From textual problems to mathematical relationships: case studies of secondary school students and the discourses at play in interpreting word problems

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Declaration

I declare that this research is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy in the Faculty of Humanities, in the School of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

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Signed this 12th day of April 2010.

To Liz, Mark and Gareth.

You believed in me, and that made all the difference.

Abstract

This study uses a discourse analysis from the perspectives of James Paul Gee (2005; 1999) in order to establish a socio-situated view of why grade 10 students often experience difficulties in representing mathematical word problems into appropriate equations and expressions that enable a solution to the problems. A discursive methodology was used to throw light on the difficulties that students experience that was different from the perspectives adopted previously, *viz*. from a view of the structure of the problems, from a pedagogic point of view and a cognitive understanding. An initial case study in one school in which four students were selected revealed that a master model existed that students were enacting when doing and talking about their experiences with word problems, *viz*. that *word problems are obfuscatory*. This master model rendered the students relatively mathematically helpless within a Discourse of school mathematics word problems. In order to more fully understand these findings an extended study was set up in which the methodology and analytic framework were refined. This extended study saw four students at each of three different sites selected to participate. The findings of this extended study were that the students enacting a *situated Discourse model* were more enabled within the Discourse of school mathematics word problems, whilst those enacting a *deficit Discourse model* were either peripheral or outside of that Discourse.

This study contributes in that the constructs for the phenomena and the analytic tools within the context of school mathematics needed to be pioneered, adapted and refined over a period of time to address aspects particular to school mathematics. This resulted in a view from a socio-situated perspective which saw a shift in seeing what students *do with the problem* to what students *do in the social setting* associated with the problem. From this shift in focus came a new understanding of student difficulties with word problems that gave rise to a sub-Discourse within the Discourse surrounding school mathematics word problems, and students finding themselves in this sub-Discourse becoming marginalised through enacting a *deficit Discourse model* because they are unable to ascribe to the success model, or *situated Discourse model*.

Key words: discourse analysis; socio-situated; secondary school mathematics; word problems; cultural models; Discourse models; marginalised

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I have often heard it said that a doctoral degree is a very lonely one. Not so with Jill's cohort. Her passion for the work that her students produce prompted her to institute "PhD Weekends", wonderfully stimulating, quarterly academic meetings that were infused with the "growth of knowledge" (one of the many *Adlerisms* that have become so much a part of me). To have been a part of Jill's initiative has been an enormous privilege!

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Chapter 1 – Background to the study

A motivation for undertaking this study

> "Let's get one thing straight from the start," said Fred belligerently, shaking a podgy finger at Rufus. "I'm only prepared to learn this new mathematics of yours if it's interesting…"
>
> "But, Fred," remonstrated Rufus gently, "mathematics isn't boring. Why, it's fascination itself."
>
> "Well, it might be for cognoscenti like you, but I can assure you that I can find more fascinating things to do than spend my time working out how many square metres of wallpaper I need to cover a two-bedroomed igloo."
>
> (Adapted from *Fred learns the new mathematics*, Continua Productions, 1978.)

As mathematics teachers this is surely a familiar interaction between student and teacher, even today. I for one share Rufus' enthusiasm for mathematics, and, like Rufus, aspire to enthusing my students as well. However, one has to concur: wallpapering an igloo…?

I have taught mathematics to secondary South African students (year 8 through 12, i.e. 14 to 18 year olds) for about twenty-five years at the time of writing this thesis. Something that has always struck me as odd is that students of mathematics in the upper secondary school often cannot interpret word problems in order to translate them into mathematical expressions or equations that enable a solution to the problem. They frequently seem to misinterpret the written problem and produce equations that bear no relationship to the problem at hand (and which are sometimes even nonsensical). Perhaps this has to do with *relevance* (supposedly a motivation for the use of word problems) and the purpose of that relevance which, in many cases, is probably artificially construed. As suggested by Julie and Mbekwa (2005): "[Mathematically] contextual situations are used as a vehicle for entry into some elementarised version of Pure Mathematics" (p.32). This suggests that as educators we might be deceiving ourselves if we think that placing the mathematics within a context necessarily makes it more relevant. Davis (2003), in an enlightening paper on how he sees teachers keeping students focused on mathematics through their own passivity, has the following thoughts on what *relevance* means in terms of school mathematics:

> "The paradigmatic popular ideological hook used in local[1] mathematics pedagogy is the notion of relevance. This rather vague notion is essentially an empty signifier, waiting to be filled with content and can be made to stand for anything at all: mathematics

[1] *Local* here refers to the South African context.

education must be relevant to the student, to the economy, to local community, in terms of the constitution… and so forth." (Davis, 2003: 4).

However, in my experience most students come to understand their classroom environments as 'construed' through attempts by educationalists to render those learning situations more credible, and despite the implausibility of many of those scenarios, most students seem to cope quite adequately in most learning situations. So, if it is not the so-called 'relevance' that word problems purport then why is it that students struggle so when it comes to word problems? I felt that there had to be something else at the heart of this conundrum.

As many teachers I am sure will attest, the enigma surrounding why students do what they do with word problems is not a new concern. So what was my motivation for taking on a study about something that many before had no doubt pursued? In my secondary school teaching I had not come across any satisfactory explanation for the seeming lack of congruence of the text book word problems and what students produced in response to them and, more importantly, despite these attempts to explain the dilemma, the problem still seemed to persist. I felt that there had to be some other way of examining secondary student responses to word problems that would help throw more light on the predicament.

Establishing a focus

I set out in this study from a perspective that sense-making and transforming of word problems into an equation (or some other mathematical form that will enable a solution) constitute some of the issues that inhibit student progress with word problems. These seemed to be linked with language issues, and by this I mean everyday language, mathematical language and the interpretations and understandings that are involved in language[2] use (Moschkovich, 1999; Pirie, 1998; Rowland, 1995). Murray (2003) cites two main reasons that *teachers give* in the South African context for learner difficulties in solving word problems:

> 1. Language related problems. The learners receive instruction in a second or third language, or they have poor reading and comprehension skills.
> 2. Many teachers believe that learners with low mathematical ability find contextualised problems more difficult than context-free problems."
>
> (Murray, 2003:39)

[2] An important aspect of language is that of multilingualism. In the South African context it is particularly pertinent since many students are learning in their second, third or even fourth language. At this stage this study attempts to establish a new perspective for understanding students' working with word problems and multilingualism possibly poses an opportunity for further research.

Yet it still seems that we are only scratching the surface. The recent curriculum change in South African mathematics education saw a shift in emphasis in mathematics to one that embraces a problem-solving, investigative learning approach. As the Learning Programme Guidelines (DoE, 2005) state, "Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives" (p. 8). This suggests that more of the problems within this curriculum will be posed within some 'real-life' context, thus making it somehow more applicable for students to use that 'understanding' in their 'daily lives'.

An examination of current outcomes based mathematics text books reveals that word problems now fall under such guises as "modelling", "solving real world problems", etc., but they are still often prevalent in their old format, or as they have appeared in text books for many years. This is possibly because mathematics educationalists still see word problems as a conduit for incorporating 'real world' problems into a Discourse of school mathematics, but in order to keep the focus on the mathematics there needs to be a 'trimming' of extraneous information and a 'neatening up' of the 'messiness' that real life mathematics usually poses. The result: contrived problems that often bear little resemblance to student experiences of their worlds. Of course, this does not answer the question of why students are still often not able to cope with these problems and, in light of the new curriculum, it renders matters in this regard more pressing.

According to Gerofsky (1999), "…discourse analysis…can provide useful and sometimes surprising new perspectives on understanding teaching and learning" (p. 36). This suggested possibilities for throwing more light on my quandary regarding student difficulties with word problems. Malcolm (1985) says that, "*Discourse* is the linguistic realization…[or] *manifestation* or substance which transmits what is being communicated, that is, what passes from the encoder to the decoder" (Malcolm, 1985: 136). By *manifestations* she means "body behaviour, gestures, audible sound/speech, visible markings/writings etc." (p. 136). Thus an hypothesis emerged: a discourse approach can help view matters related to student responses to word problems from an affective perspective, but in a non-cognitive way.

Along with this hypothesis came certain methodological challenges. From the empirical perspective, how was I to generate appropriate and relevant texts (spoken and written)? Then, from an analytical perspective, what tools could I use to make meaning of the texts that would be consistent and credible? As will be shown in subsequent chapters, these challenges necessitated the development of a methodological and analytical process, which in itself had to be refined at many points along the way to meet the needs of this discursive approach.

Choosing the appropriate theoretical grounding and analytic tools

The work of Gerofsky did not adequately address issues surrounding student affect as her focus was more on the *genre* of word problems (Gerofsky, 1996) and the teaching and learning of word problems (Gerofsky, 1999). Moschkovich (1999; 1996) also offered a discursive approach for studying students in mathematics, but her work had a focus on the teaching and learning of mathematics, with a strong emphasis on multi-lingual classrooms. Whilst South African classrooms are often multi-lingual, I wanted to hold this variable constant in this study so that matters particular to student experiences with word problems could be addressed. It was not my intention to minimalise important issues surrounding multi-lingual classrooms, nor did I wish to imply that they have no bearing on how students are able to respond to word problems. To look at student *affect* regarding word problems, however, required moving beyond language issues *per se*.

Having come across the work of James Paul Gee in my post-graduate studies I turned to his *theory and method* (Gee, 1999) for possible solutions to my methodological challenges. Gee (1999) offered a discourse analysis, with theoretical perspectives that placed meaning-making and understanding in the public domain. Thus, according to Gee (1999), texts are produced and interpreted within the social context. I began to see opportunities for understanding student sense-making through examining the social context in which that meaning-making was being construed. In addition to this, Gee (1999) also provided analytic tools for understanding how this meaning-making is constructed within that social context. As will be seen when I discuss the methodology for the study, certain contexts were set up in which I could examine how students constructed meaning. I decided to adopt Gee's notion of *Discourses*[3] (in the broader sense) in order to set up a theoretical framework for establishing a socio-situated understanding of difficulties that students encounter when attempting word problems. In other words, the students 'speak' the Discourses, but at the same time the Discourses 'speak' the students, thus giving insight into the social setting in which this

[3] Gee defines *Discourses* (with a capital D) to mean "language *plus* other stuff" (p. 17). In other words, language (written or spoken) is only one small aspect of *Discourse* – other factors include "action, interaction, values, beliefs, symbols, objects, tools and places" (Gee, 1999; 18).

meaning is made, as well as the meaning that comes to be inferred upon that social setting. From Gee's work it appeared that, through an examination of the Discourses at play, light would be shed upon what students *do* when tackling word problems because of this socio-situated approach. Furthermore, *cultural models*[4] (Gee, 1999) that underlie the students' actions in their approaches towards word problems seemed to provide an informative basis to explain why students go about solving problems in the way that they do.

With Gee (1999) providing a theoretical basis, the following research question was considered: what is it that students do that is appropriate or inappropriate in interpreting word problems?

In answering this question I considered the following research sub-questions:

1. What discourses do students use when attempting word problems, particularly in a peer setting[5]?
2. What underlying cultural models are at play when students attempt word problems?
3. How do these cultural models inform us about the students' experience and interpretation of word problems?

However, Gee's *theory and method* (1999) did not have the all-enveloping qualities that were at first perceived. For a start, it was not subject-specific, but in particular it made no provision for understanding peculiarities of mathematics (such as the register) which made mathematical texts difficult to analyse with Gee's tools. Thus there were further analytical challenges that I had not foreseen at the outset, and these will be described in the chapters that follow.

Establishing the unit of analysis

I support the view of Burton (1991) that it is salient and pertinent to understand what students do when translating English textual problems into algebraic statements. Mulligan (1992) says that,

[4] According to Gee (1999) *cultural models* are simplifications of what we hold to be *typical* or *normal* (p. 59). They are generally not consistent from one person to another, even within a particular social group, and they have a (usually) subconscious affect on interpretation of experience. Amongst other things, for Gee (1999) *cultural models* act as mediators in *Discourse*.

[5] A peer setting is important for seeing how students talk about mathematics amongst themselves and without the influence of an educational figure who is likely to cause students to give responses that they think they 'should' be giving.

> "When children experience formal instruction it cannot be assumed their conceptualisations are linked with formal mathematical ideas, or that their own strategies match those encouraged by instruction… It appears then, that the absence of these connections induces a shift from intuitive and meaningful problem-solving approaches, to mechanical and meaningless ones (Hiebert, 1984; 1990; Hughes, 1986)" (p. 25).

A shift towards algebraic methods for solving word problems is likely to elicit exactly what Mulligan (1992) is suggesting – viz. pre-empted mechanical techniques which, to the students are meaningless, except in that they assist in getting the right answer[6]. However, it is important as teachers to *know* when students are reverting to such methods. Boaler (1997) refers to *cue-based behaviour,* which leads to algorithmic procedures for solving problems and which demonstrates little actual understanding of the mathematics involved, especially with respect to applicability or relevance. If students are indeed not basing their solutions to these word problems on algorithmic-type procedures then why is it that they still, in many cases, seem to be unable to express their understanding algebraically? Here I am assuming that students understand the textual problem and what it is asking them to do and, to an extent, my assumption is confirmed by Mangan (1989):

> "…investigations in which problems have been read to pupils…difficulties in solving word problems have still been consistently reported" (p. 113)

Thus to understand what takes place from the reading of the text to the formulation of an algebraic representation I believe I needed to enter the realms of what cultural models are at play that could be influencing the student interpretation. However, this is not unequivocally attained:

> "The English sentence must first be parsed: read, understood and stored. In considering this comprehension and storage, we must posit processes that we cannot describe explicitly" (Burton, 1991: 43)

So, in order to understand this agency in student interpretation, I needed to interrogate what students do and say that reflect their cultural models. Astington (1995) says that, "…any attempt to assess young children's understanding has to be supremely sensitive to the way the children themselves talk about these things" (cited by Bruner, 1996: 109), and this sensitivity also needs to be taken into consideration when dealing with older children. In the literature review that follows it is apparent that other perspectives of student difficulties with word problems are often not sufficiently sensitive to the way in which students talk about their

[6] As shown in a study by Boaler (1997), this is not a necessary outcome. According to her study different classroom approaches result in different *kinds* of knowledge – on the one hand, knowledge that is inert and largely algorithmic, and on the other, knowledge that is flexible and more conceptual.

experiences. I now move to a discussion of some of the studies of word problems from the literature to show what work has gone before in this field and to demonstrate the affective gap in the methodological approach to these studies, which in Gee's terms, would take a socio-situated stance.

Some perspectives on word problems from the literature

Word problems have played a prominent role throughout the known history of mathematics education with little or no regard to their relevance or applicability (Toom, 1999; Gerofsky, 1996; De Corte and Verschaffel, 1989). What is particularly noticeable is that word problems have maintained a particular traditional format since ancient Babylonian and ancient Greek times (Rojano, 1996; Solomon, 1995), and in 1897 Milne (cited in Schoenfeld, 1992) produced a textbook of word problems that significantly resemble those that appear in textbooks used in current classroom practice. It has only been more recently that educationalists and researchers have begun to question the relevance, appropriateness and value of word problems in our schools. In the literature that I have encountered there is a keen, but varied interest in word problems from an educational point of view. There are many authors who question the relevance for the use of word problems in teaching (e.g. Gerofsky, 1999; Greer, 1997) and there are those who raise questions about the pertinence and fairness of the 'real-life' contexts that are ostensibly a pedagogic purpose of word problems (e.g. Cooper and Dunne, 2000; Boaler, 1997; Mangan, 1989). Another professed purpose of word problems is the enculturation of students into algebra, but Lins and Kaput (2004) question the traditional view that arithmetic necessarily precedes algebra, and Bednarz and Janvier (1996) state that the difficulties that our students have with algebra is *because* they come from an arithmetic background.

The literary review that follows was constructed from empirically reported studies that I found (mostly in journals and conference proceedings) and is complemented with some theoretical papers. The varying studies that have a bearing on the study reported here are organised below in relation to the theoretical gaze through which they view the problem of student understanding of word problems in mathematics.

The mathematical structure of word problems

At the primary school level there are a number of studies that possibly could inform secondary school practice. One such study by Mulligan (1992) undertaken as a longitudinal study in the Sydney metropolitan area of seventy children from year 2 through year 3, suggests that children can solve multiplication and division problems before they have been taught how to do so, but that they tend to use a variety of intuitive, counting-type methods. These methods were found to be lacking when problems involving larger numbers were introduced, suggesting that the students did not have coherent structural understanding of the problems. The introduction of more complex examples seemed to cause a decrease in the success rate even when problems involving smaller numbers were reintroduced.

The significance of this study by Mulligan is that intuition serves learners well in responding to word problems but eventually ‘runs out’ if and when the complexity of the problem increases.

Mangan (1989) reports that studies done on 12 to 16 year olds in England revealed that the numbers involved in word problems (particularly division problems) have a bearing on the students’ abilities to cope with the problem. Story problems involving whole numbers (as opposed to non-integral numbers) affected student understanding of the divisor and the dividend. The misconception was further exacerbated if the divisor was less than one. Such aspects of word problems as understanding the textual rendition and computational skills, although being necessary prerequisites, were ruled out as being *the* cause for lack of success in solving the problems. Rather, citing Fischbein (1985), Mangan (1989) suggests that this is more closely linked to ‘taught’ strategies of multiplication as being repeated addition, and division as being ‘sharing’[7].

Lepik (1990) undertook a quantitative study using 150 grade 8 students who worked on 35 word problems chosen for their ‘level of difficulty’. What is of interest in this discussion is that Lepik identified thirty-one linguistic variables which describe the textual statement of the problem (one of the focuses of the study). Examples of these are, the number of letters, number of words, mean length of words, etc. Whilst these linguistic variables were generally not good predictors of the number who used an appropriate solution strategy, the one variable that did show a significant positive correlation with correct solution strategy was *the average number of words for each relation* (*cf*. Lepik, 1990: 88)[8]. Lepik deduced that this seemed to

[7] An outline of several studies involving multiplication and division problems is given in Mangan (1989).
[8] Lepik was not concerned with correct answers, but rather with a correct method or procedure.

indicate that a more detailed description of the problem situation improved student understanding of the mathematical relations.

Implicit in these three studies is that a mathematical structure exists and is apparent in word problems. Murray (2003) points out that some twenty different addition and subtraction problems can be found, whilst at least three different division types exist at the primary level, each with its own, sometimes subtly different structure. Knowledge of mathematical structure is unlikely to be spontaneous in the student base knowledge. It is thus seen as a scientific concept in the Vygotskian sense (Vygotsky, 1978) and consequently needs to be 'taught'. Word problems are thus more than just applications of, for example, operational skills. Knowing how to divide numbers will not necessarily transfer to solving word problems involving division and Murray (2003) notes that the more students have had to deal with a particular structure-type the more competently they tend to be able to cope with such a problem. Word problems here are seen from a mathematical frame of reference and the learning of them is seen through a teaching perspective. The three studies cited above give us a deeper mathematical understanding of the structural make-up of word problems, as well as a pedagogic understanding of the difficulties that students have with those word problems from that structural perspective.

For example, Mangan (1989) identified 'key words' that students seemed to respond to when approaching word problems. For example, *altogether* suggested addition and *left* suggested subtraction (pp. 120 – 122), inferring that the *structure* of the problem appeared to lead to certain *actions* from the students. This brings to mind what De Corte and Verschaffel (1989) refer to as "semantically different problem types" (p. 85) to which students respond according to cues in the problem wording. Hegarty, Mayer and Monk (1995) found that unsuccessful problem-solvers are more likely to use the *direct-translation* technique, which in essence is a strategy in which 'unsuccessful' students base their solutions on numbers and key words in the problem statement. Gerofsky (1996) has the following to offer:

> "It is interesting to note that, over the course of several years, students become enculturated in the world of school mathematics, and familiar with the word problem *genre* to the extent that they are able to reproduce it." (p. 38 – my emphasis)

Here we see explications of the difficulties that students experience from a semiotic viewpoint, which attempts to give a more complete account than the mathematical and pedagogic perspectives discussed so far. My experience with mathematics textbooks used in classrooms gives credence to the concerns expressed here, and De Corte and Verschaffel (1989) state the following:

> "…instructional practice seems to be characterised by a restricted, biased, and stereotyped supply of verbal problems. It is our conviction that this situation is not only undesirable; it also involves a risk in the sense that it promotes the development in young children of rather superficial solution strategies". (p. 94)

Boaler (1997) sees one of these student strategies as being *cue-based behaviour*, which she believes can be remedied by teachers adopting a more open-ended teaching approach. Her viewpoint is essentially epistemological. There are other authors, at what I believe is the other end of the scale, who offer step-by-step strategies for achieving success in word problems. These I see as reflecting a pedagogic stance. Many of these seem to be 'quick-fix' strategies for teachers and students. Some are nevertheless informative because they sought to promote understanding of the nature of student difficulties with word problems. For example, Ferrucci, Yeap and Carter (2001) give an account of a cyclic (as opposed to a linear) model used in primary schools in Singapore in which they believe that students "develop a deeper comprehension of the problem structure" (p. 26) through a process of creating pictures and diagrams to represent the problem.

This is also a view that implies that word problems have a *structure* whilst at the same time viewing student difficulties from a pedagogic perspective. Boaler (1993) says that:

> "…problems and investigations which are structured [as opposed to open] can only demonstrate methods which are essentially impersonal" (p. 17)

Thus by adopting a structural view of word problems the difficulties that students experience can only be viewed in the limited domain of *methods*.

Kutscher and Linchevski (1997) undertook five case studies of Grade 8 students in Israel. The research was aimed at providing a procedural model that would theoretically equip teachers with an effective means of enabling students to generalise numeric relationships. The study is based on reification theory, which holds that much of mathematical understanding is achieved firstly at the 'doing' level and which then develops into 'structural conceptions'. Thus the study aimed to show that by the use of number instantations, students using the *table-filling* model (Kutscher and Linchevski, 1997) would be able to move from recognition of the structural to the procedural nature of algebraic expressions in two-step word problems.

Although vaguely described, it seems that the students found the ‘table-filling’ method tedious and tried to skip steps to generate an equation, which was mostly inaccurate. When they were guided back to the ‘table-filling’ method they usually achieved a correct equation. It seems that number instantation assists students to achieve a correct equation. The authors believe that their method has solved certain difficulties with word problems (they generalise this) but acknowledge that further study is needed to establish whether students can still generalise later on without this mediator.

All of the above studies have examined students’ interpretations of word problems by examining the underlying structure of the problems, mathematically and semiotically, and related these to pedagogy.

From structure and pedagogy to cognition and affect

Taking a cognitive perspective, Murray (2003) argues that with word problems students learn to *model* the situation, thus attributing a developmental aspect to how students come to know word problems:

> “When the following problem is posed to a young child: “*You have five sweets, and Dad gives you another 2 sweets. How many sweets do you have now?*” she may write 5 + 2 = 7. This is a model of the problem. At an earlier level she may draw 5 sweets, another 2 sweets, and count them all. The drawing is also a model. But before that, the child cannot replace the problem with a model. If she does not have the sweets physically available she cannot solve the problem.” (Murray, 2003: 41)

Christou and Philippou (1998a) conducted a study on 382 Grade 2 to Grade 4 students from seven elementary Cypriot schools. Their study aimed to investigate any developmental *schemes* according to *classes of problems* and their comparative levels of difficulty. The researchers compiled a quantitative categorisation of student success with various *problem classes* which, together with the student grade, suggests that developmental aspects exist in student ability to solve word problems. The findings seem to confirm that mental schemes exist that give students the ability to interpret word problems. Once students developed a *repertoire of schemes* they were able to identify mathematical relationships that correctly solved the problem. In addition to this at least four developmental levels were identified and these were found to be hierarchical with respect to student ability to cope with a problem.

This study gives us a psychological perspective of word problems, but more specifically it is from a developmental stance similar to that of Piaget. The study implies that student ability to cope with certain word problems of a specific structural nature is determined by their level of development and the *repertoire of schemes* that the student has.

Christou and Philippou (1998b) undertook another study that aimed at identifying a structure of beliefs that students hold about mathematics and to interrelate the components of this belief structure. Whilst this was not directed specifically at word problems it has a bearing on this study in that it helps to inform us about student beliefs which relate to cultural models. Although the authors note that there are cognitive, affective and action components of belief structures, they limit their study to cognitive aspects.

The authors use a combination of two definitions of beliefs:

> "Schoenfeld (1994) defined beliefs as an individual's understandings and feelings that shape the way the individual conceptualises and engages in mathematical behaviour, while Pehkonen (1997) explained that beliefs constitute the subjective knowledge of mathematics" (Christou and Philippou, 1998b; 192)

A random sample chosen from Grade 6 and Grade 9 Cypriot children (totalling 660 students) was used. The researchers found that student belief components can be categorised in a three-tier hierarchy. This can be broken down into two second order factors – *epistemology* and *teaching-learning* of mathematics. These second order factors can be broken down into four first order factors – *content*, *nature*, *teaching* and *learning* of mathematics. The study suggests that the structure of student beliefs across grade and gender are invariant, despite special tests that were conducted. This suggests that these belief structures are established in former years. The study also suggests that a change in belief structure is possible, but this needs to take place in the early years and can be achieved by concentrating on activities that help the students see the usefulness of mathematics.

Cortes (1998) undertook a study of three French Grade 8 classes that looked at the implicit thinking that students are involved in when translating word problems into mathematical form. The research is cognitively based and incorporates the notion of schemes from which the students can draw when writing equations from word problems. These schemes are based on *operational variants*, or implicit mathematical knowledge that enable the students to write the equation. These operational variants are *pragmatic concept of function*, *concept of numeric function* (both of which are implicit), *the concept of equivalence* and *respect for homogeneity of terms*. The authors are of the opinion that students can construct equations in three different ways and that these are implicit processes – by substituting unknowns with numbers, substituting unknowns with linear functions and by equating two linear functions.

De Bock *et al*. (2002) undertook a study of twenty 12-13 year olds and twenty 15-16 year olds from a Flemish boarding school. The aim of the study was to identify facets of student knowledge base that cause many to inappropriately apply linear or proportional reasoning. The format of the study was individual semi-structured interviews in which students were guided through up to five phases. The study showed that students rely heavily on linear modelling as a solution strategy even when strong arguments suggested that these were wrong. The authors identified four categories of student knowledge base that they deem as possibly being responsible for this student behaviour: *intuitiveness of linear relationships, illusion of linearity*, *shortcomings in geometrical knowledge* and *inadequate habits and beliefs* (p. 328).

The studies by Cortes (1998) and De Bock *et al*. (2002) view student difficulties with word problems from an epistemological and cognitive perspective. These studies suggests that students develop processes for establishing equations from word problems and that these processes are internal and latent. This relates to the study by Christou and Philipou (1998b) which examined a 'structure of beliefs' that students hold in that these processes for doing word problems are what students 'believe' they should be doing. The pedagogic implications of this are that students establish a system of beliefs that shape their actions. But, if these actions are inappropriate, how do we intervene in the belief system so as to redress these actions?

From structure, pedagogy and cognition to situated experience and action

The vast majority of studies about word problems have examined pedagogic, psychological and cognitive issues regarding student difficulties with word problems. Some studies have looked at what can be done to help students acquire the skills to generate equations successfully (for example, Ferrucci, Yeap and Carter, 2001; Kutscher and Linchevski, 1997) whilst others have looked at what students do with word problems (for example, Boaler, 1997; Mangan, 1989). Yet others have examined the nature of word problems themselves (for example, Gerofsky, 1996). The pedagogic, psychological and cognitive explications of how students tackle word problems do not seem to have fully accounted for the difficulties that students experience. These explications have contributed a great deal to illuminating why difficulties with word problems abound and have presented strategies for addressing them. Yet, the problems persist, suggesting that further research is needed.

An aspect of word problems that seems to have attracted little attention in the literature is the affective responses of students when tackling word problems. There has been a *turn to language* in recent mathematics education research that has seen a focus on socio-linguistic perspectives. However, as the term 'socio-linguistic' implies, the theoretical stance here is not simply about 'language'. It spotlights language as integral to functionality and meaning-making within a broader social context (Morgan, 2006) which has its roots in the work of Halliday (1994), and has been used in many discursive studies since then (*cf*. Case and Marshall, 2008; Setati and Barwell, 2006; Setati, 2005; 2002; 1998; Barwell 2003; Christie, 2002; Moschkovich, 1999; 1996). This *turn to language* allows the researcher a socio-cultural lens through which circumstances might be interpreted, thus allowing for affective responses to be a part of that interpretation which was not possible with research that viewed the data from a purely structural or cognitive stance. As Morgan (2006) states:

> The 'turn to language' in the theoretical perspectives adopted by researchers in mathematics education has brought with it increased attention to the nature of language and other semiotic systems used in mathematical activity and to the roles that these may play in the teaching, learning and doing of mathematics…" (p. 219)

Case and Marshall (2008) point to examples from the literature that criticise a cognitive focus on student learning saying that it "… results in a relatively asocial characterisation of the learner… [and] also tends to lead to explanations of student learning outcomes which are somewhat detached from the broader socio-cultural context" (p. 200). To me this suggested taking a socio-cultural stance in order to fill the gap. This study, through a discourse analysis, was intended to enter the realms of students' experiences when confronted with word problems and how these experiences shape their actions. Whilst the orientation of the study was promising, its interpretation into an empirical study was neither self-evident, nor were there existing studies that could easily be used to guide the process. This study has thus been empirically accompanied by a theoretical and particular methodological 'journey', the story of which follows in the subsequent chapters.

Gee (1999) sees *social languages* and *Discourse* as being useful tools of inquiry. He says:

> "…it is important to realize that, in the end, these terms are ultimately our ways as theoreticians and analysts of talking about, and, thus, constructing and construing the world." (Gee, 1999; 37)

Setati and Barwell each undertook studies of the discursive practices of teachers, the former in South Africa and the latter in the United Kingdom, and a comparison of these studies appears in Setati and Barwell (2006). The focus of these studies was on teacher discursive practices, but in essence drew on the concept of *language in use* (Gee, 1999), similarly to what my study aimed to do. According to Setati and Barwell (2006):

> Activities like mathematics have specific discursive practices that collectively make up mathematical discourse (or discourses). Discursive practices link specific moments of interaction with wider social patterns, so that interaction must be understood as situated." (p.29)

Case and Marshall (2008) conducted a study on 36 students in a third year engineering course at a large university in South Africa. The main data collection took place through semi-structured interviews and the data that emerged was analysed through a discourse analysis drawing wholly on the constructs of Gee (2005) to identify cultural models (which the authors refer to as *Discourse models* in keeping with Gee, 2005) that would help explain how the students made sense of their learning experiences. The authors found a dominant Discourse model, which they named *the no problem Discourse model*. This model was identified through the frequent occurrence in the data of *macrostructural features* (or linguistic characteristics) which appeared in different combinations and differing chronological order, but which manifested in sufficiently similar ways to be deemed to be describing the same phenomenon. This phenomenon was the tendency of the students to describe their learning experiences, firstly in optimistic terms, followed by a qualifying statement that introduced difficulties or failure, after which the latter were justified in some way and finally the talk ended on a positive note. In essence, the authors found that the *no problem Discourse model* appeared to be linked to a broader Discourse of self-actualisation which, it seemed, stemmed from the students' communities and home background. The authors also cite some possible implications for teaching and learning due to the deeply entrenched nature of the self-actualisation Discourse identified in this study.

To this end a perspective of student discourses about word problems enables us to address the affective aspects of student experiences with these problems since it examines student utterances, writings and actions, but it looks at these within the wider social scenario. In short, it looks at how the students position themselves when confronted with word problems, but it takes its stance from a socio-cultural and situated perspective. In this regard it is hoped that, complementing the work on student learning done by Case and Marshall (2008), this study will help to fill a niche in the work on word problems and contextualised mathematics that has gone before.

In the following chapter I detail the theoretical stance adopted for this study. As with Case and Marshall (2008) I have relied heavily on Gee's conception of discourse analysis (Gee, 2005; 1999). Also, it will become apparent in the discussion that follows that Gee produced a second edition of his 1999 work (*viz.* Gee, 2005) that saw a renaming of the term *cultural models* as *Discourse models*. To pre-empt any confusion that might arise from the use of the two different terminologies in this thesis, it needs to be made explicit that this was purely a name-change, and that the reader should use the two terms interchangeably to mean the same thing.

In chapter 3, a discussion of the *initial case study* is taken up. As a result of the findings of this initial case study an extended study was set up to redress certain shortcomings that became apparent. The extended study is then reported in chapter 4 onwards. The initial case study has been included in this thesis as it has significant bearing on the extended study. However, due to the fact that the analysis of the initial case study was very lengthy it has been reported in chapter 3 in an abbreviated form as it impacted the extended study. The full report of the initial case study is included digitally on a compact disc (*cf*. The full report of the initial case study.)

Chapter 2 – The orientation and rationale for the study: theory and method

Theory

As noted in the previous chapter, the process of translating the English text in word problems into algebraic symbols is twofold – it involves reading and understanding in the ordinary linguistic sense (Burton, 1991), and it involves understanding and manipulation of symbols in the mathematical sense (Pirie, 1998; Rowland, 1995). However, these activities take place within a broader social context which in this case can be seen as the context of school mathematics. In this sense we might come to regard this broader context as the *Discourse* of school mathematics.

Discourses, with a capital 'D' is how Gee (2005; 1999; 1992a; 1992b; 1990) describes the social bounds that define and give meaning to every human activity[9]. A Discourse is "an amalgam of ways of acting, interacting, talking, valuing, and thinking, with associated objects, settings, and events which are characteristic of people whose social practice it is" (Gee, 1992b: 91). Thus, Discourses are social activities, but also mental activities and material realities (Gee, 1999: 23). For Gee, "'psychological entities' are actually out *in the social world* of action and interaction" (Gee, 1992b: xvii). By this he means that experiences such as beliefs, values, meanings, etc., all exist only in the context of social activity, and hence within Discourses. Fairclough (1989) places a similar emphasis in his definition of discourse, which he says is "a mode of action, one form in which people may act upon the world and especially upon each other, as well as a mode of representation" (p. 63). This social activity takes the form of interaction between people who are fulfilling certain roles and making use of certain material objects, again, within Discourses (Gee, 2005, 1999; 1992b; 1990).

However, Fairclough (1989) envisages discourse (he does not use Gee's capitalisation) as *language as a social practice*, which is not simply language in the traditional linguistic sense. What Fairclough means by this is that language is inextricably and intrinsically embedded in society, such that it can be seen as a *process* that also incorporates non-linguistic aspects of that society.

[9] When Gee refers to *discourses* (without the capital 'D') he uses the term to mean *language in action*.

"The roles we play always involve assuming or acting as if certain sorts of people and things are 'right,' 'normal,' or 'good' and certain other sorts are 'wrong,' 'marginal,' or 'bad'" (Gee, 1992b: xvii). Van Dijk (2001) also talks of how we dichotomise with respect to 'us' and 'them' (p. 103). He asserts that there is a tendency to emphasise the 'good' in 'us' and the 'bad' in 'them' and to de-emphasise the reverse. This practice results in the marginalising of certain people or groups who do not 'fit' the Discourse, and Gee (1999) refers to this as the *colonising effect*. Jäger (2001) puts it thus:

> "Discourses exercise power as they transport knowledge on which the collective and individual consciousness feeds. This emerging knowledge is the basis of individual and collective action and the formative action that shapes reality" (p. 38)

For those that are marginalised through the inability to 'receive' the knowledge referred to by Jäger the consequence is exclusion from the Discourse.

The problem is to unravel the intricacies of Discourses in order to give meaning to the human actions and interactions that we encounter. "An important means of linking up discourses with one another is collective symbolism. Collective symbols are 'cultural stereotypes (frequently called "topoi"), which are handed down and used collectively (Drews *et. al.*, 1985: 265)" (Jäger, 2001: 35). These collective symbols enable us to interpret and hence act upon instances, or, put differently, they create meaning or 'truths' for us. They shape our actions, but they simultaneously shape the society in which they emerged (Jäger, 2001).

These social phenomena that appear to shape our behaviour are further developed by Gee (1999; 1992b; 1990). Macedo (in his provocative forward to Gee, 1992b) says that,

> "…to access the true and total meaning of the entity, we must resort to the cultural practices that mediate our access to the world semantic field and its interaction with the word semantic features. Since meaning is, at best, very leaky, we have to depend on the cultural models that contain the necessary cultural features responsible for 'our stories,' and 'often self-deceptive stories'" (in Gee, 1992b: x – xi)

Cultural models, a term adopted by Gee from Holland and Quinn (1987), represent a *simplified world view* (Gee, 1999; 1992b). For Gee, all meaning is embedded in cultural models (*cf*. Gee, 1992b: 1 – 21). In particular, cultural models can be regarded as analytic tools in a discursive research approach since, "A cultural model can be seen as a means for relating a 'word' or 'notion' within the social context that it holds for us in the world" (Gee, 1992b: 8). Put differently, "cultural models reflect social and institutional affiliations"

(Setati, 2002: 30), and hence come to give us insight as to how and why people act and interact in the light of their world experiences.

Van Dijk (2001) talks of *mental models,* which are constructed from our day-to-day experiences. "Mental models feature all personally relevant beliefs about an event, that is, knowledge as well as opinions (and probably also emotions)" (van Dijk, 2001: 112). These mental models define our roles, both in a general and localised context.

> "…social representations are 'particularized' in mental models, and it is often through mental models that they are expressed in text and talk. And conversely, it is through mental models of everyday discourse such as conversations, news reports and textbooks that we acquire our knowledge of the world, our socially shared attitudes and finally our ideologies and fundamental norms and values" (van Dijk, 2001: 114)

To pinpoint these mental models (or cultural models) is not as simple as it may at first appear. Gee's concept of cultural models is based upon the premise that meaning is a social activity which need not exist in individual thinking in any particular form, and certainly does not exist in words. "…we 'talk around' social practices, and in the act we *mean*" (Gee, 1992b: 12). Gee (1999) suggests that using a word or phrase, not in terms of its meaning, but rather *against a set of social and cultural assumptions* indicates the existence of a cultural model (p. 60).

As a premise to my epistemological stance, I look to Gee (2005; 1999; 1992b) who claims that discourse is intrinsically a social activity, which is inevitably situated in a context. From this socio-cultural perspective it is easy to be coerced into looking at the classroom as a context within a context. The implications of this, as I see it, are that students come to act within the classroom in terms of the way in which they identify with that situation, but that they also bring with them a complexity and purview of experiences (Boaler, 1997). This implies that students are not working with word problems simply at a cognitive level, but rather within a social context that goes even beyond the parameters of the classroom (Case and Marshall, 2008). Thus, in order to understand what it is that the students are doing we need to look beyond the structure of word problems, and beyond the pedagogic and developmental aspects of teaching and learning word problems. Cultural models, or *Discourse models*[10] that students hold, and the way in which these influence students as they

[10] In previous work Gee (1999; 1992b; 1992a; 1990) refers to *cultural models* following the work of D'Andrade and Strauss (1992), Holland and Quinn (1987) and others. However in his revised work Gee (2005) has come to refer to *Discourse models* because of the situated meaning surrounding the term *cultural models*.

go about translation of word problems can give insight into different student experiences that are not confined to ‘mind’, ‘ability’ or ‘teaching’, nor is it a response to the underlying structure of word problems. Discourse models are a social phenomenon (Gee, 2005). They are thus situated within a social context and therefore offer the potential to researchers for acquiring insight from a socio-cultural perspective.

The mention of the term socio-cultural brings to mind the notion of social inequality, something that is still present in post-apartheid South Africa. However, in nearly twenty-five years of teaching and experience across a range of South African secondary schools, including one exclusive, socio-economically advantaged school, my perception of the problems that students experience with word problems are very similar across all socio-economic groups, with a possible tendency for it to be more acute in certain lower socio-economic groups. As with issues surrounding language I wished to examine factors in student difficulties with word problems from the perspective of how these manifest and come to bear upon the students within their social circumstances, but not with a particular view to whether or not these circumstances are more privileged economically or materially.

Because Discourse models were borne out of a discursive paradigm it seems logical to employ a discourse analysis to examine how these come to bear upon students’ working with school word problems. It appears from the foregoing that I needed to examine student talk and actions within the social context to access the Discourse models that they hold and, consequently, understand how these impinge on their sense-making when interpreting word problems. For me this implied an approach from a discourse perspective**.** Language does not occur in isolation but within the interaction of people, even with themselves (Gee, 1992b; Bailey, 1985). Also, in analysing discourse we should consider the *context of the situation* so that we “…allow a place for the creativity of language, the constant flow of linguistic innovation that suffuses every new context in which we speak”[11] (Bailey, 1985: 4). An analysis of discourse offered me the opportunity to establish how the students position themselves or come to be positioned in relation to word problems through identifying Discourse models that students hold. In other words, the discourse, or *language in action*, would enable me to establish what the students hold to be *typical* or *normal* (Gee, 1999) about word problems. This would therefore necessarily need to be from a socio-cultural

[11] Bailey (1985) distinguishes between linguistic and non-linguistic aspects in analysis of discourse and maintains that even acts and objects can have a bearing on communication.

perspective. As discussed in chapter 1, previous studies of word problems have not used a socio-cultural paradigm and this study was therefore to provide a new methodological approach in examining student difficulties with word problems.

Moschkovich (1999) uses a discourse approach in her study of Latino mathematics students on the west coast of the USA who are learning English simultaneously with mathematics[12]. The implications in her study are somewhat different to what was sought in my study because she approached her work from a teaching and learning perspective, but there is one significant similarity that had a bearing on what I was examining: this was the notion of how students make meaning, albeit in the context of the multi-lingual mathematics classroom. In essence she believes that mathematical discourse "…involves much more than the use of technical language" (p. 11) and students sometimes build and establish meaning in more intuitive ways which can be just as valid a more formal methods. The links with *situated cognition* (Gee, 2005; 1999) seem to be apparent here. Nevertheless, one of Moschkovich's primary concerns was to focus on the students' mathematical meanings. I saw this as being pertinent to my study as it meant asking myself, does what the students *say* or *do* tell us about what they *think* and *understand* as they go about solving word problems? In answering this question I turned to an analysis of the student discourses in the broader sense that Gee (2005; 1999) proposes to shed light on how the students communicate their experiences in the parsing and translating of word problems that they undertake (Burton, 1991). This was to address research sub-question 1.

In addressing the second research sub-question I refer mainly to the writings of Gee (2005; 1999) and Setati (2002). Gee (1999) proposes that, "Cultural models [i.e. Discourse models] are an important tool of inquiry because they mediate between the 'micro' (small) level of interaction and the 'macro' (large) level of institutions" (p. 58). From this it seemed that Discourse models might divulge the underlying student beliefs, values and actions that affect their perceptions of the nature, relevance, purpose and meaning of word problems. In other words, Discourse models might provide some of the answers to understanding how students approach word problems in relation to the 'baggage' that they bring with them into the classroom. In addition to this, Discourse models could help to establish these student perceptions in relation to what they are expected to do from the school's perspective (i.e. the

[12] This again raises issues of the language of learning and teaching, which was not a focus of this study, but suggests avenues for further research.

mathematics classroom, the teacher, educational figures, etc.), and the extent to which these perceptions and expectations match or do not match their own assumptions.

Thus the examination of students' Discourse models enables us to identify the extent to which students separate or assimilate word problems in the contexts of the school and their everyday lives, which then inform us of how, or even if, they do make mathematical sense of the word problems. This addresses the way in which students talk about how Discourse models mediate in their thinking about word problems, which would enable me to answer research sub-question 3.

All of the aforegoing presupposed that the Discourse models could be identified and, as will be discussed in more detail in Chapter 4, the refinement of the analytic tools to extricate the Discourse models for the extended study provided me with additional methodological challenges. Initially I attempted to pinpoint Discourse models based on what was proposed by Gee (1999), *viz*. that Discourse models can be differentiated from each other by categorising them into espoused models, evaluative models and "models-in-(inter)action" (p. 68). The reason for this, he suggests, is so that we can more easily establish the way in which the models affect us and how we employ them in our daily lives. The Discourse model then formulates what we take to be "appropriate" (Gee, 1999) ways of viewing, judging and acting in response to a situation. It seemed to me that we could therefore link perceptive discourse to espoused Discourse models, judgemental discourse to evaluative Discourse models and resultant discourse to models-in-(inter)-action. What this linking reveals is an interpretation of the effect that the Discourse models appear to be inducing (figure 1).

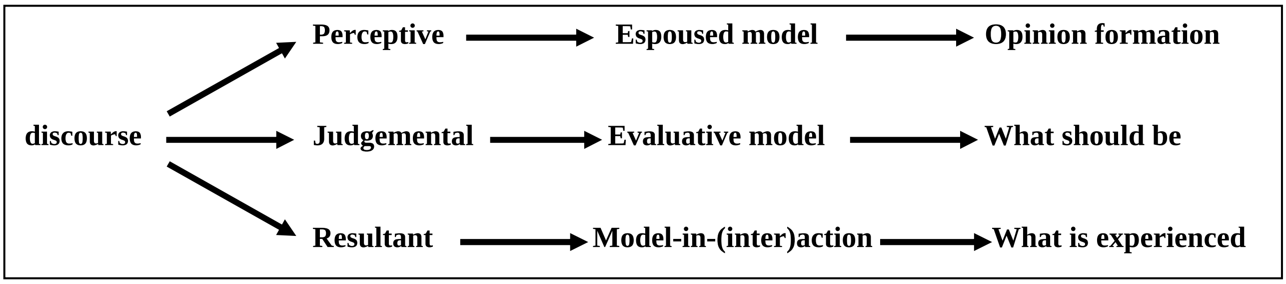


Figure 1.

This flow diagram demonstrates how the identification of the kind of discourse (language-in-action) is reflective of types of Discourse models, which in turn indicates the type of action that results.

However, it is prudent to heed Gee's advice:

> "[Discourse] models are complexly, though flexibly, organised. There are smaller models inside bigger ones. Each model triggers or is associated with others… And we can talk about "master models," that help shape and organize large and important aspects of experience…" (Gee, 1999: 69)

Grouping of Discourse models (possibly under the banner of one master model) could serve to help us understand what models individuals or groups of individuals act upon, and this informs us of how they perceive and act in a situation (which is word problems for this study), and consequently, why they act in the way that they do. In other words, a master model can indicate the existence of a particular Discourse, as was one of the findings in the study undertaken by Case and Marshall (2008).

Furthermore, the notion of conflicting Discourse models (Setati, 2002; Gee, 1999) helps to inform us of tensions that exist in individuals. Two models that are in conflict usually exist as one that is being acted upon whilst the other is merely espoused. This can result in the individual not aspiring to certain goals (as contained in the espoused Discourse model) because he is acting upon the other Discourse model. In some cases power relations exist in this form. The Discourse model that the individual is acting upon, in such a case, would serve the interests of someone else at the 'expense' of this individual. Models in conflict can thus inform us of both why a person acts in a particular way as well as who is benefiting from such actions where this is applicable.

Gee undertakes a very interesting discussion of memory (*cf*. Gee, 1992b; 53 – 64) which I believe is also significant for interpreting text, and hence identifying Discourse models. In reporting from memory (according to Gee, 1992b) the information is given as *foregrounded* or *backgrounded* events. These can be distinguished from each other by the way in which the verb is chosen. Simplistically put, a verb that names an action in the simple past tense indicates a *foregrounded event*, whilst any other use of the verb (such as one that describes a mental state, or one given in the passive voice) indicates a *backgrounded event*. Backgrounded events are important in that they give the context for interpreting the foregrounded events.

> "Whether a piece of information is foregrounded or backgrounded has to do with…how one chooses to linguistically encode the information at the time of (in the context of) telling it… Thus, foregrounding and backgrounding are not in the mind (the mental

> framework). Rather, they are in the ("real time," "on line") linguistic encoding of the report, and this encoding can change with the contexts in which [one] chooses to report the episode" (Gee, 1992b: 58)

In other words, the reported event can be seen as a social activity and not simply something that is stored in memory (Gee, 1992b: 60). This reported event occurs in response to a social situation, and the construct of the text is based upon how the reporter perceives (wittingly or unwittingly) that the receiver will interpret that episode.

What has been said here about reporting from memory, I believe, is as applicable to any text that occurs and is not limited to recall of memory. I base this supposition on the acceptance that any text (not only that which is reported from memory) is socially constructed, is situated in a context and contains within it certain intentions or motives. Accepting this, together with Gee's distinction between foregrounded and backgrounded events, we have the basis for a theoretical model within a discursive paradigm.

In addition to foregrounded and backgrounded events there are two related ways in which we can use grammatical cues to help us understand text: by using form-function analysis and by shifting between language and context (Gee, 2005: 54 – 58). In the former case we use the form (or structure, such as independent and dependent clauses, clauses and subordinate clauses, subjects and objects, passive and active voice, nouns and verbs, use of conjunction, etc.) of the language in conjunction with its function (purpose or meaning). In the latter case the utterance will be seen to take on a *situated meaning* since the meaning that the language takes on is dependent upon the context in which it is used, *and* the context determines the language that is to be used.

It is, however, arbitrary and subjective to simply do a discourse analysis using these grammatical cues. Gee (2005) offers seven building tasks that help us to implement the grammatical cues to 'build' our understanding of a situation, and these building tasks form the basis of what Gee refers to as the *situation network*. A description of the seven building tasks follows (Gee 2005, 10 – 19; 97 – 104):

1. *Building significance*

How and what significance is given to things? In other words, what meaning and values are being attached to what is being discussed? How are things made significant or insignificant through the choice of language, emphasis, gesture, etc?

2. *Building activities*

What activities do the participants put forward through their use and choice of discourse and how is language used to show what activity one is involved in? Passive voice (or in Gee's terms, 'backgrounding') could indicate a possible lack of agency (Janks, 2005).

3. *Building identities*

What identities (roles, positions) are the participants enacting and describing? How is language used to make the identity in the situation identifiable and consequential?

4. *Building relationships*

What relationships do the students see as existing in the situations that they describe? How is language used to show what relationships the participants recognise as being in place? How is language used to put forward and negotiate relationships between participants?

5. *Building politics*

What social goods are perceived by the participants? How is language used to express these social goods in terms of how they affect the participants? In what ways might these social goods benefit, advantage or inhibit the participants or those around them?

