the MTEs' professional practices, such as their pedagogical decision-making, curriculum design approaches, and the negotiation of their roles as teacher educators. Furthermore, it provided insights into how these practices were shaped by the socio-cultural dynamics, discourses, and power relations within the PLC and the broader institutional and professional contexts in which the MTEs operated.

Through the use of qualitative methods, such as interviews, observations, and document analysis, this study aimed to generate a holistic understanding of the complex interplay between the MTEs' individual experiences, their collaborative learning within the PLC, and the contextual factors influencing their professional development trajectories.

5.3 Qualitative case study research design

This study has employed a qualitative case study research design to explore and understand the complexities and nuances inherent in the professional learning of MTEs about teaching mathematics to first-year pre-service teachers. A case study approach is appropriate for this investigation as it allows for an in-depth exploration of a specific phenomenon within its real-life context (Yin, 2015). The qualitative nature of the design facilitates a rich, contextualised understanding of collaborative professional learning practices and their impact on knowledge for teaching in teacher education. By involving myself in the MTEs' professional lives, I sought to capture not just the "what" but also the "why" and the "how" of the collaborative learning processes that inform their knowledge for teaching and, ultimately, their instructional practices.

While data collection was centred on a single institution, the research constitutes a case study with multiple units of analysis (Yin, 2018). The primary case focuses on the investigation of MTE professional development within a PLC, with the four MTEs serving as distinct units of analysis. The decision to utilise a multi-unit approach further strengthens the research offering several advantages:

1. It allows for within-case and cross-unit analysis, enabling the identification of patterns across different contexts while maintaining the holistic perspective of the larger case
2. It enhances the analytical depth by examining how individual MTEs' experiences contribute to and are shaped by the broader professional learning context
3. It provides opportunities to explore both convergent and divergent experiences, strengthening the theoretical contributions of the study

The multi-unit approach resonates with Neuman's (2014) assertion that examining multiple units enables researchers to uncover both commonalities and variations within a phenomenon. This analytical strategy facilitates the identification of recurring themes and patterns while remaining attentive to instances that disrupt or challenge these patterns, thereby enriching our understanding of the complexity of professional learning. To address potential limitations associated with case study research, particularly concerns regarding methodological rigour and generalisability, several strategies such as methodological triangulation through multiple data sources, member checking to ensure accurate representation of participants' experiences, detailed documentation of research procedures and decision-making processes, rich, thick descriptions to support transferability of findings and explicit acknowledgement of researcher positionality and potential biases were implemented.

While statistical generalisation is not the primary objective of this research, the study aims to contribute to theory-building and provide insights into the complex nature of MTEs' professional learning and classroom practices. Ultimately, the choice of the qualitative case study design, particularly in its multi-unit form, was made in order to understand the impact of professional learning on knowledge construction, and instructional practice among MTEs.

5.4 Selection and description of study participants

I conducted an in-depth case study involving four male Mathematics Teacher Educators (MTEs) chosen from one of seven teacher training colleges in the Ashanti region of Ghana. The selection of participants was purposeful, meaning I specifically chose individuals who could provide rich and comprehensive information for the study. These MTEs were selected based on their extensive experience teaching algebra in pre-service teacher education for at least five years and their willingness to participate in the study. It was anticipated that the selected MTEs, with their extensive teaching experience in algebra methods and content, would play a significant role in their PLC. Their deep understanding of both the pre-service teacher education mathematics curriculum and the school-wide mathematics curriculum further reinforced their potential influence in shaping mathematics education.

Although there were 45 potential participants from the seven colleges of education across the Ashanti region, I selected four specific individuals, Sam, Dan, Nick, and Mark (pseudonyms), to participate in both phases of the study. Their selection was based on their teaching experience in mathematical content and methods in algebra and their strong enthusiasm for participating in PLCs. Additionally, the qualifications and experience of each of the four participants are summarised in Table 5.1. It's worth noting that Mark is not included in the table as he withdrew from the study, as mentioned earlier.

Table 5.1: Overview of research participants

Participants	Designation(s)	Educational background	Years of teaching experience.	Years of Teacher education experience
Sam	Member- Department of Mathematics/ICT Education, Students Affairs Officer	Teaching Cert – A B.Ed. mathematics M.Phil. Mathematics Education	25	15
Dan	Member- Department of Mathematics/ICT Education, Quality Assurance Officer	Teaching Cert – A B.Ed. mathematics MSc Mathematics Ph.D. Mathematics Education(ongoing)	20	8
Nick	Senior Tutor- Department of Mathematics and ICT Education	B.Ed. Mathematics M.Ed. Mathematics M.Phil. Mathematics Education	17	17

5.5 Research methods

Following ethical clearance, data collection for the study was conducted in two distinct phases: March to May 2022 and February to April 2023 for phases 1 and 2 respectively.

Phase 1: Initial Exploration

In the first phase, qualitative data was gathered to understand the framing of the PLC and the existing professional learning practices of MTEs at a College of Education in Mampong-Ashanti, Ghana. The analysis of data collected in Phase One informed the design and facilitation of collaborative professional learning activities and the investigation of critical friendships in Phase Two. Data sources for the first phase included the following:

Table 5.2: Summary of data generated in Phase 1

Data Source	**Description**	**Purpose**
PLC Observation	Observations of the MTEs' weekly PLC meetings by using written reflections and narratives from MTEs.	To understand the collaborative dynamics, interactions, and focus areas of discussions in their PLC regarding the teaching of algebra. To document their learning experiences, challenges, and growth throughout the first phase
Pre- and Post-Lesson Interviews	Individual interviews with MTEs before and after their lessons	To gain insights into their planning, decision-making, and reflections on their teaching practices.
Lesson Observations	Observations of MTEs' actual teaching sessions	To witness their pedagogical approaches and interactions with PSTs in action.
Focus Group Interviews	Semi-structured interviews with small groups of MTEs	To explore what knowledge MTEs consider important from their perspectives, experiences, and perceived challenges within the PLC.

Phase 2: Exploration of Critical Friendships

The second phase of the study focused on the exploration of critical friendships as collaborative professional learning activities within the PLC, specifically examining the role of critical friendships in mathematics teacher education. This phase involved three of the initial four MTE subcases, as one MTE dropped out of the study. Data gathering for Phase Two is summarised in Table 5.3

Table 5.3: Summary of data generated in Phase 2

Source of data	Description	Purpose
PLC observations/ Critical Friends Group Meeting Observation	Three observations about 120 minutes per session at a weekly interval with field notes	To explore how the framing of the PLC aids the development of knowledge for teaching algebra in pre-service teacher education. To examine how critical friendships support change in MTEs professional knowledge and practices
Pre-observation interviews	About 30 minutes of semi-structured pre-lesson interviews of each participant: audio data	To learn about what MTEs intend to teach and the preparation made for the lesson.
Lesson observations	One classroom observation (about 1 hour) of each of the 4 participants with a critical friend (CF): video data, field notes	To ascertain how MTEs draw on their Mathematical Knowledge to support algebra instruction. To explore evidence of take-up from the PLC in MTEs teaching practice.
Post lesson interviews	90 minutes of semi-structured interviews with each of the 4 participant MTEs and their critical friends (CFs)	To explore how CFs support change MTEs professional knowledge and practice, as well as understand what CFs choose to focus on during peer lesson observation. consider important for their work in pre-service teacher education. To further clarify MTEs enactment of teaching practice

The two-phase research design was strategically conceived to provide a comprehensive understanding of professional learning within the Ghanaian mathematics teacher education context. The initial phase established a baseline of existing practices, while the second phase investigated the transformative potential of collaborative learning approaches.

I took several strategic steps throughout the study design and implementation to ensure a rigorous alignment between research questions and methodological approaches. Each research question was addressed through a curated mix of data collection methods following Miles et al.'s (2014) recommendation for comprehensive qualitative inquiry. These multiple data sources (see Table 5.2 & 5.3) ensured that each research question was explored from multiple perspectives, capturing the nuances of MTEs' collaborative practices.

The analytical approach (refer to Chapter 6) further reinforced this alignment through the consistent application of inductive and deductive thematic (Braun & Clarke, 2006, 2019) analysis across all research questions. This dual approach allowed for both theory-driven and

data-driven interpretations, enabling a more comprehensive understanding of the research inquiries. Specifically, the analysis process involved iterative coding (Boyatzis, 1998), where initial theoretical frameworks were used to develop preliminary codes, while simultaneously allowing emergent themes to surface from the data. Preliminary findings were continuously compared and refined, ensuring that the analytical process remained closely tied to the original research questions. This methodical approach allowed for a systematic examination of collaborative learning, knowledge development, and professional practice integration within the professional learning community, directly addressing the study's core research objectives. Table 5.4 shows a table matrix mapping research questions, data collection methods data analysis techniques and expectations.

Table 5.4: A table matrix showing the alignment of research questions, data collection method, data analysis technique and expected outcomes.

Research Question	Data Collection Methods	Data Analysis Techniques	Expected Outcomes
1. To what extent does working collaboratively in a PLC support the development of mathematical knowledge for teaching in pre-service teacher education?	- PLC observations, pre- and post-lesson Interviews, lesson observations and focus group interviews	- Inductive and Deductive Thematic Analysis - Narrative Analysis	- Identification of collaborative learning processes - Insights into knowledge development mechanisms - Understanding of collaborative impact on teaching practices
2. What knowledge do MTEs consider important to focus on in their collaborative work in pre-service teacher education?	- Focus group interviews, PLC observations, pre-observation Interviews	- Inductive and Deductive Thematic Analysis - Content Coding - Thematic Synthesis	- Mapping of priority knowledge areas -Understanding of MTEs' professional knowledge priorities - Insights into collaborative knowledge selection
3. What counts as evidence of the integration of PLC learnings into MTEs' pre-service teacher education practices, and is there evidence of such integration?	- Lesson Observations, post-lesson interviews, critical friends group Meetings, PLC observations	- Inductive and Deductive Thematic Analysis - Pedagogical Discourse Analysis - Practice Tracing	- Documentation of learning transfer - Identification of integration mechanisms - Evaluation of PLC impact on teaching practices
4. To what extent does critical friendship support changes in MTEs' professional knowledge and practice?	- Critical friends group meeting observations, field notes, pre- and post-lesson interviews, lesson observations	- Inductive and Deductive Thematic Analysis - Reflective Practice Analysis	- Understanding of critical friendship dynamics - Mapping of professional knowledge improvement - Evaluation of peer support effectiveness

5.5.1 Lesson observation

Observation is a crucial data source in case study research, as it allows for the examination of phenomena in their natural settings (Yin, 2015). In this study, lesson observations were conducted to witness firsthand the teaching practices of MTEs and to identify evidence of the influence of their PLC on their teaching of algebra to pre-service teachers.

Observation is a deliberate, systematic, and purposeful approach to observing and documenting interactions or phenomena as such phenomena unfold in their authentic contexts (LaGarry, 2019). In this study, I assumed the role of a non-participant observer during the MTEs' lessons. The research participants were informed of my presence as an observer, and in phase two, I was accompanied by the MTEs' critical friends, who also acted as non-participant observers. Unlike interviews, which rely on participants' self-reported accounts, observation allows researchers to directly witness the complexities of a phenomenon, capturing nuanced details that might otherwise remain elusive (Morrison, 2017). Through observation, I was able to capture not only the explicit strategies and techniques employed by MTEs but also the subtle nuances of their pedagogical interactions, decision-making processes, and in-the-moment responses to students' needs.

To facilitate a more comprehensive analysis, the lessons were video recorded. As Sherin (2004) argues, video recordings in teacher education research have the advantage of enabling researchers to develop a deeper understanding of classroom practices and to engage in more nuanced interpretations and reflections through repeated viewings. The video recordings enabled me to revisit and analyse the MTEs' lessons and to notice the details of their modelling, scaffolding, and feedback approaches, which can contribute to the developing of pre-service teachers' algebraic knowledge and skills.

During Phase One, I observed three lessons taught by each participant, focusing on their teaching of algebraic concepts such as functions, integers, ratios, rates, and proportions. To triangulate the data and address potential limitations of observation and video data (Powell et al., 2003), I employed methodological triangulation by integrating additional data sources, such as interviews, as discussed in the following section.

By combining direct observation with video recordings and interviews, I was able to gain a comprehensive understanding of the MTEs' practices, capturing both the observable behaviours and the underlying decision-making processes that shaped their algebra instruction for pre-service teachers. This approach allowed me to explore the connections between the MTEs' professional learning experiences within the PLC and the enactment of their pedagogical approaches in the classroom setting.

Table 5.5: Summary of MTEs' lessons observed during data gathering in 2022 and 2023.

MTEs	Year/ semester	Dates of observation	Duration	Lesson focus
Sam	2022	28-03-2022	1:58:17	Operation and properties on integers
		18-04-2022	1:47:36	Ratio, rates and proportion
		02-05-2022	1:52:47	Identifying and evaluating the sum, difference, product, and composition of functions
	2023	28-02-2023	0:53:23	Operation and properties on integers
		10-03-2023	0:45:06	Ratio, rates and proportion
		29-03-2023	0:47:19	Identifying and evaluating the sum, difference, product and composition of functions
Dan	2022	25-03-2022	1:42:23	Operation and properties on integers
		15-04-2022	1:56:04	Ratio, rates and proportion
		29-04-2022	1:26:51	Identifying and evaluating the sum, difference, product and composition of functions
	2023	08-03-2023	O: 57:12	Operation and properties on integers
		15-03-2023	O: 48:19	Ratio, rates and proportion
		23-03-2023	1:34:00	Identifying and evaluating the sum, difference, product and composition of functions
Nick	2022	24-04-2022	1:45:46	Operation and properties on integers
		14-05-2022	1:55:02	Ratio, rates and proportion
		29-04-2022	1:12:07	Identifying and evaluating the sum, difference, product and composition of functions
	2023	28-03-2023	0:63:18	Ratio, rates and proportion
		31-03-2023	0:57:09	Identifying and evaluating the sum, difference, product and composition of functions

5.5.2 Semi-structured interviews

Semi-structured interviews were a crucial data source in this study, as they allowed me to gain insights into the thoughts, experiences, and perspectives of MTEs regarding their professional knowledge and practices. Interviews are widely recognised as a significant source of case study data (Kvale, 1996; Yin, 2015), as they enable researchers to explore the underlying motivations, beliefs, and decision-making processes that shape participants' behaviours and actions.

I conducted two semi-structured interviews with each MTE participant: 10-15 minutes pre-lesson interviews and 90-minute post-lesson interviews. The pre-lesson interviews aimed to understand the MTEs' intentions, planning, and preparation for their lessons, while the post-lesson interviews were designed to investigate and understand their decision-making processes,

perceived challenges during the actual teaching and their overall reflections on the lesson taught.

The interview questions (see Appendices A and B) were designed to explore the MTEs' thoughts and actions in relation to research questions 1, 2, and 4, focusing on their professional knowledge and practices, and on the influence of PLC activities on their teaching of algebra to PSTs. During the interviews, I followed a flexible approach, allowing for follow-up questions and prompts based on the participants' responses (Kvale, 1996). This flexibility enabled me to probe deeper into their lived experiences and perspectives, capturing nuanced details and unanticipated insights that might have been overlooked with a more rigid interview structure. All interview conversations were audio-recorded with the informed consent of the participants. This ensured an accurate and comprehensive record of the interviews, allowing me to revisit and analyse the data in depth.

While interviews are a valuable data source, they are not without limitations. Reactive effects, where participants may exhibit socially desirable behaviours due to the presence of the researcher, and investigator effects, where personal biases may influence the way questions are asked, are potential concerns (Woodside, 2010). To mitigate these limitations, I took several measures. First, I spent an extended period with the participants during lesson observations, which helped to build trust and rapport, reducing the likelihood of reactive effects. Second, at the end of each interview, I shared the main points I had captured with the participant, allowing them to confirm the accuracy of my notes or provide corrections or additions. This member-checking process helped to ensure the credibility and trustworthiness of the interview data.

Furthermore, by triangulating the interview data with other data sources, such as lesson observations and observations of the PLC, I was able to corroborate and validate the insights gained from the interviews, reducing the potential impact of reactive or investigator effects.

The semi-structured interviews provided valuable insights into the MTEs' professional knowledge, their decision-making processes during lesson planning and instruction, and their perceptions of the role played by the PLC in shaping their professional development and practices. By combining these insights with data from other sources, I was able to develop a comprehensive understanding of the complex interplay between the MTEs' individual experiences, their collaborative learning within the PLC, and the enactment of their pedagogical approaches in the classroom setting.

5.5.3 Focus group interview

Patton (2002) describes a focus group interview as an interview with a small group of individuals on a specific topic and lists the following advantages of such interviews in qualitative research: 1). data is economically gratifying; information can be gathered from several participants in a short period of time; 2). interactions among respondents may contribute to enriched data; 3). it is possible to rapidly discern whether there are unified ideas, common viewpoints, or significant differences on an issue, and 4). focus groups are usually enjoyable for participants. In this study, the focus group interview was my last interaction with participants in terms of data collection. The primary purpose of this group interview was to reflect with MTEs on what they consider important for their work in teacher education. In the discussion, I also focused on the knowledge and practices of MTEs and asked questions based on my lesson observations. During the conversation, I also explored with the group their thoughts on how collaborative professional development could support MTEs' algebra instruction in a pre-service teacher education programme.

5.6 Connecting the two Phases: How Phase One informed Phase Two

In this study, the second phase of data collection and analysis was informed by the findings of Phase One. During the Phase One focus group interview, MTEs claimed that they employed the support of critical friends in their PLC to observe their lessons and provide reflective feedback. However, there had been no evidence of this practice in my engagement with MTEs in their PLC, even though current teacher education policy mandates that this be done. For this reason, a second phase of the research was initiated in which I visited the same MTEs for another cycle of lesson observations, this time with their critical friend, and afterwards engaged with the MTE whose lesson had been observed and the critical friend to explore further what they consider important for pre-service teacher education and how to use their mathematical knowledge to support instruction in pre-service teacher education.

5.7 Exploration of critical friendships in this study

While the data collected from interviews with MTEs in Phase One indicated that the support of critical friends contributed to their professional learning, my observations and interactions with members of the PLC revealed a disconnection between this claim and reality. Despite the policy documents suggesting that MTEs should collaborate with critical friends to enhance their professional knowledge and practices, as indicated earlier, I found no evidence of such collaborative efforts during the data collection. Interestingly, at the beginning of Phase Two of the study, it became evident that some PLC members were unfamiliar with the concept of

critical friends and the purpose of critical friendships, suggesting a lack of understanding or implementation of this practice within the community.

As the facilitator of this study, I encountered initial reluctance from participants to be scrutinised by their peers and to give feedback to each other. However, by encouraging them to maintain a non-evaluative and supportive stance, I persuaded each to take on the role of a critical friend. Selecting critical friends was not complex since all the MTEs belong to the same PLC. They were familiar with each other's academic strengths and teaching goals because they taught the same course for first-year PSTs. The pairing process created synergistic relationships where MTEs could benefit from each other's expertise and perspectives (Kastberg & Grant, 2020).

Each pair consisted of an MTE and a critical friend, also an MTE, who would provide constructive (non-evaluative) feedback. The three pairs of MTEs formed a circular critical friendship triad while belonging to a larger PLC that used a plan-teach-reflect approach to members' practice-based activities. As shown in 'the curly bits' of Figure 5.1, MTEs did not necessarily receive feedback from the person to whom they provided feedback.

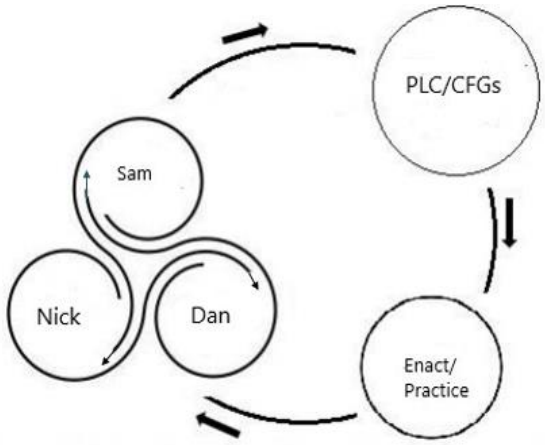


Figure 5.1 Cyclic structure to critical friendships

Certain modifications were made to accommodate the needs of participants. For instance, I introduced the cyclic approach depicted in Figure 5.1. While this approach does not align with the 'ideal' of paired reciprocity described in the literature, it enabled careful observation and constructive feedback. Also, some critical friends preferred additional guidance and structured activities during lesson observations, as they had never previously taken on such a role within mathematics teacher education. Experienced MTEs assisted first-timers in facilitating reflection and analysis, resulting in productive feedback.

During my observations, I accompanied each MTE and their critical friend individually rather than observing all three simultaneously. By rotating between critical friend pairs (Sam-Nick, Nick-Dan, and Dan-Sam), I gained insights into their approaches to mutual engagement. This immersive observation approach, together with the interviews, and observations of the larger PLC or CFGs enabled comprehensive data collection.

In addition to meeting in the PLC each MTE and critical friend met once a week for reflective dialogue and self-assessment their mutual engagement. Within the PLC (which included participants and non-participants in this study), every session began with a brief check-in, during which members shared updates on previous lessons or concerns since the last meeting. The facilitators shared a specific teaching experience, lesson plan, or instructional challenge, while members actively listened, engaged in structured conversations and activities and asked probing questions to encourage deeper reflection (Kuh, 2016). Critical friends would take turns sharing with the larger PLC group their teaching experiences, lesson plans, or any challenges they had encountered. This process continued for about three months as I engaged with MTEs and their critical friends throughout the Phase Two data collection process, prompting the critical friends to consider their strength and areas where they would like to receive support from their friend and the PLC.

5.8 The role of the researcher

My role in the research process was multifaceted, shaped by my extensive teaching experience in pre-service teacher education and my position as a mathematics teacher educator at a Ghanaian College of Education. I drew on fourteen years of experience working with in-service and pre-service teachers. Recognising my insider status within the educational community, I adopted a non-participant observer role during the observations of lessons and PLC activities. This approach aimed to enable unbiased observations of natural events without influence, in alignment with accepted practice in qualitative case study research (Yin, 2015).

Researchers are not neutral observers but active participants in the research process and are influenced by their social, cultural, and historical position concerning the research topic and participants (Foote & Bartell, 2011; Holmes, 2020; Malterud, 2001). Due to my insider identity, I had little difficulty accessing the cultural group (Ritchie et al., 2009) and I included every piece of colloquial language used by MTEs in the interview transcription. I attempted to examine my own biases, assumptions, and values, recognising their potential influence on the research process and outcomes (Bourke, 2014; Bryman, 2016), to undertake nuanced and

contextually sensitive data collection and analysis and to critically reflect on my position throughout the research process (Foote & Bartell, 2011).

I employed various strategies to navigate the complexities of positionality and reflexivity in this research. First, I kept a reflexive journal, documenting my thoughts, feelings, and personal reactions as they emerged during data collection and analysis (Holmes, 2020). Additionally, I actively sought individual and collective perspectives while engaging in critical dialogue with participants. By listening attentively to participants' voices and valuing their experiences, I hoped to minimise imposing my own preconceptions on their narratives. Furthermore, I attempted to maintain transparency and openness about my role as a researcher throughout the study.

5.9 Trustworthiness

When conducting a qualitative case study, establishing trustworthiness is essential for ensuring the credibility and validity of the research findings. Schwandt, Lincoln and Guba (2007) consider four fundamental criteria of a study's findings: credible, transferable, dependable, and confirmable.

Credibility refers to the extent to which a study's findings are believable and accurate (Yin, 2015). The credibility of this study was established through an extended data gathering process involving MTEs' classroom teaching over six months in two phases and extended engagement with MTEs in their natural environment.

Transferability refers to the extent to which the findings of a study can be generalised to other settings or contexts (Lincoln & Guba, 1985). In this study, I enhanced transferability by providing a detailed description of the context and participants, using purposive sampling to select participants who are representative of the population of interest, and using thick descriptions of the data to help readers understand the nuances of the case. However, it is essential to note that in a case study, transferability is not about generalising findings statistically, but rather about understanding the potential similarities and differences between contexts.

Dependability refers to the extent to which the study findings are consistent and reliable over time. This study established dependability using systematic and rigorous data collection and analysis methods, keeping a detailed audit trail of all the research activities. I sought the input and insights of the Wits Maths Connect - Secondary (WMC-S) team. Through discussions and critical reflections, I gained alternative viewpoints and ensured that others in the field supported

my conceptions and interpretations. Moreover, my extended engagement with MTEs in the field during the two-phase data collection process helped me reach saturation, by which I became confident that I had comprehensively explored the research topic, thereby enhancing the study's dependability.

Confirmability addresses the objectivity and neutrality of the findings, ensuring they are grounded in the data rather than the researcher's biases or preconceptions (Korstjen & Moser, 2017). Throughout the study I aimed at reflexivity. I was aware of and critically examined my assumptions and potential biases throughout the research process. I also asked the opinions of academics for constructive feedback. I maintained an audit trail during the research, making the research process transparent and enabling others to assess the confirmability of the findings.

5.10 Ethical considerations

Before entering the research site, I obtained the necessary permission to conduct research from the principal of the College of Education, where the study was carried out. Before the study began, I received informed consent from the participants regarding their involvement and the recording of the interviews and observations (Creswell, 2012; Kvale, 1996; Merriam, 1998). MTEs agreed to participate in the study, knowing that it was completely voluntary and that they could leave the study at any point without repercussions. To promote confidentiality, anonymity and respect for the participants' privacy, I omitted any identifying information regarding the precise setting and college where the research was conducted. I also gave the participants pseudonyms (Creswell, 2012; Kvale, 1996; Merriam & Tisdell, 2015). I paid close attention to debriefing the participants by encouraging them to ask questions for information whenever they felt the need. In addition, I discussed the study's potential benefits with the participants and their community of practice before the study commenced and throughout the process (Creswell, 2012; Merriam & Tisdell, 2015).

As a mathematics teacher educator, I was aware that my position could potentially influence how MTEs participated. Despite this, I made it clear to the MTEs that I was primarily interested in observing their regular teaching practices per their established curriculum. During my class visits, I scheduled my observations to align with the topics they were already teaching as part of their usual teaching schedule. I did not request the MTEs to prepare special lessons or to demonstrate specific teaching methods or approaches using particular mathematical representations.

5.11 Chapter summary

In this chapter, I have described and discussed the methods used to carry out a qualitative case study framed by social constructivist theory. The lesson observations, interviews and discussions aimed to capture MTEs' experiences and reflections on these in regard to their teaching practice, knowledge for teaching and professional learning. I gathered data for the study through video-recorded lesson observations and audio-recorded semi-structured interviews and undertook thematic analysis of the data.

CHAPTER 6

DATA ANALYSIS

6.0 Introduction

This chapter details the two-phased data analysis process in which both deductive and inductive thematic analysis approaches were employed. For Phase One, the MTEs' professional learning experiences within the PLC were first analysed deductively, using Desimone's framework, and then an inductive approach was used to identify emergent themes. For Phase Two the role of critical friendships in supporting MTEs' growth was deductively analysed by using Wenger's three-dimensional practice lens. By triangulating multiple data sources, including interviews, observations, and videos, the analysis aimed to develop a rich understanding of MTEs' professional learning, teaching practices, and the impact of critical friendships. The subsequent sections provide a detailed description of the analysis processes.

6.1 Framework selection and operationalisation

The study employed two complementary theoretical frameworks to analyse the professional learning of MTEs within their PLC. The selection and integration of Desimone's (2009) professional development framework (see Chapter 4), along with Wenger's (1998) dimensions of practice, provide complementary perspectives for understanding both the structural elements of professional development and the sociocultural aspects of learning within a PLC. Desimone's framework was applied using specific analytical criteria. The content focus component was examined by analysing discussions related to subject matter, the development of pedagogical content knowledge, and advancements in curriculum knowledge during PLC meetings. This analysis included how MTEs engaged with mathematical content and refined their teaching practices.

The collaborative learning component, which emphasises active learning and collective participation, was explored through interactive learning activities, peer feedback processes, and examples of collective problem-solving. I analysed the duration component through temporal mapping of professional learning activities, documenting patterns of sustained engagement, such as the frequency of PLC meetings and the progressive development of teaching practices. The alignment component (adapted from coherence) was assessed by examining how professional learning activities corresponded with institutional goals, curriculum objectives, and existing teaching practices, reflecting the contextual realities of the study setting.

During my research, the interactions between MTEs within their PLC revealed nuanced patterns of collaboration (e.g., critical friendship) that resonated with Wenger's conceptual lens of social learning. As I observed and analysed these critical friendships, I found myself drawn to how MTEs negotiated their professional relationships and shared understanding. The mutual engagement between these educators was far from formulaic. Conversations unfolded organically, marked by moments of genuine dialogue, collaborative problem-solving, and professional vulnerability. Critical friends would challenge each other's perspectives, share classroom experiences, and co-construct knowledge in ways that transcended simple professional courtesies.

Their joint enterprise emerged not from prescribed objectives, but from a shared commitment to improving mathematics teacher education. These MTEs were united by an implicit understanding of their collective purpose, supporting pre-service teachers' mathematical and pedagogical development. Their goals were dynamic, and continually refined through ongoing dialogue and shared reflection. The shared repertoire that developed was particularly compelling. Gradually, a common language, set of references, and collaborative practices evolved. These weren't predetermined or imposed but grew naturally from their sustained interactions—a testament to the organic nature of professional learning.

Wenger's framework provided a lens to understand these interactions, but the data spoke first. The theoretical concepts illuminated the rich, complex social learning process I observed, rather than constraining or predetermining my interpretation. The complementary nature of these frameworks offered analytical advantages. Desimone's framework provided a structured method for professional development analysis, while Wenger's framework facilitated a deeper understanding of social learning processes. Together, they offered a comprehensive lens that encompassed both individual and collective aspects of professional learning within the PLC context. This integrated theoretical approach guided the subsequent analysis processes detailed in the following sections, establishing a solid foundation for understanding both the structural and social aspects of MTEs' professional learning experiences while remaining open to emergent themes and patterns in the data.

6.2 Data analysis: Phase One

The data analysis process employed in this study involved integration of deductive and inductive approaches to enable a comprehensive examination of the qualitative data collected (Fereday & Muir-Cochrane, 2006). The deductive approach utilised predetermined categories

and themes derived from the research questions and theoretical framework which provided a structured foundation for the analysis. Simultaneously, using an inductive approach enabled the identification of emergent themes and patterns directly from the data. This integrated approach facilitated analysis aligned with the study's key areas of inquiry while remaining grounded in the participants' lived experiences and contextual realities.

6.2.1 Deductive thematic analysis

Using a deductive thematic analysis approach involved defining a set of codes and categories based on the research questions and the conceptual framework of the study (Swain 2018). The structured nature of the deductive approach assisted with organising and interpreting data in alignment with the study's key areas of inquiry. For each participant, I compiled a thematic chart in order to organise their responses from interview transcripts into relevant categories. These charts, framed by the study's theoretical underpinnings, particularly Desimone's (2009) conceptual framework for effective professional development, played a crucial role in structuring the data and making it more manageable for analysis.

The deductive thematic analysis approach, with its structured and systematic nature, offered several distinct advantages in the research process. Firstly, it facilitated the collection and integration of relevant data from multiple sources, thereby enabling a comprehensive analysis. Secondly, it allowed for a degree of systematisation, making it easier to identify patterns, relationships, and comparisons across different data types. Lastly, the sequential organisation of data from various sources streamlined the analysis process, instilling confidence in the thoroughness and robustness of the approach.

While the deductive approach provided a foundation for my analysis, I was aware of its potential limitations. To ensure I did not overlook important themes and patterns, I took proactive steps. I remained open to identifying and incorporating additional issues that surfaced during the data analysis process, allowing for a more flexible analysis beyond the predetermined themes. For example, while analysing collaborative interactions within the PLC, I discovered an unexpected theme related to exam-oriented teacher education, which was not explicitly addressed by the initial theoretical frameworks. Additionally, I conducted an inductive thematic analysis for data that did not align with the a priori themes, particularly those related to the challenges within the PLC.

6.2.2 Inductive thematic analysis

To complement the deductive approach, I employed an inductive thematic analysis (Fereday & Muir-Cochrane, 2006) to capture themes and patterns that emerged organically from the data. This approach involved an iterative process of coding, categorising, and theme development, allowing for a data-driven exploration of the participants' experiences, perspectives, and contextual factors influencing their professional learning and practices. Remaining open to the emergence of new themes, facilitated capturing at least some of the rich complexities and contextualised understandings characteristic of qualitative research.

Throughout the inductive analysis process, I compared and refined the identified themes, ensuring that they accurately represented the patterns and meanings inherent in the data. This iterative process involved revisiting the data, refining and consolidating themes, and exploring their relationships and connections to develop a coherent and comprehensive understanding of the phenomena under investigation.

6.2.3 Integrating deductive and inductive approaches

The integrated approach facilitated a comprehensive exploration of the phenomena under investigation by identifying both anticipated and emergent themes. By triangulating findings from multiple data sources and employing both analytical approaches, the study aimed to develop a rich, contextualised understanding of the professional learning and development of MTEs within their PLC context.

6.2.4 Data selection and analysis: Lesson observations

Data selection involves focusing on specific information relevant to the theoretical framework, research questions, and the chosen data collection instruments (Goldman et al., 2007). In this study, the data selection process brought into focus 'events' or 'teaching tasks' related to the different approaches and topics in teaching mathematics in pre-service mathematics teacher education. The topics selected for observation, including functions, ratios, rates, and proportions, were not intentionally chosen but were instead the were the lessons being taught during my visitation periods, as I had no intention of influencing the topics being covered. Coincidentally, these topics were being taught during both phases of data collection. In analysing data from the two phases, video clips were strategically selected to facilitate the exploration of research questions 2 and 3. The analysis of these selected clips aimed to identify whether, and if so how, the PLC activities influenced the MTEs' teaching practices and knowledge for teaching mathematics. During the analysis of data from Phase One, when the

relevant teaching practices were in focus, the teaching practices listed in Table 6.1 were identified.

Categories and codes	Sub-categories
Analysing PST's' mathematical work and thinking – **AMT** Designing, modifying and selecting mathematical tasks – **DMS** Establishing and revising mathematical goals for PSTs – **ERM** Accessing and using tools and resources for teaching – **AUR** Explaining mathematical ideas and solving mathematical problems – **EMS** Building and supporting mathematical community and discourse – **BMC**	1. Setting lesson indicators at the beginning of the lesson 2. Choosing and developing definitions useable within TE and school mathematics 3. Unpacking the definition of functions: a) Using different representations b) Using colloquial shortcuts c) Using a range of example spaces to give meaning to the definition. 4. Providing examples that support the teaching of TE and school mathematics 5. Providing a quick response to unanticipated detour /distraction 6. Inspecting equivalence in PSTs' work 7. Linking representations to underlying ideas 8. Problem-solving 9. Using mathematical symbols, notations and language 10. Unpacking the fundamental meaning of mathematical symbols and notations 11. Making a connection between function and algebra 12. Finding examples to elaborate specific mathematical points 13. Assessing the plausibility of claims made by PSTs 14. Using multiple strategies to help PSTs re-organise their understanding 15. Providing procedural and conceptual explanation/insight 16. Checking on PSTs to ascertain their level of understanding

Table 6.1 illustrates the coded categories and sub-categories of MTEs' teaching practices.

The Knowledge of Algebra for Teaching (KAT) framework helped structure the aspects of the analysis which investigated the MTEs' enactment of practice in classroom settings and what they considered important for teaching algebra in pre-service mathematics teacher education. I employed a priori themes to label the MTEs' teaching tasks/practice as captured in the video clips and transcripts. For each teaching task, I identified the use of mathematical knowledge related to what MTEs had discussed in their PLC. This approach led to examining their practice for evidence of take-up in the Phase Two observations.

6.2.5 Analysis of interviews and PLC observations

To explore research questions 1, 2, and 3, Desimone's (2009) continuous professional development framework was employed for analysing the focus group interviews and observations of the PLC in action. This framework facilitated the investigation of how MTEs construct professional knowledge within their PLC by identifying various possibilities for professional learning that could inform their practice. The framework aligns with the study's aim to understand the influence of the PLC and/or of collaborations on the MTEs' professional development and teaching practices.

The analysis of the interview and PLC observation data involved both deductive and inductive approaches to thematic analysis. First, I organised the data into patterns, emergent categories, and themes using Desimone's (2009) framework deductively. This enabled me to identify common themes in MTEs' discussions about how the PLC aided the development of their mathematical knowledge for teaching algebra in pre-service teacher education. I coded and sorted the data under the five a priori categories of Desimone's model: 1). content-focused, 2.) active learning, 3.) collective participation, 4). coherence, and 5). duration.

However, as the analysis progressed, the boundaries between active learning and collective participation activities in the PLC appeared to dissolve. Therefore, I chose to term a single combined category 'collaborative learning', which resonates with Smith and MacGregor's (1992) assumptions about collaborative learning. Additionally, the data seemed to foreground alignment rather than coherence, hence the term alignment was chosen. Next, I identified aspects of the data that could not be coded and sorted by using the framework and this identification led me to employ inductive thematic analysis. During this phase, I read the data further to identify and sort emergent ideas into themes. For example, there were sections where MTEs discussed their expectations and challenges in pre-service teacher education, which the framework did not capture. I grouped these sections together for further discussion.

Table 6.2 presents excerpts of MTEs' talk, illustrating the themes and categories that emerged from the deductive and inductive analysis of the focus group interview and PLC observation data. These excerpts were selected to represent the various aspects of the MTEs' professional learning experiences within the PLC, as well as their reflections on the broader context of pre-service teacher education.

Table 6.2: MTEs talk categories and themes extracted from focus group interviews and PLC observations.

