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The concept of effective stress in partly saturated soils

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can be used to measure β. Before admitting this it is necessary to examine the assumptions made by Croney in arriving at equation (3.5) from equation (3.4).

Coleman and Croney (1952) state that from the laws of thermodynamics equation (3.3) must give a perfect differential, i.e.

$$\left(\frac{\partial p''}{\partial V_s}\right)_{V_m} = \left(\frac{\partial p}{\partial V_m}\right)_{V_s} \quad \ldots\ldots\ldots \quad (3.6)$$

In arriving at the above relation the assumption is made that, in travelling from a fixed initial state to a fixed final state, the final state reached by the soil (in this case in terms of total volume) is independent of the path provided the changes in free energy are the same. e.g. a change in volume ΔV_s resulting from a change in free energy $p.\Delta V_s$ (V_m constant) is assumed to be the same as a change of volume ΔV_s resulting from an equal change in free energy $p''.\Delta V_m$ (p constant). This assumption is known to be valid for fully saturated soils but there is no evidence that it is correct for partly saturated soils.

It is now necessary to make the assumption that σ in equation (3.5) is a function only of V_s. Hence differentiating equation (3.5) with respect to V_m gives

$$0 = \left(\frac{\partial p}{\partial V_m}\right)_{V_s} + \beta \left(\frac{\partial p''}{\partial V_m}\right)_{V_s} \quad \ldots \quad (3.7)$$

and from equation (3.6)

$$\left(\frac{\partial p''}{\partial V_s}\right)_{V_m} = -\beta \left(\frac{\partial p''}{\partial V_m}\right)_{V_s} \quad \ldots\ldots \quad (3.8)$$

Since p'' can be expressed as a function of variables V_s and V_m only it follows algebraically that

$$dp'' = \left(\frac{\partial p''}{\partial V_s}\right)_{V_m} . d\ V_s + \left(\frac{\partial p''}{\partial V_m}\right)_{V_s} . dV_m \quad \ldots \quad (3.9)$$

In equation (3.4) p" is held constant, i.e. dp" = 0. Hence from equations (3.8) and (3.9)

$$\beta = - \frac{(\frac{\partial p''}{\partial V_s})_{V_m}}{(\frac{\partial p''}{\partial V_m})_{V_s}} = (\frac{d\ V_m}{d\ V_s})\ p''$$

It can be seen, then, that in proceeding from equation (3.4) to equation (3.5) the assumption is made that σ is a function only of V_s irrespective of the individual values of β p" and p. Since p and p" in equation (3.5) are the applied stress and pressure deficiency respectively and if it is stipulated that σ is a function only of V_s then it can be concluded that Croney's factor β is identical to Jennings' β factor. The terms σ and p in equation (3.5) can be replaced by the more conventional terms σ' and σ thereby rendering equations (3.5) and (3.2) identical.

A most important point arises out of the work just outlined. This is that when β is used in its normal effective stress context the expression $\beta = (\frac{\partial V_m}{\partial V_s})\ p''$ is only true if the principle of effective stress is valid for the soil. Hence methods for measuring β based on the above expression implicitly assume the validity of the effective stress principle as do the methods outlined by Jennings in section 3.4.

It is important to note that equation (3.5) was derived from considerations of volume change only. In fact Croney and Coleman define σ in equation (3.5) as that pressure which combined with zero pore-water pressure produces the same volume in the soil as p and p" combined. However, Croney and Coleman suggested tentatively that the shear strength of unsaturated soils is determined by an expression similar to that for determining the volume, i.e.

$$\sigma' = p + \beta' p'' \quad \ldots\ldots\ldots\ldots \quad (3.10)$$

The extent to which β and β' (in equation (3.5) and (3.10) respectively) are equivalent can only be determined by experiment.

3.6 Bishop's modified effective stress equation

Bishop (1959) discussed the question of the two phase fluid. He proposed a modified effective stress equation which included a term for the pressure in the gas phase.

$$\sigma' = \sigma - u_1 - X(u_2-u_1) \quad \ldots\ldots\ldots \quad (3.11)$$

u_1 denotes the pressure in the gas and vapour phase

u_2 denotes the pressure in the liquid phase.

This equation will be examined for various conditions of the air and water phases :

(i) Fully saturated with water : In this case there is no gas phase and u_1 falls away. Moreover since u_2 can be considered as acting over the whole of any section through the soil X must be unity. Equation (3.11) therefore reduces to

$$\sigma' = \sigma - u_2$$

(ii) Fully saturated with air : In this case there is no liquid phase and u_2 does not apply. Since u_1 can be considered as acting over the whole of any section through the soil X must be equal to zero. Equation (3.11) therefore reduces to

$$\sigma' = \sigma - u_1$$

(iii) Partly saturated with all the air in the form of occulted bubbles : It can be shown by means of a

simple capillary model similar to one used by Jennings (1960) that for this case the pressure in the liquid phase may be considered as acting over the whole area of cross section and hence X is equal to one. Equation (3.11) therefore reduces to

$$\sigma' = \sigma - u_2$$

(iv) Partly saturated with all the air voids open to the atmosphere : Since the air is in contact with the atmosphere u_1 must equal atmospheric pressure. Pore-water pressures are usually measured with respect to atmospheric pressure, i.e. u_1 is equal to zero. Equation (3.11) therefore reduces to

$$\sigma' = \sigma - X\, u_2$$

(v) Partly saturated containing both occulted and continuous air voids open to the atmosphere : In this case the equation is only meaningful if u_1 is taken as the pressure of the gas in the continuous phase. The pressure in the occulted phase varies from one bubble to another as it is dependent on the radii of curvature of the air-water interfaces. If, therefore, u_1 were to be taken as the pressure in the occulted bubbles then the expression (u_2-u_1) would vary from point to point in the soil mass. This renders the equation meaningless. Again therefore, u_1 equals atmospheric pressure and hence is equal to zero. Equation (3.11) once again reduces to

$$\sigma' = \sigma - X.u_2$$

(vi) u_1 greater than atmospheric pressure in a laboratory triaxial test : The pressure plate or pressure membrane technique described in chapter II can be employed in the triaxial testing of partly saturated soils. It appears to be specifically for this type of test that Bishop developed equation (3.11). Figure 3.4 shows a sample of soil enclosed in a rubber

sheath. The sample stands on a saturated fine porous stone or membrane through which only pore water pressures are transmitted. Measured air pressures can be applied through the end plate at the top of the sample.

