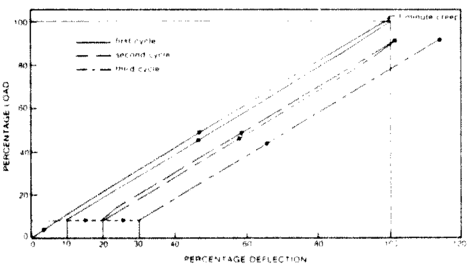
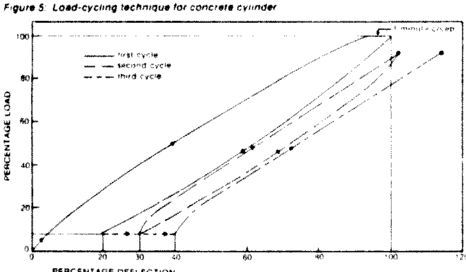


Figure 4. Load-cycling technique for concrete beam

Figure 5. Load-cycling technique for concrete cylinder



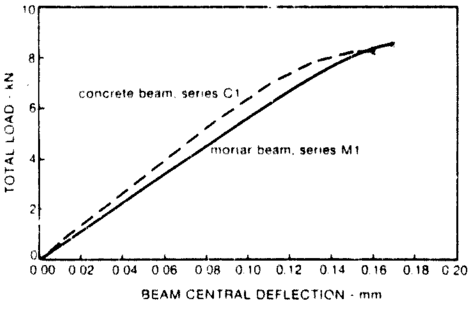
technique, together with the one minute period of creep at the peak of the initial cycle, serves to eliminate non-linearity of the loading curves.

In contrast, the percentage plot of the load-cycling of a concrete cylinder, shown in Figure 5, reveals that a large degree of hysteresis is present in cylinders, and non-linearity on the initial loading cycle is marked. However, the second and subsequent cycles follow the same loading path, and linearity of the loading portion of the curve is then established. A greater degree of creep occurs in the cylinders at the peak of the initial load cycle than in the beams, due to the greater compressive stress levels experienced in the cylinders. The same reason can be advanced for the greater hysteresis in the cylinder when compared with the beams. The convex-up initial portion of the load cycles can probably be attributed to take-up friction in the dial guage of the compressometer.

Typical load-deflection curves for a concrete and a mortar beam are shown in Figure 6. The curves represent the load cycle on which the elastic and rupture moduli were determined. The initial linearity of the curves is clearly shown, as also is the greater degree of linearity for the mortar beam when compared with the concrete beam.

The standard cylinder test for elastic modulus requires a maximum load corresponding to roughly one-third of the cube strength to be applied to the specimen. This ensures that the specimen remains in the "elastic" range. An analysis of the limits of linearity for beams, based on measured load-deflection curves, has shown that concrete beams depart from linearity at a load ratio of about 0,56 while the corresponding value for mortar beams is 0,67 for the particular materials tested[6]. Individual results with values for load ratio as low as 0,4 were obtained. For this reason, a maximum load corresponding to one-third of the estimated or measured modulus of rupture has been recommended in the present test, subject also to the proviso of repeatability of deflections for two sequential load cycles. The range of loading

Figure 6. Typical load-deflection curves for a concrete and a mortar beam.



required by the test is sufficient for a reasonable estimate of the flexural elastic modulus to be obtained without deviating from linear behaviour.

Instrumentation

The beam test involves the measurement of load and central deflection, both of which are fairly simple to obtain, electronically or mechanically. For the test geometry suggested, a deflection-measuring device reading to an accuracy of $1{,}6 \cdot 10^{-3}$ mm is required. This limit represents an extreme fibre strain of about $2 \cdot 10^{-6}$ which is considered a lower-bound increment for obtaining the linear portion of the load-deflection curve.[15] Electronic devices lend themselves to autographic recording of deflections, while mechanical gauges are normally simple to install and use. The inherent friction of dial gauges operating at very small deflections was a problem in an initial series of tests.

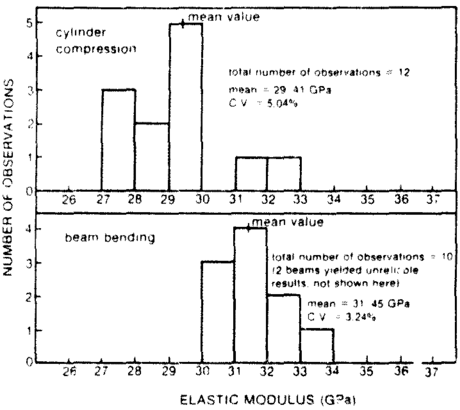
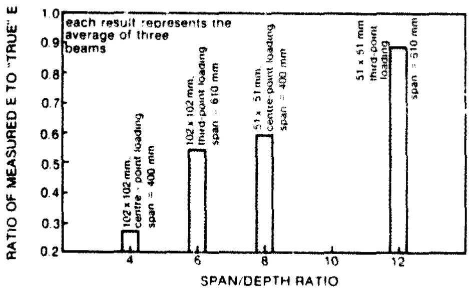


Figure 8: Elastic moduli for concrete, Series C1

All extraneous deflections, apart from shear deflections, should be eliminated from the measurements. These unwanted deflections are introduced by deformation of beam supports and loading bed, crushing of the beam at support points, etc., and can be of the same order of magnitude as the bending deflections.

The results of an earlier series of tests on mortar beams from which unwanted deflections were not excluded are shown in Figure 7. The beams were 102x102 mm or 50x50 mm in cross section, tested on a 610 mm span with third-point loading, or on a 400 mm span with centre-point loading. The span/depth ratios of the beams varied from 4 to 12. An estimate of the "true" elastic modulus for the mortar was obtained from cylinder tests and strain measurements on a number of beams. As the flexibility of the beams was increased, so a better measurement of the elastic modulus was obtained.

