supports to this remaining portion of concrete can be considered to be the compression zone of the structural element at one end, and the flexural reinforcement of the element at the other. Analysing this approximately triangular remaining portion using linear finite elements in plane stress, as indicated in Appendix G, leads to the derivation of an equivalent prismatic beam to represent this phenomenon as indicated in Figure 8.3. Since the shear displacements are proportional to the flexural displacements for the prismatic beam considered, an equivalent depth based solely on flexural parameters can be derived. The equivalent depth for the prismatic beam subjected to the effects of bending only has thus been derived from these linear finite element analyses to be 0,25u, as indicated in Figure 8.3. It follows from the observation that crack opening is caused by effects both sides of the diagonal shear crack that this equivalent prismatic beam occurs both sides of the crack and is intended to represent half the total average effect.

The second moment of area for the equivalent prismatic beam under consideration is thus given by:

$$I = b\ (0{,}25u)^3\ /12$$

thus

$$I = 0{,}0013bu^3$$

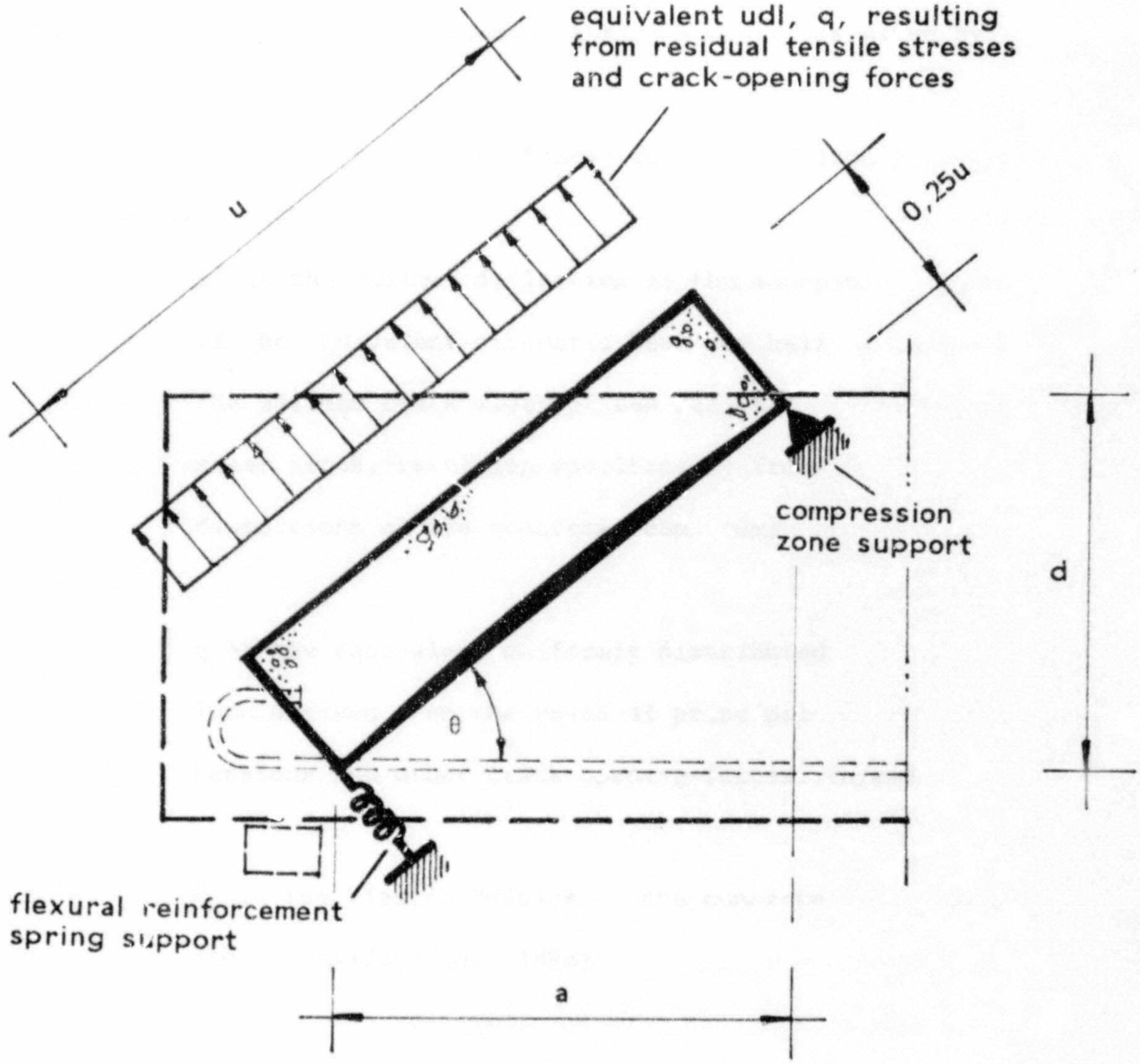


FIGURE 8.3 SIMPLIFIED EQUIVALENT PRISMATIC BEAM MODEL

The maximum deflection of this beam at midspan, neglecting provisionally the influence of the reinforcement spring support, is given by:

$$w' = (5/384)\ qu^4 /\ E_c(0{,}0013bu^3)$$

where

$w'$ is the maximum deflection at the midspan of the equivalent prismatic beam, or half the maximum crack width of the diagonal shear crack, resulting specifically from distortions of the concrete beam. (mm)

q is the equivalent uniformly distributed load derived from the residual principal tensions and other crack-opening forces. (N/mm)

$E_c$ is the Elastic Modulus of the concrete under consideration. (MPa)

The deflection (or half crack width) can thus be given by:

$$w' = 10\ qu/E_c b$$

Since q is a result of the residual principal tensions and crack-opening forces due to the aggregate interlock component of shear resistance, and because the initial shear crack commences at a theoretical slope of 45° at the neutral axis, it is evident that the average distributed load, q, varies as the slope of the diagonal shear crack. The variation observed, by assessing the average principal tensions in linear plane stress finite element analyses on a model as shown in Figure F2 of Appendix F, was approximately proportional to $\cot\theta$ for the range of $\theta$ considered in this formulation, with the unit value being assumed at a shear crack slope of 45°. The equivalent uniformly distributed load was thus assumed to be related to the aggregate interlock shear stress component by the following approximate relationship:

$$q = v_a b \ \cot\theta$$

where

$v_a$ is the aggregate interlock shear stress component of total shear resistance in MPa.

It is evident from Figure 8.1 that the idealised true crack length, u, is related to the effective depth, d, of the section under consideration by the following:

$$u = d/\sin\theta$$

Substituting the expressions for q and u into the relationship for the maximum deflection (or half maximum crack width), w', at the midspan, yields the following expression:

$$w' = 10 \ \frac{v_a b}{\tan\theta} \cdot \frac{d}{E_c b \sin\theta}$$

This expression simplifies to the following:

$$w' = 10 \frac{v_a d}{E_c} \cdot \frac{\cos\theta}{\sin^2\theta}$$

The effects of the flexural reinforcement of the structural element should now be taken into account. This reinforcement can reasonably be considered to be a spring support at one end of the equivalent prismatic concrete beam, as indicated in Figure 8.3. Electric strain gauges placed on the flexural reinforcement as in Chapter 4 indicate that it is stressed over some distance from the point where it is intersected by the diagonal shear crack and unyielded in average uniaxial tension at this point. If average uniaxial yield does occur at this point, then a flexural hinge is formed and the subsequent behaviour would not be reflected by the model which is being formulated here. Dowel bending, however, is generally evident in the region immediately adjacent to the point of intersection, particularly as the instant of maximum load is reached. Zones of exreme fibre yield of the flexural reinforcement might even be present. This effect, combined with the presence of an average uniaxial tension in the flexural reinforcement, represent the spring support indicated in Figure 8.3. A reasonable average value to consider for the zone of the steel reinforcement which is able to strain in tension with associated bond slip is related to the bond length required to develop the full tensile capacity of the flexural reinforcement. As mentioned above, the true mechanics of this spring support are further complicated by the presence of double curvature dowel bending in the close proximity of the shear crack. Rather than attempt to quantify precisely the mechanics of the reinforcement subjected to both tensile and bending stresses, which may or may not be at yield locally, an equivalent spring model based on the observed test results and also based on a simple, rational axial stiffness parameter for the flexural reinforcement has been developed.

