

https://dspace.org/

The capital asset pricing model and arbitrage pricing theory on the Johannesburg Stock Exchange

Davidson, Sinclair Richard

2015-02-20
http://hdl.handle.net/10539/17034

Downloaded from a DSpace repository. https://dspace.org/

σ_i = the standard deviation of i; and,

σ_j = the standard deviation of j.

The standard deviation is the square root of variance.

Armed with these simple statistical formulas, we are able to develop Markowitz's theory further. If R_i is the return on security i, σ_{ij} is the covariance between security i and security j, X_i is the proportion of the investors funds invested in security i and R_p is the return on the portfolio as a whole. Then we can derive the following relationships:

$$r_p = \sum_{i=1}^{N} X_i r_i$$

and,

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij}$$

These relationships define the return and risk that is associated with portfolios.

3.1.1 IS VARIANCE A GOOD MEASURE OF RISK?

There are two arguments that can be made as to whether variance is a good measure of risk:

- a philosophical argument on the merits of the concept itself; and,
- a statistical argument as to the "existence" of variance in share return distributions.

3.1.1.1 PHILOSOPHICAL ASPECTS OF VARIANCE

In his classic 1952 article, Markowitz (1952:97) writes that, "[v]arious reasons recommend the use of the expected return-variance of return rule...".[15] Within the framework described in that article, he is correct; however, a potential problem exists in his use of variance as a measure of risk. It is the purpose of this section to argue that variance is a good measure of risk and that standard deviation (the square root of variance) is an even better measure of risk.

Variance is a well known measure of dispersion about a mean. Modigliani and Pogue (1974:71) have argued that if risk is defined as it has been in the portfolio context, "it would seem to be logical to measure risk by the dispersion of the possible returns below the expected value". Variance, however, includes in its measure both deviations above and below the mean. Bowen (1984:17) has argues that "financial writers have not been completely satisfied" with this measure of risk. He further states that business executives are only concerned with down-side risk and not with upside risk, which he refers to as "upside potential".

Jones (1986:52) also mentions this potential problem, but states that if the rates of return around the mean are symmetrical (ie. the distribution function is normal) that, "the doubt would fall away"[16]. Thus, as long as share returns follow a normal distribution, variance is a relevant measure of risk. Jones (1986:57) concludes with the following:

> "What is important is that the [variance], which is a mathematically convenient concept, gives the same risk rankings for a group of portfolios as would some mathematically intractable below-the-mean

---

[15] In his Nobel Lecture, Markowitz (1991: 470) stated, "[t]he fact that the variance of the portfolio, that is the variance of a weighted sum, involved all covariance terms added to the plausibility of the approach".

[16] Modigliani and Pogue (1974:71) support this point.

measure of variability. This makes it an appropriate measure of risk." Once we accept the fact that variance is an "appropriate measure of risk", we need to deal with a real problem associated with it. Variance is a squared measure, ie. its dimension is squared rands. As most people do not relate to the concept of squared currencies, it is necessary to express the units in usual terms. This is done by taking the square root of the variance to arrive at standard deviation. Thus it can be argued that standard deviation is a superior method of measuring risk than variance.

Levy and Sarnat (1990:209) trace the development of the word, "risk" from the Latin *risicum*. They argue that the original Latin word included elements of good and bad fortune. They further argue (Levy and Sarnat 1990:209) that, while the modern meaning of the word implies "hazard or danger of loss", that MPT has returned to the original meaning of the word. Indeed, Wagner and Lau (1971:49) argue that standard deviation is the, "best measure of the total risk of a portfolio, since it identifies how the investor's wealth fluctuates over time".

3.1.1.2 STATISTICAL ASPECTS OF VARIANCE

There is a literature which suggests that reliance upon the variance of a share return distribution is inadvisable as returns are not normally distributed. This approach is especially associated with Eugene Fama, who has argued that share returns belong to a class of distribution where the mean exists, but the variance does not[17] ie. is infinite (Fama 1963, 1965a, 1965b). This has an impact on financial theories, as most of these have tended to rely on the existence of second and higher order moments.

---

[17] While Perry (1983:211) argues that while this is not fatal, as alternatives exist, much of the existing work in financial economies assumes the existence of (at least) the second moment.