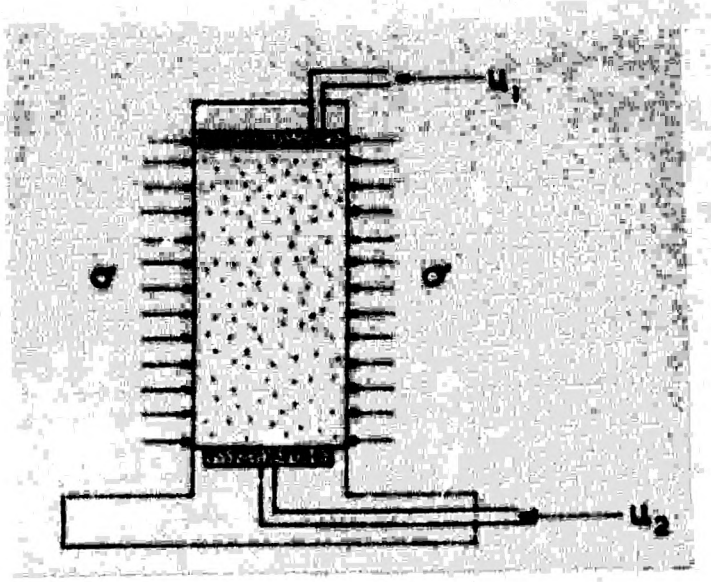


Fig. 3.4

The sample is surrounded by the cell fluid at a pressure σ. The air pressure in the sample is u_1 and the pressure in the pore water is u_2. Both u_1 and u_2 are measured with respect to atmospheric pressure.

The effective stress in the soil is given by Bishop's equation as

$$\sigma' = \sigma - u_1 - X(u_2-u_1)$$

The expression $\sigma - u_1$ is the resultant confining pressure acting on the soil sample and can therefore be replaced by the term σ_3. The expression (u_2-u_1) is the pressure deficiency p". This follows directly from the work described in chapter II where it was shown that the pressure difference across a meniscus is independent of the air pressure. Equation (3.11) can therefore be simplified to

$$\sigma' = \sigma_3 + X.p"$$

which is identical to Jennings' revised effective stress

equation.

(vii) u_1 greater than atmospheric pressure in the field. It was shown in chapter II that enclosed air voids are inherently unstable and must either eventually disappear as a result of the gas diffusing into the surrounding liquid or must break through to a nearby air filled void which is connected to the surrounding atmosphere. In considering natural soils, therefore, the air phase can generally be considered as being at atmospheric pressure. An exception to this postulate would be the case of a natural soil which is suddenly inundated or which is subjected to a rapid compression. In compacted soils the pore air pressures are almost certainly larger than atmospheric pressure immediately after placement. It is apparent, then, that the pore-air pressures might effect the value of the effective stress in some practical cases. Following the arguments presented in section 3.6(v), u_1 in equation (3.11) must refer to the pressure of the gas in the continuous phase and is measured with respect to atmospheric pressure. Equation (3.11) can be rewritten as follows

$$\sigma' = \sigma - X.u_2 - (1-X).u_1$$

Written in this form it is clear that the expression $X.u_2$ is the component of effective stress resulting from the pressure in the pore water and $(1-X).u_1$ is the component of effective stress resulting from the pressure in the pore air.

3.7 Aitchison's revised effective stress equation.

Aitchison (1960) studied the moisture stresses in unsaturated soils using theoretical and experimental methods. He deduced that pore drainage operates according to capillary laws up to pressure deficiencies approaching 1000 atmospheres. It follows then that the use of a capillary model to define the relationship between the soil moisture tension and the pore pressure in terms of the geometry of the solid, liquid and

gaseous phases is quite acceptable for soils encountered by the engineer.

Making use of such a capillary model Aitchison confirms his earlier assertion that pore pressure is considered as a stress acting over the whole area of any plane through the soil while the pressure deficiency acts only on a proportionate area given by the geometry of the system (see section 3.3). He then proposed that σ'', the effective stress due to p", should be given by

$$\sigma''/p'' = \psi$$

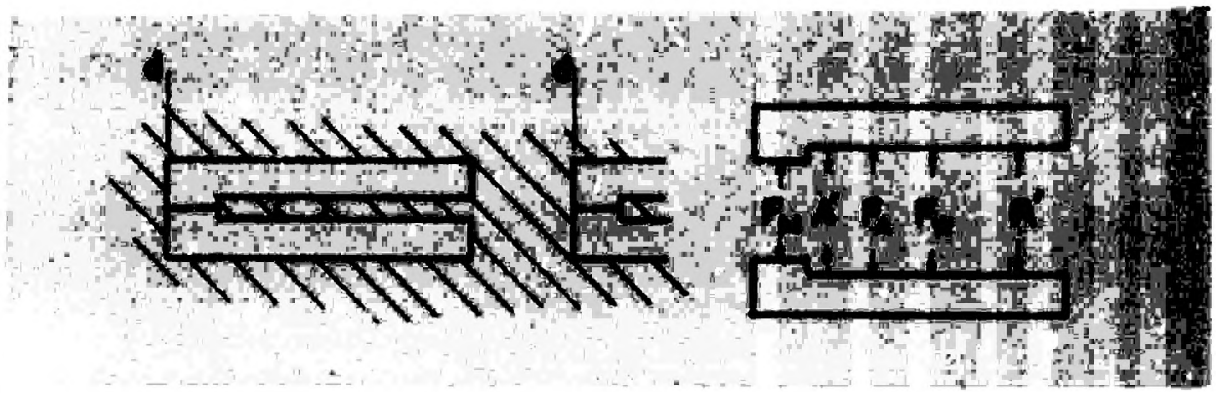
i.e. the total effective stress in a partly saturated soil is

$$\sigma' = \sigma + \sigma'' = \sigma + \psi p''$$

On comparing this equation with equation (3.2) it can be seen that both are identical and that $\psi = \beta$. Aitchison lists a number of values of ψ deduced from tests performed by Westman (1932) and Aitchison and Donald (1956).

