Profitability and Size in the Five-factor model: an African context.

MASTER OF MANAGEMENT IN FINANCE AND INVESTMENT

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Abstract

This study provides a comprehensive analysis of 18 African stock markets, employing Fama and French five-factor (FF5F) regression analysis to examine the size and profitability effects. The research specifically investigates the efficacy of both operating and gross profitability as factors within the FF5F model, finding them to be distinct but both holding explanatory power. While the study supports the relevance of the size factor in African stock markets, the data reveals inconsistencies and low statistical power, highlighting the need to further refine the analysis. This applies to the profitability factors as well. Additionally, the research explores the relationship between the business cycle and size effect, uncovering a nuanced interplay between business cycle stage, stage duration, and the size effect. The findings contribute to the literature on asset pricing models in emerging markets, particularly emphasizing the necessity for nuanced analyses that account for regional and economic specificities in the African context.

Chapter 1: Introduction

Profitability and the Size effects are two firm level variables that have been seen to have both high and low power in explaining stock returns in various markets as part of the Fama and French's Five Factor (FF5F) model. It would be interesting to see what power they have in a set of frontier and emerging markets in Africa. Various papers include data and tests on emerging markets, looking at Asia and Europe and some even including individual African countries such as South Africa (Mosoeu & Kodongo, 2020), but few attempts (Boamah, Watts, & Loudon, 2017; Mbengue, Ndiaye, & Sy, 2023; Hearn, Piesse, & Strange, 2010),have been made to consider a broad group of frontier and emerging markets consisting solely of African stock markets (ASM) Hearne and Piesse have several papers on African sub regions, West Africa, North Africa, SADC (Hearn & Piesse, 2010; Hearn & Piesse, 2012; Hearn, 2011).

Emerging markets equity returns have long posed a challenge for finance research (Claesson, 2021; Boamah et al., 2017). They are historically highly non-normal, volatile, thinly traded, and short on samples (Stocker, 2016; Leite, Klotzle, Pinto, & Da Silva, 2018; Claesson, 2021; Hearn et al., 2010); standard models are often ill suited to deal with the specific circumstances arising from these "unusual" characteristics of the typical emerging market. ASMs are categorized as emerging markets, while others are regarded as "frontier markets", but they largely conform to these stereotypical characteristics. But that's what makes it interesting to investigate, to find out if there isn't a model that can explain these quirks and challenges, such that they become idiosyncrasies that may require a unique approach but remain explicable. Size and profitability are proven factors in predicting returns in many markets. Similarly, the FF5F model is a tested model, proven to work well in a variety of cases without being too stringent or exacting; it's also broadly applicable and easily modifiable.

Across African equity markets are characteristics such as, small size, lack of trading synchronicity, weak accounting practices, and illiquidity. A lack of integration of African markets offers diversification benefits, expected returns are possibly higher but unpredictable, there's insulation from global economic shocks, and reduced cross correlation between markets (Alagidede, Panagiotidis, & Zhang, 2011; Geert & Campbell, 2002; Hearn & Piesse, 2010). But a lack of integration between countries could render Africa's FF5F factors poorly defined and insignificant, while the same factors based on country portfolios hold significance in their local markets (Boamah et al., 2017). Is there regional integration in Africa?

Given these characteristics of the ASMs, it would be interesting to establish if we can provide a clear role to the size and profitability factors. The FF5F model has shown varying effectiveness depending on the exact context of the markets in which its tested (Fama & French, 2016; Leite et al., 2018; Dwarika, 2023; Cox & Britten, 2019; Foye, 2018); therefore, it is impossible to predict the outcome without looking at the data and running the models. However, we are able to get some idea of what to expect from the extant literature, some of which have included a few ASMs (Mosoeu & Kodongo, 2020).

Thereafter it would be worth considering profitability and the size effect as factors in the African market. The Size effect has been seen to be dormant in the US market. Is it the same in Africa, should it be removed from the model or is it significant as a whole? Gross profitability has been found to be a more effective factor in reducing mispricing, especially in emerging markets. And, in theory, its anti-correlation with value means it should pair well with a value strategy. The emerging market studies are the benchmark on which to judge the performance of the FF5F in the African data set. The period under consideration is 13 years from 2010 - 2023. The expectation from previous literature is for profitability as well the size effect to find significance in the African context (Hearn, 2011; Leite et al., 2018; Taha & Elgiziry, 2016; Cox & Britten, 2019; Novy-Marx, 2013).

The size effect was discovered by Banz (1981), and refers to the observation that smaller firms tend to have higher risk-adjusted returns than large-sized firms over long horizons (Crain, 2011). This effect is considered unrelated to beta, indicating that it is not explained by systematic risk (Charteris, Rwishema, & Chidede, 2017; Segojane & Ndlovu, 2022). The size effect is nonlinear, with the main effect observed in very small firms, while there is little difference in return between average-sized and large firms (Claesson, 2021). The size effect is considered an anomaly since it holds empirically, but there is no theoretical reason for firm size to have explanatory power after controlling for risks (Claesson, 2021; Ahn, Min, & Yoon, 2019). This suggests that firm size may be a proxy for unmeasured risks related to small firms (Crain, 2011; Chan, 1985). The size premium, captured by the Small Minus Big (SMB) factor, reflects the risk exposure of small firms. Empirical studies have shown that the size premium, when in effect, has a greater impact on overall explanatory power than the premium associated with illiquidity, in emerging markets (Hearn et al., 2010). The SMB factor is a factor in the FF3F model.

The FF3F model has been seen to be limited in explaining expected returns. Fama and French added profitability and investment factors to enhance its explanatory power (Fama & French, 1993). The authors, introduce the variable of operating profitability (OP) as a factor that captures the relationship between profitability and average returns (Fama & French, 2015). The inclusion of profitability in the model is found to be important in explaining stock returns in different regions, such as North America, Europe, Asia Pacific, and emerging markets (Fama & French, 2016; Foye, 2018). The profitability factor is considered a primary driver of asset returns (Claesson, 2021). Novy-Marx (2013) suggests a gross profitability variable, measured as the ratio of (revenues minus cost of goods sold) to asset. Gross profitability, is found to have similar predictive power as the book-to-market ratio in determining average returns, and Novy-Marx (2013) argues that it is a cleaner measure of true economic profitability compared to other accounting measures. That being the case, gross profitability should have improved explanatory power over operating profitability, especially in company data oriented emerging markets (Foye, 2018; Novy-Marx, 2013). Strategies that exploit profitability by acquiring productive capacity at a lower cost generate significant abnormal returns. Controlling for profitability also dramatically increases the performance of value strategies, especially among the largest, most liquid stocks (Novy-Marx, 2013).

1.2 Context of the Study

This study aims to explore the impact of the FF5F model, specifically focusing on the size and profitability factors, in an African context. As such the state and nature of stock markets is of great importance to this study. Africa's equity markets exhibit unique characteristics that can significantly impact asset pricing models. These characteristics include the small size of the markets, low trading synchronicity, weak accounting practices, and illiquidity (Boamah et al., 2017). Additionally, these markets lack integration, integration with global markets, as well other African markets (Geert & Campbell, 2002). These characteristics inform the research design as well expected outcomes. The small size of African markets, means there's low diversity within each market itself, so in an attempt to approach the efficient frontier we look to maximise diversification, as many have done before, by taking a multimarket approach and considering as many African Stock Markets as possible (Cakici, Tang, & Yan, 2013; Boamah et al., 2017; Claesson, 2021; Foye, 2018; Mosoeu & Kodongo, 2020; Hearn et al., 2010). Many lessons are taken from literature on how to navigate this research around the nuances of ASMs. Small markets trade low volumes, that lowers their level of liquidity, creating market pressures that weaken the efficient market theory, so a loss of power is expected for any CAPM based model (Hearn et al., 2010; Stocker, 2016). Low volume trading leads to shorter trading hour, these hours are not the same across regions, which impedes synchronicity and market integration (Hearn et al., 2010). This leads to the assumption of segmented markets in this analysis, and so we follow Boamah et al's (2017) cross country pooling strategy, to form applicable regional factors. Hearn (2011; 2012; 2010) split his research, into legislatively, economically, and linguistically integrated regions, but this paper instead sides with the benefits of a diversified African portfolio (Alagidede et al., 2011). Illiquidity is not directly dealt with in this paper, capitalization, availability of funding, availability of information, access to the market, these all vary across Africa and influence the trading volumes i.e., liquidity of markets, and some papers have considered a liquidity factor to account for this (Hearn et al., 2010; Boamah et al., 2017). The effect on pricing of the liquidity is mitigated for by allowing a longer timeframe, we consider monthly rather weekly return data, such a simple approach is justified by Boamah et al (2017) who find the liquidity effects minimal in their cross-country pooled sample.

Given the limited research on African equity markets and their distinct characteristics, this study fills a critical gap in the literature. The outcomes of this research can provide valuable insights for investors, policymakers, and researchers seeking a deeper understanding of asset pricing in the unique context of Africa's diverse and dynamic markets.

1.3 Research problem

African equity markets have challenges of being small and volatile (Boamah et al., 2017; González-Sánchez, 2021) but also present diversification opportunities to international investors due to their low integration with world markets (Alagidede et al., 2011). African equity markets are also under-researched.

According to Fama and French (2017) market segmentation is a significant part of the reason why the global portfolio performs so poorly under the five-factor model. Given that their "global" portfolio comprises exclusively equities from developed markets, it is intriguing to examine whether the underwhelming performance persists when the portfolio is adjusted to include only assets in ASMs, which could potentially offer a fresh avenue for diversification (Alagidede et al., 2011; Boamah et al., 2017; Mosoeu & Kodongo, 2020). Across Africa, government, economic, legal and trade policy and language of business differs, so while it's a single geographic region, these can be differentiators that make for more significant classification, splitting Africa into multiple regions (Cooke, Kose, Otrok, & Owyang, 2015). As well as this, these policies also drive equity returns (Stocker, 2016) so market integration could be worse. With a size effect driven by business cycles, so many differentiators could lead to low cycle synchronicity across Africa, resulting in no discernible regional size effect, even if the size effect is widely observed at a local market level.

If the size effect is seen in developed markets with small and large cap stocks, can we still expect to see this effect out of developing markets, which are much smaller in magnitude and thus the difference between their "small" and "large" caps? The size effect has over time become dormant in the US but it's been seen to be active in several emerging markets, most have still shown it to be irrelevant. But this is where the combination of markets takes effect, South Africa and Egypt have featured in many emerging market studies, where the size effect is insignificant in portfolios formed across multiple nations, but a size effect has been observed in individual country portfolios of South Africa, Egypt and several other African nations. But we know that market integration across the African region is poor due to differences in trade policy, government policy, legal system, language and so on (Geert & Campbell, 2002; Cooke et al., 2015; Padilla & Otero, 2022). So, as a whole there is still no telling whether there is an African size effect.

Investors have been seen to trust in firm level data to predict stock returns, in emerging markets, this suggests profitability should prove to be powerful, and indeed a strong profitability effect has been seen in many emerging markets. So Novy-Marx's (2013) improvement on this profitability model, by using a truer representation of profit could be even more pertinent in emerging markets. But this means the quality of accounting practices becomes even more important. These characteristics could influence the degree of market frictions, a crucial foundation of the Fama-French factor models, discrepancies in market frictions may manifest in variations in empirical findings across different markets and over time within the same market (Mosoeu & Kodongo, 2020).

1.4 Research objectives

The 3 main objectives of this paper are to:

1. Ascertain whether the FF5F model holds explanatory power in a cross-section of African equity returns.
2. Establish whether the size and the profitability effects exist in Africa's equity markets.
3. Compare the effect of size and gross profitability in Africa's equity returns.

1.5 Statements of hypotheses

The FF5F often *outperforms* the FF3F model in some markets (Fama & French, 2016; Fama & French, 2015; Leite et al., 2018; Foye, 2018; Claesson, 2021). That being said, many papers point out the redundancy or ineffectiveness of SMB and HML (size and value factors), that were originally specified in the FF3F model but remain present in the FF5F model (Cox & Britten, 2019; Claesson, 2021; Ahn et al., 2019; Cakici et al., 2016; Mosoeu & Kodongo, 2020; Leite et al., 2018). The FF5F model presents two new factors investment (CMA) and profitability (RMW), CMA seems to hold little sway in emerging equity markets (Leite et al., 2018; Mosoeu & Kodongo, 2020; Foye, 2018; Fama & French, 2015; Claesson, 2021).

H1: There is no significant difference in performance between the FF5F model and the FF3F model for Africa's equity returns.

Profitability has been seen to be important to investors in emerging markets (Mosoeu & Kodongo, 2020; Fama & French, 2015; Cox & Britten, 2019; Charteris et al., 2017; Leite et al., 2018). Mosoeu & Kodongo (2020) as well as Lin & Qi (2017), attributed this functionality to emerging markets' investors trusting in accounting ratios. Lin & Qi (2017) went on to state that frequently occurring concentrated ownership in emerging markets often allows majority shareholders to interfere with management's ability to strategically reinvest, therefore past investment holds little meaning as a factor. Novy-Marx's (2013) gross-profit take on profitability should prove to be highly effective in African markets as it more directly relates performance to accounting data, that emerging market investors seem more inclined to act upon (Mosoeu & Kodongo, 2020).

H2: The gross profitability effect does not exist for the African region's equity portfolio.

The *size effect* has been often seen to be dormant (Ahn et al., 2019; Cakici et al., 2016; van Dijk, 2011; Mosoeu & Kodongo, 2020) but evidence remains for its existence in countries such as Egypt, Tunisia, Nigeria, Ghana, Tunisia, Cote d'Ivoire (Hearn, 2011; Hearn & Piesse, 2011). There has been much discourse on the lack of integration and its extent, firstly African markets are largely not integrated with developed markets (Alagidede et al., 2011). North African markets are not locally integrated, West and East African markets are not integrated with each other but they are somewhat integrated with France and the UK respectively, and Sub-Saharan markets show some level of regional integration (Hearn, 2011; Hearn & Piesse, 2011; Hearn & Piesse, 2005).

H3: There is no size effect for the African regional equity portfolio.

Leite et al., (2018) infer *preference* based on factor performance, they noticed a strong and persistent size effect, and clear but irregular profit and value effect patterns. On this basis, they advocated for the rationality of emerging market investors, acknowledging that the idiosyncrasies of emerging markets make them less efficient but asserting that assets are still being priced rationally albeit with a different factor preference (Leite et al., 2018).

H4: There is no significant difference in performance between the gross profitability factor, the size factor, and the operating profitability factor for Africa's equity returns.

1.6 Significance of the study

This research holds significant implications for various stakeholders in the financial landscape. For investors, the study provides valuable insights into the factors influencing equity returns in African markets, aiding in the development of more effective investment strategies (Boamah et al., 2017). By understanding the nuances of the size and profitability effects, investors can make more informed decisions about portfolio allocation and risk management (Hearn et al., 2010).

Policymakers can benefit from the research findings by gaining a deeper understanding of the unique dynamics of African stock markets (Geert & Campbell, 2002). This knowledge can inform the design and implementation of policies aimed at promoting market efficiency, transparency, and investor confidence (Stocker, 2016). By addressing the specific challenges and opportunities identified in the study, policymakers can foster a more conducive environment for investment and economic growth.

For researchers, this study contributes to the growing body of literature on asset pricing models in emerging markets. The focus on African stock markets, which are often underrepresented in research, fills a critical gap in the literature (Boamah et al., 2017). The findings challenge existing assumptions and highlight the need for further investigation into the complex interplay of factors influencing equity returns in this unique context. By building upon this research, future studies can delve deeper into the nuances of asset pricing in Africa, exploring regional variations, data quality issues (Kothari, Shanken, & Sloan, 1995), and the impact of changing economic conditions.

Research on emerging markets across the world is ample, therefore we have expectations on what outcomes are produced. But while African stock markets are included in some of the research, samples are never fully representative of the ASM. So, in this research, we look at a solely African dataset of 731 stocks including markets hosted in these 18 countries, Cape Verde (2), Eswatini (1), Madagascar (1), South Africa (232), Egypt (212), Morocco (17), Tunisia (22), Namibia (7), Botswana (11) , Nigeria (94), Tanzania (12), Kenya (32), Ghana (18), Uganda (5), Mauritius (39), Ivory Coast (8), Malawi (5), Zambia (12). The primary data sources used were DataStream and Bloomberg.

Chapter 2: Literature Review

2.1 The Capital Asset Pricing Model (CAPM)

Asset pricing research can be traced back to the 18th century, the emergence of an integrated global economy and the development of sophisticated financial markets have been the catalysts for its ever-increasing prominence. Asset pricing models are tools used in the undertaking of capital budgeting decisions, pricing equity, as well as determining the cost of capital. These models are underpinned by ideas regarding mean-variance optimisation, equilibrium analysis, and investor preference (Karp & van Vuuren, 2017). The main framework in modern asset pricing research has been Capital Asset Pricing Model (CAPM) since its development in the 1960s with collective contributions from Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). It was the first theoretical model built to determine the expected rate of returns on risky assets (Claesson, 2021; Taha & Elgiziry, 2016). Fundamentally, the CAPM describes the relationship between an asset's expected return and its exposure to market risk (Claesson, 2021). The theoretical underpinnings of the CAPM are grounded in Harry Markowitz's (1952) mean-variance portfolio theory. Markowitz's (1952) theory posited that assets, each with their unique risk profiles, could be combined in a portfolio to achieve an optimal risk-return trade-off. This laid the foundation for the central idea of the CAPM: that assets' expected returns are influenced by their sensitivity to market movements, known as beta (Karp & van Vuuren, 2017). A higher beta indicates more volatility compared to the market, and should thus provide a greater return, while a beta below one implies an asset is less risky than the market (Claesson, 2021). The CAPM has been built on the notion that stock returns are affected by one type of risk factor, namely systematic risk as measured by beta (market risk). The model states that the systematic risk as measured by beta is sufficient to describe cross-sectional of expected returns (Taha & Elgiziry, 2016). While CAPM has had a profound impact on asset pricing research, it is important to recognize the set of simplifying assumptions it relies on. These assumptions include the presumption of rational investors, frictionless markets, and an exclusive focus on market risk, which allow for intuitive predictions of market behaviour with respect to expected risk/return relationships, but have proven to be flawed and fall short of accurately predicting the markets actual behaviour (Fama & French, 2004; Stocker, 2016; Karp & van Vuuren, 2017; Taha & Elgiziry, 2016). The most significant critique of the CAPM revolves around its inability to explain real-world asset returns comprehensively. This is rooted in its idealistic market assumptions, assuming no transaction costs, ignoring non-market risks, and assuming investors are always rational, which may substantially influence asset pricing (Claesson, 2021; Mullins, 1982; Roll, 1977; Elton, Gruber, Brown, & Goetzmann, 2014).

The limitations of the CAPM prompted researchers to seek a more comprehensive understanding of asset pricing dynamics. Fama and French (1992) took a significant step by proposing the incorporation of two additional risk factors in their model to forecast individual stock returns: size and book-to-market ratio. This innovation was inspired by earlier research findings, such as Banz's (1981) revelation of a negative relationship between firm size and observed returns, indicating that smaller capitalization stocks tend to outperform their larger counterparts. Similarly, Stattman (1980) identified a positive correlation between average returns and book-to-market equity, suggesting that stocks with lower valuations yield higher investment returns. Subsequent studies further fuelled this research trajectory. Rouwenhorst (1999) investigated emerging market stocks, affirming that "small stocks outperform large stocks", and "value stocks outperform growth stocks." Importantly, he highlighted that the CAPM risk factor, beta, failed to provide a comprehensive explanation for asset returns, with evidence suggesting that high beta stocks do not necessarily outperform low beta stocks. Further investigations by Jegadeesh and Titman (1993) identified momentum in stock price returns as a pivotal factor, demonstrating that stocks that have performed well in the past tend to continue their positive performance. This was corroborated by Rouwenhorst (1998), who extended the notion of momentum to individual international stocks, irrespective of their country of origin. These research findings collectively laid the foundation for additional risk factors, investment and profitability, and the subsequent development of the Fama and French Five Factor Model (Fama & French, 2006; Fama & French, 2008; Fama & French, 2015).

2.2 Asset pricing theories and FF5F model

The F3FF model outperformed the CAPM and Sharpe model on a study of the US equity market and on selected international markets (Fama & French, 1993; Fama & French, 2015; Fama & French, 2016). However, the model was also found to be deficient, particularly failing to explain investment and profitability (Chen, Novy-Marx, & Zhang, 2011). Following the implications of the dividend discount model, Fama & French (2015) add two factors, profitability and investment to their model. The dividend discount model posits that the market value of a stock share is the present value of anticipated dividends per share (Fama & French, 2015).

The 5-factor model (FF5F) was shown to be able to capture the average excess returns on portfolios formed with the combination of size - value, size - operating profit and size - investment, and outperforms the FF3F model with a mixed success rate (Fama & French, 2015; Fama & French, 2016; Foye, 2018; Claesson, 2021). Fama & French (2015) also note that the FF3F may be redundant as the profitability and investment take over the value factor.

Carhart (1997) constructs a 4-factor model using FF3F model plus momentum. The author uses the previous eleven-month returns lagged one month as a proxy for momentum factor. Carhart concludes that adding one year returns to FF3F model improves the explanatory power of the model.

Novy-Marx (2013) proposes using gross profit as the basis for profitability sorts and profit factor. He suggests this on the basis that gross profit is a more direct indicator of business performance. The gross profit factor is proven effective, reducing the mispricing seen in addition to the 4-factor model as well as part of several variations of said model. Another outcome of the paper was to prove gross profit as an augmentation to a value strategy, that improves their performance. A value strategy controlling for gross profit was found to gain the full profitability premium without increasing in risk.