Content-Focused – CF	use of TLMs (artefacts and inscriptions) Multiple teaching approaches/strategies Consult experienced members for conceptual and procedural understanding Mathematical horizon Talk about TLMs (artefacts, inscriptions) Set individual and group targets based on curriculum demands Problem-solving PTs' errors and misconception / mathematical confusion Weekly session Use of mathematical language & notations Multiple examples Discuss PTs' challenges in algebra Use of multiple examples Setting group targets based on curriculum requirements Task selection and sequencing for a particular lesson Teacher education mathematics curriculum School mathematics curriculum Teacher education mathematics content Talk about the teacher education structure School mathematics content Teacher education mathematics pedagogy School mathematics pedagogy Discuss teacher education structure MTEs Reflect on practice Talk mathematics(algebra) 2 hours per session Provide cognitive support for newly appointed MTEs Topic-specific instructional strategies Using critical friends to observe lessons and provide feedback Guide novice members through team teaching Teacher education and school mathematics curricula alignment Mathematical justification
Active Learning – AL	
Collective Participation – CP	
Coherence - Co	
Duration – Du	

Figure 6.1 below, depicts the analytical framework and the final themes derived from adaptation of Desimone's (2009) model, which guided the analysis of the focus group interviews and PLC observations. This framework highlights the key components of effective professional development, including content focus, active learning, collective participation, coherence, and duration. By examining the data through this lens, the analysis aimed to understand how the PLC facilitated the MTEs' professional learning and development.

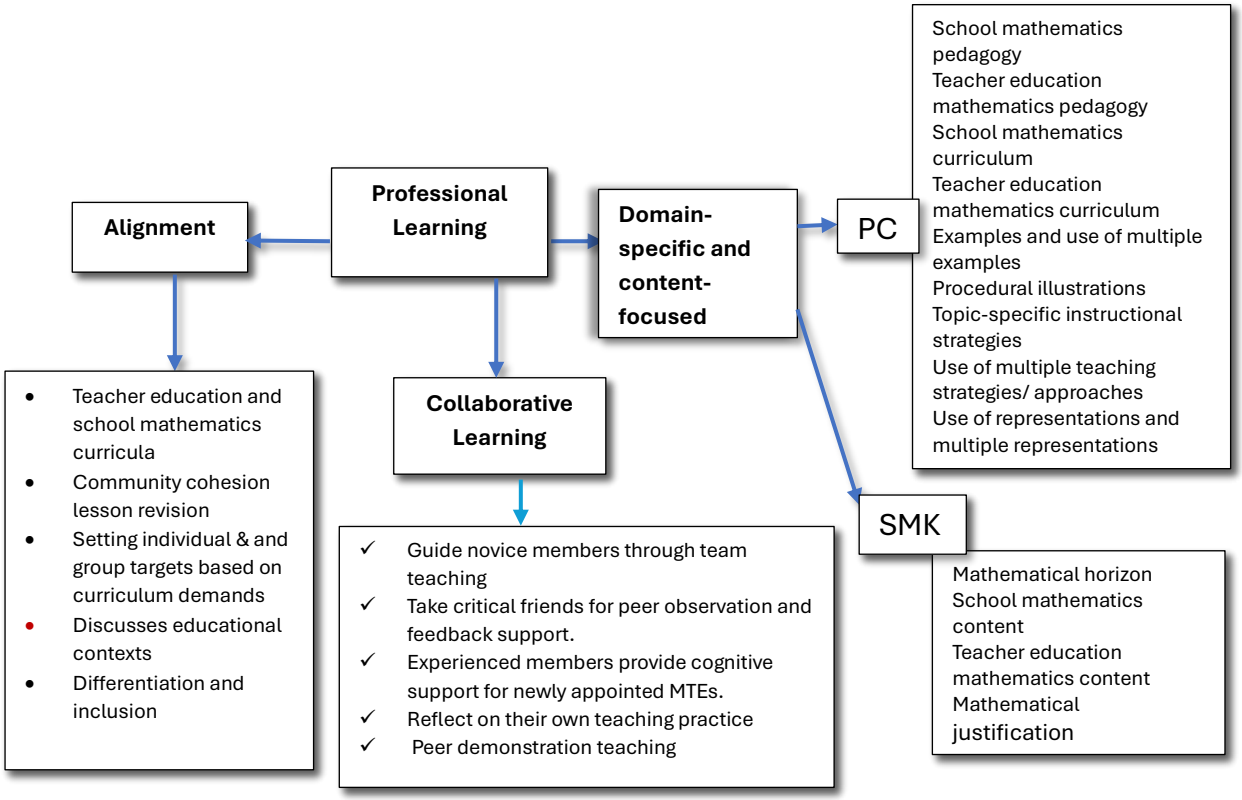


Figure 6.1: Framework for analysing data generated in Phase One

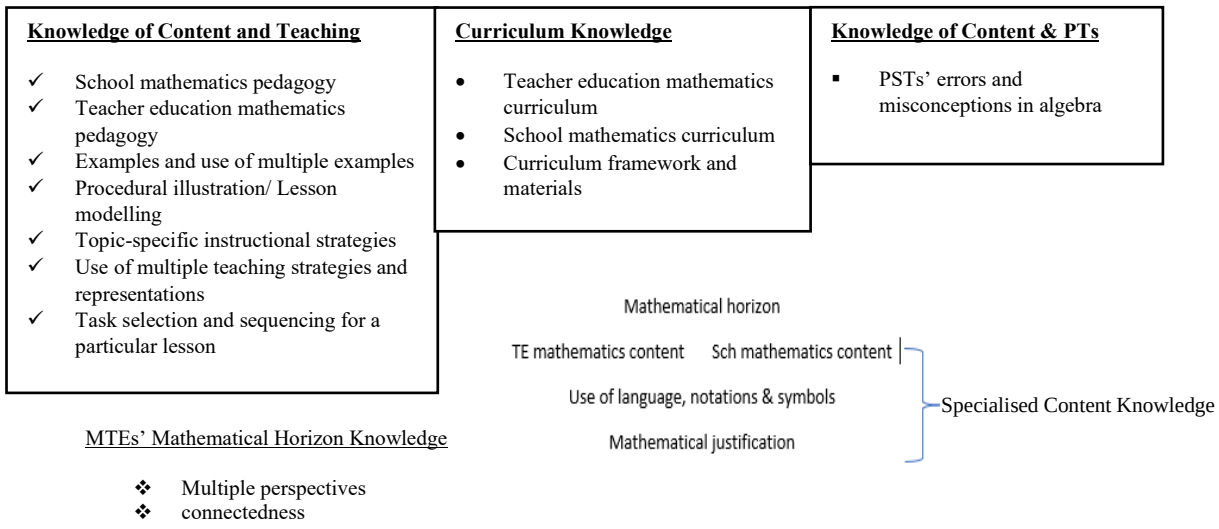


Figure 6.2: Pedagogical blocks depicting pedagogy of mathematics teacher education

Figure 6.2 illustrates the "Pedagogical Blocks" that emerged from the analysis as key aspects of the pedagogical knowledge and practices discussed and explored within the MTEs' PLC. These blocks, extracted from the broader theme of content-focused professional learning, represent the core areas of focus for the MTEs' collaborative learning and professional development efforts. The figure provides a visual representation of the interrelated components

that constitute the MTEs' pedagogical knowledge and practices in the context of pre-service mathematics teacher education.

6.3 Data preparation and selection: Phase Two

I made detailed transcriptions to capture features of the talk of MTEs and critical friends in order to understand their experiences (Kvale, 1996). The transcripts were examined line by line and coded by summing up each piece of data (Thornberg & Charmaz, 2014). I employed the repeated careful listening technique (Saldaña, 2016), listing and watching during transcription. To be sure of what I was doing, I relied on member-checking, sharing with the critical friends in this study the initial interpretation of data and providing them with the opportunity to clarify their intentions and contribute additional information if necessary. This process increased my awareness of patterns, themes and categories as I continued to familiarise myself with the data. At this stage, I organised the transcripts into files, interviews, PLC/ critical friends group observations and video lessons and immersed myself in the data to make sense of it. I read through all the transcripts repeatedly and wrote down my thoughts at the margin of the transcript as they emerged (Yin, 2015). This engagement with data and focusing on what I could see, rather than what I anticipated, revealed the recurring themes synthesised into an emerging coding system in which themes were flagged and marked for further analysis (Ritchie & Lewis, 2003).

6.4 Data analysis focused on critical friendships

Phase Two employed a deductive approach. Given the limited literature on critical friendships in mathematics teacher education, this deductive approach allowed for an exploratory examination of the data through the lens of Wenger's (1998) dimensions of practice framework which facilitated the identification of common themes in the discussions between critical friends (CFs) and MTEs regarding their mathematical practices, resources, and the support they derived from their critical friendships and the learning community. By analysing multiple data sources which included interviews, observations, and videos, I aimed to develop a rich understanding of MTEs' collaborative professional learning, teaching practices, and the influence of critical friendships on MTEs teaching knowledge.

Tables 6.3 and 6.4 present excerpts from the observations of the PLC and post-observation interviews with CFs and MTEs, respectively. These excerpts were selected to illustrate the themes and categories that emerged from the inductive analysis, focusing on the functions, purposes, and shared resources associated with the critical friendships within the PLC. The

excerpts provide insights into the various roles and contributions of the CFs in supporting the MTEs' professional learning and development.

Table 6.3: Excerpts from lesson observations and post-observation interviews.

Themes	Categories
Mutual Engagement How critical friendship functions. Thus, how MTEs/CFs interact.	***Engagement and Interaction:*** CFs focus on how MTEs' engagement and interaction with PSTs during the lesson. CFs and MTEs reflect on the opportunities that MTEs offered for PST' participation (collaborative learning, and active engagement) ***Classroom Management:*** CFs focus on and support MTEs with feedback on establishing a positive and inclusive classroom environment using effective strategies for managing PST' behaviour toward learning mathematics. CFs pay attention to MTEs' classroom management techniques. CFs' feedback focus on how MTEs foster a classroom culture that encourages PST' contributions, questioning, and peer interaction. ***Maths related Issues*** CFs focus on how MTEs promote deeper conceptual understanding of mathematical concepts among PST". CFs focus on how MTEs develop PST' problem-solving strategies, encourage critical thinking, and develop mathematical reasoning among PSTs.
Joint Enterprise Common purpose: What critical friendship is intended to achieve?	***Curriculum Alignment:*** CFs focus on the alignment between the lesson and the broader goals and objectives of the curriculum. CFS focus on alignment between teacher education curricula and school mathematics curricula. CFs focus on ascertaining if lesson covers the specified content, addresses the learning outcomes, and follows the recommended sequence related to specific lesson plan. ***Differentiation and Inclusion: (Policy driven)*** CFs focus on how MTEs address PST' diverse classroom needs and abilities. CFs focus on and provide feedback on differentiation strategies, support for struggling PSTs, gender responsiveness and challenging opportunities for advanced learners. ***Instructional Strategies:*** CFs focus on the effectiveness of the instructional strategies employed by the MTE. They provide feedback on engaging and effective teaching methods, differentiation strategies, and integrating technology or manipulatives. ***Planning and Enactment:*** CFs focus on the plausibility of strategies used during the lesson and provide feedback on their alignment with the intended learning outcomes. CFs focus on whether MTEs' choice of examples unpack the meaning of the intended mathematical concept.
Shared Repertoire What shared resources and routines? Ways of doing things	***Professional Knowledge and Expertise:*** CFs focus on the MTE's depth of content knowledge, pedagogical knowledge, and understanding of current research and best practices, emphasising pedagogical knowledge. CFs pay attention to MTEs use of instructional materials. CFs provide feedback on the accuracy and clarity of explanations and demonstrations. ***Collaboration and Support:*** CFs pay attention to MTE's ability to collaborate and support colleagues within the community of practice. They provide feedback on the educator's contributions to collaborative lesson planning and sharing of resources.

Table 6.4: Excerpts from the PLC/CFGs observations

Themes	Categories
Mutual Engagement - ME How critical friendship functions. Thus, how MTEs/CFs interact.	***These forms of participation belong to collaborative learning or active learning –I refer to them as active collaborative learning.*** Peer lesson observation Feedback on lesson observation Lesson Planning Professional learning focused on practice-based discussions Demonstration teaching Weekly meetings focused on improving pedagogical practices. Practice-based discussions Team teaching
Joint Enterprise- JE Common purpose: What critical friendship is intended to achieve?	Curriculum alignment Setting specific targets for teaching Classroom enactment of lessons Community building and support Explore instructional strategies that promote PSTs' engagement and learning.
Share Repertoire – SR What shared resources, routines etc., ways of doing things	**Sharing resources:** Content knowledge and pedagogical skills sharing MTEs and CFs exchange educational materials, lesson plans, national text documents, and other resources that are effective in their teaching practice. **Best practices:** CFs and MTEs share successful teaching strategies and appropriate techniques that have yielded positive results in teacher education. **Sharing insights/ practice-based experiences:** Reflective discussions/dialogue about teaching practice and professional experience Collective linguistic resources that provide the foundation for effective communication

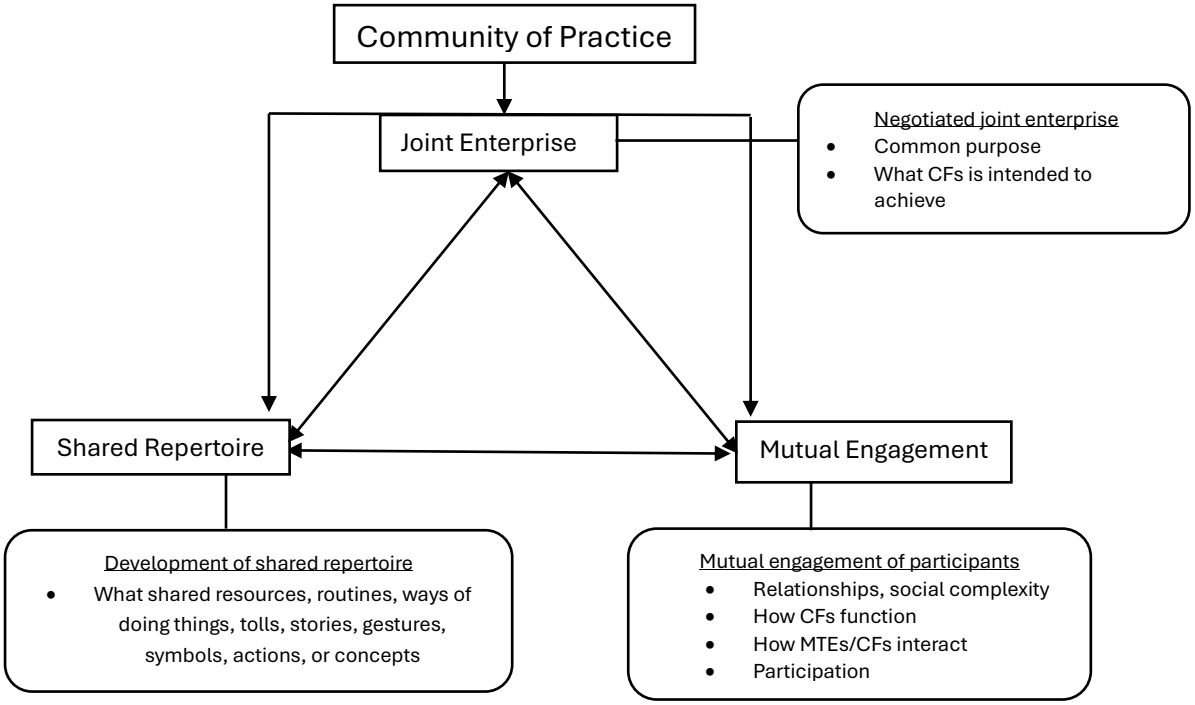


Figure 6.3: Dimensions of community of practice (An analytical framework)

Figure 6.3, inspired by Wenger's (1998) work, depicts the three dimensions of a community of practice: mutual engagement, joint enterprise, and shared repertoire. These dimensions served as an analytical framework for understanding the dynamics and functioning of the critical friendships within the PLC. By examining the data through this lens, the analysis aimed to uncover the ways in which CFs and MTEs engaged with each other, their shared goals and purposes, and the resources and routines that supported their collaborative learning and professional growth.

6.5 Chapter summary

This chapter presented an account of the data analysis process undertaken in this study. The analysis involved two distinct yet complementary phases, each employing a rigorous combination of deductive and inductive thematic analysis approaches. In Phase One, the deductive approach, guided by Desimone's (2009) framework, provided a foundation for analysing the MTEs' professional learning experiences within their PLC. The inductive approach complemented this by allowing for the identification of emergent themes directly from the data. This integration captured both anticipated and contextualised understandings.

Phase Two focused on the role of critical friendships. An inductive approach, drawing on Wenger's (1998) three-dimensional practice, facilitated an exploratory examination of the dynamics and functions of critical friendships in supporting the MTEs' professional growth. Throughout both phases the enactment of teaching practices were examined, and the analysis process triangulated findings from multiple data sources, including interviews, observations, and video recordings. This triangulation, coupled with the application of analytical frameworks, aimed to develop a rich, contextualised understanding of the phenomena under investigation. In the next three chapters, I report MTEs claim. In chapter 10, I specifically focus on evidence supporting those claims with observation of MTEs practice.

CHAPTER 7

DEVELOPING MATHEMATICAL KNOWLEDGE FOR TEACHING THROUGH COLLABORATIVE PLC ENGAGEMENT: PRESENTATION AND DISCUSSION OF FINDINGS FROM PHASE ONE

7.0 Introduction

This chapter presents and discusses findings from an analysis of data gathered from the focus group interview, professional learning community (PLC) observations, and associated field notes during Phase One of the study in order to respond to research question 1: To what extent does working collaboratively in the PLC support the development of mathematical knowledge for teaching in pre-service education?

7.1 Theme One: domain-specific and content-focused professional learning

The terms domain-specific and content-focused professional learning are used as umbrella terms for the ongoing education and training MTEs received in their specific field or domain of expertise within the PLC. While participation in the PLC activities does not always guarantee professional development (Guskey, 2002b), the findings of this study indicate that the professional learning undertaken by MTEs in the PLC is:

- Tailored to assist them in learning about teaching while concurrently enabling them to teach pre-service teachers about teaching.
- Balanced between subject-matter knowledge (algebra content) and pedagogical knowledge (effective teaching strategies)
- Domain-specific and content-focused.

These three findings are presented and discussed in sections 7.2 to 7.4

7.2 Tailored professional learning for teaching about teaching

The professional learning experiences within the PLC were carefully designed to assist MTEs in developing their own teaching abilities while equipping them to effectively teach PSTs about pedagogic practices. This dual focus on enhancing MTEs' pedagogic knowledge and skills and their capacity to model these for future teachers was evident throughout the PLC activities. For instance, in a PLC meeting on April 26, 2022, where the group analysed and discussed a short video recording of one of their colleagues teaching a previous lesson on algebraic equations to a class of PSTs, the discussion centred on the specific algebraic concepts being taught and the

instructional strategies and representations used by the MTE to facilitate PSTs' understanding. An excerpt from the conversation highlights this dual focus:

> Sam: I really liked how you used the visual model of the balance scale to represent the equations. That seemed to help the pre-service teachers grasp the concept of equivalence.
>
> Nick: Yes, and I also appreciated how you anticipated some common misconceptions students might have, like confusing the subtraction sign with a negative sign. Proactively addressing those potential stumbling blocks is so important when teaching our students [PSTs].

These affirming comments illustrate how the PLC provided a space for MTEs to critically examine and learn from each other's teaching practices, focusing on the content knowledge and the pedagogical approaches used to convey that knowledge effectively to pre-service teachers. Further evidence of tailored professional learning was found in the reflective journal entries of MTEs. For instance, Dan wrote:

> "After observing and discussing my colleague's lesson, I realised the importance of explicitly modelling the thought processes and decision-making involved in effective teaching. It's not enough to demonstrate the mathematics; we need to make our pedagogical reasoning visible to the pre-service teachers so they can learn to navigate those instructional choices themselves."

This reflection underscores the role of reflective journal entries in the evaluation process, as it prompted MTEs to examine their own teaching practices critically. However, it must be noted that Dan was the only MTE who shared his reflective journal with me. From the participants' comments it was evident that writing in their journals encouraged them to consider ways to prepare PSTs for the complexities of the classroom.

The interview data also corroborated the finding that professional learning was tailored to assist MTEs in teaching about teaching. As Dan stated:

> "The PLC has been invaluable in helping me develop a 'teacher educator' mindset. It's not just about knowing the content deeply; it's about being able to unpack that content in a way that pre-service teachers can understand and apply in their own future classrooms. The PLC has provided a supportive space to hone those specialised skills."

The evidence from various data sources, including observation notes and interviews, consistently demonstrated that the professional learning undertaken by MTEs within the PLC was carefully designed to address their roles as teacher educators. The activities were tailored to enhance their teaching abilities while concurrently developing their capacity to teach PSTs about pedagogic practices in mathematics effectively.

7.3 Balanced learning of subject-matter knowledge and pedagogic knowledge

The professional learning opportunities within the PLC struck a deliberate balance between developing MTEs' subject-matter knowledge of algebra and their pedagogic knowledge of effective teaching strategies. This balanced approach was evident in the data from multiple sources. Observational data provided insights into how this balance manifested during PLC sessions. A short extract from observation field notes (April 12, 2022) made during a PLC session highlights the dual emphasis on content and pedagogy.

> The session began with MTEs discussing the conceptual underpinnings of functions and unpacking the meaning, mathematical representations and reasoning. Then, MTEs posed probing questions to one another, challenging their content understanding.
>
> Later, the focus shifted to effectively examining different instructional approaches for teaching functions. MTEs shared concrete models, real-world examples, and technology-based visualisations they have used with PSTs. There was a rich exchange of ideas on representing these concepts in ways that resonate with learners.

These observation notes illustrate how the professional learning session struck a balance between developing the MTEs' subject-matter knowledge of functions and their pedagogic knowledge for teaching this concept effectively. In the initial part, the MTEs investigated the conceptual foundations of functions, unpacking the meaning, representations, and underlying mathematical reasoning. This exploration of the content knowledge was further reinforced through probing questions that challenged each other's understanding.

The session then transitioned to a focus on pedagogic approaches. The MTEs shared and discussed various instructional strategies they had utilised with PSTs, such as concrete models, real-world examples, and digital visualisations. This exchange fostered a rich exploration of ways to represent functions to enhance PSTs' conceptual understanding. The PLC facilitated a balanced approach to professional learning for the MTEs by dedicating time to content mastery and pedagogic techniques within the same session. This suggests that participants could

simultaneously reinforce and extend their subject expertise while expanding their repertoire of effective teaching methods, with the result being better 'equipped' educators of future teachers.

The interview data provided corroborative evidence of the balanced approach used by facilitators of the PLC sessions for MTEs as in the example below:

> Nick: The PLC helped deepen my understanding of algebraic concepts that I thought I had mastered. However, what has been genuinely transformative is learning how to unpack and represent those concepts in ways that resonate with our students [PSTs]. The PLC has helped me balance content mastery and pedagogical expertise.
>
> Dan: Sure! As teacher educators, we need to be experts in the subject matter and the teaching of that subject matter effectively. The PLC has provided a space for us to simultaneously strengthen our algebraic knowledge and our repertoire of instructional strategies for conveying that knowledge to our students.
>
> Sam: Yes, I agree with you guys. However, have you considered that the focus on subject-matter knowledge sometimes overshadows the importance of other essential teaching skills in the PLC? I think some students come with weak backgrounds, so we lose a little focus on pedagogy.

Overall, this extract from the interviews indicates that the facilitators of PLC sessions took a balanced approach to developing MTEs' subject-matter knowledge and pedagogic knowledge Nick's and Dan's comments highlight their appreciation of the PLC's role in deepening their understanding of algebraic concepts while simultaneously assisting them to represent and convey these concepts effectively to their PSTs. However, Sam raises an important point about the potential risk of over-emphasising subject-matter knowledge to the detriment of essential teaching skills within the PLC. His perspective acknowledges that some PSTs come with weak backgrounds in mathematics, necessitating a more substantial focus on pedagogic strategies to address their needs effectively.

This counterpoint from Sam underscores the complexity of achieving a truly balanced approach within a PLC. While the data suggests a concerted effort to integrate content and pedagogy, there may be instances where the scales tip more heavily towards one domain. PLC participants and collaborators must remain mindful of this potential imbalance and continuously evaluate whether the professional learning experiences adequately address subject-matter knowledge

and a comprehensive range of pedagogic skills. Regularly soliciting feedback from MTEs, like Sam, who may perceive an overemphasis on content, could help recalibrate the balance and ensure that essential teaching skills are not overlooked.

Ultimately, the interview excerpts demonstrate the PLC facilitators' commitment to a balanced approach but also highlight the need for ongoing reflection and adjustment to ensure that the unique needs of the MTEs and their PSTs are being met equitably across both content mastery and pedagogic expertise.

7.4 Domain-specific content-focused learning

The professional learning experiences within the PLC strongly emphasised domain-specific content knowledge, particularly in algebra. This content focus was evident across various data sources and activities. The detailed observation notes taken from the PLC session on March 22nd illustrate how the PLC session maintained a strong emphasis on domain-specific content knowledge within the area of algebra, mainly related to the concept of functions. This content focus was evident from the beginning, as noted in this extract: "The meeting began with the facilitator outlining the day's focus - exploring innovative pedagogic approaches for teaching functions..."

By explicitly stating the focus on functions, a key algebraic concept, the facilitator established the domain-specific and content-focused nature of the session. As the MTEs analysed sample lesson plans, their discussions further reinforced the content emphasis, as highlighted in this excerpt: "Dan argued, 'I don't see enough representation in this lesson plan.' The facilitator responded, 'That's why we're going to work in sub-groups to fine-tune it.' A discussion ensued about probing questions to build conceptual understanding versus procedural skills."

This exchange suggests how the MTEs critically examined the lesson plans through the lens of effectively representing and teaching the specific algebraic content of functions, including considerations of conceptual understanding and procedural skills. The observation notes also captured instances where MTEs directly addressed the need to connect algebraic representations with graphical representations of functions: "For instance, Nick emphasised the need to connect algebra and representations as PSTs may struggle to relate algebraic expressions to graphical representations of functions."

The excerpt above illustrates how the MTEs focused on unpacking the domain-specific content knowledge related to functions, explicitly addressing the potential challenges involved in helping PSTs understand the connections between algebraic and graphical representations.

Furthermore, the activities described in the observation notes, such as creating lesson plans, engaging in demonstration lessons, and constructing hybrid lessons, were all centred around the specific algebraic concept of functions. This consistent emphasis on a particular content area within the algebra domain highlights the domain-specific and content-focused nature of many of the professional learning experiences in the PLC.

Additionally, the focus group interview corroborated my observations of the PLC activities when MTEs stated the following:

> Nick: Though this learning community started as part of a nationwide teacher education policy initiative, I mean, err… when I joined, I asked myself, "What is there to learn?" You see, you see…. many such initiatives have failed to yield proper results. However, err…, a few months into the programme, I realised that we are learning specific content, uhm… especially to assist us as teacher educators to learn specific content and pedagogic approaches that will influence our practice in pre-service teacher education.
>
> Dan: Precisely! As MTEs, we play a crucial role in helping pre-service teachers excel in algebra. Aah... our focus should be on general pedagogy and understanding the complexities of algebra itself. Aah… this subject requires a deep understanding, and err… I mean, we should be the ones to guide our pre-service teachers in that direction.
>
> Sam: To add to what Nick mentioned, I would rather say subject-specific knowledge is vital. However, I also think we cannot ignore the broader pedagogic principles. Knowing algebra is one thing, but teaching it effectively is another. We need a balance.
>
> Nick: I agree with him [referring to Sam] here. Err… algebra is undoubtedly important, but we cannot overlook the importance of pedagogy. The way we deliver that algebra knowledge matters. So, so, it is not just about understanding algebra; it is about communicating that understanding effectively to our pre-service teachers to help them learn about teaching.

The perspectives revealed in the interview excerpt connect to ongoing debates within teacher education regarding balancing subject-specific expertise and pedagogic knowledge (Hanley & Thompson 2021). While scholars such as Ball et al. (2008) and Hill et al. (2008) have

positioned deep content knowledge as vital for effective teaching, researchers such as Clarke and Hollingsworth (2002) have cautioned that a narrow focus on content masks the complexity of teacher knowledge. Their position aligns with Sam's and Nick's balanced approach to professional development for MTEs and also echoes Shulman's seminal work on teacher knowledge frameworks (1986). Though not an MTE, Grossman (1990) found that developing pedagogic content knowledge (PCK) is key for translating subject knowledge into classroom practice. Nick's acknowledgement of the PLC's goal of enhancing how to communicate algebraic understanding links to the evolving recognition of PCK's role within teacher education.

During the interview, MTEs were explicit about what they learned in the PLC when they shared the following insights:

> Dan: Regarding what we learn in our PLC, aah… I would say that the content relates mainly to the pedagogic aspects of what we aah… need to employ to assist our pre-service teachers. Uhm… and we draw from this pedagogic basis to explain the underlying reasons of assumptions, choices and examples to our students [PSTs].
>
> Nick: I could not agree more with you. Err... after all, you see, we are teaching teachers so they must know the reasons for the choices we make and learn from our examples.
>
> Sam: In my opinion, aah…what Dan just emphasised is the most difficult part of our work. For instance, you know our students. Uhm… our students know that in real numbers, negative times negative is a positive or negative take away positive results in another negative number. But err…how do you model such concepts for pre-service teachers when they already know it by rote learning? So, you see, uhm… the importance of our work lies in our ability to model and provide basic assumptions, as they have said.

This extract and earlier extracts reveal that a significant portion of MTEs' professional learning within the PLC focuses on pedagogic aspects and strategies for effectively teaching PSTs. The findings align with the literature emphasising the importance of developing specialised content knowledge and pedagogic content knowledge in mathematics teacher education (Ball et al., 2008; Hill et al., 2008).

MTEs showed awareness of the challenges in teaching mathematical concepts to PSTs who may have previously acquired mathematical concepts through rote memorisation. This awareness aligns with the emphasis placed by researchers on the importance of developing conceptual understanding in mathematics education (Jansen et al., 2017). Rote learning, while enabling procedural fluency, often falls short of fostering a deep understanding of the underlying mathematical principles and their interconnections (Klau et al., 2020).

Assisting PSTs to develop conceptual understanding is a crucial aspect of MTE practice, as it can enhance student teachers' subsequent classroom practices (Kajander & Holm, 2013). MTEs play a pivotal role in guiding PSTs to transcend the limitations of rote learning and to develop a deep conceptual understanding of mathematical concepts. This can be achieved through various instructional approaches, such as using multiple representations, inquiry-based learning, and emphasising the connections between different mathematical ideas. By doing so, MTEs can equip PSTs with the subject knowledge and pedagogic skills necessary to become effective mathematics teachers capable of fostering meaningful learning experiences for their future students.

7.4.1 Pedagogy of mathematics teacher education

The "pedagogy of mathematics teacher education" emerged as central to domain-specific and content-focused professional learning theme. Unlike conventional pedagogies applied in school mathematics, this pedagogy is specifically tailored to the training and advancement of pre-service mathematics teachers (PSTs). Usually referred to as "second-order" pedagogy, this pedagogy plays a crucial role in shaping the instructional quality and effectiveness of MTEs (Murray & Male, 2005). Though the excitement, dedication and participation of MTEs in the PLC studied differ, there is a collective commitment among them to navigate the complexities of this second order pedagogy by aligning professional learning with the distinct needs of mathematics teacher education.

MTEs shared their views about this pedagogy as follows:

> Dan: As we have discussed, our role as MTEs demands, err…, a unique pedagogical approach specifically for training pre-service teachers. This demands that we draw from a uhm…, repertoire of knowledge to shape how our PSTs will eventually teach mathematics themselves.
>
> Nick: Exactly, Dan. It's a pedagogy that goes beyond the conventional methods we might use in a regular, err… school mathematics classroom. We are not just

teaching mathematics content; we are teaching our students [PSTs] how to teach that content effectively.

Sam: Adding to that, that's where I would say the complexities arise. This pedagogy has to be moulded and adapted based on the educational context and level. Uhm…, whether we're training PSTs for primary or junior high school. The strategies we employ can vary significantly.

Dan: I'm happy you said that, Sam. That's why gaining expertise in this pedagogy [pedagogy of mathematics teacher education] is central to our professional learning as MTEs. We must continually refine our understanding and application of the required teaching approaches.

Nick: I couldn't agree more. It's a continuous learning process for us. Every time we engage with our PSTs, we're not only imparting mathematical knowledge but err…, also modelling the very pedagogies they'll need to employ in their future classrooms.

Sam: For me, that's where the real beauty of our PLC lies. By collaborating, sharing expertise, and learning from one another, we can collectively navigate the intricacies of this pedagogy. It's a journey we're all on together, constantly evolving and improving our practice, though there are many challenges like interest in participating and others.

The statements in the extract confirm findings in the literature on the unique pedagogical demands faced by MTEs. Murray and Male (2005) introduced the concept of "second-order" pedagogy, which acknowledges the specialised nature of teaching practices required for preparing future mathematics teachers. As highlighted in the extract, MTEs must possess a deep understanding of mathematical content and the ability to model and teach the pedagogies that PSTs will employ in their future classrooms. As Nick aptly stated, "We are not just teaching mathematics content; we are teaching our students [PSTs] how to teach that content effectively."

In his first statement, Sam directly acknowledges the complexities involved in this specialised pedagogy. He highlights that the pedagogy needs to be moulded and adapted based on the specific educational context and level for which the pre-service teachers (PSTs) are being trained. His emphasis on educational context and level echoes the observations of various

scholars (e.g., ; Escudero-Ávila et al., 2021; Monroe, 2013; Zaslavsky & Sullivan, 2011) who have emphasised the need for MTEs to adapt their teaching strategies, based on the educational context and level of pre-service teacher preparation. Some of the complexities arise from the diverse backgrounds, prior knowledge, and future teaching environments of the PSTs being trained. Dan affirms that gaining expertise in the pedagogy of mathematics teacher education is a central focus of their professional learning as MTEs. They recognise the continuous need to refine their understanding and application of the teaching approaches required to effectively prepare PSTs for their future roles as mathematics educators.

Sam highlights the value of their PLC, where collaboration, sharing expertise, and collective learning allow them to navigate the intricacies of this pedagogy together. The PLC provides a supportive environment for the MTEs to continuously evolve and improve their practice despite challenges such as varying levels of interest and participation. The collective commitment and collaborative approach to navigating the details of this pedagogy through the PLC reflect the recommendations of researchers such as Owens (2016), who have advocated creating supportive learning environments that foster continuous professional growth among MTEs.

Similarly, the post-lesson observation reflection and PLC observations revealed MTEs' interest in tailoring their pedagogical approaches to the specific grade levels and contexts for which they were preparing PSTs. For instance, "Dan mentioned, "For our PSTs, I find using hands-on manipulatives and concrete models effective in building their conceptual understanding so that they can teach well in the school setting. The tactile experience seems to resonate better at that level." In another post-observation reflection, Nick highlighted that "The context where our students [PSTs] will be working is crucial in forming the approaches needed for teaching. If we train secondary PSTs, we must emphasise abstract reasoning and multiple representations to challenge their higher-level thinking skills."

These extracts from field notes highlight how MTEs like Dan and Nick recognise the need to demonstrate to PSTs teaching strategies that are responsive to students' specific educational levels and developmental needs, from concrete models to abstract reasoning and representations. Both interview data and post-lesson observation reflection entries underscored the significance of lesson modelling within this specialised pedagogy. Sam elaborated, "As an MTE, one of the most critical aspects of my practice is modelling effective lessons for my PSTs. Though sometimes time may not permit this, it's about making my pedagogical

reasoning and decision-making visible through live demonstrations. I aim to unpack my thought processes, illustrate various instructional techniques, and address potential student misconceptions during teaching. This allows PSTs to witness the nuances of effective teaching firsthand."

The PLC observational field note entries captured instances where MTEs talked about examining the pedagogical reasoning and assumptions behind specific mathematical concepts. For example, one observation note stated,

> "The discussion took an interesting turn when Sam asked, "What assumptions are we making about students' [PSTs] prior knowledge when teaching algebraic equations? Are we adequately addressing potential gaps in their understanding of variables and equality? This sparked a lively debate among the group, with MTEs sharing their experiences and strategies for addressing common misconceptions."

This excerpt illustrates how MTEs engaged in discussions about unpacking the pedagogical reasoning and assumptions underlying mathematical concepts, particularly in areas where PSTs may have misconceptions or knowledge gaps. The artefacts, such as lesson plans and instructional materials, provided concrete evidence of the emphasis on using various representations within the pedagogy of mathematics teacher education. One lesson plan (dated 22 March 2022) excerpt reads as follows:

> "Multiple representations will be utilised throughout the lesson to facilitate PSTs' understanding of linear equations. These include (algebraic, graphical, verbal, and concrete models), concrete models, such as the algebra tile, to demonstrate identity. PSTs will be able to analyse real-world situations and model them using linear equations."

This excerpt from a lesson plan shows how MTEs intentionally incorporated diverse representations, including manipulatives and real-world examples, to explain concepts on linear equations to PSTs. The strategic use of representations was a hallmark of the specialised pedagogy employed within the PLC.

In the subsequent sections, I discuss four facets of teacher-educator pedagogy.

7.4.2 Multiple teaching strategies and learning approaches

In some of their PLC sessions, the MTEs focus on understanding various instructional strategies. They recognise the need to develop the subject knowledge of PSTs through using

effective pedagogies. The vignette below, reconstructed from my observation notes on a collaborative lesson planning session on functions, indicates the MTEs' interest in integrating multiple teaching strategies and understanding instructional strategies, recognising PST needs and strong subject knowledge, and also their support of one another in doing so.

Collaborative Lesson Planning Session: Functions (Observed on 22 March 2022)

The MTEs (Sam, Dan, and Nick) planned a lesson on functions for their PSTs. The following points highlight how they integrated multiple teaching strategies while considering PST needs and subject mastery:

1. Subject Knowledge
    - Sam stressed the importance of solidifying PSTs' understanding of the formal definition of a function, its properties (one-to-one, onto, etc.), and different representations (algebraic, graphical, tabular).
    - Dan added that many PSTs struggle with domain and range, so MTEs need to plan activities and examples to clarify these ideas.
2. Multiple Strategies
    - For introducing functions, Nick suggested using real-world contexts like exploring patterns in nature or modelling situations with input-output relationships.
    - Sam proposed using graphing technology and online applets for visualising different function types and their behaviour.
    - Dan shared an activity where PSTs match function representations (equations, graphs, tables) to build connections between various forms.
3. PST Needs
    - They discussed incorporating hands-on activities with manipulatives or physical models for kinaesthetic learners when exploring transformations of functions.
    - Nick mentioned using think-pair-share and group discussions to allow PSTs to verbalise their understanding and learn from peers.
    - Sam suggested providing scaffolded examples and prompts for struggling PSTs, gradually releasing responsibility as they gain proficiency.
4. Instructional Approaches
    - The group planned to start with an exploratory approach, having PSTs investigate patterns and make conjectures about functions.