3.8 The mechanistic approach

Lambe (1960) considered the equilibrium of forces between soil particles. Figure (3.5) shows average interparticle conditions for a partly saturated soil mass.



F_m = the force where contact is mineral to mineral.
F_a = the force where contact is air to mineral.
F_w = the force where contact is water to water.
R' = electrical repulsion.
A' = electrical attraction.

Fig. 3.5

Average interparticle conditions in a partly saturated soil mass

Lambe defines σ as the combined normal stress acting on the soil. Then if 'dd' is the cross sectional area of an element of soil the following equation involving forces can be written

$$\sigma \cdot dd = F_m + F_a + F_w + R' - A'$$

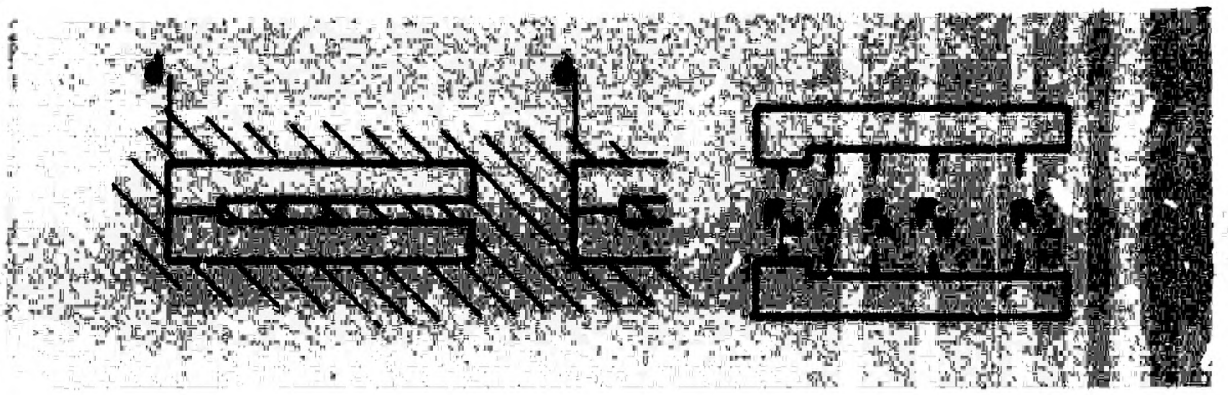
If A_m is the area of mineral contact, A_a the area of air contact, A_w the area of water contact then

$$\bar{\sigma} \cdot dd = \bar{\sigma} \cdot A_m + P_a \cdot A_a + U \cdot A_w + R' - A'$$

where $\bar{\sigma} \cdot A_m = F_m$, $P_a \cdot A_a = F_a$ and $U \cdot A_w = F_w$.

By dividing through by the area of cross section 'dd' the above equation is reduced to one involving equivalent stresses

$$\sigma = \bar{\sigma} \cdot a_m + P_a \cdot a_a + U \cdot a_w + R - A \quad . \quad . \quad . \quad . \qquad (? \ 12)$$



F_m = the force where contact is mineral to mineral.
F_a = the force where contact is air to mineral.
F_w = the force where contact is water to water.
R' = electrical repulsion.
A' = electrical attraction.

Fig. 3.5

Average interparticle conditions in a partly saturated soil mass

Lambe defines σ as the combined normal stress acting on the soil. Then if 'dd' is the cross sectional area of an element of soil the following equation involving forces can be written

$$\sigma \cdot dd = F_m + F_a + F_w + R' - A'$$

If A_m is the area of mineral contact, A_a the area of air contact, A_w the area of water contact then

$$\bar{\sigma} \cdot dd = \bar{\sigma} \cdot A_m + P_a \cdot A_a + U \cdot A_w + R' - A'$$

where $\bar{\sigma} \cdot A_m = F_m$, $P_a \cdot A_a = F_a$ and $U \cdot A_w = F_w$.

By dividing through by the area of cross section 'dd' the above equation is reduced to one involving equivalent stresses

$$\sigma = \bar{\sigma} \cdot a_m + P_a \cdot a_a + U \cdot a_w + R - A \quad . \quad . \quad . \quad . \qquad (7.12)$$

where $a_m = \frac{Am}{dd}$, $a_a = \frac{Aa}{dd}$, $a_w = \frac{Aw}{dd}$, $R = \frac{R'}{dd}$, and $A = \frac{A'}{dd}$

In a practical soil only σ , U and possibly P_a can be measured. In all except the very dryest soils there are no air-mineral contacts. This is due to the surface activity of the soil particles which will absorb water onto their surfaces in anything of humid atmosphere. We may therefore neglect the term involving P_a. Tests performed by Rendulic (1937) Henkel (1959,1960) and Bishop and Eldin (1950) give ample proof that a_m is negligible in a fully saturated soil. There appears to be no reason why this should not also be true for a partly saturated soil. Using Lambe's model one must therefore deduce that $a_w = 1$, i.e. it would appear that Lambe has not treated the unsaturated case since he considers the gas phase to be fully occulted. The unsaturated case follows simply by altering the model to represent a continuous gas phase. Clearly a_w is then less than unity and $1-a_w$ is the area of cross section of the air phase per unit total cross sectional area.

As was discussed in section 1.6 the terms $\bar{\bar{\sigma}}.a_m$, R and A cannot be evaluated quantitatively. They may, however, be grouped together and their resultant considered as an interparticle stress. Equation (3.12) may therefore be written

$$\sigma = \bar{\bar{\sigma}} + U.a_w$$

$$\text{or} \quad \bar{\bar{\sigma}} = \sigma - U.a_w \quad \ldots\ldots\ldots\ldots \quad (3.13)$$

An examination of this equation with respect to Lambe's model shows that U is the pressure in the pore water, a_w is a statistical factor relating areas of water to the total area of cross section and σ is the applied stress. Equation (3.13) is therefore identical to equation (3.2) with $\bar{\bar{\sigma}} = \sigma'$ and $a_w = \beta$.

