University of the Witwatersrand
School of Education

The Effects of Mathematical Modelling and its Application in Algebraic Functions on Grade 11 Learners' Performance.

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This Research Report is submitted to the Wits School of Education, Johannesburg in partial fulfillment of the requirement for the Master of Education by Research Degree in Mathematics Discipline.

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Declaration:

I Sibongiseni Ngubane a student number 810695 at the university of Witwatersrand School of Education hereby to declare that the Master of Education by Research (Dissertation) in Mathematics discipline titled: **“The Effects of Mathematical Modelling And its Applications in Algebraic Functions on Grade 11 learners’ Performance”** is my own work. I have acknowledged all sources of information that I have used and cited when writing this Dissertation Report and all details are provided in a reference list. In addition, this is my first-time submission of this full dissertation report at the Witwatersrand University, and I did not submit any section of it before to any institution for any other qualification for examination.

Student full Names and Surname	Signature	Date
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Abstract:

To explore the effects of mathematical modelling on Grade 11 learners' academic performance in algebraic functions, teachers' self-efficacy and their overall perceptions, is the purpose of this research study. For the achievement of this goal, the research study followed a mixed method for both collecting and analyzing data. However, a pre-test and a post-test, interviews, and a questionnaire were used to collect data. The study content was limited to mathematical modelling in algebraic functions, learners doing pure mathematics in Grade 11, and mathematics teachers in the FET phase only. Eighty-seven (87) Grade 11 learners doing mathematics from one (1) selected school under the ILembe district in KwaZulu Natal participated in the research study, where 44 (51%) formed an experimental group and the other 43 (49%) learners formed a control group. Findings revealed that the experimental group taught through modelling with a guided discovery approach outperformed the control group that was taught through a direct instruction approach to learning. Hence, the difference between the modelling and direct instruction teaching approaches is statistically significant. Six (6) learners from the experimental group were purposively sampled to participate in the semi-structured interviews for the researcher's purposes of exploring learners' perceptions about mathematical modelling. Learners reported that modelling could improve their level of cognition.

Thirty-three (33) mathematics teachers in the FET phase from fourteen (14) high schools in KwaZulu Natal participated in the study by completing a questionnaire about modelling. Findings revealed that teachers have positive attitudes, beliefs, and perceptions about modelling in mathematics education even though there are no teachers' professional development workshops provided to encourage them based on modelling. This study recommended that the department of education put more efforts into supporting and motivating teachers to implement modelling in mathematics classrooms, provide teachers with professional development workshops based on modelling, design and distribute teaching and learning support documents to all schools providing mathematics, and do follow-up to check the progress in teachers' developments.

Key Words: Mathematical modelling, learners' performance, teaching approaches, teachers' beliefs and self-efficacy, modelling perceptions.

Table of Content

List of Tables

List of Figures

Glossary

Model Is a system that represent an object, fact, thought, and framework which helps through understanding the nature of the problem(s) adapted from real-life situations, but not the original copy of the reality since models are unable to reflect all properties of the reality and they also include additional explanations in addition to target they represent.

Modelling Is the process of transferring real-life situation into language of mathematics through selection, and used of variables, expressing the relationship among them, to form models using variables as well as to test models and their practices, in terms of equations and expressions that can be used to predict and solve future unknown real-life problems.

Modelling competency Is the structure that contributes towards the identification of skills, attitude, knowledge, and understanding that one should demonstrate as indications of competent in mathematical modelling.

Beliefs Are convictions or ideas based on the nature of reality or something that a person or people accept as truth without any tangible evidence.

Constructivism Is the learning theory originated in psychology that gives the explanation on how one may learn and develop knowledge and it is rooted in observations, scientific study, and basis that learners' cognition results from mental construction of knowledge.

Constructivists beliefs Is referred to as knowledge construction that emphasis the perspective that learners need to be given opportunities of discovering their own strategies towards solving problems, develop the ability towards working as individual and independent as well as given the opportunity to present processes and solutions towards solving problems.

Curriculum Knowledge is the composition or package that involve learners, teachers, content, concepts, instructional practice methodologies, skills, beliefs, knowledge and set of objectives to be achieved in line with the social and economic needs of that era.

Curriculum is the composition of organized and channelled or directed learning proficiencies as well as envisioned objectives developed following methodical systems of knowledge reconstruction and experiences based on schools' support systems for learners' continuous development in personal and social competencies.

Transmissive belief Is the belief that teachers who are productive in teaching and learning are the ones who transmit knowledge to learners whereas learners are passive knowledge receivers, demonstrates approaches methods and approaches towards solving application problems.

Content knowledge Is the comprehension of teaching and learning theories, principles, conceptions, truths, and techniques, instead of relating abilities such as researching, reading, or writing that learners may acquire in schools.

Subject Matter Content Knowledge Is the knowledge capacity or composition that teachers have in their mind? concerned with how subject matter is presented to learners meaningfully in various Way that are promoting learners' active participation and learners' clear understanding of the concepts.

Pedagogical Content Knowledge Is about how subject matter is presented to learners meaningfully in various way that are promoting learners' active participation and learners' clear understanding of the concepts.

Self-efficacy Is the evaluation of someone's efficiency and capabilities in particular area towards the understanding and completion of tasks.

List of Acronyms

Acronym	Meaning
ATP	Annual Teaching Plan.
CAPS	Curriculum and Assessment Policy Statement.
CK	Content Knowledge.
DBE	Department of Basic Education.
DoE	Department of Education.
LC	Learner Control group
LE	Learner Experimental group
MEAs	Modelling Eliciting Activities.
MM	Mathematical modelling.
NCS	National Curriculum Statement.
NCTM	National Committee of Teaching Mathematics.
PCK	Pedagogical Content Knowledge.
PDW	Professional Development Workshops.
SBA	School Based Assessment.
SMCK	Subject Matter Content knowledge.
TEMMT	Teachers Effective Mathematical Modelling Training.

[1]CHAPTER 1

Introduction

Making connections between real-world and classroom teaching and learning has recently been reviewed as an important route towards improving learners' performance, especially in mathematics (Greefrath et al., 2022). Mathematical modelling becomes an effective approach towards effecting this connection if it can be taught using real data extracted from a real-world context. Connecting mathematics with the real world improves learners' understanding of mathematics (Dunn & Marshman, 2020), makes learners see mathematics as relevant and sensible (Geiger, Galbraith, Niss & Delzoppo, 2022), motivates learners to do mathematics, and enables learners to use mathematics knowledge to solve problems in a real-world context. Mathematical modelling in many countries (Niss & Blum, 2020; Stillman, 2019; Jojo, 2020; Baidoo, 2019) has gained attention and been recognized as the solution towards improving mathematics performance and changing the widespread negative perception that says mathematics is a challenging (Ngoepe, 2014) and critical subject (Tran, Nguyen, & Ta, 2020) for both teachers and learners.

Recognition of mathematical modelling is evidently presented by its introduction and integration in the school mathematics curriculum, both in primary and secondary school mathematics teaching and learning (Geiger, Galbraith, Niss, & Delzoppo, 2022). The main objective of mathematics education towards the introduction of modelling into the school mathematics curriculum is to improve learners' performance in mathematics. How can this objective be achieved? Since mathematical modelling is a dominant approach to teaching and learning, many countries (Chamberlin et al., 2022; Geiger et al., 2022; Jojo, 2020) are looking forward to implementing it as a promising solution to devastating learners underperformance in mathematics as compared to other subjects. The emergence, adoption, and introduction of mathematical modelling as one of the approaches that can be used in mathematics teaching and learning, is a hope of mathematics education to achieve this objective (improve learners' performance), develop a positive attitude towards doing mathematics, improve mathematics competencies, and change negative perceptions about mathematics if learners are exposed to some mathematics applications that are linked to their real contexts. However, this study contributes to innovative teaching and learning approaches that include modelling and its application in secondary mathematics education.

To teach mathematics is not easy and requires someone who is competent in mathematics concepts (Wess et al., 2021; Durandt et al., 2022; Niss & Blum, 2020), understands the basics, and is able to transmit information to another person. The goal of mathematics education is to use real-life models to solve problems in mathematics. By a model and modelling, I use Wess et al. (2021) to mean a model is a system that represent an object, fact, thought, and framework which helps towards understanding the nature of the

[1] Mathematical modelling as the scientific phenomenon that is used when creating or developing a model and it process entails the state and event to explain, use, and interpretation of a model generated (Harrison, 2001; Ornek ,2008; Ferri, 2018).

problem(s) adapted from real-life problems, but not the original copy of the reality since models are unable "to reflect all properties of the reality and they also include additional explanations in addition to target they represent" (Harrison, 2001, & Ornek,2008, p.93) whereas modelling is the scientific phenomenon that is used when creating or developing a model and it process entails the state and event to explain, use, and interpretation of a model generated (Ferri, 2018).

In South Africa, the department of education's (DoE, 2011b, p. 8) current Curriculum and Assessment Policy Statement document (CAPS) second specific aim mentions that "mathematical modelling is the focal point of the curriculum, real-life problems should be incorporated into all sections whenever appropriate, and examples used should be realistic and not contrived". One of the reasons is that modelling is grounded in an encouraging or reinforcing learner-centredness approach, as it advocates learners' opinions, expands knowledge gained, and facilitates the understanding of mathematical principles (Blum & Ferri, 2009; Niss & Blum, 2020). However, mathematical modelling in mathematics education promises to be innovatively, creatively, and interactively supportive ways of developing and examining learners' knowledge, understanding, and how they learn the concept of algebraic functions.

This current study is about exploring the effects of mathematical modelling on Grade 11 learners' performance in algebraic functions, teachers' self-efficacy, and both teachers' and learners' overall perceptions about mathematical modelling. This current study's participants were grade 11 learners doing mathematics in the FET phase who are doing algebraic functions in rural area schools and teachers who are currently teaching mathematics in the FET phase. This study explored the degree to which the use modelling approach can have an impact on improving learners' academic performance, competency, and interest through mathematical modelling in the learning of algebraic functions, namely, linear, parabola, exponential, and hyperbolic functions as well as how to actively link and apply mathematics to solving real-world context problems.

Following the study carried out by Bikić, Burgić and Kurtić (2021) in Europe about "The Effect of Mathematical Modelling in Mathematics Teaching of Linear, Quadratic, and Logarithmic Functions", that was aimed at informing high school teachers about modelling practice which aims to scale up mathematics teaching and learning, findings revealed that even though schools do prepare learners with high-level mathematics modelling tools, they still lack the knowledge to teach learners how to use those tools to solve real-life problems. However, the department of education (DoE, 2011b) in South Africa has recognized the ability of mathematical modelling to be employed to improve learning to meet curricula needs, that is grounded in social and economic needs, and that helps to make the true meaning of problems drawn from our daily lives, aiming to make mathematics relevant towards solving problems drawn from real-life context. Meanwhile, a quantitative study conducted by Julie (2020) examining competencies of mathematical modelling demonstrated by the group of learners in one of South African school mathematics programmes that was made by the group of 671 learners, where 200 and 471 were grade 12 and grade 10 learners respectively. This research study revealed that both groups of learners showed very low competencies in mathematical modelling. Thus, these results show that there is a gap that still needs attention to operationalize the DoE's vision to improve learners' academic performance through mathematical modelling.

Bikić, Burgić and Kurtić (2021) study also revealed that learners have insufficient modelling critical thinking proficiencies; teachers have insufficient skills to teach mathematical modelling application; and there is a lack of setting modelling tasks that influence learners logical reasoning ability and allow them (learners) opportunities to express their own ideas, comments, and criticisms. Bikić et al. (2021) recommended that mathematical modelling be consistently applied in lessons and that professional development workshops be held for teachers to shed light on the procedures of modelling, as well as the advantages of modelling towards upgrading learners' academic achievement. Why is mathematical modelling so important in mathematics education?

Teachers are repeatedly complaining about learners failing to preserve the mathematics skills they acquired at school and lacking the skills they acquired to solve new types of problems (Niss & Blum, 2020). This raises questions about how mathematics taught in schools is relevant to our daily activities and social needs. Ferri (2010; Wess et al., 2021) mentioned that mathematical modelling influences a special and unique high-level of cognitive demand development for both teachers and learners, which makes modelling to be trusted by the department of education as a tool that if effectively used to develop required skills and competencies, can positively impact learners' learning abilities, and improve mathematics academic results. However, how learners learn the concept of algebraic functions depends on how the teacher presents each algebraic parameter to promote modelling competence and how modelling tasks are introduced to learners, because modelling tasks are believed to reinforce a learner-centered approach that encourages learners' participation and promotes logical thinking and reasoning (Wess et al., 2021).

1.1 Problem of the study

The research problem is defined as the researcher's concern or argument of interest the researcher is investigating (Creswell, 2012). In South Africa (Durandt et al., 2022a; Jojo, 2020; Adler et al., 2017) and many other countries (Abassian et al., 2020, 2020; Niss & Blum, 2020; Stillman, 2019) in the world, high schools' curriculum is currently emphasising that mathematical modelling should be part of teaching and learning targeting to use knowledge that learners are learning from schools to solve problems that are existing in our communities. The aim of the department of education is to take teaching back to the community and setting the curriculum that meet the social needs which make mathematics much closer and relevant towards solving problems drawn from real-world contexts. There is a lack of teaching mathematics using modelling approach in mathematics classrooms (Jojo, 2020; Durandt et al., 2022a), whereas methods used to teach mathematics which include direct instruction, and learners performance in algebraic functions are some of the problems in our schools (Baidoo, 2019). In that regard, a powerful tool recognised by the department of education to close this gap is mathematical modelling. Even though mathematical modelling was introduced to school curriculum, schools are still lacking capabilities to prepare learners on how to use mathematical modelling skills and knowledge they have acquired into the real-world (Bikić et al., 2021). However, South Africa current curriculum (CAPS) specify the necessity to teach mathematics through modelling (DoE, 2011b). It is not clear if teachers are aware that the current curriculum requires mathematical modelling to be taught in mathematics classrooms or are lacking skills to teach through modelling.

It is also not clear if modelling is being implemented in schools since performance in algebraic functions is not at satisfactory (Baidoo, 2019), or tasks set by teachers are or not challenging learners to solve problems following procedures of mathematical modelling and allowing learners the opportunity to demonstrate and present their models. Hence, mathematical modelling develops learners' abilities to interpret and criticise their own results (Greefrath & Vorholter, 2016) instead of learning to pass than to know and acquiring abilities to apply knowledge acquired to solve problems adapted from real-world contexts and help them develop high level of cognition. This follows that teachers are lacking knowledge based on representing and predicting features of the model and modelling processes (Akkan et al., 2018). Thus, another challenge is that teachers' beliefs, and their self-efficacy about modelling are not clear. However, it is important for teachers to have acquired adequate mathematical model and modelling knowledge and skills to be able enhance learners modelling competencies through applying modelling approach in mathematics classrooms (Durandt et al., 2022a). The modelling approach introduction in mathematics teaching and learning (DoE, 2011b) in the presence of direct instructional approach of teaching is also not clear if can have a great impact on learners' performance as compared to direct instruction approach.

In my experience of teaching mathematics, teaching mathematical modelling, and setting of mathematical modelling tasks that make mathematics relevant towards solving problems in the real-world context, tasks `that allow learners' ideas, investigation, appreciation, and criticisms are not easy and have noticed that learners in our schools have insufficient skills to develop models, mathematising, representing and interpreting solutions to make sense of the problems adapted from real-life contexts. As a result, learners are struggling with algebraic functions. Since learners have been taught through direct instruction method their performance in algebraic functions is not at satisfactory (Baidoo, 2019). The success of modelling implementation is rooted from elevated quality teaching and learning phenomena (Blum, 2015). However, to generate learners' interest, improving their academic performance and achieving mathematical competencies demands learners to access, or to be exposed to mathematical modelling tasks that are promoting growth in levels of cognition. Thus, to adequately teach modelling requires teachers' interest, positive attitude, beliefs, skills, choice of examples and modelling competency (Geiger et al., 2022).

This study asked different questions based on: what can be done to improve learners' academic achievement in algebraic functions? how can schools prepare learners to apply mathematics tools in real-life situations to solve problems so that learners see mathematics as relevant to the real-world context? What are teachers' beliefs and self-efficacy based on mathematical modelling? How can mathematical modelling activities be best selected and used towards the gradually development of learners' modelling competencies? These are the kinds of questions this current study explored.

1.2 Purpose of the study

The purpose of this study is to explore the effects of mathematical modelling on Grade 11 learners' academic performance in algebraic functions, teachers' self-efficacy, and their overall perceptions.

1.3 Rationale of the study

The results of this study contribute to the CAPS Grade 10 -12 mathematics curriculum (see DBE, 2011, p. 8) specifically its emphasis on mathematical modelling as a possible means towards enhancing learners' mathematics interest and academic performance. Mathematical modelling significance, as stated in DoE (2011b), and its effects cannot be quantified without any enforced emphasis. So, this study aims to operationalize those statements through classroom-based teaching practices that incorporate modelling and its applications. This will help teachers realize the goal of the curriculum they are teaching, set learning objectives based on mathematical modelling, engage themselves in skills centres for training to develop relevant skills to adequately teach mathematics through modelling and set the teachers' attitude toward modelling.

The study conducted by Ferri (2018) revealed that teaching and learning through mathematical modelling demands thinking translation, whereby a teacher is required to use the language of mathematics correctly, coherently, and precisely, and provide mathematics explanations that help learners develop logical reasoning. However, the translation from traditional teaching modes to a mathematical modelling approach is basically rooted in teachers' professional development workshops (Wu, 2011). This study will inform the department of education about the gaps and need to elevate teachers' training to equip teachers with the required skills on how mathematical modelling can be applied to develop learners' interest and improve learners academic performance by using tasks that challenge learners' thinking, provide teachers with purposively generated resources to promote modelling, provide specific guidelines to appropriately use digital tools in modelling, examining, and evaluating learners' solutions, as well as giving teachers tools to track learners' growth in cognitive level resulting from the use of modelling. This form of development is important because teaching and learning changes in classrooms are fundamentally dependent on quality teachers' professional development workshops (Atteh & Andam, 2019), and thus, the department of education's reasons to adopt modelling into the curriculum and the vision to implement modelling in mathematics classrooms may not be successful (Hull, 2012).

The education reform fostered in the Curriculum and Assessment Policy Statement (CAPS) requires all learners to be well prepared to solve in-depth modelling tasks. Once the department of education is aware of the gaps in teachers' teaching and learners' learning with modelling, and sets up structured modelling workshops, modelling training will benefit teachers with skills to set tasks that provoke learners' curiosity, encourage self-sufficient thinking, co-operative arguments, and provide important themes and ideas in mathematics. Thus, this may benefit learners by expanding their level of cognition, developing flexibility to generate and solve models, and appreciating modelling. In addition, schools may then celebrate quality results and high learners' performance.

Thus, this project hopes to play a huge role in encouraging schools to apply mathematical modelling as outlined in the CAPS documents policy. Automatically, learners will develop logical reasoning skills and an interest in learning mathematics, which can effectively improve learners' performance and enable them to apply knowledge acquired at school to solve existing problems in a real-life context.

1.4 Objectives of the study

This study has the following objectives:

1. Explore how different Grade 11 learners' mean scores in algebraic functions are after being taught using a mathematical modelling approach as compared to the mean scores obtained through a traditional teaching approach.
2. Explore Grade 11 learners' perceptions about mathematics after learning algebraic functions through mathematical modelling.
3. Explore mathematics teachers' self-efficacy about the application of mathematical modelling.
4. Explore mathematics teachers' perceptions about application of mathematical modelling.

1.5 Research questions

From the above-mentioned objectives, below are the main question and the sub-questions of the current study.

What are the effects of mathematical modelling and application on grade 11 learners' academic performance in algebraic function.

Sub-questions of the study

1. How different is Grade 11 learners' mean score in algebraic functions after being taught using mathematical modelling approach as compared to the mean score obtained through traditional teaching approach?
2. What are the Grade 11 learners' perceptions about mathematics after learning algebraic functions through mathematical modelling approaches?
3. What are the mathematics teachers' self-efficacy about applications of mathematical modelling?
4. What are the mathematics teachers' perceptions about the application of mathematical modelling?

1.6 Significance of the study

Mathematics curriculum in South Africa have been undergoing significant transformation due to continual underperformance of learners in mathematics subject, which then automatically generated the stigma that mathematics is difficult (Ngoepe, 2014) and a challenging subject (Tran, Nguyen, &Ta, 2020) for both teachers and learners. This underperformance has led to the introduction of mathematical modelling by the DoE (see DoE, 2011b) in mathematics, with a strong belief that modelling can change these negative perceptions about mathematics teaching and learning. However, learners' underperformance in mathematics is still a challenge. This study hopes to alert the department of education about the effects of mathematical modelling and its application on learners' performance in certain concepts of mathematics, such as algebraic functions. Findings may also help DoE be aware of gaps based on teachers' awareness of teaching and learning through modelling. Since all mathematics teachers should be geared to designing and preparing effective lesson plans that promote meaningful learning and a conducive learning environment that allows learners to discover methods to solve problems for themselves.

This study findings contribute towards emphasising the necessity to elevate teachers' professional development workshops that are emphasising teaching through mathematical modelling aiming to enhance teachers' self-efficacy through the setting and provision of mathematical modelling-based teaching and learning support materials. Hence, these study findings may also benefit teachers of mathematics in doing reflections, evaluating the teaching methods they use, and adapting to a proposed instructional approach (modelling) in mathematics education. Furthermore, findings contribute to inform curriculum developers to make mathematical modelling an independent topic in mathematics and be emphasized in the Annual Teaching Plan (ATP). This study intervention helped learners expand their cognition and develop flexibility to generate and solve models. In addition, the findings of this study may have great effects on encouraging schools to apply mathematical modelling to improve learners' performance in mathematics education.

1.7 Conclusion

This section provided the background of the study, which included the existing negative perception that mathematics is a difficult and challenging subject for both teachers and learners. It's also provided the study's problem, which is about learners' underperformance in algebraic functions and mathematics education, where contributing factors are not clear, and that this study explored how much effect mathematical modelling with a guided discovery approach can have on improving learners' performance in comparison to the direct instruction method. The rationale, which entails contributions that this study may have in mathematics education, the main purpose, and objectives that the study aims to achieve, and the research questions asked for collecting and analyzing data towards achieving this study's objectives, were also provided.

1.8 The structure of the research report

This research study consists of six (6) chapters. The following is the format of all chapters' arrangements:

- **Chapter 1:** Provides a wide-ranging outline of the research study and introduces the study problem, rationale, significance, objectives, and research questions.
- **Chapter 2:** Is providing the reviewed significant literature to the study problem the researcher is exploring parallel to the research questions stated in Chapter 1 (see Section 1.5) above where the theoretical framework is stemmed.
- **Chapter 3:** Provides a detailed overview of the methodology for which the study was guided, from data collection up to data analysis, and a breakdown of all instruments used for data collection and data analysis.
- **Chapter 4:** Gives collected data analysis using instruments provided in the appendices table, guided by the methods that were used to guide the study.
- **Chapter 5:** Provides research findings, discussions, and analysis of the research study in accordance with the reviewed literature provided in the second chapter to support the study and answer research questions.
- **Chapter 6:** Gives an overview of the study's limiting elements, future study recommendations, and conclusions based on findings upon answering research questions to meat study's objectives.

CHAPTER 2: LITERATURE REVIEW

Introduction

This chapter provides comprehensive details about model, and modelling in mathematics education, the theory that guided this research study, and how the department of education views modelling in the mathematics classroom, including the principles guiding model-eliciting tasks. Mathematical modelling as an adopted teaching approach in mathematics education is guided by six (6) mathematics teaching and learning features and holds a strong belief that knowledge is self-constructible. Mathematical modelling significance was recognized and initiated by DBE in 2011 when the curriculum was changing from National Senior Certificate (NCS) to CAPS and trusted as one of the teaching and learning approaches to improve learners' performance in mathematics (DOE, 2011b). Characteristics of a learner who has acquired mathematical modelling sub-competencies are also included in this chapter. Modelling aims at producing learners who can construct models on their own under the teacher's guidance and solve problems related to a real-life situation. However, modelling shares common beliefs with constructivist theorists. How constructivist theorist beliefs are interconnected to modelling and the difference between constructivist, modelling and traditional learning environments are highlighted. This chapter also reveals the necessity of quality professional development workshops for teachers to enhance their effective mathematics teaching and learning, including how learners and teachers view modelling. Measures of quality teaching and learning in mathematics are also provided in this section.

2.1 Mathematical modelling

I used Ferri (2019) to define modelling as the process of transferring real-life situation into language of mathematics through selection, and used of variables, expressing the relationship among them, to form models using variables as well as to test models and their practices, in terms of equations and expressions that can be used to predict and solve future unknown real-life problems. Given that modelling is non-linear method of developing cognitive knowledge and skills of mathematics (Wess et al., 2021), it can be defined in different ways because is different from other types of modelling since is focussing on explaining processes where skills, concepts and symbols of mathematics are used such that mathematics and real living are interlinked to make connection of one another (Guzel, 2016). Furthermore, to mean mathematical modelling, Harrison, (2001) and Ornek (2008), also defined mathematical modelling as the scientific phenomenon that is used when creating or developing a model and it process entails the state and event to explain, use, and interpretation of a model generated. The mathematics education introduced modelling to enhance skills needed for learners to easily understand concepts in mathematics, understanding of maths problems, facts observations, formulation of models, solving and representation of problems resulting from situations drawn from real-life problems.

Considering the effects of modelling to enhance learners' interest in learning mathematics and improve learners' performance. Treagust (2002) mentioned that learners' ability to properly use and mathematize models requires teachers to have relevant skills in modelling to give learners adequate guidance. In this regard, it is very important that teachers are developed to acquire enough mathematical modelling knowledge and understanding. Some of the burning issues mentioned by Blum and Ferri (2009) include that the department of education is still lacking in emphasizing the use of mathematical models as well as modelling to teachers through relevant training programs, which contributes to the lack of modelling practices in schools.

The idea that mathematical modelling can improve learners' understanding of mathematical concepts, enable them to make sense of the real world through mathematics, and aid mathematics results (NCTM, 1989) gained attention from the department of education in South Africa (DoE, 2011b), which noticed the significance of employing modelling in all mathematics classes from primary to secondary schools in the newly developed or current curriculum (CAPS).

One of the reasons leading to curriculum reform is based on challenges in implementing the curriculum, making sense of the concepts before imparting knowledge of the content to learners through teaching and learning, and difficulties in finding relevant teaching methods that can improve the learners' performance in mathematics education. The low performance in mathematics education creates a devastating view of the future of the country since mathematics is recently recognized as a very significant subject and almost all human knowledge domains require the application of computational and conceptual understanding of mathematics (Atteh & Andam, 2019).

The fundamental goal that mathematics education hopes to achieve is to prepare learners through the enhancement of skills that are relevant to the current demands of life, which entail understanding, reasoning, and communicating mathematically as well as solving daily or real-life problems (DoE, 2016). This objective is to make mathematics meaningful to children and relevant to solving community problems instead of teaching it just as a subject for qualification. Mathematics without connection to reality may be meaningless (Geiger et al., 2022), develop passive participation, negative perceptions, and a negative attitude, and contribute to the current crisis of learners' underperformance in mathematics. Thus, mathematics connected to reality is believed to provide an active environment for learning, increase both learners' and teachers' interest in mathematics, and promote the development of critical thinking and logical reasoning skills (Atteh & Andam, 2019).

Teaching mathematics is not an easy task that one can do without engagement with relevant teaching theories and methodologies (Granström, 2006; Wess et al., 2021). since teaching methods used in classrooms have huge effects on learners' performance and achievement of set objectives in mathematics.

The adoption of mathematical modelling by the South African DoE (2011) into mathematics education is the gateway to enhancing learners' and teachers' interest in mathematics, making mathematics relevant to our daily practice activities since modelling tasks focus on moving from real-life problems to mathematical models, and the usage of mathematics results in solving existing problems in the real world. However, modelling is a hope of mathematics education to fight against the crisis of learners' underperformance (Durandt et al., 2022a) in mathematics, even though there is a gap in teachers' training based on mathematical modelling teaching that needs attention by the DoE. The conceptual framework that was purposively developed to give an overview of mathematical modelling steps and how modelling competencies can be acquired has been used by many researchers (Wess et al., 2021; Blum and Leib, 2007; Durandt et al., 2022; Julie, 2020; Mousoulides et al., 2008; Schukajlow et al., 2015). It was also adopted by the researcher in this study to explore the effects of modelling in algebraic functions on grade 11 learners' performance, not ignoring other contributing factors such as teachers' self-efficacy.

2.2 Theoretical framework

A theoretical framework is a formation that supports and guides all research activities, guides the researcher towards organizing and monitoring the structure of the research, and provides abstracts about analytical, epistemological, methodological, and philosophical approaches that the researcher will consider towards achieving the study objectives (Grant, Cynthia, Osanloo, & Azadeh, 2014). This research study is guided by the modelling cycle. The researcher's rationale to adopt this modelling cycle to pursue intervention in this study was rooted in developing learners' modelling competencies, understanding, skills, and abilities to translate real-world problems into mathematics language using mathematical aids, which is one of the central objectives of modelling in mathematics teaching and learning.

The researcher's purpose in using this modelling cycle was based on demonstrating modelling processes to learners, aiming at making it easy for them to develop mathematical models from unstructured real situations or problems that describe the problem in mathematical language with as much accuracy as possible. However, the modelling cycle process described by Blum and Leiss (2007) consists of seven (7) steps that one should understand to be able to do modelling tasks. The process begins with a real situation or problem that entails a realistic problem situation that requires the aid of mathematics to process and construct a cognitive situation model based on the knowledge, objectives, and interests of the modeler. Simplifying, structuring, and clarifying the problem leads to the development of a real model, where the mathematization process is translating relationships and assumptions from the real model into a mathematical model. When a mathematical model is constructed, mathematical methods are used to mathematically solve the problem to yield mathematical results that will undergo interpretation in relation to the real-world problem (Greefrath & Vorholter, 2016). Thus, the whole process needs to be repeated using an adapted model to validate the entire process. Hence, this modelling cycle gives necessary steps for learners to consider not in a linear method but serves as guidance to enable learners' translation between real-world and mathematics in both directions (Ferri, 2019). Blum & Leiss (2007) proposed a modelling cycle represented in the figure below, which shows seven (7) stages that one should follow to adequately address the mathematical concepts through modelling.

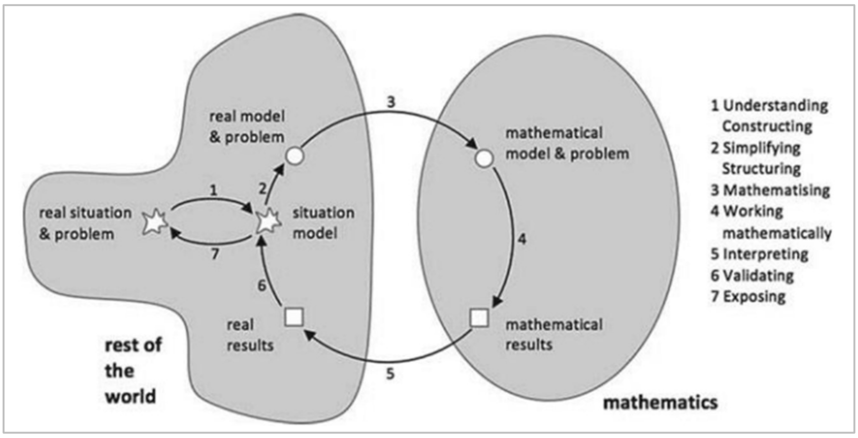


Figure 2.1 Modelling cycle by Blum and Leiss (2007, p. 225)

The following example 1 illustrates how learners would carry out a modelling task following a modelling cycle process.

Example 1: "Farming structure"
A farmer has 100 meters of wire fencing from which to build a rectangular chicken run. He intends to use two adjacent walls for sides of the rectangular enclosure. Determine the dimensions which will give a maximum enclosed area.

Firstly, learners need to understand the problem situation, which is a situation model they are required to construct. Thereafter, learners need to simplify and structure the situation more precisely, leading to the formation of a *real model* of the situation. Particularly, learners must define what "dimensions and enclosed area" should mean. In the standard model, in this case, dimensions mean "the perimeter of a rectangle" and the enclosed area, "the area of a rectangle." *Mathematizing* translates the real model into a mathematical model, which in this case entails certain equations. *Working mathematically* (solving the equations, calculations, etc.) generates *mathematical results* that require *interpretation* in relation to the real world as real results, resulting in a recommendation for the farmer on what to do (either building a rectangle or square) depending on the dimensions acquired. *Validation* of these results may indicate that it is important to consider repetition of the process (the modelling cycle), for instance, considering factors such as the shape and cost of fencing wire. Depending on the factors considered, the recommendation may be completely different. Where the teacher had to enable learners to develop modelling competencies and skills to work independently towards the construction of knowledge, process constraints, performance dashboards, and prompts were adopted from guided discovery principles (see table 3.3, p. 61) to implement modelling successfully as well as achieve the study set objectives.

This whole process of modelling begins with real-world situations that entail the authentic problem state, where the aids of mathematics are used to process them (Niss & Blum, 2020). From a real-life state, a cognitive model is generated based on the modeler's interest, aim, and knowledge. The derivative of correlation, assumption, and formulation of hypotheses about a generated real-life model or problem specification results from mental representation resulting from the clarification, structuring, and simplification stages. However, this is cognitively demanding and challenging for learners since Wess, Klork, Siller, and Greefrath (2021) mentioned that in the modelling process, every step has a potential cognitive barrier. Therefore, it is important for the teacher to lay conducive grounds for learners to develop modelling competencies adequately to increase their level of cognition.

Modelling cycle in figure 2.1 has been used by many researchers (Conmfrey & Maloney, 2006; Blum, 2015; Durandt, Blum & Lindl, 2021; Ferri, 2018; Wess, Klork, Siller & Greefrath, 2021) in mathematics education to make sense of mathematical modelling and its rationale in mathematics teaching and learning. However, since the aims and objectives of mathematics and those of mathematical modelling are not merely cognitive in nature (Durandt, Blum, & Lindl, 2021), they have no linear approach towards teaching and learning practices. Introducing learners to modelling cycle is essential for understanding the nature of modelling tasks, preparing learners' minds, and directing learners' focus to important modelling aspects since learners not only gain competencies and knowledge but also acquire relevant mental attitudes to do mathematics tasks, particularly mathematical modelling tasks, confidently.

In mathematics education, the department of education (DoE) has a goal to achieve through modelling which is the improvement of learners' performance in mathematics.

2.3 Learners' performance in algebraic functions

Algebraic functions as a topic in the grade 10 and 11 mathematics curriculum contribute 30% of the curriculum and 17% of the grade 12 syllabus (DBE, 2013). The effects of algebraic functions are not only affecting grade 11 learners' performance in mathematics but also affect the background knowledge required in grade 12, which also affects the overall learners' performance in mathematics. The DoE (2021) diagnostic report revealed that learners' performance is about 48% in algebraic functions. The Department of Education is emphasizing that teaching should enable learners' skills required for simplifying algebraic functions with understanding, logical reasoning in step presentations, and confidence (Baidoo, 2019), but algebraic functions are still one of the most difficult conceptions for learners in the mathematics curriculum. In my experience of teaching mathematics. I have noticed that the direct instruction method is dominantly used in mathematics classrooms over other teaching methods, but still, learners are not performing as expected and have challenges understanding, simplifying, and solving problems in algebraic functions in grade 11, and these challenges also impact grade 12 learners' performance. Hence, the study carried out by Baidoo (2019) with grade 10 mathematics learners in the Eastern Cape, South Africa, revealed that the teaching method used also impacts learners' acquisition of knowledge in algebraic functions.

