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Exploring the value of a teaching and learning intervention (using an assessment ***as*** learning – conceptual thread approach) in addressing acquired learning deficits in Mathematics in South Africa

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A Masters Dissertation submitted to the Wits School of Education
Faculty of Humanities, University of the Witwatersrand
August 2017

DECLARATION

I, Tracey-Anne Butchart, hereby declare that this dissertation is my own unaided work. It is submitted in fulfillment for the degree of Master in Education to the University of the Witwatersrand, Johannesburg. It has not been submitted for any other degree or for examination at any other university. Each significant contribution to, and quotation in, this dissertation from the work, or works of other people has been attributed, cited and referenced. I further declare that the information used in the dissertation, as well as the permission to use this information, was obtained while I was employed as a consultant for Uplands Outreach and that authorisation and consent from the ethics committee was granted for this research project.



1 August 2017

Tracey-Anne Butchart

Date

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- my sisters, Beth and Diane, for their continual encouragement;

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- my most special L4E Learners

Uplands Outreach Director Nicky De Bruyn concluded a 2012 L4E Funder's Annual Report with the following poem by the famous African-American poet, novelist and playwright:

> **"Because you love me, I have much achieved,**
> **Had you despised me, then I must have failed,**
> **But since I knew you trusted and believed,**
> **I could not disappoint you and so prevailed."**
> **- Paul Laurence Dunbar (1915)**

Nicky shared this poem called 'Encouraged' with the L4E learners in a Saturday assembly. It was presented to them as written by a son to his mother but relevant for a student to their teacher. When I look back on my L4E experience, this poem is most meaningful to me as if I, the teacher, had written it to these, my students – I felt their love, trust and belief and I could not disappoint them – I treasure my experience with them all and I am blessed to be witnessing their journeys to success.

ABSTRACT

Exploring the value of a teaching and learning intervention
(using an assessment ***as*** learning – conceptual thread approach)
in addressing acquired learning deficits in Mathematics in South Africa

The purpose of this study was to evaluate an **assessment *as* learning – conceptual thread** approach to the teaching and learning of Mathematics. The focus was on South African high school learners with acquired learning deficits that were causing functioning at levels as far back as Grade 2. Using a mixed-methods design, the study's key findings are: that learners can catch up between 3-6 grade levels despite limited contact time through using a diagnostic assessment ***as*** learning – conceptual thread approach for targeted intervention; that learners' acquired learning deficits are not necessarily insurmountable; and that Grade 10 is not too late to catch up on learners' grade-level achievement in Mathematics. The **assessment *as* learning – conceptual thread** approach provided value: for teachers in revealing the nature of learner difficulties; and for learners enabling their metacognitive activation and stimulating the advancement of their metacognitive skills. Learners developed skills of error detection, diagnosis and analysis; awareness of strengths, weaknesses, learning needs and opportunities; self-reflective performance analysis and self-evaluation of improvement strategies.

Keywords: formative assessment; assessment ***as*** learning; conceptual thread; conceptual landmarks; Mathematics catch-up; acquired learning deficits; metacognitive activation; metacognitive skills

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LIST OF ABBREVIATIONS

Abbreviation	Description
AAAS	American Association for the Advancement of Science
ANA	Annual National Assessment
ARG	Assessment Reform Group
B.Ed.	Bachelor of Education
B.Sc.	Bachelor of Science
CAPS	Curriculum and Assessment Policy Statement
CDE	Centre for Development and Enterprise
CCSS	Common Core State Standards
CPRE	Consortium for Policy Research in Education
DBE	Department of Basic Education
DELTA	Diagnostic E-Learning Trajectories Approach
FET	Further Education and Training
GET	General Education and Training
ID	identity
IEA	International Association for the Evaluation of Educational Achievement
ISASA	Independent Schools Association of Southern Africa
KMA	Knowledge Monitoring Ability
KMIA	Kruger-Mpumalanga International Airport
LASSI	Learning and Study Strategies Inventory
L4E	Learners For Excellence
MAAR	Metacognitive Activation & Advancement Rubric
MAI	Metacognitive Awareness Inventory
MSLQ	Motivated Strategies for Learning Questionnaire
NBT	National Benchmark Test
NEET	Not in Education, Employment, or Training
NFER	National Foundation for Educational Research
NSC	National Senior Certificate
OECD	Organisation for Economic Cooperation and Development
OGAP	Ongoing Assessment Project
OVC	Orphans and vulnerable children
PISA	Programme for International Student Assessment
STEM	Science, Technology, Engineering and Mathematics
T4E	Teachers For Excellence
TIMSS	Trends in International Mathematics and Science Study
UK	United Kingdom
UNISA	University of South Africa
USA	United States of America
VMP	Vermont Mathematics Partnership
ZPD	Zone of Proximal Development

CHAPTER ONE

Introduction to the Study

1.1 The Problem of South African Mathematics Education

Of all the Grade 12 learners writing their National Senior Certificate (NSC) examinations at the end of 2016, only 43.6% wrote Mathematics, and of these learners, only 51.1% achieved above 30% for Mathematics and thus passed the subject. The poor quality of these passes is further illustrated by the fact that only 33.5% achieved above 40% for Mathematics (Department of Basic Education - DBE, 2016). Despite this bleak level of achievement, these results are an improvement on the NSC Maths results in 2015 (DBE, 2015).

A few years earlier in a Mail & Guardian article, Nic Spaull (2013) said that existing research in South Africa, particularly in Mathematics, had identified that learners acquire learning deficits in the Foundation Phase grades and that these back-logs are cumulative in nature and are the root cause of underperformance in higher grades. This comment came after a report for the Centre for Development & Enterprise (CDE) on South Africa's education crisis, in which he said "for disadvantaged pupils, the gaps between what they *should* know and what they *do* know grow over time. This means that as time goes on, children fall further and further behind the curriculum leading to a situation where remediation is almost impossible in high school since these learning gaps have been left unaddressed for too long." (Spaull, 2013, p. 6). This culminates in poor achievement at the end of Senior Phase as evidenced by the 11% national average on the Annual National Assessment (ANA) for Grade 9 in Mathematics for 2014 (DBE, 2014). Nationally, only 3% of all Grade 9 learners achieved above 50% for Mathematics in the 2014 ANA (DBE, 2014). In November 2016, the DBE issued a circular instructing schools to condone achievement above 20% in Mathematics for Grade 9 learners who had only failed Maths so that these learner could continue into Grade 10. The proviso was that such learners would not be permitted to take Mathematics as a subject in the Further Education and Training (FET) band. This is another, and more obvious, example of how access to Maths is limited for learners in the South African education system.

The recent release of the International Association for the Evaluation of Educational Achievement's (IEA) Trends in International Mathematics and Science Study (TIMSS) results for 2015 were met with mixed responses in South Africa. While the Minister of Basic Education celebrated the results as showing that South Africa has the most improved education system in the world, the improvement from worst position to second-worst was not met with much shared enthusiasm. The results show that no comparative progress is being made in South African Maths classrooms (IEA, 2015). Grade 8 Maths results put South Africa second from the bottom of a list of 39 participating countries. A footnote to the distribution of results data highlights for the South African results "reservations about reliability because the percentage of students with achievement too low for estimation exceeds 15% but does not exceed 25%" (IEA, 2015). Grade 4 Maths achievement was no better, with South Africa second from the bottom of a list of 49 countries. It is also noteworthy that in both the Grade 8 and the Grade 4 TIMSS tests, South African learners were actually in Grade 9 and Grade 5 respectively. Of greater concern is that for Grade 4, TIMSS provided two different tests: a Fourth Grade Assessment; and a Numeracy Assessment. The latter is a less difficult version with questions requiring easier numbers and more straightforward procedures. While the bulk of the 49 countries wrote the Fourth Grade Maths Assessment only, five countries wrote both this and the Numeracy Assessment (and their results were averaged for these) but South Africa, along with Jordan, wrote *only* the Numeracy test and the results distribution list reflects our poor position despite this easier assessment.

The ultimate result of our poor Maths education is that only low numbers of learners are eligible to choose to study Mathematics to Grade 12 level and those that do choose are producing poor NSC Mathematics results. Taylor, Muller and Vinjevold (2003, p. 129) sum up the problem when they state " the … requirements of the high school curriculum make it virtually impossible for learners who have been disadvantaged by their early schooling to 'catch-up' later sufficiently to do themselves justice at the high school exit level." This is why Spaull and Kotze (2015, p. 13) call these learning deficits "insurmountable". It appears as if learners who do not master basic numeracy skills in the Foundation and Intermediate Phases have difficulty in engaging with the grade-appropriate curriculum and are unable to make progress in learning despite attending Mathematics classes.

1.2 At the Core of the Problem – Acquired Learning Deficits

Spaull and Kotze (2015, p. 15) define a learning deficit as "the difference between the actual performance of a student and some benchmark which is used as a reference category". It refers to an absence of learning and a lack of remedial attention rather than any lack of a learner's ability to learn. This is an important distinction to make as much of the global literature uses the term 'learning deficit' to refer to intellectual disabilities, sensory impairments and/or learning difficulties (neurologically-based processing problems). The commonly used term in international literature to refer to poor performance as a result of a learner's socio-economic conditions is an academic achievement gap. As Reardon (2011) discusses in his analysis of academic achievement among the rich and poor over the past 50 years, as the income gap between high- and low-income families has widened, so has the academic achievement gap. This academic achievement gap is more akin to the South African learning deficit although the socio-economic issues here stem directly from our political past and this has had a massive impact on not only learners, but also on teachers, schools and education department districts.

I am referring to learning deficits in the same way as Spaull and Kotze (2015) but I am rather using the phrase 'acquired learning deficits'. In the field of Biology the word 'acquired' refers to an environmental characteristic, one that is not an inherited characteristic. The addition of this word emphasises that the learning deficits are as a result of underperformance of the South African education system rather than any underperformance on the part of the learner. Sadly the acquired learning deficits are also accumulated learning deficits. Decades ago, Taylor in *Falling at the First Hurdle* (1989) and Taylor, Muller and Vinjevold in *Getting schools Working* (2003) were commenting that in South Africa, historical, socio-economic and geographical factors have negatively influenced the learning experiences of many learners such that they are unable to perform at the expected grade-appropriate levels.

Spaull and Kotze (2015) set out in their research to measure these learning deficits in years' worth of Mathematics learning and to see how these learning deficits change as learners proceed through Grade-levels. Their findings confirm "the learning gap between the poorest 60% of students and the wealthiest 20% of students is approximately three Grade-levels in

Grade 3, growing to four Grade-levels by Grade 9" (p. 13). In South Africa, only the top 16% of Grade 3 students are achieving at the Grade 3 level. Clearly the vast majority of South African learners are not meeting the Mathematics curriculum requirements even at the very start of their journey through the schooling system.

The early origins of mathematical difficulties have been documented in other countries too. Professor Nancy Jordan's studies in young children in the U.S.A has relevance for the South African context. Jordan specialises in early predictors and interventions for children from disadvantaged, low-income families, where their delays and deficiencies in number competence link to what she calls "diminished learning opportunities" (Jordan & Levine, 2009). Jordan's work focuses attention on the importance of developing early number competence for later success in Mathematics. Foundational number competencies develop early in life so without the help of early interventions, many disadvantaged learners are almost certain to experience what Jordan describes as ''a cascade of mathematics failure'' in elementary school and may have great difficulty catching up to their peers who have had more mathematical input in their early childhood (Jordan *et al*., 2009).

I have applied the evidence of the early emergence of learning deficits and the diminished learning opportunities they can cause to the design of the assessment ***as*** learning – conceptual thread approach. The premise is that tracing learners' origins of their mathematical difficulties can enable the teacher to facilitate the learners' going back and building their understanding from their last stable place within the conceptual thread. This can fast-track mastery of the basics of Mathematics even if interventions are made later in a learners' school career, as is often the case in the South African context.

1.3 Tackling the Problem

In 2012 I experienced first-hand the challenge of working with learners who seemingly had acquired insurmountable learning deficits since starting school. In late 2011, working as an educational consultant, I was employed by Uplands Outreach to monitor, evaluate and report on an upcoming project to be called Learners For Excellence (L4E). This project was born out

of an existing Uplands Outreach project called Teachers For Excellence (T4E) which, in 2012, was beginning its third year of existence. The T4E project was a teacher professional development project involving 45 Mathematics teachers, managed and supported by Uplands Outreach in partnership with the University of Johannesburg. Most of the Maths teachers were working in the Insikazi circuit of Mpumalanga not far from Uplands College, White River, where Uplands Outreach is based.

T4E teachers selected five Grade 10 Maths learners from each of their schools which, as there are 10 high schools in the district, resulted in 50 learners to participate in the planned, three-year Uplands Outreach Mathematics learner enrichment Saturday programme. The L4E programme was originally conceived to focus only on grade-level Maths – to supplement, improve and challenge learners to achieve Maths results at the end of Grade 12 that would enable them to enter tertiary education. My original involvement in the L4E project did not include any teaching of Mathematics. Two achievers in the T4E project (they had successfully completed an Advanced Certificate in Education in Mathematics over the two years before) were tasked with teaching Grade 10 Maths to the learners each Saturday.

An Uplands College Maths teacher was asked to provide a baseline test to administer to the L4E learners in the first meeting. I disagreed with the suggestion to use a summative Grade 9 test as a Grade 10 baseline as I believed that it would provide no diagnostic data. I was concerned that the learners would be unable to answer any of the questions and then we would be unable to pick up where they were in their Maths learning. I undertook to design a baseline test that covered the five General Education & Training (GET) Maths topics that benchmarked achievement at the Grade 3, Grade 6 and Grade 9 levels for a deeper analysis of their existing knowledge. For the purposes of a more detailed understanding of the learners' issues, I separated rational numbers from the topic Numbers, Operations & Relationships and included an additional 'topic' to test problem-solving skills.

After the learners achieved extremely disappointing results on the baseline test, my role in the project changed completely. I was tasked with designing and implementing an evidence-based teaching intervention which would address identified learning deficits and conceptual

difficulties. The challenge was to try and fix these issues in the shortest possible time as Grade 10 instruction was to proceed concurrently as planned. For this reason I was allocated only a one-hour weekly contact session with the learners.

Considering that disadvantages in early schooling cause learning deficits later (Taylor, Muller & Vinjevold, 2003; Spaull & Kotze, 2015), and my need to fast-track these Grade 10 learners, I chose a learning trajectory – learning progression approach in order to correct important conceptual landmarks. I called these trajectories or progressions 'conceptual threads'. Each of the seven conceptual threads I identified run from Grade 1 to Grade 9 through the activity classified as Mathematics. I discuss this approach in detail in Chapter Three.

As a proponent of formative assessment, my approach included using *assessment **as** learning* to fast-track learning progress along these conceptual threads, to engage learners in the analysis of their own Maths learning and empower them as partners to build on their identified strengths, correct their misconceptions and construct their own understanding.

As my component of the L4E programme had not been part of the initial project planning and as the learners were already in Grade 10, the priority was still to cover the Grade 10 content over the course of the year. There was actually no time to cater for earlier grade work. It was for this reason that I chose the strategy of *embedded* assessment ***as*** learning. This meant that self-, peer- and group assessment activities were part of every lesson. This offered time-specific benefits: as the learners were older, I believed that their metacognitive engagement and their shared responsibility would save valuable contact time and would also fast-track our progress and efficiently utilise the limited time we were given in the programme timetable. I discuss this strategy in detail in Chapter Three.

I had experienced the benefits of an embedded assessment ***as*** learning approach myself in managing large groups of learners in earlier work projects. I believed that this approach could offer great value to South African teachers. Our teachers routinely face large classes and the expectation is that the teacher should be aware of what each learner does and does not understand in order to adapt their teaching to meet specific learner's needs. With classes

sometimes of more than 50 learners, many teachers barely even know the names of the learners in their class. The assessment ***as*** learning approach promotes learners taking responsibility for their own learning and actively engaging with their own requirements to construct their own knowledge and to chart their own progress. Learners partner with their teacher in addressing their learning needs thus sharing the load and easing the burden on the teacher when the class comprises of large numbers of learners.

1.4 Focus and Purpose of the Study

The L4E programme ran for three years between February 2012 and November 2014. Data was gathered continuously throughout the course of the programme. This data was regularly analysed and reflected upon by the Uplands Outreach team. The data was used developmentally to modify and fine-tune the design of the course and activities to optimise the impact of these on learning and achievement. Over the period of the L4E intervention, my focus was on working with the learners to share this data and to use it to build their understanding of basic mathematical concepts (mostly addressing existing misconceptions) in order to make the content of FET Maths accessible to them.

This research study was undertaken retrospectively as an opportunity to provide greater understanding of what changes had actually occurred amongst the learners; specifically, their development and progress in Mathematics, and generally, their learning skills, work attitudes and behaviours. The focus and purpose of this study is to investigate the value of this assessment ***as*** learning – conceptual thread approach in catching up and fast-tracking learners with acquired learning deficits in Mathematics. My early thoughts were: are these learning deficits really “insurmountable” or not? If they can be overcome, over what time period can this be achieved? To what extent can it be achieved? What other learning benefits does this approach provide?

This pondering has been formalised in the following research questions:

What is the value of using an assessment ***as*** learning – conceptual thread approach in addressing acquired learning deficits in Mathematics in South Africa?

More specifically:

- Can a teaching and learning intervention using an assessment ***as*** learning – conceptual thread approach 'make up' acquired learning deficits in Mathematics and 'catch up' low-achieving learners?
- Which obstacles to mastering conceptual landmarks in learning about numbers and number operations are revealed by the approach followed?
- To what extent can this approach change learners' self-evaluations of their own understanding of Mathematics?

1.5 Structure of the Dissertation

I have started by describing in Chapter One the problem of widespread poor performance among our school-leaving learners and how the origins of this problem lie at the very start of learners' academic careers and cause acquired learning deficits. The L4E participants, coming from a rural and impoverished community, had experienced being at this lower end of South Africa's vast educational disparity.

In Chapter Two I tell the L4E story. The L4E programme was a life-changing experience not just for the 50 learners but also for the Uplands Outreach team and a special bond was established between us all that has lasted well past the end of the programme. I outline the programme from its origins in late 2011, its official start in 2012 through to its official end in 2014 but also continue through to the present – the beginning of 2017 as the Uplands Outreach staff and all 50 L4E participants are still in regular contact.

Chapter Three presents the assessment ***as*** learning – conceptual thread approach taken and places this in the context of published research related to the important aspects of the study. It clarifies how this research was used in developing a conceptual framework for the teaching and learning intervention in its design, implementation, evaluation and use.

As the data for this study was collected for a different purpose to that of this study, Chapter Four explains the retrospective research design, methodology of the study and discusses the which data was incorporated in the study and which was not. Data was both quantitative and qualitative and the tools, collection, analysis and comparative use of this is also included in this discussion.

Chapters Five, Six and Seven focus on results and discussion for each of the three sub-questions of the study's main research question. I have used a similar format for each chapter; presenting where the cohort of learners started, and then tracking their advancement over the study period to where they ended. Each chapter concludes with an overall comparison from beginning to end to establish their final progress.

Chapter Five presents the argument that acquired learning deficits need not be insurmountable and can be made up even at the late stage of Grade 10 using an assessment ***as*** learning – conceptual thread approach. Chapter Six presents opportunities for teachers to gain insight into learners' learning through the use of this approach. Chapter Seven presents opportunities for learners to activate and advance their metacognitive skills through using such an approach so as to set them on a road to independent, life-long learning.

The final chapter, Chapter Eight, looks back on the part of the L4E intervention that falls within the research study time constraints and addresses the overall research question and relates my findings to the broader literature. While not only reflecting on the learning made, I consider the strengths and weaknesses of the research and identify opportunities missed (particularly through this being a retrospective study). I also discuss the implications of this work for practice and highlight interesting findings that lead to further questions and inspire me to pursue further research.

CHAPTER TWO

The L4E Story

2.1 The Year 2012

2.1.1 Background to the L4E Programme

Nestled between the world-famous Kruger National Park, the town of White River and the KMIA international airport is an impoverished and neglected South African community. Education in this part of the Lowveld is provided through the Insikazi Circuit of the Ehlanzeni Region of the Mpumalanga provincial government. There are 10 high schools in this Mpumalanga Education circuit which in 2011 held the unfortunate distinction of being one of the worst performers in the province in the Grade 12 NSC results. In fact, in 2010 only 131 learners in the entire circuit passed Grade 12 NSC Mathematics. The Insikazi circuit is the target of Uplands Outreach's endeavours.

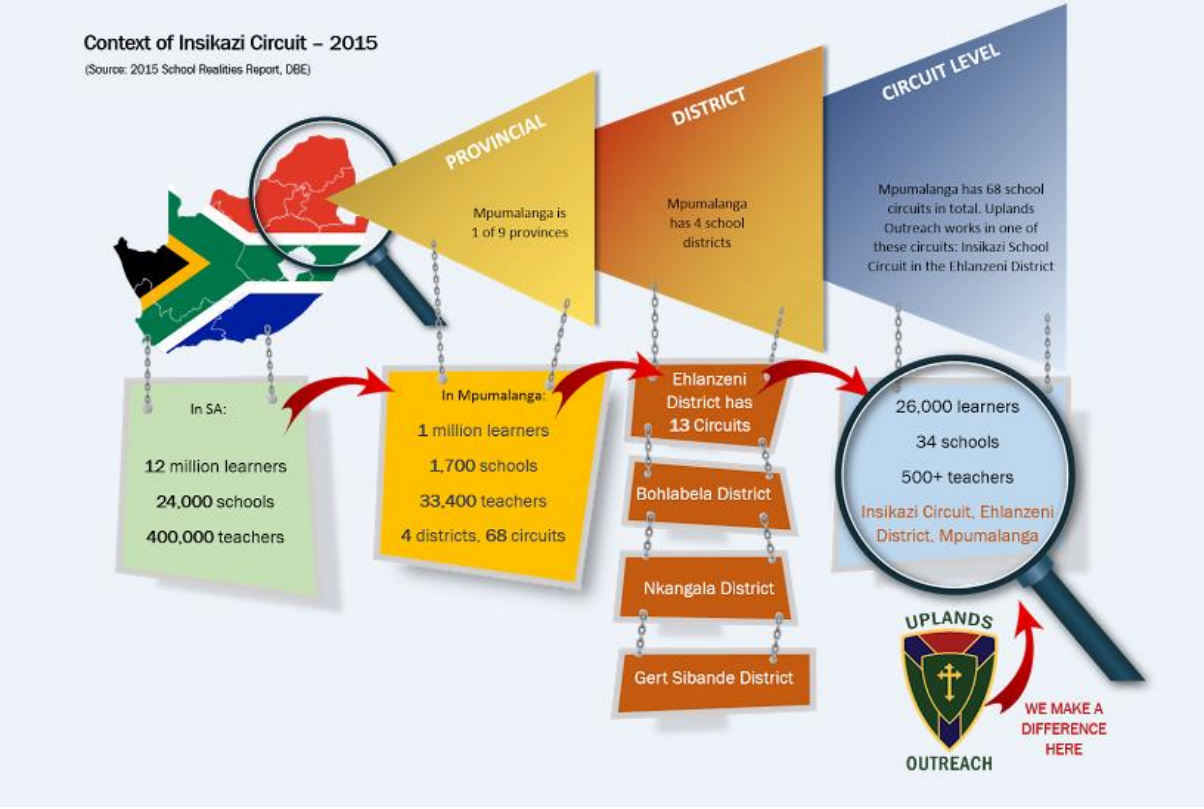


Figure 2.1 Context of Insikazi Circuit (http://www.uplandsoutreach.org/index.php/our-team-2/national-context-of-work

At the start of 2012 a new Circuit Manager had been appointed to turn around the performance of the Insikazi circuit schools (ten high schools and 24 primary schools). Uplands Outreach (in partnership with this new Circuit Manager and with funding from Rand Merchant Bank) focused attention on the design of an intervention to improve Maths proficiency in the ten Insikazi high schools to prepare learners for future tertiary studies.

Five promising Grade 10 Maths learners from each school (that is 50 learners) were targeted to participate in a 3-year Mathematics learner enrichment Saturday programme called Learners For Excellence (L4E). The programme was to begin in January 2012 and to end in November 2014 when the learners completed Grade 12. Five learners per school were nominated by their teachers who were already involved in the Uplands Outreach T4E programme. The teachers based their selections on the following provided criteria:

1. Learners must be in Grade 10 at the start of 2012;
2. Learners must demonstrate an exemplary work ethic;
3. Learners must have an appetite for success and have a solid academic record;
4. Learners must demonstrate an aptitude for Maths;
5. Learners should indicate an interest in the STEM careers: Science, Technology, Engineering, and Maths;
6. Learners must submit an application form which includes a written motivation on why the learner would like to participate in the L4E programme.

The final list of participants were mainly female (28 girls compared to 22 boys). Learners ranged in age from a very young 13 years to much older 17 years with an average age of 15.1 years in January 2012. The socio-economic challenges of the region were evident in the fact that only 2 learners had both parents in formal employment and 26 of the 50 learners qualified as Orphans and Vulnerable Children (OVC). In South Africa the classification of an orphan is applied to a person younger than 18 years of age for whom both parents are deceased or absent. A vulnerable child in South Africa is described as one who is neglected, destitute, abandoned and/or displaced; and/or one who is abused or ill-treated; has a terminally-ill parent or guardian; is born to a teenage or single mother; whose caregiver lacks income-generating opportunities; and/or is disabled (Smart, 2003).

2.1.2 Establishing a Baseline

As already mentioned, the original plan was to use an Uplands College Grade 9 summative examination produced by the Grade 9 Uplands College Maths teacher as a baseline measure for the L4E Learners. I had expressed concern that it would not provide any diagnostic data should the learners perform poorly on the test. To avoid this situation I undertook to develop a diagnostic baseline test that would identify any learning deficits just in case remediation was required.

The design of the diagnostic baseline test was to assess seven important conceptual threads that run through Primary School Mathematics. These conceptual threads will be elaborated on in Chapter Three but mastery of these is necessary for success in High School Maths and beyond. The seven conceptual threads are:

- Numbers: Concepts, Relationships & Operations;
- Fractions: Concepts, Relationships & Operations;
- Patterns, Functions & Algebra;
- Space & Shape;
- Measurement;
- Data-Handling; and
- Problem-Solving: Logical, Deductive and Adaptive Reasoning.

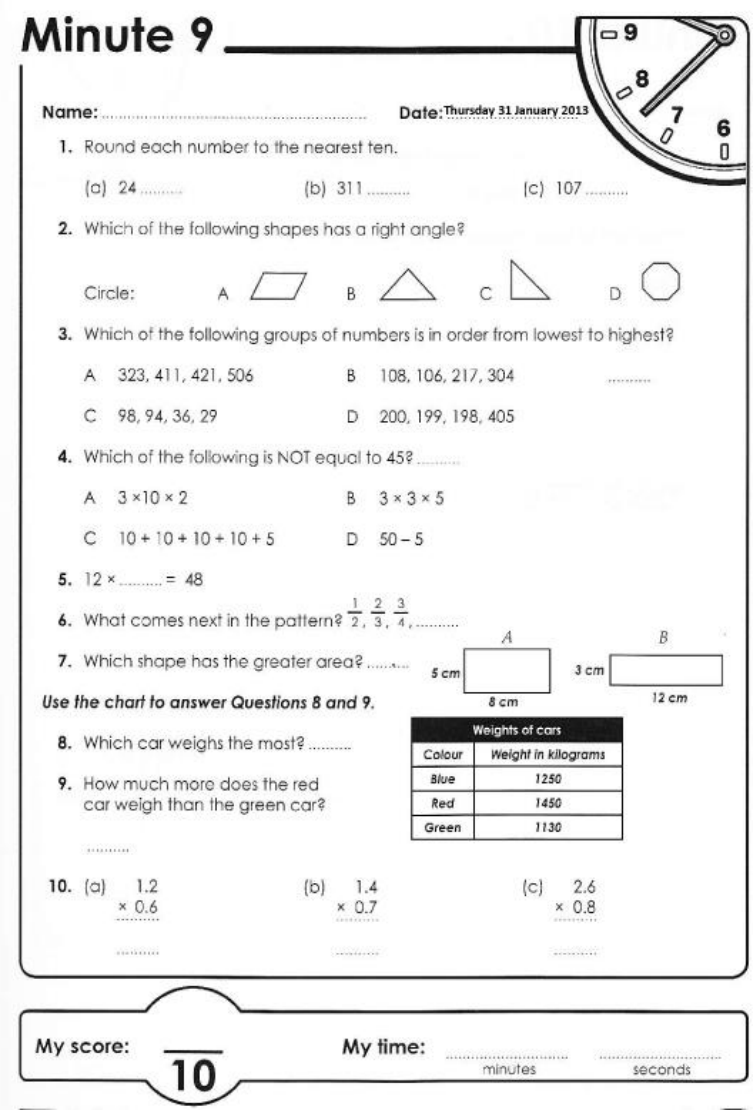
Even though the learners were in Grade 10, the questions were benchmarked against Grades 3, 6 and 9 levels to reveal the extent of any acquired learning deficits for fast and effective remediation.

The results of the baseline tests results were disappointing especially considering that these learners were chosen to participate in the L4E programme either because of their achievement, aptitude or interest in Mathematics. Overall, the learners achieved an average of just 23.8%.The highest achievement was 48% while the lowest was just 10%. In reality, this should not have come as a surprise because the South African Annual National Assessments (ANAs) had been administered for the first time in 2011 to Grade 3 and Grade 6 learners with disappointing results: 83% of both Grade 3 and Grade 6 learners in Mpumalanga were scored on the lowest level (described as 'Not Achieved') in the Mathematics ANAs. The following year when Grade 9 learners were included in the ANAs, those in Mpumalanga only achieved an average of 11.9%.

Closer scrutiny of the overall L4E results revealed that five learners (10% of the group) were performing below Grade 3 level, 38 learners (76% of the group) were performing below Grade 6 level, while seven learners (14% of the group) were performing above Grade 6 level but not yet at Grade 9 level. Results within the various conceptual threads were consistently poor with the Grade 10 learners performing between the Grade 3 and Grade 6 levels.

2.1.3 Gathering More Data

As I was following a formative assessment approach, I gathered more detailed evidence using the resource Maths minutes Book G (Stoffel, 2012). The book is a collection of short Maths exercises, each called a 'Minute' and comprising of 10 questions. I used it purely for the purpose of identifying conceptual strengths and weaknesses, not only for myself as the teacher, but mostly for the learners to realise these. Learners completed a Maths Minute exercise each day at home by themselves i.e. five Maths Minutes per week. This gave each learner the opportunity to demonstrate proficiency across the different primary school topics. Learners handed their completed work in each Saturday and this was marked by the next Saturday and returned to them.



Minute 9

Name: ..... Date: Thursday 31 January 2013

1. Round each number to the nearest ten.

   (a) 24 ..... (b) 311 ..... (c) 107 .....

2. Which of the following shapes has a right angle?

   Circle: A B C D

3. Which of the following groups of numbers is in order from lowest to highest?

   A 323, 411, 421, 506 B 108, 106, 217, 304 .....

   C 98, 94, 36, 29 D 200, 199, 198, 405

4. Which of the following is NOT equal to 45? .....

   A $3 \times 10 \times 2$ B $3 \times 3 \times 5$

   C $10 + 10 + 10 + 10 + 5$ D $50 - 5$

5. $12 \times$ ..... $= 48$

6. What comes next in the pattern? $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, ...

7. Which shape has the greater area? .....

   A: 5 cm, 8 cm B: 3 cm, 12 cm

Use the chart to answer Questions 8 and 9.

8. Which car weighs the most? .....

9. How much more does the red car weigh than the green car?

   .....

Weights of cars	
Colour	Weight in kilograms
Blue	1250
Red	1450
Green	1130

10. (a) 1.2 × 0.6 ..... (b) 1.4 × 0.7 ..... (c) 2.6 × 0.8 .....

My score: ___/10 My time: ..... minutes ..... seconds

Figure 2.2 An example of a Maths Minute daily exercise (Stoffel, 2012)

Initially I had two major challenges to overcome. Firstly, there was the learners' reliance on calculators, and secondly the issue of copying others' work. I banned calculators completely as learners were using these for the simplest of number manipulations. It was immediately apparent that, without exception, all of the learners had extremely limited knowledge of number strategies and manipulation techniques. They would add even single digit numbers on their calculators. Engaging number strategies such as decomposing, commutating, chunking, compensating, doubling and halving and presenting these as number tips, tricks and techniques would become the initial focus of the teaching intervention to secure the early conceptual landmarks along my Numbers conceptual thread. I found that their competence and confidence in manipulating numbers improved comparatively quickly and there was a

competitive enjoyment in the challenge of calculating quickly without a calculator. The other problem was more difficult to overcome. The learners' need to produce correct answers seemed to over-ride my requirement of them to 'own' their learning and make their own understanding. Many were initially copying answers from others (allegedly on the Saturday morning taxi ride to Uplands Outreach).

I spent a lot of time explaining diagnostic assessment and evidence-based teaching. I stressed that if the evidence learners presented was not reflecting their own understanding, then my intervention would not meet their needs but rather the needs of the person they were copying from. I dedicated a substantial amount of time to convincing them of the value of their errors as indicators for the path forward in their learning. I also found that most learners did not have a realistic view of their own mathematical abilities. For example, in a self-assessment of their ability to perform the four basic number operations, learners rated themselves with an overall average of 73.4%. In reality, when they performed a test of these basic number operations and scored themselves, they achieved only 42.2%. The assessment ***as*** learning approach meant that the learners were made aware of the conceptual landmarks required along the conceptual thread and they were also made aware of their own achievement of those landmarks. This resulted in much of the contact time spent analysing examples of errors and common misconceptions in order to activate and develop metacognitive skills.

Once the first 50 Maths Minutes were completed, marked and results recorded, the Saturday following the Term 1 break was allocated to performing a detailed error analysis of these Maths Minutes. I adapted a Rubricate Teaching & Learning Diagnostic Matrix Tool (an output of the Rubricate PLUS teacher development software package) to enable learners to analyse their own Maths knowledge and understanding. The full-length Saturday was used to implement a self-diagnostic activity across the primary school Maths concepts for the first 45 Maths Minutes. Each Maths Minute question was correlated to the appropriate conceptual thread i.e. ten questions per Maths Minute multiplied by the 45 Maths Minutes resulted in analysis of 450 questions.

Each learner recorded each of their incorrect answers on the Diagnostic Matrix tool by means of a tally in the appropriate box. Learners used the number of tallies in each box to classify themselves as practitioner (able to 'do' it), apprentice (beginning to 'do' it) or novice (unable to 'do' it) for each conceptual thread and colour-coded and marked these on the matrix tool.

Once all 50 Maths Minutes had been analysed, errors recorded and collated, learners identified their strengths and weaknesses and reflected on how to build on these strengths and eliminate their weaknesses. I found this process extremely valuable for providing learners with the vocabulary needed to describe their progress in learning. Learners were actively encouraged to take ownership of their own learning going forward and to pursue their own mastery of the concepts.

2.1.4 Designing the Teaching and Learning Intervention

Before designing the teaching and learning intervention, I studied the learners' self-analyses and self-reflections so as to highlight the most common areas of difficulty. I also studied the learning trajectories I had created for each conceptual thread in alignment with the CAPS curriculum when bench-marking grade-levels for the diagnostic baseline test to order areas of difficulty. This resulted in a teaching and learning plan that could be delivered weekly to address the problematic conceptual landmarks along my conceptual threads.

Table 2.1 Example of a teaching and learning plan for Numbers: Concepts, Relationships & Operations

SESSION	CONCEPTUAL LANDMARKS	NUMBER SKILLS & STRATEGIES
1	The Language of Maths – Numbers & Symbols	• subitising • identifying • comparing • ordering • estimating
2	Composition of Numbers – Place-Value	• base-10 composing • decomposing • unitising • estimating • rounding off
3	Number Operations – Adding & Subtracting	• counting (-on and -back) • skip-counting • collating • building up • breaking down • doubling • regrouping (or borrowing) • halving • chunking • scaling • known facts with fact-up and fact-down • times-table patterning • commutating • splitting • stringing • sharing • repeating • rounding up and down • approximating • estimating • compensating • using number bonds • using triples • using inverse operations • using arrays and matrices • using algorithm calculations
4	Number Operations – Multiplying by 2, 4, 10 and 5	
5	Number Operations – Multiplying by 3, 6 and 9	
6	Number Operations – Multiplying by 7, 8 and 11	
7	Number Operations – Multiplying by 2-digit and 3-digit numbers	
8	Number Operations – Dividing by 2, 4, 10 and 5	
9	Number Operations – Dividing by 3, 6 and 9	
10	Number Operations – Dividing by 7, 8 and 11	
11	Number Operations – Dividing by 2-digit and 3-digit numbers	
12	Mixed Operations – Order of Operations	

When planning the weekly sessions, the basic lesson design was as follows.

Step 1 – MY BEGINNINGS

Assessment **as** learning was practised in every lesson. Each began with a quick self-evaluation activity to establish existing knowledge and understanding around the targeted conceptual landmark. Sometimes they shared their insights with a partner and sometimes they just kept it to themselves but the purpose was always to be aware of what they already knew and where their opportunities for growth lay.

Step 2 – KICK START LEARNING

I often used a short video clip from my collection of resources to create some interest around the targeted conceptual landmark. Sometimes this posed a question that we could revisit at the end of the session. For example I downloaded a YouTube video presenting Divisibility Tests which starts by posing a problem of a group of friends splitting a restaurant bill equally between them. We paused the video to discuss ways in which we could quickly establish if a number such as 2, 3, 4, 5, 9 and 10 could divide into a large number without a remainder. After following the basic steps described below in relation to this concept, we returned to the video to compare the techniques for identifying factors of large numbers that were presented and to evaluate the solution to the friends' problem that was offered.

Step 3 – CONSTRUCT & CONCRETE

The focus was on mastering the Foundation Phase level of the targeted concept. I used concrete manipulatives to establish a basic understanding of the conceptual landmark (e.g. adding to find a total using Smartie sweets). All of the props I used appeared to be novel to the learners even though many were traditional teacher tools such as fraction circles, rods and tiles.

Step 4 – RECOGNISE & REINFORCE

The focus was on mastering Intermediate Phase level of the targeted concept. I used pen and paper exercises to apply the conceptual landmark in different kinds of activities (e.g. addition exercises presented in different written formats). A lot of attention was on strategies for solving problems which could be used in the next step. While there was sometimes drill and practise in this slot, there were also homework hand-outs for further practice which were scaled up to a Senior Phase level.

Step 5 – EXPLORE & EXCEL

The focus was on mastering Senior Phase level of the targeted concept. I presented real-life applications and problems for opportunities to interpret, solve and for investigating within the conceptual landmark.

Step 6 – MY PROGRESS

Each session ended with a self-assessment and/or self-reflection of the learning achieved. This was mostly compared with the self-evaluation at the start of the session. Sometimes this was shared with a partner, sometimes they did it on their own.

2.1.5 Implementing the Teaching and Learning Intervention

The teaching and learning intervention (then called Maths Journey) took place once a week within the L4E Saturday roster of activities. The learners were split into two groups of 25 learners in each and each group had a session of one hour contact time with me.

Classroom activities were interactive and encouraged participation by all learners. However these activities were also differentiated with supplementary practice exercises available for learners who had already mastered targeted skills so that they could choose to work independently at their more advanced level. Daily customised homework in the form of further Maths Minutes or other exercises was set for each weekday in-between the Saturday sessions at Uplands Outreach. Marked homework from the week before was returned, analysed and compared and was often the focus of the start-up self-evaluation activity. Learners were always encouraged to focus on those questions that they had got wrong and to work out why they had got it wrong for improved self-evaluation. Common misconceptions and errors were incorporated into weekly sessions where it was appropriate. Learners analysed and reflected on their feedback to develop their metacognitive skills. They were encouraged to identify, understand and conquer their own specific misconceptions and errors to attain Maths mastery.

The teaching intervention was implemented in such a way as to teach and learn through the tracking of a specific conceptual thread starting with Foundation Phase knowledge, through Intermediate Phase to mastering at Senior Phase level.

2.1.6 Measuring Catch-up

As an integral part of the teaching and learning intervention, the initial diagnostic results were shared with the learners to serve as a blueprint for personal remediation. The customised feedback and identification of individual conceptual difficulties were designed to enable metacognitive activation and facilitated the learners taking responsibility and ownership of their personal learning development.

Ongoing learner data was collected every week and shared with the learners in order for them to establish their progress along the conceptual thread. Error analysis and self-reflective skills were practiced on weekly exercises and homework. A formal test of all conceptual threads was administered in June 2012 which enabled an interim overview comparison for the progress made.

In September 2012, learners wrote a summative test which covered five of the conceptual threads. These results are presented in detail in Chapter Five to report on progress made in catching up the learners' acquired learning deficits. The L4E programme finished the first year with a 100% retention rate and a 96% attendance rate for the 27 Saturday sessions. Interestingly, at the start of the year, the average Maths term mark for the L4E learners at their schools was only 13.2% above that of peers (based on their Grade 9 final results at the ten schools), but this difference, according to their school data, had doubled to 26.8% by the end of Term 3 (September 2012). This was despite 55% of their peers dropping out of Maths due to their inability to pass it.

2.2 The Year 2013

2.2.1 Adapting the Teaching and Learning Intervention

One of the challenges the Uplands Outreach team faced at the start of the 2013 was the dilemma of what to do with two learners who had not passed Grade 10. Both were girls who had produced babies in the latter half of their Grade 10 year and both had attended L4E sessions right up until giving birth. Both girls wanted to continue attending each Saturday and so we employed a tutor to assist them with Grade 10 Maths while the rest of the learners moved onto Grade 11 content with their T4E teachers. They shared all other L4E experiences with their peers.

