Using Design-Based Research to Develop a Professional Development Intervention for Grade 9 South African Teachers' Mathematical Problem-Solving Pedagogy

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A thesis submitted to the Faculty of Humanities, University of the Witwatersrand, South Africa, in fulfilment of the requirements of the degree of Doctor of Philosophy

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Declaration

I declare that this thesis is my original work. It is submitted to the Faculty of Humanities, University of the Witwatersrand for the degree of Doctor of Philosophy. It has not been submitted before for any other degree or examination in any other university. All the sources that I have used or quoted have been indicated or acknowledged by means of comprehensive references.

_____ _____

Signature Date

Brantina Chirinda

Dedication

I dedicate this thesis to my late sister Deliwe Chirinda, who lost her battle to aortic stenosis during the writing up of this thesis.

Publications and presentations emanating from this study

Journal publications

Chirinda, B. & Barmby, P. (2018). South African Grade 9 mathematics teachers' views on the teaching of problem-solving. *African Journal of Research in Mathematics, Science and Technology Education, (22)*1, 114–124

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Refereed Papers in Conference Proceedings

Chirinda, B. & Barmby, P. (2018). Peer observation as a tool to facilitate mathematics teachers' self-reflection in a professional development intervention. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 243-250). Umeå, Sweden: PME.

Chirinda, B. (2018). Task-based interviews in grade 9 mathematics: understanding South African learners' problem-solving processes. In R. Govender, & K. Junqueira (Eds.). *Proceedings of the 24th National Congress of the Association for Mathematics Education of South Africa* (Vol. 1, pp. 92-106). Bloemfontein, South Africa: AMESA.

Conference presentations

Short paper at SAARMSTE 2018 conference in Gaborone, Botswana entitled: *Why is design-based important in a Southern Africa Mathematics context?*

Poster presentation at 2018 EARCOME 8 in Taipei, Taiwan entitled: *Design, development, enactment, evaluation and redesign of a professional development intervention for teachers' mathematical problem-solving pedagogy.*

Short report at 2017 PME 41 conference in Singapore entitled: *Generation of principles for designing a professional development intervention for mathematical problem-solving pedagogy.*

Short paper at SAERA 2017 conference in Port Elizabeth entitled: *Using audio-recordings to assist teachers' self-reflection in a professional development intervention for mathematical problem-solving pedagogy.*

Short paper at 2017 AMESA conference in Port Elizabeth entitled: *Designing a professional development intervention to support teachers in the teaching of mathematical problem-solving.*

Short paper at SAARMSTE 2017 conference in Bloemfontein entitled: *South African Grade 9 mathematics teachers' perceptions of the teaching of problem-solving*.

Abstract

This study, which is comprised of two phases, employed a mixed methods design-based research (DBR) methodological approach within a pragmatist research paradigm to develop a professional development (PD) intervention that can be used to support grade 9 mathematics teachers in the teaching of problem-solving. This study is located in the broad area of designing and developing interventions that can improve the teaching and learning of mathematics in South Africa. The first phase of this study involved a review of relevant literature, context analysis and a baseline investigation with 31 grade 9 mathematics teachers at 20 public secondary schools within a certain district in Gauteng, South Africa. During the baseline investigation, participant teachers completed an open-ended questionnaire. The purpose of the baseline study was to investigate participant teachers' views, teaching strategies and the support they required in their teaching of mathematical problem-solving. The baseline investigation revealed that teachers in this district had a partial understanding of the teaching of mathematical problem-solving and were teaching problem-solving using 'traditional' approaches that did not seem to support problem-solving processes in learners. I reached a conclusion that there was a need to develop interventions to support the grade 9 teachers in this particular context in their teaching of mathematical problem-solving.

Based on the findings from the baseline study, I developed a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy in phase two of this study through two design-enactment-evaluation-redesign cycles. These cycles took place in two successive years with a total of four grade 9 mathematics teachers and 211 learners (cycle 1: two female teachers and 115 grade 9 learners; cycle 2: one female and one male teacher and 96 grade 9 learners). Each cycle was followed by data analysis and evaluation and refinement of the prototype.

I collected data through mixed methods data collection procedures. In this regard, I analysed qualitative data through inductive data analysis techniques with constant comparisons. I analysed quantitative data using the statistical package SPSS.

Although limited to a particular context, the main goal of this study was to develop a PD intervention that can be modified and used by the South African Department of Basic Education. Furthermore, this study aimed to generate design principles that can be used by other researchers, mathematics education practitioners and teachers developing PD interventions for mathematical problem-solving pedagogy. As a result, the design principles that emerged from this study include design principles for PD in general and design principles specifically for PD for mathematical problem-solving pedagogy. Design principles for PD, in general, are that: facilitators of PD must create a positive relationship with participant teachers before implementing the intervention; participant teachers should be used as a resource and must actively participate in the implementation of a PD intervention; teachers should be physically supported in their classrooms when they implement the new ideas; respecting participants is important in an African context when implementing the PD intervention; peer observation for teaching can be used as a PD strategy to support teachers' pedagogy; and a PD intervention should be responsive to the multi-lingual needs of a particular context. Design principles for PD for teachers' mathematical problem-solving pedagogy mainly comprise of the following: a baseline investigation must be conducted to establish teachers' views and practices on the teaching of mathematical problem-solving before implementing the PD intervention; PD activities should be built from teachers' experiences and their current mathematical problem-solving pedagogy; PD should be organised around collaborative problem-solving; and task-based interviews may be used to observe learners' mathematical problem-solving behaviour in detail to evaluate the impact of teachers' pedagogy on learners' problem-solving processes.

Key words: Design-based research, design-based principles, mathematics education, mathematical problem-solving, mathematical problem-solving pedagogy, mathematical word problems, professional development intervention.

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Last but not least, to God my Lord, you are my strength. I completed this thesis because of your grace.

Abbreviations and Acronyms

ANA: Annual National Assessment

CAPS: Curriculum Assessment Policy Statement

CK: Content Knowledge

DBE: Department of Basic Education

DBR: Design Based Research

DBRC: Design-based Research Collaborative

DEER: Design, enactment, evaluation and redesign

HOD: Head of Department

IQMS: Integrated Quality Management System

LoLT: Language of Learning and Teaching

MKfT: Mathematics knowledge for teaching

MPSKT: Mathematical problem-solving knowledge for teaching

MPSSI: Mathematical problem-solving skills inventory

NCTM: National Council of Teachers of Mathematics

PCK: Pedagogical content knowledge

PCTLA: Problem-centred teaching and learning approach

PD: Professional Development

POFT: Peer observation for teaching

SACMEQ: Southern African Consortium for Monitoring Educational Quality

SPSS: Statistical Package for the Social Sciences

TIMSS: Trends in International Maths and Science Survey

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CHAPTER 1: INTRODUCTION AND OVERVIEW OF THE STUDY

1.1 Introduction and rationale

Mathematics is an essential requirement for entry into South African universities and other tertiary institutions, and it is a 'critical competency for the development of sorely-needed high-level skills' (Centre for Development and Enterprise (CDE), 2014, p.1). However, South African learners continue to perform poorly in mathematics both on local and international tests of educational achievement (Reddy et al., 2016). South Africa as a country participates in a number of national and international tests of educational achievements and these mainly include; the Southern African Consortium for Monitoring Educational Quality (SACMEQ); Trends in International Maths and Science Survey (TIMSS); and the Annual National Assessments (ANAs). Analysis of learners' achievements in these tests shows that there is a recurring poor performance by South African learners. SACMEQ II (2000) and SACMEQ III (2007) showed that there was no improvement in South African grade 6 numeracy performance over the seven-year period. By the time the grade 6 learners get to grade 9, the poor performing gap would have widened resulting in learners being approximately 4 years behind (Spaull & Kotze, 2015).

In TIMSS (1995, 1999, 2003 and 2011), South African learners were rated lowest and in 2015 were second from last in mathematics among all middle-income countries that participated in the studies, even in comparison with countries that are economically poorer than South Africa (Mullis, Martin, Foy & Arora, 2012; Reddy, 2006; Reddy, Zuze, Visser, Winnaar, & Juan, 2015; Reddy et al., 2016; Spaull, 2013). TIMSS is an international test of educational achievement for mathematics and science meant for grade 8 learners in various countries. However, it was decided that the tests were extremely challenging for grade 8 South African learners, resulting in grade 9 learners writing the tests in 2011 and 2015. The 2011 TIMSS results disclosed that in terms of learning, an average South African grade 9 mathematics learner is two years behind the

average grade 8 mathematics learner from 21 other middle-income countries. Consequently, I focused on grade 9 teachers and learners for this study.

In 2012, the South African Department of Basic Education (DBE) implemented the Annual National Assessments (ANAs) as a national standardised test of educational achievement. In 2013, the ANA results revealed that the performance levels in mathematics 'declined across the grades with progressively steeper declines from grade 6 to 9' (DBE, 2014, p.15). Furthermore, the ANA results highlighted that the Grade 9s were the worst performers. The ANAs have been discontinued since 2016 and the Minister of Basic Education announced in 2017 that they will be substituted by the National Integrated Assessment Framework (NIAF).

The DBE (2014) states that South African learner achievements in mathematics are still at an unacceptable level. This is echoed by Siyepu (2013) who posits that South African learners' achievement in mathematics is appalling. It is well-known that the bulk of South African learners perform poorly in Mathematics (Pournara, Hodgen, Adler and Pilay, 2015) and this crisis has existed for more than two decades (van Jaarsveld & Ameen, 2017). After the release of the ANA 2013 results, a diagnostic report was generated by the DBE showing that some of the numerous challenges learners experienced in certain mathematical topics were as a result of 'ineffective teaching methods' (DBE, 2014, p.7). Feza-Piyose (2012) agrees with this finding and affirms that quality of instruction is one of the contributing factors to the poor performance in mathematics by a majority of South African learners. Taylor, Van der Berg and Mabogoane (2013) echo the same sentiments by reiterating that the low achievement of South African mathematics learners can be blamed on the knowledge of practice by mathematics teachers. A number of studies (Adler & Ronda, 2014; Mji & Makgato, 2006; Mullis et al., 2012; Taylor & Vinjevold, 1999) have also outlined numerous flaws in the teaching and learning of mathematics in South Africa. In this regard, the South African Department of Education's Curriculum and Assessment Policy Statement (DBE, 2011), like other curricula (National Council of Teachers of Mathematics (NCTM), 2000; Ministry of Education Singapore, 2013; Realistic Mathematics Education (RME) of the

Netherlands, 1971), now considers problem-solving as an essential aspect of mathematics teaching and learning.

Many researchers agree that problem-solving is an important goal of mathematics teaching and have accentuated its role in improving learners' achievement in mathematics (Carpenter, Fennema, Peterson, Chiang, & Leof, 1989; Mji & Makgato, 2006; Liljedahl, Bruder, Santos Trigo, & Jurado, 2016; Schoenfeld, 1992; Polya, 1957, 1962; Posamentier & Krulik, 2008). Carpenter et al. (1989) contend that when the emphasis is positioned on problem-solving in the learning of mathematics then the following are likely to occur:

i) learners' attention on ideas and sense-making increases;
ii) there is liberty by learners to construct own strategies;
iii) mathematical problem-solving skills are enhanced; and
iv) learners' cognition increases.

NCTM (2000) identifies mathematical problem-solving as one of the five fundamental process standards. In addition, Fan and Zhu (2007) note that this is considered as part of the core of mathematics teaching and learning by the Singaporean curriculum. The NCTM (2000) and Schoenfeld (2004) recommend that mathematics should be taught from a problem-solving perspective. The DBE (2011) resonates with these statements and states that problem-solving and cognitive development should be fundamental to all mathematics teaching. 'Learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge later in their lives' (DBE, 2011, p.8). However, this is not what a number of researchers have found in South African mathematics classrooms. Jagals and van der Walt (2016) observe that mathematical problem-solving is limited in South African national examinations, classroom activities and formative assessments and Hargrove and Nietfeld (2014) perceive that this has resulted in learners' poor performance in mathematics. Taylor and Vinjevold (1999) found that some South African mathematics teachers spend a whole lesson doing problems that simply require the application of algorithms and hardly teach genuine problem-solving. Chisholm et al. (2000) lament

that South African classrooms have remained teacher-centred. Mji and Makgato (2006) further postulate that outdated teaching practices like rote learning have caused poor teaching standards in South Africa. Adler and Ronda (2014) highlight that South African teachers tend to implement traditional approaches in the classroom. The teachers start the learning day by revising the previous day's homework, demonstrating the new work with an example, and giving the learners some exercises to do. These findings seem to indicate that mathematical problem-solving has not been fully embraced in South African schools despite being emphasized in the CAPS curriculum.

In response to these identified problems, I designed a professional development (PD) intervention for grade 9 teachers' mathematical problem-solving pedagogy. In brief, a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy is a framework that can be modified and used by the South African DBE and other mathematics education practitioners to support grade 9 mathematics teachers in teaching problem-solving.

At this point, it is important to mention that, although at the beginning of the study I started with a perspective of developing a PD intervention for mathematical problem-solving pedagogy related to how problem-solving is stated in the literature (see section 2.3), I ended up developing a PD intervention tailored to the way teachers in the context of study viewed mathematical problem-solving. Out of the literature review (chapter 2), context analysis (section 1.2) and baseline investigation (chapter 4) I realised that the best pathway to answering the research questions, designing a relevant PD intervention for grade 9 teachers' mathematical problem-solving pedagogy and generating significant design principles was to take teachers' context and their views on mathematical problem-solving into greater consideration. I did not go to the teachers' context and tell them what to do, but I introduced new ideas which teachers adopted according to their context. In reality, I developed a PD intervention that can be used to support grade 9 mathematics teachers to teach word problems. I did not find open-ended tasks, which are usually used during genuine mathematical problem-solving relevant; as teachers had to eventually teach within the CAPS curriculum which has word problems.

The context of study was a crucial aspect of this study. As a result, I engaged DBR to design and develop a PD intervention that was relevant to the participant teachers. DBR as a research design was appropriate for this study since at the beginning I did not have in mind an example of a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy, but wanted to develop one that was sensitive to participant teachers' context. DBR is a flexible methodology (Wang & Hannafin, 2005) and this uniqueness gives it high a probability of generating educational interventions that are sensitive to the context (Crepon, 2014). In the next section, I discuss the context of study, which was an important aspect of this study.

1.2 Context of the study

In this section, I present the context in which this study was done. From the literature review, I realised that many researchers interpret the term context differently and Durant and Goodwin (1992) are correct when they perceive that:

> …it does not seem possible…to give a single, precise, technical definition of context, and eventually, we might have to accept that such a definition may not be possible…the term means quite different things within alternative research paradigms…(p.2)

In this study when I say 'context', I mean the many factors surrounding the teaching and learning educational environment and these include physical aspects (schools, classrooms and resources); the curriculum which comprises objectives of teaching and learning; quality of teaching and learning; and perceptions, views and beliefs of teachers. I conducted a context analysis simultaneously with the baseline investigation in March 2016. I conducted them for three reasons. The first reason was that the context within which teachers and learners teach and learn mathematics usually shapes the pedagogic style of teachers and how learners think and do mathematics. The second reason was hinged on developing a better understanding of the negatives and positives to the implementation of the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy. The third reason was for identifying areas that needed attention in order to generate some useful ideas for the design-enactment-evaluation-redesign (DEER) cycles. I describe the context in this section and return to

certain aspects throughout the thesis. Furthermore, I describe in detail the participant teachers and method of selection of participant teachers and their schools in section 1.6.2 and chapters 3, 4, 5 and 6.

I conducted this study at public South African secondary schools positioned in informal settlements. Informal settlements are the areas where people build their own temporary housing structures without the approval of local government. The South African government has often upgraded these settlements to proper residential townships by providing infrastructure. Before 1994, the system of apartheid that existed in South Africa caused large gaps between the rich and poor (Wilson & Ramphele, 1989). This system was created to ensure inequalities between Whites, Coloureds, Indians and Africans. The Merriam dictionary defines apartheid as a former South African policy of segregating and economically and politically oppressing the non-white population. During the apartheid era, only whites had democratic rights and were allocated basic services such as health, education and other crucial and necessary services. Apartheid created huge inequalities resulting in blacks being the poorest amongst all the groups. After 1994 (post-apartheid), the poorest black people resorted to staying in informal settlements since they could not afford to pay for rent, electricity and other basic services.

I worked in functional schools in informal settlements in the district under the study. Functional schools are schools where there is order as compared to dysfunctional schools where there is chaos and the principal has lost control of the school (Mawdsley, Bipath, & Mawdsley, 2014). In terms of teaching and learning environment, the analysis revealed that the classes were multi-lingual, large, overcrowded, resource-deficient in terms of technology and furniture, and with too many needy learners. Needy learners are those that come from poor homes, have or experience sexual harassment or abuse or have or experience violence at home or on their way to school. In this study, when I say multi-lingual I mean instances where learners speak more than two different languages in one class. The schools did not have enough furniture and any computers or WiFi for learners to do research. Most learners came from poor homes where there were neither desks nor electricity to do homework after school.

The legacy of apartheid extended into the education system even though the South African constitution (section 29) states that everyone has the right to basic education. South Africa continues to be among countries with the highest index of educational inequality in the world (Fleisch, 2008). During apartheid a majority of the white teachers trained at universities whilst most black teachers trained in the defunct and poorly staffed and resourced teachers' colleges. The government has in the past rolled out teacher PD courses with the objective of redressing the legacy of apartheid schooling and apartheid teacher education, but there is hardly any evidence of these courses' impact on learners' achievement in mathematics (Pournara et al., 2015). Reddy et al. (2015, p.6) while commenting on TIMSS (2015) results echo that:

> The predominant response is that very little has improved in the South African education landscape since 1994 and that resources are going to waste. The simple ranking of countries from top to bottom performers generally lends support to this perspective. As part of this narrative, South Africa's expenditure on education is contrasted with that of top-ranking developing countries that appear to be making more rapid progress. It is argued that South Africa has abandoned any educational aspirations and is effectively in a race to the bottom of virtually every educational ranking. (p.6)

The basic education received by learners in private schools or former model C schools is mostly better than the one at non-fee paying public schools in townships or informal settlements. Township schools are located in high-density areas that were situated outside the major cities during the apartheid era. Private schools are not part of the government-run school system and are usually found in the fancy and posh suburbs of South Africa. Former model C schools are former whites-only government schools during the apartheid era. These former model C schools are in rich areas of South Africa and because of the Groups Areas Act of 1950, only the affluent had access to these schools. With the Groups Areas Act of 1950 learners only had access to schools in the area they live in, and this implies that gaps created by the apartheid era still continue to exist up to today. Private schools or former model C schools' classes usually have less than 30 learners as compared to the usually more than 50 learners at government-run schools. The impact of the school type is revealed in the patterns of achievement in mathematics (Reddy et al., 2015). To close the inequality gap in

performance between the rich and poor, the government provides textbooks to all public schools. From my assessment, most textbooks provided by the DBE are of poor quality and are hardly used by the teachers and learners. All of the chapters in DBE textbooks start with a brief explanation followed by a set of routine exercises and more exercises at the end of the chapter with one or two word problems. At the two schools, where I conducted the first and second cycles of this study, the government DBE textbook boxes were left unopened (see Figure 1.1) and teachers preferred to use other textbooks like Classroom Mathematics or Platinum.

Figure 1.1: DBE textbooks which were left unopened at a public school in Gauteng.



Textbooks like Classroom Mathematics are well-written, but do not identify with the lived experiences of the black child. These textbooks are usually written by people with the middle-class learner in mind and do not respond to the black learners' context, therefore the disadvantaged learners are forced to understand the textbook out of their context.

From the context analysis, it was evident that the curriculum was strictly centralised, overloaded and teachers worked under pressure to cover a lot of material within the prescribed time. From the baseline investigation and pre-observation sessions, I could

not see the teaching of problem-solving in mathematics classrooms. The reality was that teachers ran out of time to implement mathematical problem-solving pedagogy because of the time constraints due to covering the CAPS curriculum and probably because of attitudes that teachers are known to have towards learners with low social economic status (SES). South African teachers tend to use 'traditional' methods of teaching that include memorization for low SES learners and problem-solving with high SES learners (Hoadley & Ensor, 2009).

Teachers in the district of study taught procedurally because they thought learners did not understand since they struggled with the language of learning and teaching (LoLT) and failed to execute basic mathematical problem-solving processes and skills. In this context, the teachers' views on mathematical problem, mathematical problem-solving and mathematical problem-solving pedagogy differed from what the literature states and I further discuss this in section 2.3 when I give the definitions of these terms.

According to the 2011 South African census, 9.6% of the South African population listed English as their home language (Statistics South Africa, 2011) which implies that a large population of learners is learning in a language that is not their mother tongue. English is the language of economic power (Setati, 2014) and it gives access to resources and opportunities. Understanding such inequalities that are brought by language and other aspects is important to understanding the context in which education is learnt (Taylor & Vinjevold, 1999) and in the case of this study, in which mathematical problem-solving was being taught and learnt. LoLT is noted as a factor that contributes to South African learners' poor performance in mathematics which has its own intricate language (Setati, 2008; Setati, Chitera, & Essian, 2009; Setati Phakeng & Moschkovich, 2013). Learners are confronted daily with the new complex terminology of mathematics at the same time with the complicated LoLT they struggle with.

1.3 Significance of the study

There is a dearth of literature in Southern Africa pertaining to the designing of PD interventions to support mathematics teachers in the teaching of mathematical problem-solving. This study which is located in the broad area of designing and developing

interventions that can improve mathematics teaching and learning in South Africa is the first empirical study to be done in South Africa in an attempt to design and evaluate a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy. As I have indicated before, the CAPS values the teaching of mathematical problem-solving. However, before this study, there was not a PD intervention in South Africa that could be used to support mathematics teachers in the teaching of problem-solving; therefore the key significance of this study. The major two outcomes of this study are (i) the development of a PD intervention that can be modified and used by the South African DBE and other mathematics education practitioners to support teachers in teaching mathematical problem-solving and, (ii) generation of design principles that can be used by other researchers developing PD interventions for teachers' mathematical problem-solving pedagogy. This study contributes to the mathematical problem-solving pedagogy and PD body of knowledge by working with teachers in an under-researched environment, that is, an informal settlement in Gauteng, South Africa.

The other significance of this study is that I designed a PD intervention to support teachers' mathematical pedagogy because there is evidence that PD programmes that focus on problem-solving enable teachers to become effective problem-solvers and develop their knowledge of mathematics regardless of their prior mathematical ability or problem-solving pedagogy (Guberman & Leikin 2013). At the beginning of the study, I intended to design a PD intervention that would accomplish a valued pedagogical goal and I believe this objective was achieved since I designed a PD that is relevant to teachers in the context of study.

A few South African PD programs include mathematics pedagogy and instruction (Reddy, 2006) and most PD interventions revolve around content, curriculum and assessment and not instructional challenges faced by teachers daily (Nel & Luneta, 2017). This study looked at PD for grade 9 mathematics teachers' pedagogy in problem-solving, thereby filling in the gap in the literature indicated by Nel and Luneta (2017) and contributing to policymaking and practice in the field of mathematics education and teacher PD in South Africa.

The other significance is that I wanted to demonstrate that designing educational products using DBR is possible in the context of South African schools. DBR is further explained in sections 1.4, 1.5, and 7.8 and chapter 3.

Cobb, Jackson and Dunlap (2016) note that over the years PD studies that use DBR as an approach have been focusing on teachers' involvement in the PD workshops, while they rarely focused on changing teachers' teaching practices. This study focused on teachers' participation in the PD workshops as well as supporting and changing teachers' mathematical problem-solving pedagogy, thereby filling this identified gap in the literature. This is a further noteworthy motivation for the research focus which is clearly dedicated to making a direct contribution to DBR PD studies that focus on supporting and changing teachers' mathematical problem-solving pedagogy.

1.4 Scope of the study

The motivation of the research focus emanated from when, as a mathematics teacher educator, I discovered that the South African CAPS curriculum emphasises the teaching of mathematical problem-solving; but there are few programmes in place that explicitly support teachers in teaching problem-solving or cater for ways to develop mathematical problem-solving processes in learners during lessons. In March 2016, I conducted a baseline investigation in the district of interest to explore participant teachers' views, teaching strategies and the support they required in their teaching of mathematical problem-solving (see chapter 4). The baseline investigation revealed among other issues that teachers were not teaching mathematical problem-solving because of time constraints due to covering the CAPS curriculum and many of them did not understand the problem-solving process and confused it with teaching mathematical word problems. From these findings, I realised that there was a need to design a PD intervention that can be used to support grade 9 teachers' mathematical problem-solving pedagogy. In chapter 3, I explain in detail the reasons for choosing grade 9 teachers and learners as participants.

This study was set within the overall framework of the DBR and as indicated earlier its main objective was to design and develop a PD intervention for grade 9 teachers'

mathematical problem-solving pedagogy. In addition, another objective of the study was to generate usable theoretical knowledge (Lagemann, 2002) that can be used by other practitioners in similar settings (McKenney & Reeves, 2012) who may be facing similar problems or studying analogous settings and concerns (Amiel & Reeves, 2008). Based on the baseline investigation findings, I developed a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy through two iterative cycles of DEER. I found DBR appropriate for this study because it targets solving significant problems facing the educational field by putting knowledge to use (McKenney & Reeves, 2012) and it allowed me to be connected to the research problem in its naturalistic settings, that is, schools with classrooms, teachers, and learners. Context was a crucial aspect of this study and engaging DBR allowed participant teachers to develop ideas relevant to their context and also whatever teachers did in this study was located in their context.

DBR PD studies are characterised by a researcher working and supporting teachers in their development of new pedagogical knowledge and practice (Cobb et al., 2016). My role was epitomised as that of a researcher, facilitator of the PD workshops, developer and evaluator of the PD interventions and a critical friend of the participant teachers. I further explain these roles in section 3.5. I did not impose successful teaching methods from other countries. I began with the teachers' experiences and how they were teaching problem-solving and employed the DBR approach to develop the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy relevant to the context of study. In this way, I took into consideration what were the teachers' current teaching practices as a resource for their learning and PD (Cobb et al., 2016). During lesson delivery, I encouraged teachers to act as facilitators of learning and to allow learners to actively construct knowledge individually from their prior experiences or collaboratively with their peers through discussions.

Traditionally, PD in mathematics education has normally been delivered in the form of workshops, college courses, seminars or conferences (Knight, Tait, & Yorke, 2006; Feiman-Nemser, 2001; Villegas-Reimers, 2003), and this has ultimately mirrored the traditional teaching of mathematics (Zaslavsky, Chapman, & Leikin, 2003). Zaslavsky et

al. (2003) postulate that historically, PD in mathematics education's main objective has been to transmit information, ideas, skills and techniques to teachers and not to help them to develop new forms of practice. These transmission approaches have been fiercely criticized for their ineffectiveness since they are not directly related to an individual teacher's practice (Hawley & Vali, 1999) and are detached from the teachers' actual practice and naturalistic settings. Hunzicker (2010) observes that traditional 'one-shot' and 'sit and get' approaches to PD are less effective, insufficient and unsuitable in the current educational reform environment. Hunzicker (2010) further notes that teachers forget and rarely apply in their lessons what they learn from these workshops. It is, therefore, important to design, develop and implement context-appropriate teacher PD interventions (Garaway, 2003) that form teachers' everyday professional life. For PD to be effective and successful it must be moved to the classroom (Barber & Mourshed, 2007) and must include teachers in the design and implementation of the interventions (Du Preez & Roux, 2008). This is the reason why I engaged the process of DBR; to collaboratively work with teachers in their classrooms and to tailor a PD intervention that would be appropriate for teachers in their particular local context.

The workshops in the PD intervention were different from the traditional 'one-shot' workshops in that the teachers attended three workshops; collaboratively solved word problems and worked on activities; were actively engaged in meaningful discussion, planning and practice; and learned from each other during the PD intervention (Loucks-Horsley et al., 2010). I observed and supported the teachers during the implementation phase. In this instance, I selected PD activities that offered teachers the opportunity to become actively engaged in meaningful analysis of mathematics teaching and learning. These activities are elaborated on in section 3.10

Polya's problem-solving steps (see section 2.3.4) were the backbone of this study. During the PD workshops, I worked with teachers on how to use Polya's steps in their teaching and solving of word problems. A demonstration of how I discussed and solved the problems with the teachers in the PD workshop can be found in section 5.3.2. I required teachers to implement the problem-solving pedagogy in their lessons according to Polya's steps and this is delineated in activity 3 in section 3.10. In the next

section, I describe DBR and give an overview of the data collection procedure of the baseline investigation and cycles 1 and 2.

1.5 Research Design

1.5.1 Design-based research

DBR is described in detail in chapter 3 and in this section, the adopted definition is explained and its appropriateness to my short-term PhD study is clarified. DBR is also known as design experiments (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003), development research (van den Akker, 1999), design research (Reeves, Herrington & Oliver, 2005), formative research (Newman, 1990) and educational design research (McKenney & Reeves, 2012) and is a relatively new research approach in the field of educational inquiry (Anderson & Shattuck, 2012). In this study, I prefer to call it DBR instead of any of the terms mentioned above because I discovered that this term is frequently used in the literature.

There is increasing recognition around the world of DBR as part of evidence based educational research (DBRC, 2003). Many authors have given a definition of DBR and agree that DBR is a research design that consists of a number of cycles (Barab & Squire, 2004; DBRC, 2003; Cobb et al., 2003; McKenney & Reeves, 2012; Reeves, 2006; Van den Akker et al, 2006; Van den Akker, 1999; Wang & Hannafin, 2005). For this study, I draw on Wang and Hannafin's (2005) definition which captures the overall idea, philosophical approach and procedures of DBR from different perspectives and encapsulates the key aspects of DBR that comprise this study. Wang and Hannafin (2005) define DBR as:

> a systematic but flexible methodology aimed to improve educational practices through iterative analysis, design, development, and implementation, based on collaboration among researchers and practitioners in real-world settings, and leading to contextually-sensitive design principles and theories. (p.6)

This definition implies that when one engages DBR as a methodological approach then theory development is fundamental to the research process. DBR has a 'theory-driven nature' (Wang and Hannafin, 2005, p.9) in that the designing of educational materials is interwoven with development or testing of theories (Baker & van Eerde, 2013). This makes it germane to the Southern African mathematics education context since there is a need for unique solutions and theories relevant to the educational contexts which may be very different from Western or Asian contexts. At this point, I find it appropriate to indicate that theories developed in DBR are humble and difficult to generalise in that they are 'tied to specific learning environments and learning goals' (Baker & van Eerde, 2013, p.13). Humble means that theories generated from DBR are not grand like Piaget (1929) or Newton's theories but are domain specific. DBR was relevant for this study because, as I indicated earlier, my specific interest was to design and evaluate a PD intervention on mathematical problem-solving pedagogy for grade 9 mathematics teachers in a certain district in Gauteng. I did this by collaborating with grade 9 mathematics teachers (practitioners) in their real-world settings.

DBR is a long-term approach which contains multiple cycles of design, enactment, and redesign (Amiel & Reeves, 2008), and can be challenging to use during a master's degree or PhD study (McKenney & Reeves, 2012). Pool and Laubscher (2016) argue that DBR must not be limited to long-term projects only, but researchers like graduate students, who have a short time-frame, can also engage DBR in their studies as long as they are able to follow the appropriate phases which can produce suitable interventions and generate relevant and effective design principles.

A number of researchers have successfully engaged DBR in short-term projects in Southern Africa. Firstly, Kizito (2012) used DBR to gain a deeper understanding of the extent to which Realistic Mathematics Education (RME) can be used as an instructional design theory for planning and introducing a distance teaching Calculus unit. Secondly, Mafumiko (2006) used DBR to design and evaluate a Micro-scale Chemistry Experimentation (MSCE) method that can help Tanzanian secondary school teachers in implementing practical work to promote an active learning environment in Advanced level chemistry classes, with low demand on equipment and chemicals. Thirdly, De

Villiers and Harpur (2013) engaged DBR to develop and evaluate a mobile-learning application (m-LR) for a South African tertiary education context and at the same time produced theoretical contributions to the body of educational technology. Fourthly, Pool (2014) successfully employed DBR to produce innovative interventions and significant design principles that offer valuable information for Higher Education practitioners that are in the process of adopting a blended learning approach in their teaching and learning. This study is a short-term PhD research project. Design principles were successfully generated (see 5.7 and 6.7) and a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy was designed through two iterative cycles of DEER. I had intended to design and conduct three cycles for this PhD study, but because of the time and resources, I conducted only two cycles.

1.5.2 Data collection procedure

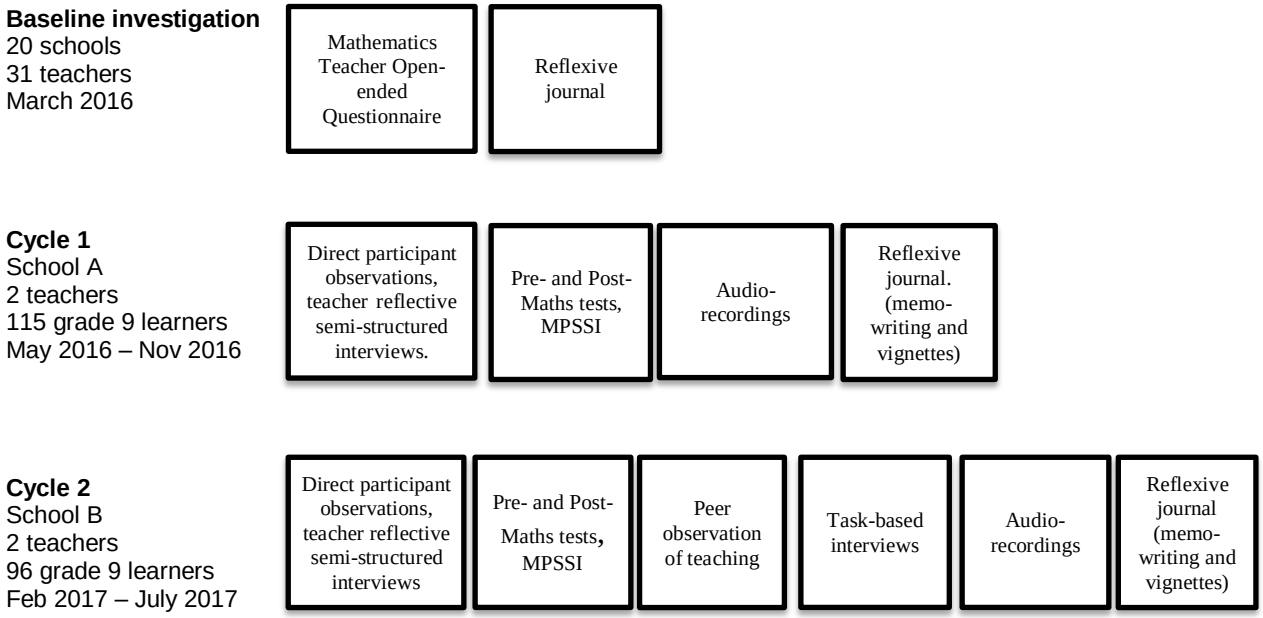
In this section, I give a synopsis of how data was collected in this study (see Figure 1.2). After identifying the need to design a PD intervention to support grade 9 teachers in teaching mathematical problem-solving pedagogy, the first phase involved a literature review (see chapter 2), context analysis (see section 1.2) and a baseline investigation (see chapter 4). The second phase involved two cycles of DEER of prototypes (see section 3.10 and Figure 3.2).

I employed a Mathematics Teacher Open-ended questionnaire to conduct the baseline investigation to establish how grade 9 mathematics teachers in the district of interest were teaching problem-solving. At the outset of the research, I kept a reflective journal where I documented memos, vignettes, my thoughts and personal reflections. Throughout the research process, I wrote extensive memos to myself in the reflective journal about the research process. I gathered qualitative data in both cycles using classroom observations, teacher semi-structured reflective interviews and learner task-based interviews (in cycle 2). I audio-recorded the classroom observations, reflective interviews and task-based interviews with participants' consent (see chapter 3).

During the first month of each of the cycles in schools A and B, as I was engaging in the process of building trust with participant teachers, I informally observed them so as to

unearth how they were teaching mathematical problem-solving. Uncovering how participant teachers were teaching problem-solving illuminated how I designed the PD intervention.

Figure 1.2: Data collection procedure for the study.



In order to get first-hand experiences when teachers implemented the problem-solving pedagogy, I observed them delivering lessons and recorded information, with their permission, on the spot as it occurred on the observation comment card (see Appendix O). After the PD workshops, I encouraged the participant teachers to implement the new ideas in their lessons. During the implementation phase, I carried out classroom observations to see if teachers were changing their teaching practices and to establish the difficulties that occurred in the classrooms. During the observations, I also supported the teachers and provided them with guidance as was necessary.

During each cycle, I held 20 minutes long semi-structured interviews (five in the first cycle and three in the second cycle) with participant teachers during their free periods. I explain in greater detail the procedure for conducting the semi-structured interviews in sections 3.8.1 and 5.4.2. I introduced learner task-based interviews in the second cycle to gain insight into the impact of the PD intervention on learners' problem-solving processes. In terms of assessing the possible impact of the intervention on learners, I

gave them pre- and post-mathematics attainment tests and a self-reporting mathematical problem-solving skills inventory (MPSSI) at the beginning and end of the intervention of both cycles.

The following are the participants for this study and their demographic details are given in detail in chapters 4, 5 and 6. 31 teachers from 20 different schools participated in the baseline investigation. In the first cycle, two female teachers and 115 grade 9 learners ranging between 14 and 16 years participated. In the second cycle, one female and one male teacher and 96 grade 9 learners ranging between 14 and 16 years participated in the study. I chose four of the learners in the second cycle to participate in the task-based interviews. The four learners were of diversified abilities and included the best, average and low-performing learners.

1.6 Research objectives and research questions

The main purpose of this study was to design and evaluate a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy relevant in a particular local context and to articulate design principles that can be used by other researchers, mathematics education practitioners and teachers developing PD interventions. Findings from this study generated usable knowledge to guide other mathematics education PD practitioners. The following were the research objectives of the study:

- Establish participant teachers' views, teaching strategies and the support required in their teaching of mathematical problem-solving in a particular local context.
- Design an effective PD intervention for grade 9 teachers' mathematical problem-solving pedagogy relevant in a particular local context.
- Explore the learning of participant teachers and learners from participating in the PD intervention.
- Examine and evaluate the potential impact of the PD intervention.

- Generate design principles that can be used to develop a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy in a particular context.

The research was guided by the following research questions:

1. What are grade 9 South African mathematics teachers' views on the teaching of problem-solving?
2. What are the design principles required to generate a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy in a particular local context?
3. What is the impact of the PD intervention on grade 9 teachers' mathematical problem-solving pedagogy in this particular context?
4. What is the impact of the PD intervention on grade 9 learners' mathematical problem-solving processes in this particular context?
5. What factors enabled the growth of teachers' mathematical problem-solving pedagogy in this particular context?
6. What factors facilitated learners' development of mathematical problem-solving processes?

1.7 Outline of chapters

This thesis consists of 8 chapters and I briefly describe them below.

Chapter 1: Introduction and overview of the study

In this first chapter, I introduce the reader to the research problem of South African mathematics learners' poor performance in local and international tests of education achievement. The introduction, rationale, context, significance and scope of the study are presented. Overviews of DBR and the data collection procedure of the study are given. The research problem is formulated and justified, and from it I derive the

objectives and research questions of the study. I conclude this chapter by presenting the outline of the chapters.

Chapter 2: Literature review and theoretical framework

In chapter 2, I give detail of the theoretical framework on which this study is grounded in. I present the literature review relating to the focus of the study. The literature review includes aspects of mathematical problem-solving, mathematics teachers' knowledge for teaching and mathematics teacher PD.

Chapter 3: Research design and methodology

In chapter 3, I outline and discuss the methodology and research design employed in the study. The rationale for selecting the DBR and mixed methods research approach is elaborated on. The sampling of participants, when and where the data was collected, procedures and techniques for data collection (Mathematics Teacher Open Questionnaire, direct participant observations, teacher reflective semi-structured interviews, learner task-based interviews, MPSSI, and pre- and post-word-problem attainment tests) and data analysis are explicated. Justifications of the data collection strategies are stated and issues of validity and reliability are addressed to a larger extent. The association between the research questions and the research methods employed to address the specific research sub-questions are clarified. The ethical procedures adopted by this study are also detailed.

Chapter 4: South African Grade 9 mathematics teachers' views on the teaching on problem-solving: The baseline investigation

In this chapter, I report on the baseline investigation which I conducted to establish Grade 9 teachers' views on the teaching of mathematical problem-solving. I discuss the participants, data collection and analysis and findings of the baseline investigation.

Chapter 5: Design, enactment, evaluation and redesign of the first cycle

In this chapter, I report on the design and findings of the first cycle of the study. Moreover, I expound on the participants and data collection and analysis procedures

and the findings of the first cycle and their implications for the second cycle. Using Guskey's (2000) Five Levels of PD Evaluation, I evaluated the PD prototype developed from this cycle. The aim of the evaluation was to establish if the prototype solved the formulated problem.

Chapter 6: Design, enactment, evaluation and redesign of the second cycle

In this chapter, I report on the design and findings of the second cycle of this study. I explain the participants, data collection and analysis procedures and the findings of the second cycle and their implications for the third cycle.

Chapter 7: Discussion

In this chapter, I discuss and synthesise the findings of the study as a whole. Context emerged as a big theme for this study. Besides context, other themes that emerged gradually during the study and are related to context are also discussed.

Chapter 8: Conclusion and recommendations

Chapter 8 is the concluding chapter and summarises the whole study. Conclusions based on the findings are discussed and interpreted with the orientation to the research objectives that are delineated in chapters 1 and 3 and related to the broader literature on teachers' mathematical problem-solving pedagogy, PD of teachers, and DBR in PD and mathematics education. Limitations of the study, recommendations for practice and areas for possible further exploration are also discussed.

1.8 Reflecting on this chapter

In this chapter, I present the rationale, scope and context of the study. DBR is introduced as the research design of the study and is further explained comprehensively in chapter 3. The purpose of the study, the research aims and research questions are specified and the outline of the chapters is given. In the next chapter, the literature review and the theoretical framework are elucidated.

Chapter 2: Review of literature

2.1 Introduction

Given that this study focused on designing and evaluating a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy, the literature review focused on three elements: what content, structure and instructional approaches best meet the objectives of the intervention as informed by the context in which I designed, enacted, tested and redesigned the PD intervention. The literature on mathematical problem-solving, teacher PD, teacher knowledge for teaching mathematics and mathematical problem-solving was expounded on. The understanding of mathematical problem-solving processes, mathematical problems, the teaching of mathematical problem-solving, factors that influence the quality of mathematical problem-solving pedagogy, teacher knowledge in mathematics and mathematical problem-solving and effective PD in general and specifically in mathematics education were imperative in shaping this study. Reviewing of literature also generated design principles that I used in the designing of the first prototype in cycle 1 (see section 5.2).

In this chapter, I start by explaining the theoretical perspective that underpins this study followed by the exploration of mathematical problem-solving in section 2.3. In section 2.4, I outline the aspects of teacher knowledge for mathematics and mathematical problem-solving pedagogy. Effective PD in teacher mathematics education is delineated in section 2.5. Finally, I give the synthesis of the literature review in section 2.6. Throughout this chapter, I identify the gaps in knowledge of PD in mathematics education, mathematical problem-solving and mathematical problem-solving pedagogy.

2.2 The theoretical framework

A theoretical framework is a group of interrelated ideas or set of theories which can be used to discover answers about a research problem at hand. It provides guidance to the study and identifies a framework for the investigation and interpretation of the findings. Underpinning the ideas discussed in this study is a social constructivist view of learning.

The social constructivist perspective to learning mainly originated from the work of Vygotsky (1978) and other theorists like Brown, Collins and Duguid (1989), Kuhn (1970), Ernest (1991) and Lave and Wenger (1991). This theory stresses the importance of social processes in an individual's acquiring of knowledge (Ernest, 1991). Social constructivism emphasises that culture and social contexts are important in understanding what occurs in the society and acquiring knowledge based on this understanding. Vygotsky (1978) argues that human interactions are a key factor in an individual's development and learning. Therefore, for meaningful learning to occur and for knowledge to be constructed, individuals need to participate in social activities. This study viewed each participant teacher as a unique individual; with unique needs and backgrounds that assist them to construct their own realities individually and socially through personal interactions with the surroundings and collaborations with others, involving reflections and adaptations (Zaslavsky et al., 2003). Participant teachers collaboratively worked on activities during PD workshops and implementation of the mathematical problem-solving pedagogy (see section 3.10). In the second cycle of the study (see section 6.3.2) teachers participated in peer observation for teaching (POFT) which allowed them to offer each other mutual support by observing each other during the implementation of the mathematical problem-solving pedagogy.

In this study, I considered learning to be situation-specific, and a context-bound activity (Woolfolk, 2007) and that participant teachers mutually build and construct their knowledge. Learning in this context is not something that is pumped into an individual as the result of teaching, but is a component of people's everyday activities as they become involved, engaged and reflect on their activities at work. This study took place in an under-resourced Southern African context, where the teachers were working with large classes, taught learners with English as a second language and the teachers had a wealth of experience of teaching mathematics based on their own cultural and social experiences. Drawing from the above discussion, I incorporated and emphasised the following assumptions in this study:

- Knowledge emanates from social interaction.

- Knowledge is socially and culturally constructed and is influenced by the group and its context.
- Learning is a social activity. This implies that meaningful learning occurs when individuals are engaged in social processes and activities. In this study, by interacting with others, teachers or learners shared their views and constructed a shared understanding connected to the concept of mathematical problem-solving pedagogy that was under discussion (see section 7.5).

The major motivation of adopting the notion of social constructivism as a theoretical perspective was that I structured this study in such a way that teacher social participation, reflections, and experiences evoked their learning of mathematical problem-solving pedagogy in their particular contexts. Teachers were required to share their experiences with other participant teachers and collaboratively learn from each other during the PD workshops, implementation of mathematical problem-solving pedagogy in their lessons and POFT. In each PD workshops, participant teachers spent considerable time (approximately 2 hours) interacting, investigating, explaining and discussing mathematics problems and mathematical problem-solving pedagogy ideas as well as solving mathematical problems with their colleagues.

In the next section, I discuss mathematical problem-solving. As I discussed earlier, it is important to note that in the context of study the teachers' views on mathematical problems, mathematical problem-solving and mathematical problem-solving pedagogy differed from what the literature states. Consequently, the PD intervention generated from this study is relevant to how teachers in this context saw mathematical problems and problem-solving and what they did on a daily basis, that is, teaching word problems found in the CAPS curriculum.

2.3 Mathematical problems, problem-solving and mathematical problem-solving pedagogy

In this section, I review the literature on mathematical problems, mathematical problem-solving and mathematical problem-solving pedagogy and give definitions of important terms used throughout the thesis. It is important to note that, I give definitions that I

found in the literature, but in reality, this is not necessarily how the teachers in this context defined these terms.

2.3.1 Mathematical problems

Over the years mathematical problems have either been viewed as routine exercises or non-routine tasks with no immediate solutions available. Routine tasks are exercises that can be solved by algorithms. An algorithm is a clearly defined set of guidelines to solve a given problem, which, if followed correctly gives a correct answer (Wilson, Fernandez, & Hadaway, 1993). This implies that, if an individual knows how to solve a problem then s/he can use an algorithm. A mathematical problem is an obstacle with no immediately noticeable solution (Blum & Niss, 1991; Polya, 1957). Lester (1983); Charles and Lester (1982) and Krulik and Rudnik (1980) posit that it is a task or situation that an individual or individuals need to find a solution for, but there is no readily available procedure or path that guarantees the solution. If the answer to a task is obvious to a solver then the particular task is not a problem to the solver. It should be noted that the complexity of a problem depends on the solver's knowledge, experience and dispositions (Blum & Niss, 1991); what is a problem for one person may not be inevitably a problem for another person. Adding two three digit numbers can be a problem to a grade two learner yet a high school learner does not see it as a problem. This is stated explicitly by Schoenfeld (1985):

> The same task that calls for significant efforts from some learners may well be routine exercises for others, and answering them may just be a matter of recall for a given mathematician. Thus being a problem is not a property inherent in a mathematical task. It is a particular relationship between the individual and the task that makes it a problem for that person. (p.74)

In this study when I say mathematical problem I am referring to a non-routine problem that a solver does not have a readily known algorithm to solve the problem. However, teachers in the context of study did not see mathematics problems this way, they had a narrower view. They viewed mathematical problems as exercises and word problems given in the textbooks and the CAPS curriculum.

For learners to be able to do mathematical problem-solving effectively, teachers need to select mathematical problems or tasks that are appropriate and worthwhile to learners. An appropriate and worthwhile problem is the one that cannot be solved by any readily available algorithm that learners already know. However, it must be connected with learners' existing ways of thinking (Hiebert et al., 1997). In essence, it should be the one that learners can problematise, must leave a residue of mathematical value, must be reasonably difficult and at the same time be within learners' reach. An appropriate and worthwhile problem should be intriguing, with a level of challenge that invites exploration and hard work (NCTM, 2000), and; should have various solutions or must allow different decisions to be taken and should be highly engaging and collaborative in nature such that it gives learners the opportunity to discuss as they try to solve it (Liljedahl, 2008). Charles and Lester (1982, p.5) state that a problem is worthwhile if it exhibits the following characteristics:

(i) The person confronting it wants or needs to find a solution.
(ii) The person has no readily available procedure for finding the solution.
(iii) The person must make an attempt to find a solution.

For this study, I adopted Van de Walle et al.'s (2013) features of worthwhile and appropriate tasks and incorporated them when developing mathematical word problems for this study. Van de Walle et al. (2013) list the following features:

- A task must begin where the learners are and must take into consideration learners' existing knowledge. Learners must have appropriate ideas, skills and tools with which they can begin to solve the problem and must at the same time find it challenging and interesting.
- The problematic or the engaging task of the problem must be due to the mathematics that learners can learn.
- Tasks should offer learners the opportunity for using multiple representations of the concept and mathematical communication that includes explanation and justification.

- For a task to be accessible to all learners, it must have multiple entry points and should promote the skilful use of mathematics.
- Appropriate problems trigger the cognitive processes of accessing prior knowledge, establishing information into knowledge that both fits into and shapes new mental models.

At this point, it is important to remind the reader that the teachers in the context of study were required to teach word problems by the CAPS curriculum and therefore, I designed word problems for this study and not open-ended tasks. Next, I briefly look at the literature on mathematical word problems. I did not drill into this literature as word problems only became part of this study because of the teachers' views on mathematical problem-solving in the context of study.

Mathematical problems are either embedded in mathematics itself or in the real world (Blum & Niss, 1991), that is, they can either be entirely mathematical or applied (Haylock & Cockburn, 2008). Applied mathematical problems are the word or story problems that are frequently used to present a context in which learners can perform problem-solving. Mathematical word problems can be defined as verbal or textual descriptions of a problem situation (Verschaffel, Greer, & De Corte, 2000) and these include pure mathematics tasks implanted in real word situations. Hence, it can be assumed that learners' ability to effectively solve mathematical word problem depends largely on their capacity to accurately understand the text of the word problem (Jitendra, & Star, 2012). To perform mathematical problem-solving, learners must be able to read and understand the given word problem. In the context of study, learners struggled with LOLT (see chapters 4, 5 and 6 and section 7.3) and this meant that I also focused on language. Word problems are usually characterised by structural components and Verschaffel et al. (2000) list them as:

i) The mathematical structure, which refers to the nature of the knowns and unknowns of the problem, as well as the kind of mathematical operation(s) by which the unknowns can be derived from the knowns.

ii) The semantic structure which alludes to the way in which an interpretation of the text points to particular mathematical relationships.

iii) The context which denotes what the problem entails.

iv) The format which refers to how the problem is constructed and presented (p. x-xi).

