DEEP-LYING HOLE STATES IN NUCLEI.

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ABSTRACT.

The strength function for deep-lying hole states in a nucleus is examined from a many-body point of view. Due to their interaction with the compound state background, such single hole excitations are interpreted as quasihole states that are not eigenstates of the nuclear Hamiltonian. These states show up as giant resonances in the strength function, with position and width determined by the real and imaginary parts of the quasihole energy. A formal theory of the strength and fragmentation of such states is developed by splitting the self-energy into background and doorway state contributions. The theory is applied to the calculation of the strength function for the isotopes of Sn using doorway states of a collective nature that consist of a hole plus collective vibrations of the target nucleus. A microscopic description of both the collective excitations and the hole state that it dresses, is given in terms of a modified Random Phase Approximation procedure that uses Green's functions for the individual single particle and single hole states that have been dressed by their interaction with the nuclear background. Specific calculations for the isotope ^{115}Sn, that are essentially free of adjustable parameters, shows excellent agreement with experiment.

I declare that the material presented in this thesis is my own work, and has not been submitted before for any degree or examination in any other University.

S. P. KLEVANSKY.

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CHAPTER 1.

INTRODUCTION.

The optical model for the nucleus [Fs 54] is now nearly 30 years old. Yet the idea of replacing the degrees of freedom of the nuclear system by their average effect on the motion of a single nucleon in the nucleus remains a very useful concept, and it is one that we shall use repeatedly throughout this thesis. The formal theoretical basis [Fs 58] of the optical model has been developed in great detail during the intervening years, amongst other things giving rise to the important concept of a quasiparticle in a nucleus. A quasiparticle is visualised as an elementary excitation mode of the system, which is, however, not an eigenstate of the full nuclear Hamiltonian. Rather, it represents an excitation mode having a finite lifetime, and can be pictured as having been established by the effect of the polarisation of the nuclear medium by a nucleon moving through it. In this connection, one of the important theoretical developments alluded to above is the application of Green's functions, and the powerful diagrammatic techniques that go with them to the problem of nuclear motion in a nucleus.

The problem we wish to study concerns itself with the

properties of nuclear excited states that are reached by removing a nucleon from deep inside the nuclear Fermi sea. Experimentally, such excited nuclear states are produced during the pickup of a nucleon from the target nucleus in reactions like (p,d), (^{3}He,α), etc. Since a single nucleon is removed from the target by this process, the resulting excited state in the nucleus containing one nucleon less is referred to as a neutron (or proton) hole state. To the extent that the nucleus can be treated as an assembly of independent nucleons moving in a common shell model potential, such a hole state continues to be a very simple excited state that is created by removing a nucleon from one of the previously occupied shell model levels. Such a picture is, of course, an extreme oversimplification; if the hole state has a large binding energy, then its creation leads to an excited state of the residual nucleus lying anywhere from 5 to 15 MeV above its ground state. This places the single hole excitation in the vicinity of the much more complicated compound nucleus excitations of the nucleus, that have an increasingly high density of levels at such excitation energies. The interactions present in the full nuclear Hamiltonian couple such a hole excitation to the many compound states that surround it, and so cause its "strength" (i.e. single hole character) to be redistributed over the many compound states in its vicinity. A theoretical and experimental measure of this redistribution of the single hole character is provided by the strength function, which measures the probability per unit energy of finding the

single hole state coupled to the compound states at a given excitation energy.

In this thesis, we study the calculation of the strength function for deep-lying hole states in particular. To this end, we exploit the important link between the single hole Green's function and the strength function, which is related to the imaginary part of the former. The construction of the Green's function, however, requires an exact solution of the full many-body problem. This is clearly a hopeless task. Therefore we have to carry out some approximate construction of the Green's function in a fashion which emphasises the main physical features of the problem. The ideas we describe next are suggested by the well-known successes of the optical model for particle scattering by atomic nuclei. In this situation one knows that the effect of the many degrees of freedom of the resulting compound nucleus can be approximately accounted for by introducing an absorptive (i.e. imaginary) component into the average field seen by the impinging nucleon. Furthermore, one knows from empirical evidence from the so called optical potentials [Ül 64] that the imaginary part of the potential is of order 3% of the real part for low energy incoming nucleons [Fs 54]. This corresponds to an excitation energy of 8 MeV in the compound nucleus, and shows that at even these excitation energies, the bulk of the (energy averaged) scattering is determined by the real part of the average nuclear

potential. Since our discussion is directed toward the study of hole states, we have to adapt these ideas to single particle motion at negative energies. The concept of an average nuclear field for a hole continues to hold of course, and we accept that this field determines the main features of the hole state (binding energy, spin and parity) in the spirit of the standard shell model. From Green's function theory, one knows that the effects of the nuclear medium on such a hole created in it are incorporated in terms of the single hole self-energy $\Sigma(\omega)$. The more formal aspects of the self-energy are discussed in chapter 2. However, it is useful at this stage to outline the physical ideas which form the basis of our approach. Consider such a single hole state in the shell model. Its self-energy $\Sigma(\omega)$, excluding that already accounted for by the average shell model potential, arises from the polarisation effects the hole causes in the nuclear medium. Microscopically, these effects come about because the hole excites compound states lying near its own excitation energy. We split the contributions of such states into two parts, viz.,

$$\Sigma(\omega) = \Sigma^{b}(\omega) + \Sigma^{d}(\omega)$$

The piece $\Sigma^{b}(\omega)$ describes the contribution to $\Sigma(\omega)$ arising from compound states that are strongly coupled to the hole state. We give arguments later to suggest that this contribution is slowly varying in energy and purely imaginary. The second piece $\Sigma^{d}(\omega)$ is the contribution from doorway states, [Fs 67]. This represents a contribution to $\Sigma(\omega)$

coming from the coupling of the hole state to the compound states <u>locally</u> via a doorway in its vicinity. This piece is expected to have both real (dispersive) and imaginary (absorptive) contributions, and to vary rapidly with energy in the vicinity of the doorway states that it includes. Since we are dealing with reasonably high excitation energies, of order of the nuclear Fermi energy, the contribution of $\Sigma^b(\omega)$ to the spreading of the single hole state is much larger than that from $\Sigma^d(\omega)$. The resulting quasihole state observed in the strength function therefore consists of a single giant resonance, whose width is determined by $Im\Sigma^b(\omega)$, possibly fragmented into one or more intermediate structures due to poles in $\Sigma^d(\omega)$. Whether or not intermediate structure states become visible in the vicinity of such a quasihole giant resonance is determined by both the widths of the doorway states it couples to, and the strength of this coupling. This latter feature suggests that only doorway states having a collective nature present the possibility of showing up as intermediate structures [Ha 75]. Thus, in examining the spreading of a single hole state, we are led to consider doorway states having a one hole plus vibration structure, where the vibrational state is one of the collective states of the nucleus. We will see that the coupling matrix elements between the quasihole and such collective states have a <u>coherent</u> character, and are thus large.

From a microscopic point of view, such doorway states

consist of a hole coupled to interacting particle-hole excitations which build up the collectivity [Br 72]. Each of these particle-hole excitations are of course, also spread out by their coupling to the nuclear compound states. Consequently, the Green's functions describing them have the same structure as the Green's function $G(\omega)$ we are trying to calculate. Thus one is faced with a self-consistency problem: the self-energy that enters $G(\omega)$ is in turn determined by the contributions $\Sigma^b(\omega)$ and $\Sigma^d(\omega)$ that in turn should enter the Green's functions determining these contributions. We break this impasse by the following argument. Since the contribution to the quasihole self-energy from $\Sigma^d(\omega)$ is much smaller than from $\Sigma^b(\omega)$ due to the localised (in energy) nature of the former, we simply neglect the contribution $\Sigma^d(\omega)$ to the dressing of the particles or holes making up the doorway state, and describe their propagation by a Green's function $G^b(\omega)$, that is only dressed with the self-energy $\Sigma^b(\omega)$.