In my Maths Journey component I felt that, of the seven conceptual threads tested at the start of the programme, five had been satisfactorily addressed in 2012. Some threads such as Numbers had taken the majority of the contact time while others such as Data-Handling had required only a session or two, proving to be quick and easy to fix. Two of the conceptual threads (Fractions and Problem-Solving) were still extremely difficult for almost all of the L4E learners. Their understanding of Fractions was so weak that I had postponed tackling this conceptual thread until we had completed Numbers. Problem-Solving I had naively believed was poor simply because the learners had not been exposed to this. As a time-saving strategy, I had decided in my early planning to introduce a 'Weekly Challenge'. Every week I provided a different type of logic or reasoning problem on a printed page. Learners took the problem/puzzle home with them and during the week they could seek assistance from family members, friends or their teacher to find the solution to the problem. They were expected to write down this solution with a clear explanation on the printed page I had provided and return it the following Saturday. I would collect their responses on arrival each week. After my sessions with each group and while they were attending their Grade 10 tuition, I would mark their responses and all correct responses would earn a large bar of chocolate at the final assembly before they boarded the taxis to go home.

While there was no penalty for not handing a puzzle in each week, I kept a record of who had completed puzzles and who had not. I also recorded who had successfully solved the puzzles and who had not. I had provided 20 weekly puzzles over the course of 2012 and was delighted that on average 94.4% of learners handed these weekly puzzles back in to be marked. This had reinforced my belief that simply being exposed to Problem-Solving would improve it. However, in this I was incorrect. On average over those 20 weekly puzzles, only 38.9% of learners solved them correctly. While this was an improvement on the 14% achievement in Problem-Solving in the diagnostic baseline, I was disappointed in the lack of progress. Along with the Uplands Outreach team, I decided to continue with my one-hour weekly contact sessions in 2013 in order to address conceptual difficulties with Fractions and Problem-Solving.

2.2.2 Addressing the More Resistant Acquired Learning Deficits

I used the same kind of teaching and learning intervention plan for Fractions as I had used for the other five conceptual threads. My lesson plan for each session moved from Foundation Phase level concrete concept building, through Intermediate Phase level reinforcement through practicing using a conceptual landmark; to Senior Phase level of application to investigate a concept more deeply. Again I found that using concrete learning aids enabled the important breakthroughs.

To address Problem-Solving I invested in a number of logic and reasoning games. I adapted these to make customised group packs that provided a start-up practice problem for the group to solve in order to understand what was required to solve the problem. Each learner was also provided with their own problem to solve and they were not allowed to ask for assistance until three minutes had elapsed. Each pack also included extra problems for the learners that had requested peer assistance to have another try at solving the problem type on their own. The games incorporated many different problem-solving skills such as visual, spatial, sequencing, conditional, eliminative, abductive and deductive logic and reasoning. Learners were organised into their school groups of five and the different games were swapped and rotated group by group each week.

2.2.3 Measuring Catch-Up

Summative data for Fractions and Problem-Solving was collected in June 2013 after these conceptual threads required further teaching and learning input to master. Comparing the results of the diagnostic assessments administered first in February 2012 and then summatively in September 2012 for five of the conceptual threads, and in June 2013 for the remaining two conceptual threads, the results were presented in project reports to show the learners' progress and the success of the self-reflective teaching and learning intervention.

I was very excited by the improvements made in such a short period and started to believe that these so-called "insurmountable" learning deficits acquired in a poorly functioning education system could actually be overcome with a self-diagnostic, reflective and metacognitively activated approach. I was encouraged by the fact that achievement in all conceptual threads was vastly improved and projections indicated that within a two year intervention, even with just one hour per week contact time, the acquired learning deficits in all conceptual threads could even be eliminated.

From the experience so far, I was convinced that I was on track, and that using embedded assessment ***as*** learning to direct and adapt teaching along learning trajectories could measurably improve Maths achievement. Assessment ***as*** learning could be used to actively engage learners in their own learning progress and to provide a time-effective method to catch-up learning deficits along learning trajectories. For the second year in a row the L4E programme finished with a 100% retention rate and a 96% attendance rate for the year's 27 Saturday sessions.

2.3 The Year 2014

At the end of 2013, I moved away from Mpumalanga to live in the Western Cape. Despite the sadness of leaving the project, I felt that the learners were no longer in need of my input in terms of making up acquired learning deficits. As they were now mostly in Grade 12, the focus of the L4E programme was shifting to exam preparations and post-school planning such as applications for tertiary study, the building of CVs and practising of interview skills. Many learners needed assistance in obtaining ID books. I continued to play a role remotely producing weekly study-aids for Maths and Physical Sciences through 2014 and I returned to Uplands College for the end of year celebrations. By the end of the programme, eight of the 28 girls had delivered babies during their FET studies but despite this, the L4E learners had achieved a 96% attendance over the three years which was an extraordinary accomplishment. Both L4E learners and Uplands Outreach staff waited in anticipation of the L4E learners' final NSC results. We were not disappointed. 98% of the learners graduated their class of 2014 with 88% of these learners achieving marks high enough to access tertiary studies. The L4E programme has been a life-changer for almost all involved. The top L4E learner achieved 94% for Mathematics and was a Top Ten achiever in the whole of Mpumalanga province (a massive achievement for the impoverished Insikazi Circuit). Three other L4E learners achieved distinctions in Maths.

In summary the achievements were:

- 96% retention rate of the cohort over three years, with a 96% attendance average, for the third year in a row;
- 98% of the cohort passed their NSC matric overall, versus 75.8% nationally;
- 74% of the cohort achieved Degree Access, versus 28% nationally. In total, 88.4% of the cohort were eligible for tertiary studies;
- In terms of Maths, 93% of the cohort passed Maths, against a declining national average of 53.5%. In addition to this, 63% of the cohort achieved Maths results higher than 50% (as an achievement mark), versus only 26.7% in the circuit;
- 9.3% of the cohort achieved a distinction in Maths, versus only 3.2% nationally.

2.4 Current Update

Much of the last year of L4E was spent trying to secure bursaries and finance for tertiary studies. Eleven learners wrote the National Benchmark Tests (NBT) to improve their opportunities. This enabled more than two-thirds of the learners to enrol in universities and colleges for 2015. The top L4E learner earned a full Glencore bursary to study Engineering at the University of Pretoria. Two of the Maths distinction achievers were accepted to study B.Sc. degrees at Wits University. The fourth Maths A-achiever was one of five learners to qualify for an Independent Schools Association of Southern Africa (ISASA) bursary for a B.Ed. and to intern at Uplands College as a Maths teacher. Other high achievers were dispersed around the country: two studying law at Rhodes University; two studying at the University of Johannesburg; others at the University of North-West, University of Mpumalanga, Tshwane University of Technology, Cape Peninsula Technicon and University of South Africa (UNISA). The greatest achievement was that all L4E learners in tertiary study (100% of them) passed their crucial first year.

In 2016, additional learners were able to access tertiary study which saw 46 of the 50 graduates enrolled in diverse fields such as Medicine, Nursing, Paramedics, Chemical Engineering, IT, Law, Human Resources, Agriculture, and, most excitingly, in Education. Eleven learners are studying to be teachers and five of these are studying to be Maths teachers. As at January 2017, the bulk of the learners are about to start their third year of tertiary study and are on track to contribute to South African Higher Education statistics with 100% retention and 100% through-put at their tertiary institutions so far. Only one of the original 50 learners is not in education, employment or training, as opposed to the appalling national NEET average of 70.4% for youth between 15 and 24 years of age. One of the learners who is in his third year of a B.Sc. in Pharmacy at Wits has started his own Non-Profit Organisation to share his L4E experience and expertise and to assist others in his community.

The success of the L4E programme, particularly the evidence that acquired learning deficits are not insurmountable and catch-up can be achieved even at the late stage of Grade 10, has motivated me to dissect and analyse the approach of the intervention. This is presented in Chapter Three which follows.

CHAPTER THREE

An Assessment *as* Learning – Conceptual Thread Approach

When comparing the L4E programme to other South African Saturday Mathematics enrichment programmes, the unique feature of L4E was the focus on developing reflective practice. This feature was dependent on the adoption of embedded formative assessment and specifically assessment ***as*** learning in order to fully engage each learner in their own learning process and progress. I pursued this approach for the benefit of metacognitively activating learners so that, through shared responsibility, their catch-up could be accelerated. I supported the assessment ***as*** learning approach with the use of conceptual threads to enable learners to see the 'big picture' of learning Maths and to be able to identify their own learning gaps within that picture and to understand how their weaknesses were impacting on other related concepts in Maths. It was a further strategy for sharing responsibility for learning to empower the learners to understand how they could build their own understanding in Maths. In this chapter, I will take each of these two aspects of the approach followed in the L4E programme and position these within the existing related body of research while also showing how it was customised for the South African context.

3.1 The Assessment *as* Learning Part of the Approach

3.1.1 Formative Assessment

The word 'assessment' comes from the Latin *assidére*, which means 'to sit by as an adviser judge'. Despite the reference to a role of advising, we have narrowed the use of the word assessment to mean to 'judge' or 'measure' something. When considering the history of assessment practices, Wilbrink (1997) traces a fascinating path through assessment traditions dating back to medieval times. With teaching and learning centred in madaris, monasteries and convents, the focus was on learning religious texts off by heart so that they could be repeated verbatim. This resulted in assessment activities requiring correct recitation and the prescribed answers to simple and straightforward questions about the texts. These

archetypes of assessment still persist today. Wilbrink describes how standardised tests only focus on and count the proportion of correct answers. He bemoans that "the difference between modern and medieval testing seems to be mainly that not the salvation of one's soul but that of one's career depends on producing the right answers" (Wilbrink, 1997 p. 33). Assessment today still largely relies on measuring the performance of learners on controlled-condition tests to determine the extent of learning by the end of a learning year (often termly too). This is summative assessment which is also known as assessment ***of*** learning. We rely on these high-stakes tests to inform our decisions about accountability. Among learners, who will pass and progress; who will fail and repeat; who will win the prize. Among teachers, who has met their performance standards and so done the job they were hired to do; and who has not.

Scriven (1966) is credited with being the first to use the word 'formative' to describe assessment that differed in its role of evaluating a process for revision and improvement before the final product would be subjected to summative assessment. More than thirty years later, Black, Harrison, Lee, Marshall & Wiliam (2002) revived this idea of assessment that is primarily designed for the purpose of promoting learners' learning. As part of the Assessment Reform Group (ARG), Black and Wiliam (1998b) reviewed existing research in formative assessment and reported that the research showed conclusively that learning was improved through formative assessment. In fact they noted that the gains in achievement were amongst the largest ever reported for educational interventions. Black and Wiliam (1998a) also claimed that assessment that is designed to promote learning is a valuable tool for empowering life-long learning. The value of assessment ***for*** learning was emphasised further when Wiliam (2011) followed up these ideas with "all teaching really boils down to three key processes: finding out where the learners are in their learning; finding out where they are going; and finding out how to get there" (p. 45). Wiliam describes assessment as the bridge between teaching and learning. This is an important reminder that just because a teacher teaches something, does not mean that a learner learns it.

The work of the ARG focused on using four main classroom behaviours to shift assessment practice in schools towards a more formative nature. These are: questioning to promote

dialogue – teachers increase wait time, prevent learners raising hands to answer questions, and create a supportive environment for providing and exploring wrong answers; feedback to cause thinking – teachers focus on comments rather than marks to identify what has been done well, what still needs improvement and guidance on how to make that improvement; peer-assessment and self-assessment to engage learners – teachers share assessment criteria and teach reflective habits and collaborative skills; the formative use of summative tests – the aftermath of a test provides formative opportunity for each learner through reflective review to actively improve their learning.

Recognition of the value of assessment ***for*** learning has grown over the past 18 years. It was in 2002 that the ARG identified assessment ***for*** learning as a research-based principle that is central to classroom practice and a key professional skill for teachers (ARG, 2002). Almost ten years later Popham (2011) claimed that formative assessment well-implemented can double the speed of student learning. Husbands and Pearce (2012) included embedded assessment ***for*** learning as one of their claims from research of what makes great pedagogy. More recently Hofman, Goodwin and Kahl (2015) described how formative assessment (stipulating again that which is well-implemented) can have a multiplier effect on learning by shrinking achievement gaps and increasing educational efficiency.

3.1.2 Formative Feedback

The shift in assessment practice to include formative activities was not confined to the UK. John Hattie (2009a) warned university lecturers that what he called the "formative assessment revolution" that had swept through New Zealand's school system was imminent in higher education. Fundamental to successful formative assessment is the quality of formative feedback provided. It is for this reason that I am discussing feedback in a separate and distinct section. Hattie has focused attention on the power of feedback from assessment and he has identified it as paramount to making a difference to student outcomes. Hattie had authority to say this based on his more than 15 years of collecting nearly 1,200 meta-analyses of 65,000 education studies focused on the learning of more than 250 million students around the world, in order to establish what has the greatest influence on student learning.

Hattie reduced his influences on learning to effect-size measures in order to compare them on one scale. He has categorised these many influences or factors into different contributors of learning. Assessment practice falls within the category of teaching approaches. The individual factors or influences researched in assessment are shown in Table 3.1.

Table 3.1 Assessment Effect Sizes on Achievement (Hattie, 2009b)

Influence	**Effect Size**
Self-reported grades	1.44
Feedback	0.72
Providing formative evaluation to teachers	0.70
Frequent/ Effects of testing	0.46
Teaching test-taking skills	0.22

The influences at the top of the list reinforce the value of formative assessment practices in general and formative feedback specifically. Self-reported grades refers to a practice of building student expectations whereby students predict how they will perform on assessment tasks and then work with the teacher to meet and exceed this. Feedback is shown as a separate powerful influencer in the table above but it takes many forms in practice. Feedback to learners should focus on what they need to do to improve, rather than on how well they have done, and should avoid comparison with others (Wiliam, 2012). The emphasis of feedback as that which a teacher provides to a learner is evident in Shute's (2007) review of research on feedback. However Hattie and Timperley (2007) emphasise that feedback is not only what students receive from their teachers. Hattie and Timperley identify the most valuable feedback as that given from the student to the teacher. This feedback allows teachers to see learning through the eyes of their students. It is this feedback *to* teachers that makes learning visible. When teachers seek, or at least are open to, feedback from students as to what students know, what they understand, where they make errors, when they have misconceptions, when they are not engaged, then the teaching process and the learning process can be synchronised and powerful. This is what Fisher and Frey (2007) called "feed forward" – feedback that teachers use to modify instruction. The influencers at the bottom of the table show that frequent testing and teaching test-taking skills do not advance learners.

3.1.3 Assessment ***as*** Learning

Thus far I have outlined two kinds of assessment practice: assessment ***of*** learning and assessment ***for*** learning. Assessment ***of*** learning is the historically familiar practice which provides teachers with evidence of learning in order to judge achievement against learning targets, standards or outcomes for public reporting and decision-making about placement and promotion. Assessment ***for*** learning enables teachers to use information about learners' knowledge, understanding and skills to inform their teaching and to provide feedback to learners about their learning.

Earl (2003) made a further distinction between *assessment **for** learning* and *assessment **as** learning.* Assessment ***as*** learning is a subset of assessment ***for*** learning in which the learner is an active participant in a transparent assessment process that develops and supports his/her metacognition.

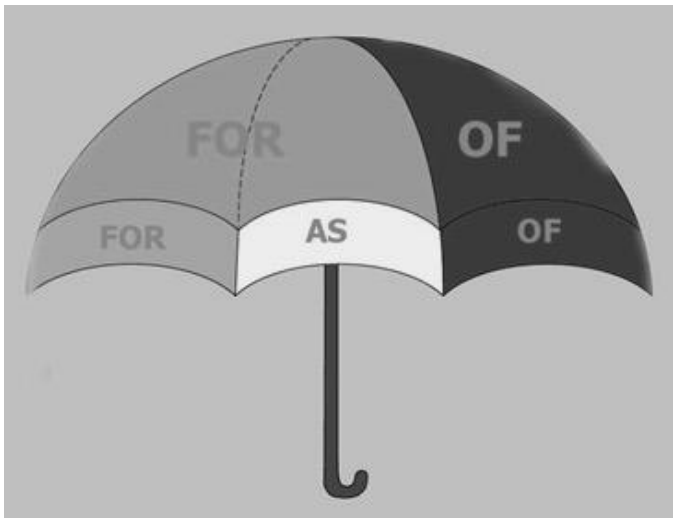


Figure 3.1 Rethinking Assessment Approaches (Earl, 2009, slide 15)

Assessment ***as*** learning involves the learners reflecting on and actively monitoring, analysing and correcting their own learning. Learners use formative feedback to readjust and improve their own learning as they develop and use metacognitive skills. According to Earl (2003), in assessment ***as*** learning, the learner is not only a contributor to the assessment and learning process but is the critical connector between these processes.

White and Frederiksen (1998) conducted a study in which groups of students participated in self-reflective assessment ***as*** learning activities in order to develop and practice metacognition. They worked with 12 classes each of 30 students in various grades who were learning physics using the same inquiry-based approach. Each class was divided into two groups, an experimental or control group. Both groups were taught a unit on force and motion. The control group spent time in each lesson discussing the work while the experimental group spent the same time considering assessment criteria and using these in self- and peer assessments. The researchers called this latter practice "reflective assessment". The results were dramatic, with the experimental group outperforming the control group on three different assessments. The greatest gains were made by those who were previously the lowest achieving students. In fact the weakest students in the experimental groups performed as well as the best students in the control groups and the other experimental students did even better. The achievement gaps in the experimental groups were reduced by half compared to the control groups. They even found that the Grade 8 experimental group performed better than the Grade 12 control group. White and Frederickson concluded that the reflective assessment approach had the valuable effect of reducing the educational disadvantage of low-achieving students while still also being beneficial for high-achieving students. They argue that their findings have great significance for what a metacognitively-focused curriculum can achieve in school settings with high proportions of disadvantaged students.

Earl's assessment ***as*** learning is differentiated by its metacognitive focus. Learners partner with their teacher to be active participants in understanding and directing their own learning processes. In fact, the ultimate goal in assessment ***as*** learning is for learners to acquire the skills and the habits of mind to be independent in their metacognitive awareness and engagement. I will discuss metacognition in more detail in the next section of this chapter.

Critical to assessment ***as*** learning's metacognitive focus is embracing error. We are all probably familiar with the variations of advice around mistakes being the stepping stones that lead to future accomplishment. However the L4E learners struggled with seeing value in their errors. In fact, as already described, initially they copied from each other to avoid presenting

errors. Kramarski and Zolden (2008) tested different metacognitive approaches with 115 9th grade Maths students in Israel. When compared with a control group that received no metacognitive instruction, they found that all students subjected to metacognitive approaches outperformed those in the control group. They also found that the students using the approach of diagnosing error outperformed their peers in reducing conceptual errors. Earl (2003) also specifically refers to the value of students' errors (including their incomplete understandings, their misconceptions and naïve interpretations of concepts) as providing clues for teachers for creating optimal conditions for learning. Embracing error was the reason that the ARG made the finding of the need of a "supportive classroom climate" where learners are comfortable giving a wrong answer, learners understand that wrong answers are as useful as correct answers and learners are happy for other learners to help explore their wrong answers further.

All of this research which has promoted the extent and speed with which learning could progress, justified the adoption of a formative assessment approach in the L4E programme. Earl's (2003) ideas of assessment ***as*** learning activating metacognitive skills for life-long learning had been influencing all of my teacher development work even in the years prior to 2012. In the L4E programme the assessment ***as*** learning approach facilitated discussion, enabled acquisition of metacognitive language and developed the ability to articulate personal understanding. Sharing understanding with others and partnering learners with their teacher in the common purpose of advancing their learning was a novel experience for the L4E learners and they responded with growing enthusiasm and engagement.

3.1.4 Metacognition and Learning

The metacognitive advantage offered by the assessment ***as*** learning approach begs further discussion around metacognition and learning. Actually metacognition is an old concept inherent in Socrates' questioning methods and explained last century by Dewey (1933) that we learn more by reflecting on our experiences than from the actual experiences themselves. The increased value in metacognition for learning is highlighted by Costa and Liebmann (1996) when they identified nine uniquely human strategies for harnessing information and making sense of the world through an intentional process of learning. The first of these strategies was

metacognition. Costa (1988) defines metacognition as awareness of our own thinking and he describes it as a characteristic of intelligent behaviour that is visible to others (such as teachers and parents) and therefore can be taught (to students).

It is the American psychologist John Flavell (1971) who is credited with the initial definition of metacognition as thinking about thinking. Later he elaborated by saying metacognition involves knowing about one's own cognitive processes and actively monitoring and regulating these processes (Flavell, 1976). This highlights his distinction of metacognition as knowledge of cognition as well as regulation of cognition. Knowledge of cognition or metacognitive knowledge is: knowing what you know and what you don't know; understanding how you learn; knowing about strategies, what strategies to use and when to use them to make understanding; being aware of your strengths and weaknesses. Regulation of cognition or metacognitive expertise is managing and improving your learning through planning, implementing, monitoring and evaluating learning strategies with self-reflection to increase cognition. Brown (1978), a contemporary of Flavell's researching the cognitive processes of reading comprehension, described metacognition as evidenced by a learner understanding a particular cognitive activity such that he or she can use it appropriately and discuss its use. Flavell (1979) further distinguished metacognition as knowledge of strategy, task and person variables. In a special edition of *Theory into Practice* entitled "Revising Blooms Taxonomy", Pintrich (2002) neatly clarified this as strategic knowledge, cognitive knowledge and self-knowledge.

Initially metacognition was studied in child development (Baker & Brown, 1984; Flavell, 1985) but then the focus shifted to studying the metacognitive thinking of experts for the purpose of teaching these thinking skills to novices to improve their learning (Hatano & Inagaki, 1986). The interest in metacognition has grown over the past decade such that in 2008 an entire issue of *Educational Psychology Review* was dedicated to metacognition, self-regulation and self-regulated learning. Alexander (2008) justifies this special issue by commenting on an inverse relationship between the growing popularity of these educational constructs and their conceptual clarity within the literature. While acknowledging the reality of conceptual cross-fertilisation, she identifies a need to establish boundaries between metacognition, self-

regulation and self-regulated learning. After tracing a historical overview of work by William James, Jean Piaget and Lev Vygotsky (Fox & Riconscente, 2008), Dinsmore, Alexander and Loughlin (2008) analysed 255 works for the use of these three terms to explore the theoretical and empirical boundaries between them. Their findings were that there was considerable overlap between the three constructs. Flavell's concept of metacognition as thinking about cognition prevailed with Bandura's concept of self-regulation (1982) shifting the cognitive focus of Flavell towards human action rather than the thinking that engendered it. There was acknowledgement of overlap between these as the original concept of metacognition was evolving over time and spilling into the concept of self-regulation. Self-regulated learning, as the newest construct, was identified as the result of bringing together the ideas of metacognition and self-regulation for an exclusive focus on academic learning. It is within this blurred picture that I am using the terms metacognition and metacognitive skills while acknowledging that while this includes self-regulating human action, this is never without cognitive awareness and engagement.

Kluwe (1982) illustrates this merging of ideas when he explained metacognitive activities as those activities where the thinking subject has knowledge of their own thinking as well as the thinking of others and may monitor and regulate the course of their own thinking. Kluwe highlights that "it is important that human beings understand themselves as agents of their own thinking" (p.222). Similarly Veenman, Van Hout-Wolters & Afflerbach (2006) highlight the complex relationship between metacognition and cognition in describing a "higher-order agent overlooking and governing the cognitive system, while simultaneously being part of it" (p. 5). They merged the two distinct concepts of knowledge of cognition and regulation of cognition into what they termed metacognitive skills. They described metacognitive skills as an "acquired repertoire of procedural knowledge for monitoring, guiding and controlling one's learning and problem-solving behavior" (p. 201). This is what I mean when I refer to metacognitive skills.

Cornford (2002) describes the range of metacognitive skills as "learning-to-learn" strategies. He stresses their importance in achieving life-long learning and cautions against confusing these with what he calls the "older study skills approach". He highlights among his learning-

to-learn strategies: students' knowledge about themselves as learners; students' knowledge about the course context and the learning tasks; and students' knowledge about what learning strategies to select and use. I would add to this the students' knowledge of what resources they need to access (including human). Interestingly Cornford links these self-knowledge abilities to cognitive development and expresses reduced expectations of their efficacy among egocentric young children and adults who lack the abilities to self-discriminate and objectively self-evaluate.

Studies into the implications of a lack of metacognitive skills by Kruger and Dunning (1999) have coined the Dunning-Kruger effect. Wikipedia puts this best: the Dunning–Kruger effect is "a cognitive bias in which low-ability individuals suffer from illusory superiority, mistakenly assessing their ability as much higher than it really is". In *Unskilled and Unaware of It* (1999), Kruger and Dunning found that people tend to hold overly favourable views of their abilities in many social and intellectual domains. This results in an overestimation of their abilities, and these people who are unskilled in these domains suffer a dual burden: not only do they "reach erroneous conclusions and make unfortunate choices, but their incompetence robs them of the metacognitive ability to realise it" (p.1121). In *Why People Fail to Recognize Their Own Incompetence* (2003), Dunning, Johnson, Ehrlinger & Kruger explored this further concluding that people tend to be blissfully unaware of their incompetence and worryingly, if people lack the skills to produce correct answers, they are also suffer an inability to know when their answers, or anyone else's, are right or wrong. Attributed to the British mathematician and philosopher, Alfred North Whitehead is the well-known quote "not ignorance, but ignorance of ignorance is the death of knowledge".

The advantages offered by a metacognitive approach in teaching and learning are substantial. For decades now metacognition has been recognised as a most relevant and powerful predictor of learning (Brown, 1978; Flavell, 1976, 1979; Glaser, 1990; Veenman & Elshout, 1999; Wang, Haertel, & Walberg, 1990). In the definitive work on cognitive science presented by Bransford, Brown and Cocking (2000) *How People Learn*, one of the three key findings was the effectiveness of a metacognitive approach to instruction. Increasing metacognitive knowledge and skills contributes to learners feeling more able, and it increases self-efficacy

which influences success in learning, motivation and interest in learning (Corno, 1986). In a classroom study, Baird and White (1984) highlight learners' metacognitive skills of self-evaluation as offering the greatest potential for improving learning. There are added bonuses. Paris and Winograd (1990a) conclude "metacognition helps students to develop intellectual curiosity and persistence, to be inventive in their pursuits of knowledge, and to be strategic in their problem-solving behaviour" (p.10). They highlight how metacognition can help learners understand that learning involves overcoming obstacles, confusion and self-doubt. This is particularly valuable for low-achieving learners who gain greater self-efficacy as they learn to understand their frustrations and realise that others share similar feelings.

Metacognitive awareness enables learners to take charge of their own learning (Hacker, 2009). A key element in metacognitive awareness is recognising the limit of one's knowledge or ability and then figuring out how to expand that knowledge or extend the ability. According to Hipkins (2014) teachers should use their expertise to set up conditions where responsibility for learning can be shared. This allows students to assemble their toolkit to go on being learners when teachers are no longer around. According to Cornford (2002), life-long learning, effective learning through one's lifespan, is dependent on knowledge centred upon cognitive and metacognitive skills. Most importantly, according to Schraw (1998), metacognitive knowledge, awareness and regulation is teachable. Tanner (2012) describes methods to promote metacognition in the classroom which include: pre-assessments – to encourage students to examine their current thinking; self-reflective questioning – to identify and share their confusions; retrospective post-assessments – to push students to recognise conceptual change; and reflective journals – to provide a forum for students to monitor their own thinking. These strategies are all fundamental in an assessment ***as*** learning approach and were adopted in the L4E programme.

3.1.5 Metacognitive Activation

I devised a new term in the L4E programme – that of metacognitive activation – which in the light of the past section, needs to be distinguished from metacognition and metacognitive skills. Cognitive activation has become a popular idea in education publications since it was identified as an important "quality characteristic" (direct translation from Klieme, Schumer &

Knoll, 2001) of effective teaching in the TIMSS-Video study of 2001. Subsequently, cognitive activation in Mathematics has been highlighted through the Organisation for Economic Cooperation and Development (OECD) Programme for International Student Assessment (PISA) COACTIV Project conducted in Germany from 2003-2004. This inspired a report for the National Foundation for Educational Research (NFER) in the UK to focus on the teaching practice of cognitive activation. The report targeted teachers and explores how cognitive activation is related to achievement, and identifies short, medium and long-term strategies for using this practice in the teaching of Mathematics. Cognitive activation is a practice in which teachers provide opportunities for learners to use thinking strategies such as summarising, questioning and predicting when solving Maths problems. This encourages learners to focus more on the methods they use to solve problems rather than just the solution to the problem (Burge, Lenkeit, & Sizmur, 2015).

I adapted this concept to describe when learners begin to question and evaluate their own thinking strategies and refer to it as ***meta***cognitive activation. I presented it to the L4E learners as a recommended learner practice to develop metacognitive awareness and improve metacognitive skills. Working in the field of Mathematics, Goos (2002) describes metacognitive processes as crucial to successful performance on mathematical tasks. She identifies metacognitive "red flags" such as lack of progress, error detection and anomalous results and identifies metacognitive success as when learners recognise these red flags and take appropriate actions to deal with the difficulties.

I am distinguishing between metacognition and metacognitive activation to refer to the latter as a specific learner practice of displaying metacognitive skills through detailed and insightful reflection, understanding and sharing of their own learning progress. Assessment ***as*** learning is perfectly placed to provide the means of initiating and sustaining metacognitive activation in the classroom and promises longer-term value because as Cornford also said "possession of effective learning-to-learn skills is an important prerequisite for effective lifelong learning to occur" (Cornford, 2002, p. 367).

3.2 The Conceptual Thread Part of the Approach

3.2.1 Social Constructivism in Mathematics

The importance of constructivism in Mathematics education is evidenced by the positive reflection of Confrey and Kazak (2006) on thirty years of a constructivist approach in Mathematics. While the term constructivism has many varying interpretations, the following is simple and often cited. “Meaning is not given ***to*** us in our encounters with the environment, but it is given ***by*** us, constructed by each of us in our own way, according to how our understanding is currently organized” (Duckworth, 1987, p. 112). In the epigraph to his book, Ausubel (1968) highlighted the importance of one’s prior learning when he wrote his often-quoted “if I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly” (p. iv). In the South African context where acquired learning deficits have caused learners to be, in many cases, multiple grades behind in their Maths learning, ascertaining prior learning becomes critical for making progress in learning.

The L4E intervention is heavily influenced by the social-constructivist ideas of Vygotsky. Vygotsky (1978) argued that the capacity to learn from others is fundamental to human intelligence, and with help from those more knowledgeable and more skilled, learners can achieve more than they can on their own. Vygotsky is well known for his concept of the zone of proximal development (ZPD), and unlike Piaget who subscribed to age-limited child development, Vygotsky believed that cognitive development could be advanced beyond expectations with the assistance of a teacher. The ZPD is the difference between the developmental level a child can achieve on his or her own and the developmental level he or she could achieve with the input of a teacher. It is the extended learning potential which, while a child may be unable to achieve on their own, is made accessible through the social interactions with a teacher or more advanced learner to co-construct knowledge. Vygotsky’s model of socially supported learning was implemented in the L4E programme where L4E learners were not only interacting with their teacher, but also working with each other in assessing their own contributions and those of their work partners and group members.

In *How People Learn*, Bransford, Brown and Cocking (2000) highlight the emphasis on actively making understanding rather than just memorising facts. This understanding is achieved by people constructing new knowledge and understanding based on what they already know and believe. This presents a new challenge. If personal experiences are integral to learning then not all learners share the same opportunities to learn. Different learners bring different backgrounds and capabilities to learning. Depending on how their previous experiences influence their thinking, they might understand a task in quite different ways, use different reasoning or skills in trying to achieve the learning, and/or understand what constitutes "success" in quite different terms. The challenge for the teacher is to offer equitable opportunities to learn in his or her classroom.

With the role of the L4E teacher as a learning partner and the L4E learners already in Grade 10, the logical extension of this was to focus on learners' existing knowledge within a 'big picture' of learning Maths and to share this with the learners. In this way learners could identify their own learning gaps and their misconceptions and play an active role in building and strengthening their earlier Maths knowledge and understanding to meet their Grade 10 needs.

3.2.2 Big Ideas in Mathematics

I have been influenced in my Maths teaching by Charles' (2005) discussion of "Big Ideas" in Maths. He defines a Big Idea as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p.10). While Charles identifies 21 of these Big Ideas, he states upfront that he is certain and even comfortable that all mathematicians and Maths educators would not agree on these. He is more interested in the beneficial discussions that Big Ideas initiate for Maths teaching and learning than in which ideas are included or not. Charles sees Big Ideas as the foundation for Mathematics content knowledge, for teachers' teaching practices, and for a Mathematics curriculum. He stresses the importance of Big Ideas to make connections and share commonalities in Maths across grades and topics. New ideas can be linked to Big Ideas connecting topics across grades, developing concepts and skills at each grade by linking to

previous and subsequent grades. In developing his Big Ideas, Charles articulated with his colleagues on a curriculum development project what he called “math understandings”. He then organised this long list of “math understandings” across content strands rather than grade levels. I followed a similar process in devising my conceptual threads with their conceptual landmarks which I will elaborate on in section 3.2.4.

3.2.3 Learning Trajectories & Learning Progressions

The relationships and connections between concepts in Maths have received more attention since Charles’ work in 2005 but others were exploring this even earlier, developing what were mostly called learning trajectories or learning progressions. The term ‘learning trajectory’ appears to have been first presented by Simon in 1995. He described a ‘hypothetical learning trajectory’ as an identified learning goal with designed learning activities taking into consideration the thinking and learning in which students might engage to achieve that goal. Unlike more recent uses of the term, Simon presented a hypothetical learning trajectory as a teaching construct – he described how he as the teacher created an initial goal and a plan for instruction, and how he continuously revised and modified this based on student responses. Simon used a hypothetical learning trajectory in order to choose a particular instructional design, making a best guess of how learning might proceed. He also noted that in this process, his own personal understanding of the mathematical connections involved was enhanced (Simon, 1995).

The term ‘conceptual trajectory’ had been used even earlier by Driver (1989) when she referred to studies in conceptual progressions (patterns of development of learners’ thinking in specific Science domains). Driver called sequences of conceptual progression a ‘conceptual trajectory’. A conceptual trajectory was quite different from the hypothetical learning trajectory of Simon and from recent ideas of learning trajectories in that her conceptual trajectories did not predict how learning might proceed but described learning after it had actually occurred.

At the Freudenthal Institute in the Netherlands, the TAL Project has been working on the development of what they call 'learning-teaching trajectories' for primary school Mathematics. Forty years down the line, they still refer to their work – Realistic Mathematics Education – as a "work in progress". Translated from the Dutch, their term learning-teaching trajectory "has three interwoven meanings: a learning trajectory that gives a general overview of the learning process of the students; a teaching trajectory, consisting of didactical indications that describe how the teaching can most effectively link up with and stimulate the learning process; and a subject matter outline, indicating which of the core elements of the mathematics curriculum should be taught" (van den Heuvel-Panhuizen, 2008, p. 13).

Over in the USA progress in Mathematics learning trajectories was being made through the work of Clements and Sarama (2004). Their definition of a learning trajectory involves describing children's thinking and learning along a conjectured route through a set of instructional tasks designed to develop progressive thinking so as to support the children in achieving specific goals in a specific mathematical domain. Clements and Sarama focused attention on trajectories as relevant to curriculum design, where the order of teaching topics over time is not determined by the logic of the discipline or the historical development of the subject but by the evidence on learners' thinking and how this develops in response to experience and teaching.

A learning trajectory or learning progression approach was the focus of two research reports for the Consortium for Policy Research in Education (CPRE) which studies alternative approaches to education reform in the USA. The first by Corcoran, Mosher and Rogat in 2009 (*Learning Progressions in Science*) and the second by Daro, Mosha and Corcoran in 2011 (*Learning Trajectories in Mathematics*) present discussion in support of such an approach to teaching and learning. The value of developing the knowledge needed to define the track that students may be on, or should be on, is presented in both reports. Similarly the value of learning trajectories/progressions to inform teachers about what to expect from their students and to provide a basis for choices about when to teach what to whom is also promoted. In both reports, there is reference to key "waypoints" along the path in which learners' knowledge and skills are likely to grow and develop in these school subjects.

The Common Core State Standards (CCSS) adopted by 42 states of the USA have a learning trajectory/progression design approach which is clearly evident and shown in Figure 3.2.

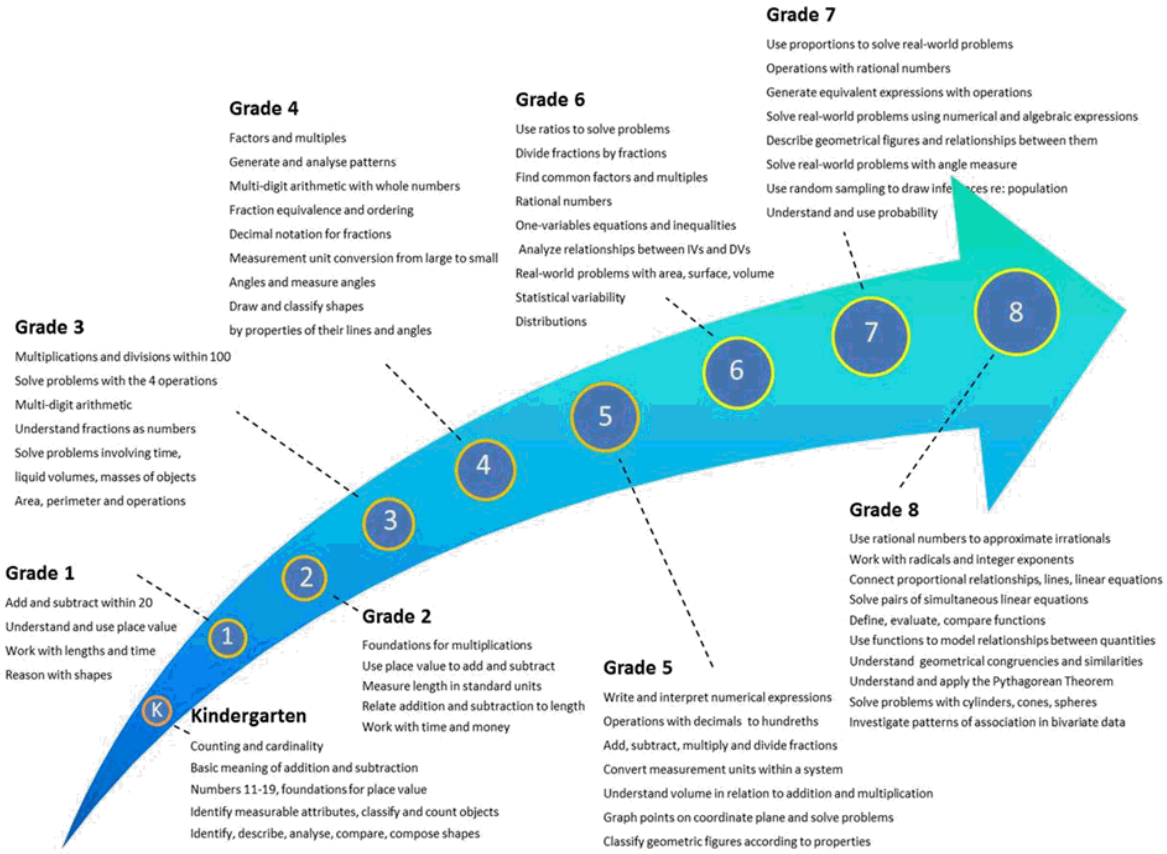


Figure 3.2 Expectation trajectory with grade targets – Common Core Standards (McLean & Rusconi, 2014 p.3)

The evolution of the language around learning trajectories and learning progressions is noticeable in the work of Confrey. In 2009 she described a learning trajectory as "a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction, and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time" (Confrey, Maloney, Nguyen, Mojica & Myers, 2009, p. 347). Confrey's writings at this time suggest that she sees learning trajectories and learning progressions as interchangeable terms for the same concept. By 2012, she simplified this description of a learning trajectory "as a path through a conceptual corridor in which there are predictable obstacles and landmarks and thus a student's particular path is an issue of expected probabilities and likelihoods" (Confrey, 2012, p.4).

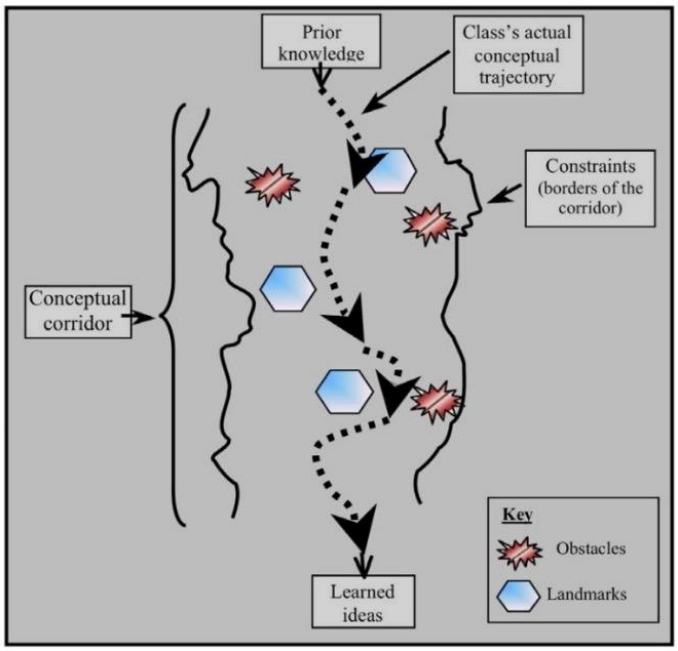


Figure 3.3 Learning trajectory within a conceptual corridor (Confrey, 2006, p. 140)

The word 'corridor' was used in a similar sense by Brown and Campione (1996) when they referred to developmental corridors. A developmental corridor is a pathway of learning across school-grades and age. A developmental corridor suggests that in the early school grades concepts are introduced and progressively refined, elaborated and extended throughout the progressive grades of school experience.