2.3 The empirical literature

Cakici et al (2013) show that Carhart's version of the Fama French model, that adds momentum sorting, seems to perform the best, outperforming the FF3F model in 3 emerging markets (Asia, Latin America and Eastern Europe). Fama & French (2016) test the FF5F and FF3F models internationally and find stronger patterns in small stocks and local data. FF5F performs well except in the case of small stocks with high profitability and aggressive investment. Mosoeu & Kodongo (2020) investigate the FF5F ability to predict returns, to get a view on emerging markets, they compare both emerging and developed markets. In so doing they find the profitability factor to be the most useful in explaining emerging market returns (Mosoeu & Kodongo, 2020). They say, this is due to its basis in accounting ratios that investors are likely to trust more due to lower levels of information efficiency in emerging markets.

According to Crain (2011), in finance literature the size effect denotes how smaller firms have higher returns than larger firms in the long-term, and as a factor (size factor) describes the contribution that firm size has in explaining stock returns. Cain notes that these effects are primarily observed in small firms rather than being scaled or distributed relative to size. Ahn et al (2019) investigate the disappearance of the size effect. They effectively prove that it is not gone but rather remains dormant. They observe the size effect to be seasonal, this seasonality is in relation to the economic business cycle, where the size effect is found to be significant only during the 'trough' stages of the cycle, both before and after its supposed disappearance. From this they theorize and prove that the size effect still persists, however a lengthening of business cycles, and thus a reduction in troughs over time, reduces its overall significance to null. And so, its presence is conditional on business cycle stage, while an unconditional effect would require a high frequency of troughs i.e., shorter business cycle. Boamah et al (2017) investigate the FF3F and Carhart models' ability to capture ASM returns at the regional level. They take special consideration for the idiosyncrasies of emerging markets, considering sparseness of data, small market size, and illiquidity. The Carhart model performed better than the FF3F, and while size and book to market effects were observed across the board, sorting relative to individual market values showed less mispricing than sorting on a combined pool. From this they conclude that African markets are poorly integrated, and this likely exaggerates the negative effects of illiquidity on describing returns. In Cooke et al's (2015) paper on regional business cycles the key takeaways relevant to this study were that, business cycles can be regional, and geography is one category of a few used to define a region. A regional business cycle consists markets with a high correlation between them i.e., any grouping of countries with similar business cycles. This co-movement comes about through market integration and low barriers to trade, which are known challenges between African markets. Factors such as legal structure, language, trade agreements, and proximity affect each nation differently and thus location is neither the only nor most accurate way to form regional cycles e.g., a francophone African nation may have greater synchronicity with France (with which it shares, language legal systems and currency) than it does with its anglophone neighbours (with which it shares proximity). In Geert & Campbell's paper (2002) a host of issues and challenges of African markets are discussed. A key takeaway was that there is significant interest in African markets, which spurs on the development of more suitable models to analyse African markets. Geert & Campbell (2002) define market integration as the state where expected returns on an asset is equal in different markets. They find that while emerging markets may find growth in market integration, this integration may reduce expected returns and diversification benefits for foreign investors. Leite et al's (2018) primary take away is that investors in emerging markets are in fact pricing assets rationally albeit with an emphasis on different factors. The FF5F outperforms the four-factor model in their tests. The value factor is once more made redundant by profitability and investment factors. Emerging markets are once again seen to be segmented, and profitability effects are unclear while there is a significant presence of size effects.

According to Hansen (1982) the Generalized Method of Moments (GMM) is a versatile statistical technique used for parameter estimation in econometric and statistical models, that addresses a wide range of issues that may arise in empirical analysis, such as endogeneity, heteroscedasticity, and serial correlation. In cases of endogeneity where the explanatory variables are correlated with the error term GMM provides consistent estimates even when OLS estimators may be biased (Hansen, 1982). Fama and French's (1993) factor models involve assessing the relationships between stock returns and underlying risk factors, these relationships often suffer from endogeneity concerns, where the risk factors influence stock returns and vice versa. GMM is well-suited to tackle endogeneity, providing unbiased estimates of factor loadings and risk premiums by accounting for the reciprocal influences between variables (Hansen, 1982). The incorporation of instrumental variables is another strength of GMM, allowing researchers to enhance parameter estimation by introducing external variables that are correlated with the endogenous regressors (Hansen, 1982). In the Fama and French framework, instrumental variables can address omitted variable bias and endogeneity concerns, enhancing the validity of the estimated relationships between stock returns and risk factors (Hansen, 1982). According to Arellano and Bond (1991) unlike the Ordinary Least Squares (OLS) method which minimizes the sum of squared residuals, GMM focuses on matching the population moments to the sample moments by defining a set of moment conditions based on the data and parameters (Arellano & Bond, 1991). This flexibility allows GMM to handle the complex model specifications inherent to Fama and French factor analyses. The method's adaptability to different moment conditions and model structures allows researchers to effectively capture time-varying factor loadings or cross-sectional correlations. Furthermore, Fama and French factor models often consider moments beyond the mean and variance, incorporating higher moments of stock return distributions (Fama & French, 1993). These models acknowledge that financial data frequently deviate from normality. GMM's ability to accommodate non-normality, heteroscedasticity and autocorrelation, and its avoidance of strict distributional assumptions make it a fitting choice for capturing the intricacies of stock return distributions (Arellano & Bond, 1991).

Boamah et al (2017) employ a cross-country asset pooling technique in their investigation of integration across African stock markets. The pooling approach involves combining stocks from multiple ASMs to create a unified dataset. This larger dataset allowed them to construct more diversified portfolios, which enhanced the statistical power of their tests and analyses. The approach also enabled the investigation of market integration across ASMs and the applicability of asset pricing models across a broader spectrum of the market. Their factor construction also used pooled portfolios, which allowed for a broader analysis of regional integration and the applicability of asset pricing models across markets.

Novy-Marx (2013) finds that stocks with high gross profitability tend to outperform low profitability stocks, that profitable stocks tend to be growth stocks. He finds this primarily through looking at mispricing in regression models sorted on past gross profitabilty data. He also finds that value and growth have a negative correlation (-0.57), allowing for investors to take on more risk without increasing their volatility, by taking a profitability strategy in addition to a value strategy (Novy-Marx, 2013). In his paper gross profits-to-assets was found to hold more predictive power than, earnings-to-book equity, and free cash flow-to-book equity. Fama - Macbeth regressions as seen in the paper were used to compare gross profits to earnings and free cashflows (Novy-Marx, 2013, p. 4). A key takeaway from Novy-Marx's paper is the relationship between value and gross profitability, although not a key objective in this research, it would be interesting to establish its position in Africa's equity returns. To test the effectiveness of gross-profit as a predictor of returns Novy-Marx (2013) justifies an alternative 4-factor model, in which he uses the market factor, value, momentum and gross profitability. Novy-Marx's (2013) 4 factor model outperforms the standard FF3F model. Mbengue et al., (2023) investigate 18 factors influencing stock returns in 13 African markets. Their study challenges the conventional asset pricing models derived from US data by demonstrating that the value factor (HML) remains significant in explaining returns, even in a five-factor model that includes profitability and investment factors. This finding contradicts the US-centric view that HML becomes redundant when these additional factors are considered. The research determines five factors Market, Size, Value, Momentum, and Profitability are able to price African stocks. However, each factor resides in a different model thus emphasizing the necessity for distinct asset pricing models tailored to the specific African context.

2.4 Gap in the literature

The literature review explores the landscape of asset pricing models, focusing on the evolution of the Fama and French Five Factor (FF5F) model. A notable gap in the literature is the limited inclusion of African stock markets in emerging market research. This is particularly evident when considering the FF5F model's applicability regionally. Latin America, Asia and Eastern Europe are frequent participants of emerging market studies, but it is difficult to tell, to what extent those findings are applicable to African stock markets. As results from these models are known to be sample specific, testing the ASMs seems to be the better approach to answering the question of how the model would perform on a solely African portfolio (Fama & French, 2015). While some studies include individual African stock markets, comprehensive examinations of African markets are few. Boamah et al., (2017) take the most extensive look at ASMs with 1531 stocks from 10 African nations, however they apply a 3 and 4 factor model, the FF5F has yet to be tested. Mbengue et al. (2023) investigate 18 factors influencing 700 stock returns in 13 African markets, they do not consider gross profitability as a factor. Profitability as in Operating profitability, is well researched in emerging markets, yet there does not seem to have been any consideration thus far of the Gross profitability factor. Mosoeu & Kodongo (2020) find profitability to be a useful factor in predicting returns in African markets, this paper follows a similar methodology to theirs, but is distinct in that the dataset is only African stocks, factors are contsructed regionally and the profitability factor under consideration is gross rather than operating profitability.

The gaps identified motivate this research to address the lack of representation of African stock markets in asset pricing studies. By conducting an in-depth analysis of 731 stocks across 18 African countries, this study aims to contribute valuable insights into the effectiveness of the FF5F model in explaining African equity returns. The unique characteristics, such as market size, volatility, and low integration, pose challenges that warrant a dedicated examination, and this research seeks to fill this gap by providing a holistic understanding of asset pricing dynamics in the African context. In addition to this, gross profitability is tested as a factor. Gross profit as a more direct indicator of profit ought to be more applicable in emerging markets, that are bank oriented and more reliant on accounting ratios to make investment decisions (Claesson, 2021; Foye, 2018; Novy-Marx, 2013). The intention is to offer practical implications for investors, policymakers, and researchers navigating the complexities of African equity markets. Mbengue et al. (2023) take a similar dataset and outlook to this paper, however their analysis is focused on factor spanning and GRS tests alone. This paper has a far broader battery of tests and further tests the FF5F model performance when modified with gross profitability. Mbengue et al. (2023) do not consider this specific profitability factor.

Chapter 3: Methodology

3.1 Data collection

The methodology consists of 3 phases. First, data collection and validation. Obtaining data to create a representative set of African stocks. The FF5F model requires both stock returns and accounting data. Stocks were filtered based on availability, length, and quality of the data. This was intended to create a representative African stock basket, with a variety of ASMs represented. To create the portfolios for most tests, stock returns were double sorted on Size-Profit, Size-Value, Size-Investment. An initial dataset of 24 countries and 3100 stocks was considered but the vast majority of stocks were filtered out due to insufficient stock market data and firm level data. Stocks were filtered based on first having no data entries and then less than a year's worth of data, 1 entry for annual data, 2 entries for semi-annual, 4 entries for quarterly, and 12 entries for monthly data. 9 categories of data were filtered in this way (Price, Book value to Market cap, Book value, Market cap, Net Operating Profit After Tax, Total Assets, Total Asset Growth, Gross Profit, and Operating Income), this presented 9 groups of stocks, between 1100 - 2000 in each category, of which 731 met all requirements simultaneously. The primary data sources used were DataStream and Bloomberg.

3.2 Data analysis

The second phase involved testing and running the models. The FF5F model was ran in-line with the procedures of Fama & French (2016) paper comparing the power of two models to explain the African markets as a whole. This paper compares the standard Fama and French Five Factor model (FF5F), to a modified version of the FF5F in which Novy Marx's (2013) specification of gross profitability (PMU) replaces Fama and French's (2015) Operating profitability (RMW). Henceforth referred to as the Novy model.

The methods used in this phase were based on those seen in Mosoeu & Kodongo (2020) and Fama & French (2016). Size, value, profitability, and investment patterns in average monthly returns in excess of the risk-free rate are examined (Mosoeu & Kodongo, 2020). Fama-French portfolios were used to evaluate the performance of the FF5F model against the Novy model for the sampled countries. The generalized method of moments (GMM) estimation method was used to regress excess portfolio returns against the five-factor model (Mosoeu & Kodongo, 2020). Using GMM in the Fama and French factor analysis can help address methodological challenges specific to financial data, enhance the validity of factor loadings and risk premium estimates, and provide more robust and reliable insights into the relationships between stock returns and risk factors. In a Fama and French factor analysis, endogeneity can be a concern when estimating factor loadings, as stock returns may be influenced by factors and, in turn, can influence factor values. Fama and French factor models can involve complex specifications, such as time-varying factor loadings or cross-sectional correlations. GMM's flexibility in handling different types of moment conditions makes it adaptable to various model specifications, allowing the intricacies of the factor-return relationships to captured effectively. Additionally, GMM techniques can account for correlated errors across different stocks or time periods. The empirical specification of the models was as follows,

Fama and French Five Factor (FF5F) model :

$$R_p = a_i + b_iMKT_t + c_iSMB_t + d_iHML_t + e_iRMW_t + f_iCMA_t + u_{it} \qquad (1)$$

The modified Fama and French Five factor (Novy) model :

$$R_p = a_i + b_iMKT_t + c_iSMB_t + d_iHML_t + e_iPMU_t + f_iCMA_t + u_{it} \qquad (2)$$

Where R_p is the excess portfolio return $R_p = (R_{it} - R_{Ft})$, where R_{it} is the monthly portfolio return, R_{Ft} is the risk-free rate of return, MKT is market factor MKT=(R_{Mt} - R_{Ft}), where R_{Mt} is the return on the value-weighted market portfolio; SMB_t is the size factor, HML_t is the value factor, RMW_t and PMU_t are the profitability factors and CMA_t is the investment factor; u_{it} is the zero-mean regression residual. If the factor exposures b_i, c_i, d_i, e_i and f_i capture all variations in expected returns, the intercept a_i in Eq. (1) and (2) should be statistically indistinguishable from zero (Mosoeu & Kodongo, 2020).

3.3 Portfolio and factor construction

9 portfolios are used as the left-hand side (LHS) assets in the regressions, this number of portfolios was chosen based on our sample size of 168 observations on 731 stocks (Boamah et al., 2017). The portfolios are double sorted on Size-BM, Size-OP, and Size-INV, based on the accounting data from the sample period. For each stock market, firms are ranked based on tercile breakpoints (30^{th} and 70^{th} percentile); a similar ranking approach is used for BM, OP, and INV (Mosoeu & Kodongo, 2020).

The factors on the right-hand side (RHS) consist of the excess market return (MKT) and portfolios formed based on Size (market capitalization), BM (book-to-market equity ratio), OP (net operating profit after tax divided by book value of equity), and INV (growth rate of total assets) (Mosoeu & Kodongo, 2020). 3 sets of factor returns, 2×3, 2×2, $2 \times 2 \times 2 \times 2$ are constructed in a way similar to (Fama & French, 2016). Details of the construction of the factors are displayed in Table 1 (Mosoeu & Kodongo, 2020)

Table 1: Factor construction

Construction of Size, BM, profitability, and investment factors. (Fama & French, 2016)

	Sort	Breakpoints	Factors and their components
2 × 3 sorts on	Size: median		SMB_{BM}= (SH + SN + SL)/3 - (BH + BN + BL)/3
Size and BM, or			SMB_{OP}= (SR + SN + SW)/3 - (BR + BN + BW)/3
Size and OP, or			SMB_{INV}= (SC + SN + SA)/3 - (BC + BN + BA)/3
Size and INV			SMB= (SMB_{BM} + SMB_{OP} + SMB_{INV})/3
	BM: 30th and 70th percentiles		HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2
	OP: 30th and 70th percentiles		RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2
	INV: 30th and 70th percentiles		CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2
2 × 2 sorts on	Size: median		SMB = (SH + SL + SR + SW + SC + SA)/6 - (BH + BL + BR + BW + BC + BA)/6
Size and BM, or	BM: median		HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2
Size and OP, or	OP: median		RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2
Size and INV	INV: median		CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2
2 × 2 × 2 × 2 sorts on	Size: median		SMB = (SHRC + SHRA + SHWC + SHWA + SLRC + SLRA + SLWC + SLWA)/8 - (BHRC + BHRA + BHWC + BHWA + BLRC + BLRA + BLWC + BLWA)/8
Size and BM, OP, and INV	BM: median INV: median		HML = (SHRC + SHRA + SHWC + SHWA + BHRC + BHRA + BHWC + BHWA)/8 - (SLRC + SLRA + SLWC + SLWA + BLRC + BLRA + BLWC + BLWA)/8
	OP: median GP: median		CMA = (SHRC + SHWC + SLRC + SLWC + BHRC + BHWC + BLRC + BLWC)/8 - (SHRA + SHWA + SLRA + SLWA + BHRA + BHWA + BLRA + BLWA)/8
			RMW = (SHRC + SHRA + SLRC + SLRA + BHRC + BHRA + BLRC + BLRA)/8 - (SHWC + SHWA + SLWC + SLWA + BHWC + BHWA + BLWC + BLWA)/8
			PMU = (SHRC + SHPA + SLPC + SLPA + BHPC + BHPA + BLPC + BLPA)/8 - (SHUC + SHUA + SLUC + SLUA + BHUC + BHUA + BLUC + BLUA)/8

This table outlines the construction of the factors. We use independent sorts to assign stocks to 2 Size groups, and 2 or 3 BM, Operating profitability (OP), and Investment (INV) groups. The Value-Weighted (VW) portfolios defined by the intersections of the groups are the building blocks for the factors. We label these portfolios with 2 or 4 letters. The first always describes the Size group, small (S) or big (B). In the 2 × 3 sorts and 2 × 2 sorts, the second letter describes the BM group, high (H), neutral(N), or low(L). The OP group, robust(R), neutral(N), or weak(W). The INV group, conservative(C), neutral(N), or aggressive(A). In the 2 × 2 × 2 × 2 sorts, the second character is BM group, the third is OP group, and the fourth is INV group. The factors are SMB (small minus big), HML (high minus low BM), RMW (robust minus weak OP), and CMA (conservative minus aggressive INV). *Source:* (Fama & French, 2016).

The Size breakpoint is the median market capitalization. The BM, OP and INV break points are the 30th and 70th percentiles of BM, OP and INV respectively, across all markets. The Size factor, SMB_{BM}, is calculated as the average of returns from three small BM portfolios subtracted by the average of returns from three large BM portfolios (Mosoeu & Kodongo, 2020). A similar operation on OP portfolios and INV portfolios provides SMB_{OP} and SMB_{INV} respectively. The Size factor for the 2×3 portfolio sorts is defined as the average of SMB_{BM} SMB_{OP} and SMB_{INV} (Fama & French, 2016).

The first approach in **Table 1**, sorts stocks into two Size groups and three BM groups referred to as 2×3 sorts. The value factor, HML, represents the disparity in average returns between two high BM portfolios and two low BM portfolios. Similarly, the profitability and investment factors, RMW and CMA, are constructed in the same manner as HML (Mosoeu & Kodongo, 2020).

In the second method (**Table 1**), we create 2×2 sorts based on Size and BM, OP, and INV, utilizing medians as breakpoints for each variable. The HML returns are formulated without adjusting for OP and INV. Consequently, returns on HML portfolios include premiums associated with BM, OP, and INV (Mosoeu & Kodongo, 2020). This pattern is also observed in the 2×2 sorted RMW and CMA portfolios, as well as in comparable portfolios constructed through the 2×3 sorting approach (Mosoeu & Kodongo, 2020).

As HML, RMW, and CMA from the 2×3 (or 2×2) sorting methods give equal weight to returns from small and large stock portfolios, they are approximately size-neutral. However, since HML is formulated without adjusting for OP and INV, it lacks neutrality concerning profitability and investment. This suggests that the average HML return encompasses a combination of premiums associated with value (BM), profitability (OP), and investment (INV). Similar considerations apply to RMW and CMA (Fama & French, 2016). To more effectively isolate the premiums in average returns linked to Size, BM, OP, and INV, the final candidate factors involve 4 sorts that jointly control for these variables. Stocks are independently sorted into 2 Size groups, 2 BM groups, 2 OP groups, and 2 INV groups using medians as breakpoints (Fama & French, 2016). The intersections of these groups result in 16 value-weighted portfolios. The Size factor SMB is determined as the average of the returns on the 8 small stock portfolios minus the average of the returns on the 8 big stock portfolios. HML is calculated similarly with 8 high BM portfolios and 8 low BM portfolios, RMW and CMA are calculated in a similar fashion (Fama & French, 2016). All profitability sorts are repeated replacing RMW (operating profit) with PMU the gross profitability factor.

The investigation was carried out at the regional level, Africa being a single region. The portfolio assets from individual countries were pooled and sorted, Fama French risk factors were regional as well. This approach enabled us to overcome the small sample problem in prior African studies, and could also indicate whether assets were priced in a regionally integrated manner across Africa (Boamah et al., 2017). For every portfolio formation date, we calculated the relative market value for each firm by adjusting the firm's size based on the cross-sectional mean of the market capitalization of listed firms on the corresponding exchange (Boamah et al., 2017).

Similarly, we estimated the BM ratio for each firm in a given country relative to the cross-sectional mean of the BM of all equities on that market[1] (Boamah et al., 2017). Normalizing firm size and BM by the cross-sectional mean guarantees that firms are assessed as either large or small in relation to their standing within their specific markets, rather than their ranking in the overall sample (Boamah et al., 2017). This was crucial for mitigating any potential confounding effects arising from size and BM variations in the combined sample. The use of relative market capitalization controls for biases that might result from differences in firm size across markets, while relative BM ratios aid in mitigating the influence of potential variations in accounting systems across the ASMs (Boamah et al., 2017).

Standardization enhances portfolio diversity by ensuring that the distribution of firms in each portfolio includes all sampled markets in proportion to the contribution of firms from each market to the overall pool. (Boamah et al., 2017); preventing the creation of portfolios with solely South African stocks, which is likely due to South Africa's dominance in stock count and capitalisation. Scaling becomes necessary only if there is market segmentation; therefore, the relative measures are considered suitable instruments for comparing ASMs, given that previous research indicates a certain level of segmentation amongst them (Boamah et al., 2017; Alagidede et al., 2011; Geert & Campbell, 2002; Cooke et al., 2015; Padilla & Otero, 2022). The scaling used in firm size and BM is used in all the other sorts as well.

