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Learning about mathematics teaching and learning from studying rituals and ritualization? A commentary

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Abstract
This is a commentary on the special issue articles on *rituals and exploration in mathematics teaching and learning.* It explores the question of whether and how we can learn from research on rituals and ritualization, when these are typically associated with poor or even no learning of mathematics. The papers in the special issue show there is indeed much to learn, and the commentary reflects on this learning together with the quandaries that arise from such engagement, in particular the quandaries of understanding and deficit discourses.

Keywords Rituals · Explorations · Quandaries · Deficits

When invited by the editors of this ESM Special Issue to provide a commentary paper, I was quick to agree. The invitation included their proposal for the special issue and initial abstracts of the papers selected. I thus had some sense of what I was agreeing to comment on. What drew my interest and attention was the suggestion of a "central remaining question" on the relationship between ritual and exploration as two types of participation mathematical learning: were these sequential or parallel? And more specifically, could a start in predominantly ritual participation evolve into exploration or did ritual participation move only in a parallel path "often leading to failure"? Here was a proposal for productive engagement with rituals/ritualized activity in school mathematics teaching and learning widespread and ever-present yet typically associated with inadequate, poor or even no learning in our field. Hence, the question mark in the title of this commentary. Can we learn from research on rituals and ritualization? Those who have read the articles in the volume before turning to this commentary will have the answer to this question. I offer some additional reflections.

The source of inspiration for the volume is the *ritual–exploration* conceptual dyad introduced by Sfard and Lavie (2005) and further elaborated in four of its articles: Lavie, Steiner, and Sfard (2018), Viirman and Nardi (2018), Nachlieli and Tabach (2018) and Heyd-Metzuyanim, Smith, Bill, and Resnick (2018), Robertson and Graven (2018) tackle a different but related conceptual dyad (*right-answerism–exploratory talk*) and the two remaining papers

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by Coles and Sinclair (2018), and McCloskey, Lloyd, and Lynch (2017) each focus on ritual/ritualization in itself. In this commentary, I engage with questions that arose for me as I read across the ideas and collective arguments these seven papers produced. In particular, I asked myself: What does the productive engagement with rituals themselves and the ritual–exploration conceptual dyad offer us? And to borrow from Sfard, what, if any, persisting *quandaries* surface when we examine what and how rituals are at work in mathematics education research? How and why might all this advance our field?

The first quandary to surface was that of *understanding*. Sfard (2008) has dealt with this extensively, drawing on Wittgenstein to argue that understanding, while something we all experience, escapes operational definition. It is routines, those patterned and repetitive actions, and observable differences between them, that can be defined and systematically described, and in turn illuminate learning. Lavie et al. (2018, this issue) build on this earlier work to describe routines as the "stuff" we live by and so learn with, and "routinization" as the discursive apparatus that will do the work of describing and explaining mathematical learning. They tease out and illustrate the distinctions between rituals, deeds and explorations. Rituals are routines that are imitative and fixed, with the focus of the agent on processes, on what to do, typically for others (i.e., social acceptance). Explorative routines, on the other hand, are those where agents are more focused on the products of their activity, in this case new mathematical narratives. Lavie et al. (2018) provide illustrations of young mathematics learners in an upper middle class rural community in Israel moving from rituals to more explorative routines as they learn number. They show that, and how, more flexible, applicable, agentic participation in mathematics begins with rituals. Fixed, imitative actions, processes or word use are necessary initial forms of participation that can, through a process of de-ritualization, enable entry into new discourses. With routines as the unit of discursive analysis, Lavie et al. (2018) argue, the pitfalls of talking about "conceptual understanding" in mathematics learning are bypassed.

If studying routines provides a means for Lavie et al. (2018) to observe and theorise a sequential learning trajectory from rituals to explorations, and avoid the pitfalls of "understanding", what then of other conceptual dyads? (e.g., procedural–conceptual or instrumental–relational understanding; Hiebert, 1986; Skemp, 1987). Notwithstanding extensive debate and indeed questioning of the distinctions between conceptual and procedural knowledge and understanding (e.g., Kieran, 2013; Star, 2005), this analytic binary remains prominent in mathematics education discourse, and often polarised. Procedural knowledge and/or instrumental understanding are constructed as inadequate, as indicative of poor or even no understanding of mathematics. Can research with routines as a unit of analysis help us to more adequately describe these ever-present practices in mathematics classrooms, across situations, contexts and conditions? The papers in this volume and their collective offerings convince me that investigations into mathematics teaching and learning through careful considerations of routines in general, and rituals more specifically, can indeed take our field forward.