Although the preceding assumptions result in an enormous simplification of the full many-body problem, their use in calculating $\Sigma^d(\omega)$ presents one with further problems, since the actual energy dependence of $G^b(\omega)$ enters the construction of the self-energy in an essential way. Therefore, to actually do such calculations, we have made appeal to the approximate procedures developed by Dover et al., [Do 72], which we have extended and developed further so as to be applicable to the much more general problem of quasihole

damping.

The use of a dressed Green's function $G^b(\omega)$ introduces new features into the construction of the collective mode part of the doorway states, which we construct in the Random Phase Approximation (RPA). The resulting RPA eigenvalue problem now develops complex eigenvalues, which in addition are a function of the energy ω, necessitating a study of this problem in conjunction with its adjoint.

We find that it is possible to implement the above procedures without making a specific choice as to how the background self-energy $\Sigma^b(\omega)$ behaves with energy, as long as this variation is a slow one. The results we obtain for the composite self-energy also retain the general features expected of it as a function of ω. In particular, we find that our approximations preserve the very important property of $Im\Sigma(\omega)$ changing sign as the energy variable ω passes through the Fermi level. The other important result that follows from using $G^b(\omega)$ in calculating the doorway state self-energy is that it automatically endows the doorway states with a width that is also determined from $\Sigma^b(\omega)$. Thus the widths of the giant resonance and the intermediate structure resonances are determined by one and the same function.

We have chosen the isotopes of Sn as a specific case to illustrate the results of this formalism, which is quite

general. The reasons for this choice are twofold: on the theoretical side, the nuclear structure of these isotopes is well understood, and their hole strength functions have been extensively investigated experimentally, most recently by Gerlic at al., [Ge 80]. The actual implementation of this formalism involves additional assumptions and technical complications due to the structure of these nuclei. The main additional assumption that must be introduced is a specific parametrisation of $Im\Sigma^b(\omega)$, and a recognition that pairing and isospin effects play an essential role in the Sn isotopes. These matters are taken up in detail in the latter part of chapter 2 and chapter 3 of this thesis. Specific results are shown in section 3(d) for the $g_{9/2}$ quasihole state in Sn. We comment there in detail on the underlying assumptions leading to these results. Suffice it to say that the fits we obtain in comparison with the experimental data are excellent, and therefore suggest that the spreading mechanism introduced into the shell model via the self-energy having background and doorway contributions, has isolated the main physical features of the problem.

CHAPTER 2.

FORMALISM.

The focus of this thesis is on the study of the properties of excited states in nuclei, that are reached, for example, by the removal of a single nucleon in a target nucleus during a pickup reaction. If the pickup occurs from valence shells (i.e. shells containing loosely bound nucleons), the resulting excited state has a strong single particle character. If, however, the pickup occurs from one of the deeper filled shells forming the core of the nucleus, the resulting excitation energy in the residual nucleus is large (of order of the Fermi energy, i.e. around 8 MeV), and the stat produced is spread over the many compound states o target nucleus that are approximately degenerate in energy with the single hole state that has been produced.

It is clearly a hopeless task to calculate in detail, the fragmentation of such a deep-lying hole state over the large number of compound states to which it is coupled. In fact, even if this were possible to implement in practise, the results would not be very useful, in the sense that the present experimental data does not contain the same amount of detail. Rather, some form

of energy averaged data is available. This comes about because the lack of infinite energy resolution in the experimental equipment does not allow for the independent resolution of the many compound states occurring at the high excitation energy of the hole. Under these circumstances, one may therefore approximate the fragmentation of the single hole state over the compound states it is coupled to, by a continous function of excitation energy or strength function. This gives the fraction of the original hole state occurring within the energy interval $d\varepsilon_s$ that surrounds an excitation energy ε_s. In the language of many body theory, such an excitation of a single hole coupled to a high density of compound states in its vicinity, represents a quasihole state, which is only an approximate eigenstate in the many - body system in which it occurs. The method of Green's functions provides a powerful tool for studying such a situation, and we follow this method of analysis.

Since the conventions used in relation to Green's functions differ widely from author to author, we establish the conventions we follow in the next section, and review any important properties as they become necessary during the development and implementation of our problem.

2.1 THE SINGLE PARTICLE GREEN'S FUNCTION.

The propagation of a single particle or hole in an interacting N particle system can be described quite generally by the time-ordered single particle Green's function,

$$iG_{12}(\omega) = \langle\Psi_o^N|Tc_1(t)c_2^+(0)|\Psi_o^N\rangle$$
$$= \langle\Psi_o^N|Tc_1(0)e^{i(E_o^N-H)t}c_2^+(0)|\Psi_o^N\rangle \qquad (2.1)$$

Here Ψ_o^N represents the ground state lying at energy E_o of the N interacting particles that are described by the many-body Hamiltonian. Since we will be discussing the nucleons in a nucleus, the $c_2^+(0)$ and $c_1(t)$ in Eq.(2.1) represent fermion creation and destruction operators for a nucleon in state "2" at time t=0, and state "1" at time t, respectively. Since they obey the usual equal time anticommutation relations $\{c_1(t),c_2^+(t)\}=\delta_{12}$, it is useful to include a sign change in the time-ordering operator T, so that

$$Tc_1(t)c_2^+(0) = \begin{cases} c_1(t)c_2^+(0) & t>0 \\ -c_2^+(0)c_1(t) & t<0 \end{cases} \qquad (2.2)$$

Our interest is in the energy spectrum of the Green's function which we obtain via its Fourier transform,

$G_{12}(\omega) = \int dt e^{i\omega t} G_{12}(t)$. It can be shown that, [Fe 71], $G_{12}(\omega)$ satisfies the following set of integral equations

$$G_{12}(\omega) = G_{12}^{(0)}(\omega) + \sum_{34} G_{13}^{(0)}(\omega)\Sigma_{34}(\omega)G_{42}(\omega) \qquad (2.3)$$

that are known as Dyson Equations. In this equation, $G_{12}^{(0)}(\omega)$ refers to the non-interacting system, while the quantity $\Sigma_{34}(\omega)$ called the proper or irreducible[1] self-energy, includes all the interaction effects exactly which change $G_{12}^{(0)}(\omega)$ into $G_{12}(\omega)$.

The labels 1,2 etc. in Eq.(2.3) are arbitrary insofar as one is free to choose the representation in which to calculate $G_{12}(\omega)$. The physical situation we have in mind in the present problem however, is how a single hole state that is created in the nucleus is affected by its interaction with the more complicated nuclear excitations to which it is coupled. Furthermore, one knows both empirically [My 57] and theoretically [Br 72] that, if possible excitations of the nuclear medium are temporarily put aside, the effect of the remaining nucleons on the motion of a nucleon in a nucleus is

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[1] We remind the reader that "proper" or "irreducible" refers to the class of diagrams that cannot be reduced to two others in the set, by the removal of a single particle line (see Eq.(2.8)).

well represented by the average field produced by these "spectator" nucleons. This field can in turn either be represented phenomenologically by a shell model potential well, or approached theoretically as in Hartree-Fock theory [Br 72] for example. In this study, we will take the latter approach in principle, and interpret the labels in Eq. (2.3) as referring to Hartree-Fock (HF) states of the nucleon. This in turn means that all effects of exciting the nuclear medium through which a nucleon or nucleon hole is moving, then reside in $\Sigma_{34}(\omega)$. In the language of Feynman diagrams, [Ma 76], this choice of representation means that all particle or hole lines are "dressed" to the extent that their interaction with the average nuclear field has been taken into account. In particular $G_{12}^{(0)}(\omega)$ then describes the propagation of a single particle or hole in the average nuclear field, and consequently must be diagonal in the labels 1 and 2::

$$G_{12}^{(0)}(\omega) = G_{1}^{(0)}(\omega)\,\delta_{12} \tag{2.4}$$

The same statement is not true for $G_{12}(\omega)$ of course, since the particle in state 2 can scatter into 1 by exciting the nuclear medium. In a spherically symmetric average field, where the labels 1 and 2 summarize the usual radial and angular momentum quantum numbers $\{n\ell jm\}$ of a single particle state in such a field, these labels can differ at most in their radial quantum numbers. However, one knows that such radial excitations involve excitation energies of order twice the average spacing,

