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by

BRIAN GAVIN HIGGINS

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A dissertation submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, for the degree of Master of Science in Engineering.

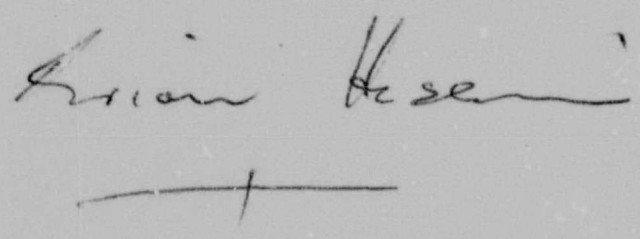
Johannesburg 1975

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Declaration

I, Brian Gavin Higgins, declare that this dissertation is my own work and has not been submitted for a degree to any other university.



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To Rosie who typed this dissertation, my sincere thanks.

ABSTRACT

Through the proper scaling of variables it is shown that laminar source flow between parallel disks may be formulated as a parameter perturbation problem to be analysed in the limit of high Reynolds numbers. The problem then reduces to an entry flow problem with three discernible regions denoted as upstream, downstream and outer. The upstream and downstream regions are similar to those found by van Dyke (1970) in his analysis of entry flow in a parallel plate channel but exhibit certain unique features: the co-ordinate for the downstream region is of $O(Re^{\frac{1}{2}})$ rather than $O(Re)$ and as a result of the adverse pressure gradient, separation can occur. It was found that for $Re \leq 60$ separation did not take place and the separation point was regular. In the outer region the radial co-ordinate is of $O(Re)$ and the asymptotic expansion proceeds in negative integer powers of Re. It reduces in the limit to the known exact solution for radial flow between parallel disks.

The transition from a turbulent to laminar flow as occurring between parallel disks was also investigated. Mean velocity profiles, radial pressure distributions, turbulent intensity profiles and spectral distributions of the longitudinal fluctuations were measured. It was found that the mechanism for reverse transition is similar to reverse transition occurring in pipe or channel flows. The decay of the turbulent fluctuations was exponential in nature and the decay rate depended on the Reynolds number. The spectral distributions were found to be similar for various radial positions. By defining a reduced Reynolds number it was possible to predict when turbulent decay would occur. This critical value was consistent with other known experimental results.

LIST OF SYMBOLS USED

A, B	hot-wire constants
E	spectral function
f	frequency
h	half the gap width
k	acceleration parameter
L	radial length scale
n	hot-wire exponent
P, p	static pressure
P_∞	atmospheric pressure
Q	volumetric flow rate
r	radial co-ordinate
Re	Reynolds number
$\hat{Re}$	reduced Reynolds number
u, U	radial velocity components
v, V	transverse velocity components
z	transverse boundary layer co-ordinate
x	transverse co-ordinate

Greek Symbols

β	Görtler's pressure function
ε	artificial parameter
η, ξ	Görtler variables
Λ	spectral length scale
μ	dynamic viscosity
ν	kinematic viscosity
ρ	radial outer co-ordinate or density
Φ, ϕ	stream function for the boundary layer and upstream region
ψ	stream function for the outer region

Superscripts

^ upstream variables

* downstream variables

- variables used in dimensional analysis or time averages of turbulent fluctuating components

Subscripts

c centreline of the channel

r,z,x,η,ξ partial differentiation

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1 INTRODUCTION

1.0 OUTLINE OF THE DISSERTATION

Various aspects of radial source flow are investigated in this dissertation covering the turbulent as well as the laminar flow regimes.

Section 1 is a review of the relevant literature as well as a general review on reverse transition. In Section 2 a critical analysis of the existing laminar theory for radial flows is presented and with the aid of some dimensional analysis a new approach to the problem is outlined in terms of asymptotic expansions. Using analytical as well as numerical techniques the analysis is carried out in Section 3.

In Section 4 an experimental investigation of reverse transition as occurring in radial source flow is described. Turbulent properties of the flow, i.e. turbulent intensity profiles and spectral distributions were measured using a hot-wire anemometer operating in the constant temperature mode. A new criterion for the inception of reverse transition is presented.