3.4 FF5F vs Novy models

Several regression analyses were run in order to assess the performance of the model and factors. Excess returns sorted on Size – Value, Size – Investment, Size – Profitability were tabulated in order to observe size and value and profitability effects in the portfolio returns, as seen in Mosoeu & Kodongo (2020, pp. 60-61). Level of mispricing in the models was considered by plotting regression intercept tables, after which the associated slope data was also tabulated to review relationships between factors (Fama & French, 2016, p. 453). Factor spanning regressions were executed as seen

[1] $s = \frac{x_{ct}}{m_{cy}}$, *"s" is the scaled variable where x is a variable in country "c" at time "t" and "*m_{cy}*" is the mean of x variables in country "c", where "y" is the annual period within which time "t" falls.*

in Fama & French (2016, p. 449), rearranging the regression model to make the factors the subject of the formula and checking the intercept values and t-stats, in order to verify that the factors had premiums of their own, independent and unexplained by the other factors in the model.

3.5 Checking factor redundancy

GRS tests are used to test redundancy in the market factors as seen in Fama & French (2015, p. 9), by looking at changes in the GRS value. The P-values of the same test were used to identify any mispricing in the regressions as seen in Fama & French (2016, p. 451). The GRS statistic follows an F-distribution and is determined as:

$$\left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-N-1}\right)\left[\frac{\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}}{1+\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}}\right] \sim F(N, T-N-L) \text{ (i)}$$

T represents the number of return observations, N the number of portfolios, and L signifies the number of factors incorporated into the valuation model (Mosoeu & Kodongo, 2020). $\hat{\alpha}$ is a vector comprising intercepts obtained through the estimation of Eq. (i), and $\bar{\mu}$ is a vector of means of factor portfolios (Mosoeu & Kodongo, 2020); $\hat{\Omega}$ is a covariance matrix of factors estimated as:

$$\hat{\Omega} = \frac{(F-\bar{F})'(F-\bar{F})}{T-1} \text{ (ii)}$$

Where F refers to the matrix of factors, which is composed of excess returns from portfolios. ∑ is the covariance matrix of residuals such that:

$$\hat{\Sigma} = \frac{\hat{e}'\hat{e}}{T-L-1} \text{ (iii)}$$

Where ê represents a vector of residuals; A lower value of the regression intercept implied a higher probability that the GRS test would not reject the five-factor model (Mosoeu & Kodongo, 2020). The GRS test checks the ability of the factor models to explain monthly excess returns in various portfolio sorts. The GRS null hypothesis tests whether the expected values of all portfolio intercept estimates are zero. Significant p-values shows presence of some mispricing in the model (Fama & French, 2016, p. 451). Applying this GRS test on portfolios sorted on size, operating profitability, book-to-market, and gross profitability, we can find the best performing model based on level of mispricing. To check for redundancy, the regression factors are regressed on each other, this is a spanning test in which a significant intercept confirms that a factor is not fully explained by the other factors (Fama & French, 2015, p. 9), as seen below:

FF5F

$$MKT = a_i + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (2.1)$$

$$SMB_t = a_i + b_iMKT + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (2.2)$$

$$HML_t = a_i + b_iMKT + s_iSMB_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (2.3)$$

$$CMA_t = a_i + b_iMKT + s_iSMB_t + h_iHML_t + r_iRMW_t + e_{it} \quad (2.4)$$

Novy

$$MKT = a_i + s_iSMB_t + h_iHML_t + r_iPMU_t + c_iCMA_t + e_{it} \quad (2.5)$$

$$SMB_t = a_i + b_iMKT + h_iHML_t + r_iPMU_t + c_iCMA_t + e_{it} \quad (2.6)$$

$$HML_t = a_i + b_iMKT + s_iSMB_t + r_iPMU_t + c_iCMA_t + e_{it} \quad (2.7)$$

$$CMA_t = a_i + b_iMKT + s_iSMB_t + h_iHML_t + r_iPMU_t + e_{it} \quad (2.8)$$

By assessing the GRS p-values, we could examine potential redundancy. If the removal of a factor from the model did not alter the GRS p-value, it indicated that the factor was redundant (Fama & French, 2016). Notably, special consideration was given to evaluating the redundancy of SMB, which served as an initial test for the existence of the size effect (Fama & French, 2016). Some extent of mispricing was expected, so a more detailed breakdown was done tabulating the intercept value of each portfolio for the individual countries, as seen in (Fama & French, 2016, p. 453). This provided a comparative view of how much mispricing was present in each of the models. The performance of FF5F model could then be benchmarked against the emerging market data from the Fama & French (2016), in terms of the magnitude of mispricing.

3.6 Extracting factor risk premia

Testing for size effects. The FF5F tests suggest the presence of the size effects, so additional testing was done following a methodology to identify the business cycle periods and phases (Ahn et al., 2019). This entailed, defining an African business cycle (Ahn et al., 2019, p. 5). The method seen in Ahn et al's (2019) paper involved identifying peaks and troughs, in economic growth and then designating the 3 months before and after (7 months in all) as peak and trough stages respectively. 2 transitory stages are defined as well, Expansion, the period from a trough to a peak, and recession, the period from a peak to a trough. Ahn et al., used precalculated business cycle data for the USA from NBER. They tested this method against a frequently used method the Hodrick-Prescott Filter (HP Filter) and found the results to be similar. In Africa precalculated business cycle turning point data is not widely available, so the HP filter method was used on basic economic data (GDP) to identify the business cycle of each country. GDP data is published for all countries, but it came with the drawback of being low frequency (annual or bi-annual data), a better indicator would have been industrial productivity index with a monthly frequency, but the data was not available for every country in this investigation.

Once the four stages of the business cycle were identified, we then needed to test for a significant return on the Size factor (SMB) in each of the 4 periods (Ahn et al., 2019, p. 5). Ahn et al., found that when restricted to Trough periods there is a significant return on size, i.e., conditional size effect. Similarly, the FF5F model was tested by setting the periods to coincide with trough, peak, recession, and expansion stages, to hopefully identify some conditional size effect (Ahn et al., 2019, p. 5).

Considering the frequency of the business cycle. As per Ahn et al. (2019) the unconditional size effect is the weighted mean of conditional size effects across various stages of the business cycle, with each stage's probability serving as the weight. Having proven the existence of conditional size effects, the next step was to find weights at which the unconditional size effect regained effectiveness i.e., the probability of business cycle stages. To explore this, a Markov chain model was employed to examine whether altering only the transition probability matrix could lead to changes in the duration of the business cycle and influence the size effect (Ahn et al., 2019). Four states of economic activity were examined, each of which represented a single phase of the business cycle (Ahn et al., 2019). The dynamic progression of the economy was characterized by a four-by-four transition matrix, indicating the likelihood of moving from one state to another (Ahn et al., 2019). Changes in the length of the business cycle, were accommodated for by changing the transition probabilities in the Markov chain (Ahn et al., 2019).

In line with Ahn et al (2019) the following procedures are adopted: an initial transition matrix for the specified time period is estimated. It was assumed that continuously compounded rates of return on small and large stocks adhere to conditional normal distributions; The means, variances, and covariance of these distributions are contingent on the different stages of the business cycle throughout the entire sample period (Ahn et al., 2019). Thus, allowing any alterations in the business cycle shape and the unconditional size effect to solely be attributed to modifications in the transition matrix (Ahn et al., 2019).

With the assumed conditional distributions for small and large stocks, coupled with the data generating process governing the economic state's evolution, a time-series of returns for both small and large stocks is simulated. These time series are generated for entire periods by utilizing the corresponding transition matrices. Subsequently, outcomes of the simulation are compared to estimates derived from the historical data.

A regime indicator variable, I_t, as defined by Ahn et al. (2019) represents each business cycle stage where:

$I_t = 1$ if time t belongs to the Trough stage,

$I_t = 2$ if time t belongs to the Expansion stage,

$I_t = 3$ if time t belongs to the Peak stage,

$I_t = 4$ if time t belongs to the Recession stage,

I_t undergoes changes following a first-order Markov process, with a transition probability matrix featuring an element:

$p_{i,j} =$ Prob $[I_t = j \mid I_{t-1} = i]$ $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$ (iv)

The construction of the transition matrix involves assigning each month to an economic state representing the four stages of the business cycle. The elements within each row of the transition matrix were determined as the proportional sample frequencies of transitioning from a specific state to each of the four states. The transition matrices for the entire sample are then estimated. (Ahn et al., 2019).

With the estimated transition probability matrix, the steady-state probability associated with each state as indicated by the matrix can be calculated. Subsequently, we verify that the steady-state simulated probabilities closely resemble those observed in historical percentages. (Ahn et al., 2019). This indicates that the Markov chain model is a reasonably accurate estimate. If there is a significant difference between the simulated and historic values it is likely due to, too small a sample size.

Next, continuously compounded returns were modelled, on small and large stocks as conditionally bivariate normal where their mean vector and covariance matrix depend on the business cycle stage.

Specifically, continuously compounded returns on small stocks ($\ln R_{S,I\,t}$) and large stocks ($\ln R_{L,I\,t}$) at state I_t , $\ln R_{I\,t}$ = ($\ln R_{S,I\,t}$, $\ln R_{L,I\,t}$), are assumed to be bivariate-normally distributed:

$$\ln R_{I\,t} \sim MV\,N_2(\mu_{I\,t}, \Omega_{I\,t}) \qquad (v)$$

where $\mu_{I\,t}$ and $\Omega_{I\,t}$ are the mean vector and the covariance matrix at state I_t, the mean vector and the covariance matrix, ($\mu_{I\,t}$, $\Omega_{I\,t}$), corresponding to each business cycle stage, I_t are estimated from the entire set of samples (Ahn et al., 2019).

Finally, the presumed conditional distributions of small and large stocks along with the calculated transition probabilities, are employed to produce time-series data on SMB generated from the corresponding transition matrices; Subsequently, we compare the statistical properties (first and second moments) of the generated data with those of the historical data (Ahn et al., 2019). The detailed procedure involves initially generating a sequence of business cycle stages, denoted as I_t, based on the transition matrix estimated from the entire sample. The sample size is selected to align with the number of monthly observations. Second, given a sequence of I_t, independent draws from a normal distribution defined in eqn. (iv) are taken to form a sequence of returns on small and large stocks. A series of SMB is calculated by taking the difference between the simulated returns on small and large stocks. Next, we iteratively generate data 1000 times. The means, standard deviations, and t-statistics for the simulated returns on small and large stocks, as well as SMB, are computed. These simulated results are then compared with the estimates obtained from the historical data (Ahn et al., 2019).

If the Markov chain model performs satisfactorily in reproducing the size effect, the next step is to analyse the sensitivity of the unconditional size effect to the duration of expansion. In other words, assuming all other factors remain constant, we vary the value of $p_{2,2}$, which determines the duration of the Expansion phase. Subsequently this analysis is repeated for all stages, and SMB returns and cycle stage returns are recorded i.e. in case of significant size effect being found in any of the other 4 business cycle stages (Ahn et al., 2019).

Finally, we plot a graph of returns vs business cycle duration in order to make inferences as to whether a change in cycle duration could result in a significant size effect, by comparing returns across various cycle structures (Ahn et al., 2019).

3.7 Gross profitability

To test the effectiveness of gross profit as a predictor of returns this paper builds the gross profitability factor as specified by Novy-Marx (2013). In his paper Novy-Marx (2013) justifies an alternative model to the FF3F model, in which he uses the market factor, value, momentum and gross profitability. Having been justified previously, this paper simply applies the factor as part of a modified Fama and French 5 Factor model, referred to in this paper as the (Novy) model. In order to fully isolate the qualities of Gross profit and Operating profitability the only difference between the models relates to the profitability factor. Unlike Fama and French's RMW constructed from Operating Profit sorts, Novy-Marx's profitability factor is constructed from Gross profit sorts. PMU - Profitable minus Unprofitable is constructed similarly to Fama French's profitability factor (RMW), the top 30% of returns sorted on gross profit are taken as (Profitable) and the bottom 30% are taken as (Unprofitable) (Novy-Marx, 2013). The regression and factor analyses are run in parallel on the FF5F and Novy model, using the same underlying data. Many other papers have similarly compared various factor models (Leite et al., 2018; Taha & Elgiziry, 2016; Foye, 2018; Claesson, 2021).

Chapter 4: Results

4.1 Description of the African stock markets

Table 2: African stock market characteristics

Characteristics of the African stock markets.

Country of stock market	Number of stocks		Mean (US$000)				Mean	
	Initial	Final	M. Cap.	BV	NOPAT	G. Profit	BV to M. Cap.	Price (US$)
Botswana	33	11	1446	155	10	38	0.84	0.69
Burkina Faso (BRVM)				-				
Cape Verde	4	2	1902	145	6	45	1.30	0.12
DRC (BRVM)	-	-	-					
Egypt	887	212	5704	3903	24	65	1.12	2.10
Eswatini	9	1	1706	135	24	58	0.91	1.18
Gabon (BRVM)	-	-	-					
Ghana	43	19	928	33	15	33	1.45	0.83
Ivory Coast (BRVM)	56	7	20415	264	45	409	1.58	10.15
Kenya	77	32	58352	205	32	103	0.62	1.44
Madagascar	1	1	60	153	-11	0	4.55	0.27
Malawi	19	6	58057	64	16	68	0.89	0.74
Mauritius	151	39	5734	203	23	97	1.19	2.26
Morocco	132	17	14833	445	51	139	0.66	28.35
Namibia	24	7	13412	477	83	534	0.87	3.39
Nigeria	328	94	56812	104	21	61	-0.85	1.49
South Africa	1063	232	20442	496	72	342	2.68	8.48
Tanzania	25	12	683145	292	107	204	0.31	1.58

Tunisia	103	22	1580	61	2	30	0.99	1.67
Uganda	10	5	285626	44	23	83	1.55	0.10
Zambia	27	12	1498	348	58	251	1.06	1.73
Restrictions								
Initial stock count	2999							
Less than 1 yrs. worth data in a dataset	1645							
No Gross Profit data	548							
Common Stocks		731						

This table details the characteristics of the candidate stock markets that for this studies Stock universe. Source: Authors' computation.

Stock return data from African Stock Markets (ASMs) spanning August 31, 2009, to June 30, 2023, forms the basis for this analysis. Although an earlier start date was initially considered, data availability restricted the final period. We began with 2999 stocks across 23 ASMs, primarily sourced from Bloomberg data **Table 2**. To mitigate survivorship bias, all listed firms on these markets, including delisted ones, were considered (Banz & Breen, 1986; Kothari et al., 1995). However, to ensure data quality and improve test power, firms lacking at least one year of consistent data (annual, semi-annual, quarterly, or monthly) were excluded (Carhart, 1997; Boamah et al., 2017). This impacted the final sample size across different data categories (e.g., Stock Price: 2000 observations, Gross Profit: 800). Maintaining a consistent stock universe across all categories further reduced the sample to a relatively small 731 stocks, Novy-Marx (2013) tests profitability on 4500 stocks, and Boamah et al. (2017) on 1500 stocks. Notably, the restriction based on Gross Profit data availability significantly reduced the sample by 548 stocks, highlighting potential data quality concerns in some markets. Interestingly, while South Africa, Egypt, and Nigeria contributed the largest number of stocks initially (32%, 29%, and 13% respectively), they also experienced the highest percentage reduction due to data restrictions (South Africa: 80%, Egypt: 75%, above the average loss of 65%). This suggests potential reporting standard issues in these larger markets despite their size (Banz & Breen, 1986; Kothari et al., 1995).

The data selection process, while prioritizing bias mitigation and data quality, resulted in a smaller sample size, potentially limiting the generalisability of findings. The significant impact of Gross Profit data availability on sample size and the disproportionate exclusion of stocks from larger markets raised concerns about data quality and sample representativeness, warranting further investigation and exploration of alternative data sources or imputation techniques.

4.2 Descriptive statistics for FF5F and Novy regression factors

Table 3: Descriptive statistics of factors

Descriptive stats for Fama and Novy regression factors															
Panel A: FF5F			2 × 2					2 × 2 × 2 × 2					2 × 3		
	MKT	SMB	HML	RMW	CMA	MKT	SMB	HML	RM W	CMA	MKT	SMB	HML	RM W	CMA
Mean (%)	-5.17	0.24	0.61	0.14	-0.01	-5.17	0.46	0.61	0.15	0.11	-5.17	0.32	1.04	0.58	0.54
St dev	2.67	2.40	2.61	2.84	3.00	2.67	1.97	1.79	2.22	2.43	2.67	2.21	2.71	3.28	3.40
t-stat	-25.1	1.3	3.0	0.6	0.0	-25.1	3.0	4.4	0.8	0.6	-25.1	1.9	5.0	2.3	2.1
Panel B: Novy			2 × 2					2 × 2 × 2 × 2					2 × 3		
	MKT	SMB	HML	PMU	CMA	MKT	SMB	HML	PMU	CMA	MKT	SMB	HML	PMU	CMA
Mean (%)	-5.17	0.26	0.61	-0.14	-0.01	-5.17	0.53	0.70	0.18	0.29	-5.17	0.09	1.04	-0.13	0.54
St dev	2.67	2.30	2.61	3.35	3.00	2.67	2.00	2.02	2.62	2.23	2.67	2.22	2.71	3.64	3.40
t-stat	-25.1	1.5	3.0	-0.6	0.0	-25.1	3.4	4.5	0.9	1.7	-25.1	0.5	5.0	-0.5	2.1

This table reports the descriptive statistics for Fama-French and Novy regression factors in emerging markets, drawing insights from Panel A (FF5F) and Panel B (Novy). In it we find partial evidence for the size and profitability premium, while there seems to be a strong value premium. Unlike the other factors the Market factor (MKT) is unchanged across all sorting regimes, and thus it is identical reporting the same set of results (return: -5.17%, standard deviation: 2.67%, t-stat: -25.1). It is notable that this factor is an order of magnitude larger than the others and this may be due to some miss specification. Source: Authors' computation.

Table 3 presents descriptive statistics for FF5F and Novy regression factors in the emerging market space, the Market factor (MKT) is unchanged cross all sorting regimes, being large and constant the discussion that follows, focuses on patterns affecting the other factors.

In Panel A, the 2 × 2 and 2 × 3 sorts offer a trade-off between diversity and significance (Mosoeu & Kodongo, 2020). While 2 × 2 sorts exhibit more diverse factor exposures with generally lower standard deviations (2.84% vs. 3.28% for RMW), they yield fewer statistically significant factors. Only the value factor (HML) with a t-statistic of 3.0 shows significance in 2 × 2 sorts, supporting the value premium (Fama & French, 1993). In contrast, all factors in the 2 × 3 sorts achieve high statistical significance alongside substantial return increases (e.g., HML: 0.61% to 1.04%). However, this comes at the cost of higher standard deviations (e.g., CMA: 2.84% to 3.28%). The $2 \times 2 \times 2 \times 2$ sort, controlling for all four factors simultaneously, appears to mitigate the standard deviation issue (SMB: 2.40% to 1.97%) while enhancing statistical significance (HML: t-stat 3.0 to 4.4). However, its impact on returns remains nuanced. HML and RMW returns remain unchanged, while SMB is maximized (0.46%, t-stat 3.0) and CMA takes a positive but non-significant value of 0.11%, much smaller than its 2×3 return (0.54%, t-stat 2.1). SMB returns (0.24% - 0.46%) lack significance in the 2×2 sort and fall short in the 2×3 sort (t-stat 1.9 vs. 1.96). Only in the $2 \times 2 \times 2 \times 2$ sort does it achieve high significance (0.46%, t-stat 3.0). HML consistently exhibits high significance (t-stat 3.0 - 5.0) and large returns (0.61% - 1.04%) across sorts, solidifying the value premium's presence in emerging markets, echoing similar observations by Fama and French (2015), Boamah et al (2017), And Leite et al (2018). RMW shows high significance in the 2×3 sorts (0.58%, t-stat 2.3), suggesting a profitability premium exists. However, its significance fades in other sorts, highlighting its conditional nature. CMA exhibits similar behaviour, being highly significant in the 2×3 sorts (0.54%, t-stat 2.1) but essentially zero in others. This conditional significance warrants further exploration.

Panel B (Novy sorts) presents similar patterns with comparable values, as expected due to the substitution of Operating profitability (RMW) with Gross profitability (PMU) (Novy-Marx, 2013). Notably, the 2×2 and 2×3 sorts where HML and CMA are independent of profitability are identical in both panels. While SMB in the 2×2 and $2 \times 2 \times 2 \times 2$ Novy sorts shows some improvement compared to its standard counterparts, coinciding with PMU's better performance, both factors essentially fall to zero in the 2×3 sort. This suggests that the profitability measure (RMW vs. PMU) can influence the significance and behaviour of these factors, requiring careful consideration (Chen et al., 2011). In the $2 \times 2 \times 2 \times 2$ Novy sorts, all factors exhibit an increase in returns compared to those in Panel A, suggesting a positive alteration in the relationship between profitability and the other factors.

Table 4 shows the stock distribution over the various portfolios used in this research. The small sample size of this research is accentuated here. Several trends emerge, the portfolios are very small only two portfolios manage an average size over 100 (Novy-Marx, 2013; Fama & French, 2016; Boamah et al., 2017). The average size 55, is lower than expected with 731 stocks and 9 portfolios per sort the average is expected to be closer to 80, this indicates a lot of unused stocks. Furthermore an uneven distribution of stocks is clearly visible in table 4, the middle portfolios are all well above average (77,78,88,105) while 6 of the extreme portfolios(Small-Aggr/Robust/Prof,Big-Cons./Weak/Unprof.) are very low (30/25/4, 34/26/8). The worst of these being the gross profitability sort portfolios with 4 and 8 stocks. The distributions were not even, which was not unexpected, however portfolios that were too small produce spurious results. The gross profitability sort as a whole is troublesome, it has the lowest total stock count of 450, its formed on 61% of available stocks. Includin the 4 and 8 stock portfolios it has 2 portfolios with 31 stocks making a total of 4 exceptionally low portfolio counts while the 2 extraordinarily high portfolio counts (105,106) are less trouble than they are a sign of trouble. Gross profitability is bad but the other 3 sorts use between 65-75% of the available stocks. Considering the quality of data observed hereafter, it's likely that significant gains could be had from further imputation of the data, so as to fully utilize the 731 stocks. Gains in both stability of the results (producing clearer trends) and in increasing the R-squareds.