$\sim 41 A^{-1/3}$ MeV, between major shells in the shell model, and this is of order 20 MeV in a typical nucleus (A=100). Since the two-nucleon scattering matrix elements are much smaller than this (typically 1 MeV), the feature of a large energy gap between shells allows us to replace $\Sigma_{34}(\omega)$ in Eq.(2.3) by its diagonal elements only:

$$\Sigma_{34}(\omega) \approx \Sigma_3(\omega)\delta_{34} \tag{2.5}$$

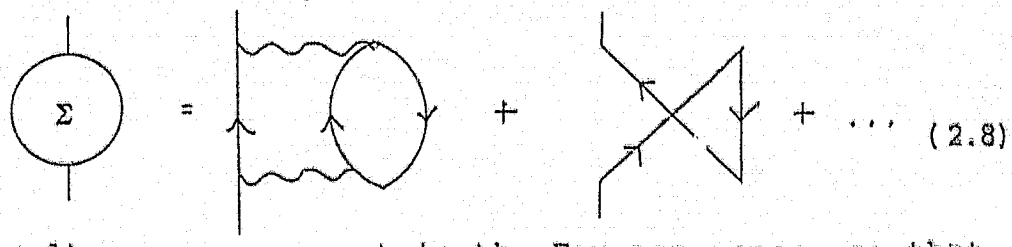
and still obtain an excellent approximation for $G_{12}(\omega)$ which is therefore now also diagonal,

$$G_{12}(\omega) = G_1(\omega)\delta_{12} \tag{2.6}$$

The resulting Dyson equation for $G_1(\omega)$ may then be solved explicitly to obtain

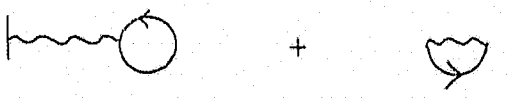
$$\left[G_1^{(0)-1}(\omega) - \Sigma_1(\omega)\right]^{-1} \tag{2.7}$$

In diagrammatic language, $\Sigma_1(\omega)$ consists of the sum of all irreducible diagrams in the following manner



(2.8)

These diagrams are meant in the Feynman sense, so that the straight lines represent a particle or a hole state (arrows up or down) interacting with each other via a static, two body interaction (wavy line). Since our labels refer to HF states, diagrams like



that generate the HF field, have to be excluded from $\Sigma_1(\omega)$, since these are already supposed to be included in the meaning of the particle and hole states.

In particular, the Green's function $G_1^{(0)}(\omega)$ for a particle or hole in the HF state 1 of energy ε_1 is given by [Fe 71]

$$G_1^{(0)}(\omega) = \left[\frac{1-n_1}{\omega-\varepsilon_1+i\eta} + \frac{n_1}{\omega-\varepsilon_1-i\eta} \right], \quad \eta \to 0^+ \tag{2.9}$$

where the HF occupation number $n_1 = 1$ or 0, depending on whether ε_1 lies below or above the Fermi surface at energy λ. As usual, the positive infinitesmal η locates the poles of $G_1^{(0)}(\omega)$ in ω-space in a manner that befits the Fourier transform of a time-ordered (i.e. acausal) propagator.

Inserting Eq.(2.9) into Eq.(2.7), we obtain the result

$$G_1(\omega) = \frac{1-n_1}{\omega-\varepsilon_1+i\eta-\Sigma_1(\omega)} + \frac{n_1}{\omega-\varepsilon_1-i\eta-\Sigma_1(\omega)} \tag{2.10}$$

We have been careful to carry the pole structure of $G_1^{(0)}(\omega)$ into Eq.(2.10). This refinement is only necessary if $\Sigma_1(\omega)$ is a real function. However, this will not be the case for the Green's function we require, As will be discussed in the next section, there is a sign change in the imaginary part of $\Sigma_1(\omega)$ as the energy ε_1 passes

from representing a hole state to representing a particle state (i.e. ε_1 moves from below to above the Fermi surface) in such a way as to reinforce the sign bought in by the infinitesmal. We may thus rewrite Eq.(2.10) more simply as:

$$G_1(\omega) = \left[\omega - \varepsilon_1 - \Sigma_1(\omega)\right]^{-1} \qquad (2.10')$$

2.2 THE STRUCTURE OF THE SELF-ENERGY $\Sigma_1(\omega)$.

The information that $G_1(\omega)$ contains can be made more explicit by returning to the defining equation, Eq.(2.1) and inserting a complete set of states of the full Hamiltonian H. The Fourier transform of Eq.(2.1) then becomes [Fe 71]

$$G_1(\omega) = \sum_{s'} \frac{|(c_1)_{s'o}|^2}{\omega - \lambda + \varepsilon_{s'} - i\eta} + \sum_{s} \frac{|(c_1^+)_{so}|^2}{\omega - \lambda - \varepsilon_s + i\eta} \tag{2.11}$$

Here ε_s ($\varepsilon_{s'}$) are the excitation energies of the N+1 (N-1) particle systems, and the sums on s and s' run over the complete sets of states of the N+1 and N-1 systems respectively. The chemical potentials $\lambda(N) = E_o^{N+1} - E_o^N$, and $\lambda(N-1)$ have been assumed equal. This assumption is reasonable for large N as the change in λ is inversely proportional to the particle number. The quantities $|(c_1^+)_{so}|^2 = |\langle \Psi_s | c_1^+ | \Psi_o \rangle|^2$ and $|(c_1)_{s'o}|^2 = |\langle \Psi_{s'} | c_1 | \Psi_o \rangle|^2$ represent the probability that the N±1 particle eigenstates $|\Psi_s\rangle$ or $|\Psi_{s'}\rangle$ consist of a particle or hole in state 1 added to $|\Psi_o\rangle$, respectively. They satisfy the sum rule $\Sigma|(c_1^+)_{so}|^2 + \Sigma|(c_1)_{s'o}|^2 = 1$ [Fe 71].

From Eq.(2.11) we may write

$$\mathrm{Im}G_1(\omega) = \begin{cases} -\pi \sum_s |(c_1^+)_{so}|^2 \delta(\omega - \lambda - \varepsilon_s) & \omega > \lambda \\ +\pi \sum_{s'} |(c_1)_{s'o}|^2 \delta(\omega - \lambda + \varepsilon_{s'}) & \omega \leq \lambda \end{cases} \tag{2.12}$$

If the density of excited states in the vicinity of ε_s and $\varepsilon_{s'}$ is high enough, the sums $\sum_s$ and $\sum_{s'}$ can be replaced by integrals over the energies ε_s and $\varepsilon_{s'}$, respectively. Then

$$\mathrm{Im}G_1(\omega) = \begin{cases} -\frac{\pi}{D_s}\,\overline{|(c_1^+)_{so}|^2}_{\omega=\lambda+\varepsilon_s} & \omega>\lambda \\ +\frac{\pi}{D_{s'}}\,\overline{|(c_1)_{s'o}|^2}_{\omega=\lambda-\varepsilon_{s'}} & \omega<\lambda \end{cases} \tag{2.13}$$

D_s, $D_{s'}$ are the average spacings of the compound levels $\varepsilon_s, \varepsilon_{s'}$.

The pole structure of $G_1(\omega)$ as given by the expansion Eq.(2.11) also shows that $\mathrm{Im}\Sigma_1(\omega)$ must have the property of changing sign as ω passes through the Fermi surface. By comparing Eq.(2.10) with Eq.(2.11) one infers that $\mathrm{Im}\Sigma_1(\omega)$ changes its sign as ω passes through λ in the following way:

$$\mathrm{Im}\Sigma_1(\omega) \lesseqgtr 0, \text{ provided } \omega \gtreqless \lambda \tag{2.14}$$

An exact calculation of $\Sigma_1(\omega)$ from first principles is clearly not possible in practice. It is certainly true that the technique of the summation of diagrams of a particular species (eg summation of ladder and bubble diagrams) has had partial success in discussing many aspects of nuclear structure (see for example [Br 72]). Nevertheless, there does not appear to be any clear cut

reason for singling out any particular set, in discussing a hole state deep in the nucleus, Consequently we follow a different approach. The success of the optical model in describing the interaction of nucleons in scattering states is well known [Fs 58]. In the following, we extend the idea of an optical potential to apply to negative energy single particle states as well.

