Evaluation of Market Risk Charge using Internal Model Approach

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DECLARATION

I, **Mvula Michael Malindi** declare that the research work reported in this dissertation is my own, except where otherwise indicated and acknowledged. It is submitted for the degree of Masters of Management in Finance and Investment as the University of the Witwatersrand, Johannesburg. This thesis has not, either in whole or in part, been submitted for a degree or diploma to any other universities.

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ABSTRACT

The revised valuation of capital requirement for market risk on non-securitizations in the trading book has become a critical field to model default events. The major reason for the process to take place is to reduce the variability in capital levels across banks and to address the flaws of the current measure. This paper provides an analysis of the impact of the new regulation framework on South Africa Equity Market across different sectors. The new measure is called the Expected Shortfall (CVar) which is the replacement of Value-at-Risk (Var) and Stressed Value-At-Risk (SVar) as suggested by Basel 2.5. The study is about the impact analysis of the new measure under Basel 4 against the current Basel 2.5 Var measure, the valuation of both metrics is done using Monte Carlo Simulation and Historical Simulation Approaches to evaluate which methodology is more efficient.

The results are as follows. First, Value-at-Risk (Var) underestimate the potential large losses of equity benchmarks with fat-tailed properties and Expected Shortfall proves to be the robust metric to capture tail risks in the distribution. Second, observation from the findings is Expected Shortfall (CVar) can also disregard tail events, unless the metric is calibrated from the stressful period. Third, Monte Carlo Approach proves to be more efficient than Historical Simulation Approach. The methodology is also easy to calibrate and economical. Last, portfolio optimisation reduces the potential loss from both measures as a result reduces the market risk capital charge.

Keywords: Market Risk, Basel 2.5, Basel 4, Value-at-Risk, Stressed Value-at-Risk, Expected Shortfall, Internal Model Approach, Monte Carlo Simulation Approach and Historical Simulation Approach.

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1 INTRODUCTION

Since the global financial credit crisis unfolded between 2007 and 2010, the importance of valuation of market risk capital charge has become of paramount importance in the banking sector. The crisis became severe and lasted longer which left big giants in the global financial sector bankrupt. Lehman Brothers is a classic example of one of the firms which were bailed out to avoid the global financial sector from collapsing.

The figure below shows that South Africa equity market was also impacted by the global financial crisis, the period from 18 July 2008 and 18 July 2009 shows big jumps in the market, which resulted in huge losses. However, there are consensus that even though we were impacted by the global financial crisis, conditions were not as bad as the Eurozone countries due to our robust exchange controls, interest rates policy and our National Credit Act.

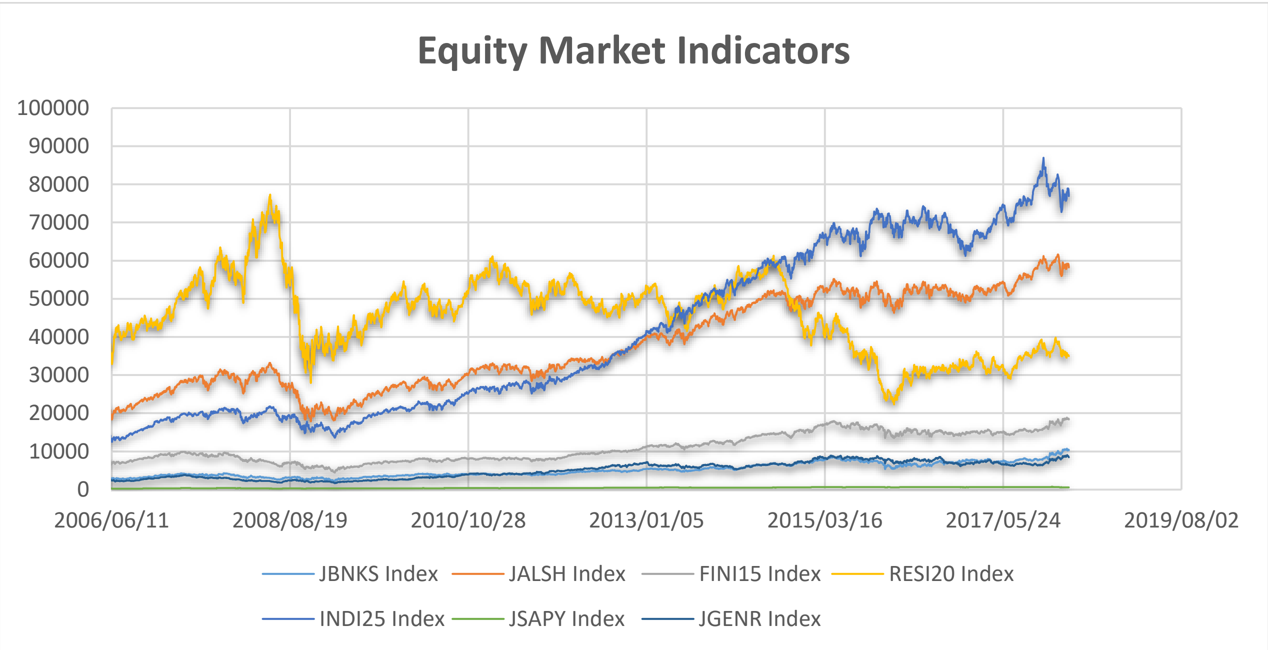

Figure 1: South African Equity Market Indicators

The global financial crisis just made an awareness that banks should be subjected to stricter regulations, which will eventually lead to a more conservative regulation framework. Uncertainties in the market encouraged Group of Ten (G10) countries to establish Basel Committee, the establishment was around 1974. The key function of the committee is simply to formulate broad supervisory standards, and it's up to each country to take ownership in terms of implementation provided they follow BCBS guidelines.

1.1 Problem Statement

The global financial sector needs a robust and sound economic regulation framework to allow a sustainable economic growth. The current measures used to calculate market risk capital charge has proven not to be robust, Studies have been conducted to prove that Value-At-Risk is not adequate to capture tail risk, which is discussed in detail on Section 1.5. The other issue is the maintenance cost associated with the Historical Simulation Approach, which is the widely used methodology in South African banks to calculate market risk capital charge. It's expensive to source historical data from vendors and the storage of a sizeable amount of data could be very expensive.

This research's focus is to evaluate the impact of the proposed market risk metric, which is the Expected Shortfall (CVar), the acronym is derived from the name, Conditional Value-At-Risk. This is the proposed market risk measure which is the replacement of Basel 2.5 market risk metrics. Most of South African Banks still use Value-At-Risk (Var) and Stressed Value-At-Risk (Var) as one of their inputs in market risk charge calculation.

After some analysis, recommendations will be made in terms of improvements on to be made on the internal model, the model that is easier and efficient will be suggested. The analysis will be done on South African equity indices across sectors under the current internal model approach and Monte Carlo Simulation Approach.

The new framework is still under testing phase for most of the banks and they want to be certain that they are not over/under capitalized. The Local regulator's plan was to implement the new framework on the 19 January 2018 and the date was extended to allow thorough testing to take place. Moreover, a lack of consistency in the model could lead on an inappropriate high capital levels for some business lines that will reduce liquidity and increase cost for the end users and, as a consequence, in this scenario some business will become uneconomic [8]. The revised capital standard for market risk under the new regulation framework focuses on the below, most critical areas:

- Revised internal models approach
- The revised boundary between trading and banking books
- Revised standardized approach (SA)

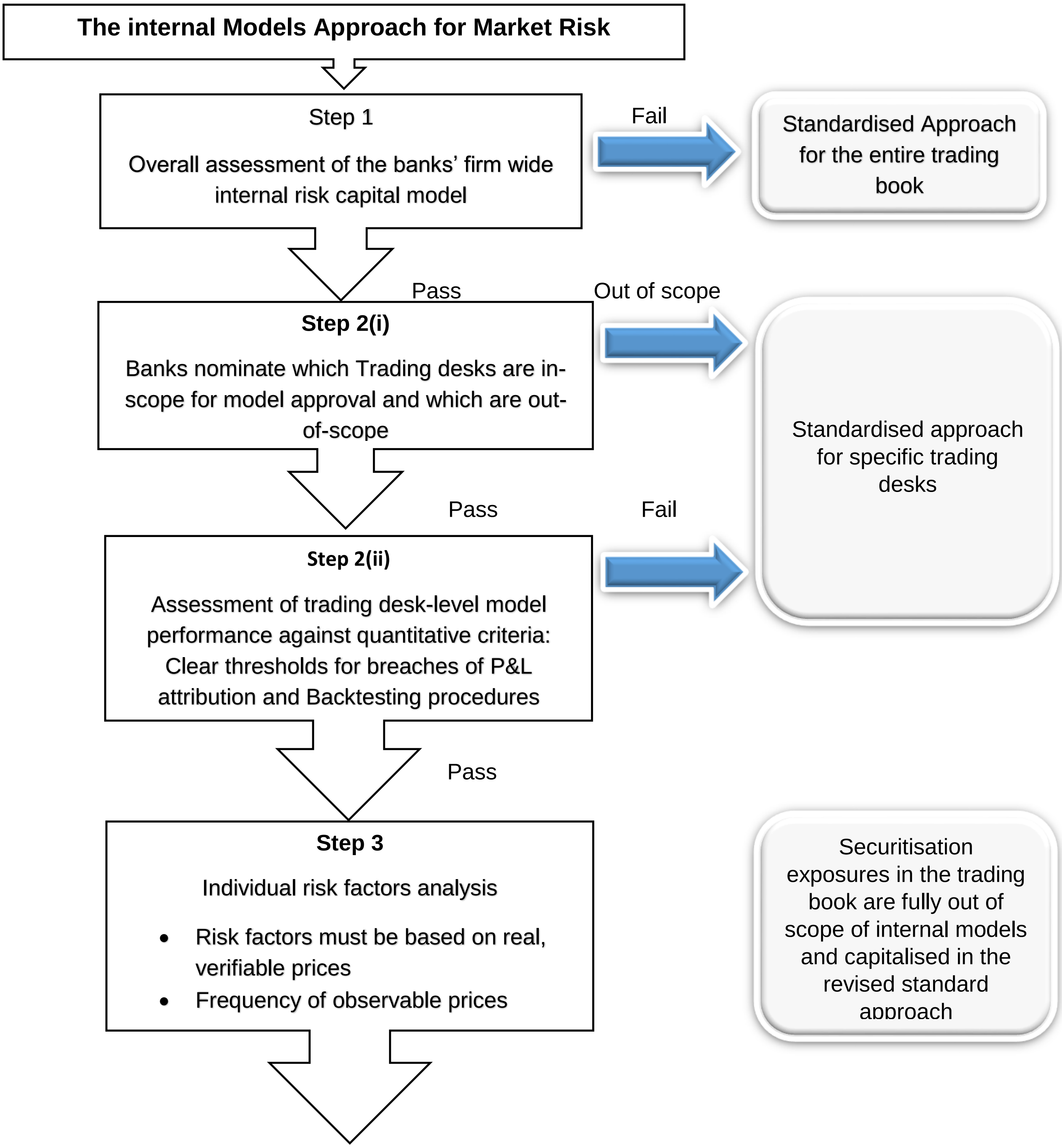

Figure 2(a): Capital Regulatory Frame – Flow chart [4]

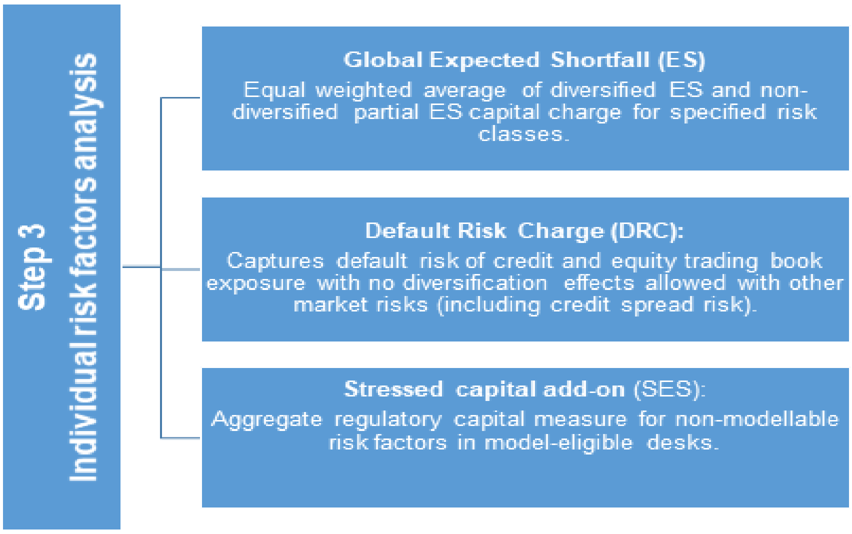

Figure 2(b): Capital Regulatory Frame – Flow chart [4]

Expected Shortfall (CVar) forms part of the inputs in the calculation of total capital charge under IMA approach. The other inputs like Default Risk Charge (DRC) don't form part of the scope for this research. The focus on this thesis is one segment of total capital charge which is the Expected shortfall (CVar), further studies can still be conducted on the other segments of total capital charge. Expected Shortfall (CVar) is a replacement of Value-At-Risk metric and Stressed Value-At-Risk metric under the new regulation framework.