3.9 The definition of pore pressure.

A study of the work described in this chapter reveals two approaches to the definition of pore pressure. The one school represented by Aitchison and Jennings considered pore pressure as a stress (equivalent or otherwise) acting over the whole area of any cross section through the soil. The other school, that of Bishop and Croney, consider pore pressure as referring to the pressure in the pore fluid, either air or water.

In a written discussion to a recent conference on Pore Pressure and Suction in soils Aitchison and Bishop attempted to reconcile the two approaches by proposing the following definitions :

Pore Pressure : The term u should only be used to define the pore-fluid pressure in saturated soils. The term should not be applied to non-saturated soils.

Pore-water Pressure : In non-saturated soils the pressure in the liquid phase of the pore fluid should be denoted by u_w.

Pore-air pressure : In non-saturated soils the pressure in the gaseous phase of the pore fluid should be denoted by the term u_a.

The definition of pore pressure given above is not very satisfactory when it is considered in relation to effective stress .

The foundation engineer is primarily concerned with the prediction of soil behaviour. As was mentioned in the introduction to this thesis the principle of effective stress has proved invaluable to the understanding of saturated soil behaviour. The extension of the principle to partly saturated soils is obviously of the greatest importance but in so doing consistency between the two cases should be maintained as far as possible.

The effective stress in a soil has always been considered as being made up of two components. One component results from the total normal pressures acting on the soil and the other component results from pressures exerted by the fluids existing within the pores of the soil. In fully saturated soils the component of effective stress resulting from the pressure of the fluid within the pores has been called the pore pressure.

In unsaturated soils the pores of the soil are filled with two fluids; air and water. The pressure in the air is often very different from the pressure in the water. Efforts to evaluate the contribution of these pressures to the effective stress has resulted in the numerous effective stress equations described in this chapter. In order to remain consistent with the traditional concept of effective stress it seems logical that the component of effective stress resulting from the pressures in the pore fluids should be called the pore pressure.

By using the term 'pore pressure' in the way suggested above the engineer can treat effective stresses in soils which change from unsaturated to saturated conditions and vice versa without having to alter terminology. It is conceivable that at some future time the contribution made to the effective stress in a partly saturated soil by the pressures exerted by the fluids in the pores will be capable of direct measurement. In fact an approximate method for doing this is outlined in a later section. If Aitchison's and Bishop's definition is accepted a new term must be found for defining this quantity. If however, pore pressure is considered as a component of effective stress no difficulty will arise.

In order to facilitate the use of effective stresses in both saturated and partly saturated soils it is suggested that the definition of pore pressure should be :

'Pore pressure is that component of effective

stress in a soil which is derived from pressures exerted by the fluids within the pores of the soil and is denoted by u. If the pore pressure is negative with respect to atmospheric pressure it may be written as $\bar{u}$.'

The above definition together with the definitions of pore-water pressure and pore-air pressure as given by Aitchison and Bishop will be used in this thesis, however the symbols p_a and p_w are preferred to the symbols u_a and u_w used by Aitchison and Bishop. The term 'pressure deficiency' will still be retained and is equal to the difference between the pore-air pressure and the pore-water pressure.

$$\text{i.e.} \qquad p'' = p_a - p_w$$

3.10 The definition of effective stress in partly saturated soils

The recent conference on Pore Pressure and Suction in Soils brought together most of the various approaches to effective stress discussed in this chapter. It can be seen that a substantial measure of agreement exists between the various expressions defining effective stress in partly saturated soils although the notation used to express the various parameters vary greatly. In order to introduce some measure of consistency Aitchison and Bishop (1960) proposed that the effective stress in a partly saturated soil should be defined by the equation

$$\sigma' = \sigma + \chi (u_w - u_a) - u_a$$

This equation can be re-written using the notation adopted in this thesis as

$$\sigma' = \sigma + \chi (p_w - p_a) - p_a \qquad (3.14)$$

The pore pressure is given by

$$-u = \bar{u} = \chi\ (p_w - p_a) - p_a$$

$$= \chi\ .p'' - p_a$$

and if the pore-water pressure is measured with respect to p_a the above expression becomes simply

$$\bar{u} = \chi\ .p''.$$

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Chapter IV

AN EXAMINATION OF THE VALIDITY OF THE EFFECTIVE STRESS PRINCIPLE IN PARTLY SATURATED SOILS.

4.1 Introduction

In Chapter III some of the better known work on partly saturated soils was discussed. The conclusions reached was that the effective stress in a partly saturated soil can be given by the equation

$$\sigma' = \sigma + \chi (p_w - p_a) - p_a \quad \ldots\ldots\ldots \quad (4.1)$$

Which reduces to

$$\sigma' = \sigma + \chi . p'' \quad \ldots\ldots\ldots\ldots \quad (4.2)$$

if the pore-water pressure and total stress are measured with respect to p_a.

At this stage it is necessary to examine what is meant by the principle of effective stress when applied to a partly saturated soil. In chapter I the two most obvious implications of the principle of effective stress for saturated soils were shown to be

(i) That volume changes and deformations in a soil depend only on the effective stress.

(ii) That the shear strength of a soil depends only on the effective stress on the plane of shearing.

The effective stress in a partly saturated soil is defined in equation (4.2) as the sum of the applied stress σ and the pore pressure $\bar{u} = \chi p''$. In order to remain consistent with the fully saturated case the principle of effective stress for a partly saturated soil implies that the volume change and shearing

characteristics of the soil are dependent purely on the effective stress as defined above, i.e. no matter how σ σ and χ .p" vary it is their sum which determines the soil behaviour. As was described in chapter I the two implications set out above have been verified experimentally for normally consolidated and over consolidated saturated soils. The question arises as to whether they can be assumed valid in partly saturated soils.

