

Revealing the embedded phase in single-pixel quantum ghost imaging

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Single-pixel quantum ghost imaging involves the exploitation of non-local photon spatial correlations to image objects with light that has not interacted with them and, using intelligent spatial scanning with projective masks, reduces detection to a single pixel. Despite many applications, extension to complex amplitude objects remains challenging. Here, we reveal that the necessary interference for phase retrieval is naturally embedded in the correlation measurements formed from traditional projective masks in bi-photon quantum ghost imaging. Using this, we develop a simple approach to obtain the full phase and amplitude information of complex objects. We demonstrate straightforward reconstruction without ambiguity using objects exhibiting spatially varying structures from phase steps to gradients as well as complex amplitudes. This technique could be an important step toward imaging the phase of light-sensitive structures in biological matter. © 2023 Optica Publishing Group under the terms of the [Optica Open Access Publishing Agreement](https://doi.org/10.1364/OPTICA.472980)

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1. INTRODUCTION

In quantum ghost imaging, one photon from an entangled pair interacts with the object and is bucket-detected (no spatial resolution) while the other non-interacting photon is directed to a spatially resolved imaging detector [1,2]. While neither photon alone has the information of the object, an image can be formed by measuring the mutual correlations through coincidences. Initially ascribed to the quantum entanglement of photons [3,4], it has since been shown that ghost imaging is possible using classical correlations too [5,6]. Ghost imaging has seen significant growth in the past two decades [7–10], with the promise of enhanced resolution [11,12] and expansion of their application to the imaging of photosensitive objects by lower photon fluxes [13,14], as well as extending into the x ray [15,16] and electron wavelength spectrum [17,18]. Imaging quality has steadily improved while acquisition times have steadily decreased, fueled by advances in computational imaging [19,20], compressive sensing [21,22], and deep learning-based methods [23–28]. Quantum imaging offers the advantage of low photon levels, typical to quantum experiments, that permit imaging of light-sensitive structures, and thereby, avoid photo-damage [7]. Additionally, a distinct embodiment of quantum ghost imaging offers special performance where it is possible to produce non-degenerate signal and idler photons. The object is probed or illuminated with one wavelength while the spatial information is recorded at another wavelength where the imaging detector is less noisy, more sensitive, or cost effective [13].

Quantum ghost imaging has traditionally been used to image amplitude objects, first with two-dimensional (2D) spatial projective masks such as Hadamard [29] and random [19], and more

recently with advanced camera systems. Despite these advances, optical phase imaging has progressed much more slowly and is a non-trivial experimental task with limited advances to date. Applications, however, range from biological [14,30] to materials [31,32] and extend into communications with quantum images [33]. Such phase imaging to date has been achieved with enhanced phase contrast [34], interferometric approaches [35,36], more recently using quantum-correlation enabled Fourier ptychography [37], and polarization enabled holography [38], all using modern cameras. While these techniques can produce a range of desired results, they have some shortcomings: the phase contrast imaging techniques return partial information, and so they only work well for objects with binary phases, the ptychography and interferometric approaches are alignment intensive, prone to instability, and costly since single photon cameras are required.

A distinct consequence of the physical correlations produced in the single-pixel environment, however, has been overlooked. We develop, for the first time, a full theoretical treatment of the correlation measurements and projections in the 2D spatial scanning technique. As a result, we reveal compelling physics, showing the traditional construction approaches using 2D masks such as the Hadamard yield a natural projection that produces phase information for free. By intelligently altering this construction, we develop a novel method, showing that the full complex amplitude image can be reconstructed where only one additional measurement per scanning mask is required. Moreover, we demonstrate that this physical principle need not be confined to one basis set of these 2D masks, illustrating the same for random masks as well. In doing so, we bypass the need for physical interferometry, which is notoriously unstable and complex to align, replacing this with

digital masks that can instead be encoded on inexpensive spatial light modulators (SLMs). Advantageously, this is easy to align in quantum experiments and readily available in most optical labs, making our approach convenient and simple. Subsequently, we both further develop the understanding of such scanning techniques and exploit it to obtain unambiguous full-phase and amplitude information. As a result, we show that conventional single-pixel ghost imaging setups are already able to see detailed phase objects without any physical adjustments and rather only an additional spatial mask projection, which we readily derive from the masks already used. Incorporation into the vanilla single-pixel setups is, thus, trivial. The advantages of single-pixel imaging are also gained here where high resolution at low flux rates is possible and inexpensive non-resolvable detectors facilitating a large range of wavelengths extend this to scenarios not viable for the sensitive, high-resolution cameras. Furthermore, it is important to note that the use of entangled photons paves the way for imaging biological samples as the lower photon levels, characteristic of quantum imaging, avoid photo-damage to light-sensitive structures present in biological matter.