Consider the HF state bound by energy ε_1. If this state is strongly coupled to the compound states surrounding it, irrespective of their excitation energy relative to ε_1, (the strong coupling limit), one can anticipate that: i) $\Sigma_1(\omega)$ will be a slowly varying function of ω, and ii) its real part should be small. This latter circumstance is brought about by the fact that, in the strong coupling approximation, the strength of the single particle state is distributed uniformly over the compound states to which it is coupled. Consequently, the contribution of compound states above and below ε_1 approximately cancel. We denote the contribution to $\Sigma_1(\omega)$ coming from compound states coupled to it in this way by $\Sigma_1^b(\omega)$, and we may thus write it as

$$\Sigma_1^b(\omega) \approx \pm iW_1(\omega) \qquad \text{for } \omega \lesssim \lambda \tag{2.15}$$

where the function $W_1(\omega)$ is always positive. The change of sign as ω passes through λ is required by the general expression Eq.(2.14).

Eq.(2.15) has the following implications: if 1 is a state with energy $\varepsilon_1 > \lambda$, call this a particle state "p". Then for $\omega > \lambda$, $W_p(\omega)$ is defined to be some as yet unspecified positive function. However, ω is a parameter, and may range over all values, greater and less than λ. Since $W_p(\omega)$ has no meaning here below λ, it is set to zero. Similarly, if 1 is a state with energy $\varepsilon_1 < \lambda$, we may call this a hole state "h". $W_h(\omega)$ is defined to be some positive function for $\omega < \lambda$. Hence, if the argument of this function ranges above λ, it is set to zero. Furthermore, we insist that $W_1(\lambda) = 0$, a condition that we borrow from Fermi Liquid theory, [Pi 66]. All this information is summarised in Fig. 2.1.

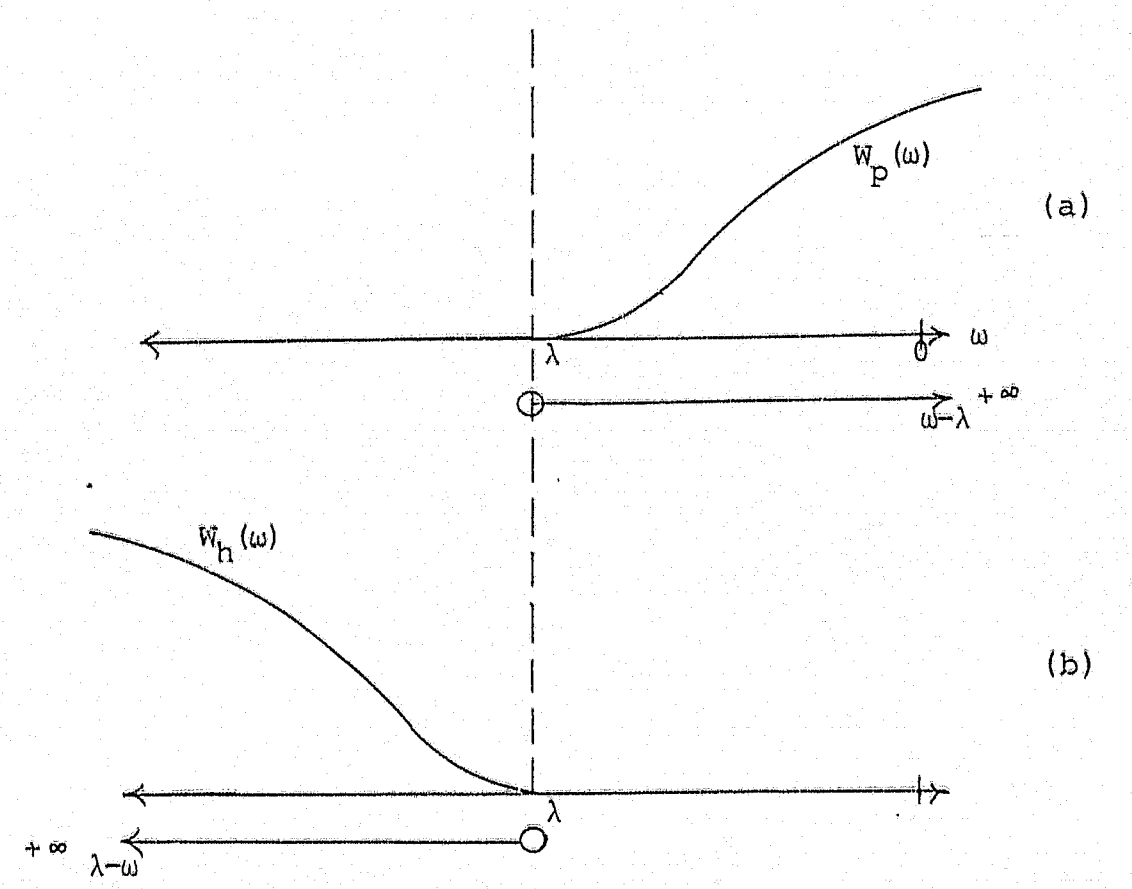


Fig. 2.1: Schematic representation of $W_p(\omega)$ $[\omega-\lambda>0]$ in (a), and $W_h(\omega)$ $[\lambda-\omega>0]$ in (b).

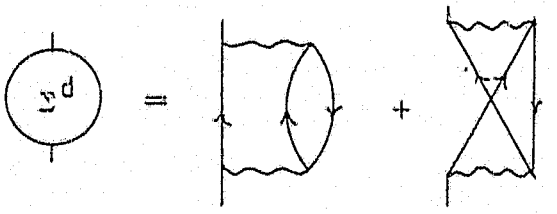
$\Sigma_1(\omega)$ can also contain a second contribution which arises in the event that there are doorway states [Fs 67] through which the state ε_1 is coupled to the compound states. Under these circumstances, another situation prevails in which the single particle excitation spreads <u>locally</u> into the compound states that it is coupled to via a doorway. The cancellation leading to a vanishing real part is not expected to occur, so that the contribution to $\Sigma_1(\omega)$ arising from this mechanism, which we denote by $\Sigma_1^d(\omega)$, will contribute to both the real and imaginary parts of $\Sigma_1(\omega)$. We thus write

$$\Sigma_1(\omega) = \Sigma_1^b(\omega) + \Sigma_1^d(\omega) \tag{2.16}$$

The information required to implement this equation in practice is, besides a more precise knowledge of $W_1(\omega)$, a microscopic description of the doorway states that contribute to $\Sigma_1^d(\omega)$.

2.3 THE SELF-ENERGY $\Sigma_1^d(\omega)$.

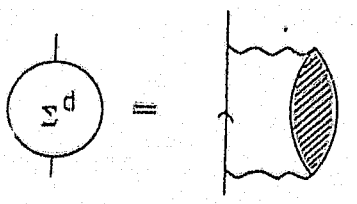
The self-energy $\Sigma_1^d(\omega)$ giving rise to doorway states consists of a sum of all proper diagrams which depict the polarisation of the medium by the nuclear particle or hole. To lowest order, this process is depicted as a single particle-hole bubble, thus :



(2.20)

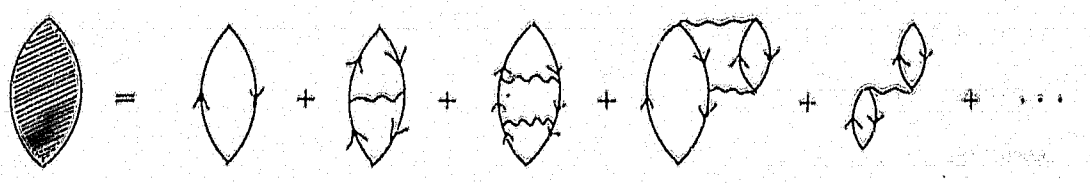
= "direct" + "exchange"

while in full generality, to obtain an exact result, all polarisation parts are included :