That *routines* as a unit of analysis for research in mathematics education has powerful possibilities is given substance in three other papers in this volume. In the first instance, these provide an indicator of the applicability of routines: the research reported in the papers operationalizes ritual and explorative routines in relation to different research question(s) and different empirical fields.

Viirman and Nardi (2018) set out to trace the interplay between ritualized and exploratory participation of tertiary biology students in the UK as they engage with tasks that involve mathematical modelling. Their study is thus located at *a different level of mathematics* from that explored in Lavie et al. (2018), and it occurs *in the context of two disciplinary discourses*—

mathematics and biology. Viirman and Nardi (2018) establish that there is indeed fluid movement from initial ritualized participation to more exploratory engagement with assumption building—a key element of mathematical modelling discourse. I use both words "participation" and "engagement" in the previous sentence intentionally. They seem to be used interchangeably in the paper. Are these one and the same? Notwithstanding this question, my all too brief description of the study does not do justice to the complex coordination of findings from which Viirman and Nardi (2018) claim that the process of de-ritualization "results in consolidated discourse (and) a … network of interlacing, partially overlapping routines" (p. 20), contributing to the story in this volume, that studying routines is productive. Their findings further enable them to make a strong case for research that appreciates the inadequacy of oppositional binaries, and they point particularly to instrumental and relational understanding. These, they claim, cannot describe the complexity and fluidity of learning, of changing participation in new discourses. Here is a positive response, from a study located in tertiary learning, and across two disciplines, to the question of whether and how studying routines, and rituals within these, advances our field.

Nachlieli and Tabach (2018) also evidence the salience of routines—here in the context of mathematics instruction. Starting from a comment that "traditional teaching is still common worldwide" (p. 1) (and thus with a similar point to one I make above), they set out to explore the possible goals that could be achieved through "ritual teaching", or what they called *ritual-enabling* opportunities to learn (OTLs). What a novel study! The empirical field was the publicly available lessons (video-records, transcripts, etc.) in English from three countries in the 1999 TIMSS study (Australia, Hong Kong and the USA) (Hiebert et al., 2005). These provided a heterogeneous and credible data corpus of lessons for secondary analysis. Nachlieli and Tabach (2018) define and then operationalise *ritual-enabling* and *exploration-requiring* OTLs, using these to analyse routines and produce a profile of these for each of the lessons studied. The profiles in juxtaposition provide vivid visual evidence of the prevalent presence of ritual-enabling routines.

With some similarity to both papers discussed so far, they suggest that ritual-enabling OTLs can serve as necessary starting points for exploration-requiring OTLs. They observe this at work for teaching goals towards object-level learning, such as opportunities to generalize from "seeking similarities" across familiar examples, as well as towards meta-level learning from attending to "distinctions". The latter emerged in a lesson on equations in Hong Kong that began with having students carry out known procedures for solving two linear equations, one with an integer solution, and the other an identity. This enabled the teacher to draw attention to the distinct results of these ritual-enabling opportunities, and have students explore the difference in the two equations and, through this, identify new mathematics objects, such as identities.

Let me deflect briefly to draw attention to the strong resonance between the *seeking similarities* and *distinctions* explorations described by Nachlieli and Tabach (2018) and the increasing interest in, and deliberate attention to, variation in mathematics teaching and learning. Variation theorists (e.g., Marton & Tsui, 2004) argue for attention to similarity and contrast as necessary for discernment and so learning. Possibilities for mathematical generalization and appreciation of structure through attention to variance amidst invariance have long been argued and illustrated by, for example, Watson (2018) and Watson and Mason (2006). In my own research related to mathematics teacher professional development in South Africa, I have argued for attention to similarity and contrast across lesson example sets, and the possibilities these open up for building generality and noticing structure (Adler & Ronda, 2015, 2017). When I bring a variation gaze to bear on the ritual-enabling example sets in the

lesson profiles presented by Nachlieli and Tabach (2018), I see deliberately organized (selected and sequenced) example sets and accompanying procedural tasks mediated through traditional forms of teaching that open possibilities/opportunities for exploration by attention to variance amidst invariance. It will be productive in future research to bring insights from studies of ritual teaching and studies related to variation, notwithstanding different theoretical orientations, into conversation.