2. METHOD

To illustrate this, we implemented an all-digital ghost imaging setup with spatial masks generated on SLMs as shown schematically in Fig. 1(a). Light from a diode laser (wavelength $\lambda = 405$ nm and radius, $w_p = 2$ mm) was then used to pump a 2 mm long temperature controlled type I PPKTP non-linear crystal (NLC). From the spontaneous parametric downconversion (SPDC) parameters, we estimate the resolution of the system to have an upper bound of ~ 487 pixels [11]. The temperature was set to optimally obtain collinear SPDC of entangled bi-photons at a wavelength of 810 nm. Any unconverted pump photons were then filtered out with a bandpass filter placed after the NLC. Next, the entangled photons were separated into two paths with a 50:50 beam splitter (BS) with the digital object encoded on an SLM in path A (object arm) and projective digital masks for imaging, also written to a SLM, in path B (image arm) with a blazed grating on each SLM. The object and image photons were coupled into single mode fibers connected to avalanche photodiodes (APDs) for detection in coincidence (C.C.). The SLMs and fibers were all positioned in the image plane of the crystal as indicated by dashed lines in the figure. We note that the pair rate of the quantum source is small and additional components such as SLMs and detectors in the single photon path reduce the overall number of photons to the detectors. Parameters such as extending the integration time until an adequate number of single photons are detected [39] and strong assumptions on the detector noise such as background count subtraction [40] may be altered to mitigate this loss. An intensity flattening technique, as described in [40], may ensure precise projective measurements when measuring in the Fourier plane.

Here we elucidated the “hidden” phase features of the objects from correlation measurements between the photons in each arm. In the discrete position basis, $\{|\mathbf{r}_j\rangle\}$, the (entangled) bi-photon state have correlations described by the following state:

$$|\Phi\rangle = \frac{1}{N} \sum_{j=0}^{N-1} |\mathbf{r}_j\rangle_A |\mathbf{r}_j\rangle_B, \quad (1)$$

where N indicates the number of discrete position basis states that are correlated in the x or y coordinate. For most ghost imaging

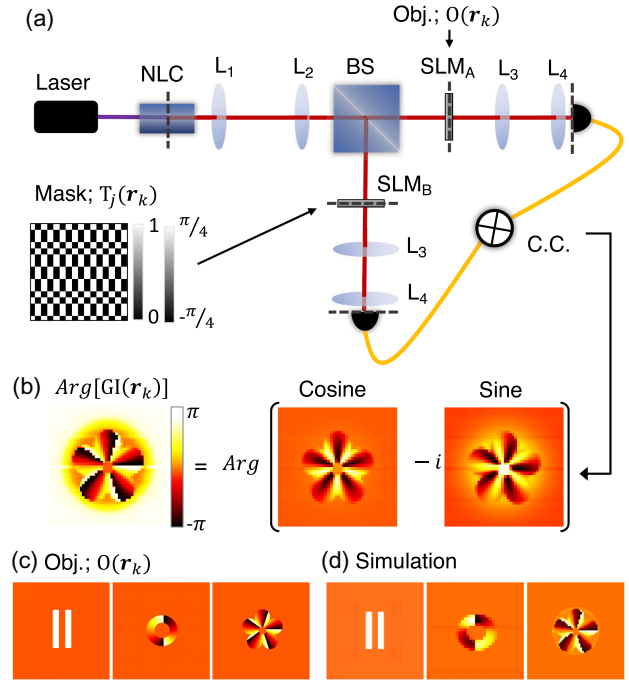


Fig. 1. Schematic diagram of the implemented quantum ghost imaging setup. (a) Entangled bi-photons are produced via spontaneous parametric downconversion (SPDC) at the NLC. The entangled photons are spatially separated, and each is imaged onto a SLM. Required holograms are displayed on each SLM. Coincidence measurements are done between both paths, and the coincidences are used to reconstruct an image of the object. For each object, two measurements are taken, as numerically simulated in (b). These two images are then combined such that the argument reveals the total phase. (c) shows the digital phase-only objects used in the experiment, while the simulated image reconstructions are shown in (d) showing how the digital objects are expected to perform. $L_1 = 50$ mm, $L_2 = 750$ mm, $L_3 = 750$ mm, $L_4 = 2$ mm.