2.3.2 Mathematical problem-solving

Over the years, several authors have written about what mathematical problem-solving is or what it entails. The purpose of this study was not to exhaust this body of knowledge, therefore a few definitions of mathematical problem-solving are provided in this section. Problem-solving has been regarded as an important characteristic of the teaching and learning of mathematics for a long time (Liljedahl et al., 2016). It is 'a process by which we use mathematical skills and knowledge in new and unfamiliar ways to find the answer to a mathematical question whose solution is not known in advance' (Freitag, 2014, p. 995) and requires higher order understanding (Jagals & van der Walt, 2016). Polya (1962) envisioned it as finding a path around a challenge or an obstacle and finding a solution to a problem that is not known. Problem-solving involves engaging in a task for which the solution method is unknown in advance (National Council of Teachers of Mathematics (NCTM), 2000) or simply the problem-solver 'does not have an algorithm for relating the data with the unknown' (Callejo & Vila, 2009, p. 112). Mayer and Wittrock (2006, p. 287) define the process of problem-solving as a 'form of cognitive processing you engage in when faced with a problem and do not have an obvious method of solution'. Schoenfeld (1992) refers to it as a process where learners encounter a mathematics task for which they have no immediate solution or an algorithm that they can directly apply to get a solution. Krulik and Rudnick (1980) recognised mathematical problem-solving as the means by which learners use previously acquired knowledge, skills, and understanding to tackle an unfamiliar situation.

From these definitions, it can be deduced that algorithms, useful as they may be, are inessential in solving new problems because they neither entail higher order thinking nor require deep understanding of the concept or problem. Algorithms can be recognized by some people as an essential part of problem-solving. However, once we

move to algorithms, even complicated ones, this can no longer be regarded as genuine mathematical problem-solving (Wilson, et al., 1993). Mathematical problem-solving requires learners to employ a variety of cognitive actions most of which are non-routine and do not depend on algorithms. Learners must construct their own meaning during the problem-solving process; a mathematics exercise requiring learners to repeatedly implement the same algorithmic process will not be regarded as a mathematical problem-solving exercise because of its routine nature. Repetitive exercises that require learners to apply memorised procedures do not support mathematical problem-solving processes in learners and fail to provide sufficient opportunities for learners to develop problem-solving skills (Schoenfeld & Floden, 2014). During the problem-solving process, previously attained experience is applied to an unfamiliar situation that contains major obstacles in order to find a solution to the problem (Killen, 2010) and learners are required to think deeply about what solution strategy to implement. The mathematical problem-solving task must promote understanding and must be 'problematic' for the learner (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997), and higher-order thinking must occur during the problem-solving process (Lester & Kehler, 2003).

Drawing from the definitions and explanations given above, when I say 'mathematical problem-solving', I am referring to the process where learners are given a mathematical problem (word problems in the context of study) to solve for which they cannot see or find an immediate solution. However, I was cautious when using this definition throughout the study as teachers in this context viewed mathematical problem-solving as learners solving word problems and finding the correct answer. As the intervention progressed, teachers' perspective changed to viewing mathematical problem-solving as learners solving a mathematical problem for which they cannot find an immediate solution. However, it was important to support these teachers according to the word problems they taught every day and not open-ended tasks that are usually used for the genuine mathematical problem-solving process. I realised that supporting teachers to teach open-ended problems would not be relevant to the context that teachers were working under, as teachers had to eventually teach in the CAPS curriculum.

2.3.3 Mathematical problem-solving pedagogy

Pedagogy is the 'science of the art of teaching' (Gage, 1985, p.25) and can be seen as 'any conscious activity by one person designed to enhance the learning of another' (Watkins and Mortimore, 1999, p.3). In simple words, pedagogy can be taken as an act, process, or art of imparting knowledge and skills. However, Siraj-Blatchford, Sylva, Muttock, Gilden & Bell (2002) refer to it as the instructional techniques and strategies which enable learning to occur as well as the interactive process between the teacher and learner. Alexander (2000) argues that pedagogy is disparate from teaching. He advocates that teaching is an act yet pedagogy is both an act and a discourse. Drawing from these definitions, I take the position that mathematical problem-solving pedagogy concentrates on the practices that teachers adopt to assist learners to understand and be able to do mathematical problem-solving. Participant teachers in the context of study viewed mathematical problem-solving pedagogy as the teaching of word problems. The teachers' views led the PD intervention to be mostly about supporting teachers in teaching word problems that are found in the CAPS curriculum.

2.3.3.1 Theories of and approaches to teaching mathematical problem-solving

In this section, I discuss the approaches to the teaching of mathematical problem-solving. For mathematical problem-solving pedagogy to be effective it must teach learners to do mathematical problem-solving. However, effective mathematical pedagogy is a contentious statement. Hiebert and Grouws (2007) argue that what is regarded as 'effective' depends on diverse factors, including teachers' views and the particular learning goals of interest. Mathematical pedagogy plays a very fundamental role in learning. Effective pedagogy is essentially concerned with how best to bring about the desired learning by some educational activity. In this study, I am going to contribute to the literature with regards to what should count as effective mathematical problem-solving pedagogy.

The traditional approach to teaching mathematical problem-solving

Murray, Olivier and Human (1998) speak of the traditional approach to teaching mathematical problem-solving and define it as when individuals perceive the teaching of problem-solving as the transmission of knowledge and learning as the absorption of knowledge. Learners are seen as 'empty vessels' and it is the teacher's duty to fill in those vessels with knowledge on how calculations are performed by standard methods and to provide practice until the learners can perform these methods accurately. The traditional approach is regarded by Wessels and Kwari (2003) as teaching algorithms in the hope that learners will use them to solve problems. Wessels and Kwari (2003) further articulate that with this approach, teaching is about problem-solving and problem-solving is not treated as a way of thinking. The teacher represents the source of all knowledge and shows learners how to do assigned exercises (Van de Walle et al., 2013). Learners work at exercises by trying to follow what was demonstrated. If a learner has difficulty, the teacher will show again how the work is to be done by breaking the mathematics into small logical pieces, explaining and drilling each piece in a sequence. The traditional method of teaching views the learner as a passive absorber of algorithms and learning through repeated practice and reinforcement. It views mathematical problem-solving as separated from the learning process, and learners expect the teacher to explain all the algorithmic rules and will not attempt to solve problems for which no solution methods have been provided. Learners believe that every problem given by the teacher has one solution and that there is only one 'right' way to approach and solve a problem (Wessels & Kwari, 2003). The traditional approach leaves most learners believing that mathematical knowledge is mysterious and past understanding (Van de Walle et al., 2013) and must only be memorised and repeated in a test without making sense of it.

The reform-oriented mathematical problem-solving teaching and learning

In the literature, I discovered that there are many definitions of reform-oriented or reform-based teaching and learning of mathematics. For this study, I decided to use the ideas that encompass the important concepts most researchers place on reform-oriented teaching. Reform-oriented teaching and learning of mathematics is a learner-

centred approach to teaching and learning of mathematics (Hiebert & Stigler, 2004); that requires learners to 'engage in problem-solving, reasoning, communicating, making connections and using multiple representations' (NCTM, 2007, p.7); promotes learner discussions during instruction (Anderson & Bobis, 2005; Brodie, 2011); fosters learners to collaboratively construct understandings of mathematical concepts (Fraivillig, Murphy, & Fuson, 1999); upholds learners to monitor their own work instead of always looking for the teacher's guidance (Anderson & Bobis, 2005) and is based on real-world mathematics application. In this study, I specifically focused on reform-oriented mathematical problem-solving teaching and learning. In reform-oriented mathematical problem-solving teaching and learning; learners continue to learn basic algorithms, but most of their time and energy is dedicated to doing and understanding problem-solving. The focus is on the problem-solving process, and not on only getting answers to a given problem. Learners actively participate and contribute meaningfully during lessons since they are no longer seen as empty vessels and the teacher is no longer the only source of knowledge. The teachers employ open-ended questions that facilitate learner thinking and problem-solving processes (Anderson & Bobis, 2005) and do not need to drill the procedure or to teach each concept step-by-step since learners are required to use the various problem-solving strategies given in section 2.3.4.2 to solve the problems. Learners are given a few problems which are of high quality for homework. This homework usually requires learners to apply multiple problem-solving strategies and the newly learnt problem-solving processes to non-routine problems.

The problem-centred teaching and learning approach

The problem-centred teaching and learning approach (PCTLA) model builds on the work of Hiebert et al. (1997), Murray et al. (1998), and Van de Walle et al. (2013) and integrates the aspect of the zone of proximal development (Vygotsky, 1978) and scaffolding (Wood, Bruner, & Ross, 1976). It incorporates key aspects of metacognition apprenticeship (Halpern, 1996), reflective thinking (Van de Walle et al., 2013) and social interaction (Schoenfeld, 2002). The PCTLA is referred to as 'teaching mathematics through problem-solving' by Murray et al. (1998) in their article 'Learning through problem-solving'. Learning mathematics takes place while learners are 'grappling' with

the problem. Learners are placed in the active role of problem solvers by being confronted with unfamiliar tasks that have no readily known procedure or algorithm. The teacher poses powerful non-routine mathematical problems to learners for solving and they are expected to justify and explain their solutions. The teacher accepts right or wrong answers in a non-evaluative way and knows the right time to intervene or step back, letting learners make their own way.

The PCTLA approach is a learner-centred educational method that uses problem-solving as the starting point for learning and as a 'vehicle for learning' (Murray et al., 1998). Murray et al. (1998) further note that learning occurs when learners actively grapple with problems for which they have no routine methods available. The PCTLA involves the learning of mathematics through real contexts, problem situations and models (Van de Walle et al., 2013). Davis (1992), cited in Murray et al. (1998), clearly states what PCTLA entails:

> Learning through problem-solving means that instead of starting with 'mathematical' ideas and then applying them, we would start with problems or tasks, and as a result of working on these problems, the children would be left with a residue of 'mathematics' and we would argue that mathematics is what you have left over after you have worked on problems. We reject the notion of 'applying' mathematics because of the suggestion that you start with the mathematics and then look around for ways to use it. (p.31)

2.3.4 Mathematical problem-solving process

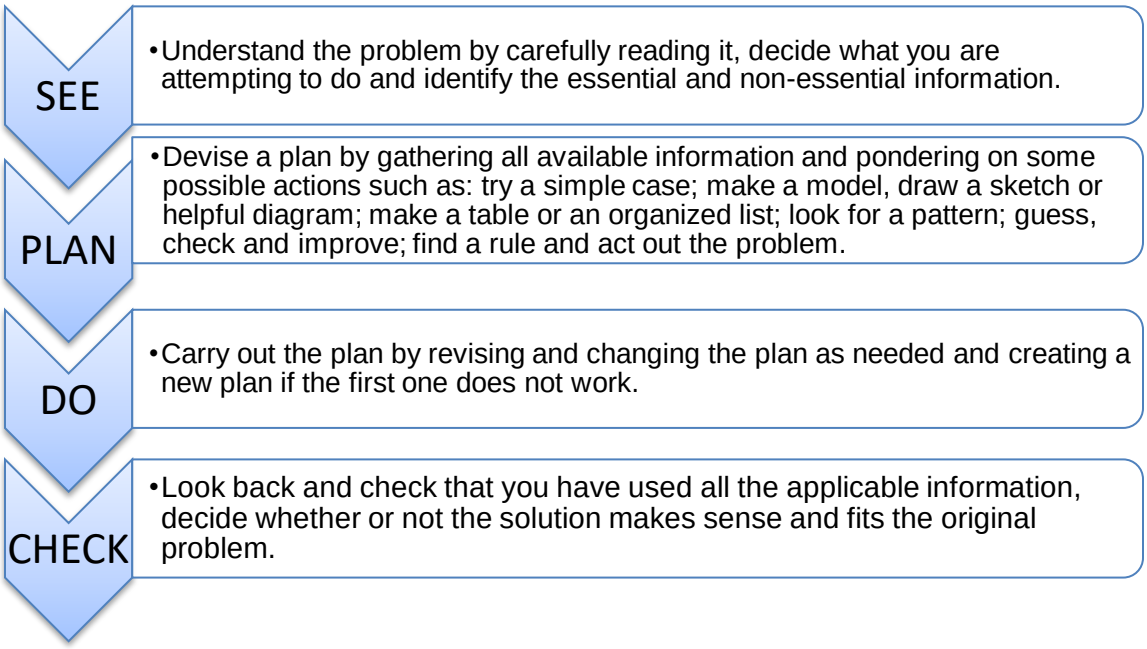
In this section, I discuss the steps involved in the mathematical problem-solving process. The DBE (2011) stresses that for learners to develop essential mathematical skills, they should learn to investigate, analyse, represent and interpret information during the problem-solving process. Dewey (1933), Polya (1957), Krulik and Rudnick (1980) and Barmby, Bolden and Thompson (2014) identified steps in the problem-solving process and I list them in Table 2.1.

Table 2.1: Problem-solving processes.

John Dewey	George Polya	Krulik and Rudnick	Barmby, Bolden and Thompson
Confront Problem	Understand the Problem (SEE)	Read	Identify the problem
Define Problem	Devise a Plan (PLAN)	Explore	Represent the problem
Inventory Several Solutions	Carry Out the Plan (DO)	Select a Strategy	Develop and carry out a solution strategy
Conjecture Consequences of Solutions	Looking back (CHECK)	Solve	Monitor progress towards the solution
Test Consequences		Review and Extend	Reflect further on the solution achieved

I put the examples of steps in problem-solving in the same table to show that they are fundamentally similar, and for the purpose of this study, I expound on George Polya's (1957) problem-solving steps. Polya (1957) proposed a four-phase problem-solving process, linear in nature, with identifiable strategies listed and explained in the text in Figure 2.1.

Figure 2.1: Polya's (1957) problem-solving process



In the following sections, mathematical problem-solving in terms of Polya's (1957) four steps of problem-solving are explicated on. Within each of the steps, I discuss the related areas.

2.3.4.1 Understanding the problem

The first step of problem-solving is making sense of the given problem (Polya, 1957; Freitag, 2014) and this may entail getting acquainted with the problem, visualising the problem and identifying the unknowns and data that are given (Barmby et al., 2014). It is important that learners first reflect on the task in order to get a firm grasp of what is known and needs to be done, and then decide what information is important and what seems unimportant. They should be clear on what may be helpful to reformulate some of the information, perhaps making a list of the knowns and unknowns or drawing pictures, charts or diagrams or 'separating different parts of the problem' (Barmby et al., 2014, p.14). At this stage, learners should represent the problem, what Barmby et al. (2014) term as 'interpreting and changing the format of the problem' (p.16).

Prior knowledge is a key element in the understanding of a given problem and consequently in the problem-solving process (Mayer, 1982; Schoenfeld, 1992, 2004).

The problem-solving process does not only involve what an individual already knows that is useful in solving a posed problem, but also how this knowledge is organised and is accessible during mathematical problem-solving episodes. Successful problem-solving involves learners coordinating and using previous knowledge, experiences and familiar representations (Lester & Kehle, 2003). Liljedahl et al. (2016) assert that learners firstly use existing knowledge to understand the given information when solving non-routine problems. Each learner comes to the classroom with a unique but rich collection of ideas. These ideas are the tools that are used to construct new concepts and procedures as learners grapple with new ideas, discuss solutions, challenge their own and others' conjectures, explain their methods and solve engaging problems. This implies that teaching should start with ideas that learners have, the ideas they will use to solve problems (Van de Walle et al., 2013). This is because learners cannot always absorb an idea as it is taught, but give meaning to unfamiliar information in terms of their existing knowledge. The connection between existing and new knowledge is an essential component of meaningful problem-solving. Learners' ability to remember new knowledge depends on what they already know (Halpern, 1996), and for this reason, it is important that teachers assess and activate learners' existing knowledge on problem-solving. Bell (1993) suggests that it is essential for teachers to start their lessons with tasks that give learners the opportunity to use and show their existing knowledge, this way they facilitate understanding in learners.

2.3.4.2 Devising a plan

In Polya's second phase of problem-solving, learners must find a link between the data and the unknown variables and must reflect on ideas that might be brought to the problem (Polya, 1957). Ideas come from the understanding of the problem, learners' common sense, mathematical knowledge, and previous experience (Freitag, 2014) and range from mathematical concepts and procedures to general processes or strategies (Kilpatrick, Swafford & Findell, 2001). It is important to remember that information that may be brought to the problem or that is needed to solve the problem is unique to each learner. Devising a plan may imply learners relating the problem at hand to the ones they have done in the past or even altering it to make it accessible (Barmby et al.,

2014). If a learner cannot solve the given problem, Liljedahl et al. (2016) recommend that the learner can try to solve some related problem or part of the problem. The related problem can be a general, special or similar problem. Liljedahl et al. further contend that if a learner decides to solve part of the problem they must drop part of the condition and determine if the unknown can then be determined or if they can discern something useful from the data.

To devise a plan to solve a given problem, individuals need to engage heuristics. The term heuristic is a Greek word, which means to discover or to find. Heuristics for solving problems are identifiable methods of approaching a task, that are completely independent of the specific topic or subject matter. They are problem-solving strategies that learners use to solve problems rather than using algorithms (Polya, 1957). For effective problem-solving learners need to have access to an organised and retrievable set of problem-solving strategies (Schoenfeld, 1992). Novotná (2017) postulates that heuristics are one of the many ways that can be used to develop learners' creative approach to problem-solving. At this point, it is important to discuss the heuristics that are commonly found in the literature. Freitag (2014, p. 44) lists the heuristics common in the literature as draw a picture, guess and check, make a list or table, work backwards, look for a pattern, use deductive reasoning, use a formula and write an equation. I discuss these heuristics and others found in the literature below.

Making a table or a chart

Charts of data, function tables, tables for operations and tables involving measurements are a major form of communication in mathematics. Use of a table or a chart is often combined with pattern searching as a means of solving problems or constructing new ideas (Van de Walle et al., 2013).

Trying a simpler form of the problem or problem reformulation

In solving a problem that appears complicated, a learner may find it helpful to start by solving simple similar problems. The given problem is reformulated to make a new problem that is easy to solve whose solution can either be directly used for the original problem or facilitates its solution (Novotná, 2017). Quantities in a problem are modified

or simplified so that the resulting task is easier to understand and analyse. Solving the easier problem may help learners to gain insight that can be used to solve the original and more complex problem. Novotná (2017) posits that if we manage to solve the simpler or similar problem then we can apply its solution strategy to the original problem.

Writing an equation or open sentence

Equations are a major communication tool in mathematics and representation that often lead to familiar procedures. A problematic situation is often translated into an equation because an equation is easy to work with, suggests a familiar pattern or is a useful way to communicate an idea. By solving the equation or inequality, the learner finds a way to the solution of the problem at hand (Lenchner, 1983).

Guessing, testing and revising

The aim of this strategy, drawing from experience, is to make a reasoned guess at the solution and then see how the guess fits the conditions. If there is a good fit, the learner may be finished, but if not, checking will tell him or her in which direction to adjust the guess (Van de Walle et al., 2013). The learner begins by guessing and then tests the guess to find out if it is good enough. If the guess is not good enough, s/he revises it and tests again.

Working backwards from the solution

Some problems may best be approached ‘through the back door’. The problems designed for this strategy usually have a series of events ending with a given conclusion. The main task is to find the starting point. This strategy starts with the goal rather than what is given because one proceeds from the end to the beginning (Novotná, 2017). In some cases, a problem may contain a series of actions that are better understood and clarified by working backwards from the end to a direct point in the action sequence.

Looking for patterns

Mathematics is the science of pattern and order (Van de Walle et al., 2013; Steen, 1988). Patterns in numbers and in operations play a huge role in helping learners learn about and master basic and advanced facts. Patterns fulfil an integral role in the discovery and application of mathematical concepts. Learners should be taught to analyse patterns and make generalisations based on their observations; to check generalisations against known information, and to construct a formal proof to verify the generalisation.

Drawing a picture and acting it out

Picturing how actions occur and how they are related in a problem may help the learner to easily find the solution. In situations where it is impossible to use objects, items may be used that represent them. Acting out the problem may in itself lead to the answer or to another strategy that may help in obtaining the answer (Lenchner, 1983).

Making a model

Modelling can refer to an object, a diagram, a pictorial representation or just a simple model. If a learner makes a model of the problem, he or she may be able to easily visualise the problem. After drawing a model, a series of logical steps may eventually lead to the solution of the problem.

Making an organised list of possible solution strategies

This strategy aims at systematically accounting for all possible outcomes in a situation, either by finding out how many possibilities there are or ensuring that all possible outcomes have been accounted for.

Changing point of view

Learners may fail to find a solution to a particular problem by believing that there is only one approach to solving the problem. In this case, it is always important to read the problem again and change one's point of view (Lenchner, 1983).

Systematic experimentation

With this strategy learners attempt to find a solution to a problem using numerous experiments (Novotná, 2017). Learners initially apply an algorithm hoping to solve the problem. Novotná suggests that learners can then systematically alter the input values of the algorithm until they get the correct solution.

2.3.4.3 Carrying out the plan

The plan can be carried out by engaging appropriate representations and making the necessary computations (Freitag, 2014). Learners follow through with the approach selected, carefully taking and checking each step along the way and making sure that it is correct. If learners get stuck on this stage, they can go back to the problem to check if it is understood correctly and search for a new way of approaching the problem. If a plan works, learners should consciously check each step and Freitag (2014) recommends that if a problem is difficult, one should leave it temporarily and work on a different problem and attempt it at a later stage.

During the execution of the planning phase, it is important for learners to self-monitor their progress and regulate the methods they will be using, that is, to engage in the metacognitive activity (Van de Walle et al., 2013). Metacognition or 'thinking about thinking' (Flavell, 1979) is regarded by Lester (1994) as the driving force in problem-solving. It refers to 'what we know about what we know' (Halpern, 1996) and the awareness of and the ability to control one's thinking processes, in particular, the selection and effective use of problem-solving strategies (Schoenfeld, 2004). It includes conscious monitoring (being aware of how and why you are doing something) of one's own thinking and self-regulation (choosing to do something or deciding to make changes) of learning. Good problem solvers automatically monitor their own thinking regularly, are deliberate about their problem-solving actions and know when they are stuck or do not fully understand, and they make conscious decisions to switch strategies, rethink the problem, search for related content knowledge that may help or they simply start afresh (Van de Walle et al., 2013). Van de Walle et al. (2013) further observe that poor problem solvers are impulsive, spend little time reflecting on a novel

problem and are unable to explain why they used the selected strategy or whether they believe it works.

Lester (1994) argues that problem-solving requires knowing what to monitor and how to monitor one's performance; therefore the provision of metacognition experiences is necessary to help learners develop their mathematical problem-solving processes and skills. Metacognition skills enable learners to strategically encode the nature of the problem by forming mental representations of the problem, select appropriate plans for solving the problem and identify and overcome obstacles to the process (Davidson & Sternberg, 1998).

The best way to assist learners to become aware of their own thinking is to create opportunities where they have to explain their thinking explicitly (Dendane, 2009). The following strategies can be used by teachers to develop learners' metacognition awareness:

- Provide learners with problems that require them to plan before solving the problem and to evaluate the solution after solving the problem.
- Encourage learners to find different ways of solving a problem and to check the appropriateness and reasonableness of a solution.
- Afford learners the opportunity to discuss how to solve a particular problem and to explain the different methods that can be used to solve the problem.

Teachers can develop learners' metacognition skills by asking them the following questions after a problem-solving session:

- What did you do that helped you understand the problem?
- Did you find any information that you did not need?
- How did you decide what to do?
- Did you think about your solution after you got it?
- How did you decide that your solution was correct?

The carrying out of the planning phase is a vital step of mathematical problem-solving; nevertheless, from my career as a mathematics educator, I have realised that the success of the whole problem-solving process also depends on the first two stages and the last stage.

2.3.4.4 Looking back

A problem should never be considered solved simply because an answer has been found, but the problem-solver needs to reflect on whether the solution makes sense (Polya, 1957). Freitag (2014) stresses that looking back encourages learners to check their solution and to evaluate and consolidate what they would have learnt. Learners must engage in the following three looking-back activities: looking at the answer; looking at the solution process or method, and looking at the problem itself.

Answer: Learners must check if the solution is correct or unreasonable and verify by checking it against all the conditions and clarifying if there are any contradictions between the answer and the conditions of the problem.

Process: Learners must justify their answers by explaining how they arrived at them. They must check if the problem could have been solved in a different, more efficient or easier ways, as well as whether the approach is transferable in solving other problems (Barmby *et al*., 2014; Freitag, 2014).

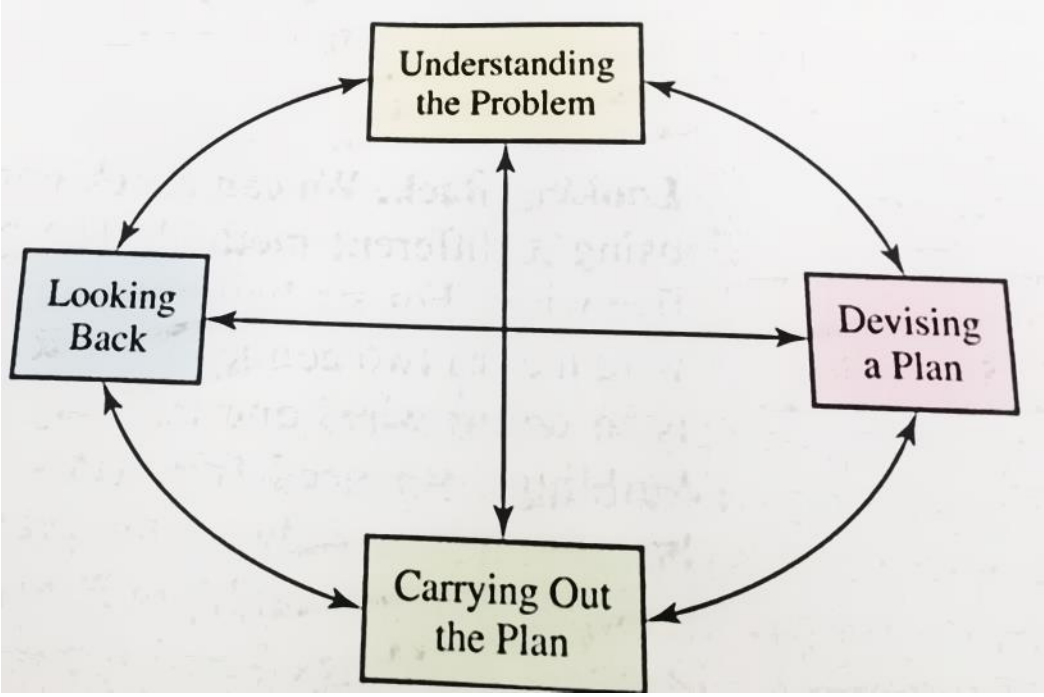
Problem: Learners must consider what was learnt from the problem itself, as well as checking if they can still solve the problem if the conditions are different or the numbers are bigger. Since it is sometimes easy to overlook certain factors, after finding an answer, learners should check if they really answered the question (Van de Walle et al., 2013).

Liljedahl et al. (2016) observe that when a learner looks back they utilise prior knowledge to solve problems. Silver (1985, p. 20) concurs with this when he states that looking back makes connections 'in memory to previously acquired knowledge….and further establishes knowledge in long-term memory that may be elaborated in later problem-solving encounters'.

2.3.5 An analysis of Polya's (1957) problem-solving steps

Polya's (1957) steps are sequential; however, Freitag (2014) posits that during the problem-solving process we often move to and fro among the steps as we solve the problem (see Figure 2.2).

Figure 2.2: Freitag's problem-solving process



(Source: Freitag, 2014, p. 45)

Freitag (2014) further argues that we occasionally solve problems linearly and we frequently skip or revisit steps. I agree with Freitag (2014) because learners usually get a better understanding of the problem when they are devising a plan and learners may need to look back at how they understood the problem or how they devised the plan when they are solving the problem. This implies that the problem-solving process is rarely sequential.

George Polya's (1957) problem-solving process is a possible backbone for the development of mathematical problem-solving skills and processes. Solving non-routine problems, and posing and answering questions to understand concepts, is generally a

form of higher-order mathematical thinking. Routine problems require one definite answer (Van de Walle et al., 2013, Murray et al., 1998) and focus on practice rather than developing new understanding (Barmby et al., 2014), yet non-routine problems with multiple solutions often challenge both teachers and learners, and this results in the development of mathematical problem-solving skills and processes.

Mathematical problem-solving skills are mental processes that allow a learner to take on a mathematical problem, choose the best of many mathematical problem-solving techniques for that particular situation, and go through the process of finding a solution to the problem. Mathematical problem-solving skills are crucial to learners because they give them the ability to face a problem head-on, use the techniques they have learnt and come to the desired outcome, thus solving the problem with the least difficulty possible and in the most effective way. In the literature (Lenchner 1983; Polya 1957; Hiebert & Wearne 1993; Van de Walle et al., 2013; Dendane 2009), I found seven mathematical problem-solving skills that I believe are of particular importance.

1. *Understanding or formulating the question in the problem***-** learners must be able to first formulate the question in the problem and make sense of it.

2. *Understanding the conditions and variables in the problem*- during the process of understanding the conditions and variables, the problem solver (learner) 'internalises the problem'. The learner develops a sense of how the conditions and variables relate to each other and clarifies the meaning of the information explicitly stated or implied in the problem.

3. *Selecting or finding the data needed to solve the problem*- the learner must be able to identify needed data, eliminate data not needed and collect and use data from a variety of sources such as graphs, maps or tables.

4. *Formulating sub-problems and selecting appropriate solution strategies to pursue*- the learner must be able to determine if there are sub-problems or sub-goals to be solved and decide which strategy or strategies to use and when.

5. *Correctly implementing the solution strategy or strategies and solving sub-problems*- the learner must know how to implement a solution or solution strategies.

6. *Giving an answer in terms of the data in the problem*- the learner must be able to give an answer in terms of the relevant features of the problem.

7. *Evaluating the reasonableness of the answer*- the learner must be able to determine whether or not the answer or solution makes sense.

2.3.6 Creating mathematical a problem-solving classroom

The purpose of this study was not to make a comparison between the PCTLA, the traditional teaching approach and the reform-oriented mathematical problem-solving teaching and learning, but to discuss and recommend how mathematical problem-solving can be taught and consequently how a mathematical problem-solving classroom can be created. The discussion I conduct in section 2.3.4 is to show that there is a number of ways to teach mathematical problem-solving. In this section, I discuss the characteristics of a mathematical problem-solving classroom and how such a class can be created.

Problematization is the main objective of a mathematical problem-solving classroom. Problematization means that instruction begins with problems, dilemmas and questions for learners (Wessels & Kwari, 2003). The teachers pose challenging non-routine problems to learners and the subject is allowed to be 'problematic' (Hiebert et al., 1997, p.12). Activities that involve learners' active learning and participation in the problem-solving process are part of mathematically powerful classrooms that encourage robust understanding (Schoenfeld, 2014). Schoenfeld (2012) notes that mathematicians are not passive receivers of information, but develop knowledge; through questioning why some concepts work and if particular examples can be extended. Schoenfeld (2012) further argues that a mathematical problem-solving classroom is 'structured to help learners to develop agency and authority' p.595. In this study, participant teachers were encouraged to plan each lesson with a focus on learners' mathematical problem-solving and to create episodes were mathematics was problematic for learners.

In a mathematical problem-solving classroom learners collaboratively solve problems with their peers and there is no right or wrong solution strategy. Another important factor for creating problem-solving classrooms is that learners must have positive attitudes and beliefs towards mathematics and their ability in problem-solving. Beliefs are the 'perspective with which one approaches mathematics and mathematical tasks' (Schoenfeld, 1985, p.45). Beliefs are important because they form attitudes and emotions that regulate the decisions made during problem-solving. When a problem is posed learners must view it as worth solving and must give it attention and genuine effort (Schoenfeld, 1992, 2004).

2.4 Teacher knowledge in mathematics

This study is about supporting teachers' mathematical problem-solving pedagogical knowledge through PD; therefore, I review three themes: mathematical problem-solving that I discuss in section 2.3; pedagogical knowledge for mathematical problem-solving that I discuss in this section and effective PD in mathematics education that I discuss in section 2.5. In order to talk about pedagogical content knowledge (PCK) for mathematical problem-solving, I need to first talk about what PCK entails; therefore I am going to look at a number of authors' work on PCK. I focus mainly on Shulman's (1986, 1987) ideas on teacher knowledge; Ball, Thames and Phelps' (2008) notion on mathematics knowledge for teaching (MKfT); and Pournara et al.'s (2015) work on PD of teachers' mathematical knowledge. Later on, I discuss pedagogical content knowledge for mathematical problem-solving in relation to PCK and MKfT. In section 2.4.5 I discuss Chapman's (2015) work on mathematical problem-solving knowledge for teaching (MPSKT) to a greater extent.

In his seminal work, Shulman (1986, 1987) defined seven categories of teacher knowledge which are: content knowledge, general pedagogical knowledge, PCK, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts and knowledge of educational ends, purposes and values. Each of this knowledge important; however this study focuses on Content Knowledge (CK) and PCK which are predominantly believed to be the backbone of teacher effectiveness (Marchant & Brodie, 2016).

2.4.1 Content knowledge

CK is the knowledge teachers have of the subject matter to be learned or taught and its organising structures (Koehler & Mishra, 2009; Shulman, 1987). It can simply be defined as the teachers' understandings of the subject they teach and Shulman (1986) perceives that knowing a subject for teaching requires knowing its facts and concepts; as well as understanding the organising principles and structures and the rules for establishing what is justifiable to do and say in a field. Teachers' CK is an important part of teaching since it affects preparation, task setting, questioning, explaining, giving feedback and assessment (Shulman, 1987). McNamara (1991) observes that teachers with strong CK employ innovative and interesting teaching techniques as compared to those with little CK who may tend to avoid more difficult aspects of the subject. Kilpatrick et al. (2001, p. 8-9) concurs with this and state that 'the effectiveness of mathematics teaching and learning is a function of teachers' knowledge and use of mathematical content…' and that teachers with strong content knowledge are able to deepen their learners' mathematical problem-solving skills and reasoning levels. Research on the relationship between teachers' mathematical knowledge and learners' achievement supports the importance of teachers' CK in learning (CDE, 2014; Taylor & Vinjevold, 1999; Taylor et al., 2013). If mathematical teachers have a conceptual understanding of mathematics they are in a position to offer learners challenging mathematics and this influences classroom instruction in a positive way. Hoover, Mosvold, Ball and Lai (2016) conducted a large-scale study that revealed a reasonable positive relationship between teachers with adequate mathematics knowledge and pedagogical training, and teaching and learning. However, my interest in this study was not to investigate what affects teacher's content knowledge in mathematics or its relationship with learner achievement, but, was to factor mathematics teachers' knowledge for teaching problem-solving into the PD intervention that I was designing.

2.4.2 Pedagogical content knowledge

In section 2.4.1, I discuss teachers' CK; however, Shulman (1986, 1987) introduced PCK as a third major component of teaching. PCK is defined by Shulman (1987) as the

knowledge of how to teach within a subject; that is; teachers' interpretations and transformations of CK in the environment of facilitating learning. PCK bridges CK and the practice of teaching (Ball et al., 2008). It is 'how teachers teach their subject by accessing what they know about the subject, the learners they are teaching, the curriculum with which they are working, and what they believe counts as good teaching in their context' (Rollnick, Bennett, Rhemtula, Dharsey & Ndlovu, 2008, p.1367). In my own words, PCK is how teachers relate what they know about teaching (pedagogical knowledge) to what they know about what they teach (CK). According to Shulman (1986), PCK also includes an understanding of what makes the learning of specific concepts easy or difficult. It represents the 'blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction' (Shulman, 1987, p. 4). Shulman advocates that the more representations and strategies teachers possess within a certain subject and the more they understand their learners' learning processes in that subject, the more effectively they can teach that subject. This is the reason why I took teachers' PCK in mathematical problem-solving into consideration when designing and developing the PD intervention for this study.

Ball et al. (2008) expand Shulman's (1987) CK and PCK and categorise them under the broad term of MKfT. Ball et al. (2008) contend that MKfT has two strands, that is, mathematical content knowledge (MCK) and PCK. Within MCK Ball et al. (2008) distinguish between Common Content Knowledge (CCK), Specialised Content Knowledge (SCK) and knowledge at the mathematical horizon. Ball et al. (2008) explain that CCK is the knowledge used by teachers in ways similar to how it is used by others who know and use mathematics; SCK is the mathematical knowledge that teachers need to teach and mathematical horizon being knowledge teachers need to link concepts at hand to learners' future mathematical learning. Within PCK, Ball et al. (2008) differentiate between knowledge of mathematics and learners, knowledge of mathematics and teaching and knowledge of the curriculum. In the next section, I discuss MKfT, which has become a crucial construct in mathematics education research today.

2.4.3 Mathematical knowledge for teaching (MKfT)

Mathematics teachers' general ability to solve mathematics problems does not imply that they can teach mathematics effectively (Ball et al., 2008), but they require a special kind of mathematical knowledge for teaching in order to teach mathematics well (Brodie, 2004). Ball and Bass (2000) stress that for mathematics teachers to be able to make mathematics accessible to learners they need to teach mathematics well by unpacking the mathematics they know. The mathematics that teachers know and use in practice is what is known as MKfT. Ball et al. (2008, p.4) state that MKfT is the 'mathematical knowledge that teachers need to carry out their work as teachers of mathematics'. This study is not focused on MKfT in general, but specifically on the knowledge teachers must have in order to be able to teach mathematical problem-solving.

2.4.4 Mathematics teachers' knowledge for teaching problem-solving

For teachers to be able to effectively teach learners mathematical problem-solving they need to have a solid knowledge of mathematical problem-solving themselves. Chapman (2015) argues that besides the ability to solve problems, teachers require a deeper and comprehensive knowledge of and for teaching problem-solving. This knowledge is what is called mathematical problem-solving knowledge for teaching (MPSKT). MPSKT is the knowledge teachers use to identify learners' mathematical problem-solving proficiency and strategies during problem-solving. It is important for teachers to identify strategies used by learners in a problem-solving episode so that they can be able to infer why a certain problem is challenging for learners. I adopted Chapman's (2015) components of MPSKT for this study because they build on important aspects of this thesis. Chapman (2015) postulates the following components of MPSKT as it relates to what the mathematics teacher must hold in order to teach mathematical problem-solving to learners effectively:

i) Problem-solving content knowledge (PSCK) which comprises of teacher's knowledge of problems, problem-solving, and problem posing. Teachers need to have knowledge of various problems relevant for mathematical problem-solving.

ii) Pedagogical problem-solving knowledge (PPSK) which involves teacher's knowledge of learners as problem solvers and knowledge for instructional practices for problem-solving.

iii) Teacher's knowledge and understanding of the positive and negative impact of the affective factors and beliefs on learning and teaching of problem-solving (p.32).

I further elaborate on the above components in table 2.2.

Table 2.2: Components of MPSKT.

Knowledge of:	Description
Mathematical problem-solving proficiency	Understanding what is needed for successful mathematical problem-solving.
Mathematical problems	Understanding of the nature of meaningful problems; structure and purpose of different types of problems; impact of problem characteristics on learners.
Mathematical problem-solving	Being proficient in problem-solving. Understanding of mathematical problem-solving as a way of thinking; problem-solving models and the meaning and use of heuristics; how to interpreting learners' unusual solutions; and implications of learners' different approaches.
Problem posing	Understanding of problem posing before, during and after problem-solving.
Learners as mathematical problem solvers	Understanding what a learner knows, can do, and is disposed to do (e.g., learners' difficulties with problem-solving; characteristics of good problem solvers; learners' problem-solving thinking) instructional practices for problem-solving Understanding how and what it means to help learners to become better problem solvers (e.g., instructional techniques for heuristics/strategies, metacognition, use of technology, and assessment of learners' problem-solving progress; when and how to intervene during learners' problem-solving).
Affective factors and beliefs	Understanding the nature and impact of productive and unproductive affective factors and beliefs on learning and teaching problem-solving and teaching.

Source: Chapman (2015, p. 31)

2.5 Professional development for teachers

PD are the 'processes and activities designed to enhance the professional knowledge, skills and attitudes of teachers so that they might, in turn, improve the learning of learners' (Guskey, 2000, p. 16). Day (1999) concurs with this definition and defines PD as a process by which, either individually or with colleagues, teachers review, renew and broaden their commitment as change agents to the moral purposes of teaching; and by which they acquire and develop critically the knowledge, skills, planning and practice with children, young people and colleagues through each phase of their teaching lives. Wei, Darling-Hammond, Andree, Richardson, and Orphanos (2009) see it as the processes and activities designed to improve teachers' knowledge, the practice of instruction, and the learning outcomes of learners.

The process of teacher development should be considered as one in which personal, professional and social development is taking place (Bell & Gilbert, 1994) and should perpetuate the emphasis of changing the philosophy of teaching, while being responsive to a particular teacher's personal, social and professional needs (Loucks-Horsley, Hewson, Love and Stiles, 2010). In the next section, I discuss effective PD and features of PD adopted for this study.

2.5.1 Effective teacher professional development

Many authors have explained what constitutes effective PD and they all agree that it is one that produces positive change in teaching practice and learners' outcomes. Hunzicker (2010, p.177) posits that effective PD is the one that involves teachers in 'learning activities that are supportive, job-embedded, instructionally focused, collaborative, and ongoing'. I found the definitions of Hunzicker (2010) and Loucks-Horsley et al. (2010) all-inclusive and relevant to this study. Loucks-Horsley et al. (2010) state that effective PD is:

> Directly aligned with learner learning needs; is intensive, ongoing and connected to practice; focuses on the teaching and learning of specific academic content; is connected to other school initiatives; provides time and opportunities for teachers to

> collaborate and build strong working relationships; and is continuously monitored and evaluated. (p. 5)

The main objective of any PD is to increase learner achievement (Quick, Holtzman & Chaney, 2009), therefore effective PD should emphasise practices 'that turn learners into critical thinkers and problem solvers' (Gulamhussein, 2013, p.6).

Effective PD should be based on social constructivism and must be perceived as a process that takes place within a particular context (Villegas-Reimers, 2003) and is situated in classroom practice (Putnam & Borko, 1997). Putnam and Borko (1997) posit that teachers should be treated as active learners who construct their own understanding for PD to be effective. Gulamhussein (2013) echoes that during the PD, teachers must be engaged through varied approaches so that they can participate actively in making sense of a new practice. For PD to be effective it must be a long-term collaborative process (Villegas-Reimers, 2003) and ongoing to give teachers ample time to learn, grapple and implement the new teaching strategies (Gulamhussein, 2013). Bell and Gilbert (1994, p.493-494) observe that teacher development should involve 'inputting of new theoretical ideas and new teaching strategies' and emphasise that 'trying out, evaluation and practice of these new theoretical ideas and new teaching methods should happen over an extended period of time in a collaborative situation.'

It is imperative that teachers are supported during the implementation stage to address the specific challenges of changing classroom practice (Gulamhussein, 2013), receive feedback and given the opportunity to reflect critically (Bell & Gilbert, 1994). Gulamhussein (2013) stresses that for PD to be effective; the content presented to teachers should not be generic, but should be grounded in the teacher's discipline. This is echoed by Villegas-Reimers (2003) who accentuates that PD is not similar in different settings, that is, what works for other teachers in a certain area may not work for other teachers in a different area. From the above literature, I concluded that reflection (see section 2.5.3), long-term support, coaching in teachers' classrooms, or ongoing interactions with colleagues and the contexts in which teachers work, all contribute to effective PD. For this study, during the PD intervention, I incorporated reflective

approaches and collaborative learning among teachers (see chapter 5 and 6) so that they could develop ideas that were relevant to their own context.

2.5.2 Effective mathematics teacher professional development

An effective mathematics teacher PD is the one where teachers experiment with new mathematical ideas in the classrooms as indicated in the literature (Guskey, 2000); work collaboratively on mathematical aspects with peers (Joubert, Back, De Geest, Hirst & Sutherland, 2009); and have time away from school to think, learn and discuss challenging mathematics concepts in similar ways they are expected to teach them (Day, 1999). Mathematics teacher PD in South Africa has mostly focused on mathematical knowledge and PCK because it is agreed that the value of instruction depends on what the teacher takes along to the classroom (Rowland & Ruthven, 2011). The other reason is because of the SACMEQ III study (2012) which unearthed that South African mathematics teachers had inadequate knowledge of what they were teaching. Therefore, it is believed that improving South African teachers' mathematical knowledge will eventually improve learners' performance in mathematics. Over the years a number of PD projects in mathematics education have been carried out in South Africa and in this section, I discuss a few.

The Wits Maths Connect Secondary (WMCS) at the University of the Witwatersrand is an example of an effective PD for grades 9 and 10 teachers in some secondary schools in Gauteng, South Africa. The project aims at improving teachers' MKfT and their teaching practices and evaluating whether the PD intervention has an impact on learners' learning outcomes (Pournara et al., 2015). WMCS is unique in that it does not copy existing PD models, but embraces several aspects of existing PD models like teacher collaborative learning. At the beginning of its project, WMCS' findings were that participant teachers lacked 'confidence in their mathematical knowledge' and that their explanations during lesson delivery 'lacked clear focus and coherence' (Adler, 2017, p.4). Adler (2017) explains that in order to cater for these shortcomings, the project then designed, developed and implemented a PD intervention that supported teachers' pedagogic strategies and enhanced their MKfT. I have indicated that this is an effective

PD intervention because its impact on learners' learning was studied in 2013 and the findings were that learners who had been taught by the participant teachers of the WMCS outperformed their peers in the same school that had been taught by non-participant teachers of the project (Pournara et al., 2015).

In 2010, two Chairs in Primary mathematics education were appointed at the University of Witwatersrand and Rhodes. These chairs focus on numeracy education and have conducted successful longitudinal in-service programmes as well as a range of classroom-based and after-school interventions. The Wits chair conducts a 20-day 'primary mathematics for teaching content knowledge' course and the **'I Hate Maths'** workshops. The Wits Maths Connect - Primary team have reported improvements in primary mathematics teaching and learning through their courses and workshops (for example Abdulhamid & Venkat, 2017). The Rhodes South African Numeracy Chair Project (SANC) runs a number of teacher PD projects that include among others: Early Number Fun teacher development programme (ENF) and Numeracy Inquiry Community of Leader Educators (NICLE). The SANC has several research outputs that indicate that the project has improved the quality of teaching of practising teachers which has resulted in improved learner performances in primary schools (Stott, Graven, Baart, Hebe, & Mofu (2017); Graven & Venkat, 2017).

Mentoring is a form of effective PD in terms of pedagogy and instruction used in mathematics education in South Africa. Mentoring is when an experienced person (teacher) or a PD practitioner is invited into the classroom to support teachers to 'rethink the teaching and learning taking place in order to devise appropriate interventions customised to the needs of teachers' (Nel & Luneta, 2017, p.1). Mentoring is found to be effective as it is done in the classroom and focuses on teachers' MCK and instructional practices. Nel and Luneta (2017) evaluated a PD mentorship programme that ran for three years in a certain rural mining district and advocate that mentoring is an effective PD method if it is informed by teachers' instructional and content knowledge needs.

2.5.3 Reflective approach to teacher professional development

Ball and Cohen (1999) advocated that for PD to be effective, teachers need experience with the tasks and perspectives that are fundamental to the instructional practice. Ball and Cohen's (1999) observation builds on the theory of teacher PD as a reflective process (Dewey, 1933; Schön, 1983, 1987). Wilson, Shulman and Richert (1987) define reflection as what a teacher does when he or she looks back at the teaching and learning that has occurred and reconstructs the events, emotions and experiences of the situation. It is the teachers' interpretation of their teaching experience and its major drive being to ascertain problems during teaching practices. Schulman (1987) proposed that reflection takes place when a teacher 'reconstructs, re-enacts and/or recaptures the events, emotions, and the accomplishment' of his or her teaching (p.19).

Reflection by teachers on their teaching experience as a technique of increasing their PD was pioneered by John Dewey and is now frequently accentuated as a fundamental feature of teachers' PD. Dewey (1916) argued that critical reflection is one of the most essential qualities that a teacher should have and that it heavily impacts on the quality of educational institutions and instruction. Dewey (1916) further advocated that reflective thinking moves people (teachers) away from routine thinking towards reflective action that involves learning from doing. Dewey (1933) noted three characteristics of reflective individuals that teachers should have, that is: open-mindedness, wholeheartedness and responsibility. Open-mindedness is having a mind receptive to new or alternative ideas or arguments. Wholeheartedness entails that teachers can overcome uncertainties to critically evaluate their practice in order to make a notable change, and responsibility encompasses being cautious of the outcome of each action.

Schön's (1983, 1987) theory of teacher reflection advocated that a good teacher incessantly reflects on his or her teaching. Schön (1983, 1987) proposed two types of reflection; that is reflection-in-action and reflection-on-action. Reflection-in-action denotes the teachers' aptitude to reflect throughout the lesson rather than after the lesson. This is regarded as a vibrant approach to the teaching and learning of

mathematics, and Schön (1987) stressed that our knowing emanates from our doing. This is reinforced by Lerman (2001, p.39) who states that 'reflective practice offers a view of how teachers act in the classroom as informed, concerned professionals and how they continue to learn about teaching and about learning, about themselves as teachers, and about their pupils as learners'. Ponte and Chapman (2006) posit that reflection-in-action is part of CK and PCK because it transpires when teachers are dealing with professional problems. Ponte and Chapman observe that when mathematics teachers reflect-in-action besides CK they need the ability to identify and solve pedagogical problems and knowing how to create knowledge. In this study reflection-in-action was very important during teachers' implementation of mathematical problem-solving pedagogy; teachers were required to reflect throughout their lessons.

Reflection-on-action encompasses thinking about, commenting, analysing and evaluating the lesson by teachers after it is completed. In this study, teachers were encouraged to reflect-in-action during their lessons and to reflect-on-action during the reflective semi-structured interviews, and in the course of and after the PD workshops. The reflective process afforded participant teachers opportunities to learn from their own pragmatic experiences. This is in line with what Wilson *et al*. (1987) noted, that there exists a phase in which teachers look back on the teaching and learning that has transpired as a way of understanding their actions and learning from their experiences. By this process, teachers can engage in the problem-solving processes which are more meaningful and thought-provoking (Schön, 1983). This was echoed by Murray (2010, p. 5) who states that reflection is important in that 'teachers can learn a great deal about the reasons behind their teaching philosophies and practices by examining their experiences and asking and answering questions about them'.

In this study, reflection helped participant teachers to make sense of their experiences in the PD intervention and this resulted in their self-improvement, PD and transformation in their mathematical problem-solving pedagogy. Teacher reflection integrates concepts of social constructivism (see section 2.2) which advocates that learning occurs within a social situation. Lerman (2001) posits that the reflective process is enhanced through an individual's social interaction with others. Participant teachers constructed their

mathematical problem-solving pedagogy knowledge through reflections, collaboration and active participation in their context.

2.5.4 Features of professional development adopted for this study

After an extensive literature review on PD, I used Desimone, Porter, Garet, Yoon & Birman's (2002) six key characteristics of effective PD. Desimone et al. (2002) propose and divide the six characteristics into two categories: structural characteristics and core characteristics of the substance of the activities (p.83).

2.5.4.1 Structural characteristics

The structural characteristics include the form of the activities:

i) whether the activities are organised as a reform type (such as a study group, teacher network, mentoring relationship, individual research project, or teacher research centre, etc) in contrast to traditional one-shot workshops, courses, or conferences;

ii) the length of the activities (the total number of contact hours that participants spend in the activity and the period of time over which the activities take place); and

iii) the extent to which the activities emphasise the collaboration of teachers from the same school, department, or grade level, as opposed to the individual participation of teachers from different schools. Collaboration can be defined as a 'systematic process in which teachers work together to analyse and improve their classroom practice' (DuFour, 2005, p. 36). Teachers in this study worked in collaboration with each other to improve their mathematical problem-solving pedagogy.