Figure 7: Effect of extraneous deflections on measured elastic modulus of mortar beams



Without taking care to eliminate unwanted deflections from the test, the measured moduli are [illegible] satisfactory. The majority of the extr[illegible] deflections were generated in the beam supports, since a number of beams with bearing plates under the support points gave similar results to beams without bearing plates. For this reason, a deflection rig supported immediately above the beam supports is required (see Figure 2). The deflection of the rig due to rotation of the beam at the supports was checked by calculation and found to be negligible. Otté[7] has used a similar rig successfully, and concluded from a finite-element analysis that rig deflections were negligible. The rig should be supported at three points using small steel balls, and held in position by spring-loaded arms.

The test is load-controlled, and accuracy of load measurement of about 0,1 kN is adequate for normal concretes and mortars. Load-deflection curves can be recorded autographically; for manual re-

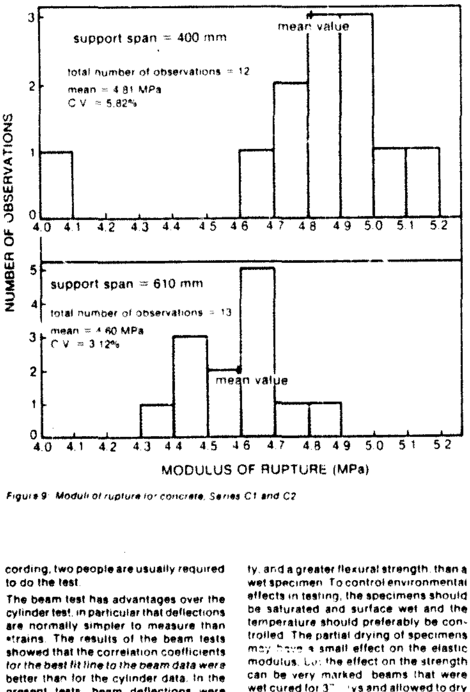


Figure 9 Moduli of rupture for concrete, Series C1 and C2

cording, two people are usually required to do the test.

The beam test has advantages over the cylinder test, in particular that deflections are normally simpler to measure than strains. The results of the beam tests showed that the correlation coefficients *for the best fit line to the beam data were* better than for the cylinder data. In the present tests, beam deflections were more accurately measured than corresponding strains on cylinders. Cylinder strains can be measured using electrical resistance gauges bonded directly to the specimen, but this method is rather cumbersome.

Environmental effects

Other researchers have noted the effect on the elastic and rupture moduli of drying of specimens; in general, a completely dry specimen has a lower elasticity, and a greater flexural strength, than a wet specimen. To control environmental effects in testing, the specimens should be saturated and surface wet and the temperature should preferably be controlled. The partial drying of specimens may have a small effect on the elastic modulus, but the effect on the strength can be very marked: beams that were wet cured for 3 days and allowed to dry for 5 days before testing gave flexural strengths as low as 44% of beams continuously wet till test. Where strains are measured by gauges bonded onto the specimen after removal from the water and just prior to test, a degree of drying may occur causing tensile shrinkage stresses in the specimen. It was noted during preliminary testing that allowing a specimen to partially dry often resulted in a higher E being measured relative to a saturated specimen. This is in contrast to remarks above, and presumably results from the additional hydration occurring before the specimen dries, but which can more than offset the reduction in E due to loss of moisture. Increases of 20% were noted in specimens allowed to dry out in this way. The effect is greater in a larger specimen due to the longer reten*tion of moisture, thus allowing continuing* hydration.

Comparison of measured elastic and rupture moduli

Figure 8 shows the results for series C1, modulus of elasticity. The beam moduli were about 7% higher than the standard cylinder values. The same effect was noticed for series M1, where the mean moduli for the three beams and the three cylinders were 28.52GPa and 26.44GPa, respectively. A significantly lower coefficient of variation occurred for the beam tests (c.v. = 3.24%) as compared with the cylinder tests (c.v. = 5.04%), indicating that beam tests had better repeatability than cylinder tests. The difference between the measured elastic moduli for the beams and cylinders is thought to result from specimen size effects, and has been discussed elsewhere[6].

For the beams of Series C1 and C2, Figure 9 indicates that the longer beams generally gave lower values of rupture modulus than the standard beams. A statistical test on the mean rupture values *showed that the values are significantly* different at the 98% significance level. The longer span beams had a single very low result, but otherwise the spread of values for both tests was similar.

The reduction in flexural strength as the beam specimens increase in length can be explained in terms of the Weibull Weakest Link Theory[16], which postulates that the probability of failure of a test specimen is dependent on the stress level in the specimen, and the volume of material under stress. For two test pieces, alike in every respect except that the one has a greater volume of material under maximum stress than the other, the theory implies that the larger test piece will fail at a lower stress than the smaller one. Since concrete fails by the successive propagation of critical cracks, larger specimens which are more likely to contain serious flaws will be more likely to fail at a lower stress than smaller specimens. Hence the 610mm span beams had a mean flexural strength of about 95% of the standards beams.

The ratios quoted above between the moduli for the different specimen types should be regarded as valid only for the particular materials that were tested; further results for other concretes and mortars would be required for a fuller description of the relationships. Nevertheless, the trend of increased elastic moduli and decreased rupture moduli for the proposed test when compared with the standard tests should exist for other normal concrete materials.

Conclusions

Within the scope of the experimental programme, the following conclusions can be drawn.

1. The proposed beam test for elastic and rupture moduli provides a useful alternative to the existing standard tests and has the advantage that a number of material parameters can be evaluated in a single test.