6. *Building Connections*

In what way are things connected or disconnected to each other? How is language used to create links or disassociations between things, events, and circumstances?

7. *Building significance for sign and knowledge*

What sign-systems or ways of knowing do the participants refer to? What language is used to show how participants know or come to know, and are able to talk about events, objects and circumstances?

From the aforegoing I propose a theoretical model for identifying Discourse models that are at play in a given context (which, for this study is in the context of students reflecting on their experiences when doing word problems). Before elucidating upon this model it is necessary to understand what Gee intends when he refers to *midlevel situated meaning*. I give a very simplistic overview here. One arrives at a midlevel situated meaning as a result of "the bottom-up action and reflection with which [one] engages the world, and the top-down guidance of the [Discourse] models [one] is developing or being apprenticed to" (Gee, 1992a: 244). Both of these are necessary. If the cultural model does not match the scenario then the circumstances surrounding that scenario are too general or peripheral to be fully embraced. If the experience of the scenario is too localised or explicit then it lacks meaning in any broader sense. If both factors are present, (i.e., a broader experiential understanding, as well as connectivity with the simplified world view) then a 'richer' midlevel situated meaning is established. I now discuss the discursive model for identifying Discourse models (*cf*. figure 2 on page 23).

The model is situated within the Discourse surrounding word problems in school mathematics. The context of action and interaction is the research situation, and in this sense we see the student as being 'in' the research situation (symbolised by the dotted rectangular outline) whilst the researcher 'looks in' from the left. The researcher's participation within the context of action and interaction is implicit and is therefore not indicated in the model since this theoretical stance seeks to explain how we come to understand that context of action and interaction as researchers.

As the student takes part in the research s/he necessarily does so as a participant in a Discourse (Gee, 2005; 1999) (labelled below the dotted rectangle), and this is *not* necessarily the Discourse surrounding word problems mentioned above – that Discourse is only observable through the Discourse models that become evident within this particular context of action and interaction (Gee, 2005; 1999). Within the context of action and interaction (the research situation) the student is involved in certain *bottom-up* action and reflection (e.g. doing word problems, past experiences with word problems, talking about these experiences, etc.), whilst the *top-down* influences take the form of the *storylines, images and explanatory frameworks* (Gee, 2005), or Discourse models, that the student holds and uses to make sense of his/her world experiences. From the bottom-up and top-down influences the student formulates a midlevel situated meaning through which s/he responds in the form of text (e.g. worked examples, written responses and talk about experiences, etc.)

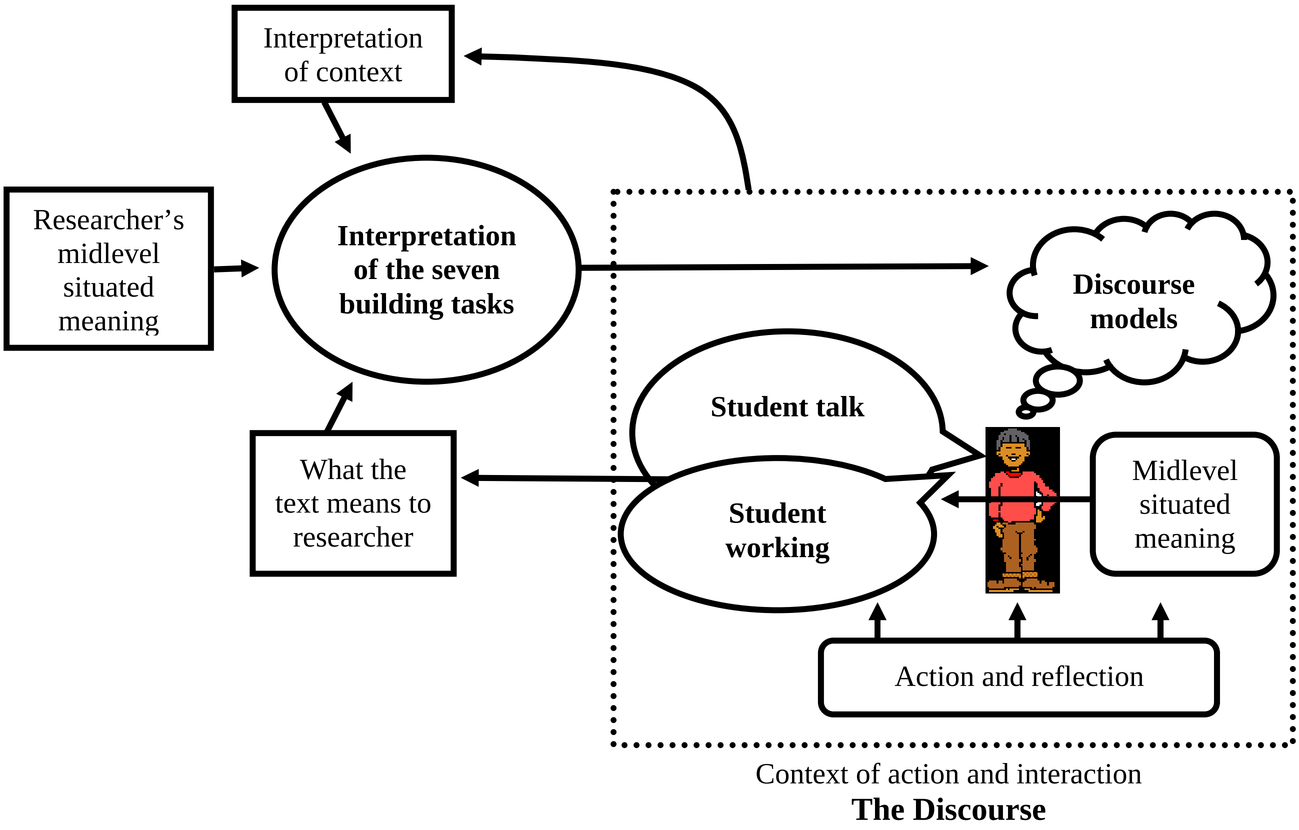


Figure 2. A discursive framework for identifying Discourse models.

From 'outside' of the context of action and interaction the researcher constructs an interpretation of the seven building tasks that the student uses to establish meaning. This interpretation comes first and foremost from the text that the student offers within the context of action and interaction, and at this level it is an amalgam of the action and reflection, and the Discourse models that served to instigate the text in the first place. What also comes to bear on the researcher's interpretation is his/her own midlevel situated meaning and in this regard there needs to be caution exercised. However, the controlling factor for the researcher comes in the interpretation of the context, or the research situation that s/he set up in the first place. Through implementing the building tasks (Gee, 2005) the researcher is guided in the interpretation of the student text so as to more reliably give access to the underlying beliefs and theories, or Discourse models that students hold that enable them to make meaning of their world.

In conclusion, the theoretical stance for this study draws on a discursive paradigm, and in particular that advocated by Gee (2005; 1999; 1992a; 1990). Whilst there are many approaches to research by discourse analysis (for example, Christie, 2002; Fairclough 2001; 1999; 1989; Jäger, 2001; Sfard, 2001; van Dijk, 2001; 1997; Jaworsky and Coupland, 1999; Schegloff, 1999; Terre Blance, 1997; Hegarty, 1995; Burman and Parker, 1993a; 1993b; Widdicombe, 1993; Parker, 1992; Potter and Wetherell, 1987; Light, 1986; Malcolm, 1985), in the context of understanding student experiences with word problems, Discourses (Gee, 2005, 1999) and the associated Discourse models offer the opportunity to understand the *simulations in the mind* (Gee, 2005: 73) that cause people to act in the way in which they do. Accessing what happens in peoples' minds is not possible. But, accessing the actions and interactions of people within certain contexts is observable, and through identifying Discourse models that are inherently at play within those actions and interactions, and understanding the possible Discourses from which they emerge, it seems that we can come to gain insight into the reasons for certain types of action that we sometimes find hard to explain.

The theoretical coherence here belies the journey that took place over a number of years (2004 – 2009). This theoretical foundation, together with the development of an appropriate methodology (both empirical and analytical) has enabled the evolution of this study. In essence I chose case studies of individuals within a particular social setting, *viz*. the research situation. The term 'individual' may appear contradictory in socio-cultural paradigm, but Gee himself acknowledges separate, idiosyncratic phenomena that enable us to regard people as 'individuals', but who function in a social world:

> "[Sociocultural practices and settings] need not … mitigate a person's own agency. Since each individual belongs to multiple sociocultural groups, the cultural models and patterns associated with each group can influence them in unique ways, depending on the different "mix" for different individuals (Kress 1985). And, of course, each individual is biologically and, in particular, neurally quite different from every other (Crick 1994).
>
> "Thus, we see that, on this perspective, talk about the mind does not lock us into a "private" world, but, rather, returns us to the social and cultural world."
>
> (Gee, 2005: 68)

I now turn to a discussion of certain guiding principles necessary for case studies and I elaborate on aspects pertaining to rigour in qualitative research that are appropriate to this socio-cultural paradigm.

Method

Case studies according to Cohen, Manion and Morrison (2000) enable the researcher to "…catch the complexity and situatedness of behaviour" (p. 79) by in-depth study of specific instances. Merriam and Simpson (1984) have the following to offer:

> "…a case study tends to be concerned with investigating many, if not all, variables in a single unit. By concentrating upon a single phenomenon or entity ("the case"), this approach seeks to uncover the interplay of significant factors that is characteristic of the phenomenon. The case study seeks holistic description and interpretation"
>
> (pp. 95 – 96)

This was particularly germane to my research since it sought to elicit cultural models through allowing respondents to interact as freely as possible. The *phenomenon* in my study was the student interpretation of word problems from textual to mathematical representation in its broadest sense. The *interplay of factors* to be uncovered comprised the discourses that revealed underlying cultural models that influence student success in interpreting these problems.

A strength of field research techniques is that the researcher "…would never for a minute rely solely on a single observation, a single instrument, a single approach. The strength of fieldwork lies in its 'triangulation'" (Wolcott, 1988: 192). My study aimed to address the three inter-related research questions, each of which was addressed to some degree by each of the data collection phases. This gave opportunity for what I called a *triumvirate* collection of data (as opposed to *triangulation* of data). I chose the term *triumvirate* since I believe that it does not purport to pinpoint some sort of 'exactness', but rather approaches the data from three different perspectives in the hope that patterns will emerge that will help to identify common factors or influences. In addition to this, discursive data can often narrow the focus as Morgan (2006) cautions:

> "… it can be argued that the results achieved in one context (such as interviewing or answering a questionnaire) offer only tangential evidence of what might be the case in a different context (such as solving a mathematical problem)." (p. 237)

The instruments that I chose (see data collection strategies) are commonly used by and appropriate to the researcher in case studies (Merriam and Simpson, 1984). They are capable of revealing rich, detailed data and they allow for flexibility and reflexivity in arriving at an understanding of student thinking. In addition to this they are varied and thus appropriate for a triumvirate collection of data, allowing for a view of individuals having their own agency, but always being, acting and discerning within a set of socio-cultural circumstances.

Considerations concerning rigour

Winter (1982), cited in Cohen, Manion and Morrison (2000), says that, “The action researcher/case study tradition does have methodology for the *creation* of data, but not (as yet) for the interpretation of data” (p. 241). In this study certain analytical devices needed to be developed so that the data could be appropriately interpreted and there were thus implications for establishing validity and reliability, terms that are generally applied to quantitative work. This is not to say that in qualitative research we can just gloss over matters of rigour (Silverman, 2001; Schumacher and McMillan, 1993; Ely, 1991) but rather that we need to establish a means of ensuring trustworthiness of the research that pertains to the methods employed. Ely (1991) advocates that we use different terminology for reliability and validity because qualitative research brings about different claims from those of other research paradigms.

Silverman (2001), citing Hammersley (1992), equates reliability in naturalistic studies with *consistency* (p. 225). This refers to the replicability of the research in different settings and/or by different researchers. This does not have the same implications as in quantitative research where one expects to arrive at the same or similar results. As far as consistency is concerned, Silverman (2000) states that, “…it is incumbent on the scientific investigator to document his or her procedure and to demonstrate that categories have been used consistently” (p. 188). This entails careful coding and classification of data gleaned from the transcripts, but in a manner that is systematic and justified to the reader. I thus believe that the qualitative data in my research can be deemed consistent since the data analysis was thorough to the point of saturation, and I make no claims about results that may occur in subsequent applications of the same methods. In the words of Merriam and Simpson (1984), “…reliability of [qualitative] results rests to a great extent with the observer” (p. 139).

Validity in qualitative studies is more appropriately viewed as *credibility*, or the extent to which the findings can be believed (Silverman, 2001). However, “Following Kirk and Miller [1986], we need to recognise that ‘the world does not tolerate all understandings of it equally’” (Silverman, 2001: 224). However, I concur with Silverman (2001) that qualitative findings *can* be established as credible if the researcher attempts every possible means to falsify assumptions and interpretations about the data. Inter-rater ‘reliability’ should be established by consultation with colleagues to ensure consistency of interpretations, and I was fortunate to have the opportunity to present aspects of my research to fellow doctoral students on a number of occasions each year.

> “Being trustworthy as a qualitative researcher means at the least that the processes of the research are carried out fairly, that the products represent as closely as possible the experiences of the people who are studied. The entire endeavour must be grounded in ethical principles about how data are collected and analysed, how one’s own assumptions and conclusions are checked, how participants are involved, and how results are communicated. Trustworthiness is, thus, more than a set of procedures. To my mind, it is a personal belief system that shapes the procedures in progress”
>
> (Ely, 1991: 93)

However, as researchers we too are situated within Discourses, and consequently hold certain cultural models about the circumstances in which we find ourselves. “Criticism must always be lodged from some set of assumed values, attitudes, beliefs, and ways of talking/writing and, thus, from within some Discourse” (Gee, 1992b; 112). As mentioned earlier, all meaning is tied up in cultural models, and therefore necessarily forms part of the Discourse, no matter who infers that meaning. Van Dijk (1997) holds that, “…social discourse [as opposed to abstract discourse] analysis defines text and talk as *situated*: discourse is described as taking place or as being accomplished ‘in’ a social situation” (p. 11). Thus, if we control the circumstances of a situation (as researchers are apt to do) then we must be aware that we also control a good deal of the structure of that Discourse (van Dijk, 1997).

The author of a text gives cues as to how the text should be interpreted by the manner in which the text is presented. However, this does not guarantee that any one particular interpretation of that text will occur and, in practice, many interpretations of the text usually take place. “The meaning is in the social practice of interpreting texts in certain ways [and that texts have the] potential to be interpreted a certain way [is] thanks to the fact that this interpretive practice *is* going on in the world” (Gee, 1992b; 18). Potter and Wetherell (1987) say that, “Discourse Analysis is not concerned with accuracy of descriptions or attitudes of the participants, but rather with the text and how this can be interpreted” (p. 160). This suggests that our task as researchers is to ‘interpret’, but in a manner that is systematic as a minimum requirement, but one which accommodates reflexivity.

Reflexivity is the means by which we as qualitative researchers are able to ensure awareness of our effects upon the research situation by problematising what we do. In other words, we reject the notion of neutral, 'non-participant', and embrace the idea of an all-encompassing social context in which the researcher acts and interacts, thus being seen as intrinsic to that situation. "…some of the enthusiasts of the postmodern in the social sciences see in [reflexivity] a way of overcoming the gulf between the individual and the social; 'postmodernists', we are told, advocate an 'anti-dualist position' (Murphy, 1988; 603). Reflexivity *appears* to provide the answer" (Parker, 1992; 93). However, every 'answer' in a postmodernist perspective brings with it a certain degree of discordance. One of the dilemmas of reflexivity is succinctly stated by Parker (1992): "In place of truth, we have perpetual reflection on the impossibility of truth" (p. 84).

However, I do not see this as a problem since, as Potter and Wetherell (1987) put it, "Reflexivity…refers to the fact that talk has the property of being both *about* actions, events and situations, and at the same time *part* of those things" (p. 182). Thus it is incumbent upon the postmodern researcher to be an active and integral part of determining the *functional* (as opposed to *constructive*) context of the participant's text (Potter and Wetherell, 1987; 165).

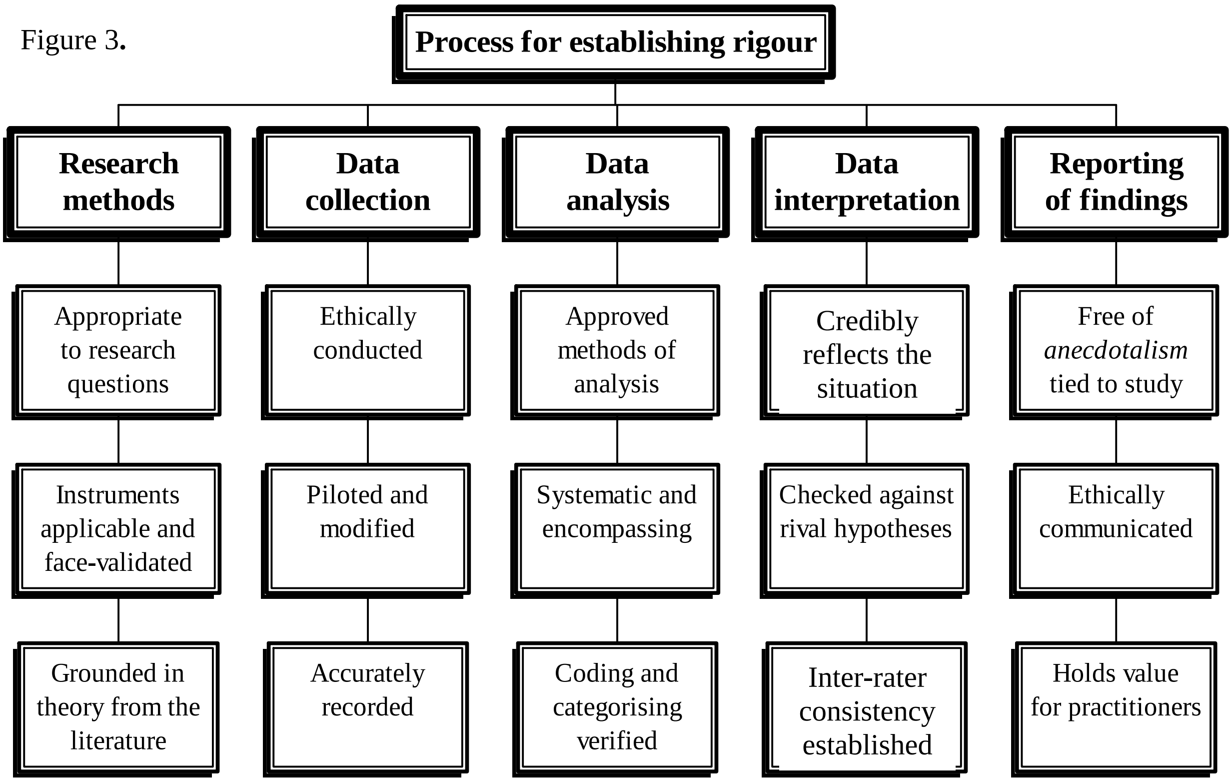
This suggests that establishing rigour is a process that is systemic to the research as a whole. I give an outline in the form of questions (adapted from Silverman, 2001) that I saw as effectively addressing issues of consistency and credibility in my study (see also figure 3):

- Are the research methods appropriate to the nature of the research questions?
- Have the research methods been adequately piloted?
- Is there a sound, applicable theoretical framework?
- Has the choice of sample been justified?
- Was the data collection and record-keeping thorough, systematic and ethical?
- Are methods of analysis acceptable, systematic and verified?
- Has the data interpretation been checked against theory from the literature and rival hypotheses?
- Has the research been accurately reported, free of *anecdotalism*[13] and ethically communicated?

[13] Silverman (2001) uses this term to refer to 'story-telling' as opposed to academic reporting.

In summary of the foregoing I include a table that depicts each aspect that forms part of the

Figure 3.



process for establishing rigour in qualitative research (see figure 3).

Having sketched the scenario that I had in mind for this study, in the chapters that follow I give an outline of the challenges that I encountered, partly because of the discursive theoretical framework that I had chosen (which is complex), but also because of the need to ensure that the analytic tools that one employs within this discursive paradigm can in fact answer the questions that the research sets out to answer. In the words of Gee (2005) I reflect:

> "… seeing that methods change with theories, it is important, as well, to see that research, whether in physics, literary criticism, or discourse analysis, is not an algorithmic procedure; it is not a set of "rules" that can be followed step-by-linear-step to get guaranteed results… [R]esearch adopts and adapts specific tools of enquiry and strategies for implementing them. These tools and strategies ultimately reside in a "community of practice" formed by those engaged in such research." (p. 6)

The development of suitable theoretical and methodological means in themselves have come to inform discursive interpretation within a mathematically-based study (in particular, student difficulties with word problems) and, as discussed earlier, I describe this journey as the thesis ensues. In the next chapter I describe how the pilot studies helped to guide the way in which I planned to collect discursive data. I then outline the instruments and data collecting strategies that I employed for the initial case study, and the remainder of chapter 3 gives a synopsis of the analysis and the findings that emerged from that initial case study. As will become apparent in chapter 4 onwards, an appreciation of this initial case study is important for understanding the reasons for, and the genesis and evolution of the extended study.

Chapter 3 – The pilot studies and the initial case study

When I set out in 2004, had I been aware of the advice of Morgan (2006) I might not have been quite so naïve in my first attempts to collect discursive data:

> "Individuals do not speak or write simply to externalise their personal understandings but to achieve effects in their social world. Studying language and its use must thus take into account both the immediate situation in which meanings are exchanged (the context of the situation) and the broader culture within which the participants are embedded (the context of culture)." (Morgan, 2006: 221)

The first pilot study

In the first pilot study Grade 10 students were clinically interviewed. They were required to do word problems and explain what they were doing, whilst at the same time being questioned about their reasons for embarking on a certain course of action. It was found that, not only do students experience difficulty in expressing their thinking, but to do this whilst having to address the mathematics proved counterproductive to both the solution process as well as the expression of their thoughts. The following is an excerpt from one of the interviews to demonstrate what I mean:

BT Let's have a look at that problem. Read the problem…

S A soccer field has a perimeter of 350 metres. The length is 20 metres shorter than twice the breadth. Find the length and breadth of the soccer field.

BT What I would like you to do is you can start to show me how you would actually solve the problem and try to talk to me if you can about what you are doing. Try to tell me what you are doing.

S Ok. Uh, soccer field. So I would use algebra by… I would make say *x* and *y* the length and the breadth. And then metres, so, I know that *xy*…no sorry, 2*x* plus 2*y* is equal to the perimeter, say. So then I've got…er…which is equal to 350 metres obviously. Length, which is 20 metres shorter than the breadth, and so… and twice the length… Oh, Ok. So then let's say that *y* is breadth, *x* is length. So 2*y* is equal to *x* minus 20. And so is that one there, presumably. Or the length combined. Ja [Yes].

BT I would say they are talking about the length.

S Ok. So…jus…I'm not too good at these ones.

BT You don't have to worry about whether you get them right or wrong.

S Oh. Ok.

BT That is not really important.

S I know there are all types of different like equations. Like if you have to work on age and stuff like that. Like ones that were, err, because… [inaudible]

This student is clearly not getting the correct solution because his thinking is being obscured by having to explain what he is doing. He is also quite threatened by the one-on-one situation. Retrospectively, I was looking for some kind of 'externalising' discourse from the students, whilst the situation quite clearly indicated that the emergent discourse was very much affected by the context of the situation (Morgan, 2006). What was needed was to allow the students time to do the problems that was separate from talking about the problem in an environment that was less intimidating. In addition to this the students needed the opportunity to develop their meta-thinking about the problems so that they could communicate their ideas more easily. I decided to undertake a second pilot study.

The second pilot study

In order to address the issues of allowing the students to do the problems and talk about them separately and to allow for the development of their meta-thinking the second pilot study comprised three phases: doing the problems, teaching the problem and talking about the problem. This pilot study was far more successful in that what emerged in the transcripts was more informative of the students' experiences of the word problems which they were able to relate separately from their experiences when doing the problems. However, in doing the problems the students were allowed to interact with each other as well as with me. I felt that there was too much interaction in the early stages that influenced the students and this needed to be redressed.

The initial case study

The second pilot study informed me of the need for activities that would elicit discourse about word problems, but that would inform me of the underlying cultural models they were at play. I also wanted to capture students 'own thoughts'[14] on how they perceived word problems before opening up avenues for interaction about these thoughts. I saw this as being addressed by ensuring that the situation was 'normal' for the students (which took the form of a test-like item and individual written responses) and that this happened prior to any interaction between the students and myself. In order to capture discourse of 'own thoughts' I included a short questionnaire and the opportunity for students to write a short paragraph giving their thoughts

[14] I use the term 'own thoughts' to mean the students' sentiments about word problems at that time, prior to any sharing of ideas between the other participants and me as the researcher.

and ideas about word problems. The actual study was then made up of three phases: the first phase incorporated doing the problems, writing about word problems and a peer group discussion about the problems; the second phase involved teaching the problems to a younger student; the third phase took the shape of a group interview about the problems and the students' experiences of the problems.

The second phase referred to above (teaching the problems to a younger student) did not elicit data that informed the study. The reason for this was that I had interacted with the students about the problems before they undertook the teaching because it would not have been ethically acceptable to have these younger students 'incorrectly' taught. Unfortunately, I could see later from the transcripts that my interaction with the students had greatly affected what they told the younger students in the 'teaching' situation, and bore very little resemblance to what they had actually done with the problems. The intention of this phase was to give the students opportunity to develop meta-thinking about the word problems that they had attempted, and to try to view this through the activity of 'teaching' a younger student, but as it turned out the data largely revealed *my* thinking about the problems. I decided not to include this phase in the analysis because of this and I have omitted further discussion of that phase in this discussion, but the transcripts are available on a compact disc for anyone wishing to view these.

The instruments were thus intended to elicit data as follows: doing the problems allowed access to student activity; the questionnaire and short paragraph gave access to 'own thoughts'; the peer discussion gave access to ideas in the 'communal space'; the focus group discussion gave access to a deeper interrogation of ideas.

The sample for the initial case study

Four grade 10 students who were deemed to be of 'average' mathematical ability by their teacher[15] were used for the initial case study. I felt that weaker mathematics students may not have been able to even begin the problem solving and this would not have given me access to any discourses or cultural models that were at play. Stronger students may have been too fluent in the solution process, which may have resulted in certain thought processes being

[15] I will address why I chose to study students and why I allowed the teacher to choose the students in more detail when I discuss the limitations and delineations of the study.

taken for granted. Put simply, to generate data that would address my research questions I needed students who would find it necessary to grapple with the problems.

The reason I chose grade 10 students is because word problems in grade 9 and grade 10 are very comparable and are only linear. Grade 11 word problems extend to more complex rate problems and some lead to quadratic equations. I also believe that grade 11 students would cope fairly easily with grade 9 and possibly even with grade 10 problems, and this could have obscured some of the discourses and cultural models that I hoped to describe. In using grade 9 word problems I also ensured that the grade 10 students would have covered the problems in the normal schooling, albeit not recently.

Data collection strategies

Initially the study focused on a single case (one school), and this is referred to as *the initial case study*. However, this proved to be insufficient for a clearer understanding of the cultural models that were at play. The study was therefore expanded to include three more cases and this is referred to as *the extended study*. The reason for this study being discussed in this format is that analytically (as will be discussed in the next chapter) the two parts are quite different, but the initial case study played an integral role in the development of appropriate tools to identify and be able to talk about cultural models that shed light on why students respond the way in which they do when it comes to word problems.

The first phase of the initial case study took place on one afternoon. That is to say that the students wrote a short paragraph about what they thought word problems are, and then attempted three problems before answering the questionnaire. This was then followed by a peer discussion at which I was not present. At this peer discussion the students were given their attempts at the problems (without pens) and were asked to discuss amongst themselves differences and similarities in approaches, how they generally felt about word problems and anything that they wanted to share about word problems. This discussion was video and audio recorded.

The next phase (the focus group discussion) could only take place the following Monday and this may have had the effect of ‘diluting’ the data since the ideas were not as fresh in the minds of the participants as I would have preferred. I now give a synopsis of the data collection.

The data collection for the first phase

The students were given booklets that guided them through the writing of a short paragraph and then doing the three problems (*cf*. annexure 2)[16]. The problems for the initial case study were selected according to three characteristic 'problem-types' (Gerofsky, 1996), *viz*. a money problem, an age problem and a speed-distance-time problem. The problems were chosen from a grade 9 text book widely in use in South Africa at the time:

Problem 1 – The watch problem

A dealer buys 200 digital watches. The cheaper watches cost R24 each and the more expensive watches cost R36 each. If the watches cost him R5 760, how many of each type does he buy?

(Laridon *et al.*, 1992: 257)

Problem 2 – The age problem

Peter is twice as old as Paul. Ten years ago Peter was three times as old as Paul. How old is Peter?

(Laridon, *et al.*, 1992: 259)

Problem 3 – The aeroplane problem

Two towns, A and B, are 195 km apart. A plane leaves town A travelling at a speed of x km/h towards town B. Another plane leaves town B at exactly the same time, travelling 60 km/h faster than the other plane, towards town A. The two planes meet after $\frac{3}{4}$ of an hour. Determine the speed of each plane.

(Laridon, *et al.*, 1992: 259)

Once all the students had attempted all three problems they were issued with the questionnaire (*cf*. annexure 3). The students then engaged in the peer discussion and the full transcription of this discussion is included on a compact disc.

Writing a paragraph

Student activity	Grade 10 students wrote a paragraph on word problems.
Researcher involvement	Booklets were handed out with guiding questions (see annexure 2). Students were asked to be as open and honest as possible.
Purpose	Establish student perceptions of word problems (research sub-questions 2 and 3).
Data collection	Students' written ideas.
Equipment	Student booklets and black pens
Instructions to students	Students were asked to write about how they feel about word problems. The questions were only there as a guide and they were encouraged to include anything else that they wanted to say.

[16] Note that annexure 2 includes the problems selected for the extended study. However, the format of the booklet was the same except for the questions.

Doing the problems

Student activity	Grade 10 Students did three Grade 9 problems from Laridon *et al* (1992). (See the problems listed above.) They worked independently showing all working.
Researcher involvement	Instructions were given to students and they were reassured that they were not being examined. I invigilated to ensure independent working but no assistance was given to students.
Purpose	What do students do with word problems when they do these on their own? (Research sub-question 1.)
Data collection	Student working and field notes taken whilst the students were working.
Equipment	Scientific calculators, answer pages with each question in the booklets and black pens.
Instructions to students	This was not a test. Students were asked to work independently. They could solve the problem in any way they wished, but were to show all working where possible. They were allowed sufficient time to do all problems.

Answering the questionnaire

Student activity	Grade 10 students responded to a questionnaire on word problems.
Researcher involvement	Sheets were handed out with guiding questions (see annexure 3). Students were asked to be as open and honest as possible.
Purpose	Establish student perceptions of word problems (research sub-questions 2 and 3).
Data collection	Students' written ideas, but in response to the problems just attempted.
Equipment	Questionnaires and black pens
Instructions to students	Students were asked to the questions about word problems, and were told that they were not obliged to answer questions if they did not wish to. They were encouraged to be as honest open as possible.

The peer discussion

Student activity	Students compared and discussed their solutions directly after phase 1, step 2.
Researcher involvement	The researcher left the students to discuss the problem solutions without his presence so as not to influence the student talk in any way.
Purpose	To establish how students talk about the problems (research questions 1 and 2).
Data collection	Audio and video recordings and subsequent transcripts, as well as any extra student working.
Equipment	Scientific calculators, the student working from phase 1, step 2, extra paper and black pens.
Instructions to students	Students were told that they would be left on their own for 10 – 15 minutes. They were to compare and discuss each other's solution to the problem, explaining what they did. They were to look for differences and similarities in approach and decide which methods worked best and why. They also discussed anything else that they felt emerged during the comparisons of workings.

The data collection for the last phase

This comprised a focus group discussion in which I compiled guiding questions from the preceding activities. The discussion was fully transcribed and, because it is rather lengthy, it has been included on the compact disc.

Student activity	A group discussion was held with certain questions compiled in advance from the first three phases to guide discussion. The Grade 10 students were encouraged to express their ideas and discuss them with the researcher and with each other.
Researcher involvement	Questions were compiled to guide discussion of particular events, talk, solution strategies, etc. The discussion was led by the researcher.
Purpose	To establish how students position themselves in relation to word problems and how cultural models mediate in the students' interpretation of word problems and their corresponding relationships.
Data collection	Audio and video recorded, with subsequent transcripts.
Equipment	Rough paper, calculators and pens and original solutions to the problems from the first phase. Pre-prepared questions to ensure that the necessary issues were addressed.
Instructions to students	Students were told that this is a 'chat' and that there were no right or wrong answers. They were encouraged to be as open as possible about their feelings and beliefs, but that they were not obliged to answer any questions or offer opinions if they did not wish to.

Delineation of the study[17]

As a secondary school mathematics teacher I am particularly interested in the incongruous mathematical statements that students sometimes make when translating from textual problems to symbolic mathematical representations. Thus my study focussed on the interpretation of word problems explicitly with respect to how they were represented algebraically or with respect to what methods and procedures were being adopted in the solution process. In other words, it was *not* concerned with the mathematical processes and manipulations that are employed in solving word problems, and it was not concerned with language in terms of the language of learning and teaching, nor of the languages of the individual participants, but was more concerned with how these relate to the interpretation and representation of the word problems into mathematical form.

Case studies are time consuming and for this reason it was necessary to restrict my research to four grade 10 (about 15 – 16 year old) students from each site. Thus the initial case study

[17] Whilst the extended study is elaborated in the next chapter and the focus here is on the initial case study, the delineation and limitations of the study discussed here hold for both, and will not be repeated later on.

comprised one case (the site) where four students were examined, and the extended study consisted of three cases (the sites) with four students being selected at each site. This, unfortunately, but necessarily did give less opportunity for cross-case study analysis, both in the initial case study and the extended study (see chapter 4).

I chose to do a discourse analysis of data. "Not only is the relationship between what is 'inside' and 'outside' language problematised by [this approach], but the very terms and tools of our inquiry and evaluation become matters of interpretation and debate" (Burman and Parker, 1993: 3). I acknowledge that discourse is highly interpretative and in this respect I invite the reader to reinterpret where they feel it to be fitting. This study aimed at providing an alternate means for examining how students confront word problems and is not prescriptive of how to interpret the results that arise from that examination.

Limitations of the study

I am a teacher of secondary school mathematics. Suffice it to say that my interest and experience lie in the teaching of mathematics to secondary students in South African schools. I have therefore limited my study to four Grade 10 students at each of the chosen sites. Thus, at most, my study can only serve to give a better understanding (Schumacher and McMillan, 1993) of how Grade 10 students might interpret word problems, and there can be no claim of generalisability of the findings. It was my intention that, "generalizability [be] related to what each user is trying to learn from the study" (Merriam and Simpson, 1984: 98).

In the initial case study it was in the interests of the students for me to allow the teacher to select the sample, based on the criteria mentioned earlier, but giving her the discretion to choose students who would not be too daunted by the circumstances of the research. This was because of the unusual data collection procedure and I felt that the teacher was in a better position to make these choices. However, this did raise the question of whether we were dealing with a 'typical' student or not. My answer to this is that the study aimed to establish links with what students do in interpreting word problems and the possible interfering factors that may be present, and that the findings can serve to establish hypotheses that promote understanding and hopefully underpin possible further research (Merriam and Simpson, 1984). Typicality of students, I believe, was not an issue. However, in the extended study I decided to remove the 'teaching' element of the data collection (for reasons that are explicated in my discussion of the data collection for the extended study) and therefore the need for the teachers to select the students no longer existed. The students were therefore randomly selected as discussed in the sampling for the extended study which is detailed in chapter 4.

According to Lave and Wenger (1991) the context or setting determines how one reacts in a situation and this could affect the way in which the students interact in the 'teaching' situation. It is possible that pedagogical cultural models may influence, and be more 'visible'[18] than the everyday attitudes and beliefs of the students. However, this may well have served to provide a richer understanding of how students perceive mathematics in relation to the pedagogy and their ordinary social setting when it comes to word problems.

A further concern was the relative linguistic competence of the students in the language of learning and teaching, especially in the context of South Africa where the predominant language of learning and teaching is English, but a very large proportion of the student population do not have English as their first language. It was necessary in this study, therefore, to be aware of these issues in terms of the meanings that students intended. However, whilst issues surrounding linguistic fluency are important, I believe that they are beyond the scope of this study and possibly open up avenues for further research from a discursive perspective.

Finally, case studies require expertise in interviewing and observing (Merriam and Simpson, 1984) and in these areas I am a novice. I believe that theoretically I was prepared, but it is only in the actual undertaking of these tasks that shortcomings become evident.

Ethical considerations

In my study there were ethical issues surrounding the integrity of the schools as well as "informed consent, guarantees of confidentiality, beneficence and non-maleficence" (Cohen, Manion and Morrison, 2000: 279) of the students that participated in the study. Ely (1991) states that, "The very naivety of many research participants makes it the more imperative that we are careful to protect them" (p. 223). As with establishing rigour, I believe that ethical issues in any research study are pervasive and that it is incumbent upon the researcher to

[18] I use the term 'visible' in this context in a similar way to Adler (2001). She found that teachers face a dilemma with respect to when to make *the mathematics* as opposed to *the mathematical language* 'visible'. As it pertains to this study, certain circumstances may predetermine that certain cultural models become more 'visible' than others.

ensure that all undertakings are ethically conceived, conducted, interpreted and reported. I now turn to a discussion of how I ensured that my research was principled, moral, fair and honourable.

I chose to undertake this study at schools other than the one at which I teach because I believe that I am too closely associated with the students at my school, and that subjectivity and bias are more likely to be prevalent. Also, students at my school may have felt compelled to participate in the study because of my status within the school. A third factor was the possibility of staff 'common-room chat' that may have resulted in inadvertent references to students and their actions whilst involved in the study. However, even in an unfamiliar school, concerns for objectivity and confidentiality are not negated. Cohen, Manion and Morrison (2000) state that "…researchers inhabit the world that they are researching, and their influence may not be neutral" (p. 315). Adler and Lerman (2003) give the following guidelines:

> "…Interpretative research is more ethically problematic than positivist research precisely because it is always ethically uncharted. It calls, therefore, for: care and reflexivity; refined notions of consent, including participation of research subjects and continual reaffirmation of consent; and a refined notion of autonomy and privacy…"
>
> (p. 9)

With regard to participants, Adler and Lerman (2003) and Silverman (2001) (amongst others) refer to *informed consent*. Silverman (2001) gives a table taken form Kent (1996: 19 – 20) that helps to elucidate aspects of informed consent that need to be borne in mind:

- Give information that is relevant to the subjects' decisions about whether to participate.
- Make sure that the subjects understand that information.
- Ensure that participation is voluntary.
- Obtain parental consent when dealing with children.

The Provincial Department of Education consented in writing to the research being done in schools under their auspices. I also obtained consent from the principals and the head of mathematics in each of the schools where I conducted, the research and this was followed up with a copy of my research proposal. I issued a consent form to be completed by students and their parents in which the following was given (*cf*. annexure 1):

- A brief outline of the research topic and aims.

- An overview of the activities in which the students were to be involved.
- An assurance that the research and the findings would in no way be any reflection of individual students, their families or their school, and that participants would be entreated to approach the study in this light.
- A guarantee of *autonomy and privacy* (Adler and Lerman, 2003) – participation in the study was purely voluntary and anonymity would be maintained, especially in the reporting of the findings.
- An explanation of how the research findings are likely to be used (Adler and Lerman, 2003).
- A set of tick-boxes in which consent could be indicated for transcripts and recordings to be used by the research team, scientific or educational publications and by other researchers (Silverman, 2001).

There are also important ethical issues surrounding the reporting of qualitative research. Citing Howe and Moses (1999), Adler and Lerman (2003) state that,

> "…there is a tension to be continually negotiated between thick description and privacy, between whose version emerges, or who owns the data interpretation, and between responsibilities to outside agencies and to the research sites" (p. 10)

To ensure integrity, reporting of the findings has been done in such a way that it ensures *confidentiality, anonymity, non-identifiabilty and non-traceability* of the participants and the schools (Cohen, Manion and Morrison, 2000: 292). All names used in the transcripts and subsequent discussions are fictitious.

In addition to this, in the initial case study I included an activity that focussed on an unusual interaction between students in which the potential for malevolence was high. This was the 'teaching' of word problems to a younger student. Therefore, it was imperative that I elicited the cooperation and empathy of the students with regard to the role that they each played in the research process. This was not difficult to achieve because of the familiarity of the context for the students, but I was present to monitor and, if necessary, check their interaction. Fortunately, this was not necessary. Since this activity was omitted in the extended study concerns over any kind of malice or rancour were considerably lessened.

The analytical framework

The analysis of the initial case study adopted a discourse perspective to ascertain influences that affect how students approach word problems. It was hoped that this would guide understanding of influences that contribute to or interfere with effective interpretation of textual word problems into mathematical representations.

Classroom discourse[19] can be seen as ranging on a continuum from *procedural discourses* to *conceptual discourses* (Setati, 2002: 30). I saw this continuum as a useful analytic tool for the first phase of the study, but I redefined these terms slightly so that they more appropriately addressed the analytical aspects of student discourse with specific reference to the thought processes and student actions involved in translating word problems into algebraic symbols. Setati (2002) believes that,

> "Procedural discourse is used to describe discourses that focus on the procedural steps to be taken to solve the problem." (p. 30).

In the context of my study procedural discourses pertain to what the students do algorithmically to achieve mathematical goals and they do not indicate or necessarily require any understanding of the mathematical processes involved. In solving word problems this may include such mechanical practices as the use of tables or formulae. In my experience teachers use a table as a means of making word problems more accessible for their students. To an extent the study undertaken by Kutscher and Linchevski (1997) could be seen in this light since they used their "table-filling" method as a means of helping students to generate correct algebraic equations. I rephrased this as *algorithmic discourse* since this study was not concerned with algebraic manipulations, but rather with the thought processes involved in the setting up of algebraic relationships by means of 'standardised' or 'taught' practices, as for example with Kutscher and Linchevski's "table filling" method. This, I believe, is more akin to Boaler's notion of *inert* knowledge[20] and this gives rise to what Boaler (1997) refers to as *cue-based behaviour*. Of course it is not to say that because students used a table, for example, to formulate their thinking that they were engaging in algorithmic discourse. It was incumbent on me to ascertain the extent to which students used the table to merely acquire the mathematical representation, or to formulate understanding that would generate a relationship.

[19] By classroom discourse I mean not only student talk, but all forms of student action relevant to the situation, such as gestures, written or diagrammatic expression, etc.
[20] The reader is referred to Boaler (1997: 96 – 109) for a more detailed account of what constitutes *inert* knowledge as opposed to *flexible* knowledge.

This distinction was achieved through an allying of what the student did and what he said. In essence, algorithmic discourse is characterised by following prescribed procedures or using a 'method' in which there is little or no understanding of the reasons for applying them. In addition to this, algorithmic discourse is recognised when the student uses the procedure or 'method' simply as a means to an end.

This brings me to the notion of *conceptual discourses*. These involve discussions or rhetoric that reflect reasons for undertaking a particular approach or an attempt to understand why and for what purpose the mathematics is being used. According to Setati (2002),

> "In conceptual discourse, the learners articulate, share, discuss, reflect upon and refine their understanding of the mathematics that is the focus of the interaction/discussion." (p. 30).

Here I preferred to use the phrase *heuristic discourse* since it more aptly described actions that emerged without a *dependence* on algorithmic processes, and it was this *action*[21] that was to inform my study. I see this form of discourse as being closely associated with the concept of *flexible* knowledge (Boaler, 1997; De Corte and Verschaffel, 1989). Student interpretation of word problems can be seen as demonstrating heuristic discourse if there is a questioning of relationships or setting up of procedures that guides understanding, such as the use of a diagram, table, or any workings that serve to guide discernment. Use of a given procedure or 'method' does not preclude heuristic discourse. The distinction lies in *how* the student applies the procedure or 'method', and this can be gauged by the extent to which he is reliant upon it as a crutch.

The analytic framework developed as the analysis of the first phase ensued. It became evident that it would be more beneficial to incorporate analyses of the peer discussion with the group discussion so that the students' elucidations could help to inform me of the cultural models, but that these needed to take the students' written responses into account as well. To elucidate on the accessibility of cultural models I quote from Setati (2002):

> "Cultural models do not reside in people's heads. They are available in people's practices and in the culture in which they live – through the media, written materials and through interactions with others in society" (p. 49).

[21] I use *action* in a socio-cultural sense. According to Gee (1992), "… concepts do not sit formed once and for all in [our] heads in grand isolation from the vicissitudes of practice and judgement" (p. 236). Individual action is situated and therefore unavoidably influenced. Analysis of discourse needs to take place through observation of interaction, and this will imply influence on action. However, what the individual expresses can inform us of what societal influences are acting upon him, thus indicating any mediating factors (see research question 3).

As mentioned in chapter 2, Gee gives three types of cultural models that can serve as analytic tools: *espoused, evaluative* and *models-in-(inter)action* (Gee, 1999: 68). I now elaborate on these with respect to the analysis of my study.

Espoused cultural models are those to which the student consciously ascribes. An example of this might be *mathematics is important for amelioration.* Whether or not this can be attributed to societal, parental or peer influence, I believe it has a bearing on what the student does with the word problem. In other words, does the student adopt algorithmic approaches simply to 'pass', or does he opt for heuristic procedures because of the inherent value that is perceived in the mathematical procedures? A connection between the approach to the problem and the espoused cultural model can thus inform us of why the student chooses to act as he does. Espoused cultural models therefore reflect the students' opinions about word problems that they see as being archetypal and standard.

Evaluative cultural models are those through which students, overtly or covertly, make judgements about their capabilities, the capabilities of others, the value of word problems, etc. An example of such a cultural model might be *word problems are difficult*. Such a cultural model may cause students to adopt an algorithmic approach (such as using a table) because there does not seem to be an alternative, or that it is perceived to be beyond their capabilities to do otherwise. Evaluative cultural models, I believe, can go a long way to explaining why students approach word problems in the way that they do because it indicates the way in which they judge the problems in terms of how they view 'what should be'.

Interactive models[22] are more specifically beliefs that arise from what students perceive they are expected to do in response to their environment. To give an example, students may hold a cultural model that *word problems don't help in solving real life predicaments*. In this situation the student probably sees little relevance in understanding the problem and the related mathematics, and therefore simply takes an algorithmic approach that is bound to get the 'right answer'. In other words, the student experiences surrounding word problems influence their actions when tackling word problems.

[22] Gee (2005; 1999) refers to these cultural models as *models-in-(inter)action* so as to incorporate aspects of *action* simultaneously with *interaction*. Whilst I concur with Gee that *action* and *interaction* in a discursive socio-cultural scenario are inseparable, I prefer the less cumbersome term *interactivel models* to denote both the action and interaction that underpin assumptions of normality and typicality.

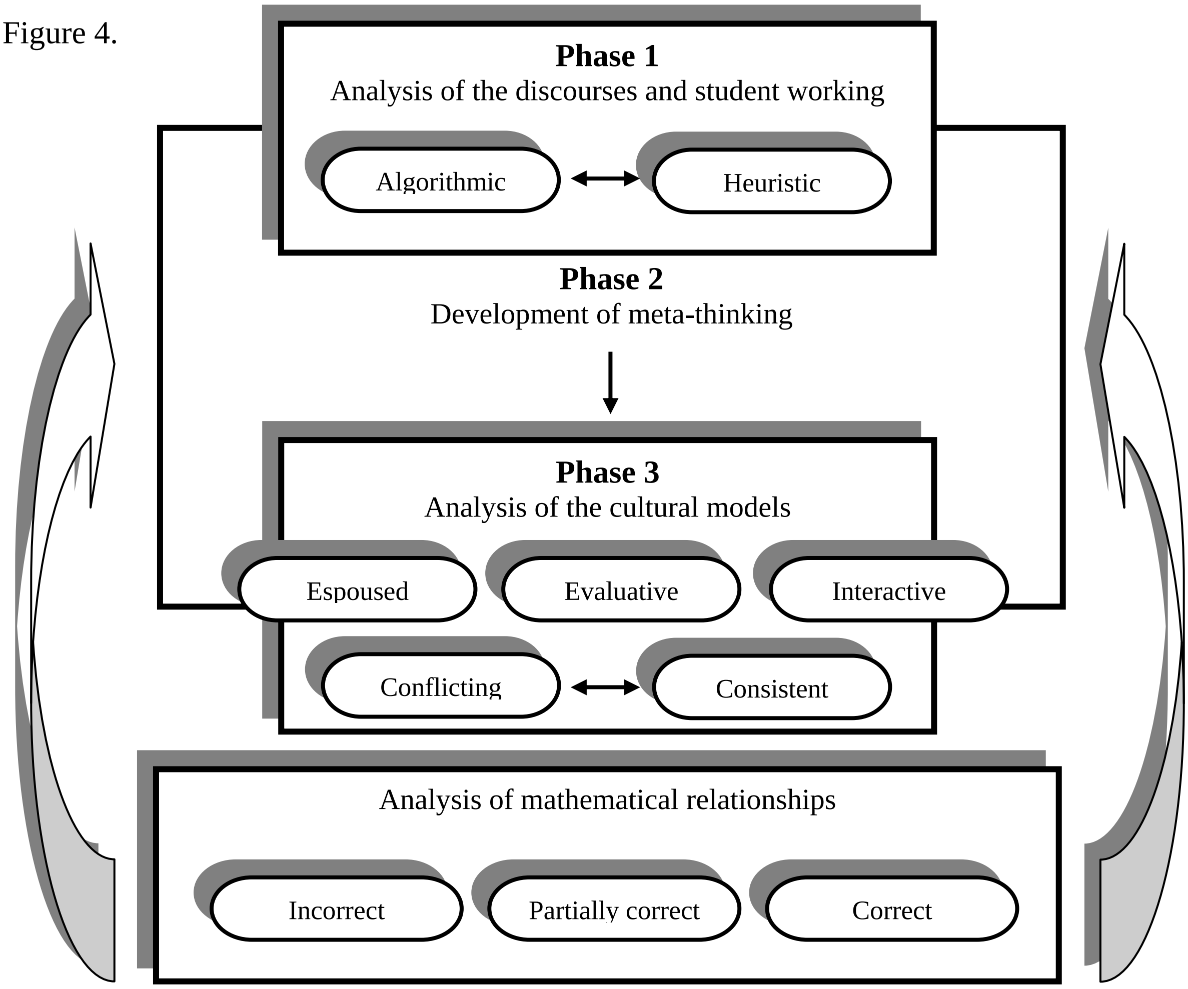
Having identified the cultural models that are at play, and categorised them as discussed above, there needed to be an analysis of these models in relation to the discourses that the students used in order to understand what it is that the students are thinking. Setati (2002) identifies cultural models that are *consistent* and those that are *conflicting*. I see these as useful tools for gauging how cultural models serve to either reinforce beliefs and attitudes, or possibly create dilemmas for students.