Table 4: Portfolio stock count

Distribution of stocks across portfolios							
Size - Value					Size - Investment		
			BM			INV	
		High		Low	Aggr.		Cons.
	Big	46	59	62	55	68	34
Size		82	88	49	59	78	62
	Small	56	59	49	30	52	55
Size - Profitability							
			OP			GP	
		Robust		Weak	Prof.		Unprof.
	Big	55	75	26	106	31	8
Size		57	77	60	31	105	46
	Small	25	49	56	4	50	77

This table describes the stock distribution amongst the 9 Left hand side portfolios, sorted on Size – Value, Size – Investment, and the two variants of profitability, Size – Operating Profitability and Size Gross Profitability. Source: Authors' computation.

4.3 Excess portfolio returns

Table 5: Excess returns

Average Monthly Excess Returns

Panel A: Average Returns										
FF5F			BM			INV			OP	
		High		Low	Aggr.		Cons.	Robust		Weak
	Big	-5.48%	-6.41%	-6.07%	-6.24%	-5.66%	-6.68%	-6.11%	-6.18%	-5.28%
Size		-5.80%	-5.28%	-7.41%	-5.90%	-6.56%	-5.85%	-6.54%	-5.45%	-7.78%
	Small	-6.00%	-5.83%	-5.70%	-8.49%	-5.75%	-5.41%	-6.38%	-7.79%	-5.34%
Novy			BM			INV			GP	
		High		Low	Aggr.		Cons.	Prof.		Unprof.
	Big	-5.48%	-6.41%	-6.07%	-6.24%	-5.66%	-6.68%	-6.18%	-5.57%	-5.92%
Size		-5.80%	-5.28%	-7.41%	-5.90%	-6.56%	-5.85%	-8.22%	-6.35%	-5.75%
	Small	-6.00%	-5.83%	-5.70%	-8.49%	-5.75%	-5.41%	-5.66%	-6.20%	-6.51%
Panel B: T-Stats										
FF5F			BM			INV			OP	
		High		Low	Aggr.		Cons.	Robust		Weak
	Big	-6.2	-10.8	-19.2	-17.2	-10.5	-12.7	-15.0	-13.5	-11.6
Size		-11.2	-6.3	-4.6	-12.9	-4.4	-11.8	-3.4	-11.5	-3.8
	Small	-14.7	-14.5	-14.1	-4.8	-14.1	-16.9	-12.1	-6.6	-13.5
Novy			BM			INV			GP	
		High		Low	Aggr.		Cons.	Prof.		Unprof.
	Big	-6.2	-10.8	-19.2	-17.2	-10.5	-12.7	-16.2	-11.0	-7.4
Size		-11.2	-6.3	-4.6	-12.9	-4.4	-11.8	-2.2	-8.4	-12.9
	Small	-14.7	-14.5	-14.1	-4.8	-14.1	-16.9	-11.3	-9.1	-9.6

Source: Authors' computation.

Table 5 presents the average monthly excess returns of value-weighted portfolios, systematically sorted into three size groups and further into either book-to-market ratio (BM), investment (INV), or operating profitability (OP), creating 27 portfolios in total for both FF5F and Novy models, following standard Fama and French portfolio formation techniques (Fama & French, 2016; Fama & French, 2015; Boamah et al., 2017; Mosoeu & Kodongo, 2020). The returns generally exhibit a vertical increase within each matrix, highlighting a size effect whereby returns tend to increase as size decreases, a phenomenon consistent with extant literature (Fama & French, 2015; van Dijk, 2011; Novy-Marx, 2013). Particularly, the Novy model substitutes operating profitability with gross profitability, introducing distinct profitability sorts, while the book-to-market and investment sorts remain identical. The data reveals that the Novy Marx's (2013) profitability effect is predominantly observed in the smallest size group within the gross profitability sort. Notably, the size effect is pronounced in low-value profitability and investment portfolios, consistent with the susceptibility of small stocks to market anomalies as indicated in previous studies (Chan, 1985; Fama & French, 1993; Fama & French, 2015; Novy-Marx, 2013; Boamah et al., 2017; Fama & French, 2016).

The returns generally increase vertically down in each matrix. Generally, as size decreases in every sort the returns increase, in the case of negative returns the returns become less negative, this trend is known as the size effect (Ahn et al., 2019; Fama & French, 2015; Crain, 2011). A similar inverse relation is seen in Size – investment portfolios and both profitability sorts. As investment becomes more conservative and profitability falls, returns become less negative. This the Novy Marx's (2013) profitability effect (higher returns in more profitable stocks) in the smallest size group within the gross profitability sort.

The size effect is particularly strong in the third columns, low value (-6.07% to -5.70%), low investment (-6.88% to -5.41%) , and low operating profitability (-7.78% to -5.34%) portfolios, and similarly small size portfolios show the most consistent change across the horizontal, which is in line with extant literature that suggests small stocks are the most susceptible to market anomalies (Carhart, 1997; Fama & French, 1993; Ahn et al., 2019; Fama & French, 2016; Fama & French, 2015). The high investment portfolios deviate from the expected size effect trend by (showing more negative returns in the small size portfolio), attributed to the notably low returns observed on high investment stocks, particularly affecting small to medium stocks, this inverse relationship between returns and investment in high investment portfolios aligns with prior findings (Fama & French, 2015). Coming to any conclusions about behavior of excess returns is difficult, the overall patterns in the data are weak and noisy (i.e., extremely non – monotic), rising and falling of returns across the groups with relatively small deltas, potentially this is a result of poor market integration, inadequate imputation of the data and the limited resolution of the 3×3 double sorts (Boamah et al., 2017). The compromise made for statistical viability is acknowledged, as a more extensive sort might have compromised the power of tests due to reduced diversification and empty portfolios. A compromise is established with the 3×3 sorts, considering the average stock count per portfolio is 55, in contrast to some studies managing over 100 stocks per portfolio (Novy-Marx, 2013; Fama & French, 2016; Boamah et al., 2017).

Portfolios formed in this study control for size and one other factor, so the stocks maybe remain highly affected by a 3rd and 4th factor as well, setting up a quadruple sort on Size -BM-INV-OP could give increased clarity and reduce any confounding effects, but the 731 stock count makes these sorts impractical (Fama & French, 2015; Fama & French, 2016; Boamah et al., 2017). With the current average of 50 stocks per portfolio in the 9 portfolios, a $4 \times 4 = 16$ or a $3 \times 3 \times 3 \times 3 = 81$

portfolios, could have greatly reduced or completely wiped out the power of the tests as portfolios would have even fewer stocks and greatly reduced diversification and a large number of empty portfolios, so the 3×3 sorts while not ideal are a compromise made for statistical viability (Fama & French, 2015; Boamah et al., 2017). Despite these limitations, this analysis provides valuable insights into the complex interplay of Fama and French factors in African markets, along with the presence of size effects and show potential differences between operating and gross profitability factors.

4.4 Test results and analysis of FF5F and Novy regressions

4.4.1 Gibbons, Ross, and Shanken F-test statistics

In **Table 6** both models do a poor job of explaining returns but as in many other papers, we focus on uncovering which model and combination of factors does the least bad job (Fama & French, 2015; Fama & French, 2016; Boamah et al., 2017; Mosoeu & Kodongo, 2020). Despite the overall poor performance, a nuanced analysis reveals intriguing patterns. Comparing Panel A (FF5F) to Panel B (Novy), the Novy model generally outperforms the FF5F model, particularly in the 2×2 and 2×3 sorts, emphasizing its relative superiority in capturing return variations. Notably, the outperformance is most pronounced in sorts related to size and profitability, especially when PMU (gross profitability) is included. A notable exception occurs in the last two models of Panel B, where the Novy models exhibit an unusual performance dip compared to their FF5F counterparts. The last to 2 models in Panel B where the 2×3 sorts generate a 48.86 and 49.68 GRS value, whereas the similar models in Panel A get a value of 40.16 and 39.92, this is also unusual because corresponding models in in Panel A and B tend to be 3 or 4 GRS value points apart, where as those two show an 8-point differential.

The FF5F model outperforms the Novy models in the $2 \times 2 \times 2 \times 2$ sorts, indicating specific scenarios where the traditional FF5F model exhibits superior explanatory power. In the $2 \times 2 \times 2 \times 2$ sorts the FF5F models generally outperform the Novy models and while $2 \times 2 \times 2 \times 2$ FF5F models perform the best in Panel A, the Novy $2 \times 2 \times 2 \times 2$ sorts show some of the poorest performance in panel B. In each factor set the first 4 factor models (MKT HML RMW /PMU CMA) achieve the lowest GRS stats, and while in general the Novy models outperform the FF5F model the absolute best model is the $2 \times 2 \times 2 \times 2$ sorted size – profit portfolios regressed on the (MKT HML RMW CMA) factors with a GRS value of 33.01, which is still extremely high.

One of the primary concerns in assessing the models is the performance of size and profitability factors. There is a ubiquitous trend as previously stated for the model without size to perform the best. As for profitability it seems the addition of it makes the model perform worse, although more so for gross profit (PMU) the same is true for (RMW). At the very least as the models were setup it is clear that when value is replaced by size or profitability the performance falls. The lower GRS stat seen in the five factor models is expected but not particularly meaningful, even if randomly selected having more factors leads to an improvement in model stats (Fama & French, 2015; Fama & French, 2016). But the size factor seems undesirable, as the best models seem to come about by excluding size, and pairing value with profitability, this seems a rather contentious result as many of the following analyses find usefulness in the SMB factor. These tests also show a preference for profitability over size.

Despite these nuances, it is crucial to acknowledge the high GRS values across all models, suggesting a systematic mispricing that remains unexplained, something broadly observed in literature on asset pricing anomalies and model limitations (Ahn et al., 2019; Novy-Marx, 2013; Cox & Britten, 2019). The analysis underscores the challenges in model selection and the nuanced impact of factors such as size and profitability on model performance, consistent with existing literature expectations (Novy-Marx, 2013; Cox & Britten, 2019; Fama & French, 1993).

Table 6: GRS test statistics

GRS test statistics						
Panel A: FF5F						
	2×2		$2 \times 2 \times 2 \times 2$		2×3	
Size-Bm	GRS	p(GRS)	GRS	p(GRS)	GRS	p(GRS)
MKT HML RMW CMA	39.86	0.000	39.79	0.000	40.57	0.000
MKT SMB HML CMA	44.11	0.000	39.33	0.000	41.50	0.000
MKT SMB RMW CMA	44.39	0.000	39.35	0.000	43.53	0.000
MKT SMB HML RMW CMA	43.88	0.000	39.07	0.000	43.42	0.000
Size-Inv						
MKT HML RMW CMA	45.07	0.000	45.35	0.000	47.24	0.000
MKT SMB HML CMA	50.45	0.000	46.02	0.000	52.79	0.000
MKT SMB RMW CMA	50.14	0.000	45.50	0.000	54.34	0.000
MKT SMB HML RMW CMA	49.72	0.000	45.53	0.000	54.30	0.000
Size-Profit						
MKT HML RMW CMA	33.86	0.000	33.01	0.000	34.03	0.000
MKT SMB HML CMA	40.45	0.000	33.24	0.000	36.82	0.000
MKT SMB RMW CMA	40.94	0.000	32.36	0.000	40.16	0.000
MKT SMB HML RMW CMA	41.03	0.000	33.51	0.000	39.92	0.000

Panel B: NOVY						
Size-Bm						
MKT HML PMU CMA	39.66	0.000	43.53	0.000	39.20	0.000
MKT SMB HML CMA	41.33	0.000	39.06	0.000	42.58	0.000
MKT SMB PMU CMA	42.95	0.000	43.67	0.000	42.50	0.000
MKT SMB HML PMU CMA	42.01	0.000	43.60	0.000	42.29	0.000
Size-Inv						
MKT HML PMU CMA	45.29	0.000	48.02	0.000	45.93	0.000
MKT SMB HML CMA	47.54	0.000	45.06	0.000	53.87	0.000
MKT SMB PMU CMA	49.22	0.000	48.97	0.000	53.59	0.000
MKT SMB HML PMU CMA	48.17	0.000	48.65	0.000	53.33	0.000
Size-Profit						
MKT HML PMU CMA	33.32	0.000	35.47	0.000	46.52	0.000
MKT SMB HML CMA	36.97	0.000	33.06	0.000	39.10	0.000
MKT SMB PMU CMA	36.76	0.000	37.18	0.000	48.86	0.000
MKT SMB HML PMU CMA	36.40	0.000	37.52	0.000	49.68	0.000

This table presents comprehensive statistics for the Fama-French five-factor (FF5F) and Novy models, analysing various combinations of size (MKT), book-to-market (HML), profitability (PMU), investment (RMW), and conservative investment (CMA) factors across different sorting dimensions. The GRS statistics, representing the Gibbons, Ross, and Shanken (1989) *test, indicate a lack of explanatory power for both models, with all p-values being 0 and GRS values ranging from 30 to 55. Source: Authors' computation.*

4.4.2 Factor spanning test

The factor spanning tests reveal if a factor's returns are fully captured by the remaining factors, indicating redundancy. The $2 \times 2 \times 2 \times 2$ sorts leave only MKT and SMB unexplained, with large significant intercepts. Notably, the market factor (MKT) exhibits a stable -5% intercept on average, with a highly significant t-statistic of -17%, indicating that the other factors absorb very little of its returns (Fama & French, 1993). The value factor (HML) generally shows no significant intercept, except in the 2×2 sorts, where the Novy model demonstrates a more negative and significant intercept. The positive slope with profitability aligns with expectations, reflecting the correlation between high profitability and high-value stocks (Fama & French, 2015; Novy-Marx, 2013). A similarly strong but negative slope from the size factor (SMB), which isn't unexpected as there is a correlation between the long side of value (High) and the short side of size (Big) i.e., big businesses tend to be more profitable. Profitability seems to be largely explained by other factors, RMW has moderate returns (0.6 - 1.0%) with low statistical significance (t-stat: 0.7 to 1.9) so they are effectively zero. It is however poorly spanned by other factors in the FF5F sorts. PMU in the 2×2 Novy sort is the only unexplained profitability factor, its intercept (2.8%) is relatively large and highly significant (4.4), despite being highly spanned by MKT (slope: 64.8%) and SMB (slope: 78.8%). These findings underscore the heterogeneity among profitability factors and the challenges in spanning them adequately (Novy-Marx, 2013; Cooke et al., 2015; Chan, 1985).

Overall, this analysis identifies varying degrees of factor explicability. While MKT remains crucial and HML shows distinctness, size and profitability factors require further investigation, considering data quality and potential model enhancements, to fully understand their roles in African markets.

Table 7: Factor spanning tests

Factor spanning tests												
Panel A: FF5F	Coefficient						T-Stat					
2×2	Int	MKT	SMB	HML	RMW	CMA	Int	MKT	SMB	HML	RMW	CMA
MKT	-5.0%		-42.6%	-29.6%	11.2%	-3.8%	-17.0		-5.8	-4.8	1.6	-0.6
SMB	-1.8%	-43.5%		-36.7%	17.0%	9.0%	-3.6	-5.8		-6.1	2.4	1.5
HML	-1.6%	-43.3%	-52.6%		-49.4%	-20.8%	-2.7	-4.8	-6.1		-6.3	-2.9
RMW	1.0%	13.5%	20.1%	-40.9%		0.8%	1.8	1.6	2.4	-6.3		0.1
CMA	-0.3%	-6.0%	13.9%	-22.5%	1.1%		-0.5	-0.6	1.5	-2.9	0.1	
2×3												
MKT	-5.0%		-42.2%	-17.5%	2.7%	12.6%	-16.6		-5.8	-3.3	0.6	2.3
SMB	-1.7%	-42.7%		-23.6%	10.9%	12.8%	-3.5	-5.8		-4.5	2.5	2.3
HML	-0.9%	-35.9%	-48.0%		7.3%	36.8%	-1.3	-3.3	-4.5		1.1	4.7
RMW	0.6%	8.0%	32.5%	10.7%		21.2%	0.7	0.6	2.5	1.1		2.2
CMA	1.1%	23.7%	23.8%	33.7%	13.3%		1.6	2.3	2.3	4.7	2.2	
$2 \times 2 \times 2 \times 2$												
MKT	-5.1%		-27.2%	-21.7%	21.3%	1.2%	-17.3		-3.2	-2.4	2.8	0.2
SMB	-1.1%	-21.7%		41.5%	32.9%	7.5%	-2.6	-3.2		5.4	5.1	1.1
HML	-0.4%	-14.7%	35.3%		21.5%	31.5%	-1.1	-2.4	5.4		3.6	5.4
RMW	1.0%	21.6%	41.7%	32.1%		30.9%	1.9	2.8	5.1	3.6		4.3
CMA	-0.1%	1.2%	9.7%	47.7%	31.4%		-0.3	0.2	1.1	5.4	4.3	
Panel B: NOVY	Coefficient						T-Stat					
2×2	Int	MKT	SMB	HML	PMU	CMA	Int	MKT	SMB	HML	PMU	CMA
MKT	-4.9%		-35.8%	-45.1%	35.6%	-12.5%	-16.9		-5.1	-7.5	6.6	-2.1
SMB	-1.6%	-40.6%		-49.4%	29.7%	0.7%	-3.0	-5.1		-7.9	5.0	0.1
HML	-2.7%	-64.9%	-62.7%		62.2%	-35.2%	-4.8	-7.5	-7.9		10.5	-5.0
PMU	2.8%	64.8%	47.7%	78.8%		31.4%	4.4	6.6	5.0	10.5		3.9
CMA	-0.9%	-20.0%	1.0%	-38.9%	27.4%		-1.4	-2.1	0.1	-5.0	3.9	

2 × 3												
MKT	-5.0%		-34.2%	-18.8%	5.9%	9.9%	-16.7		-4.3	-3.5	1.1	1.8
SMB	-1.1%	-30.0%		-32.0%	-18.4%	5.6%	-2.3	-4.3		-6.6	-3.9	1.1
HML	-1.0%	-36.8%	-71.2%		-6.6%	33.8%	-1.4	-3.5	-6.6		-0.9	4.4
PMU	0.8%	13.2%	-47.2%	-7.6%		3.2%	1.1	1.1	-3.9	-0.9		0.4
CMA	1.0%	18.6%	12.0%	32.8%	2.7%		1.4	1.8	1.1	4.4	0.4	
2 × 2 × 2 × 2												
MKT	-5.1%		-31.0%	-22.4%	26.2%	18.1%	-17.1		-3.9	-2.6	3.3	2.3
SMB	-1.3%	-28.6%		1.6%	54.6%	31.6%	-2.8	-3.9		0.2	8.2	4.5
HML	-0.4%	-17.0%	1.3%		36.0%	58.3%	-1.0	-2.6	0.2		5.8	10.7
PMU	0.9%	24.2%	54.8%	43.8%		3.4%	1.8	3.3	8.2	5.8		0.5
CMA	0.6%	16.9%	32.2%	71.9%	3.5%		1.2	2.3	4.5	10.7	0.5	

In this table factor spanning tests are conducted for the FF5F and Novy models, examining whether each factor's returns are fully explained by the other four factors. Source: Authors' computation.

4.4.3 Regression intercepts

This analysis investigates mispricing across size, value, investment, and profitability factors using the FF5F and Novy models on African stocks. Portfolio regressions reveal negative and statistically significant intercepts across all sorts, indicating potential mispricing consistent with established size, value, investment, and profitability anomalies. While both models capture similar trends, Novy regressions tend to have slightly higher mispricing, particularly for profitability sorts. Interestingly, the size effect is weakly present in FF5F regressions and largely absent in Novy models. Both models capture value and investment effects as expected, with profitability sorts showing the most significant differences between models. Novy regressions capture stronger profitability effects, suggesting gross profitability might be more relevant in the African context. However, the overall benefit of PMU over RMW as a factor remains debatable. Double sorts often exhibit predominantly non – monotonic return patterns, making it difficult to determine trends.

The value intercepts exhibit consistent negativity, indicating a prevalent undervaluation in these portfolios, supported by high t-statistics ranging from -7.5 to -19.5, highlighting the statistical significance. Notably, both Novy and FF5F models reveal nearly identical intercepts, varying from -0.050 to -0.076. Within size quintiles, mispricing tends to increase as size decreases, except for the high-value column, where an anomalous higher intercept is observed for the large size portfolios. Interestingly, the 2 × 2 sorts show slightly less mispricing compared to the

2×3 model in both Novy and FF5F. These trends suggest a weak consistency, possibly influenced by uncontrolled factors in two-factor sorts (Boamah et al., 2017). Similar weak trends are observed in size-value, size-investment, and size-profitability sorts, hinting at potential insufficiency in double sorting, a more comprehensive control of variables in a three or four-way sort may be necessary (Fama & French, 2015; Fama & French, 2016; Boamah et al., 2017).

Novy and FF5F models' R-squared values reveal distinctions. FF5F generally fit the data better, especially in the medium and high book-to-market (BM) big size 2×2 portfolios (e.g., R-squared: 0.40 and 0.27). Novy models exhibit a higher occurrence of poorly explained portfolios (13 portfolios with R-squared < 0.10) compared to FF5F (10 portfolios with R-squared < 0.10. Both models generally show a similar propensity to better describe big size portfolio (R - squared > 0.2).

Turning to size-investment intercepts **Table 9**, both Novy and FF5F models show negative values ranging from -0.06 to -0.08, with greater variability between portfolio intercepts compared to size-BM sorts. Weak size effects in aggressive and conservative investment stocks are observed, with mispricing seemingly rising with size in the size quintiles. The FF5F model generally outperforms Novy in terms of R-squared values, especially in the 2×2 sort. The exceptions to this are the 2×3 big size sort for the FF5F model, this effect seems to be exacerbated in the Novy models seen in both the big size 2×3 and $2 \times 2 \times 2 \times 2$ models. There are no anomalous mispricings. T-stats remain very high, but the magnitudes are similar to values observed in the Size – value sorts. The R-squared for the Size - investment sorts are slightly improved over the Size – value sorts with both models fitting best to the big size and conservative investment portfolios. Low R-squared values are less prevalent and once again the FF5F models seem to be the better fit, the best of which is the 2×2 sort. Novy models have a better fit in the 2×2 and 2×3 factor sorts, but the $2 \times 2 \times 2 \times 2$ sort have two regressions with 2 peculiarly low values, 0 and -0.17. The mid-size mid-investment portfolios generally show lower R-squareds, but 0 is extraordinarily low, the associated t-stats and intercepts aren't unusual though. The big/conservative portfolio R-squared is extraordinarily low, which is even more extraordinary because it usually has high value, yet again the issue doesn't seem to lie in the t-stat or intercept which appear normal.

