

FIGURE C2

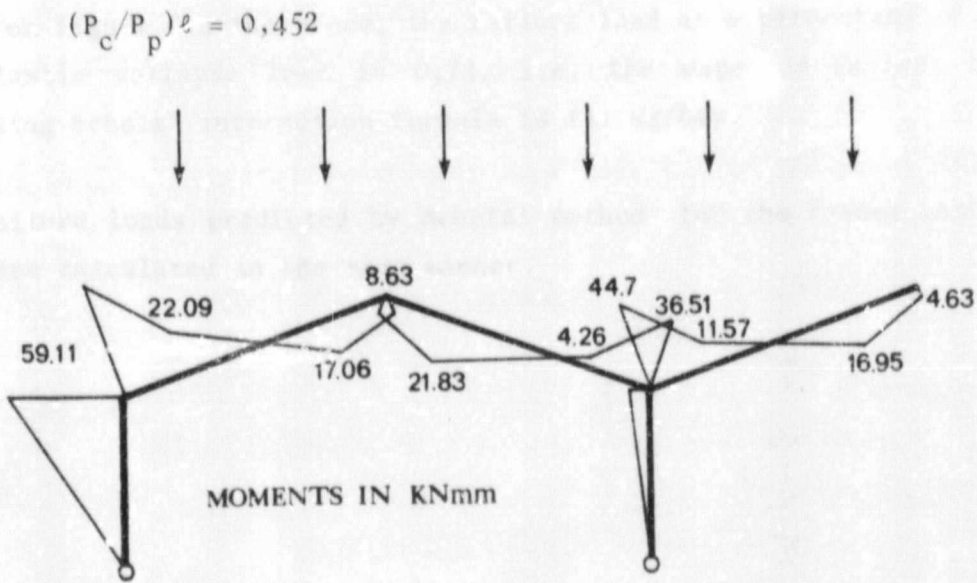


FIGURE C3

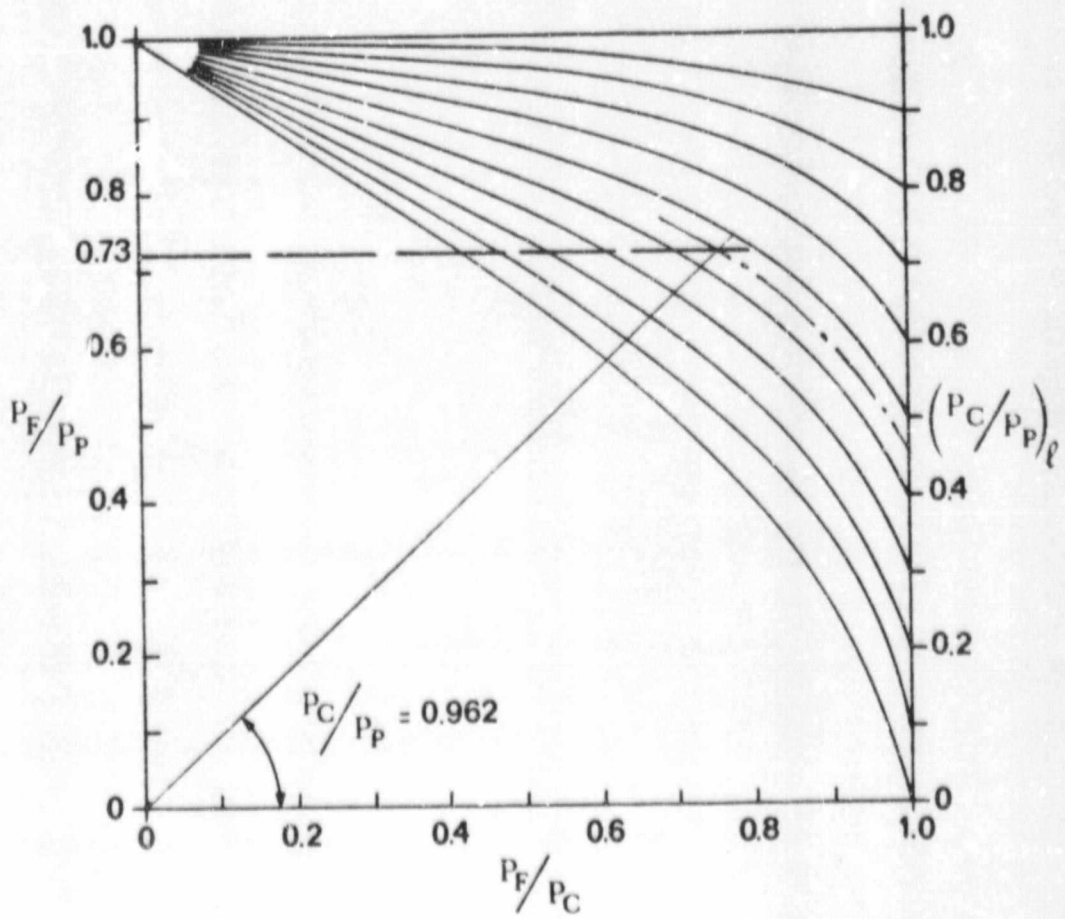


FIGURE C4



CALCULATION OF LOADING POINTS TO SIMULATE  
A UNIFORMLY DISTRIBUTED LOAD

APPENDIX D

The loading pattern illustrated in figure D.1 is used to simulate a uniformly distributed load applied to the rafters.

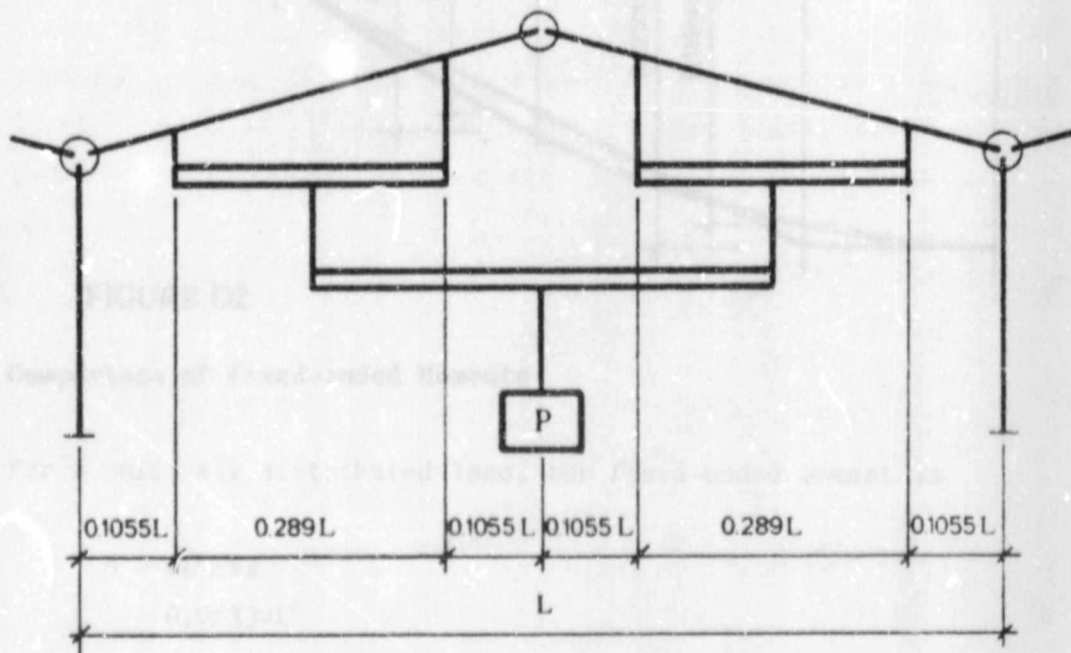


FIGURE D1

In the above diagram:

$P$  = total load on the span

=  $w \times L$

$w$  = equivalent uniformly distributed load.

A comparison of the free bending moment diagram, and fixed-ended moments will show that the loading illustrated accurately simulates a uniformly distributed

**Comparison of Free Bending Moment Diagrams**

Figure D.2 gives a comparison between bending moment under a uniformly distributed load and under the loading used in the laboratory tests.