1.1 REVIEW OF THE LITERATURE

This dissertation is concerned with source flow between two parallel stationary disks, and the review is therefore limited to work directly related to the above topic. Subjects such as flow between co-rotating or porous disks have been excluded from the review. However a general review on reverse transition is given.

a) Radial flow between parallel stationary disks

An exact solution of the Navier-Stokes equations for radial laminar flow of an incompressible fluid can be obtained by neglecting the inertia terms. The pressure then fails logarithmically in the radial direction and this phenomenon is important in the design of hydrostatic air bearings and in lubrication problems. Early workers such as Barr (1931), Kapitza (1938) and Comolet (1952) studied these effects.

When the inertia effects become important the pressure increases in the radial direction. McGinn (1955), who did some flow visualization experiments, studied inertial effects in radial flow and observed that separation could occur in diverging flows, while converging flows always remained stable. By taking a linear combination of the pressure distribution as given by the viscous and ideal flow equations he was able to derive a pressure distribution, which was, for the diverging case, in good agreement with his experimental results. For converging flow however, the agreement was poor and he attributed this to entry flow problems. Wollard (1957) obtained an expression for the pressure distribution (for source flow) using a Pohlhausen method to represent the velocity profile and Livesey (1960) using an integral approach managed to obtain a more accurate solution. This was followed by an experimental investigation of Morgan and Saunders (1960) who looked at the effects of inertia on the pressure distribution and suggested that the experimental evidence was insufficient to confirm Livesey's (1960) theoretical result. Hunt and Torbe (1962) in their studies on hydrostatic thrust bearings showed that if the Reynolds number was sufficiently small the inertia effects could be neglected. This was done by assuming a power series expansion for the velocity components.

Hagiwara (1962) in connection with pneumatic micrometers made an analytic study of the outward flow characteristic of a radial flow nozzle.

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By assuming a parabolic velocity profile he managed to obtain an approximate solution to the momentum equation. Moller (1963) published a comprehensive study on laminar as well as turbulent source flow between parallel disks. He managed to obtain an approximate solution to the momentum equation which agreed surprisingly well with his experimental results. He also suggested that reverse transition might occur when the laminar sublayer became of the same order of magnitude as half the channel width. This hypothesis was based on the fact that his measured velocity profile became progressively more parabolic with increasing radius.

Peube (1963) and Savage (1964) derived independently a pressure distribution that was in very good agreement with Moller's (1963) results. They achieved this by assuming a series expansion for the velocity and pressure components. Peube (1963) also showed that the velocity profiles could have inflexion points and,as an inflexion point in the profile suggests instability,he concluded that when the profile at a sufficiently large radius, did not have an inflexion point ,the flow would always be laminar. On the basis of this hypothesis Chen and Peube (1964) derived a criterion for reverse transition.

Jackson and Symmons (1965a) derived an expression for the pressure distribution in a hydrostatic thrust bearing by using the results of Hunt and Torbe (1962). They also showed that it was possible to obtain the same result by a simple uni-directional flow analysis. In a later paper Jackson and Symmons (1965b) carried out some experiments to test the different theoretical predictions of the pressure distributions. They found that the inertia effects were not adequately described by the existing models and suggested that the theoretical approaches required further development.

Ishizawa (1965, 1966) considered the more general problem of flow in an axisymmetric gap which for the simplified case reduces to the parallel disk situation. His analysis was concerned with the inlet as well as the downstream region and he obtained a pressure distribution that was identical to Savage's (1964) expression by perturbing the boundary layer equation.

Kreith (1965) with the aid of hot-wire measurements suggested an amendment to Chen and Peube's (1964) criterion and later Bakke and Kreith (1969) carried out the first turbulence and velocity profile measurements. This work was primarily concerned with determining a

criterion for reverse transition. Their experiments showed that reverse transition was only approached but never reached as the turbulence in the centre plane did not completely disappear in the unfavourable pressure gradient. They also found that the previously suggested transition criteria of Moller (1963) and Kreith (1965) did not agree with their experimental results. Bakke, Kreider and Kreith (1973) investigated more thoroughly the source flow problem and confirmed Bakke and Kreith's (1969) results. A more detailed exposition of this work is given by Bakke (1969).

Wilson (1972) has given some criticism on the existing theory of radial laminar flow and showed that the expansion of Savage's (1964) is singular. He suggested that in the region of non-uniformity a boundary-layer type equation might hold.