With this in mind, two categories of cultural models emerged. I did not initially anticipate these in my theoretical and analytical frameworks but they gave a means for explaining how the students were positioning themselves with regard to word problems. The first category consisted of cultural models that the students inherently ascribe to as a result of their experiences with word problems. I called these *empirical* cultural models. The second category comprised cultural models that define what the students believe teachers and other educational figures ascribe to. These I refer to as *putative* cultural models. Putative cultural models reflect the students' perceptions of what word problems are *supposed* to be and do, whereas empirical models indicate what word problems *actually* are or do for the students. These two categories of cultural models are not necessarily mutually exclusive but there does seem to be quite a rift between them in many instances in this study.

Finally the analysis examined what mathematics the students produced from the text with respect to how accurately it reflects the problem. In a cyclic manner this addressed relationships between the cultural models, the discourses and the success with which the students interpreted the problems.

I pose a model (see figure 4 on page 30) in summary of the foregoing discussion, but I acknowledge that data analysis was largely an *inductive process* (Merriam and Simpson, 1984) in which new categories emerged as the data became available for organisation. Thus, empirical and putative cultural models do not appear in the model because these became evident to me in the actual analysis, but they were informative in the way in which they manifested as being consistent or conflicting.

Figure 4.



In the data collection I used Gee (1999) as a frame of reference for viewing cultural models as tools of inquiry. However, the mathematical context necessitated that I review his original questions so that they would address my research questions as they pertained to the mathematics. I came up with the following questions:

1. *What cultural models are at play*? This question guided me to examine the discourse (in the broad sense) and to thereby identify aspects that the students hold to be *typical* or *normal* (Gee, 1999), which in turn would point me to what cultural models the students hold. This would enable me to answer research sub-question 1.

The following guiding questions would enable me to answer research sub-questions 2 and 3:

2. *What are the differences/similarities between these cultural models*? In other words, how could I group these cultural models so that they 'spoke' the same kind of message when viewed as an entity? Here I relied on Gee's description of espoused, evaluative and interactive models for my groupings (cf. Gee, 1999; 68).

3. *Are there master models at play*? Here I looked for one model that accurately encompassed the other models within a group to address Gee's notion of *models within models*.
4. *To what extent are the models consistent or conflicting*? This helped me to position the cultural models (or grouping of cultural models) in such a way that they inform us of the discord or tenacity that they engender in the students.
5. *Whose interests do the cultural models serve*? Here I was looking at whether the cultural models were putative or empirical, as this immediately identified the manner in which the student ascribes to the model. In other words, if the student does not experientially see the relevance of the cultural model, then, at least at that point, it serves the needs of someone else.

The analysis

The initial case study involved a contextual and discourse analysis of the full transcripts and all texts and as a result of this it is too lengthy to include in this report. The full analysis is available on a compact disc and a synopsis of that analysis is given here insofar as it impacts the extended study.

	more algorithmic	more heuristic
correct solution		2
makes some progress	3	2
some working, but not correct	6	1
cannot proceed	3	

Figure 5 – Overall solution strategies.

The analysis of the student working

This 'doing' phase of the research is similar to what many researchers have examined in the past in that it looks at what the students actually did with the problems. However, when we look at the discursive aspects of what the students did the data gives a different slant on the difficulties that the students experienced.

What is evident is that all four of the students demonstrate a vertical split when it comes to solution strategies. The table (see figure 5) gives an overview of all the solution strategies undertaken in this study. Here the number that appears in the table represents the number of times that all the students used a particular strategy with respect to the degree of success that they experienced in solving the problem. What is immediately noticeable is that there were only two fully correct solutions and that these were obtained through heuristic means.

The vertical split is also noticeable. Algorithmic procedures have yielded nine out of twelve attempts that are incorrect whilst heuristic methods have yielded four out of five that are fully or partially correct. However, these heuristic methods have largely been intuitive whereas the algorithmic techniques have been mostly 'taught' processes. The general lack of success with algorithmic methods demonstrates that these students are not positioned flexibly (Boaler, 1997) when it comes to word problems. They don't appear to have the confidence to solve the problems algebraically and they seem to make more progress when they use their 'own methods'. Thus it is not surprising that the students are lured into using their own methods, which is what De Bock *et al*. (2002) describe in their study as being coerced into intuitive reasoning.

The analysis of the student talk

From the analysis a master model is evident and that is *word problems are obfuscatory*. This is clearly an empirical cultural model for all the students since the discourse that underpins it is experiential, and quite emotively so.

The analysis showed that each student's experience of the perplexing nature of word problems is different. This has resulted in the development of further empirical cultural models, some of which are common to two or three of the students. In this study it appears that empirical models have 'grown' from previous models, and it seems that this is a result of repeated experiences when looking at the discourse (for example, the use of words like *always* and *usually*). In figure 1 I have attempted to represent this idea diagrammatically. The empirical cultural models 'grow' downwards from the master model. The circled cultural models are shared by two or three students. The arrows show the development of subsequent cultural models starting with the master model.

What appears to be happening with all four students is a spiralling effect in which more negative cultural models 'grow' and these are expressed with heightened emotiveness. Imran is a very good example of this as is illustrated in figure 1. For him the difficulty of the word problems manifest themselves in an illusionary quality that frustrates and terrifies him, resulting in a loss of marks in exams and the accompanying anxiety that this brings. Following Warren's and Karim's 'growth' of cultural models we see very similar trends. For Gary the difficulty about word problems is that he feels one needs proficiency in language and this results in him not being able to enjoy the working out of word problems as well as the accompanying notion that they are disliked by everyone.

Another useful way of viewing the cultural models is in the categorisation suggested by Gee (1999) since it enables us to see a 'flow of events' and this helps to give insight as to why students tackle word problems in the way that they do. It appears that there is a flow from espoused models to evaluative models and then to interactive models.

Figure 6.

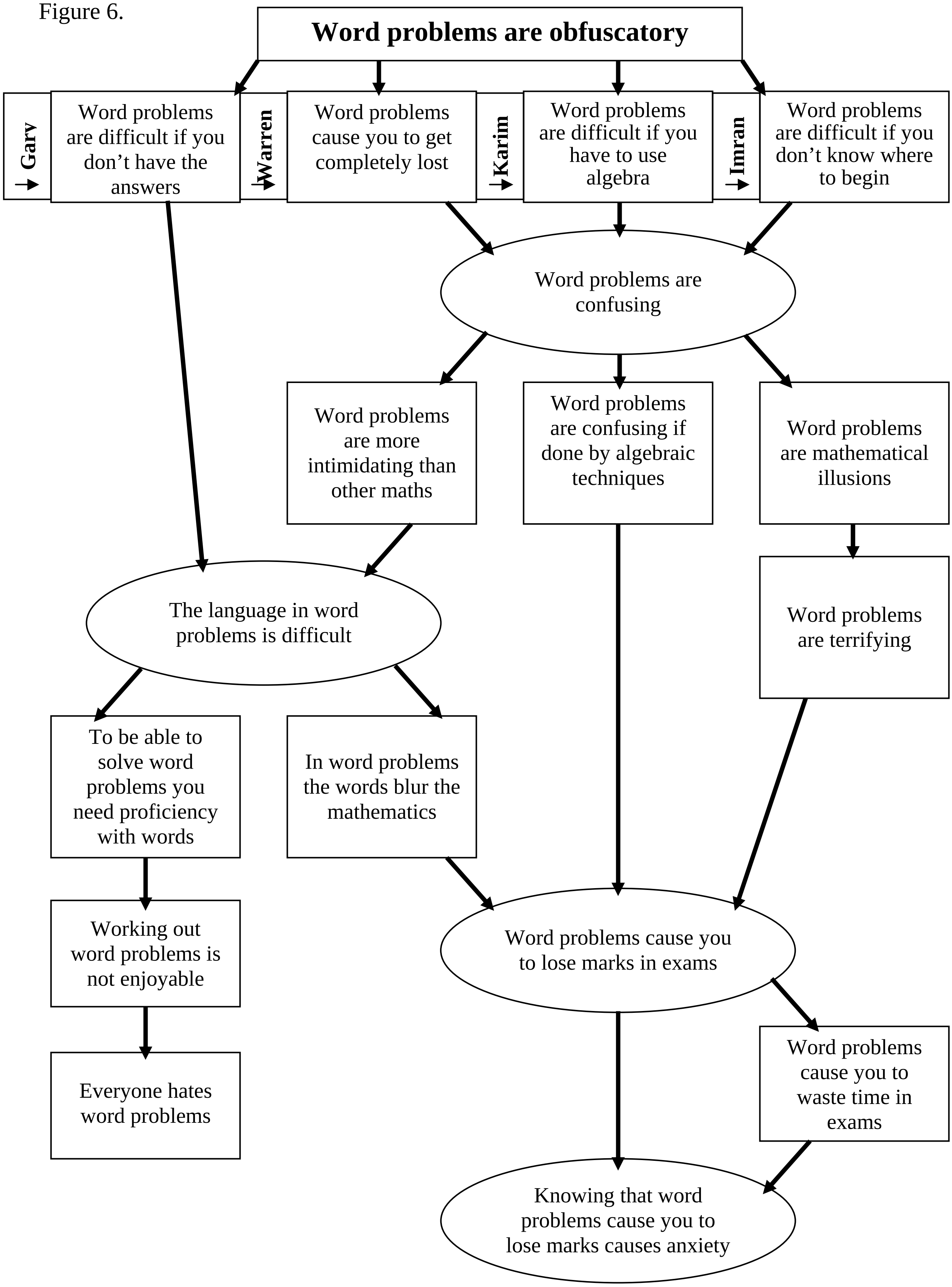
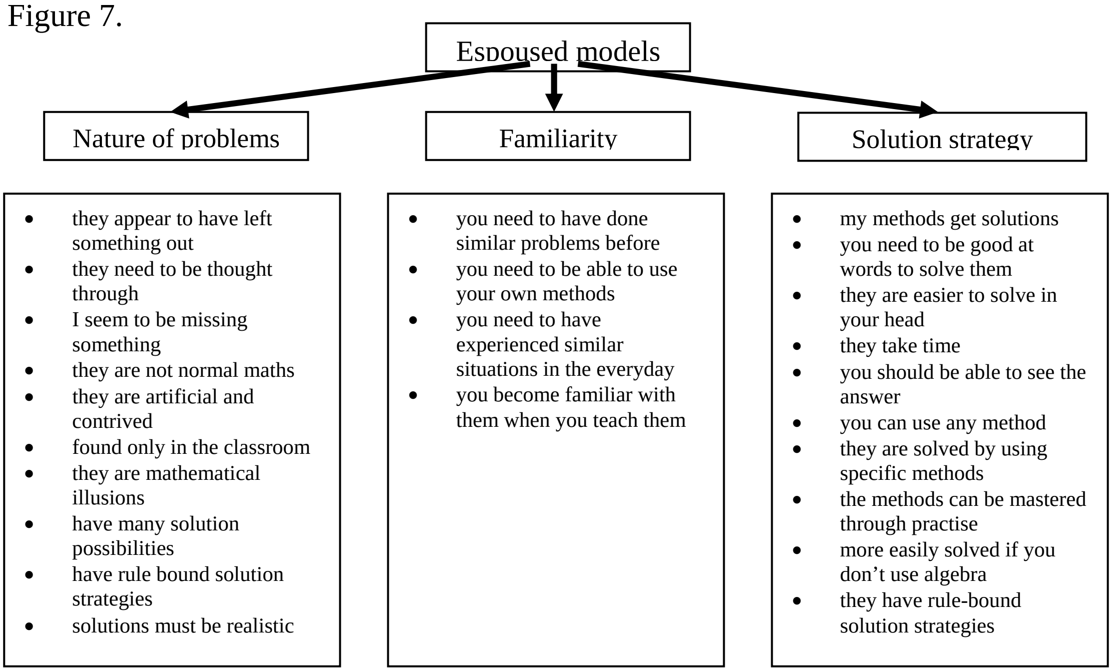

Word problems are obfuscatory
Gary
Word problems are difficult if you don't have the answers
Warren
Word problems cause you to get completely lost
Karim
Word problems are difficult if you have to use algebra
Imran
Word problems are difficult if you don't know where to begin
Word problems are confusing
Word problems are more intimidating than other maths
Word problems are confusing if done by algebraic techniques
Word problems are mathematical illusions
The language in word problems is difficult
Word problems are terrifying
To be able to solve word problems you need proficiency with words
In word problems the words blur the mathematics
Working out word problems is not enjoyable
Word problems cause you to lose marks in exams
Everyone hates word problems
Word problems cause you to waste time in exams
Knowing that word problems cause you to lose marks causes anxiety

Figure 7.



Espoused models

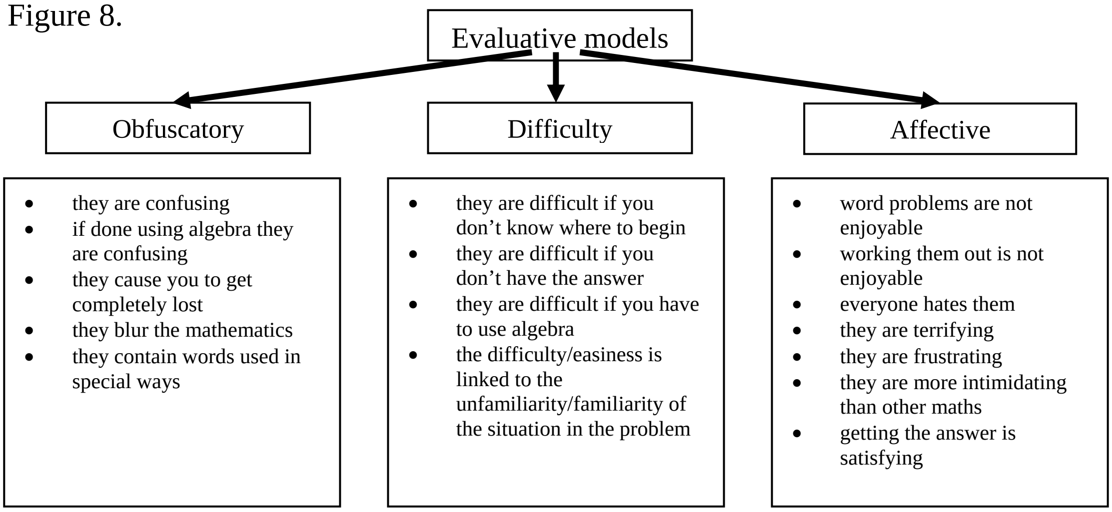
The nature of word problems – this is what the students generally think word problems are. From the discourse we can see that they perceive them to be 'fake' renditions of real life, that are encountered only in the classroom, they are illusionary and they are not normal mathematics.

Familiarity with word problems – each of the students expresses a need to be familiar with the problems to be able to cope with them. One of these notions was that word problems should be representative of real life, which is in conflict with the perceptions of the nature of word problems.

Solution strategies – here, Warren and Imran especially, each raised two conflicting cultural models: they each said that word problems have a *specific* solution strategy whilst on another occasion they said that they have *many* different solution strategies. Generally, the students see word problems as being rule-bound but that they are easier when done by their 'own' methods.

The espoused cultural models generate a fairly bleak picture of what the students hold to be normal and typical about word problems, *viz*. that they are 'fake' renditions of 'real life', and they appear to be rule-bound which makes them difficult to do. This is tied in very closely with the belief structure that was investigated by Christou and Philippou (1998b) and, if their claim that these become increasingly more difficult to change is true, the way forward for these students with respect to word problems does not look promising.

Figure 8.



Evaluative cultural models

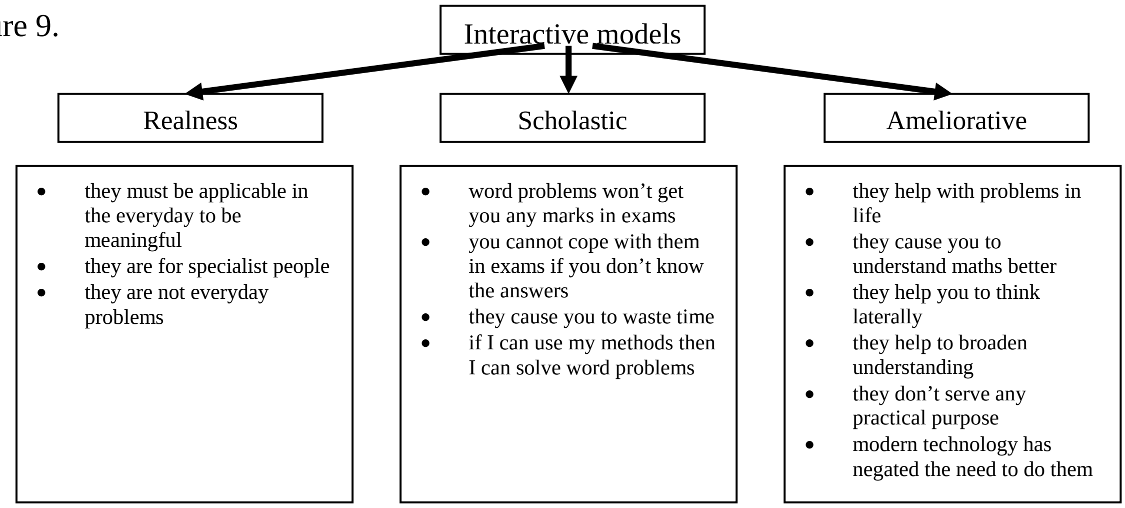
The obfuscatory effect of the models – generally, the students perceive word problems to be confusing, they blur the mathematics and they cause you to get lost.

The difficulty of word problems – each of the students experienced difficulty with word problems differently but these are largely linked to the obscurity that they experience with the problems.

The affect of word problems – generally they cause anxiety, frustration, fear, intimidation and in Gary's case, they can be satisfying.

Based on their assumptions about what word problems are the students are experiencing an overwhelming amount of negative affect. The obscurity of the problems brings about a difficulty in dealing with them and the consequences are almost always negative and very emotively expressed. The evaluative discourse gives a picture of student perception of word problems as being difficult, confusing and largely contributing to feelings of frustration and anxiety.

Figure 9.



Interactive cultural models

The realness of the problems – there is a need for the problems to be applicable in the every day, but the general perception of them is that they are not every day problems. They are for specialist people.

How word problems contribute to amelioration – they are supposed to help with problems in life, to think laterally and to do maths more successfully, but they are not actually doing this. They don't serve any real purpose and with the modern devices that exist today and therefore there is no need to do word problems.

Scholastic aspects of word problems – word problems waste time in exams and they cause you to lose marks. There is a sense of failure and inability to cope with them in the test situation. When the answer becomes apparent it tends to make the students feel 'stupid'.

The students don't perceive word problems to be real and applicable and thus cannot identify how they could improve their position with respect to advancement in life. The notion that the problems are for 'specialist' people implies that 'specialist' skills are beyond the capabilities of the 'ordinary' student. Consequently, word problems waste your time and jeopardise you in exams.

In conclusion

The cultural models that the students in general are acting upon are reflected in the progressive classification of the models (see figure 6). It helps to give a picture of why students have such a poor success rate when tackling word problems. The students firstly formulate opinions about what these problems are (the espoused models) and these form the basis of their belief structure (Christou and Philippou, 1998b) about word problems. They then make judgements about what they have conceptualised about these problems (the evaluative cultural models). Based upon their conceptualisation of the problems themselves and the judgements that they have made the students formulate opinions of how these will affect them in terms of advancement socially (the interactive models).

It appears that these students have positioned themselves in a situation of relative helplessness mathematically. They 'should' be able to do these problems and the problems are 'supposed' to be helping them to do mathematics and solve everyday problems, yet they cannot solve them for the most part. These feelings of ineptitude broaden into feelings of inadequacy that tend not to be limited only to word problems. The associated feelings of anxiety impinge on their ability to do other mathematics as well as their overall mathematics results.

The findings of the initial case study

Initially I had intended to use Gee's notion of espoused, evaluative and interactive cultural models as the primary classification for cultural models as discussed in my analytical framework. In using this categorisation of cultural models Gee was giving substance to Strauss' work on cultural models. Gee (1999) says, "…we need to distinguish between cultural models based on how they are put to use and on the effects they have on us" (p. 68). His subsequent categories for distinguishing between cultural models, Gee suggests, are not limited to the three that he gives: he says, "We can distinguish between, *at least*, the following…" (p. 68, my emphasis) and he goes on to give *espoused models*, *evaluative models* and *models-in-(inter)action*.

In the context of this study further categories were needed, and it was in the analysis of the student attempts of the problems that the notions of empirical and putative cultural models became evident. This also tied in very nicely with conflicting and consistent cultural models (Setati, 2002; Gee, 1999). This was because understanding of where cultural models appeared to be stemming from the students experiences when compared with cultural models that seemed to be gleaned from their assumptions helped to explain why conflicting (or consistent) resultant action was evident.

I suggest that where one's empirical and putative cultural models are in conflict, attaining levels of competence in working with word problems can be impeded. For example, Gary holds the cultural model that *word problems help to make maths easier*. He also holds the cultural model that *to be able to solve word problems you need proficiency with words*. For Gary, these are in conflict. Furthermore, Gary does not *act on* the former cultural model but he does on the latter one, since he presumable has experienced difficulty with the wording in the problems. The influence of the latter cultural model is that Gary's self-esteem with regard to his ability to cope with word problems is lowered since he sees himself as not having sufficient proficiency with words (*cf*. Gee, 1999: 66 – 67). This is exacerbated by the belief that word problems should be making maths easier but that this is not happening. Gary has *adopted* the putative cultural model (the former one) whilst he *acts upon* the empirical cultural model (the latter one). In this way one of the factors that underlie Gary's lack of success in solving word problems is made 'visible'.

When looking at Imran we see that he holds the same putative cultural model as Gary but his empirical cultural model is that *word problems are mathematical illusions*, and these models are in conflict. When Imran attempts a word problem he expects to encounter multiple solution possibilities because he is acting on the empirical cultural model even though he believes that word problems should be helping him mathematically.

Warren expresses a putative cultural model that *word problems serve to make a person think laterally* whilst he holds an empirical cultural model that *word problems cause you to get completely lost*. We can see that he is not acting on the putative model when he says that he cannot think laterally (phase 3, stanza 3, line 15) or in a manner that will help him to solve the problem (phase 3, stanza 6, line 28). He is acting on the empirical cultural model and, because this is in conflict with the putative model he is unable to achieve the lateral thinking that he believes word problems should be giving him.

The situation with Karim I see as being different from the other three. He holds two putative cultural models that are consistent: *word problems help with problems in life* and *for word problems to be meaningful they must be applicable in the everyday*. However, he also holds the models that *word problems are confusing if done by algebraic techniques* and *I probably wont be able to solve the problem if I have to use algebra*. Karim is acting upon these empirical cultural models which inhibit him with regard to solving the problems. However, he is also acting on the second putative cultural model in that he believes that problems 1 and 2 were very unrealistic and therefore meaningless to him in terms of helping him with everyday problems.

Thus I found the distinction between empirical and putative cultural models to be a useful tool since viewing empirical and putative cultural models that are in conflict enables us to identify which model is being acted upon and which one is being suppressed. Furthermore, I suggest that with time an empirical cultural model may become more consistent with a putative cultural model, and they may even eventually merge into one empirical model, which could alter the success or lack of success that one has in dealing with word problems. However, research undertaken by Christou and Philippou (1998b) found that students' belief structures about mathematics are established in early schooling and became increasingly difficult to alter as the students progressed through the schooling system. Cultural models are tied to this belief structure and could therefore also be subject to similar rigours. Therefore, it seems that further research is needed to establish whether empirical and putative cultural models can or do become more consistent over time.

At this point the initial case study seemed to show a Discourse of relative mathematical helplessness when students confronted word problems. However, the analytic framework did not appear to offer the tools with which to examine the underpinnings of this Discourse, which was key in answering the research questions (in particular, research sub-question 3). At the same time, this initial study illuminated a theoretical and methodological niche within the study of school mathematics word problems, but the methodology in particular had to be innovative in order to render Discourses and cultural models visible.

Hence, the initial case study showed that the students were working within a Discourse of school mathematics (more specifically, word problems) and that they were acting upon certain cultural models and not others. This helped to explicate from a different perspective why students respond in the ways that they do when attempting word problems. In this respect I was able to use the discourses that the students used when doing word problems and in the peer setting (research sub-question 1) to identify the underlying cultural models that were at play when the students attempted the problems (research sub-question 2), and this, to some extent, explained how these cultural models come to bear upon what the students do and how they experience the word problems (research sub-question 3). Thus the methodology was applicable and was able to address the research questions that I had posed.

However, one criticism of the methodology was that it yielded a myriad of cultural models which made it complex for understanding exactly which of these pinpointed the socio-situated context within which these students were operating. Furthermore, having identified that the students were operating within a Discourse of school mathematics word problems, there were no satisfactory answers as to why they appeared not to have access to this Discourse at an appropriate level in order to cope better with the problems. At the time it seemed that the methodology could be refined to delve more deeply into answering research sub-question 3 in more detail, which would give a deeper understanding of how the cultural models are at play within student performance and experience of word problems.

Thus, it was decided to take on an extended study to address these concerns. The method for collecting data, together with the analytic frame of reference were modified to enable access to this 'new' perspective. Along with this revised focus came the emergence of the second edition of Gee's work (Gee, 2005) and this led to a renaming of *cultural models* as *Discourse models*. In addition to this, the research questions were modified slightly so as to be more directed at examination of discourse that was reflective of past action by students. All of these revisions are taken up in more detail in the next chapter in which the extended study is reported.

Chapter 4 – The extended study

Gee, having adopted the term *cultural models* from Holland and Quinn (1987), and used this for some fifteen years, now recognised the need for a different name. In the words of Gee (2005):

> "… the term "cultural model" is a poor one. Not everyone who shares a given model is a member of the same cultures and not everyone in some larger culture shares all the models… "Discourse model" (with a capital "D" for "big D" Discourse) is a better term since these theories are connected to specific Discourses…" (p. 61)

It must be stressed that this was simply a name-change in which the definitions applicable to *cultural models* were conferred upon the term *Discourse models*. This simple name-change, however, brought a new clarity to my research in that understanding the theoretical basis was now no longer fraught with difficulties in defining the term *cultural models* to those who were not necessarily familiar with Gee's work. Thus explication of what I was trying to achieve became much more accessible for critique by fellow doctoral students and others concerned with my work.

For the purposes of this thesis I used the terms *cultural model* and *Discourse model* interchangeably, but chose the term *Discourse model* for the extended study as I was working largely from Gee (2005) as theoretical basis for that study.

Motivation for the extended study

From the initial case study it became evident that a myriad of cultural models are at play when students confront word problems. Two problems with the analysis and interpretation of that analysis indicated the need to extend the study.

Firstly, the analysis brought a host of cultural models under the spotlight and, whilst interesting this made it difficult to understand what any one of these cultural models contributed in answering the research question. Thus there needed to be some refinement to the methodology in order to more explicitly reveal the cultural models that would help to explain the Discourses from which the students were working, which in turn would more directly address the research question.

Secondly, the cultural models in the initial case study were used in an 'assumed' manner[23]. As was correctly pointed out, the methodology did not pinpoint what exactly I was looking for in a *cultural model*, and that clarifying this would sharpen the analytic process. Subsequent communiqués with Barwell have also helped to hone the analytic process in the extended study.

The research question was also revised slightly to make it more applicable to the unit of analysis. For the initial study the question being answered concerned what students *do* with word problems and how students *experience* word problems. However, because of the design the unit of analysis was the text put forward by students. This text comprised what students *do* with word problems (student attempts of the problems) and student *talk* about what they did with, and how they experienced word problems (the peer discussion, informed also by the paragraph and the questionnaire). The research question was modified to read: what is it that students do that is appropriate or inappropriate in interpreting word problems? In answering this question, three sub-questions were posed, and these were modified to read:

1. What discourses do students use in doing word problems and when talking about their experiences with word problems?
2. What underlying Discourse models are at play in the text that students put forward regarding their experiences with word problems?
3. How do these Discourse models inform us about the students' experiences and why they have these experiences?

The data collection

There was no change to the instruments and the data collection for the extended study except that the 'teaching' phase, as discussed earlier, was omitted completely. The format of the booklet issued to each student appears as annexure 2 and the full transcriptions of the peer and focus group discussions can be found on the compact disc. I was able to ensure that the data was collected in two sessions which took place on two consecutive afternoons at each school.

[23] In a lengthy discussion with Prof. Richard Barwell I was able to see that a more grounded methodology was needed to pinpoint cultural models.

The sample

It was decided that the extended study should incorporate diverse sites. Adler (2001) distinguishes between two different urban schools in South Africa – township schools and suburban schools. These descriptions focus more specifically on the role of language in the environment as it affects the students, but I believe that this also plays a part in establishing student points of view with respect to how language within the community is perceived and used, and how this affects their education (in particular, with respect to mathematical word problems in the classroom). The schools that were selected fall into the categories of *township schools* and *suburban schools* (Adler, 2001), but these schools, I believe, are representative of a cross-section of post-apartheid schools (as they exist at present).

However, language and text are not to be confused with one another. It is text that served as data for this study, but it may be that language in some way 'obstructed' that data. A focus on how the English language pervades the community (i.e. what Adler (2001) refers to as *language learning environments*), I believe can serve to disaggregate comprehension of word problems and interpretation of mathematical relationships. The comprehension and interpretation referred to here I see as being diametrically opposed when it comes to understanding difficulties experienced with mathematical interpretation of the problem at hand. This study sought to employ a discourse paradigm that would enable a disentangling of influences that are language-based and those that are societally-based, so that the *latter* may be examined in terms of the students' Discourse models, hence giving insight into the *Discourse*. Having said this, language is inextricably linked to society and it is for this reason that the schools were selected from different language learning environments so that the phenomena under investigation (namely interpretation of mathematical word problems) could be viewed with reference to the social structure in which they emerged. However, as stated previously, it must be stressed that language issues *per se* were not a focus of the study.

The three schools that were ultimately selected for data collection were chosen from three different language learning environments as discussed above, but they were opportunistically chosen in that they were relatively near to the researcher and someone at each of the schools was known to the researcher to provide an introduction. This was done because efforts to elicit a response at other schools that were more randomly chosen proved fruitless. A brief description of each of the schools as they impact the study follows.

Corona

This is a large co-education school that accommodates some boarding students within the approximate 1 500 student body. The school is located in a suburban, additional language learning environment (Adler, 2001) for the students selected for this study.

Duskhaven

This is a medium sized co-education school with around 1000 students. The school is located in a suburban, additional language learning environment (Adler, 2001) for the students selected for this study.

Manumission

This is a medium to large co-education school with about 800 students. The school is situated in a township, additional language learning environment.

In each school four Grade 10 students were selected. These groups were determined from the most recent, and pertinent, set of mathematics results that the school could provide for that cohort of students. By 'pertinent' I mean recent results that reflected the school's standardised summative assessment across classes within that grade.

A box and whisker analysis of the data was useful in selecting the sample groups because it distributed data values by quartiles. In this case calculations were done by medians of the complete data set, and not by medians of the frequency of data values. This divided the data set into four distinct, equal-sized groups, but the advantage was that it partitioned those data values relative to the overall median, and then relative to the first and third quartiles (or medians within each of the 'halves'). This effectively distributed the data values over four groups of equal size.

For the purposes of this study two students were randomly chosen from the data values falling between the first quartile and the median (labelled Group A figure 10), whilst the other two students were randomly chosen from the data values falling above the third quartile, but excluding 'outliers' (labelled Group B figure 10). Students were chosen in this way so that the study could examine those who struggle somewhat mathematically (as would be the case of the students falling into Group A) but would include some 'more successful' students (who would fall into Group B). In particular it was felt that students falling into the lowest quartile would possibly be struggling with mathematics to such an extent that the data gleaned for the study might not be of any use in describing the student experiences with word problems, whilst Group B members were chosen so as to possibly contrast the student experiences by looking at how ‘more successful’ students relate these.

These classifications, it must be stressed, are relative to the data from each particular school. This means that what is classified as, say, ‘more successful’ at one school may not be the same at another school. However, being classified as ‘more successful’ in any school, no matter what the criteria for that classification, brings with it certain expectations from peers, teachers, parents, etc. These societal influences form part of what this study wishes to examine and it is for this reason that I believe that the box and whisker analysis enabled a selection of participants appropriate for answering the research question. In other words, the selection of participants was such that students from the same ‘mathematical strata’ relative to each school were selected. The hypothetical box and whisker plot (see figure 10) illustrates the selection criteria.

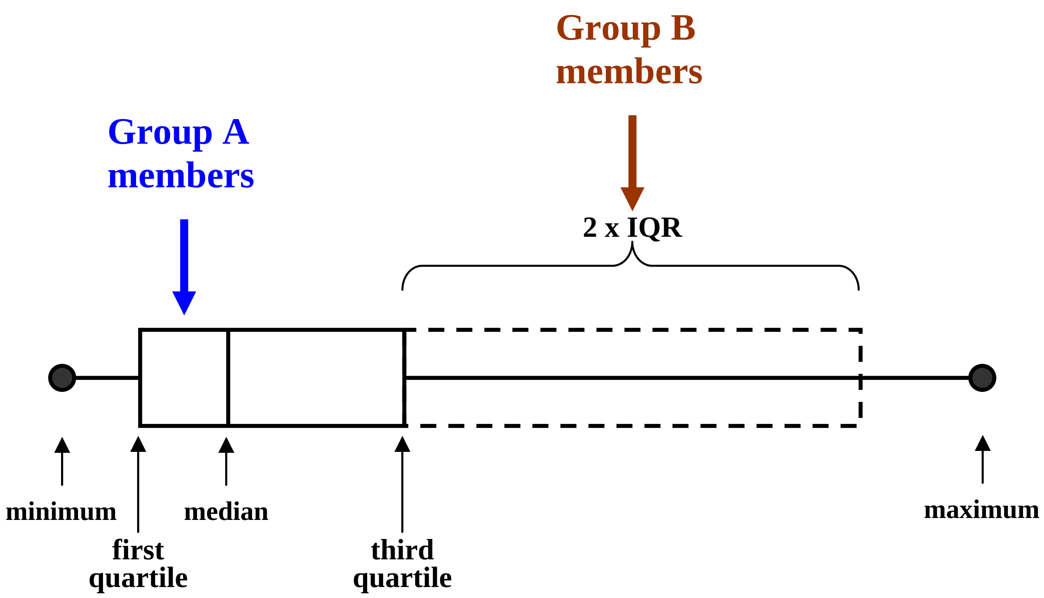


Figure 10. Diagrammatic box and whisker plot showing how data groups were selected from mathematics results of the cohort.

The ‘more successful’ students were selected so as to exclude outliers as stated earlier. I deemed mathematical marks to be outliers if they occurred beyond the third quartile by more than twice the length of the inter-quartile range (i.e. the third quartile value minus the first quartile value). An outlier is an unusually high (or low) data value for a particular data set and thus is not representative of that data set. It was felt that it was appropriate to exclude these data values for this study so that students who are representative of the ‘more successful’ students at each particular school are chosen, and not students who are unusually high achievers within each school. This was a precautionary measure and no outliers came up in the random selection of data values.

There was no indication given to the participants as to how these groups were selected or the groups from which they were selected. In addition to this the way in which the students were selected and the groups from which they were selected played no role in the study other than to establish a sample.

Below is a box and whisker analysis (figure 11) giving an indication of where the students were situated relative to their respective cohorts. This also gives a comparison of the relative positioning of students at the three schools in this study, and it also shows us a relative positioning of the students within their quartiles. The names of the students have been omitted as this was not taken into account at all after the selection of the sample, and is thus irrelevant.

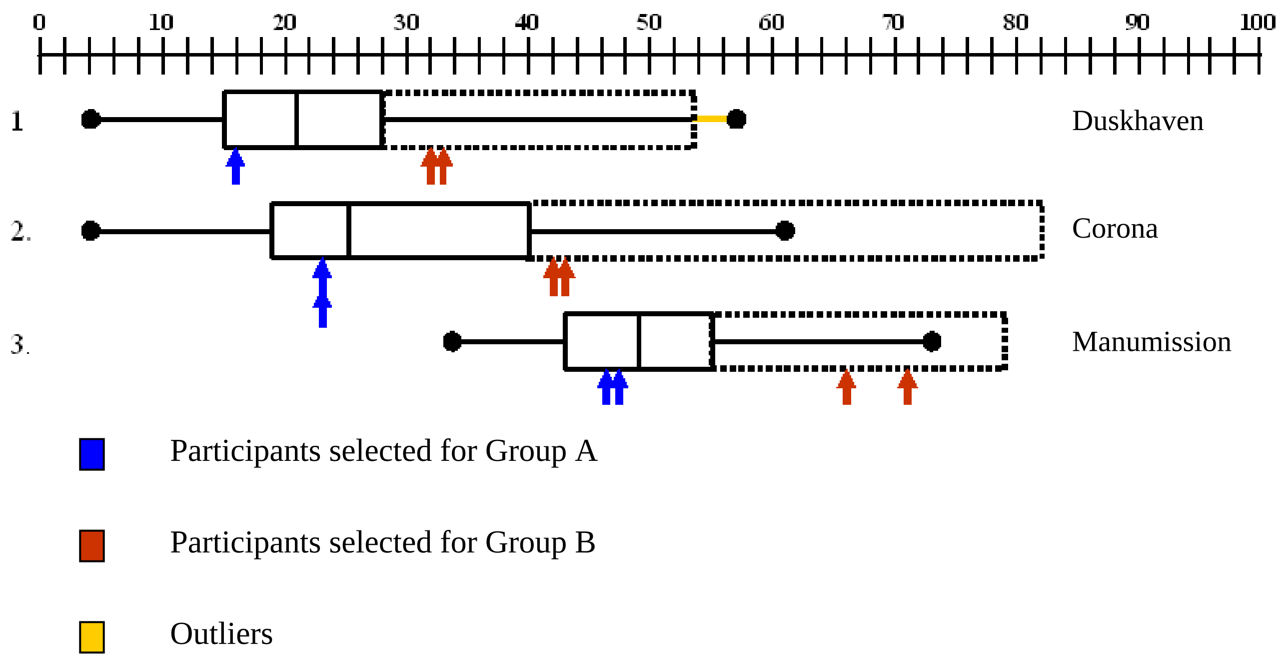


Figure 11. Box and whisker plots showing the spread of standardised, summative mathematics marks from each participant school for the grade 10 cohorts, and the selection of participants from within each school. Four random choices were made for each group, but the graph only shows the locale of the actual participants. One student did not turn out for the activities at Duskhaven.

A brief background to the students selected for the study

1. Corona

The four students chosen for this study were all girls, and all were in boarding. Three of the girls had a very good command of English, whilst Lerato was able to communicate adequately. The girls informed me that only English was spoken in the boarding facility.

Roenel came from a large township to the south west of Johannesburg where she lived with her parents and her aunt. She joined Corona in grade 8 (at about 14 years old). At home the family spoke SeSotho and she was surprised to find all the students in the hostel speaking English.

Ayanda was a year younger than her peers. She also lived in Soweto where she attended a primary school where everyone was 'forced' to communicate in English. At home, Ayanda and her family also sometimes communicated in English so she found the transition at Corona relatively easy. Ayanda did not indicate her home language.

Lerato was very shy and whenever she was called upon to speak she kept her responses very brief. She came from the Midrand area and it wasn't clear who she lived with at home or what language she spoke in the home.

Roxanne came from a large township to the south west of Johannesburg and, according to her, was placed in boarding because she watched too much television at home. There was no indication of home language spoken.

2. Duskhaven

One of the selected students did not arrive for the study. Of the three who did arrive two were boys and one a girl. Two had a poor command of the English language whilst one had a reasonable command.

Nina was born and had completed her primary schooling in Brazil. She had been in South Africa for two and a half years (when she started at Duskhaven in grade 8) and had not spoken any English before her arrival in the country. Her mother had married a South African and so they spoke English in the home.

Sipho lived in township to the east of Johannesburg with his mother. At home he spoke IsiXhosa, but had been taught English in his primary school and his communication was quite fluent. He had joined Duskhaven at the beginning of grade 8.

Gio was Portuguese speaking having lived and done his primary schooling in Angola. He had also been in South Africa for about two and a half years and his English was broken and he struggled a little to express himself at times. Gio started at Duskhaven in grade 8

3. Manumission

Of the four students chosen for the study two were boys and two were girls. The two girls had a reasonable command of English, whilst the two boys appeared to struggle to express themselves. All four of the students resided in a township to the east of Pretoria in which the school was located. All of the students had attended the local primary school and had entered Manumission in grade 8.

Danny introduced himself quite briefly, simply giving his name and the district in which he lived.

Mafifo also simply gave her name and that she was living with her mother in a suburb near the school.

Hartman said that he came from Johannesburg, but that he was living with his grandmother near the school.

Rosina mentioned that she was 15 years old (a little young for the group) and that she lived with her mother and other family members near the school.

Whilst the schools were chosen to be representative of a certain range of schools, the students were randomly chosen as discussed above. Thus the sample the sample that emerged came to have the following characteristics:

- Schools across essentially two different language infrastructures (Adler, 2001), and representing a cross-section of post-apartheid urban schools, *viz*. suburban and township, as they existed at the time in South Africa.

- Students from a variety of language groups, but none of whom had English as a home language, but having English as their medium of instruction. Interestingly, two of the students were fairly recent immigrants to South Africa.

Revised analytic framework and the need for new problems

A further complication in this study was the introduction of a new curriculum in South African secondary schools in 2005. *Curriculum 2005* (as it was known) advocated an outcomes-based approach to education. To render the study pertinent to the current South African educational context three new word problems were selected from sources claiming to embrace an outcomes-based approach. To maintain a degree of consistency in the research, the problems were chosen to include the rectangle problem which involved relationships similar to the age problem, the TV rental problem which involved money and was to an extent related to the watch problem, as well as a speed-distance-time problem.

The problems chosen were as follows:

Problem 1 – The rectangle problem

> Tseko draws a rectangle with its length 2 m more than its breadth. He then increases the length by 2 m and decreases the breadth by 1 m. He finds that the area of the new rectangle is the same as that of the first one. Find the length and breadth of Tseko's first rectangle.
>
> (Groenewald, *et al*, 2006: 104)

Problem 2 – The TV rental problem

> The Clear Vision television rental shop charges a basic fee of R150, as well as R15 per day to rent a television. The Best View television rental shop only charges a basic fee of R15 but has a daily rate of R60 per day to rent. For what number of days would it make no difference in cost as to which shop you rent from?
>
> (DoE, 2003a: 9)

Problem 3 – The speed-distance-time problem

> A boy cycles from home to school in the morning and back in the afternoon. He cycles from home to school at 32 km/h and back at 24 km/h. It takes him 15 minutes longer in the afternoon than in the morning. Find the distance between home and school.
>
> (Groenewald, *et al*, 2006: 103)

Thus, in order to maintain consistency in the shift from the initial study to the extended study the problems were chosen with two main criteria in mind: the *genre* (Gerofsky, 1996) and the *structure* of each respective problem. By *genre* I mean the type of problem that would be generally recognised within a Discourse of school mathematics (particularly in South Africa), and by *structure* I mean the mathematical requirement for solving the problem. Interestingly, problem 2 (the TV rental problem) proved to elicit very different mathematical work from the students which I had not anticipated, and this may have some bearing on assigning tasks that give access to school mathematical Discourse – this matter is taken up in more detail in the analysis and discussion that follow.

Organising the data for analysis

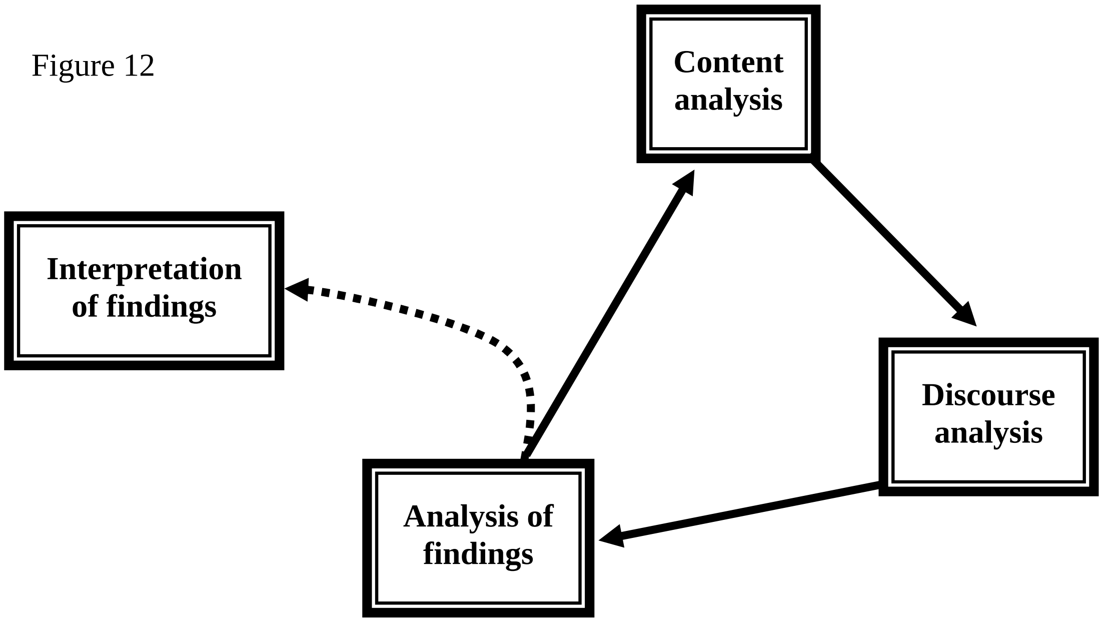
The extension of a study inevitably produces more data. However, a discursive analysis does not lend itself to a project that has generated large amounts of data. Through peer critique it became apparent that the focus group discussion tended to project a large number of my ideas since my questioning had been designed to probe the students further from information that I gleaned from the previous activities. In other words, the selection of particular discussion topics for the focus group discussion, although quite extensive, was somewhat subjective. It was felt that the peer discussion was far more reflective of the student's ideas and therefore the focus of the study was shifted so as to primarily examine what the students did with the problems and how they talked about the problems in the peer discussion, with reference to the paragraphs, the questionnaires and focus group discussion being made as and when it helped to inform the analysis[24].

The triangle model (figure 12) shows the main components of the analysis. These are the content analysis, the discourse analysis and analysis of the findings. I use a triangular model because the analysis is not linear:

> "[Qualitative data] analysis is also the process of bringing order, structure and meaning to the mass of collected data. It is a messy, ambiguous, time-consuming, creative and fascinating process. It does not proceed in a linear fashion; it is not tidy" (de Vos, 2005: 333).

[24] The full transcripts of the peer discussion and the focus group discussion are included on the compact disc.

Figure 12



Analysis of the content

This discussion is more concerned with how I undertook a content analysis to select certain excerpts from the transcripts that would be analysed in more detail through a discourse analysis.

The content of any situation serves to give meaning within which a situation can be interpreted. Without content a situation is devoid of meaning (Gee, 2005). Also, the content happens in a particular context. For this research the content needed to be separated from its context so that the essence or 'face-value' of the situation could be extracted. By examining utterances it is possible to elucidate the content by having text classified into identified categories from which trends can be observed that emerge within each of those categories.

To do this the text needed to be 'organised'. The transcripts had been numbered in *turns*, or in the order in which participants contributed. In order to establish the content I firstly identified *stanzas*, or *units of language* (Gee, 2005: 107) that incorporate several focuses into one topic or theme. From there I broke each turn into *lines*, each of which represented "one small focus or fixation" (Gee, 2005: 125) of the speaker as s/he constructs a story. A full description of all the transcriptional devices used is given at the beginning of chapter 5, where it is relevant to understanding the analysis.

In deciding what categories to use for the table of utterances quite a lot of discussion was generated amongst fellow students, which helped to give the process inter-rater consistency, and I eventually revised the categories into which the utterances were to be classified, and the criteria for deciding whether or not an utterance fitted into a particular category or not. The indicators were given in the form of key phrases that text will reflect together with examples to clarify the ‘type’ of text.

1. Structure

Key indicators: what word problems are, what they look like (e.g., a story, sentences, words), types of word problems (e.g., age problems, money problems, different types being easier or more difficult), comparisons (e.g., like/not like ones in other subject areas, comparison with other subject areas/areas in maths).

2. Affective experiences

Key indicators: how word problems make the student feel, the emotional impact of the word problem as expressed by any of the numerous human emotions or ‘mental feelings’ (e.g., hate, like, preferences, enjoyment, confusion, being lost, etc.) (Note: ‘being lost’ in this study is used figuratively to mean ‘confused’ and does not take on its usual meaning of disorientation, and is thus emotive.)

3. Difficulty with and understanding of word problems

Key indicators: understanding, misunderstanding or not understanding (e.g., finding them difficult/easy, struggling)

4. Ameliorative experiences

Key indicators: any purpose for having or reason for doing word problems (e.g., help with maths, prepare you for life), any benefits they might have (e.g., logical reasoning) or any hindrances they may impose (e.g., you lose marks)

5. Doing the problems

Key indicators: any talk about what was actually undertaken in the attempt of the problem (e.g., I added the numbers, I put it as *x*, I guessed, etc.) or any ‘theoretical’ method or suggestion for doing problems (e.g., the *x* method, table method.)

