## Chapter 6: Symmetry in Patterns in the Msithini Group

## Introduction

This chapter is devoted to an investigation of the mathematical principles present in the patterns found specifically in the Msithini group of grass mats. The fundamental aim is to elucidate and present explanations of the presence of symmetry in the patterns in this group of grass mats, applying the seven international symmetry notations for one-dimensional pattern classification. ${ }^{1}$ By doing this, an acknowledgment and enrichment of this aspect of Swazi Material Culture will be realised and links with a new mathematical disciplinary paradigm Ethnomathematics - will be made. In addition, the aims of this chapter are to draw attention to some mathematical aspects and ideas incorporated in the patterns invented by Swazi mat makers and to support further initiatives that may contribute to a fuller realization of the mathematical potential of the rural crafts made by men and women in Swaziland.

The ethnomathematical approach adopted in this chapter is a theoretical framework that questions the origins of mathematical ideas and calls for a new history of mathematics. The objective of ethnomathematical research consists of looking for possibilities of improving the teaching of mathematics by embedding it into the cultural context of students and teachers. As Prof. Gerdes (1999) explains, it is "the analysis of mathematics and mathematics education in their cultural context."

Symmetry in patterns on other parallel objects of Swazi Material culture that admit patterning can be found on clay beer pots, Ludziwo (s) Tinziwo (pl,) beaded necklaces, Ligcebesha (s) Emagcebesha (pl), grinding mats (Sitsebe) and, more

[^0]recently, baskets made from Lutindzi (a mountain grass). These will be briefly illustrated pictorially to expand and support the concepts investigated.

The theories of Professor Paulus Gerdes of the Ethnomathematics Research Project based at Mozambiques’s Universidade Pedagogica in Maputo have been closely followed in this chapter. Gerdes has written extensively on geometry in African Art (Gerdes, 1988, 1990, 1991, 1993, 1994, 1995, 1996, 1997, 1998, 1999). He has developed a complementary methodology that enables one to uncover, in traditional and material culture, "hidden moments of geometrical thinking." Therefore, ethnomathematics, according to Prof Gerdes, is:

The analysis of mathematics and mathematics education in their cultural context.

He is concerned with those spheres of African life in which geometrical ideas, geometrical considerations, geometrical explorations, and geometrical imagination are interwoven, inter-braided, inter-plaited, inter-cut, inter-coiled, inter-cised, and inter-painted.

Gerdes (1999) explains his approach in the following way:

We looked to the geometrical forms and patterns of traditional objects like baskets, mats, pots, houses, fish traps, and so forth and posed some questions: Why do these material products possess the form they have? In order to answer this question, it came out that the form of these objects is almost never arbitrary, but represents many practical advantages and is the only possible or optimal solution of a production problem. ${ }^{3}$

In relation to the application of this method, quite a lot of 'hidden' or 'frozen' mathematics was discovered and Gerdes thus substantiates:

[^1]The artisan, who imitates a known production technique, is, generally, doing some mathematics. However, the artisans, who discovered the technique, did, and invented quite a lot of mathematics, were thinking mathematically. $[s i c]^{4}$

Whilst collecting, collating, and analysing the different properties of the Swazi grass mats, it became evident that although geometric patterns existed on a large number of grass mats, the Msithini Group in particular displayed the "hidden moments of geometrical thinking" referred to by Gerdes (1995). His theoretical and empirical claims are supported by Ascher (1994), a fellow Ethnomathematician; she believes there has to be more understanding of the ideas behind the artefacts as they are culturally embedded, so that their mathematical aspects can be recognised. Weaving, for example, involves geometric visualisation. In effect, the weaver is "digitalising the pattern. The weaver expresses the visualisation through actions and materials." ${ }^{5}$

Geometrical patterns are created on the surface of the grass mat using coloured wool and a variety of shimmering sweet wrappers. The formation of geometrical patterns on the Msithini Group of mats is related to technological advancement such as the introduction of the Imbongolo the mat-making frame, (see Chapter two) (Plates 52, 53, and 54). The realisation of vertical, horizontal, and rotational symmetry is demonstrated through the patterns found on the mats.