There has been developments on Basel in the past years and this subsection discusses gaps and flaws which led to changes and developments on regulatory framework, the journey started from Basel I all the way to Basel 4. Basel 4 is in the pipeline, most banks in the emerging markets are still in the testing phase, evaluating the impact of the new market risk framework on the capital requirement, so the new framework is not yet adopted. The below sub-section takes us through the details of the developments of Basel.

1.1.1 Basel I

The global financial sector needs a robust and sound economic regulation to allow a sustainable economic growth. The existence of moral hazard and other market failures encouraged the international Basel Committee on Bank Supervision (BCBS) to formulate Base I Capital Accord in 1988 as the first major step to set a capital charged at the lowest level. Major market failures include a collapse of OPEC in early 1986 which led to a significant crude oil price decrease of more than 50% and in the same year Dow Jones Industrial Average (DJIA) dropped significantly by about 95 basis points in one day. The following year, 1987 a series of events unfolded which include Black Monday, when the equity market around the world crashed. It started with the Hong Kong market and spread to other markets. DJIA fell by about 508 basis points. In Australia and New Zealand, the crash is termed Black Tuesday due to time zone difference. However, the regulators kept a blind eye and focused on reducing credit risk. The regulation was also set for a minimum capital requirement and that was to benefit major financial institutions. The scope was limited to credit risk only with the exclusion of market and operational risk.

Market risk is defined as the losses that could be incurred given the position the bank is holding should we have severe price movements in the market.

Years later, more complex financial products and structures were introduced in the market due to innovations, and more players and financial institutions came into existence which changed the nature of financial risk and became problematic since that changed was factored in. The collapse of Barings in 1995 exposed the weakness of the regulation. The results led to a market risk amendment on Basel I which happened between 1995 and 1996. This incident revealed the importance of market risk that was out of the scope of the initial Basel proposal. The assessment of market risk in Basel included all instruments in the trading book including those that were out of balance sheet and was focused on internal models developed by own institutions in order to manage the risks that they had taken.

1.1.2 Basel II

This is the second attempt to try and amend the flaws of previous market risk frameworks. Basel I was written-off due to its inadequacy to its simplicity failed to accommodate complex financial structures. The Basel II commitment started with negotiations in 1999 and the Basel II accord was initially published in 2004 and the final draft was released in 2006. The intention of this review was to increase the capital charge to make sure banks have adequate capital for their daily exposures on landings, investments and trading activities.

The new regulation framework was built on three pillars: minimum capital for credit, market and operational risk, a supervisor process both internal as self-assessment and external that oversaw the perform of the firm, and disclosed information to let market discipline reward banks that carry out a sensible criteria management [3].

In terms of self-regulation, banks were given flexibility to build their own internal model approach and assess their own risk. Sophisticated banks with complex financial products had to decide on the methodology to capture their risk profile.

Then Basel II had to be reviewed again due to the shortcomings displayed by the 2008 financial crunch in the aftermath of Lehman Brothers collapse. The crisis revealed the inadequacy of the capital requirement framework for trading book. The lack of strict rules in terms of moving trades between books resulted in banks shifting open exposures that are held for speculation to banking books to avoid high capital charges, which in turn reduced financial buffers of capital for unforeseen circumstances. Banking books by definition should only house trades that are going to be held until expiries.

1.1.3 Basel 2.5

Basel 2.5 was meant to address burning issues that were still not addressed in Basel 2. The revision was done in 2009 on the framework as a remedial work that is how Basel 2.5 was formulated.

The key enhancements under the revised internal models approach is broadly categorised into the following:

Below is the critical refinements proposed:

(i) Replacement of Var and SVar with Expected Shortfall metric, which captures coherent and comprehensive risk.

(ii) The strict rule around the issue of portfolio diversification and hedging.

The Basel 2.5 requires calibrating Value-At-Risk (Var) under stress market conditions, known as SVar in order to alleviate the inadequate capital requirement established by the previous regulation.

1.1.4 Basel 4

At this point, regulatory background is clearly exposed as a process of continuous enhancements driven by shocks and financial crisis up to the following regulation reform that nowadays is being published, the FRTB, also known to be Basel 4. The revised capital standard for market risk is focused on key areas, namely: the revised internal models approach, the revised standardized approach and the revised boundary between trading book and banking book.

1.2 Purpose of Study

There are two key objectives of this study. Firstly, it provides the evaluation of Expected Shortfall and Value-At-Risk under HS Approach. The HS Approach is the commonly used methodology by South African banks.

The second part of this work is to employ Monte Carlo Simulation (MCS) Approach to evaluate which methodology between Historical Simulation (HS) Approach and Monte Carlo Simulation (MCS) Approach is more efficient.

Finally, the impact analysis is made on the new framework measure which is Expected Shortfall, and the recommendations will be made in terms of which methodology is more efficient and easy to implement.

1.3 Questions of Study

The research focuses on answering questions concerning the new regulation frame-work:

i. Why is the evaluation of Expected shortfall critical?
ii. What are the current methodologies to value the Expected shortfall, their advantages and drawbacks?
iii. What basic components we need to know about Expected Shortfall measure?
iv. Is the Expected Shortfall easy and efficient measure?

1.4 Significance of Study

The current market risk framework under Basel 2.5 failed to save the global financial sector, and refinement had to be done to address flaws of Value-At-Risk measure and issues around diversification. The impact analysis on the current metrics and Expected Shortfall will assist in terms of assessments. And this study will reveal which methodology is more efficient between Historical Simulation Approach and Monte Carlo Simulation Approach.

1.5 Background Literature

To achieve research key objectives a clear understanding of new capital requirement framework under Basel 4 is needed, and methodologies used in the market to evaluate the market risk charge. This thesis examines available literatures on market risk charge and literatures on methodologies used in the market. The knowledge thereafter assists in making recommendations as to which methodology is simple and easy to use for market risk capital charge.

Studies conducted in previous papers have immensely criticized Var measure concerning its ability to measure severe market conditions. Ohran and Karaahmet (2009) found out that Var performs correctly when the economy experiment smoothly moves, but fails during times of economic stress, because Var has the feature of being insensitive to the size of loss beyond the pre-specified threshold. Var measure has received generalized criticism due to its drawback that is consequence of paying attention just in a certain percentile.

Artzner, Delbaen, Eber and Heath (1999; 1997) stand out the lack of sub-additivity as the most notable flaw of this measure, whereas Pflug (2000) point out that Expected Shortfall does not have this inconvenient property by dint of looking at losses beyond Var. This is the main reason why previously most of Value-At-Risk measures of Basel 2.5 are changed and replaced by Expected Shortfall measures under Basel 4.

1.6 Outline of Study

The research comprises of five chapters, we start with the introduction. Chapter 2 focuses on literature review. Expected Shortfall and the description of methodologies used to calibrate the metric. Then weakness of Value-At-Risk are discussed which give us an opportunity to speak of the Expected Shortfall. Then a discussion around two commonly used techniques by South African banks and their applications.

Data and methodology form part of chapter three, we get to know the source of our data, how the portfolio was setup, and finally speaks of the methodologies around the calculation of Expected Shortfall. Chapter four focuses on the results and empirical analysis which is the most critical part of this work.

In chapter five, which is the last part of this research we conclude with discussions around results, findings, recommendations and limitations of the study and areas for further research.

2 LITERATURE REVIEW

The literature is about Expected Shortfall which is the new market risk framework. In this section we start by defining the metric and discuss the flaws of the current market risk measure.

2.1 Definition of Expected Shortfall

The use of a Global Expected Shortfall (CVar) is made up of equally weighted average of diversified Expected Shortfall and non-diversified partial Expected Shortfall capital charges for specified risk classes. Expected Shortfall should be calculated as the expected losses of a portfolio conditional to the fact that the loss is greater than the 97.5th percentile of the loss distribution. This risk measure must be computed on a daily basis and at a base liquidity horizon of ten days.

2.2 Why new market risk framework is needed?

Expected Shortfall is adopted to address the flaws of the current capital measure framework under Basel 2.5, which is Value-At-Risk.

2.2.1 Flaws of Value-At-Risk

The Value-At-Risk drawbacks of the Basel 2.5 reform is the weaknesses that comes with the metric. Value-At-Risk does not correctly capture huge market jumps and also creates wrong incentives. Particularly, fact is Value-At-Risk measure is not sub-additive which implies that the diversification is penalized. The Value-At-Risk metric used to capitalize trading book exposures is inadequate for capturing credit risk inherent in trading exposures. The Value-At-Risk framework just takes in consideration a percentile, without looking beyond the percentile, consequently this measure fails to capture tail risks.

2.3 How to Calibrate Expected Shortfall and Value-At-Risk?

In this subsection we firstly start by explaining a step by step process on how the returns are calculated under Historical Approach Simulation, and secondly, we look at how the returns are calibrated under Monte Carlo Simulation (MCS) Approach.

2.3.1 Historical Simulation (HS) Approach

Step 1: Use geometric rate of return on stock indices between T and T-1, going back to T-519

Step 2: Introduce logarithms to relative numbers to reduce the variability and to bring stationarity.

Step 3: Apply a hypothetical position of R1bn and use equal weights to get a profit & loss vector.

Step 4: lastly, apply percentiles to get Expected Shortfall and Value-At-Risk metrics

2.3.2 Monte Carlo Simulation (MCS) Approach

Step 1: Calibrate Mean and standard deviation from historical stock prices

Step 2: Use Geometric Brownian Motion (GBM) process to calculate daily returns, logarithms are already embedded in the GBM formula.

Step 4: Apply a hypothetical position of R1bn and use equal weights to get a profit & loss vector.

Step 5: lastly, apply percentiles to get Expected Shortfall, and Value-At-Risk metrics

3 DATA AND METHODOLOGY

3.1 Data

Historical data used is the daily price moves of South African indices. The price history is from 04 Feb 2016 to 31 Jan 2018, and the stress period data is from 18 July 2008 – 18 July 2009. The stress period is used to calculate Expected Shortfall as suggest by Basel committee. Analysis were performed to determine the most severe moves in the equity market in the past 10 years. The indices used are as follows: Banks Index, Financials 15 Index, Resources 20 Index, Industrial 25 Index, South African Listed Property Index, General Retailers Index, and the South African equity benchmark used is All Share Index. The source for our data is Bloomberg.

3.2 Portfolio Composition

A portfolio of South African Market indices is hypothetically constructed on this paper and the below is a list of all Equity Market indicators: Banks Index, Financial 15 Index, Resource 20 Index, Industrial 25 Index, South African Property Index and General Retailers Index. For the optimal asset allocation to minimise market risk which is our objective function, Treynor ratio is used to determine optimal weights that should be used. The next subsection deals with mathematical formulation of the portfolio analysis.

We start to calculate risk metrics with the below scenarios to do an impact analysis:

- **Scenario 1: Use equal weights**

$$w_i = \frac{1}{n} \quad \forall i$$

Suppose that i represents equity indices and n is the total size of the portfolio.

- **Scenario 2: Use optimised weights**

Let's start by making the following assumption, suppose the analysis is done on a Single Index Model

Capital Asset Pricing Model (CAPM)

$$\mathrm{R}_i = \alpha_i + \beta_i \mathrm{R_m} + \mathrm{e}_i \tag{1}$$

Let R_i represents the return of each index in the portfolio, α_i be the alpha, β_i be the beta, $\mathrm{R_m}$ is the return of market index, and e_i is the error term of each stock.

By assuming Ordinary Least Square (OLS), we then have the below Equations:

$$\mathrm{E}(e_i \mathrm{R_m}) = 0$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\mathrm{e}_i}^2 \tag{2}$$

The Co-Variance between asset i and asset j is:

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \tag{3}$$

We then incorporate the risk risk-free rate r_i to get the below Equation:

$$R_i - r_f Z_i \sigma_i^2 + \sum_{\substack{j=1 \\ i \neq j}}^{n} Z_j \sigma_{ij} \tag{4}$$

From OLS assumptions we now have,

$$R_i - r_f = Z_i(\beta_i^2 \sigma_m^2 + \sigma_{\mathrm{e_j}}^2) + \sum_{\substack{j=1 \\ i \neq j}}^{n} Z_j \beta_i \beta_j \sigma_m^2 \tag{5}$$

Incorporate $Z_i \beta_i^2 \sigma_m^2$ into the summation. Then solve for Z_i in Equation (5):

$$Z_i = \frac{R_i - R_f}{\sigma_{e_j}^2} - \frac{\beta_i^2 \sigma_m^2}{\sigma_{e_j}^2} \sum_{j=1}^{n} Z_j \beta_j$$

The generalised solution is as follows:

$$Z_i = \frac{\beta_i}{\sigma_{e_j}^2} \left[\frac{R_i - R_f}{\beta_i} - C^* \right] \tag{6}$$

Where, $C^* = \frac{\beta_i^2 \sigma_m^2}{\sigma_{e_j}^2} \sum_{j=1}^{n} Z_j \beta_j$, and with Treynor Ratio to be $\frac{R_i - R_f}{\beta_i}$

Therefore Z_i is the portfolio weight allocation for each index.

3.3 Methodology

3.3.1 Definition of Expected Shortfall

Expected Shortfall is calculated as the expected losses of a portfolio given the condition that the loss is greater than a certain percentile in the loss distribution. This risk measure is computed daily with a liquidity horizon of ten days.