(2.21)

where



Here all lines  refer to $G^{(0)}(\omega)$. The series  arises naturally from a special case of the particle-hole propagator, or two-particle Green's function aptly known as the polarisation propagator [Ma 76]. We define therefore, the polarisation propagator $F(4321,t)$ as

$$iF(4321,t) = \langle\Psi_o|Tc_4(t)c_3^+(t)c_2(0)c_1^+(0)|\Psi_o\rangle$$

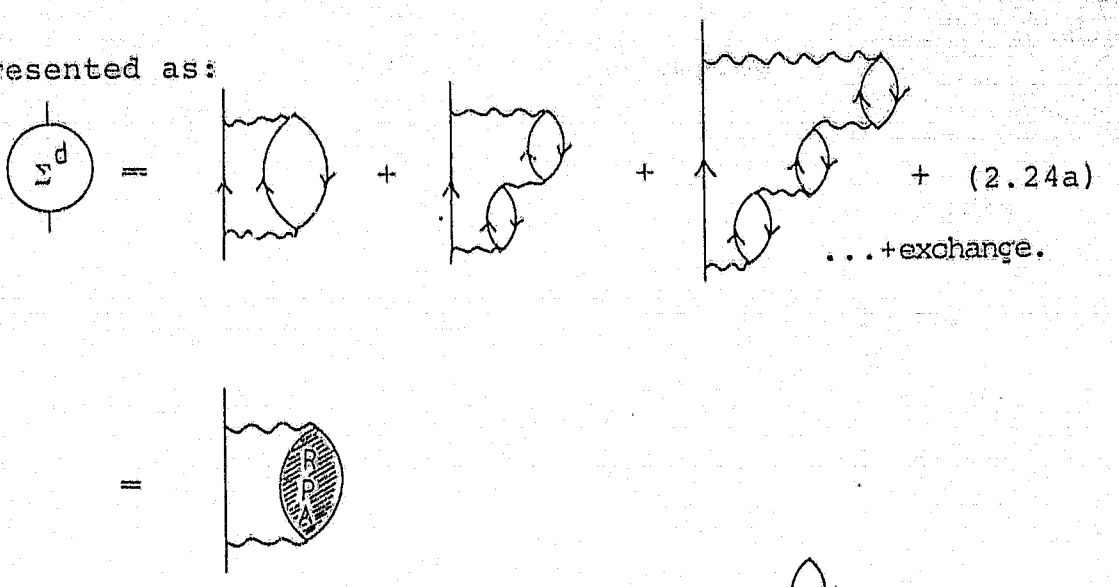
$$= \langle\Psi_o|Tc_3^+(t)c_4(t)c_1^+(0)c_2(0)|\Psi_o\rangle \qquad (2.22)$$

where all the symbols are as defined as for Eq.(2.1). F(4321,t) thus represents the probability amplitude that if a particle in state 1 and a hole in state 2 are introduced into the nuclear system at time t=0, then a particle in state 3 and a hole in state 4 will be observed at time t, for the particular time ordering t>0; otherwise the processes occur in the reverse time order. Thus F(4321,t) describes exactly what is depicted in the diagrammatic sum in Eq.(2.21). Symbolically

"F" =  (2.23)

This is all very well in principle, but it is clear that in practise, to sum the series of all possible connected diagrams beyond the HF terms to calculate $\Sigma_1^d(\omega)$ is a hopeless task, and thus some selective summation procedure must be implemented. Thus, on the basis of the mechanism giving rise to doorways, in our case, that of particle-hole excitations, we build our summation on the repetition of the simple particle-hole bubble. Once again, the simplest form of this is identical with Eq. (2.20), and in full is nothing other than the Random Phase Approximation (RPA), which is schematically

represented as:



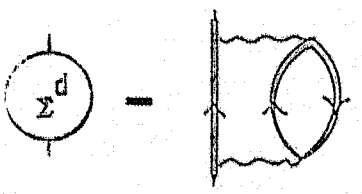
Clearly, Eq.(2.24b) is just a special selection of the diagrams from F(4321,ω).

This procedure for calculating $\Sigma_1^d(\omega)$ may be improved upon by the process of renormalisation. The equations (2.20) and (2.24) which we can calculate, require only a knowledge of $G^{(0)}(\omega)$, which according to our definition, is dressed only by the HF field. However, this ignores entirely processes such as those giving rise to $\Sigma_1^b(\omega)$, where the state's strength is spread uniformly over the compound states. This process is mainly responsible for changing the particle or hole state into a quasiparticle or quasi-hole, whereas the processes giving rise to $\Sigma_1^d(\omega)$ <u>fragment</u> the resulting particle or hole state, as might be expected from a weak-coupling effect. This physical situation therefore suggests that we include $\Sigma_1^b(\omega)$ semi-phenomenologically in our approximation to construct $\Sigma_1^d(\omega)$ via

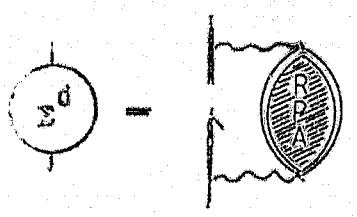
equations (2.20) or (2.24). i.e. we employ

$$G_1^b(\omega) = \left[\omega-\varepsilon_1-\Sigma_1^b(\omega)\right]^{-1} \tag{2.25}$$

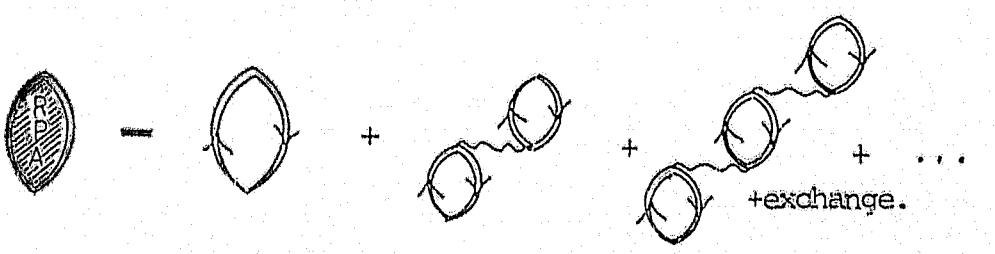
Diagrammatically, we will associate the symbol ‖ with the "better dressed" Green's function, and in so doing, this approximation replaces equations (2.20) and (2.24) by equations (2.26) and (2.27) respectively,



(2.26)



(2.27)



In the next sections, we shall direct out attention to the calculations of $\Sigma_1^d(\omega)$ by employing equations (2.26) and (2.27). However, before this, it is useful to establish the connection between the assumption Eq.(2.25) and $|(c_1^+)_{so}|^2$ and $|(c_1)_{s'o}|^2$, as appear in Eq.(2.11). The explicit forms of these may be obtained in the following way:

From equations (2.25) and (2.15), we may write

$$\mathrm{Im}G_1^b(\omega) = \frac{\mp W_1(\omega)}{(\omega-\varepsilon_1)^2 + W_1^2(\omega)} \quad , \quad \omega \gtrless \lambda \tag{2.28}$$

We now equate Eq.(2.28) with Eq.(2.13) to yield

$$\frac{\pi}{D_s}\left|\overline{(c_1^+)_{so}}\right|^2 = \frac{W_1(\varepsilon_s+\lambda)}{(\varepsilon_s-\varepsilon_1+\lambda)^2 + W_1^2(\varepsilon_s+\lambda)}$$

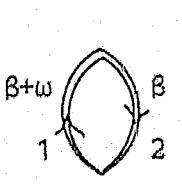
$$\frac{\pi}{D_{s'}}\left|\overline{(c_1^+)_{s'o}}\right|^2 = \frac{W_1(\lambda-\varepsilon_{s'})}{(\varepsilon_{s'}+\varepsilon_1-\lambda)^2 + W_1^2(\lambda-\varepsilon_{s'})} \tag{2.29}$$

2.4 THE SINGLE DRESSED BUBBLE.

In this, and in all subsequent sections where diagram translation has been effected, the conventions used are those of R. Mattuck [Ma 76], and are given in Appendix A. All diagrams are to be taken in the Feynman sense, unless specific indication is given to the contrary.

