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# Coherent, Correlated Phenomena resulting from the Incidence of High-energy Leptons and Photons on Oriented Crystals

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*"My mind was struck by a flash of lightning in which it's  
desire was fulfilled"...*

Dante, Paradiso Canto XXXIII - by G. Polya

I declare that this thesis is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand. It has not been submitted for any degree or examination at any other university.

Bilalayi

27<sup>TH</sup> day of FEBRUARY 1998

## Abstract

The scope of the research reported in this thesis addresses some phenomena associated with lepton-photon processes resulting from the incidence of ultra-relativistic particles upon highly ordered crystalline materials. The study of channeling phenomena were extended to investigate coherent, correlated and strong field versions of the basic QED processes at much higher energies and at incident angles slightly larger than the channeling critical angle. These investigations are based on the concept of coherence. As a result, the nature of the QED processes changes because the Lorentz boosted particles ( $\gamma \sim 10^5 - 8$ ) travelling through crystalline materials at ultra-relativistic velocities experience correlated scattering with many atomic centers. The interaction zone extends over a large region of space, the *coherence length*. This leads to coherent enhancements of the normal cross-section. Furthermore, the atomic fields reach critical values  $E \geq E_0$ , where  $E_0 = 1.32 \cdot 10^{16}$  V/cm ( $\chi \geq 1$ ). This marks the onset of strong field effects in which the radiation mechanism changes from classical synchrotron dependence  $E_\gamma^2$  ( $\chi \ll 1$ ) to the quantum synchrotron law of  $E_\gamma^{2/3}$ -dependence ( $\chi \gg 1$ ). The work of such a field  $E_0$  over the electron Compton length equals the electron mass. The formalism used to study these processes is the semi-classical Constant Field Approximation (CFA) primarily because the theoretical description of QED in such fields is beyond the framework of perturbation theory.

Thus the extra physics involved leads to either very strong enhancements or losses in cross-sections as well as dramatic changes in the shape of cross-sections. In addition, new phenomena appear which deepen an understanding of fundamental theories, and thus allowing new theoretical tests and applications. For example, an enhancement in the bremsstrahlung (BS) photon yields of more than 100 times the Bethe-Heitler cross-sections can be produced. The *polarisation and multiplicity* of these photons was

investigated.

The main thrust of this thesis, however, is the study of the enhanced pair production by aligned incidence of high energy photons ( $E_\gamma \leq 150$  GeV) in a Ge crystal at room temperature. This experimental setup was thus used as a laboratory to investigate coherent and strong field QED. The results obtained have shown that enhancements in integrated cross-sections up to four-fold the yields of the Bethe-Heitler mechanism could be obtained and these were seen to agree with the CFA theory for photons incident on the axial orientation of the germanium crystal. Furthermore, a detailed comparison of differential pair-production and the CFA calculations was carried out. This measurement is among the first in which such detailed comparisons are made. These differential pair production data also showed excellent agreement with the CFA in the whole energy range  $40 \leq E_\gamma \leq 140$  GeV.

Further investigations were then carried out to investigate the onset of the LPM effect when photons in the energy range  $10 \leq E_\gamma \leq 150$  GeV were incident on a Tungsten crystal cooled to 100 K. The LPM effect was been predicted by theory to result in suppression of pair production yields in the high-energy region of the incident photon spectrum. To date, no clear and conclusive evidence exists but the experimental data do indicate though that there could be such an inhibitive process. Taken together these results have shown that single crystals can be used to investigate and illuminate the effect of coherence and strong fields on QED processes like bremsstrahlung, pair-production and the LPM effect.

In addition, there are possibilities to investigate the applications of these systems in other physics areas. An example of this is in the induction of pair-production processes in heavy crystals in order to veto unwanted events in the upstream decay region of the CERN-NA48 CP violation experiment. Moreover, complementary studies carried out in the course of the research have alluded to the usefulness of crystals for

the bending of high-energy protons at the Large Hadron Collider.

*Ngithanda ukubonga u Mama wami ngalombhalo*

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## List of Abbreviations

- AKS : Anti-Kaon veto System
- CBS : Coherent Bremsstrahlung
- CERN : European Centre for Nuclear Research
- CFA : Constant Field Approximation
- CPP : Coherent Pair Production
- ChR : Channeling Radiation
- DC : Drift Chamber
- LG : Lead Glass
- LHC : Large Hadron Collider
- LPM : Landau Pomeranchuk Migdal effect
- MD : Multiplicity Detector (see SSD)
- MIP : Minimum Ionising Particle
- MWPC : Multi-wire Proportionality Counter
- Sc : Scintillator
- SPS : Super Proton Synchrotron
- SRCNS : Schonland Research Centre for Nuclear Sciences
- SSD : Solid State Detector (same as MD)

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# Chapter 1

## Introduction

*"Here and elsewhere, we shall not obtain the best insights into things until we actually see them growing from the beginning" ... Aristotle*

### 1.1 Historical Background

The dissertation reports on the investigation of the effects that manifest themselves when ultra-relativistic, multi-hundred GeV leptons<sup>1</sup> have oriented incidence on highly ordered crystalline matter. Over the past decade, the interaction of multi-GeV electrons and positrons with oriented crystals has been a subject of intense theoretical and experimental scrutiny.

These studies were prompted much earlier on by the discoveries of electromagnetic effects such as energy losses of MeV positive ions penetrating crystals. It was found that at incidence close to certain indices or planes, the depth penetrated into the material was greater than that obtained when the crystal was at a random orientation w.r.t. the beam [1, 2]. The conclusion reached was that there were certain orientations of crystals in which these ions experienced much lower energy loss rates. The results therefore brought about a resuscitation of a concept of the transmission of ions along open crystalline channels, which had lain dormant since Stark [3] proposed

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<sup>1</sup>This pertains specifically to electrons and positrons

it in 1912. The theoretical bedrock of these phenomena was cemented by Lindhard [5] when he conjectured a picture of channeling where ions were steered along a row of atoms, a *string*. The seminal thread in Lindhard's treatise was the concept of a continuum potential and of a criterion for channeling in the form of a critical angle of the incident beam with respect to the crystal axis or planar direction. In addition, the use of the 'transverse energy' of a channeled particle was appreciated, which together with a statistical treatment, allowed greater theoretical simplification. (The historical development of channeling is also discussed in [6, 7]). Channeling has played a pivotal role with regard to a plethora of uses like ion implantation and defect studies relevant to semi-conductor technology and well as in other fields like Particle and Astrophysics.

The channeling energies which were hitherto limited to the MeV regime were then extended into the GeV range. Investigations of various phenomena involving heavy positive and light negatively charged particles were endeavoured, amongst which were studies of the comparison of energy losses and scattering. A strong impetus to channeling was provided by *channeling radiation* [8, 9] and the subsequent experimental verification (see [11, 10] and the references therein). The promulgation of the field of channeling radiation strongly revived the field of light particle channeling where it was demonstrated how the direct electromagnetic radiation was enhanced with intensities dependent on the specific crystal potential [12]. The investigations of the influence of crystal structure on electromagnetic processes is still an enterprise of rich experimental and theoretical activity. This is evident with coherent bremsstrahlung and coherent pair-production at GeV energies which are prolific lepton-photon electromagnetic production processes. In pair production, a photon incident along a crystal axis is converted into a lepton (electron-positron) pair which may be channeled along the atomic rows and planes. Enhancements in the yields become very pronounced for GeV energies and also persist into regions far beyond the critical angles characterizing the channeling trajectories.

Recent discoveries of the enhancement in photon emissions from multi-GeV electrons incident on oriented crystals have been described within the realm of semi-classical theories, the most commonly used of which is the constant field approximation [16]. In this, there is a decoupling of the particle motion into transverse and longitudinal components. The trajectory of the particle in the longitudinal direction can be described by classical electrodynamics. This is because a full description of such a QED process is made complicated by the fact that it requires a solution of a large number of states which are proportional to the Lorentz factor ( $\gamma \approx 10^{5-6}$ ) in the Dirac equation [43]. However, notwithstanding this problem, there was a rigorous attempt by the Frankfurt group [18, 19] to provide a full QED description, invoking all the general properties of a field theory *inter alia* gauge invariance<sup>2</sup>, vacuum polarization and self-energy to provide a self consistent set of solutions. In this way, they solve the problem by invoking a kinetic equation to evaluate an electron distribution function in axially oriented single crystals. A solution was obtained based on the single-string model and the cylindrically symmetric Dirac equation. However, because the model they used is still based on a single string potential it therefore still lacks the generic features of the full lattice which will make it applicable to very interesting research areas such as the strings of strings region and planar potentials.

Consequently, the models used here are those developed by Baier [14, 16] to describe channeling radiation, and by Kononets [28, 29] which assumes channeling radiation to be basically of synchrotron nature and thereby incorporates the Constant-Field Approximation (CFA) for the evaluation of radiation probabilities. This approach is based on taking into cognizance both the coherence due to the periodicity of the crystal structure and the strong fields [13] due to the ultra-relativistic lepton incidence, and has been very successful in the description of our CERN data [21, 22, 23].

Furthermore, this investigation of the processes involved has led to further appre-

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<sup>2</sup>The term gauge invariance denotes a transformation of a physical system that obeys the symmetry of a particular mathematical group and leaves the physical state of the system unchanged.



ciation in regard to using a crystal as a laboratory to investigate the theory of strong field QED. The theory of strong field QED has attracted a lot of interest recently and turned out to be important in describing some astrophysical phenomena. Crystals have turned out to be unique for this investigation. For the incidence of GeV leptons on a crystal; the quantum parameter which sets the Schwinger Field threshold [20] is  $\chi = \gamma E/E_0$ , where  $\gamma$  is the Lorentz factor for the particle,  $E$  is the local field strength which is very high near a crystal axis and reaches values around  $10^{11}$  V/cm. Therefore, at ultra-relativistic lepton energies  $\chi$  is of the order of unity where  $E_0 = m^2 c^3 / e \hbar \approx 1.32 \cdot 10^{18}$  V/cm. For increasing incoming particle energy (increasing  $\chi$ ), the strong crystalline fields have a dramatic influence on QED processes like radiative losses which are thus turned from the classical  $E_\gamma^2$  dependence ( $\chi \ll 1$ ) to the quantum synchrotron law of  $E_\gamma^{2/3}$  dependence ( $\chi \geq 1$ ). As a result, radiation emissions are enhanced by up to two orders of magnitude, and pair production is enhanced by up to one order of magnitude. In this regime, the quantum recoil effects of the emitted hard photons have to be taken into account. However, as has been described earlier, a full QED description has proved to be very complicated. Therefore, there is a flurry of theoretical activity in the quest to discern some underlying features of these strong field effects. As an example, in the interior of compact stellar objects, [25, 26, 27], such fields are known to originate from the gravitational collapse of a star, as a result the magnetic fields get compressed along with the gravitational contraction when the star collapses, resulting in extremely intense fields.

The work described in this thesis is based on a series of experiments conducted at the European Laboratory for Particle Physics (CERN), using the NA43 detector of the North Area of the CERN-SPS when multi-GeV electrons, positrons and photons were incident on oriented single crystalline materials, namely Germanium, Silicon, Diamond and Tungsten. The physics involved therein sought to investigate the correlated versions of strong field QED as a result of strong field effects and the coherence due to the periodic nature of crystals. The uniqueness of these investigations also derived from the fact that interactions extend over a long spatial region due to coherence and

therefore certain depletions in cross-sections (Landau-Pomeranchuk-Migdal effect) can also be investigated.

## 1.2 Plan of Thesis

The thesis is structured into seven chapters as follows

The theory of classical channeling is presented in Chapter 2 and is therefore a pre-cursor to the development into the channeling radiation regime at high energies. To take the discussion beyond the channeling regime  $\psi_c$ , semi-classical formalism within the description of Baier *et al.* and Kononets is also presented whose cardinal features, *viz* coherence and strong field effects are considered. The suppression of the bremsstrahlung emissions and its time reversed process, pair-production in form of the Landau-Pomeranchuk effect are also presented. A discussion of various theoretical attempts to understand this inhibiting process will be presented.

Experimental details are dealt with in Chapter 3 where a description of the accelerator, the beam-line, the detectors, scintillators and targets are given in detail. Of importance are the position sensitive detectors, namely, drift chambers which were used for tagging, energy measurement and beam collimation in order to fulfill the stringent requirements of critical angles for studying aligned incidence phenomena in these energy regimes. The use of various aspects of the beam-line including the lead glass electromagnetic calorimeters will also be discussed. Calibration of the important detector components, namely the lead glass calorimeter, drift chambers and the magnets, is given in Chapter 4

Results of the pair-production mechanism are presented in Chapter 5. An analysis of the experimental results and an appraisal thereof with regard to the theory is presented. Various applications of pair-production are extended to include the proposed uses of single crystals for the veto of background in the CP violation experiment

in which single crystals are used to veto events which occur in the upstream fiducial decay region.

Results of studies of the LPM effect are presented in Chapter 6. Comparison with other related experiments are made and conclusions are also drawn regarding whether the results presented do indicate any evidence of the LPM suppression.

A discussion of the results of experimental findings is presented in Chapter 7 which is also a general overview of the research associated with this thesis. Following from that are conclusions and other possibilities for future (or proposed) applications of the work studied (or related to) in this thesis.

# Chapter 2

## Theory

*"The concepts initially formed by abstraction from particular situations or experimental complexes acquire a life of their own."* ... Werner Heisenberg

### 2.1 Introduction

This thesis reports on the studies of the normal QED processes like pair-production and coherent bremsstrahlung under the conditions of ultra-relativistic incidence in aligned crystalline targets. The nature of the interaction of guided particles with ordered matter is determined mainly by the coherence and strong field effects, which subsequently lead to significant changes in the shape and magnitude of the normal QED cross-sections, as well as to new effects.

In the studies of coherent correlated phenomena, enhancements in transmission yields were found to be dependent on the energy of the incident projectile as well as the orientation of the target crystal. Some of these phenomena were understood within the channeling concept of Lindhard [5], so conceived in order to describe the substantial influence of correlated scattering processes on the particle motion along an atomic chain or plane inside the crystal. In the ultra-relativistic regime, the picture changes slightly because the recoil on photon emission becomes significant and the energy dependent Lindhard critical angles  $\psi_c$  are very small (in the order of micro-radians). As the par-

ticles travel through the crystal at ultra-relativistic velocities (GeV), they experience correlated scattering with many target atoms. The interactions therein are extended over a large region of space called the coherence length. Further on, the correlation in scattering processes persist to angles much greater than  $\psi_c$ . At angles of incidence which are 5 - 10 times larger than  $\psi_c$ , the QED processes can be described as originating from a constant field and this happens just below the characteristic angle  $\Theta_0$ . Furthermore, the standard Born approximation also holds to angles much greater than  $\Theta_0$ .

Of particular interest is the investigation of QED phenomena such as pair-production and bremsstrahlung in strong fields. The concept of strong fields pertains specifically to the importance of the radiation processes for which the coherence length  $l_c$  is of the order of the field deflection length  $l_f$  (a detailed description of this concept follows in the sub-section below). This leads to a situation whereby radiation processes have a high probability of the emission of photons whose energies are in the order of radiating incident particles [21, 22, 23, 4]. Moreover, in accordance with QED cross-symmetry [42, 44, 75] the pair-production yields from incident high-energy photons will also be increased by up to an order of magnitude above that for the Bethe-Heitler process [71, 67, 66].

Various semi-classical theoretical approaches have been used to explain the experimental data and to date the most commonly used is the Constant Field Approximation of Baier *et al.* [16]. This approach provides a physically plausible, albeit quasi-classical attempt to understand the lepton-photon processes in oriented crystalline matter and to date it has shown a remarkable success in interpreting these correlated and strong field effects. In this chapter, an outline of the various aspects of the theoretical approaches used to describe the underlying phenomena is given. The starting point is the introduction of the concept of guided motion and the correlated scattering processes in the form of channeling, this is given in Section 2.3. In Section 2.4, a discussion

of the radiative processes encountered in these experiments is given, followed by an introduction to the main thrust of this thesis, namely, the study of the correlated, strong field processes in the form of the CFA. In particular, a special emphasis will be placed on the understanding of the coherence and strong field effects and how these two concepts influence the QED processes that are under investigation in this thesis. The impact of the coherence and the nature of the influence of strong fields on pair production in oriented crystals is given in Section 2.5. In this section, an analysis of the CFA is given in detail, followed by short general overview of various non exhaustive quantum mechanical attempts to interpret such processes. Furthermore, as a result of the long formation zone for these QED processes, the depletions and setting in of inelastic processes in form of the LPM effect will be studied in detail in section 2.6.

### The Coherence Length

Many electrodynamical processes which occur when an energetic particle is traversing condensed media, such as real or virtual photon emission, develop in a finite region of space along the particle trajectory. The normal QED processes there are no longer described as point interactions but happen over this large formation zone. The length of this zone is known as the *coherence length*  $l_c$ . In aligned crystalline matter the spatial extent of this coherence length can encompass many periodically arranged atoms and can result in dramatic changes to the features of ordinary QED processes like bremsstrahlung and pair production. Moreover, changes to these QED effects become more apparent when the particle (or photon) is incident at ultra-relativistic velocities on the target, as then even a (relatively) small longitudinal momentum transfer along the particle trajectory is readily observed.

For illustrative purposes, the context in which this will be looked at is quasi-classical. In the case of coherent bremsstrahlung, the coherence length  $l_c^{\text{rad}}$  can be interpreted as the length through which the de-coupling of a photon from the electron occurs. The particle and the photon are considered as separate physical entities if their

distance of separation is  $\lambda_c$ . The full separation takes place over the period (*radiation formation time*)  $\tau_c$ . The coherence length for bremsstrahlung may be defined as the distance travelled by the particle during the time  $\tau_c$ :

$$l_c^{\text{rad}} = \frac{2E(E - \hbar\omega)}{\hbar\omega mc^2} \frac{\hbar}{mc} \quad (2.1)$$

In case of pair-production;  $l_c^{\text{pair}}$  can be interpreted as the region in which the electron/positron pair goes through complete de-coupling, where the distance of separation is the Compton wavelength. The separated leptons ( $e^-/e^+$  pair) are thus only considered as separate physical entities if their distance of separation is  $\lambda_c$  and this takes place over the separation time  $\tau_c$ . The coherence length for pair-creation thus becomes  $l_c \equiv c\tau_c$  where

$$l_c^{\text{pair}}(\xi_{\pm}, \omega) \equiv c\tau(\xi_{\pm}, \omega) = 2\xi_{\pm}\xi_{-}\lambda_c\hbar\omega/2\pi mc^2. \quad (2.2)$$

The electron rest mass and the Compton wavelength have standard notation. In equation (2.2),  $\xi_{\pm} = \varepsilon_{\pm}/\hbar\omega$  is the ratio of one pair-member energy and the total incident photon energy is  $\hbar\omega$ . Note that  $l_c^{\text{pair}}$  increases with energy of the pair, whereas  $l_c^{\text{rad}}$  decreases with the energy of the radiation particle.

There is another way of looking at the coherence length from the Heisenberg uncertainty relationship: This happens when we make a supposition that the uncertainty in the incident energy during the interaction process is related to the longitudinal momentum transfer  $q_{\parallel}$  thus:

$$q_{\parallel} = p_e - p'_e - k = \sqrt{E_e^2 - m^2} - \sqrt{E'_e{}^2 - m^2} - E_{\gamma}, \quad (2.3)$$

where  $p_e, p'_e, E_e$  and  $E'_e$  are the electron momentum and energy before and after interaction, respectively.  $E_{\gamma}$  is the photon energy and, in accordance with the Heisenberg's uncertainty principle, the longitudinal momentum transfer is  $q_{\parallel} \sim \hbar/l_c$ . This equation can thus be simplified to the following formula when the incident particle is of ultra-relativistic energy, i.e. in the limit  $E_{\gamma} \ll E$ :

$$q_{\parallel} = \frac{m^2 E_{\gamma}}{2E_e(E - E_{\gamma})} \sim E_{\gamma}/2\gamma^2, \quad (2.4)$$

where  $\gamma = E_e/m$ . According to the uncertainty principle, the virtual photon exchange distance becomes finite and is given thus  $l_c = c\gamma^2/E_\gamma$ . Equation (2.4) was derived from a simplified description of the coherence length and does not depend on the influence of the medium in which radiation occurs and hence does not determine the kind of the dynamic of the reaction. For highly energetic particles and soft quanta, the coherence length may reach macroscopic dimensions. As an example, the process of emission of a 1 GeV photon by a 150 GeV electron takes place over a coherence length of 34  $\mu\text{m}$ .

A closer look at the relationship between the formation length<sup>1</sup> and the coherence length can reveal some salient differences. This relationship is strongly influenced by the kind of the reaction dynamics which take place and also on the nature of the medium in which these interactions take place. However, there is no clear cut distinction between these two concepts; what is described below is another possible way of interpreting the relationship between the two length scales:

In the presence of strong crystalline fields (in the  $\chi > 1$  regime) the coherence length is larger than the formation length. This comes about because of the periodic alignment of atoms in a crystal and also of the ultra-relativistic incidence of particles, thus as a result, the interaction lengths are larger than in amorphous matter. At  $\chi < 1$ , the formation length is once again equivalent to the coherence length. Based on these arguments, the coherence and formation lengths have to be seen in the context of understanding the underlying mechanism which are related to enhancement in radiation or pair production with respect to the random incidence.

## 2.2 Criteria for Strong Field QED

Quantum field theories can in general, provide us with the best available understanding of elementary particle interaction processes. These have resulted in models whose physical parameters agree astonishingly well with experiment. Quantum electrodynamics

<sup>1</sup>In most of the literature, the coherence length and the formation length are used interchangeably



namics (QED) is one of such theories whose overwhelming agreement with experiment has been vindicated under the most stringent of experimental conditions. In quantum electrodynamics, a transition from the neutral to a charged vacuum in the presence of strong external electromagnetic fields is predicted. This manifests itself in the form of the occurrence of spontaneous  $e^+/e^-$  pair creation. The theoretical understanding of this process as well as some attempts to successfully investigate it in strong fields will be detailed in the sections following.

### Vacuum Polarization and Pair Formation

Perturbation methods have thus far made it possible to calculate and thereby predict the behaviour of charged particles in weak electromagnetic fields. This is primarily due to the smallness of the QED coupling constant  $\alpha \sim 1/137$ . However, as stated by Greiner [33] (a detailed treatment of Strong field QED is given in this reference) physical situations exist in which this constant becomes large, for example in the presence of a large atomic nucleus with  $Z$  protons and therefore,  $Z\alpha$  can be in the order of unity [33]. In these circumstances, the predictions of the perturbation theory do not hold and therefore non-perturbative methods will have to be used in order to describe the underlying phenomena. The occurrence of a pair in this process has been mooted in heavy ion collisions in which a strong field is developed for a sufficiently long period of time. The Darmstadt group has carried out such investigations as detailed in [34]. A detailed review by A. Schäfer [35] also deals with the strong field effects.

According to the current field theories, it is possible that a vacuum can be spontaneously broken by strong external fields. In these instances the normal vacuum<sup>2</sup> state is unstable and decays into a new, charged vacuum that contains real particles. When a quantum field is considered in a strong external and classical (unquantised) field,

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<sup>2</sup>A neutral QED vacuum can be seen as negative energy states filled with electrons. The infinite charge is renormalized to zero

particles may be created spontaneously when there is a change in the vacuum caused by sufficiently strong space/time dependent external fields. However, below these field strengths, virtual particle pairs are created as fluctuations of the vacuum [41]. The Heisenberg uncertainty condition  $\Delta t \sim \hbar/2mc^2$ , none the less precludes the formation of real pairs since the lifetime is very small. In the presence of a strong field, however, the virtual pair can be separated during this short time by more than the Compton wavelength and also if the kinetic energy is greater than the pair rest mass, the pair consequently becomes real.

The external field with a significant temporal and spatial extent can lead to observable critical behaviour above which the newly charged quantum states can be described as products of a dynamically broken symmetry. According to Müller *et al.* [36, 37, 38] this product is equivalent to an excitation of the Dirac sea :

$$\hat{\Psi}(x, t=0) = \sum_p \hat{b}_p \psi_p(x) + \sum_n \hat{a}_n^\dagger \psi_n(x) \quad (2.5)$$

where  $\hat{b}_p$  and  $\hat{a}_n^\dagger$  are annihilation and creation operators for positrons and electrons, respectively. The energy of the created pair is given by the expression:

$$\hat{H}_0 = \sum_p E_p \hat{b}_p^\dagger \hat{b}_p + \sum_n |E_n| \hat{a}_n^\dagger \hat{a}_n. \quad (2.6)$$

The ground state vacuum is characterised by the Fermi-surface. A "hole" in the Dirac sea is interpreted as a positron. The Fermi surface dividing the single particle states into the electron and positron states can be determined experimentally by observing the threshold for  $e^-/e^+$  pair-production resulting in the vacuum expectation value

$$\hat{\rho}(x) = e/2 \left[ \hat{\Psi}^\dagger(x, 0), \hat{\Psi}(x, 0) \right]_- \quad (2.7)$$

and this can be described as the zero component of the current four-vector:

$$\langle \hat{j}_\mu \rangle = e/2 [\hat{\bar{\psi}}, \gamma_\mu \hat{\psi}]_- \quad (2.8)$$

From the above the expression for vacuum polarisation is given as:

$$\langle 0|\hat{\rho}|0\rangle \equiv \rho_{\text{pol}}^{\text{vac}}(x) = \frac{1}{2} \left[ \sum_n \psi_n^\dagger \psi_n(x) - \sum_p \psi_p^\dagger \psi_p(x) \right]. \quad (2.9)$$

The above expression vanishes for field-free regions because all charged states cancel each other out. There are several QED conjectures from which it can be shown that the charged vacuum state is degenerate. Such cases are encountered in  $\phi^4$  theories where there is two-fold degeneracy because both dips of the potential are energy positive:

$$\langle \phi \rangle = \pm \langle \phi \rangle_0. \quad (2.10)$$

In the supercritical case the created state is degenerate with occupied negative electron states. Hence a spontaneous creation of  $e^+e^-$  lepton pair becomes possible as the electron from the Dirac sea occupies the additional state leaving a hole in the sea. This hole escapes as a positron while the electron charge remains at the source. The newly created charged vacuum is stable and also obeys Pauli's exclusion principle.

The creation of such a pair has to be within a time-scale of  $10^{-21}$  s and can thus only be sustained in very strong (critical) external fields ( $E_0 = 1.32 \cdot 10^{16}$  V/cm). Such fields can be obtained from:

- ⇒ Heavy ion scattering processes can result in a strong field if the combined charge of the two colliding nuclei is large, i.e. ( $Ze > 173$ ), as mentioned in the discussion above.
- ⇒ A well collimated laser beam with an intensity of  $10^{21}$  W/cm can sustain a field of  $10^{14}$  V/m.
- ⇒ Single crystals which are aligned w.r.t a well collimated beam of ultra-relativistic particles can sustain a strong field. The motion of the particle in its Lorentz boosted rest-frame is steered by atoms in a plane. In crystals these fields are macroscopic and last long enough so that they can be investigated fully. A detailed account follows below:

### Why use single crystals?

Crystals are good candidates for the investigation of strong fields because probabilities for radiative electromagnetic processes have to be calculated in the electron/positron Lorentz boosted ( $\gamma \geq 10^{5-6}$ ) rest-frame of the particle which experiences a strong field over a macroscopic distance. To illustrate the point even further, take the local field on the surface of a bare nucleus<sup>3</sup> to be:  $E = Z_2 e / r^2$ , where  $r \sim 1.2 \cdot A^{1/3}$  fm and  $A \sim 5Z_2/2$ . This makes

$$E = Z_2 e / r^2 \alpha \cdot E_0 \hbar / mc, \quad (2.11)$$

where  $E_0 = 1.32 \cdot 10^{18}$  V/m is the "critical field",  $A$  the number of nucleons,  $r$  the nuclear radius and  $\alpha$  is the fine-structure constant. In such situations, the quantum parameter determining the process is given by  $\chi = \gamma \cdot E / E_0$  where  $\gamma$  is the Lorentz factor ( $\sim 10^{5-6}$ ) and  $E$  the local field for which the Lindhard continuum potential can be used. For ultra-relativistic particles, the Schwinger parameter that characterizes the influence of quantum effects on recoil  $\chi$  reaches values greater than unity. Accordingly, a particle that experiences governed motion inside the crystal thus interacts with many such nuclei since the field extends over the entire macroscopic length of the crystal. Furthermore in contrast to heavy-ion collisions, the interaction lengths (as stated before) can cover a macroscopic spatial extent of 1 - 100  $\mu$ m for incidence along the crystal axes or planes [55]. However, the theoretical difficulty is that the description of such processes in crystals is beyond the realm of fully perturbative QED because a full QED description of these effects requires a solution of the Dirac equation, and for ultra-relativistic particles, the number of quantum states ( $\propto \gamma$ ) is too large to be solved analytically and thus semi-classical methods are used.

In the sections following, an appraisal of some of the models currently being employed is detailed. The next section will be an introduction to the concept of channeling from which most of the theory originates, however we will see later that the effects of coherence and strong fields go well beyond the channeling angles.

<sup>3</sup>To simplify the argument, we will ignore screening effects

## 2.3 The Continuum description of crystalline field effects

For a particle traversing a crystal as depicted schematically in figure 2.1, directional effects start showing due to correlations between successive soft collisions with target constituents. When a charged particle moves along a low-index axial direction and passes close to one atom experiencing only a slight deflection, it will invariably pass close to the neighbouring atoms of the same row. This led Lindhard [5] to introduce the concept of a continuum potential in which the row of atoms is periodically spaced along the axis like "pearls on a string" is replaced by a continuous line of charge of the same average linear charge density. The lattice atoms in a string appear closer together (because of the Lorentz contraction of the lattice) for particles incident on a crystal at high velocities, thus the concept of a continuum potential becomes ever more realistic. The particle therefore experiences a cylindrical potential (to first order) as it travels through the crystal.

A description of this continuum processes is illustrated by figure 2.1 where it is shown how heavy, positively charged projectiles moving at non-relativistic energies undergo an angular deflection  $\Delta\psi$  associated with individual binary collisions with the target atom. These close encounters are governed by a screened Coulomb potential where the screening is due to electrons surrounding the atomic nucleus. Each  $\Delta\psi$  is small compared to the total deflection, which in itself is small for the major composition of the incident beam. The resulting motion is that of gentle steering from many atoms such that the particle leaves the "string" with the same angle as the one within which it approached it. The corresponding angular momentum transfer  $\Delta p$  is the perpendicular to the direction of motion, and this in turn, almost coincides with the direction  $z$  of the string, i.e.

$$\Delta p \sim -\frac{1}{v} \int_{-\infty}^{\infty} dz \nabla_{r_{\perp}} V(r_{\perp}, z). \quad (2.12)$$

Here  $r_{\perp}$  represents the coordinate transverse to the string and  $v$  is the velocity of

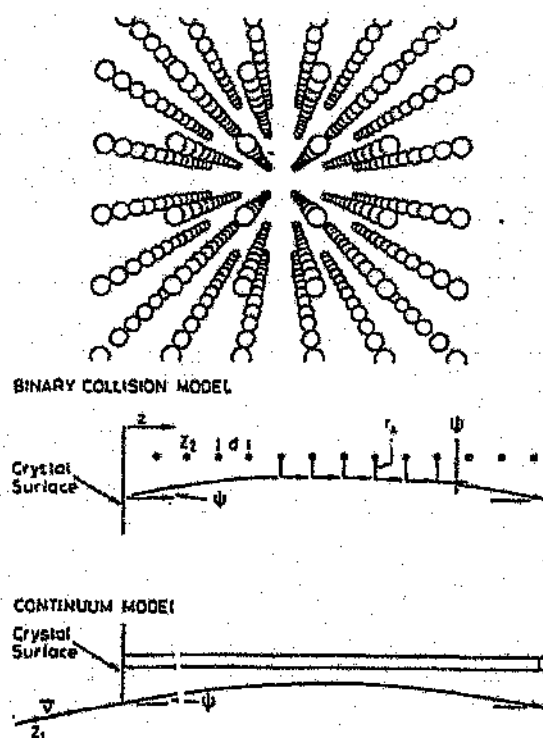


Figure 2.1: Perspective view of an axially aligned cubic crystal. The direction of observation is parallel to a major axis of the crystal. The schematic drawing also depicts the trajectory of a particle incident at small angles relative to the crystal axis where the deflection as a result of correlated binary collisions occurs and then the same deflection by a continuum row.

a particle incident on a crystal with a resulting particle-atom interaction potential represented by  $V$ . According to equation (2.12), the momentum transfer during the scattering by the single atom can be determined by the integral over a time interval  $\Delta t = d/v$  for the force obtained from the potential:

$$U(r_{\perp}) = \frac{1}{d} \int_{-\infty}^{\infty} dz V(r_{\perp}, z), \quad (2.13)$$

where  $d$  is the distance between atoms on the string. This potential, equation (2.13), corresponds to that of a charge distribution smeared in the  $z$ -direction. The motion consequently turns out to be that resembling a continuum potential obtained by a smearing of atomic potentials belonging to the same string, figure 2.1. The above argument can also be used to describe such processes for energetic charged particles incident along certain planar orientations such that the smearing gets averaged over a two dimensional continuum.