Battista (2011) provides a necessary tidy-up of the language around these ideas. He defines a learning trajectory as "a detailed description of the sequence of thoughts, ways of reasoning, and strategies that a student employs while involved in learning a topic, including specification of how the student deals with all instructional tasks and social interactions during this sequence" (Battista, 2011, p. 510). He identifies two types of learning trajectories: the hypothetical (of Simon) and the actual (of Driver). Battista distinguishes between 'learning progressions' and 'learning trajectories' by explaining that the latter includes descriptions of instruction while the former does not. He questions the influence of instruction on a trajectory (to what extent a trajectory is determined by the instructional sequence that it is embedded in). He states "a learning trajectory for a curriculum is in some sense an "average" of actual trajectories for a sample of individual students—and, as an average, it is a prediction for a target population, and thus it is necessarily hypothetical" (Battista, 2011, p. 512).

3.2.4 Conceptual Threads

In designing my teaching intervention, I identified seven conceptual threads in Mathematics running through the primary school grades preparing learners for high school Mathematics. I have used the term “conceptual thread” in a similar fashion to Maillet (2012) by referring to a cross-curricular and across-grades teaching and learning pathway identified by knowledge, skills and understandings in the ordered sequence in which these aspects of a learning area typically develop. While the learning trajectory focuses on the teaching activity and the learning progression focuses on the corresponding learning activity, I have distinguished a conceptual thread as distinct from Confrey’s descriptions of these, however my conceptual thread construct is similar to what Confrey describes as conceptual corridors. Comparable to Charles’ identification of his “math understandings”, I identified what I called “conceptual landmarks” (important concepts to master for learning progress). These conceptual landmarks were taught, learned and/or needed in Maths to achieve at Grade 9 level.

As the work of the American Association for the Advancement of Science (AAAS) post-dated my work on the L4E programme, I was not aware of their approach until L4E was well underway. Their Benchmarks Literacy Maps (AAAS, 2013) show some degree of overlap with my conceptual threads and their AAAS conceptual strand maps. Many of AAAS benchmarks along their conceptual strands are analogous to my conceptual landmarks along my conceptual threads.

The seven conceptual threads in Mathematics running through from Grade 1 to Grade 9 that I identified and used are:

- Numbers: Concepts, Relationships & Operations;
- Fractions: Concepts, Relationships & Operations;
- Patterns, Functions & Algebra;
- Space & Shape;
- Measurement;
- Data-Handling;
- Problem-Solving: Logical, Deductive and Adaptive Reasoning.

Along each conceptual thread I identified important conceptual landmarks that needed to be mastered in order to understand the conceptual thread. I correlated this with the CAPS for Mathematics for grade-specific requirements so that I could benchmark grade-level achievement at Grade 3, Grade 6 and Grade 9 levels (or, as is the case with the Numbers – Concepts, Relationships & Operations conceptual thread, only Grade 3 and Grade 6 levels).

Table 3.2 CAPS and the Numbers Conceptual Thread

CAPS Content & Skills to correlate with Conceptual Landmarks	**GRADE 3**	**GRADE 6**	**GRADE 9**
Counting forwards and backwards	Up to 1 000	Up to 9-digit numbers	Practise mastery
Skip-counting forwards and backwards	Skip counting in 2s, 3s, 5s, 10s, 20s, 25s, 50s and 100s to 1 000	Skip-counting in 2s, 3s, 5s, 10s, 25s, 50s and 100s to 9-digit numbers	Practise mastery
Recognise, identify and read numbers	Up to 1 000	Up to 9-digit numbers	Practise mastery
Describe, compare and order numbers	Up to 999	Up to 9-digit numbers	Practise mastery
Use, read and write ordinal numbers	Up to 31st	Unlimited	Practise mastery
Recognise place-value and decompose numbers	Up to 999	Up to 9-digit numbers	Practise mastery
Round off numbers	-	to the nearest 5, 10, 100, 1 000, 100 000, and 1 000 000	Practise mastery
Addition of numbers	With answers up to 999	With numbers at least up to 6-digits	Practise mastery
Subtraction	With answers below 999	With numbers up to 6-digits	Practise mastery
Multiplication	By 2, 3, 4, 5 and 10 to a total of 100 (grouping and sharing)	Up to 4-digit by up to 3-digit numbers	Practise mastery
Division	Numbers up to 100 by 2, 3, 4, 5 and 10 (grouping and sharing)	Up to 4-digit by up to 3-digit numbers	Practise mastery
Mixed Operations	-	With or without brackets	Practise mastery

3.2.5 The Numbers Conceptual Thread

In order to develop each conceptual thread and before correlating with CAPS, I studied the research around the relevant Big Ideas, learning trajectories and progressions, and even concept maps. I followed the early development of concepts through to the equivalent level of Grade 9. Considering Numbers, I looked at the following approaches.

Clements and Sarama (2009) propose that Numbers is the most important of the mathematical areas and that because of this, the learning of number and operations in the early childhood years may be the best-developed area in Mathematics education research. They present the domain of children's numerical concepts and operations as consisting of two foundational ideas, one for **number** and one for **operation**, although these are highly interrelated. Operations are not limited just to adding, subtracting, multiplying, and dividing. By "operations" they include counting, comparing, grouping, uniting, partitioning, and composing. The learning trajectories for these areas extend through Foundation Phase and build on early childhood development from as early as the first year of life.

Cross, Woods and Schweingruber (2009) proposed three early learning trajectories for Numbers. The first is **Number Core** which has four components: seeing cardinality; knowing the number word list; 1-1 correspondences when counting; and the written number symbols. Progressive steps start with connecting the number word lists with 1-1 correspondences in early counting; then connecting counting with cardinality so that the last count word tells how many there are; then connecting counting and cardinality in the opposite direction; then relating groups of tens and ones to number words and place-value notation to 100, building up to larger groups and larger place-value. The second is **Relations Core** which also has four components: using perceptual, length and density strategies to determine more than/less than; using counting and matching strategies to determine more than/less than; comparing situations with objects or in a drawing using counting and matching strategies with larger numbers; and solving comparison word problems for even larger numbers. This starts with "more than", "less than" and "equal to" on two sets of objects (with up to five entities); then matching to find left-overs and counting up to 10 entities; and leads to "the difference between" through to "add/subtract" with even larger numbers. The final trajectory is

Operations Core which is by means of "mathematising" real world situations and building language skills with word problems. It has four components: subitised and counted cardinality to solve situation and oral number word problems (Van den Heuvel-Panhuizen, 2008, calls this "two-ness, "three-ness" etc., p.25); using conceptual subitising and cardinal counting of objects or fingers to solve situation, word, and oral number word problems; using cardinal counting to solve situation, word, oral number word, and written numeral problems; and using counting-on solution procedures to solve all types of addition and subtraction word problems. Problems cover three types of add/subtract situations: change plus/change minus; put together/take apart; and comparisons with three levels in children's numerical solution methods: **direct modelling** (count all and take-away by counting things or counting fingers); **counting-on** (to find the total or an unknown addend); and **derived-fact methods** (such as doubles methods and make-a-ten methods to find total or unknown addends).

Van den Heuvel-Panhuizen (2008) presented a full learning-teaching trajectory for the domain of whole number calculation. This trajectory contains two parts: one for the lower grades and one for the upper grades. Although the learning-teaching process forms a continuous process across the grades, each part has its own characteristics. An important characteristic is that while in the lower grades, all activities with numbers can be generally labeled "arithmetic"; in the upper grades different forms of calculations can be distinguished, like mental arithmetic, estimation, column arithmetic, algorithms, and calculation by using a calculator. For progress in understanding there is a shift from informal to formal; from context-connected to model-based; from progressive complexization to progressive schematization. This last is interesting in that the traditional approach (progressive complexization) involves learning, for example division, by starting with small numbers and gradually increasing the degree of difficulty. With the progressive schematization approach, instead of a step-by-step increasing of the complexity of the problems, the problems remain the same, but the strategies become more and more advanced. This offers the advantage that students can solve the problems on their own level. They can start at a context-connected informal level, and all different levels can be accommodated in the classroom.

Another characteristic of the trajectory is that mental arithmetic plays a central role; rooted in the arithmetic work of the lower grades, it forms the backbone of the upper grades. Explicit attention is paid to numbers and number relations. The important foundation for the development of upper grade calculation skills is built through the context of numbers, their position in terms of magnitude and their internal structure in lower grades. As Van den Heuvel-Panhuizen says, “if one invests in the numbers, one gets the operations, so to say, for free” (2001, p. 31).

Having a variety of number operation strategies that a learner can use to solve problems is a contributor to achievement in Mathematics as is illustrated by the research of Gray and Tall (1994). They analysed the number strategies that young learners used to solve number operation problems. They divided their experimental group into more and less able children in the Mathematics classroom. They found that the more able children had a wider range of strategies for solving mathematical problems than the less able children. Less able children were limited to a single rote-learned operational procedure rather than having the ability to draw on a variety of different strategies depending on which best matched the problem presented. They concluded that less able children were learning a different and more difficult form of Mathematics compared to the more able children.

The importance of Numbers is evident in the South African CAPS for Mathematics and this was accommodated in my conceptual thread design. CAPS is heavily weighted in terms of time allocated in the Foundation Phase only becoming equivalent to other content areas as high school begins in Grade 8. By this stage, the expectation is that learners have mastered basic number concept, operations and relationships and are ready for the shift to more abstract algebra. Table 4.2 in the next chapter shows the weighting of Mathematics Content Areas in CAPS for the GET band.

When considering the conceptual landmarks to include in my Numbers conceptual thread, I compared to programmes based on the learning trajectories and learning progressions already described.

I also considered the work of Sherin and Fuson (2009) who developed a taxonomy for multiplication computational strategies where they describe the learning progressions of these strategies through the taxonomy ordering these by frequency of use from early to late development. I compared to Confrey *et al.* (2009a) and the DELTA project - Diagnostic E-Learning Trajectories Approach (Confrey, Penuel, Goldman, Howe & Lamb, 2009b) which focuses on rational number reasoning. Lastly, I also compared with The Vermont Mathematics Partnership Ongoing Assessment Project - VMP OGAP (Daro *et al.*, 2009) that develops tools and resources that bridge the gap between research and practice. They have produced the OGAP Frameworks for Multiplication, Division, Proportionality and Fractions. Here is a comparison summary.

Table 3.3 Comparison between the L4E Numbers Conceptual Thread and Others

Programme		**L4E**	**Clements & Sarama**	**Cross, Woods & Schweingruber**	**Van den Heuvel-Panhuizen**	**Sherin & Fuson**	**Confrey *et al.* & DELTA**	**Petit & OGAP**
Age/ Grade Range:		Grade 1 to 9	Age 0 to 8+	Age 2 to 7	K to Grade 6	Grade 3	K to Grade 8	K to Grade 8
Number Concept	Counting		x	x	x			
	Subitising		x	x	x			
	Recognising cardinality		x	x	x			
	Comparing sets of objects		x	x	x			
	Recognising numbers		x	x	x			
Number Relationships	Comparing numbers	x	x	x	x			
	Ordering numbers	x	x	x	x			
	Estimating numbers	x	x	x	x			
	Recognising place-value	x	x	x	x			
	Composing number	x	x	x	x			
Number Operations	Simple adding	x	x	x	x			
	Multi-digit adding	x	x	x	x			
	Simple subtracting	x	x		x			
	Multi-digit subtracting	x	x		x			
	Equipartitioning	x	x		x		x	
	Additive multiplying	x			x	x		x
	Multiplicative grouping	x	x	x	x	x	x	x
	Multiplicative algorithms	x			x	x		x
	Additive/ subtractive dividing	x			x			x
	Divisive grouping	x	x		x		x	x
	Divisive algorithms	x			x			x

Recognising the importance of Numbers, I started the L4E programme with this conceptual thread and the bulk of the L4E contact time in 2012 was spent on this. Learner evidence thus far was that the L4E learners definitely had acquired learning deficits and the resultant diminished learning opportunities Jordan *et al.* (2009) referred to. The prognosis was poor as

the L4E programme could not be classified as an early intervention (although by South African standards, starting in Grade 10 was a year or two earlier than most donor-funded Maths programmes of the time). I was relying on the assessment ***as*** learning – conceptual thread approach to make the difference for the L4E learners.

In summary, I chose an assessment ***as*** learning – conceptual thread approach to firstly fast-track learners' catch-up by metacognitively activating them to become agents of their own learning and empowering them with sustainable long-term learning skills. Secondly, I wanted to provide a 'big picture' of learning Mathematics so that they could see the important conceptual landmarks in Maths, how these relate to each other and to use this knowledge to identify and understand their own acquired learning deficits while actively partnering with their teacher to fix these effectively and efficiently.

CHAPTER FOUR

Research Design & Methodology

In this chapter I will outline the research methods chosen and followed, the measuring tools developed and used for each of the three separate aspects of my research question, which data was utilised to answer my research questions as well as how this data was collected and analysed over the study period.

4.1 Methods Approach

When choosing a framework for research design, I considered the recommendations of Creswell (2003). These recommendations involve three elements of inquiry: the knowledge claims made by the researcher, the strategies of inquiry informing procedures, and the methods of data collection and analysis used. I believed that a mixed methods design which would rely on both quantitative and qualitative data would best suit the research questions of this study.

The fact that this was a retrospective study necessitated a *quasi*-experimental approach as my sample was not randomised. In fact I played no part in selecting the participants in the L4E programme. The Uplands Outreach Director and the Operations Manager drafted the criteria for selection of learners and the T4E teachers in the participating schools made the selection. Quantitative data from this quasi-experimental approach and qualitative data from a narrative approach were collected concurrently from February 2012 to September 2012. Summative data for the two resistant conceptual threads of Fractions and Problem-Solving were included in the overall evaluation of the L4E programme in order to provide a full picture of the intervention's impact even though the summative data for these threads was only collected in June 2013.

The nature of the data used to answer the research questions is summarised in Table 4.1.

Table 4.1 Summary of Mixed Methods Data

QUESTION ADDRESSED	DATA SOURCE	NATURE OF DATA	PURPOSE
Can a teaching and learning intervention using an assessment *as* learning – conceptual thread approach 'make up' acquired learning deficits in Mathematics and 'catch up' low-achieving learners?	L4E Maths Baseline Term Test Scores (Feb 2012)	Quantitative (assessed by teacher)	To compare Maths achievement before and after the intervention in order to establish the extent of learning gains made in Maths through the L4E intervention.
	L4E Maths Summative Term Test Scores (Sep 2012/ Jun 2013)	Quantitative (assessed by teacher)	
Which obstacles to mastering conceptual landmarks in learning about numbers and number operations are revealed by the approach followed?	L4E Number Ops Class Test Scores (Feb 2012)	Quantitative (assessed by teacher)	To compare achievement in number operations over 7 months in order to chart progress and to identify the extent of conceptual difficulties in this aspect of the Numbers conceptual thread
	L4E Number Ops Term Test Scores (Jun 2012)	Quantitative (assessed by teacher)	
	L4E Number Ops Term Test Scores (Sep 2012)	Quantitative (assessed by teacher)	
	L4E Maths Minutes Homework Tasks 1-45 Pre Error Analysis (Apr 2012)	Quantitative (Homework tasks assessed by teacher Error Analysis assessed by learner – self-assessment)	To compare progress in the Numbers conceptual thread over 9 months in order to identify conceptual landmarks that were easier to fix and those that were more difficult and persistent
	L4E Maths Minutes Homework Tasks 76-85 Post Error Analysis (Sep 2012)	Quantitative (Homework tasks assessed by teacher Error Analysis assessed by teacher)	
To what extent can this approach change learners' self-evaluations of their own understanding of Mathematics?	L4E Number Ops Self-assessments (Feb 2012)	Quantitative (assessed by learner – self-assessment)	To compare learner self-efficacy with their actual achievement in number operations in order to gauge their initial level of metacognitive skills
	L4E Number Ops Test Results (Feb 2012)	Quantitative (assessed by teacher)	
	L4E Reflective Class Activities & Journaling (Feb 2012) (Apr 2012) (Sep 2012)	Qualitative (assessed by teacher)	To compare learners' qualitative self-analyses over time to gauge progress in their metacognitive skills

While this study tends toward being more quantitative in nature (in terms of answering the research questions) I included qualitative data to provide the depth and the detail in the exploration of the teaching and learning intervention. Throughout the L4E programme all learners kept weekly journals in which they were expected to write down not only their

thoughts about the Maths and their learning issues, but also about all aspects of the Saturday programme and their activities at school and at home between sessions. Each week the Uplands Outreach staff 'marked' these journals which entailed reading these and writing encouraging and helpful comments back to the learners. Important news, issues and concerns were shared among the staff so as to best support the learners in our individual interactions with them. The learners' response to the journaling was extremely positive with 98.6% of journals handed in weekly over the year 2012. By June 2012, it became necessary to impose a limit on the number of pages of writing as the staff were struggling with the volume of reading. These journal entries along with the self-reflective writings in the classroom sessions provided a wealth of information and insight into the learners' experiences. The method of analysis of these self-reflections is explained later in this chapter. The rationale for mixing methods is that it will be "useful to capture the best of both quantitative and qualitative approaches" (Creswell, 2003, p. 22) that were followed during the intervention.

4.2 Research Sample

The sample is composed of 50 learners in the Insikazi Circuit of the Ehlanzeni Region of Mpumalanga. These learners were drawn equally from Insikazi's ten secondary schools, through consultation with the Circuit Manager and T4E teachers at the partner schools. Each high school selected five learners to participate in this Saturday Maths programme for three years, making this an inclusive circuit programme.

South Africa's schools are divided into five categories or "quintiles" according to their poverty ranking. The poorest schools are included in Quintile 1 and the least poor in Quintile 5. The Insikazi high schools are ranked from Quintile 1 (one school), Quintile 2 (five schools) and Quintile 3 (one school) to Quintile 4 (three schools). I did not distinguish between these quintile rankings in the data collection and I did not notice any discernable difference between them. As already mentioned, selection was not random as the five learners per school were nominated by their teachers who were already involved in the Uplands Outreach T4E programme. The teachers based their selections on the criteria provided by Uplands Outreach (presented in section 2.1.2). The majority of these learners 'stood out' as learners

with potential, but were not necessarily the top-achieving learners in Grade 9 (approximately 70% of the cohort was in the top 15%). Three of the learners, while being the top achievers at their schools, had actually failed Grade 9 Mathematics. The final list of participants was made up of 28 female learners (56%) and 22 male learners (44%). Learners ranged in age from 13 years to 17 years with a mean age of 15.1 years (and mode of 15 years) in January 2012. All were South African citizens. None were English home language speakers. All were studying siSwati Home Language at their schools (along with English First Additional Language) but some learners were speaking Xitsonga at home.

4.3 Data Sources

The L4E programme officially ran for three years, from January 2012 when the learners started Grade 10 to November 2014 when the learners completed Grade 12. Quantitative and qualitative data was collected continuously over the full period of the L4E project. As this study is retrospective, I must note here that the data sources and collection of data were not for research purposes at the time but rather to inform the direction of teaching and learning over the three years. Only a portion of the data collected is used in this study and, bar a single exception of which the explanation follows, this is limited to the period from February 2012 to September 2012. My teaching contribution to the project, while initially unplanned, was extended to the second year to address what appeared to be resistant difficulties in the conceptual threads of Fractions and Problem-Solving. The summative assessments of these two threads were conducted in 2013 and are only included in the overall evaluation of the intervention (the response to my first research question). The remaining parts of this research study only consider the data collected during the period from February 2012 (when the baseline assessment was administered for all seven conceptual threads) to September 2012 (when the summative assessment was administered for five of the seven conceptual threads). Thus it presents overall results of the teaching and learning intervention across the seven conceptual threads but then analyses specifically only the first of these, and within that, number operations which were addressed in the first year of the intervention. The sources of data are summarised in Table 4.1.

Each of my three research sub-questions required not only different data sources but also a different approach in collecting and analysing the data. For this reason, I will address the methodologies of each separately, focusing on the measurement tool developed, how this was used to collect data and how the data was analysed to answer the research question.

4.4 Data Collection & Analysis: Measuring the Value in Catching Up Learners

Research sub-question: Can a teaching and learning intervention using an assessment ***as*** learning – conceptual thread approach 'make up' acquired learning deficits in Mathematics and 'catch up' low-achieving learners?

4.4.1 Developing the Measuring Tool – Maths Diagnostic Test

To understand learners' mathematical difficulties and the extent of these, I undertook to design a baseline test that not only covered the five General Education & Training (GET) Maths content areas but which was diagnostic, benchmarking achievement at Grade 3, Grade 6 and Grade 9 levels for a deeper analysis of their existing knowledge. For the purposes of a more detailed understanding of the learners' issues, I separated rational numbers from the topic Numbers, Operations & Relationships and included an additional 'topic' to test problem-solving skills. These seven components made up my conceptual threads:

- Numbers: Concepts, Relationships & Operations
- Fractions: Concepts, Relationships & Operations
- Patterns, Functions & Algebra
- Space & Shape
- Measurement
- Data-Handling
- Problem-Solving

As already discussed in Chapter Three, for each conceptual thread I identified conceptual landmarks and then prioritised these for inclusion in the L4E diagnostic test. Table 4.2 presents my lists of conceptual landmarks within each conceptual thread. The landmarks included in the L4E diagnostic test are in bold font.

Table 4.2 Conceptual Landmarks along Conceptual Threads

CONCEPTUAL THREAD	CONCEPTUAL LANDMARKS	
Numbers: Concepts, Relationships & Operations	• **comparing and ordering numbers** • recognising place value • rounding numbers • decomposing numbers • **adding numbers** • **subtracting numbers** • **multiplying numbers**	• **dividing numbers** • **mixing number operations** • **squaring and cubing numbers** • **determining square and cube roots** • working with factors and multiples • **working with exponential form** • **working with integers**
Fractions: Concepts, Relationships & Operations	• **comparing and ordering** • **recognising equivalence** • converting between fractional forms • **adding fractions** • adding decimals • subtracting fractions • **subtracting decimals**	• **multiplying fractions** • **multiplying decimals** • dividing fractions • dividing decimals • working with mixed operations • working with percentages • working with ratio and proportion
Patterns, Functions & Algebra	• **extending number patterns** • **describing and determining pattern rules** • determining input and output values • **simplifying expressions using laws** • **simplifying expressions using operations** • **simplifying expressions by factorisation**	• writing algebraic equations • **solving algebraic equations** • **substituting in algebraic equations** • drawing graphs • **interpreting linear graphs**
Space & Shape	• **identifying 2-dimensional shapes** • describing 2-dimensional shapes • **identifying 3-dimensional objects** • describing 3-dimensional objects • working with straight lines • **working with angles** • working with similarity and congruency	• working with symmetry, enlargements and reductions • working with transformations • **working with position, orientation and direction** • solving geometrical problems
Measurement	• estimating time, length, mass, volume and temperature • **measuring time, length, mass, volume and temperature** • calculating time, length, mass, volume and temperature • **converting units of measurement** • **determining perimeters** • **determining area and surface area** • determining volume	
Data-Handling	• collecting and organising data • **summarising data** (measures of central tendency and dispersion) • representing and displaying data • **interpreting data** • analysing data • drawing conclusions from data • **working with probability**	
Problem-Solving	• **completing logic patterns** • **using sequential/ conditional reasoning** • using spatial reasoning • **using deductive reasoning** • using inductive reasoning • using abductive reasoning • using creative reasoning	

The design of the diagnostic test was to distinguish between levels of achievement at Foundation Phase (Grade 3), Intermediate Phase (Grade 6) and Senior Phase (Grade 9) on the chosen conceptual landmarks. I used a 'test blueprint' to create my diagnostic test ensuring that each progressive landmark was measured on my three identified grade-levels.

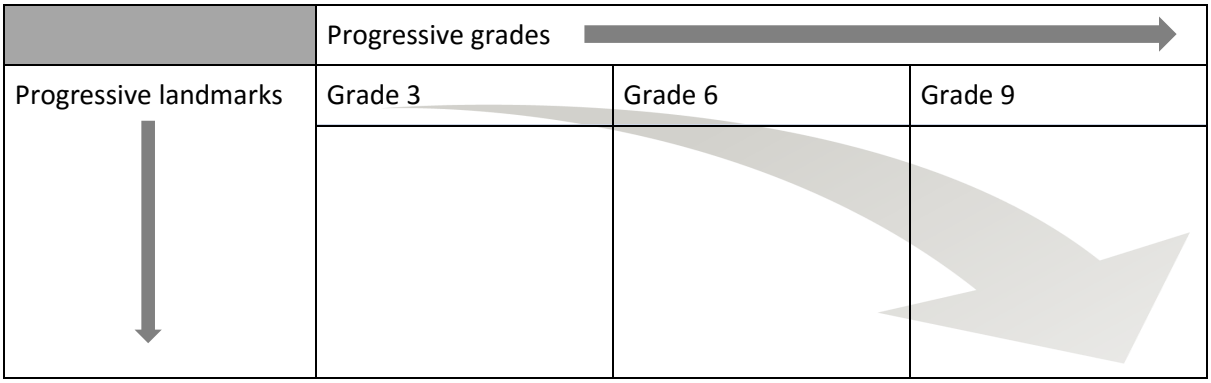


Figure 4.1 Test Blueprint for Diagnostic Design

Due to the fact that the learners were already in Grade 10, the focus of the diagnostic test was on Grade 9 level and on Numbers and Fractions (the most important component in primary school) and Algebra (an important component in high school) and this is reflected in the weighting of the conceptual threads. Considerations in the weighting of the conceptual threads were: time allocated to them in the Curriculum and Policy Statements (CAPS) from Foundation Phase through to Senior Phase; their importance for success in the Further Education & Training (FET) band and my own assumptions about early difficulties in Maths. The tables below show the weighting in CAPS Maths Content Areas in GET and FET Bands in terms of percentage of time spent on each.

Table 4.3 GET Mathematics Content Areas and Weighting in CAPS (DBE, 2011a, 2011c, 2011d)

GET Mathematics Content Areas	**Grade 1 → Grade 3** *7 hours/week*	**Grade 4 → Grade 6** *6 hours/ week*	**Grade 7 → Grade 9** *4.5 hours/ week*
Numbers, Operations & Relationships	65% → 58%	50%	30% → 15%
Patterns, Functions & Algebra	10%	10%	25% → 35%
Space & Shape (Geometry)	11% → 13%	15%	25% → 30%
Measurement	9% → 14%	15%	10%
Data-Handling (Statistics)	5%	10%	10%
Total	100%	100%	100%

Table 4.4 FET Mathematics Content Areas and Weighting in CAPS (DBE, 2011b)

FET Mathematics Content Areas	Grade 10 *4.5 hours/week*	Grade 11 *4.5 hours/ week*	Grade 12 *4.5 hours/ week*
Algebra & Equations (& Inequalities)	15% ± 1.5%	15% ± 1%	8.3% ± 1%
Patterns & Sequences	7.5% ± 1.5%	8.3% ± 1%	8.3% ± 1%
Finance & Growth (& Decay Gr 11-12)	5% ± 1.5%	5% ± 1%	5% ± 1%
Functions & Graphs	15% ± 1.5%	15% ± 1%	11.7% ± 1%
Differential Calculus	0	0	11.7% ± 1%
Probability	7.5% ± 1.5%	6.7% ± 1%	5% ± 1%
Statistics	7.5% ± 1.5%	6.7% ± 1%	6.7% ± 1%
Analytical Geometry	7.5% ± 1.5%	10% ± 1%	13.3% ± 1%
Trigonometry	20% ± 1.5%	16.7% ± 1%	13.3% ± 1%
Euclidean Geometry & Measurement	15% ± 1.5%	16.7% ± 1%	16.7% ± 1%
Total	100%	100%	100%

I considered the CAPS weighting of Content Areas in the Tables 4.3 and 4.4 when designing the L4E diagnostic test and the weighting of my conceptual threads. Diagnostic test questions were not chosen to extend or challenge learners but to assess basic competence. These questions were presented at Grade 3 level, Grade 6 level and Grade 9 level and I also weighted these grade levels. The L4E diagnostic test was made up of 100 possible scored percentage points and these were allocated to the seven conceptual threads and three grade levels as shown in Table 4.5.

Table 4.5 Weighting of L4E Mathematics Diagnostic Test

CONCEPTUAL THREADS	Grade 3-level	Grade 6-level	Grade 9-level	TOTAL %
Numbers: Concepts, Relationships & Operations	5	8	7	20
Fractions: Concepts, Relationships & Operations	1	7	2	10
Patterns, Functions & Algebra	3	0	27	30
Space & Shape	2	3	5	10
Measurement	0	5	5	10
Data-Handling	3	3	4	10
Problem-Solving	0	3	7	10
TOTAL	14	29	57	**100**

I designed the diagnostic test questions and it was moderated by two external Mathematics experts. One was a Grade 8 to Grade 12 Maths teacher with more than 20 years of classroom experience. The other was a senior education department official who had over forty years of experience in Mathematics having set provincial examinations, trained teachers and written textbooks. I therefore felt confident in the validity of the L4E diagnostic test. A copy of the L4E diagnostic test and the marking memorandum can be found in Appendix A. A more detailed analysis of CAPS aligned achievement on identified conceptual landmarks can be found in Appendix B.

4.4.2 Collecting the Data – Maths Diagnostic Test

The test was compiled as a 12-page document and a single copy was presented to each learner. On Saturday February 4, 2012, the first gathering for the L4E programme, the learners wrote the baseline diagnostic test. Learners were provided with full stationery kits but were not allowed to use provided calculators. Four L4E staff members (including myself) invigilated the test period which did not exceed 90 minutes. (Actually learners finished disappointingly quickly). Three L4E staff (including myself) marked the 50 completed test scripts. Each marker marked the same questions for all learners according to the same negotiated marking memorandum. Learners' scores for each conceptual thread as well at their overall achieved score was collected on an Excel spreadsheet.

After completing 26 weekly one-hour contact sessions focusing on catching up their basic Maths knowledge, 33 weeks later on Saturday September 15, 2012 the learners wrote the diagnostic test again in similarly controlled conditions. One difference was that the questions testing the conceptual threads Fractions and Problem-Solving were not included in the test. Formative assessment activities in the classroom had already shown that little progress had been made in these more resistant conceptual threads so it was decided to hold these over until 2013. Testing of these conceptual threads took place on June 22, 2013. All of the learners' summative scores were captured on an Excel spreadsheet for analysis.

4.4.3 Analysing the Data – Maths Diagnostic Test

To establish whether this intervention had an impact on learners' achievement in Mathematics, the baseline test scores and the summative test scores were analysed. A comparison was made between overall achievement on each test, as well as overall achievement on each conceptual thread. While the mean scores of the L4E cohort of 50 were compared for these measures, overall achievement for individual learners was also compared. All overall results were benchmarked against Grade levels – Grade 3, Grade 6 and Grade 9 levels. When I created the diagnostic test, based on my teaching experience, I pitched the questions at a level where I expected that learners who were functioning at a Grade 9 level in Maths would achieve at least 90% on the Grade 3 level questions, 80% on the Grade 6 level questions and 70% on the Grade 9 level questions. Applying this to the collected Grade 3-level, Grade 6-level and Grade 9-level questions enabled me to identify ranges of achievement on the overall test which corresponded to these benchmark levels.

Table 4.6 Grade-level Achievement on L4E Mathematics Diagnostic Test

L4E Diagnostic	Grade 3-level	Grade 6-level	Grade 9-level
Grade-level score weighting	14 out of 100	29 out of 100	57 out of 100
Expected achievement of grade-level questions	90%	80%	70%
Expected score for mastery	13	23	39
Calculated benchmark score	13	36	75
Grade-level achievement range	13-36%	36-74%	75-100%

Thus a learner achieving 75-100% was classified as on Grade 9 level, one achieving 36-74% on Grade 6 level, one achieving 13-36% on Grade 3 level and achieving less than 13% was classified as not yet at Grade 3 level. This data enabled me to ensure that the starting point of the teaching and learning intervention was at an appropriate level to build on learners' existing knowledge for greater probability of success in remediation.

These measures were used to establish the overall success of the intervention. The data was subjected to statistical measures to clarify distribution and a t-test was carried out for paired difference to calculate significance of the difference between the start and the end of the intervention. These results are presented and discussed in Chapter Five.

4.5 Data Collection & Analysis: Measuring the Value in Revealing Difficulties

Research sub-question: Which obstacles to mastering conceptual landmarks in learning about numbers and number operations are revealed by the approach followed?

4.5.1 Developing the Measuring Tools

This study focused only on the learning difficulties evident in the **one** conceptual thread Numbers, Operations and Relationships and specifically number operations. Data was selected from test questions relevant to these conceptual landmarks in the term tests written in February 2012, June 2012 and September 2012.

4.5.2 Measuring Number Operations using Test Data

With the background of Charles' Big Ideas (Charles, 2005) and Clements and Sarama's learning trajectories (Clements & Sarama, 2004), I focused on the four conceptual landmarks of number operations: adding, subtracting, multiplying and dividing. I recorded learner results for number operations questions in the term test of February 2012 (Appendix C), in a class test of February 2012 (Appendix D), the mid-year term test of June 2012 (Appendix E) as well as the summative term test of September 2012 (Appendix C).

4.5.3 Measuring Number Operations using Error Analysis Data

I adapted a Rubricate Teaching & Learning Diagnostic Matrix Tool (an output of the Rubricate PLUS teacher development software package which I had devised four years earlier) to enable learners to analyse their own Maths knowledge and understanding. The Error Analysis Diagnostic Matrix tool for the conceptual thread of Numbers is included as Appendix F. The learners used the first 1 – 45 Maths Minutes homework exercises as the source of their errors. For the purposes of this study, I used the same Error Analysis Diagnostic Matrix tool to analyse the last 76 – 85 Maths Minutes homework exercises to reveal persistent difficulties in the Numbers conceptual thread.

4.5.4 Collecting & Analysing the Test Data

All tests were written under controlled conditions and monitored by myself as the teacher. All term tests were marked by myself with two L4E staff assisting me as already described in 4.4.2. Learners' scores for the number operations conceptual landmarks were isolated and collected on an Excel spreadsheet.

4.5.5 Collecting & Analysing the Error Analysis Data

Daily Maths Minute homework exercises were used in a detailed error-analysis activity with learners which exposed the difficulties of each learner and highlighted the shared difficulties among the group of learners. These Maths Minutes exercises were completed as homework during the week. While I did have difficulties in the beginning with some learners copying work, the generally low competence of all learners resulted in little benefit from this practice. Even the weekly problem-solving puzzles I provided (which they were encouraged to discuss and solve with others, including their teachers at school) showed little evidence of any advantage in completing them beyond the classroom. I felt confident that error analysis would reveal common difficulties. All homework tasks were marked by me and returned to the learners before the error analysis activity.

A full-length Saturday was used to implement the self-diagnostic error analysis activity using the first 45 Maths Minutes homework exercises. This comprised of ten questions per homework exercise multiplied by the 45 exercises resulted in analysis of 450 questions. Each question was matched with the appropriate conceptual landmark within a conceptual thread. This study refers only to the results gathered in the Numbers conceptual thread. Appendix G shows the matching of the Maths Minutes 1 –45 questions to conceptual landmarks within the Numbers conceptual thread. Each learner focused only on their incorrect answers and recorded these on the Diagnostic Matrix Tool by means of a tally in the appropriate box. Learners used the number of tallies in each box to classify themselves as a practitioner, apprentice or novice for each conceptual landmark. Table 4.7 shows the guidelines I provided to the learners to assist them in their self-classifications. Learners drew colour-coded rings around each conceptual landmark 'cell' on their matrix tool.

Table 4.7 Learner-friendly Diagnostic Self-Classifications

Classification	**Description**
Novice	Unable to 'do' it
Apprentice	Beginning to 'do' it
Practitioner	Able to 'do' it

Once all 45 homework tasks had been analysed, errors recorded and collated, I moderated all of the learners' error analysis matrix tools to confirm their classifications of their proficiencies and recorded this data in an Excel spreadsheet. I also calculated the extent of error on the different conceptual landmarks (and calculated the percentage error occurrence) in order to reveal the conceptual landmarks with which the learners experienced the most difficulty. Afterwards learners also wrote self-reflective notes identified their strengths and weaknesses and reflected on how to build on these strengths and eliminate their weaknesses.

4.5.6 Further Analysis of Number Operations from Test Data

In revisiting the data for this study, I decided to analyse the baseline and summative term tests data further. In these tests I had presented two questions for each number operation; the first question was straightforward and involving smaller numbers while the second was a little more difficult (e.g. involving regrouping) and with larger numbers. Instead of just using a score on the tests, I studied these specific number operation questions and classified each learner as a practitioner, apprentice or novice for each operation at the start and at the end of the seven month period of the Numbers intervention. For each operation, if the learner made errors on both questions, s/he was classified as novice (unable to 'do' it); if the learner made an error on one of the two questions, s/he were classified as an apprentice (beginning to 'do' it); and if s/he made no errors on either question, s/he were classified as a practitioner (able to 'do' it). This classification gave me an opportunity to track their progress in number operations not only from the start to the finish in the tests but also through the homework exercises they completed in between this time.

4.5.7 Further Analysis of Number Operations from Error Analysis Data

For the purposes of this study, I performed the same error analysis treatment on Maths Minutes that learners completed later in 2012 to establish what the persistent conceptual challenges were for the learners. I analysed Maths Minutes 76 – 85 which were completed more than three months later in July 2012. I used the same Diagnostic Matrix Tool to match the Maths Minutes questions to my conceptual landmarks. Appendix H shows the matching of these further 100 Maths Minutes questions with the conceptual landmarks on the Numbers Diagnostic Matrix Tool.

Again, I calculated the percentage error occurrence across the different conceptual landmarks. Due to the design of the Maths Minutes exercise becoming progressively more challenging for learners, some conceptual landmarks were no longer represented in this set of Maths Minutes. For example, all place-value questions involved decimals rather than whole numbers and there were no questions that only tested the subtraction of whole numbers. Subtraction was only included in questions on mixed operations which is a separate conceptual landmark on my Numbers conceptual thread.

This additional error analysis provided the opportunity to establish which errors persisted over time and so to identify which conceptual landmarks were more resistant to mastering.

4.6 Data Collection & Analysis: Measuring the Value in Metacognitive Skills

Research sub-question: To what extent can this approach change learners' self-evaluations of their own understanding of Mathematics?

4.6.1 Choosing a Measuring Tool for Metacognition

How can we measure knowing about knowing? Almost 30 years ago, this question was presented by Garner and Alexander (1989) and the challenge it poses still exists today. Unlike the past two research sub-questions, I did not record actual measures of metacognitive skills during the period from February 2012 to September 2012. I had collected the evidence of learners' metacognitive skills in their self-reflective classroom activities and their weekly journal writings which I had read through as they were produced but the latter I had not analysed formally at the time.

In my need for a method and/or tool to analyse and measure metacognitive progress for this study, I turned to the literature. While the growth of interest in metacognition has spawned many rubrics and other assessment tools shared on the internet, almost all I studied seem to focus on a self-assessment of study skills and were not appropriate for my need.

I considered Garner's two methods to assess metacognition: interviews and think-aloud protocols (1988). While both are ways of verbalising metacognitive knowledge, the difference is that interviews are retrospective and think-alouds are concurrent. Both have the challenges of language limitations of the responders and each have compromises: interviews may suffer from processing time-delay inaccuracies, while think-alouds may interfere with cognitive activity in the moment. Both posed a problem for my retrospective design.

McCormick (2003) produced a useful overview of metacognitive assessments for use in classroom contexts. I will briefly describe each that I considered for my own use. The Learning and Study Strategies Inventory, LASSI, (Weinstein, Zimmerman and Palmer, 1988) is a broad-based measure of study skills for diagnosing students' strengths and weaknesses. It comprises of 77 items which are self-reported using a Likert-type scale with ten sub-scales – but none of which targets metacognition directly.

The Motivated Strategies for Learning Questionnaire, MSLQ, (Pintrich, Smith, Garcia & McKeachie, 1993) comprises of 81 items using a 7-point Likert-type scale and does include a subscale for metacognition as regulation of cognition but this is a minor component of the tool.

The Metacognitive Awareness Inventory, MAI (Schraw and Dennison, 1994), measures both knowledge of cognition and regulation of cognition. There are 52 items, for each of which the respondent chooses a position along a rating scale between true and false. The majority of the items had little relevance for my more specific requirements.

Some methods of measuring metacognition rely on standardised achievement scores, however this is problematic as it has not been shown that there is any direct relationship between these. Pintrich *et al.* (1993) confirmed that there was no correlation between academic achievement and metacognitive skills. Swanson (1990) showed that there was no relationship between metacognitive knowledge and intellectual aptitude. In fact Swanson found that metacognitive skills assisted lower-aptitude children in compensating for this when performing problem-solving tasks. While it is dubious practice to use academic achievement as a measure of metacognition, there is overwhelming agreement that metacognition plays an important role in student expertise and success. I could not consider any of these methods for measuring metacognition as I needed to be able to distinguish between academic achievement and metacognitive skills.

My rejection of these more established methods was reinforced by looking at research such as that of Everson and Tobias (1998) which claims accurate metacognition is associated with better academic performance. They developed the Knowledge Monitoring Ability (KMA) which sounds promising when described as measuring the difference between college students' estimates of knowledge and their actual knowledge. However it does this by means of a simple vocabulary test and then measuring words students said they knew and they did and words they said they didn't know and they didn't. I decided that it would be best to construct my own metacognitive measuring tool.

