Volatility Interconnectedness among Real Estate and Other Assets in the BRICS Countries

by

Katlego Violet Kola

Ph.D. Thesis

submitted to
University of the Witwatersrand
in the partial fulfilment of the requirements for the degree of
Doctor of Philosophy in the subject area of Econometrics

School of Construction Economics & Management
Johannesburg, 2050
South Africa

UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG

2021

Table of Contents

Declaration

I declare that this Ph.D. Thesis is my own, unaided work. It is submitted in partial fulfilment of the degree of Doctor of Philosophy in the subject area of Real Estate Finance at the University of the Witwatersrand, Johannesburg, South Africa. It has not been submitted before for any degree or examination in any other university.

Date:
16th of May 2022

Signature of Candidate:

Katlego Violet Kola (461696)

Acknowledgements

It is a genuine pleasure to express my deep sense of thanks and gratitude to my mentor, Tumellano Sebehela in the continuous support of my Ph.D. studies. His dedication and keen interest above his attitude to help his students has been greatly beneficial and responsible for the completion of my work.

A special thanks to my friends and family’s continuous support and encouragement in these very intensive academic years.

Preface

Parts of this thesis have been accepted or submitted for publication as follows:

P1: Some parts of chapters 3, 4 and 5 are published under the title "The Interdependence of Indexed Volatilities" (with Tumellano Sebehela). *Linear and Non-Linear Financial Econometrics-Theory & Practice*, chapter 7, Mehmet K Terzioğlu eds, IntechOpen Publishers, 2020, 127-154, invited.

P2: Some parts of chapters 3, 4 and 5 have been published under the title "The Capitalized Generalised Autoregressive Conditional Heteroscedasticity" (with Tumellano Sebehela). *Review of Pacific Basin Financial Markets & Policies*, 2022 (in press).

P3: Some parts of chapters 3, 4 and 5 have been published under the title "The (De)merits of using Integral Transforms in Predicting Structural Break Points" (with Tumellano Sebehela). *The International Real Estate Review*, 24(3), 405-467.

Some parts of this work were presented at the following conferences:

C1: The 27th Pacific Basin Finance, Economics, Accounting & Management Conference (National Taiwan University, Taipei, Taiwan), June 15th to 16th, 2019. Presented article entitled "The Interdependence of Indexed Volatilities".

C2: The 17th African Finance Association Conference (University of Cape Town, Cape Town, South Africa), May 11th to 12th, 2021. Presented article entitled "Alternative Model for the Generalised Heteroscedasticity of Index Movements".

Dedication

I dedicate my dissertation work to my family. A special feeling of gratitude to my parents, Thamaga and Rebecca Kola, whose words of encouragement and push for tenacity are always guiding me.

List of Tables

No table of figures entries found.

List of Figures

No table of figures entries found.

List of Abbreviations

ADF	Augmented Dickey-Fuller test
ADF-GLS	ADF test modified by Elliott, Rothenberg and Stock (1992)
ARCH	Autoregressive Conditional Heteroscedasticity
BRICS	Brazil, Russia, India, China and South Africa nation
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GFC	Global Financial Cries
GNP	Gross National Product
REIT	Real Estate Investment Trust
REOC	Real Estate Operating Company
PP	Phillips-Perron (1998) test
SA	South Africa
VAR	Vector Autoregressive
VaR	Value-at-Risk
VTT	Volatility Transfer Theory
UK	United Kingdom
U.S.	United States of America
ZA	Zivot-Andrews (1992) test

Abstract

The relationship between capital markets and geographic regional organisations has been of interest of late. One such geographical organisation is the Brazil, Russia, India, China and South Africa (from here, BRICS) nation, which was “formed” in 2010. The BRICS nation is underpinned by three traits; (i) government is very central to driving the economy, (ii) ruling elites stay in power for a longer period and (iii) government tends to combine free market and socialism. Prior studies explored the economic impact of the BRICS nation; however, it seems no study has explored volatility spillovers of the BRICS nation emanating from indices (bonds, commodities, equities and listed real estate). This thesis studies volatility spillovers of those indices; firstly, using the VAR model and validating with the Markov-Regime Switching Model. The first set of results illustrates that spillovers are non-directional and interestingly, illiquid indices converge faster than liquid indices. The second hypothesis finds that the GARCH(1,1) model has shortcomings, such as not accounting for (i) correlation coefficients of debt and equity, (ii) equity parameter, (iii) risk premium, (iv) interest rates and (v) shocks-stock markets. Then, the capitalised GARCH(1,1) is “created” to address the shortcomings of the GARCH(1,1). The results and validation of Hypothesis Two illustrate that the capitalised GARCH(1,1) outperforms and is more robust than the GARCH(1,1). Generally, structural breaks are central to risk management. Hypothesis Three addresses the latter statement by using integral transforms (i.e. Fourier and Laplace) and unit root tests (Augmented Dickey Fuller, Augmented Dickey Fuller-GLS, Phillip-Perron and Zivot-Andrews). The third hypothesis confirms that derivative structural break points precede algebraic structural break points. Similarly, unit root tests confirm the presence of structural break points. Interestingly, integral transforms confirm the presence of dozens and dozens of evident and hidden structural break points. The findings of this thesis should apply to any geographic organisation with similar characteristics to the BRICS nation. The main contributions from this study is that (i) volatility spillovers are non-directional in nature, and can be in-between or across transatlantic indices, (ii) discrete volatilities such GARCH(1,1) can be extended such it accounts for debt and equity composition and (iii) finally, there are evident and hidden structural break points. Each investment vehicle or product needs its own tailored risk management models because volatility models need to be improved from time to time. Finally, in order to detect structural break points, more advanced techniques such as integral transforms should be used in conjunction with other common diagnostic structural break tests.

Keywords: GARCH, integral transforms, Markov-regime switching, structural break points, VAR(1,1), volatility spillovers

JEL: C32, C33, C35, C58, G12

Chapter 1 Introduction ot the Thesis

1.1 Background to the Study

The formation of organisations that represents countries with similar interests or likeminded goals can be traced back many decades ago. Some of those organisations are continentally focused (i.e. African Union, former Organisation of African Unity in 1963 and European Union (from here, EU) in 1958-its original roots), while others are global (i.e. United Nations in 1945). Recently, we have seen organisations that are transatlantic; Brazil, Russia, India, China and the South African (BRICS) nation. South Africa (SA) joined BRIC countries in 2009 through invitation by other member states, while the four founding members originate from a term coined by Jim O'Neil (former Managing Partner of Goldman Sachs). While the origination of the BRIC term at the time was influenced by the economic similarities that collectively represented 15% of the Gross National Product (GNP) of the United States (U.S.), Japan, Germany, Britain, France and Italy, there are other interesting similarities within BRIC countries (Armijo, 2007).

The similarities of BRICS nations are (i) political structure-ruling parties stay in power for at least 10 years without much challenge; although, we have recently seen the rising of opposition parties or citizens, (ii) country governance-ruling elites combine free market and socialism-i.e. mixed economies (privatisation of governed owned entities is extremely rare) and (iii) economic policies-ruling parties champion economic direction and by extension, the economies of the countries (Nayyar 2016). Those three traits have a strong influence on the capital markets of those countries and form the basis to investigate sharp changes in market interdependence and associated risks. For this proposal, the special focus is on volatility spillovers among real estate and other assets (bonds, commodities, and equities) in the BRICS organisation.

The concept where volatility moves into a specific direction is commonly associated with the Volatility Transfer Theory (VTT). Roll (1984) illustrated that VTT is affected by market microstructure-it is the process where prices of assets are determined and one takes into account various parameters that affect those prices. Subsequent to VTT, numerous volatility concepts emerged, including how volatility is transferred over time. Fundamentally, VTT argues that as one becomes familiar with a firm, the volatility of that firm decreases due to a decrease in information asymmetry. By extension, the volatility risk should decrease. However, some scholars argue that

VTT does not hold in every situation (see; Peng *et al.* 2007). This is the main framework that this proposal will investigate.

Global investors continue to consistently pursue attractive investment classes to allocate to their portfolio on alternative investments. Understanding the functionality of asset classes in markets such as BRICS becomes imperative to investors, portfolio managers and policymakers. Where volatility spillover effects are seen to spread from one market to another, due to market crashes or adverse events, critical implications of portfolio allocation, hedging, diversification, and risk control can arise (Syriopoulos *et al*. 2015). For instance, previous literature documents volatility spillovers from (i) developed to emerging markets, (ii) within the same and similar capital market instruments and (iii) traditional asset multi-asset classes (stocks, bonds and money markets), not including real estate (Leachman and Francis 1996, Christiansen 2007, Dean *et al.* 2010, Garcia and Tsafack 2011, Erhmann *et al.* 2011, Zhou *et al.* 2012, Ding *et al.* 2014, Kuttu 2014 and Gamba-Santamaria *et al.* 2017). The inclusion of real estate into a portfolio of bonds, commodities, and equities grounds the investment strategy to apply to an alternative investment environment.

The Black Monday of October 1987, the U.S. born global financial crisis of 2008 and 2009, as well as the European debt crisis that occurred in late 2009, are known as some of the few financial crises in the past three decades that have resulted in the volatility of financial markets and further resulted in a widespread international crisis. These are known as co-movements of financial markets, defined as volatility spillovers from one market to another. Volatility spillover studies have come to the vanguard as they are largely associated with risks that have implications on optimal portfolio construction, financial stability and implementation of policies that may render harmful shock transmissions in financial markets. Recent studies that address the issue of volatility dynamics indicate that volatility spillover effects amongst countries or financial markets are time-varying, most importantly during times of crisis. This has particularly significant consequences for investors and policymakers. Consequently, understanding the changing aspects of volatility spillovers is imperative

Volatility spillovers are well documented across and within different traditional asset classes (i.e. stocks, bonds and money market instruments, especially cash). On the practical side, specifically among alternative asset classes, there are virtually no studies on volatility spillovers. This is the main gap that this article fills in. To put the research gap broadly, prior studies used share prices of different firms in different asset classes to explore volatility spillovers. Generally, those studies focused mainly on two assets, especially bonds and equities (see; Lee *et al.* 2008). Secondly, the countries used in the analysis are from the same continental region and/or emerging economy types. Thus, no study so far has looked at volatility spillovers emanating from bonds, commodities, equities and real estate (alternative investments) from transatlantic countries, specifically from indices. This study focuses on the BRICS countries. The main characteristics of the BRICS countries are how (i) political parties, (ii) country governance and (iii) mixed policies affect the capital markets. Thirdly, those studies used mainly univariate as opposed to multivariate models. Thus, this study explores a portfolio of volatilities based on four indices from the BRICS countries.

Fourth, this study uses integral transforms to illustrate structural break points-with this technique structural break points are many and they are interchangeable and interconnected. By extension, one illustrates the derivative and algebraic structural break points. In a 10-year data set, most studies show a few structural break points-at most 2 points. For this study, data is from 2007 to 2020-a 13 year period, while most studies use a 10 year period. Lastly, those studies never distinguished between in-sample and out-of-sample. This study takes into consideration the latter point. The last point, is the extension of the Generalised Autoregressive Conditional Heteroscedasticity GARCH(1,1) model into capitalised GARCH(1,1). The latter GARCH model is an integrated one where (i) equity component of capital structure, (ii) risk premium and (iii) interest rate contribute to discrete volatilities calculated.

1.2 Characteristics of the BRICS Nations

1.2.1 Economic Systems of the BRICS Nations

The relationship between capital markets and economic growth has been an area of long debate. As a principal, economic growth arises from the accumulation of capital or efficiency in the use of resources, whereas efficiency in the use of resources is largely influenced by economic policies and financial institutions (Gavin 2006). Previous studies argued that economic policies have an impact on capital markets. Particularly economic policies that lead to financial repression, such as negative real interest rates, lead to lower savings and in turn, result in lower investment and lower growth (Mckinnon 1973, and Shaw 1973). In this section, we look at the impact of countries implementing macroeconomic policies, particularly ones that affect financial liberalisation, for capital markets to fully benefit from and promote economic growth.

(a) Brazil

Brazil's economy experienced one of the highest growth rates in the world from 1920 to 1980. The country's strategy was based on the import substitution industrialisation policy that focused on restricting imports with domestic production. This was post-Brazil's economy that was affected by the world depression that resulted in a decrease in Brazil's import capacity. Brazil experienced high growth rates as the strategy was focused on (i) high tariffs and quantity restrictions on foreign goods, (ii) long-run credit at low interest rates for locals, (iii) state intervention through state-owned enterprises in high earning sectors and (iv) fiscal incentives for direct foreign investments (Barbosa 1998).

Post-1980, the Brazilian economy was affected by the world recession which resulted in an external debt crisis. Creditors began enforcing hard terms of debt settlement against debtors that forced developing countries to adjust economic policies to enable the full servicing of debt. In their negotiation of the debt settlement, amongst the terms of negotiation, Brazil's settlement solution included bridge loans and servicing debt from their international reserves. Despite its efforts, failure of the debt servicing resulted in government creditors proposing reductions in external debt (Dalto 2019). To counter the period of stagnation, the government implemented deregulation measures through income policies, increase in public service tariffs, the renegotiation of debts, the reform of the operation of the government bond and overnight finance markets, trade liberalisation, privatisation and aborted tax reforms (Valença 1998).

With the new government in place from 2019, structural, fiscal, and regulatory, trade liberalisation and privatisation reforms were introduced. Central changes were made on fiscal and monetary policies, as well as the exchange rate policy. These reforms allowed the federal government to address public spending issues, control inflation and promote a better environment for businesses and job creation (Dalto 2019).

(b) Russia

Smirnov (2015) studied the economic fluctuations of Russia from the 1920s to 2015. The interesting part is that Smirnov (2015) disentangled the economic fluctuations during the regional formation of the Union of Soviet Socialist Republics (USSR), all the way until the Russian Federation became a stand-alone country. Currently, USSR is pretty much the major part of Eastern Europe. Although USSR is dissimilar to the BRICS nations, one should be able to pick up some insights about the collapse of major regional nations. Furthermore, he argues that the evolution of regional nations should give insights into how financial risks are transferred in between and across different countries during the evolution of regional nations. Most of the data is from the Central Statistical Administration. Key issues noted by the author are that between 1936 and 1956, significant Eastern countries cut ties with the USSR. The notable points are that during 1920-2015, the Russian economic growth was largely driven by industrial production, agricultural products, residential construction, rail freight transportation and crude oil prices. Note that some periods were characterised by declines, while other periods by growths. Generally, declines lasted for approximately ten years from the date of the occurrence. Moreover, declines occurred during subprime crises, government infightings and perceived political interference in managing the economy. Fundamentally, Smirnov (2015) illustrated that a free market economy cannot solve all political, social and economic mayhems of Russia. This leads to the question of whether there is any room for government in growing the economy.

Mau (2018) responds indirectly to the latter question. Mau (2018) focused specifically on the key challenges when the Russian economy was contracting. He builds on from 2008-the “assumed” period of the start of the last major subprime crisis led by the U.S. economy. First, he presented three points about the economy. Firstly, developing economies such as China drove the world out of the subprime crisis. Secondly, observed growth rates were moderate-similar to ones after 1965. Thirdly, there were no V-shaped curves in financial markets that are associated with equities. Remember, most market commentators argue that the 2008/9 subprime crisis was induced by unaffordable residential housing problems in the U.S. At the same time, the world has gone through major globalisation and it has affected different people differently. Since the economic growth has been very minimal, some countries have sort out to protectionism. It can be inferred from Adewale (2017) that economic protectionism is evident among the BRICS countries.

Among the most contentious point in Russia is the relationship between monetary and budgetary methods. The role of monetary discipline is spearheaded by central banks, while the National Treasuries oversee fiscal discipline. Mau (2018) opined that the problem is not monetary policy but how far and when are reforms. In the BRICS, reforms occur at a snail’s pace. One thing that is positive about the BRICS nations is that central banks follow solely inflation targeting as a primary tool for monetary discipline. However, some Western countries such as the U.S. and New Zealand follow inflation targeting, in conjunction with supporting economic growth. On the latter point, by extension "influencing" employment levels. For those Western countries, which follow inflation targeting in conjunction with economic growth, the results have been phenomenal. To put it more precisely, the latter strategy includes government intervention. Due to different players, this government-private model leads to multipolar views, according to Mau (2018). Broadly, he argues that Keynesian economics has led to economic growth, but there is poor sharing of economic rewards. For example, since 1994, the South African economy has grown but the majority of people are still trapped in poverty. One of the discouraging things is that non-and semi-skilled compensation amounts have decreased indeed. Based on recent World Bank reports, SA is the most unequal society in the world and before then, it was Brazil. Interestingly, both countries are part of the BRICS nations. Although not explicitly stated, Mau (2018) argues that economic problems cannot be solved dogmatically, but there is a role for everyone-political and private institutions. Fundamentally, Mau (2018) argues that there is a link between political governance and financial markets. That, according to him, creates room for political governance in financial markets. "Incidentally, the problem of growth, as the experience of developed countries shows, in principle cannot be resolved exclusively by macroeconomic manipulations", Mau (2018:102).

(c) India

India's economic reforms began in 1991 after experiencing a balance of payment crisis, caused by macro-economic disparities, poor productive economic performance, rising inflation, and low foreign exchange reserves (Anand 1999). Prior to this, India's economic status could be classified into two phases. Between 1947 and 1974, India could be classified as a state-controlled and closed economy that did not promote economic growth. Particularly, this phase was a result of the anti-colonial struggle that resulted in India opposing trade liberalisation and gravitating towards a closed economy (Mukherji 2014).

The second phase was initiated between 1975 and 1990, where India allowed private enterprises, but still in a closed economy due to the previous policies that had resulted in a stagnant economy and poor human development. In this era, significant liberalisation measures were made that promoted economic growth with a mixture of socialism. However, these policies failed to engage with the global economy and India was unable to manage capital and change the economy from import towards export-led growth. One of the views that the policy failed in was that India failed to take advantage of debt-free foreign direct investment and as a result, government expenditure began exceeding its revenue (Mukherji 2014).

Post-1991, India's economic reform was geared towards more economic growth and globalisation and less of a socialist approach. India's first approach concerned a loan with the International Monetary Fund (IMF) to facilitate India's transition into a more private sector-led, open and competitive market-oriented economy. Furthermore, India implemented tighter monetary policies, import compression and the reduction of the fiscal deficit, which led to a quick recovery from its macroeconomic crisis (Wadhva 2000).

(d) China

China's economic reforms began in 1979. Prior to this, China's economic policies were considered poor, stagnant, centrally controlled, inefficient and mostly state-controlled. China has since then applied economic reforms that promote free-market policies that have made China one of the biggest economies. This was achieved by China through, manufacturing, merchandise trading and holding of foreign exchange reserves. After China's GDP began to slow down, China was set on implementing a new economic model that was less focused on merchandise trading and fixed investments and more on free-market principles, instead of state-planned (Morrison 2019).

China began implementing economic reforms that focused on rapid economic growth and capital investments. China implemented programmes such as the "Made in China" project. This plan proposed strategic goals that aimed at improving manufacturing innovation, integrating information technology and industry, strengthening the industrial base, promoting breakthroughs in key sectors and internationalising manufacturing (Mau 2018). Additionally, China's stock market was established in the early 1990s, when China was in the process of privatising state-owned ownerships to use stock market pressures to improve state-owned companies. Additionally, China loosened its monetary policies to increase bank lending to counter the effects of the Global Financial Crisis GFC (Morrison 2019).

China's current economic policies include more control of state-owned and private enterprises. Other reforms that were made particularly were on "internal and external circulation". While there are opposing opinions, under the internal circulation policy, the Chinese elaborate the policy as creating a large domestic market and demand, but while not dismissing internal-external interaction (Grieger 2020).

(e) South Africa

SA's transition into democracy has impacted the strategy around macro-economic development and has had mixed results. The government, in their strategy, was initially focused on social and physical infrastructure as growth drivers and followed with gearing growth, which involved private-sector investment through the Macroeconomic Research Group (MERG). Two years later, macroeconomic policies were introduced in 1996 to the government as a strategy to promote EAR. Particularly on targeting economic growth, the government introduced policies that were aimed at reducing fiscal deficits, lowering inflation, maintaining exchange rate stability, decreasing barriers to trade and liberalising capital flows.

Various schools of thought exist on SA's economic policy shifts, some that are surrounded by exogenous and endogenous factors. Globalisation has been the driving exogenous factor that has affected economic policies, where neo-liberal-based international institutions have driven the process and local businesses to be under pressure for being supporters of free-market forces (Nkadimeng 1999). On endogenous factors, Khamfula (2004) investigated the current economic policies employed in SA and their impact on economic growth. The study indicates that it is unclear whether or not real economic growth is more responsive to monetary or fiscal shocks. However, the study notes that the current economic policies have not produced desirable results and as a result, the government needs to take caution when making changes to both the monetary and fiscal policies.

In 2013, the SA government introduced the national development plan (NDP-2030) that was aimed at eradicating poverty and reducing inequality. Amongst its objectives, the development plan identified key constraints to faster growth and a plan for a more inclusive economy. The objectives for policymaking inherent in the NDP include (i) regulating and incentivising innovation, (ii) capitalising on recent trends and enhancing the capacity to resolve conflicts and (iii) addressing challenges at a global level (Chilenga 2017).

1.2.2 Political Economy and Capital Markets Linkages

For capital markets to work, political institutions need to be in support of the capitalism of financial markets. However, there are political factors that can affect the interaction between politics and financial markets. These political factors can range from the state's ownership of listed firms, the state's control of the banking sector or regulation of capital market activity, risk of expropriation, the extent of government intervention in business activities, corruption, cronyism and political connections, and the prevailing incentives of local politicians (Piotroski 2012). In this section, we examine political systems, and by extension, their impacts on capital market activity.

Haggard *et al.* 2008 studied the relationship between the rule of law and economic development. Note that in their study, economic development is implicitly tied with actual growth in the GDP, corporate sector, jobs and financial markets. And financial markets are largely underpinned by secondary capital markets and most secondary capital markets products were issued, initially in the primary capital markets. Therefore, economic development is inextricably linked to actual growth. The authors argue that for a country to attract investments, especially foreign direct investments, the core issues that should be addressed are property rights, contracts and economic growth. According to the authors, Western countries do enjoy positive property rights, contracting and economic growth. According to the authors, Western countries do enjoy positive investment climates because the points raised in the latter sentence, show positive signs. Note that the best stock exchanges include New York Stock Exchange, London Stock Exchange, Tokyo Stock Exchange and Hong Kong Stock Exchange. The four stock exchanges are domiciled in investor-friendly countries. The importance of the rule is that it provides security-Haggard *et al.* 2008. Furthermore, if there is corruption and laws are ineffective in dealing with corruption, then, the parliament/congress can always amend laws upwards. Another key point raised by the authors is that many economists and political scientists prefer canonical institutional formulation. In the latter case, checks and balances tend to be in place. Broadly on legality, Haggard *et al.* 2008 state that there should be judicial independence, legal origins should serve and create consistencies in laws and should be anchored on the constitution of the country. Thus, if the country is anchored on those three points, then, it will attract investors, including foreign investors and by extension, there will be solid and attractive capital markets.

1.3 Substantiation of the Problem

This study explores volatility spillovers in BRICS countries in the context of alternative investment. That is, alternative investments involve investment in bonds, commodities, equities and real estate. For this study, seems real estate is listed. On one hand, the relationship between listed real estate and unlisted real estate is a mixed bag (see; Lee *et al.* 2008, Hoesli *et al.* 2008) and on the other hand, real estate is seen as a proxy for macroeconomic risks (see; Naranjo and Ling 1997). The macroeconomic risk proxy is also evident in other industries such as commodities. Deaton (1999) argues that macroeconomic risks are evident in the commodity industry. Moreover, diversification plays a part in influencing commodity prices. Listed real estate is either Real Estate Investment Trusts (REITs) or Real Estate Operating Companies (REOCs). Further, those studies illustrate that those effects are transatlantic. The reason why cash is not analysed in this article is that cash and money asset classes have been extensively researched. For example, approximately 60% of international trade is done on U.S. dollar and currency markets are the most liquid of all capital markets (see; Kumar *et al.* 2003). For this study, it is important to drive risk management strategies, especially when information is asymmetric.

1.4 Problem Statement

Prior studies confirm contradicting views on whether transatlantic volatility spillovers can be appropriately modelled, such that the risks emanating from such portfolios are minimised (Hoesli *et al.* 2008 and Syriopoulos *et al.* 2015). Some prior studies used univariate models (i.e ARCH and GARCH) to model volatility spillovers. However, the latter models do not account for clustering and interdependence of volatilities, especially when one has a portfolio of risk (i.e. volatility). In order to do away with limitations of univariate models, some scholars use multivariate models, such as the Vector Autoregressive (VAR) model. However, VAR has been largely used on prices of assets and a limited number of indices (i.e at most two indices), generally from the same region. Then the question is, how does VAR perform when you have at least two indices from transatlantic nations, such as the BRICS. Among the things that make volatility spillovers of diversified portfolios is the inclusion of real estate (Naranjo and Ling 1997). Therefore, the research problem is “how does one appropriately model volatility spillovers such that the risk is minimised”? The implications are as follows; (i) financial institutions, such as federal banks, might put forward wrong policies to mitigate against volatility changes, and (ii) investors might invest in undesired asset classes based incorrectly on calculated risk.

1.5 Primary Research Question

Chkili and Nguyen (2014) illustrated that there is a forex volatility risk among the BRICS countries. Moreover, univariate and multivariate (i.e. VAR) analysis show that results from the same data sample are indifferent. Therefore, the main research question is; what is the best way to model volatility movements among real estate, bonds, commodities and equities in the BRICS countries, such that volatility risk is minimised?

1.6 Secondary Research Questions

From the primary question, the secondary research questions are identified as:

- What are the parameters to include in GARCH, such that it is suitable for modelling volatilities bonds, commodities, equities, and real estate?
- How best to determine structural breaks of transatlantic volatility spillovers?

1.7 Research Aim

The research aim of this study is to "appropriately model volatility spillovers in the BRICS countries, such that volatility risk is minimised".

1.8 Research Objectives

From the aim the following research objectives are identified:

- To appropriately determine and model interdependent volatility spillovers of bonds, commodities, equities and real estate among the BRICS countries.
- To extend the GARCH(1,1) model, such that it can appropriately capture volatility risk of bonds, commodities, equities and real estate among the BRICS countries.
- To determine and appropriately model structural breaks of volatility spillovers in alternative investment classes.

1.9 Research Gap

Prior studies used share prices of different companies in different asset classes to explore volatility spillovers. Moreover, those studies focused mainly on two assets, especially bonds and equities (see; Lee *et al.* 2008). Secondly, the countries used in the analysis are from the same continental region and/or emerging economy types. Thus, no study so far has looked at volatility spillovers emanating from bonds, commodities, equities and real estate (alternative investments) from transatlantic countries. Thirdly, those studies used mainly univariate, as opposed to multivariate, models. Fourth, natural structural breaks were not taken into account in their data analysis. For this study, data is from the period of 2007 to 2020-a 13 year period. Most studies use a 10 year period. Lastly, those studies never distinguished between in-sample and out-of-sample. This study takes into consideration the latter point.

Chapter 2 Theories of Risk Management

2.1 Introduction to the Theoretical Framework

The fluctuation of a variable over time is an indication of volatility and the deviation of the variable from the expected value is often used to define volatility and level of risk. Risk, in its definition, is then considered as the chance that an actual return of an investment will be different from the expected return, including the ultimate risk of loss of all of one's original investment (Shukla and Kukreja, 2014).

Risk management thus becomes a crucial process used to make investment decisions. The process requires identifying and analysing the amount of risk involved to accept or mitigate the risk. Risk management strategies can be viewed in two broad categories; (i) risk decomposition and (ii) risk aggregation. Risk aggregation is associated with identifying and handling risks individually, whereas risk aggregation is associated with reducing risks through diversification (Hull, 2015).

2.1.1 Commonly Used Risk Measures

2.1.1.1 Value-at-Risk

The Value-at-Risk (VaR) is a risk management methodology that attempts to stipulate a single number that signifies the total risk in a portfolio and is widely used and accepted by corporate treasures, fund managers and financial institutions (Hull, 2015). The VaR methodology involves measuring the risk of a portfolio so that the potential maximum expected loss of the portfolio is represented for a given time horizon and a predefined confidence level (Trenca and Dezsi, 2011). The main advantage of VaR as a risk measure is its simplistic nature and it can be utilised to inform individual positions or large financial institutions.

The VaR methodology, however, has shortcomings as there are typical approaches to estimate the VaR; the Variance-Covariance Approach and historical simulation. The Variance-Covariance Approach is based on portfolio returns following a normal distribution and the VaR percentile is based on the standard deviation of a portfolio. However, the disadvantage of this approach is its inability to cater for non-linear portfolios, while it is known that financial returns tend to be leptokurtic and fat-tailed, leading to an underestimation or overestimation of the VaR (Xu, 2017).

Under the Historical Simulation Methodology, there are minimal parametric assumptions about the distributions of risk factors, beyond assuming that the distribution of changes in the value of today's portfolio can be simulated by making draws from a historical time series of past changes in risk factors (Pritsker, 2001). The principal disadvantage of this methodology is that it assigns an equal probability weight of each return for each day, and this is equivalent to assuming that risk factors are independently and identically distributed through time. This assumption becomes unrealistic as it is known that periods of high and low volatility tend to cluster together (Pritsker, 2001).

2.1.1.2 Monte Carlo Simulation

The Monte Carlo simulation operates by creating an algorithm that generates random numbers that are used to compute a formula that does not have an analytical form. Computing the Monte Carlo is similar to the Historical Simulation, the difference being that instead of using historical data for an asset and assuming that this data will re-occur in the next interval, it generates random numbers that will be used to estimate returns of an asset at the end of the horizon. The Monte Carlo has resolved issues relating to the Historical Simulation technique, however, the methodology itself has its shortfalls. When creating experiments with Monte Carlo, each market variable must be modelled according to an estimated distribution and the relationships between the distributions (correlation, non-linear relationships, etc.). Thus, to build and keep current a Monte Carlo risk management system requires continual re-estimation, a good reserve of analytic and statistical skills and non-automated decisions (Bohdalova, 2007).

2.1.1.3 Expected Shortfall

Expected shortfall (ES), sometimes referred to as conditional tail expectation, is an alternative risk measure that is sensitive to the shape of the tail of the loss distribution and dissimilar from the VaR, the ES is able to regard the average losses beyond the VaR level. An advantage of the ES is that it is regarded as a coherent measure that satisfies the condition of subadditivity (the risk measure for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged), whereas the VaR does not meet this condition (Yamai and Yoshiba, 2005).

The expected shortfall method has better properties than the VaR as it considers the benefits of diversification, however, one disadvantage is that it does not have the simplicity of the VaR and subsequently is more difficult to understand. Despite its advantages over the VaR, expected shortfall only relies on the lower value of the tail and thus has the potential to be conservative. Another drawback of the ES is related to back-testing. Back-testing refers to validating a given estimating procedure of a risk measure using historical data. In essence, in back-testing, the prediction is an entire distribution, but the realisation is a single scenario. ES becomes difficult to back-test, due to its definition-an average of losses in excess of a given VaR level (Hull, 2015).

Another drawback of the ES is the notion that it is not elicitable and thereby not back-testable. Elicitability allows measures to have a scoring function that makes the comparison of different models possible, leading to many conclusions that it would not be back-testable. Contrasting views have been found in literature on whether expected shortfall needs to be elicitable to be back-tested (Embrechts *et al.,* 2014, Acerbi and Szekely, 2014 and Wimmerstedt, 2015).

2.1.1.4 Beta

Beta is a risk measure that assesses volatility or systematic risk of a portfolio relative to the market benchmark index. The beta can be seen as advantageous in its usage. Firstly, the beta becomes imperative where several securities are placed within a portfolio for diversification purposes, the non-diversifiable risk can be known through the beta coefficients of the individual securities and market benchmark index. Secondly, under the Equilibrium Approach, the best coefficient is shown to be a constant of proportionality between the risk premium of the individual security and risk premium of the market, under the assumption that investors borrow and lend at the same rate (Blume, 1971).

In practice, measurement issues can often lead to wrong inferences of the beta. The choice of a market index, choice of time period and choice of a return interval can affect the calculation of the beta. In terms of the market index, in reality, no index can be regarded as the ultimate market portfolio, hence different “market” portfolios can yield different betas. In terms of the choice of return interval, risk models are often silent on how long a period one needs to estimate the beta, and hence periods ranging from two to five years can yield differing beta estimates. The final choice that can affect the beta estimate is the return interval, used in measuring returns historically, whereas returns can be measured daily, weekly, monthly, quarterly or annually. By using shorter intervals, it increases the number of observations in a regression, however assets do not trade on a continuous basis and the beta estimate can be affected (Domodran, 1999). In the REIT industry, specifically negative betas have led to misunderstanding and/or misinterpretation, and by extension, that led to incorrect execution strategies (Anderson *et al.* 2009).

2.1.1.5 Expectiles

Expectiles are an alternative risk measure suggested as an alternative to the VaR and expected shortfall. Under this class of risk measures, expectiles rely fundamentally on different information to measure risk, as they are reliant on both tails of the distribution, making it a more comprehensive risk measure. Unlike the VaR and expected shortfall, expectiles have been considered a unique class of risk measure, as they combine coherence and elicitability (Chen, 2018).

Essentially, expectiles can be regarded as a special class of quantiles, however, they are determined by tail expectations, rather than tail probabilities. Expectiles work on similar properties of gain/loss ratios of financial risk management and offer advantages over quantiles, such as specific moments of the distribution of returns. For instance, in their ratio, it considers all moments of the return distribution and that is imperative as financial regulation should comprise as much information from a probability distribution (Chen, 2018).

A draw-back of the risk measure is that unlike estimation of quantiles, which uses traditional order statistics in their estimation, expectiles use a non-explicit asymmetric least square estimator, which in return makes it more difficult to estimate tail expectiles (Daouia *et al*., 2020) and as such, direct use of expectiles as a risk measure has been discouraged by the expectiles' lack of interpretation (Anders, 2012).

2.1.3 Possible Risk Measures

Volatility can be modelled both under stochastic and discrete (i.e. non-stochastic) environments. In a stochastic environment, volatility is advantageous for pricing and calibration (Horvath et al. 2021) and under discrete environments; volatility has appropriate hedging capabilities (Carr and Wu, 2014). In the context of this thesis, volatility is used for risk management purposes; applying volatility in a discrete scenario. Note that despite the latter environment being non-stochastic, the stochastic behaviour is encultured into the discrete world. To put it differently, the discrete behaviour mimics the preceding stochastic behaviour of volatility. Here is the reason why. First, stocks trade at the opening minute of each stock exchange. Thereafter, the opening prices at the first minute move to the second-minute trading levels. That is at the second trading point, the effects in price from the first point and in-between the two points (i.e. first and second), gets aggregated at the second point. Volatilities work in the same manner as volatility measures changes in stock prices over time. This is the advantage of using volatility as a risk measure in this thesis- both stochastic and discrete behaviour are accounted for during the risk management process.

Chapter 3 Literature Review to the Hypotheses

3.1 The Interdependent Volatilities

3.1.1 Introduction to the Interdepedent Volatilities

Numerous studies illustrate that volatility spillovers are common in many capital market instruments. That concept is commonly known as the VTT. VTT is well documented across and within different traditional asset classes (i.e. stocks, bonds and money market instruments, especially cash). However, some scholars argue that VTT is theoretically not well documented (Bodart and Reding, 1999). On the practical side, specifically among alternative asset classes, there are virtually no studies on VTT. This is the main gap that this study fills in. In analysis, the study draws data on bonds, commodities, equities and listed real estate from the BRICS nations. The analysis is essentially empirical. Both empirical and theoretical studies offer little, if any, insight into how volatility spillovers behave and their effects on the BRICS countries. The closest study that explores this theme is Syriopoulos *et al.* 2015. Syriopoulos *et al.* 2015 use the multivariate GARCH and disaggregated VaR to traditional asset classes. This study goes beyond traditional asset classes and uses other models such as regime-switching models. Similarly to Syriopoulos *et al.* 2015, international diversification and risk management is central to volatility spillovers in BRICS countries.

According to the policy document that was written by European Parliament in April 2012, the role of BRICS is changing the economic landscape, especially within the emerging market economies. The BRICS received over $1 billion in foreign aid in 2012 alone and the population size of all the BRICS countries is over 1.5 billion, largely made up of working individuals. The population size on its own creates opportunities for a decent working force and increased consumption rate in those countries. Despite the challenges in those countries; the investment opportunities are massive, predominantly in the primary industries. During the same period, foreign direct investments in the BRICS totalled to $688.16 billion, according to the European Parliament. That according to diversity enhances a lot of secondary economic opportunities, including the development of capital markets. In Africa, South Africa has the best capital markets in terms of regulation and advancement of technologies. The Johannesburg Stock Exchange (JSE) is ranked among the top fifteen exchanges in the world. China is the second biggest economy in the world, after the United States. More, China has been moving at least 30 million individuals out of poverty every year for the last 20 years. Those massive economic opportunities raise questions about how best can investors, including fund managers, maximise their investment returns. The GDP of the BRICS, as at 2017, is about 20% of the world GDP. This study explores maximising returns of investment portfolios in the BRICS.

Prior studies on volatility spillovers focused on trading strategies largely within the same and similar capital markets instruments. Volatility and volatility transmission are usually explored on the basis of ARCH and GARCH (Hammoudeh *et al.* 2003). One such study is Fleming *et al.* 1998, which explored volatility linkages among stocks, bonds and money markets. The latter strategy is very common for traditional asset management. This study takes that concept further and includes listed real estate among other asset classes. When investment portfolios are expanded to include listed real estate, then a different kind of investment management phenomenon arises, alternative investment. Thus, this study explores volatility spillovers in the context of alternative investments. To one's knowledge, the latter phenomenon is not yet well identified and documented. Fleming *et al.* 1998 started by building a simple trading model, assuming an autoregressive regression (i.e. AR(1,1)) process. Then progressed to a Generalised Method of Moments (GMM) model. The results illustrate that new information in derivatives markets create market linkages. Within the trading environment, the results of Fleming *et al.* 1998 support using a simple trading model. Marcato *et al.* 2018 explored volatility smiles when information is lagged in the U.S. REIT industry. Using Black-Scholes (1973) (from here, B-S) model and targeted convergence level, Marcato *et al.* 2018 illustrated that there is volatility clustering in between REIT firms. The latter phenomenon leads to premium, which increases the value of options. Probably the same phenomenon will be observable, given that the sample includes listed real estate firms.

This study explores volatility spillovers in the context of alternative investments. For this study, real estate is listed because listed real estate is so different to unlisted real estate in so many ways; (i) performance of listed real estate is similar to other capital markets, especially bonds, (ii) capital markets products are sensitive to curvature and interest rate term structures, and (iii) default risks (See; Kester 1986, Sandahl and Sjögren 2003, and Morawski *et al.* 2008). Listed real estate is either REITs or REOCs. Further, those studies illustrate that those effects are trans-Atlantic. The reason why cash is not analysed in this study is that cash and money asset classes have been extensively researched. For example, over 60% of international trade is done on U.S. dollars and currency markets are the most liquid of all capital markets (see; Kumar *et al.* 2003). For this study, it is important to drive risk management strategies, especially when information is asymmetric.

Most prior studies explored volatility spillovers; first, they focused on developed countries, second, those studies are on specific asset classes and third, the strategies derived from those studies seem to apply to the trading environment only. In this study, volatility spillovers are across several asset classes, i.e. bonds, commodities, equities and listed real estate. When the four asset classes are put together, investment management strategies in this environment tend to apply to alternative investments. Thus, this study derives a risk minimisation strategy. Secondly, most prior studies use one econometric model to illustrate global financialisation, while this study uses two econometric models; (i) VAR and (ii) regime-switching models. Thirdly, the study is on BRICS countries and BRICS combined characteristics of first and third world countries. One foresees that strategies arising from this study will be appropriate for alternative investment funds, which have explored both first and third world countries. Next, real estate (listed and unlisted) is normally seen as a safe haven for investments (see; Ziobrowski *et al.* 1997). How true is it that real estate is a safe haven for investments, especially during tough market conditions? The results of this study apply to academics and practitioners.

A study similar to this study is one by Liow (2015). First, it analyses spillovers of four major asset classes (public real estate, general equity, currency and bond). This study focuses on the four asset classes, except for the fact that currency is replaced by commodities. Liow (2015) focuses on the period of 2007-2009, while this study uses the 2007-2017 period. Given the longer period for this study, one foresees more interesting results than the ones of Loiw (2015). For modelling, Liow (2015) uses VAR and GARCH(1,1) models. This uses models used in Liow (2015), plus the regime-switching model. Liow (2015) draws data from four continents; (i) Asia emerging countries, (ii) European emerging markets (iii) Latin American emerging countries and African emerging countries. Prior studies have illustrated that emerging countries have similar economic conditions; this study focuses on the BRICS countries. Other than being emerging countries, the BRICS are similar in the sense that ruling political elites stay in power for long periods (i.e. 15 years), more, those governments have come up with organisations that are most likely to compete with established institutions, i.e. the BRICS Bank is most likely to compete with the World Bank in future. Further, there is a close political will among the BRICS, which is not prevalent among all emerging countries. As volatility spillovers are driven by financial integration, liberalisation and crises contagion (see; Liow, 2015) among other factors, the former two factors are likely to be key drivers for volatility spillovers among the BRICS countries. So far, there has not been a major crisis contagion reported in any of the BRICS countries.

Another study that is close to this study is Diebold and Yilmaz (2011). They explore spills, specifically total and directional ones based on the generalised VAR model. In order to avoid Cholesky-factor identification as in one of their papers, they disentangled spills into (i) variance, directional and net spills. The data is on the U.S. asset classes: (i) stocks, (ii) bonds, (iii) foreign exchange and (iv) commodities. The results illustrate that spills were "to" and "from" directional spills. One possible explanation is that the U.S. market is the biggest financial market in the world. Secondly, there was cyclical behaviour in the spills. Some of the spills can be explained by panics in the stock market behaviours (i.e. July-August 2007: credit crunch, January-March 2008: panic in stock and forex markets, followed by unscheduled cuts in rates by the federal bank and first half of 2009: collapse in Lehman Brothers). Thirdly, volatility spillovers stayed positive during most of the financial crisis. The results of this study illustrated that there are diversification and risk management opportunities that arise from volatility spillovers of bonds, commodities, equities and real estate indices.

3.1.2 Literature Review on Interdependent Volatilities

In criticising the prior studies, this study divides the literature review as per the four asset classes; (iii) bonds, (i) commodities, (ii) equities, and (iv) listed real estate. In this way, specific traits of each asset class are disentangled.

3.1.2.1 Bonds

Skintzi and Refenes (2006) studied volatility dynamic linkages among the European bond markets, specifically eight Euro area countries (Austria, Belgium, France, Germany, Ireland, Italy, Netherlands, and Spain) and four non-Euro area countries (Denmark, Norway, Sweden and UK). They argue that at the heart of modelling volatility spillovers, investors want to know how to diversify internationally, price securities, and build asset allocation strategies. The latter statement is synonymous with this empirical study. Among the principles that were central to Skintzi and Refenes

(2006) is one by Darbar and Deb (2002)[1]. Darbar and Dep (2002) modelled volatility spillovers using time-varying conditional correlation. To model volatility spillovers Skintzi and Refenes (2006) extend EGARCH such that it is a bivariate EGARCH model. Although the exponential feature is suitable for movements in bonds, commodities, and equities, the same feature might not be suitable for listed real estate due to the lagging effect. Other parameters include that the model should include conditional mean, lagged standardised residuals, size effect, and conditional correlation effect. Further, dummy variables are included to account for spillover variables.

The empirical results of Skintzi and Refenes (2006) illustrate that volatility spillovers in the EU zone are time-varying. To validate their results, Skintzi and Refenes (2006) used the Ljung-Box test for serial correlation and Engle and (Ng (1993)[2] asymmetric tests. The Ljung-Box test illustrated that there is no evidence of linear or no-linear dependence in the standard residuals. More, the EGARCH model captured volatility spillovers adequately, except for Belgium and France. The volatility spillovers in the UK are dependent on their past values. In the case of Euro bond markets Belgium, France, Netherlands and Spain, there were significant price spillovers, while Belgium and France exhibited reciprocal price dynamics. The Netherlands was the only country in which volatility spillovers aggregated to the Euro bond market.

Steeley (2006) uses a two-factor-no-arbitrage model to illustrate the theoretical link between stock-bond volatility in the UK market. The argument that he puts forward is that those volatility spillovers seem to be transmitted around the global trading system. He focused on spillovers between bond and stock markets, based on short and long-term volatilities. Some of the prior studies have illustrated that short-term volatilities converge to their long-term averages. In modelling the spillovers, Steeley (2006) used GARCH(1,1) that can capture the leverage effect. Given that data in this study includes listed real estate, capturing the leverage effect is paramount. To improve the efficiency of GARCH(1,1), he extends the model by incorporating multivariate extension.

The data used by Steeley (2006) is on daily observations on the FTSE-100 Share Price Index (FT100) over 15 years, representing government stocks (FTSG) and stocks (FTLG). Short-term risk-free yields were obtained from the 20-year period of June 1984-June 2004. The results of Steeley (2006) illustrate that returns on short-term and long-term bond returns were positive. The FTSE 100 experienced growth over the same period. The long-term variance was persistent in all series due to clustering effects. For all the series, there was significant order of first-order correlation, but

[1] Darbar SM and Dep P (2002). Cross-Market Correlations and Transmission of Information. *Journal of Futures Markets*, 22(11), 1059-1082.
[2] Engle RF and Ng VK (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance*, 48(5), 1749-1778.

autocorrelation decays fairly rapidly in all cases. The influence of the past long yield was evident. The correlation between stock and bond markets was negative. This is probably because of diversification effects (see, Anderson et al. 2009).

Kim *et al.* (2006) examined the dynamic relationship between bonds and stock in European countries that adopted the Euro currency (i.e. France, Germany, Italy and Spain), non-Euro countries that included the UK, and Japan and the U.S. because they are two major financial markets. Within the Eurozone, financial integration was spearheaded by the European Monetary Union (EMU). They argue that even though volatility spillovers were investigated, the co-movements in between two different asset classes were largely undocumented. On the other hand, there was practical evidence that illustrated that volatility reaction in one asset class causes movements in another asset class (see; Syriopoulos *et al.* 2015). The theoretical background on time-varying stock-bond return co-movements is based on two principles, (i) common information influencing different asset classes, and (ii) expectations in one asset class alters expectations in another asset class. The two principles fit well with volatility spillovers in the BRICS.

To deepen their analysis, Kim *et al.* 2006 used explanatory variables that included nominal short-term interest rates, inflation, real short-term interest rates, size of the trade sector, intra-regional trade integration, correlations in output, term structure and dividend yields. To account for international financial integration, they used the Chicago Board of Options Exchange (CBOE)'s Volatility Index (VIX) and the German DAX Equity Index (VDAX). The model used for the analysis is the univariate GARCH(1,1).

The preliminary results of Kim *et al.* 2006 illustrated that there are significant variations in conditional correlations of stocks and bonds during a specific period. For the global bond market, the intra-bond integration in the EU zone was on an upward trajectory. The stock market returns displayed the presence of asymmetric information, while bonds did not display the same phenomenon. The conditional variances of the two asset classes appear unrelated. EMU financial integration was evident, especially for the UK market. Finally, Kim *et al.* 20006 illustrated that diversification benefits were evident if volatility spillovers were properly mitigated.

Dean *et al.* 2010 explored volatility spillovers and return between equity and bond markets for Australia from 1992 to 2006. They argue that volatility spillovers are important for diverse purposes; (i) asset allocation, (ii) portfolio management, (iii) financial risk management, and (iv) capital market regulation. In this study, volatility spillovers are important, largely for financial risk management. Among confirmed concepts on volatility spillovers (i) hedging demands increase with price changes, (ii) positive news increases stock prices, while prices fall when the discount rate rises. Normally, an Asymmetric Price Adjustment Hypothesis (APAH) states that bad news affects bonds and stocks equally, while good news affects bonds and stocks unequally. For modelling, they used a joint process of conditional means, asymmetric Baba, Engle, Kraft and Kroner (BEKK) model, Dynamic Conditional Correlation (DCC) model and bivariate GARCH model.

The data sample is on Australian equity and government bond markets, and the equity index was on 500 companies listed on the Australian Stock Exchange. The preliminary results of Dean *et al.* 2010 illustrate that equity volatility is lowest when returns of both markets are positive, and the highest equity (bond) returns are negative (positive). More, when equity returns are negative, conditional correlation is stronger. As expected, the distribution of returns is skewed and leptokurtic. Bond (equity) markets seem to react predominantly to negative (positive) news, more than positive (negative) ones. When the bond shock moves from a negative angle to the positive side, then the equity variance surface tilts. Most volatility spillovers for equities are evident when returns are negative and vice versa for bonds. None of the used models were fully able to explain observed spillovers.

Garcia and Tsafack (2011) explored co-movements of volatilities in the international equity and bond markets. They argue that genitive returns are more common and dependent than positive returns in international equity markets. In investigating volatility spillovers, Garcia and Tsafack (2011) took into account the issue of fat tails. The data presents the dependence between two leading markets in North America (U.S. and Canada), and two major markets of the Eurozone (France and Germany). The U.S. equity index is based on the S&P500 Index and Canadian equity index returns are based on DataStream Index. The bond series are from 5-year government bond indices. The statistical tools used are exceedance correlation, Extreme Value Theory (EVT) to capture fat tails and Gaussian bivariate GARCH or regime-switching models, specifically M-GARCH because of its ability to capture many variables. Copulas are used to increase the ability to capture asymmetric dependence.

The preliminary results of Garcia and Tsafack (2011) show that there is a large, extreme dependence in international equity and bond markets while bond-equity dependence has a negative effect. The latter statement encourages international diversification and switching from equities into domestic bonds. Historically, the correlation between Canadian equity and bond markets has been relatively high. Further, results show that asymmetric regimes of dependence and negative shocks are more likely to be transmitted to other markets than positive shocks. After the introduction of the Euro, France and Germany became more dependent. Broadly, high volatilities are associated with asymmetric dependence.

3.1.2.2 Commodities

Jacks *et al.* 2011 investigated volatility spillovers in commodity markets since 1700. They argue that some authors raised questions like; have commodity prices been more volatile than manufacturing ones, any secular trend since 1700 and the relationship between globalisation and commodity volatilities. However, none of the scholars have addressed those questions using a long-term series indeed. For poor countries, Jacks *et al.* 2011 argue that volatilities for those countries should be high because those countries specialise in agriculture and mineral production. The data used in Jacks *et al.* 2011 is for the world and various trends are outlined during specific periods. This is to consolidate reasons that drove commodity prices during those periods. They calculated log prices for their study and used Dickey-Fuller and Phillips-Perron tests to validate their illustrated volatilities. The Prebisch-Singer hypothesis was central to their analysis.

Preliminary results of Jacks *et al.* 2011 show that volatilities among different commodities are different. In poor countries, volatilities tend to be higher because those countries are dependent on agriculture and mineral production. Sauerbeck-Statist shows no evidence of secular patterns from 1800 onwards. Further analysis illustrates that the French and American Revolutionary Wars, the Napoleonic Wars and the War of 1812 contributed to increases in volatilities. To test the robustness of their results, Jacks *et al.* 2011 used the GARCH(1,1) model and GARCH(1,1) confirmed that the results are robust. Seasonality also played a role in driving higher volatilities.

Antonakakis and Kizys (2015) investigated dynamic spills between commodity and currency markets. The authors argue that precious metals (gold, silver, platinum and palladium) have been seen as a safe haven during the financial crises. Further, they state that the inclusion of precious metals in equity portfolios decreases systematic risk of investments; therefore, diversification accrues in those investments. Their research is centred on these questions; (i) how time-varying spills differ among commodity and currency markets, and (ii) what is the relationship between returns and volatilities during financial transmission. In answering those questions, Antonakakis and Kizys

(2015) use the spillover index which is performed by using rolling-window Forecast Error Variance Decomposition (FEVD) by transmitters and receivers of shocks.