3.3.2 Formulation of Value-At-Risk and Expected Shortfall

In this subsection we define Expected Shortfall (CVar) and Value-At-Risk (Var), let the randomness of the distribution be defined by L. We then assume a series of losses as follows as $L_{(1)}, L_{(2)}, \ldots \ldots, L_{(n)}$, further assumption is n-losses are independent and identically distributed observations.

$$F(z) = \Pr\{L \leq z\} \tag{7}$$

The CDF of L is then defined with the below function

$$F^{-1}(\gamma) = \inf\{F(z) \geq \gamma\} \tag{8}$$

Trindade et al. (2007) defines α-Var of *L* as

$$v_\alpha = F^{-1}(\alpha) \tag{9}$$

For any $\alpha \epsilon (0,1)$,

And defined α-CVar of loss as

$$c_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 v_\beta d\beta \tag{10}$$

Pflug (2000) showed that Equation (10) can be written as the below Stochastic program:

$$c_\alpha = \inf\left\{\mathrm{p} + \frac{1}{1-\alpha}\mathrm{E}[L-\mathrm{p}]^+\right\} \tag{11}$$

Where $\mathrm{E}[L-\mathrm{p}]^+ = \max\{0, \mathrm{E}[L-\mathrm{p}]^+\}$

Let's define P as the set of optimal solutions to Equation (10)

Trindade et al. (2007) shows that $P = [v_\alpha . u_\alpha]$

Where $u_\alpha = \sup\{\mathrm{t}: \mathrm{F}(\mathrm{p}) \leq \alpha\}$.

Therefore, Equation (11) can be written as

$$c_\alpha = v_\alpha + \frac{1}{1-\alpha}\mathrm{E}[L-\mathrm{p}]^+ \tag{12}$$

$$v_\alpha = u_\alpha$$

Where *L* has positive density in the neighbourhood of v_α, from the derivation it follows that equation (11) has unique solution and it can be written as follows

$$c_\alpha = \mathrm{E}[L|L \geq v_\alpha] \tag{13}$$

Where the right-hand side of Equation (7) is the expected shortfall.

3.3.3 Monte Carlo Estimation

Suppose that $\tilde{L}$ is a vector of n independent and identically distributed (i.i.d.) observed losses. Then, the α-Var of a vector $\tilde{L}$ is estimated as

$$\hat{v}_\alpha^n = L_{[n\alpha]:n}$$

Trindade et al. (2007) suggests the estimator to be

$$\hat{c}_\alpha^n = \inf\left\{\mathrm{p} + \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}[L_i - \mathrm{p}]^+\right\} \tag{14}$$

To estimate the α-CVar of the i-th of vector $\tilde{L}$. We let

$$F_n(z) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{L_i \le z\}}$$

To be our cumulative distribution function from vector $\tilde{L}$. Where $1_{\{L_i \le z\}}$ is the indicator function of our loss distribution.

Therefore equation (13) becomes

$$\hat{c}_\alpha^n = \inf\left\{\mathrm{p} + \frac{1}{(1-\alpha)}\sum_{i=1}^{n} E[\tilde{L} - \mathrm{p}]^+\right\}$$

It follows from Equation (9) that $\hat{v}_\alpha^n = F_n^{-1}(\alpha)$, then by Equation (11) becomes

$$\hat{c}_\alpha^n = \hat{v}_\alpha^n + \mathrm{p} + \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}[L_i - \hat{v}_\alpha^n]^+ \tag{15}$$

Therefore, Equation (15) can be used as an estimator c_α, since it's simple to use as opposed to a stochastic process.

3.3.4 Asymptotic Properties

Some detailed studies have been conducted, Bahadur started the study about consistency and the normality of Value-At-Risk (Var) and Expected Shortfall (CVar) estimators.

Serfling (1980) followed and Later Trindade et al. (2007) advanced on the topic.

Assumption 1 Suppose there exists $\varepsilon > 0$ such that *L* has a positive and continuously differentiable density $f(x)$ for any $x \in (v_\alpha - \varepsilon, v_\alpha - \varepsilon)$.

Assumption 1 requires that *L* has a positive and differentiable density in a neighborhood of v_α.

It then implies that $c_\alpha = E[L/L \geq v_\alpha]$ and $F(v_\alpha) = \alpha$

Bahadur representations of $\hat{v}_\alpha^n$and $\hat{c}_\alpha^n$ are summarized in the following theorem.

Theorem 1 For a fixed $\alpha\epsilon(0,1)$, suppose that Assumption 1 is satisfied. Then

$$\hat{v}_\alpha^n = v_\alpha - \frac{1}{f(v_\alpha)}\left((1-\alpha) - \frac{1}{n}\sum_{i=1}^{n} 1_{\{L_i \leq z\}}\right) + A_n$$

And

$$\hat{c}_\alpha^n = c_\alpha + \left(\frac{1}{n}\sum_{i=1}^{n}\left[v_\alpha + \frac{1}{1-\alpha}(L_i - v_\alpha)^+\right] - c_\alpha\right) + B_n$$

Where $A_n = O_{a.s.}\left(n^{-3/4}(\log n)^{3/4}\right)$, $B_n = O_{a.s.}(n^{-1}\log n)$,

And the statement $Y_n = O_{a.s.}(\mathrm{g}(n))$ means that $Y_n/\mathrm{g}(n)$ is bounded by a constant almost surely. Consistency and asymptotic normality of $\hat{v}_\alpha^n$ and $\hat{c}_\alpha^n$ follow straightforwardly from Theorem 1. Specifically, if Assumption 1 is satisfied, then $\hat{v}_\alpha^n \to v_\alpha$ and $\hat{c}_\alpha^n \to c_\alpha$ with probability 1 (w.p.1) as $n \to \infty$.

Furthermore, if $\mathrm{E}\left(L^2\right) < \infty$, then

$$\sqrt{n}(\hat{v}_\alpha^n - v_\alpha) \Rightarrow \frac{\sqrt{\alpha(1-\alpha)}}{f(v_\alpha)} N(0,1) \quad (16)$$

And

$$\sqrt{n}(\hat{c}_\alpha^n - c_\alpha) \Rightarrow \sigma_\infty . N(0,1) \quad (17)$$

As $n \to \infty$, where "⇒" denotes "converge in distribution", $N(0,1)$ represents the standard normal random Variable, and

$$\sigma_\infty^2 = \lim_{n\to\infty} n\mathrm{Var}(\hat{c}_\alpha^n) = \frac{1}{(1-\alpha)^2} . \mathrm{Var}([L - v_\alpha]^+)$$

Theorem 2 For a fixed $\alpha \epsilon (0,1)$, suppose that Assumptions 1 and 2 are satisfied. Then,

$$\hat{v}_\alpha^{n,IS} = v_\alpha - \frac{1}{f(v_\alpha)} \left((1-\alpha) - \frac{1}{n} \sum_{i=1}^{n} 1_{\{L_i \geq v_\alpha\}} S(L_i) \right) + C_n$$

, and

$$\hat{c}_\alpha^{n,IS} = c_\alpha + \left(\frac{1}{n} \sum_{i=1}^{n} \left[v_\alpha + \frac{1}{1-\alpha} (L_i - v_\alpha)^+ S(L_i) \right] - c_\alpha \right) + D_n$$

Where $C_n = O_{a.s.}(t(n,\delta))$, with $t(n,\delta) = \max\left\{ n^{-1+2/p+\delta}, n^{-3/4 + 1/(2p) + \delta} \right\}$,

$D_n = O_{a.s.}\left(n^{-1+2/p+\delta}\right)$ for any $\delta > 0$, $C_n = o_p\left(n^{-1/2}\right)$,

And $D_n = o_p\left(n^{-1/2}\right)$. Furthermore, if $S(x) \leq C$, $\forall x \in (v_\alpha - \varepsilon, \infty)$,

Then $C_n = O_{a.s.}\left(n^{-3/4} (\log n)^{3/4}\right)$ and $D_n = O_{a.s.}(n^{-1} \log n)$.

Asymptotic normality of the estimators follows immediately from Theorem 2.

$$\sqrt{n}\left(\hat{v}_{\alpha}^{n,IS} - v_{\alpha}\right) \Rightarrow \frac{\sqrt{Var_G\left[1_{\{L\geq v_\alpha\}}S(L)\right]}}{f(v_\alpha)} N(0,1)$$

And

$$\sqrt{n}\left(\hat{c}_{\alpha}^{n,IS} - c_{\alpha}\right) \Rightarrow \frac{\sqrt{Var_G[(L-v_\alpha)^{+}S(L)]}}{1-\alpha} N(0,1)$$

As $n \to \infty$. If $S(x) < 1\ \exists\forall x \geq v_\alpha$,

Then it can be easily verified that

$Var_G\left[1_{\{L\geq v_\alpha\}}S(L)\right] < \alpha(1-\alpha)$ and $Var_G[(L-v_\alpha)S(L)] < Var[(L-v_\alpha)^{+}]$.

Then compared to Equations (16) and (17), the asymptotic Variances of the IS estimators will be smaller than those of the estimators without IS, given a sufficient condition that $S(x) < 1$ $\exists\forall\ x \geq v_\alpha$.

3.3.5 Analytical Closed-Form Expressions

It is apparent that probability distribution function and Value-At-Risk are related, and this is for a fixed $\alpha \in (0,1)$,

$$F(v_\alpha(\theta), \theta) = \alpha \tag{18}$$

We then start by analysing probability sensitivity by letting

$$q_z(\theta) = Pr\{L(\theta \leq z)\}$$

Let's make the below assumptions:

- For any $\theta \in \Theta$, $L'(\theta)$ exists w.p.1 and there exists a random Variable K, which may depend on θ,

- For any $\theta \in \Theta, F(p, \theta)$ is continuously differentiable at (z, θ). This assumption is used in pathwise derivative estimation, see, e.g., Broadie and Glasserman (1996).

Theorem 3

Suppose the mentioned two assumptions are satisfied, we then have

$$q_z'(\theta) = -\partial_z E\left[L'(\theta).1_{\{L(\theta)\leq z\}}\right]$$

Where ∂_z is the partial derivative w.r.t z. Furthermore, if $E[L'(\theta)|L(\theta) = p]$is continuous at p = z, then

$$p_z'(\theta) = -f(z,\theta)E\left[L^{'}(\theta)|L(\theta) = z\right]$$

We have now have

$$\partial_z E\big(v_\alpha(\theta),\theta\big)v_\alpha'(\theta) + \partial_\theta F(a,\theta)|_{a=v_\alpha(\theta)} = 0$$

Noting that $\partial_z E\big(v_\alpha(\theta),\theta\big) = f\big(v_\alpha(\theta),\theta\big)$ and $\partial_\theta F(a,\theta) = q_a'(\theta)$

We immediately have

$$v_\alpha'(\theta) = \frac{1}{f(v_\alpha(\theta),\theta)} q_a'(\theta)|_{a=v_\alpha(\theta)} = E[L'(\theta)|L(\theta) = v_\alpha(\theta)]$$

We then conclude by saying, Value-At-Risk can be written as a conditional-expectation.

Theorem 4

Let $z = v_\alpha(\theta)$, $E[L'(\theta)|L(\theta) = p]$ to be continuous at $p = v_\alpha(\theta)$

Therefore,

$$v_\alpha'(\theta) = E[L'(\theta)|L(\theta) = v_\alpha(\theta)]$$

The derivation of analytical closed from for Expected Shortfall from Var sensitivity, we should use:

$$c_\alpha(\theta) = v_\alpha(\theta) + \frac{1}{1-\alpha}\mathrm{E}[L(\theta) - v_\alpha(\theta)]^+$$

Theorem 5

If assumptions made are satisfied at $z = v_\alpha(\theta)$, and $v_\alpha(\theta)$ is then differentiable for any $\theta \in \Theta$. Then for any $\theta \in \Theta$,

$$v'_\alpha(\theta) = \mathrm{E}[L'(\theta)|L(\theta) \geq v_\alpha(\theta)]$$

$$v'_\alpha(\theta) = \mathrm{E}[L'(\theta)|L(\theta) \geq v_\alpha(\theta)]$$

$$c_\alpha(\theta) = \mathrm{E}[L(\theta)|L(\theta) \geq v_\alpha(\theta)] = \frac{1}{1-\alpha}\mathrm{E}\left[L(\theta).\,1_{\{L(\theta)\geq v_{\alpha(\theta)}}\right]$$

$$= \frac{1}{1-\alpha}\mathrm{E}\{\left[L(\theta) - v_\alpha(\theta)\right].1_{\{L(\theta)-v_{\alpha(\theta)}\geq 0\}}\} + v_\alpha(\theta) \quad (19)$$

For any $\theta^* \in \Theta$, because $v_\alpha(\theta)$ is differentiable at θ^*, there exist a neighbourhood of θ^*, denoted as $(a_{\theta^*}, b_{\theta^*})$, and a constant κ_{θ^*} such that $\theta^* \in (a_{\theta^*}, b_{\theta^*}), (a_{\theta^*}, b_{\theta^*}) \subset \Theta$, and

$|v_\alpha(\theta_1) - v_\alpha(\theta_2)| \leq \kappa_{\theta^*}|\theta_1 - \theta_2|$ for all $\theta_1, \theta_2 \in (a_{\theta^*}, b_{\theta^*})$.