3.1.1.2.1 SHARE RETURN DISTRIBUTIONS

Share return distributions have been found to have more observations about the mean (peaked) and in the extreme tails (fat tailed) than the normal distribution ie. share return distributions are leptokurtic (Barnea and Downes 1973:348). A test of whether the distribution is leptokurtic is to calculate the sample kurtosis (see table 3.1). There are at least three explanations for this observed behaviour. The first explanation is associated with Mandelbrot (1963) and Fama (1963, 1965a, 1965b) who have argued that share returns are drawn from a stable Paretian distribution. An alternate explanation is that returns are drawn from a mixture of normal distributions -ie. the share return distribution is said to be subordinated (Clark 1973). The last explanation states that share return distributions conform to a student distribution and that the investor can assume normality as long as his investment horizon is long enough (Hagerman 1978:1214-1215). If either of the latter explanations is true, then we can rely on the existence of the second moment as the share return distribution is either normal or becomes normal.

Using monthly return data for the period 1963-1991, taken from the I-Net service, the stable Paretian hypothesis was investigated. Following Hagerman (1978:1218), use was made of a portfolio (the ALSI) as modern portfolio theory has the portfolio as its basic unit of analysis and not the individual share.

The kurtosis of the ALSI is reported below (in this case normal i.e. no kurtosis = 0):

TABLE 3.1: KURTOSIS OF MONTHLY RETURNS ON THE JSE

PERIOD	KURTOSIS
1963-1992	.957239
1960'S	.04374715
1970'S	.7162209
1980'S	.6167346

1990'S	-.4686314

The initial evidence indicates; for the entire period, the seventies and the eighties, that the tails of the distribution are heavier than indicated by a normal distribution. The tails in the nineties have fewer outliers than indicated by the normal distribution. This is probably as a result of the short period of time under investigation in that decade.

3.1.1.2.2 THE STABLE PARETIAN HYPOTHESIS

Cootner (1964:196) explains that the distributions which are capable of explaining share returns must come from a set of distributions known as the "infinitely divisible distributions". There are a large number of distributions in this class, including the normal. When we consider the characteristics of share returns, the possible distributions are extremely limited ie. to the stable Paretian family of distributions.

The stable Paretian distribution is defined by the log of its characteristic function and has the form:

$\log f(t) = \log E(e^{iut})$

$= i\delta t - \gamma|t|^{\alpha}[1 + i\beta(t/|t|)w(t,\alpha)$

where u = random number,

t = any real number,

i = $\sqrt{-1}$, and

$$w(t,\alpha) = \begin{cases} \tan(\pi\alpha/2), & \text{if } \alpha \neq 2 \\ (2/\pi)\log|t|, & \text{if } \alpha = 1 \end{cases}$$

Fama (1965b:110) notes that this distribution has four parameters, α (characteristic exponent), β (index of skewness), δ (location parameter) and γ (scale parameter). It is α that is of interest. The characteristic exponent determines the height of the tails of the distribution. It can take on any value in the interval $0 < \alpha \leq 2$. The moments of the distribution are finite to the order

$r < \alpha$, unless in the special case of $\alpha = 2$ when all the moments are finite.

In his classic 1965 paper, Fama (1965a:66-67) presents evidence that the value of α is consistently less than two. The implication of this is that the variance of the return distribution is infinite. Cootner (1964:197) argues that there is a very simple, albeit weak, test of the Fama - infinite σ^2 hypothesis. He argues that the sample variance of a share return distribution should tend towards the population variance (ie. infinity) as the sample size increases. Perry (1983:213) argues that this test can be performed by regressing sample variances on sample sizes. If the slope coefficient (β_1) is significantly positive we can reject the null hypothesis of $\sigma^2 \neq \infty$ in favour of $\sigma^2 = \infty$.

The following procedure was performed on the ALSI index:
- sample variances were calculated for increasing time periods for the twenty eight year time period (1963-1991);
- sample variance was then regressed against corresponding sample size.

This was repeated for the period 1973-1991, in order to test the generality of the initial result. The results of this experiment are shown in table 3.2.

TABLE 3.2: RESULTS USING THE COOTNER METHOD

	β_0	β_1	R^2
1963-1991	.001017	.00002737 (11.35)	.8321
1973-1991	.01713	-.0000317 (-9.29)	.8354

When we consider the period 1963-1991, the t-value (in parenthesises) indicates that we can reject the null hypothesis and accept the stable Paretian distribution. In the latter period, however we are unable to reject the null hypothesis.

Perry (1983) points out that there are two potential problems with this approach:

- the test may not have sufficient power to avoid a type I error; and,
- the residuals in the regression coefficients may exhibit serial dependence and thus the standard errors may be biased.

Given the overwhelming evidence of this test, it seems unlikely to be faulty in respect to the standard errors.

Given these conflicting results, it seems reasonable to replicate some of the original tests that have been performed in the literature. The sequential variance approach to estimating α relies on the fact that while the population variance (σ^2) is infinite, the sample variance (s^2) will always be finite. As argued earlier, as the sample increases, so the s^2 will tend to increase in size. Fama (1965a:65) uses this knowledge to derive the following formula:

$$\alpha = \frac{2(\log n_1 - \log n_0)}{2\log s_1 - 2\log s_0 + \log n_1 - \log n_0} \qquad (3.1)$$

where: s_i = sample standard deviation; and,

n_i = sample size.