In the size-profitability sorts **Table 10**, Novy and FF5F models again display negative intercepts FF5F ranging from -0.04 – -0.06 and Novy, -0.06 – -0.08, but distinct trends emerge. Novy models show a size effect, with intercepts rising as size increases, particularly in profitable and unprofitable portfolios. In contrast, FF5F intercepts increase with size and decrease with profitability. The profitability effect is observed with returns generally increasing with profitability in Novy models, while for the FF5F models this appears to only hold true in the middle size groups. R-squared values favour FF5F, particularly in explaining big portfolios, except for the weak profitability column. The t-stats are once again highly significant.

Overall, while both models exhibit negative intercepts, FF5F consistently outperforms Novy in terms of R-squared values, suggesting a superior fit, yet in absolute terms R-squared <0.5 are quite lacklustre (Mosoeu & Kodongo, 2020; Novy-Marx, 2013; Boamah et al., 2017; Foye, 2018; Taha & Elgiziry, 2016; Claesson, 2021). The weak trends and inconsistencies across double sorts underscore the need for further refinement and additional controls in the analysis. That being said, when sorted on profitability the Novy models are the most differentiated from FF5F model, there are stronger pricing effects and better fitting of the models, which suggests the gross profitability has greater effect than operating profitability on the regressions. These differences could act in favour of PMU on closer inspection i.e., in a refined analysis, however as it stands PMU only performs better than RMW in isolation and marginally so, in full interaction with the rest of the factors RMW seems to be the better choice.

Table 8: Regression intercepts for Size-Value sorts

Intercepts and t-stats for portfolios sorted on size and value.

FF5F		a			T-stat			R^2		
			BM			BM			BM	
	2×2	High		Low	High		Low	High		Low
	Big	-0.050	-0.074	-0.076	-10.8	-12.6	-16.2	0.12	0.40	0.27
Size		-0.066	-0.057	-0.068	-13.0	-7.7	-10.9	0.14	0.08	0.10
	Small	-0.069	-0.070	-0.067	-14.4	-19.5	-13.0	0.06	0.14	0.09
	$2 \times 2 \times 2 \times 2$									
	Big	-0.051	-0.076	-0.075	-11.6	-12.7	-15.5	0.15	0.29	0.16
Size		-0.067	-0.057	-0.069	-13.3	-8.1	-10.4	0.14	0.09	0.07
	Small	-0.069	-0.070	-0.067	-14.6	-19.7	-12.8	0.08	0.15	0.09
	2×3									
	Big	-0.052	-0.076	-0.073	-12.3	-11.2	-14.8	0.19	0.15	0.27
Size		-0.070	-0.059	-0.070	-13.0	-8.5	-10.8	0.14	0.04	0.10
	Small	-0.069	-0.072	-0.066	-14.1	-18.4	-12.8	0.05	0.11	0.06
Novy	2×2									
	Big	-0.051	-0.075	-0.075	-10.7	-13.6	-16.4	0.08	0.27	0.24
Size		-0.067	-0.057	-0.069	-13.5	-8.0	-11.3	0.13	0.05	0.09
	Small	-0.069	-0.070	-0.067	-15.4	-18.4	-12.9	0.05	0.15	0.05
	$2 \times 2 \times 2 \times 2$									
	Big	-0.051	-0.074	-0.073	-11.0	-11.6	-13.6	0.06	0.17	0.19
Size		-0.070	-0.058	-0.070	-13.7	-8.0	-10.9	0.13	0.07	0.08
	Small	-0.069	-0.072	-0.066	-13.7	-18.7	-12.6	0.06	0.19	0.06
	2×3									
	Big	-0.052	-0.076	-0.074	-11.7	-11.6	-15.8	0.19	0.14	0.26
Size		-0.069	-0.058	-0.068	-12.6	-7.8	-10.9	0.15	0.04	0.13
	Small	*-0.068*	-0.071	-0.066	-13.6	-17.9	-12.4	0.05	0.12	0.06

This table presents intercepts and t-statistics for portfolios sorted on size and value across different factor models.

Source: Authors' computation.

Table 9: Regression intercepts for Size-Investment sorts

Intercepts and t-stats for portfolios sorted on size and investment.

		a			T-Stat			R^2		
			INV			INV			INV	
FF5F	2 × 2	Aggr.		Cons.	Aggr.		Cons.	Aggr.		Cons.
	Big	-0.076	-0.065	-0.071	-16.1	-12.8	-14.6	0.24	0.26	0.18
Size		-0.081	-0.059	-0.073	-13.0	-9.8	-11.9	0.13	0.14	0.16
	Small	-0.073	-0.074	-0.070	-9.3	-14.5	-15.6	0.10	0.12	0.11
	2 × 2 × 2 × 2									
	Big	-0.076	-0.066	-0.071	-16.6	-11.6	-12.3	0.23	0.11	0.04
Size		-0.081	-0.059	-0.072	-13.1	-10.6	-12.6	0.10	0.13	0.13
	Small	-0.073	-0.073	-0.069	-8.8	-14.2	-15.1	0.07	0.14	0.09
	2 × 3									
	Big	-0.073	-0.066	-0.076	-16.5	-12.3	-16.5	0.29	0.10	0.24
Size		-0.081	-0.062	-0.077	-13.7	-11.2	-12.8	0.11	0.07	0.15
	Small	-0.073	-0.074	-0.070	-9.2	-14.9	-14.5	0.08	0.09	0.07
Novy	2 × 2									
	Big	-0.076	-0.065	-0.071	-16.5	-12.8	-15.3	0.21	0.16	0.13
Size		-0.081	-0.060	-0.073	-13.2	-10.9	-12.0	0.11	0.09	0.13
	Small	-0.074	-0.074	-0.069	-9.4	-15.4	-15.5	0.09	0.11	0.11
	2 × 2 × 2 × 2									
	Big	-0.072	-0.065	-0.074	-14.0	-13.0	-13.3	0.10	0.18	-0.17
Size		-0.081	-0.062	-0.077	-13.3	-10.7	-12.4	0.12	0.00	0.04
	Small	-0.072	-0.075	-0.070	-8.7	-15.1	-15.2	0.05	0.14	0.10
	2 × 3									
	Big	-0.073	-0.066	-0.076	-15.9	-12.4	-15.8	0.25	0.10	0.23
Size		-0.080	-0.060	-0.076	-13.4	-10.6	-12.4	0.12	0.12	0.14
	Small	-0.071	-0.073	-0.070	-9.0	-14.4	-14.9	0.10	0.11	0.08

This table presents intercepts and t-statistics for portfolios sorted on size and investment across different factor models.

Source: Authors' computation.

Table 10: Regression intercepts for Size-Profitability sorts

Intercepts and t-stats for portfolios sorted on size and profitability.										
		a			T-stat			R^2		
			OP/GP			OP/GP			OP/GP	
FF5F	2×2	Robust		Weak	Robust		Weak	Robust		Weak
	Big	-0.073	-0.077	-0.058	-13.1	-14.7	-11.0	0.13	0.44	0.07
size		-0.055	-0.065	-0.071	-11.1	-12.2	-10.6	0.06	0.13	0.10
	Small	-0.076	-0.080	-0.064	-17.2	-12.2	-13.5	0.09	0.14	0.08
	$2 \times 2 \times 2 \times 2$									
	Big	-0.072	-0.077	-0.059	-12.7	-14.6	-10.7	0.09	0.28	0.05
size		-0.055	-0.066	-0.071	-11.1	-12.4	-10.7	0.06	0.10	0.14
	Small	-0.076	-0.081	-0.064	-18.0	-11.9	-13.4	0.09	0.10	0.10
	2×3									
	Big	-0.073	-0.077	-0.056	-13.3	-14.5	-10.5	0.33	0.21	0.21
size		-0.056	-0.068	-0.074	-11.7	-12.8	-11.2	0.08	0.09	0.10
	Small	-0.078	-0.081	-0.064	-16.8	-12.9	-13.5	0.08	0.10	0.09
Novy	2×2	Prof.		Unprof.	Prof.		Unprof.	Prof.		Unprof.
	Big	-0.076	-0.064	-0.075	-18.2	-15.6	-9.5	0.29	0.08	0.10
size		-0.060	-0.065	-0.081	-9.8	-13.2	-13.4	0.13	0.08	0.16
	Small	-0.066	-0.077	-0.067	-18.1	-17.0	-12.8	0.04	0.12	0.03
	$2 \times 2 \times 2 \times 2$									
	Big	-0.075	-0.062	-0.073	-17.0	-15.7	-9.9	0.27	0.08	0.07
size		-0.062	-0.068	-0.083	-10.1	-13.6	-13.1	0.15	0.00	0.10
	Small	-0.067	-0.077	-0.067	-18.6	-15.9	-12.3	0.07	0.13	0.05
	2×3									
	Big	-0.078	-0.063	-0.069	-17.2	-14.4	-9.2	0.24	0.08	0.36
size		-0.062	-0.068	-0.078	-10.4	-12.3	-13.8	0.09	0.09	0.36
	Small	-0.067	-0.076	-0.066	-17.5	-16.3	-12.3	0.03	0.13	0.04

This table presents intercepts and t-statistics for portfolios sorted on size and profitability across different factor models.

Source: Authors' computation.

4.4.4 Regression coefficient

4.4.4.1 Size coefficients

Analysing the SMB slopes, it is evident that the size effect is consistent with the literature, showing increasing SMB coefficients as size decreases. The negative coefficients in big size portfolios indicate a dominance of larger stocks in driving returns. The Novy model introduces an interesting nuance with significant positive SMB loadings in the middle size group, likely linked to the profitability component of the size factor. These findings align with the literature on asset pricing models, emphasizing the relevance of size and profitability factors in explaining stock returns (Ahn et al., 2019; Novy-Marx, 2013; Fama & French, 1993; Fama & French, 2015; Cooke et al., 2015; Cox & Britten, 2019; Mosoeu & Kodongo, 2020).

In **Table 11**, the SMB coefficients and t-stats for portfolios sorted on size and value reveal a consistent pattern across FF5F and Novy models. As anticipated, the SMB slopes generally increase as size decreases, with big size portfolios exhibiting negative slopes, particularly in the low-value category (Fama & French, 1993; Fama & French, 2015; Mosoeu & Kodongo, 2020). Notably, small size low-value portfolios consistently present negative coefficients, although these loadings are not statistically significant. The highly significant negative t-stats in big size portfolios suggest heavy investments in larger stocks, influencing returns (Fama & French, 2016). The Novy model exhibits significant positive SMB loadings in the middle size group, potentially due to the gross profitability component of the size factor (SMB_{GP}), which aligns with subsequent positive loadings in the PMU slopes, indicating increased investment in larger stocks.

Table 12 and **Table 13**, focusing on portfolios sorted on size and, investment and profitability, respectively, reveal parallel trends. In both cases, the big size portfolios consistently exhibit highly significant negative coefficients, while some mid-size portfolios in the Novy model present positive coefficients. This aligns with the expected trend, emphasizing the positive link between size and profitability (Novy-Marx, 2013).

Table 11: Size coefficients for Size-Value sorts

SMB coefficients and t-stats for portfolios sorted on size and value.

FF5F		c			T-Stat		
			BM			BM	
	2×2	High		Low	High		Low
	Big	-0.131	-0.638	-0.491	-1.6	-6.6	-4.6
Size		0.148	-0.094	0.016	1.2	-0.8	0.2
	Small	0.040	-0.022	-0.133	0.4	-0.3	-1.6
	$2 \times 2 \times 2 \times 2$						
	Big	-0.162	-0.713	-0.387	-1.5	-5.9	-3.2
Size		0.266	-0.037	0.001	1.7	-0.3	0.0
	Small	0.189	0.097	-0.071	1.8	1.1	-0.7
	2×3						
	Big	-0.323	-0.578	-0.381	-3.4	-4.3	-3.4
Size		0.105	-0.005	0.050	0.8	0.0	0.4
	Small	0.042	-0.009	-0.106	0.4	-0.1	-1.1
Novy	2×2						
	Big	-0.035	-0.473	-0.384	-0.4	-4.3	-3.4
Size		0.298	-0.046	-0.006	2.5	-0.4	-0.1
	Small	0.084	0.079	-0.096	0.9	1.1	-1.2
	$2 \times 2 \times 2 \times 2$						
	Big	-0.034	-0.416	-0.277	-0.3	-2.8	-2.3
Size		0.335	0.090	0.049	2.4	0.7	0.4
	Small	0.171	0.182	-0.099	1.6	1.9	-1.1
	2×3						
	Big	-0.308	-0.579	-0.283	-3.3	-4.4	-2.6
Size		0.054	-0.039	-0.053	0.4	-0.3	-0.4
	Small	0.038	-0.025	-0.118	0.4	-0.3	-1.2

This table presents the SMB coefficients and t-stats for portfolios sorted on size and value. Source: Authors' computation.

Table 12: Size coefficients for Size-Investment sorts

SMB coefficients and t-stats for portfolios sorted on size and investment.							
FF5F		c			T-Stat		
			INV			INV	
	2×2	Aggr.		Cons.	Aggr.		Cons.
	Big	-0.406	-0.496	-0.366	-4.7	-5.3	-3.4
Size		-0.049	0.082	0.031	-0.4	0.8	0.3
	Small	0.065	0.055	-0.093	0.6	0.6	-1.1
	$2 \times 2 \times 2 \times 2$						
	Big	-0.518	-0.348	-0.452	-5.0	-2.7	-3.0
Size		0.016	0.069	0.145	0.1	0.5	1.2
	Small	0.106	0.179	0.006	0.8	1.4	0.1
	2×3						
	Big	-0.478	-0.378	-0.435	-4.7	-3.1	-4.0
Size		0.025	0.049	0.135	0.2	0.4	1.2
	Small	0.050	0.011	-0.007	0.4	0.1	-0.1
Novy	2×2						
	Big	-0.346	-0.326	-0.289	-3.5	-2.8	-2.8
Size		-0.015	0.186	0.111	-0.1	1.8	0.9
	Small	0.063	0.156	-0.012	0.6	1.7	-0.1
	$2 \times 2 \times 2 \times 2$						
	Big	-0.323	-0.204	-0.233	-2.8	-1.8	-1.4
Size		0.064	0.245	0.199	0.5	1.7	1.4
	Small	0.000	0.285	-0.005	0.0	2.6	-0.1
	2×3						
	Big	-0.416	-0.335	-0.393	-3.9	-2.8	-3.7
Size		-0.121	0.037	0.095	-0.9	0.3	0.7
	Small	-0.062	0.048	-0.007	-0.5	0.5	-0.1

This table presents the SMB coefficients and t-stats for portfolios sorted on size and investment

Source: Authors' computation.

Table 13: Size coefficients for Size-Profitability sorts

SMB coefficients and t-stats for portfolios sorted on size and profitability.							
FF5F		c			T-Stat		
			OP/GP			OP/GP	
	2×2	Robust		Weak	Robust		Weak
	Big	-0.277	-0.754	-0.139	-3.5	-9.0	-1.3
Size		-0.113	-0.056	0.125	-1.4	-0.6	0.9
	Small	-0.077	-0.048	0.042	-0.8	-0.5	0.5
	$2 \times 2 \times 2 \times 2$						
	Big	-0.191	-0.723	-0.249	-1.9	-7.0	-1.9
Size		-0.131	0.039	0.168	-1.3	0.3	1.0
	Small	0.014	0.003	0.145	0.1	0.0	1.5
	2×3						
	Big	-0.255	-0.620	-0.373	-3.2	-5.1	-3.3
Size		-0.055	0.014	0.037	-0.6	0.1	0.2
	Small	-0.034	-0.006	0.021	-0.3	0.0	0.2
Novy	2×2						
	Big	-0.520	-0.100	-0.166	-5.9	-1.4	-0.9
Size		0.155	0.068	0.270	1.6	0.6	2.6
	Small	-0.008	0.080	0.038	-0.1	0.8	0.5
	$2 \times 2 \times 2 \times 2$						
	Big	-0.395	-0.055	-0.280	-3.5	-0.6	-1.4
Size		0.251	0.111	0.364	2.3	0.8	2.2
	Small	0.022	0.172	0.052	0.2	1.4	0.6
	2×3						
	Big	-0.536	-0.167	-0.741	-5.1	-1.9	-5.0
Size		0.101	0.013	-0.058	0.9	0.1	-0.5
	Small	-0.050	-0.038	-0.010	-0.6	-0.3	-0.1

This table presents the SMB coefficients and t-stats for portfolios sorted on size and profitability.

Source: Authors' computation.

4.4.4.2 Profitability coefficients

Table 14 - **Table 16** present profit coefficients and t-statistics for portfolios sorted on size and various factors, value (BM), investment (INV), and operating profitability (OP) to gross profitability (GP). In **Table 14**, analyzing size and value, FF5F shows negative and significant coefficients for book-to-market across all size portfolios, indicating a preference for weak profitability stocks, with larger negative coefficients for smaller size portfolios. Novy's model, on the other hand, displays significant positive coefficients, suggesting a preference for profitable stocks in the 2×2 and $2 \times 2 \times 2 \times 2$ factor sorts. The 2×3 factor sort in Novy's model exhibits mixed results. **Table 15**, focusing on size and investment, maintains a consistent trend with FF5F showing negative coefficients for book-to-market, indicative of a bias towards weak profitability, and Novy presenting positive coefficients in 2×2 and $2 \times 2 \times 2 \times 2$ sorts, emphasizing profitable stocks. Notably, investment sorts demonstrate a neutralizing effect on profitability loadings, as PMU coefficients are less positive and RMW coefficients are less negative. Examining the effect of size and, operating profitability (OP) / gross (GP) profitability sorts on profitability coefficients (RMW/PMU) in **Table 16**, both FF5F and Novy models confirm previous observations. Operating profitability (RMW) shows highly negative coefficients in weak profitability portfolios, while gross profitability (PMU) coefficients show highly positive coefficients in their profitable sorts. Both 2×3 sorts behave differently, the big and medium size robust stocks show significant positive coefficients as well as significant weak profitability coefficients. In the 2×3 Novy sort only the medium and large size unprofitable sorts show significance.

Considering the observed data, it is essential to acknowledge differences in model implications and underlying assumptions. Notably, Novy's model consistently favors profitable stocks, while FF5F exhibits variability. Further research could explore the robustness of these findings in different market conditions and geographies.

Table 14: Profit coefficients for Size-Value sorts

Profit coefficients and t-stats for portfolios sorted on size and value.							
FF5F		e			T-stat		
			BM			BM	
	2×2	High		Low	High		Low
	Big	-0.245	-0.366	-0.066	-3.1	-3.9	-0.8
Size		-0.245	-0.185	-0.148	-2.3	-2.0	-1.6
	Small	-0.113	-0.184	-0.191	-1.3	-3.0	-3.0
	$2 \times 2 \times 2 \times 2$						
	Big	-0.347	-0.445	0.169	-3.5	-3.4	1.6
Size		-0.344	-0.157	-0.155	-3.0	-1.5	-1.2
	Small	-0.192	-0.251	-0.199	-1.9	-3.1	-1.9
	2×3						
	Big	-0.070	0.065	0.061	-1.0	0.7	0.9
Size		0.029	0.067	-0.038	0.4	0.8	-0.5
	Small	0.007	0.016	-0.010	0.1	0.3	-0.1
Novy	2×2						
	Big	0.110	0.055	0.153	1.6	0.7	2.6
Size		0.088	0.041	-0.074	1.2	0.6	-0.8
	Small	0.049	0.157	0.026	0.6	2.7	0.4
	$2 \times 2 \times 2 \times 2$						
	Big	0.178	0.378	0.345	1.9	3.5	4.5
Size		0.108	0.142	-0.097	1.1	1.3	-0.8
	Small	0.143	0.234	0.051	1.4	3.1	0.6
	2×3						
	Big	-0.087	-0.012	0.123	-1.6	-0.1	2.2
Size		-0.111	-0.039	-0.198	-1.5	-0.6	-2.6
	Small	-0.053	-0.059	-0.020	-0.7	-1.1	-0.3

This table presents the RMW and PMU coefficients and t-stats for portfolios sorted on size and value. Source: Authors' computation.

Table 15: Profit coefficients for Size-Investment sorts

Profit coefficients and t-stats for portfolios sorted on size and investment.

FF5F		e			t-stat		
		INV			INV		
	2×2	Aggr.		Cons.	Aggr.		Cons.
	Big	-0.049	-0.324	-0.234	-0.6	-3.9	-2.6
Size		-0.152	-0.304	-0.239	-1.4	-3.6	-2.1
	Small	-0.168	-0.157	-0.171	-1.4	-2.1	-2.5
	$2 \times 2 \times 2 \times 2$						
	Big	0.066	-0.243	-0.226	0.7	-1.9	-1.6
Size		-0.106	-0.429	-0.327	-0.8	-4.3	-2.3
	Small	-0.239	-0.263	-0.156	-1.8	-2.7	-1.7
	2×3						
	Big	0.092	0.069	-0.032	1.3	0.9	-0.5
Size		0.063	-0.027	0.069	0.8	-0.3	0.8
	Small	0.048	-0.038	0.004	0.6	-0.5	0.1
Novy	2×2						
	Big	0.023	0.169	0.094	0.4	2.6	1.2
Size		0.043	-0.095	0.072	0.5	-1.4	0.8
	Small	-0.013	0.052	0.155	-0.1	0.7	2.4
	$2 \times 2 \times 2 \times 2$						
	Big	0.226	0.444	0.248	2.9	4.6	1.9
Size		0.107	-0.045	0.076	1.1	-0.4	0.6
	Small	0.160	0.106	0.184	1.2	1.1	2.2
	2×3						
	Big	0.024	0.084	-0.051	0.4	1.2	-0.7
Size		-0.079	-0.181	-0.042	-0.9	-2.5	-0.5
	Small	-0.165	-0.141	0.058	-2.2	-2.2	0.8

This table presents the RMW and PMU coefficients and t-stats for portfolios sorted on size and investment.