Deliberate variation is also visible in Coles and Sinclair's (2018) study of ritual teaching. Here, rituals are defined as practices that privilege certain activities over others, are embedded in a symbolically structured environment and do not make explicit the mathematical point. Coles and Sinclair (2018) thus work with a definition of ritual that does not separate knowing and thinking from doing, and so does not work with binaries. In this way, they submit, rituals are not characterised by "un-thinking-ness, rote action or submission to authority" (p. 16), nor only for social acceptance. Their empirical setting is the grade 1 mathematics classroom, one in the UK and one in Canada, with early number learning the content. The artefact on which activity was focused was a "Gattegno tens chart" (see p.6). The numbers 1 to 9 in the first row are followed in the next row by the numbers 10 to 90, then hundreds and so on forming columns of numbers 1, 10, 100, 1000 … 2, 20, 200, 2000 and so on. This was the raw material on which the authors, as the teachers, orchestrated tasks that included students reading and chorusing names of the number symbols pointed to, and identifying "one more than …". Coles and Sinclair (2018) show that students' seemly rote responses indicated thinking-ness; and note that, of course, it is the raw material, with potential or affordances condensed into it, *together with the mediation of tasks set up by a skilful teacher* that produces the "symbolically structured environment" for ritualization practices. It would be productive to study more deeply, and in a wider range of empirical settings, what and how opportunities for meaning that are condensed into a structured artefact require the work of a skilled teacher to open these up through structured ritualization.

Lurking in the background here is another binary—traditional and reform teaching. These are aptly described by Nachlieli and Tabach (2018), as they engage the quandary around reform initiatives across many contexts, and the realization of the steadfastness of traditional forms of mathematics teaching. How might the ritual–exploration dyad and its operationalization by Lavie et al. (2018, this issue) be extended to studying *mathematics teachers learning new ways of teaching* from participation in professional development (PD)? And more specifically, when there is an explicit reform agenda directed towards dialogic instruction*?* Heyd-Metzuyanim et al. (2018) take on this task, and *confirm a similar trajectory from ritual to exploration of two US teachers learning through their participation in PD over time*. They demonstrated take up of desired practices that began with forms of imitation and moved to forms evidencing more agency and flexibility with the promoted practices. Heyd-Metzuyanim et al. (2018) describe the seriousness with which both teachers attempt to enact new practices in the contexts of their work and over time. They point out further that the two selected teachers were indeed the most "successful" of the participating teachers. Their study brings to the fore the fragility of learning new teaching practices. While the teachers' learning of instructional practices proceeded from rituals to explorations "… the process of change from ritual to explorative participation tends to be slow and may never be completed …" (p. 28–29). This is sobering food for thought across much of the literature on mathematics PD that bemoans the equivocal impact of reform-oriented PD on the quality of learning and teaching mathematics in general, and teachers' instructional practices in particular.

And this brings me to the next quandary to surface in this volume: talk of deficits. Heyd-Metzuyanim et al. (2018) note that PD research, albeit unintentionally, often produces deficit discourses about mathematics teachers and teaching. Talk of deficits is also discussed by Lavie et al. (2018), with the claim that it is the salient features of routines and the fluidity of its ingredients that enable descriptions of what the child (or teacher) does rather than what he does not do or what he lacks. Here is the quandary. If we want x to learn y, and we want to show what, how and why x does or does not learn y, we first have to establish that x does not "know" y. In discursive terms, we would want to show a change in x's discourse—that intended aspects of the discourse that were not present, are now present. How do we point to that which someone is to come to know without acknowledging its initial absence, and thus a relative "lack"?

This conundrum sits inside Heyd-Metzuyanim et al.'s (2018) study that brings to light how teachers move from ritual to explorative routines as they learn new mathematics teaching practices. Heyd-Metzuyanim et al. (2018) argue that their orientation and language enable them to get at the process, the how, and the why of teachers' learning, and not only the what, and thus not fall prey to a deficit discourse. However, to describe the movement in the teachers' learning, they must start with a description of the initial teaching of M and W (the two teachers). And it is, somewhat inevitably, littered with expressions of absences as evidence of initial ritual engagement. The teachers did not have mathematical goals for the lesson; they did not alter the task to suit their concerns for their learners; they did not confront different ideas offered by learners, ultimately showing "rigid imitation of practices in the PD session" (p.19). Heyd-Metzuyanim et al. (2018), of course, do not stop here but proceed to describe a later lesson where there is flexibility and agency on the part of the teachers, bringing into view the presence over time of new ways of mathematics teaching. It is impossible in our work to point to new presences (and this pertains particularly to new teaching practices) if these are not juxtaposed with descriptions of their prior absence. How do we, as mathematics teacher educator researchers seeking to evidence change—learning of new practices, proceed professionally, responsibly and respectfully as we describe teaching practices and evidence changes in these over time?

