AN ANALYSIS OF THE AFFORDANCES FOR LEARNING PROBABILITY IN A COMMONLY USED MATHEMATICAL LITERACY GRADE 12 TEXTBOOK

Sthembile Lebohang Magwaza

(588096)

A research report submitted to the

WITS SCHOOL OF EDUCATION

At the

University of the Witwatersrand,

Johannesburg,

in partial fulfilment of the requirements for the degree of

Master of Education

Supervisor: Professor Anthony Essien

Declaration

I, **Sthembile Lebohang Magwaza,** declare that this research report is my own work, except as indicated in the acknowledgements, the text and the references. It is being submitted for the Degree of **Master of Education** at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

18 March 2024

___SL Magwaza_____ _____

Signature Date

Acknowledgements

Dear heavenly Father. Unto you who can do exceedingly above all we ask or think. To the only wise God be glory in all generations.

I want to start by appreciating my supervisor, Professor Anthony Essien. You have been supportive from the inception of my research. You offered your expertise and motivation through this tough journey. I appreciate all the guidance I received. May God continue to bless you and your family and may he extend his mercy upon you.

To my parents, thank you for being my biggest cheerleaders. Thank you for believing in me and naming me *Sthembile* meaning *you have faith in me*. Thank you for the prayers, I would not have done it alone. Thank you for teaching me about God, I have faith because of you.

To my partner, Sanele Ndlovu, thank you for understanding my journey. Thank you for your constant words of encouragement. Thank you for the critique that helped me to think objectively. Most of all thank you for loving me.

To my sister, Mbali Tlhale, thank you for the words of encouragement. Thank you for the venting sessions that allowed me to express my deepest emotions. Thank you for reminding me to always get up and try again.

To Maskew and Miller, I want to extend my heart-felt gratitude for granting me permission to use the textbook in this study. The insights and information provided in the book have been invaluable to my work.

Abstract

The purpose of this study was to explore the examples that are used to present the topic of probability in a commonly used Mathematical Literacy textbook (Platinum Mathematical Literacy) in order to explore the affordances for learning for that are made available to users of this textbook. In this research report, I combine two frameworks, namely *the framework for mathematics textbook analysis and the Variation Theory*. The Dyadic framework for comprehensive textbook analysis is a result of a combination of the two frameworks. This framework was used to qualitatively analyse the chapter on probability in the Platinum Mathematical Literacy Grade 12 textbook, a textbook commonly used in many South African schools. The results of this study showed that the structure [general structure or example space] should be set out with the intention of exposing the critical features of the object of learning. The study further provided guidelines for textbook analysis to use to select a textbook that will afford learners optimum opportunities to learn.

Key Words

Mathematical Literacy; Variation Theory; Mathematics; Mathematics Education; Teaching; Learners; Textbook Analysis; Probability.

Abbreviations/Acronyms

CAPS Curriculum and Assessment Policy Statement

DoE Department of Education

FET Further Education and Training

ML Mathematical Literacy

NSC National Senior Certificate

OBE Outcomes-Based Education

TIMSS The Third International Mathematics and Science Study

Table of Contents

List of Figures

List of Tables

CHAPTER 1: Introduction

1.1. Background to the Study

In my own teaching practice, I have found it challenging to teach learners in the absence of textbooks. Even though in some instances, the textbook acquired by the school may not provide the best support in all the topics that have to be covered, the presence of a textbook with relevant content makes a difference in both the lesson preparation and delivery. For Taylor and Vinjevold (1999), textbooks provide a 'systematic learning programme'. The systematic programme highlights the element of consistency of the content knowledge that is 'made available' or rather, accessible in classes that have textbooks (Taylor & Vinjevold, 1999).

In the year 2015, when I started practising as a teacher, the head of the Mathematics department at the school where I was working, allocated a Grade 10 Mathematical Literacy class to my timetable. At the time, my area of specialisation was Mathematics and Life Sciences, and I had no experience or exposure to Mathematical Literacy. Throughout that year, I relied on textbooks and other materials to prepare for lessons. Textbooks were the most reliable guide since they consisted of examples and explanatory texts that acted as guidelines to prepare for lessons. '*Most teachers in South African schools face the same problem since most universities do not offer Mathematical Literacy as a subject of specialisation*' (Bansilal, Webb & James, 2015, p. 8). Some teachers specialise in teaching Mathematics, however, when they are appointed in schools, they need to also teach Mathematical Literacy.

Studying the nature of classroom practice remains necessary, especially within the changing dynamics in education globally. Acknowledging the implications that the global pandemic has had on teacher and learner interactions is essential to understanding the teaching and learning issues that have emerged over this period. A textbook is an essential tool that facilitates the learning process in and out of the classroom (Basar et al., 2021). It offers a widespread range of information, different types of examples and activities that are represented in multiple formats (O'Keeffe, 2012). Moreover, textbooks illustrate representations and explanations that are pertinent to understanding mathematical topics (Ceretkova & Sedivy, 2008). In essence, textbooks mediate the curriculum from the intended to the implemented and finally the attained curriculum (Valverde et al., 2002).

Using textbooks as a resource for learning is advantageous as they can be used in and outside of classrooms. This is why continuous exploration of textbook quality is a significant and imperative research concept. Peppin and Haggerty (2001, p.74) argue that '*Concerns have been expressed about the quality of the textbooks, the way they are written, and their persuasive influence*'. This statement justifies the significance of purposefully selecting textbooks to improve the prospects of the learners to learn for understanding.

TIMSS also known as the Third International Mathematics and Science Study has provided a framework that can be used for the analysis of Mathematics textbooks (Valverde et al., 2002). The objective of the framework is to offer an analytical tool that is used to examine what textbooks offer its users. This framework can be used to determine what constitutes the structure of a textbook that gives the most support in the attainment of knowledge (O'Keeffe, 2012, p.4). The findings of the study by Valverde et al. (2002) focus on the nature of the content and the structure used to present the content. With particular reference to analysing the structure of textbooks, Valverde et al. (2002) also consider aspects like the layout of the content through definitions, examples, diagrams and explanatory texts.

Textbooks aim to translate the ideas and concepts of curriculum policy into guidelines and information that can be accessible to teachers and learners (Valverde et al., 2002). The advantage of having a textbook is their mobility if they are accessible in schools. In that, way learners and teachers may use the textbooks in class or at home to engage with the content. This statement does not imply that explanatory talk is not important, since it gives meaning to the concept displayed by the textbook (Adler & Ronda, 2017). Instead, it emphasises the need to ensure that textbooks are accessible in South African classrooms.

The presence of textbooks in South African schools is a significant indicator of developmental progress; multiple events have occurred in South Africa that justify this statement. Leshota (2015) refers to a particular political event that transpired in the year 2012, in the Limpopo province. This feud arose because parents were frustrated by the lack of delivery of textbooks to the provincial schools since the beginning of that academic year. Leshota (2015, p.3) proceeded to argue that '*Textbook delivery has since become a burning issue in South Africa, with government called upon to account for this matter continuously*'.

1.1.1. Choosing Mathematical Literacy

In 2006, Mathematical Literacy was first introduced as an alternative subject for South African learners who did not want to register for Mathematics in the Further Education and Training (FET) phase, which is Grade 10 to 12 (Graven & Venkat, 2007). This introduction was due to factors that were identified and could possibly be remediated by introducing this alternative subject. According to Bansilal et al. (2015), Mathematical Literacy is a developing subject when compared to other subjects in the country's curriculum; hence, it remains imperative to understand the subject's purpose and contribution in the pedagogic and learning experience.

There are some similarities and differences between Mathematics and Mathematical Literacy in their structures; the extent of integration between the subject and the real world is the main difference (Gal, 2009). By level of integration, we refer to the context-embedded questions. Therefore, Mathematical Literacy is regarded as having a greater integration than Mathematics (Mbekwa & Jullie, 2009). The level of integration can be attributed to Mathematical Literacy being aimed at teaching for practical relevance and applications (Graven & Venkat, 2007).

The number of learners who register for Mathematical Literacy has increased over the years, compared to those who register for Mathematics at Grade 12 level (Department of Education [DoE], 2018). This growth is attributed to concerns similar to those that resulted in the introduction of Mathematical Literacy in South African schools (DoE, 2006). Some of these concerns were the poor national results in Mathematics and second, the remains of the Outcomes-Based Education (OBE) curriculum that came before the year 2005. At the time it was not compulsory to register for Mathematics in the FET phase, thus only a few learners opted to register for Mathematics (De Clercq, 2020).

Since 2006, it has become obligatory for learners in South African schools to register for either Mathematical Literacy or Mathematics because basic calculating skills needed for the world of work remain essential (Jojo, 2019). Mathematical Literacy can be regarded as a tool that can be used to redress the past political differences that have existed in South Africa; this is done through increasing the number of learners who obtain a matric qualification (Jojo, 2019). This is done through reducing the number of subjects that learners are struggling with, such as Mathematics, while still offering learners the necessary mathematical skills (Jojo, 2019).

Mbekwa and Jullie (2009) have explored some of the research that has been conducted on Mathematics textbooks. In the paper, they classify the research into the following categories: representability, correctness of mathematical content, affordances of access to Mathematics and textbooks-in-use, and learner performance (Mbekwa & Jullie, 2009). If this research was to be classified according to Mbekwa and Jullie (2009) it would lie within the category of the affordances of access to Mathematical Literacy. This is interesting since this kind of research has not been conducted within Mathematical Literacy textbooks.

1.1.2. Selecting the topic probability

For this investigation, the purpose was to analyse the topic of probability within a Grade 12 Mathematical Literacy textbook. In the preceding section, I elaborate on this decision. Within the national curriculum statement of South Africa, probability constitutes around 8.6 % of the total workload that is to be taught in Grade 12 Mathematical Literacy (DoE, 2019). Although it may seem as if it constitutes a small percentage of the overall annual teaching plan, it is a very important section. The statement below cited from the DoE (2008, p.11) describes the learning outcomes that probability is intended to inculcate:

> *...the learner is able to collect, summarise, display and analyse data and apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions*.

Concurring with this, the study by Taylor (2011) asserts that studying the concept of probability is essential as it helps with the development of mathematical reasoning for learners. It also allows for an integration of mathematics with the real world (Brijlal, 2014). The findings of the study stipulate that probability should be taught in schools from as early as in the Foundation Phase (Taylor, 2011).

The topic of probability is essential because it allows learners to capture the concept of randomness. To support this argument Taylor (2011) asserts that when children are younger, they lack the ability to deduce patterns, and that inability makes not it easy for them to understand what it means to have random events. The older children become, the more structurally orientated their brains become, and are constantly looking for patterns to deduce from

phenomena (Taylor, 2011). Through studying probability, learners attain reasoning techniques and strategies that help them to make satisfactory decisions in situations that occur daily and in professional circumstances where the likelihood of events can be identified (Batanero, et al., 2016).

In South African schools, Grade 12 (matric) is the highest grade of basic schooling. This is the last year before learners move to institutions of higher learning. Attesting to Taylor's (2011) argument stated in the previous paragraph, at Grade 12 level the expectation is that learners would have learnt multiple procedures that may have developed their pattern deduction abilities. This is why it is interesting to study the presentation of probability in a Grade 12 textbook.

The structure of Mathematical Literacy examinations is such that probability questions do not have a specific section; they tend to be included as follow-up questions in any section. The picture below displays the first question from a November NSC exam (Paper 1 of 2021). The main theme of this question is finance. By reading through the questions, it is notable that Question 1.1.5 in the picture (**Figure 1**) below is about probability within this context. The benefit of this type of questioning is that it emphasises the idea of probability being present in any context.

1.1 TABLE 1 below shows the fuel price (in US dollars) in six African countries on 05/06/2019 and 01/03/2021 with the exchange rate per currency on 01/03/2021.

TABLE 1: FUEL PRICE (IN US DOLLARS) IN SIX AFRICAN COUNTRIES

COUNTRY	FUEL PRICE IN US$		CURRENT DOLLAR EXCHANGE RATE/CURRENCY ON 01/03/2021
	05/06/2019	01/03/2021	
South Africa	1,04	1,061	R15,36
Angola	0,47	0,254	626,41 Angolan kwanzas
Zimbabwe	0,80	1,258	$1,258
Namibia	0,88	0,796	15,36 Namibian dollars
Swaziland	0,86	0,87	15,35 Swazi emalangeni
Botswana	0,84	0,732	11,14 Botswana pulas

[Adapted from www.globalpetrolprices.com]

Use TABLE 1 to answer the questions that follow.

1.1.1 Calculate the fuel price increase for Zimbabwe from 05/06/2019 to 01/03/2021. (2)

1.1.2 Write down the current exchange rate of the Botswana pula to the US dollar in the following format:

1 Botswana pula = … US dollars (2)

1.1.3 Identify the countries which showed a decrease in the fuel price from 05/06/2019 to 01/03/2021. (2)

1.1.4 Arrange, in descending order, the fuel price in US$ for 01/03/21. (2)

1.1.5 The probability of randomly selecting a country that is not South Africa is $\frac{5}{6}$.

Write this probability in decimal form rounded to THREE decimal places. (2)

Figure 1: *Representations of probability in a context*

Source: Mathematical Literacy November paper 1 (2021)

In **Figure 1** above, Question 1.1.5 is about the representations of probability and rounding off. The figure presents information that can be analysed through different lenses. However, what resonates with this study is the need to teach learners probability so that they see it as applicable within any context.

1.1.3. Exemplification

There are different types of examples used in Mathematical Literacy classrooms. These types of examples are worked-out examples, planned examples and spontaneous examples. Worked-out examples are usually found in textbooks and worksheets. The purpose of these examples is to offer enough guidance to the Mathematical procedure, such that it allows learners to understand the illustrated basic concept. Planned examples are those that teachers select when planning lessons. They serve the purpose of demonstrating the concept to learners. The last type of

examples are spontaneous examples that arise based on questions that learners raise during lessons (Zhu & Simon, 2012).

Zodik and Zaslavsky (2008) argue that it is important to be intentional about the selection of textbook examples, and the order in which these examples are presented to learners. This purposeful selection will allow teachers to ensure that the examples presented are mathematically accurate and not misleading. There are many intricacies at play when selecting appropriate examples for lessons. In textbooks, accuracy is very important since teachers also use textbooks as a source of examples.

1.2. The purpose of this study

The purpose of this study was to explore the examples that are made available on the topic of probability in a commonly used Mathematical Literacy textbook (Platinum Mathematical Literacy) to analyse and understand the affordances made available for users of this textbook.

The significant increase in the number of learners registered for Mathematical Literacy than Mathematics in South African high schools exposes the high demand for research that looks into ways of adequately teaching and learning Mathematical Literacy. This analysis was conducted to examine the structure of a Mathematical Literacy textbook and what the affordances f learning of probability. The results obtained from this study have been used to argue that it is important to evaluate the holistic structure of Mathematical Literacy textbooks.

The analysis in this research was conducted using the combined framework that has been deduced from the Theory of Variation (Lo, 2012), in conjunction with the framework for mathematics textbooks. The framework presented by Valverde et al. (2002) was conducted by TIMMS. It is important to note that the *Platinum Mathematical Literacy* Grade 12 textbook by Frith et al. (2013) is not informed by the above-stated frameworks, rather, this study intended to raise awareness of the purposeful selection of examples as proposed by the variation theory. Moreover, the aim of this study was not to disregard the textbook but to make teachers and learners aware of the possible learning strategies that may help to improve the understanding of concepts.

1.3. Research questions

The study was informed by the following research questions:

1. How is the topic of probability presented in a commonly used Mathematical Literacy textbook?

2. Which patterns of variation can be identified in the examples and activities found in the topic of probability?

3. What affordances for learning are made available through engaging with the textbooks?

The first research question seeks to analyse the structure of the chapter of probability as presented in the textbook. The second research question looks at identifying the patterns of variation within the examples and activities presented within each unit section of the analysed chapter. Lastly, affordances for learning were analysed through observing the both the structure and the presence/absence of the patterns of variation.

1.4. Conclusion

This concludes the introductory chapter of the research; where the background that informed the study and the purpose of this study was outlined. The following section outlines what the reader can expect to read when engaging with the chapters of this study.