The calculation of the doorway self-energy relies on a calculation of the single dressed bubble. For this, according to our convention, we define

$$-i\P(12,\omega) = \tag{2.30}$$



This may be rewritten as

$$\P(12,\omega) = -\int \frac{d\beta}{2\pi i}\, G_1^b(\omega+\beta) G_2^b(\beta) \tag{2.31}$$

By inserting Eq.(2.11) for $G_1(\omega+\beta)$ and $G_2(\beta)$ into this equation, and performing the integration over β, we arrive at the still exact result

$$\P(12,\omega) = -\sum_{ss'} \frac{|(c_1^+)_{so}|^2 |(c_2)_{s'o}|^2}{\omega-\varepsilon_{s'}-\varepsilon_s+i\eta} + \sum_{ss'} \frac{|(c_1)_{s'o}|^2 |(c_2^+)_{so}|^2}{\omega+\varepsilon_s+\varepsilon_{s'}-i\eta} \tag{2.32}$$

To evaluate this, we replace the sums over s,s' by the

appropriate integrals

$$\P(12,\omega) = -\frac{\pi}{D_s}\int d\varepsilon_s \frac{\pi}{D_{s'}}\int d\varepsilon_{s'} \frac{|\overline{(c_1^+)_{so}}|^2 |\overline{(c_2)_{s'o}}|^2}{\omega-\varepsilon_s-\varepsilon_{s'}+i\eta}$$

$$+ \frac{\pi}{D_s}\int d\varepsilon_s \frac{\pi}{D_{s'}}\int d\varepsilon_{s'} \frac{|\overline{(c_1)_{s'o}}|^2 |\overline{(c_2^+)_{so}}|^2}{\omega+\varepsilon_s+\varepsilon_{s'}-i\eta} \qquad (2.33)$$

In principle, $|\overline{(c_1^+)_{so}}|^2$ and $|\overline{(c_2)_{s'o}}|^2$ are known via $G^b(\omega)$, and are given by Eq.(2.29). However, the functional dependence of W on ε_s and $\varepsilon_{s'}$ is unknown. Our philosophy is thus to evaluate the functions $W_1(\lambda+\varepsilon_s)$ and $W_1(\lambda-\varepsilon_{s'})$ occurring in $|\overline{(c_1^+)_{so}}|^2$ and $|\overline{(c_2)_{s'o}}|^2$ at the most important values of their arguments, and hold them constant over ε_s and $\varepsilon_{s'}$ throughout the evaluation of the integral. This is done in the following manner: Consider the first term in Eq.(2.33). This peaks at $\omega=\varepsilon_s+\varepsilon_{s'}$. Subject to maintaining this condition, we evaluate $W_1(\varepsilon_s+\lambda)$ which occurs in $|\overline{(c_1^+)_{so}}|^2$ at the peak value of $|\overline{(c_2)_{so}}|^2$, seen from Eq.(2.29) to be at $\varepsilon_{s'}=\lambda-\varepsilon_2$. Similarly, we evaluate $W_2(\lambda-\varepsilon_{s'})$ occurring in $|\overline{(c_2)_{s'o}}|^2$ at the point where $|\overline{(c_1^+)_{so}}|^2$ peaks, i.e. at $\varepsilon_s=\varepsilon_1-\lambda$. This means at $\varepsilon_s+\lambda=\omega+\varepsilon_2$, and $\lambda-\varepsilon_{s'}=\varepsilon_1-\omega$, resulting in the modified expressions

$$\frac{\pi}{D_s}|\overline{(c_1^+)_{so}}|^2 = \frac{W_1(\omega+\varepsilon_2)}{(\varepsilon_s-\varepsilon_1+\lambda)^2+W_1^2(\omega+\varepsilon_2)}$$

$$\frac{\pi}{D_{s'}}|\overline{(c_2)_{s'o}}|^2 = \frac{W_2(\varepsilon_1-\omega)}{(\varepsilon_{s'}+\varepsilon_2-\lambda)^2+W_2^2(\varepsilon_1-\omega)} \qquad (2.34)$$

Note that these results can only hold if 1 refers to a

particle state, and 2 to a hole state, otherwise the mechanism causing the peak from the denominators of the above expressions cannot operate. Furthermore, one sees that

$$\sum_s |(c_1^+)_{so}|^2 \cong 1-n_1$$

$$\sum_{s'} |(c_2)_{s'o}|^2 \cong n_2 \qquad (2.35)$$

by direct integration over ε_s and $\varepsilon_{s'}$ from $-\infty$ to $+\infty$, this extension of the integral to $-\infty$ being justifiable in view of the peaked nature of these distributions. The expressions Eq.(2.35) reflect the fact that the contribution due to the addition of particles below λ, and the creation of holes above λ, are excluded in the approximation Eq.(2.34).

In a similar manner, the second term of Eq.(2.33) may be analysed. This peaks at $\omega=-\varepsilon_s-\varepsilon_{s'}$. Subject to this condition being maintained, we obtain a second approximation to $\overline{|(c_2^+)_{so}|^2}$ and $\overline{|(c_1)_{s'o}|^2}$. Using Eq.(2.29) again, we evaluate $W_2(\varepsilon_2+\lambda)$ occuring in $\overline{|(c_2^+)_{so}|^2}$ at the peak of $\overline{|(c_1)_{so}|^2}$, i.e. at $\varepsilon_{s'}=\lambda-\varepsilon_1$, and $W_1(\lambda-\varepsilon_{s'})$ at the peak of $\overline{|(c_2^+)_{so}|^2}$, i.e. at $\varepsilon_s=\varepsilon_2-\lambda$. This means that $\varepsilon_s+\lambda=\varepsilon_1-\omega$, and $\lambda-\varepsilon_{s'}=\varepsilon_2+\omega$. Thus, for the evaluation of the second term in Eq.(2.34), we must employ

$$\frac{\pi}{D_s}\,|\overline{(c_2^+)_{so}}|^2 = \frac{W_2(\varepsilon_1-\omega)}{(\varepsilon_s-\varepsilon_2+\lambda)^2 + W_2^2(\varepsilon_1-\omega)}$$

and $$\frac{\pi}{D_{s'}}\,|\overline{(c_1)_{s'o}}|^2 = \frac{W_1(\varepsilon_2+\omega)}{(\varepsilon_{s'}+\varepsilon_1-\lambda)^2 + W_1^2(\varepsilon_2+\omega)} \qquad (2.36)$$

Clearly, Eq.(2.35) holds once again, in the context that the labels 1 and 2 are interchanged.

The rationale for assuming that in evaluating the integral containing a product of two peaked functions, the most important value of the first structure is at the peak of the second, and vice-versa, [Do 72], is illustrated for the case of obtaining Eq.(2.36) in Appendix B. This technique is of central importance in maintaining the generality of the yet unknown functions W, and will be used frequently in the calculations that follow.