Then by assumption,

$$|[L(\theta_2) - v_\alpha(\theta_2)] - [L(\theta_1) - v_\alpha(\theta_1)]| \leq (K + k_{\theta^*}).\,|\theta_2 - \theta_1|$$

And $\mathrm{E}(K + \kappa_{\theta^*}) = \mathrm{E}(K) + \kappa_{\theta^*} < \infty$.

$$\frac{d}{d\theta}\mathrm{E}\left\{L(\theta) - v_\alpha(\theta)].1_{\{L(\theta)-v_{\alpha(\theta)}\geq 0\}}\right\} = \frac{d}{d\theta}\mathrm{E}\left[f\left(L(\theta) - v_\alpha(\theta)\right)\right]$$

$$= \mathrm{E}\left\{L'(\theta) - v'_\alpha(\theta)].1_{\{L(\theta)-v_{\alpha(\theta)}\geq 0\}}\right\}$$

Then by Equation (12), at every n $\theta \in \left(a_{\theta^*}, b_{\theta^*}\right)$

$$c'_\alpha(\theta) = \frac{1}{1-\alpha}\mathrm{E}\left\{L'(\theta) - v'_\alpha(\theta)].1_{\{L(\theta)-v_{\alpha(\theta)} \geq 0\}}\right\} + v'_\alpha(\theta)$$

$$= \frac{1}{1-\alpha}\mathrm{E}\left[L'(\theta).\,1_{\{L(\theta)-v_{\alpha(\theta)} \geq 0\}}\right] = \mathrm{E}[L'(\theta)|L(\theta) \geq v_\alpha(\theta)]$$

Since $\theta \in (a_{\theta^*}, b_{\theta^*})$

Then

$$c'_\alpha(\theta^*) = \mathrm{E}[L'(\theta^*)|L(\theta^*) \geq v_\alpha(\theta^*)]$$

This concludes the proof of the theorem.

4 RESULTS AND EMPIRICAL ANALYSIS

In this chapter an empirical application of the proposed models is exhibited. The first part of Chapter 4, we compare Value-At-Risk and Expected Shortfall using normal market conditions Feb 2016 to 31 Jan 2018. The next step we calculate Stress Value-At-Risk and 99.75% Var on a stress period (18 July 2008-18 July 2009). We then, evaluate the impact of Expected Shortfall on Market Risk Capital charge under Historical Approach. Secondly, we repeat the same process under Monte Carlo (MC) Simulation Approach. And lastly, we do an evaluation on the impact if portfolio weights are optimised on both methodologies.

4.1 Historical Simulation Approach Results

As explained in Section 2.3, how Expected Shortfall and Value-At-Risk are calculated. The tables (Table 1 and Table 2) show the output from Historical Simulation for Expected shortfall metric and the numbers have doubled Value-At-Risk numbers. This is as expected due Expected Shortfall tracking tail events.

	1-day Value-At-Risk		
	95% Confidence Interval	99% Confidence Interval	99.9% Confidence Interval
JBNKS Index	- 4 378 431.72	- 6 792 257.92	- 9 632 826.02
FINI15 Index	- 3 144 221.06	- 5 191 894.20	- 9 273 420.21
RESI20 Index	- 4 370 776.81	- 6 292 633.20	- 7 728 767.86
INDI25 Index	- 2 564 182.10	- 3 903 212.14	- 5 428 373.68
JSAPY Index	- 2 292 054.07	- 3 867 021.58	- 6 520 472.98
JGENR Index	- 3 723 160.96	- 6 367 779.00	- 9 729 863.14
Portfolio	**- 15 040 564.32**	**- 23 930 261.25**	**- 37 242 002.45**

Table 1: Value-At-Risk: Historical Approach

The calibration of Expected shortfall outputs are moderate on Table 2, slightly higher which means South African equity market was stable around February 2016 to 31 January 2018 which is evident from Figure 1.

1-day Expected Shortfall						
	95% Confidence Interval		99% Confidence Interval		99.9% Confidence Interval	
JBNKS Index	-	6 002 947.02	-	11 447 400.88	-	13 905 997.53
FINI15 Index	-	4 652 976.14	-	10 016 512.45	-	12 976 223.08
RESI20 Index	-	5 689 565.86	-	9 911 872.98	-	11 293 049.75
INDI25 Index	-	3 549 027.37	-	6 551 401.82	-	8 013 376.08
JSAPY Index	-	3 549 693.47	-	7 314 478.59	-	10 328 209.82
JGENR Index	-	5 779 597.03	-	11 518 394.37	-	12 536 912.26
Portfolio	**-**	**21 987 037.09**	**-**	**42 231 610.83**	**-**	**47 399 185.87**

Table 2: Expected Shortfall: Historical Approach

South Africa Equity Indices captured the global financial crisis as shown in the below figure, the stress period is 2008/07/18 to 2009/07/18 as shown on Figure 2. This is the most severe market conditions in the last 10 years.

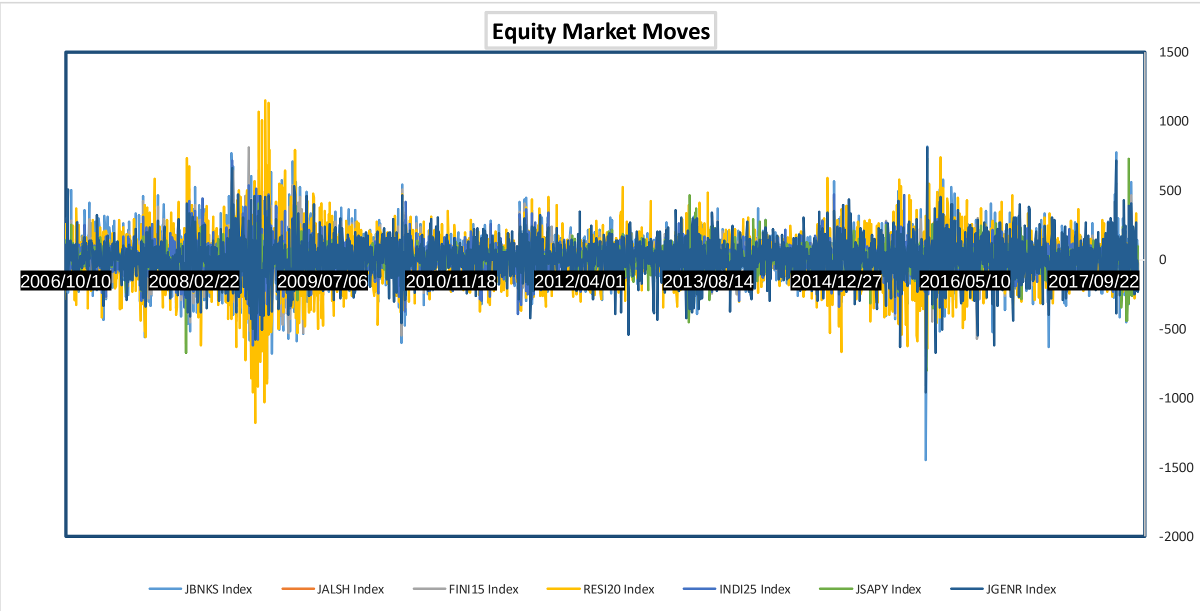

Figure 3: Equity Market Moves

The stressed Value-At-Risk on Table 3 is calculated on the most extreme market moves on a 1-year period of about 250 scenarios from 2008/07/18 to 2009/07/18, as suggested by

Figure 3.

	1-day Stressed Value-At-Risk					
	95% Confidence Interval		99% Confidence Interval		99.9% Confidence Interval	
JBNKS Index	-	7 889 303.17	-	11 502 804.52	-	15 531 564.16
FINI15 Index	-	6 696 943.35	-	8 827 709.64	-	11 629 460.87
RESI20 Index	-	9 584 119.22	-	15 667 724.75	-	19 076 803.96
INDI25 Index	-	5 513 597.24	-	7 468 577.98	-	10 190 498.59
JSAPY Index	-	3 047 260.25	-	4 745 492.12	-	6 998 682.12
JGENR Index	-	5 819 736.45	-	7 763 276.27	-	9 326 014.46
Portfolio	**-**	**31 788 947.50**	**-**	**42 140 200.84**	**-**	**60 866 759.67**

Table 3: Stressed Value-At-Risk: Historical Approach

The Expected Shortfall on Table 4 is now calculated from the most extreme market moves, a 1-year period of about 250 scenarios from 2008/07/18 to 2009/07/18. The metric captures the fat tails from financial crisis stress.

	1-day Expected Shortfall					
	95% Confidence Interval		99% Confidence Interval		99.75% Confidence Interval	
JBNKS Index	-	11 583 112.27	-	28 009 121.44	-	25 429 253.64
FINI15 Index	-	9 591 437.97	-	20 813 903.87	-	19 302 160.66
RESI20 Index	-	15 155 692.47	-	35 303 182.21	-	31 507 695.44
INDI25 Index	-	7 826 159.12	-	18 757 377.44	-	16 583 797.32
JSAPY Index	-	4 974 020.56	-	12 273 378.74	-	11 573 188.83
JGENR Index	-	7 998 772.11	-	17 558 918.55	-	15 413 084.86
Portfolio	**-**	**45 414 616.83**	**-**	**103 194 303.81**	**-**	**105 623 149.61**

Table 4: Expected Shortfall: Historical Approach (stress period)

4.2 Monte Carlo Simulation Approach Results

As explained in Section 2.3, how Expected Shortfall and Value-At-Risk are calculated. Table 5 shows the output under Monte Carlo Simulation for Expected shortfall metric to be still higher than Value-At-Risk. There's a significant reduction on Monte Carlo results compared with Historical Approach numbers on both metrics, the risk metrics stabilise as we increase number of iterations from 520 to 100 400.

520 Scenarios: Time Steps = 1							
Indices							
	JBNKS	**FINI15**	**RESI20**	**INDI25**	**JSAPY**	**JGENR**	**Portfolio**
95%Var	- 4 570 508	- 4 492 461	- 2 424 341	- 2 311 742	- 4 077 290	- 2 146 417	- 8 409 459
99% ar	- 6 412 778	- 6 044 168	- 3 395 632	- 2 901 304	- 5 866 243	- 2 955 716	- 11 145 084
99.9%Var	- 7 843 579	- 6 611 458	- 3 689 689	- 3 567 628	- 7 396 803	- 3 709 489	- 14 428 984
95%CVar	- 6 021 794	- 5 557 113	- 3 173 866	- 2 825 587	- 5 437 351	- 2 735 401	- 10 651 953
99%CVar	- 10 162 594	- 9 083 633	- 5 133 649	- 4 525 041	- 9 451 478	- 4 712 533	- 18 145 023
99.9%CVar	- 12 007 384	- 11 579 712	- 7 714 931	- 6 599 751	- 11 350 402	- 7 511 914	- 20 771 190
5 200 Scenarios: Time Steps = 10							
95%Var	- 4 388 370	- 4 377 624	- 2 278 041	- 2 347 230	- 4 145 081	- 2 126 478	- 8 097 764
99% ar	- 6 360 767	- 6 030 518	- 3 246 608	- 3 328 672	- 5 852 802	- 2 966 240	- 11 501 603
99.9%Var	- 8 296 886	- 7 379 251	- 4 521 027	- 4 245 958	- 7 902 472	- 4 015 551	- 15 434 514
95%CVar	- 5 571 856	- 5 345 617	- 2 923 096	- 2 955 370	- 5 252 417	- 2 669 361	- 10 282 784
99%CVar	- 7 354 284	- 6 706 606	- 3 955 907	- 3 850 458	- 7 045 758	- 3 474 471	- 13 518 496
99.9%CVar	- 12 674 123	- 10 682 636	- 7 315 856	- 6 503 468	- 12 416 614	- 6 115 282	- 23 898 615
10 400 Scenarios: Time Steps = 20							
95%Var	- 4 416 192	- 4 325 940	- 2 275 373	- 2 330 682	- 4 175 573	- 2 161 421	- 8 269 311
99% ar	- 6 300 539	- 6 063 516	- 3 266 957	- 3 293 118	- 5 867 252	- 3 084 003	- 11 491 735
99.9%Var	- 8 361 283	- 7 665 528	- 4 478 577	- 4 346 874	- 8 097 337	- 4 124 382	- 15 734 363
95%CVar	- 5 535 968	- 5 385 611	- 2 914 209	- 2 925 260	- 5 280 547	- 2 737 461	- 10 359 327
99%CVar	- 7 244 261	- 6 869 971	- 3 899 975	- 3 815 173	- 6 999 067	- 3 583 985	- 13 517 941
99.9%CVar	- 10 384 709	- 9 768 068	- 5 923 014	- 5 426 629	- 10 381 255	- 5 118 994	- 20 501 891
100 400 Scenarios: Time Steps = 200							
95%Var	- 4 363 588	- 4 320 004	- 2 334 131	- 2 299 594	- 4 173 029	- 2 150 057	- 8 172 868
99% ar	- 6 226 593	- 6 160 869	- 3 362 347	- 3 293 141	- 5 884 085	- 3 057 852	- 11 631 191
99.9%Var	- 8 322 431	- 8 131 684	- 4 520 769	- 4 418 048	- 7 871 400	- 4 142 089	- 15 565 634
95%CVar	- 5 370 779	- 5 304 630	- 2 888 009	- 2 826 713	- 5 091 620	- 2 643 998	- 10 072 555
99%CVar	- 6 963 140	- 6 829 356	- 3 767 720	- 3 701 119	- 6 567 432	- 3 456 619	- 13 097 378
99.9%CVar	- 9 022 429	- 8 862 761	- 4 847 445	- 4 677 893	- 8 379 826	- 4 482 300	- 17 005 393

Table 5: Expected Shortfall & Value-At-Risk: MC Simulation Approach

As explained in Section 2.3, how mean (drift) and standard deviation are calibrated to calculate daily returns from equity prices using Monte Carlo Simulation Approach. The similar approach was applied to generate results on Table 5 using the recent stress period (from 04 Feb 2016 to 31 Jan 2018). To test the impact of the severe market condition during financial crisis, we then use the stress period as suggested by Figure 3. The results are populated on Table 6, which shows big jumps on both metrics, Value-At-Risk and Expected Shortfall.