4.6.2 Creating a Measuring Tool for Metacognition

I was undaunted by my lack of success in finding an existing measuring tool to apply to my need. I am no stranger to creating assessment tools and after evaluating the existing metacognitive skills assessment tools, I felt that I could create a suitable instrument for this study.

In order to maintain a holistic view of the learners' metacognitive skills, and as a measure for this research study, I needed a tool to assist with analysing learners' metacognitive skills over the period of the study. I decided to create an instrument that would be applicable to the different criteria that matched the type of metacognitive skills embodied in my assessment activities while still reporting on the same scale.

I had made a variety of measures of metacognitive skills over the course of my involvement in the L4E programme. Only those measures made within the study period (February 2012 to September 2012) are included in this research study. Some of these measures yielded quantitative data. For example learners scored their ability to solve number operation problems and then compared these scores with their actual achievement. Learners classified their levels of proficiency on conceptual landmarks and then compared these with defined and measured levels of proficiency. I used the difference between these measures as a measure of metacognitive skill level. Some measures yielded qualitative data. For example learners produced a written description of their Maths strengths to build on and their weaknesses to overcome. In another case learners produced a written explanation for the errors that were made in a test.

I devised a researcher tool that I used to evaluate the learners' self-assessments and written responses on a single scale to standardize metacognitive measures and enable comparison of metacognitive skills over time. I chose four levels of proficiency or performance for my tool. The reasons for choosing four levels was that I believed that this many levels could be clearly and distinctly defined and could satisfactorily cover the range from very poor to excellent performance. I deliberately chose an even number of levels to avoid a middle catch-all level.

To simplify my tool I separated metacognitive skills (already discussed in Chapter Three) into:

- **metacognitive activation** – the emergence of evidence in learner practice of metacognition;
- **metacognitive awareness** – showing evidence of conscious and informed metacognition;
- **metacognitive analysis** – showing evidence of analysing own learning strengths, weaknesses, challenges and needs; and
- **metacognitive advancement** – evaluating and applying actionable strategies to improve learning.

As a starting point for writing the specific level descriptors I would need in the tool, I wrote a descriptor for each of the above metacognitive skills as reflected on each of my four levels which I labelled: absent, deficient, sufficient and proficient in terms of metacognitive skills.

Level 0 - Absent
• **Learner shows no metacognitive activation yet** – *there is no evidence that the learner realises that self- reflection could be used to advance cognition for improved learning.* • **Learner shows no metacognitive awareness yet** – *there is no evidence that the learner recognises that their learning process is any different to other learners or that they can compare their learning progress with any reference point.* • **Learner shows no ability to analyse their own cognition** – *there is no evidence that the learner has any understanding of their own learning strengths, weaknesses or needs* • **Learner shows no ability to evaluate or use learning strategies** – *there is no evidence that the learner can consider, compare, evaluate or choose any strategies to improve their learning and academic performance.*

Level 1 - Deficient
• **Learner shows limited metacognitive activation** – *there is evidence that the learner realises value in self- reflection but no evidence of how this could be used to advance cognition for improved learning.* • **Learner shows limited metacognitive awareness** – *there is evidence that the learner recognises that their learning process is personal and unique but there is no evidence that they can compare their learning progress with any reference point.* • **Learner shows limited ability to analyse their own cognition** – *there is evidence that the learner has a simple understanding of their obvious learning strengths, weaknesses and/or needs.* • **Learner shows limited ability to evaluate or use learning strategies** – *there is evidence that the learner can consider and compare strategies to improve their learning and academic performance even if they can't make an appropriate choice.*

Level 2 - Sufficient
• **Learner shows sufficient metacognitive activation** – *there is evidence that the learner realises the value of self-reflection and that this can be used to advance cognition for improved learning even if they are not using this yet.* • **Learner shows sufficient metacognitive awareness** – *there is evidence that the learner recognises and has a basic understanding of how their own learning process differs from that of other learners and can compare their learning progress with an obvious reference point.* • **Learner shows a sufficient ability to analyse their own cognition** – *there is evidence that the learner has a simple responsive and basic understanding of their own learning process, strengths, weaknesses and needs.* • **Learner shows an advanced ability to evaluate, select and use learning strategies** – *there is evidence that the learner has considered, compared, evaluated and simply chosen a strategy to improve their learning and academic performance.*

Level 3 - Proficient
• **Learner shows advanced metacognitive activation** – *there is conclusive evidence that the learner not only realises the value of self-reflection, but is actively using this to advance cognition for improved learning.* • **Learner shows advanced metacognitive awareness** – *there is conclusive evidence that the learner recognises and understands how their own learning process differs from that of other learners - they can compare their learning progress with any reference point.* • **Learner shows an advanced ability to analyse their own cognition** – *there is conclusive evidence that the learner has a responsive and actionable understanding of their own learning process, strengths, weaknesses and needs.* • **Learner shows an advanced ability to evaluate, select and use learning strategies** – *there is conclusive evidence that the learner has considered, compared, evaluated and insightfully selected best strategies to improve their learning and academic performance.*

Through this preparatory writing process I realised that it was only through learner evidence of metacognitive analysis and metacognitive advancement that I could establish the existence of metacognitive activation and metacognitive awareness. I recognised that I could simplify my metacognitive measuring tool by focusing on these first two metacognitive skills. I then brainstormed a list of criteria for metacognitive analysis and metacognitive advancement which included:

- Estimating or predicting performance or achievement
- Classifying proficiency
- Identifying, describing and discussing strengths and weaknesses
- Determining learning challenges, needs and opportunities
- Evaluating and suggesting strategies for improvement and growth

Next I looked at the self-reflective activities that learners had completed in class over the study period. I did not consider the learner responses to these, I only considered the design of each activity and from this I matched criteria to assess these metacognitive skills for each specific activity. I chose five suitable criteria and then I wrote proficiency level descriptors or performance indicators for each of the four levels of proficiency for each of my specific criteria. In this way, I created what I have called the **Metacognitive Activation and Advancement Rubric** (Table 4.8) which includes both criteria with proficiency level descriptions of quantitative measures and criteria with proficiency level descriptions of qualitative measures on the same tool. (I chose to focus on the words *activation* and *advancement* as I believed these sum up the essentials of acquiring and using metacognitive skills.)

This Metacognitive Activation & Advancement Rubric (MAAR) tool enables me to analyse and interpret the many different kinds of metacognitive evidence collected over the period of the study and assess these on the same four-level measurement scale for useful comparison. Criteria were chosen and proficiency level descriptors were written without considering learners' responses to reflective activities but rather to best match the nature and design of these activities.

Table 4.8: The Metacognitive Activation & Advancement Rubric (MAAR) Tool

Assessment Criteria	Level 0	Level 1	Level 2	Level 3
Estimating performance on a mathematical conceptual landmark (measured in percentage achievement)	Actual achievement is beyond 30% of estimated achievement.	Actual achievement is within 30% of estimated achievement.	Actual achievement is within 20% of estimated achievement.	Actual achievement is within 10% of estimated achievement.
Classifying proficiency as practitioner, apprentice and novice in error analysis of conceptual landmarks	Correct proficiency level is assigned in less than 50% of conceptual landmarks AND/OR (more than half of) these are two levels out of range.	Correct proficiency level is assigned in more than 50% of conceptual landmarks BUT (less than half of) these are two levels out of range.	Correct proficiency level is assigned in more than 50% of conceptual landmarks AND all errors are only one level out of range.	Correct proficiency level is assigned in all conceptual landmarks.
Showing awareness of strengths and weaknesses through: * identifying these * describing these * discussing these	Identified mathematical strengths and weaknesses are not clear or do not correspond with learner evidence.	Identified mathematical strength/ weakness shows some correspondence with learner evidence however core issues are overlooked.	Core mathematical strength/ weakness corresponds with obvious learner evidence.	All identified mathematical strengths and weaknesses clearly correspond with learner evidence.
	Strengths/ weaknesses are described without mathematical language and are so general that they are not actionable e.g. "writing answers"	Strengths/ weaknesses are described in simple mathematical language and are limited to conceptual threads e.g. "working with whole numbers".	Strengths/ weaknesses are described in mathematical language to the conceptual landmark level but lacking conceptual thread specifics e.g. "rounding off".	Strengths/ weaknesses are described using detailed mathematical language within conceptual landmarks e.g. "to round off whole numbers to the nearest ten".
	Discussion provides no clear actionable direction for the way forward.	Discussion focuses on attitude and effort as a solution for the way forward.	Discussion provides simple, actionable mathematical advice.	Discussion provides insightful detail of mathematical conceptual understanding.
	Writing is scanty. Single, simple words are used rather than phrases or sentences.	Writing is simple. Phrases and simple sentences used.	Writing clearly links to conceptual landmarks and provides basic detail.	Writing provides rich, mathematical self-awareness of the details of conceptual landmarks.
Suggesting strategies for improvement and growth in own mathematical conceptual development and progress	No actionable strategies for progress are communicated.	Mathematical topics or concepts are not mentioned. Suggested strategies are only attitudinal and/or related to general effort.	Mathematical topics are specifically mentioned. Suggested strategies are actionable but are limited to attitude and/or effort only.	Mathematical topics and concepts are presented in written detail. Maths-specific, actionable strategies are suggested.
Analysing own performance using metacognitive skills of: * identifying weaknesses * explaining challenges * suggesting strategies	No evidence of metacognitive skills.	Only one of identified metacognitive skills is evident in written analysis.	Two identified metacognitive skills are evident in written analysis.	All three identified metacognitive skills are evident in written analysis.

4.6.3 Collecting & Analysing the Data

Over the study period, I conducted many metacognitive skills assessments. As I did not have access after the L4E programme to many of these original learner scripts, I chose only the following six assessments (for which I did have original learner scripts) to analyse for my study.

Table 4.9: Summary of Metacognitive Skills Assessments included in this Study

Date	**Metacognitive Skill – Assessment Criteria**	**Nature of Data**
February 2012	Estimating performance on a mathematical conceptual landmark (measured in percentage achievement)	Quantitative
Mid-April 2012	Classifying performance as practitioner, apprentice and novice in error analysis of conceptual landmarks	Quantitative
April-End 2012	Describing mathematical strengths and weaknesses	Qualitative
April-End 2012	Determining learning opportunities of assessment, error analysis and self-evaluation	Qualitative
April-End 2012	Suggesting strategies for improvement and growth in own mathematical conceptual development and progress	Qualitative
June 2012	Analysing own performance using metacognitive skills of: • identifying weaknesses • explaining challenges • suggesting strategies	Qualitative

To analyse learner evidence and assess the level of metacognitive skills each learner was demonstrating, I used the Metacognitive Activation and Advancement Rubric (MAAR).

To analyse the quantitative data:

- I calculated the difference between the a learner's self-assessed, estimated score for each operation before a number operation test was conducted and the actual score the learner achieved on each operation when s/he completed the test. The range of difference was interpreted using the MAAR Tool to place each learner on one of four metacognitive levels to measure the ability to estimate performance.

- I compared a learner's self-assessed classifications as a practitioner, apprentice or novice on Numbers conceptual landmarks in the error analysis activity with my classifications of the same learner based on their Diagnostic Matrix evidence. The comparison was interpreted using the MAAR Tool to place each learner on one of four metacognitive levels to measure the ability to classify proficiency.

To analyse the qualitative data I again used the MAAR Tool to assess the metacognitive skills levels of each learner and place each on one of four levels for each of the criteria. Tables presenting examples of learner evidence corresponding to the level descriptor of proficiency on each assessment criterion are presented and discussed in Chapter Seven. The use of this MAAR tool enabled me to assess the evidence from a number of different activities and place these on the same scale to be able to compare proficiency over time and investigate progress.

4.7 Ethical Issues

One of the goals of the L4E programme was to work concurrently with the parents, educators and school leaders to provide a comprehensive support system so learners, their parents/ guardians and a staff representative from each of the ten schools involved signed a commitment agreement with Uplands Outreach at the start of the programme. While the requirement to participate in all assessment activities was conditional to inclusion in the L4E programme, learners were free to withhold the use of their assessment data in this study.

A consent letter that all data (past, present and future) could be used for research purposes was voluntarily signed by each stakeholder including the learners. The Insikazi circuit manager, in his capacity as a representative of the Mpumalanga Department of Education, issued a letter of consent to use data gathered over the course of the project for research purposes. Ms Nicolette De Bruyn, Director of Uplands Outreach, provided a letter confirming permission for me as a consultant to the project to utilise any data collected by and for Uplands Outreach. Data is lodged on the computers at the Uplands Outreach office in White River, Mpumalanga as well as on my computer in my home office in Hermanus, Western Cape. Data will be destroyed when it is no longer needed. It is at present still informing both my practice and that of Uplands Outreach.

Learners' formative and summative assessment data was made anonymous in this study by allocating codes for schools and their learners. Care has been taken to maintain anonymity but, while not mentioned by name, some learners may be identified by their success stories. Ethics clearance (protocol number: 2016ECE043M) was granted by the University of the Witwatersrand for this retrospective study.

4.8 Conclusion

One of the challenges of conducting a study after data has already been collected is the selection of which data to use and which to exclude. My decisions were restricted by the nature and extent of my assessment records. Despite having collected great volumes of data, for the purpose of this study I only used that for which I either had the learners' actual scripts or those where my recording of scores was clearly identifiable by the specific conceptual landmarks being measured. I also chose to focus on the first year of the L4E intervention despite having detailed records of both of my years of active involvement in the programme. Due to the two kinds of learner evidence I had saved after collection, i.e. achievement scores or marks and written self-reflective, self-evaluating journal entries, I chose a mixed methods approach for a richer analysis of the progress in learning rather than just using the learners' quantitative scores. The results follow in the next three chapters.

CHAPTER FIVE

Results & Discussion: Catching Up Learners

In this chapter I seek to answer the question of whether an intervention using an assessment ***as*** learning – conceptual thread approach such as that of the L4E programme can make up acquired learning deficits enabling low-achieving learners to catch up and achieve at their actual grade-level. After discussion in the last chapter of the need to design a unique diagnostic tool that could reveal the L4E learners' mathematical difficulties and identify the extent of these for remediation, this chapter focuses on the comparison between learners' achievement on that diagnostic test at the start of the intervention and at the end of the intervention in order to evaluate the impact of the approach taken.

5.1 Where We Started – L4E Pre-intervention Results

As already mentioned in Chapter Two, the baseline test results were very disappointing, especially considering that these learners were chosen to participate in the L4E programme either because of their achievement, aptitude or interest in Maths. Overall, the learners achieved only a 23.8% average.

Each learner's overall results can be seen in the 'skyscraper'graph of Figure 5.1. Learner data is collected in their school groups (five learners in each of the ten schools) and a code assigned relevant to their school. Numbering of learners was alphabetical. This means that learners F1 to F5 are from the same school etc. From the data it can be seen that there is little difference between schools in terms of overall achievement despite their varying quintile ratings.

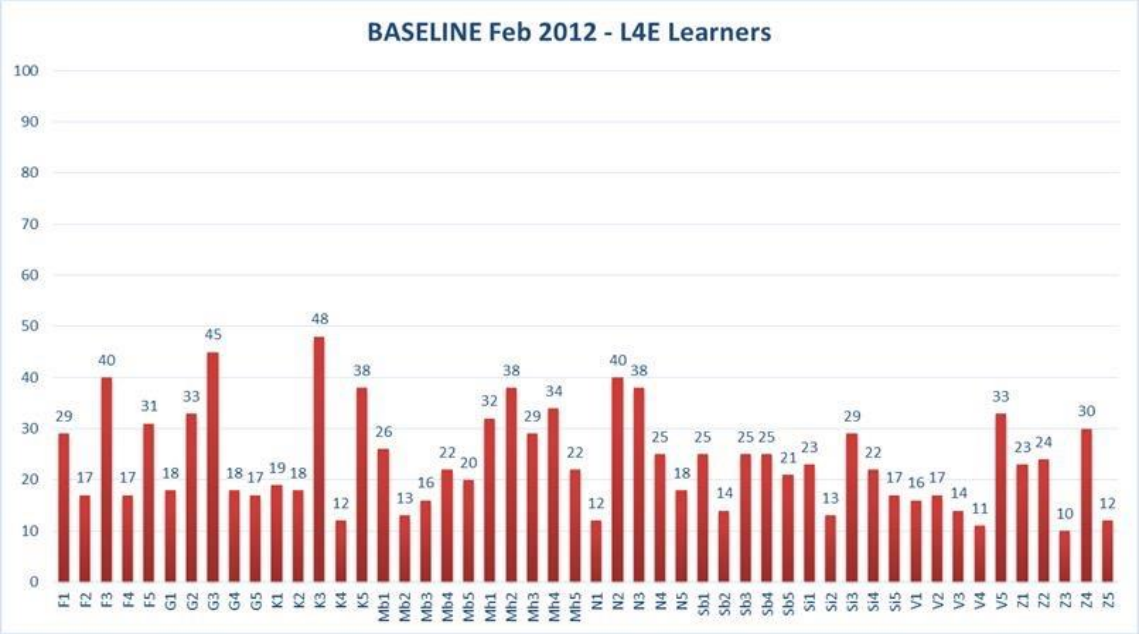


Figure 5.1 Overall Learner Achievement on L4E Pre-intervention Diagnostic Test

The highest achievement among the group of L4E learners was 48% while the lowest achievement was just 10%. This should not have been surprising as the Annual National Assessments (ANAs) had been administered for the first time in 2011 to Grade 3 and Grade 6 learners with disappointing results. 83% of both Grade 3 and Grade 6 learners in Mpumalanga were scored on Level 1 or 'Not Achieved' in the Mathematics ANAs.

Diagnostic test questions were straightforward and focused by design to establish whether a learner had mastered the specific grade level for each conceptual landmark. Based on this design, expectations were that a learner who had completed Grade 9 Maths would achieve at least 90% on the Grade 3 level questions, 80% on the Grade 6 level questions and 70% on the Grade 9 level questions. Referring to Table 4.6 in Chapter Four, this resulted in the following grade level ranges: a learner achieving 12% or less on the diagnostic test was classified as not yet at Grade 3 level; a learner achieving between 13% and 35% was classified as having mastered Grade 3 level but not yet at Grade 6 level; a learner achieving between 36% and 74% was classified as having mastered Grade 6 level but not yet at Grade 9 level; and a learner achieving 75% and above was classified at having mastered Grade 9 level.

Ordering the L4E learners' overall results and adding these Grade-Level expected benchmarks to Figure 5.1 highlighted the challenge the L4E programme was facing.

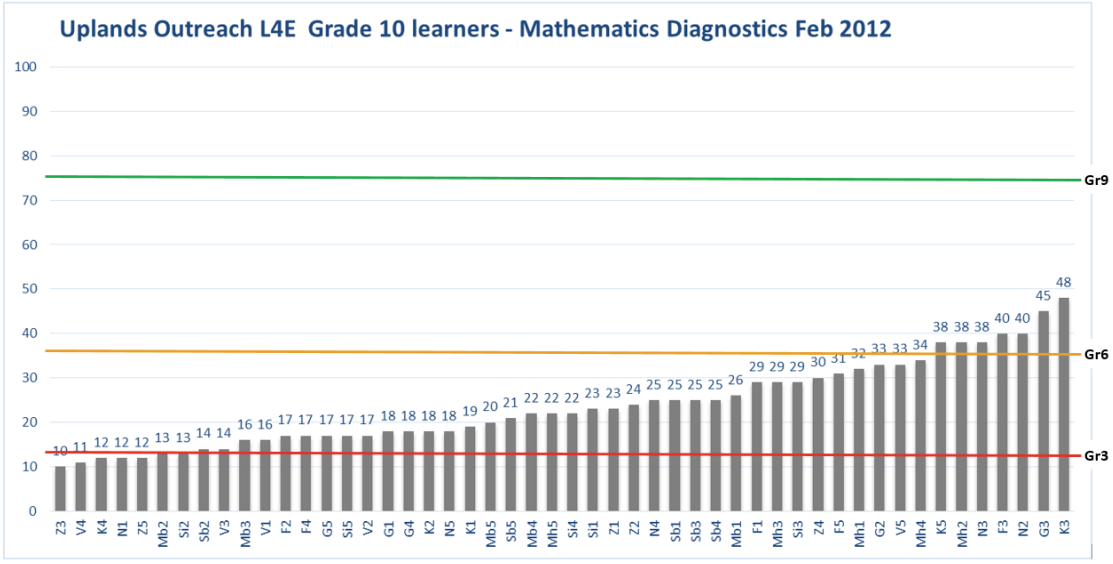


Figure 5.2 Overall Learner Achievement on L4E Pre-intervention Diagnostic Test with benchmarks

Five learners (10% of the group) were performing below Grade 3 level. Thirty-eight learners (76% of the group) were performing above Grade 3 level but below Grade 6 level. Seven learners (14% of the group) were performing above Grade 6 level but not at Grade 9 level yet. No one had mastered Grade 9 level. When considering overall learner achievement on each individual conceptual thread, strengths and weaknesses become a little more apparent.

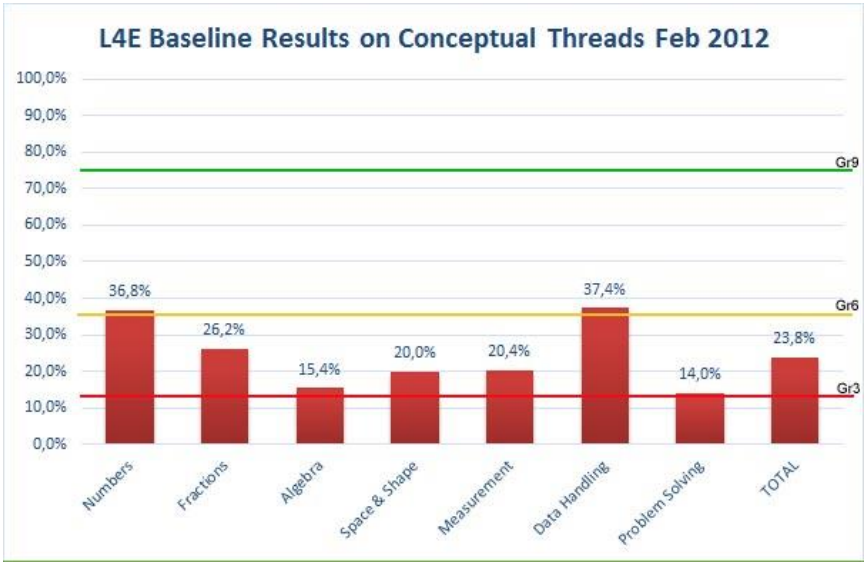


Figure 5.3 Achievement on Conceptual Threads for L4E Pre-intervention Diagnostic Test

Results across the seven conceptual threads were consistently poor. Data-Handling, with an average of 37.4% was the conceptual thread in which learners achieved the highest scores, while Problem-Solving, with an average of 14.0%, showed the lowest achievement. Algebra was particularly worrying as the focus on Algebra shifts to about 40% of time allocated in Grade 10. Overall, in most conceptual threads, learners were operating at a level between Grade 3 and Grade 6. Only in Numbers and Data-Handling were learners above the Grade 6 level, and then barely.

Deeper understanding of the levels at which the majority of learners were operating was gained by applying the same analytical approach for the overall test data to the weighting used for the individual conceptual threads and their data to establish learners' grade level of mastery of each thread. Figure 5.4 revealed the proportion of learners functioning at Grade 1-2 level (ranging from 0% in Measurement to 66% in Problem-Solving), the proportion at Grade 3-5 level (ranging from 28% in Problem-Solving to 98% in Fractions) and the proportion at Grade 6-9 level (ranging from 0% in Fractions to 34% in Patterns & Algebra) for each conceptual thread. In this way intervention priorities could be established: which conceptual thread to start with and how to order and balance the time allocated for the remaining threads within the programme constraints.

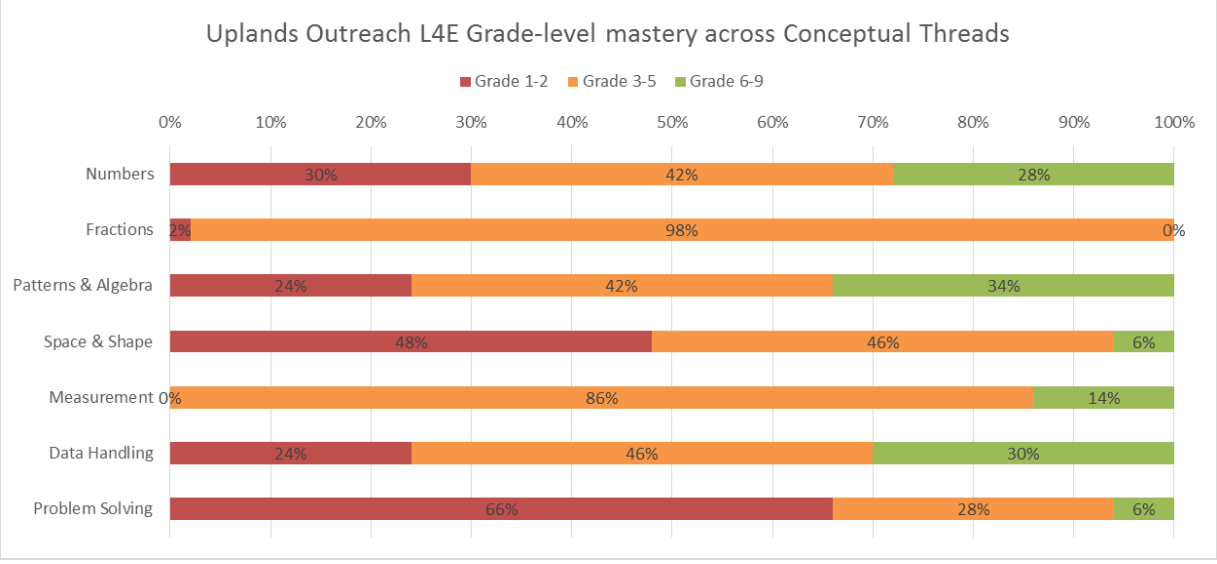


Figure 5.4 Grade-level Mastery on Conceptual Threads for L4E Pre-intervention Diagnostic Test

Although achievement in the Numbers conceptual thread was third highest, the importance of Numbers underpinning all of Maths, caused me in my teacher role to prioritise this thread. That more than 70% of learners were below Grade 6 level in this important conceptual thread was convincing that this should be the intervention starting point. I was especially concerned about the bottom 30% of learners who were performing below Grade 3 level. I justified this choice to focus on Numbers for two additional reasons: my teaching approach (see Chapter Two) provided differentiated activities and opportunities for the more able learners to work independently on more challenging tasks so the top 28% would still progress; and the CAPS curriculum extends learners in number range and operations in the Intermediate Phase (Grade 4 –6) with the bulk of Senior Phase (Grade 7 –9) involving just practising with these numbers and operations so the extra practice would be beneficial too.

The evidence of the extent of learners' difficulties with Fractions was the motivation to postpone dealing with this conceptual thread in 2012. All 50 learners were at a level below Grade 6 with two learners not even at Grade 3 level. As already mentioned in Chapter Two, I attempted to address Problem-Solving through providing weekly challenges but when I realised that learners were not improving without learning problem-solving strategies, the active teaching of this thread was postponed until 2013 too.

This baseline picture informed the focus and allocation of time to address the different conceptual threads before administering the diagnostic test as a summative measure after the seven months had passed.

5.2 Where We Ended – L4E Post-intervention Results

On the 22nd September 2012 the diagnostic test was administered again to the L4E learners under the same stringently controlled conditions and marked, collected and analysed in the same way. As described in Chapter Four, questions related to Fractions and Problem-Solving were held over until 22nd June 2013 once further teaching and learning sessions focusing on these two more problematic conceptual threads had taken place.

After administering these questions in June 2013, the Fractions and Problem-Solving results were included in the overall results and are presented in Table 5.5.

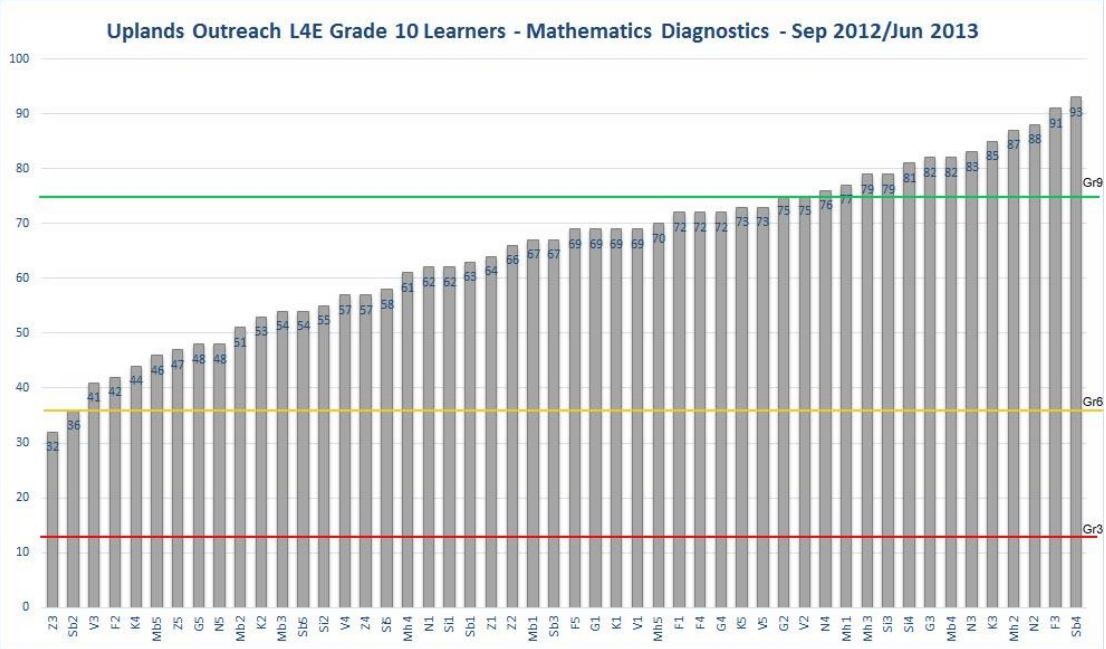


Figure 5.5 Overall Learner Achievement on L4E Post-intervention Diagnostic Test with benchmarks

These results were much improved. All learners were now above the Grade 3 level of expected achievement. One learner was still below the Grade 6 level. This was the weakest learner. She was the lowest performing learner in the baseline test (10%) and she was still the lowest performing learner in the summative test but nevertheless her result had improved to 32%. Thirty-four learners (68% of the group) were above Grade 6 level but had not yet achieved Grade 9 level after the seven month period of intervention. Fifteen learners (30% of the learners) had reached or exceeded the Grade 9 level of expected achievement.

The greatest improvement was made by learner code Sb4 who improved by 68 percentage points. He was the top learner in the summative test (achieving 93%). This was after he had been ranked 18th in the baseline test with a poor 25% result seven months earlier. The smallest improvement was made by learner codes Sb2 and Z3 who each improved by 22 percentage points. These increases were still improvements of 150% to 220%.

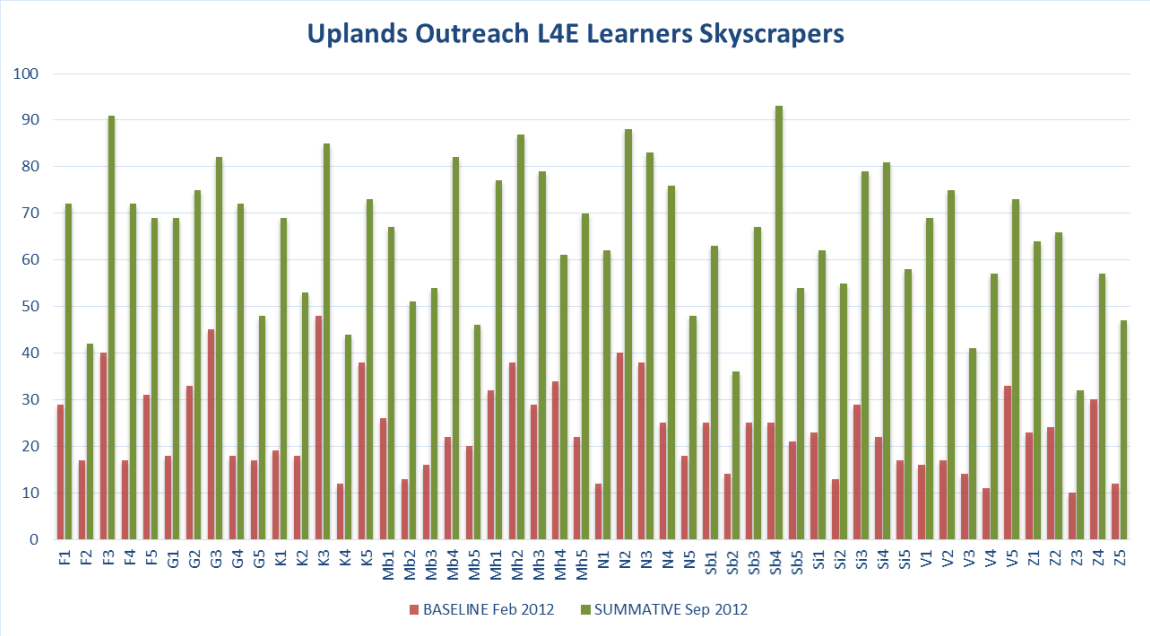


Figure 5.6 Overall Individual L4E Learner Progress

Although not all learners had reached Grade 9 level, 44 out of the 50 learners had more than doubled their baseline scores in the summative test.

To further analyse these improvements from pre-intervention to post-intervention scores, statistical measures described in Chapter Four were applied and are shown in Table 5.1.

Table 5.1 Statistical Summary of L4E Results

Statistical Measures	**Pre-intervention**	**Post-intervention**
Mean	23.8	65.5
Variance	88.6	217.7
Standard deviation	9.41	14.75
Sample within 1SD	66%	64%
Sample within 2SD	96%	98%

The data indicates a normal distribution. A paired t-test was conducted on the pre-intervention and post-intervention data and produced a t-value of 16.86. When comparing this value with those on a t-distribution table and using 49 degrees of freedom indicates with 99.9% confidence that the difference in pre- and post-intervention is statistically significant. (The t-table threshold value is 3.5004).

As is evident in Table 5.1, overall average achievement improved from 23.8% to 65.5%. Figure 5.7 compares the change in average achievement for each conceptual thread.

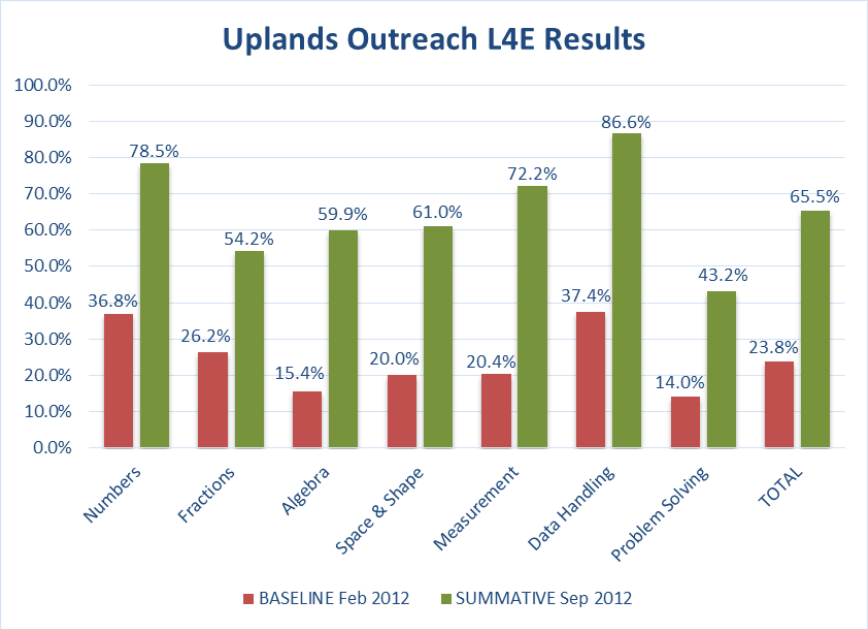


Figure 5.7 Progress in Average L4E Achievement for Conceptual Threads

Plotting the baseline results and summative test results on the same graph shows the extent of improvements in average achievement. The largest improvement was achieved in the conceptual thread of Measurement (+51.8 percentage points). The smallest improvement was achieved in the conceptual thread of Fractions (+28 percentage points) although it is still more than a 100% improvement.

Results across the seven conceptual threads were much better after the intervention with learners' average achievement on all conceptual threads well above Grade 6 level. However achievements on only two conceptual threads, Numbers and Data-Handling, were above Grade 9 level. Again Data-Handling, with an average of 86.6% was the conceptual thread showing the highest achievement, while Problem-Solving, with an average of 43.2%, showed the lowest achievement.

5.3 Conclusion

The L4E intervention with its assessment ***as*** learning – conceptual thread approach clearly made a significant difference in learner achievement on the seven conceptual threads that comprise basic Mathematics over a comparatively short period of time. However has it 'made up' acquired learning deficits and caught up learners? To some extent it has. I can conservatively assert that all of the learners caught up at least one phase (the equivalence of three years) of their Maths education. Considering each individual learner's improvement, 12 learners actually caught up in excess of two phases (progressed the equivalent of 6+ years). However only 15 learners were able to achieve overall at or above Grade 9 level. As already mentioned in Chapter 2, when comparing the L4E learners' achievement in Mathematics at their schools, they had more than doubled the difference between their results and their peers' results by the end of Term 3 (September 2012). I can conclude that all learners made some progress indicating that they were able to make up significant portions of their acquired learning deficits and to catch up to a level approaching, meeting or in some cases exceeding expectations of where they were supposed to be in terms of their Maths learning. Thus **acquired learning deficits are not necessarily insurmountable** and **Grade 10 is not too late** for learners to catch up and achieve in Mathematics.

CHAPTER SIX

Results & Discussion: Revealing Learning Difficulties

In this chapter I seek to answer the question of whether an intervention using an assessment ***as*** learning – conceptual thread approach such as that of the L4E programme, can provide insight for teachers into the occurrence and nature of learners' difficulties in Mathematics. To investigate this I have chosen to focus on one conceptual thread only, that of Numbers – Concept, Relationships and Operations. Within this conceptual thread I have only considered the conceptual landmarks in the number operations of addition, subtraction, multiplication and division. My purpose is to compare these operations and to reveal obstacles to mastering these conceptual landmarks among the L4E learners. In so doing, I can explore the value of an assessment ***as*** learning – conceptual thread approach specifically for teachers; not only providing value in revealing difficulties in Maths for the learners they may currently teach, but also through this L4E example, share understanding of some of the challenges that the majority of South African learners may face in making sense of Mathematics.

6.1 Progress in Number Operations

Achievement was measured for number operations in the three term tests conducted in February, June and September 2012. The following results were achieved by the 50 L4E learners over this period.

Table 6.1 Achievement in Number Operations over Time

	Mean Score	**Range**	
		Lowest Score	**Highest Score**
February 2012	49.5%	12.5%	100.0%
June 2012	73.2%	30.0%	100.0%
September 2012	87.2%	30.0%	100.0%

The learners' overall performance on number operations improved over time. Over seven months the class average increased from just below 50% at the start to over 85% at the end.

Using the range as a measure of dispersion does not distinguish the data much over the study period so for a deeper understanding, I calculated the percentage of learners achieving within four ranges: 0-25%, 26-50%, 51-75% and 76-100%.

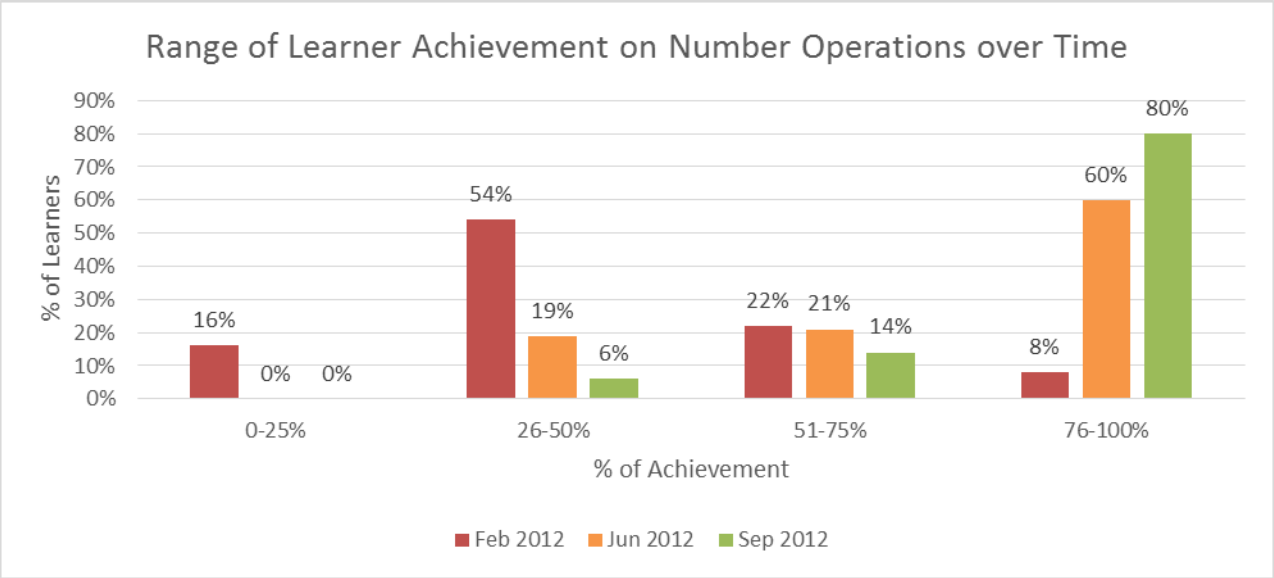


Figure 6.1 Range of Learner Achievement on Number Operations over Time

This shows that in February 2012 when the L4E programme began, 70% of the learners were achieving results below 51% on number operations. By September 2012, 80% of learners were achieving above 75% on number operations. It is interesting to see that initial progress happened quickly. By June 2012 only 19% of learners were achieving below 51% and 60% were already achieving in the highest range of 76-100%. The 6% of learners remaining below 51% in September 2012 were a concern but when I considered their individual cases, two of three learners had actually achieved 50% and so were at the top of this range. The third learner had achieved 30% in number operations. This learner was going through escalating difficulties at home that were impacting not only on his achievement in all aspects of the L4E programme but also in his behavior at L4E on Saturdays and in his school during the week.