Identifying the data to be analysed

The tables of utterances across the five categories were pulled into another table to identify trends (see figure 13 below as an example). Where there were overlaps those stanzas from the transcript were selected for more in-depth discourse analysis. Thus, in this example, stanzas 2 through 9 and stanzas 18 through 25 were selected. The stanzas in between were included in most instances to ensure continuity and that the context of the situation was not lost. Of course, this selection of excerpts from the transcripts did not preclude me from turning to stanzas that were not selected in this process if and when this became necessary during the discourse analysis.

Figure 13

	Structure	Ameliorative	Affective	Difficulty & Understanding	Doing
Peer			2		2
		3			
			9		9
					11
					12
					13
					17
	18		18	18	
				19	
		21			
			22		
			23		23
				25	25
					27

The discourse analysis

Gee makes this statement about the tools that he proposes:

> "…this book is meant to "lend" readers certain tools of inquiry, fully anticipating that these tools will [be] transformed, or even abandoned, as readers invent their own versions of them or meld them with other tools embedded in different perspectives… This book is an introduction to *one* approach to discourse analysis (the analysis of language-in-use)… Furthermore, the approach to discourse analysis taken in this book is not "mine." No set of research tools and no theory belongs to a single person, no matter how much academic style and our own egos sometimes tempt us to write that way." (Gee, 2005; 5)

I have adopted Gee's seven building tasks upon which to base the analysis. Gee himself states:

> "These building tasks involve us in using language (and other semiotic systems) to construe situations in certain ways and not in others. They are carried out all at once and together. And, they are carried out in negotiation and collaboration with others in interaction, with due regard for other related oral and written texts and situations we have encountered before." (Gee, 2005; 104)

The seven building tasks (Gee 2005, 10 – 19; 97 – 104)

1. Building significance

How and what significance is given to things? In other words, what meaning and values are being attached to what is being discussed? How are things made significant or insignificant through the choice of language, emphasis, gesture, etc?

2. Building activities

What activities do the participants put forward through their use and choice of discourse and how is language used to show what activity one is involved in? Passive voice (or in Gee's terms, 'backgrounding') could indicate a possible lack of agency (Janks, 2005).

3. Building identities

What identities (roles, positions) are the participants enacting and describing? How is language used to make the identity in the situation identifiable and consequential?

4. Building relationships

What relationships do the students see as existing in the situations that they describe? How is language used to show what relationships the participants recognise as being in place? How is language used to put forward and negotiate relationships between participants?

5. Building politics

What social goods are perceived by the participants? How is language used to express these social goods in terms of how they affect the participants? In what ways might these social goods benefit, advantage or inhibit?

6. Building Connections

In what way are things connected or disconnected to each other? How is language used to create links or disassociations between things, events, and circumstances?

7. Building significance for sign and knowledge

What sign-systems or ways of knowing do the participants refer to? What language is used to show how participants know or come to know, and are able to talk about events, objects and circumstances?

The seven building tasks from a mathematical sense-making perspective

Mathematical sense-making, particularly at secondary school level, can be seen to be happening largely during student activity, which for the purposes of this study is in the context of doing word problems. During that activity the students are making use of whatever semiotic processes, symbols and language (Morgan, 2006) with which they are familiar in order to engage with the problem. However, it is important to note that I am only able to examine how the students talk about the activity in which they were involved, and that their talk is retrospective and reflective. Through discussions with peers it became apparent that "building activity" as a building task was a difficult thing to examine in the context of what the students say since the students are talking about and recalling their 'activity' rather than being actively engaged in the problem. The way in which the students build activity is examined purely from the text that they produced when doing the problems. Thus research sub-question 1 is addressed by what the students did when they attempted the problems and their talk about what they did and about word problems in general. Research sub-question 2 is addressed by the peer discussion, with reference to the paragraphs and the questionnaires.

It is always problematic to build models that appear to be linear or even circular. Whilst the model that I propose looks linear in nature, I believe that it is necessary in any analysis to have a starting point and an end point. As Gee (2005) himself points out, all of the building tasks are interlinked and it is impossible to examine one without the others coming into play. Yet, in reality I do not think that it is practical to do an analysis that examines all of the building tasks simultaneously. Thus I have tried to adapt the analytical model so that we now firstly look at how the students build sign and knowledge systems, and how they build connections together with what they make significant. I thus came to see these three building tasks in conjunction with how the students build activity in doing the problems as forming a group of building tasks that allow me to look at the perceptions that the students have about 'things' that they encounter. I use the word 'things' here quite loosely to refer to notions, ideas and circumstances, as well as physical objects, behaviours and activities, and anything that the students may come across 'out there'. I have called this level of the analysis the *experiential* level.

At the next level of the analysis I have looked at how the students build identities and relationships, or how they talk about the way in which they perceive themselves in the world and in relation to others around them. I have called this level of the analysis the *existential* level.

The next level of the analysis looks at the social benefits or disadvantages that the students perceive, what Gee (2005) refers to as *social goods*. This stage of the analysis I have termed the *political* level.

The organisation of the analysis is in two parts for each site: firstly the analysis of student working is undertaken, followed by the analysis of student talk. For each site I begin with the analysis of how the students build activity through their working on the three problems, which I have examined through what I term *the three dimensions of student activity*, *viz.* understanding of the problem situation, use of method or procedure and use of correct mathematics. This has been to some extent modelled on the work on discursive practices done by Setati and Barwell (2006). For the purposes of the analysis of the student talk, each building task is discussed individually using Gee's descriptions (listed above) and by using the guiding questions that were developed (which are listed below). At each level of the analysis a brief discussion is undertaken, and an overall discussion for each excerpt is then done before moving on to the next excerpt.

The model is presented as a nested model to try to incorporate the interconnectedness of the different building tasks, and it is not intended to be hierarchical, although this may appear to be the case due to the 'order' in which the analysis has been undertaken. I hope that by examining the building tasks in groups (*i.e.* the three levels discussed above) I have to some extent overcome the problem of linearity.

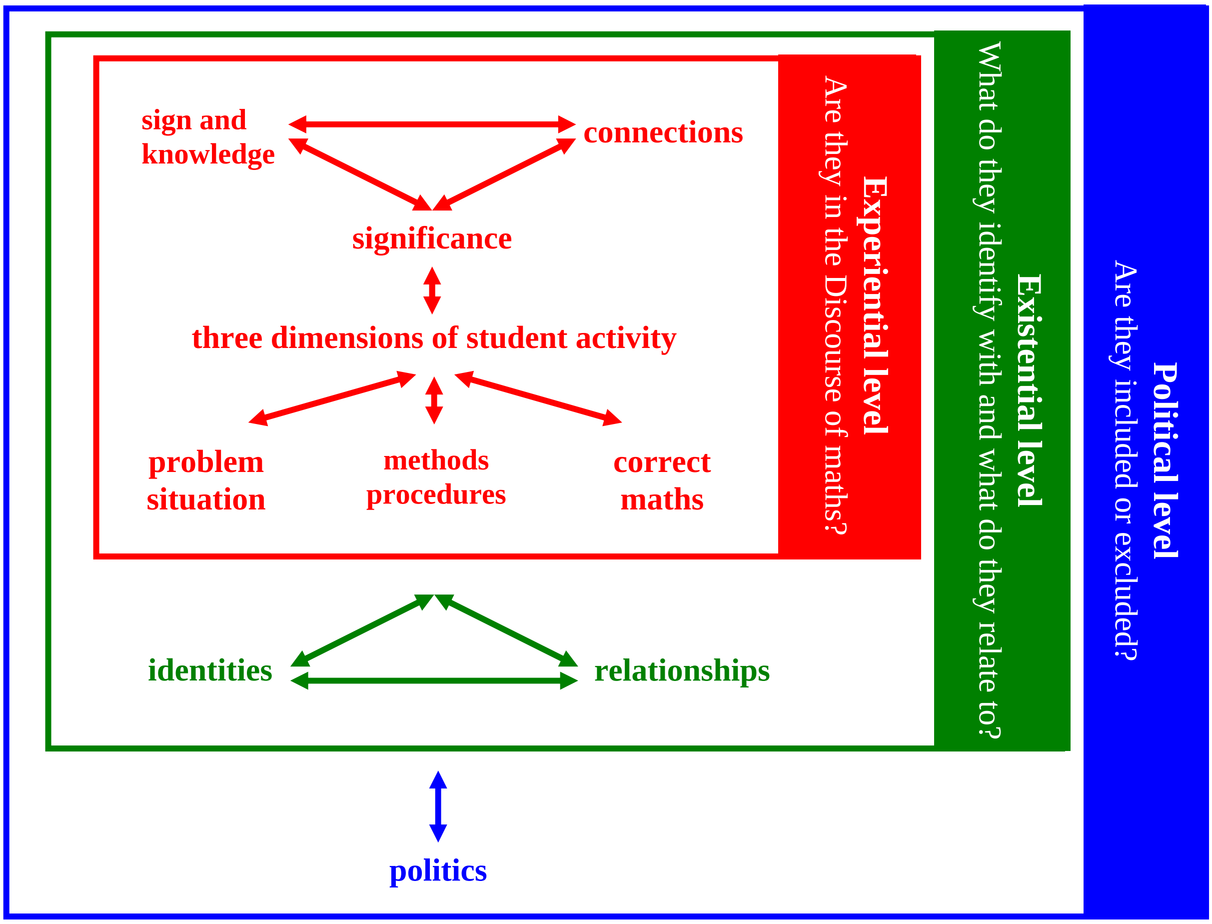


Figure 14. Model showing the three levels of analysis

Questions to guide the analytic process

What *sign and knowledge* systems are used to talk about the activity?

See this through the use of mathematical Discourse, use of symbols or unknowns, use of or reference to methods or procedures.

What *connections* (or disconnections) are evident from looking at the student discourse?

See this through the way in which the students make connections with the mathematics domain as a whole (or not), or the way in which it sets up associations with specific parts of the mathematics domain (or other domains).

What is being made *significant*?

See this through what the students bring to the fore in their talk, activity and gestures, why they make certain connections (or not) and through what they emphasise.

What *activities* are the students involved in?

See this through what the students did when attempting the problems and this is examined through the three dimensions of student activity (discussed in detail below).

Identity: how do the students position themselves with respect to word problems?

See this through the way in which students talk about their involvement in doing the problem (e.g. being in control, not coping, on the periphery, etc.) and any significance that they might attach. By 'positioning themselves' I mean how the students view themselves in relation to word problems (e.g. they are drawn to them, threatened by them), how they react to word problems (e.g. use a particular method, are at a loss, avoid them).

What *relationships* emerge with respect to peers and others involved in word problems?

See this through the way in which students put forward relative positioning of others, especially through the way in which they talk about (or do not talk about) their and others' involvement in the activity of doing word problems. By 'relative positioning' I mean how the students see themselves and others in terms of power relations (i.e. who is dominant, in control, or who is dominated).

Politics: what social goods are perceived by the students?

See this through the way in which the students see themselves positioned with respect to others in terms of the benefits or disadvantages that such a positioning might imply, and in terms of how the identity put forward enables or inhibits them from acquiring those social goods.

The three dimensions of students activity from a grounded perspective

In each of the dimensions the text, or what the students produced in attempting each of the problems, will be examined at four levels. These four levels are then used to gauge the extent to which the students have access to the Discourse surrounding school mathematics word problems in each of the three dimensions. The levels are described below in each of the dimensions, and a grading system is detailed so that a 'picture' of the relative access of the students can be created.

1. Understanding the problem situation

From the text that the students have produced in attempting the problems we can gauge the extent to which they demonstrate an understanding of the problem situation. By problem situation I mean not only the context in which the mathematics is set (colloquially referred to as ‘the story’), but also the type of problem that is perceived by the students (i.e. what Gerofsky (1996) refers to as the *genre*) and how this problem is situated within the broader context of word problems (i.e. what Gee (2005; 1999) refers to as the ‘situated meaning’).

The text will be examined for certain characteristics which can be used to infer the extent to which an understanding of the problem situation is evident. It must be noted that this analysis does not make any statement about what understanding of the problem situation might have existed, but rather serves only to gauge from *evidence in the text* what understanding is demonstrated. In other words, this is not to say that a student who produced text showing no understanding of the problem situation actually did not *have* any understanding of that problem situation; all that it tells us is that there is no *evidence* in the text that such an understanding existed.

Thus, at the most basic level the text might reflect absolutely no understanding of the problem situation. At the next level the text may show ‘vague’ references to the problem situation, or it might suggest that misinterpretations had occurred. Then, at the next level the text might indicate definite recognition of the problem situation, but be lacking in completeness. Finally, at the next level the text will indicate a clear understanding of the full problem situation (and it must be pointed out here that correctly solving the problem is not a pre-requisite for this).

Grading	Level descriptor	What the text might look like
0	Almost no understanding evident, or understanding is superficial.	Ideas simply noted down, cannot be connected to aspects of the problem, bear no relationship to the problem (other than purely numerical similarities)
1	Some evidence of understanding, but it is vague or misinterpreted.	Use of information from the problem in some way shows an understanding of the context. For example an increase is shown for the expression 'more than' or 'double', but that increase may not be correct.
2	Evidence of recognition of the problem situation, but incomplete or errors in interpretation	Calculations etc. must closely represent the problem situation. Basic operations should be correct, units should be correct, etc. with some omissions. For example, '3 more than' might be represented as '+ 3' and not as '*n* + 3'.
3	Evidence of correct interpretation of all aspects	Expressions and calculations accurately reflect the problem situation, even if the mathematics is not correct.

2. Use of methods or procedures

From the text it will be possible to gauge how the students went about doing the problem. There might be various methods or procedures evident in the text that are used to help organise ideas, and these methods and procedures may vary from being numeric to being more algebraic in nature. By methods or procedures I mean, for example, the use of formulae, tables (either numeric or algebraic), diagrams, constructive 'guess-work' (such as better approximation or trial-and-error), assigning of unknowns and establishing expressions, etc.

In this study I am interested in the degree to which students have access to a Discourse of *school mathematics*. If one were more interested in access to a Discourse of mathematics (more generally) it might be argued that more rigorous, algebraic methods or procedures would be better indicators of such an access. However, in the context of school mathematics certain numeric procedures (such as better approximation, or establishing a table of values) might be all that is needed to solve the problem. Thus in gauging students' methods and procedures I will not privilege any one approach over another. In order to gauge the methods or procedures, therefore, the text will be analysed to exhibit the extent to which it displays a systematic approach in the solution process. By systematic, I mean an approach to the problem that is deliberate and organised, and which seeks to guide the solution process.

At the most basic level the text would not show any evidence of a method or procedure. At the next level, the text will show some structured approach, which may just be in terms of layout of working, or perhaps using a diagram. One up from this we see text that shows clear methods or procedures, but that are not well organised or possibly somewhat haphazard. At the next level the text will show clear methods or procedures that are methodical, structured and guide the solution process.

Grading	Level descriptor	What the text might look like
0	No evidence of method or procedure	Working does not appear to be directed, and may seem haphazard (e.g. 'random' use of numbers from the problem).
1	Some evidence of method or procedure is present in rudimentary form	The text reflects some structure in establishing relationships, but the method or procedure is not clear.
2	Evidence of methods or procedures is clear, but they are not systematic	Clear use of a method or procedure that is valid, but implementation is not well ordered, less methodical, etc.
3	Evidence of clear methods and procedures that are systematic	Text shows organised, structured work that guides the solution process.

3. Correct mathematical working

The text reflects the extent to which the working is mathematically sound. By mathematically sound I mean working in which the mathematics is both correct *and* applicable to the problem, and this may include numerical procedures. For example, the mathematics may be absolutely correct but have no bearing on the problem (such as correctly adding the expressions for the length and breadth of a rectangle when the area was asked for).

At the basic level text will reflect mathematics that is completely incorrect (and by implication this will include mathematics that is completely inappropriate to the problem). At the next level the text shows some correct mathematics or establishing of relationships appropriate to the problem, but which are limited in that they do not enable further progress with the solution of the problem. The level up from this will have text that shows some meaningful, appropriate mathematical relationships (expressions, computations or equations) that should have enabled further progress. At the next level text will reflect correct mathematics as well as the accurate establishment of an equation or numerical situation that either did or could lead to a correct solution of the problem.

Grading	Level descriptor	What the text might look like
0	No evidence of correct or appropriate mathematics	Text might have no mathematical working, or show incorrect mathematics, or working that is inappropriate to the problem.
1	Some evidence of correct mathematics	Text reflects some working that is correct and/or appropriate, but it is not sufficient to enable further progress. Text may reflect possible conceptual errors or incorrect recall of formulae.
2	Evidence of mathematical correctness that enables some progress towards a solution	Mathematics is correct and appropriate and could have lead to a solution except for an error, omission, use of an incorrect formula, etc.
3	Text is fully correct and appropriate and leads to a solution	Text shows correct mathematics and a solution to the problem, or in cases where the problem is not fully solved, if the text shows that the mathematics generated a situation from which a solution would have emerged

The overall analytic framework thus looks as follows: an examination of the three dimensions of student activity, followed by an analysis of the seven building tasks at the levels of the experiential, the existential and the political, which enabled me to see patterns that pointed to the existence of certain Discourse models. This enabled me to address research sub-question 3.

In the next chapter the analysis (as described above) is discussed for each site. The student working and each of the excerpts are concluded with a discussion and at each site a conclusion is included to pull together all of the analysis before moving on to the next site. In view of this the next chapter is necessarily quite lengthy.

Chapter 5 – The analysis of the extended study

The focus of the analysis of the extended study has been on the student working and on the peer discussions, with some reference to the paragraphs and the questionnaires. In this analysis, the student working is examined using the three dimensions of student activity, followed by an analysis of the peer discussion using the three analytic levels discussed in the analytic framework. This was to address research sub-question 1, *viz*. what students do with the problems and how they talk about their experiences. The analysis of student working for each problem is prefaced with the word problems for ease of reference. Following each of these analyses is a discussion to identify trends that point to possible Discourse models that are at play which was to address research sub-question 2. These discussions, together with the concluding remarks at the end of the analysis of each site were intended to identify the influences that the Discourse models were having on the students as they did and spoke about their experiences with word problems, which answered research sub-question 3. What follows is a detailed discourse analysis of each of the schools which is a necessary step in examining and confirming trends that will point to Discourse models at play as the students do and talk about word problems. Before turning to the analysis a short note is given on the transcriptional devices used and why these were chosen to give the reader better access to the transcripts in the analysis.

A note on transcriptional devices (*cf.* Gee, 2005; 106 – 107)

The transcriptions were originally numbered according to turns (i.e. changes in who is putting forward ideas or contributing to the situation verbally). I have kept this original numbering because I have now selected excerpts from the transcriptions for analysis and in this way it is always possible to relate back to the original transcription. The turns have been chunked into stanzas somewhat like verses in a poem (Gee, 2005; 127). Each turn has been broken down into lines which have been sub-categorised by lower-case lettering. Each line will normally contain one piece of salient or new information (Gee, 2005; 125). Thus, I can refer to a whole stanza (e.g. Corona Focus, stanza 29), a turn (e.g. Manumission Peer, line 77) or a more specific utterance within a turn (e.g. Duskhaven Peer, line 34b).

In the transcriptions I use Gee's criteria for identifying each line as being an *idea unit* (through which people give information) and a *tone unit* (in which people stress intonation). I have therefore adopted Gee's use of underlined text to denote the part or parts of a line that carry the major stress and I will also capitalise words as Gee does where any extra emphasis is perceived.

Gee uses two periods ".." to denote a *hearable pause* or hesitation. I will use two periods to denote a short pause or hesitation and an ellipsis "…" or three periods where the pause or hesitation is perceived to be longer. An ellipsis at the end of an utterance usually means that the person indicates that they have not completed what they were going to say or that they have been interrupted. Gee uses a double forward slash "//" to indicate a tone unit that has a final contour (i.e. a rising or falling tone that indicates some kind of closure to the piece of information). I prefer to separate out a rising contour and a falling contour because of the possible impact that it might have on the interpretation of the message, so I will use a double forward slash "//" to indicate a final rising contour, and a double backward slash "\\" to indicate a final falling contour. I decided on this when I first started to add the notation to the transcripts since there seemed to be different meaning that could be attributed to an utterance if it ended on a rising contour rather that a falling one (or *vice versa*).

Non-linguistic messages, such as sighs, laughs, gestures, etc., have been described in square parentheses wherever possible. I have also included the time in the right hand margin every so often. This refers to the running time of the video recording of the discussion for ease of reference where necessary.

Setati (2003) argues that data can be *re-presented*, and that this can alter the findings and conclusions of the research[25]. Barwell (2003) also raises a concern regarding the interpretations of the researcher (particularly if that researcher is monolingual in a multilingual teaching situation). Gee (2005; 106) also makes mention of the problem of fully representing any dialogue in a transcript. In discursive analytic terms, the more detail that is provided in the transcription to indicate intentions or perceptions, the *narrower* that transcription is said to be, and Gee advocates that in determining exactly how narrow one's transcription should be one must consider "…how the transcript works together with all the other elements of the analysis to create a "trustworthy" analysis…" (Gee, 2005: 106). For the purposes of this study I have included sufficient linguistic and non-verbal detail in an attempt to render the transcript closer to the 'actual' discussion, whilst retaining a degree of

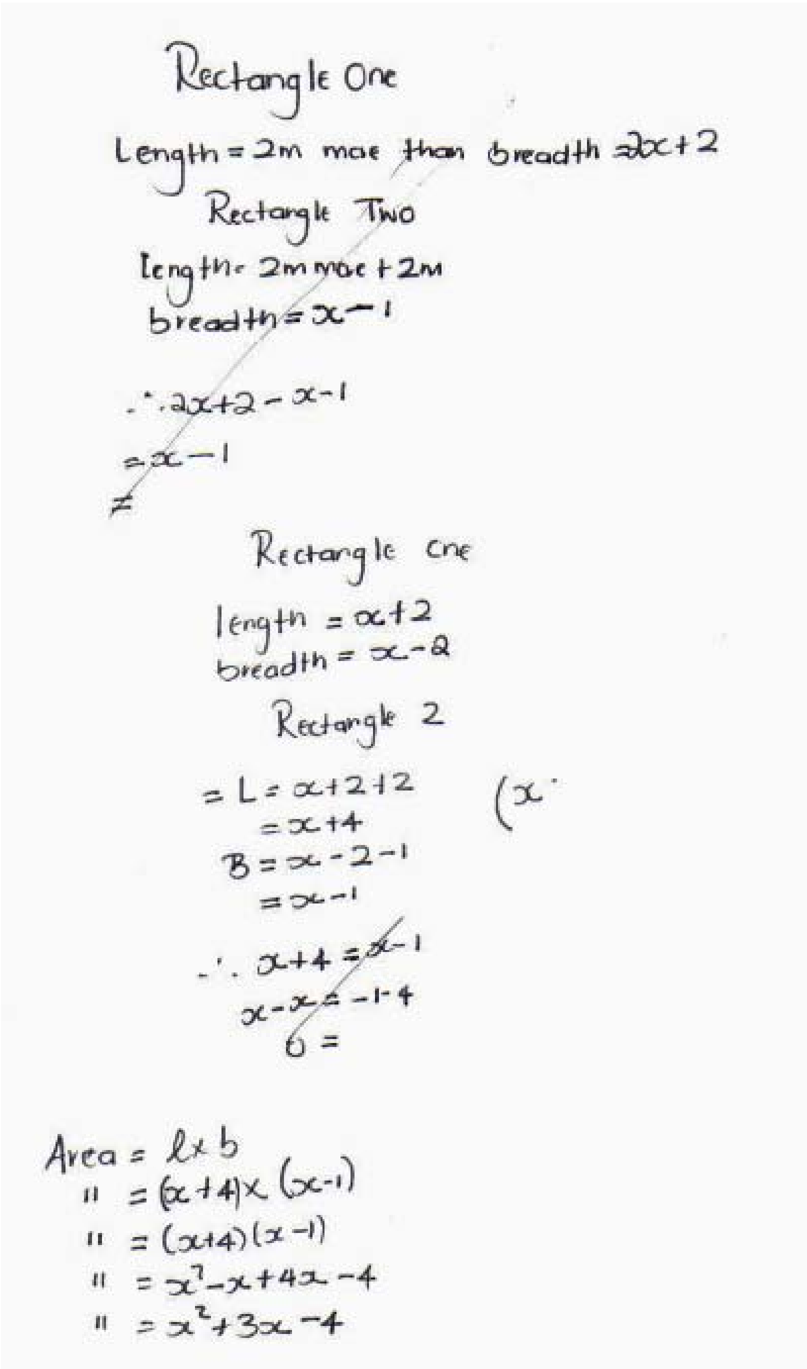
[25] Setati's research was focused more on the multilingual classroom, but it seems logical that the implications extend to the transforming of any interactive situation into a transcription.

‘readability’ in that new *re-presentation* of the transcript (Setati, 2003). In other words, I have tried to reach a compromise between a narrow transcript and legibility, both of which I hope will enhance access to the data available in this study. In the words of Gee (2005):

> “Of course, it is always open to a critic to claim that details we have left out *are* relevant. But some details will always have to be left out … and, thus, such a criticism cannot mean that we must attempt to put in all the details.”
>
> (p. 110)

Corona – The analysis of the student working

> Problem 1 – The rectangle problem
> Tseko draws a rectangle with its length 2 m more than its breadth. He then increases the length by 2 m and decreases the breadth by 1 m. He finds that the area of the new rectangle is the same as that of the first one. Find the length and breadth of Tseko’s first rectangle.


Rectangle One
Length = 2m more than breadth = 2x + 2
Rectangle Two
length = 2m more + 2m
breadth = x − 1
∴ 2x+2 − x−1
= x − 1
≠
Rectangle one
length = x+2
breadth = x−2
Rectangle 2
= L = x+2+2
= x+4
B = x−2−1
= x−1
(x
∴ x+4 = x−1
x−x = −1−4
0 =
Area = l × b
" = (x+4) × (x−1)
" = (x+4)(x−1)
" = x²−x+4x−4
" = x²+3x−4

Description

Roxanne – problem 1

1. *Understanding the problem situation*

The text indicates that Roxanne has identified that the problem involves two scenarios of a rectangle, which we see in the use of the expressions ‘Rectangle one’ and ‘Rectangle 2’. The text also reflects an understanding of the relationships between the length and breadth in both cases, but there is no indication that there was any understanding of the relationship between the areas of the two rectangles.

Grading 2: Evidence of recognition of the problem situation, but incomplete or errors in interpretation

2. *Use of method or procedure*

The text shows that an unknown *x* has been assigned, which can be inferred by the first line, “Length = 2m more than breadth = 2*x* + 2”, but what the unknown represents is not made explicit. However, from the text it is evident that Roxanne uses this unknown to establish algebraic relationships and to finally set up an expression for the area of the second rectangle. The text reflects a method that is algebraic and systematic.

Grading 3: Evidence of clear methods and procedures that are systematic.

3. *Correct mathematical working*

The text throughout suggests that Roxanne is able to set up expressions from the problem. However, there are errors. In her initial attempt Roxanne appears to have misinterpreted the length of the first rectangle to be double the breadth, and then she has added 2m to that length. That working has a line through it indicating that Roxanne wished to re-start the problem, and in her subsequent attempt we see two errors: firstly, she increases the length by 2, *and* decreases the breadth by 2, and secondly in the second rectangle she calculates the breadth to be “*x* – 2 – 1 = *x* – 1. However, apart from these errors the text reflects correct mathematics and correct application of the relationships in setting up the expression for the area of the second rectangle.

Grading 2: Evidence of mathematical correctness that enables some progress towards a solution.

Ayanda – problem 1

1. *Understanding the problem situation*

The first line of the text indicates that Ayanda was aware that the problem involved area of a rectangle, and the use of the terms ‘length’ and ‘breadth’ in the last two lines confirm that she understood that she was dealing with a rectangle, so the inclusion of the word

$l \times b$
$= 2m \times 1m$
$= \underline{2m}.$
$\therefore 2m + 2m$
$= \underline{4m}.$
∴ First triangles length = $\underline{2m}$.
First triangles breadth = $\underline{2m}$.

‘triangles’ in the last two lines appears to be an unconscious error. It also seems safe to conclude that Ayanda is aware that (at least) another rectangle was involved since she refers to “First [rectangle]…” on two occasions, but the text does not give any other indication of a second rectangle. However, the text suggests that she has misinterpreted the relationship between the length and breadth of the first rectangle (see the second line), which could be attributed to her honing in on certain key words (Hegarty, Mayer and Monk, 1995; Mangan, 1989), and there is no evidence that she has understood that there is a relationship between the areas of the two rectangles. It appears that Ayanda loses sight of the problem question since she gives her interpretation of the length and breadth of the initial rectangle in line 2. However, this differs in her final answer in that the breadth is also given as 2m. From the underlining of these figures it appears that Ayanda is satisfied with these as the dimensions of the first rectangle, but at this point it seems that she does not remember that she gave different dimensions (based on a misinterpretation) in line 2.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

The text only shows the use of a formula (*viz.* $l \times b$) to which assumed dimensions have been applied. The addition of 2m in line 4 appears to have been prompted purely by the wording of the text.

Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The text suggests that Ayanda assumed the length to be 2m and the breadth to be 1m as discussed above. Her mathematics after this is incorrect in that she arrives at an area of 2m (first dimension). She adds 2m to the length (correctly, in terms of the problem situation) to arrive at an answer of 4m. She then incorrectly attributes the length and breadth to be 2m each. Because of her initial assumption in line 2 the mathematics is also inappropriate to the problem situation.

Grading 1: Some evidence of correct mathematics.

Roenel – problem 1

$2 + x = x - 2 + x - 1$

$2 + 2 + 1 = x + x - x$

$5 = x.$

The length and breadth of the rectangle is 5m each.

OR

$2x + 2 + x - 1 = 0$

$2x + x = -2 + 1$

$\frac{3x}{3} = \frac{-1}{3}$

$x = \frac{-1}{3}.$

$2x + x = x + 2 + x - 1$

$3x = 2x + 1$

$3x - 2x = 1$

$x = 1.$

1. *Understanding the problem situation*

In line 1 the text shows some understanding of the relationships being made between the length and breadth in each rectangle. On the left hand side Roenel has "2 + *x*", which seems to indicate the length of the first rectangle being two more that its breadth, whilst the right hand sides seems to reflect some understanding of the new length and breadth of the rectangle. In the first equation, if the left hand side refers to the first rectangle and the right hand side refers to the second rectangle, it is possible that Roenel understood the relationship between the areas of the two rectangles. However, neither of the other equations appear to demonstrate any understanding of the areas of the two rectangles being equal so it is unclear where she derived these equations.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

From the use of the unknown *x* it appears that Roenel has assigned *x* to be the breadth of the first rectangle, although this is not stated directly. From the first equation it seems that Roenel has established expressions that for her relate the length and breadth of the rectangles, and these appear to have been applied directly to an equation before clarifying what they mean. There is evidence of an attempt to establish algebraic expressions and a further attempt to relate these in an equation.

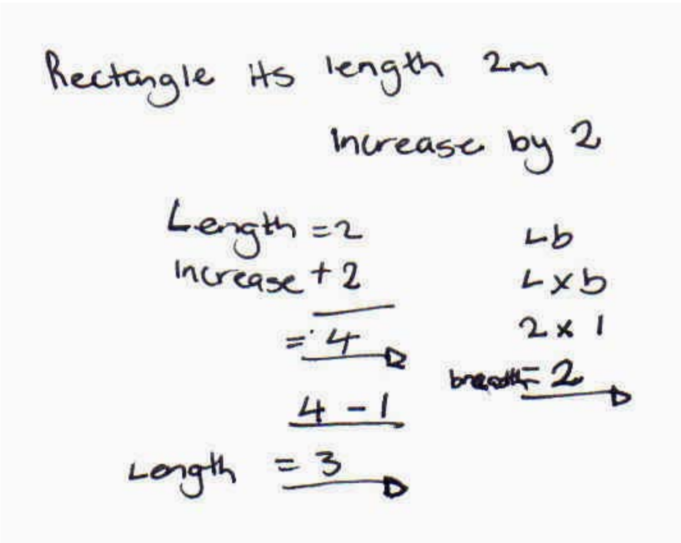
Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

From the first equation there appears to be some correct mathematics, although this is not explicit. If we take x to be the breadth of the original rectangle, then "$2 + x$" correctly expresses the length of that rectangle, and "$x - 1$" correctly expresses the reduced breadth. However, the expression "$x - 2$" and the equation itself, as well as the other two equations, make no mathematical sense in terms of the problem.

Grading 1: Some evidence of correct mathematics.

Lerato – problem 1



1. *Understanding the problem situation*

The text appears to indicate recognition of only one rectangle in the problem. From her first statement it appears that Lerato has interpreted 'two more than' to be 2m, which suggests that she might be honing in on key words in the problem (Hegarty, Mayer and Monk, 1995; Mangan, 1989). There is evidence that Lerato understood the relationship as she writes of the length, "Increase by 2", followed in her calculation by "+2" next to the word 'increase'. The text further indicates that there has been recognition of a decrease by 1, but it appears in the working and it is not clear what was reduced by 1 since Lerato has reduced her increased length by one. The text also shows that Lerato was aware that the area of a rectangle was involved since she wrote "$l \times b$" and does the calculation "2×1", the latter also indicating that she has misinterpreted the information given about the length and breadth of the original rectangle. Further to this it appears that she loses sight of the problem question since she gives the first rectangle a length of 2m in line 1, but concludes that the length (presumably of the first rectangle) is 3m at the end.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

The text indicates only the use of the formula for the area of a rectangle, into which assumed or misinterpreted values were substituted. The calculations in the text appear to have resulted from extracting certain information directly from the problem.

Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

Although the basic arithmetic is correct the mathematics is inappropriate as it does not relate to the problem at all, except possibly in the increase of the length of the first rectangle by 2.

Grading 0: No evidence of correct or appropriate mathematics.

> Problem 2 – The TV rental problem
> The Clear Vision television rental shop charges a basic fee of R150, as well as R15 per day to rent a television. The Best View television rental shop only charges a basic fee of R15 but has a daily rate of R60 per day to rent. For what number of days would it make no difference in cost as to which shop you rent from?

Roxanne – problem 2

1. *Understanding the problem situation*

The crossed-out text seems to reflect a summary of the information presented in the problem, followed by some calculations for the perceived scenario for two days of rental from each shop. However, this text shows that Roxanne did not take into account the once-off initial payment. The text that follows shows calculations of the initial payment and the daily rental

Clear Vision
= R150 and R15 per day rental

The Best View
= ~~R60 and~~ R15 basic fee and R60 per day

∴ R60 x 2 = R120 (for 2 days)
and R15 x 2 = R30 "

∴ R165 x 2 } 2 days / 165 x 3 = 495
= R330

And = R75
x

Clear Vision — Best View
= R150 + R15 — = R60 + 15
= R165 — = R75
∴ R165 x 5 — R75 x ~~H~~ 11
= R825 — = R825

∴ If you rent it from Clea Vision for 5 days = R825 and from Best View for 11 days you ~~get~~ spend the same amount of money.

for each shop, and from "R165 $\times 5$" and "R75 $\times 11$" it appears that Roxanne has again not understood the initial once-off payment for each shop. The calculations for her final answer indicates that she has also misinterpreted the problem question, but it was probably the misinterpretation of the conditions of hire that led to these calculations as it would be impossible to pay an equal amount at the two shops for a given time period using Roxanne's interpretation.

Grading 2: Evidence of recognition of the problem situation, but incomplete or errors in interpretation.

2. *Use of method or procedure*

The text to start with appears to summarise the information from the problem, and thereafter it seems from the calculations for two days, followed by the final calculations that Roxanne has used a trial-and-error approach.

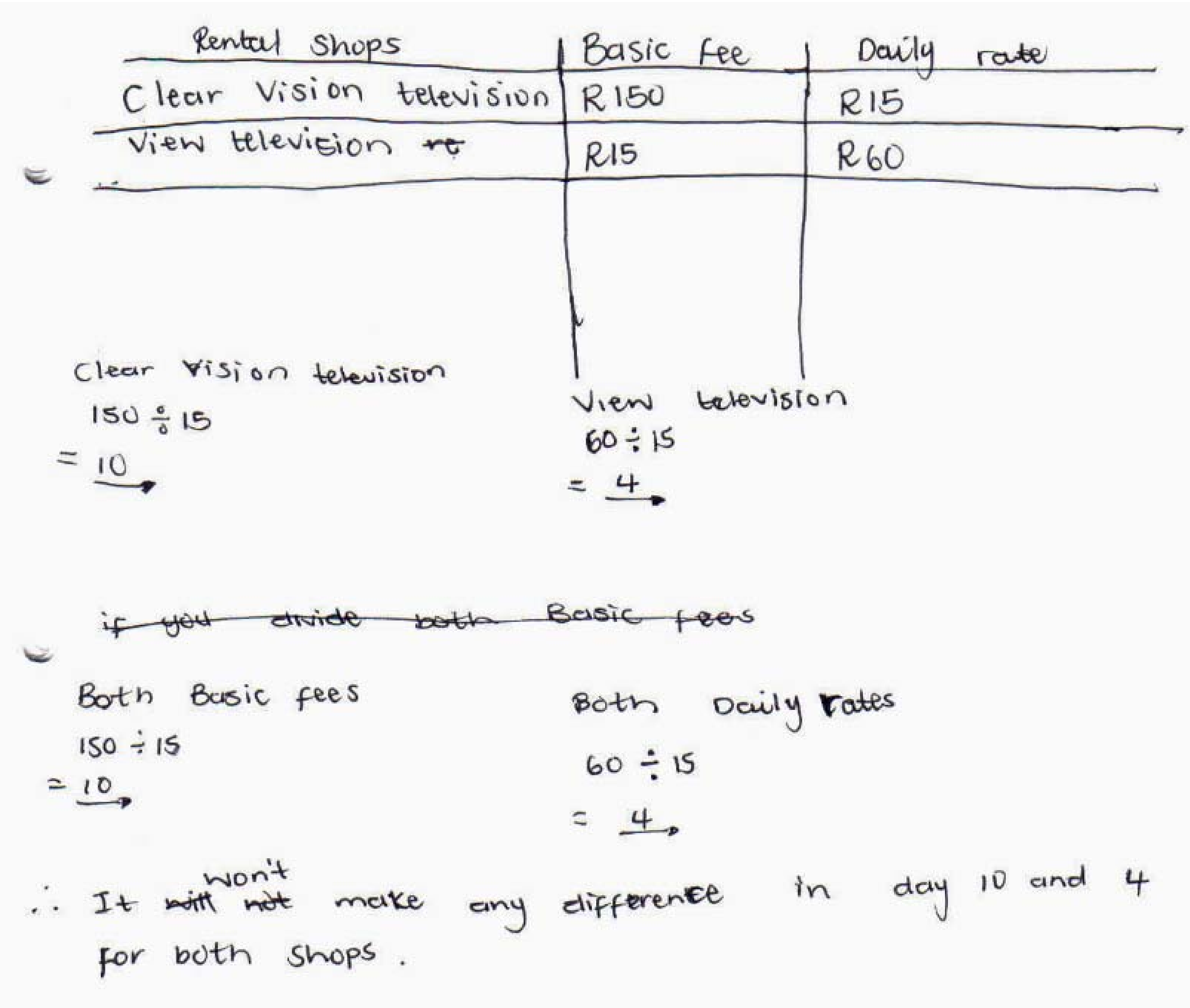
Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

The costs for one day at each of the shops have been calculated correctly. However, from the text it is apparent that the mathematics is inappropriate and this appears to have resulted from a misinterpretation of the basic fee and the daily rental.

Grading 1: Some evidence of correct mathematics.

Ayanda – problem 2



Rental shops	Basic fee	Daily rate
Clear Vision television	R150	R15
View television ~~rt~~	R15	R60

Clear Vision television
150 ÷ 15
= 10

View television
60 ÷ 15
= 4

~~if you divide both Basic fees~~

Both Basic fees
150 ÷ 15
= 10

Both Daily rates
60 ÷ 15
= 4

∴ It ~~will not~~ won't make any difference in day 10 and 4 for both shops.

1. *Understanding the problem situation*

The text reflects a clear breakdown of the basic fee and the daily rental for each shop. It is not clear what Ayanda's understanding of the problem was that resulted in her dividing "both basic fees" in the first place and then "both daily rates" in the second place. It is also not clear from the text how Ayanda is able to conclude that the answers to her calculations represent the number of days "for both shops".

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

The text shows a table that appears to simply summarise the data given in the problem. The calculations that follow seem to draw purely on the numerical data from the table, possibly from each column and, even though this does not make sense, it appears that Ayanda is trying to use the table to guide her solution strategy in that, possibly for her it gave a way of comparing the rental costs at each shop.

Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

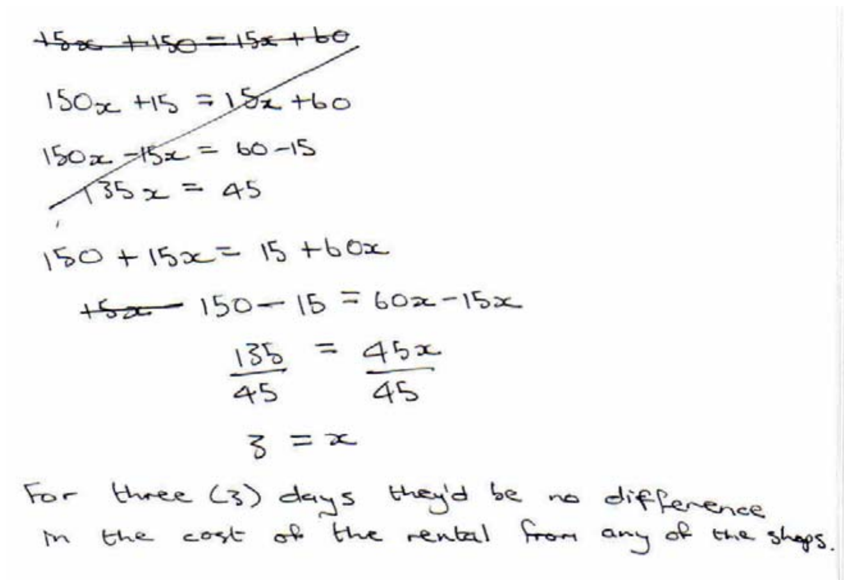
The text shows a table that does not incorporate any provision for the number of days and this appears to result in Ayanda trying to find appropriate calculations from the table to arrive at a number of days as an answer. The mathematics of dividing the two basic fees and then the two daily rates is incorrect.

Grading 0: No evidence of correct or appropriate mathematics.

Roenel – problem 2

1. *Understanding the problem situation*

In the initial text, which Roenel has crossed out, she appears to have mixed up the basic fee and the daily rate for the second shop, possibly as a result of the common amount of R15. However, the text that follows shows that she realised that there was a misinterpretation and from her equation it is evident that she fully understood the problem situation.


$\cancel{15x + 150 = 15x + 60}$

$150x + 15 = 15x + 60$

$150x - 15x = 60 - 15$

$135x = 45$

$150 + 15x = 15 + 60x$

$\cancel{+15x}\ 150 - 15 = 60x - 15x$

$\frac{135}{45} = \frac{45x}{45}$

$3 = x$

For three (3) days they'd be no difference in the cost of the rental from any of the shops.

Grading 3: Evidence of correct interpretation of all aspects.

2. *Use of method or procedure*

The text shows that an unknown was assigned, although this is not directly stated. The equations (both in the initial attempt and in line 5) appear to have been established by piecing together auxiliary expressions (Lepik, 1990) for each shop in terms of the assigned unknown, and then equating them, which demonstrates a competent algebraic approach.

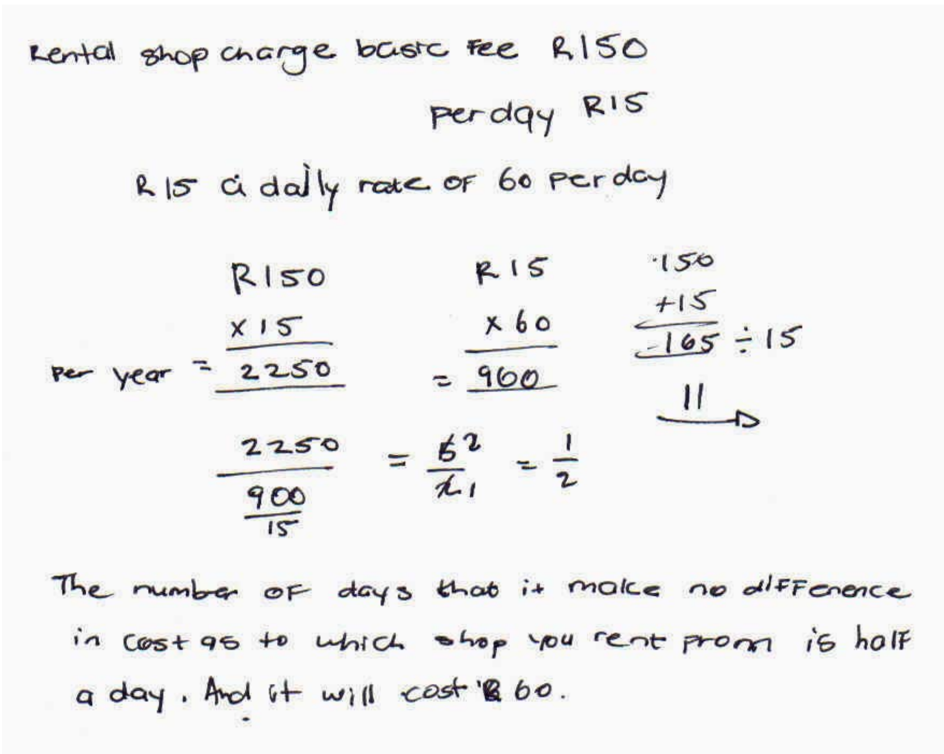
Grading 3: Evidence of clear methods and procedures that are systematic.

3. *Use of correct mathematics*

The text shows an initial error which was one of interpretation rather than a mathematical error. This was corrected and the resulting mathematics is sound, both in the inductive setting up of expressions and the equation, as well as in the algebraic manipulation that follows, and it yields a correct solution.

Grading 3: Text is fully correct and appropriate and leads to a solution.

Lerato – problem 2



1. *Understanding the problem situation*

The first three lines of the text appear to summarise the information from the problem, but from the calculations that follow there appears to be very little understanding of the problem situation. Whilst the concluding line suggests that Lerato had some understanding of what the question was asking, the fact that it is copied directly from the question may indicate that even this understanding of the problem situation was a limited one.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

The initial three lines show summarised data from the problem. Thereafter it is not clear from the text what approach Lerato is using, but it is most likely that she is simply creating calculations using the summarised data. Possibly because the resulting figures are quite large, Lerato interprets them as being annual amounts. The calculations that result in Lerato's answer of a half also appear to be simply pieced together from the figures that she arrived at before, but were probably intended to give her a way of comparing the results for each shop.

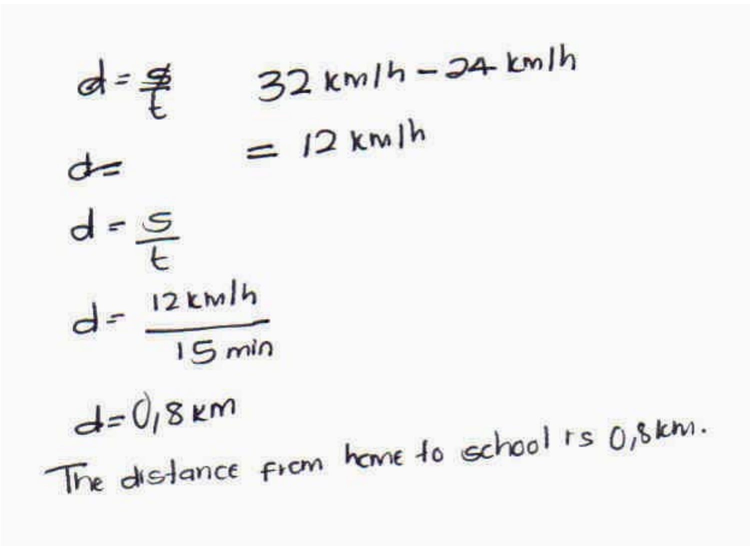
Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The mathematics demonstrated in the text appears to be inappropriate to the problem. The answers to the calculations have been stated as annual amounts which we see from the inclusion of the phrase "per year", but there is no evidence of how this relates to a year. Finally, there is a division of the two annual amounts, which seems to be an attempt to compare the rates at the two shops through an inappropriate proportional reasoning (De Bock, 2002), and if this is the case then mathematically it is incorrect since a comparison by subtraction is required in this case.

Grading 0: No evidence of correct or appropriate mathematics.

> Problem 3 – The speed-distance-time problem
> A boy cycles from home to school in the morning and back in the afternoon. He cycles from home to school at 32 km/h and back at 24 km/h. It takes him 15 minutes longer in the afternoon than in the morning. Find the distance between home and school.



$d = \frac{s}{t}$ 32 km/h − 24 km/h

$d =$ = 12 km/h

$d = \frac{s}{t}$

$d = \frac{12\text{ km/h}}{15\text{ min}}$

$d = 0{,}8$ km

The distance from home to school is 0,8km.

Roxanne – Problem 3

1. *Understanding the problem situation*

From the use of the formula in the first line it seems that Roxanne recognised the problem to be of the speed-distance-time genre (Gerofsky, 1996). In addition to this, the formula is represented with d as the subject which implies that Roxanne identified what the problem question was asking. From the text it is difficult to gauge whether or not Roxanne understood the differences between the journeys, but it is possible that she was just unable to express these mathematically.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. Use of method and procedure

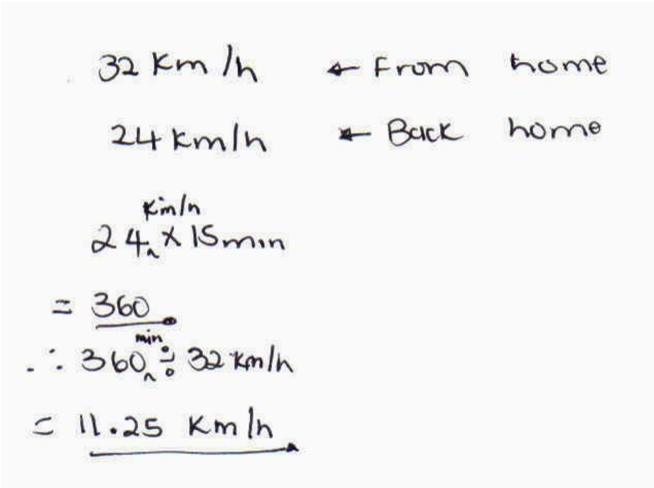
The text starts with the formula which suggests that Roxanne is accustomed to this procedure for these types of problems. The first two lines of the formula appear to be a little scratchy, which implies that Roxanne may have been battling to recall the formula, presumably not having dealt with it in a while. However, from line 4 onwards, in one sense the formula appears to enable Roxanne in that she substitutes the ‘speed’ that she arrived at in line 1, and then the time of 15 minutes (which she appears to have gleaned from the question). From there she is able to get an answer with which she seems confident, if we look at how she has expressed her result in the last line. However, in another sense, the formula may have hindered Roxanne in her solution because she appears to be using it in a rote-type manner (Boaler, 1997), which may be causing her to overlook certain anomalies in her mathematics.

Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

To start with Roxanne appears to have recalled the formula incorrectly. She has then subtracted the speeds and arrived at an answer of 12km/h, which could be as a result of her intuitively attributing linearity to the situation (De Bock, 2002). In the formula she uses the 15 minutes as the time, but does not convert this to hours so as to be compatible with the speed. Therefore, from the formula to the use of the information in the problem the mathematics is incorrect.

Grading 0: No evidence of correct or appropriate mathematics.



Ayanda – problem 3

1. *Understanding the problem situation*

The first two lines of the text indicate that Ayanda recognises that a comparison of two different journeys is described in the problem. It is also possible that the way in which she has used the 15 minutes in line 3 suggests that she understands the difference of 15 minutes applied to the journey having a speed of 24km/h. In the calculations the answer seems to be the underlined text at the end which is given in kilometres per hour. If this is the case then it is likely that Ayanda has not fully understood, or has possibly lost sight of the problem question. Furthermore, it seems that Ayanda has not recognised that the 15 minutes needed to be converted to a unit that is compatible with the speeds.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

Initially the text appears to reflect a summary of some of the information from the problem. There is no indication from the text that Ayanda was using a formula, but it is possible that in line 3 she applied her figures to an incorrect version of the speed-distance-time relationship. From the working it appears that Ayanda is trying to solve the problem through some sort of inappropriate proportional reasoning (De Bock, 2002), using inverse operations in multiplication and division.

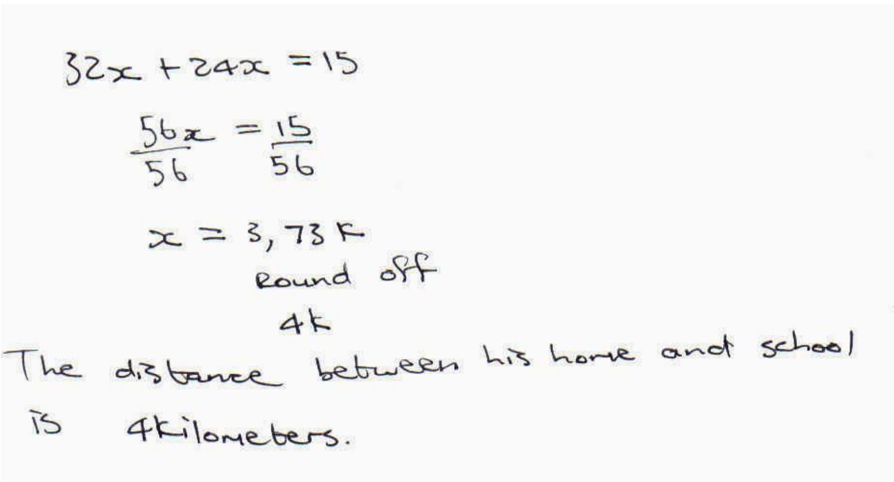
Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The calculations show speed times time giving an answer that is time, and then a time divided by speed giving an answer that is speed, so the mathematics is clearly incorrect.

Grading 0: No evidence of correct or appropriate mathematics.

Roenel – problem 3



$32x + 24x = 15$

$\frac{56x}{56} = \frac{15}{56}$

$x = 3,73k$

Round off

$4k$

The distance between his home and school is 4kilometers.

1. *Understanding the problem situation*

From the conclusion in the text Roenel seems to have a clear understanding of what the question is asking. In addition to this we see from line 1 that Roenel seems to recognise that the problem involves the speed-distance-time relationship, and she appears to be relating information from the problem to that relationship.

Grading 2: Evidence of recognition of the problem situation, but incomplete or errors in interpretation

2. *Use of method or procedure.*

From the conclusion given it seems that x was assigned as the distance, but this is not directly stated. If we take this to be the case then an auxiliary relationship (Lepik, 1990) of speed multiplied by distance appears to have been established for each journey which was probably an attempt to use the speed-distance-time relationship. In any event, it seems that Roenel has used an algebraic approach in combination with the formula to establish expressions, which she then puts into an equation. Use of this procedure appears to enable Roenel in the sense that she is able to make progress and arrives at an answer.

Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

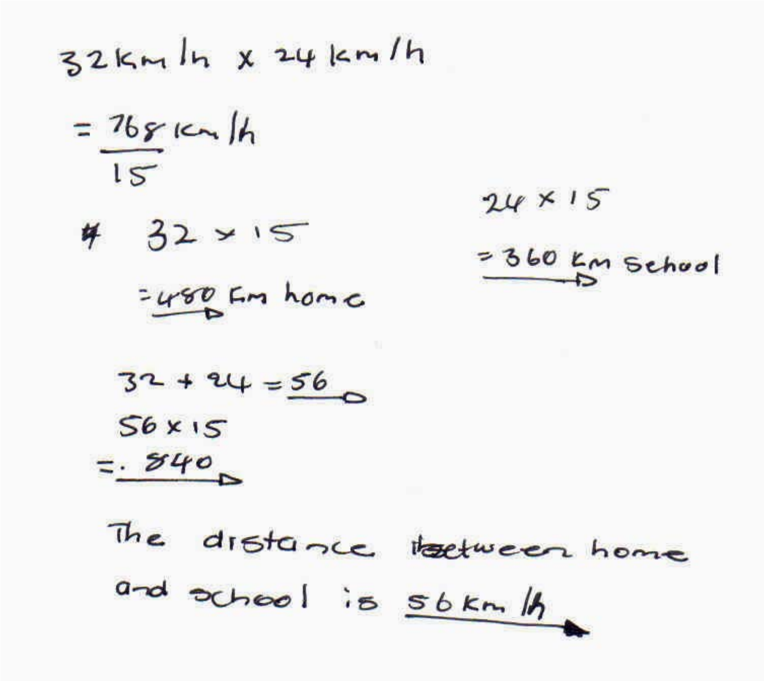
3. *Use of correct mathematics*

From the first equation it is evident that Roenel has probably used the incorrect formula or has established the relationship incorrectly. She has also used the 15 minutes incorrectly as her interpretation here would mean that the sum of the times was 15 minutes, and she has not converted the 15 minutes to comparable units. In addition to this in line 3 Roenel executes the inverse operation to arrive at her answer which may have caused her to see that answer as being more credible. The mathematics throughout is incorrect.

Grading 0: No evidence of correct or appropriate mathematics.

Lerato – problem 3

1 *Understanding the problem situation*



32km/h x 24km/h

$= \frac{768 \text{ km/h}}{15}$

32 x 15

= 480 km home

24 x 15

= 360 km school

32 + 24 = 56

56 x 15

= 840

The distance between home and school is 56km/h

From the text it appears that Lerato has not understood the problem situation other than to establish some link between the two speeds, which we see in the way in which the two speeds are operated upon seemingly to try to establish some sort of average. The final line to shows that Lerato did not understand the problem question as the answer is given as a speed but is stated as a distance between home and school. This is probably as a result of Lerato copying the wording from the problem and incorporating one of the ‘answers’ that she has arrived at in the preceding working.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

The text indicates different operations being performed on the two speeds given in the problem, and then incorporating the time of 15 minutes which, in the first calculation is divided, but elsewhere it is multiplied. It appears that Lerato is extracting information from the problem and performing various operations and has finally chosen what appears to be the most apt solution.

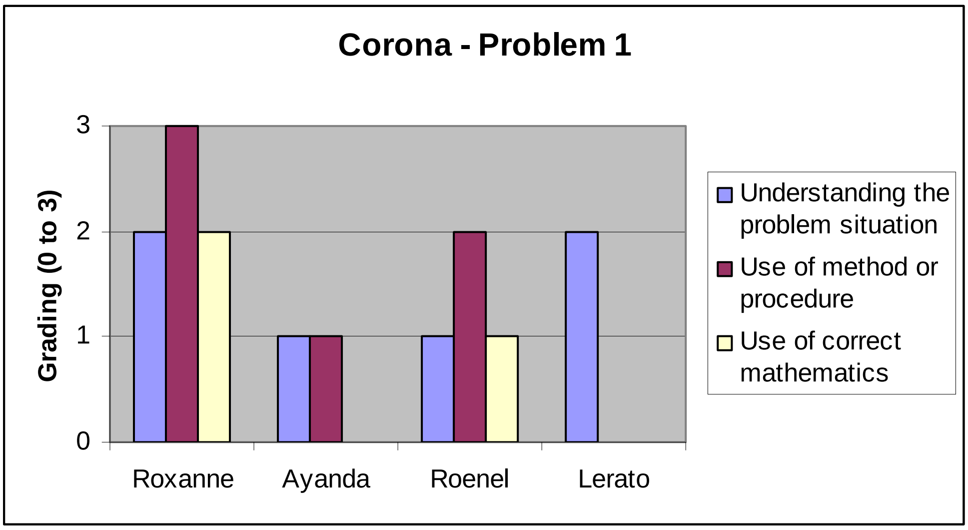
Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

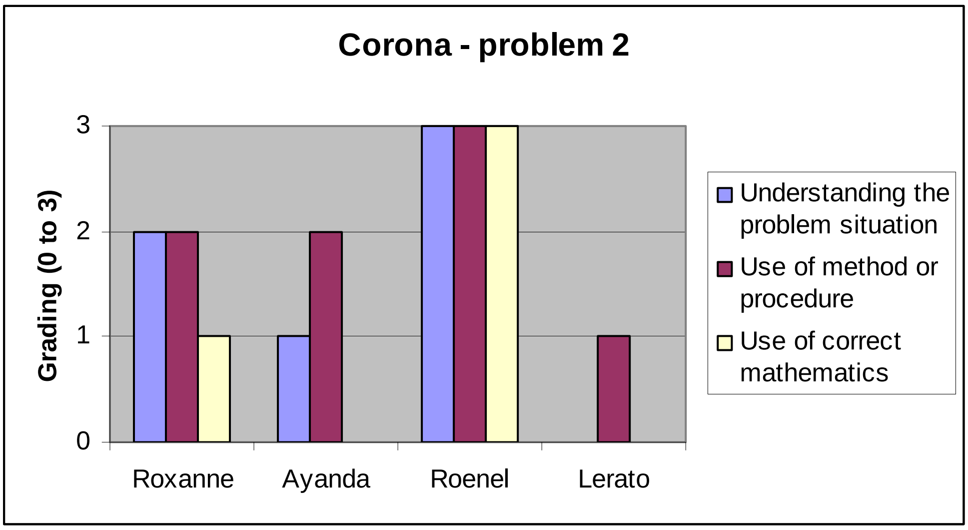
The mathematics is inappropriate and incorrect in terms of the problem context.

Grading 0: No evidence of correct or appropriate mathematics.

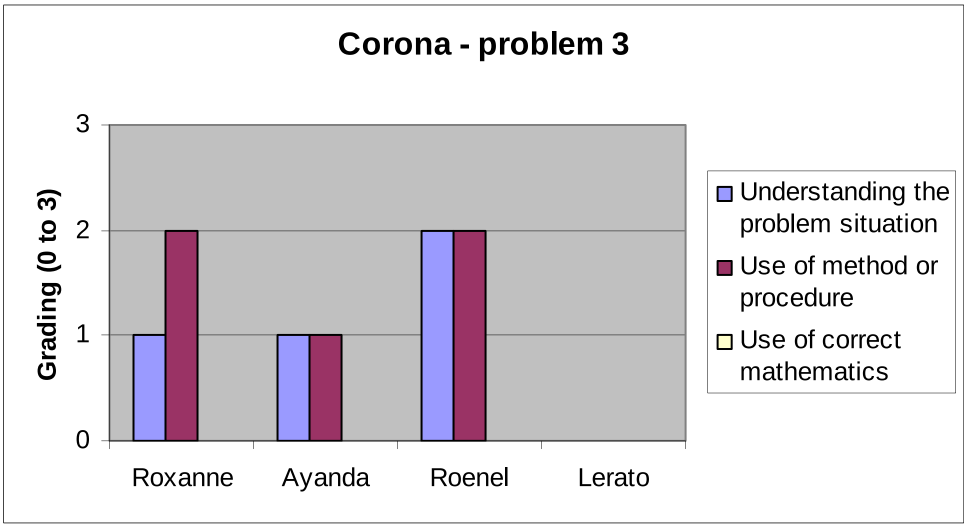
A summary of the three dimensions of student activity for Corona



Corona - Problem 1
Grading (0 to 3)
3
2
1
0
Roxanne
Ayanda
Roenel
Lerato
Understanding the problem situation
Use of method or procedure
Use of correct mathematics



Corona - problem 2
Grading (0 to 3)
3
2
1
0
Roxanne
Ayanda
Roenel
Lerato
Understanding the problem situation
Use of method or procedure
Use of correct mathematics



Corona - problem 3
Grading (0 to 3)
3
2
1
0
Roxanne
Ayanda
Roenel
Lerato
Understanding the problem situation
Use of method or procedure
Use of correct mathematics

Discussion

In nearly all of the student attempts there appears to be a reliance on some method or procedure which in most cases seems to enable the students when tackling the problem. By this I mean that it gives the student a means by which to make some progress, even though in many cases the actual working is not correct, or is based on incorrect formulae or assumptions. What is noteworthy here is that the students (with the possible exception of Lerato) are not excluded from the Discourse surrounding word problems, even though their working is sometimes off the mark, and this seems to be because they have some sort of strategy at hand that enables them to engage with the problem.

The graphs show a tendency in the students to be more successful with the television rental problem (problem 2). We see this in the generally higher bars for understanding of the problem situation as well as for the employment of methods and procedures. Of all the problems this is the only problem to be correctly solved by these students (see Roenel's solution above). The graph for the speed-distance-time problem shows that, with the exception of Lerato, the students had some understanding of the problem situation, and to an extent had some idea about a procedure to adopt for solving the problem, yet none of the students is able to make any mathematical progress.

Corona – The analysis of the student talk

In the excerpt below the students talk about their experiences when they were solving the word problems which took place prior to this discussion. They talk about whether or not the solution to a word problem is better achieved with the use of an equation or not.

Stanza 2

4.	Ayanda	a	I prefer equations	00:00:28
		b	than word problems.\\	
		c	It's much quicker	
		d	to use an equation	
		e	to solve a word problem.\\	
		f	… [talking in background]… you just balancing things.\\	
5.	Roenel	a	You need an equation	
		b	to solve word problems,	
		c	it's just one and the same thing.\\	
6.	Roxanne	a	But,	
		b	I don't need the equation	
		c	coz it's too complicated for words.//	
		d	You know some things like... [Roenel interrupts]	

Description

Sign and Knowledge systems

The students use the word 'equation' as though they have a common understanding of the concept, however none of the students questions anyone else's understanding of this notion.

Connections

There is a link between word problems and equations, but that link is different for each student. Ayanda and Roxanne seem to have a view of 'equations' as being something in their own right that might or might not be employed when doing word problems, whilst Roenel seems to have a perception of equations being more intrinsically linked with word problems. Roenel's comment, *It's just one and the same thing* (line 5c) implies something absolute about the link between the two.

Significance

Equations play a significant role for each of the students, but again, in different ways. For Ayanda it is *quicker to use an equation* (line 4c – d), and her use of the adverb 'much' (line 4c) indicates that she makes this significant. Roenel feels that equations and word problems are intrinsically related which we see when she says they are *just one and the same thing* (line 5c). Roxanne acknowledges that word problems are used in solving equations, but for her it appears to be significant that she doesn't *need the equation* (line 6b).

Experiential level

These three students appear to have experienced word problems in conjunction with equations which we see from the number of instances in which they attempt to set up equations in their working. From the rating of the student activity it seems that these students have a good degree of access to the Discourse surrounding word problems and this enables them to discuss the problems and their experiences at a fairly sophisticated level. Lerato, however, is notably absent in the student talk and her individual activity ratings demonstrate that she probably does not share the access to the Discourse that the other three students seem to have.

Identities

Ayanda distances herself from word problems to a certain extent when she says *I prefer equations than word problems* (line 4a – b), but by acknowledging that it is quicker to use an equation she shows that she sees herself as being involved in the activity of solving word problems by using equations, which appears to be through no choice of her own. Roenel seems to speak from a position of authority when she says *you need and equation...* (line 5a). Even though it is most probable that she is drawing on something like teacher-talk, she gives no indication of this in her utterance here, hence positioning herself as the authority. Roxanne expresses a position of independence when she responds to Roenel's comment with, *I don't need the equation* (line 6b).

Relationships

Ayanda's comment, *It's much quicker...* (line 4c), suggests that there is an alternate method (other than using an equation) for solving word problems, and that there appears to have been someone who showed her this 'quicker' method. We see this through the way in which this 'quicker' method seems to be a revelation to her, and in putting this forward in this manner she positions herself as the 'learner' under some 'more knowledgeable other' (Lave and Wenger, 1991) in this situation. Roenel's response is probably drawing on the teaching or advice of someone as discussed above, but she expresses this as her own. This, together with the apparent ontological view that she holds about equations and word problems as being 'out there' and unavoidably connected, suggest that she positions herself outside of any decision-making process when it comes to solving word problems. Roxanne's comment suggests that she has been exposed to solving word problems with the use of an equation by someone, but that she has found this to be *too complicated for words*. She has thus made her own decision that she can get by *without the equation* (*cf.* line 6c), and this in itself suggests that she perceives at least one alternate method for solving word problems.

Existential level

The three students talk about their experiences as a group indicates a belief that they are the ones who are expected to do the problem (Christou and Philippou, 1998b). Their conversation centres around the use of equations to solve the problem, but there is a tacit reference to some authority figure who issues the problem.

Political level

All three students acknowledge that equations can be used to solve word problems and they appear to view 'equations' as being the 'preferred method' (or in Roenel's case, the 'only method') for solving word problems within the mathematical Discourse. Ayanda seems to feel that equations advantage her when it comes to word problems (*It's much quicker* – line 4c) whereas Roxanne feels that she is disadvantaged by them (*cf.* line 6c). Roenel appears to have resigned herself to the 'fact' that there is no alternate way in which to solve word problems (*cf.* line 5a).

Discussion

All the students in this excerpt foreground equations in their talk about word problems which suggests that they hold strong links between equations and word problems. They also have an assumed common notion of the concept of equations and this seems to enable them to engage in a meaningful conversation about how to solve word problems. Even though the three girls hold different viewpoints, there is a common trend that is evident at the experiential level: *equations can be used in the solution process of word problems*. What is more, there seems to be the opinion that *the use of equations is the preferred approach to solving word problems*. It has also been argued that all three girls to an extent have acknowledged that this preferred approach comes from somewhere 'higher up' in the Discourse of school mathematics. Both Ayanda and Roxanne allude to at least one alternate method for solving word problems (other than using equations) but the actual method is backgrounded by these students.

In reflecting upon doing the problems Roenel and Ayanda in the excerpt that follows express how they felt about certain problems and Roenel then turns the focus to why she felt she was required to do the last problem. Roxanne relates how she experienced problem 1, and because of the absence of the measurement, and the limited information (she says, *they just told you*… line 10g) she *had to use all x, or two*… (line 10j).

Stanza 3

7.	Roenel	a	[Interrupting, shaking her head] I really don't like
		b	the ones with the rectangles
		c	and the...[inaudible, emphasizing with downward movement of hands]
8.	Ayanda	a	I liked the last one...
9.	Roenel	a	The last one
		b	I think
		c	he was just checking just to see if,
		d	you know,\\
		e	if you can use equations
		f	and all that,\\ [downward movement of hands in chopping-like motion]
		g	so that if [inaudible] the bottle costs <u>ten cents</u>… [gesturing with open hands and shrugging shoulders]
			[all talking together]
10.	Roxanne	a	The first one,//
		b	the problem was
		c	it didn't give you
		d	the <u>actual</u> like... [moving hands backwards and forwards on either side of her head]
		e	the <u>actual</u>

f like measurement.\\
g They just told you that
h like the length is
i like two metres more than the breadth.\\
j So you just had to use all *x*, or two…[gesturing with both hands facing forward, and then with vigorous hand movements]
[all talking together]

Description

Sign and knowledge systems

Roenel's reference to *the ones with rectangles* suggests that she perceives different categories of word problems (Gerofsky, 1999; 1996) and that something about the make-up of these problems causes her to dislike them. Roxanne's need to use an unknown implies that she tried to work in an algebraic manner. We see a certain amount of insight into the problems when Roxanne says, "…it [the word problem] didn't give you the actual… measurement" (line 10c – f), and she realises that because of this, "…you had to use all *x*…" (line 10j).

Connections

We see another connection between an equation (or at least some algebraic representation) and word problems through the activity (what they did with the problem) as well as the mathematical Discourse of Roenel (line 9c) and Roxanne (line 10j).

Significance

In line 10 Roxanne is very animated, using vigorous hand gestures, which suggests that she attaches a significance to her experience when doing problem 1. The significance seems to be connected with the way in which information is given (or not given) in the problem, and that she is therefore forced into having to employ an unknown. This follows Roenel's utterance that *he was just checking to see…if you can use equations* (line 9c – e, referring to problem 4) which she also accompanies with emphatic hand gestures, thereby suggesting that she attaches significance to this.

Experiential level

There is a sense here that word problems are set in a particular way and for a specific purpose. We also see a continued connection between word problems and equations, both in the student talk and to a lesser extent in some of their working, and the enabling effect that the students' tacit common understanding of the notion of equations has in terms of their ability to engage with each other when discussing word problems.

Identities

In the context it seems that Roenel positions herself in a subordinate role of 'being checked upon' and Roxanne appears to see herself as disempowered by the way in which the problem is presented, in that she has been 'forced' into a course of action she does not particularly like.

Relationships

Thus, both Roenel and Roxanne present a picture of some 'authority' figure. In Roenel's case, this authority figure (the researcher) has presented the problem with the purpose of *checking to see…if [she] can use equations* (line 9c – e). In Roxanne's case there is some unknown authority figure who has pre-designed the task to ensure that the only option available to her is to use an unknown.

Existential level

Roenel and Roxanne put forward two distinct groups within the Discourse surrounding word problems – those who do the problems and those who set or assign the problems. The former are explicitly the students who do word problems, but the latter is implied, except in the instance when Roenel makes reference to the researcher (line 9c – e).

Political level

In both Roenel's and Roxanne's utterances there is a sense of a lack of agency on their part. They become the 'object of the exercise' in a situation in which their control is limited. In Roenel's case, the purpose of the task was *just to see* if she could use equations, and her choice of words suggests that she could see no other purpose. In Roxanne's case (and possibly as a result of her drawing from Roenel's comments in line 9) she sees the problem as 'forcing' her to behave in a particular way.

Discussion

The students' implied common understanding of the notion of an equation that they use to discuss word problems enables them to delve quite deeply into the difficulties that face them when they do word problems, more specifically why algebraic techniques become necessary. The students foreground aspects of the problem make-up and their perceptions of the purposes behind those, and in doing so they background some authority figure behind these problems. There are two different views espoused about this authority figure behind word problems. The first has to do with the manner in which these problems are set, and the second with the reason why the problems are given, the latter of which alludes to the vague nature of the relevance of word problems discussed by Davis (2003). In both cases, however, there is a sense that the student is compelled to behave in a particular manner (viz. use an equation or an unknown), and thus the perception is that within the mathematical Discourse you do not have a choice – *equations must be used to solve word problems*.

Ayanda and Roxanne are reading their responses to the questionnaire in which they made statements about what they think word problems are and how they felt about them. Ayanda describes how she doesn't understand the problems and what she does in order to overcome this, whilst Roxanne is more explicit about misreading the question which possibly leads to a misunderstanding. In line 80, Roxanne's discussion suggests that there is an element of 'challenge' when the problems are more difficult, and that this involves her in 'thinking'.

Stanza 18

77.	Ayanda	a	I wrote here	00:09:00
		b	that word problems are//...	
		c	it's sort of like	
		d	they give you an equation//,	
		e	but they give it to you in words\\,	
		f	in writing\\.	
		g	And then... err,	
		h	what I feel about them//... err,	
		i	I really don't understand them\\,	
		j	but I really try	
		k	to understand them	
		l	and keep on learning	
		m	how to solve the problems\\.	
		n	I tried my best//.	
78.	Roxanne	a	[Reading her response on the questionnaire] I said//	00:09:22
		b	word problems are sums	
		c	given in mathematics//	
		d	by putting it in a story form//	
		e	and having to solve the…problems	
		f	and answer the required questions.	
		g	Word problems can be easy	
		h	if you understand them//	
		i	and a bit difficult	
		j	if you misread the question\\ [someone interjects with, "Ja"]	
		k	or don't understand them\\... [Roenel nods]	
79.	Lerato	a	Ja, ja...	
		b	a bit complicated\\.	
80.	Roxanne	a	For me personally//	00:09:37
		b	I would say	
		c	it depends on it's level of difficulty\\.	

d	I <u>like</u> ones
e	that are easy and straightforward//
f	and <u>also</u> the difficult ones\\... [some background comments, Lerato laughs, utterances are inaudible]
g	... as they challenge my level of thinking\\.
[Some inaudible comments, accompanied by chuckles, Roxanne thrusts her body back in an animated gesture of amusement.]

Description

Sign and knowledge systems

Ayanda talks about word problems as being given in words, whereas Roxanne describes them as 'a story' in which a problem needs to be solved. Roxanne also says, *word problems are sums* (line 78b), and here she uses the word 'sums' in a local colloquial sense to mean 'any mathematical problem' rather than in the strict sense, where it means 'addition'. By doing this she seems to place word problems clearly into the mathematical domain, and more specifically, the Discourse of school mathematics.

Connections

Ayanda makes yet another connection between equations and word problems when she says that *they give you an equation… in words* (line 77d – e). However, she says, *but they give it to you in words, in writing* (line 77e – f), which suggests that this is not the normal manner in which equations are given, but she nevertheless gives the impression that she sees word problems falling into some broader category called 'equations'. Roxanne associates word problems with 'stories', and together with her earlier reference to 'sums' suggests that this may stem from the term 'story sums', a euphemism that has sometimes been employed in an attempt to prevent students from developing negative feelings towards these problems, particularly in the earlier grades.

Significance

Ayanda says, *I really don't understand them*, (meaning word problems), and her emphasis is on the word 'don't'. By including the word 'really' and placing the emphasis on the word 'don't' she makes her apparent inability to understand the problems significant. She has made an 'odd' connection between equations and word problems (word problems are equations given in words), especially in her phrase *it's sort of like they give you an equation…*, and this serves further to make it significant that she doesn't understand them, but also that she wants to understand them. She emphasises this when she says *but I really try* and *I tried my best.* Roxanne's repetition of reference to 'the easy problems' and 'the difficult ones' (lines 78g – i and 80d – g) suggest that the level of difficulty of the problem is significant for her.

Experiential level

The students view word problems as being a mathematical thing in words or in the form of a story and some of their talk suggests that these beliefs stem from the primary school classroom (Christou and Philippou, 1998b). It is interesting that none of the students makes any connection between word problems and real-world scenarios. However, the format of the problems appears to make them difficult for the students and to an extent this increased complexity forces the students to move beyond possibly more comfortable intuitive approaches because they no longer 'work' (similar to the work discussed by Mulligan, 1992).

Identities

Ayanda positions herself as not being able to do word problems, but that she nevertheless is persevering, somehow, towards mastering them. This 'somehow' is backgrounded, and is only evident through Ayanda's references to how she "tries" to master them. Roxanne sees herself as being more in command, enjoying the easier ones and being challenged by the more difficult ones, but there is still an element of her struggling when she *misreads* or *doesn't understand* (line 77j – k) the problem. Lerato, who doesn't contribute much throughout the discussion, seems to draw on what Roxanne has said to distance herself from word problems.

Relationships

Ayanda says *they give you* (line 77d), which suggests an 'us and them' perception of the players involved in word problems. It seems that the use of 'you' in this context is referring to the students in general, and 'they' is also used more generally to refer to some authoritative, but unknown body from the mathematical community. In any event, Ayanda appears to see herself as subject to the 'they' to whom she refers. Roxanne uses the passive voice (*word problems are* (line 78b), *by putting it in a story form* (line 78d), *and having to solve the problems and answer the required question* (line78e – f)). By doing this she backgrounds any person, body or entity that set and issued the problems, and it is particularly interesting that she also doesn't acknowledge any specific person or people who do the *solving* and *answering of the required question*. Nevertheless, she implies a sense of 'those who give' and 'those who do', and that word problems are somehow important in the domain of mathematics with a definitive statement, *word problems are sums given in mathematics* (line78b – c).

Existential level

Of the three girls that feature in this excerpt, twohave given a sense that they are both embarked on a journey towards mastering word problems, and that there is some onus placed upon them to accomplish this, but it appears that Lerato is not in this journey. Within this scenario both girls seem to acknowledge a body of students who are expected to master the problems and some 'more authoritative' body who devise and issue the problems. There is no tenable link evident with anything other than 'mathematics' in what these two girls say.

Political level

Ayanda attaches worth to word problems in that she feels she needs to *really try to understand them* (line 77i – j) and *keep on learning* (line 77l). There is a sense that without these word problems she will be disadvantaged in some way, although this is not made explicit. Roxanne appears to place an import on word problems through keeping the stake-holders anonymous. This has the effect of placing word problems in some ontological realm (i.e. they exist 'out there'), and the doing of these problems becomes some necessary (possibly universal) function in life. Furthermore, she recognizes that the problems can challenge her *level of thinking* (line 80g) and that this is something desirable, but she meets with a little friendly peer ridicule when she states this.

Discussion

There are two views of the 'players involved' in word problems – one suggests an 'us-v-them' whereas the other suggests an 'ontological-v-experiential', or 'what is out there'-v-'what is experienced'. Within both perceptions there is a sense that word problems are something that are unavoidable, but nevertheless worthwhile, and worth pursuing. However, none of the girls in this excerpt make any direct links with how word problems fit into the bigger picture. Ayanda simply gives a description of what she thinks they are whilst Roxanne says they *are given in mathematics* (line 78b), thereby implying some import that they have in and of themselves.

The next excerpt follows a discussion about their experiences in other subject areas, and here they turn their attention back to mathematics (and science).

Stanza 21

87. Roenel
a Without maths,
b I don’t see you going any further…
c I seriously don’t.

88. Roxanne
a …because maths and science
b and the question
c you know
d most things,
e I mean
f you know.
g But one thing they can really help with
h is logical thinking.

89. Roenel
Ja.

Stanza 22

90. Roxanne
a Coz
b they give you a problem
c to do
d and you know
e when to solve it
f you know,
g you ..
h even if you don't know how to solve it
i but you can just find clues
j and one of the,
k what the chances are the first step
l which I don't think is right
m is probably the correct one [laughs].

91. Ayanda
a Mmmm.
b Ja,
c I know the feeling.
d Never complicate things,
e write what you think.

Stanza 23

92. Roenel
a You know//, 00:11:40
b I liked it with trigonometry\\,
c coz they just gave you an example//…
d you know
e the main story
f and all that\\...
g Then you just take this down//,
h putting them into numbers//…
i and the calculator
j and all that…

93. Roxanne
a [Interjecting] But the trigonometry word problems 00:11:50
b are easier than the algebra ones\\.
[Lerato sighs audibly]

94. Roenel
a They are much easier. [Lerato? Says something about “think” which is inaudible.]

Description

Sign and knowledge systems

Roenel in line 87 turns the discussion back to one about mathematics by raising a socially accepted perception of the importance of mathematics for future success (Setati ?). Roxanne, in line 88, also picks up on a cliché that has come to be associated with learning mathematics, which is the ability of mathematics to develop logical thinking. In the discussion Roxanne, Ayanda and Roenel display a tacit common understanding of the notion of 'a problem' in mathematics (and science). However, whilst they make distinctions between what a word problem might be in 'ordinary' mathematics as opposed to trigonometry (lines 91 – 93), they do not expand on what they understand a word problem in science to be. Roxanne and Roenel (in lines 90 and 92 respectively) are able to hone their descriptions of intricate tasks within the problems through utterances such as, *finding clues* (line 90i), *the first step* (line 90k) and *putting them into numbers* (line 92h).

Connections

Through their talk about mathematics as a gateway subject (Roenel in line 87) and its ability to help with logical thinking (Roxanne in line 88g – h) these students give mathematics exclusivity over other subject areas. Roxanne, however, brings mathematics and science under one umbrella when describing how she solves problems (lines 88 and 90), but she does not specifically refer to *word problems* in this context. Roenel appears to see word problems as being distinct from trigonometry problems. We see this when she talks of trigonometry word problems in which she does not use the expression 'word problems', but rather describes them as *story problems* (line 92e). Roxanne also seems to make a distinction between the two types of problems, but she says, "…the trigonometry word problems are easier than the algebra ones" (line 93a – b), thus acknowledging that they both belong to the category of *word problems*, but that they are nevertheless in some way fundamentally different from each other.

Significance

By including the phrase *I seriously don't* (line 87c) Roenel foregrounds that she 'buys into' the importance placed on mathematics as a gateway subject. This can be seen especially in that it is uttered after she has stated that she sees no future without mathematics, which serves to give her statement more import.

Experiential level

From what Roenel and Roxanne say in lines 87 and 88 it seems that they place quite a high importance on doing and being successful at mathematics (and by implication, word problems), which suggests that an extra pressure is present in that they are obligated to achieve in the subject (Setati,). The tacit understanding of the notion of *a problem* in science and mathematics appears to enable the students to engage in quite a rich discussion of the problems.

Identity

There is some evidence of a lack of self-confidence in the students when it comes to solving word problems. Roxanne says, "…what the chances are the first step, which I don't think is right is probably the correct one" (line 90k – m), and Ayanda positions herself similarly when she, agreeing with Roxanne says, "Write what you think" (line 91), whilst Lerato remains outside of the discussion.

Relationships

Roxanne says, "They give you a problem to do" (line 90b) and Roenel says, "They just gave you and example" (line 92c). Through these statements these two students bring into the picture some body of people within the mathematical Discourse (even though this is backgrounded) that somehow *just* (possibly arbitrarily) assign these tasks. Roxanne and Roenel also appear to identify with each other in their view of trigonometry and 'ordinary' word problems, as well as that the trigonometry word problems are 'easier'. Roxanne laughs at the end of her acknowledgement of how she was unable to know whether or not her first step was right (line 90 k – m), which suggests that she finds divulging this information discomforting. However, Ayanda collaboratively says, "Ja, I know the feeling…" and through this she is able to acknowledge that Roxanne has shared a personal experience, and at the same time offer some solidarity through her expression of what she feels are similar experiences.

Existential level

From having experienced the writing down of the 'first step', and thinking that it was incorrect, then finding that it was correct, Roxanne comes to have a lack of confidence in her ability to judge when she is right or wrong, and through her expression of this it appears to extend to Ayanda and Roenel (Lerato remains quiet on the issue). This probably stems from these students having struggled with word problems in the past. However, in another sense it suggests that they are building up a self-confidence in that they now seem able to make judgements about the validity of the mathematical statements that they come up with, even if this is only retrospectively.

Political level

When the students do talk about *they* or *them*, referring to ‘those who assign the problems’ it continues to carry a vague connotation (and sometimes no connotation) about the purpose behind the problems (Davis, 2003). The purposes that the students make explicit about word problems occur in the context of mathematics as a whole: it helps to develop logical thinking (line 88h) and without it one is not going to succeed (line 87a – b). Yet there is a strong indication from Roenel and Roxanne that mathematics is a gateway subject and that without it one is in some way ‘disadvantaged’, and these two students background the notion that word problems serve some purpose in the ‘real world’.

Discussion

When Roenel says, “Without maths I don’t see you going any further…” (line 87a –b) there is an underlying hint of anxiety brought to the fore. In the next line Roxanne seems to attempt to explain this importance of mathematics, but she is only able to come up with the ability it has to develop logical thinking (line 88g – h), whilst the balance of the excerpt reverts to the difficulties they experience in solving these word problems in comparison with the likes of science and trigonometry word problems. So whilst the students are able to talk in more detail about what they do with word problems, they appear to have a very superficial understanding of why they do these problems, or in what way these problems could benefit them, which brings us back to the relevance that these problems have in the classroom (Davis, 2003; Gerofsky, 1996).

In this next excerpt Roxanne describes how she attempted problem 1. As an exordium to this, Lerato had requested, “Lets… work out the first one together” (line 98c), to which Roxanne had responded, dropping her head onto the desk in an agonizing gesture, “I don’t want to look at the first one again” (line 100). Ayanda, however, took up Lerato’s request and started talking about *length times breadth* (line 101), to which Roxanne responded:

Stanza 25

102. Roxanne
a I know
b. the area is length times breadth\\, [talking in the background] 00:12:26
c but then the,
d the length is x plus two\\,
e right\\,
d. and then it is x plus two\\
e. but in fact//
f. they increase it by two\\
g. so we have it x plus four//.
h. And then the other one it stayed\\…
i. it was x alone\\
j. and then they decreased by one metre\\,
k. so it's x minus one\\.
l. And then if you put that in an the equation
m. it's x squared minu… plus 3 x minus… 4// [x^2+3x-4],
n. if I'm not mistaken\\.
o. Yeah,
p. and then… this answer is just WRONG\\... [Roenel shakes her head]
q. you don't get such answers in umm [slight choking, slumps forward] maths [chuckling].
r. But I just love the algebra\\...
s. the factorisations\\... [All talking together]

Description

Sign and knowledge systems

The formula for the area of a rectangle appears to be a very familiar one for Roxanne (line 102a – b), but she is able to talk about the relationships given in the problem quite eloquently, giving a paraphrasing of the question followed by an equivalent mathematical expression. For example, she says, "… and then it is x plus two, but in fact they increase it by two, so we have x plus four" (line 102f – g). Initially, Roxanne omits the assigning of x as the breadth and jumps straight into, "the length is x plus two" (line 102d) which suggests that she assumes that her peers have a similar understanding. She also does this in her working (see Roxanne – problem 1). She arrives at the expression x^2+3x-4, having done the multiplication, and again this assumes that the other students have followed what she has done. What is more noteworthy here, however, is that she calls this expression *the equation* (line 102n). She then concludes that it must be wrong (line 102r), apparently because in the context she perceives it to be out of place, or, as she stated when I spoke to her in the focus group discussion, it just *seemed wrong*.

Connections

Roxanne shows that for her the connection between the area of a rectangle and the notion *length times breadth* (line 102b) is very strong. Once she has come up with what she calls *the equation* (line 102n) she cannot proceed any further, and as a result makes reference to *factorisations* (line 102u). It appears that, because she has omitted the other side of the equation she is confronted with a quadratic trinomial, with which she seems to make a connection with factorising. It may have been that in the absence of the right hand side to the equation the only task open to her was to factorise the trinomial. It is also possible that having come up with a quadratic Roxanne might have not have been familiar with this in the context of word problems, causing her to assume that there must be an error on her part.

Significance

Roxanne foregrounds that she is familiar with the formula for the area of a rectangle through her expression, and emphasis in line 102a. Because she has launched into her explanation of doing problem 1 with "the length is x plus 2" (line 102d) she makes it significant that this is also increased by two by using the interjection "but in fact" (line 102g), which serves to differentiate between the length being two more than the breadth and the length then being increased by two, as given in the problem. Roxanne also later foregrounds that she thinks that the answer is wrong by her emphasis and the shaking of her head (line 102r) which possibly tells us that at that point (when doing the problem) she was unable to proceed. This also appears to be the case in her working.

Experiential level

From her talk and what she has done it seems that Roxanne is able to function algebraically at quite a sophisticated level. This comes through in her proficient articulation and in what she does with the problem, even though this is incomplete and with errors, but it is also evident in the manner in which she is able to omit steps (such as the assigning of the unknown, and the multiplication of $x + 4$ by $x – 1$). When Roxanne arrives at $x^2 + 3x - 4$, she is looking for an equation, but she is only able to make a connection with factorising because it is *not* an equation, and thus she is unable to proceed. Furthemore, she has now verified all the preceding algebra (both through her explanation and by assumed peer analysis) and she is unable to find an explanation as to why this is *wrong*.

Identity

Initially, Roxanne comes across very confidently mathematically, when she says, "I know…" (line 102a) and she maintains this air until she confronts what she calls *the equation* (line 102n – o). At this juncture she becomes hesitant stating, "if I'm not mistaken" (line 102p).

Her insecurity becomes more evident in her utterance, “this answer is just wrong” (line 102r), together with her chuckle in line 102s, suggesting that she has lost all confidence in her working at this point. However, we see in lines 102s – u that Roxanne seems to have confidence in her abilities with algebraic manipulations despite the difficulties that she has confronted in this problem.

Relationships

Roxanne responds to Ayanda with, “I know… [the formula for area of a rectangle]” (line 102a – b), but later she says, “…if I’m not mistaken” (line 102p). On the one hand she appears to be pushing aside (almost as trivial) that with which she is familiar, whilst on the other hand she reaches out for guidance from her peers when she confronts uncertainty (as she does when she finds *the equation* in line 102l).

Existential level

Roxanne’s confidence in her mathematical manipulations seem well grounded – she shows competence in setting up relationships from the problem. However, in line 102 she seems to use this opportunity to verify what she did when she first tackled the problem, and is then incredulous when she still finds that she is ‘wrong’. She says, “if I’m not mistaken” (line 102p) which seems to be an appeal to the others to also verify what she has done. The incredulity seems to set in because she believes that all her working is correct, yet she is still unable to solve the problem.

Political level

The importance of mathematics appears to be implicitly present in what Roxanne says in line 102t – u. By reassuring herself about her love for algebra she secures herself in the knowledge that she is included in the mathematical Discourse, despite her inability to solve the problem at hand.

Discussion

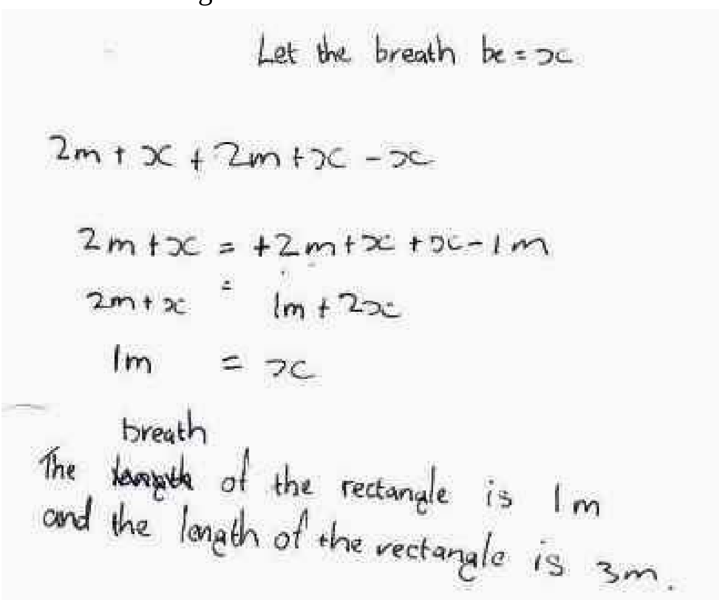
At the experiential level, Roxanne seems to think that she has done everything ‘right’, yet she finds herself in a position where she does not know how to proceed. This appears to ratify her talk about the difficulties that she experiences with word problems, and in an attempt to negate this apparent ineptitude she turns to an aspect of algebra where she is confident, *viz.* factorisations. Furthermore, her methods seem valid (both to her and her peers), yet what she arrives at *seems wrong*. Mathematically she is not that far off the mark (she needed to find another expression for the area and equate the two), but her loss in confidence appears to turn her focus to aspects of mathematics where she does experience success.

Conclusions

With the exception of Lerato, whose working demonstrates very little access to the Discourse surrounding word problems, we see evidence that the students make progress with the problems to some extent. From the student working we see that through being able to employ some sort of method or procedure the students come to be able to engage with the problem, especially with the television rental problem (problem 2), albeit that the actual working is not necessarily correct mathematically. In addition to this, we see from the student discourse that they are able to engage in discussions about the problems fairly meaningfully because of such factors as having a common understanding of what an equation is, even though this common understanding is very much assumed amongst the students. What also emerges from the discussion is that the students have some sort of common understanding of the perceived worth of word problems, even though they seem not to be able to verbalise exactly why this is seen to be. However, it is evident from the inability to clarify certain concepts (such as what an equation is) and from the partially correct mathematics that access to the Discourse surrounding word problems is ‘hard’ for all these students.

Duskhaven – The analysis of the student working

Problem 1 – The rectangle problem
Tseko draws a rectangle with its length 2 m more than its breadth. He then increases the length by 2 m and decreases the breadth by 1 m. He finds that the area of the new rectangle is the same as that of the first one. Find the length and breadth of Tseko’s first rectangle.


Let the breath be = x

$2m + x + 2m + x - x$

$2m + x = +2m + x + x - 1m$

$2m + x = 1m + 2x$

$1m = x$

The ~~length~~ breath of the rectangle is 1m and the length of the rectangle is 3m.

Description

Sipho – problem 1

1. *Understanding the problem situation*

The text demonstrates some evidence of understanding of the relationships described in the problem. For example, “2m + x” seems to indicate that Sipho recognised the increase in length by two metres, whilst “x – 1m” similarly suggests that he understood the decrease in the breadth by one metre. The text does not reflect the area of rectangles at all, but it may have been the case that Sipho’s calculations of adding what appears to be length and breadth for him represented area of the rectangle. If this is so, then the text shows that Sipho had some understanding of the areas of the initial rectangle and the resulting one being the same size.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

The text explicitly shows an assigning of x as the breadth of the rectangle. Following this the text suggests that Sipho attempted to use the assigned unknown to build expressions, which in line 3 he seems to put into the form of and equation in which he can solve for the unknown. The text therefore shows an algebraic procedure.

Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

Although parts of the expressions in the text reflect possible correct mathematics (e.g. 2m + x and x – 1m) the expression in line 2 and the equation that follows are incorrect, and it appears as though Sipho is using $l+b$ instead of $l \times b$ for the area of a rectangle. This leads to the incorrect conclusion that the breadth is one metre, but from there Sipho correctly concludes that the length is two metres more, and arrives at three metres.

Grading 1: Some evidence of correct mathematics.

Gio – problem 1

1. *Understanding the problem situation*

The text indicates that Gio probably understood that the problem involved a rectangle. In the first line it suggests that he understood that there was an increase in the

$$\begin{cases} \text{length } 2m + \\ + 2m - 1m \end{cases} \qquad (+ \; x - = +)$$

$$2m + 2m - 1m$$
$$= 4m^2 - 1m$$

the length is $4m^2$
" breadth is 1m

length of two metres, but he seems to have attributed the decrease in one metre to the length as well. There may be some connection between the notion of area and Gio arriving at $4m^2$, but this is not made explicit. There is no indication in the text that Gio has recognised two different scenarios of the rectangle or the equality of the areas.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

From the text it is difficult to discern any apparent method that Gio might have been using. From the working in line 2 it seems that Gio is trying to apply what the problem gives to an assumed original length of two metres. From the bracket containing "$+\times-=+$" it is possible that Gio is trying to apply rules that he has learnt. It does seem as though he is setting up the calculations that he generates by using key words from the problem description (Hegarty, Mayor and Monk, 1995; Mangan, 1989).

Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

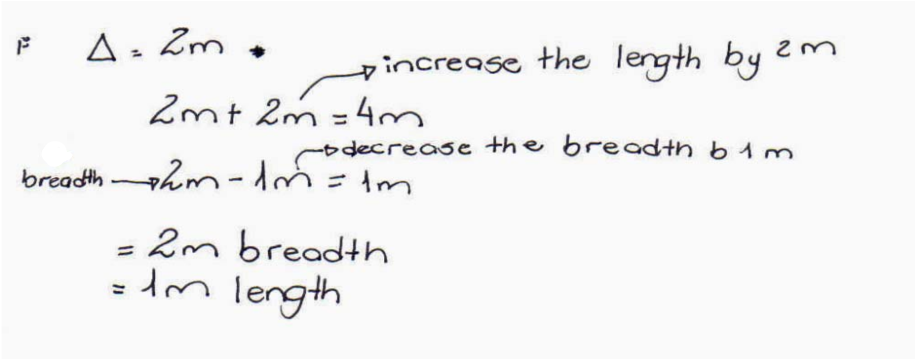
The text reflects incorrect and inappropriate mathematics. In the initial working (braced on the left) it seems that $2m$ has been assigned to the length, to which two metres has been added and then one metre subtracted. The text next to this seems to indicate "a positive multiplied by a negative gives a positive", and if this is the case Gio has recalled this rule incorrectly. In the third and fourth lines the text suggests that Gio has added $2m+2m$ and arrived at $4m^2$ which is algebraically incorrect. The conclusion uses the incorrect unit of length and it appears that Gio has taken the length and breadth from the terms of the expression he had in line 4.

Grading 0: No evidence of correct or appropriate mathematics.

Nina – problem 1

1. *Understanding the problem situation*

The text in the first line shows what appears to be the letter F to the left which possibly indicates that Nina was referring to the first rectangle, but she then uses the symbol for a triangle. However, elsewhere in the text she refers to “length” and “breadth” so it seems safe to conclude that Nina recognised that the problem was dealing with rectangles. Whilst the text suggests that Nina has understood the increase in the length and the decrease in the breadth, it appears as though she has interpreted the length to be two metres, seemingly misinterpreting “two metres more than”, and then in line 3 the breadth is indicated as being two metres as well. However, it is not clear from the text whether this is the case, since it is possible that in the first line she assigned two metres to be the breadth. The conclusion that the breadth is two metres and the length one metre does not appear to stem from Nina’s working and this may be a misinterpretation of “the length is two metres more than the breadth”. There is no indication in the text that Nina has recognised that the area of a rectangle was involved, nor that there was any equality of areas.



Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

The text shows that Nina has simply applied the wording describing the change in dimensions to the values that she has come up with for the length and breadth, and it is possible that she is also using key words as indicators of what she should be doing (Mangan, 1989).

Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

In the opening line Nina has written $\Delta = 2m$ in which she may have incorrectly used the symbol “Δ” to mean “rectangle”, or there is the possibility that she might be using it as a place-holder. However, from the next line it appears that Nina has incorrectly interpreted the length of the rectangle to be two metres, whilst in line 3 it seems that she has also incorrectly interpreted the breadth to be two metres. The text reflects correct application in the increase of length and the decrease of breadth. It is not clear from the working whether Nina arrived at her answers from her previous steps, in which case it seems that she may have taken her answer from the decreased breadth to be the length of one metre, or whether she perhaps used the misinterpreted relationship between the length and breadth. Interestingly, she has also come to a conclusion which reflects a length that is smaller than the breadth.

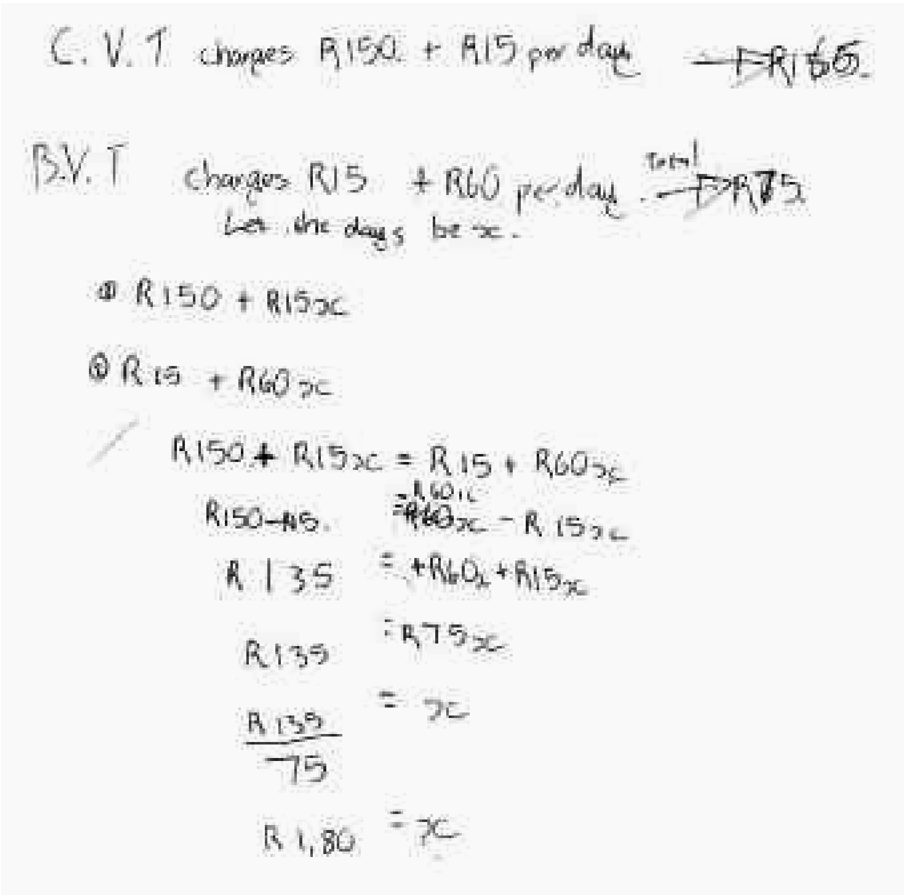
Grading 0: No evidence of correct or appropriate mathematics.

> Problem 2 – The TV rental problem
> The Clear Vision television rental shop charges a basic fee of R150, as well as R15 per day to rent a television. The Best View television rental shop only charges a basic fee of R15 but has a daily rate of R60 per day to rent. For what number of days would it make no difference in cost as to which shop you rent from?

Sipho – problem 2

1. *Understanding the problem situation*

The text in the first two lines shows that Sipho has a clear understanding of the cost structure for both shops. However, despite his statement in line 3, "Let the days be *x*", at the end of his calculation he includes an 'R' making his answer for x an amount of money. This is not to say that Sipho did not understand the problem question, but rather that he probably lost sight of it, either because of the monetary units throughout his calculation or because of the decimal nature of his answer.


C.V.T charges R150 + R15 per day → R165

B.V.T charges R15 + R60 per day total → R75

Let the days be x.

① R150 + R15x

② R15 + R60x

R150 + R15x = R15 + R60x

R150 − R15 = R60x − R15x

R135 = +R60x + R15x

R135 = R75x

R135/75 = x

R1,80 = x

Grading 3: Evidence of correct interpretation of all aspects.

2. *Use of method or procedure*

In the third line the text suggests that Sipho perceived the need to use an unknown and he introduced x. He then set up two expressions and equates them. It is clear that an algebraic approach has been used and this appears to guide Sipho in the solution process.
Grading 3: Evidence of clear methods and procedures that are systematic.

3. *Use of correct mathematics*

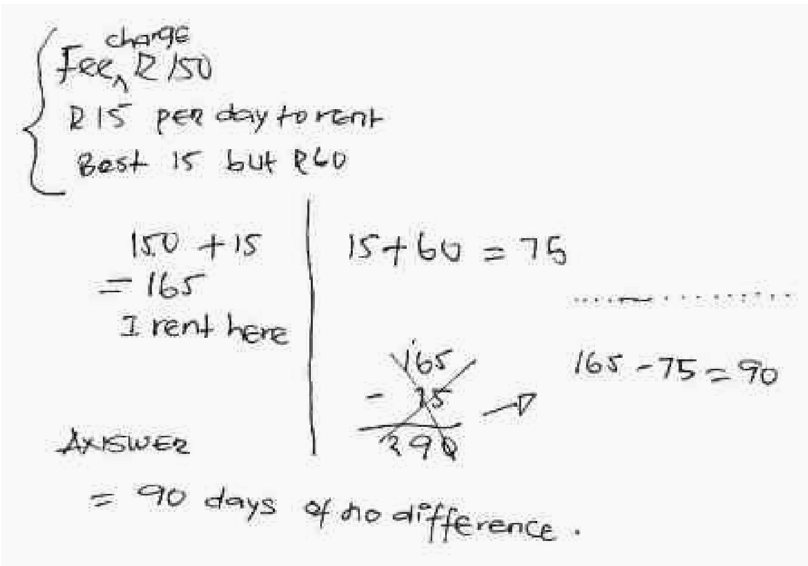
The mathematics is correct except for an incorrect sign in the second line of the equation. This resulted in a non-natural number which may be the cause for the possibly misconstruing the final result.

Grading 3: Text is fully correct and appropriate and leads to a solution.

Gio – problem 2

1. *Understanding the problem situation*

In the first three lines (braced to the left) it appears as though the problem situation is summarised, but from the tabulated calculations it seems that Gio has only understood this in the context of one day. This we see from his comment, "I rent here", which suggests that Gio perceived this to be the answer that the question sought. However, the way in which the question was phrased seems to have prompted a misunderstanding for Gio which we see from him attempting to get an answer for the phrase "no difference". The answer that he gives is "90 days of no difference", which it seems he gets from the *difference* between the two costs that he arrives at in the tabulated calculations.



Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

The text shows a summary of the problem information followed by tabulated calculations of that information, but which are not taken any further. Within the calculation to the right Gio has represented a series of dots, and three of those have been joined or crossed off. This appears to be some attempt to make sense of the information that he has at his disposal, and may be something like ‘the number of days’ in the problem context, but this is unclear. The answer appears to have been gleaned from the calculations performed in before.

Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

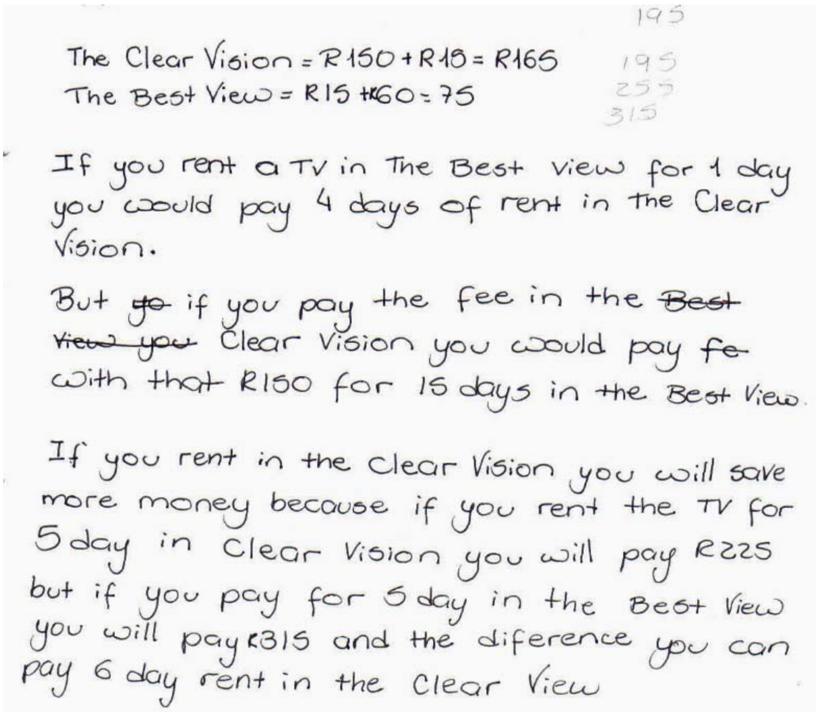
The mathematical working is correct for one day of rental form each shop. However, a solution is not possible from this alone. In addition to this, the difference in the costs for one day of rental is not relevant in the context of the problem question.

Grading 1: Some evidence of correct mathematics.

Nina – problem 2

1. *Understanding the problem situation*

From the fourth paragraph we can see that Nina has a very good understanding of the problem situation as she is able to correctly calculate the costs at each shop for five days of rental. Furthermore, the text shows that Nina is aware that the Best View shop will be cheaper under certain circumstances, whereas the Clear Vision shop will be cheaper in



195

The Clear Vision = R150 + R15 = R165 195

The Best View = R15 +R60 = 75 255

315

If you rent a TV in The Best view for 1 day you would pay 4 days of rent in the Clear Vision.

But ~~yo~~ if you pay the fee in the ~~Best View you~~ Clear Vision you would pay ~~fe~~ with that R150 for 15 days in the Best View.

If you rent in the Clear Vision you will save more money becouse if you rent the TV for 5 day in Clear Vision you will pay R225 but if you pay for 5 day in the Best View you will pay R315 and the diference you can pay 6 day rent in the Clear View other instances, and this we see emerging in the discussions in paragraphs 2 and 3. So we can conclude that Nina was probably aware of some break-even point in the problem, but she does not make this explicit.

Grading 3: Evidence of correct interpretation of all aspects.

2. Use of method and procedure

The first paragraph seems to summarise the rental costs for the two shops. Thereafter, the text reflects and investigative approach in which Nina examines different scenarios to gain insight into what is happening. Whilst her investigation informed her of certain trends at each of the shops it lacked the systematicity needed to identify the correct answer. It seems that, had Nina continued to investigate further, she may have stumbled upon the correct answer.

Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

The calculations are all correct for the problem situation.

Grading 3: Text is fully correct and appropriate and leads to a solution.

> Problem 3 – The speed-distance-time problem
> A boy cycles from home to school in the morning and back in the afternoon. He cycles from home to school at 32 km/h and back at 24 km/h. It takes him 15 minutes longer in the afternoon than in the morning. Find the distance between home and school.

Sipho – problem 3

distance = $\frac{\text{speed}}{\text{time}}$.

Let the distance be x.

$$x = \frac{32\text{km/h} + 24\text{ Km/h}}{15}$$

$x = 3.73$ km to from schoo to home and back.

1. *Understanding the problem situation*

In the text the formula is given in the first line and this suggest that Sipho might have recognised the type of problem that he was dealing with (Gerofsky, 1997). However, from line 3 where the information from the problem is substituted directly into the formula, it seems that Sipho has taken no account of the differences between the two journeys. However, from the subject of the formula, and from Sipho's answer it is apparent that he understood the problem question.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

From the text we see that the formula is used. An unknown x has been assigned in line 2 and this appears to be a taught strategy since there was no need to do this in terms of Siphos' working because he simply renamed "distance" in the formula. In line 3 it seems that Sipho is using the formula to make direct substitutions of the information given in the problem rather than using the formula to establish relationships *auxiliary relationships* that would enable an equation to emerge (Lepik, 1990).

Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

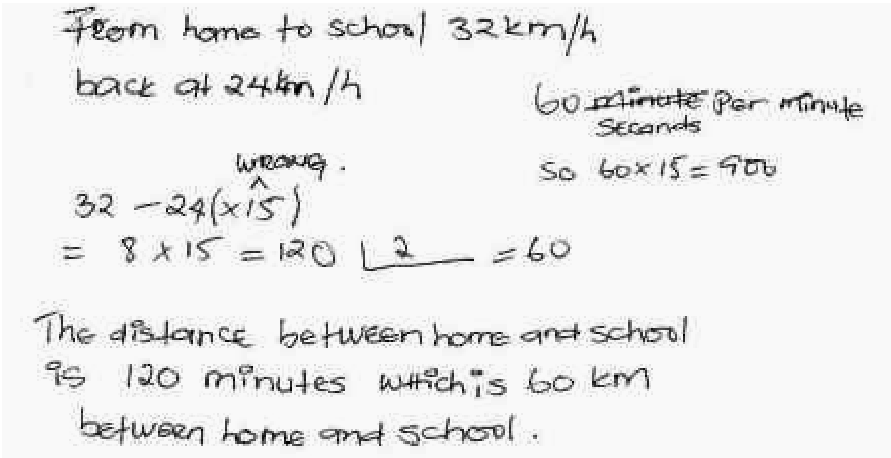
The formula has been recalled incorrectly and substituting the values into the formula as Sipho has done, presumably trying to establish some sort of average (De Bock, 2002), is also incorrect. In addition to this Sipho has not taken the units into account when using the extra time of 15 minutes.

Grading 0: No evidence of correct or appropriate mathematics.

Gio – problem 3

1. *Understanding the problem situation*

From the first two lines of the text it appears that Gio has recognised two different journeys as described in the problem. In the working in lines 3 and 4 it seems that Gio recognised something wrong with his working with 15 minutes. It is not clear at what stage he realised this or what it was about this operation that worried him, but from the calculation to the right it seems feasible to conclude that Gio realised that the 15 minutes needed to be converted in some way. From the answer given in the text it is apparent that Gio understood the problem question. We see this from the way in which he arrives at what he deems to be a time, and he then converts this to a distance since he seems to be aware that he is looking for a distance.



From home to school 32km/h
back at 24km/h

60 ~~minute~~ seconds per minute
So 60×15 = 900

wrong
32 − 24(×15)
= 8 × 15 = 120 ⌊2 = 60

The distance between home and school is 120 minutes which is 60 km between home and school.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

A first look at line 3 of this excerpt from the text suggests that Gio was using the formula for speed-distance-time relationships. However, his answer turns out to be 120 minutes, so it is more likely that he was not using the formula but rather setting up a relationship from his own interpretation of the situation. From the text it is not clear why Gio divides his answer by two in line 4, but it is possibly intended to aggregate in some way (De Bock, 2002).

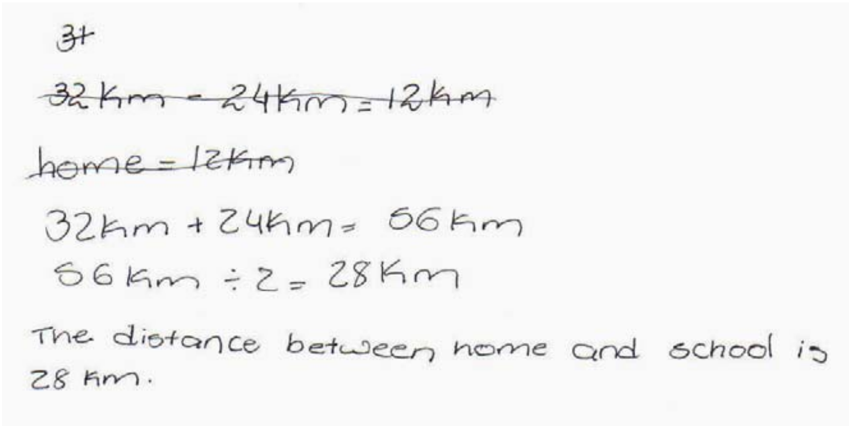
Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The text to the right shows a conversion of 15 minutes, but Gio converts to seconds instead of hours. In line 3 the calculation of speed times time would be correct in finding a distance (notwithstanding the units, or the incorrect time), but Gio has first subtracted the speeds. Gio’s answer then shows the unit to be minutes, meaning that he sees this as a time, and when he divides this by two, he deems it to be a distance (which we see from the use of "km" as the unit). The mathematics is inappropriate to the problem.

Grading 0: No evidence of correct or appropriate mathematics.

Nina – problem 3


3+
~~32km - 24km = 12km~~
home = 12km
32km + 24km = 56km
56km ÷ 2 = 28km
The distance between home and school is 28 km.

1. *Understanding the problem situation*

It appears from the use of kilometres as the unit in the text that the problem has been misinterpreted. From the calculation the speeds given in the problem appear to have been interpreted as distances, and because these distances are two distinct values it appears also that Nina has not recognised that the two journeys covered the same distance. There is no evidence to show whether or not Nina understood the difference in time of 15 minutes, but it seems that she has understood that the problem question required an answer that was a distance.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

In the initial attempt Nina appears to have taken the required distance to simply be the difference between the two, which we see from the statement. "home = 12km". However, this has been crossed out and the text that follows appears to average the two distances by adding them and dividing by two.

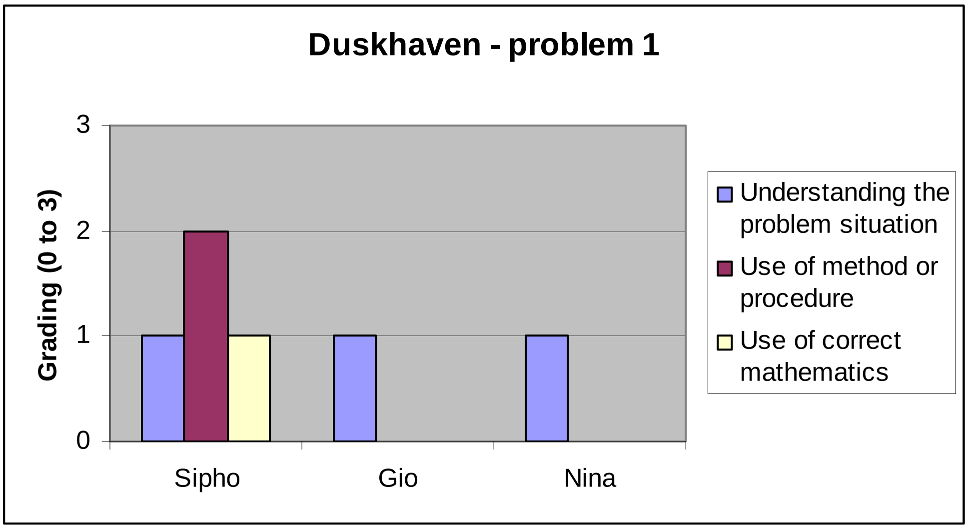
Grading 1: Some evidence of method or procedure is present in rudimentary form.

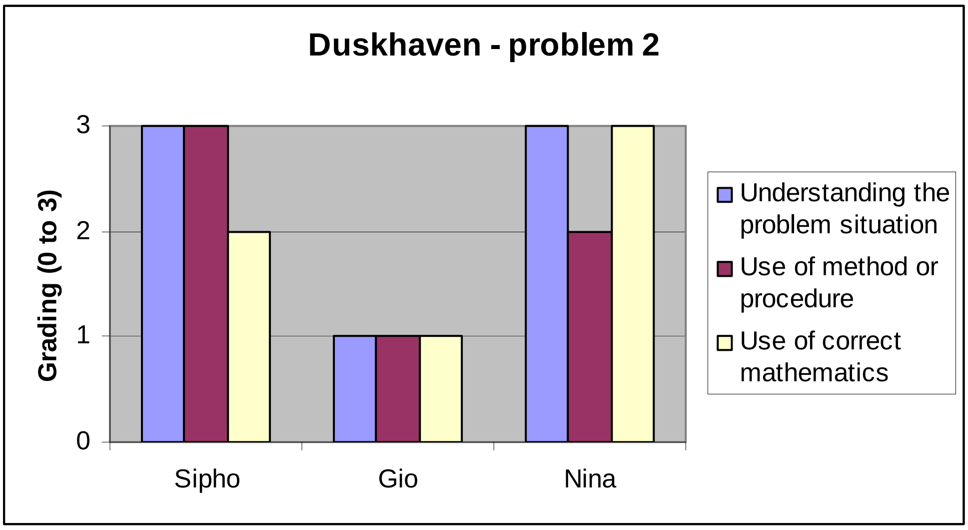
3. *Use of correct mathematics*

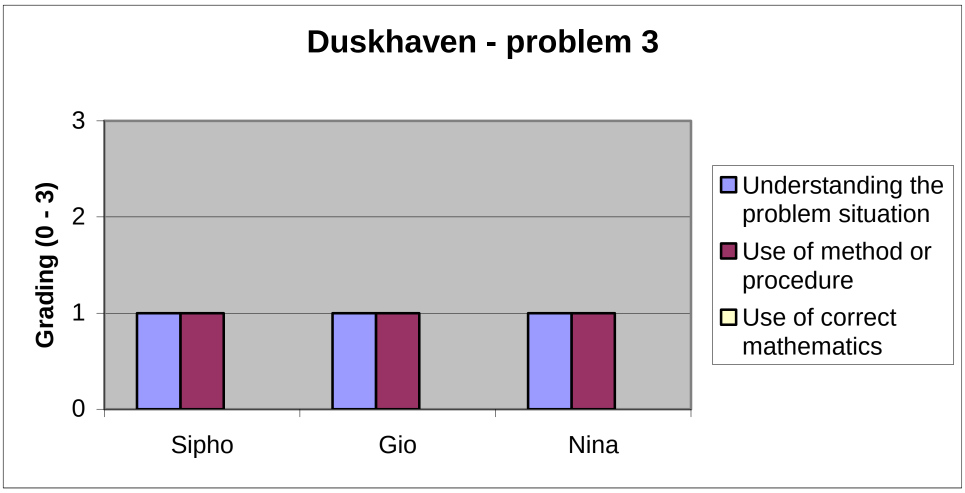
The misinterpretation of the speeds as distances makes the calculations inappropriate to the problem and it also appears that Nina has employed inappropriate linear and proportional reasoning (De Bock, 2002), firstly by adding the speeds and secondly by aggregating her result.

Grading 0: No evidence of correct or appropriate mathematics.

A summary of the three dimensions of student activity for Duskhaven







Discussion

Two main trends in students' activity tend to manifest at Duskhaven. Firstly, in some cases there appears to be some method or procedure that the students allude to, but the understanding of the problem situation is insufficient or possibly flawed in some way, and this appears to inhibit the student progress with the problem. Secondly, there are instances where the students do not have access to, or have only a superficial access to a method or procedure, and this appears to disenable the students, especially so when this is accompanied by a more superficial understanding of the problem situation.

The graphs clearly show that these students were much more successful with their attempts at the television rental problem (problem 2), but struggled much more in all respects with the other two problems.

Duskhaven – The analysis of the student talk

In the following excerpt Sipho is relating how he attempted problem 3. This follows an attempt to explain their workings by Nina and Gio, in which Nina said, "I was just guessing, I don't know." (line 53) and Gio stated the following:

> No, I did, I did one, one minute per 60 seconds, right? So I did 60 times 15 is... 900. So I took the 32 minus 24, which gave me 8, times twenty… (line 54).

Neither Nina nor Gio had managed to make any meaningful progress with the problem when Sipho offered the following:

Stanza 18

63.	Sipho	a	Okay.//	00:04:35
		b	What I did ..is, umm	
		c	totally different	
		d	to what you guys did.\\	
		e	You know, umm…	
		f	you take ..distance,	
		g	distance is equals to	
		h	speed divided by time...	
64.	Gio	a	Speed, dis…	
		b	you're talking about science	
		c	here,	
		d	right?// [Pointing at Sipho.]	
65.	Sipho	a	Yeah,	
		b	but still.//	
		c	Okay,	

d so distance is equal to
e speed divided by time,\\
f so I took, umm..
g 32 kilometres..
h and I plussed it with,
i umm ..the 24,\\ [Indicating by moving his pinky on his right hand up and down, elbow on the table]
j and <u>then</u> I took, umm\\... [At this point Sipho changes the focus of the discussion.]

Description

Sign and knowledge systems

Sipho seems to have recognised problem 3 as being a "speed-distance-time" problem and he appears to be familiar with problems of this type (Gerofsky, 1996). Gio, however, expresses surprise that Sipho is employing what he, Gio, feels is science to solve a mathematical problem. Sipho brushes this off with, "Yeah, but still." (line 65), which shows that he is probably quite comfortable in his solution strategy, even if it does seem to cut across subject areas.

Connections

By Gio's apparent surprise at Sipho's use of science we can deduce that Gio views mathematical problems and science problems as being in 'separate boxes'. Sipho appears comfortable with combining the two, even if the one is perceived as 'science' and the other as 'mathematics', or it may be that he did not really make any connection between his method and science until Gio questioned him. However, Sipho clearly sees a disconnection between what the other two have done and what he has done to solve the problem. He simply states that what he has done is *different* but he does not attempt to explain qualitatively how it was different, and thus this difference in approach is backgrounded.

Significance

Sipho foregrounds the idea that his solution is different from those of the other two by the use of the expression, "What I did... is, umm totally different…" (line 63b – c). At the same time, by the interjection, and the expression of surprise (line 64), Gio makes it significant that Sipho seems to be using science to solve the problem.

Experiential level

Sipho shows that he is familiar with the problem in that he is able to classify it as a particular type of problem (Gerofsky, 1997) and he is thus able to link it to some solution strategy. It is apparent that he is somewhat ‘rusty’ about the details, but he is able to make some progress as a result of his recognition of the problem as involving the speed-distance-time relationships. Gio, however, suggests here that his experience of the problem, having now seen Sipho’s approach, is almost as if it were a new encounter.

Identities

Sipho suggests that there is a certain autonomy by talking about the different methods used, particularly as he does not question or judge the methods used by the others except in that he acknowledges that they are *different*. He does state, however, that his method was *totally* different, thereby positioning himself almost in juxtaposition to Nina and Gio. His use of the word *totally* may give some impetus to the possibility that Sipho privately makes a judgement about the other two approaches to solving this problem, but quite clearly he does not see himself as employing that type of approach.

Relationships

Sipho established a particular identity by pointing out the *totally different* approach that he has used, and in this way he gives himself a certain amount of credibility within this group. He then refers to the formula for distance, something that in all probability he has drawn from the classroom context, but this serves to give him further credibility in this group, something which is confirmed by Gio’s surprise at his use of ‘science’. By Gio’s interjection (line 64) he foregrounds Sipho’s ‘different’ approach.

Existential level

Sipho appears confident with his approach except for one ‘niggle’ which comes through in the next excerpt. This confidence seems to stem from his recognition of the problem-type, and this is projected into the peer setting in such a way as to elevate him within the group.

Political level

Sipho’s response to Gio’s utterance, “You’re talking about science, right?” shows us that Sipho sees himself as been advantaged by this *totally different* approach. He brushes it off with the statement, “Yeah, but still…” (line 65a – b), showing that he acknowledges that the approach is *different*, and that it certainly still helped him, but in doing so he backgrounds the ‘different’ approach to allow a discussion of what he did with the problem.

Discussion

Sipho has recognised problem 3 to be a 'speed-distance-time' problem, and has made use of recall of taught procedures (viz. the formula) to tackle the problem. He begins by bringing attention to this by stating that his method is *totally different* (line 63c), to which Gio responds by asking if he is 'using science' (line 64b). The interesting thing is that through employing taught techniques he has managed to gain himself some standing within this group by seemingly using an 'out-of-the-ordinary' approach, or at least 'out-of-the ordinary' in the context of Gio, with Nina remaining quiet on the issue.

In the next excerpt Sipho turns his focus from a narrative of what he did with the problem to an analysis of an error on his part that he had alluded to earlier as *his mistake* during Nina's and Gio's accounts of doing the problem (line 56).

Stanza 19

66.	Sipho	a	My mistake was// [points to his paper with his right hand]
		b	I should have tooken,
		c	I should have made umm 15 [tapping his page with his right pinky]
		d	I should have con...,
		e	I should have converted it.\\
67.	Gio	a	Yes.
		b	15 should have been in...
68.	Sipho	a	... 15 should have been divided,\\
		b	should have been...
		c	what's 15?//...
69.	Gio	a	Should be times,//
		b	shouldn't it? [Gio looks at his answer. A long pause while they calculate]
70.	Sipho	a	15, 15, 15... mmm... [Sipho is looking at Gio and doing the calculation mentally]
		b	is equals to a <u>quarter</u>,\\
		c	divided by a quarter\\
		d	of umm 60 minutes,\\
		e	60 divided by...//
71.	Gio	a	So what's your answer?//
72.	Sipho	a	It should be 60
		b	divided by, umm,// [snaps his fingers twice]
		c	divided by <u>four</u>,\\
		d	and then this,
		e	<u>that</u> divided by... err,
		f	then you would have
		g	gotten your answer there,
		h	and then

i you take...
j umm\\... 4,\\...
k it'll be 4,//
l 4 divided by 32 kilometres
m and you'll get
n your distance
o that he had to travel.\\
p I think,//
q I'm not sure,\\
r but...
s I could have…

Description

Sign and knowledge systems

Sipho has identified his ‘mistake’, and it appears that he has recognised that he should have converted the 15 minutes into hours. He talks of the operation (division) that he should have performed when he did the question and seems to be grappling with how this conversion should have taken place. Gio tries to enter the conversation but it appears that Sipho is saying, “15 should have been divided…” whilst Gio says, “Should be times, shouldn’t it?” which in the context of what Gio did is to convert 15 minutes into seconds. It seems safe to conclude that Gio’s and Sipho’s conversations miss each other because Gio sees the conversion as being 15 times something (not realising that he is converting to seconds), whilst Sipho seems to be looking at the problem in terms of 15 minutes is what portion of an hour. As a result we see Sipho largely ignoring Gio’s input in lines 67 and 69.

Connections

There seems to be a connection that Sipho is making with these types of problems with respect to converting times that are in minutes, which we see through his repeated use of the phrase “should have”, which implies some kind of onus as a result of a pre-existing condition in the problem. The repeated use of the phrase also suggests that he is trying to recall a procedure for making this conversion. Once Sipho has tabled his problem with the conversion it appears that Gio also makes a connection with something from the classroom when he says, “15 should have been in…[hours?]” (line 67b). Neither of these boys makes any reference to why the 15 minutes needed to be converted into hours, which may provide further evidence that this might be a recalled procedure, or something that is simply ‘done’ to get you a result.

Significance

Sipho foregrounds the 'mistake' of not converting the 15 minutes into hours through his gestures: pointing to the page (line 66a), tapping the page as he describes what he should have done (line 66c) and later by snapping his fingers whilst trying to conjure up what to divide by (line 72b).

Experiential level

Both Sipho and Gio to some extent experience these problems in terms of something that has been taught in the classroom, which has a certain pre-determined 'method'. We see this through the recall of the formula (in Sipho's case) and how they both attempt to reconcile the conversion of the 15 minutes. It appears therefore that these two boys are responding to a 'type' of problem (Gerofsky, 1996) with its existing method of solution, rather than developing a particular solution strategy.

Identity

By foregrounding his 'mistake', Sipho draws Gio into the discussion, but Sipho clearly takes the lead in this discussion. This is evident on two occasions: through Sipho's interruption of Gio without any acknowledgement so that he can continue his line of thought (line 68), and again in line 70 where he completely ignores Gio's question, "Should be times, shouldn't it?" (line 69). Throughout this stanza Nina keeps completely to herself, which she further communicates by placing her chin on her hand, shifting positions and alternating hands from time to time.

Relationships

During this excerpt Nina keeps completely out of the discussion and she is not drawn in by either of the boys. Nina also seems to indicate that she does not wish to join the discussion by propping her chin in her hand and observing, shifting position from time to time and alternating hands. Sipho establishes a relationship with Gio by making eye contact, and by pointing at him during the discussion, which also seems to serve to keep Nina out of the discussion.

Existential level

In the context of this excerpt Sipho appears to see himself as almost having mastered the the problem, but for 'his mistake'. At the same time, we see Gio as only being a part of this discussion (in terms of his understanding of the need to convert the 15 minutes). In other words, Sipho appears to see the whole problem (not realising that his working with the speeds is erroneous, nor that his recall of the formula is incorrect), whilst Gio is only able to become a part of the problem through the limited context of Sipho's current focus on the conversion of 15 minutes.

Political level

Sipho repeatedly makes statements like, "I should have…" (line 66), "should have been" (line 68 and 72), whilst Gio uses similar expressions, "is should have been…" (line 67) and "Should be times…" (line 69). Both of these students appear to be trying to recall procedures that they have possibly been taught in the classroom and in this sense they allude to some accepted practice required by this type of problem.

Discussion

Gio's surprise at the use of the speed-distance-time relationship suggests that he does not have access to the Discourse of mathematics when it comes to problems of this nature. He appears to have come across it in 'science', but he does not seem to be familiar with the relationship in the context of mathematics. His own solution strategy, and his talk about his working further suggest that he is trying to make sense of the problem mathematically by associating the given numbers in any way that he can. He also seems to be unable to ascribe any possible benefits that these problems may have for him. Cumulatively, this suggests that he does not have access to the Discourse involved in these problem-types and as a result we can see his positioning relative to this problem as being peripheral in that he is unable to access the problem sufficiently to enable him to make some progress towards a solution.

Sipho's recognition of the need to use the speed-distance-time relationship suggests that he has some access to the Discourse surrounding these problem-types, even though we see through the use of an incorrect formula that his recall of this is somewhat sketchy. Furthermore, the way in which he adds the speeds together and then divides by the time, infers that he is possibly trying to apply learnt strategies (*viz.* the formula) without any further reasoning, and this is an indication that his access to the Discourse is only a little better than peripheral. In other words, Sipho is able to recognise that this problem is a particular genre of word problems (Gerofsky, 1996) with a particular 'method' for its solution, but this access to the Discourse does not enable him to move beyond this recognition in any meaningful way. This can be seen in the way in which Sipho is able to identify only one mistake (not converting the 15 minutes into hours), whilst he is oblivious of the incorrect speed-distance-time relationship, the incorrect calculation by adding the speeds as well as the disparity in the resulting answer in his discussion.

In the context of this excerpt Sipho seems to be positioned by the problem (as opposed to positioning himself relative to the problem). He therefore engages with the problem using only what the problem offers (for example he does not generate *auxiliary expressions* Lipek, 1995) for time that will help towards a solution and this does not give him sufficient resources to find a meaningful solution.

Moving into the next stanza we see that Gio turns the discussion from Sipho's attempt at problem 3 to focus on how he experienced the problems. Nina takes up this line of discussion, and both Nina and Gio talk about how difficult the problems were for them.

Stanza 20

73. Gio	a	For me//		
	b	this problems was, mmm		
	c	very difficult.\\		
74. Nina	a	Ja.\\	0:05:47
	b	For me	
	c	the last one was worst.\\ [All closing their problem booklets]	
	d	The first, [Gio talking in the background, which is inaudible]	
	e	the other ones […all talking together]...	
	f	all of them	
	g	I had to read	
	h	more than three times	
	i	to get it right.\\	
	j	Especially problem 2//	
	k	with the rent [pointing towards Gio with left index finger]	
	l	and everything,//	
	m	it was very hard..	
	n	to do.\\	

Description

Sign and knowledge systems

Nina categorises the difficulty levels of the questions by stating that the last one for her was the *worst* (line 74c), and she recognises a difficulty on her part in having to read the questions over *more than three times* (line74h) in order for her *to get it right* (line 74i), which in the context probably can be taken to mean, 'in order for her to understand the question from a mathematical perspective'. Gio, however, simply categorises all the questions as being *very difficult* for him (line 73).

Connections

Nina distinguishes between the questions in terms of the level of difficulty, but she sees them as being connected for her through a common trend, *viz*. she needs to read *all of them* (line 74f) more than three times. Gio simply sees *all* the problems as being difficult for him, which is identifiable from his generalisation in line 73b – c.

Significance

Through their emphasis of the word 'very' (line 73c and line 74m), Gio and Nina bring to the fore how difficult they found the problems (in Gio's case with all the problems, and in Nina's case mainly with problem 2). Gio says, "For me…" at the beginning of turn 73, which makes it significant that the problems were difficult for him, and that this difficulty does not necessarily extent to others.

Experiential level

We need to take cognisance of the issue of the non-English speaking students here. Both Nina and Gio had been in South Africa for about two years at the time of this data collection and in both their cases English was a completely new language to them about two and a half years prior to the collection of this data. Thus when Gio says, "For me…" (line 73a) it is possible that his experience as an immigrant has caused him to see himself differently from fellow students having to deal with word problems. Nina, positioning herself similarly to Gio as an immigrant, seems to see her difficulties as arising more explicitly from having to read the questions "more than three times" (line 74h) in order to understand them, and this 'understanding' probably refers to ordinary language as well as mathematical language (Tobias, 2003; Pirie, 1998).

Identity

From Gio's utterance in line 73 we see that Gio is positioned as being outside of the Discourse of word problems. He states that *for him* (line 73a) they are very difficult, and he does not seem able to elaborate upon this in any way, suggesting that he is not able to explain why word problems are difficult for him. In other words, he knows that word problems are difficult for him without any understanding of why this is so. Nina responds with, "Ja. For me…" (line 74 a – b) which appears to be in collaboration with Gio, but Nina is able to pinpoint reading and understanding the problem as being the factors that underlie her difficulties with word problems.

Relationships

Nina identifies with Gio's comments in line 73 and this gives her the opportunity to elaborate upon her experiences of the problems. However, both Gio and Nina confine their discussion to their personal experiences and there is no explicit acknowledgement of any other role-players in this extract.

Existential level

Gio and Nina share a common circumstance and they therefore relate this in terms of their experiences with word problems, particularly as they can identify with each other in this regard. It appears that because of this, in their perception they are separate from other students in their grade when it comes to word problems.

Political level

From Nina's perspective there is a suggestion that problems posed in this nature (*viz*. as word problems) disadvantage her because they require repeated reading, and it is this that makes the problems difficult for her. Gio is not explicit about this, but it appears that he is of the same opinion. However, Nina seems to attribute her difficulties to language and understanding, rather than from the word problems *per se*.

Discussion

Gio seems to be in a position in which he is not able to talk about the problems other than to state that they are difficult. It appears from this that Gio is therefore not a participant in the mathematical Discourse surrounding word problems. Nina is able to categorise the problems (only in terms of their difficulty levels) and is able to make certain connections and disconnections (in terms of difficulty and the necessity of having to read them repeatedly), and this gives her a means by which she is able to talk about the problems. In this extract Nina does not refer to any of the mathematics that made the problem difficult for her, and only alludes to the reading as being a possible inhibitor in terms of gaining access to the mathematics. Thus we can say that Nina has a limited access to the Discourse. It therefore appears that both Nina and Gio, because of their limited access to the Discourse are positioned by the problems as being relatively helpless (Tobias, 2006; 2005).

Once again in the next couple of stanzas we see that it is Gio who turns the focus of the discussion from Sipho's explanations of his working to a discussion of why one needs to do word problems. This leads Gio to examine what mathematics his father uses as a 'businessman', whilst Sipho attempts to construct a possible 'real-world' scenario that might give rise to such a problem.

Stanza 21

75.	Gio	a	... but anyway
		b	why do we have to use,\\
		c	have to have these questions? [Moving hands up and down emphatically whilst held out vertically in front of him]
76.	Sipho	a	Why//.. why do we have to answer them?//
77.	Gio	a	Why do they
		b	use this questions?\\
78.	Sipho	a	I, err... donno...\\
79.	Gio	a	if
		b	in the future
		c	you won't use it// [gesturing with hands open]
		d	maybe I going to... [inaudible, Sipho begins talking]
80.	Sipho	a	I think
		b	you going to use it...
		c	later\\...
81.	Nina	a	You going to be
		b	a maths teacher
		c	you going to use it//...
82.	Sipho	a	Nooo...
83.	Gio	a	Of course! ...
		b	My father's a businessman,
		c	right//.. so…
		d	he just use minus, plus, times//... [open hands moving up and down to emphasise the words minus, plus, times]
		e	he doesn't use\\,
		f	he doesn't use\\...
84.	Nina	a	... the *x*, *y*... [interjecting]
		b	triangles,
		c	circles...

Stanza 22

85.	Sipho	a	But I think,
		b	you know\\..
		c	you need
		d	a formula

		e	like//…
		f	as a businessman
		g	you'd say..
		h	like you wanna..
		i	somebody asks you
		j	and says... err,
		k	I wanna buy
		l	an orange,
		m	let’s say
		n	I wanna buy
		o	an orange,
		p	for//...
		q	this amount of money\\,
		r	how much more,
		s	how much,
		t	how much money
		u	would I need
		v	if I wanna buy,
		w	let’s say,
		x	ten oranges?//
		y	Then you gonna need to know...
86.	Nina	a	... the profit...
87.	Sipho	a	... yeah, yeah,
		b	you gonna need to know how much you gonna make… umm,
		c	like err,
		d	let's say,
		e	how many, [Nina interjecting with, ‘how much profit you’re going to make]
		f	how many,
		g	how many oranges would,
		h	how much money
		i	would I need
		j	to… [Inaudible - Nina talking at the same time]
88.	Gio	a	So,
		b	the message
		c	we need here is [Palms together, bumping the desk vertically]
		d	plus, minus, times// divided//. [Tapping his left palm with his right index finger]
		e	Okay.// [Open palms in a gesture of questioning] [all talking together]

Description

Sign and knowledge systems

Gio initiates the discussion with the question, “Why do we… have to have these questions?” (line 75). By stating it as *these questions* he intimates that word problems form a class of questions within mathematics, but he is not specific about what this class of questions might be. However, in trying to answer his own question Gio talks about how his father, as a ‘businessman’ only uses *minus*, *plus* and *times* (line 83d), and here he is presumably referring to arithmetic procedures. He tries to elucidate on what his father *does not use* (line 83 e – f) (and here we can infer that these would be the algebraic methods) and it is interesting that Nina helps him to express this by offering, "…the *x*, *y* … triangles, circles…" (line 84). Sipho attempts to give an example to answer Gio's question (lines 85 and 87) but by Gio's response in line 88 it appears that he cannot move beyond an arithmetic perception of these problems in the broader context.

Connections

Sipho's response in line 78 suggests that he has possibly never really considered the question of why one might use word problems. However, after Gio states that, "…in the future you won't use [word problems]", Sipho disagrees, saying that one will use them *later* (line 80). He then attempts to construct a problem that one might encounter in later life, but his example is quite naïve and can be more accurately associated with the types of word problems that he might have encountered in the primary school phase.

Gio appears to perceive a disconnection between what his father uses as a 'businessman' and word problems. He says that "…he just use minus, plus, times…" (line 83d), which seems to be saying that his father uses only arithmetic procedures rather than the algebraic ones required by word problems. Sipho attempts to construct a connection between 'businessmen' and word problems, but he does this to engender some credibility to the example that he offers (and this is discussed in more detail below). Nina makes a passing connection between word problems and 'becoming a teacher' (line 81) which seems likely to derive from her experience of the two in the schooling system. Sipho rejects this connection in line 82, and the way in which he does this could mean that he has no intention of becoming a teacher, or it could possible mean that the connection Nina is making is not what he and Gio had in mind.

Significance

Although Sipho initially states that he doesn't know why one would use word problems (line 78), he seems to take up a challenge to demonstrate why they might be useful (line 80) when Gio suggests that one won't use them in the future (line 79). Gio makes his point here significant by his emphatic hand gestures, and then, by the exclamation in line 83a he makes it significant that he has found an example to demonstrate that one will not use word problems in later life. It is interesting that Sipho slips in the term 'businessman' in line 85f which he appears to be using to give significance to his opinion that one might use word problems in later life. In other words, Sipho uses Gio's term 'businessman' to give weight to the counter-example that he attempts to pose for Gio's argument.

Experiential level

Both Gio and Sipho appear to have experienced problems only in the school environment and they seem unable to create any meaningful understanding of these problems in the wider context. We see through Gio's apparent naïve perception of the arithmetic nature of his father's career, and this is confirmed in his purely arithmetic working when doing the problems. It is also evident in Sipho's attempt to construct a 'real world' example to counter argue Gio, as this turns out to be very simplistic. However, Sipho manages to allude to the idea of an unknown quantity through his statement, "…for this amount of money" (line 85g), suggesting that he has a more 'algebraic' notion of what word problems are about.

Identity

In this excerpt Gio takes on the role of 'critic' of word problems, which we see firstly from his initial questioning of why we do or have these problems (lines 75, 77 and 79) and then in the discovery of an example to 'back-up' his position (line 83). Sipho seems not to have considered a critical stance (*cf.* line 78), but then feels compelled to demonstrate an example in which algebraic techniques required by word problems might be used in later life. Thus in this excerpt we see Gio and Sipho coming up against each other as antagonist and proponent (respectively) of word problems.

Relationships

Gio's positioning is such that he appears to view word problems as having been bestowed upon him by some unidentified (presumably) authoritative person or body of people. We see this in his questioning of why he has to do the problems (lines 75 and 77) which suggests that the problems have been presented for him (and other students) to do. Sipho brushes this off in line 80 by stating that one will use word problems later on, and this appears to show that he sees word problems as going beyond the schooling system, and hence not necessarily having only emerged from some educational authority.

Existential level

Gio appears to be unable to see any relevance to word problems and as a result of this comes to be positioned by word problems. Sipho, although not entirely clear about his stance, appears to attach some relevance to word problems in the wider sense, and consequently seems to be positioned with more agency in terms of being able to tackle the problems than Gio.

Political level

Gio's apparent inability to engage with word problems seems to lead to a perception of them as not serving any real purpose, which we see when he turns the conversation from Sipho's working to one in which they discuss the usefulness of the problems. However, if we look at his statements in lines 77 and 79 where he says, "Why do they use this questions... if in the future you won't use it?", we see that Gio has developed an 'assumption' that one will not use word problems in later life, and this perception leads him to view word problems as being futile. Thus we see in Gio a circular-type reasoning – he is unable to make any progress in solving the problems so he cannot see any use that the problems might serve, but because he views the problems as not being of any future value he is compelled to conclude that they are not worth doing in the first place. Sipho demonstrates a more accepting positioning with respect to word problems. In this respect it is not clear whether his perception of the problems as having some future benefit has led to this, or whether it was his more accepting disposition towards the problems that gave rise to the possible future benefits that they might have (although I think the latter is more likely from the text). Whichever this might be, it appears from this excerpt that Sipho is more enabled as a result of his positioning than Gio.

Discussion

From this excerpt we see a debate between Gio and Sipho around the usefulness of word problems. It is interesting that neither of these students manages a particularly convincing argument for his stance, but what seems to emerge is the apparent perception of the futility of word problems for Gio, whilst Sipho puts forward a more accepting disposition. In Gio's case his stance seems to leave him with limited agency when doing the problems (i.e. he becomes positioned by the problems) whilst Sipho's disposition appears to give him more agency (i.e. he is able to make some progress with the problems). Thus it seems that Gio is positioned outside of the Discourse surrounding word problems, whilst Sipho seems more enabled with that Discourse.

The next excerpt occurs at the end of the peer discussion.

Stanza 23

89.	Gio	a	So we'd been,	
		b	they making it difficult	
		c	for us//... [moving hands around vigorously from left to right]	
90.	Nina	a	I know, Gio	0:07:21
		b	you doesn't have to think	
		c	that\\.. maybe//	
		d	in our township	
		e	we would do better\\…	
		f	than here\\...	
		g	because of the language\\	
		h	we have to think	
		i	about the language.\\	
		j	I'm telling you,	
		k	if,	
		l	when I was in Brazil	
		m	my maths were,	
		n	my maths mark	
		o	was much higher	
		p	than here	
		q	in South Africa\\... [tapping the desk with fingers, palm facing inwards]	
91.	Gio	a	And also me [tapping his chest]	
		b	is the same...	
		c	is the same.	
92.	Nina	a	... the same.	
		b	When I was in Standard Five [Grade 7]	
		c	I had 70%//... [tapping the desk as before]	
		d	for maths.\\ [looking from Gio to Sipho]	
93.	Gio	a	Now for me	
		b	the same,	
		c	because I...	
		d	even before South Africa//	
		e	I got more//	
		f	what I used to get\\...	
94.	Nina	a	I got higher marks	
		b	in Brazil\\…	
95.	Sipho	a	Yes.	
		b	Let's go call him//	
		c	uh?//	

Description

Sign and knowledge systems

In this excerpt there is not much reference to the specific mathematical make-up and way of knowing and talking about word problems. However, we see again that Gio perceives them to be difficult (line 89b), but he seems unable to attach any explicit features that cause this for him, whilst Nina again attributes her experiences to ‘language’ issues (line 90g).

Connections

Nina compares her experiences to those when she was in Brazil, but these comparisons are not directly linked with word problems, but are made more broadly from a mathematical perspective.

Significance

Gio uses hand gestures again to emphasise the difficulty that he experiences with word problems (line 89c). With Nina we see by the emphasis placed on “have to” (line 90h) and “I’m telling you” (line 90j) that she foregrounds the language issues that have resulted from her move from Brazil. She makes her higher achievement in Brazil significant by emphasising the attainment of 70% (line 92c).

Experiential level

It appears that Nina sees her immigration circumstances as playing a large role in her difficulties and lack of achievement in mathematics (and by implication, in word problems). Also, whilst Gio’s own initial talk in this excerpt makes reference to his difficulties with word problems he appears to be prompted by what Nina says and this brings him to allude to his own experiences in Angola.