1.5. Outline of research

Chapter 1: Introduction

This chapter outlined the main problem that the researcher identified as one that sparked the interest in analysing a commonly used textbook. In addition, this chapter elaborated on the background of the topic chosen for the study, its objectives and research questions.

Chapter 2: Theoretical framework and literature review

In the second chapter, the reader can expect the background of the study. This was mainly to conduct research that is informed by knowledge. The section of the conceptual framework expands on the theory that guided the study. This was done by explaining what the theory is about.

Chapter 3: Methodology

This chapter provided the details of how the study was conducted. The analytical framework offered a flow diagram that shows how each aspect of the analysis is connected to the others.

Chapter 4: Analysis of results

This chapter presented and interpreted the findings that were gathered and thus formed a comprehensive analysis of the study's results.

Chapter 5: Conclusion and recommendations

The concluding remarks address the implication of the information that has been derived through analysis in the previous chapter. In addition, this chapter constructed a summary of the analysis and presented the findings. The study's limitations and recommendations for future research were provided.

CHAPTER 2: Theoretical framework and literature review

2.1. Introduction

The focus of this study was to analyse the chapter on probability in a Mathematical Literacy textbook that is commonly used to teach Grade 12 in South African schools. This chapter comprises an explanation of the theoretical underpinnings of this study to address the research objectives. The theoretical underpinnings include the Variation theory (Lo & Chick, 2016) and the framework for Mathematics Textbook Analysis (Valverde et al., 20002). After discussing the two frameworks, the research provides justifications for the merger framework used in this study. The merger framework informed the methodology in Chapter 3 and the analysis in Chapter 4. These chapters are succeded by a review of the literature that supports the study. This literature review includes research on the nature of probability, the use of textbooks in the classroom, and exemplification and understanding Mathematical Literacy.

2.2. Theoretical framework

Two frameworks – the variation theory (Watson & Mason, 2006) and the framework for Mathematics textbook analysis (Valverde et al., 2002), inform this study. In what follows, the researcher engaged with both theories and thereafter, developed a justification for the need for a combination of both frameworks.

2.2.1. Variation theory as a lens

Marton and Booth (1997) originally proposed the variation theory as a strategy intended to support learners' understanding of mathematics by exposing the critical features of the object of learning to encourage their discernment of these features. Within a set of questions, variation provides a lens to determine what changes/ varies or what remains the same/invariant (Watson & Mason, 2006). The concept of variation allows teachers to purposefully draw learners towards focusing on the idea that the teacher intends the learners to '*master*' (Adler, 2007: xii).

This theory has been applied globally to improve the understanding of mathematics. To support this argument Haung and Li (2017) refer to Gu's (1994) and Huang's (2002) dissertation studies conducted in China using the variation theory. The study intended to '*increase student's mathematics achievement on a large scale*' (Huang & Li, 2017: 3).

Variation can be observed during lesson delivery when examples are presented to the learners, it can also be perceived in written texts, which can be used in and outside the class (Adler, 2007). Procedural variation and conceptual variation are concepts that also emanate from the context in which variation teaching occurs. They are dependent on the definitions that have been given by theorists, such as Kilpatrick et al. (2001) in mathematics education. The object of learning will determine the order in which questions are varied, whether is it about conceptual or procedural understanding.

Every lesson must have an object of learning. It '… *does not mean the same thing as "learning objectives" which point to the end process of learning*' (Mhlolo, 2013, p.3). It rather refers to the start of the process of learning; in this case, the focus is particularly on comprehending what the object of learning is (Mhlolo, 2013). Discerning the object of learning is largely dependent on how the teacher sets out the lesson to purposefully increase the learners' opportunities to learn (Lo, 2012).

An object has multiple features, and these features can be perceived in various ways (Essien, 2021). Focusing on particular features influences the perception of the object of learning (Essien, 2021). When planning to teach a particular object of learning, it is important to start by identifying the critical features to facilitate teachers' understanding and to help deal with learners' differences (Lo, 2012). In that way, the examples selected are purposeful and ordered. This purposeful selection leads to understanding the patterns of variation. 'To discern the critical features of novel situations, learners must experience for themselves certain patterns of variation and invariance of these features' (Lo, 2012). This statement explains the importance of the patterns on variation in lessons.

Four patterns of variation have been proposed by the variation theory. These patterns are contrast, similarity, generalisation and fusion (Marton & Booth, 1997). Each of the patterns is explained below:

Explanation of the patterns of variation

- *Similarity* is the decision to keep certain things constant while varying others (Watson & Mason, 2006). It can also be explained as refined contrast or the separation of the critical features. This can be identified as the teacher selectively draws learners' attention to the sameness of the set of sequences (Lo, 2012).

An example of similarity in probability is keeping the numerator the same and varying the denominator i.e. $\frac{1}{2}; \frac{1}{3}; \frac{1}{4}$. The increasing denominator shows an increase in the possible outcomes. However, keeping the numerator constant allows learners to understand how the value of probability is increased or reduced by the denominator.

- *Contrast* allows learners to see the object by grounding this judgement on an observation of what the object cannot be (Lueng, 2012). To discern contrast, the critical features of one object must be compared with the critical features of other objects. An example is comparing the minimum conditions for an object to be a square by looking at what is necessary 'critical' when comparing the features of a square in relation to the critical features of a rectangle.

- *Fusion* is the integration of critical features. In this pattern, there is more than one aspect that varies at a particular time. Fusion ought to come after contrast or separation because these two focus on individual features while fusion occurs when learners' attention is on more than one feature (Watson & Mason, 2006). Using a tree diagram is an example of fusion in probability, through visually representing possible outcomes and using numerical judgements to calculate probability and done simultaneously.

- *Generalisation* is an activity of making a conjecture based on the pattern that has been identified through multiple examples that may be done using any one of the above-mentioned patterns of variation. In essence, it is a shift from the particular to the general (Mhlolo, 2013).

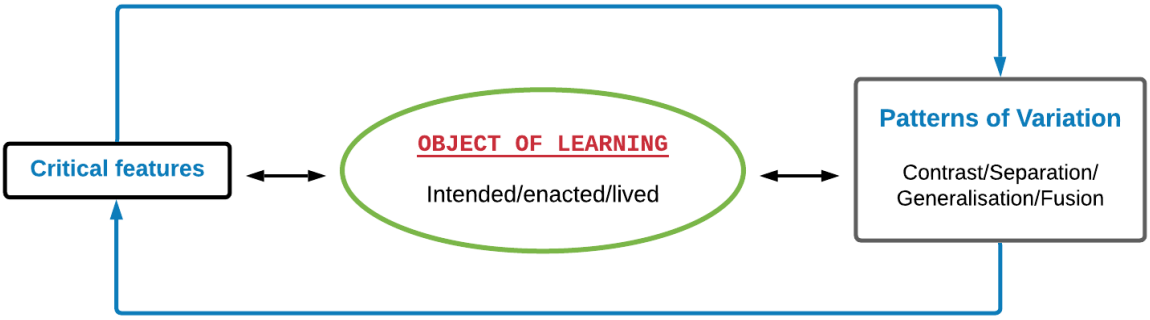


Figure 2: *The inter-connection between various concepts in the variation theory*

Source: Essien (2021p. 4)

Figure 2 above represents the core tenets of the theory of variation (Essien, 2021). These tenets include the object of learning at the centre, critical features on the left and the patterns of variation on the right. The object of learning is placed at the centre because it is what the teacher intends for the learners to understand. Essentially, it is the focus of the lesson. According to the theory of variation, the teacher needs to identify the critical features of the subject matter, these features should influence how the teacher may choose to vary the examples given (Lo & Chick, 2016). The patterns of variation are a description of how the critical features of this object of learning are varied.

2.2.2. The framework for mathematics textbook analysis

The variation theory enables the analysis of how the object of learning can be exposed through varying the critical features according to the different types of patterns of variation. What it does not do is provide a framework that engages with the layout of mathematical content through a *general structure*. The textbook analysis framework by Valverde et al. (2002) focuses on this and is therefore brought in to complement variation theory in my study.

The purpose of using the framework for mathematics textbook analysis in this study is to bridge the gap between what O'Keeffe (2012) refers to as intended and implemented curriculums. The framework for mathematics textbook analysis considers the role of the textbooks in education before expanding on how to analyse them. Figure 3 from Valverde et al, (2002) shows the

relationship between the intended, potentially implemented, implemented, as well as the attained curriculum.

- The intended curriculum comprises what is set by policy planners at the national level. This includes the specific intentions and goals that are set, based on time and topic sequence.
- The potentially implemented curriculum looks at the textbooks and other materials that are used as support for the build-up of knowledge. They are regarded as potentially implemented because not all people implement them in the same way. This is where we locate textbooks.
- What is actually implemented is the strategies, activities and practices that are used in the actual teaching of concepts in lessons.
- All these features lead to the knowledge that is finally attained by the learners, and the ideas and principles that are attained through this process.

2.2.2.1. Textbooks and tripartite model

Textbooks are a relatively small feature within the broader education system. Although they seem small, they have a very significant role in ensuring that the objectives of the education department are made accessible to the learners in the classroom (O'Keeffe, 2012). According to the tripartite model (**Figure 3**) textbooks have the potential to provide access to knowledge, ideas and constructs and achieve the curriculum (Valvelde et al, 2002). This means that textbooks are resources that need to be used by curriculum implementers (teachers) to ensure the thorough delivery of the content.

The tripartite model in (**Figure 3**) below locates the implementation strategies that aid the curriculum attainment. For the purposes of this research, the variation theory is identified as a learning strategy that can aid the attainment of knowledge. Marton and Booth (1997) originally proposed the importance of the discernment created by Variation. This discernment of the critical features of the object of learning is an essential part to learning (Lo, 2012). It assists in the navigation from intended toward the attained knowledge.

This tripartite model helps us to locate this study in the greater scheme of the education context. Since this study is on the analysis of textbooks through the lens of variation, the integration of the frameworks used shows an interest in developing ways to critically analyse learning material.

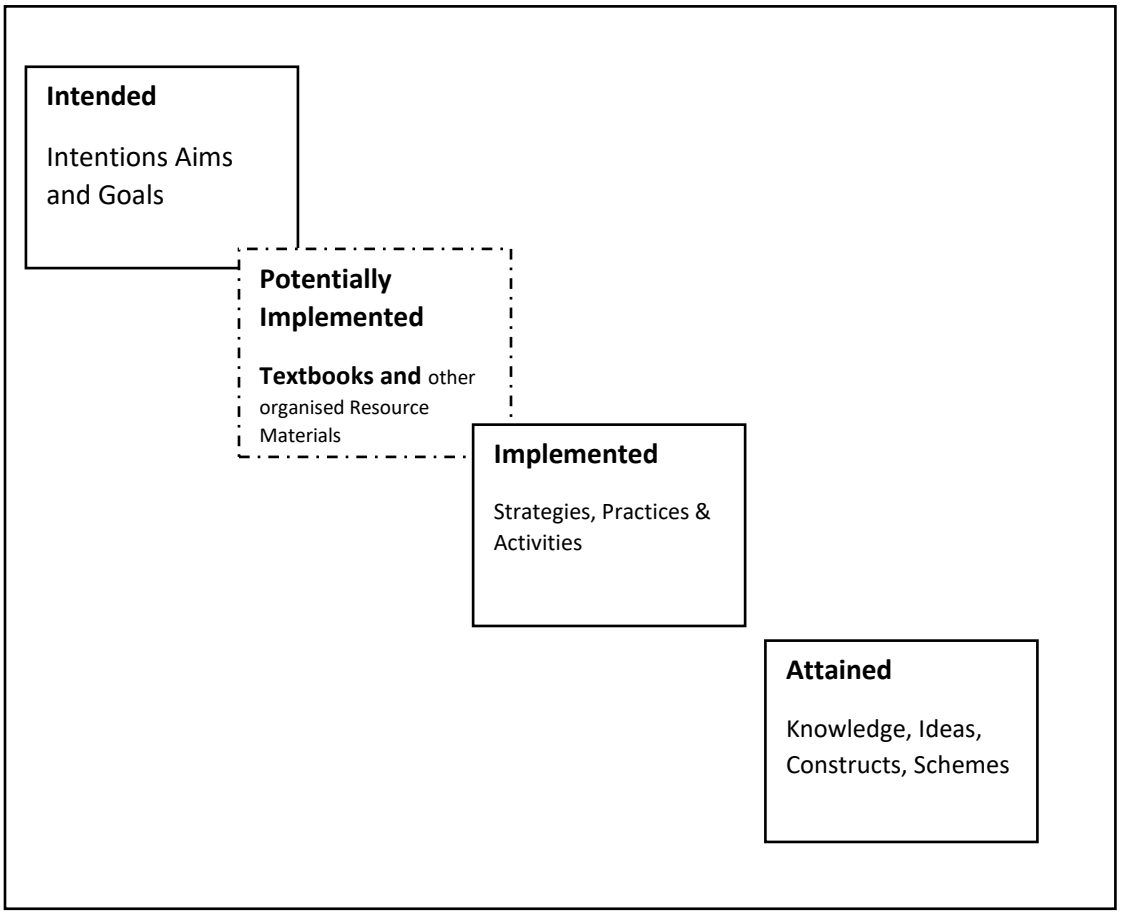


Figure 3: *Textbook and tripartite model*

Source: Valverde et al. (2002: 13)

In **Figure 3** above, the flow diagram represents the process through which the Department of Education aims to ensure that learners have access to education. This diagram is not limited to mathematics education but rather includes all subjects within the curricula of the country and helps position textbooks and teaching resources. With particular reference to the context of the curricula in South Africa, the intended goals are documented within the Curriculum Statement. From this, annual teaching plans are provided that guide teachers according to indicated times. Understanding the role of textbooks ensures that the analysis is envisioned to afford learners the best opportunities to learn.

2.2.2.2. Textbook analysis

Over the years, some researchers further refined the framework of the analysis of Mathematics textbooks (O'Keeffe, 2011, p.5) which was to '*allow for a finer grained analysis*'. The focus remains on these three main tenets, namely the structure, content and performance expectations:

- *Structure:* Is the sequence or the layout of the text. The order in which presentation formats are arranged creates a pattern that informs how a person engages with the text. The structure can be analysed by evaluating the general approach used within the textbook (Ensor et al. 2002). Quasi-deductive and quasi-inductive are the two types of approaches presented in the theory.
    - Quasi-deductive approach occurs when the textbook or teacher uses examples and definitions before giving the learners activities to engage in or apply the information they have attained (Leshota, 2017).
    - Quasi-inductive approach, on the other hand, is the direct opposite of quasi-deductive. It occurs when learners work by investigative means to make a general conjecture or discover the theory that has informed the activity (Leshota, 2017).

When analysing the structure, the focus is on presentation formats that have been purposefully included in the text to make meaning. The types of formats found in mathematics textbooks are pictures, graphs, tables, activities, information boxes and more.

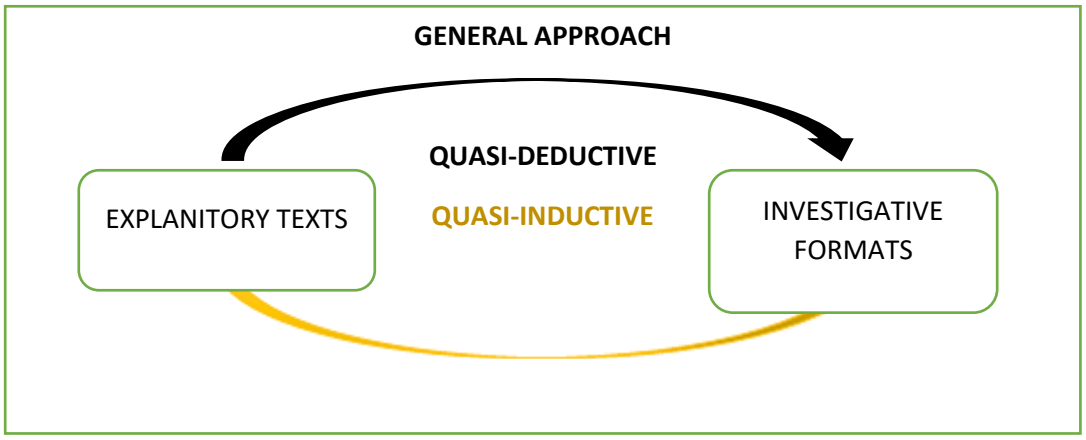


Figure 4: *The general approach used in textbooks*

Source: Ensor et al. (2013)

Figure 4 above represents the general approach used by textbooks. The black arrow describes the pathway of the quasi-deductive approach. This emphasises that explanations and definitions are the foundation of learning. The examples and exercises that follow are dependent on the definitions that are initially provided. The yellow arrow describes the quasi-inductive approach. In the deductive approach '*Activities lead to definitions and from this point opportunities may be provided for students to practice* (Ensor et al., 2013, p.23):

- *Content:* is informed by the intended curriculum. In South Africa, the production and use of textbooks are influenced by the approved National Curriculum Statement. Since the year 2013, the CAPS has been utilised across the country and textbooks have since been aligned with this statement. The National Statement gives clear directions as to the time and the sequencing of the topics and their subtopics. In analysing the content within textbooks, such information has to be considered.