All this concern with talk of deficits is indicative of our discomfort in the field of mathematics education, of pointing to absences in students or teachers, and no matter the wider socio-political-economic-cultural context of the study. For Sfard, a discursive participative paradigm bypasses this difficulty as deficits (absences) are not located in teachers' or students' "heads", but in the socio-cultural context and its dominant discourses.

For McCloskey et al. (2017), rituals do indeed transcend individuals, and thus can be productively studied at a wider instructional level. With some similar sources to those informing Coles and Sinclair (2018), they define rituals as "that aspect of action that is formalized, traditionalized, symbolic and performance" (p. 2) and the "mechanism through which participants perpetuate or resist culturally situated patterns" (p. 3). These definitions are operationalized for studying a set of lessons on fractions in a grade 5 class in the USA. They parsed classroom observational data for patterns of actions (rituals) and, through these, described cultural practices in the classroom and related tensions. They show, and this is very interesting given the traditional-reform quandary discussed earlier, that some reform-oriented practices like students showing their work and using multiple representations could ironically "undermine reasoning and sense-making"—a key goal of mathematics reform initiatives. The potential of a "ritual" lens is thus shown in this wider instructional context.

Robertson and Graven's (2018) paper gives a different level of meaning to how and why context matters in their study of mathematics instructional talk in a grade 4 classroom in South

Africa. They are not studying teacher learning and so change, but focus on teaching practices, and the constraints teachers face when they are teaching in a bi/multilingual context where the teacher and her students are working in English, and it is not their home language. The teacher struggles to teach fraction ideas in this context of enduring inequality, poor performance, limited material resources, and students with limited linguistic resources in English. It is also a context where traditional forms of teaching are associated with widespread failure in school mathematics and so an example of where rituals do lead to a parallel impoverished trajectory for students. Robertson and Graven (2018) expressed concern that their description of the prevalence of chorusing right or wrong answers and the relative absence of exploratory talk in this classroom could "invoke a deficit judgment" (p.19). They argue that drawing on socio-linguistic research, they are able to explain what they observed in non-deficit terms—that the imperative to use English as the only medium of instruction results in a "'subtractive' form of bilingualism which constrains students' opportunities to engage in more exploratory talk". The problem, as it were, lies not only in the literacy-numeracy interface, but in a complex interplay of social, economic and political factors where access to social goods is tied up with access to English—the dominant language—and to mathematics—the school subject that opens and closes doors to advancement.

In South Africa, where the majority are trapped into poverty and related poor school performance, particularly in mathematics and languages (e.g., Adler & Pillay, 2017), access to a valued social good like English literacy and mathematics has material consequences. Janks (2010) cogently argues with respect to literacy:

> Access is a type of right, the right to enter and get through the gates, the right not to be excluded. Too often language acts as a gate that sorts and selects students, with teachers performing the functions of society's gatekeepers. If what is beyond the gates are the elite literacies that are out of reach for most people, then these literacies become highly desirable, this desire is not based simply on symbolic values. Because access is a fundamental instrument of social stratification, it has material consequences that affect peoples' life changes for generations. Desire is a double-edged sword: becoming what we lack changes who we are. Something is always lost in the process. As educators changing people is our work, work that should not be done without a profound respect for the otherness of our students. Desiring what one is not should not entail giving up what one is. (p. 153)

In this and other multilingual and "developing country" contexts, the mathematics learning journey is far more complex than from proceeding from rituals to explorations.