520 Scenarios: Time Steps = 1							
Indices							
	JBNKS	**FINI15**	**RESI20**	**INDI25**	**JSAPY**	**JGENR**	**Portfolio**
95%Var	- 7 968 824	- 6 310 084	- 11 712 903	- 5 223 283	- 3 232 393	- 5 592 921	- 16 945 338
99% ar	- 11 393 596	- 8 579 675	- 15 537 267	- 7 945 228	- 4 126 708	- 7 603 583	- 25 079 837
99.9%Var	- 16 138 483	- 10 041 048	- 17 054 816	- 10 058 990	- 5 431 763	- 10 747 355	- 34 363 704
95%CVar	- 10 615 519	- 8 251 981	- 14 452 843	- 7 250 283	- 4 004 839	- 7 413 821	- 22 674 460
99%CVar	- 19 155 960	- 13 408 877	- 23 392 782	- 12 681 276	- 6 871 389	- 13 223 606	- 42 155 625
99.9%CVar	- 26 711 186	- 20 101 312	- 30 539 452	- 19 814 500	- 13 102 251	- 20 731 815	- 57 896 368
5 200 Scenarios: Time Steps = 10							
95%Var	- 7 957 815	- 6 986 607	- 10 513 298	- 5 639 692	- 3 073 507	- 5 565 865	- 17 080 784
99% ar	- 10 955 380	- 9 653 992	- 14 620 688	- 7 740 351	- 4 463 306	- 7 892 082	- 23 667 059
99.9%Var	- 14 118 586	- 12 188 024	- 18 494 714	- 10 463 710	- 5 784 406	- 10 849 102	- 30 742 823
95%CVar	- 9 849 048	- 8 682 854	- 13 010 906	- 7 032 883	- 3 913 935	- 7 096 317	- 21 505 429
99%CVar	- 12 738 618	- 11 286 458	- 16 662 845	- 9 271 983	- 5 121 809	- 9 418 430	- 27 809 709
99.9%CVar	- 22 317 933	- 20 071 185	- 27 964 767	- 16 305 284	- 8 850 237	- 16 545 145	- 48 937 373
10 400 Scenarios: Time Steps = 20							
95%Var	- 7 953 474	- 6 967 939	- 10 504 162	- 5 508 129	- 3 079 627	- 5 561 414	- 16 887 008
99% ar	- 11 022 829	- 9 902 867	- 14 805 726	- 7 740 351	- 4 409 748	- 7 834 414	- 23 667 059
99.9%Var	- 14 386 181	- 13 426 804	- 18 598 211	- 10 701 681	- 5 626 390	- 10 796 725	- 30 697 023
95%CVar	- 9 902 512	- 8 767 353	- 13 089 144	- 6 949 306	- 3 892 166	- 7 024 810	- 21 168 954
99%CVar	- 12 784 282	- 11 583 449	- 16 856 975	- 9 140 306	- 5 009 876	- 9 226 660	- 27 314 573
99.9%CVar	- 18 485 290	- 16 905 736	- 23 812 784	- 13 684 096	- 7 150 142	- 13 823 604	- 39 543 880
100 400 Scenarios: Time Steps = 200							
95%Var	- 8 038 219	- 6 924 631	- 10 546 715	- 5 534 094	- 3 119 801	- 5 558 817	- 17 233 154
99% ar	- 11 391 449	- 9 710 236	- 14 726 764	- 7 807 445	- 4 440 165	- 7 889 345	- 24 302 177
99.9%Var	- 15 258 888	- 12 857 275	- 19 654 176	- 10 521 438	- 5 802 689	- 10 536 176	- 31 997 690
95%CVar	- 9 859 894	- 8 434 435	- 12 799 309	- 6 769 624	- 3 825 523	- 6 823 322	- 21 069 977
99%CVar	- 12 741 100	- 10 845 472	- 16 428 511	- 8 733 750	- 4 940 848	- 8 876 433	- 27 123 754
99.9%CVar	- 16 477 932	- 14 010 970	- 21 224 628	- 10 949 626	- 6 286 612	- 11 524 333	- 34 080 742

Table 6: Expected Shortfall & Value-At-Risk: MC (stress period)

4.3 Portfolio Weights

The previous metrics results were calculated on an assumption that weights are the equal across sectors. The optimisation is performed to get the optimal weights needed to minimize the objective function, which is our market risk exposure on the portfolio. The below figures give the optimal weights to achieve the desired risk exposure.

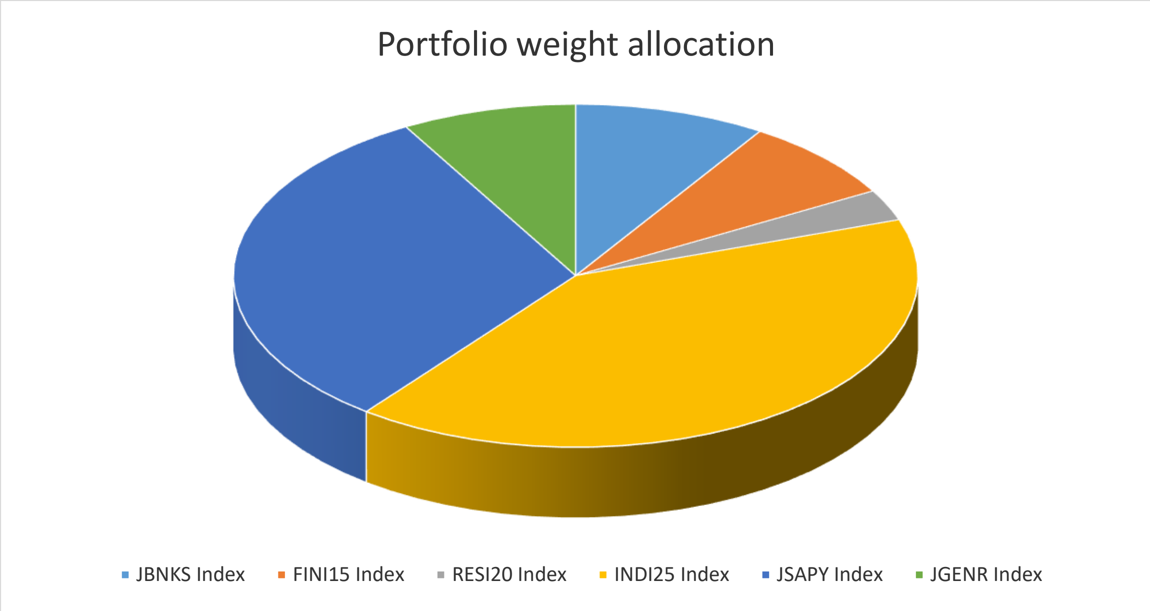

Figure 4: Portfolio Weights Allocation

4.4 Historical Simulation Approach- Modified Results

Table 5 and Table 6 show a reduction on the outputs for both metrics after the portfolio weights were optimised.

The below results were calculated after the weights were adjusted:

	1-day Value-At-Risk		
	95% Confidence Interval	99% Confidence Interval	99.9% Confidence Interval
JBNKS Index	- 2 379 596.39	- 3 691 466.14	- 5 235 262.20
FINI15 Index	- 1 455 206.51	- 2 402 909.37	- 4 291 918.80
RESI20 Index	- 770 257.25	- 1 108 943.91	- 1 362 032.37
INDI25 Index	- 6 269 316.97	- 9 543 188.87	- 13 272 144.44
JSAPY Index	- 4 301 095.82	- 7 256 561.07	- 12 235 827.87
JGENR Index	- 1 846 642.69	- 3 158 341.17	- 4 825 894.14
Portfolio	**- 13 159 837.24**	**- 20 851 794.82**	**- 33 817 680.14**

Table 7: Modified Value-At-Risk**:** Historical Approach

1-day Expected Shortfall						
	95% Confidence Interval		99% Confidence Interval		99.9% Confidence Interval	
JBNKS Index	-	3 262 490.32	-	6 221 449.97	-	8 971 595.93
FINI15 Index	-	2 153 487.63	-	4 635 836.30	-	6 625 562.93
RESI20 Index	-	1 002 666.01	-	1 746 758.61	-	3 049 784.00
INDI25 Index	-	8 677 222.06	-	16 017 900.83	-	18 002 153.05
JSAPY Index	-	6 661 087.07	-	13 725 798.93	-	15 883 292.37
JGENR Index	-	2 866 610.05	-	5 712 983.94	-	7 989 649.65
Portfolio	**-**	**19 271 710.56**	**-**	**37 820 381.52**	**-**	**39 874 523.51**

Table 8: Modified Expected Shortfall: Historical Approach

Stress Value-At-Risk values have decreased significantly as shown below on Table 9:

1-day Stressed Value-At-Risk						
	95% Confidence Interval		99% Confidence Interval		99.9% Confidence Interval	
JBNKS Index	-	4 287 689.88	-	6 251 560.82	-	8 441 116.93
FINI15 Index	-	3 099 475.32	-	4 085 635.30	-	5 382 340.13
RESI20 Index	-	1 688 998.92	-	2 761 106.11	-	3 361 884.44
INDI25 Index	-	13 480 512.45	-	18 260 357.81	-	24 915 338.78
JSAPY Index	-	5 718 258.79	-	8 905 032.65	-	13 133 199.09
JGENR Index	-	2 886 518.71	-	3 850 490.90	-	4 625 590.09
Portfolio	**-**	**26 484 301.79**	**-**	**36 782 822.27**	**-**	**49 042 440.49**

Table 9: Modified Stressed Value-At-Risk**:** Historical Approach

The output from Table 8 shows how the results under Historical Approach Simulation have improved after the portfolio weights were modified. A reduction of about 12.86% on 99% Var, and a 12.35% decrease of 95% Expected Shortfall portfolio level.

		1-day Value-At-Risk	
Names	**95% Confidence Interval**	**99% Confidence Interval**	**99.9% Confidence Interval**
JBNKS Index	-45.652%	-45.652%	-45.652%
FINI15 Index	-53.718%	-53.718%	-53.718%
RESI20 Index	-82.377%	-82.377%	-82.377%
INDI25 Index	144.496%	144.496%	144.496%
JSAPY Index	87.652%	87.652%	87.652%
JGENR Index	-50.401%	-50.401%	-50.401%
Portfolio	**-12.504%**	**-12.864%**	**-9.195%**
		1-day Expected Shortfall	
Names	**95% Confidence Interval**	**99% Confidence Interval**	**99.9% Confidence Interval**
JBNKS Index	-45.652%	-45.652%	-45.652%
FINI15 Index	-53.718%	-53.718%	-53.718%
RESI20 Index	-82.377%	-82.377%	-82.377%
INDI25 Index	144.496%	144.496%	144.496%
JSAPY Index	87.652%	87.652%	87.652%
JGENR Index	-50.401%	-50.401%	-50.401%
Portfolio	**-12.350%**	**-10.445%**	**-7.878%**

Table 10: Value-At-Risk**:** Change between Historical and Modified Approach

4.5 Monte Carlo Simulation Approach Results

The results Table 9 show how significantly the results changed under Monte Carlo Simulation Approach when optimisation has been factored in.