The weekly data in Antonakakis and Kizys (2015) is made up of the spot prices of the four precious metals, crude oil spot prices, euro (EUR/USD), Japanese yen (JPY/USD), British pound (GBP/USD) and the Swiss franc (CHF/USD) spot exchange rate, each versus the U.S. dollar. They use weekly data to synchronise data and error elimination (see; Antonakakis and Kizys, 2015). The period of the data is from January 6th, 1987 to July 22nd, 2014, totalling 1438 observations. The preliminary analysis of data illustrates that volatilities increased dramatically, especially from 2000/2001 period for the precious metals and oil, while currencies volatility decreased from 2000/2001 onwards. Moreover, preliminary analysis shows that spot prices are positively skewed, except for GBP/USD and CHF/USD. The absolute returns (volatility) for all parameters are positively skewed. And the Jarque-Berra tests confirm the non-normality of distributions. Further analysis includes using the VAR model to illustrate return transmission across all the parameters. The results of the VAR model illustrate volatility spillovers across all variables. The total spillovers index indicates a 42.41% average contribution. Most transmissions were from gold, followed by silver and then platinum. Crude oil had the lowest transmissions. On the other hand, crude oil's demand is linked to four commodities, as for production of those metals, crude oil is used. One of the notable things about Antonakakis and Kizys (2015) is that negative skewness has higher probabilities. Normally, the opposite should be true because positive skewness is more risk than negative skewness. For all variables, the curves are positively skewed and leptokurtic. The latter statement would imply that price spreads are significantly probably due to high volatilities. According to Antonakakis and Kizys (2015), volatilities in commodity and currency markets are likely to occur during less volatile episodes.

Basak and Pavlova (2016) modelled financialisation for commodities markets. According to the authors, prior studies have documented index and non-index commodities; however, the theory of financialisation, which has far-reaching implications, had limited synthetisation. The latter point is central to the study of Basak and Pavlova (2016). The main variables that were analysed in the study are (i) commodity supply shocks, (ii) commodity demand shocks, and (iii) (endogenous) changes in wealth shares of the two investor classes. The theoretical model that they built is a closed-form. Fundamentally, Basak and Pavlova (2016) argue that value assets pay off more in high-index states.

In the building model, they assumed that the model followed a Brownian Motion (BM). The model included a parameter that signals the arrival of news, supplies news of uncorrelated commodities, distinguishes between index and non-index commodities, and the inventors were accounted for; (i) normal investors and (ii) institutional investors. Moreover, the equilibrium effects of the financialisation of commodities were accounted for. Centrally to the last statement, instead of the model behaving like a trading model, it behaved like one for normal investors. Other equilibrium issues included (i) equilibrium commodity futures prices shaped on corollary, (ii) futures volatilities and correlations, and (iii) economy with demand shocks. Further, the illustrated commodity prices and inventories. For the commodity prices and inventories, they (i) incorporating storage where additional economic agents (i.e. consumers and firms) were added, (ii) equilibrium commodity prices and inventories. The second proposition is on how the discount factor is affected by institutional inventors. And finally, (iii) cross-commodity spillovers and the import of income shocks. The latter proposition is about how institutional demand increases for all assets are positively correlated with the index, especially the demand for commodity storage.

The results for Basak and Pavlova (2016) illustrated that those volatilities in futures markets do spillover into other commodities. Further, there is a trade-off between investors due to relative performance fluctuations. The latter phenomenon is consistent with what is illustrated by VIX Volatility Index (Basak and Pavlova, 2016). In addition, the model information is ‘asymmetric and investors have the same beliefs’.

Le Pen and Sévi (2017) investigated excess co-movements of commodity prices in developed (118 variables from Australia, Canada, France, Germany, Japan, the UK and the U.S.) and emerging markets (6 variables from China, Brazil, Taiwan, Mexico, etc.). They argue that prior studies illustrate that financialisation in the commodities markets led to excess price volatility. One possible reason for that is that commodities, especially of currency nature such as gold, are characterised by spikes in prices. However, not every price of every commodity is spiky. Central to their investigation is that (i) co-movements imply that ‘demands and supplies are affected by unobserved forecasts of the economic variable’ and (ii) portfolio management strategies are affected by co-movements. The latter phenomenon resonates with this study. The variables that Le Pen and Sévi (2017) investigated are (i) the U.S. index of industrial production, (ii) Consumer Price Index (CPI), (iii) effective $US exchange rate, (iv) three-month Treasury bill interest rate, (v) M1 monetary measure and (vi) S&P500 stock index.

For the commodity prices, they used wheat, copper, silver, soybeans, raw sugar, cotton, crude oil and live cattle. Further, arbitrariness and computational difficulties should be minimised. One of the ways how to avoid arbitrariness and computational difficulties is to use Principal Component Analysis (PCA) and stepwise regression, although stepwise is time-consuming when one uses many variables. In their analysis, Le Pen and Sévi (2017) focused on filtering commodity returns using large approximate factor models. And for that Le Pen and Sévi (2017) used (i) the static factor model and (ii) ARCH-LM for illustrating spillovers and (iii) the SUR model to test whether residuals are unrelated.

The preliminary analysis of Le Pen and Sévi (2017) reveal that the skewness of all commodities is positive, except wheat, which is negatively skewed. Thus, wheat should have higher volatilities than the rest of the commodities. And the Jarque-Berra test confirms non-normality for all commodities. The latter illustration is consistent with other studies on commodities. The correlation matrix shows that all commodities are correlated with one another, except with live cattle. That is, live cattle, when compared with the seven commodities, might offer diversification benefits. The results of returns show that crude oil and copper are costly correlated with variables of emerging markets. Monetary measures have more influence in emerging markets than developed countries. When they test for excess co-movement of commodity returns, results exemplify that commodity co-movements are common and influencing across all markets. Moreover, those co-movements are sampling dependent. Le Pen and Sévi (2017) state that given that speculation is rife in commodity markets, some co-movements might be driven by speculation. The OLS model confirms the presence of endogeneity.

3.1.2.3 Equities

BenSaïda *et al.* 2018 note that during tranquil times there are particular countries that are net transmitters of risk and others are net receivers of risk in global financial markets. The study particularly analyses the global financial shifts of volatility spillovers by employing the Diebold and Yilmaz (2011) forecast-error variance decomposition and incorporating a Markov switching framework that considers economic regime changes, into the generalised VAR model. The study uses the following daily stock market volatility indices as proxies of market risk; the VIX (S&P 500 volatility, U.S.), VFTSE (FTSE 100 volatility, U.K). VCAC (CAC 40 volatility, France), VDAX

(DAX 30 volatility, Germany). VAEX (AEX 25 volatility Netherlands), VSMI (SMI 20 volatility, Switzerland), VHSI (HIS 50 volatility, Hong Kong) and JNIV (Nikkei 225 volatility, Japan) for the period of 2001 to 2017. The results of the study support the theory of shock transmissions and volatility spillovers by finding that all markets are more intense and are at the frequent risk of shock transmission and reception during turbulent times.

Gamba-Santamaria *et al.* 2017 provide empirical evidence of stock market volatility spillover shifts in the U.S. and four Latin American Countries (Brazil, Chile, Colombia and Mexico). The study employs the Dynamic Conditional Correlation GARCH (DCC-GARCH), a variation of a multivariate GARCH model. The study confirms volatility spillovers in these markets and further confirms a higher volatility spillover index during the period of the beginning of the global financial crisis in 2008 to 2012. Mensi *et al.* 2018 find similar results when they investigate the volatility spillover in global and regional stock markets, as well as those of Greece, Ireland, Portugal, Spain and Italy (GIPSI) from the period 2002 to 2016. The GIPSI countries were the most affected by the Eurozone Sovereign debt crisis and more affected by the global financial crisis than other countries in the Eurozone. The study utilises an asymmetric DCC-GARCH model and finds evidence of strong volatility spillover effects amongst the markets and an increase in the volatility spillover levels during the period of both the global financial crises and the Eurozone Sovereign debt crisis.

Syriopoulos *et al.* 2015 propose a more effective way of better understanding efficient asset pricing, volatility forecasting, efficient cross-market allocation and hedging decisions along with optimal international portfolio strategies, is through understanding the stock market dynamics and volatility spillover effects of listed asset sectors individually, in particular markets. Several works of literature have focused on volatility spillovers in financial markets on a global, regional and country level. This section particularly focuses on volatility spillovers amongst equity stocks in financial markets. Cross-market volatility linkages in global developed equity markets attracted much attention in research. An earlier study by Leachman and Francis (1996) studied the return volatility dynamics and transmission among the G-7 countries' equity markets, using both the GARCH and VAR models. They find that while in these markets, domestic market shocks are the largest single source of domestic volatility variation for other markets, (apart from the UK and the U.S.) shocks to foreign markets account for a significant portion of domestic market volatility. The study provides empirical evidence of volatility spillover effects in the equity markets of these industrialised countries.

Xu *et al.* 2018 find similar results when modelling both illiquidity and volatility spillover effects through a Multiplicative Error Model (MEM) for 8 developed equity markets of the U.S., Canada, UK, Germany, France, Japan, Hong Kong and Australia from 2007 to 2016. The results also indicate that volatility spillovers in these equity markets for this period had significant changes due to the global financial crises.

Ding *et al.* 2014 analyse both implied and realised volatility linkages through a rolling correlation analysis across global equity markets. This covers the U.S., European, German, Japanese, and Swiss markets during the sample period of 1999 to 2009. Implied volatility indices provide information regarding market investors' expectations of the uncertainty of stock price movements in the future. Using the VAR method, the study indicates that both unconditional and conditional correlations for implied and realised volatility exhibit large fluctuations during that sample period. These results coincide with market fluctuations that occurred during the period of the global financial crisis.

Zhou *et al.* 2012 indicated that the U.S. was found to be the dominant transmitter of volatility during the global financial crises when investigating the volatility dynamics between China and the world equity markets (New York, London, France, Germany, Japan, India, Korea and Singapore) from 1996 to 2009. Based on Diebold and Yilmaz's (2011) forecast-error variance decompositions in a generalised VAR framework, the study found a positive impact from the Chinese equity market to other markets from 2005; however, the major correction of the Chinese market contributed to increased volatility in other markets from 2007. The study further finds that the Chinese equity market was not significantly affected by the global financial crisis.

On another standpoint, Syriopoulos *et al.* (2015) identify volatility spillover effects on a sectoral basis (industrial and financial sectors) from the U.S., as a developed country, to BRICS nations, as emerging markets, using a VAR(1)-GARCH(1,1) framework. In the industrial sector, overall results indicate that the volatility transmission from the U.S. predominantly affects Brazil, Russia and India, while in the financial sector, it predominantly affects Brazil and Russia. Dewandaru *et al.* 2017 also indicate the volatility impact from developed markets by looking at regional spillovers across transitioning emerging markets and frontier equity markets, particularly in the Middle East and Africa together, with the U.S. as the developed market. The study examines the stock markets of Saudi Arabia, UAE, South Africa and Israel from the period of 1994 to 2010 using a multi-time scale analysis using wavelet-based time and frequency distribution compositions. The study finds that the Middle Eastern countries were more susceptible to the U.S. subprime crisis as compared to South Africa, however indications of short-term shocks that produced additional vulnerability in the South African equity market before the global financial crisis are noted, which could have potentially been due to investor sentiment.

Despite the increased studies of volatility spillover analyses from developed to emerging markets, there continues to be limited cross-market studies that are undertaken in equity markets of emerging nations. The possible integration of emerging markets continues to be of great concern as theory suggests that expected returns might be expected to reduce, following a greater integration of emerging markets in the world economy. Kuttu (2014) contributes to the empirical literature of volatility spillover dynamics between equity markets by examining the returns and volatility dynamics of Ghana, Kenya, Nigeria and South Africa for the period of 2005 to 2010. The study employs a multivariate VAR-EGARCH framework and finds that Nigeria is dominant in volatility transmission to Ghana, Kenya and South Africa, while it is not a receiver of volatility from these markets. The study however found that the domestic volatility indices of these markets are the highest coefficients for all these markets, which implies that domestic shocks may impact these markets more than external shocks.

Gilenko and Fedorova (2014) analyse the external as well as the internal spillover effects for BRICS countries for the period of 2003 to 2012 using a multivariate GARCH-in-mean approach. The external spillover effects considered the developed markets of the U.S., Germany and Japan as well as the Morgan Stanley Equity Index that covers 21 markets that are currently classified as emerging markets. Amongst the BRICS markets, volatility spillovers are largely present before the global financial crises and no presence of volatility spillovers are found thereafter, however, the correlations of the BRICS nations return almost doubles in increase during this period. The study also finds strong positive influences from the external emerging markets on BRICS markets. The study however is limited to cross-market volatility spillovers only in the equity market; cross-asset volatility spillovers in those markets have not been identified as yet.

Liow (2015) investigates cross-market volatility in eight emerging markets (China, Taiwan, Malaysia, Philippines, Thailand, Argentina, Hungary and South Africa) and contributes to the body of knowledge by investigating cross-asset volatility spillovers (real estate, general equity, currency and bonds) from 2007 to 2010. This study highlights a new perspective that provides insights into the probable changing market interdependencies that may exist amongst these emerging markets by using a multi-asset class VAR model across these markets. The study finds that most of the total volatility spillover is largely due to domestic factors in these markets, however, consistent with the literature, volatility is pronounced during the global financial crises and the European debt crisis. The study further concludes that emerging markets have become more open, interactive and endogenous around the crisis periods supporting the importance of volatility spillovers investigations between the financial markets of emerging markets.

The consensus emerging from the literature on asset co-movements is that asset markets are linked internationally, and volatility is transmitted from one market to another. Earlier studies of market linkages were habitually focused on developed countries, however, due to the financial liberalisation and trade openness of emerging economies, research has also focused on investigating cross-border links in emerging economies from developed countries. Emerging markets have increasingly played an important role in financial markets and were not spared from the impact of the global financial crisis. A better understanding of how emerging markets respond to exogenous shocks can assist investors and portfolio managers to better understand if there are any diversification possibilities.

3.1.2.4 Listed Real Estate

The co-movement of real estate stocks and financial markets has been studied extensively. Previous literature has documented the theory that low correlation of an asset with other capital markets, international and domestic portfolios provides the opportunity for risk reduction and diversification in an investment (Liow *et al.* 2011). Liow and Schindler (2014) investigated the local, regional and global linkage of securitised real estate and stock markets and possible integration in nine developed markets from the three regions of North America (The U.S.), Europe (Germany, France, Netherlands and the UK) and Asia-Pacific (Japan, Hong Kong, Singapore and Australia) in the period 1990 to 2011.

The study focuses on four elements; the comparison of time-varying conditional correlation, the cross-asset market causality of returns and volatility spillovers and the long-term integration relationship, as well as return convergence between the real estate markets and stock markets as a group. The study uses a GJR-DCC method as used by Glosten *et al.* 1993 to highlight the time-varying co-movements, the residuals are then used to test the Causality-in-Mean (CIM) and Causality-in Variance (CIV) to examine the amount of spillover between public real estate and stock markets. Lastly, the study implements the Recursive Akdogan Score Analysis as a measure of long-term integration between real estate and stock markets and the Principal Component Analysis to analyse the convergence of the stock market as a group.

Time-varying conditional relationship is reported in the literature, however on a global level; real estate markets are only moderately correlated with global stock markets and could potentially provide some diversification benefits in the short run. Lead-lag linkages are found in the return and volatility spillovers but are however weaker than the contemporaneous linkages at local, regional and global levels. Another important finding was that real estate slowly becomes more integrated with the global and regional markets, and to a lesser extent with the local stock markets. Dynamic return correlations and conditional volatility spillover effects are investigated by Liow (2014) with a select group that includes real estate markets from Greater China (China, Taiwan and Hong Kong), Asian emerging markets (Malaysia, Philippines and Thailand) and the developed markets of the U.S. and Japan from 1999 to 2013.

The study employs the spillover index of Diebold and Yilmaz (2011) that produces variance decompositions that are insensitive to variable ordering by allowing correlated shocks and the historically observed distribution of the errors to account for the shocks. The spillover index is further based on a multivariate VAR that can capture market fluctuation of more than two countries concurrently, rather than bivariate models. Liow (2014) finds evidence of the following: (i) time-varying return co-movement and volatility spillovers in all markets and positive association with the global financial crisis (ii) a bi-directional and regime-dependant relationship of cross-volatility spillover effects, (iii) synchronisation between co-movements of volatility spillovers and correlation spillover cycles. Liow (2012) studies time-varying co-movements of Asian real estate and the linkages of local, regional and global stock markets throughout 1995 to 2009. Correlations of assets are interpreted to indicate co-movement and integration across financial markets. The integration of markets is also interpreted to indicate interdependence of markets, which can lead to transmission crises.

The author demonstrates the study through an Asymmetric Dynamic Conditional Correlation (ADCC) model, also a specific class of multivariate GARCH models. The author finds time-varying conditional real estate-stock correlations at local, regional and global stock markets and some asymmetry and real estate-global stock correlations are impacted significantly by volatilities at local, regional and global levels. In this period, the author also finds that real estate and stock volatilities are more substantial in influencing co-variances more than correlations during and post the global financial crises. Hoesli and Reka (2013) provide evidence on a national and international basis by investigating volatility spillovers between the U.S. and the UK real estate market, The U.S. and Australian real estate market as a national analysis and the U.S equity and real estate markets as an international analysis. The period of the study extends from 1990 to 2010 and the volatility spillovers are studied using the covariance matrix of the asymmetric t-BEKK (Baba-Engle-Kraft-Kroner) specification. On a national basis, the U.S is the net transmitter of volatility spillovers; this can be expected as the subprime crisis originated in the U.S. On an international basis, the three markets have more influence on the volatility of the global market than the reverse, indicating the importance of these developed markets. Nikbakht *et al*. 2016 contribute to this evidence by providing evidence of volatility spillovers for the pre-crisis (1999-2007) and post-crisis (2007-2011) period from the U.S to the European real estate market but none vice versa.

Lin (2013) focuses on the examination of volatility spillovers in the Asian markets of Taiwan, Japan, Malaysia, Singapore, Hong Kong and South Korea. Volatility spillovers are examined between REITs and stocks, as well as REITs and bonds through the GARCH(1,1) model. The study finds that a boom in stock markets could be able to decrease return volatility in all the six REITs markets, except for Malaysia and a positive spillover effect from bonds could potentially increase the conditional volatility of REITs in Taiwan, Malaysia and South Korea. Liow and Ibrahim (2010) take a different approach on volatility spillovers through studying volatility decomposition and correlation analysis that considers a transitory and permanent component of the real estate and stock market volatility series. Through the C-GARCH model, the study can capture the short-run and long-run volatility dynamics which are important to understand as volatility is related to information flow and provides insight into real estate asset prices. Liow and Ibrahim (2010) cover the 12 securitised real estate markets of Australia, Canada, France, Germany, Hong Kong, Netherlands, Singapore, Sweden, Switzerland, the UK and the U.S. from 1984 to 2006.

Both transitory and permanent trends are found to be significant and persistent in the volatility process, where transitory trends are more affected by external shocks. The real estate transitory and permanent volatility movements are not similar, even after common factors are considered; suggesting domestic factors need to be considered in their analysis. Lastly, the study finds significant volatility co-movement in the transitory and permanent trends of real estate and stock markets, therefore suggesting the stock markets and real estate markets are not segmented at an international level.

Liow and Ye (2017) employ both univariate and multivariate switching regime beta models from 2000 to 2015 to illustrate regime-dependent excess return distribution and volatility spillovers pre and post the global financial crises. The study examines the developed markets of the U.S., the UK, France, Germany, Australia, Japan, Hong Kong and Singapore and their linkages with the world stock market and world real estate markets. The study uses switching regime models to allow for different economic conditions, as well as to capture the changes in the stochastic volatility process driving the real estate markets. The study reports a higher volatility parameter in response to the global financial crises, as compared to the "normal" period. The real estate market linkages with the world market were affected differently by the global financial crises, however, they were amplified post-crises, particularly for the European region, while the Asian real estate markets displayed reduced volatility spillovers, with world markets in a low volatility state, post-crises.

Regime changes are associated with significant shifts in the fundamental relation between the risks and return trade-off and the probability that a switch can be initiated from one regime to another (Liow *et al.* 2005). Liow *et al.* 2011 incorporate multiple regime changes by modelling the return-volatility transmissions of real estate through the Multivariate Regime-Dependent Asymmetric Dynamic Covariance (MRDADC) model. They study the real estate markets of the U.S., the UK, Japan, Hong Kong and Singapore from 1990 to 2009. Firstly, the study finds that asymmetry, variance and covariance, associated with multiple regime changes, jointly influence return-volatility transmission in real estate markets and secondly the study finds that the five markets generally interact well with one another by finding significant mean-volatility linkages under different volatility regimes. Consequently, this has implications on the diversification benefits that these markets can offer.

3.1.2.5 Literature related to hypothesis one

Models used before in determining volatility spillovers illustrate a mixed-bag type of results' although; there are some things that are commendable including persistency of the results. However, so far, risk management is better captured by discrete models, while pricing by continuous models (Carr and Wu 2014). In the volatility space, the most commonly used model for modelling discrete volatilities is the GARCH model (Bollersev 1986). Despite its usage, GARCH(1,1) has its own weaknesses, and Hypothesis Two will deal with those weaknesses.

3.2 The Extended GARCH(1,1)

3.2.1 Introduction to the Extended GARCH(1,1)

According to the VTT, when one starts to have more information on a firm, then the volatility of that firm should decrease over time. However, it can be inferred from other studies such as Schwert (2011) VTT does not always hold all the time. Then, the question is why volatility changes are important for sell-and buy-side investors in the investment management industry. Since the late 1990s, numerous firms have built different investment strategies based on volatilities-realised variance and/or volatilities being the notable products. One of the advantages of volatility products is that they are non-linear in shape, which implies that investment firms can make more money than anticipated amounts. In managing those products, one must know how to hedge or manage risks associated with those volatility products. For hedging purposes, discrete volatilities tend to work better-Carr and Wu (2014). Discrete volatilities can be traced back to 1982 when Robert Engle pioneered the first Autoregressive Conditional Heteroscedasticity (ARCH) model. Thereafter, Tim Bollerslev generalised the effects of the ARCH model, which led to the GARCH model. The GARCH(1,1) model is widely used in academia and practice for trading and hedging because it is such an elegant model, with numerous key parameters. The GARCH(1,1) model will be discussed later in this study.

It can be inferred from Black (1986) that volatility movements are synonymous with noise. The paper by Black (1986) is theoretical and he investigates noise within three disciplines; (i) finance, (ii) econometrics and (iii) macroeconomics-this is interesting as the expanded should be usable in those three disciplines. Black (1986) stated that those three disciplines might be different but the common element among them is the casual effect from noise. He argues that noise makes the world imperfect-this study follows the same phenomenon. According to Black (1986), what drives noise is inflation and new information. Generally, when new information gets spilled in stock markets, stock prices jump. The latter movements are common in the trading environment. At the heart of his study, Black (1986) regards noise as uncertainty, and uncertainty, at least in financial markets, is associated with volatility movements. Black (1986) opined that conventional monetary and fiscal policies will be ineffective in future. This study holds a similar view in the sense that it expands the GARCH(1,1) model so that other parameters that drive volatilities are captured in the model. That should lead to volatilities mimicking symmetry distribution, given that volatilities are asymmetric, especially in the presence of leverage effects. Then the question is how different is noise in finance, econometrics and macroeconomics?

In finance, Black (1986) stated that if there is noise, there is little trading activity. He argues that traders change their exposure because of changing broad market risk levels. That might affect the amount of cash traders hold in short-term securities, money market accounts, money market mutual funds and loans backed by real estate or other assets. According to Black (1986), individuals with insight or information are more likely to trade than ones who have no information or insight. As more individuals have information, the sizes of trades increase, especially for large and mega-cap stocks. Interestingly, Black (1986) opined that traders with different beliefs create differences in information. The expanded GARCH(1,1) model will cater for differences in beliefs in information optimism versus pessimism. More, when there's a high volume of information, then it becomes costly to buy information. The latter phenomenon is one of the reasons that led to High Frequency Trading (HFT) and Proprietary (prop) Trading. According to Black (1986), during all that noise, volatilities are changing in stock markets. What makes econometrics different from the finance discipline, in terms of noise?

The econometrics field is largely centred on the utility of individuals-Black (1986). Black (1986) argues that once utility is taken into account, it is hard to know where to stop because different people have different utilities. A classic case that leads to the violation of the efficient market hypothesis is the presence of dividends. For Black (1986), share repurchase is better than dividends pay-out because of the tax laws that exist. More, he stated that dividends convey more information than what is on financial statements and in public announcements. Given that it is hard to model noise, trades adopt rules of thumb, which are not as simple as one would expect. According to Black (1986), noise is an unobservable driver of volatility, while observable drivers would include interest rates and CPI futures. When there are unobservable drivers, it is hard to estimate slopes of demand and supply curves-Black (1986). Finally, he elaborated on noise within the macroeconomics environment.

The noise in macroeconomics is explained based on business cycles. Black (1986) stated that when unanticipated shifts in price levels are caused by government spending, that noise is not necessarily uncertainty noise. However, it is caused by unanticipated shifts in the entire pattern of tastes and technologies across sectors that might be uncertainty noise. He went further and argued that more and more governments advocate shifting their economies into complex ones from simple economies. Although, that is desirable for economic reasons there are inherent issues, such as macroeconomic noise. And when those macroeconomic issues are not properly mitigated, the consequences are dire. The uncertainty noise emanating from macroeconomics tends to increase volatilities substantially.

Based on the GARCH(1,1) model, which is illustrated in the modelling section, the parameters of the GARCH model include omega (ω)-variance constant, alpha (α)-captures reaction to market events, beta (β)-volatility persistence, lambda (λ)-captures leverage, and α and β-converges of conditional volatility to the long term average volatility. Despite all that, there are certain things that the current GARCH(1,1) does not incorporate; (i) correlation coefficients of debt and equity-besides measure correlation of assets, (ii) equity parameter, (iii) risk premium, (iv) interest rate and (v) stock market shocks. The contribution of this study is extending GARCH(1,1) such that the extended model accounts for those five parameters mentioned in the latter statement. GARCH(1,1) is extended into the capitalised GARCH model. Then the question is which variables qualitatively and quantitatively are important in determining discrete volatilities movements? This study adopts the Pecking Order Theory (POT) of finance in choosing parameters. Many scholars have shown through and through that when firms finance their expansions, they follow POT of finance -first; expansion is financed through internal cash, then debt and finally, equity issuance. On the other hand, Minton and Schrand (1999) illustrated that financing of firms' expansions, especially debt and equity, influence stocks volatilities. That is, the extension of the capitalised GARCH is anchored on POT. For this study, the structure of a firm comprises debt and equity.

The results illustrate that dynamism of GARCH(1,1) is centred in the leverage parameter. Furthermore, the GARCH(1,1) model can be extended to form the capitalised GARCH. Unlike GARCH(1,1), the capitalised GARCH captures more about stock markets, including (i) correlation of capital structure, (ii) "major" stock market shocks, (iii) alpha and (iv) returns on debt issuance.

3.2.2 Literature Review on Extended GARCH(1,1)

3.2.2.1 Bonds

This section explores all products that have features and/or similarities with bonds. Heath et al. (1990) illustrated how discrete time approximation makes the original Ho and Lee (1986) model accessible to a wider audience. The original Ho and Lee (1986) model is on the trading economy and the bond is randomised, based on a single binomial process. Firstly, they start with terminologies and notation. Heath et al. 1990 created an arbitrarily trading time and family of default-free discount bonds trade. The bond price is an exponentially discounted price synonymous with the free arbitrage trading environment. On the other hand, Heath et al. 1990 argued that the free arbitrage trading environment creates a possibility that martingale measures exist. According to Heath et al. (1990), there is no need for the martingale probability to be unique. Thereafter, they moved to term structure movements. They present a family of stochastic processes exemplifying forward rate movements. The final part of the modelling is arbitrate free pricing and term structure movements. At the heart of the latter phenomenon is to show that the priced bonds satisfy the discount bonds.

Then, Heath et al. 1990 presented an example that is very similar to the Ho and Lee (1986) model. From the Ho and Lee (1986) model, they moved to an example on the constant and exponentially decaying model. What is central to the latter section is that forward rate processes are subjected to two random shocks; (i) short-term factors-dampens exponentially with time to maturity and (ii) long-term factors-affects all forward rates equally. Then, the analysis and/or illustration is limited to economies. The sampling is allowed to follow the Central Limit Theorem (CLT). To be different from the Ho and Lee (1986) model, Heath et al. 1990 implicitly estimated the parameters. Thereafter, they value a contingent claim. What is unique about the contingent valuation is that (i) original probabilities are in special cases, (ii) martingale probabilities are constant across time and (iii) variances of the changes in the forward rate process $\left(\sigma(t,T)\right)$-forward rate processes are driven by volatilities, where volatility is influenced by *tau*. Note that volatilities are discrete in this case. The results of Heath et al. 1990 show that the abstract model is made accessible to a wider audience, application of numerical approximation of continuous model in the original form and finally, Ho and Lee (1986) is parametrised, such that is applicable in the discrete trading environment.

Andersen and Bollerslev (1998) tested whether forecasts from standard volatility models are accurate. According to the authors, ex-post evaluation studies tend to produce poor forecasts. Andersen and Bollerslev (1998) stated that volatility analysis is central to option pricing and portfolio allocation. Their main contribution is to illustrate that well-specified vitality models provide accurate volatility forecasts. This includes showing how high-frequency intraday data can be appropriately modelled and illustrating inter-daily volatility measurements. Andersen and Bollerslev (1998) diverted from the common norm where the appropriateness of the model is determined by its implied practicalities.

For modelling inter-daily volatility and forecast evaluation, Andersen and Bollerslev (1998) focused on (i) daily volatility modelling and (ii) daily volatility forecast evaluation. The daily modelling is on the GARCH(1,1) model, where parameters are defined. This includes conditional variance based on information of period of time and sample frequency observations per day and volatility of the model. For this part of modelling, Andersen and Bollerslev (1998) modelled Deutschemark-U.S. dollar and Japanese Yen-U.S. spot exchange rates from October 1, 1987, to September 30, 1992. The Wald test confirms that there are no ARCH effects. More, the preliminary results show that the GARCH(1,1) model tracks the short-run inter-daily vitality dependencies. On the other hand, there was high persistence from the analysis. For daily volatility forecast evaluation, the results show volatilities can be appropriately predicted. And day-by-day movements in volatility are poorly estimated by realised squared returns. The adjusted R^2 was relatively low because of speculative returns and sample periods. They went further and modelled GARCH(1,2) and the results were not as good as in the GARCH(1,1) model.

Thereafter, Andersen and Bollerslev (1998) modelled continuous-time volatility and forecast evaluation. Interestingly, they illustrated that one can change from discrete volatilities to continuous volatilities by assuming that instantaneous returns are generated by the continuous-time martingale- a rare situation! Firstly, they assume that instantaneous returns are generated by the continuous-time martingale. Further modelling includes (i) continuous-time modelling of daily volatility and (ii) continuous-time measurement of volatility. The results show that the population $\left(R^2_{(1)1}\right)$ increases monotonically with the frequency of samples. The final part of Andersen and Bollerslev (1998) focused on intraday returns and inter-daily volatility forecast evaluation. The latter results exemplify that the frequency of the sample increases with higher correlation, while $R^2_{(1)1}$ is at five-minute levels of 0.479 and 0.392. Similarly to earlier results, GARCH(1,1) still does a good job. Overall, the prediction of volatilities is accurate indeed.

Lucas and Klaasen (2006) compared discrete and continuous default rate volatilities underpinned by switching models for portfolio credit risk. Their study is shaped on credit ratings-discrete versus continuous modelling from both analytical and empirical perspectives. According to Lucas and Klaasen (2006), the dominant focus of credit ratings, credit scoring and static assessment of default probabilities. Those ratings are equally important for trading desks, according to Lucas and Klaasen (2006). For the modelling, they start with a simple latent model where the default risk driver is a function of systematic and firm-specific risk, respectively. Thereafter, they move to show the default frequency and that the default frequency correlation is the same across obligors. For the actual analysis, they start with a simulation experiment. And that simulation experiment considers the standard CreditMetrics one-factor model at a quarterly frequency; $\begin{pmatrix} 0.96 & 0.04 \\ 0.30 & 0.70 \end{pmatrix}$. According to Lucas and Klaasen (2006), 0.96 denotes the probability of remaining in an expansion quarter. Changes in regime variability or severity (ϕ) did not affect the conditional mean.

For the empirical analysis, Lucas and Klaasen (2006) used data from CreditPro from 1981 to 2002 based on S&P and they constructed monthly, quarterly and annual time series of rating transition matrices. The estimated matrices were as follows. The monthly is $\begin{pmatrix} 98.7\% & 1.3\% \\ 9.4\% & 90.6\% \end{pmatrix}$ and quarterly is $\begin{pmatrix} 96.2\% & 3.8\% \\ 30.0\% & 70.0\% \end{pmatrix}$ where 98.7% (96.2%) shows the probability of remaining in an expansion month (quarter). When the probabilities of default increase, so are the volatilities of the assets concerned. Further, volatilities are positively correlated with default spreads.

Curci and Corsi (2012) used sine transformation to illustrate realised volatility measures. In estimating realised volatility, they used a combination of the multi-scale regression and Discrete Sine Transforms (DST) approaches. They opine that vitality analysis is central to asset allocation, risk management and option pricing. One hopes that the derived expanded GARCH(1,1) will be ideally suitable for risk management. Realised volatility is simply the square root of realised variance-sum of squared periodical returns for a particular time frame. Generally, realised volatility is computed for daily returns. Curci and Corsi (2012) argue that earlier estimators suffered from the possibility of becoming negative. While the advantage of using the DST approach is that it handles appropriately decorrelation effects. Curci and Corsi (2012: 264) stated that "the motivation for the employment of the DST approach rests in its ability to decorrelate signal for data exhibiting MA-type behaviour, which arises naturally in discrete time of models of tick-by-tick returns".

Before modelling volatility based on Monte Carlo simulations, Curci and Corsi (2012) defined and outlined the properties of DST volatility estimators. Specifically, they focused on (i) price process with microstructure noise, (ii) discrete transform, (iii) multi-scales least square estimator and (iv) multi-scales DST estimator. Curci and Corsi (2012) argued that an appropriate way of modelling various sources of microstructures is by decomposing them into the sum of two unobservable components. Therefore, a basic Stochastic Differential Equation (SDE) is suitable. More, they argue that the SDE should be able to capture and/or recover normality and homoscedasticity of asset returns. On the second point, Curci and Corsi (2012) stated that one should orthogonormalise processes such that they coincide with the DST approach. For high-frequency data, they recommend that one should use non-parameterised orthogonalisation of data. For the multi-scales least estimator, they used realised volatility and the Multi-Scales Least Square Approach. On the DST approach, they focused on the aggregation of different frequencies. Finally, they modelled data using Monte Carlo simulations. And they observed 390 observations per day. The results show that volatility is 20% on average, obtained from a frequency of 1 minute and 5 seconds, and a noise-to-signal $\left(\frac{\eta}{\sigma}\right)$ of 3.5. Broadly, the initial results varied with frequency level, local Exponential Moving Average (EMA) filter level and variance. The empirical application of Curci and Corsi (2012) is based on verified behaviour of volatility estimators of future contracts: S&P500 stock index future, 30-year U.S. Treasury Bond future and Italian stock index future FIB30. The period is from January 1998 to December 2003. Part of the investigation included disentangling the autocorrelation behaviour of tick-by-tick returns.

The main results of Curci and Corsi (2012) illustrated that autocorrelation patterns of tick-by-tick returns are consistent with being more dynamic than the Autoregressive-Moving-Average (ARMA) structure for the microstructure noise. More, the conditional mean of daily volatilities were not dissimilar to the unconditional mean of volatilities. As expected, the MS-DST approach disentangles the microstructure effects of volatilities. Returns measured over 30-minute periods show that S&P500 is leptokurtic, U.S. bond and FIB30 have fat tails.

Engle *et al.* (2013) argued that progress on studies on the variation of volatility are uneven. The first model (i.e. autoregressive conditional heteroscedasticity, from here ARCH) on volatility clustering, which is discrete, can be traced back to Engle (1982). Engle *et al.* (2013) bridged the gap between ARCH models and stochastic volatility models. They are of an opinion that there is no consensus on how to specify dynamic components of volatilities. This study incorporates certain parameters which are currently not incorporated in the GARCH(1,1) model. Their model cannot be reduced to any form and more, the model is not linked to any structure in the economy. The expanded model in this study is largely applicable to alternative investments, specifically an investment portfolio made of bonds, commodities, equities and listed real estate. The drivers of volatilities, according to Engle *et al.* (2013), are Producer Price Index (PPI) inflation, Gross Domestic Product and Industrial Production growth rate (IP).

For the modelling section, they focus on a new class of component models for stock market volatility, in particular on (i) models with realised variance, (ii) incorporating macroeconomic information directly and (iii) spline-GARCH component volatility model. Engle *et al.* (2013) opined that volatility has at least two components; (i) a component that accounts for short-run variations and (ii) a component that accounts for long-run variations. More, they assume that macroeconomic variables are sources of stock market volatility. On models with realised variance, they state that realised variance is very noisy. They specifically focus on the GARCH-MIDAS model as filter-incorporated smoothing realised variance in the spirit of MIDAS regression and MIDAS filtering. One of the advantages of GARCH-MIDAS is that it allows one to study interactions between stock market volatility and macroeconomics-Engle *et al.* (2013).

According to Engle *et al.* 2013, the measurement error in realised variance will deteriorate with time in order for the model to be an appropriate fit. For incorporating macroeconomic information variables, they included PPI inflation and IP. For the spline-GARCH component volatility model, Engle *et al.* (2013) decomposed conditional variance, into both short and long-run components. Then, they estimated the results-(i) model selection and estimation of GARCH-MIDAS models with realised variance and (ii) estimation of GARCH-MIDAS models with macroeconomic variables. The data is obtained from Schwert's website on daily U.S. stock returns from February 16, 1885, to July 2, 1962. Then, they extended data up to December 31, 2010, based on CRSP datasets. The lagged results for the GARCH(1,1) model show that volatilities are concave in shape monthly, quarterly or biannually. Notably, the sum of alpha (α) and beta (β) is less than one in all cases. That illustrates that the calculated α and β are suitable for the GARCH(1,1) model. On the other hand, those sums show that persistence is quite strong from the data used. On the question of how volatility relates to macroeconomics, the results illustrate that there were quarterly noises of the volatility and there were structural breaks. In this study, one does not illustrate structural breaks. As expected, inflation leads to higher volatilities. The IP is positive and statistically significant, while business cycles are uncertain. Volatilities tend to be higher around stock market crashes. Spline-GARCH(1,1) is a good estimator of future volatilities.

Finally, Engle *et al.* 2013 appraised the models and analysed the economic sources-(i) structural breaks and correlations and (ii) forecast comparisons. The results show that the full sample size is not immune to structural breaks. On the correlations, they only report salient features-correlations between realised volatility and macroeconomic variables with the two-sided IP level/variance model is 0.36 less than the spline-GARCH model. For forecast comparisons, the process needs to be iterated into to predict forward volatilities. More, the macroeconomic variables might not be of quality in explaining good forecasts. Overall, Engle et al. 2013 found that models with long-term components are driven by inflation IP growth and perform equally better as out-of-sample predictions over horizons of one quarter and 'outperform pure time-series statistical models at longer horizons'.

The synthetisation of bonds studies points to the fact that bonds volatilities react in risk premium, assets correlation, mean-reversion coefficient, economic growth (i.e. similar to correlation), lagging, moments, returns, shocks and bond regimes.

3.3.2.2 Commodities

Low *et al.* (1999) explored arbitrage opportunities in the commodities markets during the opening and closing of stock exchanges. At the heart of their study, is the linkages of discrete futures markets during auctions. The authors opined that arbitrage is linked with financial markets. "Classical economic theory suggests that financial markets are linked by the potential for incipient arbitrage", Low *et al.* (1999:800). According to the authors, the classical economic theory should imply that commodities traded in different markets should be linked through co-integration. To test that theory, Low *et al.* (1999) examined storable commodities traded during auctions on the Tokyo Grain Exchange (TGE) and the (now defunct) Manila International Futures Exchange (MIFE) from October 1992 to March 1994. According to Low *et al.* (1999), discrete auctions were rare events for clearing stock and commodities futures markets when they carried out their study. The source of joint dynamics, according to Low *et al.* (1999), are (i) two single fixed-price auction markets that are highly similar and (ii) TGE and MIFE are traded almost at similar times. For the literature review, the authors explored market structure.

Market structure, especially microstructure, is the whole process of determining prices of assets, parameters of those assets and pricing those assets. Prior studies illustrated that the New York Stock Exchange (NYSE) employs a "clearing mechanism" to determine prices during auctions-

“dealership” mechanism determines closing prices while TSE uses a periodical call auction (*Itayose-hoh*) at opening and closing trading periods. The *Itayose-hoh*, according to Low *et al.* (1999), trading during the day allows continuous trading (*Zaraba*) between opening and closing periods. Thus, auction prices are averages of the trading day. In order to appropriately synthesise literature on market structure, they focused on (i) single fixed price auctions and (ii) commodity futures trading on the TGE, CBT, CSCE and the MIFE. For single fixed-price auctions, Low *et al.* (1999) put forward *Walrasian tatonnement*-prices reach equilibrium in classical economic theory. The relationship between demand and supply will determine equilibrium prices. For the comparison of exchanges, this study focuses on TGE and MIFE because the data used is from those two exchanges. The main feature is that on TGE, soybeans are traded at 10:00, 11:00, 13:00 and 14:00 Tokyo time, while on MIFE; soybeans are traded at 9:45, 10:45, 13:45 and 14:45. Broadly, there is a 45-minute difference between the traded times of TGE and MIFE-MIFE is 45 minutes later than TGE.

The data used in the analysis is on TGE and MIFE soybean and sugar futures contracts over the period of October 1992 to March 1994. They covered six futures contracts. Prices from TGE are in Japanese yen and prices from MIFE are Philippine pesos. Some data was provided in U.S. dollars-this led to artificial arbitrage. Daily observations for sugar contracts ranged from 91 to 110, while for soybeans ranged between 98 and 120 contracts. Broadly, volatility in sugar contracts was higher in local currency than in U.S. dollars. According to Low *et al.* 1999, from 1993 to 1994, currency appreciation was greater against the Japanese yen than the Philippine peso. And the soybeans and sugar contracts had higher volatility in TGE than on MIFE-Tokyo is one of the biggest four exchanges including New York, London and Hong Kong. To model arbitrage, Low *et al.* 1999 used standard arbitrage theory. Thus, they considered two different futures prices at different times (i.e. current time versus future time) and presented the appropriate relationship. Then, they looked at how the continuous relationship evolved. The preliminary results showed on average, there are no arbitrage opportunities between the two exchanges. Despite that, there were quasi-arbitrage regressions for both markets based on conversations with traders from the TGE market. TGE traders acknowledged arbitrage based between TGE and the CBT as the CSCE.

Thereafter, Low *et al.* 1999 investigated the long term price dynamics (i.e. long memory) based on tests for unit roots and co-integration. The tests of unit roots results supported the presence of unit roots in prices. For the co-integration, results exemplified no co-integration between TGE and MIFE prices. Probably, the Tokyo Stock Exchange is more integrated with the three major exchanges because of its size. According to Low *et al.* 1999, those results were not surprising given that earlier studies confirmed no co-integration between cash and futures prices in other commodities markets. More, they argue that the absence of co-integration implies that (i) forecasting models should be reviewed, (ii) reviewing cross-hedging models and (iii) reviewing other economic relationships.

Pindyck (2004) investigated volatility dynamics in commodities; (i) crude oil, (ii) heating oil and (iii) gasoline. He argued that commodities prices are often associated with spikes. All those changes affect the marginal value of storage, marginal cost of production and opportunity cost of production. What is central to his study is the behaviour and role of volatility over the short run. He started by building a model for pricing those volatilities. And the modelling part includes (i) costs section, (ii) empirical specification and (iii) estimating equations. The costs formula is a linear equation made up of the direct cost of production output, the opportunity cost of production, then the risk and finally, the total marketing costs. For the opportunity costs of production, its first derivative illustrates that the opportunity costs will exceed the direct marginal cost by premium. The premium might be partly due to information asymmetry. In order to disentangle the costs formula, he used Euler principles-Euler principles are broadly used to establish relationships between functions (especially in trigonometry) and complex functions (especially exponential functions). Thus, Pindyck (2004) showed the trade-off between selling out inventory and production including minimum/maximum boundaries.

The empirical specification for the costs formula in Pindyck (2004) focused on whether the model is practically usable. In order to simplify the model, he excluded wage rates and made the marketing costs to be a proportion of the price and more, the model should have optimal function. The estimation of the formula illustrated that the parameters for all three commodities can be estimated. He used weekly data from January 1, 1984, to January 31, 2001, for crude oil and heating oil. For gasoline, he used weekly data from January 1, 1985, to January 31, 2001. The prices were modelled as being logarithmically distributed. More, Pindyck (2004) used a corresponding futures formula to calculate futures prices. Broadly speaking, the models used are Generalised Method of Moments (GMM) and he was able to pick structural errors in the modelling. Given that with those three commodities, there are numerous spikes, he lagged his data and lags ranged between 1 and 3. Those parameters that were lagged once or twice included (i) the exchange-weighted value of the U.S. dollar (EXVUS), the New York Stock Exchange Index, the three-month Treasury bill rate

(TBILL), the rate on Baa corporate bonds (CRB) and the Commodity Research Bureau's commodity price index (CRB). Overall, there were 34 instruments.

According to Pindyck (2004), the results should answer whether or not volatilities and their parameters are predictable. To strengthen the results he also used VAR relating to the markets and their parameters. The results showed that for crude and heating oil, the coefficients are positive and statistically significant. The elasticity for futures was close to one. The coefficient for gasoline is negative and statistically significant. He went further and he did simulations. Simulations focused on illustrating the effects of volatility shocks and other variables and found numerous volatility shocks.

The study of Elder and Jin (2007) is different to other studies because they investigated volatility movements in a trading environment. Their study is shaped on agricultural commodities futures. At the heart of the contribution by Elder and Jin (2007) is the Wavelet Methodology (WM) that can decompose volatility of agricultural commodities-WM is normally used in conjunction with Mean Squared Error (MSE). Secondly, Elder and Jin (2007) never 'imposed parametric restrictions on short-term volatility dynamics'-GARCH normally restricts parametric restrictions. Thirdly, Elder and Jin (2007) argue that the WM fractional integration of moving average volatilities can be applied. According to the authors, in fractional integration, time series are integrated in the order of zero or one and unit-roots are used. Normally, most prior studies illustrated that the volatility of commodity futures are dependent (See; Andersen *et al.* 2003 and Du *et al.* 2011).

According to Elder and Jin (2007) volatility of commodity futures is dependent on (i) maturity effect, (ii) underlying spot prices and (iii) sequential response of market participants with heterogeneous information. Other reasons that can explain dependence according to the authors include (i) biological factors, (ii) economic factors and (iii) weather effects. Based on prior similar studies, Elder and Jin (2007) stated that if maturity is longer, the future spot prices effects are averaged out in the future price. And those same effects diminish when the time approaches maturity-Elder and Jin (2007). The latter effect is synonymous with the Option Pricing Theory (OPT). Elder and Jin (2007) opined that supply shocks cannot be resolved at maturity, while demand shocks may be resolved. One possible explanation is that there is no substitute for commodities. Thereafter, they moved to the research methodology.

The methodology in Elder and Jin (2007) focused on (i) fractional integration, (ii) fractional integration in volatility and (iii) wavelet estimators of the fractional integration parameter.

The data used in Elder and Jin (2007) is on 14 commodities which include 7 grains (corn, Kansas City (KC) wheat, oats, soybeans, soybean meal, soybean oil and wheat), 3 soft commodities (cocoa, coffee and sugar) and four types of meat (feed cattle, live cattle, live hogs and pork bellies). The grains are traded on the Chicago Board of Trade and KC wheat is traded on the Kansas City Board of Trade. The soft commodities are traded on the New York Board of Trade and the meats are traded on the Chicago Mercantile Exchange. Daily settlement prices are five days per week, from December 1, 1993 to March 1, 2000. The used data sample is made of 4,096 observations. The preliminary statistics show that 4 commodities (corn, oats, soybean and soybean meal) have negative skewness, all 2 commodities (cocoa and sugar) and one meat (live cattle). The rest of the commodities have positive skewness. Normally, one should be able to make more from positive skewness than from negative skewness. However, if in a portfolio one has both positive and negative skewness, then that is suitable for risk minimisation-common in risk parity diversification. Similarly, volatilities in those types of portfolios will be minimal. The absolute value percentage showed that all commodities are volatile, except for meats (feed cattle, live cattle and hogs), which are less volatile. Fundamentally, the results of Elder and Jin (2007) illustrated that all commodity volatilities of futures have long memory-highly volatile.

Doran and Ronn (2008) illustrated that there is a need for negative risk in continuous option pricing. Negative risk premium works well in diversification and hedging as well, especially in delta hedging-Bakshi and Kapadia (2003). Doran and Ronn (2008) opined that futures and options markets are important for the economy; therefore, it is volatilities associated with them. Among well-documented information on issues when risk premium is high, is that their Sharpe ratios are high as well. They argue that before 2008, most studies on energy markets focused on convenience yield. According to Doran and Ronn (2008), a negative risk premium associated with the At-The-Money (ATM) implied volatilities that are normally higher than realised volatility. From prior studies, they noted the following. Higher volatilities are synonymous with higher prices, within commodities markets, beta coefficients are negative and the term structure of volatility (TSOV) is characterised by seasonality.

For modelling, Doran and Ronn (2008) used a quasi-Monte Carlo (MC) simulation. Given that energy markets are characterised by jumps, they include jumps parameters into the quasi-MC simulation with 30,000 simulations. Before computing risk premium, they simulated results based on a stochastic volatility model that has jumps (SVJ) and the Constant Elasticity of Variance (CEV) model. The results were over 22-trading days. The simulated results ratio of a strike/futures price ranged between 0.8 and 1.2-illustrating that options are In-The-Money (ITM). When risk premium is not taken into account, the jump is -2 and with risk premium, the jump is 0.5. The realised volatility shows the case of no risk premium. Thereafter, they test whether those simulations work in practice. The daily data is over 10 years (January 1995 to December 2005) of natural gas, crude oil and heating oil futures and energy option contracts. The data for options and futures is from Bloomberg and NYMEX. The sample was as follows; 23,408 for natural gas, 22,483 for heating oil and 22,406 for crude oil futures. Given that most securities, including energy commodities, are normally priced in a neutral world, Doran and Ronn (2008) computed the risk-neutral parameters.

The results from Doran and Ronn (2008) illustrated that natural gas contracts mean-reverts with a higher long-run-variance ($\Theta = 0.335$) than other commodities. More, natural gas had an economically significant TSOV time-dependent parameter, $\chi = 1.96E^{-3}$. The speed of mean-reversion for crude oil, $k = 27.74$ than the other two commodities as well as variance process, $\xi = 0.449$. More, Doran and Ronn (2008) estimated risk premium. The results for risk premium were similar to the simulated ones-λ_σ for each commodity is statistically significant and negative. More, the volatility risk premium was statistically significant and negative for all months. The robustness tests confirmed the earlier findings of Doran and Ronn (2008).

Based on prior studies on commodities, it can be inferred explicitly and implicitly that volatilities of commodities markets are driven by severe weather conditions, prices (futures and spot), inventories, convenience yield, traders heterogeneity, returns, shocks and regime changes.

3.2.2.3 Equities

Engle and Patton (2000) used daily data of the Dow Jones Industrial index, over the period 1988 to 2000 to illustrate stylised facts of volatility that need to be captured in volatility models and the ability of the GARCH-type models to capture these features. The descriptive statistics of the data over this period indicate a small positive return, a substantially negatively skewed return distribution, which is a common feature of equity, as well as a high kurtosis of the distribution. The study employs the GARCH(1,1) model on the index and reports the following; The volatility of the series returns is quite persistent with the sum of parameters $\alpha + \beta$ being 0.9904. The persistence of volatility arises from volatility clustering, whereas volatility clustering implies that volatility shocks today will influence the expectation of volatility in later future periods. However, although the persistence is high, the sum of $\alpha + \beta$ being less than 1 implies that the volatility process is mean-reverting.