When teachers give learners numerous similar problems to solve, the teacher discovers that many learners are unable to find solutions. While learners strongly believe that they understand the concept, instead, they are remembering a set of memorized facts and rules (Jojo, 2020). However, teaching methods entailing repetition of lessons to build concrete understanding of the concept do not enhance problem-solving skills; instead, they promote memorization without understanding, and the learner cannot be able to apply the same rules in different contexts (Baidoo, 2019). In that regard, I conducted this research study to explore the effects that modelling, and its application may have on learners' performance in algebraic functions compared to the direct instruction approach, since the modelling approach is one of the methods introduced by the DoE in mathematics education to supplement mathematics teaching strategies and support teachers in improving learners' performance, which may also contribute towards changing the perception that mathematics is a difficult and critical subject.

2.4 Learners' perceptions about mathematics learning

Perceptions can be defined as perspectives towards the expression of understanding a certain object or subject matter overview (Zaim et al., 2020). Perceptions develop under the influence of certain conditions, actions, the trend of certain behaviours or results, common beliefs, and continuous situations, and perceptions are the work of an individual's mindset due to contextual factors (McCoy et al., 2022; Meeran & Van Wyk, 2022; Zaim et al., 2020). It is my belief that teachers need to know and understand their learners, not only their names and surnames, but also their wellbeing, background knowledge of mathematics, attitudes, beliefs, misconceptions, and perceptions they (learners) hold about mathematics. Once teachers know and understand their learners, teaching and learning immediately begin. Learners are found to hold a strong perception, assuming that mathematics problems should be solved in very few steps using short-cut methods with a prominent goal to obtain correct solutions (Durandt et al., 2022b; Sanusi et al., 2022; Schindler & Lilienthal, 2022). Another perception learners hold is that receiving mathematics knowledge is for them (learners), and the teachers' role is knowledge transmission to learners and ensuring that all learners grasp that knowledge (Mutodi & Ngirande, 2014). These types of perceptions hinder learners from understanding that solving various mathematical problems requires many different approaches or alternative methods towards solutions, various approaches to defining concepts, and various constructions resulting from various steps taken in the beginning. It was found that many learners are relating mathematics with calculations as difficult subjects that are only for the gifted; mathematics abilities are transferrable; and more males than females believe that mathematics tasks entail processes that are not connected to the real world, such as problem solving and self-discovery (Alsina & Salgado, 2022). However, such perceptions lead learners to accept their lack of mathematical understanding as a permanent condition over which they have no control (Schindler & Lilienthal, 2022). Thus, understanding learners' perceptions may be one sign of teachers' self-efficacy in teaching mathematics.

2.5 Teachers' self- efficacy

Self-efficacy belief is about evaluating someone's efficiency in a particular area (Wess et al., 2021). However, teachers' self-efficacy is relatively concerned about evaluation of teachers' abilities to achieve set objectives for learners' learning and commitment, including learners that have difficulties or are not motivated to learn (Moran & Hoy, 2001; Tseeke, 2021). These abilities play a significant role in teaching as well as influencing learners' motivation, beliefs, and achievements. Thus, this is in line with promoting quality teaching and the adoption of mathematical modelling pedagogical knowledge (Geiger et al., 2018, 2022). Self-efficacy can also be viewed as connected to teachers' perspectives and his or her own efficiency when it comes to processes of mathematical modelling (Wess et al., 2021). To operationalize modelling in mathematics education, one of the teacher's fundamental tasks is the diagnosis of mathematical modelling processes. Following that the diagnostic element is associated with task-related and intervention knowledge aspects, the anticipation of self-efficacy about the evaluation of teachers' proficiency towards diagnosing learners' achievement potential and knowledge gain through processes of modelling can be operationalized (Borromeo Ferri, 2013; Durandt et al., 2022b; Geiger et al., 2018).

The process of modelling for learners consists of various tasks and processes of cognition in various stages (Chamberlin et al., 2022). Every stage of modelling that learners are presently working on requires various diagnostic processes to evaluate learner progress in each modelling phase (Ferri, 2018). However, this supports the belief that teachers' self-efficacy differs at each stage of modelling. Concerning learners' tasks as well as related diagnostics, it is not difficult to distinguish between phases that are specific and not specific to modelling process during the evaluation of modelling activities (diagnostic dimension). Therefore, self-efficacy about modelling in mathematics education is hypothesized to diagnose learners' accomplishment capabilities targeted for mathematical modelling and workings to solve given tasks. However, how teachers perceive mathematics also influences the methods they use to teach, the language they use to speak about mathematics, and the attitude and motivation of learners toward mathematics' significance (Mutodi & Ngirande, 2014; Zaim et al., 2020), as well as their perceptions of mathematics.

2.6 Teachers’ perceptions about mathematics

Perceptions can be defined as perspectives towards the expression of understanding a certain object or subject matter overview (Zaim et al., 2020). How teachers perceive learners' learning of mathematics has a huge impact on learners' understanding of concepts, teachers’ choice of teaching methods, and how the mathematics is defined, introduced, and presented to learners to either encourage or discourage learners from taking mathematics (Nieminen et al., 2021). Perceptions develop under the influence of certain conditions, actions, the trend of certain behaviours or results, common beliefs, and continuous situations, and perceptions are the work of an individual’s mindset due to contextual factors. One of the teachers' perceptions is that mathematics is difficult and is for only gifted learners (Meeran & Van Wyk, 2022). Such perceptions may deny learners access to knowledge basics, affect learners’ construction of knowledge, and make teachers focus more on learners who are scoring high marks in mathematics and who are active participants during teaching and learning as the ones who have mathematics genes, which may also make struggling learners lose interest in mathematics.

Teachers perceive mathematics as a stereotypically gender-based subject (McCoy, Byme, and O’Connor, 2022), where boys are regarded as the ones who do mathematics better than girls, and this perception is in line with the cultural perception that challenging activities are for males while easy or light duties are for females, since mathematics is perceived as challenging (Ngoepe, 2014) and a critical subject (Tran, Nguyen, & Ta, 2020) for both teachers and learners. These kinds of teachers’ perceptions affect female learners’ achievement of mathematics skills due to negative bias expectations of teachers that positively benefit boys in terms of receiving more attention than girls, which also affect the overall learners’ performance in mathematics. However, McCoy, Byme, and O’Connor (2022) revealed that teachers attribute learners’ underperformance who are girls to low abilities, while boys’ underperformance is due to less effort applied.

the impact of gender profile in mathematics, high performance in mathematics as genetic inheritance, I regard such perceptions as a teacher’s negative judgement and association of a subject’s hindrance towards improving learners’ performance, affecting adequate delivery of equal learners’ opportunities to do mathematics, and a barrier to building the mathematics foundation for all learners to develop the mathematics skills required since skills inequality may result from different backgrounds of mathematics exposure. In addition, such perceptions negatively impact learners’ attitudes, beliefs, competency, development of skills, and interests in mathematics, which all contribute to learners’ performance. Perceptions about mathematical modelling are that modelling may be the solution to changing negative perceptions, beliefs, and learners’ underperformance in mathematics (Chamberlin et al., 2022), and modelling beliefs about mathematics is important towards achieving the objective of elevating learners’ performance in mathematics education.

2.7 Beliefs about mathematical modelling

Mathematics teaching and learning methods transformation needs a thorough evaluation of teachers' beliefs (Lloyd, 2018; Mischo & Maaß, 2013; Zaim et al., 2020) as well as perspectives about what and how they teach (Patric & Pintrich, 2001). Specific beliefs about certain content areas can be specified or identified. As a result, Torner (2002) categorized teachers' beliefs into three hierarchical domains: globally held beliefs, domain-specific beliefs, and subject-matter beliefs. Global beliefs revolve around the enhancement of knowledge of mathematics. Domain-specific beliefs are more concerned with sub-fields of mathematics, and domain-specific beliefs are beliefs that focus on certain specific topics in mathematics, and the beliefs about mathematics terms such as expressions, mathematics objects like functions, and procedures of mathematics like simplification (Wess et al., 2021).

Mathematics teaching and learning beliefs are also affected by these above-mentioned beliefs, which involve classroom management, preferred methodologies of teaching, a set of objectives and goals, and epistemological beliefs that are about knowledge origin and frameworks. Mathematical model beliefs to change the stigma that mathematics is difficult and is for only gifted people are more rooted in the application aspects that relate mathematics' meanings and usefulness towards solving real-world problems (Wess et al., 2021). However, mathematical modelling epistemological beliefs can be conceptualized through modelling tasks practically and putting emphasis on the application aspect to make the goal of the DoE to take mathematics to the real world true and practical.

In addition, teachers' beliefs about teaching mathematics are differentiated into various approaches, which are "learner-focused approach," "content-focused approach with a core focus on conceptual understanding," and "content-focused approach with a core focus on performance" (Kuhns & Ball, 1986, p. 66). A teacher who has learner-focused beliefs normally views the teaching of mathematics as an intensive approach to constructing knowledge (Voss et al., 2013). A teacher whose beliefs are based on content-oriented beliefs depends on what is intended to be focused on, either to develop learners' proficiency to utilize procedures and rules of mathematics of the learned content or to promote conceptual understanding. Therefore, mathematics education modelling beliefs and objectives are aligned with teaching and learning beliefs.

2.8 Educational modelling

Educational modelling objectives are not only focused on the development of modelling competencies in mathematics but also on yielding competent children who will be able to serve the community with social and economic needs and to produce a generation who will meet the demands of dynamic life in this new era (Abassian et al., 2020). Julie and Muday (2007) gave emphasis to "modelling as a vehicle as well as modelling as content" in educational modelling since they emphasize the connection or correlation between mathematics and the real world as a very important component or domain that needs attention in mathematics classrooms.

Educational modelling attention for both mathematics and mathematical modelling competencies simultaneously may have a positive contribution in the development of modelling tasks that are well organized and not too complex to be dealt with as compared to disorganized authentic tasks mentioned in the realistic perspective of modelling (Stillman, 2019). However, the major objective of modelling in mathematics education is fundamentally focused on encouraging and motivating the necessity of learning mathematics (Abassian et al., 2020). The educational perspective on mathematical modelling has three main arguments that modelling can build a strong foundation for learners' good performance in mathematics, which include the following:

Firstly, modelling is a way to bridge gaps between mathematics and learners' everyday real-life experiences. Modelling is believed to be a powerful tool to motivate learners to do mathematics, direct the development of cognition skills by supporting enhancement of learners' mathematical conception, and is trusted as a fundamental approach that will place mathematics in a position of being a culture that describes and makes real life situations easily understandable (Durandt et al., 2022a; Julie, 2020; Niss & Blum, 2020; Stillman, 2019). Secondly, modelling is promising to develop learners' competencies to develop, analyse, think logically, and be able to criticize mathematics models instead of solving for correct solutions and understanding (Wess et al., 2021). This is the objective of modelling: to develop an adequately educated workforce that is needed to generate work opportunities in the community, solve the challenges of learners' underperformance in mathematics, and change the misconception that mathematics is a challenging subject. Lasty, modelling tasks set in a way that requires different levels of cognition or understanding following the modelling eliciting activities' guiding principles are believed to close that gap between reality and mathematics as well as be a powerful weapon to elevate learners' performance in mathematics (Stillman, 2017).

However, to teach mathematical modelling requires one who has undergone training on how to teach modelling (Atteh & Andam, 2019), even though it does not have a linear approach (Durandt et al., 2022a) that everyone can use, and also has the background based on how to develop modelling tasks in a manner that will improve learners' interest in doing mathematics, the development of modelling competencies, and the ability to make connections between mathematics done in class and real-world contexts (Julie, 2020).

2.9 Model eliciting activities

In the mathematical modelling perspective, it is strongly emphasized that the employment of modelling eliciting activities (MEAs) in mathematics classrooms may yield good outcomes for learners' cognitive development, logical reasoning, and critical thinking skills, which can help learners better understand and appreciate mathematics (Abassian et al., 2020). I believe that if these skills are well developed, they can improve learners' competencies in mathematics, which may result in improved mathematics results in algebraic functions, which also contribute to learners' performance in mathematics education. Linking mathematics with real-world daily activities is the main concern of the DoE. Thus, the real-world activities in modelling eliciting activities (MEAs) are developed purposefully, aiming at promoting development in mathematics. Mathematical modelling tasks are fundamentally designed to incorporate processes of mathematics into constructed or developed models to understand the real-life problem at hand and be useful towards making predictions of the new problems existing in the world (Chamberlin et al., 2022). For teachers to develop mathematical modelling tasks, they need to be competent in mathematical modelling and mathematics at large and have certain skills for modelling. However, modelling eliciting activities (MEAs) are guided by six principles, and teachers should have a thorough understanding of each phase or principle to develop modelling tasks that are coordinated with the levels of cognition of learners.

Table 2.1 Six principles that guide modelling eliciting activities (Abassian et al., 2020).

Principle	Requirement of a principle
Construction of a model	Explicitly constructing, describing, explaining, and justifying assumptions. Modelers should have acquired the ability of quantifying, coordinating, making assumptions or predictions, and identifying patterns.
Real world	The use of profound and appropriate tasks. Modelling tasks needs to be taken from reality or derived from customized real-world data. The modeler should be able to work towards solving the problem through making connection concerning reality and mathematics as well as how to use developed model that is directly constructed from the given real-world data.
Self-assessment	Selected tasks must promote selecting, refining, and elaborating constructed model using a well-defined criteria and well-defined objective of model construction from which the solution to the task is achieved. Modelers (learners) should be in position of detecting weaknesses, comparing other methods of finding solutions, integrating strengths, as well as minimizing deficiencies by the assessment of how models are adapted.
Model documentation	Learners must be given the opportunity to present their ideas based on how they developed model, and how they arrived at solutions highlighting giving overview of their hypothesis, developed objective while working towards solutions. This principle supports the development of metacognition.

Generalization or simplification	Models developed must be able to present how the problem can be solved and must not give definite answers to a particular context. Constructed model need to be clearly communicated and easy to be understood to allow other people to use it in other contexts to solve related problems.
Effective example	Models must not be complex to be understood and used but powerful enough to predict solutions. Models must have minimum steps to follow (avoiding many procedures) more specifically models that can avoid mathematics topics conceptual understanding.

Based on the principles of model construction, modelling tasks must make learners be able to explain the methods they used to develop models. After a model is constructed, to make predictions on new real-world problems in the same context, it is necessary that all modelling tasks be realistic and be derived from real-world activities. The realistic nature of modelling eliciting activities emphasizes that the settings of modelling tasks should be based on real-life activities, even though that does not necessarily mean real in an absolute sense (Borromeo Ferri, 2013). Realistic tasks: even though models may be derived from them (realistic tasks), validation and criticism are still required to be done by learners who developed those models (validation of their own work) as a requirement of the self-assessment principle. This is the same as the construction of a model principle, where constructed models should be described and justified. Documentation of a model also requires learners to give a detailed explanation or presentation of how a model was generated, not just produce a model without understanding the problem context (Schlesinger et al., 2018).

This is one of the techniques to develop learners' mathematical modelling competencies, which can assist in improving learners' performance in mathematics. Furthermore, the generalization principle is more concerned with how developed models can be used to generalize solutions mathematically in a similar real-world context or situation. Well, if the models developed are complex, meaningless, and not easy to understand, they may not be useful for generalizing situations in the real world. That is why effective example principles emphasize that models that are constructed must be easy; having few steps to solve a problem can, on the contrary, be a powerful tool of mathematics. However, these principles are in line with the modelling cycle (Abassian et al., 2020). Thus, for teachers and learners to correctly use this modelling cycle process, they need capabilities and skills from teachers to teach learners properly so that learners gain interest, confidence, and courage to solve mathematics problems. In other words, teachers should be competent in modelling for learners to understand modelling processes.

2.10 Mathematical modelling teaching

There are many experimental studies about mathematical modelling teaching and learning nowadays, some of which are closely related to my study (see Blum, 2015; Schukajlow et al., 2018; Stillman, 2015; Kaiser, 2017; Niss & Blum, 2020; Wess et al., 2021; and Durandt, Blum, & Lindl, 2022). The findings of these above-mentioned studies, which revealed significant general aspects, need to be examined to determine to what degree they can be used to generalize the situation in other teaching and learning environments since they are obtained from different cultural, societal, and organizational environments. The examination of findings is necessary before generalizations to other environments can be made because teaching and learning environments also have an impact on learners' gaining of mathematics competencies, skills, and knowledge (Brown et al., 1989), and learners' extent of acquiring mathematical modelling competencies is generally restricted by areas of mathematics and real-life context. Therefore, generalization needs serious attention and is carefully organized.

The highly significant international findings about mathematical modelling teaching indicate that teaching modelling is the same as other methods of teaching mathematics (Durandt et al., 2022b; Stillman, 2019), but it is effective only if fundamental measures of quality teaching are performed or observed. However, this does not mean that there is a linear method of teaching mathematical modelling; there are certain measures that are relevant for the achievement of modelling competencies, which include learning theories and methodologies. (Schlesinger et al., 2018) classified measures of quality teaching into fundamental quality teaching dimensions, and then Niss and Blum (2020) differentiated these measures into five quality teaching dimensions as follows:

2.10.1 Effective classroom management

This element consists of measures like developing a clear structure for lesson planning, efficient usage of timeframes, a clear difference between assessments and learning to avoid interruptions and confusion, the use of various techniques, and flexibility. However, such measures are not applicable to all subjects since teaching mathematical modelling is not linear, and for learning to happen successfully, effective classroom management is important even though it alone is not enough (Durandt, Blum, & Lindl, 2022).

2.10.2 Learner orientation

This aspect entails measures like adaptive progress, making connections between learners’ prior knowledge and new content, logically using proper mathematical language, diagnosing gaps and strengths, providing feedback and individual assistance, and using learners’ errors positively and constructively to open a room for learning opportunities so that learners gain confidence and interest in participating, as well as encouraging independent working towards finding solutions to the given tasks. However, in an independence-guided environment for learning, learners who came out with many different approaches to a solution are considered learners who acquired modelling competency skills (Schukajlow et al., 2015b).

2.10.3 Activate learners' cognition

To activate learners' cognition means to stimulate learners' mental tasks through the maintenance of constant stability among teachers' guidance and learners' independence (Aebli, 1985), and to avoid improper contrasts such as learners' individual work against teachers' guided instructions (expository learning versus direct instruction). Adaptive teachers' intervention is one of the key elements that support a constant stability among teachers' guidance and learners' independent working as it enables learners to proceed working alone without losing focus, understanding of the concepts they are dealing with, and independent working, more specifically strategic interventions (Blum, 2011; Stender & Kaiser, 2016). Bakker, Smit, and Wegerif (2015) considered adaptive intervention as a unique paradigm of scaffolding in relation to mathematical modelling tasks as the method to interrogate learners' logical reasoning when dealing with modelling activities such as "read the text carefully!, imagine the situation clearly!, what do you aim at?, what is missing?" "Are the results making sense for the real situation?" These forms of questions promote learners' logical thinking and the development of a high cognitive level of thinking, which may help to answer any real-world-based problems.

2.10.4 Activate learners' meta-cognition

To activate learners' meta-cognition, it is about stimulating supplementary and reflective measures of learners' individual learning, improving individual learning as well as working approaches. There are many studies that empirically reveal that using mathematical modelling strategies to teach mathematics has a positive impact on learners' development of knowledge and competency in mathematics learning, such as Niss & Blum (2020) and Vorhölter, Krüger, & Wendt (2019). However, Schukajlow et al. (2015) developed a solution plan that has four steps in the DISUM project and is to some extent different from the mathematical modelling cycle that is found in Wess et al. (2021), which was developed by Blum and Leib (2007).

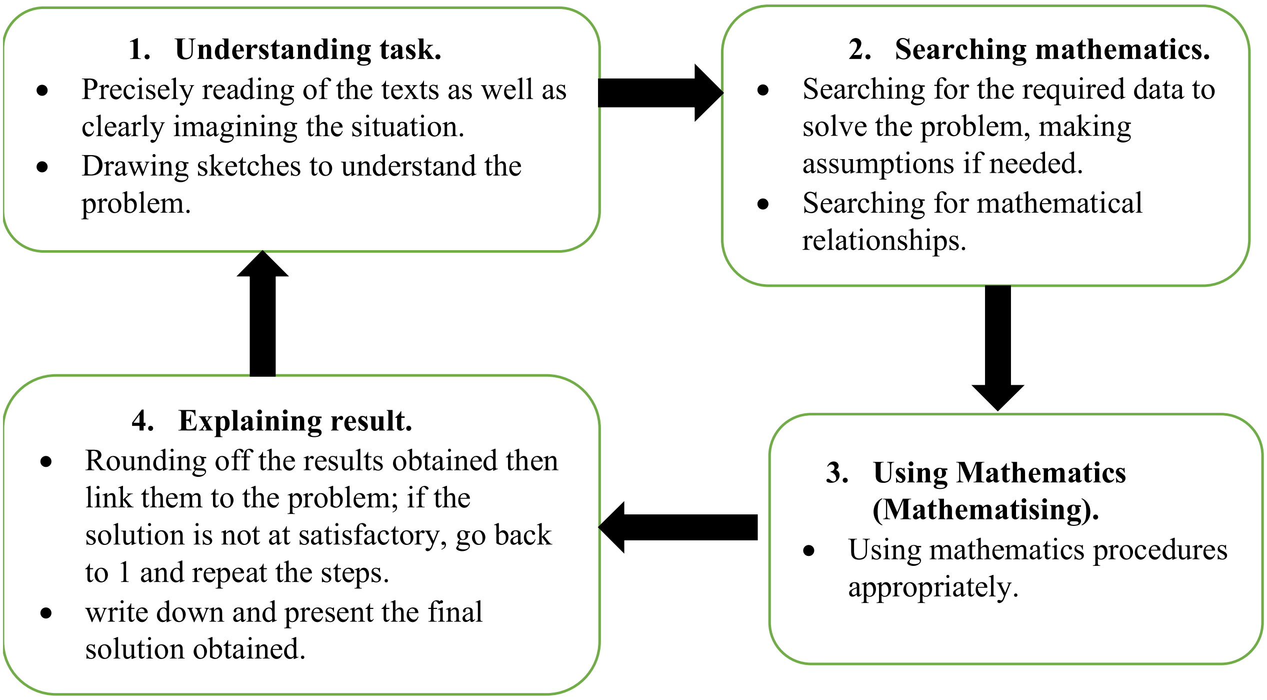


Figure 2.2. Adapted modelling cycle based on activities (Vorhölter, Krüger, & Wendt, 2019).

2.10.5 Topics demanding orchestrations

This is about creating enduring opportunities for learners' abilities that will help them (learners) develop, apply, and practice gained mathematical modelling competencies using skills such as developing tasks that promote learning with understanding, emphasis on justifying solutions with logical reasons, and the ability to draw links among topics and real-world situations. This form of quality teaching (modelling) in mathematics is introduced, but not all teachers are practicing it, not because of their resistance to using the modelling approach in their mathematics classes, but because of a lack of mathematical modelling skills.

There are many empirical studies conducted involving mathematical modelling interventions towards the improvement of mathematics results that compare the effects of different learning environments that emphasize various aspects based on quality teaching of mathematical modelling. Schukajlow (2018) conducted ten consecutive mathematical modelling intervention lessons in the DISUM project to compare two different methods of teaching mathematics, which include the operative-strategic method (an improved version of this teaching technique is method-integrative), which is also known as the independence-oriented teaching approach, and the directive method, which is also known as the teacher-guided approach. These teaching and learning approaches were based on the linear function and the theorem of Pythagoras.

The central purpose of both these approaches (method-integrative and operative-strategic methods) is to ensure the enduring equilibrium among learners' self-reliance and teachers' guidance and encourage self-dependent solutions, whereby the directive method's fundamental focus is based on teachers guiding learners, giving direction to common routes of solving problems, and the development of a common pattern of solutions (Durandt et al., 2022a). In the study of Schukajlow (2018), using the method-integrative approach, learners were provided with a solution plan, and the teacher demonstrated how to use it and how to solve mathematical modelling tasks. Cognitive development findings from Blum and Schukajlow (2018) study show that all teaching and learning methods possess similar effects of significance towards learners' practical skills in mathematics; on the contrary, only both independence-oriented methods show an effect that is significant to learners' modelling competencies.

The classes that were taught using a method-integrative approach performed very well as compared to the classes that were taught using an operative-strategic approach in both modelling and mathematics. Another study carried out to compare two different types of teaching methods (learner-centered and teacher-centered) by Parhizgar and Liljedahl (2019) based on modelling tasks and word problems examined their influence on learners' attitudes toward mathematics. The study was also based on linear functions and Theorem of Pythagoras topics, but the sample or participants of the study were grade 10 learners. The findings revealed that learners mostly enjoyed word problems, whereas intra-mathematical questions were found boring and too easy, and lastly, mathematical modelling questions were seemingly too challenging to solve. There was no statistically significant difference found between learner-centered as well as teacher-centered approaches, even though learner-centered approaches produced a little more positive effects on learners' attitudes towards mathematics, which resulted from learners' groups' working enjoyment, whereas mathematical modelling questions had great positive effects on learners' perceptions of mathematics in learner-centered and teacher-centered approaches to mathematics teaching and learning (Durandt et al., 2022a).

2.11 Learning mathematics through modelling

One of the current curriculum CAPS in South Africa's specific aims is to recognize modelling significance in mathematics education (DoE, 2011b, p. 8) due to the need to improve mathematics academic performance and to apply content knowledge acquired into real-life contexts. The mathematical modelling-specific aims "content aims" and "process-oriented aims" are more relevant to operationalize and fulfil the curriculum specific aim. This is because content aims fundamentally focus on learners' abilities towards recognition and understanding real-life processes, whereas process-oriented aims focus on training learners so that they develop problem-solving skills, a high level of cognitive demands, and an enhanced interest in mathematics (Greefrath & Vorholter, 2016).

2.12 Curriculum and Assessment Policy Statement (CAPS)

CAPS is the current curriculum in South Africa, initiated in January 2012 after the review of the previous curriculum known as NCS due to difficulties in implementation, which include unclear and complex terminologies as well as a lack of adequate training for both district officials to manage the curriculum delivery properly and teachers who should deliver the curriculum properly (Jojo, 2020). In South Africa, education reform has been a continuous activity after a certain period if the curriculum at hand yields results at an unsatisfactory level. The Outcome-Based Curriculum (OBE) was initiated in 1997, aiming at closing the gaps and overcoming the division of the past in the curriculum. This curriculum (OBE) was then revised in the year 2000 due to learners' poor performance in mathematics Grade 12 results that were speculated to be caused by inadequate implementation by teachers due to a lack of conceptual understanding, which led to the implementation of a new curriculum (Revised National Curriculum Statement) for lower grades and a National Curriculum Statement for higher grades (DoE, 2011).

CAPS is believed to be the underpinning philosophy of outcome-based education (OBE). The CAPS curriculum is being used in practice to redress the inequality and imbalance of the previous curriculum. CAPS was initiated with the objective of replacing the subject statement, learning approaches, learning programs, and subject guidelines across all phases (grades R–12). The CAPS document is composed of general aims coupled with specific aims of the curriculum's implementation. Some of CAPS' general aims involve producing learners who have acquired the abilities of:

- Working with other learners collaboratively and working as individually well effectively.
- Managing and organizing themselves (learners), taking responsibilities of their given tasks and being effective and active participants when doing activities.
- Collecting, analysing, organizing, as well as critically evaluating important data.
- Using visuals, symbolic and language skills for different way of effective communication; and
- Recognizing that the context where real-life problems are solved is not existing isolated from other learning skills and it demonstrates the appreciation of the real-world problems and interpreting real-world set of working systems using skill acquired in learning.

The quantitative study carried out by Julie (2020) examining the competencies of mathematical modelling revealed that learners showed a very minimal level of competencies in mathematical modelling. Thus, there is a need to activate the learners' high level of mathematical modelling cognitive demands, which requires teachers who hold a "high level of pedagogical content knowledge," which enables them to give individual learners good learning support. Ferri (2018) mentioned abilities aspects and different dimensions that they are considering as keys to teaching mathematical modelling, which involves conducting mathematics lessons that are based on reality. For learners to learn modelling adequately, a teacher should have enough knowledge and abilities to identify various methods to administer modelling tasks (task-related dimension). However, this is rooted in the development of modelling tasks with a specific aim and cognitive analysis, which reduces the challenges of learning mathematical modelling.

Ferri (2010) also mentioned the teaching dimension as another aspect to activate learners' cognitive thinking skills that is characterized by a well-chosen, high-quality learning environment and the preparation and accomplishment of mathematics lessons guided by theories of knowledge that are related to real-life situations. Lastly, there is the "diagnostic dimension" (Wess et al., 2021), which is more concerned with teachers' ability to classify learners' modelling tasks according to their modelling cycle phases and the ability to identify barriers or challenges that learners experience when dealing with modelling activity. Therefore, for learners to adequately learn through modelling, teachers must have pedagogical diagnostic knowledge and skills and the ability to recognize and document learners' evolution, errors, complications, and inaccuracies that they have shown during the modelling phenomenon (Ferri, 2018).

In schools, the central goal to achieve is to promote the usage of mathematics aids towards processing problems adapted from reality or real-life situations (Wess et al., 2021). To scale up the learners' interest, performance, motivation, and positive attitude in mathematics using modelling is rooted in modelling competency enhancement. For a learner to appreciate translation from real context to mathematics and vice versa, they need skills that enable them to transform or make connections between mathematics and reality. If a learner has acquired modelling competencies, they should demonstrate the following sub-competencies proposed by Greefrath and Vorholter (2016), presented in the table.

Table 2.2 Sub-competencies of modelling (Greefrath & Vorholter, 2016, p. 19)

Sub-competency	Description
Understanding	Learners are self-constructing their own mental models for a specific problem state as well as understanding the question.
Simplifying	Learners identify and split up significant and insignificant data about real life situation.
Mathematising	Learners are translating appropriately simplified real problems into mathematical model such as terms, equations, figures, diagrams, functions, etc.
Working mathematically	Learners use exploratory strategies and mathematical comprehension to solve mathematical problems.
Interpreting	Learners can refer the results acquired from the model to the real situation and therefore achieve real results.
Validating	Learner has ability to evaluate the results obtained from a real model situation for satisfactoriness.
Exposing	Learner can represent the outcomes attained from the situation model to the real situation and therefore answers the questions.

Teachers are facing difficulties in the modelling process because many learners are struggling to read and understand the activity. However, this is not caused by a lack of reading skills (Plath & Leiss, 2018), so learners are used to working and solving mathematics problems without reading and understanding the context (Blum, 2015). One of the factors is that the teaching techniques used in the classrooms do not give learners the opportunity to explore or do tasks on their own without teachers' assistance and step-by-step explanations to solve a given problem. Thus, the traditional teaching method is more dominant than modelling method of teaching and learning in mathematics classrooms.

2.13 Difference between traditional and modelling class

Herbst (2003) mentioned that teachers are providing learners with a step-by-step (routine) method to solve problems instead of giving learners the opportunity to try solving problems for themselves. In the presence of routine methods of teaching that lasted for a long period of time, there has been unsatisfactory learners' mathematics academic performance in South Africa, and this crisis has lasted for more than two decades and is still existing (Van Jaarsveld & Ameen, 2017). Hence, learners end up waiting to be provided solutions by their teachers. In addition, teachers explain the problem to learners and tell them the direct methods to find solutions. The teaching method that is more traditional or more teacher-guided is said to be the teacher-directive method, and this method is dominant in nowadays' mathematics classes in the South African context as well as in other countries (Durandt et al., 2022a). There are principles that guide the teacher-directive (direct instruction) and mathematical modelling (method-integrative) approaches to teaching mentioned by Durandt et al. (2022). The following are the key principles of the teacher-directed approach:

- The teacher is the one who is responsible for developing task patterns and common solutions (a memorandum or marking guidelines).
- systematically change among learners' doing tasks alone and teaching the whole class (in Favor of average learners) with similar activities.

Whereas in the modelling class, learners are provided the opportunity to attempt tasks first, and thereafter teachers intervene and assist. Breen (2010) mentioned that if the teacher establishes that it is the learners' responsibility to provide strategies and methods to solve a given problem, then teachers are facilitating and helping without giving the learners answers. Durandt et al. (2022) mentioned that an effective classroom that promotes learners' mathematical modelling competency is guided by the following principles:

Firstly, teacher's support focuses on learners' activeness and working on activities independently, maintaining enduring stability between learners' independence and teachers' support, as well as encouraging learners to construct or develop solutions as individuals. Secondly, understanding and maintaining the stability of learners' independence and teachers' support through adaptive teaching interventions, specifically intentional, organized, and planned interventions. Thirdly, change strategically among learners working independently in groups and whole class tasks, which includes comparing learners' discussions and individual solutions, giving learners the opportunity to present their solutions, and reflecting on learners' workings. Lastly, solution planning as a form of metacognitive support may be used to support learners in working on tasks, but not as a linear method that learners should follow to find solutions.

The mathematical modelling approach is more concerned with learners' self-construction of knowledge than the teacher being the only source of knowledge, where learners become knowledge receivers and passive participants in their learning. In modelling, the teacher becomes a facilitator of learning (Ferri, 2019); his or her understanding comes after learners have presented their own understanding of the concepts at hand. This is closely related to constructivist theory, whose beliefs about learning are rooted in the notion that knowledge, understanding, and meaning are the personal constructions of one's experience and exposure to the real-world context.

2.14 Constructivist Theory of Learning

This learning theory is fundamentally rooted in observing how learning processes occur and how learners learn and acquire knowledge. Constructivism theory believes that everyone constructs his or her own knowledge and meanings of the real world and develops the ability to understand the real world through individual personal experiences of things and reflections on each experience encountered (Bereiter, 1994). Constructivism theory believes that learners are active knowledge self-constructors and understanding agents instead of knowledge receivers and passive listeners. The modelling approach is also rooted in the notion that learners should construct models for themselves, which also leads to their own knowledge construction (Ferri, 2019).

Constructivism theory also puts more emphasis on teaching and learning in mathematics classrooms, which must not be based on one method but employ numerous different teaching techniques. However, not just any different teaching techniques or methodologies are available in the mathematics education environment, but approaches that will encourage learners to construct more knowledge through engagement in active experiments, problem-solving tasks based on real-world problems (Niss & Blum, 2020), and the ability to do reflections on that self-constructed knowledge, as well as giving learners the opportunity to present their work so that the teacher can facilitate how learners' level of understanding has changed.

During teaching and learning practice, one of the teachers' responsibilities is to ensure that they (teachers) have enough understanding of learners' pre-existing conceptions and misconceptions so that teachers can be able to guide learners' activities towards addressing them (pre-existing conceptions and misconceptions) and then building learners' understanding of them (Oliver, 2000). However, the central perspective of constructivist theory is that learning, and knowledge are personal constructs; learners construct knowledge and understanding upon underpinning previously acquired or learned knowledge (Geofrey, 2021).

A constructivist learning environment promotes learning where learners are exposed to the material for experimentation to gain their own experiences and derive meaning and understanding for themselves. The central purpose of exposing learners' self-discovery environments is to promote active learning (Atteh & Andam, 2019; Tam, 2000). Thus, mathematical modelling and constructivist theory share the common objective that the learning environment should allow learners opportunities to construct their own knowledge and understanding, where the teacher facilitates learners' construction of knowledge and promotes active learning.

2.14.1 Constructivist theory educational beliefs

To conceptualize beliefs of mathematical modelling and those of mathematics teaching and learning, these two beliefs were differentiated by Woolfolk, Davis, and Pape (2006), and the learning of mathematics beliefs emphasizing that learners are self-constructors (constructivist-oriented belief) of knowledge and learners as receivers (transmissive-oriented belief) of knowledge from the teachers as the only source of knowledge are contributing towards both mathematical modelling learning and traditional learning beliefs, respectively (Voss, Kleickmann, Kunter, & Hachfeld, 2013; Wess et al., 2021) The table below demonstrates the differences between two scales of learning mathematics: constructivist-oriented beliefs and transmissive beliefs, which contribute more to mathematics education.