520 Scenarios: Time Steps = 1							
Indices							
	JBNKS	**FINI15**	**RESI20**	**INDI25**	**JSAPY**	**JGENR**	**Portfolio**
95%Var	- 2 421 725	- 1 906 497	- 405 460	- 5 218 360	- 7 794 184	- 1 026 222	- 9 870 536
99% ar	- 3 307 241	- 2 537 312	- 579 196	- 7 379 272	- 10 425 023	- 1 430 418	- 13 928 906
99.9%Var	- 3 895 280	- 3 280 047	- 765 024	- 9 475 004	- 13 204 645	- 1 737 455	- 16 288 829
95%CVar	- 3 094 995	- 2 429 162	- 548 984	- 6 815 059	- 9 785 242	- 1 337 837	- 12 889 414
99%CVar	- 5 131 269	- 4 249 965	- 963 065	- 12 148 094	- 16 778 192	- 2 219 522	- 21 583 642
99.9%CVar	- 8 362 974	- 7 735 926	- 1 762 100	- 16 291 631	- 19 482 477	- 5 219 904	- 24 999 111
5 200 Scenarios: Time Steps = 10							
95%Var	- 2 291 984	- 1 970 246	- 407 833	- 5 539 173	- 7 922 825	- 1 059 695	- 10 003 402
99% ar	- 3 331 149	- 2 766 606	- 580 071	- 7 950 480	- 11 103 727	- 1 548 064	- 14 456 580
99.9%Var	- 4 295 850	- 3 501 099	- 750 296	- 11 131 775	- 14 539 363	- 2 012 014	- 19 136 553
95%CVar	- 2 951 235	- 2 456 878	- 510 991	- 7 104 620	- 9 939 899	- 1 358 417	- 12 836 069
99%CVar	- 3 853 313	- 3 211 299	- 671 877	- 9 403 480	- 13 081 567	- 1 827 249	- 16 891 000
99.9%CVar	- 6 624 819	- 5 668 584	- 1 157 336	- 16 582 910	- 23 882 574	- 3 116 681	- 28 655 177
10 400 Scenarios: Time Steps = 20							
95%Var	- 2 321 818	- 1 993 710	- 404 063	- 5 574 919	- 7 935 212	- 1 078 779	- 10 005 690
99% ar	- 3 281 011	- 2 870 040	- 580 071	- 8 060 072	- 11 098 490	- 1 545 840	- 14 142 326
99.9%Var	- 4 432 180	- 3 747 639	- 741 942	- 10 876 508	- 14 514 540	- 2 093 798	- 19 427 461
95%CVar	- 2 936 863	- 2 499 930	- 512 312	- 7 137 678	- 9 914 690	- 1 359 059	- 12 683 291
99%CVar	- 3 808 465	- 3 278 439	- 664 362	- 9 428 949	- 12 768 564	- 1 804 741	- 16 577 999
99.9%CVar	- 5 685 595	- 4 648 911	- 926 375	- 13 448 852	- 18 901 238	- 2 598 698	- 24 413 497
100 400 Scenarios: Time Steps = 200							
95%Var	- 2 376 597	- 2 000 621	- 407 232	- 5 644 607	- 7 824 083	- 1 066 265	- 10 030 174
99% ar	- 3 352 433	- 2 844 123	- 581 599	- 8 019 252	- 11 105 884	- 1 532 456	- 14 328 093
99.9%Var	- 4 477 755	- 3 783 445	- 774 197	- 10 664 572	- 14 781 714	- 2 034 369	- 19 407 285
95%CVar	- 2 902 522	- 2 455 062	- 502 808	- 6 944 792	- 9 590 375	- 1 315 448	- 12 418 162
99%CVar	- 3 732 139	- 3 196 821	- 654 486	- 9 007 359	- 12 434 916	- 1 715 015	- 16 177 323
99.9%CVar	- 4 870 274	- 4 168 751	- 844 773	- 11 462 499	- 15 859 846	- 2 197 058	- 20 998 644

Table 11: Modified Expected Shortfall & Value-At-Risk – MC Simulation Approach

The percentage changes on Table 10 have significantly changed the results, changed under Monte Carlo Simulation Approach when optimisation has been factored in.

520 Scenarios: Time Steps = 1							
Indices							
	JBNKS	**FINI15**	**RESI20**	**INDI25**	**JSAPY**	**JGENR**	**Portfolio**
95%Var	-47.014%	-57.562%	-83.275%	125.733%	91.161%	-52.189%	17.374%
99% ar	-48.427%	-58.020%	-82.943%	154.343%	77.712%	-51.605%	24.978%
99.9%Var	-50.338%	-50.388%	-79.266%	165.583%	78.518%	-53.162%	12.890%
95%CVar	-48.603%	-56.287%	-82.703%	141.191%	79.963%	-51.092%	21.005%
99%CVar	-49.508%	-53.213%	-81.240%	168.464%	77.519%	-52.902%	18.951%
99.9%CVar	-30.351%	-33.194%	-77.160%	146.852%	71.646%	-30.512%	20.355%
5 200 Scenarios: Time Steps = 10							
95%Var	-47.771%	-54.993%	-82.097%	135.988%	91.138%	-50.167%	23.533%
99% ar	-47.630%	-54.123%	-82.133%	138.848%	89.716%	-47.811%	25.692%
99.9%Var	-48.223%	-52.555%	-83.404%	162.173%	83.985%	-49.894%	23.985%
95%CVar	-47.033%	-54.039%	-82.519%	140.397%	89.244%	-49.111%	24.831%
99%CVar	-47.605%	-52.117%	-83.016%	144.217%	85.666%	-47.409%	24.947%
99.9%CVar	-47.730%	-46.936%	-84.180%	154.986%	92.344%	-49.035%	19.903%
10 400 Scenarios: Time Steps = 20							
95%Var	-47.425%	-53.913%	-82.242%	139.197%	90.039%	-50.089%	20.998%
99% ar	-47.925%	-52.667%	-82.244%	144.755%	89.160%	-49.876%	23.065%
99.9%Var	-46.992%	-51.110%	-83.434%	150.214%	79.251%	-49.234%	23.472%
95%CVar	-46.949%	-53.581%	-82.420%	144.002%	87.759%	-50.353%	22.434%
99%CVar	-47.428%	-52.279%	-82.965%	147.143%	82.432%	-49.644%	22.637%
99.9%CVar	-45.250%	-52.407%	-84.360%	147.831%	82.071%	-49.234%	19.079%
100 400 Scenarios: Time Steps = 200							
95%Var	-45.536%	-53.689%	-82.553%	145.461%	87.492%	-50.408%	22.725%
99% ar	-46.159%	-53.836%	-82.703%	143.514%	88.744%	-49.885%	23.187%
99.9%Var	-46.197%	-53.473%	-82.875%	141.387%	87.790%	-50.885%	24.680%
95%CVar	-45.957%	-53.719%	-82.590%	145.684%	88.356%	-50.248%	23.287%
99%CVar	-46.401%	-53.190%	-82.629%	143.369%	89.342%	-50.385%	23.516%
99.9%CVar	-46.020%	-52.963%	-82.573%	145.036%	89.262%	-50.984%	23.482%

Table 12: Change MC Simulation and the Modified Approach

5 CONCLUSIONS

5.1 Summary and Conclusions

The empirical results from chapter 4 of this paper prove that Expected Shortfall (CVar) measure to be a robust framework to address drawbacks of Value-At-Risk. The Var metric fails to capture tails in the profit and loss (P&L) distribution. The 1-year stress period (2008/07/18 to 2009/07/18) was used to calculate Stressed Value-At-Risk (SVar) which is the most severe period from the 520 scenarios used to calibrate Value-At-Risk (Var) on Historical Simulation Approach, and the Stressed Value-At-Risk (SVar) results almost double the Value-At-Risk (Var) numbers. The stress period moves are only picked-up in the Expected Shortfall (CVar) numbers, which proves the metric to be far better than Value-At-Risk (Var). The 99.75% portfolio Expected Shortfall from Table 4 under stress period is about 10% of the total investment, compared to 5% of the nominal amount invested (R1bn) from the sum of 95%Var and 99% SVar. This will protect the economy and obviously disadvantage banks because they will be expected to reserve more capital.

5.2 Recommendations

The results prove that the revised measure increases the capital charge. However, portfolio optimisation to get the optimal portfolio weights of equity indices reduces the market risk charge, this will benefit banks to save from capital charge and maximise their profits. Portfolio optimisation saves banks about 10% of the market risk capital charge, optimisation is critical since the Var metric is not sub-additive. The only disadvantage about tail risk capture is it comes with a significant cost, as data requirements and operational complexities of the Expected Shortfall measure are likely to be expensive since most of South African Banks use Historical Simulation Approach to calibrate their risk Metrics (i.e. Stress Value-At-Risk, Value-At-Risk, Expected Shortfall, & Backtesting). The other challenge with Historical Simulation Approach is it is not easy to use for portfolios without data. However, this is not an issue for

Monte Carlo Simulation Approach. You only need to calibrated two inputs into the model (i.e. standard deviation and expected return), and apply them on Geometric Brownian Motion (GBM) model to generate a vector of equity returns as suggested by John C. Hull. The other advantage about Monte Carlo Simulation Approach is, it is easy to perform a "What-if" analysis to assess the effect of alternative assumptions. Given challenges discussed a shift from Historical Approach to Monte Carlo Simulation is recommended.

The results improve and become stable as you increase the number of iterations from 520 scenarios to 100 400 scenarios, on the second attempt we used standard deviation and mean from a stress period and the results increases significantly on Table 6 compared to Table 5. The empirical results prove that high Volatilities in the market increases the market risk charge. However, this can be reduced if we use a robust methodology like Monte Carlo Simulation.

5.3 Limitations of the study and areas for further research

This study only focuses on Expected Shortfall (CVar) which is one component into the calculation of total market risk capital charge under the new regulation framework (revised IMA). Other inputs like Default Risk Charge (DRC) and Non-Modellable risk component don't form part of this research. There's an opportunity to conduct further studies on other segments of Internal Model Approach and on other pillars of the overall capital charge like the revised boundary between trading and banking books and revised standardized approach (SA).

The other thing is this paper only focuses on South African Equity Market and doesn't include other major financial markets, like foreign exchange market, commodity market, credit market and Interest rate market. Lastly, the study can be expanded beyond South African borders to other emerging markets and to other parts of the world.

In terms of Monte Carlo Simulation, the limitation is the known issue of longer computational time compared to Historical Simulation Approach.

The results show that a switch from Historical Simulation Approach to Monte Carlo Simulation Approach reduces the market risk capital charge which could be a concern to the regulator that banks are undercapitalized.

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APPENDIX

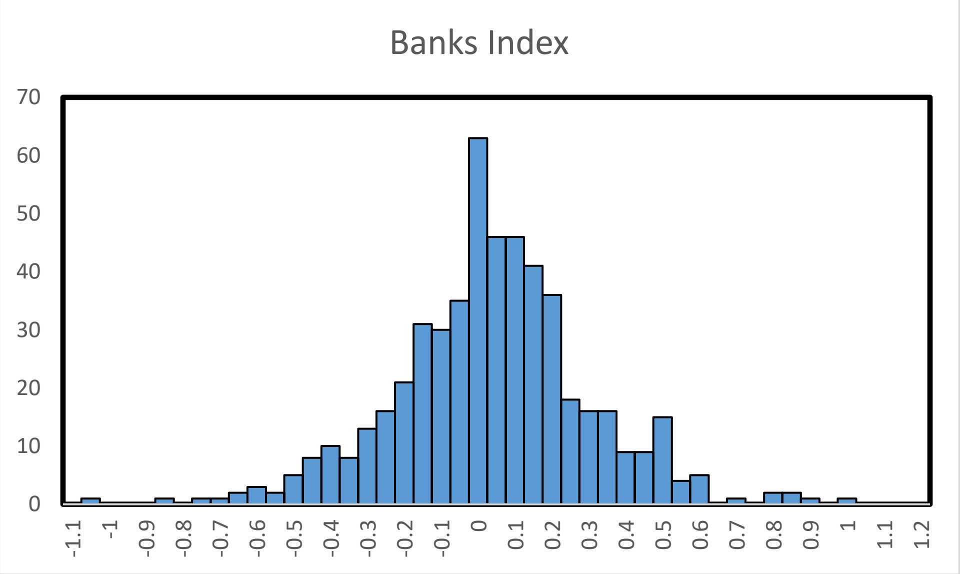

Figure 5: Banks Index- Profit and Loss distribution

Banks index		
Mean		158 292.71
Standard Error		119 061.71
Median		104 384.12
Mode		-
Standard Deviation		2 715 024.82
Sample Variance		7 371 359 759 618.86
Kurtosis		1.86
Skewness		0.16
Range		23 514 889.66
Minimum	-	10 560 899.53
Maximum		12 953 990.14
Sum		82 312 208.72
Count		520.00

Table 13: Banks Index: Descriptive Summary

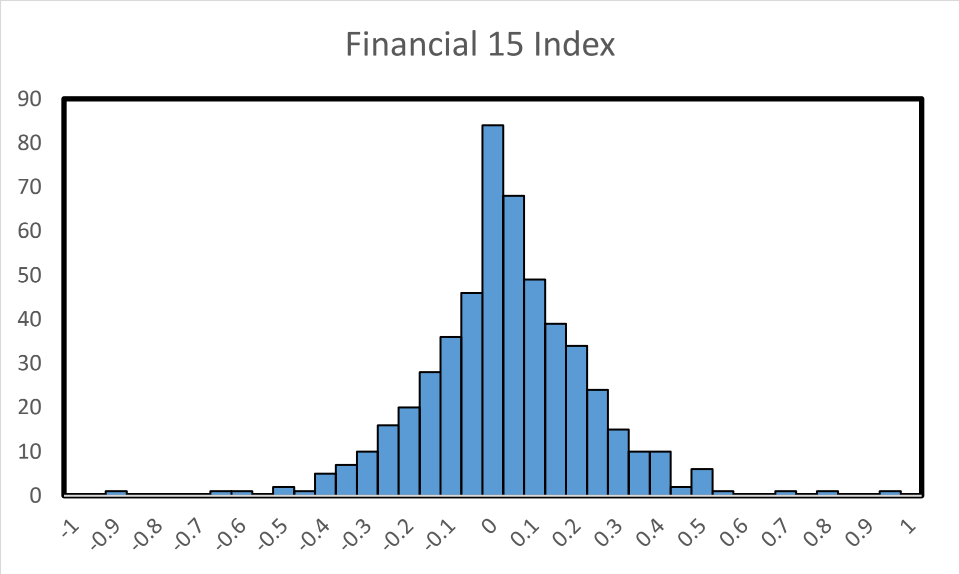

Figure 6: Financial 15 Index- Profit and Loss distribution

Financial 15 Index	
Mean	68 506.28
Standard Error	87 689.19
Median	9 418.26
Mode	-
Standard Deviatior	1 999 621.25
Sample Variance	3 998 485 156 033.89
Kurtosis	2.89
Skewness	- 0.16
Range	18 732 233.10
Minimum	- 9 519 242.08
Maximum	9 212 991.02
Sum	35 623 263.08
Count	520.00