experiments, an object is decomposed using a complete set of projection states $|M_j\rangle$ or equivalently $\mathbf{M}_j$ in matrix form. We can, thus, describe the object in this basis following the decomposition:

$$|O\rangle = \sum_{j=0}^{N-1} \sqrt{p_j} e^{i\alpha_j} |M_j\rangle. \quad (2)$$

Here p_j is the probability of projecting onto the basis state $|M_j\rangle$, and α_j represents the corresponding phases. Furthermore, the spatial profile of the object is given by $\langle \mathbf{r}_j | O \rangle = O(\mathbf{r}_j)$, corresponding to the complex matrix $\mathbf{O}$.

As an example, we will consider the basis modes constructed from the Walsh–Hadamard transform [41] with column vectors h_n . In matrix form, the projection states are $\mathbf{M}_{j \rightarrow (nm)} = h_n \otimes h_m$, which comprise arrays of ± 1 pixels (in and out of phase). After renormalization following $|T_j\rangle = 1/\sqrt{2}(|M_0\rangle + |M_j\rangle)$, where $\langle \mathbf{r}_j | M_0 \rangle = 1/\sqrt{N}$, the final projection states correspond to matrices, $\mathbf{T}_j^{\text{cos}}$, that have entries that are 0s and 1s.

Traditionally the ghost image is reconstructed from the second-order correlation function [42],

$$\mathbf{GI}_{\text{cos}} = \frac{1}{N} \sum_{j=0}^{N-1} [|\langle c_j \rangle|^2 - \langle |c_j|^2 \rangle_N] \mathbf{M}_j. \quad (3)$$

The ensemble averages are computed as $\langle A_j \rangle_N = \sum_{j=0}^{N-1} A_j / N$. While it is common to use projection masks, $\mathbf{T}_j^{\cos}$, in Eq. (3) in the reconstruction procedure, it suffices to use the Hadamard masks, $\mathbf{M}_j$, instead because $\mathbf{G}_{\cos}$ is independent of the reference mode, $\mathbf{M}_0$ (see Supplement 1). To reveal the spatial phase information in the object, we first compute $\langle |c_j|^2 \rangle_N = p_0/2 + \langle p_j \rangle_N/2 + \langle \sqrt{p_0 p_j} \cos(\Delta\alpha_j) \rangle_N$. After substitution into Eq. (3) and further simplification (see Supplement 1), the measured ghost image is

$$\mathbf{G}_{\cos} = \frac{1}{N} \left(\sqrt{p_0} \text{Re}(\mathbf{O}) + \frac{1}{2} \mathbf{H}_d \mathbf{P} \mathbf{H}_d^\dagger - g_{\cos} \mathbf{R}_0 \right), \quad (4)$$

where the first term is the real part of the object, $\text{Re}[\mathbf{O}] = |\mathbf{O}| \cos(\arg(\mathbf{O}) - \alpha_0)$. The second term is the Hadamard transform of matrix $\mathbf{P} = |\mathbf{H}_d \mathbf{O} \mathbf{H}_d^\dagger|^2$, where $|\cdot|^2$ is the element-wise absolute value squared. Each “pixel” entry in $\mathbf{P}$ maps onto a probability p_j . The last term removes the DC component by subtracting the first pixel $\mathbf{R}_0 \rightarrow \delta_{j0} |r_j\rangle$ weighted by the factor $g_{\cos}$. Moreover, the weighting, $g_{\cos}$, is a constant that depends on the ensemble average of the probabilities, p_j , and cosines of the phases (see Supplement 1). It can be shown that in some cases the second term does not overlap spatially with the desired first term and, therefore, can be removed from the constructed image by simply cropping it out (see Supplement 1).