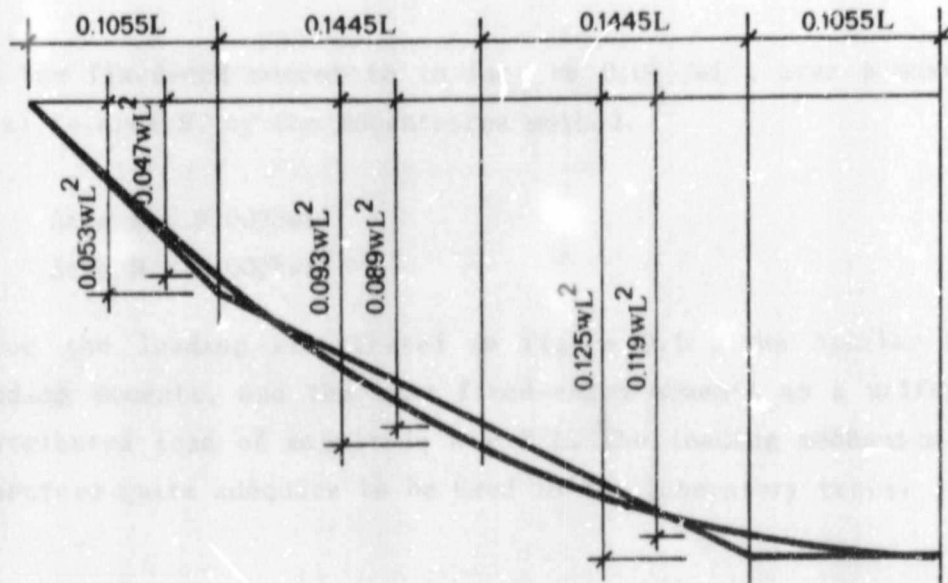


FIGURE D2

### Comparison of Fixed-ended Moments

For a uniformly distributed load, the fixed-ended moment is

$$\begin{aligned}
 M &= wL^2/12 \\
 &= 0,0833wL^2
 \end{aligned}$$

Assuming that the point loads illustrated in figure D.1 also yield a fixed-ended moment of  $0,0833wL^2$ , the bending moment diagram shown in figure D.3 is obtained.

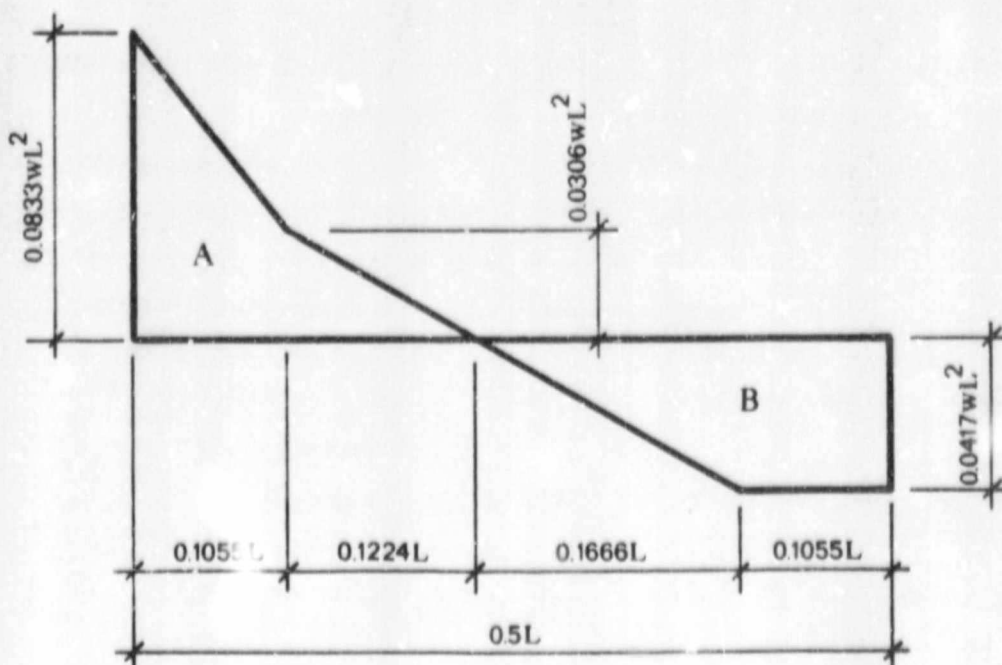


FIGURE D3

For the fixed-end moment to in fact be  $0.0833wL^2$ , area A must be equal to area B, by the moment-area method.

$$\text{Area A} = 0,0078wL^2$$

$$\text{Area B} = 0,0078wL^2$$

Hence the loading illustrated in figure D.1 gives similar free bending moments, and the same fixed-ended moments as a uniformly distributed load of magnitude  $w = P/L$ . The loading mechanism was therefore quite adequate to be used in the laboratory tests.