2.5.4.2 Core characteristics or characteristics of the substance of the activities

The core characteristics include:

i) Active learning - activities must provide teachers with opportunities to become actively engaged in the meaningful evaluation of their teaching practices and

learning, for example, in this study teachers obtained feedback during the implementation of the MPS pedagogy.

ii) Coherence in teachers' PD- activities should promote coherence in teachers' PD, by integrating experiences that are consistent with teachers' goals and encourage continuing professional communication among teachers;

iii) Content focus- the degree to which activities are focused on improving and deepening teachers' content knowledge. In this study, the activities are focused on improving teachers' mathematical problem-solving pedagogy.

2.6 Conclusion

The literature review's intention was to provide the reader with the overall picture of the motivation behind the thesis' research objectives, questions, significance, theoretical framework, methodology, analysis, conclusions and recommendations. Polya's (1957) problem processes are the backbone of this literature review and are explicated to a larger extent. The theoretical framework for this study is proposed and elucidated on. Teacher knowledge in general and MKfT are expounded on. The knowledge required of teachers in order to teach mathematical problem-solving is clarified. Effective PD is discussed and the importance of the reflective approach in PD is described. Features of PD adopted for this study are explained. The next chapter describes the methodology used in this study. DBR and mixed methods research design are expounded on. The participants, the data collection tools, the data collection procedure, the design and implementation of the PD intervention, the data analysis and issues of rigour are extensively explicated on.

Chapter 3: The research design and methodology

3.1 Introduction

This chapter starts with an in-depth description of the research questions, research paradigm, the research design, the researcher's role and the research method that was adopted to answer the research questions and the reasons why I employed this method in particular. The population sample, the participants, and the sampling procedure are also discussed. The chapter then moves on to explain the different data collection tools that I used to collect the data. Thereafter the piloting of the research tools, data analysis, the rigour in the research and the ethical considerations of this study are examined. The chapter concludes with a brief summary.

Recently researchers have begun to mix research methods within a specific methodological approach (Ivankova, 2015) and this was done in this study. This study employed a mixed methods design-based research. In this instance, I define mixed methods design-based research as research in which researchers integrate quantitative and qualitative data collection and analysis methods within a DBR methodological approach to enhance the application of DBR for developing an educational intervention. Anderson and Shattuck (2012) studied the most cited DBR articles from 2002-2011 and found that all the researchers included both quantitative and qualitative methods. In this study, I implemented, emphasised and integrated quantitative and qualitative methods within a DBR methodology thereby enhancing DBR's application in developing a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy.

3.2 The research questions

The guiding research questions for this study are found in section 1.6 and in this section, I give a table that outlines the data sources which were used to answer each of the research questions.

Table 3.1: Data sources against research questions

Research question	Data sources for answering research questions
What are grade 9 South African mathematics teachers' views on the teaching of problem-solving?	Grade 9 South African mathematics teachers' views on the teaching of problem-solving were gathered by implementing an open-ended questionnaire on 31 teachers at 20 schools in the Gauteng district.
What are the design principles required to generate a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy in a particular local context?	Design principles were generated after the completion of each cycle.
What is the impact of the PD intervention on grade 9 teachers' mathematical problem-solving pedagogy in this particular context?	I collected the evidence of the impact of the PD intervention on grade 9 teachers' mathematical problem-solving pedagogy through classroom observations, teacher reflective semi-structured interviews and POFT (in the second cycle).
What is the impact of the PD intervention on grade 9 learners' mathematical problem-solving processes in this particular context?	I used pre- and post-word problem mathematics attainment tests, learner task-based interviews and MPSSI to analyse the impact of the PD intervention on grade 9 learners' mathematical problem-solving processes.
What factors enabled the growth of teachers' mathematical problem-solving pedagogy in this particular context?	Evidence of factors that enabled the growth of participant teachers' mathematical problem-solving pedagogy was collated by observing and analyzing their teaching practices and self-reflections during the reflective interviews.
What factors facilitated learners' development of mathematical problem-solving processes?	Evidence of factors that facilitated learners' development of problem-solving processes was collated by observing learners during lessons and task-based interviews.

3.3 Research paradigm underpinning this study

A research paradigm, also known as a worldview or a philosophical assumption, is a way of looking at the world (Mertens, 2015). These are beliefs about the nature of

reality, plus how one is able to acquire knowledge about that reality (Babbie, 2010; Plano Clark & Ivankova, 2016). From the above, I conclude that the research paradigm is the worldview, ways of thinking, values, practices and assumptions in which researchers operate in their exploration of new knowledge. However, I find Dills and Romiszowski (1997)'s definition on paradigm very persuasive when they state that:

> Paradigms define how the world works, how knowledge is extracted from this world, and how one is to think, write, and talk about this knowledge. Paradigms define the types of questions to be asked and the methodologies to be used in answering them……..Paradigms structure the world of an academic worker, provide its meaning and its significance. (p. 11)

In the literature, the major paradigms are postpositivism, constructivism (interpretivism), pragmatism and transformative (Mertens, 2015), and these are distinguished in terms of ontological, methodological, axiological and epistemological assumptions (Denzin & Lincoln, 1998). Ontology, methodology, axiology and epistemology inform what one wants to research and how one does the research (Wisker, 2008).

Ontology is 'the nature of reality and its characteristics' (Cresswell, 2013, p. 20), or simply, it can be seen as one's view of reality (Wisker, 2008). Ontology poses the following questions: What is the nature of being, reality, or existence? What can be known about it? Methodology is the assumptions about what works best for attaining knowledge (Mertens, 2015). Axiology is the assumptions about ethics and values that we hold and basically focuses on questions about what ought to be. Epistemology is a set of beliefs about knowing, how individuals acquire knowledge and what kind of knowledge can be constructed through the research study. Guba and Lincoln (2005) explain that epistemology is the relationship between what is known and what is seen, and can be seen as the truths that are sought and believed by researchers. All research comes with a view of knowledge or what is knowable and worth knowing and this is termed epistemology.

The application of DBR to this study is underpinned by the pragmatist paradigm; that is known to provide a strong foundation for mixed methods research (Plano Clark &

Ivankova, 2016; Teddlie & Tashakkori, 2011) and is well aligned with DBR (Barab & Squire, 2004; Onwuegbuzie & Leech, 2005). I found pragmatism relevant to this study because it is oriented towards solving practical problems in the physical world rather than on assumptions about the nature of knowledge. I designed a PD intervention that solves a practical problem, that is, supports grade 9 teachers' mathematical problem-solving pedagogy in a particular local context. Next, I discuss pragmatism in terms of the philosophical assumptions of ontology, epistemology, axiology and methodology.

As a researcher, I took the pragmatist paradigm as being appropriate for this study. My ontological position was that there is no single reality and that all individuals understand the world in their own unique ways (Mertens, 2015) and this related to the theme of context which was central to this study. Taking a pragmatist stance implied that I assumed that knowledge was gained through cycles of independent observations and subjective constructions (Johnson & Onwuegbuzie, 2004; Morgan, 2007) and this buttressed taking the DBR approach as a research method. I developed a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy through iterative cycles of DEER. Since I took the pragmatists' position, my belief was that knowledge was acquired through experience and that meaning was negotiated by individuals based on their interpretation and understanding of reality within a social context. Consequently, I collaborated with the participant grade 9 teachers in designing the PD intervention for teachers' mathematical problem-solving pedagogy and we did this in their particular context.

The axiological position of a research underpinned by the pragmatism paradigm is to gain knowledge in pursuit of desired ends as influenced by the researcher's values and politics (Morgan, 2007). As a pragmatist, I placed emphasis on the outcomes of the research, that is, what was done, situations and consequences of inquiry rather than on existing ideas (Creswell, 2013).

Epistemologically, I looked for the best way to interact with participants in the context of study (informal settlement) I was working in to get the information I needed (Mertens, 2009). The research process was driven by the research problem and the research

objectives; that is, designing a PD intervention to support grade 9 teachers in teaching mathematical problem-solving. Since I was working within a pragmatist paradigm, I was free to do what worked to answer the research questions (Cohen, Manion, & Morrison, 2011). This meant that I had the independence to choose the methods, techniques, and procedures of the research that best met the needs and purposes of this study (Cresswell, 2013), and could do what was important and made sense to me in all given situations (Mertens, 2009; Morgan, 2007). It made sense to mix the research components in the way that I believed would work for the research context and the designing of a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy. I engaged multiple methods of data collection to best answer the research questions and to augment the credibility of the findings and conclusions. This is consistent with the vital principle of mixed methods research which advocates that qualitative and quantitative data collection and analysis methods should be mixed in a way that has complementary strengths and no overlapping weaknesses (Creswell & Plano Clark, 2011). I took a 'what works' approach to collecting learner data for this study. In the first cycle, I collected learner data using teacher-made pre- and post-attainment tests and MPSSI. However, in the second cycle, I designed my own pre- and post-attainment tests and engaged task-based interviews. The task-based interviews assisted in evaluating learners' problem-solving processes and to establish if learners were doing problem-solving differently post-intervention as compared to pre-intervention.

3.4 The research design

The research design indicates the general procedure for conducting the study. McMillan and Schumacher (2014) note that it is how the research is set up, what happens to the participants and the methods of data collection engaged. I find Plano Clark and Ivankova (2016)'s definition comprehensive; when they state that a research design is, 'a formal and defined set of procedures for collecting, analysing, and interpreting data to address a specific type of research purpose' (p.138). I found it essential to choose an appropriate research design that focused both on the research problem and the end product. Therefore, I employed the DBR design which I conducted in two phases.

Firstly, I used DBR to specify the plan for generating empirical evidence that I used to answer the research questions (McMillan & Schumacher, 2014). Secondly, DBR directed all the decisions I made during this study and the logic by which I made conclusions at the end of the study (Creswell & Plano Clark, 2011). In the next section, I discuss the DBR in detail.

3.4.1 Design-based Research

Several researchers (Anderson & Shattuck, 2012; Amiel & Reeves, 2008; Brown, 1992; Cobb et al., 2003; Collins, 1992; DBRC, 2003; Herrington, McKenney, Reeves, & Oliver, 2007; Reeves, 2006; Van den Akker et al., 2006; McKenney & Reeves, 2012) have embraced DBR as an emerging framework that can guide enhanced educational research. DBR is sometimes confused with action research because it recognises real-world problems and takes action to solve them. Action research is an inquiry in which practising teachers act as researchers in order to understand, improve and reform their practice (Cohen et al. 2011). Furthermore, in action research, teachers or practitioners initiate the research and researchers give a helping hand with the research process. However, in DBR, researchers usually take the initiative in the research process as both researchers and designers (Wang & Hannafin, 2005). Design is crucial in DBR, while action research emphasises action and change and does not focus on designing a new learning environment. With action research, teachers can work collaboratively or as individuals to inform practice at the local level; however, DBR is always collaborative with a focus on designing interventions and generating design principles. Design principles are one of the major outputs of DBR, and McKenney and Reeves (2012) term them 'theoretical insights into a phenomenon in question', which recommend how to address specific issues in a range of settings. These theoretical insights can be of value to others. Amiel and Reeves (2008) state that:

> The outcomes of DBR are a set of design principles or guidelines derived empirically and richly described, which can be implemented by others interested in studying similar settings and concerns. While the ultimate objective is the

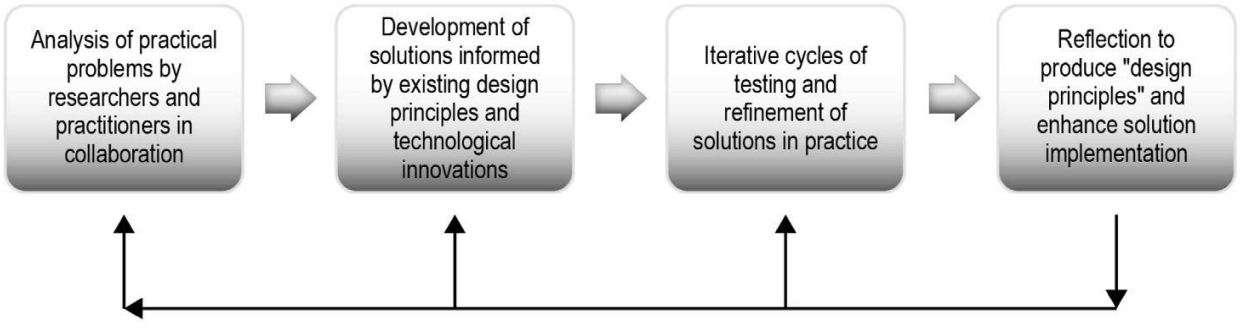
development of theory, this might only occur after long-term engagement and multiple design investigations. (p. 35)

DBR models

A number of researchers have attempted to demonstrate what DBR looks like using models. McKenney and Reeves (2012) suggested a generic model for conducting DBR in education which delineates the comprehensive cyclical nature of the DBR. McKenney and Reeves (2012) generated their generic model from other previous models and frameworks used in educational design research. This generic model consists of three main phases: analysis and exploration, design and construction, and evaluation and reflection that lead to theoretical understanding and intervention maturation.

Baker and van Eerde (2013) propose three phases in a DBR cycle, that is, preparation and design phase, teaching experiment and retrospective analysis. I applied DBR to this study as suggested by Reeves' (2006) cyclic process in Figure 3.1. Reeves (2006) suggests four phases for carrying out DBR which include; analysis of practical problems by researchers and practitioners in collaboration; development of solutions with a theoretical framework; testing and refinement of solutions in practice; and reflection to produce 'design principles'.

Figure 3.1: The design-based research approach

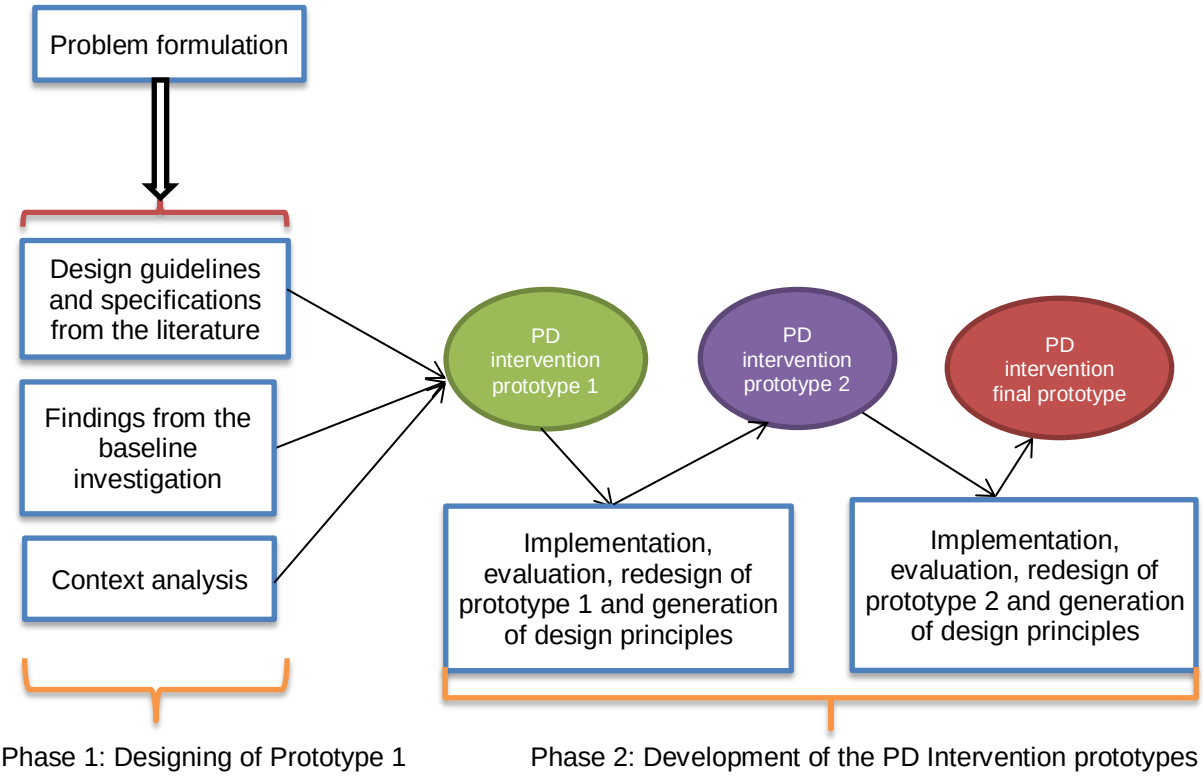


Source: Reeves (2006, p.59).

Figure 3.1 shows that in DBR, educational researchers start by analysing practical problems facing teachers and their learners. The researchers use everything available

including design principles, innovations, and practical experience with teaching and designing (Baker & van Eerde, 2013). For this study, relevant knowledge came from the baseline investigation, mathematics education, literature on problem-solving and the body of knowledge of PD. I developed prototypes and generated design principles through iterative cycles of DEER (see Figure 3.2). I found iterative cycles very important for this research as one cannot claim to produce credible results or generate solutions to a practical problem from a single study. The process of how I carried the DBR in this study is illustrated diagrammatically in Figure 3.2.

Figure 3.2: The research design of this study.



(Source: Adapted from Mafumiko, 2006, p.48)

I carried out the baseline investigation to identify the problem context (Barab & Squire, 2004), that is, to investigate participant teachers' views, teaching strategies and the support they required in their teaching of mathematical problem-solving prior to the PD implementation (see chapter 4). The second phase is named the prototyping phase by

Plomp (2007) and I designed three prototypes in this phase. Initially, I had intended to design four prototypes, but this was not possible due to the availability of time and resources. I designed each prototype in collaboration with practitioners, that is, grade 9 mathematics teachers (DBRC, 2003). I designed the first prototype for the PD intervention using the design principles from the literature and findings from the context analysis and baseline investigation. I then conducted two iterative cycles of DEER to produce prototypes for PD intervention for grade 9 teachers' mathematical problem-solving pedagogy and to generate design principles (see section 5.7 and 6.7). Evaluation and redesign of the prototypes resulted in my looking at what was working or not working and in what conditions and what needed to be changed or improved. I discuss this in greater detail in chapter 4, 5, and 6. In section 3.10, I deliberate to a greater extent on how I implemented the PD intervention in each cycle. In the next section, I outline the features of the DBR and how I conducted it for this study.

3.4.2 Outputs of DBR

Intervention

There is general agreement that DBR should generate valuable educational interventions and useful theory (Van den Akker et al, 2006). DBR uses the terminology 'intervention' to refer to the object, output, and activity or process that is designed as a possible solution to the identified problem. An intervention is defined by McKenney and Reeves (2012) as a comprehensive term used to encompass the various kinds of solutions that are designed. These various solutions can be teaching/learning strategies, educational products, processes, policies or programs. As I indicated before, a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy was designed and developed from this study.

Design principles and theory in DBR

DBR strives to make a theoretical contribution (McKenney & Reeves, 2012) and it is not conducted to empirically tune 'what works' but to develop domain-specific theories (Cobb et al., 2003). DBR does this by developing and refining new theories rather than

testing existing theories using traditional methodologies (Edelson, 2002). Edelson (2002) lists three forms of theories that can be generated from DBR and these are; domain theories, design methodologies, and design frameworks. A domain theory can be best understood as a generalisation of some segment of a problem analysis, for example, particular teachers and their teaching strategies. Meanwhile, a design methodology is a general design procedure, meaning it gives guidelines for the procedure and not the product. While a design framework is a generalised design solution. Edelson (2002) explains that design frameworks depict the attributes that a designed intervention must exhibit to achieve a particular set of objectives in a particular context. This study, therefore, generated a design framework. The design principles for the PD intervention for teachers' mathematical problem-solving pedagogy that were generated display characteristics that can be used by other PD practitioners to support grade 9 teachers in the teaching of mathematical problem-solving (see sections 5.7 and 6.7).

3.4.3 Characteristics of DBR

Cobb et al. (2003), DBRC (2003), Van den Akker et al., (2006), Van den Akker (1999) and Wang and Hannafin (2005) have each identified basic characteristics of DBR. For the purposes of this study, I adopted McKenney and Reeves' (2012) characteristics of DBR because of how they characterise the nature of theory for DBR and this was useful for this study. McKenney and Reeves (2012) state the characteristics as theoretically oriented, interventionist, collaborative, responsively grounded and iterative. In the section below, I discuss each of the given characteristics and how they are illustrated in this study.

Theoretically Oriented

Most research uses existing theory; however, with DBR, scientific understanding is used to craft both the research and to shape the design of a solution to a real problem (McKenney & Reeves, 2012). DBR uses theory to ground the design work, and the product generated then adds to a broader scientific understanding. This is the case with this study. The research was able to contribute to the building of a localised theory

through the systematic evaluation of sequential prototypes of the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy.

Interventionist

Interventionist studies interfere in what naturally happens (Baker & van Eerde, 2013). DBR is interventionist in the sense that it is undertaken to make a purposeful change in a particular functioning educational context. It focuses on designing, implementing and evaluating solutions to problems in real-world settings rather than simulated ones (McKenney & Reeves, 2012). Cobb et al. (2003, p.10) state that 'the intent is to investigate the possibilities for educational improvement by bringing about new forms of learning in order to study them'. This study is interventionist in that, I first identified an educational problem in a particular context that needed a solution; that is, the ineffective teaching of mathematical problem-solving. I am naturally an interventionist as a consequence; I played the role of researcher-developer-evaluator-facilitator and designed, implemented and evaluated a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy in the naturalist world of the participant teachers. The PD intervention and design principles were the new forms of learning.

Collaborative

DBR requires interactive collaboration among the actors (researchers, designers and practitioners) connected to the problem at hand (Cobb et al., 2003; McKenney & Reeves, 2012) and values the knowledge and skills that each can contribute to finding a solution to the problem. By collaborating with teachers and learners, I as the researcher-designer-facilitator-evaluator was able to design, develop, test and evaluate the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy. This intervention can be applied to similar or different educational contexts and the design principles can be used by other mathematical education practitioners.

Responsively grounded

DBR is responsively grounded because it engages theory, empirical evidence collected in real-world settings, and crafts wisdom to guide the work (Herrington et al., 2007;

McKenney & Reeves, 2012). McKenney and Reeves (2012) further assert that educational products generated from DBR are shaped by participant expertise, literature and principally field testing. DBR is structured to explore the complex realities of education and responds accordingly. In this study, the PD intervention evolved over the two cycles of DEER. Through this grounded process, I was able to understand teachers' mathematical problem-solving pedagogy and learners' problem-solving processes and design and redesign the PD intervention.

Iterative

With DBR, development and research takes place through multiple cycles of investigation, design, enactment, analysis and redesign (Collins, 1992; DBRC, 2003; Wang & Hannafin, 2005). For this study, DBR enabled the refining of the PD intervention based on cycles in real-world settings (see Figure 3.2). As I indicated earlier, this study adopted Reeves' (2006) model for conducting DBR (see Figure 3.1). DBR is a non-linear procedure where a single cycle of DEER phase is not anticipated to complete the design process. For this study, I used the findings from the two phases in a cyclical manner over two cycles to produce theoretical understanding, design principles and a mature PD intervention for grade 9 teachers' mathematical problem-solving pedagogy.

3.5 Researcher's role: a nuisance variable or powerful praxis?

In DBR, the researcher has a strong role in the intervention and research process; and this can occasionally be seen as problematic (Kelly, 2004). My role was simultaneously that of a researcher, developer, facilitator, and evaluator of the intervention (McKenney & Reeves, 2012) and a critical friend of participant teachers. I designed, enacted, analysed and redesigned the PD intervention throughout this study. I acted as a facilitator during the PD workshops; training the teachers and facilitating their discussions. I organised the PD activities (see section 3.10) that generated opportunities for teacher learning. Moreover, I guided teachers' discussions during collaborative problem-solving; however, during this process I was also wearing the hat of a researcher which made me cautious so as not to exhibit the image of an expert to

the teachers. As a researcher, I focused on how participant teachers received and implemented the new ideas on mathematical problem-solving pedagogy that I was facilitating.

When teachers implemented the new ideas on mathematical problem-solving pedagogy, I was with them in the classrooms as a critical friend; supporting and helping them to overcome obstacles they were experiencing. During the implementation stage, it was challenging for me to keep the critical position of a researcher and at the same time build a close relationship with the teachers as a critical friend. There were times when participant teachers lost control of the classroom in terms of discipline or were teaching learners incorrect content. In this instance, I could have easily intervened as a PD facilitator, but since I was carrying the hat of the researcher I chose not to interfere. At the beginning of each cycle, I noticed that all participant teachers struggled to sustain classroom discussions because they lacked broad MPSKT (see section 2.4.4) and struggled with making conjectures on the possible solution strategies their learners would have used to solve a given problem. To reach a balance, I only supported teachers in terms of implementing new ideas of mathematical problem-solving pedagogy.

I found my role of being a researcher-developer-facilitator-evaluator-critical friend a powerful praxis because I personally designed the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy and I witnessed its implementation by the participant teachers. My role is termed a 'nuisance variable' by McKenney and Reeves (2012, p.150). This is because DBR has generally a need for the researchers to immerse themselves in the research setting and to interact with research participants. Subsequently, researchers conducting DBR can struggle to keep objective and be unbiased. I embraced and compensated for this drawback, as recommended by McKenney and Reeves (2012) by keeping a reflexive journal and unequivocally describing my presence in the research setting and discussing my real or possible influence on the data (see section 3.8.1). Penning and tracking my thoughts in the reflexive journal increased transparency and refined my ideas, personal assumptions,

views, and reflections about the research process thereby minimising the researcher bias.

3.6 Research method

According to McMillan and Schumacher (2014), a research method refers to how data is collected and analysed. There are three major research methods: the qualitative research method, the quantitative research method and the mixed methods research method. The choice of these methods depends on the purpose of the research study. DBR requires multiple ways of evaluating the implementation process and achievement of desired outcomes. Therefore, the research method I adopted for this study was the mixed methods research design (Creswell & Plano Clark, 2011). Mixed methods enhanced the application of DBR for developing the PD intervention for mathematical problem-solving pedagogy. In the next section, I discuss the nature and origins of the mixed methods research method and the reasons why I adopted it for this study.

3.6.1 Mixed methods research design

I utilised a mixed methods research approach to collect the data for this study mainly because the iterative cycles of DBR utilises multiple sources of data. I integrated both quantitative and qualitative methods of data collection and analysis within a single study to best understand grade 9 teachers' mathematical problem-solving pedagogy (Creswell & Plano Clark, 2011; Denscombe, 2010; Johnson, Onwuegbuzie & Turner, 2007; Hearn, 2014; McMillan & Schumacher, 2014; Plano Clark & Ivankova, 2016). In order to be in a position to answer research questions for this study, I adopted the characteristics of the mixed methods research design from Creswell and Plano Clark (2011) who list that in a mixed methods research design the researcher:

- collects and analyses rigorously and persuasively both qualitative and quantitative data;
- integrates the two forms of data concurrently by merging them, or sequentially by having one build on the other or embedding one with the other;
- gives priority to one or both forms of data;

- uses these procedures in a single study or multiple phases of a programme of study;
- frames these procedures within philosophical worldviews and theoretical lenses; and
- combines the procedures into specific research designs that direct the plan for conducting the study. (p.5)

The value of using a mixed methods research design is well documented in the literature (Cresswell, 2012; Creswell & Plano Clark, 2011; Greene, 2007; Hearn, 2014; Johnson et al., 2007; Plano Clark & Ivankova, 2016). The major reason being given for the use of mixed methods research design is that it allows the researcher to incorporate the strengths of each method thereby offsetting the weaknesses and limitations of both quantitative and qualitative research. Denscombe (2010) lists the following as the benefits of engaging a mixed methods research design; increase in accuracy of the findings through the use of various methods to investigate the same subject; provides a more complete picture of the phenomenon that is being studied than would be yielded by one approach; combining different methods overcomes the weaknesses and biases of single methods and this results in more credible conclusions; enables the researcher to move the analysis forward and build on the original data; and aids sampling (Denscombe (2010) gives an example of a questionnaire which can be used to screen prospective interview participants).

The rationale for employing a mixed methods research design for this study was for two reasons; the first being for its iterative and cyclical approach to research (Teddlie & Tashakkori, 2011) thereby, fitting perfectly with DBR. Mixed methods research design has an iterative nature in that the researcher may go back and forth between quantitative and qualitative data collection. DBR researchers utilise mixed methods to analyse an intervention's outcomes and refine the intervention (DBRC, 2003). Wang and Hannafin (2005) advocate that mixed methods research design is appropriate for DBR because it augments the validity, objectivity, and reliability of an ongoing research.

The second reason for employing mixed methods research design was for triangulation and complementarity (Plano Clark & Ivankova, 2016). Triangulation means directly comparing results from quantitative methods to those drawn from qualitative methods for the purposes of convergence and divergence. Complementarity denotes engaging both quantitative and qualitative methods to obtain complete conclusions about different facets of a phenomenon (Teddlie & Tashakkori, 2009). Faced with the problem of designing a PD intervention to support grade 9 teachers in the teaching of mathematical problem-solving, I realised that the combined use of quantitative and qualitative approaches would provide more evidence and a better understanding of the possible solutions for the research problem than either approach by itself. Capitalising on the strengths of each approach facilitated this study to produce a 'stronger and more credible' PD intervention (Plano Clark & Ivankova, 2016, p.9).

Creswell and Plano Clark (2011) state six major types of mixed methods research design as the explanatory sequential design, exploratory sequential design, convergent design, embedded design, transformative design and multiphase design. I employed the convergent research design for this study and I discuss it in greater detail in the next section.

3.6.2 The convergent research design

The convergent research design also known as the concurrent Quantitative + Qualitative design (Plano Clark & Ivankova, 2016), convergent parallel design (Cresswell, 2012), concurrent triangulation design (Cresswell, 2009; Hearn, 2014), and triangulation (McMillan & Schumacher, 2014) was defined by Creswell and Plano Clark (2011) as the process in which the researcher simultaneously collects and analyses both quantitative and qualitative data during the same phase of the research process and then merges the two sets of results into an overall interpretation. This view concurs with that of McMillan and Schumacher (2014), who state that in a triangulation design, the researcher simultaneously gathers both quantitative and qualitative data, merges them using both quantitative and qualitative analysis and then interprets the findings together to provide a better understanding of the research problem. I simultaneously

collected both quantitative and qualitative data and gave them equal emphasis. This gave me the opportunity to combine the strengths of both methods. I collected and analysed the teachers' data using qualitative methods. In order to lessen possible potential alternative explanations to the findings, I concurrently gathered and analysed learner data using both qualitative and quantitative methods. In the end, I triangulated the findings from the teacher and learner data to achieve a more robust and complete understanding and development of the PD intervention for teachers' mathematical problem-solving pedagogy (Plano Clark & Ivankova, 2016).

3.7 Data collection procedure

I purposefully selected the region of Gauteng in South Africa for reasons of accessibility and convenience, a valid and useful approach pointed out by McMillian and Schumacher (2014). I employed nonprobability sampling to choose grade 9 teachers and their learners at secondary schools in a certain district in Gauteng. Furthermore, I used nonprobability sampling because I wanted to understand participant grade 9 teachers' mathematical problem-solving pedagogy without desiring to generalise all grade 9 South African mathematics teachers. Secondary schools were deliberately chosen because the ANA Diagnostic report (DBE, 2014) points out that learners' performance tends to deteriorate progressively from intermediate to senior phase.

In March 2016, I conducted a baseline investigation, of which the results are provided in chapter 4. Participants were 31 grade 9 teachers at 20 schools in Gauteng district. The baseline investigation examined how grade 9 mathematics teachers in the district of study were using problem-solving in their teaching of mathematics (see chapter 4). I 'purposefully' selected two schools out of the initial 20 schools. Purposive sampling is when a researcher intentionally selects a sample using their knowledge of a population and taking into account the purpose of the study (Babbie, 2010; Cresswell, 2012; Cohen et al., 2011; Denscombe, 2010; McMillan & Schumacher, 2014). I chose the two schools because I could conveniently access them and they represented typical South African secondary public schools which are multi-lingual, under-resourced, overcrowded and have large classes. The grade 9 participant mathematics teachers had reported on

the baseline investigation open-ended questionnaire that they were using traditional methods of teaching and had agreed to participate in this study. I named the two selected schools A and B (see section 1.5.2). The goal was to work with at least two grade 9 mathematics teachers from each school; however, this was dependent on the number of teachers teaching grade 9 mathematics at a particular school. Both the two schools I selected for this study had two teachers instructing grade 9. The limitation of focusing on one district and only two schools is that I was not able to explore how a PD intervention can be designed and implemented to support grade 9 teachers in other South African districts in teaching mathematical problem-solving. However, the main purpose of this study was not to investigate the phenomenon with all grade 9 South African mathematics teachers, but to comprehend the particulars of the cases studied in their complexity. Therefore, the results of this study can be used to understand how best to support grade 9 teachers in a particular local context to modify their mathematical problem-solving pedagogy.

Grade 9 mathematics teachers in school A participated in the first cycle and grade 9 mathematics teachers in school B participated in the second cycle (see sections 1.5.2, 5.4.1 and 6.4.1 for the explanation of participants).

3.8 The data collection tools

I collected data through a Mathematics Teacher Open-ended Questionnaire, direct participant observations, teacher reflective semi-structured interviews, memo-writing, personal reflexive journal, pre- and post-word problem mathematics attainment tests, learner task-based interviews and a MPSSI. In table 3.2, I present an overview of the timelines of the research and the extent of data collected at each point.

Table 3.2: An overview of the timelines of the research and the extent of data collected at each point.

Stage of the study	Data collection method	Extent of the data collected
Cycle 1	Pre-intervention observations	Once a week during the first month, and in total, four observations with each teacher
	Observations during the intervention	Five classroom observations
	Semi-structured reflective interviews	Five semi-structured reflective interviews
	Pre- and post-word problem mathematics attainment tests	At the beginning and end of the cycle
	MPSSI	At the beginning and end of the cycle
Cycle 2	Pre-intervention observations	Once a week during the first month, and in total, four observations with each teacher
	Observations during the intervention	Three classroom observations
	Semi-structured reflective interviews	Three semi-structured reflective interviews
	Pre- and post-word problem mathematics attainment tests	At the beginning and end of the cycle
	MPSSI	At the beginning and end of the cycle
	Learner task-based interviews	One Tuesday every two months of the cycle and in total, three task-based interviews in the cycle

3.8.1 Qualitative data collection tools

My reflexive journal

From the onset of this study, I kept a reflexive journal where I wrote in at least once a week. The journal had two purposes. First, I used it to document all my thinking processes, insights, experiences, and the events that emerged during the designing of the PD interventions, workshops, implementation of the pedagogy, classroom observations, semi-structured reflective interviews with teachers, task-based interviews with learners and data analysis. Secondly, I used it as a reflective tool to analyse and modify what was happening in each cycle so that I could feed it into the next cycle.

The other motivation for keeping a reflexive research journal throughout the research process was because comprehensive documentation has a positive correlation with the dependability of results (McMillan & Schumacher, 2014). To assist researcher reflexivity and the dependability of the findings, I included information from the reflective journal during data analysis.

The Mathematics Teacher Open-ended Questionnaire

Open-ended questionnaires 'exert the least amount of control over the respondent and can capture idiosyncratic differences (McMillan & Schumacher, 2014, p.214) and are used when a researcher needs to quickly and easily get abundant information from people in a harmless way (Mertens, 2015). Mayer (1999) perceives that teachers' self-report research tools remain the most feasible means of obtaining information about the status of mathematical pedagogy. In order to investigate participant teachers' views, teaching strategies and the support they required in their teaching of mathematical problem-solving, I administered the Mathematics Teacher Open-ended Questionnaire prior to the implementation of the PD intervention. The Mathematics Teacher Open-ended Questionnaire is a form of an open-ended questionnaire that allows teachers to self-evaluate themselves with respect to the teaching of mathematical problem-solving. It consists of two sections (see Appendix H). The first part is about the teachers' demographic information. In the second part, the questionnaire solicits participants'

ideas on the nature of mathematical problem-solving, the approaches they use in teaching problem-solving in their classes, difficulties they face with the teaching of mathematical problem-solving and the support they require. The questionnaire was open-ended because I neither wanted to assume teachers' knowledge nor direct teachers' responses to the items. This allowed participant teachers to freely express their experiences, views and attitudes on the teaching of mathematical problem-solving without being influenced by my opinions as the researcher. The other motivation for using a questionnaire was because it is economical in terms of time and money (Cohen et al., 2011).

Cohen et al. (2011) list the disadvantages of using a questionnaire as that there is a low return and the interviewer is not available to address respondents' concerns and misunderstandings. To counter these disadvantages, I travelled for two weeks to 20 secondary schools asking grade 9 mathematics teachers to respond to the questionnaire. In total, 31 grade 9 mathematics teachers responded to the questionnaire. I was present in the schools when respondents filled in the questionnaire and I was also able to address any concerns that arose and elaborated if there were misunderstandings with the questionnaire items. I pilot-tested the questionnaire before the data collection process (see section 3.9.1) in order to establish the practicability, suitability and appropriateness of the items and to enhance the dependability of the findings.

Direct teacher observation

I employed direct participant observation as a data collection tool because of its uniqueness in capitalising on the investigators' seeing and hearing things and recording these observations, instead of depending on a participant's self-report responses to questions or statements (McMillan & Schumacher, 2014). Bell (2005) and McMillan and Schumacher (2014) agree that the observation of participants uncovers physiognomies and elicits data that is almost impossible to get with other data collection methods. The major motivation for choosing direct observation as a form of inquiry was because I wanted to record teachers' mathematical problem-solving pedagogy as it occurred and

to observe their natural behaviour when teaching mathematical problem-solving. I recorded my observations on an Observation Comment Card (see Appendix O). Moreover, I adopted and modified the teacher observation form that was employed by Alsawaie (2003) when he examined the Abu Dhabi Educational zone mathematics teachers' roles and interventions with their learners' work while engaged in problem-solving. The Observation Comment Card is a valid tool for the collection of data from teachers during the teaching of mathematical problem-solving because it incorporates Polya's (1957) problem-solving processes, which forms the backbone of this study. Teachers need to assume certain roles in each problem-solving step and these are all covered in the Teacher Observation Comment Card. Classroom observations which were all audio-recorded were conducted before, during and after the PD intervention. I conducted a different number of teacher observations for each cycle and I explain this in detail in section 3.10. Audio-recordings permitted the recording of the observed lessons as they transpired and gave me the chance to re-experience the lessons at a later convenient time (Chirinda & Barmby, 2017). I also used the data from the observations to facilitate the reflective semi-structured interviews so that the findings were comprehensive.

Teacher reflective semi-structured Interviews

Semi-structured interviews are one of the 'most powerful ways in which we try to understand our fellow human beings' (Fontana & Frey, 2000, p.645). These can be taken as a dialogue between a researcher and participants, with the researcher endeavouring to understand the behaviour of the participants without imposing any *a priori* categorisation which might limit the field of enquiry (Punch, 1998). Although semi-structured interviews are known to be time-consuming, I used them as a data collection tool because they are known to be adaptable, resulting in a much higher response rate than questionnaires (McMillan & Schumacher, 2014). They are more flexible (Cohen et al., 2011), and thus facilitate broader data analysis than structured interviews. Semi-structured interviews (see Appendix P), which were conducted with all four participants in the two cycles, were extremely important as a data collection tool for this research because they drew out data which participant teachers did not display during classroom

observations. Semi-structured interviews allowed for unplanned follow-up questions which permitted me to probe teachers each time I interviewed them.

Semi-structured interviews consist of questions that are fairly specific in their intent and are phrased to permit individual responses (McMillan & Schumacher, 2014). For this study, the interview questions were open-ended to ensure stimulating interview dialogues that would enable me to hear the thinking processes, learning and teaching practices of each teacher. I also engaged semi-structured interviews so as to fully comprehend participant teachers' experiences and the sense they made from these experiences through their description of the experience and their reflection on the description (De Vos, Strydom, Fouche & Delport, 2011).

The interviews were conducted in participant teachers' classrooms during their free periods. To uphold accepted levels of accuracy in the teacher reflective semi-structured interviews, I asked teachers open-ended questions that I continuously adapted as the study progressed. These questions were not entirely pre-planned as I left room to probe participants' responses to get clarity on issues that were under discussion. I utilised the Instructions for the Interviewer used by Fennema, Carpenter and Loef (1990) in their study on Belief Interview (see Appendix P). With teachers' permission, I audio-recorded the semi-structured interviews. I engaged audio-recordings to collect more detailed information from the semi-structured interviews for subsequent analysis. Replaying the interview audio-recordings provided me with sufficient time to study and think about the data before making any conclusions. I audio-recorded the interviews in order to augment the credibility of the findings (see section 3.12.2.1). I pilot-tested the interview schedule before the data collection process (see section 3.9.1) in order to enhance the dependability of the interviews.

Learner task-based interviews

I used task-based interviews in the second cycle because they mainly focus on drawing out learners' thinking about problems, learners' growth in mathematical knowledge, learners' mathematical problem-solving processes and learners' ways of mathematical reasoning. During a task-based interview, the learner or learners being interviewed

interact with the interviewer within a task environment (Goldin, 2000). The interviews may be structured or semi-structured and should lead learners to scrutinise their problem-solving processes and to self-correct mistakes in their own solution strategies. I opted for semi-structured task-based interviews since they allowed some form of flexibility in questioning learners according to their problem-solving processes and mathematical reasoning.

Task-based interviews were an important part of this study. This is due to the realisation that they are advantageous over classroom observations, questionnaires and post-attainment tests in that the teacher or researcher can probe learners' thinking (Ginsburg, 2009) and dig deeper into their reasoning and problem-solving processes (Schoenfeld, 2002). Consequently, I introduced task-based interviews in the second cycle of this study to gain insight into learners' problem-solving processes. I did this in order for the task-based interviews to help explain why learners were doing well in the post-test and why they were giving better responses in the post-MPSSI. Learners participated in the interviews either as individuals, in pairs or as a group.

Memo writing

The memos I wrote in the reflexive journal were about my thoughts, conversations with teachers and learners, how conversations with teachers and learners were informing the data collection process, the data analysis, how themes emerging from the inductive data analysis were related to each other and how they were answering the research questions. I found memos useful in terms of helping to frame and reframe the focus of inquiry as the study evolved from the baseline investigation to the first cycle and finally to the second cycle (Miles & Huberman, 1984). Below is an example of a memo that I wrote after visiting a Grade 9 mathematics teacher at a farm school during the baseline investigation:

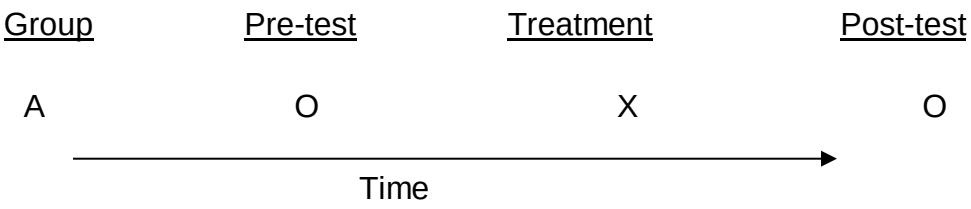
> The school is a government school at a farm that is just outside an urban area. There is only one grade 9 mathematics teacher teaching 5 classes. Each class consists of not less than 40 learners. This taught me humility. During the visit, I kept on wondering how he is able to teach 5 huge classes in one go. In his responses to the questionnaire, the

teacher noted that he taught problem-solving by introducing the topic through working examples on the board. Once he felt that learners understood the worked examples, he would give them exercises to work on. His difficulties were that his learners had no solid background in mathematics from grade 8 and it was hard to teach them problem-solving. He did not think problem-solving was applicable to him since he had large classes and needed to cover the CAPS curriculum within the prescribed time.

3.8.2 Quantitative data collection tools

I implemented pre- and post-mathematics attainment tests and the MPSSI to uncover participant grade 9 learners' performance in mathematics and their mathematical problem-solving processes. For the quantitative strand, I employed a single-group pre-test-post-test design after the baseline investigation, and this is represented in Figure 3.4 below. Single-group pre-test-post-test design is a form of pre-experimental design (McMillan & Schumacher, 2014), and its purpose is to determine cause and effect. I chose this design due to the focus of the study; learners were not randomly assigned to groups as in true experimental situations, but their classes were already organised for teaching purposes.

Figure 3.3: Single-group pre-test-post-test design.



In the above diagram, participant learners were given a pre-test (O), then taught by teachers who were receiving the PD intervention on the teaching of mathematical problem-solving (X), and given a post-test (O) at the end of the experiment. With the single-group pre-test-post-test design, a number of threats to internal validity are plausible and I expound on these in section 3.12.1. To improve internal validity for this design, McMillan and Schumacher (2014) recommend that 'adding a second pre-test and post-test of a construct similar to the one being tested' (p. 290). Hence, I used pre-

and post-mathematics attainment tests together with the pre- and post-MPSSI to test learners' problem-solving processes and skills.

Mathematical Problem-solving Skills Inventory (MPSSI)

The MPSSI is a list of items that a learner checks selectively to give a systematic self-appraisal of mathematical problem-solving processes and skills (see Appendix Q). During my master's degree studies, I developed the MPSSI based on the mathematical problem-solving skills found in the literature (Hiebert & Wearne, 1993; Polya, 1957; Van de Walle et al., 2013). As I indicated earlier, I administered the MPSSI to grade 9 mathematics learners of participant teachers in both cycles, at the beginning and at the end of the intervention. I did this in order to be in a position to establish the mathematical problem-solving processes and skills the grade 9 learners perceived as having developed after the intervention. Learners were required to evaluate their own competencies on each item on a 5-point scale. I chose the MPSSI as a data collection tool because it provided learner data that supplemented the data from pre- and post-attainment tests and task-based interviews and this allowed learners to fully contribute to the data collection process. In order to elicit honest responses from the learners, I established an atmosphere in which learners felt that their honest responses were extremely imperative to their learning of mathematics.

I calculated the Cronbach alpha for the overall mathematical problem-solving skills inventory to determine its internal consistency. Internal consistency is the most common type of reliability. Moreover, it is the consistency of items within an instrument (McMillan & Schumacher, 2014; Cohen et al., 2011), that is, how well items on an instrument measure the same construct or idea. The reliability coefficients (for pre- and post-tests) were generally above 0.7 which was sufficient for this instrument (McMillan & Schumacher, 2014). McMillan and Schumacher (2014) further state that the Cronbach alpha is generally the most appropriate type of reliability for questionnaires in which there is a range of possible answers for each item and this was the case with the MPSSI that I used for this study.

Pre- and post-word-problem attainment tests

Word-problem attainment tests are made up of items that are answered by supplying the requested information. The information may be a word, number, phrase, sentence or collection of symbols that complete a statement. Word-problem tests are useful in evaluating learners' mathematical problem-solving processes because they require learners to supply the answer thereby avoiding guesses. I used teacher-made tests for the first cycle. However, for the second cycle, I designed my own pre- and post-mathematics attainment tests that covered the topics that the learners were doing at that particular time (see Appendix S and T). The teacher-made tests used in the first cycle were not included in this thesis because the participant teachers did not consent. Tests were useful in evaluating learners' performance; however, they only provided a limited snapshot into their mathematical problem-solving processes. The questions of the pre-tests were of different content from the post-tests but were of the similar context (Cohen et al., 2011).

The mathematical pre- and post-word-problem attainment tests afforded me the opportunity to view learners' work. This provided me with a greater understanding of their progress in developing mathematical problem-solving processes and skills. To enhance validity, the mathematical tests consisted mainly of conceptual questions that had a high probability of engaging all the learners and called for learners' use of problem-solving processes and skills. I designed the conceptual questions using the guidelines on the nature of worthwhile and appropriate problems that are found in the literature and are explained in the literature review chapter in section 2.3.1.

I checked on the reliability of the pre- and post-test. The Cronbach alpha was 0.65 for both and this was acceptable. The tests were pilot-tested prior to the data collection process, and this I discuss in section 3.9.2. I worked towards the content and construct validity of the tests and this I discuss in section 3.12.1.

3.9 Pilot study

A pilot study is a mini-version of a large study that is conducted in advance to prepare for the complete study. Polit, Beck, and Hungler (2001, p.467) refer to it as a feasibility study which is a 'small-scale version, or trial run, done in preparation for the major study'. Baker (1994) remarks that a pilot study can also be the pre-testing of a particular research instrument. This is the case with this study where research instruments were tried out with a limited number of teachers and learners from the same population (district) as that for which the main study was intended and focused on.

I conducted a pilot study prior to embarking on the full-scale DBR study to identify problems, the required changes (Kumar, 2011), and suggestions to improve the reliability, clarity and format of the research instruments (McMillan & Schumacher, 2014; Mertens, 2015). De Vaus (1993) emphasises that one should not take the risk but to pilot-test first. Bell (2005, p.147) concurs with this when she stresses out that, 'however pressed for time you are, do your best to give the questionnaire a trial run' as pilot-testing predicts difficulties that may arise during the subsequent data collection. Piloting helped with the wording of the items in the instruments, ordering of question sequences in the semi-structured interviews and teacher open-ended questionnaire, and to identify unclear or ambiguously formulated items in the MPSSI. Pilot-testing is strongly recommended by Babbie (2010) as it improves reliability in that it increases the likelihood of how people understand the items or statements of research instruments.

3.9.1 Piloting of the teacher research tools

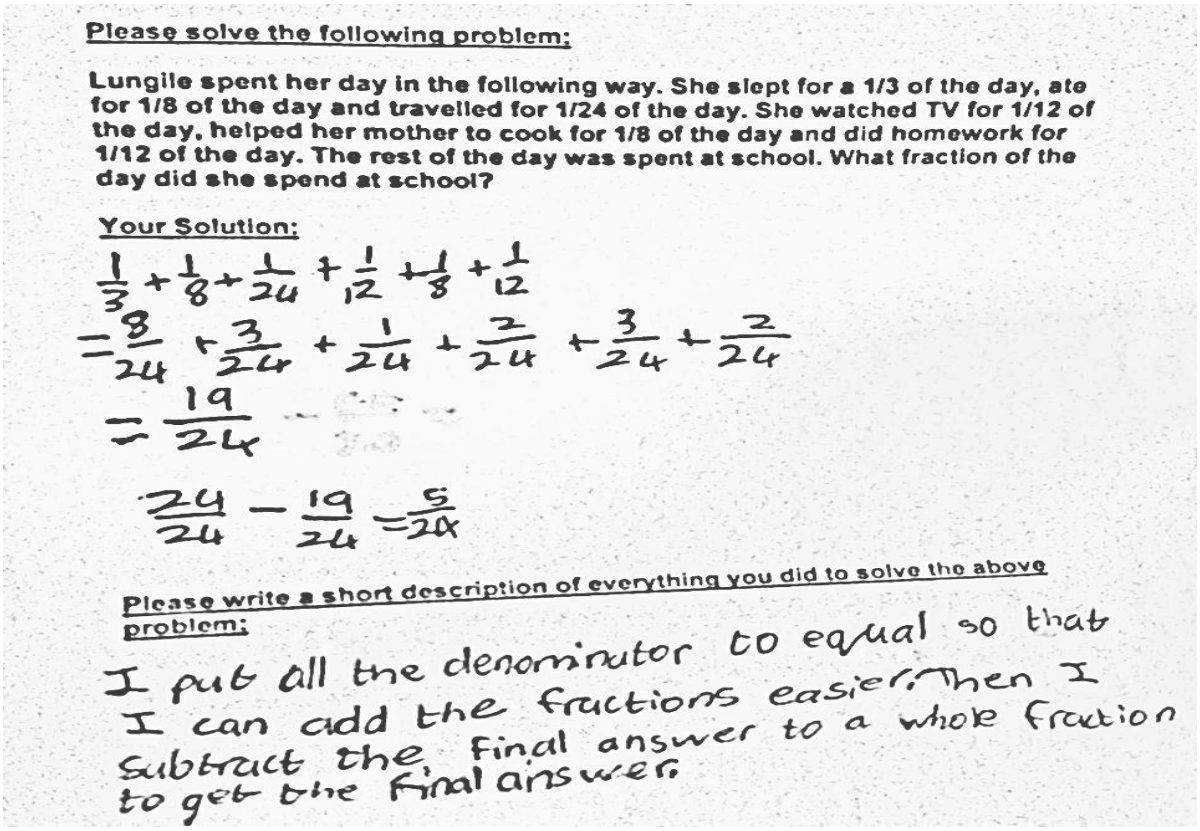
I pilot-tested the Mathematics Teacher Open-ended Questionnaire, the semi-structured interviews and the Teacher Observation Comment Card with two teachers in February 2016. These teachers were not involved in the main study although they were at a school in the same district where I conducted the main study. The main aim for piloting was to establish if the data collection tools were relevant to the context of study. Piloting also gave me an initial idea of the pattern of responses that were likely to be given in the Mathematics Teacher Open-ended Questionnaire and an estimation of the amount of time that was required to complete the semi-structured interviews. I did not modify

any items on the Mathematics Teacher Open-ended Questionnaire since I established that the instructions and content of the questions were appropriate to this study and would be in a position to provide the information sought. Piloting of the semi-structured interview questions revealed that the interviews were too long, taking between 40 and 50 minutes, that some questions were unintentionally repeated and others were ambiguous and the audio-recorder produced a poor audio quality. I deleted the duplicated and unnecessary questions from the interview schedule and I clarified the ambiguous questions were clarified thereby adjusting the interview length to between 20 and 30 minutes. I purchased a modern and advanced audio-recorder. Piloting of the Teacher Observation Comment Card gave me the opportunity to reflect on its items and what I was observing. I did not make any changes on the observation comment card as it covered all aspects of Polya's (1957) problem-solving processes.