Source: Authors' computation.

Table 16: Profit coefficients for Size-Profitability sorts

Profit coefficients and t-stats for portfolios sorted on size and profitability.							
FF5F		e			t-stat		
			OP/GP			OP/GP	
	2×2	Robust		Weak	Robust		Weak
	Big	0.085	-0.376	-0.208	1.0	-4.6	-2.4
Size		-0.025	-0.294	-0.277	-0.4	-3.1	-2.4
	Small	-0.124	-0.281	-0.151	-1.4	-3.0	-2.0
	$2 \times 2 \times 2 \times 2$						
	Big	0.200	-0.331	-0.180	1.9	-2.7	-1.7
Size		-0.018	-0.283	-0.488	-0.2	-2.4	-3.4
	Small	-0.198	-0.273	-0.217	-1.9	-2.3	-2.0
	2×3						
	Big	0.447	0.074	-0.370	6.0	0.8	-5.5
Size		0.119	0.120	-0.159	2.0	1.6	-1.5
	Small	0.046	0.094	-0.136	0.7	1.2	-2.0
Novy	2×2						
	Big	0.171	-0.089	-0.305	3.2	-1.8	-2.3
Size		0.213	-0.025	-0.233	3.2	-0.3	-2.7
	Small	0.016	0.090	0.013	0.3	1.0	0.2
	$2 \times 2 \times 2 \times 2$						
	Big	0.460	0.007	-0.208	6.1	0.1	-1.3
Size		0.276	0.036	-0.225	3.6	0.3	-1.7
	Small	0.118	0.144	0.120	1.7	1.2	1.6
	2×3						
	Big	0.113	-0.062	-0.775	1.8	-1.4	-7.4
Size		0.073	-0.054	-0.584	0.9	-0.8	-7.7
	Small	0.001	-0.147	-0.073	0.0	-2.1	-1.0

This table presents the RMW and PMU coefficients and t-stats for portfolios sorted on size and profitability.

Source: Authors' computation.

4.5 Size effect and the African Business cycle

This portioon of the study adopts Ahn et al.'s (2019) perspective on the size effect, exploring its conditional nature concerning the business cycle. The investigation simulated economic states to elucidate potential relationships between economic conditions and the observed size effect in ASMs. The size factor employed in this section, aligning with Anh et al.'s (2019) approach is formed on market capitalization and book-to-market ratio (SMB_{BM}). This is a component of the 2×3 SMB factor qouted in (Table 3), As it turned out simulating the business cycle with the $2 \times 2 \times 2 \times 2$ SMB factor did not yield any statistically significant size effects.

4.5.1 Desribing the size effect

Table 17: Descriptive statistics of SMB

Descriptive statistics of SMB.			
SMB	2 × 3	2 × 2	2 × 2 × 2 × 2
Mean	0.30%	0.20%	0.46%
t-stat	1.72	1.09	3.00

Descriptive statistics of monthly excess returns corresponding to small minus big stocks for various Fama and French sorting regimes.
Source: Authors' computation.

Table 17 presents descriptive statistics of monthly excess returns on the Small Minus Big (SMB) factor, reflecting the performance of small stocks relative to big stocks, across various Fama and French sorting regimes (Ahn et al., 2019). The mean excess returns for the 2×3, 2×2, and $2 \times 2 \times 2 \times 2$ sorting strategies are reported as 0.30%, 0.20%, and 0.46%, respectively, with corresponding t-statistics of 1.72, 1.09, and 3.00. This summary spans from August 2009 to February 2023 and serves as a foundational exploration of the size effect within ASMs. In line with prior findings in the paper, the $2 \times 2 \times 2 \times 2$ SMB factor emerges as highly significant, with an average return of 0.46% per month and a robust t-statistic of 3.00. Conversely, the 2×3 SMB, returning 0.30%, exhibits significance at a 10% level with a t-statistic of 1.72, indicating a discernible size effect in ASMs, which aligns with previous research suggesting that the size effect, while seemingly diminished in developed markets, remains active in several emerging markets (Ahn et al., 2019; Foye, 2018; Leite et al., 2018; Cox & Britten, 2019).

4.5.2 Defining the African business cycle

Table 18: Business cycles dates

Business cycles dates determined by HP filtering of quarterly GDP data.

Panel A: 2006 - 2022 Cycle dates

	Cycle 1	Cycle 2	Cycle 3	Cycle 4
Trough	2006Q1	2010Q1	2016Q1	2020Q2
Expansion	2006Q2 - 2008Q2	2010Q2 - 2014Q3	2016Q2 - 2018Q3	2020Q3 - 2022Q3
Peak	2008Q3	2014Q4	2018Q4	2022Q4
Recession	2008Q4 - 2009Q4	2015Q1 - 2015Q4	2019Q1 - 2020Q1	-

Panel B: SMB test dates (2009 July - 2023 February) Cycle stats

Stage	No. of months	
Trough	21	
Expansion	99	
Peak	21	
Recession	23	
Total	164	months

This table provides a comprehensive overview of business cycle dates determined through the Hodrick-Prescott (HP) filtering of quarterly GDP data spanning from January 2006 to December 2022. Source: Authors' computation.

Panel A of **Table 18** outlines the chronological sequence of business cycle stages, encompassing four distinct cycles labelled 1 through 4. The identification of Trough, Expansion, Peak, and Recession phases is derived from the HP filter's analysis of quarterly GDP data from 10 selected African countries, chosen based on availability of quarterly data. The HP filter identifies turning points, i.e. Peaks and Troughs; and the periods in between are designated Expansion periods (Trough - Peak) and Recession periods (Peak to Trough) (Ahn et al., 2019). It is important to note that cycle dates were expanded by two months before and after each turning point, addressing the inherent uncertainty regarding the precise timing of these transitions, following the approach by Ahn et al. (2019). Each cycle begins in the first month of a trough and ends in last month of the next recession. The HP filter identified 4 Business cycles. The business cycles demonstrate non-uniform lengths, each Trough and Peak stage lasts 7 months, while expansion periods range from 22 to 46 months, and Recession periods span 10 to 12 months. The final test period was July 2009 – February 2023, 164 months from the beginning of cycle 1 through to the end of cycle 4. The Trough and Peak stages lasted 21 months each, Expansion was 99 months long and the Recession was 23 months long.

4.5.3 The conditional size effect.

Table 19: Size effect conditional on business cycle stage

Size effect conditional on business cycle stage								
FF5F	Trough		Expansion		Peak		Recession	
	Mean	t-stat	Mean	t-stat	Mean	t-stat	Mean	t-stat
2×2 SMB	0.62%	1.3	0.28%	1.2	0.22%	0.5	0.13%	0.3
2×3 SMB	0.23%	0.5	0.08%	0.3	0.77%	1.5	0.19%	0.4
$2 \times 2 \times 2 \times 2$ SMB	0.90%	2.1	0.34%	1.6	0.47%	1.1	0.59%	1.6

This table shows the size effect conditional on different business cycle stages, utilizing the Fama and French five-factor model (FF5F). The table is organized into four distinct business cycle phases: trough, expansion, peak, and recession. Results for three SMB portfolios—2×2, 2×3, and $2 \times 2 \times 2 \times 2$ reported in terms of average returns and associated t-statistics. Source: Authors' computation.

Table 19 presents an examination of the size effect conditional on different business cycle stages. Notably, during three stages, exhibit high SMB returns (Trough: 0.90%, t-stat: 2.1), (Peak: 0.77%, t-stat: 1.5), (Recession: 0.59%, t-stat: 1.6). While the trough stage shows the only statistically significant, aligning with Ahn et al. (2019) who find the size effect to be conditional on the trough stage length. Structurally the Trough, Peak, and Recession, stages are similar in length, so we do not rule out the possibility of a significant size effect forming in the other stages. This analysis serves as a foundational exploration, with future tests directed towards scrutinizing the relationship between stage length and returns, contributing to the broader discourse on the size effect in asset pricing dynamics within African stock markets.

Table 20: Business cycle stage dummy variable regressions for the size effect.

Business cycle stage dummy variable regressions for the size effect.

		$2 \times 2 \times 2 \times 2$		2×3	
	Variable	Coefficient	t-stat	Coefficient	t-stat
Trough	Ct	0.40%	2.4	0.25%	2.0
	DT	0.50%	1.1	-0.36%	-0.7
Expansion	Ce	0.65%	2.7	0.32%	1.8
	DE	-0.31%	-1.0	0.04%	0.1
Peak	Cp	0.46%	2.8	0.31%	2.3
	DP	0.01%	0.0	-0.08%	-0.2
Recession	Cr	0.44%	2.7	0.32%	2.2
	DR	0.14%	0.3	0.19%	0.4

This table reports the estimation results when SMB is regressed on Dummy variables Dt, De, Dp, Dr, used to isolate the SMB in each business cycle stage. The regression Intercept and Dummy variable coefficients are reported along with their corresponding t-statistics. Source: Authors' computation.

To formally assess the stage-specific size effects, a set of dummy variables, $D_{(T,E,P,R),t}$ is constructed, each representing a particular stage (Trough, Expansion, Peak, or Recession) during a given time period (Ahn et al., 2019). The analysis employs the following regression equation:

$SMB_t = C_{(T,E,P,R)} + \beta\ D_{(T,E,P,R),t} + \varepsilon_t$ (4)

where the monthly return SMB_t, is regressed on the intercept term (C), which captures the size effect during non-specified stages, and the slope coefficient on the dummy variable (β), representing the incremental size effect at the relevant stage (Ahn et al., 2019).

The results, as outlined in **Table 20**, reveal consistently positive and statistically significant intercept terms (C_T, C_E, C_P, C_R), suggesting non-trivial size effects in stages other than those explicitly considered i.e., a size effect could exist in every stage. The coefficients on the dummy variables (D_T, D_E, D_P, D_R) are not statistically significant, indicating that returns within individual stages are not significantly different from each other, which again suggests the size effect isn't isolated to any one stage. This pattern is consistent across both the $2 \times 2 \times 2 \times 2$ and 2×3 SMB sorts. Notably, two dummy variables (D_E in the $2 \times 2 \times 2 \times 2$ sort and D_T in the 2×3 sort) exhibit highly negative coefficients (-0.31% and -0.30%, respectively), aligning with findings in later data plots where negative dummy variables correlate with a negative return-stage length relationship. The lack of statistically significant differences across stages suggests that the size effect is not isolated to any particular phase of the business cycle. This observation prompts consideration of the structural differences/similarities in stage durations as a potential explanation for the varying significance of the size effect. This informed the next step of this research (Ahn et al., 2019). To reshape the business cycle, by varying the duration of the Trough, Peak, and Recession and generate significant size effect in those stages, resulting in a greater overall size effect (Ahn et al., 2019).

4.5.4 Reshaping the business cycle

4.5.4.1 Transition probabilities

Ahn et al. (2019) found the size effect to be conditional on the length of the trough stage of the business cycle and thus define an unconditional size effect "SMB" as the weighted average of conditional size effects over the business cycle stages where the weight was the probability of each business cycle stage. In order to investigate the conditions under which the size effect could be significant the historically observed business cycle (cycle 0) was used to create a Markov chain probability matrix that defines the probability of remaining or transitioning to the next business cycle stage, as seen in **Table 21.** By altering this probability matrix, the study investigated the implications of different stage durations on the size effect. Twelve compound cycles were selected, and their characteristics were thoroughly examined. A nuanced approach not only considering the occurrence of stages but also their duration, selecting stable cycles with meaningful stage durations, underscored the study's robustness. The initial matrix was changed (8 times) to vary the distribution of stages. Each matrix was used to generate a series of business cycles (164 months of business stages). The criteria for selection was stage duration, to ensure stability, any series with missing stages or 1 month stage lengths were excluded. To replicate a scenario where all other variables remain consistent, during each cycle, one stage assumed either a high or low value, while the remaining three stages maintained equal values. While enforcing such an ideal but impractical constraint, an alternative approach was adopted, ensuring that the other three stages remained closely aligned, with the ratios for these "other" stages permitted to differ by no more than 4%.

Out of the (1200) series generated, 12 cycles were chosen in this manner, spanning the entire probability distribution. The 12 cycles **Table 22** mentioned hereafter represent composite cycles. These are individual sequences of business cycle stages that collectively constitute multiple business cycles occurring over a 164-month period.

Table 21: Transition probabilities

Transition probabilities						
Stages	Historic	Steady state	Probability of moving to			
	Percentages	Percentages	Trough	Expansion	Peak	Recession
Trough	13.0%	12.1%	85.7%	14.3%	0.0%	0.0%
Expansion	61.1%	57.2%	0.0%	97.0%	3.0%	0.0%
Peak	11.7%	17.3%	0.0%	0.0%	90.0%	10.0%
Recession	14.2%	13.3%	13.0%	0.0%	0.0%	87.0%

This table illustrates the transition probabilities between business cycle stages, both historically and in steady state. The steady state acts as a robustness to ensure our probability matrix is an accurate representation of the historic stages, its accurate except in the peak stage. Source: Authors' computation.

4.5.4.2 Simulated Returns

Table 23 presents the simulated SMB return data. The 2×3 SMB_{BM} based returns with statistically significant returns are reported as they are more compelling. That being said the data behaves very similarly and the $2 \times 2 \times 2 \times 2$ SMB data can be found for comparison in the Appendices. The first part of the table validates the simulated cycles by confirming that the historic full cycle SMB return matches the implied return calculated from the four stages. To validate the applicability of the business cycles, (cycle 0) was formed on an identical probability matrix as the historic data, and returns a statistically Identical (1.966%), well within one standard error from the historic value (1.959%). This consistency supports the simulation's ability to predict historic returns accurately. The second part of **Table 23** provides insights into the relationship between cycle stages and SMB returns. The data echoes the findings of Ahn et al. (2019) on the importance cycle stage returns, but supersedes it, as higher cycle durations correspond to higher SMB returns, in not just one but multiple stages. The following observations are made, implied returns range from 1.19% – 3.02%, while stage return are far more varied at 0.6% - 3.0%. Both historic and simulated stage data can be seen to yield higher returns for higher cycle duration across the stages. The stage data is rather dispersed, while the implied returns seem more clustered around the historic value 1.96%. The broad range of stage returns is due to simulating cycles with different probability matrices. Using the same probability matrix could have produced a tighter range, but testing emphasized the need for a stage-by-stage analysis, as it revealed varying sensitivities and significance for different stages. **Table 20** expresses clues to significant of returns being found in multiple stages, while subsequent data plots (Fig. 1 - Fig. 4) express the various sensitivities of stage returns to cycle duration. Having observed these relationships, a stage-wise sensitivity analysis with multiple cycles at fixed probabilities for each stage seems a warranted route for further research in order to get an idea of the magnitude of the effect each stage has on the size effect.

In the historic data the expansion stage yields 2.19% over a 99-month duration, while the recession yields 1.6% over 23 months. Clearly reflecting an increase in return magnitude with an increase in the stage length, as well as in significance 3.72 and 1.36 respectively. That being said, although a similar relationship is observed in the simulations, the rate of change of SMB return in relation to duration is clearly different for each stage as cycles with similar length stages seldom show similar returns, a typical example being cycle 11 Trough (SMB: 1.75%, N: 45), Peak (SMB: 0.73%, N: 45), Recession (SMB: 1.75%, N: 45), this seen in Fig. 1 – Fig. 4 where as a highly non uniform dispersion of data around the trend lines. This differing sensitivity of SMB:N suggests that it may be worth considering the relation on a stage-by-stage basis. **Table 24** from which Fig. 1 - Fig. 9 are derived, summarises the stage returns and confirms that stage duration and SMB are directly proportional in each stage. The recession followed by the trough stages seems to show the greatest size effect. There seems to be a significant size effect across all stages and in no instance are returns negative, and only during peak stage is it consistently insignificant.

The first 4 graphs visually represent the relationships between stage returns and durations. They demonstrate an upward trend in each stage, with the trough stage showing the highest gradient and the recession near horizontal. Regression analyses confirm positive trends, particularly in the trough stage, supporting the notion that longer durations lead to higher SMB returns. In Fig. 1 a 1.2% increase in returns is observed as the trough stage expands from 23 to 103 months in duration. This is the largest increase of the returns as well the most reliable trend with an R-squared of 23.6%. Fig. 2 and Fig. 3 show expansion and peak stage returns increasing by 0.5% over 40- and 60-month durations respectively a decent increase. Winsorizing the data improved the R-squared of Fig. 2 (expansion data) to 11.9% while Fig. 3 (peak data) was unaffected still showed a weak 6.2%. In Fig. 4 an R-squared of 0, suggests that returns do not change with duration in recession stages. The data points are highly dispersed around the trend line, the only discernible pattern is that the majority of data points fall within the 20 – 40-month duration range, and data from 40 – 100 is extremely sparse. This presents a clear opportunity to improve the trend fit by changing the stage length distributions. In Fig. 5 confirmation is found for the weak but positive trends in Fig. 1 - Fig. 4. Fig. 5 shows that on average the return to duration relationship is positive both within each stage and subsequently across the cycle.

In terms of return significance, the analysis indicates that significance is directly proportional to stage duration, considering the long business cycle this agrees with Ahn et al. (2019), because cycles are either short with high frequency (recurrence) stages or, stages' lengths are extended over long cycles. This is shown by two modes of significant stages with durations greater than 40 months, where the stages are likely to be prolonged within long cycles, or cycles are shortened and stages are highly frequent. The simulations were not set up to discriminate between the two modes, so it was not clear whether absolute duration, or the distribution of the stages has greater influence on significance. This research focused on adjusting the total duration of the stages over the 164 months allowing the length of each stage in individual business cycles to be a free variable. Earlier on it was suggested that an analysis with fixed probability matrices could help better understand return levels, but this could also allow for a more detailed look at the effect on returns of cycle structure by fixing the absolute stage duration but varying the sequencing.

When considering the relationship between stage length and the full cycle returns, the full cycle returns also follow a proportional relationship to cycle length. However, two concerns arose, the relationships while linear appeared to be weaker (R-squareds are even lower), secondly stage returns and stage length could have opposite relationships to full cycle returns, for instance, Trough stage returns decreased as Trough lengths increased, but the Full cycle returns decreased as the trough length increased. Fig. 6 shows an inverse proportionality between full cycle returns and trough stage length. This negative relationship seemed to be more reliable than the trough return/duration relationship (Fig. 1), the R-squared (53.6%) was double that of (Fig. 1). Full cycles were positively impacted by increasing expansion and peak stage durations as seen in Fig. 7 and Fig. 8, the relationship seemed weak, R-squareds are 4% and 9% respectively, but confidence was gained in this relationship as expansion and peak stage returns also showed a positive relationship with stage length. In Fig. 9 a seemingly positive relationship between full cycle returns and recession duration (R-squared: 12%), with the stage return/duration relationship being unclear (Fig. 4, R-squared: 0), the conservative opinion was that no relationship between returns and recession stage lengths existed.

The size effect in summary

In this section three links were established. The first two, Stage durations and stage returns, as seen in Fig. 1 - Fig. 4, and stage returns to full cycle returns as seen in **Table 23**, are well proven. Linking full cycle returns to stage length however proves to be challenging, full cycle returns are linked through stage returns to cycle duration. As such when stage returns and durations do not share the same relationship with the full cycle returns it becomes difficult to draw conclusions. However, it is beyond the scope and methodology of this research to determine the nature (magnitude and direction) of these relationships. A size effect is indeed observed and a positive relationship between the size effect and peak and expansion stage durations was confirmed, while the expansion and recession stage data was inconclusive.

Table 22: Markov chain simulated Business Cycles

Markov chain simulated Business Cycles				
	Stage Duration			
	Trough	Expansion	Peak	Recession
Historic Cycle	21	99	21	23
n (%)	13%	61%	12%	14%
Cycle 1	30	79	28	27
n (%)	19%	49%	17%	17%
Cycle 2	34	59	35	36
n (%)	21%	36%	22%	22%
Cycle 3	41	42	44	37
n (%)	25%	26%	27%	23%
Cycle 4	59	35	35	35
n (%)	36%	22%	22%	22%
Cycle 5	105	22	20	17
n (%)	65%	14%	12%	10%
Cycle 6	33	35	62	34
n (%)	20%	22%	38%	21%
Cycle 7	33	35	32	64
n (%)	20%	22%	20%	40%
Cycle 8	29	25	29	81
n (%)	18%	15%	18%	50%
Cycle 9	23	19	19	103
n (%)	14%	12%	12%	64%
Cycle 10	33	61	37	33
n (%)	20%	38%	23%	20%
Cycle 11	45	29	45	45
n (%)	28%	18%	28%	28%
Cycle 12	29	25	26	84
n (%)	18%	15%	16%	52%

In this table the Markov chain simulations provide an in-depth view of business cycle dynamics over a 164-month period, from July 2009 – February 2023. The duration of the stage in each cycle is reported along with the ratio of stage duration to the cycle length n (%).

Source: Authors' computation.