- *Performance expectation:* '*… are embedded throughout the textbook and will impact significantly on how students choose to deal with the topics presented*' (O'Keeffe, 2012: 7). In dealing with the presentation formats, there is an intended goal that learners are supposed to attain after engaging with the task (Valvelde et al., 2002, p.125).

  This intended goal for learners essentially means that performance expectations are the actions that learners will perform based on the presentation format. This is because these actions lead to the attainment of particular skills.

2.2.3. A merger framework for a comprehensive analysis of textbooks

The comprehensive textbook analysis framework is an integration of two frameworks that focus on important features of a textbook. In the variation theory, examples are the main feature that outlines the content, while in the framework of mathematics textbook analysis; the broader presentation formats are important which focus on the broader structure of the content. By referring to this framework as a comprehensive merger, it is informed by the observation that the two original frameworks focus on two important elements that are imperative in the analysis of textbooks. These elements complement each other to ensure a thorough comprehensive analysis of textbooks.

The similarity between the two combined frameworks is that the *object of learning* in the variation theory has similar attributes to that of the *intended curriculum* in the framework for mathematics textbook analysis. According to the variation theory, the object of learning is the knowledge that is to be taught to learners (Lo, 2012). These are the goals and aims as set by the policies that assert the suitable content in relation to the grade or stage of learning (Ensor, 2013). Textbooks are documents used to display the object of learning. Therefore, through integrating the two frameworks, the object of learning and the intended curriculum have is common attributes.

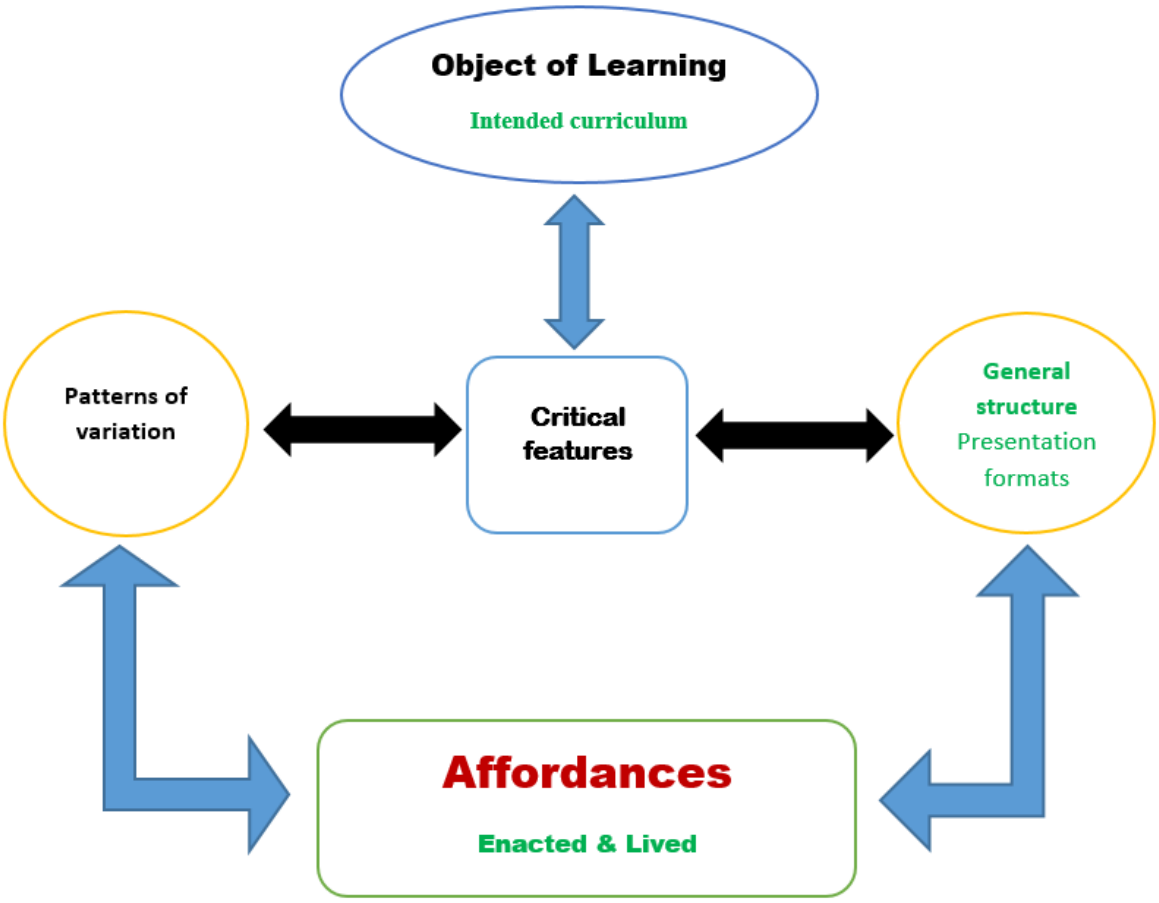


Figure 5: *Dyadic framework for a comprehensive analysis of textbooks*

The critical features are at the heart of the dyadic framework since they inform the general structure of the textbook as well as the examples used. The affordances for learning can be deduced from engaging with a particular structure; and discerning the critical features through the example space used in the textbook.

2.2.3.1. How the frameworks were used in this study

As indicated in Chapter 1, this study aimed to analyse the Platinum Mathematical Literacy textbook to identify the learning opportunities that emerge when learners engage with the textbook. The connection between the different components has been indicated with arrows which can be interpreted either from the object of learning or from the affordances for learning.

The *Platinum Mathematical Literacy* textbook was initially designed and published in 2013. All the contents of the textbook are purposefully included; therefore, it is more appropriate to analyse the textbook from the angle of the object of learning. By this statement, the researcher implies that in this study the framework is interpreted from the object of learning. This allows for an interpretation that is not biased towards any particular objective.

When creating a new textbook or material that will be used for mathematical literacy this framework can be used by starting with the affordances. This means that the structure of the document will be informed by the end goal which is the affordances that will be made available to the learners.

Each object of learning has features that separate it from other objects of learning (Essien, 2021). These features are the important characteristics of the topic (Mhlolo, 2020). The variation theory expresses that to perceive the object of learning, the perception will depend on the critical features expressed (Lo, 2012). When analysing the textbook through the merger framework, the critical features are identified in each of the units in the chapter on probability. Looking at these features allows us to look at the objective behind the general structure of the textbook.

This general approach largely focuses on identifying the order in which the presentation formats appear. This general approach can be used to conclude whether the textbook follows a quasi-deductive or quasi-inductive structure (Leshota, 2015), with the mindfulness of how the critical features of the object of learning inform the general structure. This means that the structure should be intended for use in various ways to expose the dominant features of the textbook. The textbook will be sectioned according to the presentation formats, which are explained in Chapter 3 when the recognition rule is identified. The two main categories of presentation formats are either explanatory or investigative.

When analysing the examples presented in the textbook, to determine the patterns of variation it is vital to first consider the extent to which the textbook attends to the critical features of the object of learning. This is to deduce the patterns of variation that can be identified as per the analysis structure. The purpose of this analysis is to see whether the critical features are presented through their selection and the space allowed for the examples in the textbook.

The overall focus of this study was to identify the affordances for learning that can be made available to learners by using the textbook. To identify the affordances the order of the presentation formats and the example-space are important in that they give a comprehensive analysis of the presentation of probability in the textbook. The results will be used to generalise the framework to provide an analysis for other textbooks.

2.2.3.2. The link between the variation theory and the framework for mathematics textbook analysis

The idea of how learning occurs, is the central link between the variation theory and the framework for mathematics textbooks analysis. As defined in the previous sections, both these frameworks seek to a identify key components that lead to learners understanding knowledge in a more advanced approach. In the variation theory, this occurs through exposure to critical features of the object of learning so that there is discernment that leads to learning. On the other hand, the framework for mathematics textbooks analysis presupposes that engaging with a format can encourage the development of certain skills.

The intersection between the variation theory and the framework for mathematics textbook analysis begins where the variation theory is perceived as a principle of instructional design. The learning object responds to the question asked: '*What should be learned*?' by defining the subject matter, educational objective, as well as critical features. According to Kullberg, Runesson and Marton (2017), each learner might have a unique learning objective.

Mason (2017) defines variation-pedagogy to be pedagogic acts utilised to take advantage of variation, implying that despite textbooks and materials built on variation ideals, suitable pedagogical choices necessitate an awakening of educator awareness.

Research suggests that using systematic variation among mathematical tasks may enhance educators' comprehension, as well as involvement with the content (Kullberg et al., 2017; Metz

et al., 2016). Teachers' interaction with mathematical content will become greater as they learn how to use structured variation in tasks and examples.

Through the analysis of the textbook Platinum Mathematical Literacy, the researcher in this study discovered that examples used in class can either be pre-planned or spontaneous. The planned examples can also be guided by information from textbooks. According to Zodik and Zaslavsky (2008), pre-planned examples tend to originate from resources which include textbooks, while spontaneous examples frequently arise in reaction to enquiries from learners or assertions.

The type of examples are, in addition, considered by the teachers when selecting or creating examples. Such considerations occasionally expressed pedagogical content knowledge or had been sensitive to their learners' needs, yet in every scenario, the selection heavily relies on solid mathematical understanding regarding pertinent topics at hand.

The standard method when it comes to selecting examples from textbooks is to begin with an easy or familiar scenario (Zodik & Zaslavsky, 2008) and then proceed to addressing obstacles or common mistakes made by learners. Teachers also think of how they can draw attention to pertinent features to minimise what Skemp (1987) in Zodik & Zaslavsky, 2008) alludes to as ‘noise’ which hinders the capacity to form concepts.

Professional educators use elements of variation, such as Marton's (2015) variation theory along with Watson and Mason's (2006) structured variation, to guide their teaching, regardless of a specific formula for selecting and generating examples (Zodik & Zaslavsky, 2008).

Teachers do not specifically express reflections regarding these challenges, according to Zodik and Aslavsky (2008). They thus become more cognisant about their planning, as well as on-the-spot actions by being asked to reminisce on their duties and consider what guidelines or rules of common sense they had followed. This illustrates how teachers acquire knowledge and ideas through experiences that are very similar to elements of systematic variation. It is possible to contend that by teaching mathematical literacy, teachers make use of systematic variation when teaching and encourage contemplation of their teaching practice. Teachers can not only improve their use of deliberate examples in the classroom but also learn how to express their methods

appropriately, then it can be submitted here that the textbook analysis framework interlinkage with the variation theory remains relevant.

2.3. Literature review

2.3.1. The importance of probability

According to Batanero et al,. (2016), probability is embedded in human nature, as we are constantly encountering chances of whether or not specific events may occur. The extent to which probability is studied in core Mathematics and with Mathematical literacy varies, however, what is common is the idea that probability affects learners' daily experiences (Brijlall, 2014). Some of the specific experiences in which learners encounter probability are decision-making, media and games.

The South African Department of Education has proposed these assessment standards across the FET band for Mathematical Literacy.

- 'Grade 12: *Makes comparisons between predictions and actual occurrences of independent events related to games of chance (e.g., lotto);'*(DoE, 2018)
- 'Grade 11: *Calculates experimental and theoretical probability of outcomes of independent events in terms of fractions and percentages; Makes and/or tests predictions of outcomes in the context of games, real-life situations and the work place; Uses tree diagrams to represent and work with a series of independent events.'* (DoE, 2018)

In schools, the main aim of teaching probability is to generate probabilistic reasoning among learners (Batanero et al., 2016). This reasoning is not only about the mathematical procedures but also about the ability to apply the knowledge in many contexts, such as the randomness of events in nature, and the ability to deduce patterns from particular behaviour that leads to generality for other situations (Gal, 2009). Probabilistic reasoning seeks to describe probability in an in-depth manner that surpasses observation (Batanero et al., 2016; Gal, 2009).

Batanero et al. (2016) put forward some fundamental principles that are necessary when teaching probability:

a) **Randomness of events** (experiment or random event): understanding how probability works and how we can calculate the possibility of a random event sometimes creating

misconceptions. An example of tossing a coin or rolling a dice represents many possibilities that may come in any order. This same reasoning is used in gambling, understanding one's chances of winning. The idea of the randomness of probability becomes a hindrance in explaining how the likelihood is decreasing yet the event is random.

b) **Events and sample space:** learners think in a deterministic manner, where they only focus on one event. They need to consider the possible outcomes when more events are occurring.
c) **Combinatory**: This idea is based on counting without naming the outcomes. Tools that can be used in this form of calculating are tree diagrams since there is a numerical understanding of how items are listed, and probability is calculated.
d) **Independent and conditional probability**: '*The empirical estimates of probability via frequency, since when repeating experiments, we require independence of trial.*' In compound events, some events are dependent on the other for their existence.
e) **Probability distribution and expectation**: this depends on the sample and data distribution which influences the expectation of the possible results of experiments.

An awareness of these ideas is important in teaching learners to understand the concept of probability holistically so that they are able not only to pass it as a subject but also able to apply it where necessary.

2.3.2. Mathematical Literacy in South African classrooms

Scholars perceive Mathematical Literacy in different ways. Some of the views of what the subject entails are listed below. Each scholar justifies their claims based on multiple factors. The following statements are based on the arguments posed on what Mathematical literacy means:

- '*An innovative response to the low levels of numeracy amongst adults*' (Bansilal, 2014, p.1157).
- An alternative subject for those who do not want to study core mathematics (Graven & Venkat, 2007).
- '*The Mathematical Literacy curriculum is premised on the idea that an essential goal of the education of high-school graduates from all walks of life should be to prepare them for adult life after graduation*' (Gal, 2009, p.51).

- Mathematical Literacy is a political tool used to redress past injustices that have resulted from the apartheid era (Sidiropoulos, 2008; Jojo, 2019).
- *The Mathematical Literacy curriculum for the FET phase, is aimed at preparing students to manage the mathematical demands of everyday life and professional application* (Department of Education, 2013b).

Many comparisons are done between Mathematical Literacy and Mathematics, mainly because in the FET band the choice lies between these two subjects. The issue concerns access to further studies that are created by studying Mathematics and the inability to access the same opportunities through studying Mathematical literacy (Bansilal, 2014).

Mathematical Literacy is context-dependent. Material created to teach the subject uses many common and uncommon contexts. The socio-cultural differences among the South African population are a concerning matter since learners in South Africa are not exposed to the same contexts (Bansilal, 2014). By creating a context that applies to the whole country is complex, teachers also experience this difficulty when trying to deliver these ideas during lessons. Bansilal (2014) conducted a study on the subject matter knowledge that is necessary to teach Mathematical Literacy; the study outlined that there is still a need to re-evaluate the subject as it is still developing (Bansilal, 2014).

2.3.3. Exemplification in Mathematics education

In essence, the use of examples serves the purpose of illustrating the characteristic nature of a broad rule and explains an idea or phenomenon (Mason & Pimm, 1984). Exemplification in mathematics education, therefore, from the onset involves a set of terminology of examples which can be used in educational literature to encompass teaching content within a context of textbooks, teacher narration, as well as learner explanations.