From equilibrium considerations, the tension developed in the flexural reinforcement due specifically to the effects of aggregate interlock stresses can be considered to be the reaction at that end of the equivalent prismatic beam. The tension in the flexural reinforcement is thus given by:

$$T = qu/2$$

While the reaction, T, in this equation does not represent the tension in the flexural reinforcement specifically, it certainly does contribute to this axial tension. This is consistent with the "tension shift" in flexural reinforcement noted by other researchers[32] and also reflected in other model formulations[42]. Substituting for q and u as before:

$$T = v_a \, bd/(2\tan\theta\sin\theta)$$

thus

$$T = \frac{v_a \, bd}{2} \cdot \frac{\cos\theta}{\sin^2\theta}$$

For steel reinforcement of Elastic Modulus $E_s$ and cross-sectional area $A_s$, and equivalent length x over which axial strain with bond slip occurs, the displacement, y, at the support of the prismatic concrete beam can be given by:

$$y = Tx/E_s A_s$$

Substituting for T from the above:

$$y = \frac{v_a x}{2E_s} \cdot \frac{bd}{A_s} \cdot \frac{\cos\theta}{\sin^2\theta}$$

The deflection at the midspan of the equivalent prismatic concrete beam, w", resulting specifically from the displacement, y, of the spring support at one end, is thus given by:

$$w'' = \frac{v_a x}{4E_s} \cdot \frac{bd}{A_s} \cdot \frac{\cos\theta}{\sin^2\theta}$$

If the flexural steel ratio expressed as a percentage, $100A_s/bd$, is denoted by the conventional notation, $\rho$, then the deflection at the midspan of the equivalent concrete beam can be expressed as:

$$w'' = \frac{25v_a x}{E_s} \cdot \frac{1}{\rho} \cdot \frac{\cos\theta}{\sin^2\theta}$$

A value for x corresponding to a reasonable length required to develop full average bond for a bar of average diameter has been adopted in this equation. A value of x of 400mm is thus used, based on 20 diameters of a 20mm diameter bar. This is not as unacceptably simplistic as first appears, since the intent is to include effects of dowel bending. As the bar diameter decreases, the required bond length reduces, corresponding to an apparent increase in axial stiffness. Dowel stiffness, however, reduces with reducing bar diameter. The two infuences are thus mutually compensatory and a reasonable average value is thus considered to be acceptable. The Elastic modulus for the steel reinforcement is taken to be 200GPa. Substituting these values into the above equation results in the following expression:

$$w'' = v_a \frac{0,05}{\rho} \cdot \frac{\cos\theta}{\sin^2\theta}$$

Summing the effects of the two displacements which occur at the midspan of the diagonal shear crack, the total displacement is given by:

$$w = w' + w''$$

thus

$$w = v_a \left( \frac{10d}{E_c} + \frac{0,05}{\rho} \right) \frac{\cos\theta}{\sin^2\theta}$$

From the maximum diagonal shear crack width of 1,3mm, the absolute value of the deflection, w, in the above relationship can be given as 0,65mm, as it corresponds to half the observed crack width. The above expression can thus be rewritten as:

$$v_a = \frac{0,65}{10d/E_c + 0,05/\rho} \cdot \frac{\sin^2\theta}{\cos\theta}$$

where

$v_a$ is the ultimate aggregate interlock shear stress component of shear resistance in MPa.

$E_c$ is the Elastic Modulus of the concrete used in the structural element in MPa.

$\rho$ is the flexural steel ratio expressed as a percentage. The flexural reinforcement referred to here is that which intersects the diagonal shear crack at the end remote from the compression zone and which should be anchored a full average bond length either side of this point of intersection.

$\theta$ is the slope of the idealised diagonal shear crack to the axis of the element.

*This equation* describes a model for the ultimate shear "stress" resistance, $v_a$, of the aggregate interlock component of overall shear resistance for reinforced concrete sections unreinforced for shear. Because this model represents a mean ultimate limit state phenomenon, it must be adjusted to a characteristic value and then further adjusted by a partial material factor before use in a design application. It is clear, however, that the design application of this fundamental, simple, parametric equation is virtually direct. While the number of tests in this program might not be sufficient to determine an accurate assessment of the resulting characteristic value, it would appear from Appendix E that a coefficient of variation of approximately 16% can be anticipated in the test programme. This would imply a characteristic value of approximately 75% of the mean ultimate model value.

A partial material factor, consistent with current code formulations of 1,25, could then apply to this characteristic value for direct design application.

Examination of this equation reveals that the aggregate interlock component of shear resistance increases with the Elastic Modulus of the concrete, which is traditionally regarded as being related approximately to the square root of the grade of concrete. The variability normally associated with quantifying the Elastic Modulus of concrete is consistent with the variability associated with shear test results in general. It is also evident from the equation that the aggregate interlock stress resistance decreases with increasing depth of section, d. The model is unique in this regard. The model further predicts an increase in aggregate interlock resistance with increased steel ratio, but is, however, not very sensitive to this parameter. The variation of the resistance with a/d ratio is also modelled, as the aggregate interlock stress component can be seen to depend on the variable $\theta$, where $\theta = \tan^{-1} d/a$. The model is likely to lose accuracy for very steep cracks, where a/d tends to 0, as mentioned previously. It is postulated that in this case shear behaviour will be determined primarily by particle interference and local crushing, and the evaluation of the mechanics of these phenomena[44] will model the structural element as a/d tends to 0. This type of behaviour is also likely to be true for very thin sections, where the absolute maximum crack width postulated cannot be achieved without prior particle interference and crushing.

For values of $\theta$ less than approximately $\tan^{-1}0,5$, the diagonal shear crack becomes so flat that the average diagonal tensions are no longer of sufficient magnitude to open a potential diagonal shear crack.

It has thus been demonstrated above that the aggregate interlock component of shear stress resistance for elements unreinforced for shear is dependent on four parameters, these being the effective depth, d, the shear arm ratio, a/d, the Elastic Modulus of the concrete, $E_c$ and the flexural steel ratio, $\rho$. The model proposed here for the aggregate interlock component of total shear resistance must still be augmented by the contributions from dowel action and compression zone.

### 8.1.2 THE CONTRIBUTION OF COMPRESSION ZONE

The compression zone contribution can reasonably be assumed to vary only with grade of concrete, while remaining relatively independent of the other three parameters (i.e. a/d ratio, effective depth and flexural reinforcement ratio), for elements unreinforced for shear. This can be justified in the formulation of a universal model for shear behaviour for a number of reasons.

An examination of Figures 7.1 to 7.3 in Chapter 7 reveals that as the structural element approaches the shear ultimate limit state, the diagonal shear crack penetrates well into the compression zone, leaving a compression zone area contributing to shear resistance which is virtually independent of depth of section. Further examination of the same figures reveals that this behaviour also occurs relatively independently of the slope of the crack, i.e. the a/d ratio.