2. The proposed test shows better repeatability and linearity of the load-deflection relationship than the equivalent cylinder test.

3. Failure to eliminate unwanted deflections from the results can lead to gross underestimations of the elastic modulus.

4. The mean elastic modulus measured on a particular concrete in the beam test was about 7% higher than the modulus measured in the standard cylinder test, while the mean rupture modulus was about 5% lower. These variations are thought to be attributable to specimen size effects and test geometries.

Acknowledgements

The author acknowledges the assistance given in the testing by Mr Darryl Aberdein, formerly a research student in the Department of Civil Engineering, and in the preparation of specimens by Messrs R. van der Merwe and A. Mitabine of the laboratory staff. Financial assistance for the project came from a Cement Industries Fund research grant for 1980/81.

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Discussion of the above paper is invited. A synopsis of a further paper, "Elasticity of plain concrete and mortar specimens in flexure and compression", following on the above, is included below for interest. Readers may apply to the Society for a copy if required.

The elasticity of concrete and mortar is assessed by means of cylinder and prism compression tests, and beam bending tests. A comparison between the elastic moduli measured in the different tests shows that higher moduli are measured in the smaller beams and prisms than in the larger cylinders. A statistical 'weakest link' theory is used to explain the results, and it is proposed that specimen size effects are critically important in stiffness measurements. Beams show better stress-strain linearity under load than do cylinders. A relationship is derived for the flexural tensile stresses and strains in a plain beam, and shown to be similar to previous relationships for compression. It appears that transverse strain gradients exercise a greater crack-stopping effect on concrete in tension than in compression.

Reader Contribution

Dr P W Keene, N B R I makes a contribution to the discussion of 'A simple bending test for elastic and rupture moduli for plain concrete and mortar' by M G Alexander, published in Concrete/Beton No 27, September 1982 pp 18-24

THE development of a test for modulus of elasticity using specimens in bending is to be welcomed. It would now be possible to test the same specimen for modulus of elasticity, modulus of rupture, and compressive strength on each of the two portions of the broken beam.

Several comments spring to mind on reading this interesting paper. The reasons given in the paper for the higher tensile strength in bending could be elaborated. As the author says in (i) on p 18 'the stress block at failure is approximately parabolic and not triangular as assumed.' This is because concrete is not a perfectly linear elastic material, even in tension. But there is another reason why the stress/strain relationship is not linear, and that is the Seewald effect as a result of the configuration of the loads in standard flexural test.

Details of the Seewald effect are given in Timoshenko's 'Theory of Elasticity'. It is the departure from the simple theory of bending when a moment is applied to a beam by means of a concentrated load. The effect is illustrated in Figure 1 for a beam loaded along its middle line. In the face opposite to that on which the load acts the maximum tensile stress is less than that given by the simple theory of bending. Within a short distance from the plane of application of the load the stress begins to increase relative to that given by the simple theory of bending. The deviation decreases as distance from the plane of application of the load increases, and at a distance more than the depth of the beam the deviation is insignificant.

If the span is short the supports also influence the maximum stress, and for third point loading the effect of both loads must be taken into account when applying the Seewald correction. The maximum tensile stress with third point loading is greater than that given by the simple theory of bending, as shown in Figure 2.

The Seewald correction is less important as the span/depth ratio increases as shown in the Table. If this correction is applied to the results for span/depth ratios of 4 and 6 shown in Figure 9 of the paper, the difference between the two sets of results would be reduced slightly, by about 0,03 MPa.

Another factor which would tend to reduce the difference in MR is the effect of the mass of the beams themselves. When the beam is laid on the supports before being tested, flexural stress is induced in the beam, which, at a particular point, is greater the longer the

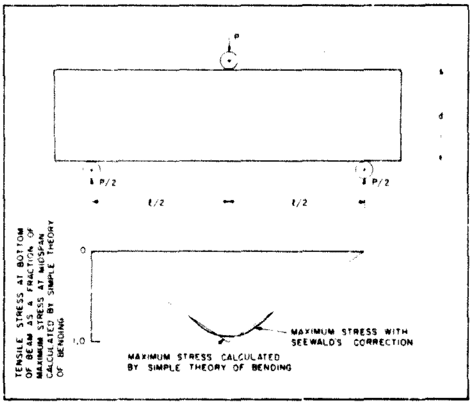
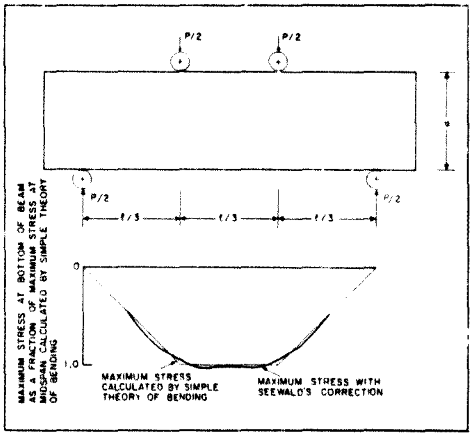


Figure 1 Maximum flexural stress for span/depth ratio of three with centre point loading

Figure 2 Maximum flexural stress for span/depth ratio of three with third point loading



span and the greater the depth of the beam. Superimposed on this stress is the stress applied by the testing machine.

The actual failure stress is greater than the indicated stress by an amount depending on the span and the depth of the beam. For the example quoted in Figure 9 of the paper, the difference could also be about 0.03 MPa. The precise value would depend on the point at which fracture occurred.

These two factors by no means explain all the difference in MR for the different span/depth ratios but at least they explain some of it. The author expresses the opinion that the lower MR of the longer specimens is because a greater quantity of material is exposed to maximum stress in the longer specimens, and failure consequently occurs at a lower stress because of the greater probability of a flaw in the area at maximum stress. The author also quotes this as an explanation for flexural strength being greater than the strength under uniform tension.

