MASS TRANSFER THROUGH A POROUS TUBE

by

B A BALE

A dissertation presented in fulfilment of the requirements for the degree of Master of Science in Engineering

UNIVERSITY OF THE WITWATERSRAND

JOHANNESBURG

SCHOOL OF MECHANICAL ENGINEERING

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MAY 1975

To

Mum and Dad

D E C L A R A T I O N

I, Barbara Anne Bale, hereby declare that this dissertation is my own work and has not been submitted by me for a degree at any other University.

B.A. Bale

ABSTRACT

The reasons for undertaking this research are explained with reference to work already attempted in the field. It was decided that the study of the flow properties associated with mass transfer through porous media required attention.

Theories by previous researchers were examined and it was realised that a better understanding of the flow phenomena could be gained with reference to a simplified mathematical model of the flow situation.

The flow field examined was produced by a cylindrical pipe along which fluid passed axially. Through the porous walls of the pipe fluid could be extracted or introduced in varying amounts. The situation was modelled using a potential flow technique involving the representation of the pipe surface by means of singularities such as sources and sinks. A finite element method was used to solve the resulting flow equations. The strength of the method lies in its generality, as the solution does not require the boundary of the pipe to be analytic in shape, and the boundary conditions also need not be analytic or continuous. Thus regions of suction or mass addition may be applied to the pipe by an inclusion of suitable boundary conditions.

An experimental test rig was designed which could accomodate porous bronze pipe specimens. Air flow rates varying in magnitude up to 1 000 m^3/hr were passed axially

through the pipe as the back pressure was varied, thus forcing different proportions of fluid to be expelled through the porous walls. Tests were performed with a range of suctions from no mass transfer to total mass extraction through the side walls Measurements were taken of the static pressure variations along the pipe, and the velocity profiles at five different axial positions. One of the velocity profiles at the beginning of the tube was measured with a pitot static tube, and at the other four locations hot-wire anemometers were used so that an estimate of the radial velocity could also be made.

Comparisons between the theory and the experiments of this work are made, together with reference to previous research. The potential flow solution provides a useful overall picture of the flow situation and predicts such properties as pressure distributions and suction variations remarkably well. The inability to simulate friction coefficients is obviously a limitation and a suggestion for future work is the adaptation of the solution to account for these effects. The experiments agree well with the few results available from other sources.

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NOTATION

ROMAN

A	Matrix of normal velocity influence coefficients
a_{ij}	Element i,j of matrix A
B	Matrix of tangential velocity influence coefficients
b_{ij}	Element i,j of matrix B
b	x co-ordinate of element's control position
C_d	Coefficient of discharge
Cp	Coefficient of pressure
Cp_k	Coefficient of pressure relative to conditons at the inlet
D	Inside diameter of the pipe
d	Half the length of a singular subelement
$E(k)$	Complete elliptic integral of the second kind
E	Voltage drop across hot-wire probe
E_o	Voltage drop across hot-wire probe at zero velocity
E_n	Voltage drop corrected for temperature
F_j	Normal velocity boundary condition at j.
J	General point inside the pipe
$K(k)$	Complete elliptic integral of the first kind
k	Constant for the hot-wire equation $U_{eff}^2 = U_{\perp}^2 + kU_{\parallel}^2$
k	Argument of the elliptic integrals
m	Ratio of d^2/D^2 for the orifice plate calculations
n	Number of elements describing the pipe
$\underline{n}$	Unit vector normal to the pipe's surface
P	Static pressure
p	Static pressure used as a variable
Px	Element's x co-ordinate
Py	Element's y co-ordinate
Q	Volume flow rate
R	Radius of the pipe
Re	Reynolds number, $(\rho V_k D/\mu)$

r	Radial co-ordinate
s	Distance along the surface of the pipe
U_{eff}	Effective cooling velocity
$U_{\perp}$	Velocity perpendicular to the hot-wire
$U_{\parallel}$	Velocity parallel to the hot-wire
V	Velocity
$V_{x\ mean}$	Mean axial velocity
$V_{x\ max}$	Maximum axial velocity
x	Co-ordinate measured in the axial direction
y	Co-ordinate measured in the radial direction
y_o	y co-ordinate of element's control point