Prominent Ethnomathematicians, Dorothy Washburn and Donald Crowe (1988), conducted a comprehensive study of symmetry in cultural objects. Using mainly art objects from Africa, such as Zulu beadwork, Zairean carvings, Nigerian weaving and Egyptian wall decorations, they offered a perceptual process of pattern recognition

[^2]and theoretical style analysis in material culture. ${ }^{6}$ Thus a unification of two normally separate disciplines was applied - mathematics and design. ${ }^{7}$

The connections between art and mathematics are well established and are firmly rooted in the history of art. ${ }^{8}$ Art historians often use the characteristics of symmetry and the rules of the Golden Mean to assess the formal qualities of a work of art. Probably the best-known connection between mathematics and art is linear perspective, the representation, or illusion, of the third dimension on a flat, twodimensional surface through a basic structure of straight lines that appear to recede to a vanishing point on the horizon. In further establishing the interrelationship between art and mathematics, Hardy (1940) compared a mathematician to a painter or a poet and a maker of patterns made with ideas. He wrote:

The mathematician's patterns, like the painter or poet's, must be beautiful. The ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics. ${ }^{9}$

In the art of the Renaissance, perspective became the convention for representing space. In Europe at the turn of the $20^{\text {th }}$ century, social and scientific activities seem to have promoted a focus on design. Because the machines of the industrial revolution could stamp and weave patterns endlessly, there was a great need to codify and order this new wealth of material. At the same time, scholars in the humanistic fields were also categorising, classifying, and bringing new order to a wealth of artefacts amassed by explorers who had begun to fill museums with objects from their world-wide travels. ${ }^{10}$

[^3]Recent research in Africa has revealed the existence of mathematical correlations. For example, Claudia Zaslavsky, in her study "Africa Counts" (1999), pays tribute to Africa's contribution to the science of mathematics. From a mathematical point of view, the most interesting find is a carved bone discovered at the fishing site of Ishango on Lake Edward, in Zaire (DRC). It is a bone tool handle with notches arranged in definite patterns; a bit of quartz is fixed in a narrow cavity in its head. It dates back to the period between 23000 and 18000 B.C. The markings on the bone represent prime numbers between 1 and 19.

In South Africa, Becker, Getz and Martinson (2001) applied the theories of Ethnomathematics to a study of "Symmetry and Pattern in Southern African Flat Beadwork Panels." In a later study, Getz (2003) used African art objects to draw a correlation between fractal geometry and Zulu copper wire baskets (izimbenge), beadwork, bowls and murals in a video entitled "Ancient Dreams in Modern Times Mathematics in Our African Heritage." Getz believes:

Instead of studying spirals and other shapes in the abstract we can root them in the world around us - not only in nature, but also in the intricate designs of our own traditional weaving and beadwork. ${ }^{11}$

The use of traditional cultures as a source for educational instruction at all levels is becoming increasingly popular. Getz (2004), in her mathematical analysis of a Northern Sotho beaded apron, found that this type of apron consists of a repetition of equilateral triangles in a variety of colours. The mathematical relevance of the design
exercise, but the designers had a practical need to organise the myriad patterns from home and afar in some systematic descriptive fashion. Although the designers saw the rhythm and repetition inherent in the patterns, they never discovered that patterns could be more systematically, precisely, and objectively described by their symmetries.
${ }^{11}$ Getz, C., 2003 'Ancient Dreams in Modern Times - Mathematics in Our African Heritage’ Video, TSFILMTV
is that it repeats itself on the plane surface and mathematicians are interested in how many variations there are in which a pattern can possibly be repeated. ${ }^{12}$

As the aim of this chapter is to establish whether there is symmetry in patterns on Swazi grass mats, we turn to Crowe (1988). In several studies of African art, he has shown that patterns can be described by their symmetries and that repeated designs occur frequently on many art objects. He asserts that symmetry is a mathematical property that generates repeated patterns. The most popular use of symmetry is bilateral symmetry - that is, mirror reflection - also evident in the mats in the Msithini Group. Symmetry analysis provides a systematic tool and invites geometrical exploration. It is a powerful tool to present these symmetrical preferences; it does not explain these preferences but it does organise the data (Washburn and Crowe 1988).

There are reasons why researchers into African art and Ethonomathematicians consider the property of symmetry in the analysis of culturally produced patterns and designs on material culture. By understanding the role of symmetry in the visual recognition process, we can better recognise its frequency throughout a number of cultural domains. Crowe (1988), in several studies of African art, has shown that patterns can be described by their symmetries and that repeated designs occur frequently on many types of objects. Crowe (1980) has drawn attention to the existence of a universal cross-cultural classification scheme for the repeated patterns occurring on such diverse media as textiles, pottery, basketry, wall decoration, and the art of M.C. Escher. Crowe (1979) has also applied the international classification for the seven one-dimensional and 17 two-dimensional patterns to the analysis of decorated pipes from Begho, Ghana.