The 'weakest link' theory may undoubtedly hold for some materials, for extremely small sizes, but there are some reasons why the theory should only be invoked with the greatest hesitation, if not rejected altogether, for concrete in sizes normally used as test specimens. Taking the example in the paper of the MR of beams spanning 4 and 6 times the depth, when the Seewald correction is applied there are only two lines at maximum tensile stress, or two narrow strips close to maximum tensile stress, in each case. There can therefore be little difference, if any, in the area at maximum stress in each case.

To take another example, there is a vast difference in volume at maximum stress between the direct tensile test and the cylinder-splitting test. In the latter there is in theory only a plane subject to maximum tensile stress. Yet Brooks and Neville have shown that there is little difference in indicated tensile strength in the two cases. At the lower end of the range of strength considered (2MPa) the two tests gave identical results. At higher levels of strength (up to 5 MPa) there was a tendency for the cylinder-splitting test to give a higher indicated strength, but if there were any truth in the weakest link theory we should expect the cylinder-splitting test to give a vastly greater indicated strength throughout the whole range, inasmuch as a volume is infinitely greater than a plane.

Probably the main reason why the flexural test gives a higher indicated tensile strength than other tests is the strain gradient, which is virtually the same as the blockage of crack propagation mentioned by the author. But strain gradient should be clearly distinguished from any statistical or weakest link effect, which, for the reasons given is, to say the least, of doubtful validity as far as normal concrete test specimens is concerned.

TABLE

Maximum stress on the underside of beams as a percentage of that given by the simple theory of bending (Naschold)

Span/depth ratio	2	3	4	6	8	10	15	20	25
Third point loading	109.80	103.62	102.17	101.45	101.09	100.87	100.58	100.43	100.35
Centre point loading	92.00	94.18	95.56	97.04	97.78	98.22	98.82	99.11	99.29

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Readers are requested to note an important change in line 5, second column, page 19, Concrete Beton Nr 27 1982 09: instead of reading "four" at the end of that line, the word is "three". To repeat: "... the ratio of support span to depth of the beam (i.e. ℓ/d) is three". The author apologises for any inconvenience this error may have caused.

Evaluating measured parameters

Reply by the author to reader contribution by Dr P W Keene NBRI, on the paper 'A simple bending test for elastic and rupture moduli for plain concrete and mortar', published in *Concrete Beton*, **No. 28, December 1983.** ***Original paper published in*** *Concrete Beton*, **No. 27, September, 1982, pp. 18-24**

by M G Alexander

The author wishes to thank Dr Keene for his comments, and, in particular, for raising the question of the effect on the measured MR of the Seewald correction. This effect was not taken into account in the original paper, and the author would like to take the opportunity to further amplify the corrections that are implied by the Seewald effect. Dr Keene has included a table in his discussion which adequately outlines the increases or decreases in maximum stress on the underside of beams due to the effect of point loads. Referring to Figure 9, the implication is that the shorter beams (support span 400 mm) should have a mean MR = 2,17 % greater than that shown, while the longer beams (support span = 610 mm) should have a mean MR 1,45 % greater than that shown. Hence, the mean MR for the shorter beams would be 4,91 MPa and that for the longer beams 4,67 MPa. This means that the difference between the two sets of results *increase* due to the Seewald effect, by about 0,04 MPa, and does not reduce, as stated by Dr Keene. The effect of the self-weight of the beams on the measured MR is, however, to reduce the difference between the two sets of results, by about 0,03 MPa in this case. The Seewald effect and the self-weight effect should, strictly speaking, be taken into account when calculating the MR of concrete beams.

The Seewald effect also influences the deflections of a pointloaded beam, and hence the measured E is influenced if this is obtained from load-deflection measurements. This effect was also omitted in the original paper. Using the expressions given in Timoshenko's 'Theory of Elasticity', we can derive an expression for the extra deflection δ at the centre of a beam under third-point loading as being:

$$\delta = \frac{23}{1296} \cdot \frac{F}{E} \frac{\ell^3}{I} \left[\frac{27}{115} \left(\frac{d}{\ell} \right)^2 (8 + 5\nu) - \frac{1}{2} \left(\frac{d}{\ell} \right)^3 \right]$$

The true E will thus be higher than that indicated by simple bending. The table below shows how the correction for E also becomes less important as the span/depth ratio increases.

TABLE: Correction factors (Seewald effect) for Young's moduli calculated from simple bending theory

Span/depth Ratio	2	3	4	6	9	10	15	20	25
Third-point loading	1 293	1 169	1 106	1 052	1 030	1 020	1 009	1 005	1 003
Centre-point loading	1 551	1 261	1 151	1 069	1 039	1 025	1 011	1 006	1 004

The table implies that the mean value for the bending E in Figure 6 should be increased by 3,2 % to 33,09 GPa, which exceeds the cylinder compression E by 12,5 %.

As can be seen from the table, the corrections are very significant at the typical span/depth ratios used in beam testing, and should be included in the calculations. It should be mentioned that, since the deflection readings are zeroed at the start of the measurements, the self-weight of the beam will not affect the slope of the measured load-deflection curve.

One further point needs to be mentioned. The expression for E given in the original paper (equation 2), includes a correction for shear deflections given by Otté, and derived from strain-energy principles. Using the classical Rankine-Grashof theory, the shear deflections will be overestimated by about 25 % relative to the more accurate strain-energy expression. Hence, the final expression for E in bending is based on simple bending theory corrected for shear and the Seewald effect.