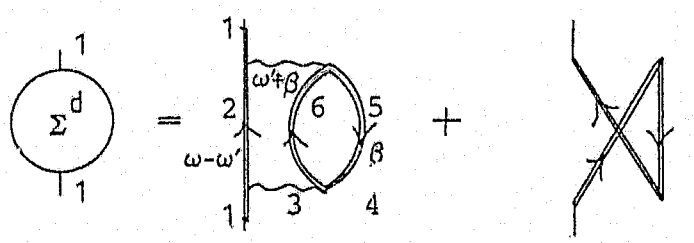
We now insert Eq.(2.34) into the first term of Eq.(2.33) bearing in mind that 1,2 refer to particle, hole states respectively, and we insert Eq.(2.36) into the second term of Eq.(2.33), bearing in mind that 1,2 refer to hole, particle states respectively. Extending the limits of the integrals to $-\infty$ in Eq.(2.33), the final result for $\P(12,\omega)$ becomes:

$$\P(12,\omega) = \frac{(1-n_1)n_2}{\varepsilon_1-\varepsilon_2-i\left[W_2(\varepsilon_1-\omega)+W_1(\varepsilon_2+\omega)\right]-\omega}$$

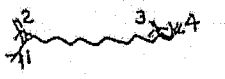
$$+ \frac{n_1(1-n_2)}{\varepsilon_2-\varepsilon_1-i\left[W_1(\varepsilon_2+\omega)+W_2(\varepsilon_1-\omega)\right]+\omega} \qquad (2.37)$$

2.5 SELF-ENERGY CALCULATION USING A SINGLE DRESSED BUBBLE.

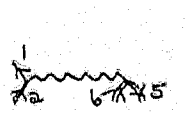
It is now possible to calculate the self-energy $\Sigma_1(\omega)$ as given by the diagrammatic representation in Eq.(2.26) i.e.:



(2.38)

Both the first (direct) and second (exchange) diagrams of this equation are included in its translation, by reinterpreting the interaction lines in the first diagram such that  means

$$(32|\tilde{V}|41) = (32|V|41) - (32|V|14) \qquad (2.39)$$
$$= \text{"direct"} - \text{"exchange"}$$

and similarly  means

$$(51|\tilde{V}|62) = (51|V|62) - (51|V|26) \qquad (2.40)$$
$$= \text{"direct"} - \text{"exchange"}$$

V is a two-body interaction potential, which will be discussed in more detail at the point of implementation.

By means of the diagram translation rules, we obtain

$$-i\Sigma_1^d(\omega) = \sum_{2345b}(-i)\,(32|\tilde{V}|41)\,(-i)\,(51|\tilde{V}|62)\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi i}iG_2^b(\omega-\omega')\,(-i)\,\P(34,\omega') \times \delta_{36}\delta_{54} \tag{2.41}$$

This may be evaluated directly by inserting Eq.(2.37) for $\P(34,\omega')$, and Eq.(2.11) for $G_2^b(\omega-\omega')$. The integral over ω' may be performed immediately, to yield

$$\Sigma_1^d(\omega) = \sum_{234}|(32|\tilde{V}|14)|^2 \times \left\{ \sum_s |(c_2^+)_{so}|^2 \frac{(1-n_3)n_4}{\omega-\lambda-\varepsilon_s-\varepsilon_3+\varepsilon_4+i[W_4(\varepsilon_3-\omega+\lambda+\varepsilon_s)+W_3(\varepsilon_4+\omega-\lambda-\varepsilon_s)]} + \sum_{s'} |(c_2)_{s'o}|^2 \frac{(1-n_4)n_3}{\omega-\lambda+\varepsilon_{s'}+\varepsilon_4-\varepsilon_3-i[W_3(\varepsilon_4+\omega-\lambda+\varepsilon_{s'})+W_4(\varepsilon_3-\omega+\lambda-\varepsilon_{s'})]} \right\} \tag{2.42}$$

We again replace the sums over ε_s and $\varepsilon_{s'}$ by the associated integrals. The resulting integrals are then evaluated in a manner similar to that employed in the previous section in the evaluation of $\P(12,\omega)$, see Appendix B. By taking the imaginary part of $\Sigma_1^d(\omega)$,

$$\mathrm{Im}\Sigma_1^d(\omega) \sim \int d\varepsilon_s |\overline{(c_2^+)_{so}}|^2 \times \text{Peaked Function}_1 + \int d\varepsilon_{s'} |\overline{(c_2)_{s'o}}|^2 \times \text{Peaked Function}_2 \tag{2.43}$$

and applying the same method as developed for $\P$, the following prescription is necessary: from Eq.(2.29) we see that $|\overline{(c_2^+)_{so}}|^2$ peaks at $\varepsilon_s=\varepsilon_1-\lambda$, and the Peaked Function$_1$, with which it is to be integrated, peaks at $\varepsilon_s=\omega-\lambda-\varepsilon_3+\varepsilon_4$. We thus evaluate $W_2(\varepsilon_s+\lambda)$ which occurs in $|\overline{(c_2^+)_{so}}|^2$ such that $\varepsilon_s=\omega-\lambda-\varepsilon_3+\varepsilon_4$, and we evaluate

both W_3 and W_4 occurring in the Peaked Function$_1$ such that $\varepsilon_s=\varepsilon_1-\lambda$. This then removes the unknown dependence of the various W's on the excitation energy ε_s. From the denominator of $\overline{|(c_2^+)_{so}|^2}$, we see too, that label 2 must refer to a particle state.

The corresponding treatment must be applied to the second term: $W_2(\lambda-\varepsilon_{s'})$ is evaluated subject to the condition $\varepsilon_{s'}=\lambda-\omega-\varepsilon_4+\varepsilon_3$, and W_3 and W_4 subject to $\varepsilon_{s'}=\lambda-\varepsilon_1$. From the structure of the denominator of $\overline{|(c_2^+)_{s'o}|^2}$, see Eq.(2.29), the label 2 must refer to a hole state.

As before, (see argument leading to Eq.(2.37)), the limits of integration are extended from $-\infty$ to $+\infty$. Inserting all this information into Eq.(2.42) gives us the expression

$$\Sigma_1^d(\omega) = \frac{1}{\pi}\sum_{234}|(32|\tilde{V}|41)|^2 \times\Bigg\{\int_{-\infty}^{\infty}d\varepsilon_s \frac{W_2(\omega-\varepsilon_3+\varepsilon_4)}{(\varepsilon_s-\varepsilon_2+\lambda)^2+W_2^2(\omega-\varepsilon_3+\varepsilon_4)} \frac{(1-n_2)(1-n_3)n_4}{\omega-\lambda-\varepsilon_s-\varepsilon_3+\varepsilon_4+i[W_4(\varepsilon_3-\omega+\lambda+\varepsilon_2)+W_3(\varepsilon_4+\omega-\lambda-\varepsilon_2)]}$$

$$\int_{-\infty}^{\infty}d\varepsilon_{s'} \frac{W_2(\omega-\varepsilon_3+\varepsilon_4)}{(\varepsilon_{s'}+\varepsilon_2-\lambda)^2+W_2^2(\omega-\varepsilon_3+\varepsilon_4)} \frac{n_2n_3(1-n_4)}{\omega-\lambda+\varepsilon_{s'}+\varepsilon_4-\varepsilon_3-i[W_3(\varepsilon_4+\omega-\lambda+\varepsilon_2)+W_4(\varepsilon_3-\omega+\lambda-\varepsilon_2)]}\Bigg\} \tag{2.44}$$

Performing the required integrals, we arrive at the final expression for $\Sigma_1(\omega)$ for a single dressed bubble:

$$\Sigma_1^d(\omega) = \sum_{234} |(32|\tilde{V}|41)|^2 \times \left\{ \frac{(1-n_2)(1-n_3)n_4}{\omega-\varepsilon_2-\varepsilon_3+\varepsilon_4+i[W_2(\omega-\varepsilon_3+\varepsilon_4)+W_3(\varepsilon_4+\omega-\varepsilon_2)+W_4(\varepsilon_3-\omega+\varepsilon_2)]} + \frac{n_2 n_3 (1-n_4)}{\omega-\varepsilon_2-\varepsilon_3+\varepsilon_4-i[W_2(\omega-\varepsilon_3+\varepsilon_4)+W_3(\varepsilon_4+\omega-\varepsilon_2)+W_4(\varepsilon_3-\omega+\varepsilon_2)]} \right\} \tag{2.45}$$

From this result, it is evident that the damping of the single particle or hole, due to the doorway mechanism, which is represented by the imaginary part of $\Sigma_1^d(\omega)$, is controlled by <u>off-shell</u>[1] arguments in W_2, W_3, and W_4. This may be understood as follows: since damping occurs through the sharing of strength of the simple excitation with the more complicated excitations which are nearly degenerate with it in energy, we expect the off-shell energy arguments in the W's to reflect energy conservation in some way. By using this argument, it is possible to intuitively verify that the energy arguments are indeed correct. Furthermore, it proves essential that the W's are in fact energy dependent; this is necessary to guarantee that the imaginary part of $\Sigma_1^d(\omega)$ changes sign as ω passes through λ, as specified by Eq.(2.14).