Robertson and Graven (2018) begin their paper, as do Nachlieli and Tabach (2018), with the traditional-reform dyad, pointing specifically to the considerable challenges reported in enacting student-centred teaching. These, they argue, are "most acute" in contexts where teaching takes place in a language that is different from the teachers' and their students' home language (L1). They investigated a related but different dyad and binary drawn from socio-linguistic research: *right-answerism–exploratory talk* (Barnes, 2010). As noted above, their study was located in a grade 4 class in South Africa, where the teacher and her learners were working on fractions and in English—a language different from their home language (L1) isiXhosa. In order to study the instructional talk in the mathematics classroom, Robertson and Graven (2018) drew on constructs in socio-linguistics, particularly Cummins' distinction between BICS (basic interpersonal communication skills) and CALP (cognitive academic language proficiency) and his framework for analysing cognitive and contextual demands in educational bilingual settings. They used these to

explore the continuum of student talk in the class from BICS-like context-embedded "everyday" talk to more CALP-like context-reduced "classroom" talk and then to relate their findings of patterns in the talk to the right-answerism–exploratory talk dyad. Like other similarly located research, they found that right-answerism was the order of the day, and exploratory talk relatively 'absent'. They suggest that development interventions into these practices would need to include support for learners' main language (their L1) and the language of instruction (here L2, English), together with ways of moving between both context-embedded, low cognitive demand tasks/talk (rituals?), to context-reduced and more cognitive demanding tasks (explorations).

The research reported here seems to run in parallel with the extensive research on teaching and learning mathematics in multilingual settings, much of which also draws on socio-linguistics (e.g., Adler, 2001; Planas, 2014; Setati Phakeng, 2012). This research has illuminated how complex is the journey from informal talk in students' main language (everyday, local or situated talk if you like) to formal talk and writing of mathematics in English, and in many classrooms remains incomplete (Setati & Adler, 2001). Research on how L1 and L2 interact as teachers and students work towards access to both mathematics and English in an English dominant society has shown, for example, that while students' L1 was drawn on as a learning resource, it was used for social solidarity (rituals?), with English privileged for mathematical talk (Setati, 2005). The insights from the exploration of right-answerism and exploratory talk adds to this complex domain, and would be productively put in conversation with this research. There are multiple tensions for teachers and teaching in this work with interesting recent research by Barwell positing an additional binary – situational-distal sources of meaning (Barwell, 2018). Barwell studied students' repertoires of meaning in a bilingual setting in Canada, showing the multiple ways students draw on sources of meaning, both situational and distal. This research reinforces the potential of orientations to interpreting practice that do not dichotomize analytic binaries, but explore their inter-relation.

The Robertson and Graven (2018) paper also drew me to thinking about its relationship to the extensive literature on education development research—an area of study that remains out of view in the mathematics education literature. I refer here to specific studies of development, often in developing countries and reported in journals such as COMPARE and the *International Journal of Educational Development*. I mention this as the dyad or binary expressed either as traditional vs reform teaching or learner- vs teacher-centred pedagogy has been the focus of intense debate following research that drew attention to the "widespread failure" of educational aid programs promoting learner-centred pedagogy (e.g., Lattimer, 2015; Schweisfurth, 2013). Development research, while sometimes about the teaching and learning of a particular subject (like mathematics), is typically focused on pedagogy more broadly. But is it useful to ask, could research inspired by the various binaries in educational discourse on the one hand, and routines as unit of analysis on the other, also benefit from engaging in conversation? I suspect so, particularly as it is in these contexts that dominant practices would typically be described as rituals/ritualized.

My more extensive engagement with Robertson and Graven's (2018) paper is, of course, a reflection of my own research history and research interests. At the same time, the discussion of their paper and its literature base reveals another challenge—perhaps a quandary—in our field. As in other papers in this volume, I have suggested the possibilities for extensions of the research through conversations with other resonating yet seemingly 'ignored' research. The challenge is whether and how, in a field where theories have proliferated, we can bring together in a more cumulative way, apparently similar research findings resting on different, even incoherent, theoretical grounds and orientations.

In conclusion, that there is much to do and that much lies ahead is wonderfully brought into view in this special issue and its focus on routines, particularly rituals. The research reported in each of these seven papers helps us understand how and why repetitive patterned actions, some of which are imitative, and thus akin to rituals, have significance in and for learning and doing mathematics, learning and doing mathematics teaching, and in illuminating tensions in mathematics classroom instruction, and across empirical sites and educational contexts.

The research studies reported here that link directly to routines, rituals and explorations confirm and strengthen the theoretical offering by Lavie et al. (2018) that the path from ritual to exploration in mathematics learning (and learning mathematics teaching) is sequential yet fluid, rather than parallel, but not necessarily so. Context and conditions can and do prevail that restrict opportunities to learn to rituals, thus limiting learning. This volume and the papers in it provide important narratives that run contrary to the frequent and typically decontextualized bad press given to these and other so-called traditional forms of mathematical activity.