CALCULATION OF THE RATIOS OF  $P_{c \text{ snap}}/P_p$  AND  
 $P_{c \text{ sway}}/P_p$ , USING THE APPROXIMATE ENERGY  
 METHOD AND HORNE'S ASSUMPTIONS

## APPENDIX E

The equation used to calculate the ratio of  $P_c/P_p$  is:

$$\frac{P_c}{P_p} = \frac{R_i \Delta}{L_i \beta_i^2} \quad (1)$$

- $P_c$  = Elastic buckling load of the frame  
 $P_p$  = Plastic collapse load  
 $R_i$  = Axial load in member at plastic collapse (newtons)  
 $L_i$  = Length of each member (millimetres)  
 $Q$  = Applied external forces which initiate the required mode of failure i.e. snap-through or sway  
 $\Delta$  = Elastic displacement as a result of  $Q$ , at the point of application of  $Q$ .  
 $\beta_i$  = Elastic rotation of each member as a result of  $Q$ .

The axial forces at plastic collapse are found by undertaking a simple plastic analysis. Values of  $\beta$  and  $\Delta$  are found by undertaking an elastic analysis of the frame with  $Q$  as the applied load. Values of  $R$ ,  $L$ ,  $Q$ ,  $\beta$  and  $\Delta$  are given in Table E1. Also illustrated is the calculation of  $P_{c \text{ sway}}/P_p$  for the case all bays equally loaded, bases pinned. Other ratios of  $P_c/P_p$  were calculated in the same way.

In calculating the ratios of  $P_c/P_p$ , the following simplifications were made. These are the same simplifications Horne used in his research of sway and snap-through stability.

- \* First order stiffnesses, i.e.  $EI/L$  were used. The stiffness reducing effect of axial forces was therefore ignored.
- \* Axial rafter forces were neglected entirely in the calculation of the  $P_{c \text{ sway}}/P_p$  ratios.
- \* Average axial rafter forces based on the undeformed

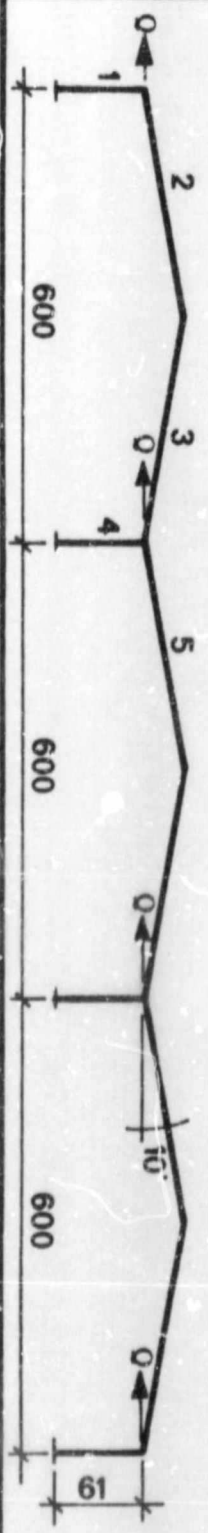
structure were used in calculating the  $P_{c \text{ snap}}/P_p$  values.

- \* The effect of rafter shortening, as a result of the frames deflection under the applied load, was not considered.
- \* Axial rafter forces at the elastic critical buckling load were assumed to be proportional to those at rigid-plastic collapse.