Another important feature taken into consideration by the study was the asymmetric feature of volatility, for equity returns it is particularly unlikely that positive and negative effects have symmetric impacts. Several researchers found that negative return shocks seem to increase volatility more than positive return shocks of the same size (Bollerslev, Chou and Kroner, 1992; Engle and Ng, 1993; Pagan and Schwert, 1990). The asymmetric feature is usually ascribed to the leverage effect and sometimes, the risk premium effect. The former theory applies to the phenomenon that as the price of a stock falls, the debt-to-equity ratio increases, thus increasing the volatility of returns to equity holders. The latter theory entails that news of increasing volatility reduces the demand for a stock due to risk aversion. The study employs the Threshold GARCH (TARCH) model in parametrising these phenomena. The results indicate the sign of an effect is central. The coefficient on negative residuals appears to be a significant influence on the volatility of returns and increases the volatility by four times as much as a positive effect of the same size.

The study also considers the impact of exogenous variables on volatility to offer a structural or economic explanation. This impact is illustrated by using the lagged level of the three-month US Treasury bill as an exogenous regressor in the model. The impact of the T-bill rate was found to be small but significant. The results indicate, through the positive coefficient sign, that high interest rates are generally associated with higher levels of equity return volatility.

Gokcan (2000) compared the modelling and forecasting ability of the linear GARCH(1,1) and non-linear Exponential GARCH (EGARCH)-introduced by Nelson (1991) versions of the GARCH model in the emerging markets (Argentina, Brazil, Colombia, Malaysia, Mexico, Philippines and Taiwan). Engle and Patton (2000) highlighted the importance of a volatility model entailing forecasting aspects of future returns. Typically, a volatility model is used to forecast the absolute magnitude of returns, as well as predict quantiles or the entire density. Such forecasts are used in risk management, derivatives pricing and hedging, market timing and portfolio selection, amongst other financial activities.

Gokcan (2000) noted that EGARCH, unlike linear GARCH models, has the ability to consider skewness in return distributions and therefore linear GARCH models can be expected to be biased with forecasting volatility for skewed time series. Using the monthly stock market portfolio returns, the study analyses the period of February 1988 to November 1997. Descriptive statistics indicate significant kurtosis for all seven markets, while skewness is present in the markets with the exception of Argentina. The data further indicated negative skewness which informs us that market declines occur more often than market increases. Gokcan (2000) began by representing the GARCH(1,1) and EGARCH model and thereafter reported the parameter estimates and Akaike Information Criterion (AIC)-an estimator of the relative quality of statistical models for a given set of data. The results indicate for all countries, the linear GARCH(1,1) model produces lower (Akaike Information Criterion) AIC values than the EGARCH(1,1) model, thus indicating that the GARCH(1,1) model outperforms the EGARCH model in terms of modelling volatility. The study further reports the forecasting ability of both models, the results indicate that the GARCH(1,1) model produces smaller forecasting errors than the EGARCH(1,1) model.

Kourtis *et al.* (2016) made a comparison of realised, implied and GARCH volatility forecasts for the period January 2000 to October 2012 for 13 equity indices across 10 countries. The study focuses on four models (i) forecasts from intraday returns involving lagged realised volatility (LRE), (ii) model-free implied volatility forecasts (MFIV), (iii) Model-free implied volatility adjusted for the volatility risk premium (C-MFIV) and (iv) the GJR-GARCH model of Glosten, Jaganathan and Runkle (1993). In defining the models, Kourtis *et al.* 2016 noted that the MFIV model is based on volatility measures implied from prices of liquid traded options and have incorporated an element to predict volatility at various horizons. Kurtosis *et al.* 2016 introduced the C-MFIV model to deal with the bias that implied volatility is a biased estimator of future volatility, due to the existence of an economically significant variance risk premium. Kurtosis *et al.* 2016 also introduced the GJR-GARCH that is able to capture serial dependencies asymmetrically. Lastly, the study introduces the Heterogeneous Autoregressive Model (HAR) to estimate and capture known stylised facts, such as long memory and fat tails in financial data.

Kurtosis *et al.* 2016 ran a univariate Mincer-Zarnowitz regression for volatility on 1-day, weekly and monthly forecast horizons and encompassing regressions. For the Mincer-Zarnowitz regressions, slope coefficient results are statistically significant, indicating that all model forecasts bear information about future volatility. The greatest explanatory power is reported for the HAR model at a daily horizon, mixed results for the GJR, HAR and MFIV at the weekly horizon and C-MFIV at the monthly horizon respectively. Results further indicate that volatility forecasts are biased to a smaller extent for the HAR model and a lesser extent for the C-MFIV model. However, results indicate that volatility risk premium significantly reduces bias in the model-free implied volatility model. Encompassing regressions are made up of competing models and can indicate which model subsumes information of the volatility forecast. Results indicate all models can explain a large fraction of the variation on a daily and weekly horizon; however, results indicate the C-MFIV as a superior model to the LRE, HAR and GJR-GARCH on a monthly horizon. The study further investigates the accuracy of the forecasting models in market turmoil. Over the selected periods of (i) pre-crisis (January 2003-July 2008), (ii) the crisis period (August 2008-December 2009) and (iii) the after-crisis period (January 2010-August 2012). The study concludes that the forecasting ability of models deteriorates during the crisis period and the models produce higher errors for the post-crisis period, as compared to the pre-crisis period. The overall findings indicate the importance of accounting for the volatility risk premium in most markets under consideration.

Horpestad *et al.* 2019 studied asymmetric volatility effects using the (i) GARCH, (ii) GJR-GARCH, (iii) log-GARCH, (iv) E-GARCH and (v) HAR-realised volatility models. The study utilises 19 equity indices from North America, Latin America, Europe, Asia and Australia comprising of realised volatility, using high-frequency data for the period of January 2000 to June 2018. In reporting the comparison of the models utilising the data, firstly the studies pair the models in the following manner (i) GARCH(1,1) and GJR-GARCH, (ii) log-GARCH and E-GARCH and (iii) HAR-realised volatility and the LHAR-realised volatility. In comparing the GARCH(1,1) and GJR-GARCH, the study indicates that both models indicate high persistence in line with existing literature, however, the asymmetrical GJR-GARCH indicates lower persistence. When comparing the AIC values for the GARCH(1,1) the GRJ-GARCH model indicates lower values, thus suggesting superiority to the GARCH model. In comparing the log-GARCH and the E-GARCH, the study reports lower AIC values for E-GARCH, thus suggesting superiority and lastly, when comparing the LHAR-realised volatility is found to be superior to the HAR-realised volatility model. Overall the study finds asymmetric effects in the data and concludes that models that allow for asymmetry fit data better than their symmetric counterparts, however, asymmetric models estimate smaller persistence.

Alexander and Lazar (2009) explored two versions of a Normal Mixture (NM) GARCH-asymmetric power GARCH (AGARCH) and GJR (Glosten, Jagannathan and Runkle) extension of NM-GARCH on four European equity indices. Versus the normal GARCH, the NM GARCH takes into consideration state probabilities of volatility and considers endogenous conditional skewness and kurtosis. The study also introduces asymmetry to account for leverage effects. The empirical study demonstrates the superiority of the two-state asymmetric NM-GARCH model to capture the characteristics of stock index returns. The study begins by estimating 15 different GARCH models, comprising of a two-state asymmetric and symmetric NM-GARCH model, symmetric and skewed-t GARCH, with both symmetric and asymmetric variance processes. The study uses five model selection criteria - Bayesian Information Criterion (BIC), Moment specification tests, Unconditional Density Fit, Autocorrelation Function (ACF) analysis and VaR-to test the fitness of the models. The results indicate that the normal GARCH is the worst fit by all the criteria. The student t-GARCH only fits well with the BIC criteria and it is noted that the model does not capture the fourth moments of the returns, according to both the unconditional density and ACF model selection. Pure NM models with different means seem to fit better than models with identical means and additionally, the consideration of asymmetric components to the model improves their fit. The two-state asymmetric NM-GARCH model, which considers persistence and dynamic, is superior to all models. The study further reports results of regime-specific volatility behaviour. The two states can be differentiated as “stable” states, which occurs more often than “stressful” or “crash” states. The report indicates that for both (AGARCH) and GJR GARCH, there is lower volatility (higher volatility) component that occurs in a stable (crash) state, thus indicating the importance of considering regimes when modelling volatility.

Baele (2005) studied volatility spillover effects in European equity markets using a regime-switching model in lieu of asymmetric GARCH models on the basis that regime-switching models produce better forecasts. The study focuses on volatility spillovers from European and U.S markets to 13 local European equity markets. Baele (2005) argues that most models using a dummy variable cannot account for volatility spillovers in a situation where the event is anticipated or needs time before becoming effective. The spillovers paper introduces a regime-dependent shock spillover methodology where (i) the intensities switch endogenously rather than exogenously (ii) probabilistic statements can be made about the relative likelihood of the spillovers states and (iii) accommodates non-linearities that occur in higher moments, such as asymmetric volatility and kurtosis. Firstly the study develops a bivariate model for the European and U.S returns with four different (bivariate) specifications for the conditional variance-covariance matrix-(i) constant-correlation method, (ii) a bivariate asymmetric BEKK model, (iii) a regime-switching normal model and (iv) a regime-switching GARCH model. Secondly, the study develops the univariate spillovers model with time-variation parameters that allow spillover intensities to switch between two states. The study finds that overall regime switches in spillover intensities are statistically and economically important. The hypothesis of constant spillover intensities is rejected in all countries, indicating that multiple regimes need to be considered for correctly measuring volatility spillovers. The study also indicates that specification tests improve drastically once multiple regimes are allowed for, especially for excess kurtosis, suggesting that allowing for non-linear shock spillover intensities contributes to a better specification of higher moments.

Catäo and Timmermann (2010) developed a regime-switching modelling framework to address three main questions; (i) whether global stock return volatility displays well-defined volatility regimes (ii) the extent to which equity market volatility is accounted for by global, country or sectorial factors, and (iii) what implications this has for the national equity market correlations and international risk diversification. The study develops the model on the basis that current volatility models, built under the Asset Pricing Theory (APT) and GARCH framework, do not provide the flexibility of modelling non-linearity features that are apparent in factor dynamics driving stock returns. The study was employed throughout 1973 to 2002, based on a data set of approximately 3951 firms in 13 developed markets. Firstly, the model entails creating country and industry portfolios through cross-sectional regressions in each firm's stock returns. In modelling for the stock market dynamics, the study follows literature that has attempted to capture periods of high and low volatility in univariate series or pairs of series (Ang and Bekart, 2002; Perez-Quiros and Timmermann, 2000) and extend this approach to multi-country/ multi-sector portfolios.

The model considers three state variables driving returns on global, industry and country portfolios, and follows a Markov chain process to allow for regime-switching, the model also implies the return process as a mixture of normal random variables to account for skewness and excess kurtosis features found in financial data. The model further considers that a state variable driving return can be common to a certain extent between industries and countries' returns. Overall the model has the mean and variance of returns constant within each state; however, the state probabilities vary gradually or rapidly when filtered state probabilities change. Descriptive statistics report a higher standard deviation average at 4.89% monthly for country portfolios and 2.96% for industry portfolios, thus verifying the findings in the literature that, on average, country factors matter more than industry factors. Country specific returns appear more positively skewed than industry returns, whereas global returns appear not to be skewed. Excess kurtosis is also reported for most of the portfolio and normality is rejected, with the exception of two countries. Firstly, the study finds evidence that there are different volatility levels for regimes in international stock returns, with low volatility regimes being two to three times more persistent when averaged over country and industry portfolios. The study reports also that allowing for time-varying factor contributions appears to characterise data better than linear models with a similar factor structure and should enable more accurate contributions to estimate various factor contributions. Secondly, the study finds that country factors explain approximately 50% of market volatility over the entire period studied, as opposed to 16% contributed by industry specific factors, however, these contributions vary across volatility states, industry specific factor contributions are more prone to rise during periods of oil shocks and IT booms. In terms of implications on international risk diversification, the study finds that when global and industry factors are in high volatility states, correlations in country portfolios become tighter and the key implication is that this undermines the benefits of cross-border diversification.

One can infer that discrete volatilities are driven by equity returns, clustering effect, risk premium, T-bill rate (interest rates), non-linearity, kurtosis, regimes, and shocks (i.e. oil, IT booms). Some of those mentioned variables or parameters are currently not accounted for in the GARCH(1,1) model.

3.2.2.4 Real Estate

Jirasakuldech *et al.* 2007 examined the conditional volatility of equity real estate returns in a GARCH context for the period of 1972-2006 and creates a comparison for "early" Equity REITs (1972-1992) and "modern" Equity REITs (1993-2006). Based on monthly returns and a set of macroeconomic variables, the study examined whether the volatility series of REITs evolves over time and measures the persistence of those changes, should they occur. Macroeconomic variable growth rates considered include the Consumer Price Index (CPI), Industrial Production (IP), the Federal Funds rate (FD), and the Risk Premium (RP). The Russell 2000 (R2000) Index is used as a proxy for the market. Descriptive statistics indicate that the market performs slightly better than the EREITs and experiences higher volatility. Both the market and the EREITs are negatively skewed, with the market projecting more skewness, indicating that the market is subject to larger infrequent negative returns than EREITs, fat tails are reported for both the market and EREITs with the indices indicating significant excess kurtosis.

The study begins by modelling the GARCH(1,1) and parameters indicate a slow mean-reversion process, as well as a high persistence, indicated the proximity of α plus β to 1. This indicates that volatility clusters over time and that the current information is useful in predicting future volatility. Structural change between the two subperiods is also tested and a difference of a 3.6% level of confidence is reported. The study then employs the GARCH-M model to investigate the effect of conditional volatility on the return-generating process. The GARCH-M model differs in the sense that it expresses present volatility as a function of past volatility and returns behaviour by introducing past conditional variance into the mean equation, and also allows the user to model a time-varying risk premium. No evidence of a positive relationship is found between the conditional variance and expected returns for both EREITS, as well as the R2000 Index.

The study also considers asymmetric or leverage effects and models the data using the Threshold ARCH (TARCH) model. In both sub-periods, none of the coefficients are statistically significant for EREITS, indicating that good or bad news does not have different impacts on volatility. These results, however, appear to be contrasting as the distribution of the EREIT returns indicates significant leptokurtosis and excessive negative skewness. In both sub-periods for R2000, the null hypothesis of no asymmetric effect is rejected, indicating that negative shocks have more of an impact than positive shocks. The study employs a VAR model to test whether a relationship exists between the macroeconomic variables and the conditional volatility of EREITS. The study finds that the CPI, IP, FD and RP all have significant explanatory power for REIT volatility for all periods, except for the R2000 Index in the pre-1993 period.

Considering the 2008 financial crisis, another strand of literature investigates the capital structure implications associated with the financial crisis. Huerta *et al.* 2016 examined the impact of liquidity crises and investor sentiment on REIT returns and volatility from 2001 to 2003. Due to the financial crises, the REIT industry experienced a significant erosion in its equity and debt capital raising ability during this period. Due to government regulations, REITs are constrained in their net income due to dividend payment requirements. Lending institutions primarily then become equally as important as capital markets during unfavourable times. The Noise Trading Theory is also considered in the study as security prices can also be affected by herding trading behaviour by individual or institutional investors. The study is based on the FTSE NAREIT US Total Return Index, which takes into account dividends and investor sentiment indices, which are retrieved from Thomson Reuters' data stream. The Fama and French (1992) factors, the default risk and term structure premiums are used as control variables. The study employs the GARCH-M to process the impact of the liquidity crisis on REIT returns and conditional variance, whereas the conditional variance equation includes a TARCH term to capture the asymmetric effects of volatility, along with changes in institutional investor sentiment and a dummy for the financial crisis period.

The study reports a significant and positive result for the crisis dummy variable in the variance equation that indicates that volatility significantly increased during the liquidity crisis. This indicates the view of the importance of capital markets or bank facilities for REITs to operate effectively. The TARCH term indicates significant and negative results, which implies that negative shocks have a larger impact on volatility than positive shocks, in terms of asymmetric effects. The model expands to include the Fama and French factor and indicates no explanatory power on REIT excess returns. The crisis coefficient remained statistically significant, even after controlling for the Fama and French bond factors. With regards to investor sentiment, the study indicates that both institutional and individual sentiments have a negative relation with volatility. This effect is reported at a larger magnitude for changes in institutional investor sentiment. This dominance is expected with institutional ownership in the REIT industry, furthermore, the market power of institutional investors. During the liquidity crises, the results indicate that institutional investor sentiment is significant and negative, whilst individual investor sentiment does not provide any explanatory power for REIT volatility.

Huerta-Sanchez and Escobari (2018) investigated a similar phenomenon and tested for asymmetric effects of individual and institutional noise traders on REIT industry returns and conditional volatility. The paper utilises De Long *et al.* 1990 and places the rationale that when the market sentiment expects upward price movements, then noise investors will significantly increase their holding on of risky assets, which raises market risk and therefore increases returns, while the effect is the opposite when expecting downward price movements. The model also believes that the magnitude and direction of the noise traders are imperative; the larger the magnitude of the change in sentiment, the larger the impact on return and volatility and the impact is larger in a bearish market than in a bullish market. Correlation analysis also indicates that there is a low correlation of 0.162 between institutional and individual investors, which suggests that the investors react differently to innovations and new information in the market, which further motivates the use of studying both in the role of noise trading on returns and volatility.

Over the sample period of 1993 to 2013, the study utilised both the FTSE NAREIT US Real Estate Index and the FTSE NAREIT US Total Return Index, where the FTSE NAREIT US Total Return Index takes into account prices and dividends. The study also used the Fama and French

(1992) factors and the Default Risk (DEF) and term structure premiums (PREM) as control variables. The model introduces a Threshold-GARCH (T-GARCH) equation to account for asymmetries, and an Exponential-GARCH (E-GARCH) equation to account for leverage effects. Results prove an asymmetric effect in bullish and bearish changes in institutional investor sentiments and none is found for changes in individual investor's sentiment. When considering heteroscedastic shocks, results indicate positive and significant effects of bearish institutional investor sentiment on excess return volatility, whereas none is found for bullish institutional investor sentiment changes. In the case of the individual investor sentiment, both bullish and bearish sentiments positively and significantly impact the excess return volatility of REITs.

Akimov *et al.* 2016 obtained similar results when investigating the impact of trading volume in REIT volatility for the period of 1990 to 2015 using Granger causality and GARCH-based tests. The results indicate that trading volume has explanatory power for REIT volatility and with the increase of the REIT maturity, an increase in volatility has also been observed. The study however also analysed the differing moments of REIT returns and volatility through monthly estimation of the first three moments. The Granger causality tests are employed on the full data set and the following sub-periods (1990-1994, 1995-1999, 2000-2004, 2005-2009 and 2010-2015). The results indicate strong evidence of volume-volatility relation, from volatility to volume and the reverse for volume to volatility, however, the latter is less marginal. The evidence is particularly strong for interactions in the post-2000 period.

The GARCH model is employed, with the variance equation adopting a volume variable. Overall, the empirical results indicate that the volume coefficient is significant, consistent with Cotter and Stevenson (2008). Furthermore, the study also investigates the relationship of skewness to volume and volatility and reports a significant relationship to not only volume but volatility. These results indicate the importance of trading volume in the REIT market. Rogers *et al.* 2014 also address the effect of trading volume, as a proxy for information, on REITs volatility within the GARCH framework in the period of 1990 to 2011. The study follows Lamoureux and Lastrapes' (1990) GARCH model that is able to incorporate a trading volume parameter. In contrast to Lamoureux and Lastrapes (1990) who study actively traded, low yield U.S. stock and Omran and McKenzie (2000) who study large UK stock, the study finds that volume does not completely suppress GARCH effects, instead, they are persistent for REITs, perhaps due to the characteristics that differentiate REITs from common stock; forced dividend pay-out, repeated issues of shares and relatively small market capitalisation.

Cotter and Stevenson (2006) modelled REIT volatility using a multivariate GARCH based model, where the advantage is using a multivariate framework that not only considers the volatility, but allows an estimation of time-varying covariance and correlations that could affect diversification opportunities. They use NAREIT daily indices from 1999 to 2003 for three sub-sectors of REITS, namely the Equity, Mortgage and Hybrid sectors. The motivation for the paper is primarily concerned with the interlinkages between REITs and other equity sectors so the following indices are also considered; S&P mid-cap Growth Index, S&P mid-cap Value Index, S&P mid-cap Growth Index and NASDAQ Composite. The S&P 500 is included as a proxy for the overall market, and particularly the large-cap sector, while the NASDAQ represents 'new economy' firms. Descriptive statistics indices indicate skewness and high kurtosis in the data set and are more pronounced for Mortgage REITs. The study carries out an empirical analysis using a multivariate GARCH model (BEKK specification) that can model interlinkages between the different assets, in terms of their second moment. Descriptive statistics of the squared residual series suggest almost normality after fitting the BEKK specification with a large reduction in skewness and kurtosis statistics. However, the kurtosis values suggest that the GARCH model is not completely able to remove the fat-tailed feature of the returns data. Overall the results indicate that general equity market sentiment plays a strong role daily, with linkages between REITs and value stocks being less evident.

Cotter and Stevenson (2007) examined the dynamics of volatility in the US equity REITS market using daily data for the period of 1992 to 2005. The sample is broken up into three sub-periods (i) the period that accounted for rapid growth in the REIT sector-1992 to 1996, (ii) the prime period of the technology boom-1997 to 2000 and (iii) the post-2000 period. A set of equity indices are included for the influence of the equity market, the S&P500 as a proxy of the overall market and the NASDAQ composite index as a proxy for new economy firms. The S&P Small-Cap Value and Growth indices are also incorporated with the notion that EREITs are mid and small-cap stocks and would be expected to have more in common with value firms. Other variables included are the one-month T-bill, as well as dummy variables to account for the market crashes of 2000 and 2001, should any structural breaks have occurred. Descriptive statistics indicate non-normality through kurtosis and the Jarque-Bera test. All the returns are time-varying with further evidence of volatility clustering and increasing levels of volatility throughout the overall sample period. Empirical results of fitting the GARCH(1,1) model variance equation indicate that none of the sectors were significant influencers in the rapid growth sub-period, while in the technology boom period, the general market sentiment was influential, and post-2000, with the exception of the small-cap growth sector, all sectors were significant. Overall, the study also reports that a significant coefficient is established between REITS and value stocks.

Considerable attention has also been given to the presence of long-term memory in volatility; however, less attention has been given to real estate. Liow (2008) studied the presence of long-term memory volatility in international securitised real estate markets from 1992 to 2005. The study considers two issues: firstly, five econometric tests (Fox and Taqqu test, GPH Procedure, Robin test, modified R/S test and FIGARCH modelling) are performed on a volatility dataset for 20 world/regional/national real estate indices and compares the differences. Secondly, the study then considers weekly hedged real estate volatility datasets and compares them to the original securitised real estate indices. The existence of long-term memory in the datasets implies autocorrelation processes, which further implies some elements of predictability in volatility behaviour.

Preliminary analysis indicates that the data for the original return distributions are significantly skewed in either positive or negative directions and further fat tails are detected with the analysis, indicating a high kurtosis value. Preliminary analysis of the hedged return distributions are reported and indicates that the standard deviation significantly reduces, as compared to the original return distributions. The Fox and Taqqu (FT) test for both the original and hedged real estate provides the best evidence in terms of long-term memory in volatilities, followed by the modified R/S method and Robin technique, whilst the GPH test provides the weakest evidence. In addition, the FT test indicates stronger evidence of long-term memory in the hedged versus the original dataset. The FT test provides an asymptotic approximation to the likelihood of an ARFIMA process in the frequency domain.

The study further employs a FIGARCH modelling on the series and finds that evidence of long-term memory in the majority of Asian markets is considered, while weak evidence is found for U.S., UK and other developed Western markets. The study also reports that in other instances across different countries, long-term volatility is weak for the original series, as compared to the hedged. Lastly the study concluded that the long-term volatility differs across countries and this might be attributable to economic and institutional differences. Overall these long-term memory effects could also be explained by factors such as bid-ask spreads, non-synchronous trading and institutional investors (Campbell *et al.* 1997 and Gabiax *et al.* 2006).

Zhou and Kang (2009) explored the topic of forecasting REIT volatility for the period of 1999 to 2008 using REIT FTSE daily returns. The study addresses models to specifically identify which one is most appropriate to predict REIT volatility. The study considers the following models (i) simple symmetric GARCH, (ii) EGARCH (iii) Stochastic Volatility (SV) (iv) Fractionally Integrated GARCH (FIGARCH) (v) Fractionally Integrated Exponential GARCH (FIEGARCH) and (vi) Fractional Integrated ARMA. The FIGARCH, FIEGARCH and ARFIMA models are included in the comparison, as they are considered long-term memory models in financial economics. In these models, the autocorrelation in volatility exists at significant levels and persists over long lags, as compared to the GARCH, EGARCH and SV models where the autocorrelation exhibits a fast decaying pattern. Descriptive statistics indicate that returns are time-varying with volatility clusters, and further indicate that volatility of REIT returns have been increasing over the sample period and one of the reasons may be due to the introduction of REIT-based derivatives, growth in the trading volume of REITs or macroeconomic fluctuations.

The study uses the Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE) models to evaluate the accuracy of the forecasts and results are based on six forecast horizons (1-,5-,10-,15-,20-, and 25 day). The best forecasts are defined as having the minimum RMSE and MAE statistics. The study finds that the ARIMA model provides the best forecasts for REIT volatility in all forecasts horizons and for both the evaluation criteria. Second in rank is the FIGARCH and thirdly, the SV model. Interestingly the asymmetric models-EGARCH and FIEGARCH are the worst performers. In conclusion, long memory models outperform shorter memory models due to their ability to approximate the true data generating process.

The prior studies imply that discrete volatilities in REITs are driven by the Consumer Price Index, industrial production, federal funds rate, risk premium, the liquidity crises, volumes (i.e. trading and stocks), non-synchronous, value stocks, equity stocks, new economy firms, and T-bill rate, which are some of the factors driving REIT volatilities. Some of those mentioned variables or parameters are currently not accounted for in the GARCH(1,1) model.

3.2.2.5 Literature related to hypothesis two

There are certain parameters that GARCH(1,1) does not take into account. Therefore, it is probably accurate to state that GARCH(1,1) does not necessarily take into account the entire volatility risk. Moreover, GARCH(1,1) model does not have any parameter that takes into account structural breaks (Bollersev 1986). Then, the question is whether structural breaks in the BRICS data exists or not, and how best to account for them.

3.3 The Integrable Structural Break Points

3.3.1 Introduction to the Integrable Structural Break Points

Both stakeholders (i.e. investor and investment manager) want to benefit from the performance of investments. One of the things that have been illustrated, that can either make or break investment returns, is *structural breaks* of any asset class (see Lumsdaine and Papell 1997; Perron 1989). Before Perron (1989), structural breaks were mostly inferred from events such as (i) the Great Depression, (ii) World Wars, (iii) breakup of colonial empires, and (iv) widespread belief in collectivism during the period of 1925 to 1975 (Avezki *et al.* 2014). Nevertheless, Perron (1989) illustrated that beyond market events, data can still be characterised by hidden structural breaks because of different reasons. This study uses data for the period of 2007 to 2017. Ten-year data is generally an acceptable timeframe for event studies (Koijen *et al.* 2017). The last economic crisis occurred in 2007/2008-the reason for this is that no scholar knows the exact date, but it is generally accepted that it was around that time. Therefore, one can state that a structural break occurred in 2007/2008. Noting from Narayan *et al.* (2013), when the exact structural break date occurred, one could simply use the Chow test to test for structural breaks. Gregory Chow pioneered the Chow test in 1960.

The line of research is nothing new; however, in the context of an alternative portfolio of assets, this type of research raises the following questions. Firstly, is there any link in different indices that leads to structural breaks? Secondly, which asset classes precede others in terms of movement when there are structural breaks and lastly, are structural breaks within or in between different asset classes in different markets? The novelty of this study is exactly responding to those questions. Most studies use mainly univariate models; ARCH and GARCH, and multivariate models, such as a VAR process to illustrate structural changes. This study uses Fourier and Laplace transforms to exemplify structural breaks. The Fourier transforms integrate values between negative and positive infinities, while Laplace transforms integrate values between zero and positive infinity. Thus, the structural breaks of returns will be illustrated using Fourier transforms, as returns can be negative. The structural breaks of volatilities will be illustrated using Laplace transforms, as volatilities cannot be negative. Fundamentally, integrals converge if there are no breaks and do not converge in the presence of breakpoints. Another compelling reason that supports the usage of the integral transforms is that normally, price changes lead to changes in returns. Thereafter, volatilities change. However, some studies separate structural breaks of returns and volatilities, although; logic tells us that there is an interconnected relationship. The closest study to this study is Enders and Hold (2012). They used the transforms for spillovers, while this study uses two integral transforms for structural breaks illustration.

The results go against the conventional findings on structural breakpoints. Firstly, there are hidden structural breakpoints in indices (i.e. bonds, commodities, equities and real estate). Moreover, hidden structural breakpoints do outnumber evident structural breakpoints. Fundamentally, structural breakpoints of index returns surpass structural breakpoints of index volatilities. Moreover, movements in index returns influence movements in index volatilities-to one's knowledge; this systematic pattern has not been established in stock returns and volatilities. This might be because indices are more encompassing than stock prices. That is, more parameters (i.e. inflation, dividends, debentures, economic growth, etc.) are included in indices, while they are not included in stock prices. Secondly, economic, political and governance structures influence the financial markets of countries-this is consistent with findings in prior microstructure studies. Thirdly, integral transforms reveal more structural breakpoints than conventional models. This is because structural breakpoints can be as high as in the twenties. Lastly, structural breakpoints from liquid (illiquid) indices are unsystematic (systematic)-this is a rare finding.

3.3.2 Literature Review to Integrable Structural Breaks

3.3.2.1 Bonds

Calice *et al.* 2013 explored liquidity spillovers of Credit Default Swaps (CDS) and sovereign bonds during the sovereign debt crisis during the EU zone. The authors argue that the debt crisis period offered an opportunity to understand spillovers during the 2010 debt crisis period. Another compelling reason that makes it interesting to investigate spillovers in the EU zone, is that numerous commentators have always argued that the borrowing levels of Greece and Italy are relatively high. Moreover, the governments of those two countries went further and borrowed money to pay off the salaries of public servants. According to their study, the events that occurred from August 2007 to May 2011, indicated the presence debt crisis in the EU. That included banks collapsing, EU countries applying for financial assistance from the IMF and countries' debt capacity increasing beyond 100%. That debt levels, go in hand with default scenarios and debt crisis capacity. One of the things that were evident during the period is that the bid-ask spread was wide and noticeable. Based on prior studies synthesised by Calice *et al.* 2013, there is a positive relationship between high bid-ask spread and default capacity.

For the modelling, Calice *et al.* 2013 used time-varying VAR (TV-VAR) and it contained four equations; two for the credit spreads in the CDS markets and bond, and two for each market's liquidity spreads. According to the authors, potential structural breaks in bond markets are unclear when diagnostic tests are used. Therefore, this study recommends not generally used techniques in identifying structural breaks, such as integral transforms. The challenge with bond data is that CDS is high frequency data; there are hidden intra-stale prices, high levels of microstructure containment and broken transaction information. Therefore, there is a chance that results will be a mixed-bag, according to the authors. They also used linear regression to calculate spreads of CDS and bonds.

The daily data on bonds is mainly on traded bonds of 5-year maturity in Western European countries (i.e. Austria, Belgium, France, Greece, Germany, Ireland, Italy, Netherlands, Portugal and Spain). The trades were from January 1, 2000, to October 1, 2010, resulting in 943 trading days. For CDS, given that some countries had incomplete information, they used the period of 2004 to 2010. In addition, the three parts of CDS data are; (i) the spread asked, (ii) spread bid and (iii) spread transacted. To improve their results and deepen the outcomes, they used a robust iterative re-estimation and Monte Carlo simulation. The descriptive results showed that post-2007, variance either decreased or increased, depending on the country. Moreover, for most countries, credit spreads increased on average and bond spreads were persistently high. In terms of first structural breaks, for most countries, the breakpoint was 2008, especially during March 2008. TV-VAR illustrated numerous structural breaks for the countries. The unit root tests revealed a mixed-bag type of results.

Furthermore, Calice *et al.* 2013 focused specifically on Greece's 5-year sovereign bond because the country had a perception of being over indebted, according to market commentators. Results revealed that Greece had deep problems of liquidity and sovereign debt crisis. The structural breaks were evident in the analysis for Greece. Given that Ireland was also experiencing debt problems, they analysed it in detail. The results on Ireland suggested temporary unobserved structural breaks. The majority of the structural breaks were observed from January 1, 2007, to October 1, 2020. The results of 5-and 10-year credit illustrated similar patterns as in the bonds markets. In Germany, Calice *et al.* (2013) suggested that its debt needs restructuring going into the future. Overall, their results revealed persistent spillovers among countries based in one region. This study explores transatlantic structural breaks.

Goda *et al.* 2013 studied how the U.S. bond affected the U.S. bond yield from 2004 to 2007. The crux of the question in Goda *et al.* 2013 is why long-term rates continue to fall despite the fact that federal funds rates have been increasing since mid-2004. The latter statement, according to the authors, illustrates the bond yield conundrum. Normally, the financial markets expected long- and short-term rates to move in tandem with each other. The prior studies synthesised by Goda *et al.* 2013 reveal contradicting views on the bond yield conundrum. The model used in their study is a multilinear formula to model macroeconomic essentials and those essentials related to financial risk on the other. The model is $y^l = f(i^s, \pi, \pi^e, g^e, rp, d)$, where y^l represents the long-term interest rate, i^s is the short-term interest rate, π is current inflation, π^e is inflation expectations, g^e is growth expectations, rp is the risk premium for the expected default risk and macroeconomic and financial volatility, and d is the investor demand for bonds. They included a further 16 variables.

The statistical tests include the Augmented Dickey-Fuller (ADF) unit root tests, stationery VAR and the Vector Error Correction Model (VECM). They estimated VAR outputs using OLS and stochastic pricing kernel. Goda *et al.* (2013) noted that other models such as the New Keynesian specification had limitations. The data used is for the period spanning from February 1994 to June 2007 on 10-year U.S. Treasury yield, 10-year agency bond yield and AAA corporate bond of Moody's bond index. Short-term changes were accounted for using a 3-month rate for Eurodollar deposits in London. Default risk was based on 5-year-ahead deficit-to-GDP expectations and Merrill Lynch Option Volatility Index (MOVE) Index. Further, on other parameters used in Goda et al. 2013, pages 119 to 120. The initial results based on unreported CUSUM and CUSUM of Squares Tests showed that there was a structural break in 1999. Other structural breaks were between 2000 and 2004. Based on numerous exposés, they believed that there was a possible structural break in November 1998. They controlled for increment in foreign and domestic 10-year Treasury bonds, as influenced by the long-term yields of bonds. The Quandt-Andrews tests showed a structural breakpoint.

The overall results from Goda *et al.* 2013 showed that an increase of foreign official holdings had a negative impact on the yields in the long-run period. The short-term interest rates had a positive effect both in the short-and long-run, but only after November 1998. The higher growth expectations increased with an increase in Treasury yields. Other variables had similar effects as the growth expectations. Goda *et al.* 2013 went further and looked at the role of agency, corporate and municipal yields. They argued that the impact of the variables is captured indirectly by the inclusion of the 10-year Treasury yields. Thus, their impact is similar to one of the 10-year Treasury yields. The bond yield, according to Goda *et al.* 2013, explained the fact that investor demand for the U.S. bonds tends to be high, furthermore, the subprime crisis that occurred in the U.S.

Tamakoshi and Hamori (2014) investigated changes in the volatility of the 10-year Greek sovereign bond index, including its structural changes. The Bai and Perron technique was used in their study. The interest in their study was triggered by shocks in the Greek sovereign bond market from the European debt crisis. At the heart of Tamakoshi and Hamori (2014) are the implications of excess volatility. Furthermore, they argue that increasing volatility disrupts the function of the financial system. The monthly data is on returns of the Greek 10-year bond index between April 1999 and March 2012-making it 156 observations. They used mainly the ADF test. The preliminary statistics illustrate that the skewness is negative-large decreases are more likely to occur than increases. The kurtosis was high and according to the authors, that kurtosis suggested frequent changes. For the actual testing of structural breaks, they used autoregressive generalised autoregressive conditional heteroscedasticity (AR-GARCH or AR-EGARCH). They employed the BP technique to incorporate structural changes.

The results of structural breaks in Tamakoshi and Hamori (2014) illustrate that occurred in April 2010, ‘when European debt crisis got exacerbated and Greek sovereign bond was rated a junk’ (P687). They claim that in that period, the Greek sovereign debt was downgraded to junk status. Another structural break was on May 2, 2010. This is when Greece was loaned a €110 billion bailout loan. Thus, based on the two days, it shows that stock market events lead to structural breaks. According to Tamakoshi and Hamori (2014), volatility increased substantially after April 2010. The results reveal that when dummies illustrate structural breaks, then one gets better estimates.

Claeys and Vašíček (2014) measured bilateral spillover based on the contagion on sovereign bond markets in the EU. The setting of Claeys and Vašíček (2014) is underpinned by the debt crisis in Europe. The salient contribution point from Claeys and Vašíček (2014) is finding out how contagious the volatility movements are. In measuring the spillover, Claeys and Vašíček (2014) used a factor-augmented version of the VAR (FAVAR) model. For structural breaks, they used the Qu and Perron (QP) technique. According to Claeys and Vašíček (2014), one of the advantages of the VAR model is that exogenous variables can be included together with control variables. On the other hand, they caution against the correlation of asset prices. Other points put forward by authors for users of FAVAR are (i) it is easier to identify structural breaks and (ii) contagion does not take into account economic behaviour. For measuring instruments, they proposed bond spreads as a measure of risk premium.

The analysed countries in Claeys and Vašíček (2014) are EMU countries (Austria, Belgium, France, Finland and the Netherlands), GIIPS countries (Greece, Ireland, Italy, Portugal and Spain) and Central European countries (the Czech Republic, Hungary and Poland) and the Eurozone 'opt-outs' (Denmark, UK and Sweden). The notable things about those countries are as follows; (i) for the EMU countries, spreads were moderate but rose since the financial crisis, (ii) for the GIIPS, spreads made refinancing impossible, (iii) for Central EU countries, their spreads were higher than EMU levels and (iv) for the 'opt-outs', their spreads were lower but volatile-Claeys and Vašíček (2014).

The basic statistics in Claeys and Vašíček (2014) illustrated that the financial crisis hit the EU in late 2008 and early 2009. Thereafter, it declined. Further analysis reveals that the spillover of those 16 EU countries amounts to a 59% variation in sovereign bond spreads. Hence, spillovers among EU countries is notable. In addition, they argue that the remaining 41% of spillovers are due to domestic factors such as spreads. The opt-outs contributed over two-thirds of changes in bond spreads. Overall, the EU bond markets are integrated and therefore, volatility spillovers of bond markets are evident. Other factors that contributed to volatility spillovers included volatility. Thus, the higher the volatility, the higher the spillover effects. The robustness tests confirmed the earlier results of FAVAR.

The study of Maghyereh and Awartani (2016) is different from other bonds studies in the sense that they focused on Sukuk (i.e. Islamic finance) and its relationship with bonds markets. Sukuk finance follows strict laws of Islam when investing. Thus, any product that is forbidden by Islam laws, implies that one should not invest in that product or related investments. According to Maghyereh and Awartani (2016), the Sukuk markets have similarities with bonds markets. Thus, ideally, the inclusion of Sukuk investments in bonds markets should not lead to diversification benefits. The crux of their study is to compare Sukuk products with bonds markets in terms of returns and volatilities. The other interesting part of their study is that the authors included two more products in their study; global equities and global Islamic stocks. Maghyereh and Awartani (2016) outlined key differences between Sukuk investments and bonds markets. For this study, the key is actual volatility transmissions as opposed to key differences. However, one should admit that their differences should influence volatility patterns.

Maghyereh and Awartani (2016) used a VAR with several variables. According to the authors, a standard Cholesky decomposition has shortcomings in exemplifying spillovers as Cholesky decomposition is affected by market orderings. The data is from the 30th of September 2005 to the 24th of February 2014. They concur that the global financial crisis started from September 2008 to December 2009. One thinks that this study will pick up the structural break that might demonstrate when the last financial crisis started. The period chosen by Maghyereh and Awartani (2016) is characterised by other sub-events-the European sovereign crisis period from April 2010 to June 2012 and the Arab spring from December 2010 to 2015. The data used for the analysis is from four indices; Dow Jones Citigroup Sukuk Index, Dow Jones Citigroup Global bond index, Dow Jones Islamic stock market index and Dow Jones Global stock market index.

The first set of results of Maghyereh and Awartani (2016), based on the Ljung-Box test, shows that Sukuk returns are highly correlated with bonds. For the forecasting part, aggregated shocks and total spillover seemed to be lower than expected. Thus, the world bonds markets contributed 3.9% spills to Sukuk, while Sukuk gives out only 0.2%. The volatility spillovers were higher between Sukuk and equities than between Sukuk and bonds. That has to do with similarities between similar assets. Generally, during markets downturn, correlation tends to be higher between different assets than similar assets. Partly, it has to do with the law of gravity. According to the authors, Sukuk are the overall net receivers of shocks. The VAR model illustrated various structural breakpoints including the overall patterns of spillovers. As part of doing robustness tests, they explored dynamic conditional correlations based on a dynamic conditional correlation model of GARCH (DCC-GARCH). They included dummy variables for structural breaks in the DCC-GARCH. The results confirmed a weak correlation between Sukuk and bonds and equities, even in the presence of structural breaks. Testing for jump and co-jumps confirmed the presence of structural breaks.

3.3.2.2 Commodities

Lumsdaine and Papell (1997) were some of the earlier scholars who included dummy variables in the specification to allow for structural breaks. According to the authors, most of the prior studies at that time focused on the endogenous variables that cause structural breaks at a specific point in time. Central to the study by Lumsdaine and Papell (1997) is using a unit-root hypothesis (especially the Dicky-Fuller 1979 test, hereafter DF) for testing for structural breaks. For the modelling, Lumsdaine and Papell (1997) allowed for two deterministic shifts with a lagged polynomial. The two dummies are represented by $DU1_t$ and $DU2_2$ occurring at times $TB1$ and $TB2$, respectively, and $DT1_t$ and $DT2_2$ are the corresponding shifts variables. To compute critical values, they used three models (i.e. one where all breakpoints are taken into account, one where all breakpoints are omitted and one where only one breakpoint is omitted), using 125 observations and 500 replications. Just like other similar studies, Lumsdaine and Papell (1997) used an autoregressive-moving-average model (ARMA), where the null hypothesis opines that there are no structural breaks. The results of Lumsdaine and Papell (1997) showed that there is a relationship between the unit-root hypothesis and structural breaks. Thus, structural breaks were common in time series data, even if there were market events. Using one endogenous variable, the results confirmed that structural breaks exist. Moreover, those structural breakpoints are reversible. Fundamentally, Lumsdaine and Papell (1997) did not compare the relative performance of models but the presence of structural breaks.

Bai and Perron (2003c) went further than Lumsdaine and Papell (1997) by investigating multiple structural change tests. That would imply that there is no specific test, which is superior to others when testing structural breaks. Some prior studies (i.e. Bai and Perron 1998) before Bai and Perron (2003c) used multi-linear regression including diagnostic tests as a way of illustrating structural breaks. Bai and Perron (2003c) focused on partial structural breaks under a general framework, which allowed ‘a subset of the parameters not to change’. One of the unique things about Bai and Perron (2003c) is that they created a trimming parameter $\left(\epsilon = \frac{h}{T}\right)$, where T is the sample size and h is the minimum permissible length of a segment. The critical value used is 5%-note according to a small percentage, such as 5%, which can lead to distortions when errors are segmented or when a serial correlation is permitted. The model used in Bai and Perron (2003c) is multi-linear regression that allowed different regimes. To maintain consistency, the authors made sure that variance breaks were at the same date as structural breaks.

To strengthen structural breaks results, Bai and Perron (2003c) tested 'a test of no breaks versus a fixed number of breaks' (i.e. used a supplementary F type of test) and double maximum tests (i.e. allows one not to have a pre-specified number of breakpoints). Moreover, they responded to surface regressions by opting for a class of nonlinear regression. A diagnostic test, such as adjusted R squared, is used. The results illustrated that time series are characterised by hidden structural breaks. The hidden notion, in this case, implies that there are no market events that occurred during the period of the time series. The trimming parameter allowed one to minimise errors, i.e. higher trimming parameter (20%) minimised heterogeneity error.

Lee and Strazicich (2003) opined that the work of Lumsdaine and Papell (1997) is derived when the null hypothesis assumes no structural breaks. That, according to Lee and Strazicich (2003) might not imply the rejection of the null hypothesis, but it might imply 'rejection of a unit without break'. Put it simply, that test in Lumsdaine and Papell (1997) leads to confusing outcomes. To do away with confusion, Lee and Strazicich (2003) argue that one should test the stationarity of breaks by allowing an endogenous two-break Lagrange multiplier (hereafter, LM) unit root test. Among the reasons put forward that support the usage of LM is that if one ignores breaks, there is a chance that they are ignoring breaks in two breaks, etc. For the modelling part, Lee and Strazicich (2003) went back to the work of Perron (1989). He used three models; (i) model A which allowed for 'crash', (ii) model B which allowed change in trend slope and (iii) model C which allowed change in both level and trend. Lee and Strazicich (2003) showed that analytically, the invariance property does not also hold for model C. The critical values are derived from 5,000,000 replications for exogenous breaks and 20,000 replications for endogenous breaks in the sample of T=100.

Furthermore, simulations are as follows. The exogenous test is replicated from 20,000 simulations and endogenous breaks are from 5,000 replications. Simulation results of exogenous breaks; firstly show that assuming that there are no breaks when those breaks do not exist is not a bad assumption. For Experiment Two of exogenous breaks, they investigated the invariance properties based on different locations (λ) and sizes (d). Lee and Strazicich (2003) stated that the null hypothesis confirmed over-rejections and those over-rejections increased with an increase in breakpoints. The authors argue that Lumsdaine and Papell (1997) (hereafter, LP) demonstrated a high power test for structural breaks. For endogenous breaks, the LM test performed well in the presence of breaks under the null hypothesis. When there are endogenous two breaks, the LM test showed important rejections. Simply, there was no systematic pattern of rejections versus breaks-it varies from one model to the next, based on certain assumptions. The lags were as much as 8. Overall, it implies that those unsystematic patterns of structural breaks endorse the notion that one should test structural breaks every time they work on time series data.

Miller and Ratti (2009) investigated stability, instability and bubbles in crude oil and stock markets. They argued that crude oil and stock markets are interconnected, predominantly oil influencing the stock markets. Some of the studies synthesised by Miller and Ratti (2009) showed that the relationship between oil and stock markets is influenced by improvement in energy markets and the conduct of fiscal and monetary policy. The authors used crude oil and international stock market data from January 1971 and March 2008. The data is on the Organisation for Economic Co-operation and Development (OECD) countries. The Vector Error Correction Model (VECM) with additional regressors is used for the modelling of that relationship. Miller and Ratti (2009; P561) opined that central to structural breaks are the ‘deterministic components of the cointegrating equations and the error correction equations’. Some of the structural breaks were due to stock market events-1973 and in 1979, stock prices fell sharply at the back of increasing oil prices. For the estimation of parameters, they used STATA software.

In order to specify and identify the modelling, Miller and Ratti (2009) chose several deterministic trends; (i) cointegrating rank, (ii) lag length, (iii) appropriate identification scheme for cointegrating space and (iv) the endogenous breakpoints. The preliminary results were as follows. There is a correct cointegrating rank selection as shown by the Hannan-Quinn criterion (HQ) and log of the Hannan-Quinn criterion (lnHQ). According to Miller and Ratti (2009), the support of the cointegration supports the usage of the VECM and VAR models. And parsimonious lag supported degrees of freedom for a selection of structural breaks. The main results revealed the following. Firstly, during the period of January 1971 and March 2008, there were no structural breaks identified. According to the authors, this is not because there are not structural breaks but long-term changes in crude oil and stock prices are synchronised and follow the same direction. However, when the full time series is divided into several mini periods; January 1971 to May 1980, June 1980 to January 1988, February 1988 to September 1999 and September 1999 to May 2008. For January 1971-May 1980 period, the authors stated that there were numerous price fluctuations in OECD countries that might have caused structural breaks. June 1980-January 1988: structural breaks might have been due to short-run coefficients, according to Miller and Ratti (2009). February 1988-September 1999: there was a steady decline in oil prices, with the exception of the Gulf War and that caused structural breaks. And finally, from September 1999 to May 2008, the natural and prevailing relationship fell apart and that led to structural breaks. Broadly, the results of Miller and Ratti (2009) illustrated that structural breaks are hidden, especially in long-term time series.

Enders and Holt (2012) explored the behaviour of commodity prices over a 50-year horizon. Their starting point is the report by the World Bank that illustrates that aggregated price indices, 1960-2010; energy, grains and oil have fat tails. Moreover, fat-tailed distributions became more prevalent in the 2000s. Some of the reasons that might have increased consumption of commodity related products are (i) high demand of growing economies (i.e. China and India) and (ii) concomitant "Westernisation"-people buying and consuming commodity products similar to trends found in Western countries. Other factors induce structural breaks in commodities, including weather changes, price speculation and expansionary monetary policies. The main contribution of Enders and Holt (2012) was the estimation of the econometric breaks dates and determining whether those breaks are stationary or not.

To model the structural breaks, Enders and Holt (2012) focused on three techniques; (i) Bai-Perron procedure, (ii) Fourier series approximation and (ii) Logistic Function components. Another complementary technique to the Fourier transform is the Laplace transform. For the study, the Laplace transform is used as it accounts only for positive parameters-remember that volatilities cannot be negative. To determine the presence of smooth shifts, they used (i) Bai-Perron stair-step function, (ii) KPSS-type test, (iii) Becker, Enders and Lee (2006) test (hereafter, BEL) and (iv) DF test. The empirical results show the following. Firstly, there are several structural breaks, especially ones resulting from the smallest residuals sum of squares. Secondly, the structural breaks occurred at specific dates. Furthermore, the mean reversion was almost twofold. Thirdly, the Fourier series showed structural breaks. Finally, they argue that the structural breaks seem to be driven by the consumption of commodity products, especially in countries such as China, India and other emerging countries.

Narayan *et al.* 2013 disentangled structural breaks in the commodity markets using dynamic trading strategies. They argued that the relationship between news (i.e. new information) and stocks markets is well documented. In investigating structural breaks in the commodities markets, they focused on two things; (i) futures markets assisting in transferring commodity price risk and (ii) spot prices can be forecasted from futures prices. In relation to the second point, it tends to be the opposite in most studies-futures prices are forecasted from spot prices. According to Narayan *et al.* (2013), structural breaks can be explained by the motivation theory and it is based on (i) interest rates, (ii) convenience yields and (iii) warehousing costs. To estimate spot prices, they use the forward price relation. Thereafter, they converted the relationship such that the spot price is the subject of the formula. Building on that latter phenomenon, they extended the relationship such that structural points can be incorporated in the pricing. Given that the structural points were known in advance, the Chow test is used. It was complemented by the *SupF* test statistic. Moreover, they used the double maximum test from Bai and Perron (1998). The historical data is on four commodities-oil, gold, platinum and silver. Note that the four commodities constitute around 75% of the commodity market-Narayan *et al.* 2013. For gold and silver, they had 11,649 observations spanning from 1st January 1908 to 22nd November 2011. Oil price, 10,418 observations, from 16th May 1983 to 11th November 2011. For platinum, 9087 observations, from 1st June 1987 to 22nd November 2011.

The results of Narayan *et al.* 2013 revealed strong evidence of predictability. Moreover, there is strong evidence of structural breaks evidence for all commodities, except for platinum. The weakness is that they limit structural breaks to one, while there could be more than one. In fact, results predict that there could be more than one structural break in each time series. Thirdly, results illustrated different changing regimes for all commodities. According to Narayan *et al.* 2013, the results confirm that structural breaks and different time series regimes influence the predictability of commodity futures returns. They went further and did a robustness test, including trading strategies based on the moving average technique. The robustness test confirmed the presence of structural breaks. Thus, structural breaks increased the profitability opportunities from trading.