In order to better understand difficulties in number operations, I considered the average achievement of the group on each of addition, subtraction, multiplication and division separately.

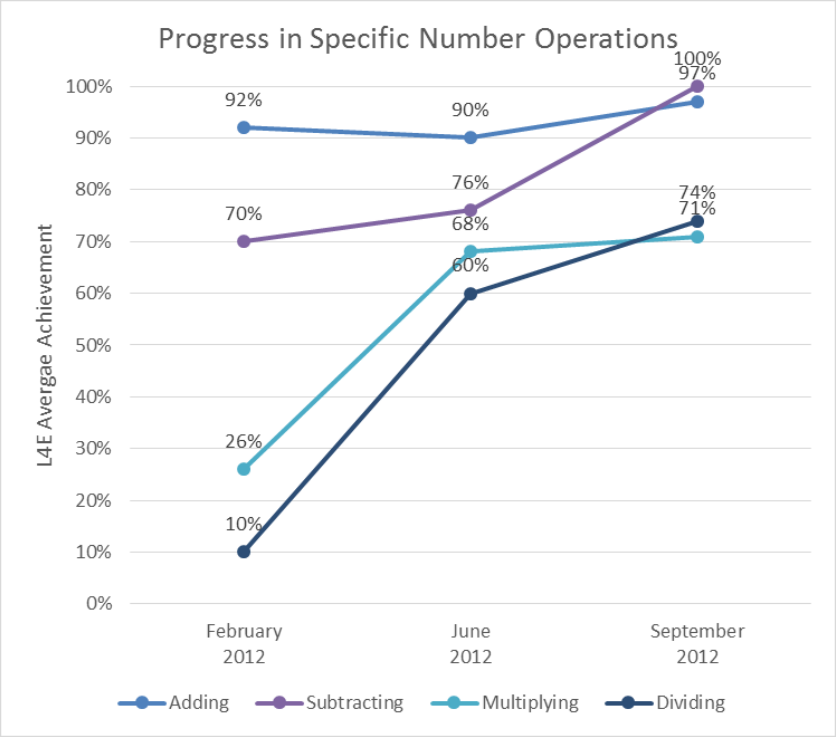


Figure 6.2 Progress in Specific Number Operations

All four number operations improved over the seven months but beyond this, Figure 6.2 shows some interesting differences in the upward patterns.

Addition of numbers was performed well by the learners from the start. Learners improved from a 92% class average to a 97% class average from February to September. There was a small but unexpected drop in the June test average but considering the high level of actual average achievement, I regarded this as most likely due to carelessness and possibly over-confidence with the operation that the learners considered easy and which had been mastered by all.

Subtraction improved from a class average of 70% to one of 100% from February to September. What is interesting is that we completed subtraction in our contact sessions well before June yet there was greater improvement from June to September than from February to June. This is at odds with the overall number operations performance pattern presented in Table 6.1. This could suggest that it takes a bit longer to fully understand this number operation. Carpenter and Moser (1984) found in a longitudinal study of learners from Grade 1 to Grade 3 that addition results in Grade 1 (almost 80% of children providing the correct

answer) were better than subtraction results in Grade 1 (just over 50% of children providing the correct answer). By Grade 3 almost 100% of these same children were providing the correct answer in both addition and subtraction. This resulted in a steeper graphed progress line for subtraction than for addition which is similar to what I found. This trend was confirmed by Fuson, Werne, Hiebert, Murray, Human, Olivier & Carpenter. (1997) who found that multi-digit subtraction seems more difficult for children to solve than multi-digit addition. Both studies presented results made at the start and then at the end of the study period without any data in between that I could compare with my L4E results. Certainly subtraction posed greater problems than addition at the start of the L4E programme and the learners were fully aware of this as we unpacked these problems in order to fix them. It seemed that this realisation of difficulties with subtraction focused the learners on this operation in their tests and their carefulness enabled a 100% achievement in September.

The number operations of multiplication and division were the poorest at the start and these improved the most over time. Average achievement in multiplication improved from 26% to 71% while average achievement in division improved from 10% to 74%. Most progress was made in the first four months which fits the patterns illustrated in Table 6.1. Similar to achievement in subtraction overtaking that in addition, achievement in division overtook that in multiplication. There seems to have been less research done on multiplication and division when compared with addition and subtraction but an experiment on the performance of university students of simple division and multiplication found that division was substantially more difficult than multiplication (Campbell, 1997). My interactions and communications with the learners in class resulted in my sense that these operations (subtraction and division) were perceived by the learners to be 'more difficult'. The results of the four number operations suggest that the learners had paid more attention to mastering subtraction and division in comparison to addition and multiplication.

6.2 Adding Whole Numbers

6.2.1 Changing Proficiencies in Addition

As described in section 4.5.3, I analysed learners' proficiencies in adding at the start of the seven month study period and at the end in September 2012 in order to understand their progress. I classified learners as practitioners if they solved both the easy and the difficult adding problems correctly. I classified them as apprentices if they could only solve one of the problems correctly and I classified them as novices if they got both problems incorrect. The fact that I didn't distinguish between learners that got the easy question wrong and the learners that got the more difficult question wrong when I recorded one out of two questions wrong is a weakness in my analysis. Careless learners who made an unexpected error in an easy question while getting a more difficult question correct, would be classified as an apprentice rather than the practitioner that they are more likely to be. The impact on my data would be more apprentices and fewer practitioners than there may actually have been. I did not worry about this compromise at the time as I was using the data to identify learners in need.

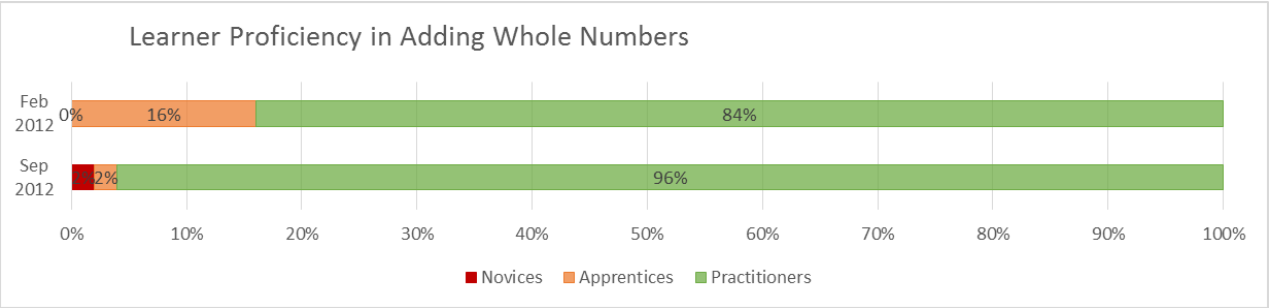
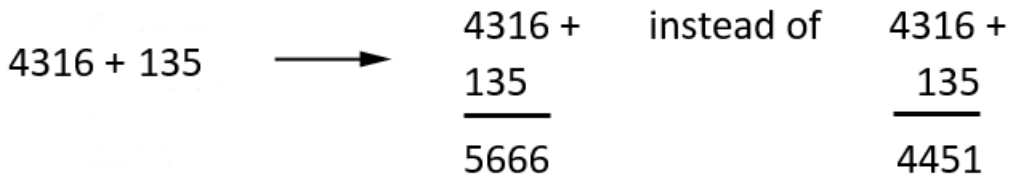


Figure 6.3 Change in Learner Proficiencies in Adding Whole Numbers

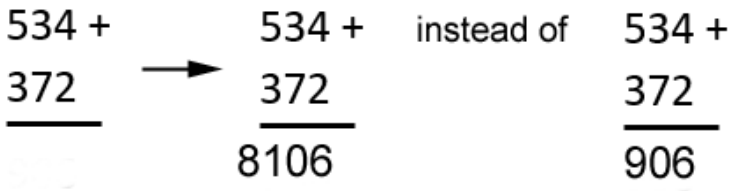
At the start of the intervention 84% of learners were already practitioners in adding whole numbers. Sixteen percent were apprentices and there were no learners who could not add whole numbers at all. Interestingly, I had a learner make errors in both addition sums in the September test resulting in his classification as a novice even though this had not been the case in February. This learner was the same learner mentioned earlier who was experiencing problems at home. Another learner remained at the level of apprentice after the seven months of intervention.

6.2.2 Common Errors in Addition

While generally learners were confident and competent in adding numbers from the start, they had limited strategies for this. They all seemed to be dependent on ordering numbers in columns for adding and some had place-value difficulties in doing this correctly. The most common error I observed was such as in the following example.



Without consideration of the place-values of digits, the learner lines up the numbers incorrectly as shown. I observed a learner adding digits by working from the left-hand column to the right-hand column. I also observed a learner lining up the numbers incorrectly as above but then adding digits from the right-hand column to the left (even placing a 0 after the 135 and below the 6 of the 4316 as a place holder before adding). In cases where learners could line up numbers correctly in place-value columns for adding and no regrouping was required then learners had no difficulty finding the correct answer. However in a sum that does require regrouping, I noticed learners making errors such as the following where ten tens are shown in the tens column instead of regrouping to form a hundred.



6.2.3 New Strategies for Addition

I do not recall any of the learners using any other strategy for adding other than in columns. It appeared as if all 50 of the learners only had this one strategy for adding numbers of two digits or more. Those who were able to add successfully were the learners correctly applying that algorithm. To develop number sense, I started with small numbers to introduce foundational strategies such as counting on and skip-counting. We explored the commutative property of addition and used number bonds to move numbers that were easier to add next to each other. Decomposing numbers was clearly a new concept for most of the learners and we practiced using this to add easily as well as the strategies of rounding and compensating. These Number Strategies for Addition are included in Appendix I.

My approach to build understanding of the traditional method of adding numbers in columns was to use place-value cards to represent the numbers involved and then to decompose the numbers to show how we were counting them up to add them. A sample of this strategy is in Appendix J.

6.3 Subtracting Whole Numbers

6.3.1 Changing Proficiencies in Subtraction

The learners' proficiency in subtracting whole numbers also improved over the seven month period.

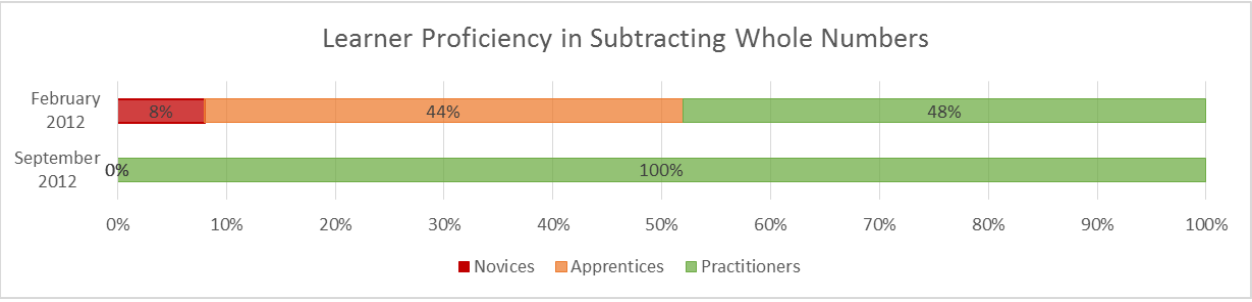


Figure 6.4 Change in Learner Proficiency in Subtracting Whole Numbers

At the start of the intervention only 48% of learners were practitioners when subtracting numbers. Forty-four percent were apprentices with 8% unable to subtract at all. By September, all learners were fully proficient in subtracting numbers (100% practitioners).

6.3.2 Common Errors in Subtraction

Again it appeared as if learners had only one strategy and that was to put the numbers in columns in order to perform subtraction. There was confusion around which number was written on top of which number. There was confusion around which digit of each number was written above or below which digit of the other number. There was confusion when confronted by a larger digit placed below a smaller digit.

Learners showed similar difficulties to adding when working in columns and using place-value correctly but the most common error I observed was simply subtracting whichever digit was larger no matter whether it was above or below the other digit in its column.

For example:

$$\begin{array}{r} 2931 - \\ 1771 \\ \hline \end{array} \longrightarrow \begin{array}{r} 2931 - \\ 1771 \\ \hline 1240 \end{array} \quad \text{instead of} \quad \begin{array}{r} 2931 - \\ 1771 \\ \hline 1160 \end{array}$$

To avoid regrouping a hundred to increase the tens when the larger number lacked tens, learners simply subtracted the tens of the larger number from the tens of the smaller number.

This was especially evident when a zero placeholder had to be bridged to subtract.

For example:

$$\begin{array}{r} 1208 - \\ 422 \\ \hline \end{array} \longrightarrow \begin{array}{r} 1208 - \\ 422 \\ \hline 1226 \end{array} \quad \text{instead of} \quad \begin{array}{r} 1208 - \\ 422 \\ \hline 786 \end{array}$$

Again, rather than regroup a hundred to ten tens in order to subtract the two tens as required, learners simply subtracted the zero tens from the two tens. It was clear from such examples (where the answer provided was larger than the original number) that the concept of subtraction was divorced from the procedure followed to produce an answer.

6.3.3 New Strategies for Subtraction

Similar to the case with adding numbers, the L4E learners appeared to use only one strategy of subtracting numbers in columns. Those who were able to subtract successfully were the learners correctly applying that algorithm. I felt that the learners lacked a basic familiarity with numbers and so we played games with adding and subtracting within the Grade 1 range of numbers (up to 99). I stressed the inverse relationship between addition and subtraction. As the numbers became larger, I focused on decomposing numbers which was clearly a novel idea for most of the learners. We started with foundational strategies such as counting back and skipping back. We practiced rounding and compensating. We focused on regrouping, first with decomposing and then when working with place-value columns. These Number Strategies for Subtraction are included in Appendix K.

6.4 Multiplying Whole Numbers

6.4.1 Changing Proficiencies in Multiplication

Learners' proficiency in multiplying numbers improved over the seven month period however only 58% were fully proficient after the intervention. Despite this 52% of the learners were able to progress from being novices at the start. More than half of these learners leap-frogged the apprentice level to achieve practitioner level by the end of the intervention.

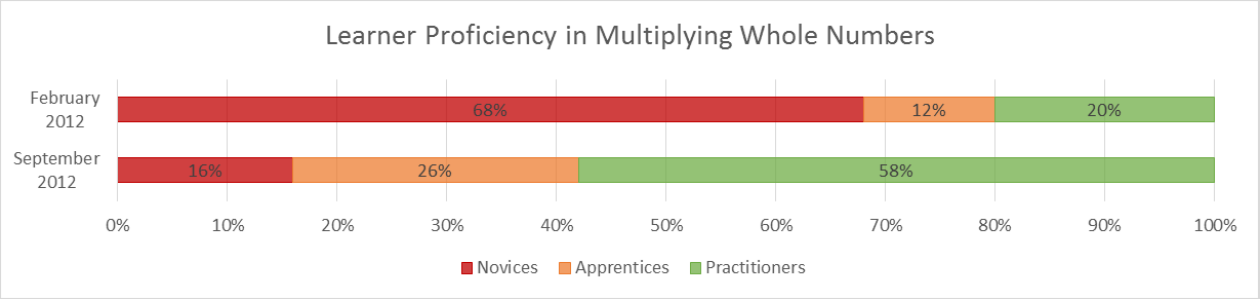


Figure 6.5 Change in Learner Proficiency in Multiplying Whole Numbers

It appeared to me that multiplying was the most challenging number operation to master and it seems to be deceptive in this. This poses a metacognitive challenge too. Time-table strategies and games had built learners' confidence in their ability to multiply but this had not

resulted in the same levels of proficiency as in the other operations. Learners' self-assessments and comments suggested that they found multiplication easier than division (as Campbell, 1997 had found) but their results did not agree with this.

It is interesting to track learner progress in practising times-tables. Initially I administered a test with 60 mixed time-table questions for the learners to answer. Learners achieved an overall average of 29%. I then decided to test each time-table separately and I interleaved these tests between February and June 2012. Tests were done in class time under time restraints (90 seconds to complete). Learners marked each other's work but I checked all marking before recording their results in my Excel spreadsheet. The L4E class average for the first and last tests for each times-table test are presented in Figure 6.5.

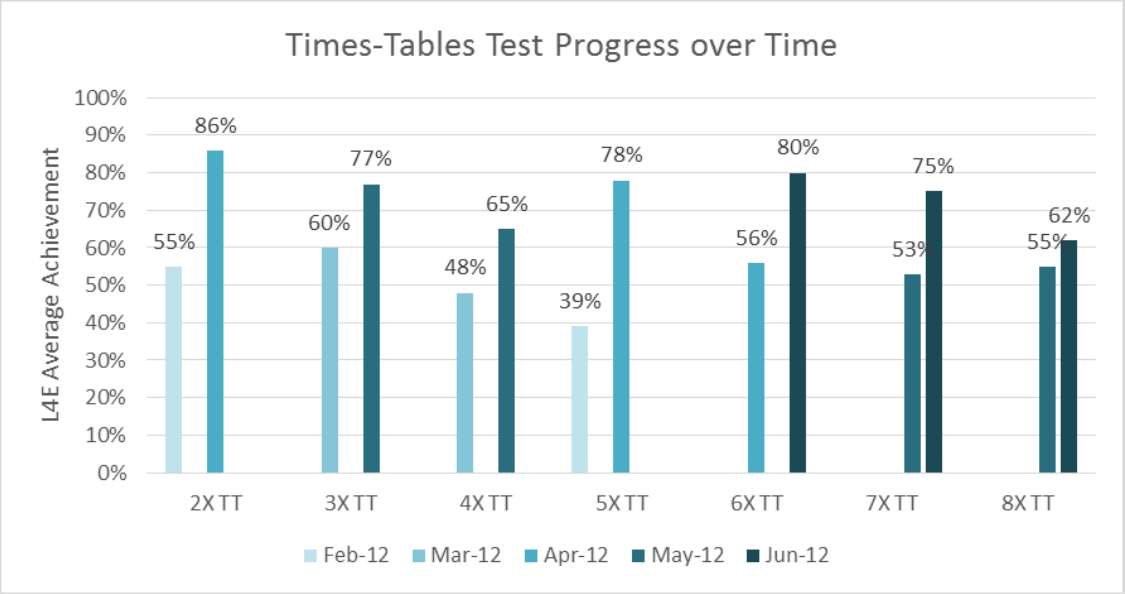


Figure 6.6 Times-Table Test Progress over Time

The 2-times- to the 6-times-table tests shown were spaced two months apart. Improvement was greatest for the 5-times- and 2-times-table tests over this time period. The 7-times- and 8-times-table tests were administered only about a month apart and although the improvement was smaller, it happened over half the time. I was surprised that the 7-times-table was improved beyond the 8-times-table as the strategy of doubling successively had been popularly received and used. This might be explained by the fact that the 4-times-table results were also low and possibly errors in doubling were increasing with each successive

iteration. I also administered a set of multiplication exercises under test conditions to gauge progress through multiplication problems of advancing complexity.

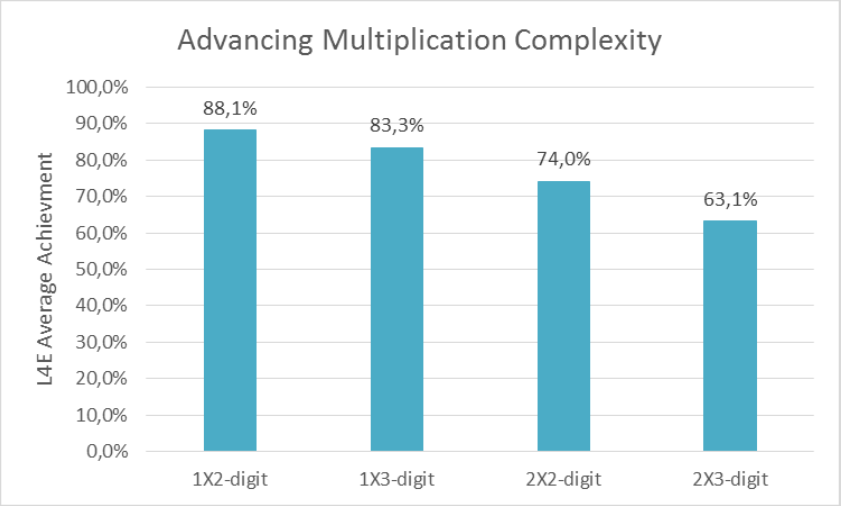


Figure 6.7 Learner Achievement with Advancing Multiplication Complexity

As I expected, performance decreased as multiplication problems became more complex and afforded greater opportunity for error.

6.4.2 Common Errors in Multiplication

Multiplying numbers was a challenge for the majority of the learners as evidenced by the achievement of a 26% average for this on the baseline test. In the learning sessions I started with Foundation Phase basics of repetitive addition. Few learners seemed to understand the concept of multiplication. Most were familiar with the algorithmic procedure of using columns to perform 'long multiplication' although they were unable to execute the procedure to produce a correct answer. The most common error was to multiply by the incorrect 'tens' digit.

For example:

211 x	instead of	211 x
15		15
10 550		1 055
211 +		2 110 +
10 761		3 165

The learners knew that they “needed to add a zero in the first row” before multiplying but they didn’t understand that the zero meant that they would then multiply by the digit in the tens column. They would add a zero as if they were multiplying by tens first and then multiply by the units and calculate the incorrect answer.

To avoid errors such as this and those detailed in the section earlier dealing with subtraction, I encouraged the use of estimation to get a rough idea beforehand of the expected answer. Estimating an answer before calculating in order to check the answer also seemed to be a new skill for the learners and one I repeatedly focused on to encourage self-detection of errors.

6.4.3 New Strategies for Multiplication

From repetitive adding and skip-counting we moved to single-digit multiplication and times-tables. I began with the 2X-tables as simply doubling. We progressed to 4X-tables and 8X-tables as doubling twice and doubling three times. Concurrently we considered the 10X-table and shifting digits from units to tens and/or from tens to hundreds and I introduced the 5X-table as halving after multiplying by 10. I showed the learners the 9X-table trick using their fingers and the patterns in the 3X-table with digits adding to multiples of 3. The 6X-table used the doubling method again and the answer check of digits adding to multiples of 3 again. It was only with the 7X-table that I introduced the ‘number fact – up or down’ strategy. The focus of all of these multiplication activities was on playing with the numbers in their heads and having fun with the challenge of learning the answers so that they always had their times-table facts at their fingertips. Many learners really enjoyed this and listed the times-table games as a highlight of the Maths Journey part of the L4E programme when they completed their term-end feedback surveys.

Once we had single-digit multiplication mastered, we moved on to bigger numbers. I presented alternative representations of the multiplication process and strategies for multiplication in the learning sessions. Again we worked with place-value cards starting with small numbers and building these up. These strategies are included in Appendix L.

6.5 Dividing Whole Numbers

6.5.1 Changing Proficiencies in Division

Learners' proficiency in dividing numbers improved most dramatically over the seven month period. This was an urgent necessity as both teachers and learners had been shocked to realise that at the start of the L4E programme only two out of the 50 learners could divide whole numbers successfully! It was extremely satisfying that 68% of the learners were able to progress off the novice level by September 2012.

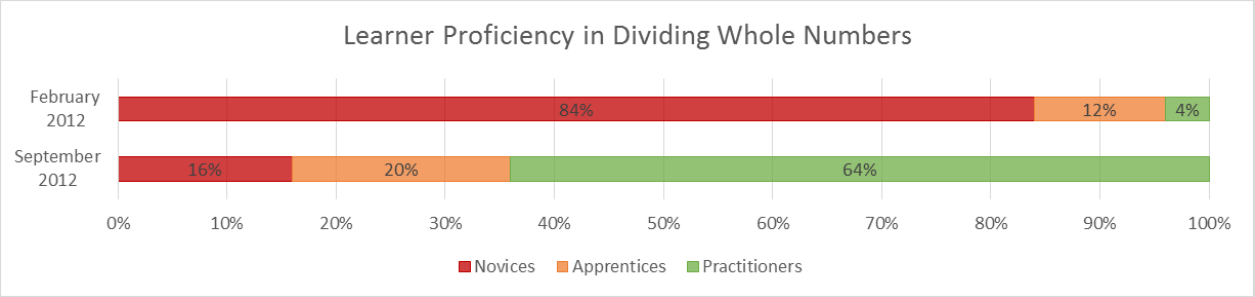


Figure 6.8 Change in Learner Proficiency in Dividing Whole Numbers

6.5.2 Common Errors in Division

Unlike the other number operations, I did not have initial opportunities to even see common division errors in learners' work. The class of learners had achieved just 10% for division on the baseline test. I had noticed while invigilating this test that most learners were using a strategy for dividing that involved drawing tally marks and then grouping these. As one division question required dividing 5 094 by 9, I assumed that learners labouring with this method were doing so because they had no other strategies to solve this problem. When I saw the learners answer sheets, I could confirm their lack of division strategies. Only two learners had both division questions correct and both had used a short-division algorithm for the more difficult question and simply written a correct answer for the easier question. Six learners had given the correct answer to the easy question only, either by simply writing the quotient, or along with the correct answer there was evidence of drawing, showing they used an equal-sharing strategy as mentioned above. The remaining 42 learners had no correct

answers to either question and no evidence of using any other strategy other than the equal-sharing/ grouping method.

The L4E Grade 10 teachers expressed great concern about how they were going to teach Grade 10 algebra to the L4E Learners. Taking out a common factor is very important in algebra. If a learner is unable to identify the factors of a whole number, even numbers that are less than 99, as was the case with the L4E learners, it would seem impossible to be able to factorise in algebra.

6.5.3 New Strategies for Division

Based on the evidence of the baseline test, I started with concrete Foundation Phase activities around grouping, again stressing the inverse relationship between dividing and multiplying. We used reverse times-table games to divide numbers less than 100 and through this process, learners could see that without realising it, they were already dividing numbers successfully. We spent some time investigating rules for divisibility by 2, 3, 5, 6, 8, 9 and 10 as I felt this would assist them as the division problems became more complex.

We used the concept of grouping to move on to repetitive subtraction and then chunked subtraction. The strategies for division are included in Appendix M. As the range of numbers we were using expanded, we looked at the short- and long-division algorithms. It seemed as if, with improved conceptual understanding of what division actually was (i.e. the splitting into equal parts or groups), there were less difficulties with these algorithms than I expected. I found it interesting that mastering the traditional long-division algorithm (refer to Appendix M) was almost a Maths status symbol for many of the learners. It was a kind of 'trophy achievement' and I had their full attention while explaining it. There was a competitiveness especially among the boys to use this procedure to successfully divide large numbers. One of the learners actually wrote me a letter at the end of the year to say that learning how to perform long-division was a highlight for him and that he had been delighted to teach the algorithm to his peers in an after-school Maths improvement club he had started at his school.

Again I administered a set of exercises under test conditions to gauge progress through problems of advancing complexity.

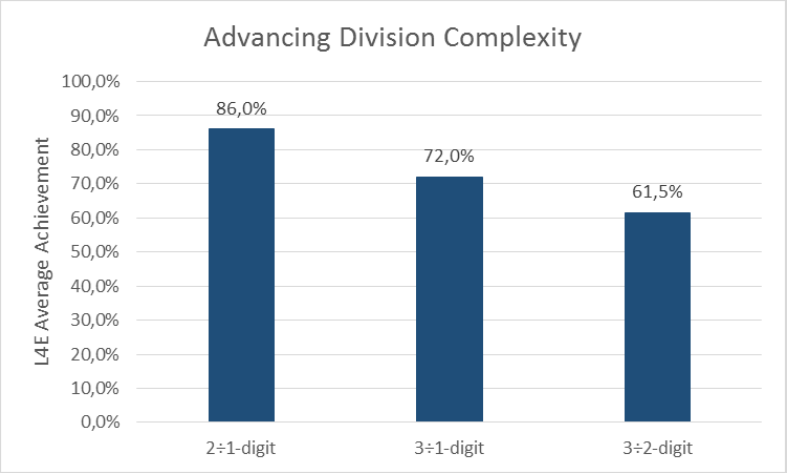


Figure 6.9 Learner Achievement with Advancing Division Complexity

These results are interesting in that I didn't expect such a drop in performance when moving from dividing a 2-digit number by a 1-digit number to dividing a 3-digit number by a 1-digit number. The lower achievement may have had something to do with the fact that the first test was completed a week before the second two tests which were completed on the same day. As was the case for multiplication, performance decreased as division problems became more complex and afforded greater opportunity for error.

6.6 Further Error Analysis

After conducting an all-day error analysis activity with the L4E learners in April 2012, I collected data from the learners' Diagnostic Matrix Tools for whole number operations of addition, subtraction, multiplication and division. The Diagnostic Matrix Tools reflected errors made in the first 45 Maths Minute homework tasks on these conceptual landmarks. From this I calculated the percentage error occurrence by dividing the number of incorrect answers provided by of the number of questions presented that corresponded to each number operation. I presented this ratio as a percentage of error occurrence for the first 45 Maths Minutes.

To establish what the persistent conceptual landmark challenges were for the learners I analysed a later set of Maths Minutes (76 – 85) in the same way. These were completed more than three months later in July 2012. I used the same Diagnostic Matrix Tool to record the occurrence of errors on these Maths Minutes questions. Certain conceptual landmarks were no longer represented in this set of Maths Minutes. For example, there were no questions that only tested the subtraction of whole numbers (subtraction was only included in questions on mixed operations which was recorded as a different conceptual landmark). Appendix G shows how the questions on the Maths Minutes 76-85 corresponded to the conceptual landmarks on the Numbers Diagnostic Matrix Tool.

Again, after collecting the data, I calculated the percentage error occurrence across the different conceptual landmarks for the later 10 Maths Minutes.

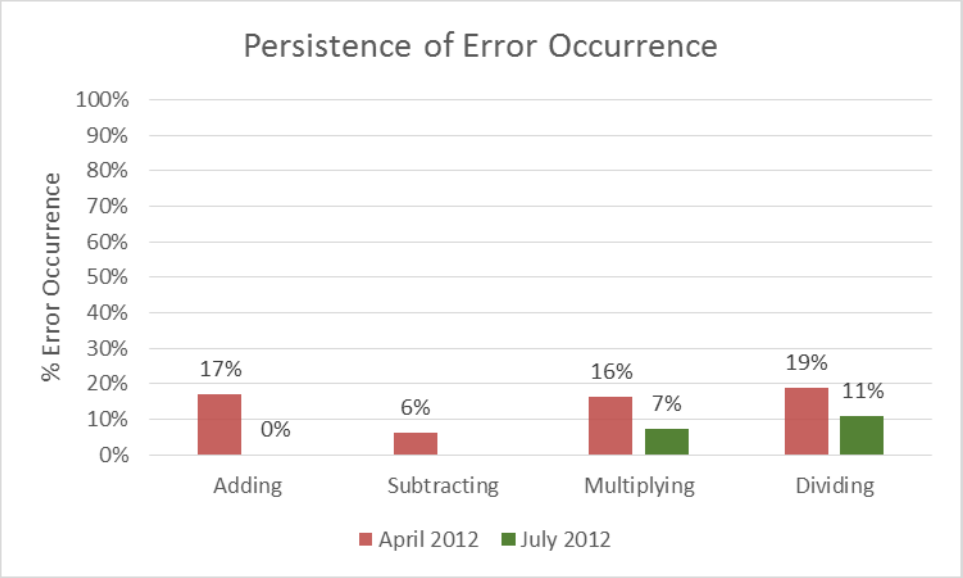


Figure 6.10 Persistence of Error Occurrence in Number Operations

It is interesting that in the April error analysis, the percentage error occurrence in adding (the operation in which learners had performed best in the tests) is just as high as that in multiplying and dividing (operations in which learners had performed more poorly in the tests). Subtracting shows the lowest error occurrence (in fact less than half of the occurrence in the other operations). When comparing error occurrence in July, errors in adding had disappeared. Errors in multiplying had dropped from 16% to 7%. Dividing numbers showed a

similar decrease (from 19% to 11%). I was not surprised to see a persistence of error in these two more difficult number operations. However it was unexpected that there was greater error occurrence in dividing compared to multiplying. In the tests, learners had achieved better post-intervention results in division when compared to multiplication. It is important to remember that the Maths Minutes were not completed under test conditions. They were homework exercises completed by the learners in the weekdays between our Saturday contact sessions. For this reason, they are less reliable than the test results which were strictly controlled by myself in class time. The other consideration is that the Maths Minutes are designed to be implemented in numerical order as they are developmental through each book. This means that the later Maths Minutes are designed so as to be more difficult than the earlier Maths Minutes potentially making a drop in error occurrence over the progression of the Maths Minutes more meaningful.

6.7 Conclusion

The research sub-question asked whether an assessment ***as*** learning – conceptual thread approach can reveal obstacles to mastering conceptual landmarks in learning about numbers, and specifically number operations. While it was the case that obstacles in performing number operations were evident through the approach, a better question would have been whether this approach provides any *greater* insight for teachers than traditional methods of assessment. The potential for this may have existed, especially as each L4E learner kept a detailed Maths journal in which they wrote weekly entries, including their issues in Maths. However, since I did not have access to all of their journals, I could not include data gathered from the journals about specific number operations difficulties even though these existed. As such, in this study assessment ***as*** learning did not offer any greater benefit to teachers than any other formative assessment approach. In fact, in this study assessment ***as*** learning did not offer any greater benefit to teachers than well-designed summative assessments that enable learners to demonstrate their mathematical thinking. In conclusion, while this assessment ***as*** learning – conceptual thread approach can (and did) reveal obstacles to mastering Maths conceptual landmarks (in this case, in number operations), this is not a benefit that is unique to this approach.

CHAPTER SEVEN

Results & Discussion: Metacognitive Activation & Advancement

In this last chapter of presenting results I explore whether an intervention using an assessment ***as*** learning – conceptual thread approach such as that of the L4E programme, can change learners' self-evaluations of their own understanding of Mathematics.

7.1 Measuring Metacognitive Skills

Through the development of a unique tool (the MAAR Tool, the development of which is described in Chapter Four and which is reproduced in Table 7.1), the quantifying of learners' self-evaluations as measures of their metacognitive skills are distinguished as:

- **metacognitive activation** – the emergence of evidence in learner practice of metacognition;
- **metacognitive awareness** – showing evidence of conscious and informed metacognition;
- **metacognitive analysis** – showing evidence of analyzing own learning strengths, weaknesses, challenges and needs; and
- **metacognitive advancement** – evaluating and applying actionable strategies to improve learning.

Using the MAAR tool to analyse the quantitative data was straightforward. For the first criterion of *estimating performance* on a mathematical conceptual landmark (measured in percentage achievement): the difference was calculated between the learner's self-assessed, estimated score for each operation before a number operation test was conducted and the actual score the learner achieved on each operation when s/he completed the test. The range of difference was interpreted using the MAAR Tool as a metacognitive level for ability to estimate performance which was recorded for each learner.

Table 7.1: The Metacognitive Activation & Advancement Rubric (MAAR) Tool

Assessment Criteria	Level 0	Level 1	Level 2	Level 3
Estimating performance on a mathematical conceptual landmark (measured in percentage achievement)	Actual achievement is beyond 30% of estimated achievement.	Actual achievement is within 30% of estimated achievement.	Actual achievement is within 20% of estimated achievement.	Actual achievement is within 10% of estimated achievement.
Classifying proficiency as practitioner, apprentice and novice in error analysis of conceptual landmarks	Correct proficiency level is assigned in less than 50% of conceptual landmarks AND/OR (more than half of) these are two levels out of range.	Correct proficiency level is assigned in more than 50% of conceptual landmarks BUT (less than half of) these are two levels out of range.	Correct proficiency level is assigned in more than 50% of conceptual landmarks AND all errors are only one level out of range.	Correct proficiency level is assigned in all conceptual landmarks.
Showing awareness of strengths and weaknesses through: * identifying these * describing these * discussing these	Identified mathematical strengths and weaknesses are not clear or do not correspond with learner evidence.	Identified mathematical strength/ weakness shows some correspondence with learner evidence however core issues are overlooked.	Core mathematical strength/ weakness corresponds with obvious learner evidence.	All identified mathematical strengths and weaknesses clearly correspond with learner evidence.
	Strengths/ weaknesses are described without mathematical language and are so general that they are not actionable e.g. "writing answers"	Strengths/ weaknesses are described in simple mathematical language and are limited to conceptual threads e.g. "working with whole numbers".	Strengths/ weaknesses are described in mathematical language to the conceptual landmark level but lacking conceptual thread specifics e.g. "rounding off".	Strengths/ weaknesses are described using detailed mathematical language within conceptual landmarks e.g. "to round off whole numbers to the nearest ten".
	Discussion provides no clear actionable direction for the way forward.	Discussion focuses on attitude and effort as a solution for the way forward.	Discussion provides simple, actionable mathematical advice.	Discussion provides insightful detail of mathematical conceptual understanding.
	Writing is scanty. Single, simple words are used rather than phrases or sentences.	Writing is simple. Phrases and simple sentences used.	Writing clearly links to conceptual landmarks and provides basic detail.	Writing provides rich, mathematical self-awareness of the details of conceptual landmarks.
Suggesting strategies for improvement and growth in own mathematical conceptual development and progress	No actionable strategies for progress are communicated.	Mathematical topics or concepts are not mentioned. Suggested strategies are only attitudinal and/or related to general effort.	Mathematical topics are specifically mentioned. Suggested strategies are actionable but are limited to attitude and/or effort only.	Mathematical topics and concepts are presented in written detail. Maths-specific, actionable strategies are suggested.
Analysing own performance using metacognitive skills of: * identifying weaknesses * explaining challenges * suggesting strategies	No evidence of metacognitive skills.	Only one of identified metacognitive skills is evident in written analysis.	Two identified metacognitive skills are evident in written analysis.	All three identified metacognitive skills are evident in written analysis.

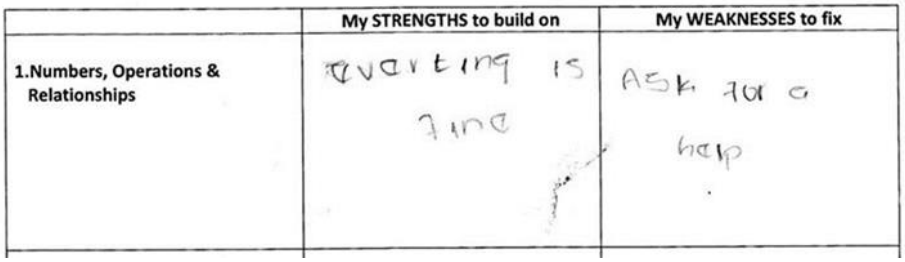
For the second criterion of *classifying proficiency* as practitioner, apprentice and novice in the error analysis of conceptual landmarks: a learner's self-assessed classifications as a practitioner, apprentice or novice on Numbers conceptual landmarks in the error analysis activity was compared with the teacher's classifications of the same learner based on their Diagnostic Matrix evidence. The comparison was interpreted using the MAAR Tool as a metacognitive level for ability to classify proficiency which was recorded for each learner.

In order to illustrate how the MAAR tool was used to analyse the qualitative data of the remaining criteria, Tables 7.2, 7.3, 7.4 and 7.5 are presented for each of the remaining assessment criteria on the MAAR tool. For each level of performance on each criterion examples of learner evidence are provided to illustrate how this evidence was interpreted and to justify the performance level assigned to the learner. Notes below these examples further clarify the reasons for scoring the learner at the designated performance level.

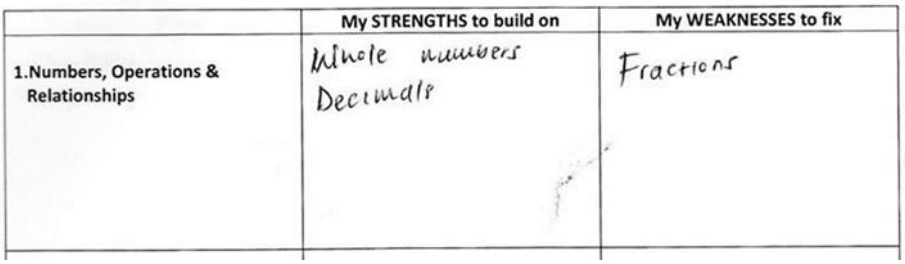
The MAAR tool enabled the evidence from a number of different activities to be assessed on the same scale in order to compare proficiency over time and investigate progress in understanding learners' strengths, weaknesses, challenges and needs in learning Mathematics. In the previous two results chapters, only quantitative data was used. This chapter draws mainly on qualitative data from self-assessments and those journal entries that were accessible after L4E finished, to provide a more comprehensive picture of metacognitive skill advancement over the study period.

In so doing, the value of an assessment ***as*** learning – conceptual thread approach for the learners is evaluated, not just to improve their achievement in Maths, but beyond this to improve their metacognitive skills which benefit a broad range of academic pursuits. This is based on the premise that the core function of the teacher is to enable a student to develop into a competent, confident, self-motivated, self-directed and independent learner. Successful teaching results in sustainable life-long learners who are able to understand their learning process and its requirements and can access and utilise the resources they need (human or otherwise) to create their own knowledge and understanding. Did the assessment ***as*** learning – conceptual thread approach provide this extra benefit to the L4E learners?