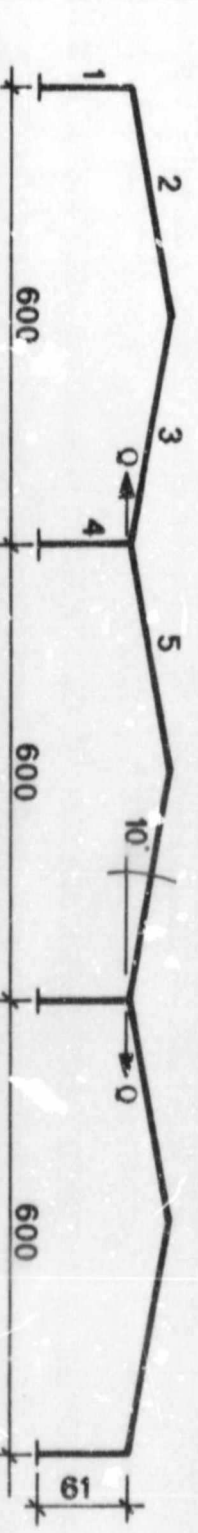


TABLE E1 : CALCULATION OF  $P_{c\ snap/P}$  and  $P_{c\ sway/P}$  RATIOS OF THE FRAMES TESTED IN THE LABORATORY

ALL BAYS EQUALLY LOADED, ALL BASES PINNED

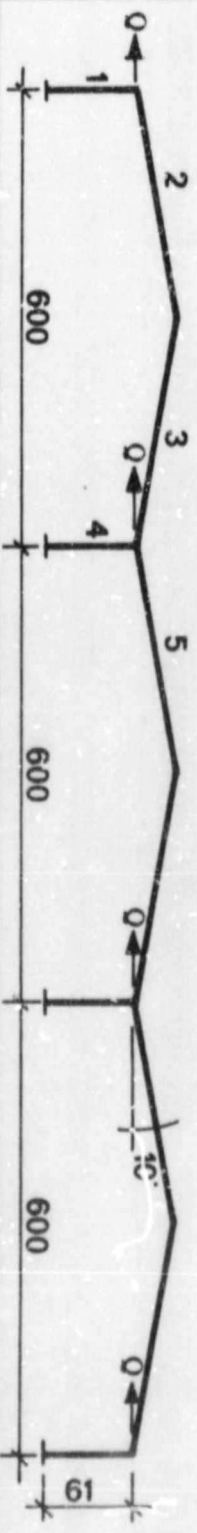


Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	P4	R5	$P_{c\ sway/P}$
2000	15.85	15.34	0.26	-	-	0.251	-	742	-	-	1454	-	$0.9[2000[15.05+15.34]]/[742 \times 61 \times 0.26^2 + 1494 \times 61 \times 0.251^2] = 6.34$

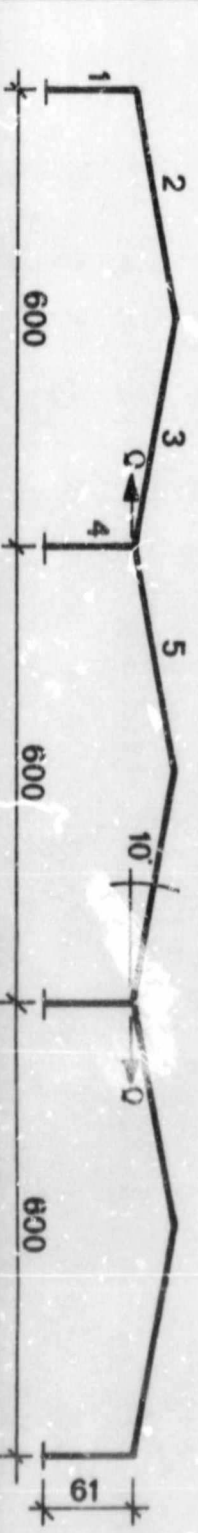


Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_{c\ snap/P}$
2000	1.30	1.81	0.021	0.005	0.005	0.030	0.034	742	739	739	1494	739	8.69

ALL BAYS EQUALLY LOADED, ALL BASES FIXED



Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_{c\ sway/P}$
2000	2.36	2.22	0.039	-	-	0.036	-	938	-	-	1875	-	34.76

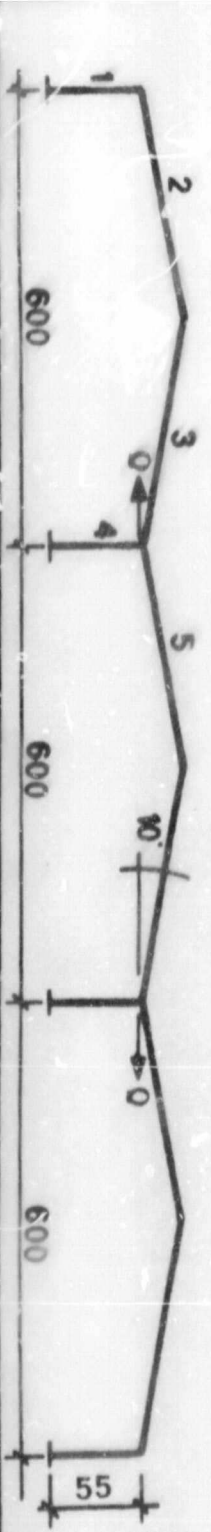


Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_{c\ snap/P}$
2000	0.33	0.84	0.005	0.005	0.005	0.014	0.016	938	1430	1430	1875	2023	7.63