- They would then transition to direct instruction and explicit modelling for formalising definitions and procedures.
- Throughout, they aimed to emphasise conceptual understanding over rote procedures, with frequent checks for PST comprehension.

5. Assessment:
    - Dan proposed using formative assessments like exit tickets or quizzes to gauge PST understanding and inform subsequent instruction.
    - They discussed creating open-ended tasks or problem-solving scenarios to assess PSTs' ability to apply their knowledge of functions in various contexts.

These reconstructed observation notes highlight how the MTEs integrated multiple teaching strategies while designing a lesson on functions for their PSTs. The multiple approaches draw on the multifaceted nature of the pedagogy of mathematics teacher education employed within the PLC. One key strategy discussed was using real-world contexts to introduce the concept of functions. As Nick suggested, "using real-world contexts like exploring patterns in nature or modelling situations with input-output relationships" can provide a concrete and relatable entry point for PSTs to grasp the fundamental ideas of functions. When MTEs ground concepts in familiar contexts, PSTs may develop a deeper conceptual understanding and appreciate the practical applications of functions.

Additionally, the MTEs recognised the value of leveraging technology as a pedagogical tool. Sam proposed "using graphing technology and online applets for visualising different function types and their behaviour." This strategy reflects contemporary teaching practices and caters to visual learners, allowing PSTs to observe and explore the behaviour of various functions. The MTEs aimed to foster connections between different representations of functions. Dan shared "an activity where PSTs match function representations (equations, graphs, tables) to build connections between various forms." This strategy addresses a common challenge PSTs face in understanding the relationships between algebraic equations, graphical representations, and tabular data when studying functions. By engaging PSTs in this matching activity, the MTEs aimed to 'solidify' the knowledge and skills needed for translating between different representations fluently.

By integrating multiple strategies into a single lesson plan the MTEs' demonstrated understanding that different teaching approaches can cater to diverse learning styles and promote a deeper understanding of the subject matter. They were able to create a rich and

multidimensional learning experience for their PSTs, thus modelling the very practices they aim for their PSTs to acquire. The focus group interview also sheds light on how the PLC supported the MTEs in their use of multiple strategies. MTEs stated the following:

Sam: The PLC is instrumental in expanding our repertoire of teaching strategies. Through our collaborative planning sessions, we get to share different approaches we've used, discuss their strengths and weaknesses, and learn from each other's experiences.

Dan: Well said, Sam. I mean you have shared my thoughts. So, err…, we encourage ourselves [MTEs] to be adaptable and err…, responsive to our student's needs. Uhm…, this means differentiating instruction, aah…, integrating technology where appropriate, and using various assessment methods to gauge understanding. Um…, if I may, err…, I would say hands-on learning experiences are invaluable, especially in this regard.

Sam: Exactly! When our students [PSTs] can see and experience the math using different strategies, it demystifies it and…, makes it more accessible. Aah…, It helps them to identify areas of improvement and fosters uhm…, a productive mindset. So, so…, teaching strategies and practical application are the bridge that connects mathematical theory to the world and uhm…, equips our students with the tools they need for success.

Researcher: So, it seems the PLC provides an enabling environment for integrating multiple teaching strategies effectively.

(MTEs in agreement) Yes, definitely. The PLC has been crucial in supporting our use of diverse instructional strategies to enhance the quality of PST training.

The emphasis of the three MTEs on the importance of adaptability, responsiveness, differentiated instruction, and varied assessment methods resonates with the principles of student-centred pedagogy and culturally responsive teaching (Gay, 2018; Starkey, 2019). These approaches recognise the diverse needs and backgrounds of PSTs, promoting equitable access to learning opportunities.

The importance of hands-on learning experiences and making abstract concepts accessible through a range of approaches is aligned with the constructivist theory of learning (Vygotsky, 1978; Richardson, 2005). Hands-on activities and concrete representations support the

construction of knowledge and facilitate the development of conceptual understanding (Boaler, 2016; Hiebert & Grouws, 2007). The metaphor of teaching strategies and practical application serving as a bridge between mathematical theory and the real world resonates with the principles of situated learning (Lave & Wenger, 1991) and the idea of authenticity in mathematics education (Boaler, 2016; Vos, 2018). These perspectives emphasise connecting mathematical concepts to real-world contexts and applications, fostering meaningful learning and preparing students for practical problem-solving.

The recognition by MTEs of the importance of adaptability, diverse teaching methods, and practical application aligns with recent studies (e.g., Mulvihill & Martin, 2019; Hong & Chai, 2017) that emphasise these aspects in mathematics education. Adaptive teaching in mathematics involves responding to various stimuli, engaging in metacognition and reflection, and taking appropriate actions (Gallagher et al., 2020). These studies advocate for the use of multiple representations, engagement in mathematical discourse, and the application of mathematical knowledge to real-world contexts.

7.4.3 Modelling teaching

A key instructional strategy emerging from the focus group interview and PLC observation data, is the MTEs' quest to model appropriate mathematics teaching practices. The term modelling is used to refer to MTEs' purposeful and explicit demonstrations to and discussion of specific teaching practices, strategies, and instructional techniques to their colleagues in the PLC. The goal is to develop PSTs' pedagogic knowledge and skills for teaching specific mathematics topics, whether or not they are mathematics majors. From observations of the PLC in action it was evident that for the facilitators modelling teaching entails targeted conversations and exemplifying the components of a robust lesson - planning, content delivery, classroom management, and assessment. To cultivate expertise in doing this, MTEs participated in demonstrative model teaching (see Chapter 8) and discussion sessions within the PLC. Below is a vignette from a lesson on integer operations and properties modelled by MTEs in their PLC on 12 April 2022. Vignette: Modelling Integer operations and properties

> MTEs gathered at a round table in their college's maths lab. After initial reflection on the previous lesson, the session focused on teaching integer operations and properties. The goal was to refine their skills in effectively delivering this crucial mathematical topic to pre-service teachers (PSTs). A facilitator led the session. He carefully walked MTEs through a lesson plan the group had already prepared. Having reminded those

present of the key properties they should emphasise, the facilitator distributed small two-coloured counters to participants. The red side represented positive integers, and the orange side negative ones.

The group modelled simple integer operations with the counters, looking for patterns. For example, when adding integers with the same sign (colour), the counters were combined, increasing the total value. The group also made connections to the properties. For example, the additive inverse property was illustrated by pairing up opposite-coloured counters to model adding an integer and its opposite.

Dan pointed out a potential misconception when they got to subtraction. What if a student [PST] tries to take away more counters than they have, like 3 - 5? Nick recreated the scenario, first placing 3 red counters and then trying to remove 5. Okay, so we cannot just take away five...what do we need to do? Add some negatives, Dan replied, carefully placing 5 orange counters. Anything minus a negative is the same as adding a positive because the two colours will form a zero pair. Nick nodded approvingly.

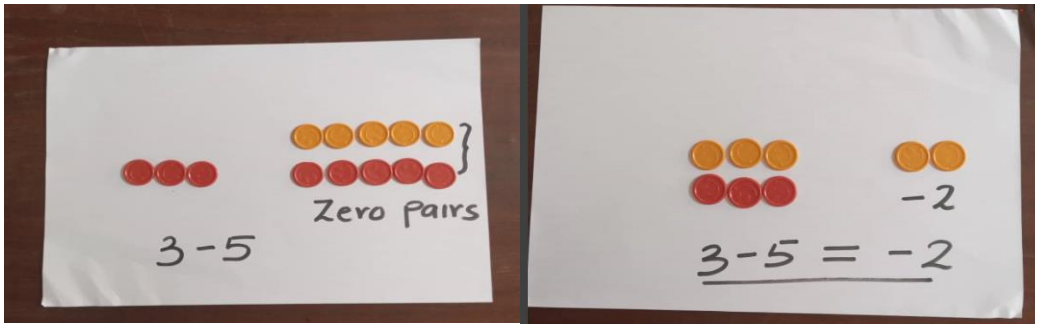


Figure 7.1: A photo taken during PLC depicting zero pairs for 3 – 5

This could be a good visual aid for developing rules for subtracting integers and showing why a negative minus a negative equals a positive. The MTEs continued modelling integer multiplication and division using a rectangular grid and working with more complex expressions. They discussed strategies such as using a number line model to reinforce the concepts of "opposites" and directionality, having students [PSTs] create story problems and visual representations and emphasising patterns and connections to the properties.

As the session wrapped up, Nick encouraged all MTEs to incorporate integer modelling in their lesson. He noted, "The more comfortable we can get working with multiple

concrete and visual representations, the better we can support our students in understanding these concepts from the ground up."

The emphasis on lesson modelling as a key instructional strategy within teacher education highlights the importance of providing PSTs with opportunities to observe and learn from experienced practitioners. This concept of modelling teaching and its potential is reflected in the results of a study by Rojas et al. (2021), who suggest modelling as a crucial concept for studying MTEs' practices in a PLC and for studying how PSTs experience these practices. By modelling effective teaching practices within the PLC, MTEs aimed to help themselves develop a deeper understanding of the pedagogic knowledge and skills required to teach specific mathematical concepts to PSTs. The collaborative nature of the PLC, where MTEs engage in joint planning, discussion, and reflection, allows them to refine their knowledge and skills in teaching about teaching. This iterative process can lead to more effective and engaging lessons for PSTs. For example, Nick noted, "This can be a good visual for developing rules for subtracting integers and showing why a negative minus a negative equal a positive." According to Ohayon and Albulescu (2023), a PLC provides a practical framework for improving educators' teaching skills. She examined whether educators who participated in a PLC improved their teaching skills compared to others who did not participate in the PLC and found that participation contributed to improving teaching skills in terms of lesson planning and aspects of classroom teaching practices.

Their use of concrete manipulatives, such as coloured chips, demonstrates the MTEs' interest in using multiple mathematical representations and hands-on learning to support PSTs' conceptual understanding of integer operations and properties. However, using such tools can also pose challenges. In Castro Superfine and Pitvorec's study, (2021), MTEs reported facing challenges such as limited resources, difficulty adapting manipulatives for different learning needs, and time constraints, highlighting unexpected barriers to the effective use of manipulatives in inclusive education programmes. The PLC in this study supported the MTEs in building a collective understanding of how to use manipulatives effectively. For instance, as illustrated in the vignette above, when Dan noted a potential misconception, Nick was ready to provide a pedagogic suggestion to address the matter. This example of addressing potential misconceptions indicates the MTEs' awareness of common PST difficulties.

In addition to the PLC observation data, the MTEs' focus group interview also included references to the importance of lesson modelling in mathematics teacher education, together with comments on some of the challenges of doing so:

> Nick: In our profession, uhm…, we teach more than abstract math concepts; aah…, we instil a practical problem-solving mindset. Uh… our PLC supports us through experienced colleagues who demonstrate effective teaching methods. So, so…, we emphasise lesson modelling per the NTS mandates to encourage conceptual understanding among students [PSTs].
>
> Sam: Uhm…, for me, while the ideal approach involves thorough lesson modelling and real-world problem-solving, err…, practical challenges arise in our exam-oriented system. So, so, balancing content coverage and meeting expectations is crucial. You see, striking a balance between lesson modelling and ensuring content delivery may require thoughtful consideration and um…, adaptation to the system's demands.
>
> Dan: I share Sam's concerns. Since our questions are from the mentoring university, we must cover the expected content. Meanwhile, we are training teachers; therefore, modelling [teaching] is an essential and invaluable strategy for our community to ensure our students are trained well.
>
> Nick: Exactly, it [modelling] holds particular significance as it provides both practical guidance and err…, as a source of inspiration for refining our teaching approaches. Aah…, through lesson modelling, we create a unique environment that um…, empowers pre-service teachers to observe, question, and err…, learn from our experiences, thereby affording them invaluable insights into the complex nature of teaching.

Analysis of this extract reveals that all three MTEs are cognisant of the practical challenges posed by the exam-oriented system, which requires a delicate balance between thorough lesson modelling and ensuring content coverage. This balance is crucial as the MTEs not only teach mathematics but also train PSTs, using lesson modelling to provide insights into the complex nature of teaching. The finding on the exam-oriented teacher education system echoes the studies of Yan (2015), who established that exam-oriented education had a negative backwash effect on the curriculum in China that teacher educators were constrained to select examination-focused content knowledge for their lessons.

While the MTEs recognised lesson modelling as an indispensable strategy in teacher education, they all highlighted the importance of covering the content prescribed by the mentoring university. The tension between modelling a more time-consuming collaborative approach that empowers PSTs to observe, question, and learn from the experiences of seasoned MTEs and that provides valuable insights into the multifaceted nature of teaching, and the constraints of addressing prescribed curriculum content, suggests that MTEs require further professional development support to prepare PSTs for success in examinations and for success as classroom teachers (Passarella, 2021).

7.4.4 Pedagogical reasoning and underlying assumptions

The findings of the study reveal that the MTEs in the PLC engaged in deliberate discussions about the pedagogic reasoning and underlying assumptions that inform their teaching practices. The analysis of classroom observation data, reflective accounts and focus group interviews with the MTEs revealed a concerted effort to make their pedagogical reasoning and underlying assumptions explicit to PSTs. Detailed observation notes and video recordings captured instances in which MTEs deliberately articulated and unpacked the rationales and principles informing their instructional choices during teaching. For example, during Dan's lesson on 25 March 2022, my field notes state: "Dan explained they used multiple real-world examples to challenge the assumption that algebra is purely abstract." Nick also explained using algebra tiles to model combining like terms, based on the assumption that connecting to representations supports comprehending the structure and properties involved, rather than rote procedural steps.

The post-lesson reflections with MTEs also highlighted their intention to communicate pedagogic reasoning. Dan reflected, "I made a point to explain my reasoning behind using representations and visuals because I wanted PSTs to understand the assumption that conceptual understanding should precede procedural fluency." In another reflection, Nick noted, "You'll notice I asked a lot of 'Why?' and 'How do you know?' questions during that example with integer multiplication and division. I reason that pressing PSTs to verbalise their justifications explicitly will reinforce the underlying quantitative reasoning, not just procedural steps."

The findings suggest MTEs made a concerted effort to explicitly communicate these assumptions to PSTs, as advocated in teacher education literature that emphasises practice-based pedagogy (Anthony & Averill, 2018; Hauser & Kavanagh, 2019). By articulating the

rationales behind their instructional moves, such as using representations before procedures or contextualising abstract algebra concepts, the MTEs modelled critical reflective practice (Loughran, 2006). Making this reasoning visible allows PSTs to examine teaching that can inform their own practice. The findings suggest that the MTEs aim to disrupt any potentially unproductive assumptions PSTs may hold, such as viewing algebra as disconnected from meaning or mathematics as solely concerned with procedures to memorise. Exposing such assumptions and contrasting them with principled reasoning opens space for PSTs to critically examine their beliefs about teaching and learning, as Cross (2009) advocates.

In the focus group interview, the MTEs expressed the following views regarding the explicit communication of their pedagogic reasoning:

> Dan: To demonstrate exemplary teaching, we clarify the 'why' behind teaching choices, making concepts clearer for us and our students. We acknowledge that they are going to teach, so that is important.
>
> Nick: I agree with that. That sums up a lot about teaching in pre-service teacher education. I mean, we were talking about modelling. Uhm..., our students [PSTs] must understand what works or not because they have chosen to be teachers, uhm…, they need to know the reasons behind basic concepts to explain to their own students when the need arises. For me, that is the difference between teaching elsewhere and teaching in teacher education.
>
> Sam: Err…, what can I say? They have said it all. And there is no doubt. So, so, that is teaching in teacher education for you. Err…, If our students are not familiar with the reasons behind our teaching assumptions, that is disastrous, and I can say our PLC is empowering our pedagogical choices and assumptions.

By overtly explaining the reasoning, assumptions, and principles behind practice, MTEs provide opportunities for PSTs to develop the reflexive and articulation abilities required for quality teaching (Appova & Taylor, 2019). Evidence from the MTE PLC in this study suggests that a PLC may serve as a fertile context for MTEs to collaboratively design lessons that optimise pedagogic reasoning as a scaffold for PSTs' understanding and application to their future practice.

7.4.5 Use of mathematical representations

The MTEs valued the use of representations in planning and teaching lessons as indicated in several of the extracts used in this chapter. Field notes from lesson observations recorded MTEs' use of such representations. For instance, during Sam's lesson on functions, the field notes read that Sam used multiple representations (verbal explanations, symbolic notation, arrow diagrams, ordered pairs) at various stages to explain the concepts of functions (see Chapter 10 for the use of representations). He explicitly pointed out the connections between these forms, enabling PSTs to extend their understanding.

The acknowledgement by the MTEs that PSTs have varied ways of understanding mathematical concepts underscores the need for flexibility in instructional approaches. For instance, Nick noted in a reflection after teaching functions, "Some students [PSTs] grasped the algebraic rules quickly, while others needed more visual cues from graphs and real-life examples to make sense of the patterns." By intentionally incorporating diverse representations in their lessons, the MTEs provided PSTs with access points aligned with their varied strengths, learning styles, and prior knowledge echoing the work of Goldin, (2020). Overall, the MTEs' discussions of the incorporation of multiple representations emerged as a critical responsive strategy within the pedagogy of mathematics teacher education, reflecting an effort to enhance the quality and inclusivity of mathematics instruction. The MTEs shared the following views in the focus group interview:

> Sam: My teaching approach is greatly influenced by our PLC. Aah…, we regularly exchange insights and strategies, emphasising the use of uhm…, various representations. So, so, personally, I incorporate visual tools such as graphs and diagrams to teach linear equations. Err…, these simultaneous graphical and algebraic representations aid students [PSTs] in grasping the connection between equations and their graphs.
>
> Dan: I agree with Sam's sentiment about, uhm…, the efficacy of visual representations in teaching. In my practice, you see, I use manipulatives, uhm…, particularly with quadratic equations. Uhm…, using algebra tiles helps students [PSTs] grasp the concept of factoring. When faced with challenges in representation, I turn to the PLC for err… valuable suggestions and support. It is essential to model our lessons for students as an example.

Nick: I agree with you, bro. I integrate practical scenarios learnt from the PLC to illustrate algebraic aah…, concepts. When covering systems of equations, I present word problems that prompt err… students (PSTs) to model the situation algebraically and solve it using equations. This contextual approach err… bridges the gap between theory and practical application.

This extract reveals that the MTEs' pedagogic approaches to teaching mathematics were informed by their participation in the PLC which fostered an environment of continued learning and growth for each MTE. They relied on the community's collective wisdom and creative ideas when faced with instructional challenges. This ongoing support appeared to sustain their professional development as teacher educators.

The data analysis suggests that the PLC studied provides a framework that supports MTEs' learning from colleagues. Crucially, the PLC is a space within which MTEs share, discuss, and refine their practices.

7.5 Chapter summary

This chapter has explored how a PLC supports MTEs development of mathematical knowledge for teaching algebra to pre-service teachers. It has identified content-focused and domain-specific professional learning as crucial components of MTEs' professional development. While participation in the PLC does not always guarantee growth, data analysed show that this tailored PLC has assisted MTEs in learning and teaching about teaching and in finding a balance between a focus on algebra content and effective teaching strategies.

Additionally, the chapter has engaged with the unique pedagogic requirements of mathematics teacher education, emphasising its multifaceted nature and the importance of various teaching strategies, contextual factors, lesson modelling, and pedagogic reasoning. The MTEs in the study value mastering this pedagogy in order to foster the growth of pre-service mathematics teachers. However, the study has also revealed that the exam-oriented nature of pre-service teacher education can sometimes pose challenges to teacher educators' endeavours.

CHAPTER 8

THE KNOWLEDGE MTES CONSIDER IMPORTANT FOR THEIR WORK IN PRE-SERVICE TEACHER EDUCATION: PRESENTATION AND DISCUSSION OF FINDINGS

8.0 Introduction

To prepare pre-service teachers (PSTs) for the challenges of teaching mathematics, mathematics teacher educators (MTEs) need to have a diverse and specialised knowledge base. Understanding what MTEs need to know and be able to do to support PSTs is crucial for the success of pre-service mathematics teacher education. This chapter focuses on the knowledge components MTEs in the study consider important for their work. The findings presented and discussed are drawn from an analysis of focus group interviews, PLC observations, and interviews with MTEs and their critical friends in both phases of the study.

The analysis revealed three prominent themes encapsulating the core knowledge domains highlighted by the MTEs: 1) pedagogic content knowledge, 2) subject-matter knowledge, and 3) knowledge of educational contexts, with a strong emphasis on pedagogic alignment, all of which were drawn from themes 1 and 3 (content-focus professional learning and alignment), respectively. These themes shed light on the specialised and context-responsive nature of the knowledge base that MTEs view as integral to effectively preparing PSTs for their work in classrooms. In the following sections, I report MTEs' professed claims and provide evidence related to these claims in chapter 10. These findings contribute to answering research question 2:

> What knowledge do MTEs consider important to focus on in their collaborative work in pre-service teacher education?

8.1 MTE – pedagogic content knowledge (MTE-PCK)

A core finding was the emphasis MTEs placed on the role of algebra pedagogic content knowledge (MTE-PCK) in their work with PSTs. The MTEs' discussions within the PLC and with their critical friends revealed how this MTE-PCK was continually being informed and refined through their collaborative reflections and exchange of practices. The MTEs recognised that developing robust MTE-PCK was essential for effectively guiding their PSTs through the complexities of algebra instruction. The knowledge that they wished to acquire or extend encompassed a deep understanding and familiarity with a repertoire of pedagogic strategies

and mathematical representations that have proved effective for teaching algebraic concepts and fostering algebraic reasoning.

The PLC served as a collaborative space where MTEs could collectively explore, challenge, and expand their MTE-PCK. Through these ongoing professional dialogues, MTEs claimed to share, analyse, and refine their approaches to teaching various algebraic topics, leveraging the collective wisdom and experiences of the group. In a focus group interview in Phase One, the MTEs said the following:

> Sam: Our knowledge of how to teach algebra concepts is constantly evolving. The PLC allows us to pool our expertise, err…, wrestle with pedagogical dilemmas, and learn from each other's successes and challenges in the classroom. So, so, regardless of the diverse backgrounds of our students [PSTs] we teach them using diverse teaching approaches.
>
> Nick: Agree, Sam. uh… the PLC makes us rethink our teaching. It's not just about what we teach, but you see, aah… it's about how we teach what we teach. Um… it's about giving PSTs not just math facts but the skills to teach algebra well.
>
> Dan: Absolutely. Guys, do you remember our conversations about um…, teaching concepts learned by rote? That was a game-changer. So …, the strategies we discussed now shape my teaching, helping me handle preconceived notions and teach algebra in uh…, a more engaging way.

The interview excerpt foregrounds the important role of the PLC in shaping and enhancing the MTEs' PCK. This finding resonates with Shulman's (1986) seminal work on pedagogic content knowledge (PCK), highlighting the importance of teachers' specialised knowledge of how to represent and convey specific content meaningfully. The MTEs emphasised that their knowledge of how to teach algebraic concepts effectively is continuously evolving, and the PLC serves as a collaborative space to collectively explore, challenge, and refine their approaches, as Sam stated. This finding is similar to that of Bragg and Nicol (2015), whose study found that collaboration in PLCs enhanced MTEs' pedagogic practices.

A common thread across the MTEs' conversations in the PLC was the central role of PCK in their teaching. Nick described PCK as "the heart of good teaching," emphasising the importance of understanding the "how" alongside the "what" in teaching algebra in the PLC – a finding similar to that of a number of researchers who have emphasised the role of MTE-

PCK in helping PSTs develop the necessary skills to teach mathematics productively (e.g., Appova & Taylor, 2019; Zazkis & Leikin, 2010).

The PLC discussions served as a platform for MTEs to share and analyse specific instructional sequences, task designs, and representations they used to support PSTs in developing a conceptual understanding of algebraic ideas. This collaborative exchange allowed them to continuously refine and expand their repertoire of pedagogic strategies and representations for teaching algebra, in line with Ball et al.'s (2008) assertion that teachers must possess a deep understanding of both content and how to teach it in ways that address common student misconceptions and difficulties.

Furthermore, the MTEs emphasised the importance of aligning their MTE-PCK with the realities of teaching algebra in school mathematics classrooms. They aimed to equip PSTs with relevant and applicable skills for effective algebra instruction by grounding their pedagogic knowledge in practical classroom contexts. This finding underscores the context-responsive and dynamic nature of the knowledge base that MTEs deem essential for their work in mathematics teacher preparation, resonating with Chapman's (2015) argument that MTEs ways of knowing, thinking, and holding knowledge are important for helping PSTs enhance their practice and student learning.

In a post-lesson observation interview with Nick and his critical friend (Sam), these MTEs expressed their thoughts on APCK as follows:

> Researcher: Sam, I would love to hear your thoughts on the algebra lesson Nick facilitated.
>
> Sam: I think it went well, for me err…, Nick's ability to integrate pedagogic strategies with mathematical concepts was impressive. The PSTs seemed engaged and understood the material well. These are some approaches we always look for as critical friends.
>
> Nick: Thank you, Sam. I have been focusing on deepening my understanding of effective instructional practices with the help of my colleagues and the PLC, and in general, I've been trying to explore how these pedagogies intersect with algebraic content.
>
> Sam: It shows. Your commitment to refining your pedagogy is evident in your teaching. In fact, I am not just patronising you.

Nick: As I said, our PLC meetings are instrumental. Sam's presence in my class feels like a guardian angel, we share resources, discuss research findings, and challenge each other's perspectives. It's a collective effort to stay at the forefront of algebra instruction.

Again, the MTEs emphasised the importance of MTE-PCK in their work with PSTs. The excerpt corroborates findings from the earlier focus group interview in which Sam praised Nick's ability to integrate pedagogic strategies, such as using artefacts effectively, with mathematical concepts (a key component of MTE-PCK). According to Appova and Taylor (2019), research has shown that often, when teaching mathematics content courses to PSTs, MTEs predominantly focus on subject matter knowledge and do not give enough attention to PCK, perhaps because they do not have expertise and teaching experience in embedding PCK-related learning into subject matter knowledge. By contrast, a finding of this study is that in the context of collaborative professional learning, MTEs put a premium on enhancing PCK. For example, Nick attributed his successful use of artefacts to the support from the PLC and Sam's presence in his class.

Chauraya and Brodie (2017), highlight the importance of collaborative learning environments for developing, refining and shifting teaching practices and the excerpt above exemplifies this collaborative approach, as Nick mentions sharing resources, discussing research findings, and challenging each other's perspectives within the PLC. Exchanges of practices, discussions about research, and challenges to perspectives create a dynamic learning environment that fosters the ongoing development of MTE-PCK which the MTEs consider very important for their work.

This finding contributes to the growing body of research on the specialised knowledge required of MTEs, underscoring the fundamental role that pedagogic content knowledge plays in this important professional domain (Castro Superfine et al., 2020). The findings could usefully inform the design of MTE professional development initiatives and mathematics teacher education programmes.

8.2 MTEs knowledge of content and teaching

In alignment with the work of Beswick and Goos (2018), who highlight the importance of MTEs possessing a specialised form of KCT specific to the preparation of teachers, the findings revealed that the participating MTEs also placed significant value on their knowledge of content and teaching (KCT) in the context of algebra instruction. KCT refers to the unique

amalgamation of content knowledge and pedagogic expertise that allows MTEs to make strategic decisions about how to represent and sequence content, as well as select appropriate instructional approaches (Ball et al., 2008).

For the MTEs in this study, KCT was considered essential for helping their PSTs develop a robust understanding of not just algebraic concepts, but also effective ways of teaching those concepts, thereby creating better learning experiences. In the focus group interview, MTEs explained the following:

Sam: What we know about the content [school and teacher education mathematics content] and teaching is the core of what we do. Uhm…, in the PLC, we often discuss how to make our teaching more robust by deepening our understanding of the subject matter and effective teaching strategies.

Nick: Absolutely. Err…, it's not just about knowing the content; it's about knowing how to teach it. Aah…, I think the PLC has been instrumental in shaping our perspectives on teaching methods that resonate with pre-service teachers. You see, it is about finding that sweet spot uhm…, between content mastery and pedagogical skill.

Dan: I agree with both of you. We strive to transform our knowledge into a form that's aah…, accessible and engaging for our pre-service teachers.

Sam: We do not mean to talk as if all is well with our PLC. You can identify some of our challenges as you have been with us for some time. But, err…, we have unique styles, but you see, the core idea is the same: using our knowledge of the content to make teaching more applicable. I focus a lot on diverse teaching approaches.

Dan: My emphasis might be more on the engagement side. How do we ensure our knowledge translates into an exciting learning experience? That is the key question I often ask myself.

The excerpt shows that MTEs recognised that content knowledge alone is insufficient; they must also deeply understand how to teach that content effectively. Their statements resonate with the work of Hallman-Thrasher et al. (2020), who found that MTEs' KCT plays a crucial role in their ability to design and implement compelling learning experiences for PSTs, ultimately impacting the quality of mathematics instruction in classrooms.

The MTEs' discussions highlighted their efforts to transform their KCT into a form that is accessible and engaging for their PSTs as Dan explained, and as is advocated by Chapman (2021) who suggests that MTEs should possess the knowledge to unpack and represent mathematics in ways that support meaningful understanding for PSTs in order to enhance their professional knowledge and practice.

While acknowledging challenges within the PLC, the MTEs emphasised their shared goal of using their knowledge of content and teaching to make instruction more applicable and engaging for PSTs. During a reflective discussion in the PLC, along with others, Dan, Nick, and Sam shared their thoughts on leveraging their KCT for algebra instruction as follows:

Dan: In my last class, as we covered quadratic functions, I realised that some students [PSTs] struggled with the procedural aspects of solving quadratic equations. So, I took a step back and used graphical representations to help them visualise the connections between the algebraic forms and their graphical behaviour.

Sam: Did that make any difference?

Dan: They seemed to be doing fine by the time I left.

Nick shares how using manipulatives allowed PSTs to explore patterns, build connections, and develop a deeper conceptual understanding of polynomials.

Nick: Leveraging our knowledge means recognising when our PSTs are struggling and having a repertoire of pedagogic strategies to address those struggles effectively.

Sam: You both make excellent points. However, I sometimes struggle to strike the right balance between content depth and pedagogic approaches.

Sam reflects on a recent lesson where he may have focused too heavily on the technical aspects of the content, losing some PSTs along the way. In closing remarks after the reflection, he said:

Sam: Today's meeting has been extremely beneficial for me. It has enabled me to reflect on my teaching practice and learn from your experiences. By sharing our teaching strategies and the challenges we face, we can continuously improve our knowledge and skills.

These excerpts indicate that MTEs' KCT goes beyond content knowledge to involve ability to unpack and represent mathematics that resonates with PSTs' current knowledge and learning styles and supports their professional development, (Chapman, 2021). As Nick mentioned,

“Leveraging our KCT means recognising when our PSTs are struggling and having a repertoire of pedagogic strategies to address those struggles effectively.” The excerpts also highlight MTEs' challenges in finding the right balance between content depth and pedagogical approaches. Sam reflected on an instance where he may have focused too heavily on the technical aspects of the content, potentially losing some PSTs along the way. This self-reflection demonstrates the importance of continuous professional growth and refinement of MTEs’ KCT. Significantly, the MTEs acknowledged the invaluable role of the PLC in helping them reflect on their practice, learn from each other's experiences, and collectively refine their KCT.

The reflective excerpts underscore the significance of MTEs’ KCT for designing and implementing appropriate learning experiences for PSTs, ultimately impacting the quality of mathematics instruction in classrooms (Hallman-Thrasher et al., 2020). The PLC is a supportive environment for MTEs to collaboratively reflect, share strategies, and continuously refine their specialised KCT, which is crucial for preparing high-quality mathematics teachers.

8.3 MTE - curriculum knowledge

Developing effective PSTs of mathematics necessitates that MTEs have a comprehensive understanding of curriculum knowledge (MTE-CK). This study’s findings underscore the pivotal role of MTE-CK in preparing competent PSTs. The data showed that MTE-CK includes an in-depth grasp of both the teacher education mathematics curriculum and the school mathematics curriculum. Importantly, it extends beyond mere content knowledge to include contextually relevant pedagogic approaches tailored to these distinct yet interrelated teaching contexts (Brown & Redmond, 2017). A strong MTE-CK facilitates the effective teaching of mathematical concepts and practices to PSTs, equipping them with the requisite knowledge and skills to become proficient mathematics teachers in school settings (Keazer & Maher, 2021). Furthermore, MTE-CK underpins the employment of appropriate assessment practices that support and enhance PST learning across various curricular disciplines (Bragg & Nicol, 2015). This integrated understanding of curriculum content, pedagogy, and assessment is paramount for teacher education programmes that aim to produce high-quality mathematics teachers. During the focus group interview in Phase One, MTEs explained the following:

> Nick: Our understanding of both teacher education and school mathematics curricula is very important for teacher training. You see, err…, this knowledge [curriculum knowledge] ensures that we teach relevant and err…, consistent

content and pedagogy, conforming with what students [PSTs] will be teaching in basic schools.

Sam: I completely agree with you. To adequately prepare future teachers; uhm…, a strong grasp of both curricula is essential. For me, err…, understanding the teacher education mathematics curriculum is foundational in the sense that it um…, offers the theoretical and pedagogic knowledge necessary for pre-service teachers' competency. So, so, the parallel insights make the curriculum knowledge complete for us as teacher educators.

Dan: Exactly! So, without this understanding, there's a risk of sending graduates into classrooms without clear expectations. So, you see, awareness of the school mathematics curriculum err…, empowers us to tailor pedagogical approaches to the system's distinct needs and goals. However, it also makes uhm…, teaching in pre-service teacher education a bit challenging because it can feel like you're not teaching one group but multiple groups, requiring constant alignment.

Nick: Allow me to emphasise again. Without the knowledge of the curricula from the two perspectives, we may end up lacking an understanding of the scope and sequence of the two curricula. The PLC has been instrumental in scaffolding our curriculum knowledge.

The extract highlights the centrality of MTE MTE-CK in preparing PSTs for teaching mathematics. Their perspectives demonstrate that MTE-CK involves an in-depth understanding of both the teacher education mathematics curriculum and the school mathematics curriculum. As Nick and Sam emphasised, grasping the teacher education curriculum is foundational for MTEs, as it provides the theoretical and pedagogic knowledge necessary for nurturing PST competency. Hill, Grudnoff, and Ell (2011) assert that MTEs must develop a deep understanding of curriculum content. This study revealed that MTEs can deepen their curriculum knowledge through their active engagement in PLC activities. As Nick highlighted, "The PLC has been instrumental in scaffolding our curriculum knowledge." This insight resonates with the work of Malambo (2023), who found that the MTEs studied acquired essential knowledge through various professional learning activities.

MTEs need a robust grasp of the school curriculum to effectively prepare PSTs (Castro Superfine & Li, 2014) and it was evident that those in this study recognised the importance of

being well-versed in the school mathematics curriculum. Dan articulated that this awareness empowers MTEs to tailor pedagogic approaches to the distinct needs and goals of the school settings in which their students will eventually teach. It is important to note that the MTEs acknowledged the challenges of navigating multiple curricula, as constant alignment efforts are necessary to meet the requirements of teacher education and school contexts.

Analysis of data from the PLC observations in Phase One also revealed that:

1. PLC meetings involved a careful look at the alignment of teacher education mathematics curriculum with current national teacher education standards documents (This involved the MTEs taking the teacher education course materials, assignments, activities, etc. and explicitly comparing and mapping them to the external curricular documents from school mathematics. This process allowed them to identify any missing areas that needed to be added or reinforced in teacher preparation).
2. Discussions centred on identifying potential differences between the curriculum for PST preparation and specific curricular materials teachers would use in schools.

This curriculum mapping and harmonisation exercise is essential for ensuring that PSTs receive comprehensive education that meets the national standards and prepares them to teach school mathematics. By comparing the teacher education curriculum to external curricular documents, MTEs positioned themselves to identify gaps or misalignments and make necessary adjustments to their programme.

By examining the curricular materials used in schools and comparing them to the content and pedagogic approaches addressed in the teacher education programme, MTEs can identify areas where PSTs may experience disconnects or feel underprepared. Such identification enables MTEs to make informed decisions about enhancing the curriculum, incorporating additional resources, or introducing new teaching strategies to better align PSTs' preparation with classroom demands.

These two focal points of the PLC discussions underscore the importance of continuously evaluating and refining the teacher education curriculum to ensure its relevance, alignment with standards, and responsiveness to the evolving needs of PSTs. By engaging in collaborative discussions and critically examining the curriculum, MTEs can make data-driven decisions to strengthen the preparation of PSTs, ultimately benefiting their future students and improving mathematics education.

In a post-lesson interview, MTEs pointed out that their colleagues who had knowledge of the curriculum required at the different levels of teacher preparation and in different school settings were better equipped to design comprehensive, relevant, and practical courses, workshops, and resources for their PSTs. During Phase Two, MTEs emphasised the following in a post-observation interview.

> A critical aspect of our work is our understanding of the curricula to connect them well. This is at the heart of what we do. When I mention connecting the curricula uhm…, I am referring to ensuring that what we teach in pre-service mathematics teacher education is …. in line with what is expected from teachers in the field. Uhm…, this connection ensures that our students [PSTs] are well-prepared for the classroom. As you saw from the lesson, the examples and teaching approach can be adapted by our students to their own teaching goals. (Dan)

Sam emphasised that their collaborative engagement helps them to review curriculum aims:

> Considering the ongoing reforms in this country. Our role is to adapt and evolve our teaching methods and content to match these curriculum frameworks and standards. This means engaging collaboratively in the PLC can help us develop and review curriculum aims. We know from the PLC experience that members who understand the specific learning objectives, content, and assessments required in different educational contexts are equipped to design relevant programmes and resources for their students [PSTs].