### Single string continuum potentials

It is obvious that, for an axially channeled ion, the angles through which it is scattered in successive collisions with crystal atoms must be small. In the classical approach, the impact parameter in each collision is very large, that is, when put in relative comparison with nuclear dimensions. The projectile thus interacts with each atom via the Coulomb potential partially screened by the electron cloud surrounding the atom as follows:

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \phi(r/a) \quad (2.14)$$

where  $\phi(r/a)$  is the screening function,  $a$  is the screening length setting the distance scale of  $\phi$ .  $Z_1 e$  and  $Z_2 e$  denote the projectile and target (nuclear) charge, respectively.

A good approximation to the function  $\phi$  is based on the Thomas-Fermi screening which uses the statistical description of the atom [30, 32] and considers the electrons from the lattice as an electron gas. The Thomas-Fermi screening length  $a_{TF}$  is :

$$a_{TF} = 0.8853 a_0 (Z_1^{2/3} + Z_2^{2/3})^{-1/2} \quad (2.15)$$

where  $a_0$  is the Bohr radius ( $a_0 = 52.9$  picometers). Solutions for  $\phi$  can only be evaluated numerically, and thus approximations are generally used. Furthermore, let us consider approximation put forward by Molière [31] which gives:

$$\phi_M(r/a) = \sum_{i=1}^3 \alpha_i \exp \frac{\beta_i r}{a}. \quad (2.16)$$

Another approximation to screened potentials is the Lindhard standard single string potential,

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \left\{ 1 - \frac{r}{(r^2 + C^2 a^2)^{1/2}} \right\} \quad (2.17)$$

In this standard approach  $C^2$  is set to 3. By insertion of equation (2.17) into equation (2.13), the standard continuum potential corresponding to a single string is obtained,

$$U(r_\perp) = \frac{Z_1 Z_2 e^2}{d} \ln \left[ 1 + \left( \frac{C a}{r_\perp} \right)^2 \right]. \quad (2.18)$$

The uncertainty in the position due to thermal vibrations of target atoms implies that a transverse smearing in addition to a longitudinal one equation (2.13) has to be taken into account. This consideration results in a modified expression for equation (2.13) as follows:

$$U(r_\perp) = \int d^2 r'_\perp P(r'_\perp) U(r_\perp - r'_\perp). \quad (2.19)$$

An harmonic approximation for the interaction forces can be applied where the probability distribution  $P(r'_\perp)$  of the target atoms is a Gaussian and is given thus:

$$P(r'_\perp) = (\pi \rho^2)^{-1} \exp(-r'^2_\perp / \rho^2), \quad (2.20)$$

where  $\rho^2$  is the two-dimensional mean square thermal displacement from the string. By insertion of equation (2.18) and equation (2.20) into equation (2.19) the following analytical form may be used:

$$U(r_\perp) = \frac{Z_1 Z_2 e^2}{d} \ln \left( 1 + \frac{C^2 a^2}{r^2_\perp + \frac{1}{2} \rho^2} \right). \quad (2.21)$$

In addition, there are other approximations for thermally averaged potentials; the most commonly used of which is the Doyle-Turner potential [40]. Doyle and Turner



| Silicon   |                   | Germanium |                   | Diamond   |                   |
|-----------|-------------------|-----------|-------------------|-----------|-------------------|
| $a_i$ (Å) | $b_i(\text{Å}^2)$ | $a_i$ Å   | $b_i(\text{Å}^2)$ | $a_i$ (Å) | $b_i(\text{Å}^2)$ |
| 2.1293    | 57.7748           | 2.446     | 55.893            | 0.70307   | 35.9951           |
| 2.5333    | 16.4756           | 2.7015    | 14.393            | 1.1951    | 11.2966           |
| 0.8349    | 2.8798            | 1.6157    | 2.4461            | 0.4563    | 2.8139            |
| 0.3216    | 0.3860            | 0.6090    | 0.3415            | 0.1247    | 0.3456            |

Table 2.1: Parameters of the Doyle-Turner potential for Si, Ge and diamond crystals

obtained analytical approximations to atomic potentials based on relativistic Hartree-Fock calculations. In their calculations, they fitted the calculated electronic-scattering form factors (in the first Born approximation, these are proportional to the Fourier transform of the potential) with a sum of four Gaussians. The continuum potential therefore becomes a sum of four Gaussians. The thermally averaged Doyle-Turner single-string potential consequently becomes

$$U(r_{\perp}) = \frac{Z_1 e^2}{a_0} \frac{2a_0^2}{d} \sum_{i=1}^4 \frac{a_i}{B_i + \rho^2} \exp\left(-\frac{r_{\perp}^2}{B_i + \rho^2}\right) \quad (2.22)$$

where  $B_i = b_i/4\pi$ . The constants  $a_i$  and  $b_i$  are given in Table 2.1. In general, it has been found that the application of the thermally averaged Doyle-Turner potential leads to very good agreements with experiment over a wide range of channeling processes [42].

### 2.3.1 Planar Channeling

The description of motion for ions penetrating an axially symmetric crystal can also be generalised to a two-dimensional case, *i.e.* motion in a planar continuum. This potential is obtained by smearing the charges of the crystal atoms into two dimensions. Therefore, by using the standard potential, equation (2.13), an expression can be derived for a single static plane as follows:

$$U(y) = 2\pi Z_1 Z_2 e^2 n d_p [y^2 + C^2 a^2]^{1/2} - y] \quad (2.23)$$

where  $y$  is the distance from the plane and  $d_p$  the inter-planar distance. The quantity  $n$  denotes the atomic density. Thermal averaging should be taken into account as was in the axial case. However, the effect of this smearing is not dramatic than in the axial situation since static planar potentials remain finite at the positions of the planes.

### 2.3.2 Critical angle for channeling

Channeling effects can be observed by physically measuring the yield of the processes which require close contact between the projectile and the constituents of the crystal target. In the event of the transverse energy of the incident particle being somewhat below the potential maximum  $U(0)$ , the positively charged particle is repelled away from the region of the string of high nuclear scattering centres. On the other hand, if the projectile moves at a transverse energy above the barrier, it would hit target atoms in essentially the same manner as in an amorphous (random orientation) material.

There is a certain optimal critical value for the transverse energy distribution in order for the channeling condition to hold and it follows from the condition:

$$\frac{1}{2} p v \psi^2 \leq \frac{1}{2} p v \psi_c^2 = U_0, \quad (2.24)$$

where  $\psi$  and is the angle of incidence to the crystallographic directions and  $p$  is the momentum of the incident particle. According to equation (2.24), the transverse energy of the incident particle must be smaller than the height of the transverse potential  $U_0$ . This designates the setting in of directional channeling effects [42, 43] such that if the incident particle is trapped in a potential well along along the direction of motion, it will be thus be guided along the crystal axis.

The characteristic angle for these directional effects is then given from equation (2.24) by

$$\psi_c \equiv \left[ \frac{4Z_1 Z_2 e^2}{p v d} \right]^{1/2} \quad (2.25)$$

| material | axis                  | $\psi_c$ ( $\mu$ rad) | plane | $\psi_p$ ( $\mu$ rad) |
|----------|-----------------------|-----------------------|-------|-----------------------|
| Si       | $\langle 110 \rangle$ | 14                    | (110) | 5                     |
| Ge       | $\langle 110 \rangle$ | 21                    | (110) | 6                     |
| W        | $\langle 110 \rangle$ | 31                    | (110) | 11                    |

Table 2.2: Typical critical angles for axial and planar channeling (in micro-radians) for a particle incident on the crystal at 1 TeV.

In this expression, the channeling angle is characteristic of an ion with charge  $Z_1e$  which is incident on a crystal with axial spacing  $d$  and the kinetic energy of the ion is  $pv/2$ . It could be pointed out that this result is an approximation of the Lindhard critical angle  $\psi_1$ . Similarly, critical angles for planar effects can also be obtained in the same way, following the previous method of generalising the axial potential into a two-dimensional potential for the planar case, thus:

$$\psi_p = (4Z_1Z_2e^2nd_pCa/pv)^{1/2}. \quad (2.26)$$

The planar continuum potential is shallower than the axial one, thus resulting in narrower critical angles for planes which are typically  $\psi_p \sim \psi_1/3$ . Values of critical angles for axial and planar channeling are tabulated as shown in table 2.2.

## 2.4 Radiation emissions

### 2.4.1 Channeling radiation

The basic theory of channeling has been described in the sections above. What is sought to be elucidated now is that the aligned motion in a crystal can subsequently give rise to coherence effects in the photon emission. When the angle of incidence is less than the critical angle for channeling, the associated radiation is called **Channeling radiation (ChR)** [49].

The structure of (ChR) is strongly dependent on the form of the crystal potential,

Channeled positive ions are repelled away from the scattering nuclear centers while negatively charged ions are focussed around rows of atomic nuclei. As a result their ChR spectra will be different since they experience different potentials. At GeV energies, particles which are nearly aligned with crystal axes or planes move along classical trajectories [52, 14, 15], however, at energies above 20 GeV a classical description is no longer valid due to the setting in of quantum recoils of the hard photon emission processes. In the next sections, it will be shown that coherent phenomena can still occur beyond the critical angle for channeling  $\psi_c$ .

### 2.4.2 Bremsstrahlung

The radiation emitted by a charged particle penetrating an atomic field is known as bremsstrahlung. For instances where the incident particle is of high-energy, the emission process can be derived from perturbative QED where, in the Born limit,  $Z\alpha \ll 1$ , and also in the regime of soft photon emission  $\hbar\omega/E \ll 1$ , the cross-section for photon emission at energy  $\hbar\omega$  upon relativistic electron/positron impact of total energy  $E$  is approximately:

$$\frac{d\sigma}{d\hbar\omega} = \frac{16}{3} Z^2 \alpha r_e^2 \frac{1}{\hbar\omega} \left( 1 - \frac{\hbar\omega}{E} + \frac{3}{4} \left( \frac{\hbar\omega}{E} \right)^2 \right) \ln(183 Z^{-1/3}) \quad (2.27)$$

where  $r_e = e^2/mc^2 = \alpha^2 a_0$  is the classical radius of the electron and  $Z$  the charge of the incident particle. However, the above equation holds only in the regime of complete atomic Thomas-Fermi screening where  $\gamma \gg 1$ . Figure 2.3 shows a typical case whereby a photon is emitted as a result of scattering from the screened field of the atomic nucleus. Furthermore, radiation may also be emitted as a result of direct scattering from target electrons, this leads to an extra contribution to the cross-section of a factor  $1/Z$  above the nuclear cross-section [42]. Equation 2.27 is referred to as the Bethe-Heitler cross-section [54].

At high-energy lepton incidence ( $E \geq 1$  GeV) the major cause for energy loss in matter is by far the bremsstrahlung emission. Therefore, a convenient approach is

| material   | $l_r(\text{equation (2.28)})(\text{cm})$ | $l_r(\text{measured})(\text{cm})$ |
|------------|------------------------------------------|-----------------------------------|
| C(diamond) | 14.8                                     | 12.2                              |
| Si         | 10.2                                     | 9.36                              |
| Ge         | 2.35                                     | 2.30                              |
| W          | 0.33                                     | 0.35                              |

Table 2.3: Comparison of the calculated and the measured values of the radiation lengths for the materials under consideration.

to introduce the radiation length  $l_r$ . This is defined as the penetration depth for which the incident particle loses all but a fraction  $1/e$  of its initial energy. An expression for the radiation length can be derived from equation (2.27):

$$1/l_r = 4Z^2 \alpha n r_e^2 \ln(183Z^{1/3}), \quad (2.28)$$

where  $n$  is the target density and the other terms are as defined in equation (2.27). The above equation is derived by integrating the differential cross-section (equation (2.27)) over all radiated energies for an amorphous material. From equation (2.28), it can be concluded that the radiation length is an energy-independent material property. Table 2.4.2 shows values of radiation lengths for materials commonly used for the studies detailed in this dissertation.

### 2.4.3 Coherent Bremsstrahlung

This phenomenon occurs when an electron crosses a single crystal in a direction which is close to major crystallographic axial and/or planar directions. The electron therefore has a greater probability of scattering coherently on a large number of target atoms along its trajectory. As a result of this coherence, the electron radiates by crossing planes of atoms as it travels through the crystal; the periodic crossing of planes results in much greater enhancements in emissions than in the amorphous (randomly oriented) target. This type of bremsstrahlung has thus been referred to as the "coher-

ent bremsstrahlung". Detailed account of explicit expressions for the cross-sections of the coherent bremsstrahlung CBS is given in a review by Diambri Palazzi [50] and Ter-Mikaelian [51]. Moreover, Sørensen and Uggerhøj [42] have also given a thorough overview of the concept of coherent bremsstrahlung. Based on the references above, it will be shown in this section how the expression of the cross-section for CBS has been derived from the tools of perturbation theory. This starts from the premise of taking bremsstrahlung emission probability to be proportional to the square of a second-order matrix element which, in turn is composed of a sum of terms, each proportional to the product of two first-order matrix elements; one for the interaction with the radiation field and another for the scattering in the atomic field.

A general expression for a series of plane waves incident on a single atomic scattering potential is:

$$d\sigma \propto \left| \int V(r) e^{i\vec{q} \cdot \vec{r}} d^3\vec{r} \right|^2, \quad (2.29)$$

where  $V$  is the atomic potential and  $\hbar q$  is the recoil momentum of the atomic nucleus. In a medium composed of  $N$  atoms, the potential  $V$  is replaced by a total interaction potential,  $V \rightarrow \sum_n V(\vec{r} - \vec{r}_n)$ , where  $\vec{r}_n$  is the position of the  $n$ 'th atom. Accordingly, equation (2.29) can be modified to give the following expression for the differential cross-section for photon production of energy  $\hbar\omega$ :

$$\left. \frac{d\sigma}{d\hbar\omega d^3\vec{q}} \right|_{N_{\text{atoms}}} = \left. \frac{d\sigma}{d\hbar\omega d^3\vec{q}} \right|_{\text{single}} \left| \sum_n e^{i\vec{q} \cdot \vec{r}_n} \right|^2. \quad (2.30)$$

However, in an amorphous medium, the last factor is  $N$  whereas for a perfect crystal (in the limit of  $N \rightarrow \infty$ ), the phases must be carefully taken into account and the expression consequently becomes:

$$\left| \sum_n e^{i\vec{q} \cdot \vec{r}_n} \right|^2 = N \frac{(2\pi)^3}{N_0 \Delta} |S(\vec{q})|^2 \sum_{\vec{g}} \delta(\vec{q} - \vec{g}). \quad (2.31)$$

In the above expression, equation (2.31),  $\vec{g}$  denotes the reciprocal lattice vector,  $S(\vec{g})$  is the structure factor and  $N_0$  the number of atoms contained in a unit cell of volume  $\Delta$ . When thermal vibrational effects are taken into account, the interference structure

given in Equations (2.30) and (2.31) becomes less prominent. As a result when thermal averaging has been factored into equation (2.30), the following expression is obtained:

$$\left\langle \left| \sum_n e^{i\vec{q} \cdot \vec{r}_n} \right|^2 \right\rangle = N[1 - \exp(-q^2 \rho_1^2)] + \exp(-q^2 \rho_1^2) \left| \sum_n e^{i\vec{q} \cdot \vec{r}_0} \right|^2, \quad (2.32)$$

where  $\vec{r}_0$  denotes the equilibrium lattice sites and  $\rho_1$  is the one-dimensional vibrational amplitude. Moreover, the Debye-Waller factor reduces the interference factor when thermal vibrational effects are taken into account. This part corresponds to the second term on the right hand side of equation (2.32) and is referred to as the *coherent part*. On the other hand, the *incoherent part* corresponds to the first term on the right hand side of equation (2.32).

Following from the above, explicit expressions for the amorphous and the interference cross-sections may be deduced. As an example, Ter-Mikaelian [51] gives the following form:

$$\frac{d\sigma}{d\eta} = z^2 \alpha_e^2 \frac{1}{\eta} [(1 + (1 - \eta)^2)(\psi_1^{\text{am}} + \psi_1^{\text{int}}) - 2/3(1 - \eta)(\psi_2^{\text{am}} + \psi_2^{\text{int}})], \quad (2.33)$$

In the above expression, the fractional energy of the photon is denoted by  $\eta = \hbar\omega/E$ . Furthermore, expression for  $\psi_1^{\text{int}}$  and  $\psi_2^{\text{int}}$  are:

$$\psi_1^{\text{int}} = \frac{4\delta(2\pi)^2}{N_0\Delta} \sum_{\vec{g}} |S(\vec{g})|^2 \exp(-g^2 \rho_1^2) F^2(g) \left( \frac{g^2}{g_{\parallel}^2} - 1 \right) \quad (2.34)$$

and

$$\psi_2^{\text{int}} = \frac{4\delta(2\pi)^2}{N_0\Delta} \sum_{\vec{g}} |S(\vec{g})|^2 \exp(-g^2 \rho_1^2) F^2(g) \left( \frac{g^2}{g_{\parallel}^2} - 1 \right) \left\{ 5 \frac{\delta}{g_{\parallel}} \left( 1 - \frac{\delta}{g_{\parallel}} \right) \right\}, \quad (2.35)$$

respectively, where the factor  $F$  is proportional to the Fourier transform of the atomic potential:

$$F(k) = \frac{1}{(4\pi Ze)} \int e^{i\vec{k} \cdot \vec{r}} V(r) d^3r, \quad (2.36)$$

the prime in the summation indicates that all terms are summed according to the kinematic restriction which limits them to the component of  $\vec{g}$  along the motion of the

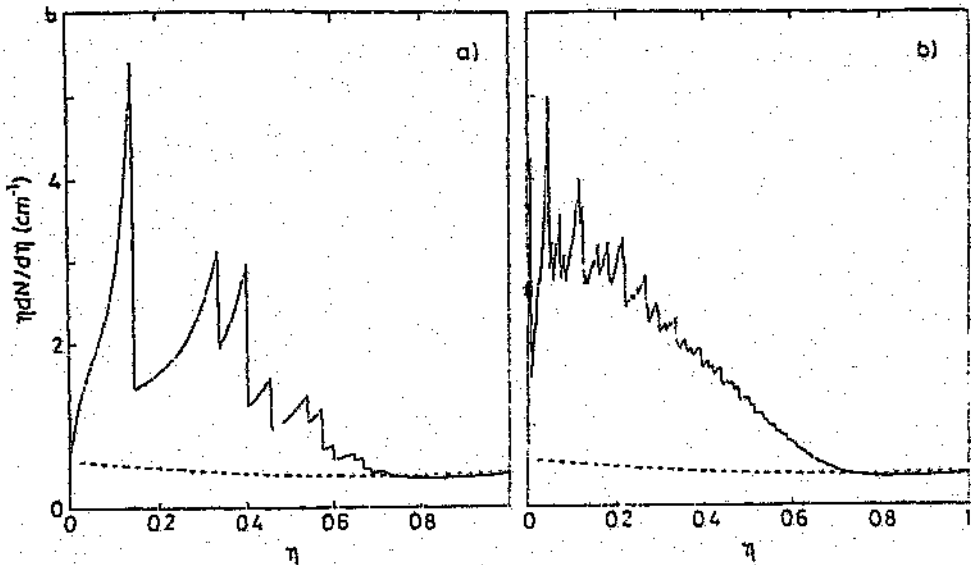


Figure 2.2: CBS as obtained from the Born approximation for 10 GeV electrons and positrons incident at 1 mrad from the  $\langle 110 \rangle$  axis of germanium kept at room temperature. In the power spectra shown above, a) the beam is incident in a direction parallel to the (110) plane whereas in b) a random value of the azimuthal angle is chosen.

projectile.  $g_{\parallel}$  in which  $g_{\parallel} \geq \delta$ . Figure 2.2 shows examples of coherent bremsstrahlung spectra computed for high-energy electrons near the  $\langle 110 \rangle$  axis in a germanium crystal. As shown in the figure, there is a strong enhancement of the radiation yield in comparison to the Bethe-Heitler yield (as given in equation (2.27)). In addition, for incidence in a plane, strong interference peaks show up due to periodic crossing of different axes [42].

## 2.5 Coherence and Strong Field Effects

The basic radiation and pair-creation effects described in this thesis are related, ... is best illustrated by the Feynman diagrams in figure 2.3. It can also be said that at these high energies, the transverse excursions of the radiating projectile are negli-



gible in comparison with those along the atomic planes/axes. The particle therefore emits radiation in such a way that it appears to be radiating as a result of a constant electromagnetic field [13, 14] and the characteristic angle was given by Baier, i.e.  $\Theta_0$  which can be assigned to signify these effects can be derived from the conservation of transverse energy as following:

$$\frac{p^2}{2\gamma m} \psi_0^2 = \frac{p^2}{2\gamma m} (\psi_0 + \Delta\psi)^2 - U_0, \quad (2.37)$$

where  $\psi_0$  and  $\Delta\psi$  are the incident and deflection angle, respectively,  $U_0$  is the scale of the axial/planar potential and  $m$  the electron mass. For small transverse excursions;  $\Delta\psi \ll \psi_0$ , we get

$$\Delta\psi = \frac{1}{\gamma} \frac{U_0}{mc^2 \psi_0}. \quad (2.38)$$

Finally, we can deduce the following expression for the Baier angle:

$$\Theta_0 = \frac{U_0}{mc^2}. \quad (2.39)$$

For incident angles less than the Lindhard critical angle, namely,  $\Theta \leq \psi_1$ , a strong channeling effect appears where negatively charged particles are focussed around atomic nuclei and positively charged ones are trapped into atomic dips. This therefore leads to enhanced yields for electrons.

For multi-GeV  $e^+/e^-$ , the Lindhard angles are (100 – 200) micro-radians, whilst the Baier angle  $\Theta_0$  is 5 – 10 times larger and “independent” of particle energy. At  $\Theta \gg \Theta_0$ , the standard Born approximation holds. However, for  $\Theta \ll \Theta_0$ , the process can be described as originating from a constant field. For  $\Theta = \Theta_0$ , a full quantum description must be performed and has turned out to be very complicated. By taking a prompt from classical physics where an electrically charged particle emits electromagnetic radiation during its acceleration, the instantaneously emitted power  $P$  is proportional to the square of the acceleration viz  $P \propto (\gamma F/M)^2$ . Hence, we may show that electrons and positrons are of practical interest as radiators.

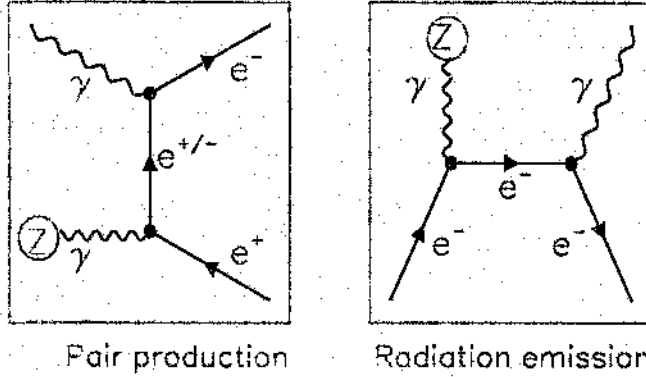


Figure 2.3: First order QED description of the bremsstrahlung and pair-production mechanisms.

### 2.5.1 Pair-Production (Semi-classical approximation)

Detailed below is a derivation by Kononets [44] for an expression of the space-energy density distribution of the  $e^+e^-$  pairs created per unit path by a non-polarised beam of photons in a single axially oriented crystal [45]. A method is developed in which the Baier-Katkov semi-classical equation [46] may be written as an integral of time  $\tau$  relative to the singled out axial family, using classical co-ordinates  $r_{\perp}^{\pm}$  and velocities  $v_{\perp}^{\pm}$ <sup>4</sup> of one of the pair members as follows:

$$\frac{d^2 N_p}{d\xi_{\pm} dS_{\perp}^0} = \frac{\alpha}{2\pi S_{\perp}^0 L_c(\xi_{\pm}, \omega)} \left\{ (\xi_{+}^2 + \xi_{-}^2) \frac{\gamma_{\pm}^2}{c^2} \int_0^{\infty} [v_{\perp}^{\pm}(\tau) - v_{\perp}^{\pm}(0)]^2 \sin A_{\pm}(\tau) \frac{d\tau}{\tau} + 4\xi_{+}\xi_{-} \left( \frac{\pi}{2} - \int_0^{\infty} \sin A_{\pm}(\tau) \frac{d\tau}{\tau} \right) \right\}, \quad (2.40)$$

where

$$A_{\pm}(\tau) = \frac{1}{\tau_c(\xi_{\pm}, \omega)} \left\{ \tau + \frac{\gamma_{\pm}^2}{c^2} \left[ \int_0^{\tau} [v_{\perp}^{\pm}(\tau')]^2 d\tau' - \frac{1}{\tau} [r_{\perp}^{\pm}(\tau) - r_{\perp}^{\pm}(0)]^2 \right] \right\}. \quad (2.41)$$

In the above expression,  $\xi_{\pm} = \varepsilon_{\pm}/\hbar\omega$  is the ratio of one pair-member energy and the total  $\gamma$  photon energy,  $\gamma_{\pm} = \varepsilon_{\pm}/mc^2$ ,  $S_{\perp}^0$  is the unit-cell area for the 2D lattice

<sup>4</sup>Note that these quantities are vectors

disregarding (within the quasi-classical phenomenology) the longitudinal contribution which is treated classically.

It can be seen from equation (2.40) and equation (2.41) that the longitudinal forces play no major role for the transverse motion. Moreover, the particle at very high energies is confined to a very small forward angle such that emissions take place within the emission cone  $1/\gamma$ . An integration of equation (2.40) over all possible particle trajectories in the transverse plane and in the process also taking into cognizance the 2-D lattice density distribution and the transverse scattering results in the following expression [44]:

$$\frac{dN_p}{d\xi_{\pm} dS_1^0} = \frac{\alpha}{2\pi S_1^0 c \tau_c} \{ (\xi_+^2 + \xi_-^2) \int_0^\infty \left( \frac{\tau}{\tau_{\gamma s}} + \frac{\tau^2}{\tau_{\gamma f}^2} \right) \sin \left( \frac{\tau}{\tau_c} + \frac{\tau^2}{6\tau_c \tau_{\gamma s}} + \frac{\tau^3}{12\tau_c \tau_{\gamma f}^2} \right) \frac{d\tau}{\tau} + 4\xi_+ \xi_- \left[ \frac{\pi}{2} - \int_0^\infty \sin \left( \frac{\tau}{\tau_c} + \frac{\tau^2}{6\tau_c \tau_{\gamma s}} + \frac{\tau^3}{12\tau_c \tau_{\gamma f}^2} \right) \frac{d\tau}{\tau} \right] \}. \quad (2.42)$$

In the above expression equation (2.42), information about the balance (or competition) between the aligned (coherent) and random (incoherent) effects is clearly elucidated. To illustrate this point further: the incoherent type of deflection is represented by  $\tau_{\gamma s}$ , and is independent of  $\tau_{\gamma f}$ , whereas the coherent type is determined by  $\tau_{\gamma f}$  and independent of  $\tau_{\gamma s}$  - these effects (coherent and incoherent) interlace with each other in a single common expression, (equation (2.42)). A highly relativistic photon (or particle) will interact coherently with many periodically aligned nuclear scattering centers along its trajectory in the crystal and this can result in the radiation length being shorter by 3 orders of magnitude (we will discuss this effect in greater detail later in the section that deals with the LPM). It is therefore worthy of note that the corresponding formation time is very short in aligned crystals

$$\tau_{\gamma f} = m_e / F_{\perp} \quad (2.43)$$

because of the strong field  $U_0$  that the particle experiences inside the crystalline channels and planes:

$$F_{\perp} = V_0 / a_{TF} \quad (2.44)$$

where  $a_{TF}$  is the screening length for the Thomas-Fermi potential.

### Pair Production in Amorphous Targets

An amorphous target corresponds to the limiting case when:

$$l_c \ll l_{\gamma s} \equiv c\tau_{\gamma s}. \quad (2.45)$$

Here,  $l_{\gamma s}$  is the multiple-scattering length over which an electron has passed from the creation point, and the corresponding mean squared scattering angle becomes:

$$\langle \Delta^2(l_{\gamma s}) \rangle = \frac{1}{\gamma^2} \quad (2.46)$$

The above equation shows that the opening angle is inversely proportional to the Lorentz factor, hence for highly Lorentz-boosted projectiles, the emissions are generally confined to a small frontal cone.

The equation for the energy distribution of  $e^+/e^-$  pairs produced by a single high-energy photon per unit path in an amorphous target is

$$\frac{dN_{\text{pair}}}{d\xi_{\pm}} = \frac{1}{X_0} (\xi_+^2 + \xi_-^2 + \frac{2}{3}\xi_+\xi_-), \quad (2.47)$$

which is the Bethe-Heitler formula. In this expression  $X_0$  is the radiation length related to  $l_{\gamma s}$  as following:

$$l_{\gamma s} = \frac{\alpha}{4\pi} X_0. \quad (2.48)$$

Integrating equation (2.47) gives the total yield:

$$N_{\text{pair}} = \int_0^1 \frac{dN_{\text{pair}}}{d\xi_{\pm}} d\xi_{\pm} = \frac{7}{9} \frac{1}{X_0}. \quad (2.49)$$

### Pair production on aligned single crystals

The cross-sections for the formation of a pair in crystalline matter can be obtained within the definitions of the constant field approximation. In this case the crystal is envisaged as being aligned near an axial direction and the corresponding limiting case thereby being  $\tau_{\gamma f} = \infty$ . It follows therefore from equation (2.42) that in this case the pair-creation processes resemble a purely coherent character. A simple derivation based on Kononets [44] is presented in the following section:

$$\int_0^\infty \sin\left(\frac{\tau}{\tau_c} + \frac{\tau^3}{12\tau_c\tau_{rf}^2}\right) d\tau = \frac{4}{\sqrt{3}} K_{2/3}\left(\frac{4\tau_{rf}}{3\tau_c}\right), \quad (2.50)$$

$$\int_0^\infty \sin\left(\frac{\tau}{\tau_c} + \frac{\tau^3}{12\tau_c\tau_{rf}^2}\right) \frac{d\tau}{\tau} = \frac{\pi}{2} - \frac{1}{\sqrt{3}} \int_{\frac{4\tau_{rf}}{3\tau_c}}^\infty K_{1/3}(y) dy. \quad (2.51)$$

In Eqs. (2.50) and (2.51) the factor  $K_\mu(z)$  is a modified Bessel function of the third kind for the  $\mu$ th order [47] and  $\tau_f$  is the pair formation time; the size of a time domain which gives a main contribution to the integrals of equation (2.42). The longitudinal path  $l_f = c\tau_f$  passed by the particle during the time  $\tau_f$  give the pair formation length which, according to equation (2.42) has a single common value for coherent and incoherent pair production processes. On substitution of the above equations into equation (2.42) one obtains the following constant field approximation solution:

$$\frac{d^2 N_p}{d\xi_{\pm} dS_{\perp}^0} = \frac{2\alpha}{\sqrt{3}\pi l_c S_{\perp}^0} \left[ (\xi_+^2 + \xi_-^2) (K_{2/3}) \frac{4l_{rf}}{3l_c} + \xi_+ \xi_- \int_{\frac{4\tau_{rf}}{3\tau_c}}^\infty K_{1/3}(y) dy \right]. \quad (2.52)$$

Sørensen [43] arrives at a somewhat similar equation by the inverse method of calculating radiation yields since there is a cross-symmetry between bremsstrahlung emissions and pair-production as shown in figure 2.3 where these two processes are related via the rotation of the diagrams by a quarter turn.