GREEK

α	Ratio of fluid extracted to axial inflow
β	Mass transfer coefficient $(V_r/V_{x\ max})$
β_o	Initial mass transfer coefficient (V_r/V_k)
β_{av}	Average mass transfer coefficient $\left(\frac{\text{Volume transferred}}{\text{porous surface area}}\right)$
Δs	Length of element
λ	Radial Reynolds number, $(\rho V_r D/\mu)$
μ	Viscosity
ν	Dynamic viscosity
ρ	Density
$\sigma(j)$	Source strength at position j
ϕ	Potential
ϕ_{ij}	Potential at j due to element i
Φ	Matrix of potential coefficient
ψ	Stream function

SUBSCRIPTS

a	Atmospheric conditions
k	Axial conditions at the entrance to the pipe
n	Referring to the n^{th} element
r	In the radial direction

w	Properties of water
x	In the axial direction
y	In the y (radial) direction
⊥	Perpendicular
∥	Parallel

CHAPTER 1

INTRODUCTION

In the search for effective cooling techniques, attention has been directed towards a method known as transpiration cooling. This process involves manufacturing the surfaces to be cooled from a porous material. A cold fluid is ejected through the surface to form a protective layer along the wall and isolate it from the influence of a hot fluid stream. This method is used on forward areas of high-speed aircraft and missiles as a protection against aerodynamic heating. Another important application of this method of cooling is in relation to propulsive devices. The performance of rocket and jet engines is limited by the maximum temperatures at which the device can operate, due to the thermal stresses experienced by such components as turbine blades and combustion chambers. Cooling of rocket and jet motors may be achieved by the diffusion of cooling fluids through porous metal combustion chamber liners or through porous turbine blades.

Porous surfaces with suction are also used on airfoils and bodies of aircraft to delay boundary layer separation or transition to turbulence.

As a result of the above applications numerous studies of the flow and heat transfer connected with fluid ejection or suction through porous surfaces have been made and they are discussed in Chapter 2 of this dissertation

The theoretical results obtained by previous researchers in the field were often limited by the methods of solution and results were only valid for small ranges of boundary conditions. The relative merits of the various theories could not be discussed without making comparisons with experimental data of which there was not much available. It was decided that an experimental test rig would be designed and built that would be as versatile as possible.

The design of the experimental apparatus is discussed in Chapter 5 and the scope of its applications is mentioned. To test the apparatus it was decided to perform experiments with pure mass transfer and leave heat transfer tests for a later research programme. The porous material that is to be examined is sintered bronze in the form of a pipe. The effects of sucking air out of the pipe, on an axial flow through the pipe, will be studied.

The solutions obtained from previous theories have been hampered by numerous assumptions and mathematical limitations, these are discussed in Chapter 2. It was thought necessary to avoid these totally mathematical solutions of the real flow problem and seek a more flexible form of solution. A potential flow solution is formalised and examined in Chapters 3 and 4.

Results from the potential flow technique and the experimental work are presented in Chapter 6 together with comparisons between work done by other authors and the present work.

The references follow Chapter 8 and then the Appendices. These contain aspects of work which the author considered would detract from the smooth reading of the thesis but were necessary to include for a deeper understanding of finer points involved.

CHAPTER 2

REVIEW OF THE LITERATURE ON PIPE FLOW WITH RADIAL MASS TRANSFER

2.1 Introduction

The purpose of this chapter is to present, in outline, the main properties of flow through porous media. A considerable amount of research has been done involving this type of flow and it may be divided into three main categories, i e

(i) flow through a porous flat plate, references [1-3]

(ii) flow through a porous channel, references [4-7]

(iii) flow through a porous pipe, references [8-25]

In essence the three are related, however, during the solution of the resulting equations, their similarity is lost. The first is useful in enabling the mechanism of mass transfer to be studied, but it is essentially a two dimensional problem with no bounding walls and so results obtained for flow properties such as pressure distributions [1], skin friction [2,3] and especially velocity distributions [1] are invalid for contained flows. The second has been studied mainly as a mathematical problem [6,7]. Channel flows enable the direct use of Cartesian co-ordinates [5] and although the flow is now contained between walls, the results cannot be applied to pipe flows because of the effects of the different geometries especially on the velocity distribution. Porous pipe flow has been examined by several authors both theoretically and experimentally. Correlations between the results obtained in both are rare, partly due to the mathematical limitations placed in the theoretical solutions and partly due to the practical limitations imposed by inadequate experimental equipment.

2.2 Theoretical Approaches

The first flows in pipes, concerned with mass addition or subtraction, to be studied in detail were in the late 1940's. The problem was referred to as 'The Manifold Problem'. These papers examined the flow in a pipe with discrete discharge openings, rather than uniform porosity, distributed along the manifold, or pipe length.