Compared to other knowledge disciplines or domains, research on mathematics education has had to place more emphasis on the process of exemplification because of the critical role that examples serve throughout the discipline's development and instruction (Watson & Mason, 2005). There have been various interpretations of the concept in tandem with the growing interest of researchers in this matter.

Examples are described as a tool for demonstrating mathematical knowledge (Bills et al., 2006). They are also a type of activity technique that enables theoretical communication between the

teacher and the learner (Peled & Zaslavsky, 1997). A selected but most importantly unique representation for a general principle of exploration or elaboration (Watson & Chick, 2011), in addition to content that enables the general ideas to be perceived in the specific examples (Mason & Pimm, 1984), are a few examples of the representations found within the literature regarding mathematics education. When the traits that each of these elaborations, for example. Are combined, it becomes clear that examples may serve a variety of purposes, including clarification, affirmation, interpretation, as well as summarisation.

To comprehend the qualities of examples throughout mathematics education, several classifications for examples were established. Examples were initially separated into two main strands: 1) general examples (which attempted to clarify the subject itself); and, 2) particular/specified examples (which were limited and applied to give meaning to more specific scenarios) (Mason & Pim, 1984). The work of Watson and Mason (2005) offered a similar viewpoint and recommended readings that highlight a concept's essential elements while outlining boundary examples.

Moreover, Bills et al. (2006) created a classification scheme that is frequently used in mathematics education studies, by dividing examples into three strands: 1) generic examples; 2) non-examples; and 3) counter-examples. Non-examples may be used to define the parameters of a concept, whereas contrary to that counter-examples serve the purpose of contesting a statement or hypothesis (Bills et al., 2006).

According to Rowland (2008), who examined how teachers used examples under the headings of representations, sequencing, variables, as well as learning objectives, inexperienced teachers require particular support and guidance to recognise the various functions that examples play in the teaching of mathematics.

Three additional special labels of example types, namely prototype, start-up, and proof, are also observed independently by researchers during the teaching process and fall under these categories of examples (Peled & Zaslavsky, 1997; Watson & Mason, 2005; Sevimli, 2016 Zaslavsky, 2019).

On the other hand, it is imperative to understand that some researchers, namely Yopp and Ely (2016) built their categorisation on the use of examples throughout the processes of generalising

and disproving, even though there has been an increase in studies in previous years regarding the examples' usage as an augmentation or evidence tool (Rø & Arnesen, 2020).

The technique of teaching and presentation of mathematics tasks by teachers using examples is a characteristic feature of categorisation (Bentley & Stylianides, 2017). For instance, there can be an example that symbolises teacher-centred strategies, where the teacher becomes dominant throughout the content-sharing process or examples may take the form of learner-generated strategies, where learners are pro-actively involved in the process of interpreting, establishing, and solving problems using examples (Bentley & Stylianides, 2017). In this case, it can also be argued that the related literature has highlighted that it is pertinent for learners to establish or generate examples with the ultimate goal of solving problems using such examples. This includes a variety of examples that are presented in the classroom setting (Bentley & Stylianides, 2017; Watson & Chick, 2011).

Current research on the process of exemplification in mathematics education focuses primarily on two areas; 1) identifying the qualities of the type of examples that are used during learning and teaching in the classroom setting (Sinclair et al., 2011; Bills et al., 2006; Watson & Chick, 2011); and 2) assessing the impact of the process of exemplification on reasoning and producing evidence of mathematics reasoning (Peled & Zaslavsky, 1997; Bentley & Stylianides, 2017). Numerous studies have addressed these issues.

Nevertheless, there are no prior studies in the literature on the types of modifications that have taken place in the selection of examples in learning environments, particularly in the South African public schooling system. This includes a lack of research regarding how the reflection process on the knowledge ought to be taught/conferred (knowledge of textbook) to actual knowledge taught/conferred (teacher practice) shapes teachers' selection of examples.

In other words, examples serve as essential reference materials for understanding the information taught by instructors in the classroom and for determining the specific functions of various sources during the didactic transposition concerning scholarly knowledge and pedagogy of mathematics (Chevallard & Bosch, 2013).

2.3.4. Variation as a learning theory focused on examples: The case for Mathematical Literacy

It is crucial to understand that the difficulty lies in determining the essential elements of a given learning object, that is, what needs to change and what needs to stay constant throughout the experience of the learner. According to the variation theory, one way to bring about a change in understanding is to focus on certain important facets of the object of learning and introduce variation to them, while maintaining consistency in other aspects.

The teacher is instructed by the variation theory to concentrate on the crucial component of the learning object (for example, a mathematical literacy concept), recognise varying conceptual levels, and extract from each of these notions the essential components (core ideas) that will change and those that are unlikely to. All such invariants are typically the concept's characteristics in Mathematical Literacy.

The ability to decipher and understand mathematical problems defines Mathematical Literacy. In the words of Lange (2006), Mathematical Literacy entails a broad understanding of mathematics, in addition to the capacity to perform calculations. Furthermore, Stacey (2011) suggests learners' capacity to apply modelling in solving mathematical concepts defines mathematical literacy. The capacity to formulate problems in terms of mathematics and interpret and assess solutions is critical to improving one's ability to resolve mathematical scenarios in real-world situations because the application of mathematical literacy has an impact on people's lives.

According to Bansilal et al. (2015), ideas about the objectives of Mathematical Literacy arguably form the basis of teachers' priorities, the origin of the issue and the intended curriculum(Kristayulita et al., 2018). Items from Mathematical Literacy tests may be used to gauge learners' aptitude for solving problems. Learners must be able to analyse, formulate and effectively communicate ideas to complete test items that involve interpreting, defining and resolving mathematical problems in a variety of contexts (Thomson, Hillman & Bortoli, 2013). According to Masjaya and Wardono (2018), two factors determine learners' proficiency in mathematics: internal and external. In addition to the non-cognitive components of interest and enthusiasm, internal variables involve cognitive elements, verbal skills, as well as numerical skills.

To further elaborate on the significance of using examples at the Mathematical Literacy level, Annisavitri et al. (2020) observed, analysed, and outlined the patterns of problem-solving of learners during Mathematical Literacy activities/exercises. This was a further endeavour to explain exemplification as an aspect of variation theory within mathematical literacy learning as follows:

Learners carry out fundamental tasks with integers, decimals, fractions, and straightforward graphics, including counting, choosing, and performing a basic arithmetic activity. In addition, they recognise data and respond to enquiries on the assessment items. Further, learners address issues using pertinent data (Annisavitri et al., 2020).

Learners work with integers, which are divided into fractions, decimals, as well as graphs as part of a basic mathematical procedure. In addition, learners solve puzzles that call for straightforward procedures and fundamental mathematics skills. supplying suitable justification. The learners can apply their foundational mathematics skills in a variety of mathematical literacy contexts by having the capacity to decipher and interpret data into tables or graphs. Integer, decimal, fractional, per cent, straightforward measuring, spatial representation, estimations, and basic data interpretation are among the procedures that are normally intricate in mathematics problems. In addition, learners establish basic communication based on the deductions drawn from analysis and deduction (Annisavitri et al., 2020).

Learners possess the ability to resolve issues of their understanding of geometry, spatial visualisation, and event possibilities. Moreover, they solve mathematical puzzles by creating patterns and proportions that can be stated verbally, in terms of numbers or both. Furthermore, they apply their mathematical knowledge in the form of opinions based on various points of view. Constructing and expressing answers based on logic and interpretations (Annisavitri et al., 2020).

The learners can apply their understanding and perspectives to moderately complex issues and circumstances. Applying the knowledge to resolve issues involving various numbers, as well as operations. In the same sense, this level of complexity necessitates more complex modelling, pertinent processes, and a variety of steps and approaches to problem solving. It is also imperative to note that learners link symbol representation and relevant knowledge through the

application of thinking that is more comprehensive and reasoning exercises (Annisavitri et al., 2020).

Learners are more capable of reasoning and thinking mathematically; possessing the capacity to solve linear equation problems, draw generalisations, and apply and clarify concepts and information in a variety of contexts. applying their understanding of geometry and measurements to the resolution of challenging issues. Also, learners create intricate illustrations, create mathematical representations, and conduct critical evaluations. analyse the connections between results, interpret, and have a viewpoint in light of the circumstances at hand (Annisavitri et al., 2020).

To further elaborate on variation as a learning theory, the focus was on examples. In the case of this study on Mathematical Literacy, the perspectives of certain researchers in constructing variation theory's meaning can be taken into account subsequently as follows:

Table 1: Further perspectives on the variation theory

Author/Researcher	**Explanation**
Marton and Pang (2016)	Simply put, variation theory is an increasingly logical, clear, and meticulous structure for dealing with variation, as well as invariance.
Askew (2012)	Variation theory serves as a prerequisite for discerning.
Akerlind (2015)	

	Acquiring knowledge using the variation theory is viewed as a growth in awareness, through which learners evolve into cognisant minds regarding essential components concerning a disciplinary notion, ability, or practice of which they were previously unaware of.
Kullberg et al. (2017)	Variation theory-based mathematics teaching strategy that emphasises variation as a necessary condition for learners to be able to discern new aspects of an object of learning.

2.4. Conclusion

Chapter 2 commences with a review of the literature that informs the study. Furthermore, a theoretical grounding is explained; learning the development of the dyadic framework for comprehensive textbook analysis. This framework will inspire the methodology of the research. This will be made applicable in the analysis and discussion stage.

CHAPTER 3: METHODOLOGY

3.1. Introduction

A detailed description of the methodology used to collect the data for this study is explained in this chapter. This is followed by a justification of the textbook selected and the chapter and units analysed. The justification of the sample led to a discussion about the reliability and validity of the study. Ethical considerations were mentioned together with limitations that have been identified in this study. The section that follows is a description of how the limitations were addressed. This chapter concludes with a summary and brief explanation of what to expect in the next chapter.

3.2. Methodology and research design

To analyse the presentation of the topic of the *Platinum Mathematical Literacy* textbook, the study did not require human participants, thus the focus was the interpretation of the textbook based on the conceptual framework mentioned in Chapter 2. This research applied a qualitative approach to address the research questions, where the focus was on interpreting and describing data rather than giving a numerical representation of the data (Adler, 2017). Elements of a quantitative study will be used in the analysis. This may comes from the research questions, where there was a need to measure the number (the patterns of variation) within the chapter of analysis.

The textbook is *Platinum Mathematical Literacy* (Frith et al., 2013). The analysis was completed using a combined framework of the theory of variation (Zodik & Zaslavsky, 2008) and the framework for analysing mathematics textbooks (Valverde et al., 2002) through which the core tenets of these combined theories were observed.

3.2.1 Sampling

a) Choosing Platinum Mathematical Literacy

Many factors were considered to select the textbook that was analysed. First, few textbooks have been published on the subject of Mathematical Literacy as it is a fairly new subject still being developed (Bansilal et al., 2014). From 2006 to 2013 Mathematical Literacy was advanced through the lens of the National Senior Certificate (the curriculum statement at the time). There were further developments to the subject from 2013 to 2019 as informed by CAPS (the

curriculum statement at the time). The changes that were implemented in 2019 have provided a clear structure to assessments and topics.

Some of the most common authors/publishers in Mathematics only produced Mathematical Literacy textbooks up to Grade 11 and not Grade 12. The other factor that makes some Mathematical Literacy textbooks to be more common than others is the date when the textbook was published. Textbooks that were published up to Grade 12 around 2006 to 2013 when Mathematical Literacy was introduced, have a greater chance of being utilised in schools for longer.

Once textbooks are procured in a public school, they may be issued to learners and retrieved at the end of the year. They are then issued to learners in the same grade in the following year. Unless the curriculum statement changes or the textbook is declared to be no longer useful, schools are most likely to use these books for a long time. The wear and tear of textbooks reduces the number of textbooks available in the school; schools may use their budget to augment the textbook that already exists. These are some of the reasons why the *Platinum Mathematical Literacy* textbook is still commonly used in many South African schools.

b) Choosing probability

Probability is one of the common topics that are studied in both Mathematics and Mathematical Literacy (Gal, 2009). Mathematical Literacy is about the practical application of mathematical ideas into real-life contexts (Ogbonnaya & Awuah, 2019). Through the topic of probability, mathematical expressions of everyday life are expressed. This gives learners the ability to learn formal school knowledge and apply it.

Probability is a common topic that is taught from Grade 8 to 12, this implies that all learners engage with the topic at different levels. Questions in Mathematical Literacy are phrased with words because Mathematical Literacy focuses on using language to explain mathematics. For this study, probability was selected because of its features that would allow an analysis using variation.

This topic is also multifaceted because learners apply the skills of calculating, drawing and interpretation. In addition, they experience probability in many everyday situations like games, sports and decision-making (Gal, 2009).

c) Unit analysis

In the *Platinum Mathematical Literacy* Grade 12 textbook (Frith et al., 2013), the section that was analysed was separated into four units. The units are Unit 1: the expressions of probability, Unit 2: prediction, Unit 3: determining possible outcomes, and Unit 4: expressions involving probability. For this analysis, only Units 1, 3 and 4 were analysed. Unit 2: Prediction is excluded from the analysis because the nature of the questions and examples are open-ended. With the other units, the answers are controlled and allow us to cast a theoretical lens on the questions.

Based on the literature review in Chapter 2, it is evident that although it is important to rely on the observation of probability at Grade 12 level, learners must be able to understand the meaning of probability more deeply (Batanero et al., 2016). This justified the reason to exclude Unit 2: prediction in the analysis since it still only focuses on determining probability through experiments.

3.3. Data analysis

The data were analysed in two sections. Each section focused a component of the analytical framework , which was guided by the research question. The initial research question was to determine the presentation of how the topic of probability in the textbook was addressed by analysing the general approach of the textbook. The second section of the analysis considered the exemplification. In exemplification, the core tenets of the variation theory were used to analyse the extent to which the exercises attend to the critical features identified. Once all the data were collected, each phase had implications about the affordances for learning.

3.3.1. The general approach

The textbook was sectioned according to presentation formats. **Table 2** below lists and describes the presentation formats that are found in the Platinum textbook. The order of the presentation formats was to be determined. This sequencing was used to determine the general approach of the textbook that could either be quasi-deductive or quasi-inductive.

*A **Quasi-deductive** textbook can be identified when the textbook starts by explaining the content, thereafter an example is given to illustrate the explanation, and an exercise is given to the learners to complete using the knowledge attained from the explanation (Leshota, 2015).*

*A **Quasi-inductive** textbook can be identified if the approach is investigative. An activity is given first for students to discover the content through exploration. Thereafter, summary notes and information about the nature of the content are given (Leshota, 2015).*

Table 2 illustrates some of the presentation formats that are found in the textbook. The diagram also gives a detailed description that helps understand how it can be identified within the textbook.

Table 2: Description of the presentation formats

Presentation format	Description
Explanatory text	This can be a statement or paragraph that explains Probability
Note box	Information in a box, usually on the side of the page, it has information that is of importance to the example or exercise
Revision table	A set of table/tables that summarises revision material, relevant to the chapter that is supposed to have been
Probability number line	A diagrammatic representation of the meaning of the likelihood in probability.
Example	Commonly worked examples are found before activities. This is so as to paint out a picture of how learners need to engage with the task at hand.
Exercise	A set of exercises that afford learners an opportunity to engage with the aspects being dealt with in the examples and descriptive texts that the textbook has.
Summary table	Provides a summary of the information that is important within this chapter.
Check your skills/Revision activity	A revision activity is a means to summarise all the information that has been learnt throughout the chapter.
Key words	Important and terminology that is used multiple times in the chapter. Thus, these words are critical in understanding the topic

Each unit analysed has a broad topic. Instead of referring to the topic as the object of learning, the topic was referred to as the core idea of the unit section analysed. Thereafter, the critical features were identified in each unit section. These critical features were identified in the introductory section of each unit. This section includes the definitions, narrative texts and worked examples. Thereafter, the exercises were analysed to see the extent to which the exercises attend to the critical features. Each object of learning or core idea identified in the textbook has numerous critical features (Lo, 2012); these features can be identified by considering the following key items:

a) A set of competencies that are required to comprehend the object of learning/core idea.
b) A concept that underpins the object of learning/ core idea.