Shalini and Prassana (2016) explored structural breaks in the Indian commodity markets during the financial crisis. In analysing structural breaks, they focused on dynamic; (i) volatility clustering and persistence, (ii) leverage effect and (iii) long memory. Long memory should usually decrease with maturity to time to expiration. At the heart of the argument by Shalini and Prassana (2016) is that complex channelisation is misunderstood. Interestingly, India is one of the major consumers of commodities and related products. On the structural breaks, the focus is on how different commodity regimes influence breakpoints using Markov regime-switching models. Other techniques used to illustrate structural breaks include Iterative Cumulative Sum of Squares (ICSS), wavelet analysis, and wavelet exponential Generalised Autoregressive Conditional Heteroscedasticity (wavelet-EGARCH, hereafter). The data is on spot prices of commodities including energy, precious metals, industrial metals and agricultural metals from 2005 to 2012. Note that they use logarithmic prices.

The results of Shalini and Prassana (2016), firstly, for the Markov regime model, show that structural breaks are influenced by different regimes. Furthermore, the first regimes are longer than the second regimes for most of the commodities. Interestingly, it took longer for most prices to converge to their long-term averages. One possible reason is that India uses most commodities-generally, prices of commodities should converge much quicker. For energy, structural breaks were evident during the 2008-2009 crisis periods, except for natural gas. The same phenomenon was evident for both industrial and precious metals during the same period. However, according to Shalini and Prassana (2016), industrial and precious metals had dissimilar convergence periods. Agricultural products were more characterised by numerous structural breaks than most commodities analysed in this study. Indices showed evidence of structural breaks.

Shalini and Prassana (2016) went further and explored volatility dynamics. Moreover, they found out that for energy there was persistence that increased to be negative over time. Post-crisis period, the results showed that energy did not show any significant signs of persistence or leverage effect. According to Shalini and Prassana (2016), energy shocks are short-lived. The results of metals were a mixed bag. The authors argue that this is because metals have different investment and industry uses. Moreover, some metals are substitutes while others have complimentary use. For agricultural products, there were significant persistence levels, except for castor oil-it is highly consumed in India, according to Shalini and Prassana (2016). The highly traded and constituted indices such as the S&P500 and Nifty 500 had high volatility persistence. Finally, long-term memory was not evident in most commodities.

3.3.2.3 Equities

Structural breaks can occur in parameters related to stock returns and state variables due to changes in market sentiment, speculation bubbles, regime changes in monetary policies, debt management policies and instability in predictive regression models. Rapach and Wohar (2006) examined the structural stability of predictive regression models using bivariate and multivariate predictive models of US stock returns from 1953-2000. The study employs the Andrews (1993) SupF statistic, Hensen (2000) Heteroscedastic Fixed-Regressor Bootstrap, the Bai (1997) and the J Statistic of Elliot and Muller (2003) in their analysis. Andrews (1993) developed a limiting distribution of the Supremum of the F_K Statistic that indicates that the statistic is non-standard and depends on a trimming parameter. For a certain value of the trimming parameter, the null hypothesis of no structural breaks can be tested. If the null hypothesis is rejected, the breakpoint can be estimated. Unlike the Chow (1960) procedure, this model can be utilised when a structural break is unknown. The disadvantage of the Andrews (1993) model was, due to non-stationarities, its inability to distinguish between the conditional distribution of the regression and the parameters of the regression model as well as the marginal distributional of the regressors. Bai (1997) then further developed a subsampling procedure that is able to detect and locate multiple structural breaks-however it is noted that some bias could arise from the model due to low power estimated in multiple structural breaks.

Non-stationarity becomes problematic as financial variables may have non-stationarity characteristics. Hansen (2001) introduces a heteroscedastic fixed regressor bootstrap procedure that accounts for the non-stationarity in the $\boldsymbol{F_K}$ Statistic. The bootstrap procedure treats regressors as though they are fixed to achieve size properties and allows for arbitrary structural breaks in regressors. The study augments both the Hansen (2001) and Bai (1997) methodology. Strong evidence is found for both bivariate and multivariate predictive regression models. Where significant evidence is found, the degree of predictability of stock returns can vary widely across regimes. Structural breaks also become imperative when modelling volatility as results from models could lead to wrong inferences.

Andreou and Ghysels (2002) evaluated tests for structural breaks by applying them to ARCH and SV-type processes. The empirical application considers the equity index, as well as the FX stock market. Firstly, the study considers the Kokoszka and Leipus (2000) test which can be regarded as a Cumsum-type test, where the Cumsum estimator $\boldsymbol{\hat{k}}$ is a consistent estimator of an unknown break in the presence of a single break. Secondly, the study considers the Lavielle and Moulines (2000) test, considered as a Least Squares type test, which uses a modified version of the Schwarz criterion that yields a consistent estimator for structural breaks. Both tests can be applied to testing for multiple structural breaks, however, the K&L test follows a sequential sample segmentation approach like Inclan and Tiao (1994) and Bai (1997), whereas the L&M test directly models multiple structural breaks.

The study proposes three types of extensions; (i) application of the Vector Autoregression Heteroscedasticity Autocorrelation Consistent (VARHAC) estimator (ii) application of tests to more precise measures of volatility, including high frequency data and (iii) sample performance via Monte Carlo simulation. The simulation design firstly compares the K&L test and the Inclan and Tiao (1994) test. Both tests have good power properties in detecting changes in GARCH parameters; however, they both suffer from some size distortion in the persistent GARCH case. However, the K&L test proves to be robust to outliers, unlike the I&T test that is adversely affected by outliers, where in most cases, change points are found in outliers. The simulation design further indicates that both the Cumsum-type test and Least Squares test are characterised by computational simplicity and have good power properties when considering a multiple break hypothesis, especially the L&M test.

Chen *et al.* 2017 tested structural breaks in factor augmented models where information from a large set of macroeconomic variables were parsimoniously modelled. The study considered the structural breaks of Andrews (1993) and Andrews and Ploberger (1994), where they propose the supreme Wald test statistics (supW), the mean Wald statistic (meanW) and the exponential Wald statistics (expW), however, these models are infeasible to factor augmented models due to the presence of unobservable factors. The study proposes a two-stage procedure that firstly estimates factors through principal component analysis (PCA) and utilises the retained components that are used as the underlying factors in the supW, meanW and expW to test for structural breaks. The study indicates that the proposed testing method produces the same asymptotic null distributions as observable factors.

When considering the stock market volatility, Eizaguirre *et al.* 2004 considered both transitory and structural volatility in the Spain stock market. According to Eizaguirre *et al.* 2004, for the past six decades, the Spanish stock market has indicated ARCH-type effects, not only due to the arrival of new information but also due to shifts in the unconditional volatility. Thus, suggesting the presence of structural breaks. The study implements methodologies based on the estimation of endogenous breakpoints. The baseline GARCH is considered first to test for structural breakpoints at unknown times in the parameters of the variance equation. In order to test structural breaks, the study considers the sequential application of Bai and Perron (1998, 2003a, b) consisting of locating breaks one at a time, conditional on the breaks that have already been located. The results indicate that the model captures the behaviour before the break, the increase in unconditional variance at the time of the break and volatility persistence.

For robustness checks, the study considers the two CUMSUM tests of Kokoszka and Leipus (2000) and Inclan and Tiao (1994). Both tests have the ability to test for structural breaks in the unconditional variance of a series. The findings indicate that both tests locate structural breaks at a similar time as the $Sup\ LR$ test.

Boamah et al. 2017 explored structural breaks in African equity markets and their response to global, country and industry factors. The study uses Quandt's test for unknown structural breaks by selecting an arbitrarily chosen potential breakpoint and thereafter adopts the Bai and Perron (1998) methodology to test for multiple structural breaks. Critical values are used to test the significance of Quandt's statistic. The study considers the Hansen (2001) procedure to estimate critical values through bootstrap replications. The Andrews (1993) critical values were considered, however it has been shown that the values could possibly lead to distorted inferences if there is a structural break in the mean or the variance of the regressors. Multiple structural breaks are found for three majorities of the countries and the global industry factor around the GFC and AFC period.

Kuttu (2018) examined long memory features in the volatility of the equity markets in Africa and used structural breaks to isolate spurious long memory. The study uses the FIEGARCH model that emanates from extending the FIGARCH and incorporating the EGARCH's asymmetric feature. The model does not need to satisfy non-negativity constraints on the estimate parameters for the model to be well specified. The study considers the three different statistics of Bai and Perron (1998, 2003c) consisting of (i) the supF test of no structural break $(\mathrm{m} = 0)$ versus $\mathrm{m} = \mathrm{k}$ breaks (ii) $\mathrm{supF_T}\ (\mathrm{l} + \frac{1}{\mathrm{l}})$ that considers l versus $\mathrm{l} + 1$ and (iii) the double maximum tests: $\mathrm{UD\ maxF_T}\ (\mathrm{M}, \mathrm{q})$ and $\mathrm{WD\ maxF_T}\ (\mathrm{M}, \mathrm{q})$. The three tests are based on three different information criteria (i) Bayesian Information Criterion (BIC), (ii) Modified Schwarz Criterion (LWZ) and sequential estimates. The Bai and Perron (2003c) methodology is based on sequential estimates of the $\mathrm{supF_T}\left(\mathrm{l} + \frac{1}{\mathrm{l}}\right)$ due to spurious results of the LWC and BIC test in the presence of serial correlation errors. The study finds long memory variance in all four markets, however after structural breaks were accounted for, no evidence of spurious long memory was found.

Tsuji (2018) considered structural breaks in the context of volatility persistence of stock returns in the US and UK markets from 2000 to 2018. The study applies both the GARCH and EGARCH models and uses dummy variables to investigate unknown structural breaks. To detect structural breaks, the study employs the Iterative Cumulative Sum of Squares (ICSS) algorithm, where the procedure is a systematic search for possible change points from the extremes towards the middle of the series (Inclan and Tiao, 1994). When considering both the GARCH and EGARCH, the volatility persistence parameter values decline when structural breaks are considered; thereby indicating that the non-consideration of structural breaks might lead to parameter overestimation of GARCH models.

Another approach captured in literature has been to account for structural breaks in the parameters of the models. The interface between unit roots, structural breaks and long-term memory is often important. Structural breaks have been documented to possibly explain the extreme persistence of volatility and can generate a series that "appears" to have a long memory. Unit root tests can also be viewed as a specific form of long-term memory and persistence tests can often be distorted in terms of their size and power properties in a series that contains structural breaks (Banerjee and Urga, 2005). Baillie and Morana (2009) introduced a long memory volatility process, which is designed to account for long memory and structural change in conditional variance processes. The study accounts for long-term memory and structural breaks in volatility processes using the FIGARCH and the Augmented FIGARCH (A-FIGARCH) model. The A-FIGARCH is considered an advantageous model due to its ability to allow for a stochastic long memory component and time-varying intercept that allows for structural breaks cycles and changes in drift. The structural break is particularly modelled by allowing an intercept to follow a slowly varying function. Their results indicate that the A-FIGARCH is superior to the standard FIGARCH when structural change is present and is inferior when there is no break.

Accounting for structural breaks in volatility forecasting becomes important as it can assist with improving forecasting methods. Babikir *et al.* 2012 considered the relevance of structural breaks in forecasting predictability in South Africa using both in-sample and out-of-sample tests on the Johannesburg Stock Exchange index. The study considers the GARCH model estimated using a rolling window and benchmarks it against three models; (i) GJR-GARCH that considers the leverage effect, (ii) MS-GARCH with transition probabilities and (iii) the FIGARCH. The ICSS algorithm of Inclan and Tiao (1994) is also incorporated as a competing model; however, the model can be oversized when following a GARCH process, due to it being designed to follow an i.i.d process. The study adjusts this by allowing the unconditional variance to follow a variety of processes by using a nonparametric adjustment, based on the Bartlett Kernel. The second competing model is the moving average model that is based on the average of squared returns over the previous 250 days to form the volatility for day t: $h_{t|t-1,MA} = \widehat{(\frac{1}{250})} \sum_{t=1}^{250} z_{t-\iota}^2$. The in-sample results reveal two structural breaks identified by the ICSS algorithm. The out-of-sample results indicate that the benchmarking models' ability to account for structural breaks, when forecasting, is statistically insignificant relative to the GARCH model with the expanding window. Furthermore, the recursive estimation of the GARCH(1,1) model is sufficient to account for structural breaks in parameter estimates.

According to Hillebrand (2005), structural breaks in volatility cause an upward bias in estimated volatility persistence and contribute to a higher estimated kurtosis that then produces non-normality in the data. The study considers that risk models are primarily concerned with extreme events and subsequently it becomes imperative that they can account for structural breaks. Hood and Malik (2018) use the VaR as a primary measure of downside risk and argues that a forecasting method that does not account for structural breaks will lead to incorrect inferences on forecasts of VaR. Through Monte Carlo simulations, the study explores the effect of a structural break in the unconditional variance on the non-normality of returns, estimated volatility persistence. The simulations indicate that inducing a structural break in normally distributed data causes excess kurtosis that drives the non-normality in returns. Furthermore, the simulations indicate that ignoring structural breaks in volatility produces misleading results and that accounting for these structural breaks, remedies the problem.

The study uses the ICSS algorithm of Inclan and Tiao (1994) to test for multiple breaks in the unconditional variance, however, the study also adjusts the algorithm through a non-parametric adjustment as the normal statistic can be oversized when applied to processes such as the GARCH. Findings of the ICSS algorithm identify six breakpoints and thereafter, incorporate these breakpoints in the GARCH by including a set of dummy variables in the variance equation and the semi-parametric Filtering Historical Simulation (FHS) model. Results also indicate that volatility persistence reduces considerably when structural breaks are accounted for in the GARCH model. The models are then used to compute out-of-sample forecasts of VaR. Overall the results indicate that incorporating structural breaks in both the GARCH and FHS models results in improved downside risk forecast.

Yin (2019) investigated parameter instability and model selection when forecasting the equity premium. Based on the study of Goyal and Welch (2008), weak results were obtained in out-of-sample forecasts. According to Yin (2019), the results obtained might be due to the uncertainty of model selection and parameter instability surrounding the data generating process. The study employs a stable linear regression model $Y_t = \beta_0 + \beta_1 X_{t-1}$, where Y_t is the premium and the parameter vector β is subject to a one-time discrete break at an unknown time τ bounded from both ends of the sample. For the structural break detection, the study uses a combination of SupF-type tests of Andrews (1993), Andrews and Ploberger (1994) and Hansen (2001). Methods such as the post-break window estimation are generally used to construct forecasts with estimated break dates; however, only observations detected after the latest structural break are used to train the predictive model. According to Yin (2019), break dates and size estimates are very likely to be imprecise, particularly when the sample size is small relative to the number of parameters. The study proposes using the estimator of Pearson *et al.* 2013-the robust optimal weights that integrate the optimal weights concerning uniformly distributed break dates and avoid uncertainty regarding the timing of parameter instability.

Empirical findings indicate that the structural break tests reject the null hypothesis of no structural breaks and overall the structural break tests indicate that only a subset of linear predictive regression models are subject to parameter instability. The study concludes that structural breaks cannot fully explain the weak predictive performance for all forecasting models considered in Goyal and Welch (2008).

Turtle and Zhang (2015) investigated portfolio performance in global equity markets in the presence of structural breaks, based on the notion that, in general, structural breaks produce erroneous inferences and portfolio decisions due to model misspecification. Using a general multivariate method of Qu and Perron (2007) of the $supLR_T$, the study examines diversification benefits in international market portfolios.

The sample data used consists of observations from 1988 to 2010 on the US market, developed markets and emerging markets. Firstly, the study begins with a multivariate structural investigation into the number of regimes detected in these markets with no accompanying tests of portfolio restrictions. The study reports a high $supLR_T$ that strongly rejects the hypothesis of no structural breaks and two breaks are identified in the series of all markets. The study finds that significant performances to a well-diversified portfolio can be obtained, however, structural breaks need to be considered.

3.3.2.4 Real Estate

Gerlach *et al.* 2006 investigated the impact of structural breaks on diversification within the Asian-Pacific real estate markets. The impact of structural breaks becomes imperative in the role of diversification; the existence of a structural break may disguise the true nature of the relationship between assets within a portfolio and thus has implications for diversification strategies. Gerlach *et al.* 2006 focused on the 1997 Asian financial crises and the interdependence amongst Asian-pacific real estate markets. The study uses cointegration tests that also account for structural breaks following Inoue (1999). The Inoue (1999) procedure allows for a test of cointegration rank within the presence of a mean- or trend- break. The test appealed to the authors due to its ability to analyse, in one pass, the number of common linkages that may exist amongst real estate markets of different countries, given the presence of an unknown structural break. The Inoue (1999) procedure resembles Johnson type tests of VAR models, however, the study extends to include trace and maximum eigenvalue statistics and provides asymptotic critical values for these to test if they perform better than Gregory and Hansen (1996) and the standard Johansen methodology.

In contrast with previous studies within the same region, findings indicate that there is no evidence of cointegrating vectors over the full sample period. According to Gerlach *et al.* 2006, a possible explanation of why no integration might be undetected is that structural breaks within the sample period may have distorted long-run trends that may be within the property markets. The study proceeds to implement the Zivot-Andrews(ZA) test that allows for the testing of a unit root in the presence of structural breaks in the series. The findings indicate that all the significant test statistics indicate a break date in 1997 that coincide with the Asian financial crises. These findings indicate that breaks in the series can have implications in co-integrating relationships and explain why no long-run relationships were discovered. When splitting the data into subsamples considering the break, evidence of cointegration is found.

Assaf (2015) investigated long memory and level shifts in REIT returns and volatility. The presence of long memory in asset pricing causes complications regarding the econometric modelling of asset pricing, statistical testing of pricing models, pricing of derivatives and tests of efficiency and rationality. According to previous literature, long memory is often confused with structural breaks. Long memory suggests constant unconditional volatility, while structural changes assume a significant change in the unconditional variance of a series. The study follows the two proposed tests of Shimotsu (2006) to distinguish between long memory and structural breaks.

The first test consists of estimating the long memory parameter over the full sample using the Wald test and over different subsamples and investigates whether the estimate of the full sample long memory parameter is equal to one of each of the subsamples. The second test requires the estimation of the long memory parameter and uses it to take the difference of the considered return series and further tests for stationarity of the different series and its partial sum using the Phillips-Perron (1998) (PP) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests. The empirical analysis is based on five real estate markets of the U.S., UK, Japan, Australia and Hong Kong over the sample period of 1993 to 2013. The study begins by employing the unit root test of Saikkonen and Lütkepohlrial (2002) and Lanne *et al.* (2002) and the null hypothesis of non-stationarity is rejected. The results further indicate breaks of data points for 2008 to 2009 and the ADF test, with and without time trends, reject the null hypothesis. Using the Bai and Perron (1998) statistics, the study finds weak evidence of structural breaks in the mean of the REIT series, except for the U.S., where five breaks were identified. Overall the results indicate that the REIT series is not characterised by multiple regime changes and match those results obtained from unit root tests with structural breaks. In terms of long memory, the overall findings indicate that REITs have an underlying fractal structure and does not follow the hypothesis of market efficiency.

Several authors have documented the interface between financial markets and economic variables; however, the majority of these studies have been carried out assuming that there is no structural break. Kim *et al.* 2007 investigated the empirical relationship between REITs and other macroeconomic variables. The study employs a VAR model and uses the $SupLM$ test of Andrews (1993) to test for structural breaks in the underlying coefficients of the VAR. The empirical analysis comprises of REITs, equity and macroeconomic variables returns for the period of 1971 to 2004. The $SupLM$ is firstly examined and concludes the presence of a structural break and the break date coincides with a monetary policy change that was implemented in October 1980. The authors proceeded to test for multiple structural breaks for each subsample period and findings indicate that the LM statistic is a less than critical value at 90% of the asymptotic distribution, thereby indicating no presence of any structural breaks. The study considers testing for structural breaks in a reduced VAR by reducing the endogenous variables and the evidence supports the existence of a structural break between two assets. Fundamentally, the monetary policy change of October 1980 accounts for structural breaks.

Wilson *et al.* 2007 explored the interdependence across real estate markets. Their starting point, before undertaking structural time series, was performing cointegration tests to determine any long run relationships that may exist in the markets. According to the authors, concerning the presence of integration, common trends in a series and previous research has indicated that structural breaks can yield misleading results in the search of common trends. The study employs the Inoue (1999) methodology that takes into consideration the possibility of trend breaks in the cointegration relationship; the procedure sequentially estimates a model that incorporates a step dummy to account for a structural break and the break date. Subsequently, the restriction is added to the structural time series model.

The empirical results show the following. The results indicate that the series are cointegrated with a possible break in trend, implying that there is a common stochastic trend in these markets over the long term, even though there might be short-run departures. The structural time series model is then used to undertake a simulation analysis to determine the spillover effects from hypothetical scenarios on stress levels from each property market. According to the authors, developed property markets are less likely to be impacted to the least extent through stress in the international property market and developed markets have the highest impact across the system.

Okunev *et al.* 2000 investigated the causal relationship between real estate and trends within the stock market. While previous research is unclear on whether real estate and stock markets are integrated or segmented, the study undertakes a comprehensive analysis on the distributional analysis of equity and real estate, the economic and statistically significant structural breaks within the series and the causal relationship between the markets.

Firstly, the study uses the ADF test to test the stationarity of the series and which indicates returns are stationary. A cointegration Engle and Granger (1987) test indicates no evidence of linear cointegration; however, through the non-linear cointegration test of Okunev and Wilson (1997), the markets are fractionally integrated. The ZA test for structural breaks is then implemented. If no structural break test is considered, linear causality tests will produce spurious results as the parameters' stability becomes questionable over the whole sample period. A significant structural break is identified in August 1989. The structural break coincides with prior studies that relate to two possible incidences in that period; (i) real estate returns were highly correlated with changes in global and domestic GDP and (ii) The S&L crises that occurred in 1989, which resulted in shifts in the financing of real assets.

Cheong (2009) explored the importance that explanatory factors have in driving the long-run trend of listed real estate. Similarly, the study follows the Inoue (1994) that model cointegration accounts for structural breaks. The study further combines this method with Granger and Gonzalo

(1995) and Johansen (1991) to test for permanent and transitory components in vector autoregressive systems.

The Inoue (1994) procedure allows simultaneous testing of hypotheses on the cointegrating rank of the system, compared to the Johansen (1988) procedure, the methodology estimates eigenvalue and trace statistics to be compared against critical values in order to determine the rank of the system, but in a complex manner. Reviewing the Johansen (1988) procedure, an autoregressive order is selected for the model and the output residuals are retained. The level series is then regressed on lags of the different series and then the output residuals are retained. In the last step, the squares of the canonical correlations between the two sets of output residuals are calculated and then used in the calculation of both the trace and maximum eigenvalues. In the Inoue (1994) procedure each time point is considered to contain a potential break and test statistics are calculated for each point in the series. The largest test statistic is then defined as the new trace or maximum eigenvalue statistic. Test statistics of the trace and eigenvalues for the Johansen and Eigen procedures are reported. When comparing the Johansen results to the Inoue statistics, it is evident that ignoring structural breaks can lead to an erroneous conclusion that there is no long-run relationship between any of the series, regardless of which property index is being examined. Furthermore, empirical results on the Inoue (1994) procedure identify a structural break in the period of 1996/7, which coincides with a period where there was a turnaround from there being a bull to bear market for securitised real estate assets.

Lee *et al.* 2012 applied the Granger causality test of Today and Yamamoto (1995; hereafter TY) to investigate the causal relationship between Taiwan REITs (T-REITs) and weighted stock prices. The TY (1995) is considered a modified WALD (MWALD) test in a VAR framework and the advantage of the test is that it does not require knowledge as to the cointegration properties of the system and can be applied in the absence of cointegration. To facilitate testing for Granger non-causality in level VARs, TY (1995) sets up a two-stage procedure. Firstly, augmented$(k + d)$ VAR is estimated, where d is the maximal order of integration. Secondly, the Wald test is applied to the first k VAR coefficient. According to TY (1995), this procedure minimises the risks perhaps associated with misidentifying the orders of integration of the series, or the presence of cointegration, while additionally, it minimises the possibility of distorting the test size that frequently results from pretesting.

The study begins by testing for unit roots in the presence of structural breaks. The study firstly considers using the extension of Zivot and Andrews (1992) of the ADF-type tests, however, it is indicated that ADF-type endogenous break unit root tests have a potential problem of deriving their critical values, assuming no breaks under the null. According to Nunes *et al.* 1997, this assumption can lead to size distortions in the presence of a unit root. The study also considers the two-break LM unit root statistic of Lee and Strazicich (2003) and results are reported for both methods. The results of both tests indicate that structural breaks occurred in three periods during 2007 to 2009. These periods are found to be linked to critical economic events, as well as the global financial crises. Zhou (2011) addresses two questions (i) whether observed long memory is genuine or caused by structural changes, and (ii) whether estimated long memory parameters are sensitive to the data sampling frequency. It has been suggested that structural breaks can induce strong persistence and thus appear to have a long memory.

In addressing the first question, they study two proposed tests of Shimotsu (2006). The two tests are based on the fundamental properties of the $I(d)$ process, where d is a long memory parameter. The first test is based on subsampling the properties of d. In splitting the sample, which would lead to the same d for each subsample as for the full sample. However, this property does not hold for spurious long memory processes, where the ds from the subsample is different from d of the full sample and differences increase as the degree of sample splitting increases. The second test is based on differentiating the properties of the $I(d)$ process, whereby the dth differentiated series would be stationary, but for a spurious process, it would be non-stationary. According to the authors, the proposed tests used are advantageous in comparison to methods that employ a two-step procedure by firstly avoiding directly detecting breaks when the underlying data generating process is unknown. The reason is that, when the underlying series is a genuine long memory process, spurious structural breaks could be detected and that would invalidate any analysis using a filtered break-free process for long memory testing. Overall findings indicate that the markets under study show the coexistence of pure long memory and structural breaks and varies across markets.

Liow (2009) investigated the long memory property in REIT returns. The study considers five econometric tests (i) Fox and Taqque test (ii) GPH fractional differencing (iii) Robinson test (iv) Modified R/S test and (v) FIGARCH test. Those tests require the tests of stationarity as non-stationarity can lead to the rejection of the null hypothesis of no long-term memory. Subsequently, the study employs the ZA test to consider structural breaks in our volatility series. Overall, the ZA procedure indicates that all unconditional variance and the majority of the conditional variance is trend-stationary over the period and models such as the FIGARCH long memory inferences are to be approached with caution, as such models assume strict stationarity. Zhou and Kang (2011) compared the relative performance of models in the forecasting of REIT volatility. The study considers the following models (i) GARCH (ii) EGARCH (iii) Stochastic Volatility (SV) (iv) FIGARCH and (v) FIEGARCH (vi) ARFIMA. Concerning the GARCH class models, the study employs both the symmetric and the asymmetric GARCH (EGARCH). The remainder of the models SV, FIGARCH, FIEGARCH and ARFIMA are considered post GARCH models, where the SV model differs from the GARCH models by following a stochastic process and the FIGARCH, FIEGARCH and ARFIMA are commonly referred to as long memory models.

The sample period of the study spans from 1999 to 2008 and in this period REIT volatility might have experienced structural shifts. To explore the structural breaks in REIT, the study employs Bai and Perron (1998) to detect multiple structural breaks. The procedure is applied to realised volatility, as per Zeiles *et al.* 2007, which by construction proxies the true volatility. The results indicate two breakpoints in 2002 and 2006 which correspond to the technology bubbles that occurred in 2002 and the onset of the subprime mortgage crises. The results indicate that long memory models outperform the short memory models in the forecasting of REIT volatility and the reason is that the strength of long memory models lies in their ability to capture long-term dependence in volatility.

Zhou (2016) tested the two hypotheses of the leverage effect and volatility feedback effect in the real estate market in both a linear and non-linear fashion. The econometric methodology considers the fact that leverage effect is return driven, whereas the volatility is volatility driven. The study uses firstly the linear Granger causality test, implemented in a VAR framework. In order to detect the nonlinear causality, the study follows the procedure of Baek and Brock (1992), which uses a nonparametric setting. Findings indicated in the linear Granger test that both the leverage and volatility effect are present, while the nonlinear test also indicated that the leverage and volatility effect exist in a nonlinear fashion. The study further analysed the causality of nonlinearity and considered multiple sources, such as regime-switching, structural breaks and outliers. In testing for structural breaks, the study employed the Double Weighted maximum test proposed by Qu and Perron (2007), which is different from other structural breaks tests, which can only be applied to single-equation regressions.

Findings indicate that structural breaks are amongst the multiple sources that cause linearity. Leverage and volatility effects are found in the REIT market and are found to be nonlinear. Moreover, the study finds that the leverage effect has a dominant casual effect. Liow *et al.* 2011 explored the importance of a multiple-regime time-varying asymmetric variance and covariance approach in modelling real estate security. The study begins by employing Bai and Perron (2003c) to identify multiple regime changes in the return and volatility series in REITs. Relative to other structural break tests, the study favours the Bai and Perron (2003c) procedure, as it allows for general specifications when computing test statistics and confidence interval levels for the break dates and regression coefficients. These specifications include autocorrelations and heteroscedasticity in the regression coefficients, as well as different matrices for the regressor in the different regimes.

In addition, the authors use the Multivariate Regime-Dependent Asymmetric Dynamic Covariance (MRDADC) model to detect the presence of mean-volatility linkages in real estate markets. Findings indicate the presence of structural breaks in the real estate market and using the $SupF_{i,T}(l+1|l)$ statistics, no evidence of more than one break is found in the series. The MRDADC findings indicate that there are volatility-linkages across real estate markets, which further have important implications for real estate valuations and portfolios.

3.3.2.5 Literature related to hypothesis three

The structural breaks determined in Hypothesis Three are during volatility spillovers. Thus, a few studies focused on structural breaks during volatility spillovers. And those studies used share prices instead of indices. Therefore, there remains a gap in how structural breaks of volatility spillovers can be modelled, especially volatility spillovers of indices. This is the question for future research agendas.

Chapter 4 Research Design

4.1 Research Philosophy and Approach

The aim of this study is to "appropriately model volatility spillovers in the BRICS countries, such that volatility risk is minimised". From the aim of the research the following objectives are identified; (i) To appropriately determine and model interdependent volatility spillovers of bonds, commodities, equities and real estate among the BRICS countries, (ii) To extend the GARCH(1,1) model, such that it can appropriately capture volatility risk of bonds, commodities, equities and real estate among the BRICS countries and (iii) to determine and appropriately model structural breaks of volatility spillovers in alternative investment classes.

This research involves analysing and interpreting data from bonds, commodities, equities and real estate industries. Under the positivist research approach, the role of the researcher is limited to data collection and interpretation, objectively. Furthermore, the positivism depends on quantifiable observations that lead to statistical analyses, where the researcher is independent of the study. According to Bisaro *et al.* 2018, it aims to provide an "objective and observer-independent knowledge" solution, backed by quantitative answers (P36).

4.2 Methodology

4.2.1 Hypothesis One

(a) VAR Model

Volatility and volatility transmission can be illustrated using most econometric models, including the VAR model (Liow 2015). According to Hammoudeh *et al.* 2003 and Syriopoulos *et al.* 2015, VAR is a widely used model illustrating volatility spillovers. The formula for VAR model is:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \mathrm{e}_t \qquad (4.2.1)$$

Where the l-periods back observation y_{t-1} is called the l-th lag of l-th lag of y, c is a $k * 1$ vector of constants (intercepts), A_j is the time-invariant $k * k$ matrix and e_t is a $k * 1$ vector of error terms satisfying $\mathrm{E}(\mathrm{e}_t) = 0$, every error term has mean zero. $\mathrm{E}(\mathrm{e}_t \mathrm{e}_t') = \Omega$, the contemporaneous covariance matrix of error terms is Ω (a $k * k$ positive-semidefinite matrix. $\mathrm{E}(\mathrm{e}_t \mathrm{e}_{t-k}') = 0$, for any non-zero k, there is no correlation across time; in particular, no serial correlation in individual error terms. In order to have a deeper insight in volatility spillovers, this article proposes using regime-switching models in order to capture different spills regimes.

(b) Robustness Test

Besides deepening the results from the VAR model, the Markov-Regime switching model illustrates the robustness of the results from the first hypothesis. The common model used for regime-switching variables is the Markov Switching Model (Fatnassi *et al.* 2014). The simple Markov model of conditional mean is presented when s_t denotes an unobservable state variable, assuming the value one or zero. A simple switching model for the variable z_t involves two AR specifications:

$$z_t = \begin{cases} \alpha_0 + \beta z_t + \varepsilon_t, s_t = 0, \\ \alpha_0 + \alpha_1 + \beta z_t + \varepsilon_t, s_t = 1, \end{cases} \tag{4.2.2}$$

Where $|\beta| < 1$ and ε_t are i.i.d. random variables with mean zero and variance σ_ε^2. This is a statitionay AR(1) process with the mean $\frac{\alpha_0}{1-\beta}$ when $s_t = 0$, and it switches to another stationary AR(1) process with the mean $\frac{\alpha_{0+\alpha_1}}{1-\beta}$ when $s_t = 1$. If $\alpha_1 \neq 0$ then the model admits two dynamic structures at different levels, depending on the value of the state variable s_t. In this case, z_t are governed by two regimes with distinct means, and s_t determines switching between two different regimes. The transition matrix for the Markov is:

$$\mathcal{P} = \begin{bmatrix} IP(s_t = 0|s_{t-1} = 0) IP(s_t = 1|s_{t-1} = 0) \\ IP(s_t = 0|s_{t-1} = 1) IP(s_t = 1|s_{t-1} = 1) \end{bmatrix} \tag{4.2.3a}$$

and

$$= \begin{bmatrix} p_{00} p_{01} \\ p_{10} p_{11} \end{bmatrix}, \tag{4.2.3b}$$

Where p_{ij} $(i, j = 0,1)$ denotes the transition probabilities of $s_t = j$ given that $s_t = i$. The transition probabilities satisfy $p_{i0} + p_{i1} = 1$. The matrix governs the random behavior of the state variable, and it contains two parameters (p_{00} and p_{11}). One can extend model (5) such that a more general dynamic structure is captured. Then model (4.2.2) is extended into:

$$z_t = \alpha_0 + \alpha_1 s_t + \beta_1 z_{t-1} + \cdots + \beta_k z_{t-k} + \varepsilon_t \tag{4.2.4}$$

Where $s_t = 0{,}1$ are still the Markovian state variables with the transition matrix (3a) and ε_t are i.i.d. random variables with zero and variance σ_ε^2. This is a model with a general $AR(k)$ dynamic structure and switching intercepts. For the d-dimensional time series $\{z_t\}$, eq. (3.2.4) can be re-written as:

$$z_t = \alpha_0 + \alpha_1 s_t + B_1 z_{t-1} + \cdots + B_k z_{t-k} + \varepsilon_t \tag{4.2.5}$$

While $s_t = 0{,}1$ are still the Markovian state variables with the transition matrix (3a), $B_i(i = 1, \dots, k)$ are matrices of parameters, and ε_t are i.i.d. random vectors with zero and variance-covariance matrix $\sum_0$. Eq. (4.2.5) is a VAR model with switch intercepts. Although generalisation is easy, some parameters, such as d variables, might be difficult to estimate. Fundamentally, the Markov-regime switching model has switch intercepts, which by extension, those intercepts should have different means and variances. Moreover, based on the supporting empirical analysis, means and variances of those regimes are different. The y_t and z_t mean the same variable, but this thesis used different notations for the same variable in two different models.

4.2.2 Hypothesis Two

(a) Introduction

The four sections of the literature review show that discrete volatilities are driven by risk premium, correlation, mean-reversion coefficient, economic growth, moments, regimes (bonds, commodities, equities and real estate), weather conditions, inventories, convenience yield and trading (heterogeneity, lagging, non-synchronisation and volumes). Other variables include clustering effects, risk premium, T-bill rate (interest rates), non-linearity, shocks (i.e., oil, IT booms), Consumer Price Index, industrial production, federal funds rate, liquidity crises, and new economy firms. Some of those mentioned variables are currently not accounted for in the GARCH(1,1) model. All used variables influence volatility movements in bonds, commodities, equities and listed real estate. In case the former statement does not hold, variables are chosen based on descending order of influence-from more to less industries. Therefore, the extended model will be widely applicable to the greater capital markets. Note that all chosen variables or parameters affecting dispersion of prices from their average price-this is what volatility is about. The expansion of GARCH(1,1) is based on economic heuristics.

(b) GARCH(1,1) Model

The formula for GARCH(1,1) is as follows:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4.3.1}$$

Where $\omega > 0$, α, β and $\lambda \geq 0$. And ω is a constant variance coefficient, α captures reaction of the volatility to market events, β determines the persistence in volatility and $\alpha + \beta$ determines the rate of converge of the conditional volatility to the long-term average level. Further, $\alpha + \beta = 1$ otherwise the model "explodes" and σ_{t-1}^2 is the spot variance. Note that eq. (1) does not account for (i) correlation coefficients of debt and equity, (ii) equity parameter, (iii) risk premium, (iv) interest rates and (v) shocks-stock markets. The capitalised GARCH will account for the shortcomings listed in the former statement. When GARCH(1,1) includes leverage (λ), the formula is:

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1} - \lambda)^2 + \beta\sigma_{t-1}^2 \tag{4.3.2}$$

Where λ captures the leverage effect. The variables that are included in expansion, based on economic heuristics are; (i) correlation coefficients of debt and equity-besides measure correlation of assets, correlations accounted for (de)smoothing (Cho *et al.* 2003), (ii) equity parameter-captures movements in equities, including subsequent shocks (Black 1986), (iii) risk premium-measures returns (especially expected and implied of any asset). (iv) interest rate is charged by debt lenders to earn extra money on debt issued. The interest rate referred to here is a risk-free interest rate. Further, the reason will become clearer as one goes through the expansion of GARCH(1,1). Note all added parameters are allowed to vary over time. Lu *et al.* 2014 illustrated that parameters time-vary show the appropriate scenarios; although, parameters are discrete. Sometimes, leverage is expressed as a ratio, commonly debt: equity ratio. This study adopts the same principle in the inclusion of equity parameter (δ). Thus, eq. (4.3.2) becomes:

$$\sigma_t^2 = \omega + \alpha\left(\varepsilon_{t-1} - \frac{\lambda}{\delta}\right)^2 + \beta\sigma_{t-1}^2 \tag{4.3.3}$$

The key issue in eq. (4.3.3) is that the original GARCH(1,1) still maintains all ratio symbolism for different parameters. Put differently, GARCH(1,1) is slightly tilted but it still maintains in its originality. The shocks due to δ will be explained in the latter part of the modelling section. On the other hand, the model captures movement in capital structures of firms. To one's knowledge, this is the first GARCH(1,1) type model that captures movements in capital structure, when the capital structure is made up of debt and equity.

The correlation relationship of debt and equity is simply illustrated as $\rho_{D:E}$. In the context of the real estate industry, correlation coefficient is affected by (de)smoothing or unsmoothing. Cho *et al.* 2003 studied (de)smoothing of commercial property using the Fisher-Geltner-Webb (1994) (FGW, hereafter) technique. The FGW technique assumes mean reverting log prices and accordingly adjusting weights. The latter statement resonates with prices plugged into the GARCH(1,1) model. The actual prices and returns that are used in the FGW model, in terms of parameters, are similar to GARCH (see Cho *et al.* 2003, 394-395). Although, Cho *et al.* 2003 did not state the composition of the capital structure of a firm, it seems that the extension of the FGW model is insensitive to the composition of capital structures. The lagging effect is exciting because GARCH(1,1) is generally lagged, especially for 1 period-most common. In order to strengthen the FGW model further, Cho *et al.* 2003 unsmoothed the index-results and showed that the unsmoothed index is very close to the index indeed. McConnell and Servaes (1995) investigated the relationship between equity and debt. They found that for high (low) growth firms, debt and equity are negatively (positively) correlated with each other. According to McConnell and Servaes (1995), those findings imply that debt and equity policy matters. Therefore, this study adopts a similar phenomenon-$\frac{\lambda}{\delta}$ is multiplied by $\rho_{D:E}$.

$$\sigma_t^2 = \omega + \alpha\left[\varepsilon_{t-1} - \frac{\lambda}{\delta}(\rho_{D:E})\right]^2 + \beta\sigma_{t-1}^2 \tag{4.3.4}$$

The second parameter that one adds to GARCH(1,1) is the risk premium. Given the discrete nature of GARCH(1,1), one uses the constant risk premium; $r_t = \mu + \eta_{t+1}$ where $\eta_{t+1} = \sigma_t \epsilon_{t+1}$. Note that risk premium can be either negative or positive. Negative risk premiums are black swans. Normally, they occur during great depression when firms are performing poorly-for example, during the 1960s depression in the United States. Pettengill *et al.* 1995 explored the conditional relation between beta and returns. Just like the conditional beta, Pettengill *et al.* 1995 used the Sharpe-Litner-Black (SLB) model to show dynamism of betas and returns. According to the authors, risk-adjusted returns (in principle risk premium) tend to be positive. Although not explicitly stated, positive (negative) risk-adjusted return is due to increase (decrease) in risk. This study adopts the latter principle. Some of the prices of securities included in Pettengill *et al.* 1995 were from firms whose capital structure comprised of debt and equity. Probably one of the reasons why the risk premium increases is due to the capital structure made of debt and equity. The third parameter that is added to GARCH(1,1) is r_t:

$$\sigma_t^2 = \omega + \alpha\left[\varepsilon_{t-1} - \frac{\lambda}{\delta}(\rho_{D:E})r_t\right]^2 + \beta\sigma_{t-1}^2 \tag{4.3.5}$$

Those interest rates differ from country to country and from institution to institution. Merton (1974) disentangled interest rate term structure in relation to corporate debt level. Note that corporate debt of firms used in GARCH(1,1) is normally reflected in prices. Moreover, overview from Merton (1974) is that the term structure gets more complex with increments in debt levels. At some point in time, corporates reach maturity levels in terms of amount of debt that they take on. In the context of GARCH modelling, it implies that interest rate (R) is earned on leverage level. Therefore, for extension reasons, the leverage parameter is multiplied by interest rate earned:

$$\sigma_t^2 = \omega + \alpha\left[\varepsilon_{t-1} - \frac{\lambda(R)}{\delta}(\rho_{D:E})r_t\right]^2 + \beta\sigma_{t-1}^2 \tag{4.3.6}$$

Eq. (4.3.6) is the expanded GARCH(1,1); namely capitalised GARCH(1,1). Given that GARCH(1,1) parameters are estimated from prices of listed securities, this study dwells on the same principle. The shocks of a firm are captured by its capture structure. Hence, GARCH models should incorporate changes in capital structures. The elegance of capitalised GARCH is that it captures all valuable parameters or information, which is central to making investment and business decisions. Secondly, the model is not country or sector specific flexibility embedded in the capitalised GARCH. Thirdly, it can be used both by buy and sell-side investors, and intraday and long-term investors. The only probable shortcoming of the capitalised GARCH is that it is discrete in nature; therefore, it is suitable for risk management and hedging. Generally, for pricing options, one needs continuous models.

(b) Robustness Test(s)

For Hypothesis Two, the robustness tests are based on family GARCH models; specifically, GARCH(1,1)-eq. (4.3.1) and spot volatility. Note that spot volatility, technically has no “formula” and it is the volatility that appeared in the market at that point in time. For example, spot volatility at time 2, is the current volatility that appears at time 2. Given that one wants spot volatility for the 2007-2017 period, in principle, one is looking for historical volatility of the same period. The advantage of mapping the relationship of spot volatility to any type of volatility is that hedging patterns can be picked up-Todorov (2019).

3.2.3 Hypothesis Three

(a) Integral Transforms

This study assumes that volatilities and returns in Fourier and Laplace transforms, respectively are represented by s parameter. The returns and volatilities are based on logarithms in order to be continuous because suppose function (f) is continuous and piecewise smooth. Given that returns and volatilities are calculated based on logarithms, one wants to continue in keeping them (i.e., returns and volatilities) in that state. Therefore, both $X(t)$ and $f(s)$ are e^{st}. The exponent (e^{t}) keeps functions in a logarithmic state. Note that in the e^{st}, one does not account for time in order for time not to influence structural breaks. Therefore, t is left out and e^{st} is simply written as e^{s}.The $f\colon \mathbb{R} \to \mathbb{R}$ is integrable over the real line $\mathbb{R}$, then the Fourier transform $\mathcal{F}[f(x)] \coloneqq \hat{f}\colon \mathbb{R} \to \mathbb{R}$ of f is defined by:

$$\hat{f}(x) \coloneqq \mathcal{F}[f(s)](x) \coloneqq \int_{-\infty}^{+\infty} e^{-isx} f(s)d(s) \tag{4.4.1a}$$

$$\coloneqq \mathcal{F}[e^{s}](x) \coloneqq -\sqrt{\pi}e^{s/2} \tag{4.4.1b}$$

And the inverse Fourier transform is given by the following formula:

$$f(x) = \mathcal{F}^{-1}\big[\hat{f}(s)\big](x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathcal{F}(s)e^{isx}ds \tag{4.4.2a}$$

$$= s \quad \underbrace{\hat{f}(x)}_{\substack{Fourier \\ Transform}} \tag{4.4.2b}$$

The formula for Laplace transform is:

$$\bar{X}(s) \coloneqq \mathcal{L}[X(t)](s) \coloneqq \int_{-\infty}^{+\infty} e^{-st} X(t)dt \tag{4.4.3a}$$

$$= \mathcal{L}[e^{at}](s) = \frac{1}{s-a} \tag{4.4.3b}$$

And the formula for inverse Laplace is:

$$f(t) = \mathcal{L}^{-1}[\mathcal{F}(s)] = \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] \tag{4.4.4a}$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{st} = e^{s} \quad (4.4.4b)$$

Where function $X(t)$ is defined for $t \geq 0$. Function $\bar{X}(s)$ is defined for all values of such that the integral converges, and one derives the function with respect to (w.r.t.) any variable of s. Eqs. (4.4.1b) and (4.4.2b) will be used to calculate structural break points of returns-illustrating both the algebraic and derivative structural breaks. Eqs. (4.4.3b) and (4.4.4b) will be used to exemplify structural break points of volatilities-illustrating both the algebraic and derivative structural breaks. Note, one assumes that structural breaks occur at maturity of some event. Similar inferences are drawn from Gurrib (2008). Note that Fourier and Laplace transforms are derivatives, while their inverses are algebraic formulas. In the context of this study, the inverses illustrate the structural break points, while Fourier and Laplace transforms will show the structural break point underpinned by the preceding structural break point. Note that for eqs. (4.4.1b), (4.4.2b), (4.4.3b) and (4.4.4b) s is chosen such that transforms converge. Sebehela (2014) used the same phenomenon. For those formulas that have a parameter just like eq. (4.4.3b), a parameter is the long-term average, given that both returns, and volatilities converge to their long-term average (Straub and Werning 2020, and Hasbrouck 2018). Given that eqs. (4.4.1b) and (4.4.3b) are Fourier and Laplace transforms, respectively are derivatives formulas, then calculated solutions/answers will be forward looking. For example, solutions will be telling us the future structural break points. Eqs. (4.4.2b) and (4.4.4b) are inverse Fourier and inverse Laplace transforms, respectively are algebraic formulas, then the calculated solutions will be current events.

(b) Robustness Test(s)

In order to test the robustness of the results of integral transforms, one uses the commonly used structural breaks tests to verify the presence of break points; (i) Augmented Dickey Fuller (ADF) , (ii) ADF test modified by Elliott, Rothenberg and Stock (1992) (ADF-GLS), (ii) Phillips-Perron (PP) and (iv) Zivot Andrews (ZA). The traditional view of a unit root hypothesis is based on the theory that current shocks have a temporary effect, and the long-run movements are not affected by such shocks. However, Perron (1989) challenges these findings and argues that the standard ADF tests are biased towards the non-rejection of the null hypothesis, based on the notion that most time series are not characterised by unit roots but rather persistence arises only from large and infrequent shocks. The ADF-GLS test is also a modification of the ADF test, whereas in this test it considers a series featuring deterministic components in the form of a constant or linear trend. The testing procedure allows for de-trending a series to estimate the parameters of the series. This procedure assists in removing the means and linear trends for a series that is not far from the non-stationary region point.

The PP test can be viewed as a modified DF unit root test that has been made robust to serial correlation in the error term by utilising the Newey-West (1987) heteroscedasticity and autocorrelation- consistent covariance matrix estimator. Under the PP test, similar to the ADF test, the null hypothesis is that of a unit root being present. The ZA is a proposed variation of the PP test that assumes that the exact time of the breakpoint is unknown. The ZA break date is where the t-statistic is most significant; this is where the t-statistic from the ADF test of a unit root is at its minimum. This is the break point where there is strongest evidence against the null hypothesis of a unit root.

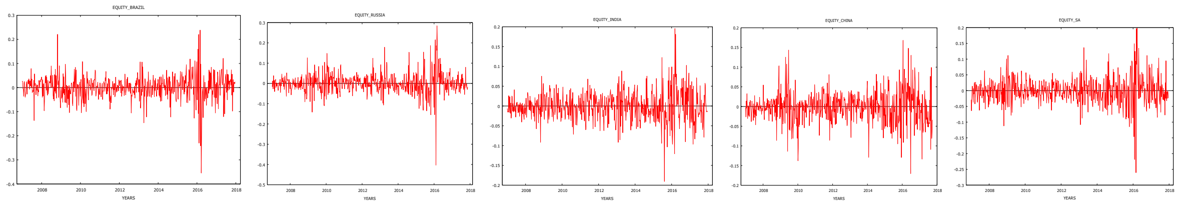
Chapter 5 Data

5.1 Background on Data

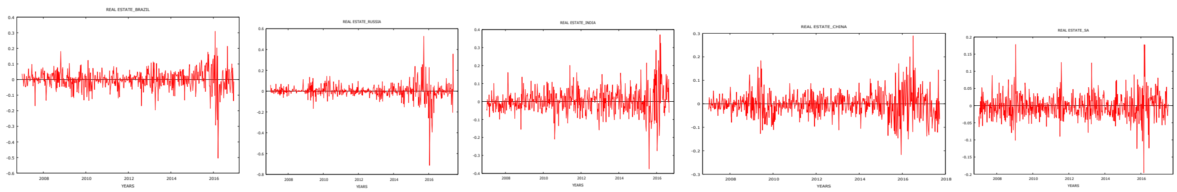
The weekly data is for the five BRICS countries (general equities, real estate, commodities and bonds) for the period of 1 January 2007 to 31 December 2017, obtained from Bloomberg. The out-of-sample is from 2007-2017 and in-sample from 2012-2017. The in-sample is for parameters estimation and out-of-sample for evaluating forecasting performance-Haigh and Holt (2002). The use of weekly data ameliorates concerns over non-synchronicities and bid-ask effects in daily data (Antonakakis and Kizys, 2015). The phenomenon of using returns to illustrate the descriptive nature of volatility spillovers is synonymous with Liow (2015) and Mensi *et al.* 2018. The returns are logarithm returns and they are consistent with the VAR model. All returns are calculated based on the indices of those countries. The indices are as follows; (i) general equities, Brazil IBRX 50 for Brazil, Moex Russian index for Russia, Nifty 50 for India, SSE50 for China and JSE top 40 index for South Africa, (ii) listed real estate, IMOB for Brazil, for Russia the index is created based on PIKK Group, PJSC LSR Group, World Trade Centre "ordinary shares" and World Trade Centre "preferred shares", because Russia does not have a listed real estate index-the market capitalisations of those firms where aggregated over time, Nifty Realty for India, SHROP for China and all Property index (J803) for South Africa, (iii) commodities, BM&F BOVESPA for Brazil, MICEX Oil and Gas Index-from the Moscow exchange for Russia, Nifty Commodities for India, CCI for China and JCGMSAG (gold mining index) for South Africa and (iv) bonds, for Brazil-Brazil 8 7/8 04/15/24 bond, Russia-RFLB 08/29/18 bond, India-Nifty 10yr benchmark, China-GTUSDCN15yr bond and South Africa-SAGB 10 ½ 12/26 bond. Skintzi and Refenes (2006) used indices to investigate regional and country shocks. This article is the first one that uses indices to illustrate shocks in the BRICS countries. According to Skinzi and Refenes (2006) one of the advantages of modelling volatility shocks using indices is that shocks are captured both as endogenous and exogenous variables. Just like Liow (2015) and Mensi *et al.* 2018 this article presents diagnostic analysis based on graphs as part of volatilities transmission investigation. The data is analysed using Gretl software.