### 2.5.2 General overview of pair creation in a constant field

When a high-energy photon interacts with a single crystal, the mechanism of pair-creation is modified considerably in comparison with that in an amorphous medium. The coherence and correlated effects start manifesting themselves when the particle is incident on a target that is characterized by a periodicity of the structure. Hence, at definite incident angles and photon energies, the cross-sections for the pair-creation processes differs from that of scattering from independent, isolated scattering centers (Bethe-Heitler mechanism). The process of pair-creation in a constant field has been detailed in the literature [63]. It has been shown that the Constant Field Approximation

[63] is applicable if  $\Theta \ll U_0/m$ . Theoretically, the treatment of pair-creation is parametrised by the setting in of the following scale factor<sup>5</sup>:

$$\kappa = \frac{e}{m^3} \sqrt{|F_{\mu\nu} k^\nu|^2} \longrightarrow \frac{\omega}{m} \frac{E}{E_0}, \quad (2.53)$$

where,  $k^\nu(\omega, \mathbf{k})$  is the four-momentum of a photon,  $F_{\mu\nu}$  is the electromagnetic field tensor,  $m$  the electron mass,  $E$  the local electric field of the crystalline plane/axis and finally  $E_0 = m^2/e$  is the critical field. Many peculiarities of pair-production in single crystals bear similarities with those of pair-creation in the field of a plane electromagnetic wave. This comes as a result of the fact that for photons (or ultra-relativistic particles) travelling in the vicinity of the axes (planes), the crystalline field can be reduced to a flux of equivalent photons. This concept was originally proposed by Weizsäcker and Williams and since been known as the Weizsäcker-Williams [69] method of virtual quanta.

### 2.5.3 Interlude on formalism of the pair-creation processes

In this section is given a general field-theoretic derivation of the pair-production mechanism, following from which will be a description of the criterion for required parameters of pair-production. It is rather not an exhaustive treatise on the theory but what is sought to be shown is a standard textbook approach to find if there are any connections to the CFA. In the initial analysis will be shown how the interaction with the external fields yields a simple example of a dynamical system.

#### Standard quantum field approach

It is well known from the quantum field theoretic formulation that the interaction of photons with an external field yields a simple example of a dynamical system. There are several examples that serve to illustrate such processes, for example, radiation from

<sup>5</sup>Here, relativistic units are used, i.e.  $\hbar = c = 1$

a classical source. Here, we deal largely with a fermionic systems and therefore, a physical counterpart herein is a process of pair-creation by an external electromagnetic field.

The lepton-photon processes are studied on the basis of some insight of the role of the Dirac equation of the quantized field  $\psi$ :

$$[i \not{\partial} - e \not{A}^\mu(x) - m]\psi(x) = 0, \quad (2.54)$$

where  $\not{A}^\mu(x)$  is the gauge external field.

The equation corresponds to interaction with the Lagrangian

$$\mathcal{L}_I = -\mathcal{H}_I = -e\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu. \quad (2.55)$$

The premise for such processes starts from the basic tools of perturbation theory, in which all steps leading to the matrix are known:

$$\begin{aligned} S &= \exp \left[ -i \int d^4x A_{in}(x) j(x) \right] \\ &= \exp \left[ -i \int d^4x A_{out}(x) j(x) \right]. \end{aligned} \quad (2.56)$$

The above expression is a Lagrangian which is quadratic in the fields, rather than linear and therefore the commutator of the two currents  $\bar{\psi}\gamma\psi$  is not a  $c$  number. However, Wick's theorem gives the probability amplitude of emitting no pair to be:

$$\begin{aligned} S_0(A) &= \langle 0_{in} | S | 0_{in} \rangle = \sum_{n=0}^{\infty} \frac{(-ie)^n}{n!} \int dx_1 \dots dx_n \langle 0 | T [\bar{\psi}(x_1) \not{A}(x_1) \psi(x_1) \dots \\ &\quad \bar{\psi}(x_n) \not{A}(x_n) \psi(x_n)] | 0 \rangle \end{aligned} \quad (2.57)$$

From the algebra<sup>6</sup> the final answer gives [41], after some truncation on the terms of the  $S$  matrix:

$$S_0(A) = e \text{Det}(I - \Gamma) \exp [\text{Tr} \ln(I - \Gamma)]. \quad (2.58)$$

<sup>6</sup>The "in" subscripts have been omitted in equation (2.57), moreover, the coupling gets switched off at large  $T$

where

$$\Gamma = e \not{A}(x) \frac{1}{\not{P} - m - i\varepsilon}. \quad (2.59)$$

Therefore,

$$\begin{aligned} S_0(A) &= \text{cDet} \left[ I - e \not{A}(x) \frac{1}{\not{P} - m - i\varepsilon} \right] \\ &= \exp \text{Tr} \left[ (\not{P} - m - i\varepsilon) \frac{1}{\not{P} - e \not{A}(x) - m - i\varepsilon} \right]. \end{aligned} \quad (2.60)$$

Furthermore,

$$\ln S_0(A) = \text{Tr} \ln \left( [\not{P} - e \not{A}(x) - m + i\varepsilon] \frac{1}{\not{P} - m + i\varepsilon} \right). \quad (2.61)$$

Application of charge conjugation yields:

$$\ln S_0(A) = \text{Tr} \ln \left( [\not{P} - e \not{A}(x) + m - i\varepsilon] \frac{1}{\not{P} + m - i\varepsilon} \right). \quad (2.62)$$

Summing equation(2.61) and equation(2.62) gives

$$2 \ln S_0(A) = \text{Tr} \ln \left( [\not{P} - e \not{A}(x)]^2 - m^2 + i\varepsilon \frac{1}{\not{P}^2 - m^2 + i\varepsilon} \right). \quad (2.63)$$

Utilizing the useful identity,

$$\ln \frac{a}{b} = \int_0^\infty \frac{ds}{s} (e^{is(b+i\varepsilon)} - e^{is(a+i\varepsilon)}) \quad (2.64)$$

makes it possible for the probability of pair-creation to be derived:

$$w(x) = -\frac{1}{(2\pi)^2} \int_0^\infty \frac{ds}{s} \left[ eE \coth(eEs) - \frac{1}{s} \right] \Re(i e^{is(m^2 - i\varepsilon)}) \quad (2.65)$$

where the term  $1/s$  corresponds to the subtraction at  $e = 0$ . To be noted also is that the infra-red divergences at  $s \rightarrow \infty$  are insured by the  $m^2 - i\varepsilon$  prescription. Moreover, at  $s \rightarrow 0$ , the bracket vanishes like  $s$  and the integrand is finite. Therefore, the probability of creating a pair varies as an exponential<sup>7</sup>, where it is seen that the probability for producing a pair in a unit volume per unit time is reduced to the following expression:

$$w(x) = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{|eE|}\right) \quad (2.66)$$

<sup>7</sup>Again it is seen from this "naive" view that in order to induce spontaneous pair-creation, very large fields are required.



### Onset of criticality

The parameter which determines the behaviour of pair creation in a constant field is given in equation (2.53). At  $\kappa \ll 1$  the probability for creating a pair is low. This ties in well with equation (2.66); as it can be noted that the probability for QED pair-creation is very small since  $eE$  is negligible in comparison with  $m^2$ . The parameter  $\kappa \propto E/E_0$  sets a minimum threshold below which pair-creation is not probable<sup>a</sup>. However, with  $\kappa \sim 1$ , the probability for pair creation in a constant crystalline field increases rapidly and becomes equal to the probability of the Bethe-Heitler mechanism. The field threshold or conversely the energy threshold above which pair-production is real,  $\omega \geq \omega_t$ , is determined by, among other effects, crystal parameters like thermal vibrations as follows:

$$\omega_t \sim \frac{m^3 \rho d}{Z\alpha}. \quad (2.67)$$

In equation (2.67),  $\rho$  is the temperature dependent thermal vibrational amplitude,  $d$  the parameter that defines the inter-atomic distance for a crystal of atomic number  $Z$  and  $\alpha$  is the fine structure constant. Another approach that can be attempted is one in which the minimum field is set by the parameter  $\chi = \gamma E/E_0$ . This sets a criterion for quantum recoil effects to become significant. In the photon energy regime under consideration  $20 \leq E \leq E_{\max} \sim 140$  GeV, the strong field effects and quantum recoils ( $\chi \geq 1$ ) can start manifesting themselves, as a result, crystal induced pair creation can be observed. Moreover, the creation of a pair occurs over a finite spatial extent which is characterised by the formation zone. A detailed derivation has been given in [63] where it is shown that the maximum excess of the pair-creation probability in the axis field over the pair creation in the amorphous medium (Bethe-Heitler process) is designated by the parameter

$$r^{\max} \sim \frac{1}{3} \frac{a_s m}{Z\alpha \ln(183Z^{1/3})}, \quad (2.68)$$

where  $a_s$  is the effective screening radius of the string potential. From the above equation it can be deduced that  $r^{\max}$  is larger for small  $Z$  and large  $a_s$ . Furthermore,

<sup>a</sup>For example; atomic field strength on a Bohr orbit  $E_{\text{atom}}$  is of the order  $\sim eE_{\text{atom}} \sim m^2\alpha^3$

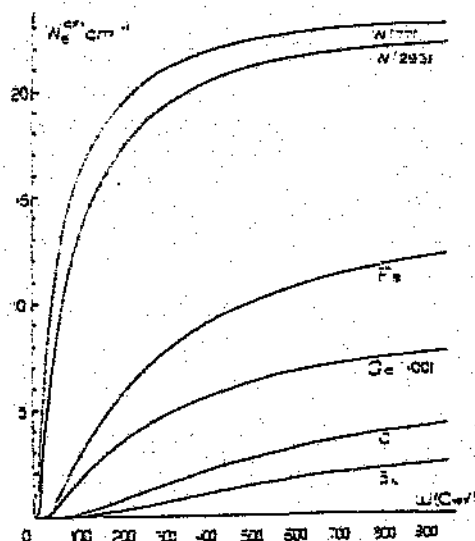


Figure 2.4: The probability for pair creation by a photon incident on the  $\langle 111 \rangle$  axis of W, Fe, C and Si and the  $\langle 110 \rangle$  axis for Ge. The numbers in brackets indicate the temperature (in Kelvins) of the crystal. The temperature, unless stated otherwise is the room temperature. These values are based on CFA calculations of Baier *et al.* [63].

the formation length is related to the Compton wavelength, as follows [66]:

$$l_f = \lambda_c / \chi. \quad (2.69)$$

By the combination of equation (2.69) and the Schwinger criterion  $\chi$ , it can be seen that the enhancement is proportional to  $\lambda_c E / \gamma E_0$ . In other words, it is inversely proportional to the Lorentz factor  $\gamma$ . However, the strong field in the rest frame of the electron extends over many nuclear scattering centers. Conversely, by taking the situation where the field is  $E$  in the laboratory frame instead of the Lorentz boosted  $\gamma E$  electron frame leads to a constant enhancement, namely,  $\lambda_c E / E_0$ . However, the saturation in this case happens at energies well above the threshold. This is illustrated in figure 2.4 where the behaviour of pair creation probability as a function of photon energy is shown. In the figure, all materials under consideration are shown.

The analysis that follows is based on the crystals whose parameters are tabulated as shown in Table 2.4:

| material   | axis                  | $\rho(\text{\AA})$ | $d(\text{\AA})$ | $\omega_t(\text{GeV})$ |
|------------|-----------------------|--------------------|-----------------|------------------------|
| C(diamond) | $\langle 111 \rangle$ | 0.040              | 3.567           | 90                     |
| W(293)     | $\langle 110 \rangle$ | 0.050              | 3.165           | 22                     |
| W(77)      | $\langle 110 \rangle$ | 0.030              | 3.165           | 13                     |
| Ge(293)    | $\langle 110 \rangle$ | 0.085              | 4.00            | 80                     |
| Ge(100)    | $\langle 110 \rangle$ | 0.054              | 4.00            | 50                     |

Table 2.4: Parameters used in the thesis to calculate the threshold energies for pair-production. In the table,  $d$  and  $\omega_t$  are the lattice spacing and the photon energy threshold, respectively. The threshold energies  $\omega_t$  are calculated using equation (2.67).

## 2.6 The Landau Pomeranchuk Migdal Effect

The afore-described phenomena depict a semi-classical description of pair-production and bremsstrahlung emissions both for the random and oriented incidence of leptons and photons on crystals. It should be noted, however, that the picture changes dramatically due to the setting in of processes that result in the suppression of radiation probabilities (depletions in cross-sections) for bremsstrahlung and conversely, for pair-formation. An *a priori* conjecture was forwarded in the seminal paper of 1953 by Landau and Pomeranchuk [53]. Their thesis was argued largely on the basis of the classical theory in which it was stated that the expected increase of bremsstrahlung emission actually changed to a depletion when additional inelastic interference processes are taken into account.

### 2.6.1 Basic theoretical principles of the LPM

The importance of coherence time effects on the production and absorption of field quanta from the motion of source particles in non-equilibrium dense media has been a subject of intense study since Landau, Pomeranchuk and Migdal formulated their hypothesis in the early 1950's [53, 48]. These studies were contextualised on the ba-

sis of bremsstrahlung from ultra-relativistic electrons undergoing multiple scattering from Coulomb scattering centers. The formalism which follows below is for radiation emission, however following from that will be a derivation for the LPM suppression of PP processes. We will re-visit the discussion of the coherence length in order to show its importance in dealing with the LPM effect, as has been described, it is a finite region in space at which the stripping of the photon from the electron takes place. The separated electron-photon are thus only considered as separate physical entities if their distance of separation is  $\lambda \sim 1/\omega$  and thus this takes place over the separation time

$$\Delta t = \lambda/v, \quad (2.70)$$

where the relative velocity is given as:

$$v_r = 1 - v \cos \theta_B \quad (2.71)$$

thus making the coherence length to be

$$l_c = v\Delta t \sim 2E^2/m^2\omega. \quad (2.72)$$

Here, an assumption<sup>9</sup> is made that  $\theta_B \ll 1$  and  $1-v$  is replaced by  $m^2/2E^2$ . The above arguments therefore allude to the fact that bremsstrahlung (and pair-production) do not occur at a single point (typified by the QED vertex) but extend over a finite zone in space-time. Dyson and Überall [83] formulated a similar hypothesis in the 1950's in which they studied the anisotropy of these processes in crystals. The coherent radiation was found to be influenced by the LPM at low energy emissions.

### General formulation

The hypothesis is basically that if all scattering centers are isotropic, then the standard notation for the radiation is well known and given by Jackson [85]:

$$\frac{dI}{d^3k} = \frac{\alpha}{(2\pi)^2} \left[ \int_{-\infty}^{+\infty} dt e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \underline{n} \wedge \underline{v} \right]^2, \quad (2.73)$$

<sup>9</sup> Relativistic units viz  $\hbar = c = 1$  are used

where  $\underline{r}(t)$  denotes the trajectory of the charged particle through the dense medium,  $\underline{v}(t)$  its velocity and  $\underline{n} = \underline{k}/\omega$  is a unit vector directed along the photon momentum. This equation has its equivalence in QED language wherein it can be described as a square of two Feynman propagators [84], namely:

$$\begin{aligned} \frac{dI}{d^3k} &= \frac{\alpha}{(2\pi)^2} \left[ \int_{-\infty}^0 dt e^{i(\omega t - \underline{k} \cdot \underline{r})} \underline{n} \wedge \underline{v}_0 + \int_0^{+\infty} dt e^{i(\omega t - \underline{k} \cdot \underline{r})} \underline{n} \wedge \underline{v}_1 \right]^2 \\ &= \frac{\alpha}{(2\pi)^2} \left[ \frac{\underline{n} \wedge \underline{v}_0}{\omega - \underline{v}_0 \cdot \underline{k}} - \frac{\underline{n} \wedge \underline{v}_1}{\omega - \underline{v}_1 \cdot \underline{k}} \right]^2, \end{aligned} \quad (2.74)$$

where the QED propagator is

$$\begin{aligned} (p + k)^2 - m^2 &= 2p_0 k_0 - 2\underline{p} \cdot \underline{k} \\ &= 2p_0(\omega - \underline{v} \cdot \underline{k}). \end{aligned} \quad (2.75)$$

Following from Cleymans [84], radiation yields can thus be averaged over all possible final velocities  $\underline{v}$ , as an example, by considering all  $s$ -scattering, isotropic processes where the distribution of final velocities is isotropic and independent of final velocity. Furthermore, equation (2.73) can be generalised over all segments of the locus of the particle trajectory

$$\begin{aligned} \frac{dI}{d^3k} &= \frac{\alpha}{(2\pi)^2} \left[ \int_{-\infty}^{t_1} dt e^{i(\omega t - \underline{k} \cdot \underline{r})} \underline{n} \wedge \underline{v}_0 + \int_{t_1}^{t_2} dt e^{i(\omega t - \underline{k} \cdot \underline{r})} \underline{n} \wedge \underline{v}_1 \right. \\ &\quad \left. + \dots + \int_{t_n}^{t_{n+1}} dt e^{i(\omega t - \underline{k} \cdot \underline{r})} \underline{n} \wedge \underline{v}_n + \dots \right]^2. \end{aligned} \quad (2.76)$$

The premise of these calculations emanates from the assumption that the velocity between collisions is constant:

$$\begin{aligned} \frac{dI}{d^3k} &= \frac{\alpha}{(2\pi)^2} \left[ \frac{v_0^2 - (\underline{n} \cdot \underline{v}_0)^2}{(\omega - \underline{v}_0 \cdot \underline{k})^2} + \left\langle \frac{v_n^2 - (\underline{n} \cdot \underline{v}_n)^2}{(\omega - \underline{v}_n \cdot \underline{k})^2} \right\rangle \right. \\ &\quad \left. + \sum_{j=1}^{n-1} \left\langle \frac{v_0 - (\underline{n} \cdot \underline{v}_j)^2}{(\omega - \underline{v}_j \cdot \underline{k})^2} (2 - e^{i\xi_j(\omega - \underline{k} \cdot \underline{v}_j)} - e^{i\xi_j(\omega + \underline{k} \cdot \underline{v}_j)}) \right\rangle \right]^2, \end{aligned} \quad (2.77)$$

where  $\xi_j$  is the medium dependent step term. Averaging over equation (2.76), one obtains an expression in which the first two terms correspond to the radiation emitted

by a single scatterer of a charged particle from  $v_0$  to  $v_n$ . This part is meant to diverge for  $\omega \rightarrow 0$  in much the same way as ordinary Bremsstrahlung. The third term gives the solution and also designates the medium modification of bremsstrahlung.

Another way of looking into the problem is by setting the average time between collisions to be  $a^{-1}$ . Thus after averaging, the time can be obtained from the exponential distribution function as follows:

$$\frac{d\omega}{d\xi_j} = ae^{\xi_j a} \quad (2.78)$$

The constant  $a$  is a material property that sets an *a priori* dependence of the physics on the material. The integral over time between collisions,  $\xi_j$  is derived as follows:

$$a \int_0^\infty d\xi_j e^{-\xi_j a} [1 \cos \xi_j b] = \frac{b^2}{a^2 + b^2} \quad (2.79)$$

The above action effectively replaces the propagator factor  $1/(\omega - \underline{k} \cdot \underline{v})^2$  with  $1/(\omega - \underline{k} \cdot \underline{v})^2 + a^2$  and gives in a final analysis, a modification of the photon emissions which resembles the following form:

$$d\sigma^\gamma/d\omega \sim \sqrt{\omega}, \quad \text{if } l_c \gg a^{-1}. \quad (2.80)$$

The photon emission spectrum described by the Bethe-Heitler formula extends over the whole frequency range without showing any energy dependence; however, the LPM effect changes qualitatively the results, leading to a  $\sqrt{\omega}$  behaviour.

For a good understanding of the method, Migdal [48] sought to use the procedure of averaging over the transition rates. This method is a mathematically involved quantum field formulation albeit yielding the same results in the limit of soft-photon emission. The solution, as in all cases, is reduced to the Fokker-Planck equation in the limiting case of small angle scattering. Accordingly, the typical angles of scattering are small in comparison with  $m/E$ , and the function which was used to determine the photon distribution was [86]

$$\phi(s) = 24s^2 \int_0^\infty dx \left( \frac{1}{\tanh x} - \frac{1}{x} \right) e^{-2sx} \sin 2sx, \quad (2.81)$$

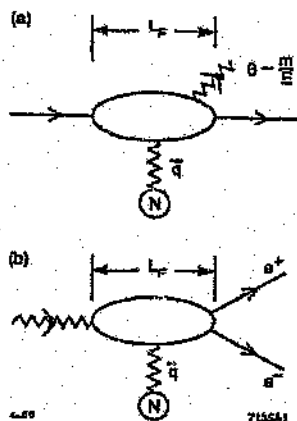


Figure 2.5: Simple Feynman diagrams used for the illustrative purposes of depicting the LPM effect for Bremsstrahlung emission (a) and pair-production (b). The finite extent of the formation length is shown.

where the Migdal parameter,  $s \approx 1/8(m^2/p^2)\sqrt{\omega}/q$ . Figure 2.5 is a graphical illustration of the LPM for both these QED processes.

Bell [86], also proposed (following from the assumption which was also implicit in the work of Migdal) that typical angles of single scattering are small compared with  $m/E$ . Following from that, the average energy radiated per collision<sup>10</sup> was then reduced to

$$\frac{2e^2}{6\pi} \left( \frac{E}{m} \right)^2 \overline{\Theta^2}, \quad (2.82)$$

where  $\overline{\Theta^2}$  is the mean square angle of scattering in a single collision. This expression for the energy radiated gives a classical approximation to the Bethe-Heitler approximation<sup>11</sup> for the emission at low frequencies. Furthermore, Bell has also stated that frequencies of order  $1/(\tau\gamma)$  correspond to photon energies  $\hbar\omega$  such that  $\hbar\omega =$

<sup>10</sup>Interference effects are ignored.

<sup>11</sup>Strictly speaking, these considerations apply only to hypothetical atoms with very large nuclei, however, it is probable that the results are qualitatively relevant to the real case

$$(\hbar/mr)/(E/m) \approx 10^{-3} E/m.$$

### Pair production

Following from Kononets [45], we can derive an expression for the LPM suppression in pair-production which arises under the condition:

$$l_c \gg l_{\gamma s}, \quad (2.83)$$

where  $l_c$  and  $l_{\gamma s}$  are the coherence (formation) and the mean free path (m.f.p.) lengths respectively. Furthermore, equation (2.83) can also be represented in terms of angles as follows:

$$\langle \Delta^2(l_{\gamma c}) \rangle \gg \frac{1}{\gamma^2} \quad (2.84)$$

Simplification can be obtained in the limit of  $\tau_{\gamma s} \rightarrow 0$ , by making use of the following relations:

$$\int_0^\infty \sin\left(\frac{\tau}{\tau_c} + \frac{\tau^3}{4\tau_c\tau_{\gamma s}^2}\right) d\tau = \sqrt{\frac{3\pi\tau_{\gamma s}}{4\tau_c}} \quad (2.85)$$

$$\int_0^\infty \sin\left(\frac{\tau}{\tau_c} + \frac{\tau^3}{12\tau_c\tau_{\gamma s}^2}\right) \frac{d\tau}{\tau} = \frac{\pi}{4}, \quad (2.86)$$

Now, we can introduce the ratio  $R$  which is given by the expression:

$$R = \frac{\ln \theta_{\max}^f / \theta_{\min}^f}{\ln 183 Z^{-1/3}}, \quad (2.87)$$

where  $\theta_{\max}^f$  and  $\theta_{\min}^f$  are maximum and minimum angles for scattering on an atom with nucleus of charge number  $Z$ , respectively. This discussion is based on the fact that in high energy pair-production processes, scattering angles which exceed the mean squared scattering angles are ignored. In other words, the large angle scattering gives a negligible contribution to pair production processes; all motion is confined to a very narrow kinematic cone. The factor  $R$  is obviously a very small parameter for the kind of investigations carried out in this thesis. The following expression can be deduced for the time interval,  $\tau_{\gamma s}$ :

$$\tau_{\gamma s} = \frac{2}{R} \tau_{\gamma s}^0, \quad (2.88)$$



where  $\tau_{\gamma s}^0$  the time interval through which the mean squared angle of ordinary multiple scattering is  $1/\gamma$ .

Using the relations (2.85) and (2.86) and also taking into account equation (2.88) and equation (2.48), gives the result:

$$\frac{d^2 N_p}{d\xi_{\pm}} = \frac{R(s)}{X_0} \left[ (\xi_+^2 \xi_-^2) \sqrt{6\pi s \xi_+ \xi_-} - 8\pi s^2 \right]. \quad (2.89)$$

In equation (2.89),  $s$  is the Migdal parameter[48] which couples the mean free path to the coherence length as following:

$$s = \sqrt{\frac{l_{\gamma s}}{l_c}}. \quad (2.90)$$

Equation(2.90) fixes  $s$  as a function of the scattering parameters and energies. It follows therefore from Eqs. (2.85) and equation (2.86) that the pair formation length under the LPM conditions is:

$$(l_f)_{\text{LPM}} \sim \sqrt{6l_c l_{\gamma s}} \sim l_{\gamma s}/s \quad (2.91)$$

It can be deduced from above that the radiation length shortens with decreasing  $s$ . From equation (2.90), it can also be seen that the LPM suppression becomes substantial at  $s \leq 8$  and leads to decrease in the pair production rate with increasing energy  $\omega$  of the incident photon as following:

$$\frac{d^2 N_p}{d\xi_{\pm}} \propto \omega^{-1/2} \quad (2.92)$$

Moreover, equation (2.92) is a condition which sets a requirement for very high incident photon energies:  $\hbar\omega \geq \hbar\omega_{\text{LPM}} \equiv 8\alpha X_0 m^2 c^3 / \hbar\pi$ . As an example, in amorphous tungsten where  $X_0 = 3.25$  mm, the LPM effect will start showing up only when the energy of the photon is 80 TeV. However, when dealing with aligned crystals, there is a strong reduction in multiple scattering length near atomic axes, this length may reduced to  $10^{-3}$  of the value of  $l_{\gamma s}$ . For example, along the  $\langle 110 \rangle$  axis in tungsten at 77 K, the length has a reduction factor of  $9.8 \cdot 10^{-4}$ . Therefore, the LPM frequency  $\hbar\omega_{\text{LPM}}$  gets lowered by three orders of magnitude. This could go down to the value  $\hbar\omega_{\text{LPM}} = 80$  GeV, hence the LPM suppression will appear in the yield of coherent processes at about  $\hbar\omega \geq 80$  GeV.

## 2.7 Summary

The theoretical outline which was detailed in this chapter will be used in the interpretation of the data. The starting point of the study of the coherent, correlated strong field processes is in the use of the semi-classical predictions and comparison with the experimental results for pair-production in an axially aligned Ge  $\langle 110 \rangle$  crystal at room temperature. More importantly, the use of equation (2.67) which determines the threshold for pair creation will, in the initial analysis give the value of the photon energy in which enhancements above the Beth-Heitler cross-sections equation (2.27) occur. The semi-classical calculations used in section 2.5 were based on the formalism of Kononets, in which very accurate predictions of the differential distribution of the pair-production rate are made, which will later on be compared with the experimental data obtained in this thesis.

The predictions of the LPM both for bremsstrahlung and pair-production will be used to evaluate if they also are consistent with the experimental results given in chapter 6. More importantly in this case we will see if the  $\sqrt{\omega}$  deviation at low photon emissions, predicted in (equation (2.80)) can be observed. Finally, the measurement of the suppression in pair-production yields will also be compared with the theoretical predictions of Kononets.

## Chapter 3

# The NA43 Experiment

*"Ce qui le surpris davantage, et qui leu fit plus de plaisir, ce fut le palais de sciences, dans lequel il vit une galerie de deux milles pas, toute pleine d'instruments de mathematique et de physique"... Voltaire - 1759*

### 3.1 Introduction

The experimental setup for the investigation of the high-energy lepton-photon processes is described in this chapter. The principal purpose herein is to elucidate various key facets of the NA43 experiment together with some detailed overview of the operation of the CERN-SPS. Also described are various aspects of beam production and transport including many important characteristics that determine the quality of the beam *inter alia* intensity, diameter and angular divergence. It is these features which characterise the quality of the beam and thus have a profound influence on the nature of the physics to be investigated. This can be appreciated from the fact that at ultra-relativistic energies, requirements for channeling and other aligned CFA effects are severe and thus beam divergences have to be kept within acceptable limits. Moreover, the use of position sensitive detectors (drift-chambers) has played a pivotal role in the experiment. Other detector components, magnets and scintillators are also described in detail as well as the data acquisition and trigger settings.

## 3.2 The CERN-SPS

The Super Proton Synchrotron (SPS) accelerator is a proton accelerator in the European Laboratory for Particle Physics (CERN), Geneva, Switzerland. The accelerator complex and the injectors therein straddle the Swiss-French border at mean depth of 40 m. The diameter of the SPS is 2.2 km. A plan view of the entire complex is shown in figure 3.1.

### 3.2.1 The H2 beam-line of the North Area Hall of the SPS

A 450 GeV/c primary proton beam of nominal intensity  $1 - 5 \cdot 10^{12}$  particles per spill is supplied by the proton accelerator complex, the CERN-SPS, and directed onto the North Area targets T2, T4 and T6 at fixed energy and percentage distributions. All beams in the North Hall are provided with sophisticated particle identification and momentum measuring equipment. T2 and T4 provide proton beams for ENH1 (Experimental Hall North 1) whilst T6 produces a muon beam for ENH2. The production target centres are located in a hall 12 m below the ground surface. The beams are transported upwards so as to be a few meters below ground before emerging into the experimental halls. This set-up ensures that radiation levels on the surface are kept well below the acceptable limit and, furthermore, the muon background at the experiments is also greatly reduced in the process.