Table 23: Simulated moments of size effect

Simulated moments of size effect		Historic SMB_{BM}	Implied	SMB Trough	Expansion	Peak	Recession
Mean		1.959%	1.959%	2.23%	2.19%	0.99%	1.60%
Std err		0.44%		1.12%	0.59%	1.25%	1.17%
t-stat		4.41		1.98	3.72	0.79	1.36
N		164		21	99	21	23
Simulated Size effect							
cycle 0	SMB		1.966%	2.14%	2.20%	1.00%	1.67%
	St err			1.98%	0.91%	1.99%	1.82%
	t-stat			1.08	2.42	0.50	0.92
	N			21	99	21	23
cycle 1	SMB		1.968%	1.06%	2.46%	2.56%	0.92%
	St err			1.69%	1.02%	1.78%	1.62%
	t-stat			0.63	2.42	1.44	0.57
	N			30	79	28	27
cycle 2	SMB		1.969%	1.98%	1.89%	1.70%	2.35%
	St err			1.50%	1.22%	1.58%	1.53%
	t-stat			1.32	1.55	1.08	1.54
	N			34	59	35	36
cycle 3	SMB		1.968%	3.00%	1.88%	1.44%	1.55%
	St err			1.44%	1.35%	1.38%	1.48%
	t-stat			2.09	1.39	1.04	1.05
	N			41	42	44	37
cycle 4	SMB		3.024%	3.03%	3.03%	3.02%	3.01%
	St err			1.21%	1.56%	1.56%	1.56%
	t-stat			2.50	1.95	1.95	1.93
	N			59	35	35	35
cycle 5	SMB		1.969%	2.19%	1.86%	0.51%	2.45%
	St err			0.90%	1.92%	1.93%	2.06%
	t-stat			2.43	0.97	0.26	1.19
	N			105	22	20	17

cycle 6	SMB	1.952%	1.88%	2.14%	2.34%	1.12%
	St err		1.44%	1.63%	1.23%	1.54%
	t-stat		1.30	1.31	1.90	0.73
	N		33	35	62	34
cycle 7	SMB	1.954%	1.20%	1.94%	1.49%	2.58%
	St err		1.59%	1.53%	1.65%	1.08%
	t-stat		0.76	1.27	0.90	2.39
	N		33	35	32	64
cycle 8	SMB	1.194%	1.18%	1.21%	1.20%	1.20%
	St err		1.68%	1.82%	1.69%	1.03%
	t-stat		0.70	0.66	0.71	1.17
	N		29	25	29	81
cycle 9	SMB	1.977%	0.62%	0.89%	1.56%	2.56%
	St err		1.80%	2.03%	2.06%	0.90%
	t-stat		0.34	0.44	0.76	2.85
	N		23	19	19	103
cycle 10	SMB	1.952%	1.95%	1.40%	2.25%	2.65%
	St err		1.50%	1.18%	1.59%	1.61%
	t-stat		1.30	1.19	1.41	1.65
	N		33	61	37	33
cycle 11	SMB	1.971%	1.75%	2.25%	0.73%	3.25%
	St err		1.37%	1.67%	1.44%	1.29%
	t-stat		1.28	1.35	0.51	2.51
	N		45	29	45	45
cycle 12	SMB	1.720%	1.71%	1.73%	1.71%	1.72%
	St err		1.68%	1.81%	1.77%	1.01%
	t-stat		1.02	0.96	0.96	1.71
	N		29	25	26	84

*This table presents simulated moments of the size effect, comparing historic and implied SMB (Small Minus Big) returns over different business cycle stages (Trough, Expansion, Peak, and Recession). The mean returns and standard errors, along with t-statistics and sample sizes, are reported for each stage. Reported are the returns of the observed (historic) data over the full cycle and during the 4 stages. The weighted average is used to calculate an Implied size effect from the Stage returns. The weights are calculated as (N/164) i.e., n (%) in (**Table 22**). Historic full cycle and the Implied SMB calculated from the 4 stages are identical (1.959%), this proves the validity of the stage returns as accurate predictors of the full cycle SMB return (Ahn et al., 2019). The second part of table reports the simulated cycle stage returns, from which Implied full cycle returns are calculated. Source: Authors' computation.*

Table 24: SMB and Stage durations summary.

SMB and Stage durations summary.

	SMB				N			
Cycles	Trough	Expansion	Peak	Recession	Trough	Expansion	Peak	Recession
1	1.06%	2.46%	2.56%	0.92%	30	79	28	27
2	1.98%	1.89%	1.70%	2.35%	34	59	35	36
3	3.00%	1.88%	1.44%	1.55%	41	42	44	37
4	3.03%	3.03%	3.02%	3.01%	59	35	35	35
5	2.19%	1.86%	0.51%	2.45%	105	22	20	17
6	1.88%	2.14%	2.34%	1.12%	33	35	62	34
7	1.20%	1.94%	1.49%	2.58%	33	35	32	64
8	1.18%	1.21%	1.20%	1.20%	29	25	29	81
9	0.62%	0.89%	1.56%	2.56%	23	19	19	103
10	1.95%	1.40%	2.25%	2.65%	33	61	37	33
11	1.75%	2.25%	0.73%	3.25%	45	29	45	45
12	1.71%	1.73%	1.71%	1.72%	29	25	26	84
Avg.	1.80%	1.89%	1.71%	2.11%	41.2	38.8	34.3	49.7

This table shows a summary of stage returns and stage durations. Source: Authors' computation.

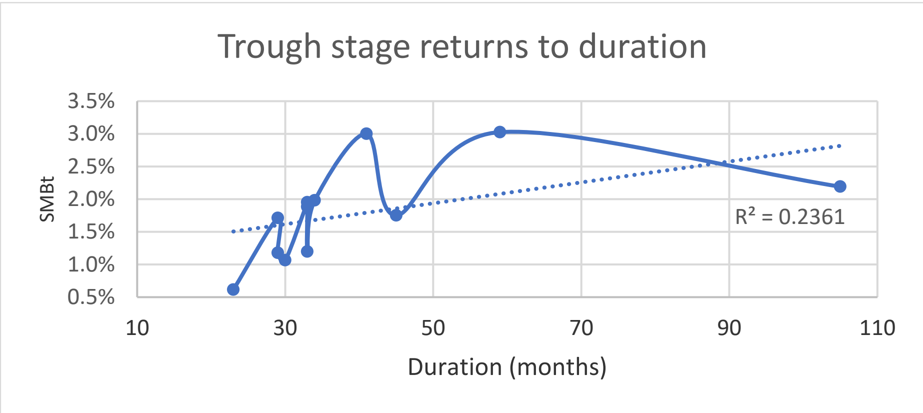

Fig. 1: Trough stage returns over varying stage durations. Source: Authors' plots.

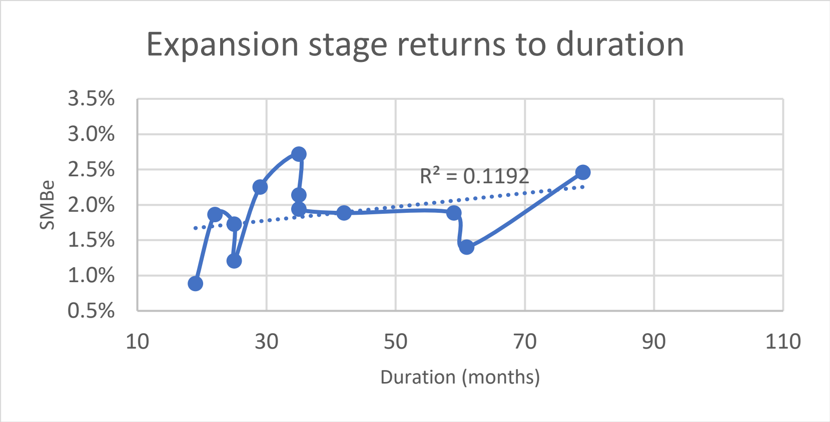

Fig. 2: Expansion stage returns over varying stage durations. Source: Authors' plots.

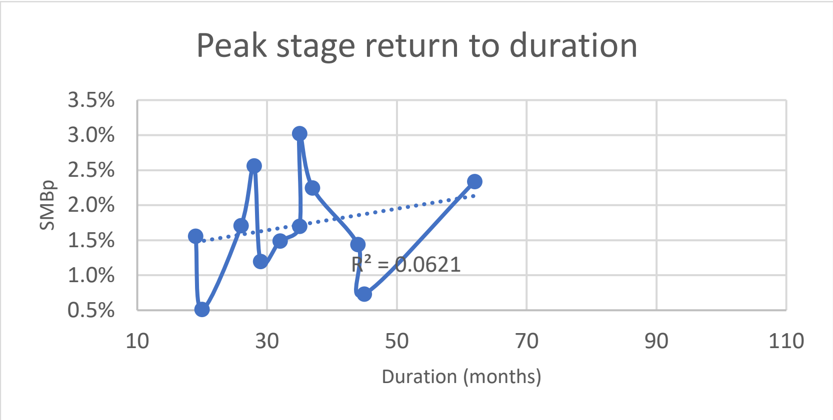

Fig. 3: Peak stage returns over varying stage durations. Source: Authors' plots.

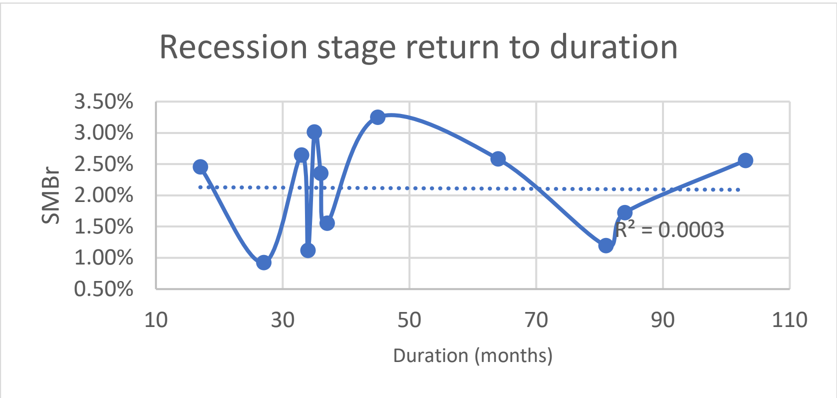

Fig. 4: Recession stage returns over varying stage durations. Source: Authors' plots.

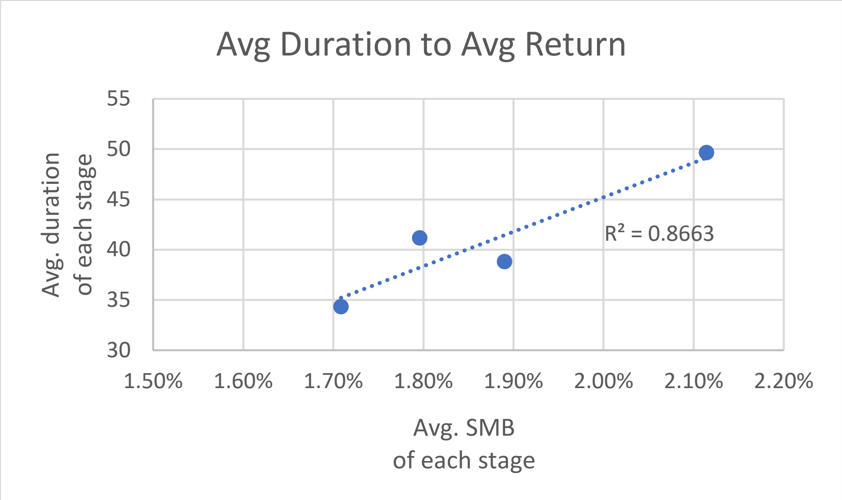

Fig. 5: Comparison of average duration of the stages to their returns. Source: Authors' plots.

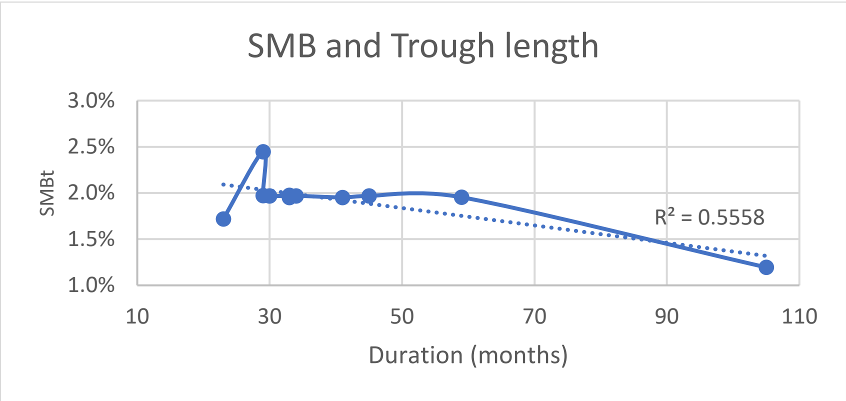

Fig. 6: Full cycle 2×3 *SMB returns compared to trough stage durations. Source: Authors' plots.*

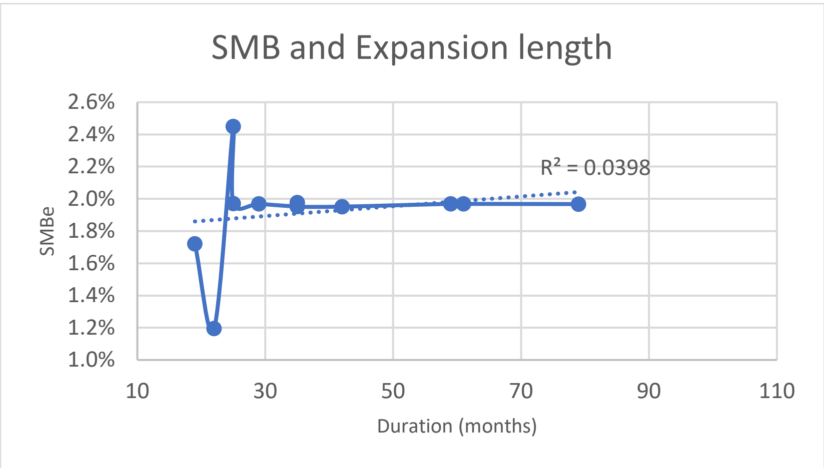

Fig. 7: Full cycle 2×3 *SMB returns compared to expansion stage durations. Source: Authors' plots.*

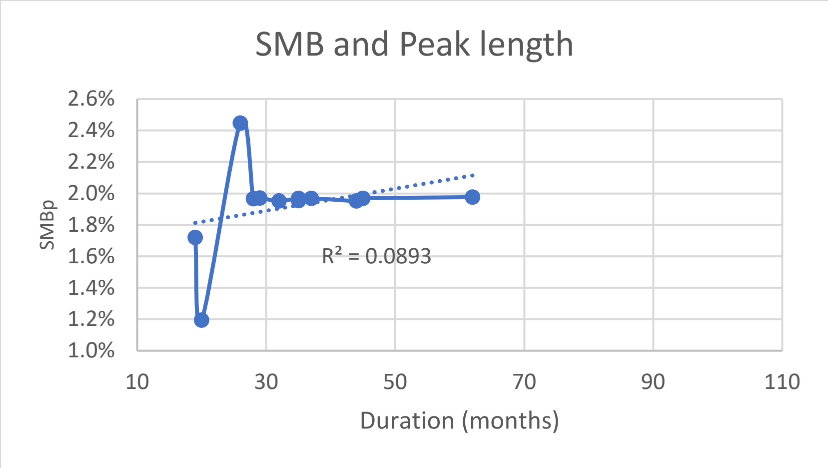

Fig. 8: Full cycle 2 × 3 *SMB returns compared to Peak stage durations. Source: Authors' plots.*

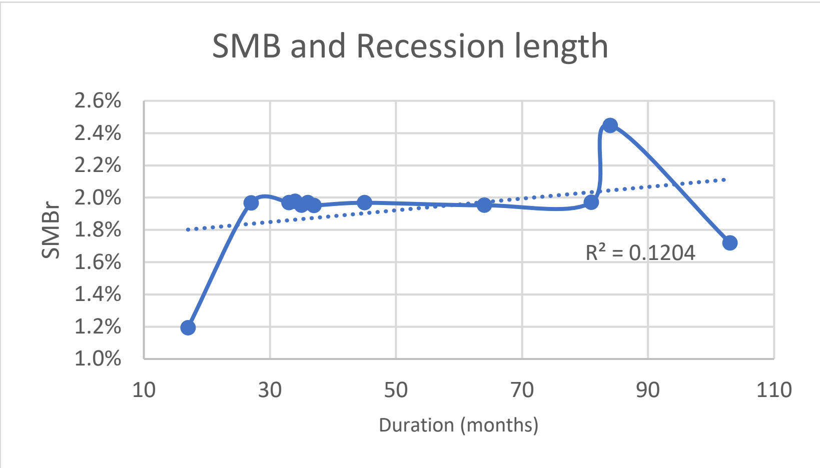

Fig. 9: Full cycle 2 × 3 *SMB returns compared to Recession stage durations. Source: Authors' plots.*

Chapter 5: Discussion and Conclusions

5.1 Discussion of Results

In this comprehensive analysis of 18 African stock markets, we find that the Fama-French five-factor (FF5F) model, while not fully explanatory, offers valuable insights into the region's unique asset pricing dynamics. The model's performance varied across different factor combinations and sorting regimes, with the Novy model, incorporating gross profitability (PMU), often outperforming the traditional FF5F model, particularly in capturing variations related to size and profitability. Both size and profitability effects where observed, with size effects being somewhat more pronounced, however the most pronounced size effects occurred when controlling for gross profitability. This observation is in favour of gross profitability being a more relevant factor than operating profitability in the African context, potentially due to these markets lacking cheap accessible information and thus relying more on accounting ratios to make investment decisions (Claesson, 2021; Foye, 2018; Novy-Marx, 2013). However, the FF5F model often demonstrated superior explanatory power over the Novy model, highlighting the nuanced interplay of factors and aligning with prior research emphasizing the importance of considering regional variations in African asset pricing dynamics (Alagidede et al., 2011; Boamah et al., 2017). Gross profitability proved a promising factor, combining it with a different set of factors could result in a more effective African asset pricing model (Novy-Marx, 2013; Boamah et al., 2017; Mbengue et al., 2023). The prevalence of mispricing in African markets could be due to several factors, including market segmentation (Hearn et al., 2010; Stocker, 2016), information asymmetry (Boamah et al., 2017), limited institutional investor participation (Chirwa, 2012), and behavioural biases (Mosoeu & Kodongo, 2020). The observed lacklustre R-squared values, often below 0.5, further underscore the presence of unexplained systematic risk factors in these markets (Mosoeu & Kodongo, 2020; Novy-Marx, 2013; Boamah et al., 2017; Foye, 2018; Taha & Elgiziry, 2016; Claesson, 2021). The persistent presence of negative and significant intercepts across FF5F and Novy models suggests systematic mispricing in African markets, aligning with observed anomalies in other emerging markets (Fama & French, 2016).

The heterogeneous nature of African markets is highlighted by the distinct behaviour of profitability factors, particularly RMW (operating profitability) and PMU (gross profitability). While RMW showed greater significance in the 2×3 sorts, PMU's performance fluctuated, suggesting that the choice of profitability measure significantly impacts model outcomes (Novy-Marx, 2013). This variability could be due to differences in accounting standards, industry composition, or specific economic contexts, such as high inflation or volatile exchange rates, where gross profitability might be a more reliable indicator than operating profitability.

Spanning tests further underscore this heterogeneity, aligning with Novy-Marx's (2013) findings. Inconsistent results between FF5F and Novy models, with FF5F generally outperforming in R-squared values, contribute to the ongoing discourse on the diverse nature of profitability factors in asset pricing models (Chan, 1985; Cakici et al., 2016; Cakici et al., 2013; Foye, 2018; Leite et al., 2018; Ahn et al., 2019; van Dijk, 2011). A deeper understanding of these factors could refine profitability measures and improve model explanatory power. However, inconsistencies and weak trends in double sorts, coupled with challenges in conducting quadruple sorts due to limited sample size, underscore the need for further refinement and additional controls in the analysis.

The examination of SMB in African markets also revealed intriguing patterns. While significance was found in some but not all sorts, contrasting with developed market views that deny the size effect almost entirely (Fama & French, 2015). This suggests that the size effect in African markets may be conditional on additional factors like liquidity (Boamah et al., 2017), market volatility, or specific firm characteristics (Cakici et al., 2013; Ahn et al., 2019). The observed inconsistencies in small portfolio return trends, factor significance, and SMB coefficient trends across various tests and sorting regimes call for a more refined analysis, potentially involving three or four-way sorts, as recommended by Cakici et al. (2013), Fama & French (2015), and Lin & Qi (2017). However, spanning tests strongly support the distinctiveness of the size factor, with SMB consistently showing unexplained returns. While regression intercepts are not a perfect expression of the size effect, they do highlight the challenges in capturing it comprehensively (Cakici, et al., 2013; Ahn et al., 2019; van Dijk, 2011; Crain, 2011). Nevertheless, concerns about data quality and sample representativeness arise due to exceptionally low portfolio counts and the disproportionate exclusion of stocks from larger markets due to data limitations, potentially impacting the robustness of the size effect findings.

The size effect, while not consistently significant across all portfolios, emerged as a relevant factor, especially in specific market conditions and business cycle phases. The exploration of stage durations, transition probabilities, and simulated moments reveals a dynamic relationship between the size effect and business cycle phases. The analysis of the relationship between the size effect and the business cycle revealed that the size effect is not isolated to any particular phase of the business cycle. This suggests that the size effect in African markets is a persistent phenomenon, although its magnitude and significance may vary depending on the specific economic conditions and market environment. This dynamic relationship could be due to the fact that during economic expansions, investors may be more willing to take on the risk associated with small-cap stocks, leading to a stronger size effect. Conversely, during economic contractions, investors may prefer larger, more stable companies, leading to a weaker size effect (Ahn et al., 2019). These findings support the relevance of the size effect in African stock markets, albeit with variations across different economic conditions and market environments (Leite et al., 2018; Boamah et al., 2017; Cox & Britten, 2019; Taha & Elgiziry, 2016). However, the lack of statistically significant differences in returns across stages and the potential confounding effect of uncontrolled factors warrants further investigation to fully understand the complex interplay between the size effect and business cycle phases.