Symmetry in its various forms is one of the most fundamental features of the designs on the Msithini Group of grass mats. A possible reason could be because the shape

[^4]of the rectangular mat is conducive to being divided equally both horizontally and vertically. This is what Gerdes calls the "practical advantage of a production problem;" that is, the limitation of the technology applied.

The seven one-dimensional pattern classifications proposed by Washburn are applied to analyse the Msithini Group of mat patterns. The Msithini Group has been chosen for this analysis because, while some mats in the General Group exhibit geometric patterns, these are single motif patterns, whereas the Msithini Group comprises onedimensional patterns. Thus, this group can be analysed using the international seven one-dimensional pattern classifications.

Mathematicians are interested in how a pattern can possibly be repeated. It is a geometrical fact that there are only four possible rigid motions of a plane (distancepreserving transformation of the plane onto itself), 'plane' here referring to the 'Euclidean plane,' the plane of everyday experience: the tabletop, a stretched canvas, the unrolled surface of a cylindrical pot, the flat woven fabric that is imagined to extend to infinity in all directions (Washburn \& Crowe 1988). These are reflection, translation, rotation and glide-reflection. For this reason it is not surprising that any one-dimensional pattern admits one of only seven different admissible rigid motions; that is, there are only seven one-dimensional patterns. Similarly, there are only 17 two-dimensional patterns.

Getz (2004) explains rotations, translations, reflections, and glide reflections simply as imagining tracing a repeated pattern onto a large sheet of tracing paper. The ways in which you move the tracing paper so that the original and the traced patterns exactly coincide are obtained through the following possibilities:
I. slide the paper in a straight line - a translation
II. rotate the paper about a point - a rotation
III. rotate the tracing paper through $180^{\circ}$ about a straight line - a reflection
IV. combine a translation and a reflection to give - a glide reflection Therefore, the only way a periodic pattern can repeat itself is by means of some (or all) of the above 'rigid motions.' The following diagram illustrates the Seven Notations for one-dimensional patterns: ${ }^{13}$

$$
1.1
$$



| 1.1 | Notation 1 <br> pmm2 | Symmetries <br> Vertical, horizontal and rotational <br> symmetry of $180^{\circ}$ | Pattern invariant under <br> Vertical reflection, horizontal <br> reflection and rotation through an <br> angle of $180^{\circ}$ |
| :--- | :--- | :--- | :--- |



| 1.2 | Notation 2 <br> pma2 | Symmetries <br> Vertical, translational-reflected and <br> rotational symmetry of $180^{\circ}$ | Pattern invariant under <br> Vertical reflection, glide reflection <br> and rotation through an angle of <br> $180^{\circ}$ |
| :--- | :--- | :--- | :--- |



[^5]| 1.4 | Notation 4 <br> $p 1 m 1$ | Symmetries <br> Horizontal symmetry | Pattern invariant under <br> Horizontal reflection |
| :--- | :--- | :--- | :--- |



| 1.5 | Notation 5 <br> $p 112$ | Symmetries <br> Rotational symmetry of $180^{\circ}$ | Pattern invariant under <br> Rotation through an angle of $180^{\circ}$ |
| :--- | :--- | :--- | :--- |



| 1.7 | Notation 7 <br> p111 | Symmetries <br> Only translational symmetry | Pattern invariant under <br> Only translation |
| :--- | :--- | :--- | :--- |

A detailed discussion of all the possible variations of the Msithini Patterns will follow. The patterns to be analysed on the Msithini Group of mats will always be thought of as lying on a plane. First, a brief explanation of the special kinds of design that are normally referred to as periodic or repeated pattern is in order. A repeated pattern on the plane (flat surface) may repeat in only one direction (like a border, a strip or a band) or in more than one direction (like a fabric design that covers the entire surface or the hexagons on a tortoise shell.) The former are called onedimensional patterns; the latter are two-dimensional patterns. Another way of describing this difference is to say that a one-dimensional pattern can be slid along itself, in exactly one direction; in such a way, that in its resulting position it cannot be
noticed to have shifted. Such a pattern admits a translation in exactly one direction. A two-dimensional pattern admits translation in more than one direction. Twodimensional patterns may also be called "all over" patterns (Crowe 1980).

The Msithini Group of grass mats has invited a geometrical examination, and the explorations of the designs are placed in the one-dimensional pattern analysis because they are repeated patterns in one direction only.