> Similarly, in another post-lesson interview, Nick reiterated the role of the PLC in ensuring coherence when he said:

> I want to emphasise that our knowledge of the curriculum influences our selection and modelling of instructional strategies and classroom management approaches. So…, through our interactions in the PLC, we get to discuss and collaborate on strategies to improve curriculum coherence. Err... the collaborative effort empowers us to make informed decisions and ensure that our students [PSTs] receive the best education possible. (Nick)

Again, in a post-lesson interview, Dan emphasised that:

> In the lesson, I used examples and text materials to emphasise the importance of connecting our knowledge, skills and teaching practices with the expectations set by

the curriculum. We are not merely teaching algebraic concepts, but we are also responsible for shaping the future of mathematics education in Ghana. This is a responsibility we take very seriously.

The excerpts highlight the importance the MTEs place on deep understanding of the curriculum at various levels and within different school settings. This understanding allows them to design effective courses, workshops, and resources that directly connect to the demands placed on practising teachers, echoing Putnam (2020), who found that understanding teacher education and school mathematics curricula is important for MTEs to enhance teaching practices and align with current school mathematics teaching approaches. The MTEs in this study acknowledge that their ability to link what is taught in pre-service programmes to the realities of the classroom is "at the heart of what we do." The excerpts suggest that MTEs achieve this by ensuring their lessons and approaches are adaptable, allowing PSTs to tailor them to their specific teaching goals, as advocated by Gallagher et al. (2022).

Furthermore, the excerpts show the important role of the PLC in keeping MTEs abreast of curriculum reforms. Collaborative engagement within the PLC allows MTEs to review curriculum aims and ensure their teaching remains relevant.

8.4 MTE- Subject Matter Knowledge – (MTEs-SMK)

Subject Matter Knowledge (MTEs-SMK) is a pivotal domain for MTEs, encompassing their understanding of their teaching content. SMK transcends merely knowing algebraic facts and concepts; it entails the ability to forge connections across different mathematical topics, apply knowledge in diverse contexts, and facilitate meaningful learning experiences for PSTs. The analysis of data from the collaborative professional learning space highlights two key components of MTEs-SMK:

1. MTEs' Specialised Content Knowledge (MTEs-SCK): This component includes MTEs' in-depth mastery of both teacher education algebra content and school-level algebra content. Further, it includes their fluency in mathematical language, notation, and symbols and their proficiency in providing rigorous mathematical justifications and explanations.
2. MTEs' Mathematical Horizon Knowledge (MTEs-MHK): This aspect of SMK pertains to MTEs' broader understanding of the intricate connections and relationships among various mathematical concepts and topics, both within algebra and across other mathematical domains. It enables MTEs to contextualise algebraic concepts within a

broader mathematical landscape, fostering a comprehensive and cohesive understanding for their PSTs.

8.5 MTEs specialised content knowledge of algebra (MTEs-SCK)

Observations undertaken within the collaborative professional learning space provided valuable insights into the specialised content knowledge (SCK) that MTEs need and deem important for teaching mathematics effectively to PSTs. During the PLC observations, the MTEs expressed their dedication to improving their subject matter knowledge.

MTEs recognised the need to build a solid foundation in understanding algebraic concepts for PSTs, as pointed out by Sam, "having a strong conceptual understanding of algebraic concepts like quadratics is necessary for us as MTEs. We have to be able to break concepts down, represent ideas in multiple ways, and get to the core of why things work the way they do." This statement is consistent with Webb's (2018) finding that MTEs break down concepts by unpacking core practices, engaging in rehearsals, and supporting PSTs in enacting ambitious teaching to enhance student learning and understanding.

Effective teaching of algebra requires not only content knowledge but also pedagogic strategies. Dan stated, "Knowing the common misconceptions PSTs tend to have about variables and expressions allows us to design instruction that gets ahead of those issues from the start using appropriate pedagogic strategies." This statement echoes the finding of Li and Castro Superfine (2018) that MTEs use their specialised content knowledge to support PSTs in understanding and addressing children's errors, adapting tasks, and considering common conceptions in teaching mathematics.

During the PLC observations, I noticed that mathematical language and symbols were occasionally discussed intentionally with a view to supporting PSTs' understanding of the algebra concepts they need to teach in schools, as advocated by Adams et al. (2016). For example, Mark explained that his PSTs work through lesson study cycles in his classes, where they are given scaffolded learning activities that gradually increase in complexity. This approach enables them to build their understanding of algebraic language and symbols over time, even in their first year. In another observation, Nick said, "Using precise mathematical vocabulary, notation, and symbols with intentionality is hugely important. If we lack that rigour in our communication, we will never develop it in our students [PSTs]."

The MTEs discussed strategies for assisting PSTs to better understand the progression of algebraic concepts. They believe this understanding will prepare PSTs for teaching algebra in schools. According to Sam, "It is essential we map out and understand the progression of how algebraic concepts build on each other. That coherent trajectory from simple to complex allows us to systematically develop our students' [PSTs] mathematical understanding over time."
Dan also highlighted the importance of justifying and unpacking the conceptual reasoning behind mathematical procedures when he said: "We cannot just demonstrate procedures. Modelling how to logically justify each step by unpacking its conceptual reasoning is key for fostering true understanding in our students [PSTs]."

In the focus group interview in Phase One, MTEs shared similar insights to those made in the PLC sessions:

Sam: In my view, we possess a unique algebra knowledge specific to teaching in teacher education. Uhm…, it concerns having a deep understanding that allows us to aah…, guide pre-service teachers effectively.

Dan: Absolutely, and I think, err…, it's not just about knowing the content but understanding how to teach it. And you see, this knowledge involves a mix of pedagogic strategies and a keen awareness of how students [PSTs] grasp algebraic concepts.

Nick: I'd add that it's also about understanding the, uhm…, language and symbols of mathematics. We need to be able to communicate these concepts clearly and assist them [PSTs] to, to develop the same skills.

Sam: Exactly, if any of us acquires the mathematical language, the content, the strategy for teaching algebra, then it means that person has acquired the skills needed to train somebody to understand algebra and be able to teach algebra as well.

Researcher: Excellent points. Could you share some specific examples of aspects of MTE Knowledge that emerged in your discussions within the PLC?

Nick: Well, we focus on teacher education mathematics content and school mathematics content. Bridging the gap between theoretical knowledge and what is taught in schools is crucial.

Dan: Sure. During our discussions, it becomes clear that what we strive to know involves aah…, not just knowing how to teach algebra but understanding the progression of concepts. You see, err…, we need to guide teachers [PSTs] on how to build on foundational knowledge for teaching.

Sam: Let me also attempt to answer your question. I found that our discussions emphasised uhm…, the importance of mathematical justifications. We are teaching students from various senior high school backgrounds, which is important.

Dan: Well said! It's interesting how our expertise in these areas [school mathematics content and teacher education mathematics content] extends our knowledge beyond uhm..., what we teach. I mean, just as we engage pedagogy from both sides, so do we incorporate content from both sides. Aah…, this implies navigating a broader mathematical concept and teaching what is important.

The extracts from both the observation notes and the interview highlight the multifaceted nature of MTEs' SCK of mathematics. Researchers have long recognised that MTE knowledge goes beyond content expertise and encompasses a unique blend of mathematical, pedagogical, and pedagogic content knowledge (Ball et al., 2008; Castro Superfine & Li, 2014). As the MTEs in the focus group highlighted, their knowledge involves an understanding of mathematics that allows them to guide PSTs effectively.

The extracts illustrate that for these MTEs, SCK comprises:

1. An understanding of algebraic concepts that allows them to guide pre-service teachers.
2. Knowledge of pedagogical strategies and a keen awareness of how students (pre-service teachers) grasp algebraic concepts.
3. Understanding mathematical language and symbols to communicate concepts clearly and assist pre-service teachers in developing the same knowledge and skills.
4. Knowledge of the progression of algebraic concepts, enabling them to guide pre-service teachers in building upon foundational knowledge for effective teaching.
5. Ability to provide mathematical justifications beyond mere problem-solving, fostering a deeper understanding of algebraic concepts among pre-service teachers.

The MTEs' emphasis on understanding the progression of algebraic concepts and providing mathematical justifications reflects their commitment to nurturing a deeper conceptual

understanding among PSTs and is in line with Ma's (1999) call for developing a profound understanding of fundamental mathematics among educators. Moreover, it resonates with the literature on the role of MTEs in fostering PSTs' mathematical reasoning and sense-making (Stylianides & Stylianides, 2010; Zaslavsky et al., 2012). More importantly, it reflects the importance of the MTEs' knowledge of PSTs' thinking and learning trajectories (Sztajn et al., 2012; Wilson et al., 2013). This specialised knowledge should enable MTEs to support PSTs in developing a deep, conceptual understanding of algebra rather than just procedural fluency.

8.6 MTEs mathematical horizon knowledge (MHK)

Analysis of the data also highlighted the importance of MTEs' mathematical horizon knowledge (MHK) in their work with PSTs. This aspect of subject matter knowledge involves MTEs' thorough understanding of fundamental and advanced algebraic concepts and mathematical knowledge unique to the teaching profession. As the literature suggests, MTEs need to have a profound understanding of the algebra content that is the goal of instruction for their PSTs, as well as advanced mathematical knowledge that goes beyond the immediate curricular content (Masingila et al., 2018). This broader mathematical understanding enables MTEs to provide a richer and more nuanced educational experience for their PSTs and facilitates the development of a solid conceptual understanding of mathematics and its applications.

Findings from the study reveal that MTEs recognise the necessity of cultivating this MHK within their PLC. Collaborative learning enables MTEs to refine their subject matter knowledge, address misconceptions, and develop a deeper understanding of the interconnections between various mathematical topics. During the focus group interview, MTEs said the following:

> Dan: Our knowledge extends beyond immediate algebra content aah…, in pre-service teacher education. You know, it includes a strong awareness of advanced mathematical concepts. And it aah…, forms the foundation for effective teaching practices. Providing not just 'what' but 'why' we teach empowers pre-service teachers with a deeper understanding.
>
> Nick: I couldn't agree more. Uhm…, our role demands a comprehensive understanding of err…, broader mathematical concepts. So, qualifications and ongoing professional learning are essential. You see, through our PLC, we enhance our understanding, uhm…, extending beyond both teacher education

and school mathematics contents. I mean, this knowledge enables us to bridge the gap between err…, what we know and what we teach to fostering aah…, a holistic understanding of mathematics among pre-service teachers.

Dan: Allow me to add to it. For me, aah…, I have realised that our knowledge of the broader context of mathematics is important, especially in teaching first-year students who have transitioned from senior high school to pre-service teacher education. The implication is that we draw from different sources to improve teaching and understanding.

Sam: Guys, guys, in as much as I would want to agree with you, I think aah…, the training we receive does not equip us enough to, … foreground our knowledge of a broader context of mathematics. In my candid opinion, uhm…, the content we learn in teacher education is limited. You and I know that aah…, the focus on pedagogy is too much. It takes a lot of experience and learning to achieve an understanding of aah…, the broader context of mathematics. That is why some teacher education institutions still want to have pure and applied mathematicians among us.

The extract reveals a dissenting voice as Sam expresses concerns about the limitations of their preparation as teacher educators in equipping them with a broader understanding of the context of mathematics. This highlights the challenge that some MTEs may face in developing the necessary MHK, which could require significant self-directed learning and professional experiences beyond their academic studies and initial teacher education.

The MTEs emphasise the importance of ongoing collaborative professional learning within the PLC for enhancing their MHK and for grasping the interconnections between different mathematical concepts and topics, as well as for fostering an appreciation for the overarching structure and organisation of mathematical knowledge (Vale et al., 2011). The findings suggest that developing this broader understanding of how mathematical ideas relate to each other and how they coalesce into a coherent discipline is an essential aspect of building robust horizon content knowledge (HCK) among MTEs, especially, during the process of assisting PSTs transition from senior high schools to pre-service teacher education (Fernánez & Figueiras, 2014)

During the PLC observations, the MTEs emphasised the significance of their MHK by discussing the importance of algebra in the wider context of mathematics. Sam stated that

although their area of expertise is algebra, they cannot overlook the fact that algebra concepts are interconnected with other math domains such as arithmetic and geometry. Dan agreed and added that having a broader perspective allows them to demonstrate to pre-service teachers how mathematics is an integrated discipline, rather than a collection of isolated topics. Nick pointed out that it is crucial to not only understand the algebra being taught but also its relevance to future studies such as calculus. Preparing students for that mathematical horizon is key.

From these exchanges it is evident that the MTEs view MHK as important for:

1. Making connections between algebraic concepts and other mathematical topics to provide cohesion.
2. Contextualising algebra learning within a broader mathematical landscape.
3. Foreshadowing more advanced mathematical concepts that build upon algebra to instil a further learning trajectory.

The observations in the PLC underscore the special role of MHK in enabling MTEs to assist PSTs in developing a cohesive and progressive understanding of mathematics. As Sam emphasised, MTEs cannot disregard the interconnectedness of algebraic concepts with other mathematical domains such as arithmetic and geometry. This statement resonates with the work of Zazkis and Mamolo (2011), who highlight the importance of MHK in helping teachers recognise the "horizontal" connections between various mathematical topics. Moreover, Dan explained that possessing a broad MHK perspective allows MTEs to demonstrate to PSTs how mathematics is an integrated field rather than a collection of isolated topics. Dan's words align with Oehrtman et al.'s (2008) assertion that a robust MHK enables teachers to present mathematics as a coherent body of knowledge, fostering a more holistic understanding in students.

Notably, Nick's insights on the relevance of MHK for future studies align with the views of Mosvold et al., (2009), who emphasise the importance of MHK in helping teachers anticipate and prepare students for future mathematical concepts. Additionally, Plotz et al. (2012) argue that teachers' mathematical knowledge should encompass both the specific content they teach and the broader mathematical context in which it is situated. This holistic understanding of mathematics is crucial for effective teaching and for fostering a comprehensive understanding among PSTs.

In summary, findings from an analysis of MTE PLC observations and from literature underscore the significance of MTEs' MHK in fostering a cohesive, integrated, and progressive understanding of mathematics for PSTs. By making connections across mathematical domains, contextualising algebra within the broader mathematical landscape, and foreshadowing future concepts, MTEs can better equip PSTs with a robust mathematical foundation.

8.7 MTEs' knowledge of educational contexts

Analysis of the data revealed that collaborative professional learning for MTEs within the PLC deliberately focused on assisting MTEs to develop a robust understanding of the educational contexts in which they operate as teacher educators and the contexts in which PSTs will eventually teach. This focus on understanding educational contexts emerged as a significant finding, highlighting its importance in preparing PSTs. During a PLC discussion on focused curriculum materials, Nick stated, "If we are not preparing our students [PSTs] for the real contexts they will be teaching in, then we are failing them. We have to keep the school realities at the forefront." During the observation, Sam also said, "I always try to bring in examples from the classes I observe during teaching practice supervision and the teachers I work with. Seeing that connection to practice is so valuable for our PSTs."

These excerpts suggest that MTEs are conscious of the educational contexts in which they operate and this consciousness can help them to tailor their teaching approaches and content to the specific needs and challenges faced by PSTs and the schools they will eventually teach in. This contextualised understanding helps MTEs to provide PSTs with relevant and applicable knowledge, equipping them with the skills and strategies necessary to navigate the complexities of diverse educational environments, reaffirming the work of Monroe (2013) who argued that contextual knowledge can shape educator's understanding of practice to improve practice. As Peter (a member of the PLC but not a participant in the study) emphasised, "The concepts we cover are important, but how we teach them has to model what works in today's mathematics classrooms across different settings."

In developing a grounded understanding of educational contexts, alignment emerges as a critical component in ensuring that the professional learning experiences of MTE translate into practical knowledge and skills to effectively prepare PSTs for the realities of school mathematics classrooms. In the following sub-sections, I elaborate on alignment.

8.7.1 Alignment

For this study, alignment refers to the coherence and congruence between what is investigated by the MTEs during collaborative professional learning and the practical knowledge, skills, and competencies required to effectively support PSTs in preparing for teaching mathematics in school settings. Analysis of the data revealed that for a comprehensive understanding of educational contexts, MTEs placed significant importance on aligning the curriculum, pedagogy, and, to some extent, specific content with the realities of school mathematics classrooms. This alignment was seen as important for MTE in their quest to ensure that what is taught in teacher education programmes resonates with the experiences and challenges PSTs will face in their future teaching roles.

The data also revealed that a focus on alignment within MTEs' professional learning involved a concerted effort among PLC members. For instance, during an interview with Dan and his critical friend (Nick), Dan expressed that "after a professional learning session, we would get together as critical friends to discuss how to integrate what we learned into our specific content and ask Nick to observe and provide feedback. He would suggest changes to assignments, readings, and classroom activities when we meet before the general PLC meeting – it has been a collective process of figuring out how to ensure coherence." Nick also expressed that they engage in activities like observations to 'safeguard' coherence and consistency.

The findings highlight the collaborative efforts made by the MTE community to foster alignment between the curriculum, pedagogy, and specific content addressed during their professional learning sessions and the demands of school mathematics classrooms.

8.7.2 Curricular alignment

I refer to curricular alignment as a deliberate process of ensuring that the expectations outlined in the teacher education mathematics curriculum 'match' the teaching practices and assessment strategies employed by the school mathematics curriculum. This process is a collaborative endeavour of MTEs to ensure that their efforts align with the desired academic outcomes for PSTs. It is a complex process that strategically connects teacher education and school mathematics curricula, highlighting MTEs' knowledge of educational contexts.

During a PLC observation in Phase One, I wrote a summary of an important discussion around the need for alignment between teacher education mathematics and school mathematics curricula. The key points raised by MTEs included:

1. Understanding both curricula is essential for the success of PSTs in their preparation.

2. The content and pedagogical approaches covered in the teacher education programme should be directly relevant to what PSTs will be teaching in schools.
3. Aligning instruction with the school mathematics curriculum can help PSTs develop a relational understanding of key concepts.
4. Navigating the complexities of the two distinct curricula is challenging, but a holistic approach that considers both is important for effective teacher education practices.
5. Tailoring instruction to meet the unique needs of PSTs while considering both curricula can better prepare them for success in teacher education and school mathematics classrooms.

MTEs recognise the need for PSTs to understand both the teacher education mathematics curriculum and the school mathematics curriculum. Connecting teacher education courses with the school curriculum helps PSTs develop relational understanding, as Dan observed "In recent times, I have noticed that when we align our instruction with the school mathematics curriculum, our PSTs seem to develop a relational understanding of the concepts we are teaching them in their practice." This observation is similar to the finding of Cross (2009) that aligning teacher education with school curricula can deepen PSTs' conceptual understanding and help them connect content and pedagogy.

The content and pedagogy in mathematics teacher education programmes, if possible, needs to relate as far as possible to what PSTs will teach in schools, as Nick stated: "The content and pedagogical approaches we cover in our programme need to be directly relevant to what our students [PSTs] will be teaching in schools. Otherwise, we risk sending them into classrooms unprepared." This statement echoes the work of Lindvall and Ryve (2019), who argues that effective professional learning programmes must provide coherent learning experiences that connect coursework to the realities of classroom practice.

For MTEs navigating the two distinct curricula is challenging but important for effective teacher education practices. As Dan expressed, "It's not always easy, but I believe this holistic approach is important for our practice."

These findings suggest that the MTEs' curriculum knowledge is not limited to the acquisition of content knowledge but rather involves a deep understanding of the pedagogical approaches and the scope and sequencing of the school mathematics curriculum in order to effectively prepare PSTs to become competent and successful mathematics teachers in school settings. The MTEs model examples of effective school classroom practices. For example, Dan shared their

experience of using a think-pair-share as a PST engagement activity to introduce the concept of fractions to a class of PSTs. He explained, “By allowing students [PSTs] first to think individually, then discuss their thoughts with a partner, and finally share their ideas with the whole class, I noticed a significant increase in their participation and engagement.” Nick also highlighted the effectiveness of incorporating manipulatives and hands-on activities in teaching algebraic concepts. Citing an example of using algebraic tiles to teach algebraic expressions and equations, he noted that “students said that when they can physically manipulate objects or engage in kinaesthetic activities, they develop a deeper understanding of abstract concepts.”

MTEs stressed the need to address diverse learning needs and styles to ensure that PSTs develop the ability to differentiate their instruction effectively. For instance, Sam shared a strategy he employed in his classroom: “Knowing that my PSTs were admitted from different backgrounds, I group them based on their readiness levels and provide them with different entry points or scaffolding to access the same mathematical concept. This way, I can challenge those ready for more complex tasks while providing additional support to those who need it.” Mark highlighted the importance of offering multiple representations and modalities in teaching mathematics. For instance, he said, “Some students [PSTs] grasp concepts better through visual aids, while others respond better to auditory approaches,” this suggests that by incorporating a variety of representations and teaching strategies, MTEs can cater to different learning styles and ensure that all PSTs have an opportunity to engage with the content.

In relation to curriculum knowledge, MTEs highlighted the significance of understanding various assessment methods and their alignment with learning objectives, enabling them to gauge PSTs’ progress and inform their teaching practices. Nick shared an example of using formative assessments, such as exit tickets or warm-up questions, to gauge students' understanding during a lesson. He explained that “these quick assessments allow me to adjust my instruction on the fly and address any misconceptions or gaps in understanding.” These are also useful examples of assessment for teachers to use in schools.

Mark discussed the value of incorporating project-based assessments or performance tasks that require PSTs to apply their mathematical knowledge and skills to real-world scenarios. He said that “these authentic assessments not only help PSTs’ to master content but also support their ability to think critically, problem-solve, and communicate their reasoning.” In addition, they provide examples of how teachers can assess learners’ work in the future.

An important element of MTE-CK is creating a supportive and inclusive classroom atmosphere that is conducive to learning. Doing this in teacher education also provides models for PSTs for the future. Dan emphasised the importance of promoting a collaborative and inclusive learning environment. He explained, "By encouraging students [PSTs] to work together, share their thinking processes, and respect diverse perspectives, we create a safe space where everyone feels valued and supported." Analysis of observation notes made during PLC sessions indicates that collaborative discussions have been instrumental in shaping and refining MTE-CK and that MTEs are continuing to learn about teaching while teaching about teaching in pre-service mathematics teacher education.

During the focus group interview in Phase One, MTEs expressed their views about curriculum alignment as follows:

Sam: Ensuring a connection between the teacher education curriculum and the school mathematics curriculum guides what and how we teach our pre-service teachers. It allows us to use effective approaches to ensure they [PSTs] grasp the necessary knowledge and skills for their future students.

Dan Well said, Sam, the curricular connection is a roadmap for us amidst educational reforms and challenges. Ensuring harmony between them better prepares pre-service teachers to make an impact in the classroom.

Nick: Sure, that's a good point. I want to emphasise that we are not just doing this for our work as MTEs. We are doing it for the students who will be taught by our students [PSTs]. We want to ensure that these students receive the best possible education in schools. That is why we take the time to review and align the curricula carefully. It's about creating a ripple effect of quality education in our schools.

Sam: Adding on to that, our responsibility is to design mathematics teacher education programmes relevant to school mathematics classrooms, ensuring coherence.

Nick: Dan spoke about challenges. May I speak about them?

Researcher: Sure!

Nick: In fact, we have a challenge as a PLC. We have our individual and collective challenges concerning safeguarding coherence. Despite our agreement to work

> together, some members do not make themselves available for collaboration. It becomes difficult to achieve coherence when we have different philosophies. Additionally, the constantly evolving nature of our work adds to the challenge.

This extract supports the findings from initial interviews with individual MTEs. MTEs explained that their understanding of curriculum alignment enables them to teach appropriate content in teacher education and to design teacher education programmes that are coherent with the actual needs of school mathematics classrooms. By understanding the content and pedagogical requirements of the school mathematics curriculum, MTEs can focus on teaching PSTs the mathematics concepts and skills that are most "relevant" for their future roles as teachers using culturally responsive pedagogy Nagasawa (2020).

Analysis of the extract above reveals that MTEs face some challenges within a PLC. Despite initial agreement to collaborate, the lack of consistent participation from certain members posed an obstacle. When individuals fail to make themselves available for joint efforts, it becomes increasingly difficult to maintain a cohesive and unified approach towards achieving the PLC's objectives. Divergent philosophies among MTEs in the PLC add a layer of complexity to the pursuit of curriculum alignment. These philosophical differences may stem from varying backgrounds, experiences, or perspectives, leading to contrasting viewpoints on teaching in teacher education. Such disparities can hinder the development of a shared vision and impede the alignment of individual efforts towards common goals.

Adding to these challenges is the ever-evolving nature of the work of MTEs in pre-service teacher education. The literature emphasises that as education policies shift and evolve, so the need for adaptability and flexibility in teacher education becomes paramount (Loughland, 2019). However, this changing policy environment can also contribute to the erosion of coherence, as members may struggle to maintain a consistent and coordinated approach amidst constant change.

8.7.3 Pedagogic alignment

Another key finding of the study is the emphasis MTEs placed on aligning the pedagogical approaches modelled and taught in the teacher education programme with effective instructional strategies and practices in successful school mathematics classrooms. The data showed that MTEs strove to ensure consistency between the pedagogy PSTs experienced and the realities of how algebra is and should be taught in school settings. For instance, in a PLC discussion, Sam said, "We cannot just lecture them about good teaching. We have to model and

engage them in the practices we want them to use." Nick also highlighted that "we cannot just give them [PSTs] lectures about good teaching. We must model and engage them in the powerful practices we want them to use." In the PLC, some sessions focussed on the following:

1. In-depth analysis of high-leverage pedagogical practices and how to incorporate these into teaching algebra. (e.g., guiding PSTs in creating and interpreting mathematical representations)
2. Modelling of research-based instructional approaches such as discourse facilitation, questioning sequences, representation use, etc.
3. Collaborative designing of lesson plans and activities that immersed PSTs in reform-oriented, student-centred pedagogical practices aligned with national teacher education curriculum framework recommendations.

Dan stated emphatically, "If we are not talking about how we expect them [PSTs] to teach mathematics, then we are just a bigger part of the problem." The findings clearly indicate that pedagogical alignment and modelling were crucial for MTEs. There was a deliberate effort to create coherence between the teaching methods being advocated and those being experienced by PSTs. By focusing on pedagogic alignment, MTEs aimed to equip PSTs with knowledge of teaching algebra and first-hand skilful practice using the techniques demonstrated by the teacher educators. This alignment work positioned PSTs for smoother transitions into enacting reform-oriented, rigorous mathematics instruction from their first teaching days.

The findings underscore the deliberate efforts by MTEs to create a solid pedagogical alignment between the instructional approaches modelled in the teacher education programme and the effective teaching practices found in successful school mathematics classrooms. This alignment work aimed to bridge the theory-practice divide that is often cited as a challenge in teacher preparation (Burn & Mutton, 2015; Jenset et al., 2018). The PLC discussions focusing on analysing high-leverage practices and how to incorporate them into algebra teaching accord with recommendations for engaging PSTs in the core tasks of teaching (Grossman et al., 2018; Wilburne et al., 2018). By unpacking practices such as facilitating mathematical discourse and using representations, MTEs provided approximations of the knowledge and skills required by PSTs. The intentional modelling of research-based approaches by MTEs resonates with the notion of providing PSTs with concrete experiences of the practices they are expected to embrace (Fortney & Atwood, 2019). As Nick and Sam emphasised, simply lecturing about pedagogy is insufficient - PSTs must experience powerful teaching practices authentically.

The collaborative design of lesson plans immersed PSTs in reform-oriented, student-centred pedagogies, reflecting principles of practice-based teacher education (Grossman et al., 2018; McDonald et al., 2014). The MTEs aimed to create coherence between PSTs' preparation and classroom realities by aligning teaching with national curriculum frameworks. The findings reveal MTEs' conviction that this pedagogical alignment and modelling helps to create coherence and bridges gaps between PSTs' training and enactment of effective mathematics instruction, in line with literature that emphasises the role of approximations and rehearsals of practice in impactful teacher education (Grossman et al., 2009; McDonald et al., 2013). By prioritising pedagogical alignment focused on high-leverage practices, MTEs position PSTs for smoother transitions into classrooms, supporting their ability to engage in the complex work of ambitious, reform-oriented mathematics teaching from the start (Ball & Cohen, 1999; Grossman et al., 2018).

In the Phase One focus group interview, MTEs also emphasised the need to align the pedagogy and content taught in both mathematics teacher education and schools. They stated the following:

> Dan: By knowing the content and pedagogical requirements of the school mathematics curriculum, we can focus our efforts on teaching them [PSTs] the mathematics concepts and skills that are most relevant for their future roles as teachers.
>
> Sam: Exactly. If we want our students [PSTs] to implement things like inquiry-based learning or using multiple [mathematical] representations effectively, we must practice what we preach. The pedagogy we use has to mirror what good mathematics instruction looks like in schools.
>
> Nick: Well said. We can't just teach our students [PSTs] theoretical approaches disconnected from the challenges of classrooms. We need to model the specific instructional strategies and techniques they will use when teaching the school mathematics curriculum.

These excerpts underscore the MTEs' conviction that:

1. Understanding the school math curriculum enables them to target the most relevant math concepts/skills for PSTs.

2. The pedagogy used in teacher education cannot be disconnected from the authentic classroom realities PSTs will face.

Jenset et al. (2018) found that coherence between teacher education curricula and school curricula positively impacts beginning teachers' preparedness. Similarly, for Justino and Rafael (2019) a focus on constructive alignment in professional learning supports the transfer of learning to practice, and a pedagogic strategy based on alignment enhances learning transfer to practice. By grounding pedagogies in authentic classroom contexts, the MTEs aimed to bridge the oft-cited theory-practice divide in teacher education (Burn & Mutton, 2015; Jenset et al., 2018). The findings of this study resonate with shifts towards practice-based teacher education, which prioritises authenticity, coherence, and direct engagement with core teaching tasks (et al., 2014).

8.8 Chapter summary

This chapter has explored the knowledge that MTEs consider important for their work in pre-service teacher education, based on data from focus group interviews, PLC observations, and interviews with MTEs and their critical friends. Analysis of this data revealed prominent themes regarding the core knowledge domains emphasised by the MTEs: 1) pedagogic content knowledge (MTE-PCK) - MTEs stressed the importance of deep pedagogic content knowledge specific to teaching algebra, including effective pedagogic strategies, mathematical representations, and ability to address student misconceptions. 2) subject-matter knowledge (MTEs-SMK) which includes two components: a) Specialised content knowledge of algebra concepts for teaching, mathematical language/symbols, and providing justifications, and b) Mathematical horizon knowledge - a broader understanding of connections across mathematical topics and advanced concepts, and 3) Knowledge of educational contexts and of alignment between teacher education and school.

These findings connect with the earlier discussion in Chapter 6 about the content focus of MTEs' professional learning. The findings from these two chapters suggest that content focus is deliberate and tailored to assist MTEs learn what they deem important for pre-service teacher education.

CHAPTER 9

THEME THREE COLLABORATIVE LEARNING - PRESENTATION OF FINDINGS AND DISCUSSION

9.0 Introduction

This chapter presents the findings and discussion on collaborative learning within a professional learning community (PLCs). It examines how collaborative engagements support mathematics teacher educators' (MTEs) professional practices and teaching knowledge, specifically in teaching algebra. The chapter explores collaboration as a fundamental activity within a PLC that shapes MTEs' approaches to teaching algebra, It is based on an extensive analysis of a focus group interview and PLC observations conducted during the first phase of the study. The findings presented are interpreted through a sociocultural lens. By highlighting the insights gained through collaborative engagement, the chapter foregrounds the impact of collaboration on MTEs' acquisition of enriched content and pedagogic knowledge. The chapter also addresses research question one, a question that is not only academically significant but also holds practical implications for the field of mathematics teacher education:

1. To what extent does working collaboratively in a PLC support the development of mathematical knowledge for teaching in pre-service teacher education?

As elaborated in Chapter 3, the learning journey of MTEs is not solitary. It is mediated and facilitated by various agents, which can be human, cultural, or material artefacts. For Kozulin (2003) organised learning activities for educators can mediate their appropriate use of knowledge in their classrooms. One of the most important findings relating to how the PLC supports the MTEs is the evidence of collaboration within it. Analysis of the data revealed that the MTEs viewed collaborative learning and sharing of expertise within their PLC as very beneficial for their growth as teacher educators. They appreciated the opportunity to engage in recurring learning cycles with colleagues, applying new practices, reflecting, and refining their pedagogies. The PLC provided a rich environment for this collaborative professional inquiry. In separate post-observation reflections, MTEs emphasised that:

> Collaboration is at the heart of our professional growth. We believe that every institution has a treasure trove of knowledge in the form of experienced faculty members. Moreover, such people are waiting to be explored. Through collaboration,

> we tap into this collective expertise and share our knowledge. It is a symbiotic relationship. (Dan)

Another MTE said that:

> Our PLC activities are well-structured, and as you can see, we engage in various activities, discussions [practice-based], lesson demonstrations, critical friendships, and more. The idea is to create a platform to learn from each other's experiences and expertise. (Nick)

Similarly, Sam described the essence of their collaboration as:

> We are not hoarding knowledge but instead using it to benefit our students [PSTs]. That is the essence of partnership for us. (Sam)

Through an analysis of the interview extracts and a close observation of the activities within the PLC, I identified several key collaborative efforts undertaken by the MTEs. These efforts were crucial in ensuring that the objectives of the PLC were met. They included:

1. Sharing practice-based experiences
2. Reflecting on practice
3. Team teaching
4. Demonstrative teaching
5. Critical Friendships

These collaborative activities act as mediators in supporting the development of mathematical knowledge for teaching PSTs. In the subsequent sections of this chapter, the first four identified collaborative efforts are discussed. I have excluded critical friendships since they were not evident in my engagement with MTEs during the first data collection phase, even though MTEs claimed they used their support. The focus of data collection in Phase Two was the affordances of critical friendship which are discussed in Chapter 11.

9.1 Learning from shared practice-based experiences

The MTEs believed that sharing their practice-based experiences enriched their professional learning and helped them build a collective knowledge base for preparing PSTs for teaching in schools. They achieved this by exchanging insights, challenges, and practical knowledge from their teaching practices. The MTEs considered the PLC to be a productive setting for their continuing professional learning, regardless of their specific level of involvement. Sam noted,

“I am one of the more vocal ones in our PLC meetings, as you have seen by now. I enjoy sharing techniques or asking questions. It is a way of getting feedback from everyone.” Patrick, (not a participant in the case study) also said in conversation, “To be honest, I do not talk much during the group discussions. However, I find it important to listen and absorb most of my colleagues’ ideas based on their classroom experiences with the students [PSTs].” These comments suggest that MTEs varied in their degree of active engagement and participation in collaborative activities. Some were more inclined to share their experiences and insights frequently, while others preferred to observe and learn from others.

Observation of the PLC during a debriefing session on March 22, 2022, corroborated my earlier understanding of how learning from practice-based experience supports the MTEs’ ongoing professional development. In the debriefing session, Nick noted, “This PLC provides a safe space to openly discuss what worked well and what did not in our classrooms. Hearing about your [MTEs] approaches and how you navigated different situations is insightful and crucial for our growth.”

Patrick said “I often felt isolated in my work, but the PLC made me realise that many of the struggles I face are common among our group. Sharing our experiences helps normalise these challenges and gives us ideas for addressing them collectively.” Patrick’s words illustrate the importance of creating a supportive and non-judgmental environment within PLCs in which MTEs feel comfortable to openly discuss their teaching experiences, both successes achieved and challenges faced. According to Vangrieken et al. (2017), effective PLCs foster a culture of trust, respect, and open dialogue, which is essential for meaningful professional learning. The collaborative sharing of practice-based experiences reduces feelings of isolation, provides validation that pedagogical struggles are common, and allows MTEs to learn from each other’s approaches. This finding resonates with studies by Dogan et al. (2016) and Brodie (2013), which highlight how PLCs can mitigate the isolation often experienced by teachers and create a sense of community where challenges are normalised and collective problem-solving occurs. Ultimately, this exchange of knowledge and strategies can contribute to the continuous growth and development of MTE practice and teaching knowledge.

The MTEs highlighted that sharing practice-based experiences provided insights into using and adapting concrete teaching materials, lesson plans, and assessment strategies. This use of resources was precious for novice MTEs. Mark emphasised, “I can see the newcomers among us benefit from the collective lesson planning, assessment approaches and the specific materials

we use in teaching. I wish I had had this opportunity when I joined the staff." Mark's comment underscores how newcomers to the MTE role could significantly benefit from accessing the collective wisdom and resources curated by the experienced members of the PLC. Access to exemplary lesson plans, instructional materials, and proven assessment approaches can provide a solid foundation and guidance for these novice MTEs as they navigate the complexities of mathematics teacher education.

This finding echoes recommendations in the literature on providing structured support and mentorship for early-career teacher educators (Suppa et al., 2020). Sharing instructional resources and practical knowledge within PLCs can serve as a form of induction and professional socialisation, helping novice MTEs develop their pedagogical repertoire and learn from the expertise of veteran colleagues (Eshchar et al., 2021).

Additionally, during the Phase One focus group interview, MTEs shared their thoughts on learning from their colleagues' practical experiences as follows:

> Dan: We gather to collaborate, share experiences and insights, and overcome challenges. Together, err…, we complete the puzzle of teaching in teacher education.
>
> Sam: Exactly! You may ask how we do that. However, learning from each other's experiences is our uhm…, shortcut to building a collective understanding of practice. Some of us have been in this field for years, and err…, we have faced all sorts of situations. Sharing those experiences with less experienced colleagues is invaluable as it improves their teaching approaches. It feels like passing on the torch.
>
> Nick: You guys have said it all. But for me, aah…, the classroom is our testing ground. Seeing our students [PSTs] grappling with mathematical concepts is also, err…, a learning opportunity. We bring those experiences back to the PLC, discuss them, and learn from them. Our growth is directly tied to this sharing of experiences.

The extracts indicate that, as Dan stated, PLCs serve as a valuable forum for MTEs to gather, collaborate, share insights and experiences, and collectively work to overcome the challenges faced in teacher education. These statements affirm the collaborative and supportive nature of

the PLC and its role in bringing together MTEs to pool their collective knowledge and experiences.

The findings align with and build upon the existing literature on the role of PLCs in collaborative professional development. As evidenced in this study, the collaborative nature of PLCs can foster a sense of collective responsibility for both collegial and PST learning. The iterative learning process mirrors the principles of job-embedded professional learning (Zepeda, 2014) by situating learning within the authentic context of educators' work, promoting knowledge transfer into practice and fostering sustained changes in instructional practices (Desimone & Garet, 2015; Opfer & Pedder, 2011).