Following the same arguments, the sine of the phase, corresponding to the “imaginary part” of the ghost image, can be computed by applying the same analysis but with an adjustment to the projection masks, i.e., the new projections are $\mathbf{T}_j^{\sin} = \frac{1}{\sqrt{2}} (\mathbf{M}_j + i \mathbf{M}_0)$, having a relative phase of $\pi/2$. The resulting detection probabilities are then $|c_j|^2 = p_0/2 + p_j + \sqrt{p_0 p_j} \sin(\Delta\alpha_j)$. Accordingly, the detected ghost image becomes

$$\mathbf{G}_{\sin} = \frac{1}{N} \left(\sqrt{p_0} \text{Im}(\mathbf{O}) + \frac{1}{2} \mathbf{H}_d \mathbf{P} \mathbf{H}_d^\dagger - g_{\sin} \mathbf{R}_0 \right), \quad (5)$$

having an embedded imaginary part of the object profile, where $\text{Im}(\mathbf{O}) = |\mathbf{O}| \sin(\arg(\mathbf{O}) - \alpha_0)$. The constant $g_{\sin}$ weights the DC component to be subtracted and depends on the ensemble average of the probabilities, p_j , as well as the sine of the phases (see Supplement 1). Realization of imaginary image reconstruction is then achieved using Eq. (3) where the projected sine probabilities c_j are paired with the corresponding Hadamard masks $\mathbf{M}_j$ such that a real image is formed with the required sine phase information.

Accordingly, two image reconstructions (cosine and sine) are required to fully reconstruct a phase object, as illustrated in Fig. 1(b). Applying this to three phase objects, shown in Fig. 1(c), results in the simulated outcomes shown in Fig. 1(d). Experimentally, the cosine projection was obtained by displaying masks on the SLM_B in which the blazed grating was turned off (amplitude of 0) in certain pixels and on (amplitude of 1) in others. The location of the “on” or “off” pixels was determined by the specific mask in the basis set (for full details of the scanning technique, see [27,28]). The sine projection was then obtained by displaying the same masks with an identical pixel arrangement; however, the “on” and “off” pixel states were instead replaced with $1 + i$ and $1 - i$ phase states, respectively. Furthermore, we emphasise that both reconstructed images contain the amplitude of the object, i.e., $|\mathbf{O}|$, in addition to the phase dependent components, enabling the

extension of our technique to complex amplitude objects using quantum ghost imaging.

3. RESULTS

Initially, we show the experimental reconstruction of two amplitude-only objects in Fig. 2, namely two vertical slits [Fig. 2(a)] and an annular ring [Fig. 2(b)] with 64×64 scanned superpixels (yielding a scanning resolution of $128 \mu\text{m}$). These were achieved by modulating only the amplitude of the photons with SLM_A (leaving a flat phase) and using the cosine projection masks on SLM_B to detect and, thus, reconstruct the amplitude object according to the algorithm in Eq. (3). Insets in the corners of the figure show the transmission masks used. A good correlation between the object and detected distribution can, thus, be seen.

By converting the amplitude objects in Fig. 2 to pure phase objects shown in Fig. 3, we show that phase information is also embedded in the ghost imaging technique and retrieved by an additional sine projection. Here, we illustratively compare the numerical simulation (Sim) with the experimentally (Exp) reconstructed phase-only π -phase slit and the azimuthal phase ring. Insets at the bottom of Fig. 3 show both the experimentally reconstructed cosine and sine projections. While, previously, we focused on the Walsh–Hadamard basis, here we also show that this phase retrieval can be extended to the pseudo-complete random basis. As can be seen, the total phase of the object is recovered for both the Walsh–Hadamard in Fig. 3(a) and random masks in Fig. 3(b), showing not only phase steps for the π -phase slit but also contrast in the phase gradient by the spiral features with the azimuthal phase ring. Apart from noise distortion, we show good visual agreement between the numerical simulation and the reconstructed experimental images. In these acquisitions, single photon counts ranging between 40k to 100k, yielding coincidences between 500 to 2000, were obtained in these measurements. It may also be noted that a flat circular phase background persists within the detected region on the SLMs that was illuminated with the SPDC photons. Past this region, a larger variation of noise persists due to lack of signal. We have, therefore, suppressed the noise in the region where there is no signal, illustrated by a dashed white circle.