Identity

From line 89 it seems that Gio perceives himself as a ‘victim’ which results in him experiencing these problems as being difficult, whilst Nina appears to see herself as a ‘victim’ of having had to emigrate from Brazil.

Relationships

In line 89b, Gio attributes the difficulties that he experiences with word problems to the devices of some other ‘they’. Interestingly, he uses the first person plural (line 89a and line 89c), thereby generalising his experience somewhat, but this appears to be only to students in the same immigration circumstances, and may even be completely limited to himself and Nina. In the excerpt we see Nina responding to this talk in a collaborative manner with, “I know, Gio…” (line 90a), and her reference to ‘we’ in line 90e – h appears also to be collaborative. This apparent collaborative talk appears to exclude Sipho from the discussion (being a non-immigrant) and he makes this clear when he terminates the discussion in line 95.

Existential level

Nina and Gio both position themselves as victims when it comes to word problems (or more generally mathematics, in Nina's case). As a result of this they come to be positioned by the word problems and this positioning renders them relatively helpless (Tobias, 2006; 2005).

Political level

Gio implies some kind of disadvantage being bestowed upon him in his statement, "…they making it difficult for us…" (line 89b – c). Prior to this he said, "So we'd been… (line 89a), and then changes his phrasing, but this suggests that he sees himself (and the rest of 'we', or the other immigrants) as victims in the circumstances, and therefore in a position of disadvantage. Of relevance to this study is that, in the peer discussion it seems that Gio's perceptions of his disadvantage occurring through his immigrant status may have derived from intertextuality (Gee, 2005) from the talk that Nina had put forward earlier in the discussion. Nina's collaborative talk (line 90a and line 92a) may confirm this.

Discussion

In this excerpt Gio's and Nina's experiences of their difficulties with word problems come to have a collaborative meaning for both of them. Whilst Gio in previous talk seemed unable to express the exact nature of the difficulties that he experienced, he now comes to see them in the light of being a 'victim', although he is not explicit about this. Nina makes her status as a 'victim' somewhat more explicit as she discusses her difficulties more with mathematics in general than specifically with word problems. Nevertheless, their talk seems to show that both these students are positioned outside of the Discourse surrounding word problems and as a result of this they are relatively helpless when it comes to tackling these problems.

Conclusions

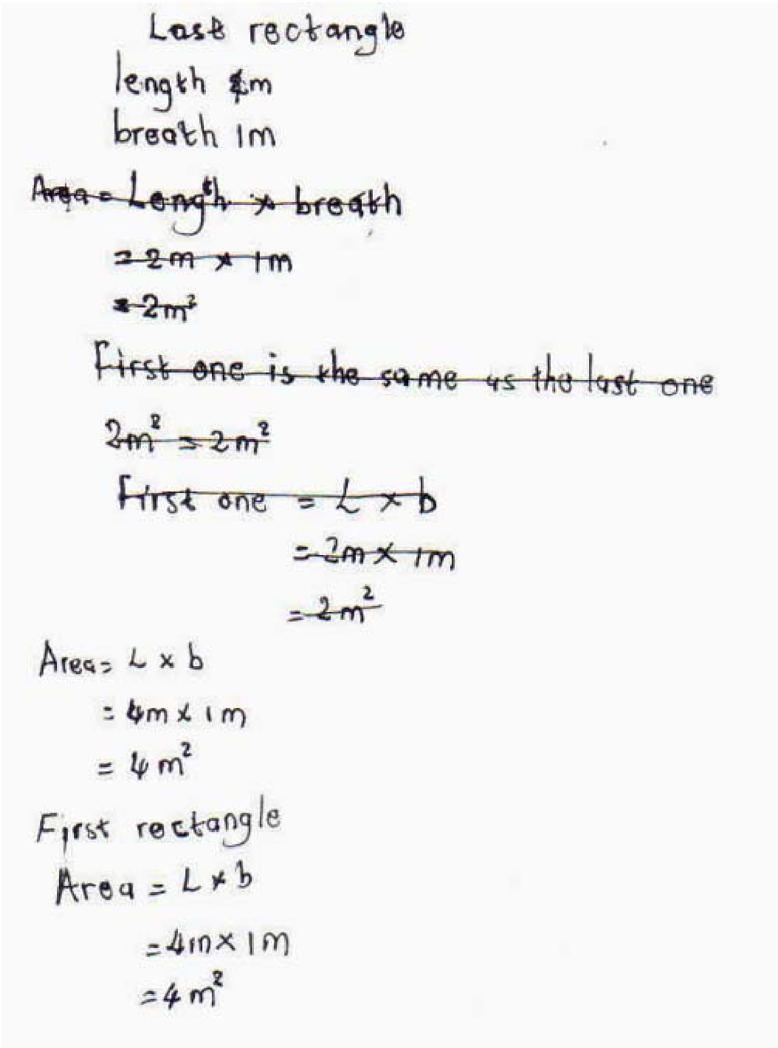
In one case (viz. Sipho) it appears that a taught strategy gives a certain amount of access to the problem, even if this only seems to enable the student to perceive that s/he is making some progress. However, from Sipho's working it seems that, without the deeper understanding of the problem situation, he becomes lulled into a false sense of 'being right' (in problem 3 he only saw the conversion of the 15 minutes as being incorrect, whereas all his working is actually incorrect). In Sipho's case with problem 3 his 'access' to the problem appears to come about because of his recognition of the problem type (Gerofsky, 1996), but when this recognition is not present it seems that accessing the problem becomes more difficult. In either case, these students appear to be positioned by the problem in that they either respond to the problem on the basis of taught rote-like strategies, or they are unable to do anything meaningful with it.

Manumission – The analysis of the student working

> Problem 1 – The rectangle problem
> Tseko draws a rectangle with its length 2 m more than its breadth. He then increases the length by 2 m and decreases the breadth by 1 m. He finds that the area of the new rectangle is the same as that of the first one. Find the length and breadth of Tseko's first rectangle.

Description

Mafifo – problem 1



Last rectangle
length 4m
breath 1m
~~Area = Length × breath~~
~~= 2m × 1m~~
~~= 2m²~~
~~First one is the same as the last one~~
~~2m² = 2m²~~
~~First one = L × b~~
~~= 2m × 1m~~
~~= 2m²~~
Area = L × b
= 4m × 1m
= 4 m²
First rectangle
Area = L × b
= 4m × 1m
= 4m²

1. *Understanding the problem situation*

The text shows that Mafifo has recognised that there are two scenarios of the area of the rectangle which is evident from her reference to "Last rectangle" and "First one". It also appears from the statement, "First one is the same as the last one" together with her workings (which show length times breadth) that she has understood that the areas of the two rectangles are the same. However, there is no indication in the text that Mafifo has understood the increase in the length and the decrease in the breadth to arrive at the second scenario of the rectangle and in her initial working (crossed out from line 4 onwards) that she misinterpreted the length to be 2m and the breadth to be 1m.

In her final working below it may be that Mafifo arrives at the 4m by adding 2m to her initial assumed length of 2m, but this is not made explicit.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

From the text it appears that Mafifo's recognition that areas of rectangles were involved prompted her to use the area formula for a rectangle. From the line "$2m^2 = 2m^2$" and the statement that precedes this it seems that Mafifo is using the fact that the areas are the same as a basis for establishing the dimensions of the original rectangle. Furthemore, it appears that she does this through a process of better approximation, whish is suggested by the change of 2m to 4m in the second line and the calculations which follow.

Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The mathematics is inappropriate to the problem which probably stems from the initial misinterpretation of the original dimensions of the rectangle.

Grading 0: No evidence of correct or appropriate mathematics.

Danny – problem 1

I think Tseko's first rectangle the length is 2m and the breadth is 1m because Tseko's length is more that breath.

1. *Understanding the problem situation*

The text suggests that Danny is aware of a relationship between the length and breadth of the initial rectangle, but he has misinterpreted it to mean that the length is 2m and the breadth 1m. From his sentence we can deduce that he has understood what the question is asking, but that in view of what he has come up with he has the answer, *viz*. the length and breadth of the first rectangle. It is interesting that Danny gives as a reason for his answer that the "length is more that (sic) the breadth." This may stem from the use of language in the problem, in this case, "the length is 2m *more* than the breadth", suggesting the possible use of key words from the problem statement (Hegarty, Mayer and Monk, 1995; Mangan, 1989). Danny may have taken the length to be 2m, and interpreted that it is *more* than the breadth, and therefore arrived at 1m for the breath.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

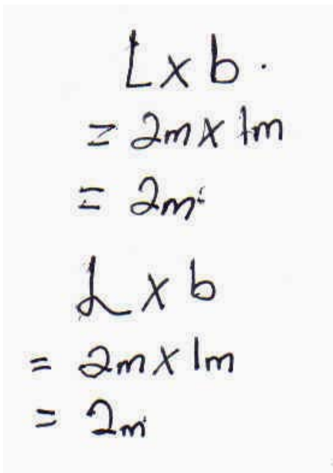
The text does not reflect any method or procedure for arriving at this conclusion other than pure interpretation of the problem statement.

Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

No mathematical working is evident in the text.

Grading 0: No evidence of correct or appropriate mathematics.



L x b.
= 2m x 1m
= 2m²
L x b
= 2m x 1m
= 2m

Hartman – problem 1

1. *Understanding the problem situation*

From the text it seems that Hartman has recognised that the problem involved area of a rectangle which we see in his use of the formula $l \times b$. From the fact that two calculations have been done it is possible that Hartman also recognised that two scenarios of the rectangle were being referred to, and the fact that the calculations are the same might indicate that he had some understanding of the areas being equal. The calculation "$2m \times 1m$" suggest that Hartman has misinterpreted the phrase "length [is] 2m more that its breadth" to mean that the length is 2m, and that the length is double the breadth.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

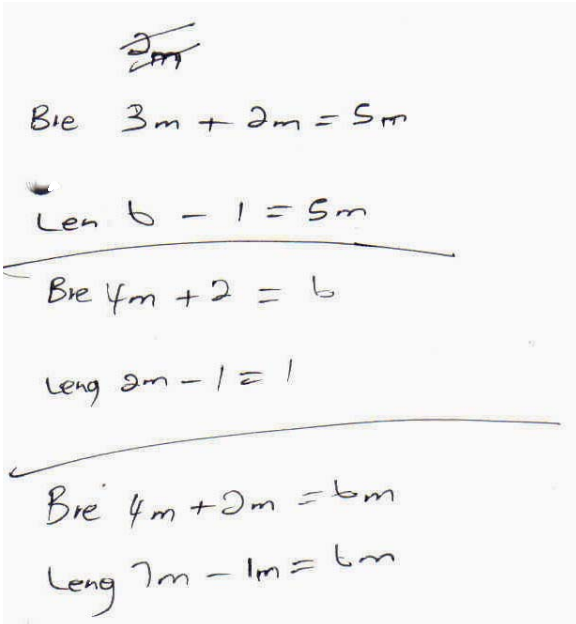
The text shows that Hartman has applied the area formula for a rectangle. No other method or procedure is evident.

Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The mathematics in the text is inappropriate to the problem.

Grading 0: No evidence of correct or appropriate mathematics.



Rosina – problem 1

1. *Understanding the problem situation*

The text indicates that Rosina has recognised that the length has been increased by 2m and the breadth decreased by 1m which we see in all three scenarios presented by her. The fact that she has recorded the breadth to be 3m and the length to be 6 in the first instance, and the length to be 4m and the breadth 2m in the second instance, possibly means that she has misinterpreted "2m more than the breadth" to mean "length is double the breadth". What is interesting is that 2m at the top of the working has been crossed out, which suggests that Rosina resisted the urge to think of the length as 2m and the breadth as 1m. In the first and third calculations both the length and breadth end up being the same, and it seems from this that Rosina has interpreted the question to be asking for manipulations of the length and breadth that will give her the same measure, rather than the same area.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

From a possible initial misinterpretation it appears that Rosina is using a trial and error approach to arrive at the same length and breadth. It seems in the third calculation that she has had to abandon her initial premise that the length is double the breadth in order to achieve this.

Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

In all three calculations Rosina adds 2m to the length and subtracts 1m from the breadth, which is correct in terms of the altered rectangle in the problem. Otherwise, the mathematics is inappropriate to the problem.

Grading 0: No evidence of correct or appropriate mathematics.

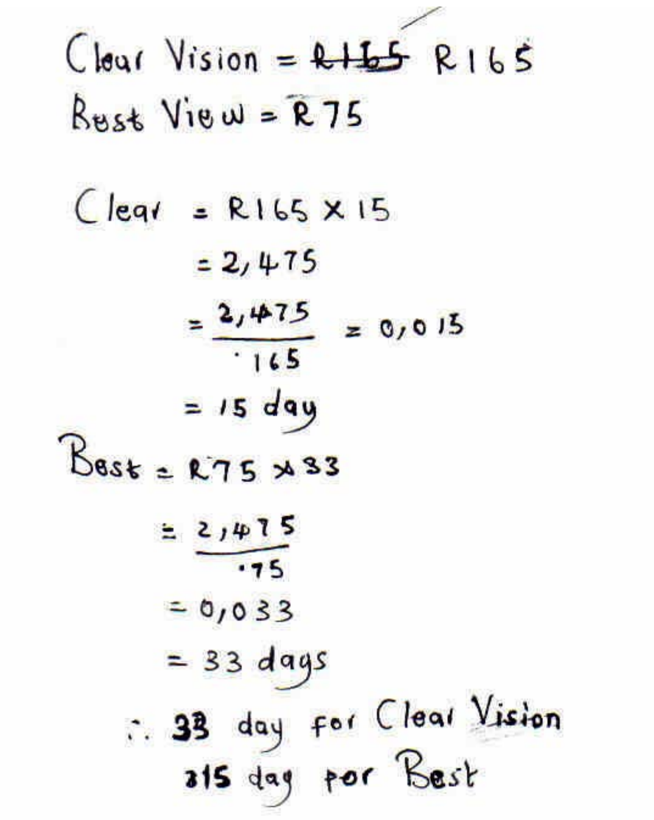
> Problem 2 – The TV rental problem
> The Clear Vision television rental shop charges a basic fee of R150, as well as R15 per day to rent a television. The Best View television rental shop only charges a basic fee of R15 but has a daily rate of R60 per day to rent. For what number of days would it make no difference in cost as to which shop you rent from?

Mafifo – problem 2

1. *Understanding the problem situation*

From the first two lines of the text it is apparent that Mafifo has interpreted the daily cost to be a total of the basic fee and the daily rate. The text shows that she has interpreted the question to mean that there would be a different number of days of rental at each shop, but this is probably as a result of her interpretation of the cost structure as she would not have been able to arrive at the same cost for a given number of days using her interpretation.

Grading 2: Evidence of recognition of the problem situation, but incomplete or errors in interpretation.


Clear Vision = ~~R165~~ R165
Best View = R75

Clear = R165 × 15
= 2,475
= $\frac{2,475}{\cdot 165}$ = 0,015
= 15 day

Best = R75 × 33
= $\frac{2,475}{\cdot 75}$
= 0,033
= 33 days

∴ 33 day for Clear Vision
315 day for Best

2. *Use of method or procedure*

The text shows that costs have been established for each shop and then it appears as though a trial-and-error approach has been used to arrive at the answers of 15 and 33 days, probably using the calculator which may explain why the working is not shown.

Grading 2: Evidence of methods or procedures is clear, but they are not systematic.

3. *Use of correct mathematics*

In Mafifo's answer the results appear to have been transposed. However, in view of the misinterpretation of the problem information the mathematics is inappropriate to the problem.

Grading 1: Some evidence of correct mathematics.

Danny – problem 2

The clear vision television rental shop becouse if you pay a feel of R150 in the day rent television you could not pay a lot money it is only R15 per day and in is better that R60 per day.

1. *Understanding the problem situation*

The text appears to give the answer as "The clear vision television rental shop" which we see from the conjunction 'because' that follows it. This shows that Danny probably did not understand what the problem question was asking. There seems to be some indication that Danny understood the breakdown in payment as he notes the large discrepancy between the R15 and R60 payable per day for each shop.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

The text does not indicate any particular use of method or procedure other than to arrive at an intuitive conclusion based on what information is given in the problem.

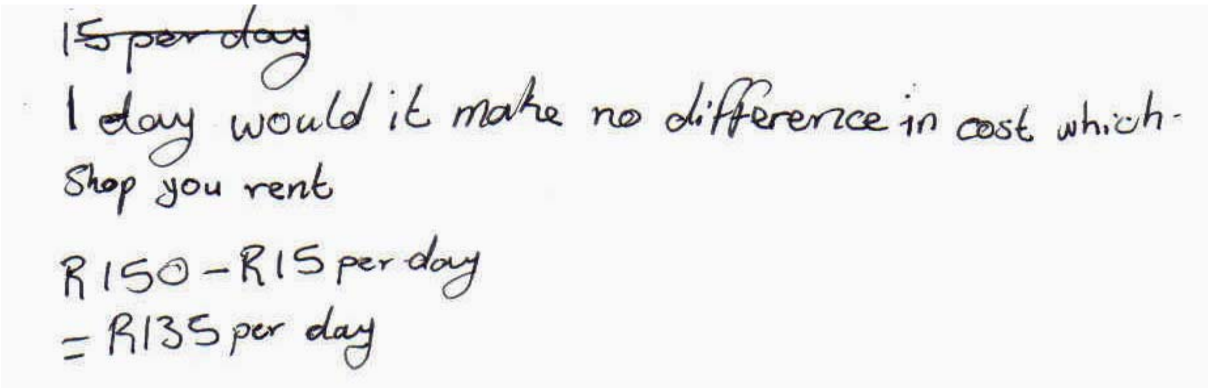
Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

The only mathematics that the text implies is a comparison between the daily amounts payable at each shop.

Grading 0: No evidence of correct or appropriate mathematics.

Hartman – problem 2


~~15 per day~~
1 day would it make no difference in cost which Shop you rent
R150 – R15 per day
= R135 per day

1. *Understanding the problem situation*

It is not apparent from the text where Hartman gets his answer of “1 day”, but it seems that he had no understanding of the problem situation, other than possibly that a number of days was being sought. However, if we note the semantics of the answer given, *viz*. “1 day would it make no difference…”, we see that it has probably been copied directly from the question, indicating that Hartman may not have even understood the problem question. The last two lines indicate that Hartman had no understanding of the payment structure. We see this from the subtraction of the daily amount from the initial amount payable, which makes no sense in terms of the problem context. In addition to this the text only shows calculations for one rental shop, and yet the answer given implies a comparison of two shops.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

The text begins with “15 per day” which is then crossed out, and this may indicate an attempt to summarise the information from the problem. However, in the next line an answer to the problem is posed, followed by a calculation that does not make any sense in light of the problem, nor does it in any way seem to be an attempt to validate the proposed answer. Together these seem to suggest that there is no method or procedure being used.

Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

The mathematics is inappropriate to the problem.

Grading 0: No evidence of correct or appropriate mathematics.

Rosina – problem 2

1 Day 165	1 Day 75
2 Day 180	2 Day 135
3 Day 195	3 Day 195
4 Day 210	

It would in day 3 becouse amount is the some.

1. *Understanding the problem situation*

The text shows correct calculations which indicate that Rosina understood the payment structure at the rental shops. The way in which she answers the question, i.e. rewording what was required, indicates that she fully understood what was required by the problem question.

Grading 3: Evidence of correct interpretation of all aspects.

2. *Use of method or procedure*

The text shows tabulated data (the table is not drawn) and there has been a progression calculated for the cost at each shop for the first few days. This indicates that Rosina has used a systematic numeric approach to find the solution.

Grading 3: Evidence of clear methods and procedures that are systematic.

3. *Use of correct mathematics*

Although purely numerical, the mathematics is sound and yielded the correct answer.

Grading 3: Text is fully correct and appropriate and leads to a solution.

> Problem 3 – The speed-distance-time problem
> A boy cycles from home to school in the morning and back in the afternoon. He cycles from home to school at 32 km/h and back at 24 km/h. It takes him 15 minutes longer in the afternoon than in the morning. Find the distance between home and school.

Mafifo – problem 3

Home = 32 km

School = 24 + 15 minutes

$32 + 24 = 56$

$56 \div 15 = 3{,}1$ km

$\frac{24}{15}$

$= 1{,}6$ km

Home $= \frac{32}{15}$

$= 2{,}1$ km

1. *Understanding the problem situation*

From the text in the first two lines it appears that Mafifo has not understood the description of the two journeys, other than that the journey home is ‘longer’, which we see with the addition of 15 minutes in line 2. The first line states “Home = 32km” which seems to be a misinterpretation of the distance from home to school (which, if this were the case, would answer the problem question), and the second line states, “School = 24 + 15 minutes”, in which it is not clear whether this represents a distance or a time for Mafifo.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

In the first two lines it appears that Mafifo has attempted to summarise the information from the problem. What follows below and to the right are three different calculations in which 15 is divided into the speeds or the sum of the speeds. There is no indication of any formula in the text, but it is possible that these calculations stem from incorrect recall of the speed-distance-time relationship. No indication is given as to the relevance of each calculation, but the inclusion of the word “Home” next to the final calculation seems to indicate that Mafifo intended this as her answer.

Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

The division of speed by time does not yield a distance and Mafifo appears not to have recognised that the 15 minutes needs to be converted into a unit compatible with the speeds. The mathematics is incorrect.

Grading 0: No evidence of correct or appropriate mathematics.

Danny – problem 3

In the morning I think is he trevels in 8 minutes
because in the two kilomerers 32 km/m and 24 km/h
is in the morning he trevels at a shoter time and
in the longer it takes him many time travel.

$$\begin{array}{r} 32\text{ km/h} \\ -\ 24\text{ km/h} \\ \hline 8\text{ minutes} \\ \hline \end{array}$$

1. *Understanding the problem situation*

It appears from the text that Danny has misinterpreted the problem question as he seems to be giving "8 minutes" as his answer, which we see from the first line and the calculation at the end. From the paragraph that he wrote it seems that Danny is aware of the two different journeys, and what the different speeds meant for the journeys in terms of time. But from the calculation at the end it appears that he does not understand the speeds in terms of the problem situation.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. *Use of method or procedure*

The text seems to show that Danny came up with a solution of '8 minutes' by subtracting the two speeds given in the problem. From the way in which the text is presented, i.e. with the answer first, followed by a calculation to justify the proposed answer, it appears that Danny may have come up with the solution whilst reading the problem, written this down and then given an explanation of how he 'saw' this answer.

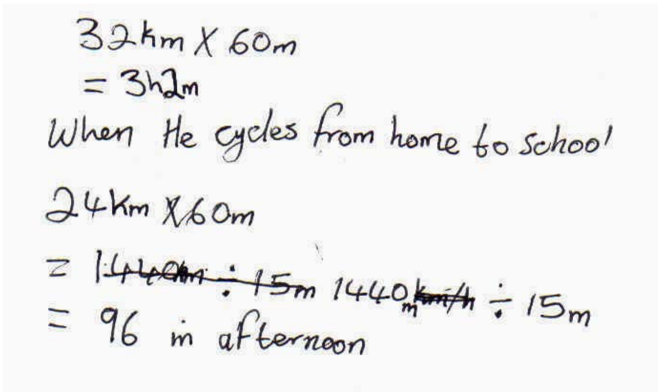
Grading 0: No evidence of method or procedure.

3. *Use of correct mathematics*

Danny appears to refer to the two speeds as “the two kilometres” and he writes “32km/m”. The latter is possibly a simple transcription error, but the former might be significant mathematically since it then refers to a distance rather than a speed. Danny’s calculation is a subtraction of two speeds, which he sees as resulting in a time, and this is mathematically incorrect.

Grading 0: No evidence of correct or appropriate mathematics.

Hartman – problem 3


32km X 60m
= 3h2m
When He cycles from home to school
24Km X 60m
= ~~1440m ÷ 15m~~ 1440m ~~km/h~~ ÷ 15m
= 96 in afternoon

1. *Understanding the problem situation*

Initially, from the text it appears as though Hartman may have misconstrued the speeds given in the problem to be distances as he recorded them as 32km and 24km respectively in his working. However, the text does not make it clear what the ‘60m’ means that Hartman has included in his calculations, nor where he derived this figure. From his use of ‘15m’ in line 5 it appears that the ‘m’ stands for minutes, since 15 minutes was given in the problem. Thus it may be that ‘60m’ represents one hour, and that Hartman has interpreted 32km/h to be ‘32km/60m’, which he then construes to be ‘32km$\times$60m’. If this is the case he does a similar manipulation with 24km/h. Whilst the two calculations show that Hartman to some extent understood the two different journeys (which we note by the inclusion of the 15 minutes into the calculations performed on the 24km/h scenario) it seems that he did not understand how the information in the problem came to bear upon these two situations.

Grading 1: Some evidence of understanding, but it is vague or misinterpreted.

2. *Use of method or procedure*

If we interpret the calculations as speed times time (i.e. that Hartman has simply stated the speeds as 32km and 24km) it may be possible that the formula was being used to calculate a distance. However, this is not made explicit and no indication of the formula is given. In addition to this the answers are given in terms of times and Hartman later divides by the 15 minutes, which does not suggest use of the formula. Thus it seems more likely that Hartman is simply using the information in the problem and performing operations according to his understanding of the problem situation.

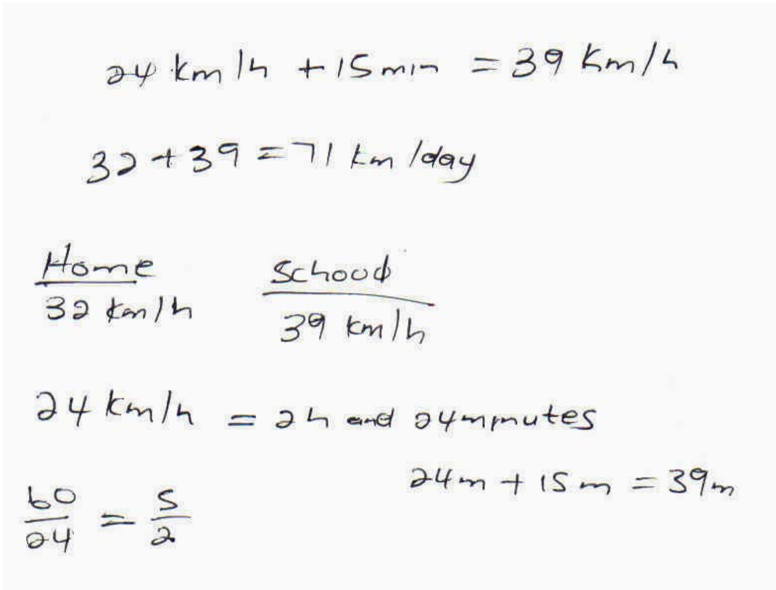
Grading 1: Some evidence of method or procedure is present in rudimentary form.

3. *Use of correct mathematics*

Whatever interpretations that we take for the working in the text the mathematics is incorrect in terms of the problem situation.

Grading 0: No evidence of correct or appropriate mathematics.

Rosina – problem 3



24 km/h + 15 min = 39 km/h

32 + 39 = 71 km/day

Home	School
32 km/h	39 km/h

24 km/h = 2 h and 24 minutes

24 m + 15 m = 39 m

$\frac{60}{24} = \frac{5}{2}$

1. Understanding the problem situation

From the first line we can see that Rosina understood that there were two journeys described in the problem and that the journey home from school took longer. However, the speeds that she generates in the tabulated information in line 3 give the speed from school to home as 39km/h, which indicates that Rosina had lost sight of whatever understanding she may have had initially, probably as a result of her calculation in line 1. In line 4 it seems that Rosina is trying to establish a distance by first arriving at the time it took, thus suggesting that she had some understanding of what the problem question was asking.

Grading 0: Almost no understanding evident, or understanding is superficial.

2. Use of method and procedure

The calculation in line 1 suggests that Rosina was using the figures from the problem together with what she perceived to be an appropriate operation (15 minutes longer may suggest adding) to generate answers. Then in line 4 the text seems to show a move to try to establish the time taken for the journey from the speed. The manipulation of the different units shows a tendency for Rosina to 'convert' the figures into units that will be appropriate for her purposes.

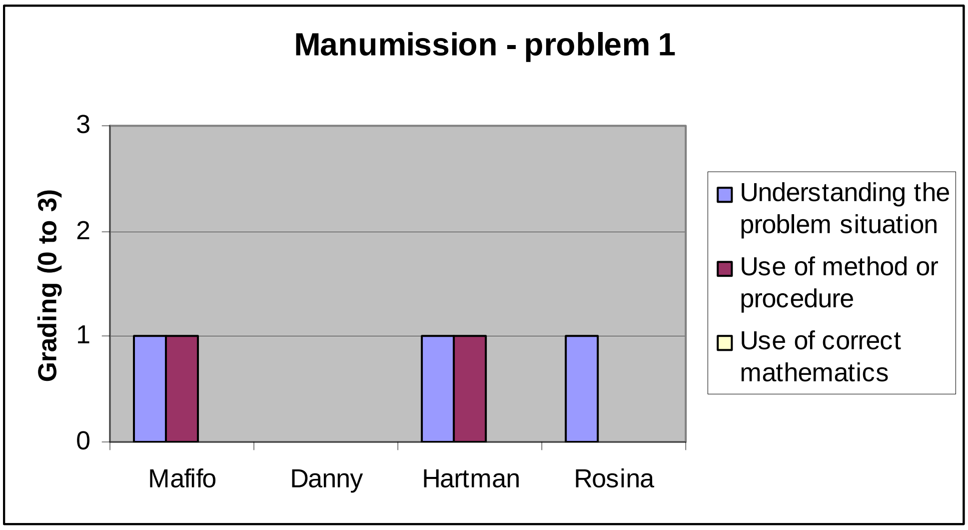
Grading 1: Some evidence of method or procedure is present in rudimentary form.

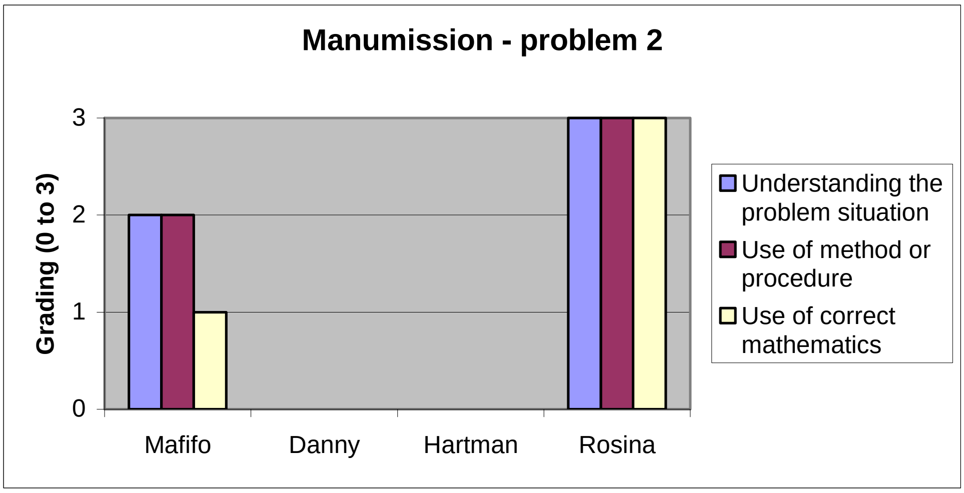
3. *Use of correct mathematics*

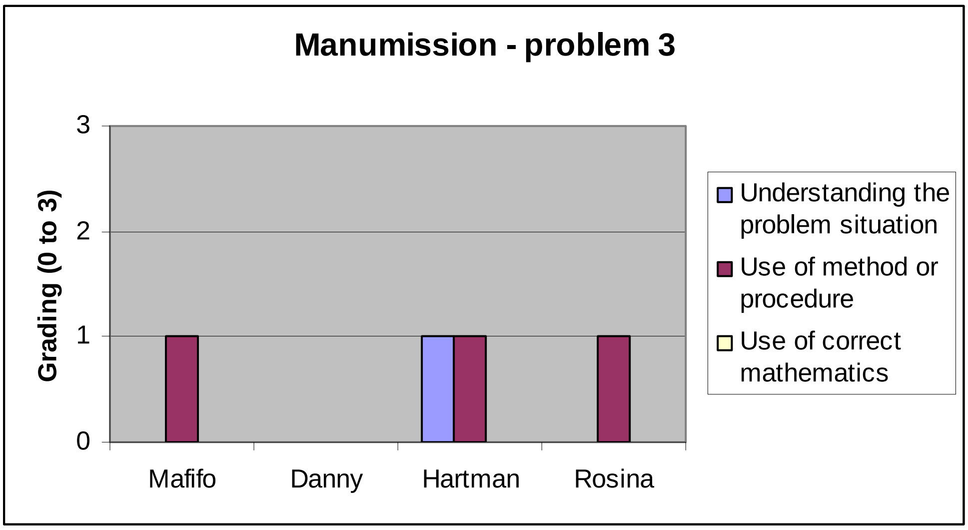
The mathematics is incorrect, for example the addition of speed and time giving a speed (line 1), and "24km/h = 2h and 24 minutes" (line 4). It is also not clear how Rosina gets to "24m + 15m = 39m" but it seems that this has been derived somehow from 24km/h and 15 minutes as stated in line 1.

Grading 0: No evidence of correct or appropriate mathematics.

A summary of the three dimensions of student activity for Manumission







Discussion

From the student working it is apparent that a lack of understanding of the problem situation in many cases inhibits any meaningful progress with the problem. Even when some recognition of a method or procedure is evident, the students still seem to be unable to engage with the problem, and this seems to be as a result of their superficial or lack of understanding of the problem situation. In problem 2 both the attempts by Mafifo and Rosina show a fairly good to excellent understanding of the problem situation, and they are able to employ some sort of strategy, and both students make good progress with the problem. In general, however, it seems that understanding of the problem situation, together with what appears to be a poor resource of methods or procedures, positions the students outside of the Discourse surrounding word problems.

Although these students were generally unsuccessful in solving these problems, from the graphs we see a much more meaningful attempt at the television rental problem (problem 2), and Rosina managed to solve the problem successfully (the only problem to be correctly solved by these students). In terms of understanding the problem situation the graphs suggest that these students battled more in the speed-distance-time problem (problem 3).

Manumission – The analysis of the student talk

In the excerpt that follows the students are examining what they did to and how they experienced problem 3.

Stanza 13

66. Rosina — Number terii.
Nna ke ne ke sa itse go dirahalang.
a Number 3.
b I didn't know
c what was going on.

67.
a [All read the question, not very clearly.] A boy cycles
b from home to school
c in the morning
d and back home
e in the afternoon.
f He cycles
g from home to school
h at 32 km/h
i and back home
j at 24 km/h.
k It takes him
l 15 minutes longer
m in the afternoon
n than in the morning.
o Find the distance
p between home and school.

68. Rosina — Nna ke ne ke sa itse go dirahalang moo.
a I didn't know
b what was going on
c here.

69. Danny — Mare aowa, mare Question ee e hlaka-hlakane.
a But,
b but this question
c is confusing.

Stanza 14

70. Mafifo — Haa tloha ko habo a ya sekolong ke 32 ha ke re?
a From home to school
b is 32,
c is it?

71. Danny Kilometres.

a Kilometres.

72. Rosina Ya haa tloha ko habo, haa tloha ko habo a ya ko sekolong and then haa boya sekolong ke 24 long mare ha ba e…?

a When he leaves home to school,
b when he leaves home to school
c and then
d from school to home
e is 24,
f but…?

73. Mafifo

a He takes
b 15 minutes longer.

74. Mafifo wa bona e hlaka-hlakane.

a You see
b it's confusing.

Stanza 15

75. Rosina Maybe u tswanetse ho plusha 15 ee ka 24 eene ba re ke awara, ha ke re …? [Interrupted] Nna ke ne sa itse hore … ? [interrupted].

a Maybe
b you must add this 15 and 24
c and they say
d is an hour,
e is it... ? [Interrupted]
f I didn't know… ? [Interrupted].

76. Danny ke Nna wa itse ke dirileng ke itse 32 wa shapa 24 ya mpha 8 minutes, Ka re may be ya from home to school.

a You know what I did,
b I said
c 32 subtract 24,
d which gave me 8 minutes,
e I said
f Maybe
g that's from home to school.

77. Rosina Nna ha ke itse, ya itse?

a You know,
b I don't know?

Stanza 16

78. Danny nako Mare question ee e hlaka-hlakane. Moo o tsamaya dikilomitare tse dintsi mare ee e nyane, mare moo o tsamaya tse nyane mare he takes 15 minutes long, wa bona e hlaka-hlakane.

a But
b this question is confusing,
c here he walks many kilometers
d but time is short,
e but here he walks less kilometers
f but he takes 15 minutes longer,

g you see
h it's confusing.

79. Rosina — Le nna mola ke ngotse, mola mola di-questioneng tsela , ke ngotse ka re aowa e yona le khale... ba be more specific hore ho dirahalang hape nna ha ke utlwisise.
a I also wrote there,
b there,
c there in those questions,
d I wrote
e that this one
f no, no,…
g they should be more specific
h about what is happening.
i Again
j I don't understand.

Description

Sign and knowledge systems

Mafifo talks about, "From home to school is 32" (line 70) but she does not state that it was at an average speed of 32 kilometres per hour. Rosina makes a similar statement in line 72d – e in which the unit of measure is omitted. Again in line 75b we see Rosina wanting to add the 15 minutes to the 24km/h, but she has simply stated, "Maybe you must add this 15 and 24." Danny talks about a similar operation that he performed in the task when he says, "32 subtract 24, which gave me 8 minutes" (line 76c – d). In all of these examples it appears as though the students are attempting to access the problem through the numbers that are given. However, because they omit the unit of measure they lose the context in which the number is situated which leads to a lack of understanding or a misunderstanding of what the numbers mean within the problem. This leads the students to simply 'use' the numbers in any way that they can so as to arrive at an 'answer'.

Connections

In this excerpt there are no apparent connections that the students are making. Their utterances remain focused on what they did in trying to solve the problem interspersed with comments on the confusion or lack of understanding that they were experiencing.

Significance

In this discussion it appears as though the students bring to the fore how confusing the problems were for them. We see this in the way in which many of the turns end with statements like *it's confusing* (line 74b and 78h), *I didn't know what was going on* (line 66b – c) or *I don't understand* (line 79j).

Experiential level

The student talk in this excerpt brings to the fore many experiences of confusion and inability to understand when it come so doing word problems. It is possible that their apparent glib use of units of measure when talking about the problem situation contributes to this, but from other talk we see that there are also language issues involved since the students talk about their inability to understand (e.g. Mafifo in line 70, Rosina in line 72 and Danny in line 78).

Identity

The students expressions of confusion and lack of understanding indicate that they are positioned by the question in such a way that they are unable to solve the problem. Statements such as “I didn’t know what was going on” (lines 66 and 70) and “…it’s confusing” (lines 74b and 78h) show that the students are positioned *by* the problem rather than positioning themselves relative to the problem.

Relationships

Most of the talk in this excerpt does not give any direct indication of the relationships that the students perceive. However, Rosina says, “… they should be more specific about what is happening” (line 79g – h). By *they* she implies some authority figure (possibly an institution such as the education department) who compiles or issues these questions, and by the use of the word *should* she intimates that the question is set in such a way as to obscure what is happening. The students do however express a sense of collaboration through their common talk about being confused and not understanding the problem.

Existential level

These students are positioned outside of the Discourse surrounding word problems. We see this from their inability to move beyond a summary of what they understand the problem situation to be (both in their utterances and in their working). Their talk is also often interspersed with comments that express how confused they are or that they do not understand.

Politics

The students are very aware of their inability to access the problems and Rosina seems to attribute this to the design of the question, saying that "…they should be more specific" (line 79g), and this suggests that inability (or that of the students in general) is possibly as a result of the way in which questions are posed.

Discussion

Because of their positioning the students are unable to enter meaningful dialogue around the solution of the problem and many of the attempts end in statements of futility. The students' inability to access the content of the question creates feelings of confusion and highlights ineptitudes that the students are experiencing with the problem. Since they seem acutely aware of these 'shortcomings' it positions these students as relatively helpless (Tobias, 2006; 2005) and therefore at a disadvantage in the broader political sense (Gee, 2005; 1999).

In the excerpt that follows the students are discussing how they tried to do problem 1 and the associated experiences that they had.

Stanza 17

80. Danny — Enaaa… Problem 1…
a This one…
b problem 1… [Reads the question inaudibly.]

81. Rosina — wa itse ba nyakang?
a Do you know
b what is required?

82. Danny — … he then increases the length by 2 metre. Wa bona jwale nou if u tjhentjha length ka 2 metre which means e tlobe 4 metre.
a … he then increases the length
b by 2 metres.
c You see now
d if you change length
e by 2 metres,
f which means
g it will be 4 metres.

83. Mafifo — … and then ho etsahalang moo?
a … and then
b what is happening
c here?

84. Danny — And then a decrease ela ka wane.
a And then
b he decrease that one
c by 1.

85. Danny Ka wane, ha ho sale selo moss…

a By one,

b nothing is left…

86. Mafifo Kapa … [interrupted].

a But … [interrupted].

87. Rosina wa itse Mafifo uena , moo Danny, ha ba re fa di metres tsa breadth ha ba re fa tsona le tsa length ha ba re fa tsona hore re bone hore na motho eo o dirileng a dira, nna ke dirile janong, ke no re 4 metres, nna ken e ke nahanetse ka re 4 metres ka plusha ka 2 ee ya ba 6 metres and then ya breadth e le 7m mare a minasa ka 1m ea mo fa 6m. ke dirile jwalo.

a You know Mafifo…,

b here Danny,

c they didn't give us

d the measurements

e for breadth and length

f so that we could see

g what this person did.

h What I did is,

i I said 4 metres,

j I thought,

k and said 4m

l plus this 2m

m is 6m,

n and then

o breadth was 7m,

p but he subtracted 1m

q which gave him 6m.

r I did it like that.

Stanza 18

88. Danny

a [Reads inaudibly] … he builds

b first

c a rectangle field…

89. Rosina Rectangle, haa yona eo…

a Rectangle,

b yes that one…

90. Mafifo ya mathomo.

a The first one.

91. Rosina ya mathomo, ya e dirileng la mathomo.

a The first one,

b the one he did first.

92. Mafifo Ya, and then a dira ya bobeli.

a Yes,

b and then

c he did the second one.

93. Danny nna ke itse … [interrupted].

a I said ... [interrupted].

94. Rosina … eene ya bobeli a e thomara moo ela ya mathomo a increasa e ngwe e iwane.
a … and started the second one
b on the first
c by adding another one.

95. Mafifo Mare nna ke ne ke sa e utlwisise.
a But
b I did not understand.

Description

Sign and knowledge

The three students engaged in this discussion (Hartman does not contribute anything in this excerpt) give the impression that they have a common understanding of what a rectangle is together with its dimensions of length and breadth. However, their interpretation of the relationships given for the length and breadth of the rectangle in the problem vary considerably. It appears as though Danny has interpreted the rectangle to be 2 metre by 1 metre (line 84). Mafifo questions something about Danny's explanation (line 85), and from Danny's response in line 86 and 87 it seems as though she is pointing out to Danny that the breadth cannot be 1 metre if it is to be reduced by 1 metre and still produce a rectangle. Rosina then points out in line 89c – e that the measurements of the length and breadth are not given in the problem, but in her attempt at a solution we see that she has also assigned a value, *viz*. 4 metres for the length. From her working and her explanation this appears to be a guess, but her reasoning behind the original breadth of 7 metres is not clear either in her working or in her explanation in line 89. The three students then move on to a discussion about how the rectangle changes, but they don't seem to be able to make any further progress regarding the solution to the problem.

Connections

Only Danny makes one reference to a rectangular *field* (line 90c), but it seems that none of the students make use of this context to help them proceed with the question, for example by using a sketch. In fact, when Danny makes reference to the *field*, Rosina refocuses the discussion on the rectangle by saying, "Rectangle, yes that one." (line 91) and she seems to wittingly omit the context of the field. Of course, the context of the field in this case might not have been of any direct help, but none of the students even attempts a diagram to help them understand the relationships between the length and breadth of the rectangle, and so it seems as though they have disassociated the rectangle from any physical thing (such as a field) or any representation (such as a sketch) and are working purely theoretically.

Significance

Danny's expression of, "Enaaa… Problem 1." (line 82) emphasises the difficulty that he experienced with that problem. Later on in his explanation he repeats the expression *by one* thereby reinforcing the anomaly that he comes up with when reducing the breadth by one metre, and this appears to be why he feels he has struggled with the problem. Rosina makes significant the fact that the length and breadth are not given by drawing Danny and Mafifo into the conversation by name (line 89a – b).

Experiential level

The three students involved in the discussion of problem 1 have had different experiences of the problem which can be seem by their different interpretations of the given dimensions of the rectangle in the problem. They all appear to have disregarded the context of the field – even though Danny does allude to it he doesn't make any use of it or refer to it anywhere else. Only Rosina seems to have been able to make some progress with the problem by getting an initial length of 4 metres and an adjusted length of 6 metres (which is the correct answer), but it is not clear whether this was a guess or whether it was as a result of some intuitive reasoning that was not shown in her working and not discussed in this excerpt. Danny, because his initial breadth reduces to zero appears to be questioning whether the given information is perhaps incorrect.

Identity

Danny seems to be talking from a position in which he lacks confidence. He starts his explanation with a statement that suggests that the problem has confused him (line 82). In his discussion of the solution the problem reduces to an impossible situation, and Danny expresses his confusion by stating, "… nothing is left…" (line 87b). Rosina, however, is initially more positive about her approach. She asserts that the measurements for length and breadth were not given (line 89c – e), but she becomes a little more tentative as she explains her working. Finally she concludes with, "I did it like that" (line 89r), which, coupled with the fact that she makes no attempt to verify her results, gives the impression that she is not at all sure about what she has done.

Relationships

Danny makes references to *he* (lines 84a and 86b) which refers to Tsepo, the person in the problem, and he makes reference to *you* (line 84d) which seems to be the person or people (students) who are doing the problem. He uses *you* in the utterance, “you change length by 2 metres” (line 84d – e) whereas he uses *he* when he says, “he decreases by 1” (line 86b – c). In the first instance Danny does not seem to see any problem with his assumption of the length being 2 metres, but when the breadth is reduced by 1 metre a problem does occur, so he changes the subject of his utterance to *he*, implying some sort of blame that is attached to this fictitious person.

Rosina refers to *they* (line 98c) when talking about the length and breadth having not been given, but she changes to *this person* when talking about what happened in the problem. She then reverts to the first person singular in explaining what she did, and ends with, “… but he subtracted 1 metre which gave him 6 metres” (lines 89p – q), where *he* and *him* presumably refer to *this person* in the problem.

Existential level

Danny, and to a lesser extent Rosina, show a lack of confidence when they present their solutions to the problem. This results in the students attributing certain actions to different ‘players’ as they explain their solution. By doing this they can be seen to only take responsibility for certain parts of the solution, whilst other actions (the more confusing or problematic ones) are attributed to others, and are possibly seen as being out of their control.

Political level

Danny finds that he cannot do the problem and he seems to attribute this to something within the problem itself (*viz*. the reduction of the breadth by 1 metre) instead of the misinterpretation that he made initially (i.e. the original dimension of the rectangle). As a result of this he is positioned by the problem as relatively helpless, and is unable to proceed. Rosina states, “… they didn’t give us the measurements… so that we could see what this person did” (Lines 89c – g). She intimates here that this disadvantages her, but she nevertheless went on to do some calculations and gets the correct lengths of 4 metres and 6 metres. However, the comment, “I said 4 metres” (line 89i) suggests that this was a guess. So, to a lesser extent Rosina is also positioned as relatively helpless by the question in that she can only proceed with a guess.

Discussion

Both Rosina and Danny seem to ignore the context of the question and work with it from a more theoretical perspective (i.e. without any diagram or context to help with interpretation).

Furthermore, they do not make use of an unknown to express the relationship between the length and the breadth, but rather rely on supposition. This leads them to arrive at calculations that are problematic (in the case of Danny) and unverified (in the case of Rosina), and we see them questioning the problem as a result of this. Both of these students can be seen as being positioned outside of the Discourse surrounding word problems because they appear to be unable to access the necessary interpretation techniques that would allow them to make more meaningful progress towards a solution. From Danny's and Rosina's perspective it appears as though they see this word problem as not providing vital information or possibly giving incorrect information, and this inhibits them from making any further progress.

Conclusions

From the student working it seems that the students become bogged down by the textual rendition of the problem. In many cases they can only proceed by implementing some seemingly arbitrary operation on the numbers that they find within the problem. According to Bednarz and Janvier (1996), our students in South Africa come from a arithmetic background and thus find the transition to working with variables difficult. In addition to this, the evidence above suggests that in many instances the students do not seem to be able to understand the problem situation sufficiently which gives rise to arbitrary operations on the given numbers based on certain key words (Mangan, 1989). It is possible that the students thus are unable to recognise 'problem-types' (Gerofsky, 1996) and therefore have no available strategy for coping with the problem. In any event, in most cases it seems that the students do not have any method or procedure available to enable them to engage with the problem other than to simply do operations on the available numbers.

From the student talk there seems to be very little discussion about the problems from a mathematical perspective, with the utterances focusing more on the 'confusion' that they are experiencing, how they 'do not know' how to proceed and the 'lack' of information given in the problem. There is also no talk by these students of any possible purposes that there might be in doing such problems, or even any link with anything other than the activity in which they were involved (*viz*. doing the problems). The apparent lack of resources for accessing the problem situation may give rise to, or may simply be accompanied by no particular method or procedure with which to engage the problems, and this appears to place the students outside of the Discourse surrounding word problems.

Chapter 6 – The findings

Not all word problems are equal

I begin the discussion of the findings with something that emerged incidentally. From the student working it is clear that some students make progress, whilst others do not, but this progress is not uniform across the three problems. Closer inspection of the graphs of the student working shows that the students at all three sites make more progress with problem 2 (the TV rental problem) than with the other two problems. The students who used numeric procedures were not generally too successful when applying those techniques to problems 1 and 3, yet those same numeric-type processes enabled them when it came to problem 2. This caused me to look into what it is about the different problems that 'enables' or 'does not enable' students, something which was not a focus in this study, but which is nevertheless connected and interesting in light of the findings.