Table 14: Financial 15 Index: Descriptive Summary

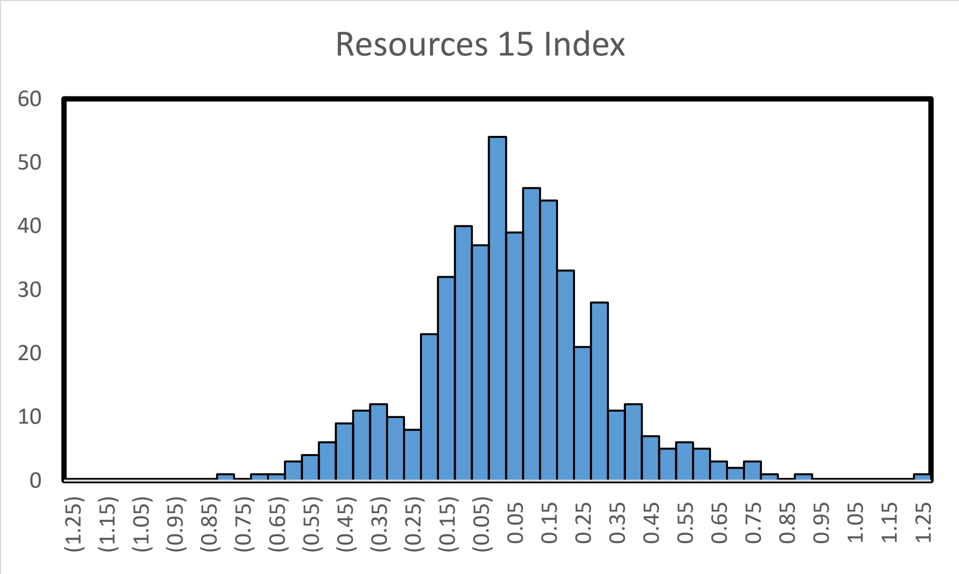

Figure 7: Resources 15 Index- Profit and Loss distribution

Resources 20 Index		
Mean		141 181.31
Standard Error		116 002.45
Median		117 683.52
Mode		-
Standard Deviatior		2 645 262.84
Sample Variance		6 997 415 486 587.55
Kurtosis		1.17
Skewness		0.18
Range		20 594 114.23
Minimum	-	8 275 395.75
Maximum		12 318 718.48
Sum		73 414 283.72
Count		520.00

Table 15: Resource 20 Index: Descriptive Summary

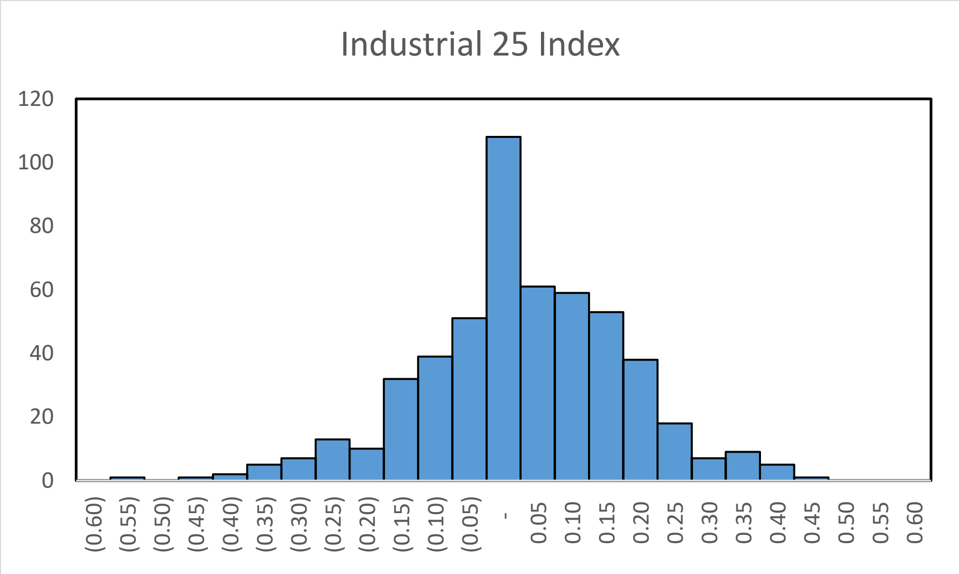

Figure 8: Industrial 25 Index- Profit and Loss distribution

Industrial 25 Index		
Mean		43 271.86
Standard Error		65 140.00
Median		-
Mode		-
Standard Deviation		1 485 420.59
Sample Variance		2 206 474 316 131.87
Kurtosis		0.79
Skewness	-	0.28
Range		10 045 725.36
Minimum	-	5 914 644.08
Maximum		4 131 081.28
Sum		22 501 365.24
Count		520.00

Table 16: Industrial 25 Index: Descriptive Summary

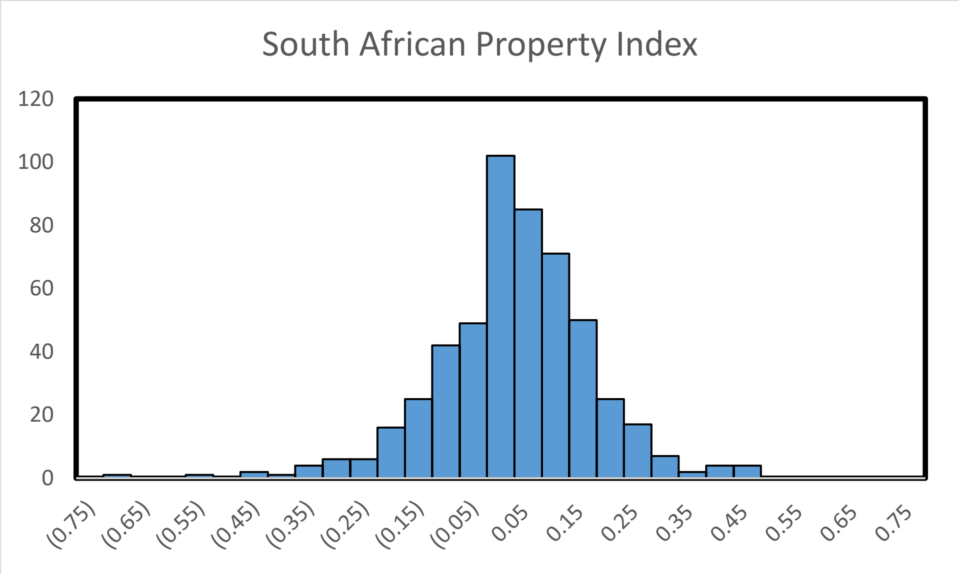

Figure 9: South African Property Index - Profit and Loss distribution

SA Listed Property Index		
Mean		19 042.00
Standard Error		62 344.79
Median		27 521.85
Mode		-
Standard Deviation		1 421 680.01
Sample Variance		2 021 174 059 024.77
Kurtosis		2.42
Skewness	-	0.47
Range		11 730 084.77
Minimum	-	7 328 011.82
Maximum		4 402 072.95
Sum		9 901 838.24
Count		520.00

Table 17: SA Listed Property Index: Descriptive Summary

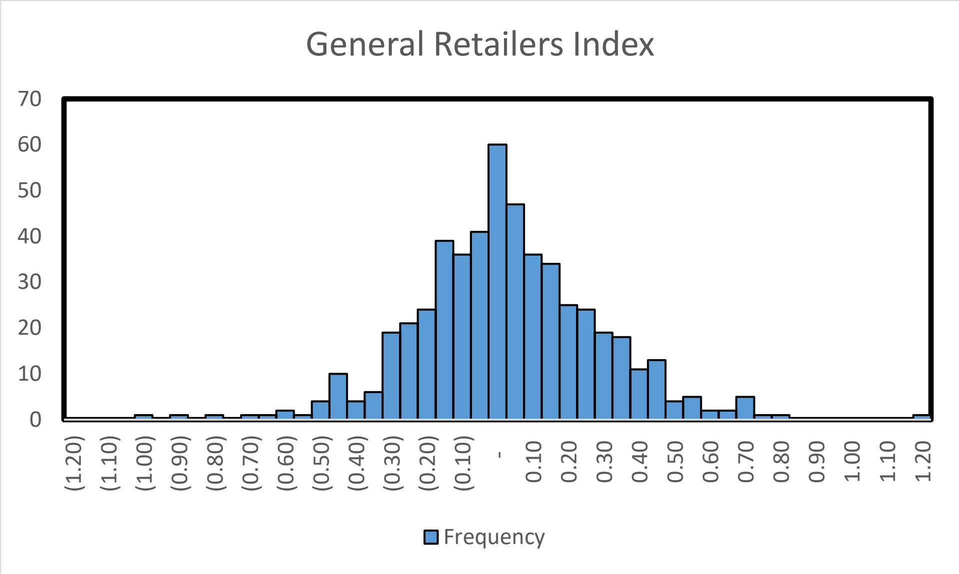

Figure 10: General Retailers Index- Profit and Loss distribution

General Retail Index	
Mean	55 017.20
Standard Error	113 359.26
Median	-
Mode	-
Standard Deviatio	2 584 988.89
Sample Variance	6 682 167 565 304.17
Kurtosis	1.53
Skewness	0.09
Range	22 268 893.28
Minimum	- 10 377 039.26
Maximum	11 891 854.02
Sum	28 608 945.16
Count	520.00

Table 18: General Retail Index: Descriptive Summary

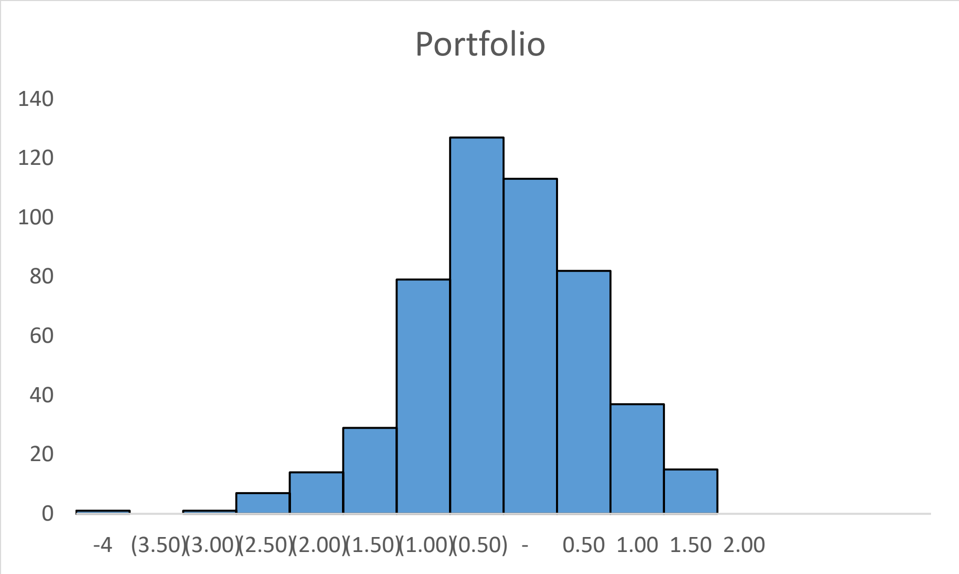

Figure 11: Portfolio - Profit and Loss distribution

Portfolio		
Mean		471 965.80
Standard Error		405 748.11
Median		54 691.13
Mode		-
Standard Deviation		9 252 480.46
Sample Variance		85 608 394 686 988.10
Kurtosis		1.58
Skewness	-	0.07
Range		72 630 116.92
Minimum	-	37 120 741.60
Maximum		35 509 375.31
Sum		245 422 216.06
Count		520.00

Table 19: Portfolio: Descriptive Summary

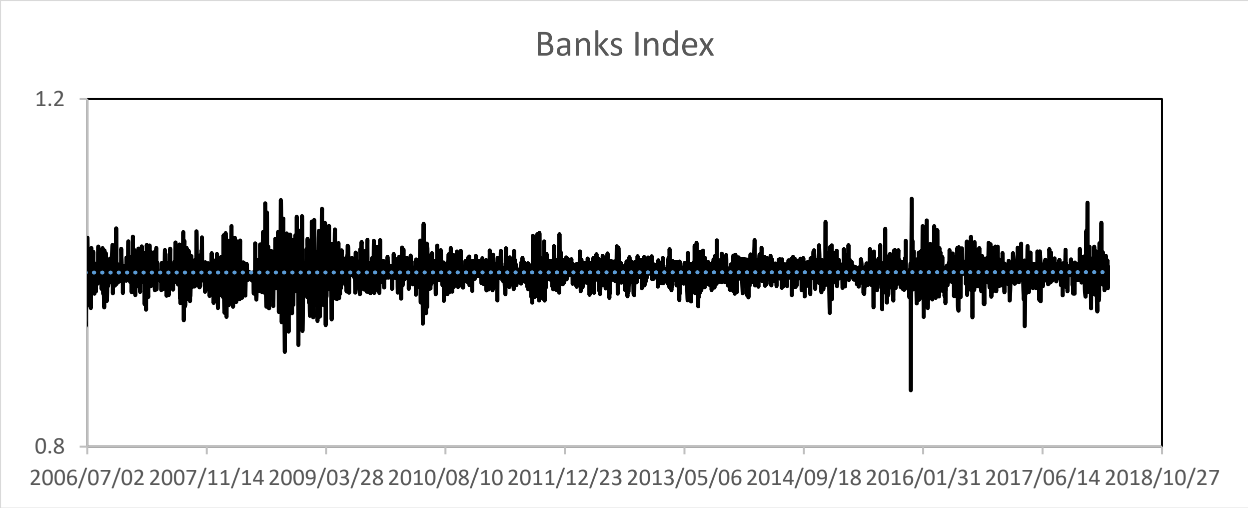

Figure 12: Banks Index (daily moves)