3.9.2 Piloting of the learner research tools

Initially, I had designed a learner open-ended questionnaire to gather learners' mathematical problem-solving approaches. I pilot-tested it with ten learners, at the same time and school as that of the two teachers. The pilot-test revealed that the open-ended questionnaire would not provide the required information on learners' mathematical problem-solving processes and skills. A sample is shown in Figure 3.4 on how learners responded to the open-ended questionnaire. All ten learners did not explain the problem-solving process behind the calculation, but explained how they added the fractions.

Figure 3.4: A sample of learners' responses to the learners' open-ended questionnaire.



Please solve the following problem:

Lungile spent her day in the following way. She slept for a 1/3 of the day, ate for 1/8 of the day and travelled for 1/24 of the day. She watched TV for 1/12 of the day, helped her mother to cook for 1/8 of the day and did homework for 1/12 of the day. The rest of the day was spent at school. What fraction of the day did she spend at school?

Your Solution:

$\frac{1}{3}+\frac{1}{8}+\frac{1}{24}+\frac{1}{12}+\frac{1}{8}+\frac{1}{12}$

$=\frac{8}{24}+\frac{3}{24}+\frac{1}{24}+\frac{2}{24}+\frac{3}{24}+\frac{2}{24}$

$=\frac{19}{24}$

$\frac{24}{24}-\frac{19}{24}=\frac{5}{24}$

Please write a short description of everything you did to solve the above problem:

I put all the denominator to equal so that I can add the fractions easier. Then I subtract the final answer to a whole fraction to get the final answer.

In this regard, I decided to adopt the MPSSI that I had designed during my master's degree studies. I wrote the MPSSI items in a simple, clear and understandable language for learners. I pilot-tested the MPSSI at the end of February 2016 to identify and refine difficult items; to record the time taken to complete it by learners; to decide whether it was reasonable; and to find out the clarity of the instructions and whether the layout was clear (Bell, 2005). Learners followed the instructions correctly, completed the inventory within a reasonable timeframe and had no problem in understanding most items except item 5 which stated: 'I can divide the problem into smaller parts'. Most learners left it blank or put a comment next to the statement that indicated that they did not understand what it meant. I then replaced the statement with 'I can separate different parts of the problem' (Barmby *et al*., 2014, p.14). When I pilot-tested this statement, the learners understood and completed it without any difficulty.

I pilot-tested the task-based interviews with a group of three learners. With the permission of the school, parents/guardians and learners, I video-recorded the episodes. However, in the main study learners were only audio-recorded because I could not get consent from the school to video-record them (see section 6.3.3).

3.10 Design and implementation of the professional development intervention

This study focused on the designing, implementing, evaluating, and redesigning of a PD intervention that can be used to support grade 9 South African teachers in teaching mathematical problem-solving. The first prototype of the PD intervention (see chapter 5) was designed from the design principles generated from the literature review, context analysis and baseline investigation. I implemented the first prototype in the first cycle. I designed the second prototype from the design principles generated from the first cycle and I implemented it in the second cycle. From the design principles generated in the second cycle I redesigned the second prototype to develop the third prototype (see Figure 3.2). Because of time and resources, I was unable to implement and evaluate the third prototype.

I designed the cycles to take place within a period of six months. The chief goals I set for the PD intervention prototypes were to support teachers' mathematical problem-solving pedagogy and improve learners' problem-solving processes. PD took place during the process of classroom instruction in order to link with classroom teaching (Barber & Mourshed, 2007). In the first cycle, I conducted three PD workshops with participant teachers on the last Wednesday of the first, third and fifth months of the 6-month period (see section 5.3). In the second cycle, the workshops were on the last Friday of the first, third and fifth months of the 6-month intervention period (see section 6.3). The workshops were from 12 pm to 3 pm and that worked out to three hours per workshop which resulted in a total of nine hours of training for participant teachers in each cycle. I carried out the training at a community centre in the district of study. I received funding from a non-governmental organisation and gave participant teachers a stipend to arrange work for their classes before leaving the school for the program and to cover their lunch and transport costs.

Next, I discuss how I conducted the PD with the teachers. The activities I implemented were theoretically informed by the literature and my personal preferences as well as the context of the study. The first workshop of both cycles had five main activities which were 30 minutes long with a 30-minute break between the third and fourth activities. I initially presented the workshop's contents to the participant teachers and what they were expected to do. Below I discuss the activities that teachers participated in during the PD workshops.

Activity 1: Teachers watched two short videos either from Japan, Singapore or the USA on mathematical problem-solving pedagogy. The baseline investigation I conducted revealed that participant teachers had a partial understanding or misconceptions of what mathematical problem-solving pedagogy entails (see chapter 4). Participant teachers believed that the teaching of mathematical problem-solving was about explaining to learners each and every concept, step-by-step, breaking down the topic, working out examples on the chalkboard and giving learners practice exercises to work on. This study is about supporting teachers' mathematical problem-solving pedagogy using Polya's (1957) problem-solving framework, therefore I chose videos that clearly showed teachers teaching mathematics using Polya's (1957) four problem-solving processes (see section 2.3.4). The videos, though from different contexts, were to prompt discussions about mathematical problem-solving pedagogy; to contrast and to compare the teaching of mathematical problem-solving; and to inspire and model the teaching of problem-solving to participant teachers. I discussed the videos with the teachers, focusing on what genuine mathematical problem-solving pedagogy entails and how to apply Polya's (1957) four steps of problem-solving as a teaching process. Heuristics (see section 2.3.4.2) and their integration into teaching problem-solving were introduced during this discussion and examined in detail. I provided teachers with Polya's *How to solve it* (1957) book at the beginning of the workshop and we used it during the discussions.

Activity 2: This activity was about collaborative problem-solving. I first explained to teachers the aspects and advantages of collaborative learning and clearly defined roles each teacher had to play. Among several aspects, I discussed with teachers that with

collaborative learning teachers should have mutual respect and be willing to learn from each other. Teachers collaboratively solved at least two word problems relating to the topic they were teaching at the time. These were problems teachers could implement in their lessons and that I had experienced as a mathematics teacher educator that they were engaging to learners. I explained to teachers that they had to share ownership of the problem, its solution strategy and the final answer. Teachers expounded on ways of introducing or posing these problems in such a way that learners understood the given problems. Teachers also collaboratively devised how to teach these problems to their learners.

Activity 3: We discussed a structure of implementing the mathematical problem-solving pedagogy in the lesson and we came up with guidelines that we adapted from Polya's (1957) problem-solving steps (see section 2.3.4):

- Reviewing the previous lesson or homework.
- Lesson starts with a problem (preferably word problem) in the textbook or the teacher poses a problem related to those found in the CAPS curriculum.
- Learners read the problem as individuals or in pairs and the teacher determines if all learners understand the problem. If some learners do not understand the problem at hand, the teacher assists learners to understand by using approaches in section 2.3.4.1. These problems include teacher stressing to learners the importance of reading and re-reading mathematical problems carefully with the aim of understanding before solving them. I further worked with teachers on how they could work with learners on understanding given word problems. I encouraged teachers to ask learners during lessons questions like: Do you understand what the problem is looking for? Do you know all the words? Can you repeat the problem in your own words?
- This stage involves the devising of strategies to solve the problem (see section 2.3.4.2). In this instance, I discussed with teachers how they could help learners to create a plan to solve a given problem. Furthermore, I encouraged teachers to

ask learners questions like: What operation are you going to engage? Do you need to draw a picture? A table? Would you use an equation?

- This stage involves finding a solution to the problem by learners on their own, in pairs or as a group. I encouraged teachers to assist learners in carrying out the plan, that is, to do calculations. I discussed ways teachers could help learners to persist with a chosen plan or if a plan does not work to discard it and choose another one. After getting a solution to the given problems, I persuaded teachers to always help learners to review their answers by reflecting and looking back at what worked and did not work. The teacher during this process circulates in the classroom asking learners questions about their work, clarifying misunderstandings, giving suggestions and helping or giving hints to learners that get stuck. Moreover, the teacher may repeat the understanding process with the stuck learners if necessary. Concurrently, the teacher looks for learners who have interesting solutions with the objective of asking them to explain their solutions in a certain order during the whole class discussions. At this stage, once learners get solutions, they are encouraged to look back and try to find alternative solution strategies.

- At this point, the class conducts a whole class discussion where learners present their ideas and solutions as well as listen to others. Learners are required to look at the similarities and differences among the solutions being presented by their peers and to realise that there are multiple solution strategies to some given problems.

- The teacher sums up the solutions presented by learners, highlights the main points and concludes the lesson or if there is still time, learners solve another problem.

Activity 4: During the fourth activity, I supported teachers on how to implement mathematical problem-solving pedagogy in a multi-lingual context. In the first cycle, I focused on code-switching (see section 5.3.1) and in the second cycle I focused on

translanguaging (see section 6.3.1) and other various strategies that are explicated in section 6.7.

Activity 5: In the last activity of the workshop, I explained and emphasised the importance of reflection by teachers during the implementation of the mathematical problem-solving pedagogy and their life as mathematics teachers. I encouraged teachers to reflect-in-action and reflect-on-action (see section 2.5.3).

I concluded the workshop by asking teachers what they had learnt and requiring them to summarise what they were taking from the workshop.

After attending the first PD workshop, I encouraged the participant teachers to go and implement the mathematical problem-solving pedagogy in their lessons for two months. During this implementation stage, I observed, supported and guided them as was necessary and audio-taped the lessons. It is imperative that teachers are supported during the implementation stage so as to address the specific challenges that may come as a result of their changing classroom practice (Gulamhussein, 2013); receive feedback; and be given the opportunity to reflect critically (Bell & Gilbert, 1994). Throughout the implementation phase, I encouraged participant teachers to continuously reflection-on-action and reflection-in-action on their experiences as learners of the PD program and on their classroom practices.

After the first implementation, I conducted the second workshop, at the end of the third month, where the aim was for teachers to further collaboratively reflect on their teaching experiences and to review the audio-recordings of the observed lessons. I selected crucial and relevant audio-recordings that foregrounded participant teachers' use of mathematical problem-solving in their teaching. Together with participant teachers we analysed the videos and reflected on how teachers had taught mathematical problem-solving. After the reflective process, teachers watched two more videos showing mathematical problem-solving pedagogy. Once again teachers collaboratively solved mathematical word problems relating to what they were teaching and planned how to teach similar problems to their learners. I also discussed with participant teachers their

experiences with learners' language during the implementation and linguistic strategies appropriate to support learners in their context (see section 5.3.1 and 6.3.1).

After the second workshop, the teachers implemented the new ideas for two months whilst I observed, supported, audio-taped and interviewed them. I also encouraged them to continuously reflect on their experiences and classroom practices. The third workshop took place at the end of the fifth month, and its activities were similar to the second workshop. Since the PD intervention was six months long; it meant that after the third workshop I only observed and supported teachers for a month.

3.11 Data analysis

Since I employed mixed methods strategies to gather data, I independently analysed the quantitative data using quantitative methods and the qualitative data using qualitative methods and then merged the two databases (Creswell & Plano-Clark, 2011). In this study, it was challenging for me to manage a huge amount of data that I had collected over approximately two years. The raw data included teacher responses on the open-ended questionnaire, recorded notes on the observation comment card, audio-tapes from the classroom observations, teacher semi-structured interviews and learner task-based interviews, memos, learner test marks, responses from the MPSSI and the reflexive journal that I kept throughout this study. I Initially transcribed verbatim into written notes the audio-recordings from the pre-observations of the first cycle, but I realised that it was cumbersome and required extensive detail such that I would lose sight of the bigger picture of the study (Cohen et al., 2011). Therefore, I wrote analysis of data directly from the audio-recordings of classroom observations. This process was of value because it enabled me to handpick the important ideas directly from the original sources rather than from the transcripts. I handpicked part of the audio-recordings where the teachers and learners focused on mathematical problem-solving. However, I transcribed the audio-recordings of the semi-structured interviews and the task-based interviews.

McMillan and Schumacher (2014) define qualitative data analysis as chiefly an inductive process of organising data into categories and classifying patterns and relationships among the categories. This concurs with Creswell and Plano-Clark (2011, p.208) who state that qualitative data analysis involves 'coding the data, dividing the text into small units, assigning a label to each unit and then grouping the codes into themes.' I started the qualitative data analysis immediately after the qualitative data collection such that the two were interwoven. The qualitative data analysis was an ongoing, cyclical process that was integrated into all the phases of the qualitative strand. I continually adapted and sharpened the research tools during the intervention.

I employed inductive data analysis to analyse the qualitative data from the open-ended questionnaire, reflexive journal, classroom observations, learner task-based interviews, memos, and the semi-structured interviews (Mertens, 2015). The inductive data analysis allowed me to move from specific data to general categories and patterns and this resulted in themes and conclusions emerging from the data rather than from predetermined categories (McMillan & Schumacher, 2014). During inductive data analysis, I endeavoured to have a conversation with the data (Merriam, 1998). I made notes of emerging themes and asked questions about 'how frequently and why'. I named the themes that were emerging from the data from three sources: the literature review, participant teachers and myself as the researcher (Merriam, 1998).

I used the statistical software package SPSS to analyse the learner quantitative data from pre- and post-mathematical attainment tests and the MPSSI. A t-test for paired samples was used because the same group of learners was tested on the pre- and post-mathematical attainment tests and the MPSSI. After analysing the data separately, I analysed the quantitative data together with the qualitative data using 'side-by-side comparison of the merged data' (Creswell & Plano Clark, 2011, p.223). After that, I gave an interpretation of how the merged results had answered the research questions.

3.12 Rigour in research

Rigour in research deals with generating and sustaining the reader's trust and confidence. I endeavoured to achieve rigour in this study so as to prove that the outcomes of this study were indeed due to the PD intervention. Rigour in mixed method design-based research is intricate because two different data sets are collected, analysed and mixed (Teddlie & Tashakkori, 2011). Like any other research designs, there are various aspects that threaten the rigour of the DBR process or its findings (DBRC, 2003) such that objectivity, validity and reliability are all indispensable in a DBR study in order to meet acceptable standards. DBR literature does not provide clear standards for assessing the rigour of the DBR process or its findings (Plomp, 2007). Kelly (2004) recommends that DBR researchers should establish a clear criterion for ensuring objectivity, validity, and reliability in their DBR process and findings. Consequently, in order to establish rigour in this study, I differentiated between the rigour in quantitative and qualitative methods and data as well as the rigour of the results produced by integrating these data (Greene, 2007). I initially evaluated the quality of data and results in the quantitative and qualitative strands using the quality standards adopted in basic quantitative and qualitative research respectively (Cresswell & Plano Clark, 2011; Greene, 2007).

3.12.1 Rigour in the quantitative strand

Objectivity, reliability and validity are three standards used to judge the quality of the quantitative research (Mertens, 2015).

Objectivity

Throughout this study, I endeavoured to remain distanced from the research process so that the findings and conclusions would be an outcome of what was studied rather than a result of my personality, beliefs or values. I kept a reflexive journal throughout this study in order to uphold objectivity (see section 3.8.1).

Reliability

Reliability refers to consistency or precision in measurement (McMillan & Schumacher, 2014; Cresswell, 2012). This implies that if the same research is done again under

similar conditions, the researcher or different researchers will obtain the same results and not erratic or inconsistent results (Babbie, 2010; Mertens, 2015). A data collection tool or a research technique is said to be reliable if its scores are consistent and stable even if it is administered to a different group of participants under different conditions such as time and venue. Therefore, reliability can be taken as the core requirement for quantitative research tools. To address this issue, the research tools were pilot-tested prior to the data collection process (see section 3.9). I also worked towards the reliability of each quantitative research tool of the study and this I explain in section 3.8.2. To address the reliability of the whole study, I employed triangulation by using multiple sources of data (DBRC, 2003) and I engaged iterations to design and develop the final prototype of the PD intervention that can be used to support grade 9 South African teachers in the teaching of mathematical problem-solving.

Validity in the quantitative strand

Validity is the extent to which inferences and uses made on the basis of numerical scores are proper, meaningful, valuable and valid (Babbie, 2010; McMillan & Schumacher, 2014) and its purpose is 'to check the quality of the data, the results and the interpretations' (Creswell & Plano Clark, 2011, p.210). Validity determines whether the research truly measures that which it is intended to measure and is vital in evaluating the relevance of the gathered data in answering the quantitative research questions (Babbie, 2010; Cresswell, 2012; McMillan & Schumacher, 2014; Plano Clark & Ivankova, 2016). There are several kinds of validity; however, I found only internal validity, external validity, content validity, construct validity and criterion-related validity applicable to this study.

Internal validity is the extent to which the researcher can reach a conclusion that there is a 'cause and effect relationship among variables' (Creswell & Plano Clark 2011, p.211). This means that the changes observed in the dependent variable are only due to the effect of the independent variable and not to extraneous variables or other alternative explanations (Mertens, 2015). In quantitative research, internal validity asks the question: are we measuring what we think we are measuring? In this case, internal

invalidity is the possibility that the conclusions drawn from the intervention may not truly reflect what has transpired in the intervention itself (Babbie, 2010). For this study, I worked towards the following to minimise the threats to internal invalidity.

History: I concluded that there was no known history threat to internal validity for this study because there were neither extraneous nor confounding events that occurred during the data collection period (Babbie, 2010; Cohen et al., 2011; Cresswell, 2012; De Vos et al., 2011, McMillan & Schumacher, 2014; Mertens, 2015)..

Maturation: In longer studies, participants grow, change or get more coordinated and in shorter studies, participants can get tired, bored, discouraged, and inattentive or change their behaviour (De Vos et al., 2011; McMillan & Schumacher, 2014). This study's cycles were 6 months long; therefore, there was no threat from the participant teachers' and learners' maturation. Participant teachers actively participated during workshops and in this way; I concluded that they were not bored. I kept the semi-structured interviews less than 20 minutes long and the length of task-based interviews was equivalent to one lesson so that the participants would neither get bored nor tired.

Statistical regression denotes the tendency of participants who score very high or low on a pretest, but regress to the mean on the post-test regardless of the effects of the intervention (Cohen et al., 2011; McMillan & Schumacher, 2014). McMillan and Schumacher (2014) further note that regression is problematic when participant groups are selected on the basis of extremely high or low scores. Statistical regression was not a threat to internal validity since no learner classes were chosen on the basis of extremely high or low scores.

Testing and retesting has a possibility of influencing participants' behaviour. Participants can become 'test-wise' by taking a pre-test that is similar to a post-test (Mertens, 2015). In the post-test, they may give responses that they think the researcher requires and that will make them look good (Babbie, 2010). I minimised the threat from retesting by varying the content of the questions in pre- and post- attainment tests.

Instrumentation: I reduced the threat to internal validity from instrumentation by using different but similar pre- and post-attainment tests. These tests were comparable because their content was the same and the questions had the same level of difficulty; however, the context of the questions was different. Instrumentation is problematic in observational research in that the observers may become tired, bored or change in some way and this affects the scoring of data (McMillan & Schumacher, 2014). I was the only observer in this study and I tried to be consistent in the recording of the data throughout the research process.

Selection refers to the way the participants are selected or how groups are composed which may introduce threats that influence the outcomes of the study (Cohen et al., 2011; Cresswell, 2012; De Vos et al., 2011; McMillan & Schumacher, 2014; Mertens, 2015). Selection was not a threat to internal validity for this study because classes were already organised for teaching purposes.

Experimental mortality (attrition) means that participants may fail to complete a study due to lack of time, death, immigration or may simply drop out (De Vos et al., 2011; McMillan & Schumacher, 2014; Mertens, 2015). This threat may affect the findings and conclusions of a study. The cycles were 6-months long and there was no experimental mortality that occurred during this study. I ruled out experimental mortality in this study because I had a pre-test and a pre-MPSSI that I would have used to determine if there were systematic differences in the characteristics of the learners who would have completed this study and those who would have dropped out of this study (Mertens, 2015).

External validity

To reduce the threat to external validity, I chose schools for the research that were a representative sample of a typical South African public school. However, this was a DBR study and generalisability was not the intent of this study. Kelly (2004) notes that, when one foregoes experimental controls, it is impossible to generalise the findings to other settings irrespective of how rich the local descriptions can be.

Content validity

Content validity is concerned with whether a measure is representative of all possible items within a concept (Babbie, 2010; De Vos et al., 2011). Creswell (2012) postulates that scores from the data collection tool must illustrate that the test's content relates to what the test is proposed to measure. Content validity is concerned with coverage and representativeness rather than with the pattern of responses (Cohen et al., 2011). For this study, as it regards content validity, all the mathematical problem-solving processes and other factors explored in the literature review were all represented by the items in the different sections of the open-ended questionnaire, task-based interviews, observation comment card, the mathematical problem-solving skills inventory and pre- and post- attainment tests.

Construct validity

A construct is an abstract and construct validity concerns the extent to which a particular data collection instrument conforms to the theoretical context in which it is positioned (Cohen et al., 2011). Construct validity asks the question: is the data collection tool measuring the correct construct or something else? For this study construct validity was based on whether the data collection tools were indeed measuring learners' mathematical problem-solving processes and skills. Mertens (2009) explains that construct validity depicts the degree to which all the accumulated evidence supports the intended interpretation of scores for the proposed purpose. Cohen et al. (2011) recommend that a researcher must demonstrate construct validity by confirming their construction with that given in the relevant literature and giving counterexamples that may falsify their construction. Cohen et al. (2011) further note that construct validity is threatened by the underrepresentation of the construct for example when a test is not broad enough and fails to cover significant aspects of a construct. To strengthen construct validity of this study various methods of assessment, that is, the MPSSI, task-based interviews and pre- and post-attainment tests were closely aligned with Polya's (1957) problem-solving processes.

3.12.2 Rigour in the qualitative strand

In qualitative research, rigour is assessed based on 'whether the findings are accurate from the standpoint of the researcher, the participant, or readers of an account' (Cresswell, 2014, p.201). Lincoln and Guba (1985) compare the criteria for judging rigour in qualitative research versus quantitative research. They equate credibility with internal validity, transferability with external validity, dependability with reliability and confirmability with objectivity in quantitative research. In this study, rigour in the qualitative strand was supported through strategies that I discuss below.

3.12.2.1 Credibility

I was cautious that researchers and readers are concerned with the trustworthiness and credibility of the qualitative findings of this study and their interpretations (Creswell & Plano Clark, 2011). As a result, one criterion of trustworthiness that I adopted was credibility and this refers to the extent to which the qualitative findings are true and recognised as precisely communicating participants' experience (Mertens, 2015; Plano Clark & Ivankova, 2016). I adopted multiple strategies to enhance the credibility of this study and I discuss them next.

Data triangulation

I collected data from various sources and through multiple methods (Marshall & Rossman, 2011). I drew conclusions based on converging data from teacher observations, semi-structured interviews, audio-recordings, memos, researcher's reflexive journal and learner task-based interviews. The multiple data collection methods produced data that were more or less the same and this made me confident in the results and findings from the study. I also used data triangulation to demonstrate concurrent validity in the qualitative strand of this study (Cohen et al., 2011).

Prolonged engagement in the field and persistent observation

Researchers must stay long enough in the field in order to make accurate conclusions and must observe participants sufficiently in order to have an accurate picture (Mertens,

2015). I spent a lot of time building trust, observing and interviewing participant teachers and learners in their naturalist settings. This is emphasised by Creswell (2009) who states that the more time the researcher spends with participants the more valid their findings are. Prolonged time in the field and persistent observation assisted me to develop an in-depth understanding of how teachers were teaching mathematical problem-solving before, during and after the PD intervention.

Member checking

Member checking is the most critical procedure for establishing credibility (Mertens, 2009, 2015; Guba and Lincoln, 1989). Therefore, I took the data interpretations and the final research report to the participants so that they could verify if the findings and interpretations reflected their position (Cohen et al., 2011; Cresswell, 2013; Cresswell, 2012; Cresswell and Plano Clark, 2011; Marshall & Rossman, 2011; McMillan & Schumacher, 2014). The participant teachers served as a check throughout the data collection process. An on-going conversation regarding my interpretation of their mathematical problem-solving pedagogy ensured my inferences were valid. After each semi-structured reflective interview, I summarised the participant teacher's responses and asked if the notes accurately encapsulated the teacher's mathematical problem-solving pedagogy. I went back to the schools after completing the data analysis of each cycle and presented summaries of the findings to the participant teachers. We discussed if the findings were truly what they experienced during the data collection process. No changes were provided by the participants to the data that were presented to them during the member checking process.

Peer debriefing

A post-doctoral fellow served as a peer examiner for this study. I discussed the study with him to test authenticity, working hypothesis, and subsequent steps in the research process (Cohen et al., 2011) and to ensure that the analyses were grounded in the data (Marshall & Rossman, 2011). The peer examiner reviewed and contributed to this study by challenging me at times when I failed to keep an open mind or when I was unaware of my own biases. This process augmented the accuracy of the qualitative account. As

recommended by Lincoln and Guba (1985), the peer examiner and I both kept written accounts of the peer debriefing sessions that we conducted.

Mechanically recorded data

I used tape recorders to record classroom observations and semi-structured reflective interviews and I took pictures of learners' work during task-based interviews. This provided me with precise and reasonably complete records (McMillan & Schumacher, 2014).

3.12.2.2 Confirmability

Confirmability means that the data, findings and conclusions are not fabrications of the researcher's illusions (Guba & Lincoln, 1985; Mertens, 2009, 2015). Guba and Lincoln (1985) recommend a confirmability audit to prove that the data can be traced back to the original sources, findings are rooted in contexts and persons apart from the researcher and that the process of synthesising data to reach conclusions can be established. I employed excerpts from field notes, reflexive journal and the peer examiner as the confirmability audit.

Excerpts from field notes

In the final report, I used participant teachers' original quotes from the interviews and open-ended questionnaires to support the themes that emerged from the data analysis. I did this in order to demonstrate that the findings of this study were not a product of my imaginations or bias (Mertens, 2009).

Reflexivity

I reflected critically on myself as the researcher in the reflexive journal and stated clearly the bias, values and experiences that I could have brought to the study (Creswell, 2009; Cresswell, 2013; Guba & Lincoln, 2005; McMillan & Schumacher, 2014; Mertens, 2015). Throughout the study, I posed difficult questions to myself about my ideas, experiences, assumptions and biases. I reflected critically in the reflexive journal (see section 3.8.1) on my role as the human instrument in this study and articulated the researcher bias

under the section on the researcher's role (see section 3.5). I included comments on how my background as a mathematics learner, teacher, teacher educator and researcher shaped the interpretation of the findings.

Peer examiner

I asked the peer examiner to evaluate the open-ended questionnaire transcripts, field notes on the observation comment card, interview transcripts and field notes from the task-based interviews and reflective journal and determine if the findings and conclusions of this study were supported by the data. The peer examiner also made a judgment on the conclusions to check for neutrality and traces of biases and concluded that this study was reasonably confirmable. This process saved as a confirmability audit for this study.

3.12.2.3 Transferability

Transferability implies the extent to which the results can be generalised to other situations. In qualitative research it is the responsibility of the reader to determine the external validity of a study and the researcher is required to give enough detail to enable the reader to make such a judgment (Mertens, 2015). I provided clear, rich and thick descriptions of the data in order to show readers that the findings of this study are applicable or transferable to other contexts.

Rich and thick descriptions

I provided detailed descriptions of the participant teachers and learners, schools, classrooms, PD training workshops and themes generated from this study to communicate the findings. I carried out a context analysis before conducting this study, and in section 1.2, I give a rich description of the informal settlement in which the participant schools were located. I also give an extensive description of how the classrooms were set up, how learners were overcrowded and how teachers were teaching mathematical problem-solving. I believe this rich and thick explanation makes the results more realistic and richer to the readers (Cresswell, 2009) who can then be

able to transfer this information to other settings (Cresswell, 2013) or compare the similarities of their circumstances with that of this study.

3.12.2.4 Dependability

A dependable study needs to be stable over time and not erratic. Change in qualitative research is practical and possible; however, it should be tracked, documented and openly inspectable (Mertens, 2009, 2015). This process is called the dependability audit by Guba and Lincoln (1985). Thus for the purposes of this study, I kept a reflexive journal throughout this study in order to track the events that emerged during the research process.

3.12.3 Rigour in mixed methods DBR generated inferences

Assessing the level to which the integration of the quantitative and qualitative methods in a mixed methods DBR study produces quality inferences is vital. Inference quality in mixed method research design is defined by Teddlie and Tashakkori (2009) as 'standards for evaluation of the quality of conclusions that are made on the basis of research findings from the entire study' (p.287). Teddlie and Tashakkori (2009) further contend that researchers of mixed methods studies should make sure that their conclusions can be applied to comparable settings, environments, and people.

Bearing in mind that inferences in mixed methods design-based research that are inconsistent and below standard in terms of quality have no significance, I incorporated a number of strategies that I discuss above that enhanced the objectivity, validity and reliability of this study's conclusions. I discuss how I achieved objectivity in the quantitative strand and confirmability in the qualitative strand of this study in sections 3.12.1 and 3.12.2.2 respectively. However, it is well-known that achieving objectivity and confirmability in DBR is difficult (DBRC, 2003) and this study was not an exception because I immersed myself in the context of study; interacted extensively with the participant teachers and learners and had several roles (see section 3.5). Triangulation is known to increase the objectivity (Wang & Hannafin, 2005) as well as the reliability of the DBR findings (DBRC, 2003). I used triangulation in terms of multiple data sources (see section 3.8) and researchers' views, data analysis and interpretation (i.e. using a

peer reviewer). However, I did not focus on external validity and generalizability of the findings because it is impossible to generalise DBR findings from a sample to a large population (Barab & Squire, 2004; Kelly, 2004; Plomp, 2007). In section 3.12.1, I discuss to a larger extent how I minimised the threat to internal validity in the quantitative strand of this study and in section 3.12.2.1; I describe how I enhanced the credibility of the qualitative findings. I strengthened the internal validity of the DBR findings and enhanced their credibility by going through two cycles of DEER in order to design the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy.

Throughout this study, I endeavoured to produce quality inferences from the design of this study to the interpretation of the results. I thoroughly documented the research process and how I drew the conclusions from the data. I did this to allow the readers to track the learning trajectory and to re-experience the cyclic nature of this study like their own (Baker & van Eerde, 2013).

3.13 Ethical procedures

Ethics deal with the beliefs or guidelines about what is right or wrong, proper or improper, good or bad from a moral perspective (McMillan & Schumacher, 2014). In this section, I discuss how throughout this study I endeavoured to protect participants' rights and welfare, including physical and mental discomfort, harm and danger. In compliance with the Gauteng Department of Education and the University of the Witwatersrand research ethical policy, I took precautions to protect the autonomy and privacy of participants. The following were the key issues concerning this study around ethics: informed consent, voluntary participation, privacy and reciprocity.

3.13.1 Obtaining informed consent

In order to ensure informed consent, I took several steps to make sure that the participants had a choice of either participating or not participating in the research after a complete understanding of their role, the procedures and purpose of the research and likely risks involved in this study (Babbie, 2010; Cohen et al., 2011; Cresswell, 2012; De

Vos et al., 2011; McMillan & Schumacher, 2014). Concerning obtaining informed consent to collect data, I first sought permission from the Gauteng Department of Education (GDE) (see Appendix B) which issued an approval letter with protocol number D2016/373AA (see Appendix C) that gave me the permission to gain entry into the Johannesburg north district of education. Before entering a particular school, I wrote letters to the school principal and school governing board (see Appendices E & F) seeking permission to collect research data from their schools. All school principals and the school governing boards that I applied to willingly gave approval. However, the letters were not included in the appendices for the sake of anonymity and confidentiality. I applied for a research ethical clearance certificate which was granted by the Witwatersrand ethics committee with protocol number 2016ECE002D (see Appendix A). The research ethical clearance certificate that was issued by the university implied that there were no ethical issues, no deceptions and no possible risks or danger for human participants in this study.

At the onset of both cycles, I wrote letters to the participant teacher and participant learners' parents or guardians (see Appendices I & K) that clearly explained among other issues the purpose of this study, their role, and voluntary participation. After understanding the content of the letters, they all agreed to participate in this study and gave their written consent through signing the letters. I got 'assent' from the participant learners (Mertens, 2015, p.352) by explaining clearly the study to them and getting them to sign the consent letters as a way of showing that they voluntarily agreed to participate in the study.

3.13.2 Voluntary participation

Voluntary participation denotes that participants are not obliged, coerced, or required to participate in a research (Cohen et al., 2011; McMillan & Schumacher, 2014). To ensure that participants freely chose to participate (or not) in the research, I made sure that the participants were well informed about the purpose of the research, the procedures of the data collection process and the research's possible impact on them. I clearly explained to participants that they were free to decide whether they wanted to

participate in the research or not and had the freedom to withdraw from the research or stop the interviews and audio-recordings at any time without incurring any negative consequences, although none did. Participants were not deceived in any way and I was open and honest about all aspects related to the study.

3.13.3 Privacy

Privacy is ensured by using three practices which are confidentiality, anonymity and appropriate storage of data (McMillan & Schumacher, 2014). In this study, I informed and assured participants that their confidentiality would be respected and that their responses would be used only for the purposes of this study. I made reasonable efforts and necessary precautions to maintain complete participant confidentiality by not disclosing the identity of the participants. The school names, principal names, mathematics teacher names and learner names were eliminated from all reports. As another precautionary measure, pseudonyms were assigned such that participants' real names were/are unidentifiable and non-traceable (Babbie, 2010; Cohen et al., 2011). McMillan and Schumacher (2014) stress that the settings and participants should not be identified in print. I was and I am still the only one who has access to the data and the participant names (McMillan & Schumacher, 2014). During the data collection process, data was never shared with other participants or other people outside this study.

Anonymity implies that the names of the participants where data is obtained are unknown (Babbie, 2010) and the researcher or another person cannot identify the participants from the information provided (Cohen et al., 2011; McMillan & Schumacher, 2014; Mertens, 2015). Even though pseudonyms were assigned to the participants, complete anonymity was not possible in this study since I carried out interventions, observations and face-to-face interviews with the participants. Data was stored in password-protected folders on my password-protected MacBook such that no-one can have access to it except me. I keep the hard copies of the open-ended questionnaires, observation comment card scripts, learners' test answer scripts, responses on the MPSSI, reflexive journal and memos in a drawer in my bedroom that is locked at all times.

3.13.4 Reciprocity

I reciprocated the participant teachers and learners for their willingness to provide data and how they adjusted their priorities and routines to tolerate my presence in their schools and classrooms. I provided the research findings to the GDE, Johannesburg North district department of education and the participant schools. I did not pay teachers because I did not want them to be bound to the research because of monetary incentive, but to participate freely in the research.

3.14 Linking the theoretical framework, research questions, literature review, and methodology.

In this chapter, I outline the mixed methods DBR that I adopted as a research design for this study. In this section, I focus on how the literature and theory were applied within the mixed methods DBR. In the introductory chapter, I highlighted the importance of the context of study. The importance of context compelled me to take a social-constructivist perspective, the pragmatic paradigm and the DBR approach.

I used the literature on mathematical problem-solving, mathematics teachers' knowledge for teaching problem-solving and effective mathematics teacher PD as organising frames for the formulation of the research questions (see section 1.6). I used Polya's (1957) framework on problem-solving processes (see section 2.3.4) to design the observation comment card (see Appendix O) and to analyse teachers' problem-solving pedagogy during the classroom observations. I designed the Mathematics Teacher Open-ended Questionnaire using the literature on mathematical problem-solving. I incorporated the reflective approach in teacher PD (see section 2.5.3) as an organising frame to design the questions of the semi-structured reflective interviews and to analyse teachers' responses during the interviews. I used the literature on mathematical problem-solving skills and processes to frame the MPSSI. I fed the literature on worthwhile mathematical problems (see section 2.3.1) into the designing of the pre- and post-word-problem attainment tests.

More broadly a social constructivist perspective to learning (see section 2.2), Polya's (1957) framework on problem-solving processes and the literature on effective mathematics teacher PD and teacher knowledge in mathematics fed into the different features of the design and implementation of the PD intervention (see section 3.10).

3.15 Conclusion

This is a mixed methods DBR study and extensive data collection, time and resources were required. Data collection took approximately two years and I conducted the study at various schools. The resources were adequately available as I was partly funded by the National Research Foundation (NRF). It is argued in the literature that the researcher's competence, experience and knowledge may not be adequate to conduct both mixed methods research design and DBR in a single study. However, I had attained a deeper knowledge to adequately conduct mixed methods research design during my master's degree studies and I attended several seminars and conferences about DBR throughout my PhD studies.

This chapter justifies why I adopted mixed methods DBR methodology. Characteristics of the DBR and mixed methods research design are expounded on. The participants, data collection tools, pilot study, ethical procedures, and data collection and analysis procedures are described in detail. Various ways of how I upheld rigour in the research are accentuated. In the next chapter, I present the baseline investigation and its findings.

Chapter 4: South African Grade 9 mathematics teachers' views of the teaching of problem-solving: The baseline investigation

4.1 Introduction

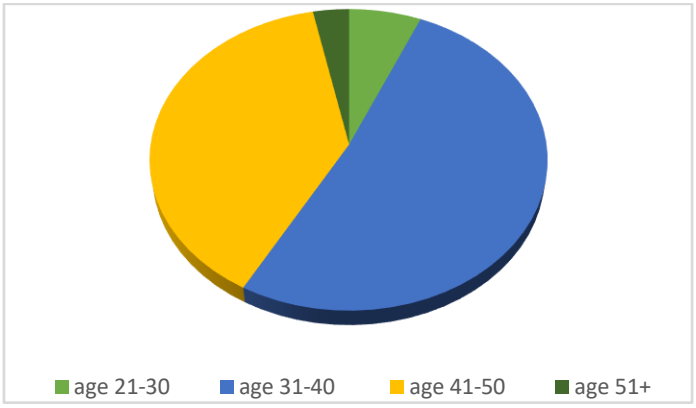
Context is an important theme in this study. As such, when I decided to embark on my PhD study, I realised that I did not know grade 9 South African teachers' views on the teaching of mathematical problem-solving in their particular context. It was important for me to establish participant teachers' views, teaching strategies and the support required in their teaching of mathematical problem-solving before implementing the PD intervention. To unearth these views, I conducted a baseline investigation in March 2016 and this involved travelling to 20 schools in the Gauteng district to collect data from 31 teachers using an open-ended questionnaire and classroom observations (see section 3.8.1). I used the classroom observation data to exemplify and expand on issues that were emerging from the questionnaire.

In this chapter, I start by giving participants' biographical data, then move on to the description of the baseline investigation, data analysis, findings and discussion and lastly I conclude with the implications of the findings for design of the intervention for the first cycle.

4.2 Participants' biographical data

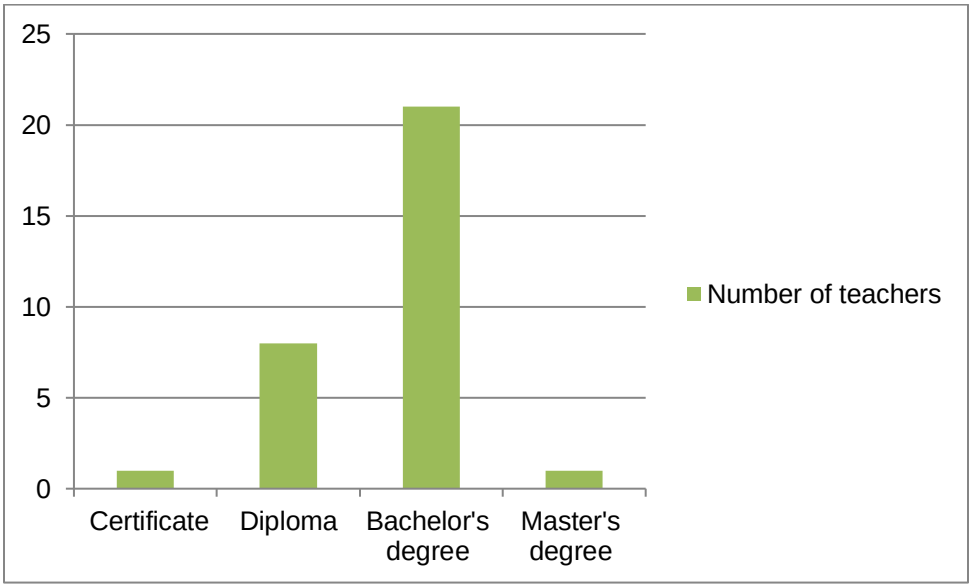
I conducted the baseline study with a sample of grade 9 teachers (n = 31, 19 female and 12 male) at 20 secondary schools in the district of Gauteng. I selected the secondary schools on the basis of their convenient location and accessibility (McMillan & Schumacher, 2014). As demonstrated in Figure 4.1, participant teachers were between the ages of 21-60 years, with one teacher above the age of 51 years meanwhile 28 of the participant teachers were between 31-50 years.

Figure 4.1: Distribution of teachers' age (n=31).



The study revealed that the participant teachers were well-qualified to teach grade 9 mathematics; with one of the teachers having received a master's degree in mathematics education and 21 of the teachers having a bachelor's degree as the highest qualification. Only one female teacher had a Certificate in Education as her highest qualification and eight of the teachers had a Diploma in Education.

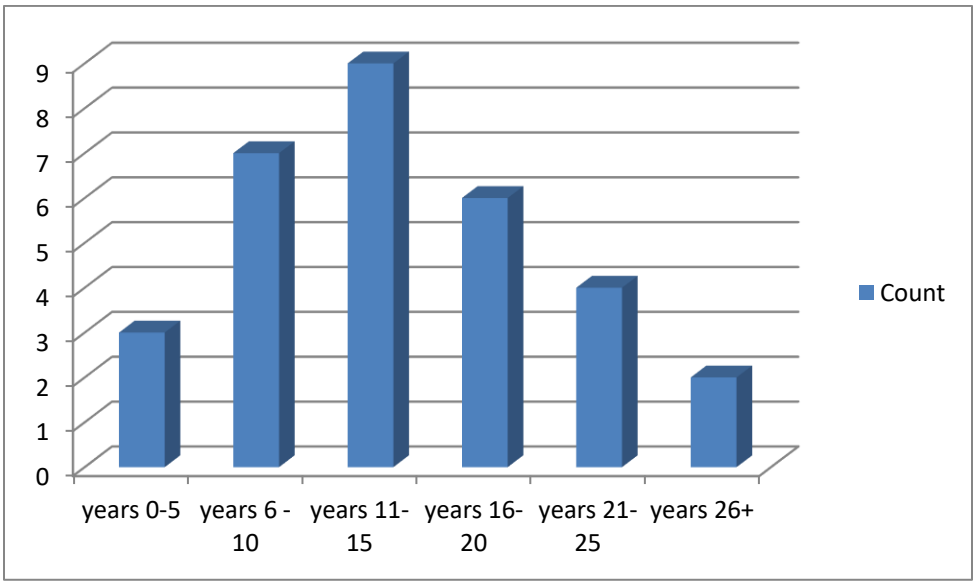
Figure 4.2: Participant teachers' qualifications.



The participant teachers' teaching experience ranged from two to 25 years. In this instance, 28 of the teachers had more than 6 years of teaching experience. On analysis

of these numbers, I concluded that the participant teachers were adequately experienced to teach mathematics in grade 9.

Figure 4.3: Participant teachers' teaching experience.



For the subsequent classroom observations, I selected two female teachers from the original sample (n = 31). I purposefully selected the two teachers because they had reported on the open-ended questionnaire that they were using which I termed 'traditional' methods of teaching problem-solving (see section 2.3.3.1). These two teachers were also the participants in the first cycle of the subsequent PD intervention.

4.3 Describing the baseline investigation

Over the years, a number of studies have been conducted to establish mathematics teachers' beliefs or views on mathematical problem-solving. Leikin and Levav-Waynberg (2007) investigated teachers' thoughts about alternative solution methods in problem-solving. Their findings revealed that participant teachers had positive attitudes towards the use of multiple solution methods in problem-solving and valued learners' development of mathematical thinking, reasoning and problem-solving processes. Siswono, Kohar, Kurniasari and Astuti (2016) investigated mathematics secondary teachers' understanding and beliefs towards mathematical problem-solving. Findings

revealed that the participant teachers understood mathematical problem-solving pedagogy and its implementation in their teaching practice. However, participant teachers frequently failed to identify correct solution strategies when carrying out problem-solving processes and indicated that the most difficult step of carrying out problem-solving was devising a plan. Anderson, White and Sullivan (2004) carried out a large study on teachers' beliefs and practices on mathematical problem-solving. Primary and secondary school teachers participating in the study viewed problem-solving as something extra that they had to do in the curriculum and believed that learners should learn algorithms in order to solve problems. The above studies on teachers' views and beliefs about problem-solving were conducted in the USA, Australia, and Asia. However, the limited literature on mathematics teachers' views on the teaching of problem-solving in South Africa only identifies that teachers find the implementation of the problem-solving approach problematic (Naroth & Luneta, 2015). Therefore, I felt compelled to know about teachers' views on the teaching of mathematical problem-solving in a South African context, i.e. in classes that are often large, multi-lingual and under-resourced before implementing the PD intervention. Specifically, I formulated the following research question for the baseline investigation, which is the first research question of this study (see section 1.6):

What are grade 9 South African mathematics teachers' views on the teaching of mathematical problem-solving?

As I indicated in section 3.8.1, the open-ended questionnaire was the main data source for the baseline investigation. In this process, I selected and observed two teachers delivering lessons and recorded information as it occurred on the observation comment card (see Appendix O). With the observations, I also wanted to investigate if the teaching approaches participant teachers indicated in the questionnaire supported learners' problem-solving processes. I observed each teacher once a week for a month, and in total, I had four observations with each teacher. Data from the classroom observations were in the form of field notes that I made on the observation comment card during classroom observations and listening to the audio-recordings. These observations also worked as pre-intervention observations for the first cycle (see

section 3.8.1). Audio-recordings, which were done with the teachers' consent, provided me with the opportunity to analyse the lessons at a later convenient time. I described the classroom observations in detail in vignettes in the reflexive journal and I provide an example below.

Figure 4.4: Vignette of the classroom pre-observation

Problems: Teachers used non-routine problems from the textbook. In this particular lesson, the following problem was one of the problems used by the teachers: You are given the following relationship: y= 8x +3. Calculate y if x has the value of (i) 4 (ii) -3 (iii) 0.

Learners' role: Learners sat in rows copying what the teacher was writing on the board. Half of the class looked uninterested in solving the given problems. After the teacher demonstrated how to find y when the value of x was 4 the learners could not identify data needed to solve (ii) and (iii). They could neither think of a useful strategy nor attempt the question without the teacher's help.

Teacher's role: The teacher stood in front of the classroom and worked examples on the board and did not require learners to formulate a plan before solving the given problems. If learners struggled to follow the steps the teacher had shown on the board, the teacher would show them again without establishing the reasons why they were struggling.

4.4 Data analysis

I transcribed verbatim the audio-recordings from the classroom observations. I employed inductive data analysis of the teacher questionnaire responses, field notes on the observation comment card and transcripts from the audio-recordings (McMillan & Schumacher, 2014). However, I did not employ an existing framework for the problem-solving process as a basis for data analysis because I did not want to predetermine what I was going to find from the questionnaire and classroom observations. To determine the different themes that were emerging from the data, I first read and reread several times the responses on the questionnaires, transcripts from audio-recordings and text on the field notes, and this I named early coding. I did this by way of circling and examining a chunk of text, and by writing down the name of the theme that I felt it

belonged to. If a chunk of data did not fit into any particular theme I created a new theme.

During early coding, I conducted what Corbin and Strauss (2008) term ‘constant comparative analysis’. I broke down the data into discrete parts and constantly compared these for similarities and differences so as to extract the phenomena that were embedded within the information. To determine consistency in the way I was coding, I frequently compared and contrasted the themes that were emerging. I went through the process of repetitive coding several times and concluded early coding when I felt that there were no new relevant major themes coming from the data. This process is named saturation by Corbin and Strauss (2008). After early coding, I conducted what I called secondary coding. During this stage, I put the data back together by making connections between themes (De Vos et al., 2011; Mertens, 2015). I searched for themes that I suspected to be core to the phenomena involved and connected these to each other. After this, I conducted the final coding by selecting what I felt were the core themes. I systematically related these core themes to other themes and used them to explain my understanding of participant teachers’ views of the teaching of mathematical problem-solving. Table 1 shows an example of how I coded the data from the open-ended questionnaire.

Table 4.1: A sample of how data was coded.

Raw data	Early coding	Secondary coding	Final coding
Q1 Can you please say something about the teaching of mathematical problem-solving.	Teaching using textbooks and chalkboard	Step-by-step explanation to learners using textbooks and working out examples.	Nature of sense-making
Q2 How do you approach the teaching of problem-solving in your classes?	Step-by-step explanation, doing examples on the board		
Q3 What difficulties do you face with teaching mathematical problem-solving?	Language	Language	
Q4 What difficulties do your learners face with mathematical problem-solving?	Learners have no solid background in mathematics	Lack of learners' basic skills in mathematics	
Q5 What support would you like in your teaching of problem-solving?	Language		
	Workshops and training		

4.5 Reliability and validity in the baseline investigation

Issues of rigour in the study as a whole were thoroughly explicated in section 3.12. In this section, I expound on how I worked towards reliability and validity in the baseline investigation. I used an open-ended questionnaire that gave the teachers the opportunity to express their views rather than constraining them to closed questions, thus supporting the validity of the approach. I also selected and observed the two teachers as a way of triangulating the data from the open-ended questionnaire, again supporting the validity of the questionnaire findings. I spent a reasonable amount of time observing the two teachers delivering lessons in their naturalist settings; contributing to the credibility of the observation process. I took the transcripts from the audio-recordings to the two participant teachers for checking. During data collection and analysis, I maintained an open mind in order to eliminate bias. I used participant

teachers' original quotes from the open-ended questionnaire to support the themes that emerged from the data analysis (see section 3.12.2.1).

4.6 Findings and discussion

The findings from the open-ended questionnaire and classroom observations are presented under the three themes that emerged: ownership of mathematical problem-solving, mathematical problem-solving pathway and the nature of sense-making. I discuss each of the final themes in the next sections and I replaced the names of all the respondents with pseudonyms.