BAYS UNEQUALLY LOADED, ALL BASES PINNED

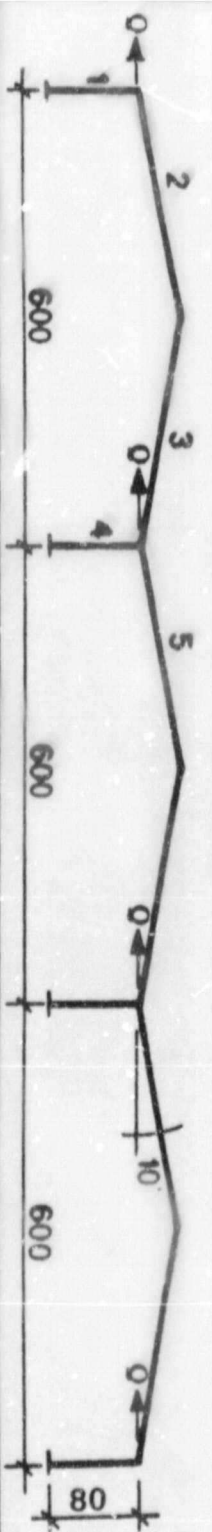


Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_c \text{ sway/Pd}$
2000	12.68	12.11	0.231	-	-	0.220	-	695	-	-	1516	-	7.35

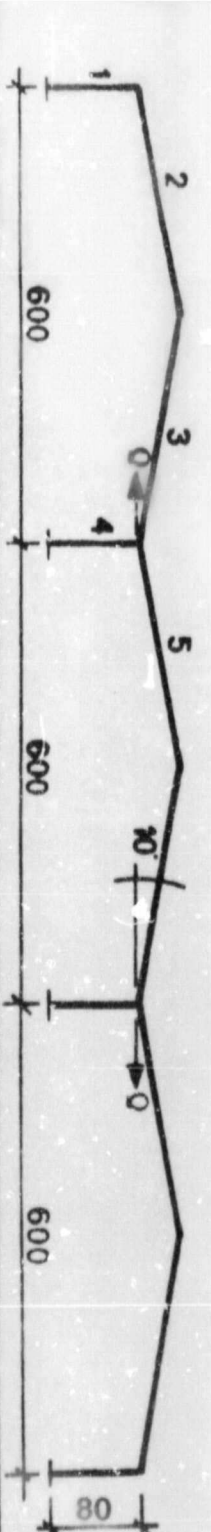


Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_c \text{ sway/Pd}$
2000	1.16	1.63	0.021	0.005	0.005	0.030	0.031	695	804	804	1516	1372	5.91

BAYS UNEQUALLY LOADED, ALL BASES FIXED



Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_c \text{ sway/Pd}$
2000	4.85	4.68	0.061	-	-	0.059	-	791	-	-	1788	-	23.73



Q (N)	$\Delta_1$ (mm)	$\Delta_4$ (mm)	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	R1	R2	R3	R4	R5	$P_c \text{ sway/Pd}$
2000	0.64	1.27	0.008	0.006	0.006	0.016	0.024	791	1093	1093	1788	1925	5.70

EXAMPLE OF THE INTERACTION EQUATION IN  
BS5950, TO PREVENT MEMBER INSTABILITY<sup>1</sup>

APPENDIX F

To prevent individual member instability, the following interaction equation must be applied to critical members within a frame.

$$\frac{m.M_x}{Max} + \frac{m.M_y}{May} \leq 1 \quad \dots\dots(1)$$

- $m$  = uniform moment factor obtained from Table 18 in BS5950  
 $M_x$  = applied moment about the major axis at the critical section  
 $M_y$  = applied moment about the minor axis at the critical section  
 $Max$  = maximum allowable buckling moment about the major axis in the presence of axial load  
 $May$  = maximum allowable buckling moment about the minor axis in the presence of axial load

Since all frames tested in the laboratory were bent about the minor axis only, the interaction equation reduces to:

$$\frac{m.M_y}{May} \leq 1 \quad \dots\dots(2)$$

$May$  is calculated using equation (3)

$$May = M_{cy} \left( \frac{1 - R/R_{cy}}{1 + 0,5R/R_{cy}} \right) \quad \dots\dots(3)$$

- $R$  = applied axial load in the critical section  
 $R_{cy}$  = compressive resistance of the section about the minor axis  
 $M_{cy}$  = plastic moment capacity of the section calculated from clause 4.2.5 or 4.2.6 in BS5950.