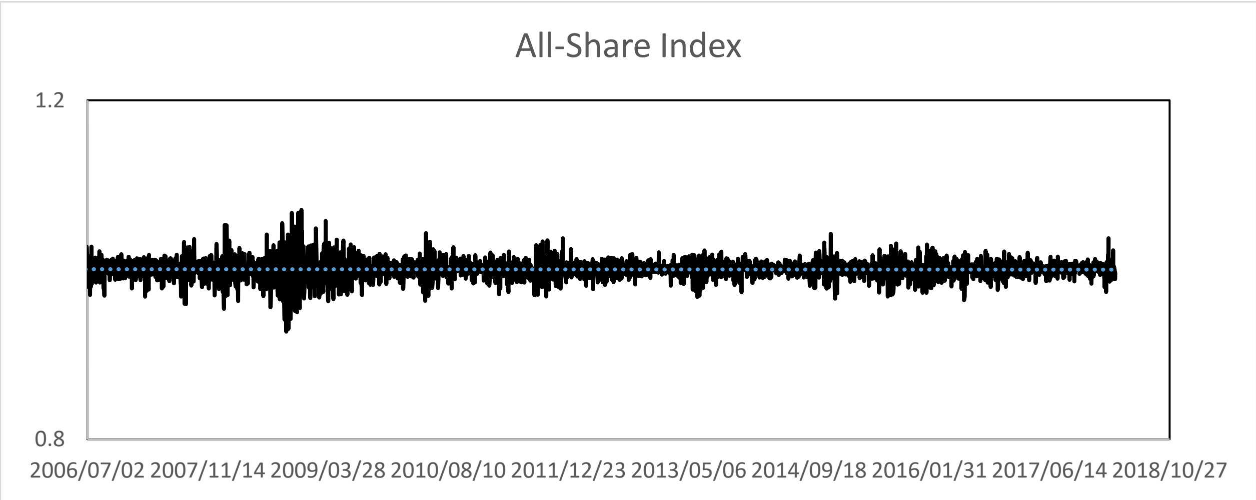

Figure 13: All-Share Index (daily moves)

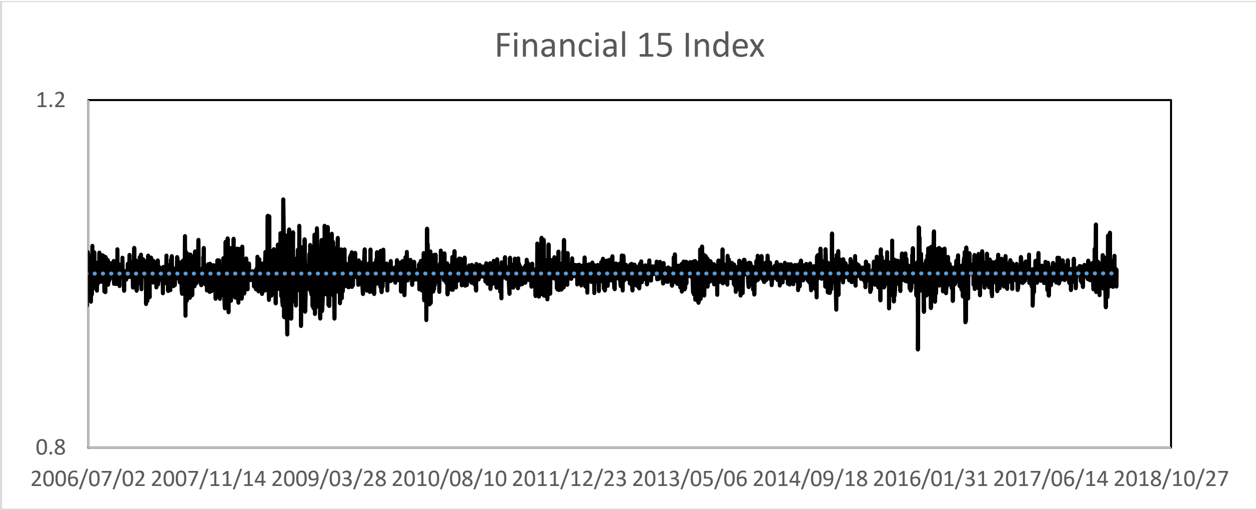

Figure 14: Financial 15 Index (daily moves)

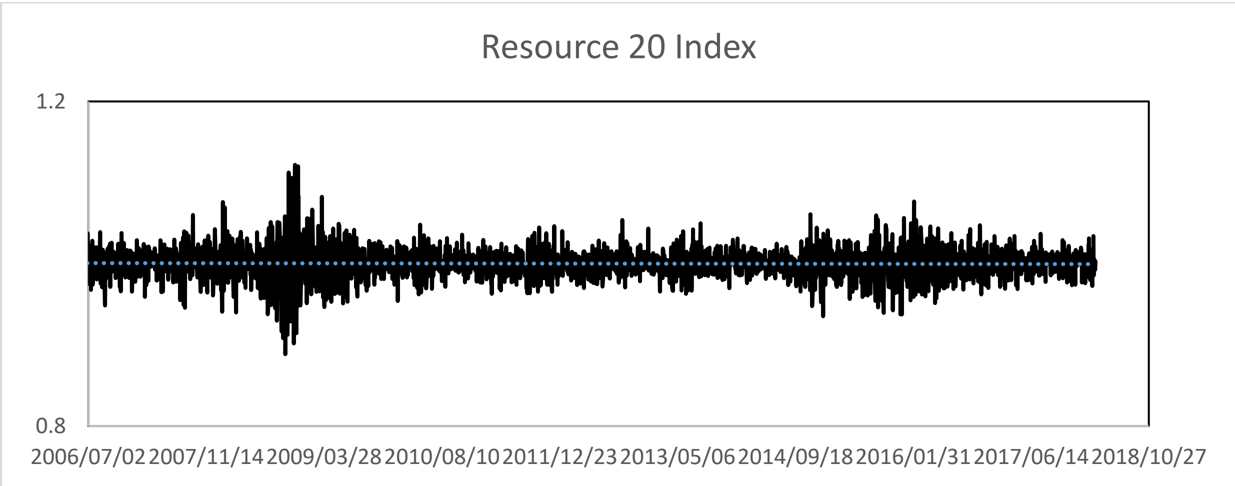

Figure 15: Resource 20 Index (daily moves)

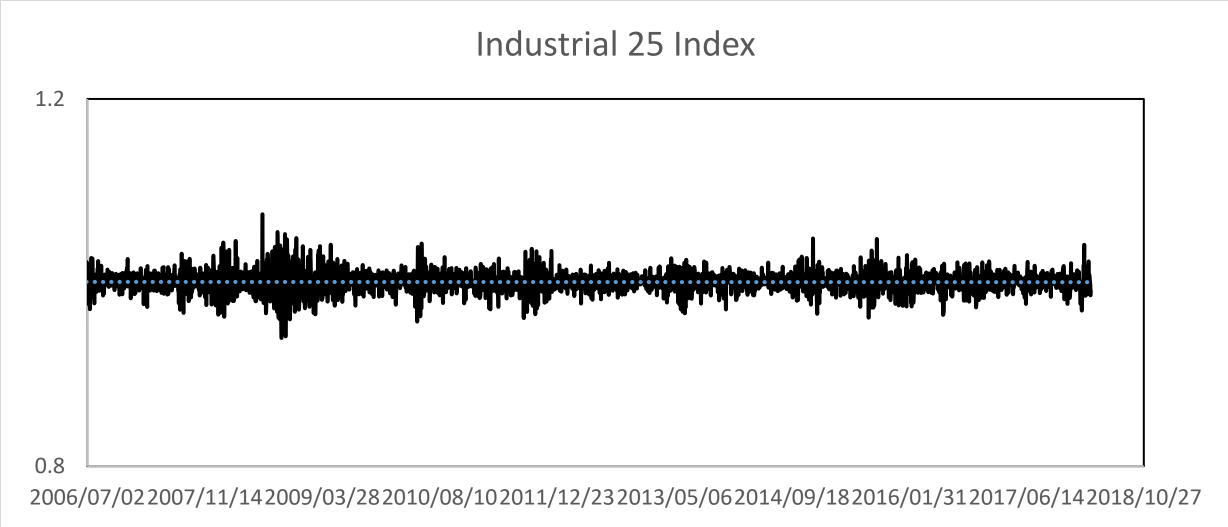

Figure 16: Industrial 25 Index (daily moves)

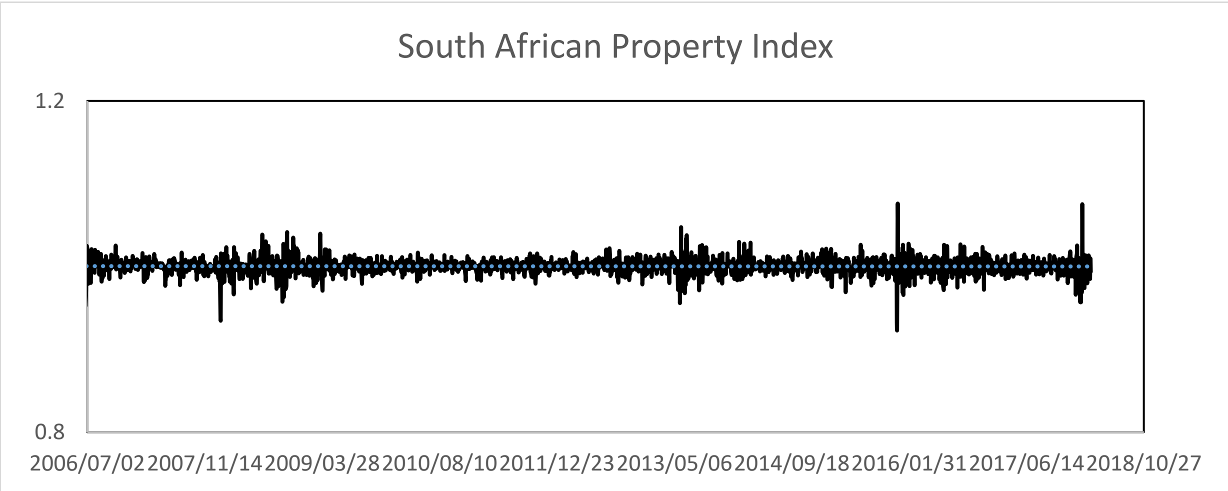

Figure 17: SA Listed Property Index (daily moves)

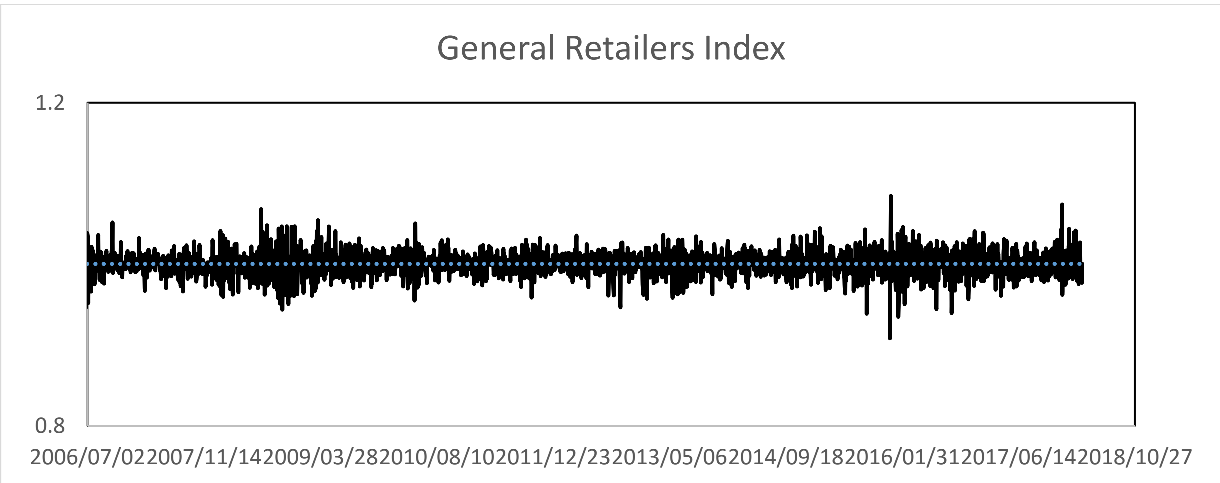

Figure 18: General Retailers Index (daily moves)