3.3.3. Patterns of variation

The worked examples and exercises were analysed. Since Mathematical Literacy is very descriptive, it was not easy to determine the patterns of variation by looking at the question posed. What was done was to look at the solutions to the questions to see the patterns. **Table 3** below shows the recognition rule used to analyse the solutions given in each of the sections analysed.

Table 3: The Patterns of Variation

Pattern of variation	**Recognition rule**
Similarity	These are questions with responses that have a similar structure
Contrast	These are questions with more than one feature that is varied in a comparative manner.
Fusion	When two critical features can be discerned within one question, then there is fusion.
Generalisation	While observing the features that vary, generality is focused on the invariant features; these allow learners to deduce certain patterns about the nature of the data given.
No variation	These are a group of questions that do address similar critical feature/s

Table 3 above is used to determine the patterns of variation. The critical features of the object of learning needed to be identified because the patterns of variation can be seen if the critical features are varied.

3.4. Validity and reliability

To reduce any risk associated with this research I considered the validity and reliability of this research concept I drew on Maxwell's (1992) thinking of the trustworthiness of the research.

3.4.1. Validity of data collection

This qualitative study was a textbook analysis. It is vital to mention that the theoretical framework that underpins this study has not been used in this particular context. To advocate for the legitimacy of the data collection method, Maxwell (1992) put forward different types of validity and the purpose of 'justifying' the data collection method. The argument is informed by the objective nature of a qualitative study which can be analysed in various ways.

The framework for comprehensive textbook analysis has been established to analyse the textbook to collect data and interpret it. Maxwell (1992) refers to this type of validity as factual data collection. The textbook was analysed using a framework, therefore, the results obtained are authentic to the lens of the framework used. The results obtained in this study are not intended to discredit the textbook. They are rather intended to identify techniques which can improve the teaching and learning of Mathematica Literacy.

3.4.2. Validity of data analysis

A recognition rule is established based on the two frameworks that inform the study. These recognition rules are explained for clarity. To safeguard the interpretive validity of the data collected I ensured that the results were interpreted based on the framework and not on my opinion (Maxwell, 1992).

3.5. Limitation of the study

There were no human participants in this study which was a limitation as this study only focused on the exploration of the structure of the textbook. In the theoretical framework section, research shows that textbooks are a tool used to improve learning. Therefore, it is a limited suggestion to make judgements about the affordances that the textbook may or may not offer if there is no consideration of how it is utilised in a classroom setting. If teachers were to be made aware of this study, it would raise an awareness of the sequencing of examples in lesson preparation and lesson delivery. Teachers may bring a different understanding that may enhance learners' understanding of what is taught.

Since this study was a textbook analysis, it would have been interesting to find out how this study would have resulted if it were conducted in a classroom setup. This would have allowed us to see more about the application framework that has been developed. It would have also allowed us to see how teachers are currently using these textbooks to teach the topic of probability.

3.6. Ethical considerations

Since there were no human participants, it reduced the risk of disclosure of information (Protection Of Personal Information Act). Furthermore, there was an acknowledgement by the researcher that ethical considerations go beyond having human participants in the study. Applications for permission to make observations and recommendations on published material were made. The ethics waiver was applied for and permission was granted to proceed with the research concept.

The results of the ethical clearance application showed there was a limited risk with the results of the study. Additionally, permission was requested from Maskew and Miller the publishers of the Platinum Mathematical Literacy Grade 12 textbook. The permission granted the use of all the written material excluding images of people and objects.

The textbook is not a variation-informed textbook. Analysing the textbook through the lens of variation does not imply that the textbook is not theory-informed. The intention is to rather show how variation would offer an alternative view of the delivery of content.

3.7. Conclusion

With the use of the conceptual framework, this chapter has expounded on the methodology that was used to gather data. The following chapter explores the actual data that was collected showing how to source results from the analysis conducted.

CHAPTER 4: Analysis

4.1. Introduction

This chapter aims to provide an in-depth analysis of the topic of probability in a Grade 12 Mathematical Literacy textbook (*Platinum Mathematical Literacy)* to delineate the affordances that this textbook makes available to learners:

To reiterate, this investigation is informed by the following research questions:

1. How is probability as a topic presented in the commonly used Grade 12 Platinum Mathematical Literacy textbook?

2. How are the examples structured in the textbook available to learners?

3. What affordances do these examples offer to learners for the learning of probability?

The dyadic framework for a comprehensive analysis of textbooks as mentioned in Chapter 2, informs the structure of the analysis. This framework is a combination of the two prominent theories that inform the study, the framework for Mathematics textbook analysis and the variation theory. The analysis is set out in the following order:

- The general approach: This section will analyse the order in which the presentation formats appear to deduce the general structure of the textbook
- Exemplification: This section identifies the core ideas and the critical features on which to focus and the extent to which the exercise examples attend to the core ideas and their critical features.

The initial section of the analysis responds to the first research question on determining the general structure of the Mathematical Literacy textbook; by analysing the general approach of the textbook this structure is made explicit. Thereafter, the analysis drew on two important tenets of the variation theory, the first being the *critical features* presented in each of the units of the chapter, and the second tenet is the *nature and order of examples* presented.

The objective of this research was to determine the affordances /competencies that are made available to the learners when they engage with the textbook. These affordances can be identified

in each of the sections of the analysis. In all of the sections of analysing the textbook, there is an anticipation of identifying affordances and some possible areas for improvement.

4.2. The general approach

This phase of analysis focuses on determining the general approach of the textbook. As explained in Chapters 2 and 3, the approach can be quasi-deductive which is regarded as a narrative approach or quasi-inductive regarded as an investigative approach (Valverde et al., 2002). To determine the general approach, some extracts from the textbook are labelled below. The labels on the right-hand side identify some of the presentation formats as they appear in the Platinum textbook. To illustrate the order in which the presentation formats appear, the figure below is from Unit 3 of the chapter on probability. A tabular explanation of the unit comes after the pictorial analysis.

The following figures show the presentation formats identified in Unit 3.

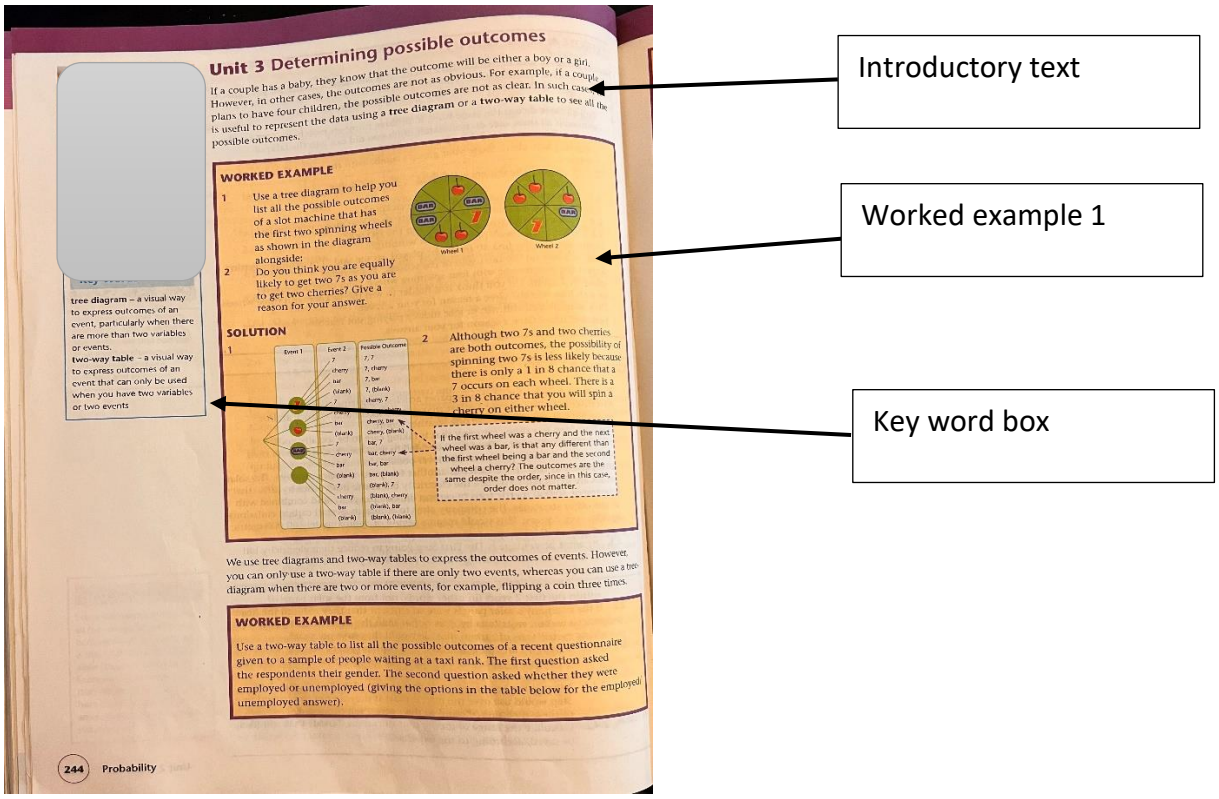


Figure 6: Determining possible outcomes from multiple events

Source: Reproduced with permission from Maskew Miller Learning

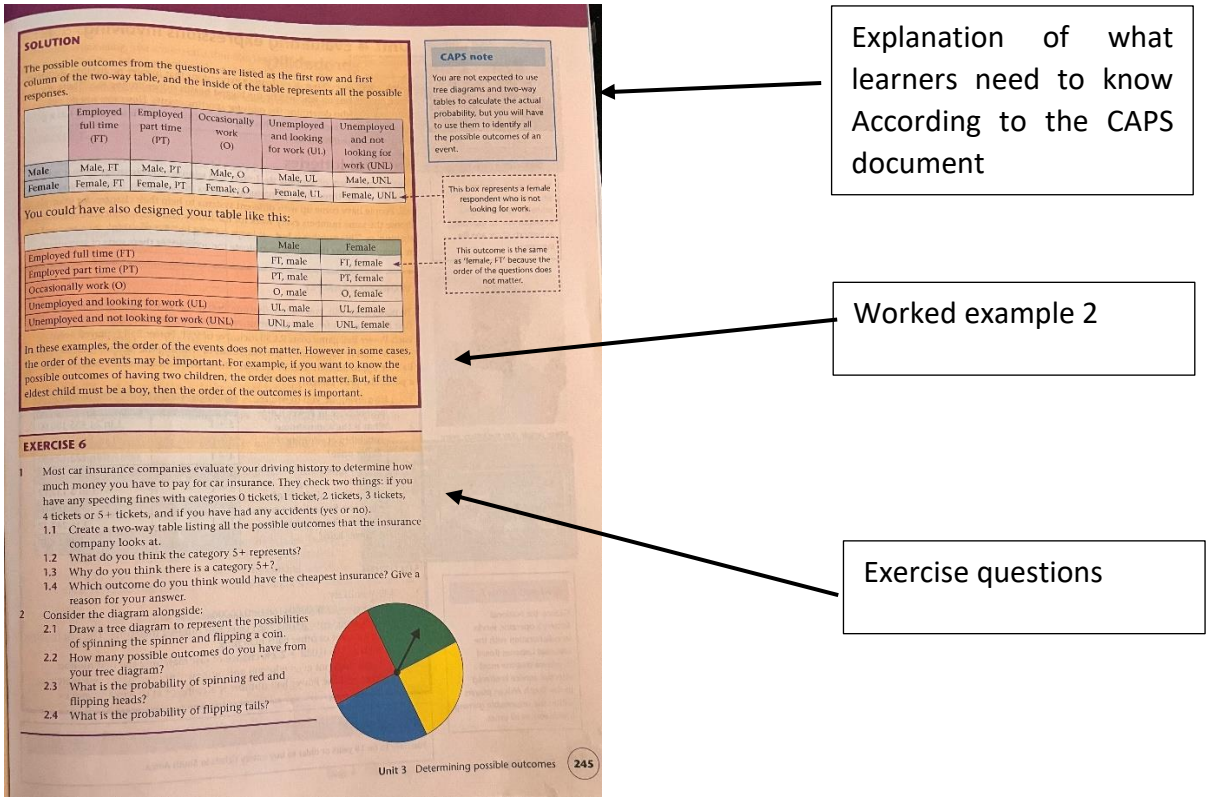


Figure 7: Determining possible outcomes continued

Source: Reproduced with permission from Maskew Miller Learning

The first presentation format is a narrative format to introduce the concept. This is followed by an investigative format of a worked example that exposes learners to questions and answers on the topic. A keyword box on the side defines the keywords that learners will see in this section. A second worked example precedes the exercise that learners must complete to check their understanding of the topic. By observing the pattern above, one can deduce that the textbook is largely quasi-deductive. The narrative presentation formats precede the investigative formats. When engaging with the textbook it is evident that there is a narrative introduction of the content. The application is required once a foundation is presented.

Table 4 below shows the order in which the presentation formats appear in the chapter of probability. The three tables below are set up to explain the presentation formats in each unit section that was analysed. Table 4 describes presentation format found in Unit 1.

Table 4: Expressions of probability; Order of presentation formats

Table 4 Expressions of probability: Order of presentation formats	
Presentation format	**Explanation**
1. Introductory text	Explanation of the concept of probability.
2. Keyword box	Highlighting the most important words that learners may expect to see within the chapter.
3. Worked example	Example of the representation of probability.
4. Picture of multiple choice	Visual representation of an assessment technique where there are multiple possibilities.
5. Information text	A text showing and explaining the different representations of probability.
6. Probability number line	An indication of the likelihood of events occurring based on the chance because of the probability.
7. Worked example	A worked example of the probability number line
8. Exercise	This activity allows learners to apply the knowledge they have acquired in the other presentation formats.

The quasi-deductive approach is evident in Unit 1: The expressions of probability. The narrative presentation formats do not only precede the investigative presentation formats, but they also inform the type of questions found in the investigative formats. If one refers to Table 4.1 above one will notice that the worked examples and exercises are informed by what is presented in the introductory text or information text. For the learners to engage with the last presentation format Number 8 (exercise) in Table 4, they must have engaged with the presentation formats that come before the exercise.

Table 5 below shows the order of the presentation formats in unit 3: Determining possible outcomes. This table identifies and explains the presentation formats as they appear in the unit.

Table 5: Determining possible outcomes: Order of presentation formats

Table 5 Determining possible outcomes: Order of presentation formats	
Presentation format	**Explanation**
1. Introductory text	Explanation of the diagrams used to represent the probability of multiple events.
2. Keyword box	Explanation of the term, a tree diagram, and the term, a two-way table.
3. Worked example	Illustration of a tree diagram drawn using a spinning wheel.
4. Information text	A description of the conditions that are necessary to draw either the probability number line or the two-way table.
5. Worked example	Worked example showing different ways of drawing a two-way table.
6. Exercise	This activity allows learners to apply the knowledge they have attained from the other presentation formats.

This unit 1 is also largely quasi-deductive. Similar to Table 4, Table 5 also identifies and explains the presentation formats as they appear in the unit of *determining possible outcomes*. A deduction can be made; this is to say that the investigative formats in this unit are informed by the information that is explained in the narrative formats.

Table 6 below illustrates and explains the presentation formats as they appear in Unit 4 of the chapter on probability. The title of the chapter is evaluating expressions involving probability.

Table 6: Evaluating expressions involving probability; Order of presentation formats

Table 6 Evaluating expressions involving probability: Order of presentation formats	
Presentation format	**Explanation**
1. Introductory text	An explanation of gambling and the National Lottery.
2. Keyword box	A description of the keyword used in the unit.
3. Worked example	An illustration of how to respond to questions that ask about the context of gambling.
4. Fun fact (did you know)	These are important facts that learners need to be aware of about gambling.
5. Information text	A description of the context of a scratch card.
6. Exercise	This activity allows learners to apply the knowledge they have attained from the other presentation formats.

What is common in all three units is that the narrative presentation of the formats constitutes the introduction of the core idea presented in the chapter. The units analysed have uncovered that they all conclude with an exercise that is intended to evaluate the learners' ability to apply the knowledge that they have acquired through engaging with the text. This analysis is about the broader structure of the chapter. An example-specific explanation follows in the exemplification section. This will expound on the extent to which the investigative formats are informed by the narrative presentation formats.