Aitchison and Donald (1956) assumed that the principle of effective stress was valid for a partly saturated soil when they suggested that the shear strength and the void ratio could be used as a measure of the effective stress. Jennings (1960) also assumed its validity when he outlined methods for measuring β . Croney et al (1958) stated that σ' in equation (4.2) was an equivalent stress which determines both the volume and the shear strength of a partly saturated soil. Bishop (1960) wrote :

'There appears to be no reason why the principle of effective stress should not be equally valid for negative pore pressures or suctions.'

Skempton (1960) defined σ' as the stress controlling changes in volume and shear in a soil and stated that it was reasonable to presume that for partially saturated soils the effective stress could be given by equation (4.1).

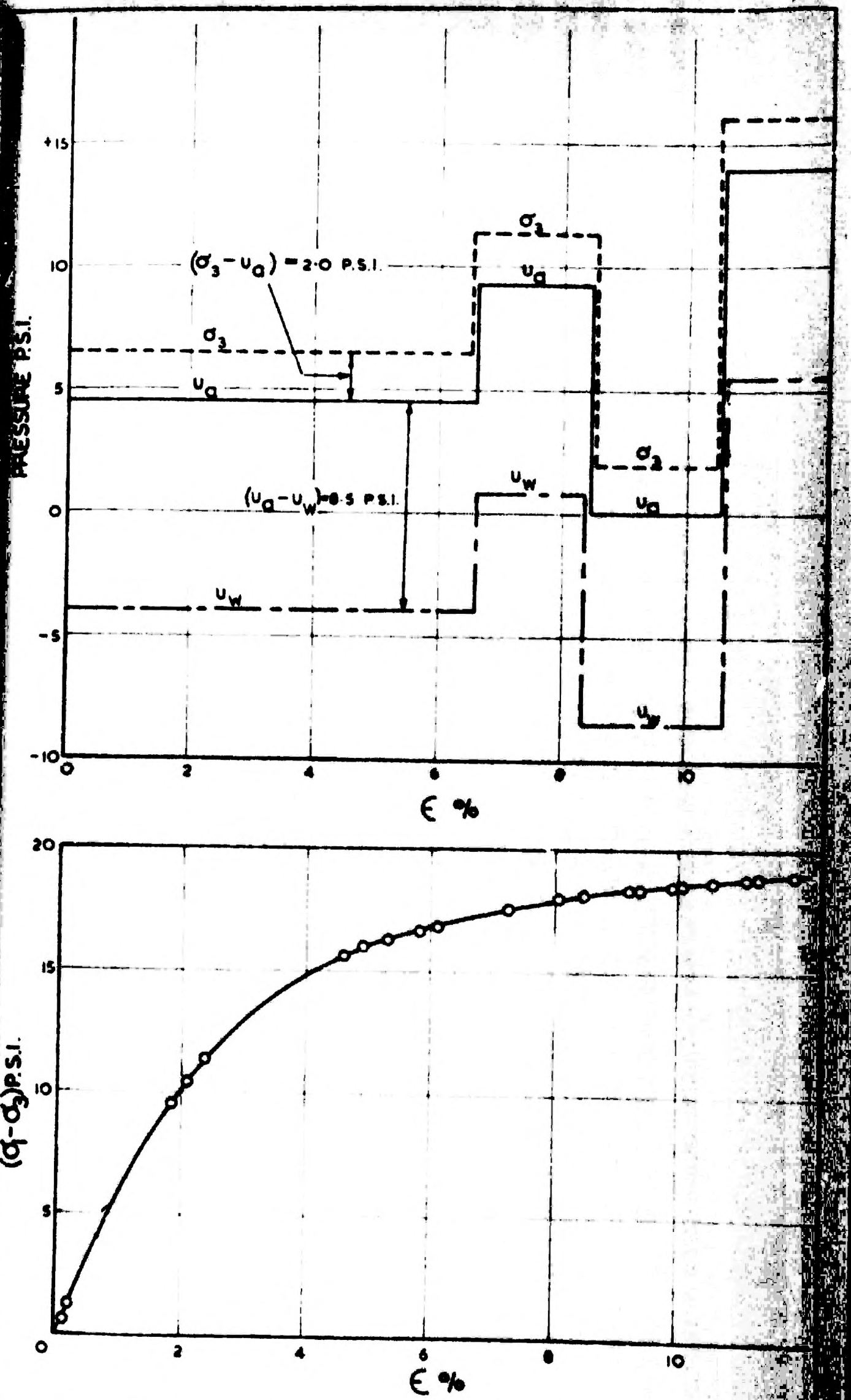
It is apparent that all the work on partly saturated soils has assumed the validity of the effective stress principle and has concerned itself with deriving a suitable expression for defining effective stress. It is important to note that the validity of equation (4.1) does not, of its own, validate the principle of effective stress (see section 1.2). Before the effective stress principle for a partly saturated soil can be taken as valid the volume change and shearing characteristics of the soil must be shown to depend purely on the effective stress.

Bishop and Donald (1961) carried out triaxial shear tests on a silt in order to check the validity of equation (4.1). In these tests the cell pressure, the pore-water pressure and the pore-air pressure could be varied, and measured, independently. In figure 4.0 is shown Bishop's and Donald's results of a drained test in which σ_3, p_a and p_w were all varied during shear in such a way that (σ_3-p_a) and (p_a-p_w) were kept constant. It can be seen that the shear strength is unaffected by these changes. Bishop and Donald deduced from this that the general form of equation (4.1) is correct.

The above test cannot be interpreted as proving the validity of the principle of effective stress in partly saturated soils. In section 3.6(vi) it was shown that (σ_3-p_a) is the resultant confining pressure acting on the sample and that (p_a-p_w) is the pressure deficiency p". The tests performed by Bishop and Donald show that p" is independent of the absolute value of air pressure and that the confining pressure depends only on σ_3 and p_a.