When choosing the problems for the extended study I attempted to find problems that paralleled those used in the initial case study in terms of genre and structure. Thus, the TV rental problem was chosen as the parallel of the watch problem because they were both 'money-type relationships' that required (from a mathematical point of view) linear relationships for their solutions. Yet in the extended study the students were able to make progress with problem 2 by using techniques involving better approximation, or a simple table of values, and only very elementary mathematical knowledge was required (although, of course, it could also be solved using more formal algebra). Thus it can be said that problem 2 could be solved by students who did not necessarily have access to the Discourse surrounding school mathematics word problems, and, given the new curriculum in South Africa, this is perhaps a more fitting problem in the context of mathematical literacy.

On the other hand, problem 1 (the rectangle problem) and problem 3 (the speed-distance-time problem) required students to have some access to the Discourse surrounding word problems in order for them to make some progress towards a solution of those problems. Although this was not a specific focus of this study, it seems that different problems are more (or less) able to elicit responses in students that indicate whether (or not) they have access to a school mathematics Discourse around word problems.

Problem 2 was drawn from a Common Task for Assessment of grade 9 learners in the new General Education and Training band (*cf*. DoE 2003a) which was compiled at a time in the changing South African curriculum when, "… a discourse belonging to mathematics education and critical applied mathematics [was being] recontextualised to the classroom" (Chritiansen, 2006; 11). Consequently, what emerged from the findings with regard to problem 2 is that the students are able to make more progress with the *English*, and not necessarily the mathematics *per se*. Thus, retrospectively, this was probably not a good question to use to gauge the extent to which students have access to a Discourse of school mathematics (as opposed to a Discourse of mathematical literacy) as they did not necessarily need to be particularly versed in a Discourse of school mathematics to cope with this particular problem because of its more functional make-up. It also may say something about what needs to be incorporated into a mathematics curriculum in order to promote access to a Discourse of school mathematics, however, that goes beyond the scope of this study.

The graphs also bring to the fore what appears to be a large discrepancy between the structural make-up of the television rental problem (problem 2) and the other two problems. As mentioned above, the students at all three sites made more progress with the television rental problem than with the other two problems. In particular, two of the students were able to access the problem quite successfully through trial-and-error or better-approximation techniques, whilst only one employed a more formal algebraic approach in solving this problem. This was not anticipated when the problems were selected, and retrospectively, it appears that the problem discourse in the television rental problem in the form of language use and register (Tobias, 2003) was easier for the students to access. This seems to be mainly because the relational aspects of the problem were less complex than the other two problems. In addition to this, the problem situation in both the rectangle problem (problem 1) and the speed-distance-time problem (problem 3) were more 'mathematical' in the sense implied in the context of the current South African curriculum[26]. By this I mean that they required more formal strategies and were not easily dealt with through intuitive approaches, which Graven and Venkat (2007) refer to as *maths for induction into mathematical working*. The television rental problem (problem 2) on the other hand lent itself to intuitive, investigative techniques which are less mathematically rigorous, and we could see this problem as, "… relevant and practical [having] utilitarian value and [able to] be applied to many aspects of life" (Graven

[26] See DoE (2003b:9) for a more detailed discussion of what is expected from a student of mathematical literacy as opposed to a student of mathematics in the current South African curriculum.

and Venkat, 2007: 68 – 69; DoE, 2003b). This classification of the problem would orientate it as having a *greater integration between mathematics and the context*, which, according to Graven and Venkat (2007) places the problem more within a mathematical literacy curriculum rather than mathematics in the new educational reform in South Africa. This also confirms what emerged in the study by Mulligan (1992) that students coped adequately with problems in which they could implement intuitive notions, but when the problems became more complex (possibly requiring higher mathematical demand) the students' intuition 'runs out'.

Whilst this phenomenon emerged through the analysis, the methodology presented here does not cater for a fuller investigation of the nature of word problems in terms of 'enabling' or 'not enabling' students within the mathematical Discourse surrounding school word problems. However, I suggest that this has implications for curriculum design and classroom practice, especially in the context of the new curriculum reform in South Africa with respect to mathematics and mathematical literacy, and this possibly deserves further investigation within a discursive paradigm.

The discourse

Here I consider the 'small d' discourse (the use of language *per se*, and, in the context of mathematics, the use of mathematical register) to put forward what students use in doing and talking about their experiences with word problems (*viz*. the findings relative to research sub-question 1). I discuss these discourses with specific reference to, firstly, what enabled students to make progress with word problems, and secondly, what inhibited progress with the problems.

1. *The discourses of students who made at least some meaningful progress*

Those students who show some sort of reliance on a method or procedure (using a table, or assigning an unknown) were more enabled when doing the problems. Coupled to this is the recognition of a particular problem genre (Gerofsky, 1996), and the recognition of the problem structure, or what is required mathematically (setting up auxiliary expressions (Lepik, 1990) or use of a formula), the latter of which is related to the student ability to recognise the problem situation and hence, at least to some degree, correctly set up the auxiliary expressions.

With respect to student talk about word problems we see, at the experiential level, students made links between these word problems and other learning areas (even within mathematics), they talked about mathematical ‘objects’ (such as equations) and attached significance in what they did with the problems (both in terms of what they could do and in terms of where they perceived to fail at the problem). At the existential level, students were able to situate themselves and others as active role players (subordinate or otherwise) in the doing of word problems, and it can be said that they saw themselves as having some agency when it came to word problems (i.e. they were not just passive recipients of a problem with no course of action open to them). At the political level students attached some ‘worth’ to word problems (more often quite simply stated, such as ‘they help you with maths’) and hence there was a need to master them.

2. *The discourses of students who were unable to make progress*

Some of the students in doing the problems showed no indication of using any method or procedure and were consequently unable to make any meaningful progress. Others who could not make progress had used some method or procedure (such as a table) but the implementation of this inhibited (rather than enabled) a solution of the problem (because, for example, elements of the table were incorrectly assigned or related). There were still others whose working did not demonstrate any apparent method or procedure, which were in some cases what appeared to be ‘random’ operations on the numbers given in the problems, or consisted of a written response purporting to answer the problem question (as for example with Hartman). Some of these ‘unsuccessful’ students did recognise a genre and even the structure of the problem (evidenced in, for example, their recall of a formula or the use of a particular table) but they were unable to make use of these to help in the solution of the problems. In all of the cases in which the students made no meaningful progress there was, at most, a rudimentary understanding of the problem situation.

In terms of the student talk, at the experiential level these students either made no reference to, or were not able to see the relevance of links between word problems and other aspects of mathematics or learning areas more generally. Their talk demonstrated either limited, or was devoid of appropriate mathematical register, and the significance that they attached to word problems in all cases was exasperation at their inability to do the problems. At the existential level, most of these students situated themselves as subordinate within the context of word problems (sometimes even to their peers). At the political level the students expressed feelings of disempowerment, and all except one of the students made no mention of any perceived worth that word problems might have, with Gio explicitly stating that word problems would not be of any use in later life.

Much of what is discussed here about the discourses is not entirely unexpected. Students who are more enabled when it comes to doing word problems have no doubt been schooled in doing word problems and have developed a register with which to talk about the problems, and strategies with which to do the problems. As a natural consequence students would become less averse to the problems, experience feelings of accomplishment and thus see themselves as having the necessary agency when confronted with a word problem. Students who were unable to make progress with the problems may have been schooled in the problems, but will not have mastered the register, and thus not have the tools to engage with the problems at a suitable level that would enable a solution. The natural consequence would be feelings of inadequacy and ineptitude, together with feelings of disempowerment.

What is pertinent to this study is that the discourses of such students are made explicit in order to understand the Discourse models that underpin such discourses, so that a socio-situated perspective of why students are 'enabled' or 'not enabled' can be understood.

The Discourse models

From the discourses that emerged here it is possible to discern a number of Discourse models at play, as resulted in the initial case study. However, using the lens that the seven building tasks (Gee, 2005) have now enabled, two master models came to the fore (*viz*. the findings for research sub-question 2). I now turn to Moschkovich (1996) to develop a language to describe these master models.

Moschkovich (1996) used the terms *discontinuity model* and *situated model* to describe how, in a discourse analysis, language is perceived to mediate between talk and the learning that is taking place in students who are learning mathematics in more than one language. The discontinuity model came to be viewed as a deficit model in essence because the students' experiences and use of language were seen to be 'obstacles' to learning. The situated model as a view of learning and teaching mathematics in this multilingual context, was posed as a view that incorporated such factors as the problem context, representation and how things are done, together with the social context, identities of those involved, etc.

I draw on Moschkovich's terminology as a means of articulating observations from what emerges in the analysis. This requires that we shift the focus from a discursive view of learning and teaching (Moschkovich, 1999; 1996) to one that views students enacting a Discourse (Gee, 2005; 1999) with its associated Discourse models that guide how we make sense of the world within the context of that Discourse.

With this in mind, the analysis revealed two broad categorisations of the students who took part in this research. Roxanne, Ayanda and Roenel (from Corona) along with Sipho (from Duskhaven) each (in their own way) do and say things that suggest that they are 'enabled' in their approach to word problems. As discussed earlier, this has nothing to do with them correctly solving the problem, but rather refers to the extent to which they come to be able to engage the problem. The evidence from the discourses indicates that: firstly, these students locate the problems within the Discourse of school word problems, sometimes according to genre, (Gerofsky, 1996); secondly, that they have certain tools with which they are able to engage the problem in some mathematically meaningful manner (*viz*. some strategy, method or procedure); and thirdly, they perceive some relevance or meaning attached to the problems. In this sense these students react to word problems based upon their understanding of where and how these problems fit into the Discourse (the problem context), how they might handle the problems (the representation) and how they see them in the 'real world' (the social context). In view of the congruence that this holds with Moschkovich's view of learning it seems appropriate to talk about the 'belief structures' upon which these students appear to be acting as a *situated Discourse model*.

The second broad categorisation of students involves Gio and Nina (from Duskhaven) and Mafifo, Danny, Hartman and Rosina (from Manumission). The evidence from the working and talk from these students suggests that they generally do not have adequate methods or procedures at their disposal in order to engage the problems, and in many instances they do not understand or they misinterpret the problem context, and they either do not refer to or have naïve conceptions of the relevance or any possible benefits that the problems may have. In this case there is not a direct mapping with the discontinuity model described by Moschkovich (1996), but there is a definite link with the notion of 'what is missing' and 'what obstructs', and it therefore seems appropriate to talk about the 'belief structures' upon which these students appear to be acting as a *deficit Discourse model*.

One student that has not been located within these two categorisations is Lerato (from Corona). Limited evidence in the form of workings and some of her verbal contributions indicate that Lerato may have been located as responding to a *deficit Discourse model*. However, there is other evidence that suggests that she was more peripheral to the Discourse surrounding word problems, and was enacting some sort of *peer acceptance Discourse model* (in the face of contributions made by the others in the group) that, for her, would not render her as being outside of the Discourse. Of course, it is likely that all of the students are responding to some sort of *peer acceptance Discourse model*, perhaps not quite in the same way as Lerato, but this was not a focus of this study.

The Discourse models and the socio-situatedness of the students

From the foregoing, it can be said that the students responding to a *situated Discourse model* become empowered within the Discourse surrounding school-type word problems, even if this is only in the limited capacity of allowing them to make seemingly (and especially from their perspective) meaningful progress with the problem. Those students in this study responding to a *deficit Discourse model* become positioned outside of the Discourse surrounding word problems since they come to see themselves as lacking the tools (the methods or procedures) and the discourse (the mathematical register) with which to engage the problems. The important point here is that *they come to see themselves* in this light! In other words, they are well aware of their 'exclusion' from the Discourse and we see this particularly in many instances in what the students say. However, it is not as simple as just *being excluded* from the Discourse. These students are compelled to participate, despite their possible exclusion, by virtue of them being students within the Discourse of schooling, with its associated Discourse models. Therefore these students become part of a sub-Discourse (within the Discourse surrounding word problems) based upon the *deficit Discourse model* at play.

At Duskhaven, Nina and Gio allude to being 'victims' of having had to emigrate from their country of birth where they both perceived themselves as having coped reasonably well in mathematics. This is evident in Nina's utterances, and confirmed by the way in which Gio responds. Whatever the circumstances, Nina and Gio collaborate on this issue and come to see themselves as having 'missed out'.

At Manumission, we see talk about the confusion created by the problems, the omissions in the problems, the lack of clarity in the problems, all of which lead to the students' apparent inability to understand those problems. This talk alludes to authority figures within the Discourse surrounding word problems (such as teachers, curriculum designers and even the researcher in this instance), but they suggest that something has been 'left out' of the problem, or the wording is not clear that gives rise to their lack of understanding and confusion. This also happens in the initial case study and it interestingly comes through from a different analytical focus. The students, through an empirical perception of the Discourse model *word problems are obfuscatory* come to be positioned in the Discourse of school mathematics word problems as being relatively mathematically helpless (Tobias, 2006; 2005; 2004).

With the conception of a *situated Discourse model* comes a background in which daily activity and expectations derive from a success model (similar to that described by Boaler, 1996). To paraphrase Gee (2005), for those students, "… daily observations and social practices reinforce explicit ideological learning in regard to the [*situated Discourse model*]" (p. 82). The students come to see themselves (their participation) in terms of the *situated Discourse model*, because they judge themselves by the success of others, and the expectations of the 'school' (Gee, 2005: 83). When they find themselves 'lacking' with regard to the *situated Discourse model* (i.e. they do not have the tools or the register, and cannot locate the problems within the Discourse), they come to enact the *deficit Discourse model*. Thus, the students in this study who are positioned outside of the Discourse surrounding word problems for the most part appear to be marginalized. It may be more severe – they may have been 'colonized' by the Discourse model (Gee, 2005; 1999). This is to say that they become participants in a Discourse that they see as an *inescapable fact of life* (Gee, 2005: 82). In addition to this, the *deficit Discourse model* appears to evolve around a similar frame of reference to that found in the Discourse model, *word problems are obfuscatory*, and in order to justify and rationalise this, the students come to position themselves as 'victims' of circumstances. In the initial case study the students became victims of the obfuscatory word problems, which gave rise to feelings of ineptitude, and which in turn came to be perceived as inadequacies. In the case of Nina and Gio they see themselves as being victims of having had to emigrate from the country of origin where they saw themselves as 'successful' mathematically. In the case of the Manumission students they see themselves as 'victims' by exclusion. In all of these cases, however, the students come to be positioned outside of the Discourse surrounding word problems in that they are relatively mathematically helpless (Tobias, 2006; 2005; 2004).

The findings with respect to empirical and putative cultural models in the initial case study help to elucidate this scenario. Empirical Discourse models (to incorporate Gee's amended terminology) are those models that the students enact, whilst the putative Discourse models are those models that the students merely espouse and do not act upon. When these models are in conflict with one another we have the potential for the students to become marginalised. The students who are outside of the Discourse surrounding word problems hold a putative Discourse model in the form of the *situated Discourse model* which they cannot enact because they lack the tools in the form of methods and procedures, the disourses in the form of the mathematical register and the ability to talk about the problems, as well as the notion of how the word problems fit into 'the bigger picture'. The *situated Discourse model* becomes for the students what is 'normal' or 'right' (Gee, 1992), yet they are unable to enact this success model, and hence they find themselves entrapped by the *deficit Discourse model*, which gives rise to a sub-Discourse within the Discourse of school word problems. This sub-Discourse into which the students have become marginalised elicits student behavious that attempts to negate the *situated Discourse model* by emphasising the 'good' in what these students do and the 'bad' in others (van Dijk, 2001), which manifests in this study as a form of victimisation. Consequently, the *situated Discourse model* as a success model becomes more elusive for these students and they are thus in danger of becoming 'colonized' (Gee, 2005) or unable to escape the sub-Discourse in which they find themselves.

It is also interesting to note that the students mentioned here come from three different schools, whose development over the past fifteen years have taken quite different paths since the introduction of democracy in South Africa. There might be aspects of these different developmental paths that help elucidate this colonising effect upon the students. However, all that we can deduce from what emerges in the analysis is that the colonising effect appears to manifest in very similar ways, *viz*. through a perception of relative mathematical helplessness, despite the differences in the schools.

Student interpretation of word problems

This study was not geared to address issues of marginalisation or colonisation. The research question was: what is it that students do that is appropriate or inappropriate in interpreting word problems? From a socio-situated perspective, this study has shown that the student activity referred to in this question is what the student 'does' in the social setting that is appropriate (or inappropriate), as apposed to what the student does with the word problem *per se*. In other words, how the student acts and responds in the social setting has a significant bearing on how s/he will cope with word problems. That is not to say that how students cope with word problems has no bearing on how they come to be positioned in the social setting. These two perspectives are inextricably linked, and it is in fact the latter that has made the former visible in this study.

However, the socio-situated perspective that emerges from this study of students who do not have access to the Discourse surrounding word problems is a disturbing one, mainly because there are no immediate solutions to the problems. From a more positive position, though, in making this problem 'visible' from a socio-situated perspective we now have another explanation of student difficulties with word problems that will hopefully open avenues for new investigative approaches to this seeming impasse.

This study raises previously asked questions such as, "Are these word problems worth knowing?" (Gerofsky, 1996), but we now see these questions in the light of how word problems form part of school mathematics within the current reform curriculum in South Africa. As mentioned in chapter 1, the problems still appear in the newer text books, even though they sometimes go by different 'names', and that these problems are nearly always very similar in structure to the word problems that appeared prior to the new curriculum in South Africa. In cases where the problems have been geared to more life-like situations (such as the TV rental problem) it appears that they may have limited worth in giving students access to a Discourse of mathematics because they can often be done more intuitively without the algebraic manipulations that underscore a Discourse of mathematics.

Chapter 7 – Concluding remarks

In the initial case study, student ‘beliefs’ about word problems were examined in depth (at one site) using groupings of cultural models (Gee, 1999) as tools for that analysis. From this analysis a complex web of cultural models and sub-models emerged that in certain ways helped to explain what the students were experiencing with word problems, and how they came to be positioned with respect to these problems. However, even though this gave insight into *how* students responded to word problems, it did not adequately interrogate the more illusive *why* they responded in this way, which is ultimately of more concern to us as researchers and educational practitioners more generally.

In the extended study the field was expanded and an elaborated and refined analytic structure was adopted to redress the shortcomings found in the initial case study. Gee’s building tasks provided this different lens for examining the text produced by the students, but the extended study also provided the opportunity to examine students in different learning environments (Adler, 2001) that opened the way to seeing the commonly held beliefs and assumptions to which the students were responding. These, of course, are the Discourse models referred to by Gee (2005) that lie at the heart of *why* students do what they do when it comes to word problems. As stated in chapter 6, the historical differences between the schools could have impacted the students in different ways (and this was not a focus of the study), but what did emerge from the different sites was a similar pattern of marginalized behaviour from the students.

As much as these Discourse models exist in the social interaction (in this case the research situation), they are phenomena and emerge as such, without any particular way in which to speak about them. As Gee (2005) himself points out, they are more often unconscious in the minds of those acting upon them, so how much more so is this for those trying to research these phenomena. This was a difficulty in the analytic process, as well as in the reporting of the study, particularly in terms of how to reify the phenomena so as to be able to address the affects that they might be having upon the students.

What was needed was a ‘name’ to describe the Discourse models that were at play. In this instance the work done by Moschkovich (1996) paralleled what was emerging in this study, even though her ideas were situated in a theory of learning and teaching. The congruence was such that it fruitfully transferred to a theory of how students act within a particular Discourse, and hence aptly described the Discourse models at play, *viz*. a *situated Discourse model* and a *deficit Discourse model*.

Methodologically, the initial case study shed light on how students respond to word problems. Many would probably react by saying that we already know *how* students respond to word problems, and I would concur. However, the insight that the initial case study provided for this research went a little further than just answering the 'how'. It clarified this through the identification of a myriad of *basic assumptions* (Gee, 1999) underlying the student positioning with respect to word problems that we do not ordinarily see, and it was through this insight that this study was able to begin to address the 'why'.

The extended study has begun to address *why* students do what they do with word problems from a socio-situated perspective. It has by no means answered all the questions, but it has taken a peek at the problem from a somewhat different perspective to what seems to have gone before.

Observations

It is not surprising that in the analysis of the extended study those students who had a better understanding of the problem situation were generally able to make more progress with the problems than those who seemed to battle with the context of the problem. However, we see from the worked problems and the student talk, that access to the Discourse surrounding word problems (within the Discourse of school mathematics) seems to enable students, both in the ability to make progress with the problem as well as to engage in meaningful discussions about the problems. Those students who, from their discourse appear to be more peripheral to the Discourse surrounding word problems tend to employ seemingly arbitrary operations on given numbers, or are not able to engage the problem at all, and their talk about the problems tends to be more superficial and focused on the 'lack' of information in the problem or their confusion by the make-up of the problem.

From the analysis of the extended study it became apparent that access to a Discourse of school mathematics entails having certain 'tools' with which to engage the problems. These tools come through in the analysis as a particular method or solution strategy, which to an extent (but not totally) was envisaged as 'algorithmic procedure' in the analysis of the initial case study. Contrary to the assumptions made in the initial case study it appears from the analysis of the extended study that these ‘tools’ enable students within the Discourse surrounding word problems. From the graphs we can see that those students who had access to certain methods or procedures, or who even attempted to employ these, generally were more able to make progress with the problem (particularly problems 1 and 3) than those who appeared to have no course of action available to them.

An issue that has been raised earlier in this report is the one about language. This appears to emerge again in the context of the television rental problem. By the term ‘language’ I include aspects of non-English language learning situations (Setati and Barwell, 2006; Barwell, 2003; Moschkovich, 1999; 1996) and also issues surrounding the multi-lingual learners and the language of learning and teaching in the South African setting (Setati, 2005; 2002; 1998; Adler, 2001). Although language issues *per se* were not an intended focus of this study they are unavoidably present in the analysis in three distinct ways: the students from the initial case study were all English first language learners; Gio and Nina (in the extended study) were immigrants to South Africa and were learning English as a relatively new language at the time of the data collection; all the other students in the extended study were part of the complex multi-lingual make-up of South African students learning in English as the language of learning and teaching, which in most cases was probably their third or fourth language. Access to the English language is thus a highly complex variable that undoubtedly has significant affects on student ability to access the problems as well as their consequent performance with the problems. However, this study was set up to examine the Discourse models at play that affect students when they attempt word problems and therefore the instruments were not designed to capture aspects of access to the English language *per se*, but rather the sociological phenomena that influence students when it comes to doing word problems. The emergence of language issues in the study are nevertheless interesting and certainly open avenues for further investigation, and it was not the intention of this study to ignore or even to subvert these issues.

The contribution of this study

In the words of Gee (2005): “We are creatures of language. Evolution has seen to that.” (Preface, xii)

Implicit in this statement is how the ability to use complex language appears to make communication simpler, yet at the same time renders understanding of that communication a more intricate matter. As educationalists and researchers we cannot dispute that language is integral to the teaching and learning of our subject because it affords us the opportunity to communicate ideas. Yet that communication upon which we rely proves to be very slippery territory when examined from a discursive perspective.

As discussed in the introductory chapter, studies located in the structural make-up of word problems, the pedagogic and the cognitive aspects of solving word problems shed light on the problems that students face from those particular points of view. From a socio-situated perspective studies that limit themselves in these aforementioned ways hold too many of the variables constant. It must be acknowledged that researchers cannot possibly examine all variables in a research situation simultaneously (and in many situations the research circumstances predetermine what will be examined). However, a socio-situated lens (appropriately implemented) does appear to provide the researcher with the flexibility to adapt and/or adopt analytic tools to provide a different interpretation of the research circumstances to some of the more traditional perspectives.

Adopting and adapting the analytic tools for this study turned out to be more intricate that originally thought. Gee's building tasks (2005; 2009) initially appeared to provide illuminating tools for understanding how the students build meaning. However, when they were applied in the analysis it became apparent that they were not sufficiently well defined to capture what was happening mathematically in the student meaning-making and this led to necessary revisions and the development of a nested model that would enable me to capture the mathematical elements around meaning making in the context of school word problems. Although the development of the tools was a lengthy process the results that have emerged from this study made the effort worthwhile as it has enabled us to understand student difficulties with word problems from a socio-situated perspective.

In the main this methodological approach has enabled us to see that students enacting a *situated Discourse model* appear to become enabled within the Discourse surrounding word problems (and will presumably continue to become more enabled as time goes by), whilst those ascribing to a *deficit Discourse model* appear to be peripheral or even outside of the Discourse surrounding word problems (which it seems is unlikely to improve without some sort of intervention). Therefore one scenario that this methodological approach does seem to be able to uncover is that students enact a particular Discourse as a result of how enabled they have become within that Discourse, as demonstrated by the Discourse model that they enact. On the surface this may seem to be a very simplistic notion of enabling students through learning–teaching, but when viewed in terms of how students understand how the problem 'fits in' (i.e. to the Discourse), how the problem can be manipulated (in terms of the tools available) and what the problem means to the student in their everyday lives (or the ameliorative connotations that derive from the Discourse model), we begin to see that this is not just a repeat of your everyday teaching-learning scenario.

The students enacting a *deficit Discourse model* are doing so because their understanding of these aforementioned factors is rudimentary at best, but what is disturbing is that these students are in danger of becoming marginalised through this 'learning' process (whether this is intentional or unintentional). That these same students in some instances can employ intuitive techniques indicates that language *per se* (as indicated by Murray, 2003) is not the issue in any simple sense, and that there are other issues around the mathematical demand of the word problem. The implications of this, as I see them, are that teachers need to be acutely aware of exactly what it is that students need to know to be able to become included in the Discourse surrounding word problems. This research therefore seems to give impetus to the research into areas such as *pedagogic content knowledge* (Shulman, 1987) since it alludes to the possible dangers of 'uninformed' pedagogic practices that, I suggest, are part and parcel of many of our current South African mathematics classrooms.

Going forward

Viewing students who have or do not have access to the Discourse of school mathematics word problems from a socio-situated perspective requires that we understand that they are responding (respectively) to a *situated Discourse model* and a *deficit Discourse model*. Thus we have come to see the problem of student difficulties with word problems through a different lens to that provided by studies examining the structural make-up of word problems, the pedagogy of word problems and student cognitive functioning with respect to word problems. This socio-situated lens, however, does not (as yet) provide answers for what is to be done with this different understanding of student difficulties with word problems, especially in regard to improving student access to the Discourse of school mathematics word problems.

In conclusion

This study shows that a discourse analysis is capable of spotlighting problems from a different perspective. At this point it only highlights this different perspective and does not purport to offer solutions to the problem *per se*. However, what has also been demonstrated in this study is that the methodology and the analytic tools can be adapted and refined so as to more appropriately address the research question and this bodes well for potential discursive research around addressing more practical solutions to the phenomena revealed by this study. What has also emerged from this study, though, is that this methodological and analytical refinement is no simple matter because the constructs in any discursive study are not easily communicated, making it at times a frustrating and time-consuming process to identify, and put the necessary language in place in order to report on the phenomena. But then again, understanding human action and interaction has fascinated people through the ages *because* it is rich and complex. In the words of Gee (2005):

> "…speakers and writers use the resources of grammar to *design* their sentences and texts in ways that communicate their perspectives on reality, carry out various social activities, … and allow them to enact different social identities… We are all designers – artists, in a sense – in this respect. Our medium is language." (p. 5)

As discursive researchers we deliver critiques of the pictures that are painted in language in order to better understand how those pictures were created.

List of references

Adler, J. (2001). *Teaching and learning mathematics in multilingual classrooms.* Dordrecht, Netherlands: Kluwer Academic Publishers.

Adler, J. and Lerman, S. (2003). Getting the description right and making it count: Ethical practice in mathematics education research. In Bishop, A., Clements, K., Kietel, C., Kilpatrick, J. and Leung, F. (Eds.) *Second International Handbook of Mathematics Education*. Dordrecht, Netherlans: Kluwer Academic Publishers, pp. 441 – 470.

Bailey, R.W. (1985). Negotiation and meaning: revisiting the context of situation. In Benson, J.D. and Greaves, W.S. (Eds.) *Systemic perspectives on discourse*, Volume 2. Norwood, New Jersey: Ablex Publishing, pp. 1 – 17.

Barwell R (2003). Patterns of attention in the interaction of primary school mathematics students with English as an additional language. *Educational Studies in Mathematics,* 53, 35-59.

Bednarz N & Janvier B (1996). Emergence and development of algebra as a problem-solving tool: Continuities and discontinuities with arithmetic. In Bednarz, N, Kieran, C & Lee, L (eds). *Approaches to algebra: perspectives for research and teaching*. Dordrecht: Kluwer Academic Publishers, pp. 115 – 136.

Boaler, J. (1993). The role of contexts in the mathematical classroom: Do they make mathematics real? *For the learning of Mathematics* 13 (2), pp. 12 – 17.

Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham: Open University Press.

Bruner, J. (1996). *The culture of education*. Massachusetts: Harvard University Press.

Burton, M. (1991). Grammatical translation-inhibitors in two classic word problem sentences. *For the Learning of Mathematics* 11(1), pp. 43 – 46.

Burman, E. and Parker, I. (1993a). Introduction – discourse analysis: The turn to the text. In Burman, E. and Parker, I (Eds.) *Discourse analytic research: repertoires and readings of texts in action*. New York: Routledge, pp. 1 – 13.

Burman, E. and Parker, I. (1993b). Against discursive imperialism, empiricism and constructionism: Thirty-two problems with discourse analysis. In Burman, E. and Parker, I. (Eds.) *Discourse analytic research: repertoires and readings of text in action*. London: Routledge, pp. 155 – 172.

Case, J. and Marshall, D. (2008). The 'no problem' Discourse model: Exploring an alternative way of researching student learning. *International Journal of Educational Research* 47, pp. 200 – 207.

Christiansen, I. (2006). Mathematical literacy as a school subject: Failing the progressive vision? *Pythagoras* 64 (4), pp. 6 – 13.

Christie, F. (2002). *Classroom discourse analysis*. London: Continuum.

Christou, C. and Philippou, G. (1998a). The developmental nature of ability to solve one-step word problems. *Journal for Research in Mathematics Education* 29 (4), pp. 436 – 442.

Christou, C. and Philippou, G. (1998b). The structure of students' beliefs towards the teaching of mathematics: Proposing and testing a structural model. In Olivier, A. and Newstead, K. (Eds.) *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics education*. Volume 2. Stellenbosch: South Africa, pp. 192 – 199.

Cohen, L., Manion, L. and Morrison, K. (2000). *Research methods in education*. London: Routledge Falmer.

Cooper, B. and Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham: Open University Press.

Cortes A (1998). Implicit cognitive work in putting word problems into equation form. In Olivier, A &

Newstead, K (eds). *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education*, Volume 2. South Africa: University of Stellenbosch, pp. 208 – 215.

D'Andrade, R. and Strauss, C. (Eds.) (1992). *Human motives and cultural models*. Cambridge: Cambridge University Press.

Davis, Z. (2003). Free associations: From canned laughter to relevant mathematics to the tamagochi. *Pythagoras* 58(3), pp. 2 - 6.

De Bock, D., van Dooren, W., Janssens, D. and Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics* 50, pp. 311 – 334.

De Corte, E. and Verschaffel, L. (1989). Teaching word problems in the primary school: What research has to say to the teacher. In Greer, B. and Mulhern, G. (Eds.) *New directions in mathematics education*. London: Routledge, pp. 85 – 106.

DoE. (2003a). *Mathematical literacy, mathematics and mathematical sciences: Common task for assessment – grade 9, Section B*. Pretoria: Department of Education.

DoE. (2003b). *National curriculum statement grades 10 – 12: Mathematical literacy, Section B*. Pretoria: Department of Education.

DoE. (2005). *National curriculum statement grades 10 – 12: Learning programme guidelines, mathematics*. Pretoria: Department of Education.

Ely, M. (1991). *Doing qualitative research*. London: The Falmer Press.

Fairclough, N. (1989). *Language and power*. London: Longman.

Fairclough, N. (1999). Linguistic and intertextual analysis within discourse analysis. In Jaworski, A. and Coupland, N. (Eds.) *The discourse reader*. London: Routledge, pp 183 – 211.

Fairclough, N. (2001). Critical discourse analysis as a method in social scientific research. In Wodak, R & Meyer, M (eds). *Methods of critical discourse analysis*. London: SAGE Publishers, pp. 121 – 138.

Ferrucci, B., Yeap, B. and Carter, J. (2001). Achieving understanding in solving word problems. *Pythagoras* 54(1), pp. 26 – 29.

Gee, J. P. (1990). *Social linguistics and literacies: Ideology in Discourses*. London: Falmer Press.

Gee, J. P. (1992a). Thinking, learning, and reading: The situated sociocultural mind. In Duranti, A & Goodwin, C (eds). *Rethinking context: language as an interactive phenomena*. New York: Cambridge University Press, pp. 235 – 257.

Gee, J. P. (1992b). *The social mind: Language, ideology and social practice*. New York: Bergin and Garvey.

Gee, J. P. (1999). *An introduction to discourse analysis: Theory and method*. London: Routledge.

Gee, J. P. (2001). Identity as an analytic lens for research in education. In Secada, W. (Ed.) *Review of research in Education*, Volume 2. Washington, DC: American Research Association, pp. 99 – 125.

Gee, J. P. (2005). *An introduction to discourse analysis: Theory and method*, 2nd ed. London: Routledge.

Gerofsky, S. (1996). A linguistic and narrative view of word problems in mathematics education. *For the Learning of Mathematics* 16(2), pp. 36 – 45.

Gerofsky, S. (1999). Genre analysis as a way of understanding pedagogy in mathematics education. *For the Learning of Mathematics* 19(3), pp. 36 – 46.

Graven, M. and Venkat, H. (2007). Emerging pedagogic agendas in the teaching of mathematical literacy. *African Journal of Research in Mathematics, Science and Technology Education* 11 (2), pp. 67 – 84.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction* 7 (4), pp. 293 – 307.

Groenewald, M., Minshall, G., Otto, H., Roos, H. and van der Westhuizen, G. (2006). *Maths today, grade 9*. Cape Town: Maskew Miller Longman.

Halliday, M. A. K. (1994). *An introduction to functional grammar*, 2nd ed. London: Edward Arnold.

Hegarty, M., Mayer, R.E. and Monk, C.A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of educational Psychology*, 87(1), pp. 18 – 32.

Holland, D. and Quinn, N. (Eds.). (1987). *Cultural models in language and thought*. Cambridge, UK: MIT Press.

Jäger, S. (2001). Discourse and knowledge: Theoretical and methodological aspects of a critical discourse and dispositive analysis. In Wodak, R & Meyer, M (eds). *Methods of critical discourse analysis*. London: SAGE Publishers, pp. 32 – 62.

Janks, H. (2005). Language and the design of texts. *English Teaching: Practice and Critique*, *4*(3), 97 – 110.

Jaworski, A. and Coupland, N. (1999). Introduction: Perspectives on discourse analysis. In Jaworski, A. and Coupland, N. (Eds.) *The discourse reader*. London: Routledge, pp 1 – 4.

Julie, C. and Mbekwa, M. (2005). What would grade 8 to 10 learners prefer as context for mathematical literacy? The case of Masilakele Secondary School. *Perspectives in Education* 23(3), pp 31 – 43.

Kutscher, B. and Linchevski, H. (1997). Number instantation as mediators in solving word problems. In Pehkonen, E. (Ed.) *The Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, Volume 3. Finland: University of Helsinki, pp. 168 – 175.

Laridon, P., Burgess, A., Jawurek, A., Kitto, A., Pike, M., Strauss, J., Strimling, L. and Wilson, H. (1992). *Classroom mathematics standard 7*, second edition. Johannesburg: Lexicon Publishers, pp. 257 – 259.

Lave, J. and Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Lepik, M. (1990). Algebraic word problems: Role of linguistic and structural variables. *Educational Studies in Mathematics* 21, pp. 83 – 90.

Light, P. (1986). Context, conservation and conversation. In Richards, M. and Light, P. (Eds.) Children of social worlds. Cambridge: Polity Press, pp. 170 - 190.

Lins, R., and Kaput, J. (2004). The early development of algebraic reasoning: The current state of the field. In Stacey, K Chick, H & Kendal M (eds). *The future of the teaching and learning of algebra: The 12th ICMI Study*. Dordrecht: Kluwer Academic Publishers, pp. 47 – 70.

Malcolm, K. (1985). Communication linguistics: A sample analysis. In Benson, J.D. and Greaves, W.S. (Eds.) *Systemic perspectives on discourse*, Volume 2. Norwood, New Jersey: Ablex Publishing, pp. 136 – 151.

Mangan, C. (1989). Multiplication and division as models of situations: What research has to say to the teacher. In Greer, B. and Mulhern, G. (Eds.) *New directions in mathematics education*. London: Routledge, pp. 107 – 128.

Merriam, S.B. and Simpson, E.L. (1984). *A guide to research educators and trainers of adults*. Malabar, Florida: Robert E. Krieger Publishing Company.

Milne, W.J. (1987). *A mental arithmetic*. New York: NY American Book.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics* 61, pp. 219 – 245.

Moschkovich, J. (1996). Learning math in two languages. In Puig, L. and Gutierez, A. (Eds.) *Proceedings of the 20th Conference of the International Group of the Psychology of Mathematics Education, Volume 4*. Valencia: University of Valentia, pp. 27 – 34.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the learning of mathematics* 19(1), pp. 11 – 19.

Mulligan, J. (1992). Children's solutions to multiplication and division word problems: A longitudinal study. *Mathematics Educational Research Journal* 4(1), pp. 24 – 41.

Murray, H. (2003). Word problems: A problem! *Pythagoras* 58(3), pp. 39 – 41.

Parker, I. (1992). Discourse discourse: Social psychology and postmodernity. In Doherty, J., Graham, E and Malek, M. (Eds.) *Postmodernism and the social sciences*. London: MacMillan Academic and Professional Ltd., pp. 80 – 94.

Pirie, S. (1998). Crossing the gulf between thought and symbol: Language as (slippery) stepping stones. In Steinbring, H., Bussi, B. and Sierpinska, A. (Eds.) *Language and communication in the mathematics classroom*. Reston, Virginia: NCTM, pp. 7 – 29.

Potter, J. and Wetherell, M. (1987). *Discourse and social psychology*. London: SAGE Publishers.

Rojano, T. (1996). The role of problems and problem solving in the development of algebra. In Bednarz, N Kieran, C and Lee, L (eds). *Approaches to algebra:perspectives for research and teaching.* Dordrecht: Kluwer Academic Publishers, pp. 55 – 62.

Roux, A. (2003). The impact of language proficiency on geometric thinking. In Jaffer, S. and Burgess, L. (Eds.) Proceedings of the Ninth National Congress of the Association of Mathematics Education of South Africa. Volume 1, South Africa: Cape Town, pp. 362 - 371.

Rowland, T. (1995). Between the lines: The languages of mathematics. In Anghileri, J. (Ed.) *Children's mathematical thinking in the primary years*. London: Cassell, pp. 54 – 73.

Schegloff, E.A. (1999). Talk and social structure. In Jaworski, A. and Coupland, N. (Eds.) *The discourse reader*. London: Routledge, pp 107 – 120.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In Grouws, DA (ed). *Handbook of research on mathematics teaching and learning*. New York: Macmillan Publishing Company, pp. 334 – 370.

Schumacher, S. and McMillan, J. (1993). *Research in education*, third edition. New York: Harper Collins College Publishers.

Setati, M. (1998). Code-switching in a senior primary class of second language learners. *For the Learning of Mathematics*, 18(2), pp. 114 – 160.

Setati, M. (2002). *Language practices in intermediate multilingual mathematics classrooms*. Unpublished Ph. D. Thesis. Johannesburg: University of the Witwatersrand.

Setati, M. (2003). Re-presenting multilingual data. In Putsoa, B., Dlamini, M., Dlamini, B. and Kelly, V. (Eds.) *Proceedings of the 11th Annual Conference of the Southern African Association of Research in Mathematics, Science and Technology Education*. Swaziland: University of Swaziland.

Setati, M. (2005). Teaching mathematics in a multilingual primary classroom. *Journal for Research in Mathematics Education*, 25, pp 20 – 23.

Setati, M. and Barwell, R. (2006). Discursive practices in multilingual mathematics classrooms: An international comparison. *African Journal of Research into Science, Mathematics and Technology Education* 10(2), pp. 27 – 38.

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematics learning. *Educational Studies in Mathematics*, 46, pp. 13 – 57.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57 (1), pp. 1 - 21.

Silverman, D. (2000). *Doing qualitative research: A practical handbook*. London: Sage Publications.

Silverman, D. (2001). *Interpreting qualitative data: Methods for analysing talk, text and interaction.* London: Sage Publications.

Solomon, R. C. (1995). *Pure mathematics*. London: John Murray Publishers.

Stillman, G. (1996). Mathematical processing and cognitive demand in problem solving. *Mathematics Education Research Journal*, 8 (2), pp. 174 – 197.

Terre Blanche, M.J. (1997). 'The knowledge one seeks to disinter': Psychiatry and the discourse of discourse analysis. In Levett, A., Knotter, A., Burman, E. and Parker, I. (Eds.) *Culture, power and difference: discourse analysis in South Africa*. London: Zed Books Ltd.

Tobias, B. (2003). Do you speak mathematics? *Pythagoras* 58(3), pp. 21 – 26.

Tobias, B. (2004). From textual problems to mathematical relationships: Case studies of secondary school boys and the discourses at play in interpreting word problems. A paper presented at *the Annual Conference of the Association of Mathematics Educators of South Africa, University of the North West.*

Tobias, B. (2005). Mathematical word problems: The student's voice. *Perspectives in Education*, 23(3), pp.61 - 75.

Tobias, B. (2006). Mathematical word problems: Understanding how secondary students position themselves. *African Journal of Research in Mathematics, Science and Technology Education*, 10(2), 1 – 14.

Toom, A. (1999). Word problems: Applications or mental manipulatives. *For the Learning of Mathematics* 19(1), pp. 36 – 38.

Van Dijk, T. A. (1997). Discourse as interaction in Society. In Van Dijk, TA (ed). *Discourse as social interaction*. London: SAGE Publishers, pp. 1 – 37.

Van Dijk, T. A. (2001). Multidisciplinary CDA: A plea for diversity. In Wodak, R & Meyer, M (eds). *Methods of critical discourse analysis*. London: SAGE Publishers, pp. 95 – 120.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Widdicombe, S. (1993). Autobiography and change: Rhetoric and authenticity of 'Gothic' style. In Burman, E. and Parker, I. (Eds.) *Discourse analytic research: repertoires and readings of text in action*. London: Routledge, pp. 94 – 113.

Wolcott, H.F. (1988). Ethnographic research in education. In Jaeger, R.M. (Ed.) *Complementary methods for research in education*. Washington D.C.: American Educational Research Association, pp. 187 – 206.

Annexure 1 – Letter of consent sent out to participants and their parents

17 August 2005

Dear Mr and Mrs __________________ and __________________

Request for your son/daughter to participate in a research project

I have been a secondary school mathematics teacher for the past twenty years and I am currently reading for a PhD at the University of the Witwatersrand. My thesis examines the influences that are at play when grade 10 learners interpret word-type problems into mathematical relationships.

Your principal, Mr ***** and the Gauteng Department of Education have given me permission to do research at your son's / daughter's school. This does not mean that your son/daughter has to participate in this study and there will be no recriminations should he/she choose not to participate. However, should you and you son/daughter decide to grant permission to participate I will give the following guarantees of anonymity and confidentiality (which means that I will use a different name and not give your child's responses to unauthorised people):

- all learners' contributions will be given alternate names before the data analysis begins
- any people from the school referred to during the research activities will be given alternate names
- the school and any reference to named places within the school will also be given different names.

The research at (school's name) will be made up of four case studies of learners from the grade 10 year. These learners have been chosen randomly. They will be involved in some maths activities after school and I will need to see them as a group for not more than one and a half hours after school on Monday 22 August and Tuesday 23 August straight after school. Mr. *****, the Head of Mathematics at the school, has kindly agreed to organise the learners and the venue.

I now give a brief outline of what the students will be required to do. They will firstly be told about the research and then asked to write a short paragraph. Thereafter they will be asked to try a set of maths problems as best they can, but with no time limit. After this the students will be given a short questionnaire about the word problems. To end off the first afternoon the four learners will hold a discussion amongst themselves. This discussion will be video and audio recorded.

On the second afternoon I will hold a general discussion with all four learners and this will also be video and audio recorded. This discussion will be fairly informal and I will guide the discussion.

As a researcher I would like to help teachers and learners gain a better understanding of why word problems are so difficult for learners. To do this I will need to use the data from this research, and for this I will need the permission of the learners and their parents. I ask therefore that you please complete and return the attached form, whether or not your son/daughter will be participating.

I would like to inform you that I undertook three pilot studies and I have already completed the activities at two other schools for this research, and all the learners enjoyed having the chance to speak freely about their mathematical experiences. I am sure that your son/daughter will also gain from this experience, but I assure you that he/she will not have to answer any questions that he/she does not wish to answer.

Should you have any queries or need clarity on any of the issues mentioned above please feel free to contact me. I hope that I shall have the opportunity of meeting (child's name) in the research situation.

Yours faithfully

BRUCE TOBIAS

E-mail: tobias@stjohns.wits.ac.za
Cellphone: 082 877 3708

Consent form for participants and their parents

Learner: __________________________ Date: ________________

Father: ______________________________________

Mother: ______________________________________

Please tick (√) the boxes next to the statements with which you agree and place a cross (X) where you disagree:

I give consent for __________________ to participate in the study subject to the conditions contained in the letter, and subject to my specifications indicated below:

☐

Subject to the conditions contained in the above letter I give consent for the audio/video transcripts and student working and responses to questionnaires to be used in the following manner:

for research purposes by the research team ☐

for research purposes by other university research students ☐

at educational workshops, symposia or conferences ☐

in educational publications ☐

Learner's signature: ___________________________

Father's signature: ___________________________

Mother's signature: ___________________________

Annexure 2 – Student booklet for the first phase of the data collection

Grade 10

School:

Date:

Name:

Write a paragraph to explain how you feel about *word problems* in mathematics. Start by saying what you think *word problems* are, and then explain how you feel about them.

Please do not turn the page until you are asked to do so

Please read these instructions carefully:

On the following pages there are three problems.

Try to solve each of the problems.

You may try to solve the problems in any way that you wish.

You may use a calculator.

There is no time limit – you may work on the problems for as long as you need, and you may return to problems if you wish to make amendments.

Please try to show as much working as you can.

Don't scratch out your working or use correction fluid. If you wish to indicate that the working is incorrect please put a neat line through that working.

This is *not* a test. Your work will not count in any way towards marks.

Show all working on the page below the problem, and you may use the reverse of the page if you need more space.

Please only work in the black pen that has been issued to you.

Problem 1

Tseko draws a rectangle with its length 2 m more than its breadth. He then increases the length by 2 m and decreases the breadth by 1 m. He finds that the area of the new rectangle is the same as that of the first one. Find the length and breadth of Tseko's first rectangle.

Problem 2

The Clear Vision television rental shop charges a basic fee of R150, as well as R15 per day to rent a television. The Best View television rental shop only charges a basic fee of R15 but has a daily rate of R60 per day to rent. For what number of days would it make no difference in cost as to which shop you rent from?

Problem 3

A boy cycles from home to school in the morning and back in the afternoon. He cycles from home to school at 32 km/h and back at 24 km/h. It takes him 15 minutes longer in the afternoon than in the morning. Find the distance between home and school.

Name: ____________________________

You are invited to respond to the questions below. You **do not** have to write a response if you do not wish to, but it would be helpful if you could think of how you felt, or what you did when you solved the problems.

What did you think of the problems that were asked in this session?

Choose one of the problems and explain what you did to try to solve it.

Explain *how* you felt when you tried to solve this problem, and *why* you think that you felt this way.

Why do you think that word problems like these are given to learners at school?

Please write down any other comments that you would like to make about word problems.

Reflective commentary

The problems for this study were chosen from three typical genres of school mathematics word problems. More specifically, they were chosen for the way in which they placed a particular demand on the students to formulate mathematical expressions and then equate these so that a solution could be achieved. In analyzing the data it soon became apparent that many of the students had employed 'alternate' techniques to solve the problems, and in some cases these alternate procedures proved successful.

In the analytic framework I discussed how students were to be graded for the use of methods and procedures in their working, and I stated the following:

> In this study I am interested in the degree to which students have access to a Discourse of *school mathematics*. If one were more interested in access to a Discourse of mathematics (more generally) it might be argued that more rigorous, algebraic methods or procedures would be better indicators of such access. However, in the context of school mathematics certain numeric procedures (such as better approximation, or establishing a table of values) might be all that is needed to solve the problem. Thus in gauging students' methods and procedures I will not privilege any one approach over another. (*cf.* pp81 – 82)

In an attempt to achieve these ends I employed an analysis that sought to grade the student working according to how systematic it was.

Retrospectively, it appears to me that algebraic approaches offer much easier indicators of systematicity. For example, we can look for cues such as the assigning of an unknown, relating that unknown in setting up expressions and establishing equality of expressions. However, alternate solution strategies are more difficult for establishing how systematic such an approach might be, especially, for example when a student employs a trial-and-error technique. It may have been helpful for me to have included in my analytic structure some categorisation of the alternate methods into, for example, trial-and-error, better approximation, numeric tabulation, etc. With an understanding of where a particular solution strategy fits in terms of the categorisations, I feel, could have enabled me to better gauge the systematicity of such an approach through a fundamental understanding of what each different approach entails. This may also have enabled me to avoid the perceived discrepancies that have emerged as a result of my grading of student approaches to the solution of word problems in the analysis.

I also believe that a more structured categorization of solution strategies (particularly for 'alternate' methods), together with an understanding of how to gauge sytematicity of each method, would be beneficial to practicing teachers. If teachers are able to understand student learning of mathematics in terms of the extent to which they have access to the Discourse surrounding that mathematics, they could be in a better position to make better decisions about how to assist those students. A classification of solution strategies provides an initial step for teachers to understand student working, but coupled with a means to gauge how systematic such an approach might (or might not) be could offer the teacher valuable insight into student access to the Discourse of that mathematics (in this instance, the mathematics surrounding school word problems). I therefore see this as a potential tool in helping teachers to gain insight into their students' learning.

Bruce Tobias
April, 2010