#### Production of secondary lepton beams

A beam of 450 GeV protons, fired on a fixed target (T2) made of 300 mm thick Be, produces a high-energy secondary hadron beam [57]. This was then injected into the H2 beam-line shown schematically in figure 3.2. The beam-line could also be alternatively used to transport an attenuated primary beam of protons, electrons from  $\gamma$  conversions and polarized protons from  $\Lambda$ -decay. However, the selection of various beam species was enabled by means of an array of various magnets and targets as shown in figure 3.2. The electron beam is filtered from hadronic debris emanating from the

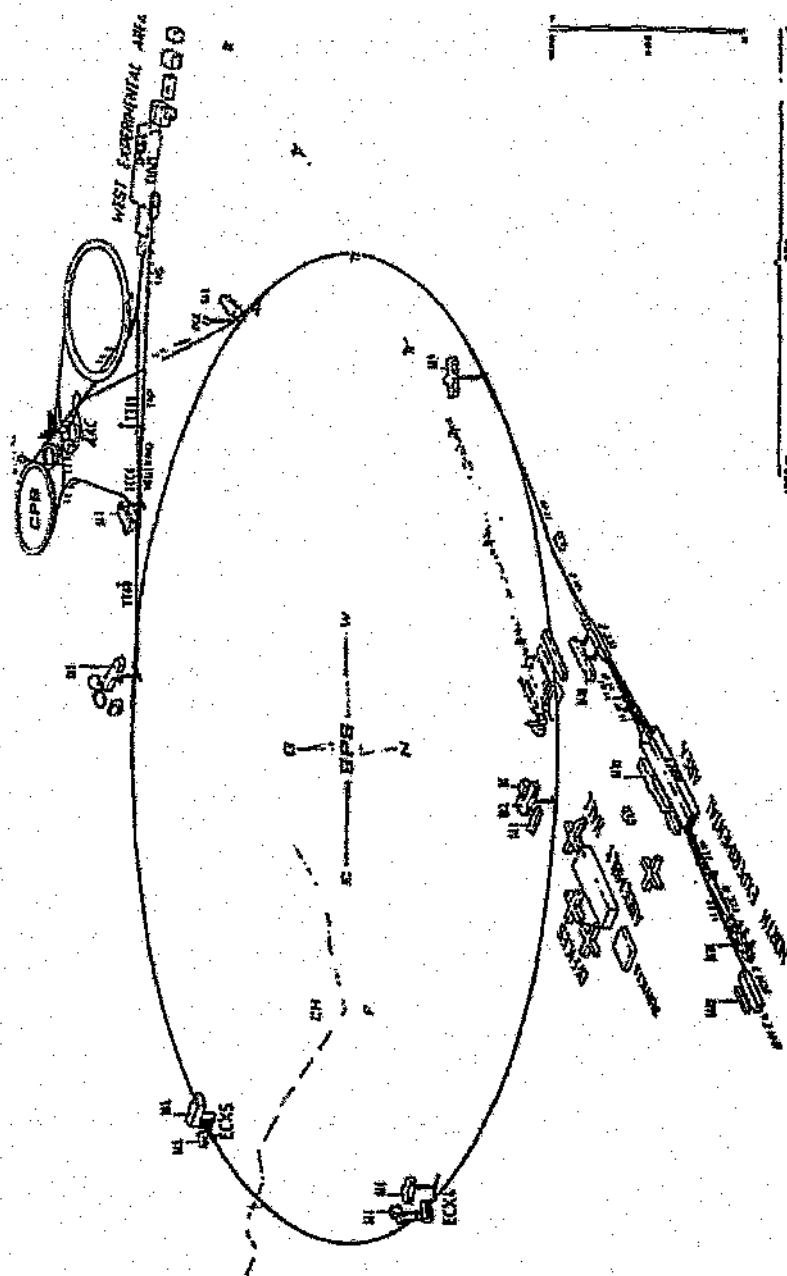


Figure 3.1: Diagram of the 450 GeV Super Proton Synchrotron. The 26 GeV proton synchrotron (PS) serves as an injector for the main ring of diameter of 2.2 km

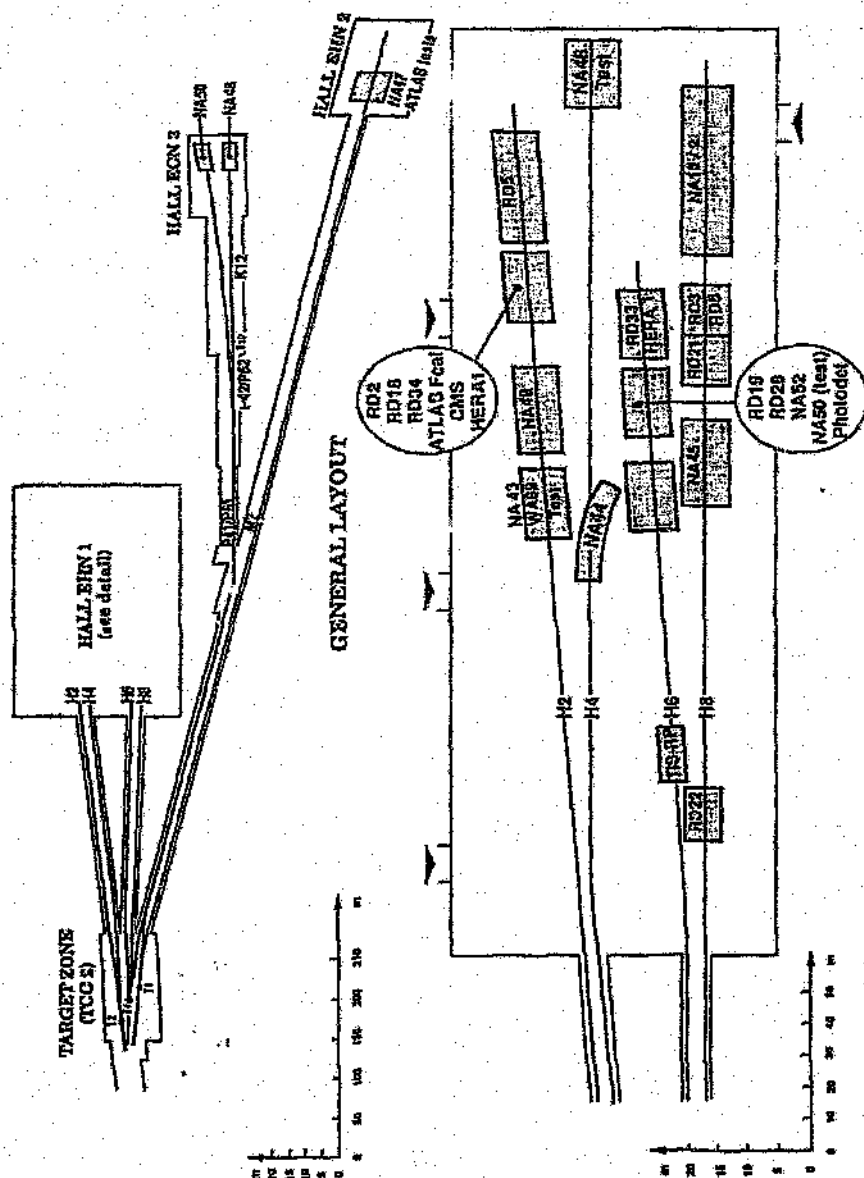


Figure 3.2: Plan view showing the general layout of the H2 and other beamlines in the experimental North Area (ENH2)

T2 target. The various stages of the process are given by the series of equations below:

$$p^+ + \text{Be} \rightarrow \pi^0, K^0, n, \Lambda \quad (3.1)$$

All charged particles are swept away from the beam-line with the help of various magnets along the beam-line. However, neutral particles pass through undeflected. The electron beam is produced indirectly from the decay of  $\pi^0 \rightarrow 2\gamma$ , the photons therein are converted into a  $e^+/e^-$  pair upon incidence on a 6 cm thick Pb target just downstream of the T2 target. The  $e^+/e^-$  have energies up to 300 GeV.

Finally, a parallel secondary beam of 150 GeV/c electrons or positrons with nominal intensity of  $\sim 4 \cdot 10^4$  particles/spill is transported 500m down to NA43. The beam is also very carefully guided by a set of bending magnets optimally positioned such that low-energy leptons are filtered away. The resulting r.m.s. beam divergences were 40  $\mu\text{m}$  and 50  $\mu\text{m}$ ; in the horizontal and vertical plane, respectively. The momentum resolution of the beam was  $\Delta p/p \leq 1\%$ .

Radiation due to Bremsstrahlung from scintillators and drift-chambers located between the bends could be discarded as the positioning of these bends ensured that the radiated photons due to their production angle, would escape detection in the lead-glass array (see figure 3.4).

It also should be noted that the beams suffered some contamination from upstream hadronic debris. In figure 3.3 is shown the various fractional distributions of hadrons, as a function of energy for the electrons and positrons in the H2 beamline. This contamination appears as grass<sup>1</sup> in the lead glass spectrum. This proton contamination was estimated to be 35%, the origin of which was attributed to the decay of a neutral  $\Lambda$  baryon thus

$$\Lambda^0 \rightarrow p^+ + \pi^- \quad (3.2)$$

<sup>1</sup>This pertains to a flat background. Negative and positive contamination appear in the lead glass as well defined MIP signals

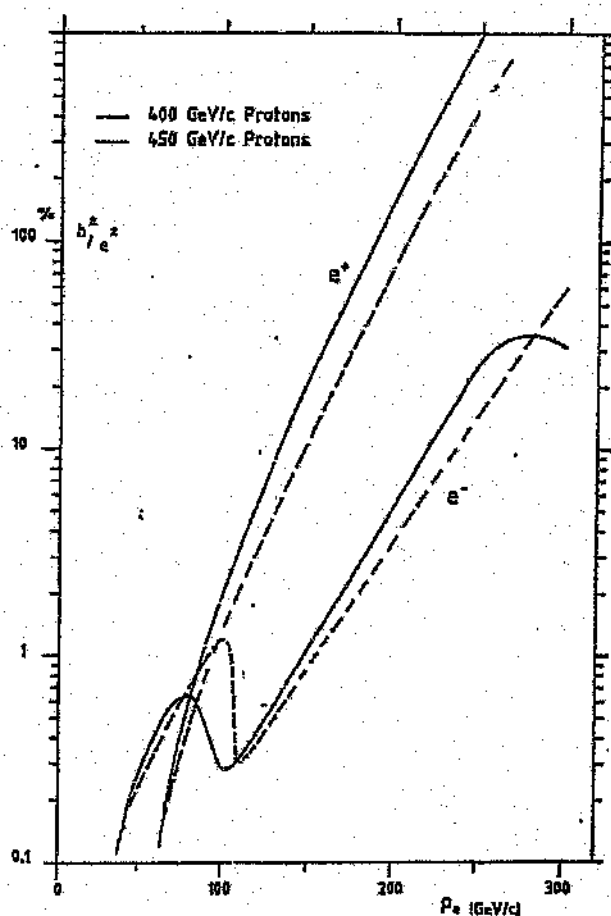


Figure 3.3: Hadronic contamination of the electron and positron beams. The variation of the fractional contamination as a function of energy is given for the two nominal energy settings of the SPS.



## NA43

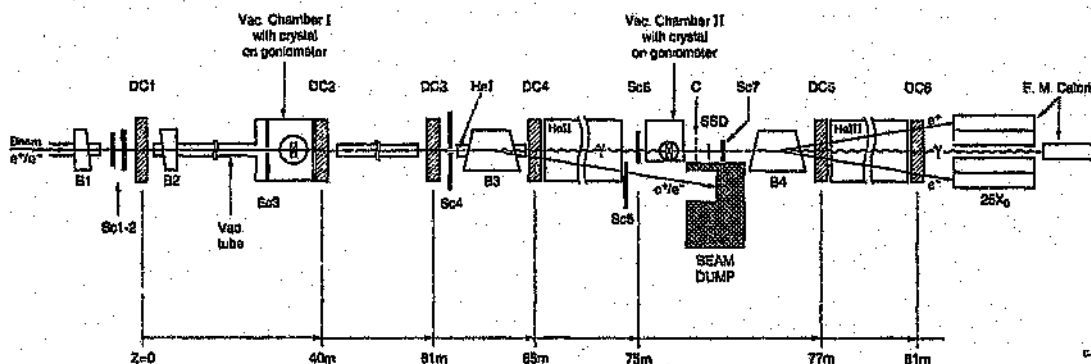


Figure 3.4: Typical arrangement of the NA43 beam line. This configuration was the arrangement used to investigate pair production

### 3.3 The NA43 beam-line

The NA43 beam-line and detector system is a multi-purpose system in which various aspects of strong-field QED physics can be investigated. The NA43 detector set-up spanned a length of more than 80m. Figure 3.4 is a schematic representation of the beam-line and the detectors therein. However, cognizance should be taken of the fact that the set-up changed according to the physics under investigation, an example of which was in the investigation of pair-production in heavy crystals wherein the anti-Kaon veto (AKS)-detector array was installed [59]. An overview and detailed description of various segments and components of the NA43 detector will be given in the sections following:

#### 3.3.1 Target Vacuum Chambers and Crystal targets

These were used in temperature stabilised huts for the measurement of pair-production at 100K (these are shown figure 3.4 and are represented as Vac. Chamber I, and II, respectively). The photon beam entered this vacuum chamber through an aluminised

mylar window  $\sim 20 \mu\text{m}$  thick. The crystalline targets were used in order to investigate the QED effects described in the thesis. These targets formed the core of the entire experimental project. They were placed in two vacuum chambers, designated by labels I and II in figure 3.4. Inside the vacuum chambers were computer controlled goniometers whose purpose was to align the crystal to within the severe orientation requirements of the various critical angles. The first crystal was placed in a goniometer which had a step size of  $17 \mu\text{rad}$ . For the purposes of simplification, the experiment can be zoned into regions "upstream" and the "downstream" of bend 8. The radiator crystal is placed in the upstream region whereas the converting crystal is placed in the vacuum chamber that is in the downstream region. Figure 3.5 and figure 3.6 will serve best to illustrate the exact location, detailed descriptions of each region and the exact dimensions thereof.

### Alignment of Crystals

Accurate alignment of the crystal w.r.t. the incident beam had to be obtained in order to observe all the crystal dependent effects described in this thesis. A well pre-aligned crystal was useful in the sense that it would take less time to align. In order to ensure that the crystal was mounted correctly (without losing pre-alignment), a laser was used to perform "in-situ" pre-alignment, that is, aligning the crystal with respect to the nominal beam. The converting crystal, which in this case is a  $0.6 \text{ mm}$  Ge target could thus be aligned with a goniometer which had a step size of  $20 \mu\text{rad}$ . A converter of  $5 \text{ mm}$  Pb was then placed in front of the alignment scintillator Sc 11 so as to convert photons emitted by the crystal. Alignment would thus be confirmed by observing maximum yields in photon conversions. Moreover, data was taken at different angular settings (crystallographic orientations for the converter crystal).

### Helium tanks

The nature of the physics to be investigated was such that the whole beam-line would not be under continuous vacuum conditions. Therefore, Helium tanks played a pivotal

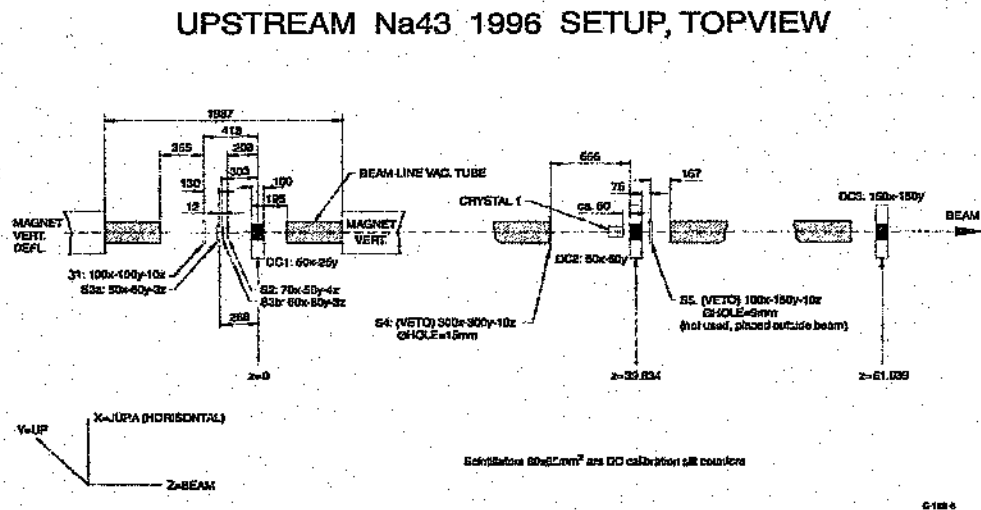


Figure 3.5: Detailed plan view showing the upstream region of the experimental set-up

## Na43 1996 DOWNSTREAM SET-UP, TOP-VIEW

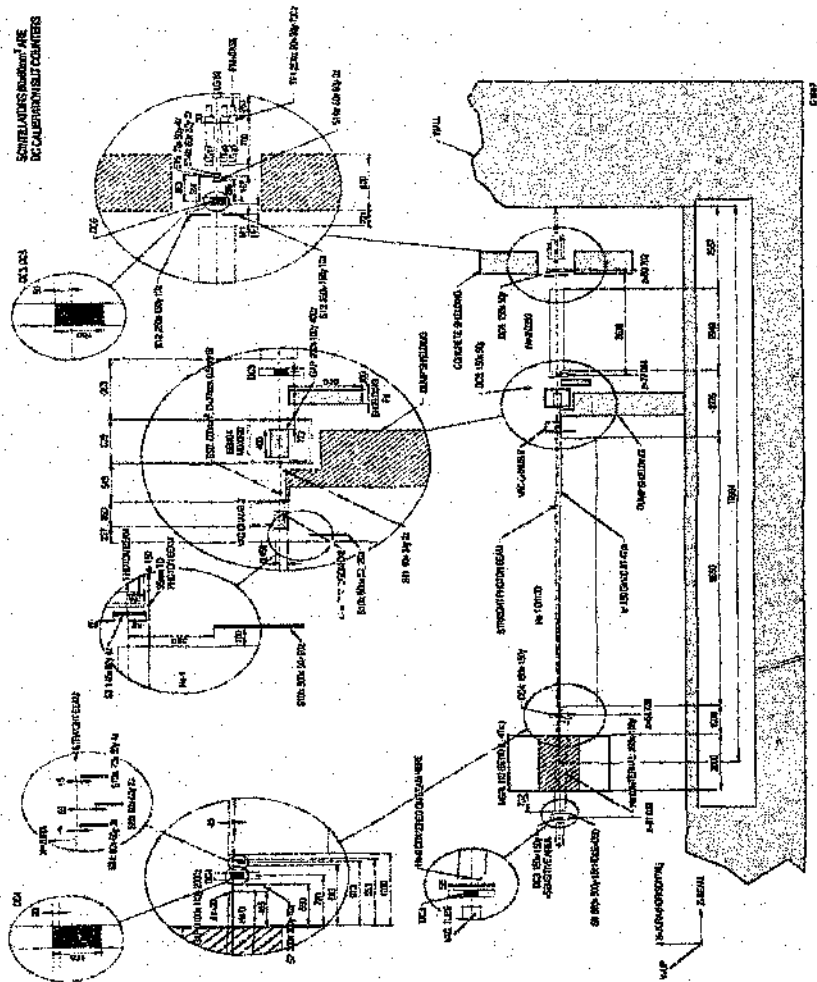


Figure 3.6: Detailed plan view showing the downstream region of the experimental set-up

role in the reduction of material along the beam line. The purpose of the He-tanks therefore, was material minimisation along the beam-line so as to reduce, as much as possible, multiple coulomb scattering (m.c.s.) and conversions of radiated photons and radiation photons by electrons and positrons.

Inside the bending magnet (Bend 8) (see figure 3.6) a rectangular shaped He tank was placed, positioned in a skewed way such that high-energetic particles could not hit the walls of the tank and thence contaminate the beam from the showers that would be produced in that process.

### 3.3.2 Drift chambers

The description and technology of the construction of the position sensitive detectors, namely, drift chambers, is well detailed in literature and is still a subject of continuing investigation and development to this day. A comprehensive study is given by Sauli [60]. What is largely detailed in this section is a brief historical overview and a global description of the means of operation of the drift chambers.

It was realised during the early stages of the development of multi-wire proportional counters (MWPC) that spatial information could also be obtained by measuring the drift time of the electrons coming from an ionising event. If the trigger is available to signal the arrival of a particle and also if the drift velocity through a gas mixture is known, then the distance from the sensing wire to the origin of the electron can be deduced. It is highly desirable to have a constant drift speed and hence a constant drift field so as to obtain a constant relationship between the time of drift and the distance travelled [62]. Figure 3.7 shows a schematic view of the drift chamber, in which the particle crosses a gas and ionises a certain number of molecules, creating primary electrons and positive ions. In the presence of a constant electric field, electrons will move with constant drift velocity towards the wire with positive tension (*sense wire*) and thereby creating an avalanche. The position of the ionising particle comes therefore at

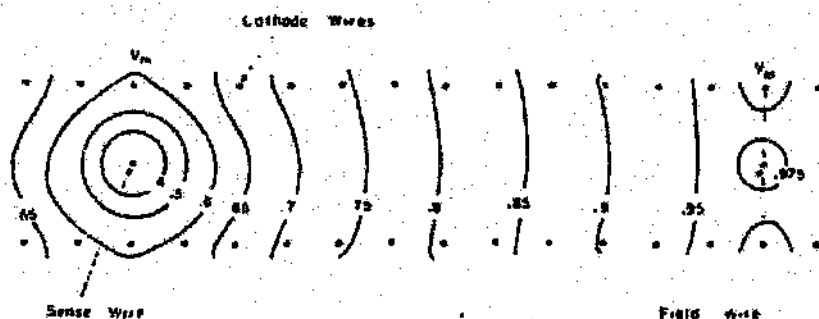


Figure 3.7: Schematic view of the basic principles of operation of a drift chamber. The length cell (distance between the field wire and the anode(sense wire) is 25 mm. In the centre is placed the anode wire of 20  $\mu\text{m}$  diameter connected to a positive potential  $+HV_2$  which is uniformly distributed on the cathode wire plane. The negative high tension drops regularly from 0 V (in front of the anode wire) to  $-HV$  in front of the field wire.

intervals of time which between the arrival of a signal and the position of the ionising particle. The voltage in various wires is regulated such that there is a uniform electric field throughout the drift region.

For the efficient operation of a drift chamber, it is therefore necessary that a precise knowledge of drift velocity is ascertained. This is accomplished by making a strict selection of the gas to fill the drift region. The choice of a filling gas used is in turn, governed by several factors among which are:

- ⇒ low-working voltage
- ⇒ high-gain
- ⇒ good proportionality
- ⇒ high-rate capacity

In general, these conditions are fulfilled by a using a gas mixture rather than a

pure gas. In order to obtain a desirable drift velocity of  $\sim 5 \text{ cm}/\mu\text{s}$ , a gas mixture of 31% argon/methynal, 42% argon and 26% isobutane quenching gas subjected to an electric field of  $\sim +2\text{kV}$  is essential. Methynal of concentration  $\sim 2.5\%$  is added in order to keep wires clean and thus prolong the lifetime of the chambers. Moreover, the spatial resolution of a drift chamber depends on how well the relation between drift-time and space co-ordinates is known. Furthermore, the spatial resolution is also dependent on the amount of diffusion suffered by the electrons during the course of their drift through the gas. The latter factor, however, depends on the length of the drift region. For a charged beam traversing through a gas at constant drift velocity  $\mu$ , the spread in the electron cloud after a distance  $x$  is:

$$\sigma = \sqrt{\frac{2Dx}{\mu E}}, \quad (3.3)$$

where  $D$  is the diffusion constant and  $E$  is the electric field. Therefore, one needs small diffusion for good spatial resolution. In the NA43 experiment, the drift chambers in use were characterised by a maximum drift region<sup>2</sup> of 2.5 cm giving a spatial resolution of  $100 \mu\text{m}$ . The intrinsic accuracy is however, limited by the fact that as electrons drift around the anode, a cloud spreads out due to diffusion thus resulting in multiple scattering and secondary ionisation.

The placing of field wires between sense wires ensures that the fields are uniformly shaped across the drift chamber. Moreover, in order to ensure maximum efficiency within the given strengths, it was necessary to use shorter path lengths to minimise the effects of diffusion. The drift velocity is characteristically  $5 \text{ cm}/\mu\text{s}$ , thus giving drift times of  $1\text{--}2 \mu\text{s}$  (the *memory time*). The wider surface area of  $15 \text{ cm}$  was covered by using 6 adjacent drift cells. Each of these cells were set in such a way that there was an alternation in the orientation of drift velocities from cell to cell.

### The Tagging System

The radiated photon was tagged by measuring the deflection of the radiating electron through B8 (4.05Tm). This was particularly important when the photon converted

<sup>2</sup>This is also known as a cell of a drift chamber

into an electron/positron pair which would often not reach the LG array when the pair spectrometer was on, therefore making the precise photon energy measurement dependent on the tagger. As can be seen in figure 3.6, the dotted line shows a straight photon beam and the solid line displays a track of a charged article (electron) emerging from DC4. The magnets were tuned and optimised in such a way that all the electrons were swept away from the photon beam. The fact that the lepton-photon opening angle is very small, i.e.  $\theta \sim 1/\gamma$  was the reason behind the use of a relatively strong magnetic field.

### The pair spectrometer

The pair spectrometer set-up was so conceived in order to measure the single photon energy. In this arrangement, DC5 and DC6 were used in conjunction with DC4 to measure the energy of the tagged photon which upon incidence on the second crystal would have converted into a lepton pair. A spectrometer magnet (MDX100) was used to separate the  $e^-/e^+$  pair, the tracked charged particles would then be reconstructed using the recorded hits on DC5 and DC6. Since the tagged photons were of variable energy, the consequence was that low and high-energy pairs were picked up by drift-chambers 5 and 6, respectively. It is by employing this technique that an accurate measurement of pair-production in aligned crystal was achieved.

### 3.3.3 Electromagnetic Calorimeters

The electromagnetic calorimeter is a total absorption device used to detect and measure energies of photons and electrons/positrons over a wide range of energies<sup>3</sup>. This calorimeter is in the form of a lead glass (LG) array with effective thickness of 25 radiation lengths ( $X_0$ ). These are stacked as shown in figs. 3.4 to 3.6. The array is designated as LGa1 to LGb2 as shown in figure 3.6. LGb2, placed downstream of the

<sup>3</sup>The calorimeters can measure energies in the range keV to multi-GeV, so a cut-off has to be set in the lower scale. The sole purpose is to set a restriction on useless events that enter the Pb glass array



array is in direct line of straight electron and photon beams. This arrangement was such that photons fired the most downstream component whilst lepton pair triggers the nearest adjacent blocks.

All the calorimeter blocks are characterised by the fact that they have a generally good energy resolution ( $\sigma_E/E \sim 5\%/\sqrt{E}$ , where  $E$  is the energy of the electron/positron and photon in GeV). The energy deposited is practically equal to the energy of the incident particle and proportional to the number of Cherenkov photons produced by the charged particles (electrons and positrons) in the electromagnetic shower inside the lead glass. In this way, very good energy detection efficiency could be obtained.

### 3.3.4 Event Trigger and selection

This was facilitated by means of a plastic scintillator array set in coincidence with one of the other detectors shown in figures 3.5-3.6. The incident beam was of approximately Gaussian profile with FWHM = 10 mm. The useful part of the beam was selected by a scintillator (Sc4) with a hole of  $\approx 15$  mm diameter which served to "sweep" away the flanks off the beam. The beam particle was therefore defined by the coincidence (Sc1\*Sc2\*Sc4). The Sc1\*Sc2 trigger was needed in order to take away "accidentals" in the form of noise whose rate was estimated to be  $\sim 100 \rightarrow 1000s^{-1}$ . In this way, 70 % of the incident beam was rejected. The data acquisition logic was set such that four main triggers were used, namely

- Normalisation = (Sc1\*Sc2\*Sc4)
- Radiation = (Sc1\*Sc2\*Sc4)(Sc11 > 1.5 Mip + LG > 10 GeV)
- Pair = Radiation\*(DC5Y1+DC5Y2)
- Calibration: was in used to calibrate the drift chambers.

The selection of useful events was facilitated by the array of scintillators. Mentioned specifically here is the counter Sc11, among its versatile purposes were the following:

- To give timing signals to drift chambers.
- Helped duplicate multiplicity detector (MD) information when set into coincidence with that MD detector
- It was also used to trigger conversions when alignment scans on the crystal were conducted

Sc10 was also useful, it was positioned such that it matched the acceptance of the DC4 calibration slit scintillator Sc8. Thus, information on Sc10 could also be used to reflect on the secondaries in goniometer 2 downstream. A similar purpose was served by Sc15 with regard to the calibration slit counters Sc14a and Sc14b downstream.

### 3.3.5 Data acquisition and online reduction

A block diagram of the electronic arrangement used for this project is shown in figure 3.8. Amplified analog outputs were generated by the detectors, *viz* solid-state detector, scintillators and the calorimeter. The signals were then processed by the ADC's<sup>4</sup> of the CAMAC system. The drift time (proportional to the distance) in the drift chambers was taken between the Sc trigger and the anode wire signals and these were processed by the TDC's<sup>5</sup>. A PC based interactive programme DUMLE [39] was used to retrieve converted signals from the CAMAC for online reduction and analysis and in list mode storage. The CAMAC system gives a BUSY signal which prevents an avalanche of events from getting into the system; the events that come in during the BUSY time window are thus thrown away. To minimise saturation, the events are written in the PC disc space only in between bursts. Moreover, the triggers are optimised with re-

<sup>4</sup>Analogue to Digital Converter

<sup>5</sup>Time to Digital Converter

spect to maximum rejection of uninteresting events.

It also has to be noted that drift chambers were all synchronised by a simultaneous START signal from Sc1\*Sc2. However, the STOP signal was not timed in a similar way because the leptons did not register hits<sup>6</sup> simultaneously across all drift chambers. An exclusion of two non-simultaneous hits in one drift cell was facilitated by a mechanism that rejects hits that are not in the same time window for processing by the TDCs. All DC pulses were set such as to be in coincidence in the same gate. The 'doublehit' timer was set up such that if two particles which are close enough in time to produce hits which are inside the 1  $\mu$ s time window, then these triggers produced would be rejected. In other words, any registered doublehit within the time given by the timer was cleared by the CAMAC. Otherwise one would not be able to reconstruct the event(s).

Pre-scaling [58] was a module used to scale the number of signals that were generated by all the physics events. When the prescaling on the NORM triggers was set to  $2^N$ , this means that  $1/2^N$  of the NORM triggers would not be rejected. Figure 3.9 shows the pattern after all the pre-scaling was put into effect. All triggers go into the pattern bit module which is read out and used in the (PC based) analysis.

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<sup>6</sup>The term hit denotes a signal which is registered by the *sense wire* of a drift-chamber

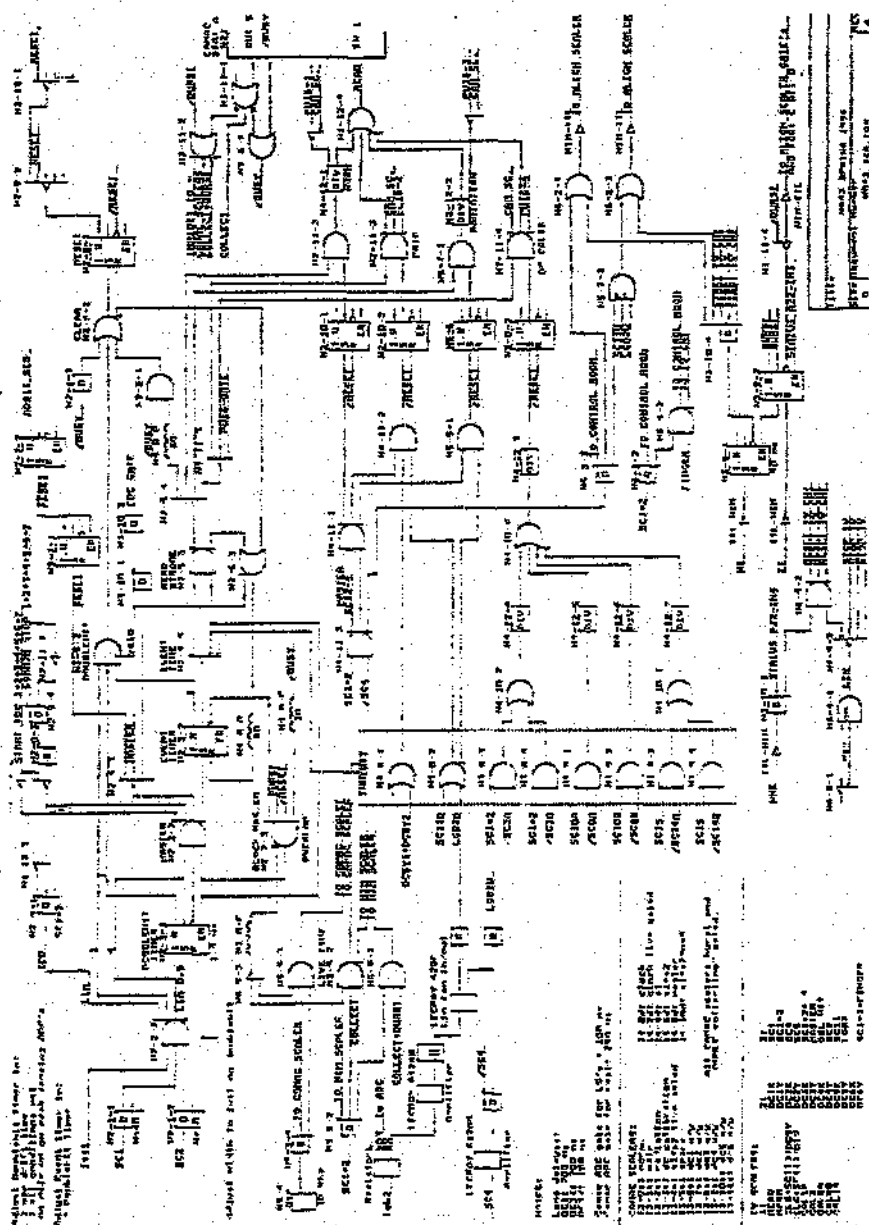


Figure 3.8: Electronic circuitry for the NA43 experiments

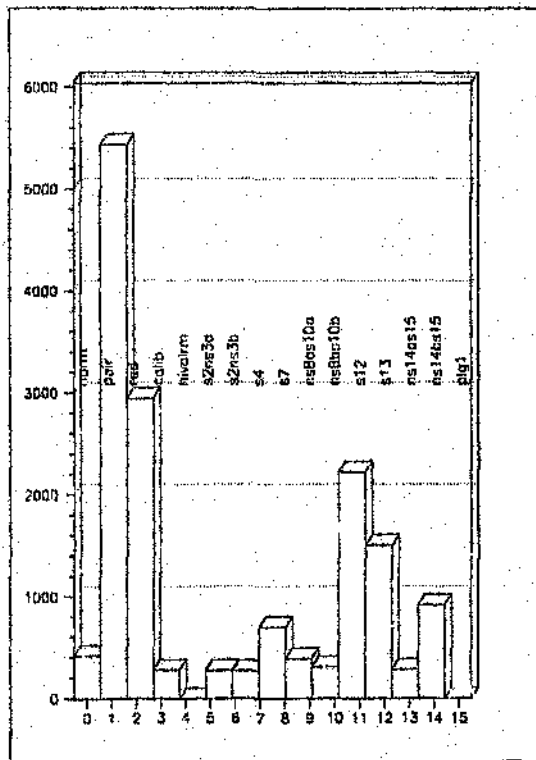


Figure 3.9: An example of the bit pattern used for event selection. The histograms show various triggers which are in use during the experiment.