In order to show that the principle of effective stress is valid it is necessary to show that the soil behaviour is unaffected by changes in (σ_3-p_a) and $\chi\,(p_w-p_a)$ such that their sum $(= \sigma')$ is constant. Such a test would be very difficult to perform since it would require a knowledge of χ which, at present, can only be measured using methods which tacitly assume the validity of the effective stress principle.

In view of the difficulty just mentioned it is not possible to test the validity of the principle of effective stress in unsaturated soils rigorously. However there is a limited amount of experimental data available which can be used to assess, in a preliminary fashion, the validity of the principle. Some of this data will be analysed in this chapter.



DRAINED TEST ON PARTLY SATURATED LOOSE SILT IN WHICH σ_3, u_a AND u_w ARE VARIED SO AS TO KEEP $(\sigma_3 - u_a)$ AND $(u_a - u_w)$ CONSTANT

AFTER BISHOP AND DONALD (1961)

4.2 An examination of experimental data obtained by Aitchison

Aitchison and Donald (1956) performed some shear and consolidation tests on a limited range of unsaturated soils. These tests were described in the previous chapter. They found that in the case of incompressible sands the principle of effective stress is quite valid. Similarly, for very compressible clays they were able to show that the principle is valid over the range of pressure deficiency pF0 to pF5.

As was mentioned previously the sand used in the tests was of very uniform grain size and hence tended to take up a very stable pack. In practice such uniformity is only likely to be found in fairly coarse granular materials. In such cases the effective stresses resulting from pressure deficiencies are negligible by comparison with externally applied loads. In granular soils in which the grains are very much smaller the effects of grain size gradation are more pronounced and the material is usually fairly compressible. Such soils are of interest to the engineer as they are very common and the component of effective stress due to p" can become quite large. These soils have not been treated by Aitchison and Donald and since they are compressible their geometric methods cannot be used. These soils fall in the range between a silty sand and a clayey silt, i.e. the major proportion of soils likely to be encountered by the engineer.

The clays used by Aitchison and Donald were very compressible and all of them had very high -2μ fractions (65%-85%). Such soils are encountered by the engineer but not often. It is apparent that Aitchison's and Donald's conclusions regarding the validity of the principle of effective stress in unsaturated soils are based on tests which covered a very limited range of practical soils.

Aitchison and Donald performed one test on a silty sand. They found that the material was fairly

compressible when subjected to applied loads but very incompressible when subjected to pressure deficiencies.

There are only two conclusions that can be drawn from such a result :

(i) The effective stress resulting from an increase of pressure deficiency remains approximately constant once the soil has become unsaturated. Such a result is quite possible in a uniform coarse granular material but is most unlikely in a well graded silty sand.

(ii) Stress application due to an increase in applied pressure results in structural changes which differ from those due to an increase in pressure deficiency. This conclusion is in conflict with the principle of effective stress which implies that the behaviour of the soil is independent of whether effective stresses result from applied loads or pressure deficiencies.

4.3 The phenomenon of collapse of structure on wetting.

The phenomenon of collapsing sands has recently received a great deal of attention in South Africa (Jennings and Knight (1957)). The problem is one of rapid change of volume of a loaded soil when wetted. This behaviour will be considered in relation to the principle of effective stress.

An element of soil in its natural condition is acted on by a vertic pressure σ_{on} due to the weight of soil above it and a pore pressure $\bar{u}_{on}$, which results from a pressure deficiency p''_{on}. The effective stress in the soil in its natural condition is therefore given by

$$\sigma'_{on} = \sigma_{on} + \bar{u}_{on} = \sigma_{on} + \chi_{on} p''_{on}$$

The void ratio corresponding to this effective stress is e_{on} and the soil in its natural condition may be represented by point A on a void ratio: log effective stress plot (see fig. 4.1.)

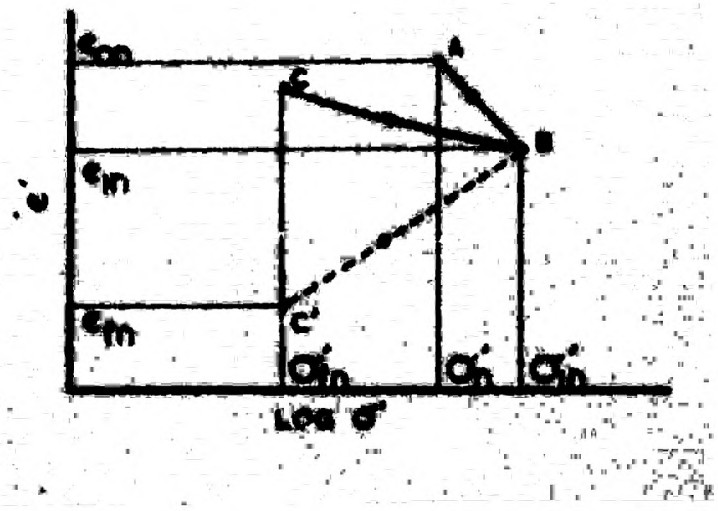


Fig. 4.1

Relationship between void ratio and effective pressure in a collapsing soil.

A pressure increment $\Delta\sigma$, resulting from a surface load, is now applied to the soil. The new effective stress is

$$\sigma'_{1n} = \sigma_{on} + \chi_{1n} \cdot p''_{1n} + \Delta\sigma$$

where χ and p" have changed a small amount because of the change of applied pressure $\Delta\sigma$. The void ratio corresponding to this new effective stress is e_1 and the soil may now be represented by point B in figure 4.1.

If the soil is saturated under the new condition of applied load $\chi_{1n} \cdot p''_{1n}$ will become zero, since $\sigma \longrightarrow 1$ and $p'' \longrightarrow o$ at saturation. The effective stress therefore becomes

$$\sigma'_{fn} = \sigma_{on} + \Delta\sigma$$

where σ'_{fn} is less than σ'_{1n}.

If the soil were to behave in accordance with principle of effective stress it would be expected bound along a curve BC after saturation had occurred. This is because a reduction in effective stress is associated with an increase in volume. However it has been observed that, far from expanding, soil actually undergoes a large reduction in volume. Instead of swelling along a curve BC it collapses to a point C'. Such behaviour is not consistent with the principle of effective stress since volume changes due to reduction in negative pore pressure are very different from those resulting from an equivalent reduction in applied pressure.