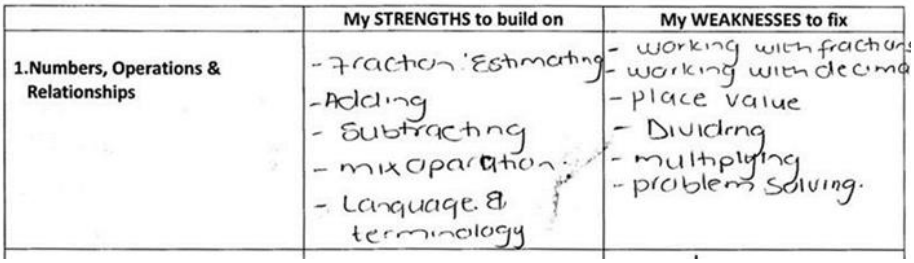
Table 7.2 Using the MAAR Tool to assess the describing of mathematical strengths and weaknesses

<table>
<tr><td colspan="2">Assessment Criterion: Describing mathematical strengths and weaknesses</td></tr>
<tr><td>Level 0
Strengths/ weaknesses are described without mathematical language and are so general that they are not actionable e.g. “writing answers”</td><td>For example:


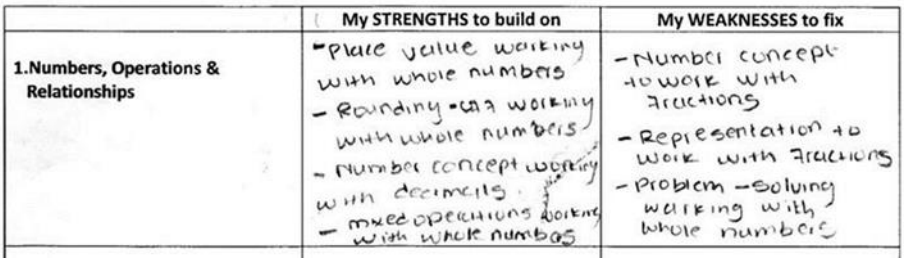
No strengths nor weaknesses are described. This learner does not even refer specifically to Maths at all.</td></tr>
<tr><td>Level 1

Strengths/ weaknesses are described in simple mathematical language limited to conceptual threads e.g. “working with whole numbers”.</td><td>For example:


Strengths and weaknesses are specified at a conceptual thread level only. No conceptual landmarks are identified among strengths or weaknesses.</td></tr>
<tr><td>Level 2

Strengths/ weaknesses are described in mathematical language to the conceptual landmark level but lacking conceptual thread specifics e.g. “rounding off”.</td><td>For example:


Strengths and weaknesses are specified to conceptual landmark level but descriptions are incomplete omitting conceptual thread information that makes these difficult to properly identify.</td></tr>
<tr><td>Level 3

Strengths/ weaknesses are described using detailed mathematical language within conceptual landmarks e.g. “to round off whole numbers to the nearest ten”.</td><td>For example:


Strengths and weaknesses are identified clearly enough to distinguish between the level of conceptual threads and conceptual landmarks for focused remediation.</td></tr>
</table>

Table 7.3 Using the MAAR Tool to assess the determining of learning opportunities of assessment, error analysis and self-evaluation

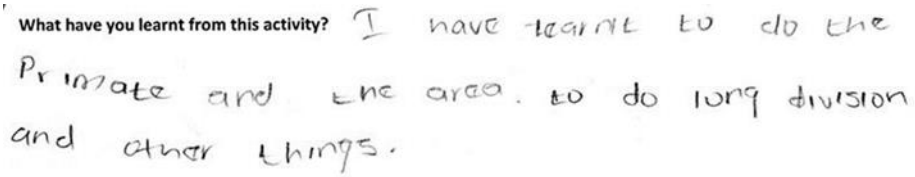
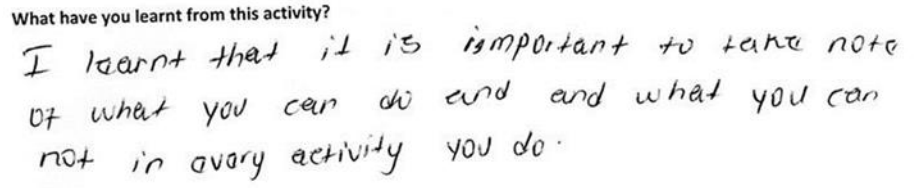
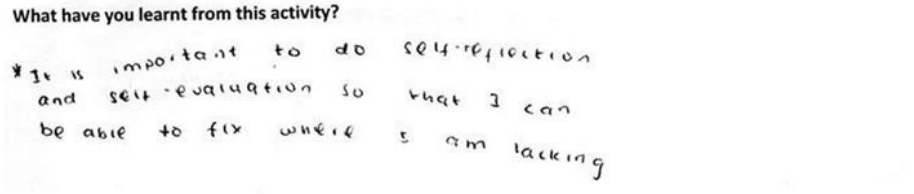
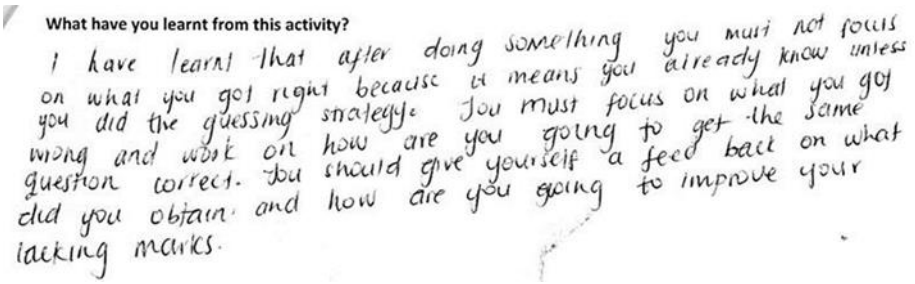
Assessment Criterion: **Determining learning opportunities of assessment, error analysis and self-evaluation**	
Level 0 Language difficulties, misinterpretation and/or brevity results in no clear comment on what has been learnt.	For example:  What have you learnt from this activity? I have learnt to do the Primate and the area. to do long division and other things. It is not clear that the learner has learned anything from the activity.
Level 1 Comment is limited to learning only that errors, strengths and/or weaknesses have been identified.	For example:  What have you learnt from this activity? I learnt that it is important to take note of what you can do and and what you can not in avary activity you do. Learning is limited only to rcognition of the value of errors . No specific learning opportunities that these afford are mentioned.
Level 2 Comment makes a basic link between errors, strengths and/or weaknesses to a general learning opportunity e.g. added effort.	For example:  What have you learnt from this activity? It is important to do self-reflection and self-evaluation so that I can be able to fix where I am lacking This learner not only shows recognition of the value of the errors revealed but refers the learning benefit that the activity provides.
Level 3 Comment links errors, strengths and/or weaknesses to more detailed and specific learning strategies.	For example:  What have you learnt from this activity? I have learnt that after doing something you must not focus on what you got right because it means you already know unless you did the guessing strategy. You must focus on what you got wrong and work on how are you going to get the same question correct. You should give yourself a feed back on what did you obtain and how are you going to improve your lacking marks. The learner clearly explains the learning opportunities and benefits afforded by the activity while also presenting strategies (e.g. feedback) for progress going forward.

Table 7.4 Using the MAAR Tool to assess the suggesting of strategies for improvement and growth

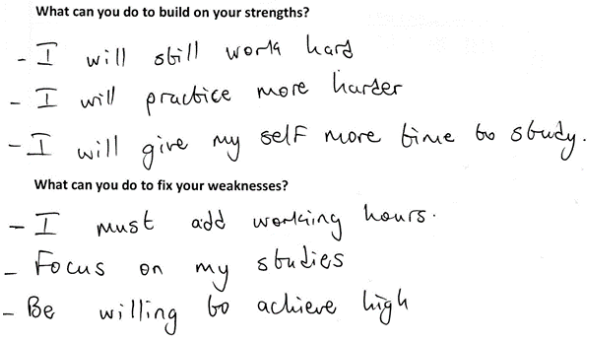
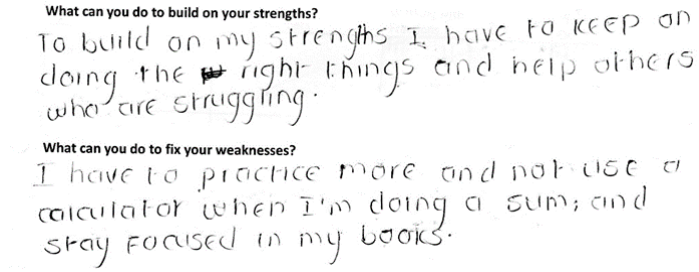
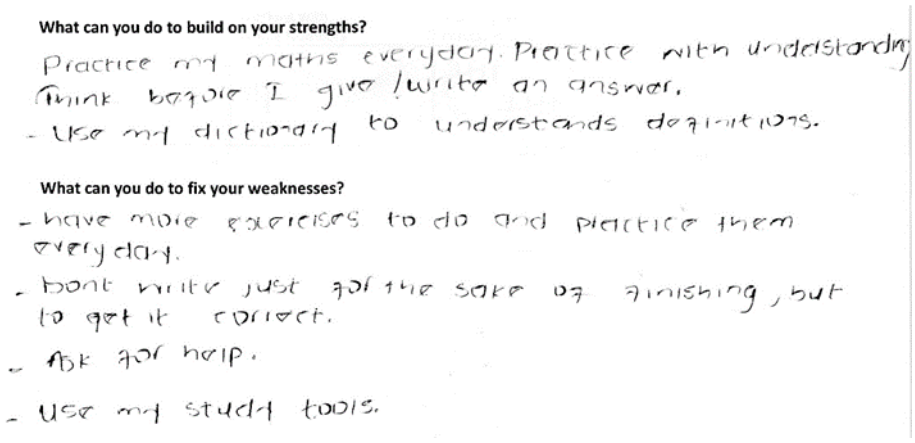
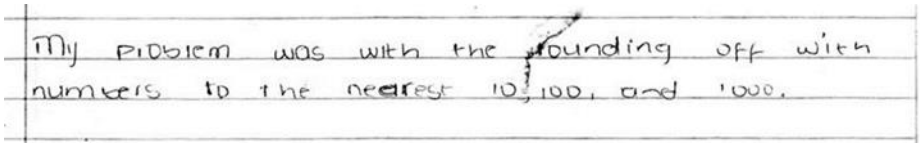
Assessment Criterion: **Suggesting improvement strategies and growth opportunities in own mathematical conceptual development and progress**	
Level 0 No clear or actionable strategy for progress is communicated.	There were no examples of Level 0 performance for this criterion. All learners were able to articulate a strategy of some kind.
Level 1 General study strategies are suggested which are limited to attitude changes, practice and effort and/or time management.	For example:  What can you do to build on your strengths? - I will still work hard - I will practice more harder - I will give my self more time to study. What can you do to fix your weaknesses? - I must add working hours. - Focus on my studies - Be willing to achieve high This learner suggests strategies for improvement but these involve general study behaviours rather than any Maths-specific strategies.
Level 2 Basic suggestions provide simple, but relevant advice for Maths-specific progress.	For example:  What can you do to build on your strengths? To build on my strengths I have to keep on doing the right things and help others who are struggling. What can you do to fix your weaknesses? I have to practice more and not use a calculator when I'm doing a sum; and stay focused in my books. The learner suggests simple strategies for improvement and does refer specifically to opportunities to advance Maths-specific development and progress.
Level 3 Insightful and useful Maths-relevant, strategies are suggested.	For example:  What can you do to build on your strengths? Practice my maths everyday. Practice with understanding think before I give/write an answer. - Use my dictionary to understands definitions. What can you do to fix your weaknesses? - have more exercises to do and practice them everyday. - Dont write just for the sake of finishing, but to get it correct. - Ask for help. - Use my study tools. This learner suggests many varied Maths-specific strategies to not only fix weaknesses but also to build on her strengths.

Table 7.5: Using the MAAR Tool to assess the analysing of own performance

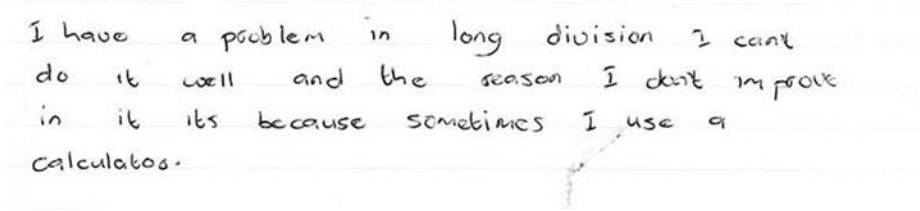
<table>
<tr><td colspan="2">Assessment Criterion: Analysing own performance using metacognitive skills of:

• identifying weaknesses
• explaining challenges
• suggesting strategies</td></tr>
<tr><td>Level 0

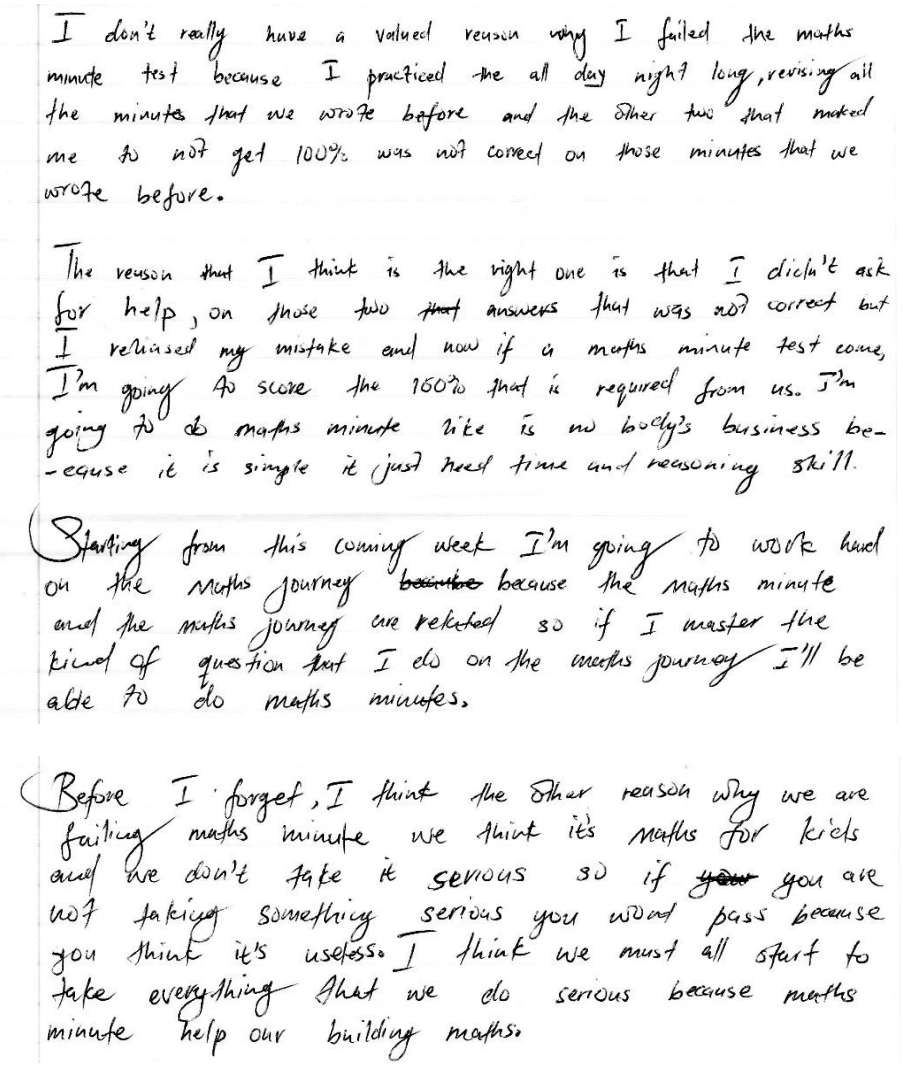
No evidence of any of these three metacognitive skills.</td><td>There were no examples of Level 0 performance on this criterion. All learners were able to analyse their own performance showing some level of metacognitive skill (either identifying weaknesses and/or explaining challenges and/or suggesting strategies).</td></tr>
<tr><td>Level 1

Only one of the identified metacognitive skills is evident in written analysis.</td><td>For example:


This learner only presented evidence of one metacognitive skill i.e. that of identifying her weaknesses. She does not explain why she is struggling with this or what she can do to improve going forward.</td></tr>
<tr><td>Level 2

Two identified metacognitive skills are evident in written analysis.</td><td>For example:


This learner identifies her weaknesses and explains why she thinks she had this difficulty. Her solution to the problem is not explicit so she is scored on Level 2 rather than level 3.</td></tr>
</table>

Level 3 All three identified metacognitive skills are evident in written analysis.	For example:  This learner identifies the fact that he got the same questions wrong in the Maths Journey class test that he got wrong in the Maths Minute exercises. As he did not ask for help on these when he was revising the Maths Minute exercises, he got them wrong again in the test. He explains that now if he makes sure that he understands what he did wrong in the Maths Journey test, he can go back to the Maths Minutes exercises and he will know how to answer those questions.

7.2 Results: Distinguishing Between Estimated and Actual Achievement

In February 2012, soon after the baseline diagnostic test had been completed and analysed, the learners were asked to rate their abilities to solve number operation problems on a scale of 0 to 10. Appendix N shows the rating scale tool the learners used for this self-assessment. After this they were tested on this and their self-assessment scores were compared with their actual achieved scores on the test. Appendix O shows an example of a learner's rating scale with the estimated score and actual achieved score for each number operation recorded.

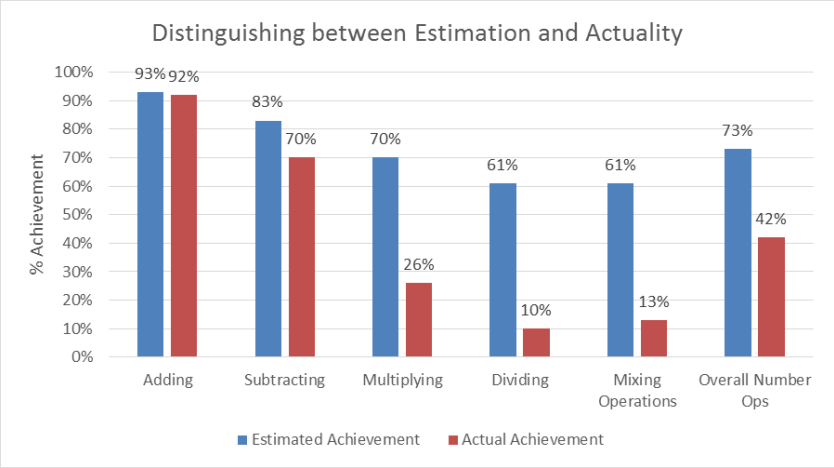


Figure 7.1 Comparison of Learners' Estimated Achievement with Actual Achievement (Feb 2012)

The overall class average for the estimated scores (pre-test self-assessment) was calculated at 73%. Not surprisingly, addition was the number operation rated highest with an average of 93% across the 50 learners with division rated lowest with an average of 61%.

Learners' actual overall measured performance on number operations produced an average of only 42%. In line with their predictions, addition was measured with an average of 92% across the 50 learners with division measured lowest with an average of just 10%. There was a big drop between actual achievements when comparing the addition/subtraction pair of operations with the multiplication/division pair of operations. Although the learners predicted the multiplication and division scores would be lower than the addition and subtraction scores, the difference between the pairs was large. It is also interesting to note that both estimated and actual achievements on mixed operations closely matched that of division which was the number operation with the poorest results. While this would be

expected this to be the case for actual achievement (that the mixed operations results would be limited by the weakest operation), it is surprising that the learners had predicted this in their estimations.

Comparing these two sets of data provides some insight into learners' initial level of metacognitive awareness. While there is little difference between the average self-assessment and actual achievement in addition, the difference grows as performance drops. It reaches a maximum difference of 51% for division. This task reinforced research by Miller and Geraci (2011) which showed that students' predictions were almost always higher than the grade they earned. This was particularly true for low-performing students. It also confirms the Dunning-Kruger Effect (Kruger and Dunning, 1999) – a cognitive bias in which low-ability individuals suffer from illusory superiority, mistakenly assessing their ability as much higher than it really is. In the case of the L4E learners it is not fair, and not true, to refer to them as low-performing students but certainly they were students who had acquired learning deficits due to their circumstances and past school experiences.

When considering the individual learner's estimated achievements and actual achievements, the range of differences went from a minimum of a 2% difference to a maximum of an 84% difference. The latter was consistently the lowest performer in the class. Only 6% of learners under-estimated their performance on number operations while the remaining 94% over-estimated their performance.

Although these scores were made on a 10-point scale (this for the learners' ease of use, relating it to percentage with which they were familiar), it was the difference between their self-assessment and the teacher-assessment that was used to measure their metacognitive skill level on the four-level MAAR tool. Using the MAAR Tool level descriptors, each learner can be measured on one of the four metacognitive skills levels based on the size of the difference between their estimated and actual achievement.

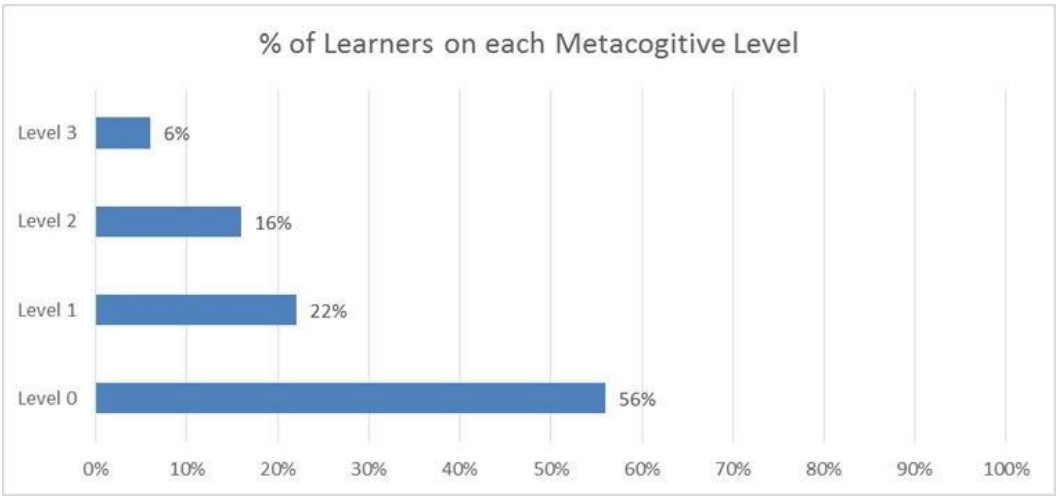


Figure 7.2 Metacognitive Levels based on Estimating Achievement (Feb 2012)

More than half of the L4E learners (56%) were placed on Level 0, i.e. not yet metacognitively activated nor metacognitively aware. Interestingly of the three learners who under-estimated their abilities, one of the learners under-estimated himself within Metacognitive Level 0 while the other learners were at Level 2 and Level 3 and under-estimated themselves at lower levels than they actually achieved.

7.3 Results: Classifying Proficiency

In April 2012, the L4E learners participated in an all-day error analysis activity in which they recorded all of their errors made on 450 Math Minutes questions they had completed for homework since the start of the programme. Errors were recorded on a Diagnostic Matrix Tool (Appendix F). Learners were then asked to look at each conceptual landmark on the Diagnostic Matrix Tool and classify themselves as a practitioner, apprentice or novice. They indicated this by drawing a colour-coded ring around each conceptual landmark corresponding to their chosen proficiency level. The learners were then classified by their teacher. The Math Minute exercises and questions were analysed and each question was correlated with the conceptual landmark it referred to. If a learner made errors on 50% of these questions or higher (i.e. they achieved less than 50%), they were classified by the using as a novice. If a learner only made errors on 25% of the questions i.e. they got 75% or more of them correct, they were classified as a practitioner. Achievement in between of 51% to 74% earned a classification of apprentice. While these categories may not seem to be in line

with the CAPS curriculum which requires only 30% to pass, this raised benchmark of 50% was chosen to distinguish the novices from those achieving above 50%. This resulted in the novice category being broader, making more novices than apprentices and practitioners. This category design was in line with my teaching and assessment activities with the learners in class where the focus was not on 'passing' at 30% (or even at 50%) but on 'catching up' to 100% on their completed grades' work. The categories are summarised in Table 7.1.

Table 7.6: Proficiency Classification Scoring Guide for Math Minutes 1 – 45

CONCEPTUAL LANDMARKS	Number of Questions	Number of Errors		
		Practitioner Level	Apprentice Level	Novice Level
Adding whole numbers	8	0-2	3	4-8
Subtracting whole numbers	12	0-3	4-5	6-12
Multiplying whole numbers	18	0-4	5-8	9-18
Dividing whole numbers	22	0-5	6-10	11-22
Mixing operations with whole numbers	14	0-3	4-6	7-14
Problem-solving with operations	13	0-3	4-6	7-13

The teacher-assessed level of proficiency for each learner on each conceptual landmark was then compared with each learner-assessed classification of themselves on that landmark. Using the MAAR Tool, each learner was measured on one of the four metacognitive skills levels based on the degree of their classifications matching the teacher-assessed classifications.

A month later than when the first metacognitive measure had been made, there were only 4% of learners on Level 0 and the bulk of learners (57%) were now on Level 2.

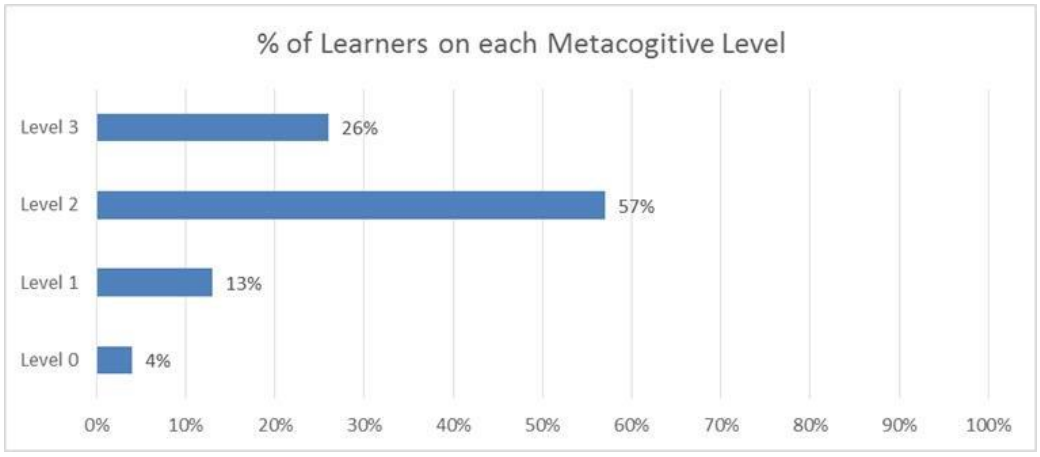


Figure 7.3 Metacognitive Levels based on Classifying Proficiency (mid-Apr 2012)

Most interesting was that most learners now *under-estimated* their achievement rather than over-estimated as they had done just a month before. Mathematical achievement was increasing and this correlates with research conducted by McCormick (2013) who found that while less competent students tend to over-estimate their performance, more competent students tended to under-estimate how well they had performed.

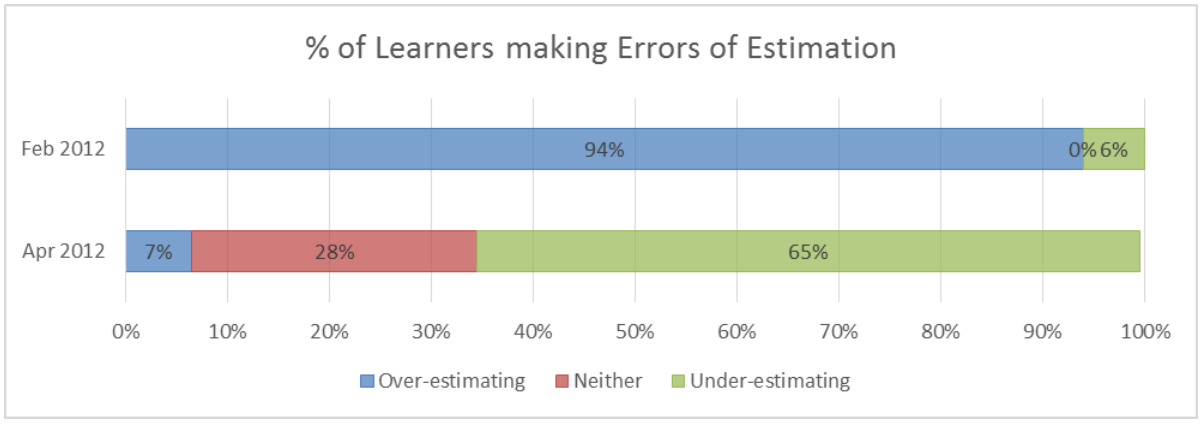


Figure 7.4 Percentage of Learners making Errors of Estimation over Time

Figure 7.4 shows quite clearly how the bulk of the learners have shifted from over-estimating their proficiency (94%) to either correctly estimating or under-estimating their proficiency (93%) in just two months. Based on McCormick's research, I propose that the shift from over-estimating to under-estimating marks the occurrence of metacognitive activation in learners. The correction of under-estimation back towards more accurate estimation illustrates growing metacognitive awareness and the advancement of metacognitive skills.

7.4 Results: Describing Mathematical Strengths and Weaknesses

After the mid-April error analysis activity, learners were instructed to reflect on the activity as a whole and their strengths and weaknesses revealed by the activity. They were expected to complete a self-reflection table as homework during the following week. This process proved valuable for providing learners with the vocabulary needed to describe their progress in learning. Learners referred back to the conceptual landmarks on the Diagnostic Matrix tool when describing their strengths and weaknesses distinguishing between whole numbers, integers, common fractions and decimals. Written responses were better than when they had completed similar tasks earlier in the year without the guidance of the Diagnostic Matrix tool. This qualitative data from each learner was analysed and, using the MAAR Tool, each learner was assigned to a level of metacognitive skill for this specific assessment criterion. (The MAAR Tool with examples of the interpretation of learner evidence for this criterion is presented in Table7.2.)

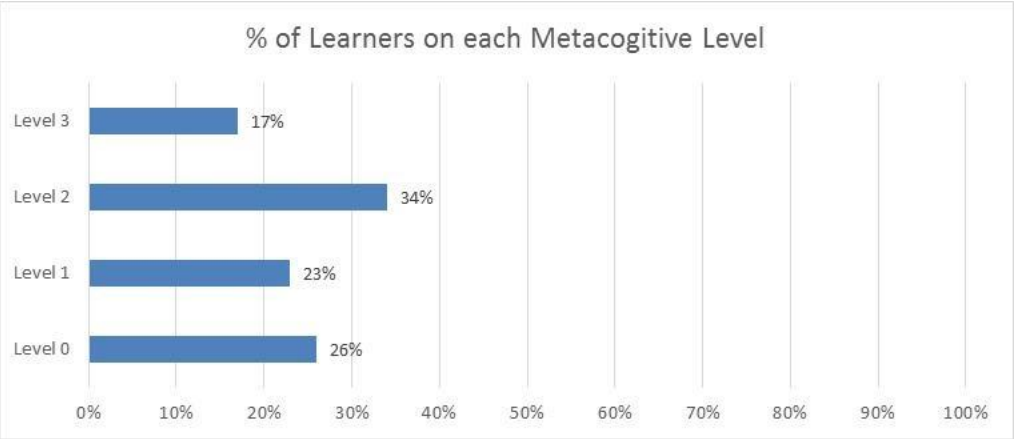


Figure 7.5 Metacognitive Levels based on Describing Strengths and Weaknesses (late-Apr 2012)

There were more learners on Level 0 compared to the previous activity. The learner evidence for this criterion required written descriptions from the learners. Many of the learners achieved on low levels due to not completing this (almost 20% of the learners) yet all of these learners answered the rest of their self-evaluation questions in the same task. I interpreted this as an inability to describe strengths and weaknesses and recorded them on Level 0. Those learners on Level 0 who did write a response showed a lack of detail (in fact, in many cases a lack of words) in their writing. This is evident in the example of learner evidence in Table 7.2.

7.5 Results: Determining Learning Opportunities and Suggesting Strategies for Improvement & Growth

In the latter half of April 2012, the learners were presented with the following questions:

- What have you learnt from this (error-analysis) activity?
- What can you do to build on your strengths?
- What can you do to fix your weaknesses?

In their written responses they were required to determine learning opportunities afforded by their assessments, error analysis activity and self-evaluations. They were also required to suggest actionable strategies for improvement and growth in their own mathematical development and progress. Again, the MAAR Tool was used to analyse their written responses and assign levels of proficiency of metacognitive skills for these specific assessment criterion. (The MAAR Tool with examples of the interpretation of learner evidence for this criterion is presented in Table7.3 and Table 7.4.)

Learner results for metacognitive levels measured on these criteria were very similar to each other even though learner responses for measuring on these criteria was obviously different. This can be seen when comparing Figure 7.6 and Figure 7.7. By the end of April 2012, it appeared that all but one learner had become metacognitively activated.

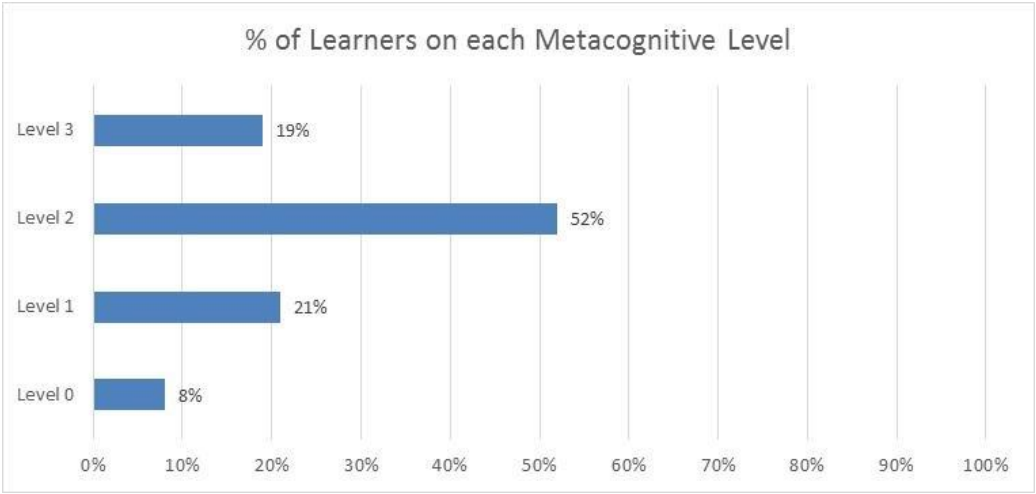


Figure 7.6 Metacognitive Levels based on Determining Learning Opportunities (late-Apr 2012)

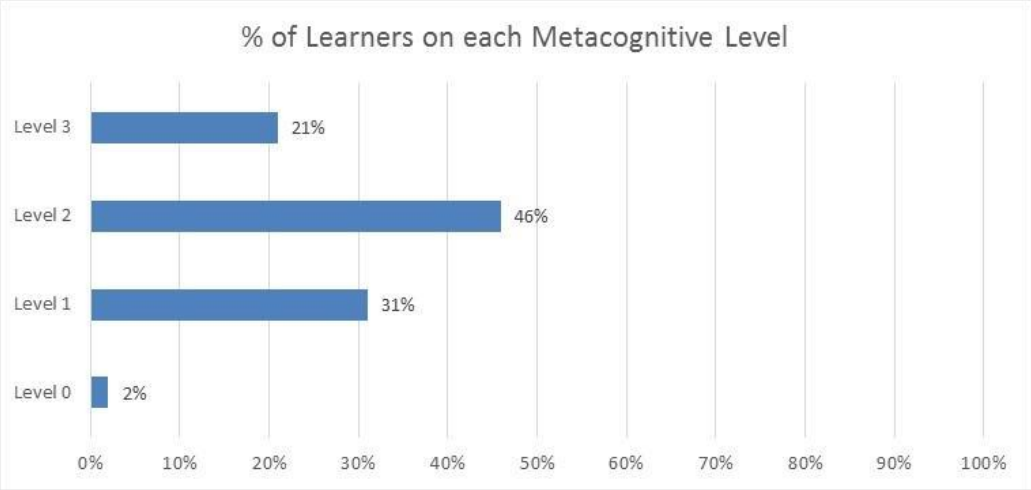


Figure 7.7 Metacognitive Levels based on Suggesting Improvement & Growth Strategies (late-Apr 2012)

7.6 Results: Analysing Performance

Two months later, in June I compiled a class test from ten Math Minute questions that had been the most poorly answered by the group of learners in their homework exercises. I told the learners that I expected that none of them would fail to achieve 100% on this test as they had answered all of the questions before, and that if they had made errors with any of these in past, those questions would have been the subject of their error-analysis in the April activity. I administered the test at the start of a Saturday contact session and marked their scripts to return to them the following Saturday.

The results were disappointing the learners, achieved an overall average of 60%. Each learner was then tasked with writing a letter to explain what had gone wrong and what they could do to improve their performance. Learners' self-analyses have been assessed using the MAAR Tool. (The MAAR Tool with examples of the interpretation of learner evidence for this criterion is presented in Table7.5.)

This enabled a level of achievement of metacognitive skills for this specific assessment criterion to be assigned to each learner.

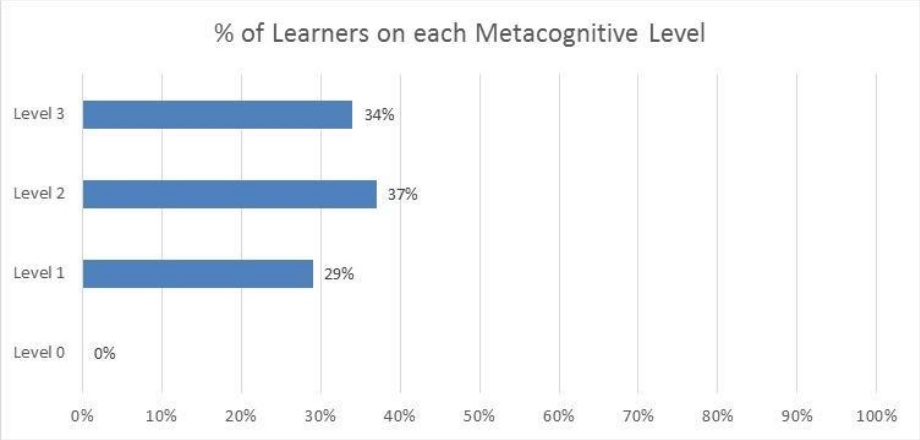
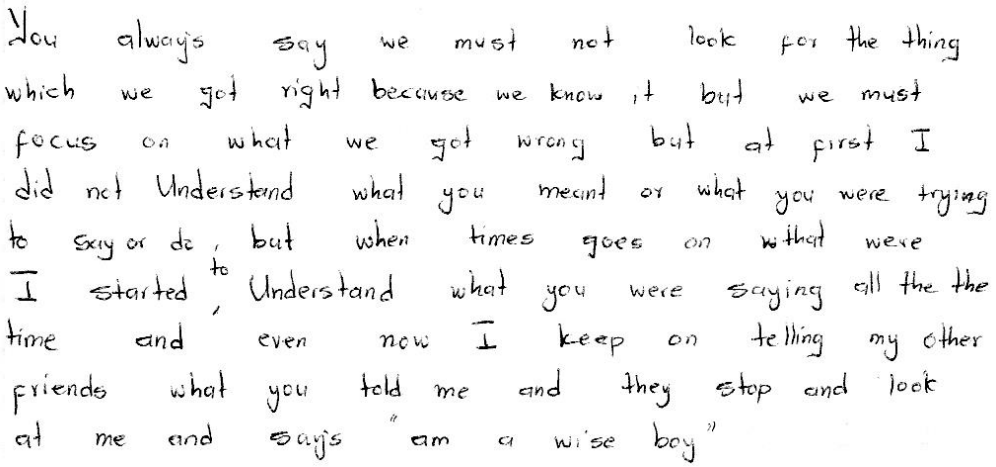


Figure 7.8 Metacognitive Levels based on Analysing Own Performance (Jun 2012)

Although the actual test results were disappointing, the quality of the learners' written responses was better. Greater detail and improved insight suggested evidence of metacognitive activation.


You always say we must not look for the thing which we got right because we know it but we must focus on what we got wrong but at first I did not Understand what you meant or what you were trying to say or do, but when times goes on what were I started to Understand what you were saying all the the time and even now I keep on telling my other friends what you told me and they stop and look at me and says "am a wise boy"

It appeared that all learners were metacognitively activated and metacognitively aware now and at progressive levels of metacognitive advancement above Level 0.

7.7 The Activation and Advancement of Metacognitive Skills

When considering the time period over which evidence of metacognitive advancement was collected, a clear pattern of change is evident.