The behaviour of the compression zone with respect to the shear capacity of reinforced concrete beams unreinforced for shear has been studied in considerable detail by Taylor[40,41], and for specimens with a/d ratios in excess of 2, this study indicated that a compression zone contribution of the order of 20% could be anticipated. It is postulated within the scope of this thesis that the absolute value of compression zone resistance remains approximately constant with reducing a/d ratio and effective depth. This implies that the relative contribution will reduce significantly with these two parameters, because absolute aggregate interlock increases rapidly with reduction in these two parameters, as indicated in the parametric equation developed in

Chapter 8.1.1. The overall implication of this evaluation is that compression zone contribution will be a maximum of 20% and will often be even less for elements unreinforced for shear. For elements reinforced for shear, the contribution of compression will be further reduced. It is thus evident that the proposed model for shear will be very insensitive to errors in the assessment of compression zone contribution in general. Because of this, the mechanics of the compression zone resistance are not of significance in terms of the proposed model for shear, and have thus not been studied in further detail in this work. It is also apparent from the tests carried out in this work that a significant proportion of the compression zone contribution is lost abruptly, simultaneously with the aggregate interlock component, and only a friction component is active in the post-peak zone of the shear load-deflection relationship.

In the formulation of the proposed universal model for shear, the absolute compression zone contribution has thus been assumed to vary with the square root of mean concrete cube strength, and is assumed to remain constant for the other three parameters which influence shear behaviour of elements unreinforced for shear.

### 8.1.3 THE CONTRIBUTION OF DOWEL ACTION

The dowel action component of ultimate shear resistance has been studied by various researchers[12,14,44], and is dependent on a large number of parameters. Dowel behaviour is also dependent on the presence or absence of vertical links which encompass the flexural reinforcement. Fundamentally, however, it can be assumed that the ultimate dowel capacity generally varies approximately with the square of the dowel diameter and the square or cube root of the concrete grade. It is thus evident that it is far more sensitive to flexural steel ratio than any of the other parameters and as a generalisation can therefore b[illegible]d to vary only with this parameter in terms of this model [illegible]. Although this is a considerable simplification it is justified by the fact that the contribution of dowel action to overall shear resistance, as in the case of the compression zone contribution, will generally be less than 20% for normal reinforced concrete construction. For certain structural elements, in particular in precast work as indicated in Chapter 3, a more accurate assessment of dowel action might be warranted.

### 8.1.4 COMBINATION OF THE COMPONENTS

From the concept of the "standard beam", introduced in Chapter 7, the overall model for shear within the definition of this thesis can be formulated. As determined previously, the standard beam, unreinforced for shear, can be considered to possess the following properties:

Effective depth of 500mm

Concrete of 25MPa mean cube strength.

Flexural steel ratio of 1%

Shear arm ratio of 2 (unconstrained shear crack)

The proportions for the various contributions of shear resistance for the standard beam were also determined in Chapter 7, and for the purpose of the formulation of the complete model for shear proposed within the scope of this work, are given by:

Dowel Action 20%

Compression Zone 20%

Aggregate Interlock 60%

By making use of these proportions and the parametric equations described previously for the contributions of the various components of shear resistance, the proposed model for shear can be

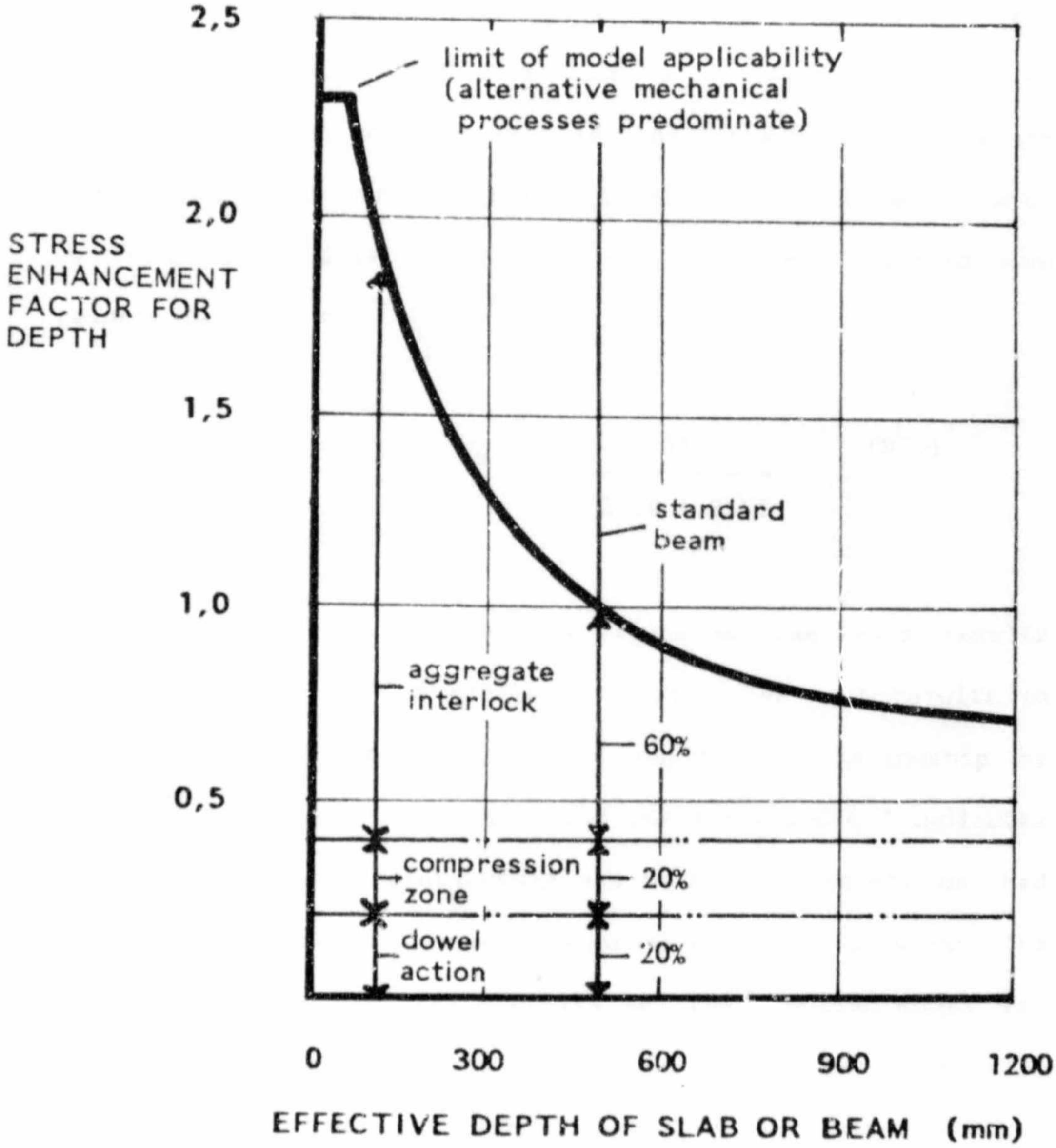


FIGURE 8.4 MODEL VARIATION OF ULTIMATE SHEAR STRESS RESISTANT CAPACITY WITH EFFECTIVE DEPTH OF SECTION FOR ELEMENTS UNREINFORCED FOR SHEAR

depicted graphically as resistant shear stress for each of the parameters which are known to influence shear stress performance. On this basis, the variation of overall resistant shear stress for reinforced concrete structural elements unreinforced for shear with effective depth of section is indicated in Figure 8.4. In this case, the equation defining the contribution of the aggregate interlock contribution to total shear resistance, with all other parameters remaining constant at their standard beam values, is given by:

$$v_a = \frac{0,15}{d/2500 + 0,05} \quad \text{(MPa)}$$

The good correlation between this figure and the tests results shown in Figure 4.7 is clearly evident. The test results in Figure 4.7 are represented by an empirical relationship of $\sqrt[3]{(500/d)}$. A direct comparison of Figures 8.4 and 4.7 indicates that this empirical relationship has been very closely matched by the rational formulation of the proposed model for shear. The good agreement between test results and the proposed model for variation in effective depth for a wide variety of elements is further demonstrated in Appendices A to C.