Since the case all bays equally loaded all bases pinned, was considered in the other appendices, the interaction equation will be applied to this frame. The bending moment diagram under the

rigid-plastic collapse load is given in figure F.1.

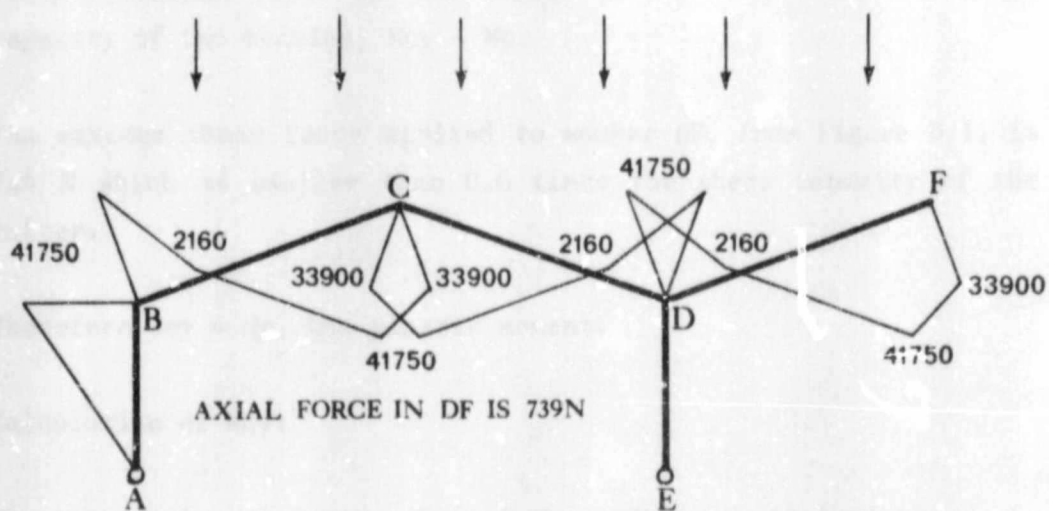


FIGURE F1

The critical member in the frame is DF as illustrated. The axial force in this member is 739 N.

#### Calculation of the Numerator in Equation (2)

From Table 18 of BS5950,  $m = 0,43$

Since the greatest moment in the critical section is  $M_p$  the numerator of equation (2) is  $0,43M_p$

#### Calculation of the Denominator in Equation (2)

Calculation of  $M_{cy}$ :

The value of  $M_{cy}$  depends on the shear capacity of the section.

$$\text{Shear capacity} = 0.6 \times \sigma_y \times A_v$$

$$A_v = 0.9 A_g = 0.9 \times 20 \times 5 = 90 \text{mm}^2$$

Hence shear capacity is:

$$0.6 \times 334 \times 90 = 18036 \text{ N}$$

If the maximum shear in the rafter is  $< 0.6$  times the shear capacity of the section;  $M_{cy} = M_p$ .

The maximum shear force applied to member DF, from figure D.1, is 746 N which is smaller than 0.6 times the shear capacity of the rafter.

Therefore  $M_{cy} = M_p$ , the plastic moment.

Calculation of  $R_{cy}$ :

The compressive resistance  $R_{cy}$  of the rafter is obtained from:

$$R_{cy} = A_g \cdot \sigma_{cy}$$

$\sigma_{cy}$  = allowable compressive stress of the section about the minor axis.

$\sigma_{cy}$  depends on the yield stress  $\sigma_y$  and the slenderness of the member. Assuming an effective length factor of 1, table 27(b) of BS5950 gives for a yield stress of 334 N/mm<sup>2</sup>:

$$\sigma_{cy} = 41.5 \text{ N/mm}^2$$

Therefore  $R_{cy}$  is:

$$100 \times 41.5 = 4150 \text{ N}$$

Substituting  $R = 739 \text{ N}$ ,  $R_{cy} = 4150 \text{ N}$  and  $M_{cy} = M_p$  into equation (3) gives for  $M_{ay}$ :

$$M_{ay} = 0.755 M_p$$

Substituting  $M_{ay}$  into equation (2) gives:

$$\frac{0.43 M_p}{0.755 M_p} = 0.57 \quad \therefore \quad \text{O.K.}$$

From the interaction equation it appears that member instability is not a problem. This type of failure should not occur in the given frame.

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