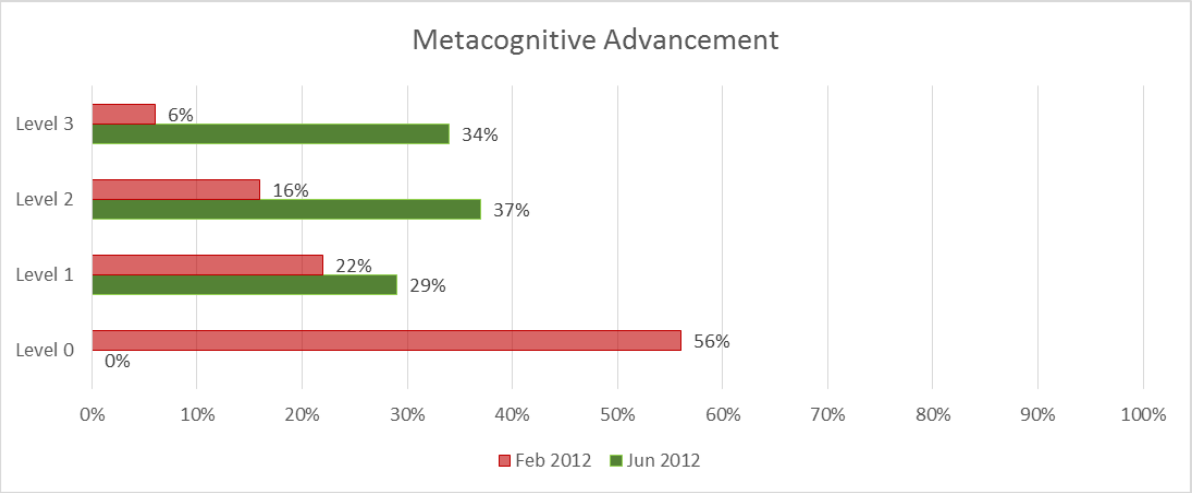


Figure 7.9 Percentage of L4E Learners per Metacognitive Level over Time

In February 2012, more than half (56%) of the L4E learners were on Level 0 in terms of their metacognitive skills. This means they were not yet metacognitively activated nor metacognitively aware. The remaining 44% of learners were metacognitively activated and aware to different degrees: 22% of learners were on Level 1, 16% on Level 2 and only 6% on Level 3.

By June 2012 all learners (100%) were metacognitively activated with no learners left on Level 0 at all. Twenty-nine percent of learners were on Level 1, 37% on Level 2 and 34% on Level 3. It is interesting to compare these overall results with the learners' individual measures of metacognitive skill level. I have taken the same two measures to compare: the first measure of metacognitive skill level (recorded in February 2012) and my last measure of metacognitive skill level (recorded in June 2012) to create a more detailed picture of progress.

Figure 7.10 shows learners' first and last measures of metacognitive skills. The green bars indicate baseline levels so missing bars correspond to an assessment on Level 0.

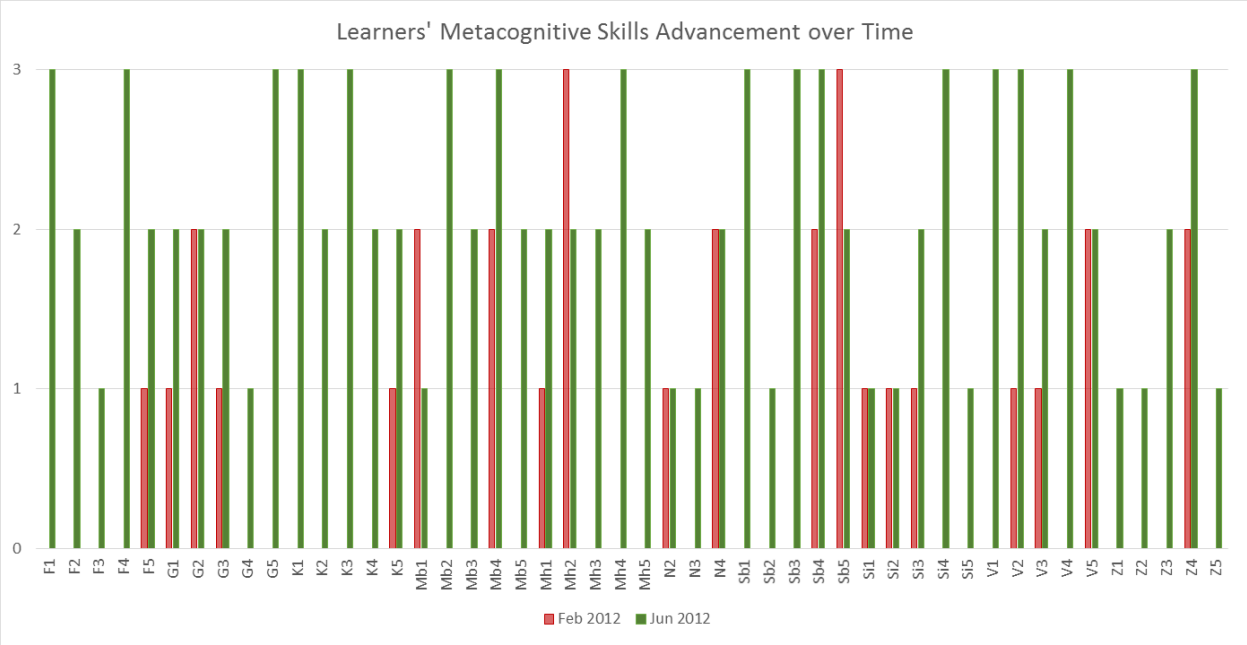


Figure 7.10 L4E Learners' Metacognitive Skills Levels over Time

The collections of green bars with no red bars in between indicate the learners who started out in February 2012 on Level 0. Many learners (24% of the learners) advanced from Level 0 all the way up to Level 3 in the five month period.

There were three learners (Mb1, Mh2 and Sb5) who scored more poorly in June compared to February. Progressive measures in metacognitive skills over this time period had required increased written language skills due to the nature of the assessment criteria (again refer to the MAAR Tool on page 74). I surmised that learners with poor English language and communication skills might perform worse on successive criteria. To confirm this I compared these learners' English Language scores in the L4E programme in order to compare them with the others. We used a reading programme called Readers are Leaders which measured a learner's Grade level of language comprehension in reading English to monitor progress in English. The L4E average level in June 2012 was 4.4 (between Grade 4 and Grade 5 level). Learner Mb1 was on Grade 2 level and learners Mh2 and Sb5 were both on Grade 3 level so all were below average.

7.8 Conclusion

My third research sub-question was: to what extent can this approach change learners' self-evaluations of their own understanding of Mathematics? When I formulated this question I was careful not to use the words 'metacognition' and 'metacognitive'. At the time, being aware of the minefield of literature around these words, I purposefully avoided them, uncertain of how I could measure these if I committed to them. After investigating the tools already developed (in order to address my need to assess change in learners' self-evaluations), I was disappointed with the quality of these tools and undertook to develop my own. In the process I stepped into the measuring metacognition arena and by the end of this endeavour, I felt confident that my tool (the Metacognitive Activation and Advancement Rubric) was useful for my purposes. The tool, in using the same four levels of performance, enabled a standardised way of measuring metacognitive skills level of each learner despite a variety of reflective activities. In fact developing this tool clarified my own initial distinctions between metacognitive activation, metacognitive awareness and metacognitive advancement and the progression of these. Considering the data collected over the five month period of February to June, there is clear evidence that the learners' level of metacognitive skills improved over time. Learners were better able to understand what it was that they understood and what they did not understand, to identify strengths to build on and weaknesses to fix, to explain the challenges they were facing in mastering problematic conceptual landmarks and to suggest strategies to overcome their learning difficulties. The assessment ***as*** learning – conceptual thread approach was able to provide an added benefit to the learners of, not only fast-tracking their catch-up of acquired learning deficits in Maths and improving their achievement, but also to advance the metacognitive skills needed to become their own agents of learning.

CHAPTER EIGHT

Conclusion to the Study

8.1 Overview of the Study

This study set out to evaluate an **assessment *as* learning – conceptual thread** approach to the teaching and learning of Mathematics. A key feature of this approach is developing reflective practice. This is achieved by using embedded formative assessment, and specifically assessment ***as*** learning, in order to engage each learner in self-assessment, self-reflection and self-evaluation of their own learning process and progress. The purpose of the approach is to metacognitively activate learners so that, through shared responsibility, their learning can be accelerated. This was a key need in this intervention as the focus was on 'catching-up' learners' acquired learning deficits to fast-track grade-level Maths teaching and learning. The assessment ***as*** learning approach is supported using identified conceptual threads running through from Grade 1 to Grade 9. These enable learners to comprehend the relatedness of concepts in Maths. Each of the seven conceptual threads is populated by conceptual landmarks which distinguish required learning. This 'route map' design assists learners in identifying their own learning gaps within a broader Maths 'picture' and to understand how their weaknesses impact on related topics in Maths.

The **assessment *as* learning – conceptual thread** approach was conducted as part of an intervention with 50 Grade 10 learners with acquired learning deficits that were causing functioning at levels as far back as Grade 2. These learners were participants in a donor-funded Maths programme called Learners For Excellence (L4E) which was initiated and implemented by Uplands Outreach in Mpumalanga. Focused on making an impact in the impoverished and poor-performing circuit of Insikazi, L4E launched in February 2012. Its aim was to increase both the numbers of learners who pass Maths as well as the quality of Maths results through a weekly Saturday Maths enrichment programme that ran over three years. The 50 learners were selected from the ten Insikazi high schools by their teachers. Selection criteria favoured learners with an interest and/or aptitude for Maths, a positive attitude and

a good work ethic. A baseline assessment conducted in the first L4E session produced disappointing results. Due to the diagnostic design of this assessment, which benchmarked achievement at the successive levels of Grade 3, Grade 6 and Grade 9 (even though the learners were in Grade 10), individual learner's acquired learning deficits were identified. This served as a personal audit of each learner's existing Maths knowledge and understanding as well as a gauge of the cohort's difficulties in Maths. The learners' poor results caused a change in the planned L4E programme. It was decided to include an hour-long weekly class in which an **assessment *as* learning – conceptual thread** approach would be employed to 'make up' acquired learning deficits and fast-track learners to actual grade-level expectations of performance in the shortest possible time. The diagnostic baseline results were shared with the learners as a personal blueprint for required remediation.

Embedded assessment ***as*** learning practice directed an adaptive teaching intervention through 2012 (and the first half of 2013) within the L4E programme. Conceptual threads were addressed in such a way as to develop understanding from Foundation Phase level along a learning trajectory through to Senior Phase level. Activities were varied and interactive, and required focused participation of all learners. Daily customised homework was set for each weekday in-between the Saturday sessions at Uplands Outreach. Marked homework was analysed by the learners through self-reflective activities to develop their metacognitive skills. Learners were encouraged to identify, understand and conquer their own specific misconceptions and errors to reach Grade 10 levels of achievement.

While the evidence of learner progress was collected to inform and direct the L4E intervention at the time, it was important to document this experience in the light of the widely-held view of the poor state of Mathematics education in South Africa. It was this that motivated me to take on a retrospective research study of the experience. The study was conducted after the three-year L4E programme was completed and data included was collected between February 2012 and September 2012. Summative data for two conceptual threads, Fractions and Problem-Solving, obtained in June 2013, is also included (in order to complete the picture of overall achievement on conceptual threads).

A mixed methods design using both quantitative and qualitative data was followed to answer the following research question:

what is the value of using an assessment *as* learning – conceptual thread approach in addressing acquired learning deficits in Mathematics in South Africa?

To answer this general question I asked more specifically:

Sub-question: Can a teaching and learning intervention using an assessment **as** learning – conceptual thread approach 'make up' acquired learning deficits in Mathematics and 'catch up' low-achieving learners?

This question focuses on the value of this approach for the learners to improve their achievement in Mathematics.

Sub-question: Which obstacles to mastering conceptual landmarks in learning about numbers and number operations are revealed by the approach followed?

This question focuses on the value of this approach for the teacher in order to provide greater understanding of learner difficulties in Maths and insight into where these difficulties commonly lie, what these difficulties are and how these can be addressed successfully. The conceptual thread of Numbers was studied as an example of this.

Sub-question: To what extent can this approach change learners' self-evaluations of their own understanding of Mathematics?

This question focuses on the value of an 'added benefit' for each learner to advance his/her metacognitive skills, engaging and equipping a learner to play an active part in their own learning process and so become self-motivated, self-directed and independent learners.

8.2 Research Findings

With respect to the value of this approach in 'making up' acquired learning deficits in Mathematics and 'catching up' low-achieving learners:

The L4E intervention with its assessment ***as*** learning – conceptual thread approach achieved a significant improvement in learner results on the seven conceptual threads that comprise basic Mathematics. It is important to remember that the learners were actually in Grade 10 and this learner achievement was measured up to Grade 9 level. However the approach has 'made up' acquired learning deficits and 'caught up learners' to varying degrees. As already stated in Chapter Five, all 50 learners caught up at least three grade levels of Maths education with some learners making even larger gains of up to six grade levels in a one year period with limited contact time. With an overall average improvement of over 40 percentage points, by the end of the intervention all learners were either approaching, meeting or exceeding grade-level achievement for the end of Senior Phase. To reconsider research already presented in Chapter One where high school remediation is described as "almost impossible" Spaull (2013) and "virtually impossible" Taylor, et.al (2003), the assessment ***as*** learning – conceptual approach resulted in significant remediation. This suggests that **acquired learning deficits are not necessarily insurmountable** and that **Grade 10 is not necessarily too late** for learners to catch up and achieve in Mathematics.

With respect to the value of this approach in revealing obstacles and difficulties in Mathematics for greater insight into learning for teachers:

On reflection, this research sub-question was weak in that it didn't specify what any greater insight for teachers provided by this approach would be compared to. As the teacher in the L4E programme, I certainly gained more insight into the difficulties learners were facing when they added, subtracted, multiplied and divided numbers. The reflective nature of the assessment ***as*** learning – conceptual thread approach broadened and extended my understanding of the troublesome conceptual landmarks along the conceptual threads and the common causes of difficulties in order to focus my attention on these. My learning as the teacher in this intervention would most definitely influence how I would introduce these conceptual landmarks to younger learners after the L4E experience. All of this is supported

by the work of Hattie and Timperley (2007) that the feedback a teacher receives from learners through formative assessment makes "learning visible". Similarly it is supported by the value of the "feed forward" of Fisher and Frey (2007) that enables modifying instruction. This study doesn't only evaluate formative assessment though, it evaluates a more specific formative assessment practice, that of "assessment ***as*** learning". While the assessment ***as*** learning – conceptual thread approach did **reveal difficulties that learners experienced** and **provided insight** into the way learners make sense of the Numbers conceptual landmarks, this was not necessarily unique to this approach.

With respect to the value of this approach in changing learners' self-evaluations of their own understanding of Mathematics:

Considering data collected over the five month period of February to June, there is evidence that the learners' self-evaluations of their own understanding of Mathematics changed. These changes in self-evaluations suggest changes in their metacognitive skills. In February 2012, on the first self-evaluation activity, 94% of the learners over-estimated their performance. Two months later in April, this was turned around with 93% of learners under-estimating their performance. Based on existing research (Kruger & Dunning, 1999; Dunning et al., 2003; Miller & Geraci, 2011; McCormick, 2013), I propose that the shift from over-estimating to under-estimating provides evidence of metacognitive activation in learners. This means that the period between February 2012 and April 2012 marks the metacognitive activation of the majority of the L4E learners. The correction of under-estimation back towards more accurate estimation (which matches actual achievement) illustrates metacognitive awareness (as a result of metacognitive activation) progressing through to advanced metacognitive skills. Using the MAAR tool devised to analyse learners' metacognitive skill level (from Level 0 to Level 3), it is evident that over time, learners developed skills of error detection, diagnosis and analysis; awareness of strengths, weaknesses, learning needs and opportunities; self-reflective performance analysis and self-evaluation of improvement strategies. The assessment ***as*** learning – conceptual thread approach provided an added benefit to the learners of, not only fast-tracking their catch-up of acquired learning deficits in Maths and improving their achievement, but also **to advance the metacognitive skills that they could use to become their own agents of learning**.

8.3 Research Reflections

The fact that this study was conducted after the L4E intervention was already complete resulted in compromises in the design of my study. These compromises were mainly due to the fact that at the time my focus was on collecting data to use in designing activities and directing the intervention rather than for research purposes. Consequently I could not include many assessment activities because I did not have the necessary qualitative data from learners' scripts but only had records of their scores. If I had planned this as a research activity from the outset I would have gathered more learner evidence particularly more of the qualitative data accumulated in their Maths journals. Looking back specifically in the Numbers conceptual thread, I had witnessed many learner difficulties involving place-value. These place-value difficulties impacted on other conceptual landmarks such as rounding numbers. The lack of data of these difficulties and the failure to isolate more conceptual landmarks recorded in my assessment record prevented me from including these issues in this study. I also noticed difficulties with divisibility especially with identifying factors and prime factors of large numbers. While I had recorded specific changes in dividing numbers which were clearly affecting learners' progress with factors, I had not isolated and recorded the details of these difficulties with factors.

Analysing the learners' metacognitive skill development retrospectively was a challenge that resulted in the need to develop a new instrument, the Metacognitive Activation and Advancement Rubric. While the development of the MAAR tool clarified and improved my understanding about metacognition in the classroom, this tool needs to be tested and verified further.

My focus in this study was therefore to some degree directed by the data available to me through my work on the intervention and this was not ideal. I would certainly think twice about embarking on a retrospective study again. At the same time as saying that, I feel that due to the limitations and the difficulties of the retrospective nature, I too have been 'activated' to see research opportunities which I would not even have considered before taking on this study. I am grateful that despite a lack of detail in some of my data, I collected a lot of evidence over the course of my involvement with the L4E learners. For this reason, I

had sufficient data in the areas I did focus on to make findings about the value of implementing an assessment ***as*** learning – conceptual thread approach in the classroom and to make learning progress myself through the research study.

8.4 Implications of the Study

As already mentioned, it was the self-reflective nature of the assessment ***as*** learning – conceptual thread approach of the L4E programme that made it different to most Saturday Maths enrichment programmes. Embedding formative assessment practice in classroom activities with the active engagement of the learners in these activities was a unique experience for both the L4E learners as well as the other Uplands Outreach staff members and teachers involved in the programme. This study reinforces the research of White and Frederiksen (1998) which concluded that a reflective assessment approach (discussed in Chapter Three) had the valuable effect of reducing the educational disadvantage of low-achieving students. White and Frederiksen's approach shares commonalities with this assessment ***as*** learning – conceptual thread approach, specifically in the use of assessment ***as*** learning to develop and practice metacognition. This study strengthens White and Frederiksen's argument for a metacognitively-focused curriculum by providing further evidence of this as an effective strategy for catching up the acquired learning deficits prevalent in Maths in South Africa.

8.4.1 Implications for Practice

I would argue that diagnostic assessment (***as*** learning) needs to be a first step to remediation. This study shows that while learners may actually be in Grade 10, they may be functioning mathematically below a Grade 3 level. This has implications for their teachers. Faced with a learner struggling to understand a concept, a teacher may revise the previous year's work. The L4E evidence shows that this may have little impact if the learner's existing knowledge is at a level too far back for them to build their new knowledge on. While acquired learning deficits are prevalent in South Africa, even learners who have experienced a privileged

education may still have deficits. Just because a teacher has 'taught' a topic doesn't mean that all learners have 'learned' it.

Addressing each learners' acquired learning deficits in a class of learners is a challenge for a single teacher, especially if there are many learners in the class. Using assessment ***as*** learning empowers the learners to participate in their learning process and provides the added benefit for learners of metacognitive activation and advancement of metacognitive skills. Assessment ***as*** learning can fast-track remediation. The L4E programme design of an hour-long contact session once a week suggests that a weekly after-school Maths 'club' form of engagement can provide sufficient time to make an impact on catching up learners to grade-level performance.

An encouraging message to come out of the L4E intervention documented for teachers is not to give up on poor-performing Maths learners. It *is* possible to catch-up learning deficits of even more than six years and at a late intervention stage, and for learners to pass and achieve at expected grade-level in Maths.

8.4.2 Implications for Policy Makers

As a result of this study, I would propose that formative assessment and specifically assessment ***as*** learning should be a major component of teacher training courses. As already mentioned in Chapter Three, this is supported by research that formative assessment is recognised as a key professional skill for teachers (ARG, 2002; Wiliam, 2011; Husbands and Pearce, 2012). I would argue that despite the L4E learners being chosen for the programme based on their aptitude and interest in Maths, their poor baseline results and revealed learning deficits were not very different from the data at the time of Grade 9 Maths ANAs, numbers of learners in FET Maths and Grade 12 NSC Maths results. With more than half of the L4E learners classified as OVC, these were not privileged learners. The scalability of the assessment as learning – conceptual thread approach beyond L4E could provide a broader impact in South Africa. Of course, even though it has not been in focus in this study, the

Mathematics teachers' knowledge and skill is critical in the success of any teaching and learning approach.

Since the launch of the Data Driven Districts programme in 2012, the DBE is following a path of data-driven practice. There is an opportunity to extend this practice to the classroom through inclusions in teacher pre- and in-service training of how to design and implement diagnostic and formative assessment and how to interpret learner results in order to direct classroom practice and meet learners learning needs. This presents a growth opportunity in teacher development for the acquisition, practice and advancement of skills of data analysis and interpretation.

8.4.3 Implications for Further Study

An assessment ***as*** learning – conceptual thread approach establishes a partnership between the teacher and learner which relies on feedback from teacher to learner and from learner to teacher. This increases opportunities for the production and collection of diagnostic data. There is value in this accumulated data for studies of conceptual difficulties for learners in Maths not only for teachers in their classrooms but also for researchers in further study. When I started this study, I chose to focus on only one conceptual thread, that of Numbers. At that point I thought the next step after completing this study would be to focus on another conceptual thread such as Problem-Solving. However, the exploration of metacognition and the development of the MAAR tool has opened a more tempting learning path for me.

The questions I am asking are:

- how could the MAAR tool be used in other Maths assessment ***as*** learning interventions to identify the point of metacognitive activation of learners?
- could diagnostic assessment with a self-reflective component and the subsequent sharing of explicit feedback alone facilitate metacognitive activation (without a specific teaching intervention)?
- what are the impacts of different types and levels of Maths teaching inputs on metacognitive activation and the development of metacognitive skills?

While it appears that the majority of studies on metacognition are published in the journals of psychology rather than of education, this changed with the publishing of a revision of Bloom's Taxonomy (Anderson and Krathwohl, 2001) which included metacognitive knowledge in the knowledge domain. From general studies to more specific, there are a number of researchers focusing on metacognition within the Maths classroom (Garafalo and Lester, 1985, Goos, 2002, Stillman and Mevarech, 2010). In most cases this involves identifying learners' metacognitive skills in the practice of strategic knowledge in problem-solving. Earl's assessment ***as*** learning approach provides opportunities to learn more about learners' metacognitive skills in the practice of strategic knowledge, cognitive knowledge *and* self-knowledge.

8.5 A Final Word

In final conclusion, discussions about the state of Maths education in South African schools are usually woeful. The media reinforce a hopeless story with headlines such as "South Africa has one of the world's worst education systems" (The Economist, 2017). While this unfortunately may reflect the view of many South Africans, it is not the only one. The 50 young South Africans of Uplands Outreach L4E who, at the start of the programme, were little different to the majority of South African Grade 10 learners have shown that, with a diagnostic, self-reflective, data-driven intervention, and learner engagement and perseverance, acquired learning deficits can be overcome in time for academic achievement and success in high school Mathematics.

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APPENDICES

APPENDIX A

LEARNERS FOR EXCELLENCE PROGRAMME
Mathematics Test

NAME: _____ **SCHOOL:** _____

INSTRUCTIONS: **TIME @ START:** _____

- Answer all the questions on the paper.
- Show ALL working out.
- Write clearly and legibly.
- Calculators may **NOT** be used.

Gr. 3
Gr. 6
Gr. 9

QUESTION 1 – Numbers, Operations & Relationships (30 marks)

Numbers
Variables

1.1 Which number is **bigger**? In each case, draw a circle around the **bigger** number.

A.	110	or	101	✓
B.	101	or	1·1	✓
C.	½	or	¼	✓
D.	3¾	or	4¼	✓
E.	-2	or	-6	✓
F.	-30	or	-3	✓
G.	0,405	or	0,41	✓
H.	12,3	or	2,34	✓
I.	$\sqrt{64}$	or	$\sqrt[3]{64}$	✓
J.	5^2	or	2^5	✓

1.2 Is this **TRUE** or **FALSE**? In each case, draw a circle around the correct answer.

A.	109 < 190	TRUE	or	FALSE	✓
B.	0 > -2	TRUE	or	FALSE	✓
C.	3,5 = 3⅕	TRUE	or	FALSE	✓
D.	50% = ½	TRUE	or	FALSE	✓

1.3 Work out the following and write your answer clearly:

A. $44 + 21 = 65$ ✓

B.
$$\begin{array}{r} 905 \\ +\ 137 \\ \hline 1042 \end{array}$$
✓

1.4 Work out the following and write your answer clearly:

A. $44 - 21 = 23$ ✓

B.
$$\begin{array}{r} {}^{8}\cancel{9}{}^{9}\cancel{0}{}^{1}5 \\ -\ 137 \\ \hline 768 \end{array}$$
✓

1.5 Work out the following and write your answer clearly:

A. $13 \times 3 = 39$ ✓

B.
$$\begin{array}{r} 905 \\ \times\ 13 \\ \hline 9050 \\ 2715 \\ \hline 11765 \end{array}$$
✓

1.6 Work out the following and write your answer clearly:

A. $56 \div 4 = 14$ ✓

B.
$$\begin{array}{r|l} & 566 \\ \hline 9 & 5094 \\ & \underline{45}\downarrow \\ & 59 \\ & \underline{54}\downarrow \\ & 54 \\ & \underline{54} \\ & 0 \end{array}$$
✓

1.7 Work out the following and write your answer clearly:

A. $13 - (2 \times 4) = 13 - 8 = 5$ ✓

B. $8 + (16 \div 4) = 8 + 4 = 12$ ✓

1.8 Work out the following and write your answer clearly:

A. $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$ ✓

B. $\frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$ (or $\frac{5}{9}$) ✓

C. $0,2 - 0,15 = 0,05$ ✓

D. $1,1 \times 0,2 = 0,22$ ✓

E. $5^2 + 5^3 = 25 + 125 = 150$ ✓

F. $\sqrt{225} = 15$ ✓

QUESTION 2 – Patterns, Functions & Algebra (30 marks)

2.1 Study the following number pattern: 0 ; 3 ; 6 ; ____ ; 12 ; 15 ; 18 ; 21 ...

A. What is the missing number? 9 ✓

B. What is the next number after the last number shown in the pattern? 24 ✓

C. What is the rule of this pattern? add 3 ✓

2.2 Write down the answers to the following questions in the space provided.

A. If $a = 4$ what is the value of $\frac{1}{a}$?

$\frac{1}{(4)} = \frac{1}{4}$ ✓

B. If $a = 4$ what is the value of $2a^2$?

$2(4)^2 = 32$ ✓

C. If $a = ¼$ what is the value of $2a^2$?

$2\left(\frac{1}{4}\right)^2 = \frac{1}{8}$ ✓

D. If $a = ¼$ what is the value of $\frac{1}{a}$?

$\frac{1}{\left(\frac{1}{4}\right)} = 4$ ✓

2.3 Solve for x:

A. $\frac{3x}{3} = \frac{12}{3}$

$x = 4$ ✓

B. $x + 2 = 12$

$x + 2 - 2 = 12 - 2$

$x = 10$ ✓

C. $2x - 1 = 23$

$2x - 1 + 1 = 23 + 1$

$\frac{2x}{2} = \frac{24}{2}$

$x = 12$ ✓

D. $\frac{x}{4} = 5$

$\frac{4}{1} \cdot \frac{x}{4} = \frac{5}{1} \cdot \frac{4}{1}$

$x = 20$ ✓

E. $\frac{2x}{5} - 1 = 9$

$\frac{2x}{5} - 1 + 1 = 9 + 1$

$\frac{5}{2} \times \frac{2x}{5} = 10 \times \frac{5}{2}$

$x = 25$ ✓

F. $\frac{x}{4} = \frac{x}{3} - \frac{1}{2}$

$\frac{x}{4} - \frac{x}{3} = \frac{x}{3} - \frac{1}{2} - \frac{x}{3}$

$\frac{x}{4} - \frac{x}{3} = -\frac{1}{2}$

$\frac{3x}{12} - \frac{4x}{12} = -\frac{1}{2}$

$-\frac{12}{1} \times -\frac{x}{12} = -\frac{1}{2} \times -\frac{12}{1}$

$x = 6$ ✓

2.4 Simplify the following:

A. $2x + 5x$

$= 7x$ ✓

B. $2x \cdot 5x$

$= 10x^2$ ✓

C. $2x^3 . 3x^2$

$= 6x^5$ ✓

D. $\frac{2x^2}{x}$

$= 2x$ ✓

E. $\frac{x}{5} + \frac{x}{3}$

$= \frac{3x}{15} + \frac{5x}{15}$

$= \frac{8x}{15}$ ✓

F. $\frac{6x^4y^2}{24x^5y}$

$= \frac{y}{4x}$ ✓

G. $2x(5x + 3y)$

$= 10x^2 + 6xy$ ✓

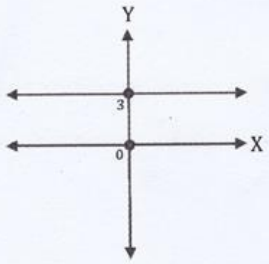
H. $(2x - 3y)(4x - y)$

$= 8x^2 - 12xy - 2xy + 3y^2$

$= 8x^2 - 14xy + 3y^2$ ✓

2.5 What is the equation of each of the following lines? Circle the correct answer.

A.



a. $x = 0$

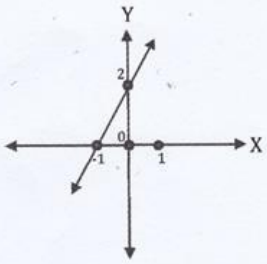
b. $y = 0$

c. $x = 3$

(d.) $y = 3$

e. $y = x + 3$ ✓

B.



a. $x = -1$

b. $y = 2$

c. $y = x + 2$

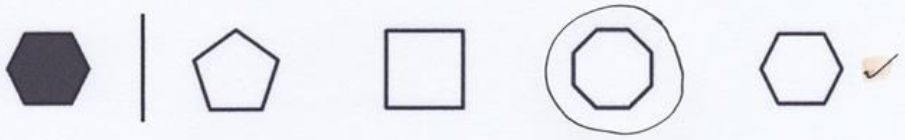
d. $y = -2x$

(e.) $y = 2x + 2$ ✓

QUESTION 3 – Space & Shape (Geometry) (10 marks)

3.1 Study the following geometric figures. Draw a circle around the shape that is the same kind of shape as the first shape in each row.

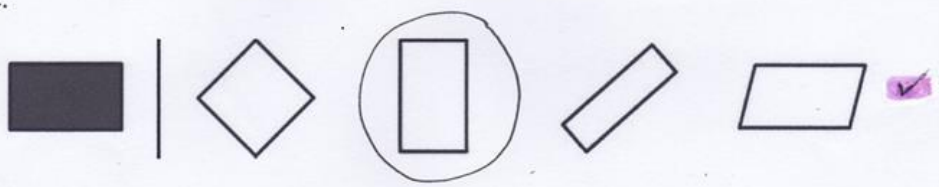
A.



B.



C.



3.2 Identify these 2-dimensional shapes. Write the name on the line below the diagram.

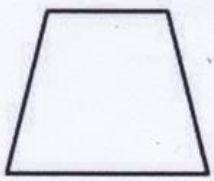
A.



rhombus ✓

(quadrilateral) ½

B.

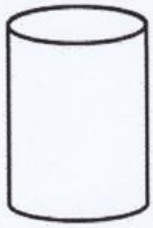


trapezium ✓

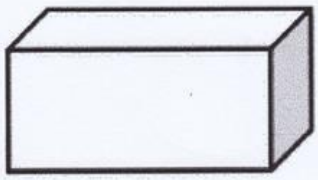
(quadrilateral) ½

3.3 Identify these 3-dimensional objects. Write the name on the line below the diagram.

A.

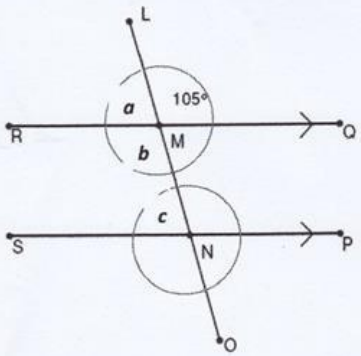


B.



cylinder ✓ rectangular prism/cuboid ✓

3.3 Study the following drawings of angles. What is the size of the marked angle? Write your answer down below.



A. Angle **a** is: 75 ° ✓

B. Angle **b** is: 105 ° ✓

C. Angle **c** is: 75 ° ✓

QUESTION 4 – Measurement (10 marks)

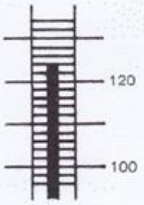
4.1 What are the readings on the following scales?
Write the answer below each diagram

A.



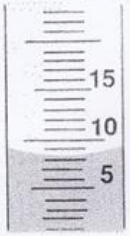
4 units ✓

B.



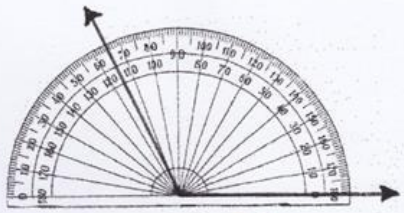
124 units ✓

C.



8 units ✓

D.



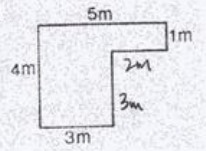
115° ✓

4.2 Which measurement is **smaller**? Draw a circle around the **smaller** measurement.

A. 1 centimetre OR (1 millimetre) ✓

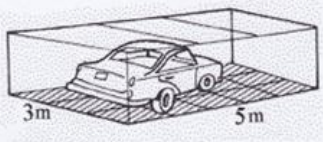
B. (500 grams) OR 0,6 kilograms ✓

4.2 What length of wire fencing is required to enclose this space?
(Calculate the perimeter of the shape.)



P = 18 m ✓✓ or ✓ if calculating P with missing values

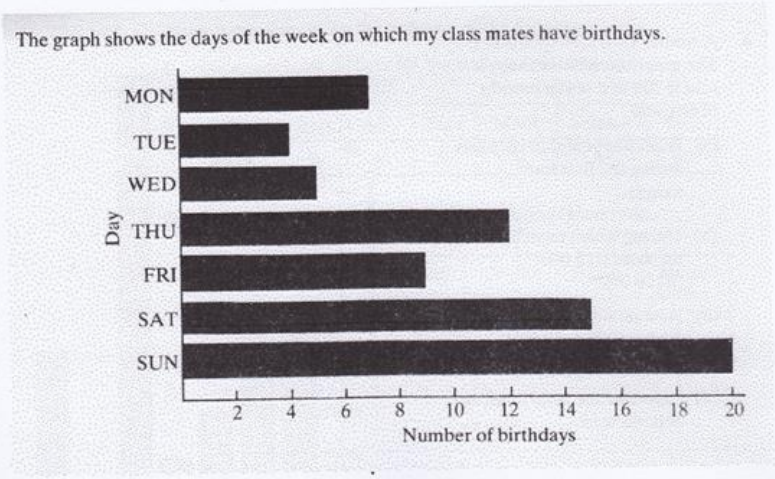
4.3 How much flooring is needed to cover this garage floor?
(Calculate the area of the shape.)



A = 15 m² ✓✓

QUESTION 5 – Data-Handling (10 marks)

5.1 Look at the following graph of data.



A. On which day do the fewest (least number of) people have their birthdays?

Tuesday ✓

B. On which day are there 12 birthdays?

Thursday ✓

C. How many learners have birthdays on:

i) Sunday?

20 ✓

ii) Monday?

7 ✓

D. How many learners are there in the class?

7 + 4 + 5 + 12 + 9 + 15 + 20 = 72 ✓

5.2 Thembi achieves the following results for her languages in the first term of Grade 10:

Subject	Mark	Class Average
English	68%	53%
Siswati	75%	70%

A. In which language did Thembi achieved the highest mark?

Siswati ✓

B. In which language did Thembi achieve the most compared to her class mates?

English ✓

C. What is Thembi's **average mark** for the two languages (the mean)?

68 +
75
143 ÷ 2 = 71,5 % ✓

5.3



A bag contains 5 red balls and 20 blue balls.

What is the probability (chance) of:

A. pulling a red ball out of this bag?

$\frac{5}{25}$ or $\frac{1}{5}$ or (1 in 5 chance) ✓

B. NOT pulling a red ball out of the bag?

$\frac{20}{25}$ or $\frac{4}{5}$ or (4 in 5 chance) ✓

QUESTION 6 – Problem-Solving (10 marks)

6.1 Each learner needs three books for each of their seven subjects plus one extra book for managing their homework. If there are 50 learners in the class, how many books must the teacher hand out to the class at the start of the term?

each learner needs $(3 \times 7) + 1 = 22$ books

$\therefore$ 50 learners need $50 \times 22 = 1100$ books ✓✓

6.2 Seven men build a house in 280 days. How many men, working at the same rate, can build the house in 70 days?

$\frac{70}{280} = \frac{1}{4}$ of the time $\therefore$ need 4 x number of men: $4 \times 7 = 28$ men ✓✓

6.3 Three children (Sipho, Thembi and Anna) eat three different kinds of chocolate (Bar One, Kit Kat and Flake). Each child eats only one kind of chocolate and no two children eat the same kind of chocolate. If:

- Sipho hates Bar Ones
- Thembi only eats Kit Kats
- Anna hates Flake

	Bar One	KitKat	Flake
Sipho	X	X	✓
Thembi	X	✓	X
Anna	✓	X	X

A. Which chocolate does Sipho eat?

Flake ✓

B. Which chocolate does Anna eat?

Bar One ✓

6.4 In the yard there are only two kinds of animals: chickens and dogs. Each chicken has two eyes and two feet. Each dog has two eyes and four feet. Altogether, the animals have 58 eyes and 84 feet.