## Chapter 4

# Detector Calibration

*"Take half from a foot-long stick each day you will not exhaust it in a million years."*  
... Kungsun Lung, in Chuang Chou (ca. 300 B.C.)

### 4.1 Introduction

A detailed description of the calibration of the various detector components is given. The main detector components whose calibration was particularly important were the drift chambers and the calorimeter. Together they were used to give accurate measurement of the tagged photon energy, the energy of one of the pairs and the energy that could be radiated inside the crystal by one of the pairs produced inside the crystal. In addition, magnetic field strengths of the bending magnet B8 and the spectrometer (trim 6) had to be optimized accurately so as to achieve accurate deflections. Drift chambers and the lead glass could thus be positioned in such a way so as to achieve maximum acceptance of the  $e^-/e^+$  pairs.

### 4.2 The Electromagnetic Calorimeter

The lead glass calibration method was conducted during data taking by allowing a straight beam of primary electrons to hit the calorimeter at different energies. Each energy setting therefore corresponded to a data point. In a typical course of an ex-

periment, the energy settings were 10, 22 35, 70, 150 and 200 GeV. Lead glass spectra from each of these energies are fitted with gaussian peaks in order to locate the mean position (in Channels). These mean positions would thus be plotted with the nominal electron energies and the parameters for a polynomial calibration curve extracted.

Furthermore, the energy resolution of a lead glass electromagnetic calorimeter is energy dependent [64]. It was thus imperative that the calibrations were taken over as wide an energy range as possible. These methods used in the calibration processes were of crucial importance in establishing consistency in the ensuing physics investigations. The calibrations were carried out regularly during the course of the experimental data taking in the eventuality of drifts in voltages and other systematic changes to the apparatus. In this way the systematics, which could have serious implications for the physics under investigation were corrected.

As can be seen in figure 3.4, the array was stacked such that the straight beam hit the central lead glass component (LG b2) only. The other components of the calorimeter, namely, LG a1, LG a2 and LG b1 had to be physically moved into the beam-line so as to be in the line of direct hits by the straight beam. A detailed analysis of the calibration process is also given in reference [75]. The three components of the calorimeter, namely, LG a1, LG a2 and LG b1 were fitted using the a quadratic fit as following:  $E = \alpha C + \beta C^2$ , where,  $E$  is the particle energy (in GeV),  $C$  the ADC channel for the Lead glass,  $\alpha$  and  $\beta$  are the calibration (fitting) constants. However, a fourth-order polynomial was used to extract fitting parameters for the downstream component (LG b2). Furthermore, the feature of energy resolution of the calorimeter had also to be taken into serious cognizance. The resolution of the calorimeter is usually dominated by sampling fluctuations, leading to a fractional energy resolution  $\sigma/\sqrt{E}$  scaling as an inverse root of the particle energy. It is also a well established fact that at high-energies, deviations from  $1/\sqrt{E}$  occur because of noise, calibration errors, pedestal fluctuations and incomplete shower containment. Therefore, an addition of a

constant term to  $\sigma/E$  [64], either in quadrature or directly, ostensibly takes all these effects into account. The resolution of the lead glass in this experiment (NA43) was found to be  $\sigma/E \sim 20\%/\sqrt{E}$  at  $E = 150$  GeV.

## 4.3 Drift Chambers

The mechanism of operation of the drift chamber detectors is described in detail in Chapter 3. In this section, the methods of calibration of the drift chambers are described. The time signal registered by the charged particle is nominally given in TDC channels<sup>1</sup> These signals have to be converted into physically more relevant spatial distances, therefore, calibration of the drift chambers is required so that the path of a charged particle can be reconstructed correctly. Also, using the method of event reconstruction and particle tracking, we could effectively use the drift chambers as part of a spectrometer set-up in order to measure the energy of the electron/positron pair. The details of which will be given in the sections following:

### 4.3.1 Method of calibration

The calibration was facilitated by means of a trigger made up of scintillators placed on the various drift chambers. These calibration scintillators (also known as slit counters) were placed downstream of drift chambers 1, 3, 4 and 6. Figure 4.1 shows a profile of the beam after going through the calibration slit. The other chambers, namely, 2 and 5 were calibrated by reconstruction of the geometry of the path of the charged track from the hits on (DCs 1 and 3) and (3 and 6), respectively. These hits were then mapped onto the relevant drift chambers by a method of the extrapolation of impact position. From a simple geometrical re-construction of the path, the hit positions on the chambers could be estimated. Since the spatial location of the drift chambers is known, therefore, the calibration of the drift chambers could be deduced. During data

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<sup>1</sup>The TDC (time to digital converter) is a module that is used to measure a time interval between analog pulses. The operation is similar to that of an ADC.



taking, the slit scintillators were moved out of the beamline so that they would not contribute to the background events due to the scattering of particles or conversions of photons. The summary of the triggers used for drift chamber calibrations is listed thus:

⇒ DC1 calibration:  $Sc2 * \overline{Sc3}$

⇒ DC3 calibration:  $Sc2 * \overline{Sc5}$

⇒ DC4 calibration:  $Sc10 (*\overline{Sc8a} + \overline{Sc8b})$

⇒ DC5 calibration:  $Sc15 (*\overline{Sc14a} + \overline{Sc14b})$

The inclusion of one of the above in the pattern bit (depending on which drift chamber is to be analysed) can be seen to be typically of the profile shown in figure 4.1. The calibration of drift chambers 4, 5 and 6 is made complicated by the fact that the beam profile is no longer a Gaussian that hits only one cell but is smeared over the whole sensitive part of the drift chamber. This meant that cognizance would have to be taken of the region in between cells. This area is also characterised by the fact that the efficiency, near the wires is reduced to less than 50% [62]. Furthermore, the space between sense wires is known as a dead zone. By implication, therefore, events in this region were nominally flagged as non-events. This problem provided an obstacle which was circumvented by calibrating each active cell independently of the others. The space in between was bridged by including it in the calibration constants as an off-set. The wire thickness was also taken into account and thus was included in the calibration constants.

The calibration of the entire chamber surface was facilitated by the use of two slit counters, placed such that the cells closest to the largest part of the beam (inner cells) and those on the fringes of the beam (outer cells) were covered. For example,  $Sc8a$  and  $Sc8b$  were set into coincidence with  $Sc10$  so as to calibrate the inner and the outer cells of drift chamber 4, respectively. Figure. 4.2 is a plot of TDC channels

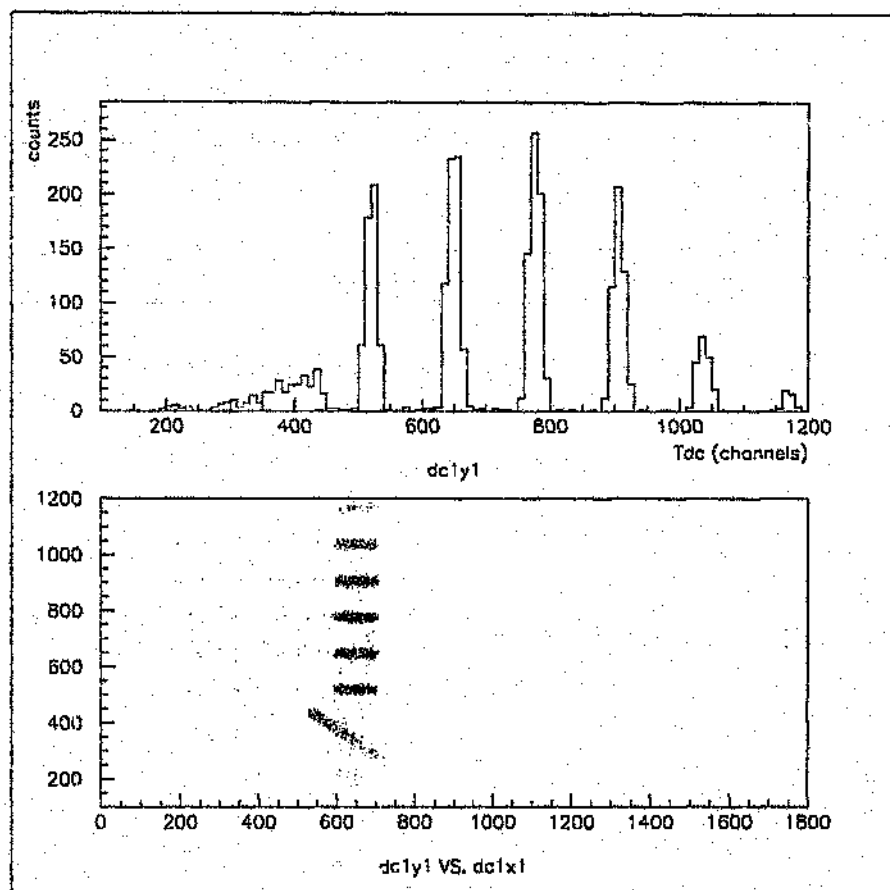


Figure 4.1: Profile of the "grating" slit pattern showing the spatial distribution of uniformly separated peaks (top diagram) as recorded by the active cell (y1) of DC1. The slits are separated by a distance of 3 mm. The bottom diagram shows a two dimensional profile viewed from DC1; both for the x and y distributions of DC1.

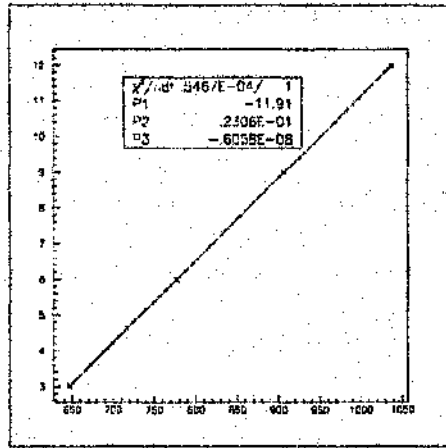


Figure 4.2: Space-to-time coordinate plot showing a quadratic calibration of the drift chambers. The slit position (mm) is plotted against the drift chamber hits in Tdc channels. Shown in the insert are the parameters which were extracted from the fitting process.

and position of an event in a drift chamber. Typically, a quadratic fit was found to give more accurate values of fit constants. The knowledge thus obtained from the calibration of drift chambers could be useful also when the entrance and exit angles on the radiating crystal are deduced as shown below:

$$\Theta_{\text{in}} = \frac{X_2 - X_1}{L_{12}} \quad (4.1)$$

and for the entrance in the y-direction

$$\Theta_{\text{in}} = \frac{Y_2 - Y_1}{L_{12}} \quad (4.2)$$

where  $L_{12}$  is the distance separating the drift chambers 1 and 2. The exit angle was worked out using a similar approach from the extrapolated impact position on DC2-DC3 hit positions as shown in fig. 4.3.

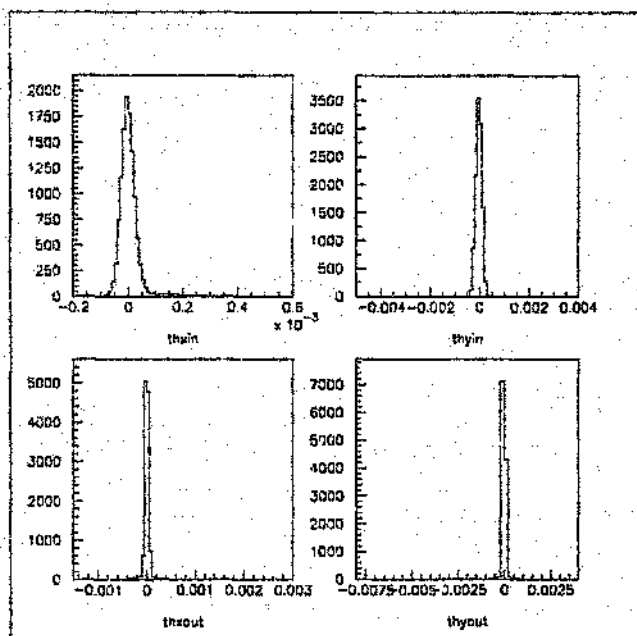


Figure 4.3: Beam profiles as a function of the calculated angles.

## 4.4 The Pair spectrometer and tagging system

Two magnets came into valuable use during the experiment. These are Bend 8 and the spectrometer magnet Trim 6. Their uses have been detailed in the previous chapter, however, what is sought to be briefly outlined here are the various techniques used to optimise and calibrate these magnets.

### Tagging system

As has been described in the previous chapter, the use of tagging was made imperative by the fact that the accurate measurement of the incident photon energy required the use of DC4 and Bend 8 (as shown in figure 3.6). In this process, the radiating electron was deflected by Bend 8 such that the registered hit on drift chamber 4 would correspond to the energy of the incident photon downstream. This could be accomplished by making an accurate and proper calibration of the various properties of the bending

magnet, viz current and magnetic field to be applied so as to achieve the desired deflection of the electron. Low energy particles are deflected the most and thus the accurate tuning of the magnets was such that as many electron momenta as possible had to be within the detector (DC4) acceptance. This is illustrated in figure 4.4 where is seen that low energy tagging electrons  $\leq 10$  GeV are outside the range of DC4 acceptance.

### Magnetic Spectrometer

The spectrometer was used to separate the lepton pair. Due to the fact that the angle between the pairs is very small  $\sim 1/\gamma$  where  $\gamma$  is  $10^5-6$  for ultra-relativistic leptons. Therefore, it can be seen that the lepton opening angle is very small, thus for high energy photons, the magnetic fields had to be strong. However, at low photon energies the magnetic field could deflect the pair such that none of the electrons/positrons could be detected either by the calorimeter or by any of the downstream drift chambers, namely, DC5 and 6. It is these factors which had to be taken into account when the magnets were put into the beamline. Shown in figure 4.5 and 4.6 are the illustrations of the deflections one of the pairs produced in the crystal and their acceptance in DC5 and DC6, respectively.

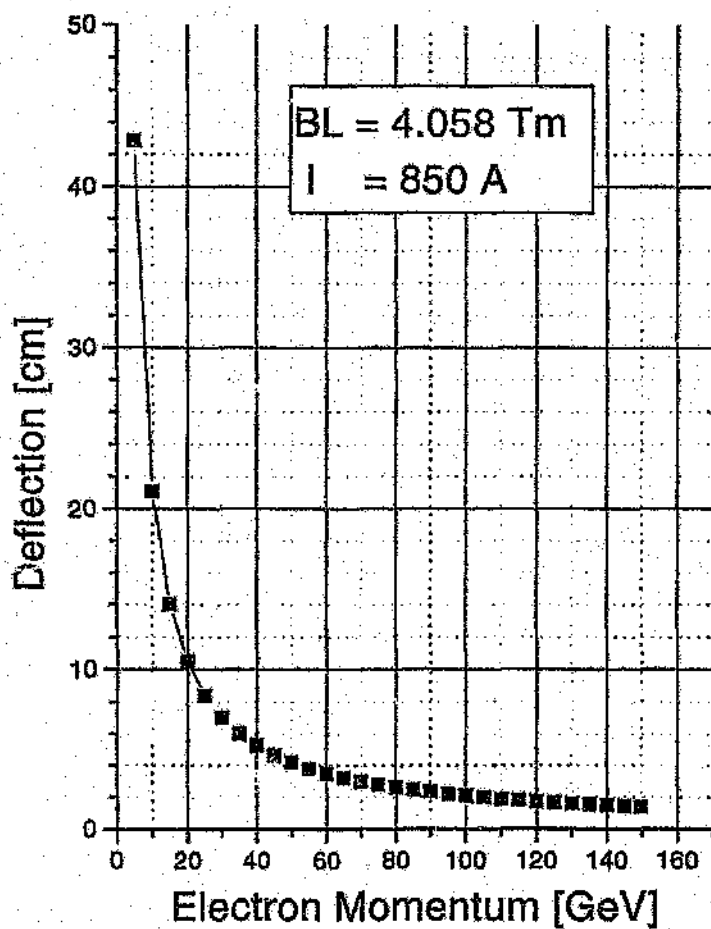


Figure 4.4: Deflection of the tagging electron as a function of electron momentum for the given current and field in the magnet.

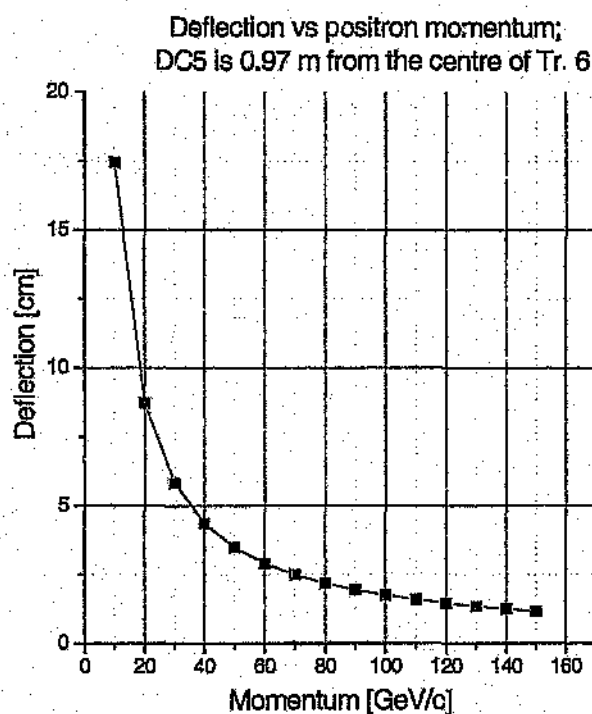


Figure 4.5: Deflection as a function of electron/positron momentum in DC5

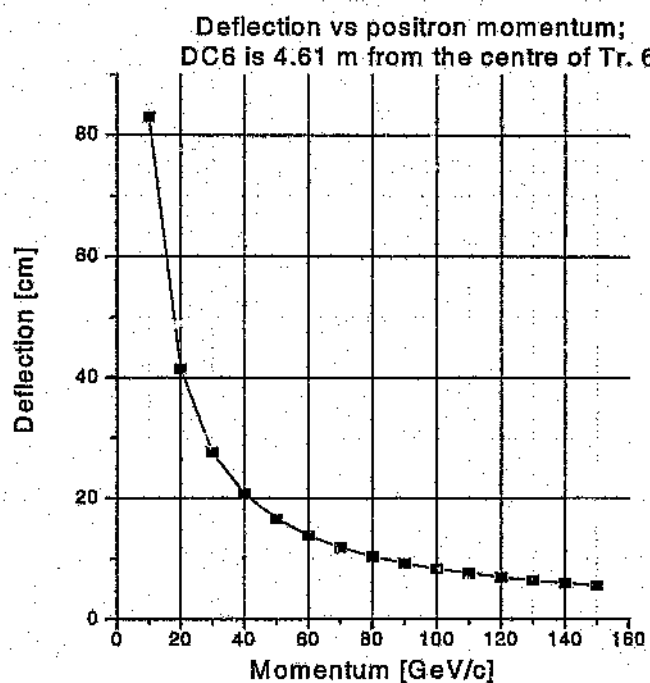


Figure 4.6: Deflection as a function of electron/positron momentum in DC6



## Chapter 5

# Pair-Production in a Ge Crystal

*"It was a saying of the ancients, that 'truth lies in a well'; and to carry out the metaphor, we may justly say that logic applies to us with steps whereby we go down to reach the water" ... Isaac Watts*

### 5.1 Introduction

In this chapter, results of the analysis of the measurement of pair-production on a 6mm thick Germanium crystal are presented. Essentially, the measurements were taken for a crystal that was at  $T = 293$  K, namely, room temperature. These results complement those that were undertaken when the crystal was cooled to 100 K [66]. Complications that do arise as a result of thermal vibrations have to be integrated into the continuum potential, this was explained in general terms in Chapter 2, section 2.3. A detailed treatment of thermal effects and their influence on the threshold for pair-production is also given in Chapter 2, section 2.5. The motivation for undertaking these measurements at room temperature is primarily because temperature dependence does not greatly influence yields along the atomic planes. However, along the axis (in the presence of axial strings; in which the fields are very strong<sup>1</sup>) thermal vibrations could influence pair-production yields; these factors will also be investigated in this chapter. Furthermore, the threshold for pair production on the axis of a warm crystal (293 K)

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<sup>1</sup>The maximum averaged fields in crystals are attained near crystal axes.

is higher than that on a cooled crystal. This was calculated from Eq. (2.67) which is based on detailed derivations given in reference [63] and more qualitatively in [71]. From these calculations it can be seen that there is a reasonably large energy span in which the pair production mechanism as a result of crystalline fields can be observed and from which reliable conclusions can also be drawn. The main thrust of the analysis therefore sought to investigate and gain insight into these quantum electrodynamical processes at room temperature ( $T = 293$  K). The investigations carried out in this thesis could be compared with the measurements and the analysis of Belkacem [67, 68] and more accurate subsequent measurements by the CERN-Aarhus channeling group [71]. The latter of these measurements was based on techniques developed from ultra-relativistic channeling experiments in which the crystal was positioned in a divergent beam. These measurements were shown to yield results which were consistent with the predictions of the Constant Field Approximation.

The analysis carried out in the sections following was based on a detailed overview of the theory of pair-production that was presented in Chapter 2. Following from that, a detailed analysis of 1996 data taken at the NA43 detector of the CERN-SPS will be presented.

## 5.2 Analysis of Experimental Data

### 5.2.1 Pre-selection of useful events

During the course of the analysis, certain cuts were imposed on the raw data set. The initial selection was dictated by the electronic thresholds and nominal detector acceptance and thus taken as *a priori* and written below:

#### Quality cuts

- A selection of signals on the calorimeter above threshold, viz  $E \geq 5$  GeV as designated by the geometrical limit on the acceptance of DC5. Furthermore, the

calorimeter is a composite of one central Pb-glass detector, two Pb-glass detectors on the electron side and one Pb-glass detector on the positron side; therefore the nominal detector acceptance threshold for positrons and electrons is 7.5 GeV and 5 GeV, respectively.

- ✧ All unwanted downstream particles in the beam are vetoed out by means of the scintillator Sc9. A rejection of all pair events above the pedestal, namely 1 MIP was made. This procedure could thus make it possible that only photons were allowed to enter the converting crystal.
- ✧ Pair production is defined by a trigger that also includes events above 3/2 MIPS on Sc11. This sets a condition for a "true" pair-event. Similar cuts were also imposed on the SSD detector.

The electronic pre-selection mentioned above was made possible due to the setting up of the bit pattern (see Figure 3.9) such that most of the events would be attributed to pair-production. Also, as can be seen in figure 3.9, these event triggers ensured that a self-consistent normalisation between runs was made such that measurements of absolute cross-sections<sup>2</sup> for enhancements could be carried out. A nominal electron beam of 150 GeV was used to produce tagged photons that were in the energy range 10-140 GeV. In a typical experimental data analysis, an estimate of about 10% out of 32,000 events are selected. The analysis code of S. Ballestrero, namely **BOOK95** [73] was used to select pre-calibrated data. This code follows a data translation and reduction procedure and is an easily modifiable and flexible program that transforms raw ADC and TDC data into physically meaningful N-tuples which can be fed into the highly interactive CERN analysis code (PAW) [74]. Sophisticated off-line analysis could thus be carried out using PAW.

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<sup>2</sup>The term absolute is used liberally here because these measurements are primarily about macroscopic effects, where the overall behaviour, "gross" structure (or trend) is investigated. The particle interacts (inside the crystal) with many atoms over a large macroscopic distance

### 5.3 Background Study and multi-photon effects

Several processes are known to have signatures similar to those of the required events and thus are required to be considered as backgrounds. Taking cognizance of the possibilities of multi-photon effects can also lead to the reduction of unwanted events. The multi-photon effects happen when more than one photon convert as a result of the relatively large emitter foil. The events in which more than one photon convert can not be reconstructed in the pair spectrometer and thus were rejected in the analysis. The tagged photon beam was produced from bremsstrahlung of the incident electron beam in a 0.8 mm Germanium target. The value of the thickness, is meant, in addition to 2 mm copper, to balance the need for substantially large photon intensities whilst at the same time, keeping the emission of two photons (multi-photons) by the same electrons at low levels. A selection of events was thus made in order to discriminate between those events that are real and those that are from the background processes: This is based on the following expression for the measured experimental yields,  $\eta$ :

$$\eta = \frac{\sigma - \sigma_{\text{background}}}{\sigma_{\text{random}} - \sigma_{\text{background}}} \quad (5.1)$$

In the above expression,  $\sigma$  is the measured probability for pair creation in the crystal. Precise measurement of the total background could not be ascertained because of the paucity of "no target" data. In this case it would be possible to evaluate the background events coming from other materials and subtract that from the crystal related events. Moreover, a selection based on scanning of the crystal was attempted; this however could have helped if the cross-section of the beam was slightly larger than the surface area of the crystal, this, however, proved not to be the case. Also this method was not very economical because of the loss of statistics, see figure 5.1. The area outside the assumed crystalline surface would be equivalent to the "no target" situation: of course, one also has to take into account conversions due to the metal holder onto which the crystal was mounted. However, when various sources of backgrounds were identified, these were then treated as given in Eq. 5.1.

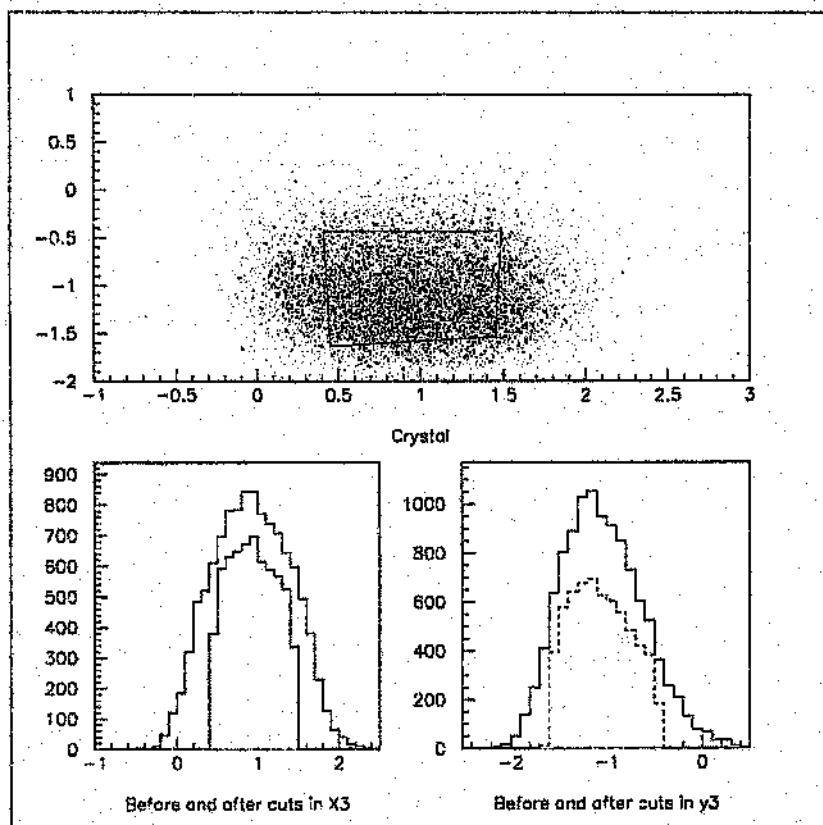


Figure 5.1: Depiction of the profile of the beam that is incident on the crystal target. In the figure are shown various attempts at making selections in both the x and y directions.

### Rejection of unwanted events in the upstream region

Unwanted charged particles could be rejected before entering the crystal by means of veto scintillator<sup>3</sup>. Moreover, Sc7, placed in front of the DC4 frame (see figure 3.6), was used in the vetoing of electrons which could also contribute to backgrounds. These particles are low energy electrons (which have radiated  $\gamma$  photons) which upon deflection by B8 would hit the DC4 frame. These electrons could be deflected back into the main part of the beam and as a consequence, register false events in the lead glass calorimeter downstream.

### Background from the radiator and the converter

The first crystal (the converter at random) gave, in addition to the 2 mm Cu, a total background of  $\sim 20\%$ . Contributions to the backgrounds could also be attributed to the solid state detector. This could be handled by selecting events above  $3/2$  MIPS and only these events would be considered as "real". These backgrounds were noted during some stages of the data taking when the spectrum had shown an unusually large number of events for the SSD with respect to the Sc11. An estimate of  $\sim 13\%$  of the total number of events could be pairs not detected in the multiplicity detector. This, however would give MIP signals on Sc 11 and on the lead-glass calorimeter. In other words, photons that did not convert in the crystal will be converted in the silicon surface of solid state detector and the scintillator. Furthermore, there was an additional 0.5 mm copper converter placed in front of the SSD when measurements for random orientation were made. These were found to contribute  $\sim 0.035 X_0$  to the total number of background events; where  $X_0$  is an expression which is generally used to define the radiation length (in centimeters) of the material under consideration.

All the above mentioned factors were taken into the background subtraction algorithm. A summary of the main sources of background events could be listed thus:

⇒ Radiator: 0.8 mm Ge in random ( $0.036X_0$  without any material) and an addi-

<sup>3</sup>This could also be susceptible to systematic inaccuracies since the detector Sc9 inefficiency could affect the results, detailed study of the functioning of Sc9 was not carried out.

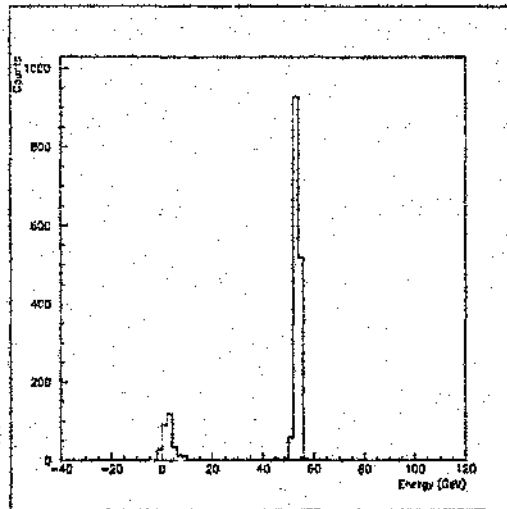


Figure 5.2: Energy distribution of muon background.

tional 2 mm Cu radiator + background gives a total of  $0.058 X_0$ .

- ⇒ Target made of a (0.6 mm Ge converter) was estimated to contribute ( $\sim 0.026X_0$ ) of the total background. These were based on a reference to the theoretical conversion probability.