Mathematical Literacy is a context-imbedded subject that requires a substantive amount of narrative presentation formats; this is to allow learners from different parts of South Africa to understand the various contexts. This may not necessarily expose the learners to all the

mathematical literacy contexts, but it should give learners exposure to the cognitive abilities that will be required to assess different contexts.

Every approach has its benefits and limitations. The benefit of using the quasi-deductive approach is that it allows for scaffolding. This means that the content pieces have been sectioned into simpler smaller narrative formats before engaging with investigation. The limitation of using this approach is that the end goal is for the textbook users to complete the exercise, thereafter there is rarely an explanation. The advantage of a quasi-inductive approach is that new information is learnt through deduction. It requires higher-order thinking. This approach aims for learning by drawing conclusions.

Throughout the chapter, it can be noted that there are no summary texts for learners after they engage with the investigative presentation formats, there is no reinforcement of what they could have learnt through engaging with the textbook. This conclusion is predisposed since there are few cases where there is an explanatory text after an exercise, it seems that the purpose is to prepare the learners for the next exercise.

4.3. Exemplification

This section of the analysis intends to use the theory of variation as a theoretical lens to analyse exemplification within the *Platinum Mathematical Literacy Grade 12* textbook. According to Valverde et al. (2002), textbooks are designed to provide opportunities to learn. This statement addresses the purpose of this study, which is to analyse the affordances for learning that may be inferred through engaging with the *Platinum Mathematical Literacy* textbook. Throughout the exemplification section, the core ideas in each unit are lifted to determine the extent to which the textbook attends to the critical features.

The presentation formats as explained in previous sections are grouped into two categories. These are either narrative or investigative. In this part of the analysis, the gist is to thoroughly analyse the exercises, which is a type of investigative presentation format. The narrative formats are used to determine the core idea and its critical features. Thereafter, the exercises are considered to firstly analyse the extent to which the critical features are made explicit to the learners and secondly, consider how a variation-informed approach would improve the understanding of the topic.

4.3.1. Expressions of probability: Analysis of Unit 1

Expressions of probability is the core idea presented in the first unit. The textbook defines probability as the likelihood or chance that an event will occur (Frith et al.2013, p.238-251). This is expressed as a formula that is used as a mathematical representation.

$$Probability = \frac{favourable\ outcomes}{total\ possible\ outcomes}$$

The critical features of the expressions of probability are inferred from the above-mentioned formula. These features are the ability to express probability in three ways, a fraction, a decimal and a percentage. The contextual meaning of the value of probability is the second critical feature which represents the likelihood of an event occurring. To illustrate this Figure 8 below is an extract from the textbook to illustrate likelihood through a probability number line. The number line uses probability terminologies that express the chances of an event occurring. Where 0% is impossible and 100% is certain.

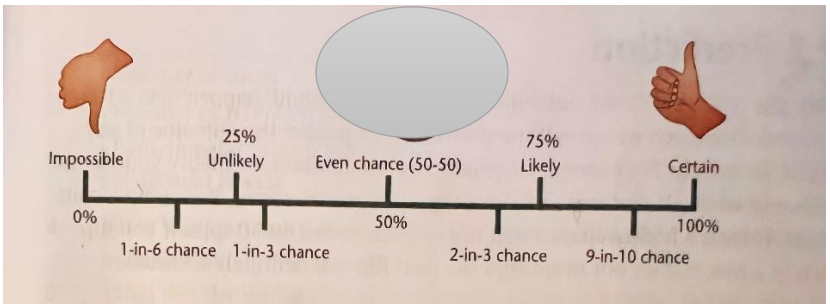


Figure 8: Probability number line

Source: Reproduced with permission from Maskew Miller Learning

The table below represents a summary of the critical features of the core idea of expressions of probability. To evaluate the extent to which the critical features attend to the core idea, the critical features must be clearly outlined.

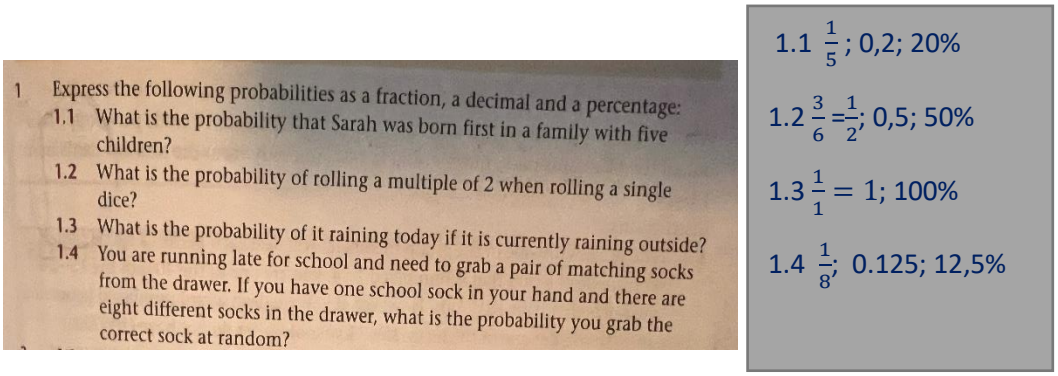
Table 7: The critical features of expressions of probability

Core Idea	**Expressions of probability**
Critical features	• The representation of probability as a fraction, decimal or percentage. • The representation of likelihood on a probability number line. • Probability increases from 0 to 1.

The critical features of probability are centred on the formula stated above. This table indicates that the fraction can be simplified to a decimal or percentage that represents the same value. From the fraction, decimal or percentage the value of the probability can be interpreted to explain the chances of the event happening.

In the next section, I engage with how the examples and the sequencing of examples attend to this critical feature(s).

a) Micro analysis: using Variation as a lens Exercise 1


1 Express the following probabilities as a fraction, a decimal and a percentage:
1.1 What is the probability that Sarah was born first in a family with five children?
1.2 What is the probability of rolling a multiple of 2 when rolling a single dice?
1.3 What is the probability of it raining today if it is currently raining outside?
1.4 You are running late for school and need to grab a pair of matching socks from the drawer. If you have one school sock in your hand and there are eight different socks in the drawer, what is the probability you grab the correct sock at random?

1.1 $\frac{1}{5}$; 0,2; 20%

1.2 $\frac{3}{6} = \frac{1}{2}$; 0,5; 50%

1.3 $\frac{1}{1} = 1$; 100%

1.4 $\frac{1}{8}$; 0.125; 12,5%

Figure 9: Questions and answers of Exercise 1

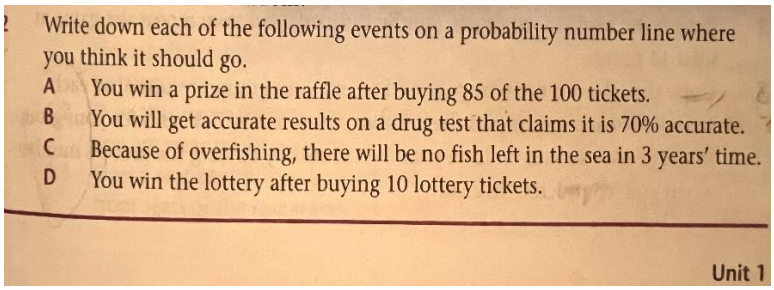
Source: Reproduced with permission from Maskew Miller Learning

Mathematical Literacy questions are written in long sentences that require answers in the form of mathematical expressions. To cast variation as a lens, the answers to the questions are provided in the grey box. It is evident that throughout Question 1 the requirement is to express the probability of events occurring. The requirement is to express probability as a fraction, decimal or percentage.

The observation is that the numerator of all the simplified answers is one but the denominators vary according to the total possible outcomes. This leads us to reflect on the critical features of expressions of probability. First, probability can be expressed in three different ways, as a fraction, decimal or a percentage. The other feature is that the larger the denominator, the lower the probability and that decreases the likelihood of an event occurring.

From the answers in the grey textbox, we can deduce that there is no particular pattern used to order the questions. The example space does not show an increase or decrease in probability through the order in which the denominators vary. The example space created does not seem intentional. A variation-informed textbook would have reordered the questions such that there is a gradual increase of the dominator to show the decrease in probability. In that way, the focus is not only on expressing probability in three ways, but it is also on the meaning that is drawn from intentionally ordering questions. If the questions can be reordered from 1.3 to 1.2 then 1.1 and 1.4, this change would allow learners the opportunity to discern the decrease in the likelihood of an event occurring. A variation-informed textbook would order the questions such that all the critical features are attended to.

Exercise 1 continued



Write down each of the following events on a probability number line where you think it should go.

A You win a prize in the raffle after buying 85 of the 100 tickets.

B You will get accurate results on a drug test that claims it is 70% accurate.

C Because of overfishing, there will be no fish left in the sea in 3 years' time.

D You win the lottery after buying 10 lottery tickets.

Unit 1

A. Likely

B. Likely

C. impossible

D. Unlikely

Figure 10: Question and answers continued

Source: Reproduced with permission from Maskew Miller Learning

This is an extract from Question 2 of Exercise 1, which is a continuation of Exercise 1. This means that the critical features explained in Unit 1, expressions of probability still inform the requirements of this question. This question requires learners to draw a probability number line to place the events in relation to each other. The questions can be answered by placing the question number (alphabet) on the number line to represent the likelihood of events happening.

The first two answers, A and B, will be positioned towards the far right and the last two answers will be positioned towards the far left. These questions in all likelihood allow learners to discern the difference.

4.3.2. Determining possible outcomes: Analysis of Unit 3

In this unit, the core idea is determining possible outcomes of compound events (in mathematical literacy compound events are two or more events occurring simultaneously or consecutively). The critical features of this core idea are based on drawing a two-way table and a tree diagram. These are visual representations showing how to determine the possible outcomes. See Figure 11 below:

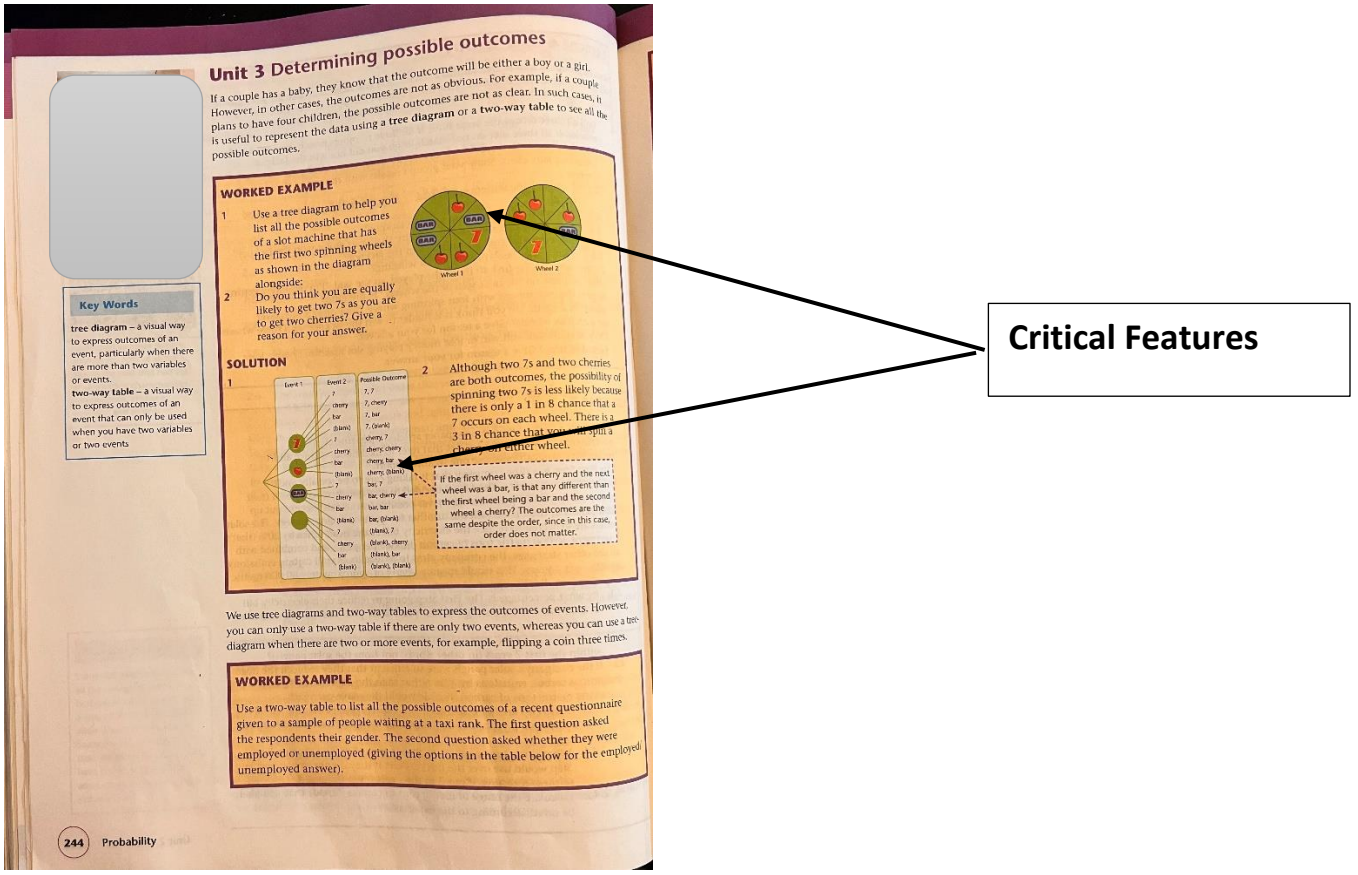


Figure 11: Critical features of determining possible outcomes

Source: Reproduced with permission from Maskew Miller Learning

Figure 11 identifies the critical features of Unit 3. These features are listed in Table 8 below.

Table 8: The Critical features determining possible outcomes

Core idea	**Determining possible outcomes**
Critical features	1. Representing the probability of multiple events. 2. Determining the **total** possible outcomes of multiple events. 3. Determining the possible outcomes. 4. The two-way table can only represent the possible outcomes of two events whereas a tree diagram can represent more than two events. 5. The principles of drawing a tree diagram and a two-way table.

In the next section, I engage in how the exercises attend to the critical features of the core idea.

Analysing Exercise 6

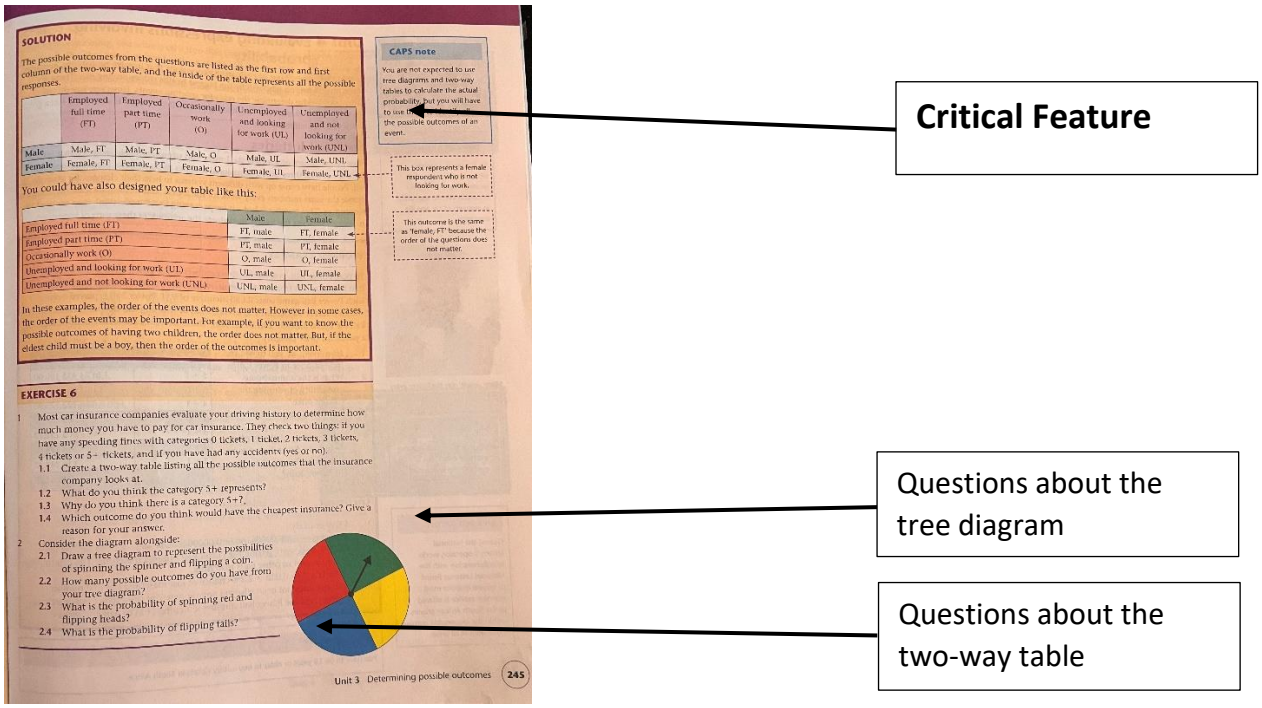


Figure 12: Analysing Exercise 6

Source: Reproduced with permission from Maskew Miller Learning

An analysis of exercise 6

1. The first question is about drawing a two-way table and interpreting the table. The context of the question is about car insurance. All the questions thereafter are about insurance rather than a mathematical interpretation of the information. There is a misalignment of the critical features emphasised. Question 1.4 is more about understanding the context of car insurance than it is about probability.