---

[1] By off-shell, we mean, as usual, that the energy argument of a function like $W_1(\omega)$, does not equal the energy eigenvalue ε_1 of the state 1.

Let us first deal with the issue of the sign change of the imaginary part of $\Sigma_1^d(\omega)$, when ω passes through λ. In doing this, we examine each term in Eq.(2.45) individually.

The first term of Eq.(2.45) demands that labels 2 and 3 refer to particle states, and 4 refers to a hole state. Since we adopt the general convention that labels m,n,p refer to particle states, and i,j,h refer to hole states, let us designate $2 \equiv m$, $3 \equiv n$, and $4 \equiv j$. Schematically, we may represent the energies associated with these states as depicted in Fig. 2.2.

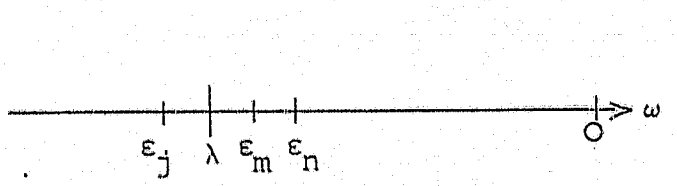
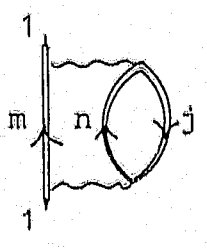


Fig. 2.2: Schematic representation of the particle energies ε_m and ε_n, and the hole energy ε_j.

Clearly, at this stage, we cannot decide whether $\varepsilon_m > \varepsilon_n$, or vice-versa. We need only bear in mind that ε_m and ε_n lie to the right of λ, and ε_j to the left. This first term in Eq.(2.45), which we are considering, corresponds to the specific diagram (to be understood in the Goldstone sense - see Appendix A),



where the exchange term is included in the sketch. To analyse whether any contributions to the imaginary part of $\Sigma_1^d(\omega)$ will whether any contributions to the imaginary part of $\Sigma_1^d(\omega)$ will

arise from W_m, W_n, and W_j, we continually make use of the definition of W for a particle and for a hole, as was discussed in Sec.2.2. From this, it is evident that W_m gives a non-zero contribution to the imaginary part of $\Sigma_1^d(\omega)$ only if its argument, $\omega-\varepsilon_n+\varepsilon_j$ satisfies the condition

$$\omega-\varepsilon_n+\varepsilon_j>\lambda$$

$$\text{i.e.} \qquad \omega>\lambda+(\varepsilon_n-\varepsilon_j)$$

Since the particle-hole excitation energy $\varepsilon_n-\varepsilon_j=E_{nj}>0$, it is clear that W_m can contribute only if $\omega>E_{nj}+\lambda$.

Similarly, W_n gives a contribution for all ω, such that

$$\omega+\varepsilon_j-\varepsilon_m>\lambda$$

$$\text{i.e.} \qquad \omega>\lambda+(\varepsilon_m-\varepsilon_j)$$

$$\text{or} \qquad \omega>\lambda+E_{mj}, \quad E_{mj}>0$$

The third possible contributor to the imaginary part, W_j, will do so provided that

$$\varepsilon_n-\omega+\varepsilon_m<\lambda$$

$$\text{i.e.} \qquad \omega>\varepsilon_m+\varepsilon_n-\lambda$$

which falls also only to the right of λ.

In summary, all three W's, i.e. W_m, W_n, and W_j, have the potential for switching on only at values of ω satisfying the condition $\omega>\lambda$, with the further restrictions indicated

in equations (2.46), (2.47), and (2.48) depending on the values of E_{nj}, E_{mj}, and $\varepsilon_n+\varepsilon_m$. Schematically, this is represented by Fig. 2.3.

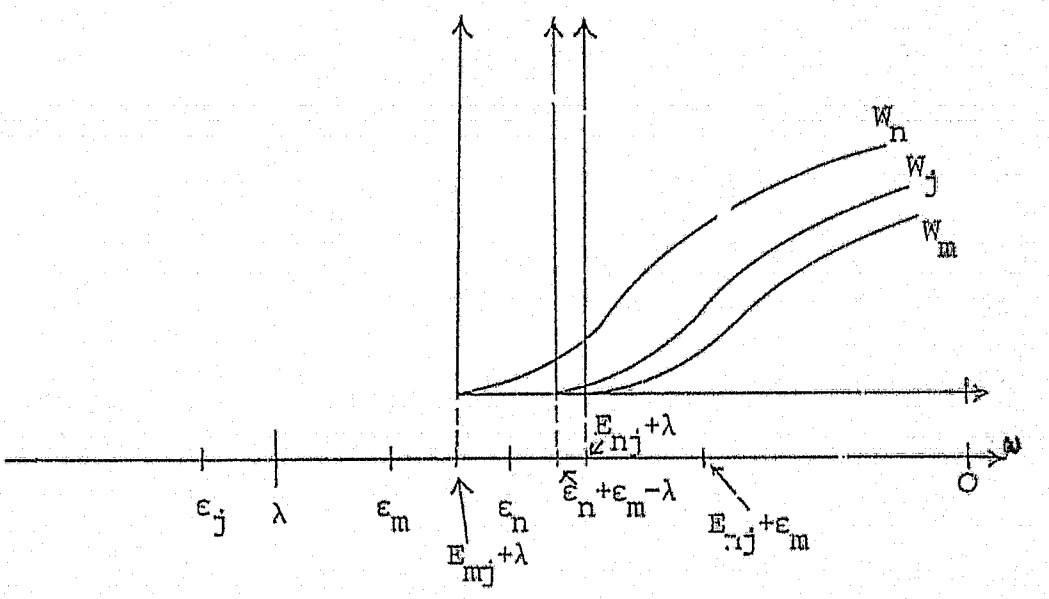
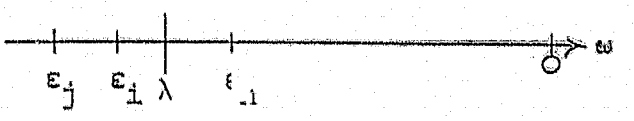


Fig.2.3: The contributions of W_m, W_n, and W_j to the first term of $\Sigma_1^d(\omega)$ are indicated schematically for and arbitrary choice of $\varepsilon_m, \varepsilon_n$, and ε_j.

The second term of Eq.(2.45) for $\Sigma_1^d(\omega)$ may be analysed in a similar manner. Since the labels 2 and 3 refer to hole states, and 4 refers to a particle state, designate $2\equiv i$, $3\equiv j$, and $4\equiv n$. The energies associated with these three states are drawn schematically in Fig.2.4.

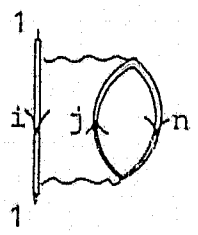
Fig.2.4: Schematic representation of the particle energy ε_n, and the hole energies ε_i and ε_j.



As before, the important information conveyed in this

diagram is that ε_n lies to the right of λ, and ε_i and ε_j (in whichever order they occur) lie to the left.

In similar vein to the description of the first term of Eq.(2.45), the second corresponds to the specific diagram in the Goldstone sense (including the exchange term):



Furthermore, using the same method as before for analysing the W's, we note that contributions to the imaginary part of the second term of $\Sigma_1^d(\omega)$ occur for:

W_i, provided that $\omega-\varepsilon_j+\varepsilon_n<\lambda$

i.e. $\omega<\lambda-E_{nj}$

W_j, provided that $\omega+\varepsilon_n-\varepsilon_i<\lambda$

i.e. $\omega<\lambda-E_{ni}$

W_n, provided that $\varepsilon_j-\omega+\varepsilon_i>\lambda$

i.e. $\omega<\varepsilon_j+\varepsilon_i-\lambda$

Thus, these three equations indicate that the W's in question will all contribute to the imaginary part of $\Sigma_1^d(\omega)$ for values of $\omega<\lambda$ starting at specified points determined by E_{ni}, E_{nj}, and $\varepsilon_i+\varepsilon_j$. This information is represented schematically in Fig.2.5.