Fama (1965a:66) argues that this is a very weak test as the α-estimate is sensitive to the ending and starting sample points. The results of this approach are as follows:

TABLE 3.3: α-VALUES GIVEN VARIOUS n_0 AND n_1

$n_0 \backslash n_1$	337	252	168	84
12	1.84	1.82	1.81	1.91
84	1.75	1.69	1.58	
168	1.95	1.90		
252	2.02			

Table 3.3 reveals a tendency for the ALSI-α to fall within the range $0 < \alpha \leq 2$.

In a later paper, Fama and Roll (1971:333) develop the following estimate of α:

$$z_f = (.827)\frac{x_f - x_{1-f}}{x_{.72} - x_{.28}} \qquad (3.2)$$

where: $x_i - x_{1-i}$ = an interfractile range; and,

z_f = an estimator of the f-fractile of the standardized symmetric stable distribution with α. (Fama and Roll 1968:822-823 calculated tables where α can be looked up for a given z_f-value.)

Using Monte Carlo simulation, Fama and Roll (1972:334-335) report that (3.2) is robust when $.95 \leq f \leq .97$. Using the .95 fractile, for the period 1963-1991, $\alpha = 1.3$; for the period 1980-1991, $\alpha = 1.5$.

The evidence in favour of the infinite variance hypothesis is respectable, but not overwhelming. The tests are inconclusive, but tend towards acceptance of the stable Paretian hypothesis. This implies that the mean exists but that the variance does not. In the next section we will investigate the normal share return hypotheses.

3.1.1.2.3 THE NORMAL RETURN HYPOTHESES

The stable distribution hypothesis has not gone unchallenged. Blattberg and Gonedes (1974:244) (in addition to Hagerman 1978) argue that the Student t-distribution is also capable of explaining the observed fat tails in return distributions. As the degrees of freedom in a t-distribution tend toward infinity the distribution tends towards normality.

An alternate explanation associated with Clark (1973) is that share returns follow a subordinated stochastic process. If we accept the EMH as being valid (as established in the previous chapter) then there is a relationship between the arrival of information in the market and share returns. Share returns are then "driven" by two processes:

- a subordinated (conditional) price change process; and,
- driving process (Castanias 1979:442).

The subordinated process describes the chronological behaviour[18] of the market and the driving process describes the impact of the arrival of new information.

Castanias (1979:443) describes the processes as follows:

$R(t) \sim N(0, s^2\mu_g)$, where $g(t) \sim LN(\mu_g, s^2_g)$

where: R(t) is the observed, continually compounded rate of return; and,

g(t) describes the arrival of information in the market.

The distribution of R(t) conditional on g(t) is normal, as follows:

$R(t|g(t)) \sim N(0, g(t)s^2)$.

Castanias (1979:443) argues that the higher order moments of g(t) do not affect the variance of R(t) and that the observed R(t) will have fat tails, but that share returns are normally distributed.

The following tests of normality were applied to the ALSI index monthly data for the period 1963-1991:

- Martinez and Iglewicz (1981) test for normality;
- Kolmogorov-Smirnov test; and,
- D'Agostino-Pearson Omnibus K^2 test (D'Agostino, Belanger and D'Agostino 1990).

The results of these test are displayed in table 3.4. It can be seen that the three tests clearly indicate that the share returns are normally distributed. The only data series where there could be some doubt is for entire period 1963-1991, the tests (not shown) indicate that the entire period returns are marginally non-normal. The data which could be a cause of this is the 1960's. A Kolmogorov-Smirnov test comparison (not shown) of the returns indicates that the period of the 1960's does differ from the other decades. The nature and causes of the difference are as yet unknown, but is a potential area of

---

[18] Investors should earn a return that is related to time. This rate would probably approximate the risk free rate.

further research.

TABLE 3.4: RESULTS OF NORMALITY TESTS

MARTINEZ-IGLEWICZ NORMALITY TEST			
	TEST VALUE	10% REJECTION	5% REJECTION
1960's	.994	1.068	1.106
1970's	1.059881	1.043*	1.068
1980's	1.038003	1.043	1.068
1990's	1.000907	1.182	1.289
KOLMOGOROV-SMIRNOV TEST FOR NORMALITY			
	TEST VALUE	10% REJECTION	5% REJECTION
1960's	.06923	.095	.105
1970's	.07136	.074	.081
1980's	.03182	.074	.081
1990's	.08573	.162	.178
D'AGOSTINO-PEARSON OMNIBUS K^2 NORMALITY TEST			
	TEST VALUE	Pr LEVEL	DECISION
1960's	.1	.9535	NORMAL
1970's	2.7	.2534	NORMAL
1980's	3.4	.1801	NORMAL
1990's	.3	.8812	NORMAL

Note: The * value indicates non-normality.