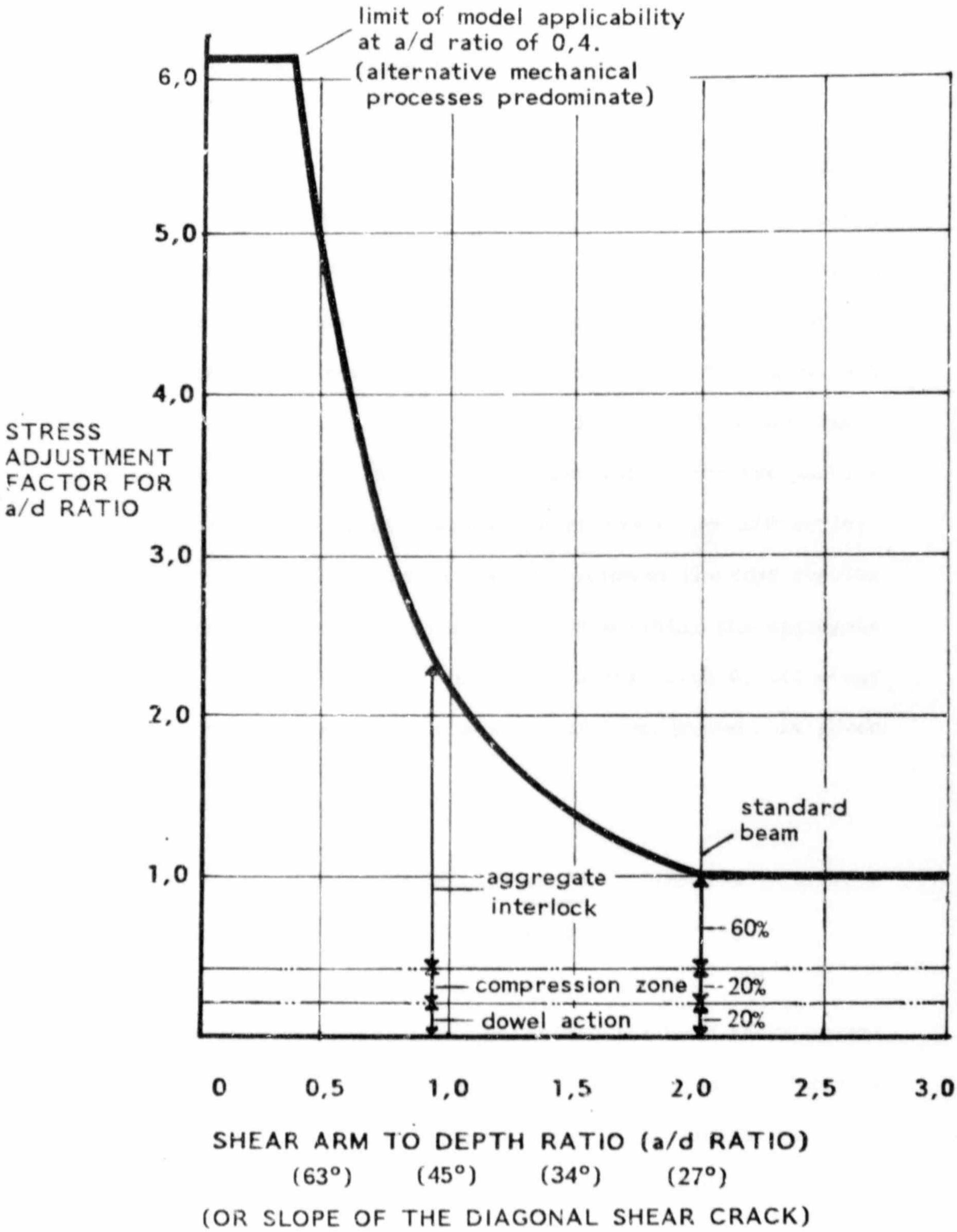


FIGURE 8.5 MODEL VARIATION OF STRESS ADJUSTMENT FACTOR WITH a/d RATIO FOR ELEMENTS UNREINFORCED FOR SHEAR

The curve of the variation of total ultimate shear stress resistance with a/d ratio is depicted in Figure 8.5 on the same basis. The good correlation between this curve and that observed in tests in Figure 4.8 is also evident. The empirical relationship for the factor by which resistant shear stress must be multiplied to take a/d ratio into account is 2d/a for both CP110 and BS0000. A direct comparison between this empirical curve in Figure 4.8 and the curve for the proposed model in Figure 8.5, which has a rational basis, indicates the close correlation for the particular parameter of slope of diagonal shear crack (or a/d ratio). This is further demonstrated in the tabulation of the test results in Appendices A to C. The relationship describing the aggregate interlock component of resistant shear stress with θ, all other parameters being constant at the standard beam values, is given by:

$$v_a = 2,6 \frac{\sin^2\theta}{\cos\theta} \qquad \text{(MPa)}$$

It is clear that the model has limitations for both these parameters as they tend to zero. The practical considerations of depths of section that tend to zero are such that this limitation is is of no consequence generally. The extreme of an a/d ratio that tends to zero, however, has a number of practical applications, such such as web-flange interfaces and checks at column faces for shear in general. The model depicted in these curves, particularly the variation with a/d ratio, will thus be influenced near the vertical axis by the values for shear capacity determined by Walraven and Reinhardt for elements having an a/d ratio of zero. It is important to note that in their tests, they determined the contribution of dowel action to be of the order of 6% for the specimens with zero a/d ratio. This observation is totally consistent wi'l the model depicted in Figure 8.5, where it is evident that the contribution of dowel action reduces significantly as the a/d ratio tends to zero. If a limitation of a/d of 0,4 (corresponding to an angle $\theta$ of 68°) is placed on the model proposed here, then the model predicts results similar to those of Walraven and Reinhardt and can therefore be used to give an approximate evaluation of the ultimate shear capacity of structural systems having zero a/d ratio, such as web-flange interfaces. The proposed cut-off is shown in Figure 8.5. A limitation in terms of the proposed model for shear is obviously also required with respect to depth. A cut-off has thus been proposed at an effective depth of section of the order of 50mm, (corresponding closely to the shallowest sections tested in this programme) as shown in Figure 8.4. This limitation does not have particular relevance to practical reinforced concrete construction, as it would be rather unusual to make use of such shallow sections normally. The fundamental observation (which is well modelled) of increasing resistant shear stress with reducing depth is, however, significant with respect to research, where inaccurate predictions