Alongside the discussion of the different papers has been a reflection on some of the quandaries in our field: understanding, deficit discourses and reform. The papers, separately and collectively will, I am sure, spur further engagement with the meanings of these in further research on rituals, ritualization and de-ritualization and exploration, and the role these can and do play in mathematics learning, mathematics teaching and learning mathematics teaching.

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Compliance with ethical standards

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References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht: Kluwer Academic Publishers.

Adler, J., & Pillay, V. (2017). Mathematics education in South Africa. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 9–24). Abingdon: Routledge.

Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education*, *19*(3), 237–254. https://doi.org/10.1080/10288457.2015.1089677.

Adler, J., & Ronda, E. (2017). A lesson to learn from: From research insights to teaching a lesson. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 133–143). Abingdon: Routledge.

Barnes, D. (2010). Why talk is important. *English Teaching: Practice and Critique, 9*(2), 7–10.

Barwell, R. (2018). Repertoires of three second language learners of mathematics: Distal and situational sources of meaning. *Research in Mathematics Education*, 1–17. https://doi.org/10.1080/14794802.2018.1490664

Coles, A., & Sinclair, N. (2018). Ritualisation in early number work. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-018-9815-6.

Heyd-Metzuyanim, E., Smith, M., Bill, V., & Resnick, L. B. (2018). From ritual to explorative participation in discourse-rich instructional practices: A case study of teacher learning through professional development. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-018-9849-9.

Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., & Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis, 27*(2), 111–132.

Janks, H. (2010). *Literacy and power*. New York: Routledge.
Kieran, C. (2013). The false dichotomy in mathematics education between conceptual understanding and procedural skills: An example from algebra. In K. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 153–171). Dordrecht: Springer.
Lattimer, H. (2015). Translating theory into practice: Making meaning of learner centered education frameworks for classroom-based practitioners. *International Journal of Educational Development, 45*, 65–76.
Lavie, I., Steiner, A., & Sfard, A. (2018). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-018-9817-4.
Marton, F., & Tsui, A. B. M. (2004). *Classroom discourse and the space of learning*. New Jersey: Lawrence Erlbaum Associates.
McCloskey, A., Lloyd, G., & Lynch, C. (2017). Theorizing mathematics instruction using ritual: Tensions in teaching fractions in a fifth-grade classroom. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-017-9779-y.
Nachlieli, T., & Tabach, M. (2018). Ritual-enabling opportunities-to-learn in mathematics classrooms. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-018-9848-x.
Planas, N. (2014). On the process of gaining language as a resource in mathematics education. *ZDM, 46*(6), 883–893.
Robertson, S. A., & Graven, M. (2018). Exploratory mathematics talk in a second language: A sociolinguistic perspective. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-018-9840-5.
Schweisfurth, M. (2013). *Learner-centred education in international perspective: Whose pedagogy for whose development?* London: Routledge.
Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*(5), 447–466.
Setati, M., & Adler, J. (2001). Between languages and discourses: Code-switching practices in primary mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*, 243–269.
Setati Phakeng, M. (2012). Mathematics in multilingual classrooms in South Africa: From understanding the problem to exploring possible solutions. In B. Herbel-Eisenmann, J. Choppin, & D. Wagner (Eds.), *Equity in discourse for mathematics education: Theories, practices and policies* (pp. 125–145). Switzerland: Springer.
Sfard, A. (2008). *Thinking as communicating human development, the growth of discourses, and mathematizing*. New York: Cambridge University Press.
Sfard, A., & Lavie, I. (2005). Why cannot children see as the same what grown-ups cannot see as different?: Early numerical thinking revisited. *Cognition and Instruction, 23*(2), 237–309.
Skemp, R. R. (1987). Relational understanding and instrumental understanding. *Mathematics teaching in the middle school (MTMS)., 77*, 20–26.
Star, J. (2005). Reconceptualising procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404–411.
Viirman, O., & Nardi, E. (2018). Negotiating different disciplinary discourses: Biology students' ritualized and exploratory participation in Mathematical Modelling activities. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-018-9861-0.
Watson, A. (Ed.) (2018). *Variation in mathematics teaching and learning.* Derby: Association of Teachers of Mathematics.
Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning, 8*(2), 91–111.

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