2. The second question is about drawing a tree diagram based on the spinner given along the questions. The questions thereafter are about the possible outcomes. Throughout all the questions in Unit 3, this is the first account that attends to the critical features of determining possible outcomes. The last two questions add the context of a coin about the already given context of a spinner. The inclusion of the context of the coin is not clear. See Questions 2.3 and 2.4. It only states 'flipping heads' and 'flipping tails', it does not say on a coin.

   Perhaps if the main question had included the spinner and the coin to show compound events, it would have firstly attended to the created features and secondly, it would have cleared the confusion that is currently in the question.

4.3.3 Expressions involving probability: Analysis of Unit 4

In this section, the core idea is 'probability in everyday life'. This section is an application of the other core ideas that have been discussed in the previous units. These core ideas are the expression of probability and determining possible outcomes. The everyday life expressions are set out to expose the critical features of odds in insurance and gambling.

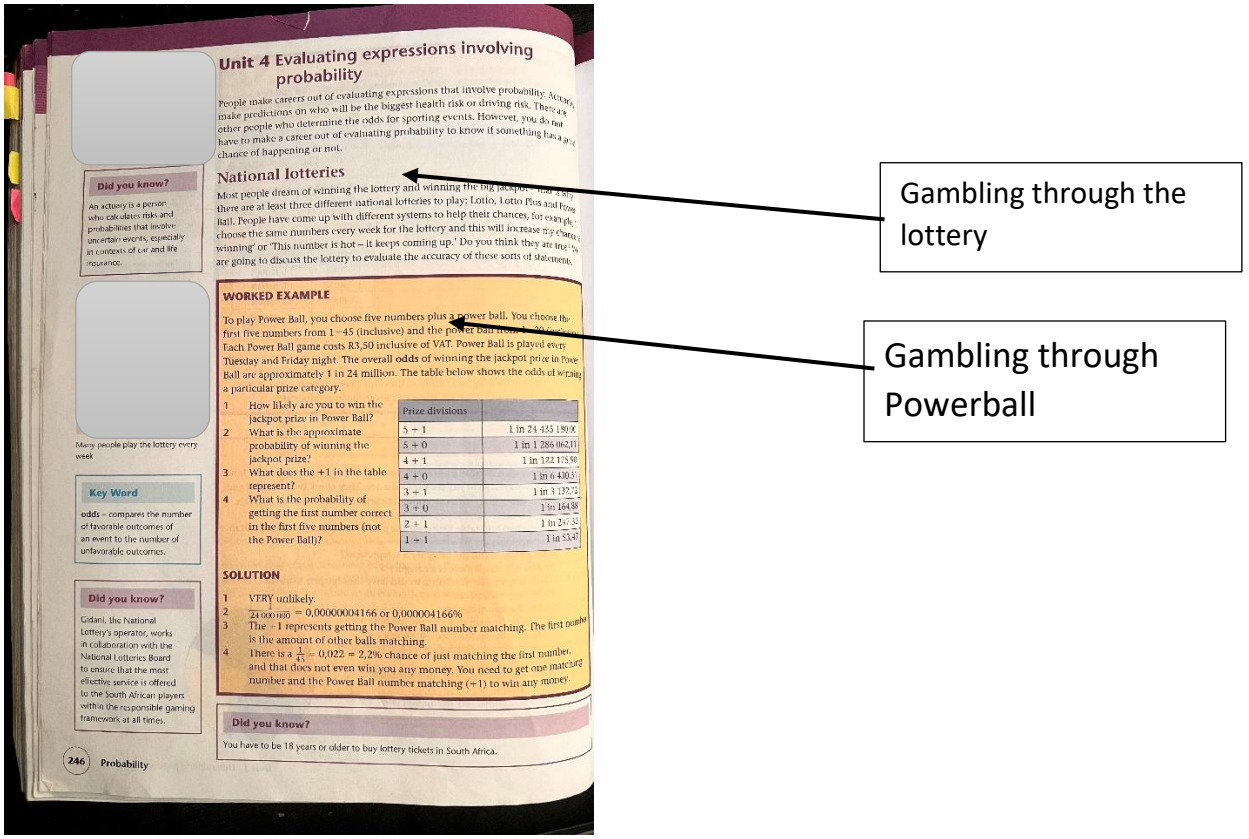


Figure 13: The critical features of expressions involving probability

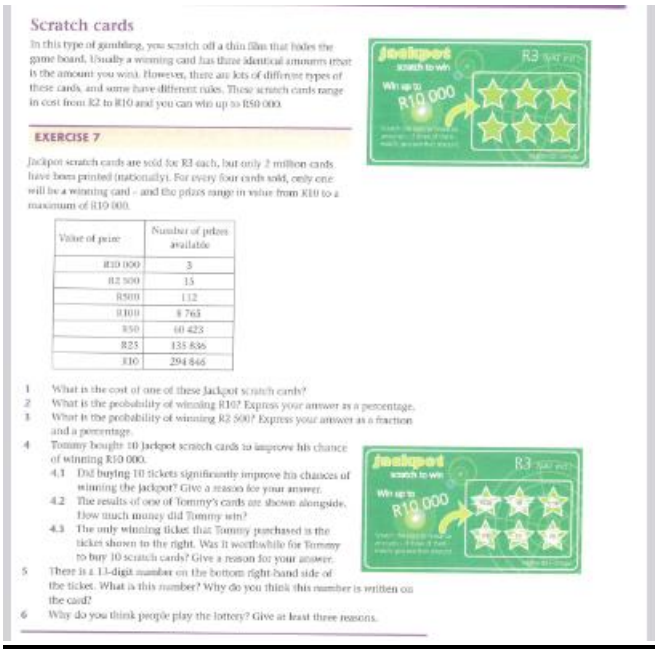
Source: Reproduced with permission from Maskew Miller Learning

Figure 13 above shows probability in the context of gambling. This is one of the critical features of this unit section. The critical features of this core idea are listed below.

Table 9: The Critical features of expressions involving probability

Core Idea	**Expressions involving probability**
Critical features	• Probability in everyday life contexts (Gambling and insurance). • Odds versus the likelihood of an event occurring.

The following section shows how Exercise 7 attends to the core ideas and its critical features of this unit.



Scratch cards

In this type of gambling, you scratch off a thin film that hides the game board. Usually a winning card has three identical amounts: what is the amount you won. However, there are lots of different types of these cards, and some have different rules. These scratch cards range in cost from R2 to R10 and you can win up to R50 000.

EXERCISE 7

Jackpot scratch cards are sold for R3 each, but only 2 million cards have been printed (nationally). For every four cards sold, only one will be a winning card – and the prizes range in value from R10 to a maximum of R10 000.

Value of prize	Number of prizes available
R10 000	3
R2 500	15
R500	112
R100	8 763
R50	40 423
R25	135 836
R10	294 846

1 What is the cost of one of these Jackpot scratch cards?
2 What is the probability of winning R10? Express your answer as a percentage.
3 What is the probability of winning R2 500? Express your answer as a fraction and a percentage.
4 Tommy bought 10 Jackpot scratch cards to improve his chance of winning R10 000.
4.1 Did buying 10 tickets significantly improve his chances of winning the jackpot? Give a reason for your answer.
4.2 The results of one of Tommy's cards are shown alongside. How much money did Tommy win?
4.3 The only winning ticket that Tommy purchased is the ticket shown to the right. Was it worthwhile for Tommy to buy 10 scratch cards? Give a reason for your answer.
5 There is a 13-digit number on the bottom right-hand side of the ticket. What is this number? Why do you think this number is written on the card?
6 Why do you think people play the lottery? Give at least three reasons.

Figure 14: Analysing Exercise 7

Source: Reproduced with permission from Maskew Miller Learning

This exercise is based on gambling using scratch cards. Out of a total of 2 million cards that can be purchased. there is a table provided that shows the number of prizes available according to the amount that can be won. These prizes range from 3 prizes worth R10 000 to R294 846 prizes worth R10. This table expresses the limited odds of winning R10 000 and the high chances of winning R10.

The context is very important in analysing Mathematical Literacy questions. Some of the questions that asked focus on the particular context used. The learners need to engage with determining the probability of winning based on the possible tickets that can be bought. However the focus of this question is only on the value of tickets bought.

4.4. The patterns of variation

In the preceding section, the paper engaged with how the examples and the sequencing of examples attend to this critical feature(s). This section further expands on the component of

variation by identifying the patterns of variation in the three units that have been analysed. *Platinum Mathematical Literacy* is not a variation-informed textbook, thus identifying the patterns is more about explaining how the examples are ordered to enhance understanding. As explained in Chapter 2, the patterns of variation are a description of the order in which examples are ordered. This section will consider both worked examples and exercise examples.

Worked example 1

There is no variation because there is only one question.

Worked example 2

In this example, the four questions show probability as a fraction, decimal or percentage. These values are then interpreted in terms of the likelihood/ chance of occurrence. This allows learners to contrast two critical features. This example shows the pattern of **contrast.**

Worked example 3

This example shows a pattern of **similarity** because there is a comparison of the same critical feature which is the total possible outcomes. The learners are not allowed to contrast this feature with other features.

Worked example 4

This example also shows **similarity**. The central critical feature that is addressed in the example is determining the total possible outcomes. This critical feature is the same throughout the example given.

Worked example 5

This example compares more than one critical feature. The first question is about determining the likelihood of an event occurring. The second question is about representing probability as a fraction and as a percentage. The third question is about interpreting the table in the given context. The last question is also about expressing the probability.

Worked example 6

The critical feature of this unit section is about the application of probability in everyday life contexts. This example does not address any of the critical features that have been identified in the entire chapter. This example is about interpreting the monetary value.

4.4. SUMMARY: The patterns of variation

Understanding that the textbook was not informed by the theory of variations is important to observe what the results tell us about our framework. The results of the analysis above show that few examples show variation. Table 10 below illustrates the possible affordances that may be a result of considering variation in a textbook.

The dyadic framework for comprehensive textbook analysis considered the general approach and the extent to which the textbook attended to the critical features of the object of learning. Table 10 below is a deduction that can be used to generalise the possible affordances that may be attained if engaging with a theory-informed textbook.

A theory informed textbook

General approach / Variation	**Deductive**	**Inductive**
Variation informed	a) Deductive +Variation	b) Inductive + Variation
Non-variation informed	c) Non-variation informed + Deductive	d) Non-Variation informed Inductive

Table 10: General approach +patterns of variation

a) A textbook that is quasi-deductive and variation-informed

The affordances of a deductive textbook are that the examples give learners information that is necessary to understand the topic before they engage with questions that require them to apply their knowledge. The affordances of a variation-informed textbook are that it allows learners to discern the critical features of the object of learning. Therefore, a textbook that is quasi-deductive and variation-informed is intentional about all the information presented whether it is a narrative

or investigative presentation format. Therefore, the narrative texts and examples are sequenced through the patterns of variation to create discernment of the critical features.

b) A textbook that is quasi-inductive and variation-informed

An inductive textbook is more investigative which requires thoroughness from the learners. The inductive and variation-informed textbook should have well-structured questions that build to generality. The questions should allow learners to discern the critical features of the object of learning as they engage with them. This will encourage in-depth reasoning that is structured towards the attainment of the new information.

c) A textbook that is quasi-deductive and not informed by variation

A textbook that is deductive and not informed by variation is a narrative textbook that may be informed by other theories. The focus of this textbook is to give foregrounding information that is necessary for the learners to understand the object of learning. If examples are varied, they are not intentionally structured to bring out the different critical features. *The Platinum Mathematical Literacy textbook is quasi-deductive and not informed by variation.*

d) A textbook that is quasi-inductive and not informed by variation

A textbook that is investigative and not informed by variation may not necessarily be structured to bring out the critical features but the textbook will encourage higher-order thinking from the learners. The difficulty with this textbook is how the critical features may be exposed by the investigative questions if they are not structured.

CHAPTER 5: Conclusion and recommendations

5.1. Introduction

The desire to conduct this research emanated from challenges that I have faced as a Mathematics teacher having to teach Mathematical Literacy without having acquired training in pedagogical content knowledge required to teach the subject. As it is the case for many other teachers in South Africa, my concern was to find ways to deliver the content accurately to the learners. By having to refer to textbooks and online material I was exposed to several resources. I felt the need to find a framework that can be used to assist with carefully selecting a textbook that will accurately convey the mathematical literacy content to the learners. Thus, the objective of the study was to investigate the affordances for learning probability that are made available by engaging with the *Platinum Mathematical Literacy* Grade 12 textbook.

Three research questions that informed the study have been unpacked in various ways:

1. How is the topic of probability presented in a commonly used Mathematical Literacy textbook?

The presentation of the topic of probability was analysed by investigating the general approach of the textbook. This general approach explains the overall structure of the textbook based on the order of presentation formats identified in the analysed chapter. From the findings, the textbook leans towards using a quasi-deductive approach to present the topic of probability.

2. Which patterns of variation can be identified within the examples found in the chapter on probability?

The theory of variation has three core tenets. Before identifying the patterns of variation, it is important to understand that the object of learning in this paper is referred to as the core ideas of probability. Once that was established, the critical features were identified; this was to establish the extent to which the exercises attend to the critical features identified. Identifying the patterns of variation within the examples is not an independent activity, it is dependent on the variation of the same critical features that were identified.

3. What affordances for learning are made available through engaging with the textbooks?

In each of the phases/sections of the analysis of the textbook, affordances have been identified. Drawing on literature a description of the affordances that would occur in a variation-informed textbook has also been stipulated.

5.2. Findings and recommendations

The findings of this study are based on the order in which the research was conducted. The dyadic framework for comprehensive textbook analysis informed this order of research. The three main findings are based on the following ideas; the general approach, developing an understanding of the critical features, and lastly understanding how the example space created helps to create discernment.

It is important to consider the general approach of a textbook to encourage a particular type of reasoning. If there is a need to encourage investigative thinking, one should consider a textbook that is quasi-inductive. However, if you want to encourage deductive reasoning a teacher should consider using a quasi-deductive textbook. For the purpose of this research, the Platinum textbook was identified as a quasi-deductive textbook.

The quasi-deductive approach is relevant to the structure of schooling in South Africa, where learners are allowed to change from Mathematics to Mathematical Literacy until Grade 11. A deductive textbook is effective in encouraging a narrative explanation of the content, this helps to bridge the gap between the knowledge that learners should have learnt in Grades 10 and 11.

An understanding of critical features should be encouraged to create better opportunities to learn. For learners to discern the critical features of an object, the features must first be exposed. This would mean that effective learning material must expose its critical features. The Platinum textbook does expose the critical features. However, the only concern is that the exercises are not intended to allow learners to engage with all the critical features and secondly, there are very few patterns of variation.

When analyzing the patterns of variation, a lens is cast on the order in which the critical features vary. Therefore, an awareness of the critical features is an important aspect to consider when selecting a textbook. Second, when choosing examples for teaching, intentionally sequencing examples is vital to expose the critical feature to encourage discernment of these features.