To assess the degree of agreement between the numerical simulations and experimental phase reconstructions in Fig. 3, cross section plots are given in Fig. 4. The phase values per pixel of the slits are shown in Fig. 4(a) for a representative horizontal cross section. Random fluctuations in the phase values near the edges of the profile may be noted due to low signal as a result of being near the end of the detection region. We show good agreement between

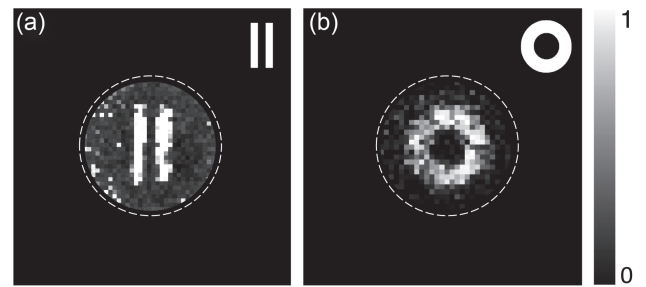


Fig. 2. Reconstructed amplitude-only images using the Walsh–Hadamard masks for (a) an intensity slit and (b) an intensity ring. The outer area of the dashed white circle indicates the region in which noise was suppressed due to lack of SPDC signal. The amplitude-only objects are shown as insets.

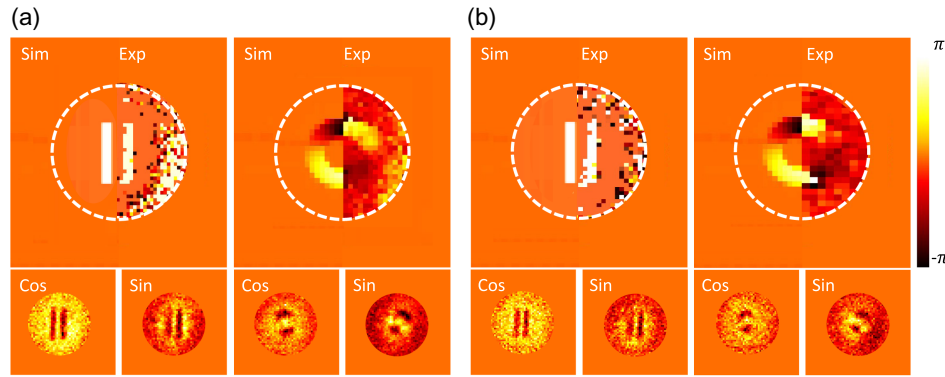


Fig. 3. Numerical simulations (Sim) showing excellent agreement with the experimental reconstructions (Exp) for the π -phase slit and the azimuthal gradient ring using (a) Walsh–Hadamard and (b) random masks. Experimental images were denoised with image processing tools and contrast adjustment after reconstruction. The outer area of the dashed white circle indicates the region in which noise was suppressed due to lack of SPDC signal. Insets show the corresponding cosine and sine components of the experimental reconstructions.

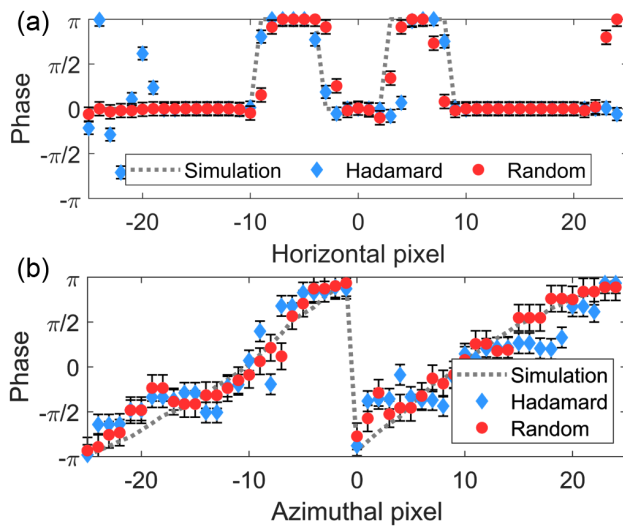


Fig. 4. Phase image cross sections showing the phase value per pixel for the numerically simulated reconstruction given by the gray dotted line and experimental reconstructions for both the Walsh–Hadamard (blue diamonds) and random (red dots) basis reconstructions. Similarly, in Fig. 4(b), the phase values per pixel are given in the azimuth direction for a set radius within the annular ring. Again, we show excellent agreement between the numerical simulation and experimental reconstructions for both scanning methods.