Ownership of mathematical problem-solving

The notion of the ownership of the process of mathematical problem-solving emerged in various ways. A spectrum from tight teacher ownership to full learner ownership is represented in the data. For example, Deliwe intends to keep control over the process of mathematical problem-solving herself:

> I explain to learners how to solve problems by doing examples on the board with them. I then give learners several questions to solve and I check if they do the calculations as well as apply the formulas like I taught them. If learners find difficulties when solving a problem I explain in details. I have 55 learners so I do not let them discuss as I may lose class control.

As with Deliwe, several other respondents considered the transfer of ownership to the learners in the teaching of mathematical problem-solving as impractical since they were uncertain how they could control learners in overcrowded classrooms. This is in agreement with the concerns about classroom control put across by teachers in Anderson et al.'s (2004) study. Teachers in the former study also feared losing classroom control although their reason was that they were uncertain of what to instruct learners during mathematical problem-solving.

In contrast, a number of teachers believed that learners have the task of identifying the similarity of a problem with the example the teacher would have shown previously.

Although some responsibility is given to the learners, it is limited to recognition and does not extend to creativity, and this was articulated below by Lindiwe:

> When I teach problem-solving I move from simple to difficult problems because for a learner to be able to solve problems, they must have seen it before. Learners should see how a new problem is like or is not like the one they have solved before.

Lindiwe's articulation corroborates with the classroom observations. The two teachers emphasised the similarity of consecutive examples and encouraged learners to use the same algorithms they had used before to solve the problem at hand or to complete homework. After demonstrating examples on the board, teachers would give learners homework from the prescribed textbook and expected learners to complete the work following earlier examples.

Full ownership over mathematical problem-solving by learners was assumed by Marie and Ruggy when they stated their views on teaching mathematical problem-solving. Megan expected learners themselves to devise alternative ways of solving the given problem:

> Most of my learners fail to identify the correct methods to solve problems. If learners are stuck they fail to think of any other way to solve the given problem and wait for me to help them.

Megan assumed that learners should take ownership of mathematical problem-solving and was frustrated that learners were not taking initiative to find alternative ways to solve a given problem. This corroborates with what I observed that if learners got stuck they did not try an alternative method but called the teacher for help. I observed in one instance when learners did not understand how to solve $\frac{5}{8} \div \frac{2}{3}$, the teacher circulated in the classroom showing all learners individually how to solve the problem. Learners did not think of working backwards or using a different strategy; they relied predominantly on the teacher as the only source of knowledge and did not want to take ownership of mathematical problem-solving.

Ruggy indicated that he required learners to take full ownership of mathematical problem-solving:

> I tell my learners to draw pictures or diagrams when solving problems. If at one moment they cannot solve a problem with the chosen strategy I encourage them to try a different way.

As Deliwe pointed out in the first quote, teachers feel compelled to tightly hold on to the ownership of mathematical problem-solving in large classes. Participant teachers were unclear what the teaching of mathematical problem-solving involved in overcrowded classes where discipline is a challenge - they required practical demonstration *in the context of their classes*. This was captured in Malvern and Adonia's comments respectively:

Malvern:

> At the moment I am unclear about the teaching of problem-solving, it can be very helpful if I can see another knowledgeable teacher practically teaching problem-solving in an overcrowded classroom.

Adonia:

> Lack of discipline and overcrowded classrooms are huge problems when I teach. I am unsure of how to control my learners during problem-solving. I need a workshop that guides me on how I can effectively teach problem-solving in such a situation.

A second factor determining the ownership of the mathematical problem-solving process is the multi-lingual nature of the classrooms. Even if teachers were willing to let learners take ownership of mathematical problem-solving it was considered impossible because English as the language of instruction emerged as a barrier in the teaching and learning of mathematical problem-solving. This predicament was articulated by Edmund and Ruth respectively:

> The English language is a problem as learners do not even understand the given problems in order to solve them.

> Learners fail to comprehend questions and end up putting effort on what the question never asked for.

Mathematical problem-solving pathway

28 participant teachers' responses indicated that they used mathematics textbooks as their main planning and teaching tool. There was no indication of teachers doing extra research in order to understand the topics beyond what the learners were required to know; they purely replicated examples in the textbook. This was stated explicitly by Debra:

> I always plan the next lesson using the mathematics textbook. I teach for 20 minutes and give learners 20 minutes classwork from the textbook. I make sure learners leave my class with a lot of homework every day. I give learners' weekly tests that I compile using the textbook - it gives a true reflection of their understanding.

And also echoed by Stan:

> I initially give a clear explanation of unfamiliar concepts in the new topic. I then work out examples on the board and after this; I give learners relevant problems in the textbook regarding the topic to check if they really understood the new topic. If learners fail to do the given problems I work out more examples on the board and ask them to do the problems again.

This resonates with what I observed in the classrooms. Classroom observations revealed that both participant teachers taught the day's work by demonstrating the examples in the textbook on the board and there was no establishment of learners' prior knowledge by both teachers. The teachers and learners were focused on getting the correct answer to given problems and did not put much emphasis on the solution strategy. I observed that after solving the problems on the board, the two teachers did not evaluate or reflect on the answer or the solution method. Teachers did not give learners the opportunity to look back at their solution, strategy or the problem itself. On analysis of the responses to the items on the open-ended questionnaire, there were no

instances where reflection or evaluation was articulated by participant teachers. I concluded that the teachers were oblivious to the latter and did not value the complete process of teaching mathematical problem-solving. These findings resonate with the conclusions by Adler and Ronda (2014) that South African teachers have a tendency of implementing traditional approaches (see section 2.3.3.1) rather than more authentic mathematical problem-solving strategies.

The quotes by Debra and Stan denote that participant teachers believed that successful replication of tasks in the textbook by the learners implied understanding. I observed that during the lessons the two teachers did not pose non-routine problems to their learners. Problematisation (see section 2.3.6) seemed to have been lacking in the observed lessons because most problems in the prescribed textbook are exercises. The DBE textbook has a few non-routine problems at the end of each chapter that could be used by teachers to engage learners in the process of mathematical problem-solving, but I realised that teachers avoided these problems. I asked the teachers; however, I could not get a straight answer from them to establish why they omitted non-routine problems in their teaching. Maybe it could be that teachers perceived these problems as either difficult or time-consuming. This finding seems to also imply that teachers did not know what to tell learners during the teaching of mathematical problem-solving or simply lacked confidence in teaching mathematical problem-solving. I felt that if participant teachers got support in teaching mathematical problem-solving this would buttress their confidence. This is confirmed by teachers' requests for workshops on heuristics and on designing of tasks, as expressed by Mary:

> I really want to attend workshops based on different strategies for teaching mathematical problem-solving. These workshops should also teach me to design problem-solving activities that can shift the learners' mindset to love mathematics.

This finding is in agreement with the findings by Anderson et al. (2004) and Siswono et al. (2016). Participant teachers in the former study required support in choosing appropriate solution strategies during teaching. Similarly, participant teachers in

Siswono et al.'s (2016) study had challenges in designing context-based tasks and lacked understanding of problem-solving strategies.

In contrast, three of the 31 participant teachers viewed the teaching of mathematical problem-solving as facilitating learners to understand a given problem (as opposed to an exercise) and to use all the given components of the problem in order to find a strategy or a solution, as articulated by Owen:

> Learners are supposed to firstly read and understand a given problem. They must be given a chance to explain what the problem. After understanding the problem learners can then choose and apply a method to solve the problem.

Owen's mathematical problem-solving pathway was unusual and not observed in the lessons because all 31 participant teachers lamented that their learners were under-prepared, did not have a good foundation in mathematics and lacked basic mathematical problem-solving skills. Teachers realise that learners need prior knowledge and adequate preparation from previous grades in order to be able to engage in productive mathematical problem-solving. Teachers acknowledged that their learners' content knowledge was very weak and that most of their learners failed to interpret the given problems. The following quotes by Edmond and Debra respectively are examples of their statements:

> Learners are under-prepared from primary school such that they are unable to do basic problem-solving. This is a very difficult situation for me because learners fail to link previous knowledge to current knowledge during problem-solving.

> I need help on how I can teach problem-solving to my learners who are underprepared from grade 8 and cannot do even simple problem-solving.

The above quotes indicate that if learners are underprepared from the previous grades they have inadequate prior knowledge to perform mathematical problem-solving in grade 9 and teachers need assistance with mathematical problem-solving teaching strategies taking these gaps in learners' knowledge and skills into account.

Nature of sense-making

20 teachers responded that the teaching of mathematical problem-solving was breaking down the topic at hand into smaller chunks and explaining concepts to learners step-by-step as enunciated by Gloria:

> If I teach a new topic and learners find it difficult, I break it into smaller portions and I make them practice it until they can do it perfectly.

The perception that sense-making in mathematics is about replicating routine algorithms 'perfectly' in small steps directs teachers to the use of exercises rather than worthwhile mathematical problems (see section 2.3.1). In addition, during classroom observations, I observed that learners' lack of fluency in algorithms they should master from earlier years could be a second reason for teachers considering the teaching of mathematical problem-solving as teaching a series of small steps; so that learners could be supported on the specific step they did not master.

More explicitly, 13 teachers believed that teaching mathematical problem-solving was teaching learners to master mathematical concepts, calculations and application of formulas, as was articulated by Tony:

> Problem-solving in mathematics is about practice. I give learners tasks daily to keep them busy. To be good problem solvers; learners must know formulas and how to calculate.

These findings corroborate with the classroom observations. Learners solved problems by looking at the examples on the board or in the prescribed textbook as stated by Bernard:

> If I give learners problems and they cannot solve them with the equation I have shown them, I do it again on the board until they can imitate the equation.

There were limited instances when teachers allowed for brief group discussions. The discussions were centred on finding one correct solution strategy and not multiple ways to solve the given problem.

All 31 teachers felt that mathematical problem-solving was something extra they had to do and very time-consuming. Teachers did not want to spend a lot of time helping learners to understand the given problems - they chiefly focused on covering the prescribed CAPS curriculum within the stipulated time. Teachers taught huge classes, often with more than 55 learners, and felt that it was impossible to have every learner understand all the problems that they demonstrated on the board. This was stated explicitly by Yolanda:

> I do not have time to explain in detail a given problem to learners because the syllabus is too long and my learners are many. It is impossible to make each one of the 60 learners understand the concepts I will be teaching in one lesson.

This finding reveals that participant teachers' views on mathematical problem-solving are closely related to the context a particular teacher works in. Teachers in this study worked within a prescribed curriculum which emphasises the teaching of mathematical problem-solving. They teach mathematics to large, resource-deficient and multi-lingual classes (see section 1.2). Participant teachers seemed to view the teaching of mathematical problem-solving as a time consuming added extra they had to integrate into their teaching and would take up most of their time resulting in them struggling to cover the prescribed syllabus material within the stipulated time. This finding is in agreement with Silver, Ghousseini, Gosen, Charalambous and Font Strawhun's (2005) and Anderson et al.'s (2004) findings with teachers in both studies indicating that they did not have adequate time to implement new ideas and felt that it was impossible to implement mathematical problem-solving in an already packed curriculum.

4.7 Conclusion

The purpose of the baseline investigation was to find out mathematics teachers' views on problem-solving in this particular context. The baseline investigation revealed that three participant teachers seemed to understand what the teaching of mathematical problem-solving entails except that they did not mention any instances of looking back as a specific step in mathematical problem-solving. The remaining 28 participant teachers had a partial understanding of the teaching of mathematical problem-solving

and held more ‘traditional’ perspectives (see section 2.3.3.1). These teachers believed that teaching mathematical problem-solving was demonstrating examples to learners and giving them practice exercises to work on. They believed that learners were doing mathematical problem-solving each time they found a solution to a given mathematical problem. In all of the observed lessons when the learners asked for help the teachers told them what to do; there were no instances where teachers asked learners to explain the plans they devised and executed.

Before conducting the baseline investigation, I had suspected that teachers in this district are still practising what I referred to as ‘traditional approaches’ to teaching mathematical problem-solving in the literature review section, and this was confirmed by the findings. Having found that teachers in this district are teaching mathematical problem-solving in a traditional way, I observed them delivering lessons. There is a mismatch between Polya’s (1957) mathematical problem-solving processes and participant teachers’ views and practices. The teachers’ practices, i.e. the ‘traditional’ approach to teaching, did not seem to encourage mathematical problem-solving processes in the teaching and learning of mathematics. The lessons were textbook-bound, and there seemed to be no evidence of the teachers supporting learners to understand the given problems, devise and carry out a plan during mathematical problem-solving or to reflect after solving problems. The learners were more reliant on the teachers for support and I did not observe any discussions taking place among them. From these findings, I concluded that this traditional approach to the teaching of mathematics does not seem to stimulate mathematical problem-solving processes in learning. This kind of teaching is believed to leave the majority of learners believing that mathematical knowledge is inexplicable and beyond understanding (Van de Walle et al., 2013). Participant teachers stated that they required support on the teaching of mathematical problem-solving that included the practical demonstration of mathematical problem-solving pedagogy in the context of their classes where discipline is a challenge and learners are under-prepared in mathematics, lack basic mathematical problem-solving skills and struggle with the LoLT.

I am cautious not to generalise the findings from the observations since I observed only two teachers. However, I concluded that there is a need to develop interventions to support mathematics teachers in this particular context in the teaching of mathematical problem-solving. The participant teachers wanted to see the teaching of mathematical problem-solving in action, and therefore; I suggest that teachers can be supported by showing them mathematical problem-solving pedagogy videos of actual interactions in overcrowded, under-resourced and multi-lingual classrooms during PD interventions. There is also a need for factoring in the specific issues of discipline and language into the PD intervention as I did in the first and second cycles of the study (see chapter 5 and 6).

The literature review that I conducted divulged that there is limited research on teachers' views on the teaching of mathematical problem-solving in South Africa. The baseline investigation, therefore, helped to fill in the gaps in the literature on teachers' views on mathematical problem-solving pedagogy in under-resourced, overcrowded and multi-lingual classes. The findings indicate that overcrowded classes and learners' problems with English as the LoLT compel teachers to retain strict ownership of the mathematical problem-solving process. The findings also uncovered that teachers' adoption of the mathematical problem-solving as a step-by-step guided sense-making strategy is a response to the need to support learners on the basic algorithms and specific problem-solving steps they do not master. Further research can be done on the teachers' role and how they can control learners during mathematical problem-solving in large and overcrowded classes. Schettino (2003) analysed the challenges in teaching problem-solving and observed that teachers' reluctance to implement mathematical problem-solving in their lessons may partly be due to limited classroom management skills in such contexts.

4.8 Implications for design of the intervention for the first cycle

As I concluded in section 4.7, grade 9 South African mathematics teachers in the district of study still teach mathematical problem-solving in a traditional way. From the findings, I concluded that the traditional way of teaching does not support the development of

mathematical problem-solving processes in learners. The main findings from the baseline investigation were:

- Participant teachers were text-book bound and believed that teaching mathematical problem-solving was impractical in their context of huge classes, where learners struggled with English, the LoLT and lacked basic mathematical problem-solving skills and processes from lower grades.

- Participant teachers were more comfortable to teach learners procedurally and believed that teaching mathematical problem-solving would consume most of their time and would fail to cover the prescribed syllabus material within the required time.

These findings implied that in the first cycle of the study I worked on strategies with teachers that applied to overcrowded and multi-lingual classes and that took into account the pressures of the curriculum. These strategies included among others group discussion, code-switching and using problem-solving to introduce topics.

All participants in the study stated that they required support on the teaching of mathematical problem-solving that included the practical demonstration of mathematical problem-solving pedagogy in the context of their classes. Therefore in the first cycle, I supported teachers through showing them mathematical problem-solving pedagogy videos during PD intervention. The participant teachers raised issues of classroom control and LoLT challenges and this implied that there was a need to factor in the issues of discipline and language into the PD intervention. All these factors shaped the basis of the design of the intervention of the first cycle discussed in the next chapter.

In this chapter, I present the baseline investigation. I used the findings from the baseline investigation; together with the literature review (see chapter 2) and findings from the context analysis (see section 1.2); to design the first cycle of the PD intervention and I discuss this in the next chapter.

Chapter 5: Design, enactment, evaluation and redesign of the first prototype

5.1 Introduction

In this chapter, I report on the design and development of the first prototype and the findings from the first cycle of the study. Since there were no design principles to design the first prototype, it was essential to derive them from existing PD studies. Consequently, the context analysis (section 1.2), the baseline investigation (chapter 4) and the literature review (chapter 2) informed the design guidelines and preliminary design specifications for the first prototype that I developed in this first cycle.

The context analysis assisted with gaining insight into the possible positives or negatives to the implementation of the PD intervention. From the context analysis, it was evident that the schools in the district of study were under-resourced and overcrowded, with teachers teaching classes that had an average of 60 learners. The baseline investigation established the PD needs of the grade 9 participant teachers. I conducted the literature review to identify initial design principles that had the potential to address the PD in mathematical problem-solving pedagogy. The literature review also divulged how effective PD for mathematics teachers can be conducted. In this cycle, I worked with two experienced grade 9 mathematics female teachers and their learners in school A (see sections 1.5.2 and 3.10). Findings from the data are discussed in light of their implications for the second cycle and other PD studies.

5.2 Design specifications for the first prototype

I designed the PD Intervention to take place within a period of 6 months and its goals were:

- To support grade 9 teachers' mathematical problem-solving pedagogy.
- To improve learners' performance in mathematics.

- The facilitator to explain and agree with teachers what mathematical problem-solving pedagogy is and what it is not in their context. This is because the baseline investigation revealed that most participant teachers had a partial understanding or had misconceptions about what constitutes mathematical problem-solving pedagogy (see section 4.6).

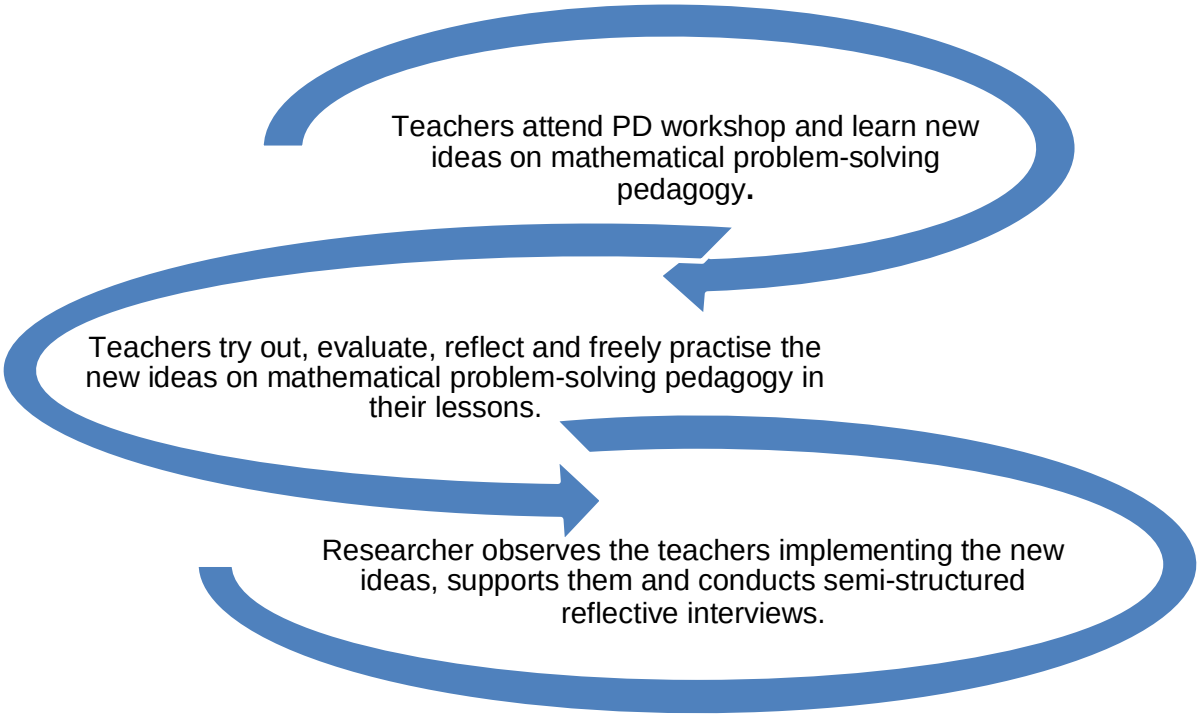
- To take place during the process of classroom instruction in order to link with classroom teaching (Barber & Mourshed, 2007; Desimone et al., 2002).

- Participant teachers to be actively engaged in meaningful discussion, planning and practice (Loucks-Horsley et al., 2010).

- PD activities to offer teachers the opportunity to become actively engaged in the meaningful analysis of teaching and learning (Desimone et al., 2002).

The PD intervention had three PD workshops. After each PD workshop, teachers were required to implement the mathematical problem-solving pedagogy and to reflect on their teaching practice, as explained in section 3.10 and illustrated below in Figure 5.1.

Figure 5.1: The professional development intervention of this study



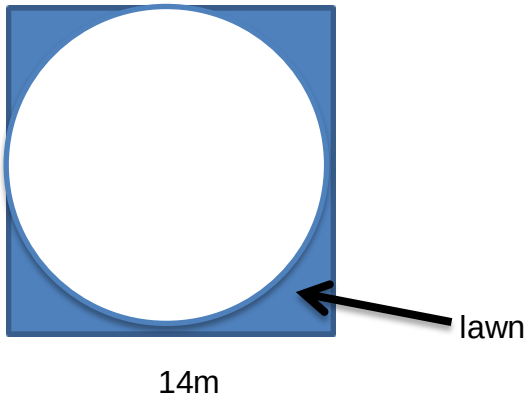
5.3 Implementation of the intervention

I conducted three PD workshops with teachers on the last Wednesday of the first, third and fifth months of the 6-month intervention period (see section 3.10). As I indicated in section 3.10, teachers watched two short-videos that showed the teaching of mathematical problem-solving in action. The videos were to prompt discussions about mathematical problem-solving pedagogy. We discussed the videos, focusing on what mathematical problem-solving pedagogy entails and how Polya's (1957) four steps of problem-solving (see section 2.3.4) were applied by the teachers in the videos. From the discussions of the videos, the key issue that emerged was that the context in which the videos were recorded was different from the one participant teachers were working in. Participant teachers felt that they could not imitate problem-solving pedagogy that was demonstrated in the videos because of the time constraints due to the prescribed curriculum and that the context they were working in had large and overcrowded classes. These aspects of context were very important when teachers implemented the

new ideas on mathematical problem-solving in their classes. In order to take context into consideration, teachers worked with word problems found in the CAPS curriculum during the PD workshops and lesson delivery. I worked with teachers on how to use Polya's (1957) steps in their teaching (see section 2.3.4). The following is a demonstration of how we solved and discussed word problems in the workshop:

Sipho built a circular patio that fits perfectly inside his father's square yard measuring 14m on the sides. If he needs to plant a lawn in the four corners of the square, how many square meters of the lawn does he need to buy?

Figure 5.2: Circular patio fitted in a square yard.



Understanding the problem: The question requires us to find the amount of lawn needed to be planted on the four corners of the yard. This area is inside the square but outside the circle. The square's sides are 14m long. The radius of the circle is not given, but we know the circle fits perfectly inside the square.

Devising a plan: We can find the area of the square and circle by using formulas. Area of square = s^2 and area of a circle is $A = \pi r^2$. By finding these 2 areas, we can get the amount of lawn by subtracting the area of the circle from that of a square.

Carrying out the plan: Side of square= 14m

$A_{square} = s^2 = 14^2 = 196m^2$

The circle fits perfectly inside the square, therefore the radius of the circle is half the side of the square, r = 7m.

$A_{circle} = \pi r^2 = \pi 7^2 = 49\pi = 153{,}86m^2 = 154m^2$

Sipho must buy $A_{square} – A_{circle} = 196 – 154= 42m^2$

Looking back: Estimation. The shaded region looks like a quarter of the square area therefore $42m^2$ is reasonable.

5.3.1 Code-switching

The baseline investigation and discussions during the first workshop of the PD revealed that language featured as an obstacle to the teaching and learning of mathematics in the context of study. As a consequence, I proposed the process of code-switching to the teachers. Code-switching is widely known to be the utterance of two or more languages in the same conversation (Adler, 2001). Gardner- Chloros (2009) defines it as the alternate usage of more than one language in the course of a single utterance, conversational exchange or speech event. We discussed that during the lessons, the teachers or their learners could code-switch between the LoLT and learners' home languages.

After attending the first PD workshop, I encouraged teachers to go and implement the mathematical problem-solving pedagogy and new ideas on code-switching in their lessons for a month (see section 3.10). During this first implementation, we had a discussion about language with the teachers and code-switching proved to be difficult as the participant teachers did not share home languages with all the learners. In the context of study, learners spoke all eleven different home languages in one class, resulting in teachers not being able to speak most learners' home languages. The other challenge was that teachers did not have the correct mathematics register to code-switch in many instances. Mathematics register is generally the language used to convey mathematical concepts. During the semi-structured reflective interviews, teachers revealed that the challenge was they were confused on the perfect time they

were required to code-switch and unfortunately during the first PD workshop discussion I had assumed that the teachers would know. Consequently, in the second PD workshop, I discussed with teachers instances where they could practice code-switching. In doing so, I suggested that teachers could code-switch when they were introducing new concepts or to re-explain difficult concepts. Furthermore, I proposed that teachers would introduce new concepts in English and further explain in learners' home language and this they practised during the second and third implementation.

5.3.2 An example of how Polya's (1957) problem-solving framework was implemented by the teachers in their lessons

As indicated in section 3.10, I observed teachers implementing ideas from the workshops in their teaching of mathematical problem-solving and in this section I discuss, in a vignette (Figure 5.3), how Mrs. Y discussed with her learners each step of Polya's (1957) 4-step problem-solving processes while solving the following word problem:

Reverend Joseph of the Methodist church of South Africa has passed on and his wish before he died was that people should wear only white, red and pink hats at his funeral. 171 women and 93 men attend the funeral. His wife made 13 dozen white hats, five dozen red hats and three dozen pink hats for the guests. At the end of the funeral, 11 hats were not worn. How many hats were worn by the guests?

Figure 5.3: Implementation of the mathematical problem-solving pedagogy by Mrs. Y.

Understanding the Problem

The first step was for learners to understand the problem. Mrs. Y established first if learners understood what was being asked for in the problem by asking them to do the following:

* State the problem in their own words.
* Identify the unknowns.
* Decide what information was important and irrelevant to the problem.

In the given problem I observed that it took all the learners time to realize that they did not need the information about the colour of hats and gender of the guests because it was irrelevant. The teacher worked with learners to understand that the problem being asked in the question was: How many hats were worn by the guests?

Devising a Plan

After understanding the problem, the teacher required learners to devise a plan to solve the problem by considering various strategies (see section 2.3.4.2) like:

* Making a table, diagram or chart
* Trying a simpler form of the problem
* Writing an equation
* Guessing and checking
* Looking for pattern

In the given problem Mrs. Y and her learners had to figure out the total number of hats made before determining the number of hats worn by the guests.

After trying so many strategies Mrs. Y agreed with the learners that writing an equation to show the unknown was the best way to solve the problem: (13 dozen + 5 dozen + 3 dozen) - 11 = number worn

Carrying out the plan

During carrying out the plan Mrs. Y required learners to implement the chosen strategy from the second step. I observed that half of the class struggled to find how many hats were made in total by the Reverend's wife because they did not have the prior knowledge that 1 dozen equals 12. After Mrs. Y assisted learners in finding the initial hats, learners needed to know how many were worn if only 11 remained after the funeral. To find out, Mrs. Y worked with the learners to write an equation that would be used to solve the problem and finding the answer:

So, (13 dozen + 5 dozen + 3 dozen) - 11 = number worn.

13 x 12 = 156; 5 x 12 = 60, and 3 x 12 = 36, so we have (156 + 60+ 36) - 11 = number worn.

252 - 11 = 241

The number of worn hats = 241.

Looking back

Looking back implies that the learners must check their solution strategy and the solution itself. I observed that learners in Mrs. Y's class preferred to skip this step and felt that it was unnecessary. As per our discussions in the workshop, Mrs. Y explained to the learners that looking back was important in that sometimes you can add when you are supposed to subtract which results in a wrong answer. She stressed to learners that checking the answer would help to identify the error. Mrs. Y then discussed with the learners if the answer to the given problem made sense. The above problem required learners to find the hats worn out of a total of 252. Mrs. Y explained that getting 241 as the number of the worn hats made sense since it was less than 252 and it would not have made sense if they had got an answer bigger than 252. After her explanation, I observed that learners understood that guests would not have worn more hats than those that were made and therefore the answer was reasonable.

5.4 Research design

5.4.1 Participants

Two female teachers aged between 31 and 40 years participated in the first cycle (see section 1.5.2). Both teachers had Bachelors' degrees in Mathematics Education. Mrs. X had 13 years' experience and Mrs. Y had 19 years' experience in teaching high school mathematics. As indicated in section 1.5.2, 115 grade 9 learners ranging between 14 and 16 years participated in this cycle. The two teachers were chosen because I could conveniently access them, had reported on the open-ended questionnaire that they were using what I termed 'traditional' methods of teaching in section 2.3.3.1 and had shown interest in mathematical problem-solving pedagogy during the baseline investigation informal discussions (see chapter 4).

5.4.2 Data sources

I collected teacher data through classroom observations and semi-structured reflective interviews (see section 3.8.1). I conducted and audio-taped the classroom observations before and during the PD intervention. I had four pre-observations (see section 4.3) and I was at the school on one Wednesday every month and observed Mrs. X's and Mrs. Y's lessons consecutively. After each classroom observations, I conducted a semi-structured reflective interview with each teacher in their classrooms since it was during their free periods. In total, I had 5 classroom observations and 5 interviews with each teacher. I conducted the first interview at the beginning of the intervention as a starting point to establish how the teachers taught mathematical problem-solving, the difficulties they encountered in the teaching of mathematical problem-solving, and what they hoped to get from the training. The second, third and fourth interviews, which I conducted at the beginning of the third, fourth and fifth months of the intervention respectively, were conducted with each participant mathematics teacher on the lessons observed and were facilitated by playing the teachers selected audio-recordings of their lessons. I held the last interview at the end of the intervention to establish the impact of the PD intervention on participants.

Moreover, teachers were required to reflect on their experiences from participating in the PD intervention and state what aspects in the PD intervention they had found most or least useful. In addition, I also required teachers to evaluate if they felt that they had acquired and were applying any new knowledge and skills in the teaching of mathematical problem-solving. Teachers were also required to state any barriers that they had experienced during the intervention and what they felt could have been done better by me, the facilitator and researcher. In terms of assessing the possible impact of the PD intervention on participant learners, I gave them pre- and post- mathematics attainment tests and a self-reporting MPSSI at the beginning and at the end of the intervention (see section 3.8.2).

I used teacher-made pre- and post-mathematics attainment tests that covered the topics that the learners were doing at the time (geometry and data handling). Both tests had 15 questions and I converted the marks to a percentage. The pre-test and post-test focused on the same concepts of geometry and data handling, but the teachers used different questions. In order to measure the comparability of the pre- and post-test, I combined the two tests and asked 15 learners from the second cycle to complete it. Analysis of the answer scripts for the combined test revealed that the learners had very similar scores on both sets of questions. This finding supported the fact that the teacher-made pre- and post-mathematics attainment tests were likely measuring the same construct.

5.4.3 Data analysis

In this section, the raw data included recorded notes on the observation comment card, audio-tapes from the classroom observations and semi-structured interviews, test marks and responses from the MPSSI. I transcribed audio-tapes from the semi-structured interviews verbatim into written notes in order to be able to identify common patterns. I employed inductive data analysis to analyse the teacher qualitative data from the observations and the semi-structured interviews. Constant comparison (Corbin & Strauss, 2008) of new data against existing data and existing codes against new codes and emerging categories occurred throughout the data analysis process.

Table 5.1: Coding of data from the classroom observations.

Raw data	Early coding	Secondary coding	Final coding
What teacher or learners did before learners started solving the posed problem.	Teacher poses a problem to learners orally or in a written form. Teacher reads the problem. Teacher asks a learner to read the problem. Teacher makes sure that learners understand the problem before they begin to work on it. Teacher asks learners to identify what is asked for in the problem. Teacher discusses any unfamiliar terms in the problem with learners. Teacher asks learners to paraphrase the problem.	Posing the problem Reading to understand the problem Clarifying the task at hand. Understanding of unfamiliar terms.	Understanding the problem
What happened when learners were solving the problem?	Learners brainstorm possible solution strategies in groups. Teacher arranges learners in pairs or groups for discussions Learners are given the opportunity to draw sketches or diagrams and look for patterns. Learners generate solutions collaboratively. Teacher allows learners to share their ideas and strategies. Teacher probes learners how the problem is similar to and different from problems they previously solved. If learners' chosen strategies fail, the teacher encourages them to try another one. Teacher requests learners to communicate their ideas in words, diagrams, or pictures	Brainstorming or working in pairs or groups. Learners generating solutions collaboratively Learners share ideas. Devising a plan Carrying out a plan.	Collaborative learning Devising and carrying out a plan
Events that transpired after learners got solutions to the given problems	The teacher encourages learners to reflect on their solutions and on the processes they used. Invite learners to justify their solutions. Have learners discuss the critical aspects of the problem and solution. Summarise the chief points of the discussion and establish that all learners comprehend them.	Reflection on solutions Justifying solutions Discussing critical aspects of the problem, solution strategy and answer.	Looking back

Table 5.1 above shows how I coded data from the classroom observations, audio-recordings and field notes which I had recorded on the observation comment card. I used the statistical software package SPSS to analyse the learner quantitative data from the pre- and post-tests and the MPSSI.

5.5 Findings

5.5.1 Learners

The MPSSI results exhibited substantial gains in percentages for each question. Table 5.2 shows the pre- and post-intervention percentages for each of the 13 items on the MPSSI, showing the percentage of learners that agreed or strongly agreed with each item. For example, on Q1: I always read the problem carefully to understand it, 34% of the learners agreed or strongly agreed with this statement pre-intervention, and this figure rose to 86% post-intervention. I concluded that the learners perceived an increase in their mathematical problem-solving skills after participating in the intervention.

Table 5.2: Results from the mathematical problem-solving skills inventory (MPSSI).

Test	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13
Pre-intervention	34	10	2	10	18	32	18	14	26	14	28	22	22
Post- intervention	86	54	42	48	76	72	54	70	80	72	66	86	84

Figures 5.4 and 5.5 below show that the average response by each learner to all the questions on the pre-test and post-test of the MPSSI was 2.4 and 3.8 respectively. This indicates that learners were more likely to agree to the questions in the post-test as compared to the pre-test.

Figure 5.4: Average pre-test mean for MPSSI.

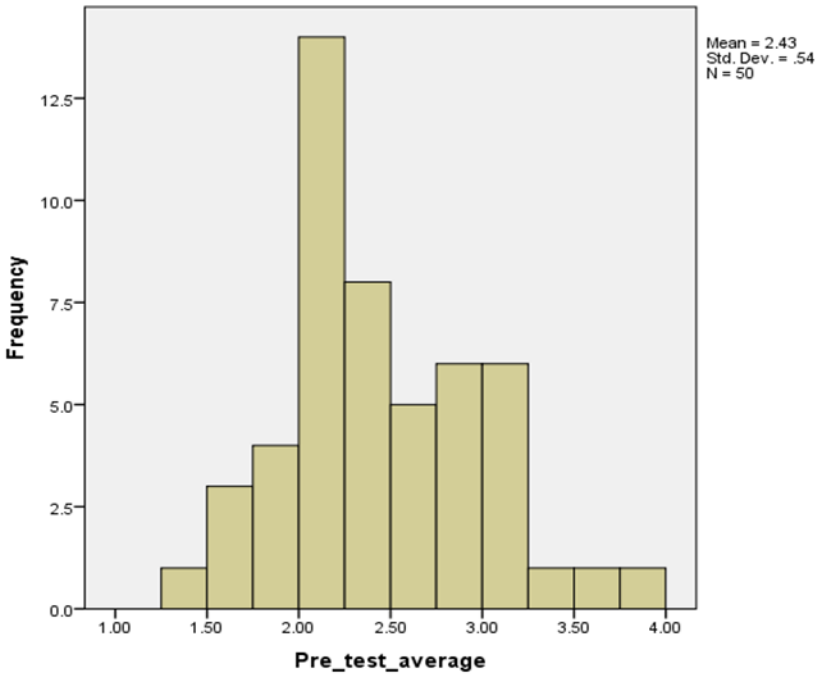
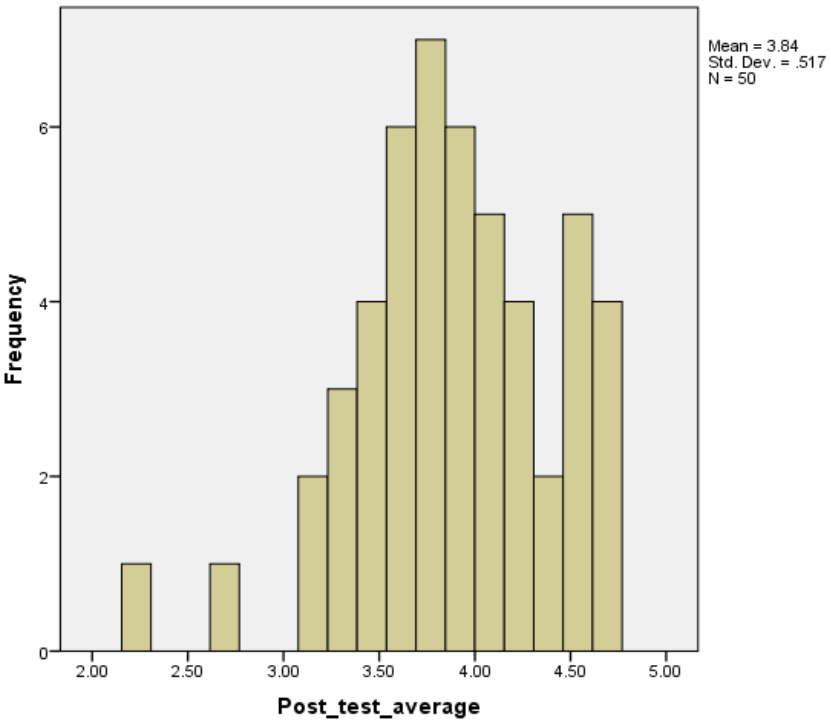


Figure 5.5: Average post-test mean for MPSSI.



I conducted a paired samples t-test to compare learners' responses on the pre-test MPSSI and the post-test MPSSI. Results (Table 5.3 and 5.4) showed that the difference in the scores was statistically significantly different in the average scores for the pre-test MPSSI (M= 2.43, SD= 0.54) and post-test MPSSI (M=3.84, SD= 0.52) conditions; t (49) = 17.93, $p < 0.05$. These results suggest that the responses of the learners changed significantly to agreeing in the post-test MPSSI as compared to the pre-test MPSSI.

Table 5.3: Paired Samples Statistics.

		Mean	**N**	**Std. Deviation**	**Std. Error Mean**
Pair 1	Pre-test average	2.43	50	0.54	0.076
	Post-test average	3.84	50	0.52	0.073

Learner attainment tests

There was an increase in the learner test scores and this seemed to indicate that there were gains in learner attainment. Table 5.4 shows the average percentages on the pre- and post-attainment tests with the standard deviation of each set of marks. I used these results to conclude that the PD intervention implementation seemed to have had a positive impact on learners' performance. Statistical analysis was not done on the learner pre- and post-attainment tests because I had used two different teacher-made tests (see section 3.8.2); therefore, I was not totally certain if this was a genuine change. As a result in the second cycle, I designed my own pre- and post-attainment tests.

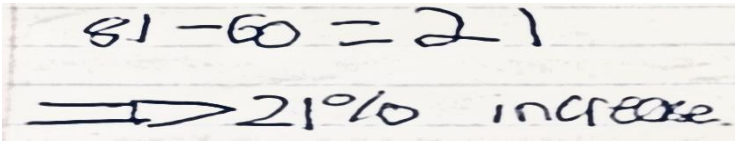
Table 5.4: Results from the pre- and post- attainment tests.

Test	**Average percentage on test**	**Standard Deviation**
Pre-test	19,2	16.2
Post-test	37.8	17.5

I took a closer look at the learners' scripts and analysis of learners' solution strategies showed noteworthy gains in their problem-solving processes in the post-test as compared to the pre-test. Below are Sihle's solutions to question number 4 in the pre-test and question number 2 in the post-test respectively. I compared question 4 and 2 because they were similar, although of different contexts.

Cycle 1 Pre-test question number 4: Calculate the percentage increase if the price of an Uber taxi of R60 is increased to R81.

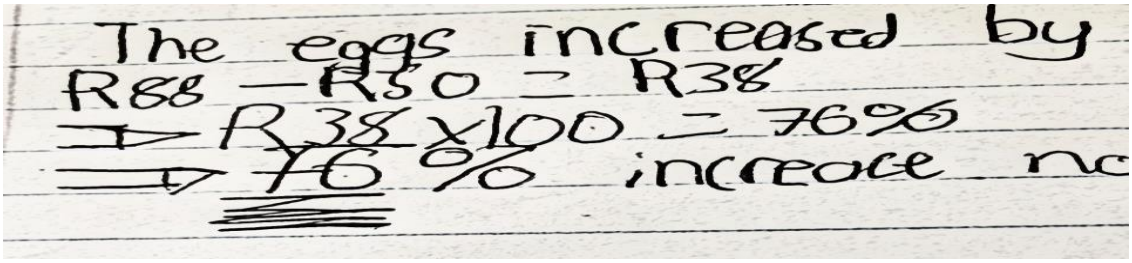
Figure 5.5: Cycle 1 pre-test question number 4 sample solution



After marking Sihle's work, I had an opportunity to ask him why he had computed his solution by only subtraction. The response that Sihle gave was that it was because the question was asking for the taxi fare increase.

Cycle 1 Post-test question number 2: Busi is sent by her mother to Shoprite supermarket to buy eggs. A crate of eggs has gone up from R50 to R88. Calculate the percentage increase.

Figure 5.6: Cycle 1 ost-test question number 2 sample solution



The above computation, although containing errors, indicates that Sihle exhibited notable gains in his problem-solving processes in the post-test. In the pre-test, Sihle only subtracted the given number and gave it as a percentage. In the post-test, Sihle could reason that he was required to calculate the percentage increase. Because of

time, resources and access to the school, I did not have the opportunity to investigate further if these gains were genuine and were indeed a result of the intervention. This is the reason why I engaged learner task-based interviews in the second cycle of the study (see section 6.3.3).

5.5.2 Teachers

5.5.2.1 Findings from teacher observations

The findings from the teacher classroom observations are presented under the three themes: understanding the problem, collaborative learning, devising and carrying out a plan and looking back.

(i) Understanding the problem

The observations I conducted before implementation of the intervention unearthed that the participant teachers relied solely on the textbooks. They introduced the topic by demonstrating the examples in the textbook on the board and there was no clarification of tasks at hand. The teachers did not verify if learners understood what was being demonstrated on the board. After demonstrating examples on the board, teachers would give learners homework from the textbook. After the first workshop, teachers began to gradually implement what they had learned from the mathematical problem-solving pedagogy training. Teachers began to encourage learners to read and understand the given problem before attempting it. They also required learners to paraphrase the problem at hand in their own words as a way of demonstrating that they indeed understood. English language, the language of instruction, proved to be an obstruction to learners' understanding. I, therefore, encouraged the teachers to code-switch as was necessary (see section 5.3.1).

As the intervention progressed, teachers assisted learners to understand the given problems by discussing any unfamiliar terms in the problems and required them to underline keywords in a given problem. In one instance, during a data handling lesson, Mrs. Y posed a problem on the total number of fatal accidents in each province on South African roads. In this problem, learners were required to find measures of central

tendency and dispersion. Mrs. Y required the learners to firstly underline the keywords in the problem and discuss in pairs the unfamiliar terms, that is, ‘measures of central tendency’ and ‘measures of dispersion’. Learners were required to verbalise and restate the given problems in their own words to prove that they really understood the posed problem. In one lesson, I observed that Mrs. X encouraged learners to verbalise their thinking processes during an investigation to establish the minimum conditions for congruent triangles.

(ii) Collaborative learning

As the intervention progressed, teachers arranged learners in pairs or groups of three or four for discussions. Learners were given the opportunity to brainstorm ideas and solution strategies, draw sketches or diagrams, look for patterns and to generate solutions collaboratively. I observed that Mrs. X put her learners into groups of three or four when they were investigating how doubling any or all of the dimensions of right prisms and cylinders affects their volume.

(iii) Devising and carrying out a plan

As the intervention progressed, participant teachers began to probe the learners to identify how the given problems were similar to or different from the problems they had previously solved. I observed a learning episode where learners in Mrs. Y’s class were investigating the properties of quadrilaterals. Learners had investigated the properties of a parallelogram the previous day and on that particular day, they were required to investigate the properties of a rectangle and a square. Mrs. Y probed learners on how the given problem was similar to the previous day’s problem. It was interesting to observe that most learners could deduce from what they had learnt from the previous grades that rectangles and squares are parallelograms. Participant teachers frequently requested learners to communicate their ideas in words, diagrams, or pictures. I noted that the participant teachers required learners to draw either a tree diagram or a two-way table when finding probabilities of compound events. Mrs. X posed the following probability problem and it was mandatory for learners to draw a tree diagram before solving the problem:

A coin is tossed two times:

1. Determine the possible outcomes for the event.
2. What is the probability of scoring tails as one of the outcomes?
3. Determine the probability of scoring two heads.

If learners' chosen strategies failed, teachers encouraged them to try another one.

(iv) Looking back

As the intervention progressed, teachers required that learners look back and reflect after getting solutions to check if the way they had solved the given problems was the best way, to clarify if there were other solutions other than what they had found and to establish if their solutions were reasonable. In one instance Mrs. Y required her learners to solve the following question and to look back at the solution process and the answer:

The monthly average for Gauteng's rainfall is in the table below. Represent this information in a format that is easy to interpret.

Table 5.5: Monthly average rainfall for Gauteng

Month	**Rainfall in mm**
January	95
February	40
March	10
April	0
May	45
June	5
July	0
August	0
September	45
October	70
November	85
December	90

I observed that 50% of the learners represented the information in bar graphs, 45% drew pie charts and 5% drew histograms. After finding solutions, the learners were required to look back and reflect on their solutions and think if they could represent the information in a different format. It was interesting to observe that learners who had drawn bar graphs began to think of drawing pie charts and broken-line graphs. Learners that had initially represented information on the pie charts had what I called an 'aha' moment and realised that they could also represent the information in a bar graph or in a broken-line graph. One of the learners that had represented the information in a histogram was asked to show his solution on the board and the whole class was asked to discuss his solution. It was interesting to observe that most learners understood that it was an unreasonable form in which to represent this kind of data because histograms are used to represent continuous data.

5.5.2.2 Findings from the teacher semi-structured interviews

I present the findings from the teacher semi-structured interviews under the four themes that emerged from the interview data. The four themes are: changes in views about mathematical problem-solving pedagogy; appreciation of collaborative learning; increased awareness of learners' needs; and PD activities that had a positive impact on teachers' PD.

(i) Changes in views about mathematical problem-solving pedagogy

Initially, teachers viewed the teaching of problem-solving as a step-by-step explanation, doing examples on the board and giving learners practice exercises. When I introduced mathematical problem-solving pedagogy to the teachers, teachers perceived it as time-consuming and asserted that they had a lot of prescribed teaching material to cover. However, as the intervention progressed they began to understand and appreciate the teaching of mathematical problem-solving as a process as illustrated by the quote from Mrs. Y below:

> Initially, I was confused and showed learners how to work a problem using drilling methods, but now I know how to teach mathematical problem as a process.

(ii) Appreciation of collaborative learning

Both teachers valued collaborative learning as made explicit by Mrs. X:

> Working with my colleague during the workshops and teaching was superb because we could support each other and we shared ideas and obstacles. Collaboration made learning effortless. I liked it. I became a better teacher by learning from my colleague.

As the intervention progressed, participant teachers began to implement collaboration in their own teaching. Learners were required to work in pairs or in groups and this kept them on-task. Teachers were no longer the only source of knowledge and this helped with classroom management as learners had a responsibility of completing tasks in pairs or groups, as was highlighted by Mrs. Y in one of the interviews:

> As I arranged learners to work in pairs, they were no longer moving in and out of the classroom. Pairing them helped me with discipline and class control.

(iii) Increased awareness of learners' needs

The two participant teachers revealed that they had become more aware that they had to clarify the task at hand and make sure the learners understood before solving the given problem. This is illustrated by these quotes from Mrs. Y:

> This professional development intervention opened my eyes. I used to assume that learners understood the given questions, but now I check if they really understand. If they don't we read the problem together, paraphrase the questions and I sometimes require learners to verbally tell me what the question would be asking for.

> If the learners do not understand a given problem, I now re-read the problem for them or use their mother tongue to explain. This is to make sure that they understand the given problem.

(iv) PD activities that had a positive impact on teachers' professional development

Videos on mathematical problem-solving pedagogy that were shown during the workshops were very positive for teachers, as articulated by Mrs. Y:

> To be able to see mathematical problem-solving teaching in action was intriguing. I saw where I was getting it wrong. It helped me to realise areas that needed improvement in my own teaching.

In addition, the teachers appreciated that during the PD intervention I was respectful to them and acknowledged their experiences, as expressed by Mrs. X:

> You treated me like an adult during the intervention. I have been to other workshops where I was treated like a child who knows nothing. You showed me respect. You were aware that I had fully trained to be a teacher and that I teach large classes.

Finally, the teachers valued solving problems and simulating how to teach problem-solving during the workshops as explained by Mrs. Y:

> To practically learn how to teach mathematical problem-solving during the workshops was helpful for me because it built my confidence before implementing the new ideas in the classroom.

5.6 Discussion of findings, conclusions, and implications

5.6.1 Factors that influenced change in learners

The post-test and post-MPSSI demonstrated that the PD intervention had a positive impact on learners' performance. This finding resonates with Barber and Mourshed's (2007) results from their research of 25 national school systems, that there is a positive correlation between teachers' PD and learners' achievement. Classroom observations confirmed that teachers were working in a different way with learners and emphasising problem-solving approaches. Teachers were encouraging collaborative learning amongst learners. When learners grappled with the problems in pairs or groups, it

allowed for richer and more worthwhile discussions by the whole class. After solving problems, teachers encouraged learners to look back on the answer, the solution process or method and looking at the problem itself. This is the final and important step of mathematical problem-solving. When learners were required to look back and justify their solutions, they were able to come up with solutions in many ways, were able to correct themselves and could easily discern if their solutions were appropriate.

5.6.2 Factors that influenced change in teachers' pedagogy

The pre-intervention observations revealed that the lessons were teacher-led and teachers implemented traditional methods of teaching. However, as the intervention progressed, I noticed that the teachers became more of facilitators rather than teachers, giving learners the opportunity to grapple with the given problems. There were more group work and discussions among learners, and learners began to work with and listen to each other. Teachers became more focused on working with learners to understand the given problems. This was an important step since understanding is the first important step in the teaching of mathematical problem-solving. In situations where learners did not understand the given problems because of the language, teachers code-switched.

This part of the study fills in a gap in the literature on PD for mathematical problem-solving pedagogy as it unearthed that language and code-switching are important aspects to be considered when implementing a PD intervention for mathematical problem-solving pedagogy in a multi-lingual context. I recommend that a PD programme should comprise of a segment that supports teachers on how to appropriately conduct code-switching and to support learners with their language. This concurs with Setati (2008) who advocates that the use of learners' first language in the learning of mathematics has been found to be a beneficial resource for instruction.