While individual work allows for practice and consolidation of knowledge (Hargreaves & O'Connor, 2018), the opportunity to collectively analyse experiences within a PLC fosters an understanding and refinement of pedagogic approaches. This understanding was evident in MTEs' unanimous appreciation for learning from practice-based experience within the PLC, suggesting its significant contribution to MTEs' teaching knowledge and classroom practice.

9.2 Learning through reflection on practice

Within the PLC MTEs deliberately and systematically engaged in reflective discussions to enhance the complex knowledge required for teaching teachers. To translate their learning into classroom practice, the PLC adopted a process of reflection embedded in a plan-teach-reflect structure, which involved thoughtful analysis of their teaching practice during discussions with colleagues. As Dan noted during a reflective session in the PLC, "regularly taking time to think on how my lessons went, what worked well, and what could be improved is important for us. It, uhm…, allows me to constantly refine my practice based on evidence rather than just gut instinct." I noticed that reflection was an activity of the MTEs' PLC in which they regularly assessed their strengths and weaknesses as teacher educators, particularly at the beginning and end of each section of the curriculum. This reflective process allowed them to make informed decisions about what aspects of their practice required refinement to enhance their teaching and PSTs' learning experiences.

During a PLC observation on April 26, 2022, to encourage members who showed limited interest in reflection, Nick remarked: "The reflections we do as a group provide such rich insights. Hearing your different perspectives pushes me to consider my practice from new angles I may have missed." Sam explained that reflecting and analysing their teaching experiences deepens their understanding of practice. He encouraged colleagues by saying "At

first, I admit I was sceptical about devoting so much time to reflection. However, now I see it makes much sense. The insights have helped evolve my teaching in ways I did not anticipate." Reflective conversations within the PLC provided a dedicated space for MTEs to examine their practices critically, uncover blind spots, and explore alternative pedagogical strategies (Slavit et al., 2011). As highlighted by Sam's comment, engaging in collaborative reflection deepened the MTEs' understanding of their practice. It promoted a mindset of ongoing development and consideration for PSTs' needs (Timperley et al., 2019).

The value of reflection was affirmed in a focus group interview with the MTEs:

> Sam: Reflection is crucial to our growth as teacher educators within this PLC. We intentionally self-assess our teaching practices and interactions with students [PSTs]. It is like looking in the mirror and asking ourselves tough questions. Did our lessons meet their objectives? Were our pedagogic strategies effective? Through this reflective process, we dig deeper into mathematical concepts and pedagogical approaches, uncovering areas where we can improve our teaching.
>
> Dan: Exactly! Reflection gives us a sense of direction for navigating the complexities of our practice. Not only do the reflections help improve my current teaching, but they have also made me much more reflective as a practitioner in general. I now instinctively analyse situations and think critically about how to address challenges.
>
> Nick: The reflection sessions are valuable because they provide a structured time and space to think deeply about our practice. With all the daily demands of teaching, it is easy to keep pushing forward without pausing to reflect.

The excerpts indicate that reflection was central to the MTEs' professional learning and growth. Findings from analysis of both interview transcripts and observation field notes are that structured reflective practice enabled the MTEs to:

1. Intentionally and systematically self-assess their teaching practices, interactions with PSTs, and effectiveness of pedagogical strategies.
2. Evaluate their lessons to see if the goals were reached and pinpoint areas where the pedagogy and mathematics instruction concepts need improvement.
3. Gain deeper insights into the complexities of their practice as teacher educators through thoughtful examination of challenges faced.

4. Develop an ongoing reflective mindset that extends beyond the PLC sessions into their daily work as MTEs.

The excerpts and the summary illustrate how structured reflection provided opportunities for MTEs to evaluate their work, pinpoint strengths and weaknesses, and refine approaches based on evidence rather than instinct alone. Engaging in both individual and collective reflection appeared highly beneficial. Individual reflection allowed space for deep introspection, while group reflections leveraged the PLC's collective wisdom and diverse perspectives to shed light on blind spots (Slavit et al., 2011; Fullard & Watts, 2022). By continuously self-studying their practice through a reflective lens, the MTEs can be better equipped to model practical teaching approaches and can prepare PSTs to become reflective practitioners (Thomas & Correa Molina, 2020).

Overall, the findings suggest that ample opportunities for systematic individual and group reflection were critical to enabling the MTEs to translate their learning from the PLC into tangible growth and improvements in their teaching practices. Developing habits of consistent, intentional reflection fostered an evolving reflective practitioner mindset.

9.3 Learning from team teaching

In the context of the study, team teaching is a collaborative approach in which two or more MTEs collaborate to plan and deliver instructional content to PSTs and reflect on this teaching within the PLC. Although I did not observe the MTEs team teach in their classrooms, planning sessions before teaching and reflective discussions during debriefing showed that team teaching was a collaborative practice within the PLC designed to improve the MTEs' teaching knowledge and practices. I noticed that the more experienced MTEs scaffolded the work of less experienced colleagues to aid them in reaching a certain level of development and coherence in mathematical instruction.

The MTEs revealed that they work as a team to review the curriculum goals and instructional materials for mathematics teacher education programmes. They pool their expertise to create well-structured and engaging learning experiences for PSTs. Sam noted during a planning session, "Having multiple perspectives during the planning phase is necessary. My colleague often caught instances where the materials I selected did not fully align with the curriculum goals or missed opportunities to highlight certain pedagogic strategies." Likewise, Nick expressed, "Team teaching means we must get on the same page about the key takeaways and learning outcomes we want our students [PSTs] to walk away with. This thorough search of

our curriculum goals helps us streamline our instruction and materials to focus on the essentials."

Sam's comment underscores the value of having multiple perspectives during the planning phase. Collaborating with colleagues helped identify instances where the selected materials may not fully align with curriculum goals or miss opportunities to highlight specific pedagogical strategies. This collaborative review process contributes to better alignment between instructional materials and intended learning outcomes. Nick's statement further emphasises the importance of achieving a shared understanding of the key takeaways and learning objectives for PSTs.

In a PLC debriefing session, Peter and Patrick shared with the PLC their recent experience of team teaching the concept of modular arithmetic to their PSTs. Here is a summarised field note:

> Peter started by providing an overview of the lesson and areas they identified for potential improvement: "Overall, we felt the hands-on clock activity worked well for illustrating why modular arithmetic is used for telling time. The students [PSTs] stayed engaged, working through examples in small groups. However, we noticed some struggled to generalise the process for doing operations within different modular systems. We may need to be more explicit about deriving those general processes next time."
>
> Patrick then weighed in on the sequencing and pacing of their examples: "Starting with easier modulo examples like 5 or 10 before jumping into modulo 12 for the clocks may have been a better progression. The leap to Modulo 12 initially seemed to lose some students [PSTs]. We also wish we had allocated more time for the application examples at the end rather than rushing through them."
>
> The two opened the floor for feedback and suggestions from other PLC members who have also taught the same lesson. Nick recommended more robust activation of PSTs' prior knowledge of related concepts: "It may help to prime them more by first surfacing connections to familiar ideas like even/odd numbers and divisibility rules they have seen before. Modular arithmetic is abstract, so bridging to that prior knowledge could provide an entry point, knowing they come from different backgrounds."
>
> Other MTEs also shared their thoughts, prompting Peter and Patrick to reflect on improving their lesson pacing, examples, and prerequisite knowledge and to leverage

their instructional styles more effectively. Peter and Patrick concluded by thanking the group for the valuable feedback and committing to incorporate suggestions as they reworked plans to teach the modular arithmetic lesson again in the future.

The debriefing extract exemplifies how the PLC provided a space for Peter and Patrick to engage in reflective practice, receive constructive feedback, and refine their pedagogic approaches. The PLC encouraged systematic analysis through individual reflection and leveraging the community's diverse expertise and perspectives. Recognising that PSTs come from varied backgrounds, the PLC members emphasised building on PSTs' prior knowledge as an essential entry point for understanding mathematical concepts like modular arithmetic.

Additionally, during the focus group interview, the MTEs reported that team teaching provided opportunities for mutual learning, peer feedback, and the development of shared understandings about effective mathematics teaching practices:

Dan: Collaborating with my colleagues through team teaching has been an enlightening experience. We exchange ideas, experiment with different teaching methods, and learn from each other's strengths in real-time.

Nick: Exactly! Planning and teaching together allowed us to unpack the rationale behind our pedagogical choices and delve deeper into the mathematical content knowledge required for teaching specific topics effectively.

Sam: Sure! Though I don't have a team to teach with, team teaching fosters a sense of accountability and collective responsibility for student [PSTs] learning. The debriefing sessions after teaching or peer observation were productive because I learned from them. The opportunity to analyse and rationalise pedagogic choices facilitated a deeper understanding of effective mathematics teaching practices for PSTs.

The extracts suggest that collaborative teaching can promote the development of pedagogical content knowledge (PCK) among MTEs and their PSTs, which is crucial for mathematics instruction (Muir et al., 2017). As MTEs collaborate and reflect on their teaching practices, they can provide PSTs with examples of teaching to be emulated. It should be noted that teacher educators (and teachers) do not always have a 'partner' available to work with. In this study, even though Sam had no partner for team teaching, he remained an important part of the community by participating in the reflection and analysis of his colleagues' collaborative

teaching experiences. Improving practice through team teaching requires thorough planning, clear communication, and a shared vision among the collaborating teacher educators (Marzocchi et al., 2021). Establishing clear roles, responsibilities, and decision-making processes can help address potential challenges and ensure a positive and productive team-teaching experience. Overall, the findings suggest that team teaching, as a collaborative activity within the PLC, can support MTEs' teaching knowledge and development by fostering shared understandings, mutual accountability, and opportunities for peer feedback and reflection on teaching practices.

9.4 Learning from demonstration lessons

In the context of the PLC, demonstrating their teaching to one another is a collaborative practice where MTEs observe and learn from their colleagues' teaching practices. The data analysis revealed that demonstration lessons, where one member of the PLC modelled an exemplary instructional strategy, were prevalent and important activities within the community. In the focus group interview, MTEs reported that these demonstrations allowed them to witness effective mathematics teaching practices in action and gain insights into how to implement specific pedagogic approaches:

> Nick: Watching my colleagues deliver demonstrations was like having, err…, a front-row seat to a masterclass in mathematics instruction. I could see firsthand how they structured their lessons, managed classroom dynamics, and responded to students' [PSTs] questions or misconceptions.
>
> Dan: The demonstrative teaching sessions were beneficial for me regardless of the years I have taught as a mathematics educator. I observed experienced colleagues and learned from their expertise in areas where I felt less confident, such as facilitating productive mathematical discussions or effectively incorporating technology.
>
> Sam: The discussions after the demonstrations were constructive and supportive. We respectfully provided feedback to our colleagues while learning from their expertise.
>
> Dan: Thanks for bringing this up, Sam. The post-demonstration reflections were just as valuable as the demonstrations themselves. Having that dedicated time to unpack what we observed, ask questions, and analyse the pedagogical choices made was crucial for maximising our learning.

This interview extract shows that demonstration lessons and post-demonstration discussions provided a rich learning experience for the MTEs. Observing experienced colleagues in action allowed them to witness first hand exemplary mathematics instruction practices for PSTs.

Dan's acknowledgement of the benefits of observing experienced colleagues, irrespective of one's experience level as an MTE, underscores the significance of continuing professional development and the value of opportunities to learn from peers' expertise. This acknowledgement indicates that even for seasoned educators, there are always areas where they can feel less confident or seek improvement.

For Sam, the discussions that followed the demonstrations created a supportive and collaborative learning environment. The respectful exchange of feedback and the opportunity to analyse and justify teaching choices helped the MTEs better understand effective mathematics teaching practices, preparing them to guide PSTs in the mathematics teacher education classroom. I acknowledge that collaboration was not perfect within the PLC, as there were different levels of participation and commitment. However, I observed that demonstration teaching within the PLCs was a reciprocal process, allowing members to demonstrate and observe at different sessions. Different MTEs facilitated the demonstrations at different times, creating an environment where knowledge and experiences are shared and appreciated among all members in a mutually respectful learning culture.

For instance, during a PLC observation on April 12, 2022, Dan demonstrated a lesson on properties and operations on functions. For a certain period, he focused on the concept of composition of functions, knowing that this could be challenging for many PSTs in their first year. To make this abstract concept more accessible, he shared the following approach with his colleagues:

He suggested beginning with a clear overview of the concept and then breaking it into smaller, more understandable parts. He used visual aids, such as diagrams and graphs, to illustrate the process of composing functions, which he envisioned could make the abstract concept more tangible for the PSTs. Additionally, he incorporated real-world examples, such as calculating distance travelled when given velocity and time functions or the determination of the final cost of an item after applying multiple discounts, to help PSTs connect the mathematical concept to relatable situations. During the demonstration, the other MTEs observed, taking notes and afterwards asking clarifying questions. Some of the questions raised were:

1. How do you plan to address potential misconceptions related to the order of function composition?
2. What strategies will you employ to ensure PSTs can differentiate between the composition of functions and the algebraic combination of functions?

After the demonstration, the MTEs engaged in a reflective discussion, sharing insights and suggestions for improvement. Some of the recommendations made included:

1. To gradually build PSTs' understanding more scaffolding techniques should be incorporated, such as providing guided practice examples before moving to independent practice.
2. Emphasising the importance of precise mathematical notation and language when working with function composition to avoid ambiguity.
3. Exploring alternative representations, such as arrow diagrams, to cater to different learning styles and reinforce the concept from multiple perspectives.
4. Consider using formative assessments during the lesson to gauge PSTs' understanding and promptly address any lingering misconceptions.

The recommendations made by the MTEs after the demonstration further exemplify the value of the PLC as a space for professional growth and continuous improvement. Suggestions such as incorporating scaffolding techniques, emphasising precise mathematical language, exploring alternative representations, and utilising formative assessments address the challenges of teaching complex mathematical concepts to PSTs. These recommendations demonstrate the MTEs' commitment to adopting evidence-based instructional strategies and catering to diverse learning needs (Anhalt & Cortez, 2016).

Additionally, the reflective discussion among MTEs underscores the value of continuous professional development and of sharing expertise within the PLC. Through critical analysis of Dan's demonstration and by offering constructive feedback, the MTEs engaged in collective learning and knowledge co-construction.

9.5 Chapter summary

This chapter has examined how collaborative learning within a PLC supported the development of MTEs' knowledge for teaching algebra. Key collaborative activities identified were sharing practice-based experiences, reflection on practice, team teaching, and demonstration lessons / post lesson discussions. These collaborative efforts enabled MTEs to exchange insights from classroom experiences, critically analyse their practices through reflective discussions, learn

from peer observations and feedback, and co-construct effective pedagogical approaches. The PLC provided an environment conducive to continuous improvement by fostering inquiry, shared expertise, and collective reflection. The findings underscore the importance of cultivating trust and openness within PLCs to maximise the benefits of collaborative professional learning. The PLC in this study facilitated MTEs' growth by enabling knowledge exchange, peer learning, and the collaborative refinement of robust mathematics teaching practices.

CHAPTER 10

OBSERVATION OF CLASSROOM PRACTICES: EXPLORING EVIDENCE OF PLC INTEGRATION - FINDINGS AND DISCUSSIONS

10.0 Introduction

In Chapters 7 to 9, I presented and discussed the claims made by Mathematics Teacher Educators (MTEs) during interviews and discussions. This chapter presents and discusses evidence related to these claims that was observed in the teaching practices of these MTEs, given that the professional development of MTEs goes beyond acquiring deepened mathematical knowledge to integrating such knowledge, derived from participation in the PLC, into their classroom practices. In the context of this study, the pre-service teacher education classroom serves as the crucial testing ground for refining and applying this knowledge. The discussion in this chapter is based on the key themes and sub-themes that emerged in Chapters 7 to 9 and addresses the research question three: *What counts as evidence of the integration of PLC learning into MTEs' pre-service teacher education practices, and is there evidence of such integration*?

10.1 Exploring the evidence of take-up from PLC in MTEs classroom practice

In the methodology chapter, I outlined a data collection strategy that included multiple lesson observations for each MTE and pre- and post-lesson interviews. Table 5.5 presents the topics and number of lessons observed for each MTE. Here, is a summary of classroom practice.

Table: 10.1 Summary of classroom enactment of practice

Theme	Sam	Nick	Similarities	Differences
Alignment with NTECF standards	Explicitly stated lesson objectives aligned with NTECF	Did not write objectives on board but verbally communicated them	Both aimed to align with NTECF standards	Sam is more explicit in stating objectives
Use of multiple representations	Used verbal explanations, arrow diagrams, ordered pairs, and symbolic notations	Used arrow diagrams, ordered pairs, graphs, and symbolic notations	Both used multiple representations	Nick more effectively helped PSTs make connections between representations
Lesson modelling	Mostly teacher-centred, limited PST engagement	Encouraged PST participation and understanding	Both attempted to model teaching practices	Nick's approach was more participatory
Scaffolding and example spaces	Progressed from simple to complex examples	Used real-world scenarios and relatable language	Both used scaffolding techniques	Nick's examples were more contextualised
Use of artefacts	Prioritised pace over use of artefacts	Did not use specific artefacts, but used board effectively	Neither used physical artefacts despite PLC discussions	Sam explicitly mentioned time constraints as a reason
PST engagement	Limited engagement, mostly chorused responses	Encouraged PSTs to share interpretations and ideas	Both attempted to engage PSTs	Nick's approach led to more active PST participation
Fostering diverse perspectives	Limited evidence	Explicitly encouraged diverse problem-solving approaches	Both are aware of the importance	Nick more explicitly fostered diverse perspectives

To facilitate analysis of the data, observational data and interview transcripts were first coded for recurring patterns. These codes were then grouped into broader categories, from which the final themes emerged. The insights presented in this chapter are drawn from this process.

Two explicit themes emerged from this process: 1) the alignment between school and teacher education mathematics curricula and 2) the pedagogy of mathematics teacher education. The latter includes representations, lesson modelling, strategic use of teaching tools, and the implementation of student-centred approaches. These themes, particularly the pedagogy of mathematics teacher education, build upon concepts introduced in Chapters 7 and 8, in which MTEs' perceptions of their professional development and the role of PLCs in shaping their teaching practices were explored. Before presenting specific evidence (such as board work, comments, etc.) linked to each MTE, I provide a summary of how each MTE enacted teaching on functions. I then connect these summarised teaching practices to what MTEs said in relation to the two themes.

10.2 Overview of classroom observations

This chapter examines the teaching practices of MTEs. I initially observed three MTEs. However, as a result of data saturation, my analysis focuses on two MTEs (Sam and Nick). After a comprehensive examination of the first two MTEs' pedagogical approaches, I found that subsequent data analysis from the third MTE (Dan) yielded no substantial new insights into the research questions. In the following section, I begin with Sam's case, as it offers a particularly rich example of function teaching that aligns closely with the themes identified in earlier chapters.

10.3 Sam's teaching of functions

The lesson analysed here is part of a series of observations conducted over the semester. This particular lesson on functions was chosen for its clear demonstration of concepts discussed in the PLC and its representation of Sam's typical teaching approach.

Sam initiated the lesson by informing the PSTs that they would learn about functions. He assumed some prior knowledge, asking, "Who can tell me what they remember about relations and mappings from school mathematics?" This question was an attempt to engage PSTs to respond with their existing knowledge. Despite their initial reluctance, Sam persisted in fostering a participatory environment. For example, he asked, "Can anyone give me an example of a relation from everyday life?" When met with silence, he offered, "Think about the relation

between your age and your height. How might we represent that?" At the outset, Sam wrote the lesson objectives on the board:

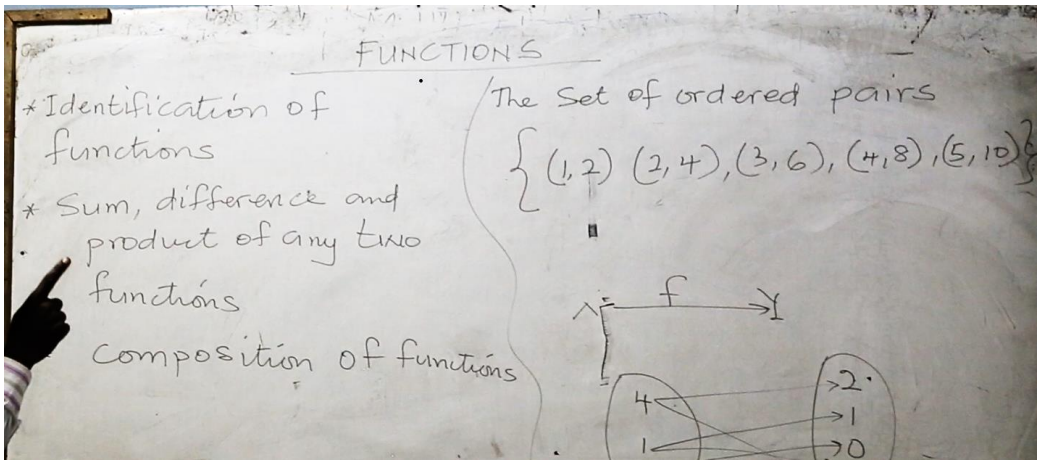


Figure 10.1: Depicting Sam's lesson objectives

Sam defined relations and mappings, explaining, "A relation is any association between two sets, while a mapping requires each element in the domain to have a unique image in the range." He then employed arrow diagrams to visually represent functions. For instance, he drew a diagram showing the mapping of days of the week to their first letters, emphasising that each day maps to one and only one letter. Sam's use of multiple representations reflected a key emphasis in the PLC discussions. He transitioned between arrow diagrams, ordered pairs, and algebraic notation. He emphasised that no two ordered pairs should have the same first component in a function, reinforcing the uniqueness of mapping in a function. This approach suggests Sam's knowledge of multiple representations that his participation in the PLC had enhanced.

Throughout the lesson, Sam encouraged participation. When introducing symbolic notation for functions, he asked, "What do you think $f(x)$ means in this context?" One PST hesitantly suggested, "Is it like an input-output machine?" Sam built on this, saying, "Excellent analogy! Can anyone expand on that?"

Sam addressed PSTs' queries, particularly around distinguishing between relations and functions. One student asked, "Isn't every function also a relation?" Sam used this as an opportunity for class discussion, eventually clarifying, "All functions are relations, but not all relations are functions. Can anyone tell me why?" To challenge the PSTs, Sam provided examples like $f(x) = x^2$ and $g(x) = \sqrt{x}$, asking them to determine the domain and range of

each. This revealed some misconceptions, particularly about the domain of $g(x)$, providing an opportunity for Sam to address gaps in understanding.

Based on his informal discussions with PSTs before the lesson, Sam found that approximately 60% were familiar with basic function concepts from high school, while for others, it was new material. He skilfully balanced review and new content to engage all learners. In the latter part of the lesson, he guided PSTs through increasingly complex operations on functions, including addition, multiplication, and composition. He consistently referred back to real-world applications, such as combining functions representing distance and time to calculate speed. Sam concluded by revisiting the lesson objectives, asking PSTs to reflect on their learning. He emphasised, "Understanding functions is crucial not just for your own mathematical knowledge, but for how you'll teach these concepts in the future."

In Sam's teaching there was evidence of the following:

1. Alignment with what had been discussed in PLC sessions: Sam's emphasis on multiple representations and real-world applications directly reflected topics discussed in the PLC.
2. A PST-centred approach: Despite initial reluctance, Sam consistently encouraged PSTs' participation and built on their contributions.
3. Scaffolding of complex concepts: Sam moved from basic definitions to complex operations, providing support at each stage.
4. Addressing misconceptions: Sam used PST questions and errors as learning opportunities for the whole class.

While this analysis focuses on one lesson, these approaches were consistent across multiple observations of Sam's teaching and provide concrete evidence of how what had been learned through participation in the PLC was integrated into classroom practice, demonstrating the impact of collaborative professional development on one MTE's teaching strategies.

10.4 Evidence of Alignment with NTECF Standards

Sam's teaching demonstrated alignment with the learning objectives outlined in the National Teacher Education Curriculum Framework (NTECF). This alignment is evident in how he communicated with PSTs and in how he structured his lesson as demonstrated in the extract from a videoed lesson observation:

> Sam: So, by the end of the lesson, I'm expecting each one of you to be able to identify what a function is. And then also find that sum, the difference and product of

any two functions and then also composition of functions. So, I'm expecting each one of you to be able to achieve these objectives. [He writes on the board...]

This extract, which occurred early in the 90-minute session, provides insight into Sam's approach to curriculum alignment. By explicitly stating these objectives, Sam sets clear expectations for the lesson, aligning with best practices in mathematics education. As Hiebert et al. (2007) argue, clear learning goals are essential for effective mathematics instruction, providing both teachers and students with a shared understanding of the lesson's purpose. These objectives correspond directly to key components of the NTECF, particularly in the area of algebraic understanding and manipulation. Throughout the lesson, Sam's teaching showed connections to these stated objectives. For example: When introducing function identification, Sam used multiple representations (arrow diagrams, ordered pairs, algebraic notation) to help PSTs recognise functions in various forms as recommended by Goldin (2020), who emphasises the importance of using multiple representations in mathematics education to develop a comprehensive understanding of mathematical concepts. For function operations, Sam provided examples like: "If $f(x) = 2x + 1$ and $g(x) = x^2$, how would we find $(f + g)(x)$?" When addressing function composition, he used real-world scenarios: "Consider $f(x)$ as the cost of x items and $g(x)$ as the number of items sold in x days. What might $(f \circ g)(x)$ represent?"

These examples demonstrate how Sam structured his lesson to address each stated objective, thus providing evidence of curriculum alignment. In a post-lesson interview, Sam stated, "I always try to align my teaching with the NTECF. It's something we've discussed extensively in our PLCs." During a PLC session observed earlier in the semester, Dan also shared, "I've been working on explicitly stating lesson objectives that match the curriculum framework. It helps both me and the PSTs stay focused."

While Sam's teaching shows a clear intention to align with curriculum standards, the impact on PSTs' learning outcomes wasn't directly assessed in this research as this was not what I sought to explore. Sam's approach to curriculum alignment was evident in the lesson's structure. He began with basic function identification, progressed to operations on functions, and concluded with the more complex concept of function composition. This progression mirrors the structure suggested in the NTECF, indicating Sam's understanding of the curriculum's intended flow. This structured approach aligns with the cognitive apprenticeship

model proposed by Collins et al. (1991), which emphasises the importance of sequencing in instruction, moving from simple to complex tasks.

10.5 Teacher education mathematics pedagogy: Scaffolding and use of example spaces

Sam's teaching method incorporated scaffolding and the deliberate selection of example spaces to enhance PSTs' understanding of mathematical concepts. This aligns with essential components of mathematics teacher education pedagogy discussed in the PLC sessions (see Chapter 9, p. 167).

10.5.1 Scaffolded approach

Sam adopted a scaffolded approach, progressing from foundational concepts to more complex ideas. This approach allowed PSTs to build on their prior knowledge from school mathematics as illustrated in the following exchange from the lesson observation transcript:

> Sam: We are going to learn about functions. Moreover, I presume that uh…, you have heard about relations and mappings before.
>
> Sam: You have learnt about relations and mapping before you know how to define a relation, uhm…, and define a mapping. So, so… what is a relation? When we say relation, what is it about? Aah…, in mathematics, when we say there is a relationship between set A and set B, what do we mean?

This extract demonstrates Sam's use of questioning to activate PSTs' prior knowledge, creating a bridge to the new concept of functions. By prompting PSTs to recall and articulate their understanding of relations, Sam facilitated a structured learning experience. Bakker et al., (2015) emphasise the importance of scaffolding in mathematics education, particularly in bridging the gap between informal and formal mathematical knowledge. They argue that scaffolding helps students connect their intuitive understanding with more abstract mathematical concepts. The extract reinforces the work of Rachamim et al. (2022), who assert that professional development programs can assist MTEs in creating effective mathematical tasks through scaffolding techniques such as modelling, reflection, and worked examples for PSTs.

10.5.2 Use of example spaces

Sam's example spaces progressed from simple to complex, building on PSTs' existing knowledge. For instance, he presented the following example to explain a function:

Sam: Now look at this [referring to a question on the marker board]. Determine whether or not this is a function. {(5,2), (3,5), (-2,3), (-5,1)}.

Sam: Look at it carefully. consider the property we have defined about a function and tell me whether or not this relation is a function.

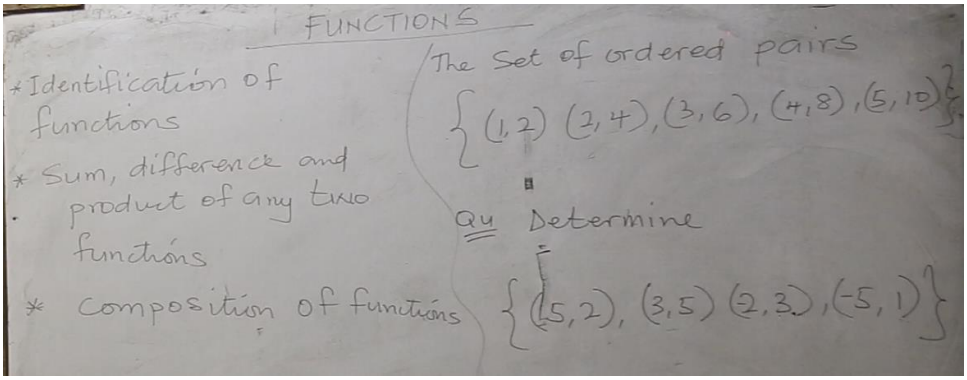


Figure 10.2: Boardwork of some examples

Sam's choice of example spaces was not only evident in this lesson but also in other lessons I observed during the two phases of the study. His use of examples was purposeful and aligns with the goal of preparing PSTs for both teacher education and school mathematics classrooms. The examples he used, such as set notation {(5,2), (3,5), (-2,3), (-5,1)}, could be employed effectively in both contexts. The strategic use of examples in mathematics education has been a focus of research. Goldenberg and Mason (2008) introduced the concept of "example spaces" as a tool for understanding how teachers and learners conceptualise mathematical ideas. By prompting PSTs to analyse whether the given relation is a function, Sam provided opportunities for them to apply newly learned properties and definitions. Sam's progression from simple to complex examples aligns with the principles of variation theory in mathematics education, as discussed by Kullberg et al. (2017).

The variety of examples used at different stages of the lesson (as shown in Figure 11.1) suggests intentional selection and sequencing to reinforce PSTs' understanding of algebraic concepts. This strategic use of multiple example spaces allowed Sam to highlight different aspects of the concept and cater to diverse PSTs' learning needs. This approach enables learners to discern critical aspects of the concept by experiencing systematic variations in examples. As Sam highlighted in the focus group interview (p. 141), MTEs use multiple strategies, including varied examples, to enhance PSTs' comprehension of mathematical concepts. This approach was evident in PLC discussions and collaborative lesson planning (p. 139). The alignment

between Sam's classroom practice and what was discussed in the PLC demonstrates the impact of collaborative professional development on teaching strategies.

In conclusion, Sam's use of scaffolding and strategically selected example spaces reflects key principles of mathematics teacher education pedagogy. This approach not only supports PSTs' immediate learning but also provides them with a model for their future teaching practice, bridging the gap between theory and practice in mathematics education.

10.5.3 Use of mathematical representations

In Sam's teaching of functions (observed in Lesson 3 of the semester), he employed multiple representations to support PSTs' understanding of algebraic concepts.

Sam: Now look at the second one I'm going to draw. look at it whether or not it defines a function. Let's see. In the first set this is {2, 1, 0, -1, -2} and the second set {4, 1, 0}. Look at the sets carefully. Now, let's map.

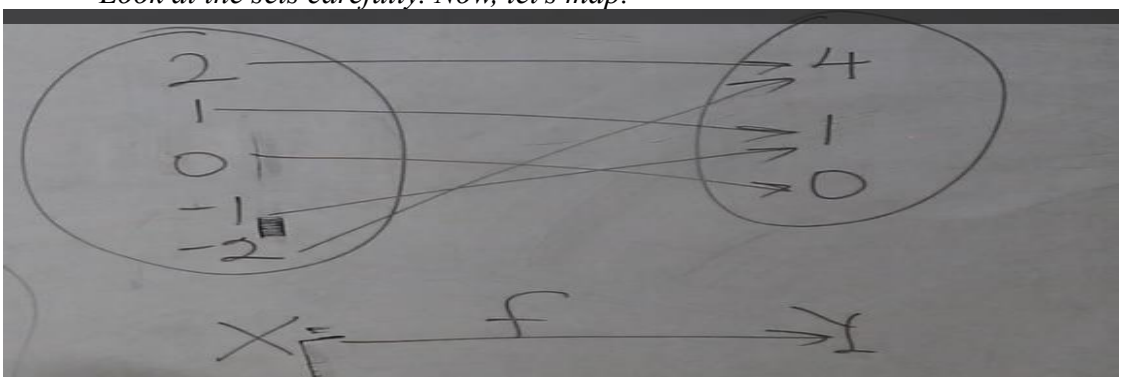


Figure 10.3: Sam's board work on the arrow diagram

Sam: Now, the definition, the arrow diagram, and the ordered pairs are all means of identifying a function. The next thing we want to look at in our activity is symbolic notation.

Sam: Now, if I say symbolic notation, it means I am not using a rule diagram. I am not using a set of order either.

Sam: I use the notation f(x) so that x maps onto y

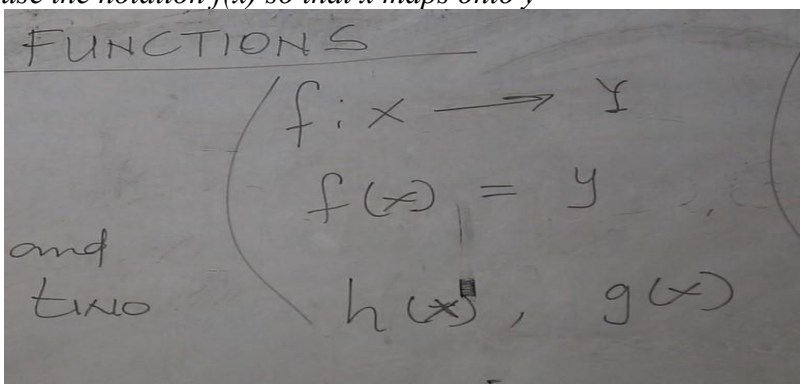


Figure 10.4: Use of multiple notations

Sam introduced the concept of functions using verbal explanations, arrow diagrams, ordered pairs, and symbolic notations. Dreher et al. (2016) emphasise the importance of connecting various representations to develop a comprehensive understanding of mathematical concepts as Sam did when he said "So, basically, when we say function, err… we are referring to the dependence of one variable on the other. Let us use an arrow diagram to support this." Sam then progressed through different representations:

Sam: Now, the definition, the arrow diagram, and the ordered pairs are all means of identifying a function. The next thing we want to look at in our activity is symbolic notation.

Sam: Now, using symbolic notation, if I say symbolic notation, it means I am not using a rule diagram. I am not using a set of orders, either."

Sam: I use the notation f(x) so that x maps onto y.

This progression suggests Sam's intention to scaffold PSTs' learning across different representational forms. The use of multiple representations in mathematics education has been shown to enhance conceptual understanding (Mainali, 2021). However, the effectiveness depends on how well students can make connections between these representations. It's important to note that the majority of the classroom discussions came from Sam, with PSTs primarily responding in short phrases or chorused answers. For instance:

Sam: Now, let us check whether this mapping obeys the definition. 2 is mapped onto 4, only 4, right? 1 is also mapped onto 1, and 0 is also mapped onto 0, right? So, is any element in the domain mapped onto more than one element in the range?

PSTs: No.

Sam: So, each of them has one and only one image. Do you see that? So, this mapping defines a function. Is that clear?

While Sam's approach reflects some aspects of effective pedagogical practices in mathematics teacher education, there are notable limitations. This is not meant to sound judgmental, but the lesson appears to be largely teacher centred. For Selling (2016), the opportunities for PSTs to engage with and discuss multiple representations can substantially enhance their mathematical knowledge for teaching. Although he initiated PSTs' engagement, there were limited opportunities for PSTs to actively engage with the material or demonstrate their ability to make connections between different representations.

This observation raises questions about the alignment between Sam's classroom practice and the pedagogic approaches emphasised in PLC discussions and policy documents. As shown in Chapter 8, MTEs, including Sam, stressed the importance of modelling appropriate pedagogy for PSTs in their mathematics classes (Chapter 8, p. 161). However, the observed lesson suggests a discrepancy between these stated intentions and actual practice. The discrepancy between stated intentions and actual practice highlights the challenges of translating theoretical knowledge into classroom practice, a phenomenon well-documented in mathematics teacher education research (Castro Superfine & Li, 2014). The limited opportunities for PST participation also suggest a need to examine the choice of tasks or lesson plans used. The

current structure seems to prioritise Sam's explanations over PST exploration and discussion. This raises questions about how lesson plans are developed within the PLC and whether they adequately support the pedagogical approaches advocated in policy documents and PLC discussions.

In conclusion, while Sam demonstrates intentional use of multiple representations, there's a noticeable gap between his approach and the student-centred, participatory pedagogy emphasised in teacher education policies. This discrepancy highlights the complex nature of translating theoretical knowledge and intentions into classroom practice, a theme that warrants further exploration in the context of mathematics teacher education.

10.5.4 Lesson modelling: Instruction and PST interactions

The Ghanaian National Teachers' Standards (MoE, 2017a, p. 17) emphasise the need for MTEs to develop skills in observing student teachers, providing constructive feedback, modelling teaching, monitoring, and assessing student teachers' progress. Sam's teaching demonstrated efforts to model effective teaching practices for pre-service teachers (PSTs).