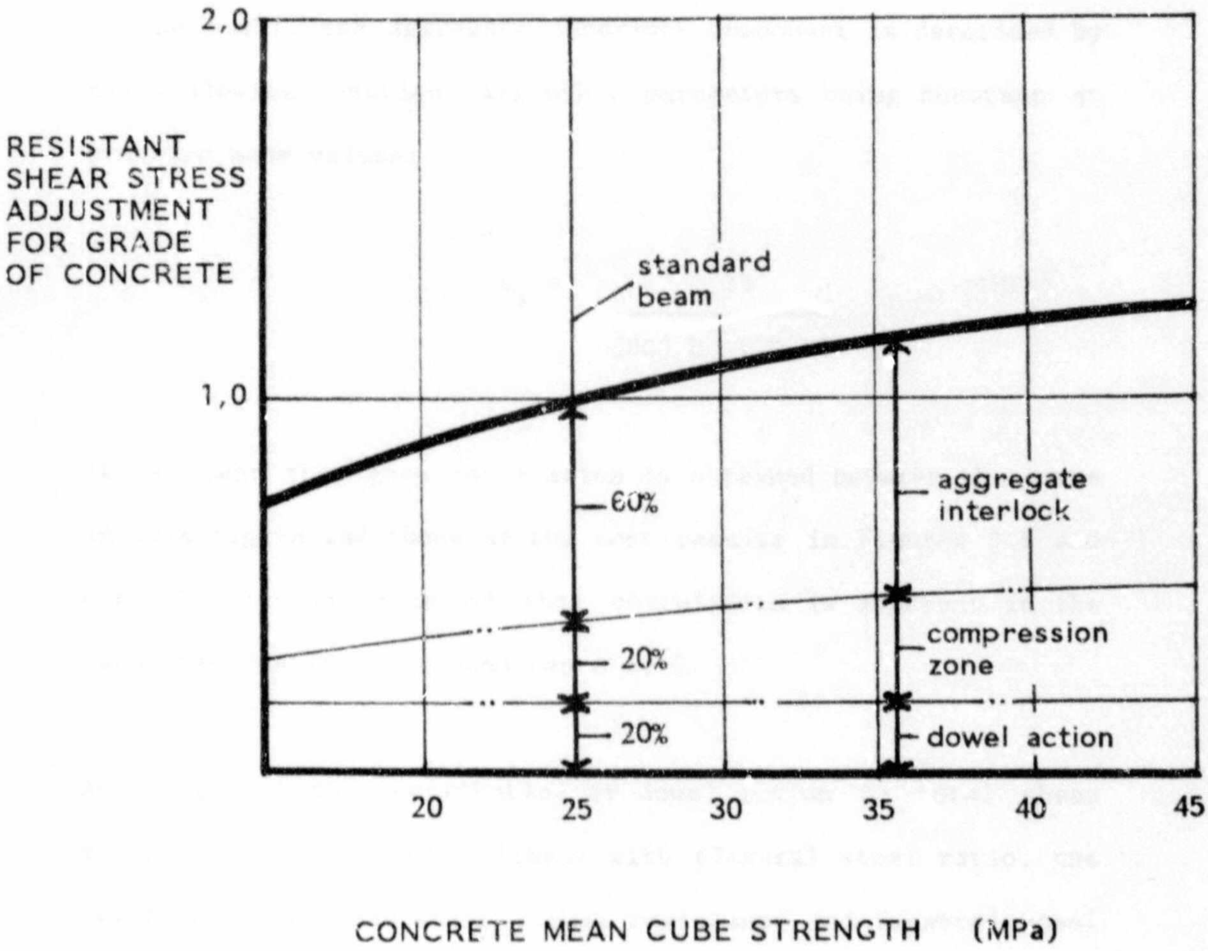


FIGURE 8.6 MODEL VARIATION OF ULTIMATE SHEAR STRESS RESISTANT CAPACITY WITH GRADE OF CONCRETE FOR ELEMENTS UNREINFORCED FOR SHEAR

of shear behaviour can been made owing to the convenient use of very thin elements in laboratory tests.

The model variation of ultimate shear stress resistance with grade of concrete is indicated in Figure 8.6. In this depiction of the model, the aggregate interlock component is described by the following function, all other parameters being constant at standard beam values:

$$v_a = \frac{0,15}{5000/E_c + 0,05} \qquad \text{(MPa)}$$

It is clear that good correlation is obtained between the curve in this figure and those of the test results in Figures 3.9 and 4.6. Further evidence of this correlation is apparent in the tabulated results in Appendices A to C.

Assuming that the contribution of dowel action to total shear resistance is virtually linear with flexural steel ratio, the relationship between shear stress resistance and flexural steel ratio, $\rho$, is depicted in Figure 8.7. The contribution of aggregate interlock in this case, with all other parameters constant at standard beam values, is given by:

$$v_a = \frac{0,15}{0,2 + 0,05/\rho} \qquad \text{(MPa)}$$

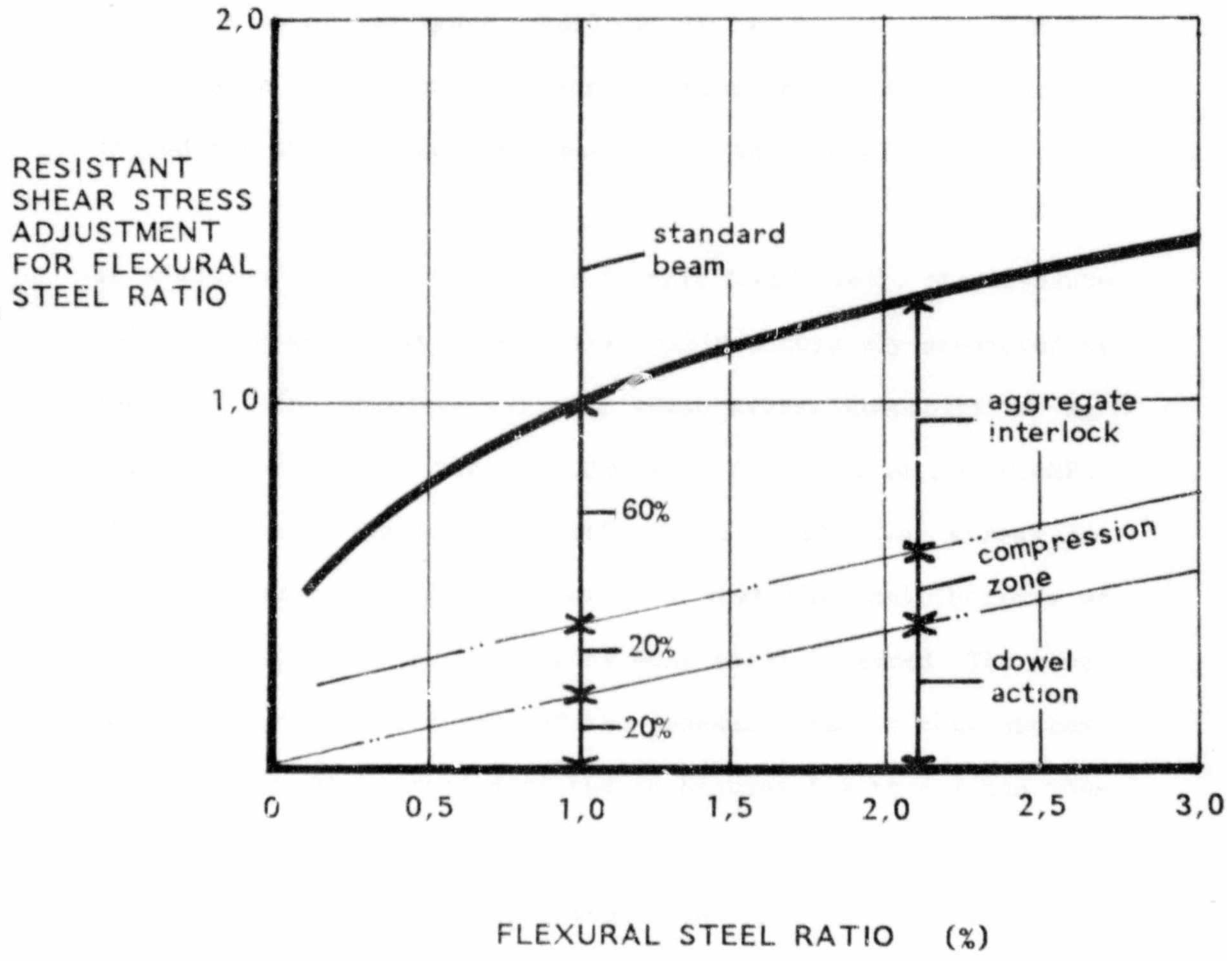


FIGURE 8.7 MODEL VARIATION OF ULTIMATE SHEAR STRESS RESISTANT CAPACITY WITH FLEXURAL STEEL RATIO FOR ELEMENTS UNREINFORCED FOR SHEAR

The good correlation between this curve and those of Figures 3.9 and 4.7 is also evident. Further evidence of this correlation is given in Appendices A to C. In making use of the equilibrium equation which sums the three contributions to total shear resistance and the new mathematical relationship for the particular contribution of aggregate interlock, it can be concluded that the performance of reinforced concrete structural elements unreinforced for shear is well represented by this model.