Keller [8] was interested in configurations of manifolds which would produce uniform discharge along the pipe. Equations were developed which equated the pressure variation along the tube with the inertia and frictional forces. The result was a second order non-linear differential equation which was solved by means of a point-by-point numerical method, the step sizes being determined by the distances between the adjacent discharge holes. Results of pressure variation, and hence discharge, were obtained for assumed values of friction factor and discharge coefficients. These were not compared with experimental tests, however, a configuration for uniform discharge was designed and qualitatively examined, giving good results. It is thought that this simple theory is not sufficient to analyse more complicated flow situations due to many simplifications in the analysis. Keller assumed that the friction factor for the pipe was unaffected by mass extraction at the walls. This seems unlikely but there is no experimental data to substantiate this. The velocity distribution across the pipe was ignored and a flat profile was assumed, thus giving an overestimation of the longitudinal momentum being lost at the wall. The third and perhaps not quite so serious assumption was that the coefficients of discharge of the orifices were independent of Reynolds number.

In trying to remove the mathematical difficulties encountered when dealing with discrete orifices Olson [9] treated the manifold as though it was uniformly porous along its length. The cross sectional area of the pipe was circular and did not vary with length. A differential equation was developed connecting the rate of lateral discharge to the pressure drop in the pipe. The relation between these quantities was obtained in terms of elliptic functions and because of this no results or graphs were given. The method for calculating results, given certain boundary conditions, was indicated, but no comparison made with other theories. Although fluid friction was taken into account, it was assumed that mass extraction at the walls did not affect the friction factor or the velocity profile, which was considered parabolic, and so the solution was only valid for the limiting case of negligible discharge. No account was taken of the momentum decrease in the pipe due to the mass extraction at the wall.

Van Der Hegge Zijnen [10] developed a theory which accounted for the drag due to the fluid leaving at the side walls as well as fluid friction. He did, however, neglect any boundary layer effects other than the shear stress at the wall. He assumed that the laws for fluid friction in continuous pipes also held for 'tapped' pipes. Solutions for uniform discharge (which means that the static pressure was assumed to remain constant over the entire length of the pipe) were obtained for laminar and turbulent flow. Both results proved to be trivial, since for laminar flow the length to diameter ratio was of the order of hundreds and the Reynolds number required for the turbulent flow case was far below the critical Reynolds number and hence the flow was not turbulent. Van Der Hegge Zijnen [10] also developed a theory for varying orifice spacing and changes in pipe area in order to achieve uniform discharge.

Late in the 1950's work was published which attempted to solve the Navier-Stokes equations for uniformly porous pipe flows. Yuan and Finkelstein [11] used the equations in cylindrical co-ordinates. As their method was used as a basis for further research it has been described in reasonable detail.

In cylindrical co-ordinates the Navier-Stokes and continuity equations become:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2}\right] \quad . \; . \; . \quad (2.2.1)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right] \quad (2.2.2)$$

and $$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2.2.3)$$

For a circular pipe with a porous wall, through which uniform fluid injection or suction is applied, the boundary conditions are:

at $\quad r = 0 : \quad v = \frac{\partial u}{\partial r} = 0$

at $\quad r = R : \quad u = 0, v = -v_o = \text{const}$

The authors then proposed a similarity transform involving the stream function ψ, of the following form:

$$\psi = [A + B\frac{x}{R}\, f(\eta)]$$

where $\quad \eta = (\frac{r}{R})^2$

The constants A & B are derived from the boundary conditions and mass flow considerations. If u_o is the maximum velocity at the start of mass transfer, where Poiseuille flow exists then the expression becomes:

$$\psi = \frac{R^2}{2}\left[\frac{u_o}{f'(o)} + 4v_o \frac{x}{R}\right]f(\eta) \qquad . \; . \; . \; . \; . \; . \qquad (2.2.4)$$

and the velocity components are given by:

$$u = u_o\left[\frac{1}{f'(o)} + 4\frac{\lambda}{Re}\frac{x}{R}\right]f(\eta) \qquad . \; . \; . \; . \; . \; . \qquad (2.2.5)$$

$$v = -2v_o\frac{f(\eta)}{\sqrt{\eta}} \qquad . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (2.2.6)$$

When equations (2.2.5) and (2.2.6) are substituted into equations (2.2.1) and (2.2.2) suitable differentiation and integration yield

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \qquad . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (2.2.7)$$

$$\eta f''' + f'' - \lambda((f')^2 - ff'') = c \qquad . \; . \; . \qquad (2.2.8)$$

for $\lambda \le 1$ and

$$(f')^2 - ff'' - \frac{1}{\lambda}(\eta f''' + f'') = k \qquad . \; . \; . \qquad (2.2.9)$$

for $\lambda > 1$.