Table 2.3 Constructivists and transmissive beliefs in mathematics education (Wess et al., 2021)

Constructivists-oriented	**Transmissive-Oriented**
Beliefs are grounded on the application of mathematical modelling: - Various mathematical modelling domains emphasize practical applications or direct applications references.	Beliefs about transmission of knowledge: - The teacher who is effective in teaching and learning of mathematics is the one who demonstrates methods and approaches towards solving application problems. - Teachers are the knowledge pourers whereby learners are knowledge receivers.
Beliefs that are based on classrooms practising mathematical modelling approach: - The emphasis is that mathematical modelling must be the one of the approaches to be used in mathematics education.	
Beliefs about knowledge construction: - Learners are well learning mathematics if are exposed to the environment where they discover methods of solving problems for themselves.	

Constructivist beliefs about learning mathematics through a modelling approach have been classified as the approach with the most positive effect on promoting modelling competencies in both teachers and learners. In the study carried out by Voss, Kleickmann, Kunter, and Hachfeld (2013), they also adopted these constructivist and submissive beliefs in the COACTIV research about mathematics teaching and learning after they emerged from Staub and Stern (2002). Beliefs based on knowledge construction emphasize the perspective that learners need to be given opportunities to discover approaches to solving problems for themselves, develop the ability to work as individuals and independently, and be given the opportunity to present processes and solutions to solving problems. In contrast, constructivist beliefs support learning through modelling in the mathematics classroom, whereas transmissive beliefs seem to oppose and argue against modelling in mathematics education (Schwarz, Wissmach, & Kaiser, 2008).

2.14.2 Constructivist learning environment fundamental characteristics

The constructivist learning environment is characterized by the consideration and implementation of the four fundamental teaching and learning constructivist strategies proposed by Tam (2000).

1. The teacher and the learner are sharing knowledge.
2. The teacher and the learner are sharing teaching and learning authority.
3. A teacher's responsibility is to facilitate as well as guide learners' active engagement in activities and learning.
4. Setting of learning groups in such a way that all groups are made up of heterogeneous learners in terms of cognitive levels, avoiding having too active and too passive groups of learners who cannot develop one another.

2.14.3 The learning environment pedagogical objectives for constructivist learning

There are seven (7) proposed learning objectives directed to the teaching and learning environment of constructivism theory.

- Providing experiences through processes of knowledge construction (learners are the ones who are determining)
- Providing experiences and understanding for several perceptions (evaluating multiple and alternate solutions)
- to provide learning that is embedded in a realistic context (authentic activities).
- Encouraging learners' voice and ownership of the processes of learning (a learner-centered approach to learning)
- ensuring that learning is embedded in social experiences (collaboration).

- encouraging the use of various kinds of lesson presentations (videos, audio text, etc.).
- Encourage the recognition of phenomena used to construct knowledge (reflections and meta-cognitions).

2.14.4 Constructivism theory of learning benefits

The difference between constructivist and traditional classroom

Constructivist classrooms are more focused on the movement from teacher-centered learning to learner-centered learning. In constructivist classrooms, there is no point where teachers are the only experts who transfer knowledge to passive learners who are waiting as blank-minded "empty vessels" for teachers to fill with knowledge. Instead, during learning processes, learners are forced through teaching and learning strategies to become active participants, whereas teachers' responsibility is to become facilitators who are coaching, mediating, prompting, and scaffolding learners towards developing and assessing their understanding as well as their learning. Furthermore, in the constructivist's classrooms, both teachers and learners view knowledge as a dynamic, continual change perspective of the real world as well as their effective capability of stretching and exploring the dynamics of the world we live in, not as inert facts for memorization (Bada, 2015). The following table demonstrates the difference between traditional learning approaches and constructivist theory perspectives about learners, learning, and knowledge.

Table 2.4 Constructivists and traditional classrooms for learning (Bada, 2015, p. 68)

Constructivists classrooms	Traditional classrooms
The curriculum puts more emphasis on large learning concepts, starting with the whole as well as expanding by including the parts.	The curriculum emphasizes basic skills and starts with the parts of the whole.
Learners' interest and questions are valued	Fixing the curriculum is greatly valued.
Teaching and learning is rooted in learners' interactions and the construction of knowledge based on their prior knowledge.	Teaching and learning focus on the repetition of concepts.
Teaching and learning resources entail manipulative materials and their primary sources.	Teaching and learning materials are basically workbooks and textbooks.
The teacher interacts with learners and assists learners in the personal construction of their own knowledge.	The teacher is the expert who is disseminating information to learners, and learners are passive knowledge receivers.
The role of the teacher is to be interactive, and learning is grounded in negotiations with learners.	The role of the teacher is to be directive, and learning is grounded in the fact that teachers are the only authorities on knowledge.
Learners' assessments involve learners' work, observation, perspectives, and tests. The processes towards obtaining correct solutions are as important as the solutions.	Learners' assessments are based on testing for the correct solution.
Knowledge is continuous, changes through our daily experiences and is dynamic.	Knowledge is unique and inert.
Learners are primary working in groups.	Learners are primarily working alone.

Due to these differences in teaching methodologies to transcend from traditional to modelling classes and the demands of modelling tasks, training for teachers to conduct modelling classes properly is important. Because the correct implementation of modelling tasks that enhance the development of high levels of cognition will benefit learners, poor implementation may limit learners' growth in thinking capabilities.

2.15 Teachers' effective mathematical modelling training (TEMMT)

The mathematics curriculum that is taught in schools has undergone numerous changes over certain periods due to curriculum developments aimed at meeting the issues and demands of society at that time. This curriculum evolution demands teachers' empowerment programs to develop new skills in line with the curriculum demands at hand (Atteh & Andam, 2019). These empowerment programs play a significant role in preventing teachers' limited understanding of concepts from becoming barriers to learners' learning of mathematics.

New curriculum development also affects mathematics teaching and learning methodologies, meaning that teachers' professional development workshops must be in place; otherwise, learning will promote learners' passive participation, increasing algorithm memorization, and increasing the power of teachers to become knowledge disseminators instead of developing a learning environment where teachers are facilitators and learners are knowledge constructors (Mousoulides et al., 2008). This means that teachers need to acquire comprehensive knowledge about the present content of mathematics, different topics, pedagogical principles, and learning support materials informing the scope of work. Well, teachers' professional development workshops help them gain teaching and learning skills that are relevant to the new curriculum at hand.

2.16 Teachers' professional development workshops

Why are teacher development workshops so important in mathematics education? If teachers' knowledge is limited to concept definitions, teaching and learning may not be as productive as expected. Thus, teachers also need to understand concepts by having adequate content knowledge for subject matter (SMCK), pedagogical content knowledge (PCK), and curriculum knowledge (CK) (Atteh & Andam, 2019).

2.16.1 Subject matter content knowledge

The SMCK is the knowledge capacity or composition that teachers have in their minds (Shulman, 1986). The SMCK has no need to be restricted only to facts, knowledge, and certain procedures to follow when solving the problem instead of having enough understanding. Clearly understanding the subject matter enables teachers to use the teaching and learning support material given properly and to make corrections on mistakes that learners may have by considering, analyzing, and justifying their solutions (Atteh & Andam, 2019). How teachers use teaching and learning support material, methodologies they are using when teaching and assessing learners' progress, and mathematics concepts are sequenced in consecutive lessons they are teaching to develop learners' understanding is basically rooted in their content knowledge of mathematics. Asiedu-Addo and Yidana (2004) believe that teachers' methods of teaching mathematics may be more dynamic, lessons may be presented in various effective ways, and learners may be encouraged and given adequate responses to their questions and comments if teachers' knowledge is expressed more explicitly and if teachers are well equipped with relevant teaching skills for the curriculum at hand (Atteh & Andam, 2019; Eroğlu & Donmus Kaya, 2021). Conversely, if teachers have limited mathematics content knowledge, their methods of teaching will only rely on textbook examples, teaching concepts as the only facts that learners should know, limiting learners' ideas, questions, and comments, not giving learners the opportunity to work for themselves trying to solve the problem, and relying on memoranda or rubrics to evaluate learners' workings and solutions without focusing on how a model is constructed and solved (Ball, Hill, & Bass, 2005).

Lack of adequate content knowledge of the subject may be dangerous and lead to learners' underperformance in mathematics education because, if teachers lack CK, they may not be able to explore different teaching techniques or methodologies due to inferiority and mislead learners. Even if learners are producing correct solutions using their self-discovered method, their solutions will not be considered valid. Teachers' subject matter knowledge deficiency makes it practically impossible for the teacher to set a conducive and effective environment for learners to learn as well as teach mathematics effectively.

2.16.2 Pedagogical content knowledge

Pedagogical content knowledge (PCK) is concerned with how subject matter is presented to learners meaningfully in various ways that promote learners' active participation and their clear understanding of the concepts. PCK is about how pedagogies of teaching and learning and mathematics content are combined effectively (Shulman, 2000). PCK entails teachers' knowledge of adequate methods to be used to address certain content or concepts since there is no linear method to teach all mathematics concepts (Wess et al., 2019), and knowledge of how to arrange content elements in an easy way for learners to follow to avoid confusion during teaching and learning.

I believe that this kind of knowledge (PCK) also entails teachers' understanding of strategies of teaching that can make learning of a particular topic difficult, understanding of preconceptions and conceptions that learners from different backgrounds or environments of learning may have, and also understanding that learners' levels of cognition at different ages and grades are not the same, meaning that teachers should be able to identify teaching approaches based on learners' levels of cognition without reducing learners' opportunities to learn or construct knowledge for themselves. However, PCK promotes teachers' elevated level of anticipating learners' learning challenges and providing immediate support to remove those learning barriers (Andam, Atteh, Obeng, & Denteh, 2016). This means that teachers have to know the weaknesses and strengths of all learners they teach individually, have the ability to identify mathematical modelling activities enhancing learning through understanding instead of learning to produce correct solutions, activities that develop learners' positive perceptions towards mathematics, and have the ability to identify effective strategies to teach mathematics effectively that are not promoting routine learning (Andam, Atteh, Obeng, & Denteh, 2016) and those strategies that may form part of improving mathematics quality teaching and learning.

2.16.3 Curriculum Knowledge

Curriculum may be defined as the composition or package that involves learners, teachers, content, concepts, instructional practice methodologies, skills, beliefs, knowledge, and a set of objectives to be achieved in line with the social and economic needs of that era (Smith & Morgan, 2016). Curriculum is the composition of organized and channelled or directed learning proficiencies as well as envisioned objectives developed following methodical systems of knowledge reconstruction and experiences based on schools' support systems for learners' continuous development in personal and social competencies (Atteh & Andam, 2019; Mareku & Agbemaka, 2009). More specifically, the curriculum for mathematics consists of structured programs developed specifically to teach mathematics topics per grade, with set objectives to be achieved per topic and per grade level.

It is very important for all teachers to undergo curriculum development workshops or training sessions where they can be equipped with adequate and relevant skills for teaching mathematics concepts. Since curriculum development is an ongoing process, the curriculum changes continuously after a certain period, even when teachers are not yet ready to teach the present curriculum. Training based on mathematical modelling should be taken into consideration to equip teachers with modelling competencies. Minding the gap between 2011 when CAPS was introduced with modelling as a specific aim to the curriculum and the year 2022 (almost eleven (11) years), which is above a decade period, in my experience of teaching as a qualified mathematics teacher, there are still no specific professional development workshops in place for training teachers, and there are no teaching and learning support documents provided to schools by the DBE towards supporting mathematical modelling teaching and learning; hence, learners' performance in mathematics is still a pandemic.

Lack of resources available for teachers and learners to teach and learn mathematics through modelling, respectively, may limit the level of improvement in mathematics performance. For the teacher to be able to perform quality and effective teaching of mathematics, they must have acquired substantial compassion and knowledge of the available present curriculum teaching and learning materials so that they can make them available for learners to learn concepts from themselves during and after teaching and learning sessions (Atteh & Andam, 2019). One of the focal points of the mathematical modelling process is that teachers' professional development must be based on the interaction between learners and teachers (Galbraith, Stillman, & Brown, 2010).

Training should provide teachers with techniques to scaffold and provide help that will enable learners to complete tasks while avoiding reducing task complexity (Henningsen & Stein, 1997) and allowing learners an adequate timeframe for completing tasks. This is because the inadequate timeframe given to learners to complete tasks can shift their attention from key concepts towards striving for correct answers without conceptual understanding (Galbraith, Stillman, & Brown, 2010). In addition, training should equip teachers with skills to develop high-quality modelling tasks because if tasks are very easy or very difficult, not built from learners' previously learned ideas, and have unclear instructions, such tasks cannot effectively increase learners' understanding, interest, and performance in mathematics.

2.17 How do learners do mathematical modelling tasks?

Learners around the world are struggling to solve problems involving mathematical modelling tasks (PISA, 2006; OECD, 2007; Abassian et al., 2020; Atteh & Andam, 2019; Chamberlin et al., 2022; Geiger et al., 2022; Stillman, 2019). One of the reasons why learners find it difficult to do mathematical modelling tasks is because they are cognitively demanding and require that learners acquire modelling competency skills (Wess et al., 2021) so that they can be able to solve problems related to mathematical modelling. There are some identified challenges that learners have when it comes to solving modelling tasks identified by Galbraith and Stillman (2006), which are said to be cognitive barriers for our learners and have a negative impact on their mathematics performance. Learners have challenges when it comes to the "construction" of a model, from reading through to understanding the real-world context problem given to their inability to identify important aspects that can be applied to a mathematical model. Secondly, even if learners had abilities towards the construction of correct and appropriate models to represent the situation and problem(s) to be solved, they would find it difficult to "simplify" the problem and make predictions.

Lastly, learners do not check (validate) their solutions to see if they make sense or are appropriate to answer the question; their teachers do not emphasize the necessity to validate solutions obtained; they just mark if the solution is mathematically correct. However, these challenges also influence learners' perceptions about mathematics, which end up affecting their academic performance in mathematics. Ferri (2007) mentioned that the learners' individual route to solve modelling problems, as routes vary from one learner to another, is called the "modelling route." This means that learners start with different phases and follow them with other phases, which make them produce various solutions, given that there is no linear method of solving problems through modelling approaches (Wess et al., 2021).

To learn how to solve modelling tasks, how learners are taught modelling influences their ways and levels of cognition or thinking ability and knowledge development, as well as their concrete understanding of mathematical modelling cycle application (Ferri, 2018). The effects of teachers' self-efficacy about mathematical modelling, skills, and knowledge to track learners' progress and recognize challenges and errors that learners are experiencing during the process of modelling (Ferri, 2018) are also part of the elements that contribute to learners' academic performance in mathematics.

2.18 How teachers treat modelling in the classroom?

Teachers are crucial pillars when it comes to learning mathematical modelling in school mathematics. Ferri (2009) reported that learners working alone are different from learners working self-reliantly under the support of the teacher. Even though this difference may sound insignificant and thin, it plays a huge role in learners' development of high levels of cognitive demand and mathematical competencies that influence their academic performance.

Figure 2.3 Teacher-centred learning (Ferri, 2009)

In figure 2.3, the teacher is assisting learners to work out the problem and find out the solutions in his presentation; learners are not given the opportunity to discuss independently so that they come up with their own solutions from their own understanding of the problem. They end up using the route preferred by the teacher.

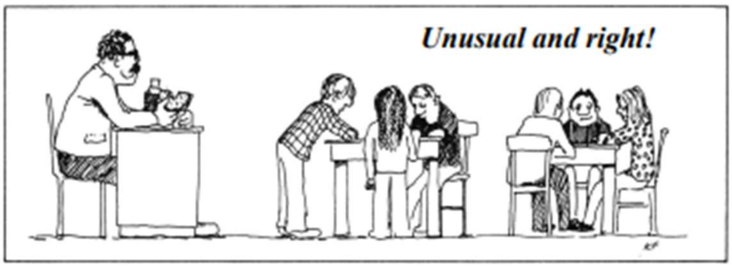


Figure 2.4 Learner-centred learning approach (Ferri, 2009)

In the above figure, learners are working independently with the teacher's support. They discuss the problem independently, and there is no direct influence from the teacher that the learners must follow to solve the problem. Hence, the solutions obtained represent a true reflection of learners' competencies and level of development in cognition and reasoning skills.

The effects of mathematical modelling on learners' academic performance are evidently said to result fundamentally from quality teaching of mathematics (Ferri, 2009). The type of quality teaching of mathematics that is rooted in providing learners with enough chances to develop mathematical modelling competencies and the ability to establish interconnections in the content and context of mathematics.

2.19 Quality teaching of mathematics requirements

One of the requirements of quality teaching of mathematics is the maintenance of a constant equilibrium between a low rate of teacher guidance and a high rate of learners' independence if learners are doing modelling tasks (Ferri, 2009). To adequately achieve this equilibrium, there is a need to initiate frequent strategic interventions that provide learners with hints to solve problems rather than giving solutions on a meta-cognitive level, such as questions that advocate learners' focus on the statement, like "Imagine the situation," "What is the problem about?" What did you obtain, and what do you think is missing? Are the results meaningful enough to make clear sense of the real-world problem and answer the question, etc.? This type of quality teaching in school mathematics is minimal (Ferri, 2009). Thus, it is not clear if minimal quality teaching is due to teachers' lack of understanding of the richness of modelling tasks, which makes them end up reducing the tasks' complexity, a lack of teachers' professional development workshops that emphasize applying a modelling approach when teaching mathematics in schools, or a lack of resources. However, for learners to acquire mathematical modelling competencies, it is rooted in teachers' understanding of the tasks, the choice of examples, and how learners are being taught.

2.20 Teaching mathematics through real data

Teaching mathematics using real-world data or context is advantageous towards improving the mathematics skills required to elevate mathematics performance in algebraic functions (Dunn & Marshman, 2020). Questions involving real-world data may emphasize real-world problem solving skills rather than computational or just theoretical problems (Hand, Daly, Lunn, McConway, & Ostrowski, 1996), have a positive effect on increasing learners' commitment and interest in mathematical-based activities (Aliaga, Cobb, Cuff, Garfield, Gould, Lock, Moore, Rossman, Stephenson, Utts, Velleman, & Witmer, 2005), may help to dispute the perception that mathematics is a difficult subject, enhance learners' positive attitude towards mathematics, and assist learners to have a clear understanding of why they do mathematics rather than how to do mathematics (Dunn & Marshman, 2020), and assist learners to have a clear understanding of why they do mathematics (Dunn & Marshman, 2020 Thus, the use of real-world situations in mathematics classrooms may undoubtedly contribute towards improving learners' performance in mathematics because mathematical modelling emphasizes the translation from real-world situations to mathematics, which makes a good connection between mathematics and real-world daily activities. Can modelling change the stigma that mathematics is difficult? Is it a relevant and effective method to improve learners' performance in mathematics? Why is mathematical modelling so important nowadays?

Learners have shown a very resilient overall tendency to exclude knowledge about the real world and lack skills to make sense of problems when they are required to solve questions if problems are extracted from real-world situations (Chamberlin et al., 2022; Greefrath et al., 2022). However, these are not triggered by a deficiency of unfamiliar or underdeveloped cognitive level; rather, seemingly learners' beliefs in relation to mathematics problems that are based on real-world situations and how real-word problems must be answered in mathematics classrooms are one of the causes that influence learners' attitude and performance in mathematics (Dewolf, Dooren, Cimen, & Verschaffel, 2014). Hence, mathematics learned at school can play a huge role in solving community problems. For these reasons, mathematical modelling indeed has an important task towards improving mathematics teaching methods, which may also have perspectives on how learners recognize the relevance of mathematics to their learning and their daily lives.

Conclusion

The review of the above literature revealed modelling as a dominant approach to teaching and learning that many countries (Chamberlin et al., 2022; Geiger et al., 2022; Jojo, 2020) are looking forward to implementing as a promising solution to devastating learners' underperformance in mathematics as compared to other subjects. Many factors were revealed to have an impact on mathematics quality teaching and learning as well as the relevance of mathematics teaching techniques and approaches that affect both teachers' and learners' perceptions (Mutodi & Ngirande, 2014), beliefs, teaching and learning approaches, effective classroom management, activation of learners' cognition, teachers' self-efficacy (Tseeke, 2021), and the ability to develop a positive attitude and interest towards mathematics (Nieminen et al., 2021; Tseeke, 2021). Considering that learners have shown a very resilient overall tendency of excluding knowledge about the real world and lacking critical thinking and logical reasoning skills to understand the problem if they are required to answer questions if problems are extracted from real-world situations, it is for this reason that mathematical modelling brings hope to mathematics learners' performance because modelling deals with making connections between mathematics and daily real-life situations, which develops a strong belief that learners can see the necessity to do mathematics when they see its relevance towards solving existing social and economic problems.

CHAPTER 3: RESEARCH METHODOLOGY.

Introduction

Research methodology is a collection of philosophical guidelines for the whole research phenomenon, including research design, methods used in the achievement of study objectives, fulfilment of the study purpose, procedures followed throughout participant sampling, and techniques used to collect and analyse data (Kabir, 2016). This chapter gives a full description of the methodology applied by the researcher when conducting this study. It also gives full details on how data was collected and how each instrument was applied to collect data with respect to the set objectives to be achieved, the study population (participants), and the method used for sampling. The study was conducted using primary data collected by the researcher at one public technical school in Kwa-Zulu Natal.

The purpose of this study is to explore the effects of mathematical modelling on Grade 11 learners' academic performance in algebraic functions, teachers' self-efficacy, and their overall perceptions. The central purpose of the research methodology is to assist the researchers in understanding study processes by giving clear guidance on methods to collect and analyse data and answering study questions (Cohen, Manion, & Morrison, 2000). Understanding all study processes is directed by the various approaches employed for the data collection process and the strategies the researcher used to familiarize himself with the gathered data. For this study's purpose and achievement of objectives, the researcher used a set of study questions to guide the study that is based on the effects of mathematical modelling in algebraic functions on grade 11 learners' performance (see section Research questions).

3.1 Research design

Research design refers to a direction, technique, or tool employed to generate meaningful, precise, and ethical data based on the study, including strategies applied for manipulation of the collected data (Asenahabi, 2019). The research design is fundamentally influenced by the research objectives, purpose, problem, and research questions to be investigated (Jilcha, 2019). Additionally, it serves as a strategic framework responsible for all actions that serve as the bridge between the implementation of research instruments, answering research questions, and controlling all study activities.

The researcher, after intensive consideration of the problem of the study, objectives, and questions to be explored, found the design favouring a two-fold method since it is basically rooted in the exploration of two teaching approaches (direct instruction and mathematical modelling) and has a clear understanding of both teachers' and learners' levels of awareness, beliefs, competencies, and overall perceptions about mathematical modelling. He described the study as one that follows an experimental design complemented by the sequential cooperation of quantitative and qualitative methods of data collection and analysis. However, the composition of both quantitative and qualitative methods is known as the mixed method approach. The researchers' choice of employing quantitative and qualitative methods for data collection and analysis is because quantitative methods alone are unable to demonstrate an in-depth perspective of the context, whereas qualitative methods alone seem insufficient to be used alone since the researcher's personal interpretation of the results might contain bias (Creswell, 2013), which may be the result of the limited number of participants in the research study. Using a mixed method allowed the researcher to gather and analyse quantitative and qualitative data collectively. The type of mixed method the researcher adopted is known as sequential explanatory design. The diagram below illustrates the sequence in which the study was carried out. "QUAN" and "QUAL" stand for quantitative and qualitative, respectively. A "$\longrightarrow$" represents a sequential data collection method where the qualitative method builds on the quantitative method.

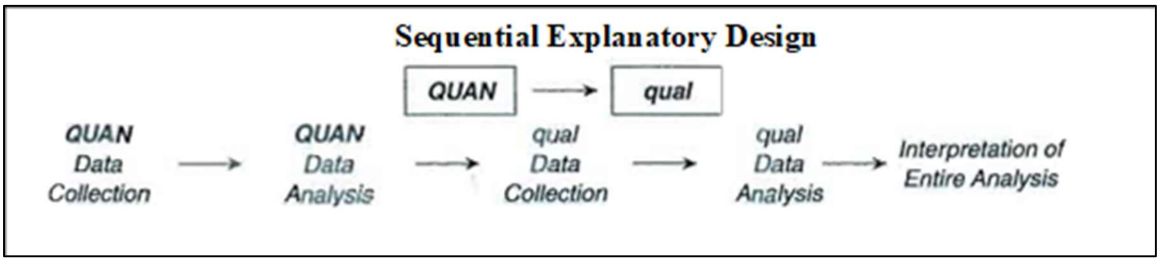


Figure 3.1 Sequential explanatory design (Creswell, 2009)

This type of mixed method is evident in the study design, following which the researcher began with gathering quantitative data followed by collecting qualitative data because qualitative data were to assist in clarifying or elaborating on the results obtained from quantitative data. Full account details on how qualitative and quantitative data were collected are provided in sections 3.1.1–3.1.6 below.

3.1.1 Quantitative data collection

The formation of two groups (control and experimental) was necessary for both collecting and analyzing the data. An identical pre-test and post-test were conducted in both the experimental and control groups to get comprehensive evidence based on the effects that mathematical modelling has on the Grade 11 learners' performance in algebraic functions.

Table 3.1 Overview of participants into the mathematical modelling intervention.

Groups	**Numbers**	**Pre-test**	**Intervention**	**Post-test**
Experimental group	Teachers: 01 Learners: 44	Mathematics: Algebraic function test 1	Mathematical modelling approach	Mathematics: Algebraic function test 2
Control group	Teachers: 01 Learners: 43		Traditional approach	

3.1.2 Qualitative data collection

After the intervention had occurred, six learners were purposefully selected by the researcher based on their academic performance using post-test results according to the groupings or categories mentioned in Table 3.1 below.

Table 3.2 Semi-structure interview description of participants (Leedy & Ormrod, 2005)

Groups	Quantity	Categories	Activity
Experimental group	Learners: 06 Teacher: 01	1. Lowest score marks (minimum score or percentage in a test). 2. Average score marks (closest or equal to the mean score). 3. Highest score marks (maximum score or percentage).	Individual semi-structured interview

Drawing from Table 3.2 above, the purposeful selection of participants for the interview was done following the learners' performance after the intervention. From the experimental group, two learners who scored the lowest marks percentage (minimum score or percentage in a test), average score (a score that is closest or equal to the class mean percentage in a test), and highest score (referred to as a maximum score or percentage in a test) in a post-test were taken for interviews, respectively. This method of selection was due to finding learners' perceptions after experiencing the mathematical modelling approach of learning based on the structure of questions in tasks and modelling language, and to finding background factors that influenced their performance to get insight based on the occurrence. Towards answering study questions, the researcher explored the effects of modelling on learners' performance as compared to a direct instruction approach, learners' perceptions about modelling, teachers' self-efficacy, and teachers' perceptions. Different test instruments were used to achieve the research objectives.

3.1.3 To explore learners' performance in modelling

This study employed the Blum and Ferri (2009) technique, which involves a pre-test that had six (6) questions containing a total of 30 marks, where learners were given 40 minutes to write, and a post-test that had four questions having an overall total of 30 marks, and learners were given 40 minutes to write a post-test after the intervention had taken place. Pen and paper tests were set by the researcher based on grade 11 algebraic functions in line with the CAPS Annual Teaching Plan (ATP). All questions were adapted from two Grade 11 mathematics textbooks. A pre-test set for both the control and experimental groups was administered to get learners' understanding of algebraic functions before the intervention occurred. A total of 87 learners participated in the study that took place in about two (2) classes, where one had 44 (who were selected to form the experimental group) and the other had 43 (who were selected to form the control group) learners. Both tests were conducted for the comparison of learners' performance before and after the intervention. Intervention (teaching and learning using different approaches) was intended to explore the effects of the mathematical modelling approach against the direct instruction approach of teaching mathematics on learners' performance. This research's findings about the effects of modelling on learners' performance as compared to a direct instruction approach are presented under the data analysis chapter (see Chapter 4, Tables 4.1.1–4.1.6).

3.1.4 Learners' perceptions about modelling

To explore learners' perceptions about modelling, six (6) learners were purposefully selected from the experimental (modelling and guided discovery) group for interviews. Learners interviewed were two (2) learners who scored the lowest mark (minimum score or percentage in a test), two (2) learners who scored the average mark (closest to the class mean score or percentage), and two (2) learners who scored the highest marks (maximum score or percentage in a test) from experimental groups. Semi-structured interviews consisting of six interview questions were used to explore learners' engagement with the tasks during the experiment and their beliefs and perceptions about modelling in mathematics immediately after the experiment period. Interview questions were sufficiently open-ended to allow learners' sharing of the intervention experience, elaborate on the other factors that influenced their performance in tests they wrote, and share their perceptions about mathematical modelling in algebraic functions and mathematics at large. A full account of details about the structure of the interview schedule is attached under the appendices section (see Appendix M), how interviews were conducted is presented in Section 3.4.2.5, and the study findings based on learners' perceptions about mathematical modelling are presented under data analysis in Chapter 4 (see Section 4.2).

3.1.5 Teachers' self-efficacy

To explore teachers' self-efficacy about modelling, thirty-three (33) current mathematics teachers in the FET phase from fourteen (14) targeted different secondary schools providing mathematics, which include twelve (12) secondary and two (2) technical high schools. All thirty-three (33) teachers who participated in the study are qualified mathematics teachers who are currently teaching mathematics in the FET phase. A questionnaire consisting of 61 test questions was adapted from Wess et al. (2021) with a 5-point Likert scale for data collection from all participating teachers in this study. Teachers were given a questionnaire to complete and return to the researcher, which took approximately 30–45 minutes to complete. Teachers were given a maximum of two weeks to complete and return the questionnaire to the researcher. Giving teachers two weeks to complete a 30- to 45-minute questionnaire was to allow them enough time to respond to questions effectively, and the researcher did not want to rush teachers since schools were open at the time, so to avoid interrupting them during working hours, teachers had to complete a questionnaire at their own leisure to reduce their chances of responding for the sake of submission. The full account details about the structure of a questionnaire are provided in Section 3.4.2.3; the questionnaire is attached under the Appendices section (see Appendix J); and the study findings about teachers' self-efficacy are presented under the Data Analysis section (see Section 4.3).

3.1.6 Teachers' perceptions about mathematical modelling

To explore teachers' perceptions about mathematical modelling, the researcher conducted a semi-structured interview with the teacher who taught modelling and guided discovery groups to explore mathematical modelling and modelling teaching approach perceptions the teacher held after being trained for three (3) consecutive sessions and after the teacher had taught learners using modelling and guided discovery approaches for the period of two weeks (8 consecutive lessons). A detailed semi-structured interview schedule with six questions that took about 15–30 minutes to explore the teacher's perception of modelling is attached in the appendices section (see Appendix N). In addition, another set of data was collected on teachers' perceptions about mathematical modelling through a questionnaire. There was a comment box provided at the end of the questionnaire for teachers to give their perceptions about modelling after they had enough time to think about it during the time they were completing a questionnaire that challenged their knowledge and understanding of modelling in the curriculum they teach every day and the relevance of their teaching and learning methods towards achieving curriculum objectives. Teachers' responses are attached under the appendices section (see Appendix Q). Full account details on how interviews (qualitative data) were conducted (see Section 3.5) and the findings of the research study are presented under Data Analysis Chapter 4 (see Section 4.4).

3.2 Population and sampling

In contrast to population and sample, population is a collection of all elements that conform to the criteria of the researcher's interest in generalizing the research findings (McMillan & Schumacher, 2010). Alternatively, in the field of research, "population" is a whole group where the research finding may be generalized, and the group of elements should have similar characteristics (homogenous) to those of the researcher's interest for generalization to be more relevant and accurate. However, a sample is a group of representatives selected from the whole population of the researcher's concern (McLeod, 2014). In the field of research, a sample is a group of elements or people taking part in the study as a sample who are also known as participants. However, with regards to this research study, the group of all learners taking mathematics in Grade 11 is the population to which generalizations of the results of this research study may be made, whereas the group of Grade 11 learners doing mathematics from one selected school for the study to take place is a sample. In addition, mathematics teachers in the FET phase are the population, whereas FET phases from 14 different schools that participated in this study form a sample of the entire population.

The sampling method used to conduct this research study is a purposive non-probability sampling method. Purposive sampling is an intentional selection of participants, a non-random method that has no guiding theories, no minimum or maximum number of participants, and follows the study's required qualities or characteristics and the interest of the researcher towards achieving the research study objectives (Etikan, Musa, & Alkassim, 2016). Regarding this research study, the researchers decided on the sample based on characteristics required for the achievement of the study purpose, which entails Grade 11 learners taking mathematics as well as teachers teaching mathematics in the FET phase. The advantage of using the purposive sampling technique is that it enables a researcher to generalize from the investigated sample regardless of whether the generalization is logical, analytical, or theoretical (Palinkas, Horwitz, Green, Wisdom, Duan, & Hoagwood, 2015). The primary data was collected from the purposively sampled 87 Grade 11 learners from a selected public technical high school under the ILembe district in the Gingindlovu area of KwaZulu Natal, South Africa. The total of 87 learners who participated in the study were taken from two (2) classes, where one had 44 (who were taken to form the experimental group) and the other had 43 (who were taken to form the control group). The researcher had no control over the classroom settings since the aim was not to interfere with the school rules, settings, and programs, except that the targeted group of grade 11 learners doing mathematics would not be compromised.

The non-probability sampling technique consists of a reduced number of objectives as compared to the probability sampling method (Stratton, 2021), and this is a method of sampling where the researcher applies a sampling procedure that does not provide an opportunity for every member of the targeted population to participate in the study but selects participants according to the researcher’s characteristics of interest (Creswell, 2009; Creswell & Creswell, 2017). This type of sampling method allows for the determination of statistical sample size, sampling error, and its precision or findings, which allow for conclusions and inferences towards a target group or population (Creswell & Creswell, 2017). However, it is not possible for this study to generalize findings to the whole population because of limitations, which include learners from one school, one teacher who taught learners through a modelling approach, one topic (algebraic functions), and a duration of two weeks for the intervention limited to eight lessons. According to the researcher’s characteristics of interest, participants of this study were Grade 11 learners taking pure mathematics and thirty-three (33) current mathematics teachers in FET phase from fourteen (14) targeted different secondary schools providing mathematics, which include twelve (12) secondary and two (2) technical high schools. These schools were targeted because they have a huge number of learners, which is parallel to the number of teachers teaching similar subjects.

All grade 11 learners doing mathematics in a selected school would have participated in the study, but due to non-probability sampling only those with favourable characteristics (Creswell & Creswell, 2017) were allowed to participate in the study. Hence, all 87 learners sampled for the purpose of the study were learners who were doing Grade 11 for the first time since Grade 10. All learners who participated in this study were taken from one technical school. The reason for targeting newcomers in Grade 11 was to avoid the effects of doing algebraic functions for the second or more time (avoid learners who once learned algebraic functions) because for those learners, the learning would be a form of revision while for the other learners, it would be their first-time experience. All thirty-three (33) teachers who participated in the study were qualified mathematics teachers who were currently teaching mathematics in the FET phase. More details about teachers' demographics are presented in Table 4.3.1. The reason to use this sample size of participants is that for statistical analysis, the number is large enough, while on the other hand, it is sufficiently small to manage for both cost and time constraints.

3.3 Research study variables

The following are the variables explored for the quantitative part of the study.