The creation of the symmetrical patterns on the Msithini Group is largely dependent on the organised manipulation of coloured strings whilst making the mat on the Imbongolo. Many variations are possible (diagrams below represent these variations). At the simplest level, the pattern-making is comparable to a knitting pattern: the knitter memorises the pattern and whatever takes place on the left half of the mat is mirrored on the right half of the mat. For the grass mat, an uneven number of vertical strings will ensure a vertical line of symmetry. From the collection of the Msithini Group of 53 grass mats, (see Figures MM01 - MM53 on pages 134-135 and Appendix B, B37-B43) eight have been selected (for the eighth mat, Msithini was commissioned to make a mat with a pattern that was translation only). The primary reason for this was to complete the set of visual aids necessary to facilitate teaching this concept.

The patterns chosen were all selected from the central panel, each labelled ML1a to ML8a (ML denoting Msithini Licansi, the SiSwati word for mat). The central panel pattern was converted into a linear diagram, ML2b. For ML1c to ML8c, the patterns were rotated $90^{\circ}$ to the right for further observations of the presence of symmetry. For patterns MLIa, ML2a and ML7a, subjacent symmetries were sought (subjacent symmetries are symmetries within the existing pattern that only become prominent once that pattern is ungrouped). This allowed for a further analysis of line behaviour, increasing the symmetrical properties within pattern ML7. In the diagrams below, dotted lines are used to represent lines of symmetry:


If MLP1e is rotated $180^{\circ}$, it will map onto MLP1f


Patterns with double symmetry enjoy a remarkable property. If one rotates such a pattern through a straight angle $\left(180^{\circ}\right)$ about the point of intersection of the horizontal axis with one of the vertical axes, exactly the same pattern is obtained (MLP1g, illustrates this).


MLP2d may be rotated $180^{\circ}$ to map onto MLP2e, MLP2f \& MLP2g (repeat action for MLP2e, MLP2f and MLP2g)



MLP5c-p1m1


MLP6c - pmm2 (p112)


MLP7a

(Rotate MLP7b 90 ${ }^{\circ}$ to the right)
MLP7c - p1m1


## MLP7d

## Subjacent symmetries

In this case, the symmetries become stronger if MLP7b is separated. Thus MLP7d - pmm2


MLP7e-pmm2


MLP7g

When MLP7d is separated further, the behaviour of the pattern changes again. MLP7g - p1a1 (Translational-reflected symmetry) and pma2


MLP7h - p1a1 and pma2


MLP7i - p1a1 and pma2


MLP8a
(Rotate MLP8b 90 ${ }^{\circ}$ to the right)
MLP8b MLP8c p111

Other Swazi Material Culture items that admit patterning including the following examples of pattern behaviour that are derived from Sitsebe (a grinding mat) Tinziwo (beer-pots), a basket made from Lutindzi (mountain grass) and Emagcebesha (beaded necklaces).


SP1b
(SP1b is two-dimensional)


SP1a

pm11

In this frieze design drawn from a Swazi beer pot, the pattern has vertical lines of symmetry A frieze pattern is one consisting of a motif that is repeated at regular intervals along a straight line. Frieze groups are symmetry groups - they include translations that are parallel.


The pattern on this basket made from Lutindzi represents a 4-fold rotational symmetry. The motif appears in 4 different positions. From one position to the next, the motif rotates through $1 / 4$ of a complete turn, through an angle of $90^{\circ}$.


EP1 - pmm2, p112


EP2-pm11


EP3-none

The patterns on these Emagcebesha, beaded necklaces, represent both vertical and horizontal lines of symmetry. ${ }^{14}$

[^6]The chart below records and places the various patterns within the seven notations for one-dimensional patterns:

|  | Notation | Symmetries | Pattern invariant under | Patterns from Swazi, mats, beer-pots, beaded necklaces. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | pmm2 | Vertical, horizontal <br> and rotational <br> symmetry of $180^{\circ}$  | Vertical reflection, horizontal reflection and rotation through an angle of $180^{\circ}$ | MLP1b, $\quad$ MLP1c, $\quad$ MLP2b, MLP2c, $\quad$ MLP4b, MLP64c, MLP66, LP7f EP1 |
| 2 | pma2 | Vertical, translationalreflected and rotational symmetry of $180^{\circ}$ | Vertical reflection, glide reflection and rotation through an angle of $180^{\circ}$ | MLP1d, $\quad$ MLP2d, MLP2e, <br> MLP2f, MLP2g, MLP7g, <br> MLP7h, MLP7i  |
| 3 | pm11 | Vertical symmetry | Vertical reflection | MLP1e, MLP1f, MLP3b, MLP5b, MLP7b, MTP1c, MTP2b, EP2 |
| 4 | p1m1 | Horizontal symmetry | Horizontal reflection | MLP3c, MLP5c, MLP7c, TP1b |
| 5 | p112 | Rotational symmetry of $180^{\circ}$ | Rotation through an angle of $180^{\circ}$ | MLP1g, MLP6c, |
| 6 | p1a1 | Translationalreflected symmetry | Glide reflection/reflected translation | MLP1d, MLP7g, MLP7h, <br> MLP7i, MLP2d, MLP2e, <br> MLP2f, MLP2g  |
| 7 | p111 | Only translational symmetry | Only translation | MLP8b ML8c |