In addition to modelling the parametric trends well, the absolute value of shear stress is also reasonably accurately predicted by the model. The absolute ultimate shear stress component of aggregate interlock for the standard beam, for the model, is 0,6MPa. This represents approximately 60% of the total shear stress resistance, taking into consideration that the contributions of dowel action and compression zone must still be added. The ultimate resistant shear stress of the standard beam is thus approximately 1MPa. The curves depicted in Figures 8.4 to 8.7 can thus be read off directly in units of stress in MPa. The model depicted by these figures represents the mean ultimate resistant shear stress. Adjusting to a characteristic value of 75% of the mean ultimate model resistance and further reducing this by a partial material factor of 1,25, gives a design ultimate resistant shear stress for the standard beam, unreinforced for shear, of approximately 0,6MPa, which is in very close agreement with both CP110 and BS0000.

The modelling of elements unreinforced for shear thus appears to be extremely satisfactory, and an evaluation of reinforced concrete structural elements reinforced for shear is now required within the definition of this model.

## 8.2 ELEMENTS WHICH ARE REINFORCED FOR SHEAR

### 8.2.1 VERTICAL LINK REINFORCEMENT

Where use is made of vertical links as shear reinforcement in reinforced concrete construction, these will generally be small diameter bars at fairly close centres. A typical portion of such an element is indicated in Figure 8.8, and a reasonable approximation of the influence of the links on the diagonal shear crack shown in this figure can be made in terms of a uniformly distributed load. It is proposed, within the scope of this model for shear, that the links influence shear performance in two ways. Firstly, they influence the fundamental equilibrium equation of shear resistance which recognises the existence of the contributions of dowel action, compression zone and aggregate interlock. The equation for vertical equilibrium now includes the contribution of the link reinforcement, such that:

$$V_R = V_d + V_c + V_a + V_r$$

In this equation all the terms are as previously defined, with the exception of the term, $V_r$, which is defined as the contribution of the link reinforcement. At the instant of shear failure,

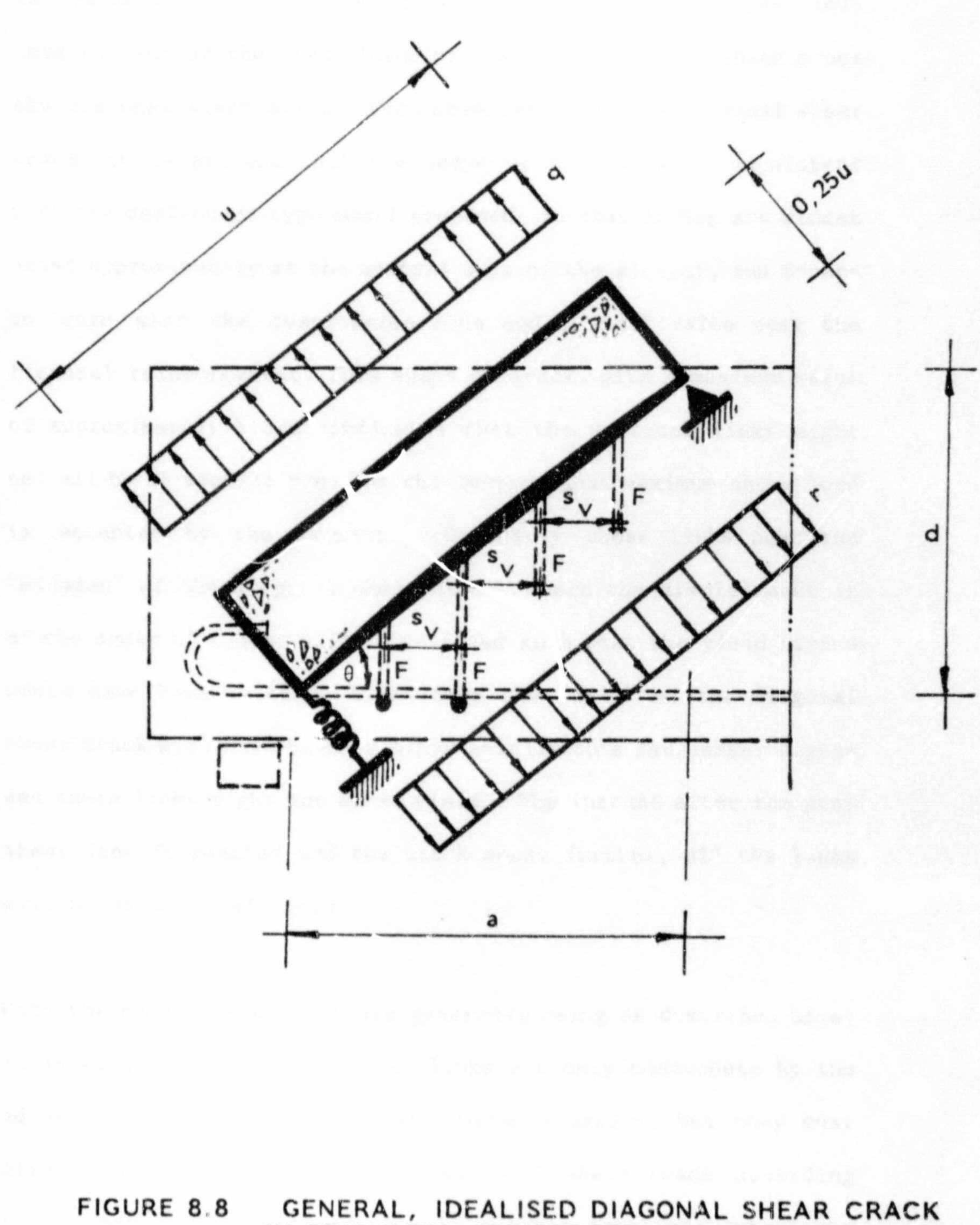


FIGURE 8.8 GENERAL, IDEALISED DIAGONAL SHEAR CRACK IN PRISMATIC REINFORCED CONCRETE ELEMENT REINFORCED FOR SHEAR WITH VERTICAL LINKS

the value of $V_r$ in the case of vertical links will be just less than the sum of the yield loads of the vertical links which cross the diagonal shear crack. From observation of the diagonal shear crack, it is evident that the shape of the crack is consistent with the deflection type model proposed, in that it has its widest point approximately at the neutral axis of the element, and tapers to zero near the compression zone and a small value near the flexural reinforcement. This shape of crack, with a maximum value of approximately 1,3mm, indicates that the vertical links might not all be at tensile yield at the instant that maximum shear load is accepted by the element. Certainly those links near the "midspan" of the diagonal shear crack, where the displacement is of the order of 1,3mm, will be strained such that the yield stress would have been reached. Links near the "ends" of the diagonal shear crack will have been strained axially to a far lesser degree and these links might not be at yield. The instant after the peak shear load is reached and the crack opens further, all the links will be in uniaxial yield.

With the mode of shear failure generally being as described here, it is evident that the vertical links not only contribute by the adjustment to the vertical equilibrium equation, but they must also inhibit the opening of the diagonal shear crack according to the proposed model, and thereby enhance aggregate interlock. It is postulated that it is this dual role of the links that results in the existing truss analogy models appearing to be "conservative", especially for structural elements which are lightly reinforced for shear. The links thus influence the aggregate interlock component of total shear resistance. The manner in which the aggregate interlock resistant stress, $v_a$, is defined in terms of the influence of the vertical links, is identical to the approach proposed for structural elements unreinforced for shear.