Using the given boundary conditions, in terms of λ, the authors obtained solutions of equations (2.2.8) and (2.2.9) using λ as a perturbation parameter, for small and large values of λ. Flow properties such as velocity distribution, pressure gradients and skin friction were calculated and plotted for given values of Re x/R and λ. No comparisons were made with experimental results and the values of λ were limited. The main conclusions drawn were that the maximum axial velocity increased with the increase of fluid injection and decreased with increase in suction. The axial pressure drop became appreciably larger, even for very small fluid injection, than in the Poiseuille flow case, and appreciably less for the small suction case. The effect of injection into the pipe was to increase the wall frictional coefficient and with suction to decrease it.

Eckert et al [12] used the same similarity equations as Yuan and Finkelstein and solved them using a five point numerical integration technique, the only alteration made was the use of a boundary condition expressed in a slightly different form from that used in the previous solution. Results obtained were in reasonable agreement with the perturbation method, however, difficulty was encountered in the higher suction range. Two solutions were found at $\lambda = +2$ and $\lambda = +10$. One of the solutions at $\lambda = +2$ seemed to agree with extrapolation of the other results but both of the solutions at $\lambda = +10$ were vastly different from any reasonable extrapolation of the curve obtained from the other solutions. The maximum axial velocity was found not to be at the centre of the channel. No solutions could be found in the range $2 < \lambda < 10$. It was thought that a possible explanation for the difficulty was that in this range there was an inflexion in the velocity profile and the same difficulties were being experienced as in the solution of the boundary layer equations near the separation profile. Yuan and Finkelstein's perturbation method does not detect these dual solutions and this indicates, perhaps, an insensitivity in their method.

Berman [13] also solved the differential equation derived by Yuan and Finkelstein by a numerical method and it is concluded from this work that the highest derivative term, which was dropped from the differential equation in Yuan's treatment and which became less important with increasing injection could not be neglected in the neighbourhood of the wall for large suction. The validity of Yuan's approximation for large Reynolds numbers is thus again in doubt.

F M White [14] in a short paper tried to resolve the difficulties encountered by Eckert et al by giving, in power series form, the complete solution for the uniformly porous tube. He used the identical forms of the reduced Navier Stokes equations as Yuan and Eckert. He discovered that equation (2.2.8) could be solved by a power series solution in the form:

$$f(\eta) = d_1\eta + d_2\eta^2 + \ldots\ldots d_n\eta^n + \ldots\ldots$$

When this series was substituted into equation (2.2.8) recurrence relations were obtained for d_n. The results could be classed into three categories:

(i) The injection solutions, which were well-behaved and stable.

(ii) 'Regime one' wall suction solutions. These included suction Reynolds numbers from zero to approximately two. Double solutions existed for all these values of Reynolds number.

(iii) 'Regime two' wall suction solutions. Here double solutions existed for Reynolds numbers between approximately ten and infinity.

White could find no solutions between Reynolds numbers of two and ten thus substantiating the solutions of Eckert et al.

It is thought that these mathematical solutions would be difficult to realise in the laboratory due to the unstable nature of the suction. A paper by Weissberg [15] examines the flow development in the inlet, using an initially parabolic profile. Weissberg concluded from this mathematical analysis that the suction solutions couldn't be fully developed for values of λ greater than 2,3.

Morduchow [16] used the differential equation of Yuan and Finkelstein and applied a method of averages to find solutions for the total range of injection Reynolds numbers.

In principle this method uses equation (2.2.8) in its definite form, i e $c = 0$ and the limits on the left hand side are set from $\eta = 0$ to 1. A solution is assumed in the form of a fourth degree polynomial, the coefficients being derived from the original equations. The solution was found to agree exactly with the small-perturbation solution and agreed fairly well with the exact asymptotic solution as λ tended to minus infinity. Morduchow only applied this method to injection solutions and did not detect any dual solutions, which therefore makes the method doubtful.