### Other backgrounds

An estimate of the other background events could also be established. These could be conversions in the air ( $X_0 = 30420$  cm), the scintillator wrappings, mylar windows (in He-tubes) and the muons. However, as shown in figure 5.2 the muon events can be estimated to contribute less than a fraction of a percent of the background events. This is a result of proper beam filtering in the upstream region. In total, the background associated with materials crossed by the electron beam during its trajectory along the beamline was estimated to be  $0.022 \pm .003 X_0$  [66].

### 5.3.1 Selection of tagged events

The tagging system is described in Chapter 3 and the calibrations are shown in Chapter 4. It can be shown that DC4 acceptance is mapped onto the photon conversion downstream from the following formula:

$$E_t = E_{e^-} + E_{\text{calo}}. \quad (5.2)$$

Here,  $E_t$  is the nominal electron beam energy,  $E_{e^-}$  the energy registered on the tagging system and the energy  $E_{\text{calo}}$  is registered in the calorimeter. Figure 5.3 shows the plot of the electron hit on DC4 and the photon energy. It should be noted that electrons which radiate more energy experience a greater deflection when they travel through a magnetic field. Accordingly, the shape of the Calorimeter-DC4 plot takes the shape  $1/(149.1 - E_{\text{calo}})^4$ . Events that are not in the locus defined by the tagger and the photon energy are rejected. Reconstruction of the energy of the incoming photon was deduced from the general expression:  $p = 300/Bl.L_{\text{DC}}x$ , where  $Bl$  is the integrated field strength for (see Chapter 4),  $L_{\text{DC}}$  is the distance from the centre of the magnet to the drift chambers 5 and 6 and  $x$  the position of the hit on a drift chamber.

One fact that has to be noted is that the sole use of the tagger to define the incident photon energy has a serious shortcoming in the sense that the tagging chamber (DC4) has a limited range of acceptance of electrons; this means that the tagged photon energies are restricted to  $E_\gamma \leq 130$  GeV. The determination therefore, of the highest energy bin at the upper end of the spectrum is difficult to obtain. The energy binning in this case has to be done by using the deposition on the calorimeter. However, one major drawback of this procedure is that some energy might be lost due to the incomplete shower containment in the lead glass calorimeter.

A selection of backgrounds coming from the tagger is shown in figure 5.4. It can be seen that the selection was made in such a way that events which fall outside

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<sup>4</sup>Because of energy loss as a result of synchrotron radiation, the nominal beam energy, 150 GeV is changed to 149.1 GeV.



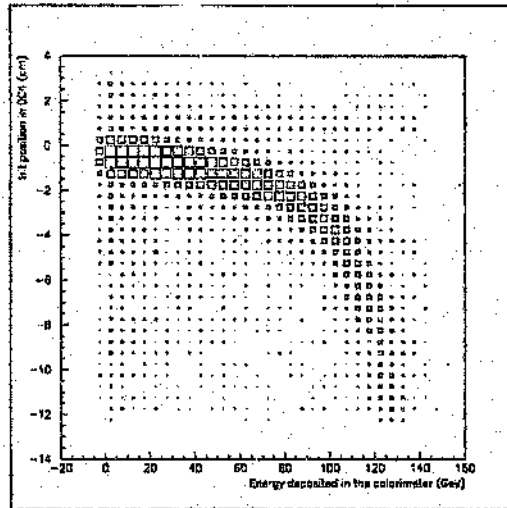


Figure 5.3: Position of the electron hit in DC4 as a function of the energy deposited in the lead glass calorimeter.

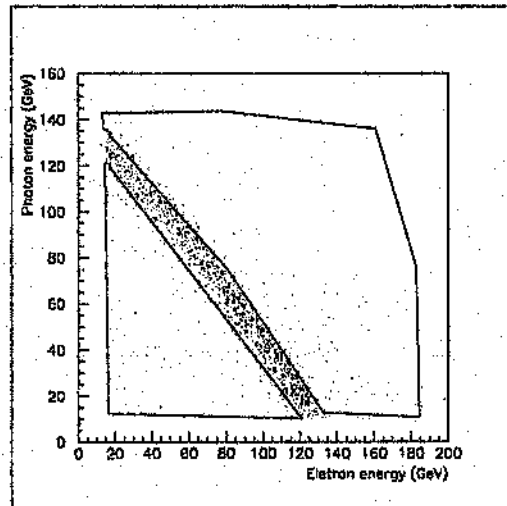


Figure 5.4: Cuts on the plot of tagged photon energy and the energy deposited by the electron emerging from the radiator.

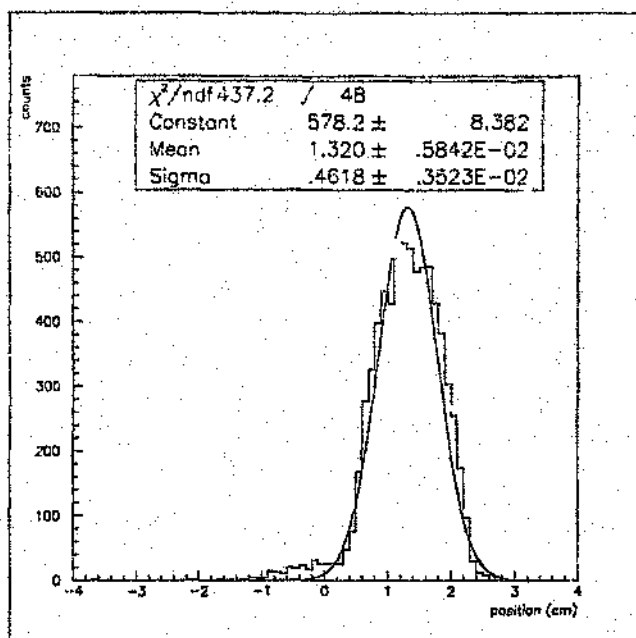


Figure 5.5: Reconstruction of the pair vertex.

the locus defined by a straight line<sup>5</sup> are rejected. Further useful cuts in the vertex position of the pair were made. The vertex is attributed to the point in the direction perpendicular to the beam where the electron and the positron de-couple from each other. Figure 5.5 shows a Gaussian fit function that was used to locate the mean position of the pair-vertex; in this way the track of each particle could be monitored, a selection was made such that events falling outside the coincidence of this vertex and other cuts were rejected, i.e. events falling outside two standard deviations of the Gaussian fit ( $2\sigma$ ) are thus disregarded. This is in addition to selections the opening angle of the pair(s) where we track the particle originating from the electron/positron pair vertex and then set a cut in angle space. Thus an event would be considered real if it fell within the coincidence defined by the selections mentioned above.

<sup>5</sup>This is a region defined by the coincidence of a tagger and the spectrometer.

### Radiative losses

Energy loss for GeV electrons/positrons is due to radiation emission inside the crystal. Detailed investigations of such radiative losses for 150 GeV electrons have been carried out [21]. A detailed theoretical discussion of these processes will be given in Chapter 7.

### 5.3.2 Comparison of experimental and theoretical random yields.

A precise cross check was conducted in order to convince ourselves that the nominal position of the "random" is in a region far away from the crystallographic indices such that the crystal resembles an amorphous converter. One method which could be endeavoured thereby was by making a detailed comparison of the value of differential pair production yields for a crystal resembling such a converter with that of the theoretical Bethe-Heitler cross-sections. In other words, a photon beam coming out of the radiator upstream (0.8 mm Ge + 2 mm Cu), with an energy spectrum which follows the Bethe-Heitler radiative distribution as given by equation (2.27), will upon incidence on the target downstream, produce pairs in a fashion similar to that predicted by differential distribution (equation(2.47)).

Shown in figure 5.6 are the data of differential yields for the crystal oriented in a random<sup>6</sup> direction. The theoretical yields are constant for all selected energy bins and thus accordingly, the ratio of experimental differential yields and theoretical yields should<sup>7</sup> result into an energy invariant "flat" spectrum with a value averaging to unity. As seen in the figure, there are slight deviations from the theory. These deviations are more glaring at high incident photon energies and these were also noted in the analysis given in reference [66]. One possible explanation for the deviation is that the pairs produced in the various incident energy photon ranges can experience incomplete shower containment in the Pb converter.

<sup>6</sup>It has to be noted that background yields have also been subtracted.

<sup>7</sup>We also have factored in the radiation length of the converter.

## Ge (110), Random orientation

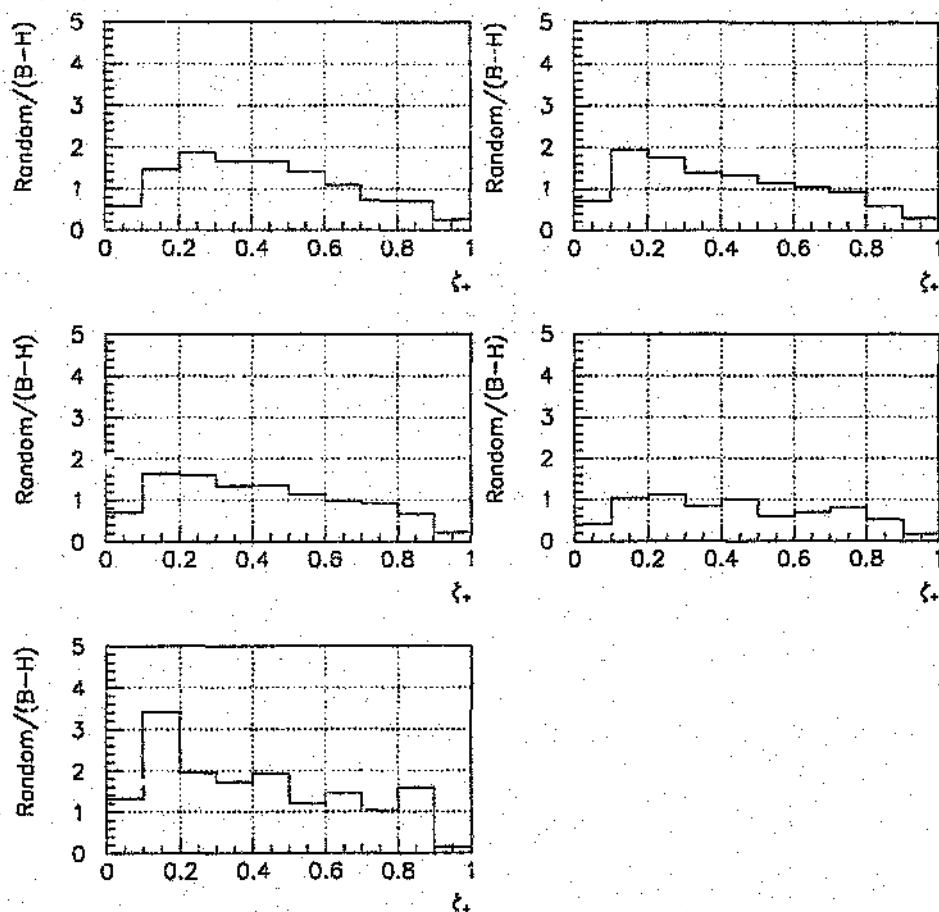


Figure 5.6: Comparison of experimental data with respect to the Bethe-Heitler cross-sections for photons incident on a 6 mm Ge (110) crystal oriented at a crystallographic position far away from planes and axes w.r.t. the incident photon beam. The various energy bins for the photon are: (i) Top left: 40-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. The data are scaled with the Bethe-Heitler theory, given by Eq. (2.47).

## 5.4 Results

The magnitude of the observed pair-creation enhancement is shown in figure 5.7. These enhancements have been well explained by theoretical predictions in which the crystal is described as a source of strong external fields. What transpires therefore is the fact that the periodicity of crystalline planes has a strong bearing on the nature of the physics (and the strong field energy dependent effects therein) that can be studied. The angles of incidence of the photon will determine whether the Born-approximation holds in such instances or not, notwithstanding that there are many planar and axial effects that start showing up as well. The following cardinal points were taken into consideration during the analysis:

- ⇒ The crystal in this case is at room temperature, so it has to be noted that for a given axial direction, the depth of the potential is shallower due to a larger amplitude of oscillatory motion of the atoms in a plane or axial string, hence there should be a damping of the effects compared to the case where the crystal been cooled to 100 K.
- ⇒ The results, though, do nonetheless signify a substantial coherent enhancement to pair creation probability at room temperature.
- ⇒ Radiative losses by the electron/positron inside the crystal are less than in the cooled case. This is because of the fact that the particles produced are not trapped very deep in the well. Escape by de-channeling could thus mean that there are instances where the particle emerges from the crystal with less radiative losses than in the cooled case.

### 5.4.1 Total enhancement (CFA)

Shown in figure 5.7 is the energy dependence of the pair-creation probability expressed in units of the random (Bethe-Heitler) values. The total enhancement was calculated by integration over  $\xi$ , of the conversion probabilities; both for the aligned and the

amorphous incidence of photons. All the energy bins  $\Delta E_\gamma$  are weighted according to the statistics in each of the bins, which in most cases is the centroid of the bin. The photon intensity spectrum coming from the radiator is primarily a Bethe-Heitler profile. The energy distribution of these photons was selected from the calorimeter. In the analysis, the spectrometer and the calorimeter were used to measure photon energies. Residual forward emitted photons due to radiative emissions inside the converter were re-coupled by the central Pb-glass block. Also shown in the figure are data points taken by direct measurement of taking counts using the solid state detector and Sc11 in coincidence (inverted triangles in figure 5.7). As a result, all the energy bins are different from those of the taken by using the spectrometer. The general expression for the pair-production enhancement is given by:

$$\eta(E_\gamma) = \frac{X_0(E_\gamma)}{X_{\text{aligned}}(E_\gamma)}. \quad (5.3)$$

The radiation length is related to the conversion probability  $p(E_\gamma)$ , for a material of thickness  $t$  as following:

$$p(E_\gamma) = 1 - \exp(-7t/9X_{\text{aligned}}(E_\gamma)). \quad (5.4)$$

It also has to be noted that multi-photons are accounted for more accurately using the pair spectrometer; however, the drawback of the spectrometer is that sometimes good events can be rejected due to the ambiguity in the event reconstruction.

The CFA predicts the measurements for total enhancement very well. Furthermore, the two sets of data agree relatively well with each other. The consistency of the CFA with experimental data extends to very heavy crystalline targets, as is shown in reference [59] where Tungsten was used. This has led the proposal of heavy targets in order to induce pair-production in the upstream decay fiducial region of the newly commissioned NA48 detector of the CERN-SPS. The purpose thereof is to veto events resulting from the  $2\pi \rightarrow 4\gamma$  decays. This technique utilises the fact that crystalline fields are very strong along and near axis and also taking into account the fact that the crystal is heavy and hence the threshold for pair-production is low, irrespective of

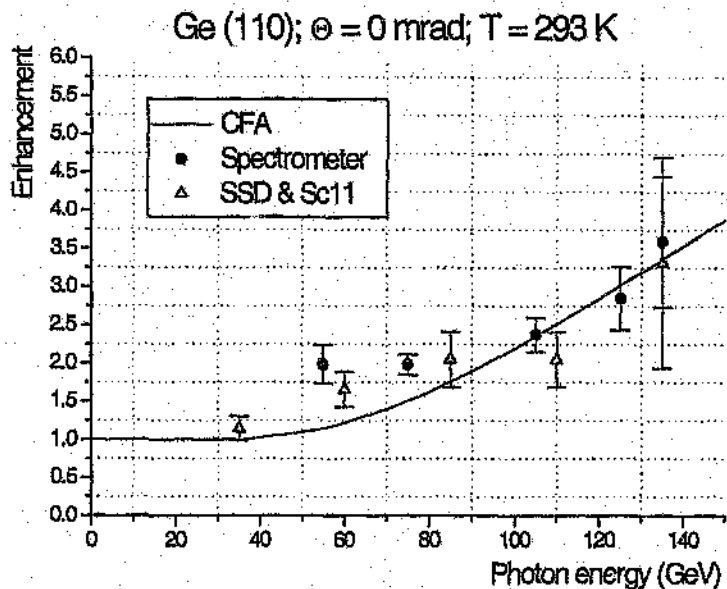


Figure 5.5: Total enhancement,  $\eta$ , as a function of photon energy,  $E_\gamma$ . The results are compared with the predictions of the Constant Field Approximation, represented by the full drawn curve.

thermal effects. A much closer scrutiny however, shows that measurements on W and Ir crystals [55] were found to disagree with calculations by a factor of 2 along the axis but do display a better agreement for incidences along planes. A qualitative note has been taken of the fact that these crystals are thicker and the consequence therefore is that already in random 50% of all photons do convert. In this case it was anticipated that a good set of results would be obtained if a thinner crystal 0.6 mm (with lesser radiative losses) was used. This expectation is clearly shown in the results presented in the thesis.

The experimental data show a characteristic increase of the total enhancement as

a function of energy. The increase is more prominent at energies above the designated threshold. This general trend is predicted in the literature [13, 65]. These approximations are based on the synthesis of the continuum model (see Chapter 2) and the Constant field Approximation [63] in which the crystalline fields are calculated from the thermally averaged Doyle-Turner potential.

#### Total enhancements away from the regime of the CFA

There were other data sets in which integrated pair-production yields were measured at angles away from the regime of the CFA (i.e. at  $\theta \gg \theta_0$ ). The angular span covered herein was in the range 0.5 - 3.0 mrad. The theoretical calculations used were based on the semi-classical formalism of Kononets [28, 45]. The results and the comparison with theory are shown in figure 5.8. Furthermore, it has to be noted that the data shown in the figure were measured using the SSD and Sc11 counters. It can be seen that the structure of the energy dependence of pair production changes dramatically on moving away from the axis. More specifically, the distribution of pair production yields becomes somewhat flatter and the enhancements therein less prominent than in the case of close encounter of photons w.r.t. to the crystalline axial orientation.

#### 5.4.2 The differential Spectra

Shown in the figures following are the results of differential enhancements in the aligned Ge  $\langle 110 \rangle$  crystal. Figures 5.9 - 5.14 show the measured differential pair-production probability within the incident angle range 0 - 3 mrad from the  $\langle 110 \rangle$  along (110) plane. The measurement of the differential distribution in yields has so far been investigated by Bak *et al.* [71]. In these experiments, the analysis was done using the Constant Field Approximation for photons aligned with axis. Furthermore, calculations using the semi-classical approximation based on the Coherent Pair Production (CPP) for the photon incident in the angle range 2.75 to 3.25 mrad to the  $\langle 110 \rangle$  axis along the (110) plane showed a good agreement with the experimental data. The measurements that are reported in this thesis, which as described in the preceding



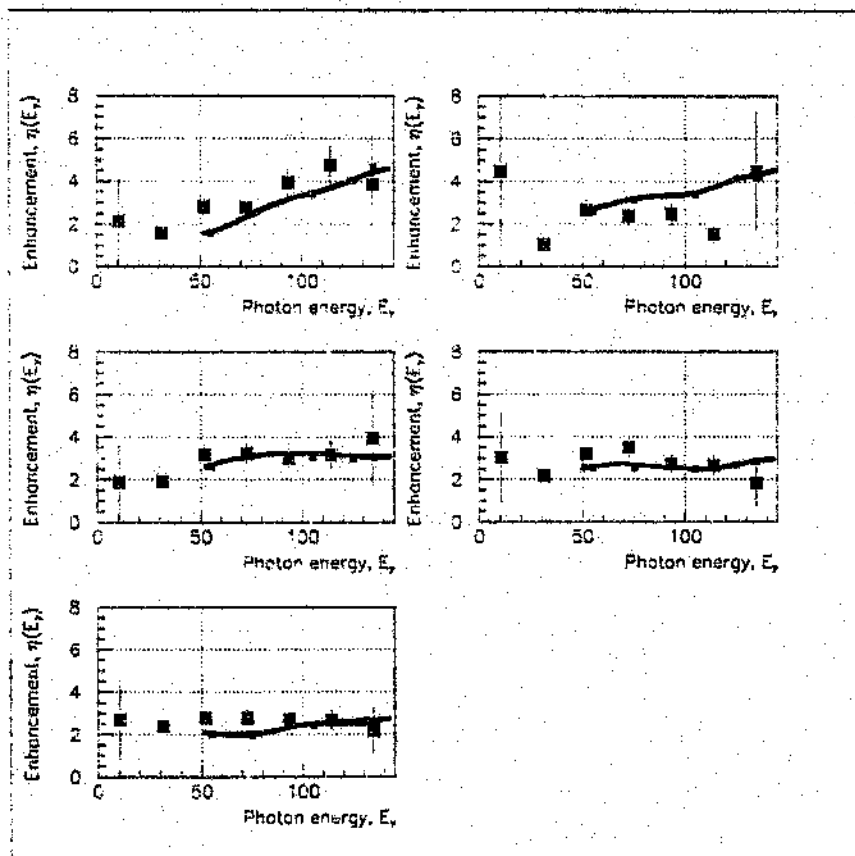


Figure 5.8: Total enhancement,  $\eta$ , as a function of photon energy,  $E_\gamma$ . In the figure is shown the energy dependence of the measured pair-production rate when photons are incident at various angles from the  $\langle 110 \rangle$  axial direction along the (110) plane of 0.6 mm Ge left at  $T = 293$  K. These data were taken using the SSD and Sc11 counters. The angular settings for the incident photon are: (i) Top left: 0.5 mrad; (ii) Top right 1 mrad; (iii) Middle left: 1.5 mrad; (iv) Middle right: 2 mrad (v) Bottom left 3 mrad. Here, the data (filled squares with statistical error bars) are compared with the semi-classical calculations represented by full-drawn curves based on reference [28, 45].

sections, were carried out on a crystal at room temperature. These results are shown in figures 5.9 - 5.14 for photons aligned with the  $\langle 110 \rangle$  axis and the incident angle range 0.5 to 3 mrad from the  $\langle 110 \rangle$  axis along the (110) plane. In the figures, the full drawn curves are the semi-classical calculations of Kononets [28, 45]. As can be seen, on the axis (figure 5.9), the distribution is a dome like structure centered around  $\xi_+ = 1/2$  as predicted by the CFA theory. This behaviour is consistent throughout the incident energy photon range.

Moving out from the  $\langle 110 \rangle$  axis along the (110) plane, one notices that the distribution changes and becomes more structured. At 0.5 mrad (figure 5.10), one is close to the string of strings orientation of the target crystal. The string of strings effect comes as a result of a unique angle space orientation in which coherence occurs as a result of periodic crossing of axial rows of atoms. Lindhard *circa* 1965 hypothesized the string of strings effect as just a sheet of atomic planes. The string of strings effect in this case is interpreted as being analogous to the case of coherent bremsstrahlung CBS and coherent pair production CPP. The difference though is that there is a crossing of atomic planes in the CBS case. As stated earlier and as has been emphasised through the thesis, fields are stronger along axes than on planes, thus the string of strings effect will result in stronger field effects (enhanced yields) than the CPP. Unfortunately though, for the measurements reported in this thesis there was no angular setting which was accurately fixed to exact position of the string of strings effects for Ge  $\langle 110 \rangle$ . From 1.5 mrad to 3 mrad from the  $\langle 110 \rangle$  axis along the (110) plane, we can see that the distribution is more spread out and somewhat flatter and also the data is consistent with the semi-classical predictions for the incident energy photons 55 to 135 GeV.

For low photon energies in figure (5.11), differential pair-production yields are seen to peak at  $\xi_+ = \frac{1}{2}$ , where  $\xi_+$ , which as described in Chapter 2, is the ratio of the positron energy and the incident photon energy. The spectra are normalised to

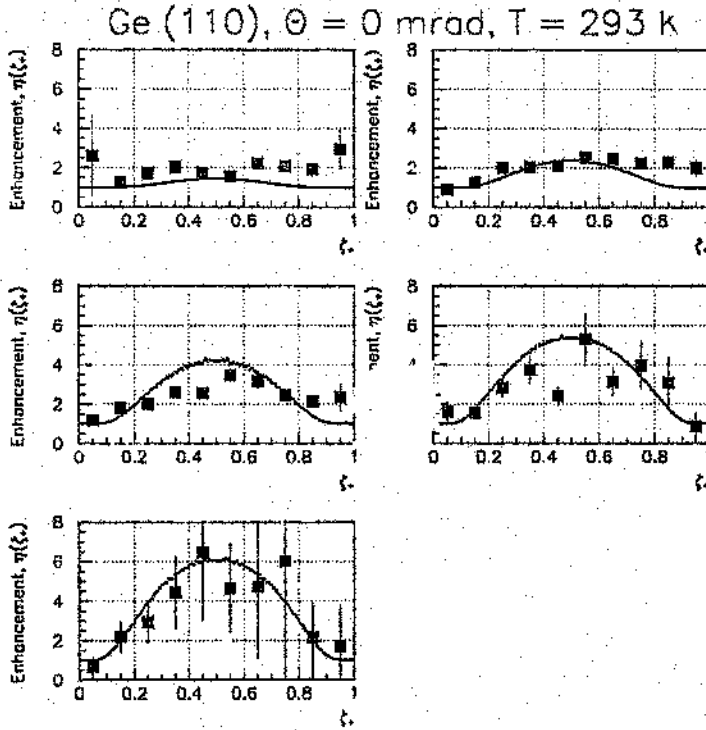


Figure 5.9: Differential enhancement,  $\eta$  as a function of the relative positron energy,  $\xi_+$ . The photon energy is incident along the direction of orientation corresponding to the  $\langle 110 \rangle$  axis of Ge. In the figure are various energy bins for the incident photon: (i) Top left: 50-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. Here, the data (filled squares) are compared with the semi-classical CFA calculations of Kononets [28, 45].

the Bethe-Heitler distributions, which are essentially constant over a large part of the spectrum. The distribution is narrower for incident photon energies near the threshold (75 GeV). This is consistent with the theoretical predictions [63]. At higher incident energies, the peak at  $\xi_{\pm} = \frac{1}{2}$  is seen to gradually spread out into what resembles a dip which is similar to the Bethe-Heitler distribution, however, the structure becomes more defined. Interestingly though, is the fact that the theory has predicted the behaviour very well at energies just above threshold.

To be noted however, is the fact that the data do not seem to agree very well with theory at 1 mrad. As can be seen in figure 5.11, the data are over-predicted by theory at incident photon energies larger than 60 GeV.

However, since there is no extensive literature on the LPM, it is difficult to quantify and put a certainty for this suppression. Another explanation we could seek is that of incomplete shower containment in the lead glass calorimeter. Also, when a closer analysis of radiative losses is taken into account as described earlier on, we can therefore assume that particles could have lost energy radiatively (taking into account the fact that the maximum  $\Delta E$  losses happen at 130 GeV and that could be responsible for the suppression. To this end, these are just speculative accounts.

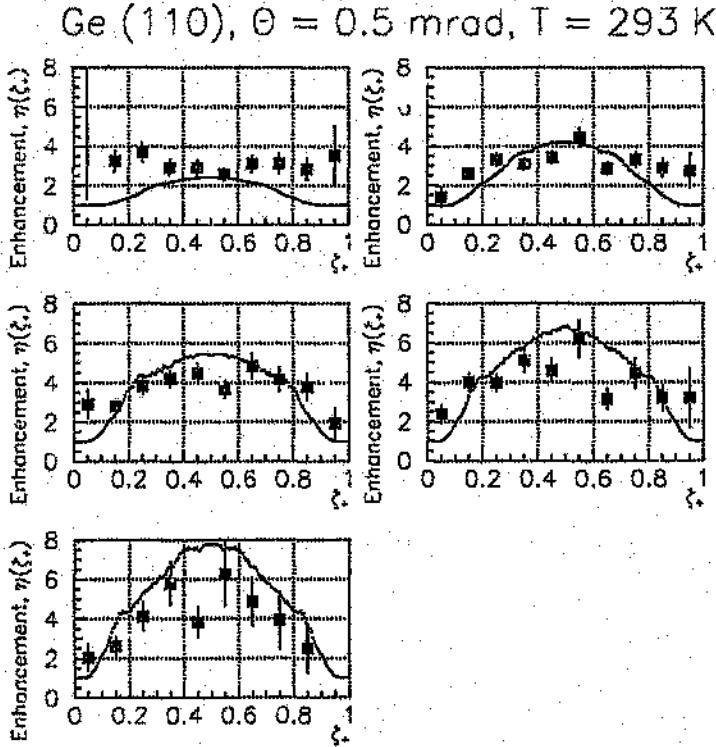


Figure 5.10: Differential enhancement,  $\eta$  as a function of the relative positron energy,  $\xi_+$ . The photon energy is incident along the direction of orientation corresponding to 0.5 mrad off the  $\langle 110 \rangle$  axis along the (110) plane in Ge. In the figure are various energy bins for the incident photon: (i) Top left: 50-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. The data (filled squares with statistical error bars) are compared with the semi-classical calculations of Kononets [28, 45].

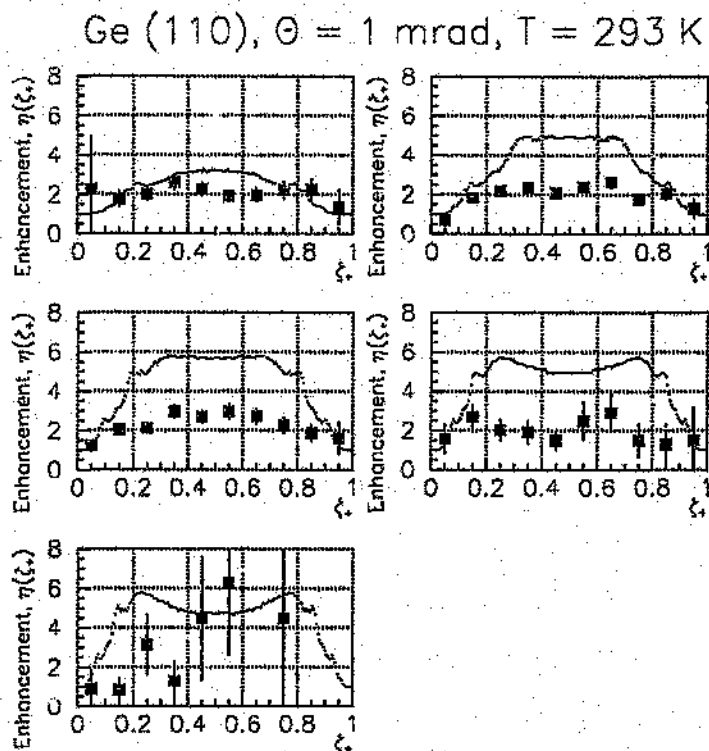


Figure 5.11: Differential enhancement,  $\eta$  as a function of the relative positron energy,  $\xi_+$ . The photon energy is incident along the direction of orientation corresponding to 1.0 mrad off the  $\langle 110 \rangle$  axis along the (110) plane in Ge. In the figure are various energy bins for the incident photon: (i) Top left: 50-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. The data (filled squares with statistical error bars) are compared with the semi-classical calculations of Kononets [28, 45].

Ge (110),  $\Theta = 1.5$  mrad,  $T = 293$  K

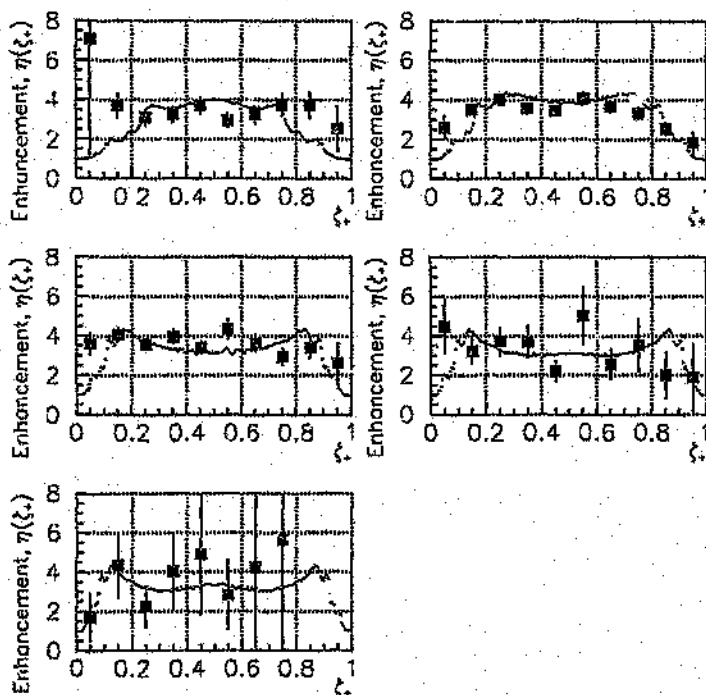


Figure 5.12: Differential enhancement,  $\eta$  as a function of the relative positron energy,  $\xi_+$ . The photon energy is incident along the direction of orientation corresponding to 1.5 mrad off the  $\langle 110 \rangle$  axis along the (110) plane in Ge. In the figure are various energy bins for the incident photon: (i) Top left: 50-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. The data (filled squares with statistical error bars) are compared with the semi-classical calculations of Kononets [28, 45].