Figure 5. 1: BRICS Log Returns

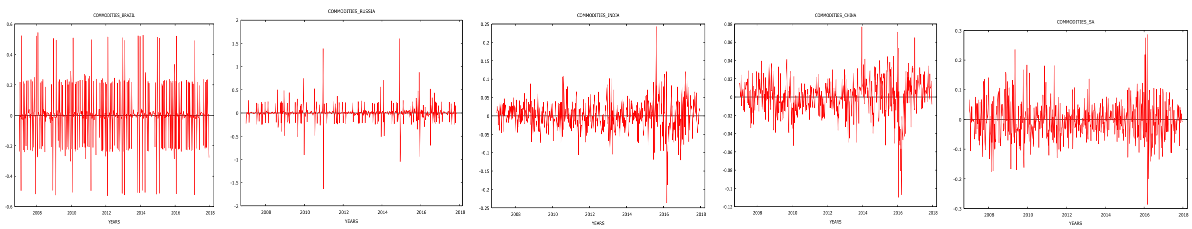
Equities



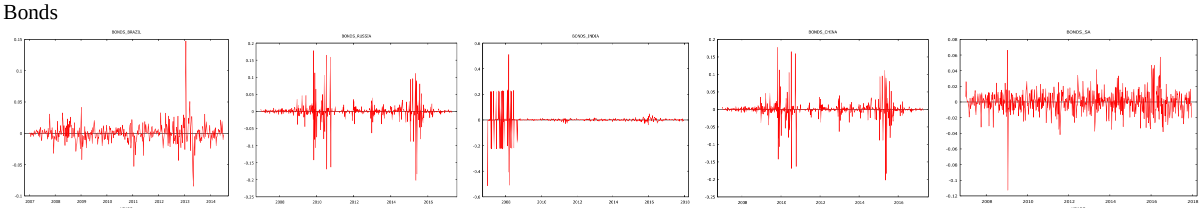
Real Estate



Commodities



Bonds



For every index per row, the first country is Brazil, followed by Russia, then India; thereafter, China and finally South Africa. A close inspection of fig.5.1 illustrates that the log returns of BRICS countries, as shown by different graphs, BRICS returns were characterised by spikes during the 2007-2017 period. The latter statement might be interpreted as the presence of changing volatility patterns and probably spillovers. Similar arguments were put forward by Liow (2015) and Mensi *et al.* 2018 on return patterns. The years are on the x-axis and the log-returns are on the y-axis. During 2008/2009, there was a global financial crisis that mainly affected Western countries-western Europe and U.S. were the hardest hit by that subprime crisis. According to the Bank of International Settlements (BIS), Brazil only reacted to the global subprime crisis after Lehman Brothers collapsed. Due to that reaction, there was panic in Brazil, which lead to the property market falling, but IBOVESPA rose by 20%-in local currency; local capital issuance stood around $165 billion-around 5.6% of Brazilian GDP. And bank credit increased to 36% from 32% during that period. Although, there were still spikes after 2009, they hovered around same levels until 2017. For Russia, one sees a similar pattern to Brazil. For both countries-Brazil and Russia, during subprime crises, real estate reacts more than other indices.

Does that imply that during subprime crises, volatilities are much higher in real estate? Volatility modelling will provide answer(s) to that. Similar patterns are observable about Indian and Chinese indices. However, India and China have very strong capital markets and those countries are self-reliant on financing countries' infrastructure. It seems that India and China tend to be insulated from external capital shocks (Biancomi *et al.* 2013). South Africa is a unique member of the BRICS, which joined through invitation. During the year of 2008, South African indices reacted to global capital markets movement; however, there was no subprime crisis effects felt in South Africa (See; Sebehela 2009). A study by PricewaterhouseCoopers (PWC) South Africa in 2016 illustrated that there was (i) a decline in new equity capital raised in South Africa, (ii) active and growing bond market in South Africa and (iii) a number of corporate transactions decreased in South Africa. The decline in commodities index during 2014-2016 can be attributed to a decline in commodities price and demand in commodities by South African trading partners. All those graphs illustrated diagnostic analysis on volatility spillovers. Now, the article takes the analysis further and it explores formative assessment of global transmission in the BRICS countries. The next section presents descriptive statistics of indices of the BRICS countries.

5.2 Data Description and Preliminary Statistics

Table 5.1 provides the descriptive statistics of the returns of general equities, real estate, commodities and bonds.

Table 5. 1: Descriptive Statistics

Description	Mean	Minimum	Maximum	SD	Kurtosis	Skewness	JB
Panel 1: General Equity							
Brazil	0.0007	-0.3547	0.2385	0.051	7.0571	-0.6941	1235.05
Russia	0.0009	-0.4031	0.2841	0.052	9.1638	-0.0484	1987.64
India	-0.001	-0.1906	0.1956	0.037	3.2816	0.2699	264.06
China	-0.001	-0.1704	0.168	0.04	2.1323	0.0068	106.28
South Africa	-6E-04	-0.2606	0.1984	0.043	5.1509	-0.0227	633.49
Panel 2: Real Estate							
Brazil	-0.002	-0.5044	0.3097	0.066	8.8189	-1.0306	1780.53
Russia	-0.001	-0.7145	0.5282	0.077	22.483	-1.3087	11613.2
India	0.003	-0.3752	0.3719	0.072	4.0292	0.2754	384.67
China	-0.002	-0.2161	0.2894	0.054	2.6673	0.3788	179.68
South Africa	-0.001	-0.1961	0.1781	0.037	4.2404	0.4225	428.42
Panel 3: Commodities							
Brazil	0.0002	-0.529	0.5435	0.177	2.0477	0.0046	100.11
Russia	-8E-04	-1.6035	1.6337	0.199	22.8521	-0.2209	12385
India	0.0009	-0.2369	0.2432	0.042	3.8767	-0.0754	359.35
China	0.0005	-0.1096	0.0768	0.02	4.0293	-0.7607	442.87
South Africa	-0.002	-0.2866	0.286	0.065	2.0176	0.3055	106.1
Panel 4: Bonds							
Brazil	0.0001	-0.0845	0.1474	0.016	21.5203	1.6562	7763.32
Russia	-2E-04	-0.2019	0.1777	0.029	20.4736	-0.7622	9466
India	-3E-04	-0.5151	0.5113	0.065	26.9833	-1.1651	17455.1
China	-3E-04	-0.1126	0.0661	0.015	7.2105	-0.5118	1266.32
South Africa	-1E-04	-0.2019	0.1777	0.029	20.4549	-0.7619	9431.2

Note: SD stands for standard deviation and JB for Jarque-Bera test for the return normality.

Panel 1 of table 5.1 indicates the equity information across all countries. Russia leads with the highest return at 28.41%, while China has the lowest maximum return of 16.80%. Over the full period, Russian equities are also the most volatile, with a standard deviation of 5.20% and the lowest volatile equities being that of China, at 3.72%. The distribution of returns over time is negatively skewed with the exception of India and China. In addition, for all countries, the excess kurtosis exceeds 3, indicating that the return series is leptokurtic, which is inconsistent with a normal distribution. The real estate data in Panel 2 for the five countries indicates Russia with the highest return of 52.82%, while South Africa closed off with a lowest maximum return of 17.81% return. Russia is the most volatile with a weekly standard deviation of 7.65%, while South Africa reports the lowest standard deviation of 3.68%. All five countries exceed the kurtosis of 3 and with the exception of Brazil and Russia, the data is positively skewed.

For commodities indicated in Panel 3, Russia reports the highest maximum of 163.37% in returns, while India reports the lowest at 7.68%. Russia commodity stocks are more volatile, with a standard deviation of 19.87% and the Chinese stocks are the least volatile at the standard deviation of 2.02%. All countries exceed the kurtosis of 3 and the data is negatively skewed, with the exception of Brazil and South Africa. In the bonds market, indicated in Panel 4, India has the highest return at 51.13%, while China has the lowest maximum return at 6.61%. India is also the most volatile with a standard deviation of 6.50% and China. The data is also leptokurtic and is negatively skewed, with the exception of Brazil. JB values in all panels (i.e. 1-4) illustrate that the four indices are abnormal and that can be interpreted as the presence of shocks. Liow (2015) stated the same view on JB values. The skewness values show that some countries have negative skews, while others have positive skews for different capital markets. That mixture of different skewness assists in hedging volatility, while positive skewness assists in generating high alpha and/or arbitrage opportunities (See; Sebehela 2016). The former phenomenon is ideal for risk managers, while the latter phenomenon is suitable for intraday investors-traders.

Chapter 6 Data Analysis

6.1 Analysis for Hypothesis One

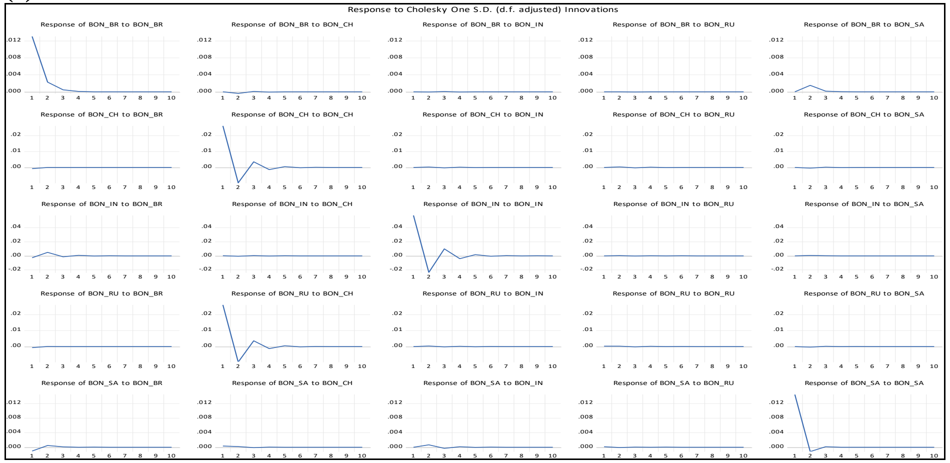
6.1.1 Results for Hypothesis One

In presenting the empirical results, the article starts with VAR calculations. Thereafter, Markov regime-switching results are presented in order to explore if one can infer interdependence of volatilities regimes.

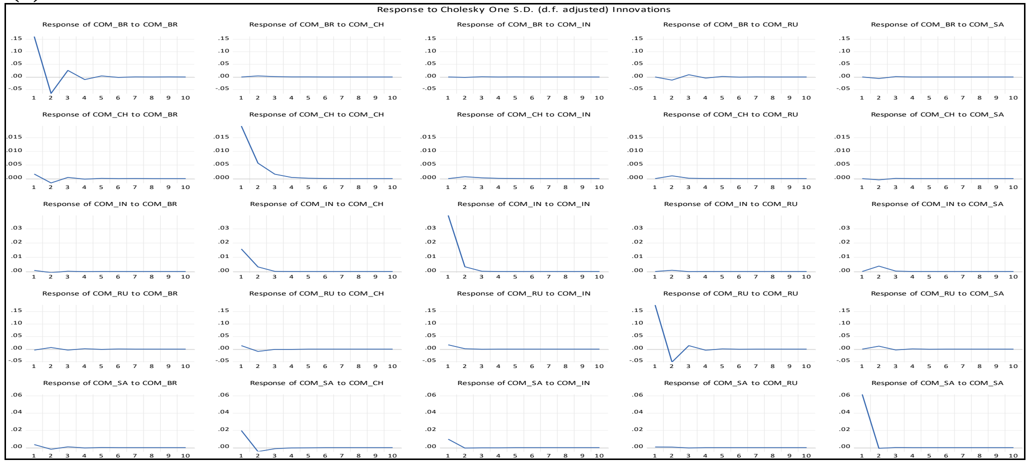
In order to verify which VAR is suitable, the first and second order tests (i.e. residual serial correlation) testing validity are used. Thereafter, a lag-length criterion is used. All those tests confirmed the appropriateness of the VAR(1,1) model. Further, in order to interpret the results, the Cholesky decomposition is used. Generally, when using the Cholesky decomposition, the order of VAR parameters matters. The BRICS countries are inputted in alphabetical order because that order coincides with normal writing order. Although, VAR results might be different when one inputs them in a different format, one views normal order as an appropriate one. It can be inferred from Van Griensven *et al.* 2006 that alphabetical order modelling leads to better estimates. In Tables 5.1 and 5.2 all variables highlighted in grey are statistically significant for VAR values, as they are at least 2, irrespective of being negative or positive. The F-statistic is basically Anova values and one reads them in the following manner. Assume the following inequality $F(2{,}12) = 22.59, p < 0.05)$, the 2 is the degrees of freedom numerator, 12 is the total observations of freedom denominator, 22.59 is the calculated Anova value and 0.05 is alpha (i.e. significance level). This article assumes that both the degrees of freedom numerator and total observations of freedom denominator are infinities in order to illustrate the best case scenario. In the latter situation, the critical value is 1.22. Thus, F-statistic values highlighted in grey fall within the non-rejection (i.e., acceptable) regions, while values which are not highlighted fall within rejection regions. That is, latter values exemplify auto-correlation for those VAR(1,1) models. Note that the VAR(1,1) results should be read in conjunction with the Cholesky decomposition.

Figure 6. 1: Out-of-sample Cholesky Decomposition

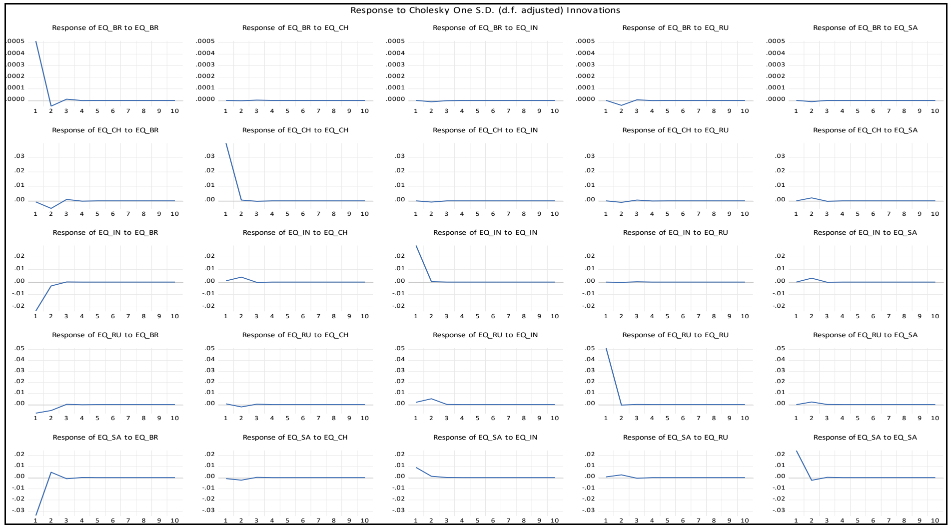
(a) Bonds



(b) Commodities



(c) Equities



(d) Real Estate

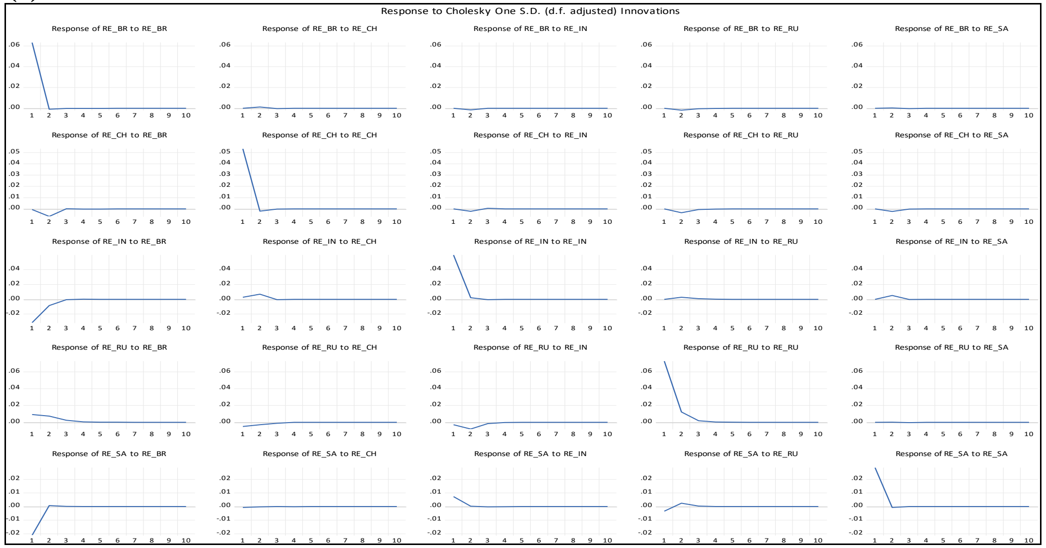


Table 6. 1: VAR(1,1): Out-of-sample Period (2007-2017)

Panel 5: Bonds					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.1833 (4.4095)	-0.0202 (-0.2429)	0.2881 (1.5718)	-0.0201 (-0.24301)	0.0332 (0.7153)
China	0.0077 (0.0038)	1.7959 (-0.4477)	-0.8782 (-0.0993)	-1.4175 (-0.3533)	0.2652 (0.1185)
India	-0.0004 (-0.0440)	0.0050 (0.2812)	-0.4140 (-10.5584)	0.0049 (0.2801)	0.0114 (1.1494)
Russia	-0.0226 (-0.0113)	1.4237 (0.3549)	0.8546 (0.0966)	1.0447 (0.2605)	-0.2573 (-0.1150)
South Africa	0.1055 (2.7839)	-0.0280 (-0.3701)	0.0383 (0.2292)	-0.0280 (-0.3697)	-0.0003 (-1.9232)
F-Statistic	5.2479	17.9005	23.0227	17.9331	1.1701
Akaike AIC	-5.8292)	-4.4438	-2.8618	-4.4433	-5.6106
Schwarz SC	-5.7830	-4.3973	-2.8157	-4.3907	-5.5643
Panel 6: Commodities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.4092 (-10.5168)	-0.0125 (-2.6652)	-0.0063 (-0.6138)	0.0339 (0.7887)	-0.0089 (-0.5621)
China	0.3609 (0.9718)	0.2844 (6.3448)	0.0436 (0.4442)	-0.5516 (-1.3461)	-0.2129 (-1.4044)
India	0.0028 (0.0162)	0.0159 (0.7539)	0.0670 (1.4443)	0.1208 (0.6232)	-0.0092 (-0.1288)
Russia	-0.0701 (-1.9894)	0.0059 (1.3848)	0.0048 (0.5099)	-0.2919 (-7.5058)	0.0033 (0.2270)
South Africa	-0.0943 (-0.8626)	-0.0078 (-0.5876)	0.0627 (2.1706)	0.1935 (1.6049)	-0.0114 (-0.2545)
F-Statistic	22.7883	11.7035	2.3085	12.2434	0.7016
Akaike AIC	-0.8059	-5.0351	-3.4670	-0.6091	-2.5984
Schwarz SC	-0.7597	-4.9888	-3.4208	-0.5629	-2.5522
Panel 7: Equities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.1469 (-2.0726)	-7.4597 (-1.3644)	1.2495 (0.2433)	2.5543 (0.3575)	7.1695 (1.2132)
China	0.0000 (-0.0920)	0.0178 (0.4235)	0.1028 (2.5982)	-0.0552 (-1.0039)	-0.0616 (-1.3525)
India	-0.0002 (-0.3129)	-0.0531 (-0.8988)	-0.0257 (-0.4625)	0.1499 (1.9421)	0.0724 (1.1336)
Russia	-0.0008 (-2.0001)	-0.0213 (-0.6589)	-0.0082 (-0.2684)	-0.0094 (-0.2233)	0.0520 (1.4883)
South Africa	-0.0004 (-0.4955)	0.0826 (1.2342)	0.1263 (2.0106)	0.0982 (1.1239)	-0.0959 (-1.3261)
F-Statistic	1.9912	2.5000	2.7793	2.7791	2.7064
Akaike AIC	-12.2983	-3.6080	-3.7334	-3.0728	-3.4525
Schwarz SC	-12.2520	-3.5618	-3.6871	-3.0266	-3.4062
Panel 8: Real Estate					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.0197 (-0.3642)	-0.1362 (-2.9988)	-0.0720 (-1.2489)	0.0276 (0.4412)	0.0031 (0.1009)
China	0.0236 (0.4708)	-0.0399 (-0.9456)	0.1344 (2.5099)	-0.0325 (-0.5584)	-0.0011 (-0.0391)
India	-0.0259 (-0.5738)	-0.0286 (-0.7487)	0.0162 (0.3345)	-0.1256 (-2.3889)	0.0097 (0.3717)
Russia	-0.0229 (-0.6359)	-0.0504 (-1.6582)	0.0466 (1.2075)	0.1679 (4.0062)	0.0332 (1.6033)
South Africa	0.0107 (0.1145)	-0.0780 (-0.9927)	0.1783 (1.7871)	0.0010 (0.0093)	-0.0233 (-0.0449)
F-Statistic	0.2134)	2.6753	3.8118	5.8949	0.6335
Akaike AIC	-2.6706	-3.0149	-2.5382	-2.3731	-3.7809
Schwarz SC	-2.6244	-2.9687	-2.4919	-2.3269	-3.7347

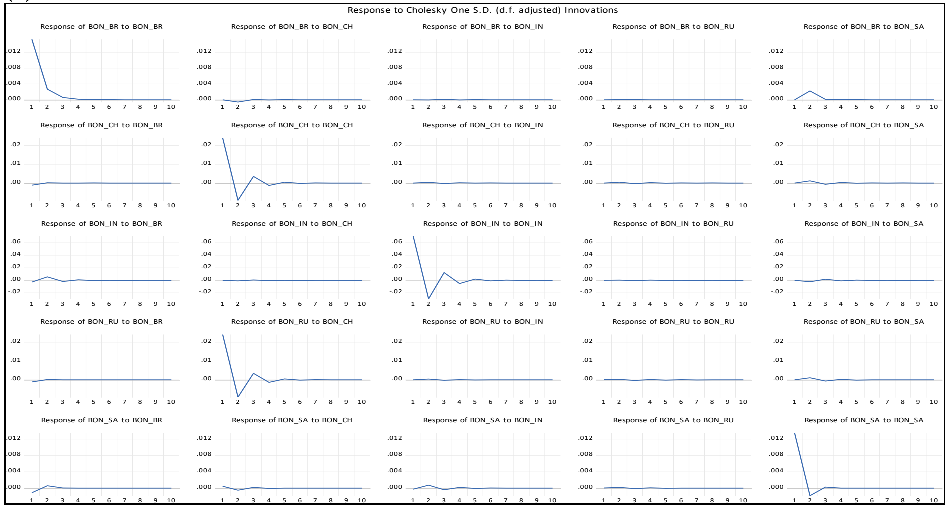
Note: in each cell, the first number is the coefficient and the number in brackets is the t-test. All variables highlighted in grey are statistically significant for VAR values as they are at least 2 irrespective of being negative or positive. The VAR results should read in conjunction with Cholesky decomposition as illustrated in fig. 5.1.

The results in Panel 5 of Table 6.1 illustrate that one-lag in Brazilian indexed volatility of bonds causes one-lag in Brazilian indexed volatility of bonds by 0.18 units. The latter statement is sensible given that what happens in one market should have similar effects in the short run-regimes show that regimes stay just over 2 weeks. Similarly, a one-lag in Brazilian indexed volatility of bonds causes one-lag in South African indexed volatility of bonds-this is probably because of similarities between the two countries, i.e., ruling political parties stay in power much longer; historically, Brazil and South Africa have good trade relations and further, the BRICS formation is strengthening that relationship even more. The one-lag in Indian volatility of bonds causes one-lag in Indian volatility of bonds. The phenomenon is similar with the one of Brazilian lags; however, the Indian one is negative, while the Brazilian one is positive. One possible explanation for India’s negative lag is that in India, the government is highly involved in driving economic growth, more than in Brazil. All other indexed volatilities of bonds in other BRICS countries are statistically insignificant. However, those latter results should be read with caution, using the Cholesky decomposition for curves for those countries to start at zero. Panel 6 of Table 6.1 illustrates results for commodities indexed volatilities. The statistically significant results are for Brazil and Brazil-this is for the same reasons as in Panel 5, Brazil and China-Brazil is the producer of commodities, while China is a consumer. This implies that the one-lag in producer of commodities indexed volatilities causes one-lag in consumer indexed volatilities, but not vice versa. More, the coefficient is negative because the effects spillover to the consumer from the producer. The results for China lags can be explained by the same reasons as the Brazil lags. Similarly, the one-lag in Indian index volatility causes a one-lag in South African indexed commodities volatility-the same as the Brazil and China one-lags. The Russian lags are the same as China lags. Note that China's lag with itself is positive, while Russia's lag with itself is negative. The positive lag for China's lag with itself is probably due to the economic influence that China has on the major world issues. The influence of Russia on major economic issues is limited. Thus, it might imply that South Africa needs to establish itself globally before the South African government can play a major role in South African economic issues.

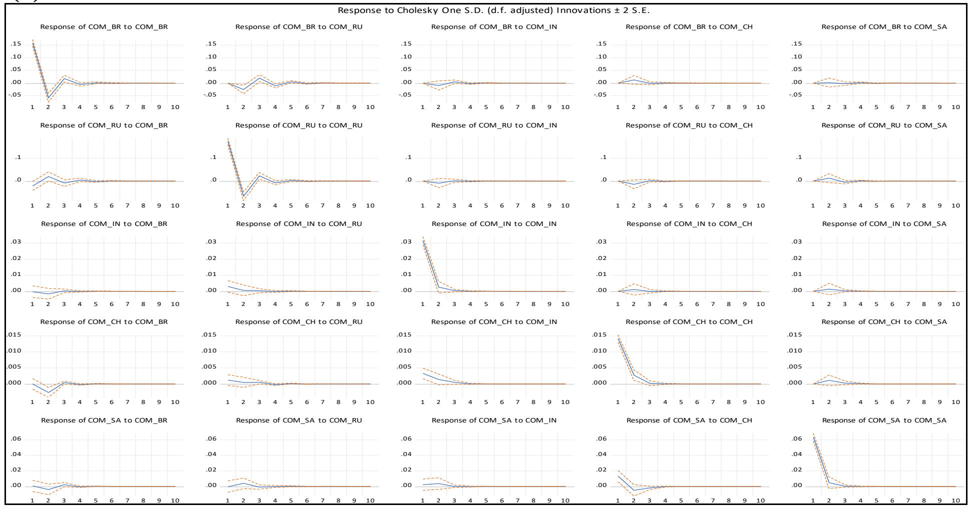
Panel 7 shows that spillovers which are statistically significant are for Brazil lags with itself-this pattern has been explained before, Brazil lag with Russian lag-in both countries, commodities firms are the main constituents of equities indices. And the causal relationship is slightly negative. Thus, 1 unit lag in Brazilian indexed volatilities emanating from equities causes a -0.0008 lag in Russian indexed volatility of the same index. The latter strategy is synonymous with hedging and speculation in equity markets. More, straddles work in a similar manner. Panel 8 shows the results of lags in real estate indices. The statistically significant lags are for Russia with itself-that pattern has been explained before, China and Brazil-Brazil is probably the most powerful economy in South America, while China is the second biggest economy after the U.S. China has been on major infrastructure projects including real estate, and many academics and practitioners have questioned whether the bubble is on the horizon in China. The negative coefficient is probably due to "overbuilding" in China. Indian lagged volatility causes a positive lag in China. The latter finding is probably due to ruling parties' influences in managing their economies. Interestingly, one-lag in Russian volatility causes one-lag in India. Normally, the collapse of currencies and commodity markets precede other capital markets products. Overall, one can see that volatility spillovers in the BRICS countries, based on four indices during the 2007-2017 period, exemplify opportunities to diversification opportunities-when indexed volatilities move in different directions and risk management opportunities-when indexed volatilities move the same direction.

Figure 6. 2: In-Sample Cholesky Decomposition

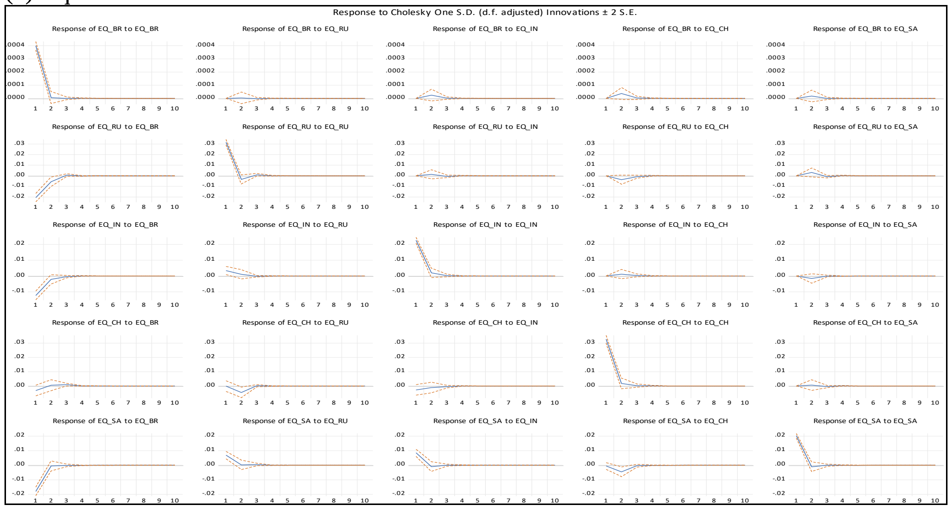
(a) Bonds



(b) Commodities



(c) Equities



(d) Real Estate

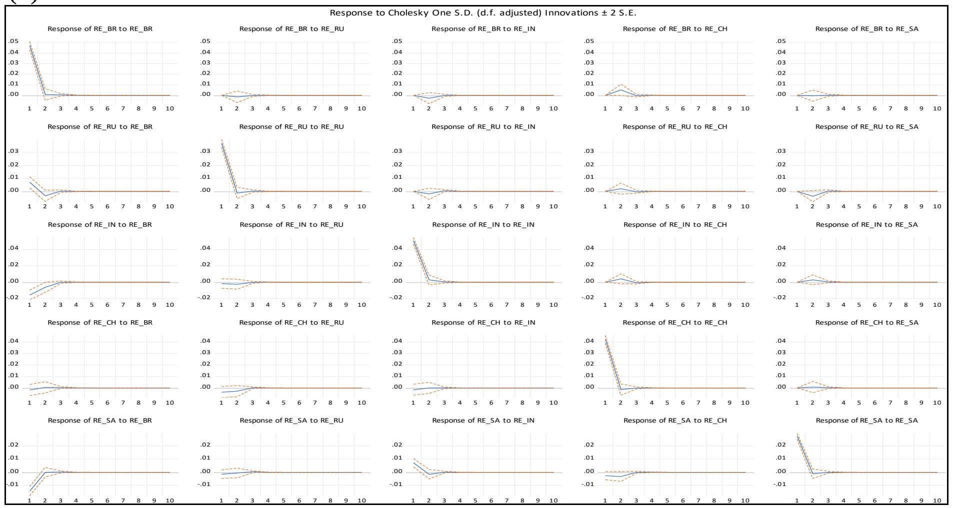


Table 6. 2: VAR(1,1): In-Sample Period (2012-2017)

Panel 9: Bonds					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.1876 (3.8632)	-0.0120 (-0.1567)	0.2688 (1.1956)	-0.0121 (-0.1578)	0.0278 (0.6435)
China	-0.0817 (-0.0304)	-2.0308 (0.4779)	-1.6449 (-0.1322)	-1.5335 (-0.3608)	-0.7866 (-0.3289)
India	0.0001 (0.0114)	0.0056 (0.3399)	-0.4205 (-8.7141)	0.0056 (0.3386)	0.0099 (1.0653)
Russia	0.0553 (0.0259)	1.6457 (0.3874)	1.6068 (0.1292)	1.1477 (0.2701)	0.7689 (0.3216)
South Africa	0.1664 (3.0025)	0.08445 (0.9643)	-0.1679 (-0.6544)	0.0837 (0.9552)	-0.1354 (-2.7462)
F-Statistic	4.5645	14.0761	15.7002	14.1098	2.0675
Akaike AIC	-5.5172	-4.6011	-2.4522	-4.6006	5.7506
Schwarz SC	-5.4585	-4.5423	-2.3935	-4.5419	-5.6919
Panel 10: Commodities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.3855 (-7.4290)	-0.0166 (3.4736)	-0.0097 (-0.9339)	0.0746 (1.2989)	(-1.0249)
China	0.8517 (1.3579)	0.1761 (3.0423)	0.0542 (0.4329)	-1.2171 (-1.7521)	(-1.6324)
India	-0.3983 (-1.3512)	0.0242 (0.8910)	0.0728 (1.2363)	-0.1864 (-0.5711)	(1.2772)
Russia	-0.1453 (-3.0541)	0.0010 (0.2295)	0.0015 (0.1561)	-0.3677 (-6.9804)	(1.1867)
South Africa	0.0280 (0.1981)	0.0177 (1.3538)	0.0216 (0.7648)	0.1967 (1.2551)	(1.2906)
F-Statistic	12.6282	5.3503	0.7579	11.8936	0.0651
Akaike AIC	-0.8229	-5.5888	-4.0445	-0.6187	-2.6074
Schwarz SC	-0.7506	-5.5165	-3.9722	-0.5464	-2.5350
Panel 11: Equites					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.0795 (1.0222)	-6.1630 (-0.9579)	-3.1423 (-0.6267)	-15.9395 (-2.1685)	-4.7373 (-0.8339)
China	0.0011 (1.6271)	0.0537 (0.9391)	0.0342 (0.7611)	-0.1100 (-1.6849)	-0.1395 (-2.7636)
India	0.0009 (0.8208)	-0.0566 (-0.6371)	0.1208 (1.7324)	-0.0125 (-0.1229)	-0.0386 (-0.4921)
Russia	-0.0002 (-0.2543)	-0.1442 (-2.3803)	0.0363 (0.7626)	-0.1418 (-2.0488)	0.0223 (0.4165)
South Africa	0.0009 (0.7747)	0.0287 (0.3102)	-0.0790 (-1.0876)	0.1526 (1.4447)	-0.0512 (-0.6272)
F-Statistic	0.8648	1.5195	1.2508	3.1221	1.6589
Akaike AIC	-12.7912	-3.9589	-4.4415	-3.6926	-4.2078
Schwarz SC	-12.7188	-3.8867	-4.3692	-3.6203	-4.1355
Panel 12: Real Estate					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.0041 (0.0635)	0.0284 (0.4825)	-0.0729 (-0.9954)	-0.1126 (-2.1609)	-0.0207 (-0.4699)
China	0.1213 (1.8983)	-0.0306 (-0.5308)	0.1037 (1.4455)	0.0393 (0.7707)	-0.0794 (-1.8378)
India	-0.0482 (-0.8873)	-0.0046 (-0.0943)	0.0426 (0.6990)	-0.0174 (-0.4013)	-0.0282 (-0.7678)
Russia	-0.0309 (-0.4209)	-0.0771 (-1.1644)	-0.0512 (-0.6213)	-0.0362 (-0.6179)	-0.0266 (-0.5363)
South Africa	-0.0128 (-0.1304)	0.0293 (0.3308)	0.1061 (0.9652)	-0.1355 (-1.7336)	-0.0415 (-0.6271)
F-Statistic	1.0215	0.3552	0.0529	1.5103	0.8958
Akaike AIC	-3.2510	-3.4551	-3.0200	-3.7027	-4.0345
Schwarz SC	-3.1787	-3.3828	-2.9477	-3.6305	-3.9622

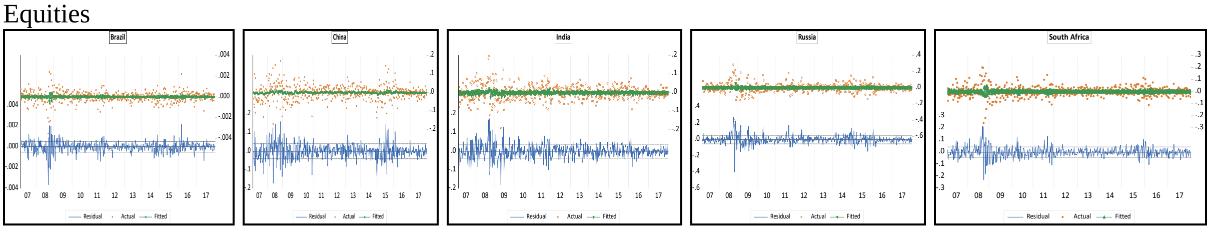
Note: in each cell, the first number is the coefficient and the number in brackets is the t-test. All variables highlighted in grey are statistically significant for VAR values as they are at least 2 irrespective of being negative or positive. The VAR results should read in conjunction with Cholesky decomposition as illustrated in fig. 5.2.

The influence of Brazil's lag to South African's lag during the period of 2012-2017 is the same as during the 2007-2017 period, as illustrated in Panel 9. The period of 2012-2017 was largely a bull market, while 2007-2017 had some bearish years, i.e. the 2008/2009 period. This implies that indexed volatilities of bonds during out-of-sample reflect similar patterns as in-sample periods. The sample phenomenon can be advocated on the influence of the one-lag of Indian volatility on the lag of India. The interesting result in Panel 9 is the one-lag of SA with itself- during the in-sample period, the lag is influential. During 2012-2017, the South

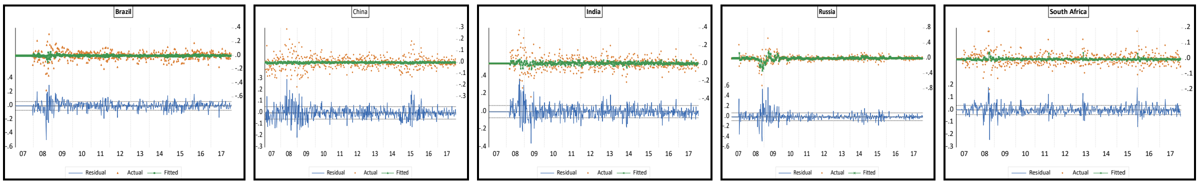
African long-and short-term yields were on an upward trajectory. This is probably why one-lag for SA during the in-sample period had a casual effect. For commodity indices-Panel 10, the results are the same as in Table 6.2, except for the one-lag of Brazilian volatility on one-lag of Russia. During 2012-2017, commodity prices were stable. In Panel 11, one-lag of China has influence on one-lag of Russia and one-lag of SA had an influence on one-lag on China-all the lags are negative. This is probably due to the declining consumption of commodity products by China. The rest of the results in Panel 11 are the same as in Panel 7. For Panel 12, only one-lag on Brazil has a negative influence on one-lag of Brazil. In short-run volatilities tend to be spikier than in the long run. That is, the volatility spillovers might be temporary. The filtred Markov-regime probabilities as shown in fig.6.3, shows how regimes moved over time.

Figure 6. 3: Filtered Regime Probabilities-Out Sample: 2007-2017

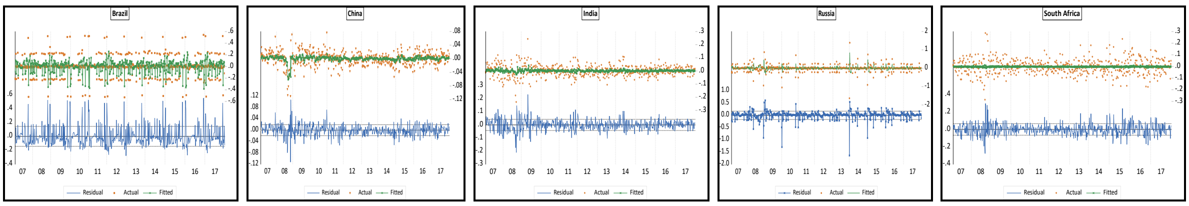
Equities



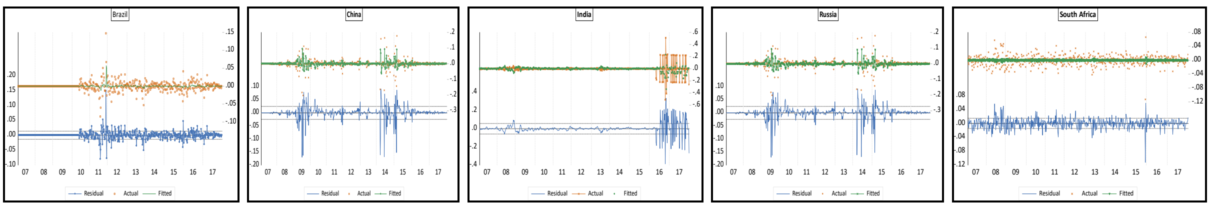
Real Estate



Commodities



Bonds



For every index type in fig. 6.3 in every row, the first country is Brazil followed by China and then India; thereafter Russia. The last country is always SA. For equity indices, all the five countries have main shocks in the 2007-2008 period, as illustrated by residuals. This is the period of the last subprime crisis. However, the actual date reveals a similar picture. The upward regimes in all countries were during the 2007-2009 period. It can be inferred from Liow (2015) and Mensi *et al.* 2018 that when indices move in the same direction, the volatilities should follow a similar pattern. However, those graphs do not tell one from and to which are the volatilities. The equities volatilities in Brazil and South Africa seem to hover around the same level during the entire out-of-sample. Sebehela (2009) illustrated that the subprime effects of 2007-2009 in South Africa were minimal. From what was reported in media, Brazil never suffered much from the subprime effects of 2007-2009. Real estate indices show similar patterns as equities indices, except for China and India. Normally, during subprime crises, equities movements preceded real estate movements (see; Phillips and Yu 2011). The real estate indices of China and India show similar and strong patterns. One of the reasons for that is the BRIC relations between those two countries precedes the establishment of the BRIC countries. More, they have large populations and their respective governments are at the heart of driving those economies.

For the commodity indices, Brazil and South Africa have the most and similarly volatile indice patterns. One of the reasons for that is that Brazil and SA are rich in mineral resources. On the other hand, China and India consume most commodities products. Surprisingly, Russia had the most stable commodity index during the 2007-2017 period. Unlike Brazil and SA, Russia is mainly rich in oil, while the other two countries are rich in minerals. The bonds indices show similar patterns to real estate indices. Numerous studies illustrate that listed real estate exhibits traits of other capital markets, especially bonds. The patterns of bonds indices are dissimilar, except for China and Russia. It can be inferred that bonds volatilities of those two countries follow in the same direction. The graphs show diagnostic patterns and in order to have more depth, this article illustrates Markov transitions, as shown in Table 6.3, as part of robustness check. In most studies, transition probabilities and expected durations are used to illustrate Markov transitions.

6.1.2 Robustness Check for Hypothesis One

Table 6. 3: Markov Transition-Out Sample: 2007-2017

Panel 13: Equities										
Parameter	Brazil		China		India		Russia		South Africa	
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.5703	0.4297	0.5009	0.4991	0.4943	0.5057	0.5058	0.4942	0.5661	0.4339
2	0.4993	0.5107	0.5126	0.4874	0.5034	0.4967	0.5216	0.4784	0.6033	0.3967
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	2.3274	2.0436	2.0034	1.9507	1.9775	1.9864	2.0233	1.9171	2.3047	1.6577
Panel 14: Real Estate										
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.4983	0.5017	0.0000	1.0000	0.9827	0.0173	0.4689	0.5311	0.3442	0.6558
2	0.4893	0.5107	0.0244	0.9756	0.8896	0.1004	0.0210	0.9789	0.0074	0.9926
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	1.9932	2.0439	1.0000	40.9669	57.7020	1.1116	1.8827	47.5343	1.5248	134.6585
Panel 15: Commodities										
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.5206	0.4795	0.9093	0.0907	0.4737	0.5263	0.4965	0.5035	0.0000	1.0000
2	0.5024	0.4976	0.0021	0.9979	0.4544	0.5456	0.4998	0.5002	0.0141	0.9859
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	2.0857	1.9903	11.0248	485.6439	1.8999	2.2007	1.9859	2.0007	1.0000	70.8325
Panel 16: Bonds										
Constant Transition Probabilities	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**
1	0.4959	0.5040	0.2862	0.7138	0.9919	0.0081	0.4817	0.5185	0.5067	0.4933
2	0.0018	0.9985	0.4598	0.5402	0.7747	0.2253	0.3799	0.6201	0.4853	0.5147
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	1.9841	556.1991	1.4009	2.1748	123.8068	1.2908	1.9285	2.6323	2.0270	2.0605

Panel 13 (14) illustrates Markov transitions for equities (real estate), while Panel 15 (16) shows Markov transitions for commodities (bonds). For equities indices, for the four countries; Brazil, China, India and Russia, there is considerable transition dependence between the two regimes, as the original regimes start from as low as 0.50 and increases to as high as 0.57. The non-original regimes are as low as 0.50. Although the original regime for SA is 0.56 (relatively high) but the non-origin regime seems less dependent on the original regime. It can be inferred from Hamilton and Lin (1996) that volatilities exemplify when there are Markov transitions. The expected durations of all countries are approximately 2 weeks. The quickly changing patterns in equities would be expected, given that equities markets are quite volatile, as compared to other capital markets. For real estate indices, China and SA show an interesting pattern-the original regimes are very low but the non-original regimes are highly dependent on the original regimes. That rare scenario is hardly observable in most countries in the world. That could possibly be due to the influence of governments, which translates into financial markets in those countries. For Brazil, India and Russia, the two regimes seem to be dependent on each other. The expected durations for real estate indices show interesting results-the expected durations are shorter than their equities counter-parts, mostly for first regimes. That is highly unexpected. One possible explanation is that real estate indices in those countries are quite thin and represent a few constituencies. Similar inferences can be made from Byström (2006). For commodities indices, the original regimes and non-original regimes are dependent. South Africa is the only country that illustrates a unique regime-non original regime is not so much dependent on original regime. All the regimes, with the exception of China and SA, last for a few weeks. The reason why China and SA have longer accepted durations is because China consumes most commodities in the world, while South Africa is a country rich in minerals. The regimes of bonds indices of all countries seem to be dependent. One possible explanation for that is that bonds are the oldest market in the capital markets. More, bonds are used mostly in those countries to finance private and public infrastructure. The expected durations of Brazil and China are entirely longer. Probably those two countries use their bond markets frequently for their capital markets offerings.

Figure 6. 4: Filtered Regime Probabilities-In Sample: 2012-2017

For every index type in fig. 6.4 in every row, the first country is Brazil followed by China and then India; thereafter Russia. The last country is always SA. For equities indices, the later periods of China, Russia and SA show similar regime patterns. Thus, there is a possibility that equities indexed volatilities of those move from and to with each other. For all the five countries, in year 2014, equities indexed volatilities show similar movements. Most of the 2014 year was characterised by bull markets in most countries throughout the world. At that time, probably volatilities spillover among and in between each other. The real estate indices for all of the countries, with the exception of Brazil exemplify the same pattern. One possible reason is that listed real estate mimics similar movements (Titman and Warga 1986). So far, the diagnostic assessments illustrate that there is some relationship between indexed volatilities of equities (real estate). This might imply that volatilities of indices move together during bullish periods, more than bearish periods.

The indexed commodities volatilities of Brazil, China, India and SA exemplify similar movements. Brazil and SA are some of the main producers of minerals, while China and India are some of the main consumers of mineral products. The bonds volatilities in all countries show different movements. The indexed volatilities for the in-sample period seem to be more spiky than ones of the out-of-sample period. It can be inferred from Borenstein (2005) that volatilities flatten out in the long-run because of diversification benefits, which are more prevalent in the long-run. Broadly, the graphs of in-sample regimes are similar to ones of out-of-sample. Just like in the out-of-sample analysis for indexed volatilities, Markov transitions are calculated in order to deepen the insights on how indexed volatilities during the in-sample period behave.

Table 6. 4: Markov Transition-In Sample: 2012-2017

Panel 17: Equities										
Parameter	Brazil		China		India		Russia		South Africa	
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.5147	0.4853	0.4494	0.5506	0.9731	0.0269	0.9756	0.0244	0.0000	1.0000
2	0.5159	0.4841	0.5897	0.4102	0.9977	0.0023	0.6888	0.3112	0.3045	0.6955
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	2.0605	1.9382	1.8161	1.6955	37.1527	1.0023	40.9446	1.4518	1.0000	3.2839
Panel 18: Real Estate										
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.0000	1.0000	0.9847	0.0153	0.0000	1.0000	0.1127	0.8873	0.9934	0.0066
2	0.0544	0.9456	0.5366	0.4634	0.0343	0.9657	0.0744	0.9256	1.0000	0.0000
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	1.0000	18.3795	65.2176	1.8635	1.0000	29.1281	1.1271	13.4442	153.0187	1.0000
Panel 19: Commodities										
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.0000	1.0000	0.3956	0.6044	0.1097	0.8903	0.0189	0.9811	0.4853	0.5147
2	1.0000	0.0000	0.0466	0.9534	0.9999	0.0000	0.0000	0.9997	0.3379	0.6621
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	1.0000	1.0000	1.6545	21.4513	1.1232	1.0000	1.0193	3 059.4310	1.9430	2.9595
Panel 20: Bonds										
Constant Transition Probabilities	1	2	1	2	1	2	1	2	1	2
1	0.9738	0.0262	0.0000	0.9999	0.5174	0.4826	0.0000	1.0000	0.0000	1.0000
2	1.0000	0.0000	1.0000	0.0000	0.5776	0.4224	0.0197	0.9802	0.0033	0.9967
Constant Expected Durations	1	2	1	2	1	2	1	2	1	2
	38.2366	1.0000	1.0000	1.0000	2.0720	1.7314	1.0000	50.6053	1.0000	303.1733

Panel 17 of Table 6.4 illustrates that non-original regimes are dependent for indexed volatilities; however, the original regimes are not necessarily trend setters. One of the reasons that might explain that pattern is that during the 2012-2017 period, most equities markets experienced bull phase. The expected durations for all equities volatilities are fairly short, with the exception of the Russian market. Panel 18 illustrates the same pattern as Panel 17 except, in the case of South Africa. Surprisingly, expected durations of real estate are far shorter than ones of equities. The patterns of regimes in Panels 19 and 20 show similar patterns as in Table 6.4. The interesting part is that expected durations for Russia are fairly long. Normally, currencies markets lead movements in stock markets, followed by equities, then bonds and finally, the real estate markets. Based on the latter principle, in Russian commodities Markov transitions are longer because of the long-expected duration of the Russian bond index, which was preceded by equities volatilities. Similarly, real estate volatilities follow the same pattern. The Russian commodities volatilities are higher because Russia is a major player in the commodities market in the world.

6.1.3 Conclusion for Hypothesis One

To sum up, this study illustrates that; firstly, there are spillovers that happen across, in-between and within bonds, commodities, equities and real estate indices. Secondly, sometimes the illiquid indices contribute more to volatility spillovers than liquid indices. Thirdly, expected durations of illiquid indices have shorter time spans than liquid indices. Fourth, in most cases, the volatility spillovers patterns during the out-of-sample period are similar to ones emanating during the in-sample period. Finally, periodical movement patterns vary across, in-between and within bonds, commodities, equities and real estate indices.

The implications from this study are as follows. Firstly, similar governmental formations should be encouraged throughout the world, provided that there are economic benefits associated with those formations. Secondly, investing in different indices should be encouraged diversification pays. Thirdly, there are risk management strategies that one can design based on volatility spillovers across, in-between and within bonds, commodities, equities and real estate indices. Fourth, the BRICS formation has indirectly influenced how capital markets (i.e. bonds, commodities, equities and real estate indices) behave. Finally, investment managers can build based on volatility spillovers numerous investment strategies.