Terrill and Thomas [17] also used a numerical technique to solve equation (2.2.8) and its associated boundary conditions. They realised that to solve it using the boundary conditions in the given form, (i e a boundary value problem) would require a double infinity of integrations for a prescribed Reynolds number. They overcame this by making a suitable transformation and allowing the Reynolds number to be determined by integration. The transformation used was:

$$f(\eta) = ag(\xi)$$

$$\eta = \xi/b$$

where a and b were constants to be determined.

The problem then reduced to one which could be solved by one integration using a step by step method. Difficulties were encountered in the vicinity of $\xi = 0$, so in this region Taylor's expansion was used and the Runge Kutta procedure thereafter. Solutions were found for the whole range of Reynolds numbers, i e $-\infty < R < +\infty$, with the remarkable results that dual solutions existed everywhere, even for injection, except in the range $2,3 < \lambda < 9,1$ where no solutions were found.

The authors attempted to explain the physics of the flow associated with the various solutions and it is relevant to note their explanations.

(i) Section I ($\lambda = -\infty$ to $\lambda = 2,3$) contained all the well behaved solutions for injection and the solutions for small suction up to the point where there was an inflexion in the profile.

(ii) Section II ($\lambda = 0$ to $2,3$) included small suction solutions which had reverse axial flow near the wall of the tube.

(iii) Section III ($\lambda = -\infty$ to $\lambda = 0$) covered the solutions for small and large injection, the velocity profiles being characterised by a region of reverse flow at the centre of the pipe.

(iv) Section IV ($\lambda = 21,2$ to $\lambda = \infty$) the velocity profiles had two turning points and a minimum between the axis and the pipe wall.

(v) Section V covered all the remaining solutions, the velocity profiles being characterised by a single point of inflexion.

The above papers have attempted to solve the laminar pipe flow by using the Navier Stokes equations. One simple theory has been developed by Berger and Blake [18]. They merely applied Newton's second law of motion to a control volume within the pipe. They used the extremely dubious assumption that the axial velocity remained constant across the duct, thus obtaining an overestimate of the axial momentum being transferred through the wall. They assumed an equivalent friction factor for the flow expressed in the form given by Silver and Wallis [19] for outward flow from the pipe and they did not take into account the fact that they would be using this as a friction factor for injection as well. It was mentioned, however, that frictional effects were of relatively small importance. The differential equation obtained was solved by means of a finite difference technique for the pressure distribution along the tube.

These solutions were compared with experimental results, given in the same paper. The assumption of a flat velocity profile in the derivation of the theoretical equation was found to underestimate the change in pressure along the tube.

So far in this chapter no papers have been discussed which allow for a variable wall suction along the lenth of the tube. Galowin et al in a recent paper [20] have attempted to solve the problem. A Karman - Polhausen integral momentum technique was adopted and fully developed Poiseuille pipe flow was postulated at the inlet. A quadratic velocity profile was assumed and the resulting differential equation reduced to a nonlinear ordinary differential equation. The magnitude of the suction velocity at the wall was small and varied with the axial co-ordinate only. Darcy's Law was assumed to describe the flow through the porous wall i e:

$$-\frac{\partial p}{\partial r} = \frac{\mu}{c}v$$

A fourth-order Runge-Kutta technique was used to integrate the second order, ordinary nonlinear differential equation subject to the normal boundary conditions and the condition that the axial velocity vanished at the end wall of the porous tube section, indicating complete extraction of the fluid.

The theoretical results showed good agreement over a limited range of suction Reynolds numbers with available experimental data. It seems that the analysis provides more accurate predictions of the flow properties than the previously developed similarity solutions.

It is difficult to objectively discuss the relative merits of the various theories examined without comparing the results with experimental data. Due to restrictions imposed by experimental techniques there is a severe shortage of experimental data that is directly relevant to the porous pipe flow problem.

2.3 Experimental Results

The first experiments, that the author is aware of, were performed by Wallis [21].

The apparatus used was a length of porous canvas hose with provision for measuring the inlet and outlet flow rates and recording the pressures along the tube at various intervals. Wallis supposed that by having large pressure drops across the porous walls, as compared with an axial pressure gradient, uniform mass extraction could be ensured. This assumption was examined by Galowin et al [20], as previously mentioned, and seems to have a noticeable effect on theoretical results. Static pressure tappings were introduced into the wall of the hose thus effecting its permeability and there is no mention in the report on the accuracy of using static pressure tappings when the flow direction is indeterminate. It seems that the pressure readings obtained will probably be overestimating the actual static pressure due to stagnation effects.