A. How many animals are in the yard?

$\frac{58 \text{ eyes}}{2} = 29$ animals ✓

B. How many of the animals are dogs?

84 feet $-$ (2×29) feet $= 26$ extra feet

each dog has 2 extra feet $\therefore \frac{26}{2} = 13$ dogs ✓✓

C. How many of the animals are chickens?

29 animals $-$ 13 dogs $= 16$ chickens ✓

TIME @ END: _____

TIME TAKEN: _____

TEST ANALYSIS

Conceptual Thread	Marks Available	Marks Achieved
Numbers: Concept, Operations & Relationships (Whole Numbers and Integers)	20	5 8 7
Numbers: Concept, Operations & Relationships (Fractions)	10	1 7 2
Patterns,Functions & Algebra	30	3 0 27
Space & Shape (Geometry)	10	2 3 5
Measurement	10	0 5 5
Data-handling	10	3 3 4
Problem-solving	10	0 3 7
TOTAL	100	14 29 57

90% 80% 70%

13 23 39

13 36 75

APPENDIX B

Numbers: Concepts, Relationships & Operations (CAPS and the Conceptual Thread)

	GRADE 3	GRADE 6	GRADE 9
Counting forwards and backwards	Up to 1 000	Up to 9-digit numbers	Practise mastery
Skip-counting forwards and backwards	Skip counting in 2s, 3s, 5s, 10s, 20s, 25s, 50s and 100s to 1 000	Skip-counting in 2s, 3s, 5s, 10s, 25s, 50s and 100s to 9-digit numbers	Practise mastery
Recognise, identify and read numbers	Up to 1 000	Up to 9-digit numbers	Practise mastery
Describe, compare and order numbers	Up to 999	Up to 9-digit numbers	Practise mastery
Use, read and write ordinal numbers	Up to 31st	Unlimited	Practise mastery
Recognise place-value and decompose numbers	Up to 999	Up to 9-digit numbers	Practise mastery
Round off numbers	-	to the nearest 5, 10, 100, 1 000, 100 000, and 1 000 000	Practise mastery
Addition of numbers	With answers up to 999	With numbers at least up to 6-digits	Practise mastery
Subtraction	With answers below 999	With numbers up to 6-digits	Practise mastery
Multiplication	By 2, 3, 4, 5 and 10 to a total of 100 (grouping and sharing)	Up to 4-digit by up to 3-digit numbers	Practise mastery
Division	Numbers up to 100 by 2, 3, 4, 5 and 10 (grouping and sharing)	Up to 4-digit by up to 3-digit numbers	Practise mastery
Mixed Operations	-	With or without brackets	Practise mastery
Represent prime numbers	-	Up to 100	Practise mastery
Identify and determine Factors	-	Of 2-digit and 3-digit numbers	Practise mastery
Identify and determine multiples	-	Of 2-digit and 3-digit numbers	Practise mastery
Identify and determine prime factors	-	Of numbers to at least 100	Of number up to 3-digits

	GRADE 3	GRADE 6	GRADE 9
Determine squares, cubes and roots of whole numbers	Informally 2^2, 3^2, 4^2, 5^2 and 10^2	Formally up to 12^2	Up to 12^2 and 6^3
Compare and represent numbers in exponential form	-	-	With positive and negative exponents
Calculate using numbers in exponential form	-	-	Using integer exponents
Compare and order integers	-	-	Up to 9-digit numbers
Calculate using integers	-	-	Up to 9-digit numbers
Determine squares, cubes and roots of integers	-	-	Up to 9-digit numbers

Fractions: Concepts, Relationships & Operations (CAPS and the Conceptual Thread)

	GRADE 3	GRADE 6	GRADE 9
Use and name common fractions	Unitary and non-unitary fractions of halves, quarters, eighths, thirds, sixths, fifths	With denominators up to hundredths	With denominators beyond thousandths
Describe, compare and order common fractions	With same denominators	With different denominators up to hundredths	With different denominators beyond thousandths
Recognise equivalent common fractions	Wholes as two-halves, three-thirds etc. and a half as two-quarters	With common 1-digit or 2-digit denominators where one is a multiple of other	With up to 3-digit denominators where one is the multiple of another
Recognise equivalence of common fractions with decimal fractions	-	Common fraction and decimal form equivalence up to hundredths	Common fraction and decimal form equivalence up to thousandths
Recognise equivalence of common fractions with percentages	-	Common fraction, decimal form and percentage form up to hundredths	Common fraction, decimal form and percentage form up to thousandths
Simplify common fractions	-	Use factors and multiples to write in simplest form (within hundredths)	Use factors and multiples to write in simplest form (beyond thousandths)
Describe, compare and order decimal fractions	-	With digits of numbers up to 2 decimal places	With digits of numbers up to 3 decimal places
Recognise place-value of decimal fractions	-	With digits of numbers up to 2 decimal places	With digits of numbers up to 3 decimal places
Round off decimal fractions	-	To whole numbers	To one, two or three decimal places
Convert mixed numbers to improper fractions and vice versa for calculations	-	With denominators up to hundredths	With denominators beyond thousandths
Addition of common fractions	-	With different denominators where one is a multiple of another (including mixed numbers)	With denominators where one is not a multiple of another (including mixed numbers)
Subtraction of common fractions	-	With different denominators where one is a multiple of another (including mixed numbers)	With denominators where one is not a multiple of another (including mixed numbers)
Addition of decimal fractions	-	With digits of numbers up to two decimal places	With digits of numbers beyond three decimal places
Subtraction of decimal fractions	-	With digits of numbers up to two decimal places	With digits of numbers beyond three decimal places

	GRADE 3	GRADE 6	GRADE 9
Determine percentages of whole numbers	-	With numbers up to six digits	With numbers up to nine digits
Multiplication of common fractions	-	Limited to grouping and sharing techniques	With denominators up to thousandths where one is not a multiple of another (including mixed numbers)
Multiplication of decimal fractions	-	By 10 and 100	With digits of numbers to three decimal places
Division of common fractions	-	Limited to grouping and sharing techniques	With denominators up to thousandths
Division of decimal fractions	-	-	By whole numbers and decimal fractions
Mixed operations with common factions	-	-	All four operations with or without brackets
Mixed Operations with decimal fractions	-	-	All four operations with or without brackets
Calculate with percentage increase and decrease	-	-	Both percentage increase/decrease from amount and amount from increase/decrease
Determine squares, cubes and roots of common fractions	-	-	With denominators up to thousandths
Determine squares, cubes and roots of decimal fractions	-	-	With digits of numbers up to three decimal places

Patterns, Functions & Algebra (CAPS and the Conceptual Thread)

	GRADE 3	GRADE 6	GRADE 9
Copy, extend and create patterns	With physical objects, drawings and simple number sequences up to 1000	With sequences not limited to constant difference and constant ratio	With sequences not limited to constant difference and constant ratio
Describe patterns	In words	In words	In words and using algebraic language
Determine rules of patterns	Only informally	In words, using sequences, flow diagrams and tables	In words and algebraic language, using sequences, flow diagrams, tables
Determine input and output values	-	Using sequences, flow diagrams and tables	Using flow diagrams, tables, formulae and equations
Determine equivalence	-	Verbally, in a flow diagram, table and number sentence	Verbally, in flow diagrams, tables, by formulae, equations and graphs on a Cartesian plane
Use algebraic language	-	-	Recognise variables, constants, coefficients, exponents, like and unlike terms and use algebraic conventions
Simplify algebraic expressions using laws	-	-	Expand and simplify algebraic expressions using distributive, associative and commutative laws and laws of exponents
Simplify algebraic expressions using operations	-	-	Simplify algebraic expressions by adding, subtracting, multiplying and dividing (up to trinomials by monomials)
Simplify algebraic expressions by factorisation	-	-	Factorise with common factors, difference of squares and simple trinomials including fractions
Determine squares, cubes and roots	-	-	Of single algebraic terms up to squaring binomials and including fractions

	GRADE 3	GRADE 6	GRADE 9
Write algebraic equations	-	Limited to writing simple number sentences to describe problems	Write algebraic equations to describe problems
Solve algebraic equations	-	Limited to solving and completing simple number sentences by inspection and trial and improvement	Solve equations by inspection including multiplicative and additive inverses and laws of exponents
Substitute numerically	-	To check number sentence solution	To determine numerical values and to generate tables of ordered pairs
Analyse graphs	-	-	Focus on linear or non-linear; constant, increasing or decreasing; maximum or minimum; discrete or continuous
Interpret linear graphs	-	-	Find x-intercept, y-intercept, and gradient up to determining equations from graphs
Draw linear graphs	-	-	Plot ordered pairs on a Cartesian plane up to drawing graphs from given equations

Space & Shape (CAPS and the Conceptual Thread)

	GRADE 3	GRADE 6	GRADE 9
Identify 2-D shapes	Recognise, identify, name and sort circles, triangles, squares, rectangles	Recognise, identify, name and sort regular and irregular polygons: triangles, squares, rectangles, parallelograms, other quadrilaterals, pentagons, hexagons, heptagons, octagons and circles	Recognise, identify and classify: equilateral, isosceles, right-angled triangles and quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium and kite)
Describe 2-D shapes	Describe and compare characteristics of circles, triangles, squares and rectangles	Describe and compare characteristics of regular and irregular polygons using number of sides, lengths of sides and sizes of angles	Describe, compare and define the different triangles and quadrilaterals and identify their properties
Work with similarity	-	-	Establish minimum conditions for similarity
Work with congruency	-	-	Establish minimum conditions for congruency
Solve 2-D geometric problems	-	-	Determine unknown sides and angles
Create 2-D shapes	Draw circles, triangles, squares and rectangles	Draw (on grid paper) regular and irregular polygons: triangles, squares, rectangles, parallelograms, other quadrilaterals, pentagons, hexagons, heptagons, octagons and circles (with compasses)	Construct: ° geometric figures; ° angles of 30^o, 45^o, 60^o and their multiples without a protractor; ° bisected angles of a triangle
Recognise 3-D objects	Recognise, identify and sort spheres, prisms, cylinders, pyramids and cones	Recognise, identify and sort rectangular prisms, cubes, tetrahedrons and pyramids (i.t.o shape and number of faces, vertices and edges)	Recognise, identify and sort the 5 Platonic solids (i.t.o shape, number of faces, vertices and edges) and spheres and cylinders
Describe 3-D objects	Describe and compare spheres, prisms, cylinders, pyramids and cones	Describe and compare rectangular prisms, cubes, tetrahedrons and pyramids	Describe the properties of the 5 Platonic solids, spheres and pyramids
Create 3-D objects	Build spheres, prisms, cylinders, pyramids and cones from cut-out shapes and boxes etc.	Build rectangular prisms, cubes, tetrahedrons and pyramids from nets, straws and sticks	Build cubes, prisms, pyramids and cylinders from nets

	GRADE 3	GRADE 6	GRADE 9
Work with straight lines	Informally only as sides of some figures	Only as sides of some figures	Recognise, identify and define straight lines, rays, line segments, parallel and perpendicular lines
Work with angles	-	Recognise, identify and define angles: acute, right, obtuse, straight, reflex and revolution	Recognise, identify and describe relationships between angles formed by perpendicular lines, intersecting lines and parallel lines cut by a transversal
Solve straight-line geometric problems	-	-	Use angle relationships to solve geometric problems
Symmetry	Recognise lines of symmetry in geometrical and non-geometrical shapes	Recognise, draw and describe lines of symmetry in specified 2-D shapes	Recognise, draw and describe lines of symmetry in specified 2-D shapes
Work with transformations	-	Use transformations to make: ◦ composite shapes ◦ tessellations ◦ enlargements ◦ reductions and ◦ to describe patterns	Recognise, describe and perform transformations with points, line segments and simple geometric figures on a co-ordinate plane: ◦ reflection in x-axis or y-axis ◦ translation within or across quadrants ◦ reflection in the line y = x
Investigate with enlargements and reductions	-	Investigate only size and shape	Investigate effects on area and perimeter and co-ordinates of vertices
Position, orientation and views	Identify, match and find different views of the same everyday object	Identify, match and find different views of the same single or composite geometric object and related to viewer position	Mastered
Locate position	Find objects on simple informal maps	Locate positions on a grid or map using alpha-numeric references	Mastered
Use direction	Follow and give directions to move and change positions in classroom and school	Follow and give directions to move and change positions using alpha-numeric grid or map references	Mastered

Measurement (CAPS and the Conceptual Thread)

	GRADE 3	GRADE 6	GRADE 9
Measuring time	In days on calendar 12-hour time in hours, half-hours, quarter-hours and minutes	12-hour and 24-hour time in hours, minutes and seconds	Mastered
Calculating with time	° Passing time in hours, half-hours, quarter-hours and minutes ° Passing time in days, weeks and months	Time differences with calendars and time-zone maps	Mastered
Converting units of time	° Converting days and weeks ° Converting weeks and months	Converting between seconds, minutes, hours, days, weeks, months, years, decades, centuries	Mastered
Measuring temperature	-	In degrees Celsius with analogue and digital thermometers	Mastered
Calculating with temperature	-	Limited to positive whole numbers	Mastered
Measuring length	° Using informal units ° Using formal units of metres and centimetres	Of 2-D shapes and 3-D objects using millimetres, centimetres, metres and kilometres	Mastered
Comparing length	Using language – longer, shorter, taller, wider	Using measurements	Mastered
Estimating length	° Using informal units ° Using formal units of metres and centimetres	Of 2-D shapes and 3-D objects using millimetres, centimetres, metres and kilometres	Mastered
Calculating with length	Informally	Simple contextual problems with conversions	Mastered
Converting units of length	-	Converting between millimetres, centimetres, metres and kilometres	Mastered
Measuring mass	° Using informal units ° Using formal units of kilograms and grams	Of 3-D objects using kilograms and grams	Mastered
Comparing mass	Using language – light, lighter, heavy, heavier	Using measurements	Mastered
Estimating mass	° Using informal units ° Using formal units of kilograms and grams	Of 3-D objects using kilograms and grams	Mastered
Calculating with mass		Simple contextual problems with conversions	Mastered
Converting units of mass	-	Converting between kilograms and grams	Mastered

	GRADE 3	GRADE 6	GRADE 9
Measuring volume	Limited to capacity ° Using informal units ° Using formal units of litres, half-litres and quarter-litres (linking to standard cups and teaspoons)	Capacity and volume of 3-D objects using millilitres, litres and kilolitres	Mastered
Comparing volume	Using language – full, empty, more than, less than	Using measurements	Mastered
Estimating volume	Limited to capacity ° Using informal units ° Using formal units of litres, half-litres and quarter-litres (linking to standard cups and teaspoons)	Of 3-D objects using millilitres, litres and kilolitres	Mastered
Calculating with volume	-	Simple contextual problems with conversions	Mastered
Converting units of volume	-	Converting between kilolitres, litres and millilitres	Mastered
Determine perimeter	Investigate informally with string	Measure with ruler or tape measure	Calculate with formulae for polygons and circles
Determine area	Investigate informally with tiling	Measure with grid squares Derive formula of l x b for squares and rectangles	Calculate with formulae for polygons and circles
Determine surface area	-	Investigate relationship with volume	Calculate for cubes, rectangular prisms, triangular prisms and cylinders
Determine volume	-	Investigate formula of l x b x h for rectangular prisms	Calculate for cubes, rectangular prisms, triangular prisms and cylinders
Using Theorum of Pythagorus	-	-	To calculate unknowns in figures including right-angled triangles

Data-Handling (CAPS and the Conceptual Thread)

	GRADE 3	GRADE 6	GRADE 9
Collect data	◦ Collect simple data about class or school	◦ Collect local data using simple questionnaires	◦ Pose questions related to social, economic and environmental issues ◦ Select and justify: ◦ sources of data ◦ samples of data ◦ collection methods
Organise data	◦ Use lists, tally marks and tables	◦ Use tally marks and tables ◦ Order from smallest group to largest group	◦ Use tallies, tables, stem-and-leaf displays and group in intervals
Summarise data	-	◦ Use central tendencies of mode and median	◦ Use measures of central tendency (mean, median and mode) ◦ Use measures of dispersion (range and extremes)
Represent data	Draw: ◦ pictographs (limited to one-to-one correspondence) ◦ simple bar graphs	Draw: ◦ pictographs (many-to-one correspondence) ◦ bar graphs and ◦ double bar graphs	Draw by hand/ technology: ◦ bar graphs ◦ double bar graphs ◦ histograms (with given and own intervals) ◦ pie-charts and ◦ broken-line graphs
Interpret data	Answer simple questions from: ◦ pictographs and ◦ bar graphs	From: ◦ words ◦ pictographs ◦ bar graphs ◦ double bar graphs ◦ pie charts	◦ From a wide variety of representations including: ◦ histograms ◦ broken-line graphs and others ◦ Compare two sets of related data
Analyse data	Answer simple questions	Answer questions about: ◦ data categories and ◦ intervals ◦ data sources and contexts ◦ central tendencies (mode and median) even in ungrouped data	Answer questions about: ◦ data collection methods ◦ summary of data ◦ sources of error and bias

	GRADE 3	GRADE 6	GRADE 9
Report data	Verbally answer simple questions	Verbally and short written paragraphs to: ◦ draw simple conclusions ◦ make simple predictions	Written paragraphs to: ◦ draw conclusions ◦ make predictions ◦ make comparisons ◦ discuss error and bias ◦ choose appropriate summary statistics ◦ discuss role of extremes and outliers
Probability	-	◦ Perform simple repeated events ◦ Record outcomes ◦ Compare frequencies up to 50 trials	◦ For equal probable outcomes, determine probabilities for simple and compound events ◦ Predict relative frequency ◦ Compare and explain possible differences

Problem-Solving

	GRADE 3	GRADE 6	GRADE 9
Complete logic patterns	Complete simple logic patterns with one criterion	Complete simple logic patterns with two criteria	Complete logic patterns with at least three criteria
Use visual reasoning	Use simple visual recognition and identification	Use simple visual identification and discrimination	Use visual discrimination, analysis and interpretation
Use spatial reasoning	Use simple spatial recognition	Use simple spatial and directional discrimination	Use spatial, directional and orientational discrimination, analysis and interpretation
Use conditional reasoning	Use simple if … then conditional recognition	Use simple if … then conditional discrimination	Use conditional discrimination, analysis and interpretation
Use sequential reasoning	Use simple next-step sequential recognition	Use simple sequential thinking for up to three steps ahead	Use sequential planning, analysis and thinking for more than three steps ahead
Use abductive reasoning	Use simple abductive reasoning to make a likely hypothesis	Use simple abductive reasoning to make an testable hypothesis	Use abductive reasoning make a meaningful hypothesis
Use deductive reasoning	Use reasoning to make most simple and obvious deductions	Use simple reasoning to make plausible deductions	Use reasoning to make insightful deductions
Use inductive reasoning	Use reasoning to make a most simple and obvious generalisation	Use reasoning to make a plausible generalisation	Use reasoning to make meaningful generalisations
Use creative reasoning	Use simple creative reasoning to provide a possible solution	Uses creative reasoning to provide a plausible solution	Use creative reasoning to provide highly insightful solutions

APPENDIX C

1.3 Work out the following and write your answer clearly:

A. $44 + 21 =$

B. $\begin{array}{r} 905 \\ +\ 137 \\ \hline \end{array}$

1.4 Work out the following and write your answer clearly:

A. $44 - 21 =$

B. $\begin{array}{r} 905 \\ -\ 137 \\ \hline \end{array}$

1.5 Work out the following and write your answer clearly:

A. $13 \times 3 =$

B. $\begin{array}{r} 905 \\ \times\ 13 \\ \hline \end{array}$

1.6 Work out the following and write your answer clearly:

A. $56 \div 4 =$

B. $9\overline{)5094}$

1.7 Work out the following and write your answer clearly:

A. $13 - 2 \times 4 =$

B. $8 + 16 \div 4 =$

APPENDIX D

Basic Number Operations Test

Name: _____ **School:** _____

a. $4\,026 + 1\,273$

b. $2\,373 + 638$

c. $7\,752 - 1\,342$

d. $4\,121 - 218$

e. $1\,544 \times 20$

f. 353×35

g. $8\overline{)680}$

h. $12\overline{)14\,412}$

i. $55 + 33 \div 11$

j. $90 - 30 \times 3$

Extra Space for Working out:

APPENDIX E

QUESTION 2 – I Can ADD! **(10 marks)**

Work out the following and write your answer clearly:

2.1 27 + 3 =

2.2 24 + 64 =

2.3
$$\begin{array}{r} 74 \\ +\ 27 \\ \hline \end{array}$$

2.4
$$\begin{array}{r} 194 \\ +\ 748 \\ \hline \end{array}$$

2.5
$$\begin{array}{r} 80 \\ 9626 \\ 935 \\ +\ 8103 \\ \hline \end{array}$$

QUESTION 3 – I Can SUBTRACT! **(10 marks)**

Work out the following and write your answer clearly:

3.1 13 - 8 =

3.2 48 - 34 =

3.3
$$\begin{array}{r} 97 \\ -\ 78 \\ \hline \end{array}$$

3.4
$$\begin{array}{r} 405 \\ -\ 123 \\ \hline \end{array}$$

3.5
$$\begin{array}{r} 2542 \\ -\ 1703 \\ \hline \end{array}$$

QUESTION 4 – I Can MULTIPLY! **(10 marks)**

Work out the following and write your answer clearly:

4.1 $7 \times 8 =$

4.2 $13 \times 3 =$

4.3
$$\begin{array}{r} 95 \\ \times\ 6 \\ \hline \end{array}$$

4.4
$$\begin{array}{r} 88 \\ \times\ 21 \\ \hline \end{array}$$

4.5
$$\begin{array}{r} 2375 \\ \times\ 83 \\ \hline \end{array}$$

QUESTION 5 – I Can DIVIDE! **(10 marks)**

Work out the following and write your answer clearly:

5.1 $24 \div 3 =$

5.2 $48 \div 11 =$

5.3 $6\overline{)96}$

5.4 $7\overline{)509}$

5.5 $12\overline{)5088}$

APPENDIX F

1. **Numbers, Operations & Relationships**

	Working with Whole Numbers	Working with Fractions	Working with Decimals
Number Concept (identifying, comparing, ordering numbers)			
Number Representations (visual - graphic and symbolic)			
Place value			
Rounding			
Estimating			
Adding			
Subtracting			
Multiplying			
Dividing			
Mixed Operations			
Word Problem Solving			
Issues of Language & Terminology			
Working with Percentages			

APPENDIX G

1. Numbers, Operations & Relationships

	Working with Whole Numbers
Number Concept (identifying, comparing, ordering numbers)	9.3 10.3
Number Representations (visual - graphic and symbolic)	26.3
Place value	1.1 19.8 40.1 4.1 34.1 42.1 11.1 35.1 14.1 37.5 16.1 39.1
Rounding	9.1 40.10 13.1 41.1 23.1 40.8 40.9
Estimating	41.8 41.9 41.10 44.6 44.8
Adding	1.5 18.9 4.9 26.10 4.10 27.10 6.10 11.10
Subtracting	5.9 17.10 27.9 13.10 18.10 36.9 14.5 21.9 15.9 21.10 16.5 24.5
Multiplying	1.9 11.9 24.9 39.6 2.9 18.8 24.10 42.6 3.9 19.9 29.10 42.10 5.10 22.10 35.9 8.10 23.9 38.9
Dividing	1.10 10.5 22.9 31.10 2.10 14.10 23.10 34.5 3.10 15.10 25.9 35.7 7.10 19.6 26.9 45.10 9.5 19.10 31.6
Mixed Operations	2.5 20.10 26.5 4.5 21.8 27.5 5.6 22.5 30.6 13.7 23.5 40.7 17.5 24.1
Word Problem Solving	3.5 37.6 9.4 38.7 10.1 39.5 25.5 40.5 34.8 42.5
Issues of Language & Terminology	

APPENDIX H

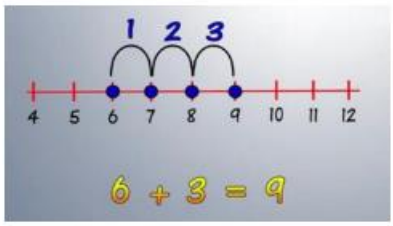
1. Numbers, Operations & Relationships

	Working with Whole Numbers
Number Concept (identifying, comparing, ordering numbers)	77.6 81.2 77.7 83.10 77.8 84.4 78.6 78.7
Number Representations (visual - graphic and symbolic)	
Place value	
Rounding	
Estimating	
Adding	81.4
Subtracting	
Multiplying	76.5 78.9 82.10
Dividing	78.10 81.8 85.8 81.9 83.4 83.5
Mixed Operations	76.6 81.5 81.6 81.7 82.5
Word Problem Solving	77.5 79.7 79.1 79.10 79.4 80.1 79.5 81.1 79.6 84.10
Issues of Language & Terminology	HCF 83.6 84.7 85.7

APPENDIX I

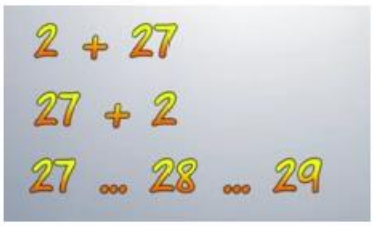
Number Strategies for Addition

Counting On



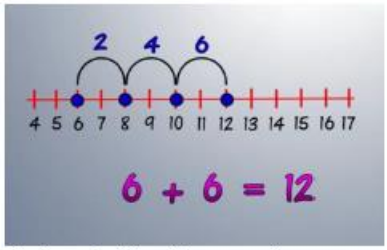
Count on from the larger number upwards to find the sum.

Commutating

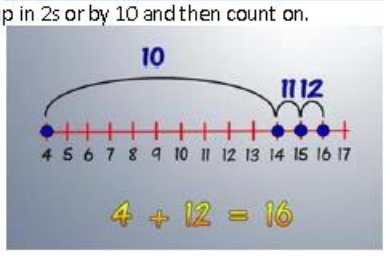


Move numbers so that those that are easier to add are next to each other then add.

Skip-counting

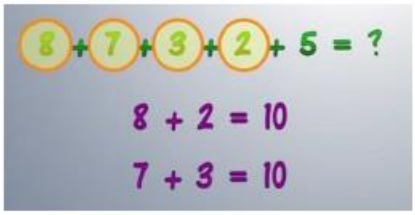


Skip up in 2s or by 10 and then count on.



Using Number Bonds to 10

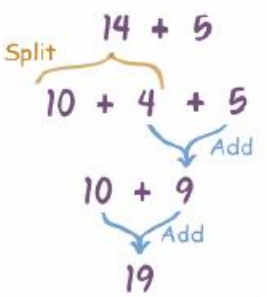






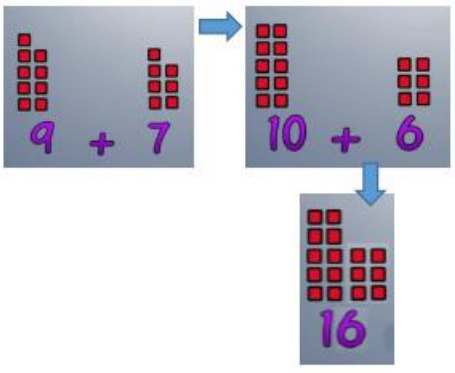
Move numbers that add up to ten next to each other before adding.

Decomposing



Break numbers into tens and units, add units and then add tens afterwards.

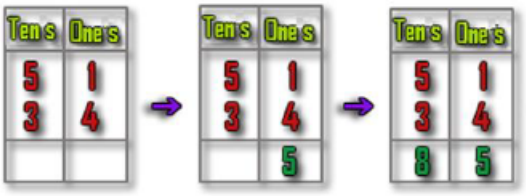
Rounding and Compensating



Round up or down to 10 and compensate by adding or subtracting to the other number then add.

Adding in Place-Value Columns (without regrouping)

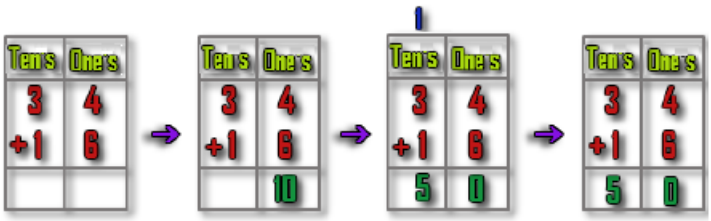
51 + 34 =



Write numbers below each other so that the tens and units of each number are in the same columns. Add the units and then add the tens separately to find the total.

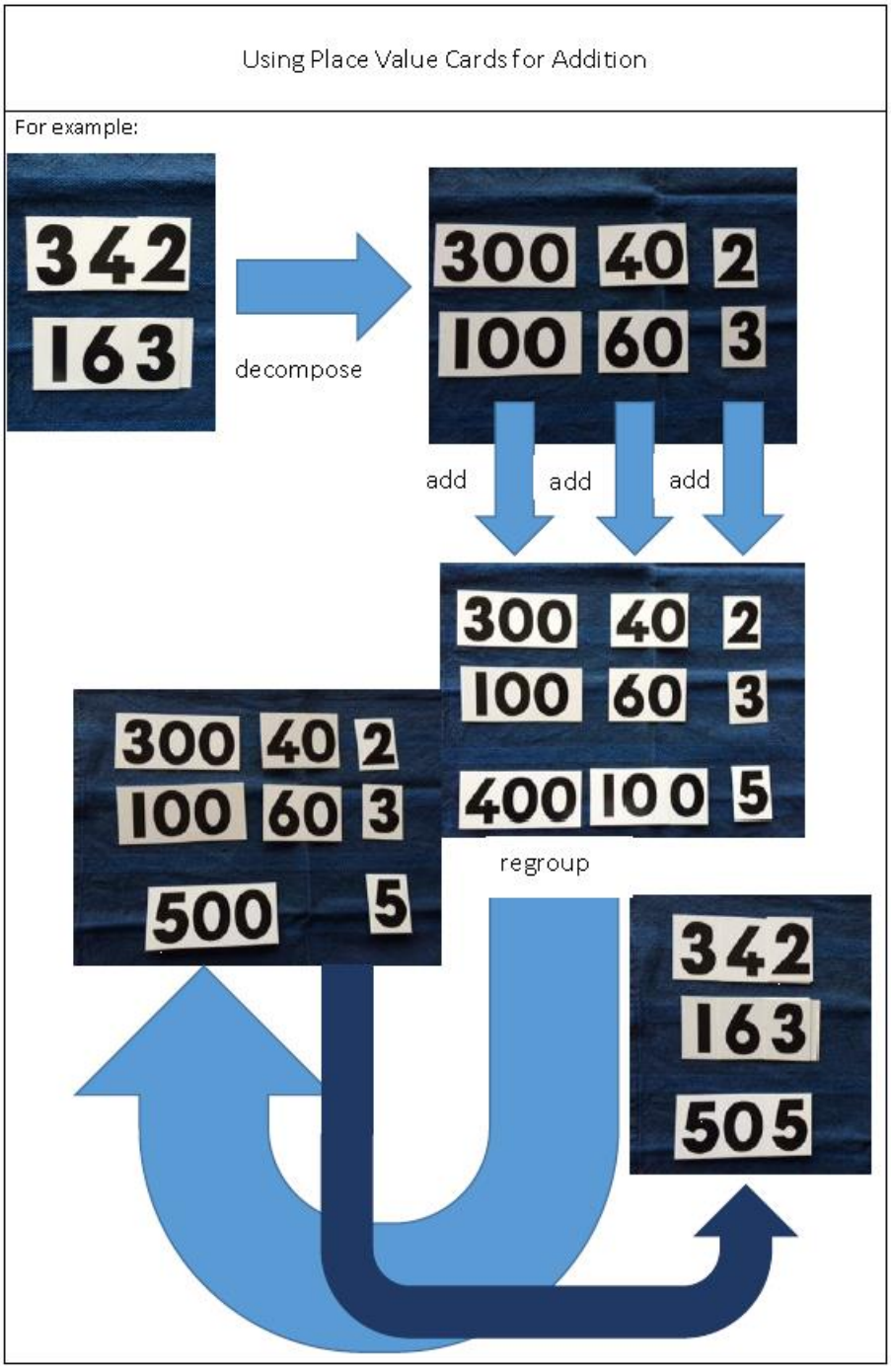
Adding in Place-Value Columns (with regrouping)

34 + 16 =



If adding the units results in a number greater than ten, make a new group of ten from this, add the group of ten to the tens column and record the remaining units in the units column.

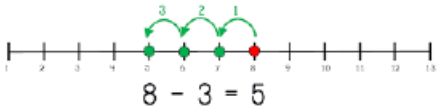
APPENDIX J


Using Place Value Cards for Addition
For example:
342
163
decompose
300 40 2
100 60 3
add
add
add
300 40 2
100 60 3
400 100 5
regroup
300 40 2
100 60 3
500 5
342
163
505

APPENDIX K

Number Strategies for Subtraction

Counting Back



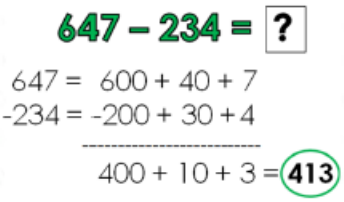
Count from the first number backwards to find the difference.

Skipping Back



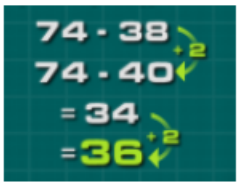
Count back in twos or tens and then in units to find the difference.

Decomposing (without regrouping)



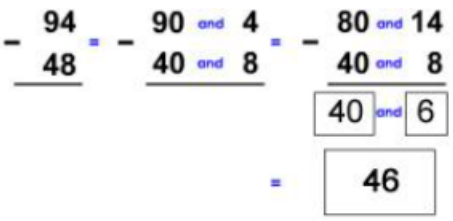
Break numbers into hundreds, tens and units, subtract units, then tens and then hundreds.

Rounding and Compensating



Round up or down to a multiple of 10 and compensate by adding or subtracting to the other number.

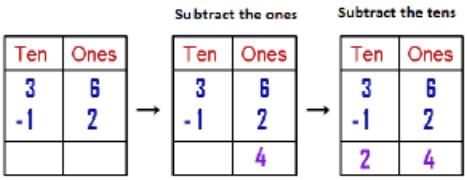
Decomposing (with regrouping)



Break numbers into tens and units. When there are not enough units to subtract from, regroup one ten as ten units and add to existing units. Now subtract units then subtract tens.

Subtracting in Columns (without regrouping)

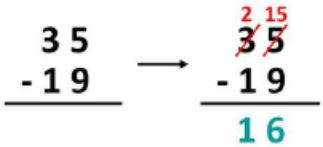
36 – 12 =



Write numbers below each other so that the tens and units (ones) of each number are in the same columns. Subtract the units and then subtract the tens to find the difference

Subtracting in Columns (with regrouping)

35 –19 =



Write numbers below each other so that the tens and units of each number are in the same columns. When there are not enough units to subtract from, regroup one ten as ten units and add to the existing units. Now subtract the units and then the tens to find the difference.

Regrouping across Zeros

4 000 – 2 896 =



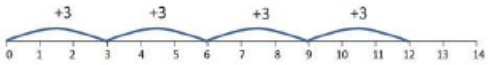
Subtracting numbers with zeros can be solved with sequential regrouping. Start with the non-zero digit and regroup each column to the right of this non-zero digit in order to subtract in each column as required.

APPENDIX L

Number Strategies for Multiplication

Repeated Addition

3 x 4 is the same as 3 + 3 + 3 + 3



Skip Counting

2, 4, 6, 8, 10, 12
3, 6, 9, 12, 15, 18
5, 10, 15, 20, 25, 30
10, 20, 30, 40, 50, 60
100, 200, 300, 400, 500

Counting up in equal groups.

Doubling and More

24 x 2 → Doubling
21 x 4 → Double, double again
24 x 8 → Double, double and double again

Shifting Place Value

204 x 10 → Shift place value once to make 10X larger
32 x 100 → Shift place value two places
43 x 1000 → Shift place value three places
74 x 5 → X 10 and halve

Partitioning (for partial products)

36 x 3

→ 30 x 3 = 90 → 90 +
6 x 3 = 18 18
108

Decompose numbers and multiply each part by each other and add partial products for a full product.

Rounding and Compensating

36 x 3

→ 36 + 4 → 40 x 3 = 120 –
→ 4 x 3 = 12
108

Round up or down, multiply this number and multiply the rounding difference and then subtract or add the rounding product.

Using Area Models

36 x 12

	30	+ 6
10	30 x 10	6 x 10
+ 2	30 x 2	6 x 2

30 x 10 = 300 +
30 x 2 = 60 +
6 x 10 = 60 +
6 x 2 = 12 +
432

This visual representation of multiplication facilitates understanding of partitioning and partial products for a full product.

Using Arrays



4 x 6 = 24 6 x 4 = 24

This visual representation of multiplication facilitates understanding of commutative properties and factors of a product.

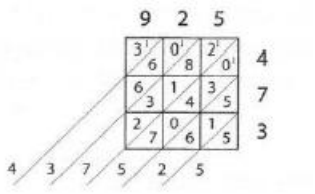
Using Grids

X		132		
		100	30	2
452	400	40 000	12 000	800
	50	5 000	1 500	100
	2	200	60	4

40 000 + 12 000 + 5 000 + 1 500 + 800 + 200 + 100 + 60 + 4 = 59 664

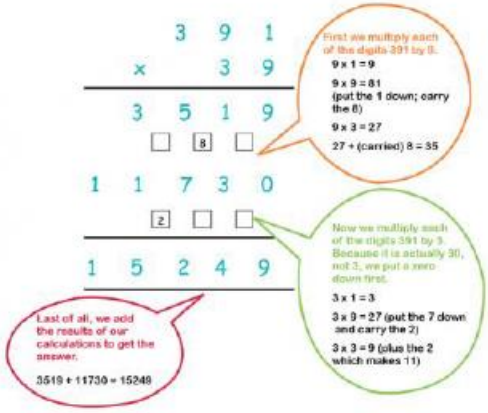
This visual representation of multiplication focuses partitioning on using place value and partial products for a full product.

Using lattices



Explaining how multiplication lattices work is an advanced learning strategy.

Long Multiplication Algorithm

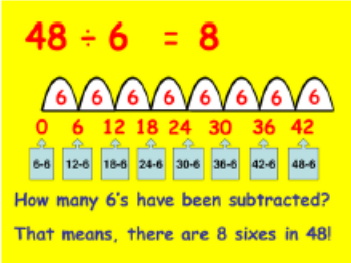


9 x 1 =	9+
9 x 90 =	810+
9 x 300 =	2 700+
30 x 1 =	30+
30 x 90 =	2 700+
30 x 300 =	9 000+
	15 249

APPENDIX M

Number Strategies for Division

Repeated Subtraction



Skip Counting

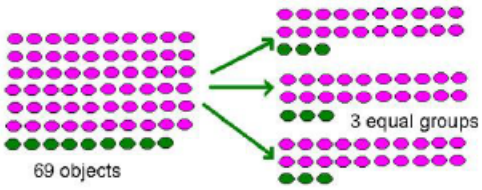
30 / 5 = 6

5, 10, 15, 20, 25, 30

1 2 3 4 5 6

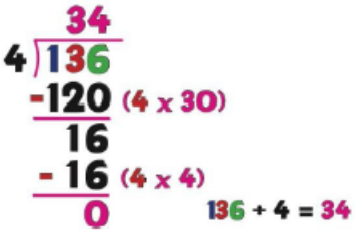
How many times does 5 divide into 30? Count in five on fingers: 5, 10, 15, 20, 25, 30. How many fingers are counted? 5 divides into 30 six times.

Equal Partitioning



Dividing 69 into three equal sized groups shows 23 in each group. Therefore 69 divided by 3 equals 23.

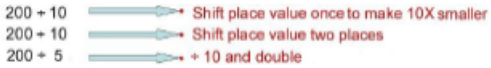
Chunking



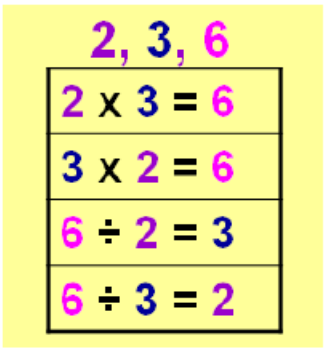
Halving and More



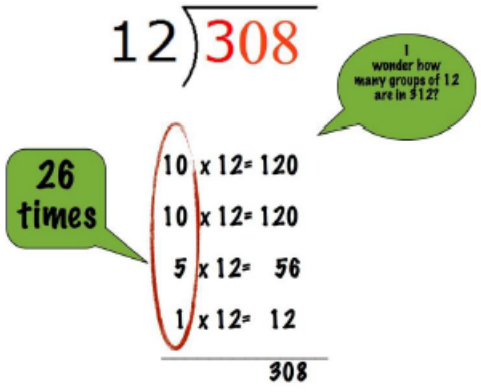
Shifting Place Value



Inverse Times-Tables (aka Fact Families)



Multiplying Up



Long Division Algorithm

$$
\begin{array}{r}
1813 \\
13\overline{)\,23576} \\
13 \\
\hline
105 \\
104 \\
\hline
17 \\
13 \\
\hline
46 \\
39 \\
\hline
7
\end{array}
$$

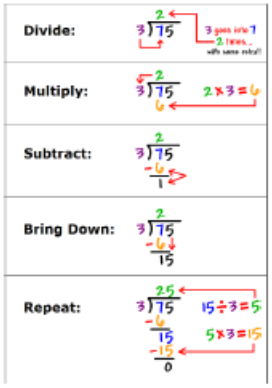
$23 = 1 \times 13 + 10$

$105 = 8 \times 13 + 1$

$17 = 1 \times 13 + 4$

$46 = 3 \times 13 + 7$

13 divides into 23576, 1 813 times with a remainder of 7 left over.



APPENDIX N

Analysing my Numeracy Skills

Name: _____ **School:** _____

On a scale of 0 to 10 (where 10 is a perfect score) please rate your own numeracy skills by circling the number on the scale which best reflects your ability to perform the following operations.

I can add

0 1 2 3 4 5 6 7 8 9 10

I can subtract

0 1 2 3 4 5 6 7 8 9 10

I can multiply

0 1 2 3 4 5 6 7 8 9 10

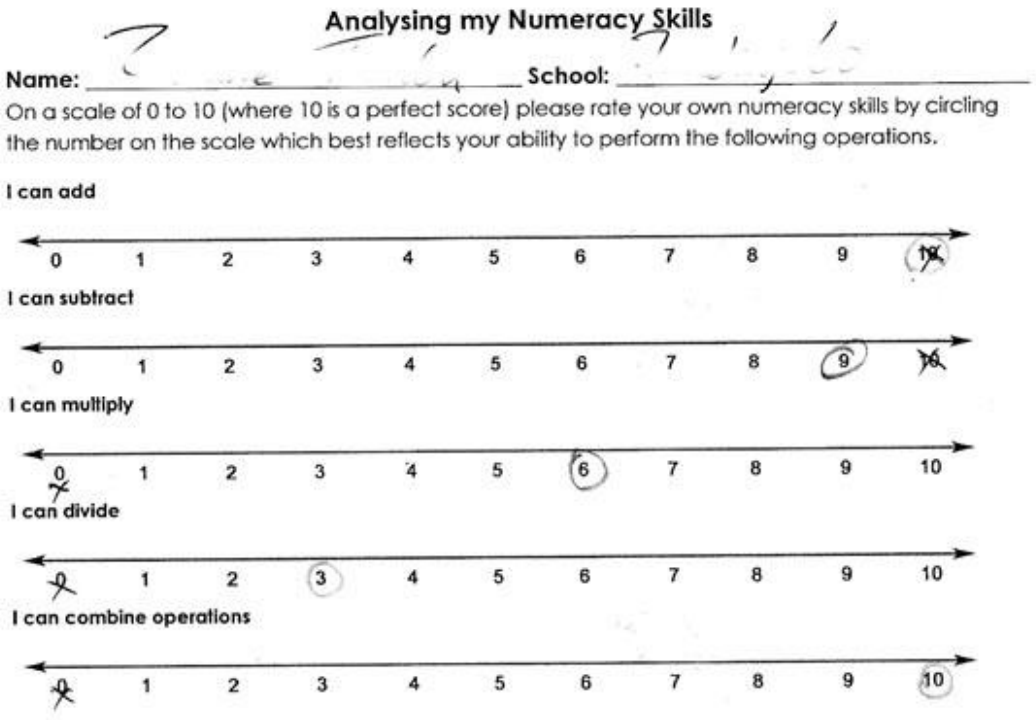
I can divide

0 1 2 3 4 5 6 7 8 9 10

I can combine operations

0 1 2 3 4 5 6 7 8 9 10

Sample showing self-assessment (pencil-drawn circles) and real achievement (pen-drawn crosses)



APPENDIX O

FOUNDATIONS IN MATHEMATICS

Self-Evaluation Activity

Name: _____

School: _____

1. Numbers, Operations & Relationships

	Working with Whole Numbers	Working with Fractions	Working with Decimals
Number Concept			
Representations			
Place value			
Rounding-off			
Estimating			
Adding			
Subtracting			
Multiplying			
Dividing			
Mixed Operations			
Problem-solving			
Language & Terminology			
Working with Percentages			

APPENDIX P

	My STRENGTHS to build on	My WEAKNESSES to fix
1.Numbers, Operations & Relationships		
2. Patterns, Functions & Algebra		
3. Space & Shape		
4. Measurement		
5. Data-handling		

APPENDIX Q

What have you learnt from this activity?

What can you do to build on your strengths?

What can you do to fix your weaknesses?

THE END