1. **Independent variables**: Teaching and learning approaches which include.
    i. Mathematical modelling approach
    ii. Traditional approach
2. **Dependent variables**: learners' performance in algebraic functions.
    i. Test scores serve as a measuring tool to test learners' performance and the effects of teaching and learning approaches used in algebraic functions.
    ii. Learners' perceptions about learning approaches' effects on their performance

3.4 Research instruments and data collection

This section gives full details of the tools and techniques the researcher used for data collection. This study took about 2 weeks, 8 successive lessons, and each lesson took 30 minutes (4 lessons per week) of teaching, learning, and 2 assessments (pre-test and post-test) of learners participating in the study. Individual face-to-face audio-recorded semi-structured interviews were conducted for insight into the possible effects emerging from certain factors that contributed to or affected learners' performance. All activities of this study were conducted within the school premises but did not contribute towards learners' school-based assessment (SBA), promotion, or grading purposes. All activities were only used to achieve the purpose of the study; however, learners who did not participate in the study were not disadvantaged anyway.

3.4.1 Data collection tool’s purpose

Choosing research instruments was driven by the problem of the study, the set of objectives to be achieved, and the study purpose in relation to the research questions of the researcher’s concern. This section outlines the purpose of each instrument used to collect data towards achieving the objectives of the study and answering research questions. A pre-test set for both the control and experimental groups was administered to get learners' understanding of algebraic functions before the intervention occurred. This was done to compare learners’ performance before intervention, which assisted the researcher in distinguishing learners’ performance before and after the intervention occurred. Intervention (teaching and learning using different approaches) was intended to explore the effects of the mathematical modelling approach against the traditional approach to teaching mathematics. A post-test was also given to learners after the intervention to evaluate the significance of the study intervention and the effects or contributions that both teaching approaches had towards improving learners’ performance in algebraic functions. Lesson observations were conducted for verbal actions and physical conduct both learners and teachers had during mathematical modelling teaching and learning and how learners engaged with the tasks. Observations also avoided the teacher’s claim to incorporate a mathematical modelling approach during teaching and learning and helped the researcher have positive interactions with the teacher before and after teaching and learning sessions without influencing the teacher’s teaching but keeping focus on the theory that guides modelling environments (see table 2.4).

The use of a questionnaire to collect data from teachers was to open a room for teachers to think and relate their teaching and learning everyday practices with mathematical modelling level of awareness and practice, knowledge about modelling, self-efficacy, and beliefs they have about learning mathematics through modelling, teaching methods guiding a conducive learning environment, and their teaching competencies in mathematics education. The researcher aimed to explore teachers’ perceptions about mathematical modelling after they had enough time to think about it while completing a questionnaire that challenged their knowledge and understanding of the curriculum they teach every day and the relevance of their teaching and learning methods towards achieving curriculum objectives. Data collection instruments were developed in two folds, such that pre-test, post-test, and questionnaires were used to collect qualitative data, whereas semi-structured interviews and observations were developed to collect qualitative data. However, full account details about the composition of each data collection instrument are provided in Section 3.4.2 below.

3.4.2 Quantitative data collection

This study employed the Blum and Ferri (2009) technique, which involves a pre-test that had six (6) questions containing a total of 30 marks, where learners were given 40 minutes to write a test, and a post-test that consisted of four (4) questions with an overall total of 30 marks, and learners were given 40 minutes to write a post-test at the end of the intervention. Pen and paper tests were set by the researcher based on grade 11 algebraic functions in line with the CAPS Annual Teaching Plan (ATP). All questions were adapted from two Grade 11 mathematics textbooks. These two forms of tasks involving basic algebraic functions were given to both the treatment and control groups, but these tests did not contribute to their school-based assessment (SBA). The treatment group was made up of 44 Grade 11 learners, whereas the control group was made up of 43 Grade 11 learners who were currently doing mathematics, to balance the experimental and controlled groups with the aim of avoiding the skewness of the data because it may influence the research results. More detailed question papers are attached in the list of appendices (see Appendix K).

3.4.2.1 Intervention overview

Contact class sessions were conducted as an intervention. The experimental group of learners was taught using a modelling and guided discovery approach, whereas the control group was also taught using a direct instruction approach. Sequentially, learners from both groups first wrote a pre-test, then were engaged in the teaching and learning intervention with different teaching approaches that consisted of 8 lessons. Following the intervention, learners wrote a post-test. All these activities were taking place under the observation of COVID-19 safety regulations, which include a 1.5-meter physical distance, always wearing masks, and sanitizing at their entry point.

Test (pre and post) marks were used for comparing learners' performance prior to and after being taught using modelling with guided discovery in comparison with a direct instruction approach. The researcher was not teaching any of the groups but was responsible for all activities throughout the treatment process and did survey observations to track if learners understood the problem, were able to decide on relevant information to generate model(s) to answer the problem given, were able to check (validate) whether answers that they produced made sense to answer the problem, etc. during peer discussions. Full details and sequenced observation questions are presented in the observation sheet (see Appendix J).

3.4.2.2 Brief procedure overview of how data was collected through intervention

The researcher trained the teacher who taught the experimental group using the modelling method of teaching and learning. Why was the training of the teacher for the experimental group necessary? Training was necessary because modelling is unlike traditional teaching and learning methods, and to develop modelling competencies requires a transformation of the teacher's role in the mathematics classroom (Burkhardt, 2006). With regards to transformation, learners must be provided enough time and develop positive self-confidence to effectively solve given problems for themselves. Interaction between teacher and learner should be supportive to build knowledge but not directive to give ways of doing tasks, meaning that interaction should be strategic, avoiding detailed propositions of how the problem may be solved. The teacher is required to broadly and deeply understand the diversities of teaching methods that learners may possibly pursue during the modelling process.

A teacher's supportive and knowledge constructive intervention is significant during the learning process of mathematical modelling (Wess et al., 2021). Training was based on understanding the modelling cycle processes of translation from real-life situation problems to mathematical model construction. Teaching and learning training is also directed by the Guided Discovery Theory (GDT) for clear focus, understanding of the requirement for proper implementation of the modelling approach, and understanding the role of the teacher against the learner to make sure that the teacher implements modelling properly. Guided Discovery Theorists De Jiong and Lazonder (2014) classified the type of assistance that teachers must give to learners as a supportive measure for discovery learning phenomena into six GDT principles as follows:

Table 3.3 Principles of Guided Discovery Theory (De Jiong & Lazonder, 2014).

Guided Discovery Theory (GDT) principles	**Principle description**
Process constraints	Entails a reduction in learners' possible number of options they may pursue when solving problems at hand by reducing the complexity of the learning phenomenon being discovered.
Performance Dashboard	Assistant provides learners with an outline based on their learning progress as well as the quality work they do. The topic of discussion focuses on what is covered during teaching and learning as well as its effects on producing possible solutions to given tasks.
Prompts	Observed appropriate guidance and clues aimed at reminding learners about performing certain actions, such as telling learners what is required to be done but not how it should be done.
Heuristics	Are unlike prompts because heuristics provide learners with both hints of what and how problems may be solved. However, this form of assistance is required and provided in a state where learners are clueless about what to do and where to start the phenomenon.
Scaffolds	They structure solution processes by giving learners the entire set of elements they need to find solutions to tasks. This form of assistance is practically applied in situations where learners are unable to carry out

	processes to solve the problem independently and if the given task is too complex for them to solve.
Direct Presentation of Instruction	This help entails direct instruction on what is needed and how to solve the problem by providing all the content components required. This is more meaningful if it is used to address those learners' lack of knowledge and their inability to find task components for themselves.

The type of assistance required to be provided depends on the learners' deficiencies. Meaning that if learners manage their learning phenomena properly, very little can be done by the teacher to assist. Even though the principles of GDT mentioned (see Table 3.3) are necessary, none of them promise profound success in learning (Lazonder & Harmsen, 2016). However, helping learners has a positive effect on learning processes rather than providing no assistance. For the treatment group where the teacher had to enable learners to develop modelling competencies and skills to work independently towards the construction of knowledge, process constraints, and performance Dashboards and prompts were adopted, aiming at training the teacher to get the required skills to implement modelling successfully as well as achieving the study set objectives..

The adoption of these principles aims at helping learners develop skills for discovering and constructing knowledge for themselves. Knowledge as self-construct is one of the constructivist beliefs about learning, and to operationalize modelling beliefs in mathematics, two scales of beliefs emerged from an epistemological perspective among teaching and learning mathematical modelling, which are transmissive and constructivist scales (Voss, Kleickmann, Kunter, & Hachfeld, 2013). Epistemological belief is defined as the framework and the beginning of knowledge that can be operationalized through schematic aspects, process aspects, application aspects, and formalism aspects of mathematics teaching and learning (Buel & Alexander, 2001; Wess et al., 2021).

Applications aspects are appropriate to operationalize beliefs based on epistemology about modelling in mathematics education because applications aspects have their own different application references, which are more relevant beliefs regarding the construction of knowledge as emphasized by constructivist theorists. For the experimental group's constructivist-driven belief that learners are better at learning mathematics if they discover methods of solving problems for themselves, mathematical modelling has to be adopted in all mathematics classrooms (Voss, Kleickmann, Kunter, & Hachfeld, 2013; Wess et al., 2021), and promoting practical activities where learners are the ones who solve problems and the teacher's role is to facilitate was adopted (see table 2.3).

In contrast, the group taught through the traditional method of learning was taught following transmissive-driven beliefs that fundamentally believe that the teacher who is effective in the mathematics classroom is the one who is demonstrating the right methods and how to solve application-based problems, and learners should follow the teacher's method (see table 2.3). However, these two scales of teaching and learning mathematical modelling were also adopted in the COACTIV research carried out by Voss et al. (2013) from Staub and Stern (2002) for the success of properly teaching modelling in the mathematics classroom. Thus, the development of these two scales of learning mathematics was based on making a difference between traditional and modelling classes in mathematics education, where key role players are constructivist-oriented beliefs and transmissive-oriented beliefs about mathematics teaching and learning contexts. Training of the teacher who was assisting the researcher was necessary and effective towards the success of the intervention program, avoiding confusion and internal conflicts between the traditional method of teaching he normally used and the new mode of teaching and learning. It is more important to note that both teachers who taught the experimental and control groups were qualified mathematics teachers, and both have been teaching mathematics in grade 11 for more than 5 years. Considering experienced teachers for the intervention was to avoid a lack of understanding of the content of algebraic functions, which would have influenced the quality of results in the post-test if teaching qualifications and experience were not observed.

3.4.2.3 Questionnaire for teachers

The study questionnaire was used for data collection about teachers' mathematical modelling awareness, perceptions, beliefs, and self-efficacy. The entire group of teachers who participated as participants in this research were given a questionnaire containing 61 questions that could take approximately 30-45 minutes to complete, but teachers were given a maximum of two weeks to complete and send the questionnaire back to the researcher. In case of unforeseen circumstances, an online Google document link (see Appendix N) was sent out via emails and WhatsApp platforms to those participants who were not reachable to complete a questionnaire. Hence, most of the teachers preferred online links, mentioning that hard papers are easily misplaced and require time to return to the researcher, whereas an online link, once completed and submitted, allows the researcher to receive it immediately and saves time.

Structure of a questionnaire

A questionnaire was more of a 5-point Likert-scale question; closed questions in the study helped participants easily point out their agreement degree with a particular statement (Bertram, 2006). This research study used a 5-point Likert-scale entailing coded options starting from one (1) and increasing up to five (5), where 1 strongly disagrees, 2 disagrees, 3 is neutral, 4 agrees, and 5 strongly agrees. This Likert-scale allowed the treatment of questions as individual and evaluated separately, but alike questions were gathered to generate a combined score for such questions (Mcleoud, 2008).

Sub-titles in a questionnaire towards answering the research questions are as follow.

1. **Demographics of the participants (teachers).**
   This section included teachers' gender, age group, highest level of qualifications they have in mathematics, number of years teaching mathematics (teaching experience), as well as the average class size they are teaching in their schools.

2. **Teachers' awareness of modelling.**
   The researcher used ten (10) test questions to measure teachers' level of awareness about mathematical modelling in the current syllabus they teach. This includes awareness of the introduction of modelling in the curriculum, specific aims, and the implementation of modelling in their everyday teaching practice.

3. **Professional development about mathematical modelling.**
   To get insight on the level of professional development training workshops provided to teachers as to operationalise modelling approach techniques, the researcher used four (4) test questions. These questions were based on finding if teachers do attend professional development workshops if workshops they attend do emphasis teaching through modelling and provide modelling teaching and learning support material or documents (see Appendix I).

4. **Teacher beliefs about mathematical modelling.**
   Beliefs that teachers hold based on mathematics teaching and learning through modelling were tested using sixteen (16) test questions. These questions were adopted from Wess et al. (2021). Teachers' beliefs in relation to certain teaching methods have a great effect on their choice of techniques and contribute to learners' performance. More detailed questions asked based on teachers' beliefs are attached under the appendices section (see Appendix I).

5. **Teachers' Self-Efficacy (SEF)**
   To explore the teacher's SEF about mathematical modelling, the test questions were two-fold. On the one hand, twelve (12) test questions tested teachers' competence towards recognizing learners' different abilities they demonstrate when modelling and their abilities to teach through a modelling approach. On the other hand, thirteen (13) test questions were used to explore the challenges they experience in relation to their modelling competencies. The total number of test questions is 25.

6. **Teachers' perceptions about mathematical modelling in mathematics.**

Regarding getting insight on teachers' perceptions about mathematical modelling, a comment section was provided at the end of a questionnaire in both hard copy (see Appendix I) and soft copy (see Appendix N). The teacher who taught modelling class was interviewed to get insight on any perceptions he might have about teaching and learning through modelling. All his responses are attached (see Appendix P).

3.5 Qualitative method of data collection

Individual face-to-face audio-recorded semi-structured interviews that took approximately 15–30 minutes were conducted after the experiment to explore learners' and teachers' perceptions about mathematical modelling learning, incorporating the effect of modelling on learners' interest, benefits, and attitudes towards modelling. Six (6) learners were purposefully selected from the experimental (modelling and guided discovery) group for interviews. Learners interviewed were two (2) learners who scored the lowest mark, two (2) learners who scored an average mark, and two (2) learners who scored the highest marks from the experimental groups. Not taking learners from the traditional method group was because there was no intervention; it was normal teaching and learning with traditional methods as usual. Interviews were based on mathematical modelling interventions with learners. All research activities (interviews) took place within the school premises. The teacher who taught modelling and guided discovery groups was also interviewed. An interview schedule with six questions was also administered to find out the teacher's perception of the teaching approach (modelling and guided discovery) he used.

Semi-structured interviews consisting of six interview questions were used to explore learners' engagement with the tasks during the experiment and their beliefs and perceptions about modelling in mathematics immediately after the experiment period. However, the researcher prepared these interviews in advance of the intervention, where dialogue-form interactions were more employed for conversional methods of questioning participants, and there was a possibility for the emergence of unstructured questions along the interview process (Lupahla, 2014). Interview questions were sufficiently open-ended to allow learners' sharing of the intervention experience, elaborate on the other factors that influenced their performance in tests they wrote, and share their perceptions about mathematical modelling in algebraic functions and mathematics at large. All teachers and learners participated in the study voluntarily.

3.6 Data Analysis

Microsoft Excel and IBM Statistical Version 23 Package for the Social Sciences (SPSS) used by George (2016) were used for coding and analyzing the raw data collected. To explore the effects of mathematical modelling in algebraic functions, cross-tabulation with chi-square, t-test, percentages, mean, and frequencies was employed by the researcher. The researcher also used inferential analysis to evaluate the correlation between the average or mean scores and the standard deviation, and descriptive analysis was also employed.

3.6.1 Qualitative data analysis

The researcher employed thematic analysis to analyse the qualitative part of the study because thematic analysis is frequently used for qualitative studies (Cristal & George, 2017). This is an approach to describing data that also entails interpreting data during the coding process as well as theme construction. Kiger and Varpio (2020) defined thematic analysis as the analytical framework that involves searching for themes across the collected data, identifying, analyzing, verifying, and reporting commonalities and differences repeated in the collected data, and defining themes as responses' patterns or meanings found in the data set informing research questions.

Identification of research themes is differentiated into two folds: deductive and inductive approaches. The deductive approach is normally used in pre-existing theories, frameworks, or other researcher-driven attention, whereas the inductive approach is frequently used to derive themes directly from the researcher's collected data of interest (Varpio, Young, Uijtdehaage, & Paradis, 2019). However, it is possible for themes to not precisely reflect the research questions' interest since themes remain data-oriented because participants might not have a clear understanding of the content and context of the research concept. In contrast, Braun, and Clarke (2012) revealed thematic analysis as the most appropriate system for acquiring insight as well as analyzing participants' perceptions established at the core of the research questions. The researcher used an inductive approach because it is said to give a broad and more extensive whole-data analysis since themes are extracted directly from the researcher's collected data. Thematic analysis is guided by six steps towards analyzing qualitative data, as found in Table 3.4 below.

Table 3.4 Steps guiding thematic analysis.

Thematic analysis steps	**Analysis method**	**Description on how the researcher followed each analysis step.**
Familiarizing yourself with data	The researcher became familiar with the collected data. This includes re-reading the data to have a clear understanding of all points revealed during interviews, focus groups, observations, notes taken, etc. (Nowell, Norris, White, & Moules, 2017)	The researcher used audio-recorded interviews to collect data. To familiarize himself, the researcher listened to audio and did a pen-and-paper transcription of the data. This was done because some participants used mixed languages during interviews, which the researcher had to translate into English. After transcribing, he kept listening to the audio to ensure that there were no missing words or points mentioned by participants. The researcher was able to well familiarize himself with the whole set of data.
Generation of initial codes	To code data assists the researcher towards organizing data through noting data of the researcher's potential interest, identifying connections between participants' responses, recording emerged ideas, and grouping similar patterns according to research questions.	The codes list was generated by organizing similar patterns from the data set about the effects that modelling approaches have on the teaching and learning of algebraic functions established from the conceptual framework adopted by the researcher and the guiding theory employed. Generated codes were clearly grouped, adequately well-defined for the application, and categorized by means of avoiding overlapping with other codes.
Searching for themes	This step entails evaluation of generated codes to extract themes of potential broader significance from the collected data.	The researcher identified data patterns that corresponded to a particular theme and grouped them accordingly. The researcher noted all themes identified for potential significance, regardless of their direct or indirect relation to research questions.
Themes review	The researcher reviews all themes generated and makes decisions about whether each theme makes sufficient sense and substantially represents the entire data set in a coherent manner.	The researcher reviewed the generated codes and searched for connections with identified themes to determine if they were properly meaningful and sufficient to answer research questions based on the study purpose and objectives. The researcher

		also checked if themes supported the data adequately and if the data involved supported themes coherently.
Demarcating and labelling themes	At this stage of analysis, the researcher derives definitions and narrative descriptions of themes individually and gives justifications for their importance in answering research questions.	The researcher defined themes based on codes generated to make meaningful connections between themes and research questions.
Producing the report	Reporting refers to the continuation of analyzing and interpreting the data set in all stages mentioned above (King, 2004; Kiger & Varpio, 2020), but this step goes beyond the descriptions of the generated codes and themes of the study. and the researcher writes up a full description of the findings and analysis.	The researcher aligned all generated codes and themes for the entire data set, reported interconnections between codes, themes, and research questions, and analysed, interpreted, and justified correlations that emerged. The researcher then reported findings for which hypotheses were approved or falsified based on learners' perceptions about the mathematical modelling approach in algebraic functions.

2

2 Thematic analyses are defined as the analytical framework that involves searching for themes across the collected data, identifying, analyzing, verifying, and reporting commonalities and differences repeated in the collected data, and as responses' patterns or meanings found in the data set that are informing the research questions (Kiger and Varpio, 2020).

3.7 Ethical Consideration

The researcher was guided by the WSoE Ethics Committee's set rules and regulations as stipulated in the research ethics document to conduct research. He was also guided by the DoE research policy guidelines as stipulated in the permission letter from KZNDoE (see Appendix A). All research study activities were conducted under clearance number H22/07/30.

3.7.1 Informed Consent and Assent forms

The researcher followed all ethics guidelines to conduct the research study. The researcher applied for permission to do research with the KwaZulu Natal Department of Basic Education (KZNDBE) in selected schools and with mathematics teachers. Further arrangements for the study with the principal of the school, teachers, and learners targeted for the research study were made in advance (see appendices).

3.7.1.1 Learners

Since this study involved Grade 11 learners, some of whom were above and others were under the age of 18, assent forms and information sheets about the study were sent out to learners' parents through the learners for all learners who were below the age of 18. Only learners whose parents signed their consent forms were permitted to contribute as participants, even if learners themselves wanted to participate. Learners were not allowed to participate without their parents' consent. All learners above the age of 18 were given consent forms to sign before participating in the study. The data collection process that involved physical contact with the learners and with the teachers was done with observance of COVID-19 SOPs, namely that learners were divided into small groups to keep a physical distance of 1.5 meters during the experiment. Leaners were also subjected to sanitizing their hands at the entrance of the venue and were always obliged to wear masks at all stages of the intervention while inside the classroom.

3.7.1.2 Teachers

All participating teachers were above the age of 18 years, which is considered adult. All teachers were provided study information sheets for them to understand the nature of the study, including their roles to play, and consent forms were also given to sign for those who were willing to participate. Two teachers assisted the researchers by teaching both the experimental and control groups using different teaching and learning methods. Teachers who assisted the researcher with data collection also ensured that COVID-19 regulations were observed, and learners were always treated with care and respect. For example, learners were allowed to withdraw their participation from the study, and there were no implications for their withdrawals.

3.7.1.3 Parents

All parents whose children were participants in the study signed assent forms acknowledging and allowing their children to be part of the study. The researcher also took full responsibility to inform parents in advance about the times of the week for which the researcher needs their children as study participants and the duration of the research in which their children participated to maintain transparency between the researcher, and their parents. All study activities involving intervention took place within the school premises and during hours agreed upon with the learners, learners' parents, the school governing body, and the school principal. Permission from the school principal was sought before the researcher began collecting data in the school. Samples of the permission letters to all relevant bodies for the study to be transparently known and authorized were attached to the ethics application form and are also attached at the end of the report study (see appendices section). In summary, ethical procedures were followed to respect participants' dignity because that is critical for every research study.

3.7.2 Voluntary participation

Every participant in the study has the right to decide whether they are willing to participate and is free to withdraw their participation at any research phase (Cahana & Hurst, 2008). This study respected this right by giving informed consent to the participants prior to their participation. This was done to give an overview of what is expected from the participant and the risks of the study before they commit themselves. Samples of informed consent forms and assent forms are attached (see the appendices section). All participants were advised that they could not continue to participate in any study phase without providing any reason for withdrawal other than acknowledging the researcher. Hence, all detailed study information sheets are attached under the appendices section.

3.7.3 Privacy and confidentiality

Confidentiality entails the researcher's high level of commitment or the entire effort the researcher puts into avoiding disclosing the identification details of the participants. This is informed by the anonymity rule, which emphasizes that during the reporting stage of the research study, the researcher must not include participants' real names or any identifying details (Wiersma & Jur, 2009). In this study, confidentiality, and privacy rights for every participant, which entail data protection regarding participants' personal information, were seriously considered. The research participants were alerted and assured before the commencement of the study activities that they would remain anonymous, and that no information would give hints or track any identifying details in all activities; instead, coded names were used. The data collected were used for the purpose of this study only, were not given to a third party, and were kept on a password-locked computer. Because in the writing of a report and analysis, no personal details should be involved (Yin, 2013).

3.7.4 Honesty

All researchers are urged to ensure honesty by not changing observations and data (Resnik, 2011) to ensure study quality and integrity. The researcher refrained from data misrepresentation and fabrication to obey ethical protocols. Two (2) teachers were requested to teach the experiment group and control group to avoid bias and power relations that the researcher may have if the researcher was the one who was teaching the targeted classes for the study.

3.7.8 Credibility and trustworthiness

Credibility is known as the level of the research study's presentation of trusted data collected meaningfully, showing the researcher's familiarity with, and understanding of the collected data's strengths and gaps. Credibility is one of the foremost study ethics objectives. The researcher ensured that bias during all stages of collecting, analyzing, and interpreting data, as well as in all other aspects of research, was avoided and integrity was maintained. The study conducted intervention, pre-test and pot-test, interviews with students and the teacher who taught the experimental group, and then a questionnaire was also used for teachers. After intervention and test writing, semi-structured interviews were conducted with learners to help the researcher cross-examine learners' performance integrity and evaluate all other factors that might have affected their performance, which would have negatively affected the study findings, as well as achieve a deeper comprehension of the process on which the research study focused. The researcher also ensured that the research study was free from misleading and false data. It was also emphasized that all research activities should be kept in good records, free from negligence and careless errors (Crow & Wiles, 2008). In that regard, the researcher strived for carefulness at all stages of the research study.

3.7.9 Trustworthiness

Trustworthiness is referred to as the constancy of the study outcomes per time taken (Korstjens & Moser, 2018). The researcher ensured dependability and quality through the establishment of the following: all research study stages were described transparently from the beginning of the study to the end, including study development and report writing of findings (see Section 3). Secondly, all intervention tests (pre-test and post-test) were administered under the researcher's observation and support. Thirdly, semi-structured interviews conducted directly by the researcher where all interview questions were asked in the order stipulated in the interview schedule (see appendices J and P) without changing the sequence of questions or wording.

3.7.10 Confirmability

Confirmability is the degree to which other researchers may verify the findings of the research study (Korstjens & Moser, 2018). To achieve confirmability, the researcher employed the triangulation facet to lower the effects of bias during the presentation of the research findings. However, triangulation is defined as the method of ensuring the maintenance of the study findings' consistency and interconnection with different methods used during data collection processes (Pandey & Patraik, 2014). However, pre-test and post-test were used by the researcher for data collection about the effects of that mathematical modelling approach on learners' performance in mathematics subjects, specifically algebraic functions, where learners' mean scores (percentage) in the experimental group were evaluated against those of the control group. Semi-structured interviews were conducted to get learners' perceptions about learning through mathematical modelling to supplement their marks scored when writing pre-test and post-test for the researcher to find underlying emerging factors contributing towards their performance (marks scored). It is possible for learners to claim to have benefited (acquired some skills) or not gained skills to solve problems, but through observations and interviews, it is only found that they largely gained problem-solving skills or neither.

3.7.11 Study limitations

This research study is limited to thirty-three (33) mathematics teachers who are teaching mathematics in the FET phase from fourteen (14) different schools of the same district, ILembe only. Due to time constraints for the study, 87 Grade 11 learners participated, with 44 learners forming the experimental group (modelling and guided discovery method) and 43 learners in the control group (direct instruction). This study is limited to learners taken from one school, specifically Grade 11 mathematics learners, and is only focusing on one topic (algebraic functions).

3.8 Conclusion

The methodology chapter provided full details about how the study was designed and the methods used for collecting and analyzing data. Research design and research methods were structured in a manner that led towards answering set research study questions. Full details of participants' characteristics and the researcher's interest, including how participants in the study were identified and the method of sampling, are provided (see Section 3.4). A thorough description of details for each data collection tool utilized (pre-test, post-test, semi-structured interview schedule, and questionnaire) is attached under the appendices section. Data analysis methods and theories that guided the analysis of the collected data are provided, along with a full account of details (see Section 3.6). The research study could not be well conducted in a meaningful manner if the research methodology was not substantial.

CHAPTER 4: DATA ANALYSIS

Results Presentation

The research study was conducted in one secondary school with learners who were doing pure mathematics in Grade 11. There were 87 learners who participated in the study. The experimental group entailed 44 (51% of the sample group) learners, and the control group also entailed 43 (49% of the sample group) learners. There was an imbalance between the number of boys and girls who participated in the study; the researcher had no control over that since she was working with learners available at school, but she ensured that participating learners were doing pure mathematics. The demographic data of the participants in this study is represented in Table 4.1 below.

Table 4.1 Demographic of the participants (learners).

Gender	Experimental group	Control group	Frequency	Percentage
Male	21	18	39	44.8
Female	23	25	48	55.2
Total	**44**	**43**	**87**	**100**
		Age		
15 - 17 years	35	31	66	58.6
18 years and above	9	12	21	41.4
Total	**44**	**43**	**87**	**100**

The table above represents the demographic data of all learners who participated in the study. Findings reveal that 39 (44.8%) were male learners and 48 (55.2%) were female learners. Among those 39 male learners, 21 (53.8%) were in the experimental group, and 18 (46.2%) were male learners in the control group. The groups of 48 female learners consist of 23 (47.9%) learners in the experimental group and the other 25 (52.1%) learners in the control group. One may agree that in the intervention program, the number of male learners is 39 (44.8%), which is lower than the number of female learners, 48 (55.2%). It is also noted that most learners (66 = 58.6%) who participated in the study were between 15 and 17 years old, whereas 21 (41.4%) participants were 18 years old and above.

Intervention implementation

Two groups were developed for the achievement of the study objectives. Both the experimental and control groups of learners were subjected to writing an identical pre-test in a similar order, and they were given an equal time frame to complete the task. The experimental group and control group wrote a pre-test on the same day at the same time to avoid leakage of the test, which would have implicated the results of the study if learners wrote at different time slots, and the same procedure applied to the post-test. The content of the tasks (pre-test and post-test) was the same (algebraic functions), and the questions were in the same order. After writing a pre-test, an intervention occurred where the researcher organized learners randomly without looking for any characteristics except that all participants were Grade 11 mathematics learners. Both the experimental and control groups were taught by different teachers. The researcher was assisted by two qualified and experienced teachers with more than six (6) years of teaching mathematics, even though it was not one of the aspects of the researcher's criteria, but it was advantageous for the quality of teaching and learning during the intervention. Both mathematics teachers were from one school where the intervention as a means of data collection was conducted.

The researcher organized a workshop with two selected teachers before intervention occurred to prepare them so that they would familiarize themselves with the material they would use during intervention. The guided discovery theory was adopted to guide the whole process of the intervention program. The researcher did this (identifying and requesting two teachers to teach on his behalf) because he wanted to avoid power relations during the experiment. As a result, the researcher was the observer and the facilitator of the intervention, but he did not interfere with teaching and learning occurrences other than having met every day before and after each lesson towards quality teaching and learning improvement. One of the two teachers who were teaching both study groups, the one who taught the experimental group, was trained by the researcher himself to close gaps between modelling and mathematics teaching knowledge (modelling and guided discovery approach training). There was no class taught by the researcher. The content of all eight consecutive lessons conducted is given below.

Lessons 1 and 2: The teacher introduced a mathematical modelling cycle to all experimental group participants (learners) and explicitly engaged learners in all aspects of the modelling cycle and how to use a modelling cycle to understand or make sense of problems adapted from real-life contexts. The core focus was on how to translate the real-life situation problem into mathematical language, making assumptions and predictions to develop a meaningful and real mathematical model.

Lesson 3, 4, and 5: The focus was on the movement from the real model to the mathematization of the problem and writing out the problem mathematically. e.g., the question that was used dealt with finding the dimensions that would give the maximum enclosed area. Learners had to work mathematically so that they could find the mathematical results. The example of a farmer having 100 meters of wire fencing from which to build a rectangular chicken run is from Phillips, Basson, and Botha (2014).

Lesson 6: In this lesson, the theme was to interpret and validate the mathematical results as well as graphically represent the results. Learners were required to graphically represent mathematical results and interpret them in relation to the given real-life problem. Interpretation and validation of the results were conducted using examples from lessons 1 through 5.

Lesson 7: Learners were given a more complex task to do that was dealt with comprehensively. The task was about the bricklayer and his apprentice building a wall within 24 days, taken from Bradley, Campbell, and McPetrie (2013). See figure 4.1 below. Learners were given a more complex task to do that was dealt with comprehensively. The task was about the bricklayer and his apprentice building a wall within 24 days, taken from Bradley, Campbell, and McPetrie (2013). See figure 4.1 below.

"A bricklayer and his apprentice build a wall in 24 days. When each person works separately, the apprentice takes 20 days longer than the bricklayer to complete the job. Calculate the number of days each person takes to complete the job on his own."

Figure 4.1 Grade 11 Platinum Mathematics textbook (Bradley, Campbell, & McPetrie, 2013, p.33)

Learners were required to use the modelling cycle's following stages to answer the given problem. The aim was to track if learners acquired the skill to transcend an unstructured real-life problem using a modelling cycle to the point where they make a meaningful interpretation of the mathematical results to answer the actual given real-life problem situation.

Lesson 8: Two tasks were treated: the task of a group of Grade 11 learners who went out for lunch to celebrate their exam results (Bradley, Campbell, & McPetrie, 2013, p. 32) and the problem of the rectangular parking area having a diameter of 50 meters by 120 meters (p. 34). Hence, these tasks were treated purposefully as revising for a post-test with the same content as the pre-test but with different questions.

Every lesson conducted was thoroughly prepared by the researcher and both teachers who assisted with teaching and learning. The reason to prepare lessons together was to avoid imbalance in teaching due to the teacher's competencies and shift in core focus and objectives to be met. The researcher is an experienced mathematics teacher and has experience in mathematical modelling acquired at a high institution of learning since he was taught a mathematical modelling course in his postgraduate degree. Teachers who assisted the researcher during the intervention also had experience teaching mathematics in the FET phase. To close the gap in mathematical modelling experience, the assisting teacher was trained by the researcher in three (3) constant sessions before the beginning of the intervention and in everyday meetings prior to the beginning of each lesson and after the lesson, when they (the researcher and teachers) reviewed the lessons of the day and progressed. Towards the end of the intervention, the control and experimental groups wrote a post-test on the same day to evaluate the significance of the intervention.

Report of the intervention implementation

Learners, when they were solving modelling tasks, followed the mathematical modelling cycle process and went through all modelling stages depending on the context of the modelling tasks. However, the modelling tasks given were different but contained the same content (algebraic functions). The variation of the modelling tasks process has different objectives (Ferrie, 2006; Kaiser & Stender, 2013), and this process was employed because it is appropriate and complete to examine how learners approach modelling tasks. The process entails five different stages, which are not meant to be in a linear direction.

(i) a real-life situation problem analysed and made simpler to generate a real-life model,
(ii) the generated real-life model undergoes mathematization to develop mathematical model,
(iii) a developed mathematical model is worked mathematically to obtain mathematics results,
(iv) the interpretation of mathematics results to make true meaning to the real-life situation problem, and
(v) results obtained are validated with the real-life problem or with the real model to illustrate the result or solution as well as to reiterate the whole process.

The whole intervention process was aimed at answering research questions accordingly.

Data Analysis Report

The findings of the study are arranged according to the research questions of this study.

4.1 How different is Grade 11 learners' mean score in algebraic functions after being taught using mathematical modelling approach as compared to the mean score obtained through traditional teaching approach?

The table below shows the statistical summary of the data set for both the experimental group and the control group of the learners who were participating in the study. Fortunately, it can be observed that the number of learners who participated in writing a pre-test from both the experimental group and the control group is the same as the number of learners who wrote a post-test. This is because no learners withdrew their participation from the study before the writing of a post-test. Thus, all participants were alerted that their participation is completely voluntary and that they can withdraw at any stage of the research (Cahana & Hurst, 2008) without giving any reason but notifying the researcher. The researcher obeyed research protocol by not seeking reasons from learners and their parents who did not participate in that study. However, the number of participants in the control and experimental groups was balanced since the imbalance would have influenced the skewness of the data, which eventually would have led to misleading data and results. The outputs of the independent sample t-procedure are displayed below. Figures 4.1.1–4.1.3 are the outputs of the data collected at the beginning of the study. Figures 4.1.4–4.1.6 are the outputs of data collected at the end of the teaching intervention (see details of the intervention under methodology section 3).

4.1.1 Group statistics: pre-test (experimental versus control group)

Table 4.1.1 displays the descriptive statistics, including the sample size, mean, standard deviation, and standard error mean for each of the teaching approach groups in the pre-test. As shown in Table 4.2, the teacher-centered group obtained a slightly higher mean score ($M = 28.06, SD = 8.80$) than the modelling and guided discovery group ($M = 26.67, SD = 7.50$). Whether the difference between the two groups is large enough to be statistically significant is discussed in the next section.

Table 4.1.1 Description of modelling and Guided discovery and direct instruction teaching approaches.