Initially, teachers were reluctant to participate in the intervention and implement mathematical problem-solving pedagogy. They indicated that they were worried that they would fail to cover the CAPS syllabus within the prescribed time. This finding aligns with what Slattery (2013) states; that the syllabus and task completion places pressure

on teachers. As the intervention progressed, teachers began to understand and became comfortable with mathematical problem-solving pedagogy and started implementing the new ideas.

I observed the participant teachers for a month before implementing the intervention and this created trust between me and the teachers, and moreover resulted in teachers working comfortably with me. Therefore, I recommend creating a positive relationship with teachers before implementing a PD intervention and building on teachers' experiences when training them. Teachers appreciated that I respected them and acknowledged their experiences. It was important that I was responsive to respecting the participant teachers because respect is a key aspect of the African culture. I further explain the issue of respect in section 7.2. Collaborative learning was beneficial to both participant teachers and learners. I was not surprised by the findings on the benefits of collaboration as various authors have discussed the value of collaborative learning during teachers' PD and this I discuss in section 7.5.

The semi-structured reflective interviews with participant teachers were also imperative to the teachers' growth in mathematical problem-solving pedagogy and this I further discuss in section 7.4. Following up on participant teachers in the classrooms to check if they were correctly implementing the mathematical problem-solving pedagogy was advantageous as I was able to support them as was necessary. This is different from the traditional 'one-shot' workshops and I recommend that PD practitioners should support the teachers in the classroom during the implementation stage. It is imperative that teachers are supported during the implementation stage to address the specific challenges of changing classroom practice (Gulamhussein, 2013), receive feedback and are given the opportunity to reflect critically (Bell & Gilbert, 1994).

5.7 Design principles

Design principles are one of the major outputs of DBR. McKenney and Reeves (2012) term them theoretical insights into a phenomenon in question, which recommend how to address specific issues in a range of settings. While I acknowledge that I had gone

through only one cycle of the DBR, the first cycle generated a number of design principles relating to the designing of a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy. These principles informed the designing of the second prototype in the second cycle of the study and are imperative to mathematics education and PD practitioners who are in the process of designing PD interventions. The design principles include design principles for PD in general and design principles for PD for mathematical problem-solving pedagogy

Design principles for professional development in general

- Facilitators of PD must create a positive relationship with participant teachers before implementing the intervention.
- PD should be built from teachers' experiences and current knowledge.
- Teachers should be used as a resource and must actively participate in the implementation of a PD intervention
- Respecting participants is important in an African context when implementing the PD intervention.
- PD should support teachers on how to implement pedagogy in a multi-lingual context.
- Teacher reflection is an important aspect of the PD intervention.

Design principles for professional development for mathematical problem-solving pedagogy

- A baseline investigation must be conducted to establish teachers' views and practices on the teaching of mathematical problem-solving before implementing the PD intervention.
- PD for teachers' mathematical problem-solving pedagogy should be organised around collaborative problem-solving. Collaborative problem-solving in the context of this study is when teachers solve mathematical problems and negotiate meaning in social settings.

- Facilitators of PD must observe teachers practically implementing the mathematical problem-solving pedagogy and support them as necessary.

5.8 Implications of the findings for the second cycle

Implications for theory

One of the most significant outcomes from this cycle was that language stands so much in the way of learners' learning of mathematics. Polya's (1957) first step in mathematical problem-solving is understanding the problem, and if learners struggle with LoLT, it means that they do not even understand the given problems. Language and the problem-solving process cannot be separated (see section 7.3 for further discussion). Therefore in the second cycle, I did a lot more with teachers on how they could support learners' difficulties with problem-solving, English language and code-switching (see chapter 6).

Implications for design

- In this cycle, I used participant teacher-made tests to act as pre- and post-attainment tests, but in the second cycle I designed my own tests so that I could easily check on the reliability of these tests.

- In the second cycle, I introduced learner task-based interviews to help explain why learners were doing well in the post-test and why they were giving better responses in the post- MPSSI. I anticipated that the learner task-based interviews would explain if learners' improvements were real and if they were doing problem-solving differently from the way they were doing it pre-intervention.

- In the second cycle participant teachers were observed delivering lessons by other participant teachers and were given feedback. This development emerged from the usefulness of teachers observing how other teachers taught problem-solving in the videos shown in the first cycle and the usefulness of working on and reflecting collaboratively on lessons.

5.9 Evaluation of the first prototype

The purpose of this section is to evaluate the first prototype (the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy) which was generated from this first cycle. Evaluating the prototypes was imperative to the study since during the evaluation phase, 'design ideas and prototype solutions are empirically tested and the findings are reflected upon, with the aim of refining (theoretical) understanding about, if, how, and why intervention features work' (McKenney & Reeves, 2012, p. 133). I used Guskey's (2000) model of five critical levels of PD evaluation (Table 5.6) to evaluate the prototype generated from this cycle. I used Guskey's (2000) model because it is more relevant to educational research in general and for this study.

Level 1: Participants' reactions

Guskey (2000) advises that information at this level should be gathered at the end of the PD session. Questions addressed at this level include whether; the participants enjoyed the event, thought it was useful, addressed their needs, was well-presented and well organised. I carried out post-semi structured reflective interviews (see Appendices P and R) with the participant teachers after the PD intervention implementation and the findings were that participant teachers felt that the intervention was well-organised, helpful and applicable to their context.

Level 2: Participants' learning

This level measures the new knowledge and skills that participants gained from the PD intervention. I assessed participant teachers through classroom observations and semi-structured reflective interviews. The findings seemed to indicate that participant teachers exhibited positive gains in their mathematical problem-solving pedagogy.

Table 5.6: Guskey's (2000) five critical levels of professional development evaluation.

Evaluation Level	**What Questions Are Addressed?**	**How Will Information Be Gathered?**	**What Is Measured or Assessed?**	**How Will Information Be Used?**
1. Participants' Reactions	Did they like it? Was their time well spent? Did the material make sense? Will it be useful?	Questionnaires administered at the end of the session	Initial satisfaction with the experience	To improve program design and delivery
2. Participants' Learning	Did participants acquire the intended knowledge and skills?	Paper-and-pencil instruments Simulations Demonstrations Participant reflections (oral and/or written)…	New knowledge and skills of participants	To improve program content, format, and organization
3. Organization Support & Change	Was implementation advocated, facilitated, and supported? Was the support public and overt? Were problems addressed quickly and efficiently?...	District and school records Minutes from follow-up meetings Questionnaires Structured interviews with participants and district or school administrators…	The organization's advocacy, support, accommodation, facilitation, and recognition	To document and improve organization support To inform future change efforts
4. Participants' Use of New Knowledge and Skills	Did participants effectively apply the new knowledge and skills?	Questionnaires, Structured interviews with participants and their supervisors, Participant reflections (oral and/or written), Participant portfolios, Direct observations, Video or audio-tapes	Degree and quality of implementation	To document and improve the implementation of program content
5. Learner Learning Outcomes	What was the impact on learners? Did it affect learner performance or achievement?...	Learner records School records Questionnaires Structured interviews with learners, teachers…..	Learner learning outcomes….	To focus and improve all aspects of program design, implementation, and follow-up. To demonstrate the overall impact of PD.

Level three: Organisation Support & Change

For this level, I focused on the following questions:

1) Did the activities promote changes that were aligned with the mission of the school?

2) Were changes at the individual level encouraged and supported at all levels?

3) Were sufficient resources made available, including time for sharing and reflection?

4) Were successes recognised and shared?

I used this information to document and improve organisation support and to inform the designing of the second prototype.

Level 4: Participants' use of new knowledge and skills

During classroom observations and semi-structured reflective interviews, I evaluated if the new knowledge and skills that participants learned made a difference in their teaching of mathematical word problem-solving and professional practice in general. The findings seemed to imply that the PD intervention had a positive impact on participant teachers' mathematical problem-solving pedagogy. I used the gathered information to document and improve the implementation of the PD intervention in the second cycle.

Level 5: Learners learning outcomes

At this level, I assessed the impact of the PD intervention on learners' learning outcomes that included cognitive (performance & achievement), affective (attitudes and dispositions), psychomotor (skills and behaviour). The findings from the tests and MPSSI revealed that the intervention had a positive impact on participant learners' achievement in mathematics. However, this assessment did not reveal enough information on learners' problem-solving processes such I implemented task-based

interviews in the second cycle. I used the information I obtained from this level to demonstrate the overall impact of the first cycle of PD intervention and to focus and improve all aspects of the PD intervention in the second cycle.

5.10 Conclusion

I used the design principles in section 5.7 to design the second prototype of the study. In the next chapter, I discuss the design, development, enactment, evaluation and redesign of the second prototype in the second cycle of the study.

Chapter 6: Design, enactment, evaluation and redesign of the second prototype

6.1 Introduction

In this chapter, I report on the design and findings of the second cycle of the study. I used the design principles generated from the first cycle (see section 5.7) as the design specifications for the second prototype. It emerged that the design specifications for the first cycle (see section 5.2) were important in designing PD of mathematics teachers' problem-solving pedagogy and any PD intervention in general. Consequently, I also incorporated these in the design of the second prototype.

In this cycle, I worked with teachers on how they could support learners' difficulties with problem-solving, English language and translanguaging. I used POFT as a PD strategy to support participant teachers' mathematical problem-solving pedagogy. I engaged task-based interviews to provide further evidence on the impact of the PD intervention on participant learners' performance and mathematical problem-solving processes.

6.2 Design specifications for the second prototype

I designed the PD Intervention to take place within a period of 6 months and its goals are indicated in section 5.2 and they also included the following:

- To conduct a baseline investigation to establish participant teachers' views and practices on the teaching of mathematical problem-solving before implementing the PD intervention.
- To create a positive relationship with participant teachers before implementing the PD intervention for mathematical problem-solving pedagogy.
- To build PD from teachers' experiences and current knowledge of mathematical problem-solving pedagogy.

- To use teachers as a resource and give them opportunities to actively participate in the implementation of a PD intervention for mathematical problem-solving pedagogy.
- To support teachers on how to implement mathematical problem-solving pedagogy in a multi-lingual context.
- To integrate teachers' reflection as an important aspect of the PD intervention.
- PD for mathematical problem-solving pedagogy should be organised around collaborative problem-solving.
- PD facilitators to observe teachers practically implementing the mathematical problem-solving pedagogy and support them as necessary.
- Respecting participants is an important aspect in an African context when implementing the PD intervention.
- To use POFT as a PD strategy to support participant teachers.
- To use task-based interviews to assess the impact of the PD intervention on learners' performance in mathematics and to evaluate learners' mathematical problem-solving processes.

6.3 Implementation of the intervention

The PD intervention, which was completed within a period of six months, was implemented just as in the first cycle (see section 5.3 and 3.10). The intervention had three PD workshops which I conducted with teachers on the last Friday of the first, third and fifth months of the 6-month period. As I indicate in section 3.10, the workshops were from 12 p.m. to 3 p.m., which resulted in a total of nine hours of training for participant teachers.

One of the most significant outcomes from the first cycle was that language stands so much in the way of learners' learning of mathematics (see section 5.8). Polya's (1957) first step in mathematical problem-solving is understanding of the problem, and if learners struggle with language, it means that they do not even understand the given word problems. It is impossible to separate the language of instruction and the mathematical problem-solving process (Chirinda & Barmby, 2017).

6.3.1 Language

In the first cycle, I encouraged teachers to code-switch, but I realised that code-switching was opportunistic and not very effective as teachers were often unable to find words or phrases in their languages that corresponded to certain mathematical concepts. The practice of code-switching was also challenging in that I was working in schools where learners were speaking all eleven different languages as their mother tongue. Therefore, it was necessary to practice translanguaging. Translanguaging is 'the flexible and meaningful actions through which bilinguals select features in their linguistic repertoire in order to communicate appropriately' (Velasco & Garcia, 2014, p. 7), and this includes learners receiving information in one language and repeating it in another language (Baker, 2006).

In the first PD workshop, I discussed with teachers how they could think about mathematical terminology and explain it to learners before they attempted the given problems. Participant teachers practiced how to present problems in English and one other language that they were comfortable with. I suggested that participant teachers could put learners in groups according to their common languages where they would translate to one another and thereby creating peer-to-peer interaction (Anton & DiCamilla, 1998). Anton and DiCamilla (1998) observe that peer-to-peer interaction creates a social space which empowers learners to work collaboratively on problems or activities.

In the second workshop, the teachers reflected on the use of translanguaging in their lessons during the implementation and we deliberated on how teachers could encourage and support learners to translate given problems for each other in their languages and have learners explain back to the teachers in English what the problem required.

From the classroom observations, I observed that translanguaging facilitated learners' understanding of the given problems because it made the language to become more transparent to learners. I also perceived that translanguaging was advantageous in that as learners translated to their peers it reinforced their understanding of the given

problems and teachers could easily identify their misunderstandings and misconceptions. Translanguaging had many benefits in this study; however it had its own limitations. There were instances where learners could not translate complicated mathematical words between English and their home language or it would be a case that the maths word did not exist in the learners' home language or had different meanings. For example, learners found it hard to translate the following word problem:

> There are 23 learners in a grade 10 mathematics class. There are 3 more boys than girls in the class. How many girls are in the class?

I had initially thought the above problem was relatively easy; however, during the semi-structured interviews, the teachers revealed to me that the difficulty with this question arose from the fact that the words like more, above, over, etc have one meaning in South African languages.

In the third workshop, we took into consideration ways in which teachers could use language as a resource, for example looking at a word in a task and allowing the structure of the word to help learners understand, for example, the word transformation. We pondered on ways in which teachers could change and simplify the mathematical language if learners were struggling; for example, if learners were struggling with algebra language, the teachers could change it to arithmetic language. During the final semi-structured interview of this cycle Ms. N affirmed to me that she had found translanguaging very helpful; however, she was worried that its excessive use would undermine the learners' learning of LoLT and their achievement during national examinations which are always in English. As the PD facilitator in this study, I was left wondering if Ms. N's concerns were something to worry about; her concern is called the dilemma of code-switching by Adler (2001). Ms. N's concerns are observed by Howie (2003) who states that the use of home language as a mode of instruction hinders learners' proficiency in English language. As a mitigation measure, I discussed with Ms. N that in this study it was important that the learners understood the given word problems and were able to express themselves. Language was not the main focus of this study; therefore, I did not investigate this issue further.

6.3.2 Peer observation for teaching

In this second cycle of the study, I decided to use POFT as a PD strategy to support participant teachers' self-reflection, because of the usefulness that emerged from teachers observing practice in videos in the first cycle. The aim of POFT was to test and evaluate the benefits it would bring in a PD intervention for teachers' mathematical problem-solving pedagogy. POFT is primarily focused on teachers observing each other's practice and learning from as well as providing descriptive feedback to one another (Chamberlain, D'Artrey & Rowe, 2011) without necessarily judging each other (Hendry, Bell & Thomson, 2014) and the overall goal being to advance teaching practice (Hendry & Oliver, 2012). Chamberlain et al. (2011) state that peer observation is intended for PD and frequently for quality assurance and reward, for example in the peer observation of teaching Integrated Quality Management System (IQMS). IQMS (Department of Basic Education, 2014) is a quality and performance instrument that was introduced in 2003 to measure and improve the quality of teaching and learning in South African schools. There are two key forms of peer observation, that is; formative and summative review (Rowe, Solomonides & Handal, 2010). Rowe et al. (2010) explain that formative review focuses on the on-going enhancement of teachers' practice and summative review is usually for promotion or formal teaching evaluation.

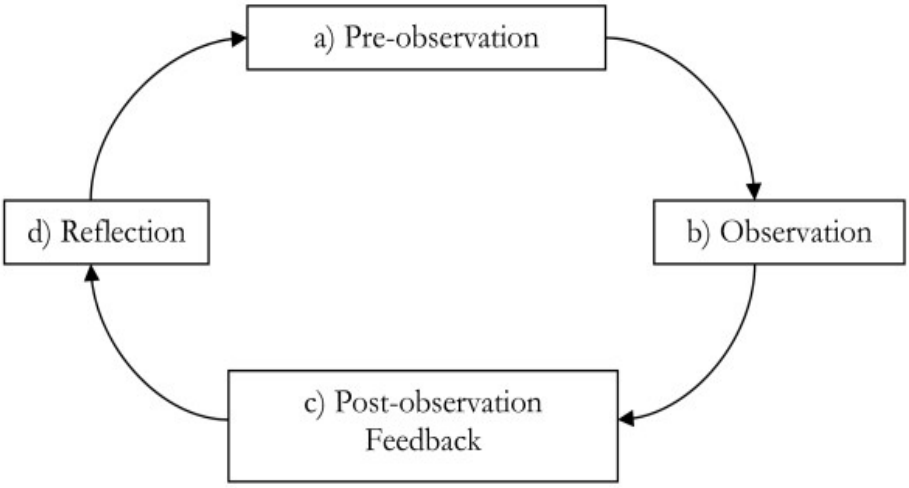
Peer observation has been used successfully at universities in Asia and Australia (Bell & Cooper, 2013; Bolt & Atkinson, 2010; Engin & Priest, 2014; Hendry et al., 2014; Hendry & Oliver, 2012; Pereira, 2014). However, there is not much documented in the literature on peer observation in South African institutions, although it is a component of the IQMS. Therefore, there is a dearth of research in South Africa on how POFT can be used to support mathematics teachers' PD. Hence, the significance of this section of my PhD studies.

Peer observation is a powerful vehicle for teacher PD (Bell & Mladenovic, 2008) that focuses on teachers' exclusive needs and the opportunity to both learn from one another's practice and to offer constructive feedback to one another (Bell, 2002). Bell and Mladenovic (2008) are of the opinion that peer observation is profitable if conducted

in well-planned and supportive conditions. This is echoed by Pereira (2014) who posits that a suitably structured peer observation of a teaching programme is capable of providing valuable feedback to teachers and has a positive impact on teaching and learning. In this study, POFT was appropriately structured and well thought out. During the first PD workshop, I explained and clarified the process and objectives of POFT to the participant teachers. Furthermore, I explained to the teachers that with POFT, the feedback is non-judgmental, non-evaluative, descriptive, formative, and solely for teachers' mathematical problem-solving pedagogy advancement (Bell & Mladenovic, 2008). As suggested by Engin and Priest (2014), I explained to the participant teachers that the observer was rather supposed to evaluate and reflect on their own mathematical problem-solving teaching in the light of observation. Participant teachers understood that they were carrying out the observations neither to critique nor to discover if they had more advanced mathematical problem-solving pedagogy than the observed. Bell and Cooper (2013) recommend that the relationship between the observer and the observed should be that of critical friends where feedback is given as a dialogue and not a judgment and I encouraged this amongst the teachers. After understanding the process, the two teachers agreed to participate in the POFT.

A buddy system was employed in this study and the two participant grade 9 teachers acted as the observer and observed with reciprocation of roles after each observation. One teacher Mr. M was an experienced teacher and Ms N. was a novice (see section 6.4.1). Pairing experienced and novice teachers is beneficial since it holds the promise of merging expertise and knowledge transfer (Bolt & Atkinson, 2010). Each teacher observed or was observed once every two months in a 6-month intervention. In the second and third observation sessions, the observer was required to check for transformation in the observed's practice as per the previous post-observation discussions. At the end of the 6-month cycle teachers had been observed or had observed three times. The POFT involved four phases: a pre-observation discussion, observation, post-observation feedback and reflection (Bell, 2002).

Figure 6.1: The POFT process



(Source: Bell, 2002, 8)

6.3.2.1 Pre-observation discussion

Participant teachers met for no more than 10 minutes before each observation episode to establish objectives of each observation and to agree on the date and time. Both teachers were teaching grade 9 mathematics, therefore the content was the same. However, the observed was required to provide a brief background and context of the lesson and what he or she expected and required from the observation.

6.3.2.2 Observation

The focus of the observation was on the observed teachers' mathematical problem-solving pedagogy. During observation, the observer was required to take notes on the mathematical content, mathematical problem-solving pedagogy and learners' problem-solving processes. The observer was cautioned not to disrupt the learning process and was not allowed to interact with the observed and their learners during the lesson. The objective was to observe in a natural rather than intrusive manner (Bolt & Atkinson, 2010).

6.3.2.3 Post-observation feedback

A post-observation discussion, which was not longer than 10 minutes, occurred immediately and briefly after the observed lesson. The post-observation discussion had

to be brief because in most cases teachers had to go to the next lesson. The observer was required to share their recorded notes with the observed. The observed also shared their experience of being observed. The purpose of the post-observation discussion was to identify the strengths and weaknesses of the observed lesson and to share ideas on how both teachers could improve their mathematical problem-solving pedagogy and learners' learning of mathematical problem-solving.

6.3.2.4 Reflection

I carried out three reflective interviews during the 6-month cycle, that is, once every 2 months (see section 6.4.2). During the semi-structured interviews, I required the teachers to reflect on their experience of observing and being observed and the feedback received during the post-observation discussion. To assist teachers' self-reflection, I asked them the following umbrella questions, which were pilot-tested with a pair of grade 9 teachers at a neighbouring school:

- What did you observe that you would like to replicate in your teaching of mathematical problem-solving? Why?
- What was the observed doing that seemed central in enhancing learners' understanding and problem-solving processes? Why?
- What have you learnt from participating in the POFT?
- What is working with the process of POFT? Why?
- How is POFT affecting learners' learning of problem-solving? Why?
- How are you going to implement what you have gained from the POFT process in your next lessons?
- What can I do to help?
- Mr. M or Ms. N said this, what do you think of that?

The above questions were for guiding purposes as there were many emergent questions during the interviews.

6.3.3 Task-based interviews

Task-based interviews which are a form of clinical interviews; are interviews in which a participant learner or group of participant learners have a discussion while solving a

mathematical problem or problems (Maher & Sigley, 2013). Task-based interviews are one of the most powerful ways of exploring learners' mathematical thinking and problem-solving processes (Dunphy, 2010) and offer great benefits such that they are now increasingly being used as a research method for the study of mathematical learning and problem-solving (Dunphy, 2010; Goldin, 2000; Heng & Sudarshan, 2013; Jenkins, 2010; Maher, Powell, & Uptegrove, 2011). Heng and Sudarshan (2013) unearthed that task-based interviews lead to better comprehension of learners' thinking, learning and problem-solving processes and skills in mathematics. Jenkins (2010) found that questions that were asked by prospective teachers who participated in the task-based interviews study prompted learners to clarify their thought processes. Pre-service Irish teachers in Dunphy's (2010) study accentuated that task-based interviews facilitated them to gain insight into young learners' thinking processes when doing mathematics. In McDonough, Clarke and Clarke (2002) study, Australian pre-service teachers comprehended young children's mathematical thinking and problem-solving processes through task-based interviews. McDonough et al. (2002) concluded that the pre-service teachers in their study capitalised from task-based interviews such that the former was incorporated in the curriculum for the training of pre-service teachers. As exemplified by these studies, I found it relevant to use task-based interviews to investigate the impact of the PD intervention on grade 9 participant learners' performance and problem-solving processes in mathematics. The goals of the interviews were to observe learners' mathematical problem-solving behaviour in detail (Goldin, 2000) and to gain insight into learners' existing and developing mathematical problem-solving processes.

The grade 9 learners in this cycle normally had mathematics extra lessons on Tuesdays between 14h30 and 15h30 and I was allowed by the school to use this time to conduct the task-based interviews. I conducted the task-based interviews one Tuesday every two months and in total, I carried out three interviews with a group of 4 learners of diverse abilities (see section 3.8.1) whom I gained consent to interview. Pseudonyms were assigned to the learners and they were known as James, Tshedza, Peter and Rashid.

The interviews were approximately 40 minutes long, which was equivalent to one class period. Three questions were used in each interview and their level of difficulty increased as the intervention progressed. The tasks were taken from the topics the learners were doing at the time: number operations, patterns, functions, algebra, financial mathematics and geometry. I required learners to solve the first problem independently without disruption, and then allowed them to discuss the second and third problem with their peers in the group. For me to be in a position to access learners' thinking, I required them to talk about their ideas as they solved the problems (thinking aloud). However, I realised that the learners were hesitant or shy to think aloud unless I encouraged them. To encourage learners to think aloud, I frequently arranged them to work in pairs so that they could talk to each other whilst solving problems. As learners discussed problems aloud, I observed that they became more aware of the information that they were using to solve the problems and became more conscious of their solution strategy.

The interviews focused on the strategies that the learners were using to solve problems, the reason a learner would have taken a certain step, and learners' mathematical problem-solving processes. I asked learners questions that are indicated in Appendix U to clarify my understanding of learners' problem-solving processes and to give learners the opportunity to verbalise their thought processes. Through questioning, I encouraged learners to self-correct when they made mistakes, to extend their thinking in circumstances where they got stuck, to examine their solution strategies and problem-solving processes and to justify their own solutions. At times I pretended not to understand the problems and required learners to explain the problems to me. In this way, I was able to evaluate a learner's understanding of the problem. Where learners did not understand the given problems, I assisted by getting them to read and explain the problem to me. In all instances, I ensured that learners understood the posed problems; however, I avoided helping them to solve the problems. I observed how the learners were approaching the problems and asked them to verbalise their thinking. During the interactions, I wrote notes about learners' problem-solving processes and sometimes took pictures of learners' work which I used during data analysis. Task-based interview sessions were audio-recorded with teachers', learners' and learners'

guardians or parents' permission. I had wanted to video record the task-based interview episodes, but I did not get clearance from the school because one of the participant learners was an orphan that is owned by the state.

6.3.3.1 Selection of tasks

I carefully constructed the tasks for the task-based interviews, because this is a key component of the task-based interviews in mathematics education (Maher, Powell & Uptegrove, 2011). When the first task-based interview was conducted, learners were doing numbers, ratio and rate and the problems that were in the interviews are in Appendix V.

6.4 Research design

6.4.1 Participants

One female and male teacher aged between 21 and 30 years participated in this second cycle. The male teacher (Mr. M) had a diploma in Mathematics Education and the female teacher (Ms. N) had master's degree in Mathematics Education. Mr. M had six years' experience teaching high school mathematics and Ms. N had one year's experience of teaching high school mathematics. This demographic information is tabulated in Table 6.1 below.

Table 6.1: Demographic information of the second cycle participants

Variable	Mr. M	Ms. N
Sex	Male	Female
Age	30	27
Qualification	Diploma	Masters' degrees
Number of years teaching	6	1
Class size	45	51

96 grade 9 learners ranging between 14 and 16 years participated in this cycle.

6.4.2 Data sources

In this cycle, I collected teacher data through classroom observations, semi-structured reflective interviews, teachers' field notes from the observations and teachers' audio-recordings from the post-observation feedback sessions. Classroom observations which were audio-taped were conducted before, during and after the PD intervention. Before the intervention, I observed each teacher once a week for a month. In total, I had four pre-intervention classroom observations with the teachers. After the pre-intervention observations, I was at the school one Tuesday every two months to observe Ms. N, and one Friday every two months to observe Mr. M. In total, I observed the participant teachers three times during this 6-month cycle. I conducted three semi-structured reflective interviews with each participant teacher just after lesson observations. Like in the first cycle, I conducted the interviews in teachers' classrooms since it was during their free periods. The semi-structured reflective interviews looked at teachers' experiences with the POFT and the PD intervention.

In terms of assessing the possible impact of the PD intervention on participant learners, I gave them pre- and post- mathematics attainment tests and a self-reporting MPSSI (see section 3.8.2) at the beginning and at the end of the intervention. I designed pre- and post- mathematics attainment tests that covered the topics that the learners were doing (number operations and relations; patterns, functions, financial mathematics, algebra and geometry). The questions of the pre-tests were similar to the post-tests, but were of different context. Both tests had 10 questions and the marks were converted to a percentage. Furthermore, I also used task-based interviews (see section 3.8.1) as a way to explain why learners were possibly doing well in the post-attainment test and why they were giving better responses in the post-MPSSI.

6.4.3 Data analysis

The raw data included recorded notes on the observation comment card, audio-tapes from the classroom observations, task-based and semi-structured reflective interviews,

field notes from the learner task-based interviews, learners' work from the task-based interviews, test marks and responses from the MPSSI. I conducted data analysis just like in the first cycle, that is, inductive data analysis for qualitative data and used direct quotations to give insight into when some remarks were made. I then used the statistical software package SPSS to analyse the learner quantitative data.

6.5 Findings

6.5.1 Teachers

As I indicated before, in this cycle teacher data were collected through classroom observations, semi-structured reflective interviews and POFT. In the next section, I discuss the findings from the teacher data.

6.5.1.1 Findings from the teacher observations and semi-structured reflective interviews

Three themes emerged from the data from classroom observations and these include; understanding the problem, collaborative learning, devising and carrying out a plan and looking back. The following themes from the semi-structured interviews were identified: changes in perceptions about mathematical problem-solving pedagogy; appreciation of collaborative learning; increased awareness of learners' needs; and PD activities that had a positive impact on teachers' PD. These themes were explicated on in section 5.5.2.1 and section 5.5.2.2 and thus, are not covered further in this section. Rather, I focus on the findings from the POFT.

6.5.1.2 Findings from peer observation for teaching

It is essential that teachers reflect on their mathematical problem-solving pedagogy in light of the feedback from the observer and their experience of observing. During semi-structured interviews (see section 6.4.2), I reflected with the teachers on their experiences with the POFT process. The reflections from the semi-structured interviews were analysed using inductive data analysis techniques (see section 6.4.3). Themes that emerged from POFT were: PD in mathematical problem-solving pedagogy, non-threatening process and safe environment, valuable and necessary process and

increased awareness in mathematical problem-solving pedagogy and valuable feedback.

Professional development in mathematical problem-solving pedagogy

During the semi-structured interviews, teachers reflected that they learnt a lot about their own teaching and the teaching of mathematical problem-solving by observing another teacher. Teachers acknowledged that by merely observing their colleague teaching, they were able to identify areas of their pedagogy that needed improvement. This was explained by Ms. N during the semi-structured reflective interview:

> As a beginner teacher, peer observation is the best way to improve myself and to improve my teaching of mathematical problem-solving. I learnt new skills from Mr. M, not only about teaching problem-solving but about planning, classroom control and discipline which is a major challenge at our school.

Teachers reflected that observing their peers reinforced what they had learnt in the PD workshops as reflected by Mr. M:

> It helped me to see Ms. N encouraging and helping her learners to look back and evaluate their answers after solving given problems. We had done this in the workshop, but to see her doing it in the classroom facilitated my understanding.

Participant teachers realised that each teacher interacts with learners in unique ways. Mr. M admitted that he observed Ms. N teaching functions using a game and was inspired to see that games can be incorporated into the teaching and learning of functions successfully and can facilitate learners to solve problems in a ‘funny’ way. Mr. M also noted that he learnt new techniques about teaching learners how to understand a given problem:

> At the PD program, we learnt how to make our learners understand a given problem. It was worthwhile to see Ms. N practically incorporating the different ways we had learnt in the workshop to make learners understand the given

> problems. I will now let learners read problems to each other in their own languages during the next lessons.

Mr. M's reflections indicated that he was going to incorporate translanguaging in his lessons. We had done translanguaging in the PD workshops, but it turned out that Mr. M became more comfortable in integrating the process in his lessons after observing Ms. N practising it in her lessons.

Non-threatening process

Participant teachers reflected that the POFT was non-threatening but rather beneficial. However, they acknowledged that POFT has a potential of intimidating especially if done by a teacher that they are not comfortable with. Below are the responses from the participant teachers.

Ms. N:

> As a beginning teacher, it was initially unsettling to be observed; however, it was a great privilege to work with my colleague. To have real conversations with him before and after the observations benefitted me professionally in that he is familiar with the subject, CAPS curriculum and mathematics problem-solving teaching strategies.

Mr. M:

> The way Ms. N conducted her observations was considerate and non-intrusive.

During the interview, I probed Mr. M further to elaborate on what he implied by being considerate and non-intrusive. Below is how he responded:

> She always came in quietly such that I did not feel her presence. However, I work with Ms. N frequently since we teach the same grade; such that I question and wonder if I would have felt the same if it was a different teacher.

Valuable and necessary process

Mr. M acknowledged in one semi-structured interview that POFT was a valuable and necessary process:

> It is always advantageous to see the teaching of mathematical problem-solving by a different person. I found it valuable to observe my colleague teaching mathematical problem-solving to her learners. I think it is necessary and needs to be done by all teachers in the school.

Ms. N appreciated the POFT process and indicated that it made her think carefully about every step in the teaching of mathematical problem-solving.

Increased awareness in mathematical problem-solving pedagogy

Mrs. N acknowledged that observing Mr. M in action made her more flexible as a maths teacher. She indicated that after observing Mr. M she went home and reflected on and reassessed her own mathematical problem-solving pedagogy.

> I began to fully understand myself as a teacher and the way I teach mathematical problem-solving. I became flexible in incorporating Polya's (1957) problem-solving steps in my teaching. By observing Mr. M teaching I realized that learners must reflect on their answer after solving a problem. Each day I observed Mr. M I went home and reflected on what I had learnt by observing him.

Valuable feedback

Participant teachers treasured the feedback they received from their peers and indicated that it improved their mathematical problem-solving pedagogy. This was acknowledged by Mr. M:

> I had never received feedback on my teaching for the 6 years that I have been teaching. It really made me see areas in my teaching that required improvement. I liked that I received the feedback within a short time just after completing the lesson.

6.5.2 Learners

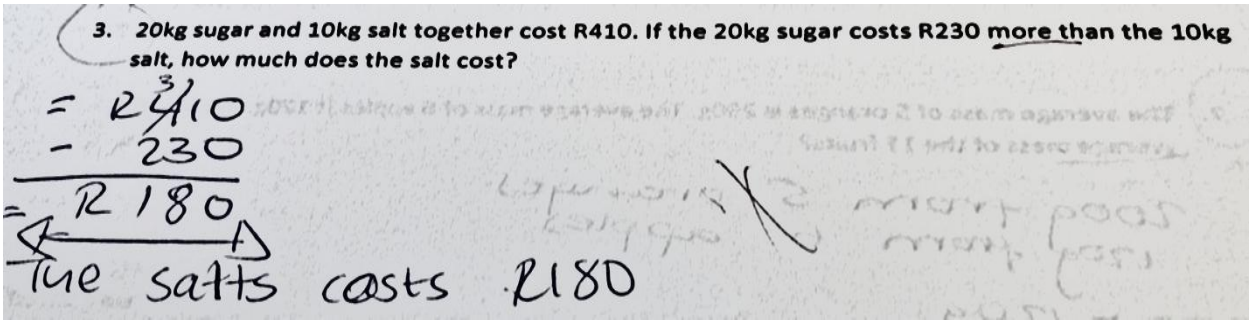
6.5.2.1 Findings from the mathematics attainment tests and MPSSI

I conducted a paired samples t-test to compare learners' responses on the pre-MPSSI and the post-MPSSI. I combined learners from the two classes for analysis. Results showed that the difference in the scores was statistically significantly different in the average scores for the pre-MPSSI (M= 1.6, SD= 0.24) and post-MPSSI (M=4.11, SD= 0.58) conditions; t (95) = 42.77, $p < 0.05$. As in the first cycle (see section 5.5.1), these results suggested that the responses of the learners changed significantly to agreeing in the post-MPSSI as compared to the pre-MPSSI.

Furthermore, I also conducted a paired samples t-test to compare learners' performance on the pre-attainment word problem test and the post-attainment word problem test. The results showed that the difference in the scores was statistically significantly different in the average scores for the pre-test (M= 1.58, SD= 0.24) and post-test (M= 4.25, SD= 0.58) conditions; t (95) = 18.41, $p < 0.05$. These results suggest that the learners performed better in the post-attainment test as compared to the pre-attainment test. However, it was difficult to compare this improvement as I did not have similar data from a control group. To cater for this gap, I analysed learners' solutions in the pre- and post-tests and below are a few images of their work.

This was Zain's solution to question 3 on the pre-test (see Appendix S).

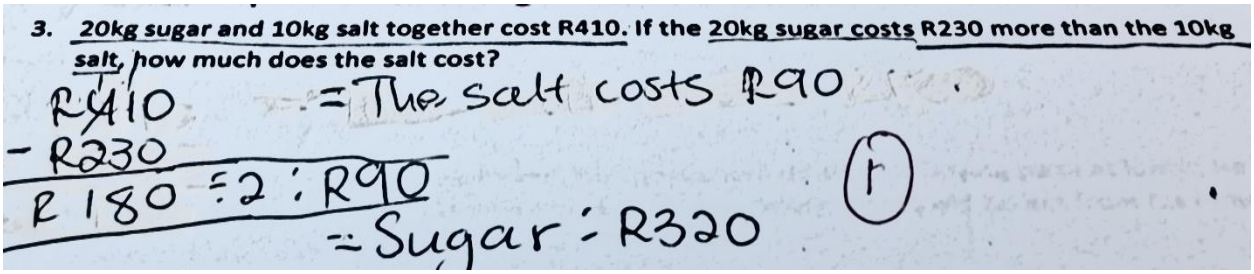
Figure 6.2: Pre-test question 3, sample solution.



Zain, like all learners in his class, could not understand the 'more than' in the question and thought sugar costs R230. As the intervention progressed, learners were taught to

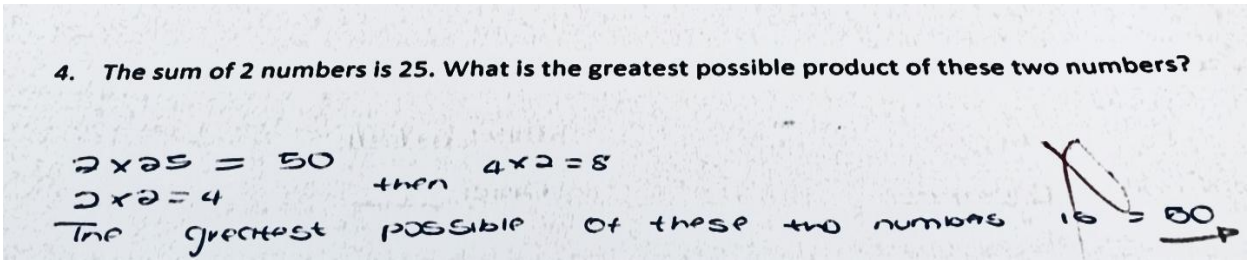
understand a given problem by re-reading and underlining key words, etc (see section 2.3.4.1 and 3.10). Below is Zain's solution to post-test question 3. The solution is correct and shows that learners were now underlining key words and focused on understanding before solving the problem at hand.

Figure 6.3: Post-test question 3, sample solution.



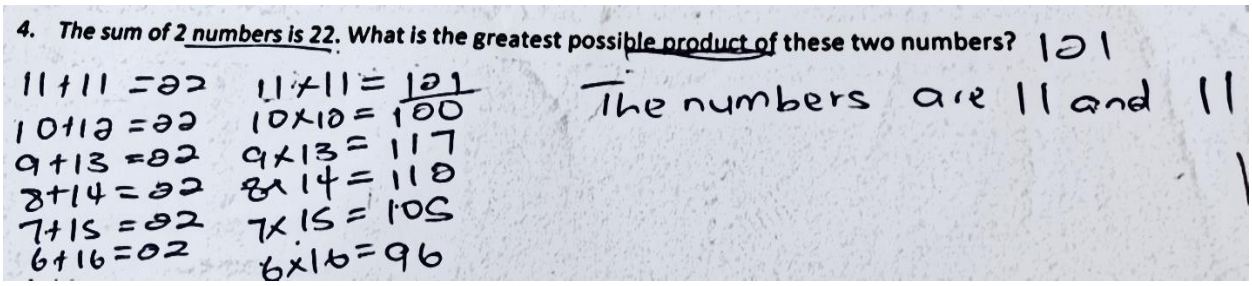
Katy solved question 4 of the pre-test as shown in Figure 6.4. On analysing her solution, I could see that she multiplied the two numbers that were in the question, although I failed to deduce her reasoning of multiplying 4 by 2. I did not interview Katy in order to investigate the thinking behind her computation because of accessibility to learners, time, and resources and also that this study was not focusing on face-to-face interviews with learners. However, during data analysis, I felt that I should have asked for consent from the school to interview the learners after they wrote the tests in order to get explanations of their solution strategies.

Figure 6.4: Pre-test question 4, sample solution.



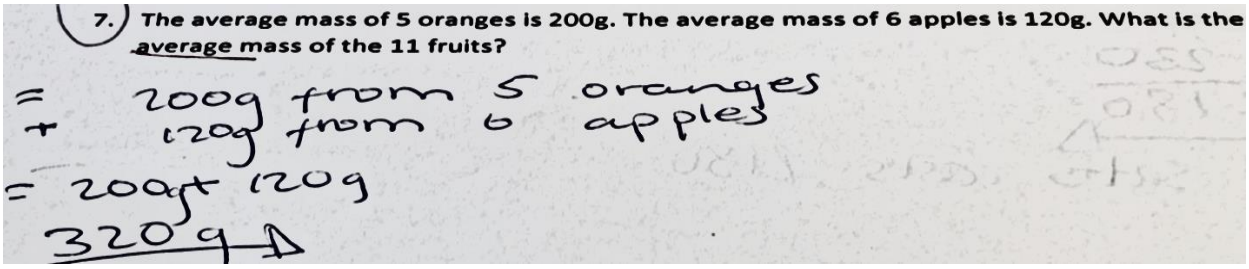
As the intervention progressed, from the classroom observations I noticed that Katy displayed noteworthy gains in her problem-solving processes and this was also exhibited by her solution to question 4 of the post-test in Figure 6.5. This solution displays that Katy solved the problem using the guess and check strategy.

Figure 6.5: Post-test question 4, sample solution.



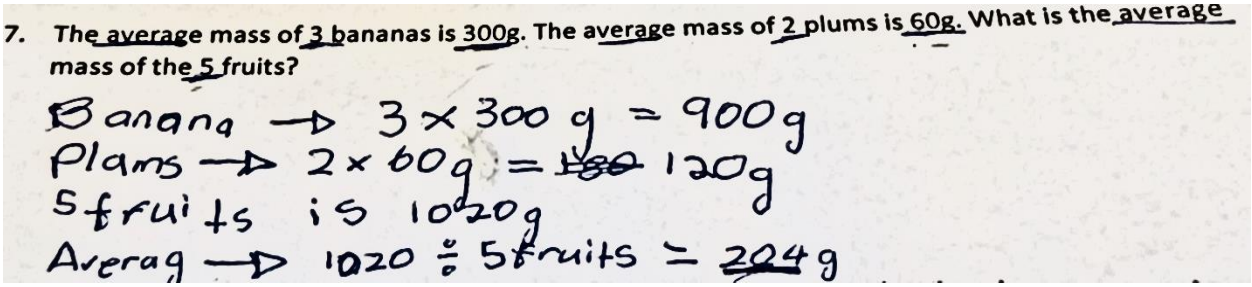
At the beginning of the intervention, Siyekha solved question 7 from the pre-test by multiplying and adding numbers in the problem, without demonstrating neither reasoning nor problem-solving processes (see Figure 6.6).

Figure 6.6: Pre-test question 7, sample solution.



In Figure 6.7 below I present Siyekha's solution to question 7 of the post-test. Siyekha underlined the keywords to show that he tried to understand the problem before solving it. His solution indicated that his mathematical problem-solving processes in had improved at the end of the intervention.

Figure 6.7: Post-test question 7, sample solution.



From the above findings, I concluded that the PD intervention implementation, just like in the first cycle (see section 5.5.1), seemed to have had a positive impact on learners' performance and mathematical problem-solving processes. This was reflected in the noticeable improvement between the pre-test and the post-test. However, since there was no control group, I was cautious that maybe the gains could have just happened over time. This is why delving deeper with the task-based interviews was extremely important for this study.

6.5.2.2 Findings from the task-based interviews

As I indicated previously, the purpose of the task-based interviews was to assess the impact of the PD intervention on learners' performance and problem-solving processes in mathematics. The task-based interviews provided me with the opportunity to go beyond the correct solutions to tasks and understand the learners' thinking (Goldin, 2000) and problem-solving processes. The sample of four learners used for the task-based interviews was small, such that I was cautious not to generalize the findings to the 96 participant learners in the cycle. However, these findings illustrated the possible learner gains in mathematical problem-solving and which were shown for the cohort in the MPSSI and the test results. Therefore, in this section (6.5.2.2), when I say learners I am referring to the four that participated in the task-based interviews. Based on the audio-recordings and field notes from the task-based interviews, I identified the following themes from the data: understanding the problem, language challenges, and devising and carrying out a plan and looking back.

Understanding the problem

The major focus of the task-based interviews was to draw extrapolations on how and why participant learners' mathematical problem-solving processes developed during the six-month intervention. During the first task-based interviews, which I conducted at the beginning of the PD intervention, learners exhibited very limited mathematical problem-solving processes. By allowing learners to work independently, I noticed that they faced major challenges in understanding the problems. Learners frequently guessed at what the problem was asking for by using a few words in the problem that they understood.

For example, on the first question in task-based interviews 1, one learner multiplied 17 by 2.69 without understanding why he was doing it. Another learner added 17 and 2.69 and then the other two learners divided 17 by 2.69. I concluded that the learners seemed to attempt problems without really understanding what was required. This was also verified by their responses during the task-based interviews.

Interviewer: Why did you multiply 17 by 2.69?

Rashid: Ummm…I don't know…. (*then silence*).

Rashid had correctly solved the problem and got the correct answer. However, he did not know why he had performed the solution strategy. He was also not sure if his answer was correct.

Interviewer: Why did you divide 17 by 2.69?

Peter: Because it makes sense to divide a big number by a small number.

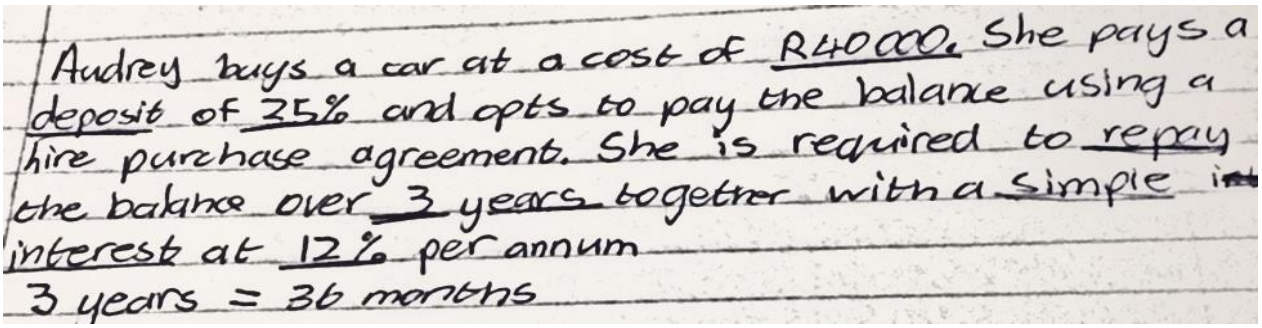
James had added the two numbers without a valid reason.

Interviewer: Why did you add 17 and 2.69?

James: Umm I do not know….we usually add when we have two numbers in a given problem.

As the PD intervention progressed, teachers' mathematical problem-solving pedagogy improved and this resulted in learners becoming more comfortable with problem-solving. Learners began to apply what they were learning in their lessons and focused on understanding the problem before attempting it. For example, learners began to underline keywords in a given problem that would help to solve the problem as they had been taught by their teachers in the lessons. Below is an extract of how Rashid attempted question (c) in task-based interviews 2 (see appendix V).

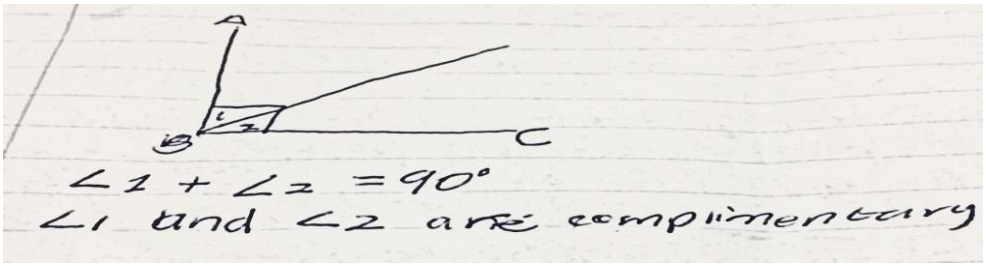
Figure 6.8: Task-based interviews 2, question (c) sample solution



I noticed that Rashid discussed with Peter making sure that they both understood the main words, all the words used in stating the problem and what the problem was looking for. This resonated with what was happening in their classrooms. Teachers were now encouraging learners to understand the given problems before attempting to solve them. Classroom discussions were also now being supported by the teachers to a large extent. The two learners both agreed that it was necessary to convert 3 years to 36 months in order to solve iv in (c).

As the intervention progressed, participant teachers often asked learners to think of a picture or a diagram to help them understand the given problems. I noticed that during the task-based interviews, the four learners incorporated what they were learning in their lessons. For example, this is how Tshedza attempted question (c) in task-based interviews 3. He drew the diagram below trying to show how complementary angles looked like. Using this diagram, Tshedza could easily explain to James what he understood by the term complementary angles.

Figure 6.9: Task-based interviews 3, question (c) sample solution



Language challenges

During the first task-based interviews, learners had challenges in verbalising their thought processes. They struggled to answer the interview questions. As the intervention progressed teachers gave learners linguistic support as indicated in section 6.3.1 and learners could easily understand the given problems. During task-based interviews 2, I could see learners translating problems to each other when they were working in pairs. The quality of explanations by learners had improved during task-based interviews 3. They were comfortable with the LoLT, and had become fluent and confident in their explanations. Below is an excerpt of a tasked-based interview 3 question d (i) with Peter;

> Interviewer: Why do you say it is true that if two triangles are congruent, then they are also similar?
>
> Peter: If two angles pairs are the same, then the third is also equal. Last week I learnt that when three angle pairs are all equal it means that the three pairs of sides are in proportion. If all three sides are in proportion then the triangles are similar.

Devising and carrying out a plan and looking back

During the first task-based interviews learners were stuck and could not think of ways to solve the given problems. As the PD intervention progressed and as teachers implemented what they were learning in the workshops, I noticed that during task-based interviews 2 and 3 learners began to consider ways of solving the given problems like guess and check, drawing a table, making lists, formulating an equation, thinking of a different approach and trying it out, etc. These were the problem-solving strategies that the teachers were using in the lessons. In turn, this indicated that what the teachers were doing in the classroom had an influence on learners' mathematical problem-solving processes. James, as indicated in the image below, answered question (a) in task-based interviews 2 using two strategies; guess and check and an equation.

Figure 6.10: Task-based interviews 2, question (a) sample solution.