Throughout the lesson, Sam modelled explicit instruction by breaking down the complex concept of functions into smaller, manageable parts. He explained some step in detail, using clear language to enable PSTs to understand the rationale behind each step. The following excerpt illustrates how Sam modelled the concept of functions, incorporating both mathematical and colloquial language while also engaging with PST responses:

> PST 1: The set of second elements in the ordered pairs are even numbers.
>
> Sam: [Sam repeats] He says all the second elements are even numbers. Thank you; that is a good observation. What again do you see? Let us hear from the gentleman at the back.
>
> PST 2: The first elements in the ordered pairs have a difference of one.
>
> Sam: I see. There is a difference of one. I see! That is true. The central concept is that if we use this to define a function, no two, listen, and no two ordered pairs must have the same first components. What did I say?
>
> PSTs [chorus]: No two ordered pairs must have the same first components.
>
> Sam: Meaning that the first components in any two of them are different. Have you seen that? If you choose any two of the ordered pairs, no two ordered pairs have

the same first components. Have you seen that? When we identify that, then we can say that this set defines a function. Is that clear?

Sam: A function is a pairing in which no two ordered pairs have the same first component. Have you seen that?

In this excerpt, Sam incorporated PST responses and used a combination of mathematical terminology and colloquial language. However, several critical points emerge: 1). While Sam acknowledged PST observations, their contributions were brief. The interaction pattern remained teacher-centred, with Sam doing most of the talking, 2). Sam presented the condition for a function "*No two ordered pairs must have the same first components"* as a fact without providing mathematical justification. This approach may lead to a procedural understanding rather than a deeper conceptual grasp (Star & Stylianides, 2013), 3). Sam frequently repeated statements and asked, "Have you seen that?" without providing opportunities for PSTs to elaborate on their understanding and 4). While Sam's approach implicitly demonstrated some teaching practices (e.g., breaking down concepts, using clear mathematical and colloquial language), he did not explicitly draw attention to these pedagogic strategies for PSTs to observe and reflect on.

This analysis reveals a discrepancy between the NTS's emphasis on active PST engagement and Sam's largely teacher-centred approach. This discrepancy highlights the persistent challenge of translating policy expectations into classroom practice, a phenomenon well-documented in recent teacher education research (Cochran-Smith et al., 2016). While Sam made efforts to model instruction, the limited opportunities for PST participation and the focus on procedural understanding rather than conceptual depth suggest areas for improvement. Again, I emphasise that available data do not provide evidence of PSTs actively recognising or reflecting on Sam's teaching practices as I chose to concentrate on MTEs rather than PSTs.

In relation to the broader context of the PLC discussions and policy expectations (as discussed in Chapter 8, p. 161), Sam's approach in this lesson suggests a gap between the espoused theories of student-centred, participatory pedagogy and the theories-in-use within the classroom. This discrepancy highlights the challenges MTEs face in translating theoretical knowledge and intentions into practice. To better align with the NTS and the pedagogic approaches emphasised in PLC discussions, Sam could have considered 1). Providing more opportunities for PSTs to engage in extended discussion and problem-solving, 2). Explicitly discussing his teaching strategies with PSTs to help them recognise and reflect on effective

practices and 3). Focusing more on conceptual understanding by encouraging PSTs to justify mathematical statements rather than presenting them as facts to be memorised.

While Sam's efforts to model explicit instruction are evident, there remains significant room for improvement in terms of PST engagement, depth of conceptual understanding, and explicit modelling of effective teaching practices. Rojas et al. (2021) propose modelling as a crucial concept for studying mathematics teacher educators' practices and promoting professional learning among PSTs. The analysis of available data underscores the complex nature of implementing effective mathematics teacher education and the ongoing need for professional development aligned with policy expectations and research-based best practices.

10.6 Prioritising pace over use of artefacts

Despite the emphasis in PLC sessions on incorporating artefacts into lesson modelling, Sam's lesson on functions relied primarily on mathematical inscriptions which he believed to have worked for him. In a post-lesson interview (31/03/2023), Sam explained his decision to forego the use of teaching artefacts:

> "You know, err… Using artefacts, as a matter of fact, somehow delays the lesson's pace. Umm... I mean, it slows me down, though ideally, it is what I am supposed to do to help them [PSTs] learn about teaching."

This statement reveals Sam's awareness of the potential benefits of artefacts in supporting PSTs' learning about effective teaching practices. However, it also highlights a perceived trade-off between incorporating artefacts and maintaining a desired lesson pace. This tension between content coverage and depth of instruction is a well-documented challenge in mathematics education (Charalambous & Hill, 2012). It's important to note that Sam's decision should be contextualised within broader institutional and curricular demands. Recent research highlights how systemic factors can impact instructional decisions in mathematics teacher education (Ko et al., 2023). The pressure to complete the syllabus or maintain a specific lesson structure may have influenced his choice to adopt a more 'streamlined' teaching method. This approach is streamlined in two ways: the absence of artefacts and the predominance of teacher talk, minimising time spent on extended PST discussions.

During PLC discussions (28 March 2023), MTEs had explored potential artefacts for teaching functions, including function machines, graphing calculators, and real-world data sets. However, Sam did not use any of these in the observed lesson. When asked about available artefacts, Sam mentioned (31 March 2023):

> "We have some manipulatives in the department, like function machines, but setting them up takes time. I felt we could cover more ground without them today."

While Sam's approach may have allowed for more content delivery, it potentially limited opportunities for PSTs to observe and experience the use of teaching artefacts as part of lesson modelling. In the context of teaching functions, incorporating artefacts such as function machines or graphing technology could have provided concrete representations of abstract concepts, potentially deepening PSTs' understanding and modelling how to use such tools in their future classrooms.

The challenge of balancing artefact use with lesson pacing was a common concern among MTEs in the PLC. This challenge reflects broader issues of time management and curriculum coverage in mathematics teacher education programmes (Koponen et al., 2016). For instance, during a PLC observation (26 April 2022), another MTE, Dan, shared: "I tried using the function machine in my lesson last week. The PSTs were engaged, but we only covered half of what I'd planned. It's a constant struggle between depth and breadth." This comment resonates with Sam's concerns and suggests that the pressure to cover content can sometimes conflict with the goal of modelling diverse teaching strategies. To provide a more comprehensive view of MTE perspectives on this issue, it should be noted that in an earlier PLC session (4, 26 April 2022), Sam had shared the worrying reality that the exam-oriented nature of current teacher education sometimes prevented them from practising what they talked about (see extract from p 125). The impact of high-stakes assessments on instructional practices in mathematics education remains a significant concern (Kitchen et al., 2016).

Sam's concerns highlight the complex decision-making process MTEs engage in when balancing pedagogical ideals with practical constraints. While Sam's lesson on functions did not incorporate physical artefacts, his use of multiple representations (verbal, symbolic, and diagrammatic) could be seen as a compromise between the ideals discussed in PLC sessions and the perceived need for efficient content coverage.

In conclusion, Sam's decision to prioritise pace over artefact use reflects broader tensions within mathematics teacher education. It underscores the need for ongoing discussions within PLCs about strategies to effectively integrate artefacts without sacrificing content coverage, and potentially, for a reassessment of curriculum pacing and assessment practices to allow for more diverse pedagogic approaches and greater take-up from the PLCs into classroom contexts.

10.7 PST engagement: Attempts and limitations

Throughout the observed lesson, Sam made efforts to engage PSTs in the learning process, though the extent and depth of this engagement were limited. While Sam's approach included some elements of interaction, it would be inaccurate to characterise it as a "PST-centred teaching approach" based on what was observed. One instance of Sam's attempt to involve PSTs through questioning occurred in this excerpt:

PST: The set of second elements in the ordered pairs are even numbers.

Sam: [Sam repeats] He says all the second elements are even numbers. Thank you; we can't rule it out. What again do you see? Let us hear from the gentleman at the back.

PST: The first elements in the ordered pairs have one difference.

Sam: I see. There is a difference of one. I see! That is true.

While Sam acknowledged PST observations, these interactions were brief and did not lead to extended discussions or deeper analytical thinking. The majority of the discussion remained MTE-centred, with Sam doing most of the talking. Another example of Sam's efforts to promote PST engagement can be seen in the following excerpt:

Sam: The central concept is that if we are using this to define a function, then no two, listen, no two ordered pairs must have the same first components. What did I say?

PSTs: No two ordered pairs must have the same first components.

Sam: Meaning that the first components in any two of them are different. Have you seen that? If you choose any two of the ordered pairs, no two ordered pairs have the same first components. Have you seen that? When we identify that, then we can say for sure that this set defines a function. Is that clear?

In this exchange, PSTs' responses were limited to repeating Sam's statements, with no evidence of critical thinking or analytical reasoning. A comparison of Sam's practice with the teaching approaches of other participants Nick and Dan reveals significant differences in the level of PST engagement and opportunities for active learning. While Sam made some attempts to involve PSTs, the overall structure of the lesson remained predominantly teacher centred. In conclusion, the evidence suggests that Sam's classroom enactment shows limited take-up from the PLC discussions on student-centred learning and active engagement. While he made efforts

to involve PSTs through questioning, the depth of engagement and opportunities for building and supporting mathematical community and discourse were restricted. The lesson structure primarily favoured teacher exposition over student exploration and discussion. In the next section, I present and discuss how Nick enacted his lesson on functions.

10.8 Nick's teaching of functions

In the observed lesson, Nick demonstrated effective teaching of functions, blending clarity, engagement, and practicality. He began the lesson by assuming that PSTs knew the concept of functions from school mathematics. Unlike Sam, Nick did not write the lesson objective on the board. However, he explicitly told the PSTs the lesson objectives. He began by asking the PSTs to define a function and give an example. To help them understand the difference between a function and a relation, he used the idea of a relation as a guide. He defined a function as a rule that assigns each input value to a unique output value and highlighted the importance of this definition in understanding functions.

Nick introduced arrow diagrams and asked the PSTs to identify whether his examples illustrated a function or a relation. He then demonstrated the mapping of one set to another and ensured that every element in the domain mapped to a unique element in the range. The PSTs then confirmed whether the mapping defined a function or not. Nick emphasised that "all functions are relations, but not all relations are functions," and explained the significance of this statement by introducing symbolic notation for functions. Nick used multiple representations, such as ordered pairs and graphs, to explain the concept at various stages of the lesson. He used relatable language to explain operations and functions and encouraged the PSTs to share their interpretations and ideas. It's worth noting that his use of visual representations played a significant role in explaining concepts. For Nick's board on multiple representations (see Figure 10. 4).

Nick's examples were suitable for teaching algebra in both school and teacher education in mathematics. His use of inscriptions helped the PSTs follow along and reinforced vital concepts. This teaching approach facilitated PSTs' note-taking and provided a clear structure to the lesson. Nick transitioned from one concept of functions (addition, subtraction, multiplication, and composition) to another, visually representing them on the whiteboard. Furthermore, he encouraged PSTs to verbalise their understanding of these operations,

highlighting that mathematical concepts are not abstract but can be made tangible through discussion and visualisation.

Nick integrated practical problem-solving into the lesson by presenting real-world scenarios where PSTs had to apply these operations to functions. For example, he presented a scenario involving the calculation of total cost based on a fixed fee and per-unit price, requiring PSTs to create and compose functions. Another example involved modelling the growth of a bacterial population, where PSTs had to apply exponential functions. These approaches demonstrated the practical relevance of functions in everyday life, which would be valuable for the PSTs when they become teachers. Nick concluded the lesson by restating the lesson objectives and asking if they had been achieved, to which the PSTs responded affirmatively. While the PSTs affirmed their understanding, it's worth noting that Nick did not employ specific techniques to individually assess comprehension beyond verbal confirmation. This raises questions about the depth of understanding achieved and whether there was a genuine opportunity for PSTs to express any lingering confusion

10.9 Pedagogy of Mathematics Teacher Education: The use of multiple mathematical representations

A crucial aspect of Nick's pedagogic approach was his incorporation of various mathematical representations to support PSTs' understanding of functions, and more importantly, his efforts to help PSTs make connections between these representations. This strategy aligns with what is currently considered to be effective pedagogy in mathematics teacher education, as it not only facilitates comprehension of abstract concepts but also emphasises the relationships between different forms of representation. When providing an example, {(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)}, Nick used arrow diagrams to illustrate the mapping of elements from the domain to the range, highlighting the unique correspondence required for a function:

> Nick: In this example, we can see that each element in the domain is mapped to one and only one element in the range. This defines a function; however, if I have a case where one element in the domain maps to multiple elements in the range, then it is a relation but not a function.

Nick then transitioned to other representations, attempting to help PSTs make connections:

> Nick: Let us represent this function using ordered pairs...
> [Nick writes down a set of ordered pairs on the board]
> Nick: Now, let us graph this function on the coordinate plane...

[Nick plots the points and draws the graph on the board]

Nick: As you can see, we can represent functions in various ways, such as arrow diagrams, ordered pairs, and graphs. Let's discuss how these representations relate to each other.

Occasions on which PSTs successfully made connections between representations were observed during the lesson. For example:

PST1: I can see how the arrows in the diagram correspond to the points in the ordered pairs. Each arrow is like a pair!

PST2: And on the graph, each point represents one of those ordered pairs. The x-coordinate is from the domain, and the y-coordinate is from the range.

Nick: Excellent observations! Can anyone explain how the uniqueness property of functions is represented in each form?

PST3: In the arrow diagram, each element in the domain has only one arrow coming out of it. In the ordered pairs, no x-value is repeated. And on the graph, if we draw a vertical line, it would only intersect the function once.

This exchange demonstrates that at least some of the PSTs were able to articulate the connections between different representations and apply the concept of functions across these representations.

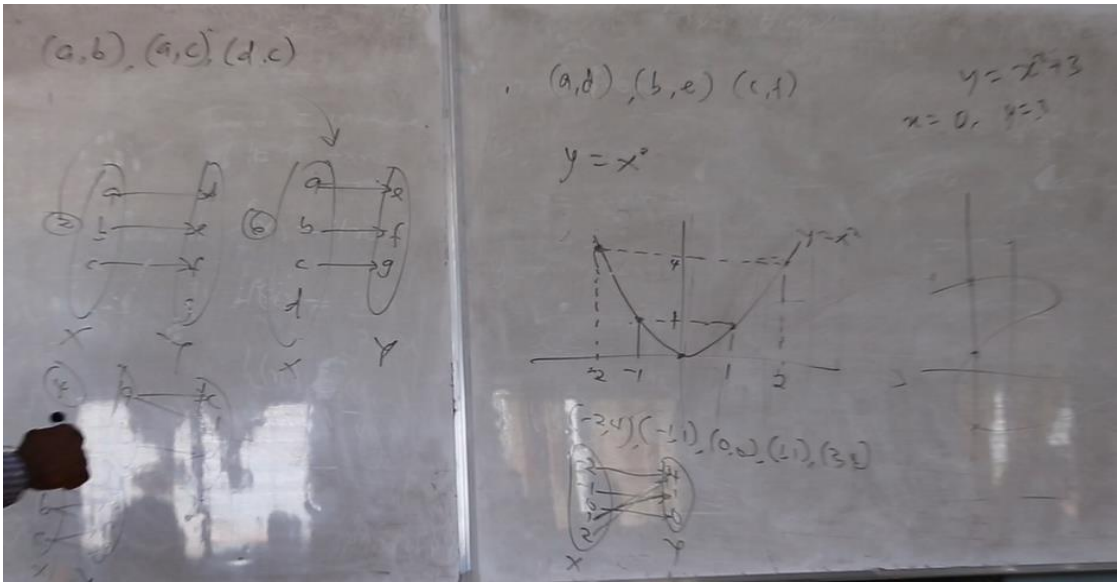


Figure10.5: Nick's board work depicting multiple representations

The board work in Figure 10.5 provides insights into Nick's approach. By using different representations side by side, Nick enabled PSTs to compare and contrast, facilitating their ability to make connections. This approach enriched PSTs' understanding by providing diverse perspectives on the same concept, as evidenced by their comments and observations.

Nick's usage of symbolic notation to portray functions and their operations at various stages of the lesson underscores his understanding of mathematical conventions. For example, he used symbols such as $(f + g)(x)$ for the sum of two functions or $(f \circ g)(x)$ for composition, he introduced PSTs to mathematical conventions and language. This introduction to mathematical language was integrated with the multiple representations:

Nick: How does our symbolic notation f(x) relate to our graph?

PST4: The f(x) represents the y-value for any given x on the graph. It's like the output of our function for each input.

This exchange shows how Nick's use of symbolic notation was tied to the pedagogy of mathematics teacher education, helping PSTs connect formal notation with graphical representation. Nick's board work, as seen in Figure 10.6, reveals his use of functional notations on the markerboard as a way of bringing mathematics to life. It exemplifies the use of inscriptions and examples in mathematics teaching.

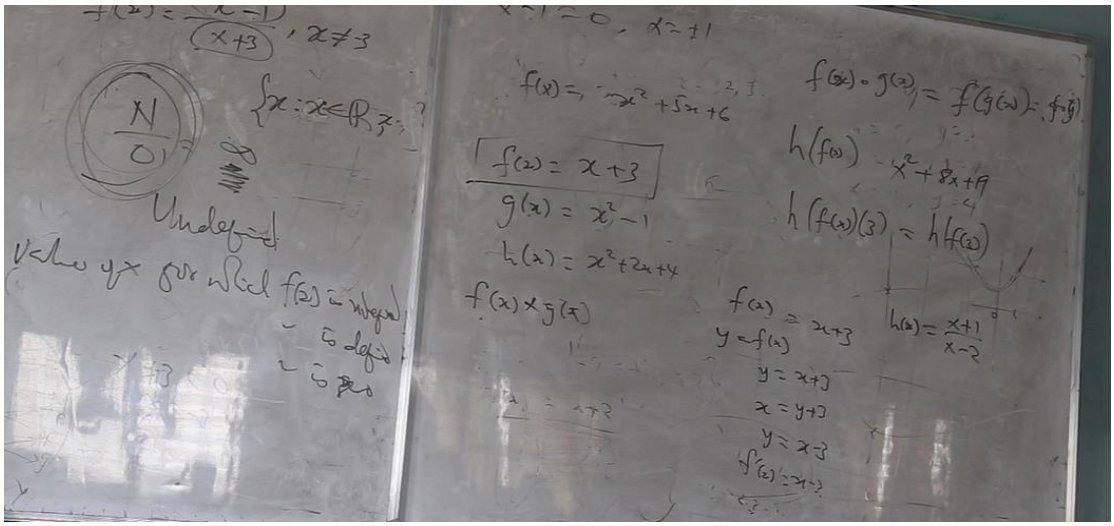


Figure 10.6: Board work 2

The evidence presented demonstrates Nick's effective use of multiple representations and his success in guiding PSTs to make connections between these representations. This approach not only taught the mathematical content but also modelled effective strategies for future

mathematics teachers. By engaging PSTs in articulating these connections, Nick fostered a deeper understanding of functions and provided a model for how they might approach this topic in their future classrooms. This finding highlights the importance of not only using multiple representations but also of explicitly helping PSTs make connections between these representations thus demonstrating Nick's conscious effort to teach about teaching. Nick's approach shows promise in this area, effectively supporting PSTs in developing a connected understanding of mathematical concepts.

10.9.1 Lesson modelling: Encouraging PST participation and understanding

In addition to using multiple representations, Nick's teaching approach prioritised active engagement and participation of the pre-service teachers (PSTs). He consistently encouraged PSTs to share their interpretations, ideas, and understanding of the concepts being taught, fostering an environment conducive to peer-to-peer learning and deeper comprehension. Nick elicited PST participation by prompting students to define and provide examples of key concepts. For instance, he began the lesson by asking:

Nick: All right let's start by defining a function. Can anyone give me the definition of a function and provide an example?

PST 1: A function is a relation that assigns each input value to a unique output value.

Nick: Excellent, thank you. [Nick writes the definition on the board] Can someone give me an example of a function?

PST 2: For example, the function $f(x) = 2x + 3$, where for any input x, we get a unique output value.

By inviting PSTs to articulate their understanding from the outset and then writing key points on the board, Nick could assess their prior knowledge and create opportunities for them to engage with the content, setting the stage for a participatory learning environment. Nick consistently encouraged PSTs to verbalise their thought processes and understanding throughout the lesson. When introducing operations on functions, such as addition, subtraction, multiplication, and composition, he prompted PSTs to explain their interpretations:

Nick: Can someone explain what happens when we add two functions together? How would you represent the addition of two functions symbolically?

PST 3: If we have functions *f(x)* and *g(x),* then the addition of these two functions would be represented as *(f + g)(x) = f(x) + g(x).*

Nick: Excellent explanation! [Nick writes the symbolic representation on the board] Now, Let us go through an example to solidify our understanding...

This exchange suggests that Nick was treating the lesson as a recap of concepts the PSTs had previously encountered. However, it's important to note that while the PSTs demonstrated familiarity with the content, here, Nick's approach was not explicitly focused on teaching about teaching. Instead, he was primarily reinforcing mathematical content knowledge.

By inviting PST explanations and building upon their contributions, Nick fostered an environment where PSTs could learn from one another and actively co-construct their understanding of the concepts being taught. Nick's use of relatable language and his emphasis on making mathematical concepts more concrete through discussion and visualisation encouraged PSTs to engage with the content in a more accessible way. For example, when discussing the concept of function composition, Nick used a real-world scenario:

Nick: Let's consider a real-life application of function composition. Imagine you're uhm… buying a car. The first function, $f(x)$, calculates the price of the car including taxes, where x is the base price. The second function, $g(x)$, calculates er… the monthly payment based on the total price. How would we represent this situation using function composition?

[after several unsuccessful attempts],

PST 7: We could use $g(f(x))$ to represent this. First, $f(x)$ calculates the total price, and then g takes that result to calculate the monthly payment.

Nick: Well done! This is a great example of how function composition applies in real-world scenarios.

This example demonstrates how Nick connected abstract concepts with real-world applications, making the mathematics more tangible and relevant to the PSTs. This participatory approach aligns with aspects of the pedagogy of mathematics teacher education, which was a critical focus of discussions in the PLC. However, just like Sam, Nick did not use any specific artefacts to explicitly teach about teaching, even though their PLC discussions centred around the use of learning materials in teaching about teaching. This indicates that in classroom practice, MTEs may not be fully implementing the approaches they discuss in the PLC.

Nonetheless, Nick's teaching of functions demonstrates several features of effective mathematics teaching practices discussed in the PLC. These features are evident in his lesson modelling, use of multiple mathematical representations, and efforts to connect abstract concepts with real-world applications. While these practices are valuable for mathematics

instruction in general, his future lessons could benefit from more explicit focus on teaching about teaching, in line with the goals of mathematics teacher education.

10.10 Alignment with NTECF standards: fostering diverse perspectives in mathematical problem solving

The National Teacher Education Curriculum Framework (NTECF) emphasises various aspects of inclusive education, including the importance of addressing diverse learning needs. While Nick's observed lesson doesn't directly address issues of learner diversity or broader inclusive education as outlined in the NTECF, it does demonstrate an approach that values diverse perspectives in mathematical problem-solving. Nick's teaching approach fostered an environment that encouraged PSTs to share different viewpoints and methods. This was evident in a brief exchange during the Functions lesson observed [29 April 2022]:

Nick: You see, umm... I might not know much about your academic background, so this is the place for us to explore and learn from each other.

PST 4: (Raising a hand) Could you please explain what you mean by diverse perspectives?

Nick: That is a great question! You see, err... when I say diverse perspectives, I mean there can be many ways to approach and understand mathematical concepts. Umm… I am unsure who taught you about functions and how much you already know. That is why I encourage you to share your thoughts, ask questions, and try different approaches, even if they are not the same as what you are used to.

Nick: For example, when we discuss operations on functions, feel free to share your approach as you have been taught in school mathematics. We can all benefit from each other's insights and expand our knowledge to solve problems.

PST 5: (Eagerly) So, there is no one correct way to do it?

Nick: (Smiling) Precisely! Err… While there are standard methods, what makes mathematics so beautiful is error, that it often allows for many paths to arrive at a solution. You see, by encouraging discussions and embracing different approaches, we are getting ready to be adaptable and uhm… resourceful so that when you meet your students with different learning styles, it does not become a problem.

This exchange demonstrates Nick's intention to create a classroom environment that values diverse perspectives. Following this exchange, Nick's actions did not fully align with his stated intention to explore diverse approaches. Instead, he referred PSTs to the mathematical representations he had used earlier in the lesson:

> Nick: Remember the different ways we represented functions earlier - we used algebraic expressions, arrow diagrams, ordered pairs, and graphs. These are all valid ways to understand and work with functions.

Nick explained the connections between these representations:

> Nick: Did you notice how each representation gives us a different way to understand the same function. The algebraic form gives us a rule, the arrow diagram shows the mapping, the ordered pairs give us specific examples, and the graph gives us a visual of all possible input-output pairs.

This use of multiple representations demonstrates Nick's practical implementation of diverse approaches to understanding functions. It aligns with his earlier statement about the value of different perspectives in mathematics.

During the lesson, Nick encouraged PSTs to engage with these different representations:

> Nick: "Now, let's take the function $g(x) = x^2$. Can someone suggest how we might represent this using one of the methods we've just seen?"
>
> PST 3 suggested using ordered pairs, while PST 5 proposed sketching the graph. Nick incorporated both of these suggestions, demonstrating how different representations can provide unique insights into the function's behaviour.

This excerpt shows that while Nick may not have explicitly used the term "multiple approaches," his use of various representations effectively provided diverse perspectives on the concept of functions. This practice aligns with his stated intention of encouraging different ways to approach and understand mathematical concepts. Nick's emphasis on creating a learning environment that welcomes diverse viewpoints aligns with some aspects emphasised in the NTECF (MoE, 2017b, p. 37), particularly the value placed on diverse experiences and approaches. He explicitly acknowledges that PSTs may come from various academic backgrounds and encourages them to share their interpretations and explore different methods.

Nick's choice of words foregrounds his teaching philosophy, highlighting the importance of adaptability discussed in the PLC. He emphasised that there is often more than one right way to approach mathematical problems, which indicates to PSTs that they will need to be flexible and well-equipped to meet the diverse learning needs of their students. His words also indicate that while teaching functions, he was also trying to teach about teaching. By encouraging discussions and exchanging insights, Nick promoted the collaborative learning environment that the MTEs embraced as part of their participation in PLC activities. This collaborative approach not only enriches the learning experience of PSTs but also reflects the importance of peer-to-peer learning and knowledge sharing in pre-service mathematics teacher education.

Nick's acknowledgement of his limited knowledge about the PSTs' academic backgrounds is a significant one. It demonstrates respect for the unique experiences and prior knowledge each PST brings to the teacher education mathematics classroom. In the extract, the language used by Nick indicates an intention to create an inclusive learning environment where diverse perspectives of PSTs are valued. This empathetic approach can enhance the lecturer-PST relationship and create a more inviting and supportive classroom atmosphere. The content of the dialogue indicates Nick's commitment to creating an inclusive, adaptable, and equitable learning environment where diverse perspectives are valued. I see substantial evidence of alignment with the NTECF here. (see MoE, 2017b, p. 38 - 40). These principles are essential in preparing future mathematics teachers to be empathetic and versatile in their teaching roles.

10.11 Chapter Summary

This chapter presented and discussed evidence of how the knowledge gained by MTEs from participating in PLCs is implemented in their actual classroom teaching practices. The focus was on investigating the integration of theory and practice, with the classroom serving as a testing ground for refining and applying what had been introduced in the PLC. The chapter presented detailed observations and analysis of lessons taught by two of the three MTE participants in the study (Nick and Sam). It highlights how these MTEs attempted to align their instruction with the content and learning objectives outlined in the NTECF. Their pedagogy involved using multiple representations, lesson modelling, eliciting PST participation and engagement, scaffolding learning by building on prior knowledge, and strategic use of example spaces progressing from simple to complex.

Nick, in particular, created an inclusive learning environment by welcoming diverse perspectives, acknowledging PSTs' varied backgrounds, and encouraging different problem-

solving approaches. However, despite the PLC's emphasis on using artefacts, Sam prioritised maintaining lesson pace over incorporating artefacts due to time constraints.

The chapter highlighted differences between the ways in which two MTEs translated the collective knowledge developed in the PLC into their classroom practices, reflecting both adherence to discussed principles and adaptations based on practical constraints or perhaps incomplete internalisation of new knowledge. Analysis of the lesson observations suggests that while the MTEs demonstrated several elements of effective pedagogy aligned with what had been discussed and modelled in the PLC, there were also instances of deviation or incomplete implementation.

CHAPTER 11

PHASE TWO

ANALYSIS OF CRITICAL FRIENDSHIPS: FINDINGS AND DISCUSSION

"Wishing to be friends is quick work, but friendship is a slow-ripening fruit."

Aristotle:384 BC – 322 BC

11.0 Introduction

This chapter presents and discusses the findings from the second phase of the study, which was conducted in order to explore critical friendships among mathematics teacher educators (MTEs) in Ghana and their possible benefits to participants in a PLC for MTEs. The study was guided by research question four: To what extent does critical friendship inform the development of MTEs' professional knowledge and practice? To answer this question, I undertook an in-depth inquiry into the interactions unfolding within critical friendship groups (CFGs). Data was gathered through observations of CFG meetings, which included debriefing and reflections, and post-lesson observation interviews with three pairs of critical friends from phase two. To understand how collaboration works in this situation, I analysed the data through the conceptual lens of Wenger's (1998) framework of dimensions of practice. This framework has three key elements: mutual engagement, joint enterprise, and shared repertoire. The data analysed revealed the nature of academic friendships and their influence on MTEs' reflections, learning processes, and teaching approaches.

The findings indicate that peer lesson observations, constructive feedback, collaborative development of teaching strategies, and resource sharing can enhance pre-service teacher education practices. While challenges such as superficial participation, relationship complacency, and time competition surfaced for the critical friends, the key finding of this part of the study is that when rooted in a commitment to mutual growth, critical friendships can create non-judgmental spaces for risk-taking, open reflection, and collaborative teaching techniques and thus be valuable for collaborative knowledge construction for all involved in teacher education.

11.1 Wenger's dimension of practice framework

The analytical framework, illustrated in Figure 6.3 (see Chapter 6), outlines the specific criteria associated with each dimension of practice, thereby demarcating analytical boundaries. This framework is derived from Wenger's dimensions of community of practice. As discussed in Chapter 4, Wenger's conceptual framework serves as a foundational underpinning for understanding the dynamics of collaborative learning. It emphasises that, to varying extents, collaborative groups manifest characteristics of mutual engagement, joint enterprise, and shared repertoire.

The analytical framework presented in Figure 6.3 describes and offers a nuanced and interpretive perspective on the dynamics of critical friendships within mathematics teacher education. Mutual engagement by participants emphasises shared ownership of and responsibility for collaborative learning. This engagement entails a multifaceted understanding of collaboration, participation, and active learning.

Examining the joint enterprise dimension extends my comprehension of critical friendships through a focus on the collective purpose at the core of these relationships. Here, the convergence of curriculum goals, the focus on differentiation and inclusion, and the joint exploration of instructional strategies emerge as integral components. This dimension invites the use of an interpretive lens, in order to understand how shared goals and purposes drive collaborative efforts among critical friends in a relationship that aims to foster a more holistic and impactful teacher education experience. Finally, a focus on the shared repertoire dimension enables insights into the extensive repository of shared resources, routines, and methodologies cultivated within the critical friendship framework.

11.2 Mutual engagement of participants (MEP)

Analysis of data from observations, post-lesson observation interviews, and debriefing sessions with the CFGs revealed rich examples of mutual engagement among the critical friends that can support change in MTEs' professional knowledge and practice, characterised by active participation, open 238, and a shared effort to further their professional growth. The key facilitators of engagement and interaction identified in the study are demonstration teaching, lesson observation, lesson planning, and debriefing/ reflective discussions. In a post-lesson interview with Sam and Nick, they discussed what they do as critical friends and how such activities impact their professional practices and teaching knowledge:

Sam: In our CFG, we err…, engage in collaborative lesson planning, which, uhm…, I find beneficial, though I was initially reluctant. Our role revolves around observing each other's lessons uhm…, providing constructive feedback, and discussing how we can improve. It's a continuous cycle of learning and, err…, refining our teaching methods, as you have witnessed.

Nick: Absolutely, uhm…, you know, it goes beyond the classroom. Uh…, you know, discussions allow us to bring different perspectives to the table and challenge uhm…, each other's thinking. Additionally, we engage in team teaching if necessary, which is, err…, a unique opportunity to apply what we've discussed in an actual classroom setting.

Sam: One significant impact has been the emphasis on reflective practice. …, the regular peer observations and reflection sessions have made me more err…, conscious of my teaching methods, although I was initially reluctant, as I said earlier. I have realised that it's not about criticism but err…, about growth. For this reason uhm…, I've become more open to trying new approaches and adjusting, err…, my teaching style based on feedback.

Nick: I completely agree with Sam. Uh…, our friendship has given me a space to experiment with teaching strategies in a supportive environment. Uhm…, the peer observation experiences, err…, have allowed me to witness different teaching styles and adapt elements that resonate with my approach.

Sam's experience with reflective practice resonates with current research (e.g., Bognar & Krumes, 2017; Smith, 2019; Fuentealba & Russell, 2022) in teacher education. Körkkö et al. (2016) argue that reflective practice is a crucial component of professional development for teachers. Their study demonstrates how structured reflection can lead to increased self-awareness and openness to new teaching approaches, mirroring Sam's reported growth through the CFG process.

Through my observation of CFG meetings and discussions, I witnessed a remarkable effort to engage in peer observation and reflection. The MTEs owned it and were not just doing it because they participated in the study. Both MTEs and their critical friends reflected on lesson observations. One instance that stood out was when Dan was observed by his critical friend, Nick, during a lesson on integers. Nick shared his feedback with the larger CFG by praising Dan's use of formative assessment techniques to assess PSTs' understanding. However, he also

provided constructive feedback on incorporating more PST-led discussions to deepen conceptual understanding. During the CFGs debriefing meeting observation (28 March 2023), Nick said the following:

> I enjoyed how formative assessment techniques guided your lesson. Err…, I agree there's room for improvement, particularly by shifting towards a more student-centred approach. Let's explore ways to err…, involve students (PSTs) in leading discussions for better conceptual understanding in future lessons.

This feedback then sparked a meaningful discussion on actionable strategies, with Dan later sharing:

> For now, uhm…, I think peer observation has become an integral part of my professional development, though I took it for granted at first. Nick's insights positively challenged my teaching methods. I implemented his suggestions in subsequent lessons, and the impact on student [PSTs] engagement and comprehension was notable.

Researcher: Can we get down to the specifics?

> After observing my lesson, Nick encouraged me to highlight patterns and properties of integer operations explicitly. For instance, focusing on the consistent rules for adding/subtracting integers or demonstrating the commutative and associative properties using examples could deepen PSTs' conceptual understanding. I took Nick's advice in my next lesson on integer multiplication with another group. Instead of just proceeding through examples, I intentionally highlighted the patterns - like how multiplying integers with the same sign gives a positive product, but integers with different signs gives a negative product. I used visuals and had PSTs identify these patterns themselves through guided examples.
>
> For the associative property, I had PSTs multiply sets of 3 integers, first grouping the first two then the last two. They saw that the product stayed the same regardless of how they grouped. This allowed them to discover and articulate the property themselves. Compared to my previous lessons, the PSTs seemed more engaged in constructing their own understanding of these concepts through the patterns and properties.

> During the exercises, I also noticed improved comprehension - PSTs were better able to explain their reasoning instead of just doing the calculations procedurally. So, Nick's suggestions about explicitly highlighting key patterns and properties, rather than just presenting procedures, did seem to foster deeper conceptual understanding and engagement among the PSTs when I applied it in subsequent lessons.

This extract highlights the practical benefits of collaborative and peer observation within CFGs that can support professional change. It emphasises that shared insights can be practically applied to improve teaching practices and promote ongoing growth among MTEs. Stolle et al. (2019) caution that while peer observation can be a powerful tool for professional development, it is a complex process that requires careful navigation of interpersonal dynamics and institutional contexts. The initial hesitation and subsequent positive outcomes reported by Dan reflect the nuanced nature of peer observation processes highlighted in Stolle et al's study. However, while Dan felt Nick's suggestions improved engagement and comprehension, my own observations of that integer multiplication lesson did not show as notable an impact. Many PSTs still struggled to articulate the reasoning behind the patterns, and there was significant procedural confusion during the practice problems. The adjustments may have been a step in the right direction, but more work is still needed to build a conceptual understanding.

I do not intend to give the impression that everything about mutual engagement in critical friendship goes smoothly. For example, while Dan and Nick's friendship thrived, the friendships of other MTEs revealed potential tensions arising from attempting to balance close interpersonal familiarity and maintaining objectivity and also to allocating time to the activities involved. During one of the CFG meetings, Sam expressed concern about providing timely feedback to his critical friend Nick, stating:

> May I request some time to meet with Nick? You know, I have been busy with my teaching schedule for the past two weeks, so I haven't been able to provide him with feedback. Uhm…, I want to reflect on my observations with him. (Sam)

This excerpt suggests that the demands of their regular teaching responsibilities occasionally conflicted with the time and effort required to support each other within their critical friendship. Analysis of the data revealed instances where close interpersonal familiarity among critical friends could potentially override objectivity and lead to complacency. For example, during an

interview with Nick and Dan (23 March 2023), Nick initially expressed hesitation in providing critical feedback to Dan regarding his lesson on Functions:

> Nick: I noticed some areas for improvement in your lesson, but I didn't want to come across as too critical...
>
> Dan: No, you could have shared your thoughts honestly. That's the whole point of our critical friendship.
>
> Nick: It was too early to be critical because we had just begun our engagement.

This exchange demonstrates Nick's initial reluctance, potentially stemming from a desire to preserve their close relationship. However, by insisting on receiving honest feedback Dan recognises that maintaining objectivity and rigorous analysis is crucial for meaningful collaboration and professional growth within their critical friendship dynamic. Richards and Shiver (2020) address this common challenge in critical friendships, emphasising the importance of establishing clear norms and expectations for feedback. They suggest that successful critical friendships require a delicate balance of trust-building and constructive critique, which aligns with the ongoing negotiation of these dynamics observed in this study.