5.2 Conclusion of the Study

In conclusion, this study contributes to the growing body of literature on asset pricing in emerging markets, particularly highlighting the unique dynamics of African stock markets. The findings underscore the need for nuanced analyses that account for regional and economic specificities, challenging existing assumptions and paving the way for further research in this under-explored area. The study's findings have important implications for investors, policymakers, and researchers. The distinctiveness of profitability factors further reinforces the heterogeneity in asset pricing models in the African context (Boamah et al., 2017; Cakici et al., 2013). Investors should consider the conditional nature of the size effect and profitability factors when developing investment strategies for African markets (Boamah et al., 2017; Cakici et al., 2013). Diversification across different size and profitability portfolios could be beneficial, given the observed variations in factor performance. Additionally, investors should be aware of the potential for mispricing in African markets and consider investing across the region to earn diversification benefits from the lack of integration within the continent.

Policymakers should focus on improving market integration, transparency, and data quality to enhance the efficiency and attractiveness of African stock markets (Hearn & Piesse, 2005; Stocker, 2016). Addressing the specific challenges identified in the study, such as the small sample size and data quality issues, can enhance the generalizability and robustness of future research findings. For instance, the lack of gross profitability data greatly limited the number of stocks in this study, uniform reporting standards that include gross profitability, would allow for a much larger samples, more powerful tests and more generalizable outcomes. This would also allow for greater comparability of stocks across the region. Strong research assertions can in turn stimulate investor interest.

Researchers should delve deeper into the nuances of asset pricing in Africa, exploring regional variations, the impact of changing economic conditions, and the potential for refining existing models. Investigating the relationship between the size effect and business cycle phases, as well as the distinct behaviour of different profitability measures, could yield valuable insights for both academics and practitioners. Future research could also explore Gross profit alongside additional factors, such as liquidity, volatility, or momentum, to improve the model's explanatory power in the African context. Momentum as a factor in the Carhart and Fama and French 6 factor models has proven useful in some emerging market tests (Charteris et al., 2017; Cakici et al., 2013) This is further corroborated by Mbengue et al. (2023) who came to a similar conclusion finding Momentum, and Profitability (ROE) to be useful factors to combine in a model. Therefore, momentum seems like a good choice to further improve the model, by increasing the factor count and thus likely increasing explanatory power, while using a factor known to be priced into African stocks. Additionally, future research could investigate the impact of market integration and regulatory reforms on the performance of the FF5F and Novy models in African markets. Further refinement of the analysis, including additional control variables, and exploring the implications of changing business cycle characteristics on the size effect, offer promising directions for future research.

Appendices

Table A 1

Simulated moments of size effect.

		Implied SMB	SMB			
			Trough	Expansion	Peak	Recession
cycle 1	Mean	2.50%	0.34%	2.25%	4.96%	3.10%
	Std dev		0.06	0.03	0.05	0.05
	t-stat		0.0	0.7	1.0	0.6
	N	164	30	79	28	27
cycle 2	Mean	2.27%	3.19%	4.97%	-0.33%	-0.50%
	Std dev		0.05	0.04	0.05	0.05
	t-stat		0.7	1.3	-0.1	-0.1
	N	164	34	59	35	36
cycle 3	Mean	2.56%	6.17%	-0.70%	0.42%	4.82%
	Std dev		0.05	0.05	0.04	0.05
	t-stat		1.4	-0.1	0.1	1.0
	N	164	41	42	44	37
cycle 4	Mean	2.83%	4.67%	-3.33%	6.20%	2.50%
	Std dev		0.04	0.04	0.05	0.05
	t-stat		1.3	-0.8	1.2	0.5
	N	164	59	35	35	35
cycle 5	Mean	2.40%	2.74%	6.49%	-0.69%	-1.32%
	Std dev		0.03	0.07	0.06	0.06
	t-stat		0.9	1.0	-0.1	-0.2
	N	164	105	22	20	17
cycle 6	Mean	2.35%	7.42%	2.64%	2.51%	-3.17%
	Std dev		0.05	0.06	0.04	0.04
	t-stat		1.4	0.5	0.7	-0.7
	N	164	33	35	62	34
cycle 7	Mean	2.51%	3.50%	-0.10%	3.36%	3.00%
	Std dev		0.05	0.05	0.04	0.04
	t-stat		0.7	0.0	0.9	0.8
	N	164	33	35	32	64
cycle 8	Mean	2.64%	2.04%	-3.08%	8.27%	2.60%
	Std dev		0.05	0.05	0.06	0.03
	t-stat		0.5	-0.6	1.5	0.8
	N	164	29	25	29	81

cycle 9	Mean	2.44%	3.38%	-1.49%	0.95%	3.23%
	Std dev		0.06	0.07	0.08	0.03
	t-stat		0.6	-0.2	0.1	1.2
	N	164	23	19	19	103
cycle 10	Mean	2.34%	4.65%	1.73%	-0.37%	4.17%
	Std dev		0.06	0.04	0.05	0.05
	t-stat		0.9	0.4	-0.1	0.9
	N	164	33	61	37	33
cycle 11	Mean	2.70%	-0.40%	4.22%	1.92%	5.60%
	Std dev		0.04	0.05	0.04	0.04
	t-stat		-0.1	1.0	0.4	1.3
	N	164	45	29	45	45
cycle 12	Mean	-0.45%	-0.45%	-0.44%	-0.34%	-0.48%
	Std dev		0.05	0.06	0.06	0.03
	t-stat		-0.1	-0.1	0.0	-0.1
	N	164	29	25	26	84

Simulation data of the SMB 2 × 2 × 2 × 2 *sorts. Source: Authors' computation.*

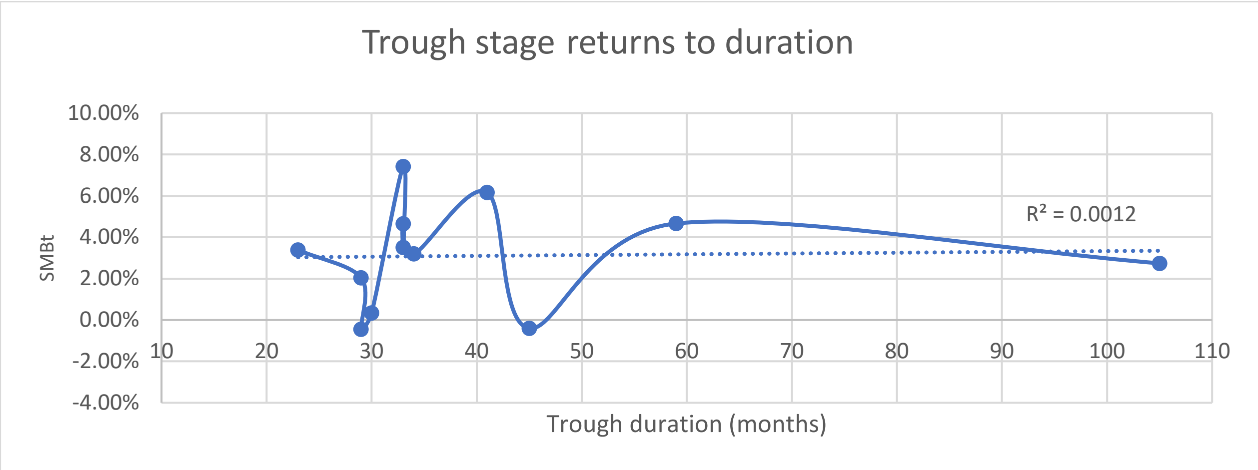

Fig. A 1: 2 × 2 × 2 × 2 *Trough stage returns over varying stage durations. Source: Authors' plots.*

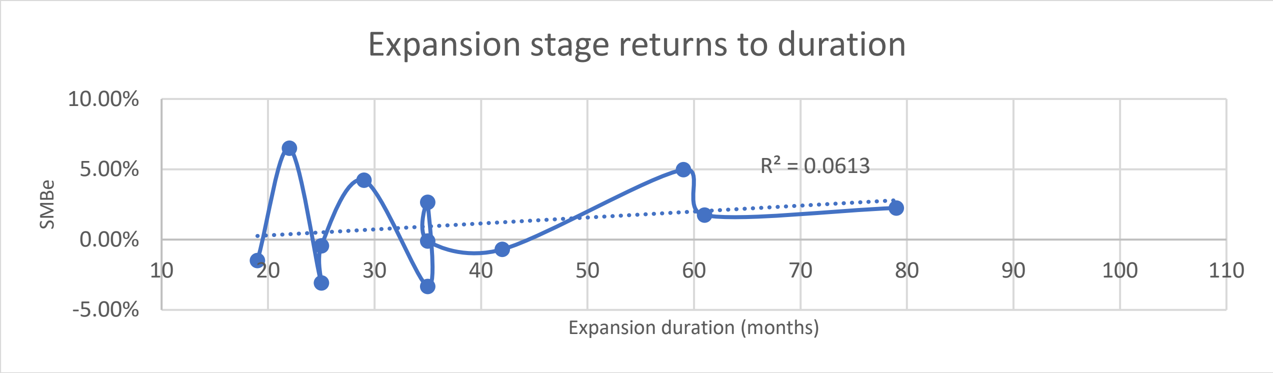

Fig. A 2: 2 × 2 × 2 × 2 Expansion stage returns over varying stage durations. Source: Authors' plots.

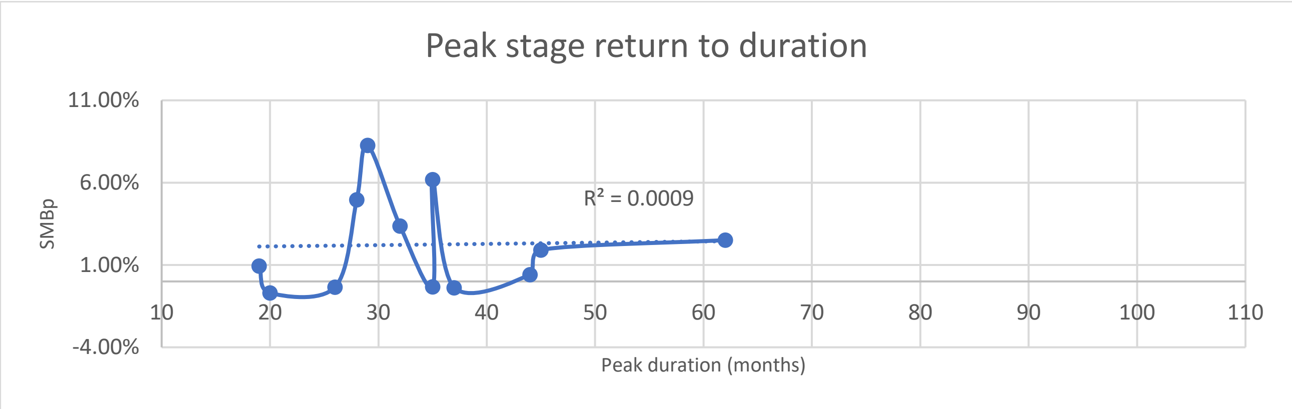

Fig. A 3: 2 × 2 × 2 × 2 Peak stage returns over varying stage durations. Source: Authors' plots.

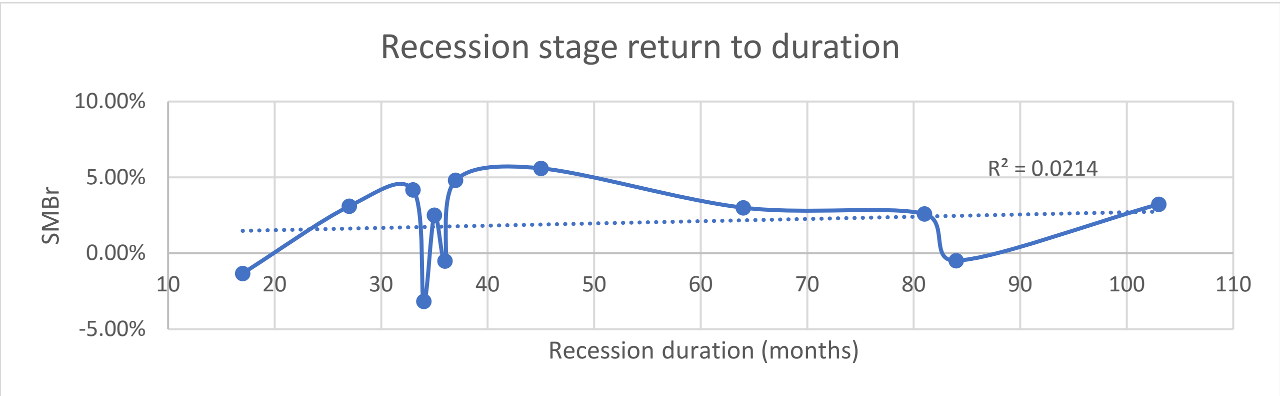

Fig. A 4: 2 × 2 × 2 × 2 Recession stage returns over varying stage durations. Source: Authors' plots.

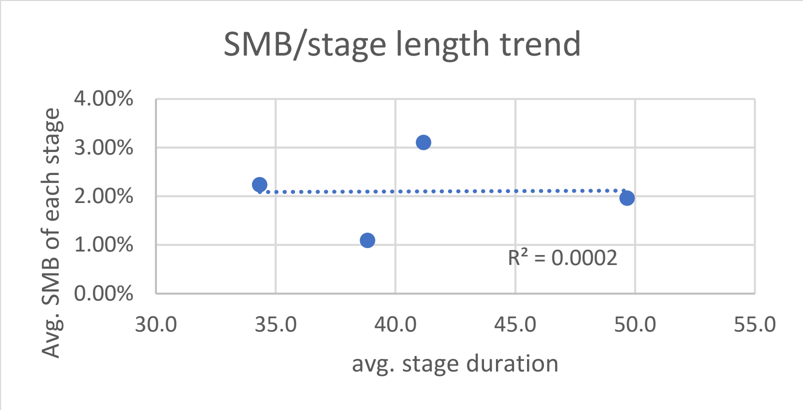

Fig. A 5: Comparison of average duration of the stages to their $2 \times 2 \times 2 \times 2$ *sorted SMB returns. Source: Authors' plots.*

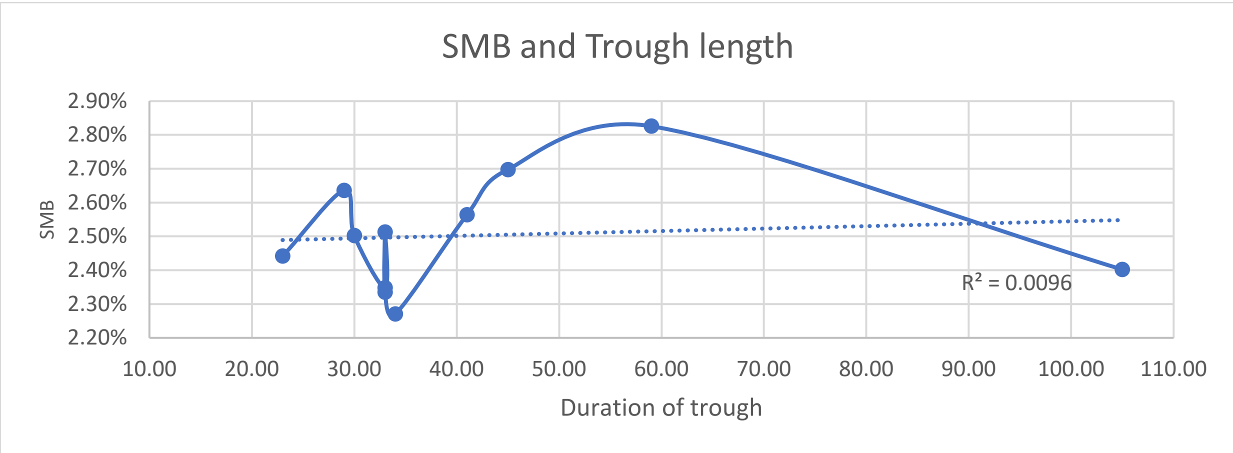

Fig. A 6: Full cycle $2 \times 2 \times 2 \times 2$ *SMB returns compared to trough stage durations. Source: Authors' plots.*

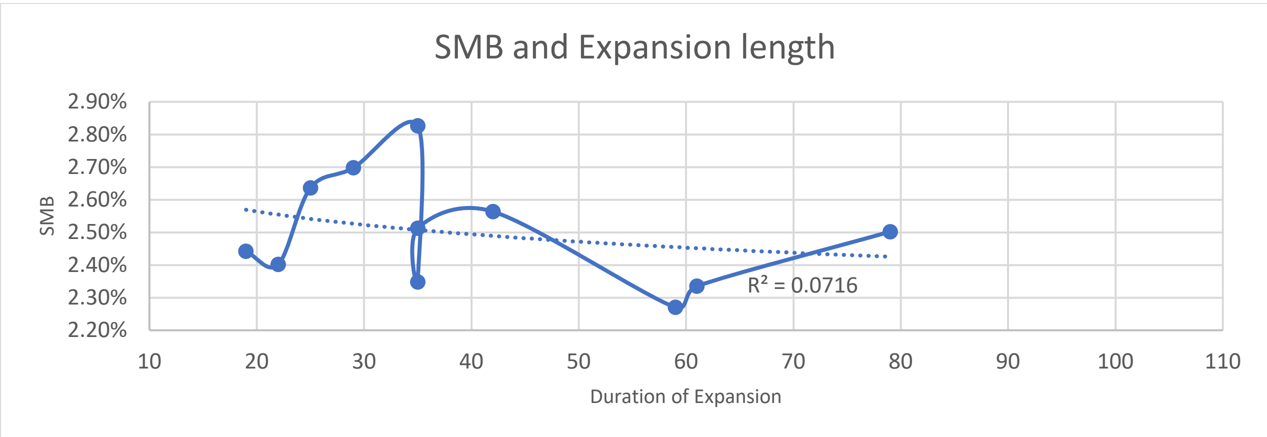

Fig. A 7: Full cycle $2 \times 2 \times 2 \times 2$ *SMB returns compared to expansion stage durations Source: Authors' plots.*

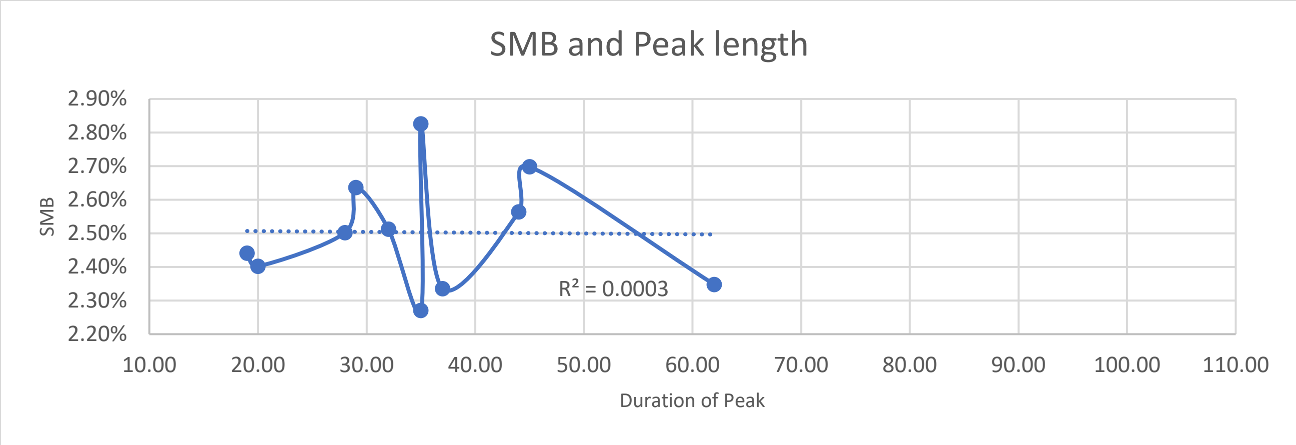

Fig. A 8: Full cycle $2 \times 2 \times 2 \times 2$ *SMB returns compared to peak stage durations. Source: Authors' plots.*

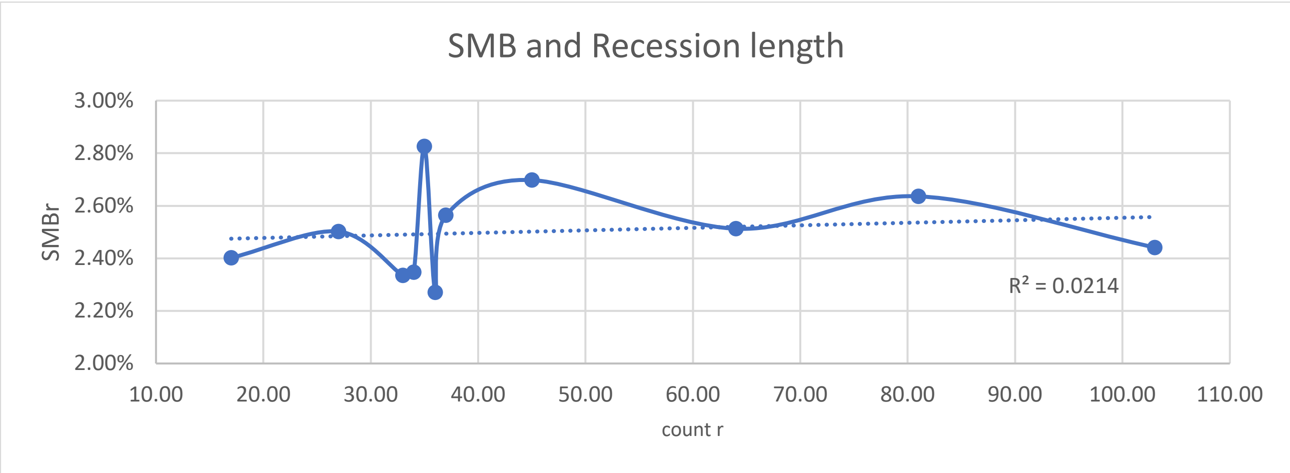

Fig. A 9: Full cycle $2 \times 2 \times 2 \times 2$ *SMB returns compared to recession stage durations. Source: Authors' plots.*

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