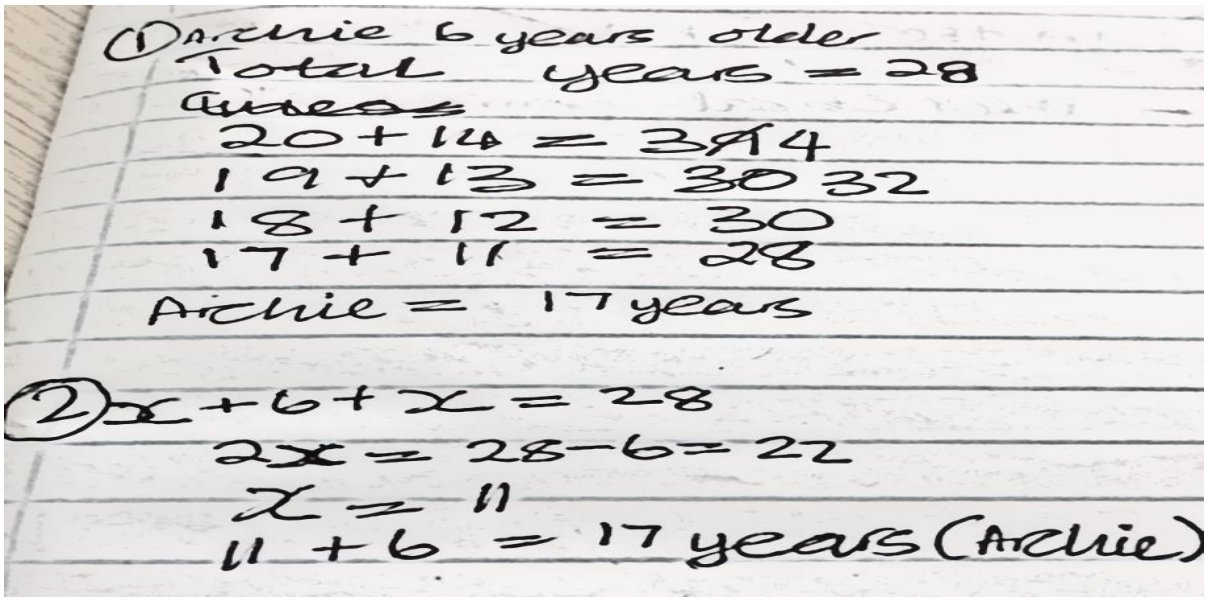
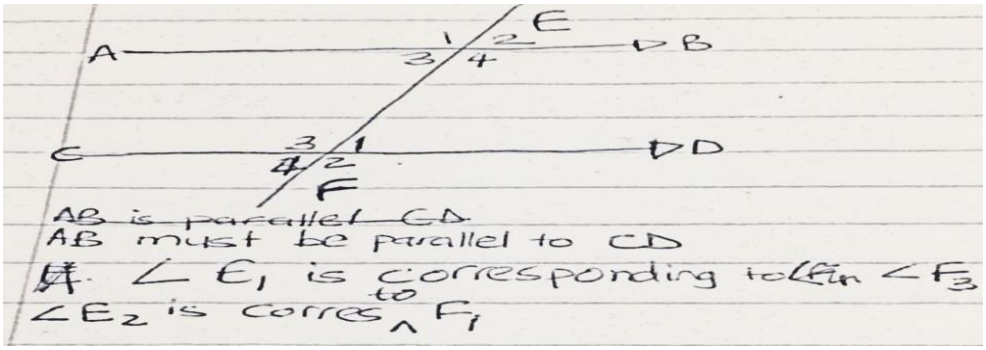


Figure 6.11 below shows how Peter and Rashid solved question (b) in task-based interviews 3. The two learners still made minor mistakes, but it was clear that they were now devising ways of solving a given problem; in this case a diagram.

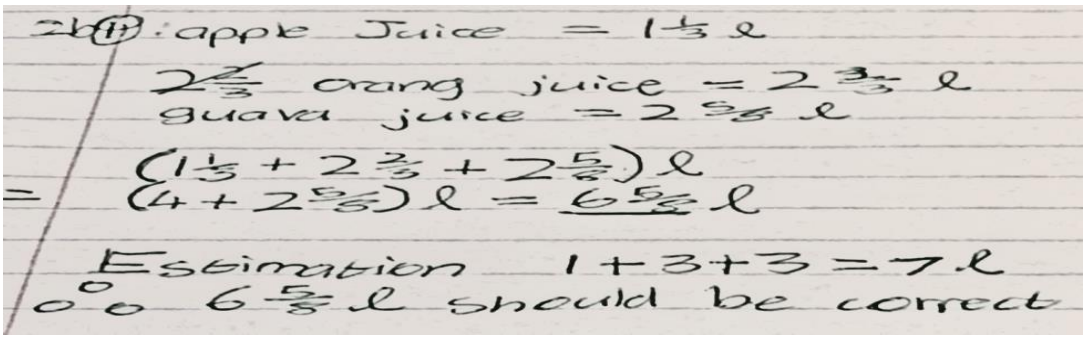
Figure 6.11: Task-based interviews 3, question (b) sample solution.



I also observed that when learners were working on question (d) in task-based interviews 3, they all started by drawing diagrams of triangles, parallelograms and kites and then proceeded to explain them. This demonstrates that learners were integrating what they were learning in their lessons; that is, to first think carefully about the operation they could use before they solved the problem.

It was pleasing to see that during task-based interviews 2, learners began to look back and reflect on their solution strategies just like how they had been taught by their teachers during the lessons. For example in question (b), learners reviewed to see if their answers made sense by quickly estimating the answer. Below is an image on how Rashid solved question b (ii):

Figure 6.12: Task-based interviews 2, question b (ii) sample solution



6.6 Discussion of findings and conclusions

The POFT process was non-threatening but useful and constructive to teachers' PD. Participant teachers reflected that the process was indeed a source of PD for them and was valuable for both the observer and the observed. The observer developed an increased awareness of how mathematical problem-solving was being taught and learned by learners. By observing one another, participant teachers not only learnt new ideas that they could apply in their own teaching of mathematical problem-solving but aspects of planning and classroom management. This finding resonates with what Bolt and Atkinson (2010) noted in their study on academics from Curtin Business School who participated in a peer review of face-to-face teaching. Bolt and Atkinson (2010) found that the process of peer observation increases the skills of the observer especially when they observe other teachers in action. This also concurs with what Bell and Mladenovic (2008) found in a peer review program; that peer observation of teaching is valuable to the participant teachers and gives them the opportunity to emulate the teaching techniques of others. Engin and Priest (2014) assert that peer

observation has an immediate impact on teachers' practices in that the observer may go to their next class and implement strategies from the observation.

The process of POFT mainly focused on the observer's mathematical problem-solving pedagogy; however, from listening to the audio-recordings of the post-observation discussions, I realised that the observed became aware if their teaching of mathematical problem-solving was clear and understandable to learners and revised their own teaching techniques. This finding is in agreement with Bell and Cooper (2013), who used peer observation process at an Australian university and found out that, participants in the study improved their teaching skills and confidence in teaching. It was of particular interest that early career academics in the study benefitted the most from the process. This implies that it is worthwhile to pair the experienced with the novice for POFT. For this study, Ms. N, who only had one-year of teaching experience in mathematics felt that she benefitted a lot by observing Mr. M.

Participant teachers valued feedback from their peers and felt that the feedback improved their performance. This finding matches with what Hendry and Oliver (2012) found in their study of Foundation lecturers that participated in a peer observation of teaching that was part of a formal teacher-development programme. Lecturers in the programme found the feedback from the observer generally useful and learnt new skills by observing a peer teach.

The findings suggest that POFT reinforced and enhanced the correct way of teaching mathematical problem-solving in the context of study, which in turn resulted in an increased focus on learners' problem-solving processes during lessons. This finding is in agreement with two studies that were conducted in Indonesia by Zwart, Wubbels, Bolhuis and Bergen (2007, 2008) which showed that peer observation results in the growth of teacher learning.

POFT was conducted successfully with two teachers in this study and the results seem to indicate that the process can be used as a PD strategy to support South African teachers' mathematical problem-solving pedagogy. However, it is essential that teachers freely participate in the peer observation process (Engin & Priest, 2014) and

do not feel that the process is one more aspect of the PD intervention they have to do; a waste of their time or a judgmental process. In order to avoid this, I explained to the teachers that the observations were an enriching part of the PD intervention and not an evaluative process.

The limitations of peer observation are that the observer can be unintentionally subjective (Engin & Priest, 2014) and the presence of the observer can distract the learners and the observed can feel very uncomfortable and fail to teach their normal way. As per teachers' reflections, in this study learners were not distracted because they knew the observer and that s/he also taught other grade 9 classes. Teachers were both grade 9 teachers and usually planned together and were attending the PD intervention together, therefore they were comfortable to enter POFT partnerships and understood the importance of being objective during observations and when giving feedback. I noted that for POFT to be successful, it needs to be non-evaluative and non-judgmental; otherwise teachers become anxious and view it as an intimidating process and may refuse to participate. Furthermore, I also noted that if participant teachers lack training and experience in observation and feedback this can cause drawbacks and ineffectiveness of the process. For this study, teachers were continuously trained of the POFT process during the PD workshops and understood the process. Teachers were also aware that the main purpose of POFT was for the observer to self-evaluate and improve their mathematical problem-solving pedagogy and not to judge the observed.

The post-attainment tests and post-MPSSI demonstrated that the PD intervention had a positive impact on learners' performance. The goal of task-based interviews was to evaluate if teachers' participation in the PD intervention had an effect on learners' performance and problem-solving processes in mathematics. The findings from the task-based interviews indicate that the PD intervention had a positive impact on learners' problem-solving processes. Although their solution strategies were not sophisticated, learners displayed greater gains in their problem-solving processes as the PD intervention for their teachers progressed. This finding confirms that learners' improvements in post-attainment test and post-MPSSI were genuine. Learners were

actually doing problem-solving differently from the way they were doing it prior to the intervention. This outcome addressed the fourth research question of this study which states that: *What is the impact of the PD intervention on grade 9 learners' problem-solving processes in this particular context?*

From the findings, it is proper to claim that the task-based interviews can be used to assess learners' performance and problem-solving processes in mathematics. This resonates with Maher et al. (2011) who state that task-based interviews can be used to investigate learners' existing and developing mathematical knowledge, reasoning habits and problem-solving processes and skills. From the task-interviews, I noticed that learners' problem-solving processes also improved because of the opportunity which was given to them to solve word problems and the chance they got to think about their problem-solving processes and explain the former to me or their peers. This is in agreement with Mcdonough et al's (2002) and Heng and Sudarshan's (2013) findings. Mcdonough et al. 2002) uncovered that task-based interviews help learners to display mathematical thinking and problem-solving processes initially believed to be beyond their capabilities. Heng and Sudarshan (2013) found out that practising teachers were able to assist learners to explain, substantiate, justify and verbalise their mathematical ideas and solution strategies through participating in task-based interviews.

The task-based interviews were of limited scope and generalisability was not the aim of the study. The findings may inform South African teachers and other researchers who work with learners, who have no or little background in mathematical problem-solving. It can be challenging for teachers to implement the task-based interviews in their lessons simply because of the CAPs syllabus time constraints. However, I recommend that teachers can create groups of four to six learners and carry out the task-based interviews. Groups save time and learners can learn from each other as they solve problems and verbalise their thought processes. In this study, learners did not enjoy thinking-aloud when working on problems unless I encouraged them. This created a limitation in that sometimes I missed their reasoning processes when they were solving problems. However, arranging learners to work in pairs helped a lot as they began to talk to each other whilst solving problems. Unfortunately, I could not get the consent to

videotape the learners and this resulted in a constraint in that I was unable to analyse learners' actions in detail after the task-based interview episodes.

6.7 Design Principles

In addition to the design principles indicated in section 5.7, the second cycle generated the following design principles:

- POFT can be used as a PD strategy to support grade 9 South African teachers' mathematical problem-solving pedagogy.
- Task-based interviews can be used to observe learners' mathematical problem-solving behaviour in detail and to gain insight into their problem-solving processes.
- PD should support teachers on how to implement mathematical problem-solving pedagogy in a multi-lingual context and the following principles should be factored into the support given to teachers. Teachers must;
  a) think about mathematical terminology that can be challenging to learners and explain before word problem is given to learners;
  b) present the mathematical word problem in English and one other language that they are comfortable with;
  c) encourage learners to translate the word problems for each other in their languages;
  d) get learners to explain back to them what the word problem asks for;
  e) allow code switching when explaining;
  f) use language as a resource, for example, looking at a word in a word problem and allow the structure of the word to help learners understanding it for example the word transformation;
  g) change the representation if it is difficult for learners to understand graphs; and
  h) change and simplify the mathematical language if learners are struggling, for example, if learners are struggling with algebra language then the teachers may change it to arithmetic language.

6.8 Implications of the findings for the third cycle

At the beginning of my PhD study, I had intended to conduct three cycles but because of time and resources, I only managed to complete two cycles. In this section, I give the implications for the third cycle which I may conduct in the future after my PhD studies or in case another researcher may want to extend the study. In the second cycle, I realised that teachers were more productive in the second and third PD workshops. Therefore, in the 3rd cycle contact time should be increased before teachers can go and implement the new ideas in their lessons.

I worked with two participant teachers in both the first and second cycle. The number of participant teachers can be increased to eight or ten in the third cycle. This is to see the implications of increasing the number of teachers in the intervention. The increase may reveal aspects that may suffer if a PD intervention for mathematical problem-solving pedagogy involves many teachers. Since the number of the participant teachers increases, it entails that researchers cannot carry out intense observations with each teacher. Therefore teachers can audio/video record themselves teaching and the researcher can analyse the audio/video recordings with them. Moreover, the researcher may only observe and support participant teachers once in the classroom during the 6-month cycle. This exemplifies a situation where a big organisation decides to implement the PD intervention with many teachers at various schools.

In the first cycle, I showed teachers videos; they made sense of them and found them useful. I thought teachers could observe each other in the second cycle and this worked. In the third cycle, teachers may conduct POFT without the researcher's supervision. For instance, the researcher may only guide teachers on what they should be observing and looking for. Teachers may be provided with POFT forms and the researcher may reflect with them on the POFT process during reflective interviews. In addition, teachers may be required to audio or video-record the POFT process so that the researcher can be certain that teachers will be genuinely conducting the process. I was the driver of this intervention; therefore it will be a situation of removing the driver from the process and implementing the intervention and checking its failures and successes.

In the PD workshops when we were devising and carrying out plans to solve word problems, I realised that teachers found other strategies easier to use than others. Easier strategies were guessing and checking and working backwards. Other strategies needed more time and practice before they could be used spontaneously by teachers.

6.9 Evaluation of the second prototype

The second PD prototype generated from this cycle was evaluated using Guskey's (2000) five critical levels of PD evaluation. I followed the same procedure as in section 5.9 to evaluate this prototype; however, I do not discuss it here.

6.10 Conclusion

I implemented POFT in the second cycle of this study. POFT proved that it is a valuable PD strategy that can be used to support grade 9 South African teachers' mathematical problem-solving pedagogy. The study having been done with two teachers it is, therefore, unjustifiable to generalise the findings to a larger population. However, the findings fill in the gap in the literature on POFT since there is limited literature on successful POFT experiences with grade 9 South African mathematics teachers.

Task-based interviews were used successfully in this study; however, in normal circumstances, teachers may not know how to interview learners appropriately as purported by Labinowicz (1985):

> It takes considerable sensitivity, experience, and skill to become a good interviewer. A raw beginner might not uncover any more information than a standardised achievement test could. This might be due to either a failure to probe or an overbearing manner that inhibits the child (p. 28).

The task-based interviews were intended to be part of a PD intervention that supports grade 9 teachers in the teaching of mathematical problem-solving. I trained teachers on the interview techniques, how to craft proper word problems, how to pose problems in a manner understandable to participant learners and how to ask suitable probing questions. If task-based interviews are to be implemented in a classroom, then the

teachers concerned, need to be trained on how to conduct task-based interviews properly. Teachers can also be trained on how to take all-inclusive notes on what learners do and say during the task-based interviews. This is because learners' actions and utterances are important during the problem-solving processes. Furthermore, teachers can also be trained on how to create a less-threatening environment to learners during interviews and how best to assess learners' thinking and problem-solving processes without guiding or showing learners how to solve the problem. McDonough et al. (2002) stress that teachers must always be conscious that their purpose in task-based interviews is to pose tasks and not to teach learners how to solve them. In the next chapter, I discuss the findings of the study as a whole and I report on the themes that emerged from the study.

CHAPTER 7: Discussion

7.1 Introduction

This chapter is the apex of this thesis as I discuss the findings of the study as a whole. Context emerged as a big theme for this study. Consequently, I viewed the research process through the lens of a particular social context. Besides context, other themes that emerged gradually during the study and are related to context are language considerations in a Southern African context, POFT as a PD strategy for mathematical problem-solving pedagogy, collaborative learning by teachers, teachers' views on teaching of problem-solving, teachers' self-reflection as a tool to enhance their PD and why we need to use DBR approaches in a South African context. In the following sections, l discuss these themes.

7.2 The importance of context in research and teacher professional development

In section 1.2, I listed the factors that make up the context. These factors include the physical aspects; goals and quality of teaching and learning; and teachers' perceptions, views and beliefs. All these factors usually affect one's education and development, therefore context needs to be at the forefront when one is doing educational research. By going through the baseline investigation and DBR cycles of this study, I realised that just like mathematics teachers who are encouraged to use realistic contexts so as to make mathematics relevant and accessible for all learners (Sullivan, Zevenbergen, & Mousley, 2003), PD facilitators need to take context into greater consideration in order to make PD meaningful and applicable to teachers. Clarke and Hollingsworth (2002, p.947) posit that, 'If we are to facilitate the PD of teachers, we must understand the process by which teachers grow professionally and the conditions that support and promote that growth'. Context is one of the conditions that are known to support teachers' growth in PD and consequently the importance of context was ubiquitous throughout this study, which was primarily grounded within the social constructivist perspective. Vygotsky (1978) advocates that learning cannot be separated from the

social context. Therefore, I endeavoured to design a PD intervention that was relevant to the teachers' experiences, beliefs, views and context they were working in. I found it easy to develop or enhance participant teachers' mathematical problem-solving pedagogy through the implementation of activities that were relevant and meaningful to their context. Consequently, teachers reflected during the interviews that they found the PD programme fulfilling and meaningful because I had recognised their context of prescribed curriculum and huge, overcrowded and multi-lingual classes with many needy learners.

As indicated in section 4.3, I conducted pre-observations before implementing the PD intervention. During pre-observations, all the four participant teachers indicated that they had previously participated in several 'one-shot' PD programmes (Hunzicker, 2010) and viewed PD as not resulting in either improvement of their own pedagogy or leading to promotion. Teachers had developed negative attitudes towards these PD programmes which they indicated were short-term, often lasting not more than one day and believed that they were designed and conducted by facilitators with no teaching experience in their particular contexts. Teachers were frustrated that the PD programmes were unrelated to their professions and real classroom challenges. Teachers viewed PD facilitators as authorities who came to impart new knowledge to them, but seemed not to understand who they were, what context they were working in, what they experienced every day when they were working in overcrowded and multi-lingual classes and how they were supposed to learn to enhance their practice. Teachers' exhaustion from low quality and irrelevant PD programmes was my main motivation to engage DBR as a research methodology. I did not go into teachers' contexts as a knowledgeable person on mathematical problem-solving pedagogy, but I did a baseline investigation and started the PD intervention from the teachers' views and their experiences of teaching mathematical problem-solving in their particular context. This way I created an environment where teachers felt that they were in control and had ownership of the PD intervention. Collaboration among researchers and practitioners in real-world settings is an important aspect of DBR (Wang & Hannafin, 2005) and with teachers' input; I was able to develop the PD intervention that was relevant to their lived experiences and context.

As indicated in section 5.6.2, respect is a key component of the African culture and I was responsive to respecting participant teachers in the context of study. Teachers indicated during the reflective interviews that they valued that I respected them. This finding helps to fill in the gap in the literature on respect as being an important aspect of the African culture and the fact that it should be considered when supporting African teachers in a PD programme for grade 9 teachers' mathematical problem-solving pedagogy.

At the start of each cycle, participant teachers in the context of study did not believe that their learners could learn mathematical problem-solving as they did not have basic problem-solving processes and skills from previous grades. Teachers believed that they could not give learners full ownership of problem-solving, but had to provide clear, step-by-step demonstrations of each procedure to the learners and in turn give the learners the opportunity to practice the procedures (Chirinda & Barmby, 2018). Teachers believed that they had the responsibility of re-explaining procedures in response to learners' questions as they felt that learners were underprepared from previous grades and had difficulty in grasping grade 9 concepts. Subsequently, during PD workshops I worked with teachers on how they could teach the skill of problem-solving to under-prepared learners. Furthermore, I proposed that during exposition teachers could explain to learners how to understand the given problem and how to break the problem into doable chunks. After that, the teachers could discuss the different problem-solving strategies applicable to the given problem, how to eventually solve it and reflect on the solution. This illustration could then be followed by a class exercise for learners to practice the problem-solving processes and the teacher could then move around as a facilitator assisting learners as was necessary.

I found the context very significant in this study such that I concluded that, without attending to educational context and how it interacts with PD interventions, PD programmes are likely to be unsuccessful or to underachieve their planned objectives.

7.3 Language in multi-lingual contexts

Further, in relation to context, from the very beginning of this study, the baseline investigation revealed that language should be considered as an important aspect of teacher PD (see chapter 4). As I conducted the PD intervention, I discovered that despite the apparent success of encouraging the problem-solving pedagogy in teachers and problem-solving processes in learners, English as LoLT was a major obstacle to learners' grasping of the given word problems and understanding what the teacher was teaching. The finding on language as an obstacle to learning in South African mathematics classrooms is in agreement with what a number of researchers have disclosed in the past (Setati et al., 2009; Setati Phakeng & Moschkovich, 2013; Webb & Webb, 2008). Polya's (1957) first step in mathematical problem-solving is 'understanding the problem' (see section 2.3.4.1), and if learners struggle with language, it means that they do not even understand the given word problems. Understanding the problem has a direct connection to the language of instruction (Chirinda & Barmby, 2017) since language creates access to mathematical concepts (Setati, 2014). Teachers cannot expect learners to simply understand a given word problem when learners do not have access to meaning because the English language is a barrier to their learning. Bernardo (2005) notes that learners whose first language is not English may struggle to understand the written content (in case of this study 'word problems') and require additional support from the teacher.

Teachers realise that, in these multi-lingual contexts, giving ownership to learners from the beginning of the mathematical problem-solving process is ineffective. It is, therefore, difficult to separate issues with the language of instruction from the teaching of mathematical word problem-solving. Polya (1957) did not incorporate issues of language when he was explaining the section on understanding the given problem. I do not wish to change Polya's (1957) problem-solving framework, but I would like to extend it by adding issues of language and context. This implies that when working in the Southern African context, it is imperative to add the notion of multi-lingualism when conducting PD interventions for grade 9 teachers' mathematical problem-solving pedagogy.

In this regard, LOLT was a focal point during the designing of the PD intervention in this study. In the first cycle, teachers code-switched as was necessary, but during the implementation phase, I realised that teachers did not speak all the learners' home languages, struggled to find the correct register or failed to code-switch at appropriate times during lesson delivery (see section 5.3.1). For this reason, I implemented translanguaging in the second cycle. I explain its benefits and limitations are in section 6.3.1. My main goal for engaging translanguaging was to give learners the opportunity to understand the LoLT, mathematics language, mathematics and mathematical problem-solving they were learning. With translanguaging, learners were able to explain or re-explain the mathematical concepts or difficult concepts to each other in their home language(s). They were no longer worried about the LoLT as a hindrance when solving problems. The focus was no longer on the language of instruction but on mathematics only. Learners spent part of the lessons translating given problems, such that initially teachers complained that the process was consuming most of their instruction time. However, as the intervention progressed, I observed that challenging concepts were easily conveyed to the learners when the teachers encouraged translanguaging and in turn teachers realised that it was beneficial to teach learners who understood the given problems, as they were not required to explain concepts repeatedly. Teachers appreciated that when learners read the problems back to them, they were able to pick parts that were difficult to learners and easily assist them with the mathematics, mathematical problem-solving, mathematics language and LOLT. The benefits of translanguaging outweighed its drawback of being time-consuming because learners' understanding of the given problems substantially resulted in their development of problem-solving processes. The value of making LoLT transparent and accessible to learners is observed by Durkin (1991) who declares that, 'mathematics education begins in language, it advances and stumbles because of language, and its outcomes are often assessed in language' (p.3).

Initially, I was worried that translanguaging would not work since teachers did not speak all the eleven different languages spoken in their classes. However, by working with teachers, I realised that teachers do not need to be multi-lingual because when mathematics is being instructed, the focus is on 'mathematics proficiency and not

language fluency' (Setati Phakeng & Moschkovich, 2013, p.4). Setati Phakeng and Moschkovich (2013) posit that what is important is for teachers to understand the complications of 'working across multiple languages and know how to support learners who struggle with comprehension of mathematics instruction in English' (p.4). I discussed with teachers during the workshops on how to work in multi-lingual contexts and how to support learners that struggled with LoLT (see section 6.3.1).

7.4 Teachers' self-reflections as a tool to enhance their professional development

Moon (2008) lists writing reflective journals, peer observation of (for) teaching, lesson reports, teaching portfolios, etc as approaches that can be used to facilitate reflection in order to enhance teachers' practices. In both the first and second cycles, I conducted the second and third workshops where the aim was for teachers to collaboratively reflect on their teaching experiences by reviewing and self-reflecting on the audio-tapes of the observed lessons (see section 3.10). I used semi-structured reflective interviews to assist teachers to self-reflect on the mathematical problem-solving pedagogy of the implemented lesson, POFT, the lessons in general and learners' problem-solving processes.

During the workshops and semi-structured reflective interviews, I noticed that as teachers looked back on the classroom events and made critical judgements about them, they modified their teaching behaviour and this resulted in them constructing knowledge about themselves, their teaching practices and their learners (Schunk, 2012). As participant teachers knowingly and systematically reflected on their teaching experiences (Farrell, 2007), I realised that they made sense of the actions they had taken during teaching and were able to extract meaning from those actions and consequently were consciously able to improve their own teaching. This procedure by teachers; of thinking about what they were doing and why they were doing it; turned their experiences into meaningful learning in their own context. In this case, learning by teachers did not just happen, but was derived from them self-enquiring and constructing sense from their experiences and particular contexts. This finding is articulated by Moon

(2008) who advocates that self-reflection leads to self-awareness and self-improvement. Self-reflection as a means of teacher PD is recommended by Muir and Beswick (2007) who advocate that to improve teachers' classroom practice, emphasis should be placed on teachers' learning and self-reflection. Drago-Severson (2007) concurs with this and observes that;

> 'when teachers … engage in reflective practice, they have the opportunity to become aware of their own and others' thinking and assumptions. This awareness can, in turn, clarify thinking and help us better understand our behaviours, leading to growth.' p.155

Self-reflection during the workshops and reflective interviews forced participant teachers to confront and become critical of their views on mathematical problem-solving and mathematical problem-solving pedagogy. As teachers became critical of their views on what the teaching of problems-solving implied, they were able to self-correct and improve their own mathematical problem-solving pedagogy. During the reflective interviews, I learnt that teachers do not reflect in a vacuum (Reiman 1999), but that context is an important aspect. When teachers reflect, they adapt their ideas to their particular contexts; and for this study, it gave teachers the opportunity to function better in their context and became more confident in their teaching of mathematical problem-solving. In this study, I believe that self-reflection was an important aspect that enhanced teachers' PD.

7.5 Teacher collaboration in professional development

I required teachers to work collaboratively during PD workshops and implementation of the mathematical problem-solving pedagogy (see section 3.10, chapters 5 and 6). Introducing collaboration at the beginning of each cycle was challenging as teachers were used to working as individuals and not sharing expertise with their colleagues. Teachers felt as if their practice they had kept personal and invisible for a long time was now being made visible to their colleagues. After explaining the characteristics of collaboration (see section 3.10), participant teachers in each cycle rose up to the

challenge and accepted the joint responsibility of working on the word problems and other activities during the workshops, implementing the pedagogy and making POFT successful in the second cycle.

It is well documented in the literature that collaboration brings teachers together to implement new teaching strategies and reflect on their own practices. There is an African proverb that says, 'the whole is greater than the sum of the parts' and this was true for this study when teachers worked collaboratively. As teachers worked together during the intervention, they were able to look back and reflect on their own teaching. Collaboration assisted teachers to easily accept the new ideas on mathematical problem-solving pedagogy they were learning because during the implementation stage they tried them out through the support of a colleague with whom they observed each other's successes, challenges and failures. My observations during the workshops were that; as teachers collaboratively solved word problems and planned how to teach these problems to their learners they reached a shared understanding of how to implement Polya's (1957) four-steps of problem-solving processes in their lessons. As Vygotsky (1978) theorises, when participant teachers in this study worked on activities collaboratively during the PD workshops it challenged their thinking and this resulted in the advancement of their mathematical problem-solving pedagogy. In the second cycle, teachers could easily discuss the mathematical problem-solving pedagogy with each other during the post-observation feedback of the POFT process and in turn, had a shared understanding.

The reflective interviews gave insight into participant teachers' perceptions and opinions about the process of collaboration during the PD intervention. Participant teachers in both cycles embraced collaboration and valued the discussions they carried out with their colleague on how to teach certain mathematics problems to learners. Teachers reflected that it was easy to teach problem-solving after they had discussed it with a colleague and this benefitted learners and in turn improved their problem-solving processes. The outcome of collaborative learning concurs with Cordingley, Bell, Thomason and Firth's (2005) finding after systematically reviewing 17 studies of collaborative and or sustained continuous PD in diverse contexts. Cordingley et al.

(2005) discovered that when teachers engage in collaborative PD, there are vast improvements in learning outcomes and behaviour, and in teachers' practices. It is well documented in the literature that teacher collaboration results in effective PD and improved instructional quality and learning outcomes (Loucks-Horsley et al., 2010; Wei et al., 2009).

In the second cycle, Ms. N a novice teacher with one year of teaching experience reflected that collaborating with her colleague during the PD intervention had vastly improved her mathematical problem-solving pedagogy and eliminated feelings of loneliness. As a newcomer both in the profession and in the school, Ms. N confessed to me that she had felt alone. She treasured collaborating with Mr. M as she felt she had someone to work with and no longer felt isolated. This implies that collaboration eradicates feelings of isolation that beginner teachers usually find themselves experiencing. Cookson (2005) observes that overloaded curriculum and tight school timetable like in the context of study limits teacher collaboration and hampers the possibilities of teachers participating in peer observation of (for) teaching thereby resulting in teacher isolation.

Ms. N also reflected that collaborating with Mr. M during the workshops made her realise that grade 9 learners normally have the same misconceptions and having discussions with Mr. M on how to use problem-solving to teach some word problems helped her to see how she could address these misconceptions. From these discussions with Ms. N, I concluded that collaboration resulted in effective PD for participant teachers' mathematical problem-solving pedagogy in this particular context.

7.6 Teachers views on teaching problem-solving

At the beginning of each cycle, participant teachers exhibited what I termed 'traditional' perspectives to teaching mathematical problem-solving (see section 2.3.3.1). During lessons, learners had to listen carefully as the teacher demonstrated the method to solve the given problems and then worked to practice the teacher's method until they acquired the skills. Teachers' reasons for adopting the traditional method of teaching

was that they felt that learners lacked basic problem-solving skills and processes (see section 4.6) and had pressure to finish the syllabus within the prescribed time (see section 5.6.2). In the workshops, I discussed with teachers so that they could realise that when teaching problem-solving, it is not compulsory to complete the given tasks on the same day, but the task can still be completed the next day. We agreed that the important issue was to enhance learners' mathematical understanding and problem-solving processes. Initially, teachers were missing the point that the knowledge gained and the procedures used in solving a specific problem may be applied to other problems and thereby, partly addressing the time factor. However, by participating in the PD intervention teachers' perspectives on mathematical problem-solving moved from traditional to reform-oriented perspective (see section 2.3.3.1).

Open-ended problems are problems that can be solved using different strategies (Sole, 2016) and are suitable to be used during the problem-solving process. As I conducted this study, I discovered that the definition of problem-solving depends on the context in which it is being practised. Participant teachers in this study worked within the CAPS curriculum which has word problems and believed that problem-solving is when learners solve and find solutions to the given mathematics word problems. Therefore, during problem-solving, I used word problems which were relevant to the teachers' context. Taking context into consideration was very important because if I had used open-ended tasks, it would have implied that the PD intervention would have been disconnected from teachers' practice.

7.7 Peer observation for teaching as a professional development strategy for mathematical problem-solving pedagogy

I introduced POFT in the second cycle of this study. I implemented it in such a way that the purpose of the observer was not to judge the observed, but to learn and reflect on their own mathematical problem-solving pedagogy from the observations they were making. Peer observation of teaching is commonly conducted so that the observer provides feedback for the observed. However, POFT was appropriate for this study because usually teachers in this particular context are never observed teaching such

that they become nervous when they get observed. Teachers were comfortable with this arrangement since they felt that POFT was not being done to judge them, but to self-improve the observer's mathematical problem-solving pedagogy. POFT is explained in detail in section 6.3.2 and; I concluded that it was relevant to this study since it really focused on teachers' unique needs and thereby dealing directly with the context.

POFT was effective as a tool for self-reflection in this study because I was working within the social-constructivist paradigm. This is in agreement with Engin and Priest (2014) who argue that for POFT to be successful there is a need of a context of shared understanding and collaboration. POFT promoted collegiality between participant teachers. Collegiality involves collaborative interaction and shared responsibility among teachers in joint work associated with their core tasks in school (Little, 1990). Drago-Severson (2007) posits that collegiality occurs when teachers:

1. Talk about practice;
2. Share expertise knowledge;
3. Observe one another; and
4. Help one another (p. 73).

The above four points were experienced by the participant teachers during POFT. Collegiality encouraged the development of positive interdependence between participant teachers which is known to deepen learning (Drago-Severson, 2007). Drawing from the literature, I concluded that positive interdependence is where participants with shared understanding believe that working collaboratively is both individually and co-operatively valuable. Both teachers reflected that POFT was beneficial to them as individuals as well as collaboratively and hence being dependent on each other during workshops and implementation of the mathematical problem-solving pedagogy.

As much as collegiality was an important aspect of POFT, the recognition by teachers on each other's context was imperative to the success of POFT. Teachers in this study talked about their mathematical problem-solving pedagogy; shared knowledge during workshops and on what was working (or not working) during the implementation;

observed each other and gave each other feedback during the post-observation feedback and helped one another during POFT and PD workshops. Teachers understood each other since they were working in the same context.

7.8 The importance of DBR in the Southern African context

DBR as a research method adapts to a particular context and develops theories in context using the successes and failures of various iterations. As indicated in section 7.2, context was an important theme in this study and this is why I adopted DBR as a research method. If we say the idea of context is vitally important when we are developing interventions that support teachers' pedagogy, then we cannot have interventions that neither take context into account nor are sensitive to context. This is why DBR is important in research contexts like Southern Africa. As I indicated in section 1.1 and 1.5.2, DBR generates interventions, design principles and theories that are highly sensitive to the context (Crepon, 2014; Wang & Hannafin, 2005).

Education research has been criticised for failing to focus on problems and issues that confront everyday practice (DBRC, 2003) and not generating useful knowledge that helps to inform the development of new innovations and reform (Van den Akker, 1999). The DBRC (2003) further states that educational researchers and practitioners agree that educational research is frequently unconnected to educational problems and issues of everyday practice. However, DBR aims to address and solve real-world problems by designing, enacting and sustaining interventions (Van den Akker et al., 2006), focuses on building a solid link between educational research and real-world challenges (Amiel & Reeves, 2008) while expanding theories or developing proto-theories and generating and refining design principles (DBRC, 2003; Herrington et al., 2007; Wang & Hannafin, 2005). This is the reason why researchers for Mathematics education trying to develop PD interventions in a Southern African context and are not using DBR are never going to succeed since they will not have taken context into account. If a researcher is not taking context into consideration it implies that they are using their own viewpoints regardless of the context of study.

As I indicated above, DBR is grounded in context and its main feature is the collaboration between researchers and practitioners. DBR researchers work in collaboration with teachers (practitioners) to develop interventions that address the specific needs of the teachers and the learners in a particular context. Herrington et al. (2007, p. 4090) state that DBR addresses 'complex problems in real contexts in collaboration with practitioners'. In this study, I worked with a team of teachers to provide solutions to practical challenges in the teaching of mathematical problem-solving that was being faced in a particular educational context. This is imperative to the South African context because without collaboration; interventions doubtfully effect changes in the real world context (DBRC, 2003; Wang & Hannafin, 2005). In this study, DBR allowed participant teachers to play an active role in their own PD, thereby developing ideas relevant in their context. Furthermore, DBR also permitted the refining and redesigning of the PD intervention based on multiple cycles in naturalistic settings. The naturalistic settings were the public schools and the grade 9 classrooms of the participant teachers. Designing the PD intervention for mathematical problem-solving pedagogy in naturalistic settings helped to 'build a stronger connection between educational research and real-world problems' (Amiel & Reeves, 2008, p. 34) for this particular context.

The other motivation for choosing DBR as research design was because of the notion that researchers in Southern Africa normally test existing theories through artificial treatments in controlled contexts. However, DBR dissociates itself from solely testing whether or not a theory works (Van den Akker, 1999). Rather DBR focuses on designing practical educational interventions for a particular local context and this is what differentiates it from other research approaches like the experimental design. I designed a PD intervention that can be used by DBE to support grade 9 mathematics teachers in teaching problem-solving and generated design principles that can be used by other mathematics education research practitioners interested in designing PD interventions in particular contexts (see section 5.7 and 6.7). DBR recognises that neither theory nor intervention alone is adequate and this is what we need in the South African context. We need to generate interventions and theories relevant to our own contexts and thereby making noteworthy contributions to the mathematics education

field. DBR involves theory development alongside solution development, and this is important because a lot of theories are developed outside of the Southern African mathematics education context. DBR has the capacity of generating information that provides important solutions for various design and development problems in usually unclear contexts (Van den Akker, 1999) like those frequently found in Southern Africa and in particular the context of informal settlements I worked in.

In this study, I established that context must be taken into account in educational research. Subsequently, methods such as DBR are not just important; they are necessary. Excessively focusing on randomised controlled trials and not taking contexts into account results in research being alienated from problems that face everyday practice.

7.9 Conclusion

In this chapter, I discuss the themes that progressively emerged from the study. Context was a major theme of the study and its significance in the development of the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy is discussed. Language considerations in PD in the Southern African context, POFT, teacher collaborative learning, teachers' views of teaching mathematical problem-solving, teachers' self-reflections as a tool to enhance their PD and the importance of DBR approaches in the South African context are discussed. In the next chapter, I summarise the study and review how the research questions were answered by the study. I reflect on the theoretical perspective informing the study, literature review, the mixed methods DBR research approach and my role in the study. Lastly, I discuss the recommendations, limitations of the study and areas for further possible research.

Chapter 8 Summary, Conclusions and Recommendations

8.1 Introduction

Using a mixed methods DBR methodological approach within a pragmatic paradigm, I had two objectives in this study. The first objective was to design a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy that can be used by the South African DBE and PD practitioners to support grade 9 mathematics teachers in the teaching of problem-solving. Secondly, it was to generate usable theoretical knowledge, in the form of design principles that can be used by other mathematics education practitioners in similar settings to develop PD interventions for mathematical problem-solving pedagogy. I achieved these objectives by carrying out a baseline investigation and developing prototypes of the PD intervention with two pairs of different grade 9 mathematics teachers and their learners. The findings from the baseline investigation, the literature review and context analysis shaped the designing of the first prototype. I conducted the first cycle with two female teachers and the findings had implications for both theory and design such that in the second cycle, I made changes to a number of aspects and extensively documented the effects of the changes.

Meeting the indicated two objectives helped to fill in the gap in the literature on:

(i) teachers' views of mathematical problem-solving pedagogy in under-resourced, overcrowded and multi-lingual classes with needy learners;

(ii) PD for mathematical problem-solving pedagogy in a particular local context; and

(iii) design principles for designing PD interventions for grade 9 teachers' mathematical problem-solving pedagogy in a particular local context. The design principles generated in this study are expected to eventually lead to theoretical maturation if continually refined and redesigned.

Methodologically, the study contributes to how DBR can be done in short-term projects such as master's degree and PhD studies. Some literature on DBR suggests that outputs and results of DBR are only valid if many cycles are conducted in a research project. Due to time and resources constraints, I only carried out two cycles in this study. Two prototypes were designed and redesigned to generate design principles and to develop the third and final prototype. One may debate that if many cycles had been conducted and evaluated for this study then the process would have been more sophisticated and momentous. However, I argue that regardless of the few cycles and the short duration of my PhD study, the design principles and the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy developed are valid and can be built upon. This is because I followed all the relevant phases of DBR as suggested by Reeves (2006) and worked in partnership with practitioners (teachers). Reeves (2006) proposes that one must start with the analysis of a practical problem, then develop solutions by testing and refining prototypes and reflecting on these to produce design principles, all in collaboration with practitioners; and this I followed.

DBR has commonly been used to study learners' response to interventions or innovations (Bakker & van Eeden, 2013) whereas the use of DBR to study how teachers learn and then develop interventions based on this is uncommon in the Southern African context. Methodologically, the study offers insight into how important DBR is as a research inquiry in the Southern African context. As education researchers, we sometimes do not involve teachers (practitioners) directly in the design process and only want them to use the products of the research process. This implies that we may not be developing what practitioners find useful and relevant to their contexts, yet effective teacher learning is highly contextual (Lampert, 2009). DBR gave participant teachers the opportunity to be involved in the designing and developing of the PD intervention. I viewed participant teachers as valuable collaborators from the very beginning and this resulted in the research process and teachers' practice being intertwined. Consequently, the findings of the study seem to reveal that DBR is imperative as a research methodology in the Southern African mathematics education context. This is because it ties neatly with effective PD as interventions are developed after various iterations and in collaboration with teachers. Collaborating with teachers in

designing and developing the PD intervention for mathematical problem-solving pedagogy; helped to reveal teachers' thinking, needs and dispositions. By gaining insight into teachers' thinking, I was in a position to develop a PD intervention that was relevant to teachers in the context of study. This is different from many PD programmes that have been criticised for being unconnected to teachers and classroom practice. Gulamhussein (2013) laments that, 'most of the PD interventions today are too divorced from teachers' contexts, such that they neither change teaching practices nor improve learners' learning'.

Theoretically, the study critiques the work of Polya's (1957) problem-solving processes. Polya's (1957) problem-solving processes only provide guidance on how teachers can teach learners problem-solving, but do not provide ways teachers can identify learners' difficulties in problem-solving or problem-solving skills required in successful problem-solving. I believe that it is important for teachers to be able to identify learners' difficulties during mathematical word problem-solving. When Polya (1957) was explaining the notion of understanding of the problem, he did not incorporate issues of language, yet language and the process of mathematical word problem-solving are not disparate (see section 5.8). Learners need to understand the problem first before attempting to solve it, yet LoLT is a challenge in the South African context.

In this chapter, I summarise and reflect on the study as a whole. I start by discussing the summary of the study and explaining how the study answered the research questions. I reflect on the mixed methods DBR as a research process and on my role in the study. Furthermore, I discuss the limitations of the study and state some implications for both research and practice that emanated from this study. Lastly, I conclude the study by emphasising how important the PD intervention and the design principles generated from the study are important for the South African mathematics education context.

8.2 Summary of the study

The South African CAPS curriculum emphasises the teaching of mathematical problem-solving. However, there are no programmes in place to support teachers in this regard.

The baseline study revealed that grade 9 mathematics teachers had partial understanding or misconceptions about the teaching of mathematical problem-solving. It was against this background that the purpose of this study was to develop a PD intervention that can be used to support South African grade 9 mathematics teachers in the teaching of mathematical problem-solving, as well as to generate design principles that can be used by other mathematics PD practitioners that are in the process of designing their own PD interventions. The summary of the main findings of this study are given as a summary of the answers to the research questions.

1. *What are South African grade 9 mathematics teachers' views on the teaching of problem-solving?*

I answered the first research question by carrying out a baseline investigation to establish grade 9 South African mathematics teachers' views, teaching strategies and the support required in their teaching of mathematical problem-solving (see chapter 4). The findings unearthed by the investigation are as follows. Firstly, there was a huge mismatch between participant teachers' definition of mathematical problem-solving and the typical definition found in the literature. Secondly, the study uncovered that overcrowded classes, learners' struggles with LoLT and a prescribed and overloaded curriculum compel teachers to retain strict ownership of the mathematical problem-solving process. Thirdly, the findings revealed that teachers' adoption of mathematical problem-solving as a step-by-step guided sense-making strategy is a response to the need to support learners who lack basic mathematical problem-solving processes from previous grades. Therefore, participant teachers required support on the teaching of mathematical problem-solving that included the practical demonstration of mathematical problem-solving pedagogy in the context of their classes.

2 *What are the design principles required to generate a professional development intervention for grade 9 teachers' mathematical problem-solving pedagogy in a particular local context?*

The second research question focused on the design principles that can be used by other research practitioners, PD facilitators or mathematics educators in similar settings

to generate a PD intervention to support mathematics teachers in teaching problem-solving. The study generated a handful of design principles for PD in general and specific design principles for PD for mathematical problem-solving pedagogy. These design principles are articulated in sections 5.7 and 6.7 and will not be elaborated on in this section.

3 *What is the impact of the PD intervention on grade 9 teachers' mathematical problem-solving pedagogy in this particular context?*

The third research question focused on the impact of the PD intervention on grade 9 teachers' mathematical problem-solving pedagogy. The findings suggested that the PD intervention had a positive impact on participant teachers' mathematical problem-solving pedagogy. The classroom observations revealed that at the beginning of each cycle, teachers struggled with teaching mathematical problem-solving and in making conjectures on the possible solution strategies their learners would use to solve a given problem. As the intervention progressed, teachers learnt how to teach the process of mathematical problem-solving and the various heuristics that can be used to solve problems in mathematics (see section 2.3.3.2). Subsequently, this resulted in teachers improving notably in their pedagogy. For example, during the implementation phase (see section 3.10), I observed that all participant teachers struggled to sustain classroom discussions because they lacked broad mathematical content and MPSKT (see section 2.4.4). In one instance, in the first cycle, I noted one participant teacher teaching learners incorrect content. At the end of the intervention, it appeared that the participant teachers experienced what was intended by the study because their MPSKT had significantly markedly improved and could easily sustain mathematics discussions with learners during problem-solving.

4 *What is the impact of the PD intervention on grade 9 learners' mathematical problem-solving processes in this particular context?*

The PD intervention had a positive impact on learners' performance and mathematical problem-solving processes (see section 5.5.2 and 6.5.2). At the beginning of each cycle, I observed that grade 9 learners paralleled mathematical successes with getting

the correct answer, teacher-reliance and attentiveness during the lesson rather than sense-making and genuine mathematical problem-solving (Graven & Heyd-Metzuyanim, 2014). This had changed at the end of each cycle as participant learners demonstrated considerable gains in their mathematical problem-solving processes and were using different heuristics to solve given problems. Learners began to value understanding and creating a plan before attempting to solve given problems. After solving the problems learners were looking back at their solution strategies and solutions to check if they were correct or made any sense.

5 *What factors enabled the growth of teachers' mathematical problem-solving pedagogy in this particular context?*

After each PD intervention, it was clear that there was growth in participant teachers' mathematical problem-solving pedagogy. This was evident in participant teachers' teaching practices and self-reflections during the interviews. In this section, I recap on some of the factors that promoted the indicated growth in teachers.

- In each cycle, I created a positive relationship with participant teachers before implementing the PD intervention and this resulted in teachers being comfortable with me and having a positive attitude towards the intervention.
- I built the PD workshops from participant teachers' experiences and knowledge such that teachers were able to correct their misconceptions or misunderstandings. Teachers' ability to self-correct resulted in tremendous growth of their mathematical problem-solving pedagogy.
- Collaborative participation was beneficial to participant teachers in that, each time they communicated their ideas to one another they became confident in teaching mathematical-problem-solving. Working collaboratively on mathematical problems during workshops afforded teachers the opportunities to discuss different heuristics to teach various word problems and this facilitated growth of their mathematical problem-solving pedagogy.
- The self-reflections by the participant teachers throughout the study pushed them to explore how they could have implemented the new ideas better during

implementation and this process allowed for improvement in their mathematical problem-solving pedagogy. As participant teachers meaningfully reflected on their pedagogy during lessons, I observed that they were able to self-correct themselves. Self-correction resulted in teachers consciously improving their own teaching practices. During semi-structured reflective interviews, teachers' challenges became more visible since they reflected on problematic areas of their lessons and areas of their mathematical problem-solving pedagogy that needed attention. When these challenges became more noticeable I was able to support the teachers as was necessary during the PD workshop and implementation of the pedagogy in the classroom. For example, in the second cycle participant teachers reflected that they did not know the perfect time to code-switch during lesson delivery. During PD workshops, I suggested to the teachers that they could code-switch when introducing new concepts or to re-explain difficult concepts (see section 5.3.1).

- Supporting teachers during the implementation stage reinforced the growth of their mathematical problem-solving pedagogy. Thus, I was able to address the specific challenges teachers faced during the process of changing classroom practice.
- POFT facilitated teachers' growth in their pedagogy in that they were able to observe the teaching of problem-solving by a colleague in a context that was relevant to them. The observer developed an increased awareness of how mathematical problem-solving was being taught and learned. The observed valued feedback from the observer and became aware of their mathematical problem-solving pedagogy and improved their own teaching strategies.

6 What factors facilitated learners' development of problem-solving processes?

At the start of each intervention, learners struggled with understanding given problems because of LoLT. In order to support participant learners with LOLT, teachers implemented code-switching in the first cycle and translanguaging in the second cycle. Although I am cautious, during classroom observations, I noticed that translanguaging

was better than code-switching in terms of learner behaviour and learners' understanding. I was working in schools where learners were speaking more than eight different languages as their mother tongue; therefore it was difficult for teachers to code-switch. Code-switching could have worked well if the classes were bilingual, that is, if only two different languages were being spoken by both the teacher and learners.

Teachers' proper facilitation of mathematical problem-solving pedagogy enabled the growth of learners' problem-solving processes in that learners could understand the given problems, knew how to develop and execute a plan and to look back after solving a given problem. I observed that collaborative learning facilitated learners' development of mathematical problem-solving processes, because as learners tackled problems in pairs or groups they were able to formulate, verbalise and discuss their ideas and received instant feedback from their peers.

8.3 Reflections on the theoretical perspective and literature review

A social-constructivist perspective views social interaction in a culturally-organised context as the important mechanism in learning. In this study, teacher collaborative participation evoked their learning of mathematical problem-solving pedagogy in their particular contexts, hence fitting perfectly with the social-constructivist perspective to teaching and learning.

This study focused on designing and evaluating a PD intervention for mathematical pedagogy. Therefore, the literature on mathematical problem-solving, teacher PD, teacher knowledge for teaching mathematics and problem-solving was examined. The design principles of the first prototype were generated from this literature review. In addition, the literature review on mathematical problem-solving knowledge for teaching (MPSKT) shaped the classroom observations in that I knew what I was looking for from teachers' mathematical problem-solving pedagogy. The review of the literature on effective PD moulded the activities that I implemented during the PD workshops.

8.4 Reflections on the DBR

This study engaged DBR and the PD intervention for teachers' mathematical problem-solving pedagogy was developed through iterative cycles of DEER. I used a mixed methods research design to gather the data. The type of mixed methods research design that I employed for this study was the convergent research design (see section 3.6.2). Convergent research design requires much effort and expertise since equal emphasis is given to both the quantitative and qualitative data (Cresswell & Plano Clark, 2011). DBR may be a challenging research methodology (Bakker & van Eerde, 2013) in that it requires skills, expertise and it is time-consuming. For me, these challenges were offset by the rewards and insights that I gained from the research process as well as the research outputs of the study; that is, the PD intervention and the design principles that were generated from the study.

At the beginning of this study, like any other researcher, I started with both a theoretical and a researcher's perspective. The DBR paradigm assisted me to move towards the practitioners' perspective as I worked in collaboration with participant teachers throughout the study. However, with DBR you do not move completely to the practitioners' perspectives, rather you move towards the middle ground by working with different participants through the cycles. As a DBR researcher in this study, I endeavoured to move towards the context of the teachers and their perspectives towards mathematics problems, mathematical problem-solving and mathematical problem-solving pedagogy. In the end, I developed a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy that have a balance between the researcher and the practitioners' perspective. Henceforth, I believe that the PD intervention is more beneficial as compared to interventions that emanate from laboratories under controlled environments

8.5 Reflections on my role in the study

My role in this study was that of a researcher, developer, facilitator, and an evaluator of the intervention and a critical friend of the participant teachers (see section 3.5). When I was doing presentations at PhD weekends and conferences, I got challenged by other

researchers that if one designs, implements and evaluates their PD interventions, there are biases. It was argued that since I was too involved in the project it was difficult for me to protect myself from finding what I wanted to find and that it would be hard to maintain the objectivity researchers need to maintain. To decrease researcher bias, I maintained an open mind throughout the study and kept a reflexive journal where I documented, critiqued, re-examined, reflected and looked at everything that occurred in the study from various angles (see section 3.8.1). During the semi-structured interviews, I stuck to the established interview protocol, so as not to influence the responses of the participants. I engaged a peer examiner (see section 3.12.2.2) who looked at the research process, data sets, analysis and interpretation. In this way, I used triangulation in terms of researchers' views and data analysis and interpretation.