the numerical simulation (gray dotted line) and the experimental images for both the Walsh–Hadamard (blue diamonds) and random (red circles) basis reconstructions. Similarly, in Fig. 4(b), the phase values per pixel are given in the azimuth direction for a set radius within the annular ring. Again, we show excellent agreement between the numerical simulation and experimental reconstructions for both scanning methods.

Lastly, in Fig. 5(a), we show the experimental reconstruction for the detailed spiral phase flower, which was numerically demonstrated in Fig. 1(b). Here, both the cosine and sine projections (middle and right of Fig. 5) are given, which were used to reconstruct the full phase profile (left of Fig. 5). Using Walsh–Hadamard masks, we imaged at a higher resolution of 128×128 superpixels ($60 \mu\text{m}$ scanning resolution), with counts ranging between 100k to 350k and coincidences between 1500 to 3000 across the projections. Importantly, we see that the use of 2D spatial projective masks reveals the entire phase structure including all phase steps and phase gradients, albeit with the presence of

noise. Furthermore, in Fig. 5(b), we apply an additional amplitude modulation such that the flat phase components instead have an amplitude of 0 with 1 elsewhere, as illustrated in the intensity of the left image (Complex Obj.). Using the same mask projections as in Fig. 5(a), the spatial structures of both the amplitude ($A = |GI_{(\cos)} + iGI_{(\sin)}|$) and phase of this object were reconstructed with the numerically simulated outcomes in the upper-right corners and raw reconstructed structures rendered in three-dimensions below. Thresholding the amplitude in the right of Fig. 5(b) shows a good the comparative structure of the retrieved intensity with respect to the numerically simulated result. We note fluctuations in the intensity were seen due to notable sensitivity of the amplitude reconstruction to noise and additional experimental imperfections, making this approach non-trivial in accurately determining this component of the distribution. Nonetheless, an excellent reconstruction of the phase is possible, and definite similarity between the encoded, simulated, and experimental data is present for the amplitude, showing it is possible to retrieve the information of a complex object using our approach.

In contrast to other approaches, we have reduced the number of necessary measured variables to two (cosine and sine), and we require n^2 measurements for each variable, where $n \times n$ pixels represent the image resolution. We showed that the required interference naturally arises from the correlation measurements without interferometry. While calculated projective measurements in pursuit of phase have found development in classical single-pixel demonstrations [43,44], in such approaches, four-step phase shifting is still needed, requiring multiple measurements to reconstruct the desired phase information. In our approach, we reveal that traditional second-order quantum correlations allow the phase information to be available for free and to be acquired with a single adjustment in each digital mask. Our work makes two advancements: (i) reveals the previously overlooked phase response of observables in single-pixel quantum ghost imaging and (ii) offers an intelligible means to extracting the desired phase with controlled projective measurements. We have, therefore, developed and implemented a stable, cost-effective quantum ghost imaging technique to reconstruct the full phase profile of a phase object by measuring a smaller number of variables while retrieving phase steps and phase gradients with the use of single-pixel detectors and no need for computationally intense iterative algorithms to extract the phase.