6.2 Analysis for Hypothesis Two

6.2.1 Results for Hypothesis Two

Note that there is no amount of debt and equity or correlation coefficients given on indices. Therefore, one assumes when correlation is +1 firstly, then 0 and finally -1. Risk premium is calculated based on the Capital Asset Pricing Model (CAPM), as CAPM is widely used in calculated equity premium $\left[K_e = R_f + \beta\left(R_m - R_f\right)\right]$. Thus, K_e for different countries; (i) Brazil (equities=0.49%, real estate=-1.19%, commodities=0.15% and bonds=0.07%), (ii) Russia (equities=-0.62%, real estate=-1.82%, commodities=0.56% and bonds=-0.01%), (iii) India (equities=0.76%, real estate=-2.19%, commodities=0.63% and bonds=0.21%), (iv) China (equities=0.82%, real estate=1.57%, commodities=0.35% and bonds=-0.11%) and (v) South Africa (equities=0.42%, real estate=0.95%, commodities=-1.57% and bonds=0.22%). Table 5.5 illustrates the results of GARCH(1,1) and the capitalised GARCH of indices (i.e. bonds, commodities, equities and real estate) of the BRICS countries.

The betas are assumed to be 1, as by definition, the beta of the market is 1. R_f is the yield of government bond with the shortest maturity during those chosen periods (i.e. Brazil=6.07%, Russia=6.47%, India=3.01%, China=3.23% and SA=7.32%). One uses government bonds maturing in a 5-year time period, during the 2007-2017 period. The reason for using a 5-year bond is a standard practice (See; Altman 1989). R_m is based on the average total index return per year (Brazil: equities=0.49%, real estate=-1.19%, commodities=0.15% and bonds=0.07%; Russia: equities=-0.62%, real estate=-1.82%, commodities=0.56% and bonds=-0.10%; India: equities=0.76%, real estate=-2.19%, commodities=0.63% and bonds=-0.21%; China: equities=0.82%, real estate=1.57%, commodities=0.35% and bonds=-0.11% and South Africa: equities=0.42%, real estate=0.95%, commodities=-1.57% and bonds=-0.22%). Different countries have different indices that represent their different stock market constituencies. The closest index that represents constituencies of the BRICS countries is the S&P500.

Moreover, the S&P500 includes constituencies of those four industries. Based on the S&P500 weekly data from January 2007 to December 2017, its total index return annually is 1.42%. Therefore, this article takes market return as 1.42% for all the BRICS countries during the 2007-2017 period. This partly assists for consistency reasons. Table 5.5 compares GARCH(1,1) and the capitalised GARCH values of the indices (i.e. bonds, commodities, equities and real estate) of the BRICS countries.

Table 6. 5: GARCH(1,1) versus capitalised GARCH

Panel A: Bonds					
Parameter	Brazil	Russia	India	China	South Africa
ω	0.0001 (0.8304)	8.8410 (0.0000)***	0.0015 (0.0000)***	5.8915 (0.0000)***	0.0005 (0.3737)
α	2.76e-05 (0.0344)**	1.04e-05 (0.0000)***	1.09e-05 (0.0054)***	9.02e-05 (0.0000)***	8.65e-05 (0.0094)***
β	0.3215 (0.0006)***	1.0505 (0.0000)***	0.2564 (0.0000)***	1.0052 (0.0000)***	0.2214 (0.0011)***
λ	0.5791 (0.0000)***	-0.0152 (0.7098)	0.7436 (0.0000)***	-0.0071 (0.9101)	0.2250 (0.3389)
$\alpha+\beta$	0.3215	1.0505	0.2564	1.0052	0.2214
GACH(1,1)	0.5671	3.1451	0.5078	2.6262	0.5220
capitalized GARCH	0.5671$^{\#,\dagger,\ddagger}$	3.1451$^{\#,\dagger,\ddagger}$	0.5078$^{\#,\dagger,\ddagger}$	2.6262$^{\#,\dagger,\ddagger}$	0.4711$^{\#,\dagger,\ddagger}$
Akakike	-2.2882	-2.1443	-2.3278	-2.3383	-2.3383
Schwarz	-2.2525	-2.1360	-2.2920	-2.3025	-2.3035
Hannan-Quinn	-2.2740	-2.1413	-2.3136	-2.3241	-2.3241
Panel B: Commodities					
ω	0.0069 (0.3000)	0.0140 (0.0261)**	0.0012 (0.3604)	0.0008 (0.2506)	-0.0025 (0.2290)
α	0.0213 (0.1771)	0.0168 (0.0000)***	6.27e-05 (0.0176)**	1.96e-05 (0.0592)*	3.49e-05 (0.1747)
β	0.2106 (0.0002)***	0.6111 (0.0000)***	0.1710 (0.0000)***	0.1139 (0.0000)***	0.0833 (0.0000)***
λ	0.1014 (0.2149)	0.1083 (0.0061)***	0.7935 (0.0000)***	0.8306 (0.3598)	0.9102 (0.0000)***
$\alpha+\beta$	0.2319	0.6279	0.1710	0.1139	0.0833
GARCH(1,1)	0.4666	0.7908	0.4150	0.3387	0.2843
capitalized GARCH	0.4663$^{\#}$ 0.4597† 0.4887‡	0.3496$^{\#}$ 0.3497† 0.3498‡	0.3771$^{\#,\dagger,\ddagger}$	0.3387$^{\#,\dagger,\ddagger}$	0.2843$^{\#,\dagger,\ddagger}$
Akakike	-0.3768	-0.3944	-2.1695	-2.9873	-1.6265
Schwarz	-0.3378	-0.3553	-2.1304	-2.9482	-1.5874
Hannan-Quinn	-0.3616	-0.3792	-2.1543	-2.9720	-1.6112
Panel C: Equities					
ω	-0.0010 (0.3540)	0.0008 (0.4938)	0.0014 (0.1308)	0.0018 (0.1592)	9.79e-05 (0.91410
α	1.16e-05 (0.1239)	3.61e-05 (0.0519)*	1.29e-05 (0.0651)*	2.49e-05 (0.0397)**	5.27e-05 (0.0321)**
β	0.0639 (0.0003)***	0.2132 (0.0000)***	0.0813 (0.0000)***	0.1204 (0.0000)***	0.1149 (0.0006)***
λ	0.9252 (0.0000)***	0.7867 (0.0781)***	0.9047 (0.0000)***	0.8734 (0.0000)***	0.7946 (0.0000)***
$\alpha+\beta$	0.0639	0.2132	0.0813	0.1204	0.1149
GARCH(1,1)	0.2508	0.4626	0.2876	0.3496	0.3390
capitalized GARCH	0.2508$^{\#}$ 0.2526†,‡	0.4626$^{\#,\dagger,\ddagger}$	0.2876$^{\#,\dagger,\ddagger}$	0.3496$^{\#,\dagger,\ddagger}$	0.3390$^{\#,\dagger,\ddagger}$
Akakike	-2.3816	-2.0161	-2.4997)	-2.1233	-2.6033
Schwarz	-2.3426	-1.9771	-2.4607	-2.0843	-2.5644
Hannan-Quinn	-2.3663	-2.0008	-2.4845	-2.1081	-2.5881
Panel D: Real Estate					
ω	-0.0021 (0.2619)	0.0002 (0.91410)	-0.0019 (0.3983)	0.0008 (0.6116)	0.0030 (0.0028)**
α	4.28e-05 (0.1637)	0.0007 (0.0011)***	0.0001 (0.0754)*	5.27e-05 (0.0646)*	0.0002 (0.0016)***
β	0.0773 (0.0002)***	0.6975 (0.0000)***	0.0967 (0.0008)***	0.1088 (0.0000)***	0.2882 (0.0000)***
λ	0.9097 (0.0000)***	0.3024 (0.0000)***	0.8706 (0.0000)***	0.8754 (0.0000)***	0.4934 (0.0000)***
$\alpha+\beta$	0.0773	0.6982	0.0968	0.1088	0.2884
GARCH(1,1)	0.2742	0.8353	0.3080	0.3311	0.5397
capitalized GARCH	0.2777$^{\#,\dagger,\ddagger}$	0.8353$^{\#,\dagger,\ddagger}$	0.3079$^{\#,\dagger,\ddagger}$	0.3311$^{\#,\dagger,\ddagger}$	0.5396$^{\#,\dagger,\ddagger}$
Akakike	-1.6605	-1.3388	-1.4678	-1.6866	-2.2125
Schwarz	-1.6224	-1.3008	-1.4298	-1.6485	-2.1745
Hannan-Quinn	-1.6456	-1.3239	-1.4529	-1.6717	-2.1976

Note: ***, ** and * illustrate α at 1%, 5% and 10, respectively. And $^{\#}$, † and ‡ represent capitalised GARCH values when $\rho_{D:E}=+1$, $\rho_{D:E}=0$ and $\rho_{D:E}=-1$, respectively. Omega is represented by ω, alpha by α, beta by β and lambda by λ. Mathematically, the highest correlation between any two variables is 100% (as a ratio +1) and the lowest correlation between any parameters is -100% (as a ratio -1) and the neutral correlation by any two parameters is 0% (as ratio 0); therefore, this thesis takes on the similar approach, in finding the best, neutral and worst scenarios in demonstrating volatilities. Hence, the usage of the window (-1,0,+1). The reason why of not using implied volatilities for this case is because implied volatilities are disentangled at other cases; hence, it is better to look at different, but similar measures.

The calculations of GARCH(1,1) versus the capitalised GARCH have been rounded to four decimal places-for spacing reasons. However, if calculations are rounded to seven decimal places, the salient point is that GARCH(1,1) differs with the capitalised GARCH by 1.1e-06 on average, irrespective of the industry focus. Although, that difference is minute, given the sizes of the indices, the difference of GARCH(1,1) and capitalised GARCH values is significant-this is the main finding of this article, which is demonstrated both theoretically and empirically. The diagnostic tests (Akakike, Schwarz and Hannan-Quinn) for Panels A and C are virtually normally distributed as they are within an acceptable range (i.e. 1.7-2.6). The robustness of the models is shown by diagnostic tests. The diagnostic tests of Panels B and D are not normally distributed. This might be due to the fact that commodities are spiky and real estate is heterogeneous and affected by the lagging effect. This might be partly because bond and equities markets are the oldest securitised markets, which are mechanised in the world. In Panel A, the alphas of the five countries are statistically significant-this confirms that those bonds indices react to market events (Chkili 2016). Similarly, betas of the BRICS illustrate that there is volatility persistence captured. Only the lambdas of Brazil and India are statistically significant and positive. This might be due to the fact that during the analysis period, Brazil and India have been on massive expansion trajectories, including numerous projects, some being financed through debt. The sum of alpha and beta in Panels A to D add up to one, except for the alpha and beta of Russia. Thus, models are appropriate for this action. GARCH(1,1) and capitalised GARCH values are similar; however, there is some difference for reason(s) explained earlier in this section. Thus, capitalised GARCH values are smaller than GARCH(1,1) values-this implies the capitalised GARCH converges more than GARCH(1,1). By extension, capitalised GARCH would be more suitable for discrete hedging than GARCH(1,1). Lastly, correlation has no effect on the capitalised GARCH. This might be due to common stability in bonds markets.

In Panel B, GARCH(1,1) parameters for most of the BRICS are positive and statistically significant-this is synonymous with the GARCH(1,1) model. Despite the general expected findings, there are unique revelations. India and China are predominantly main importers of commodities in the world. Therefore, they react to both volatility market events and persistence, while exporting countries (i.e. Brazil, Russia and SA) only capture volatility persistence, except for Russia. It accounts for volatility market events and persistence. Russia mainly produces energy and oil, not many metals. The studies have shown that volatilities of energy and oil are quite spiky. For commodities, the capitalised GARCH values are less than GARCH(1,1)-similar pattern in Panel A. Interestingly, correlation has an effect on the capitalised GARCH values-positive correlation decreases the capitalised GARCH values, while negative correlation increases the capitalised GARCH values. That finding is similar to winner's curse. Put it simply, the winner is less risky than the loser is. Back to volatility analysis, probably when margins are positive, one has more money to cover for losses than vice versa.

Panel C illustrates the same patterns as in Panels A and B, except that correlation has an effect on the capitalised GARCH values of Brazil, not for the other four countries. Why? According to stock market commentators, the Brazilian equities markets increased 40-fold between 1990 and 2007, generating a 25% annual return. Since 2007, Brazilian equities lost half of their value and stabilised at low levels in 2016. That would imply that, correlation in equities has an effect on the capitalised GARCH during declining phase(s). This can be explained by *Newton's Law of Universal Gravitation*. Remember that law from high school Physics? Lastly, it seems that when equities are stable (i.e. BRICS countries), GARCH(1,1) and the capitalised GARCH values converge to the same levels. Thus, GARCH(1,1) appears not to necessarily capture all information during declining phases of the economy-this is the second main finding of this study. To one's knowledge, no study has illustrated the weaknesses of the GARCH(1,1) model during subprime crises. During the 2007/2008 subprime period, the GARCH(1,1) model was widely used in practice for volatility analysis and it is still, even now.

Panel D illustrates the same findings as shown in Panels A to C. The main interesting finding in panel D is that the GARCH(1,1) values are less than the capitalised GARCH-opposite to the values in Panels A to C. One possible explanation might be that not enough leverage is captured by lambda. This inclusion $\rho_{D:E}$ leads to more capturing of "leverage". This is the third main finding of this article. This begs the question, how leverage must be captured by lambda. The answer is beyond the scope of this study; however, the latter question is a point to note for investigation for future research.

6.2.2 Robustness Test for Hypothesis Two

Given that discrete movements are central to volatility risk management processes, then one compares different volatilities as part of the robustness test. The hedging, based on volatilities, is shown by the relationship between spot volatilities and their long-term average volatilities (i.e. GARCHs). Note that there is no formula for spot volatility, the general principle is that the volatility should be current at that point in time. For this article one uses the historical volatility at the points in time to illustrate the spot volatilities. Table 6.6 illustrates the relationship between spot (i.e. current market volatility) and GARCH volatilities:

Table 6. 6: Spot versus GARCH Volatilities

Parameter	Brazil	Russia	India	China	South Africa
Panel E: Bonds					
σ	0.2.76e-05 (0.0304)***	1.09e-06 (0.0054)***	1.04e-05 (0.0000)***	9.02e-05 (0.0000)***	8.65e-05 (0.0094)****
GACH(1,1)	0.5671	3.1451	0.5078	2.6262	0.5220
capitalised GARCH	$0.5671^{\#,\dagger,\ddagger}$	$3.1451^{\#,\dagger,\ddagger}$	$0.5078^{\#,\dagger,\ddagger}$	$2.6262^{\#,\dagger,\ddagger}$	$0.4711^{\#,\dagger,\ddagger}$
Panel F: Commodities					
σ	0.0213 (0.0000)***	0.0168 (0.0000)***	6.28e-05 (0.0176)**	1.93e-05 (0.0592)*	3.46e-05 (0.1747)
GARCH(1,1)	0.4666	0.7908	0.4150	0.3387	0.2843
capitalised GARCH	$0.4663^{\#}$ $0.4597^{\dagger}$ $0.4887^{\ddagger}$	$0.3496^{\#}$ $0.3497^{\dagger}$ $0.3498^{\ddagger}$	$0.3771^{\#,\dagger,\ddagger}$	$0.3387^{\#,\dagger,\ddagger}$	$0.2843^{\#,\dagger,\ddagger}$
Panel G: Equities					
σ	1.16e-05 (0.1239)	3.61e-05 (0.0519)*	1.25e-0.5 (0.0651)*	02.49e-05 (0.0397)***	5.27e-05 (0.0321)**
GARCH(1,1)	0.2508	0.4626	0.2876	0.3496	0.3390
capitalised GARCH	$0.2508^{\#}$ $0.2526^{\dagger,\ddagger}$	$0.4626^{\#,\dagger,\ddagger}$	$0.2876^{\#,\dagger,\ddagger}$	$0.3496^{\#,\dagger,\ddagger}$	$0.3390^{\#,\dagger,\ddagger}$
Panel H: Real Estate					
σ	4.27e-05 (0.1637)	0.0007 (0.0017)**	5.14e-05 (0.0746)*	5.27e-05 (0.0646)*	0.0002 (0.0017)**
GARCH(1,1)	0.2742	0.8353	0.3080	0.3311	0.5397
capitalised GARCH	$0.2777^{\#,\dagger,\ddagger}$	$0.8353^{\#,\dagger,\ddagger}$	$0.3079^{\#,\dagger,\ddagger}$	$0.3311^{\#,\dagger,\ddagger}$	$0.5396^{\#,\dagger,\ddagger}$

Note: ***, ** and * illustrate α at 1%, 5% and 10, respectively. And $^{\#}$, † and ‡ represent the capitalised GARCH values when $\rho_{D:E} = +1$, $\rho_{D:E} = 0$ and $\rho_{D:E} = -1$, respectively. Mathematically, the highest correlation between any two variables is 100% (as a ratio +1) and the lowest correlation between any parameters is -100% (as a ratio -1) and the neutral correlation by any two parameters is 0% (as ratio 0); therefore, this thesis takes on the similar approach, in finding the best, neutral and worst scenarios in demonstrating volatilities. Hence, the usage of the window (-1,0,+1). The reason why of not using implied volatilities for this case is because implied volatilities are disentangled at other cases; hence, it is better to look at different, but similar measures.

The rule of thumb on volatilities is that, the spot volatilities should converge to their long-term average volatilities in the long-run. Hence, one should follow the latter phenomenon in hedging volatilities. That is, one is trading spot volatility in relation to its long-term average volatility. Based on your calculations at the beginning of time to expiration, when spot volatilities converge from the top, in relation to the long-term average volatility, one shorts (i.e. sell) the long-term average volatility and longs (i.e. buy) spot volatility. Similarly, when spot volatility converges from the bottom in relation to the long-term average volatility, one longs the long-term average volatility and shorts spot volatility. Fundamentally, the latter statement would imply bearish conditions, while the former illustrates bullish conditions.

Panel E in Table 6.6 illustrates that spot volatilities for bond indices for the BRICS countries are statistically significant. For the five countries, volatilities are less than their long-term average volatilities, as illustrated by GARCH models. That is, despite the fact that the capitalised GARCH and GARCH values are different, they illustrate the same implication in terms of hedging volatilities-short long term average volatilities and long spot volatilities. It can be inferred from Buraschi *et al.* 2014 that different volatilities lead to alpha for some investors, such as intraday investors. Furthermore, the authors find that, the more the correlation, the higher the alpha. Given similarities between the BRICS countries, one can assume that there is a higher presence of correlation. By extension, the alpha should be evident. Panels F, G and H illustrate that spot volatilities converge from the bottom for commodities, equities and real estate, respectively. Therefore; in terms of volatility hedging, one would implement the same strategy as in hedging in Panel E (i.e. bond index).

6.2.3 Conclusion for Hypothesis Two

This article illustrates the following. Firstly, GARCH(1,1) can be extended into capitalised GARCH. Secondly, the capitalised GARCH has more parameters than the GARCH(1,1) model; (i) correlation of debt and equity, (ii) equity parameter, (iii) risk premium, (iv) interest rate on debt and (v) “major” shocks, as illustrated by equity parameter. Thirdly, the capitalised GARCH enhances hedging or risk management of volatility. Finally, one can improve the shortcomings of financial models, for example the GARCH(1,1) model.

The implications from this article are as follows. Firstly, financial models are not definitive in nature. Secondly, the dynamism inherent in models makes them better and more effective and efficient. Thirdly, risk managers and hedgers can minimise risk in investment portfolios by applying risk minimisation strategies based on the capitalised GARCH model. Finally, one should always find better ways of enhancing financial models.

6.3 Analysis for Hypothesis Three

6.3.1 Results for Hypothesis Three

(a) Structural Break Points

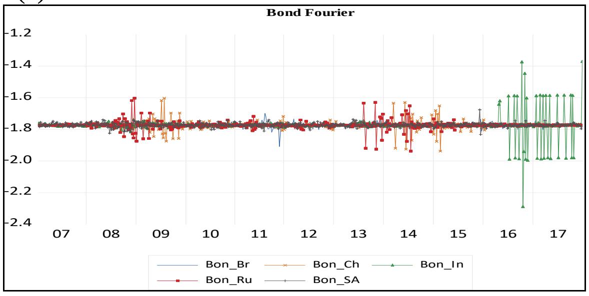
For structural breaks, the breaks based on returns are calculated based on the out-and-in sample sizes, while volatility breaks are based only on the in-sample. This is because one has to calculate volatilities based on returns. Moreover, it should be based on a reasonable period of a dataset. Thus, from 2007 to 2012 is 6 years and based on weekly dates, it makes 312 data points for calculating volatilities. In principle, volatilities are for the in-sample period. This study did not calculate volatilities during the 2007 to 2011 period because the volatilities for that period should be calculated based on returns prior to the 2007 period. That will make those volatilities incomparable.

For every graph illustrated, Bon is for bonds, Com is commodity, Eq is equity and RE is real estate. Furthermore, Br is for Brazil, Ch is China, In is India, Ru is Russia and SA is for South Africa. Evident break point implies that structural break points, as per integral transforms, can be easily picked up. Hidden point in case means that those structural points are not easily picked up; however, integral transforms illustrate them; although not so easily. Broadly, there is interconnection between structural break points, irrespective of whether those points are on upward and/or downward trajectory. Interestingly, one can infer that there were technical recessions and expansions that are invisible to market commentators and policy makers. That might imply that some policies and/or strategies incorporated poor understanding of unfolding in stock markets. Results for every formula, based on each index, are synthesised below in the respective tables. The salient points in the tables are summarised versions of the graphs.

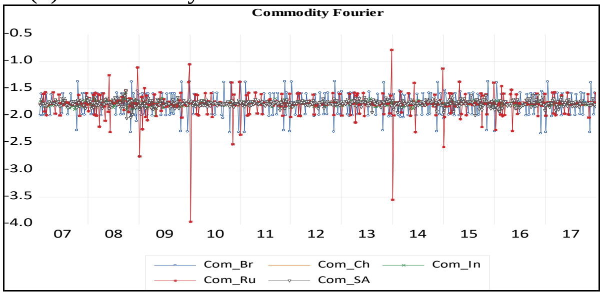
[1] Fourier Transform

Figure 6. 5: Fourier Transform

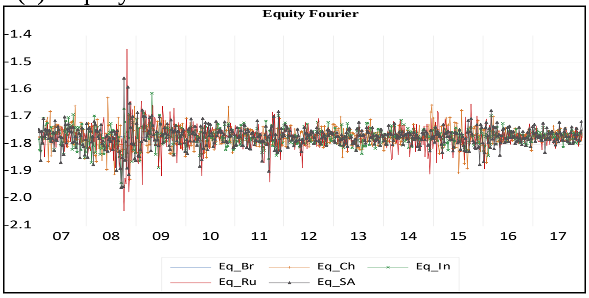
1(a): Bond



1(b): Commodity



1(c): Equity



1(d): Real Estate

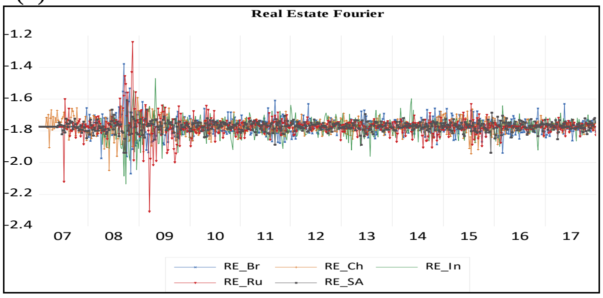


Table 6. 7: Fourier Transform

Parameter	Country	Brazil	Russia	India	China	South Africa	Parameter	Country	Brazil	Russia	India	China
Bond	**Evident Break Point**	23rd Jun 2008, 29th Sep 2008, 17th Nov 2008, 25th Jul 2011, 15th Aug 2011, 3rd Oct 2011, 28th Nov 2011, 10th Jun 2013, 7th Oct 2013, 7th Dec 2015, 14th Dec 2015, 14th Mar 2016, 22nd Aug 2016, 7th Nov 2016 and 27th Mar 2017	30th Apr 2007, 6th Aug 2007, 1st Oct 2007, 4th Feb 2008, 30th Jun 2008, 27th Oct 2008, 22nd Dec 2008, 27th Dec 2010, 16th Jul 2012, 19th Jan 2015, 7th Dec 2015, 14th Dec 2015, 2nd May 2016, 4th Jul 2016, 15th Aug 2016, 5th Sep 2016, 10th Oct 2016, 31st Oct 2016, 14th Nov 2016, 14th Nov 2016, 23rd Jan 2017, 20th Feb 2017, 3rd Apr 2017, 1st May 2017, 26th Jun 2017, 14th Aug 2017, 2nd Oct 2017 and 25th Dec 2017	28th Nov 2011	10th Mar 2008, 19th May 2008, 23rd Jun 2008, 27th Oct 2008, 22nd Dec 2008, 4th May 2009, 11th May 2009, 6th Jul 2009, 20th Jul 2009, 14th Sep 2009, 28th Sep 2009, 9th Nov 2009, 16th Nov 2009, 20th Jun 2011, 5th Dec 2011, 19th Dec 2011, 26th Nov 2011, 12th Mar 2012, 10th Dec 2012, 21st Jan 2013, 17th Mar 2014, 31st Mar 2014, 9th Jun 2014, 16th Jun 2014, 4th Aug 2014, 1st Sep 2014, 6th Oct 2014, 22nd Dec 2014, 5th Jan 2015, 26th Jan 2015, 23rd Feb 2015, 20th Jul 2015, 10th Aug 2015, 5th Oct 2015, 4th Jan 2016 and 11th Jan 2016	23rd Jun 2008, 22nd Sep 2008, 17th Nov 2008, 12th Sep 2009, 17th Jun 2013, 9th Sep 2013, 12th Jan 2015, 2nd Mar 2015, 7th Dec 2015, 14th Dec 2015, 22nd Aug 2016, 7th Nov 2016 and 27th Mar 2017	Commodity	**Evident Break Point**	17th Dec 2007, 14th Jan 2008, 21st Jan 2008, 18th Feb 2008, 14th Apr 2008, 19th May 2008, 21st Jul 2008, 27th Oct 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 15th Oct 2009, 19th Apr 2010, 4th Oct 2010, 25th Oct 2010, 17th Jan 2011, 25th Jul 2011, 12th Sep 2011, 7th Nov 2011, 5th Mar 2012, 2nd Apr 2012, 7th May 2012, 26th Jan 2012, 27th May 2013, 4th Nov 2013, 20th Jan 2014, 8/9/2014, 26th Jan 2015, 16th Mar 2015, 29th Feb 2016 and 16th May 2016	5th Mar 2007, 23rd Apr 2007, 4th Jun 2007, 22nd Oct 2007, 25th Feb 2008, 21st Apr 2008, 2nd Jun 2008, 22nd Dec 2008, 9th Feb 2009, 26th Oct 2009, 28th Dec 2009, 1st Mar 2010, 26th Apr 2010, 7th Jun 2010, 1st Nov 2010, 27th Dec 2010, 7th Mar 2011, 25th Apr 2011, 6th Jun 2011, 31st Oct 2011, 26th Dec 2011, 27th Feb 2012, 11th Jun 2012, 29th Oct 2012, 10th Dec 2012, 11th Mar 2013, 11th Apr 2013, 30th Nov 2013, 6th Jan 2014, 27th Jan 2014, 27th Jan 2014, 28th Apr 2014, 16th Jun 2014, 29/12/2014, 16th Feb 2015, 27th Apr 2015, 8th Jun 2015, 28th Sep 2015, 5th Oct 2015, 9th Nov 2015, 14th Mar 2016, 2nd May 2016, 6th Jun 2016, 7th Nov 2016, 20th Feb 2017, 24th Apr 2017, 15th May 2017, 5th Jun 2017, 12th Jun 2017, 30th Oct 2017, 6th Nov 2017 and 25th Dec 2007	None	28th Nov 2011
	Hidden Break Point	14th May 2007, 6th Aug 2007, 19th Nov 2007, 28th Apr 2008, 16th Dec 2008, 15th Jun 2009, 26th Oct 2009, 26th Apr 2010, 26th Jul 2010, 25th Oct 2010, 28th Mar 2011, 12th Mar 2012, 27th Aug 2012, 19th Nov 2012, 26th May 2014, 26th Jan 2015, 14th Sep 2015, 13th Jun 2016, 3rd Jul 2017, 30th Oct 2017 and 18th Dec 2017	19th May 2008, 23rd Dec 2008, 15th Jun 2009 and 7th Mar 2016	26th Feb 2007, 28th May 2007, 17th Sep 2007, 14th Jan 2008, 11th Jan 2010, 11th Jul 2011, 27th Feb 2012, 21st May 2012, 10th Aug 2012, 1st Jul 2013, 25th Nov 2013, 22nd Sep 2013, 22nd Sep 2014, 4th May 2015, 31st Aug 2015, 11th Jan 2016 and 19th Dec 2016	21st May 2007, 20th Aug 2007, 12th Nov 2007, 17th Jan 2011, 28th Mar 2011, 26th Apr 2010, 30th May 2016, 29th Aug 2016 and 25th Dec 2017	29th Jan 2007, 19th Mar 2007, 30th Apr 2007, 9th Jul 2007, 30th Jul 2007, 24th Sep 2007, 31st Mar 2008, 19th May 2008, 8th Jun 2009, 29th Sep 2009, 12th Apr 2010, 26th Jul 2010, 26th Sep 2011, 2nd Jul 2012, 11th Nov 2013, 2nd Jun 2014, 24th Aug 2015, 20th Jun 2016, 10th Jul 2017, 16th Oct 2017 and 18th Dec 2017		**Hidden Break Point**	2nd Nov 2009, 29th Mar 2010, 25th Apr 2011, 10th Dec 2012, 25th May 2015 and 7th Mar 2016	1st Sep 2008, 26th Sep 2009, 20th Apr 2009, 1st Jun 2009, 7th May 2012, 8th Sep 2014, 3rd Nov 2014 and 11th Sep 2017	19th Mar 2007, 18th Jun 2007, 1st Oct 2007, 7th Jul 2008, 22nd Jun 2009, 25th Jan 2010, 19th Jul 2010, 8th Nov 2010, 31st Jan 2011, 18th Jul 2011, 3rd Oct 2011, 18th Mar 2013, 9th Sep 2013, 25th Nov 2013, 2nd Jun 2014, 13th Jul 2015, 8th Feb 2016, 4th Jul 2016, 19th Sep 2016 and 18th Dec 2017	26th Feb 2007, 28th May 2007, 17th Sep 2007, 14th Jan 2008, 11th Jan 2010, 11th Jul 2011, 27th Feb 2012, 21st May 2012, 10th Aug 2012, 1st Jul 2013, 25th Nov 2013, 22nd Sep 2013, 22nd Sep 2014, 4th May 2015, 31st Aug 2015, 11th Jan 2016 and 19th Dec 2016
Equities	**Evident Break Point**	17th Dec 2007, 14th Jan 2008, 21st Jan 2008, 18th Feb 2008, 14th Apr 2008, 19th May 2008, 21st Jul 2008, 27th Oct 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 15th Oct 2009, 19th Apr 2010, 4th Oct 2010, 25th Oct 2010, 17th Jan 2011, 25th Jul 2011, 12th Sep 2011, 7th Nov 2011, 5th Mar 2012, 2nd Apr 2012, 7th May 2012, 26th Jan 2012, 27th May 2013, 4th Nov 2013, 20th Jan 2014, 8/9/2014, 26th Jan 2015, 16th Mar 2015, 29th Feb 2016 and 16th May 2016	16th Jul 2007, 13th Aug 2007, 29th Oct 2007, 3rd Mar 2008, 18th Aug 2008, 22nd Sep 2008, 17th Nov 2008, 8th Dec 2008, 16th Mar 2009, 13th Apr 2009, 18th May 2009, 15th Jun 2009, 6th Jul 2009, 12th Oct 2009, 1st Feb 2010, 30th Aug 2010, 1st Aug 2011, 24th Dec 2012, 18th Mar 2013, 29th Apr 2013, 16th Dec 2013, 3rd Feb 2014, 3rd Mar 2014, 9th Jun 2014, 11th Aug 2014, 29th Sep 2014, 13th Oct 2014, 17th Nov 2014, 20th Jul 2015, 14th Sep 2015, 9th Nov 2015, 19th Dec 2016, 6th Feb 2017, 22nd May 2017 and 26th Jun 2017	22nd Jan 2007, 19th Feb 2007, 20th Aug 2007, 26th Nov 2007, 3rd Mar 2008, 14th Apr 2008, 12th May 2008, 10th Jun 2008, 30th Jun 2008, 27th Apr 2009, 22nd Jun 2009, 21st Nov 2011, 9th Jan 2012, 19th Nov 2012, 4th Mar 2013, 15th Apr 2013, 29th Jul 2013, 23rd Sep 2013, 16th Dec 2013, 12th May 2014, 2nd Jun 2014, 21st Jul 2014, 20th Oct 2014, 18th May 2015, 14th Sep 2015, 14th Mar /2016 and 9th Oct 2017	9th Apr 2007, 21st May 2007, 3rd Sep 2007, 8th Oct 2007, 11th Feb 2008, 2nd Jun 2008, 7th Jul 2008, 28th Jul 2008, 22nd Dec 2008, 7th Jun 2010, 7th Jun 2010, 1st Nov 2010, 15th Nov 2010, 28th Nov 2011, 19th Mar 2012, 21st Jan 2013, 6th May 2013, 15th Dec 2014, 23rd Feb 2015, 22nd Jun 2015, 20th Jul 2015, 7th Sep 2015 and 14th Mar 2016	25th Jun 2007, 23rd Jul 2007, 14th Dec 2007, 28th Jan 2008, 11th Aug 2008, 6th Oct 2008, 29th Sep 2008, 3rd Nov 2008, 17th Nov 2008, 12th Jan 2009, 9th Mar 2009, 22nd Jun 2009, 13th Jul 2009, 14th Sep 2009, 14th Dec 2009, 18th Dec 2010, 20th Jun 2011, 30th Jan 2012, 27th Aug 2012, 8th Oct 2012, 8th Apr 2013, 10th Feb 2014, 21st Apr 2014, 8th Dec 2014, 26th Jan 2015, 16th Mar 2015, 17th Aug 2015, 5th Oct 2015, 22nd Feb 2016, 13th Mar 2017, 10th Jul 2017 and 14th Aug 2017	Real Estate	**Evident Break Point**	17th Dec 2007, 14th Jan 2008, 21st Jan 2008, 18th Feb 2008, 14th Apr 2008, 19th May 2008, 21st Jul 2008, 27th Oct 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 15th Oct 2009, 19th Apr 2010, 4th Oct 2010, 25th Oct 2010, 17th Jan 2011, 25th Jul 2011, 12th Sep 2011, 7th Nov 2011, 5th Mar 2012, 2nd Apr 2012, 7th May 2012, 26th Jan 2012, 27th May 2013, 4th Nov 2013, 20th Jan 2014, 8th Sep 2014, 26th Jan 2015, 16th Mar 2015, 29th Feb 2016 and 16th May 2016	16th Jul 2007, 13th Aug 2007, 29th Oct 2007, 3rd Mar 2008, 18th Aug 2008, 22nd Sep 2008, 17th Nov 2008, 8th Dec 2008, 16th Mar 2009, 13th Apr 2009, 18th May 2009, 15th Jun 2009, 6th Jul 2009, 12th Oct 2009, 1st Feb 2010, 30th Aug 2010, 1st Aug 2011, 24th Dec 2012, 18th Mar 2013, 29th Apr 2013, 16th Nov 2013, 3rd Feb 2014, 3rd Mar 2014, 9th Jun 2014, 11th Aug 2014, 29th Sep 2014, 13th Oct 2014, 17th Nov 2014, 20th Jul 2015, 14th Sep 2015, 9th Nov 2015, 19th Dec 2016, 6th Feb 2017, 22nd May 2017 and 26th Jun 2017	14th Apr 2008, 19th May 2008, 2nd Jun 2008, 4th Aug 2008, 15th Sep 2008, 29th Sep 2008, 27th Oct 2008, 15th Dec 2008, 12th Jan 2009, 16th Mar 2009, 27th Apr 2009, 22nd Jun 2009, 3rd Aug 2009, 9th Nov 2009, 31st May 2010, 13th Sep 2010, 13th Nov 2010, 21st Feb 2011, 14th Mar 2011, 17th Aug 2011, 22nd Aug 2011, 2nd Jan 2012, 6th Feb 2012, 18th Nov 2012, 14th Jan 2013, 4th Mar 2013, 9th Sep 2013, 7th Oct 2013, 3rd Mar 2014, 19th May 2014, 27th Jun 2016, 7th Nov 2016, 8th Dec 2016, 29th Dec 2016, 30th Jan 2017, 17th Apr 2017, 14th Aug 2017 and 30th Oct 2017	9th Apr 2007, 21st May 2007, 3rd Sep 2007, 8th Oct 2007, 11th Feb 2008, 2nd Jun 2008, 7th Jul 2008, 28th Jul 2008, 22nd Feb 2008, 7th Jun 2010, 22nd Oct 2008, 7th Jun 2010, 1st Nov 2010, 15th Nov 2010, 28th Nov 2011, 19th Mar 2012, 21st Dec 2013, 6th May 2013, 15th Dec 2014, 23rd Feb 2015, 22nd Jun 2015, 20th Jul 2015, 7th Sep 2015 and 14th Mar 2016
	Hidden Break Point	29th Oct 2007, 1st Mar 2009, 21st Mar 2011, 6th Jun 2011, 9th Jan 2012, 6th Feb 2012, 9th Jan 2012, 6th Feb 2012, 14th Jul 2014, 28th Jul 2014, 8th Dec 2014, 27th Apr 2015, 8th Aug 2016, 2nd Jan 2017, 30th Jan 2017, 25th Sep 2017 and 27th Nov 2017	12th May 2008, 21st Jul 2008, 12th Jan 2009, 16th Feb 2009, 24th Aug 2009, 26th Jul 2010, 27th Sep 2010, 27th Dec 2010, 7th Mar 2011, 2nd Mar 2011, 27th Jun 2011, 22nd Aug 2011, 12th Sep 2011, 3rd Oct 2011, 31st Oct 2011, 5th Dec 2011, 30th Jan 2012, 30th Apr 2012, 24th Sep 2012, 8th Jul 2013, 4th Nov 2013, 2nd Dec 2013, 15th Dec 2014, 9th Feb 2015, 23rd Mar 2015, 18th Jul 2015, 4th Sep 2017 and 11th Dec 2017	14th May 2007, 19th Jan 2009, 14th Sep 2009, 25th Jan 2010, 26th Apr 2010, 13th Feb 2012, 26th Mar 2012, 21st May 2012, 30th Jul 2012, 24th Sep 2012, 3rd Dec 2012, 20th May 2013, 13th Jan 2014, 21st Jul 2014, 23rd Feb 2015, 11th Jul 2016, 26th Sep 2016, 20th Feb 2017 and 22nd May 2017	21st Jan 2008, 6th Apr 2009, 20th Jul 2009, 21st Sep 2009, 23rd Nov 2009, 11th Jan 2010, 20th Feb 2012, 13th Aug 2012, 3rd Sep 2012, 22nd Oct 2012, 4th Nov 2013, 10th Feb 2014, 17th Mar 2014, 6th Apr 2015, 23rd Nov 2015, 4th Nov 2016, 23rd May 2016, 29th Aug 2016, 1st May 2017 and 2nd Oct 2017	22nd Jan 2007, 12th Feb 2007, 30th Apr 2007, 9th Jul 2007, 10th Sep 2007, 8th Oct 2007, 12th Nov 2007, 28th Jul 2008, 16th Mar 2009, 18th Jan 2010, 5th Apr 2010, 13th Sep 2010, 27th Dec 2010, 9th May 2011, 15th Aug 2011, 31st Oct 2011, 16th Jan 2012, 13th Feb 2012, 9th Apr 2012, 30th Apr 2012, 14th Jan 2013, 30th Jun 2014, 28th Jul 2014, 13th Oct 2014, 4th Jun 2015, 13th Jul 2015, 13th Jul 2015, 28th Mar 2016, 8th Aug 2016, 7th Nov 2016, 12th Dec 2016, 9th Jan 2017, 10th Apr 2017, 29th May 2017, 9th Oct 2017 and 4th Dec 2017		**Hidden Break Point**	29th Oct 2007, 1st Mar 2019, 21st Mar 2011, 6th Jun 2011, 9th Jan 2012, 6th Feb 2012, 9th Jan 2012, 6th Feb 2012, 14th Jul 2014, 28th Jul 2014, 8th Dec 2014, 27th Apr 2015, 8th Aug 2016, 2nd Jan 2017, 30th Jan 2017, 25th Sep 2017 and 27th Nov 2017	12th May 2008, 21st Jul 2008, 12th Jan 2009, 16th Feb 2009, 24th Aug 2009, 26th Jul 2010, 27th Sep 2010, 27th Dec 2010, 7th Mar 2011, 2nd May 2011, 27th Jun 2011, 22nd Aug 2011, 12th Sep 2011, 3rd Oct 2011, 31st Oct 2011, 5th Dec 2011, 30th Jan 2012, 30th Apr 2012, 24th Sep 2012, 8th Jul 2013, 4th Nov 2013, 2nd Dec 2013, 15th Dec 2014, 9th Feb 2015, 23rd Mar 2015, 18th Jun 2015, 4th Sep 2017 and 11th Nov 2017	7th Apr 2007, 14th Dec 2009, 15th Dec 2009, 22nd Mar 2010, 27th Jun 2011, 28th May 2012, 23rd Jul 2012, 13th May 2013, 29th Jul 2013, 16th Dec 2013, 31st Mar 2014, 14th Jul 2014, 12th Jan 2015, 6th Apr 2015, 5th Oct 2015, 5th Sep 2016, 2nd Jan 2017 and 3rd Jul 2017	21st Jan 2008, 6th Apr 2009, 20th Jul 2009, 21st Sep 2009, 23rd Nov 2009, 23rd Nov 2009, 11th Jan 2010, 20th Feb 2012, 13th Aug 2012, 3rd Sep 2012, 22nd Oct 2012, 4th Nov 2013, 10th Feb 2014, 17th Mar 2014, 6th Apr 2015, 23rd Nov 2015, 4th Nov 2016, 23rd May 2016, 29th Aug 2016, 1st May 2017 and 2nd Oct 2017

Given that Fourier transform generates a derivative, it implies that Fourier transforms of bonds, commodities, equities and listed real estate symbolise casual effects. During the period of 2007-2017, the Brazilian bond market had small proceeds and short maturity, according to market commentators. Moreover, during that time, less than half of bonds were investment-grade and at least 50% were not traded at all. Those listed traits should cause movements in bonds. Evident structural break points were in 2008, 2011, 2013 and 2017, while hidden points were at different points between 2007 to 2017. Hidden structural break points are more than evident ones. This might be because Brazil had good relations with many Western countries before the election of President Jair Bolsonaro. The listed real estate market of Brazil mimics the same pattern as the bonds, in terms of break points sequence. Nevertheless, evident break points equal to hidden break points. It can be inferred from Okunev *et al.* 2000 that real estate has stable patterns. Among the macroeconomic factors that contributed to structural break points (i.e. evident and hidden); include sovereign borrowing according to OECD Sovereign Borrowing Outlook 2017.

According to market commentators, during 2008/2009, Russia experienced recession. Given that its economy is highly dependent on commodities prices, the effects of global subprime crises were evident in Russia. The Russian state divested in thousands of Russian firms. During that time both the bond market, and Russian Rubble collapsed. During 2011, market commentators stated that Russia held the lowest $\frac{debt}{gross\ domestic\ product\ (GDP)}$ among most emerging markets. In 2016, Russia started using short-term debt paper that matures in a month. In 2008, 2011, 2015 and 2016, structural break points were evident. However, during 2007, 2009, 2010, 2012, 2015 and including periods of evident break points, there were hidden structural break points. It seems that evident and hidden structural break points are interrelated-to one’s knowledge; this is the first article to show interrelated structural break points. During 2007 to 2017, it seems Russian interest rates, budget deficit, GDP growth, inflation, exchange rate movements, interbank interest rates, repo rate and political environment affected the major parts of the Russian economy. This is interesting that Fourier transforms can capture structural break points caused by different macroeconomic variables. The listed real estate index for Russia illustrates similar structural break points, such as the bond index. Numerous prior studies illustrate that listed real estate mimics the bond market because of the similarity in their traits; (i) convexity, (ii) sensitivity to same term structures and (iii) yield patterns.

The Indian bond market has traded at low yields during the 2007-2017 period-hovering around 5%. Akram and Das (2019) illustrated that Indian bonds are influenced by interest rates and monetary policy. Moreover, they found that government policy is influenced by government fiscal variables (Akram and Das, 2019; page 168). Just like Brazil, there are more hidden structural break points than evident structural break points. This is probably due to the influence of political parties on the economy. For the real estate market, there are more evident structural break points than hidden structural break points. Both hidden and evident structural break points can be spotted throughout the 2007-2017 period. India uses nominal values in doing property valuations (Abidoye and Chan 2017). It is well known that nominal values do not necessarily reflect real values in property markets. The latter statement might explain some structural break points (evident and hidden). Note for the Indian bonds, there was one evident break point and the rest are hidden points. For Indian real estate, there are numerous evident and hidden points. Evident break points out-number hidden break points.

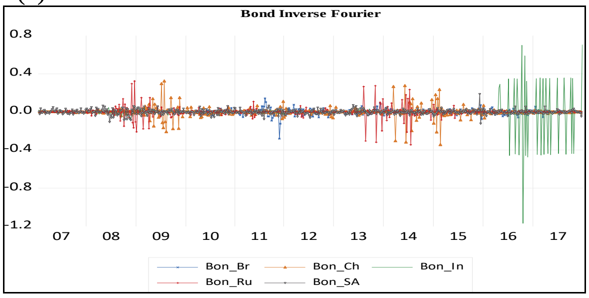
For the Chinese bond market, there are more evident structural break points than hidden ones. China is known to have bought the largest quantity of bond assets and some of those bond assets are denominated in U.S. dollars. This might lead to China influencing the global bond market. In terms of the 2008/2009 global financial crisis, its impact on China was not as severe as in the Western countries. According to market commentators, China has become a major contributor to global trade and product integration in the global bond market. The Chinese real estate market is integral to the financial system of China. Therefore, it partly explains why break points (i.e. evident and hidden) of bonds and real estate are similar. At the heart of Chinese property, is the housing market, which is influenced by the government. SA is largely a net importer of skills and services because of the history of this country. Therefore, its bond yields are relatively high when compared to other emerging countries. Moreover, market commentators tend to advocate government intervention that should lead to everyone benefiting from the economic growth.

The equities markets of the BRICS have been volatile throughout the 2007-2017 period. For SA, its equities move in tandem with European markets. Given that Europe felt the 2008/2009 subprime crisis effects, so did SA. Although, the effects were minimal on SA. Brazil, Russia, China and India are among the major consumers of equity products. Furthermore, some major corporates from those countries have dual listing in developed countries. For the four BRICS countries, except for Brazil, hidden structural break points out-number evident break points. Overall, for the BRICS nations, there are more structural break points in equities than ones shown by bonds and/or real estate indices. The commodity market illustrates interesting patterns. For the commodities consumers (i.e. India and China), there are virtually no evident structural break points, while for commodities producers (i.e. Brazil, Russia and SA), both evident and hidden points are there-Brazil and Russia have more evident points than hidden points, and SA has more hidden points than evident break points. This might be due to Russia and Brazil having more sizable economies in the world than SA. Moreover, commodities including oil and gas are highly influenced by government policies.