A method of testing the validity of the principle of effective stress, by means of the double oedometer test.

The phenomenon of heave can be attributed to the ingress of moisture by an initially unsaturated clay. In 1961 Jennings and Knight put forward a modified consolidation theory whereby the prediction of total heave can be made using the 'double oedometer test'. This test consists of running concurrently two conventional oedometer tests on samples of the same soil. The first sample is tested at its natural moisture content while the other sample is tested in a soaked condition. Since the test was proposed a large amount of experimental data has been collected. This data takes the form of natural moisture content and soaked void ratio/applied pressure curves for numerous partly saturated soils. A section dealing with the validity of the effective stress principle in partly saturated soils would hardly be complete without at least a brief study of such a valuable source.

In figure 4.2 are shown some typical e: log p curves obtained from a double oedometer test on an expansive soil. In general the initial void ratios of the two samples are not the same. This is due to variations in the natural soil and difficulties in measuring the initial void ratios accurately.

Jennings and Knight have confirmed observations by Taylor and others that despite these initial variations adjacent soil samples tested under identical conditions give curves which, though displaced, are of the same form. i.e. If the curves are transferred so that their initial void ratios coincide they are then approximately coincident.

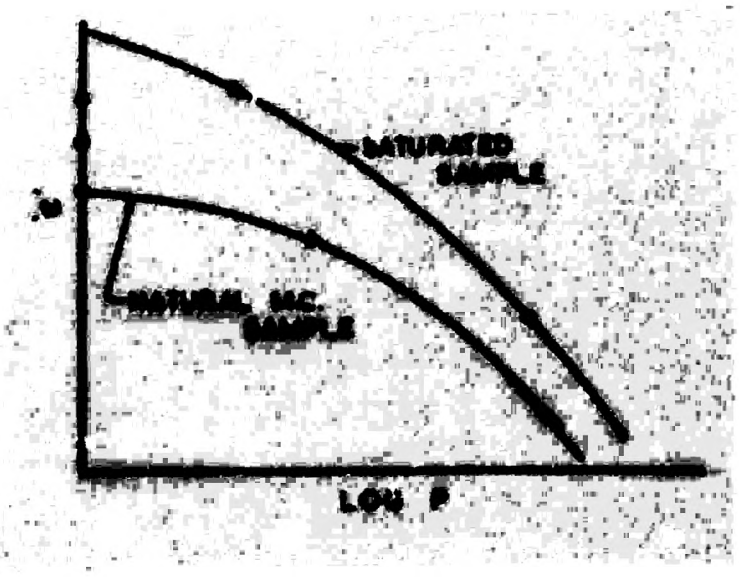


Fig. 4.2

Typical double oedometer curves for an expansive clay.

Classical consolidation practice has shown that a preconsolidated soil will return to a virgin compression curve, which is unique for that soil, when the effective stresses are re-applied. Jennings and Knight found that, in spite of using the mean of the void ratios of the two samples as the origin, the curves frequently do not coincide over the straight line portion. This has been attributed to local variations in initial moisture contents affecting the swell of the saturated sample. The curves are therefore adjusted to bring the straight line portions coincident. Undoubtedly local variations in the initial moisture content will have some influence on the relative positions of the two curves. However it is also felt that the phenomenon might be attributed to a change of soil structure resulting from the saturation of the soil. This effect would be similar to the collapse phenomenon encountered in collapsing sands but any tendency to

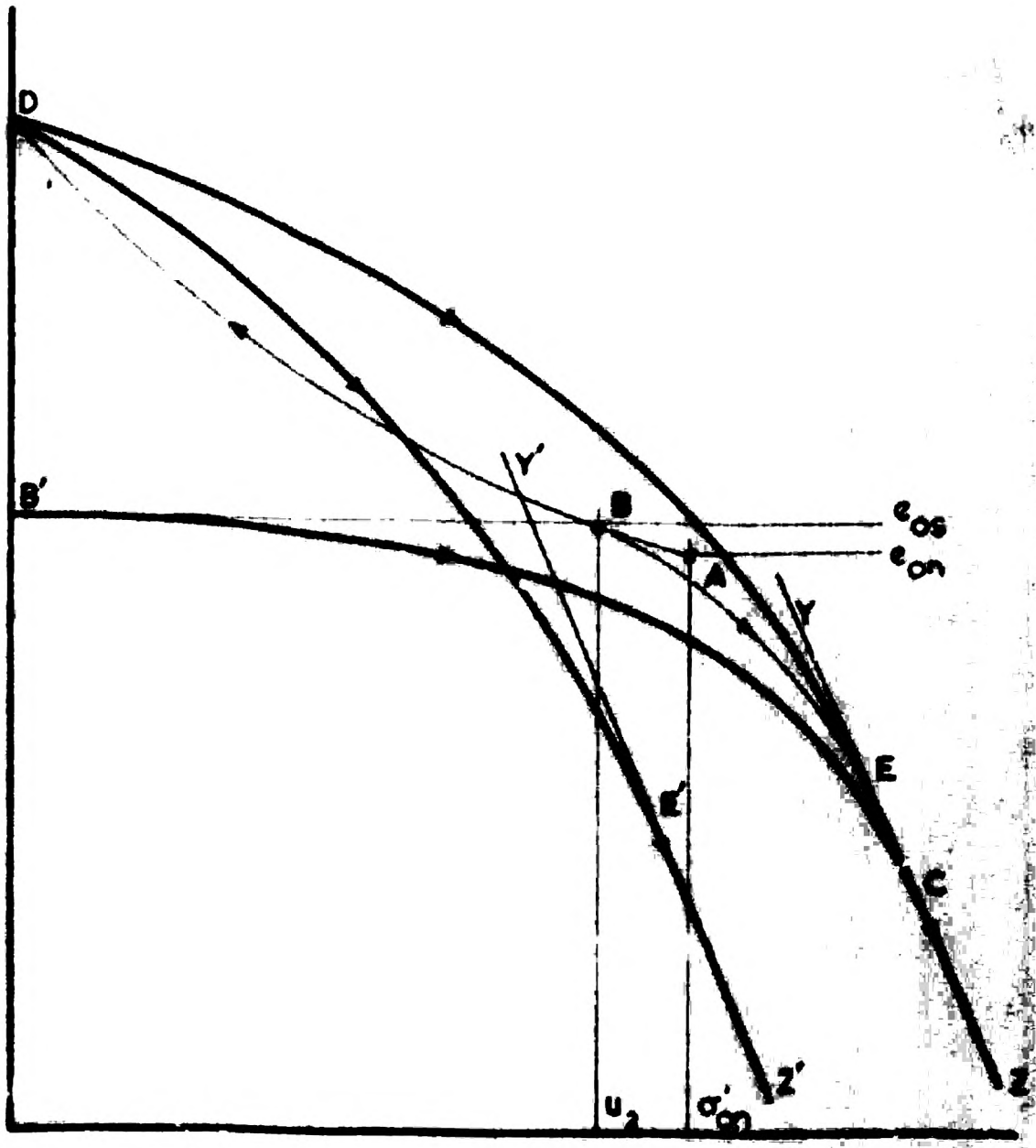
collapse would be offset by the expansive characteristics of the soil.