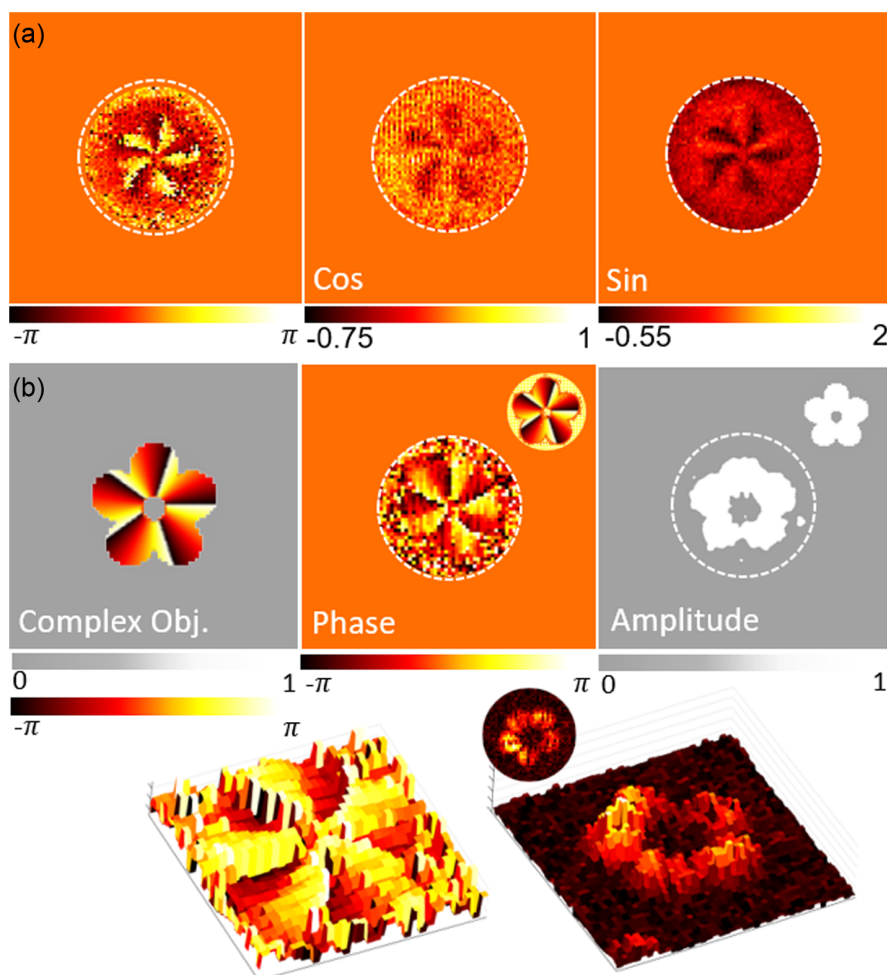


Fig. 5. Experimental ghost image of (a) pure phase and (b) complex amplitude gradient spiral flower using Walsh–Hadamard masks. The full phase profile was denoised after reconstruction by image processing tools and contrast adjustment. The cosine (middle) and sine (right) components are shown for completeness in (a). The complex amplitude object (left) was reconstructed to give phase (middle) and threshold amplitude (right) in (b). Three-dimensional renders of the raw reconstruction data are shown below.

4. CONCLUSION

In conclusion, we have shown, that with our new approach, the 2D spatial masks used in traditional ghost imaging hold sufficient information to not only fully reconstruct and reveal complex phase structures but the full complex amplitude as well. Here we demonstrated clear phase steps and azimuthal phase gradients in the image reconstructions. Our cosine and sine projective masks reconstructed both counterparts of the image, which when combined revealed the entire phase profile of our encoded phase-only objects. By comparing the simulated and experimental phase values per pixel, we have shown good agreement, albeit with noise distortions present. Furthermore, we show the intrinsic nature of our technique and the versatility thereof by imaging with both the Walsh–Hadamard and random spatial bases. We speculate that this naturally present interference, arising from our 2D spatial mask correlation measurements, has been overlooked due to the swift transition to sophisticated cameras. Nevertheless, this all-digital setup allows full amplitude and phase retrieval without complicated interferometric or computational techniques, and it can be enhanced further by a judicious choice of mask type and image processing tools. A larger sensitivity in the amplitude reconstruction is noted, however, requiring additional care. While

we demonstrated this technique on digital complex amplitude objects, it is well suited to achieve simple and stable quantum imaging of complex objects where the low photon numbers, typical in quantum experiments, are useful for applications in both the physical and biomedical domains. Additionally, the technique can be extended into the non-degenerate regime where biological samples can be probed with one wavelength (i.e., infrared), while the spatial information of the twin photon is recorded at another wavelength (which can be either visible or near-infrared).

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Data availability. Data underlying the results presented in this paper may be obtained from the authors upon reasonable request.

Supplemental document. See [Supplement 1](#) for supporting content.

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