Observational notes from PLC discussions further corroborated the participants' understanding of the need to balance supportive relationships and constructive critique. During an open PLC discussion, Peter, a non-selected participant member of the PLC, remarked:

> Our collaborations should be built on trust and respect, but that doesn't mean we should avoid tough conversations or justify mediocre practices. We need to hold each other accountable to improve our teaching continuously." (Peter)

This view was evident throughout the PLC discussions. However, analysis of the data revealed that on occasion, members disregarded problematic practices or rationalised deficiencies in each other's teaching when critical friendships became overly comfortable leading to mutual engagement without accountability. For instance, during a post-observation debriefing session, Sam appeared to downplay concerns raised by Dan about his lesson pacing:

> "I think the pacing was fine. Our students are used to that instructional style, and we shouldn't try to fix what isn't broken." (Sam)

This rationalisation could severely impede the genuine growth and development intended within these collaborative partnerships. While critical friendships fostered supportive

environments for professional growth, the findings underscore the importance of maintaining rigorous critical analysis and holding each other accountable. Letting superficial participation or rationalisations of mediocre instruction slide could undermine the honesty, shared responsibility, and commitment to improvement that should be foundational to these partnerships.

While collaborative practices such as peer observation, feedback exchange, and joint planning can potentially enhance professional growth when fully embraced, the MTEs continued to acknowledge that dedicating the requisite time and effort for critical friendships often competed with other responsibilities and personal obligations. For instance, Sam's request above for more time highlights how even when the underlying intention to engage in rigorous partnership exists, the practical time demands of regular teaching duties can make it challenging to fully uphold the mutual engagement and accountability expected of critical friends.

Similarly, in a CFG meeting discussing their peer observation cycle, Dan noted the general time pressure:

> "With everything on our plates - teaching courses, grading, department meetings – it's been tough to carve out enough time to do thorough peer observations every week as we'd hoped. Sometimes it feels like the critical friendships get shortchanged."

These examples underscore that while the MTEs recognised and valued the potential benefits of critical friendships, fully actualising those benefits was an ongoing challenge given the competing demands on their time and effort as teacher educators. This tension between aspirational goals and practical realities is well-documented in recent literature. Margolis et al. (2017) highlight the challenges of implementing collaborative professional development models within existing workloads in education settings. Their study suggests that successful implementation often requires institutional support and strategic allocation of resources, factors that may need to be considered in future development of CFG initiatives.

11. 3 Negotiated joint enterprise

The data analysis revealed several recurrent themes on which critical friends focused in an attempt to support themselves to improve practice. These included curricular alignment, differentiation and inclusion, exploring instructional strategies to enhance PSTs' engagement and learning, and enacting lessons. For instance, in a post-lesson observation interview, Dan noted "We planned the lesson together, so we knew the broader goal and what we intended to achieve. Standing at the back of his class, I asked myself, "Is the lesson going according to

plan?" Are the instructional strategies effective? Has he ensured alignment? These are some of the things that I was looking for."

Despite the MTEs' diverse backgrounds and experiences, the data analysed revealed a collaborative effort to improve their professional knowledge and practice even though the level of engagement differed within and across the CFGs. During a CFGs meeting observation (4 March 2023), Sam highlighted the essence of a common purpose when he stated:

> We know that our collaboration is focused on improving practice through various engagements like lesson planning. This collaborative effort… involves each member contributing their insights and ideas. This exercise proves instrumental in err…, realigning our teaching strategies and fostering pedagogical cohesion, ultimately aiding us in achieving our educational goals. Uhm…, thank you for your support in reinforcing our critical friendship groups, as we now recognise the significance we had previously overlooked.

Sam's remarks shed light on the collaborative negotiations underpinning the critical friends' shared understanding of their joint enterprise. His statement, "We know that our collaboration is focused on improving practice through various engagements like lesson planning," highlights the collective recognition of their main purpose – enhancing their professional practices as MTEs. Sam's words, "This collaborative effort… involves each member contributing their insights and ideas," underscore the negotiated nature of their joint enterprise. By actively participating in collaborative practices such as lesson planning, the critical friends can contribute their perspectives and expertise, fostering a shared vision of and coherence in their teaching strategies.

Finally, Sam's acknowledgement, "as we now recognise the significance we had previously overlooked," corroborates my earlier claim that during Phase One of the study there was no evidence of critical friendships despite the policy requirement. The newfound recognition evident in Phase Two of the study highlights the transformative nature of their critical friendships which enable MTEs to engage collaboratively in professional development.

11.3.1 Curricular alignment

Analysis of the data from CFG meetings, PLC observations, and post-lesson observation interviews highlighted the critical friends' efforts to align their lessons with the prescribed curriculum standards and learning objectives to be achieved by their PSTs. The MTEs expressed a specific interest in aligning teacher education mathematics with the school

mathematics curricula to support PSTs' transition from teacher education to school classrooms Critical friends were asked about what they chose to focus on when observing their colleagues' lessons. Dan expressed that:

> You see, one primary area of focus is the alignment of teacher education mathematics with the school mathematics curriculum. Uhm…, we believe that this alignment is crucial for curriculum coherence. So… during lesson planning and observation, I closely examine how the content and pedagogical approaches resonate with the broader objectives of school mathematics.

In another interview with Nick and Dan, Nick shared the same sentiments when he expressed that:

> What we do in our friendship groups is align methods with pedagogic principles and adapt them to the diverse needs of our PSTs. Err…, this collective effort elevates the quality of teaching within our community, fostering an environment where ah…, innovative approaches are not just embraced but uhm…, are continuously optimised for the best possible engagement and learning outcomes. (Nick)

In a separate interview with Sam and Nick, Sam presented a contrasting view on curriculum alignment, emphasising that distinct objectives must apply to mathematics curricula at different levels of teaching. He expressed his perspective by stating:

> As much as I agree with what we do as collaborators to ensure coherence, I believe that teaching mathematics does not follow a one-size-fits-all approach. You see, … consequently, different school levels should have distinct objectives for their mathematics curriculum, and ah... conformity may not be a requirement.

The extracts emphasise the role of critical friendships in reflecting on the complex issue of alignment between teacher education mathematics and school mathematics.

In a reflective discussion during the CFGs meeting, MTEs said the following. regarding the role of their critical friendships in peer lesson observation.

> Dan: Coherence in our teaching practices is crucial for maintaining consistency and providing a unified learning experience for students [PSTs].

Nick: Our collaboration in the PLC contributes to a unified approach, enhancing individual teaching methods, ensuring curriculum alignment, and improving the overall educational experience for students (PSTs).

Sam: Yes, I agree that collaboration can provide quality education to our students [PSTs]. However, my concerns are regarding the alignment of teaching practices. While alignment is important, it can limit creativity and innovation in the classroom. If we stick to the same teaching practices, we might miss out on new and better ways of teaching.

Sam's contrasting perspective aligns with ongoing discussions in mathematics education literature. For instance, Confrey et al. (2017) argue for a more nuanced approach to curriculum alignment that considers the specific needs and contexts of different educational levels. This tension between coherence and individual innovation resonates with Wenger's (1998) suggestion that joint enterprise is not necessarily about complete agreement on every undertaking of the group, but rather, it is the result of social negotiation. More recent work by Farnsworth et al., (2016) further develops this idea, emphasising the dynamic and negotiated nature of joint enterprise in professional communities.

11.3.2 Exploration of instructional strategies

One of the core foci of the CFG discussions centred on exploring and refining instructional approaches to enhance PST learning. Through sharing their expertise and distinct perspectives, the MTEs and critical friends collaboratively identified innovative teaching methods, unpacked their underlying pedagogic principles, assessed effectiveness across different PST needs, and engaged in a continuous cycle of improvement for optimal engagement and outcomes.

In a CFGs lesson planning session, Nick highlighted this pursuit: "We will explore new strategies to ensure our teaching is responsive to students' [PSTs] needs. Uhm…, it's our collective effort to provide the best learning experiences for our students (PSTs)." Sam elaborated further in a post-observation interview:

> In CFGs, we err…, witness a collective process of identifying innovative teaching methods and experimenting with them... It's not a surface-level adoption; …, it's a continuous exploration based on shared expertise and perspectives. We go beyond merely pointing out what works to include the underlying pedagogic principles and, uhm…, assessing how these methods cater to diverse learner needs. It's our effort to

ensure optimal engagement and …, learning outcomes for our students [PSTs], even though some of us were lazy with the whole process. (Sam)

Similarly, during a post-lesson reflection with Dan and Sam (31 March 2023), I noticed how MTEs and their CFs spoke about whether certain instructional strategies worked in their classrooms:

Dan: I commend you for anticipating and addressing common misconceptions during the lesson. Explicitly discussing the difference between function composition and multiplication really seemed to clear up some confusion.

Sam: Thanks, Dan. That's definitely an area where PSTs often struggle, so I made sure to emphasise those distinctions. I also tried incorporating real-world examples, like determining the total cost after applying multiple discounts or calculating the final position after a series of movements.

Dan: Using relatable contexts like that is so important. It helps the students [PSTs] see the practical applications of these mathematical concepts beyond just abstract symbols and operations. We can continue to share and build upon these insights and strategies within our CFG. Observing each other's lessons and having these debriefing discussions can enhance our expertise.

During a CFG meeting observation, Sam expressed mixed feelings. He is satisfied with the current teaching methods but questions the need for overwhelming changes in pre-service mathematics teacher education classrooms. He highlighted that:

I think ah…, the current techniques that are being employed in our classroom are already effective, so err…, why overwhelm ourselves with a lot more in the pre-service mathematics teacher education classrooms?

Sam's perspective suggests a more cautious approach, acknowledging satisfaction with existing techniques and raising questions about the added value of introducing an overwhelming number of new methods. This stance aligns with some research that cautions against inundating educators with a constant influx of new strategies instead of developing mastery of core pedagogical practices (Korthagen, 2017; Silver et al., 2019).

Scholars like Korthagen (2017) argue that rather than overwhelming early-career educators with an overabundance of new methods, teacher education programmes should prioritise

facilitating the development of a stable core repertoire of high-leverage practices first. Introducing too many techniques too soon can result in cognitive overload and surface-level implementation (Liston et al., 2006). From this perspective, Sam's reservations raise valid considerations about finding the proper balance and sequencing when incorporating new instructional innovations.

However, perspectives like Dan's align with the view that continuous refinement and expansion of one's pedagogical repertoire is crucial for developing adaptive expertise to meet the evolving needs of diverse learners (Le Fevre, 2023; McDonald et al., 2013). This view sees pre-service teacher education as the initiation of a lifelong process of learning and upgrading one's practices based on emerging research and experiential insights.

McDonald et al. (2013) argue that effective teacher education programmes should embrace a comprehensive vision of ambitious instruction, equipping novices with a range of student-centred, culturally responsive strategies from the outset. This contrasts with more gradual or minimalist approaches advocated by Korthagen and others. The discussions within the CFGs highlight these contrasting perspectives on optimal instructional practices in pre-service teacher education**.**

11.3.3 Differentiation and inclusion

As part of supporting each other's overall professional growth, critical friends (CFs) demonstrated a keen interest in how MTEs addressed the diverse needs and abilities of pre-service teachers (PSTs) within their classrooms. As noted in Chapter 3, Ghana's recent National Teacher Education Curriculum Framework (NTECF) prioritises promoting inclusive, differentiated teaching practices to meet the varied learning needs of all PSTs. Recent research supports the importance of differentiation and inclusion in teacher education. For instance, Florian and Camedda (2020) argue that inclusive pedagogies should be at the core of teacher education programmes. This focus on inclusive practices in teacher education is crucial not only for addressing the needs of diverse PSTs but also for preparing them to address diverse learning needs in their future classrooms effectively. While maintaining a non-evaluative stance, the critical friends seemed to prioritise assessing the MTEs' ability to create inclusive learning environments that catered to the individual strengths of and challenges faced by their PSTs. The MTEs revealed that gaining specific insights and recommendations from their critical friends about how to facilitate differentiation and inclusion in their lessons informed helpful adjustments to their teaching approaches and classroom management strategies in

subsequent lessons. For example, after Sam observed Nick's lesson (31 March 2023), he suggested to Nick that he could work more inclusively by incorporating more opportunities for PSTs to work in small groups tailored to their varying proficiency levels. Nick shared:

> "Sam pointed out that while I used good motivating examples, I tended to jump too quickly into whole-class instruction. He suggested alternating those examples with short small-group activities where PSTs could grapple with the concepts at their own level first before reconvening. I tried that the next class, and it enabled me to better meet PSTs where they were at before progressing the lesson."

This example illustrates how the critical friends' targeted feedback prompted tangible adjustments to the MTEs' instruction that enabled them to better differentiate and create inclusive opportunities aligned with the vision of the NTECF. Similarly, during another post-lesson observation interview with Nick and Dan, the MTEs reflected that:

Nick: We are very concerned about meeting our students' [PSTs] diverse needs and abilities (PSTs) in our classrooms. One of the main reasons I was in class was to provide him with constructive feedback on cross-cutting issues like inclusion. Our meeting emphasised our shared understanding of the importance of not leaving any student [PST] behind.

Dan Exactly; our supportive colleagues prioritise our ability to create fair learning environments that cater to the individual strengths and challenges of our students [PSTs] without being judgmental. This focus aligns with the broader educational goals of providing quality instruction and growth opportunities to all learners, regardless of their unique attributes.

Though this focus aligns with a broader goal, in a separate reflective session during a CFGs meeting, Dan expressed mixed feelings about how productive teaching may be when taking student diversity into account:

> The emphasis on these crosscutting issues may not be productive for teaching mathematics in teacher education because of time and resources. Ah…, I prefer to focus on what I need to teach and not focus on gender issues and the like. (Dan)

Despite recognising the importance of inclusivity, some MTEs like Dan expressed apprehension about whether an intensive focus on "cross-cutting issues" like gender

differentiation is ultimately productive, citing limitations in time and resources for teacher education coursework. While the MTEs and their critical friends shared a philosophy of advancing equitable learning opportunities, some felt constrained by what they considered to be teaching realities. Nonetheless, the critical friendships provided a productive forum for sustained reflection on these challenges and for refining approaches that foster more inclusive learning environments in pre-service mathematics teacher education.

11.3.4 Enactment of lessons in the context of improving practice

Another goal of the critical friendship groups (CFGs) was to provide targeted, practice-based feedback to support continuous improvement of each other's teaching practices. The MTEs emphasised that their CFG collaborations played a deliberate and impactful role in elevating the quality of instruction and student learning in their teacher education courses.

As Dan explained in a post lesson observation:

> "One key reason for engaging critical friends is providing practice-based feedback to support each other's improvement. During peer observations, we focus on specific aspects of teaching and give constructive feedback…. It's not just pointing out what could be better, but a targeted process aimed at ensuring that our practices are in-tune with the intended outcomes for our PSTs and curricular objectives."

This approach to practice-based feedback aligns with recent research on effective professional development in education. Darling-Hammond et al. (2017) emphasise the importance of job-embedded, collaborative professional learning that focuses on educators' real work with students. The CFGs' approach to peer observation and feedback exemplifies this type of contextualised professional learning. Nick elaborated on this mechanism:

> "If one of us is trying a new instructional strategy, our critical friends observe, analyse, and provide feedback. It's not just saying if it worked or not, but understanding the reasoning behind the choices and ensuring they align with our goals and students' [PSTs] needs. It's an intentional process geared towards improving practice."

The depth of analysis described by Nick reflects what Timperley et al. (2017) term 'adaptive expertise' in teaching. These authors argue that effective professional learning helps educators develop this adaptive expertise, enabling them to make informed decisions about their practice based on student needs and learning goals. Nick's comments suggest that feedback

conversations moved beyond binary evaluations to unpack the "why" driving pedagogical decisions in MTE classrooms.

While all recognised the importance of the feedback loop, I witnessed that participation levels varied among CFs suggesting that commitment to a common purpose also differed. Sam acknowledged, "Getting feedback is immensely impactful, but I'm guilty of not providing it consistently…. I've realised practice-based feedback fosters better teaching and learning for us and our students [PSTs]."

Nonetheless, the data revealed the CFGs can establish a structured, collaborative space for this valuable exchange to unfold in the service of professional growth. As Sam summarised, "For us, it's a continuous cycle of collaboration and improvement, and the CFG serves as the forum for exchanging insights."

Through careful planning, thoughtful negotiation, and targeted feedback, critical friendships fostered within the CFGs' collaborative dynamics can become a key driver for advancing the overall quality of MTE practices. By anchoring the feedback process in classroom realities, these partnerships have the potential to iteratively refine instructional strategies to better align with educational theories, curricula and PSTs' specific needs.

11.5 Development of a shared repertoire

Under this key theme, the data analysed revealed the importance of developing a shared repertoire of knowledge and resources and of practices centred around community building and support among MTEs.

11.5.1 Repertoire of professional knowledge: from individual insights to developing shared resources

The data analysed revealed that as MTEs engaged with teaching resources and each other within CFGs, their understanding coalesced into a collective knowledge base which consisted of pooled knowledge resources; 2) co-constructed pedagogic techniques; 3) integration of diverse viewpoints; and 4) iteratively refined practice.

11.5.1.1 Pooling knowledge resources

There was evidence in the data of the consolidation of distinct content resources, instructional narratives, and contextual insights from individual CFG members into an expansive shared repertoire. This pooling of resources through co-contributing practice-based materials (lesson plans, national text documents and other resources) enabled members to develop and extend their cumulative knowledge. Nick described the outcomes of this pooling process:

> We each bring stuff to share and...it's like having all these extra lesson plans and activities and just knowledge about trainees' backgrounds from contexts we have taught before. This knowledge one person wouldn't normally have. (Nick)

The composite resource base fuelling such expertise arises through reciprocal exchange. Members discuss and collectively determine resources missing from the group repertoire to expand their collective understanding. By requesting specific activities from collaborators' unique practice settings and integrating additions into the jointly built library, CFG participation has the potential to expand contextualised knowledge as Sam emphasised:

> You know, like when Dan talked about those multipurpose charts that he did in our group meeting, ...we said it would be cool to try, and the next time, he sent all the materials he made. Now, that is something we can all use for the training of our students [PSTs]. (Sam)

While sharing specific instances or experiences that highlight this transformation of individual understanding into a collective one, Dan made the following observation:

> In our discussions…. It became apparent that contributions from each member, like unique experiences and personal interpretations, greatly enrich our professional knowledge. For instance, during the uhm…, practice-based discussion, we actively exchange insights and engage in constructive criticism. Err…, through these mechanisms, not only do we refine our own understanding, ah…, but we also solidify shared concepts that can be applied in our practice.

The extract indicates that the robust shared foundations constructed through pooling enabled members to hone training approaches reflecting different realities that PSTs would be likely to encounter, as Nick described:

> With all of us putting stuff out there to the group...I can show students [PSTs]different ways to teach one thing. It makes me better at helping them tailor teaching to their needs. (Nick)

This comment suggests that by consolidating member contributions into collective capital, MTEs and their critical friends facilitate access to the cumulative, practice-connected repository of resources, narratives and techniques for their own growth and to enhance mathematics teacher preparation. While describing the impact of this collaborative process on

the development of collective resources within critical friendships in mathematics teacher education, Sam highlighted that:

> The idea of collective resources is quite palpable. Uhm…, it's like we're building this pool of resources, narratives, and methodologies. Err…, our diverse contributions, discussions, and debates create ah…, a resource of knowledge that goes beyond what any individual could bring to teacher education. You see, you see,…, this common resource then becomes a valuable asset for all of us, uhm…, enhancing our teaching practices and contributing to the ah…, the broader field of mathematics teacher education.

The extracts are indicative of the transformative potential of CFGs in mathematics teacher education, emphasising the reciprocal exchange of content knowledge and pedagogic skills, the enrichment of collective understanding through diverse contributions, and the tangible development of a valuable pool of resources and methodologies. The collaborative process not only benefits individual MTEs but also contributes significantly to the advancement of mathematics teacher education in general.

While observing CFGs meetings, I found that MTEs shared resources including exchanging educational materials, lesson plans, national text documents, and other resources that had proved to be effective in their teaching practices and supported their professional knowledge For instance, during one of the CFG meetings observations, Dan encouraged his colleagues to share resources they found to be productive for the teaching of integers when the stressed that:

> Per the reflective feedback section in our CFGs meetings, uhm…, we all know that some low or no-cost artefacts that we develop and use have proven to be effective. So, when we go back to our classrooms, ah…, let's not forget to use manipulatives such as multi-purpose grids to explore operations on integers.

> Reflection played a vital role in the meetings of critical friends. Reflecting on past projects, successes, and challenges enabled MTEs to identify patterns, extract valuable lessons, and refine their practices, which were examples of what Rienties and Kinchin (2014) term 'networked learning' in higher education. These authors argue that such collaborative knowledge construction is crucial for developing adaptive expertise in teaching.

11.5.1.2 Co-constructing pedagogic techniques

A second important aspect of the work of the CFGs involved jointly conceptualising, creating, and refining instructional approaches attuned to contemporary classroom needs. As noted by Dan:

> We are always developing new ways for students [PSTs] to get in there and practice key ideas they will need as real teachers, new ways of modelling or assessing students before they even set foot in a school. (Dan)

In another interview, Nick stated the following:

> What we've observed is that within CFGs… there is a reciprocal exchange of knowledge not just content knowledge but also pedagogic skills. Uhm…, as we engage with the teaching and learning materials and interact with each other, our understanding begins ah…, to transform into something collective. You see, it is like a merging of perspectives and skills that benefits everyone involved. (Nick)

The co-construction of pedagogic techniques by participants in this study aligns with Grossman et al.'s (2009) view that such collaborative development of teaching practices is essential for preparing PSTs to adapt to diverse classroom contexts. As Dan described in a PLC planning session:

> If I design a lesson plan alone, I might only picture how I taught functions in senior high school back in the day. Together, it is this imaginary future classroom we envision for the next generation of teachers and students. (Dan)

The collaborative development process included open critique, redesigning of activities to expand applicability, and walk-throughs of approaches to refine flow and scaffolding. The workshop environment in which the MTE PLC, and the CFGs within it, enabled the evolution of versatile, anticipatory training techniques for MTEs as they positioned PSTs for entry into 21st-century educational settings. Overall, the findings revealed shared conceptualising and building of instructional approaches in the service of future-focused, responsive teaching methodologies.

11.5.1.3 Integrating diverse viewpoints

CFG participation provided a forum in which to voice contextual insights, theoretical orientations, and practice-based perspectives. The deliberate surfacing of diverse viewpoints

fuelled the examination of personal assumptions underpinning training approaches. As Sam expressed:

> We all bring our own experiences, you know. So, Sam mostly would say, "When I taught secondary school mathematics, I did..." uhm…, and that makes me say 'hmm, I never considered that angle before for my pre-service teachers'.

Findings illustrate how exposure to alternate standpoints spurred reflection on previously tacit beliefs shaping instructional decisions. Through debate and collaborative analysis of practice experiences, MTEs collectively explored and synthesised elements of these varied perspectives. Research has demonstrated that when cognitive diversity among community members is combined with integrative methods such as open communication and conflict resolution, it can enhance performance (Ven et al., 2008). However, the advantages of diversity are only fully realized when team members are completely integrated, which facilitates equitable information exchange (Smith-Doerr et al., 2017) During a CFG observation, Sam expressed that:

> You see, my own thinking shifts... there is this blended perspective that we create together beyond just what I came in with or what I still hold on to. (Sam)

In an interview, Nick also highlighted the significance of engaging with differing education and training orientations within the group, as MTEs have had differing education experiences. Nick expressed the following:

> Within our group, we have recognised the value of confronting diverse training orientations. Uhm…, it's not about sticking to one rigid approach but, uhm…, rather opening ourselves to different perspectives. This deliberate confrontation ah…, allows us to continually question and expand err…, our own assumptions and biases. It's a reflective process that goes beyond our comfort ah…, zones but is essential for personal and professional growth. When we integrate these err…, diverse viewpoints, it forms the foundation for ah…, developing more responsive teaching approaches. You see, uhm…, it's not just about teaching methods; ah…, it is about understanding the varied training orientations and adapting our approaches to meet the diverse needs of our students. Uhm…, this constant questioning and integration have been instrumental in our collective journey toward improving our teaching practices. (Nick)

The extracts suggest that making space for multiplicity and blending of once-disparate mindsets allows transformation toward expertise attuned to meeting pre-service teachers' needs. By implication, MTEs must let go of all the personal prejudice associated with practice and be more open to diverse perspectives regarding practice. A collaborative approach to developing professional knowledge and resources can support improving mathematics teacher education practices.

11.5.2 Community building and support

The participants frequently noted how the sense of community and trust that developed through their critical friendships enabled them to share their knowledge and classroom practices. As Dan described:

> Having that safe space where I knew my vulnerabilities or struggles wouldn't be judged allowed me to lay everything out on the table when we were collaborating or exchanging resources.

In particular, the non-evaluative stance embodied in their critical friend relationships facilitated the exchange of knowledge around areas in which the participants felt less confident or competent. Sam explained how he was struggling with integrating technology into his instruction, and he expressed:

> I would have been embarrassed for anyone else to see that, but with my critical friend, I could expose those weaknesses and get input that has helped me grow. (Sam)

The mutual support and reassurance around vulnerabilities thus enabled the participants to pool their knowledge and classroom practices more extensively. Developing trust and providing safe spaces for vulnerability are practices associated with effective professional learning communities. Vangrieken et al. (2017) emphasise the importance of psychological safety in fostering genuine collaboration and knowledge sharing among educators. I witnessed that over time, the MTEs' respect for one another enabled the co-construction of a shared set of instructional strategies, educational resources, and professional development goals that were discussed and experimented with to improve their collective capacity as MTEs, allowing the shared repertoire that emerged from their community-building and knowledge exchange to mediate and support their continued professional growth within the group.

11.5.2.1 Developing shared linguistic resources

A notable finding was that the critical friendship groups (CFGs) among the MTEs created discourse communities devoted to improving algebraic instruction. A prominent feature of these communities was the development and utilisation of shared linguistic resources and specialised vocabulary related specifically to the teaching and learning of algebra. Through regular collaborative discussions and co-planning within their CFGs, the MTEs co-constructed a unique shared professional language for discussing algebraic pedagogy.

During CFG meeting observations, I noticed that MTEs used specialised mathematical language. When asked about the significance of these unique linguistic resources, Dan emphasised:

> In our CFG, err…, we develop and use a unique professional language that serves as a common ground for our discussions. This err…, shared language is crucial because it allows us to communicate effectively about practice. For instance, when discussing algebraic instruction, uhm…, there is a specific set of terms and phrases that we consistently use. These terms include problem-solving, algebraic reasoning, and conceptual understanding. You see, you see, these words are integral to our conversations, forming ah…, the basis of our shared language. (Dan)

Building on this idea, Nick added:

> Exactly! The use of specialised terminology creates a sense of …, belonging within our group. When we talk about problem-solving or algebraic reasoning, uhm…, we all know exactly what we mean because err…, we have discussed these terms. You know, uhm…, it facilitates a deeper understanding of, err…, the challenges and strategies involved in teaching algebra to our students [PSTs]. It is not just about using the same words but rather sharing the understanding that comes with, err…, these terms enhance the quality of our collaboration. (Nick)

The significance of these shared linguistic resources was evident during a CFG meeting observation focused on using teaching and learning materials (TLMs) in algebraic instruction. Dan encouraged his colleagues:

> "As we engage with our students during the coming week, let us not teach for teaching's sake. Let us ensure conceptual understanding among our students [PSTs] using artefacts."

In the extract, though the emphasis was on artefacts, Dan uses conceptual understanding to emphasise the need for PSTs to understand the underlying principles rather than just memorising procedures. This choice of language was not arbitrary; it was a deliberate use of a shared linguistic resource. The extract suggests the significance of MTEs' shared linguistic resources in fostering meaningful dialogue and collaboration within the CFGs.

Sam further illustrated this point during another CFG meeting:

> "We all started talking about 'covariational reasoning' as this essential goal for developing deep algebraic understanding, and what each of us meant by that phrase really aligned with all our readings and discussions. It gave us clarity and unity when we collaborated on algebra lessons to have that common language." (Sam)

This development of shared linguistic resources was crucial in forming a sense of professional belonging centred around improving algebraic instruction within their CFGs. The specialised terminology facilitated more efficient and meaningful discussions about classroom practices and instructional decisions. As Nick reflected:

> "The language we developed together through this collaborative process binds us together around this common purpose of getting better at teaching algebra."

Overall, the findings suggest that constructing shared ways of communicating, particularly through specialised terminology, strengthened the discourse and collaboration within these critical friend groups among mathematics teacher educators. This shared language not only facilitated more effective communication but also contributed to a sense of community and shared purpose among the MTEs.

11.6 Chapter Summary

This chapter has examined how engagement in critical friendships can shape professional knowledge and practice among MTEs. Utilising a qualitative methodology, including interviews and observations, I analysed collaborative dynamics within critical friendship groups using Wenger's communities of practice framework. Key findings are that the relationships that developed in the CFGs were enabling for MTEs' improved alignment of teacher education and school mathematics curricula, co-construction of pedagogies and resource sharing, with all of these contributing to professional growth despite tensions sometimes arising in regard to time constraints, criticality and participation. Ultimately, these partnerships fostered reciprocal expertise development, supported vulnerable reflection on

practice, and catalysed pedagogic innovation, all grounded in a shared commitment to advancement in mathematics education. The study concludes that realising the possibilities of critical friendships in enhancing teacher education necessitates sincere engagement and willingness to challenge entrenched assumptions within a supportive environment and over a considerable period of time.

CHAPTER 12

CONCLUSION

12.0 Introduction

There is a critical need to improve the quality of pre-service mathematics teacher education in Ghana (Adu-Gyamfi & Otami, 2020). This study was driven by the recognition that MTEs play a pivotal role in shaping the next generation of mathematics teachers, yet their professional development needs and the specific knowledge required for their work remain underexplored. The lack of research focused on whether, and if so, how, collaborative professional learning experiences support the development of MTEs' knowledge and practices for teaching algebra in pre-service teacher education programmes provided the rationale for this study.

The professional learning community at the heart of both phases of the study provided a structured environment in which MTEs could collectively develop their knowledge and practices, while the critical friendships explored in the second phase potentially enabled both risk taking and critical reflection.

The study was guided by the following research questions:

1. To what extent does working collaboratively in a PCL support the development of mathematical knowledge for teaching in pre-service teacher education?

2. What knowledge do MTEs consider important to focus on in their collaborative work in pre-service teacher education?

3. What counts as evidence of the integration of PLC learnings into MTEs' pre-service teacher education practices, and is there evidence of such integration?

4. To what extent does critical friendship support changes in MTEs' professional knowledge and practice?

This study is distinctive in its focus on MTEs in Ghana, a context that has been underrepresented in international research on mathematics teacher education. By examining collaborative professional learning in this setting, the study not only addresses a local need but also contributes to the global understanding of MTE development in diverse educational contexts.

The significance of this study lies in the potential of its findings to inform and improve collaborative professional learning practices for MTEs, with implications for the field of

mathematics education in Ghana and potentially in other contexts. The Ghanaian situation provides a productive context in which to explore MTE professional development in the Global South, which may inform similar future work in various educational settings.

This research aimed to contribute to understanding effective approaches for MTE professional growth by exploring collaborative learning dynamics within a PLC and within professional critical friendships. By identifying the specific knowledge domains that MTEs consider crucial for their work, the study offers insights for curriculum design in teacher education programmes at a time of global emphasis on improving mathematics education and of recognition that teacher quality is a key factor in student achievement (Darling-Hammond, 2017). By focusing on the professional learning of those who educate teachers, this study addresses a critical link in the chain of educational improvement. It aligns with and informs Ghana's national strategy for the development of MTEs, potentially offering insights that could be adapted for MTE professional development initiatives in other contexts.

In the following sections, I summarise the key findings of the research, discuss their theoretical and practical implications, reflect on the research design, acknowledge the study's limitations, and propose directions for future research.

12.1 Summary of the research: theories, methods, and key findings

12.1.1 Theories and methods

The study is grounded in a sociocultural perspective on learning, primarily drawing from Vygotsky's (1978) sociocultural theory. This main framework was complemented by three key conceptual models: Desimone's (2009) framework for effective continuous professional development (CPD), which guided the analysis of professional learning experiences; Wenger's (1998) dimensions of practice framework, which provided a lens for understanding the dynamics of critical friendships. Additionally, McCrory et al.'s (2012) knowledge of algebra for teaching (KAT) framework informed my understanding of the specialised knowledge required for teaching mathematics, though it played a less central role in the overall theoretical framework and analysis of data.

These theoretical frames enabled analysis of complex collaborative learning processes and knowledge development among MTEs. By combining sociocultural perspectives with domain-specific frameworks, the study offers a holistic view of how MTEs develop their professional knowledge and practices within a PLC.

A qualitative case study (Yin, 2015) was conducted in two phases. Phase One focused on exploring collaborative practices within a PLC of MTEs teaching algebra concepts in pre-service teacher education in Ghana. Phase Two examined the role of critical friendships in supporting MTEs' professional growth. Data collection methods included focus group interviews, PLC observations, individual interviews with MTEs and their critical friends, and classroom observations. Data analysis involved both deductive and inductive thematic analysis. For Phase One, MTEs' professional learning experiences were analysed using Desimone's framework, followed by inductive analysis to identify emerging themes. Phase Two analysis employed Wenger's three-dimensional practice lens to examine the role of critical friendships.

12.1.2 Key findings

12.1.2.1 Collaborative learning in PLC

In addressing research question 1, content-focused and domain-specific professional learning emerged as crucial components of MTEs' professional learning in alignment with previous findings on effective professional development (e.g., Desimone, 2009; Kennedy, 2016). The PLC provided tailored professional learning experiences for MTEs that were specific to their domain and balanced the extension of MTEs' subject-matter knowledge and pedagogic knowledge. These experiences supported MTEs in continually learning about teaching while teaching about teaching, echoing the importance of ongoing, focused professional development (Darling-Hammond et al., 2017). For example, MTEs collaboratively analysed student work to identify common misconceptions in algebraic thinking, leading to discussions on how to address these in their teaching.

While lesson modelling was a planned component of the PLC design, the research revealed that MTEs engaged in deliberate discussions about its implementation as a key instructional strategy. These discussions included MTEs making explicit their pedagogic reasoning. For instance, one MTE shared how they used multiple representations (symbolic, graphical, and verbal) when introducing linear equations, explaining the rationale behind this approach.

The PLC fostered an environment of continued learning and growth for MTEs. However, a critical finding was that varying levels of participation and commitment led to uneven professional growth. Some MTEs attended sporadically, citing time constraints, while others engaged consistently but struggled to implement new strategies due to institutional pressures or perhaps to their incomplete internalisation of these strategies. There is a complex interplay between individual motivation, institutional support, and the effectiveness of professional development initiatives

12.1.2.2 Knowledge domains for MTEs

To address question 2, the study identified three core knowledge domains crucial for MTEs' work in pre-service teacher education:

- Pedagogic content knowledge: PCK includes understanding effective pedagogic strategies for teaching mathematics, knowledge of mathematical representations, and addressing student misconceptions. For example, MTEs discussed strategies for helping PSTs understand the transition from arithmetical to algebraic thinking.
- Subject-matter Knowledge: SMK comprises specialised Content Knowledge of algebra concepts and mathematical horizon knowledge. MTEs emphasised the importance of understanding both advanced algebraic concepts and their connections to school algebra.
- Knowledge of educational contexts: MTEs stressed the importance of understanding both teacher education and school mathematics curricula, highlighting the need for alignment between the two in preparation of student teachers for their work in classrooms.

These domains align with and extend frameworks of mathematical knowledge for teaching (Ball et al., 2008), specifically addressing the unique context of teacher education.

A key finding of the study is that collaborative professional learning within the MTE PLC contributed to enhanced knowledge in each domain. MTEs emphasised the importance of curriculum and pedagogic alignment and of modelling pedagogic approaches that PSTs should employ in their future classrooms. A key finding was the value the MTEs placed on bridging the theory-practice divide in teacher education by grounding their teaching in real classroom contexts. For instance, MTEs incorporated real classroom scenarios and PST work samples into their discussions to demonstrate how theoretical concepts apply in practice.

However, challenges were noted in achieving coherence within the PLC. Inconsistent participation and differing teaching philosophies among members sometimes hindered progress. Some MTEs advocated a more theoretical approach, while others pushed for a purely practical focus. This contrasted with the balanced approach emphasised in national policy documents, which call for integrating theory and practice in teacher education. For example, one MTE consistently promoted a problem-solving approach, while another focused primarily on procedural fluency, leading to tensions in collaborative planning sessions.

12.1.2.3 Evidence of integration of PLC learning into MTE teaching

The teaching of the MTEs is generally aligned with the National Teacher Education Curriculum Framework. In response to question 3, the Pedagogic strategies discussed in the PLC and observed in MTE practice included:

- Using multiple representations (e.g., graphical, symbolic, and verbal representations of algebraic concepts)
- Lesson modelling (demonstrating effective teaching strategies for PSTs to observe and analyse)
- Eliciting PST participation (encouraging active engagement through questioning and discussion)
- Scaffolding learning (providing structured support to help PSTs develop algebraic thinking)
- Strategic use of example spaces (carefully selecting and sequencing algebraic problems to highlight key concepts)

However, some challenges in implementing PLC learnings were noted, often related to time constraints. For instance, the exam-oriented nature of Ghana's current teacher education system prevented some MTEs from fully implementing PLC learnings. As one MTE stated, "I want to spend more time modelling with artefacts, but I need to cover the course content quickly to prepare students for exams.".

12.1.2.4 Critical friendships

To address research question 4, critical friendship pairs were created based on complementary expertise and a shared willingness to engage in mutual observation and feedback. Data was collected through interviews with the critical friends and by observing their feedback within a larger group of critical friends. A key finding is that these relationships fostered reciprocal development of expertise, supported vulnerable reflection on practice, and catalysed pedagogic innovation. I found that these partnerships among MTEs enabled the exploration of pedagogies, curricular alignment, and resource sharing. However, as in any other friendship, there were challenges, including time constraints, maintaining criticality, and ensuring consistent participation.

These findings collectively demonstrate that collaborative professional learning experiences can enhance MTEs' knowledge and practices for teaching mathematics in pre-service teacher education. The study reveals:

1. The interconnected nature of MTE professional development involves content knowledge, pedagogic skills, and reflective practice.
2. The importance of content-focused learning, peer collaboration, and critical reflection.

It also underscores the challenges in translating collaborative learning into individual practice. For example, some MTEs struggled to implement new strategies due to the exam-oriented nature of ITE. As detailed in Chapter 7, one MTE initially resisted using multiple representations in their teaching, citing time constraints, but gradually incorporated this approach after observing its effectiveness in a colleague's class. These insights point to areas for further development in MTE professional growth, such as addressing institutional barriers and providing ongoing support for implementing new practices.