Graphs of pressure drop along the tube were plotted against axial distance for various suction rates. The results seemed to satisfy a simple theory developed in the report. Wallis developed two expressions for the pressure drop along the pipe,

(i) it was assumed that the fluid was removed from the side walls with no momentum loss in the axial direction;

(ii) It was assumed that mass was removed from the side walls with a velocity equal to the maximum axial velocity.

Expression (i) is an underestimate of axial momentum and it is expected that the latter is an overestimate.

Wallis stated that the experimental results should lie between the two regions which within the limited experimental accuracy they appeared to do.

A similar experiment was performed more recently by Quaile and Levy [22] who, for their porous pipe used a sintered bronze with a permeability, they supposed, small enough to ensure uniform suction along the wall. They did not have an outflow from the pipe, instead all the fluid was made to flow through the porous wall. Again, like Wallis, pressure tappings were introduced into the porous wall, affecting the permeability and also giving dubious readings because of the flow direction. A parabolic inlet profile was obtained by having a suitably long inlet pipe. Results of pressure drop versus axial distance were plotted for various suction Reynolds numbers and compared with known similarity solutions and 'inlet region' solutions as developed by reference [15]. It was found that in the range $2,4 < \lambda < 5,0$ there was good agreement between [15] and the experimental results, however the similarity solutions for $0 < \lambda < 2,3$ seemed to be grossly in error, as is expected from their irregular behaviour in this region.

Berger and Blake [23] were the first authors to study different porous specimens in the same test rig. One specimen was porous bronze, of the type used by Quail and Levy, and the other was a woven steel mesh. It was found that, due to its large permeability, it was impossible to achieve uniform suction through the steel mesh, and in some regions there was even reversed flow through the wall. The static and dynamic pressures along the axis of the pipe were measured by means of a pitot static probe. Graphs of comparisons between the theoretical pressure drop given by reference [18] and the experimental results were reasonable for the bronze tube but inaccurate for the steel tube due to reasons already mentioned. This is not surprising as the theory assumed a flat velocity profile. It would have been interesting to have studied a velocity profile from the experiment but no traverses were made.

Turbulent flow with mass addition was studied by Pappas and Okuno [24]. They measured the effects of injecting a gas through the surface of a cone which was situated in a main stream of mach number which was varied from 0,3 to 3,5. They were primarily interested in skin friction measurements and the effects of introducing different types of gases through the surface. The method used for measuring pressures (static pressure tappings) was again subject to errors for reasons already discussed. The main conclusion drawn from their results was the reduction in skin friction due to the injection of a foreign gas into the main stream. This need not necessarily be the case for pipe flow as the flow rate in a pipe is controlled and hence the velocity profile will change with fluid injection in a different manner to the cone flow, which is an external flow.

The first comprehensive experiments performed on turbulent porous pipe flow with uniform suction are described in a recent paper by Aggarwal et al [25]. The apparatus was essentially a bronze porous pipe surrounded by a housing divided into four chambers, through which the mass extraction could be controlled and hence uniform suction down the pipe could be approximated. Air from a mains supply was delivered to the test section through a settling tube of 48 diameters in length, thus ensuring fully developed turbulent flow.

Velocity profiles in the inlet plane were measured with a pitot tube and in the exit with a hot wire probe. At three intervals along the porous tube measurements were taken of the pressure by means of static pressure tappings. The authors acknowledged the difficulties encountered by previous experimenters in the measurement of static pressure when the flow direction is indeterminate, so they forced the flow to remain parallel to the wall in the vicinity of of the tapping by applying a small quantity of paraffin wax to the porous surface. It is not known to what extent this may have affected the results. The experiments covered an inlet Reynolds number range of 1,1 x 10^4 - 10,1 x 10^4 with a ratio of the transverse velocity at the wall to the mean axial velocity at inlet from zero to about 0,027. The main conclusions drawn from the results of the experiments may be summarised as follows:

(i) The net effect of suction was to make the velocity profile fuller at modest suction, and to make it more peaked as the suction rate became greater, (this was also experienced by Wallis [21]).

(ii) The relative turbulence level was found to increase with suction at all radii, save for some reduction in the region of the wall at very low rates of suction.

(iii) At a fixed inlet Reynolds number average friction factors increased markedly with suction but decreased along the tube.

(iv) In the radial direction suction created a definite, but insignificant, gradient of static pressure.

(v) Pressure coefficients were positive and showed a steady rise both with suction and distance along the tube, except for very low values of suction where negative values were experienced.