5.3. Limitations to the study

Language issues in mathematics education is a growing field of study. Some studies have interrogated the use of language in mathematics, as to how that limits the access learners have to the content since they may grapple with the language before they understand the meaning in the context of mathematics. This study does not consider the issue of the language used with the topic analysed. There remains a need to consider the linguistic implications of analysing a mathematical literacy textbook that uses more English to put across its ideas.

The second limitation of the study is that there are no human participants in this study. This would have added a layer of benefit to the research. By allowing learners and teachers to engage with the textbook, the idea of theory in practice would have been unpacked. This limitation is valid, but before analysing the textbook in the classroom context, it is important to analyse the affordances that the textbook offers.

Hence, that may be regarded as a possible future research topic.

5.4. How the research has influenced my practice

Researching textbooks has made me more aware of the nature of the textbook used. As a teacher, I have now become more purposeful in my lesson planning and delivery so that it is not just about giving learners a lot of questions, it is about how these questions will expose the critical features of the object of learning.

Before my understanding of the variation theory, I would just use the questions as they appear in the textbook. I was not exposed to a framework that explains the need to be intentional with the work that I give my learners. Second, I always used the deductive method of teaching in my class. This creates a sense of dependency of the learners. It reduces the learners' opportunity to independently draw conclusions.

5.5. What does this imply for the Platinum Mathematical Literacy Grade 12 textbook?

A textbook is said to be deductive when the author carefully explains ideas, presents fresh ideas, provides definitions, and then sets exercises. In adopting a deductive strategy, researchers will begin with a strong hypothesis and use evidence to examine its implications (Harel & Weber, 2020). Expressed differently, going from broader to more specific areas. In light of this, the

deductive reasoning of the textbook: P*latinum Mathematical Literacy* is most commonly linked to scientific research. It presents information about what other people have done, examines accepted theories about the subject matter under analysis, and assesses concept-derived assumptions in a manner that makes its questions clear. As clearly stated at the beginning, the *Platinum Mathematical Literacy* textbook is not informed by the variation theory.

5.6. What opportunities 'Affordances' does it allow the learners?

In the context of teaching mathematics, the affordances a textbook's deductive approach offers learners are that, by definition, deductive inference remains a conclusion derived from premises from which there are reasonable grounds to conclude that the conclusion is required (Stylianides, Stylianides & Weber, 2017). In other words, the conclusion cannot be untrue when the premises are true. Most significantly, the textbook's rational foundations for Mathematical literacy indicate that new knowledge is obtained from a set of premises through a series of logical conclusions.

5.7. What would it mean if the textbook was inductive?

Should an inductive method be presented, it implies that the writer applies an interpretation towards the solution to a problem (Břehovský, 2010). A new problem arises that is further defined; the author addresses the difficulties without providing the reader with any space; formulae are derived and so on. Validating the hypothesis or via return deduction, is the final step in this inductive process (Břehovský, 2010). It is obvious that the inductive approach takes a lot longer and is more challenging for educators and learners than the conventional approach. However, it also includes valuable and significant learner activities that focus on empowering learners to take an active role in their education. These activities include research, formulation and solution of issues, observation or creation and verification of assumptions.

5.9. Final remarks

The dyadic framework for comprehensive textbook analysis was successful in helping to analyse the *Platinum Mathematical Literacy* textbook. The results obtained from this study do not only inform readers about the trend of the book but also allow for refining the theoretical lens for analysing textbooks. The limitations of the study create an eagerness to conduct a follow up

study in the classroom context. This research has not only informed me about the findings of the structure of probability but also influenced my own practice as a Mathematical Literacy teacher.

References

Adler, J., & Ronda, E. (2017). Mathematical discourse in instruction matters. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 64–81). New York, NY: Routledge.

Akerlind, G. (2015). From phenomenography to variation theory: A review of the development of the variation theory of learning and implications for pedagogical design in higher education. *Review of Higher Education*, 2, 5-26.

Annisavitri, R., Sa'dijah, C., Qohar, A., Sa'diyah, M. & Anwar, L. (2020). Analysis of mathematical literacy test as a problem-solving ability assessment of junior high school students. *AIP Conference Proceedings*, 2215.

Askew, M. (2012). *Transforming Primary Mathematics*. New York: Routledge.

Bansilal, S., Webb, L. & James, A. (2015). Teacher training for mathematical literacy: A case study taking the past into the future. *South African Journal of Education*, 35, 1-10.

Başar, S. (2021). Exploring EFL teachers' decision-making skills: departure from lesson plans. *ELT Res. J.* 10, 61–82.

Batanero, C., Godino, J. D. & Roa, R. (2004). Training Teachers to Teach Probability. *Journal of Statistics Education*, 12:1. htpps://doi.org/ 10.1080/10691898.2004.11910715

Bentley, J. & Stylianides, G. J. (2017). Drawing inferences from learners' examples and questions to inform task design and develop learners' spatial knowledge. *Journal of Mathematical Behavior*, 47, 35–53.

Bills, E., Dreyfus, T., Mason, J., Tsamir, P., Watson, A. & Zaslavsky, O. (2006).Exemplification in mathematics education. In J. Novotna (Ed.), *Proceedings of the 30th PME Conference* (pp. 126–154).

Břehovský, J. (2010). *Inductive and Deductive Approach in Mathematical Education*. Ph.D.dissertation, Palacký University Olomouc, Department of Algebra and Geometry, Czech Republic.

Bowie, L. & Frith, V. (2006). Concerns about the South African Mathematical Literacy curriculum arising from experience in materials development. *Pythagoras Journal*, 64, pp. 2936.

Brijlal P 2014. An exploration of the contribution of the Advanced Certificate in Education in Mathematical Literacy programme towards the professional development of teachers in KwaZulu-Natal.

Ceretkova, S. & Sedivy, O. (2008). *The Role and Assessment of Textbooks in Mathematics Education*. Palacky University in Olomouc, Czech Republic.

Chevallard, Y. & Bosch, M. (2013). Didactic transposition in mathematics education. In S. Lerman (Ed.) *Encyclopedia of Mathematics Education.* Springer.

De Clercq, F. (2020). The Persistence of South African Educational Inequalities: The Need for Understanding and Relying on Analytical Frameworks. *Education as Change*, 24. https://doi.org/10.25159/1947-9417/7234

De Lange, J. (2006). Mathematical literacy for living from OECD-PISA perspective. *Tsukuba Journal of Educational Study in Mathematics*, 25, 13-35.

Department of Basic Education. (2010) *Diagnostic Report: Annual assessment national assessment.* Pretoria: DBE

Department of Basic Education. (2012) *Diagnostic Report: Annual National Assessment 2012*. Pretoria: DBE

Department of Basic Education. (2013). *Diagnostic Report: National senior certificate examination 2013*. Pretoria: DBE

Department of Basic Education. (2014a). *Diagnostic Report: National senior certificate examination 2014*. Pretoria: DBE

Essien, A. A. (2021). Understanding the choice and use of examples in mathematics teacher education multilingual classrooms. ZDM Mathematics Education. https://doi.org/10.1007/s11858-021-01241-6.

Frith, Jakins, Winfield, & Yeo, (2013). Platinum Mathematical Literacy Grade 12 learners; book. Pearson South Africa (Pty) Ltd. http://schools.pearsons.co.za

Gal, I. (2009). South Africa's Mathematical Literacy and Mathematics curricula: Is probability literacy given a fair chance? *African Journal of Research in Mathematics, Science and Technology Education, 13*(1), 50–61.

Graven, M. & Venkat, H. (2007). Emerging pedagogic agendas in the teaching of Mathematical Literacy. *African Journal of Research in SMT Education*, 11(2) 2007, pp. 67-86.

Huang, R. & Li, Y. (2017)Teaching and learning through Variation. Vol (2) Confutation *Heritage meets Western theories*: Sense Publishers

Jojo, 2019, Z. Jojo (2019) Mathematics education system in South Africa Education systems around the world

Kilpatrick, J., Swafford, J. & Bradford Findell, B. (eds.). (1992) *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academy Press, 2001), pp. 116, 118, 121, 124, 127, 129, 131.

Harel, G. & Weber, K. (2020). Deductive Reasoning in Mathematics Education. In: Lerman, S. (eds). *Encyclopedia of Mathematics Education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_43

Krista Yulita, K., Nusantara, T., As'ari, A. R., & Sa'dijah, C. (2018). Identification of Students Errors in Solving Indirect Analogical Problems Based on Analogical Reasoning Components. *Journal of Physics*: Conference Series (Vol. 1028, p. 012154). IOP Publishing.

Kullberg, A., Kempe, U. R. & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM Mathematics Education*, 49, 559 – 69.

Leshota, M. (2015). *The relationship between textbook affordances and mathematics teachers pedagogical design capacity (PDC).* Doctoral Thesis, University of the Witwatersrand, Johannesburg.

Leshota, M. (2017). *The framework for determining the embedded instructional approach in mathematics textbooks (3, pp. 25-32).* Pretoria, South Africa; PME

Lo, M. L. & Chik, P. P. M. (2016). Two Horizons of Fusion. *Scandinavian Journal of Educational Research, 60*(3), 296-308. htpps://doi.org/10.1080/00313831.2015.1119730

Lo, M. L. (2012). *Variation theory and the improvement of teaching and learning*

Göteborg, Sweden: Acta Universitatis Gothoburgensis.

Leung, A. (2012). Variation and mathematics pedagogy. In J. Dindyal, L.P. Cheng, & S.F. Ng (Eds.), *Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 433–440). Singapore: MERGA. Available from

Marton, F. & Booth, S. (1997). *Learning and Awareness*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Marton, F., & Tsui, A. (2004). *Classroom Discourse and the Space of Learning*. Mahwah, NJ: Lawrence Erlbaum Associates.

Marton, F. (2015). *Necessary conditions of learning*. New York: Routledge.

Marton, F & Pang, M.F. (2016). On some necessary conditions of learning. *The Journal of the Learning Sciences*, 15, 193-220.

Mason, J. & Pimm, D. (1984). Generic examples: Seeing the general in the particular. *Educational Studies in Mathematics*, 15(3), 277–290.

Mason, J. (2017). Issues in variation theory and how it could inform pedagogical choices.

Maxwell, J. (1992). Understanding and validity in qualitative research. Harvard Educational review, 62(3), 279-301.

Mbekwa, M. (2006). Teachers' views on mathematical literacy and their experiences as students of the course. Pythagoras 63, June, 2006, pp.22-29.

Huang, & Li, Y. (Eds.), *Teaching and learning mathematics through variation* (pp. 407- 438). Sense Publishers.

Masjaya, M. & Wardono, A. (2018). AUN-QA, PRISMA, Prosiding Seminar Nasional Matematika: PRISMA. *Unnes Journal of Mathematics Education*, 7(1), 568-57414.

Metz, M., Preciado-Babb, A. P., Sabbaghan, S., Davis, B., Pinchbeck, G. & Aljarrah, A. (2016). Transcending traditional/reform dichotomies in mathematics education. In M. B. Turner, E. E. Turner, M. Civil & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. (pp. 1252-1258). Tuscon, AZ: The University of Arizona.

Mhlolo, M.(2021) The merits for teaching mathematics with variation. *Pythagorus*, 34(2)Art #233, 8.

O'Keefe, L. (2011). *The use of evidence based Research on Mathematics textbooks to increase Student Conceptual understanding.* Research gate: University of South Australia.

O'Keefe, L. (2012). *A Framework for Textbook Analysis.* Research gate: University of South Australia.

Peled, I. & Zaslavsky, O. (1997). Counter-examples that (only) prove and counter-examples that (also) explain. *FOCUS on Learning Problems in Mathematics*, 19(3), 49–61.

Rowland, T. (2008). The purpose, design and use of examples in the teaching of elementary mathematics. *Educational Studies in Mathematics*, 69(2), 149–163.

Sevimli, E. (2016). Investigation of structural features of examples which textbook located and lecturers preferred in Calculus. *Education and Science*, 41(183), 319–339.

Sinclair, N., Watson, A., Zazkis, R. & Mason, J. (2011). The structuring of personal example spaces. *Journal of Mathematical Behavior*, 30(4), 291–303.

Stacey K 2011 The PISA View of Mathematical Literacy in Indonesia. Journal Mathematics Education 2 pp 95-126

Stylianides, G., Stylianides, A., &Weber, K. (2017). Research on the teaching and learning of proof: taking stock and moving forward. In: Cai J (ed) *Compendium for research in mathematics education.* Reston: National Council of Teachers of Mathematics.

Thomson, S., Hillman, K. & Bortoli, L. (2013). *Teacher's Guide to PISA Mathematical Literacy* Adelaide: ACER Press.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H. & Houang, R. T. (2002). *According to the book: Using TIMMS to investigate the translation of policy into practice through the world of textbooks*. The Nertherlands: Kluwer

Vithal, R. Bishop, A (2006) Mathematical Literacy: A new literacy or a new mathematics? *Pythagoras,* 64, pp. 2-5.

Watson, A. & Mason, J. (2006). Seeing an Exercise as a Single Mathematical Object: Using Variation to Structure Sense-Making. Mathematical Thinking and Learning, 8(2), 91111.

Watson, A. & Chick, H. (2011). Qualities of examples in learning and teaching. *ZDM-Mathematics Education*, 43(2), 283–294.

Watson, A. & Mason, J. (2005). *Mathematics as a Constructive Activity: Learners Generating Examples*. Lawrence Erlbaum.

Watson, A. & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning*, 8(2), 91-111.

Yopp, D. A. & Ely, R. (2016). When does an argument use a generic example? *Educational Study in Mathematics*, 91, 37–53.

Zaslavsky, O. (2010). The Explanatory Power of Examples in Mathematics: Challenges for Teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp.107-128): Springer US

Zaslavsky, O. (2019). There is more to examples than meets the eye: Thinking with and through mathematical examples. *Journal of Mathematical Behavior*, 53, 245–255.

Zodik, I. & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics*, 69, 165-182'

APPENDICES

Appendix A: Turnitin report

Appendix B: Ethical Clearance

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2022ECE008M**
PROJECT TITLE	An Analysis of the Affordances for learning Probability in two commonly used Grade 12 Mathematical Literacy textbooks
INVESTIGATOR	**STHEMBILE LEBOHANG MAGWAZA**
SCHOOL/DEPARTMENT OF INVESTIGATOR	**WSoE**
DATE CONSIDERED	07 FEBRUARY 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	NO RISK
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	**CHAIRPERSON** Dr. Batseba Mofolo-Mbokane

cc: Prof. Anthony Essien

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

_____ **Signature**

_____/_____/_____ **Date**

maskew miller learning

Maskew Miller Learning
10 Freedom Way, Milnerton,
Cape Town 7441, South Africa
T: +27 (0)21 532 6000

Permission to use copyrighted material

18 June 2024

Dear Ms. Magwaza

We refer to your request dated 5 April 2024 to use extracts from copyrighted material belonging to Maskew Miller Learning namely, *Platinum Mathematical Literacy Grade 12_9780636143357*, in your study called Expressions of probability, determining possible outcomes and Evaluating expressions involving probability.

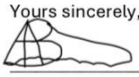
It is our pleasure to hereby grant you permission to use extracts from pages 239, 244 and 245 from the above-mentioned title, subject to the standard terms and conditions contained in Schedule A, attached hereto, as well as the following special conditions:

- Proper acknowledgement of the Publisher, the source and the authors.
- Language(s): English
- Territory(s): South Africa
- Fee: None
- No Fee is payable.
  Conditions: The content will only be used in the mentioned study and in a presentation at the AMESA conference in June 2024.

Kindly sign this letter and the attached Schedule A to indicate your acceptance of the terms and conditions pertaining to the grant of the permission and return it to us within 30 (thirty) days of date hereof

Please note that the permission contained herein shall only become effective once we receive the signed copies of this letter and Schedule A.

Should you have any queries, please contact our *Rights and Permissions* Team at rightsgranting@mml.co.za
Yours sincerely,



On behalf of Maskew Miller Learning (Pty) Ltd
Per: Dr Benadette Aineamani
Executive: Product and Services