All the corrections dealt with represent refinements which, in the final analysis, must be viewed against the background that concrete is a highly variable material whose response to stress is non-linear and whose mode of failure is essentially tensile in nature. Consequently, the measured parameters must be evaluated relative to the particular problem in hand.

APPENDIX D

LOAD-DEFLECTION DATA FOR SELECTED FRACTURE BEAMS

Table D1 gives the coordinates of the load-deflection records (F,δ) for the nineteen "typical" fracture beams noted in 5.3.

It should be noted that the tables include the following points:-

1. The point of initial departure from linearity, represented by the first coordinated point after 0,0.

2. The point of maximum load and corresponding deflection (F_{max}, δ_{Fmax}). This point is indicated by being enclosed in a block.

3. The deflection at zero load, δ_m, and the deflection at ultimate fracture, δ_u (for series F3 only).

(Note - Experimental data of all the beams is available if required).

Table D1 Coordinates of load-deflection records for selected "typical" beams

(a) Series F1

F1/2 (500/0,2)		F1/4 (500/0,4)		F1/6 (300/0,2)		F1/8 (200/0,4)		F1/14 (100/0,4)	
F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)
0	0	0	0	0	0	0	0	0	0
9,24	0,052	6,16	0,072	9,24	0,057	2,57	0,033	2,69	0,025
13,34	0,080	9,24	0,119	13,34	0,092	3,59	0,055	3,59	0,040
16,42	0,115	10,26	0,142	14,37	0,109	3,98	0,078	4,11	0,054
16,78	0,126	10,78	0,166	14,88	0,136	4,04	0,102	4,31	0,068
15,91	0,178	10,88	0,195	14,37	0,151	3,98	0,128	4,11	0,075
14,78	0,209	10,78	0,226	13,34	0,163	3,59	0,159	3,59	0,086
13,24	0,269	10,26	0,269	11,29	0,186	2,95	0,193	2,82	0,109
11,29	0,284	9,24	0,322	8,21	0,245	2,05	0,285	2,31	0,128
8,83	0,292	6,16	0,473	5,13	0,367	1,28	0,432	1,80	0,161
5,95	0,364	3,08	0,748	3,08	0,505	0,90	0,557	1,28	0,216
3,70	0,447	2,05	0,865	1,54	0,738	0,51	0,844	0,77	0,327
1,74	0,619	0,87	1,200	0,72	1,090	0,21	1,322	0,38	0,557
0,82	0,859	0,54	1,295	0	1,465	0	1,934	0,22	0,767
0	1,203	0	1,509					0,13	0,968
								0	1,398

(b) Series F2

F2/1 (800/0,2)		F2/3 (800/0,4)		F2/7 (100/0,25)		F2/10 (100/0,4)		F2/25 (100/0,5)	
F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)
0	0	0	0	0	0	0	0	0	0
18,48	0,241	5,65	0,083	2,95	0,019	2,44	0,029	0,69	0,077
20,94	0,275	8,72	0,133	5,00	0,041	3,08	0,042	0,87	0,132
26,69	0,355	10,78	0,176	5,38	0,060	3,39	0,062	0,90	0,160
28,12	0,398	11,06	0,183	5,00	0,075	3,08	0,079	0,87	0,189
26,69	0,414	12,68	0,219	4,23	0,086	2,05	0,150	0,51	0,361
22,58	0,412	13,06	0,235	3,08	0,126	1,54	0,196	0,21	0,647
No further		12,11	0,269	1,92	0,208	0,90	0,294	0,04	1,134
record -		11,29	0,282	1,21	0,301	0,38	0,637	0	1,556
spontaneous		8,21	0,291	0,90	0,415	0,18	1,119		
crack growth		4,11	0,317	0,27	0,874	0	1,509		
		2,05	0,345	0	1,804				
		0,62	0,388						
		0	0,558						

Table Dl Coordinates of load-deflection records for selected "typical" beams

(a) Series F1

F1/2 (500/0,2)		F1/4 (500/0,4)		F1/6 (300/0,2)		F1/8 (200/0,4)		F1/14 (100/0,4)	
F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)
0	0	0	0	0	0	0	0	0	0
9,24	0,052	6,16	0,072	9,24	0,057	2,57	0,033	2,69	0,025
13,34	0,080	9,24	0,119	13,34	0.092	3,59	0,055	3,59	0,040
16,42	0,115	10,26	0,142	14,37	0,109	3,98	0,078	4,11	0,054
16,78	0,126	10,78	0,166	14,88	0,136	4,04	0,102	4,31	0,068
15,91	0,178	10,88	0,195	14,37	0,151	3,98	0,128	4,11	0,075
14,78	0,209	10,78	0,226	13,34	0,163	3,59	0,159	3,59	0,086
13,24	0,269	10,26	0,269	11,29	0,186	2,95	0,193	2,82	0,1[illegible]
11,29	0,284	9,[illegible]	0,322	8,21	0,245	2,05	0,285	2,31	0,[illegible]
8,83	0,292	6,1[illegible]	0,473	5,13	0,367	1,28	0,432	1,80	0,161
5,95	0,364	3,08	0,748	3,08	0,505	0,90	0,557	1,28	0,216
3,70	0,447	2,05	0,865	1,54	0,738	0,51	0,844	0,77	0,327
1,74	0,619	0,87	1,200	0,72	1.090	0,21	1,322	0,38	0,557
0,82	0,859	0,54	1,295	0	1,465	0	1,934	0,22	0,767
0	1,203	0	1,509					0,13	0,968
								0	1,398