12.1.3 Contextualising findings within the larger scholarly conversation

This study's findings contribute to the ongoing scholarly discourse on mathematics teacher education, particularly in professional development for MTEs. For example, they support and extend the work of scholars like Jaworski et al. (2017) and Robutti et al. (2016), who have emphasised the value of collaborative inquiry in mathematics education. However, my focus on MTEs specifically, adds a new dimension to this conversation, highlighting the unique challenges and opportunities of collaborative learning models for those who educate mathematics teachers.

Identifying core knowledge domains for MTEs contributes to the ongoing dialogue about the nature of mathematical knowledge for teaching teachers (Masingila et al., 2018; Jankvist et al., 2020; Castro Superfine et al., 2020). While scholars like Ball et al. (2008) have extensively explored mathematical knowledge for teaching at the school level, this study extends this conversation to the tertiary level, specifically in the context of teacher education. The emphasis on pedagogic content knowledge, subject-matter knowledge, and knowledge of educational contexts aligns with but also extends existing frameworks (e.g., Castro Superfine et al., 2020), highlighting the specialised knowledge required for teaching future teachers.

This study's exploration of critical friendships connects with the broader literature on peer coaching and mentoring in education (e.g., Burley & Pomphrey, 2011; Jones & McNulty, 2023). However, my focus on MTEs offers specific insights into how these relationships function in the context of mathematics teacher education. The findings on reciprocal expertise development and vulnerable reflection on practice contribute to the understanding of how experienced educators can continue to grow and innovate in their practice.

This study also engages with the ongoing conversation about the theory-practice divide in teacher education. The challenges I observed in translating PLC learning into classroom practice echo concerns scholars like Cochran-Smith and Lytle (1999) raised about the complexities of connecting professional development experiences with day-to-day teaching realities. The findings offer insights into these challenges, specifically in the context of mathematics teacher education.

This study's emphasis on pedagogic alignment across knowledge domains connects with broader discussions about coherence in teacher education programmes (e.g., Floden et al., 2021; Levine et al., 2022). The study's findings suggest that this alignment is not just important at the programme level but also within the practice of individual MTEs.

Finally, this research study contributes to the growing body of literature on mathematics education in African contexts (e.g., Bethell, 2016; Mosvold, 2022). While it was conducted in Ghana, its findings contribute to the international discourse on mathematics teacher education. The challenges and opportunities identified in MTE professional development likely resonate with other developing country educational contexts but also with other contexts that have a much longer tradition of research in mathematics teacher education such as Germany (Prediger et al., 2022).

Moreover, the study's insights into the specialised knowledge required for teaching teachers and the potential of collaborative learning models are relevant to mathematics teacher education globally, expanding perspectives on MTE professional growth that can inform practice and research in diverse international settings.

12.2 Conclusions

The study concludes that collaborative professional learning, both through PLCs and critical friendships, can substantially enhance MTEs' knowledge and practices for teaching mathematics in pre-service teacher education. However, the effectiveness of these approaches relies on several factors, including the level of engagement of the MTEs, their willingness to question their own assumptions as well as those of others, and the support from their institutions.

The research underscores the specialised nature of the knowledge required for teaching teachers and demonstrates the potential of professional collaborative learning communities for supporting the development of this specialised knowledge. It also highlights the complexities

involved in translating collective knowledge into individual classroom practices, pointing to areas for future research and development in MTE professional growth.

These findings have implications for the field of mathematics teacher education. They suggest that effective MTE professional development requires a multifaceted approach that goes beyond traditional content-focused education and training. The study highlights the potential of collaborative learning communities for fostering not only individual growth but also the development of a shared professional knowledge base among MTEs. Moreover, it underscores the need for institutional support and shifts in policy to create environments conducive to sustained professional learning.

12.3 Contributions

This study has aimed to contribute to mathematics teacher education, both in terms of the theoretical linkages made and used in new ways, and in terms of its empirical findings.

12.3.1 Theoretical contributions

A synthesis of the theoretical work of Vygotsky, Wenger and Desimone enabled a multi-dimensional analysis of MTE learning and development. Vygotsky's sociocultural theory is foundational for understanding learning as a socially mediated process, particularly relevant in collaborative professional development. Wenger's dimensions of practice offer a structured way to analyse the dynamics within critical friendships which include mutual engagement, joint enterprise and shared repertoire. Desimone's framework highlights the critical features of effective professional development, while McCrory et al.'s KAT framework provides a subject-specific lens for examining mathematical knowledge in practice.

By demonstrating how these theories can be synthesised and applied to the specific context of MTEs' professional development, I offer a new theoretical model for future research in this field. This integrated framework allows for a more nuanced understanding of how MTEs develop their professional knowledge and practices through social interaction, community engagement, and focused professional development experiences. It provides a theoretical basis for examining not only what MTEs learn, but how they learn, and how this learning is situated within their professional context.

This approach to integrating theoretical perspectives could be adapted for other topics in mathematics education research, where McCrory et al.'s framework might be replaced or supplemented with other domain-specific frameworks. While this study does not fully develop a new theoretical framework, it demonstrates the potential of combining existing theories to

provide fresh insights into MTE professional development. It suggests that a nuanced, multi-layered approach might be beneficial for understanding the complexities of teaching teachers to teach.

12.3.2 Methodological contributions

This study makes a substantial methodological contribution through its approach to implementing and studying critical friendships among MTEs. The research design addressed real-world challenges in establishing critical friendships within a professional learning community in which members were unfamiliar with what enacting such critical friendships entailed, even though doing so was a policy requirement.

Firstly, the study developed a cyclic structure for critical friendships (as illustrated in Figure 5.1 in the methodology chapter). This approach diverged from the traditional paired reciprocity model often described in the literature (e.g., Bognar, Branko, Krumes &Irena, 2017, Jones & McNulty, 2023). Instead, a circular critical friendship triad was created within the larger PLC. This structure allowed for a broader exchange of perspectives and expertise, potentially enriching the feedback process and expanding the learning opportunities for each MTE.

Secondly, the research demonstrated a flexible and responsive approach to implementing critical friendships. After recognising participants' initial reluctance and unfamiliarity with the concept, a scaffolded approach was adopted and included providing additional guidance and structured activities for lesson observations, particularly for first-time critical friends. The involvement of experienced MTEs in facilitating reflection and analysis for newcomers to the role was another part of this scaffolded approach.

Thirdly, the study showcased a method for integrating critical friendships within an existing PLC structure. This integration allowed for the examination of how critical friendships function within a broader collaborative learning environment, providing insights into the interplay between different forms of professional development.

Lastly, the research methodology highlighted the role of the researcher as a facilitator in establishing critical friendships. The study demonstrated how researchers can actively contribute to creating conditions conducive to effective critical friendships by encouraging a non-evaluative and supportive stance, while still maintaining their role as observers and analysts (see my role as a researcher in Chapter 5).

The approach taken for this study offers a model for future research on critical friendships, particularly in contexts where such practices are not yet established. It demonstrates how researchers can adapt theoretical concepts to practical realities, creating opportunities for meaningful professional development even when participants are initially unfamiliar or hesitant about the process. The study's methodology thus contributes to the broader field of educational research by offering an example of a flexible, context-sensitive approach to implementing and studying collaborative professional development strategies. The methodological approach developed in this study has potential applications beyond the context of mathematics teacher education. Researchers in other educational fields could adapt this model of implementing and studying critical friendships, particularly in contexts where collaborative professional development is not yet established. The cyclic structure and scaffolded approach to critical friendships could be especially valuable in studying professional development in other specialised or emerging educational roles.

12.4 Implications of the study

The findings of this research have important implications for various stakeholders in mathematics education. While there are numerous potential implications, this section focuses on three key areas: practice, policy and future research.

12.4.1 Implications for practice in mathematics teacher education

This study's findings strongly suggest the need for a rethinking of professional development programmes for MTEs. Research has long shown that traditional one-off workshops or isolated training sessions are insufficient for developing the complex knowledge and skills required for effective teaching (Darling-Hammond et al., 2017). This study suggests that the same applies to MTEs. Instead, institutions should consider enabling and supporting sustained, collaborative professional development opportunities such as PLCs and critical friendship networks for MTEs that are designed to foster long-term engagement and continuous learning, encourage reflective practice and peer feedback, focus on content-specific pedagogic knowledge (particularly in challenging areas like algebra), and engage MTEs in collaboratively analysing and improving their teaching practices.

In resource-constrained environments like Ghana, these findings suggest the need for creative and cost-effective approaches to MTE professional development. Institutions might consider using technology for virtual PLCs, implementing peer-led workshops based on the critical friendship model, or developing mentorship programmes that pair experienced MTEs with

novices. These approaches could provide sustained, collaborative learning opportunities without requiring significant financial investment

The study's findings on effective pedagogic strategies have direct implications for MTE practice. MTEs should be encouraged to use multiple representations when teaching mathematical concepts, employ lesson modelling to demonstrate effective teaching strategies, elicit and build upon pre-service teachers' existing knowledge and misconceptions, scaffold learning experiences to bridge theoretical understanding and practical application, and strategically use example spaces, progressing from simple to complex problems. My identification of core knowledge domains for MTEs implies the need for targeted development in these areas. Teacher education programmes should provide opportunities for MTEs to deepen their algebra pedagogic content knowledge, advanced algebra subject-matter knowledge, and understanding of educational contexts, particularly those relevant to pre-service and novice teachers. Institutions should create structures and allocate resources to support the development of critical friendships among MTEs. This might involve providing time and space for regular peer observations and feedback sessions, offering training on effective peer coaching and mentoring techniques, and encouraging a culture of openness, trust, and constructive criticism.

12.4.2 Implications for policy in teacher education

The study's findings on the specialised knowledge required for teaching teachers suggest the need for curriculum reform in MTE preparation programmes. Policymakers should consider developing standards for MTE preparation that reflect the complex knowledge domains identified in this study, strengthening collaborative learning opportunities in the curriculum for MTE training, and ensuring that MTE preparation programmes focus on advanced mathematical content knowledge as well as pedagogic content knowledge. This implication extends beyond mathematics to all teacher educators.

There are implications for the development of professional standards for MTEs. These standards should recognise the unique role of MTEs and the specialised knowledge they require, include expectations for ongoing professional development and collaborative learning, and emphasise the importance of research-informed practice in mathematics teacher education.

The study's findings on the benefits of PLCs and critical friendships imply the need for targeted resource allocation. Policymakers should consider providing funding for sustained, collaborative professional development programmes for MTEs, allocating time within MTE

workloads for participation in PLCs and critical friendship activities and investing in technology and infrastructure to support collaborative learning, especially in resource-constrained contexts.

The findings of this study call for systemic changes in how MTE professional development is conceptualised and implemented. Policymakers should consider creating structures that support ongoing, collaborative professional learning as an integral part of MTE practice. This may involve rethinking workload allocations, assessment criteria for MTEs, and the integration of research and practice in teacher education institutions.

12.4.3 Implications for future research

There is a need for longitudinal research to track the long-term impact of collaborative learning on MTE practice, examine how changes in MTE practice influence the quality of initial teacher education, and investigate the sustainability of PLCs and critical friendships over extended periods. Given that this study was conducted in Ghana, there are implications for cross-cultural research. Future studies should investigate how the research methodology and findings translate to different cultural and educational contexts, examine the universality and cultural specificity of effective MTE practices, and explore how collaborative learning models can be adapted to diverse educational systems. Future research should explore the potential of technology to support and enhance collaborative learning among MTEs, the use of online platforms to facilitate PLCs and critical friendships (especially in geographically dispersed or resource-constrained contexts), and the development of digital tools to support the specialised teaching practices of MTEs.

There is a need for research that traces the impact of MTE professional development through to student outcomes. This includes investigating how changes in MTE practice influence the teaching practices of pre-service teachers and examining the ultimate impact on learning in school mathematics classrooms. Research is needed on strategies for scaling up successful PLC and critical friendship interventions. This could involve investigating the factors that support or hinder the wider implementation of such interventions, examining how to maintain quality and effectiveness if implemented at scale, and exploring the role of policy and institutional leadership in supporting large-scale implementation.

This study's integrated theoretical framework opens avenues for further theoretical work, including an exploration of how the framework could be extended to other subject areas in

teacher education, and an investigation of the interplay between individual and collective learning in MTE professional development.

12.5 Limitations of the study

While this study has yielded significant insights into the professional learning and development of MTEs, it is important to acknowledge its limitations. Recognising these limitations not only contributes to transparency in the research process, but also provides context for interpreting the findings and suggests avenues for future research.

There are both strengths and limitations to the study having been conducted in Ghana. A significant strength is Ghana's national focus on Initial Teacher Education (ITE) and the professional development of MTEs in ITE. This context provided a rare opportunity to study MTE professional development within a national framework that expects and supports MTEs' participation and growth. Such a comprehensive, nationally driven approach to MTE development may not be common in other countries, which enhances the study's contribution to the field while also limiting direct comparability to contexts without such initiatives.

While this study provides insights into mathematics teacher education in a developing country context, it also raises questions about the applicability of the findings to other cultural and educational settings. The specific challenges and opportunities faced by MTEs in Ghana, shaped by this national focus on ITE, may not be representative of those in other countries, particularly in educational systems without such targeted initiatives for MTEs. Therefore, caution should be exercised when applying these findings to significantly different contexts.

Furthermore, the study focused on a specific group of MTEs teaching algebra in pre-service teacher education. While algebra is a crucial area of mathematics, the findings may not fully capture the experiences and professional development needs of MTEs specialising in other mathematical domains. The unique challenges of teaching mathematics may have influenced the nature of the collaborative learning and knowledge development observed in this study. These contextual factors, while limiting broad generalisability, also highlight the value of this research in providing insights into MTE professional development within a supportive national framework - an aspect that could inform policy and practice in other countries seeking to enhance their teacher education systems.

12.5.1 Methodological and scope limitations

While this study's qualitative nature provides rich, in-depth data, it also has limitations. The small sample size, while appropriate for a case study approach, may not represent the

experiences and perspectives of all MTEs, even within the Ghanaian context. The reliance on self-reported data through interviews and focus groups introduces the potential for social desirability bias. Participants may have presented their experiences and practices more positively or aligned their responses with what they perceived to be the researcher's expectations. While classroom observations were used to triangulate data, the presence of an observer may have influenced the MTEs' behaviour, potentially affecting the authenticity of the observed practices.

While the study's duration allowed for an in-depth exploration of MTE professional development, it was not long enough to capture the full trajectory of change in MTE knowledge and practice. A longer-term longitudinal study might have revealed additional insights into the sustainability and long-term impact of collaborative learning initiatives.

It should also be noted that the study focused primarily on the experiences and practices of MTEs, with only limited exploration of the impact on pre-service teachers. Future studies could benefit from larger sample sizes, cross-cultural comparisons, and longer-term longitudinal designs, allowing for a greater diversity of perspectives and experiences to extend the insights gained from this research.

12.6 Concluding reflection: A journey through mathematics teacher education

This research journey was driven by one question: How can we improve mathematics education in Ghana? As a mathematics educator myself, I had seen firsthand the struggles of students and teachers alike. I knew that to make a real difference, we needed to start at the source - with those who teach the teachers. This study has been more than just an academic exercise. It has been a personal exploration of the challenges and opportunities in mathematics teacher education. As I reflect on the years spent immersed in this research, I am struck by how much I have learned, not just about MTEs and their professional development, but about the nature of teaching and learning.

I have learned that teaching teachers requires a unique set of skills and knowledge that goes beyond what is needed to teach mathematics to students. The MTEs I worked with showed me that to be effective, one must not only understand mathematics deeply but also grasp how to cultivate that understanding in future teachers. This realisation has transformed my own approach to teaching and mentoring.

The role of collaboration has been a recurring theme throughout this study. Watching MTEs come together in PLC meetings and sharing their challenges and triumphs, has been truly

inspiring. It has reinforced my belief that we are stronger together, that the isolation often felt in teaching can be overcome through purposeful collaboration something that I personally experienced through working with the Wits Maths Connect–Secondary project team. This insight will shape my future work in education, as I seek to create more opportunities for educators to learn from and support each other.

One of the most profound lessons from this journey has been the importance of context in education. Ghana's unique educational landscape, with its challenges and opportunities, has shaped every aspect of this study. I have come to understand that while we can learn from global best practices, true progress comes from solutions tailored to our specific context. This realisation inspires me to continue working within the Ghanaian education system, leveraging our strengths and addressing our unique challenges.

As I look beyond this PhD, I see a world of possibilities in mathematics education in Ghana. This study has shown me the critical role that MTEs play in shaping the future of mathematics teaching. It's clear to me now that investing in their professional development is not only about improving teacher education programmes but also about elevating the entire field of mathematics education in my country. I am inspired to work towards creating a national framework for MTE professional development, one that recognises their unique role and provides sustained support for their growth. I envision a system where collaborative learning is not just encouraged but embedded in the fabric of teacher education institutions.

This journey has also highlighted areas where we need to do more. The gap between teacher education and classroom reality is still too wide. Moving forward, I am committed to finding ways to bridge this gap, perhaps through closer partnerships between universities and schools or through innovative practicum experiences for pre-service teachers.

Moreover, this study has opened my eyes to the global conversations on mathematics education. While our context in Ghana is unique, many of the challenges we face are universal. I'm excited about the potential to contribute to this global dialogue, sharing our experiences and learning from others around the world. As I conclude this chapter of my academic journey, I am filled with gratitude for the MTEs who shared their experiences with me, the PSTs who remind me why this work is so important, and the broader community of mathematics educators who continue to strive for excellence.

This PhD has been more than a research project; it has been a transformative experience that has shaped my understanding of mathematics education and my role within it. As I look to the

future, I am energised by the possibilities ahead. I see a future where every mathematics classroom in Ghana is led by a teacher who has been prepared by knowledgeable, reflective, and collaborative MTEs. I see a future where mathematics is not a subject to be feared, but one that opens doors to opportunity for our students.

The journey continues. In many ways, this study is just a beginning. Armed with the insights from it and a renewed passion for mathematics teacher education, I'm ready to take the next steps. Whether through further research, policy advocacy, or hands-on work with MTEs and teachers, I am committed to continuing this important work. As I close this chapter, I am reminded of why I started this journey in the first place - the belief that every student deserves a quality mathematics education, and that the key to achieving this lies in how we prepare their teachers. This belief has only grown stronger, and it will continue to guide me in the years to come.

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Appendices

Appendix A: Sample of MTEs focused group interview protocol (generic) – A

Introduction

Thank you for having me in the group. Please feel free to answer the questions. The aim is not to evaluate your knowledge but to collect information that will help in *getting an in-depth understanding of the mathematical knowledge for teaching that MTEs require for teaching algebra in pre-service teacher education, as well as what they do that facilitates their learning as they work together in a professional learning community*. Would you mind if I recorded the interview? The recording will help me to ensure the accuracy of what we discuss. I assure you that I will take precautionary measures not to disclose to anyone else any part of the data that is linkable to a participant's identity. If you have any questions, please ask. I would like you to read and sign this consent form before we begin. Feel free to do so if you do not wish to answer any questions or discontinue this interview at any point. Do you have any questions you would like to ask before we begin?

1. How long have you been teaching algebra in pre-service teacher education?
2. If you had to lecture your students on algebra, what makes algebra different from other areas such as geometry, calculus, and the like?
3. Did your teacher education training focus on content and pedagogy in algebra? If so, could you provide a little detail?
4. In your experience as an MTE, what knowledge do you think you need as a mathematics teacher educator? Better still, what are the components of mathematical knowledge that you require to teach algebra in a way pre-service teachers will understand as they develop their own MKT?
5. a. Can you recall a time in your pre-service teacher education career when teaching algebra was unclear to you? individual
   b. What did you do to overcome that challenge from your PLC perspective?
6. One typical characteristic of PLC is its members' varying levels of expertise. I would very much appreciate it if you could share some of your experiences in this community. How helpful has the PLC been in sharing any algebra instructional challenges or victories you encountered?

7. If I may ask, what do you do in the PLC at all that facilitates learning and informs your practice? By do, I mean what you talk about. Does it concern knowledge about specific subject matter in teacher education, knowledge of how to teach particular subject matter or any other issue?
8. Are there things you would like PLC to address regarding your practice?
9. the PLC program is explicitly designed to improve your practice. What are the key focus areas for your professional learning in the PLC, and how have participating in PLC activities contributed to developing your knowledge of teaching algebra in pre-service teacher education?
    a. how do you learn to teach teachers by working together in the PLC?
    b. Why do you think so?
    c. To what extent do you use what you learn from the PLC in your algebra instructions?
10. can you elaborate on how helpful working together in a PLC has been to you in developing your mathematical knowledge for teaching algebra in pre-service teacher education?

Post-lesson interview Protocol (individual MTEs) - B

Thank you for having me in your classes. Please feel free to answer the questions; the aim is not to evaluate your knowledge but to collect information that will help me understand what counts as evidence of the integration of PLC learning in your classroom practice. I want to ask a few questions regarding the lesson I just observed and some general questions. Would you mind if I recorded the interview? The recording will help me to ensure the accuracy of what we discuss. I assure you that I will take precautionary measures not to disclose to anyone else any part of the data that is linkable to a participant's identity. If you have any questions, please ask. I would like you to read and sign this consent form before we begin. Feel free to do so if you do not wish to answer any questions or discontinue this interview at any point. Do you have any questions you would like to ask before we begin?

Algebra content knowledge

1. What teaching materials and strategies do you use to teach specific algebraic concepts? Why? (Referring to the session and topic observed)
2. In your view, Is there any particular area in the algebra you teach that you could use additional help from the PLC?

Knowledge in use (pedagogical knowledge)

1. How did you select your teaching/learning materials? Why?
2. How do you ensure that pre-service teachers understand the algebra you teach?
3. Do you think any of your pre-service teachers found the lesson difficult or did not fully understand what was taught? How did you attend to them?
4. If you get to re-teach the lesson, would you make any changes? Why?
5. What misconceptions might students have in algebra, and how would you overcome those?

Thank you very much for taking part in this study. If there is anything I could do for you, please feel free to contact me.

Appendix B: Ethics approval

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Research Office

HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)
R14/49 Nkrumah

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: H21/11/60**
PROJECT TITLE	Exploring mathematics teacher educators' knowledge for teaching Algebra in pre-service teacher education: A multicase study in Ghana
INVESTIGATOR(S)	Mr F Nkrumah
SCHOOL/DEPARTMENT	Wits School of Education/
DATE CONSIDERED	26 November 2021
DECISION OF THE COMMITTEE	Approved Risk Level: Minimal
EXPIRY DATE	23 March 2025
DATE 24 March 2022	**CHAIRPERSON** _____ (Professor J Watermeyer)

cc: Supervisor : Professor C Pournara

DECLARATION OF INVESTIGATOR(S)

To be completed in duplicate and **ONE COPY** returned to the Secretary at Room 10004, 10th Floor, Senate House, University. Unreported changes to the application may invalidate the clearance given by the HREC (Non-Medical)

I/We fully understand the conditions under which I am/we are authorized to carry out the abovementioned research and I/we guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to submit an amendment of the protocol to the Committee. **I agree to completion of a regular progress report. For Minimal and Low studies, this is due annually on 31 December. For Medium and High Risk studies, this is due twice annually on 30 June and 31 December.**

_____ _____/_____/_____
Signature **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

Appendix C: Interview guide (Phase Two) – Critical friendship

Thank you for having me in the group. Please feel free to answer the questions. The aim is not to evaluate your knowledge but to collect information that will help in *getting an in-depth understanding of the mathematical knowledge for teaching that MTEs require for teaching algebra in pre-service teacher education, as well as what they do that facilitates their learning as they work together in a professional learning community*. I assure you that I will take precautionary measures not to disclose to anyone else any part of the data that is linkable to a participant's identity. If you have any questions, please ask. I would like you to read and sign this consent form before we begin. Feel free to do so if you do not wish to answer any questions or discontinue this interview at any point. Do you have any questions you would like to ask before we begin?

Section 1: Introduction and Background Information

a. Can you briefly describe your role as a mathematics teacher educator? - MTE/CF

b. How long have you been involved in mathematics teacher education? – MTE/CF

c. Have you participated in any critical friendship groups or professional learning communities related to mathematics education? If yes, please provide some details about your experience. - MTE/CF

Section 2: Critical Friendship and Knowledge for Teaching

d. So what do you do in critical friendships – MTE/CF

e. How has critical friendship influenced your understanding of pedagogical strategies and approaches in mathematics education? MTE

f. Have you observed any changes or improvement in your ability to interpret and respond to PSTs' mathematical thinking due to participating in critical friendship groups? - MTE

g. What mathematical knowledge for teacher education do you consider important for your work in pre-service teacher education? MTE/CF

Section 3: Impact on Teaching Practices

h. Can you share specific examples of teaching practices that you believe have been informed or improved through your engagement with your critical friend? - MTE

i. How has critical friendship informed your approach to lesson planning and curriculum design in mathematics teacher education? - MTE/CF

j. Have you experienced any changes in your teaching methods or instructional strategies due to collaborating with your critical friend? - CF

k. When you stand at the back of your colleague's classroom, what do you choose to focus on? - CF

Section 4: Perceptions of Professional Growth

l. In what ways do you believe critical friendship has contributed to your professional growth as a mathematics teacher educator? MTE

m. Can you describe any challenges or obstacles you have faced during your participation in critical friendship groups and how you overcame them? - MTE/CF

Section 5: Reflection and Concluding Thoughts

n. How do you perceive the role of collaboration in developing your knowledge for teaching mathematics? - CF/MTE

o. Has critical friendship influenced your openness to feedback and willingness to engage in reflective practices regarding your teaching? – MTE/CF

p. Based on your experience with critical friendship, what recommendations would you give to other mathematics teacher educators interested in engaging in similar professional development opportunities? – MTE/CF

q. Is there anything else you would like to share regarding the impact of critical friendship on your knowledge and teaching practices in mathematics teacher education?

Appendix D: Participant information letters and consent forms



Study title: Exploring mathematics teacher educators' knowledge for teaching algebra in pre-service teacher education: A case study in Ghana

STUDY INFORMATION SHEET MTEs

Dear MTE

My name is Frank Kwarteng Nkrumah, and I am a PhD student in Mathematics Education at the University of the Witwatersrand, Johannesburg. My supervisor is Prof. Craig Pournara. My research study explores collaborative professional learning as a potential means for developing mathematics teacher educators' (MTEs) professional practices and knowledge for teaching algebra in pre-service teacher education.

Invitation to Participate:

I would like to invite you to participate in this research study. Should you agree to take part in this study, you will be participating in a case study design that will involve lesson observations and face-to-face interviews (both individual and focus group) with the researcher, which will last approximately 30 minutes after each lesson and 1.5 hours for the focus group. Interviews will be scheduled at a date, time and venue convenient to you.

Methods of Communication:

This study is mindful of all COVID-19 protocols: All communication, including consent processes, will be done via email at the initial stages. Regarding faca-to-face interviews and lesson observations, all COVID protocals will be strictly adhered to in the teaching and in conducting this study.

Benefits of Participating in the Study:

I cannot guarantee that you will directly benefit from this study, and research does not involve payment. However, your participation, feedback and shared reflection in a professional learning community and on how MTEs integrate PLC learnings into their classroom practice.

Privacy Protection:

Your name and identity will be kept confidential at all times and in all academic writing about this study. You may revoke data at any time that you do not wish to be published. When referring to you in the thesis and publications emanating from this study, I will use a pseudonym to refer to your written work or responses to interviews.

Rights of Participants:

Note that your participation is voluntary. signing and participating in the study, you do not waive any of your legal rights to revoke your consent and withdraw from the study at any time. You may refuse to answer any questions which makes you feel uncomfortable.

Data Storage:

Only the researcher will have access to the confidential information being collected. The researcher will make the necessary arrangements to have the raw data stored in a restricted access area for two years if the research report is published and six years if no publications emanate. Data will then be destroyed to ensure confidentiality.

Availability of the Study:

This thesis will be publicly available on the world-wide web through Wired Space accessed through the Wits library website.

Who to Contact if you Have Questions:

If at any time you have questions regarding the research or your participation, please feel free to contact me at the details listed below. Should you be interested in a summary of the research/research results, it will be made available on request once the study has been completed.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Yours sincerely,

Researcher: Frank Kwarteng Nkrumah

Email: 2509410@students.wits.ac.za

Contacts: +27639941312 / +233548489102

Supervisor: Prof. Craig Pournara

Email: Craig.Pournara@wits.ac.za

+27(11) 717 3253

Study title: Exploring mathematics teacher educators' knowledge for teaching Algebra in pre-service teacher education: A case study in Ghana

Frank Kwarteng Nkrumah

Consent form – focus group interviews

I, ……., (full name) agree to participate in this research project. The research has been explained to me and I understand what my participation will involve. I agree to the following:

(Please circle the relevant options below).

1.	I agree that I have read and understood the research information sheet	YES	NO
2.	I agree that my identity will remain anonymous	YES	NO
3.	I agree that the researcher may use anonymous quotes in his research report	YES	NO
4.	I agree to be video and audio recorded in interviews	Y ES	NO
5.	I am aware that my participation in a professional learning community may be used for research purposes	YES	NO

Participant Name: …… Signature: …… Date: …….

Researcher Name: …… Signature: …… Date: ……

Study title: Exploring mathematics teacher educators' knowledge for teaching Algebra in pre-service teacher education: A case study in Ghana

Frank Kwarteng Nkrumah

Consent form PLC/CFGs observation - MTEs

I, ……., (full name) agree to participate in this research project. The research has been explained to me and I understand what my participation will involve. I agree to the following:

(Please circle the relevant options below).

1.	I agree that I have read and understood the research information sheet	YES	NO
2.	I agree that my identity will remain anonymous	YES	NO
3.	I agree that the researcher may use anonymous quotes in his / her research report	YES	NO
4.	I agree to be video and audio recorded in interviews	Y ES	NO
5.	I am aware that my participation in a professional learning community may be used for research purposes	YES	NO

Participant Name: …… Signature: …… Date: ……

Researcher Name: …… Signature: …… Date: ……

Study title: Exploring mathematics teacher educators' knowledge for teaching Algebra in pre-service teacher education: A case study in Ghana

Frank Kwarteng Nkrumah

Consent form lesson observations-

I, ……., (full name) agree to participate in this research project. The research has been explained to me and I understand what my participation will involve. I agree to the following:

(Please circle the relevant options below).

1.	I agree that I have read and understood the research information sheet	YES	NO
2.	I agree that my identity will remain anonymous	YES	NO
3.	I agree that the researcher may use anonymous quotes in his / her research report	YES	NO
4.	I agree to be video and audio recorded in interviews	Y ES	NO
5.	I am aware that my participation in a professional learning community may be used for research purposes	YES	NO

Participant Name: …… Signature: …… Date: ……

Researcher Name: …… Signature: …… Date: ……

Study title: Exploring mathematics teacher educators' knowledge for teaching Algebra in pre-service teacher education: A case study in Ghana

Frank Kwarteng Nkrumah

Consent form individual interviews- MTEs

I, ……., (full name) agree to participate in this research project. The research has been explained to me and I understand what my participation will involve. I agree to the following:

(Please circle the relevant options below).

1.	I agree that my identity will remain anonymous	YES	NO
2.	I agree that the researcher may use anonymous quotes in his research report	YES	NO
3.	I agree to be video recorded in my lesson delivery	Y ES	NO
4.	I am aware that I may withdraw from the research at any stage if I so desire	YES	NO
5.	I agree that I have read and understood the research information sheet	YES	NO

Participant Name: …… Signature: …… Date: ……

Researcher Name: …… Signature: …… Date: ……

Study title: Exploring mathematics teacher educators' knowledge for teaching algebra in pre-service teacher education: A case study in Ghana

STUDY INFORMATION - PSTS

Dear PST

My name is Frank Kwarteng Nkrumah, and I am a PhD student in Mathematics Education at the University of the Witwatersrand, Johannesburg. My supervisor is Prof. Craig Pournara. My research study explores collaborative professional learning as a potential means for developing mathematics teacher educators' (MTEs) professional practices and knowledge for teaching algebra in pre-service teacher education.

Invitation to Participate:

I would like to invite you to participate in this research study. Should you agree to take part in this study, you will be participating in a case study design that will involve your tutors' lesson observations of which you will be part as a pre-service teacher. My observation will last for 2 hours per lesson and specific days and date for observation will be communicated to you. I will make sure my presence does not intimidate you in anyway.

Methods of Communication:

This study is mindful of all COVID-19 protocols and as during the lesson observation all COVID protocals will be strictly adhered to in the teaching and in conducting this study.

Benefits of Participating in the Study:

I cannot guarantee that you will directly benefit from this study, and research does not involve payment. However, your participation in your lecturer's lesson will provide an in-depth understanding of how MTEs integrates PLC learnings in their classroom practice.

Privacy Protection:

Your name and identity will be kept confidential at all times and in all academic writing about this study. You may revoke data at any time that you do not wish to be published. When referring to you in the thesis and publications emanating from this study, I will use a pseudonym to refer to your written work or responses to interviews.

Rights of Participants:

Note that your participation is voluntary. signing and participating in the study, you do not waive any of your legal rights to revoke your consent and withdraw from the study at any time. You may refuse to answer any questions which makes you feel uncomfortable.

Data Storage:

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If at any time you have questions regarding the research or your participation, please feel free to contact me at the details listed below. Should you be interested in a summary of the research/research results, it will be made available on request once the study has been completed.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Yours sincerely,

Researcher: Frank Kwarteng Nkrumah

Email: 2509410@students.wits.ac.za

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Supervisor: Prof. Craig Pournara

Email: Craig.Pournara@wits.ac.za ,

+27(11) 717 3253

Study title: Exploring mathematics teacher educators' knowledge for teaching Algebra in pre-service teacher education: A case study in Ghana

Frank Kwarteng Nkrumah

Consent form – lesson observations (PSTs)

I, ……., (full name) agree to participate in this research project. The research has been explained to me and I understand what my participation will involve. I agree to the following:

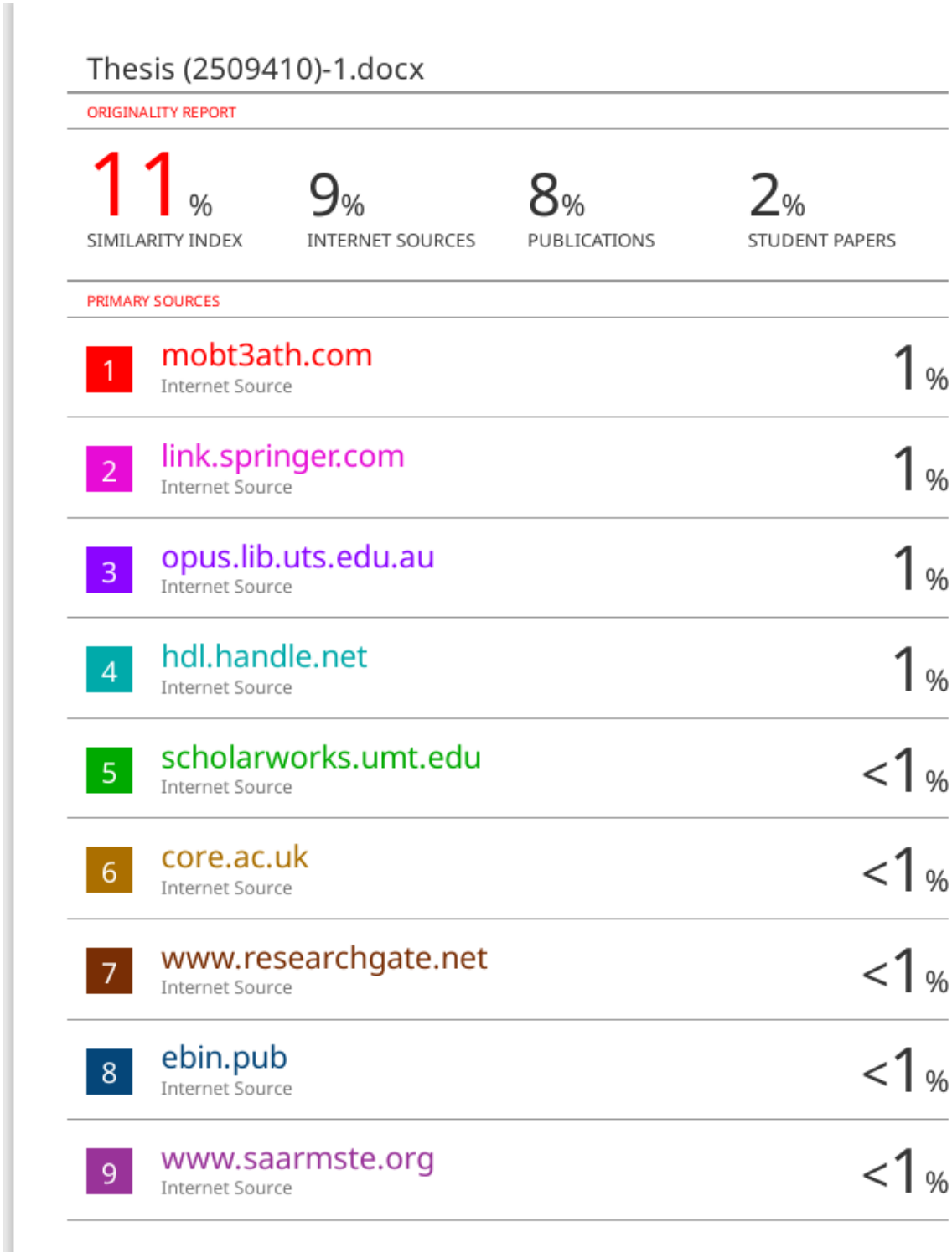
(Please circle the relevant options below).

1.	I agree that I have read and understood the research information sheet	YES	NO
2.	I agree that my identity will remain anonymous	YES	NO
3.	I agree that the researcher may use anonymous quotes in his / her research report	YES	NO
4.	I agree to be video and audio recorded in interviews	Y ES	NO
5.	I am aware that my participation in a professional learning community may be used for research purposes	YES	NO

Participant Name: …… Signature: …… Date: ……

Researcher Name: …… Signature: …… Date: ……

Appendix E: Similarity report



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ORIGINALITY REPORT

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