	Teaching approach	N	Mean	Std. Deviation	Std. Error Mean
Assessment score	Modelling and Guided discovery	44	26.67	7.50	1.13
	Teacher centred teaching approach	43	28.06	8.80	1.34

Group statistics, including the sample size, mean, and standard deviation, for the experimental and control groups. The experimental group of students was taught based on modelling and guided discovery principles, whereas the control group was based on the traditional teacher-centered approach.

Independent Samples Test

Table 4.1.2 presents results for Levene's test for the equality of variances, followed by the t-test for the equality of means. Levene's test for the equality of variances tests whether the population variances for the two groups are equal. Equal variances are a necessary assumption for the independent sample test. The null and alternative hypotheses for Levene's test are, respectively, that the population variances are equal for the two groups and that the population variances are not equal for the two groups, i.e., $H_0 : \sigma^2\ (Modelling\ and\ guided\ discovery) = \sigma^2\ (Teacher\ centred)$, and $H_1 : \sigma^2\ (Modelling\ and\ guided\ discovery) \neq \sigma^2\ (Teacher\ centred)$.

Table 4.1.2 Levene's test for the equality of Variances, and the t-test for the equality of means.

		Levene's Test for Equality of Variances				***t*-test for the Equality of Means**				**95% Confidence interval of the difference**	
						Significance		Mean Difference	Standard Error Difference		
		F	Sig.	*t*	Df	One-sided *p*	Two-sided *p*			Lower	Upper
Assessment Score	Equal variance assumed	0.622	0.433	-0.797	85	0.214	0.428	-1.39550	1.75170	-4.87835	2.08736
	Equal variance not assumed			-0.795	82.321	0.214	0.429	-1.39550	1.75491	-4.88638	2.09538

The assumption of equal variances is assessed by examining the p-value (Sig.) reported under Levene's test for equality of variances in table 4.1.2. The decision criteria are as follows: If $p \leq 0.05$, the null hypothesis is rejected, and it is assumed that the population variances are not equal. However, if $p > 0.05$, the null hypothesis is not rejected, and it is assumed that the population variances are equal. The results from Table 4.1.2 show that Levene's test produces an F value of 0.622 with a $p-$value of 0.433. Since the p-value of 0.433 is greater than 0.05, the null hypothesis that the variances are equal is not rejected. Based on Levene's test, the researcher assumed that the population variances for the two groups were equal.

$t - test$ for the equality of Means.

The t-test for equality of means answers the question of whether there is a difference in the mean scores for the two groups of learners. Since Levene's test is not significant, we use the values from the first row in table 4.1.2, labelled *Equal variance assumed*; otherwise, we use the second row if Levene's test is significant. The test of the null hypothesis that the two means are equal is provided by the value of $|t| = 0.797$. With 87 participants, the degrees of freedom (df.) are 85 (i.e., number of participants -2), with a corresponding p $-$value of 0.428. Because the p-value of 0.428 is greater than 0.05, the null hypothesis that the means are equal is not rejected, and it is concluded that the two means are not significantly different. The researcher assumes that any difference between the two means in the group statistics was due to sampling error.

Independent Samples Effect Sizes.

Hypothesis testing indicates that the groups are different or not different; however, it does not indicate how different the groups are. In other words, hypothesis testing does not indicate whether the difference is small, medium, or large. One way to describe the degree of difference between the groups is by calculating the effect size. A popular estimate of the effect size for the independent samples *t*-test is given by d, where $= t\sqrt{(N_1 + N_2)/(N_1 N_2)}$, N_1 and N_2 are the samples for group 1 and 2, and t is the value of the *t*-statistics reported in Table 4.1.3. Cohen's (1988) estimates values of d of 0.2, 0.5, and 0.8 as corresponding to *small*, *medium*, and *large effect* sizes, respectively. Based on Cohen's (1988) guidelines, the effect size $|d| = 0.17$ is considered small in practice and indicates that at the pre-test stage, the teacher-centred group of learners had scores that were 0.17 standard deviations higher than the modelling and guided discovery group.

Table 4.1.3 Effect size indicates the magnitude of the results.

	Standardizer	Standardizer**	Point Estimate	**95% Confidence interval**	
				Lower	Upper
Assessment score	Cohen's *d*	8.16886	-0.171	-0.591	0.251
	Hedges' correction	8.24183	-0.169	-0.586	0.249
	Glass's delta	8.79660	-0.159	-0.579	0.264

**The denominator used in estimating the effect sizes

Cohen's ***d*** uses the pooled standard deviation.

Hedges' correction uses the pooled standard deviation, plus a correction factor.

Glass's delta uses the sample standard deviation of the control group.

Summary of pre-test results.

At the pre-test assessment, the modelling and guided discovery group did not have a significantly different mean ($M = 26.67, SD = 7.50$) than the teacher centred group ($M = 26.67, SD = 7.50$), $t(85) = .797$, $p = .428 > .05$, $d = .17$.

4.1.2 Post-test results: Experimental vs Control group.

The outputs of the independent samples t-procedure are displayed below. Tables 4.1.4 - 4.1.6 are the outputs of data collected at the end of the teaching intervention (see details of the teaching intervention under methodology section 3).

Group statistics

Table 4.1.4 provides the descriptive statistics for the post-test data, including the sample size, mean, standard deviation, and standard error for the experimental and control groups. As shown in the table 4.1.4, the modelling and guided discovery group obtained a higher mean score ($M = 55.30, SD = 11.99$) than the teacher-centred group ($M = 37.52,\ SD = 10.29$). Whether the difference between the two groups is large enough to be considered statistically significant is discussed in the next section.

Table 4.1.4 Group statistics including the sample size, mean, standard deviation, and standard error for the post-test data.

	Teaching approach	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
Assessment score	Modelling and guided discovery	44	55.30	11.99	1.81
	Direct instruction	43	37.52	10.29	1.57

Independent Samples Test

Table 4.1.4 presents results for Levene's test for the equality of variances, followed by the t-test for the equality of means. Levene's test for the equality of variances tests whether the population variances are equal for the two groups. Equal variances are a necessary assumption for the independent sample test. The null and alternative hypotheses for Levene's test are, respectively, that the population variances are equal for the two groups and that the population variances are not equal for the two groups. i.e.,, $H_0 : \sigma^2\ (Modelling\ and\ guided\ discovery) = \sigma^2\ (Teacher\ centred),$ and $H_1 : \sigma^2\ (Modelling\ and\ guided\ discovery) \neq \sigma^2\ (Teacher\ centred).$

Table 4.1.5 Levene's test for the equality of Variances, and the t-test for the equality of Means for post-test scores.

		Levene's Test for Equality of Variances				***t*-test for the Equality of Means**				**95% Confidence interval of the difference**	
						Significance		Mean difference	Standard error difference		
		F	Sig	*T*	Df	One-sided *p*	Two-sided *p*			Lower	Upper
Assessment Score	Equal variance assumed	0.687	0.409	7.42	85	<0.001	<0.001	17.78	2.398	13.015	25.551
	Equal variance not assumed			7.43	83.61	<0.001	<0.001	17.78	2.394	13.022	25.544

The assumption of equal variances is assessed by examining the p-value (Sig.) reported under Levene's test for equality of variances in Table 4.1.5. The decision criteria are as follows: If $p \leq 0.05$, the null hypothesis is rejected, and it is assumed that the population variances are not equal. However, if $p > 0.05$, the null hypothesis is not rejected, and it is assumed that the population variances are equal. The results from Table 4.1.5 show that Levene's test produces an F value of 0.687, with a $p -$value of 0.001. Since the *p*-value of 0.001 is less than 0.05, the null hypothesis that the variances are equal is rejected. Based on Levene's test, then, we assume that the population variances are equal.

The t-Test for the equality of Means.

The t-test for equality of means answers the question of whether there is a difference in the mean scores for the two groups of learners. Since the Levene's test is significant, the researcher used the values from the first row in Table 4.1.5 labelled *Equal variance assumed,* otherwise, the researcher would use the second row if Levene's test was not significant. The null and alternative hypotheses for the independent *t*-test are: $H_0 : \mu\ (Modelling\ and\ guided\ discovery) = \mu\ (Direct\ instruction)$, and $H_1 : \mu\ (Modelling\ and\ guided\ discovery) \neq \mu\ (Direct\ instruction)$ The test of the null hypothesis that the two means are equal is provided by the value of $t = 7.42$. With 87 participants, the degrees of freedom (df.) is 85 (i.e., number of participants −2) with a corresponding p-value of 0.001. Because the *p*-value of 0.001 is less than 0.05, the null hypothesis that the means are equal is rejected, and it is concluded that the two means are significantly different. The researcher assumed that any difference between the two means in the group statistics was not due to sampling error.

Independent Samples Effect Sizes

Hypothesis testing indicates that the groups are different (or not different); however, it fails to indicate how different the groups are. In other words, hypothesis testing does not indicate whether the difference between the groups is small, medium, or large. One way to describe the degree of difference between the groups is by calculating the effect size. A popular estimate of the effect size for the independent samples *t*-test is given by *d*, where, $= t\sqrt{(N_1 + N_2)/(N_1 N_2)}$, $N_1$and N_2 are the sample sizes for group 1 and 2, and t is the value of the *t*-statistic reported in table 4.1.5 Cohen's (1988) estimates values of *d* of 0.2, 0.5, and 0.8 as corresponding to small, medium, and large effect sizes, respectively. Based on Cohen's (1988) guidelines, the effect size *d* of 1.59 is considered very large in practice and indicates that at the post-test, the modelling and guided discovery group obtained scores that were 1.59 standard deviations higher than the teacher direct instruction group.

Table 4.1.6 Effect sizes indicate the magnitude of the results.

	Standardizer**	Standardizer	Point Estimate	**95% Confidence interval**	
				Lower	Upper
Assessment score	Cohen's *d*	11.18	1.59	1.10	2.07
	Hedges' correction	11.28	1.58	1.09	2.05
	Glass's delta	10.29	1.73	1.16	2.28

**The denominator used in estimating the effect sizes

Cohen's ***d*** uses the pooled standard deviation.

Hedges' correction uses the pooled standard deviation, plus a correction factor.

Glass's delta uses the sample standard deviation of the control group.

Summary of post-test results.

In the post-test assessment, the modelling and guided discovery group obtained a significantly higher than the mean ($M = 55.30, SD = 11.99$) than the teacher centred group ($M = 37.52, SD = 10.29$), $t(85) = 7.42,\ p < 0.001,\ d = 1.59$.

Assumptions

The assumptions of the independent *t*-test are:

a. *The observations are independent.*

The independence assumption should be satisfied by designing the study so that participants do not influence each other, for example, by working together when answering assessment questions. Failure to adhere to these assumptions leads to inaccurate results.

b. *The dependent variable is normally distributed.*

The normality assumption means that the assessment scores for both groups should be normally distributed in the population (i.e., the distribution should look like a bell-shaped curve when plotted). The normality test can be done by using statistical software to plot a histogram of the assessment scores. However, for moderate ($N \geq 30$) to large sample sizes, most non-normal distributions tend to have a relatively minimal impact on the accuracy of the *t*-test. In general, using a large sample size in the study mitigates against possible violation of the normality assumption.

c. *The variances for each of the groups are equal in the population.*

The equal variance assumption should be carefully monitored as violating, it can compromise the accuracy of the independent sample t-test, particularly when the sample sizes are not equal. Interpreting the results of Levene's test in SPSS and reading the appropriate results for th*e* t from the output answers this assumption.

4.1.3 Comparison of the experimental (Modelling and Guided discovery) and control (Direct instruction) group performance

Considering the data set above (see Tables 4.1.1 and 4.1.4) represents the comparison of two teaching and learning approaches, which include teacher-centered teaching approaches and modelling and guided discovery approaches, the mean score (x =28.06) of the control group is slightly greater than the mean score (x =26,67) of the experimental group in the pre-test. Two groups were selected randomly without looking at the learners' capabilities. Since all participants who wrote a pre-test had never been exposed to any intervention regarding modelling, functions, or graphs, there was no bias, advantage, or disadvantaged group. These results led to the hypothesis, which states that there is no difference in performance between learners in the control group and those in the experimental group in the pre-test. It can also be observed that the mean score (x=55,30) of the learners in the experimental group in the post-test was greater than that of the teacher-centered approach mean (x=37,52). However, the hypothesis that there is no difference in performance between learners in the control group and those in the experimental group is rejected by their performance in the post-test. This is because the experimental group also had a greater actual mean as compared to the control group of learners' performance mean scores after the intervention (see tables 4.1.1 and 4.1.6). The figure below shows the actual mean scores (learners' performance) for the pre-test and post-test for both the experimental and control groups.

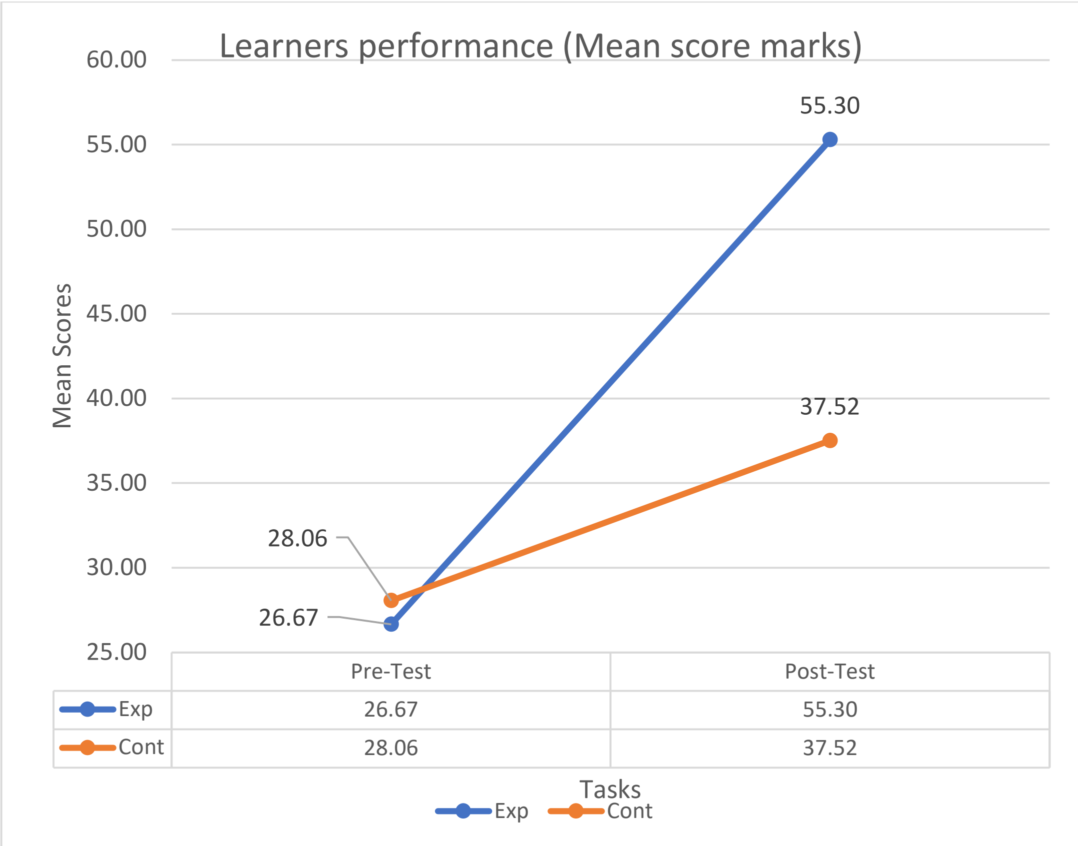


Figure 4.1.1 Modelling and Guided Discovery versus Teacher-centred teaching approaches.

Figure 4.1.1 indicates that the experimental group as well as the control group have changed at various rates between pre-test and post-test. The trend lines represent the trends between pre-test and post-test for both the experimental and control groups. The end points of each trend line indicate the mean test scores for each group at the indicated session. Considering the figure above, observing the diagram and then generalizing the results without any interpretation may be dangerous and misleading (McElreath, 2020). One may assume that the intersection between trend lines for the experimental and control groups indicates that there was a time when both groups were receiving the same treatment. However, the intersection represents that there are learners who scored equal marks in both the experimental and control groups.

During the time when learners were writing a pre-test of mathematical modelling, both groups, the experimental and the control group, were at the same ground level of mathematical modelling since they had never been engaged in modelling before and in Grade 11 algebraic functions. Comparing the mean scores of the control group and that of the experimental group in the pre-test, it can be observed that the control group performed better than the experimental group (see figure 4.1.2).

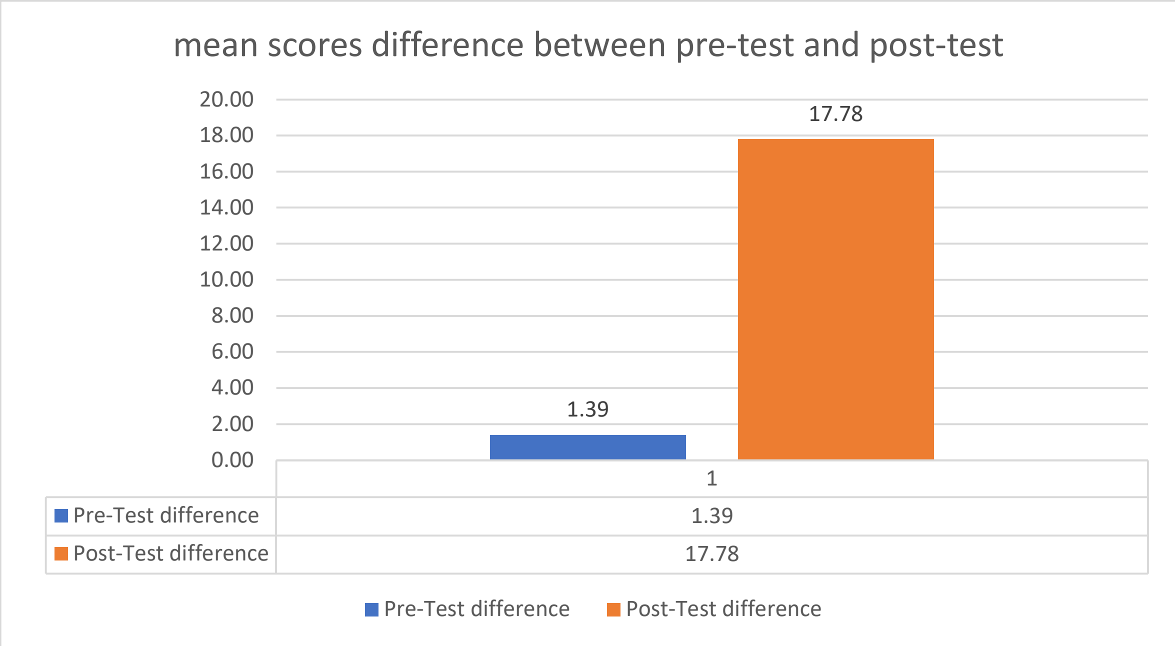


Figure 4.1.2 Difference between learners' performance in a pre-test and post-test among experimental and control groups

Figure 4.1.2 represents the range concerning the mean scores pre-test of both the experimental and control groups, as well as the difference in the post-test for both the experimental and control groups. The difference between their (experimental and control group) mean scores in the pre-test is x=1.39 which means that the difference is not significant (see Table 4.1.1) since both groups' levels of performance are more equal. After teaching and learning through teacher-centered (control group) and modeling and guided discovery (experimental group) approaches, there is a massive rate of change in learners' performance when comparing the experimental group against the control group; the difference in mean scores is x=17,78. This difference is statistically significant enough to allow the researcher to draw the conclusion that the modeling and guided discovery approach is a more effective teaching and learning approach as compared to direct instruction (teacher-centered) in mathematics classrooms.

The assumption may be that indeed the control group has a greater mean in the pre-test, meaning that the control group consists of stronger learners as compared to the experimental group. This assumption leads to the hypothesis that the group taught through direct instruction will outperform the group taught through modelling and guided discovery in both sessions (pre-test and post-test). After intervention had occurred, both the control and experimental groups wrote a similar post-test. The modelling and guided discovery approach group outperformed the direct instruction approach group and improved by more than 28%, whereas the control group improved by less than 10%. Hence, this difference is statistically significant enough to reject the hypothesis that learners who were taught through traditional methods of teaching would outperform learners who were taught through a modelling approach. meaning that the experimental group that received mathematical modelling treatment has acquired more skills than learners in the control group who were taught through a traditional approach.

These findings mean that there is a statistically significant positive difference based on learners' performance in the experimental group. However, one of the abilities required to understand and be able to do modelling tasks is a good understanding of the modelling cycle processes developed by Blum and Leib (see chapter 2, figure 2.1), since Wess et al. (2021) mentioned that in the modelling process every step has a potential cognitive barrier. Findings in figure 4.1.2 above are a clear indication that the learners' performance in the experimental group improved at a higher rate as compared to that of the control group. Therefore, these forms of results reveal that the teacher who taught modelling group was able to lay conducive grounds for learners to develop modelling competencies adapted from the understanding of how to develop a mathematical model from an unstructured real situation, be able to mathematize to acquire mathematics results, and then learners had to interpret them (results) in relation to real situation problems with understanding. Learners' gain in mathematical modelling competencies is evidently represented by learners' performance after being taught through a modelling approach. This improvement is significantly differentiating the teaching methods used for this study since the group taught through a direct instruction approach to teaching and learning has a lower rate of change in the mean percentage than those learners in the experimental group who were taught using a modelling and guided discovery approach. The diagram below illustrates the difference between learners' performance (mean scores of the pre-test and post-test) in both the control and experimental groups.

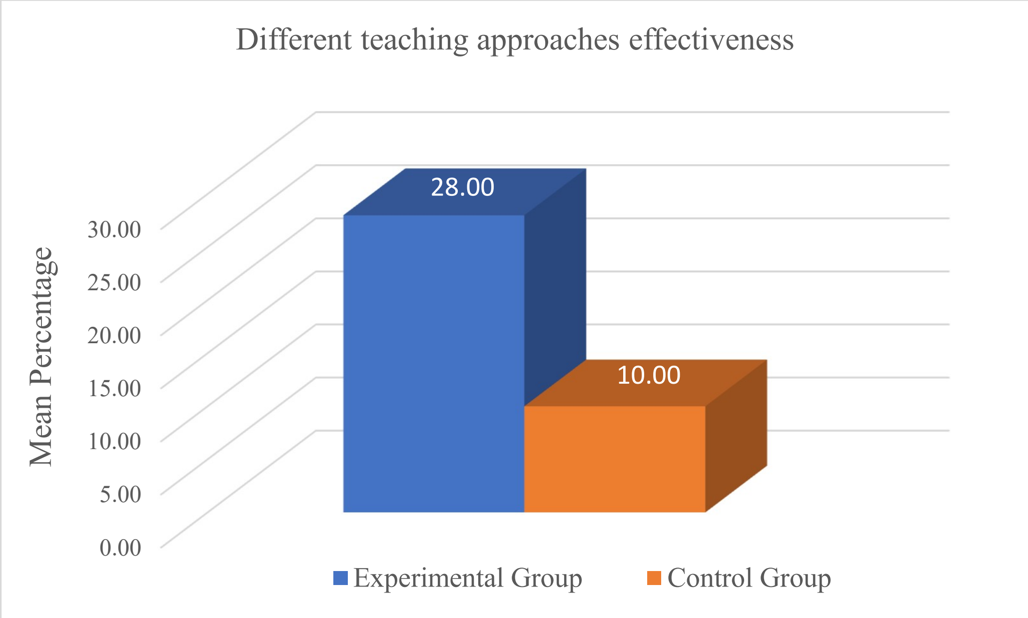


Figure 4.1.3 The effectiveness of different teaching approaches

In figure 4.1.3, a histogram graph illustrates the difference between modelling and guided discovery and traditional (teacher-centered) approaches' effectiveness and how learners' performance changed from pre-test to post-test. Figure 4.1.3 shows that the modelling and guided discovery approach was more effective as compared to the traditional (teacher-centered) approach to teaching and learning. However, variation among experimental and control group mean scores in the post-test is x=18. After the intervention, learners in the experimental group showed much greater gains in mathematical modelling skills than those in the control group. These results reject the researcher's hypothesis that learners in the modelling and guided discovery group will underperform learners in the direct instruction group because mathematical modelling is a difficult and unusual method of teaching and learning that requires specific skills that teachers may not have acquired. Even though the group taught through direct instruction's performance is lower than the performance of learners taught through modelling and guided discovery, there is a great improvement in terms of knowledge and understanding of algebraic functions (10%), even though there is a significant difference in learners' performance in the experimental group as compared to the control group.

The histogram graph (see figure 4.1.3) above illustrates that the academic performance of both groups of learners changed at different rates. Comparing the effectiveness of traditional and modelling approaches, a generalization that mathematical modelling had a greater positive impact on learners' knowledge may result. However, the range of the learners' performance mean score from the pre-test and post-test is significant to measure the level of improvement between the two groups investigated. Lastly, in the post-test assessment, the modelling and guided discovery group obtained a significantly different mean

(M=55.30, SD=11.99) than the direct instruction group (M=37.52, SD=10.29., t (85) =7.42, p<0.001, d=1.59).

4.2 What are the Grade 11 learners' perceptions about mathematics after learning algebraic functions through mathematical modelling approaches?

After the intervention had taken place, the researcher performed individual semi-structured interviews involving six (6) learners chosen purposefully. All six (100%) learners were taken from the experimental group, which received mathematical modelling and a guided discovery approach. Learners who participated in the interviews were selected according to their level of performance, which entails the learner who scored the lowest marks (minimum score or percentage), the learner who scored average marks (closest or equal to the mean score or percentage), and the learner who scored the highest marks (maximum score or percentage) from the experimental group. Interviews were audio recorded to avoid missing words and important points from the participants.

Semi-structured interview questions were asked in relation to the pre-test, post-test, and intervention. The interviews were conducted to get a clear perspective directly from learners about possible factors that influenced their results. Learners were asked different questions, and their responses per question asked are summarized, even though some are brought up or quoted as being about the whole intervention program. Learners were asked about how the language used in mathematical modelling affected their ability to solve the given real-world problem and, in answering the question, their ability to make sense of the real-world situation at hand.

To answer the research question about learners' perceptions, the researcher generated themes to group learners' common responses together (see Thematic Analysis in Table 3.4). To represent learners in the experimental group, the researcher generated the acronyms "LE" (e.g., LE1 stands for learner number 1 in the experimental group) and "LC" (e.g., LC1 stands for learner number one from the control group). The first theme was the language barrier.

4.2.1 Language barrier to mathematics performance

Considering these learners' responses above, language barrier is one of the factors affecting mathematics performance. LE1*: "...modelling English is stronger than mathematics English; I even failed to understand the problem."* LE2*: "English is a problem to me; I find it difficult to understand the statement."* LE3*: "The phrasing of the question was difficult since I am not good in English."* One may agree that mathematics results are not only rooted in the quality of mathematics teaching and learning but also that the language used in mathematics to ask questions has an impact on learners' performance.

Considering the results of this study, learners revealed that English as the Language of Teaching and Learning (LoLT) is one of the contributing factors towards understanding mathematics problems, which also contributes to their mathematics performance. The rationale for introducing learners to mathematical modelling processes was to develop a certain cognitive level or focused thinking ability, aiming to make it easy for learners to understand mathematical modelling language. Mathematical modelling language entails the structure of questions, a method to analyse questions logically, not in a linear way, and the ability to translate real-world situations into mathematical expressions or equations. Thus, if learners have acquired modelling language, using the modelling cycle to generate mathematical models from unstructured real situations or problems that describe the problem with as much accuracy as possible becomes easy. The mathematical language used in the context of mathematics significantly suggests the variance between “mathematics English (ME) and ordinary English (OE)” (Mapolelo, 2009, p. 7).

The mathematics English (ME) language entails the actual English language; this combination of languages produces a completely different form of language from the present languages of communication (Mapolelo, 2009). However, mathematics in the English language often involves exposing learners to the use or manipulation of symbols. Since the group of learners participating in the study are learning English as a First Additional Language (ENGFAL) and not their home language, learners have different levels of understanding depending on their different backgrounds in the English language. More specifically, one learner reported that “English is not my home language, which makes it difficult to understand properly, so it was difficult." "English is a problem to me; I find it difficult to understand the statement.” This factor of mathematics language is not a new or initial discovery of this study. Riccomini et al. (2015) revealed that one of the difficulties learners are facing when learning is that mathematical language is complex and needs teachers to use formal mathematics language to avoid under-developing correct use of language by learners and understanding of mathematical concepts. Different teachers with different levels of mathematics understanding, training, proficiency, and experience, who are teaching learners mathematics at different grades and levels of schools, clearly cannot perform equally due to various learning environments and mathematics backgrounds. This means that learners are not equally proficient in the English language. Hence, language proficiency is also one of the factors affecting learners’ performance in mathematics.

The language of mathematics is an important aspect that should not be ignored to scale up learners’ performance in algebraic functions because it can have a negative effect on understanding mathematics and the development of positive perceptions about mathematics. It is very important that mathematics teachers adopt techniques of teaching mathematics that develop mathematics language proficiency because to understand mathematics vocabulary, it must be easy to learn, follow instructions, answer synthesis questions, and enable learners to answer questions with understanding (Riccomini et al., 2015).

However, the emphasis on language that is significant to learners' performance in mathematics may need attention since mathematical modelling requires both learners and teachers to be able to make sense of the statement problems drawn from real-life situations. Without enough vocabulary in mathematics, answering questions may be difficult. The issue of language in mathematics as a contributing factor to learners' underperformance resulting from learners' lack of understanding how the given problems can be solved is not new since Secanda (1992, p. 639) revealed that "language proficiency, no matter how it is measured, ... is strongly related to mathematics achievement." Learners also mentioned that the language used in mathematical modelling tasks seems quite different from the English they are exposed to in class. However, some learners may have a quality background in mathematics in the English (ME) language acquired either from previous grades, pre-schools, or even at home since 50% of learners (3 out of 6 interviewed) indicated that understanding the question is not a problem to them (learners can understand English), but the challenge is when they should start to develop a mathematical model (translation from a word problem to a mathematical equation or expression).

To achieve the goal of improving mathematics results and to operationalize the Department of Basic Education's specific aim about mathematical modelling in mathematics, learners should be able to effortlessly use, understand, and develop models on their own, apply mathematics signs and symbols, and be able to graphically present the problem when doing mathematical modelling tasks. These results illustrate that effective teaching and learning of mathematics and language proficiency are closely related and have a huge impact on mathematics results.

4.2.2 Prior knowledge about modelling and mathematics learning

Prior or background knowledge is the existing knowledge that learners have acquired in previous grades or previous lessons that helps them make connections between new information and experiences that help them conceptualize new knowledge and understanding (Mabonga Geofrey, 2021). The gap between mathematical modelling, the guided discovery method of teaching, and actual teaching in mathematics classrooms is one of the contributing factors to learners' performance. Learners reported that they had never seen this type of mathematics learning approach, and it was their first experience learning and finding solutions on their own. Hence, learners depend on their teachers for the correct solutions. As *LE4: "It was difficult since it was my first time to find a question where I am the one who should come up with solutions..." LE1; "... very difficult. I think it's because it was my first time to come across this type of mathematics, and the way it's done…"* Considering the above learners' report about residing around the school, this is a clear indication that modelling is not emphasized at the lower grades (LE6: "*…I have never seen this kind of question before, even from one of the previous and current exam papers.…*". These findings reveal that learners have never experienced modelling in their mathematics classroom, including in previous grades. Hence, learners were not able to translate from an unstructured real-word problem to a mathematical model (expression or equation), which would have allowed learners to mathematize the problem and find mathematics results, which would have assisted learners in interpreting results in relation to real-world situations as stipulated in the modelling cycle. This means that the modelling and guided discovery approaches were only introduced in the CAPS document but not implemented. Simple prior knowledge has an impact on the learners' appreciation of new knowledge and helps to improve learners' participation during teaching and learning phenomena because they can make interrelationships between new concepts and previously learned experiences or knowledge (Mabonga, 2021).

4.2.3 Translation from real-life situation problem to mathematics

In contrast, modelling is defined as "transferring real-life problems into the language of mathematics" (Ferri, 2019). Learners reported that they were frustrated when they were required to translate real-world word problems into algebraic equations, and they also believed that mathematical modelling tasks (tests) contained the most difficult questions they had ever solved in classroom or school mathematics examinations. However, LE4: reported that "*.... the challenge was the translation from English words to mathematical equations or formulae; it was difficult for me." LE3:" ... I found it very difficult to move from words to mathematics. I had clues on how to write words mathematically…" and the LE2 said, "... "But the challenge was where to start to answer the questions, from words to mathematics…"* Learners were having a common problem moving from real-life situations to mathematical language. However, understanding the concept is one of the mathematical modelling competencies that a learner should have, and learners are self-constructing their own mental models for a specific problem state as well as understanding the question and translating a real-world problem into a mathematical model. One of the challenges learners faces when they are translating appropriately simplified real problems into mathematical models such as terms, equations, figures, diagrams, and functions is mathematizing.

4.2.4 Disjoint between modelling and mathematics classrooms

Learners were asked to compare the problems that they do in class with those in the research, their responses and perceptions indicate that there is a disjoint between the modelling and mathematics they do in their classrooms. Learners' perceptions indicate that there is a gap between modelling tasks and their everyday learning of mathematics. Learners reported a common point about the structure of questions. *"...these of the research are phrased in an unusual manner…"* emphasising that modelling tasks questions are not straight forward as these they normally encounter in their normal basis mathematics classrooms. Learners believe that modelling is a form of mathematics that is not part of their syllabus since they never seen it before LE6: "*…I never saw any of this kind of question before, even* *from one of the previous and current exam papers…". LE3 reported that "there is a very big difference because here questions are in words even though they give mathematical practical equations, unlike in the classroom."* and is challenging for them to solve. Learners strongly believe that modelling is not part of their learning. They also reported that there was a lot to consider or that they had to remember many different methods they had learned before to find solutions to tasks. However, for learners to remember techniques to answer questions indicates that learning is occurring.

Understanding mathematics is limited to sequences or expressions, equations, and formulae that are straight-forward to solve only to find correct answers instead of application and understanding the nature of the question. Learners' common sense about mathematics is based more on "how" to solve mathematics problems than "why." Learners also revealed that "…*the research task was very different from the classroom tasks because the research task had no straightforward instruction, unlike in the class.…*" (LE3). This learners' perspective about the structure of mathematics questions being more straight-forward than exploratory was also reported by students in the studies carried out by Brown, Cooney, and Jones (1990); Mapolelo (2009). Learners, when doing mathematics, want questions with precise instruction and activities that are closely prescribed and easy to follow. Knowing "why" as an important factor that enables the application of mathematics knowledge acquired from school is seemingly superseded by knowing "how." This is an indication that for learners to understand mathematics, it is basically rooted in remembering procedures to use when solving problems and scoring high marks in tests and examinations, instead of investigating different methods or routes to solve mathematics problems in different contexts, which would develop skills to apply knowledge even in the real-world context.

4.2.5 Improvement in level of cognition

Modelling is believed to be a powerful tool to motivate learners to do mathematics, direct the development of cognition skills by supporting enhancement of learners' mathematical conception, and is trusted as a fundamental approach that will place mathematics in a position of being a culture that describes and makes real-life situations easily understandable (Stillman, 2017). However, the modelling and guided discovery approach in mathematics education is promising to have great effects on the development of learners' competencies and to enhance skills to analyse, think logically, and be able to criticize mathematics models instead of solving for correct solutions rather than understanding (Abassian et al., 2020).

Despite that, learners focus more on scoring high marks in mathematics than understanding the concepts involved and the true meaning entailed. In this study, learners also reported that. *"... this form of mathematics needs one to use the whole brain, bring all focus to the question, and it takes much time to* *make sense such that you can solve the problem."* (LE5). This complaint about the time required to think abstractly to solve the problem indicates that learners believe that the exploratory method (working out the problem and coming up with your own method to solve it without being given a formula or linear route to follow) is wasting time. Such perceptions about the method used to learn mathematics were also reported by students in the study carried out by Mapolelo (2009).