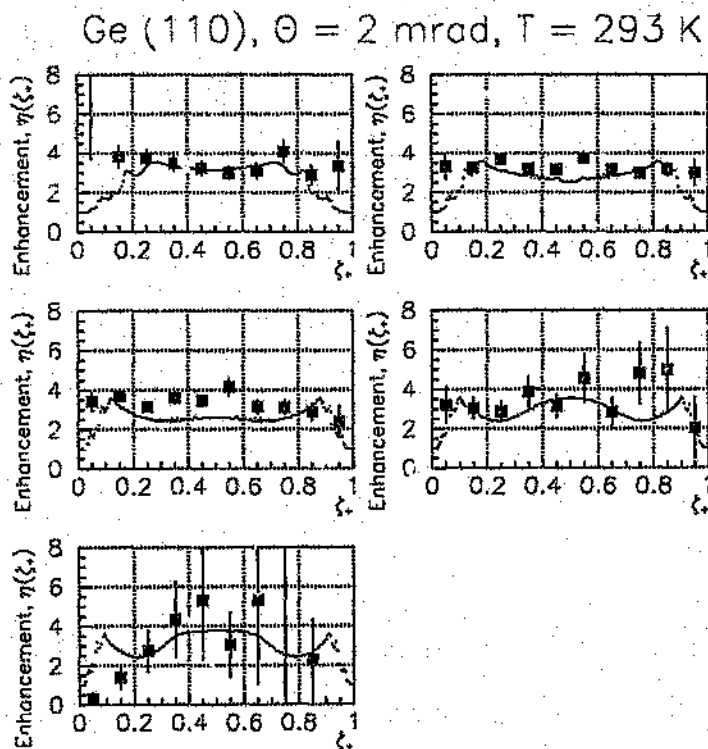


Figure 5.13: Differential enhancement,  $\eta$  as a function of the relative positron energy,  $\xi_+$ . The photon energy is incident along the direction of orientation corresponding to 2 mrad off the  $\langle 110 \rangle$  axis along the (110) plane in Ge. In the figure are various energy bins for the incident photon: (i) Top left: 50-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. The data (filled squares with statistical error bars) are compared with the semi-classical calculations of Kononets [28, 45].



Ge (110),  $\Theta = 3$  mrad,  $T = 293$  K

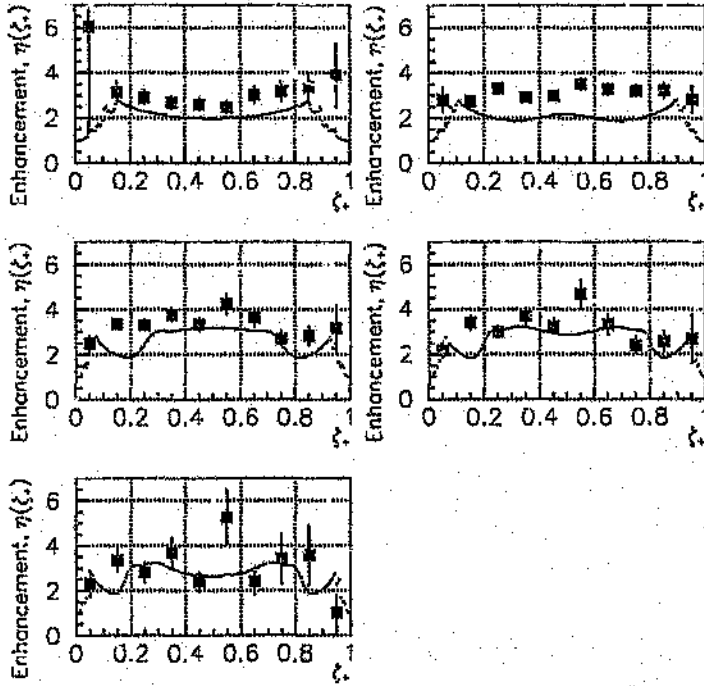


Figure 5.14: Differential enhancement,  $\eta$  as a function of the relative positron energy,  $\xi_+$ . The photon energy is incident along the direction of orientation corresponding to 3 mrad off the  $\langle 110 \rangle$  axis along the (110) plane in Ge. In the figure are various energy bins for the incident photon: (i) Top left: 50-60 GeV; (ii) Top right: 60-90 GeV; (iii) Middle left: 90-120 GeV; (iv) Middle right: 120-130 GeV (v) Bottom left: 130-140 GeV. The data (filled squares with statistical error bars) are compared with the semi-classical calculations of Kononets [28, 45].

### 5.4.3 Angular distributions

The angular variation of the pair-production mechanism was also investigated. The distribution also shows the energy dependence of the total yields (the different energy settings are shown in the legend). Figure 5.15 shows the structure of the distribution of the yields with respect to the orientation of the crystal for total yields in pair-production. The experimental results correspond to the different intervals of energies of the incident photons indicated in the diagram. At  $\theta \sim 0.5$  mrad, the yields start rising; this is the region around the string of strings in Germanium. The results stated here also vindicate the previous measurements [71] in which it was sought to explain the data on the basis of coherent pair-production. In accordance with the Born-approximation, the data are meant to drop to zero at axial incidence but they reach values which considerably exceed the incoherent background. According to the theoretical predictions of Kononets and Tupitsyn [44] the angular position of maximum pair-creation yield is

$$\theta(\omega) = \frac{4a_{TF}mc^2}{\lambda\hbar\omega}. \quad (5.5)$$

As seen in figure 5.15, the maximum yield occurs at 0.7 mrad. Unfortunately, there was no data point taken exactly at the position, however, one could estimate (through interpolation) from the picture that the maximum is at the designated angular position. This value is slightly larger than the one predicted by Baier *et al.*, viz 0.6 mrad. Accordingly, the calculation of peaks in the coherent bremsstrahlung is determined by the condition

$$\lambda_{\text{coherent}}(\xi_+, \omega)\theta \equiv \frac{d_a}{2\pi i} \quad (5.6)$$

where  $i$  is an integer and  $\theta$  is the angle of incidence w.r.t the axial orientation. This however, is restricted by the condition which is invoked in order not to destroy the coherence as a result of the photon divergence upon incidence on the crystal. Finally, as predicted by theoretical calculations, the maximum enhancement appears for lower incident angles and higher photon energies as prescribed by Eq. (5.5). We can also see from the figure that there is also another "resonance peak" at 2.0 mrad. This observation parallels a similar measurement done by U. Mikkelsen (page 77 of reference

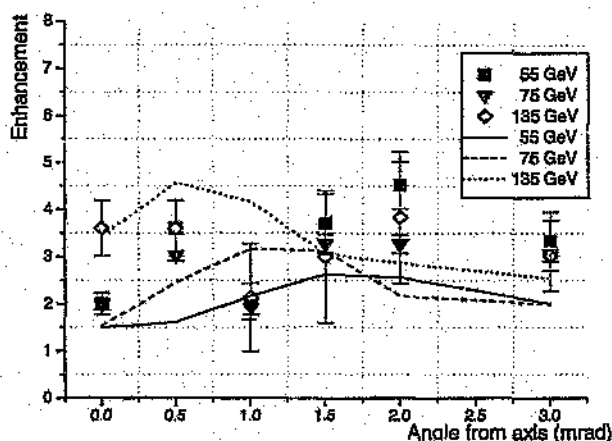


Figure 5.15: Angular dependence of the pair-production enhancement for photons incident at various angles with respect to the  $\langle 110 \rangle$  axis of Ge left at room temperature (293 K). In the legend are the different energy bins (centroid) of the incident photon. The data points are denoted by the different symbols with error bars showing statistical uncertainty, these were obtained from the spectrometer, i.e. by direct integration of figures 5.9 - 5.14. The full drawn theoretical curve are calculated with the semi-classical approximation [28, 45]

[66]), where for low incident energy photons the peak is at a region around 2 mrad. Following from Kononets' theory, the maximum enhancement appears at larger angles for lower incident energies of the photon. This can be qualitatively shown by the expression:  $\theta_{\max} \sim 0.7 \text{ mrad} \cdot 150 / h\nu$ . To elaborate on this point; if the energy of the photon is 55 GeV, then the corresponding maximum in pair production yields will show at  $\theta_{\max} = 1.9 \text{ mrad}$ .

#### 5.4.4 Summary

In this chapter, results of pair production in a warm ( $T = 293$  K) Ge crystal were presented. The starting point was to define a method of selecting good data, followed by a good background evaluation process. In the end, the spectrometer, tagger and the calorimeter were used to measure the required pair events. Furthermore, radiative losses as a result of the electron being trapped closer to atomic nuclei have been accounted for by using the calorimeter to detect the forward emitted radiation. With all the various concerns taken into account, the experimental data and theory are in very good agreement. Moreover, two methods were used to measure the PP processes on axis, namely, using the spectrometer and the SSD, Sc11 counters. Both of them agree remarkably well with each other and also with the CFA for axial incidence of the photons.

## Chapter 6

# The Landau-Pomeranchuk-Migdal Effect

*"The cause is hidden, but the effect is known." ... Ovid*

### 6.1 Introduction

A brief overview of the Landau-Pomeranchuk Migdal effect has already been given in Chapter 2. In this chapter, however, an attempt will be made to review and provide a synthesis of the subject on account of the various manifestations of the LPM effect with respect to the different areas of physics ranging from simple lepton-photon processes to that of the quark-gluon plasma. Furthermore, this study is quite general and extends over many physical problems where a source couples weakly to a wave field. Examples therein include the production of a photon, a dilepton (pair) from dense matter, or a hadron gas formed in high-energy nuclear collisions and the gluon-parton radiation (such problems are discussed in [76, 77]). These examples serve to illustrate the recent interest of the LPM from QCD field theoreticians in their quest to discern the LPM effect in nuclear matter for QCD. The discourse therefore is to generalise the derivation of the gluon (photon) energy spectrum in analogy to the QED LPM [82]. In this case, the emission of soft photons (gluons) is considered in the context of treating the scattering centers as independent of each other and therefore greatly simplifies all formulae

reducing the problem into a picture of classical propagation of a relativistic particle traversing an infinitely dense medium. Blankenbecler *et al.* [90] have understood the concept in terms of the quantum treatment of multiple scattering with the purpose of providing a proper physical interpretation of the suppression effect and thus redefine the model of multiple scattering. Furthermore, the manifestations of the LPM effect also have influences on the neutrino and axion radiation from supernovas [78, 79] and on several other condensed matter phenomena [80]. A detailed and more comprehensive overview of other manifestations and applications of the bremsstrahlung effects in dense media is given by Knoll and Levin [81]. Nevertheless, the details laid out in this chapter are the basics of the QED versions of the LPM and they will be considered.

## 6.2 Implications with respect to NA43

It is pointed out here how the theoretical analysis given in Chapter 2, section 2.6 relates to the NA43 experimental data that has been under consideration. What is detailed here are the influences of the parameters which set conditions for the manifestation of the LPM with respect to the Bremsstrahlung emissions and pair-production. Another important factor that can also be highlighted is that of medium dependence. For example, inside a dense medium and in the event of the incidence of highly-energetic projectiles; the mean free path will then exceed the coherence length and, as a result, the medium will act as a sum of highly concentrated scattering centres. Interference within the limits of the coherence length starts to occur, yielding to the modification of the cross-section by a factor  $1/(al_c)^2$ . As noted in equation (2.82), the opening angle is inversely proportional to the Lorentz factor. Therefore, for multi-GeV energy regimes, the opening angles are small. Below is written the influence of the size of the target crystal in relation to the formation length  $l_\gamma$ , since it is shown in Chapter 2 that the LPM suppression appears when  $l_\gamma \ll l_c$ .

### Medium dependence

In section 2.6, a discussion of various factors that influence the onset of the LPM was given. Now the argument can be taken further so that we can now relate these factors to our experimental results. Accordingly, it becomes clear that the LPM effect is dependent on the medium in which the radiative processes take place. For instance, the radiation lengths of small  $Z$  crystals are longer than that of heavy crystalline targets, therefore it follows that the most suitable candidate for the measurement of the LPM is a heavy target. As an example consider the two extreme cases, *viz* diamond and Tungsten. The radiation lengths of the two crystals are given below:

⇒ Diamond  $l_{\text{rad}}$  : 120 mm

⇒ Tungsten  $l_{\text{rad}}$  : 3.3 mm

The above facts thus give justification to the fact that heavy-atom crystalline targets have been suitable for the investigation of the LPM. Moreover, in aligned crystals, the radiation lengths also gets drastically reduced. This fact will be considered in greater detail in the sections which follow.

## 6.3 Experimental results to date

The measurements that were endeavoured in this thesis followed from the measurements of R. Moore *et al.* in which a suppression was observed in the analysis of the data for the measurement of pair-production when 10-150 GeV photons were incident on a 3.2 mm  $< 100 >$  aligned Tungsten crystal. As shown in figure 6.1, the CFA was seen to over predict the data at high incident photon energies. It was thus thought that the effect of the LPM could also be measured more carefully on cooled crystals, the reason being that in cooled crystals, the threshold are reduced and thus can obtain yields in pair production over a large energy span. Furthermore, the deeper transverse potential implies that high crystalline fields will be obtained as well and thus the Schwinger parameter ( $\chi \geq 1$ ) at lower values of  $\gamma$ . Also to be taken into account is

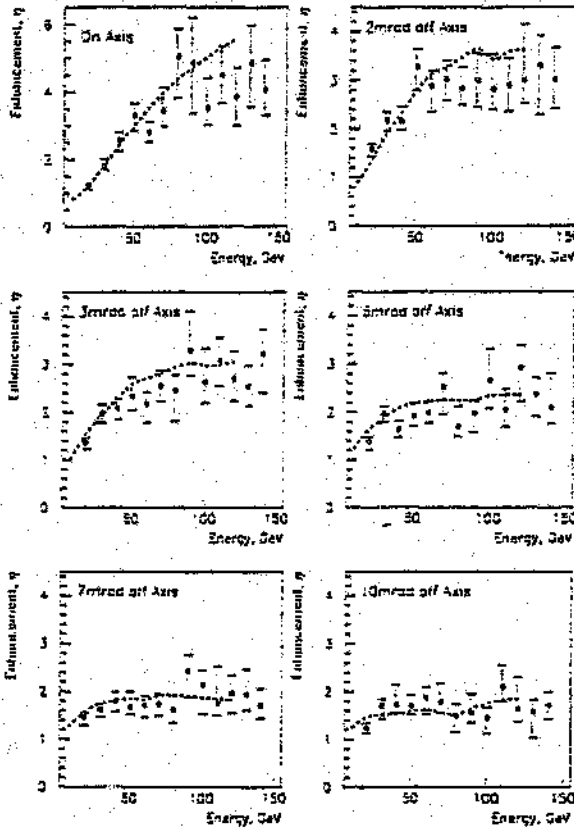


Figure 6.1: Enhancement in pair-production yields (scaled to the Bethe-Heitler yields) for photons incident along the 3.2 mm  $\langle 110 \rangle$  Tungsten crystal; the predictions of the CFA are shown in dotted lines. These results are taken from reference [59].



the fact that the LPM predictions of Kononets [45] are based on the assumption that the crystal is cooled to 77 K, which is close enough to the temperatures which could be attained at the NA43 experiment, namely 100 K.

The work reported in the sections which follow below will show the results of the latest measurements.

#### Radiation suppression

This effect was investigated for the first time by the Aarhus group [87]. A more comprehensive set of measurements of the LPM effect were undertaken at SLAC [88, 89] (and the references therein) in which 8 - 50 GeV electrons were incident on thin gold and carbon targets; the purpose was to study in detail the emissions of soft photons in the range 5 to 500 MeV. In these measurements, a suppression of production rates was observed and the results were within an accuracy of 5%. Moreover, the results did confirm Migdal's theory of the LPM effect, in which it is stated that the photon suppression rates vary with both the target thickness and the energy of the incident particle<sup>1</sup>.

Subsequently, experiments have been undertaken at CERN-NA43, in which the electron beams were incident at higher energies ( $0 \leq E \leq 149.1$  GeV) [55]. These electrons were directed on a 0.2 mm W  $\langle 111 \rangle$  target; a study of soft photon emissions was carried out and thus far, measurements to date have indicated that there is a soft photon suppression, as predicted by the original LPM hypothesis. The results of these measurements are shown in figure 6.2. Shown in the figure is enhancement of radiation yields when electrons are incident on the randomly oriented target (top) and along the  $\langle 111 \rangle$  axis of the aligned target (bottom). The results indicate that there

<sup>1</sup>As mentioned in the theoretical outline, there are certain boundary limits which can influence these effects; for example, we have assumed that the targets are relatively extended, because the formalism depends very critically on the edge effects as stated in Cleymans' work which was used extensively in the theoretical analysis.

is a large suppression compared to the Bethe-Heitler prediction. It is within this soft photon limit that the emitted power is modified from being a constant value of the Bethe-Heitler to a square root behaviour as predicted by equation (2.80).

It can be seen in figure 6.2 that the low end of the radiated photon spectrum shows a deviation from the Bethe-Heitler prediction and tails off in a  $\sqrt{\omega}$  fashion. This fact is one of the main predictions stated in equation (2.80), therefore, one can deduce that the results indicate evidence for the LPM effect for 150 GeV electrons incident on a W-crystal. Moreover, it should be noted that a very pronounced reduction of photon yields for energies up to 30 GeV are observed.

### Pair-production

The investigations were also extended to measure the LPM effect for the time reversed process, namely pair production. In accordance with cross-symmetry, the coherence for pair-production is proportional to the energy of the pair produced and thus the LPM suppression (see Chapter 2, section 2.1) will appear in the high energy end of the spectrum. Since the LPM appears in the soft photon limit for radiation emissions, it is expected to appear at the higher energy limit in pair-production, this gives an interesting complementary measurement and a more complete picture of the LPM manifestations w.r.t. to both processes will be obtained. In the analysis which follows, the following factors are taken into account and are listed thus:

- The LPM suppression for pair-production arises under condition:  $l_c \gg l_{\gamma\gamma}$ .
- The mean free path and the coherence length are coupled together by the Migdal parameter  $s$ , which is a scaling factor that could be used to determine the onset of the LPM and is given by the following expression:  $s = \sqrt{\frac{l_{\gamma\gamma}}{l_c}}$ .

From the above argument, it follows therefore that the LPM suppression occurs at values for  $s$  when  $s \leq 1/8$  (see equation (2.90)), or conversely, when the mean free

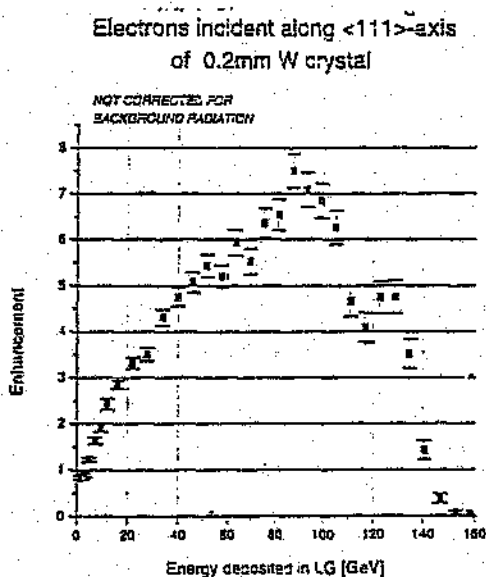
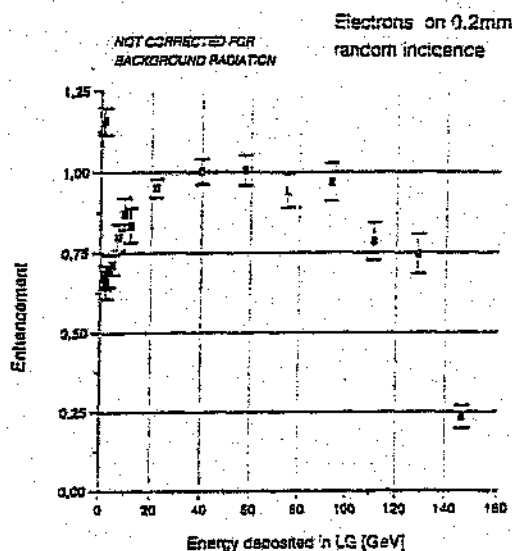


Figure 6.2: Enhancement as a function of radiated energy when 150 GeV electrons are incident on a 0.2 mm W  $\langle 111 \rangle$  target, oriented at random (top) and along the axis [55].

path (m.f.p.) is less than the pair creation length. This leads to a decrease in pair production rate with increasing energy  $\omega$  of the incident photon. Again, following from Kononets [45], the LPM suppression for aligned crystals will appear at lower energies than in amorphous targets. In aligned crystals, there is a strong reduction in multiple scattering length near atomic axes and this length may be reduced to  $10^{-3}$  of the value of the m.f.p. Hence along the  $\langle 110 \rangle$  axis of Tungsten at 77 K, the threshold for the LPM suppression get lowered to  $\hbar\omega \geq 80$  GeV.

Furthermore, in accordance with equation (2.67), the threshold for pair-production is lowered considerably. However, the lower threshold for heavy crystals is accompanied by a lower maximum yield [63]. Hence as is shown in figure 6.3, the threshold is lower, *viz*  $\hbar\omega \geq 15$  GeV (this is also stated in table 2.4) and this also compares favourably with published results [59].

The results shown in figure 6.3 are very consistent with the calculations of Kononets, because at incident photon energies larger than 80 GeV, there is an observed drop in pair production yields<sup>2</sup>. This deviation from the characteristic CFA rise (cf p101) shown in figure 6.1 is also observed for different angular orientation of the W crystal. Taking this observation into account, it can thus be deduced that these results have given substantial evidence of the LPM suppression for pair production in aligned crystals.

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<sup>2</sup>Of course, there could be other effects like the mosaic spread as was interpreted by R. Moore [59]

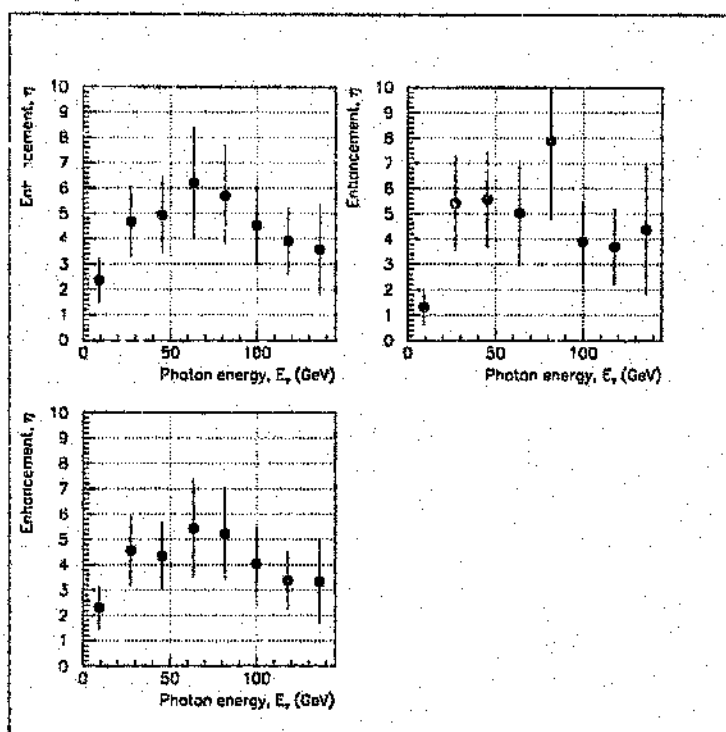


Figure 6.3: Pair-production enhancement as a function of photon energy in 3 mm Tungsten crystal cooled to 100 K for the following angles of incidence: Top left; 0 mrad; top right 0.2 mrad and bottom left, 0.5 mrad.

# Chapter 7

## Discussion

*"If a man begins with certainties, he shall end with doubts; but if he will be content with doubts, he shall end in certainties." ... Francis Bacon*

### 7.1 Overview

A general, comprehensive study into the investigation of lepton-photon processes was carried out. In these studies, ultra-relativistic particles and photons incident on oriented crystalline targets in high-symmetry directions interact with electromagnetic fields from atomic rows and planes. Two features make these investigations very interesting: these are coherence and strong fields:

- ⇒ **Coherence:** The Lorentz boosted particle incident at ultra-relativistic speed ( $\gamma \geq 10^5-6$ ) experience correlated, coherent scattering with a continuum of many atomic centers along the trajectory of its guided motion in the crystal. The electromagnetic processes therein happen over an extended zone and this is known as the **coherence length**. The coherence length for the processes described in this thesis can reach macroscopic values ( $l_c \sim 100\mu\text{ m}$ ). Hence, processes which ordinarily occur at a singular point in space now get extended over a long spatial distance. As a result, the QED processes are changed dramatically, resulting into enhancements in yields for pair production and radiation emission. Furthermore,

the long coherence length can also result in the setting in of inhibiting processes like the Landau Pomeranchuk Migdal effect. A detailed account of the LPM effect has been given in (chapter 6).

- ⇒ **Strong fields:** When the particles are incident at ultra-relativistic energies on oriented crystals, the large Lorentz factors enlarge the fields along the crystalline axes and planes. In the Lorentz boosted rest frame of the particle, the crystalline fields (which are proportional to  $\gamma$ ) become large and subsequently attain values which are in the range of criticality, i.e. the regime of  $(\chi \equiv \gamma E/E_0 \geq 1)$ , where  $E_0 = m^2 c^3 / e \hbar$ . The onset of strong fields is thus reached in which dramatic effects on QED processes occur where the radiation mechanism changes from the classical to quantum synchrotron law.

It is under the above mentioned conditions that crystalline targets have turned out to be useful in order to investigate the theory of coherent and strong field QED. Under these conditions, the phenomena mentioned above are characterised by interaction regions of macroscopic dimensions encompassing a large number of periodically arranged target atoms, as a result, the coherence and strong field effects are encountered. For ultra-relativistic electrons and positrons, the parameter  $\chi$  is used to characterise the influence of quantum recoil effects during photon emission. At  $\chi \geq 1$ , the quantum recoil effects have to be taken into account; this is the region of hard photon emission and thus the theoretical description of QED in such fields is beyond the framework of perturbation theory.

Also to be noted is that the coherence in scattering processes persist to angles much greater than the channeling critical angle and the QED processes therein can be described as originating from a constant field. This is done by using semi-classical approaches in the form of the constant field approximation (CFA), which thus far has proved to be very successful in the description of these effects.

### 7.1.1 The analysis of pair production

The periodicity of the crystalline matter together with the extension of coherence lengths at higher energies resulted in the coherence in scattering: this leads to enhancements in radiation/pair production more intense than for the amorphous target. By taking into cognizance the intensity of the field and the nature of the coherence, a methodology of describing these electrodynamical processes has largely been semi-classical and is based on the Constant Field Approximation (CFA). This is a good approximation for small incidence angles to the crystalline planes and axes. The effective critical angle is  $\Theta_0 = U/mc^2$ , where  $U$  is the depth of the planar/axial potential and  $m$  is the electron rest mass.

The investigations carried out in this thesis were with regard to the measurement of pair-production on a 6 mm thick Ge  $\langle 110 \rangle$  crystal. However, these measurements were part of a broader study extending over many years. Some aspects of this study are reported wherein they relate to the main ideas covered in this thesis. In the pair production measurements reported in this thesis, enhancements of up to values  $\leq 4$  above the Bethe-Heitler yields were observed for a crystal at room temperature ( $T = 293$  K). Differential pair-production spectra were also investigated in detail. These were taken over an angle span of 0 - 3 mrad from the  $\langle 110 \rangle$  axis along the (110) plane. Furthermore, a detailed analysis of the enhancement in pair-production yields with respect to the change in the angle of incidence of the photon beam was also carried out.

All the results mentioned above were found to agree very well with the theoretical predictions of the constant field approximation. In fact, given the many approximations in the construction of the CFA (and other semi-classical conjectures) which limit its applicability, it is astonishing how well the theory describes this comprehensive body of data.



### 7.1.2 Systematics which could affect the data

Various factors which could have influence on the results were taken into account during the analysis and are discussed in detail as following:

- The idea of the mosaic spread which could wash out the coherence was considered by R. Moore *et al.* [59] to be the cause for the “drop” in yields. This factor was also considered here. However, it is difficult to see why the lower energy CFA predictions (irrespective of the angular orientation: on which the effect of the mosaic spread is dependent on) agree so well with experimental data. Mosaic spread should wash all coherence over the entire energy range, not just the high energy region. Crystals are very thin and the mosaic spread therefore is in the  $\mu$ rad range. For example, an estimate of the mosaic spread for Tungsten is in the order of 10 micro-radians. This is way smaller than the angle span covered in this work, which goes up to 3 m radians.
- Another reason for the suppression in pair production yields could be the result of radiative losses inside the crystal. The use of the CFA for calculating these radiated photon spectra at high energies is connected to the fact that at  $E = 150$  GeV, photons with energy less than 3 GeV contribute to the CFA only to 4% of the total energy loss. With increasing energy, the portion of photons with energy  $\omega \leq E$  decreases because of the transformation from a synchrotron radiation spectrum with increasing  $\chi$ , leading to the quantum approximation  $\gamma^{2/3}$  [72]. A detailed study of radiative losses in Ge < 110 > at  $T = 293$  K was undertaken [24] in which it was noted that there is a region of maximum depth in the Doyle-Turner potential at 150 GeV. In this region, energy multiple scattering exceeds radiation cooling by more than one order of magnitude and hence electrons, experiencing strong nuclear scattering along the axis, leave the area of maximum potential gradient over a depth interval in a few micrometers.

This redistribution lessens the steepness of the potential, as a result the largest

radiative losses happen at maximum values of  $\chi$ . The width of the distribution in transverse energy space is thus greater than the average change. This results in the decrease in radiation due to de-channeling which is compensated by the increased radiation from above-barrier electrons as well as with electrons of small transverse energy  $E_{\perp}$ , thus making the population of this area higher than the initial distribution in transverse space<sup>1</sup>. Therefore, maximum radiative energy losses become peaked at  $\hbar\omega \sim 0.8$  [22, 4]. Owing largely to the fact that electrons are trapped close to nuclear scattering centers along the row and positrons repelled away, it follows therefore, that electrons lose energy more than positrons due to the re-distribution of channeling yields [4, 71, 56].

The first observed influence of these radiative processes in pair creation experimental yields was observed in measurements of pair-production by Belkacaem *et al.* [67]. In these measurements, it was noted that a positron that loses energy inside the crystal by radiative losses has a higher probability of emerging from the crystal with energy less than the experimental cut-off. This experimental factor lead to substantial reduction in yields and hence a severe disagreement with CFA theory was recorded.

In the analysis detailed in this thesis, the region of maximum energy loss for Ge was estimated to be twice the threshold energy for pair production in  $\text{Ge} < 110 >$  at room temperature. The use of the tagger/spectrometer and calorimeter coincidence ostensibly took all the radiative effects into account; energy radiated within the crystal was picked up by the calorimeter where only events above 10 GeV were considered. An estimate of radiative losses based on reference [66] was about 20 %. This however, could only happen at maximum values for  $\chi$ , namely  $E \geq 130$  GeV, which is nearly beyond the range of energies recorded in this

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<sup>1</sup>For a detailed derivation and description of transverse space distribution of populations and energies, see reference [5, 42].

thesis. Moreover, it is doubtful whether the radiative losses could also persist so strongly into angular regions far away from the axis (as seen in figures 5.12 - 5.14). The fact that warmer crystals will experience lower radiative losses could thus give credence to the conclusion that these losses could not fully account for the changes in pair-production yields that could be observed in this thesis.