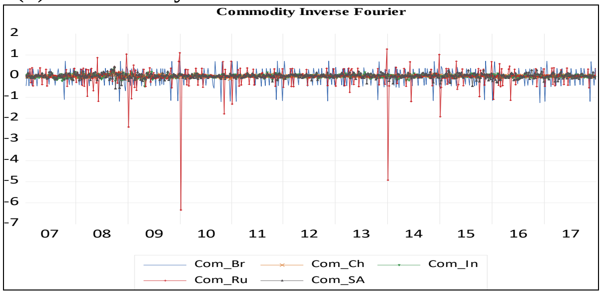
[2] Inverse Fourier Transform

Figure 6. 6: Inverse Fourier Transform

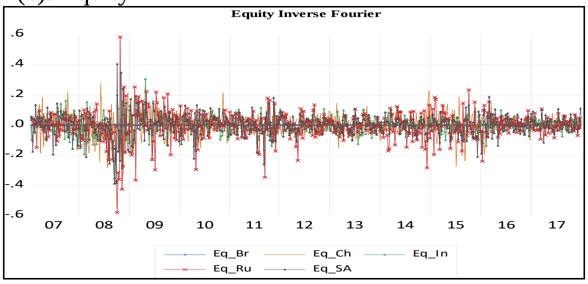
2(a): Bond



2(b): Commodity



2(c): Equity



4(d): Real Estate

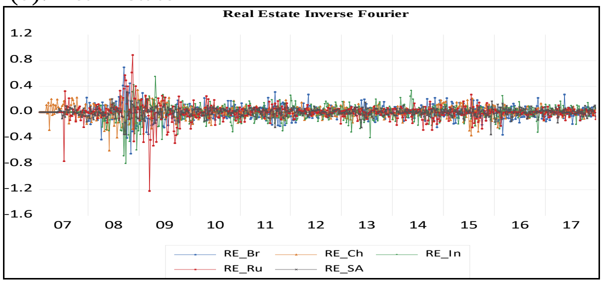


Table 6. 8: Inverse Fourier Transform

Parameter	Country	Brazil	Russia	India	China	South Africa	Parameter	Country	Brazil
Bond	**Evident Break Point**	15th Aug 2011, 12th Sep 2011, 26th Sep 2011, 3rd Oct 2011 and 28th Nov 2011	6th Oct 2008, 1st Dec 2008, 15th Dec 2008, 22/12/2008, 9/2/2009, 13/4/2009, 18/5/2009, 6/7/2009, 7/9/2009, 23/5/2011, 12/8/2012, 4/12/2013, 9/12/2013, 30/12/2013, 19/5/2014, 16/6/2014 and 14/7/2014	2nd May 2016, 4th Jul 2016, 11th Jul 2016, 15th Aug 2016, 22nd Aug 2016, 5th Sep 2016, 19th Sep 2016, 10th Oct 2016, 31st Oct 2016, 23rd Jan 2017, 20th Feb 2017, 13th Mar 2017, 3rd Apr 2017, 1st May 2017, 26th Jun 2017, 14th Aug 2017, 2nd Oct 2017, 16th Oct 2017 and 25th Dec 2017	4th May 2009, 11th May 2009, 6th Jul 2009, 20th Jul 2009, 27th Jul 2009, 10th Aug 2009, 14th Sep 2009, 28th Sep 2009, 16th Nov 2009, 9th Nov 2011, 26th Dec 2011, 17th Mar 2014, 31st Mar 2014, 9th Jun 2014, 16th Jun 2014, 4th Aug 2014, 22nd Dec 2014, 5th Jan 2015, 19th Jan 2015, 26th Jan 2015, 16th Feb 2015 and 23rd Feb 2015	23rd Jun 2008, 13th Oct 2008 and 17th Nov 2008	Commodity	**Evident Break Point**	15th Jan 2007, 2nd Apr 2007, 25th Jun 2007, 6th Aug 2007, 8th Oct 2007, 3rd Dec 2007, 11th Feb 2008, 7th Apr 2008, 21st Jul 2008, 3rd Nov 2008, 15th Dec 2008, 6th Apr 2008, 8th Jun 2009, 13th Jul 2009, 10th Aug 2009, 26t Oct 2009, 14th Dec 2009, 1st Feb 2010, 30th Aug 2010, 18th Oct 2010, 7th Feb 2011, 1st Aug 2011, 29th Aug 2011, 26th Sep 2011, 14th Nov 2011, 26th Dec 2011, 6th Feb 2012, 16th Apr 2012, 18th Jun 2012, 23rd Jul 2012, 27th Aug 2012, 17th Sep 2012, 5th Nov 2012, 3rd Dec 2012, 18th Feb 2013, 25th Mar 2013, 15th Jul 2013, 9th Sep 2013, 28th Oct 2013, 9th Dec 2013, 3rd Mar 2014, 24th Mar 2014, 28th Apr 2014, 4th Aug 2014, 29th Sep 2014, 17th Nov 2014, 8th Dec 2014, 9th Feb 2015, 23rd Mar 2015, 27th Apr 2015, 9th Nov 2015, 4th Jan 2016, 27th Jun 2016, 15th Aug 2016, 5th Sep 2016, 10th Oct 2016, 2nd Jan 2017, 13th Feb 2017, 24th Apr 2017, 26th Jun 2017, 31st Jul 2017, 4th Sep 2017, 25th Sep 2017, 13th Nov 2017 and 11th Dec 2017
	Hidden Break Point	15th Jan 2007, 6th Dec 2010, 7th Feb 2011, 7th Mar 2011, 25th Jul 2011, 15th Aug 2011, 12th Mar 2012, 19th Mar 2012, 9th Jul 2012, 30th Jul 2012, 10th Sep 2012, 15th Oct 2012, 19th Nov 2012, 20th Jul 2015, 12th Oct 2015, 7th Mar 2016 and 23rd May 2016	11th Feb 2008, 4th Aug 2008, 8th Sep 2008, 20th Sep 2010, 2nd May 2011, 16th May 2011, 14th May 2012, 11th Jun 2012, 29th Oct 2012, 27th Jan 2014, 3rd Mar 2014, 15th Dec 2014, 5th Jan 2015, 2rd Mar 2015, 1st Jan 2015 and 8th Jun 2015	15th Jan 2007, 18th Jun 2007, 7th Apr 2008, 16th Jun 2008, 27th Jul 2008, 25th Jan 2010 and 30th Jul 2012	15th Jun 2009, 2nd Nov 2009, 18th Jan 2010, 15th Feb 2010, 28th Jun 2010, 20th Jun 2011, 16th Jan 2012, 3rd Dec 2012, 24th Dec 2012, 21st Feb 2013, 23rd Mar 2015, 20th Jul 2015, 7th Sep 2015, 5th Oct 2015, 4th Jan 2016 and 11th Jan 2016	22nd Jan 2007, 30th Apr 2007, 28th May 2007, 19th Nov 2007, 11th Feb 2008, 5th May 2008, 15th Dec 2008, 12th Jul 2010, 30th Aug 2010, 8th Nov 2010, 8th Aug 2011, 9th Jul 2012, 7th Nov 2016, 27th Mar 2017, 23rd Oct 2017 and 18th Dec 2017		**Hidden Break Point**	None
Equities	**Evident Break Point**	None	26th Feb 2007, 21st May 2007, 12th May 2008, 6th Oct 2008, 27th Oct 2008, 9th Feb 2008, 27th Apr 2009, 13th Jul 2009, 5th Oct 2009, 23rd Nov 2009, 11th Jan 2010, 1st Mar 2010, 29th Nov 2010, 28th Mar 2011, 16th May 2011, 27th Jun 2011, 10th Oct 2011, 20th Feb 2012, 4th Jun 2012, 10th Sep 2012, 17th Mar 2014, 5th May 2014, 1st Sep 2014, 27th Oct 2014, 2nd Feb 2015, 6th Apr 2015, 24th Aug 2015, 5th Oct 2015 and 25th Jan 2016	20th Aug 2007, 24th Sep 2007, 3rd Mar 2008, 30th Jun 2008, 27th Apr 2008, 25th May 2009, 19th Nov 2012, 15th Apr 2013, 29th Jul 2013, 9th Sep 2013, 2nd Dec 2013, 30th Jun 2014, 20th Apr 2015, 8th Feb 2016 and 23rd May 2016	19th Mar 2007, 9th Apr 2007, 8th Oct 2007, 9th Jun 2008, 25th Aug 2008, 22nd Dec 2008, 30th Mar 2009, 8th Jun 2009, 8th Mar 2009, 7th Jun 2010, 15th Nov 2010, 25th Feb 2013, 27th May 2013 and 20th Sep 2013	6th Oct 2008, 3rd Nov 2008, 12th Jan 2009, 13th Apr 2009, 17th Aug 2009, 14th Sep 2009, 21st Apr 2010, 10th May 2010, 5th Oct 2015, 25th Jan 2016 and 29th Feb 2016	Real Estate	**Evident Break Point**	17th Dec 2007, 7th Apr 2008, 15th Sep 2008, 27th Oct 2008, 3rd Nov 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 5th Oct 2009, 19tyh Apr 2010, 27th Sep 2010, 25th Oct 2010, 17th Jan 2011, 12th Sep 2011, 2nd Apr 2012, 7th May 2012, 27th May 2013, 8th Sep 2014, 3rd Nov 2014, 26th Jan 2015, 7th Nov 2016 and 15th May 2017
	Hidden Break Point	12th May 2008, 15th Sep 2008, 23rd Jan 2012, 7th May 2012, 27th May 2013 and 19th May 2014	20th Aug 2007, 8th Oct 2007, 10th Dec 2007, 11th Feb 2008, 16th Jun 2008, 11th Aug 2008, 10th May 2010, 19th Jul 2010, 19th Nov 2012, 29th Apr 2013, 16th Sep 2013, 23rd Jun 2014, 8th Jun 2015, 23rd May 2016, 5th Dec 2016, 13th Mar 2017, 10th Jul 2017, 6th Nov 2017 and 27th Nov 2017	19th Mar 2007, 16th Apr 2007, 26th Nov 2007, 30th Aug 2010, 15th Nov 2010, 2nd Feb 2011, 22nd Aug 2011, 10th Oct 2011, 7th Nov 2011, 9th Jan 2012, 16th Jul 2012, 10th Dec 2012, 4th Mar 2013, 3rd Mar 2014, 20th Feb 2017, 7th Aug 2017 and 6th Nov 2017	19th Nov 2007, 24th Mar 2008, 3rd Aug 2009, 23rd Dec 2009, 2nd Aug 2010, 27th Sep 2010, 13th Dec 2010, 2nd May 2011, 4th Jul 2011, 19th Dec 2011, 13th Dec 2012, 14th May 2012, 1st Oct 2012, 22nd Oct 2012, 24th Dec 2012, 6th Jan 2014, 22nd Sep 2014, 2nd Mar 2014, 4th May 2015, 22nd Jun 2015, 2nd Nov 2015, 14th Mar 2016, 19th Dec 2016 and 29th May 2017	14th May 2007, 23rd Jul 2007, 20th Aug 2007, 7th Jan 2008, 21st Apr 2008, 28th Jul 2008, 2nd Nov 2009, 25th Jan 2010, 21st Jun 2010, 30th Aug 2010, 24th Jan 2011, 14th Jan 2011, 23rd May 2011, 3rd Oct 2011, 21st Nov 2011, 12th Dec 2011, 9th Apr 2012, 16th Jul 2012, 22nd Apr 2013, 24th Jun 2013, 14th Oct 2013, 16th Dec 2013, 24th Mar 2014, 27th Oct 2014, 15th Dec 2014, 16th Mar 2015, 20th Apr 2015, 13th Jul 2015, 11th Jul 2016, 19th Sep 2016, 9th Jan 2017, 10th Apr 2017, 10th Jul 2017, 9th Oct 2017 and 18th Dec 2017		**Hidden Break Point**	14th Jan 2008, 19th May 2008, 21st Jul 2008, 21st Jun 2010, 28th Mar 2011, 9th Jan 2012, 12th May 2012, 26th Nov 2012, 4th Nov 2013, 16th Dec 2013, 16th Mar 2015, 31st Aug 2015, 29th Feb 2016, 9th May 2016, 30th Jan 2017 and 23rd Oct 2017

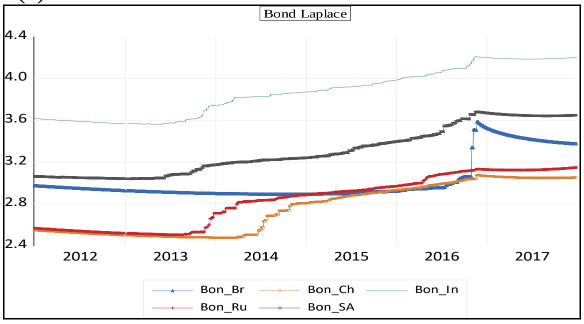
The inverse Fourier transform gives rise to a linear equation. This implies that results of inverse Fourier transform illustrate the impact of each variable in a given situation. The Inverse Fourier transforms for bond indices for the BRICS countries show the opposite of the Fourier transforms for bond indices for the BRICS countries. Firstly, on aggregate, there are more evident structural break points than hidden structural break points, except for Brazil and SA-both countries are based in the Southern hemisphere. Secondly; for Russia, India and China, it seems that evident break points lead to hidden break points. It is the opposite for Brazil and SA. The real estate indices for the five countries exhibit same pattern as the bonds. Similarly, the evident break points out-number hidden break points for Brazil and China. Thus, inverse Fourier transforms for bonds show similar patterns to inverse Fourier transforms for real estate. The equities of the BRICS show that India, China and SA have more hidden break points than evident break points. Brazil has no evident break points and Russia has more evident break points than hidden break points. Thus, the results of equities show a mixed-bag pattern. This could be due to the fact that equities are the largest asset classes, at least in the BRICS countries.

For commodities; India, China and SA have no evident break points but they have hidden break points. For SA, this is surprising given that the country is a major contributor to the commodities market. One thing notable about SA is that despite its massive commodities contribution to the world, SA does not get involved in setting up commodities prices. Moreover, SA does not have any sit on any organisation that sets up prices of any commodity in the world. Probably, SA should get involved in setting up commodities prices going forward. Given that the indices used in the inverse Fourier transform are the same ones used in the Fourier transform, this study assumes that the earlier stated macroeconomic variables have a casual effect in the inverse Fourier transform. One should note the following going on Laplace and inverse Laplace transforms. Index returns use the Fourier transform, while index volatilities use the Laplace transform. Naturally, stock prices change, leading to changes in returns and then volatilities. To put it simply, changes in stock returns should supersede changes in stock volatilities. By extension, most impact should be felt in returns, as opposed to volatilities. Laplace transforms will either confirm or disconfirm that logical thinking.

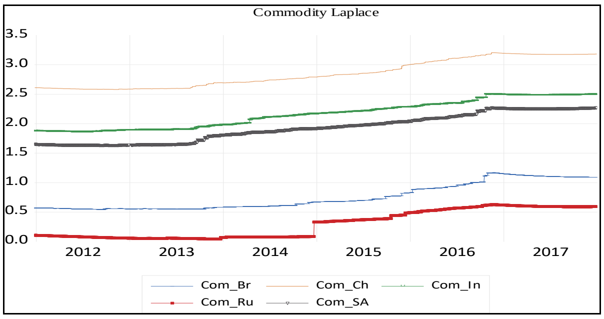
[3] Laplace Transform

Figure 6. 7: Laplace Transform

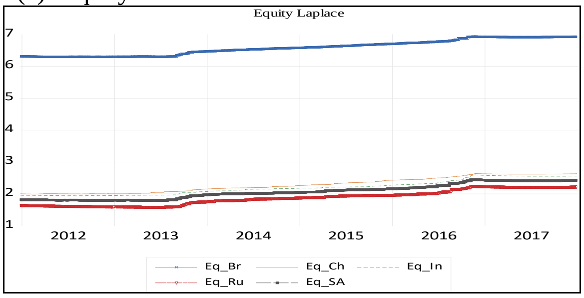
3(a): Bond



4(b): Commodity



3(c): Equity



3(d): Real Estate

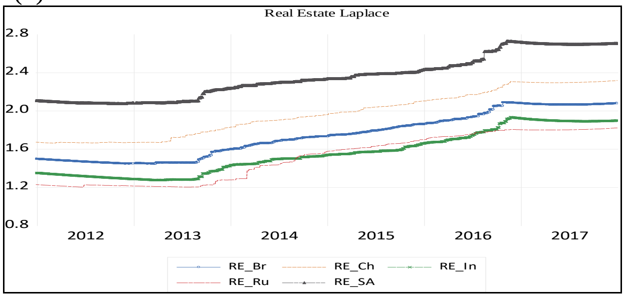


Table 6. 9: Laplace Transform

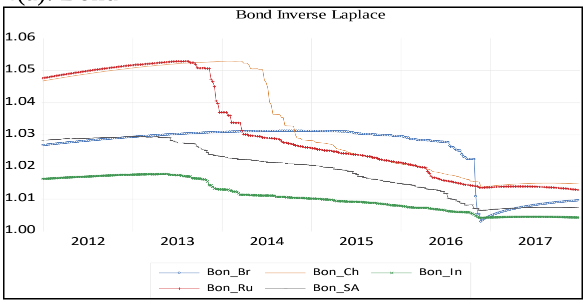
Parameter	Bond		Equities	
Country	**Evident Break Point**	**Hidden Break Point**	**Evident Break Point**	**Hidden Break Point**
Brazil	15th Jul 2016, 28th Oct2016 and 28th Nov 2016	18th Apr 2016, 20th Sep 2016, 4th Nov 2016 and 18th Nov 2016	23rd Aug 2013, 8th Nov 2013 and 10th Nov 2016	28th Mar 2012, 6th Jul 2012, 4th Oct 2012, 27th Dec 2012, 21st Sep 2016 and 9th Oct 2017
Russia	13th Nov 2013, 3th Apr 2014, 15th Apr 2016, 11th May 2016 and 18th Nov 2016	21st Nov 2013, 20th Dec 2013, 31st Jan 2014, 20th Jun 2016 and 23rd Nov 2016	7th Mar 2013, 26th Jul 2013, 14th Nov 2013 and 22nd Nov 2016	27th Jan 2012, 27th Mar 2012, 22nd Sep 2014 and 4th Dec 2015
India	11th Oct 2013, 5th Dec 2013, 18th Aug 2014, 7th Oct 2016 and 11th Nov 2016	30th Jul 2013, 28th Nov 2013, 26 h Dec 2014, 26th Mar 2015, 22nd Nov 2016 and 18th Aug 2016	28th Aug 2013, 19th Sep 2013, 10th Jun 2014, 13th Jul 2016, 21st Nov 2016 and 18th Jan 2017	13th Apr 2012, 31st Jan 2013, 13th Sep 2013 and 21st Apr 2015
China	6th May 2014, 18th Jun 2014, 1st Aug 2014, 11st Sep 2014, 25th Sep 2014, 24th Oct 2014, 13th Nov 2014, 17th Nov 2016 and 29th Nov 2016	27th Feb 2014, 11th Jul 2014, 29th Aug 2014, 23rd Jan 2015 and 5th Apr 2016	15th Dec 2012, 8th Aug 2013, 4th Nov 2013, 27th Mar 2015, 6th Jul 2015, 17th Aug 2015, 3rd Nov 2016 and 16th Nov 2017	28th Mar 2012, 22nd Oct 2012, 13th Sep 2013, 27th May 2015, 19th Jan 2017 and 6th Apr 2017
South Africa	28th Jun 2013, 18th Nov 2013, 15th Jul 2016 and 24th Nov 2016	18th Oct 2013 and 21st Nov 2016	13th Jul 2013, 7th Nov 2013, 10th Apr 2015, 1st Jul 2016, 18th Aug 2016, 18th Nov 2016 and 19th May 2017	2nd Mar 2012, 11th May 2012, 1st Aug 2012, 24th Oct 2012, 19th Mar 2013, 26th Jul 2013, 31st Jan 2014, 25th Jul 2014 and 20th Jul 2016
Parameter	**Commodity**		**Real Estate**	
Country	**Evident Break Point**	**Hidden Break Point**	**Evident Break Point**	**Hidden Break Point**
Brazil	24th Oct 2013, 9th Oct 2014, 28th Nov 2014, 25th Sep 2015, 27th Nov 2015, 7th Dec 2015, 18th Jan 2016, 14th Oct 2016, 20th Oct 2016 and 7th Nov 2016	27th Sep 2012, 11th Jan 2013, 8th May 2013, 14th Oct 2014, 8th Dec 2014, 30th Sep 2015, 8th Jan 2016, 28th Oct 2016 and 25th Nov 2016	25th Oct 2013, 16th Jun 2014, 6th May 2016, 25th Jul 2016, 27th Sep 2016 and 28th Oct 2016	18th Jan 2013, 27th Mar 2013, 20th Feb 2014 and 11th Oct 2016
Russia	20th Dec 2013, 19th Dec 2014, 23rd Dec 2014, 16th Oct 2015, 19th Oct 2015, 16th Dec 2015 and 7th Dec 2016	2nd Apr 2013, 10th Jun 2013, 26th Jul 2013, 22nd Oct 2013, 24th Oct 2014 and 20th May 2016	22nd Jun 2012, 27th Nov 2013, 17th Jan 2014, 28th Feb 2014, 13th Mar 2014, 23rd Apr 2014, 3rd Jul 2014, 28th Aug 2014, 8th Oct 2014, 25th Dec 2014, 8th May 2015, 22nd Jun 2015, 20th Aug 2015, 4th Nov 2015, 12th Feb 2016, 18th Jul 2014, 8th Aug 2015, 22nd Jun 2015, 20th Aug 2015, 4th Nov 2015, 12nd Feb 2016, 18th Jul 2016 and 29th Sep 2016	25th Jun 2013, 4th Apr 2014 and 17th Feb 2016
India	26th Sep 2013, 14th Apr 2014, 21st Oct 2016 and 28th Oct 2016	30th Jul 2013, 28th Nov 2013, 26th Dec 2014, 26th Mar 2015, 22nd Nov 2016 and 18th Aug 2016	27th Aug 2013, 20th Sep 2013, 7th Nov 2013, 6th Nov 2013, 16th Apr 2014, 15th May 2014, 5th Jun 2014, 19th Sep 2014, 17th Dec 2014, 16th Oct 2015, 27th Jan 2016, 17th Jun 2016, 4th Aug 2016, 29th Sep 2016 and 25th Nov 2016	8th Apr 2013, 28th Jun 2013, 17th Apr 2013 and 31st Mar 2016
China	2nd Sep 2013, 5th Dec 2013, 27th Nov 2015, 4th Dec 2015 and 28th Nov 2016	26th Nov 2013, 25th Oct 2013 and 3rd Mar 2016	17th May 2013, 17th Oct 2014, 20th Mar 2014, 26th May 2015, 16th Jun 2016 and 28th Nov 2016	22nd Mar 2012, 15th Jul 2013, 12th Sep 2014 and 10th Jul 2016
South Africa	28th Jun 2013, 18th Nov 2013, 15th Jul 2016 and 24th Nov 2016	18th Oct 2013 and 21st Nov 2016	7th Feb 2013, 5th Feb 2012, 27th Mar 2013, 29th Mar 2013, 26th Sep 2013, 18th Feb 2014, 19th Sep 2014, 3rd Apr 2013, 16th Sep 2015, 4th Dec 2015, 29th Dec 2015, 12th Apr 2016, 12th Aug 2016 and 8th Nov 2016	25th May 2012, 7th Aug 2012, 21st Sep 2012, 10th Jun 2013, 13th Sep 2013, 10th Apr 2014 and 1st Jul 2016

The volatility curves (i.e. Fourier transform) are slightly flatter when compared with the return curves (i.e. Laplace transform). Furthermore, there are less structural break points in volatilities than break points illustrated by returns. One possible explanation for the latter statement might be that some volatility effects might be that returns captured those shocks earlier. As stated earlier, movements in returns should supersede and give direction to volatility movements. In terms of movements, volatilities show an upward trajectory over the 2007-2017 period for the four indices of the BRICS countries. When one revisits the return curves, returns become spikier with time. This confirms the earlier statement that volatilities follow the preceding movements of returns. For Laplace transform of bonds, evident and hidden break points are almost equally spread. For real estate indices, evident structural break points are more than hidden break points. For equities indices, the evident break points are out-numbered by the hidden break points-this point was illustrated and debated earlier. Nevertheless, the break points of commodities are almost equally spread between evident and hidden points. From this section, one can see that if returns are correctly modelled, then by extension, volatilities should be correctly captured. This partly explains why strategies like risk parity have not been as successful as traditional return strategies.

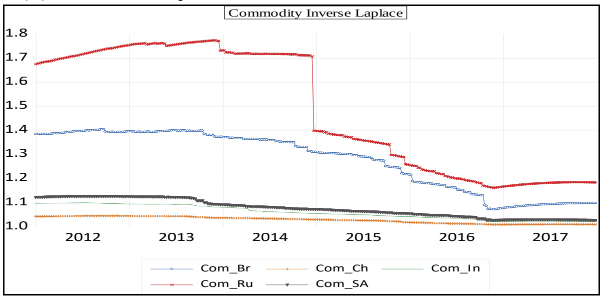
[4] Inverse Laplace Transform

Figure 6. 8: Inverse Laplace Transform

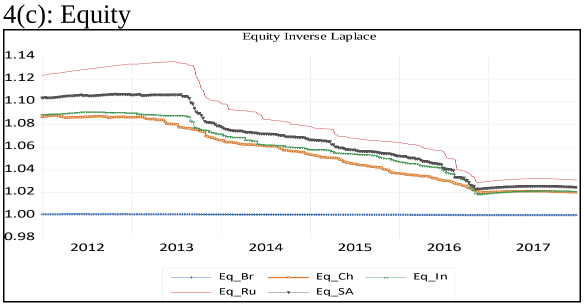
4(a): Bond



4(b): Commodity



4(c): Equity



4(d): Real Estate

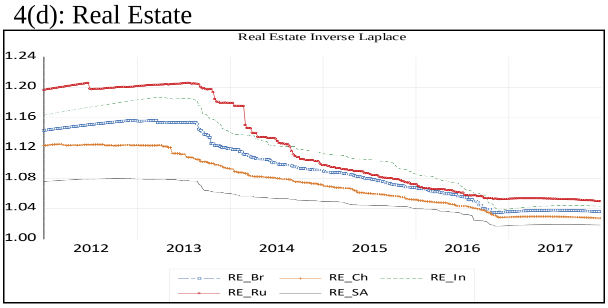


Table 6. 10: Inverse Laplace Transform

Parameter	**Bond**		**Equities**	
Country	**Evident Break Point**	**Hidden Break Point**	**Evident Break Point**	**Hidden Break Point**
Brazil	8th Jun 2015, 29th Jun 2015, 7th Sep 2015, 23rd Nov 2015, 15th Feb 2016, 11th Jul 2016, 1st Aug 2016, 29th Aug 2016, 19th Sep 2016, 24th Oct 2016, 14th Nov 2016 and 21st Nov 2016	24th Apr 2014, 28th Jul 2014, 29th Dec 2014, 23rd Mar 2015, 18st Jan 2016 and 14th Mar 2016	None	11th Jun 2012, 5th May 2014 and 8th Aug 2016
Russia	19th Aug 2013, 2nd Sep 2013, 9th Sep 2013, 30th Sep 2013, 28th Oct 2013, 11th Nov 2013, 9th Dec 2013, 30th Dec 2013, 20th Jan 2014, 3rd Feb 2014, 10th Feb 2014, 17th Mar 2014, 31st Mar 2014, 21st Apr 2014, 9th Jun 2014, 25th Aug 2014, 13th Oct 2014, 7th Sep 2015, 28th Sep 2015, 29th Feb 2016, 11th Apr 2016, 16th Jan 2015, 19th Sep 2016, 14th Nov 2016 and 21st Nov 2016	1st Jul 2013, 25th Nov 2013, 29th Sep 2014, 26th Jan 2015 and 23rd Nov 2015	18th Feb 2013, 1st Jul 2013, 12th Aug 2013, 2nd Sep 2013, 11th Nov 2013, 30th Dec 2013, 20th Jan 2014, 10th Dec 2014, 14th Apr 2014, 26th May 2014, 26th May 2014, 23rd Jun 2014, 1st Sep 2014, 15th Sep 2014, 13th Oct 2014, 1st Apr 2015, 6th Apr 2015, 20th Apr 2015, 20th Jul 2015, 21st Mar 2016, 11th Apr 2016, 27th Jun 2016, 18th Jul 2016, 15th Aug 2016, 15th Aug 2016, 22nd Aug 2016, 26th Sep 2016 and 14th Nov 2016	6th Feb 2012, 17th Dec 2012, 13th Oct 2016, 29th May 2017 and 18th Sep 2017
India	16th Sep 2013, 7th Apr 2014, 12th Sep 2016 and 17th Oct 2016	27th Feb 2012, 10th Sep 2012, 18th Feb 2013 and 26th May 2014	20th Feb 2012, 4th Mar 2012, 6th May 2013, 28th Oct 2013, 18th Aug 2014, 15th Apr 2014, 8th Dec 2014, 23rd Feb 2015, 27th Apr 2015, 25th May 2015, 19th Oct 2015, 7th Dec 2015, 18th Jul 2015 and 21st Nov 2016	16th Apr 2012, 24th Sep 2012, 20th Jan 2014, 6th Jul 2015 and 2nd Oct 2017
China	17th Mar 2014, 14th Apr 2014, 5th May /2014, 26th May 2014, 16th Jun 2014, 23rd Jun 2014, 14th Jul 2014, 28th Jul 2014, 25th Aug 2014, 8th Sep 2014, 22nd Sep 2014, 20th Oct 2014, 10th Nov 2014, 1st Dec 2014, 19th Jan 2015, 30th Mar 2015, 27th Apr 2015, 18th May 2015, 7th Mar 2015, 11th Apr 2016, 9th May 2016, 27th Jun 2016, 19th Sep 2016 and 14th Nov 2016	7th Jul 2014, 13th Oct 2014, 17th Nov 2014 and 23rd Nov 2015-*Russia has the same hidden date point.*	20th Feb 2012, 4th Mar 2012, 6th May 2013, 28th Oct 2013, 18th Aug 2014, 15th Apr 2014, 8th Dec 2014, 23rd Feb 2015, 27th Apr 2015, 25th May 2015, 19th Oct 2015, 7th Dec 2015, 18th Jul 2015 and 21st Nov 2016	16th Apr 2012, 24th Sep 2012, 20th Jan 2014, 6th Jul 2015 and 2nd Oct 2017
South Africa	18th Feb 2013, 15teh Apr 2013, 3rd Jun 2013, 8th Jul 2013, 12th Aug 2013, 16th Sep 2013, 28th Oct 2013, 18th Nov 2013, 30/03/2015, 15/06/2015, 20/07/2015, 29th Feb 2016, 11th Apr 2016, 30th May 2016, 20th Jun 2016, 4th Jul 2016, 11th Jul 2016, 8th Aug 2016, 19th Sep 2016, 17th Oct 2016, 31st Oct 2016, 7th Nov 2016, 21st Nov 2016 and 12th Jun 2017	9th Apr 2012, 10th Sep 2012, 17th Jun 2013, 5th Aug 2013, 7th Oct 2013, 19th May 2014 and 20th Apr 2015	9th Apr 2012, 9th Jul 2012, 19th Aug 2013, 30th Sep 2013, 11th Nov 2013, 3rd Mar 2014, 7th Apr 2014, 7th Jul 2014, 11th Aug 2014, 22nd Dec 2014, 16th Mar 2015, 16th Nov 2015, 21st Mar 2016, 20th Jun 2016, 27th Jun 2016, 8th Aug 2016, 15th Aug 2016 and 19th Sep 2016	2nd Nov 2012, 2nd Sep 2013 and 17th Oct 2016
Parameter	**Commodity**		**Real Estate**	
Country	**Evident Break Point**	**Hidden Break Point**	**Evident Break Point**	**Hidden Break Point**
Brazil	12th Mar 2012, 17th Sep 2012, 1st Oct 2012, 1st Apr 2013, 8th Jul 2013, 14th Oct 2013, 6th Oct 2014, 20th Oct 2014, 24th Nov 2014, 1st Dec 2014, 12th Jan 2015, 18th May 2015, 15th Jun 2015, 3rd Aug 2015, 17th Aug 2015, 21st Sep 2015, 5th Oct 2015, 23rd Nov 2015, 30th Nov 2015, 4th Jan 2016, 11th Jan 2016, 23th May 2016, 29th Aug 2016, 10th Oct 2016, 17th Oct 2016 and 31st Oct 2016	11th Nov 2013, 9th Dec 2013, 26th May 2016, 25th Aug 2014 and 28th Nov 2016	18th Mar 2013, 19th Aug 2013, 9th Sep 2013, 30th Sep 2013, 21st Oct 2013, 9th Dec 2013, 3rd Feb 2014, 14th Apr 2014, 9th Jun 2014, 7th Jul 2014, 25th Aug 2014, 5th Jan 2015, 20th Apr 2015, 23rd Nov 2015, 15th Feb 2016, 18th Apr 2016, 18th Jul 2016, 22nd Aug 2016, 29th Sep 2016, 17th Oct 2016 and 24th Oct 2016	3rd Dec 2012, 11th Aug 2014, 23rd May 2016 and 19th Jun 2017
Russia	4th Mar 2013, 18th Mar 2013, 22nd Apr 2013, 20th May 2013, 27th May 2013, 7th Oct 2013, 16th Dec 2013, 6th Jan 2014, 20th Jan 2014, 10th Feb 2014, 17/02/2014, 8th Dec 2014, 22nd Dec 2014, 20th Apr 2015, 25th May 2015, 12th Oct 2015, 19th Oct 2015, 7th Dec 2015, 21st Dec 2015, 1st Feb 2016 and 28th Nov 2016	4th Feb 2013, 15the Jul 2013, 14th Apr 2014, 29th Sep 2014, 4th Apr 2016, 23rd May 2016 and 3rd Oct 2016	18th Feb 2012, 2nd Jul 2012, 02/09/2013, 23rd Sep 2013, 21st Oct 2013, 25th Nov 2013, 20th Jan 2014, 24th Feb 2014, 3rd Mar 2014, 7th Apr 2014, 14th Apr 2014, 28th Apr 2014, 26th May 2014, 3rd Jun 2014, 11th Aug 2014, 22nd Sep 2014, 29th Sep 2014, 6th Oct 2014, 8th Dec 2014, 5th Jan 2015, 15th Jun 2015, 10th Aug 2015, 24th Aug 2015, 2nd Nov 2015, 30th Nov 2015, 21st Dec 2015, 4th Nov 2016, 25th Jan 2016, 22nd Feb 2016, 23rd May 2016, 11th Jul 2016 and 19th Sep 2016	23rd Apr 2012, 12th Nov 2012, 29th Jul 2013, 23rd Dec 2013, 11th May 2015, 22nd Aug 2016 and 16th Jan 2017
India	16th Sep 2013, 7th Apr 2014, 12th Sep 2016 and 17th Oct 2016	27th Feb 2012, 10th Aug 2012, 18th Feb 2013 and 26th May 2014	1st Apr 2013, 27th May 2013, 12th Aug 2013, 26th Aug 2013, 9th Sep 2013, 23rd Sep 2013, 21st Oct 2013, 25th Nov 2013, 30th Dec /2013, 27th Nov 2014, 7th Apr 2014, 21st Apr 2014, 15th May 2014, 26th May 2014, 9th Jun 2014, 15th Sep 2014, 8th Dec 2014, 5th Jan 2015, 30th Mar 2015, 13st Jul 2015, 28th Sep 2015, 26th Oct 2015, 30th Nov 2015, 21st Dec 2015, 25th Jan 2016, 14th Mar 2016, 13th Jun 2016, 25th Jul 2016, 8th Aug 2016, 5th Sep 2016, 26th Sep 2016, 24/10/2016 and 28th Nov 2016	20th Aug 2012, 2nd Dec 2013, 3rd Mar 2014, 28th Jul 2014, 10th Nov 2014, 9th May 2016 and 7th Nov 2016
China	26th Mar 2013, 28th Oct 2013, 10th Feb 2014, 1st Dec 2014, 15th Jun 2015, 30th Nov 2015 and 7th Oct2016	13th May 2013, 9th Jun 2014, 25th Aug 2014 and 16th Feb 2015	24th Sep 2013, 15th Apr 2013, 15th Apr 2013, 13th May 2013, 27th May 2013, 1st Jul 2013, 8th Jul 2013, 22nd Jul 2013, 16th Sep 2013, 9th Dec 2013, 13th Jan 2014, 17th Mar 2014, 8th Sep 2014, 5th Jan 2014, 16th Feb 2015, 27th Apr 2015, 25th May 2015, 23rd Nov 2015, 13th Jun 2016, 25th Jul 2016, 26th Sep 2016, 24th Oct 2016 and 21st Nov 2016	12th Mar 2013, 2nd Jul 2013, 28th Oct 2013, 24th Feb 2013, 21st Apr 2014, 20th Oct 2014, 28th Sep 2015, 2nd May 2016, 29th Aug 2016 and 26th Jun 2017
South Africa	29th Sep 2014, 24th Nov 2014, 1st Dec 2014, 15th Jun 2015, 14th May 2015, 27th Jul 2015, 28th Sep 2015, 16th Nov 2015, 30th Nov 2015, 28th Nov 2015, 18th Jan 2016, 4th Apr 2016, 1st Aug 2016, 29th Aug 2016, 10th Oct 2016 and 31st Oct 2016	19th Mar 2012, 10th Sep 2012, 12th Nov 2012, 1st Jul 2013, 26th Aug 2013 and 16th May 2016	26th Aug 2013, 3rd Sep 2013, 28th Oct 2013, 10th Feb 2014, 7th Apr 2014, 22nd Sep 2014, 30th Mar 2015, 7th Dec 2015, 4th Apr 2016, 4th Jul 2016, 8th Aug 2016, 22nd Aug 2016, 31st Oct 2016 and 7th Nov /2016	3rd Dec 2012, 11th Mar 2013, 6th May 2013, 17th Jun 2013, 23rd Dec 2013, 11th Aug 2014, 29th Dec 2014, 28th Sep 2015 and 3rd Apr 2017

The inverse Laplace transforms for the four indices of the BRICS countries illustrate that there are less break points of volatilities than points of returns. However, unlike Laplace transforms, inverse Laplace transforms illustrate that evident break points surpass hidden points for the four indices of the BRICS countries. Interestingly for bonds, China and Russia have a break points at the same date-23rd November 2015. According to Malle (2017), during that period, China and Russia made political and economic agreements on a number of fields: '(i) energy, (ii) arms production, (iii) trade in national currencies, and (iv) strategic projects in transport and supporting infrastructure" (P136). Therefore, some structural break points are interrelated and caused by political, economic and corporate governance agreements. One doubts that the three latter issues were considered in the formation of the BRICS countries. Moreover, one doubts that their implications for the financial markets were taken into account. Fundamentally, it seems that in liquid indices, evident break points lead to hidden points, while in illiquid indices, patterns are indefinite.

6.3.2 Robustness Test for Hypothesis Three

In order to test the robustness of the results of integral transforms, one uses the commonly used structural breaks tests to verify the presence of break points; (i) ADF, (ii) ADF-GLS, (ii) PP and (iv) ZA. The traditional view of a unit root hypothesis is based on the theory that current shocks have a temporary effect and the long-run movements are not affected by such shocks. However, Perron (1989) challenges these findings and argues that the standard ADF tests are biased towards the non-rejection of the null hypothesis, based on the notion that most time series are not characterised by unit roots but rather persistence arises only from large and infrequent shocks. The ADF-GLS test is also a modification of the ADF test, whereas in this test it considers a series featuring deterministic components in the form of a constant or linear trend. The testing procedure allows for de-trending a series to estimate the parameters of the series. This procedure assists in removing the means and linear trends for a series that is not far from the non-stationary region point.

The PP test can be viewed as a modified Dickey-Fuller (DF) unit root test that has been made robust to serial correlation in the error term by utilising the Newey-West (1987) heteroscedasticity and autocorrelation-consistent covariance matrix estimator. Under the PP test, similar to the ADF test, the null hypothesis is that of a unit root being present. The ZA is a proposed variation of the PP test that assumes the exact time of the breakpoint is unknown. The ZA break date is where the t-statistic is most significant; this is where the t-statistic from the ADF test of a unit root is at its minimum. This is the break point where there is strongest evidence against the null hypothesis of a unit root.

Table 6. 11: Robustness Results

	In Sample: 2012-2017				**Out Sample: 2007-2017**			
	Panel 1: General Equities				Panel 5: General Equities			
Country	ADF	ADF_GLS	PP	ZA	ADF	ADF_GLS	PP	ZA
Brazil	-17.27328 (0.0000)***	-3.2126 (0.0000)***	-17.39733 (0.0000)***	-18.1513 (0.000)***	-26.1141 (0.0000)***	-4.7406 (0.0010)***	-26.0006 (0.0000)***	-27.1956 (0.0963)*
Russia	-17.58079 (0.0000)***	-16.6755 (0.0000)***	-17.5814 (0.0000)***	-18.0856 (0.0006)***	-23.5397 (0.0000)***	-5.7182 (0.0070)***	-23.6378 (0.0000)***	-24.7129 (0.0000)***
India	-15.73895 (0.0000)***	-1.3263 (0.1860)	-15.6445 (0.0000)***	-16.1302 (0.0000)***	-14.5895 (0.0000)***	-7.2046 (0.0020)***	-22.4309 (0.0000)***	-23.2648 (0.0000)***
China	-16.65792 (0.0000)***	-5.1898 (0.0020)***	-16.8115 (0.0000)***	-17.5101 (0.000)***	-23.6047 (0.0000)***	-7.4981 (0.0000)***	-23.8296 (0.0000)***	-24.5763 (0.0000)***
South Africa	-12.82296 (0.0000)***	-3.7492 (0.0040)***	-17.8370 (0.0000)***	-18.1426 (0.0019)***	-26.6127 (0.0000)***	-4.9341 (0.0020)***	-23.6127 (0.0000)***	-27.7004 (0.0000)***
	Panel 2: Real Estate				Panel 6: Real Estate			
Brazil	-17.30683 (0.0000)***	-2.9227 (0.0000)***	-17.4133 (0.0000)***	-18.0025 (0.0000)***	-15.1422 (0.0000)***	-3.9855 (0.0030)***	-24.3789 (0.0000)***	-25.5267 (0.0000)***
Russia	-18.27117 (0.0000)***	-14.3830 (0.0000)***	-18.2910 (0.0000)***	-18.7428 (0.0003)***	-19.7322 (0.0000)***	-6.9952 (0.0000)***	-21.0643 (0.0000)***	-21.5215 (0.0000)***
India	-16.1277 (0.0000)***	-17.7402 (0.0000)***	-16.1465 (0.000)***	-16.6302 (0.0001)***	-21.7249 (0.0000)***	-5.1461 (0.0010)***	-21.7161 (0.0000)***	-22.4564 (0.0000)***
China	-18.02238 (0.0000)***	-10.2775 (0.0000)***	-18.0307 (0.0000)***	-18.7896 (0.0000)***	-24.5635 (0.0000)***	-10.1463 (0.0000)***	-24.5892 (0.0000)***	-25.3948 (0.0050)***
South Africa	-18.07221 (0.0000)***	-17.8131 (0.0000)***	-18.0729 (0.0000)***	-18.2630 (0.0288)**	-24.2428 (0.0000)***	-17.8184 (0.0000)***	-24.2428 (0.0000)***	-24.9301 (0.0000)***
	Panel 3: Commodities				Panel 7: Commodities			
Brazil	-13.79419 (0.0000)***	-0.1055 (0.9160)	-65.1962 (0.0000)***	-26.3433 (0.1156)7	-19.2138 (0.0000)***	-0.1696 (0.8660)	-36.4268 (0.0001)***	-36.4268 (0.0719)*
Russia	-11.89339 (0.0000)***	-0.4808 (0.6310)	-77.6376 (0.0000)***	-26.6164 (0.0004)***	-14.8599 (0.0000)***	-0.5089 (0.6110)	-89.6084 (0.0001)***	-33.5459 (0.0017)***
India	-16.19014 (0.0000)***	-5.0916 (0.0000)***	-16.2187 (0.0000)***	-16.7072 (0.0000)***	-14.0478 (0.0000)***	-5.2404 (0.0000)***	-21.4659 (0.000)***	-22.2974 (0.0000)***
China	-14.56573 (0.0000)***	-1.4849 (0.1390)***	-15.2546 (0.0000)***	-15.6712 (0.0000)***	-9.2169 (0.0000)***	-1.3740 (0.1705)	-20.3033 (0.0000)***	-19.4907 (0.0000)***
South Africa	-16.6124 (0.0000)***	-5.4620 (0.0000)***	-16.6399 (0.0000)***	-17.0895 (0.0005)***	-24.4568 (0.0000)***	-5.5415 (0.0000)***	-24.4785 (0.0000)***	-24.7321 (0.0000)***
	Panel 4: Bonds				Panel 8: Bonds			
Brazil	-18.02346 (0.0000)***	-5.7488 (0.0000)***	-18.0466 (0.0000)***	-18.7128 (0.0000)***	-19.9030 (0.0000)***	-5.8396 (0.0000)***	-20.2053 (0.0000)***	-20.1958 (0.0003)***
Russia	-18.4531 (0.0000)***	-18.3137 (0.0000)***	-39.1954 (0.0000)***	-27.2352 (0.0000)***	-27.0774 (0.0000)***	-5.1180 (0.0000)***	-45.5733 (0.0001)***	-35.3488 (0.0000)***
India	-17.93421 (0.0000)***	-16.3294 (0.0000)***	-39.6338 (0.0000)***	-26.1177 (0.0004)***	-22.1265 (0.0000)***	-16.5746 (0.0000)***	-43.0290 (0.0000)***	-34.5019 (0.2991)
China	-18.5369 (0.0000)***	-1.0673 (0.0000)***	-39.8675 (0.0000)***	-27.3225 (0.0000)***	-27.0955 (0.0000)***	-4.2682 (0.0000)***	-45.5372 (0.0000)***	-35.2522 (0.0000)***
South Africa	-19.7802 (0.0000)***	-38.3220 (0.0020)***	-19.8445 (0.0000)***	-19.7802 (0.0079)***	-25.6287 (0.0000)***	-3.8638 (0.0001)***	-25.6144 (0.0000)***	-25.8981 (0.0206)**

Note: In every cell, the first variable is the test value and the one in the bracket is the p-value or significance level. The asterisks; ***, ** and * represent significance levels at 1%, 5% and 10%, respectively. Critical values for the ADF test -3.451, -2.870651, -2.5716 at 1%, 5% and 10% significance level, respectively. Critical values for the ADF-GLS are -2.57, -2.89, and -3.48 at 1%, 5% and 10% significance level, respectively. Critical values for the Phillip test -3.451, -2.870651, -2.5716 at 1%, 5% and 10% significance level, respectively. Critical values for the ZA test are -5.57, -5.08, and -4.82 at 1%, 5% and 10% significance level, respectively. ADF stands for Augmented Dickey Fuller, (ii) ADF-GLS is ADF test modified by Elliott, Rothenberg, Stock (ERS) in 1992, (iii) PP is Phillips Perron 1988 test and (iv) ZA is for Zivot-Andrews 1992 test.

For the in-sample, ADF results confirm the presence of the structural break points during the 2012-2019 period, as test values are statistically significant. That, it is true for the four indices for every BRICS country. This is consistent with the findings of the integral transforms (i.e. Fourier and Laplace). Still on the 2012-2017 period, for the four indices, all structural break points are confirmed by the ADF-GLS tests, except for general equities and commodities indices for India and Russia, respectively. The reason for consistent performance in Indian general equities might be due to the liberalisation of the market. According to Bekaert *et al.* 2003, the Indian general equities were liberalised since the late 1990s. For Russian commodities markets, it might be due to the fact that Russia is next exporter of commodities, especially oil and gas. The PP and ZA tests confirm the structural break points for the four indices of the BRICS nations during the 2012-2017 period. The results of the same four indices for in-sample are replicated by the out-of-sample period. The main difference is that the out-of-sample data confirms the presence of structural break points more than the in-sample data. That might be due to the fact that structural break points tend to pick up over a longer period than a shorter period (Holmes 2011). For the ZA tests, the study also explored the graphs of the structural break points. The salient points from the graphs are as follows, starting with the in-sample period; (i) the bonds show that there is at least one structural break point up to five. Russia has two and SA has five break points-the bond market for SA is very volatile because many capital projects are financed through the bond market. Secondly, the commodities market structural break points range from three to six points. Thirdly, equities have one to two structural break points. Fourth, the real estate structural break points range from one to nine-only for SA. The salient point for the ZA analysis for the out-of-sample is that there are more structural break points over the 2007-2017 period than the 2012-2017 period. This is consistent with the notion that, the longer the period, the more one can pick other structural break points. Fundamentally, the years that were picked earlier as periods of structural breaks by integral transforms, ZA graphs confirm those years as periods of structural breaks.

6.3.3 Conclusion for Hypothesis Three

This study illustrates the following. Firstly, integral transforms capture more structural break points than uni-and-multivariate models. In integral transforms, structural break points can be anything form tens to twenties in numbers, while uni-and-multivariate models tend to have a few structural break points. Secondly, integral transforms illustrate systematic structural break point pattern(s). Thus, break points in returns lead to break points in volatilities. In addition, returns tend to have more structural break points than volatilities. Thirdly, economic, political and governance agreements cause a linkage between transatlantic structural break points. Lastly, in bonds, commodities, equities and real estate, evident structural break points lead to hidden break points.

The implications of this study are as follows. Firstly, it is commendable to use integral transforms in illustrating structural break points because integrals capture more break points, including hidden structural break points. Secondly, in capturing structural break points, one should follow systematic patterns. Thirdly, agreements between countries have implications for the financial markets. Lastly, there are also hidden break points, which are normally led by evident structural break points.

Chapter 7 Conclusions, Recommendations and Further Work

7.1 Overall Conclusion

Prior studies have studied volatility risk using mainly univariate risk models. Furthermore, most of those studies used prices to calculate risk, in particular volatility. On the other hand, prior studies focused on two assets, predominantly bonds and equities. Lastly, the geographic focus is usually on one continent and/or region. Although their findings are interesting, those findings have limitations-prices only embed price sensitive information, bonds and equities are fairly traditional asset classes; therefore, findings on bonds and equities do not necessarily apply to all alternative asset classes. In addition, a continent and/or region tends to be mainly one dimensional, i.e. driven by region desires, which tends to be similar.

This thesis uses an organisation which is transatlantic in nature-BRICS nations. BRICS has one American country; Brazil, one European country; Russia, two Asian giants; India and China and one, probably the most advanced economy in Africa; SA. Thus, the BRICS offer a rare laboratory in terms of its set up. To one's knowledge, there is no transatlantic organisation that has similar characterises to the BRICS nations. Secondly, this thesis focuses on indices as opposed to prices. Indices are more encompassing in terms of parameters and constituencies than prices (Miao et al. 2017). Thus, all major economic parameters are captured by indices than in prices.

Further, indices are modelled within a portfolio framework. Markowitz (1952) illustrated that the best portfolio is determined by the relationship of stocks in a portfolio. That is, there is no best way to model a portfolio. The composition of assets in one portfolio will determine the best combination, in terms of strategy, holding and execution style. The results of this thesis, in particular for hypothesis one, illustrate that when volatility of indices from bonds, commodities, equities and listed real estate are modelled using the VAR model, there is no definitive pattern and "better" strategy. Moreover, the second hypothesis shows that even the widely used technique for modelling in practice and academics, have their own shortcomings. Finally, the third hypothesis exemplifies "other" structural break points that are not captured by the widely used acceptable models and techniques. Fundamentally, this thesis illustrates that volatility risk is dynamic and needs complex understanding and modelling, and each situation needs its own customised modelling.

7.2 Applied Techniques

For the first hypothesis, the thesis first, presents log returns on graphs of indices-equities, listed real estate, commodities and bonds. Those graphs illustrate that the log returns over a selected period are stochastic, with distinct spikes at certain points in time. Moreover, the stochastic patterns are unevenly distributed over time. That leads to the question, why are patterns unevenly distributed for every index used. Thereafter, Hypothesis One uses basic statistic measures (mean also known as average, minimum, maximum, standard deviation, kurtosis, skewness and Jarque-Bera). The basic statistics confirmed the presence of skewness, kurtosis and uneven distribution curves of the indices. The main test for Hypothesis One is based on the VAR model. The VAR results confirmed that spillovers are both in between and across indices and countries-BRICS nations. Fundamentally, indices spillover patterns are a mixed-bag, which implies it is a challenge to appropriately model those spillovers. To validate and/or confirm the robustness of Hypothesis One, the thesis used the Markov-Regime Switching Model. The Markov-Regime Switching Model revealed interesting findings-illiquid indices converge faster than liquid assets. Furthermore, illiquid regimes stay for shorter periods than liquid regimes. Prior studies illustrated that liquid assets converge faster than illiquid assets, especially when prices are used as measures. That would imply that illiquid indices lead to trends for illiquid assets. One possible reason is the willingness of investors to incur more costs in protecting their illiquid investments. According to the Ang et al. (2014:2737), "investors are willing to forgo 2% of their wealth to hedge illiquidity crises occurring once every 10 years. That would make sense for assets such as listed real estate. Note that all models used in Hypothesis One were used without any alteration(s) and/or extension(s).

The second hypothesis uses the original GARCH(1,1); however, one found that the original GARCH(1,1) does not account for (i) correlation coefficient of debt and equity, (ii) equity parameter, (iii) risk premium, (iv) interest rates and (v) shocks-stock markets. Then, the second hypothesis extends the original GARCH(1,1) to form the capitalised GARCH(1,1). Thereafter, empirical analysis carried out is based on the four indices of the BRICS nations. Empirically, the capitalised GARCH(1,1) outperforms the GARCH(1,1) in terms of "accuracy" and convergence of parameters. To one's knowledge, there is no GARCH family and/or similar model to the capitalised GARCH(1,1). Ardia et al. 2018 illustrated that the addition of input parameters to Markov switching GARCH models improves the accuracy and convergence of calculated output parameters. In order to validate the results, Hypothesis Two maps spot volatilities with GARCH(1,1) volatilities-both for out-of-sample and in-sample time series. The robustness results confirm that spot volatilities can be hedged against GARCH(1,1) volatilities-both for out-of-sample and in-sample. In terms of contextual findings, the robustness results confirmed the main results of Hypothesis Two. Note that models for robustness tests are used in their original form.

Hypothesis Three illustrates both evident and hidden structural break points throughout the entire time series. Prior studies, such as Ewing and Malik (2013), show that over a time series, there are one to three detected structural break points. Moreover, structural break points use pretty much univariate and multivariate volatility models. Hypothesis Three goes into unchartered territory in illustrating structural break points. It uses integral transforms, specifically Fourier and Laplace transforms including their inverses. In principle, Hypothesis Three illustrates structural break points of both algebraic and derivative natures. To one's knowledge, this is novel! Fundamentally, the findings show that derivative structural break points set up trends for algebraic structural break points. So far, studies on structural break points have never confirmed, whether structural break points are algebraic or derivative in nature. Thereafter, commonly used diagnostic structural break tests (ADF, ADF-GLS, PP 1988 and ZA 1992) to confirm the presence of structural break points.

7.3 Contributions from the Hypotheses

7.3.1 Findings of Hypothesis One

The crux of Hypothesis one is modelling volatility of indices as one portfolio, as opposed to as a single asset, as is done in univariate models. In the context of a portfolio, Hypothesis One illustrates that volatility spillovers move in between and across indices, irrespective of a country of the BRICS nations. Prior studies mostly model volatility risk using a univariate model; therefore, knowledge on how volatility behaves in a portfolio is minute, especially within the transatlantic geographical setting. This is the focus point of Hypothesis One and its fact finding mission. Fundamentally, Hypothesis One showed that the volatility spillovers are non-directional. Interestingly, a mixed-bag type of results exists, irrespective of the index type and the geographic position of a country in the BRICS nations.

The focus of Hypothesis One is different from prior studies because of geographic focus-BRICS countries, a transatlantic nation that encompasses five countries from five continents and those countries that have similar political governance, similar policies and economic policies. In principle, most of those economic policies are pretty much a mixture of a free market system and socialism. On the other hand, the ruling elites in the BRICS nations tend to stay in power for longer periods, in certain cases “forever”. The Markov-Regime Switching Model also presents interesting results-illiquid indices converge faster than liquid indices. Moreover, regimes of illiquid indices are shorter than regimes of liquid indices. Although, the latter findings are interesting, Hypothesis One never went further to find out why.