A brief survey on the more practical aspects of radial flow now follows. Moller (1966a, 1966b) has done a comprehensive study on various flow situations that can occur in radial channels. This includes incompressible flow with and without swirl, compressible flow with swirl and the effects of chocking at the inlet of the channel. Jansen (1964) in connection with radial vaneless diffusers has taken extensive pressure readings and has also looked at situations where stall occurs.

Kawaguchi (1971) studied the effect of disk spacing and the radius of the rounded corner on entrance losses,while Garcia (1971) made measurements of inward and outward flows and found that in both cases the flow was unsteady, the outward flow being more unsteady as a result of the diffusive nature of the flow.

b) Reverse transition occurring in situations other than parallel disks

Reverse transition, relaminarization, laminarization are terms used to describe the phenomenon when a turbulent flow reverts back to the laminar state. Steinberg (1954) and Sergienko and Gretsov (1959) were first to observe this phenomenon in accelerating boundary layers but undertook no quantitative measurements. Since then reverse transition has been known to occur in at least three ways:

i) by strong acceleration,

ii) by decelerating a flow through a critical Reynolds number,

iii) the application of body forces, e.g. centrifugal

Laufer (1962), Sibulkin (1962) and Badri Narayanan (1968) studied reverse transition in duct flows. They achieved this by suddenly increasing the cross-sectional area (i.e. reducing the Reynolds number) and found that the decay of turbulence was exponential in nature. Also during the decay $\overline{u^2}$ and the spectrum of $\overline{u^2}$ exhibited similarity behaviour.

Reverse transition induced by accelerating boundary layers has been extensively studied in recent years. Morreti and Kays (1965) noticed that the turbulent heat transfer rate near the wall increased substantially when the acceleration parameter

$$K > \frac{\nu}{\overline{u^2}} \frac{du}{dx} = 3{,}5 \times 10^{-6}.$$

Patel and Head (1968) noticed large departures from the universal inner law velocity distribution when turbulent boundary layers were accelerated and suggested another parameter

$$\Delta > \frac{\nu}{\rho U_\tau^3} \frac{dp}{dx} = 0{,}02$$

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i) the disappearance of the large eddy structure near the wall when

$$K > 3 \times 10^{-6},$$

ii) the breakdown of the law of the wall at some critical value of Δ,

iii) the decay of turbulent intensity starting at some critical Reynolds number.

Subsequently Bradshaw (1969) showed that it was possible to deduce a general Reynolds number criterion which was consistent with the results of Patel and Head (1968) and Badri Narayanan and Ramjee (1969).

Launder and Jones (1969a) and Jones and Launder (1972) have studied sink flows where the acceleration parameter K remains invariant. They showed that it was possible to sustain a turbulent boundary layer at a value of

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but were unable to ascertain the maximum level of acceleration needed to

cause reverse transition.

At the Minneapolis Heat Transfer Conference in 1969 various papers were presented on different aspects of reverse transition. Launder and Jones (1969b) proposed a generalization of Bradshaw's (1969) hypothesis while Jones and Launder (1969) presented some progress in the development of a prediction procedure for reverse transition. Black, Cuffel and Massier (1969) measured heat transfer coefficients in conical diffusers and nozzles and confirmed Morretti and Kay's (1965) results.

Backston (1970) investigated reverse transition induced by temperature gradients and noticed some similarity between accelerating boundary layers. Blackwelder and Kovasznay (1972) looked at the large eddy structure during reverse transition and observed that the space-time correlation of the normal velocity component in accelerating flows was significantly different from that found in non-accelerating flows. Kim, Kline and Reynolds (1971) used flow visualization techniques to study the mechanism of turbulent production near a smooth wall. Their results showed that most of the turbulent production occurs during the intermittent bursts, and in accelerating flows these bursts cease once the parameter K was

$$K > 3 \times 10^{-6}.$$

Narasimha and Sreenivasan (1973) have made a theoretical study of reverse transition. Their results are in excellent agreement with known experimental results and they concluded that reverse transition is essentially due to the domination of pressure forces over Reynolds stresses.

1.2 SUMMARY

The theoretical treatment of the inertia effects in radial incompressible flow is incomplete. The only systematic approach is essentially due to Savage (1964), but as shown by various authors the same results can be achieved by assuming a simple uni-direction flow model. This is rather surprising since Savage's method assumes the flow to be two dimensional. For small radii the theory is not in good agreement with experimental results. Wilson attributed this fact to the singularity of the expansion, but to date no further work has been done on improving Savage's theory.

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