(b) Series F2

F2/1 (800/0,2)		F2/3 (800/0,4)		F2/7 (100/0,25)		F2/10 (100/0,4)		F2/25 (100/0,5)	
F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)
0	0	0	0	0	0	0	0	0	0
18,48	0,241	5,65	0,083	2,95	0,019	2,44	0,029	0,69	0,077
20,94	0,275	8,72	0,133	5,00	0,041	3,08	0,042	0,87	0,132
26,69	0,355	10,78	0,176	5,38	0,060	3,39	0,062	0,90	0,160
28,12	0,398	11,06	0,183	5,00	0,075	3,08	0,079	0,87	0,189
26,69	0,414	12,68	0,219	4,23	0,086	2,05	0,150	0,51	0,361
22,58	0,412	13,06	0,235	3,08	0,126	1,54	0,196	0,21	0,647
No further		12,11	0,269	1,92	0,208	0,90	0,294	0,04	1,134
record –		11,29	0,282	1,21	0,301	0,38	0,637	0	1,556
spontaneous		8,21	0,291	0,90	0,415	0,18	1,119		
crack growth		4,11	0,317	0,27	0,874	0	1,509		
		2,05	0,345	0	1,804				
		0,62	0,388						
		0	0,558						

Table D1 (Continued)

(c) Series F3

F3/2 (500/0,2)		F3/4 (500/0,4)		F3/6 (300/0,2)		F3/8 (300/0,25)		F3/11 (300/0,4)	
F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)
0	0	0	0	0	0	0	0	0	0
12,5	0,168	4,7	0,085	4,9	0,068	5,0	0,068	2,93	0,050
16,0	0,222	7,0	0,129	8,03	0,113	7,75	0,117	4,25	0,079
17,0	0,244	8,0	0,159	9,75	0,146	8,3	0,136	5,31	0,107
18,5	0,289	8,88	0,222	$\boxed{10,15}$	$\boxed{0,164}$	8,8	0,156	5,61	0,129
$\boxed{18,74}$	$\boxed{0,308}$	$\boxed{9,03}$	$\boxed{0,251}$	8,95	0,215	9,0	0,174	$\boxed{5,71}$	$\boxed{0,148}$
17,9	0,340	8,0	0,286	7,63	0,239	$\boxed{9,03}$	$\boxed{0,187}$	5,50	0,171
15,1	0,391	5,9	0,319	4,01	0,341	8,4	0,209	4,84	0,193
9,55	0,429	4,7	0,367	3,21	0,416	6,8	0,240	3,63	0,251
4,4	0,544	3,93	0,433	2,58	0,518	5,4	0,282	2,33	0,382
1,45	0,856	1,38	0,705	2,06	0,629	3,95	0,372	1,81	0,492
0,5	1,133	0,33	1,125	0,97	0,877	2,9	0,513	1,19	0,687
0	1,297	0	1,403	0	2,381	1,1	1,054	1,00	0,787
0	2,506	0	2,945	0	4,511	0	2,722	0,13	1,150
						0	3,822	0	2,763
								0	3,759

(c) Series F3 (Continued)

F3/14 (200/0,2)		F3/15 (200/0,4)		F3/19 (100/0,2)		F3/26 (100/0,4)	
F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)	F (kN)	δ (mm)
0	0	0	0	0	0	0	0
4,75	0,029	2,88	0,033	1,84	0,015	2,21	0,025
6,00	0,041	4,05	0,055	4,30	0,044	2,95	0,038
7,45	0,053	4,43	0,067	$\boxed{4,92}$	$\boxed{0,066}$	3,07	0,044
8,35	0,063	4,75	0,087	4,30	0,099	$\boxed{3,17}$	$\boxed{0,052}$
8,80	0,079	$\boxed{4,84}$	$\boxed{0,102}$	2,98	0,127	3,12	0,072
$\boxed{8,95}$	[illegible]	4,75	0,122	2,15	0,167	2,37	0,136
8,5	0,114	4,55	0,138	1,84	0,201	1,64	0,206
6,8	0,131	3,81	0,190	1,17	0,281	0,93	0,379
5,5	0,155	2,83	0,302	1,07	0,290	0,30	0,883
4,4	0,212	2,05	0,416	0,46	0,726	0,07	2,268
3,38	0,315	1,13	0,801	0,15	1,470	0	3,477
2,5	0,414	0,5	1,164	0	4,135	0	4,135
1,8	0,524	0,13	2,056	0	6,140		
1,2	0,824	0	2,318				
0,5	1,433	0	3,527				
0	2,512						
0	3,759						

APPENDIX E

DERIVATION OF THEORETICAL COMPLIANCE - NOTCH DEPTH RELATIONSHIPS, AND EFFECTIVE SPAN/DEPTH RATIO FOR VARIABLE SECTION BEAMS

E.1 CONSTANT SECTION BEAMS

Geometry:- Width constant at b = 0,1m.
Span/depth ratio constant at S/d = 4.
Crack depth c

Loading:- Mid-span point load F, producing moment M

The compliance λ is defined as:-

$$\lambda = \frac{\delta}{F/b}$$

where δ = beam deflection under the load point.

The following three basic equations are used:-

The expression for K:- $$K = Y \frac{6M\sqrt{c}}{bd^2} \qquad \text{(E.1)}$$

where $Y = A_0 + A_1\,(c/d) + A_2\,(c/d)^2 + A_3\,(c/d)^3 + A_4\,(c/d)^4$.

(Equations (3.2) and (3.3) in main text).

The expression for G in terms of the rate of change of compliance with crack length:-

$$G = \frac{1}{2}\left(\frac{F}{b}\right)^2 \frac{d\lambda}{dc} \qquad (E.2)$$

(Equation (3.22) in main text).

and the LEFM relationship between K and G:-

$$K^2 = \frac{EG}{(1-\nu^2)} \qquad (E.3)$$

(Equation (3.20) in main text).

The values of the coefficients A are given in table 3.1 in the main text.