7.3.2 Findings of Hypothesis Two

Hypothesis Two illustrates that the GARCH(1,1) model has short comings. And, in order to address the short comings of the GARCH(1,1) model, this thesis extends the latter model in order to “create” the capitalised GARCH(1,1) model. The latter model is more advantageous than the GARCH(1,1) model because the capitalised GARCH(1,1) accounts for the following; correlation effects between debt and equity, uniqueness of equity parameter is captured and the debt parameter debt is already accounted for in the GARCH(1,1), the risk premium, interest rate effect and stock market shocks. The risk premium is important in determining the risk level (i.e. volatility) an investor is willing to take and/or accept. Conrad *et al.* 2018 illustrated that there is a positive relationship between risk premium and volatility levels. The effects of other variables; (i) interest rate and (ii) stock market shocks are captured in the debt parameter but presumably not in full. Equity, just like debt, gets affected by stock market shocks. The interest rate does influence equity investments (see; Lian *et al.* 2019). Based on the improvements of the capitalised GARCH(1,1) it outperforms the GARC(1,1), in particular in terms of performance. The latter finding is novel. Convergence is central to pricing, hedging and calibration in capital markets. Furthermore, Hypothesis Two shows that the GARCH(1,1) volatilities can be hedged against the spot volatilities.

7.3.3 Findings of Hypothesis Three

The third and final hypothesis focused on (de)merits of using integral transforms in determining structural break points. The integral transforms used here are Fourier and Laplace transforms, including their inverse transforms. Note the final functions of Fourier and Laplace transforms are derivatives, while the final functions of inverse Fourier and Laplace transforms are algebraic functions. Thus, Hypothesis Three illustrates structural break points of algebraic and derivative nature. To one’s knowledge, no prior study has undertaken such kind of assignment. Most prior studies use unit root tests to illustrate structural break points. And in most cases, structural break points detected by unit root tests are fewer. On the other hand, the integral transforms illustrate that structural break points run into dozens. Furthermore, integral transforms show that derivative-type structural break points precede algebraic-type structural break points. To put it differently, derivative-type structural break points set up a trend for algebraic-type structural break points. However, the third hypothesis never tested whether integral transform structural break points are real or artificial. Moreover, what is their impact on indices? So far, all discussed under section 6.3.2 is different from prior studies. What is not new is that commonly used unit root tests (ADF, ADF-GLS, PP 1988 and ZA 1992) confirmed the presence of structural break points.

7.4 Future Implications and Future Research Areas

7.4.1 Future Implications

The implications of this thesis are as follows. It is generally insightful to model volatility spillovers as a portfolio, rather than individual assets. This is because in a portfolio, one is able to see dynamism of spillovers. Furthermore, modelling volatility spillovers in a portfolio is complex, as the patterns of spillovers are non-directional. On the other hand, spillovers from indices reveal unconventional results-faster convergence is more evident in illiquid indices than in liquid indices. That is, in cases like the one illustrated by Hypothesis One, one should use advanced volatility risk management techniques in order to appropriately model spillovers.

Secondly, even the best and most widely acceptable discrete volatility models have some shortcomings inherent in them. Those shortcomings might be due to the exclusion of some parameters and/or inflexibility when used on certain asset classes or industries. It could also be because of the geographic position of those industries-this has not be proven in this thesis. Therefore, volatility models should be improved from time to time.

Thirdly, over any time series, in particular when the time series is at least 10 years long, one is most likely to find more structural break points than ones detected by unit root tests. However, at the moment, one cannot tell whether some of those dozens of structural break points are either artificial or real. Fundamentally, in time series analysis, one should always test structural break points, even if they seem not to be there.

7.4.2 Future Research Areas

- Volatility clustering among different indices

  The study illustrates that when volatility movements occur, they do not have a specific pattern and they are non-directional. Therefore, that makes it difficult to model volatilities, be it in pricing volatility related products or hedging volatility related products. Then, the question is how one should model appropriately non-directional volatilities, such that volatility movements, including patterns, are being captured in models.

- Volatility clustering: indices versus prices

  Hypothesis One illustrated that findings of volatilities from indices of the BRICS nations are different from what is usually found in stock prices. Then the question is: why? One of the potential reasons might be information asymmetry and it would be insightful to explore whether unique patterns and behaviour of volatility spillovers from indices have anything to do with information asymmetry in indices.

- The unconventional relationship between illiquid and liquid indices

  Hypothesis One found that illiquid regimes converge faster than liquid regimes; moreover, illiquid regimes have a shorter lifespan than liquid regimes. However, the reasons leading to such findings have not been explored. On the other hand, those findings go against prior studies and general held norms, when comparing illiquid and liquid assets.

- The ultimate accuracy of volatility models

  The second hypothesis illustrated that the capitalised GARCH(1,1) outperforms the GARCH(1,1), especially in terms of convergence given that error is lower in the capitalised GARCH(1,1). However, Hypothesis Two never verified that the capitalised GARCH(1,1) is the best model when calculating discrete volatilities. The theme in subtitle talks to major challenges to many mathematical models-they have shortcomings and their shortcomings need to be improved from time to time.

- The parameters of the GARCH family models

  Hypothesis Two "created" the capitalised GARCH(1,1) models and the latter model has more parameters than the GARCH(1,1) model. So far, the GARCH(1,1) model parameters are widely understood and the impact of those parameters is pretty much clear to econometrics society. However, additional parameters, due to the extension of the GRACH(1,1) into the capitalised GARCH(1,1), have never been thoroughly tested and/or understood. Thus, in order to properly understand and use appropriately the capitalised GARCH(1,1), one should test the additional parameters of the capitalised GARCH(1,1) model.

- The uniqueness of integral structural break points

  Hypothesis Three illustrated that integral transforms show more structural break points than commonly used unit root tests. However, the third hypothesis never tested whether those integral transform structural break points are real or artificial. Moreover, their impact in capital markets has not been verified. Fundamentally, integral transform structural break points need to be understood in greater detail.

- Artificial versus real structural break points of indices

  The third hypothesis shows that the structural break points of integral transforms and their inverse functions run into dozens. Then, the question is why, if there are so many structural break points, do practitioners not pick them up? The possible answer lies in the confirmation of whether those dozens of hidden structural break points are artificial or not. Therefore, the authenticity and veracity of the structural break points detected by the integral transforms and their inverse functions need to be tested.

- Linear capitalised GARCH

The advantage of linear models is that they describe a continuous response of variable(s) as a function of at least one predicting variable. According to James (2002), one of the advantages of continuous modelling is that, one can capture the curve of movements over a given period. The capitalised GARCH, in its current form is discrete; therefore, based on the argument of James (2002), one cannot capture the curvature of volatility spillovers. Furthermore, it can be inferred from James (2002) that continuous models are better than discrete models in predicting ability. Therefore, this thesis recommends that in the future the capitalised GARCH should be extended into a linear form-linear capitalised GARCH.

7.5 Recommendations

Recommendations emerging from this thesis are as follows.

- It is generally insightful to model volatility as a portfolio, rather than for individual assets when investing at least two assets in one portfolio.
- In modelling volatility spillovers, one should use advanced volatility models and techniques.
- Illiquid indices should not be ignored as they have some advantages over liquid indices. Therefore, one should invest some money into illiquid indices.
- Volatility models should always be tested and one should add "new" parameters in order to strengthen the performance of those models. Therefore, there is no such thing called the best volatility model, as each has its own strengths and weaknesses.
- One can always use one type of volatility to hedge against other types of volatilities. For example, Hypothesis Two illustrates that spot volatilities can be hedged against the GARCH(1,1) volatilities.
- Structural break points should be thoroughly tested, as one might ignore non-evident structural break points.
- Integral transforms illustrated a broader view on structural break points because integral transforms can be used interchangeably between algebraic and derivatives formulas.

7.6 Limitations of the Study

Although, this thesis offered several contributions to the research field of volatility analysis of indices (bonds, commodities, equities and listed real estate), there are some limitations emanating from this thesis. Firstly, the data is on indices and indices encompass a lot of parameters and constituencies than just share prices. Therefore, findings of this thesis might not necessarily apply to share price situations. This is because share prices have less constituencies than indices (Gao et al. 2020). By extension, indices have less information asymmetry than share prices. Secondly, the BRICS nations, in terms of governance structures of capital markets, are characterised by the following traits; (i) political elites driving the economy, (ii) ruling political parties stay in power for longer and (iii) mixed-type of economic policies-free economy and socialism.

Thirdly, Russia does not have an active listed real estate index. Thus, one constructed the Russian listed real estate index based on a few Russian listed real estate companies. Therefore, the Russian listed real estate index is not a true listed real estate index. This is because, any listed firm encompasses information that affects that firm and may be, to some extent, industry information. Fourth, the GARCH(1,1) model is specifically a discrete volatility model; hence, the results of Hypothesis Two are most likely to apply to a discrete environment. However, for pricing securities, especially derivatives, it is better to price them in a continuous context (Bravo and Nunes 2021). Therefore, the findings from this thesis, when applied in a continuous environment, should be done in a cautious manner.

Finally, the structural break points are calculated using integral transforms. One of the conditions that should exist in order to appropriately use integral transforms, parameters should exist in a continuous form, while unit root tests illustrate discrete structural break points. That is, it is better when one calculates integral transform-type of structural break points in conjunction with discrete structural break points (usually detected by unit root tests), because 'structural break points tend to be discrete in nature' (Jiang *et al.* 2018).

References

Abidoye RB and Chan AP (2017). Valuers' Receptiveness to the Application of Artificial Intelligence in Property Valuation. *Pacific Rim Property Research Journal*, 23(2), 175-193.

Abken PA and Nandi S (1996). Options and Volatility. *Economic Review*, 81(3-6), 21-35.

Acerbi C and Szekely B (2014). Back-Testing Expected Shortfall. *Risk*, 27(11), 76-81.

Adewale AR (2017). Import Substitution Industrialisation and Economic Growth-Evidence from the Group of BRICS Countries. *Future Business Journal*, 3(2), 138-158.

Alberola E, Chevallier J and Chèze B (2008). Price Drivers and Structural Breaks in European Carbon Prices 2005-2007. *Energy Policy*, 36(2), 787-797.

Alexander C and Lazar E (2009). Modelling Regime-Specific Stock Price Volatility. *Oxford Bulletin of Economics & Statistic*, 71(6), 761-797.

Altman EI (1989). Measuring Corporate Bond Mortality and Performance. *Journal of Finance*, 44(4), 909-922.

Akimov A, Hutson E, and Stevenson S (2016). The Interaction of Volatility, Volume and Skewness: Empirical Evidence from REITs. *Journal of Real Estate Portfolio Management*, 22(1), 1-17.

Amkram T and Das A (2019). The Long-Run Determinants of Indian Government Bond Yields. *Asian Development Review*, 36(1), 168-205.

Anand BP (1999). India's Economic Policy Forms: A Review. *Journal of Economic Studies,* 26(3), 241-257.

Andreou E and Ghysels E (2002). Detecting Multiple Breaks in Financial Market Volatility Dynamics. *Journal of Applied Econometrics*, 17(5), 579-600.

Andrews DWK (1993). Tests for Parameter Instability and Structural Change with Unknown Change Point. *Econometrica*, 61(4), 821-856.

Andrews DWK and Ploberger W (1994). Optimal tests When a Nuisance Parameter is Present Only under the Alternative. *Econometrica.* 62(6), 1383-1414

Anderson RW (1985). Some Determinants of the Volatility of Futures Prices. *Journal of Futures Markets*, 5(3), 331-348.

Andersen TG and Bollerslev T (1998). Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts. *International Economic Review*, 39(4), Symposium on Forecasting and Empirical Methods in Macroeconomics & Finance (Nov. 1998), 885-905.

Andersen TG, Bollerslev T, Diebold FX and Labys P (2003). Modeling and Forecasting Realized Volatility. *Econometrica*, 71(2), 579-625.

Anderson RI, Benefield JD and Zumpano LV (2009). Performance Difference in Property-Type Diversified versus Specialized Real Estate Investment Trusts (REITs). *Review of Financial Economics*, 18(2), 70.79.

Anderson AL (2012). A Study on Expectiles: Measuring Risk in Finance. M.Sc. Thesis. University of Georgia, USA.

Ang A and Bekaert (2002). Regime Switches in Interest Rates. *Journal of Business & Econoimc Statistics*, 20 (2), 163-182.

Ang A, Papanikolaou D and Westerfield MM (2014). Portfolio Choice with Illiquid Assets. *Management Science*, 60(11), 2737-2761.

Antonakakis N and Kizys R (2015). Dynamic Spillovers between Commodity and Currency Markets. *International Review of Financial Analysis*, 41, 303-319.

Ardia D, Bluteau K, Boudt K and Catania L (2018). Forecasting Risk with Markov-switching GARCH Models: A Large-Scale Performance Study. *International Journal of Forecasting*, 34(4), 733-747.

Armijo LE (2007). The BRICs Countries (Brazil, Russia, India, and China) as Analytical Category: Mirage or Insight? *Asian Perspective*, 31(4), 7-42.
Assaf A (2015). Long Memory and Level Shifts in REIT Returns and Volatility. *International Review of Financial Analysis*, 42, 172-182.
Awartani BMA and Corradi V (2005). Predicting the Volatility of the S&P-500 Stock Index via GARCH models: the Role of Asymmetries. *International Journal of Forecasting*, 21(1), 167-183.
Avezki R, Hadri K, Loungani P and Rao Y (2014). Testing the Prebisch-Singer Hypothesis Since 1650: Evidence from Panel Techniques that Allow for Multiple Breaks. *Journal of International Money & Finance*, 42, 208-223.
Basak S and Pavlova A (2016). A Model of Financialization of Commodities. *Journal of Finance*, LXXI(4), 1511-1556.
BenSaïda A, Litimi H and Abdallah O (2018). Volatility Spillover Shifts in Global Financial Markets. *Economic Modelling*, 73, 1-11.
Baele L (2005). Volatility Spillovers Effects in European Equity Markets. *Journal of Financial & Quantitative Analysis*, 40 (2), 373-401.
Bakshi G and Kapadia N (2003). Delta-Hedged Gains and the Negative Market Volatility Risk Premium. *Review of Financial Studies*, 16(2), 527-566.
Barsky RB and Kilian L (2004). Oil and the Macroeconomy since the 1970s. *Journal of Economic Perspectives*, 18(4), 115-134.
Babikir A, Gupta R, Mwabutwa C and Owsu-Sekyere E (2012). Structural Breaks and GARCH Models of Stock Return Volatility: the Case of South Africa. *Economic Modelling*, 29(6), 2435-2443.
Bai J (1997). Estimation of a Change Point in Multiple Regressions. *Review of Economics & Statistics*, 79(4), 551-563.
Bai J and Perron P (1998). Estimating and Testing Linear Models with Multiple Structural Changes. *Econometrica*, 66(1), 47-78.
Bai J and Perron P (2003a). Computation and Analysis of Multiple Structural Change Models. *Journal of Applied Econometrics*, 18(1), 1-22.
Bai J and Perron P (2003b). Multiple Structural Change Models: A Simulation Analysis, Corbeam D, Durlauf S, Hansen BE (eds). Econometric Essays in Honor of Peter Phillips. *Cambridge University Press, Cambridge Press*, 212-237.
Bai J and Perron P (2003c). Critical Values for Multiple Structural Change Tests. *Econometrics Journal*, 6(1), 72-78.
Baillie RT and Morana C (2009). Modelling Long Memory and Structural Breaks in Conditional Variances: An Adaptive FIGARCH Approach. *Journal of Economic Dynamics & Control*, 33(8), 1577-1592.
Banerjee A and Urga G (2005). Modelling Structural Breaks, Long Memory and Stock Market Volatility: An Overview. *Journal of Econometrics*, 129(1-2), 1-34.
Barabashev AG and Klimenko AV (2017). Russian Governance Changes and Performance. *Chinese Political Science Review*, 2(1), 22-39.
Barbosa HF (1998). Economic Development: the Brazilian Experience. In: Hosono A, Saavedra-Rivano N (eds) Development Strategies in East Asia and Latin America. Palgrave Macmillan, London.
Bardhan P (2009). India and China: Governance Issues and Development. *Journal of Asian Studies*, 68(2), 347-357.

Bekaert G, Harvey CR and Lundblad CT (2003). Equity Market Liberalization in Emerging Markets. *Journal of Financial Research*, XXVI(3), 275-299.
Bianconi M, Yoshino JA and De Sousa MOM (2013). BRIC and the US financial Crisis: An Empirical Investigation of Stock and Bond Markets. *Emerging Markets Review*, 14, 76-109.
Bisaro A, Roggero M and Villamayor-Tomas S (2018). Institutional Analysis in Climate Change Adaptation Research: A Systematic Literature Review. *Ecological Economics*, 151, 34-43.
Black F (1986). Noise. *Journal of Finance*, XLI(3), 589-543.
Blume M (1971). On The Assessment of Risk. *Journal of Finance*, 26 (1), 1-10
Boamah NA, Loudon G and Watts EJ (2017). Structural Breaks in the Relative Importance of Country and Industry Factors in African Stock Returns. *Quarterly Review of Economics & Finance*, 63, 79-88.
Bodart V and Reding P (1999). Exchange Rate Regime, Volatility and International Correlations on Bond and Stock Markets. *Journal of International Money & Finance*, 18(1), 133-151.
Bollerslev T, Chou RY and Kroner KF (1992). ARCH modelling in Finance-A Review of the Theory and Empirical Evidence. *Journal of Econometrics*, 52(1-2), 5-59.
Bohdalová M (2007). A Comparison of Value-at-Risk Methods for Measurement of the Financial Risk. Comenius University, Bratislava, Slovakia, Working Paper.
Borenstein S (2005). The Long-Run Efficiency of Real-Time Electricity Pricing. *Energy Journal*, 63(3), 93-116.
Bravo JM and Nunes JPV (2021). Pricing Longevity Derivatives via Fourier Transforms. *Insurance: Mathematics & Economics*, 96, 81-97.
Buraschi A, Trojani F and Velodin A (2014). When Uncertainty Blows in the Orchard: Comovement and Equilibrium Volatility Risk Premia. *Journal of Finance*, LXIX(1), 101-137.
Byström H (2006). CreditGrades and the iTraxx CDS index market. *Financial Analysts Journal*, 62(6), 65-76.
Calice G, Chen J and Williams J (2013). Liquidity Spillovers in Sovereign Bond and CDS Markets: An analysis of the Eurozone Sovereign Debt Crisis. *Journal of Economic Behavior & Organization*, 85, 122-143.
Calvert L and Fisher A (2004). How to Forecast Long-run Volatility: Regime Switching and the Estimation of Multifractal Processes. *Journal of Financial Econometrics*,2 ,49-83.
Camarero M, Ez JO and Tamarit CR (200)2. Tests for Interest Rate Convergence and Structural Breaks in the EMS: Further Analysis. *Applied Financial Economics*, 12(6), 447-456.
Carr P and Wu L (2014). Static Hedging of Standard Options. *Journal Financial Econometrics*, 12(1), 3-46.
Catao L and Timmermann A (2009). Volatility Regimes and Global Equity Returns. In Bollerslev T, Russell J and Watson, M. (eds.), Volatility and Time Series Econometrics: Essays in Honor of Robert Engle. Oxford University Press, Oxford University Press.
Chen S, Cui G and Zhang J (2017). On Testing for Structural Break of Coefficients in Factor Augmented Regression Models. *Economic Letters*, 161, 141-145.
Chen MJ (2018). On Exactitude in Financial Regulation: Value-at-Risk, Expected Shortfall, and Expectiles. *Risks*, 6(61), 1-28
Cheong CS, Gerlach R., Stevenson S and Zurbruegg R (2009). Equity and Fixed Income Markets as Drivers of Securitised Real Estate. *Review of Financial Economics*, 18(2), 103-111.
Chilenga TJ (2017). Practicalities of the National Development Plan: Prospects and Challenges, using Rural Economy as a Case Study. *South African Review of Sociology*. 48(2), 87-105.

Chkili W and Nguyen DK (2014). Exchange Rate Movements and Stock Market Returns in a Regime-Switching Environment: Evidence for BRICS Countries. *Research in International Business & Finance*, 31, 46-56.

Chkili W (2016). Dynamic Correlations and Hedging Effectiveness Between Gold and Stock Markets: Evidence for BRICS Countries. *Research in International Business & Finance*, *38*, 22-34.

Cho H, Kawaguchi Y Shilling JD (2003). Unsmoothing Commercial Property Returns: A Revision to Fisher-Geltner-Webb's Unsmoothing Methodology. *Journal of Real Estate Finance & Economics*, 27(3), 393-405.

Chow GC (1960). Tests of Equality Between Sets of Coefficients in Two Linear Regressions. *Econometrica*, 28(3), 591-605.

Christiansen C (2007). Volatility-Spillover Effects in European Bond Markets. *European Financial Management*, 13(5), 923-948.

Claeys P and Vašíček B (2014). Measuring Bilateral Spillover and Testing Contagion on Sovereign Bond Markets in Europe. *Journal of Banking & Finance*, 46, 151-165.

Conrad C, Custovic A and Ghysels E (2018). Long-and Short-Term Cryptocurrency Volatility Components: A GARCH-MIDAS Analysis. *Journal of Risk & Financial Management*, 11(2), 1-12.

Cordis AS and Kirby (2014). Discrete Stochastic Autoregressive Volatility. *Journal of Banking & Finance,* 43, 160-178.

Cotter J and Stevenson S (2006). Multivariate Modelling of Daily Volatility. *Journal of Real Estate Finance & Economics*, 32(3), *305-325.*

Cotter J and Stevenson S (2007). Uncovering Volatility Dynamics in Daily REIT Returns. *Journal of Real Estate Portfolio Management*, 13 (2), 119-128.

Cotter J and Stevenson S (2008). Modeling Long Memory in REITs. *Real Estate Economics*, 36, 533-554

Curci G and Corsi F (2012). Discrete Sine Transform for Multi-Scale Realized Volatility Measures. *Quantitative Finance*, 12(2), 263-279.

Dalto FAS (2019). Brazilian financial Crisis in The 1980s: Historical Precedent of an Economy Governed by Financial Interest. *Journal of Contemporary Finance*. 23(3), 1-25.

Damodaran A (1999). Estimating Risk Parameters. Stern School of Business at NYU, Finance Working Paper: FIN-99-019, 1-31.

Dean WG, Faff RW and Loundon GF (2010). Asymmetry in Return and Volatility Spillover between Equity and Bonds Markets in Australia. *Pacific-Basin Finance Journal*, 18(3), 272-289.

Deaton A (1999). Commodity Prices and Growth in Africa. *Journal of Economic Perspectives*, 13(3), 23-40.

Dewandaru G, Masih R and Masih M (2017). Regional Spillovers Across Transitioning Emerging and Frontier Equity Markets: A Multi-Time Scale Wavelet Analysis. *Economic Modelling*, 65, 30-40.

Diebold F and Mariano R (1995). Comparing Predictive Accuracy. *Journal of Business and Economic Statistics*, 13, 253-263

Diebold FX and Yilmaz K (2011). Better to Give than to Receive: Predictive Directional Measurement of Volatility Spillovers. *International Journal of Forecasting*, 28(1), 57-66.

Ding L, Huang Y and Pu X (2014). Volatility Linkage across Equity Markets. *Global Finance Journal*, 25, 71-89.

Domodran A (1999). Financing Innovation and Capital Structure Choices. *Journal of Applied Corporate Finance*, 12, 28-39.

Doran JS and Ronn EI (2008). Computing the Market Price of Volatility Risk in the Energy Commodity Markets. *Journal of Banking & Finance*, 32(12), 2541-2552.
Du X, Cindy LY and Hayes DJ (2011). Speculation and Volatility Spillover in the Crude Oil and Agricultural Commodity Markets: A Bayesian Analysis. *Energy Economics*, 33(3), 497-503.
Ehrmann M, Fratzsher M and Rigobon R (2011). Stocks, Bonds, Money Markets and Exchange Rates: Measuring International Financial Transmission. *Journal of Applied Econometrics*, 26(6), 948-974.
Eizaguirre JC, Biscarri JG and Gracia Hidalgo FP (2004). Structural Changes in Volatility and Stock Market Development: Evidence for Spain. *Journal of Banking & Finance*, 28, 1745-1773.
Elder J and Jin HJ (2007). Long Memory in Commodity Futures Volatility: A Wavelet Perspective. *Journal of Futures Markets*, 27(5), 411-437.
Embrechts P, Puccetti G, Rüschendorf L, Wang R and Beleraj A (2014). An Academic Response to Basel 3.5. *Risks*, 2(1), 25-48.
Enders W and Holt MT (2012). Sharp Breaks or Smooth Shifts? An Investigation of the Evolution of Primary Commodity Prices. *American Journal of Agricultural Economics*, 94(3), 659-673.
Engle RF (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica: Journal of the Econometric Society*, 50(4), 987-1007.
Engle RF and Ng VK (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance,* 48 (5), 1749-1778.
Engle RF and Patton AJ (2000). What is a Good Volatility Model? *Quantitative Finance*, 1, 237-245.
Engle RF, Ghysels E and Sohn B (2013). Stock Market Volatility and Macroeconomic Fundamentals. *Review of Economics & Statistics*, 95(3), 776-797.
Ewing BT and Malik F (2013). Volatility Transmission between Gold and Oil Futures under Structural Breaks. *International Review of Economics & Finance*, 25,113-121.
Fatnassi I, Slim C, Ftiti Z and Maatoug AB (2014). Effects of monetary policy on the REIT returns: Evidence from the United Kingdom. *Research in International Business & Finance,* 32, 15-26.
Fisher JD, Geltner DM and Webb RD (1994). Value Indices of Commercial Real Estate: A Comparison of Index Construction Methods. *Journal of Real Estate Finance & Economics*, 9(2), 137-164.
Fleming J, Kirby C and Ostdiek B (1998). Information and Volatility Linkages in the Stock, Bond and Money Markets. *Journal of Financial Economics*, 49(1), 111-137.
Franses PH, Van der Leij M and Paap R (2008). A Simple Test for GARCH against a Stochastic Volatility Model. *Journal of Financial Econometrics*, 6 (3), 291-306.
Gamba-Santamaria S, Gomez-Gonzalez JE, Hurtado-Guarin JL and Melo-Velandia LF (2017). Stock Market Volatility Spillovers: Evidence for Latin America. *Finance Research Letters*, 20, 207-216.
Gao H, L, K and Ma Y (2020). Stakeholder Orientation and the Cost of Debt: Evidence from State-Level Adoption of Constituency Statutes. *Journal of Financial & Quantitative Analysis*, 1-37.
Garcia R and Tsafack G (20111). Dependence Structure and Extreme Comovements in International Equity and Bond Markets. *Journal of Banking & Finance*, 35(8), 1954-1970.
Gel'Man V (2008). Party politics in Russia: From Competition to Hierarchy. *Europe-Asia Studies*, 60(6), 913-930.

Gerlach R, Wilson P and Zurbruegg R (2006). Structural Breaks and Diversification: The Impact of the 1997 Asian Financial Crisis on the Integration of Asia-Pacific Real Estate Markets. *Journal of International Money & Finance,* 25(6), 974-991.
Geweke J and Porter-Hudak S (1993). The Estimation and Application of Long Memory Time Series Models. *Journal of Time Series Analysis*, 4(4), 221-238.
Gilenko E and Fedorova E (2014). Internal and External Spillover Effects for The BRIC Countries: Multivariate GARCH-in-mean approach. *Research in International Business & Finance*, 31, 32-45.
Gokcan S (2000). Forecasting Volatility of Emerging Stock Markets: Linear versus Non-linear GARCH Models. *Journal of Forecasting*, 19(6), 499-504.
Goda T, Lysandrou P and Stewart C (2013). The Contribution of US Bond Demand to the US Bond Yield Conundrum of 2004-2007: An Empirical Investigation. *Journal of International Financial Markets, Institutions & Money*, 27,113-136.
Gregory AW and Hansen BE (1996). Residual-Based Tests for Cointegration in Models with Regime Shifts. *Journal of Econometrics*, 70(1), 99-126.
Grieger G (2020). China's Economic Recovery and Dual Circulation Model. European Parliamentary Research Service. Retrieved from https://www.europarl.europa.eu/RegData/etudes/BRIE/2020/659407/EPRS_BRI(2020)659407_EN.pdf
Güloğlu B, Kaya P and Aydemir R (2016). Volatility Transmission Among Latin American Stock Markets under Structural Breaks. *Physic A*, 462(15), 330-340.
Gurrib I (2008). Do Large Hedgers and Speculators React to Events? A Stability and Events Analysis. *Applied Financial Economics Letters*, 4(4), 259-267.
Haggard S, MacIntyre A and Tiede L (2008). The Rule of Law and Economic Development. *Annual Review of Political Science*, 11, 205-234.
Haigh MS and Holt MT (2002). Crack Spread Hedging: Accounting for Time-Varying Volatility Spillovers in the Energy Futures Markets. *Journal of Applied Econometrics*, 17(3), 269-289.
Hamilton JD and Lin G (1996). Stock Market Volatility and the Business Cycle. *Journal of Applied Econometrics*, 11(5), 573-593.
Hammoudeh S, Li H and Jeon B (2003). Causality and Volatility Spillovers among Petroleum Prices of WTI, Gasoline and Heating Oil in Different Locations. *North American Journal of Economics & Finance*, 14(1), 89-114.
Hansen PR and Lunde A (2005). A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH (1, 1)? *Journal of Applied Econometrics*, 20(7), 873-889.
Hadri K and Rao Y *(2008). Panel Stationarity Test with Structural Breaks. Oxford Bulletin of Economics & Statistics, 70(2), 245-269.*
Hansen BE (2001). The New Econometrics of Structural Change: Dating Breaks in US Labor Productivity. *Journal of Economic Perspectives,* 15(4), 117-128.
Hasbrouck J (2018). High-Frequency Quoting: Short-Term Volatility in Bids and Offers. *Journal of Financial & Quantitative Analysis*, 53(2), 613-641.
Heath D, Jarrow R and Morton A (1990). Bond Pricing and the Term Structure of Interest Rates: A Discrete Time Approximation. *Journal of Financial & Quantitative Analysis*, 25(4), 419-440.
Hillebrand E (2005). Neglecting Parameter Changes in GARCH Models. *Journal of Econometrics,* 129(1-2), 121-138.
Ho TS and Lee SB (1986). Term Structure Movements and Pricing Interest Rate Contingent Claims. *Journal of Finance*, 41(5), 1011-1029.

Hoesli M, Lizieri C and MacGregor B (2008). The Inflation Hedging Characteristics of US and UK Investments: a Multi-Factor Error Correction Approach. *Journal of Real Estate Finance & Economics*, *36*(2), pp.183-206.
Hoesli M and Reka K (2013). Volatility Spillovers, Co-movements and Contagion in Securitised Real Estate Markets. *Journal of Real Estate Finance & Economics*, 47(1), 1-35.
Hoevenaars RP, Molenaar RD, Schotman PC and Steenkamp TB (2008). Strategic Asset Allocation with Liabilities: Beyond Stocks and Bonds. *Journal of Economic Dynamics & Control*, 32(9), 2939-2970.
Holmes MJ (2011). Threshold Cointegration and the Short-Run Dynamics of Twin Deficit Behaviour. *Research in Economics*, *65*(3), 271-277.
Hood M and Malik F (2018). Estimating Downside Risk in Stock Returns under Structural Breaks. *International Review of Economics & Finance*, 58, 102-112.
Horpestad JB, Lyocsa S, Molnar P and Olsen TB (2019). Asymmetric Volatility in Equity Markets around the World. *North American Journal of Economics & Finance*, 48, 540-554.
Horvath B, Muguruza A and Tomas M (2021). Deep Learning Volatility: A Deep Neural Network Perspective on Pricing and Calibration in (Rough) Volatility Models. *Quantitative Finance*, 21(1), 11-27.
Huerta-Sanchez D and Escobari D (2018). Changes in Sentiment on REIT Industry Excess Returns and Volatility. *Financial Markets & Portfolio Management*, 32(3), 239-274.
Hull JC (2015). Risk Management and Financial Institutions, 4th ed., Wiley Finance Publishers, New Jersey, USA.
Inclan C and Tiao GC (1994). Use of Cumulative Sums of Squares for Retrospective Detection of Changes of Variance. *Journal of the American Statistical Association*, 89 (427), 913-923.
Jacks DS, O'Rourke KH and Williamson JG (2011). Commodity Price volatility and World Market Integration Since 1700. *Review of Economics & Statistics*, 93(3), 800-813.
James GM (2002). Generalized Linear Models with Functional Predictors. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64(3), 411-432.
Jiang L, Wang X and Yu J (2018). New Distribution Theory for the Estimation of Structural Break Point in Mean. *Journal of Econometrics*, 205(1), 156-176.
Jirasakuldech B, Robert D and Riza E (2009). Conditional Volatility of Equity Real Estate Investment Trust Returns: A Pre- and Post-1993 Comparison. *Journal of Real Estate Finance & Economics*, 38(2), 137-154.
Kester WC (1986). Capital and Ownership Structure: A Comparison of United States of Japanese Manufacturing Corporations. *Financial Management*, 15(1), 5-16.
Khamfula Y (2004). Macroeconomic Policies, Shocks and Economic Growth in South Africa. SEBS Working Paper, WITS University, South Africa. Retrieved from https://www.imf.org/external/np/res/seminars/2005/macro/pdf/khamfu.pdf
Kim S-J, Moshirian F and Wu E (2006). Evolution in International Stock and Bond Market Integration: Influence of the European Monetary Union. *Journal of Banking & Finance*, 30(5), 1507-1534.
Kim JW, Leatham DJ and Bessler DA (2007). REITs' Dynamics under Structural Change with Unknown Break Points. *Journal of Housing Economics*, 16(1), 37-58.
Koijen RS, Lustig H and Van Nieuwerburgh S (2017). The Cross-Section and Time Series of Stock and Bond Returns. *Journal of Monetary Economics*, 88, 50-69.
Kokoszka P and Leipus P (2000). Change-Point Estimation in ARCH Models. *Bernoulli*, 6(3), 1-28.
Koma SB (2010). The State of Local Government in South Africa: Issues, Trends and Options. *Journal of Public Administration*, 45(si-1), 111-120.

Kumar M, Moorthy U and Perraudin W (2003). Predicting emerging market currency crashes. *Journal of Empirical Finance*, 10(4), 427-454.

Kurtosis A, Markellos RN and Symeonidis L (2016). An International Comparison of Implied, Realized, and GARCH Volatility Forecasts. *Journal of Future Markets*, 36 (12), 1164-1193.

Kuttu S (2014). Return and Volatility Dynamics among Four African Equity Markets: A Multivariate VAR-EGARCH Analysis. *Global Finance Journal*, 25, 56-69.

Kuttu S (2018). Modelling Long Memory Volatility in Sub-Saharan African Equity Markets. *Research in International Business & Finance*, 44, 176-185.

Lavielle M and Moulines E (2000). Least-Squares Estimation of an Unknown Number of Shifts in Time Series. *Journal of Time Series Analysis*, 20(1), 33-60.

Le Pen Y and Sévi B (2017). Futures Trading and the Excess Co-Movement of Commodity Prices. *Review of Finance*, 22(1), 381-418.

Leachman LL and Francis B (1996). Equity Market Return Volatility: Dynamics and Transmission among the G-7 Countries. *Global Finance Journal*, 7(1), 27-52.

Lee J and Strazicich MC (2003). Minimum Lagrange Multiplier Unit Root Test with Two Structural Breaks. *Review of Economics & Statistics*, 85(4), 1082-1089.

Lee ML, Lee MT and Chiang KC (2008). Real Estate Risk Exposure of Equity Real Estate Investment Trusts. *Journal of Real Estate Finance & Economics*, 36(2), 165-181.

Lee C, Chein M and Lin TC (2012). Dynamic Modelling of Real Estate Investment Trusts and Stock Markets. *Economic Modelling*, 29(2), 395-407.

Levy C (2012). Social Movements and Political Parties in Brazil: Expanding Democracy, the 'Struggle for the Possible' and the Reproduction of Power Structures. *Globalizations*, 9(6), 783-798.

Li Y and Giles DE (2015). Modelling Volatility Spillover Effects between Developed Stock Markets and Asian Emerging Stock Markets. *International Journal of Finance & Economics*, 20(2), 155-177.

Lian C, Ma Y and Wang C (2019). Low Interest Rates and Risk-Taking: Evidence from Individual Investment Decisions. *Review of Financial Studies*, 32(6), 2107-2148.

Lin P (2013). Examining Volatility Spillover in Asian REIT Markets. *Applied Financial Economics*, 23(22), 1701-1705.

Liow KH, Zhu H, Ho D and Kwame Al (2005). Regime Changes in International Securitized Property Markets. *Journal of Real Estate Portfolio Management*, 11(2), 147-167.

Liow KH (2008). Long-term Memory in Volatility: Some Evidence from International Securitized Real Estate Markets. *Journal of Real Estate Finance & Economics*, 39(4), 415-438

Liow KH (2009). Long-Term Memory in Volatility: Some Evidence from International Securitised Real Estate Markets. *Journal of Real Estate Finance & Economics*, 39(4), 415-438.

Liow KH, Chen Z and Jingran L (2011). Multiple Regimes and Volatility Transmission in Securitised Real Estate Markets. *Journal of Real Estate Finance & Economics*, 42(3), 295-328.

Lumsdaine RL and Papell DH (1997). Multiple Trend Breaks and the Unit-Root Hypothesis. *Review of Economics & Statistics*, 79(2), 212-218.

Low AHW, Muthuswamy J and Webb RI (1999). Arbitrage, Cointegration, and the Joint Dynamics of Prices across Discrete Commodity Auctions. *Journal of Futures Markets*, 19(7), 799-815.

Liow KH and Ibrahim MF (2010). Volatility Decomposition and Correlation in International Securitized Real Estate Markets. *Journal of Real Estate Finance & Economics*, 40(2), 221-243.

Liow KH, Chen Z and Liu J (2011). Multiple Regimes and Volatility Transmission in Securitized Real Estate Markets. *Journal of Real Estate Finance & Economics*, 42(3), 295-328.

Liow KH (2012). Co-movements and Correlations Across Asian Securitized Real Estate and Stock Markets. *Real Estate Economics,* 40(1), 97-129.
Liow KH (2014). The Dynamics of Return Co-Movements and Volatility Spillover Effects in Greater China Public Property Markets and International Linkages. *Journal of Property Investment & Finance*, 32(6), 610-641.
Liow KH and Schindler F (2014). An Assessment of the Relationship between Public Real Estate and Stock Markets at the Local, Regional and Global Levels. *International Real Estate Review*, 17(2), 157-202.
Liow KH (2015). Conditional Volatility Spillover Effects across Emerging Financial Markets. *Asia-Pacific Journal of Financial Studies,* 44, 215-245.
Liow KH and Ye Q (2017). Switching Regime Beta Analysis of Global Financial Crisis: Evidence from International Public Real Estate Markets. *Journal of Real Estate Research,* 39(1), 127-164.
Lucas A and Klaassen P (2006). Discrete Versus Continuous State Switching Models for Portfolio Credit Risk. *Journal of Banking & Finance*, 30(1), 23-35.
Lu XF, Lai KK and Liang L (2014). Portfolio Value-at-Risk Estimation in Energy Futures Markets with Time-VaryingCcopula-GARCH Model. *Annals of Operations Research*, 219(1), 333-357.
Maghyereh AI and Awartani B (2016). Dynamic Transmissions between Sukuk and Bond Markets. *Research in International Business & Finance*, 38, 246-261.
Malle S (2017). Russian and China in the 21st Century: Moving Towards Cooperative Behaviour. *Journal of Eurasian Studies*, 8, 136-150.
Marcato G, Sebehela T and Campani CH (2018). Volatility Smiles When Information is Lagged in Prices. *North American Journal of Economics & Finance,* 46, 151-165.
Markowitz H (1952). Portfolio Selection. *Journal of Finance*, 7(10, 77-91.
Mau V (2018). Russian Economic Policy: Challenges of Growth. *Russian Journal of Economics*, 4, 87-107.
McConnell JJ and Servaes H (1995). Equity Ownership and the Two Faces of Debt. *Journal of Financial Economics*, 39(1), 131-157.
Mei B, Clutter ML and Harris TG (2013). Timberland Return Drivers and Timberland Returns and Risks: A Simulation Approach. *Southern Journal of Applied Forestry*, 37(1), 18-25.
Mensi W, Boubaker FZ, Al-Yahyaee KH and King SH (2018). Dynamic Volatility Spillovers and Connectedness between Global, Regional and GIPSI Stock Markets. *Finance Research Letters*, 25, 230-238.
Merton RC (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance*, 29(2), 449-470.
Miao H, Ramchander S, Wang T and Yang D (2017). Role of Index Futures on China's Stock Markets: Evidence from Price Discovery and Volatility Spillover. *Pacific-Basin Finance Journal*, 44, 13-26.
Miller JI and Ratti RA (2009). Crude Oil and Stock Markets: Stability, Instability and Bubbles. *Energy Economics*, 31(4), 559-568.
Minton BA and Schrand C (1999). The Impact of Cash Flow Volatility on Discretionary Investment and The Costs of Debt and Equity Financing. *Journal of Financial Economics*, 54(3), 423-460.
Morawski J and Rekugler H and Füss R (2008). The Nature of Listed Real Companies: Property or Equity Market. *Financial Markets & Portfolio Management*, 22(2), 101-126.

Morrison W (2019). China’s Economic Rise: History, Trends, Challenges and Implications for the United States. Available from: https://crsreports.congress.gov/product/pdf/RL/RL33534 [accessed 20 April 2021]

Mukherji R (2014). The Political Economy of Development in India. Oxford India Short Introductions. Oxford University Press.

Naranjo A and Ling DC (1997). Economic Risk Factors and Commercial Real Estate Returns. *Journal of Real Estate Finance & Economics,* 14(3), 283-307.

Narayan PK, Narayan S and Sharma SS (2013). An Analysis of Commodity Markets: What Gain for Investors? *Journal of Banking & Finance*, 37(10), 3878-3889.

Nayyar D (2016). BRICS, Developing Countries and Global governance. *Third World Quarterly*, 37(4), 575-591.

Nelson B (1991). ‘Conditional heteroscedasticity in asset returns: a new approach. *Econometricia*, 59(2), 347-370.

Nikbakht E, Shahroki M and Spieler AC (2016). An International Perspective of Volatility Spillover Effect: The Case for REITs. *International Journal of Business*, 21(4), 283.

Nkadimeng MG (1999). From Miracle to Mirage: The ANC’s Economic Policy Shift from Merg to Gear. M.Sc. Research Report, WITS University, South Africa.

Nguyen D (2008). An Empirical Analysis of Structural Changes in Emerging Market Volatility. *Economics Bulletin,* 6(10), 1-10.

Okunev J and Wilson P (1997). Using Nonlinear Tests to Examine Integration between Real Estate and Stock Markets. *Real Estate Economics,* 25(3), 487-503.

Okunev J, Wilson P and Zurbruegg R (2000). The Causal Relationship between Real Estate and Stock Markets. *Journal of Real Estate Finance & Economics,* 21(3), 251-261.

Omran M and McKenzie E (2000). Heteroscedasticity in Stock Returns Data revisted: Volume versus GARCH Effects. *Applied Financial Ecomomics,* 10, 553-560.

Pagan A and Schwert G (1990). Alternative Models for Conditional Stock Volatility. *Journal of Econometrics,* 45 (1-2), 267-290.

Pettengill GN, Sundaram S and Mathur I (1995). The Conditional Relation between Beta and Returns. *Journal of Financial & Quantitative Analysis*, 30(1), 101-116.

Perez-Quiros G and Timmermann A (2000). Firm Size and Cyclical Variations in Stock Returns. *Journal of Finance,* 55 (3), 1229-1262.

Pindyck RS (2004). Volatility and Commodity Price Dynamics. *Journal of Futures Markets*, 24(1), 1029-1047.

Piotroski JD (2012). The Influence of Political Forces on Financial Reporting and Capital Market Activity. *European Accounting Review,* 21(3), 649-650.

Pritsker M (2001). The Hidden Dangers of Historical Simulation. *Journal of Banking & Finance,* 561-582

Pavlova I, Cho JH, Parhizgari AM and Hardin III WG. (2014). Long Memory in REIT Volatility and Changes in the Unconditional Mean; a Modified FIGARCH Approach. *Journal of Property Research*, 31(4), 315-332.

Peng L, Xiong W and Bollerslev T (2007). Investor Attention and Time-Varying Comovements. *European Financial Management*, 13(3), 394-422.

Perron P (1989). The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis. *Econometrica,* 57(4), 1361-1401.

Phillips P and Yu J (2011). Dating the Timeline of Financial Bubbles During the Subprime Crisis. *Quantitative Economics*, 2, 455-491.

Qu Z and Perron P (2007). Estimating and Testing Structural Changes in Multivariate Regressions. *Econometrica*, 75 (2), 459-502.
Rapach DE and Wohar ME (2006). Structural Breaks and Predictive Regression Models of Aggregate US Stock Returns. *Journal of Financial Econometrics*, 4(2), 238-274.
Rogers N, Winson-Gelderman K and Karafiath I (2014). The Impact of Trading Volume on REIT Volatility using the GARCH model. *Journal of Real Estate Portfolio Management,* 20 (3), 167-178.
Roll R (1984): A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. *Journal of Finance*, 39(4), 1127-1139.
Sandahl G and Sjögren S (2003). Capital Budgeting Methods Among Sweden's Largest Groups of Companies. The State of the Art and a Comparison with Earlier Studies. *International Journal of Production Economics*, 84(1), 51-69.
Schwert GW (2011). Stock Volatility During the Recent Financial Crisis. *European Financial Management*, 17(5), 789-805.
Sebehela T (2014). The "Delta" of the Margrabe Formula. *Annals of Financial Economics* 9(3), 1450007.
Sebehela T (2009). The Impact of the Subprime Mortgage Financial Crisis on Housing Finance in South Africa. *Housing Finance International* 23(4), 44-46.
Sebehela T. (2016). Portfolio Formation Memory. *Annals of Financial Economics* 11(2), 1650010.
Shalini V and Prassanna K (2016). Impact of the Financial Crisis on Indian Commodity Markets: Structural Breaks and Volatility Dynamics. *Energy Economics*, 53, 40-57.
Shih CW (1963). Co-Operatives and Communes in Chinese Communist Fiction. *China Quarterly*, 13, 195-211.
Shukla A and Kukreja M (2014). A Study of Financial Risk Management. *International Journal on Concept Management*, 3(1), 1-9.
Skintzi VD and Refenes AN (2006). Volatility Spillovers and Dynamic Correlation in European Bond Markets. *Journal of International Financial Markets, Institutions & Money*, 16(1), 23-40.
Smirnov S (2015). Economic Fluctuations in Russia (from the late 1920s to 2015). *Russian Journal of Economics*, 1(2), 130-153.
Solnik B, Boucrelle C and Le Fur Y (1996). International Market Correlation and Volatility. *Financial Analysts Journal*, 52(5), 17-34.
Steeley JM (2006). Volatility Transmission Between Stock and Bond Markets. *Journal of International Financial Markets, Institutions & Money*, 16(1), 71-86.
Straub L and Werning I (2020). Positive Long-Run Capital Taxation: Chamley-Judd Revisited. *American Economic Review*, 110(1), 86-119.
Suri KC, Elliott C and Hund, D (2016). Democracy, Governance and Political Parties in India: An Introduction. *Studies in Indian Politics*, 4(1), 1-7.
Syriopoulos T, Markam B and Boubaker A (2015). Stock Market Volatility Spillovers and Portfolio Hedging: BRICS and the Financial Crisis. *International Review of Financial Analysis*, 39, 7-18.
Tamakoshi G and Hamori S (2014). Greek Sovereign Bond Index, Volatility, and Structural Breaks. *Journal of Economics & Finance*, 38(4), 687-697.
Taylor I (2009). The South Will Rise Again? New Alliances and Global Governance: The India-Brazil-South Africa Dialogue Forum. *Politikon*, 36(1), 45-58.
Taylor SJ (1994). Modeling Stochastic Volatility: A Review and Comparative Study. *Mathematical Finance*, 4(2), 183-204.

Thao TP and Daly K. (2012). The Impacts of the Global Financial Crisis on Southeast Asian Equity Markets Integration. *International Journal of Trade, Economics & Financial,* 3(4), 299-304.
Titman S and Warga A (1986). Risk and the Performance of Real Estate Investment Trusts: A Multiple Index Approach. *Real Estate Economics*, 14(3), 414-431.
Thompson P and Wissink H (2018). Recalibrating South Africa's Political Economy: Challenges in Building a Developmental and Competition State. *African Studies Quarterly,* 18(1), 31-48.
Todorov V (2019). Nonparametric Spot Volatility from Options. *Annals of Applied Probability,* 29(6), 3590-3636.
Trenca L and Dezsi (2011). Advantages and Limitations of VaR Models Used in Managing Market Risk in Banks. *Finance-Challenges of the Future*, 13, 32-43.
Tsuji C (2018). Structural Breaks and Volatility Persistence of Stock Returns: Evidence from the US and UK Equity Markets. *Applied Economics & Finance*, 5(6), 76-83.
Turtle HJ and Zhang C (2015). Structural Breaks and Portfolio Performance in Global Equity Markets. *Quantitative Finance*, 15(6), 909-922.
Turnbull SM and Wakeman LM (1991). A Quick Algorithm for Pricing European Average Options. *Journal of Financial & Quantitative Analysis*, 26(3), 377-389.
Van Griensven A, Meixner T, Grunwald S, Bishop T, Diluzio M and Srinivasan R (2006). A Global Sensitivity Analysis Tool for the Parameters of Multi-Variable Catchment Models. *Journal of Hydrology*, 324(1-4), 10-23.
Wadhva PC (2000). Political Economy of post-1991 Economic reforms in India. *South Asia: Journal of South Asian Studies,* 23(1), 207-220.
We S (1963). Political Parties in Communist China. *Asian Survey*, 3(3), 157-164.
Wilson PJ, Stevenson S and Zurbruegg R (2007). Foreign Property Shocks and the Impact on Domestic Securitised Real Estate Market: An Unobserved Components Approach. *Journal of Real Estate Finance & Economics,* 34(3), 407-424.
Wimmerstedt L (2015). Backtesting Expected Shortfall: The Design and Implementation of Different Backstests. M.Sc. Thesis, KTH Royal Institute of Technology, Sweden.
Winniford M (2003) Real Estate Investment Trusts and Seasonal Volatility: a Periodic GARCH Model. Working Paper, Duke University, USA.
Xu S (2017). A VaR Assuming Student t Distribution not significantly Different from a VaR Assuming Normal Distribution. *Risk Management,* 19(3), 189-201
Xu Y, Taylor N and Lu W (2018). Illiquidity and Volatility Spillover Effects in Equity Markets during and After the Global Financial Crisis: An MEM Approach. *International Review of Financial Analysis,* 56, 208-220.
Xu K (2013). Powerful Tests for Structural Changes in Volatility. *Journal of Econometrics,* 173(1), 126-142.
Yamai Y and Yoshiba T (2005). Value-at-Risk versus Expected Shortfall: A Practical Perspective. *Journal of Banking & Finance*, 29, 997-1015.
Yin A (2019). Out-of-Sample Equity Premium Prediction in the Presence of Structural Breaks. *International Review of Financial Analysis,* 65, 1-16.
Zhao Y and Peters BG (2009). The State of the State: Comparing governance in China and the United States. *Public Administration Review*, 69(1), 122-128.
Zhou J and Kang Z (2011). A Comparison of Alternative Forecast Models of REIT Volatility. *Journal of Real Estate Finance & Economics,* 42(3), 275-294.
Zhou X, Zhang W and Zhang J (2012). Volatility Spillovers between Chinese and World Equity Markets. *Pacific-Basin Finance Journal,* 20, 247-270.

Zhou J (2011). Long Memory in REIT Volatility Revisited: Genuine or Spurious and Self-Similar? *Journal of Property Research*, 28(3), 213-232.

Zhou X, Zhang W and Zhang J (2012). Volatility Spillovers between Chinese and World Equity Markets. Pacific-Basin Finance Journal, 20, 247-270.

Zhou J (2016). A High-Frequency Analysis of the Interactions between REIT Return and Volatility. *Economic Modelling*, 56, 102-108.

Zivot E and Andrews DWK (1992). Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. *Journal of Business & Economic Statistics*, 10, 251-270.

Ziobrowski AJ, Ziobrowski BJ and Rosenberg S (1997). Currency Swaps and International Real Estate Investment. *Real Estate Economics*, 25(2), 223-251.