Following this perception of learning mathematics, one of the reasons why mathematical modelling was introduced by the DoE to mathematics education is to expose learners to a solution-self-discovery environment, develop critical thinking skills and the ability to think outside the box, and promote learning independence rather than waiting for the teacher to give answers or hints to solutions without individual engagement with problems, which may lead to correct answers without understanding the concepts. However, the study findings suggest that modelling indeed develops learners' independence, abstract thinking, and expansion, or improves learners' level of cognition.

When learners were asked to raise their perceptions about whether modelling is needed in algebraic functions and mathematics, the following is their report: LE1 "*"Yes, we need modelling in mathematics because it helps to check the level of people's thinking capability… modelling will help to check our psychology and how we process problems to give solutions." LE2: "Yeah, modelling should be involved because it helps to expand the thinking capacity, even though it is challenging, but it makes a person think outside the box." LE5: "Yes, we need it... "... can improve the level of thinking and expand the level of thinking, and that is good for learning and logical reasoning." and LE6: "Yes, it should be taught because... it helps us to think critically. Learning this kind of mathematics in class can also help learners understand how they are going to use the mathematics knowledge they learned in class to solve problems in the real world."* These findings indicated that the lack of teaching through modelling and guided discovery at school is limiting the learners' ability to acquire great skills, knowledge, and even improve their mathematics performance.

Findings of this study follow the results of the study conducted by Julie (2020), which also revealed that modelling is promising to improve learners' interest in doing mathematics, the development of modelling competencies, and the ability to make connections between mathematics done in class and real-world contexts.

The teaching and learning method where the step-by-step techniques to be followed based on "how" to solve problems are the main topic of discussion is known as procedural discourse, whereas the method where the main topic of conversation is the discussion based on reasons as "why" calculations in a particular approach is known as conceptual discourse. However, modelling, and guided discovery approaches to learning should be emphasized. There is a great necessity for DoE to do follow-up if modelling is implemented in mathematics education because if modelling and the guided discovery approach can be well implemented, the widespread perception among learners that mathematics is a challenging (Ngoepe, 2014) and critical subject (Tran, Nguyen, & Ta, 2020) for both teachers and learners may come to an end since modelling is promising to be a powerful tool that, if well utilized, can yield amazing results in mathematics education.

4.3 What are the mathematics teachers' self-efficacy about applications of mathematical modelling in CAPS curriculum?

The research study targeted two different types of participants, which include grade 11 mathematics learners and mathematics teachers, superficially FET phase teachers. The researcher targeted getting 50 EFT-phase mathematics teachers, unfortunately, the target was not met, but the researcher managed to have thirty-three (33) FET phase teachers. All thirty-three (33) mathematics teachers (100%) participated in the study from the beginning up to the end. A full account of participants' (teachers') demographics is provided in the table below.

Table 4.3.1 Teachers' demographics data.

Factors	Definitions (code)	Frequency (%)
Gender	Male (1)	19 (57,6)
	Female (2)	14 (42,4)
Qualification level	≤Bachelor's degree (1)	27 (81,8)
	> Bachelor's degree (2)	6 (18,2)
Age group	≤ 30-years	12 (36,4)
	> 30-years	21 (63,6)
Teaching experience	≤ 10-years	23 (69,7)
	> 10-years	10 (30.3)
Attending professional development workshops	Yes (1)	31 (93.9)
	No (2)	2 (6,1)

The above table 4.3.1 represents the demographics of all teachers who participated in the study. It can be observed that the study was dominated by 19 (57.6%) male teachers, whereas 14 (42.4%) of the participants were female teachers. Only 6 (18.2%) teachers reported having a bachelor's degree in mathematics and sciences as their highest level of qualification, whereas 27 (81.8%) reported having a bachelor's degree as their highest qualification. Observing from Table 4.3.1 above, it is evident that the majority of 21 (63.6%) teachers are found in brackets of 30 years old and below, whereas 12 (36.6%) of teachers are above 30 years old. When teachers were asked to indicate their number of years teaching mathematics in the FET phase, 10 (30.3%) teachers reported to have above 10 years of teaching experience, whereas the remaining 23 (69.7%) teachers have taught mathematics for less than 10 years.

This is an indication that most of participated teachers in this study are more experienced when it comes to mathematics teaching and learning.

4.3.1 Teachers' professional development

Teaching experience is referred to as "culminated skills," being exposed or trained for a certain time range in a particular context to enable one to prepare lessons, teach accordingly, and switch from one teaching approach to another if necessary, or the number of years being exposed to the same content and grade applying various teaching approaches emerged, including being familiar with different teaching and learning support materials. Table 4.3.1 also demonstrates that 31 (93.9%) teachers do attend teachers' professional development workshops organized to revive their teaching strategies and equipment relevant to the curriculum demands and modify their teaching approaches towards certain topic areas.

Towards responding to the research questions, the table 4.3.1 above revealed that 31 (93.9%) teachers do attend all professional development workshops (PDWs) organized by mathematics subject advisors, whereas 2 (6.1%) do not attend workshops. The researcher explored teachers' awareness of the necessity and demands of the CAPS curriculum and the extent to which PDWs put more emphasis on operationalizing the CAPS curriculum's specific aim to teach mathematics through a modelling approach. The researcher used a questionnaire with closed-question (yes or no) responses to explore teachers' awareness. Findings on these aspects are presented in Table 4.3.2 below.

Table 4.3.2 Teachers' awareness of mathematical modelling in CAPS curriculum.

Questions	**Yes**	**NO**	**Mean**
	frequency (%)	**frequency (%)**	$\bar{x}$
Have you seen CAPS document syllabus or scheme of work?	33(100)	0(0)	1.00
Are you aware of the difference between old curriculum (NCS) and new curriculum (CAPS)?	7(21,2)	26(78,8)	1.79
Can you correctly state the philosophy for the new curriculum (CAPS)?	25(75,8)	8(24,2)	1.24
Have you ever heard about mathematical modelling?	27(81,8)	6(18,2)	1.18
Have you seen CAPS specific aims of teaching and learning?	28(84,8)	5(15,2)	1.15
Are you aware that modelling is introduced in CAPS syllabus?	29(87,9)	4(12,1)	1.15
Are you aware that to teach mathematics through modelling is compulsory across the grades?	28(84,8)	5(15,2)	1.85
Have you taught mathematics through modelling processes?	3(9,1)	30(90,9)	1.91
Have you received any training or workshop about implementation of modelling in mathematics teaching and learning?	3(9,1)	30(90,9)	1.91

$mean\ (\bar{x}) rating\ scale; 1 = yes,\quad and\ \ 2 = no$

Considering the findings presented in Table 4.3.2 above, the findings revealed that all 33 (100%) mathematics teachers have seen the CAPS document as one of the guiding teaching and learning policies. Among those 33 teachers, only 7 (21.2%) indicated that they knew how CAPS was different from the

NCS curriculum. Hence, the mean (x=1,79) also confirmed that many teachers are not aware of the variation among NCS and CAPS curricula, even though 28 (84,8%) teachers have seen CAPS Specific aims, meaning that teachers are aware that modelling was introduced in mathematics education and the mean ($\bar{x} = 1{,}15$) is closer to one.

When teachers were asked to respond to whether they had taught mathematics through modelling, only 3 (9.1%) out of 33 indicated that they had once used a modelling approach in their teaching of mathematics. However, noting that 30 (90,9%) teachers indicated that they never received any training based on how to implement modelling in the teaching and learning of mathematics (x=1,91), even though teachers do attend PDWs, teachers' training to apply modelling is not adequately emphasized. One of the major necessities for PDWs is to equip teachers with substantial and relevant skills and competencies to meet the demands of the curriculum at hand (Atteh & Andam, 2019). Lack of training in response to curriculum transformation to meet demands of the newly developed curriculum may also have a huge effect on learners' performance in mathematics and teachers' self-efficacy. Thus, there is a lot to be done to operationalize teaching through modelling if 30 (90,9%) of the sampled teachers indicate that they never received any training on modelling while modelling was introduced over a decade ago (see DoE, 2011b, and CAPS curriculum).

Nowadays, there has been a significant technological and scientific transformation. However, this rapid transformation phenomenon influences changes in various fields (Atteh & Andam, 2019). One of the affected areas is the education system. Since transformation comes with new demands, for the system of education to meet the high demands of transformation, it is required to ensure the readiness of all its structures, such as learners, educators, required infrastructure, and curriculum. However, curriculum changes to meet standards of living call for teachers' professional development training to scale up teachers' quality.

To improve the quality of educators is witnessed as the key element to improving quality education, following that quality teaching, and learning largely influence learners' performance and knowledge achievement (Muthanna et al., 2022). Hence, PDWs form one of the significant factors in every teacher's fulfilment of their responsibilities to operationalize curriculum specific aims, objectives, and demands to improve learners' performance. The researcher further explored the extent to which teachers are furnished with mathematical modelling for professional development and modelling teaching and learning support material using closed questions. The table 4.3.3 below gives a full account of the detailed questions asked and responses given by teachers to each question about how often they undergo PDWs.

Table 4.3.3 Frequency at which teachers attend professional development workshops.

Questions	Never	Sometimes	Always	Mean
Indicate the rate at which modelling support is provided	**Frequency (%)**	**frequency (%)**	**frequency (%)**	$\bar{x}$
Do you attend teachers' professional development workshops?	2 (6,1)	2 (6,1)	29 (87,9)	2,82
Do professional development workshops emphasis teaching through modelling?	0 (0)	0 (0)	0 (0)	1
Have you received mathematical modelling support documents?	0 (0)	0 (0)	0 (0)	1
Have you received training on how to set mathematical modelling tasks?	0 (0)	2 (6,1)	0 (0)	1,03

3-point rating scale: 1 – never, 2 – sometimes, 3 – always

The table 4.3.3 above illustrates the frequency at which mathematical modelling support is rendered to mathematics educators to operationalize DoE objectives to aid learners' performance in mathematics education. The researcher used a scale of 1 to 3 to measure the level of PDWs' support, where 1-never, 2-sometimes, and 3-always getting support. An average mean was calculated for each level to explore the strengths of PDWs. The average mean for teachers' attendance at PDWs is x=2.82 PDWs emphasize teaching through modelling x=1, the receipt of modelling teaching and learning support material is x=1, and the training based on the setting of mathematical modelling activities is x=1. These findings indicate that there is huge gap between the modelling approach and its implementation, which is to operationalize the DoE's specific aim to teach mathematics through modelling, since the study revealed that there are no PDWs that specifically address modelling, no support material for modelling, and no training for teachers based on modelling.

Teachers were asked about their frequency of attendance at PDWs, and it can be observed that all 31 (93,9%) teachers do attend PDWs even though their frequency of attendance is not the same. Thus, 29 (87.9%) of teachers always attend PDWs if they are informed, but there is nothing being said based on modelling. This study revealed that all 33 (100%) teachers who participated did not receive any PDWs based on modelling approach and never received even one mathematical modelling support document for themselves or for the learners as a result mean score ($\bar{x} = 1$) for both emphasis and provision of modelling-based learning material. Professional developments for teachers are very important to improve teachers' skills and knowledge, and updating teachers with recently introduced or discovered approaches improves instructional practice and learners' performance (Eroğlu & Donmus Kaya, 2021). Teachers were further asked if they had ever been trained on how to set mathematical modelling tasks, only 2 (6,1%) teachers received modelling training. One educator learned modelling in one of the courses at the high institution level, and the other educator was trained by the researcher when conducting this research study. If it were not for intervention, they would not have experienced mathematical modelling.

4.3.2 Teachers' beliefs

Teachers have different beliefs about the learning of mathematics through modelling, and beliefs have a high effect on learners' performance. The table 4.3.4 below illustrates some teachers' responses based on their beliefs about modelling.

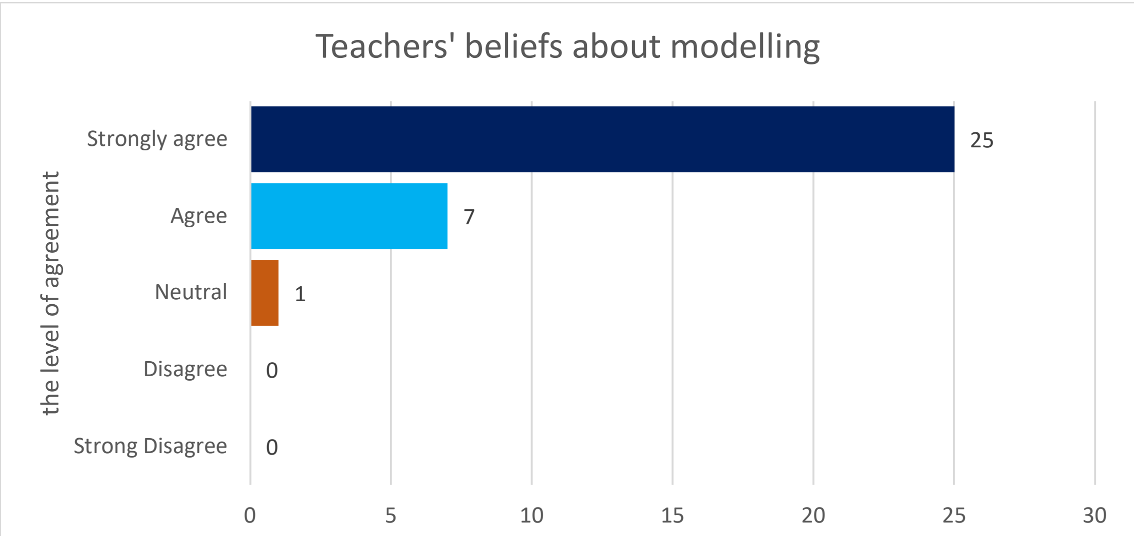
Table 4.3.4 Teacher's beliefs about mathematical modelling.

To wat extent do you agree to the following about mathematical modelling?	Strongly Disagree f(%)	Disagree f(%)	Neutral f(%)	Agree f(%)	Strongly agree f(%)	$\bar{x}$
Mathematical modelling should be a part of mathematics education.	0(0)	0(0)	1(3,00)	7(21,2)	25(75,6)	4,73
Results of mathematical modelling have a general, fundamental benefit for society.	0(0)	0(0)	1(3,0)	15	17(51,5)	4,46
Learners should be given the opportunity to do mathematical modelling in mathematics classrooms.	0(0)	0(0)	1(3,0)	18(54,5)	14(42,4)	4,46
Learners learn best through self-discovery of ways to solve problems.	0(0)	0(0)	0(0)	11(33,3)	22(66.7)	4,46
Learners must work on tasks before the teacher's intervention	0(0)	2(6,1)	2(6,1)	14(42,4)	15(45,5)	4,24
Mathematical modelling competencies should be taught in the classroom.	0(0)	0(0)	6(18,2)	13(39,4)	14(42,4)	4,19
Mathematics should be taught at school in such a way that learners can discover connections on their own.	0(0)	1(3,0)	3(9,1)	13(39,4)	16(48,5)	4,33
Mathematical modelling should be a specific component in mathematics education.	0(0)	0(0)	8(24,2)	12(36,4)	13(39,4)	4,18
Modelling helps students to understand mathematics when they are asked to discuss their own solution ideas.	0(0)	1(3,0)	8(24,2)	14(42,2)	10(30,3)	4,00

5-point scale: 1-strongly disagree, 2-disagree, 3-neutral, 4-agree, and 5-strongly agree.

The table 4.3.4 above reveals some of the teachers' beliefs about modelling in mathematics classrooms. The study revealed that among the participated teachers, none of the 33 (100%) teachers shown a strong disagreement about using a modelling approach in mathematics teaching and learning. Among 33 participated teachers, 32 (97%) of teachers were positive and believe that mathematical modelling should form part of mathematics education and the mean score $\bar{x} = 4{,}73$ is closer to the rating scale that reflects a very strong agreement about modelling in mathematics. When teachers were asked about learning through modelling in the mathematics classroom, 32 (97%) teachers agreed that learners must be provided a chance to learn through modelling in the mathematics classroom, and the mean score of $\bar{x} = 4{,}46$. All beliefs about using modelling approach in mathematics teaching and learning were averaged to explore the extent to which teachers' beliefs about modelling. The average mean signified that learners best learn mathematics through self-discovery of methods to solve given problems is $\bar{x} = 4{,}46$, learners to work on task individually or collaboratively before the teacher's intervention $\bar{x} = 4{,}24$ where 2 (6%) teachers disagreed and another 2(6%) were neutral. The average mean for that modelling competencies should be taught at school is $\bar{x} = 4{,}19$ due to 6(18,2%) teachers were neutral, and the other 27(81,8%) teachers agreed, mathematics should be taught in a sense that enables learners to make connections for themselves to solve classroom mathematics activities and existing problems in the real-world context is $\bar{x} = 4{,}33$, necessity of making modelling a stand-alone topic like other topics in mathematics is $\bar{x} = 4{,}18$, and teachers emphasis that learners well understand mathematics if are given the opportunity to discuss their own ideas about learning

All mean scores about teachers' beliefs based on the use of modelling in mathematics teaching and learning is $\bar{x} > 4$, therefore, these findings indicate that teachers have positive attitude towards teaching and learning using modelling approach. The overall mean of means about teachers' beliefs is 4,33, which indicates that teachers hold positive beliefs about modelling in mathematics education. However, teachers were further asked to rate their level of agreement that modelling should be part of mathematics education. The figure below shows teachers' beliefs about learning through modelling.



	Strong Disagree	Disagree	Neutral	Agree	Strongly agree
Series1	0	0	1	7	25

Figure 4.3.1 Teachers' beliefs about modelling.

The figure above represents teachers' beliefs that modelling should indeed be part of mathematics education and be taught in everyday mathematics. It can be observed that 25 (75,76%) of teachers strongly agree that modelling should form part of mathematics education, and their average mean is 4,73. Hence, this shows that most of the sampled teachers 32 (97%) showed strong beliefs about teaching and learning using a modelling approach, but the challenge is that all teachers mention that there is no professional development workshop they receive and schedule for at least once a year. 7 (21.21%) of teachers also agreed, but the remaining 1 (3%) teacher was neutral, having no idea at all. As for what mathematics should emphasize and its benefits towards improving learners' performance. However, not only beliefs influence learners' performance but also teachers' efficiency or self-efficacy. The figure below reveals the findings about teachers' self-efficacy. The researcher used two forms of test questions to explore teachers' efficacy in mathematical modelling.

4.3.3 Teachers' self-efficacy about mathematical modelling

Teachers were asked to rate their level of performance in mathematical modelling using a 5-lickert scale with closed questions to rate their abilities to operationalize mathematical modelling teaching and learning methods aimed at changing the stigma that says mathematics is a challenging subject. Teachers were asked about their level of abilities towards the identification and recognition of learners' various abilities to solve given problems.

Table 4.3.5 Teachers' modelling self-efficacy.

It is easy for me to recognise different abilities of the leaners using	Strongly disagree	Disagree	Neutral	Agree	Strongly agree	$\bar{x}$
	f (%)	*f (%)*	*f (%)*	*f (%)*	*f (%)*	
... their translation of mathematical results into reality.	0(0)	7 (21,2)	9(27,3)	16(48,5)	1(3,0)	3.33
... their written solutions when modelling.	3(9,1)	2 (6,1)	3(9.1)	23(70,7)	5(15,1)	3.64
... the recording of the plausibility test of their solution.	0(0)	2 (6,1)	12(36,4)	14(42,4)	3(9,1)	3.46
... the adequate assessment of the relationship between the mathematical result and reality that they produce.	0(0)	7 (21,2)	12(36,4)	14(42,4)	0(0)	3.21
... the recording of the solutions they create for modelling tasks.	0(0)	5 (15,2)	16(48,5)	12(36,4)	0(0)	3.24
... the mathematical models they chose when modelling.	0(0)	6 (18,2)	11(33,3)	15(45,5)	1(3,0)	3.24
... the recording of their mathematical results in the modelling process.	0(0)	6 (18,2)	9 (27,3)	16(48,5)	2(6,1)	3.42
... the mathematical formulae and symbols they used in the modelling process.	0(0)	3 (9,1)	14(42,4)	18(54,5)	1(3,0)	3.42
... the determination of their improvement of the established models presented during modelling.	0(0)	8 (24,2)	12(36,4)	11(33,3)	2(6,1)	3.24
... the assumptions they made when modelling.	0(0)	5 (15,2)	14(42,4)	11(33,3)	3(9,1)	3.36
... the students' solutions when modelling.	0(0)	11 (33,3)	9 (27,3)	10(30,3)	3(9,1)	3.15
... their handling of the mathematical symbols and operators used in modelling.	0(0)	3(9,1)	17(51,5)	11(33.3)	2(6,1)	3.36

Five points scale: 1 – Strongly disagree, 2 – disagree, 3 – neutral, 4 – agree, 5 – strongly agree.

To explore teachers' self-efficacy about teaching and learning mathematical modelling, considering the set of questions in the table above and their responses. Teachers' responses based on one question were grouped together, and all responses were averaged to demonstrate a clear understanding and to get the overall stance of teachers using a scale from 1 to 5 (see table 4.3.5). Teachers were asked to rate their competence to recognize learners' various abilities through the use of learners' translations from mathematics to real-life problems is $\bar{x}$ = 3,33, the mean score for all participated teachers about to explore learners' answers when modelling is $\bar{x}$ = 3,64. The researcher asked teachers if they are able to recognize learners' abilities using adequate evaluations through the interconnection that arises between the mathematics solutions produced by the learners and the mean score is $\bar{x}$ = 3,21, learners record of solution produced during solving a modelling tasks, $\bar{x}$ = 3,64 ,mathematical models that learners generated or constructed and make use of them to real-life problem situation is $\bar{x}$ = 3,6, using learners records of their solutions acquired during is $\bar{x}$ = 3,42. The mean score to recognize learners' various abilities through learners' usage of mathematics symbols and formulae during modelling phenomena is $\bar{x}$ = 3,42, the learners assumptions or hypothesis they made during modelling process is $\bar{x}$ = 3,36, evaluating answers that learners produced is $\bar{x}$ = 3,15, and learners treatment of mathematics symbols when modelling took place is $\bar{x}$ = 3,36.

Considering the findings presented in Table 4.3.5 above, since the rating scale was between 1 and 5, the average score for all components ranges between 3 and 4, where three (3) represents a neutral rating and four (4) represents an agree rating. Since all mean scores are $3 < \bar{x} < 4$ the scores, this is an indication that teachers are neutral about modelling but are slightly positive about how it may benefit learners. In addition, the overall mean of means is 3,33, which is closer to neutral (Neutral = 3), this indicates that teachers are not confident about their efficacy in the application of modelling in mathematics classrooms. Nevertheless, ignoring the fact that not all teachers rated themselves in the range of 3 and 4, due to the average calculation, all their responses fell into one bracket. There is a lot to be done by the DoE to scale teachers' modelling competencies for the enjoyment of the modelling approach as one of the productive teaching methods, and since modelling was introduced to the current curriculum as one of the specific aims that stand as a component to be achieved. The figure below represents teachers' ability to recognize learners' competencies to interpret and translate their mathematics results to answer the real-life problem statement.

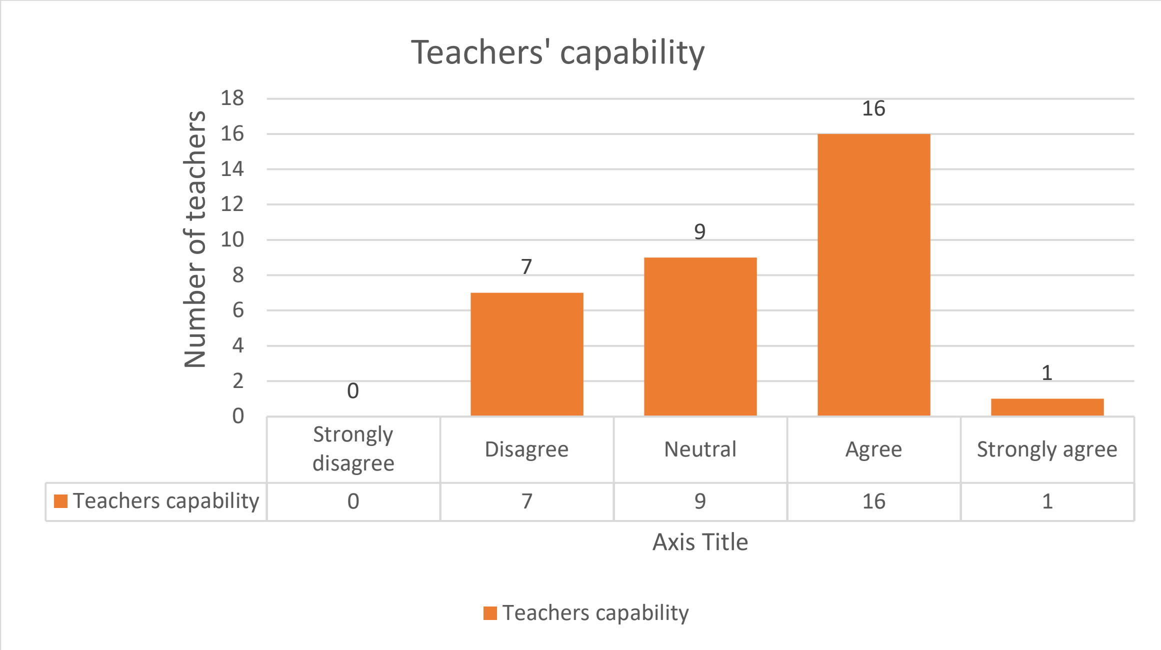


Figure 4.3.2 Teachers' ability to recognise learners' competence

Considering the figure above, it is a representation of teachers' responses based on their capabilities to recognize learners' abilities through evaluating how learners are interpreting and translating their obtained mathematical solutions into reality (answering real-world problems). Among the 33 participated teachers, 7 (21,2%) reported that they could not diagnose if learners were able to answer real-life questions properly even after they had obtained mathematical solutions. However, 9 (27.3%) of 33 rated themselves as neutral members, meaning that all 9 teachers cannot even tell if they are able or not to diagnose if learners do have acquired modelling skills. This is an indication of the extent to which PDWs are required. Thus, 16 (48,5%) out of 33 teachers reported having the capability to evaluate learners' interpretations of the mathematical solution they (learners) obtain after mathematizing in response to real-world situations. Lastly, only 1 (3,0%) out of 33 teachers who undoubtedly reported to have strong modelling competencies examined learners' interpretations of solutions with regards to answering problems related to real-world activities. In addition, the 1 (3,0%) teacher is the one who was helping the researcher with teaching modelling (the experimental group), and following that, he had undergone training based on mathematical modelling by the researcher.

Table 4.3.6 Difficulties in modelling.

It is difficult for me to recognize the different abilities of the students using ...	**Strongly disagree**	**Disagree**	**Neutral**	**Agree**	**Strongly agree**	$\overline{x}$
	f (%)	***f (%)***	***f (%)***	***f (%)***	***f (%)***	
... their translation of mathematical results into reality.	0 (0)	17(51,5)	16(48,5)	0(0)	0(0)	2.49
.. the assumptions they have made in the modelling process	2 (6,1)	19(57,6)	11(33,3)	1(3,0)	0(0)	2.33
... the plausibility checks of their solution	3 (9,1)	17(51,5)	12(36,4)	1(3,0)	0(0)	2.33

... the recording of the real-world restructuring they have undertaken in modelling.	0 (0)	22(66,7)	10(30,3)	1(3,0)	0(0)	2.36
... the mathematical models they chose when modelling.	1 (3,0)	21(63,6)	9(27,3)	2(6,1)	0(0)	2.36
.. the recording of the relationship between mathematical result and reality that they produce when modelling	2 (6,1)	15(45,5)	14(42,4)	2(6,1)	0(0)	2.49
.. the recording of the relationship between mathematical result and reality that they produce when modelling	1 (3,0)	19(57,6)	9(27,3)	4(12,1)	0(0)	2.52
... their handling of the mathematical formulae and symbols used in modelling.	1 (3,0)	21(63,6)	9(27,3)	1(3,0)	1(3,0)	2.42
... their written solution when modelling.	2 (6,1)	21(63,5)	9(27,3)	0(0)	1(3,0)	2.30
... their evaluation of the established models in the modelling process.	4 (12,1)	16(51,5)	8(24,2)	3(9,1)	1(3,0)	2.39
... the simplifications and structures they have undertaken in the modelling process	4 (12,1)	17(51,5)	9(27,3)	3(9,1)	0(0)	2.33
... their mathematisation of a real situation.	2 (6,1)	21(63,6)	8(24,1)	2(6,1)	0(0)	2.30
... the recording of their mathematical results in modelling	5 (15,1)	18(54,5)	8(24,1)	1(3,0)	1(3,0)	2.24

Agreement scale: 1 - Strongly disagree, 2 - Disagree, 3 - Neutral, 4 - Agree, 5 - Strongly agree.

The table above represents teachers' responses about difficulties they experience when teaching modelling. Teachers reported that they never received any training based on modelling, however, their average mean of difficulties they may encounter when modelling is 2,31. This average mean is closer to a scale (Disagree = 2) where teachers disagree that modelling can challenge them even though they are formal or fully exposed to it since it is not catered to in the PDWs they attend every term. These findings show that teachers are positive towards the employment of modelling and are not resistant to adapting new modes of teaching and learning.

The researcher explored the difference between activities that are administered by teachers during normal teaching and learning in relation to the tasks given to learners as part of the study, including intervention activities at large. Learners had different views about modelling tasks given as part of the research study compared to the activities they do every day in mathematics classrooms. All 100% of the learners interviewed (6 out of 6) reported that the mathematical modelling tasks they had done in a pre-test, during intervention, and post-test had a huge difference from the activities they do in mathematics class.

4.4 What are the mathematics teachers' perceptions about the application of mathematical modelling?

Perceptions can be defined as perspectives towards the expression of understanding a certain object or subject matter overview (Zaim et al., 2020). Perceptions develop under the influence of certain conditions, actions, the trend of certain behaviours or results, common beliefs, and continuous situations, and perceptions are the work of an individual's mindset due to contextual factors. The goal of education is the achievement of excellence in all subjects provided at all levels and in all schools. The main character and the determinant of the achievement of all set goals, ambitions, and objectives is the teacher, who is the key to knowledge and understanding for every learner. However, teachers' beliefs and perceptions are more powerful than the knowledge they have about subject content, including what they should teach (Zaim et al., 2020). However, this research study explores teachers' perceptions about using the modelling and guided discovery approach in mathematics teaching and learning as the method introduced or adopted by the DoE in 2011, which was then initiated in 2012 during the time when the NCS curriculum was phased out. Towards answering the research question, the researcher developed themes from participants' (teachers') responses and then aligned them (responses) accordingly to report on teachers' perceptions about the awareness, teaching and learning they receive and the support they receive based on modelling.

4.4.1 Modelling as stand-alone Topic

Findings from this research revealed that teachers do know that modelling is one of the teaching approaches adopted in mathematics classrooms. One of the challenges teachers are facing based on modelling is that they cannot distinguish modelling from other topics in mathematics. *"Mathematical modelling is not clearly stated or introduced as a stand-alone topic in mathematics…it difficult to teach and even answer questions that require mathematical modelling skills*" T1, "*modelling is not clearly introduced for us as educators to know and teach our learners*" T5. Teachers believe that modelling should be an independent topic like other topics in mathematics, meaning that it must not be infused with other topics, so they will treat it equally with other mathematics topics. Teachers believe that all content to be covered for the entire year of teaching and learning is covered and given direction as to which topic to be taught in which term, this information is given in the Annual Teaching Plan (ATP). Teachers raised their concerns that modelling should be one of the topics to be dealt with in Annual Teaching Plan (ATP). T4 reported that "*…it is not there in the scope of teaching and learning*." Another teacher (T12) also reported that "*…is not there in annual teaching plans (ATPs) for every year ever-since it was introduced…",* T13 revealed that *"…there is no topic in the ATP that is about modelling…*" T2, also complained about modelling not being the topic on its own and not emphasised in the ATP as other topics. T2 "*...in the ATP there is nothing talking about mathematical modelling*." Considering teachers' responses, even if teachers are willing to teach mathematical modelling and to teach using a modelling approach, there is still a gap between the teaching plan and modelling. What makes teachers aware and prepares them with skills and knowledge about the dynamics of the curriculum are teachers' development workshops.

4.4.2 Lack of training

Teachers reported that they are not receiving substantial modelling development workshops like other sections in mathematics. Teachers' development workshops are important (Eroğlu & Donmus Kaya, 2021). Teachers reported they never received any training based on modelling. T2 mentioned that "*…there is no even one day where we were trained about modelling..” T3 “…there is nothing that is emphasised in our workshops that we attend about mathematical modelling… there is neither formal nor informal training based on modelling." T6:” I never receive any training based on modelling” T11 reported that “it is a pity that modelling is not emphasized by the department of education to teachers because there are no specific workshops based on modelling that are conducted to train teachers how to teach modelling and how to teach through a modelling approach."*

Mathematical modelling questions are closely related to problem solving (word problems), which makes it difficult for teachers to make distinctions between two sections and even more difficult to solve. T10 mentioned that *“Modelling is not easy to teach since we are not receiving any training based on it and it is difficult even to tell if the question is about modelling or not because it is not clearly revealed as other topics.”* teachers also reported that they are not provided with study material that is specifically based on modelling. "*…there is no workshop specifically for modelling, no study guides ... What I am sure about it is that there are word problems that are used to be tested in question papers involving real life situation, but I am not sure if they are the part of mathematical modelling*" T4. This is a clear indication that there is a lot that needs to be done by the Department of Education to emphasize modelling in schools.

4.4.3 Teachers' beliefs about modelling

When teachers were asked about their perceptions based on teaching and learning mathematics through modelling reported that the mathematics crisis of underperformance may come to an end. One of the teachers, T10, reported that *“maths classrooms will never be the same as today if modelling can be correctly implemented across grades*". T12 also mentioned that "*I strongly believe that modelling can change the state of the art that say mathematics is an underperforming subject...*", T17, also reported that "*I believe leaners can do better in mathematics if modelling can be adopted by all maths teacher”*. Reasons for mathematical modelling to gain so much trust by teachers is because modelling makes mathematics practically connected to our everyday activities, and "*mathematical modelling is a good approach towards improving mathematics results in school…*" (T23) and another teacher revealed that modelling links mathematics to our daily lives' activities, which is good for learners to know that they are not just learning mathematics to pass but also to solve daily problems.

CHAPTER 5: DISCUSSIONS OF STUDY RESULTS

The aim of this study was to explore the effects of mathematical modelling on Grade 11 learners' academic performance in algebraic functions, teachers' self-efficacy, and their overall perceptions. This was guided by four set objectives for all research activities undertaken. To achieve these research objectives, the researcher used a mixed method involving both quantitative and qualitative instruments. For the quantitative part of the research study, there was an intervention involving teaching and learning with two methods of teaching against each other (the modelling and guided discovery method versus the direct instruction method), where learners were the main participants of the intervention. Thus, the qualitative data was collected through interviews with the teacher who taught modelling and guided discovery methods (the experimental group), and six (6) learners from the experimental group were selected according to their levels of performance in a post-test. This chapter gives a comprehensive discussion of the study's findings.

5.1 Study objectives achievement

This research entailed four main objectives (see Section 1.4) aiming to explore the effects of mathematical modelling and its applications in algebraic functions on grade 11 learners' performance. The researcher has undertaken more activities.

5.1.1 The first objective of the study: Learners' performance

The first study objective was developed to explore the extent to which Grade 11 learners' performance is different after being taught algebraic functions using mathematical modelling approaches compared to traditional teaching approaches. Exploring learners' performance was done through conducting a pre-test and post-test, prior to and after the intervention processes. Learners were subjected to a pre-test that was identical for both the modeling and guided discovery methods and the traditional (direct instructional) method covering algebraic functions. When learners wrote a pre-test, they were all at the same level of understanding of algebraic functions since they had not been engaged in the teaching and learning of algebraic functions before.