For the elements unreinforced for shear, opening of the diagonal shear crack was caused primarily by the equivalent distributed load, q. For the case of shear link reinforcement, the hypothetical equivalent prismatic beam is subjected to the restraining influence of the links in addition to the opening load, q. From Figure 8.8, it is evident that restraining equivalent uniformly distributed load, r, resulting from the effects of the vertical links, is given by:

$$r = \frac{F}{s_v} \cos^2\theta$$

where

F is the average force per link (or pair of legs) in N.

$s_v$ is the spacing of the links along the axis of the structural element in mm.

$\theta$ is the slope of the diagonal shear crack as defined previously. The derivation of the expression $\cos^2\theta$ is identical to that of the evaluation of reinforcement performance for yield line theory.

It can be assumed from the shape of the diagonal shear crack just prior to shear peak load being attained that the majority of the links are likely to be at yield. The peripheral links are also likely to be reasonably stressed, so that an average load per link across the full length of the diagonal shear crack of $0,9F_y$, where $F_y$ is the yield force in each link (or pair of legs), appears to be a reasonable assumption. While this assumption should not be significantly inaccurate, this is an area where more specific tests could be undertaken to confirm this assumed value and is thus an avenue of further research. On the basis of the assumed value, however, the equivalent distributed load can be given as:

$$r = \frac{0,9F_y}{s_v} \cos^2\theta$$

The yield force, $F_y$, can be rewritten as $f_y A_{sv}$, where $f_y$ is the yield stress of the link and $A_{sv}$ is the cross-sectional area of the link (usually two legs). The equivalent uniformly distributed load from the links acting on the prismatic concrete beam can thus be rewritten as:

$$r = 0{,}9f_y \frac{A_{sv}}{bs_v} b\cos^2\theta$$

thus

$$r = 0{,}9f_y \ \mu \ b\cos^2\theta$$

where $\mu$ is the (vertical) link reinforcement ratio

From the model for structural elements unreinforced for shear derived previously, the aggregate interlock component of shear stress resistance is given by:

$$v_a = \frac{0{,}65}{10d/E_c + 0{,}05/\rho} \cdot \frac{\sin^2\theta}{\cos\theta}$$

The equivalent uniformly distributed load resulting from the residual tensions derived from the aggregate interlock component of shear stress, as before, is given by:

$$q = \frac{v_a\, b}{\tan\theta}$$

The effective uniformly distributed load applied to the hypothetical equivalent prismatic concrete beam is given by the difference between the two effects. The adjusted equivalent distributed load is thus given by q-r. Reconstituting the expression for the aggregate interlock component of shear stress exactly as before, under the influence of the revised equivalent distributed load, q-r, results in the following relationship:

$$v_{a'} = \frac{0,65}{10d/E_c + 0,05/\rho} \cdot \frac{\sin^2\theta}{\cos\theta} + \frac{r\tan\theta}{b}$$

Making the appropriate substitution for the uniformly distributed load due to the links, r, results in the following relationship for the contribution of aggregate interlock stress to total shear resistance:

$$v_{a'} = \frac{0,65}{10d/E_c + 0,05/\rho} \cdot \frac{\sin^2\theta}{\cos\theta} + 0,9f_y\mu\sin\theta\cos\theta$$

where

$v_{a'}$ is the enhanced aggregate interlock shear stress component of total shear resistance in MPa.

$E_c$ is the Elastic Modulus of the concrete of the structural element in MPa.

$\rho$ is the flexural steel ratio of the structural element expressed as a percentage

$\theta$ is the slope of the idealised diagonal shear crack to the axis of the element.

$f_y$ is the mean yield stress of the vertical link shear reinforcement in MPa.

$\mu$ is the (vertical) link reinforcement ratio expressed as a ratio.

*This equation* defines the contribution of the aggregate interlock component of resistant shear stress for structural concrete elements which are reinforced for shear with vertical links. The effect of the links in this regard is thus to inhibit opening of the diagonal shear crack in terms of the proposed model, thereby enhancing aggregate interlock along this crack. The overall shear resistance of the structural element is still composed of the contributions of the four components, dowel action, compression zone, aggregate interlock and vertical reinforcement which crosses the diagonal shear crack. The value of aggregate interlock in this case, however, is enhanced relative to elements unreinforced for shear.

For the standard beam unreinforced for shear, the value of the aggregate interlock component of shear stress is given by:

$$v_a = 0,6 \text{ MPa}$$

The enhanced values for aggregate interlock resistant stress for a "standard beam", reinforced for shear with vertical mild steel links of average yield stress 280MPa in uniaxial tension, would be as follows:

$$v_{a'} = 0,8 \text{ MPa for a link ratio, } \mu \text{, of } 0,002$$

$$v_{a'} = 2,6 \text{ MPa for a link ratio, } \mu \text{, of } 0,01$$

$$v_{a'} = 3,6 \text{ MPa for a link ratio, } \mu \text{, of } 0,03$$

For structural elements reinforced for shear with vertical links which rely specifically on shear ductility, and not the attainment of flexural ductility prior to shear failure which itself might remain brittle, even though the element is reinforced for shear, the introduction of a new concept is proposed. It is thus proposed that for specimens of this nature, the concept of "residual ductility", with specific reference to shear flow, becomes significant. It is clear from a large number of tests conducted in this programme on various specimens which failed in a distinct shear mode, that the shear resistance attains a peak value, even for elements reinforced for shear, before dropping to some "residual" ductile plateau, which depends for its value on the vertical link reinforcement ratio, $\mu$, and the ductile contribution of dowel action (although this contribution is generally small). For structural elements with a high $\mu$ (greater than about 2% for mild steel links) and a slope of the diagonal shear crack approaching the minimum value ($\theta$ approximately 27°), the residual ductile plateau value will begin to approach the peak shear value. The peak value is, however, always enhanced by the presence of link reinforcement by virtue of increasing the aggregate interlock component of overall shear resistance. The residual ductile plateau occurs once aggregate interlock has been overcome, with the maximum diagonal shear crack width exceeding approximately 1,3mm. The full shear load is then taken by the sum of the vertical uniaxial yield loads of the links which cross the diagonal shear crack (and the ductile component of dowel action).

The proposed model for shear for elements reinforced for shear is thus described more fully by the following equations:

The peak ultimate shear resistance of the structural element prior to loss of aggregate interlock is given by:

$$V_{peak} = V_d + V_c + V_{a'} + V_r \quad ( = V_R)$$

where

$V_{a'}$ is the enhanced aggregate interlock component. (N)

$V_r$ is the vertical link contribution, prior to loss of aggregate interlock, where not all links are yielded. (N)

$V_d$ and $V_c$ are the unchanged contributions of dowel action and compression zone derived previously. (N)

After the peak shear value has been attained, the component of aggregate interlock is abruptly lost as before. The compression zone component is simultaneously reduced to a negligible friction contribution. The ultimate shear resistance of the structural element is thus given by the following:

$$V_{duct} = V_d + V_{ry}$$

where

$V_{duct}$ is the residual ductile plateau value of shear resistance. (N)

$V_d$ is the ductile dowel action contribution. (N)

$V_{ry}$ is the sum of the uniaxial tensile yield values of the vertical links that cross the diagonal shear crack. (N)

For structural elements reinforced for shear with vertical links which attain the flexural ultimate limit state prior to reaching the peak shear value predicted above, the concept of the residual ductile plateau is obviously of no consequence. For elements reinforced for shear which still fail in a shear mode prior to the attainment of the flexural ultimate limit state, this concept is of considerable importance in defining overall structural ductility.