The bending moment at the mid-span of the beam, M, for three-point bending is:-

$$M = \frac{FS}{4} \qquad (E.4)$$

From equation (E.2),

$$\frac{d\lambda}{dc} = \frac{2b^2G}{F^2}$$

Substituting for G from equation (E.3) and then for K from equation (E.1):-

$$\frac{d\lambda}{dc} = \frac{2b^2}{F^2}\,\frac{(1-\nu^2)}{E}\left[Y\,\frac{6M\sqrt{c}}{bd^2}\right]^2$$

By substituting for M this reduces to

$$\frac{d\lambda}{dc} = \frac{9(1-\nu^2)S^2}{2E}\,\frac{Y^2c}{d^4} \qquad (E.5)$$

Integrating with respect to c:-

$$\lambda = \frac{9(1-\nu^2)S^2}{2Ed^3}\int Y^2\,\frac{c}{d}\,dc \qquad (E.6)$$

The integral $\int Y^2 \frac{c}{d} dc$ is an integral of a polynomial function in the variable (c/d).

$$\text{Let} \quad \int Y^2 \frac{c}{d} dc = Z \qquad \text{(E.7)}$$

From elementary beam theory, an expression for E can be obtained for an unnotched beam in three-point bending:-

$$E = \frac{1}{48} \frac{F}{\delta} \frac{S^3}{I} \qquad \text{(E.8)}$$

where (F/δ) is the slope of the initial linear portion of the load-deflection curve, and I is the second moment of area of the section. Writing the unnotched compliance λ_0 as:-

$\lambda_0 = \frac{\delta}{F} b$, equation (E.8) can be re-written as follows:-

$E = \frac{1}{48} \frac{b}{\lambda_0} \frac{S^3}{I}$, and since $I = bd^3/12$ this reduces to:-

$$2Ed^3 = \frac{S^3}{2\lambda_0} \qquad \text{(E.9)}$$

From equations (E.6), (E.7) and (E.9) a relationship between compliance ratio λ/λ_0 and crack depth ratio c/d, where c/d is included in the integral Z, can now be written down as follows:-

$$\lambda = \frac{9(1-\nu^2)S^2 Z}{S^3/2\lambda_0}$$

which finally reduces to

$$\frac{\lambda}{\lambda_0} = \frac{18(1-\nu^2)}{S} Z \qquad \text{(E.10)}$$

Equations (E.6) and (E.10) represent the required compliance-notch depth relationships.

The integral Z depends on the specimen dimensions, including the crack depth ratio c/d, and on the type of bending in the specimen. It also includes a constant which can be evaluated from boundary conditions. It is preferable to express Z entirely in terms of geometrical ratios, in which case equation (E.10) reduces to:-

$$\frac{\lambda}{\lambda_0} = \frac{18(1-\nu^2)}{S/d} Z' \qquad (E.11)$$

where

$$Z' = \int Y^2 k dk \qquad (E.12)$$

and $k = c/d$

For the particular test geometry:- $S/d = 4$. Using this, values of Z' as functions of c/d and ν obtained from integrating the expression in equation (E.12) and inserting the boundary conditions, appear in table E1.

With the values of Z' evaluated, it is now possible to evaluate the compliance - notch depth relationship (E.11). The values of λ/λ_0 versus c/d for different ν values are given in table E2, and also graphically in the main text in figure 6.2.

E.2 VARIABLE SECTION BEAMS

Geometry:- Width constant at b = 0,1m
Beam section variable.
Equivalent span/depth ratio taken as 5,45 (see later).

The first step is to derive a load-deflection relationship for an unnotched beam, taking the varying cross-section into account. Refer to figure 4.1, and consider the beam to be acted on by a centrally applied point load F. Using the Moment-Area Theorem it is possible to derive the F-δ relationship by considering the values of M/EI at the points of abrupt change of cross-section. The Moment-Area Theorem states that the deflection at a given point relative to the tangent at another point equals the moment about the first point of the area of the M/EI diagram between the two points. On this basis, it is possible to plot an M/EI diagram taking the support and mid-span as the two points. The moment of the area of the M/EI diagram can then be numerically assessed. This results in the following dimensionless expression:-

$$\frac{bE\delta}{F} = 40{,}495 \tag{E.13}$$

The compliance - notch depth relationship may now be derived by following the identical approach given for rectangular beams up to equations (E.6) and (E.7).

The expression for E for the particular geometry can be obtained by re-writing equation (E.13) as

$$E = 40{,}495\ \frac{(F/b)}{\delta}$$

and since $\lambda_0 = \dfrac{\delta}{F/b}$ this can further be written as

$$\lambda_0 = \frac{40{,}495}{E} \tag{E.14}$$

Substituting equation (E.14) in (E.6) we can write:-

$$\frac{\lambda}{\lambda_0} = \frac{9(1-\nu^2)S^2}{2\ d^3}\ \frac{Z}{40{,}495}$$

$$\frac{\lambda}{\lambda_0} = \frac{9}{80{,}99}\ \frac{(1-\nu^2)S^2}{d^3}\ Z \tag{E.15}$$

Equation (E.15) is the required compliance - notch depth relationship.

The integral Z can be evaluated similarly to the case for constant section beams given previously. In particular, defining Z' in terms of geometrical ratios only, we get:-

$$Z' = \int Y^2 k dk \quad \text{where } k = \frac{c}{d} \qquad (E.16)$$

Values of Z' are given in table E3. The compliance - notch depth relationship is given in table E4, and also graphically in the main text in figure 6.2.

E.3 EFFECTIVE SPAN/DEPTH RATIO OF VARIABLE SECTION FRACTURE BEAMS

The effective span/depth ratio of the variable section beams is found by using an effective depth which is equal to the depth of a constant section beam which would exhibit the identical load-deflection behaviour as the variable section beams (for unnotched beams). The procedure is therefore to use simple elastic deflection theory to match the deflection response of the two types of beam under identical loading.