Since I was implementing the PD intervention and at the same time interviewing the teachers, I was apprehensive about the truthfulness of the responses provided by the participant teachers. During the pre-intervention observations, I observed that sometimes teachers gave socially desirable answers that they thought I wanted to hear. To minimise this bias, I built trust with participant teachers before implementing the PD intervention. I tried to create an environment where teachers and learners felt that they had ownership of the research process, felt safe and comfortable to be truthful. I explained to participant teachers and learners that critical feedback was constructive and made sure that they understood the value of the PD intervention in supporting grade 9 teachers in the teaching of mathematical problem-solving. I emphasized to teachers the need to give honest responses throughout the intervention. I also corroborated the teachers' interview responses with classroom observations. On the learners' part, to eliminate bias, I triangulated classroom observations with learners' test marks, responses on the MPSSI and findings from the task-based interviews.

8.6 Limitations of the study

This study is a mixed methods DBR study which occurred in real-life education situations in collaboration with teachers, thus jettisoning the gap between education research and education practice. Bakker and van Eerde (2013, p.2) postulate that DBR

closes the gap between educational practice and theory in that it has a twofold objective of 'developing theories about domain-specific learning and the means that are designed to support that learning'. This implies that DBR generates interventions (in the case of this study the intervention is a PD intervention for grade 9 teachers' mathematical problem-solving pedagogy) that are accompanied by insights (for example in this study these are teacher activities that can be used by PD practitioners in PD workshops) on how these interventions can be implemented in education (McKenney & Reeves, 2012). DBR is an impact-inspired inquiry in that it targets real-world problems. However, like any other study, this study had limitations. The first limitation of the study was that the findings cannot be generalised to other contexts because they are drawn from a small sample of four teachers. It is impossible to establish the degree to which the four participant teachers are representative of South African in-service teachers. The PD workshop or classroom activities designed in this study may be impossible to directly repeat in other contexts and may need to be adjusted to suit particular conditions of certain contexts. However, DBR does not aim for generalisability because 'the effectiveness of a design is no guarantee of its effectiveness in other settings' (Collins, Joseph, & Bielaczyc, 2004, p.18). Consequently, generaliability was not the focus of this study. I was interested in developing an intervention that can be used to support grade 9 mathematics teachers in a particular context in teaching problem-solving. The context of this study is typical of South African townships schools. Therefore, the study gave insight into how grade 9 South African teachers at public secondary schools can be supported in teaching problem-solving.

DBR is time-consuming since it has an iterative nature. This study involved a baseline investigation and two iterative cycles of DEER. Finding adequate research time in the classrooms was challenging. The curriculum was overloaded and the school day was short. Some school terms were full of public holidays. The second cycle fell into the first and second term of the year where there were public holidays like Human Rights Day, Easter holidays, Freedom Day, Workers' Day and Youth Day, and as a result, I lost some of the research time. During this study, I also discovered that the classroom environment is very intricate in that there are social and cultural issues that clash with the psychological aspects of the PD intervention for example there were many

disruptions and obstructions that ranged from learner absenteeism and rowdiness. Nonetheless, each time I got time for research purposes I used it effectively. For example; on the Tuesdays set for task-based interviews, I made sure that I covered all the prepared interview questions with participant learners. I carefully used time during semi-structured interviews with participant teachers and avoided personal or non-relevant discussions.

DBR generates vast quantities of data due to its iterative nature and Brown (1992) articulates that if DBR researchers are not careful; data selection can be a likely limitation of DBR. In each cycle, I had lots of data from several classroom observations, teacher reflective semi-structured interviews, memo-writing, pre- and post- word problem mathematics attainment tests, learner task-based interviews, MPSSI, peer observations and the reflexive journal. I only transcribed the audio-recordings from the pre-observations of the first cycle and realised that it was arduous since I had done many classroom observations and the lessons were between 40-45 minutes long. I then decided to write the analysis of the data directly from the audio-recordings (see section 3.11). Not transcribing all the audio-recordings proved to be a limitation to the study since I had no access to transcripts which subsequently meant that I did not have direct access to the data from classroom observations. I could have used the transcripts as a reference during semi-structured interviews with the participant teachers. I also felt that if I had the transcripts from the classroom observations audio-recordings, I could have gone over them several times during data interpretation, in order to examine the classroom events in detail. However, it is well-documented in the literature that DBR researchers usually fall into the trap of insignificant contributions to the educational body of knowledge but immense effort on data collection, analysis and interpretation. I believe I avoided this situation by not transcribing the audio-recordings from the classroom observations, which could have taken much of my research time.

I could neither get clearance to video record the teachers or learners during the classroom observations nor learners during the task-based interviews. Taking audio-recordings only implied that I was unable to capture participants' actions in a live setting for analysis at a later convenient time. Video recordings could have given me the

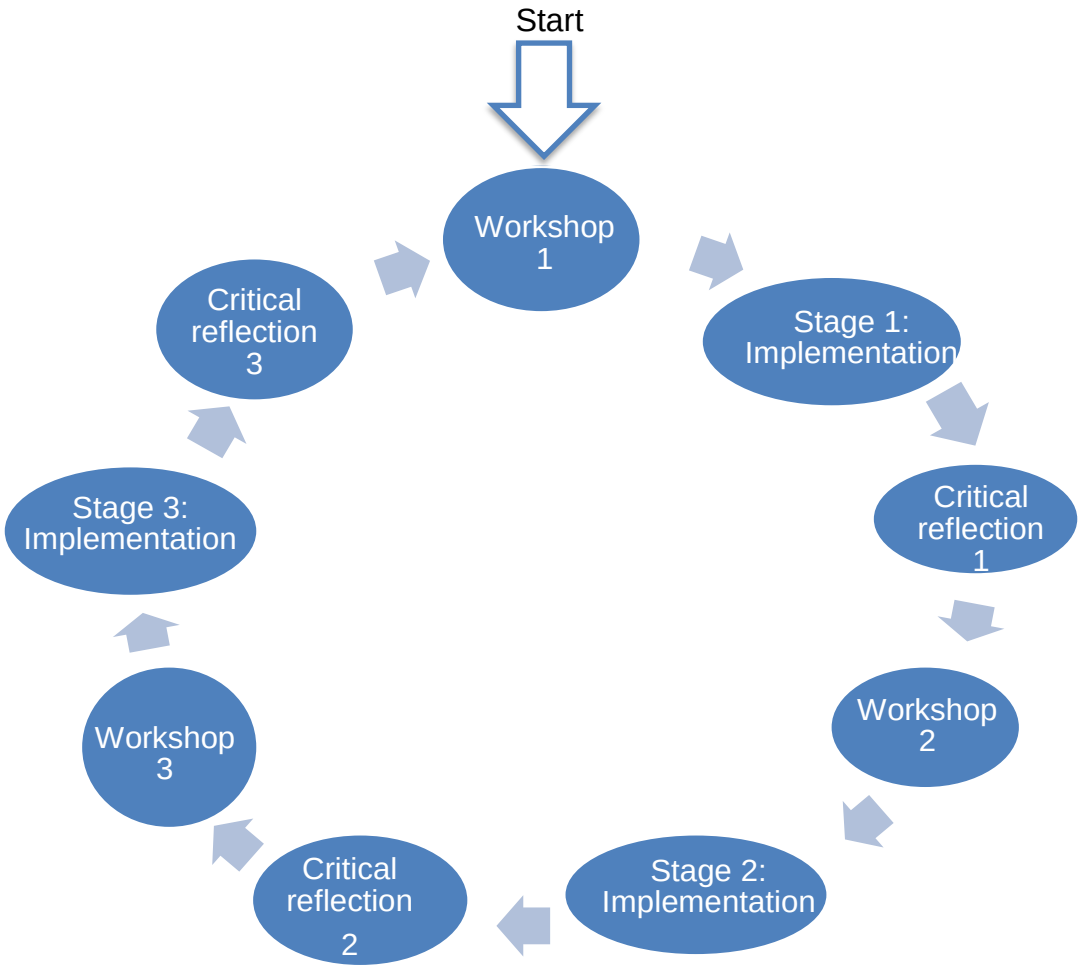
opportunity to verify the classroom observations during the data analysis phase. However, I used teachers' responses from the reflective interviews to verify the audio-recordings from the classroom observations. I was unable to video-record participant learners during task-based interviews and this was a huge drawback as I could not capture learners' actions and be in a position to re-visit them after the task-based interview episodes had transpired. Videos offer data that is irreplaceable and enables one to capture phenomena that one might otherwise miss (Schoenfled, 2016).

During the task-based interviews in the second cycle, learners did not like to think-aloud when working individually on problems unless I encouraged them or when they were working in pairs or groups. This resulted in identifying this as a limitation in that; sometimes I missed their reasoning and thinking processes during problem-solving.

8.7 Recommendations

The findings from this study may prompt other researchers to develop PD interventions in their local contexts. The study, having been done with a few schools, means that the transferability of the findings to larger contexts can be challenging. Findings from DBR cannot be generalised from a sample to a large population (Barab & Squire, 2004). However, I hope that the design principles generated from this study are going to be useful to primary stakeholders like researchers, teacher educators, or PD facilitators wishing to support teachers in teaching mathematical problem-solving. I will give the PD intervention for grade 9 teachers' mathematical problem-solving pedagogy developed from this study to the South African DBE and I recommend that it should be used to support grade 9 mathematics teachers in teaching problem-solving. The PD intervention is illustrated in Figure 8.1 and explained in the section that follows.

Figure 8.1: The professional development intervention developed from this study



The baseline investigation should be conducted before implementing the PD intervention and its purpose being to establish teachers' knowledge, views and practices on the teaching of mathematical problem-solving and for PD facilitators to create a positive relationship with the teachers before implementing the intervention.

The six-month-long PD intervention consists of three phases. Each phase consists of one three-hour workshop, an implementation stage and a reflection episode. The first workshop comprises of four main activities.

- The introductory activity consists of videos to show teachers mathematical problem-solving pedagogy in a context relevant to them.

- The second activity comprises of the PD facilitator and the teachers discussing the videos and what mathematical problem-solving pedagogy entails. At this moment, the facilitator introduces and explains heuristics and how teachers can implement them in their teaching. The facilitator can be guided by the elaboration on heuristics in section 2.3.4.2.

- After teachers understand mathematical problem-solving pedagogy, the third activity involves teachers collaboratively solving and discussing open-ended mathematical problems (in this study teachers worked with word problems which were relevant to their context). Teachers also discuss, simulate and plan how to teach these problems to their learners. The PD facilitator must agree with teachers on a structure of implementing the mathematical problem-solving pedagogy in their classroom or they can adopt one given in section 3.10.

- The fourth activity should support teachers on how to implement mathematical problem-solving pedagogy in a multi-lingual context and the facilitator can use strategies documented in section 6.7.

The PD facilitator summarises the workshop by explaining and emphasising the importance of reflection by teachers during the implementation of the mathematical problem-solving pedagogy. The facilitator concludes the workshop by encouraging teachers to reflect-in-action and reflect-on-action (see section 2.5.3 for the explanation of terms). After the workshop teachers are encouraged to go and implement the new ideas for two months and should audio or video-record themselves in action. During the implementation stage, teachers participate in the POFT and video or audio-record their meetings, discussions and practices. PD facilitators must provide teachers with peer observation forms and explain clearly the POFT process to the teachers as indicated in section 6.3.2, guiding them on what they should be observing and looking for. The PD facilitators should observe the participant teachers at least once in the classroom during the implementation stage. During the classroom observation, the PD facilitators can reflect with the teachers on the difficulties or successes they will be experiencing during the implementation of the pedagogy and support them as necessary.

After two months of implementation, teachers attend the second workshop. The first activity of the workshop is reviewing and reflecting on the audio or video-tapes with the teachers. Teachers collaboratively reflect on their experiences from the POFT process and the implementation of the mathematical problem-solving pedagogy. The discussions must centre on how teachers could have done or could do better in terms of implementing the pedagogy, supporting learners in terms of language, the process of POFT and the reflective process. If necessary, teachers are shown the second set of videos showing mathematical problem-solving pedagogy relevant to their context. The second activity involves teachers collaboratively solving word problems relating to what they will be teaching. Teachers discuss and plan on how to teach similar tasks to their learners. After the second workshop, the teachers go to implement the mathematical problem-solving pedagogy for two months. As in the first implementation, teachers participate in POFT, reflect on their mathematical problem-solving pedagogy, audio or video-record their teaching, and the PD facilitator observes and supports them as necessary.

After two months, teachers attend the third workshop which has similar activities as that of the second workshop. After the third workshop, teachers go and implement the mathematical problem-solving pedagogy like in the second implementation stage. The PD facilitator observes and supports the teachers and after six months if s/he sees that teachers are not teaching mathematical problem-solving correctly or competently s/he recommends that the teacher(s) start the training again, that is, start with the first workshop.

On top of the PD intervention for mathematical problem-solving pedagogy stated above, there are numerous recommendations from the study that I give to the Department of Basic Education, PD practitioners and mathematics teachers. These I explain in the next three sections.

8.7.1 The Department of Basic Education

I recommend the following to the Department of Basic Education:

- The baseline investigation (see chapter 4) revealed that the majority of South African teachers possibly have a partial understanding of what mathematical problem-solving pedagogy entails and hold more 'traditional' perspectives to the teaching of problem-solving. It is against this notion that I recommend that DBE should allocate part of its budget to rolling out frequent programmes of training teachers on how to implement mathematical problem-solving pedagogy. These programmes can be in the form of workshops and refresher courses. The PD intervention generated from this study can be adapted to support the teachers.
- Mathematical problem-solving gives learners the opportunity to investigate and to play with the concepts, but teachers have no time due to the CAPS curriculum being overloaded. School mathematics curriculum designers and policymakers should design a curriculum that promotes and accommodates the implementation of mathematical problem-solving pedagogy so that teachers do not see it as time-consuming and 'something extra' they have to do.
- This study evidenced that learners benefit from methodically planned problem-solving teaching. I recommend that the DBE should integrate problem-solving in the curriculum. Currently, the CAPS curriculum has examples of story problems and suggestions for classroom activities, but no programmes on how to teach problem-solving.
- Teachers must be given adequate preparation and teaching time as teaching mathematical problem-solving takes time. Therefore, I recommend that extra mathematics lessons should be arranged in schools during afternoons, weekends or school holidays.
- The South African DBE uses peer observation of teaching for quality assurance and teacher promotion. I implemented POFT in the second cycle of the study to assist teachers' self-reflection and the results seemed to indicate that it is a valuable PD strategy that can be used to support grade 9 South African teachers' mathematical problem-solving pedagogy. POFT buttressed and enhanced the

correct way of teaching mathematical problem-solving. Thus, I recommend that all South African education institutions should consider implementing POFT for formative purposes and it should be used as a PD tool to support grade 9 teachers' mathematical problem-solving pedagogy.

- Since most learners in public schools in poor areas of South Africa struggle with the LOLT, I recommend the production of mathematics teaching and learning materials in different home languages. In-service teachers must be provided with PD programmes that help them with code-switching and translanguaging. South African teacher preparation programmes must help pre-service teachers with the former and the latter.

8.7.2 Mathematics teachers

Task-based interviews were performed in this study and became an integral part of assessing the impact of the PD intervention on learners' mathematical problem-solving processes. I was able to understand grade 9 participant learners' problem-solving processes by listening to explanations of their solution strategies and looking at their actions when solving problems. Task-based interviews can be an important strategy for teaching mathematics and for extending learners' mathematical thinking. It is against this background that I recommend that South African mathematics teachers should implement task-based interviews in their teaching. The way task-based interviews can be implemented is discussed in detail in chapter 6.

In this study, POFT had an immediate impact on teachers' mathematical problem-solving pedagogy in that after observing their peers teaching, teachers went to their next class and implemented problem-solving teaching strategies from the observation. I recommend that South African teachers should practice peer observation of (for) teaching as a way of improving their mathematical problem-solving pedagogy or teaching practices in general.

8.7.3 Professional development facilitators

Language proved to be an extremely important aspect PD intervention in a South African context. As such, I recommend that teacher PD practitioners should consider PD programmes that comprise a segment that guides teachers on how to support learners that struggle with LoLT.

After conducting the PD, PD practitioners should support teachers during the implementation stage to address any challenges that may arise. This is supported by Kazemi and Hubbard (2008) who postulate that it is erroneous to assume that what teachers learn in a PD workshop gets transferred intact into their teaching practices; therefore, teachers need to be supported when they return back to their teaching contexts.

This study proved that when POFT is used as a PD strategy to support teachers' self-reflections the benefits are noteworthy. Hence, I recommend that PD interventions in South Africa that support the teaching of mathematical problem-solving should include POFT as a tool to assist teachers' self-reflections. However, the PD interventions must take into account the time constraints of the South African CAPS curriculum as POFT can be time-consuming.

8.8 Areas for possible further research

As I indicated in section 1.3, I know no other study of a similar nature in South Africa that has designed and developed a PD intervention that can be used to support grade 9 teachers in teaching mathematical problem-solving. If such an intervention existed, I would not have needed to embark on this study. Although I am confident that this study generated a valid PD intervention, the results can be regarded as tentative and I recommend that a large and extensive study with multiple cycles should be conducted involving a larger sample of grade 9 mathematics teachers so that the results can be generalised.

The baseline study investigated participant teachers' views on the teaching of mathematical problem-solving. Participant teachers had a partial understanding of mathematical problem-solving pedagogy in their context of overcrowded and multi-lingual classrooms and wanted to see it in action. During the PD intervention, I could only get videos from USA, Singapore or Japan showing the teaching of mathematical problem-solving such that I concluded that no such videos exist in South Africa. Consequently, further research can be done with a large number of teachers on the teachers' role and how learners can be controlled during the teaching of mathematical problem-solving in large, overcrowded and multi-lingual classes, where discipline is a challenge. Another study can also be done on different approaches that can be used to teach problem-solving effectively in the context of study.

During the study, I realised that teachers found some heuristics easier to use during the workshops and to explain to learners during the implementation phase. Furthermore, research can be conducted with South African teachers to investigate the heuristics teachers find easy to personally use or teach; and why they find them easy. The findings from the study can then be taken into account when introducing heuristics to South African teachers and learners.

I recommend that a more rigorous qualitative study should be conducted on how best South African grade 9 mathematics teachers can conduct task-based interviews. This should involve teachers video-recording themselves during the learner task-based interviews. Meanwhile, on the other hand, researchers or PD practitioners can analyse the videos with the teachers and discuss ways to effectively interpret learners' work and how best teachers can help, question and probe learners during the task-based interviews. Other research can be done on how to effectively extend South African learners' mathematical thinking using task-based interviews.

I successfully introduced POFT with two participant teachers in the second cycle of the study. However, further studies can be done in the future with a large number of South African Grade 9 mathematics teachers that are a representative sample and the results can be compared with my findings. Moreover, a study is also required that considers

how teachers can be trained to give constructive and non-judgmental feedback to the observed during the POFT process.

Translanguaging was introduced in the second cycle and seemed to have been more successful than code-switching. Further research can be done with a larger sample of grade 9 mathematics teachers and learners over a longer period of time to investigate and compare the effectiveness of code-switching and translanguaging in multi-lingual classes.

8.9 Concluding remarks

The South African curriculum emphasises the teaching of problem-solving in mathematics. However, the CAPS curriculum is deficient of problems that require to be solved by problem-solving. Non-routine problems and problem-solving are only covered by a tenth of the curriculum (DBE, 2011). There are no programmes that support mathematics teachers in the teaching of problem-solving. This study developed a PD intervention that can be modified and used to support grade 9 South African mathematics teachers in similar contexts in teaching problem-solving. Design principles were generated from the study and these can be used by other PD practitioners, researchers, mathematics education practitioners and teachers developing PD interventions. I am confident that the PD intervention and design principles generated from this study will be implemented by other researchers and will evolve over time.

I could not manage to go back to the field after six months or 1 year to assess the long-term effect of the PD intervention on teachers' mathematical problem-solving pedagogy because of participant accessibility, ethics clearance, resource constraints and length of the study. Subsequently, I evaluated the effectiveness of the PD interventions generated in each cycle during the development of the prototypes and I have not had the opportunity to visit the participant teachers. This leaves me wondering how they are doing at the moment!

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10. Appendices

10.1 Appendix A: The University of the Witwatersrand clearance certificate

Wits School of Education



27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

29 February 2016

Student Number: 1405647

Protocol Number: 2016ECE002D

Dear Brantina Chirinda

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

An exploration of how a professional development intervention can be designed to promote the development of grade 9 South African educators' mathematical problem solving pedagogy.

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Mabeta

Wits School of Education

011 717-3416

cc Supervisor – Dr Patrick Barmby

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10.2 Appendix B: Letter to GDE seeking approval to collect data in secondary schools

03 August 2015
The Director: Knowledge Management and Research
Gauteng Department of Education
P O BOX 7710
Johannesburg
2000

Dear Sir/ Madam

Application for permission to conduct research in secondary schools

My name is Brantina Chirinda. I am currently enrolled in a PhD specialising in mathematics education at the University of Witwatersrand and I am in the process of collecting empirical data for my thesis. I hereby apply for permission to conduct research in a number of secondary schools in Johannesburg North District as part of my study.

I envisage conducting a study on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy. The purpose of this study is to examine how grade 9 mathematics teachers in this district are currently teaching problem-solving; to design a professional development intervention for South African teachers on the teaching of mathematical problem-solving and to help participant teachers to implement the new teaching ideas.

The study will employ a classroom-based design research, and convergent mixed methods strategies will be used to gather the data. Participant teachers will be required to fill in a teacher open-ended questionnaire at the onset of the study; attend voluntary,

face-to-face professional development workshops on the teaching of mathematical problem-solving and to participate in teaching observations and teacher reflective semi-structured interviews. Participant learners will be required to write pre- and post-word problem tests and fill in a Mathematical Problem-solving Skills Inventory at the onset and at the end of the study. The University of Witwatersrand ethics policy requires that I get permission from the Department of Education to conduct this study in public schools and that the participants of this study be protected in terms of keeping their identity anonymous and their information to be kept confidential. Kindly note that the participants who choose to participate in the study can withdraw from the research at anytime without incurring any negative consequences.

The Department of Education will benefit in that the research will inform policy makers to make informed decisions on the designing and implementation of professional development intervention on mathematical problem-solving pedagogy for South African teachers. Upon completion of this study, a copy of the report will be made available to the Department of Education and other educational agencies.

Kindest Regards



Brantina Chirinda

10.3 Appendix C: GDE Approval Letter for the first cycle

For administrative use:
Reference no. D2016 / 373 AA
Enquiries: Diane Buntting 011 843 6503

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GDE AMENDED RESEARCH APPROVAL LETTER

Date:	**18 January 2016**
Validity of Research Approval:	**8 February 2016 to 30 September 2016**
Previous GDE Research Approval letter reference number	**D2016 / 336A dated 13 November 2015 and D2016 / 212 dated 18 August 2015**
Name of Researcher:	**Chirinda B.**
Address of Researcher:	**240 Anita Avenue; Northwold; 2188**
Telephone / Fax Number/s:	**083 538 7075**
Email address:	**brantinac@yahoo.com**
Research Topic:	**An exploration of how a professional development intervention can be designed to promote the development of Grade 9 South African educators' mathematical problem solving pedagogy.**
Number and type of schools:	**TWENTY Secondary Schools**
District/s/HO	**Johannesburg North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved. A separate copy of this letter must be presented to the Principal, SGB and the relevant District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted. However participation is VOLUNTARY.

The following conditions apply to GDE research. The researcher has agreed to and may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

CONDITIONS FOR CONDUCTING RESEARCH IN GDE

Mkhado
2016/01/20

Office of the Director: Education Research and Knowledge Management ER&KM)
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The District/Head Office Senior Manager/s concerned, the Principal/s and the chairperson/s of the School Governing Body (SGB.) must be presented with a copy of this letter.*
2. *The Researcher will make every effort to obtain the goodwill and co-operation of the GDE District officials, principals, SGBs, teachers, parents and learners involved. Participation is voluntary and additional remuneration will not be paid;*
3. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal and/or Director must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
4. *Research may only commence from the second week of February and must be concluded by the end of the THIRD quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
5. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
6. *It is the researcher's responsibility to obtain written consent from the SGB/s; principal/s, educator/s, parents and learners, as applicable, before commencing with research.*
7. *The researcher is responsible for supplying and utilizing his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institution/s, staff and/or the office/s visited for supplying such resources.*
8. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research title, report or summary.*
9. *On completion of the study the researcher must supply the Director: Education Research and Knowledge Management, with electronic copies of the Research Report, Thesis, Dissertation as well as a Research Summary (on the GDE Summary template). Failure to submit your Research Report, Thesis, Dissertation and Research Summary on completion of your studies / project – a month after graduation or project completion - may result in permission being withheld from you and your Supervisor in future.*
10. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned;*
11. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director/s and school/s concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

Makhado

Dr David Makhado

Director: Education Research and Knowledge Management

DATE: 2016/01/20

Making education a societal priority

Office of the Director: Education Research and Knowledge Management ER&KM)
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

10.4 Appendix D: GDE Approval Letter for the second cycle

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE AMENDED RESEARCH APPROVAL LETTER

Date:	**30 May 2017**
Validity of Research Approval:	**06 February 2017 – 29 September 2017** **D2016/373AAA**
Name of Researcher:	**Chirinda B.**
Address of Researcher:	**240 Anita Avenue**
	Northwold
	2188
Telephone Number:	**083 538 7075**
Email address:	**brantinac@yahoo.com**
Research Topic:	**An exploration of how a professional development intervention can be designed to promote the development of Grade 9 South African educators' mathematical problem solving pedagogy**
Number and type of schools:	**Twenty Secondary Schools**
District/s/HO	**Johannesburg North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

30/05/2017

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Ms Faith Tshabalala
CES: Education Research and Knowledge Management

DATE: 30/05/2017

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

10.5 Appendix E: Letter to school principals

18 January 2016
The Principal
XXX Secondary School
P O BOX 000
Johannesburg

Dear Sir / Madam

Application for permission to conduct research at your institution

I hereby apply for permission to conduct research at your school as part of a PhD specialising in mathematics education for which I am enrolled at the University of Witwatersrand. The Gauteng Department of Education approved my request to collect data for this project at public schools in Johannesburg North. Please see the letters of approval attached.

I envisage conducting a study on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy. The purpose of this study is to examine how grade 9 mathematics teachers in this district are currently teaching problem-solving; to design a professional development intervention for South African teachers on the teaching of mathematical problem-solving and to help participant teachers to implement the new teaching ideas.

I hope that the school will afford me the opportunity to collect data from the grade 9 mathematics teachers and their learners. If approval is granted, all learners who will participate in this study will be given a consent form to be signed by their parents or guardians to allow me to conduct the research. These consent forms should be returned to me as the researcher at the onset of the data collection process. Before the

start of the study, I will explain the procedures of the study to the participant teachers and learners. Participant teachers will be required to fill in a teacher open-ended questionnaire at the onset of the study; attend voluntary, face-to-face professional development workshops on the teaching of mathematical problem-solving and to participate in teaching observations and teacher reflective semi-structured interviews. Participant learners will be required to write pre- and post-word problem tests and fill in a Mathematical Problem-solving Skills Inventory at the onset and at the end of the study.

The University of Witwatersrand ethics policy requires that I get permission from your institution before conducting this study and that the participants of this study be protected in terms of keeping their identity anonymous and their information to be kept confidential. The school's participation in this study is voluntary and confidential. At no time will the name of your school or an individual participant be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or the individual participants. Kindly note that your school or the participants who choose to participate in the study can freely withdraw from the research at anytime without incurring any negative consequences.

Your approval to conduct this study will be greatly appreciated as I firmly believe that both your school and I will benefit from this research relationship. I will follow up this communication with a telephone call and will be happy to answer any questions or concerns that you may have. If you agree, kindly organise a signed letter of permission on your school's letterhead acknowledging your consent and permission for me to conduct this study at your institution.

Kindest Regards



Brantina Chirinda

10.6 Appendix F: Letter to the School Governing Board

18 January 2016
The School Governing Board
XXX Secondary School
P O BOX 000
Johannesburg

Dear Sir / Madam

Application for permission to conduct research at your institution

I hereby apply for permission to conduct research at your school as part of a PhD specialising in mathematics education for which I am enrolled at the University of Witwatersrand. The Gauteng Department of Education and the Johannesburg North District Department of Education approved my request to collect data for this project at public schools in Johannesburg North. Please see the letters of approval attached.

I envisage conducting a study on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy. The purpose of this study is to examine how grade 9 mathematics teachers in this district are currently teaching problem-solving; to design a professional development intervention for South African teachers on the teaching of mathematical problem-solving and to help participant teachers to implement the new teaching ideas.

I hope that the school will afford me the opportunity to collect data from the grade 9 mathematics teachers and their learners. If approval is granted, all learners who will participate in this study will be given a consent form to be signed by their parents or guardians to allow me to conduct the research. These consent forms should be returned to me as the researcher at the onset of the data collection process. Before the start of

the study, I will explain the procedures of the study to the participant teachers and learners. Participant teachers will be required to fill in a teacher open-ended questionnaire at the onset of the study; attend voluntary, face-to-face professional development workshops on the teaching of mathematical problem-solving and to participate in teaching observations and teacher reflective semi-structured interviews. Participant learners will be required to write pre- and post-word problem tests and fill in a Mathematical Problem-solving Skills Inventory at the onset and at the end of the study.

The University of Witwatersrand ethics policy requires that I get permission from your institution before conducting this study and that the participants of this study be protected in terms of keeping their identity anonymous and their information to be kept confidential. The school's participation in this study is voluntary and confidential. At no time will the name of your school or an individual participant be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or the individual participants. Kindly note that your school or the participants who choose to participate in the study can freely withdraw from the research at anytime without incurring any negative consequences.

Your approval to conduct this study will be greatly appreciated as I firmly believe that both your school and I will benefit from this research relationship. I will follow up this communication with a telephone call and will be happy to answer any questions or concerns that you may have. If you agree, kindly organise a signed letter of permission on your school's letterhead acknowledging your consent and permission for me to conduct this study at your institution.

Kindest Regards



Brantina Chirinda

10.7 Appendix G: Letter to the HOD

DATE: 18 January 2016

Dear Sir/ Madam

I hereby apply for permission to conduct research at your school as part of a PhD specialising in mathematics education for which I am enrolled in the School of Education at the University of Witwatersrand. The Gauteng Department of Education and approved my request to collect data for this project at public schools in Johannesburg North. Please see the letters of approval attached.

I envisage conducting a study on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy. The purpose of this study is to examine how grade 9 mathematics teachers in this district are currently teaching problem-solving; to design a professional development intervention for South African teachers on the teaching of mathematical problem-solving and to help participant teachers to implement the new teaching ideas.

The reason why I have chosen your school is that it is a public school, located in an informal settlement and with large overcrowded classes that are under-resourced. I hope that the school will afford me the opportunity to collect data from grade 9 mathematics teachers and their learners. If approval is granted, all learners who will participate in this study will be given a consent form to be signed by their parents or guardians to allow me to conduct the research. These consent forms should be returned to me as the researcher at the onset of the data collection process. Before the start of the study, I will explain the procedures of the study to the participant teachers and students. Participant teachers will be required to fill in a teacher open-ended questionnaire at the onset of the study; attend voluntary, face-to-face professional development workshops on the teaching of mathematical problem-solving and to

participate in teaching observations and teacher reflective semi-structured interviews. Participant learners will be required to write pre- and post-word problem tests and fill in a Mathematical Problem-solving Skills Inventory at the onset and at the end of the study.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and the identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.
Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

NAME: Brantina Chirinda



EMAIL: brantinac@yahoo.com

10.8 Appendix H: Teacher open-ended questionnaire

Dear Grade 9 Mathematics teacher

My name is Brantina Chirinda. I am currently enrolled in a PhD specialising in mathematics education at the University of Witwatersrand, and I am in the process of collecting empirical data for my thesis. I hereby invite you to participate in a research project on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy.

The enclosed open-ended questionnaire is designed to attain information about you and your Mathematics teaching in grade 9 this year. Your completion of the questionnaire will be much appreciated as the data gathered will assist in enhancing the teaching of Mathematics problem-solving which results in our students achieving better in Mathematics. I understand that you are very busy; however, the 15 minutes it will take you to complete the questionnaire are very important as this will help us understand how to improve the teaching and learning of Mathematics in Gauteng, South Africa.

Thank you in advance for your participation. Your participation in this research project is voluntary and confidential. At no time will the name of your school or your name be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or you as a participant. After completing this questionnaire there is a possibility that you will be chosen for a follow-up project. This project will be explained to you in detail if you become part of it.

Kindly note that if you choose to participate, you can withdraw from the research at anytime without incurring any negative consequences. If you have questions about the study or any items in the questionnaire, I hope you will not hesitate to contact me on my e-mail address: brantinac@yahoo.com.

Yours in mathematics education



Brantina Chirinda

This Teacher Open-ended Questionnaire requires you to describe your current grade 9 mathematical problem-solving teaching. There are no correct or wrong answers. Please give an answer to every question by clearly explaining your teaching practices or ticking an option that applies to you. If you make a mistake cross it out and choose the correct option.

A. Demographic Details

1. What is your sex? _____
2. How many years have you taught? _____

B. Teacher Education

3. Do you have each of the following degrees?

[1] Certificate [2] Diploma [3] Bachelors [4] Masters [5] Doctorate

Please indicate the subject(s) for each of your degrees.

		Certificate	Diploma	Bachelors	Master	Doctorate
4	Mathematics	1	2	3	4	5
5	Computer Science	1	2	3	4	5
6	Mathematics Education	1	2	3	4	5
7	Science/Science Education	1	2	3	4	5
8	Secondary Education	1	2	3	4	5
9	Other Education (e.g.,	1	2	3	4	5
10	Others, please specify	1	2	3	4	5

C. Mathematics Instruction

1. Can you please say something about your mathematical problem-solving teaching ideas?

2. How do you approach the teaching of problem-solving in your classes?

3. What difficulties do you face with teaching mathematical problem-solving?

4. What difficulties do your learners face with mathematical problem-solving?

5. What support would you like in your teaching of mathematical problem-solving?

Thank you so much!!

10.9 Appendix I: Information sheet for grade 9 mathematics teachers

18 January 2016

Dear Grade 9 Mathematics teacher

My name is Brantina Chirinda. I am currently doing a PhD in mathematics education with the School of Education at the University of Witwatersrand. I hereby invite you to participate in a study on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy. The purpose of this study is to examine how grade 9 mathematics teachers in your district are currently using problem-solving in the teaching of Mathematics; to design a professional development intervention for South African teachers on the teaching of mathematical problem-solving and to help participant teachers to implement new teaching ideas.

The reason why I have chosen your school is that it is a public school, located in an informal settlement and with large overcrowded classes that are under-resourced. If you are willing to participate in this study, during the data collection process you will be required to fill in an open-ended questionnaire; attend voluntary, face-to-face professional development workshops on the teaching of mathematical problem-solving and to participate in teaching observations and teacher reflective semi-structured interviews. The teaching observations and the reflective interviews will be audio-recorded. Participant learners will be required to write pre- and post-word problem tests and fill in a Mathematical Problem-solving Skills Inventory at the onset and at the end of the study. If a learner is not taking part in this project s/he will not be required to fill in anything. The study will take a maximum of 6 months at your school.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written

data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Please let me know if you require any further information.

Thank you very much for your help. If you are interested please fill in the attached consent form.

Yours sincerely,



NAME: BRANTINA CHIRINDA
EMAIL: brantinac@yahoo.com

10.10 Appendix J: Grade 9 mathematics teacher's consent form

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project called:
Using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy.

I, _____ give my consent for the following:

Permission to review/collect documents/artefacts	**Circle one**
I agree that reflective journals can be used for this study only.	YES/NO
Permission to observe you in class	
I agree to be observed in class.	YES/NO
Permission to be audio-taped	
I agree to be audio-taped during the interview or observation lesson	YES/NO
I know that the audio-tapes will be used for this project only	YES/NO
Permission to be interviewed	
I would like to be interviewed for this study.	YES/NO
I know that I can stop the interview at any time and don't have to answer all the questions asked.	YES/NO
Permission for questionnaire/test	
I agree to fill in a question and answer sheet or write a test for this study.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audio-taped.
- all the data collected during this study will be destroyed within 3-5 years after the completion of the project.

Sign_____ Date_____

10.11 Appendix K: Information sheet for grade 9 mathematics learners' parents or guardians

DATE: 18 January 2016

Dear Parent/ Guardian

My name is Brantina Chirinda. I am currently doing a PhD in mathematics education with the School of Education at the University of Witwatersrand. I am doing research on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy.

My research involves looking at how grade 9 mathematics teachers in your district are currently using problem-solving in their teaching of mathematics. The reason for looking at this is to be able to design a professional development intervention for South African teachers on the teaching of problem-solving and to help participant teachers to implement new teaching ideas. During the study, which will take a maximum of 6 months at your child's school teachers will be required to fill in an open-ended questionnaire; attend voluntary, face-to-face professional development workshops on the teaching of mathematical problem-solving and to participate in teaching observations and teacher reflective semi-structured interviews. The teaching observations and the reflective interviews will be audio-recorded.

The reason why I have chosen your child's class is that it is a grade 9 class and students tend to perform poorly in mathematics once they get to grade 9. I was wondering whether you would mind giving consent for your child to participate in this study. If you give consent for your child to participate, your child will be required to write pre- and post-word problem tests and fill in a Mathematical Problem-solving Skills Inventory at the onset and at the end of the study. If your child is not taking part in this project s/he will not be required to fill in anything.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help. If you are interested please fill in the attached consent form.

Yours sincerely,



NAME: Brantina Chirinda

EMAIL: brantinac@yahoo.com

10.12 Appendix L: Parent's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called:
Using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy.
I, _____ the parent of _____

Circle one

Permission to observe my child in class

I agree that my child may be observed in the class. YES/NO

Permission to be audio-taped

I agree that my child may be audio-taped during classroom observations. YES/NO

I know that the audio-tapes will be used for this project only.
YES/NO

Permission for questionnaire/test

I agree that my child may fill in a question and answer sheet or write a test for this study YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- he/she does not have to answer every question and can withdraw from the study at any time.
- he/she can ask not to be audio-taped
- all the data collected during this study will be destroyed within 3-5 years after the completion of the project.

Sign_____ Date_____

10.13 Appendix M: Information sheet for grade 9 mathematics learners

DATE: 18 January 2016

Dear Learner

My name is Brantina Chirinda. I am currently doing a PhD in mathematics education with the School of Education at the University of Witwatersrand. I hereby invite you to participate in a study on using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy.

My investigation involves looking at how grade 9 mathematics teachers in your district are currently using problem-solving in the teaching of mathematics. The reason for looking at this is to be able to develop a professional development intervention for South African teachers on the teaching of mathematical problem-solving and to help participant teachers to implement the new teaching ideas.

I was wondering whether you would mind writing a pre-word problem mathematical tests and fill in a Mathematical Problem-solving Skills Inventory at the beginning of the study. At the end of the study, you will be required to write a post-test and to fill in the Mathematical Problem-solving Skills Inventory once again. The research will take a maximum of 6 months at your school. If you choose not to take part in this project you will not be required to fill in anything. Remember, this is not a test, it is not for marks and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your own name, but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day, it is your decision to join us in the study. I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you



NAME: Brantina Chirinda

EMAIL: brantinac@yahoo.com

10.14 Appendix N: Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

Using Design-Based Research to develop a professional development intervention for Grade 9 South African teachers' mathematical problem-solving pedagogy.

My name is: _____

Circle one

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be audio-taped

I agree to be audio-taped during the classroom observation lesson YES/NO

I know that the audio-tapes will be used for this project only YES/NO

Permission for questionnaire/test

I agree to fill in a question and answer sheet or write a test for this study. YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audio-taped.
- all the data collected during this study will be destroyed within 3-5 years after the completion of the project.

Sign_____ Date_____

10.15 Appendix O: Teacher observation comment card

Stage	What teacher should do	What teacher really did (evidence/indicators)	Comments
Before	-Give instructional strategies and activities that incorporate students' prior knowledge. -Pose the problem to students orally or in a written form. - Read or have a student read the problem. - Make sure that students understand the problem before they begin to work on it, for example, asking students to identify what is asked for in the problem, discussing any unfamiliar terms in the problem, and/or asking students to paraphrase the problem in their own words. - Brainstorm possible solution strategies. -Clarify the task at hand.		
During	-Move among students. -Listen prudently to students' thoughts and discussions. - Embrace the dynamics of group work. -Question individual students or groups about the strategy they are using and their findings. -Probing students with suitable questions to assist them to clarify the direction they are taking in solving the problem. -Provide hints to students who require them -Persuade students to seek and value alternative modes of problem-solving. - Encourage students to make extensions or generalisations.		
After	-Encourage students to reflect on their solutions and on the processes they used. -Invite students to justify their solutions. -Engage the class in productive discourse by letting students communicate their ideas in words, diagrams, and allowing them to share their ideas and strategies. - Probe the students how the problem is similar to and different from problems they previously solved. - Have students discuss the critical aspects of the problem. -Summarise the chief points of the discussion and establish that all students comprehend them.		

10.16 Appendix P: Teacher reflective semi-structured interview protocol

Instructions for Interviewer

1. Be sure that you and the teacher are alone and in a quiet room where responses can be recorded.
2. Remember to follow the written protocol loyally. You should always probe once if you think that the teacher has not answered the question asked by using one of the following: Anything else? Can you tell me more about----? Or you could simply rephrase the question.
3. In responding to some questions, the teacher may describe one aspect of the question in depth while not addressing the breadth of the question. As you listen to the teacher's responses probe him or her so that he or she gives a complete response to the question. You may ask say, 'What about ----?' or 'How important is----?' If the teacher has already answered a question you are about to ask you should say: 'The next question is----'. I think you have already answered it. Do you think you have answered it? Is there anything else you want to add?
4. Your responses to the teacher's statements should be non-committal and non-judgmental. Use responses such as 'Thanks,' 'Thats fine', 'Alright,' and 'Okay.'
5. If you forget to ask a question, make sure that you go back and ask it even if it is out of order.
6. Thank the teacher for his or her time.

Introduction by the interviewer

Thank you for allowing me to observe your class. I have some questions that I would like to ask you related to this lesson. Kindly allow me to tape the interview. It will assist me to stay focused on our conversation and it will guarantee that I have a precise record of what we will discuss. If applicable, ask: Can I have a copy of the instructional materials you used for this lesson?

Guiding interview questions

1. I would like to know a bit more about the learners in this class. Tell me about the mathematical problem-solving ability levels of learners in this class. How do they compare to learners in the school as a whole?
2. What was the specific purpose of today's lesson?
3. How do you feel about how the lesson played out? What do you think were the strengths and weaknesses of your lesson?
4. If you are to teach the lesson again what would you do differently?
5. With reference to your classroom tasks, why did you choose those tasks to support your mathematical problem-solving teaching?
6. Tell me about the mathematics problem-solving skills of this task.
7. Did you feel that the tasks achieved their intended purpose in the lesson? If so, how?
8. What documents/textbooks/other sources do you use to inform your selection of classroom tasks?
9. What are some of the difficulties you experience in designing or selecting tasks for your mathematical problem-solving teaching lesson?
10. Do you think the problem-solving approach you took today facilitated learners to understand the mathematical concepts? If so how?
11. Was your philosophy of teaching reflected in the lesson? How do you feel about teaching with this problem-solving pedagogy?
12. What changes would you like to make in your mathematical problem-solving teaching?
13. Did you discover anything new or fascinating about mathematical problem-solving?
14. If it is a second reflective interview: We spoke about reflection, have you been reflecting? If yes, how has that assisted you in your professional development as a teacher? If no, we discuss the importance of reflection and how to reflect properly.
15. Any other comments related to your lesson?

10.17 Appendix Q: Mathematical Problem-solving Skills Inventory (MPSSI)

	Mathematical problem-solving skills inventory	1	2	3	4	5
1	I am able to work out the question in a given problem or task					
2	I can divide a task into parts in order to explore, organise, plan, construct or deconstruct ideas or relationships					
3	I can easily explain, conjecture, describe, interpret, classify and communicate my ideas.					
4	I can construct my own problem-solving strategies when given an unfamiliar problem					
5	I can easily generate ideas through brainstorming, visualising, connecting and re-connecting ideas.					
6	I am able to justify my solution strategy or a course of action I take to solve a problem					
7	I can evaluate the reasonableness of the answer and can tell if the answer is 'correct'					
8	I am capable of implementing, carrying out, using or executing my existing knowledge in familiar or unfamiliar situations					
9	I can try a different solution strategy when stuck without the teacher's help					
10	I can easily reflect after a learning episode or a mathematical problem session					

10.18 Appendix R: Professional development evaluation interview protocol

The focus of this post-interview is to inspect what changes have taken place as a result of the professional development and determining whether intended goals have been achieved. Professional development is about transformation. The purpose of this professional development was to improve learner outcomes by changing the instructional behaviour of teachers. The intended goal was to train grade 9 teachers to teach mathematical problem-solving.

Guiding interview questions

1. Did you like the PD intervention? YES/ NO. Why?
2. Was your time used effectively?
3. How useful was the PD intervention?
4. What have been the most useful aspects in the PD intervention? Why?
5. What have been the least useful aspects in the PD intervention? Why?
6. Did you acquire any new knowledge and skills?
7. Are you applying the new knowledge and skills? Give an example.
8. How do you think your teaching behaviour has improved or changed? What changed it?
9. What do you think has been the impact of the PD intervention on the students?
10. There seemed to be an impact on learners' test results& on their problem-solving questionnaire that I gave them. What do you think might have caused this?
11. What impact did you see on learner actions in the classroom?
12. What are the barriers that teachers and learners are facing?
13. What could have been done better? Any suggestions?

10.19 Appendix S: Cycle 2 Pre-test

Date: _____

Name: _____ **Grade:** _____

Dear learner

Answer the following questions on the spaces provided. Each question is worth 2 marks.

Total marks: 20

1. A mathematics lesson is 55 minutes long and ends at 10:25. At what time did it start?

2. Write the next 2 numbers in the following sequence:
   3; 9; 15; 21; 27; …?

3. 20kg sugar and 10kg salt together cost R410. If the 20kg sugar costs R230 more than the 10kg salt, how much does the salt cost?

4. The sum of 2 numbers is 25. What is the greatest possible product of these two numbers?

5. In 4 different Maths tests Peter got 63%; 52%; 60%; and 48%. What mark did he get in the 5th test if his average mark is 59%?

6. 3000 people were at the shopping mall on Christmas Day. This was 30% more than the number of people on New Years' Day. How many people were at the shopping mall on New Year's Day?

7. The average mass of 5 oranges is 200g. The average mass of 6 apples is 120g. What is the average mass of the 11 fruits?

8. Bread costs R12. This price includes 14% Value Added Tax (VAT). What does bread cost without VAT?

9. Lisa gets pocket money from her father. She spends three eighth of it buying lunch and decides to saves the rest. She has R25 left. How much pocket money did she get from her father?

10. Which of these statements is false?

    (A) All squares are rectangles
    (B) All rectangles are quadrilaterals
    (C) All rectangles are parallelograms
    (D) All squares are quadrilaterals
    (E) All rectangles are squares

Thank you!!

10.20 Appendix T: Cycle 2 Post-test

Date: _____

Name: _____ **Grade:** _____

Dear student

Answer the following questions on the spaces provided. Each question is worth 2 marks.

Total marks: 20

1. A double mathematics lesson is 1 hour 20 minutes long and ends at 11:35. At what time did it start?

2. Write the next 2 numbers in the following sequence:
   2; 9; 16; 23; 30; …?

3. 20kg sugar and 10kg salt together cost R410. If the 20kg sugar costs R230 more than the 10kg salt, how much does the salt cost?

4. The sum of 2 numbers is 22. What is the greatest possible product of these two numbers?

5. In 3 different Maths tests, Peter got 52%; 60%; and 48%. What mark did he get in the 4th test if his average mark was 55.5%?

6. 200 people attended church on Sunday. This was 40% more than the people that attended the mid-week church service. How many people attended the mid-week church service?

7. The average mass of 3 bananas is 300g. The average mass of 2 plums is 60g. What is the average mass of the 5 fruits?

8. 2kg sugar costs R30, this price excludes 14% Value Added Tax (VAT). What does sugar cost with VAT?

9. John gets pocket money from his father. He spends two-thirds of it buying pizza at lunch. He decides to save the rest. He has R20 left. How much pocket money did he get from his father?

10. Which of these statements is false?

    (A) All squares are rectangles
    (B) All rectangles are quadrilaterals
    (C) All rectangles are parallelograms
    (D) All squares are quadrilaterals
    (E) All rectangles are squares

Thank you!!

10.21 Appendix U: Learner task-based interview questions

- Explain to me what the question means.
- How did you work out the meaning of this question?
- Explain what you are doing and why are you doing that?
- How did you decide on the operations to use?
- Why did you choose this strategy?
- Does this problem remind you of any other problems you have done before?
- Were you happy with your answer at the end? And why?
- How do you know how to solve this problem?

10.22 Appendix V: Learner task-based interview problems

Task-based interviews 1

a) Mrs Dube bakes biscuits and sells them at R17 per kg. How much will you pay for 2.69 kg?
b) The side length of a square pool area is 6m and the side length of a square tile is 200mm.
    i) Express the lengths as a ratio.
    ii) Express the area of the floor of the pool and the area of one tile as a ratio.
    iii) How many tiles are required to cover the floor?
c) The school bought a water tank this term. The water tank has the capacity of 10 000 litres. It takes 8 hours for the school's water system to fill this tank with water.
    i) Determine the rate at which water flows into the water tank in litres per hour.
    ii) Determine the rate in litres per minute.

Task-based interviews 2

Task-based interviews 2 occurred when learners had covered financial mathematics, fractions, numeric and geometric patterns, functions and relationships and algebraic expressions and equations. The following were the problems in the interview:

a) Archie is 6 years older than Adrian. The total of their ages is 28. How old is Archie?
b) Mrs Nkosi prepares a fruit cocktail for his son's 21st birthday.
    i) She mixes 1⅓ litres of apple juice and 2⅔ litres of orange juice. How much juice is this altogether?
    ii) If she adds 2⅝ litres of guava juice how much juice did she make?

c) Audrey buys a car at a cost of R40 000. She pays a deposit of 25% and opts to pay the balance using a hire purchase agreement. She is required to repay the balance over 3 years together with a simple interest of at 12% per annum.

   i) Calculate the deposit she pays in cash.
   ii) Determine the amount of the balance to be paid as a loan.
   iii) Determine the amount of money to be repaid by Audrey including the interest over the period of 3 years.
   iv) Calculate her monthly payments.

Task-based interviews 3

Task-based interviews 3 transpired when learners were doing the section on geometry.

a) Your teacher says that all squares are similar? What is your response to this? Write down a reason for your answer.
b) What is meant by corresponding angles? Verbally explain to the person sitting next to you and write down your explanation.
c) What is meant by complementary angles? Verbally explain to the person sitting next to you and write down your explanation.
d) State with reasons whether the following statements are true or false:
   i) If two triangles are congruent, then they are also similar.
   ii) All right-angled triangles are similar.
   iii) A rectangle is a parallelogram.
   iv) The diagonals of a kite are equal in length.