The acceptable ratio between the residual ductile plateau and the peak shear value is a subjective issue which may depend on the overall structural performance requirements. For the ductile plateau value to approach the peak value such that the transition will be acceptably smooth for most structural situations, the vertical link ratio, $\mu$, needs to be of the order of 2%, or greater, for mild steel reinforcement. The range of $\mu$ for which specific shear ductility is thus attained for vertical link reinforcement is rather narrow, as the model also predicts that the resistant shear stress approaches the concrete crushing strength for values of $\mu$ of the order of 3% to 4% for mild steel links. At this link ratio, web compression crushing is thus predicted by the model, and this shear failure mode would again be non-ductile. Excluding the achievement of flexural ductile failure, it is thus evident that specific shear ductility for elements reinforced for shear with vertical links, only occurs for a narrow range of $\mu$ ($2\% < \mu < 4\%$ for mild steel vertical links and corresponding reduced percentages for high yield links).

The stress-deflection curves for specific shear failure modes as predicted by this proposed model for shear for structural elements reinforced for shear with mild steel vertical links of average yield stress 280MPa are given in Figure 8.9. The stress

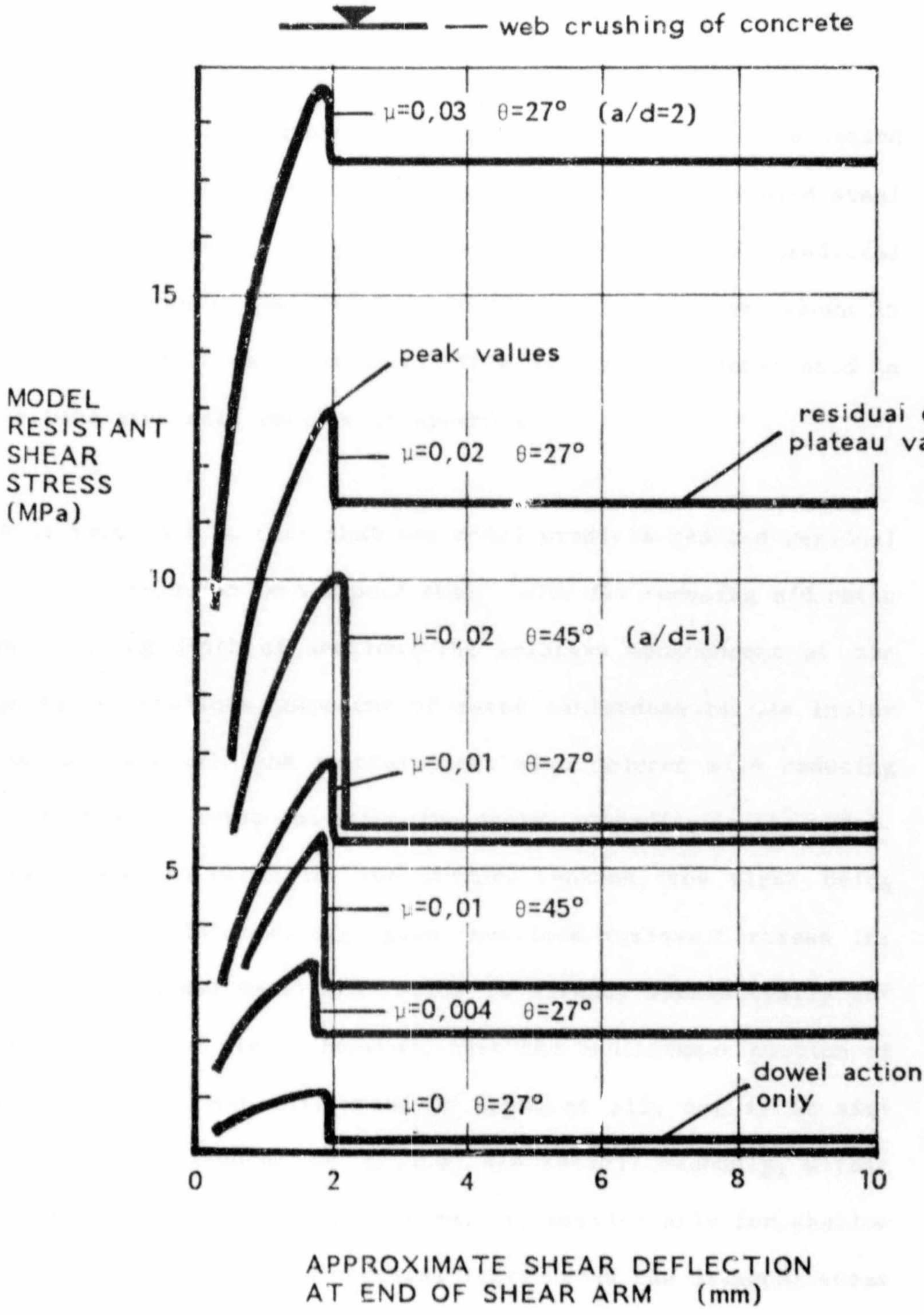


FIGURE 8.9 MODEL STRESS-DEFLECTION CURVES FOR SHEAR FAILURE MODES FOR THE "STANDARD BEAM" REINFORCED FOR SHEAR WITH MILD STEEL VERTICAL LINKS OF VARYING PERCENTAGE

values indicated in these curves are derived from an evaluation of the standard beam with varying ratios of vertical mild steel link reinforcement. The good correlation between these predicted curves and the curves observed in the test programme, shown in Figure 6.3, is clearly evident. This is further demonstrated in the tabulated test results in Appendix D.

It is important to note that the model predicts reduced residual ductility relative to the peak shear value for reducing a/d ratio and reducing depth of section. The relative enhancement of the aggregate interlock component of shear resistance by the inclusion of vertical link reinforcement also reduces with reducing parameters a/d ratio and effective depth, according to the model. These phenomena occur for two primary reasons, the first being that the unreinforced aggregate interlock resistant stress for thin sections and small a/d ratios is already substantially increased. It is clear, however, that the enhancement portion of the equation is not influenced by depth at all, nor is it significantly influenced by $\theta$ (i.e. a/d ratio). Secondly, within normal practical detailing constraints, particularly for shallow sections, relatively few vertical links cross the diagonal shear crack because the horizontal length of the diagonal crack, being related to the effective depth of the section with a maximum value of 2d, is actually physically small.

The model thus predicts the phenomenon observed in tests, and noted in codes of practice[5,13], that link reinforcement is neither practical, nor particularly effective, in thin sections.

The fundamental model, derived on the basis of a structural element unreinforced for shear, has thus been extended to quantify the performance of structural elements reinforced for shear with vertical links in a consistent and simple manner. The model predictions of the shear resistance of such elements correlate well with the test results and observations of this work and those of other researchers in this field.

### 8.2.1 HORIZONTAL LINK REINFORCEMENT

Whereas the modelling of structural elements (primarily beams) reinforced for shear with vertical links has received wide attention and considerable development, there is no universal current model which includes a technique for quantifying the performance of horizontal link shear reinforcement. The model proposed within the scope of this work aims at extending the principles adopted thus far to include an evaluation of the performance of horizontal links in a fully consistent approach.

As for vertical links, where use is made of horizontal link reinforcement in reinforced concrete construction, these will generally be small diameter bars at fairly close centres. A reasonable approximation of the influence of the horizontal links on the diagonal shear crack can thus again be made in terms of a uniformly distributed load. As before, the horizontal links influence overall shear performance in two ways. Firstly, they influence the fundamental equilibrium equation which recognises the contributions of the various components of shear resistance as before. In this case, however, the contribution of the link reinforcement is the sum of the dowel action capacities of all the links that cross the diagonal shear crack, and is consequently a considerably smaller contribution than that of vertical links in this equilibrium equation. Because the horizontal links will generally be remote from the edges of the structural element, dowel splitting and spalling of the concrete cover will not occur, and this contribution will thus generally be ductile in nature, even though of considerably smaller magnitude than that of vertical links.

An identical equilibrium equation to that used previously for sections unreinforced for shear can thus be derived, with the term, $V_d$, somewhat increased, but still significantly less than the component, $V_r$, for vertical links.