Findings of the pre-test revealed that learners in the group that was taught using direct instruction had a mean score that was greater than that of the experimental group (modelling and guided discovery approach). However, that yields a hypothetical statement that the control group of learners (the group that received direct instruction) would outperform the experimental group in the post-test because the control group seemed to be the stronger group. However, the difference in terms of mean scores was not statistically significant since the significance value was greater than ($p>0.05$). In addition, the hypothesis that the experimental group would outperform the experimental group was tested, but then hypothesis testing indicated that groups were different, or not different, but the degree at which they were different, whether small, medium, or large, was responsible for describing the effect size. However, there was no statistically significant difference observed. Smith & Morgan (2016) mentioned that making connections between mathematics and the real world is very important. The intervention occurred over a period of two weeks and involved eight consecutive lessons with the same content of algebraic functions and data related to real-world situations.

Wess et al. (2021) mentioned that teaching modelling is the same as other methods of teaching mathematics, but it is effective only if fundamental measures of quality teaching are performed or observed. After the intervention, all learners who participated in the study were also subjected to one task (a post-test) with the same content as algebraic functions to measure the effects of mathematical modelling in algebraic functions. After the intervention has occurred, the mean scores of the experimental (modelling and guided discovery methods) group are found to be greater than the mean of the controlled group. Then the hypothesis that control group learners would outperform experimental group learners was rejected since the mean average was far greater than that of the control group after the t-test for equality of means answered the question of whether there was a difference in the means scores for the two groups of learners. The statistical difference was tested, and it was the statistical significance value used to measure the study's significance where $p<0.05$ represents that the difference is significant. However, the significance value ($p<0.001$) meaning that the difference was statistically significant between two methods of teaching and learning. In contrast, the modelling and guided discovery methods were found to be more effective in the teaching and learning of mathematics.

5.1.2 The second study objectives: Learners' perceptions

The study undergone semi-structured interviews to explore Grade 11 learners' perceptions about mathematics after learning algebraic functions through mathematical modelling. Interviews with learners were conducted individually and were audio-recoded for depth and entire data capture, avoiding data loss. Learners had different perceptions about learning mathematics through modelling and factors affecting learners' results also emerged during interviews, including language barriers, prior knowledge based on modelling, translation from real-life problems to mathematics equations, disjoint between modelling and the mathematics classroom, and it was found that modelling plays a role in improving level of cognition. Learners' extent of acquiring mathematical modelling competencies is generally restricted by areas of mathematics and real-life context (Wess et al., 2021).

5.1.3 The effect of language

Learners mentioned that the phrasing of mathematical modelling questions is different and difficult for them to understand, and they reported that modelling is not original English and believe that it is a combination of mathematics English and ordinary English languages. This variation of language used was also mentioned in the study conducted by Mapolelo (2009), which confirmed that the language of teaching and learning also contributes to the learners' performance in mathematics and revealed that the combination of languages produces a completely different form of language from other languages of communication. Learners who participated in the study were learning English as their first additional language, and they found it difficult to move from word problems to mathematics equations. However, this factor was also revealed by Riccomini et al. (2015) that mathematics language is complex and needs teachers to use formal mathematics language to avoid under-developing correct use of language by learners and understanding of mathematical concepts. Additionally, Secanda (1992, p. 639) revealed that "language proficiency, no matter how it is measured…. But is strongly related to mathematics achievement." To operationalize the modelling approach in mathematics teaching and learning, results illustrate that effective teaching and learning of mathematics and language proficiency are closely related and have a huge impact on mathematics results.

Prior knowledge about modelling and mathematics learning, as well as the knowledge that learners already have acquired in previous grades, also has effects on learning, meaning that it sets grounds for making connections between new information and previous experiences, which help learners easily grasp knowledge and understanding (Mabonga & Geofrey, 2021). Learners reported that they had never seen this type of mathematics learning approach before and that it was their first experience learning and finding solutions on their own before the teacher's intervention. Hence, this means that learners depend on their teachers for correct solutions. "…I have never seen this kind of question before, even from one of the previous and current papers…" said the learner from the experimental group (LE6). This report shows that learners have never experienced modelling in their mathematics classroom, including in previous grades. These findings reveal that mathematical modelling was only introduced in the CAPS document for teachers but not implemented. Since learners in grade 11 are from grade 10, if learners never saw and heard about modelling, it is an indication that even in lower grades, modelling is not implemented while it is a prescribed approach to easy mathematics learning. Underperformance in mathematics that is seen in Grade 12 as a fault of mathematics teachers' incompetence to make learners pass mathematics is the result of injustice done in lower grades.

5.1.4 Translations from real-life situation problem to mathematical model

This research report reports that learners were frustrated when required to translate real-world problems to algebraic functions and that mathematical modelling tasks contained the most difficult questions they ever solved in classroom or school mathematics examinations. LE3 said, "…I found it very difficult to move from words to mathematics." In contrast, Ferri (2009) defined modelling as the method of "transferring real-life problems into the language of mathematics." Understanding the concept is one of the mathematical modelling competencies that a learner should have, as is the ability to self-construct their own mental models for a specific problem state as well as understanding the question. Leaners' lack of ability to translate from word problem to mathematical model results from the disjoint between modelling and the mathematics classroom.

5.1.5 Disjoint between modelling and mathematics classroom

When learners were asked to compare the problems, they were doing in mathematics classrooms with the problems they were doing during the intervention program. Learners reported that the problems in the research study were completely different from those they used to solve in the mathematics classroom. LE4 mentioned that *"There was a huge difference, is because in the classroom during learning the teacher is able to give us formula that may be used to solve the problem…*". As a result, they found it very difficult to workout algebraic functions through modelling and the guided discovery method, even though it is not part of school mathematics because modelling emphasizes learners' independence. Hence, learners' common sense about mathematics is based more on "how" to solve mathematics problems than "why." This indicates that for learners to understand mathematics, it is basically rooted in remembering procedures to use when solving problems and scoring high marks in tests and examinations, instead of investigating different methods or routes to solve mathematics problems in different contexts, which would develop skills to apply knowledge even in the real-world context. The findings of this research study report that learners also revealed that "*…the research task was very different from the classroom tasks because research task had no straightforward instruction unlike in the class…*" (**LE3**). This learners' perspective about the structure of mathematics questions being more straight forward than exploratory was also reported by students in the studies carried out by Brown, Cooney, and Jones (1990) and Mapolelo (2009). Not only negative perceptions were revealed by learners who participated in this study, but learners also mentioned that modelling improved their level of cognition.

5.1.6 Level of cognition improvement

The findings of this study also reveal that modelling and guided discovery methods are powerful methods to improve learners' levels of cognition. LE2 reported that "*yeah, modelling should be involved because it helps to expand the thinking capacity, even though it is challenging but it makes a person to think* *outside the box."* These results confirm that modelling indeed develops learners' independence, abstract thinking, and expansion, or improves learners' level of cognition (Wess et al., 2021). When learners were asked to raise their perceptions of whether modelling is needed in algebraic functions and mathematics. This study's results follow the results of the study conducted by Julie (2020), which also revealed that modelling is promising to improve learners' interest in doing mathematics, the development of modelling competencies, and the ability to make connections between mathematics done in class and real-world contexts.

5.1.7 The third study objective: Teachers' self-efficacy

The third objective was to explore mathematics teachers' self-efficacy about the application of mathematical modelling. A questionnaire was used to first explore teachers' level of awareness about modelling in the curriculum they present to learners every day. The study revealed that indeed teachers know that modelling is one of the most adopted and introduced teaching methods in mathematics education, however, there is a lack of emphasis by the DoE through school leadership and teachers' development workshops to emphasize the implementation of modelling approach in mathematics classrooms. A teacher's efficacy is acquired through thorough engagement with professional development workshops or training based on the content or subject of the teacher's interest, and the efficacy is measured through the results the teacher or teachers are producing in terms of learners' performance.

This study's results revealed that there is a lack of training in response to curriculum transformation to meet the demands of the newly developed curriculum, which may also have a huge effect on learners' performance in mathematics and teachers' self-efficacy. However, improving the quality of teachers is seen as the key element to improving quality education, given that quality teaching and learning largely influence learners' performance and knowledge achievement (Muthanna et al., 2022). Therefore, PDWs form one of the significant factors in every teacher's fulfilment of their responsibilities to operationalize curriculum-specific aims, objectives, and demands to improve learners' performance. Thus, findings indicate that there is a huge gap between the modelling approach and its implementation, which is to operationalize the DoE's specific aim to teach mathematics through modelling, since the study revealed that there are no PDWs that specifically address modelling, no support material for modelling and no training for teachers based on modelling.

Teachers' beliefs also influence learners' performance. Findings revealed that teachers strongly believe that if modelling as one of mathematics' teaching and learning methods can be emphasized and teachers are exposed to professional development workshops, it may benefit them in terms of knowledge and improve learners' performance in mathematics. Teachers also show a positive attitude towards modelling, emphasizing that modelling should be taught in a way that enables learners to make connections between previously learned concepts and new problems in mathematics classrooms and in the real-world context. During the interviews with the teacher who taught modelling and guided discovery class (experimental group), mentioned that modelling promotes learners' centeredness, trains learners to work independently towards finding possible solutions, and allows learners to share ideas and work together to reach a common understanding based on the specific problem they are given under the modelling cycle, and believe that modelling can bail out many teachers.

5.1.8 The fourth objective: Teachers' perceptions

The researcher also explored mathematics teachers' perceptions about the application of mathematical modelling. The findings of this study revealed that teachers do know about modelling as one of the teaching approaches adopted in mathematics education. One of the challenges reported by teachers has been making distinctions between modelling and other topics in mathematics. One of the teachers' perceptions about modelling is that if modelling can be an independent topic in mathematics, all teachers can take it seriously to learn this teaching approach and to teach it as they do in other topics.

Teachers also reported that there is no topic in the ATP that is about mathematical modelling, and that modelling is not within the scope of teaching and learning. Results of this research also figured out a negative perception that teachers have about modelling, even though they do not know much about it since all participants (33) reported that there are no teachers' development workshops based on modelling. This perception is that "modelling is difficult to teach, to learn, and even to solve modelling problems is a challenge, and learners are failing to answer word problems, how much more with modelling is…" T7, and another teacher's reported perception is that "some teachers do not teach word problems because they are difficult even for them to understand." However, teachers reported not having PDWs to train them on modelling. Teachers' holding these perceptions is an indication of low self-esteem or confidence towards modelling and word problem solving, and these perceptions are hindrances towards the implementation of modelling since teachers' perceptions are more powerful than the knowledge they have about subject content, including what they should teach (Zaim et al., 2020).

5.1.9 Conclusion.

This chapter provided a discussion of the results of this research in proportion to answering the study questions and achieving the research study objectives. Research outcomes based on the intervention, which entailed teaching and learning through a modelling and guided discovery approach and a traditional (direct instruction) approach, revealed that the modelling approach was significantly effective in learners' accumulation of knowledge and skills and contributed a lot towards learners' performance (see Section 4, Table 4.2.2). Learners' perceptions after learning through modelling in support of guided discovery were positive about modelling. In contrast, the modelling approach improved learners' level of cognition, and enhanced learners' positive attitude and interest towards learning mathematics through modelling. Teachers' outcry about the lack of teachers' professional development workshops revealed the key aspect towards the implementation of the modelling method in mathematics classrooms. However, teachers' self-efficacy and beliefs were also explored, and findings were positive about the modelling approach (see tables 4.3.4 and 4.3.5), and their perceptions are discussed.

CHAPTER 6: CONCLUSION, LIMITATIONS, AND RECOMMENDATIONS.

This section covers the summary of the whole research study, the limiting factors of the study, and the researcher's recommendations with respect to the study findings.

6.1 Conclusion

The study explored the effects of mathematical modelling and its applications in algebraic functions on Grade 11 learners' performance. The study employed two-fold methods of data collection, which entailed quantitative and qualitative methods using a sequential explanatory design. Quantitative data was collected through a pre-test that was followed by the intervention, which involved modelling and the guided discovery method for the experimental group and the traditional (direct instruction) method for the control group. After the intervention, learners were subjected to the writing of a post-test to make a distinction between modelling and direct instruction. The findings revealed that modelling and guided discovery methods were significantly more effective as compared to the direct instruction approach. A questionnaire was also used to explore teachers' awareness of modelling, beliefs, and self-efficacy (see Appendix J). For qualitative data collection, semi-structured individual interviews were conducted to find learners' and teachers' perceptions about modelling, and the findings revealed that learners do believe that modelling is a good method of teaching mathematics that benefits them in terms of knowledge and improves their level of cognition.

Teachers reported that modelling is not emphasized, and as a result, positive and negative perceptions about modelling were raised. On the one hand, a positive perception is that modelling can prevent most mathematics teachers from accounting for learners' poor performance and can improve mathematics results. On the other hand, modelling is difficult to teach and learn, and even solving modelling problems is a challenge. Learners are failing to answer word problems, how much more with modelling? However, teachers reported that there are no teachers' professional development workshops specifically for modelling, even in the present PDWs, modelling is emphasized. The absence of teachers' professional development workshops on modelling is hindering teachers from developing modelling competencies. This indicates that teachers have not applied the modelling approach in their mathematics teaching and learning because the modelling approach requires an understanding of the modelling cycle process (Wess et al., 2021). Hence, the modelling cycle enhances the ability to understand unstructured real situation problems where the aids of mathematics are used to process them (Niss & Blum, 2020), the ability to generate a mathematical model from an unstructured real situation problem, and working with a constructed model mathematically to acquire mathematical results that require interpretation and validation (Greefrath & Vorholter, 2016) in relation to real situation problems, which enables a modeler to solve the problem at hand and be able to use a model to predict future results of problems of the same nature (Wess et al., 2021). Thus, the application of mathematical modelling in mathematics classrooms to improve learners' academic performance may not be initiated if the modelling approach is not emphasized, and the DoE objective to improve learners' performance in mathematics through modelling may not be achieved if there are no organized interventions that are specifically preparing or training teachers to apply mathematical modelling in their mathematics classrooms. Quantitative data was analysed using SPSS version 23 (see Section 3.6), and qualitative data was analysed through thematic analysis (see Table 3.4).

6.2 Study limitations

This research study was limited to only eighty-seven (87) grade 11 learners, who were sampled from one public technical high school. Among those 87 learners, only 44 formed the experimental group, which was taught through modelling and guided discovery methods (see Section 3.4.2.1), and the other 43 formed the control group. This study was also limited to one subject (mathematics only) and focused on algebraic functions only. For data collection and in-depth analysis in proportion to time constraints, the research only managed to have thirty-three (33) mathematics teachers who are teaching mathematics in the FET phase from fourteen (14) different schools of the same district, ILembe, KwaZulu Natal, South Africa. The employment of two methods of data collection and analysis, which included quantitative and qualitative methods, was to first find quantitative data, then follow up with qualitative methods, which also involved pre-test, intervention, and post-test for the quantitative part, and semi-structured interviews with learners for the insight of responses provided through pre-test and post-test after the intervention program. The researcher further interviewed one educator (a teacher who taught modelling and guided discovery groups) to get his perception about modelling after teaching during the whole intervention process. The above-mentioned limitations mean that the researcher could provide excessive data based on the intervention, learners' perceptions, and teachers' efficacy, knowledge, and beliefs based on modelling.

6.3 Recommendations

Due to the limited number of learners presented in Section 6.2 above, the purpose of the study was to explore the effects of mathematical modelling and its application to algebraic functions on grade 11 learners' performance. Many factors were observed during study processes that were reported to have effects on learners' performance. This study found that learners have no idea about modelling and are not taught through a modelling approach. This means teachers are still using the traditional (direct instruction) method, where the teacher is the only master of knowledge. I recommend that modelling be used to teach mathematics to develop learners' adequate skills, improve learners' meta-cognition, interest, and positive attitude toward mathematics, and improve learners' skills to solve real-life problems in mathematics and real-life situations. Teachers reported that they are not receiving any training based on modelling, and modelling in mathematics is not clearly stated, there are no teaching and learning materials supporting modelling, and mathematical modelling is not part of the set scope of the year

(ATPs). I therefore recommend that the department of education put more effort into supporting and motivating teachers to implement modelling, provide teachers with professional development workshops based on modelling, design and distribute teaching and learning support documents to all schools providing mathematics, and do follow-up to check progress in teachers' developments. Lastly, I also recommend that mathematical modelling be set and fitted into the ATP as an independent topic in the mathematics scope of work to be covered.

Learners in this study specified the English language as a strong barrier to understanding mathematics concepts and having a huge negative impact on their academic performance. Moschkovich (2012) mentioned that to support learners in achieving English language proficiency, especially in mathematics, there are two general domains: viewing language as a resource, not a deficit, and emphasizing academic achievement more than learning English as a language separately. Therefore, this study recommends that to assist learners in understanding mathematics language, mathematics teachers should expose learners to mathematics discussions drawn on various resources that focus on important concepts of mathematics that challenge learners' development of logical reasoning skills and create a conducive learning environment for peer discussions that may also have the potential to develop learners' ability to describe patterns, generalize, and use presentations to support their statements or claims. In addition, teachers may also use mathematics tasks that develop and maintain high cognitive demand, activities that allow learners opportunities to engage in mathematical practices, abstract reasoning, constructing feasible arguments, evaluating peer reasoning, modelling with mathematics, strategically using teaching and learning resources, and approaches that are appropriate to develop learners' confidence to solve mathematics problems extracted from real situations and develop learners' beliefs that mathematics is rational, meaningful, and doable.

6.4 For future studies focus

I recommend that the future study may increase the number of learners to participate since this study was limited to 87 learners, and the number of teachers to teach modelling groups also be increased following that this study had one teacher who taught modelling, to allow sharing of ideas between teachers teaching modelling and the researcher. Future studies may also focus on learners in Grade 10 and below to examine teachers' implementation of modelling when teaching mathematics because learners reported that they had never been exposed to modelling before and showed no background knowledge based on modelling. It is not clear why teachers are not implementing modelling in the mathematics classroom. For future studies, I recommend that future studies also interrogate barriers to teaching mathematics through modelling. It is not clear why teachers' professional development workshops are not emphasizing mathematical modelling.

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APPENDICES

Appendix A: KwaZulu Natal department of basic education permission letter.

KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE HEAD OF DEPARTMENT

Private Bag X9137, PIETERMARITZBURG, 3200
Anton Lembede Building, 247 Burger Street, Pietermaritzburg, 3201
Tel: 033 392 1063
Email: Phindile.duma@kzndoe.gov.za

Enquiries: Phindile Duma **Ref.**:2/4/8/4102

Mr S Ngubane
PO Box 161
KRANSKOP
3268

Dear Mr Ngubane

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"EXPLORING THE EFFECT OF MATHEMATICAL MODELLING AND ITS APPLICATION IN ALGEBRAIC FUNCTIONS ON GRADE 11 LEARNERS' PERFORMANCE, TEACHERS' SELF-EFFICACY AND THEIR OVERALL PERCEPTIONS"**, in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 20 July 2022 to 30 June 2025.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

ILEMBE DISTRICT

Mr GN Ngcobo
Head of Department: Education
Date: 22 July 2022

GROWING KWAZULU-NATAL TOGETHER

Appendix B Learners' participants information sheet.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

Learner's Information Sheet

Dear Sir / Madam

My name is Mr. Ngubane Sibongiseni. I am a registered student for a Master of Education by Research in Mathematics discipline in the Wits School of Education (WSoE) at the University of the Witwatersrand, Johannesburg. My supervisor is Dr G Ekol. I am conducting research study about mathematical modelling. The study title is **"The Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions"**.

I am inviting you to take part in the study about mathematical modelling. If you agree, you will write a pre-test for 40 minutes, then followed by 8 successive lessons using mathematical modelling cycle and GeoGebra for a selected group, thereafter, will write a post-test (same content as pre-test) after intervention programme to identify the difference between learners' performance before and after intervention lessons. After writing a post-test, six (6) selected learners will undergo individual face-to-face audio-recorded interview that will take 15 – 30 minutes based on mathematical modelling intervention only, and your identifying details will not be used and that of the school during interviews. Each lesson will take about 40 minutes after normal school teaching and learning time. All study activities will take place within school premises for 40 minutes after school hours (14h30 to 15h10) for a maximum of two weeks (school days only). All activities of the study will not contribute to your School-Based Assessment.

Participation to the study is completely voluntarily and confidentially. All data collected and used in this study will be kept safe in a password-locked computer and will be used for my study purposes only. When I share the results of the research study, I will not include your name or anything else that could identify you. With your permission, other researchers may use the data collected from this research study, but your name and any other personal information will not be used or passed on to the third party.

If you agree to participate into the study, you can stop being in the study at any time without any penalty and giving any reason for withdrawal from the study, but you are kindly requested to inform the researcher about your withdrawal. You will not get any direct benefits by joining the research study. You will not lose any services, benefits, or rights you would normally have if you refused to join the study.

There are no foreseeable risks in participating in this study. All participants will not be paid for this study and will not pay anything to participate in the study. If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Your participation will be greatly appreciated.

Yours sincerely,
Mr. Sibongiseni Ngubane

Should you need any further information, please do not hesitate to contact me or my supervisor using the contacts provided below:

Name	**Contact no.**	**Email address**
Mr S. Ngubane (Student)	082 794 0598	810695@students.wits.ac.za
Dr G. Ekol (Supervisor)	011 717 3055	george.ekol@wits.ac.za

Appendix C Learner accent.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

LEARNER'S ASSENT FORM

Title: ***Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy, and their Overall Perceptions.***

Researcher: Mr Sibongiseni Ngubane

I..………………………………………………..... (learner's name) agree to participate in this research project. The research was explained to me, and I understand what my participation will involve. I agree to the following:

No.		Yes	No
1.	The research study was explained to me. I understand what this study is about.		
2.	I agree to participate in all study activities stated by the researcher for a maximum of two (2) weeks.		
3.	I understand that all research activities will take place within school premises for 40 minutes after school hours.		
4.	I understand that participation to the study will not contribute to my School-Based Assessment and will not disturb my studies.		
5.	I understand that my participation to the study is voluntary.		
6.	I understand that I will not be paid or pay anything to participate to the study.		
7.	I agree that the interview may be audio-recorded without using any other my identification details.		
8.	I agree that direct quotations from my interview may be used anonymously by the researcher in their research report.		
9.	I agree that my participation will remain confidential and anonymous (no personal details will be used by the researcher in their research report).		
10.	I agree that other researchers may use the information I will provide during interview (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on.		

I have read this assent form and have been given the opportunity to ask questions. I will also be given a copy of this assent form for my records. I assent to participate in the research study described from the information sheet.

________________	_______________________________	____/____/2022
Learner's Signature	Learner's Printed Name	Date
________________	_______________________________	____/____ /2022
Researchers' Signature	Researchers' Printed Name	Date

Appendix D: Parents participants information sheet

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

Parent's Information Sheet

Dear Sir / Madam

My name is Mr. Ngubane Sibongiseni. I am a registered student for a Master of Education by Research in Mathematics discipline in the Wits School of Education (WSoE) at the University of the Witwatersrand, Johannesburg. My supervisor is Dr G Ekol. I am conducting a research study about mathematical modelling. The study title is **"Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy and their Overall Perceptions"**.

I am inviting your child to take part in the study about mathematical modelling. If you agree, your child will write a pre-test for 40 minutes, then followed by 8 successive lesson lessons using mathematical modelling cycle and GeoGebra for a selected group, thereafter, will write a post-test (same content as pre-test but) after intervention programme to identify the difference between learners' performance before and after intervention lessons. After writing a post-test six (6) selected learners will undergo individual face-to-face audio recorded interview that will take 15 – 30 minutes based on mathematical modelling intervention only, and there will be no identifying details of your child will used and that of the school during interviews. Each lesson will take about 40 minutes after normal school teaching and learning time. All study activities will take place within school premisses for 40 minutes after school hours (14h30 to 15h10) for a maximum of two weeks (school days only). All activities of the study will not contribute to learners School Based Assessment.

Participation to the study is completely voluntarily and confidentially. All data collected and used in this study will be kept safe in a password-locked computer and will be used for my study purposes only. When I share the results of the research study, I will not include your child's name or anything else that could identify your child. With your permission, other researchers may use the data collected from this research study, but your child's name and any other personal information will not be used or passed on to the third party.

If you agree that your child can participate into the study, your child can stop being in the study at any time without any penalty and giving any reason for withdrawal from the study, but you are kindly requested to inform the researcher about your child's withdrawal. Your child will not get any direct benefits by joining the research study. Your child will not lose any services, benefits, or rights they would normally have if your child does not join the study.

There are no foreseeable risks in participating in this study. All participants will not be paid for this study and will not pay anything to participate in the study. If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Your participation will be greatly appreciated.

Yours sincerely,
Mr. Sibongiseni Ngubane

Should you need any further information, please do not hesitate to contact me or my supervisor using the contacts provided below:

Name	Contact no.	Email address
Mr S. Ngubane (Student)	082 794 0598	810695@students.wits.ac.za
Dr G. Ekol (Supervisor)	011 717 3055	george.ekol@wits.ac.za

Appendix E: Parents' consent form.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

PARENT' CONSENT FORM

Title: ***Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy, and their Overall Perceptions.***

Researcher : Mr Sibongiseni Ngubane

I..………………………………………………...., (parent/guardian's name) agree that my child……………………………………………… (learner's name) will participate in this research project. The research has been explained to me and I understand what my child's participation will involve. I agree to the following:

No.		**Yes**	**No**
1.	The research study was explained to me. I understand what this study is about.		
2.	I agree that my child will participate in all study activities stated by the researcher for a maximum of two (2) weeks.		
3.	I understand that all research activities will be done within school premises for 40 minutes after school hours.		
4.	I understand that participation to the study will not contribute to my child's School-Based Assessment and will not disturb my child's study.		
5.	I understand that my child's participation to the study is voluntary.		
6.	I understand that my child will not be paid or pay anything to participate to the study.		
7.	I agree that the interview may be audio recorded without using any other identification details of my child.		
8.	I agree that direct quotations from my child's interview may be used anonymously by the researcher in their research report.		
9.	I agree that my child's participation will remain confidential and anonymous (no personal details will be used by the researcher in their research report).		
10.	I agree that other researchers may use the information my child provides during interview (depending on their own ethics clearance being obtained) but my child's name and any personal information will not be used or passed on.		

I have read this consent form and have been given the opportunity to ask questions. I will also be given a copy of this consent form for my records. I consent to my child's participation in the research study described from the information sheet.

________________	________________________________	____/____/2022
Parent's/Guardian's Signature	Parent's/Guardian's Printed Name	Date
________________	________________________________	____/____/2022
Learner's Signature	Learner's Printed Name	Date
________________	________________________________	____ /____ /2022
Researchers' Signature	Researchers' Printed Name	Date

Appendix F: Principals' information sheet

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

RE: Request permission to conduct a research study at your school

The principal

Dear Sir / Madam

My name is Ngubane Sibongiseni. I am registered for a Master of Education by Research in Mathematics discipline at tl School of Education in the University of the Witwatersrand, Johannesburg. My supervisor is Dr G. Ekol (the senior lectur in Mathematics Division at WSoE).

I am conducting research study on this title: **"Exploring the Effect of Mathematical Modelling and i Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy and the Overall Perceptions".**
The aim of this study is to explore the effect of mathematical modelling on Grade learners' performance in algebra functions, teachers' self-efficacy and both learners and teachers' overall perceptions about mathematical modelling inli with the CAPS curriculum.

I kindly request you to grant me the opportunity to conduct the above-mentioned research in your school and yo permission to invite teachers who are currently teaching mathematics in senior and FET phase as well as Grade 11 learne doing mathematics to participate in the study. If they agree, teachers will have to answer a questionnaire about the knowledge of Mathematical Modelling that will take approximately 15-30 minutes of their spare time, wherea learners will be engaged in teaching and learning through modelling, writing pre-test, and post-test. If learne do not like to be taught by the researcher class observations with the teacher teaching will be conducted. Tes will not contribute to learners' marks School-Based Assessment (SBA), will be used for the research purpos only. Thereafter, with your permission I would like to also conduct individual face-to-face semi-structure interviews with 6 learners at school after normal teaching and learning hours which will take about 15-3 minutes, without seeking any identifying details of both learners and of the school.

All teachers and learners above 18 years who will agree to participate will be required to sign a written conse form before they participate to the study. For learners under the age of 18 years, their parents will be requeste to give permission to their children to participate in the study by signing a consent form, then leaners will als sign assent forms if they agree to participate to the study. Learners under the age of 18 years will not be allowe to participate to the study if their parent did not give a consent even if those learners insist to participate.

This research study will take a maximum of two (2) weeks. It will start in a third term of schooling, and class will not interfere with normal teaching and learning hours as they will be conducted forty (40) minutes aft school hours inside school premises. Participants will be asked to give their written consent before the researc begins. Participants' responses will be treated confidentially, and identities **(names and the name of the schoo** will be anonymous unless otherwise expressly indicated. Individual privacy will be maintained in all publishe and written data resulting from the study.

Data that will be collected in this study once are recognized, possible strategies to lighten the challenges will l recommended that will assist the Department of Basic Education CAPS curriculum, teachers, and learners : improving the teaching and learning of mathematical modelling. All data collected will be stored in a passwo locked computer and only the researcher will have access to it. The research participants will not be advantage or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time durir this project without any penalty. There are no foreseeable risks in participating in this study. All participants wi not be paid for this study and will not pay anything to participate in the study. This research study will be writt up as a research report. Feedback procedure will involve providing your school with a copy (email) of the research stuc written report upon request.

I kindly request your permission in writing to conduct my research at your school. The permission letter should be on your school's headed paper, signed, dated, and specifically referring to myself by name and the title of my study.

Your permission to conduct this study in your school will be greatly appreciated.

Your sincerely,
Mr Sibongiseni Ngubane
Should you need any further information, please do not hesitate to contact me or my supervisor using the contact details below:

Name	Contact no.	Email address
Mr S. Ngubane (Student)	082 794 0598	810695@students.wits.ac.za
Dr G. Ekol (Supervisor)	011 717 3055	george.ekol@wits.ac.za

Appendix G   Schools' permission letter

INGOBAMAKHOSI HIGH SCHOOL

Enquiries:
The Principal
Tel: 0793148527

PO Box 279
Gingindlovu
3800
Email: ingobamakhosi@webmail.co.za

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193
Date: 23 August 2022

Dear Mr. Ngubane Sibongiseni

PERMISSION TO CONDUCT RESEARCH IN ABOVE HEADED SCHOOL

Your application to conduct research entitled: **"Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy, and their Overall Perceptions"** has been approved. The conditions of the approval are as follows:

- The research study was explained to me. I understand what this study is about.
- The researcher will make all the arrangements concerning the research and interviews.
- The researcher must ensure that educators and learning programs are not interrupted.
- The research activities are not conducted during the time of writing examinations in the school.
- Participation will remain confidential and anonymous (No learners' and teachers' identifying details and that of the school will be used by the researcher in their research report in any way from the results of the research).
- Participation is voluntarily. Educators, and learners are under **NO** obligation to participate or assist you in your investigation.
- The period of investigation is limited to two (2) weeks from the starting date of the investigation.
- The researcher cannot start data collection without providing Ethical Clearance Letter from the WSoE Ethics Committee.
- The researchers may use the data collected from this proposed study and interviews depending on their own ethics clearance being obtained but teachers, learners and school identifying details will not be used or passed on.

Sincerely,

Mr M.C Mazibuko (Principal)

ISIFUNDAZWE SAKWAZULU-NATAL
INGOBAMAKHOSI HIGH SCHOOL
23 AUG 2022
PO BOX 279 GINGINDLOVU 3800
PROVINCE OF KWAZULU-NATAL

1 | Page

ISANDLWANA TECHNICAL HIGH SCHOOL

P. O. Box 951
Gingingdlovu
3800

Tel/Fax: 035 337 1590

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193
Date: 20 August 2022

Dear Mr. Ngubane Sibongiseni

PERMISSION TO CONDUCT RESEARCH IN ABOVE HEADED SCHOOL

Your application to conduct research entitled: **"Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy, and their Overall Perceptions"** has been approved. The conditions of the approval are as follows:

- The research study was explained to me. I understand what this study is about.
- The researcher will make all the arrangements concerning the research and interviews.
- The researcher must ensure that educators and learning programs are not interrupted.
- The research activities are not conducted during the time of writing examinations in the school.
- Participation will remain confidential and anonymous (No learners' and teachers' identifying details and that of the school will be used by the researcher in their research report in any way from the results of the research).
- Participation is voluntarily. Educators, and learners are under **NO** obligation to participate or assist you in your investigation.
- The period of investigation is limited to two (2) weeks from the starting date of the investigation.
- The researcher cannot start data collection without providing Ethical Clearance Letter from the WSoE Ethics Committee.
- The researchers may use the data collected from this proposed study and interviews depending on their own ethics clearance being obtained but teachers, learners and school identifying details will

Sincerely,

Mr.M.S.Mjadu Deputy Principal)

DEPARTMENT OF EDUCATION
ISANDLWANA HIGH SCHOOL
P.O BOX 758 GINGINDLOVU 3800
TEL/FAX: 035 337 1590
KWAZULU NATAL PROVINCE

ISINYABUSI
TECHNICAL HIGH SCHOOL

Postal Address: P.O. Box 471, Gingindlovu, 3800
EMIS: 168 3
For enquires: W. B Lus iozi : 079 676 3931

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193
Date: 17 August 2022

Dear Mr. Ngubane Sibongiseni

PERMISSION TO CONDUCT RESEARCH IN ABOVE HEADED SCHOOL

Your application to conduct research entitled: **"Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Alg braic Functions, Techers' Self-Efficacy, and their Overall Perceptions"** has been approved. The condi ons of the approval are as follows:

- The research study was explained to me. I understand what this study is about.
- The researcher will make all the arrangements concer ing the research and interviews.
- The researcher must ensure that educators and learnin programs are not interrupted.
- The research activities are not conducted during the ti ne of writing examinations in the school.
- Participation will remain confidential and anonymous (No learners' and teachers' identifying details and that of the school will be used by the researcher in their research report in any way from the results of the research).
- Participation is voluntarily. Educators, and learners are under **NO** obligation to participate or assist you in your investigation.
- The period of investigation is limited to two (2) weeks from the starting date of the investigation.
- The researcher can not start data collection without providing Ethical Clearance Letter from the WSoE Ethics Committee.
- The researchers may use the data collected from this proposed study and interviews depending on their own ethics clearance being obtained but teacher , learners and school identifying details will not be used or passed on.

Sincerely,

W. B. Lushozi	[signature]	17/08/2022	DEPARTMENT OF EDUCATION ISINYABUSI TECHNICAL H. SCHOOL 2022 -08- 17 P. O. BOX 4 1 GINGINDLOVU 3800
Principal's initial & surname	Principal's signature	Date	school stamp

MANQAMU HIGH SCHOOL
Postal Address: P. O. Box 10124, Meerensee, 3901
EMIS: 500197284
Tel/Fax: 035 773 0655

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193
Date: 22 September 2022

Dear Mr. Ngubane Sibongiseni

PERMISSION TO CONDUCT RESEARCH IN ABOVE HEADED SCHOOL

Your application to conduct research entitled: **"Exploring the Effect of Mathematical Modelling and its Applications in Algebraic Functions on Grade 11 learners' Performance,"** has been approved. The conditions of the approval are as follows:

- The research study was explained to me. I understand what this study is about.
- The researcher will make all the arrangements concerning the research and interviews.
- The researcher must ensure that educators and learning programs are not interrupted.
- The research activities are not conducted during the time of writing examinations in the school.
- Participation will remain confidential and anonymous (No learners' and teachers' identifying details and that of the school will be used by the researcher in their research report in any way from the results of the research).
- Participation is voluntarily. Educators, and learners are under **NO** obligation to participate or assist you in your investigation.
- The period of investigation is limited to two (2) weeks from the starting date of the investigation.
- The researcher cannot start data collection without providing Ethical Clearance Letter from the WSoE Ethics Committee.
- The researchers may use the data collected from this proposed study and interviews depending on their own ethics clearance being obtained but teachers, learners and school identifying details will not be used or passed on.