Consider the problem in relation to effective pressures. An element of soil in its natural state having a void ratio e_{on} and acted on by an applied vertical pressure σ_{on} and a pore pressure $\bar{u}_{on}$ may be represented by point A on the void ratio : log pressure plot in figure 4.3. (Point A is not necessarily on the virgin compression curve).

At the instant of sampling the soil will rebound, without change in moisture content, to a point B which is close to point A. The new void ratio is e_{os}. Since the applied stress is zero the effective stress corresponding to this void ratio is equal to a new pore pressure $\bar{u}_2$. The position of point B cannot be found directly because the magnitude of $\bar{u}_2$ is not known. If, however, applied stresses are considered instead of effective stresses the soil may be represented by point b'.

If a sample of the soil is tested at natural moisture content in the oedometer it will consolidate along a curve BC where C lies on a straight line YZ. At point C the pore-water pressure is probably zero and compression beyond C involves expulsion of water. If the process is considered in terms of applied stress curve BC is stretched out to give curve B'C.

A sample of the same soil is now subjected to an oedometer test under soaked conditions. On soaking under a very small applied load the soil will rebound along curve BD to D. This curve is assumed to be in terms of effective stress and is unknown because of the indeterminate effective stress condition represented by point B. In terms of applied stresses the same rebound takes place along the line B'D. The same final point D is reached in both cases. If the soil obeys the effective stress principle re-application of the effective stress by means of applied pressures will bring the sample back to a point E on the straight



POSSIBLE RELATIONSHIPS BETWEEN APPLIED PRESSURE AND VOID RATIO FOR NATURAL MOISTURE CONTENT AND SOAKED SAMPLES IN THE DOUBLE OEDOMETER TEST.

line YZ. If, however, application of the effective stress, by means of applied loads only, causes changes in structure which are different from those induced by a combination of applied pressure and pore pressure the sample will consolidate along a line CE' to join up with a saturated virgin compression curve Y'Z' which is displaced from YZ. At points E and E' the void ratios and structures of the two samples might very well be identical but the changes in stress necessary to bring this about will be different depending on the mecharism of stress application.

The displacement between lines YZ and Y'Z' will give some measure of the validity of the effective stress principle as applied to the soil when it changes from natural moisture content to saturation. By examining a number of curves it should be possible to arrive at some conclusion regarding the trend of curve DE in relation to curve B'C. If the effective stress principle is valid it can be expected that the curve Y'Z' will, in general, be close to YZ. Slight variations in initial water contents might cause the two curves to be displaced a small amount. However, the effect will be random and over a number of samples will tend to cancel out, i.e. Y'Z' will tend to take up random positions a small distance on either side of YZ. If, however, saturation of the soil results in behaviour which is not in accordance with the effective stress principle Y'Z' will be displaced from YZ in a reasonably consistent manner.

4.5 The analysis of some double oedometer results.

A number of double oedometer results, obtained for Free State clay from three different sites, were analysed with the natural moisture content and soaked curves adjusted so that the initial void ratios were equal. For each set of curves the displacement of Y'Z' below YZ was measured in terms of change in void ratio. In table 4.1 the displacement between Y'Z' and YZ at various depths is given for the three profiles. A positive displacement indicates that YZ lies above Y'Z'.

Table 4.1

The displacement between natural moisture content and soaked 'virgin' consolidometer curves at various depths

Vereeniging			Coalbrook			Odendaalsrust		
Depth	Δe	S_r%	Depth	Δe	S_r%	Depth	Δe	S_r%
5'	-0.061	76	5'	-0.115	79	6'	-0.068	63
8'	0.000	93	8'	0.000	75	7'	-0.040	65
10'	-0.062	90	12'	0.000	97	11'	-0.032	89
12'	-0.020	94	14'	0.000	96	16'	-0.030	86
15'	0.000	98	21'	-0.035	93			
19'	0.000	101	32'	-0.045	87			

It can be seen that in all except one case (Vereeniging - 12' Y'Z' lies below or coincident with YZ. In general this displacement is more significant at shallow depths and low degrees of saturation.

In figures 4.4 to 4.6 are shown some typical double oedometer curves which were used in the above analysis. It can be observed that the natural moisture content curves for samples near the surface are initially flat and then bend down fairly sharply to join the straight line YZ. Lower down the profile, however, the initial slopes of the natural moisture content curves are quite steep and the curvatures are much less pronounced. This indicates that the soil near the surface has been heavily precompressed while at greater depths this precompression is much less. Obviously the precompression has not resulted from a previously existing overburden pressure as the lower layers are unaffected. The only other alternative is that the precompression resulted from high pressure deficiencies near the surface. It is over this range that the principle of effective stress appears to be invalid.

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