The findings of the analysis of patterns ML1 to ML8 indicate the most popular notation to be No. 1 pmm2, closely followed by No. 2 pma2, and No. 3 pm11. The realisation of Notation No. 6 was largely achieved through subjacent symmetry and No. 7 p111 is not inherent but superficially created to show the concept of translation only.

Further analysis of the Msithini patterns involves establishing how many variations exist for the central panel patterns, the outer panel patterns, and full patterns. To determine this, the mats were divided first into two different zones, Msithini Central Pattern Panel (MCPP) and Msithini Outer Pattern Panel (MOPP). In addition, the other patterns were placed as Msithini Full Mat Patterns (MFMP) and Msithini Additional Mat Pattern (MAMP).

The whole group of mats have been entered into a spreadsheet in order of date of purchase; each mat has been allocated a number in that order MM1 to MM53 (where MM denotes Miriam Msithini) The patterns have been sorted into linear diagrams and all the possible variations of patterns recorded and designated a number, MCPP1 to MCPP24, MOPP1 to MOPP14 and MFMP1 to MFMP4. This breakdown was necessary in order to ease the process of pattern recognition and collation. For example, from the Msithini Group consisting of 53 mats there are 24 types of Central Panel Patterns, 14 Outer Panel Patterns, 4 Full Mat Patterns and one Additional Mat Pattern. These abbreviations also appear in (Appendix B, B35-B36). For additional information concerning this group of mats, (see Appendix B, B37-B43). The diagrams below show symmetrical properties with possibilities for further investigation:

## MSITHINI CENTRAL PANEL PATTERNS (MCPP)



MCPP 1


MCPP 3



MCPP 2



MCPP 7


MCPP 9



MCPP 8


MCPP 10



MCPP 13


MCPP 17


MCPP 14


MCPP18


MCPP 21



MCPP 22


MSITHINI OUTER PANELS PATTERNS (MOPP)


MOPP 1


MOPP 3




MOPP 13


MOPP 14

MSITHINI FULL MAT PATTERNS (MFMP)


MSITHINI ADDITIONAL MAT PATTERN


MAMP 1


[^0]:    ${ }^{1}$ During the $19^{\text {th }}$ century, crystallographers were attempting to fully describe and categorise the different ways in which the molecules in a crystal are packed. The Russian scientist E. S. Fedorov (1891) was able to solve this problem by means of symmetry groups. Washburn D \& D Crowe, 1988:59.

[^1]:    ${ }^{2}$ Gerdes, P. 1999:3-50
    ${ }^{3}$ Gerdes, P.1997: 227-228

[^2]:    ${ }^{4}$ Gerdes, P. 1998:228
    ${ }^{5}$ Marcia Ascher 1994:36-43

[^3]:    ${ }^{6}$ Washburn and Crowe offered a course at the University of Wisconsin called 'Symmetries of Culture.'
    ${ }^{7}$ Washburn and Crowe, 1988:3
    ${ }^{8}$ See works of Renaissance artists in particular Leonardo Da Vinci
    ${ }^{9}$ Hardy 1940:p/uk
    ${ }^{10}$ Washburn, D \& D Crowe 1988:7. It is interesting to note that during this period, although both crystallographers and designers were describing repeated patterns, neither seemed to take cognisance of the others' work. The crystallographers derived the geometry of crystal structure as a mathematical

[^4]:    ${ }^{12}$ Getz, C., 2004:60-60

[^5]:    ${ }^{13}$ (For Notations for One-Colour, Two-Dimensional Patterns, Two-Colour, One-Dimensional Patterns and Two-Colour, Two-Dimensional Patterns see Washburn \& Crowe 1988)

[^6]:    ${ }^{14}$ An opportunity exists for an in depth symmetrical analysis of the baskets made from Lutindzi, Tinziwo, and Emagcebesha, as well as for the two dimensional patterns found on the Sitsebe.