Sincerely,

DEPARTMENT OF EDUCATION
MANQAMU HIGH SCHOOL
22 - 09 - 2022
P.O BOX 10124 MEERENSEE 3902
TEL/FAX: 035 773 0655
KWAZULU NATAL PROVINCE

Administrator' Signature 22/09/2022 Date

1 | Page

LAMBOTHI HIGH SCHOOL
P/Bag 2104 Cell: 083 249 5242
NYONI
3802
Email: lambothischool@gmail.com

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193
Date: 24 August 2022

Dear Mr. Ngubane Sibongiseni

PERMISSION TO CONDUCT RESEARCH IN ABOVE HEADED SCHOOL

Your application to conduct research entitled**: "Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy, and their Overall Perceptions"** has been approved. The conditions of the approval are as follows:

- The research study was explained to me. I understand what this study is about.
- The researcher will make all the arrangements concerning the research and interviews.
- The researcher must ensure that educators and learning programs are not interrupted.
- The research activities are not conducted during the time of writing examinations in the school.
- Participation will remain confidential and anonymous (No learners' and teachers' identifying details and that of the school will be used by the researcher in their research report in any way from the results of the research).
- Participation is voluntarily. Educators, and learners are under **NO** obligation to participate or assist you in your investigation.
- The period of investigation is limited to two (2) weeks from the starting date of the investigation.
- The researcher cannot start data collection without providing Ethical Clearance Letter from the WSoE Ethics Committee.
- The researchers may use the data collected from this proposed study and interviews depending on their own ethics clearance being obtained but teachers, learners and school identifying details will not be used or passed on.

Lambothi
High
School
Physical Address: Macambini Tribal Authority
24 AUG 2022
Next to MLOTSHWA CASH STORE
NYONI 3802
Email: lambothischool@gmail.com
PHONE: 068 568 3356

Sincerely,

Mr E.N Ndlovu (School Principal)

Appendix H: Teachers' information sheet.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

Teacher's Information Sheet

Dear Sir / Madam

My name is Ngubane Sibongiseni. I am a registered student for a Master of Education by Research in Mathematics discipline in the Wits School of Education (WSoE) at the University of the Witwatersrand, Johannesburg. My supervisor is Dr G Ekol (senior lecturer in Mathematics Division). I am conducting a research study about mathematical modelling. The study title is **"Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy and their Overall Perceptions"**.

I am inviting you to take part in the study by answering a questionnaire about mathematical modelling. If you decide to take part, your participation in this research study will last about 15–30 minutes of your spare time. The completion of a questionnaire will take place at your own convenient place and time.

The questionnaire will be confidential and anonymous. When I share the results of the research study, I will not include your name or anything else that could identify you. The data collected will be used for this study purposes only. With your permission, other researchers may use the data collected from this research study, but your name and any personal information will not be used or passed on to the third party.

If you decide to take part in the research study, it should be because you want to volunteer. You can stop being in the study at any time without any penalty and giving any reason for your withdrawal from the study but please inform the researcher about your withdrawal. You do not have to answer any questions if you do not want to. You will not get any direct benefits if you choose to join the research study. You will not lose any services, benefits, or rights you would normally have if you decided not to join.

There are no foreseeable risks in participating in this study. All participants will not be paid for this study and will not pay anything to participate in the study. This research study will be written up as a research report. Feedback procedure will involve providing your school with a copy (email) of the research study written report upon request.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Your participation will be greatly appreciated.

Yours sincerely,
Mr. Sibongiseni Ngubane

Should you need any further information, please do not hesitate to contact me or my supervisor usin
he contact details below:

Name	Contact no.	Email address
Mr S. Ngubane (Student)	082 794 0598	810695@students.wits.ac.za

Appendix I: Teacher consent form.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

TEACHERS' CONSENT FORM

Exploring the Effect of Mathematical Modelling and its Applications on Grade 11 learners' Performance in Algebraic Functions, Techers' Self-Efficacy, and their Overall Perceptions

Researcher : Mr Sibongiseni Ngubane
Supervisor : Dr George Ekol

No.		**Yes**	**No**
1.	The research study was explained to me. I understand what this study is about.		
2.	I understand I will have to complete a test questionnaire of approximately 15 - 30 minutes.		
3.	I understand that I can volunteer to take part in the study.		
4.	I agree that the interview may be audio recorded.		
5.	I agree that direct quotations from my interview may be used anonymously by the researcher in their research report.		
6.	I agree that my participation will remain confidential and anonymous (my personal details will not be used by the researcher in their research report)		
7.	I agree that other researchers may use the information I provide in my interview (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on.		

I have read this consent form and have been given the opportunity to ask questions. I will also be given a copy of this consent form for my records. I consent to my participation in the research study described from the information sheet.

___________________	________________________________	____/____/2022
Participant's Signature	Participant's Printed Name	Date
___________________	________________________________	____/____/2022
Researchers' Signature	Researchers' Printed Name	Date

Appendix J: Questionnaire.

1. BACKGROUND INFORMATION

1.1 Gender: Male ☐ Female ☐

1.2 Your age group: below 26 or < 26 ☐ 26-30 ☐ 31-35 ☐ 36-40 ☐ 41-45 ☐ 46-50 ☐ above 50 ☐

1.3 Level of qualification: Metric ☐ Diploma ☐ Degree ☐ Honours ☐ Masters ☐ PhD ☐

1.4 Your years of teaching experience:

5or < 5 years ☐ 6-10 years ☐ 11-15 years ☐ 16-20 years ☐ 20 or >20 years ☐

1.5 Specify your mathematics teaching experience (e.g. 3 years):…………………………

1.6 The average class size: 10-15 ☐ 15-20 ☐ 21-25 ☐ 26-30 ☐ 31-35 ☐ 35-40 ☐ 41-45 ☐ 46-50 ☐ above 50 ☐

2. Teachers' awareness of modelling in CAPS

Please check one box next to each statement provided below.

Questions	**Yes (aware)**	**No (unaware)**
Have you seen CAPS document syllabus or scheme of work?	☐	☐
Are you aware of the difference between old curriculum (NCS) and new curriculum (CAPS)?	☐	☐
Can you correctly state the philosophy for the new curriculum (CAPS)?	☐	☐
Have you ever heard about mathematical modelling?	☐	☐
Have you seen CAPS specific aims of teaching and learning?	☐	☐
Are you aware that modelling is introduced in CAPS syllabus?	☐	☐
Are you aware that to teach mathematics through modelling is compulsory across the grades?	☐	☐
Have you taught mathematics through modelling processes?	☐	☐
Have you received any training or workshop about implementation of modelling in mathematics teaching and learning?	☐	☐

3. Professional development about CAPS

Please check one box next to each statement provided below.

Questions	Never	Sometimes	Always
Do you attend teachers' professional development workshops?	☐	☐	☐
Do professional workshop emphasis teaching through modelling?	☐	☐	☐
Have you received mathematical modelling support documents?	☐	☐	☐
Have you received training on how to set mathematical modelling tasks?	☐	☐	☐

4. Teachers' beliefs about mathematical modelling.

Please check the one box next to one statement below

To what extent do you agree with the following statements?	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
Mathematical modelling should be a part of mathematics education.	☐	☐	☐	☐	☐
Results of mathematical modelling have a general, fundamental benefit for society.	☐	☐	☐	☐	☐
Learners should be given the opportunity to do mathematical modelling in mathematics education.	☐	☐	☐	☐	☐
Learners learn mathematics best by discovering ways to solve problems themselves.	☐	☐	☐	☐	☐
Mathematical modelling is a futile game, an engagement with objects with no concrete relation to reality.	☐	☐	☐	☐	☐
Effective teachers demonstrate the right way and methods to solve an application problem.	☐	☐	☐	☐	☐
Learners should usually be required to solve tasks in the way they were taught in class.	☐	☐	☐	☐	☐
You should allow learners to come up with their own ways of solving application problems before the teacher shows how to solve them.	☐	☐	☐	☐	☐
Among other competences, the competence for mathematical modelling should be taught in the classroom.	☐	☐	☐	☐	☐
Learners should often have the opportunity to follow their teacher's model solutions.	☐	☐	☐	☐	☐
Mathematics should be taught at school in such a way that learners can discover connections on their own.	☐	☐	☐	☐	☐
Many aspects of mathematical modelling have a practical use or a direct application reference.	☐	☐	☐	☐	☐
Mathematical modelling should be a specific component in mathematics education.	☐	☐	☐	☐	☐
It helps students to understand mathematics when they are asked to discuss their own solution ideas.	☐	☐	☐	☐	☐
In mathematical modelling, you work on tasks that have practical value.	☐	☐	☐	☐	☐
Teachers should provide detailed procedures for solving application problems.	☐	☐	☐	☐	☐

5. Teachers' Self-efficacy

Please check one box in each line

To what extent do you agree with the following statements?	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
It is easy for me to recognise the different abilities of the learners using…					
... their translation of mathematical results into reality.	☐	☐	☐	☐	☐
... their written solutions when modelling.	☐	☐	☐	☐	☐
... the recording of the plausibility test of their solution.	☐	☐	☐	☐	☐
... the adequate assessment of the relationship between the mathematical result and reality that they produce.	☐	☐	☐	☐	☐
... the recording of the solutions they create for modelling tasks.	☐	☐	☐	☐	☐
... the mathematical models they chose when modelling.	☐	☐	☐	☐	☐
... the recording of their mathematical results in the modelling process.	☐	☐	☐	☐	☐
... the mathematical formulae and symbols they used in the modelling process.	☐	☐	☐	☐	☐
... the determination of their improvement of the established models presented during modelling.	☐	☐	☐	☐	☐
... the assumptions they made when modelling.	☐	☐	☐	☐	☐
... the students' solutions when modelling.	☐	☐	☐	☐	☐
... their handling of the mathematical symbols and operators used in modelling.	☐	☐	☐	☐	☐

To what extent do you agree with the following statements?	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
It is easy for me to recognise the different abilities of the learners using…					
... their translation of mathematical results into reality.	☐	☐	☐	☐	☐
... their written solutions when modelling.	☐	☐	☐	☐	☐
... the recording of the plausibility test of their solution.	☐	☐	☐	☐	☐
... the adequate assessment of the relationship between the mathematical result and reality that they produce.	☐	☐	☐	☐	☐
... the recording of the solutions they create for modelling tasks.	☐	☐	☐	☐	☐
... the mathematical models they chose when modelling.					

... the recording of their mathematical results in the modelling process.	☐ ☐ ☐ ☐ ☐
... the mathematical formulae and symbols they used in the modelling process.	
... the determination of their improvement of the established models presented during modelling.	
... the assumptions they made when modelling.	☐ ☐ ☐ ☐ ☐
... the students' solutions when modelling.	☐ ☐ ☐ ☐ ☐
... their handling of the mathematical symbols and operators used in modelling.	

Comments (for additional information about modelling)

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The end!

Appendix K: Observation sheet.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

Observation sheet

No	Observation questions	Yes / No
1	Do learners understand or show understanding of the problem?	
2	Are learners able to scan the problem to identify the key words or aspects of the problem?	
3	Can learners decide which information is relevant or irrelevant towards answering the question?	
4	Are learners able to generate a model (equation) using mathematical symbols that can help them to solve the question?	
5	Are learners able to re-organise the problem into smaller parts to start solving the problem?	
6	Are the learners able use the generated model to solve the problem?	
7	Can learners think of what mathematics can be used to solve problem?	
8	Can learners check whether the answer makes sense?	
9	Can learners give the answer in a way that relates to the problem asked?	
10	Are learners able to present a problem graphically, interpret a graph and give the correct answer relative to the problem at hand?	

______________________ ______________________
Researcher' signature | Date

Contact details:
Researcher: Mr Ngubane Sibongiseni
Contact number: 0827 794 0598
Email address: 810695@students.wits.ac.za

Appendix L: Pre-test and Post-test.

Research Title: The Effects of Mathematical Modelling and its Application in Algebraic Functions on Grade 11 Learners' Performance.

Researcher: Mr Ngubane Sibongiseni

Supervisor: Dr. G. Ekol

Mathematical Modelling Task

CODE:

Grade: 11
Task: Pre-test
Section: Algebraic functions
Total: 30 Marks
Duration: 40 minutes

1. A farmer has 100 metres of wire fencing from which to build a rectangular chicken run. He intends using two adjacent walls for two sides of the rectangular enclosure.

   (a) Determine a formula for the enclosed area in terms of x.

   (b) Determine the dimensions which will give a maximum enclosed area.

   (c) Hence or otherwise, determine the maximum enclosed area graphically, and show all workings

2. The area of a classroom is 2 20m. If the length is increased by 3m and the width is increased by 1m, the classroom will double in area. Determine the new dimensions of the new classroom, new equation of the area, and find maximum enclosed area graphically.

3. A bricklayer and his apprentice build a wall in 24 days. When each person works separately, the apprentice takes 20 days longer than the bricklayer to complete the job.

Calculate the number of days each person takes to complete the job on his own.

4. The area of a rectangle is 2 96m and has a perimeter of 39,2m. Determine the dimensions of the rectangle that will give a maximum area graphically.

5. A lady travels 180km from her wine farm to Ceres. On the return journey she travels at night and travels 30km/h slower. She lost 1 hour on the return journey by travelling slower.
At what speed did she travel back home?

6. Mr B is now twice as old as his younger brother. Eight years ago, he was three times as old as his younger brother was then. How old is the younger brother now?

The End!

Research Title: The Effects of Mathematical Modelling and its Application in Algebraic Functions on Grade 11 Learners' Performance.

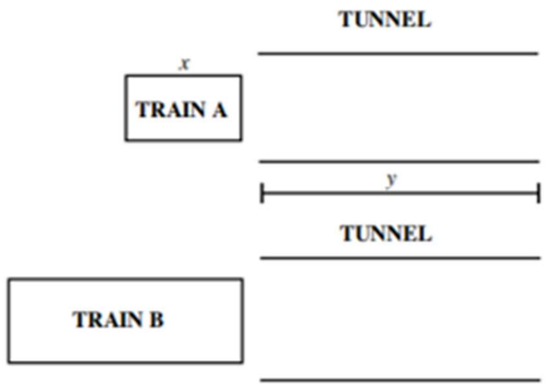
Researcher: Mr Ngubane Sibongiseni

Supervisor: Dr. G. Ekol

CODE:

Grade: 11
Task: Post Test
Section: Algebraic functions
Total: 30 Marks
Duration: 40 Minutes

1. TRAIN A passes completely through a tunnel in 4 minutes. A second train, TRAIN B, twice as long as the first, passes completely through the tunnel in 5 minutes. Both trains are travelling at the same speed, namely, 48km/h. Consider the diagram below. In the diagram y represents the length of the tunnel and x the length of **TRAIN A**.



In the diagram y represents the length of the tunnel and x the length of TRAIN A.
[Additional information: $\text{TIME} = \dfrac{\text{DISTANCE}}{\text{SPEED}}$]

(a) Determine the length of TRAIN A in km

(b) Determine the length of the tunnel in km

2. This question is designed to show you that you can set up your own model from raw data. The data in the table was used to generate the graphs you used in Question 2.

Year	Number of years since 1996	Number of deaths per year in South Africa
1997	1	316 559
1998	2	365 109
1999	3	381 037
2000	4	414 768
2001	5	500 082
2002	6	554 199
2003	7	572 620
2004	8	593 337
2005	9	605 408

a. Using your calculator or Excel, generate each of these models: $f(x) = a + bx$; $g(x) = a + bx + cx^2$; $h(x) = ab^x$.

b. Determine the number of years since 1996 if the year is 2020.

c. Now determine the anticipated number of deaths in 2020 for each different model.

d. Briefly discuss your findings and comment on the different outcomes.

3. Vincent has money to invest and considers two options:

Option 1: Simple interest at a rate of 13,5% using the formula $A = P(1 + in)$

Option 2: Compound interest at a rate of 7,5% p.a. compounded annually using the formula $A = P(1 + i)^n$

a. Vincent wants to know how long it will take for the two options to become equal.

i. Set up an equation which would help him to work out how long it would take.

ii. Explain why is makes no difference how much money he invests at the start.

b. Consider the formula: $A = 1 + in$ | Ignore P as it does not affect the calculation.

i. Rewrite the formula to express it in terms of x and y.

ii. Is the graph a straight-line graph or an exponential graph?

c. Consider the formula: $A = (1 + i)^n$ | ignore P as it does not affect the calculation.

i. Determine $(1 + i)$ and give your answer in its simplest form

ii. Rewrite the formula to express it in terms of x and y.

iii. Is the graph a straight-line graph or an exponential graph?

d. Sketch both graphs on the same system of axes, clearly labelling the graphs and indicating the points used to plot the graph.

e. Use your graph to estimate the number of years it will take for the investments to be equal. Give your answer to the nearest year.

4. Nyati makes up his own graph application question for a maths project. He decides to compare the height of a stone that is thrown up into the air, against the time taken by the stone to reach that height. After several trials he records these results, with t being the time in seconds and h the height of the stone in metres.

t	0	1	2	3	4	5
h	15	20	21	18	11	0

a. How long did it take for the stone to hit the ground?

b. Draw a graph to represent the information in the table.

c. Determine an equation which models the relationship and state your answer in the form

$h = \cdots$

d. After how many seconds did the stone reach its maximum height?

e. What was the maximum height reached?

The end!

Appendix M: Teachers' questionnaire online link.

https://docs.google.com/forms/d/e/1FAIpQLScTRXMAj_05_0-30kH2GRlhEiSyPBQlJHtaOdGHw7ApU_GxHQ/viewform?fbzx=-3363522437497371781&pli=1

Appendix N: Interview schedule for the teacher.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Witwatersrand University
27 St Andrews Rd,
Park town, Johannesburg,
2193

Interview questions for the experimental group teacher.

Question 1: What were you satisfied with during the teaching sessions?

Question 2: What would you change if you were to teach the sessions again?

Question 3: What specific challenges did you face teaching using modelling and guided discovery and how did you overcome them?

Question 4: Any other observation/ recommendation you want to make about your experience teaching the sessions?

Question 5: Overall how would rate your level of satisfaction with the teaching sessions including modelling and guided discovery?

Question 6: Do you think you need modelling in algebraic functions and mathematics as a whole and why?

Please, use a scale of 1, 2,3, 4,…,10. 1 is the lowest grade, and 10 is the highest (maximum grade)

___________________ ___________________
Teacher's signature Date

___________________ ___________________
Researcher' signature Date

Contact details:
Researcher: Mr Ngubane Sibongiseni
Contact number: 0827 794 0598
Email address: 810695@students.wits.ac.za

Appendix O: Interview transcript for the teacher.

Interviews transcript with the teacher who taught experimental group.

Duration: Lasted for 24 minutes and 32 seconds.

Question 1: What were you satisfied with during teaching sessions?

Respondent:

- Firstly, I knew this method of modelling cycle is of the very important method to be used in teaching mathematics from CAPS document but then, I never been exposed on it. I never applied it when teaching or during teaching and learning process. So, to me this was a great experience I have gained. It was like, I am new in this method, yet I once saw it, even at tertiary level, I never been exposed to it, so it could not be easy for me to use it when teaching mathematics. So, I was very much interested on it, and I saw that it encourages learners to think independently, and I have been working as a facilitator, so to me it's a great pleasure to be exposed or to be part of these lesson or training.

Question 2 what would you change if you were to teach sessions again?

Respondent:

- The first thing I noticed , this is general but specifically towards modelling cycle, the first thing I can change is the sitting plan or sitting arrangement since learners were grouped, when I group learners there must be a balance in terms of understanding of learners, remember there is fast learners, mediocre learner and slow learner, for those who are slow learners to gain confidence and easily understand the process, there must be mixed with those who are high performers so that it balance and they are able to share information and understanding when they share ideas to grasp the understanding of the context. Also what I can change is length of time on this method of teaching, it requires more time for the learners to understand each an every step when they do it because this is not just a method but it is a process, so for the learner to understand step by step , they must be given enough time to think so that they are able to think and share ideas up until they come up with the solution because what also important is that learners must understand the concept or the process not just knowing the solution, if they know the solution but not understanding the process, it wont be easy for them to arrive at the possible solution.

Question 3. What specific challenges did you face during using modelling and guide discovery and how did you overcome them?

Respondent:

- The first thing, I think the challenge is the language barrier, when I say language barrier I don't mean just English, but how learner should aline themselves so that they understand this process,

- the language which support modelling cycle is much important so that learners will understand what they are expected to do to solve these problems. So I think to deal with that guidance is much important and learners must be given hints, some sort of guiding questions so that learners are able to understand what they are expected, if I can mange to teach them these small skills they will understand the language why are they doing modelling cycle, so that will help them to be more interested and gain confidence in working under this type of learning method.

- The other thing I can say is that, even after the learners got correct answers or possible solutions but they were still failing to relate ,remember this solution were in the form of functions so they were failing to relate it to the original question or even to the real life situation where they practically apply those functions or equations that they developed . that is the of the challenges I experiences. I think to overcome it, I must guid them, guidance is very much important. They must be able to sketch, they must be able to analyse the components of these functions as their solutions. Once they analyse, they understand the concept functions, so they know components of a function, by so they will make meaning of what they are doing and why are they doing it.

Interviewer's follow up question: *To elaborate on the language barrier you have mentioned that there is a language barrier, not necessary meaning that learner they fail to understand English. Did learners manged to answer or understand the question or needed you to assist them?*

Respondent:

- I was needed to assist them in answering as the guidance, some learners show lack of understanding why they are arriving to this solution. I think is still a challenge I needed to address that after finding out this solution. So, I think it is still a challenge that I need to address that after finding out this solution, they understand how they arrived at it and how does it link with the original question. Through understanding these components, then it means they understand the language because the language has to do with the whole process not just knowing English but to understand what steps lead to which step and what will be the possible solution and how does it link to the original question so I think that the language they need to speak so that independently they can work to find possible solutions

Questions 4: Any other observations/ recommendations you want to make about your experience teaching the sessions including modelling and guided discovery?

Respondent:

The first observation, some of the learners had a thought that what we are doing today does not belong to their grade, is like it is too complex for them and I also observe that some they lack interest because they think it is too difficult for them, some they had interest, were interested in

- working through modelling cycle method, such that they were curious to arrive at the answer and to see how is this going to help them in terms of understanding functions because solutions were in a form of functions , some they shown interest at the end where they got a solution and like wow! this is what we were looking for, but also I have notice that they need guidance through out while they are working together sharing ideas, but there must be in a form of leading questions to guide them.

Interviewer: A follow up question emerges from the response. *Was there anything that you noticed in terms of teaching method that you were using since using modelling guided discovery, how learners were interacting with one another?*

Responded:

- At time depends, some they were interacting positively, some were showing less interest but to those who were showing lot of interest they were sharing ideas, asking from me as a teacher and facilitator, they were able to move on and tell, hear we are stacked, how can we move?

Interviewer: Are there any recommendations that you may like to recommend.

Respondent:

- I noticed that this emphasised the problem solving where learners work together and being assisted or facilitated by the teacher. I think I can recommend this teaching method to be adopted so that we teach learners to work independently and gain inner trust and be able to share information and understanding.

Question 5. Overall satisfaction, how would you rate your level of satisfaction with teaching sessions including modelling and guided discovery? Use a scale from 1 to 10, where 1-not satisfied and 10-very satisfied.

Responded:

- I can rate me at level 6, since this was my first time experience, this was a first time training and I needed lot of attention to understand and ensure that there are no mistakes along the process but also modelling cycle and guided discovery principle they are much important to me, in a as much as they seem to be new but I think I am still learning from them and I was developing some skills to lead the learners while working independently alone to find at the possible solution, I also needed to adopt scaffolding principle so that I help my learners where I restructure question, I separate into pieces for them to move correctly following necessary steps and don't mix these steps by rushing into a final answer.

Question 6: *Own your own after you have received training on how to teach learners using modelling and guided discovery approach, you taught learners for eight consecutive lessons*. **Do you think you need modelling in algebraic functions and mathematics, and why?**

Respondent:

- The answer is yes, we need modelling cycle in both algebraic function and other mathematics topics or sections. The reason for that, modelling promote learner centredness. It means that it trains learners to work independently to find possible solutions, also learners can share ideas and work together to find one common understanding based on that specific problem they are given under modelling cycle, and I believe that modelling can bail out the majority of maths teachers from accounting from poor performance of learners, because learner will no longer depend from their teachers to come up with solutions, they learn to be independent and even if the teacher is not Infront of them but they will be able solve different problems and even also to apply these skills they are learning from modelling cycle to solve real problems outside the world.

The end.

Appendix P: Learners' interviews questions.

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Learners' Interview questions

Question 1: How did you find the task? Was it easy or difficulty?

Question 2: Can you describe the problem, which one was the most difficult problem to answer and why doo you think it was difficult?

Question 3: Which problem did you find it easy to answer and do you think it was easy?

Question 4: How did language affected you to solve the problem and in answering the main question?

Question 5: Comparing the problems that you do in class and these of the research, what was the difference?

Question 6: Did you visualise any of the problems you did before identifying the keywords, for example, drawing the diagrams?

Question 7: In respect to the problems, are there specific mathematical symbols, equations or concept to follow to solve a problem?

Question 8: Do you think you need modelling in algebraic functions and mathematics as a whole and why?

Learner's signature	Date
Researcher' signature	Date

Contact details:

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Appendix Q: Transcription of learners' interviews.

INTERVIEWS TRANSCRIPTS FOR LEARNERS

QUESTION 1: HOW DID YOU FIND THE TASK? WAS IT EASY OR DIFFICULT?

LE1: It was difficulty.

LE2: I was able to write though it had that thing of being mathematics, which require to use your brain too much and somewhere somehow, it was challenging.

LE3: I find it difficult since we had to take words, interpret them, then develop a mathematical equation, so some of people are not good in translation words to number, its same thing to me also, I found it very difficult move from words to mathematics. I was clues on how to write words mathematically.

LE4: firstly, I would say it was difficulty since it was my first time to find a question where I am the one who should come up with solutions and there were formulae to use. That was the challenge but if the teacher started to do it, I saw that there was nothing seriously difficult, the I started to understand.

LE5: it was difficult, because this kind of mathematics uses lot of word than symbols and is the methods to use are very hidden or unclear which direction to take.

LE6: I would say it was fair, it was a decent paper, it was fair in some of the questions, I couldn't understand some of them.

QUESTION 2: CAN YOU DESCRIBE THE PROBLEM, WHICH ONE WAS THE MOST DIFFICULT PROBLEM TO ANSWER AND WHY DO YOU THINK IT WAS DIFFICULT?

LE1: The first question was very difficult I think it's because maybe it was my first time to come across this type of mathematics, and the way is done.

LE2: It was the task of the bricklayer and apprentice where they were required to build a wall because, when we got answer, we got two days, then be confused that how can one person build a wall within two days? The stress began.

LE3: The first question of the task I found it difficult, since the farmer wanted to build a chicken run, I had no idea about the structure of it since I never saw it before, I found it difficulty since I grew up in township, I had no idea about building kraal.

LE4: The first question challenged me since was my first time doing this kind of question, I was lost on how I am going to find x and y values since the questions looked like measurements but then it was not and not clear of what first step one need to follow even though there were shapes mentioned (rectangle).

LE5: Bricklayer number 3, I don't know how to put it but, the way in which the question was phrased is more based on real life situations. It is not a normal mathematics that we do in our daily basis and that question was very confusing for me, it took quiet sometime to understand if one wall may be built in one day or more than that, and how may walls in total needed to be built. I was also confused to make a difference between bricklayer and apprentice, the statement was full of unfamiliar words for me, and that took me a lot of my time to think about where to start.

LE6: The first question that was talking about meter, the reason why I didn't understand is because, my teacher didn't teach us more about the measurements, so it looked like measurements even though it's not clear what is needed to answer the question, the challenge is the result of lacking skills in measurements.

QUESTION 3: WHICH PROBLEM DID YOU FIND IT EASY AND WHY DO YOU THINK IT WAS EASY?

LE1: The second problem, because I noticed that it's related to measurements of which is the topic that we have done in class when I was doing grade 10, since it was asking to determine the new dimensions of the new class if the width of the class was increased.

LE2: The easy question was that of a farmer having a fence to which to build a chicken run, even though was challenging because, initially I struggled to make sense of the question then after I manged to use x and y s to develop a model, but the to answer the question after creating a model was very easy.

LE3: The one for the bricklayer and apprentice, I cannot say it was too simple and easy to answer but for me it was easy to follow since the phrasing was not complex and enough information was given though you had to translate from word problem to mathematics equation.

LE4: Question 1 was pre much easier, because I was taught that firstly I must look out for what the given variables are, and then since we were given km/h there required us to solve the length of a tunnel, then apart from mathematics the statement already given a hint of what the formula might look like to solve the problem.

LE5: Question 6 was easy for me, because it was based on years of people, then for me it was the simpler than all questions.

LE6: None. All questions were challenging to me 4.

QUESTION 4: HOW DID THE LANGUAGE AFFECTED YOU TO SOLVE THE PROBLEM AND IN ANSWERING THE MAIN QUESTION?

LE1: The way questions are asked is different since the phrasing and language used is different, questions of the research were not straight forward, I had to think, and they need more time to be understood.

LE2: Yes, English is not my home language which make it difficult to understand properly, so it was difficulty. the language affected me, the phrasing of questions was difficult since I am not good in English.

LE3: In a pre-test there were equal chances to get wrong and right answer, for instance the question for the bricklayer, I was confused either I should start to calculate the number of days separately (bricklayer and apprentice) or group them together in a one equation. But all in all, questions were understandable, but the challenge was where to start.

LE4: No, the language was not a problem. It was very easy to understand the question, but the challenge was where to start to answer the questions.

LE5: English, I have no problem about it, but the challenge was the translation from English words to mathematics symbols or formulae it was difficult for me.

LE6: I don t think the language affected me because every question was asked in English, and I understand English very well.

QUESTION 5: COMPARING THE PROBLEMS THAT YOU DO IN CLASS AND THESE OF THE RESEARCH, WHAT WAS THE DIFFERENCE?

LE1: There was a very big difference, research questions were very challenging because I had to read everything properly, couldn't answer the question since I was confused actual about what exactly the question wants me to do. I can say it was my first time being asked this type of questions.

LE2: Yes, there is a huge difference. Questions of the research are phrased in an unusual manner, whereas those in class are more related and based on what we already have; learnt, even though these also were requiring us to apply the classroom knowledge, but then we took it as that this is research now, that alone make you think your own perspective not related to knowledge learner in class.

LE3: There is a very big difference because research questions are in words even though they give mathematical practical equations, unlike in the classroom because you are taught things and certain methods are used to solve problems, and in the class, you can easily identify entities needed and start working easily, and even given a complete equation where you need to solve a straightforward question. The research task was very different from the classroom tasks because research task had no straightforward instruction unlike in the class. Due to the given information, you can easily differentiate that this is calculus, trigonometry, or even finance due to the structure of the question. The research task is not giving even a clue of what and how the problem should be solved.

LE4: There was a huge difference, because in the classroom during learning the teacher can give us formula that may be used to solve the problem, but in this research, we had to think out of the box so that we would be able to solve problems, so I find research questions very different.

LE5: Too much difference, I can say mathematics that we are doing in our daily basis in class, no matter how solution are hidden or difficult to find methods are easy to follow if we do questions together, but this form of mathematics need one to use the whole brain, bring all focus to the question and it takes much time to make sense in a way that you can be able solve the problem. In most cases you end up not getting the correct solution or straight forward method to solve the problem.

LE6: There is a difference in some of the questions, in a pre-test everything was like general knowledge about not everything but almost everything was about general knowledge, measurements. In some of the questions required general knowledge of measurements, but in class we do activities that are found in textbooks, teachers address thing that are examinable only but thee one of the research projects I never seen any of this kind of questions even from one of the previous and current exam papers.

QUESTION 6: DID YOU VISUALISE ANY OF THE PROBLEMS YOU DID BEFORE IDENTIFYING THE KEYWORDS, FOR EXAMPLE, DRAWING THE DIAGRAMS?

LE1: Yes, question one we ended up drawing a parabolic function that was negative and positive.

LE2: The one for the farmer having a wire to build a chicken run, required us to draw a rectangle, indicate the sides to be covered by the fence, and the part to be covered by the wall.

LE3: Yes, almost all questions were requiring us to draw graphs at the end, even during the time when I was working, it was helpful to draw diagrams to understand and have a full picture of what is happening.

LE4: No, I had no idea of diagram that may help me, I was totally lost where to start.

LE5: number 6 was easy and I was able to do it in my head like all concept was done in my head, but number 5 was not easy and needed me to write down some pieces of rough work to make sense of the question then the answer came out, but my mind was taking very low, so the I drew a diagram to answer number 5 and that helped me.

LE6: I could say no.

QUESTION 7: IN RESPECT TO THE PROBLEMS, ARE THERE SPECIFIC MATHEMATICAL SYMBOLS, EQUATIONS OR CONCEPT TO FOLLOW TO SOLVE A PROBLEM?

LE1: Yes, in question two we used the surface area equations to calculate the area of the class and we used the area are equation to calculate the area of the class if its width was increased by 3 meters.

LE2: We were just doing it generally, and there was no specific method of answering the question/s.

LE3: No, there were no specific method to solve the problem but everyone came up with his/her own method which led to different solutions while doing one task and the questions were not specific, like there was no hint that say now a graph, a rectangle, or use a quadratic equation etc, but when

I was writing down given data some ideas automatically came out and started to try answering questions.

LE4: Yes, I don’t remember well, but there some specific mathematical symbols we used.

LE5: For me, I can say I had to use algebraic functions methods much more because it was easy method for me and which I think helped to solve some problems.

LE6: Yes, I think the instruction was clear enough for one to develop a formula but there was no specific method and direction to which formula can be used to solve, all you do was needed to take it from your mind.

QUESTION 8: DO YOU THINK YOU NEED MODELLING IN ALGEBRAIC FUNCTIONS, MATHEMATICS AS A WHOLE, AND WHY?

LE1: Yes, we need modelling in mathematics because it helps to check the level of people thinking capability, how they manage to solve problem using mathematics, because I believe mathematics does no not solve community problems, the modelling will help to check our psychology and how we process problems to give solutions.

LE2: yeah, modelling should be involved because it helps to expand the thinking capacity, even though it is challenging but it makes a person to think outside the box.

LE3: No, this type of mathematics is too difficult. So, I don’t think that learners can pass it easily because you need to have too much time to solve one question.

LE4: No, I don’t think modelling should be form part of mathematics to be learnt by learners in school because it might be possible for teachers do not cover the scope of work then learner write their exams with outstanding topics then fail. This because modelling is much confusing because you need to find formulae for yourself, since we are used in pure mathematics straight forward questions.

LE5: Yes we need it, because this form of mathematics, modelling, I can assure you that even the person who underperforms can improve the level of thinking since each and every one will come out with different metho to solve one problem then if combining together those ideas at the end everyone has gained some skill and its expands the level of thinking and that is good for learning and logical reason, so I prefer that modelling should be taught and can help learners to gain interest in mathematics subject as a whole.

LE6: I think it should be taught because, like I can say it relates to what is happening in the real world outside the school learning and can help us on how are we going to solve problems in the real world like when we solve for x in class, we are learning on how to solve problems in the real world, t help us to think critically, learning this kind of mathematics in class can also help learner to understand how are they going to use mathematics knowledge they learnt in class to solve problems in the real world.

THE END!