Given that the ALSI index returns appear normally distributed, we can accept *as a working assumption* that the variances of share returns in a portfolio context are finite. This does not necessarily imply that all individual shares will conform to normality, but it does indicate that the stable Paretian distribution

is not a valid working assumption for South African data.

3.1.2 DIVERSIFICATION

In a previous section, we saw Markowitz's (1952) contribution to understanding the risk of a portfolio. Unfortunately, he (Markowitz 1952:89) asserts that, "[d]iversification is both observable and sensible", but does not prove this statement. It is the purpose of this section to show that diversification is both desirable and sensible.

We saw that the variance of a portfolio is a function of three factors, viz. the proportions of the stock invested in the portfolio, their variances and their covariances with other stocks in the portfolio. When one of these changes, the risk of the portfolio changes. Statman (1987:353) writes that, "it is generally true that when stocks are randomly selected and combined in equal proportions into a portfolio, the risk of a portfolio declines as the number of different stocks in it increases".

The proof of this is statement is taken from Elton and Gruber (1991:30-31). In the previous section we saw that the variance of a portfolio may be written as follows:

$$\sigma^2_p = \Sigma X^2_i \sigma^2_i + \Sigma X_i \Sigma X_j \sigma_{ij} \qquad (3.3)$$

In order to escape from the fact that by investing in different proportions of securities we can influence risk, we assume that equal investments in N securities is made. Equation (3.3) becomes:

$$\sigma^2_p = \Sigma(1/N)^2 \sigma^2_i + \Sigma(1/N)_i \Sigma(1/N)_j \sigma_{ij} \text{ for } i \neq j$$

By factoring the above equation, we can derive:

$$\sigma^2_p = (1/N)\Sigma[\sigma^2_i/N] + (N-1)/N\Sigma[\sigma_{ij}/N(N-1)] \qquad (3.4)$$

The terms in square brackets are averages, thus (3.4) can be further simplified to:

$$\sigma^2_p = (1/N)_a \sigma^2_i + [(N-1)/N]_a \sigma_{ij} \qquad (3.5)$$

where:

$_a\sigma^2_i$ = the average variance of the securities in the portfolio; and,

$_a\sigma_{ij}$ = the average covariance of the securities in the portfolio.

As N tends to infinity, so the average term of the securities in the portfolio tends to zero and the average covariance of the portfolio tends towards the average for all securities. On any stock market, there are not an infinite number of securities, so as N tends to the total number of securities so the variance of the portfolio tends toward the average covariance of the securities in the market.

It is possible to reduce risk through diversifying across securities and by diversifying across markets. Bhana (1987:74) states that, "[t]he true market portfolio should be a total-world portfolio to derive maximum benefits from diversification." If it can be shown that the relationship between domestic and foreign portfolios are characterised by low covariances, then it is rational for the investor to hold an internationally diversified portfolio.

Bradfield (1990:2) has calculated the correlation coefficients for the ten year period ending 1987, between the JSE and three of the major stock markets.

TABLE 3.5. STOCK MARKET CORRELATIONS (LONDON, NEW YORK AND TOKYO)

	JSE	LSE	TSE	NYSE
JSE	1.000			
LSE	0.181	1.000		
TSE	0.205	0.370	1.000	
NYSE	0.268	0.639	0.423	1.000

Source: Bradfield (1990:2).

Bhana (1987) investigated the potential benefits of international diversification to South African investors and concluded that, "South African investors are bearing a high cost for the inability to attain effective diversification as a result

Author Davidson S R
Name of thesis The capital asset pricing model and arbitage pricing theory on the Johannesburg Stock Exchange 1993

PUBLISHER:
University of the Witwatersrand, Johannesburg
©2013

LEGAL NOTICES:

Copyright Notice: All materials on the University of the Witwatersrand, Johannesburg Library website are protected by South African copyright law and may not be distributed, transmitted, displayed, or otherwise published in any format, without the prior written permission of the copyright owner.

Disclaimer and Terms of Use: Provided that you maintain all copyright and other notices contained therein, you may download material (one machine readable copy and one print copy per page) for your personal and/or educational non-commercial use only.

The University of the Witwatersrand, Johannesburg, is not responsible for any errors or omissions and excludes any and all liability for any errors in or omissions from the information on the Library website.