- A study of systematic factors like the background and poor shower containment could also be looked at. This was also mooted in the analysis of R. Moore [59]. The background events should have the effect of also washing out the coherence. However, based on the fact that in the theoretical predictions, no background events are considered and the structure is therefore highly pronounced. In the analysis carried out in this report, the results have more or less reproduced the structure predicted by theory, which gives credence to the fact that background study was undertaken to exhaustive detail. Furthermore, the background events are not variable with energy, and hence the energy dependent anomaly observed thus rules out background related effects. The energy resolution of the calorimeter, which as described in chapter 4 could also result in poor shower containment at high energies. This is illustrated by the fact that the energy resolution at 10 GeV is  $\sigma/E \leq 11\%/\sqrt{E}$ , which is considerably better than the resolution in the high energy end, namely,  $\sigma/E \leq 20\%/\sqrt{E}$ . Moreover, material background in the beam-line could also give the beam an energy spread which could affect the energy resolution of the calorimeter.

### The LPM effect

A detailed analysis (both theoretical and experimental) of the LPM was endeavoured. The results thus far have served to indicate that there could be a suppression of radiation yields at high-energies of incident photons. Furthermore, the resurgence of interest in the LPM effect has promised to unravel some new insights into the manifestations and implications of the LPM effect with regard to a wide range of experimental and theoretical studies in high-energy physics. A lot of questions still remain concerning the

consistency of the theories used to interpret the LPM effect. Paramount among these are all the various proposed developments; both experimental and theoretical, for further applications of the LPM effect in many areas of high-energy physics. In this study, a comprehensive investigation and comparison of both the bremsstrahlung LPM and pair-production LPM was carried out. The thesis, though mainly reported on the data for pair production although a comparison of both processes was made. The difference between these processes comes from the fact that, as a result of cross-symmetry, the LPM suppression occurs at the low energy end for bremsstrahlung whereas it appears at the highest possible energy end for pair production. The reason for this being that the coherence length (as described in chapter 2) is proportional to the inverse of the energy of the emitted photon and the energy of the pairs emitted, respectively.

## 7.2 Conclusions

The work described in this thesis has dealt largely with the experimental investigation of channeling, channeling radiation (ChR), coherent bremsstrahlung (CBS) and pair production when multi-GeV electrons/positrons and photons were incident on oriented crystals. What has emerged is that the effects under consideration could be well described using a semi-classical constant field approximation. Some attempts at using a quantum mechanical approach have also been attempted. The CFA (and other similar semi-classical methods) have thus far been more successful in predicting the observed phenomena than any other theory which has hitherto been attempted.

The NA43 experimental endeavours were a comprehensive programme that covered many aspects of both coherent and strong field QED processes resulting from the incidence of ultra-relativistic particles on highly ordered crystalline matter. The focus of this thesis was based on the investigation of pair-production on Ge  $\langle 110 \rangle$ , where the enhancements above the Bethe-Heitler cross-sections were observed. These enhancements rise from 1 at threshold to 3.5 for the axial incidence of photons with energy  $E_\gamma \sim 135$  GeV. These results agree very well with the CFA calculations of Kononets *et al.* [44].

Furthermore a study of the measurement of the differential distribution of pair production was also made. This work was preceded by a similar measurement carried out by Bak *et al.* [71] in which good agreement with the Constant Field Approximation for photons incident along the axis as well as the with the Coherent Pair production calculations was found. The measurements that were reported in this thesis were also complementary to recent similar measurements in which the Ge crystal was cooled to 100 K [66]. The results presented here are among the first of the measurements in which a detailed comparison with the CFA calculations of yields in differential distributions for an axially aligned crystal is made. Furthermore, the experimental differential distribution in yields were found to agree well with the theoretical semi-classical calcu-

lations of Kononets [28, 29, 45]. This agreement was found to be relatively consistent throughout the range of incident photons (0.5-3 mrad) along the (110) plane off the  $\langle 110 \rangle$  axis.

Further investigations involving a detailed analysis of the LPM effect were carried out. In these measurements, the investigation of pair production on a cooled Tungsten crystal was endeavoured. As predicted by theory, there was an observed suppression in pair-production yields in the region of threshold for the emergence of the LPM, namely 80 GeV. This analysis has thus provided what appears to be evidence of the LPM. Further analysis (both experimental and theoretical) of the LPM is in progress and more results which will yield more information about this effect are in the process of being completed. Cognizance has to be taken of the fact that there is no clear and conclusive evidence of the LPM. However, the results shown in this thesis are also a very important complement to the ongoing study of the LPM effect.

Various poignant questions as far as the study of the strong field effect is concerned still need to be answered. Pertinent among them is the possible existence of single photons, which have proved to be very elusive to detect. However, recent measurements of the NA43 CERN-Aarhus group have a promise of yielding good results in the detection of the single photon spectra. The success of the CFA in explaining the data using a semi-classical formalism has led to attempts by the Frankfurt group to use a full quantum description of these effects. These attempts, however, still use approximations for axially oriented strings only, and therefore can only be relevant to well-channeled particles. However, the nature of the physics being investigated requires that the theories should also account for incident angles up to the Baier angle. Furthermore, investigations of various strong field QED effects like photon splitting could also provide deep insights about the nature of higher order processes in QED and thus help concretise the knowledge of the subject that we have to date.

## 7.3 Applications

The study of channeling and related effects has led to many applications. A discussion of the use of bent crystals is one example that has already been detailed in this thesis. What follows below is a summary of other possible applications of multi-GeV channeling techniques:

### Cascade showers

The combination of pair-production and bremsstrahlung emissions for high-energy electrons and photons can result in large cascade showers. By taking into account the radiation length  $l_r$ , one can study the characteristics of shower development in crystals. Along crystalline planes, the radiation lengths are reduced 20-50 times as compared to amorphous targets. A closer study of these shower formation mechanism promises to yield information about compact (lightweight) calorimeters for very high-energy electrons, positrons and gamma rays with an angular acceptance of 1 mrad and shows promise to be used in high-energy physics experiments.

There are also possibilities of constructing a compact direction sensitive electromagnetic calorimeter for gamma ray astronomy [99]. This detector could have an area less than the ground based detectors used so far. Using either the Cerenkov light or multiplicity of charged particles which develop in the aligned crystal, such a detector will allow safe identification of celestial very high-energy ( $\geq 100$  GeV)  $\gamma$ -ray sources better than the present 3-5 standard deviations. This is largely due to a very good hadron rejection of such a detector.

### Lifetime measurements

Single crystals could also find use in the search for short lived ( $10^{-14} - 10^{-15}$  sec) neutral particles. When a 300-500 GeV electron or photon beam is incident along the axis,

the coherence results in strong enhancements of particle production. For example, the beam sees 25-30 radiation lengths in a few mm thick W crystal. The path length of the produced particles is still large in comparison to the crystal thickness and particles will decay within a few meters behind the target. Such a technique of using crystals as targets could turn out to be a simple method in searches for rare particles produced in electromagnetic showers.

#### Other applications

The study of these high-energy correlated scattering processes has proved to be practically useful for the bending of high-energy hadrons. This has been proposed for applications at the Large Hadron Collider (LHC). Finally, the technique of channeling radiation and pair conversion has also found use for the vetoing of unwanted photons in the upstream decay fiducial region of the NA48 CP-violation experiment.

## 7.4 Outlook

The study of the LPM also promises to result in many applications, both theoretical and experimental. Crystals appear to provide an accessible laboratory to study the LPM effect. Examples where such knowledge is useful are:

In the study of cosmic ray showers, particles have energies up to  $10^{20}$  GeV [100], therefore one would expect a very strong LPM suppression to show up in the detection of the air showers. Furthermore, there is also a very strong interest from high-energy physics studies of the quark-gluon plasma. In this case the LPM crosses the QED domain into QCD. This obviously poses some theoretical challenges because the explanation in QED is simplified by the fact that there is no self coupling of the photon, however, when a particle crosses a gluon field (which is self coupling) the formalism therein becomes more complicated). An attempt to investigate this effect in hadronic



studies is given in a paper by Gyulassy *et al.* [76] and also in a more simplified overview by Cleymans [84].

---

## Appendix A

### Other related experiments and applications

---

#### A.1 Hard photon emission and polarisation

In addition, the studies of coherent, correlated mechanisms have been pursued to detail, more specifically the measurement of photon intensity spectra [4, 21, 22]. The measurement of these photon emissions are related to the investigations of high-energy, nearly mono-energetic photons. Initial conjectures were put forward which, in accordance with semi-classical theories predicted that single hard photons were responsible for the production of the nearly mono-energetic peak seen in figure A.1. However, in the case of thick crystals ( $> 200 \mu\text{m}$ ), the high-energy peak was found [75] to display a multiplicity greater than three. This means that in order to measure single photon spectrum one would require very thin crystals, e.g.  $\leq 50 \mu\text{m}$  for diamond. This however, would destroy some of the coherence because the crystal will in this case be shorter than the coherence length. This could be illustrated by the fact that the process of emission of 1 GeV photons by 200 GeV electrons incident on diamond occurs over a coherence length of  $64 \mu\text{m}$ . Thus far, severe experimental constraints have precluded the measurements of single photons.

A deeper investigation of these emitted photons has promises to yield information about their polarisability as predicted by theories. In actual fact, what arises from the preliminary analysis is that there is a degree of linear polarisation. This polarisation, was proposed by Cabbibo [97] to manifest itself in the form of photon attenuation through pair-production (or conversion probability). Pair-production of high-energy photons in crystalline matter has been theoretically predicted to be dependent on linear polarisation. Moreover, the high intensity, spectral density as well as the linear polarisation of these ultra-hard photons will have uses in many other branches of physics research. A lot of research is also currently under way in an effort to investigate the circular polarisation of these radiation processes [98].

## A.2 Crystal bending

The uniqueness of the detector setup at the NA43 meant that a wide variety of measurements on strong field effects could be made. The investigations that form part of this thesis also have a strong influence on a whole body of research that covers the use of crystalline targets for bending of high-energy proton beams for fixed targets. These results are discussed extensively in the literature [91, 92, 94]. The basis of this use of the crystals is essentially channeling. As has been described in Chapter 2, positive particles are trapped in potential dips and negative ones experience an inverted potential, thus making it possible to use bent crystals for beam extraction and bending. This technique also benefits from utilising the effect of strong fields and thus crystals can be equivalent to the use of 1000 Tesla magnets (details are given in reference [93]). Channeling in a bent crystal with constant curvature is described in the continuum potential by the introduction of a centrifugal force, giving rise to an effective potential:

$$Y_{\text{eff}}(x) = Y(x) - \kappa p v x \quad (\text{A.1})$$

where  $x$  is the distance from the line that is between atomic planes. The curvature is given by  $\kappa = 1/R$  where  $R$  is the radius of curvature. The effective potential is thus

150 GeV electrons incident on  
1.5 mm Diamond, 0.3 mrad off  $\langle 100 \rangle$  - axis

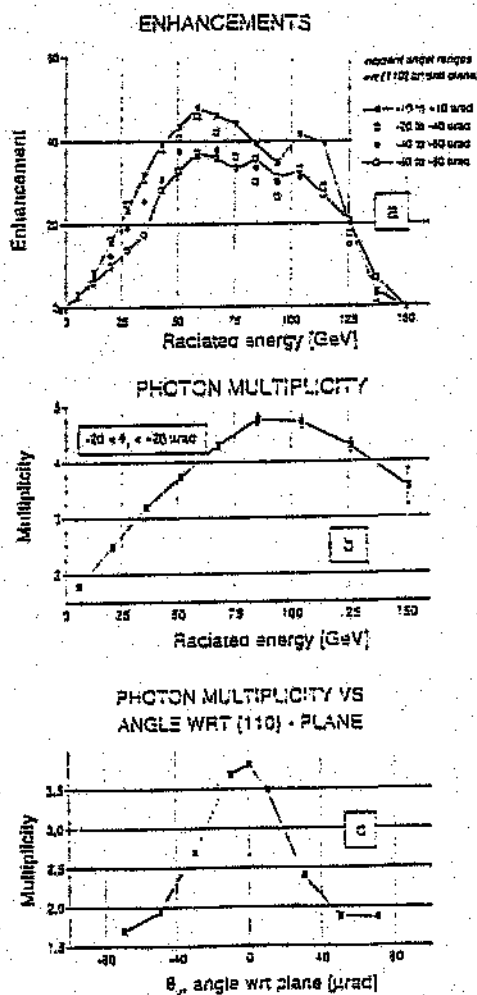


Figure A.1: Photon spectra showing (a) the enhancement in radiation as a function of relative photon energy. The photon spectra have been normalised to the Bethe-Heitler yields. The change in photon multiplicity w.r.t. energy is shown in diagram (b) and in (c) the photon multiplicity is plotted as a function of the orientation of the target crystal [75].

deflection of a parallel 450 GeV proton beam by means of (111) planar channeling in a bent silicon crystal

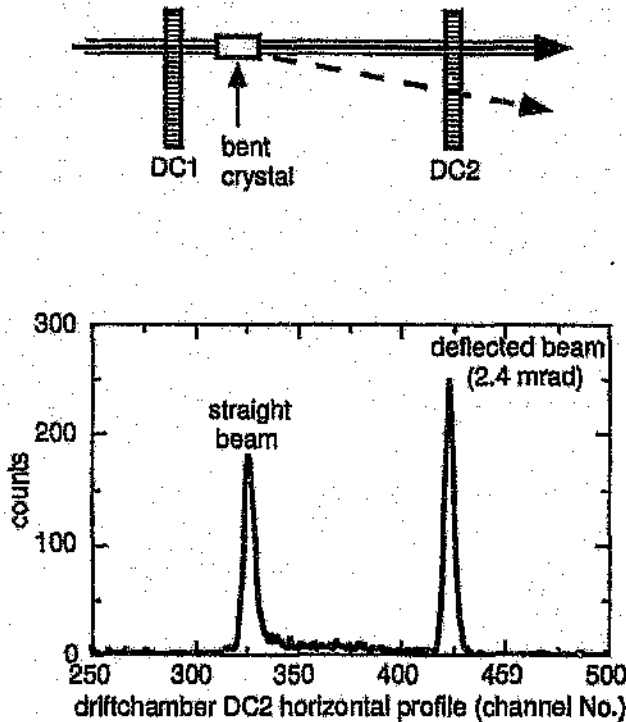


Figure A.2: Simple layout of bending experiments, shown also is the drift chamber spectrum in which the profile of the deflected beam appears.

reduced in the process, such that only particles with low transverse energy  $E_{\perp}$  can be trapped in the well. Bending efficiency will thus be determined by the length and the maximal curvature of the crystal. The measurements given in [96] provided additional information on the data for 450 GeV/c CERN bending experiments using planar channeling in a bent silicon crystal. The experimental set-up for these experiments is shown in figure A.2

Axial bending [95] has been investigated for both positive and negative particles.

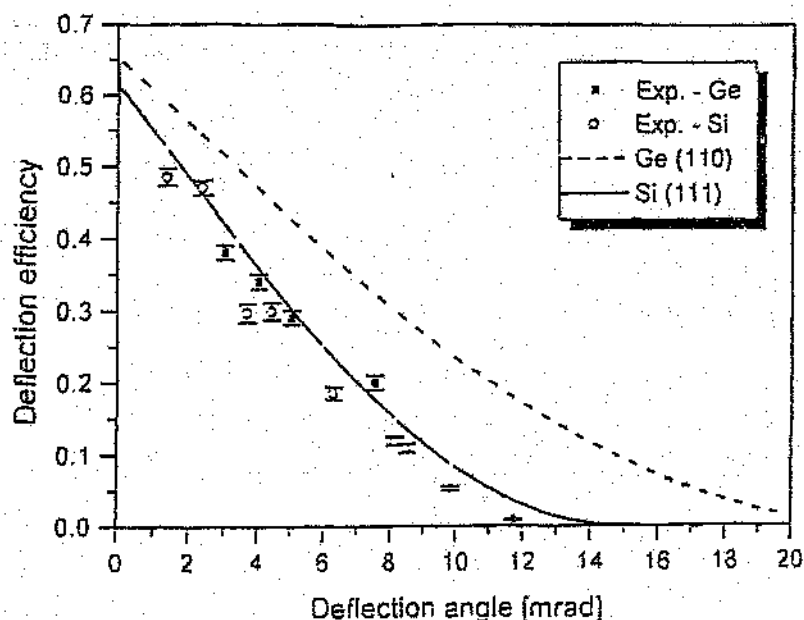


Figure A.3: Deflection efficiencies for the (110) planes in Germanium and Silicon. A comparison with a calculation that includes the curvature of the crystal is also shown.

A deflection measurement of a 450 GeV/c proton beam by means of a crystal heavier than silicon, namely germanium, was performed as well. A comparison therein indicated no overall advantage of using a higher-Z crystal. Positive results of the technique have been obtained and high bending efficiencies have also been obtained. The remaining investigations will deal largely with stretching the technique towards measurements of beam bending when curvatures are large. The results of bending efficiencies of both germanium and silicon are shown in fig.A.3.

## Appendix B

### Standard quantum field calculations

---

Wick's theorem states that each term of the  $S$  matrix is a sum of products of contractions of the form:

$$\langle 0|TA(x_k)\psi(x_k)\bar{\psi}(x_l)|0\rangle \quad (\text{B.1})$$

For  $x_k$  and  $x_l$  given, let us introduce the 4 by 4 matrix as follows:

$$c(\alpha_k, x_k; \alpha_l, x_l) = -ie \sum_{\sigma} \langle 0|T[A_{\alpha_k}(x_k)\bar{\psi}_{\alpha_l}(x_l)]|0\rangle \quad (\text{B.2})$$

In terms of  $c$ ,  $S_0(A)$  reads:

$$S_0(A) = \sum_n \frac{(-1)^n}{n!} \int d_1 \dots d_n \sum_p \varepsilon_p \sum_{\alpha_1 \dots \alpha_n} C((\alpha_1, x_1; \alpha_{p1}, x_{p1}) \dots (\alpha_n, x_n; \alpha_{pn}, x_{pn})) \quad (\text{B.3})$$

The introduction of the matrix  $\Gamma$  in the term:

$$\langle x, \alpha | \Gamma | y, \beta \rangle = c(x, \alpha; y, \beta) \quad (\text{B.4})$$

gives

$$S_0(A) = \text{Det}(1 - \Gamma) \exp[\text{Tr} \ln(1 - \Gamma)] \quad (\text{B.5})$$

Using the matrix notation together with the expression of the Dirac propagator gives:

$$T\psi_{\xi}(x)\bar{\psi}_{\alpha'}(y) = \Theta(x^0 - y^0)\psi_{\xi}(x)\bar{\psi}_{\alpha'}(y) - \Theta(y^0 - x^0)\bar{\psi}_{\xi}(y)\psi_{\xi}(x) \quad (\text{B.6})$$

such that:

$$\langle 0|T\psi_x i(x)\bar{\psi}_\xi(y)|0\rangle = iS(x-y)_{xix\xi}, \quad (\text{B.7})$$

where

$$S(x-y) = \int \frac{d^4}{(2\pi)^4} e^{ik(x-y)} \frac{k+m}{k^2-m^2+i\varepsilon}. \quad (\text{B.8})$$



# Bibliography

- [1] G. R. Piercy, F. Brown, J. A. Davies and McCargo, Phys. Rev Lett 10, 399 (1963)
- [2] R. S. Nelson, M. W. Thompson, Philosophical Magazine 8, 1677 (1963)
- [3] J. Stark, Physik. Zeitsch 13, 973 (1912)
- [4] A. Belkacaem *et al.*, Phys. Lett. B 177, 211 (1985)
- [5] J. Lindhard, Kgl. Dan Vid Selsk, Mat. Fys. Medd. 34 No. 14, (1964)  
J. Lindhard Phys. Lett. 12, 126 (1965)
- [6] D. S. Gemmel, Rev. Mod. Phys. 46, 129 (1974)
- [7] M. W. Thompson in Channeling, Ed: D. V. Morgan (J Wiley, London, 1973)  
p 1
- [8] A. A. Vorbiev, V. V. Kaplin, S. A. Vorbiev Nucl. Instrum. Methods. 127, 41 (1975)
- [9] M. A. Kumakhov, Phys Lett 57, 17 (1977)  
Phys. Stat Sol. (b) 84, 41 (1977)
- [10] R. L. Swent *et al.* Phys. Rev. Lett. 43, 1723 (1979)
- [11] J. U. Andersen and E. Lægård, Phys. Rev. Lett. 44, 1079 (1980)
- [12] J. U. Andersen, E. Bonderup and R. H. Panthell, Ann. Rev. Nucl. Part. Sci. 33, 453 (1983)

- [13] J. C. Kimball and N. Cue, Phys. Rep. 125, 69 (1985)
- [14] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Phys. Lett A 114, 511 (1986)
- [15] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Phys. Lett A 117, 251 (1986)
- [16] V. N. Baier, V. M. Katkov and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz 92, 1228 (1987)
- [17] V. N. Baier, V. M. Katkov, V. M. Strakhovenko, Nucl. Sci. Appl. 3, 245 (1989)
- [18] L. Augustin, A. Schäfer and W. Greiner, Phys. Rev. A 51, 1367 (1995)
- [19] A. Schäfer, Nucl. Instrum. Methods B 119, 48 (1996)
- [20] J. Schwinger, Phys. Rev. 75, 1912 (1949)  
J. Schwinger, Phys. Rev D 7, 1696 (1973)
- [21] R. Medenwaldt *et al.*, Phys. Lett, B 242, 517 (1990)
- [22] R. Medenwaldt *et al.*, Phys. Lett. B 260, 235 (1991)  
R. Medenwaldt *et al.*, Nucl. Instr. Methods B 103, 139 (1995)
- [23] R. Medenwaldt *et al.*, Phys. Lett. B 281, 153 (1992)
- [24] V. V. Beloshitsky, M. A. Khumakhov and A. Kh. Khokonov, Nucl. Instr. and Meth. B 62, 207 (1991)
- [25] C. Graziani, A. K. Harding and R. Sina, Phys. Rev. D 51, 7097 (1995)
- [26] A. K. Harding, M. G. Baring and P. L. Gonthoer, Astr. Astrophys. Suppl. Ser. 120 111 (1996)  
R. Araya and A. K. Harding, Astrophys. Journ. 463, L33 (1996)

- [27] D. F. Helfand, R. H. Becker and R. L. White, *Astrophys. Journ.* **453**, 741 (1995)
- [28] Yu. V Kononets and V A Ryabov, *Nucl. Instrum. Methods B* **48**, 269 (1990)  
Yu. V Kononets and V A Ryabov, *Nucl Instrum. Methods B* **48**, 271 (1990)
- [29] Yu. V Kononets, *J. Moscow Phy. Soc.* **2**, 71 (1992)
- [30] J. Lindhard, M. Scharff and H. E. Schiøtt, *Matt Fys Medd dan Vid Selsk* **14**, 33 (1963)
- [31] G. Molière, *Z Naturforschung* **2a**, 133, 1430 (1971)
- [32] P. Gombás, *Handbuch der Physik* **36**, (Springer, Berlin, 1956) p 109
- [33] W. Greiner, B. Müller and J. Rafelski, in *Quantum Electrodynamics of Strong Fields*, Springer-Verlag, 1985
- [34] G. Bassompierre *et al.*, *Phys. Lett. B* **355**, 584 (1995)
- [35] A Schäfer, *J. Phys. G: Nucl. Part. Phys.* **373**, (1989)
- [36] B. Müller, H. Peitz, J. Rafelski, W. Greiner, *Phys. Rev. Lett.* **28**, 1235 (1972)
- [37] B. Müller, J. Rafelski, W. Greiner, *Z Phys* **257**, 62 (1972)
- [38] B. Müller, J. Rafelski, W. Greiner, *Z Phys* **257**, 183 (1972)
- [39] T. Worm, *Dumle Programme*, Aarhus University, unpublished, 1996.
- [40] P. A. Doyle and P. S. Turner, *Acta Cryst. A* **24**, 390 (1968)
- [41] C. Itzykson and Jean-Bernard Zuber, in *Quantum Field Theory*, McGraw-Hill, 224
- [42] A. H. Sørensen and E. Uggerhøj, *Nuclear Science Applications*, Vol. 3, pp 147-205 (1989)

- [43] A. H. Sørensen, Nucl. Instr. and Meth. B 119, 3 (1996)  
A. H. Sørensen, in Proc. of a NATO Advanced Study Institute on Vacuum Structure in Intense Fields, 91 (1990)
- [44] Yu. V. Kononets and I. S. Tupitsyn, Pis'ma Zh. Eksp. Teor. Fys. 57, No 3, 148-153 (1993)
- [45] Yu. V. Kononets, private communication (1997)
- [46] V. N. Baier, V. M. Katkov and V. M. Strakhovenko, in *Electromagnetic Processes at High-Energy in Oriented Single Crystals* (Nauka, Novosibirsk, 1989)
- [47] Handbook of Mathematical Functions, Edited by M. Abramowitz and I. A. Stegun (National Bureau of Standards, Washington, 1966)
- [48] A. B. Migdal, Phys. Rev. 103, 1811 (1956)
- [49] Coherent Radiation Sources, ed. A. W. Sáenz and Überall, Springer Verlag, 1985
- [50] G. Diambrini Palazzi, Rev. Mod. Phys. 40, 611 (1968)
- [51] M. L. Ter-Mikaelian, in *High-Energy Electromagnetic Processes in Condensed Media*, Wiley, New York, 1972
- [52] J. Bak *et al.*, Nucl. Phys. B 254 (1985) 491,  
J. Bak *et al.* Nucl. Phys. B 302, 525 (1985)
- [53] L. D. Landau and I. J. Pomeranchuk, Dokl. Akad. Nauk. SSSR 92, 535 (1953)  
L. D. Landau and I. J. Pomeranchuk, Dokl. Akad. Nauk. SSSR 92, 735 (1953)
- [54] W. Heitler, in *Quantum theory of radiation*, John Wiley, 1975
- [55] K. Kirsebom *et al.*, CERN/SPSLC/ Note 96-5, unpublished.

- [56] K. Kirsebom *et al.*, Nucl. Instr. Meth. B 119, 79 (1996)
- [57] K. Elsener, private communication (1997)
- [58] K. Kirsebom and P. Christensen, private communication (1997)
- [59] R. Moore *et al.*, Nucl. Instr. Meth. B 119, 149 (1996)
- [60] F. Sauli, in Principles of Operation of Multiwire Proportional and Drift Chambers, CERN Yellow Report 77-09 (1977)
- [61] W. R. Leo, in "Techniques for Nuclear and Particle Physics Experiments", 151, Springer-Verlag, 1994
- [62] G. Charpak, F. Sauli and W. Duinker, Nucl. Instr. and Meth. 108, 413 (1973)  
A Breskin, G. Charpak, G Babioud, F. Sauli, N. Trautner, W Duinker and G Schultz, Nucl. Instr. and Meth. 119, 9 (1974)
- [63] V. N. Baier, V. M. Katkov and V. M. Strakhovenko, Phys. Lett. A 104, 231 (1984)  
V. N. Baier, V. M. Katkov and V. M. Strakhovenko, Dokl. Acad. Nauk (SSR), 282, 851 (1985)  
V. N. Baier, V. M. Katkov and V. M. Strakhovenko, Nucl. Instr. and Meth. B 16, 5 (1986)
- [64] A. Addis *et al.*, Phys. Rev. D 50, Review of Particle Properties, 1261 (1994)
- [65] V. B. Baryshevskii and V. V. Tikhomirov, Sov. J. Nucl. Phys. 36 (1982)  
V. B. Baryshevskii and V. V. Tikhomirov, Phys. Lett. 90 A, 153 (1982)
- [66] U. Mikkelsen, Ph. D. Thesis, University of Aarhus (1997), unpublished
- [67] A. Belkacaem *et al.*, Nucl. Instr. and Meth. B 13, 9 (1986)  
A. Belkacaem *et al.*, Phys. Rev. Lett 53, 2371 (1984); erratum 54

- [68] A. Belkacem *et al.*, Phys. Rev. Lett. **58**, 1196 (1987)
- [69] E. J. Williams, Kgl. Dansk. Vid. Selsk. **13** (1935), No. 4  
C. F. Weizsäcker, Z. Phys. **88**, 612 (1934)
- [70] E. Uggerhoj in, *Relativistic Channeling* eds. R. A. Carrigan jr. and J. A. Ellison (Plenum, New York, 1987)
- [71] J. F. Bak *et al.*, Phys. Lett B **202**, 615 (1988)
- [72] V. I. Ritus, Nucl. Instr. and Meth. **B48**, 278 (1990)
- [73] S. Ballestrero, Book95, University of Florence and CERN, unpublished.
- [74] CERN Program Library Long Writeup - Q121, CERN, Geneva 1997
- [75] J. Romano, Tesi di Laurea, University of Rome (La Sapienza) (1995) unpublished
- [76] M. Gyulassy and X. N. Wang, Nucl. Phys. B **420**, 583, (1994)
- [77] E. Stenlund, *et. al.*, Nucl. Phys. B **566**, 1c (1994)
- [78] S. L Shapiro and S. A. Teuklosky, in "Black holes, White dwarfs, and Neutron stars: The Physics of compact objects," Chapter 10, Wiley, New York, 1983.
- [79] A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rep. **192**, 179 (1990)
- [80] V. Spicka and P. Lipavsky, Phys. Rev. Lett. **73**, 3439 (1994)  
V. Spicka and P. Lipavsky, Phys. Rev. B **52**, 14615 (1995)  
P. Lipavsky, V. Spicka and B. Velicky, Phys. Rev. B **34**, 6933 (1986)
- [81] J. Knoll and D. N. Voskresensky, Annals of Physics **249**, 532 (1996)
- [82] E. Levin, Phys. Lett. B **380**, 399 (1996)
- [83] F. J. Dyson and H. Überall, Phys. Rev. **99**, 604 (1955)

- [84] J. Cleymans, V. V. Golizivninm and K. Redlich, Phys. Rev D 47, 173, 989 (1993)
- [85] J. D. Jackson, in Classical Electrodynamics, John Wiley, 671 (1975)
- [86] J. S. Bell, Nucl. Phys. 8, 613 (1958)
- [87] E Uggerhøj, Rad. Eff and Defects in Solids 25 3 (1993)
- [88] P. L. Anthony *et al.*, Phys. Rev. Lett. 10, 1949 (1995)
- [89] M. L. Perl, in *Proceedings of the 1994 Les rencontres de Physique de al Vallee D'Aoste*, edited by M. Grego (Editors Frontieres, Gif-sur-Yvette, France, 1994), p. 567
- [90] R. Blankenbecler and S. D. Drell, SLAC-PUB-95-6944, 1994, unpublished  
R. Blankenbecler and S. D. Drell, Phys. Rev. D 36, 277 (1987)
- [91] E. N. Tsyganov, Fermilab TM-682, TM -684, Batavia, (1976) (unpublished)
- [92] A. F. Elishev *et al.*, Phys. Lett. B 88, 387 (1979) and the references therein
- [93] S. P. Møller, Lecture delivered at CAS, 9th John Adams Memorial lecture, CERN (1993)
- [94] S. P. Møller *et al.*, Nucl. Instrum. Methods B 84, 434 (1994)
- [95] J. F. Bak *et al.*, Nucl. Phys. B 242, 1 (1984)  
J. F. Bak *et al.*, Phys. Lett. B 63, 387 (1980)
- [96] A. Baurichter *et al.*, Nucl. Instrum. Methods B 119, 172 (1996)
- [97] N. Cabibbo *et al.*, Nuovo Cimento 28, 435 (1963)  
N. Cabibbo *et al.*, Phys. Rev. Lett. 9, 270 (1962)  
G. Barbiellini *et al.*, Nuovo Cimento 28, 435 (1963)
- [98] R. O. Avakian *et al.*, Nucl. Intr. Meth. A 357, 238 (1995)

- [99] K. Elsener *et al.*, Phys. Lett. A 212, 537 (1988)
- [100] L. G. Dedenko *et al.*, JETP Lett. 61, 241 (1995)



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