From expression (E.13), the derived load-deflection relationship for the unnotched variable section beams is:-

$$\frac{bE\delta}{F} = 40{,}495$$

For a constant section beam with span/depth = 4 and a mid-span point load F, we can use elastic deflection theory to derive an expression for the mid-span deflection δ_D as follows:-

$\delta_D = \dfrac{FS^3}{48EI}$ Since $I = \dfrac{bd^3}{12}$, this can be written as:-

$$\delta_D = \frac{F}{b}\,\frac{(S/d)^3}{4E} \qquad \text{(E.17)}$$

Comparing (E.17) with (E.13) we get:-

$\dfrac{(S/d)^3}{4} = 40{,}495$ from which the effective S/d is:-

$$\left(\frac{S}{d}\right)_{eff} = \sqrt[3]{4.40{,}495}$$
$$= 5{,}45$$

Note also that the ratio of the maximum depth of a variable section beam to the depth of an equivalent constant section beam is:-

$$\left(\frac{s}{d}\right)_{eff} \left(\frac{s}{d}\right)^{-1}_{var.sec.} = 5{,}45/4$$
$$= 1{,}36$$

Table E1 Values of Z' for constant section beams

	Values of Z'		
c/d	ν=0,1	ν=0,15	ν=0,2
0,0	0,2245	0,2273	0,2315
0,1	0,2405	0,2433	0,2475
0,2	0,2854	0,2882	0,2924
0,3	0,3653	0,3681	0,3723
0,4	0,4995	0,5024	0,5065
0,5	0,7338	0,7367	0,7408
0,6	1,1824	1,1853	1,1894
0,7	2,1417	2,1446	2,1487
0,8	4,3786	4,3815	4,3856

Table E2 Compliance - notch depth relationship for constant section beams

	Values of λ/λ_o		
c/d	ν=0,1	ν=0,5	ν=0,2
0,0	1,0	1,0	1,0
0,1	1,071	1,070	1,069
0,2	1,271	1,268	1,263
0,3	1,627	1,619	1,608
0,4	2,225	2,210	2,188
0,5	3,269	3,241	3,200
0,6	5,268	5,214	5,138
0,7	9,541	9,433	9,283
0,8	19,507	19,273	18,946

Table E3 Values of Z' for variable beams

	Values of Z'		
c/d	ν=0,1	ν=0,15	ν=0,2
0,0	0,5681	0,5754	0,5859
0,1	0,5841	0,5914	0,6019
0,2	0,6290	0,6363	0,6468
0,3	0,7089	0,7162	0,7267
0,4	0,8432	0,8505	0,8610
0,5	1,0775	1,0848	1,0953
0,6	1,5260	1,5333	1,5438
0,7	2,4853	2,4926	2,5031
0,8	4,7220	4,7292	4,7397

Table E4 Compliance - notch depth relationship for variable section beams

	Values of λ/λ_o		
c/d	ν=0,1	ν=0,5	ν=0,2
0,0	1,00	1,00	1,00
0,1	1,028	1,028	1,027
0,2	1,107	1,106	1,104
0,3	1,248	1,245	1,240
0,4	1,484	1,478	1,470
0,5	1,897	1,885	1,870
0,6	2,686	2,665	2,635
0,7	4,374	4,332	4,273
0,8	8,3105	8,2190	8,0896

APPENDIX F

CALCULATION OF MEAN COEFFICIENT OF FRICTION FOR FIXED ROLLER BEAMS OF SERIES F3

An estimate of the mean coefficient of friction $\bar{\mu}$ for the fixed roller 100mm beams of series F3 can be obtained by considering the influence on applied moment of the frictional forces acting at the supports. Consider figure 5.10 and assume a vertical mid-span load is applied resulting in horizontal friction forces acting at the supports. Designate the applied load as F' to indicate that the load to produce any given mid-span moment is greater than the load that would be required for the same moment if friction were absent. Note that the frictional forces $F_f = \mu F'$. Taking moments of the frictional force about beam neutral axis, an approximate expression for the total mid-span moment M' is:-

$$M' = \frac{F'S}{4} - \frac{\mu F'(d-d')}{2}$$

$$= \frac{F'}{2}\left[\frac{S}{2}-\mu(d-d')\right] \qquad (F.1)$$

where d' is the depth of the neutral axis below the upper face, and the other symbols are defined in figure 5.10.

Now, for the same mid-span moment M in the absence of friction, we have:-

$$M = \frac{FS}{4} \qquad (F.2)$$

where F is the applied load in the absence of friction.

Equating moments, we have from equations (F.1) and (F.2):-

$$\frac{F'}{2}\left[\frac{S-\mu(d-d')}{2}\right] = \frac{FS}{4} \qquad \text{from which:-}$$

$$\frac{F'}{F} = \frac{S/2}{S/2-\mu(d-d')} \qquad (F.3)$$

The ratio F'/F will be equal on average to the ratio of γ' values for fixed and free roller beams. Taking such a ratio for the 100mm beams of series F1/F2, and series F3, we have:-

Mean γ' for series F3 = 112,7N/m
Mean γ' for series F1/F2 = 88,9N/m

Ratio of γ' values = 112,7/88,9
= 1,27

Solving equation (F.3) using a value of $F'/F = 1,27$ gives $\bar{\mu} \approx 0,5$ for typical values of $(d-d')$. This value represents the average coefficient of friction during the fracture process. Since μ will vary from zero at the start of the test to a value approaching unity in the the final stages of fracture (see 5.5), the average value of 0,5 appears reasonable.

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